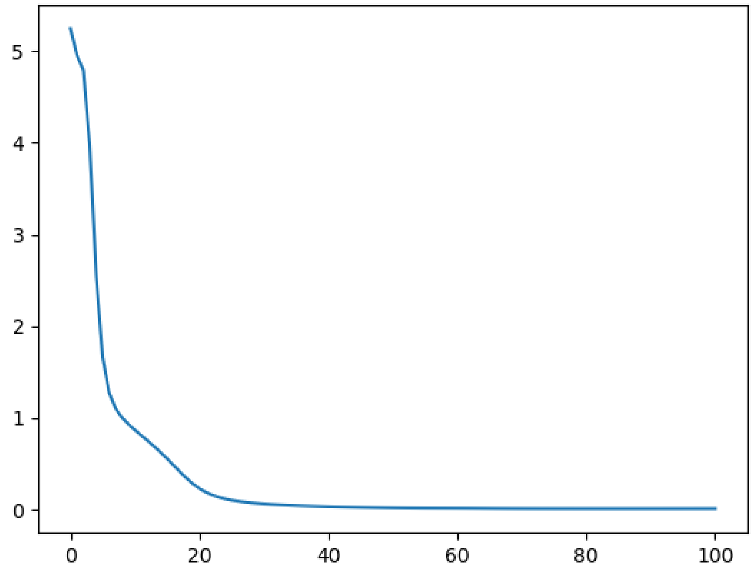
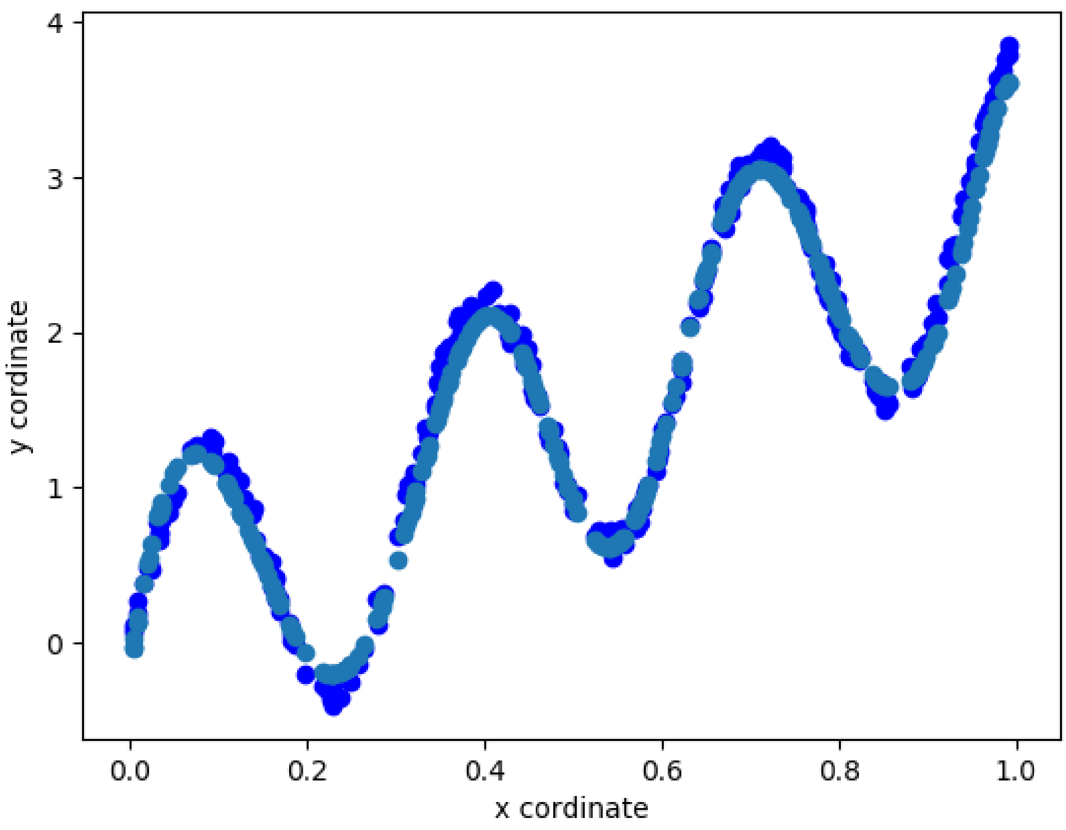
CS -559 – Neural Netwok , Fall -02018

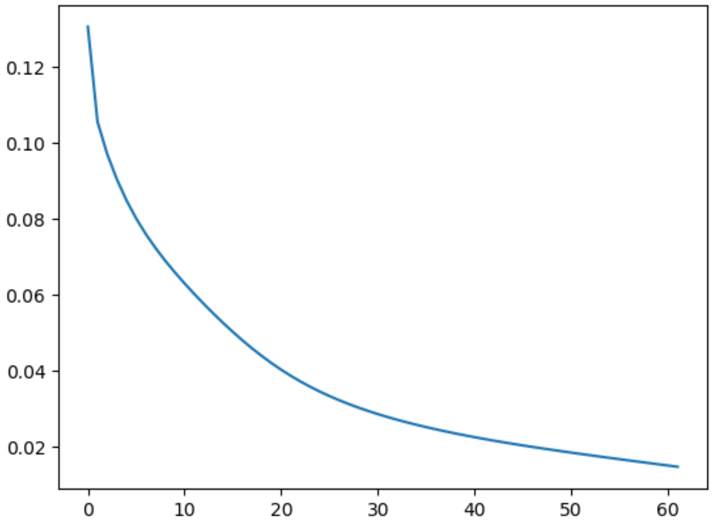
Q1) d) Plot of some of the trials on different set of initial weights



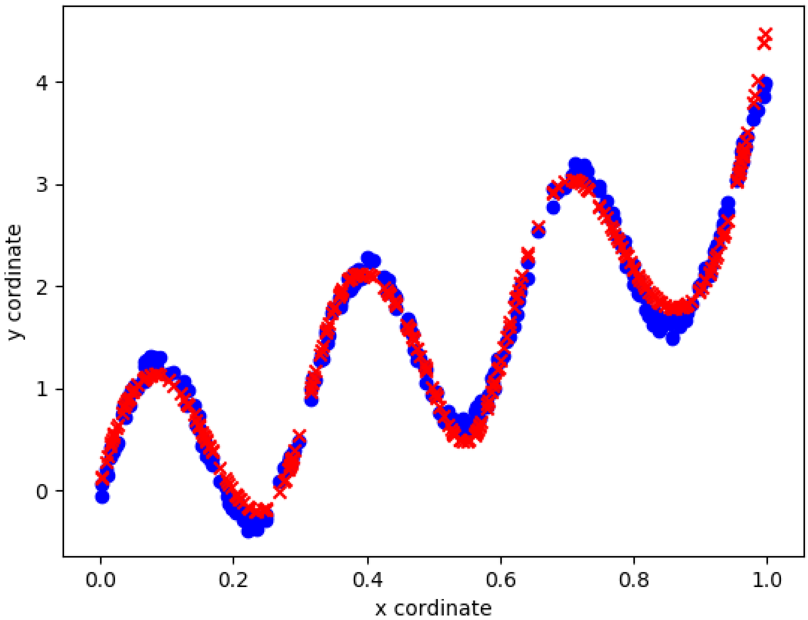
Error vs Epoch



Fit of the curve with the original



Error vs Epoch



Fit of the curve with the original

Source Code:

```
import numpy as np
import random
import math
import matplotlib.pyplot as plt
from random import shuffle


N = 300         # no of points in 2 layer

# for a specific case for q1 ; will update for general case
class Neural_Network:
   def __init__(self):
      self.X = self.get_random_points(0, 1, N)
      self.V = self.get_random_points(-1 / 10, 1 / 10, N)
      self.D = self.get_desired_output(self.X, self.V)

   # initilaze the weights
   def get_weights(self,rows,column):
      if ( type(rows) is int and type(column) is int):
         W = np.empty((0, (column)))
         for i in range(rows):
            w = self.get_random_points(-15, 15, column)
            W = np.vstack((W, w))
         return W
      else:
         print("specify correct value for inpur and output layer nodes (int)")

   # Updated the previous function to randomly assign randomNeural_Network
   # points of any length with the specified range
   def get_random_points(self,a,b,n):
      x = list()
      for i in range(n):
         temp = random.uniform(a,b)
         x.append(temp)
      return x

   def get_desired_output(self,X,V):
      D= list()
      for x,v in zip(X,V):
         d = math.sin(20 * x) + 3 * x + v
         D.append(d)
      return D

   # forward pass
   # getting the individual induced local field and output
   # returns induced local field and outputs
   def get_Output(self,x,W,A):
      I= []  # induced local field
      Z = [] # output field
      x = np.array(np.insert(x,0,1)).reshape(1,-1)
      for idx,(a,w) in enumerate(zip(A,W)):
         if (idx is 0):
            u = np.dot(w,x.T)
         else:
            u = np.insert(u,0,1)
            u = np.dot(w,u.T)
         I.append(np.array(u))
         u = np.array(self.get_activation(a,u))
         Z.append(u)
      return (I,Z)

   # new function for update using equations
   def get_backpropagation_update(self,x,d,W,no_of_layers,A,rate):
      # for l in range(no_of_layers):
      L = no_of_layers
      I, Z = self.get_Output(x, W, A)
```

```
        Delta = 0
        for i in reversed(range(L)):
            if i is (L-1):
                Delta = np.multiply((d - Z[i]),
self.get_derivative_activation(A[i],I[i]))[0] #
            else:
                W_n = np.delete(W[i+1],0,1)
                Delta = np.multiply(np.dot(W_n.T, Delta) ,
self.get_derivative_activation(A[i],I[i]))
            if i is 0:
                Z_n = np.insert(np.array([x]), 0, 1).reshape(1,-1)
                Delta = Delta.reshape(1,-1)
                W[i] = W[i] + (rate) * np.dot((Delta).T, (Z_n))
            else:
                Z_n = np.insert(Z[i - 1], 0, 1)
                W[i] = W[i] + (rate) * np.dot((Delta), (Z_n))
        return (W)

    def softmax(self,X):
        return (np.exp(X)/np.sum(np.exp(X)))

    def softmax_grad(self,s):
        jacobian_m = np.diag(s)
        for i in range(len(jacobian_m)):
            for j in range(len(jacobian_m)):
                if i == j:
                    jacobian_m[i][j] = s[i] * (1 - s[i])
                else:
                    jacobian_m[i][j] = -s[i] * s[j]
        return jacobian_m

    def get_activation(self, a, X):
        tanh = np.vectorize(lambda x:math.tanh(x))
        relu = np.vectorize(lambda x:x)
        step = np.vectorize(lambda x:1 if x>=0 else 0)
        sigmoid = np.vectorize(lambda x: (math.exp(x)/ (1 + math.exp(x))))
        if a is 'tanh':
            y = tanh(X)
        elif a is 'relu':
            y = relu(X)
        elif a is 'step':
            y = step(X)
        elif a is 'softmax':
            y = self.softmax(X)
        elif a is 'sigmoid':
            y = sigmoid(X)
        return y

    def get_derivative_activation(self,a,Y):
        der_tanh = np.vectorize(lambda x:(1-math.tanh(x)**2))
        der_relu  = np.vectorize(lambda x:1)
        sigmoid = np.vectorize(lambda x: (math.exp(x) / (1 + math.exp(x))))
        if a is 'tanh':
            q = der_tanh(Y)
        elif a is 'relu':
            q = der_relu(Y)
        elif a is 'softmax':
            q = self.softmax_grad(Y)
        elif a is 'sigmoid':
            q =  (sigmoid(Y) * (1-(sigmoid(Y)**2)))
        return q

    # @param X array , Y array
    def graph(self,X,D,Y_out):
        plt.scatter(X, D, color='b',marker='o')
        plt.scatter(X,Y_out,color ='r',marker='x')
        plt.xlabel('x cordinate')
        plt.ylabel('y cordinate')
```

```
        # plt.figure(num=None, figsize=(8, 6), dpi=80, facecolor='w', edgecolor='k')
        plt.show()

    def calculate_output_vector(self,W,X,A):
        Y = []
        for x in X:
            _,y = self.get_Output(x,W,A)
            Y.append(y[1])
        return Y

    def get_MSE(self,D,Y):
        MSE = 0
        for d,y in zip(D,Y):
            MSE += (d - y)**2
        return MSE/len(D)

if __name__ == '__main__':
    ob = Neural_Network()
    D = ob.D
    X = ob.X
    X_axis = [i for i in range(N)]
    # ob.graph(X,D, type = 'scatter')  # @TODO remove comment

    W_final = []
    # fwd propagating to the next layers

    no_of_nodes_hidden = 24
    W1 = ob.get_weights(no_of_nodes_hidden, 2)  # first layer has one input and we
have
    W2 = ob.get_weights(1, (no_of_nodes_hidden + 1))  # output layer has one
neuron,bias and previour layer with N inputs
    W_final.append(W1)
    W_final.append(W2)

    A = ['tanh','relu']

    MSE = []
    ### calculate desired output
    Y = ob.calculate_output_vector(W_final, X, A)

    e = 0.015
    epoch = 0
    while True:
        for x,d in zip(X,D):
            W_final = ob.get_backpropagation_update(x,d,W_final,no_of_layers=2,A =A,
rate=0.01)
        Y = ob.calculate_output_vector(W_final,X,A)
        mse = ob.get_MSE(D,Y)
        print(mse)
        MSE.append(mse)
        epoch += 1
        if ((MSE[epoch - 1] <= e) or (epoch>100)):      # break if value decreases
below that value
            break

    range_epoch = [i for i in range(epoch)]
    plt.plot(range_epoch,MSE)
    plt.show()

    Y = ob.calculate_output_vector(W_final, X, A)
    ob.graph(X,D,Y)
    plt.show()
```

Q2) Network Topology

Architecture Details

For the digit classification of MNIST dataset, I have used 2 layer neural network, 784 * 24 * 10.
So the output vector represents a vector for the digits Eg ([1 0 0 ..]) for 0. Activation function is sigmoid activation function as represents the value within the range of 0-1 and in our case we require the output vector to be between 0-1. Furthermore, sigmoid gives a probalistic values for the input within (0 1), hence making it easier to classify digits. The no of nodes for the hidden layer has been based on the previous setup. I have also normalized the input vector as sigmoid function was giving math error with raw input as pixel value range from 0-255.

Design Process:

So initially I started with un-normalized data that led to math error in case of sigmoid function. Also, initially I took step function but that had zero gradient so the back propagation algorithm could not work in that scenario. So I adjusted the learning rate to 0.1 – 0.001 as initially the learning rate was too high that led to overflow of values and the it was converging.

Source Code:

```
import numpy as np
import random
import math
import matplotlib.pyplot as plt
import MNIST
import backPropagation


class image:
   def __init__(self):
      ob = MNIST.Mnist()
      self.trainingImgs = ob.trainingImgs
      self.trainingLabels = ob.trainingLabels

   # initilaze the weights
   def get_weights(self, rows, column):
      if (type(rows) is int and type(column) is int):
         W = np.empty((0, (column)))
         for i in range(rows):
            w = self.get_random_points(-15, 15, column)
            W = np.vstack((W, w))
         return W
      else:
            print("specify correct value for inpur and output layer nodes (int)")

   # Updated the previous function to randomly assign randomNeural_Network
   # points of any length with the specified range
   def get_random_points(self, a, b, n):
      x = list()
      for i in range(n):
         temp = random.uniform(a, b)
         x.append(temp)
      return x

   def calculate_output_vector(self,W,X,A):
      Y = []
      for x in X:
         _,y = self.get_Output(x,W,A)
         Y.append(y[1])
      return Y

   def encode_labels(self):
      labels = self.trainingLabels
      D = []  # desired output
      d = np.array([])
      for label in labels:
         label = int(label)
         if label == 0:
            d = np.array([[1], [0], [0], [0], [0], [0], [0], [0], [0], [0]])
         elif label is 1:
            d = np.array([[0], [1], [0], [0], [0], [0], [0], [0], [0], [0]])
         elif label == 2:
            d = np.array([[0], [0], [1], [0], [0], [0], [0], [0], [0], [0]])
         elif label == 3:
            d = np.array([[0], [0], [0], [1], [0], [0], [0], [0], [0], [0]])
         elif label == 4:
            d = np.array([[0], [0], [0], [0], [1], [0], [0], [0], [0], [0]])
         elif label == 5:
            d = np.array([[0], [0], [0], [0], [0], [1], [0], [0], [0], [0]])
         elif label == 6:
            d = np.array([[0], [0], [0], [0], [0], [0], [1], [0], [0], [0]])
         elif label == 7:
            d = np.array([[0], [0], [0], [0], [0], [0], [0], [1], [0], [0]])
         elif label == 8:
            d = np.array([[0], [0], [0], [0], [0], [0], [0], [0], [1], [0]])
         elif label == 9:
```

```
            d = np.array([[0], [0], [0], [0], [0], [0], [0], [0], [0], [1]])
        D.append(d)
    return D

    # forward pass
    # getting the individual induced local field and output
    # returns induced local field and outputs
    def get_Output(self, x, W, A):
        I = []  # induced local field
        Z = []  # output field
        x = np.array(np.insert(x, 0, 1)).reshape(1, -1)
        for idx, (a, w) in enumerate(zip(A, W)):
            if (idx is 0):
                u = np.dot(w, x.T)
            else:
                u = np.insert(u, 0, 1)
                u = np.dot(w, u.T)
            I.append(np.array(u))
            u = np.matrix(self.get_activation(a, u))   # to change it to 2-D
            Z.append(u)
        return (I, Z)

    # new function for update using equations
    def get_backpropagation_update(self, x, d, W, no_of_layers, A, rate):
        # for l in range(no_of_layers):
        L = no_of_layers
        I, Z = self.get_Output(x, W, A)
        for i in reversed(range(L)):
            if i is (L - 1):
                Delta = np.multiply((d - Z[i]), self.get_derivative_activation(A[i],
I[i]))  #
            else:
                W_n = np.delete(W[i + 1], 0, 1)
                Delta = np.multiply(np.dot(W_n.T, Delta),
self.get_derivative_activation(A[i], I[i]))
            if i is 0:
                Z_n = np.insert(np.array([x]), 0, 1).reshape(1, -1)
                W[i] = W[i] + (rate) * np.dot((Delta), (Z_n))
            else:
                Z_n = np.insert(Z[i - 1], 0, 1)
                W[i] = W[i] + (rate) * np.dot((Delta), (Z_n))
        return (W)

    def get_activation(self, a, X):
        tanh = np.vectorize(lambda x:math.tanh(x))
        relu = np.vectorize(lambda x:x)
        step = np.vectorize(lambda x:1 if x>=0 else 0)
        sigmoid = np.vectorize(lambda x: (math.exp(x)/ (1 + math.exp(x))))
        if a is 'tanh':
            y = tanh(X)
        elif a is 'relu':
            y = relu(X)
        elif a is 'step':
            y = step(X)
        elif a is 'softmax':
            y = self.softmax(X)
        elif a is 'sigmoid':
            y = sigmoid(X)
        return y

    def get_derivative_activation(self,a,Y):
        der_tanh = np.vectorize(lambda x:(1-math.tanh(x)**2))
        der_relu  = np.vectorize(lambda x:1)
        sigmoid = np.vectorize(lambda x: (math.exp(x) / (1 + math.exp(x))))
        if a is 'tanh':
            q = der_tanh(Y)
        elif a is 'relu':
            q = der_relu(Y)
```

```
      elif a is 'softmax':
         q = self.softmax_grad(Y)
      elif a is 'sigmoid':
         q =  (sigmoid(Y) * (1-(sigmoid(Y)**2)))
      return q

   def normalize_input(self,X):
      normalize = np.vectorize(lambda x:x/255)
      return (normalize(X))

   def sigmoid(self,X):
      return (np.exp(X)/ (1+ np.exp(X)))

   def softmax(self,X):
      return (np.exp(X)/np.sum(np.exp(X)))

   def softmax_grad(self,s):
      jacobian_m = np.diag(s)
      for i in range(len(jacobian_m)):
         for j in range(len(jacobian_m)):
            if i == j:
               jacobian_m[i][j] = s[i] * (1 - s[i])
            else:
               jacobian_m[i][j] = -s[i] * s[j]
      return jacobian_m

   def graph(self,X,D,Y_out):
      plt.scatter(X, D, color='b',marker='o')
      plt.scatter(X,Y_out,color ='r',marker='x')
      plt.xlabel('x cordinate')
      plt.ylabel('y cordinate')
      # plt.figure(num=None, figsize=(8, 6), dpi=80, facecolor='w', edgecolor='k')
      plt.show()

   # @param self , output vector
   def get_mse_mnist(self,Y):
      error = 0
      labels = self.trainingLabels
      for label,y in zip(labels,Y):
         j = np.argmax(y,axis=1)
         if label != j[0]:
            error += 1
      return (error/len(Y))

if __name__ == "__main__":
   ob = image()
   ob1 = backPropagation.Neural_Network()
   images = ob.trainingImgs
   labels = ob.trainingLabels

   W_final = []
   N = 24              # hidden layer
   D = ob.encode_labels()
   images_n = []
   for image in images:
      image = ob.normalize_input(image)
      images_n.append(image)
   W1 = ob1.get_weights(N,(784+1))
   W2 = ob1.get_weights(10,(N+1))
   W_final.append(W1)
   W_final.append(W2)

   e = 0.015
   epoch = 0
   A = ['sigmoid','sigmoid']
   MSE = []
   while True:
      for idx,(x,d) in enumerate(zip(images_n,D)):
```

```
         W_final = ob.get_backpropagation_update(x, d, W_final, no_of_layers=2,
A=['sigmoid','sigmoid'], rate=0.0009)

      Y = ob.calculate_output_vector(W_final, images_n, A=['sigmoid','sigmoid'])
      mse = ob.get_mse_mnist(Y)
      epoch = epoch + 1
      print(mse)
      print(epoch)
      MSE.append(mse)
      if ((MSE[epoch - 1] <= e) or (epoch>100)):      # break if value decreases
below that value
         break
   # if ((MSE[epoch - 1] <= e) or (epoch > 100)):  # break if value decreases below
that value
   #  break
```