end sem set 2

All questions are compulsory, Q13 is a bonus question, it will be checked only when all the other
questions are attempted

* Required

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1

Q1 Find the acute angle θ between the curves $y^2 = 4x$ and $y = e^{-\frac{x}{2}}$

(3 marks)

a) $\theta = 0$ b) $\theta = \frac{\pi}{8}$ c) $\theta = \frac{\pi}{4}$

d) $\theta = \frac{3\pi}{4}$ e) $\theta = \frac{\pi}{2}$ f) $\theta = \pi$

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

2

Q2 Find the points on the curve $y = x^3 - x^2 - x + 3$, where the tangents are parallel to x-axis

(3 marks)

a) $(1, 2)$ b) $(-1, -2)$ c) $(1, 3)$

d) $(-\frac{1}{3}, \frac{70}{27})$ e) $(\frac{1}{3}, -\frac{70}{27})$ f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

3

Q3 The shortest distance between the lines x - y = 0 = 2x + z and x + y - 2 = 0 = 3x - y + z - 1 is

(3 marks)

a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $\frac{1}{2}$

d) 1 e) 2 f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

4

Q4 Find the curvature κ for the space curves

$$r(t) = (\cos^3 t)\vec{i} + (\sin^3 t)\vec{j}, \quad 0 < t < \frac{\pi}{2}$$

(3 marks)

a) $\kappa = \frac{1}{\cos t \sin t}$ b) $\kappa = \frac{1}{2\cos t \sin t}$ c) $\kappa = \frac{1}{3\cos t \sin t}$

d) $\kappa = \frac{1}{\cos t \tan t}$ e) $\kappa = \frac{1}{2\sin t \tan t}$ f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

5

Q5 Find the maximum rate of change of the function at the indicated point

$$f(x, y) = \sqrt{x^2 + y^4} \text{ at } (-2, 3)$$

(3 marks)

a) 5.86 b) 7.3 c) 9.25

d) 10.03 e) 11.7 f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

6

Q6 Find the maximum or minimum values of the function f(x, y) = 2 xy subject to the constraint $x^2 + y^2 = 1$

(3 marks)

a) maximum of f(x, y) is 1 at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

b) maximum of f(x, y) is 1 at $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

c) maximum of f(x, y) is 1 at $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

d) minimum of f(x, y) is -1 at $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

e) minimum of f(x, y) is -1 at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

f) minimum of f(x, y) is -1 at $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

7

Q7 Determine the area of the region bounded by the given set of curves

$$y = 4x + 3, y = 6 - x - 2x^2, x = -4 \text{ and } x = 2$$

(3 marks)

a) $\frac{43}{2}$

b) $\frac{97}{6}$

c) $\frac{143}{6}$

d) $\frac{243}{12}$

e) $\frac{343}{12}$

f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

8

Q8 Find the centre of mass of the region $x^2 + y^2 \leq R^2, y \geq 0$ with the given mass density $\delta(x, y) = y$

(3 marks)

a) $\left(-\frac{3R}{16}, -\frac{3\pi R}{16}\right)$ b) $\left(\frac{3R}{16}, \frac{3\pi R}{16}\right)$ c) $\left(-\frac{5R}{8}, \frac{5\pi R}{8}\right)$

d) $\left(-\frac{5R}{8}, -\frac{5\pi R}{8}\right)$ e) $\left(-\frac{7R}{16}, \frac{5\pi R}{16}\right)$ f) $\left(\frac{7R}{16}, -\frac{5\pi R}{16}\right)$

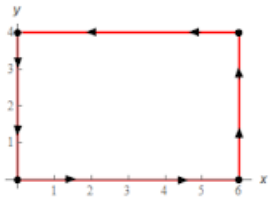
- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

9

Q9 Compute

$$I = \oint_C (y^4 - 2y)dx - (6x - 4xy^3)dy, \qquad \text{where C is shown below}$$

(4 marks)



a) I = 0 b) I = -6 c) I = -11

d) I = 18 e) I = -76 f) I = -96

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

10

Q10 Evaluate the surface integral $\iint_S F.n \, dA$, where $F = 6z\vec{i} + 6\vec{j} + 3y\vec{k}$ and S is the portion of the plane 2 x + 3y + 4z = 12, which is in the first octant

(4 marks)

2

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a) 28 b) 138 c) 148

d) 208 e) 373 f) 379

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

11

Q11 Find the volume of the solid in the first octant bounded by the paraboloid $z = 36 - 4x^2 - 9y^2$

(4 marks)

a) 2π b) 3π c) 9π

d) 19π e) 27π f) 31π

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

12

Q12 Evaluate $\iint_S (\text{Curl } F.N)dS$ for the vector field $F(x, y, z) = z\vec{i} + 3x\vec{j} + 2z\vec{k}$ and surface S is $z = 1 - x^2 - 2y^2, z \geq 0$, C is boundary circle $x^2 + y^2 = 1$ and S is oriented in the positive z- direction

(4 marks)

a) 0 b) π c) 3π

d) $\frac{5\pi}{2}$ e) $\frac{7\pi}{2}$ f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

13

Q13(Bonus question) Evaluate I = $\iint_R (x+y)dA$ where R is the trapezoidal region with vertices given by (0, 0), (5, 0), $\left(\frac{5}{2}, \frac{5}{2}\right)$ and $\left(\frac{5}{2}, -\frac{5}{2}\right)$ using the transformation x = 2 u + 3 v and y = 2u -3 v

(4 marks)

a) $\frac{25}{4}$ b) $\frac{75}{8}$ c) $\frac{121}{16}$

d) $\frac{125}{4}$ e) $\frac{225}{4}$ f) None of these

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

Submit