Hybrid models

TURBULENCE THEORY & MODELLING

Outline

- RANS vs. LES vs. DNS
- Hybrid models
  – Why and how to hybrid?
  – Type of hybrid models
    » Pros /Cons
- Literature

1. Fröhlich, J., von Terzi, D. Hybrid LES/RANS methods for the simulation of turbulent flows, Progr. in Aerosp. Sci., 44 (2008), 349-377
2. Spalart, P.R. Detached-Eddy Simulation, Annu. Rev. Fluid Mech. 2009, 41:181-202

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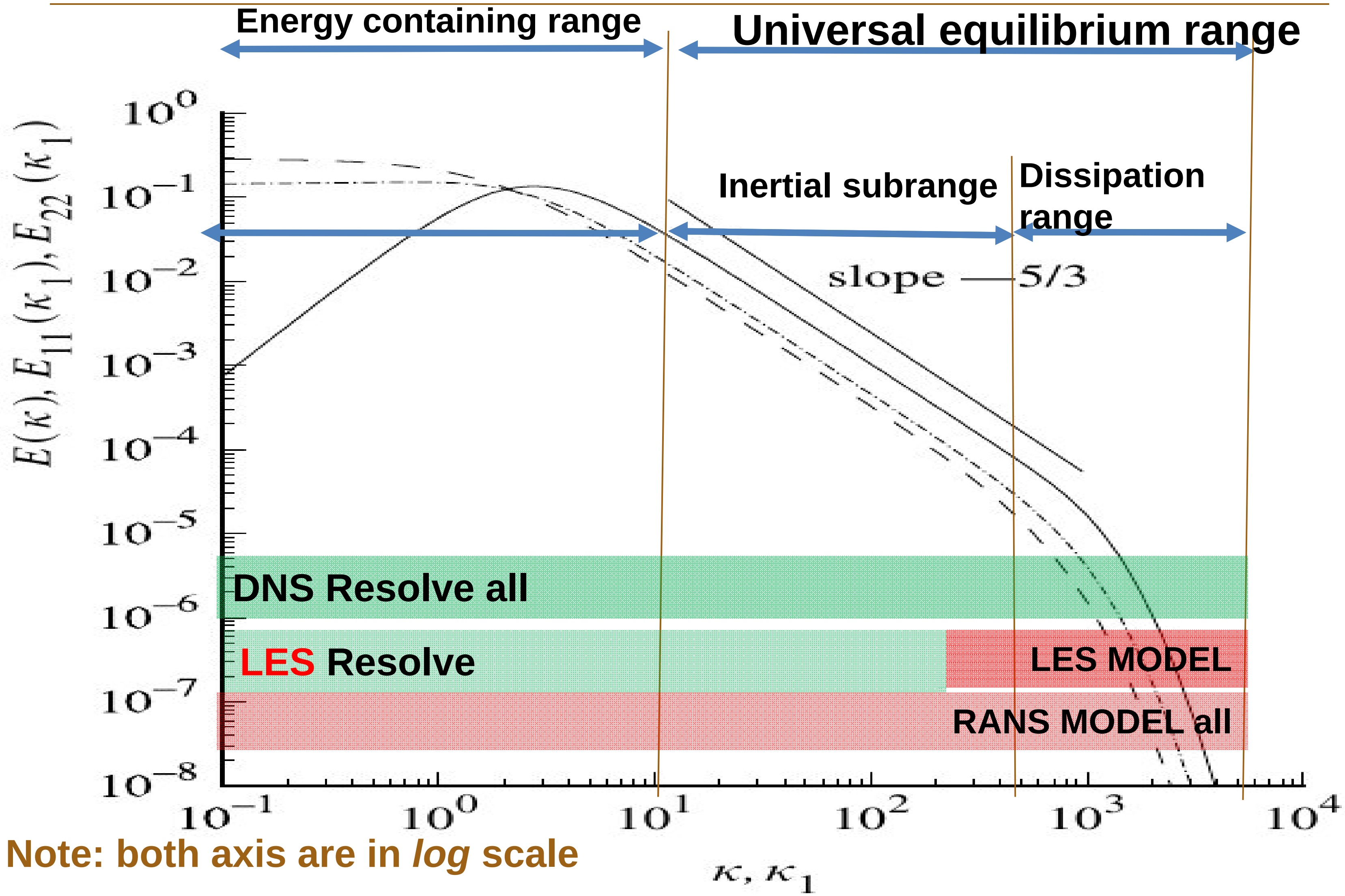
RANS vs. LES vs. DNS

(Averaged) vs. (filtered) vs. (raw) naiver stokes equations



Note: both axis are in *log* scale

RANS vs. LES vs. DNS

(Averaged) vs. (filtered) vs. (raw) naiver stokes equations

Possible issues

- Accuracy/fidelity ↑, Cost ↑
- Ease of implement/use ↑
- discretization schemes ↑
  - » DNS favor high order scheme
- Stability ↑↓
  - » RSM can be unstable due complicated eq.s.
  - » DNS is on a edge trying to minimize number of grid cells.
- Boundary conditions
  - » P.D.E.s ↓
  - » Inflow/outflow fluctuation↑
- Data Post-processing, analysis ↑

All models solve a similar form of 3 momentum +1 continuity equations. The unresolved Reynolds or residual stresses can all be modeled by eddy viscosity assumption as:

- RANS

$$\nu_{eff} = \nu_{molec} + \nu_{RANS}$$

- LES

$$\nu_{eff} = \nu_{molec} + \nu_{LES}$$

- Why not combine modeling concepts?

$$\nu_{eff} = \nu_{molec} + \nu_{model}$$

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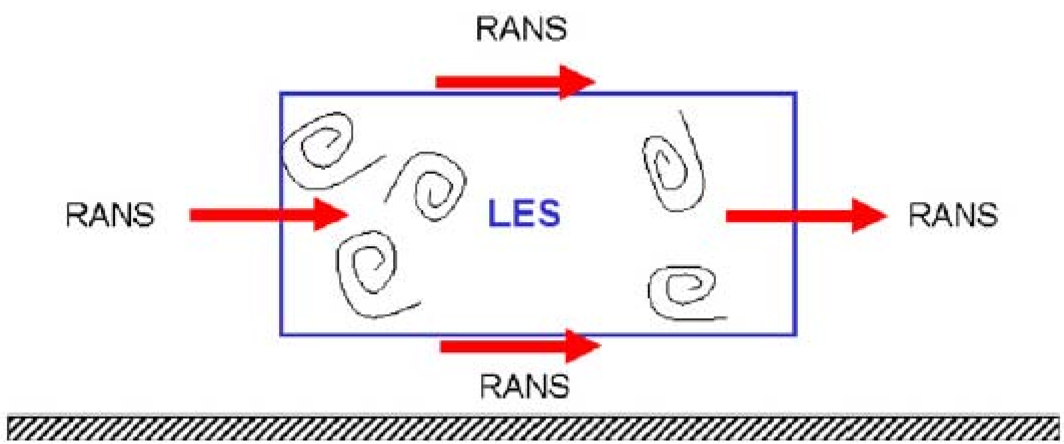
Why NOT?

- Conceptual issues (Reynolds average vs. spatial filtering vs. raw)
- 2 models more difficult than 1 (e.g. different model assumptions and asymptotic behaviors)

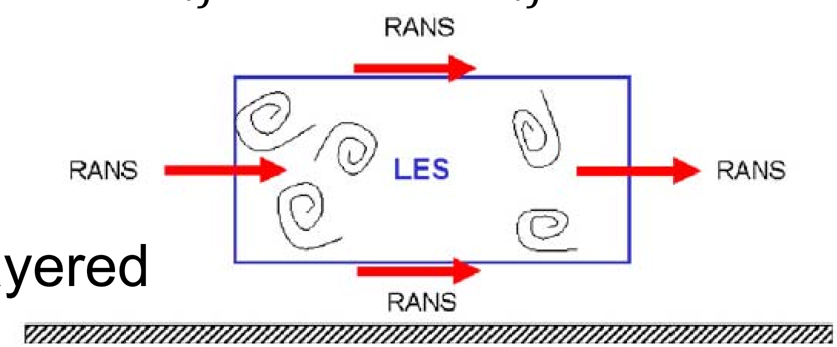
- How to separate?
- How to connect?



[Frohlich2008]

Coupling strategies

- $\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_i u_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j}\tau_{ij}^{Model}$
- Unified
  - blending $\quad \tau_{ij}^{model} = f^{RANS}\tau_{ij}^{RANS} + f^{LES}\tau_{ij}^{LES}$
  - adjusting
- Segregated
  - Interfaced/ Embedded/Layered
    - » Hard (constant in time)
    - » Soft



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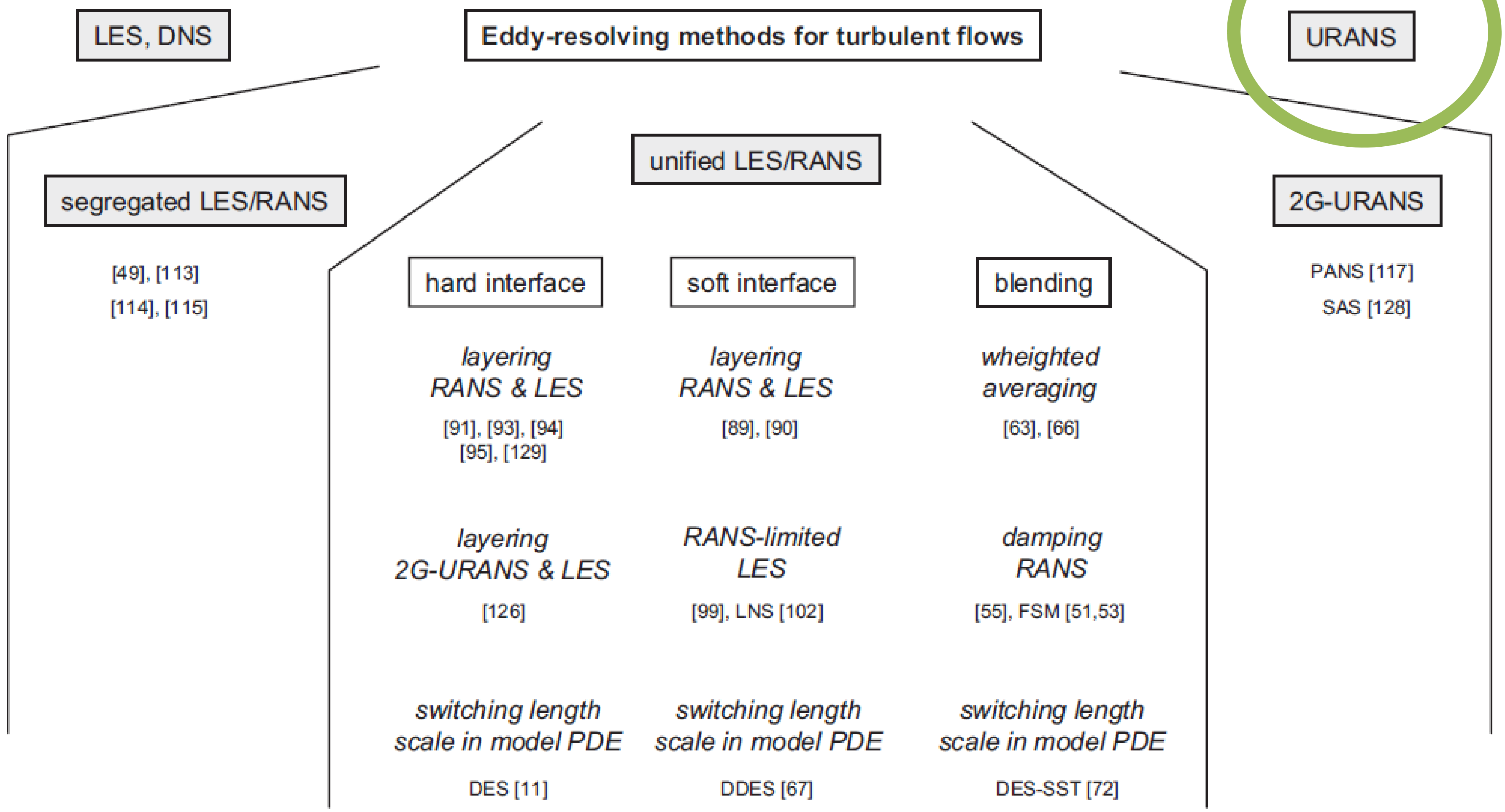
How?



[Frohlich2008]

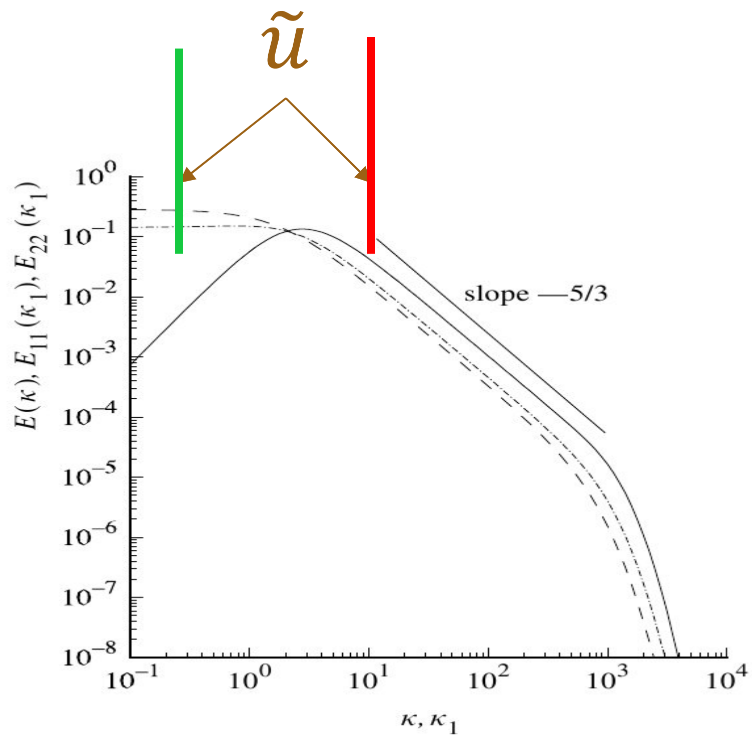
A spectrum of hybrid methods

URANS

- RANS = steady flows
- URANS = Unsteady RANS

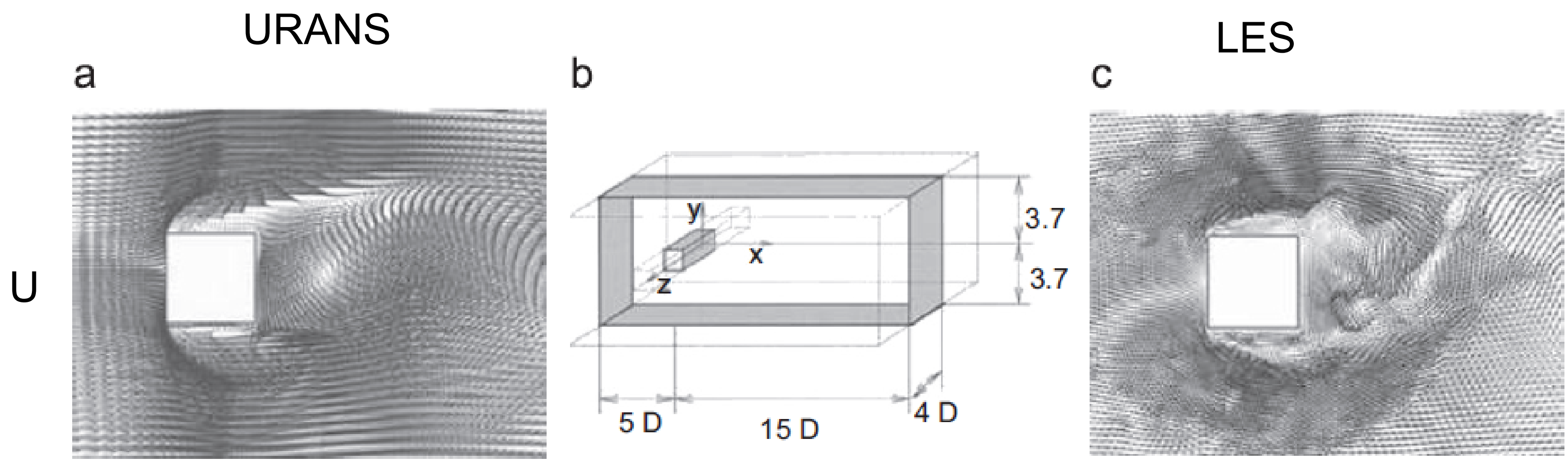
$$u = \bar{u} + \tilde{u} + u'$$

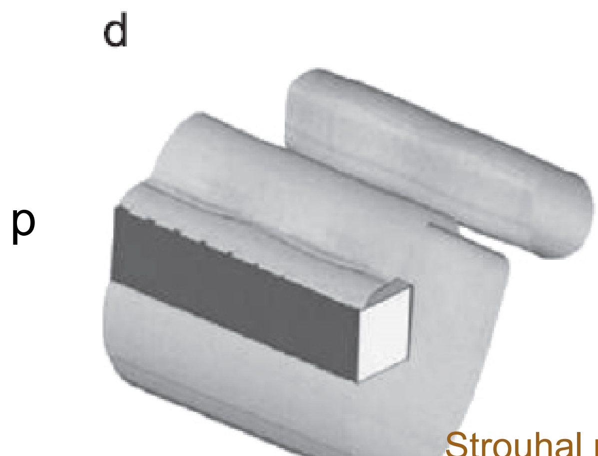
- $\tilde{u}$ = “phase average or the average conditioned on some slowly varying quantity’
- Source of unsteadiness
  - External / BC
  - Internal / flow instability
- Add time-dependent term
- No change in turb. model



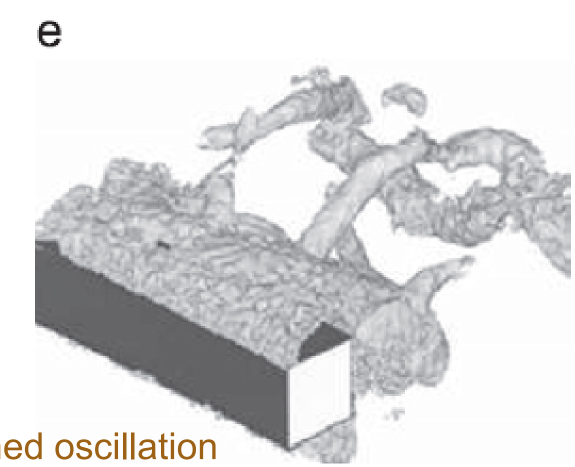
- **Conceptually OK only if resolved timescale is not within turbulent range**

E.g. flow over a square cylinder





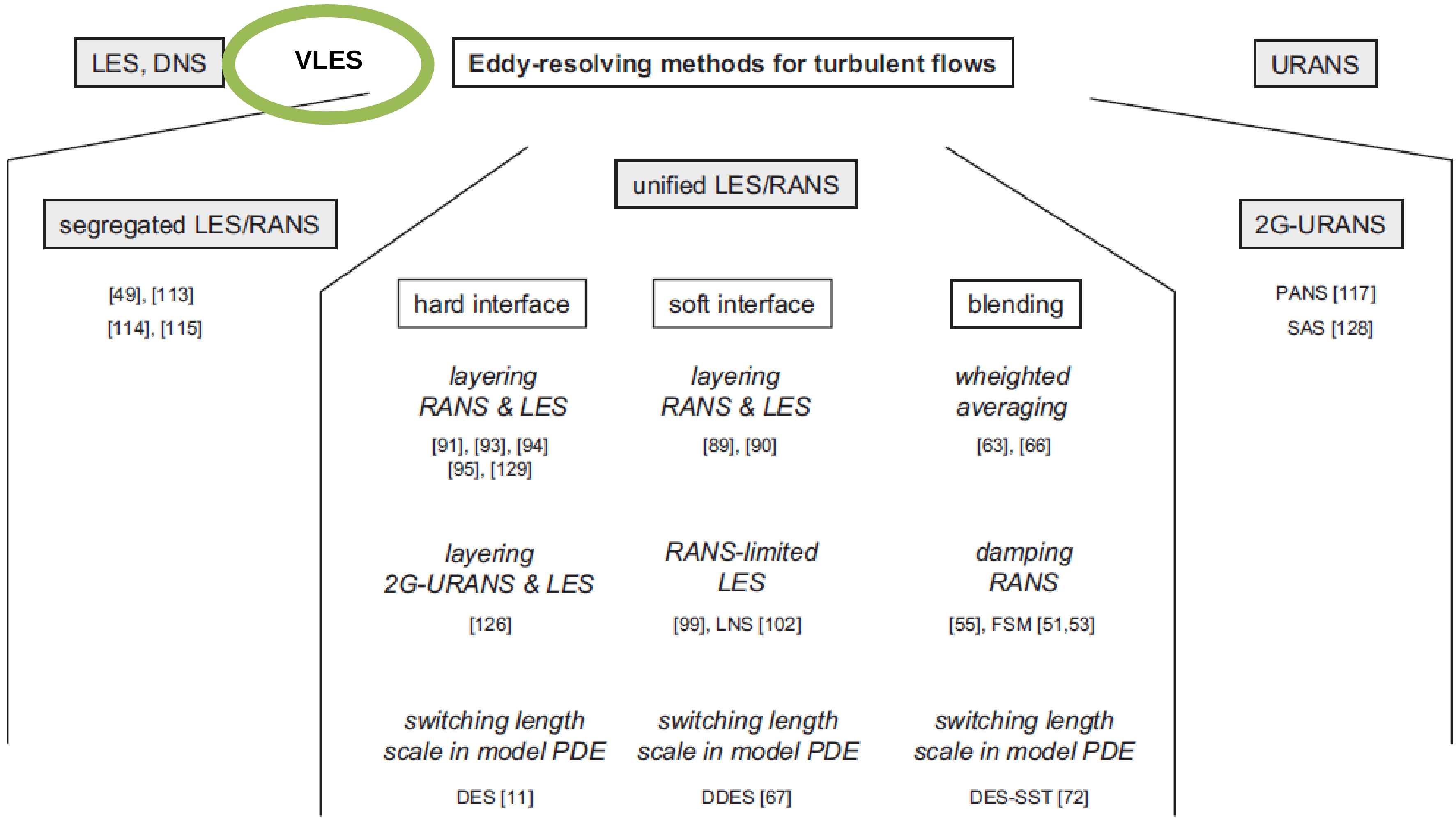
$St_{URANS} = 0.121$
$St_{LES} = 0.144$
$St_{EXP} = 0.143$



Strouhal number describes the non-dimentioned oscillation

[Frohlich2008]

How?



[Frohlich2008]

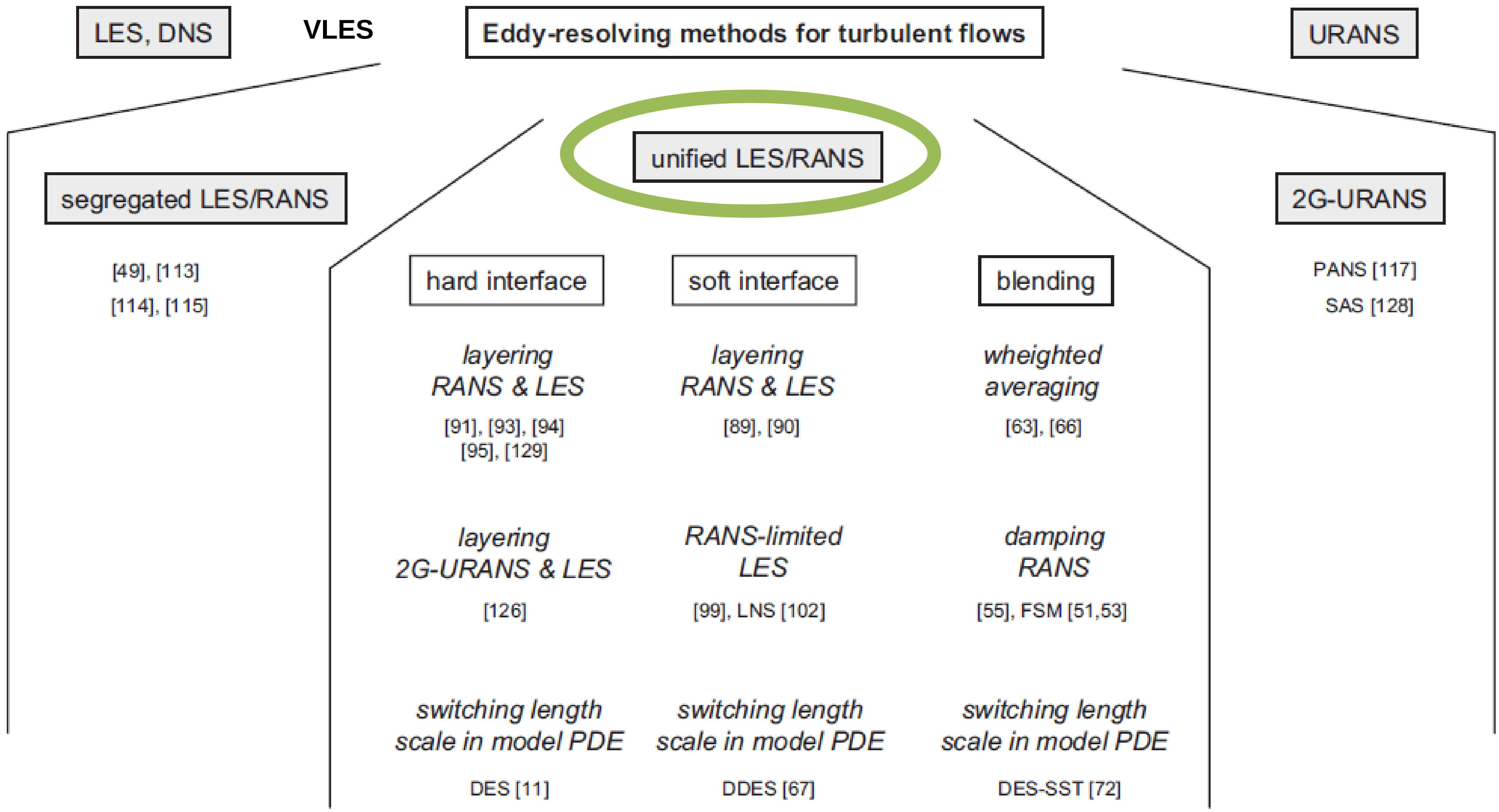
Very Large Eddy Simulation (VLES)

LES

- Resolve at least 80% of kinetic energy
- Filter cutoff within inertial subrange

- VLES
  - Several methods are called VLES
  - Too coarse LES
  - No adjustment of the SGS model
  - Problems obtaining physical results
  - Used e.g. for inflow data generation

How?



[Frohlich2008]

Categorization/terminology not established yet!

Unified turbulence models

- Blending: $\tau_{ij}^{model} = f^{RANS}\tau_{ij}^{RANS} + f^{LES,DNS}\tau_{ij}^{LES,DNS}$
- Requirements [Speziale]
  1. For coarse grids -> RANS
  2. For fine grids -> DNS (possibility to turn off)
  3. No explicit filtering (make it easy in complex situations)
- Examples
  – Damping of a RANS model
  – Weighted sum of RANS and LES

Damping of a RANS model (FSM)

- FSM = Flow Simulation Methodology
- $\tau_{ij}^{model} = f_\Delta\left(\frac{\Delta}{l_K}\right)\tau_{ij}^{RANS} \quad 0 \le f_\Delta \le 1$

contribution function

Damp model when part of the turbulence is resolved

Δ – grid size

$l_K = \frac{\nu^{3/4}}{\epsilon^{1/4}}$ (Kolmogorov scale)

$$f_\Delta\left(\frac{\Delta}{l_K}\right) = \left(1 - e^{-\beta\frac{\Delta}{l_K}}\right)^n$$

- Issues of consistency

$u_{\text{FSM model}} \neq \langle u_i \rangle_{Reynold\ average}$

  – How to compute τ_{ij}^{RANS}?

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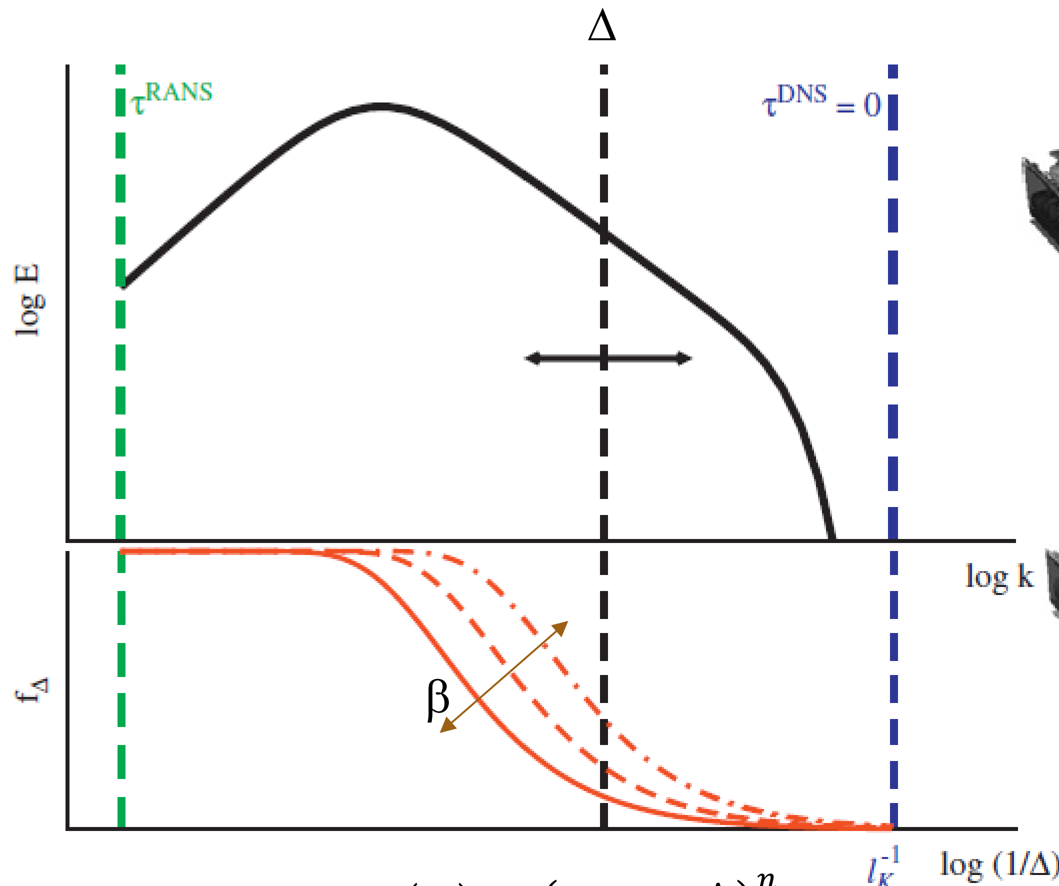
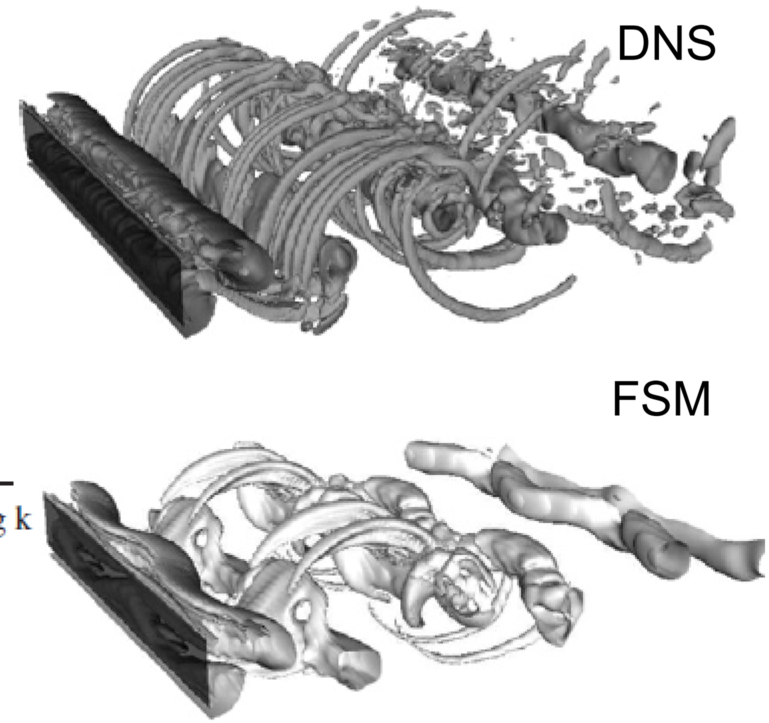
Flow Simulation Methodology(FSM)



[Frohlich2008]

$$f_\Delta\left(\frac{\Delta}{l_K}\right) = \left(1 - e^{-\beta\frac{\Delta}{l_K}}\right)^n$$



$$\tau_{ij}^{model} = f_\Delta\left(\frac{\Delta}{l_K}\right)\tau_{ij}^{RANS}$$

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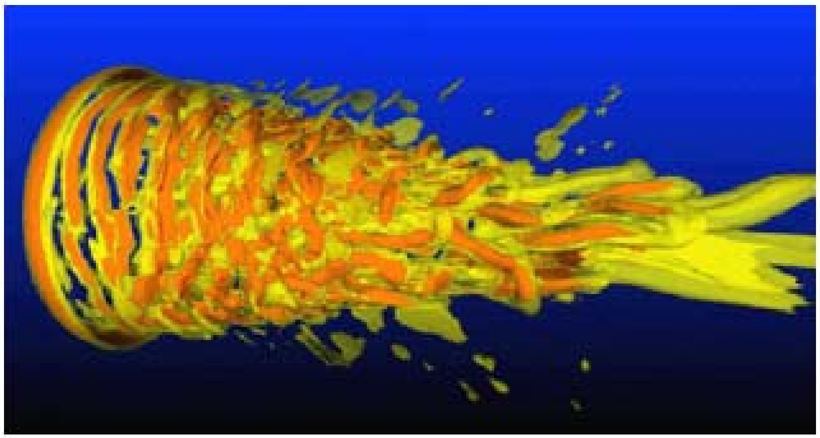
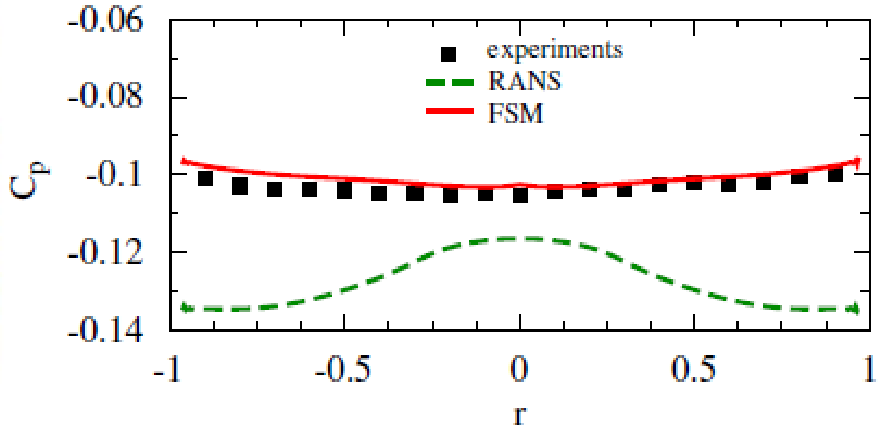
Flow Simulation Methodology(FSM)





[Frohlich2008]

- Supersonic flow computed with FSM

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Comparisons of governing equations solved in RANS and in LES
(bearing in mind the difference in spatial filtering and Reynolds average)

$$\tau_{ij}^{model} = f^{RANS}\tau_{ij}^{RANS} + f^{LES,DNS}\tau_{ij}^{LES,DNS}$$

Mass: $$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

Momentum: $$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}^{LES,RANS}}{\partial x_j}$$

RANS: solve additional transport equations

$$\boxed{\frac{\partial \phi}{\partial t} + \frac{\partial \bar{u}_j \phi}{\partial x_j} = \frac{\nu_T}{C_\phi}\frac{\partial^2 \phi}{\partial x_j \partial x_j} + P_\phi - \epsilon_\phi + R_\phi}$$

$$\phi \in \begin{bmatrix} 0 \\ \nu_T, l_s, k \,.. \\ \varepsilon, \omega, .. \\ k_l, \theta, \gamma, \ldots \\ \tau_{ij}^{RANS} \,.. \end{bmatrix} \begin{array}{l} \text{0-eq} \\ \text{1-eq.} \\ \text{2-eq.} \\ \text{3,4-eq.} \\ \text{7-eqs.RSM} \end{array}$$

Boussinesq eddy viscosity assumptions (Algebric)

$$\tau_{ij}^{RANS} - \frac{2}{3}k\delta_{ij} = 2\nu_T \bar{S}_{ij}$$

LES: no need for new transport eq.s.

Note: Some LES SGS models do involve new transport equations, but the transporting effects are of minor importance than their role in RANS modelling)

Smagrinsky eddy viscosity model (Algebric)

$$\tau_{ij}^{LES} - \frac{2}{3}k\delta_{ij} = 2\nu_r \bar{S}_{ij} = 2C_s^2\Delta^2 \bar{S}\,\bar{S}_{ij}$$

Near wall boundary treatment:

Using a blending function $f(\mathrm{d})$: d is the distance to wall

$$F(\phi) = f \cdot F^{Interior}(\phi) + (1-f)F^{NearWall}(\phi)$$

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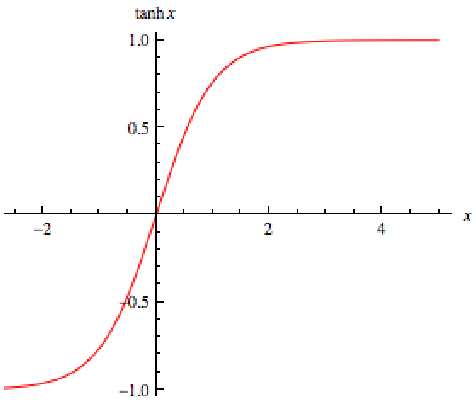
Filter or grid cell size

Weighted sum of LES and RANS

- E.g. using the Shear-Stress Transport (SST) model (solve K, ω)
- $\nu_t = f\nu_t^{RANS} + (1-f)\nu_t^{LES} = f\frac{K}{\omega} + (1-f)C_S\sqrt{K}\Delta$
- $\epsilon = f\epsilon^{RANS} + (1-f)\epsilon^{LES} = f\beta^* K\omega + (1-f)C_S\frac{K^{3/2}}{\Delta}$

$$\boxed{\nu_{LES} \sim C_S^2\Delta^2\overline{S} \sim C_S\sqrt{k_r}\Delta}$$

- f – modification of the SST blending function
- $f = \tanh(\eta^4)$; $\eta = \frac{1}{\omega}max\left\{\frac{500\nu}{d^2}; \frac{\sqrt{K}}{C_\mu d}\right\}$
- Close to wall d→0: RANS; far away d→∞: LES
- Issues
  - Can generate unphysical flow structures
  - URANS issues
  - Narrow blending region -> reverts to interfacing



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Interfacing RANS and LES

- **Detached Eddy Simulation (DES)**
- **Layering RANS and LES**
- RANS limited LES
- Limited numerical scales

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1-eq.
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Detached Eddy Simulation (DES)

- For details see [Spalart 2008]
- Started from the 1-eq. Spalart-Allmaras RANS model

$$\partial_t \tilde{\nu} + \langle u_j \rangle \partial_{x_j} \tilde{\nu} = c_{b1} \tilde{S} \tilde{\nu} + \frac{1}{\sigma_{\tilde{\nu}}} \left[\partial_{x_j} \left((\nu + \tilde{\nu}) \partial_{x_j} \tilde{\nu} \right) + c_{b2} (\partial_{x_j} \tilde{\nu})^2 \right] - c_{w1} f_w \left(\frac{\tilde{\nu}}{d} \right)^2 .$$

$$\tilde{\nu} = \nu_t / f_{v1}(y^+)$$

Distruction term, depends on the wall distance

- Idea: make it depend on the grid size, replace d with
- $\tilde{d} = \min\{d; C_{DES}\Delta\}$; $\Delta = \max\{\Delta_x; \Delta_y; \Delta_z\}$
- The basic idea can be combined with other RANS models as well

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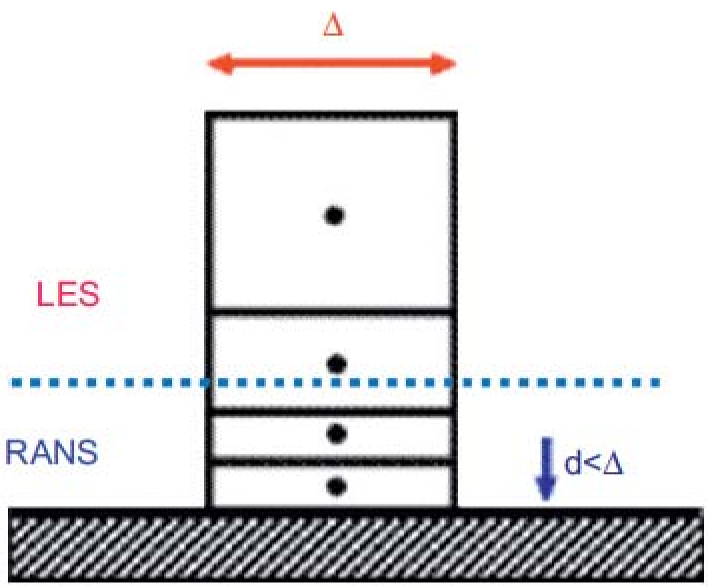
DES



- Smooth changes (only source term is changed in different regions)
- Even RANS part will fluctuate due to outer region fluctuations
- Can be used as a wall model
- Issues (see [Spalart 2008])
  - Modeled-Stress Depletion
  - Grid Induced Separation
  - Log-layer mismatch
  - Slow LES development in mixing layers
- There are improved versions

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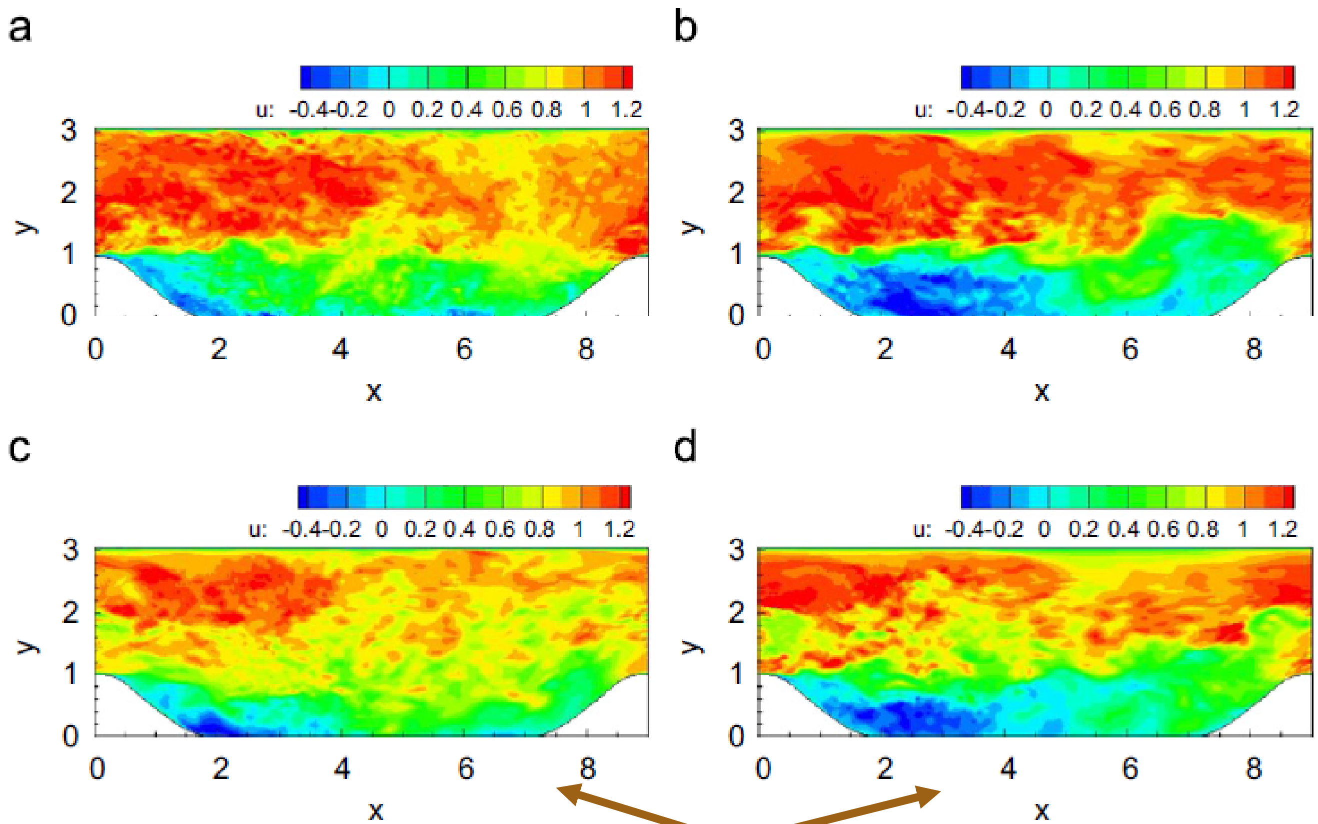
Sample DES results

LES

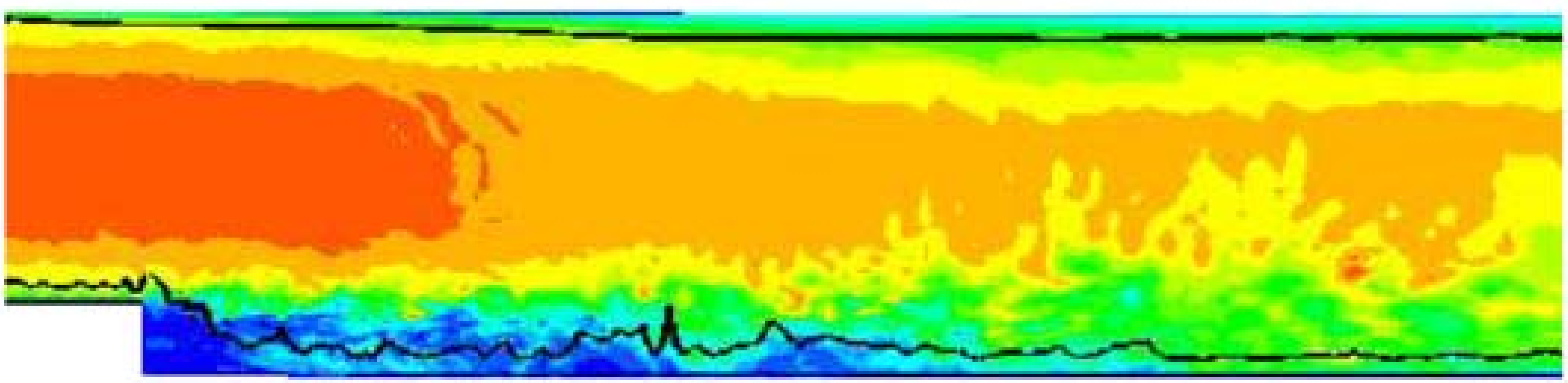


DES

DDES (Delayed DES)
With different initial
conditions

Layering RANS and LES

- Typically near walls
- Can be **hard** or **soft** interface
- Matching models or quantities at the interface (not a term in a transport eq. like in DES)
- Interface position
  - – Grid line
  - – Wall distance y or y+
    - » Problem @ separation/reattachment
- Usually discontinuities appear



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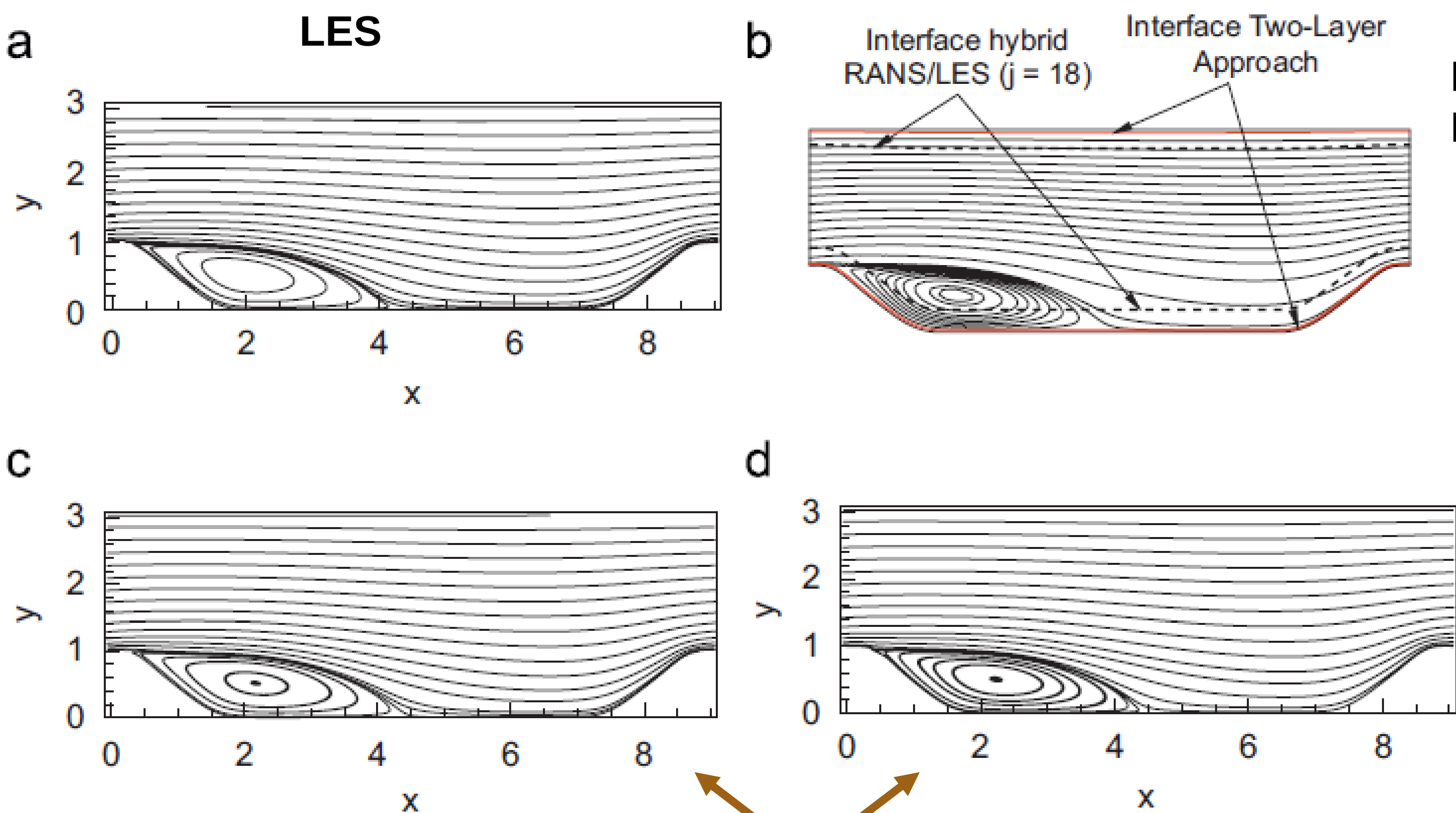
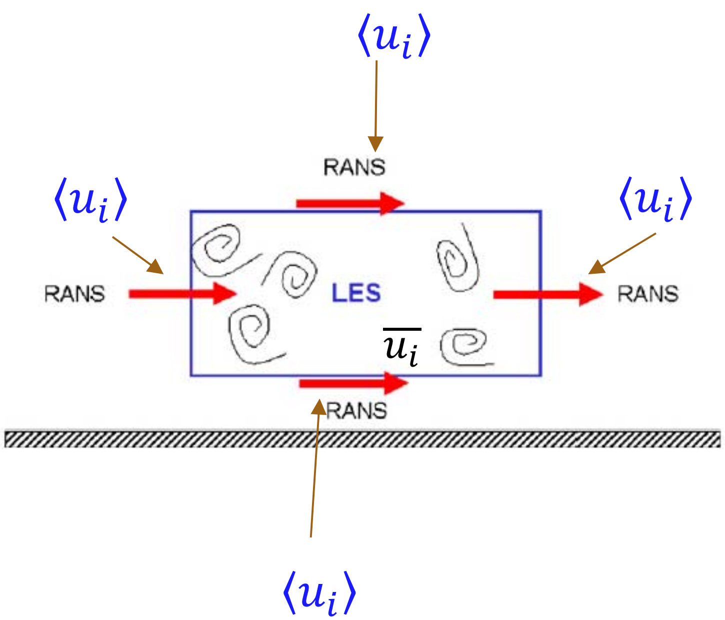
Layering RANS and LES



LES+RANS
Hard interface

LES+RANS
Soft interface

Segregated modeling



- Geometry divided before the computations are started
- Flow variables (u, p) matched at the interface
  - $\langle u \rangle|_{interface} \approx \langle \overline{u} \rangle|_{interface}$, …
- Should be two-way coupling
  - Governing eq.s are P.D.Es
- Typical interface scenarios
  - Inflow coupling
  - Outflow coupling
  - Tangential coupling

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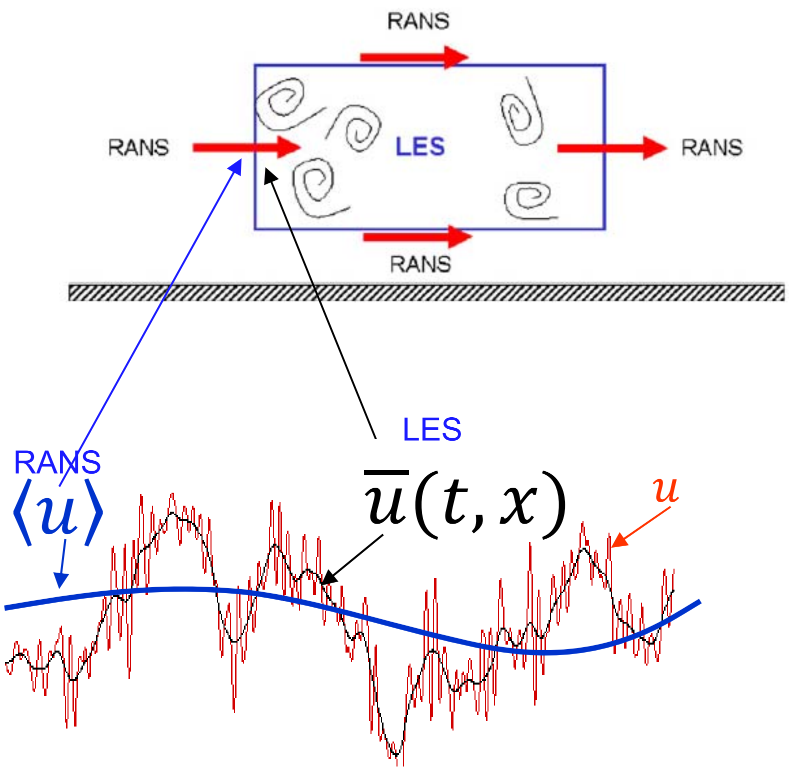
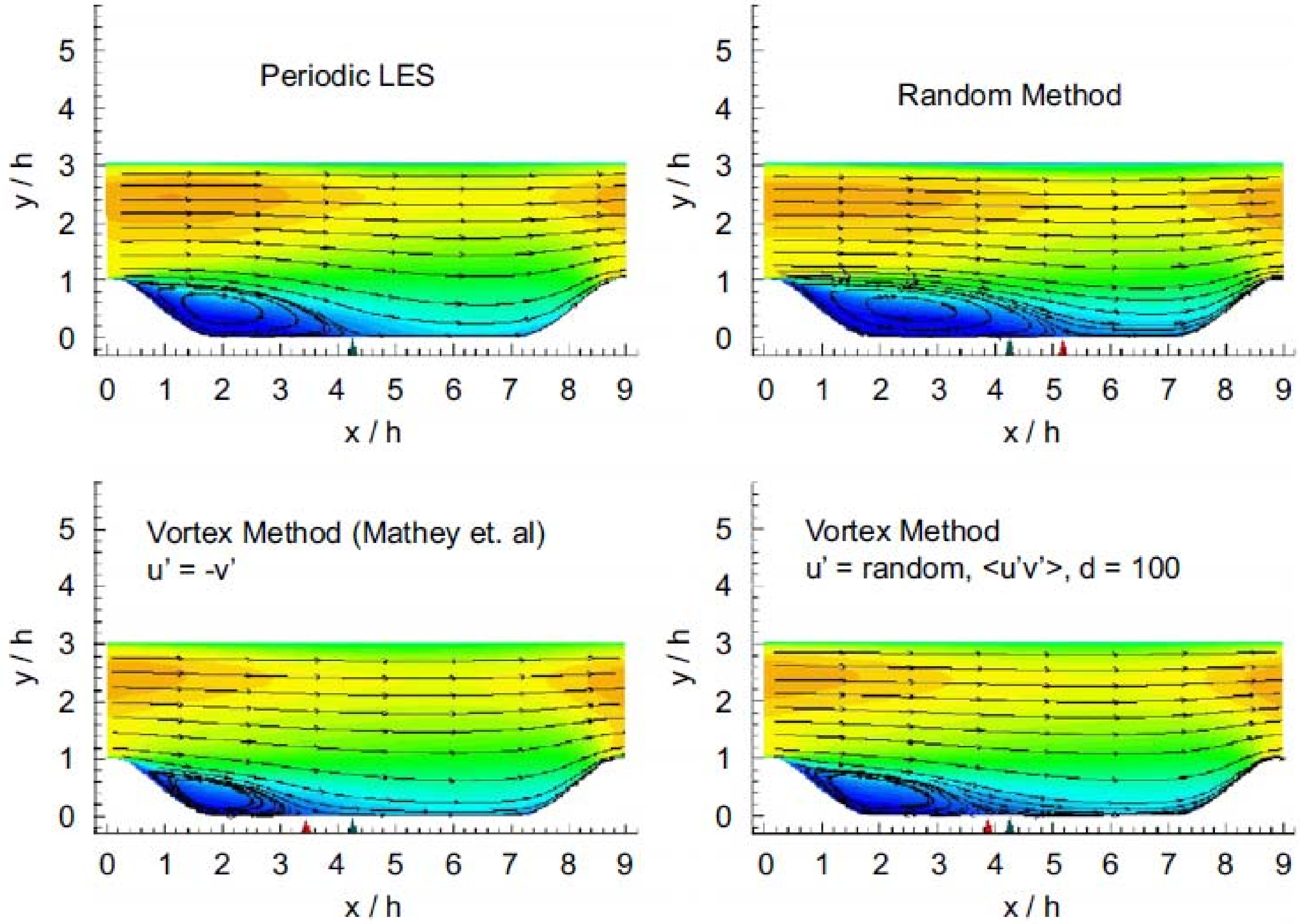
Inflow coupling

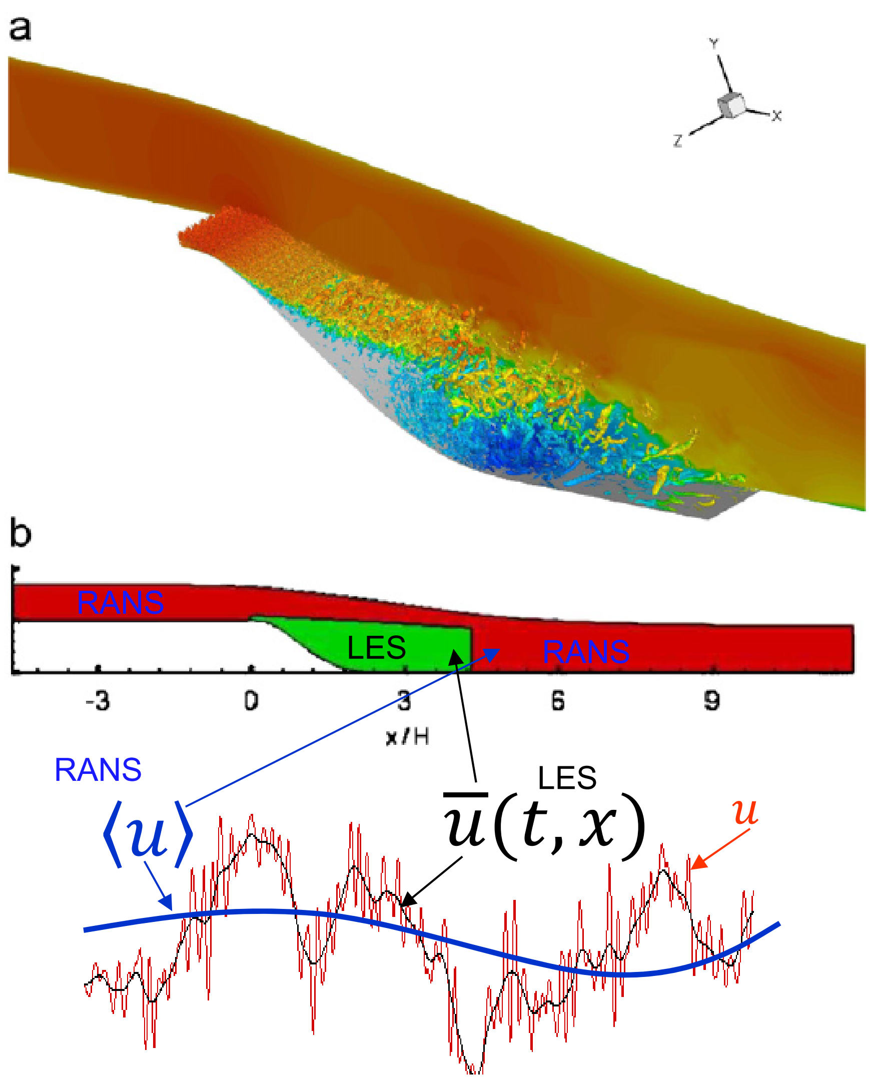


- RANS provides no fluctuations
  - $\langle u \rangle$, $\langle u_i' u_j' \rangle$ are just statistics
- Generate in the LES domain
  - $\overline{u}(t,x)$ should fluctuate in space and time
- Impose at coupling
  - Real fluctuations
    - » Precomputed, database
    - » Synthetic

Inflow coupling



Outflow and tangential coupling



- RANS should propagate mean flow information upstream
  - Incompressible N-S is elliptical
  - An expensive full LES will waste many cells to do this job.
- LES should deliver average flow data
- Fluctuations should leave the LES domain without reflections
  - Unless you had wrong implementation

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2^{nd} generation URANS models

- Aim to solve a *substantial part of the energy spectrum*
- 2G-URANS vs LES: No dependence on grid size
- 2G-URANS vs URANS: contain a term **sensing the** amount of **resolved fluctuations**
- Examples
  - Partially Averaged Navier Stokes (PANS)
  - Scale Adaptive Simulation (SAS)

PANS

- Solve a standard RANS equations
  - Define two ratios:
    - » $f_K = \frac{K_\tau}{K} \quad f_\epsilon = \frac{\epsilon_\tau}{\epsilon} \quad 0 \le f_K \le f_\epsilon \le 1$
      - $K, \varepsilon,$ starndard turbulent kinetic energy and dissipation rate
      - $K_\tau,\ \epsilon_\tau$ - **unresolved part**
    - » User decides ratio a priori, will be applied everywhere
      - Say, you to resolve 60%, f_K=f_ϵ=0.4
    - » $\nu_t = C_{\mu\tau} \frac{K_\tau^2}{\epsilon_\tau} = C_\mu \frac{f_K^2}{f_\epsilon} \frac{K^2}{\epsilon}$
- Easy to implement into existing 2-equation model
- The user needs to specify appropriate grid resolution

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Scale Adaptive Simulation (SAS)

- Definition of SAS
  - Contains two lengthscales
    - » First derivative of the velocity
    - » Higher order derivative of the velocity
  - Reverts to RANS in stable flow regions
  - Allows the breakup of large unsteady structures (no explicit grid or timestep dependency)
    - » Proper damping of resolved turbulence at the resolution limit of the grid.

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Scale Adaptive Simulation (SAS)

- Idea started from the K-KL model
- $\nu_t = \frac{\mu_t}{\rho} = c_\mu^{1/4}\Phi.$ $\Phi = \sqrt{K}L$

$$\frac{\partial(\rho K)}{\partial t} + \frac{\partial(\rho U_j K)}{\partial x_j} = P_K - c_\mu^{3/4}\rho\frac{K^{3/2}}{L} + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_K}\frac{\partial K}{\partial x_j}\right), \quad (52)$$

$$\frac{\partial(\rho\Phi)}{\partial t} + \frac{\partial(\rho U_j\Phi)}{\partial x_j} = \frac{\Phi}{K}P_K\left(\zeta_1 - \zeta_2\left(\frac{L}{L_{\nu K}}\right)^2\right) - \zeta_3\rho K + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\Phi}\frac{\partial\Phi}{\partial x_j}\right), \quad (53)$$

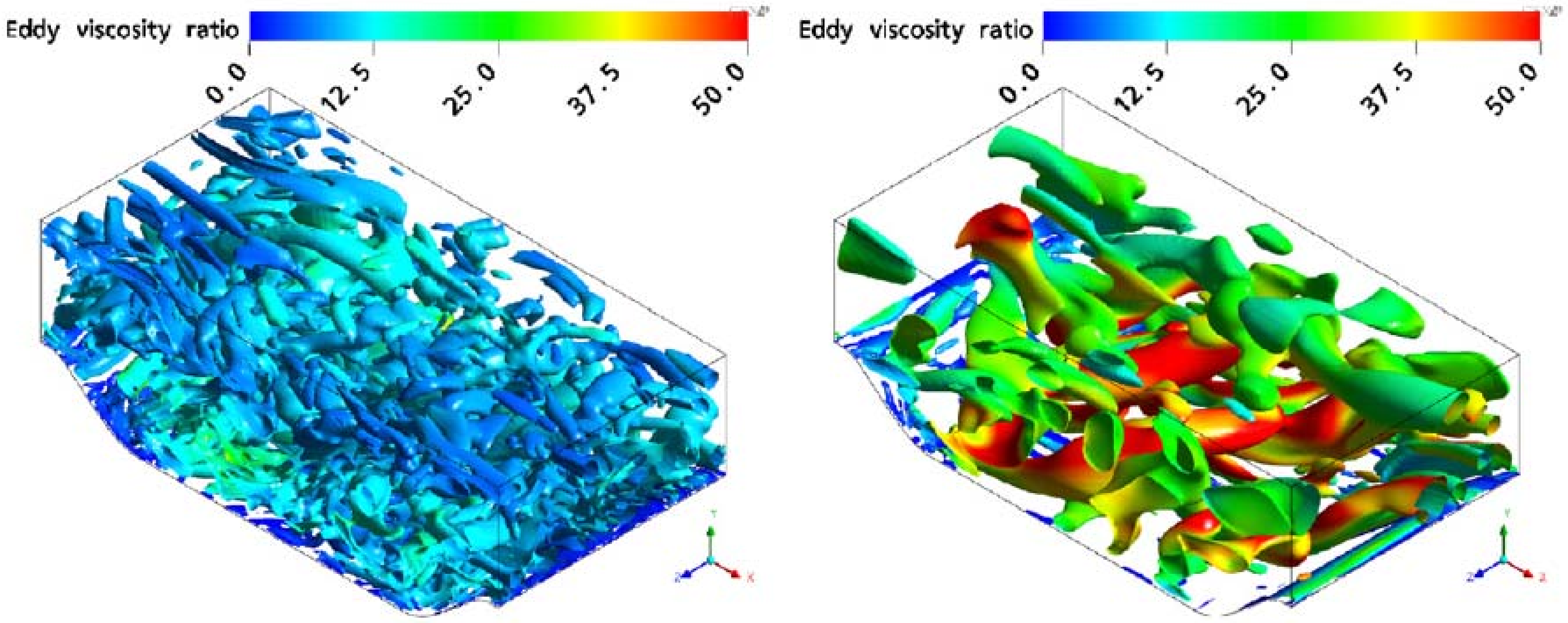
$$L_{\nu K} = \kappa\frac{|U'|}{|U''|}, \quad |U'| = \sqrt{\frac{\partial U_i}{\partial x_j}\frac{\partial U_i}{\partial x_j}}, \quad |U''| = \sqrt{\frac{\partial^2 U_i}{\partial x_j\partial x_j}\frac{\partial^2 U_i}{\partial x_k\partial x_k}},$$

$$P_K = \mu_t S^2, \quad S = \sqrt{2S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right).$$

The larger of $L_{\nu K,}$ the less production of viscosity ϕ, allowing the flow more susceptible to flow instability and break up.

Sample SAS results

ν_T/ν



- Different timesteps

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