Chapter 4 Graphing Relations and Functions

What You'll Learn

- **Lessons 4-1, 4-4, and 4-5** Graph ordered pairs, relations, and equations.
- **Lesson 4-2** Transform figures on a coordinate plane.
- **Lesson 4-3** Find the inverse of a relation.
- **Lesson 4-6** Determine whether a relation is a function.
- **Lessons 4-7 and 4-8** Look for patterns and write formulas for sequences.

Key Vocabulary

- coordinate plane (p. 192)
- transformation (p. 197)
- inverse (p. 206)
- function (p. 226)
- arithmetic sequence (p. 233)

Why It's Important

The concept of a function is used throughout higher mathematics, from algebra to calculus. A function is a rule or a formula. You can use a function to describe real-world situations like converting between currencies. For example, if you are in Mexico, you can calculate that an item that costs 100 pesos is equivalent to about 11 U.S. dollars. *You will learn how to convert different currencies in Lesson 4-4.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 4.

For Lesson 4-1 — Graph Real Numbers

Graph each set of numbers. *(For review, see Lesson 2-1.)*

1. $\{1, 3, 5, 7\}$ **2.** $\{-3, 0, 1, 4\}$ **3.** $\{-8, -5, -2, 1\}$ **4.** $\left\{\frac{1}{2}, 1, 1\frac{1}{2}, 2\right\}$

For Lesson 4-2 — Distributive Property

Rewrite each expression using the Distributive Property. *(For review, see Lesson 1-5.)*

5. $3(7 - t)$ **6.** $-4(w + 2)$ **7.** $-5(3b - 2)$ **8.** $\frac{1}{2}(2z + 4)$

For Lessons 4-4 and 4-5 — Solve Equations for a Specific Variable

Solve each equation for y. *(For review, see Lesson 3-8.)*

9. $2x + y = 1$ **10.** $x = 8 - y$ **11.** $6x - 3y = 12$

12. $2x + 3y = 9$ **13.** $9 - \frac{1}{2}y = 4x$ **14.** $\frac{y + 5}{3} = x + 2$

For Lesson 4-6 — Evaluate Expressions

Evaluate each expression if $a = -1$, $b = 4$, and $c = -3$. *(For review, see Lesson 2-3.)*

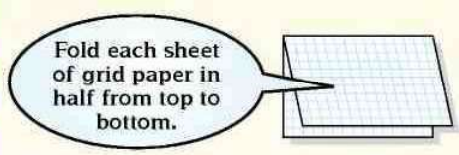
15. $a + b - c$ **16.** $2c - b$ **17.** $c - 3a$

18. $3a - 6b - 2c$ **19.** $8a + \frac{1}{2}b - 3c$ **20.** $6a + 8b + \frac{2}{3}c$

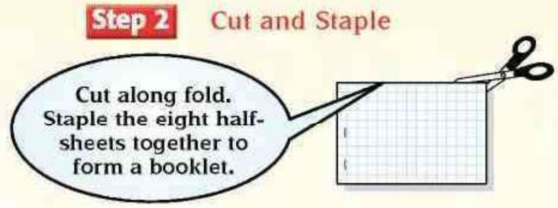
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your notes about graphing relations and functions. Begin with four sheets of grid paper.

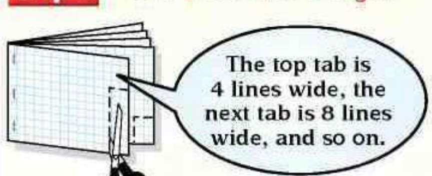
Step 1 Fold



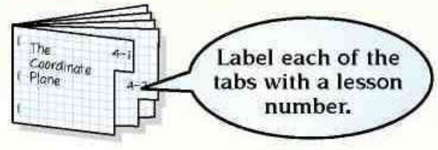
Step 2 Cut and Staple



Step 3 Cut Tabs into Margin



Step 4 Label



Reading and Writing As you read and study the chapter, use each page to write notes and to graph examples.

4-1 The Coordinate Plane

What You'll Learn

- Locate points on the coordinate plane.
- Graph points on a coordinate plane.

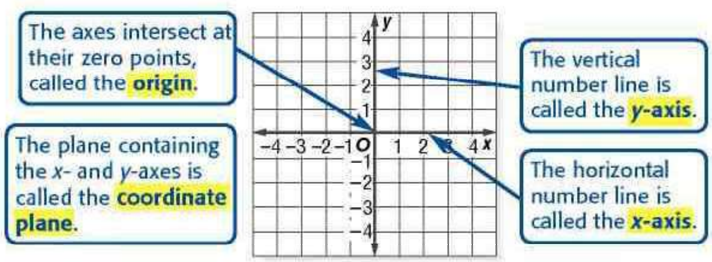
How do archaeologists use coordinate systems?

Underwater archaeologists use a grid system to map excavation sites of sunken ships. The grid is used as a point of reference on the ocean floor. The coordinate system is also used to record the location of objects they find. Knowing the position of each object helps archaeologists reconstruct how the ship sank and where to find other artifacts.

Vocabulary

- axes
- origin
- coordinate plane
- y-axis
- x-axis
- x-coordinate
- y-coordinate
- quadrant
- graph

IDENTIFY POINTS In mathematics, points are located in reference to two perpendicular number lines called **axes**.



Study Tip

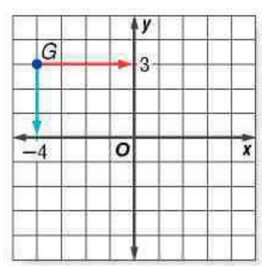
Reading Math
The x-coordinate is called the *abscissa*. The y-coordinate is called the *ordinate*.

Points in the coordinate plane are named by ordered pairs of the form (x, y). The first number, or **x-coordinate**, corresponds to the numbers on the x-axis. The second number, or **y-coordinate**, corresponds to the numbers on the y-axis. The origin, labeled O, has coordinates $(0, 0)$.

Example 1 Name an Ordered Pair

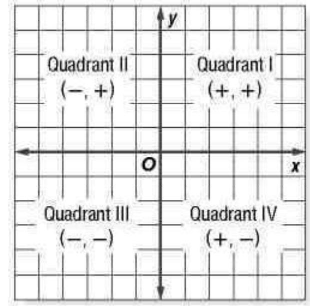
Write the ordered pair for point G.

- Follow along a vertical line through the point to find the x-coordinate on the x-axis. The x-coordinate is -4.
- Follow along a horizontal line through the point to find the y-coordinate on the y-axis. The y-coordinate is 3.
- So, the ordered pair for point G is $(-4, 3)$. This can also be written as $G(-4, 3)$.



Unless marked otherwise, you can assume that each division on the axes represents 1 unit.

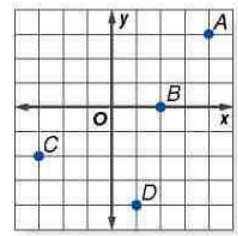
The x-axis and y-axis separate the coordinate plane into four regions, called **quadrants**. Notice which quadrants contain positive and negative x-coordinates and which quadrants contain positive and negative y-coordinates. The axes are not located in any of the quadrants.



Example 2 Identify Quadrants

Write ordered pairs for points *A*, *B*, *C*, and *D*. Name the quadrant in which each point is located.

Use a table to help find the coordinates of each point.



Point	x-Coordinate	y-Coordinate	Ordered Pair	Quadrant
A	4	3	(4, 3)	I
B	2	0	(2, 0)	none
C	−3	−2	(−3, −2)	III
D	1	−4	(1, −4)	IV

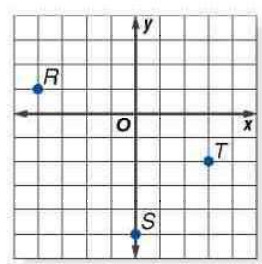
GRAPH POINTS To **graph** an ordered pair means to draw a dot at the point on the coordinate plane that corresponds to the ordered pair. This is sometimes called *plotting a point*. When graphing an ordered pair, start at the origin. The x-coordinate indicates how many units to move right (positive) or left (negative). The y-coordinate indicates how many units to move up (positive) or down (negative).

Example 3 Graph Points

Plot each point on a coordinate plane.

a. $R(-4, 1)$

- Start at the origin.
- Move left 4 units since the x-coordinate is −4.
- Move up 1 unit since the y-coordinate is 1.
- Draw a dot and label it R.



b. $S(0, -5)$

- Start at the origin.
- Since the x-coordinate is 0, the point will be located on the y-axis.
- Move down 5 units.
- Draw a dot and label it S.

c. $T(3, -2)$

- Start at the origin.
- Move right 3 units and down 2 units.
- Draw a dot and label it T.

CONTENTS

More About. . .

Geography

The prime meridian, 0° longitude, passes through London's Greenwich Observatory. A metal marker indicates its exact location.

Source: www.encarta.msn.com

Example 4 Use a Coordinate System

GEOGRAPHY **Latitude and longitude lines form a system of coordinates to designate locations on Earth. Latitude lines run east and west and are the first coordinate of the ordered pairs. Longitude lines run north and south and are the second coordinate of the ordered pairs.**

a. **Name the city at (40°, 105°).**
Locate the latitude line at 40°. Follow the line until it intersects with the longitude line at 105°. The city is Denver.

b. **Estimate the latitude and longitude of Washington, D.C.**
Locate Washington, D.C., on the map. It is close to 40° latitude and 75° longitude. There are 5° between each line, so a good estimate is 39° for the latitude and 77° for the longitude.

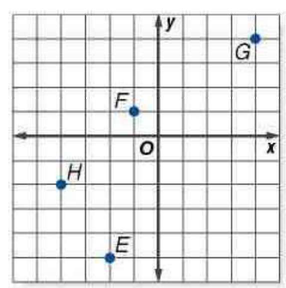
Check for Understanding

Concept Check

1. **Draw** a coordinate plane. Label the origin, x-axis, y-axis, and the quadrants.
2. **Explain** why $(-1, 4)$ does not name the same point as $(4, -1)$.
3. **OPEN ENDED** Give the coordinates of a point for each quadrant in the coordinate plane.

Guided Practice

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.



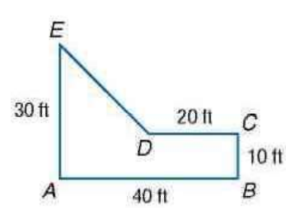
4. E
5. F
6. G
7. H

Plot each point on a coordinate plane.

8. $J(2, 5)$
9. $K(-1, 4)$
10. $L(0, -3)$
11. $M(-2, -2)$

Application

12. **ARCHITECTURE** Chun Wei has sketched the southern view of a building. If A is located on a coordinate system at $(-40, 10)$, locate the coordinates of the other vertices.



Practice and Apply

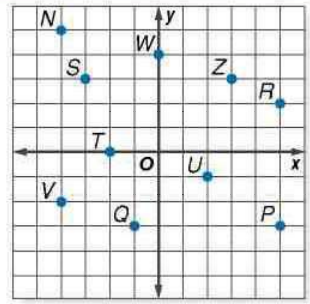
Homework Help

For Exercises	See Examples
13–24, 39	1, 2
25–36	3
37, 38, 40–43	4

Extra Practice

See page 828.

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.



13. N **14.** P

15. Q **16.** R

17. S **18.** T

19. U **20.** V

21. W **22.** Z

23. Write the ordered pair that describes a point 12 units down from and 7 units to the right of the origin.

24. Write the ordered pair for a point that is 9 units to the left of the origin and lies on the x-axis.

Plot each point on a coordinate plane.

25. $A(3, 5)$ **26.** $B(-2, 2)$ **27.** $C(4, -2)$ **28.** $D(0, -1)$

29. $E(-2, 5)$ **30.** $F(-3, -4)$ **31.** $G(4, 4)$ **32.** $H(-4, 4)$

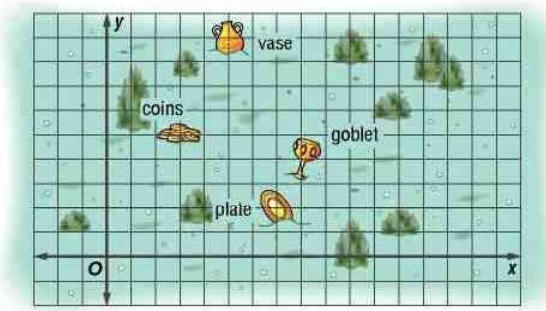
33. $I(3, 1)$ **34.** $J(-1, -3)$ **35.** $K(-4, 0)$ **36.** $L(2, -4)$

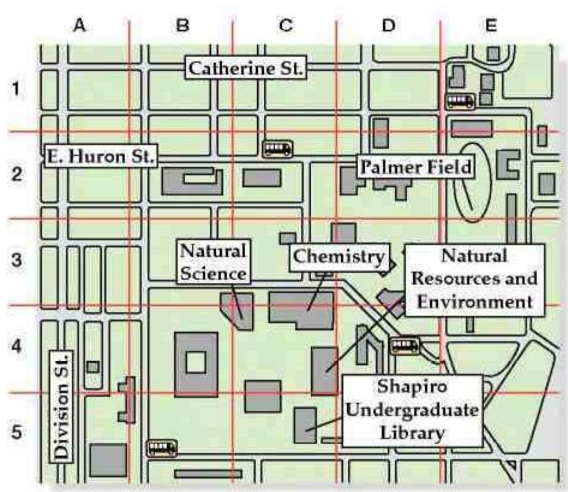
GEOGRAPHY **For Exercises 37 and 38, use the map on page 194.**

37. Name two cities that have approximately the same latitude.

38. Name two cities that have approximately the same longitude.

39. ARCHAEOLOGY The diagram at the right shows the positions of artifacts found on the ocean floor. Write the coordinates of the location for each object: coins, plate, goblet, and vase.





MAPS **For Exercises 40–43, use the map at the left.** On many maps, letters and numbers are used to define a region or sector. For example, Palmer Field is located in sector E2. Rogelio is a guide for new students at the University of Michigan. He has selected campus landmarks to show the students.

40. In what sector is the Undergraduate Library?

41. In what sector are most of the science buildings?

42. Which street goes from sector (A, 2) to (D, 2)?

43. Name the sectors that have bus stops.

44. CRITICAL THINKING Describe the possible locations, in terms of quadrants or axes, for the graph of (x, y) given each condition.

a. $xy > 0$ **b.** $xy < 0$ **c.** $xy = 0$

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

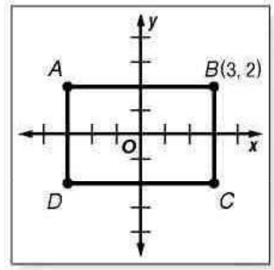
How do archaeologists use coordinate systems?

Include the following in your answer:

- an explanation of how dividing an excavation site into sectors can be helpful in excavating a site, and
- a reason why recording the exact location of an artifact is important.

Standardized Test Practice

For Exercises 46 and 47, refer to the figure at the right.



46. *ABCD* is a rectangle with its center at the origin. If the coordinates of vertex *B* are (3, 2), what are the coordinates of vertex *A*?

Ⓐ $(-3, -2)$ Ⓑ $(3, -2)$

Ⓒ $(-3, 2)$ Ⓓ $(3, 2)$

47. What is the length of $\overline{AD}$?

Ⓐ 6 units Ⓑ 4 units

Ⓒ 5 units Ⓓ 3 units

Extending the Lesson

The midpoint of a line segment is the point that lies exactly halfway between the two endpoints. The midpoint of a line segment whose endpoints are at (a, b) and (c, d) is at $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. Find the midpoint of each line segment whose endpoints are given.

48. $(7, 1)$ and $(-3, 1)$ 49. $(5, -2)$ and $(9, -8)$ 50. $(-4, 4)$ and $(4, -4)$

Maintain Your Skills

Mixed Review

51. **AIRPLANES** At 1:30 P.M., an airplane leaves Tucson for Baltimore, a distance of 2240 miles. The plane flies at 280 miles per hour. A second airplane leaves Tucson at 2:15 P.M. and is scheduled to land in Baltimore 15 minutes before the first airplane. At what rate must the second airplane travel to arrive on schedule? *(Lesson 3-9)*

Solve each equation or formula for the variable specified. *(Lesson 3-8)*

52. $3x + b = 2x + 5$ for x 53. $10c = 2(2d + 3c)$ for d

54. $6w - 3h = b$ for h 55. $\frac{3(a-t)}{4} = 2t$ for t

Find each square root. Round to the nearest hundredth if necessary. *(Lesson 2-7)*

56. $-\sqrt{81}$ 57. $\sqrt{63}$ 58. $\sqrt{180}$ 59. $-\sqrt{256}$

Evaluate each expression. *(Lesson 2-1)*

60. $52 + |18 - 7|$ 61. $|81 - 47| + 17$ 62. $42 - |60 - 74|$

63. $36 - |15 - 21|$ 64. $|10 - 16 + 27|$ 65. $|38 - 65 - 21|$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Rewrite each expression using the Distributive Property. Then simplify.** *(To review the **Distributive Property**, see Lesson 1-5.)*

66. $4(x + y)$ 67. $-1(x + 3)$ 68. $3(1 - 6y)$

69. $-3(2x - 5)$ 70. $\frac{1}{3}(2x + 6y)$ 71. $\frac{1}{4}(5x - 2y)$

4-2 Transformations on the Coordinate Plane

What You'll Learn

- Transform figures by using reflections, translations, dilations, and rotations.
- Transform figures on a coordinate plane by using reflections, translations, dilations, and rotations.

Vocabulary

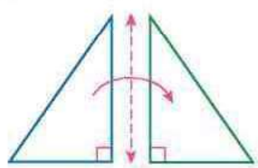
- transformation
- preimage
- image
- reflection
- translation
- dilation
- rotation

How are transformations used in computer graphics?

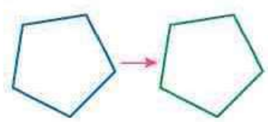
Computer programs can create movement that mimic real-life situations. A new CD-ROM-based flight simulator replicates an actual flight experience so closely that the U.S. Navy is using it for all of their student aviators. The movements of the on-screen graphics are accomplished by using mathematical transformations.

TRANSFORM FIGURES **Transformations** are movements of geometric figures. The **preimage** is the position of the figure before the transformation, and the **image** is the position of the figure after the transformation.

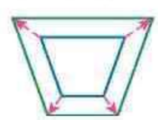
reflection
a figure is flipped over a line



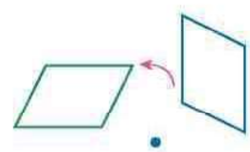
translation
a figure is slid in any direction



dilation
a figure is enlarged or reduced



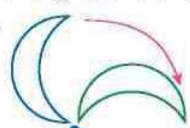
rotation
a figure is turned around a point



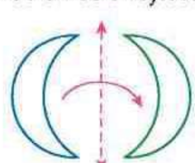
Example 1 Identify Transformations

Identify each transformation as a *reflection, translation, dilation,* or *rotation.*

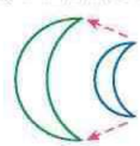
a.



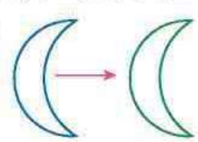
b.



c.



d.



a. The figure has been turned around a point. This is a rotation.

b. The figure has been flipped over a line. This is a reflection.

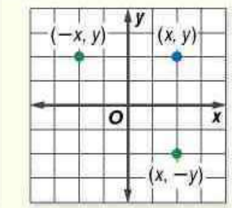
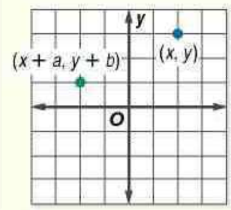
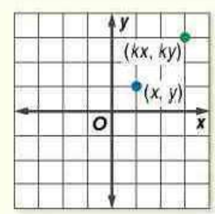
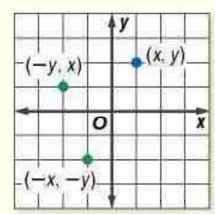
c. The figure has been increased in size. This is a dilation.

d. The figure has been shifted horizontally to the right. This is a translation.

CONTENTS

TRANSFORM FIGURES ON THE COORDINATE PLANE You can perform transformations on a coordinate plane by changing the coordinates of the points on a figure. The points on the translated figure are indicated by the prime symbol ′ to distinguish them from the original points.

Key Concept *Transformations on the Coordinate Plane*

Name	Words	Symbols	Model
Reflection	To reflect a point over the x-axis, multiply the y-coordinate by -1. To reflect a point over the y-axis, multiply the x-coordinate by -1.	reflection over x-axis: $(x, y) \rightarrow (x, -y)$ reflection over y-axis: $(x, y) \rightarrow (-x, y)$	
Translation	To translate a point by an ordered pair (a, b), add a to the x-coordinate and b to the y-coordinate.	$(x, y) \rightarrow (x + a, y + b)$	
Dilation	To dilate a figure by a scale factor k, multiply both coordinates by k. If $k > 1$, the figure is enlarged. If $0 < k < 1$, the figure is reduced.	$(x, y) \rightarrow (kx, ky)$	
Rotation	To rotate a figure 90° *counterclockwise* about the origin, switch the coordinates of each point and then multiply the new first coordinate by -1. To rotate a figure 180° about the origin, multiply both coordinates of each point by -1.	90° rotation: $(x, y) \rightarrow (-y, x)$ 180° rotation: $(x, y) \rightarrow (-x, -y)$	

Study Tip

Reading Math
The *vertices* of a polygon are the endpoints of the angles.

Example 2 Reflection

A parallelogram has vertices $A(-4, 3)$, $B(1, 3)$, $C(0, 1)$, and $D(-5, 1)$.

a. Parallelogram $ABCD$ is reflected over the x-axis. Find the coordinates of the vertices of the image.

To reflect the figure over the x-axis, multiply each y-coordinate by -1.

$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (x, -y)$
$A(-4, 3) \rightarrow A'(-4, -3)$	$C(0, 1) \rightarrow C'(0, -1)$
$B(1, 3) \rightarrow B'(1, -3)$	$D(-5, 1) \rightarrow D'(-5, -1)$

The coordinates of the vertices of the image are $A'(-4, -3)$, $B'(1, -3)$, $C'(0, -1)$, and $D'(-5, -1)$.

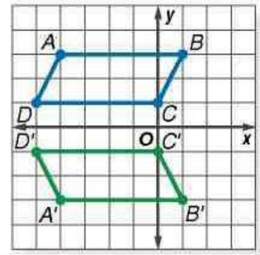
Study Tip

Reading Math
Parallelogram *ABCD* and its image *A′B′C′D′* are said to be **symmetric**. The *x*-axis is called the **line of symmetry**.

b. Graph parallelogram $ABCD$ and its image $A'B'C'D'$.

Graph each vertex of the parallelogram $ABCD$. Connect the points.

Graph each vertex of the reflected image $A'B'C'D'$. Connect the points.



Example 3 Translation

Triangle ABC has vertices $A(-2, 3)$, $B(4, 0)$, and $C(2, -5)$.

a. Find the coordinates of the vertices of the image if it is translated 3 units to the left and 2 units down.

To translate the triangle 3 units to the left, add -3 to the x-coordinate of each vertex. To translate the triangle 2 units down, add -2 to the y-coordinate of each vertex.

$$(x, y) \rightarrow (x - 3, y - 2)$$

$$A(-2, 3) \rightarrow A'(-2 - 3, 3 - 2) \rightarrow A'(-5, 1)$$

$$B(4, 0) \rightarrow B'(4 - 3, 0 - 2) \rightarrow B'(1, -2)$$

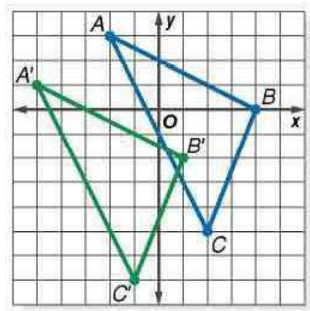
$$C(2, -5) \rightarrow C'(2 - 3, -5 - 2) \rightarrow C'(-1, -7)$$

The coordinates of the vertices of the image are $A'(-5, 1)$, $B'(1, -2)$, and $C'(-1, -7)$.

b. Graph triangle ABC and its image.

The preimage is $\triangle ABC$.

The translated image is $\triangle A'B'C'$.



Example 4 Dilation

A trapezoid has vertices $L(-4, 1)$, $M(1, 4)$, $N(7, 0)$, and $P(-3, -6)$.

a. Find the coordinates of the dilated trapezoid $L'M'N'P'$ if the scale factor is $\frac{3}{4}$.

To dilate the figure multiply the coordinates of each vertex by $\frac{3}{4}$.

$$(x, y) \rightarrow \left(\frac{3}{4}x, \frac{3}{4}y\right)$$

$$L(-4, 1) \rightarrow L'\left(\frac{3}{4} \cdot (-4), \frac{3}{4} \cdot 1\right) \rightarrow L'\left(-3, \frac{3}{4}\right)$$

$$M(1, 4) \rightarrow M'\left(\frac{3}{4} \cdot 1, \frac{3}{4} \cdot 4\right) \rightarrow M'\left(\frac{3}{4}, 3\right)$$

$$N(7, 0) \rightarrow N'\left(\frac{3}{4} \cdot 7, \frac{3}{4} \cdot 0\right) \rightarrow N'\left(5\frac{1}{4}, 0\right)$$

$$P(-3, -6) \rightarrow P'\left(\frac{3}{4} \cdot (-3), \frac{3}{4} \cdot (-6)\right) \rightarrow P'\left(-2\frac{1}{4}, -4\frac{1}{2}\right)$$

The coordinates of the vertices of the image are $L'\left(-3, \frac{3}{4}\right)$, $M'\left(\frac{3}{4}, 3\right)$, $N'\left(5\frac{1}{4}, 0\right)$, and $P'\left(-2\frac{1}{4}, -4\frac{1}{2}\right)$.

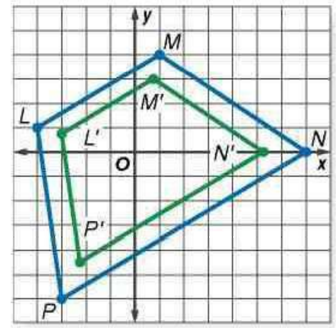
(continued on the next page)

b. Graph the preimage and its image.

The preimage is trapezoid $LMNP$.

The image is trapezoid $L'M'N'P'$.

Notice that the image has sides that are three-fourths the length of the sides of the original figure.



Example 5 Rotation

Triangle XYZ has vertices $X(1, 5)$, $Y(5, 2)$, and $Z(-1, 2)$.

a. Find the coordinates of the image of $\triangle XYZ$ after it is rotated 90° counterclockwise about the origin.

To find the coordinates of the vertices after a 90° rotation, switch the coordinates of each point and then multiply the new first coordinate by -1.

$(x, y) \rightarrow (-y, x)$

$X(1, 5) \rightarrow X'(-5, 1)$

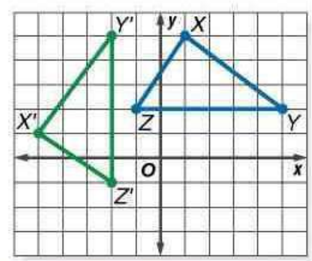
$Y(5, 2) \rightarrow Y'(-2, 5)$

$Z(-1, 2) \rightarrow Z'(-2, -1)$

b. Graph the preimage and its image.

The image is $\triangle XYZ$.

The rotated image is $\triangle X'Y'Z'$.



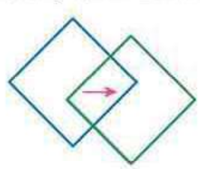
Check for Understanding

Concept Check

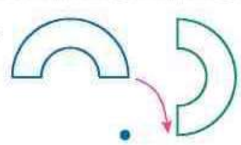
1. **Compare and contrast** the size, shape, and orientation of a preimage and an image for each type of transformation.
2. **OPEN ENDED** Draw a figure on the coordinate plane. Then show a dilation of the object that is an enlargement and a dilation of the object that is a reduction.

Guided Practice **Identify each transformation as a *reflection, translation, dilation,* or *rotation*.**

3.



4.



Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image.

5. triangle PQR with $P(1, 2)$, $Q(4, 4)$, and $R(2, -3)$ reflected over the x-axis
6. quadrilateral $ABCD$ with $A(4, 2)$, $B(4, -2)$, $C(-1, -3)$, and $D(-3, 2)$ translated 3 units up
7. parallelogram $EFGH$ with $E(-1, 4)$, $F(5, -1)$, $G(2, -4)$, and $H(-4, 1)$ dilated by a scale factor of 2
8. triangle JKL with $J(0, 0)$, $K(-2, -5)$, and $L(-4, 5)$ rotated 90° counterclockwise about the origin

Application **NAVIGATION** **For Exercises 9 and 10, use the following information.**
A ship was heading on a chartered route when it was blown off course by a storm. The ship is now ten miles west and seven miles south of its original destination.

9. Using a coordinate grid, make a drawing to show the original destination A and the current position B of the ship.

10. Using coordinates (x, y) to represent the original destination of the ship, write an expression to show its current location.

Practice and Apply

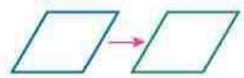
Homework Help

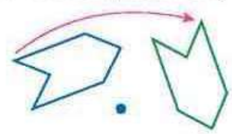
For Exercises	See Examples
11–16, 37, 38	1
17–36	2–5

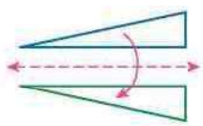
Extra Practice

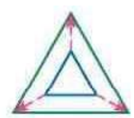
See page 828.

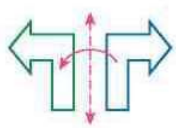
Identify each transformation as a *reflection, translation, dilation,* or *rotation.*

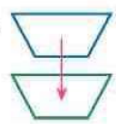
11. 

12. 

13. 

14. 

15. 

16. 

For Exercises 17–26, complete parts a and b.

a. Find the coordinates of the vertices of each figure after the given transformation is performed.

b. Graph the preimage and its image.

17. triangle RST with $R(2, 0)$, $S(-2, -3)$, and $T(-2, 3)$ reflected over the y-axis

18. trapezoid $ABCD$ with $A(2, 3)$, $B(5, 3)$, $C(6, 1)$, and $D(-2, 1)$ reflected over the x-axis

19. quadrilateral $RSTU$ with $R(-6, 3)$, $S(-4, 2)$, $T(-1, 5)$, and $U(-3, 7)$ translated 8 units right

20. parallelogram $MNOP$ with $M(-6, 0)$, $N(-4, 3)$, $O(-1, 3)$, and $P(-3, 0)$ translated 3 units right and 2 units down

21. trapezoid $JKLM$ with $J(-4, 2)$, $K(-2, 4)$, $L(4, 4)$, and $M(-4, -4)$ dilated by a scale factor of $\frac{1}{2}$

22. square $ABCD$ with $A(-2, 1)$, $B(2, 2)$, $C(3, -2)$, and $D(-1, -3)$ dilated by a scale factor of 3

23. triangle FGH with $F(-3, 2)$, $G(2, 5)$, and $H(6, 3)$ rotated 180° about the origin

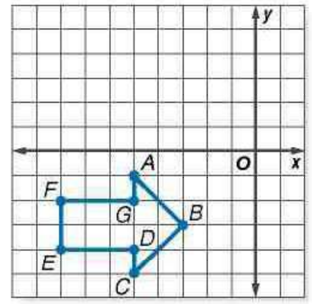
24. quadrilateral $TUVW$ with $T(-4, 2)$, $U(-2, 4)$, $V(0, 2)$, and $W(-2, -4)$ rotated 90° counterclockwise about the origin

25. parallelogram $WXYZ$ with $W(-1, 2)$, $X(3, 2)$, $Y(0, -4)$, and $Z(-4, -4)$ reflected over the y-axis, then rotated 180° about the origin

26. pentagon $PQRST$ with $P(0, 5)$, $Q(3, 4)$, $R(2, 1)$, $S(-2, 1)$, and $T(-3, 4)$ reflected over the x-axis, then translated 2 units left and 1 unit up

ANIMATION **For Exercises 27–29, use the diagram at the right.**

An animator places an arrow representing an airplane on a coordinate grid. She wants to move the arrow 2 units right and then reflect it across the x-axis.



27. Write the coordinates for the vertices of the arrow.

28. Find the coordinates of the final position of the arrow.

29. Graph the image.

30. Trapezoid $JKLM$ with $J(-6, 0)$, $K(-1, 5)$, $L(-1, 1)$, and $M(-3, -1)$ is translated to $J'K'L'M'$ with $J'(-3, -2)$, $K'(2, 3)$, $L'(2, -1)$, $M'(0, -3)$. Describe this translation.

31. Triangle QRS with vertices $Q(-2, 6)$, $R(8, 0)$, and $S(6, 4)$ is dilated. If the image $Q'R'S'$ has vertices $Q'(-1, 3)$, $R'(4, 0)$, and $S'(3, 2)$, what is the scale factor?

32. Describe the transformation of parallelogram $WXYZ$ with $W(-5, 3)$, $X(-2, 5)$, $Y(0, 3)$, and $Z(-3, 1)$ if the coordinates of its image are $W'(5, 3)$, $X'(2, 5)$, $Y'(0, 3)$, and $Z'(3, 1)$.

33. Describe the transformation of triangle XYZ with $X(2, -1)$, $Y(-5, 3)$, and $Z(4, 0)$ if the coordinates of its image are $X'(1, 2)$, $Y'(-3, -5)$, and $Z'(0, 4)$.

DIGITAL PHOTOGRAPHY **For Exercises 34–36, use the following information.**

Soto wants to enlarge a digital photograph that is 1800 pixels wide and 1600 pixels high (1800×1600) by a scale factor of $2\frac{1}{2}$.

34. What will be the dimensions of the new digital photograph?

35. Use a coordinate grid to draw a picture representing the 1800×1600 digital photograph. Place one corner of the photograph at the origin and write the coordinates of the other three vertices.

36. Draw the enlarged photograph and write its coordinates.

More About . . .

Digital Photography

Digital photographs contain hundreds of thousands or millions of tiny squares called pixels.

Source: www.shortcourses.com

ART **For Exercises 37 and 38, use the following information.**

On grid paper, draw a regular octagon like the one shown.

37. Reflect the octagon over each of its sides. Describe the pattern that results.

38. Could this same pattern be drawn using any of the other transformations? If so, which kind?

39. **CRITICAL THINKING** Make a conjecture about the coordinates of a point (x, y) that has been rotated 90° *clockwise* about the origin.

40. **CRITICAL THINKING** Determine whether the following statement is *sometimes*, *always*, or *never* true.

A reflection over the x-axis followed by a reflection over the y-axis gives the same result as a rotation of 180°.

41. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How are transformations used in computer graphics?

Include the following in your answer:
- examples of movements that could be simulated by transformations, and
- types of other industries that might use transformations in computer graphics to simulate movement.

Standardized Test Practice

42. The coordinates of the vertices of quadrilateral $QRST$ are $Q(-2, 4)$, $R(3, 7)$, $S(4, -2)$, and $T(-5, -3)$. If the quadrilateral is moved up 3 units and right 1 unit, which point below has the correct coordinates?

Ⓐ $Q'(1, 5)$ Ⓑ $R'(4, 4)$ Ⓒ $S'(5, 1)$ Ⓓ $T'(-6, 0)$

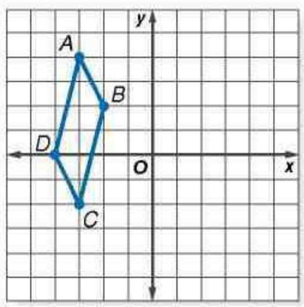
43. x is $\frac{2}{3}$ of y and y is $\frac{1}{4}$ of z. If $x = 14$, then $z =$

Ⓐ 48. Ⓑ 72. Ⓒ 84. Ⓓ 96.

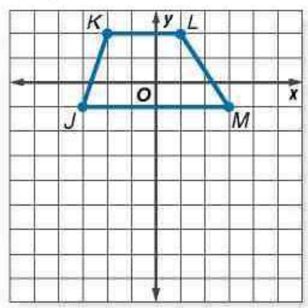
Extending the Lesson

Graph the image of each figure after a reflection over the graph of the given equation. Find the coordinates of the vertices.

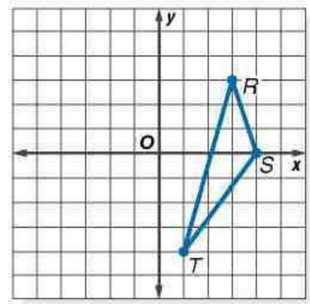
44. $x = 0$



45. $y = -3$



46. $y = x$



Maintain Your Skills

Mixed Review

Plot each point on a coordinate plane. *(Lesson 4-1)*

47. $A(2, -1)$
48. $B(-4, 0)$
49. $C(1, 5)$
50. $D(-1, -1)$
51. $E(-2, 3)$
52. $F(4, -3)$

53. **CHEMISTRY** Jamaal needs a 25% solution of nitric acid. He has 20 milliliters of a 30% solution. How many milliliters of a 15% solution should he add to obtain the required 25% solution? *(Lesson 3-9)*

Two dice are rolled and their sum is recorded. Find each probability. *(Lesson 2-6)*

54. P(sum is less than 9)
55. P(sum is greater than 10)
56. P(sum is less than 7)
57. P(sum is greater than 4)

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write a set of ordered pairs that represents the data in the table.** *(To review **ordered pairs**, see Lesson 1-8.)*

58.

Number of toppings	1	2	3	4	5	6
Cost of large pizza ($)	9.95	11.45	12.95	14.45	15.95	17.45

59.

Time (minutes)	0	5	10	15	20	25	30
Temperature of boiled water as it cools (°C)	100	90	81	73	66	60	55

Graphing Calculator Investigation

A Preview of Lesson 4-3

Graphs of Relations

You can represent a relation as a graph using a TI-83 Plus graphing calculator.

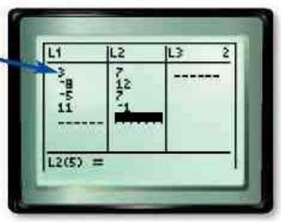
Graph the relation $\{(3, 7), (-8, 12), (-5, 7), (11, -1)\}$.

Step 1 *Enter the data.*

- Enter the x-coordinates in L1 and the y-coordinates in L2.

KEYSTROKES: STAT ENTER 3 ENTER −8 ENTER −5 ENTER 11 ENTER ▶ 7 ENTER 12 ENTER 7 ENTER −1 ENTER

The first ordered pair is (3, 7).



Step 2 *Format the graph.*

- Turn on the statistical plot.

KEYSTROKES: 2nd STAT PLOT ENTER ENTER

- Select the scatter plot, L1 as the Xlist and L2 as the Ylist.

KEYSTROKES: ▼ ENTER ▼ 2nd L1 ENTER 2nd L2 ENTER

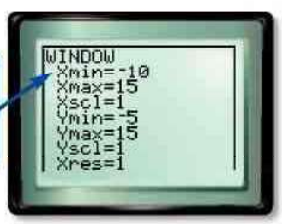
Step 3 *Choose the viewing window.*

- Be sure you can see all of the points.

[−10, 15] scl: 1 by [−5, 15] scl: 1

KEYSTROKES: WINDOW −10 ENTER 15 ENTER 1 ENTER −5 ENTER 15 ENTER 1

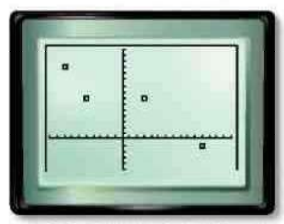
The x-axis will go from −10 to 15 with a tick mark at every unit.



Step 4 *Graph the relation.*

- Display the graph.

KEYSTROKES: GRAPH



[−10, 15] scl: 1 by [−5, 15] scl: 1

Exercises

Graph each relation. Sketch the result.

1. $\{(10, 10), (0, -6), (4, 7), (5, -2)\}$

2. $\{(-4, 1), (3, -5), (4, 5), (-5, 1)\}$

3. $\{(12, 15), (10, -16), (11, 7), (-14, -19)\}$

4. $\{(45, 10), (23, 18), (22, 26), (35, 26)\}$

5. MAKE A CONJECTURE How are the values of the domain and range used to determine the scale of the viewing window?

www.algebra1.com/other_calculator_keystrokes

4-3 Relations

What You'll Learn

- Represent relations as sets of ordered pairs, tables, mappings, and graphs.
- Find the inverse of a relation.

Vocabulary

- mapping
- inverse

How can relations be used to represent baseball statistics?

Ken Griffey, Jr.'s, batting statistics for home runs and strikeouts can be represented as a set of ordered pairs. These statistics are shown in the table at the right, where the first coordinates represent the number of home runs and the second coordinates represent the number of strikeouts. You can plot the ordered pairs on a graph to look for patterns in the distribution of the points.

Ken Griffey, Jr.

Year	Home Runs	Strikeouts
1994	40	73
1995	17	53
1996	49	104
1997	56	121
1998	56	121
1999	48	108
2000	40	117
2001	22	72

REPRESENT RELATIONS Recall that a *relation* is a set of ordered pairs. A relation can be represented by a set of ordered pairs, a table, a graph, or a **mapping**. A mapping illustrates how each element of the domain is paired with an element in the range. Study the different representations of the same relation below.

Study Tip

Look Back
To review **relations**, see Lesson 1-8.

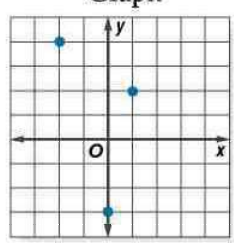
Ordered Pairs

(1, 2)
(−2, 4)
(0, −3)

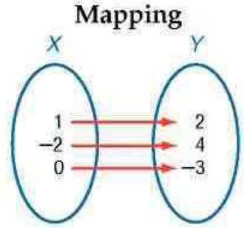
Table

x	y
1	2
−2	4
0	−3

Graph



Mapping



Example 1 Represent a Relation

a. Express the relation {(3, 2), (−1, 4), (0, −3), (−3, 4), (−2, −2)} as a table, a graph, and a mapping.

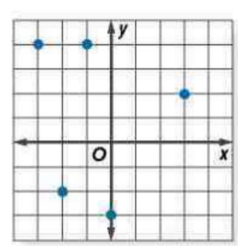
Table

List the set of x-coordinates in the first column and the corresponding y-coordinates in the second column.

x	y
3	2
−1	4
0	−3
−3	4
−2	−2

Graph

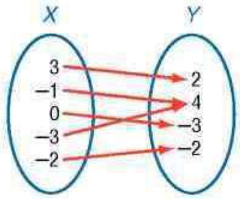
Graph each ordered pair on a coordinate plane.



(continued on the next page)

Mapping

List the x values in set X and the y values in set Y. Draw an arrow from each x value in X to the corresponding y value in Y.



b. Determine the domain and range.
The domain for this relation is $\{-3, -2, -1, 0, 3\}$.
The range is $\{-3, -2, 2, 4\}$.

When graphing relations that represent real-life situations, you may need to select values for the x- or y-axis that do not begin with 0 and do not have units of 1.

Example 2 Use a Relation

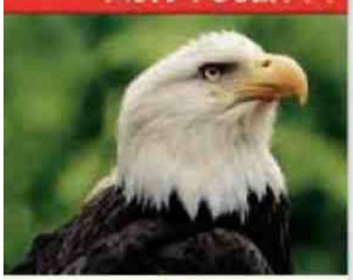
BALD EAGLES **In 1990, New York purchased 12,000 acres for the protection of bald eagles. The table shows the number of eagles observed in New York during the annual mid-winter bald eagle survey from 1993 to 2000.**

Bald Eagle Survey

Year	1993	1994	1995	1996	1997	1998	1999	2000
Number of Eagles	102	116	144	174	175	177	244	350

Source: New York Department of Environmental Conservation

More About. . .



Bald Eagles

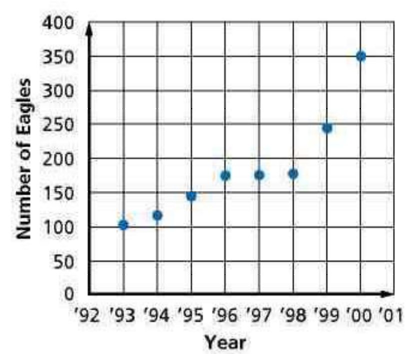
The bald eagle is not really bald. Its name comes from the Old English meaning of bald, "having white feathers on the head."

Source: *Webster's Dictionary*

a. Determine the domain and range of the relation.
The domain is {1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000}.
The range is {102, 116, 144, 174, 175, 177, 244, 350}.

b. Graph the data.

- The values of the x-axis need to go from 1993 to 2000. It is not practical to begin the scale at 0. Begin at 1992 and extend to 2001 to include all of the data. The units can be 1 unit per grid square.
- The values on the y-axis need to go from 102 to 350. In this case, it is possible to begin the scale at 0. Begin at 0 and extend to 400. You can use units of 50.



c. What conclusions might you make from the graph of the data?
The number of eagles has increased each year. This may be due to the efforts of those who are protecting the eagles in New York.

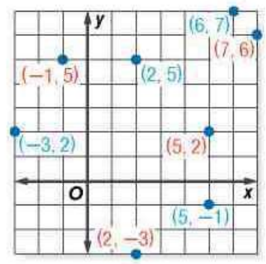
INVERSE RELATIONS The **inverse** of any relation is obtained by switching the coordinates in each ordered pair.

Key Concept *Inverse of a Relation*

Relation Q is the inverse of relation S if and only if for every ordered pair (a, b) in S, there is an ordered pair (b, a) in Q.

Relation	Inverse
(2, 5)	(5, 2)
(−3, 2)	(2, −3)
(6, 7)	(7, 6)
(5, −1)	(−1, 5)

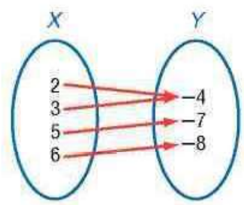
Notice that the domain of a relation becomes the range of the inverse and the range of a relation becomes the domain of the inverse.



Example 3 Inverse Relation

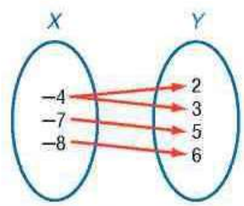
Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.

Relation Notice that both 2 and 3 in the domain are paired with −4 in the range.
$\{(2, -4), (3, -4), (5, -7), (6, -8)\}$



Inverse Exchange x and y in each ordered pair to write the inverse relation.
$\{(-4, 2), (-4, 3), (-7, 5), (-8, 6)\}$

The mapping of the inverse is shown at the right. Compare this to the mapping of the relation.



Algebra Activity

Relations and Inverses

- Graph the relation $\{(3, 4), (-2, 5), (-4, -3), (5, -6), (-1, 0), (0, 2)\}$ on grid paper using a colored pencil. Connect the points in order using the same colored pencil.
- Use a different colored pencil to graph the inverse of the relation, connecting the points in order.
- Fold the grid paper through the origin so that the positive y-axis lies on top of the positive x-axis. Hold the paper up to a light so that you can see all of the points you graphed.

Analyze

1. What do you notice about the location of the points you graphed when you looked at the folded paper?
2. Unfold the paper. Describe the transformation of each point and its inverse.
3. What do you think are the ordered pairs that represent the points on the fold line? Describe these in terms of x and y.

Make a Conjecture

4. How could you graph the inverse of a function without writing ordered pairs first?

Check for Understanding

Concept Check

1. **Describe** the different ways a relation can be represented.
2. **OPEN ENDED** Give an example of a set of ordered pairs that has five elements in its domain and four elements in its range.
3. **State** the relationship between the domain and range of a relation and the domain and range of its inverse.

Guided Practice

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

4. $\{(5, -2), (8, 3), (-7, 1)\}$
5. $\{(6, 4), (3, -3), (-1, 9), (5, -3)\}$
6. $\{(7, 1), (3, 0), (-2, 5)\}$
7. $\{(-4, 8), (-1, 9), (-4, 7), (6, 9)\}$

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

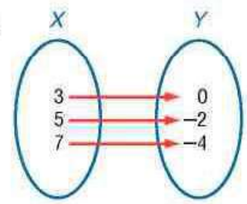
8.

x	y
3	−2
−6	7
4	3
−6	5

9.

x	y
−4	9
2	5
−2	−2
11	12

10.



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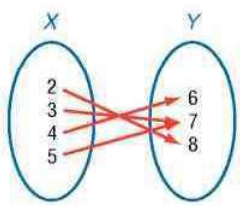

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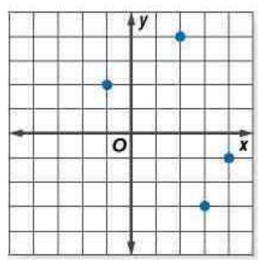

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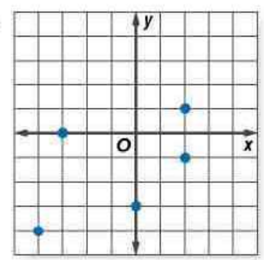
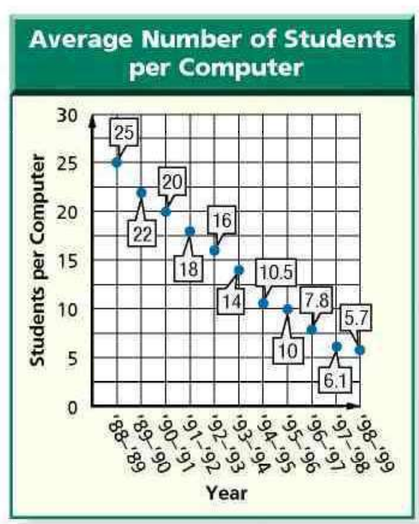


Application

TECHNOLOGY For Exercises 14–17, use the graph of the average number of students per computer in U.S. public schools.

14. Name three ordered pairs from the graph.
15. Determine the domain of the relation.
16. Estimate the least value and the greatest value in the range.
17. What conclusions can you make from the graph of the data?



Source: Quality Education Data

Online Research **Data Update** What is the average number of students per computer in your state? Visit www.algebra1.com/data_update to learn more.

Practice and Apply

Homework Help

For Exercises	See Examples
18–25	1
26–37	3
38–48	2

Extra Practice

See page 829.

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

18. {(4, 3), (1, −7), (1, 3), (2, 9)}

19. {(5, 2), (−5, 0), (6, 4), (2, 7)}

20. {(0, 0), (6, −1), (5, 6), (4, 2)}

21. {(3, 8), (3, 7), (2, −9), (1, −9)}

22. {(4, −2), (3, 4), (1, −2), (6, 4)}

23. {(0, 2), (−5, 1), (0, 6), (−1, 9)}

24. {(3, 4), (4, 3), (2, 2), (5, −4), (−4, 5)}

25. {(7, 6), (3, 4), (4, 5), (−2, 6), (−3, 2)}

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

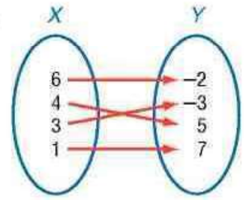
26.

x	y
1	2
3	4
5	6
7	8

27.

x	y
0	3
−5	2
4	7
−3	2

28.



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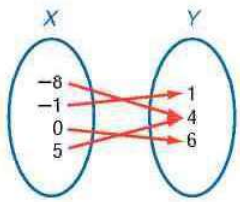

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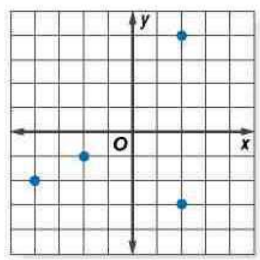
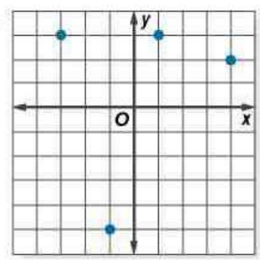


31.



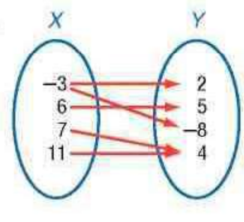
32.

x	y
0	0
4	7
8	10.5
12	13
16	14.5

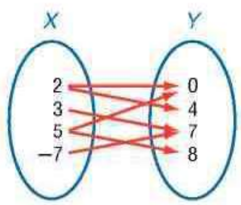
33.

x	y
1	16.50
1.75	28.30
2.5	49.10
3.25	87.60
4	103.40

34.



35.



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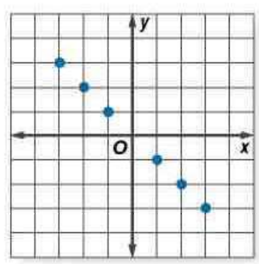

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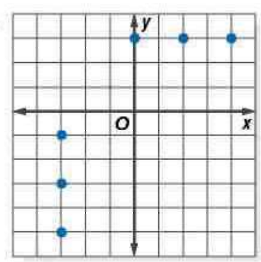


COOKING **For Exercises 38–40, use the table that shows the boiling point of water at various altitudes.** Many recipes have different cooking times for high altitudes. This is due to the fact that water boils at a lower temperature in higher altitudes.

38. Graph the relation.

39. Write the inverse as a set of ordered pairs.

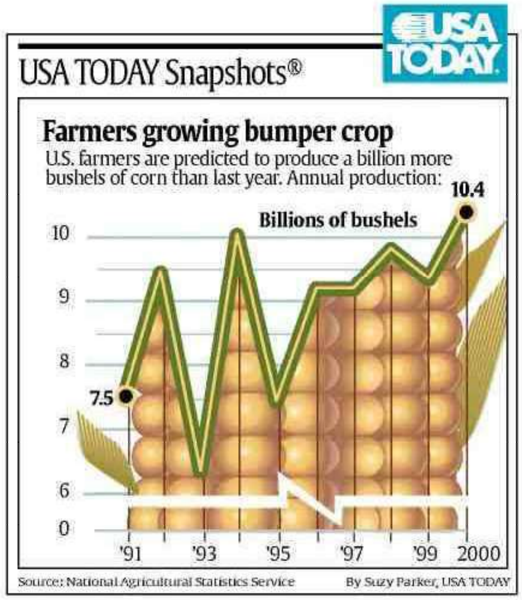
40. How could you estimate your altitude by finding the boiling point of water at your location?

Altitude (feet)	Boiling Point of Water (°F)
0	212.0
1000	210.2
2000	208.4
3000	206.5
5000	201.9
10,000	193.7

Source: Stevens Institute of Technology

FOOD **For Exercises 41–43, use the graph that shows the annual production of corn from 1991–2000.**

41. Estimate the domain and range of the relation.
42. Which year had the lowest production? the highest?
43. Describe any pattern you see.



HEALTH **For Exercises 44–48, use the following information.**
A person's muscle weight is about 2 pounds of muscle for each 5 pounds of body weight.

44. Make a table to show the relation between body and muscle weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds.
45. What are the domain and range?
46. Graph the relation.
47. What are the domain and range of the inverse?
48. Graph the inverse relation.

49. **CRITICAL THINKING** Find a counterexample to disprove the following.

The domain of relation F contains the same elements as the range of relation G. The range of relation F contains the same elements as the domain of relation G. Therefore, relation G must be the inverse of relation F.

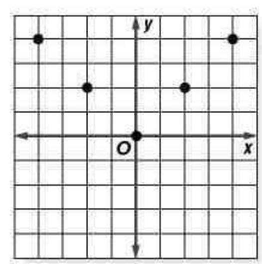
50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can relations be used to represent baseball statistics?

Include the following in your answer:
- a graph of the relation of the number of Ken Griffey, Jr.'s, home runs and his strikeouts, and
- an explanation of any relationship between the number of home runs hit and the number of strikeouts.

Standardized Test Practice

For Exercises 51 and 52, use the graph at the right.



51. State the domain and range of the relation.

(A) D = {0, 2, 4}; R = {−4, −2, 0, 2, 4}
(B) D = {−4, −2, 0, 2, 4}; R = {0, 2, 4}
(C) D = {0, 2, 4}; R = {−4, −2, 0}
(D) D = {−4, −2, 0, 2, 4}; R = {−4, −2, 0, 2, 4}

52. **SHORT RESPONSE** Graph the inverse of the relation.

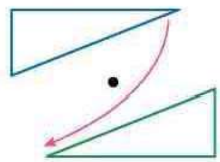
Graphing Calculator

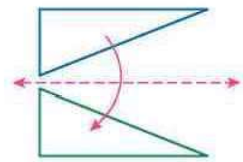
For Exercises 53–56, use a graphing calculator.
a. Graph each relation.
b. State the WINDOW settings that you used.
c. Write the coordinates of the inverse. Then graph the inverse.
d. Name the quadrant in which each point of the relation and its inverse lies.

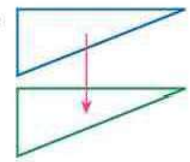
53. $\{(0, 10), (2, -8), (6, 6), (9, -4)\}$
54. $\{(-1, 18), (-2, 23), (-3, 28), (-4, 33)\}$
55. $\{(35, 12), (48, 25), (60, 52)\}$
56. $\{(-92, -77), (-93, 200), (19, -50)\}$

Maintain Your Skills

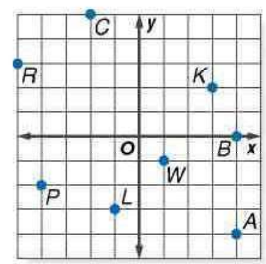
Mixed Review **Identify each transformation as a *reflection, translation, dilation,* or *rotation.*** *(Lesson 4-2)*

57. 

58. 

59. 

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located. *(Lesson 4-1)*



60. A	**61.** K
62. L	**63.** W
64. B	**65.** P
66. R	**67.** C

68. HOURLY PAY Dominique earns \$9.75 per hour. Her employer is increasing her hourly rate to \$10.15 per hour. What is the percent of increase in her salary? *(Lesson 3-7)*

Simplify each expression. *(Lesson 2-4)*

69. $72 \div 9$

70. $105 \div 15$

71. $3 \div \frac{1}{3}$

72. $16 \div \frac{1}{4}$

73. $\frac{54n + 78}{6}$

74. $\frac{98x - 35y}{7}$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Find the solution set for each equation if the replacement set is {3, 4, 5, 6, 7, 8}.** *(To review **solution sets**, see Lesson 1-3.)*

75. $a + 15 = 20$

76. $r - 6 = 2$

77. $9 = 5n - 6$

78. $3 + 8w = 35$

79. $\frac{g}{3} + 15 = 17$

80. $\frac{m}{5} + \frac{3}{5} = 2$

Practice Quiz 1

Lessons 4-1 through 4-3

Plot each point on a coordinate plane. *(Lesson 4-1)*

1. $Q(2, 3)$ **2.** $R(-4, -4)$ **3.** $S(5, -1)$ **4.** $T(-1, 3)$

Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image. *(Lesson 4-2)*

5. triangle ABC with $A(4, 8)$, $B(7, 5)$, and $C(2, -1)$ reflected over the x-axis

6. quadrilateral $WXYZ$ with $W(1, 0)$, $X(2, 3)$, $Y(4, 1)$, and $Z(3, -3)$ translated 5 units to the left and 4 units down

State the domain, range, and inverse of each relation. *(Lesson 4-3)*

7. $\{(1, 3), (4, 6), (2, 3), (1, 5)\}$

8. $\{(-2, 6), (0, 3), (4, 2), (8, -5)\}$

9. $\{(11, 5), (15, 3), (-8, 22), (11, 31)\}$

10. $\{(-5, 8), (-1, 0), (-1, 4), (2, 7), (6, 3)\}$

4-4 Equations as Relations

What You'll Learn

- Use an equation to determine the range for a given domain.
- Graph the solution set for a given domain.

Vocabulary

- equation in two variables
- solution

Why are equations of relations important in traveling?

During the summer, Eric will be taking a trip to England. He has saved \$500 for his trip, and he wants to find how much that will be worth in British pounds sterling. The exchange rate today is 1 dollar = 0.69 pound. Eric can use the equation $p = 0.69d$ to convert dollars d to pounds p.

SOLVE EQUATIONS The equation $p = 0.69d$ is an example of an **equation in two variables**. A **solution** of an equation in two variables is an ordered pair that results in a true statement when substituted into the equation.

Example 1 Solve Using a Replacement Set

Find the solution set for $y = 2x + 3$, given the replacement set $\{(-2, -1), (-1, 3), (0, 4), (3, 9)\}$.

Make a table. Substitute each ordered pair into the equation.

x	y	$y = 2x + 3$	True or False?
−2	−1	$-1 = 2(-2) + 3$ $-1 = -1$	true ✓
−1	3	$3 = 2(-1) + 3$ $3 = 1$	false
0	4	$4 = 2(0) + 3$ $4 = 3$	false
3	9	$9 = 2(3) + 3$ $9 = 9$	true ✓

The ordered pairs $(-2, -1)$ and $(3, 9)$ result in true statements. The solution set is $\{(-2, -1), (3, 9)\}$.

Since the solutions of an equation in two variables are ordered pairs, the equation describes a relation. So, in an equation involving x and y, the set of x values is the domain, and the corresponding set of y values is the range.

Study Tip

Variables

Unless the variables are chosen to represent real quantities, when variables other than x and y are used in an equation, assume that the letter that comes first in the alphabet is the domain.

Example 2 Solve Using a Given Domain

Solve $b = a + 5$ if the domain is $\{-3, -1, 0, 2, 4\}$.

Make a table. The values of a come from the domain. Substitute each value of a into the equation to determine the values of b in the range.

a	$a + 5$	b	(a, b)
−3	−3 + 5	2	(−3, 2)
−1	−1 + 5	4	(−1, 4)
0	0 + 5	5	(0, 5)
2	2 + 5	7	(2, 7)
4	4 + 5	9	(4, 9)

The solution set is $\{(-3, 2), (-1, 4), (0, 5), (2, 7), (4, 9)\}$.

GRAPH SOLUTION SETS You can graph the ordered pairs in the solution set for an equation in two variables. The domain contains values represented by the *independent variable*. The range contains the corresponding value represented by the *dependent variable*.

Study Tip

Look Back
To review **independent and dependent variables**, see Lesson 1-8.

Example 3 Solve and Graph the Solution Set

Solve $4x + 2y = 10$ if the domain is $\{-1, 0, 2, 4\}$. Graph the solution set.

First solve the equation for y in terms of x. This makes creating a table of values easier.

$4x + 2y = 10$ Original equation

$4x + 2y - 4x = 10 - 4x$ Subtract $4x$ from each side.

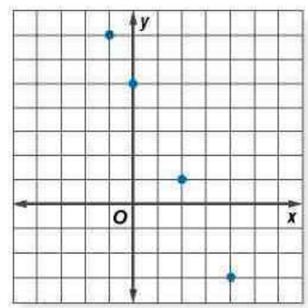
$2y = 10 - 4x$ Simplify.

$\frac{2y}{2} = \frac{10 - 4x}{2}$ Divide each side by 2.

$y = 5 - 2x$ Simplify.

Substitute each value of x from the domain to determine the corresponding values of y in the range.

x	$5 - 2x$	y	(x, y)
-1	$5 - 2(-1)$	7	$(-1, 7)$
0	$5 - 2(0)$	5	$(0, 5)$
2	$5 - 2(2)$	1	$(2, 1)$
4	$5 - 2(4)$	-3	$(4, -3)$



Graph the solution set $\{(-1, 7), (0, 5), (2, 1), (4, -3)\}$.

When you solve an equation for a given variable, that variable becomes the dependent variable. That is, its value depends upon the domain values chosen for the other variable.

Example 4 Solve for a Dependent Variable

Refer to the application at the beginning of the lesson. Eric has made a list of the expenses he plans to incur while in England. Use the conversion rate to find the equivalent U.S. dollars for these amounts given in pounds (£) and graph the ordered pairs.

Explore In the equation $p = 0.69d$, d represents U.S. dollars and p represents British pounds. However, we are given values in pounds and want to find values in dollars. Solve the equation for d since the values of d depend on the given values of p.

$p = 0.69d$ Original equation

$\frac{p}{0.69} = \frac{0.69d}{0.69}$ Divide each side by 0.69.

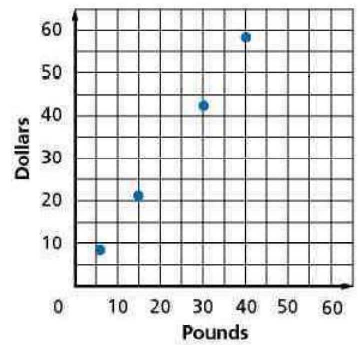
$1.45p = d$ Simplify and round to the nearest hundredth.

(continued on the next page)

Plan The values of p, {40, 30, 15, 6}, are the domain. Use the equation $d = 1.45p$ to find the values for the range.

Solve Make a table of values. Substitute each value of p from the domain to determine the corresponding values of d. Round to the nearest dollar.

p	$1.45p$	d	(p, d)
40	1.45(40)	58.00	(40, 58)
30	1.45(30)	43.50	(30, 44)
15	1.45(15)	21.75	(15, 22)
6	1.45(6)	8.70	(6, 9)



Graph the ordered pairs. Notice that the values for the independent variable p are graphed along the horizontal axis, and the values for dependent variable d are graphed along the vertical axis.

Examine Look at the values in the range. The cost in dollars is higher than the cost in pounds. Do the results make sense?

Expense	Pounds	Dollars
Hotel	40	58
Meals	30	43
Entertainment	15	22
Transportation	6	9

Check for Understanding

Concept Check

1. **Describe** how to find the domain of an equation if you are given the range.
2. **OPEN ENDED** Give an example of an equation in two variables and state two solutions for your equation.
3. **FIND THE ERROR** Malena says that (5, 1) is a solution of $y = 2x + 3$. Bryan says it is not a solution.

Malena	Bryan
$y = 2x + 3$	$y = 2x + 3$
$5 = 2(1) + 3$	$1 = 2(5) + 3$
$5 = 5$	$1 \neq 13$

Who is correct? Explain your reasoning.

Guided Practice **Find the solution set for each equation, given the replacement set.**

4. $y = 3x + 4$; {(−1, 1), (2, 10), (3, 12), (7, 1)}
5. $2x - 5y = 1$; {(−7, −3), (7, 3), (2, 1), (−2, −1)}

Solve each equation if the domain is {−3, −1, 0, 2}.

6. $y = 2x - 1$
7. $y = 4 - x$
8. $2y + 2x = 12$
9. $3x + 2y = 13$

Solve each equation for the given domain. Graph the solution set.

10. $y = 3x$ for $x = \{-3, -2, -1, 0, 1, 2, 3\}$
11. $2y = x + 2$ for $x = \{-4, -2, 0, 2, 4\}$

Application **JEWELRY** **For Exercises 12 and 13, use the following information.**
Since pure gold is very soft, other metals are often added to it to make an alloy that is stronger and more durable. The relative amount of gold in a piece of jewelry is measured in karats. The formula for the relationship is $g = \frac{25k}{6}$, where k is the number of karats and g is the percent of gold in the jewelry.

12. Find the percent of gold if the domain is {10, 14, 18, 24}. Make a table of values and graph the function.

13. How many karats are in a ring that is 50% gold?

Practice and Apply

Homework Help

For Exercises	See Examples
14–19	1
20–31	2
32–39	3
40–45	4

Extra Practice
See page 829.

Find the solution set for each equation, given the replacement set.

14. $y = 4x + 1$; {(2, −1), (1, 5), (9, 2), (0, 1)}

15. $y = 8 - 3x$; {(4, −4), (8, 0), (2, 2), (3, 3)}

16. $x - 3y = -7$; {(−1, 2), (2, −1), (2, 4), (2, 3)}

17. $2x + 2y = 6$; {(3, 0), (2, 1), (−2, −1), (4, −1)}

18. $3x - 8y = -4$; {(0, 0.5), (4, 1), (2, 0.75), (2, 4)}

19. $2y + 4x = 8$; {(0, 2), (−3, 0.5), (0.25, 3.5), (1, 2)}

Solve each equation if the domain is {−2, −1, 1, 3, 4}.

20. $y = 4 - 5x$
21. $y = 2x + 3$
22. $x = y + 4$
23. $x = 7 - y$
24. $6x - 3y = 18$
25. $6x - y = -3$
26. $8x + 4y = 12$
27. $2x - 2y = 0$
28. $5x - 10y = 20$
29. $3x + 2y = 14$
30. $x + \frac{1}{2}y = 8$
31. $2x - \frac{1}{3}y = 4$

Solve each equation for the given domain. Graph the solution set.

32. $y = 2x + 3$ for $x = \{-3, -2, -1, 1, 2, 3\}$

33. $y = 3x - 1$ for $x = \{-5, -2, 1, 3, 4\}$

34. $3x - 2y = 5$ for $x = \{-3, -1, 2, 4, 5\}$

35. $5x + 4y = 8$ for $x = \{-4, -1, 0, 2, 4, 6\}$

36. $\frac{1}{2}x + y = 2$ for $x = \{-4, -1, 1, 4, 7, 8\}$

37. $y = \frac{1}{4}x - 3$ for $x = \{-4, -2, 0, 2, 4, 6\}$

38. The domain for $3x + y = 8$ is {−1, 2, 5, 8}. Find the range.

39. The range for $2y - x = 6$ is {−4, −3, 1, 6, 7}. Find the domain.

TRAVEL **For Exercises 40 and 41, use the following information.**
Heinrich and his brother live in Germany. They are taking a trip to the United States and have been checking the average temperatures in different U.S. cities for the month they will be traveling. They are unfamiliar with the Fahrenheit scale, so they would like to convert the temperatures to Celsius. The equation $F = 1.8C + 32$ relates the temperature in degrees Celsius C to degrees Fahrenheit F.

City	Temperature (°F)
New York	34
Chicago	23
San Francisco	55
Miami	72
Washington, D.C.	40

40. Solve the equation for C.

41. Find the temperatures in degrees Celsius for each city.

GEOMETRY **For Exercises 42–44, use the following information.**
The equation for the perimeter of a rectangle is $P = 2\ell + 2w$. Suppose the perimeter of rectangle $ABCD$ is 24 centimeters.

42. Solve the equation for ℓ.

43. State the independent and dependent variables.

44. Choose five values for w and find the corresponding values of ℓ.

45. **ANTHROPOLOGY** When the remains of ancient people are discovered, usually only a few bones are found. Anthropologists can determine a person's height by using a formula that relates the length of the tibia T (shin bone) to the person's height H, both measured in centimeters. The formula for males is $H = 81.7 + 2.4T$ and for females is $H = 72.6 + 2.5T$. Copy and complete the tables below. Then graph each set of ordered pairs.

Male		
Length of Tibia (cm)	Height (cm)	(T, H)
30.5		
34.8		
36.3		
37.9		

Female		
Length of Tibia (cm)	Height (cm)	(T, H)
30.5		
34.8		
36.3		
37.9		

Career Choices

Forensic Anthropologist
Forensic anthropologists assist police investigations. They can determine the age and stature of a victim by examining dental wear on the teeth and measuring certain bones.

Online Research
For information about a career as a forensic anthropologist, visit: www.algebra1.com/careers

46. **RESEARCH** Choose a country that you would like to visit. Use the Internet or other reference to find the cost of various services such as hotels, meals, and transportation. Use the currency exchange rate to determine how much money in U.S. dollars you will need on your trip.

47. **CRITICAL THINKING** Find the domain values of each relation if the range is $\{0, 16, 36\}$.

a. $y = x^2$ **b.** $y = |4x| - 16$ **c.** $y = |4x - 16|$

48. **CRITICAL THINKING** Select five values for the domain and find the range of $y = x + 4$. Then look at the range and domain of the inverse relation. Make a conjecture about the equation that represents the inverse relation.

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are equations of relations important in traveling?

Include the following in your answer:

- an example of how you would keep track of how much you were spending in pounds and the equivalent amount in dollars, and
- an explanation of your spending power if the currency exchange rate is 0.90 pound compared to one U.S. dollar or 1.04 pounds compared to one dollar.

Standardized Test Practice

50. If $3x - y = 18$ and $y = 3$, then $x =$

(A) 4. (B) 5. (C) 6. (D) 7.

51. If the perimeter of a rectangle is 14 units and the area is 12 square units, what are the dimensions of the rectangle?

(A) 2×6 (B) 3×3
(C) 3×4 (D) 1×12

Graphing Calculator

TABLE FEATURE You can enter selected x values in the TABLE feature of a graphing calculator, and it will calculate the corresponding y values for a given equation. To do this, enter an equation into the Y= list. Go to TBLSET and highlight Ask under the Independent variable. Now you can use the TABLE function to enter any domain value and the corresponding range value will appear in the second column.

Use a graphing calculator to find the solution set for the given equation and domain.

52. $y = 3x - 4$; $x = \{-11, 15, 23, 44\}$

53. $y = -6.5x + 42$; $x = \{-8, -5, 0, 3, 7, 12\}$

54. $y = 3x + 12$ for $x = \{0.4, 0.6, 1.8, 2.2, 3.1\}$

55. $y = 1.4x - 0.76$ for $x = \{-2.5, -1.75, 0, 1.25, 3.33\}$

Maintain Your Skills

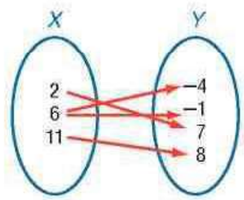
Mixed Review

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation. *(Lesson 4-3)*

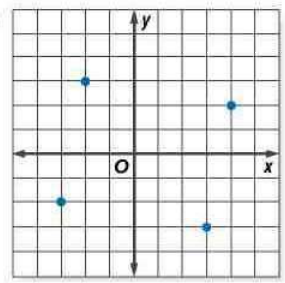
56.

x	y
4	9
3	−2
1	5
−4	2

57.



58.



Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image. *(Lesson 4-2)*

59. triangle XYZ with $X(-6, 4)$, $Y(-5, 0)$, and $Z(3, 3)$ reflected over the y-axis

60. quadrilateral $QRST$ with $Q(2, 2)$, $R(3, -3)$, $S(-1, -4)$ and $T(-4, -3)$ rotated 90° counterclockwise about the origin

Use cross products to determine whether each pair of ratios forms a proportion. Write *yes* or *no*. *(Lesson 3-6)*

61. $\frac{6}{15}, \frac{18}{45}$

62. $\frac{11}{12}, \frac{33}{34}$

63. $\frac{8}{22}, \frac{20}{55}$

64. $\frac{6}{8}, \frac{3}{4}$

65. $\frac{3}{5}, \frac{9}{25}$

66. $\frac{26}{35}, \frac{12}{15}$

Identify the hypothesis and conclusion of each statement. *(Lesson 1-7)*

67. If it is hot, then we will go swimming.

68. If you do your chores, then you get an allowance.

69. If $3n - 7 = 17$, then $n = 8$.

70. If $a > b$ and $b > c$, then $a > c$.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** *(To review **solving equations**, see Lesson 3-4.)*

71. $a + 15 = 20$

72. $r - 9 = 12$

73. $-4 = 5n + 6$

74. $3 - 8w = 35$

75. $\frac{g}{4} + 2 = 5$

76. $\frac{m}{5} + \frac{3}{5} = 2$

4-5 Graphing Linear Equations

What You'll Learn

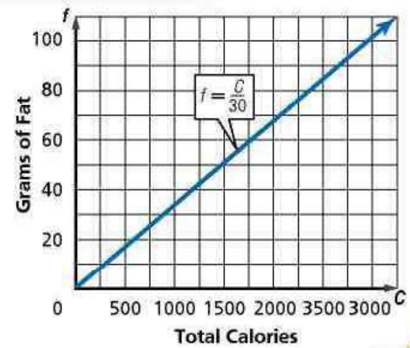
- Determine whether an equation is linear.
- Graph linear equations.

Vocabulary

- linear equation
- standard form
- *x*-intercept
- *y*-intercept

How can linear equations be used in nutrition?

Nutritionists recommend that no more than 30% of a person's daily caloric intake come from fat. Each gram of fat contains nine Calories. To determine the most grams of fat f you should have, find the total number of Calories C you consume each day and use the equation $f = 0.3\left(\frac{C}{9}\right)$ or $f = \left(\frac{C}{30}\right)$. The graph of this equation shows the maximum number of grams of fat you can consume based on the total number of Calories consumed.



IDENTIFY LINEAR EQUATIONS A **linear equation** can be written in the form $Ax + By = C$. This is called the **standard form** of a linear equation.

Key Concept — Standard Form of a Linear Equation

The standard form of a linear equation is

$$Ax + By = C,$$

where $A \geq 0$, A and B are not both zero, and A, B, and C are integers whose greatest common factor is 1.

Example 1 Identify Linear Equations

Determine whether each equation is a linear equation. If so, write the equation in standard form.

a. $y = 5 - 2x$

First rewrite the equation so that both variables are on the same side of the equation.

$y = 5 - 2x$	Original equation
$y + 2x = 5 - 2x + 2x$	Add $2x$ to each side.
$2x + y = 5$	Simplify.

The equation is now in standard form where $A = 2$, $B = 1$, and $C = 5$. This is a linear equation.

b. $2xy - 5y = 6$

Since the term $2xy$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

c. $3x + 9y = 15$

Since the GCF of 3, 9, and 15 is not 1, the equation is not written in standard form. Divide each side by the GCF.

$3x + 9y = 15$	Original equation
$3(x + 3y) = 15$	Factor the GCF.
$\frac{3(x + 3y)}{3} = \frac{15}{3}$	Divide each side by 3.
$x + 3y = 5$	Simplify.

The equation is now in standard form where $A = 1$, $B = 3$, and $C = 5$.

d. $\frac{1}{3}y = -1$

To write the equation with integer coefficients, multiply each term by 3.

$\frac{1}{3}y = -1$	Original equation
$3\left(\frac{1}{3}\right)y = 3(-1)$	Multiply each side of the equation by 3.
$y = -3$	Simplify.

The equation $y = -3$ can be written as $0x + y = -3$. Therefore, it is a linear equation in standard form where $A = 0$, $B = 1$, and $C = -3$.

GRAPH LINEAR EQUATIONS The graph of a linear equation is a line. The line represents all the solutions of the linear equation. Also, every ordered pair on this line satisfies the equation.

Example 2 Graph by Making a Table

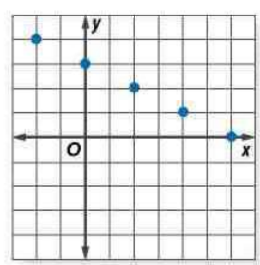
Graph $x + 2y = 6$.

In order to find values for y more easily, solve the equation for y.

$x + 2y = 6$	Original equation
$x + 2y - x = 6 - x$	Subtract x from each side.
$2y = 6 - x$	Simplify.
$\frac{2y}{2} = \frac{6 - x}{2}$	Divide each side by 2.
$y = 3 - \frac{1}{2}x$	Simplify.

Select five values for the domain and make a table. Then graph the ordered pairs.

x	$3 - \frac{1}{2}x$	y	(x, y)
-2	$3 - \frac{1}{2}(-2)$	4	$(-2, 4)$
0	$3 - \frac{1}{2}(0)$	3	$(0, 3)$
2	$3 - \frac{1}{2}(2)$	2	$(2, 2)$
4	$3 - \frac{1}{2}(4)$	1	$(4, 1)$
6	$3 - \frac{1}{2}(6)$	0	$(6, 0)$



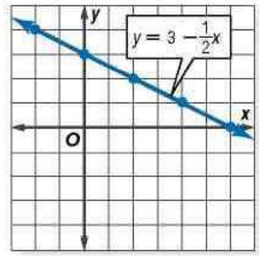
(continued on the next page)

Study Tip

Graphing Equations

When you graph an equation, use arrows at both ends to show that the graph continues. You should also label the graph with the equation.

When you graph the ordered pairs, a pattern begins to form. The domain of $y = 3 - \frac{1}{2}x$ is the set of all real numbers, so there are an infinite number of solutions of the equation. Draw a line through the points. This line represents all of the solutions of $y = 3 - \frac{1}{2}x$.



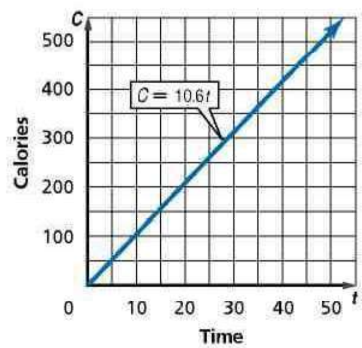
Example 3 Use the Graph of a Linear Equation

PHYSICAL FITNESS **Carlos swims every day. He burns approximately 10.6 Calories per minute when swimming laps.**

a. Graph the equation $C = 10.6t$, where C represents the number of Calories burned and t represents the time in minutes spent swimming.

Select five values for t and make a table. Graph the ordered pairs and connect them to draw a line.

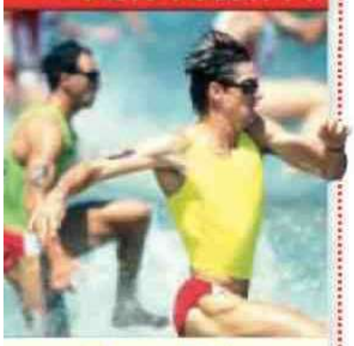
t	$10.6t$	C	(t, C)
10	10.6(10)	106	(10, 106)
15	10.6(15)	159	(15, 159)
20	10.6(20)	212	(20, 212)
30	10.6(30)	318	(30, 318)



b. Suppose Carlos wanted to burn 350 Calories. Approximately how long should he swim?

Since any point on the line is a solution of the equation, use the graph to estimate the value of the x-coordinate in the ordered pair that contains 350 as the y-coordinate. The ordered pair (33, 350) appears to be on the line so Carlos should swim for 33 minutes to burn 350 Calories. *Check this solution algebraically by substituting (33, 350) into the original equation.*

More About. . .



Physical Fitness

In a triathlon competition, athletes swim 1.5 kilometers, bicycle 40 kilometers, and run 10 kilometers.

Source: www.usatriathlon.org

Since two points determine a line, a simple method of graphing a linear equation is to find the points where the graph crosses the x-axis and the y-axis. The x-coordinate of the point at which it crosses the x-axis is the **x-intercept**, and the y-coordinate of the point at which the graph crosses the y-axis is called the **y-intercept**.

Example 4 Graph Using Intercepts

Determine the x-intercept and y-intercept of $3x + 2y = 9$. Then graph the equation.

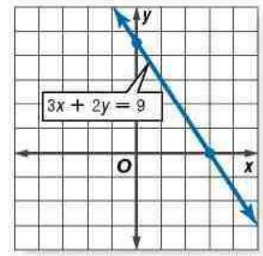
To find the x-intercept, let $y = 0$.

$3x + 2y = 9$	Original equation
$3x + 2(0) = 9$	Replace y with 0.
$3x = 9$	Divide each side by 3.
$x = 3$	

To find the y-intercept, let $x = 0$.

$3x + 2y = 9$	Original equation
$3(0) + 2y = 9$	Replace x with 0.
$2y = 9$	Divide each side by 2.
$y = 4.5$	

The x-intercept is 3, so the graph intersects the x-axis at (3, 0). The y-intercept is 4.5, so the graph intersects the y-axis at (0, 4.5). Plot these points. Then draw a line that connects them.



Check for Understanding

Concept Check

1. **Explain** how the graph of $y = 2x + 1$ for the domain {1, 2, 3, 4} differs from the graph of $y = 2x + 1$ for the domain of all real numbers.
2. **OPEN ENDED** Give an example of a linear equation in the form $Ax + By = C$ for each of the following conditions.
   a. $A = 0$ **b.** $B = 0$ **c.** $C = 0$
3. **Explain** how to graph an equation using the x- and y-intercepts.

Guided Practice

Determine whether each equation is a linear equation. If so, write the equation in standard form.

4. $x + y^2 = 25$
5. $3y + 2 = 0$
6. $\frac{3}{5}x - \frac{2}{5}y = 5$
7. $x + \frac{1}{y} = 7$

Graph each equation.

8. $x = 3$
9. $x - y = 0$
10. $y = 2x + 8$
11. $y = -3 - x$
12. $x + 4y = 10$
13. $4x + 3y = 12$

Application

TAXI FARE **For Exercises 14 and 15, use the following information.**
A taxi company charges a fare of \$2.25 plus \$0.75 per mile traveled. The cost of the fare c can be described by the equation $c = 0.75m + 2.25$, where m is the number of miles traveled.

14. Graph the equation.
15. If you need to travel 18 miles, how much will the taxi fare cost?

Practice and Apply

Homework Help

For Exercises	See Examples
16–25	1
26–45	2, 4
46–56	3

Extra Practice
See page 829.

Determine whether each equation is a linear equation. If so, write the equation in standard form.

16. $3x = 5y$
17. $6 - y = 2x$
18. $6xy + 3x = 4$
19. $y + 5 = 0$
20. $7y = 2x + 5x$
21. $y = 4x^2 - 1$
22. $\frac{3}{x} + \frac{4}{y} = 2$
23. $\frac{x}{2} = 10 + \frac{2y}{3}$
24. $7n - 8m = 4 - 2m$
25. $3a + b - 2 = b$

Graph each equation.

26. $y = -1$
27. $y = 2x$
28. $y = 5 - x$
29. $y = 2x - 8$
30. $y = 4 - 3x$
31. $y = x - 6$
32. $x = 3y$
33. $x = 4y - 6$
34. $x - y = -3$
35. $x + 3y = 9$
36. $4x + 6y = 8$
37. $3x - 2y = 15$

Graph each equation.

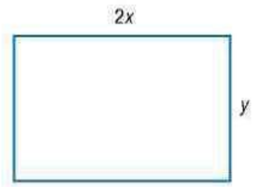
38. $1.5x + y = 4$ **39.** $2.5x + 5y = 75$ **40.** $\frac{1}{2}x + y = 4$

41. $x - \frac{2}{3}y = 1$ **42.** $\frac{4x}{3} = \frac{3y}{4} + 1$ **43.** $y + \frac{1}{3} = \frac{1}{4}x - 3$

44. Find the x- and y-intercept of the graph of $4x - 7y = 14$.

45. Write an equation in standard form of the line with an x-intercept of 3 and a y-intercept of 5.

GEOMETRY **For Exercises 46–48, refer to the figure.** The perimeter P of a rectangle is given by $2\ell + 2w = P$, where ℓ is the length of the rectangle and w is the width.



46. If the perimeter of the rectangle is 30 inches, write an equation for the perimeter in standard form.

47. What are the x- and y-intercepts of the graph of the equation?

48. Graph the equation.

METEOROLOGY **For Exercises 49–51, use the following information.** As a thunderstorm approaches, you see lightning as it occurs, but you hear the accompanying sound of thunder a short time afterward. The distance d in miles that sound travels in t seconds is given by the equation $d = 0.21t$.

49. Make a table of values.

50. Graph the equation.

51. Estimate how long it will take to hear the thunder from a storm 3 miles away.

More About. . .

Oceanography

How heavy is air? The atmospheric pressure is a measure of the weight of air. At sea level, air pressure is 14.7 pounds per square inch.

Source: www.brittanica.com

BIOLOGY **For Exercises 52 and 53, use the following information.** The amount of blood in the body can be predicted by the equation $y = 0.07w$, where y is the number of pints of blood and w is the weight of a person in pounds.

52. Graph the equation.

53. Predict the weight of a person whose body holds 12 pints of blood.

OCEANOGRAPHY **For Exercises 54–56, use the information at left and below.** Under water, pressure increases 4.3 pounds per square inch (psi) for every 10 feet you descend. This can be expressed by the equation $p = 0.43d + 14.7$, where p is the pressure in pounds per square inch and d is the depth in feet.

54. Graph the equation.

55. Divers cannot work at depths below about 400 feet. What is the pressure at this depth?

56. How many times as great is the pressure at 400 feet as the pressure at sea level?

57. **CRITICAL THINKING** Explain how you can determine whether a point at (x, y) is *above*, *below*, or *on* the line given by $2x - y = 8$ without graphing it. Give an example of each.

58. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can linear equations be used in nutrition?

Include the following in your answer:

- an explanation of how you could use the Nutrition Information labels on packages to limit your fat intake, and
- an equation you could use to find how many grams of protein you should have each day if you wanted 10% of your diet to consist of protein. (*Hint*: Protein contains 4 Calories per gram.)

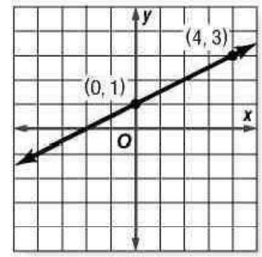
Standardized Test Practice

59. Which point lies on the line given by $y = 3x - 5$?

Ⓐ $(1, -2)$ Ⓑ $(0, 5)$ Ⓒ $(1, 2)$ Ⓓ $(4, 3)$

60. In the graph at the right, $(0, 1)$ and $(4, 3)$ lie on the line. Which ordered pair also lies on the line?

Ⓐ $(1, 1)$ Ⓑ $(2, 2)$
Ⓒ $(3, 3)$ Ⓓ $(4, 4)$



Maintain Your Skills

Mixed Review

Solve each equation if the domain is {−3, −1, 2, 5, 8}. *(Lesson 4-4)*

61. $y = x - 5$
62. $y = 2x + 1$
63. $3x + y = 12$
64. $2x - y = -3$
65. $3x - \frac{1}{2}y = 6$
66. $-2x + \frac{1}{3}y = 4$

Express each relation as a table, a graph, and a mapping. Then determine the domain and range. *(Lesson 4-3)*

67. $\{(3, 5), (-4, -1), (-3, 2), (3, 1)\}$
68. $\{(4, 0), (2, -3), (-1, -3), (4, 4)\}$
69. $\{(1, 4), (3, 0), (-1, -1), (3, 5)\}$
70. $\{(4, 5), (2, 5), (4, -1), (3, 2)\}$

Solve each equation. Then check your solution. *(Lesson 3-5)*

71. $2(x - 2) = 3x - (4x - 5)$
72. $3a + 8 = 2a - 4$
73. $3n - 12 = 5n - 20$
74. $6(x + 3) = 3x$

ANIMALS **For Exercises 75–78, use the table below that shows the average life spans of 20 different animals.** *(Lesson 2-5)*

Animal	Life Span (years)	Animal	Life Span (years)	Animal	Life Span (years)
Baboon	20	Lion	15	Squirrel	10
Camel	12	Monkey	15	Tiger	16
Cow	15	Mouse	3	Wolf	5
Elephant	40	Opossum	1	Zebra	15
Fox	7	Pig	10		
Gorilla	20	Rabbit	5		
Hippopotamus	25	Sea Lion	12		
Kangaroo	7	Sheep	12		

75. Make a line plot of the average life spans of the animals in the table.
76. How many animals live between 7 and 16 years?
77. Which number occurred most frequently?
78. How many animals live at least 20 years?

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
(To review ***evaluating expressions****, see Lesson 1-2.)*

79. $19 + 5 \cdot 4$
80. $(25 - 4) \div (2^2 - 1^3)$
81. $12 \div 4 + 15 \cdot 3$
82. $12(19 - 15) - 3 \cdot 8$
83. $6(4^3 + 2^2)$
84. $7[4^3 - 2(4 + 3)] \div 7 + 2$

CONTENTS

Graphing Calculator

A Follow-Up of Lesson 4-5

Graphing Linear Equations

The power of a graphing calculator is the ability to graph different types of equations accurately and quickly. Often linear equations are graphed in the standard viewing window. The **standard viewing window** is $[-10, 10]$ by $[-10, 10]$ with a scale of 1 on both axes. To quickly choose the standard viewing window on a TI-83 Plus, press ZOOM 6.

Example 1

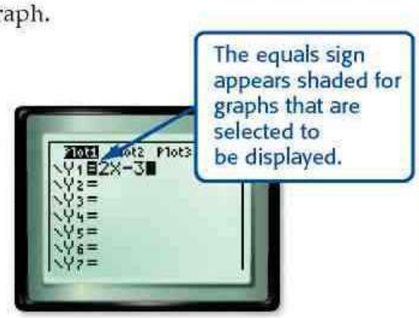
Graph $2x - y = 3$ on a TI-83 Plus graphing calculator.

Step 1 *Enter the equation in the Y= list.*

- The Y= list shows the equation or equations that you will graph.
- Equations must be entered with the y isolated on one side of the equation. Solve the equation for y, then enter it into the calculator.

$2x - y = 3$		Original equation
$2x - y - 2x = 3 - 2x$		Subtract $2x$ from each side.
$-y = -2x + 3$		Simplify.
$y = 2x - 3$		Multiply each side by -1.

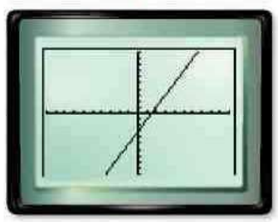
KEYSTROKES: Y= 2 X,T,θ,n − 3



Step 2 *Graph the equation in the standard viewing window.*

Graph the selected equations.

KEYSTROKES: ZOOM 6



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Notice that the graph of $2x - y = 3$ above is a complete graph because all of these points are visible.

Sometimes a complete graph is not displayed using the standard viewing window. A **complete graph** includes all of the important characteristics of the graph on the screen. These include the origin, and the x- and y-intercepts.

When a complete graph is not displayed using the standard viewing window, you will need to change the viewing window to accommodate these important features. You can use what you have learned about intercepts to help you choose an appropriate viewing window.

www.algebra1.com/other_calculator_keystrokes

Investigation

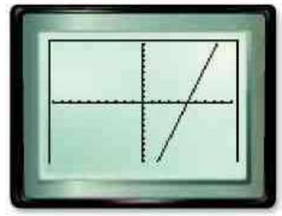
Example 2

Graph $y = 3x - 15$ on a graphing calculator.

Step 1 *Enter the equation in the Y= list and graph in the standard viewing window.*

Clear the previous equation from the Y= list. Then enter the new equation and graph.

KEYSTROKES: [Y=] [CLEAR] 3 [X,T,θ,n] [−] 15 [ZOOM] 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 2 *Modify the viewing window and graph again.*

The origin and the x-intercept are displayed in the standard viewing window. But notice that the y-intercept is outside of the viewing window. Find the y-intercept.

$y = 3x - 15$ Original equation

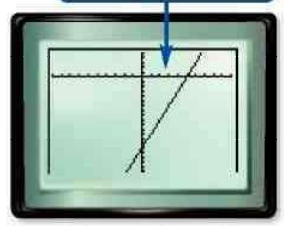
$y = 3(0) - 15$ Replace x with 0.

$y = -15$ Simplify.

Since the y-intercept is -15, choose a viewing window that includes a number less than -15. The window $[-10, 10]$ by $[-20, 5]$ with a scale of 1 on each axis is a good choice.

KEYSTROKES: [WINDOW] −10 [ENTER] 10 [ENTER] 1 [ENTER] −20 [ENTER] 5 [ENTER] 1 [GRAPH]

This window allows the complete graph, including the y-intercept, to be displayed.



[−10, 10] scl: 1 by [−20, 5] scl: 1

Exercises

Use a graphing calculator to graph each equation in the standard viewing window. Sketch the result.

1. $y = x + 2$ **2.** $y = 4x + 5$ **3.** $y = 6 - 5x$

4. $2x + y = 6$ **5.** $x + y = -2$ **6.** $x - 4y = 8$

Graph each linear equation in the standard viewing window. Determine whether the graph is complete. If the graph is not complete, choose a viewing window that will show a complete graph and graph the equation again.

7. $y = 5x + 9$ **8.** $y = 10x - 6$ **9.** $y = 3x - 18$

10. $3x - y = 12$ **11.** $4x + 2y = 21$ **12.** $3x + 5y = -45$

For Exercises 13–15, consider the linear equation $y = 2x + b$.

13. Choose several different positive and negative values for b. Graph each equation in the standard viewing window.

14. For which values of b is the complete graph in the standard viewing window?

15. How is the value of b related to the y-intercept of the graph of $y = 2x + b$?

4-6 Functions

What You'll Learn

- Determine whether a relation is a function.
- Find function values.

Vocabulary

- function
- vertical line test
- function notation

How are functions used in meteorology?

The table shows barometric pressures and temperatures recorded by the National Climatic Data Center over a three-day period.

Pressure (millibars)	1013	1006	997	995	995	1000	1006	1011	1016	1019
Temperature (°C)	3	4	10	13	8	4	1	−2	−6	−9

Notice that when the pressure is 995 and 1006 millibars, there is more than one value for the temperature.

IDENTIFY FUNCTIONS Recall that relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

Study Tip

Functions
In a function, knowing the value of x tells you the value of y.

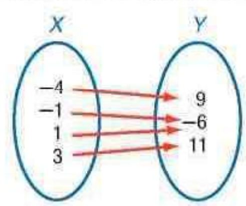
Key Concept — Function

A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

Example 1 Identify Functions

Determine whether each relation is a function. Explain.

a.



This mapping represents a function since, for each element of the domain, there is only one corresponding element in the range. It does not matter if two elements of the domain are paired with the same element in the range.

b.

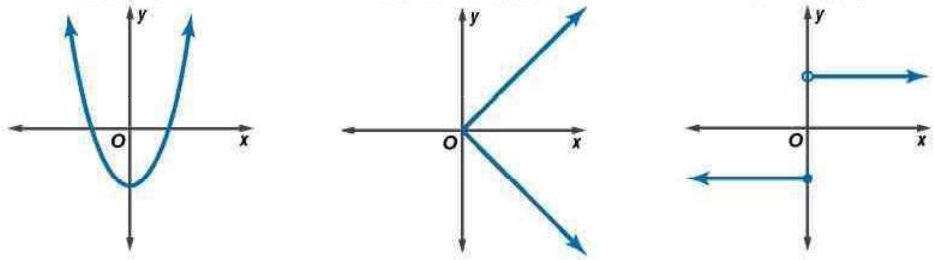
x	y
−3	6
2	5
3	1
2	4

This table represents a relation that is not a function. The element 2 in the domain is paired with both 5 and 4 in the range. If you are given that x is 2, you cannot determine the value of y.

c. $\{(-2, 4), (1, 5), (3, 6), (5, 8), (7, 10)\}$

Since each element of the domain is paired with exactly one element of the range, this relation is a function. If you are given that x is -3, you can determine that the value of y is 6 since 6 is the only value of y that is paired with $x = 3$.

You can use the **vertical line test** to see if a graph represents a function. If no vertical line can be drawn so that it intersects the graph more than once, then the graph is a function. If a vertical line can be drawn so that it intersects the graph at two or more points, the relation is not a function.

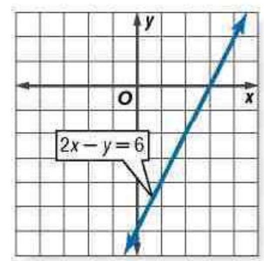


One way to perform the vertical line test is to use a pencil.

Example 2 Equations as Functions

Determine whether $2x - y = 6$ is a function.

Graph the equation using the x- and y-intercepts.



Since the equation is in the form $Ax + By = C$, the graph of the equation will be a line. Place your pencil at the left of the graph to represent a vertical line. Slowly move the pencil to the right across the graph.

For each value of x, this vertical line passes through no more than one point on the graph. Thus, the line represents a function.

FUNCTION VALUES Equations that are functions can be written in a form called **function notation**. For example, consider $y = 3x - 8$.

equation	function notation
$y = 3x - 8$	$f(x) = 3x - 8$

Study Tip

Reading Math
The symbol $f(x)$ is read *f of x*.

In a function, x represents the elements of the domain, and $f(x)$ represents the elements of the range. Suppose you want to find the value in the range that corresponds to the element 5 in the domain. This is written $f(5)$ and is read "f of 5." The value $f(5)$ is found by substituting 5 for x in the equation.

Example 3 Function Values

If $f(x) = 2x + 5$, find each value.

a. $f(-2)$

$$\begin{aligned} f(-2) &= 2(-2) + 5 && \text{Replace } x \text{ with } -2.\\ &= -4 + 5 && \text{Multiply.}\\ &= 1 && \text{Add.} \end{aligned}$$

b. $f(1) + 4$

$$\begin{aligned} f(1) + 4 &= [2(1) + 5] + 4 && \text{Replace } x \text{ with 1.}\\ &= 7 + 4 && \text{Simplify.}\\ &= 11 && \text{Add.} \end{aligned}$$

www.algebra1.com/extra_examples

c. $f(x + 3)$

$f(x + 3) = 2(x + 3) + 5$ Replace x with $x + 3$.

$= 2x + 6 + 5$ Distributive Property

$= 2x + 11$ Simplify.

The functions we have studied thus far have been linear functions. However, many functions are not linear. You can find the value of these functions in the same way.

Example 4 Nonlinear Function Values

Study Tip

Reading Math
Other letters such as g and h can be used to represent functions, for example, $g(x)$ or $h(z)$.

If $h(z) = z^2 + 3z - 4$, find each value.

a. $h(-4)$

$h(-4) = (-4)^2 + 3(-4) - 4$ Replace z with -4.

$= 16 - 12 - 4$ Multiply.

$= 0$ Simplify.

b. $h(5a)$

$h(5a) = (5a)^2 + 3(5a) - 4$ Replace z with $5a$.

$= 25a^2 + 15a - 4$ Simplify.

c. $2[h(g)]$

$2[h(g)] = 2[(g)^2 + 3(g) - 4]$ Evaluate $h(g)$ by replacing z with g.

$= 2(g^2 + 3g - 4)$ Multiply the value of $h(g)$ by 2.

$= 2g^2 + 6g - 8$ Simplify.

On some standardized tests, an arbitrary symbol may be used to represent a function.

Standardized Test Practice

Example 5 Nonstandard Function Notation

Multiple-Choice Test Item

If $\ll x \gg = x^2 - 4x + 2$, then $\ll 3 \gg =$

(A) -2. (B) -1. (C) 1. (D) 2.

Read the Test Item

The symbol $\ll x \gg$ is just a different notation for $f(x)$.

Solve the Test Item

Replace x with 3.

$\ll x \gg = x^2 - 4x + 2$ Think: $\ll x \gg = f(x)$

$\ll 3 \gg = (3)^2 - 4(3) + 2$ Replace x with 3.

$= 9 - 12 + 2$ or -1 The answer is B.

The Princeton Review
Test-Taking Tip
If the nonstandard function notation confuses you, replace the arbitrary symbol with $f(x)$.

Check for Understanding

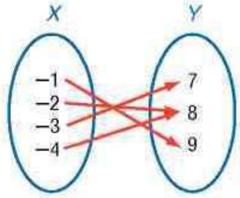
Concept Check

1. **Study** the following set of ordered pairs that describe a relation between x and y: $\{(1, -1), (-1, 2), (4, -3), (3, 2), (-2, 4), (3, -3)\}$. Is y a function of x? Is x a function of y? Explain your answer.
2. **OPEN ENDED** Define a function using nonstandard function notation.
3. **Find a counterexample** to disprove the following statement.
All linear equations are functions.

Guided Practice

Determine whether each relation is a function.

4.



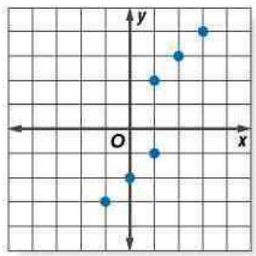
5.

x	y
−3	0
2	1
2	4
6	5

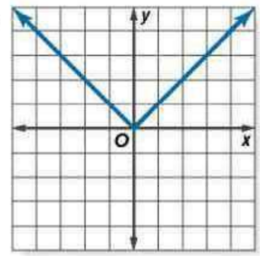
6. {(24, 1), (21, 4), (3, 22), (24, 5)}

7. $y = x + 3$

8.



9.



If $f(x) = 4x - 5$ and $g(x) = x^2 + 1$, find each value.

10. $f(2)$

11. $g(-1)$

12. $f(c)$

13. $g(t) - 4$

14. $f(3a^2)$

15. $f(x + 5)$

Standardized Test Practice

16. If $x^{**} = 2x - 1$, then $5^{**} - 2^{**} =$

(A) 3. (B) 4. (C) 5. (D) 6.

Practice and Apply

Homework Help

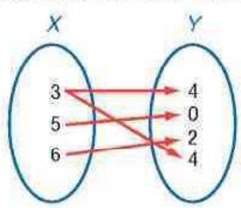
For Exercises	See Examples
17–31, 44	1, 2
32–43, 45–51	3–5

Extra Practice

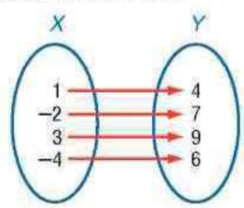
See page 830.

Determine whether each relation is a function.

17.



18.



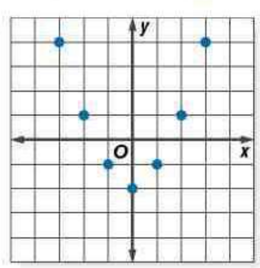
19.

x	y
2	7
4	9
5	5
8	−1

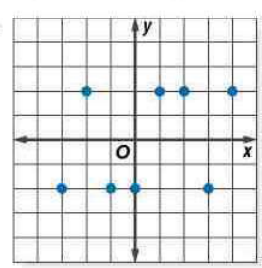
20.

x	y
−9	−5
−4	0
3	6
7	1
6	−5
3	2

21.



22.



23. {(5, −7), (6, −7), (−8, −1), (0, −1)}

24. {(4, 5), (3, −2), (−2, 5), (4, 7)}

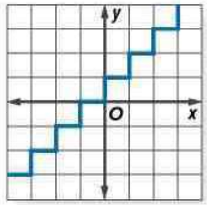
25. $y = -8$

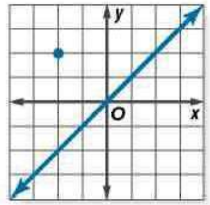
26. $x = 15$

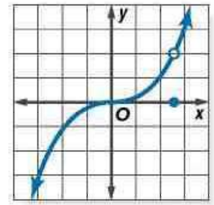
27. $y = 3x - 2$

28. $y = 3x + 2y$

Determine whether each relation is a function.

29. 

30. 

31. 

WebQuest

A graph of the winning Olympic swimming times will help you determine whether the winning time is a function of the year. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

If $f(x) = 3x + 7$ and $g(x) = x^2 - 2x$, find each value.

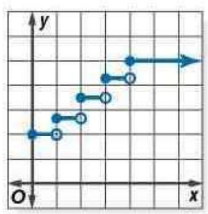
32. $f(3)$ **33.** $f(-2)$ **34.** $g(5)$

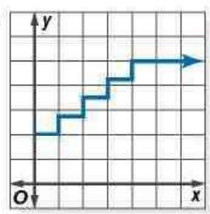
35. $g(0)$ **36.** $g(-3) + 1$ **37.** $f(8) - 5$

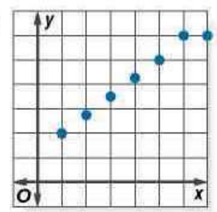
38. $g(2c)$ **39.** $f(a^2)$ **40.** $f(k + 2)$

41. $f(2m - 5)$ **42.** $3[g(x) + 4]$ **43.** $2[f(x^2) - 5]$

44. PARKING The rates for a parking garage are as follows: \$2.00 for the first hour; \$2.75 for the second hour; \$3.50 for the third hour; \$4.25 for the fourth hour; and \$5.00 for any time over four hours. Choose the graph that best represents the information given and determine whether the graph represents a function. Explain your reasoning.

a. 

b. 

c. 

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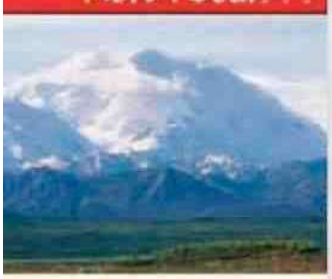


Climate

Earth's average land surface temperature has risen 0.8–1.0°F in the last century. Scientists believe it could rise 1–4.5°F in the next fifty years and 2.2–10°F in the next century.

Source: Environmental Protection Agency

CLIMATE **For Exercises 45–48, use the following information.**
The temperature of the atmosphere decreases about 5°F for every 1000 feet increase in altitude. Thus, if the temperature at ground level is 77°F, the temperature at a given altitude is found by using the equation $t = 77 - 0.005h$, where h is the height in feet.

45. Write the equation in function notation.

46. Find $f(100)$, $f(200)$, and $f(1000)$.

47. Graph the equation.

48. Use the graph of the function to determine the temperature at 4000 feet.

EDUCATION **For Exercises 49–51, use the following information.**
The National Assessment of Educational Progress tests 4th, 8th, and 12th graders in the United States. The average math test scores for 17-year-olds can be represented as a function of the science scores by $f(s) = 0.8s + 72$, where $f(s)$ is the math score and s is the science score.

49. Graph this function.

50. What is the science score that corresponds to a math score of 308?

51. Krista scored 260 in science and 320 in math. How does her math score compare to the average score of other students who scored 260 in science? Explain your answer.

52. CRITICAL THINKING State whether the following is *sometimes*, *always*, or *never* true.

The inverse of a function is also a function.

53. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are functions used in meteorology?

Include the following in your answer:

- a description of the relationship between pressure and temperature, and
- an explanation of whether the relation is a function.

Standardized Test Practice

54. If $f(x) = 20 - 2x$, find $f(7)$.

(A) 6 (B) 7 (C) 13 (D) 14

55. If $f(x) = 2x$, which of the following statements must be true?

I. $f(3x) = 3[f(x)]$

II. $f(x + 3) = f(x) + 3$

III. $f(x^2) = [f(x)]^2$

(A) I only (B) II only (C) I and II only (D) I, II, and III

Maintain Your Skills

Mixed Review

Graph each equation. *(Lesson 4-5)*

56. $y = x + 3$ 57. $y = 2x - 4$ 58. $2x + 5y = 10$

Find the solution set for each equation, given the replacement set. *(Lesson 4-4)*

59. $y = 5x - 3$; $\{(3, 12), (1, -2), (-2, -7), (-1, -8)\}$

60. $y = 2x + 6$; $\{(3, 0), (-1, 4), (6, 0), (5, -1)\}$

61. **RUNNING** Adam is training for an upcoming 26-mile marathon. He can run a 10K race (about 6.2 miles) in 45 minutes. If he runs the marathon at the same pace, how long will it take him to finish? *(Lesson 3-6)*

Name the property used in each equation. Then find the value of n. *(Lesson 1-4)*

62. $16 = n + 16$ 63. $3.5 + 6 = n + 6$ 64. $\frac{3}{5}n = \frac{3}{5}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each difference.**
*(To review **subtracting integers**, see Lesson 2-2.)*

65. $12 - 16$ 66. $-5 - (-8)$ 67. $16 - (-4)$

68. $-9 - 6$ 69. $\frac{3}{4} - \frac{1}{8}$ 70. $3\frac{1}{2} - \left(-1\frac{2}{3}\right)$

Practice Quiz 2 — Lessons 4-4 through 4-6

Solve each equation if the domain is $\{-3, -1, 0, 2, 4\}$. *(Lesson 4-4)*

1. $y = x + 5$ 2. $y = 3x + 4$ 3. $x + 2y = 8$

Graph each equation. *(Lesson 4-5)*

4. $y = x - 2$ 5. $3x + 2y = 6$

Determine whether each relation is a function. *(Lesson 4-6)*

6. $\{(3, 4), (5, 3), (-1, 4), (6, 2)\}$ 7. $\{(-1, 4), (-2, 5), (7, 2), (3, 9), (-2, 1)\}$

If $f(x) = 3x + 5$, find each value. *(Lesson 4-6)*

8. $f(-4)$ 9. $f(2a)$ 10. $f(x + 2)$

Spreadsheet Investigation

A Preview of Lesson 4-7

Number Sequences

You can use a spreadsheet to generate number sequences and patterns. The simplest type of sequence is one in which the difference between successive terms is constant. This type of sequence is called an **arithmetic sequence**.

Example

Use a spreadsheet to generate a sequence of numbers from an initial value of 10 to 90 with a fixed interval of 8.

Step 1 Enter the initial value 10 in cell A1.

Step 2 Highlight the cells in column A. Under the Edit menu, choose the Fill option and then Series.

Step 3 A command box will appear on the screen asking for the Step value and the Stop value. The Step value is the fixed interval between each number, which in this case is 8. The Stop value is the last number in your sequence, 90. Enter these numbers and click OK. The column is filled with the numbers in the sequence from 10 to 90 at intervals of 8.

	A	B
1	10	
2	18	
3	26	
4	34	
5	42	
6	50	
7	58	
8	66	
9	74	
10	82	
11	90	
12		
13		
14		

Exercises

For Exercises 1–5, use a sequence of numbers from 7 to 63 with a fixed interval of 4.

1. Use a spreadsheet to generate the sequence. Write the numbers in the sequence.
2. How many numbers are in the sequence?

MAKE A CONJECTURE Let a_n represent each number in a sequence if n is the position of the number in the sequence. For example, a_1 = the first number in the sequence, a_2 = the second number, a_3 = the third number, and so on.

3. Write a formula for a_2 in terms of a_1. Write similar formulas for a_3 and a_4 in terms of a_1.
4. Look for a pattern. Write an equation that can be used to find the nth term of a sequence.
5. Use the equation from Exercise 4 to find the 21st term in the sequence.

4-7 Arithmetic Sequences

What You'll Learn

- Recognize arithmetic sequences.
- Extend and write formulas for arithmetic sequences.

Vocabulary

- sequence
- terms
- arithmetic sequence
- common difference

How are arithmetic sequences used to solve problems in science?

A probe to measure air quality is attached to a hot-air balloon. The probe has an altitude of 6.3 feet after the first second, 14.5 feet after the next second, 22.7 feet after the third second, and so on. You can make a table and look for a pattern in the data.

Time (s)	1	2	3	4	5	6	7	8
Altitude (ft)	6.3	14.5	22.7	30.9	39.1	47.3	55.5	63.7

+ 8.2 + 8.2 + 8.2 + 8.2 + 8.2 + 8.2 + 8.2

RECOGNIZE ARITHMETIC SEQUENCES A **sequence** is a set of numbers in a specific order. The numbers in the sequence are called **terms**. If the difference between successive terms is constant, then it is called an **arithmetic sequence**. The difference between the terms is called the **common difference**.

terms

7 12 17 22 27

+ 5 + 5 + 5 + 5

common difference

Key Concept — *Arithmetic Sequence*

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.

Example 1 *Identify Arithmetic Sequences*

Determine whether each sequence is arithmetic. Justify your answer.

a. 1, 2, 4, 8, …

1 2 4 8

+ 1 + 2 + 4

This is not an arithmetic sequence because the difference between terms is not constant.

b. $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \ldots$

$\frac{1}{2} \quad \frac{1}{4} \quad 0 \quad -\frac{1}{4}$

$-\frac{1}{4} \quad -\frac{1}{4} \quad -\frac{1}{4}$

This is an arithmetic sequence because the difference between terms is constant.

Study Tip

Reading Math
The three dots after the last number in a sequence are called an *ellipsis*. The ellipsis indicates that there are more terms in the sequence that are not listed.

WRITE ARITHMETIC SEQUENCES You can use the common difference of an arithmetic sequence to find the next term in the sequence.

Key Concept — *Writing Arithmetic Sequences*

- **Words** Each term of an arithmetic sequence after the first term can be found by adding the common difference to the preceding term.
- **Symbols** An arithmetic sequence can be found as follows

$$a_1, a_1 + d, a_2 + d, a_3 + d, \ldots,$$

where d is the common difference, a_1 is the first term, a_2 is the second term, and so on.

Example 2 *Extend a Sequence*

Find the next three terms of the arithmetic sequence 74, 67, 60, 53, …

Find the common difference by subtracting successive terms.

$$74 \xrightarrow{-7} 67 \xrightarrow{-7} 60 \xrightarrow{-7} 53 \xrightarrow{-7} ? \xrightarrow{-7} ? \xrightarrow{-7} ?$$

The common difference is -7.

Add -7 to the last term of the sequence to get the next term in the sequence. Continue adding -7 until the next three terms are found.

$$53 \xrightarrow{-7} 46 \xrightarrow{-7} 39 \xrightarrow{-7} 32$$

The next three terms are 46, 39, 32.

Each term in an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Term	Symbol	In Terms of a_1 and d	Numbers
first term	a_1	a_1	8
second term	a_2	$a_1 + d$	$8 + 1(3) = 11$
third term	a_3	$a_1 + 2d$	$8 + 2(3) = 14$
fourth term	a_4	$a_1 + 3d$	$8 + 3(3) = 17$
⋮	⋮	⋮	⋮
nth term	a_n	$a_1 + (n - 1)d$	$8 + (n - 1)(3)$

Study Tip

Reading Math
The formula for the nth term of an arithmetic sequence is called a *recursive formula.* This means that each succeeding term is formulated from one or more of the previous terms.

The following formula generalizes this pattern and can be used to find any term in an arithmetic sequence.

Key Concept — *nth Term of an Arithmetic Sequence*

The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by

$$a_n = a_1 + (n - 1)d,$$

where n is a positive integer.

Example 3 Find a Specific Term

Find the 14th term in the arithmetic sequence 9, 17, 25, 33, …

In this sequence, the first term, a_1, is 9. You want to find the 14th term, so $n = 14$. Find the common difference.

9 17 25 33

+ 8 + 8 + 8 The common difference is 8.

Use the formula for the nth term of an arithmetic sequence.

$a_n = a_1 + (n - 1)d$	Formula for the nth term
$a_{14} = 9 + (14 - 1)8$	$a_1 = 9$, $n = 14$, $d = 8$
$a_{14} = 9 + 104$	Simplify.
$a_{14} = 113$	The 14th term in the sequence is 113.

Example 4 Write an Equation for a Sequence

Consider the arithmetic sequence 12, 23, 34, 45, …

a. Write an equation for the nth term of the sequence.

In this sequence, the first term, a_1, is 12. Find the common difference.

12 23 34 45

+ 11 + 11 + 11 The common difference is 11.

Use the formula for the nth term to write an equation.

$a_n = a_1 + (n - 1)d$	Formula for nth term
$a_n = 12 + (n - 1)11$	$a_1 = 12$, $d = 11$
$a_n = 12 + 11n - 11$	Distributive Property
$a_n = 11n + 1$	Simplify.

CHECK For $n = 1$, $11(1) + 1 = 12$.
For $n = 2$, $11(2) + 1 = 23$.
For $n = 3$, $11(3) + 1 = 34$, and so on.

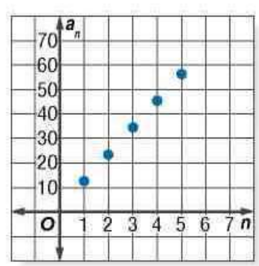
b. Find the 10th term in the sequence.

Replace n with 10 in the equation written in part **a**.

$a_n = 11n + 1$	Equation for the nth term
$a_{10} = 11(10) + 1$	Replace n with 10.
$a_{10} = 111$	Simplify.

c. Graph the first five terms of the sequence.

n	$11n + 1$	a_n	(n, a_n)
1	$11(1) + 1$	12	(1, 12)
2	$11(2) + 1$	23	(2, 23)
3	$11(3) + 1$	34	(3, 34)
4	$11(4) + 1$	45	(4, 45)
5	$11(5) + 1$	56	(5, 56)



Notice that the points fall on a line. The graph of an arithmetic sequence is linear.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an arithmetic sequence whose common difference is -10.
2. **Find** the common difference and the first term in the sequence defined by $a_n = 5n + 2$.
3. **FIND THE ERROR** Marisela and Richard are finding the common difference for the arithmetic sequence $-44, -32, -20, -8$.

Marisela	Richard
$-32 - (-44) = 12$	$-44 - (-32) = -12$
$-20 - (-32) = 12$	$-32 - (-20) = -12$
$-8 - (-20) = 12$	$-20 - (-8) = -12$

Who is correct? Explain your reasoning.

Guided Practice

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

4. $24, 16, 8, 0, \ldots$
5. $3, 6, 12, 24, \ldots$

Find the next three terms of each arithmetic sequence.

6. $7, 14, 21, 28, \ldots$
7. $34, 29, 24, 19, \ldots$

Find the *n*th term of each arithmetic sequence described.

8. $a_1 = 3, d = 4, n = 8$
9. $a_1 = 10, d = -5, n = 21$
10. $23, 25, 27, 29, \ldots$ for $n = 12$
11. $-27, -19, -11, -3, \ldots$ for $n = 17$

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

12. $6, 12, 18, 24, \ldots$
13. $12, 17, 22, 27, \ldots$

Application

14. **FITNESS** Latisha is beginning an exercise program that calls for 20 minutes of walking each day for the first week. Each week thereafter, she has to increase her walking by 7 minutes a day. Which week of her exercise program will be the first one in which she will walk over an hour a day?

Practice and Apply

Homework Help

For Exercises	See Examples
15–20, 43, 44	1
21–26	2
27–38, 45–49, 54, 55	3
39–42, 50–53	4

Extra Practice

See page 830.

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

15. $7, 6, 5, 4, \ldots$
16. $10, 12, 15, 18, \ldots$
17. $9, 5, -1, -5, \ldots$
18. $-15, -11, -7, -3, \ldots$
19. $-0.3, 0.2, 0.7, 1.2, \ldots$
20. $2.1, 4.2, 8.4, 17.6, \ldots$

Find the next three terms of each arithmetic sequence.

21. $4, 7, 10, 13, \ldots$
22. $18, 24, 30, 36, \ldots$
23. $-66, -70, -74, -78, \ldots$
24. $-31, -22, -13, -4, \ldots$
25. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$
26. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \ldots$

Find the nth term of each arithmetic sequence described.

27. $a_1 = 5, d = 5, n = 25$

28. $a_1 = 8, d = 3, n = 16$

29. $a_1 = 52, d = 12, n = 102$

30. $a_1 = 34, d = 15, n = 200$

31. $a_1 = \frac{5}{8}, d = \frac{1}{8}, n = 22$

32. $a_1 = 1\frac{1}{2}, d = 2\frac{1}{4}, n = 39$

33. $-9, -7, -5, -3, \ldots$ for $n = 18$

34. $-7, -3, 1, 5, \ldots$ for $n = 35$

35. $0.5, 1, 1.5, 2, \ldots$ for $n = 50$

36. $5.3, 5.9, 6.5, 7.1, \ldots$ for $n = 12$

37. 200 is the __?__th term of 24, 35, 46, 57, …

38. -34 is the __?__th term of 30, 22, 14, 6, …

Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms in the sequence.

39. $-3, -6, -9, -12, \ldots$

40. $8, 9, 10, 11, \ldots$

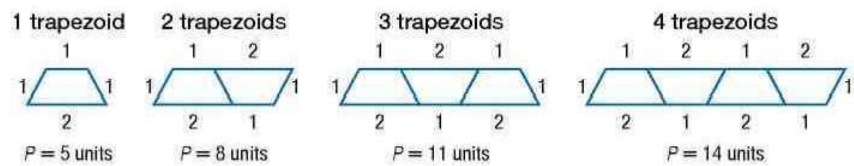
41. $2, 8, 14, 20, \ldots$

42. $-18, -16, -14, -12, \ldots$

43. Find the value of y that makes $y + 4, 6, y, \ldots$ an arithmetic sequence.

44. Find the value of y that makes $y + 8, 4y + 6, 3y, \ldots$ an arithmetic sequence.

GEOMETRY **For Exercises 45 and 46, use the diagram below that shows the perimeter of the pattern consisting of trapezoids.**



45. Write a formula that can be used to find the perimeter of a pattern containing n trapezoids.

46. What is the perimeter of the pattern containing 12 trapezoids?

More About. . .

Theater

The open-air theaters of ancient Greece held about 20,000 people. They became the models for amphitheaters, Roman coliseums, and modern sports arenas.

Source: www.encarta.msn.com

THEATER **For Exercises 47–49, use the following information.**
The Coral Gables Actors' Playhouse has 76 seats in the last row of the orchestra section of the theater, 68 seats in the next row, 60 seats in the next row, and so on. There are 7 rows of seats in the section. On opening night, 368 tickets were sold for the orchestra section.

47. Write a formula to find the number of seats in any given row of the orchestra section of the theater.

48. How many seats are in the first row?

49. Was this section oversold?

PHYSICAL SCIENCE **For Exercises 50–53, use the following information.**
Taylor and Brooklyn are recording how far a ball rolls down a ramp during each second. The table below shows the data they have collected.

Time (s)	1	2	3	4	5	6
Distance traveled (cm)	9	13	17	21	25	29

50. Do the distances traveled by the ball form an arithmetic sequence? Justify your answer.

51. Write an equation for the sequence.

52. How far will the ball travel during the 35th second?

53. Graph the sequence.

GAMES **For Exercises 54 and 55, use the following information.**
Contestants on a game show win money by answering 10 questions. The value of each question increases by \$1500.

54. If the first question is worth \$2500, find the value of the 10th question.

55. If the contestant answers all ten questions correctly, how much money will he or she win?

56. **CRITICAL THINKING** Is $2x + 5, 4x + 5, 6x + 5, 8x + 5 \ldots$ an arithmetic sequence? Explain your answer.

57. **CRITICAL THINKING** Use an arithmetic sequence to find how many multiples of 7 are between 29 and 344.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are arithmetic sequences used to solve problems in science?

Include the following in your answer:
- a formula for the arithmetic sequence that represents the altitude of the probe after each second, and
- an explanation of how you could use this information to predict the altitude of the probe after 15 seconds.

Standardized Test Practice

59. Luis puts \$25 a week into a savings account from his part-time job. If he has \$350 in savings now, how much will he have 12 weeks from now?

Ⓐ \$600　Ⓑ \$625　Ⓒ \$650　Ⓓ \$675

60. In an arithmetic sequence a_n, if $a_1 = 2$ and $a_4 = 11$, find a_{20}.

Ⓐ 40　Ⓑ 59　Ⓒ 78　Ⓓ 97

Maintain Your Skills

Mixed Review

If $f(x) = 3x - 2$ and $g(x) = x^2 - 5$, find each value. *(Lesson 4-6)*

61. $f(4)$　**62.** $g(-3)$　**63.** $2[f(6)]$

Determine whether each equation is a linear equation. If so, write the equation in standard form. *(Lesson 4-5)*

64. $x^2 + 3x - y = 8$　**65.** $y - 8 = 10 - x$　**66.** $2y = y + 2x - 3$

Translate each sentence into an algebraic equation. *(Lesson 3-1)*

67. Two hundred minus three times x is equal to nine.

68. The sum of twice r and three times s is identical to thirteen.

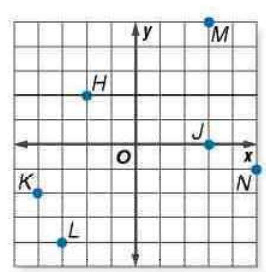
Find each product. *(Lesson 2-3)*

69. $7(-3)$　**70.** $-11 \cdot 15$　**71.** $-8(-1.5)$

72. $6\left(\frac{2}{3}\right)$　**73.** $\left(-\frac{5}{8}\right)\left(\frac{4}{7}\right)$　**74.** $5 \cdot 3\frac{1}{2}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write the ordered pair for each point shown at the right.**
*(To review **graphing points**, see Lesson 4-1.)*



75. H　**76.** J

77. K　**78.** L

79. M　**80.** N

Reading Mathematics

Reasoning Skills

Throughout your life, you have used reasoning skills, possibly without even knowing it. As a child, you used inductive reasoning to conclude that your hand would hurt if you touched the stove while it was hot. Now, you use inductive reasoning when you decide, after many trials, that one of the worst ways to prepare for an exam is by studying only an hour before you take it. **Inductive reasoning** is used to derive a general rule after observing many individual events.

Inductive reasoning involves . . .

- observing many examples
- looking for a pattern
- making a conjecture
- checking the conjecture
- discovering a likely conclusion

With **deductive reasoning**, you use a general rule to help you decide about a specific event. You come to a conclusion by accepting facts. There is no conjecturing involved. Read the two statements below.

1) If a person wants to play varsity sports, he or she must have a C average in academic classes.

2) Jolene is playing on the varsity tennis team.

If these two statements are accepted as facts, then the obvious conclusion is that Jolene has at least a C average in her academic classes. This is an example of deductive reasoning.

Reading to Learn

1. Explain the difference between *inductive* and *deductive* reasoning. Then give an example of each.

2. When Sherlock Holmes reaches a conclusion about a murderer's height because he knows the relationship between a man's height and the distance between his footprints, what kind of reasoning is he using? Explain.

3. When you examine a sequence of numbers and decide that it is an arithmetic sequence, what kind of reasoning are you using? Explain.

4. Once you have found the common difference for an arithmetic sequence, what kind of reasoning do you use to find the 100th term in the sequence?

5. **a.** Copy and complete the following table.

3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9
3	9	27						

b. Write the sequence of numbers representing the numbers in the ones place.

c. Find the number in the ones place for the value of 3^{100}. Explain your reasoning. State the type of reasoning that you used.

6. A sequence contains all numbers less than 50 that are divisible by 5. You conclude that 35 is in the sequence. Is this an example of inductive or deductive reasoning? Explain.

CONTENTS

4-8 Writing Equations from Patterns

What You'll Learn

- Look for a pattern.
- Write an equation given some of the solutions.

Vocabulary

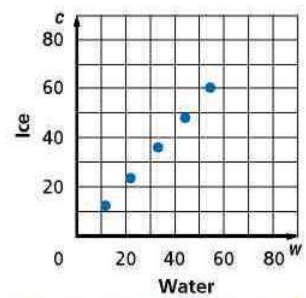
- look for a pattern
- inductive reasoning

Why is writing equations from patterns important in science?

Water is one of the few substances that expands when it freezes. The table shows different volumes of water and the corresponding volumes of ice.

Volume of Water (ft^3)	11	22	33	44	55
Volume of Ice (ft^3)	12	24	36	48	60

The relation in the table can be represented by a graph. Let w represent the volume of water, and let c represent the volume of ice. When the ordered pairs are graphed, they form a linear pattern. This pattern can be described by an equation.



Study Tip

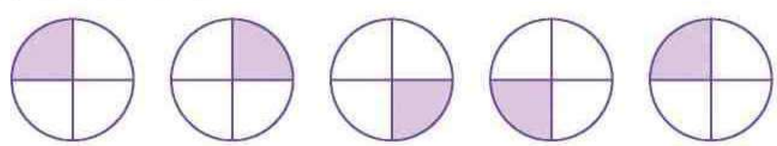
Look Back

To review **deductive reasoning**, see Lesson 1-7.

LOOK FOR PATTERNS A very useful problem-solving strategy is **look for a pattern**. When you make a conclusion based on a pattern of examples, you are using **inductive reasoning**. Recall that *deductive reasoning* uses facts, rules, or definitions to reach a conclusion.

Example 1 Extend a Pattern

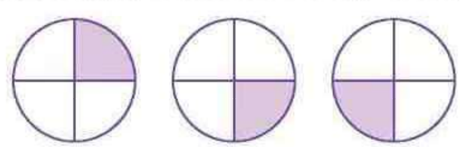
Study the pattern below.



1 2 3 4 5

a. Draw the next three figures in the pattern.

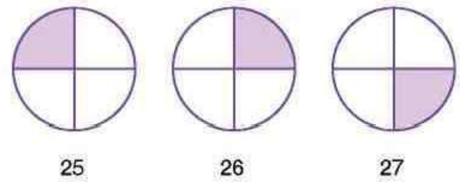
The pattern consists of circles with one-fourth shaded. The section that is shaded is rotated in a clockwise direction. The next three figures are shown.



6 7 8

b. Draw the 27th circle in the pattern.

The pattern repeats every fourth design. Therefore designs 4, 8, 12, 16, and so on, will all be the same. Since 24 is the greatest number less than 27 that is a multiple of 4, the 25th circle in the pattern will be the same as the first circle.



Other sequences besides arithmetic sequences can follow a pattern.

Example 2 Patterns in a Sequence

Find the next three terms in the sequence 3, 6, 12, 24, … .

Study the pattern in the sequence.

$$3 \xrightarrow{+3} 6 \xrightarrow{+6} 12 \xrightarrow{+12} 24$$

You can use inductive reasoning to find the next term in a sequence. Notice the pattern 3, 6, 12, … The difference between each term doubles in each successive term. To find the next three terms in the sequence, continue doubling each successive difference. Add 24, 48, and 96.

$$3 \xrightarrow{+3} 6 \xrightarrow{+6} 12 \xrightarrow{+12} 24 \xrightarrow{+24} 48 \xrightarrow{+48} 96 \xrightarrow{+96} 192$$

The next three terms are 48, 96, and 192.

Algebra Activity

Looking for Patterns

- You will need several pieces of string.
- Loop a piece of string around one of the cutting edges of the scissors and cut. How many pieces of string do you have as a result of this cut? Discard those pieces.
- Use another piece of string to make 2 loops around the scissors and cut. How many pieces of string result?
- Continue making loops and cutting until you see a pattern.

Analyze

1. Describe the pattern and write a sequence that describes the number of loops and the number of pieces of string.
2. Write an expression that you could use to find the number of pieces of string you would have if you made *n* loops.
3. How many pieces of string would you have if you made 20 loops?

WRITE EQUATIONS Sometimes a pattern can lead to a general rule. If the relationship between the domain and range of a relation is linear, the relationship can be described by a linear equation.

CONTENTS

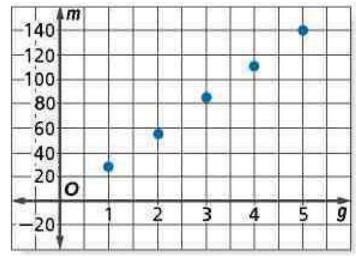
Example 3 Write an Equation from Data

FUEL ECONOMY **The table below shows the average amount of gas Rogelio's car uses depending on how many miles he drives.**

Gallons of gasoline	1	2	3	4	5
Miles driven	28	56	84	112	140

a. Graph the data. What conclusion can you make about the relationship between the number of gallons used and the number of miles driven?

The graph shows a linear relationship between the number of gallons used g and the number of miles driven m.



b. Write an equation to describe this relationship.

Look at the relationship between the domain and range to find a pattern that can be described by an equation.

+1 +1 +1 +1

Gallons of gasoline	1	2	3	4	5
Miles driven	28	56	84	112	140

+28 +28 +28 +28

Since this is a linear relationship, the ratio of the range values to the domain values is constant. The difference of the values for g is 1, and the difference of the values for m is 28. This suggests that $m = 28g$. Check to see if this equation is correct by substituting values of g into the equation.

CHECK If $g = 1$, then $m = 28(1)$ or 28. ✓
If $g = 2$, then $m = 28(5)$ or 56. ✓
If $g = 3$, then $m = 28(3)$ or 84. ✓

The equation checks. Since this relation is also a function, we can write the equation as $f(g) = 0.28g$, where $f(g)$ represents the number of miles driven.

More About. . .

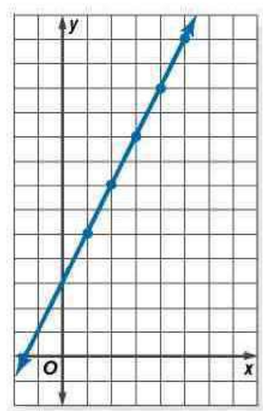
Fuel Economy

Not all cars use gasoline. Many alternative fuels are being used today in place of fossil fuels like oil. In 2001, there were 456,306 alternative fuel vehicles (AFVs) on the road in the U.S.

Source: U.S. Department of Energy

Example 4 Write an Equation with a Constant

Write an equation in function notation for the relation graphed at the right.



Make a table of ordered pairs for several points on the graph.

+1 +1 +1 +1

x	1	2	3	4	5
y	5	7	9	11	13

+2 +2 +2 +2

The difference of the x values is 1, and the difference of the y values is 2. The difference in y values is twice the difference of x values. This suggests that $y = 2x$. Check this equation.

CONTENTS

CHECK If $x = 1$, then $y = 2(1)$ or 2. But the y value for $x = 1$ is 5. This is a difference of 3. Try some other values in the domain to see if the same difference occurs.

x	1	2	3	4	5
$2x$	2	4	6	8	10
y	5	7	9	11	13

y is always 3 more than $2x$.

This pattern suggests that 3 should be added to one side of the equation in order to correctly describe the relation. Check $y = 2x + 3$.

If $x = 2$, then $y = 2(2) + 3$ or 7.
If $x = 3$, then $y = 2(3) + 3$, or 9.

Thus, $y = 2x + 3$ correctly describes this relation. Since this relation is also a function, we can write the equation in function notation as $f(x) = 2x + 3$.

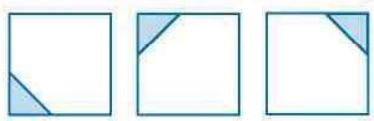
Check for Understanding

Concept Check

1. **Explain** how you can use inductive reasoning to write an equation from a pattern.
2. **OPEN ENDED** Write a sequence for which the first term is 4 and the second term is 8. Explain the pattern that you used.
3. **Explain** how you can determine whether an equation correctly represents a relation given in a table.

Guided Practice

4. Find the next two items for the pattern. Then find the 16th figure in the pattern.

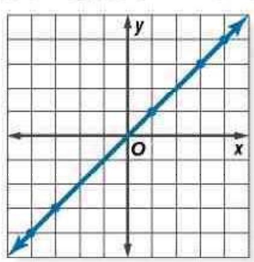


Find the next three terms in each sequence.

5. 1, 2, 4, 7, 11, …
6. 5, 9, 6, 10, 7, 11, …

Write an equation in function notation for each relation.

7.



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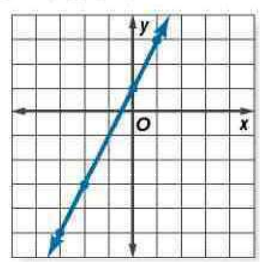


Application

GEOLOGY **For Exercises 9–11, use the table below that shows the underground temperature of rocks at various depths below Earth's surface.**

Depth (km)	1	2	3	4	5	6
Temperature (°C)	55	90	125	160	195	230

9. Graph the data.
10. Write an equation in function notation for the relation.
11. Find the temperature of a rock that is 10 kilometers below the surface.

Practice and Apply

Homework Help

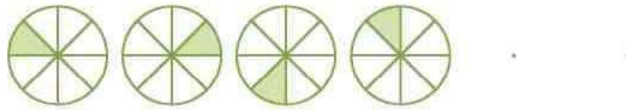
For Exercises	See Examples
12, 13, 26	1
14–19, 27, 28	2
20–25	4
29, 30	5

Extra Practice

See page 830.

Find the next two items for each pattern. Then find the 21st figure in the pattern.

12. 

13. 

Find the next three terms in each sequence.

14. 0, 2, 6, 12, 20, …

15. 9, 7, 10, 8, 11, 9, 12, …

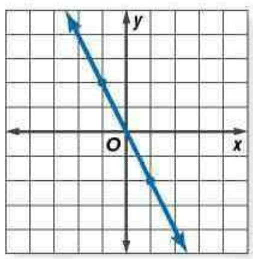
16. 1, 4, 9, 16, …

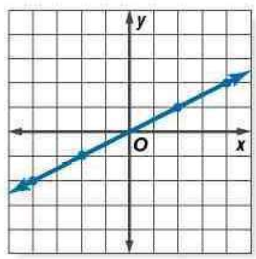
17. 0, 2, 5, 9, 14, 20, …

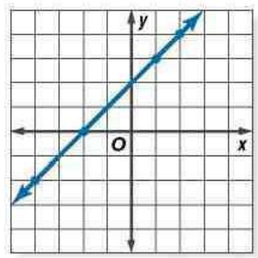
18. $a + 1, a + 2, a + 3, \ldots$

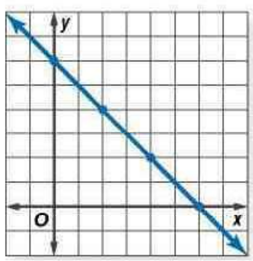
19. $x + 1, 2x + 1, 3x + 1, \ldots$

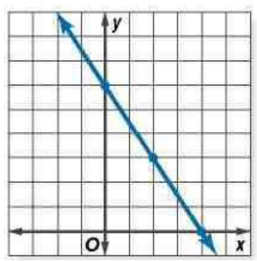
Write an equation in function notation for each relation.

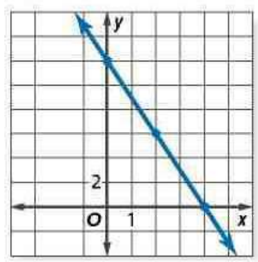
20. 

21. 

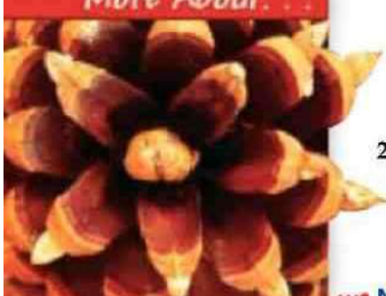
22. 

23. 

24. 

25. 

More About. . .



Number Theory

Fibonacci numbers occur in many areas of nature, including pine cones, shell spirals, flower petals, branching plants, and many fruits and vegetables.

26. **TRAVEL** On an island cruise in Hawaii, each passenger is given a flower chain. A crew member hands out 3 red, 3 blue, and 3 green chains in that order. If this pattern is repeated, what color chain will the 50th person receive?

NUMBER THEORY **For Exercises 27 and 28, use the following information.**
In 1201, Leonardo Fibonacci introduced his now famous pattern of numbers called the Fibonacci sequence.

$$1, 1, 2, 3, 5, 8, 13, \ldots$$

Notice the pattern in this sequence. After the second number, each number in the sequence is the sum of the two numbers that precede it. That is $2 = 1 + 1$, $3 = 2 + 1$, $5 = 3 + 2$, and so on.

27. Write the first 12 terms of the Fibonacci sequence.

28. Notice that every third term is divisible by 2. What do you notice about every fourth term? every fifth term?

FITNESS **For Exercises 29 and 30, use the table below that shows the maximum heart rate to maintain, for different ages, during aerobic activities such as running, biking, or swimming.**

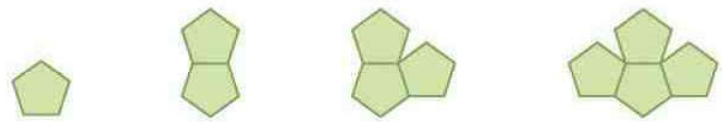
Age (yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

Source: Ontario Association of Sport and Exercise Sciences

29. Write an equation in function notation for the relation.

30. What would be the maximum heart rate to maintain in aerobic training for a 10-year old? an 80-year old?

CRITICAL THINKING **For Exercises 31–33, use the following information.**
Suppose you arrange a number of regular pentagons so that only one side of each pentagon touches. Each side of each pentagon is 1 centimeter.



1 pentagon **2 pentagons** **3 pentagons** **4 pentagons**

31. For each arrangement of pentagons, compute the perimeter.

32. Write an equation in function form to represent the perimeter $f(n)$ of n pentagons.

33. What is the perimeter if 24 pentagons are used?

34. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

Why is writing equations from patterns important in science?

Include the following in your answer:
- an explanation of the relationship between the volume of water and the volume of ice, and
- a reasonable estimate of the size of a container that had 99 cubic feet of water, if it was going to be frozen.

Standardized Test Practice

35. Find the next two terms in the sequence 3, 4, 6, 9, … .

Ⓐ 12, 15 Ⓑ 13, 18 Ⓒ 14, 19 Ⓓ 15, 21

36. After P pieces of candy are divided equally among 5 children, 4 pieces remain. How many would remain if $P + 4$ pieces of candy were divided equally among the 5 children?

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 3

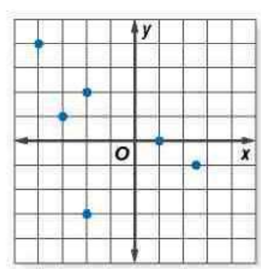
Maintain Your Skills

Mixed Review **Find the next three terms of each arithmetic sequence.** *(Lesson 4-7)*

37. 1, 4, 7, 10, … **38.** 9, 5, 1, −3, …

39. −25, −19, −13, −7, … **40.** 22, 34, 46, 58, …

41. Determine whether the relation graphed at the right is a function. *(Lesson 4-6)*



42. **GEOGRAPHY** The world's tallest waterfall is Angel Falls in Venezuela at 3212 feet. It is 102 feet higher than Tulega Falls in South Africa. How high is Tulega Falls? *(Lesson 3-2)*

Chapter 4

Study Guide and Review

Vocabulary and Concept Check

arithmetic sequence (p. 233)
axes (p. 192)
common difference (p. 233)
coordinate plane (p. 192)
dilation (p. 197)
equation in two variables (p. 212)
function (p. 226)
function notation (p. 227)
graph (p. 193)
image (p. 197)
inductive reasoning (p. 240)
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look for a pattern (p. 240)
mapping (p. 205)
origin (p. 192)
preimage (p. 197)
quadrant (p. 193)
reflection (p. 197)
rotation (p. 197)
sequence (p. 233)
solution (p. 212)
standard form (p. 218)
terms (p. 233)
transformation (p. 197)
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vertical line test (p. 227)
x-axis (p. 192)
x-coordinate (p. 192)
x-intercept (p. 220)
y-axis (p. 192)
y-coordinate (p. 192)
y-intercept (p. 220)

Choose the letter of the term that best matches each statement or phrase.

1. In the coordinate plane, the axes intersect at the __?__.
2. A(n) __?__ is a set of ordered pairs.
3. A(n) __?__ flips a figure over a line.
4. In a coordinate system, the __?__ is a horizontal number line.
5. In the ordered pair, $A(2, 7)$, 7 is the __?__.
6. The coordinate axes separate a plane into four __?__.
7. A(n) __?__ has a graph that is a nonvertical straight line.
8. In the relation $\{(4, -2), (0, 5), (6, 2), (-1, 8)\}$, the __?__ is $\{-1, 0, 4, 6\}$.
9. A(n) __?__ enlarges or reduces a figure.
10. In a coordinate system, the __?__ is a vertical number line.

a. domain
b. dilation
c. linear function
d. reflection
e. origin
f. quadrants
g. relation
h. *x*-axis
i. *y*-axis
j. *x*-coordinate
k. *y*-coordinate

Lesson-by-Lesson Review

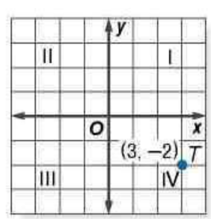
4-1 The Coordinate Plane

See pages 192–196.

Concept Summary

- The first number, or *x*-coordinate, corresponds to the numbers on the *x*-axis.
- The second number, or *y*-coordinate, corresponds to the numbers on the *y*-axis.

Example **Plot $T(3, -2)$ on a coordinate plane. Name the quadrant in which the point is located.**



$T(3, -2)$ is located in Quadrant IV.

Exercises **Plot each point on a coordinate plane.** *See Example 3 on page 193.*

11. $A(4, 2)$	**12.** $B(-1, 3)$	**13.** $C(0, -5)$
14. $D(-3, -2)$	**15.** $E(-4, 0)$	**16.** $F(2, -1)$

www.algebra1.com/vocabulary_review
CONTENTS

4-2 Transformations on the Coordinate Plane

See pages 197–203.

Concept Summary

- A reflection is a flip.
- A translation is a slide.
- A dilation is a reduction or enlargement.
- A rotation is a turn.

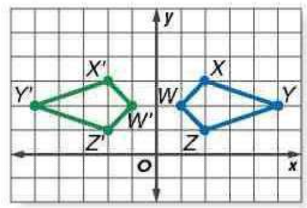
Example **A quadrilateral with vertices $W(1, 2)$, $X(2, 3)$, $Y(5, 2)$, and $Z(2, 1)$ is reflected over the y-axis. Find the coordinates of the vertices of the image. Then graph quadrilateral $WXYZ$ and its image $W'X'Y'Z'$.**

Multiply each x-coordinate by –1.

$W(1, 2) \rightarrow W'(-1, 2)$ $\quad Y(5, 2) \rightarrow Y'(-5, 2)$

$X(2, 3) \rightarrow X'(-2, 3)$ $\quad Z(2, 1) \rightarrow Z'(-2, 1)$

The coordinates of the image are $W'(-1, 2)$, $X'(-2, 3)$, $Y'(-5, 2)$, and $Z'(-2, 1)$.



Exercises **Find the coordinates of the vertices of each figure after the given transformation is performed. Then graph the preimage and its image.**

See Examples 2–5 on pages 198–200.

17. triangle ABC with $A(3, 3)$, $B(5, 4)$, and $C(4, -3)$ reflected over the x-axis

18. quadrilateral $PQRS$ with $P(-2, 4)$, $Q(0, 6)$, $R(3, 3)$, and $S(-1, -4)$ translated 3 units down

19. parallelogram $GHIJ$ with $G(2, 2)$, $H(6, 0)$, $I(6, 2)$, and $J(2, 4)$ dilated by a scale factor of $\frac{1}{2}$

20. trapezoid $MNOP$ with $M(2, 0)$, $N(4, 3)$, $O(6, 3)$, and $P(8, 0)$ rotated 90° counterclockwise about the origin

4-3 Relations

See pages 205–211.

Concept Summary

- A relation can be expressed as a set of ordered pairs, a table, a graph, or a mapping.

Example **Express the relation $\{(3, 2), (5, 3), (4, 3), (5, 2)\}$ as a table, a graph, and a mapping.**

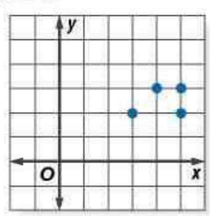
Table

List the set of x-coordinates and corresponding y-coordinates.

x	y
3	2
5	3
4	3
5	2

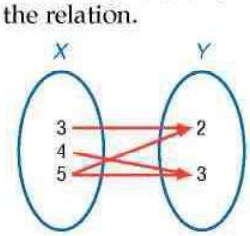
Graph

Graph each ordered pair on a coordinate plane.



Mapping

List the x and y values. Draw arrows to show the relation.



Exercises **Express each relation as a table, a graph, and a mapping. Then determine the domain and range.** *See Example 1 on page 205.*

21. $\{(-2, 6), (3, -2), (3, 0), (4, 6)\}$ **22.** $\{(-1, 0), (3, 0), (6, 2)\}$

23. $\{(3, 8), (9, 3), (-3, 8), (5, 3)\}$ **24.** $\{(2, 5), (-3, 1), (4, -2), (2, 3)\}$

4-4 Equations as Relations

See pages 212–217.

Concept Summary

- In an equation involving x and y, the set of x values is the domain, and the corresponding set of y values is the range.

Example **Solve $2x + y = 8$ if the domain is $\{3, 2, 1\}$. Graph the solution set.**

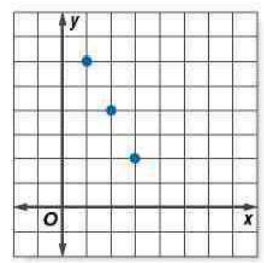
First solve the equation for y in terms of x.

$2x + y = 8$ Original equation

$y = 8 - 2x$ Subtract $2x$ from each side.

Substitute each value of x from the domain to determine the corresponding values of y in the range. Then graph the solution set $\{(3, 2), (2, 4), (1, 6)\}$.

x	$8 - 2x$	y	(x, y)
3	$8 - 2(3)$	2	(3, 2)
2	$8 - 2(2)$	4	(2, 4)
1	$8 - 2(1)$	6	(1, 6)



Exercises **Solve each equation if the domain is $\{-4, -2, 0, 2, 4\}$. Graph the solution set.** *See Example 3 on page 213.*

25. $y = x - 9$ **26.** $y = 4 - 2x$ **27.** $4x - y = -5$

28. $2x + y = 8$ **29.** $3x + 2y = 9$ **30.** $4x - 3y = 0$

4-5 Graphing Linear Equations

See pages 218–223.

Concept Summary

- Standard form: $Ax + By = C$, where $A \geq 0$ and A and B are not both zero
- To find the x-intercept, let $y = 0$. To find the y-intercept, let $x = 0$.

Example **Determine the x- and y-intercepts of $3x - y = 4$. Then graph the equation.**

To find the x-intercept, let $y = 0$.

$3x - y = 4$ Original equation

$3x - 0 = 4$ Replace y with 0.

$3x = 4$ Simplify.

$x = \frac{4}{3}$ Divide each side by 3.

To find the y-intercept, let $x = 0$.

$3x - y = 4$ Original equation

$3(0) - y = 4$ Replace x with 0.

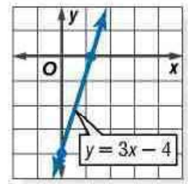
$-y = 4$ Simplify.

$y = -4$ Divide each side by -1.

The x-intercept is $\frac{4}{3}$, so the graph intersects the x-axis at $\left(\frac{4}{3}, 0\right)$.

The y-intercept is -4, so the graph intersects the y-axis at $(0, -4)$.

Plot these points, then draw a line that connects them.



Exercises **Graph each equation.** *See Examples 2 and 4 on pages 219 and 220.*

31. $y = -x + 2$ **32.** $x + 5y = 4$ **33.** $2x - 3y = 6$

34. $5x + 2y = 10$ **35.** $\frac{1}{2}x + \frac{1}{3}y = 3$ **36.** $y - \frac{1}{3} = \frac{1}{3}x + \frac{2}{3}$

4-6 Functions

See pages 226–231.

Concept Summary

- A relation is a function if each element of the domain is paired with exactly one element of the range.
- Substitute values for x to determine $f(x)$ for a specific value.

Examples

1 **Determine whether the relation $\{(0, -4), (1, -1), (2, 2), (6, 3)\}$ is a function.**

Since each element of the domain is paired with exactly one element of the range, the relation is a function.

2 **If $g(x) = 2x - 1$, find $g(-6)$.**

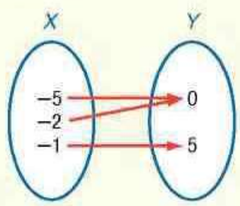
$g(-6) = 2(-6) - 1$ Replace x with -6.

$= -12 - 1$ Multiply.

$= -13$ Subtract.

Exercises **Determine whether each relation is a function.** *See Example 1 on page 226.*

37.



38.

x	y
5	3
1	4
−6	5
1	6
−2	7

39. $\{(2, 3), (-3, -4), (-1, 3)\}$

If $g(x) = x^2 - x + 1$, find each value. *See Examples 3 and 4 on pages 227 and 228.*

40. $g(2)$ **41.** $g(-1)$ **42.** $g\left(\frac{1}{2}\right)$

43. $g(5) - 3$ **44.** $g(a + 1)$ **45.** $g(-2a)$

4-7 Arithmetic Sequences

See pages 233–238.

Concept Summary

- An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.
- To find the next term in an arithmetic sequence, add the common difference to the last term.

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Chapter 4 **For More ...**

- Extra Practice, see pages 828–830.
- Mixed Problem Solving, see page 856.

Example **Find the next three terms of the arithmetic sequence 10, 23, 36, 49, … .**

Find the common difference.

10 23 36 49

+ 13 + 13 + 13

So, $d = 13$.

Add 13 to the last term of the sequence to get the next term. Continue adding 13 until the next three terms are found.

49 62 75 88

+ 13 + 13 + 13

The next three terms are 62, 75, and 88.

Exercises **Find the next three terms of each arithmetic sequence.**
See Example 2 on page 234.

46. 9, 18, 27, 36, …
47. 6, 11, 16, 21, …
48. 10, 21, 32, 43, …
49. 14, 12, 10, 8, …
50. −3, −11, −19, −27, …
51. −35, −29, −23, −17, …

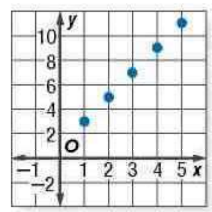
4-8 Writing Equations from Patterns

See pages 240–245.

Concept Summary

- Look for a pattern in data. If the relationship between the domain and range is linear, the relationship can be described by an equation.

Example **Write an equation in function notation for the relation graphed at the right.**



Make a table of ordered pairs for several points on the graph.

x	1	2	3	4	5
y	3	5	7	9	11

The difference in y values is twice the difference of x values. This suggests that $y = 2x$. However, $3 \neq 2(1)$. Compare the values of y to the values of $2x$.

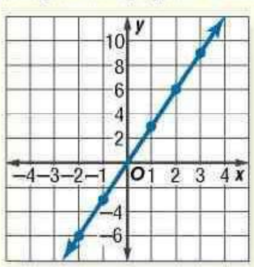
x	1	2	3	4	5
2x	2	4	6	8	10
y	3	5	7	9	11

y is always 3 more than 2*x*.

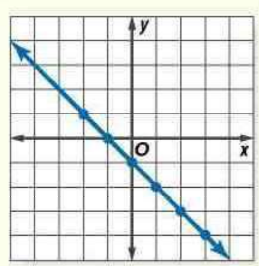
The difference between y and $2x$ is always 1. So the equation is $y = 2x + 1$. Since this relation is also a function, it can be written as $f(x) = 2x + 1$.

Exercises **Write an equation in function notation for each relation.**
See Example 4 on pages 242 and 243.

52.



53.



Chapter 4 Practice Test

Vocabulary and Concepts

Choose the letter that best matches each description.

1. a figure turned around a point
2. a figure slid horizontally, vertically, or both
3. a figure flipped over a line

a. reflection
b. rotation
c. translation

Skills and Applications

4. Graph $K(0, -5)$, $M(3, -5)$, and $N(-2, -3)$.
5. Name the quadrant in which $P(25, 1)$ is located.

For Exercises 6 and 7, use the following information.
A parallelogram has vertices $H(-2, -2)$, $I(-4, -6)$, $J(-5, -5)$, and $K(-3, -1)$.

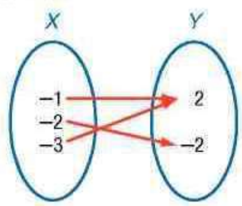
6. Reflect parallelogram *HIJK* over the *y*-axis and graph its image.
7. Translate parallelogram *HIJK* up 2 units and graph its image.

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

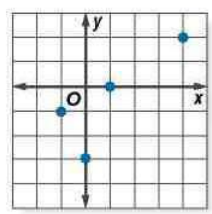
8.

x	$f(x)$
0	−1
2	4
4	5
6	10

9.



10.



Solve each equation if the domain is {−2, −1, 0, 2, 4}. Graph the solution set.

11. $y = -4x + 10$ **12.** $3x - y = 10$ **13.** $\frac{1}{2}x - y = 5$

Graph each equation.

14. $y = x + 2$ **15.** $x + 2y = -1$ **16.** $-3x = 5 - y$

Determine whether each relation is a function.

17. $\{(2, 4), (3, 2), (4, 6), (5, 4)\}$ **18.** $\{(3, 1), (2, 5), (4, 0), (3, -2)\}$ **19.** $8y = 7 + 3x$

If $f(x) = -2x + 5$ and $g(x) = x^2 - 4x + 1$, find each value.

20. $g(-2)$ **21.** $f\left(\frac{1}{2}\right)$ **22.** $g(3a) + 1$ **23.** $f(x + 2)$

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

24. 16, 24, 32, 40, … **25.** 99, 87, 76, 65, … **26.** 5, 17, 29, 41, …

Find the next three terms in each sequence.

27. 5, −10, 15, −20, 25, … **28.** 5, 5, 6, 8, 11, 15, …

29. **TEMPERATURE** The equation to convert Celsius temperature to Kelvin temperature is $K = C + 273$. Solve the equation for C. State the independent and dependent variables. Choose five values for K and their corresponding values for C.

30. **STANDARDIZED TEST PRACTICE** If $f(x) = 3x - 2$, find $f(8) - f(-5)$.

Ⓐ 7 Ⓑ 9 Ⓒ 37 Ⓓ 39

Chapter 4 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The number of students in Highview School is currently 315. The school population is predicted to increase by 2% next year. According to the prediction, how many students will attend next year? (Prerequisite Skill)

Ⓐ 317 Ⓑ 321

Ⓒ 378 Ⓓ 630

2. In 2001, two women skied 1675 miles in 89 days across the land mass of Antarctica. They still had to ski 508 miles across the Ross Ice Shelf to reach McMurdo Station. About what percent of their total distance remained? (Prerequisite Skill)

Ⓐ 2% Ⓑ 17%

Ⓒ 23% Ⓓ 30%

3. Only 2 out of 5 students surveyed said they eat five servings of fruits or vegetables daily. If there are 470 students in a school, how many would you predict eat five servings of fruits or vegetables daily? (Lesson 2-6)

Ⓐ 94 Ⓑ 188

Ⓒ 235 Ⓓ 282

4. Solve $13x = 2(5x + 3)$ for x. (Lesson 3-4)

Ⓐ 0 Ⓑ 2 Ⓒ 3 Ⓓ 4

The Princeton Review Test-Taking Tip

Questions 4 and 14 Some multiple-choice questions ask you to solve an equation or inequality. You can check your solution by replacing the variable in the equation or inequality with your answer. The answer choice that results in a true statement is the correct answer.

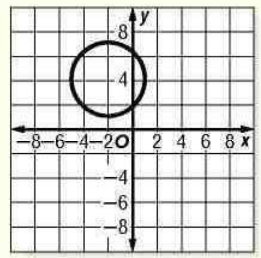
5. The circle shown below passes through points at $(1, 4)$, $(-2, 1)$, $(-5, 4)$, and $(-2, 7)$. Which point represents the center of the circle? (Lesson 4-1)

Ⓐ $(-2, -4)$

Ⓑ $(-2, 4)$

Ⓒ $(-4, 2)$

Ⓓ $(4, -2)$



6. Which value of x would cause the relation $\{(2, 5), (x, 8), (7, 10)\}$ *not* to be a function? (Lesson 4-4)

Ⓐ 1 Ⓑ 2 Ⓒ 5 Ⓓ 8

7. Which ordered pair (x, y) is a solution of $3x + 4y = 12$? (Lesson 4-4)

Ⓐ $(-2, 4)$ Ⓑ $(0, -3)$

Ⓒ $(1, 2)$ Ⓓ $(4, 0)$

8. Which missing value for y would make this relation a linear relation? (Lesson 4-7)

Ⓐ -2

Ⓑ 0

Ⓒ 1

Ⓓ 2

x	y
1	−3
2	−1
3	?
4	3

9. Which equation describes the data in the table? (Lesson 4-8)

Ⓐ $y = -2x + 1$

Ⓑ $y = x + 1$

Ⓒ $y = -x + 3$

Ⓓ $y = x - 5$

x	y
−2	5
1	2
4	−1
6	−3

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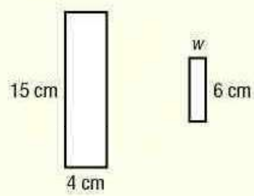
Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. The lengths of the corresponding sides of these two rectangles are proportional. What is the width w? (Lesson 2-6)



11. The PTA at Fletcher's school sold raffle tickets for a television set. Two thousand raffle tickets were sold. Fletcher's family bought 25 raffle tickets. What is the probability that his family will win the television? Express the answer as a percent. (Lesson 2-7)

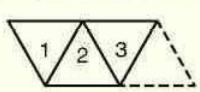
12. The sum of three integers is 52. The second integer is 3 more than the first. The third integer is 1 more than twice the first. What are the integers? (Lessons 3-1 and 3-4)

13. Solve $5(x - 2) - 3(x + 4) = 10$ for x. (Lesson 3-4)

14. A CD player originally cost \$160. It is now on sale for \$120. What is the percent of decrease in its price? (Lesson 3-5)

15. A swimming pool holds 1800 cubic feet of water. It is 6 feet deep and 20 feet long. How many feet wide is the pool? ($V = \ell wh$) (Lesson 3-8)

16. Garth used toothpicks to form a pattern of triangles as shown below. If he continues this pattern, what is the total number of toothpicks that he will use to form a pattern of 7 triangles? (Lessons 4-7 and 4-8)



Part 3 Quantitative Comparison

Compare the quantity in Column A and in Column B. Then determine whether:

Ⓐ the quantity in Column A is greater,

Ⓑ the quantity in Column B is greater,

Ⓒ the two quantities are equal, or

Ⓓ the relationship cannot be determined from the information given.

	Column A	Column B
17.	$4^2 \div 16(2 + 5) \cdot 3$	$\frac{60 - 2^3 \cdot 3 + 6}{4^3 - 62}$
		(Lesson 1-2)
18.	$\left(\frac{2}{3}\right)\left(\frac{15}{8}\right)\left(\frac{1}{9}\right)$	$7\left(\frac{3}{4}\right)\left(\frac{1}{14}\right)$
		(Lesson 2-3)
19.	x if $6x - 15 = -3x + 75$	y if $3y - 32 = 7y - 74$
		(Lesson 3-5)
20.	$f(-10)$ if $f(x) = 37 + 10x$	$g(-15)$ if $g(x) = 9x - 7$
		(Lesson 4-6)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

21. Latoya bought 48 one-foot-long sections of fencing. She plans to use the fencing to enclose a rectangular area for a garden. (Lesson 3-8)

a. Using ℓ for the length and w for the width of the garden, write an equation for its perimeter.

b. If the length ℓ in feet and width w in feet are whole numbers, what is the greatest possible area of this garden?

Chapter 5

Analyzing Linear Equations

What You'll Learn

- **Lesson 5-1** Find the slope of a line.
- **Lesson 5-2** Write direct variation equations.
- **Lessons 5-3 through 5-5** Write linear equations in slope-intercept and point-slope forms.
- **Lesson 5-6** Write equations for parallel and perpendicular lines.
- **Lesson 5-7** Draw a scatter plot and write the equations of a line of fit.

Key Vocabulary

- slope (p. 256)
- rate of change (p. 258)
- direct variation (p. 264)
- slope-intercept form (p. 272)
- point-slope form (p. 286)

Why It's Important

Linear equations are used to model a variety of real-world situations. The concept of slope allows you to analyze how a quantity changes over time.

You can use a linear equation to model the cost of the space program. The United States began its exploration of space in January, 1958, when it launched its first satellite into orbit. In the 1970s, NASA developed the space shuttle to reduce costs by inventing the first reusable spacecraft. *You will use a linear equation to model the cost of the space program in Lesson 5-7.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lesson 5-1 — Simplify Fractions

Simplify. *(For review, see pages 798 and 799.)*

1. $\frac{2}{10}$ **2.** $\frac{8}{12}$ **3.** $\frac{2}{-8}$ **4.** $\frac{-4}{8}$

5. $\frac{-5}{-15}$ **6.** $\frac{-7}{-28}$ **7.** $\frac{9}{3}$ **8.** $\frac{18}{12}$

For Lesson 5-2 — Evaluate Expressions

Evaluate $\frac{a-b}{c-d}$ for each set of values. *(For review, see Lesson 1-2.)*

9. $a = 6, b = 5, c = 8, d = 4$

10. $a = 5, b = -1, c = 2, d = -1$

11. $a = -2, b = 1, c = 4, d = 0$

12. $a = 8, b = -2, c = -1, d = 1$

13. $a = -3, b = -3, c = 4, d = 7$

14. $a = \frac{1}{2}, b = \frac{3}{2}, c = 7, d = 9$

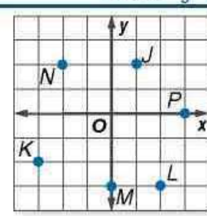
For Lessons 5-3 through 5-7 — Identify Points on a Coordinate Plane

Write the ordered pair for each point.
(For review, see Lesson 4-1.)

15. J **16.** K

17. L **18.** M

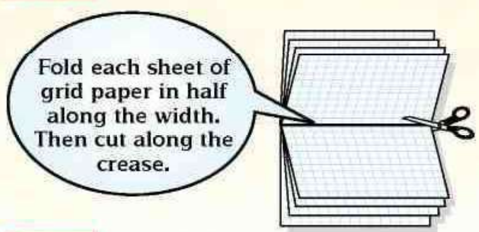
19. N **20.** P



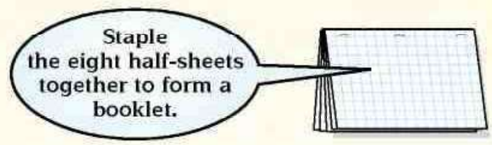
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about writing linear equations. Begin with four sheets of grid paper.

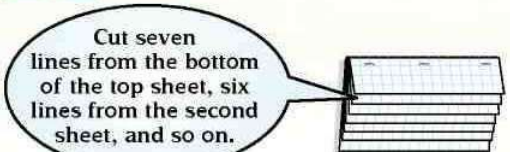
Step 1 Fold and Cut



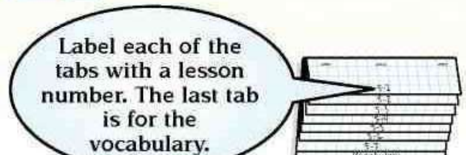
Step 2 Staple



Step 3 Cut Tabs



Step 4 Label



Reading and Writing As you read and study the chapter, use each page to write notes and to graph examples for each lesson.

5-1 Slope

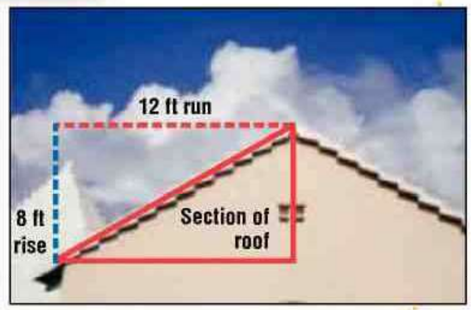
What You'll Learn

- Find the slope of a line.
- Use rate of change to solve problems.

Why is slope important in architecture?

The slope of a roof describes how steep it is. It is the number of units the roof rises for each unit of run. In the photo, the roof rises 8 feet for each 12 feet of run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{8}{12} \text{ or } \frac{2}{3}$$



Vocabulary

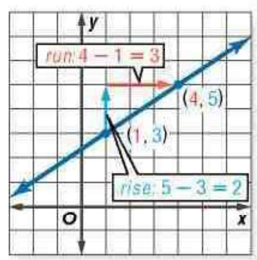
- slope
- rate of change

FIND SLOPE The **slope** of a line is a number determined by any two points on the line. This number describes how steep the line is. The greater the absolute value of the slope, the steeper the line. Slope is the ratio of the change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move from one point to the other.

The graph shows a line that passes through (1, 3) and (4, 5).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}} = \frac{5-3}{4-1} \text{ or } \frac{2}{3}$$

So, the slope of the line is $\frac{2}{3}$.

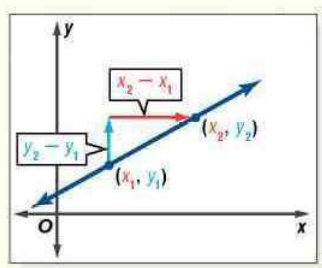


Key Concept — Slope of a Line

- **Words** The slope of a line is the ratio of the rise to the run.
- **Symbols** The slope m of a nonvertical line through any two points, (x_1, y_1) and (x_2, y_2), can be found as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \leftarrow \text{change in } y$$

- **Model**



Study Tip

Reading Math

In x_1, the 1 is called a *subscript*. It is read *x sub 1*.

Study Tip

Common Misconception
It may make your calculations easier to choose the point on the left as (x_1, y_1). However, *either point* may be chosen as (x_1, y_1).

Example 1 Positive Slope

Find the slope of the line that passes through (−1, 2) and (3, 4).

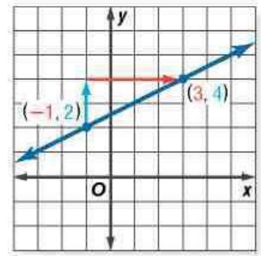
Let $(-1, 2) = (x_1, y_1)$ and $(3, 4) = (x_2, y_2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{\text{rise}}{\text{run}}$$

$$= \frac{4 - 2}{3 - (-1)} \quad \text{Substitute.}$$

$$= \frac{2}{4} \text{ or } \frac{1}{2} \quad \text{Simplify.}$$

The slope is $\frac{1}{2}$.



Example 2 Negative Slope

Find the slope of the line that passes through (−1, −2) and (−4, 1).

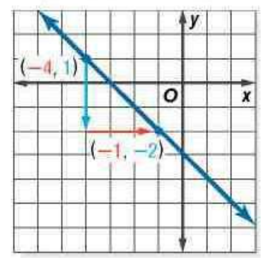
Let $(-1, -2) = (x_1, y_1)$ and $(-4, 1) = (x_2, y_2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{\text{rise}}{\text{run}}$$

$$= \frac{1 - (-2)}{-4 - (-1)} \quad \text{Substitute.}$$

$$= \frac{3}{-3} \text{ or } -1 \quad \text{Simplify.}$$

The slope is −1.



Example 3 Zero Slope

Find the slope of the line that passes through (1, 2) and (−1, 2).

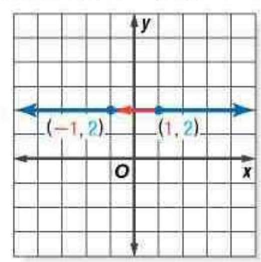
Let $(1, 2) = (x_1, y_1)$ and $(-1, 2) = (x_2, y_2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{\text{rise}}{\text{run}}$$

$$= \frac{2 - 2}{-1 - 1} \quad \text{Substitute.}$$

$$= \frac{0}{-2} \text{ or } 0 \quad \text{Simplify.}$$

The slope is zero.



Example 4 Undefined Slope

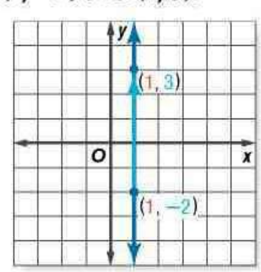
Find the slope of the line that passes through (1, −2) and (1, 3).

Let $(1, -2) = (x_1, y_1)$ and $(1, 3) = (x_2, y_2)$.

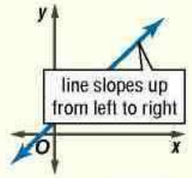
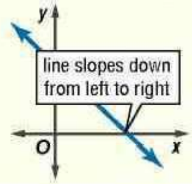
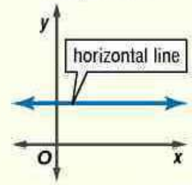
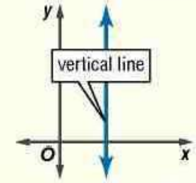
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{\text{rise}}{\text{run}}$$

$$= \frac{3 - (-2)}{1 - 1} \text{ or } \cancel{\frac{5}{0}}$$

Since division by zero is undefined, the slope is undefined.



Concept Summary — Classifying Lines

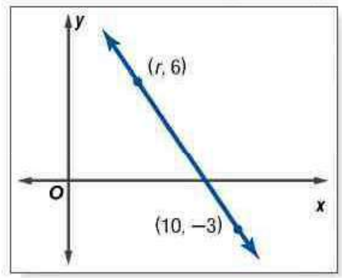
Positive Slope	Negative Slope	Slope of 0	Undefined Slope
			

If you know the slope of a line and the coordinates of one of the points on a line, you can find the coordinates of other points on the line.

Example 5 Find Coordinates Given Slope

Find the value of r so that the line through $(r, 6)$ and $(10, -3)$ has a slope of $-\frac{3}{2}$.

Let $(r, 6) = (x_1, y_1)$ and $(10, -3) = (x_2, y_2)$.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$-\frac{3}{2} = \frac{-3 - 6}{10 - r}$ Substitute.

$-\frac{3}{2} = \frac{-9}{10 - r}$ Subtract.

$-3(10 - r) = 2(-9)$ Find the cross products.

$-30 + 3r = -18$ Simplify.

$-30 + 3r + 30 = -18 + 30$ Add 30 to each side.

$3r = 12$ Simplify.

$\frac{3r}{3} = \frac{12}{3}$ Divide each side by 3.

$r = 4$ Simplify.

Study Tip

Look Back

To review **cross products**, see Lesson 3-6.

RATE OF CHANGE Slope can be used to describe a rate of change. The **rate of change** tells, on average, how a quantity is changing over time.

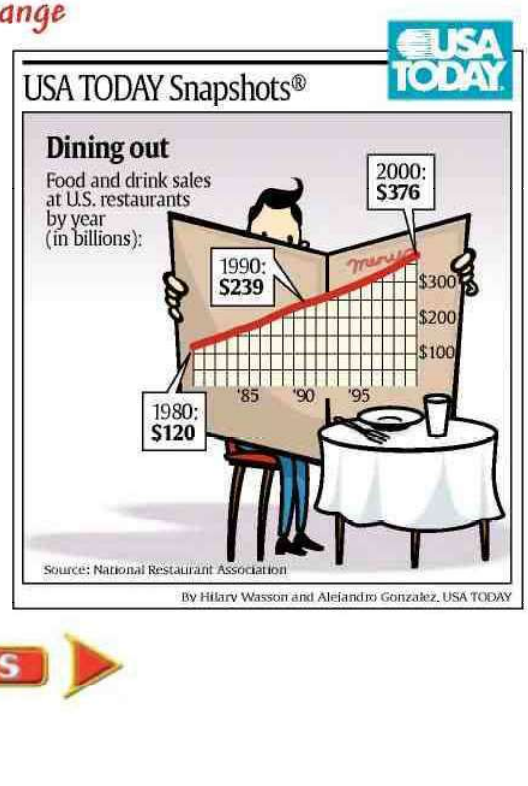
Example 6 Find a Rate of Change

DINING OUT **The graph shows the amount spent on food and drink at U.S. restaurants in recent years.**

a. Find the rates of change for 1980–1990 and 1990–2000.

Use the formula for slope.

$\frac{\text{rise}}{\text{run}} = \frac{\text{change in quantity}}{\text{change in time}}$ ← billion \$ ← years



Log on for:
- Updated data
- More activities on rate of change

www.algebra1.com/usa_today

1980–1990: $\frac{\text{change in quantity}}{\text{change in time}} = \frac{239 - 120}{1990 - 1980}$ Substitute.

$= \frac{119}{10}$ or 11.9 Simplify.

Spending on food and drink increased by $119 billion in a 10-year period for a rate of change of $11.9 billion per year.

1990–2000: $\frac{\text{change in quantity}}{\text{change in time}} = \frac{376 - 239}{2000 - 1990}$ Substitute.

$= \frac{137}{10}$ or 13.7 Simplify.

Over this 10-year period, spending increased by $137 billion, for a rate of change of $13.7 billion per year.

b. Explain the meaning of the slope in each case.

For 1980–1990, on average, $11.9 billion more was spent each year than the last. For 1990–2000, on average, $13.7 billion more was spent each year than the last.

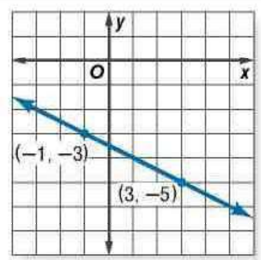
c. How are the different rates of change shown on the graph?

There is a greater vertical change for 1990–2000 than for 1980–1990. Therefore, the section of the graph for 1990–2000 has a steeper slope.

Check for Understanding

Concept Check

1. **Explain** how you would find the slope of the line at the right.



2. **OPEN ENDED** Draw the graph of a line having each slope.
   a. positive slope **b.** negative slope
   c. slope of 0 **d.** undefined slope
3. **Explain** why the formula for determining slope using the coordinates of two points does not apply to vertical lines.
4. **FIND THE ERROR** Carlos and Allison are finding the slope of the line that passes through (2, 6) and (5, 3).

Carlos	Allison
$\frac{3-6}{5-2} = \frac{-3}{3}$ or -1	$\frac{6-3}{5-2} = \frac{3}{3}$ or 1

Who is correct? Explain your reasoning.

Guided Practice

Find the slope of the line that passes through each pair of points.

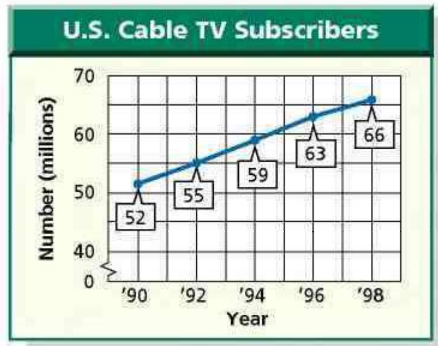
5. (1, 1), (3, 4)
6. (0, 0), (5, 4)
7. (−2, 2), (−1, −2)
8. (7, −4), (9, −1)
9. (3, 5), (−2, 5)
10. (−1, 3), (−1, 0)

Find the value of *r* so the line that passes through each pair of points has the given slope.

11. (6, −2), (r, −6), $m = 4$
12. (9, r), (6, 3), $m = -\frac{1}{3}$

Application **CABLE TV** **For Exercises 13 and 14, use the graph at the right.**

13. Find the rate of change for 1990–1992.
14. Without calculating, find a 2-year period that had a greater rate of change than 1990–1992. Explain your reasoning.



Practice and Apply

Homework Help

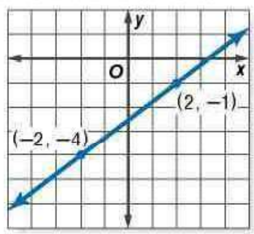
For Exercises	See Examples
15–34	1–4
41–48	5
53–57	6

Extra Practice

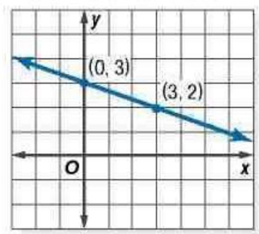
See page 831.

Find the slope of the line that passes through each pair of points.

15.



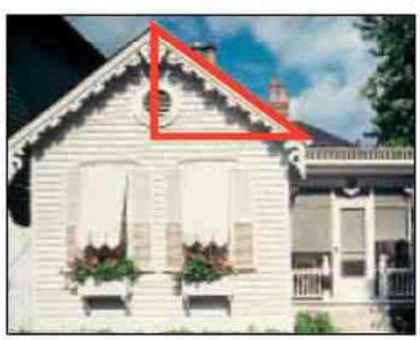
16.



17. $(-4, -1), (-3, -3)$
18. $(-3, 3), (1, 3)$
19. $(-2, 1), (-2, 3)$
20. $(2, 3), (9, 7)$
21. $(5, 7), (-2, -3)$
22. $(-3, 6), (2, 4)$
23. $(-3, -4), (5, -1)$
24. $(2, -1), (5, -3)$
25. $(-5, 4), (-5, -1)$
26. $(2, 6), (-1, 3)$
27. $(-2, 3), (8, 3)$
28. $(-3, 9), (-7, 6)$
29. $(-8, 3), (-6, 2)$
30. $(-2, 0), (1, -1)$
31. $(4.5, -1), (5.3, 2)$
32. $(0.75, 1), (0.75, -1)$
33. $\left(2\frac{1}{2}, -1\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right)$
34. $\left(\frac{3}{4}, 1\frac{1}{4}\right), \left(-\frac{1}{2}, -1\right)$

ARCHITECTURE **Use a ruler to estimate the slope of each roof.**

35.



36.

37. Find the slope of the line that passes through the origin and (r, s).
38. What is the slope of the line that passes through (a, b) and $(a, -b)$?
39. **PAINTING** A ladder reaches a height of 16 feet on a wall. If the bottom of the ladder is placed 4 feet away from the wall, what is the slope of the ladder as a positive number?

40. **PART-TIME JOBS** In 1991, the federal minimum wage rate was \$4.25 per hour. In 1997, it was increased to \$5.15. Find the annual rate of change in the federal minimum wage rate from 1991 to 1997.

Find the value of r so the line that passes through each pair of points has the given slope.

41. $(6, 2), (9, r), m = -1$
42. $(4, -5), (3, r), m = 8$
43. $(5, r), (2, -3), m = \frac{4}{3}$
44. $(-2, 7), (r, 3), m = \frac{4}{3}$
45. $\left(\frac{1}{2}, -\frac{1}{4}\right), \left(r, -\frac{5}{4}\right), m = 4$
46. $\left(\frac{2}{3}, r\right), \left(1, \frac{1}{2}\right), m = \frac{1}{2}$
47. $(4, r), (r, 2), m = -\frac{5}{3}$
48. $(r, 5), (-2, r), m = -\frac{2}{9}$

49. **CRITICAL THINKING** Explain how you know that the slope of the line through $(-4, -5)$ and $(4, 5)$ is positive without calculating.

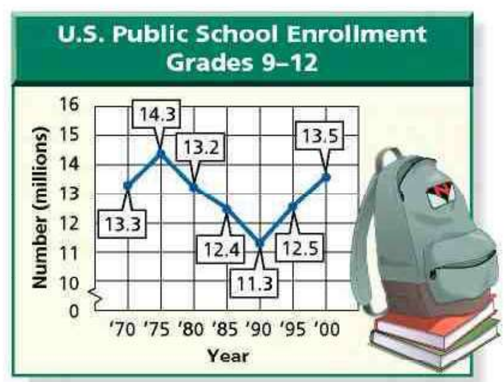
HEALTH **For Exercises 50–52, use the table that shows Karen's height from age 12 to age 20.**

Age (years)	12	14	16	18	20
Height (inches)	60	64	66	67	67

50. Make a broken-line graph of the data.
51. Use the graph to determine the two-year period when Karen grew the fastest. Explain your reasoning.
52. Explain the meaning of the horizontal section of the graph.

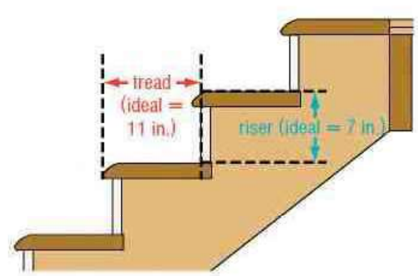
SCHOOL **For Exercises 53–55, use the graph that shows public school enrollment.**

53. For which 5-year period was the rate of change the greatest? When was the rate of change the least?
54. Find the rate of change from 1985 to 1990.
55. Explain the meaning of the part of the graph with a negative slope.



56. **RESEARCH** Use the Internet or other reference to find the population of your city or town in 1930, 1940, . . . , 2000. For which decade was the rate of change the greatest?

57. **CONSTRUCTION** The slope of a stairway determines how easy it is to climb the stairs. Suppose the vertical distance between two floors is 8 feet 9 inches. Find the total run of the ideal stairway in feet and inches.



58. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

Why is slope important in architecture?

Include the following in your answer:

- an explanation of how to find the slope of a roof, and
- a comparison of the appearance of roofs with different slopes.

Standardized Test Practice

59. The slope of the line passing through (5, −4) and (5, −10) is

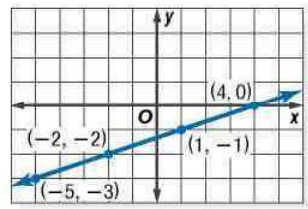
Ⓐ positive. Ⓑ negative. Ⓒ zero. Ⓓ undefined.

60. The slope of the line passing through (a, b) and (c, d) is

Ⓐ $\frac{d-c}{b-a}$. Ⓑ $\frac{b-d}{a-c}$. Ⓒ $\frac{d-b}{a-c}$. Ⓓ $\frac{a-c}{b-d}$.

Extending the Lesson

61. Choose four different pairs of points from those labeled on the graph. Find the slope of the line using the coordinates of each pair of points. Describe your findings.



62. **MAKE A CONJECTURE** Determine whether $Q(2, 3)$, $R(-1, -1)$, and $S(-4, -2)$ lie on the same line. Explain your reasoning.

Maintain Your Skills

Mixed Review

Write an equation for each relation. *(Lesson 4-6)*

63.

x	1	2	3	4	5
$f(x)$	5	10	15	20	25

64.

x	−2	−1	1	2	4
$f(x)$	13	12	10	9	7

Determine whether each relation is a function. *(Lesson 4-5)*

65. $y = -15$

66. $x = 5$

67. $\{(1, 0), (1, 4), (-1, 1)\}$

68. $\{(6, 3), (5, -2), (2, 3)\}$

69. Graph $x - y = 0$. *(Lesson 4-4)*

70. What number is 40% of 37.5? *(Lesson 3-4)*

Find each product. *(Lesson 2-4)*

71. $7(-3)$

72. $(-4)(-2)$

73. $(9)(-4)$

74. $(-8)(3.7)$

75. $\left(-\frac{7}{8}\right)\left(\frac{1}{3}\right)$

76. $\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)(-14)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each quotient.**

*(To review **dividing fractions**, see pages 800 and 801.)*

77. $6 \div \frac{2}{3}$

78. $12 \div \frac{1}{4}$

79. $10 \div \frac{3}{8}$

80. $\frac{1}{2} \div \frac{1}{3}$

81. $\frac{3}{4} \div \frac{1}{6}$

82. $\frac{3}{4} \div 6$

83. $18 \div \frac{7}{8}$

84. $\frac{3}{8} \div \frac{2}{5}$

85. $2\frac{2}{3} \div \frac{1}{4}$

Reading Mathematics

Mathematical Words and Everyday Words

You may have noticed that many words used in mathematics are also used in everyday language. You can use the everyday meaning of these words to better understand their mathematical meaning. The table shows two mathematical words along with their everyday and mathematical meanings.

Word	Everyday Meaning	Mathematical Meaning
expression	**1.** something that expresses or communicates in words, art, music, or movement **2.** the manner in which one expresses oneself, especially in speaking, depicting, or performing	one or more numbers or variables along with one or more arithmetic operations
function	**1.** the action for which one is particularly fitted or employed **2.** an official ceremony or a formal social occasion **3.** something closely related to another thing and dependent on it for its existence, value, or significance	a relationship in which the output depends upon the input

Source: *The American Heritage Dictionary of the English Language*

Notice that the mathematical meaning is more specific, but related to the everyday meaning. For example, the mathematical meaning of *expression* is closely related to the first everyday definition. In mathematics, an expression communicates using symbols.

Reading to Learn

1. How does the mathematical meaning of *function* compare to the everyday meaning?

2. **RESEARCH** Use the Internet or other reference to find the everyday meaning of each word below. How might these words apply to mathematics? Make a table like the one above and note the mathematical meanings that you learn as you study Chapter 5.

a. slope **b.** intercept **c.** parallel

5-2 Slope and Direct Variation

What You'll Learn

- Write and graph direct variation equations.
- Solve problems involving direct variation.

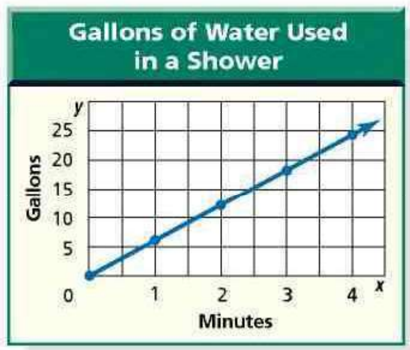
Vocabulary

- direct variation
- constant of variation
- family of graphs
- parent graph

How is slope related to your shower?

A standard showerhead uses about 6 gallons of water per minute. If you graph the ordered pairs from the table, the slope of the line is 6.

x (minutes)	y (gallons)
0	0
1	6
2	12
3	18
4	24



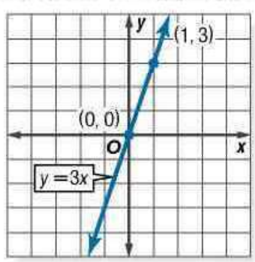
The equation is $y = 6x$. The number of gallons of water y depends *directly* on the amount of time in the shower x.

DIRECT VARIATION A **direct variation** is described by an equation of the form $y = kx$, where $k \neq 0$. We say that *y varies directly with x* or *y varies directly as x*. In the equation $y = kx$, k is the **constant of variation**.

Example 1 Slope and Constant of Variation

Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

a.



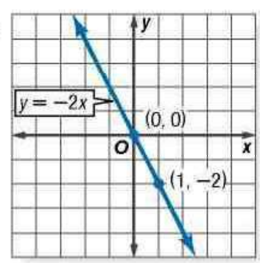
The constant of variation is 3.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{3 - 0}{1 - 0}$ $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 3)$

$m = 3$ The slope is 3.

b.



The constant of variation is -2.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{-2 - 0}{1 - 0}$ $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, -2)$

$m = -2$ The slope is -2.

Compare the constant of variation with the slope of the graph for each example. Notice that the slope of the graph of $y = kx$ is k.

The ordered pair (0, 0) is a solution of $y = kx$. Therefore, the graph of $y = kx$ passes through the origin. You can use this information to graph direct variation equations.

Example 2 Direct Variation with $k > 0$

Graph $y = 4x$.

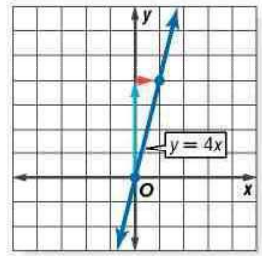
Step 1 Write the slope as a ratio.

$$4 = \frac{4}{1} \quad \frac{\text{rise}}{\text{run}}$$

Step 2 Graph (0, 0).

Step 3 From the point (0, 0), move up 4 units and right 1 unit. Draw a dot.

Step 4 Draw a line containing the points.



Example 3 Direct Variation with $k < 0$

Graph $y = -\frac{1}{3}x$.

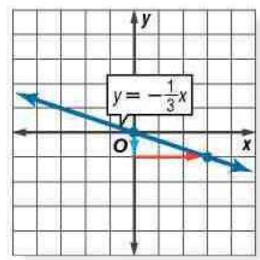
Step 1 Write the slope as a ratio.

$$-\frac{1}{3} = \frac{-1}{3} \quad \frac{\text{rise}}{\text{run}}$$

Step 2 Graph (0, 0).

Step 3 From the point (0, 0), move down 1 unit and right 3 units. Draw a dot.

Step 4 Draw a line containing the points.

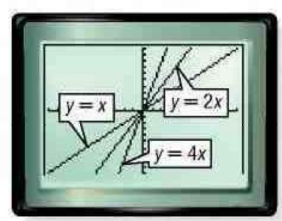


A **family of graphs** includes graphs and equations of graphs that have at least one characteristic in common. The **parent graph** is the simplest graph in a family.

Graphing Calculator Investigation

Family of Graphs

The calculator screen shows the graphs of $y = x$, $y = 2x$, and $y = 4x$.



[−10, 10] scl: 1 by [−10, 10] scl: 1

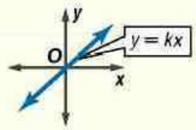
Think and Discuss

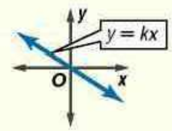
1. Describe any similarities among the graphs.
2. Describe any differences among the graphs.
3. Write an equation whose graph has a steeper slope than $y = 4x$. Check your answer by graphing $y = 4x$ and your equation.
4. Write an equation whose graph lies between the graphs of $y = x$ and $y = 2x$. Check your answer by graphing the equations.
5. Write a description of this family of graphs. What characteristics do the graphs have in common? How are they different?
6. The equations whose graphs are in this family are all of the form $y = mx$. How does the graph change as the absolute value of m increases?

Concept Summary — Direct Variation Graphs

- Direct variation equations are of the form $y = kx$, where $k \neq 0$.
- The graph of $y = kx$ always passes through the origin.

• The slope can be positive. $k > 0$	• The slope can be negative. $k < 0$





If you know that y varies directly as x, you can write a direct variation equation that relates the two quantities.

Example 4 Write and Solve a Direct Variation Equation

Suppose y varies directly as x, and $y = 28$ when $x = 7$.

a. Write a direct variation equation that relates x and y.

Find the value of k.

$y = kx$ Direct variation formula

$28 = k(7)$ Replace y with 28 and x with 7.

$\frac{28}{7} = \frac{k(7)}{7}$ Divide each side by 7.

$4 = k$ Simplify.

Therefore, $y = 4x$.

b. Use the direct variation equation to find x when $y = 52$.

$y = 4x$ Direct variation equation

$52 = 4x$ Replace y with 52.

$\frac{52}{4} = \frac{4x}{4}$ Divide each side by 4.

$13 = x$ Simplify.

Therefore, $x = 13$ when $y = 52$.

SOLVE PROBLEMS One of the most common uses of direct variation is the formula for distance, $d = rt$. In the formula, distance d varies directly as time t, and the rate r is the constant of variation.

Example 5 Direct Variation Equation

BIOLOGY A flock of snow geese migrated 375 miles in 7.5 hours.

a. Write a direct variation equation for the distance flown in any time.

Words The distance traveled is 375 miles, and the time is 7.5 hours.

Variables Let r = rate.

Equation $\overbrace{375 \text{ mi}}^{\text{Distance}} \quad \overbrace{=}^{\text{equals}} \quad \overbrace{r}^{\text{rate}} \quad \overbrace{\times}^{\text{times}} \quad \overbrace{7.5 \text{ h}}^{\text{time.}}$

Solve for the rate.

$375 = r(7.5)$ Original equation

$\frac{375}{7.5} = \frac{r(7.5)}{7.5}$ Divide each side by 7.5.

$50 = r$ Simplify.

Therefore, the direct variation equation is $d = 50t$.

More About. . .

Biology

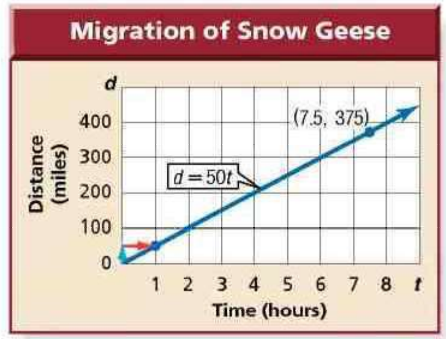
Snow geese migrate more than 3000 miles from their winter home in the southwest United States to their summer home in the Canadian arctic.

Source: Audubon Society

b. Graph the equation.

The graph of $d = 50t$ passes through the origin with slope 50.

$m = \frac{50}{1}$ $\frac{\text{rise}}{\text{run}}$



c. Estimate how many hours of flying time it would take the geese to migrate 3000 miles.

$d = 50t$	Original equation
$3000 = 50t$	Replace d with 3000.
$\frac{3000}{50} = \frac{50t}{50}$	Divide each side by 50.
$t = 60$	Simplify.

At this rate, it will take 60 hours of flying time to migrate 3000 miles.

Check for Understanding

Concept Check

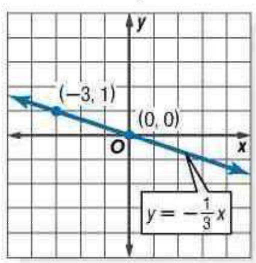
1. **OPEN ENDED** Write a general equation for *y varies directly as x*.
2. **Choose** the equations that represent direct variations. Then find the constant of variation for each direct variation.

   a. $15 = rs$ **b.** $4a = b$ **c.** $z = \frac{1}{3}x$ **d.** $s = \frac{9}{t}$
3. **Explain** how the constant of variation and the slope are related in a direct variation equation.

Guided Practice

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

4.



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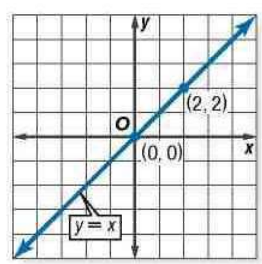


Graph each equation.

6. $y = 2x$ 7. $y = -3x$ 8. $y = \frac{1}{2}x$

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve.

9. If $y = 27$ when $x = 6$, find x when $y = 45$.
10. If $y = 10$ when $x = 9$, find x when $y = 9$.
11. If $y = -7$ when $x = -14$, find y when $x = 20$.

Application

JOBS For Exercises 12–14, use the following information.
Suppose you work at a job where your pay varies directly as the number of hours you work. Your pay for 7.5 hours is \$45.

12. Write a direct variation equation relating your pay to the hours worked.
13. Graph the equation.
14. Find your pay if you work 30 hours.

Practice and Apply

Homework Help

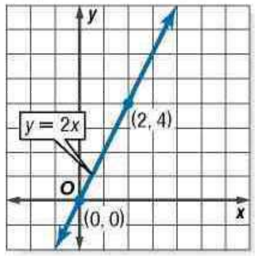
For Exercises	See Examples
15–32	1–3
33–42	4
43–46, 52–55	5

Extra Practice

See page 831.

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

15.



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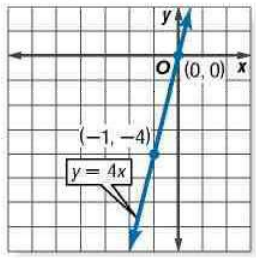

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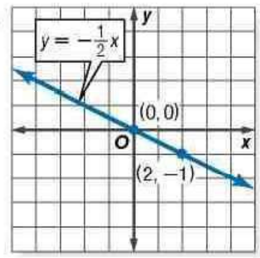

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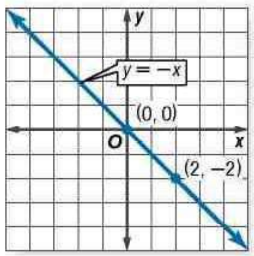

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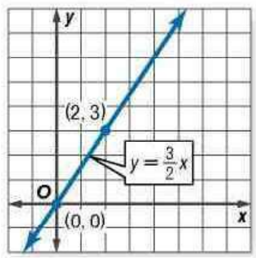

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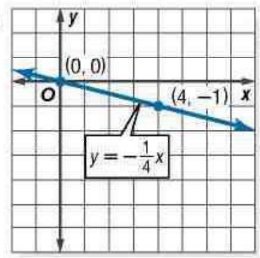


Graph each equation.

21. $y = x$ **22.** $y = 3x$ **23.** $y = -x$ **24.** $y = -4x$

25. $y = \frac{1}{4}x$ **26.** $y = \frac{3}{5}x$ **27.** $y = \frac{5}{2}x$ **28.** $y = \frac{7}{5}x$

29. $y = \frac{1}{5}x$ **30.** $y = -\frac{2}{3}x$ **31.** $y = -\frac{4}{3}x$ **32.** $y = -\frac{9}{2}x$

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve.

33. If $y = 8$ when $x = 4$, find y when $x = 5$.

34. If $y = 36$ when $x = 6$, find x when $y = 42$.

35. If $y = -16$ when $x = 4$, find x when $y = 20$.

36. If $y = -18$ when $x = 6$, find x when $y = 6$.

37. If $y = 4$ when $x = 12$, find y when $x = -24$.

38. If $y = 12$ when $x = 15$, find x when $y = 21$.

39. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

40. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

41. If $y = 2\frac{2}{3}$ when $x = \frac{1}{4}$, find y when $x = 1\frac{1}{8}$.

42. If $y = 6$ when $x = \frac{2}{3}$, find x when $y = 12$.

Write a direct variation equation that relates the variables. Then graph the equation.

43. GEOMETRY The circumference C of a circle is about 3.14 times the diameter d.

44. GEOMETRY The perimeter P of a square is 4 times the length of a side s.

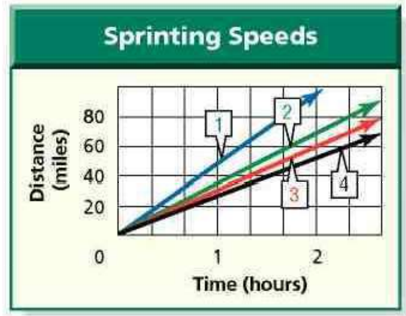
45. SEWING The total cost is C for n yards of ribbon priced at \$0.99 per yard.

46. RETAIL Kona coffee beans are \$14.49 per pound. The total cost of p pounds is C.

47. **CRITICAL THINKING** Suppose y varies directly as x. If the value of x is doubled, what happens to the value of y? Explain.

BIOLOGY **Which line in the graph represents the sprinting speeds of each animal?**

48. elephant, 25 mph
49. reindeer, 32 mph
50. lion, 50 mph
51. grizzly bear, 30 mph



Career Choices

Veterinary Medicine

Veterinarians compare the age of an animal to the age of a human on the basis of bone and tooth growth.

Online Research For information about a career as a veterinarian, visit: www.algebra1.com/careers

SPACE **For Exercises 52 and 53, use the following information.**
The weight of an object on the moon varies directly with its weight on Earth. With all of his equipment, astronaut Neil Armstrong weighed 360 pounds on Earth, but weighed only 60 pounds on the moon.

52. Write an equation that relates weight on the moon m with weight on Earth e.
53. Suppose you weigh 138 pounds on Earth. What would you weigh on the moon?

ANIMALS **For Exercises 54 and 55, use the following information.**
Most animals age more rapidly than humans do. The chart shows equivalent ages for horses and humans.

Horse age (x)	0	1	2	3	4	5
Human age (y)	0	3	6	9	12	15

54. Write an equation that relates human age to horse age.
55. Find the equivalent horse age for a human who is 16 years old.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

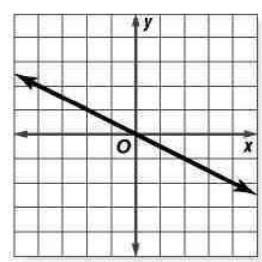
How is slope related to your shower?

Include the following in your answer:
- an equation that relates the number of gallons y to the time spent in the shower x for a low-flow showerhead that uses only 2.5 gallons of water per minute, and
- a comparison of the steepness of the graph of this equation to the graph at the top of page 268.

Standardized Test Practice

57. Which equation best describes the graph at the right?

Ⓐ $y = 2x$ Ⓑ $y = -2x$
Ⓒ $y = \frac{1}{2}x$ Ⓓ $y = -\frac{1}{2}x$



58. Which equation does *not* model a direct variation?

Ⓐ $y = 4x$ Ⓑ $y = 22x$
Ⓒ $y = 3x + 1$ Ⓓ $y = \frac{1}{2}x$

Graphing Calculator

FAMILIES OF GRAPHS **For Exercises 59–62, use the graphs of $y = -1x$, $y = -2x$, and $y = -4x$ which form a family of graphs.**

59. Graph $y = -1x$, $y = -2x$, and $y = -4x$ on the same screen.
60. How are these graphs similar to the graphs in the Graphing Calculator Investigation on page 265? How are they different?

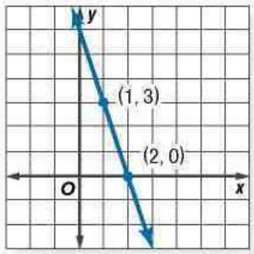
61. Write an equation whose graph has a steeper slope than $y = -4x$.

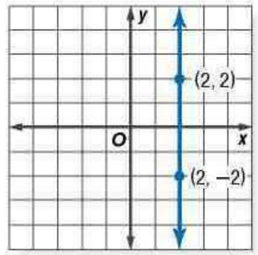
62. **MAKE A CONJECTURE** Explain how you can tell without graphing which of two direct variation equations has the graph with a steeper slope.

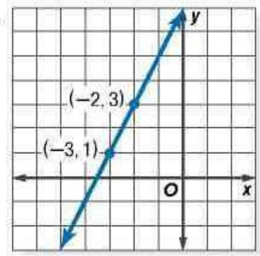
Maintain Your Skills

Mixed Review

Find the slope of the line that passes through each pair of points. *(Lesson 5-1)*

63.


64.


65.


66. Find the value of r so that the line that passes through (1, 7) and (r, 3) has a slope of 2. *(Lesson 5-1)*

Each table below represents points on a linear graph. Copy and complete each table. *(Lesson 4-8)*

67.

x	0	1	2	3	4	5
y	1		9	13		21

68.

x	2	4	6	8	10	12
y			4	2		−2

Add or subtract. *(Lesson 2-3)*

69. $15 + (-12)$ 70. $8 - (-5)$ 71. $-9 - 6$ 72. $-18 - 12$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation for y.**
*(To review **rewriting equations**, see Lesson 3-8.)*

73. $-3x + y = 8$ 74. $2x + y = 7$ 75. $4x = y + 3$

76. $2y = 4x + 10$ 77. $9x + 3y = 12$ 78. $x - 2y = 5$

Practice Quiz 1

Lessons 5-1 and 5-2

Find the slope of the line that passes through each pair of points. *(Lesson 5-1)*

1. $(-4, -6), (-3, -8)$ 2. $(8, 3), (-11, 3)$ 3. $(-4, 8), (5, 9)$ 4. $(0, 1), (7, 11)$

Find the value of r so the line that passes through each pair of points has the given slope. *(Lesson 5-1)*

5. $(5, -3), (r, -5), m = 2$ 6. $(6, r), (-4, 9), m = \frac{3}{2}$

Graph each equation. *(Lesson 5-2)*

7. $y = -7x$ 8. $y = \frac{3}{4}x$

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve. *(Lesson 5-2)*

9. If $y = 24$ when $x = 8$, find y when $x = -3$. 10. If $y = -10$ when $x = 15$, find x when $y = -6$.

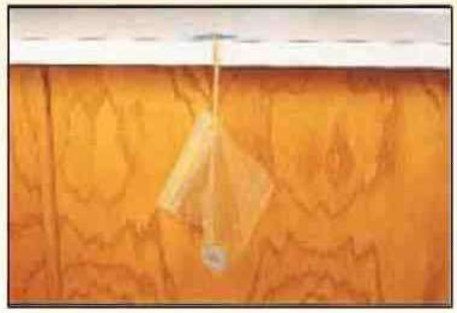
Algebra Activity

A Preview of Lesson 5-3

Investigating Slope-Intercept Form

Collect the Data

- Cut a small hole in a top corner of a plastic sandwich bag. Loop a long rubber band through the hole.
- Tape the free end of the rubber band to the desktop.
- Use a centimeter ruler to measure the distance from the desktop to the end of the bag. Record this distance for 0 washers in the bag using a table like the one below.



Number of Washers x	Distance y
0	
1	

- Place one washer in the plastic bag. Then measure and record the new distance from the desktop to the end of the bag.
- Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the distance from the desktop to the end of the bag.

Analyze the Data

1. The domain contains values represented by the independent variable, washers. The range contains values represented by the dependent variable, distance. On grid paper, graph the ordered pairs (washers, distance).

2. Write a sentence that describes the points on the graph.

3. Describe the point that represents the trial with no washers in the bag.

4. The rate of change can be found by using the formula for slope.

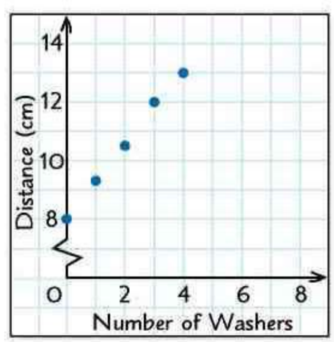
$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in distance}}{\text{change in number of washers}}$$

Find the rate of change in the distance from the desktop to the end of the bag as more washers are added.

5. Explain how the rate of change is shown on the graph.

Make a Conjecture

The graph shows sample data from a rubber band experiment. Draw a graph for each situation.



6. A bag that hangs 10.5 centimeters from the desktop when empty and lengthens at the rate of the sample.

7. A bag that has the same length when empty as the sample and lengthens at a faster rate.

8. A bag that has the same length when empty as the sample and lengthens at a slower rate.

5-3 Slope-Intercept Form

What You'll Learn

- Write and graph linear equations in slope-intercept form.
- Model real-world data with an equation in slope-intercept form.

Vocabulary

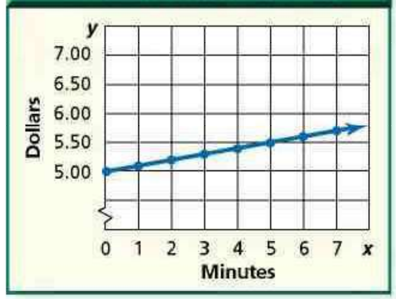
- slope-intercept form

How is a y-intercept related to a flat fee?

A cellular phone service provider charges \$0.10 per minute plus a flat fee of \$5.00 each month.

x (minutes)	y (dollars)
0	5.00
1	5.10
2	5.20
3	5.30
4	5.40
5	5.50
6	5.60
7	5.70

Total Cost of Cellular Phone Service



The slope of the line is 0.1. It crosses the y-axis at (0, 5).
The equation of the line is $y = 0.1x + 5$.

charge per minute, \$0.10 — flat fee, \$5.00

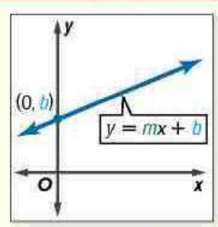
SLOPE-INTERCEPT FORM An equation of the form $y = mx + b$ is in **slope-intercept form**. When an equation is written in this form, you can identify the slope and y-intercept of its graph.

Study Tip

Look Back
To review **intercepts**, see Lesson 4-5.

Key Concept — Slope-Intercept Form

- **Words** The linear equation $y = mx + b$ is written in slope-intercept form, where m is the slope and b is the y-intercept.
- **Symbols** $y = mx + b$
  slope — y-intercept
- **Model**



Example 1 Write an Equation Given Slope and y-Intercept

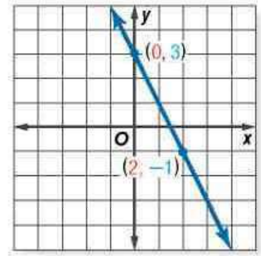
Write an equation of the line whose slope is 3 and whose y-intercept is 5.

$y = mx + b$ Slope-intercept form

$y = 3x + 5$ Replace m with 3 and b with 5.

Example 2 Write an Equation Given Two Points

Write an equation of the line shown in the graph.



Step 1 You know the coordinates of two points on the line. Find the slope. Let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (2, -1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{\text{rise}}{\text{run}}$

$m = \frac{-1 - 3}{2 - 0}$ $x_1 = 0, x_2 = 2$; $y_1 = 3, y_2 = -1$

$m = \frac{-4}{2}$ or -2 Simplify.

The slope is -2.

Step 2 The line crosses the y-axis at (0, 3). So, the y-intercept is 3.

Step 3 Finally, write the equation.

$y = mx + b$ Slope-intercept form

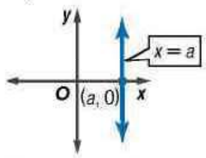
$y = -2x + 3$ Replace m with -2 and b with 3.

The equation of the line is $y = -2x + 3$.

Study Tip

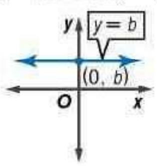
Vertical Lines

The equation of a vertical line *cannot* be written in slope-intercept form. *Why?*



Horizontal Lines

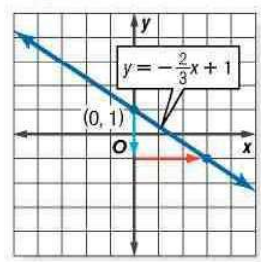
The equation of a horizontal line *can* be written in slope-intercept form as $y = 0x + b$ or $y = b$.



One advantage of the slope-intercept form is that it allows you to graph an equation quickly.

Example 3 Graph an Equation in Slope-Intercept Form

Graph $y = -\frac{2}{3}x + 1$.



Step 1 The y-intercept is 1. So, graph (0, 1).

Step 2 The slope is $-\frac{2}{3}$ or $\frac{-2}{3}$. $\frac{\text{rise}}{\text{run}}$

From (0, 1), move down 2 units and right 3 units. Draw a dot.

Step 3 Draw a line connecting the points.

Example 4 Graph an Equation in Standard Form

Graph $5x - 3y = 6$.

Step 1 Solve for y to find the slope-intercept form.

$5x - 3y = 6$ Original equation

$5x - 3y - 5x = 6 - 5x$ Subtract $5x$ from each side.

$-3y = 6 - 5x$ Simplify.

$-3y = -5x + 6$ $6 - 5x = 6 + (-5x)$ or $-5x + 6$

$\frac{-3y}{-3} = \frac{-5x + 6}{-3}$ Divide each side by -3.

$\frac{-3y}{-3} = \frac{-5x}{-3} + \frac{6}{-3}$ Divide each term in the numerator by -3.

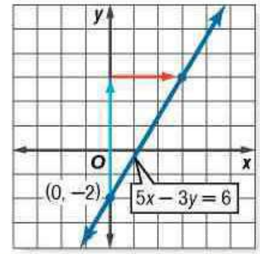
$y = \frac{5}{3}x - 2$ Simplify.

(continued on the next page)

Step 2 The y-intercept of $y = \frac{5}{3}x - 2$ is -2. So, graph $(0, -2)$.

Step 3 The slope is $\frac{5}{3}$. From $(0, -2)$, move up 5 units and right 3 units. Draw a dot.

Step 4 Draw a line containing the points.



MODEL REAL-WORLD DATA If a quantity changes at a constant rate over time, it can be modeled by a linear equation. The y-intercept represents a starting point, and the slope represents the rate of change.

Example 5 Write an Equation in Slope-Intercept Form

AGRICULTURE The natural sweeteners used in foods include sugar, corn sweeteners, syrup, and honey. Use the information at the left about natural sweeteners.

a. The amount of natural sweeteners consumed has increased by an average of 2.6 pounds per year. Write a linear equation to find the average consumption of natural sweeteners in any year after 1989.

Words The consumption increased 2.6 pounds per year, so the rate of change is 2.6 pounds per year. In the first year, the average consumption was 133 pounds.

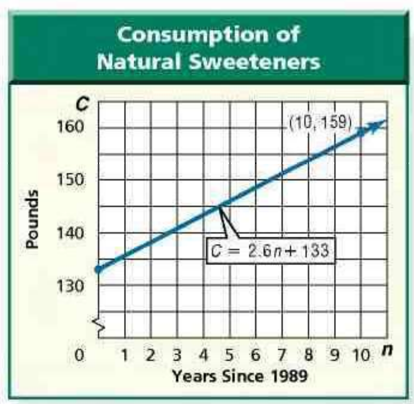
Variables Let C = average consumption.
Let n = number of years after 1989.

Equation

Average consumption	equals	rate of change	times	number of years after 1989	plus	amount at start.
C	$=$	2.6	$\cdot$	n	$+$	133

b. Graph the equation.

The graph passes through (0, 133) with slope 2.6.



c. Find the number of pounds of natural sweeteners consumed by each person in 1999.

The year 1999 is 10 years after 1989. So, $n = 10$.

$C = 2.6n + 133$ Consumption equation

$C = 2.6(10) + 133$ Replace n with 10.

$C = 159$ Simplify.

So, the average person consumed 159 pounds of natural sweeteners in 1999.

CHECK Notice that (10, 159) lies on the graph.

More About...

Agriculture

In 1989, each person in the United States consumed an average of 133 pounds of natural sweeteners.

Source: USDA *Agricultural Outlook*

Check for Understanding

Concept Check

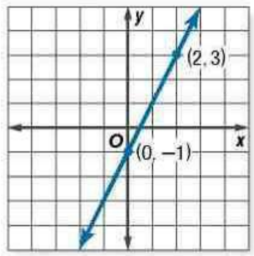
1. **OPEN ENDED** Write an equation for a line with a slope of 7.
2. **Explain** why equations of vertical lines cannot be written in slope-intercept form, but equations of horizontal lines can.
3. **Tell** which part of the slope-intercept form represents the rate of change.

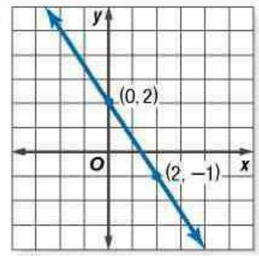
Guided Practice

Write an equation of the line with the given slope and y-intercept.

4. slope: -3, y-intercept: 1
5. slope: 4, y-intercept: -2

Write an equation of the line shown in each graph.

6. 

7. 

Graph each equation.

8. $y = 2x - 3$
9. $y = -3x + 1$
10. $2x + y = 5$

Application

MONEY **For Exercises 11–13, use the following information.**
Suppose you have already saved \$50 toward the cost of a new television set. You plan to save \$5 more each week for the next several weeks.

11. Write an equation for the total amount T you will have w weeks from now.
12. Graph the equation.
13. Find the total amount saved after 7 weeks.

Practice and Apply

Homework Help

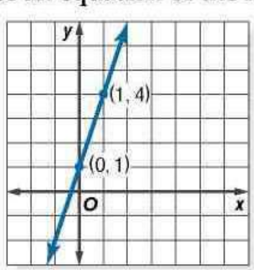
For Exercises	See Examples
14–19	1
20–27	2
28–39	3, 4
40–43	5

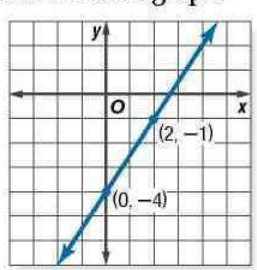
Extra Practice
See page 831.

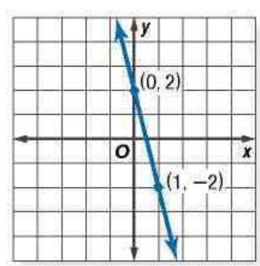
Write an equation of the line with the given slope and y-intercept.

14. slope: 2, y-intercept: -6
15. slope: 3, y-intercept: -5
16. slope: $\frac{1}{2}$, y-intercept: 3
17. slope: $-\frac{3}{5}$, y-intercept: 0
18. slope: -1, y-intercept: 10
19. slope: 0.5; y-intercept: 7.5

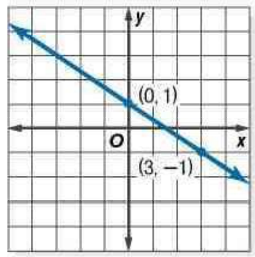
Write an equation of the line shown in each graph.

20. 

21. 

22. 

Write an equation of the line shown in each graph.

23.


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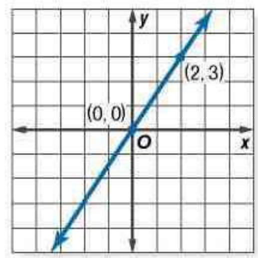

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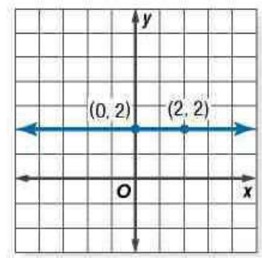


26. Write an equation of a horizontal line that crosses the y-axis at $(0, -5)$.

27. Write an equation of a line that passes through the origin with slope 3.

Graph each equation.

28. $y = 3x + 1$ **29.** $y = x - 2$ **30.** $y = -4x + 1$

31. $y = -x + 2$ **32.** $y = \frac{1}{2}x + 4$ **33.** $y = -\frac{1}{3}x - 3$

34. $3x + y = -2$ **35.** $2x - y = -3$ **36.** $3y = 2x + 3$

37. $-2y = 6x - 4$ **38.** $2x + 3y = 6$ **39.** $4x - 3y = 3$

Write a linear equation in slope-intercept form to model each situation.

40. You rent a bicycle for \$20 plus \$2 per hour.

41. An auto repair shop charges \$50 plus \$25 per hour.

42. A candle is 6 inches tall and burns at a rate of $\frac{1}{2}$ inch per hour.

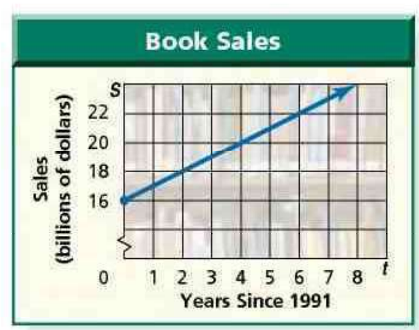
43. The temperature is 15° and is expected to fall 2° each hour during the night.

44. **CRITICAL THINKING** The equations $y = 2x + 3$, $y = 4x + 3$, $y = -x + 3$, and $y = -10x + 3$ form a family of graphs. What characteristic do their graphs have in common?

SALES **For Exercises 45 and 46, use the following information and the graph at the right.**
In 1991, book sales in the United States totaled \$16 billion. Sales increased by about \$1 billion each year until 1999.

45. Write an equation to find the total sales S for any year t between 1991 and 1999.

46. If the trend continues, what will sales be in 2005?



Source: Association of American Publishers

TRAFFIC **For Exercises 47–49, use the following information.**
In 1966, the traffic fatality rate in the United States was 5.5 fatalities per 100 million vehicle miles traveled. Between 1966 and 1999, the rate decreased by about 0.12 each year.

47. Write an equation to find the fatality rate R for any year t between 1966 and 1999.

48. Graph the equation.

49. Find the fatality rate in 1999.

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is a y-intercept related to a flat fee?

Include the following in your answer:

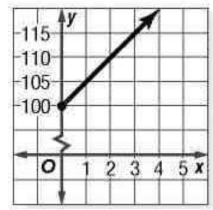
- the point at which the graph would cross the y-axis if your cellular phone service provider charges a rate of \$0.07 per minute plus a flat fee of \$5.99,
- and a description of a situation in which the y-intercept of its graph is \$25.

Standardized Test Practice

51. Which equation does *not* have a y-intercept of 5?

(A) $2x = y - 5$ (B) $3x + y = 5$
(C) $y = x + 5$ (D) $2x - y = 5$

52. Which situation below is modeled by the graph?



(A) You have \$100 and plan to spend \$5 each week.
(B) You have \$100 and plan to save \$5 each week.
(C) You need \$100 for a new CD player and plan to save \$5 each week.
(D) You need \$100 for a new CD player and plan to spend \$5 each week.

Extending the Lesson

53. The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers, $A \geq 0$, and A and B are not both zero. Solve $Ax + By = C$ for y. *Your answer is written in slope-intercept form.*

54. Use the slope-intercept equation in Exercise 53 to write expressions for the slope and y-intercept in terms of A, B, and C.

55. Use the expressions in Exercise 54 to find the slope and y-intercept of each equation.

a. $2x + y = -4$ **b.** $3x + 4y = 12$ **c.** $2x - 3y = 9$

Maintain Your Skills

Mixed Review

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve. *(Lesson 5-2)*

56. If $y = 45$ when $x = 60$, find x when $y = 8$.

57. If $y = 15$ when $x = 4$, find y when $x = 10$.

Find the slope of the line that passes through each pair of points. *(Lesson 5-1)*

58. $(-3, 0), (-4, 6)$ 59. $(3, -1), (3, -4)$ 60. $(5, -5), (9, 2)$

61. Write the numbers $2.5, \frac{3}{4}, -0.5, \frac{7}{8}$ in order from least to greatest. *(Lesson 2-4)*

Solve each equation. *(Lesson 1-3)*

62. $x = \frac{15 - 9}{2}$ 63. $3(7) + 2 = b$ 64. $q = 6^2 - 2^2$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the slope of the line that passes through each pair of points.** *(To review **slope**, see Lesson 5-1.)*

65. $(-1, 2), (1, -2)$ 66. $(5, 8), (-2, 8)$ 67. $(1, -1), (10, -13)$

Graphing Calculator

A Follow-Up of Lesson 5-3

Families of Linear Graphs

A family of people is a group of people related by birth, marriage, or adoption. Recall that a *family of graphs* includes graphs and equations of graphs that have at least one characteristic in common.

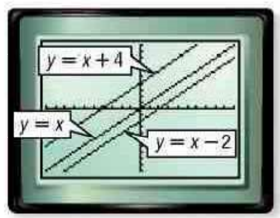
Families of linear graphs fall into two categories—those with the same slope and those with the same y-intercept. A graphing calculator is a useful tool for studying a group of graphs to determine whether they form a family.

Example 1

Graph $y = x$, $y = x + 4$, and $y = x - 2$ in the standard viewing window. Describe any similarities and differences among the graphs. Write a description of the family.

Enter the equations in the Y= list as Y1, Y2, and Y3. Then graph the equations.

KEYSTROKES: *Review graphing on pages 224 and 225.*



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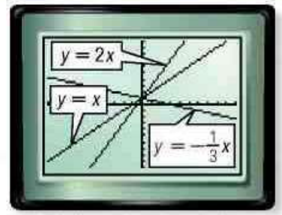
- The graph of $y = x$ has a slope of 1 and a y-intercept of 0.
- The graph of $y = x + 4$ has a slope of 1 and a y-intercept of 4.
- The graph of $y = x - 2$ has a slope of 1 and a y-intercept of -2.

Notice that the graph of $y = x + 4$ is the same as the graph of $y = x$, moved 4 units up. Also, the graph of $y = x - 2$ is the same as the graph of $y = x$, moved 2 units down. All graphs have the same slope and different intercepts.

Because they all have the same slope, this family of graphs can be described as linear graphs with a slope of 1.

Example 2

Graph $y = x + 1$, $y = 2x + 1$, and $y = -\frac{1}{3}x + 1$ in the standard viewing window. Describe any similarities and differences among the graphs. Write a description of the family.



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Enter the equations in the Y= list and graph.

- The graph of $y = x + 1$ has a slope of 1 and a y-intercept of 1.

www.algebra1.com/other_calculator_keystrokes

Investigation

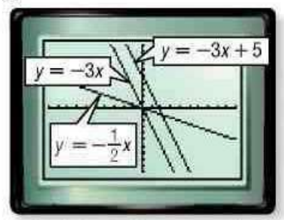
- The graph of $y = 2x + 1$ has a slope of 2 and a y-intercept of 1.
- The graph of $y = -\frac{1}{3}x + 1$ has a slope of $-\frac{1}{3}$ and a y-intercept of 1.

These graphs have the same intercept and different slopes. This family of graphs can be described as linear graphs with a y-intercept of 1.

Sometimes a common characteristic is not enough to determine that a group of equations describes a family of graphs.

Example 3

Graph $y = -3x$, $y = -3x + 5$, and $y = -\frac{1}{2}x$ in the standard viewing window. Describe any similarities and differences among the graphs.



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- The graph of $y = -3x$ has slope -3 and y-intercept 0.
- The graph of $y = -3x + 5$ has slope -3 and y-intercept 5.
- The graph of $y = -\frac{1}{2}x$ has slope $-\frac{1}{2}$ and y-intercept 0.

These equations are similar in that they all have negative slope. However since the slopes are different and the y-intercepts are different, these graphs are not all in the same family.

Exercises

Graph each set of equations on the same screen. Describe any similarities or differences among the graphs. If the graphs are part of the same family, describe the family.

1. $y = -4$
$y = 0$
$y = 7$

2. $y = -x + 1$
$y = 2x + 1$
$y = \frac{1}{4}x + 1$

3. $y = x + 4$
$y = 2x + 4$
$y = 2x - 4$

4. $y = \frac{1}{2}x + 2$
$y = \frac{1}{3}x + 3$
$y = \frac{1}{4}x + 4$

5. $y = -2x - 2$
$y = 2x - 2$
$y = \frac{1}{2}x - 2$

6. $y = 3x$
$y = 3x + 6$
$y = 3x - 7$

7. **MAKE A CONJECTURE** Write a paragraph explaining how the values of m and b in the slope-intercept form affect the graph of the equation.

8. Families of graphs are also called **classes of functions**. Describe the similarities and differences in the class of functions $f(x) = x + c$, where c is any real number.

9. Graph $y = |x|$. Make a conjecture about the transformations of the parent graph, $y = |x| + c$ and, $y = |x + c|$. Use a graphing calculator with different values of c to test your conjecture.

5-4 Writing Equations in Slope-Intercept Form

What You'll Learn

- Write an equation of a line given the slope and one point on a line.
- Write an equation of a line given two points on the line.

Vocabulary

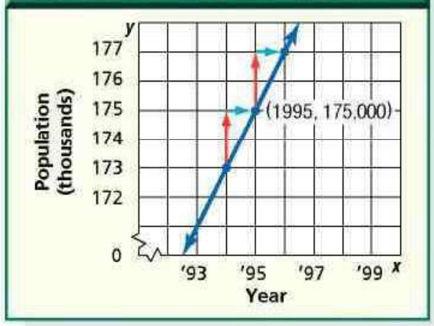
- linear extrapolation

How can slope-intercept form be used to make predictions?

In 1995, the population of Orlando, Florida, was about 175,000. At that time, the population was growing at a rate of about 2000 per year.

x (year)	y (population)
⋮	⋮
1994	173,000
1995	175,000
1996	177,000
⋮	⋮

Population of Orlando, Florida



If you could write an equation based on the slope, 2000, and the point (1995, 175,000), you could predict the population for another year.

WRITE AN EQUATION GIVEN THE SLOPE AND ONE POINT

You have learned how to write an equation of a line when you know the slope and a specific point, the y-intercept. The following example shows how to write an equation when you know the slope and any point on the line.

Example 1 Write an Equation Given Slope and One Point

Write an equation of a line that passes through (1, 5) with slope 2.

Step 1 The line has slope 2. To find the y-intercept, replace m with 2 and (x, y) with (1, 5) in the slope-intercept form. Then, solve for b.

$y = mx + b$ Slope-intercept form

$5 = 2(1) + b$ Replace m with 2, y with 5, and x with 1.

$5 = 2 + b$ Multiply.

$5 - 2 = 2 + b - 2$ Subtract 2 from each side.

$3 = b$ Simplify.

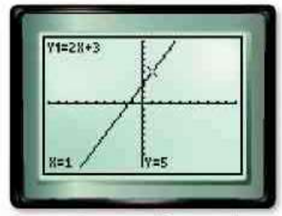
Step 2 Write the slope-intercept form using $m = 2$ and $b = 3$.

$y = mx + b$ Slope-intercept form

$y = 2x + 3$ Replace m with 2 and b with 3.

Therefore, the equation is $y = 2x + 3$.

CHECK You can check your result by graphing $y = 2x + 3$ on a graphing calculator. Use the CALC menu to verify that it passes through (1, 5).



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WRITE AN EQUATION GIVEN TWO POINTS Sometimes you do not know the slope of a line, but you know two points on the line. In this case, find the slope of the line. Then follow the steps in Example 1.

Standardized Test Practice

Example 2 Write an Equation Given Two Points

Multiple-Choice Test Item

The table of ordered pairs shows the coordinates of the two points on the graph of a function. Which equation describes the function?

x	y
−3	−1
6	−4

(A) $y = -\frac{1}{3}x - 2$ (B) $y = 3x - 2$

(C) $y = -\frac{1}{3}x + 2$ (D) $y = \frac{1}{3}x - 2$

Read the Test Item

The table represents the ordered pairs (−3, −1) and (6, −4).

Solve the Test Item

Step 1 Find the slope of the line containing the points. Let $(x_1, y_1) = (-3, -1)$ and $(x_2, y_2) = (6, -4)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{-4 - (-1)}{6 - (-3)}$ $x_1 = -3, x_2 = 6, y_1 = -1, y_2 = -4$

$m = \frac{-3}{9}$ or $-\frac{1}{3}$ Simplify.

Step 2 You know the slope and two points. Choose one point and find the y-intercept. In this case, we chose (6, −4).

$y = mx + b$ Slope-intercept form

$-4 = -\frac{1}{3}(6) + b$ Replace m with $-\frac{1}{3}$, x with 6, and y with −4.

$-4 = -2 + b$ Multiply.

$-4 + 2 = -2 + b + 2$ Add 2 to each side.

$-2 = b$ Simplify.

Step 3 Write the slope-intercept form using $m = -\frac{1}{3}$ and $b = -2$.

$y = mx + b$ Slope-intercept form

$y = -\frac{1}{3}x - 2$ Replace m with $-\frac{1}{3}$ and b with −2.

Therefore, the equation is $y = -\frac{1}{3}x - 2$. The answer is A.

The Princeton Review

Test-Taking Tip

You can check your result by graphing. The line should pass through (−3, −1) and (6, −4).

Example 3 Write an Equation to Solve a Problem

More About. . .

Baseball

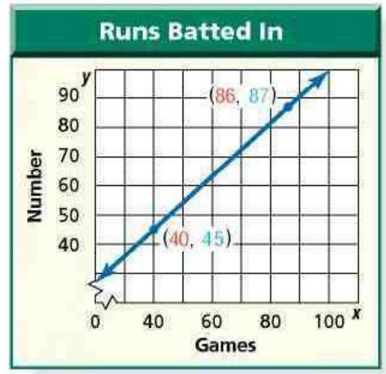
Mark McGwire is best known for breaking Roger Maris' single-season home run record of 61. In the 1998 season, McGwire hit 70 home runs.

Source: *USA TODAY*

BASEBALL **In the middle of the 1998 baseball season, Mark McGwire seemed to be on track to break the record for most runs batted in. After 40 games, McGwire had 45 runs batted in. After 86 games, he had 87 runs batted in. Write a linear equation to estimate the number of runs batted in for any number of games that season.**

Explore You know the number of runs batted in after 40 and 86 games.

Plan Let x represent the number of games. Let y represent the number of runs batted in. Write an equation of the line that passes through (40, 45) and (86, 87).



Solve Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{87 - 45}{86 - 40}$ Let $(x_1, y_1) = (40, 45)$ and $(x_2, y_2) = (86, 87)$.

$m = \frac{42}{46}$ or about 0.91 Simplify.

Choose (40, 45) and find the y-intercept of the line.

$y = mx + b$ Slope-intercept form

$45 = 0.91(40) + b$ Replace m with 0.91, x with 40, and y with 45.

$45 = 36.4 + b$ Multiply.

$45 - 36.4 = 36.4 + b - 36.4$ Subtract 36.4 from each side.

$8.6 = b$ Simplify.

Write the slope-intercept form using $m = 0.91$, and $b = 8.6$.

$y = mx + b$ Slope-intercept form

$y = 0.91x + 8.6$ Replace m with 0.91 and b with 8.6.

Therefore, the equation is $y = 0.91x + 8.6$.

Examine Check your result by substituting the coordinates of the point not chosen, (86, 87), into the equation.

$y = 0.91x + 8.6$ Original equation

$87 \stackrel{?}{=} 0.91(86) + 8.6$ Replace y with 87 and x with 86.

$87 \stackrel{?}{=} 78.26 + 8.6$ Multiply.

$87 \approx 86.86$ ✓ The slope was rounded, so the answers vary slightly.

Concept Summary — Writing Equations

Given the Slope and One Point	Given Two Points
Step 1 Substitute the values of *m*, *x*, and *y* into the slope-intercept form and solve for *b*.	**Step 1** Find the slope.
Step 2 Write the slope-intercept form using the values of *m* and *b*.	**Step 2** Choose one of the two points to use.
	Step 3 Then, follow the steps for writing an equation given the slope and one point.

When you use a linear equation to predict values that are beyond the range of the data, you are using **linear extrapolation**.

Example 4 Linear Extrapolation

SPORTS The record for most runs batted in during a single season is 190. Use the equation in Example 3 to decide whether a baseball fan following the 1998 season would have expected McGwire to break the record in the 162 games played that year.

$y = 0.91x + 8.6$ Original equation

$y = 0.91(162) + 8.6$ Replace *x* with 162.

$y \approx 156$ Simplify.

Since the record is 190 runs batted in, a fan would have predicted that Mark McGwire would not break the record.

Be cautious when making a prediction using just two given points. The model may be *approximately* correct, but still give inaccurate predictions. For example, in 1998, Mark McGwire had 147 runs batted in, which was nine less than the prediction.

Check for Understanding

Concept Check

1. **Compare and contrast** the process used to write an equation given the slope and one point with the process used for two points.
2. **OPEN ENDED** Write an equation in slope-intercept form of a line that has a *y*-intercept of 3.
3. **Tell** whether the statement is *sometimes*, *always*, or *never* true. Explain. *You can write the equation of a line given its x- and y-intercepts.*

Guided Practice

Write an equation of the line that passes through each point with the given slope.

4. $(4, -2)$, $m = 2$
5. $(3, 7)$, $m = -3$
6. $(-3, 5)$, $m = -1$

Write an equation of the line that passes through each pair of points.

7. $(5, 1)$, $(8, -2)$
8. $(6, 0)$, $(0, 4)$
9. $(5, 2)$, $(-7, -4)$

Standardized Test Practice

10. The table of ordered pairs shows the coordinates of the two points on the graph of a function. Which equation describes the function?

x	y
−5	2
0	7

Ⓐ $y = x + 7$ Ⓑ $y = x - 7$

Ⓒ $y = -5x + 2$ Ⓓ $y = 5x + 2$

Practice and Apply

Homework Help

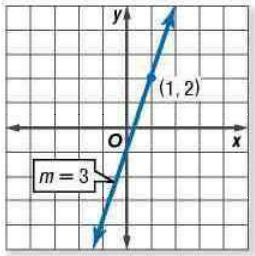
For Exercises	See Examples
11–18	1
19–29	2
34–39	3, 4

Extra Practice

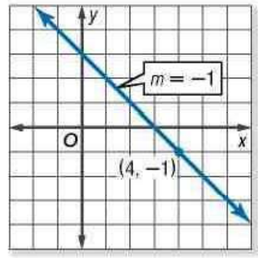
See page 832.

Write an equation of the line that passes through each point with the given slope.

11.



12.

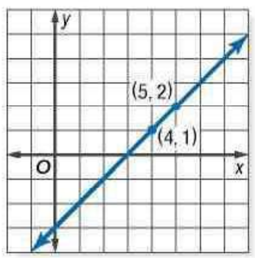


13. $(5, -2)$, $m = 3$ **14.** $(5, 4)$, $m = -5$ **15.** $(3, 0)$, $m = -2$

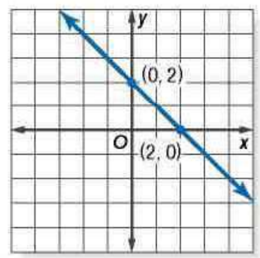
16. $(5, 3)$, $m = \frac{1}{2}$ **17.** $(-3, -1)$, $m = -\frac{2}{3}$ **18.** $(-3, -5)$, $m = -\frac{5}{3}$

Write an equation of the line that passes through each pair of points.

19.



20.



21. $(4, 2), (-2, -4)$ **22.** $(3, -2), (6, 4)$ **23.** $(-1, 3), (2, -3)$

24. $(2, -2), (3, 2)$ **25.** $(7, -2), (-4, -2)$ **26.** $(0, 5), (-3, 5)$

27. $(1, 1), (7, 4)$ **28.** $(5, 7), (0, 6)$ **29.** $\left(-\frac{5}{4}, 1\right), \left(-\frac{1}{4}, \frac{3}{4}\right)$

Write an equation of the line that has each pair of intercepts.

30. x-intercept: -3, y-intercept: 5 **31.** x-intercept: 3, y-intercept: 4

32. x-intercept: 6, y-intercept: 3 **33.** x-intercept: 2, y-intercept: -2

MARRIAGE AGE **For Exercises 34–37, use the information in the graphic.**

34. Write a linear equation to predict the median age that men marry M for any year t.

35. Use the equation to predict the median age of men who marry for the first time in 2005.

36. Write a linear equation to predict the median age that women marry W for any year t.

37. Use the equation to predict the median age of women who marry for the first time in 2005.

POPULATION **For Exercises 38 and 39, use the data at the top of page 280.**

38. Write a linear equation to find Orlando's population for any year.

39. Predict what Orlando's population will be in 2010.

40. **CANOE RENTAL** If you rent a canoe for 3 hours, you will pay \$45. Write a linear equation to find the total cost C of renting the canoe for h hours.

For Exercises 41–43, consider line ℓ that passes through (14, 2) and (27, 24).

41. Write an equation for line ℓ.

42. What is the slope of line ℓ?

43. Where does line ℓ intersect the x-axis? the y-axis?

44. **CRITICAL THINKING** The x-intercept of a line is p, and the y-intercept is q. Write an equation of the line.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can slope-intercept form be used to make predictions?

Include the following in your answer:
- a definition of linear extrapolation, and
- an explanation of how slope-intercept form is used in linear extrapolation.

Standardized Test Practice

46. Which is an equation for the line with slope $\frac{1}{3}$ through $(-2, 1)$?

Ⓐ $y = \frac{1}{3}x + 1$ Ⓑ $y = \frac{1}{3}x + \frac{5}{3}$ Ⓒ $y = \frac{1}{3}x - \frac{5}{3}$ Ⓓ $y = \frac{1}{3}x + \frac{1}{3}$

47. About 20,000 fewer babies were born in California in 1996 than in 1995. In 1995, about 560,000 babies were born. Which equation can be used to predict the number of babies y (in thousands), born x years after 1995?

Ⓐ $y = 20x + 560$ Ⓑ $y = -20x + 560$
Ⓒ $y = -20x - 560$ Ⓓ $y = 20x - 560$

Maintain Your Skills

Mixed Review

Graph each equation. *(Lesson 5-3)*

48. $y = 3x - 2$ 49. $x + y = 6$ 50. $x + 2y = 8$

51. **HEALTH** Each time your heart beats, it pumps 2.5 ounces of blood through your heart. Write a direct variation equation that relates the total volume of blood V with the number of times your heart beats b. *(Lesson 5-2)*

State the domain of each relation. *(Lesson 4-3)*

52. $\{(0, 8), (9, -2), (4, 2)\}$ 53. $\{(-2, 1), (5, 1), (-2, 7), (0, -3)\}$

Replace each ● with <, >, or = to make a true sentence. *(Lesson 2-4)*

54. $-3 \bullet -5$ 55. $4 \bullet \frac{16}{3}$ 56. $\frac{3}{4} \bullet \frac{2}{3}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each difference.**
*(To review **subtracting integers**, see Lesson 2-3.)*

57. $4 - 7$ 58. $5 - 12$ 59. $2 - (-3)$

60. $-1 - 4$ 61. $-7 - 8$ 62. $-5 - (-2)$

5-5 Writing Equations in Point-Slope Form

What You'll Learn

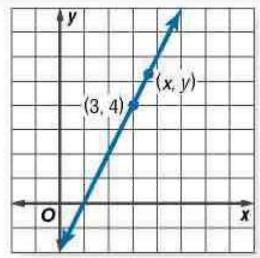
- Write the equation of a line in point-slope form.
- Write linear equations in different forms.

Vocabulary

- point-slope form

How can you use the slope formula to write an equation of a line?

The graph shows a line with slope 2 that passes through (3, 4). Another point on the line is (x, y).



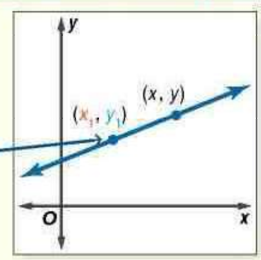
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$2 = \dfrac{y - 4}{x - 3}$	$(x_2, y_2) = (x, y)$ $(x_1, y_1) = (3, 4)$
$2(x - 3) = \dfrac{y - 4}{x - 3}(x - 3)$	Multiply each side by $(x - 3)$.
$2(x - 3) = y - 4$	Simplify.
$y - 4 = 2(x - 3)$	Symmetric Property of Equality

In $y - 4 = 2(x - 3)$: 4 is the y-coordinate, 2 is the slope, 3 is the x-coordinate.

POINT-SLOPE FORM The equation above was generated using the coordinates of a known point and the slope of the line. It is written in **point-slope form**.

Key Concept — *Point-Slope Form*

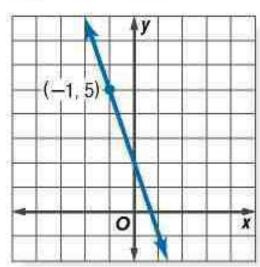
- **Words** The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.
- **Symbols** $y - y_1 = m(x - x_1)$ given point
- **Model**



> **Study Tip**
>
> *Point-Slope Form*
> Remember, (x_1, y_1) represents the *given* point, and (x, y) represents *any other* point on the line.

Example 1 Write an Equation Given Slope and a Point

Write the point-slope form of an equation for a line that passes through (−1, 5) with slope −3.



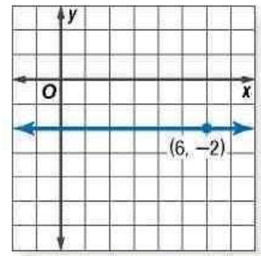
$y - y_1 = m(x - x_1)$	Point-slope form
$y - 5 = -3[x - (-1)]$	$(x_1, y_1) = (-1, 5)$
$y - 5 = -3(x + 1)$	Simplify.

Therefore, the equation is $y - 5 = -3(x + 1)$.

Vertical lines cannot be written in point-slope form because the slope is undefined. However, since the slope of a horizontal line is 0, horizontal lines can be written in point-slope form.

Example 2 Write an Equation of a Horizontal Line

Write the point-slope form of an equation for a horizontal line that passes through (6, −2).



$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = 0(x - 6)$ $(x_1, y_1) = (6, -2)$

$y + 2 = 0$ Simplify.

Therefore, the equation is $y + 2 = 0$.

FORMS OF LINEAR EQUATIONS You have learned about three of the most common forms of linear equations.

Concept Summary — Forms of Linear Equations

Form	Equation	Description
Slope-Intercept	$y = mx + b$	m is the slope, and b is the y-intercept.
Point-Slope	$y - y_1 = m(x - x_1)$	m is the slope and (x_1, y_1) is a given point.
Standard	$Ax + By = C$	A and B are not both zero. Usually A is nonnegative and A, B, and C are integers whose greatest common factor is 1.

Study Tip

Look Back
To review **standard form**, see Lesson 4-5.

Linear equations in point-slope form can be written in slope-intercept or standard form.

Example 3 Write an Equation in Standard Form

Write $y + 5 = -\frac{5}{4}(x - 2)$ in standard form.

In standard form, the variables are on the left side of the equation. A, B, and C are all integers.

$y + 5 = -\frac{5}{4}(x - 2)$ Original equation

$4(y + 5) = 4\left(-\frac{5}{4}\right)(x - 2)$ Multiply each side by 4 to eliminate the fraction.

$4y + 20 = -5(x - 2)$ Distributive Property

$4y + 20 = -5x + 10$ Distributive Property

$4y + 20 - 20 = -5x + 10 - 20$ Subtract 20 from each side.

$4y = -5x - 10$ Simplify.

$4y + 5x = -5x - 10 + 5x$ Add $5x$ to each side.

$5x + 4y = -10$ Simplify.

The standard form of the equation is $5x + 4y = -10$.

Example 4 Write an Equation in Slope-Intercept Form

Write $y - 2 = \frac{1}{2}(x + 5)$ in slope-intercept form.

In slope-intercept form, y is on the left side of the equation. The constant and x are on the right side.

$$y - 2 = \frac{1}{2}(x + 5) \quad \text{Original equation}$$

$$y - 2 = \frac{1}{2}x + \frac{5}{2} \quad \text{Distributive Property}$$

$$y - 2 + 2 = \frac{1}{2}x + \frac{5}{2} + 2 \quad \text{Add 2 to each side.}$$

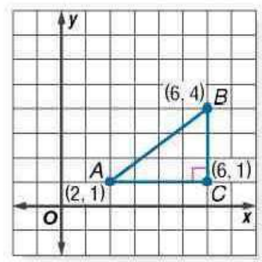
$$y = \frac{1}{2}x + \frac{9}{2} \quad 2 = \frac{4}{2} \text{ and } \frac{4}{2} + \frac{5}{2} = \frac{9}{2}$$

The slope-intercept form of the equation is $y = \frac{1}{2}x + \frac{9}{2}$.

You can draw geometric figures on a coordinate plane and use the point-slope form to write equations of the lines.

Example 5 Write an Equation in Point-Slope Form

GEOMETRY **The figure shows right triangle ABC.**



a. Write the point-slope form of the line containing the hypotenuse $\overline{AB}$.

Study Tip

Geometry

The *hypotenuse* is the side of a right triangle opposite the right angle.

Step 1 First, find the slope of $\overline{AB}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$= \frac{4 - 1}{6 - 2} \text{ or } \frac{3}{4} \quad (x_1, y_1) = (2, 1), (x_2, y_2) = (6, 4)$$

Step 2 You can use either point for (x_1, y_1) in the point-slope form.

Method 1 Use (6, 4).

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{3}{4}(x - 6)$$

Method 2 Use (2, 1).

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{4}(x - 2)$$

b. Write each equation in standard form.

$y - 4 = \frac{3}{4}(x - 6)$	Original equation	$y - 1 = \frac{3}{4}(x - 2)$
$4(y - 4) = 4\left(\frac{3}{4}\right)(x - 6)$	Multiply each side by 4.	$4(y - 1) = 4\left(\frac{3}{4}\right)(x - 2)$
$4y - 16 = 3(x - 6)$	Multiply.	$4y - 4 = 3(x - 2)$
$4y - 16 = 3x - 18$	Distributive Property	$4y - 4 = 3x - 6$
$4y = 3x - 2$	Add to each side.	$4y = 3x - 2$
$-3x + 4y = -2$	Subtract $3x$ from each side.	$-3x + 4y = -2$
$3x - 4y = 2$	Multiply each side by -1.	$3x - 4y = 2$

Regardless of which point was used to find the point-slope form, the standard form results in the same equation.

Check for Understanding

Concept Check

1. **Explain** what x_1 and y_1 in the point-slope form of an equation represent.
2. **FIND THE ERROR** Tanya and Akira wrote the point-slope form of an equation for a line that passes through $(-2, -6)$ and $(1, 6)$. Tanya says that Akira's equation is wrong. Akira says they are both correct.

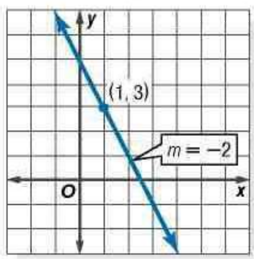
Tanya	Akira
$y + 6 = 4(x + 2)$	$y - 6 = 4(x - 1)$

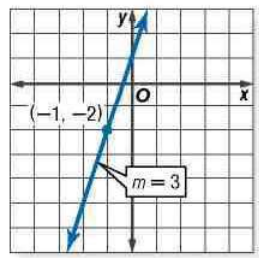
Who is correct? Explain your reasoning.

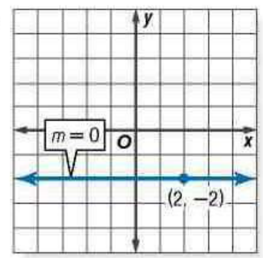
3. **OPEN ENDED** Write an equation in point-slope form. Then write an equation for the same line in slope-intercept form.

Guided Practice

Write the point-slope form of an equation for a line that passes through each point with the given slope.

4. 

5. 

6. 

Write each equation in standard form.

7. $y - 5 = 4(x + 2)$
8. $y + 3 = -\frac{3}{4}(x - 1)$
9. $y - 3 = 2.5(x + 1)$

Write each equation in slope-intercept form.

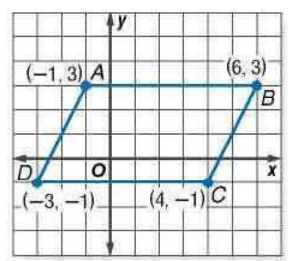
10. $y + 6 = 2(x - 2)$
11. $y + 3 = -\frac{2}{3}(x - 6)$
12. $y - \frac{7}{2} = \frac{1}{2}(x - 4)$

Application

GEOMETRY **For Exercises 13 and 14, use parallelogram *ABCD*.**

A parallelogram has opposite sides parallel.

13. Write the point-slope form of the line containing $\overline{AD}$.
14. Write the standard form of the line containing $\overline{AD}$.



Practice and Apply

Homework Help

For Exercises	See Examples
15–26	1
27–28	2
29–40	3
41–52	4

Extra Practice

See page 832.

Write the point-slope form of an equation for a line that passes through each point with the given slope.

15. $(3, 8)$, $m = 2$
16. $(-4, -3)$, $m = 1$
17. $(-2, 4)$, $m = -3$
18. $(-6, 1)$, $m = -4$
19. $(-3, 6)$, $m = 0$
20. $(9, 1)$, $m = \frac{2}{3}$
21. $(8, -3)$, $m = \frac{3}{4}$
22. $(-6, 3)$, $m = -\frac{2}{3}$
23. $(1, -3)$, $m = -\frac{5}{8}$
24. $(9, -5)$, $m = 0$
25. $(-4, 8)$, $m = \frac{7}{2}$
26. $(1, -4)$, $m = -\frac{8}{3}$

27. Write the point-slope form of an equation for a horizontal line that passes through $(5, -9)$.

28. A horizontal line passes through $(0, 7)$. Write the point-slope form of its equation.

Write each equation in standard form.

29. $y - 13 = 4(x - 2)$
30. $y + 3 = 3(x + 5)$
31. $y - 5 = -2(x + 6)$
32. $y + 3 = -5(x + 1)$
33. $y + 7 = \frac{1}{2}(x + 2)$
34. $y - 1 = \frac{5}{6}(x - 4)$
35. $y - 2 = -\frac{2}{5}(x - 8)$
36. $y + 4 = -\frac{1}{3}(x - 12)$
37. $y + 2 = \frac{5}{3}(x + 6)$
38. $y + 6 = \frac{3}{2}(x - 4)$
39. $y - 6 = 1.3(x + 7)$
40. $y - 2 = -2.5(x - 1)$

Write each equation in slope-intercept form.

41. $y - 2 = 3(x - 1)$
42. $y - 5 = 6(x + 1)$
43. $y + 2 = -2(x - 5)$
44. $y - 1 = -7(x - 3)$
45. $y + 3 = \frac{1}{2}(x + 4)$
46. $y - 1 = \frac{2}{3}(x + 9)$
47. $y + 3 = -\frac{1}{4}(x + 2)$
48. $y - 5 = -\frac{2}{5}(x + 15)$
49. $y + \frac{1}{2} = x - \frac{1}{2}$
50. $y - \frac{1}{3} = -2\left(x + \frac{1}{3}\right)$
51. $y + \frac{1}{4} = -3\left(x + \frac{1}{2}\right)$
52. $y + \frac{3}{5} = -4\left(x - \frac{1}{2}\right)$

53. Write the point-slope form, slope-intercept form, and standard form of an equation for a line that passes through $(5, -3)$ with slope 10.

54. Line ℓ passes through $(1, -6)$ with slope $\frac{3}{2}$. Write the point-slope form, slope-intercept form, and standard form of an equation for line ℓ.

BUSINESS For Exercises 55–57, use the following information.
A home security company provides security systems for \$5 per week, plus an installation fee. The total fee for 12 weeks of service is \$210.

55. Write the point-slope form of an equation to find the total fee y for any number of weeks x.
56. Write the equation in slope-intercept form.
57. What is the flat fee for installation?

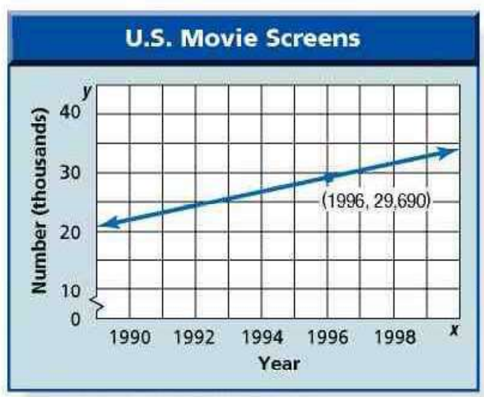
More About . . .

Movies

In 1907, movie theaters were called nickelodeons. There were about 5000 movie screens, and the average movie ticket cost 5 cents.

Source: National Association of Theatre Owners

MOVIES For Exercises 58–60, use the following information.
Between 1990 and 1999, the number of movie screens in the United States increased by about 1500 each year. In 1996, there were 29,690 movie screens.

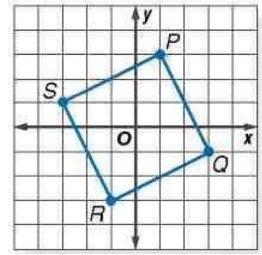


Source: Motion Picture Association of America

58. Write the point-slope form of an equation to find the total number of screens y for any year x.
59. Write the equation in slope-intercept form.
60. Predict the number of movie screens in the United States in 2005.

Online Research **Data Update** What has happened to the number of movie screens since 1999? Visit www.algebra1.com/data_update to learn more.

GEOMETRY **For Exercises 61–63, use square *PQRS*.**



61. Write a point-slope equation of the line containing each side.
62. Write the slope-intercept form of each equation.
63. Write the standard form of each equation.
64. **CRITICAL THINKING** A line contains the points (9, 1) and (5, 5). Write a convincing argument that the same line intersects the x-axis at (10, 0).
65. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

    How can you use the slope formula to write an equation of a line?

    Include the following in your answer:

    - an explanation of how you can use the slope formula to write the point-slope form.

Standardized Test Practice

66. Which equation represents a line that *neither* passes through (0, 1) *nor* has a slope of 3?

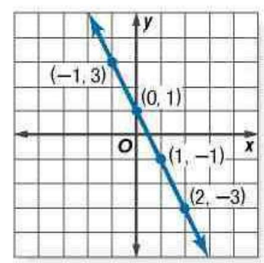
    (A) $-2x + y = 1$ (B) $y + 1 = 3(x + 6)$

    (C) $y - 3 = 3(x - 6)$ (D) $x - 3y = -15$

67. **OPEN ENDED** Write the slope-intercept form of an equation of a line that passes through (2, −5).

Extending the Lesson

For Exercises 68–71, use the graph at the right.



68. Choose three different pairs of points from the graph. Write the slope-intercept form of the line using each pair.
69. Describe how the equations are related.
70. Choose a different pair of points from the graph and predict the equation of the line determined by these points. Check your conjecture by finding the equation.
71. **MAKE A CONJECTURE** What conclusion can you draw from this activity?

Maintain Your Skills

Mixed Review

Write the slope-intercept form of an equation of the line that satisfies each condition. *(Lessons 5-3 and 5-4)*

72. slope −2 and y-intercept −5
73. passes through (−2, 4) with slope 3
74. passes through (2, −4) and (0, 6)
75. a horizontal line through (1, −1)

Solve each equation. *(Lesson 3-3)*

76. $4a - 5 = 15$
77. $7 + 3c = -11$
78. $\frac{2}{9}v - 6 = 14$

79. Evaluate $(25 - 4) \div (2^2 - 1^3)$. *(Lesson 1-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write the multiplicative inverse of each number.**
*(For review of **multiplicative inverses**, see pages 800 and 801.)*

80. 2
81. 10
82. 1
83. −1
84. $\frac{2}{3}$
85. $-\frac{1}{9}$
86. $\frac{5}{2}$
87. $-\frac{2}{3}$

5-6 Geometry: Parallel and Perpendicular Lines

What You'll Learn

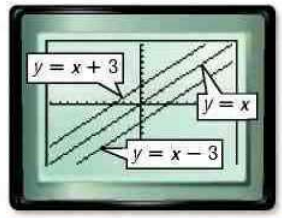
- Write an equation of the line that passes through a given point, parallel to a given line.
- Write an equation of the line that passes through a given point, perpendicular to a given line.

Vocabulary

- parallel lines
- perpendicular lines

How can you determine whether two lines are parallel?

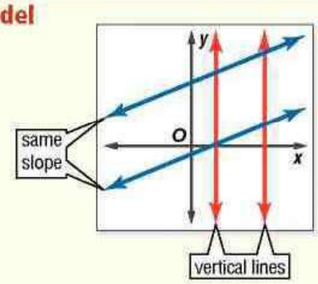
The graphing calculator screen shows a family of linear graphs whose slope is 1. Notice that the lines do not appear to intersect.



PARALLEL LINES Lines in the same plane that do not intersect are called **parallel lines**. Parallel lines have the same slope.

Key Concept — Parallel Lines in a Coordinate Plane

- **Words** Two nonvertical lines are parallel if they have the same slope. All vertical lines are parallel.
- **Model**



You can write the equation of a line parallel to a given line if you know a point on the line and an equation of the given line.

Example 1 Parallel Line Through a Given Point

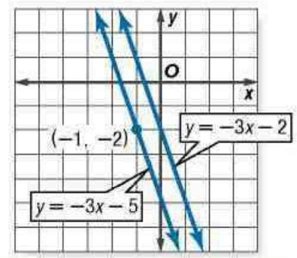
Write the slope-intercept form of an equation for the line that passes through $(-1, -2)$ and is parallel to the graph of $y = -3x - 2$.

The line parallel to $y = -3x - 2$ has the same slope, -3. Replace m with -3, and (x_1, y_1) with $(-1, -2)$ in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = -3[x - (-1)]$	Replace m with -3, y with -2, and x with -1.
$y + 2 = -3(x + 1)$	Simplify.
$y + 2 = -3x - 3$	Distributive Property
$y + 2 - 2 = -3x - 3 - 2$	Subtract 2 from each side.
$y = -3x - 5$	Write the equation in slope-intercept form.

Therefore, the equation is $y = -3x - 5$.

CHECK You can check your result by graphing both equations. The lines appear to be parallel. The graph of $y = -3x - 5$ passes through $(-1, -2)$.



PERPENDICULAR LINES

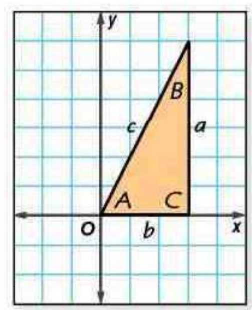
Lines that intersect at right angles are called **perpendicular lines**. There is a relationship between the slopes of perpendicular lines.

Algebra Activity

Perpendicular Lines

Model

- A scalene triangle is one in which no two sides are equal. Cut out a scalene right triangle *ABC* so that $\angle C$ is a right angle. Label the vertices and the sides as shown.
- Draw a coordinate plane on grid paper. Place $\triangle ABC$ on the coordinate plane so that *A* is at the origin and side *b* lies along the positive *x*-axis.



Analyze

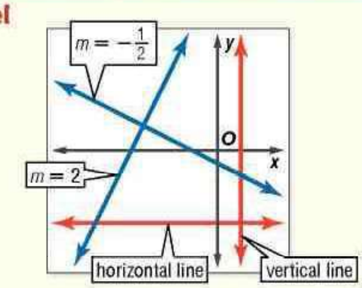
1. Name the coordinates of *B*.
2. What is the slope of side *c*?
3. Rotate the triangle 90° counterclockwise so that *A* is still at the origin and side *b* is along the positive *y*-axis. Name the coordinates of *B*.
4. What is the slope of side *c*?
5. Repeat the activity for two other different scalene triangles.
6. For each triangle and its rotation, what is the relationship between the first position of side *c* and the second?
7. For each triangle and its rotation, describe the relationship between the coordinates of *B* in the first and second positions.
8. Describe the relationship between the slopes of *c* in each position.

Make a Conjecture

9. Describe the relationship between the slopes of any two perpendicular lines.

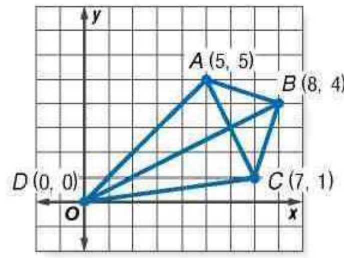
Key Concept — *Perpendicular Lines in a Coordinate Plane*

- **Words** Two lines are perpendicular if the product of their slopes is -1. That is, the slopes are *opposite reciprocals* of each other. Vertical lines and horizontal lines are also perpendicular.
- **Model**



Example 2 Determine Whether Lines are Perpendicular

KITES The outline of a kite is shown on a coordinate plane. Determine whether $\overline{AC}$ is perpendicular to $\overline{BD}$.



Find the slope of each segment.

Slope of $\overline{AC}$: $m = \frac{5-1}{5-7}$ or -2

Slope of $\overline{BD}$: $m = \frac{4-0}{8-0}$ or $\frac{1}{2}$

The line segments are perpendicular because $\frac{1}{2}(-2) = -1$.

More About...

Kites

In India, kite festivals mark *Makar Sankranti*, when the Sun moves into the northern hemisphere.

Source: www.cam-india.com

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

Example 3 Perpendicular Line Through a Given Point

Write the slope-intercept form for an equation of a line that passes through $(-3, -2)$ and is perpendicular to the graph of $x + 4y = 12$.

Step 1 Find the slope of the given line.

$x + 4y = 12$	Original equation
$x + 4y - x = 12 - x$	Subtract 1x from each side.
$4y = -1x + 12$	Simplify.
$\frac{4y}{4} = \frac{-1x + 12}{4}$	Divide each side by 4.
$y = -\frac{1}{4}x + 3$	Simplify.

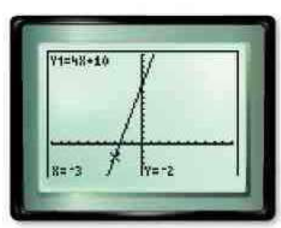
Step 2 The slope of the given line is $-\frac{1}{4}$. So, the slope of the line perpendicular to this line is the opposite reciprocal of $-\frac{1}{4}$, or 4.

Step 3 Use the point-slope form to find the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 4[x - (-3)]$	$(x_1, y_1) = (-3, -2)$ and $m = 4$
$y + 2 = 4(x + 3)$	Simplify.
$y + 2 = 4x + 12$	Distributive Property
$y + 2 - 2 = 4x + 12 - 2$	Subtract 2 from each side.
$y = 4x + 10$	Simplify.

Therefore, the equation of the line is $y = 4x + 10$.

CHECK You can check your result by graphing both equations on a graphing calculator. Use the CALC menu to verify that $y = 4x + 10$ passes through $(-3, -2)$.



$[-15.16..., 15.16...]$ scl: 1 by $[-10, 10]$ scl: 1

Study Tip

Graphing Calculator

The lines will not appear to be perpendicular on a graphing calculator if the scales on the axes are not set correctly. After graphing, press ZOOM 5 to set the axes for a correct representation.

Example 4 Perpendicular Line Through a Given Point

Write the slope-intercept form for an equation of a line perpendicular to the graph of $y = -\frac{1}{3}x + 2$ and passes through the x-intercept of that line.

Step 1 Find the slope of the perpendicular line. The slope of the given line is $-\frac{1}{3}$, therefore a perpendicular line has slope 3 because $-\frac{1}{3} \cdot 3 = -1$.

Step 2 Find the x-intercept of the given line.

$y = -\frac{1}{3}x + 2$ Original equation

$0 = -\frac{1}{3}x + 2$ Replace y with 0.

$-2 = -\frac{1}{3}x$ Subtract 2 from each side.

$6 = x$ Multiply each side by -3.

The x-intercept is at (6, 0).

Step 3 Substitute the slope and the given point into the point-slope form of a linear equation. Then write the equation in slope-intercept form.

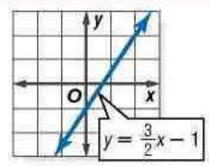
$y - y_1 = m(x - x_1)$ Point-slope form

$y - 0 = 3(x - 6)$ Replace x with 6, y with 0, and m with 3.

$y = 3x - 18$ Distributive Property

Check for Understanding

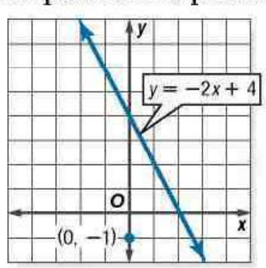
Concept Check

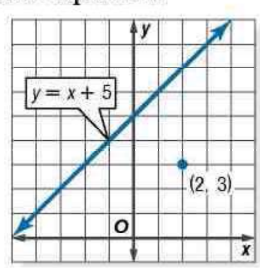


1. **Explain** how to find the slope of a line that is perpendicular to the line shown in the graph.
2. **OPEN ENDED** **Give an example** of two numbers that are negative reciprocals.
3. **Define** *parallel lines* and *perpendicular lines*.

Guided Practice

Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation.

4. 

5. 

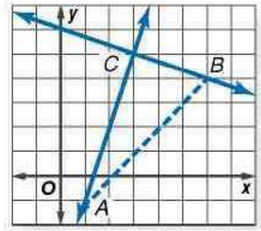
6. $(1, -3), y = 2x - 1$
7. $(-2, 2), -3x + y = 4$

8. **GEOMETRY** Quadrilateral $ABCD$ has vertices $A(-2, 1)$, $B(3, 3)$, $C(5, 7)$, and $D(0, 5)$. Determine whether $\overline{AC}$ is perpendicular to $\overline{BD}$.

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.

9. $(-3, 1), y = \frac{1}{3}x + 2$
10. $(6, -2), y = \frac{3}{5}x - 4$
11. $(2, -2), 2x + y = 5$

Application 12. **GEOMETRY** The line with equation $y = 3x - 4$ contains side $\overline{AC}$ of right triangle ABC. If the vertex of the right angle C is at $(3, 5)$, what is an equation of the line that contains side $\overline{BC}$?



Practice and Apply

Homework Help

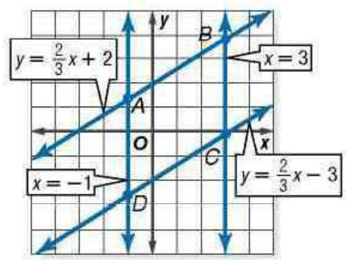
For Exercises	See Examples
13–24	1
26	2
28–39	3, 4

Extra Practice
See page 832.

Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation.

13. $(2, -7), y = x - 2$
14. $(2, -1), y = 2x + 2$
15. $(-3, 2), y = x - 6$
16. $(4, -1), y = 2x + 1$
17. $(-5, -4), y = \frac{1}{2}x + 1$
18. $(3, 3), y = \frac{2}{3}x - 1$
19. $(-4, -3), y = -\frac{1}{3}x + 3$
20. $(-1, 2), y = -\frac{1}{2}x - 4$
21. $(-3, 0), 2y = x - 1$
22. $(2, 2), 3y = -2x + 6$
23. $(-2, 3), 6x + y = 4$
24. $(2, 2), 3x - 4y = -4$

25. **GEOMETRY** A *parallelogram* is a quadrilateral in which opposite sides are parallel. Is $ABCD$ a parallelogram? Explain.



26. Write an equation of the line parallel to the graph of $y = 5x - 3$ and through the origin.

27. Write an equation of the line that has y-intercept -6 and is parallel to the graph of $x - 3y = 8$.

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.

28. $(-2, 0), y = x - 6$
29. $(1, 1), y = 4x + 6$
30. $(-3, 1), y = -3x + 7$
31. $(0, 5), y = -8x + 4$
32. $(1, -3), y = \frac{1}{2}x + 4$
33. $(4, 7), y = \frac{2}{3}x - 1$
34. $(0, 4), 3x + 8y = 4$
35. $(-2, 7), 2x - 5y = 3$
36. $(6, -1), 3y + x = 3$
37. $(0, -1), 5x - y = 3$
38. $(8, -2), 5x - 7 = 3y$
39. $(3, -3), 3x + 7 = 2x$

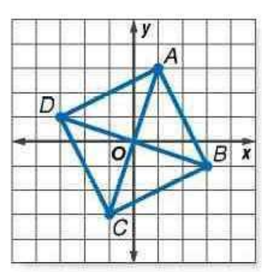
40. Find an equation of the line that has a y-intercept of -2 and is perpendicular to the graph of $3x + 6y = 2$.

41. Write an equation of the line that is perpendicular to the line through $(9, 10)$ and $(3, -2)$ and passes through the x-intercept of that line.

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

42. $y = -2x + 11$
$y + 2x = 23$
43. $3y = 2x + 14$
$2x - 3y = 2$
44. $y = -5x$
$y = 5x - 18$

45. **GEOMETRY** The diagonals of a square are segments that connect the opposite vertices. Determine the relationship between the diagonals $\overline{AC}$ and $\overline{BD}$ of square $ABCD$.



46. **CRITICAL THINKING** What is a if the lines with equations $y = ax + 5$ and $2y = (a + 4)x - 1$ are parallel?

47. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you determine whether two lines are parallel?

Include the following in your answer:

- an equation whose graph is parallel to the graph of $y = -5x$, with an explanation of your reasoning, and
- an equation whose graph is perpendicular to the graph of $y = -5x$, with an explanation of your reasoning.

Standardized Test Practice

48. What is the slope of a line perpendicular to the graph of $3x + 4y = 24$?

Ⓐ $-\frac{4}{3}$ Ⓑ $-\frac{3}{4}$ Ⓒ $\frac{3}{4}$ Ⓓ $\frac{4}{3}$

49. How can the graph of $y = 3x + 4$ be used to graph $y = 3x + 2$?

Ⓐ Move the graph of the line right 2 units.
Ⓑ Change the slope of the graph from 4 to 2.
Ⓒ Change the y-intercept from 4 to 2.
Ⓓ Move the graph of the line left 2 units.

Maintain Your Skills

Mixed Review

Write the point-slope form of an equation for a line that passes through each point with the given slope. *(Lesson 5-5)*

50. $(3, 5), m = -2$ **51.** $(-4, 7), m = 5$ **52.** $(-1, -3), m = -\frac{1}{2}$

TELEPHONE For Exercises 53 and 54, use the following information.
An international calling plan charges a rate per minute plus a flat fee. A 10-minute call to the Czech Republic costs \$3.19. A 15-minute call costs \$4.29. *(Lesson 5-4)*

53. Write a linear equation in slope-intercept form to find the total cost C of an m-minute call.

54. Find the cost of a 12-minute call.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write the slope-intercept form of an equation of the line that passes through each pair of points. *(To review **slope-intercept form**, see Lesson 5-3.)*

55. $(5, -1), (-3, 3)$ **56.** $(0, 2), (8, 0)$ **57.** $(2, 1), (3, -4)$

58. $(5, 5), (8, -1)$ **59.** $(6, 9), (4, 9)$ **60.** $(-6, 4), (2, -2)$

Practice Quiz 2

Lessons 5-3 through 5-6

Write the slope-intercept form for an equation of the line that satisfies each condition.

1. slope 4 and y-intercept -3 *(Lesson 5-3)*
2. passes through $(1, -3)$ with slope 2 *(Lesson 5-4)*
3. passes through $(-1, -2)$ and $(1, 3)$ *(Lesson 5-4)*
4. parallel to the graph of $y = 2x - 2$ and passes through $(-2, 3)$ *(Lesson 5-6)*
5. Write $y - 4 = \frac{1}{2}(x + 3)$ in standard form and in slope-intercept form. *(Lesson 5-5)*

5-7 Statistics: Scatter Plots and Lines of Fit

What You'll Learn

- Interpret points on a scatter plot.
- Write equations for lines of fit.

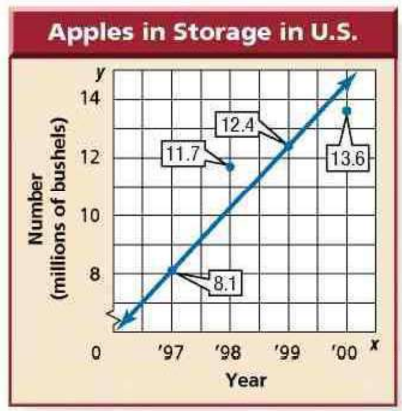
Vocabulary

- scatter plot
- positive correlation
- negative correlation
- line of fit
- best-fit line
- linear interpolation

How do scatter plots help identify trends in data?

The points of a set of real-world data do not always lie on one line. But, you may be able to draw a line that seems to be close to all the points.

The line in the graph shows a linear relationship between the year x and the number of bushels of apples y. As the years increase, the number of bushels of apples also increases.



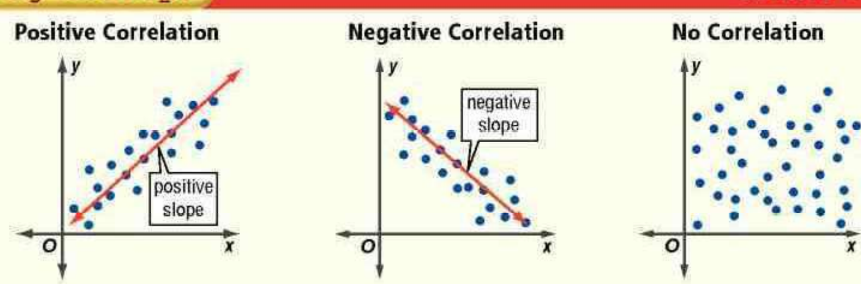
Source: U.S. Apple Association

INTERPRET POINTS ON A SCATTER PLOT A **scatter plot** is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.

- In the first graph below, there is a **positive correlation** between x and y. That is, as x increases, y increases.
- In the second graph below, there is a **negative correlation** between x and y. That is, as x increases, y decreases.
- In the third graph below, there is *no correlation* between x and y. That is, x and y are not related.

If the pattern in a scatter plot is linear, you can draw a line to summarize the data. This can help identify trends in the data.

Key Concept — Scatter Plots

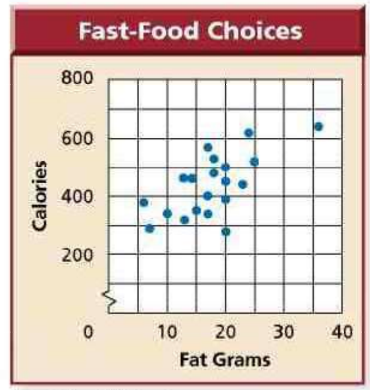


Example 1 Analyze Scatter Plots

Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation.

a. NUTRITION The graph shows fat grams and Calories for selected choices at a fast-food restaurant.

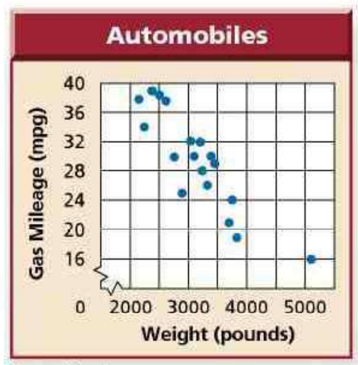
The graph shows a positive correlation. As the number of fat grams increases, the number of Calories increases.



Source: Olen Publishing Co.

b. CARS The graph shows the weight and the highway gas mileage of selected cars.

The graph shows a negative correlation. As the weight of the automobile increases, the gas mileage decreases.



Source: Yahoo!

Is there a relationship between the length of a person's foot and his or her height? Make a scatter plot and then look for a pattern.

Algebra Activity

Making Predictions

Collect the Data

- Measure your partner's foot and height in centimeters. Then trade places.
- Add the points (foot length, height) to a class scatter plot.

Analyze the Data

1. Is there a correlation between foot length and height for the members of your class? If so, describe it.
2. Draw a line that summarizes the data and shows how the height changes as the foot length changes.

Make a Conjecture

3. Use the line to predict the height of a person whose foot length is 25 centimeters. Explain your method.

LINES OF FIT If the data points do not all lie on a line, but are close to a line, you can draw a **line of fit**. This line describes the trend of the data. Once you have a line of fit, you can find an equation of the line.

In this lesson, you will use a graphical method to find a line of fit. In the follow-up to Lesson 5-7, you will use a graphing calculator to find a line of fit. The calculator uses a statistical method to find the line that most closely approximates the data. This line is called the **best-fit line**.

More About. . .

Birds

The bald eagle was listed as an endangered species in 1963, when the number of breeding pairs had dropped below 500.

Source: U.S. Fish and Wildlife Service

Example 2 Find a Line of Fit

BIRDS **The table shows an estimate for the number of bald eagle pairs in the United States for certain years since 1985.**

Years since 1985	3	5	7	9	11	14
Bald Eagle Pairs	2500	3000	3700	4500	5000	5800

Source: U.S. Fish and Wildlife Service

a. Draw a scatter plot and determine what relationship exists, if any, in the data.

Let the independent variable x be the number of years since 1985, and let the dependent variable y be the number of bald eagle pairs.

The scatter plot seems to indicate that as the number of years increases, the number of bald eagle pairs increases. There is a positive correlation between the two variables.

b. Draw a line of fit for the scatter plot.

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown in the scatter plot at the right.

c. Write the slope-intercept form of an equation for the line of fit.

The line of fit shown above passes through the data points (5, 3000) and (9, 4500).

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{4500 - 3000}{9 - 5}$ Let $(x_1, y_1) = (5, 3000)$ and $(x_2, y_2) = (9, 4500)$.

$m = \frac{1500}{4}$ or 375 Simplify.

Step 2 Use $m = 375$ and either the point-slope form or the slope-intercept form to write the equation. You can use either data point. We chose (5, 3000).

Point-slope form	Slope-intercept form
$y - y_1 = m(x - x_1)$	$y = mx + b$
$y - 3000 = 375(x - 5)$	$3000 = 375(5) + b$
$y - 3000 = 375x - 1875$	$3000 = 1875 + b$
$y = 375x + 1125$	$1125 = b$
	$y = 375x + 1125$

Using either method, $y = 375x + 1125$.

Study Tip

Lines of Fit

When you use the graphical method, the line of fit is an approximation. So, you may draw another line of fit using other points that is equally valid. Some valid lines of fit may not contain any of the data points.

CONTENTS

CHECK Check your result by substituting (9, 4500) into $y = 375x + 1125$.

$y = 375x + 1125$	Line of fit equation
$4500 \stackrel{?}{=} 375(9) + 1125$	Replace x with 9 and y with 4500.
$4500 \stackrel{?}{=} 3375 + 1125$	Multiply.
$4500 = 4500$ ✓	Add.

The solution checks.

In Lesson 5-4, you learned about linear extrapolation, which is predicting values that are *outside* the range of the data. You can also use a linear equation to predict values that are *inside* the range of the data. This is called **linear interpolation**.

Example 3 Linear Interpolation

BIRDS Use the equation for the line of fit in Example 2 to estimate the number of bald eagle pairs in 1998.

Use the equation $y = 375x + 1125$, where x is the number of years since 1985 and y is the number of bald eagle pairs.

$y = 375x + 1125$	Original equation
$y = 375(13) + 1125$	Replace x with 1998 − 1985 or 13.
$y = 6000$	Simplify.

There were about 6000 bald eagle pairs in 1998.

Check for Understanding

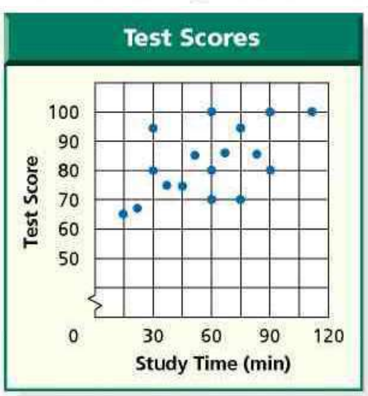
Concept Check

1. **Explain** how to determine whether a scatter plot has a positive or negative correlation.
2. **OPEN ENDED** Sketch scatter plots that have each type of correlation.
   a. positive **b.** negative **c.** no correlation
3. **Compare and contrast** linear interpolation and linear extrapolation.

Guided Practice

Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation.

4.



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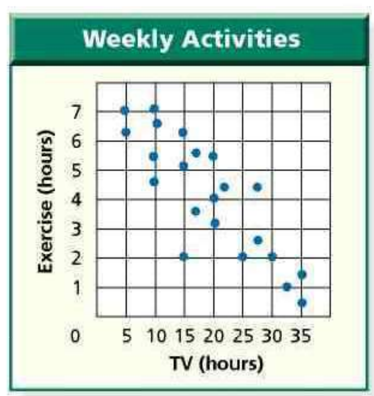


Application **BIOLOGY** **For Exercises 6–9, use the table that shows the average body temperature in degrees Celsius of 9 insects at a given air temperature.**

	Temperature (°C)								
Air	25.7	30.4	28.7	31.2	31.5	26.2	30.1	31.5	18.2
Body	27.0	31.5	28.9	31.0	31.5	25.6	28.4	31.7	18.7

6. Draw a scatter plot and determine what relationship exists, if any, in the data.
7. Draw a line of fit for the scatter plot.
8. Write the slope-intercept form of an equation for the line of fit.
9. Predict the body temperature of an insect if the air temperature is 40.2°F.

Practice and Apply

Homework Help

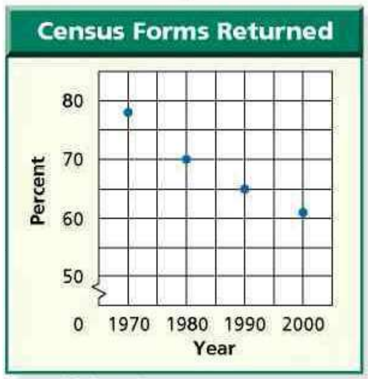
For Exercises	See Examples
10–13	1
14–33	2, 3

Extra Practice

See page 833.

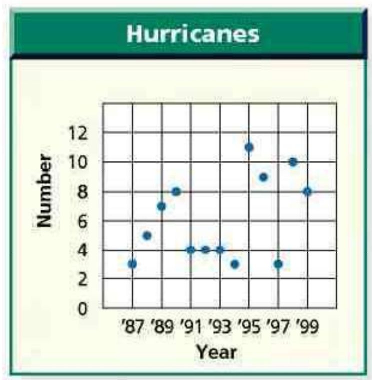
Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation.

10.



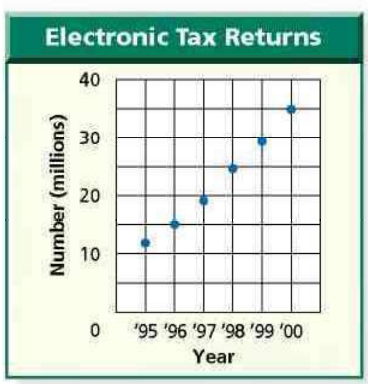
Source: U.S. Census Bureau

11.



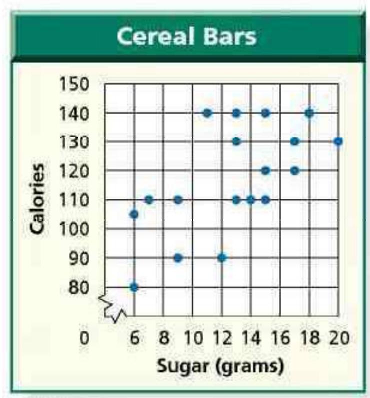
Source: *USA TODAY*

12.



Source: IRS

13.



Source: *Vitality*

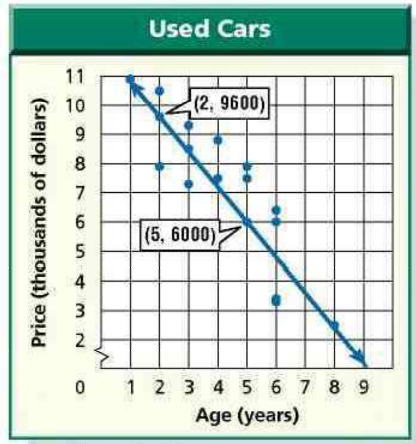
FARMING **For Exercises 14 and 15, refer to the graph at the top of page 298 about apple storage.**

14. Use the points (1997, 8.1) and (1999, 12.4) to write the slope-intercept form of an equation for the line of fit.
15. Predict the number of bushels of apples in storage in 2002.

USED CARS **For Exercises 16 and 17, use the scatter plot that shows the ages and prices of used cars from classified ads.**

16. Use the points (2, 9600) and (5, 6000) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.

17. Predict the price of a car that is 7 years old.



Source: *Columbus Dispatch*

Career Choices

Aerospace Engineer

Aerospace engineers design, develop, and test aircraft and spacecraft. Many specialize in a particular type of aerospace product, such as commercial airplanes, military fighter jets, helicopters, or spacecraft.

Online Research For information about a career as an aerospace engineer, visit: www.algebra1.com/careers

PHYSICAL SCIENCE **For Exercises 18–23, use the following information.** Hydrocarbons like methane, ethane, propane, and butane are composed of only carbon and hydrogen atoms. The table gives the number of carbon atoms and the boiling points for several hydrocarbons.

Hydrocarbons			
Name	Formula	Number of Carbon Atoms	Boiling Point (°C)
Ethane	C_2H_6	2	−89
Propane	C_3H_8	3	−42
Butane	C_4H_{10}	4	−1
Hexane	C_6H_{12}	6	69
Octane	C_8H_{18}	8	126

18. Draw a scatter plot comparing the numbers of carbon atoms to the boiling points.

19. Draw a line of fit for the data.

20. Write the slope-intercept form of an equation for the line of fit.

21. Predict the boiling point for methane (CH_4), which has 1 carbon atom.

22. Predict the boiling point for pentane (C_5H_{12}), which has 5 carbon atoms.

23. The boiling point of heptane is 98.4°C. Use the equation of the line of fit to predict the number of carbon atoms in heptane.

SPACE **For Exercises 24–28, use the table that shows the amount the United States government has spent on space and other technologies in selected years.**

Federal Spending on Space and Other Technologies								
Year	1980	1985	1990	1995	1996	1997	1998	1999
Spending (billions of dollars)	4.5	6.6	11.6	12.6	12.7	13.1	12.9	12.4

Source: U.S. Office of Management and Budget

24. Draw a scatter plot and determine what relationship, if any, exists in the data.

25. Draw a line of fit for the scatter plot.

26. Let x represent the number of years since 1980. Let y represent the spending in billions of dollars. Write the slope-intercept form of the equation for the line of fit.

27. Predict the amount that will be spent on space and other technologies in 2005.

28. The government projects spending of \$14.3 billion in space and other technologies in 2005. How does this compare to your prediction?

FORESTRY For Exercises 29–33, use the table that shows the number of acres burned by wildfires in Florida each year and the corresponding number of inches of spring rainfall.

Florida's Burned Acreage and Spring Rainfall

Year	Rainfall (inches)	Acres (thousands)	Year	Rainfall (inches)	Acres (thousands)
1988	17.5	194	1994	18.1	180
1989	12.0	645	1995	16.3	46
1990	14.0	250	1996	20.4	94
1991	30.1	87	1997	18.5	146
1992	16.0	83	1998	22.2	507
1993	19.6	80	1999	12.7	340

Source: Florida Division of Forestry

29. Draw a scatter plot with rainfall on the x-axis and acres on the y-axis.
30. Draw a line of fit for the data.
31. Write the slope-intercept form of an equation for the line of fit.
32. In 2000, there was only 8.25 inches of spring rainfall. Estimate the number of acres burned by wildfires in 2000.
33. In 1998, there was 22.2 inches of rainfall, yet 507,000 acres were burned. Where was this data graphed in the scatter plot? How did this affect the line of fit?

Online Research **Data Update** What has happened to the number of acres burned by wildfires in Florida since 1999? Visit www.algebra1.com/data_update to learn more.

WebQuest

You can use a line of fit to describe the trend in winning Olympic times. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

34. **CRITICAL THINKING** A test contains 20 true-false questions. Draw a scatter plot that shows the relationship between the number of correct answers x and the number of incorrect answers y.

RESEARCH For Exercises 35 and 36, choose a topic to research that you believe may be correlated, such as arm span and height. Find existing data or collect your own.

35. Draw a line of fit line for the data.
36. Use the line to make a prediction about the data.

37. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

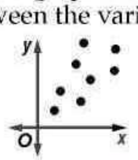
How do scatter plots help identify trends in data?

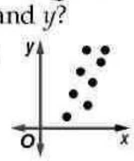
Include the following in your answer:

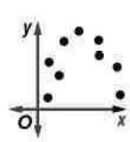
- a scatter plot that shows a person's height and his or her age, with a description of any trends, and
- an explanation of how you could use the scatter plot to predict a person's age given his or her height.

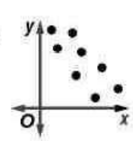
Standardized Test Practice

38. Which graph is the best example of data that show a negative linear relationship between the variables x and y?

(A) 

(B) 

(C) 

(D) 

CONTENTS

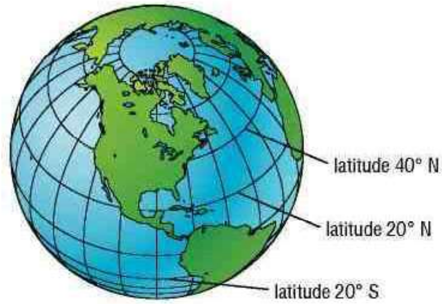
39. Choose the equation for the line that best fits the data in the table at the right.

Ⓐ $y = x + 4$
Ⓑ $y = 2x + 3$
Ⓒ $y = 7$
Ⓓ $y = 4x - 5$

x	y
1	5
2	7
3	7
4	11

Extending the Lesson

GEOGRAPHY **For Exercises 40–44, use the following information.**
The *latitude* of a place on Earth is the measure of its distance from the equator.



40. **MAKE A CONJECTURE** What do you think is the relationship between a city's latitude and its January temperature?

41. **RESEARCH** Use the Internet or other reference to find the latitude of 15 cities in the northern hemisphere and the corresponding January mean temperatures.

42. Make a scatter plot and draw a line of fit for the data.

43. Write an equation for the line of fit.

44. **MAKE A CONJECTURE** Find the latitude of your city and use the equation to predict its mean January temperature. Check your prediction by using another source such as the newspaper.

Maintain Your Skills

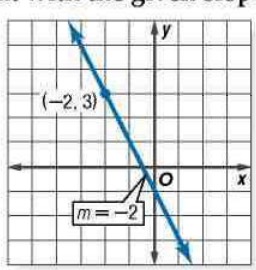
Mixed Review

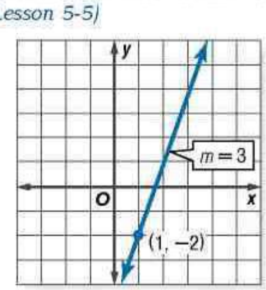
Write the slope-intercept form of an equation for the line that satisfies each condition. *(Lesson 5-6)*

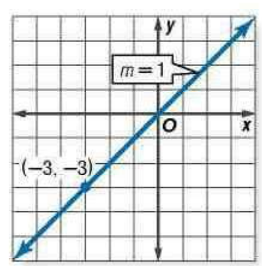
45. parallel to the graph of $y = -4x + 5$ and passes through $(-2, 5)$

46. perpendicular to the graph of $y = 2x + 3$ and passes through $(0, 0)$

Write the point-slope form of an equation for a line that passes through each point with the given slope. *(Lesson 5-5)*

47. 

48. 

49. 

Find the x- and y-intercepts of the graph of each equation. *(Lesson 4-5)*

50. $3x + 4y = 12$

51. $2x - 5y = 8$

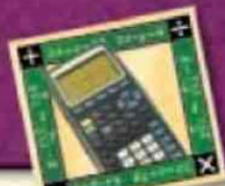
52. $y = 3x + 6$

Solve each equation. Then check your solution. *(Lesson 3-4)*

53. $\frac{r+7}{-4} = \frac{r+2}{6}$

54. $\frac{n-(-4)}{-3} = 7$

55. $\frac{2x-1}{5} = \frac{4x-5}{7}$



Graphing Calculator

A Follow-Up of Lesson 5-7

Regression and Median-Fit Lines

One type of equation of best-fit you can find is a linear **regression equation**.

EARNINGS **The table shows the average hourly earnings of U.S. production workers for selected years.**

Year	1960	1965	1970	1975	1980	1985	1990	1995	1999
Earnings	$2.09	2.46	3.23	4.53	6.66	8.57	10.01	11.43	13.24

Source: Bureau of Labor Statistics

Find and graph a linear regression equation. Then predict the average hourly earnings in 2010.

Step 1 *Find a regression equation.*

- Enter the years in L1 and the earnings in L2.
  KEYSTROKES: *Review entering a list on page 204.*
- Find the regression equation by selecting LinReg(*ax*+*b*) on the STAT CALC menu.
  KEYSTROKES: STAT ▶ 4 ENTER

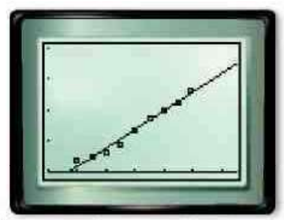
The equation is in the form $y = ax + b$.

The equation is about $y = 0.30x - 588.35$.

r is the **linear correlation coefficient**. The closer the absolute value of r is to 1, the better the equation models the data. Because the r value is close to 1, the model fits the data well.

Step 2 *Graph the regression equation.*

- Use STAT PLOT to graph the scatter plot.
  KEYSTROKES: *Review statistical plots on page 204.*
- Copy the equation to the Y= list and graph.
  KEYSTROKES: Y= VARS 5 ▶ ▶ 1 GRAPH

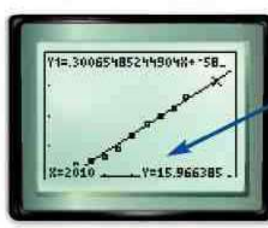


[1950, 2000] scl: 10 by [0, 20] scl: 5

Step 3 *Predict using the regression equation.*

- Find y when $x = 2010$ using value on the CALC menu.
  KEYSTROKES: 2nd [CALC] 1 2010 ENTER

According to the regression equation, the average hourly earnings in 2010 will be about $15.97.



The graph and the coordinates of the point are shown.

www.algebra1.com/other_calculator_keystrokes

Investigation

A second type of best-fit line that can be found using a graphing calculator is a **median-fit line**. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.

Find and graph a median-fit equation for the data on hourly earnings. Then predict the average hourly earnings in 2010. Compare this prediction to the one made using the regression equation.

Step 1 *Find a median-fit equation.*

- The data are already in Lists 1 and 2. Find the median-fit equation by using Med-Med on the STAT CALC menu.

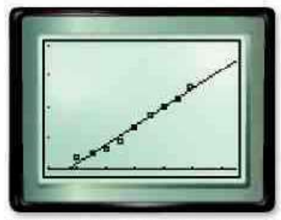
KEYSTROKES: STAT ▶ 3 ENTER

The median-fit equation is $y = 0.299x - 585.17$.

Step 2 *Graph the median-fit equation.*

- Copy the equation to the Y= list and graph.

KEYSTROKES: Y= VARS 5 ▶ ▶ 1 GRAPH

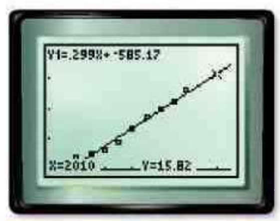


[1950, 2010] scl: 10 by [0, 20] scl: 5

Step 3 *Predict using the median-fit equation.*

KEYSTROKES: 2nd [CALC] 1 2010 ENTER

According to the median-fit equation, the average hourly earnings in 2010 will be about \$15.82. This is slightly less than the predicted value found using the regression equation.



Exercises

Refer to the data on bald eagles in Example 2 on pages 300 and 301.

1. Find regression and median-fit equations for the data.

2. What is the correlation coefficient of the regression equation? What does it tell you about the data?

3. Use the regression and median-fit equations to predict the number of bald eagle pairs in 1998. Compare these to the number found in Example 3 on page 301.

For Exercises 4 and 5, use the table that shows the number of votes cast for the Democratic presidential candidate in selected North Carolina counties in the 1996 and 2000 elections.

4. Find regression and median-fit equations for the data.

5. In 1996, New Hanover County had 22,839 votes for the Democratic candidate. Use the regression and median-fit equations to estimate the number of votes for the Democratic candidate in that county in 2000. How do the predictions compare to the actual number of 29,292?

1996	2000
14,447	16,284
19,458	19,281
28,674	30,921
31,658	38,545
32,739	38,626
46,543	52,457
49,186	53,907
69,208	80,787
103,429	126,911
103,574	123,466

Source: NC State Board of Elections

Chapter 5

Study Guide and Review

Vocabulary and Concept Check

best-fit line (p. 300)
constant of variation (p. 264)
direct variation (p. 264)
family of graphs (p. 265)
linear extrapolation (p. 283)
linear interpolation (p. 301)
line of fit (p. 300)
negative correlation (p. 298)
parallel lines (p. 292)
parent graph (p. 265)
perpendicular lines (p. 293)
point-slope form (p. 286)
positive correlation (p. 298)
rate of change (p. 258)
scatter plot (p. 298)
slope (p. 256)
slope-intercept form (p. 272)

Exercises **Choose the correct term to complete each sentence.**

1. An equation of the form $y = kx$ describes a *(direct variation, linear extrapolation)*.
2. The ratio of *(rise, run)*, or vertical change, to the *(rise, run)*, or horizontal change, as you move from one point on a line to another, is the slope of the line.
3. The lines with equations $y = -2x + 7$ and $y = -2x - 6$ are *(parallel, perpendicular)* lines.
4. The equation $y - 2 = -3(x - 1)$ is written in *(point-slope, slope-intercept)* form.
5. The equation $y = -\frac{1}{3}x + 6$ is written in *(slope-intercept, standard)* form.
6. The *(x-intercept, y-intercept)* of the equation $-x - 4y = 2$ is $-\frac{1}{2}$.

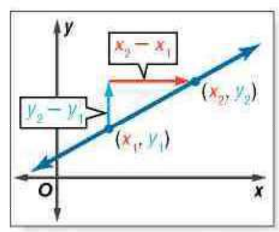
Lesson-by-Lesson Review

5-1 Slope

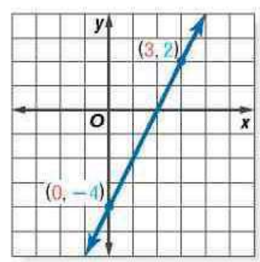
See pages 256–262.

Concept Summary

- The slope of a line is the ratio of the rise to the run.
- $m = \frac{y_2 - y_1}{x_2 - x_1}$



Example **Determine the slope of the line that passes through (0, −4) and (3, 2).**



Let $(0, -4) = (x_1, y_1)$ and $(3, 2) = (x_2, y_2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{2 - (-4)}{3 - 0}$ $x_1 = 0, x_2 = 3, y_1 = -4, y_2 = 2$

$m = \frac{6}{3}$ or 2 Simplify.

Exercises **Find the slope of the line that passes through each pair of points.**
See Examples 1–4 on page 257.

7. (1, 3), (−2, −6)
8. (0, 5), (6, 2)
9. (−6, 4), (−6, −2)
10. (8, −3), (−2, −3)
11. (2.9, 4.7), (0.5, 1.1)
12. $\left(\frac{1}{2}, 1\right), \left(-1, \frac{2}{3}\right)$

www.algebra1.com/vocabulary_review

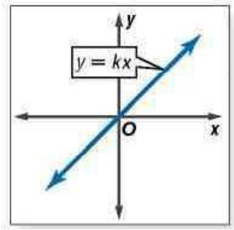
CONTENTS

5-2 Slope and Direct Variation

See pages 264–270.

Concept Summary

- A direct variation is described by an equation of the form $y = kx$, where $k \neq 0$.
- In $y = kx$, k is the constant of variation. It is also the slope of the related graph.



Example **Suppose y varies directly as x, and $y = -24$ when $x = 8$. Write a direct variation equation that relates x and y.**

$y = kx$ Direct variation equation

$-24 = k(8)$ Replace y with -24 and x with 8.

$\frac{-24}{8} = \frac{k(8)}{8}$ Divide each side by 8.

$-3 = k$ Simplify.

Therefore, $y = -3x$.

Exercises **Graph each equation.** *See Examples 2 and 3 on page 265.*

13. $y = 2x$ **14.** $y = -4x$ **15.** $y = \frac{1}{3}x$

16. $y = -\frac{1}{4}x$ **17.** $y = \frac{3}{2}x$ **18.** $y = -\frac{4}{3}x$

Suppose y varies directly as x. Write a direct variation equation that relates x and y. *See Example 4 on page 266.*

19. $y = -6$ when $x = 9$ **20.** $y = 15$ when $x = 2$ **21.** $y = 4$ when $x = -4$

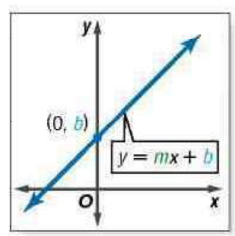
22. $y = -6$ when $x = -18$ **23.** $y = -10$ when $x = 5$ **24.** $y = 7$ when $x = -14$

5-3 Slope-Intercept Form

See pages 272–277.

Concept Summary

- The linear equation $y = mx + b$ is written in slope-intercept form, where m is the slope, and b is the y-intercept.
- Slope-intercept form allows you to graph an equation quickly.



Example **Graph $-3x + y = -1$.**

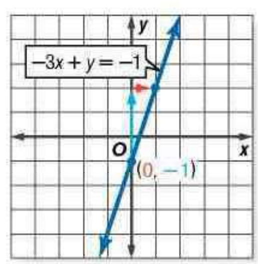
$-3x + y = -1$ Original equation

$-3x + y + 3x = -1 + 3x$ Add $3x$ to each side.

$y = 3x - 1$ Simplify.

Step 1 The y-intercept is -1. So, graph $(0, -1)$.

Step 2 The slope is 3 or $\frac{3}{1}$. From $(0, -1)$, move up 3 units and right 1 unit. Then draw a line.



CONTENTS

Exercises **Write an equation of the line with the given slope and y-intercept.**
See Examples 1 and 2 on pages 272 and 273.

25. slope: 3, y-intercept: 2
26. slope: 1, y-intercept: -3
27. slope: 0, y-intercept: 4
28. slope: $\frac{1}{3}$, y-intercept: 2
29. slope: 0.5, y-intercept: -0.3
30. slope: -1.3, y-intercept: 0.4

Graph each equation. *See Examples 3 and 4 on pages 273 and 274.*

31. $y = 2x + 1$
32. $y = -x + 5$
33. $y = \frac{1}{2}x + 3$
34. $y = -\frac{4}{3}x - 1$
35. $5x - 3y = -3$
36. $6x + 2y = 9$

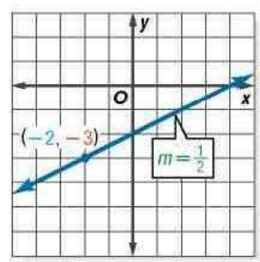
5-4 Writing Equations in Slope-Intercept Form

See pages 280–285.

Concept Summary

- To write an equation given the slope and one point, substitute the values of m, x, and y into the slope-intercept form and solve for b. Then, write the slope-intercept form using the values of m and b.
- To write an equation given two points, find the slope. Then follow the steps above.

Example **Write an equation of a line that passes through $(-2, -3)$ with slope $\frac{1}{2}$.**



$y = mx + b$	Slope-intercept form
$-3 = \frac{1}{2}(-2) + b$	Replace m with $\frac{1}{2}$, y with -3, and x with -2.
$-3 = -1 + b$	Multiply.
$-3 + 1 = -1 + b + 1$	Add 1 to each side.
$-2 = b$	Simplify.

Therefore, the equation is $y = \frac{1}{2}x - 2$.

Exercises **Write an equation of the line that satisfies each condition.**
See Examples 1 and 2 on pages 280 and 281.

37. passes through $(-3, 3)$ with slope 1
38. passes through $(0, 6)$ with slope -2
39. passes through $(1, 6)$ with slope $\frac{1}{2}$
40. passes through $(4, -3)$ with slope $-\frac{3}{5}$
41. passes through $(-4, 2)$ and $(1, 12)$
42. passes through $(5, 0)$ and $(4, 5)$
43. passes through $(8, -1)$ with slope 0
44. passes through $(4, 6)$ and has slope 0

5-5 Writing Equations in Point-Slope Form

See pages 286–291.

Concept Summary

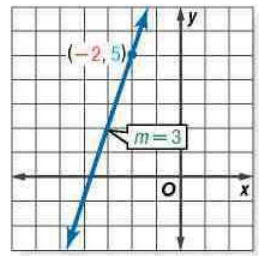
- The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope.

Example **Write the point-slope form of an equation for a line that passes through (−2, 5) with slope 3.**

$y - y_1 = m(x - x_1)$ Use the point-slope form.

$y - 5 = 3[x - (-2)]$ $(x_1, y_1) = (-2, 5)$

$y - 5 = 3(x + 2)$ Subtract.



Exercises **Write the point-slope form of an equation for a line that passes through each point with the given slope.** *See Example 2 on page 287.*

45. $(4, 6), m = 5$ **46.** $(-1, 4), m = -2$ **47.** $(5, -3), m = \frac{1}{2}$

48. $(1, -4), m = -\frac{5}{2}$ **49.** $\left(\frac{1}{4}, -2\right), m = 3$ **50.** $(4, -2), m = 0$

Write each equation in standard form. *See Example 3 on page 287.*

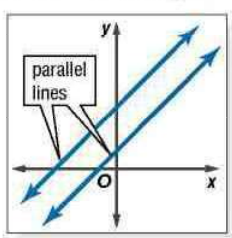
51. $y - 1 = 2(x + 1)$ **52.** $y + 6 = \frac{1}{3}(x - 9)$ **53.** $y + 4 = 1.5(x - 4)$

5-6 Geometry: Parallel and Perpendicular Lines

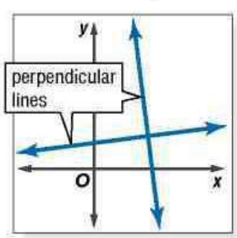
See pages 292–297.

Concept Summary

- Two nonvertical lines are parallel if they have the same slope.



- Two lines are perpendicular if the product of their slopes is −1.



Example **Write the slope-intercept form for an equation of the line that passes through (5, −2) and is parallel to $y = 2x + 7$.**

The line parallel to $y = 2x + 7$ has the same slope, 2.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = 2(x - 5)$ Replace m with 2, y with −2, and x with 5.

$y + 2 = 2x - 10$ Simplify.

$y = 2x - 12$ Subtract 2 from each side.

Chapter 5 For More ...

- Extra Practice, see pages 831–833.
- Mixed Problem Solving, see page 857.

Exercises **Write the slope-intercept form for an equation of the line parallel to the given equation and passing through the given point.**
See Example 1 on page 292.

54. $y = 3x - 2, (4, 6)$ **55.** $y = -2x + 4, (6, -6)$ **56.** $y = -6x - 1, (1, 2)$

57. $y = \frac{5}{12}x + 2, (0, 4)$ **58.** $4x - y = 7, (2, -1)$ **59.** $3x + 9y = 1, (3, 0)$

Write the slope-intercept form for an equation of the line perpendicular to the given equation and passing through the given point. *See Example 3 on page 294.*

60. $y = 4x + 2, (1, 3)$ **61.** $y = -2x - 7, (0, -3)$ **62.** $y = 0.4x + 1, (2, -5)$

63. $2x - 7y = 1, (-4, 0)$ **64.** $8x - 3y = 7, (4, 5)$ **65.** $5y = -x + 1, (2, -5)$

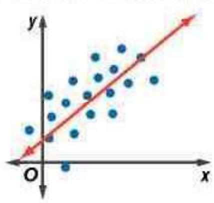
5-7 Statistics: Scatter Plots and Lines of Fit

See pages 298–305.

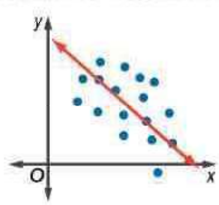
Concept Summary

- If y increases as x increases, then there is a positive correlation between x and y.
- If y decreases as x increases, then there is a negative correlation between x and y.
- If there is no relationship between x and y, then there is no correlation between x and y.
- A line of fit describes the trend of the data.
- You can use the equation of a line of fit to make predictions about the data.

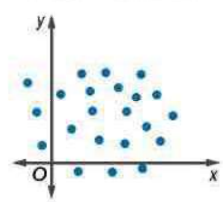
Positive Correlation



Negative Correlation



No Correlation



Exercises **For Exercises 66–70, use the table that shows the length and weight of several humpback whales.** *See Examples 2 and 3 on pages 300 and 301.*

Length (ft)	40	42	45	46	50	52	55
Weight (long tons)	25	29	34	35	43	45	51

66. Draw a scatter plot with length on the x-axis and weight on the y-axis.

67. Draw a line of fit for the data.

68. Write the slope-intercept form of an equation for the line of fit.

69. Predict the weight of a 48-foot humpback whale.

70. Most newborn humpback whales are about 12 feet in length. Use the equation of the line of fit to predict the weight of a newborn humpback whale. Do you think your prediction is accurate? Explain.

Chapter 5 Practice Test

Vocabulary and Concepts

1. **Explain** why the equation of a vertical line cannot be in slope-intercept form.
2. **Draw** a scatter plot that shows a positive correlation.
3. **Name** the part of the slope-intercept form that represents the rate of change.

Skills and Applications

Find the slope of the line that passes through each pair of points.

4. $(5, 8), (-3, 7)$
5. $(5, -2), (3, -2)$
6. $(6, -3), (6, 4)$

7. **BUSINESS** A web design company advertises that it will design and maintain a website for your business for $9.95 per month. Write a direct variation equation to find the total cost C for any number of months m.

Graph each equation.

8. $y = 3x - 1$
9. $y = 2x + 3$
10. $2x + 3y = 9$

11. **WEATHER** The temperature is 16°F at midnight and is expected to fall 2° each hour during the night. Write the slope-intercept form of an equation to find the temperature T for any hour h after midnight.

Suppose y varies directly as x. Write a direct variation equation that relates x and y.

12. $y = 6$ when $x = 9$
13. $y = -12$ when $x = 4$
14. $y = -8$ when $x = 8$

Write the slope-intercept form of an equation of the line that satisfies each condition.

15. has slope -4 and y-intercept 3
16. passes through $(-2, -5)$ and $(8, -3)$
17. parallel to $3x + 7y = 4$ and passes through $(5, -2)$
18. a horizontal line passing through $(5, -8)$
19. perpendicular to the graph of $5x - 3y = 9$ and passes through the origin
20. Write the point-slope form of an equation for a line that passes through $(-4, 3)$ with slope -2.

ANIMALS **For Exercises 21–24, use the table that shows the relationship between dog years and human years.**

Dog Years	1	2	3	4	5	6	7
Human Years	15	24	28	32	37	42	47

21. Draw a scatter plot and determine what relationship, if any, exists in the data.
22. Draw a line of fit for the scatter plot.
23. Write the slope-intercept form of an equation for the line of fit.
24. Determine how many human years are comparable to 13 dog years.

25. **STANDARDIZED TEST PRACTICE** A line passes through (0, 4) and (3, 0). Which equation does *not* represent the equation of this line?

Ⓐ $y - 4 = -\frac{4}{3}(x - 0)$ Ⓑ $y = -\frac{4}{3}x + 3$ Ⓒ $\frac{x}{3} + \frac{y}{4} = 1$

Ⓓ $y - 0 = -\frac{4}{3}(x - 3)$ Ⓔ $4x + 3y = 12$

Chapter 5 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If a person's weekly salary is $\$x$ and she saves $\$y$, what fraction of her weekly salary does she spend? (Lesson 1-1)

(A) $\frac{x}{y}$ (B) $\frac{x-y}{x}$

(C) $\frac{x-y}{y}$ (D) $\frac{y-x}{x}$

2. Evaluate $-2x + 7y$ if $x = -5$ and $y = 4$. (Lesson 2-6)

(A) 38 (B) 43

(C) 227 (D) 243

3. Find x, if $5x + 6 = 10$. (Lesson 3-3)

(A) $-\frac{5}{4}$ (B) $\frac{1}{10}$

(C) $\frac{5}{16}$ (D) $\frac{4}{5}$

4. According to the data in the table, which of the following statements is true? (Lesson 3-7)

Age	Frequency
8	1
10	3
14	2
16	1
17	2

(A) mean age = median age

(B) mean age > median age

(C) mean age < median age

(D) median age < mode age

5. What relationship exists between the x- and y-coordinates of each of the data points shown in the table? (Lesson 4-1)

x	y
−3	4
−2	3
0	1
1	0
3	−2
5	−4

(A) x and y are opposites.

(B) The sum of x and y is 2.

(C) The y-coordinate is 1 more than the square of the x-coordinate.

(D) The y-coordinate is 1 more than the opposite of the x-coordinate.

6. What is the y-intercept of the line with equation $\frac{x}{3} - \frac{y}{2} = 1$? (Lesson 4-5)

(A) -3 (B) -2

(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

7. Find the slope of a line that passes through (2, 4) and (24, 7). (Lesson 5-1)

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) -2 (D) 2

8. Which equation represents the line that passes through (3, 7) and (21, 21)? (Lesson 5-4)

(A) $x + y = 10$ (B) $y = \frac{1}{2}x + \frac{11}{2}$

(C) $y = 2x + 1$ (D) $y = 3x - 2$

9. Choose the equation of a line parallel to the graph of $y = 3x + 4$. (Lesson 5-6)

(A) $y = -\frac{1}{3}x + 4$ (B) $y = -3x + 4$

(C) $y = -x + 1$ (D) $y = 3x + 5$

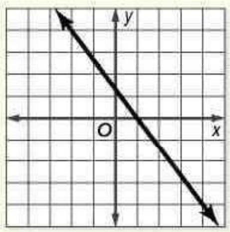
Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. While playing a game with her friends, Ellen scored 12 points less than twice the lowest score. She scored 98. What was the lowest score in the game? (Lesson 3-4)

11. The graph of $3x + 2y = 3$ is shown at the right. What is the y-intercept? (Lesson 5-3)



12. The table of ordered pairs shows the coordinates of some of the points on the graph of a function.

x	y
−1	6
0	4
1	2
2	0
3	−2

What is the y-coordinate of a point $(5, y)$ that lies on the graph of the function? (Lesson 5-4)

13. The equation $y - 3 = -2(x + 5)$ is written in point-slope form. What is the slope of the line? (Lesson 5-5)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the two quantities are equal, or**

Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
14.	$2(x + 6)$	$2x + 6$

(Lesson 1-2)

	Column A	Column B
15.	$\|x\|$	$\|x + 1\|$

(Lesson 2-1)

	Column A	Column B
16.	the slope of any nonvertical line	the slope of the line parallel to the line in Column A

(Lesson 5-6)

	Column A	Column B
17.	the slope of $y = -2x$	the slope of the line perpendicular to $y = -2x$

(Lesson 5-6)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

18. A friend wants to enroll for cellular phone service. Three different plans are available. (Lesson 5-5)

Plan 1 charges \$0.59 per minute.

Plan 2 charges a monthly fee of \$10, plus \$0.39 per minute.

Plan 3 charges a monthly fee of \$59.95.

a. For each plan, write an equation that represents the monthly cost C for m number of minutes per month.

b. Graph each of the three equations.

c. Your friend expects to use 100 minutes per month. In which plan do you think that your friend should enroll? Explain.

The Princeton Review Test-Taking Tip

Questions 14–17 Before you choose answer A, B, or C on quantitative comparison questions, ask yourself: "Is this always the case?" If not, mark D.

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