# Spatial Heterogeneity of Air Pollution Statistics in Europe

## *Unveiling the Dynamics of Extremes and Variability*

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## INTRODUCTION

Outdoor air pollution is estimated to cause a huge number of premature deaths worldwide, it catalyses many diseases on a variety of time scales, and it has a detrimental effect on the environment. In light of these impacts, it is necessary to obtain a better understanding of the dynamics and statistics of measured air pollution concentrations, including temporal fluctuations of observed concentrations and spatial heterogeneities.

Figure 1: We arrive at 3544 sites with data that meet our criteria before we proceed with our statistical analysis.

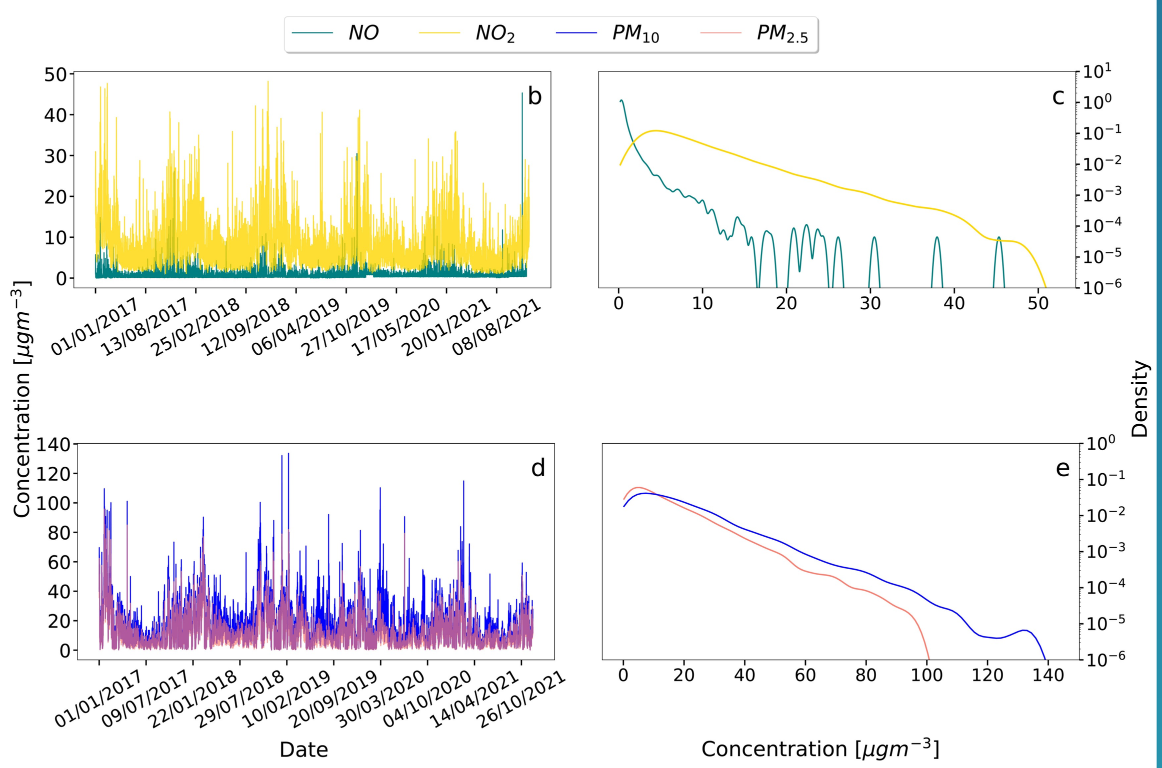


Figure 2: Our interest is in the behaviour of the tails of the PDFs. These tails correspond to high pollution states and are most damaging to health.

## METHODS

- Local exponential distribution: $p_\beta(x) = \beta e^{-\beta x}, x \geq 0$, with exponential decay $\beta$. (1)
- The fluctuations of $\beta$ take place on a long-time scale, much longer than local air pollution concentration fluctuations. We observe a superposition of two statistics in the marginal distribution: $p(x) = \int_0^\infty p_\beta(x) f(\beta)\, d\beta$, (2)
- Assume that exponents follow: $f(\beta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)} \left(\frac{n}{2\beta_0}\right)^{\frac{n}{2}} \beta^{\frac{n}{2}-1} e^{\frac{-n\beta}{2\beta_0}}$, (3)
- By describing the long-term behaviour of the air pollution concentration dynamics, is then a $q$-exponential[1,2,3]:

  $p_{q,\lambda}(x) = (2-q)\lambda[1-\lambda(1-q)x]^{\frac{1}{1-q}}$ for $1-\lambda(1-q)x \geq 0, x > 0$, (4)

  where $q$ is the entropic index[1,4,5], $\lambda$ is a positive width parameter and $x$ denotes the air pollutant concentration.
- Long-time scale $T$ : Time-dependent kurtosis $\kappa(\tau)$, then $\kappa(T) = \kappa(exp) = 9$. (5)

## RESULTS

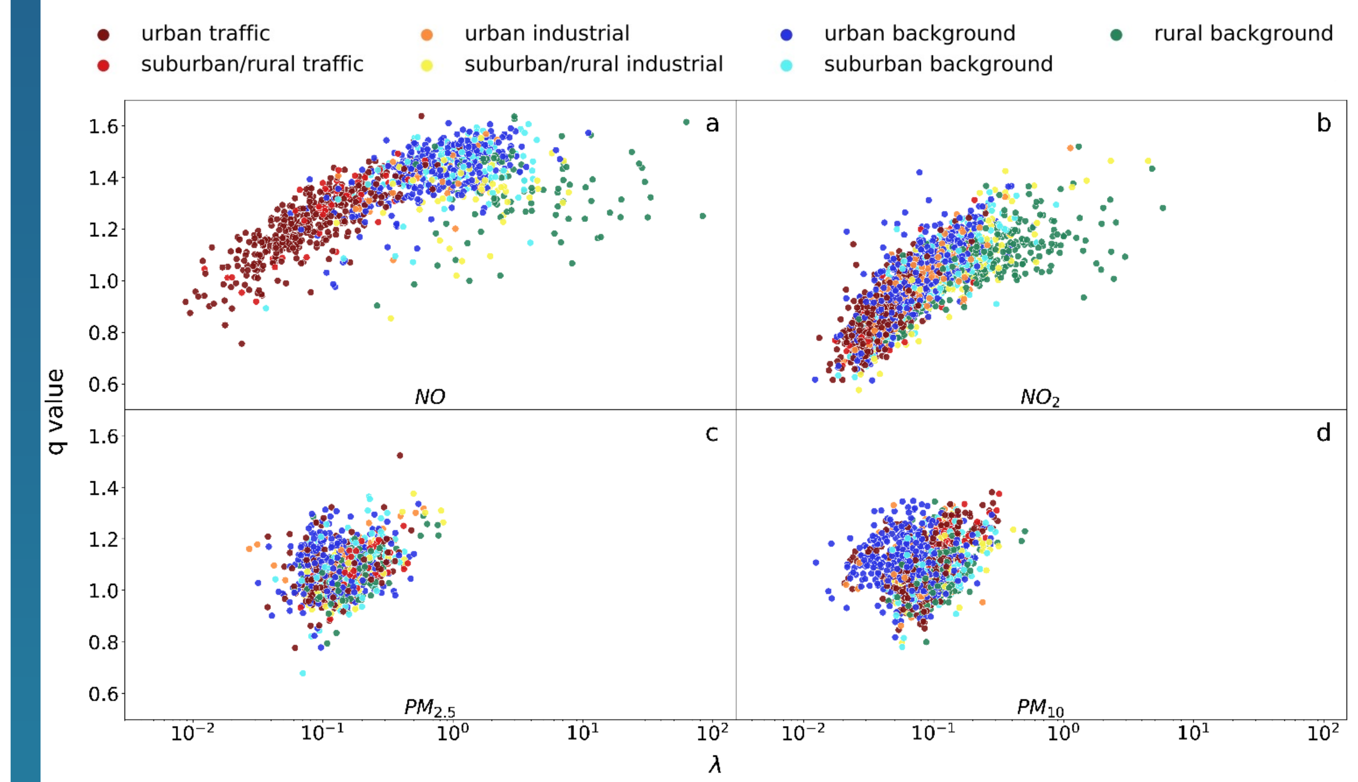


Figure 3: Best-fitting parameters of $q$-exponentials at the various measuring stations.

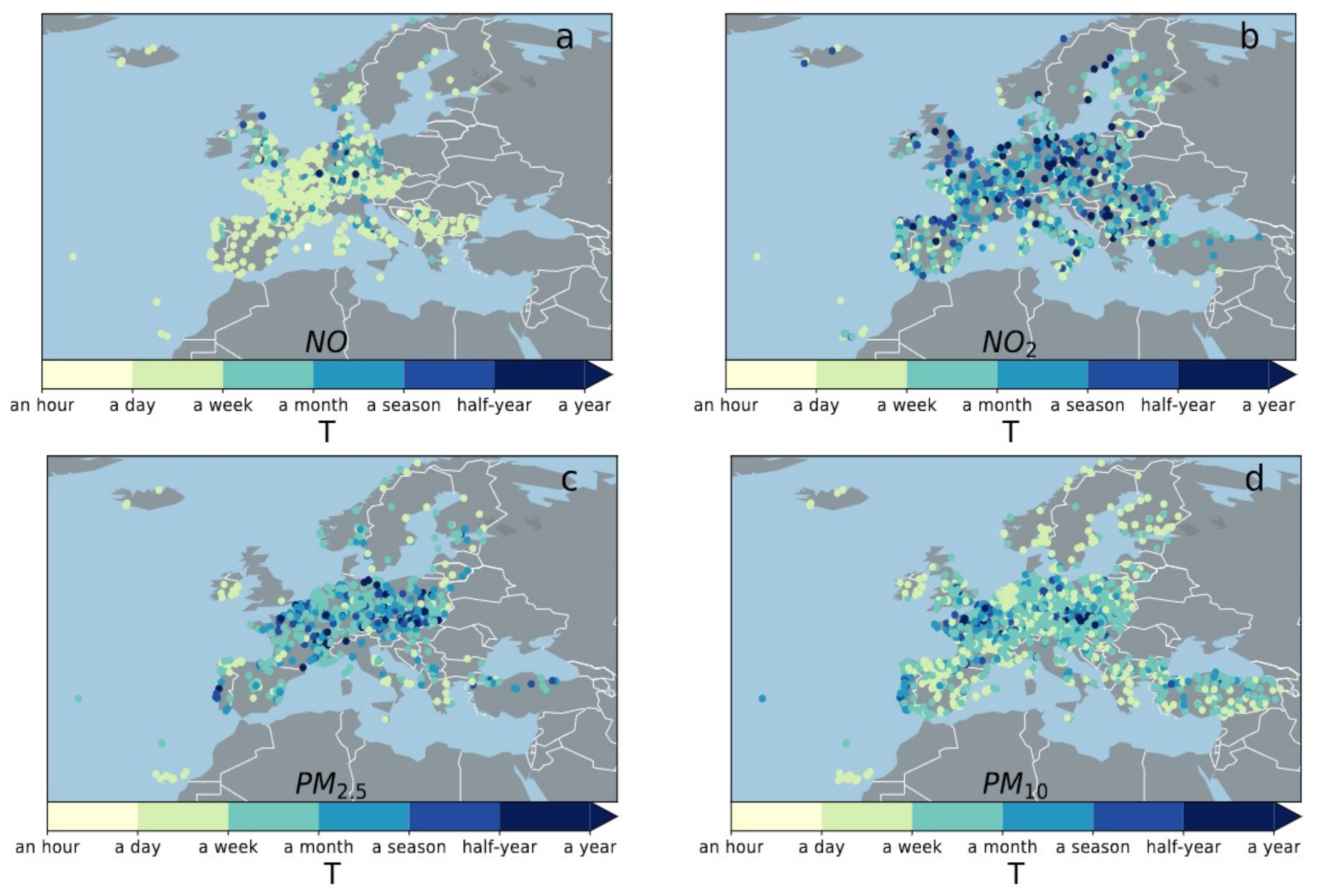


Figure 4: The long time scales $T$ describing the scale of typical changes of the temporal mean and variance of the measured time series.

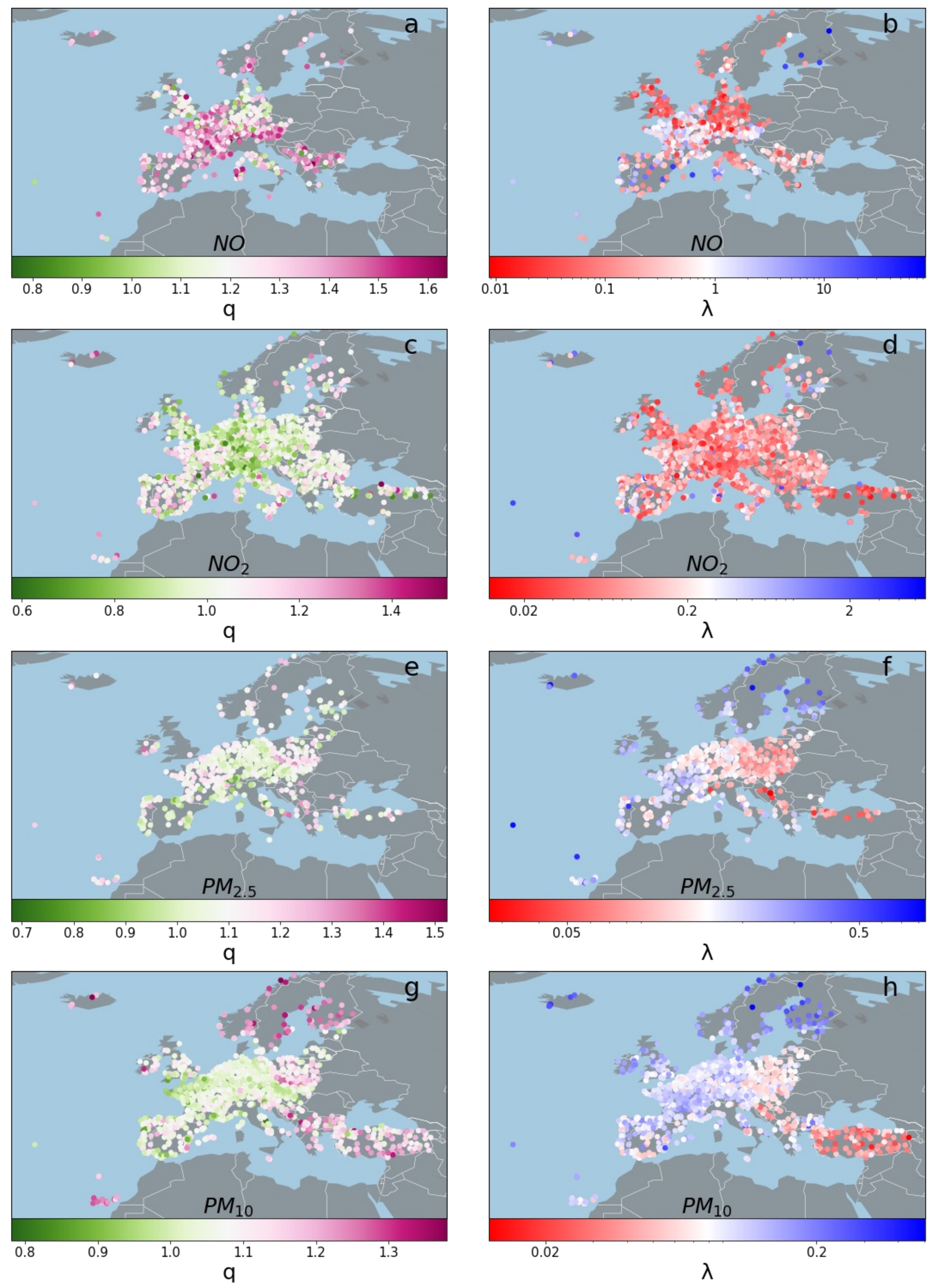


Figure 5: Spatial distribution of best-fitting parameters $(q,\lambda)$ characterizing the measured PDFs of $NOx$ and $PM$ pollutants across Europe.

## CONCLUSIONS & OUTLOOK

- Clear evidence[6] that generically PDFs of air pollution concentrations do not decay in an exponential way. A much better fit is given by $q$-exponential, which asymptotically decay as a power-law if $q > 1$, with exponent $-\frac{1}{(q-1)}$.
- We observe an immensely large range of values of the parameter $\lambda$ for the various measuring stations. The $(q,\lambda)$ planes depend on the type of pollutants and on the environmental characteristics.
- We observe a significant spatial heterogeneity in the best-fitting shape parameters of the PDFs. There are strong variations of PDFs at local level as well.
- The long Superstatistical time scales $T$ show distinct temporal dynamics at different spatial locations and for different pollutants.

Further local-level statistical analysis could support policymakers to produce more precise rules and thresholds for individual types of environmental conditions and meteorological conditions, accounting for fluctuations, extreme events, and variations of concentrations on different time scales. Our analysis could also be extended to other substances, such as sulfur oxide, carbon oxide and ozone.

## REFERENCES

[1] Tsallis, Constantino. (2009) Introduction to nonextensive statistical mechanics: approaching a complex world, Springer, 1(1), 2–1. Springer.
[2] Beck, C. and Cohen, Ezechiel G D. (2003) Superstatistics, Physica A: Statistical mechanics and its applications, 322, 267–275. Elsevier.
[3] Beck, Christian and Cohen, Ezechiel G D. and Swinney, Harry L. (2005) From time series to superstatistics, Physical Review E, 72(5), 056133. APS.
[4] Hanel, Rudolf and Thurner, Stefan and Gell-Mann, Murray. (2011) Generalized entropies and the transformation group of superstatistics, Proceedings of the National Academy of Sciences, 108(16), 6390–6394. National Acad Sciences.
[5] Jizba, Petr and Korbel, Jan. (2019) Maximum Entropy Principle in Statistical Inference: Case for Non-Shannonian Entropies, Physical Review Letters, 122(12), 120601. American Physical Society.
[6] He, H., Schäfer, B., & Beck, C. (2022) Spatial heterogeneity of air pollution statistics in Europe, Sci Rep, 12, 12215. DOI:10.1038/s41598-022-16109-2.

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