Holistic AI × The Alan Turing Institute

Bias in Clustering Systems Part III

Sachin Beepath
Assurance Officer, Holistic AI

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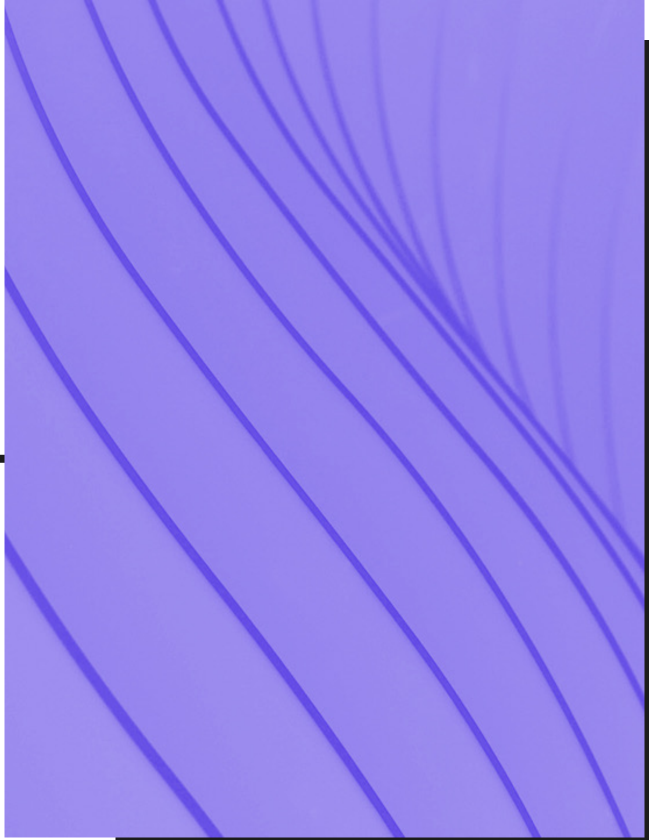
Measuring Bias in Clustering



1. Formally introduce several group and individual fairness metrics.
2. Demonstrate how to measure fairness and bias in clustering systems.

Balance

For m protected groups and k clusters, we can define two ratios:

- r, the proportion of samples in the data set that belong to the protected group b
- r_a, the proportion of samples in cluster a belonging to group b

The balance is then defined over all clusters and protected groups as:

$$\min_{a \in [k], b \in [m]} \min\{R_{a,b}, \frac{1}{R_{a,b}}\}, R_{a,b} = \frac{r}{r_a},$$

- Balance can take values between 0 and 1.
- The closer the balance is to 1, the more fair the clustering is.

Balance (Example)

– Consider $m = 2$ protected groups and $k = 3$ clusters

– For group A we have 10 data points with cluster assignments: [1,2,1,3,2,1,1,1,2,3]

– For group B we have 8 data points with with cluster assignments: [2,3,1,2,1,2,3,3]

Balance (Example)

The proportions of group A are:

Entire dataset: $\frac{10}{18} = 0.56$

Cluster 1: $\frac{5}{7} = 0.71$

Cluster 2: $\frac{3}{6} = 0.5$

Cluster 3: $\frac{2}{5} = 0.4$

The proportions of group B are:

Entire dataset: $\frac{8}{18} = 0.44$

Cluster 1: $\frac{2}{7} = 0.29$

Cluster 2: $\frac{3}{6} = 0.5$

Cluster 3: $\frac{3}{5} = 0.6$

Balance (Example)

The ratios group group A are:

- $R_{A,1} = \frac{0.56}{0.71} = 0.79$
- $R_{A,2} = \frac{0.5}{0.56} = 0.89$
- $R_{A,3} = \frac{0.4}{0.56} = 0.71$

The ratios of group B are:

- $R_{B,1} = \frac{0.29}{0.44} = 0.66$
- $R_{B,2} = \frac{0.44}{0.5} = 0.88$
- $R_{B,3} = \frac{0.44}{0.6} = 0.73$

The balance of the clustering then 0.66

Cluster Distribution KL Divergence

- For $m = 2$ protected groups and k clusters, we can calculate the distribution of clusters amongst each group and calculate the KL divergence between the two distributions:

$$KL(P||Q) = \sum_k P(k) \log(\frac{P(k)}{Q(k)})$$

- Where $P(k)$ represents the cluster distribution for the first group and $Q(k)$ represents the cluster distribution for the second group.
- The more similar the distributions are, the fairer the system is.
- An ideal value is 0.

Cluster Distribution KL Divergence

- Consider $m = 2$ protected groups and $k = 3$ clusters.
- For group A we have 10 data points with cluster assignments: [1,2,1,3,2,1,1,1,2,3]
- For group B we have 8 data points with with cluster assignments: [2,3,1,2,1,2,3,3]
- $P(k) = [0.71, 0.5, 0.4]$
- $Q(k) = [0.29, 0.5, 0.6]$
- $KL(P||Q) = 0.71 \log \frac{0.71}{0.29} + 0.5 \log \frac{0.5}{0.5} + 0.6 \log \frac{0.6}{0.4} = 0.88$

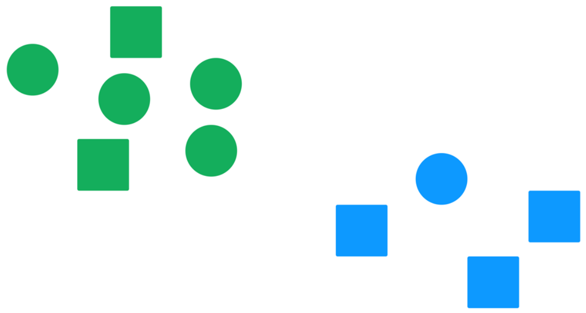
Part 3 Question 1

What is the cluster balance of the system?

Total: 5 Squares, 5 Circles

Blue: 3 Squares, 1 Circle

Green: 2 Squares, 4 Circles



Social Fairness

– The clustering cost O for a set cluster centres $U = \{U_1, \ldots, U_k\}$ and the input dataset X is defined as:

$$O(U, X) = \sum_{x \in X} \min_{u \in U} \|x - u\|^2$$

For m protected groups, let X_a be the samples of X that belong to protected group a, the social fairness cost is then:

$$\max_{a \in [m]} \frac{O(U, X_a)}{|X_a|}$$

Social Fairness

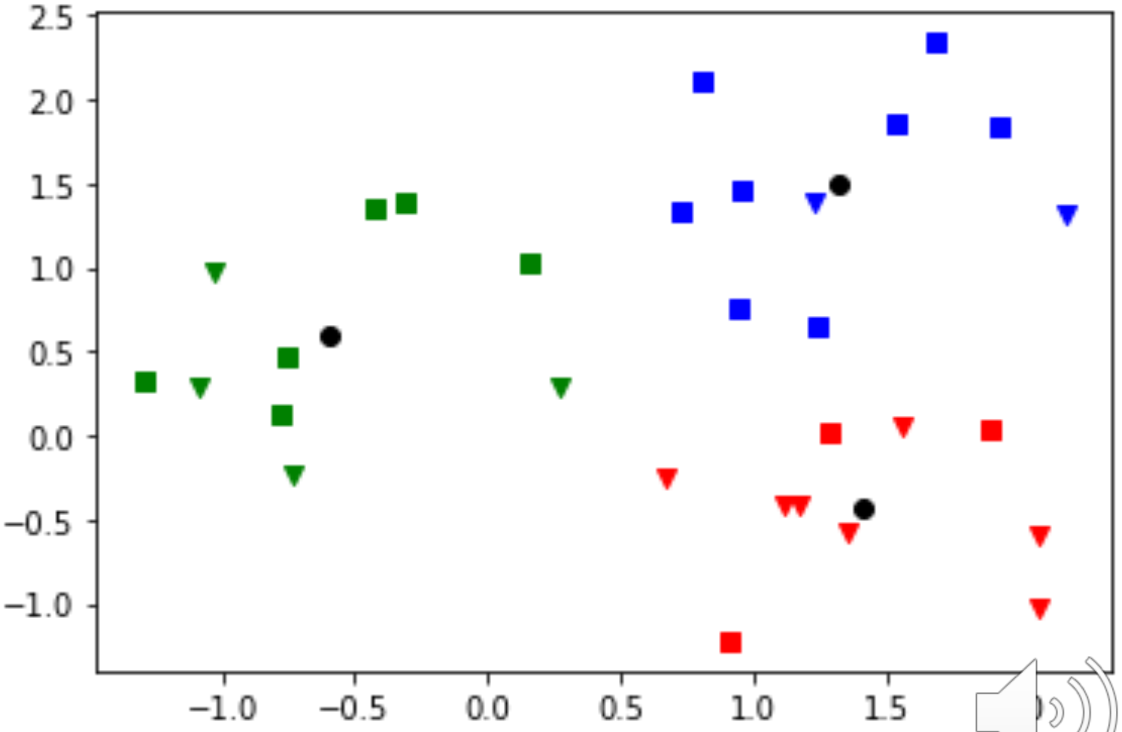
Clustering Costs:

– Squares: 8.52

– Triangles: 5.27

Social Fairness:

$$\max\left(\frac{8.52}{17}, \frac{5.27}{13}\right) = 0.501$$



Silhouette Difference

- For $m = 2$ protected groups and k clusters, the silhouette difference is the difference in silhouette scores for each protected group. For a given group, the silhouette score is:

$$\frac{d_b - d_a}{\max(d_a, d_b)}$$

where d_b is the mean nearest-cluster distance and d_a is the mean intra-cluster distance. The silhouette difference is defined as:

$$\frac{S_a - S_b}{2}$$

Silhouette Score and Silhouette Difference are bound between -1 and 1.

Silhouette Difference

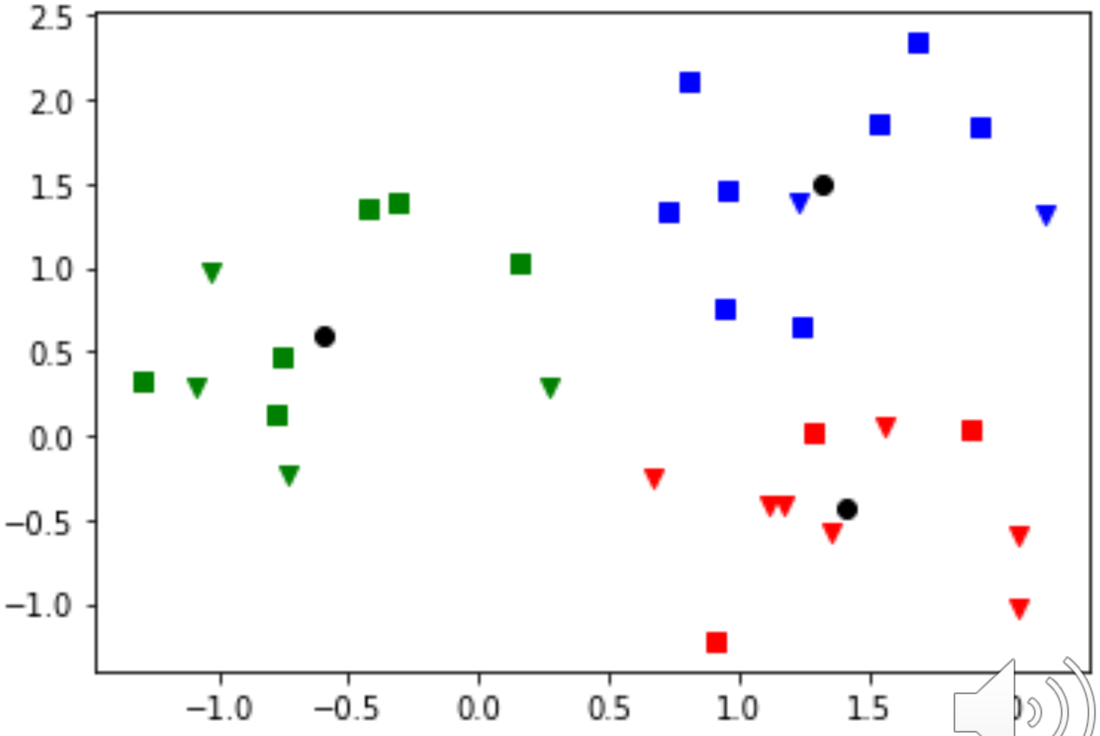
Average Silhouette Scores:

– Squares: 0.480

– Triangles: 0.552

Silhouette Difference:

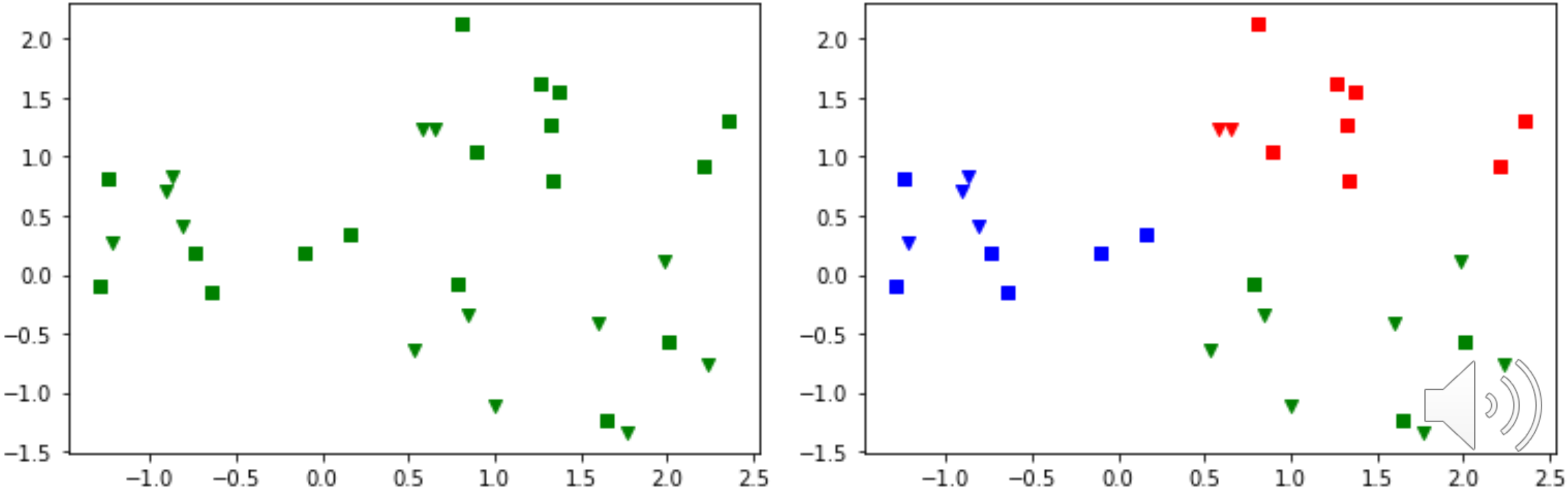
$$\frac{0.480 - 0.552}{2} = -0.036$$



Toy Example

– Use k-means to label data into 3 clusters

– 2 protected groups: Triangles and Squares



Toy Example (Cont.)

Metric	Value
Cluster Balance	0.462
Cluster Distribution KL Divergence	0.378
Cluster Social Fairness Ratio	1.022
Cluster Silhouette Difference	0.002

Individual Fairness Metrics

Proportionality

- For n data points and k clusters, any $\frac{n}{k}$ points are entitled to form their own cluster if there is another centre that is closer in distance for all $\frac{n}{k}$ points.

Aggregate Fairness

- Requires the distance of a point from its centroid to be at most α times the average distance of the points in its cluster to its centroid.

Conclusion

– Defined several group and individual fairness metrics in the context of clustering.

– Demonstrated how to measure fairness and bias using group fairness metrics.

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