COMP 790-124, HW3

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November 9, 2012

Deadline: 11/9/2012 11:59PM EST
Submit `hw3.pdf` by e-mail, `mailto:vjojic+comp790+hw3@cs.unc.edu`.

We will assume existence of an N long backbone sequence s in an N. In this assignment the alphabet will be of size 4, corresponding to nucleotides. We will construct a hidden Markov model that generates a shorter sequence from the backbone sequence. The shorter sequence will consist of two parts of equal length L. First part of sequence corresponds to offsets, 1 through L, and the second part of the sequence corresponds to offsets, $L+1$ to $2L$. With each offset i in the two-part sequence we will associate a hidden variable that points to a position in the backbone sequence. The probability of a letter x_i, given pointer h_i is

$$p(x_i|h_i) = \begin{cases} 0.99, & \text{if } x_i = s_{h_i} \\ \frac{0.01}{a-1}, & \text{if } x_i \neq s_{h_i}. \end{cases}$$

Finally, we define transition probability on the h_i

$$h_{i+1}|h_i \propto \begin{cases} \text{TruncPoiss}(h_{i+1} - h_i), & \text{if } i = L \\ \pi_{\text{ins}}, & \text{if } h_{i+1} = h_i, i \neq L \\ \pi_{\text{del}}, & \text{if } h_{i+1} = h_i + 2, i \neq L, h_i + 2 \leq N \\ \pi_{\text{copy}}, & \text{if } h_{i+1} = h_i + 1, i \neq L, h_i + 1 \leq N. \end{cases}$$

where TruncPoiss denotes a truncated poisson, given by

$$\text{TruncPoiss}(l) \propto \begin{cases} \text{Poiss}(l, \lambda = 100), & \text{if } 90 \leq l \leq 110 \\ 0, & \text{otherwise} \end{cases}$$

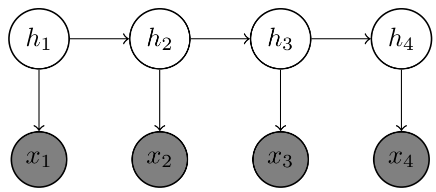
Hence, we move the pointer forward with an occasional skip (deletion) or lag (insert), except when we reach offset L where we make a leap of approximately 100 positions.

We will use a `logsum` function

```
function s = logsum(vec)
m = max(vec);
s = log(sum(exp(vec - m))) + m;
```

and `logProbPoiss`

```
function logProb = logProbTruncPoiss(i,lambda)
logProb = log(lambda)*i + (-lambda) - sum(log(1:i));
```



Problem 1(3pt) Given the HMM model specification above implement a forward pass in the HMM. Storing the transition matrix explicitly would be too costly. But the matrix is very sparse. With the exception of $i = L$, from a given offset h_i we can transition to *at most* three different offsets. Hence computation of the forward pass will require us to explicitly account for these possibilities. Note that if $h_i = N$ then the only possible transition is to $h_{i+1} = N$.

Note that transition from h_L to h_{L+1} is made according to a Poisson.

```
function m_f = fw(s,x,pins,pdel,pcopy)
N = length(s);
L = length(x)/2;
m_f = -realmax*ones(N,2*L);

logpins = log(pins);
logpdel = log(pdel);
logpcopy = log(pcopy);
for a=1:4
    for b=1:4
        logmut(a,b) = log(0.99)*(a==b) + log(0.01)*(a~=b);
    end
end

m_f(1:N,1) = -log(N) + logmut(s,x(1));
for i=2:2*L
    for prev=1:N
        if i~=L+1
            % insert
            vala = m_f(prev,i);
            valb = m_f(prev,i-1)+ logmut(s(prev),x(i)) + logpins;
            m_f(prev,i) = logsum([vala valb]);
            if prev<=N-2
                % delete
                vala = m_f(prev+2,i);
                valb = ...
```

```
                m_f(prev+2,i) = logsum([vala valb]);
            end
            if prev<=N-1
                % copy
                vala = m_f(prev+1,i);
                valb = ...
                m_f(prev+1,i) = logsum([vala valb]);
            end
        else
            if prev+90<=N
                for next=prev+90:min(prev+110,N)
                    vala = m_f(next,i);
                    valb = ...;
                    m_f(next,i) = logsum([vala valb]);
                end
            end
        end
    end
end
```

Run forward pass on inputs stored in `hw3.mat` and run this script

```
m_f = fw(seq,x,0.005,0.005,0.99);
logProb = logsum(m_f(:,end))
```

The resulting `logProb` is `answer`

Problem 2(3pt) Implement a backward pass.

```
function m_b = bw(s,x,pins,pdel,pcopy)
N = length(s);
L = length(x)/2;
m_b = -realmax*ones(N,2*L);

logpins = log(pins);
logpdel = log(pdel);
logpcopy = log(pcopy);
for a=1:4
    for b=1:4
        logmut(a,b) = log(0.99)*(a==b) + log(0.01)*(a~=b);
    end
end

m_b(:,2*L) = 0;
for i=2*L-1:-1:1
    i
    for next=1:N
```

```
        if i~=L
            % insert
            vala = m_b(next,i);
            valb = m_b(next,i+1)+ logmut(s(next),x(i+1)) + logpins;
            m_b(next,i) = logsum([vala valb]);
            if next-2>=1
                % delete
                vala = m_b(next-2,i);
                valb = ...
                m_b(next-2,i) = logsum([vala valb]);
            end
            if next-1>=1
                % copy
                vala = m_b(next-1,i);
                valb = ...
                m_b(next-1,i) = logsum([vala valb]);
            end
        else
            if next-90>=1
                for prev=max(1,next-110):next-90
                    vala = m_b(prev,i);
                    valb = ...;
                    m_b(prev,i) = logsum([vala valb]);
                end
            end
        end
    end
end
```

To check your implementation run following code

```
m_f = fw(seq,x,0.005,0.005,0.99);
m_b = bw(seq,x,0.005,0.005,0.99);
mm = m_f + m_b;
logsum(mm(:,1))
logsum(mm(:,end))
```

If the two logsum calls output different values, you have a bug.

Problem 3(3pt) Implement Viterbi forward and backward pass. To do this transform your forward pass and backward by replacing the `logsum` with `max`

Problem 4(2pt) Use MATLAB commands `tic` and `toc` to measure the time that it takes to run standard forward pass and Viterbi forward pass to complete. The ratio of the these times is $\boxed{\texttt{answer}}$. Is one of these functions faster? If so, why?

Hint: One way to answer this question is to use MATLAB's profiler. Before calling a function you want to profile do folowing

```
profile clear
profile on
```

Run your code and once done (or once you interrupt it)

```
profile viewer
```

Rest ought to be self-explanatory.

Problem 5(3pt) Modify your Viterbi forward pass to also record which of the states h_{i-1} was the most likely to have given rise to the current state h_i. This is sometimes called trace or backward pointers.

Implement code that starting with the state h_L with highest probability in the forward pass, backtracks according to the trace recording the path. Apply this procedure to long sequence `seq` and short sequence `x` in `hw3.mat`.

Paste most likely sequence of offsets in `seq` used to generate `x`

```
...
```