



**Holistic AI**



**The  
Alan Turing  
Institute**

# **Bias in Multiclass Classification Part II**

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# Contents

- Part I – Introduction to Multiclass Classification
- **Part II – Fairness in Multiclass Classification**
- Part III – Measuring Bias in Multiclass Classification
- Part IV – Mitigating Bias in Multiclass Classification



# II – Fairness in Multiclass Classification

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- 1) Introduce some taxonomy around fairness in Multiclass setting
- 2) Introduce different fairness notions in the Multiclass setting



# Fairness Taxonomy

- As with other ML tasks, we usually split fairness notions into two main categories: **Equality of Outcome** and **Equality of Opportunity**.
- Equal Opportunity: We want the algorithm to **perform** similarly for different groups.
- Equal Outcome: we want the algorithm to **behave** similarly for different groups.



# Equality of Outcome Notions

- All notions we introduce are from [Putzel et al, 2022](#)



# Frequency Matrix

- A useful concept for Equality of Outcome fairness notions is the Frequency Matrix.
- It is a matrix indexed on groups and classes (shape  $M \times N$ ) with the  $g, i$  entry being the proportion of group  $g$  that is allocated to class  $i$ .
- In equations,  $FM_{gi} = P(Y_{pred} = i | \mathcal{P} = g)$



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# Multiclass Statistical Parity

- A multiclass predictor satisfies demographic parity if the protected group conditional class probabilities are equal across groups
- In equations  $FM_g = FM_h$  for any two groups  $g$  and  $h$ .
- Note that this can be computed even if we don't have true labels!
- We can think of these as allocations of members of a group to the different classes.





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# Equality of Opportunity Notions

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# Confusion Matrix

- Recall that in the Multiclass setting, we have a prediction  $Y_{pred}$  belonging to a collection of discrete and mutually exclusive outcomes, we can name  $1, 2, \dots, N$ . Suppose we also have the true labels  $Y_{true}$ .
- We allow our predictor to be probabilistic so we may write the confusion matrix as  $CM_{ij} = P(Y_{pred} = i | Y_{true} = j)$
- Notice this is not the same confusion matrix as in the Introduction, it is now **normalised** over columns (predictions).
- From now on we will only speak of the confusion matrix as a normalised one!



# Conditional Confusion Matrices

- All Equality of Opportunity fairness notions make use of Conditional Confusion Matrices.
- We remind you that the protected attribute  $\mathcal{P}$  (e.g. ethnicity) can belong to a collection of discrete and mutually exclusive groups, we name  $1, 2, \dots, M$ .
- We now define the conditional confusion matrices for each group as
- $CM_{ij}^g = P(Y_{pred} = i | Y_{true} = j, \mathcal{P} = g)$



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# Term-by-Term Equality of Odds

- The first fairness notion is the strongest possible notion.
- This notion ensures all rates of all types of errors are exactly the same for all groups.
- In mathematical terms, all conditional confusion matrices are equal.
- $CM^g = CM^h$  for any two groups  $g$  and  $h$ .



# Classwise Equality of Odds

- For this notion we must define the **Conditional False Detection Rate** for a group  $g$ . It is defined as  $FDR_i^g = P(Y_{pred} = i | Y_{true} \neq i, \mathcal{P} = g)$ .
- Notice this is a vector, not a matrix.
- The classwise equality of odds ensures that all diagonals of the conditional confusion matrices are equal **AND** all conditional false detection rates are equal for all groups.
- In equations,  $diag(CM^g) = diag(CM^h)$  **and**  $FDR^g = FDR^h$  for any two groups  $g$  and  $h$ .



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# Multiclass Equality of True Rates

- This notion is a relaxation on the previous two metrics.
- This time we only ensure that the diagonals of the Conditional Confusion Matrices are equal.
- In equations,  $\text{diag}(CM^g) = \text{diag}(CM^h)$  for any two groups  $g$  and  $h$ .



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# References

- [1] Putzel et al, Blackbox Postprocessing for Multiclass Fairness (<https://arxiv.org/abs/2201.04461>)