

The Alan Turing Institute

Bias in Multiclass Classification Part III

Content by: Sachin Beepath, Giulio Filippi, Cristian Munoz, Roseline Polle, Nigel Kingsman, Sara Zannone

Speaker: Sara Zannone



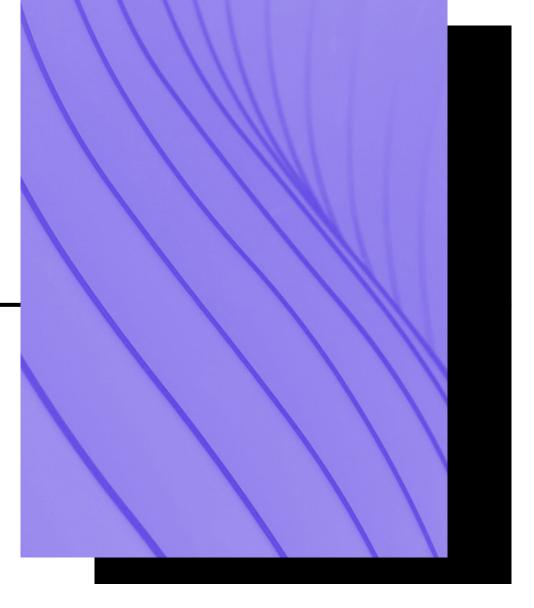
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- Part I Introduction to Multiclass Classification
- Part II Fairness in Multiclass Classification
- Part III Measuring Bias in Multiclass Classification
- Part IV Mitigating Bias in Multiclass Classification



III – Measuring Bias in Multiclass Classification

- 1) Introduce one Equality of Outcome Metric
- 2) Introduce one Equality of Opportunity Metric
- 3) Explain how the holistical library can help in computing bias metrics in multiclass setting





Equality of Outcome Metric



Reminder - Frequency Matrix

- The frequency Matrix is a matrix indexed on groups and classes (shape $M \times N$) with the g, i entry being the proportion of group g that is allocated to class i.
- In equations, $FM_{gi} = P(Y_{pred} = i | \mathcal{P} = g)$
- All equality of outcome metrics start from the Frequency Matrix!



Multiclass Statistical Parity (Step 1)

- We start by computing the frequency matrix.
- Recall that each row is a distribution (the allocation of a group to classes).
- We will usually need a way to compare these matrices row by row.
- In the most general setting, we can use any distance on distributions.
- In practice we use the total variation distance.
- In equations $d(g,h) = \frac{1}{2}\sum_{i}|FM_{gi} FM_{hi}|$



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Multiclass Statistical Parity (Step 2)

- Once we have the $\frac{M(M-1)}{2}$ distances d(g,h) between group allocation distributions.
- We need a way of aggregating these scores
- Usually, we use a maximum to get an idea of the worst-case scenario.
- We can also use an average or a weighed average with weights being the importance of each pair of groups being similarly treated.



Multiclass Statistical Parity (Example)

• Consider the following example with M = 3 groups (A, B, C) and N = 3 classes (1,2,3).

$$Data = \begin{bmatrix} A & A & A & A & B & B & B & B & C & C \\ 0 & 1 & 2 & 0 & 1 & 2 & 0 & 1 & 2 & 0 \end{bmatrix}$$

• We compute the Frequency Matrix

$$\bullet FM = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$



Multiclass Statistical Parity (Example)

- We then compute the distances between the rows of the confusion matrix
- d(row1, row2) = 0.5(|0.5 0.25| + |0.25 0.5| + |0.25 0.25|) = 0.25
- d(row2, row3) = 0.5(|0.25 0.5| + |0.5 0| + |0.25 0.5|) = 0.5
- d(row1, row3) = 0.5(|0.5 0.5| + |0.25 0| + |0.25 0.5|) = 0.25



Multiclass Statistical Parity (Example)

- With a maximum approach we get the following metric
- max(0.5, 0.25, 0.25) = 0.5
- With a mean approach we get the following metric
- $mean(0.5, 0.25, 0.25) = \frac{1}{3}$



Equality of Opportunity Metric



Reminder - Conditional Confusion Matrices

- Recall the conditional confusion matrices for each group is defined as
- $CM_{ij}^g = P(Y_{pred} = i | Y_{true} = j, \mathcal{P} = g)$
- All equality of opportunity metrics start from the Conditional Confusion Matrices (we will later call this the confusion tensor)!



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Multiclass Equality of Odds (Step 1)

- The first step is computing the conditional confusion matrix of each group CM^g .
- Next we compute a pairwise distance between these matrices.
- We could define any distance we like. But in this case we will take a mean average deviation approach.
- In equations $\frac{1}{2N}\sum_{ij}\left|CM_{ij}^g-CM_{ij}^h\right|=d(g,h)$
- If different misclassifications $i \to j$ have different levels of importance, we can weigh this sum $\frac{1}{2N\sum_{ij}w(i,j)}\sum_{ij}\left|CM_{ij}^g-CM_{ij}^h\right|w(i,j)=d(g,h)$



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Multiclass Equality of Odds (Step 2)

- Once we have the $\frac{M(M-1)}{2}$ distances d(g,h) between between group conditional confusion matrices.
- We can aggregate them with a max approach to get an idea of the worst-case scenario.
- Or we can aggregate them with a mean.



Multiclass Equality of Odds (Example)

• Consider the following example with M = 3 groups (A, B, C) and N = 3 classes (1,2,3). In the following matrix, the first row is the groups, the second row is the predictions, and the third row is the true values.

•
$$Data = \begin{bmatrix} A & A & A & A & A & B & B & B & B & C & C & C \\ 0 & 1 & 2 & 0 & 0 & 1 & 2 & 0 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 2 & 1 & 0 & 2 & 0 & 2 & 1 & 0 \end{bmatrix}$$

• The first step is computing the Conditional Confusion Matrices.

•
$$CM^A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2/3 & 0 \\ 0 & 1/3 & 0 \end{bmatrix}$$
, $CM^B = \begin{bmatrix} 0.5 & 0 & 1 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0 \end{bmatrix}$, $CM^C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Multiclass Equality of Odds (Example)

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$$CM^A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2/3 & 0 \\ 0 & 1/3 & 0 \end{bmatrix}$$
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We take a traditional mean absolute deviation distance

•
$$d(A,B) = \frac{1}{6} \left(0.5 + 0.5 + \frac{1}{3} + \frac{1}{3} \right) = \frac{5}{24}$$

•
$$d(B,C) = \frac{1}{6}(0.5 + 0.5 + 1 + 1 + 1 + 1) = \frac{5}{6}$$

•
$$d(A,C) = \frac{1}{6} \left(1 + \frac{2}{3} + \frac{1}{3} + 1 + 1 \right) = \frac{4}{6}$$



Multiclass Equality of Odds (Example)

- $distances = [\frac{5}{24}, \frac{5}{6}, \frac{4}{6}]$
- If we take a max aggregation approach, we get 5/6.
- If we take a mean aggregation approach, we get 0.57.



Metrics in Python



Installing the holistical library

- Documentation for multiclass metrics can be found here https://holisticai.readthedocs.io/en/latest/metrics.html#multiclass-classification.
- First step is installing the library, this can be done via pip. The following is run in a jupyter cell.

```
!pip install holisticai

✓ 0.2s
```



Computing Bias Metrics with holistical library

 Suppose we have some multiclass data and we would like to compute the frequency matrix / conditional confusion matrices.

```
import numpy as np
import pandas as pd
from holisticai.bias.metrics import frequency_matrix, confusion_tensor

✓ 0.2s
```



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```
import numpy as np
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from holisticai.bias.metrics import frequency_matrix, confusion_tensor

✓ 0.2s
```



Frequency Matrix with holistical library

We can compute the Frequency Matrix as follows.

```
frequency_matrix(p_attr, y_pred, normalize='class')

    0.3s
```

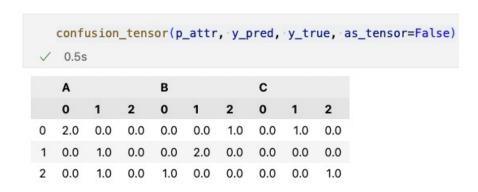
| | 0 | 1 | 2 |
|---|------|----------|----------|
| Α | 0.50 | 0.333333 | 0.333333 |
| В | 0.25 | 0.666667 | 0.333333 |
| С | 0.25 | 0.000000 | 0.333333 |

• Note that we add a normalize over class parameter. This is because there is also the option of normalising over group or None (no normalisation).



Confusion Tensor with holistical library

- Note that what we previously called the conditional confusion matrices, is what we named the confusion tensor in the code.
- We give the option of outputting it as a tensor or as a multi-level pandas DataFrame.





Multiclass Statistical Parity with holisticai library

 Suppose we wish to compute the multiclass statistical parity with both the max and mean aggregation methods.



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References

• [1] Putzel et al, Blackbox Postprocessing for Multiclass Fairness (https://arxiv.org/abs/2201.04461)