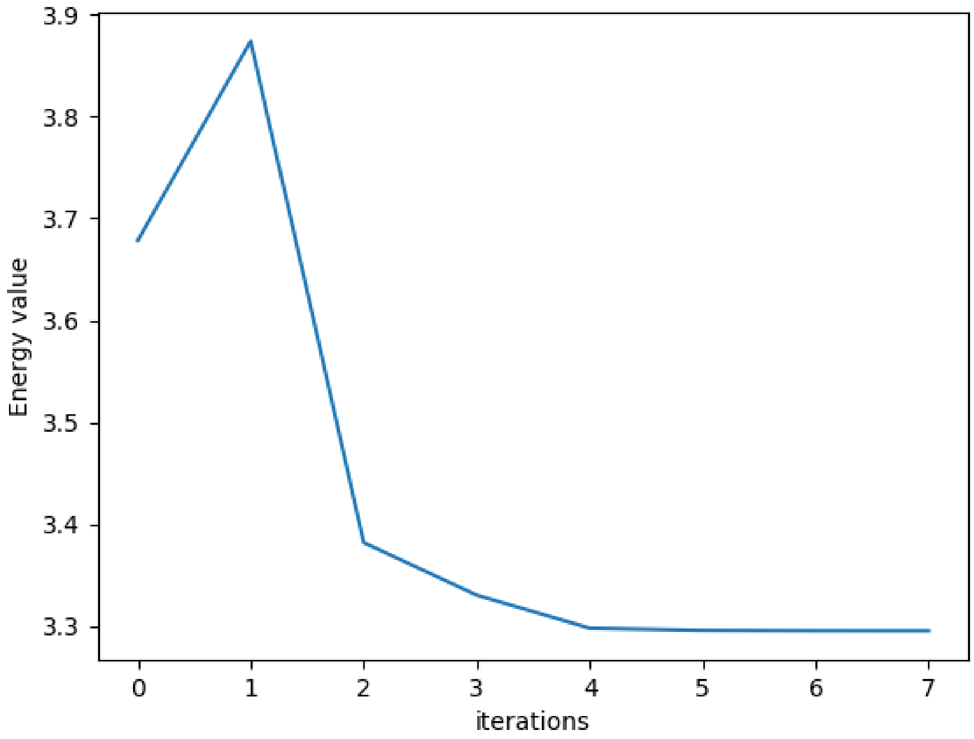
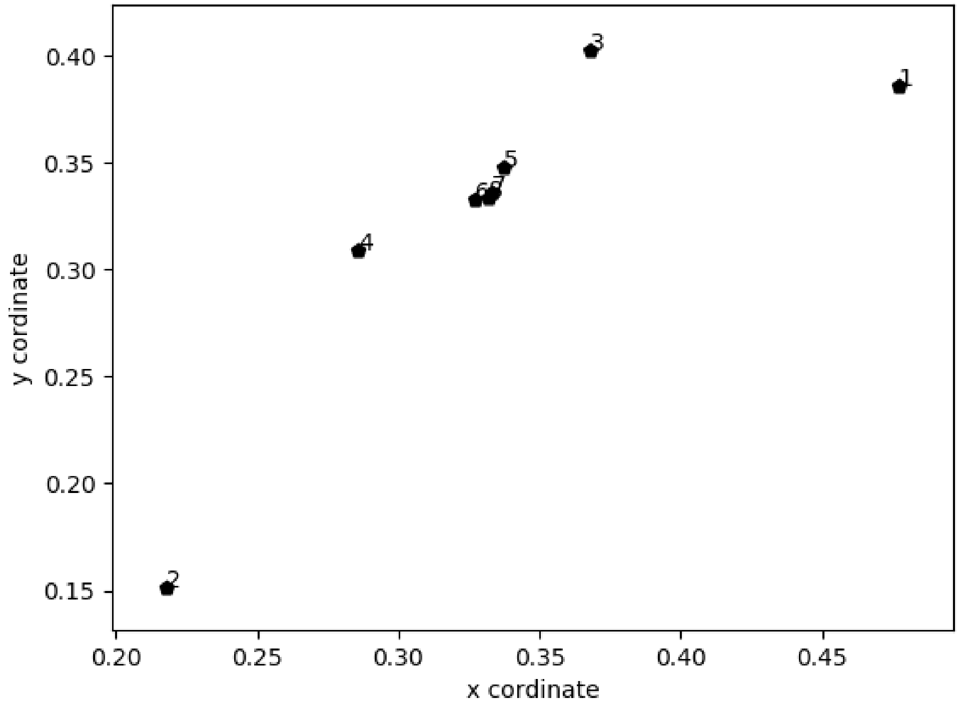
Q2 b)

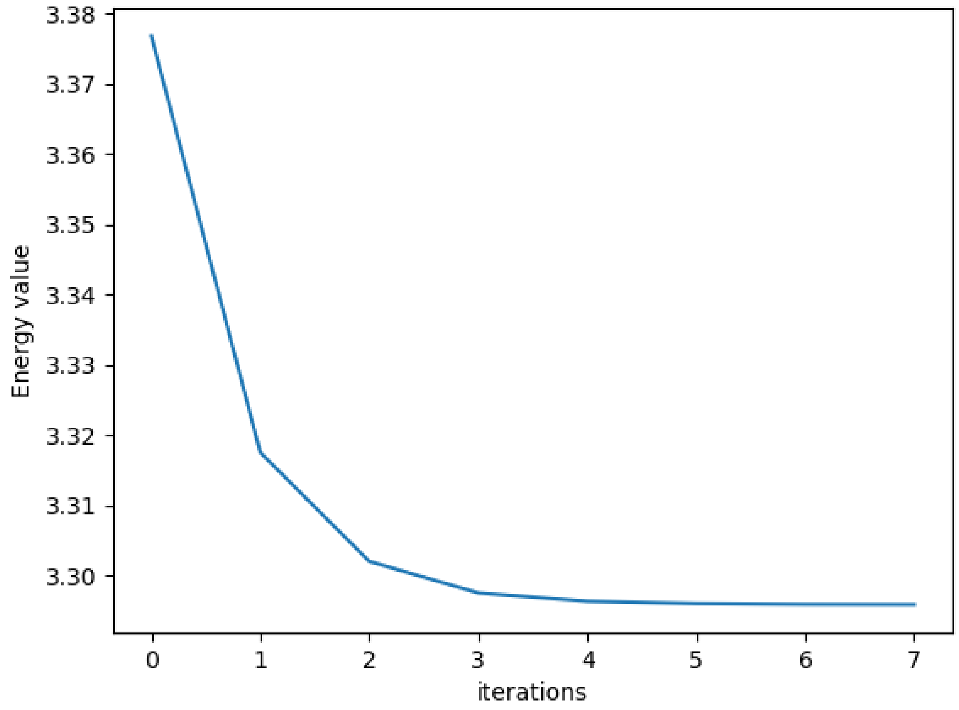
Initial w_o = [0.28332678,0.57361592]

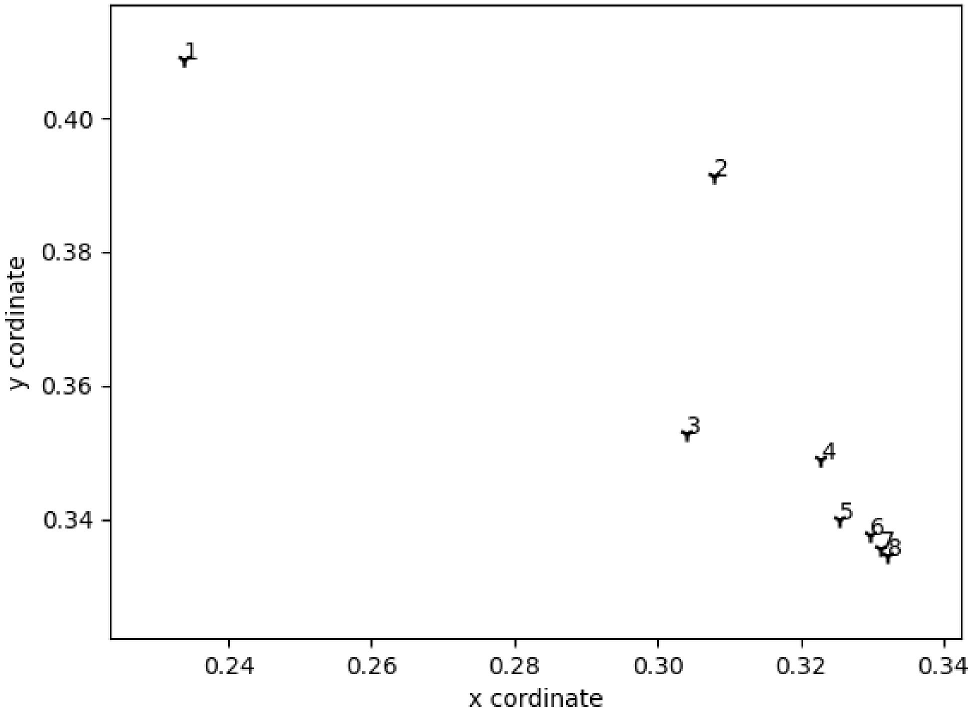
learning rate = 0.05





c) Initial w_o = [0.28332678,0.57361592]
learning rate = 0.05





d) Newton method converges slightly better than the gradient descent. As we can see in the above example that Newton method converges within approximately 6 iteration and gradient descent converges 7 iterations. As the function that we were trying to converge had only small domain so both methods perform almost similarly. Newton method converges slightly faster than the gradient descent. For some initial weights (Eg [0.19333942, 0.13318579]) where gradient descent converged, Newton method was not converging at all even with same value of learning rate. Furthermore, the function must be twice differentiable as well in case of Newton method to work, which is not always the case.

Source Code:

```
import numpy as np
import random
import matplotlib.pyplot as plt

class Gradient:
   def __init__(self):
      # self.W = self.getInputPoints()
      # self.W = np.array([0.36298233,0.20383835])
      self.W = np.array([0.19333942, 0.13318579])
   def getInputPoints(self):
      x1 = random.uniform(0,1)
      x2 = random.uniform(0,1)
      while (x1 + x2 >= 1 ):
         x1 = random.uniform(0,1)
         x2 = random.uniform(0,1)
      X = np.array([x1,x2])       # weight vector Ω
      return X

   # Perceptron training algoritm
   def weightUpdate(self,rate,W,type= 'grad'):
      if type == 'grad':
         W = W - rate * self.gradient(W)
      elif type == 'hessian':
         W = W - rate * np.linalg.inv(self.hessian(W)) @ self.gradient(W)
      return(W)

   def energy(self,W):
      x1 = W[0]
      x2 = W[1]
      E = - np.log(1-x1-x2) - np.log(x1) - np.log(x2)
      return E

   def gradient(self,W):
      x1 = W[0]
      x2 = W[1]
      dw1 = (1/(1-x1-x2) - 1/x1)
      dw2 = (1/(1-x1-x2) - 1/x2)
      grad = np.array([dw1,dw2])
      return grad

   def hessian(self,W):
      w1 = W[0]
      w2 = W[1]
      dw11 = ((1/(1-w1-w2)**2 )+ 1/(w2**2))
      dw12 = (1/(1-w1-w2)**2)
      dw21 = (1 / (1 - w1 - w2)**2)
      dw22 = ( (1 / (1 - w1 - w2)**2) + 1 / (w2**2))
      hessian = np.array([[dw11,dw12],[dw21,dw22]])
      return hessian

   def graphEnergy(self,epoch,energy):
      plt.plot(np.array(range(epoch)),energy)
      plt.xlabel('iterations')
      plt.ylabel('Energy value')
      plt.show()

   def graphWeights(self,xpoints,ypoints):
      fig, ax = plt.subplots()
      markers =
[".",",","o","v","^","<",">","1","2","3","4","8","s","p","P","*","h","H","+","x","X
","D","d","|","_"]
      ax.scatter(xpoints, ypoints,color='#000000',marker=np.random.choice(markers))
      for i in range(len(xpoints)):
         ax.annotate(i+1,(xpoints[i],ypoints[i]))
      plt.xlabel('x cordinate')
      plt.ylabel('y cordinate')
      plt.show()

def descentAlgo(ob,rate,W0,type='gradient'):
   W = ob.weightUpdate(rate, W0,type=type)
   Energy = []
   xpoints = []
   ypoints = []
   for i in range(8):
      W = ob.weightUpdate(rate, W)
      xpoints.append(W[0])
      ypoints.append(W[1])
      Energy.append(ob.energy(W))
   ob.graphEnergy(8, Energy)
   ob.graphWeights(xpoints,ypoints)

if __name__ == "__main__":
   rate = 0.05
   ob = Gradient()
   W0 = ob.W
   print(ob.W)
   descentAlgo(ob,rate,W0,type='grad')
   descentAlgo(ob,rate,W0,type='hessian')
```

Q3 c) To minimize the equation $\sum_{i=1}^{50}(y_i - (w_0 + w_1 x_i))^2.$ We can calculate using the pseudo inverse. As the equation is of the form :

||D – W X||^2 = DX$^+$

Or D * X.T (X * X.T)$^{-1}$

So based on the idea we can calculate the final set of weights.

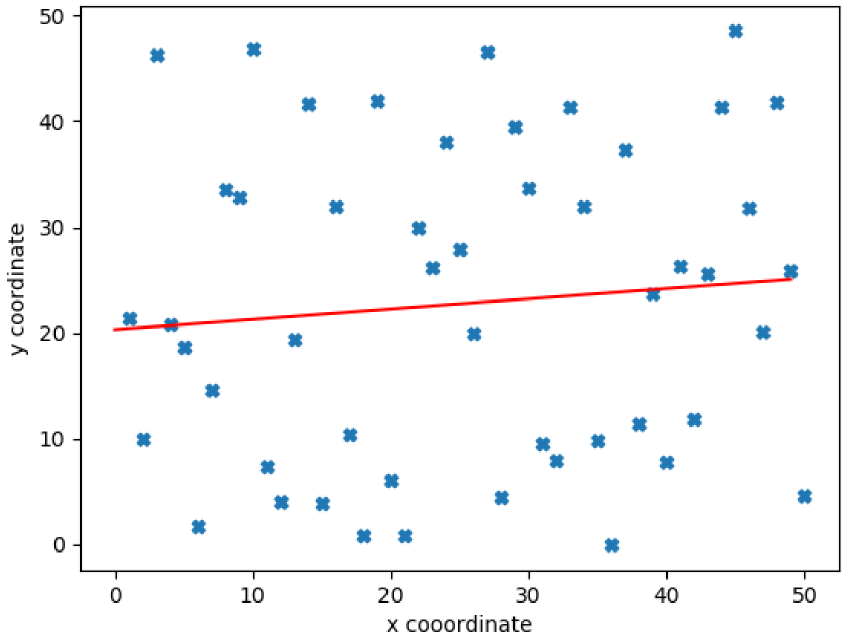
Source Code:

```
import numpy as np
import random
import matplotlib.pyplot as plt

threshold = 0.7
class LMS:
    def __init__(self):
        self.X ,self.Y = self.getInputPoints()
        self.W = self.getRandomWeights()

    def getRandomWeights(self):
        w0 = random.uniform(0,1)
        w1 = random.uniform(0,1)
        return (np.array([w0,w1]))

    def getInputPoints(self):
        X = []
        Y = []
        for i in range(50):
            u = random.uniform(-1,1)
            y = u + random.uniform(0,50)
            Y.append(y)
        X = np.array([(i+1) for i in range(50)])
        Y = np.array(Y)
        return (X,Y)

    #for closed form
    def getXY(self):
        X = np.vstack((np.array(self.X), np.ones(50)))
        Y = np.array(self.Y)
        return(X,Y)

    def closedForm(self):
        X,Y = self.getXY()
        X_pseduo = X.T @ np.linalg.inv(X @ X.T)
        W = Y @ X_pseduo
        return W

    def graphPlot(self,X,Y,formula):
        plt.scatter(X,Y,marker='X')
        #plotting our equation of line
        x = np.arange(50)
        y = formula(x)
        plt.plot(x,y,'r')
        plt.xlabel('x cooordinate')
        plt.ylabel('y coordinate')
        plt.show()

    def energy(self,X,Y,W):
        w0 = W[0]
        w1 = W[1]
        E = 0
        for i in range(len(X)):
           E += (Y[i] - (w0 + w1* X[i]))**2
        return E

     # Perceptron training algoritm
     def weightUpdate(self,rate,W,X,Y,type='grad'):
        if type == 'grad':
           W = W - rate * self.gradient(W,X,Y)
        elif type == 'hessian':
           W = W - (rate * np.linalg.inv(self.newton(W,X,Y)) @
  self.gradient(W,X,Y)).T
        print("weight= ", W)
        return (W)

     def graphEnergy(self,epoch,energy):
        plt.plot(np.array(range(epoch)),energy)
        plt.xlabel('iterations')
        plt.ylabel('Energy value')
        plt.show()

  if __name__ == "__main__":
     ob = LMS()
     W = ob.closedForm()
     print("For the closed form weight= ", W)
     ob.graphPlot(ob.X,ob.Y,(lambda x: (W[0]*x + W[1])))
```

So Final Set of weights obtained are:

[0.07259726 22.23369444]

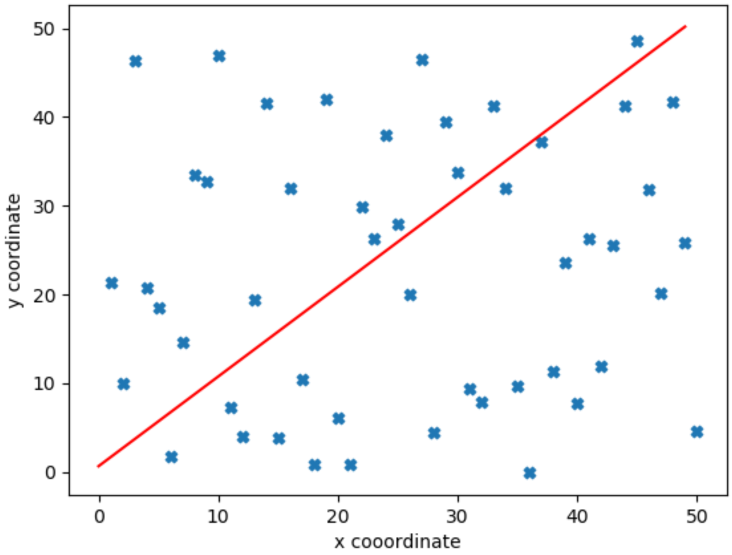
d)



f) Using gradient descent to find the least square fit we get different set of weights.

Weights obtained: [1.01059951 0.66322798]

```
With learning rate = 0.000001
```



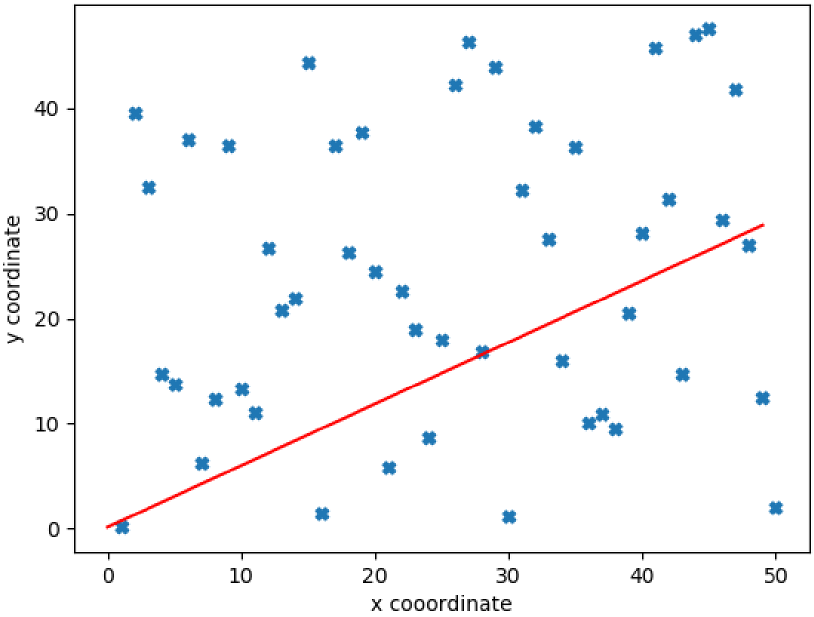
With gradient descent we get different set of weights and it is very sensitive to the initial set of weights. It converge to different set of weights based on the initial points and the learning rate. With the threshold value = 0.7 (based on the differrence in energy) it takes lot of iterations to converge every time and might not lead to global optimum.

Source Code:

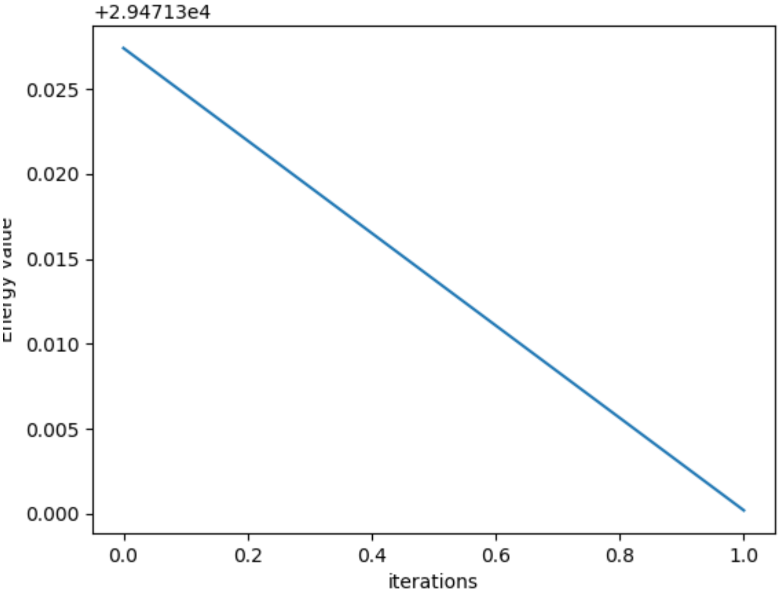
```
import numpy as np
import random
import matplotlib.pyplot as plt

threshold = 0.7
class LMS:
   def __init__(self):
      self.X ,self.Y = self.getInputPoints()
      self.W = self.getRandomWeights()

   def getRandomWeights(self):
      w0 = random.uniform(0,1)
      w1 = random.uniform(0,1)
      return (np.array([w0,w1]))

   def getInputPoints(self):
      X = []
      Y = []
      for i in range(50):
         u = random.uniform(-1,1)
         y = u + random.uniform(0,50)
         Y.append(y)
      X = np.array([(i+1) for i in range(50)])
      Y = np.array(Y)
      return (X,Y)

   #for closed form
   def getXY(self):
      X = np.vstack((np.array(self.X), np.ones(50)))
      Y = np.array(self.Y)
      return(X,Y)

   def closedForm(self):
      X,Y = self.getXY()
      X_pseduo = X.T @ np.linalg.inv(X @ X.T)
      W = Y @ X_pseduo
      return W

   def graphPlot(self,X,Y,formula):
      plt.scatter(X,Y,marker='X')
      #plotting our equation of line
      x = np.arange(50)
      y = formula(x)
      plt.plot(x,y,'r')
      plt.xlabel('x cooordinate')
      plt.ylabel('y coordinate')
      plt.show()

   def energy(self,X,Y,W):
      w0 = W[0]
      w1 = W[1]
      E = 0
      for i in range(len(X)):
         E += (Y[i] - (w0 + w1* X[i]))**2
      return E

   def gradient(self,W,X,Y):
      w0 = W[0]
      w1 = W[1]
      dw0 = 0
      dw1 = 0
      for i in range(len(X)):
         dw0 += ( Y[i] - (w0 + w1 * X[i]))   * (-2)
         dw1 += (( Y[i] - (w0 + w1* X[i])) * X[i])* (-2)
      return (np.array([dw0,dw1]))

   def newton(self,W,X,Y):
      w0 = W[0]
      w1 = W[1]
      dw11 = 0
      dw12 = 0
      dw21 = 0
      dw22 = 0
      for i in range(len(X)):
         dw11 = 2
         dw12 += 2 * (X[i])
         dw21 += 2 * (X[i])
         dw22 += 2 *((X[i])**2)
      hessian = np.array([[dw11,dw12],[dw21,dw22]])
      return (hessian)


   # Perceptron training algoritm
   def weightUpdate(self,rate,W,X,Y,type='grad'):
      if type == 'grad':
         W = W - rate * self.gradient(W,X,Y)
      elif type == 'hessian':
         W = W - (rate * np.linalg.inv(self.newton(W,X,Y)) @
self.gradient(W,X,Y)).T
      return (W)

   def graphEnergy(self,epoch,energy):
      plt.plot(np.array(range(epoch)),energy)
      plt.xlabel('iterations')
      plt.ylabel('Energy value')
```

```python
        plt.show()

if __name__ == "__main__":
    ob = LMS()
    W = ob.closedForm()
    print("For the closed form weight= ", W)
    ob.graphPlot(ob.X,ob.Y,(lambda x: (W[0]*x + W[1])))

    #gradient descent
    rate = 0.000001
    dE = 1
    count = 0
    W1 = ob.W
    Energy = []
    Energy.append(ob.energy(ob.X,ob.Y,W1))
    while dE > threshold:
        E0 = ob.energy(ob.X,ob.Y,W1)
        W1 = ob.weightUpdate(rate,W1,ob.X,ob.Y,type='grad')
        E1 = ob.energy(ob.X,ob.Y,W1)
        Energy.append(E1)
        dE = abs(E1 - E0)
        count += 1
    print(W1)
    ob.graphPlot(ob.X,ob.Y,(lambda x: (W1[0]*x + W1[1])))
```

g) Taking initial weight =[1,1] and learning rate =1



Newton method converges in one iteration. Above is the solution obtained from newton method.



It converges in one iteration. Above is the graph of energy vs iterations.