Chapter 6

Solving Linear Inequalities

What You'll Learn

- **Lessons 6-1 through 6-3** Solve linear inequalities.
- **Lesson 6-4** Solve compound inequalities and graph their solution sets.
- **Lesson 6-5** Solve absolute value equations and inequalities.
- **Lesson 6-6** Graph inequalities in the coordinate plane.

Key Vocabulary

- set-builder notation (p. 319)
- compound inequality (p. 339)
- intersection (p. 339)
- union (p. 340)
- half-plane (p. 353)

Why It's Important

Inequalities are used to represent various real-world situations in which a quantity must fall within a range of possible values. For example, figure skaters and gymnasts frequently want to know what they need to score to win a competition. That score can be represented by an inequality. *You will learn how a competitor can determine what score is needed to win in Lesson 6-1.*

CONTENTS

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-3 — Solve Equations

Solve each equation. *(For review, see Lessons 3-2, 3-4, and 3-5.)*

1. $t + 31 = 84$ **2.** $b - 17 = 23$ **3.** $18 = 27 + f$ **4.** $d - \frac{2}{3} = \frac{1}{2}$

5. $3r - 45 = 4r$ **6.** $5m + 7 = 4m - 12$ **7.** $3y + 4 = 16$ **8.** $2a + 5 - 3a = 4$

9. $\frac{1}{2}k - 4 = 7$ **10.** $4.3b + 1.8 = 8.25$ **11.** $6s - 12 = 2(s + 2)$ **12.** $n - 3 = \frac{n+1}{2}$

For Lesson 6-5 — Evaluate Absolute Values

Find each value. *(For review, see Lesson 2-1.)*

13. $|-8|$ **14.** $|20|$ **15.** $|-30|$ **16.** $|-1.5|$

17. $|14 - 7|$ **18.** $|1 - 16|$ **19.** $|2 - 3|$ **20.** $|7 - 10|$

For Lesson 6-6 — Graph Equations with Two Variables

Graph each equation. *(For review, see Lesson 4-5.)*

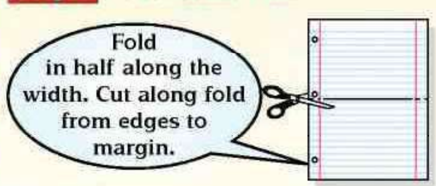
21. $2x + 2y = 6$ **22.** $x - 3y = -3$ **23.** $y = 2x - 3$ **24.** $y = -4$

25. $x = -\frac{1}{2}y$ **26.** $3x - 6 = 2y$ **27.** $15 = 3(x + y)$ **28.** $2 - x = 2y$

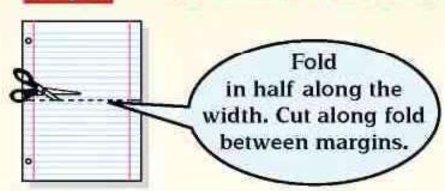
FOLDABLES™ Study Organizer

Make this Foldable to record information about solving linear inequalities. Begin with two sheets of notebook paper.

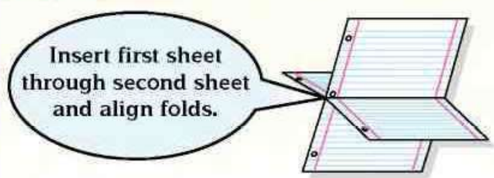
Step 1 Fold and Cut



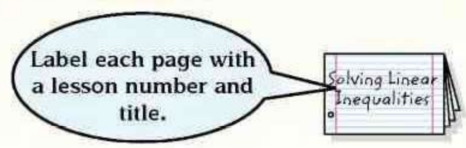
Step 2 Fold a New Paper and Cut



Step 3 Fold



Step 4 Label



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples of linear inequalities.

6-1 Solving Inequalities by Addition and Subtraction

What You'll Learn

- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

Vocabulary

- set-builder notation

How are inequalities used to describe school sports?

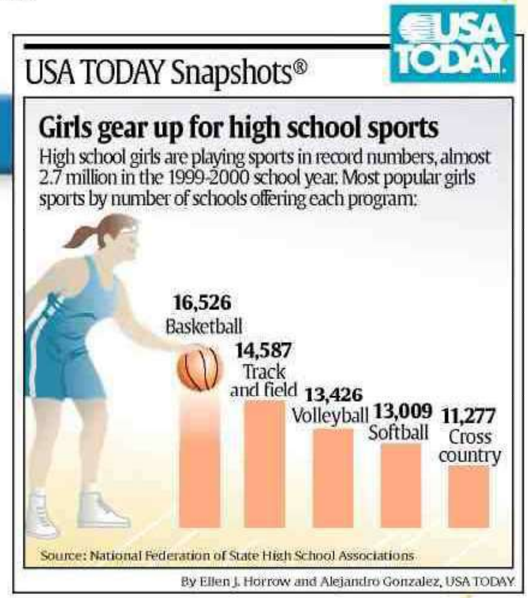
In the 1999–2000 school year, more high schools offered girls' track and field than girls' volleyball.

$$14{,}587 > 13{,}426$$

If 20 schools added girls' track and field and 20 schools added girls' volleyball the next school year, there would still be more schools offering girls' track and field than schools offering girls' volleyball.

$$14{,}587 + 20 \ \underline{\ ?\ }\ 13{,}426 + 20$$

$$14{,}607 > 13{,}446$$



Study Tip

Look Back
To review **inequalities**, see Lesson 1-3.

SOLVE INEQUALITIES BY ADDITION Recall that statements with greater than (>), less than (<), greater than or equal to (≥), or less than or equal to (≤) are *inequalities*. The sports application illustrates the **Addition Property of Inequalities**.

Key Concept — Addition Property of Inequalities

- **Words** If any number is added to each side of a true inequality, the resulting inequality is also true.
- **Symbols** For all numbers a, b, and c, the following are true.
  1. If $a > b$, then $a + c > b + c$.
  2. If $a < b$, then $a + c < b + c$.
- **Example**
  $2 < 7$
  $2 + 6 < 7 + 6$
  $8 < 13$

This property is also true when > and < are replaced with ≥ and ≤.

Example 1 Solve by Adding

Solve $t - 45 \le 13$. Then check your solution.

$t - 45 \le 13$	Original inequality
$t - 45 + 45 \le 13 + 45$	Add 45 to each side.
$t \le 58$	This means all numbers less than or equal to 58.

CHECK Substitute 58, a number less than 58, and a number greater than 58.

Let $t = 58$.	Let $t = 50$.	Let $t = 60$.
$58 - 45 \stackrel{?}{\le} 13$	$50 - 45 \stackrel{?}{\le} 13$	$60 - 45 \stackrel{?}{\le} 13$
$13 \le 13$ ✓	$5 \le 13$ ✓	$15 \not\le 13$

The solution is the set {all numbers less than or equal to 58}.

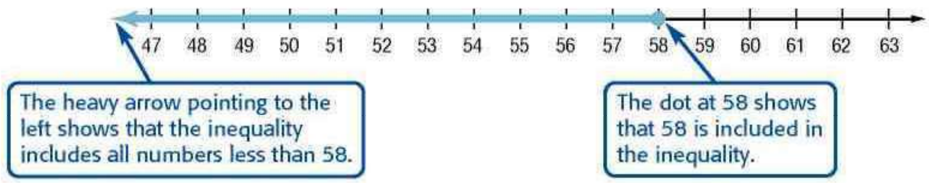
Study Tip

Reading Math

$\{t \mid t \leq 58\}$ is read the set of all numbers t such that t is less than or equal to 58.

The solution of the inequality in Example 1 was expressed as a set. A more concise way of writing a solution set is to use **set-builder notation**. The solution in set-builder notation is $\{t \mid t \leq 58\}$.

The solution to Example 1 can also be represented on a number line.



Example 2 Graph the Solution

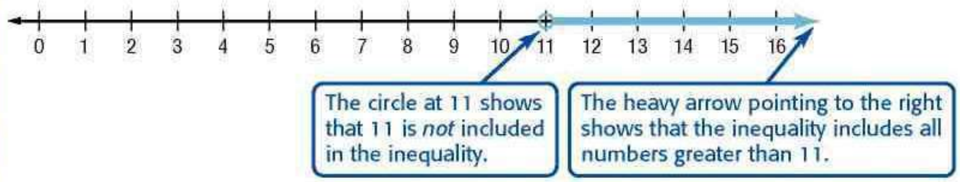
Solve $7 < x - 4$. Then graph it on a number line.

$7 < x - 4$ Original inequality

$7 + 4 < x - 4 + 4$ Add 4 to each side.

$11 < x$ Simplify.

Since $11 < x$ is the same as $x > 11$, the solution set is $\{x \mid x > 11\}$.



SOLVE INEQUALITIES BY SUBTRACTION Subtraction can also be used to solve inequalities.

Key Concept — Subtraction Property of Inequalities

- **Words** If any number is subtracted from each side of a true inequality, the resulting inequality is also true.
- **Symbols** For all numbers a, b, and c, the following are true.
  1. If $a > b$, then $a - c > b - c$.
  2. If $a < b$, then $a - c < b - c$.
- **Example** $17 > 8$; $17 - 5 > 8 - 5$; $12 > 3$

This property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

Example 3 Solve by Subtracting

Solve $19 + r \geq 16$. Then graph the solution.

$19 + r \geq 16$ Original inequality

$19 + r - 19 \geq 16 - 19$ Subtract 19 from each side.

$r \geq -3$ Simplify.

The solution set is $\{r \mid r \geq -3\}$.

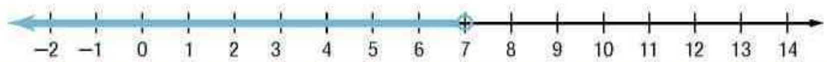
Terms with variables can also be subtracted from each side to solve inequalities.

Example 4 Variables on Both Sides

Solve $5p + 7 > 6p$. Then graph the solution.

$5p + 7 > 6p$	Original inequality
$5p + 7 - 5p > 6p - 5p$	Subtract $5p$ from each side.
$7 > p$	Simplify.

Since $7 > p$ is the same as $p < 7$, the solution set is $\{p \mid p < 7\}$.



Verbal problems containing phrases like *greater than* or *less than* can often be solved by using inequalities. The following chart shows some other phrases that indicate inequalities.

Inequalities			
<	>	≤	≥
• less than • fewer than	• greater than • more than	• at most • no more than • less than or equal to	• at least • no less than • greater than or equal to

Example 5 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

Four times a number is no more than three times that number plus eight.

$$\underbrace{4n}_{\text{Four times a number}} \quad \underbrace{\leq}_{\text{is no more than}} \quad \underbrace{3n}_{\text{three times that number}} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{8}_{\text{eight.}}$$

$4n \leq 3n + 8$	Original inequality
$4n - 3n \leq 3n + 8 - 3n$	Subtract $3n$ from each side.
$n \leq 8$	Simplify.

The solution set is $\{n \mid n \leq 8\}$.

Example 6 Write an Inequality to Solve a Problem

OLYMPICS **Yulia Raskina scored a total of 39.548 points in the four events of rhythmic gymnastics. Yulia Barsukova scored 9.883 in the rope competition, 9.900 in the hoop competition, and 9.916 in the ball competition. How many points did Barsukova need to score in the ribbon competition to surpass Raskina and win the gold medal?**

Words Barsukova's total must be greater than Raskina's total.

Variable Let r = Barsukova's score in the ribbon competition.

Inequality $\underbrace{9.883 + 9.900 + 9.916 + r}_{\text{Barsukova's total}} \quad \underbrace{>}_{\text{is greater than}} \quad \underbrace{39.548}_{\text{Raskina's total.}}$

More About. . .

Olympics

Yulia Barsukova of the Russian Federation won the gold medal in rhythmic gymnastics at the 2000 Summer Olympics in Sydney, and Yulia Raskina of Belarus won the silver medal.

Source: www.olympic.org

Solve the inequality.

$$9.883 + 9.900 + 9.916 + r > 39.548$$ Original inequality

$$29.699 + r > 39.548$$ Simplify.

$$29.699 + r - 29.699 > 39.548 - 29.699$$ Subtract 29.699 from each side.

$$r > 9.849$$ Simplify.

Barsukova needed to score more than 9.849 points to win the gold medal.

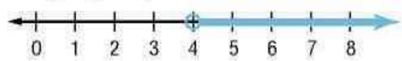
Check for Understanding

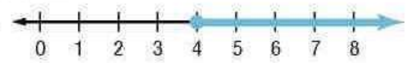
Concept Check

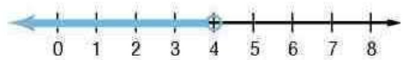
1. **OPEN ENDED** List three inequalities that are equivalent to $y < -3$.
2. **Compare and contrast** the graphs of $a < 4$ and $a \le 4$.
3. **Explain** what $\{b \mid b \ge -5\}$ means.

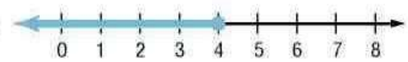
Guided Practice

4. Which graph represents the solution of $m + 3 > 7$?

a. 

b. 

c. 

d. 

Solve each inequality. Then check your solution, and graph it on a number line.

5. $a + 4 < 2$
6. $9 \le b + 4$
7. $t - 7 \ge 5$
8. $y - 2.5 > 3.1$
9. $5.2r + 6.7 \ge 6.2r$
10. $7p \le 6p - 2$

Define a variable, write an inequality, and solve each problem. Then check your solution.

11. A number decreased by 8 is at most 14.
12. A number plus 7 is greater than 2.

Application

13. **HEALTH** Chapa's doctor recommended that she limit her fat intake to no more than 60 grams per day. This morning, she ate two breakfast bars with 3 grams of fat each. For lunch she ate pizza with 21 grams of fat. If she follows her doctor's advice, how many grams of fat can she have during the rest of the day?

Practice and Apply

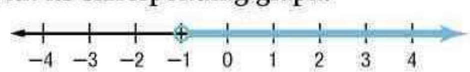
Homework Help

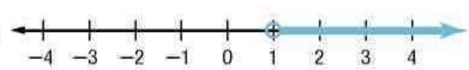
For Exercises	See Examples
14–39	1–4
40–45	5
46–55	6

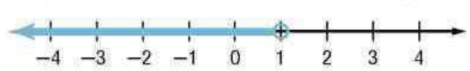
Extra Practice

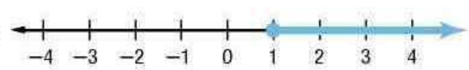
See page 833.

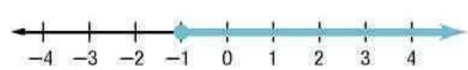
Match each inequality with its corresponding graph.

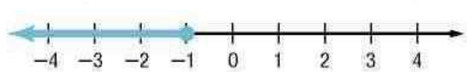
14. $x - 3 \ge -2$ **a.** 

15. $x + 7 \le 6$ **b.** 

16. $4x > 3x - 1$ **c.** 

17. $8 + x < 9$ **d.** 

18. $5 \le x + 6$ **e.** 

19. $x - 1 > 0$ **f.** 

Solve each inequality. Then check your solution, and graph it on a number line.

20. $t + 14 \geq 18$	**21.** $d + 5 \leq 7$	**22.** $n - 7 < -3$
23. $s - 5 > -1$	**24.** $5 < 3 + g$	**25.** $4 > 8 + r$
26. $-3 \geq q - 7$	**27.** $2 \leq m - 1$	**28.** $2y > -8 + y$
29. $3f < -3 + 2f$	**30.** $3b \leq 2b - 5$	**31.** $4w \geq 3w + 1$
32. $v - (-4) > 3$	**33.** $a - (-2) \leq -3$	**34.** $-0.23 < h - (-0.13)$
35. $x + 1.7 \geq 2.3$	**36.** $a + \frac{1}{4} > \frac{1}{8}$	**37.** $p - \frac{2}{3} \leq \frac{4}{9}$

38. If $d + 5 \geq 17$, then complete each inequality.

a. $d \geq$ ___?___ **b.** $d +$ ___?___ ≥ 20 **c.** $d - 5 \geq$ ___?___

39. If $z - 2 \leq 10$, then complete each inequality.

a. $z \leq$ ___?___ **b.** $z -$ ___?___ ≤ 5 **c.** $z + 4 \leq$ ___?___

Define a variable, write an inequality, and solve each problem. Then check your solution.

40. The sum of a number and 13 is at least 27.

41. A number decreased by 5 is less than 33.

42. Thirty is no greater than the sum of a number and -8.

43. Twice a number is more than the sum of that number and 14.

44. The sum of two numbers is at most 18, and one of the numbers is -7.

45. Four times a number is less than or equal to the sum of three times the number and -2.

46. **BIOLOGY** Adult Nile crocodiles weigh up to 2200 pounds. If a young Nile crocodile weighs 157 pounds, how many pounds might it be expected to gain in its lifetime?

47. **ASTRONOMY** There are at least 200 billion stars in the Milky Way. If 1100 of these stars can be seen in a rural area without the aid of a telescope, how many stars in the galaxy cannot be seen in this way?

More About. . .

Biology

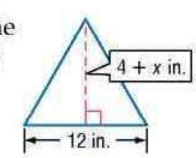
One common species of bees is the honeybee. A honeybee colony may have 60,000 to 80,000 bees.

Source: Penn State, Cooperative Extension Service

48. **BIOLOGY** There are 3500 species of bees and more than 600,000 species of insects. How many species of insects are not bees?

49. **BANKING** City Bank requires a minimum balance of \$1500 to maintain free checking services. If Mr. Hayashi knows he must write checks for \$1300 and \$947, how much money should he have in his account before writing the checks?

50. **GEOMETRY** The length of the base of the triangle at the right is less than the height of the triangle. What are the possible values of x?



51. **SHOPPING** Terrell has \$65 to spend at the mall. He bought a T-shirt for \$18 and a belt for \$14. If Terrell still wants to buy a pair of jeans, how much can he spend on the jeans?

52. **SOCCER** The Centerville High School soccer team plays 18 games in the season. The team has a goal of winning at least 60% of its games. After the first three weeks of the season, the team has won 4 games. How many more games must the team win to meet their goal?

53. **CRITICAL THINKING** Determine whether each statement is *always*, *sometimes*, or *never* true.

a. If $a < b$ and $c < d$, then $a + c < b + d$.

b. If $a < b$ and $c < d$, then $a + c \geq b + d$.

c. If $a < b$ and $c < d$, then $a - c = b - d$.

HEALTH **For Exercises 54 and 55, use the following information.**
Hector's doctor told him that his cholesterol level should be below 200. Hector's cholesterol is 225.

54. Let p represent the number of points Hector should lower his cholesterol. Write an inequality with $225 - p$ on one side.

55. Solve the inequality.

56. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are inequalities used to describe school sports?

Include the following in your answer:

- an inequality describing the number of schools needed to add girls' track and field so that the number is greater than the number of schools currently participating in girls' basketball.

Standardized Test Practice

57. Which inequality is *not* equivalent to $x \leq 12$?

(A) $x - 7 \leq 5$ (B) $x + 4 \leq 16$ (C) $x - 1 \leq 13$ (D) $12 \geq x$

58. Which statement is modeled by $n + 6 \geq 5$?

(A) The sum of a number and six is at least five.
(B) The sum of a number and six is at most five.
(C) The sum of a number and six is greater than five.
(D) The sum of a number and six is no greater than five.

Maintain Your Skills

Mixed Review

59. Would a scatter plot for the relationship of a person's height to the person's grade on the last math test show a *positive*, *negative*, or *no correlation*? *(Lesson 5-7)*

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of each equation. *(Lesson 5-6)*

60. $(1, -3)$; $y = 3x - 2$ 61. $(0, 4)$; $x + y = -3$ 62. $(-1, 2)$; $2x - y = 1$

Find the next two terms in each sequence. *(Lesson 4-8)*

63. 7, 13, 19, 25, … 64. 243, 81, 27, 9, … 65. 3, 6, 12, 24, …

Solve each equation if the domain is $\{-1, 3, 5\}$. *(Lesson 4-4)*

66. $y = -2x$ 67. $y = 7 - x$ 68. $2x - y = 6$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.**
*(For review of **multiplication and division equations**, see Lesson 3-3.)*

69. $6g = 42$ 70. $\frac{t}{9} = 14$ 71. $\frac{2}{3}y = 14$ 72. $3m = 435$

73. $\frac{4}{7}x = 28$ 74. $5.3g = 11.13$ 75. $\frac{a}{3.5} = 7$ 76. $8p = 35$

Algebra Activity

A Preview of Lesson 6-2

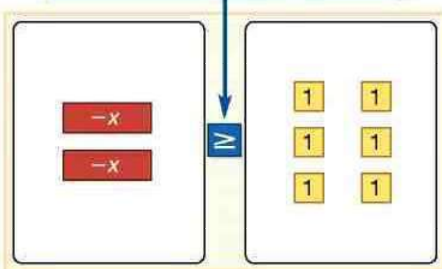
Solving Inequalities

You can use algebra tiles to solve inequalities.

Solve $-2x \geq 6$.

Step 1 *Model the inequality.*

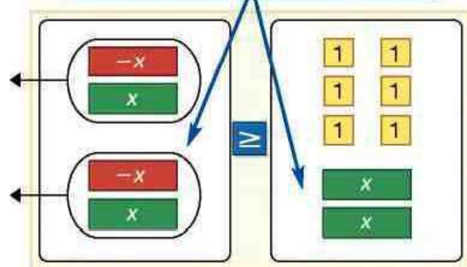
Use a self-adhesive note to cover the equals sign on the equation mat. Then write a ≥ symbol on the note. Model the inequality.



$-2x \geq 6$

Step 2 *Remove zero pairs.*

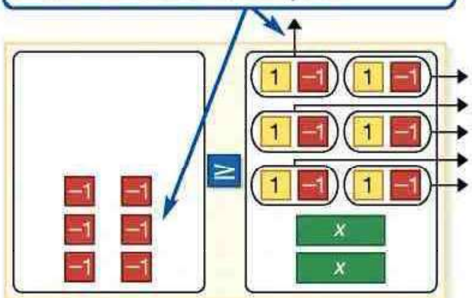
Since you do not want to solve for a negative *x* tile, eliminate the negative *x* tiles by adding 2 positive *x* tiles to each side. Remove the zero pairs.



$-2x + 2x \geq 6 + 2x$

Step 3 *Remove zero pairs.*

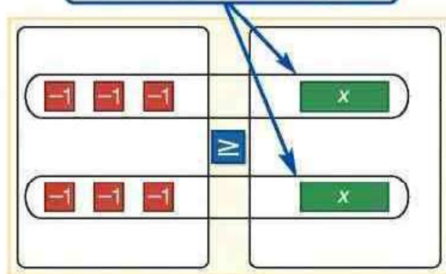
Add 6 negative 1 tiles to each side to isolate the *x* tiles. Remove the zero pairs.



$-6 \geq 2x$

Step 4 *Group the tiles.*

Separate the tiles into 2 groups.



$-3 \geq x$ or $x \leq -3$

Model and Analyze

Use algebra tiles to solve each inequality.

1. $-4x < 12$ **2.** $-2x > 8$ **3.** $-3x \geq -6$ **4.** $-5x \leq -5$

5. In Exercises 1–4, is the coefficient of x in each inequality positive or negative?

6. Compare the inequality symbols and locations of the variable in Exercises 1–4 with those in their solutions. What do you find?

7. Model the solution for $2x \geq 6$. What do you find? How is this different from solving $-2x \geq 6$?

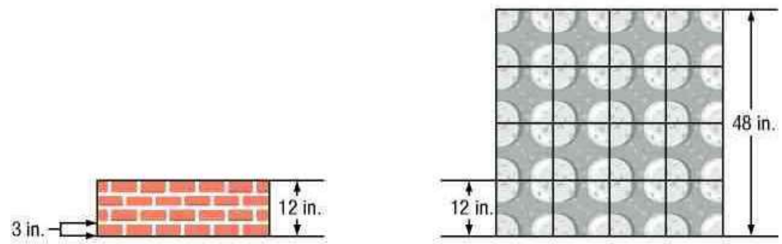
6-2 Solving Inequalities by Multiplication and Division

What You'll Learn

- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

Why are inequalities important in landscaping?

Isabel Franco is a landscape architect. To beautify a garden, she plans to build a decorative wall of either bricks or blocks. Each brick is 3 inches high, and each block is 12 inches high. Notice that $3 < 12$.



A wall 4 bricks high would be lower than a wall 4 blocks high.

$$3 \times 4 \ \underline{\ ?\ }\ 12 \times 4$$
$$12 < 48$$

SOLVE INEQUALITIES BY MULTIPLICATION If each side of an inequality is multiplied by a positive number, the inequality remains true.

$8 > 5$		$5 < 9$	
$8(2) \ \underline{\ ?\ }\ 5(2)$	Multiply each side by 2.	$5(4) \ \underline{\ ?\ }\ 9(4)$	Multiply each side by 4.
$16 > 10$		$20 < 36$	

This is *not* true when multiplying by negative numbers.

$5 > 3$		$-6 < 8$	
$5(-2) \ \underline{\ ?\ }\ 3(-2)$	Multiply each side by -2.	$-6(-5) \ \underline{\ ?\ }\ 8(-5)$	Multiply each side by -5.
$-10 < -6$		$30 > -40$	

If each side of an inequality is multiplied by a negative number, the direction of the inequality symbol changes. These examples illustrate the **Multiplication Property of Inequalities**.

Key Concept — Multiplying by a Positive Number

- **Words** If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a positive number, the following are true.
  If $a > b$, then $ac > bc$, and if $a < b$, then $ac < bc$.

Key Concept — *Multiplying by a Negative Number*

- **Words** If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a negative number, the following are true.
  If $a > b$, then $ac < bc$, and if $a < b$, then $ac > bc$.

This property also holds for inequalities involving $\geq$ and $\leq$.

You can use this property to solve inequalities.

Example 1 Multiply by a Positive Number

Solve $\frac{b}{7} \geq 25$. Then check your solution.

$\frac{b}{7} \geq 25$ Original inequality

$(7)\frac{b}{7} \geq (7)25$ Multiply each side by 7. Since we multiplied by a positive number, the inequality symbol stays the same.

$b \geq 175$

CHECK To check this solution, substitute 175, a number less than 175, and a number greater than 175 into the inequality.

Let $b = 175$.	Let $b = 140$.	Let $b = 210$.
$\frac{175}{7} \overset{?}{\geq} 25$	$\frac{140}{7} \overset{?}{\geq} 25$	$\frac{210}{7} \overset{?}{\geq} 25$
$25 \geq 25$ ✓	$20 \ngeq 25$	$30 \geq 25$ ✓

The solution set is $\{b \mid b \geq 175\}$.

Study Tip

Common Misconception

A negative sign in an inequality does not necessarily mean that the direction of the inequality should change. For example, when solving $\frac{x}{6} > -3$, do *not* change the direction of the inequality.

Example 2 Multiply by a Negative Number

Solve $-\frac{2}{5}p < -14$.

$-\frac{2}{5}p < -14$ Original inequality

$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}p\right) > \left(-\frac{5}{2}\right)(-14)$ Multiply each side by $-\frac{5}{2}$ and change $<$ to $>$.

$p > 35$ The solution set is $\{p \mid p > 35\}$.

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

One fourth of a number is less than −7.

One fourth	of	a number	is less than	−7.
$\frac{1}{4}$	$\times$	n	$<$	-7

$\frac{1}{4}n < -7$ Original inequality

$(4)\frac{1}{4}n < (4)(-7)$ Multiply each side by 4 and do not change the inequality's direction.

$n < -28$ The solution set is $\{n \mid n < -28\}$.

SOLVE INEQUALITIES BY DIVISION Dividing each side of an inequality by the same number is similar to multiplying each side of an equality by the same number. Consider the inequality $6 < 15$.

Divide each side by 3.

$$6 < 15$$
$$6 \div 3 \ \underline{\ ?\ }\ 15 \div 3$$
$$2 < 5$$

Since each side is divided by a positive number, the direction of the inequality symbol remains the same.

Divide each side by -3.

$$6 < 15$$
$$6 \div (-3) \ \underline{\ ?\ }\ 15 \div (-3)$$
$$-2 > -5$$

Since each side is divided by a negative number, the direction of the inequality symbol is reversed.

These examples illustrate the **Division Property of Inequalities**.

Key Concept — Dividing by a Positive Number

- **Words** If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a positive number, the following are true.

  If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

Dividing by a Negative Number

- **Words** If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a negative number, the following are true.

  If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

This property also holds for inequalities involving $\geq$ and $\leq$.

Example 4 Divide by a Positive Number

Solve $14h > 91$.

$14h > 91$ Original inequality

$\frac{14h}{14} > \frac{91}{14}$ Divide each side by 14 and do not change the direction of the inequality sign.

$h > 6.5$

CHECK

Let $h = 6.5$.	Let $h = 7$.	Let $h = 6$.
$14h > 91$	$14h > 91$	$14h > 91$
$14(6.5) \overset{?}{>} 91$	$14(7) \overset{?}{>} 91$	$14(6) \overset{?}{>} 91$
$91 \not> 91$	$98 > 91$ ✓	$84 \not> 91$

The solution set is $\{h \mid h > 6.5\}$.

Since dividing is the same as multiplying by the reciprocal, there are two methods to solve an inequality that involve multiplication.

Example 5 Divide by a Negative Number

Solve $-5t \geq 275$ using two methods.

Method 1 Divide.

$-5t \geq 275$ Original inequality

$\frac{-5t}{-5} \leq \frac{275}{-5}$ Divide each side by -5 and change $\geq$ to $\leq$.

$t \leq -55$ Simplify.

Method 2 Multiply by the multiplicative inverse.

$-5t \geq 275$ Original inequality

$\left(-\frac{1}{5}\right)(-5t) \leq \left(-\frac{1}{5}\right)275$ Multiply each side by $-\frac{1}{5}$ and change $\geq$ to $\leq$.

$t \leq -55$ Simplify.

The solution set is $\{t \mid t \leq -55\}$.

You can use the Multiplication Property and the Division Property for Inequalities to solve standardized test questions.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

Example 6 The Word "not"

Multiple-Choice Test Item

Which inequality does *not* have the solution $\{y \mid y \leq -5\}$?

Ⓐ $-7y \geq 35$ Ⓑ $2y \leq -10$ Ⓒ $\frac{7}{5}y \geq -7$ Ⓓ $-\frac{y}{4} \geq \frac{5}{4}$

Read the Test Item

You want to find the inequality that does *not* have the solution set $\{y \mid y \leq -5\}$.

Solve the Test Item

Consider each possible choice.

Ⓐ $-7y \geq 35$

$\frac{-7y}{-7} \leq \frac{35}{-7}$

$y \leq -5$ ✓

Ⓑ $2y \leq -10$

$\frac{2y}{2} \leq \frac{-10}{2}$

$y \leq -5$ ✓

Ⓒ $\frac{7}{5}y \geq -7$

$\left(\frac{5}{7}\right)\frac{7}{5}y \geq \left(\frac{5}{7}\right)(-7)$

$y \geq -5$

Ⓓ $-\frac{y}{4} \geq \frac{5}{4}$

$(-4)\left(-\frac{y}{4}\right) \leq (-4)\frac{5}{4}$

$y \leq -5$ ✓

The answer is C.

The Princeton Review

Test-Taking Tip

Always look for the word *not* in the questions. This indicates that you are looking for the one incorrect answer, rather than looking for the one correct answer. The word *not* is usually in italics or uppercase letters to draw your attention to it.

Check for Understanding

Concept Check

1. **Explain** why you can use either the Multiplication Property of Inequalities or the Division Property of Inequalities to solve $-7r \leq 28$.

2. **OPEN ENDED** Write a problem that can be represented by the inequality $\frac{3}{4}c > 9$.

3. **FIND THE ERROR** Ilonia and Zachary are solving $-9b \leq 18$.

Ilonia	Zachary
$-9b \leq 18$	$-9b \leq 18$
$\frac{-9b}{-9} \geq \frac{18}{-9}$	$\frac{-9b}{-9} \leq \frac{18}{-9}$
$b \geq -2$	$b \leq -2$

Who is correct? Explain your reasoning.

Guided Practice

4. Which statement is represented by $7n \geq 14$?
   a. Seven times a number is at least 14.
   b. Seven times a number is greater than 14.
   c. Seven times a number is at most 14.
   d. Seven times a number is less than 14.

5. Which inequality represents *five times a number is less than 25*?
   a. $5n > 25$ b. $5n \geq 25$ c. $5n < 25$ d. $5n \leq 25$

Solve each inequality. Then check your solution.

6. $-15g > 75$ 7. $\frac{t}{9} < -12$ 8. $-\frac{2}{3}b \leq -9$ 9. $25f \geq 9$

Define a variable, write an inequality, and solve each problem. Then check your solution.

10. The opposite of four times a number is more than 12.
11. Half of a number is at least 26.

Standardized Test Practice

12. Which inequality does *not* have the solution $\{x \mid x > 4\}$?
   Ⓐ $-5x < -20$ Ⓑ $6x < 24$ Ⓒ $\frac{1}{5}x > \frac{4}{5}$ Ⓓ $-\frac{3}{4}x < -3$

Practice and Apply

Homework Help

For Exercises	See Examples
13–18, 39–44	3
19–38	1, 2, 4, 5
45–51	6

Extra Practice

See page 833.

Match each inequality with its corresponding statement.

13. $\frac{1}{5}n > 10$ a. Five times a number is less than or equal to ten.
14. $5n \leq 10$ b. One fifth of a number is no less than ten.
15. $5n > 10$ c. Five times a number is less than ten.
16. $-5n < 10$ d. One fifth of a number is greater than ten.
17. $\frac{1}{5}n \geq 10$ e. Five times a number is greater than ten.
18. $5n < 10$ f. Negative five times a number is less than ten.

Solve each inequality. Then check your solution.

19. $6g \leq 144$ 20. $7t > 84$ 21. $-14d \geq 84$ 22. $-16z \leq -64$
23. $\frac{m}{5} \geq 7$ 24. $\frac{b}{10} \leq 5$ 25. $-\frac{r}{7} < -7$ 26. $-\frac{a}{11} > 9$
27. $\frac{5}{8}y \geq -15$ 28. $\frac{2}{3}v < 6$ 29. $-\frac{3}{4}q \leq -33$ 30. $-\frac{2}{5}p > 10$
31. $-2.5w < 6.8$ 32. $-0.8s > 6.4$ 33. $\frac{15c}{-7} > \frac{3}{14}$ 34. $\frac{4m}{5} < \frac{-3}{15}$

35. Solve $-\frac{y}{8} > \frac{1}{2}$. Then graph the solution.

36. Solve $-\frac{m}{9} \leq -\frac{1}{3}$. Then graph the solution.

37. If $2a \geq 7$, then complete each inequality.

a. $a \geq$ __?__ b. $-4a \leq$ __?__ c. __?__ $a \leq -21$

38. If $4t < -2$, then complete each inequality.

a. $t <$ __?__ b. $-8t >$ __?__ c. __?__ $t > 14$

Define a variable, write an inequality, and solve each problem. Then check your solution.

39. Seven times a number is greater than 28.

40. Negative seven times a number is at least 14.

41. Twenty-four is at most a third of a number.

42. Two thirds of a number is less than -15.

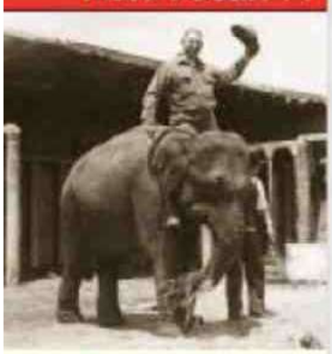
43. Twenty-five percent of a number is greater than or equal to 90.

44. Forty percent of a number is less than or equal to 45.

45. **GEOMETRY** The area of a rectangle is less than 85 square feet. The length of the rectangle is 20 feet. What is the width of the rectangle?

46. **FUND-RAISING** The Middletown Marching Mustangs want to make at least \$2000 on their annual mulch sale. The band makes \$2.50 on each bag of mulch that is sold. How many bags of mulch should the band sell?

More About. . .



Zoos

Dr. Harry Wegeforth founded the San Diego Zoo in 1916 with just 50 animals. Today, the zoo has over 3800 animals.

Source: www.sandiegozoo.org

47. **LONG-DISTANCE COSTS** Juan's long-distance phone company charges him 9¢ for each minute or any part of a minute. He wants to call his friend, but he does not want to spend more than \$2.50 on the call. How long can he talk to his friend?

48. **EVENT PLANNING** The Country Corner Reception Hall does not charge a rental fee as long as at least \$4000 is spent on food. Shaniqua is planning a class reunion. If she has chosen a buffet that costs \$28.95 per person, how many people must attend the reunion to avoid a rental fee for the hall?

49. **LANDSCAPING** Matthew is planning a circular flower garden with a low fence around the border. If he can use up to 38 feet of fence, what radius can he use for the garden? *(Hint:* $C = 2\pi r$)

50. **DRIVING** Average speed is calculated by dividing distance by time. If the speed limit on the interstate is 65 miles per hour, how far can a person travel legally in $1\frac{1}{2}$ hours?

51. **ZOOS** The yearly membership to the San Diego Zoo for a family with 2 adults and 2 children is \$144. The regular admission to the zoo is \$18 for each adult and \$8 for each child. How many times should such a family plan to visit the zoo in a year to make a membership less expensive than paying regular admission?

52. **CRITICAL THINKING** Give a counterexample to show that each statement is not always true.

a. If $a > b$, then $a^2 > b^2$. b. If $a < b$ and $c < d$, then $ac < bd$.

53. **CITY PLANNING** The city of Santa Clarita requires that a parking lot can have no more than 20% of the parking spaces limited to compact cars. If a certain parking lot has 35 spaces for compact cars, how many spaces must the lot have to conform to the code?

54. **CIVICS** For a candidate to run for a county office, he or she must submit a petition with at least 6000 signatures of registered voters. Usually only 85% of the signatures are valid. How many signatures should a candidate seek on a petition?

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

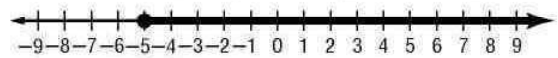
Why are inequalities important in landscaping?

Include the following in your answer:

- an inequality representing a brick wall that can be no higher than 4 feet, and
- an explanation of how to solve the inequality.

Standardized Test Practice

56. The solution set for which inequality is *not* represented by the following graph?



(A) $-\frac{x}{5} \le 1$ (B) $\frac{x}{5} \le -1$ (C) $-9x \le 45$ (D) $2.5x \ge -12.5$

57. Solve $-\frac{7}{8}t < \frac{14}{15}$.

(A) $\left\{t \,\middle|\, t > \frac{16}{15}\right\}$ (B) $\left\{t \,\middle|\, t < \frac{16}{15}\right\}$ (C) $\left\{t \,\middle|\, t > -\frac{16}{15}\right\}$ (D) $\left\{t \,\middle|\, t < -\frac{16}{15}\right\}$

Maintain Your Skills

Mixed Review

Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

58. $s - 7 < 12$ 59. $g + 3 \le -4$ 60. $7 > n + 2$

61. Draw a scatter plot that shows a positive correlation. *(Lesson 5-7)*

Write an equation in standard form for a line that passes through each pair of points. *(Lesson 5-4)*

62. $(-1, 3), (2, 4)$ 63. $(5, -2), (-1, -2)$ 64. $(3, 3), (-1, 2)$

If $h(x) = 3x + 2$, find each value. *(Lesson 4-6)*

65. $h(-4)$ 66. $h(2)$ 67. $h(w)$ 68. $h(r - 6)$

Solve each proportion. *(Lesson 3-6)*

69. $\frac{3}{4} = \frac{x}{8}$ 70. $\frac{t}{1.5} = \frac{2.4}{1.6}$ 71. $\frac{w + 2}{5} = \frac{7}{5}$ 72. $\frac{x}{3} = \frac{x + 5}{15}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.**
*(To review **multi-step equations**, see Lessons 3-4 and 3-5.)*

73. $5x - 3 = 32$ 74. $4t + 9 = 14$ 75. $6y - 1 = 4y + 23$

76. $\frac{14g + 5}{6} = 9$ 77. $5a + 6 = 9a - (7a + 18)$ 78. $2(p - 4) = 7(p + 3)$

Practice Quiz 1

Lessons 6-1 and 6-2

Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

1. $h - 16 > -13$ 2. $r + 3 \le -1$ 3. $4 \ge p + 9$ 4. $-3 < a - 5$ 5. $7g \le 6g - 1$

Solve each inequality. Then check your solution. *(Lesson 6-2)*

6. $15z \ge 105$ 7. $\frac{v}{5} < 7$ 8. $-\frac{3}{7}q > 15$ 9. $-156 < 12r$ 10. $-\frac{2}{5}w \le -\frac{1}{2}$

6-3 Solving Multi-Step Inequalities

What You'll Learn

- Solve linear inequalities involving more than one operation.
- Solve linear inequalities involving the Distributive Property.

How are linear inequalities used in science?

The boiling point of a substance is the temperature at which the element changes from a liquid to a gas. The boiling point of chlorine is $-31°F$. That means chlorine will be a gas for all temperatures greater than $-31°F$. If F represents temperature in degrees Fahrenheit, the inequality $F > -31$ represents the temperatures for which chlorine is a gas.

If C represents degrees Celsius, then $F = \frac{9}{5}C + 32$. You can solve $\frac{9}{5}C + 32 > -31$ to find the temperatures in degrees Celsius for which chlorine is a gas.

Boiling Points

argon	$-303°F$
chlorine	$-31°F$
bromine	$138°F$
water	$212°F$
iodine	$363°F$

Source: *World Book Encyclopedia*

SOLVE MULTI-STEP INEQUALITIES The inequality $\frac{9}{5}C + 32 > -31$ involves more than one operation. It can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

Example 1 Solve a Real-World Problem

SCIENCE Find the temperatures in degrees Celsius for which chlorine is a gas.

$$\frac{9}{5}C + 32 > -31 \quad \text{Original inequality}$$

$$\frac{9}{5}C + 32 - 32 > -31 - 32 \quad \text{Subtract 32 from each side.}$$

$$\frac{9}{5}C > -63 \quad \text{Simplify.}$$

$$\left(\frac{5}{9}\right)\frac{9}{5}C > \left(\frac{5}{9}\right)(-63) \quad \text{Multiply each side by } \frac{5}{9}.$$

$$C > -35 \quad \text{Simplify.}$$

Chlorine will be a gas for all temperatures greater than $-35°C$.

When working with inequalities, do not forget to reverse the inequality sign whenever you multiply or divide each side by a negative number.

Example 2 Inequality Involving a Negative Coefficient

Solve $-7b + 19 < -16$. Then check your solution.

$$-7b + 19 < -16 \quad \text{Original inequality}$$

$$-7b + 19 - 19 < -16 - 19 \quad \text{Subtract 19 from each side.}$$

$$-7b < -35 \quad \text{Simplify.}$$

$$\frac{-7b}{-7} > \frac{-35}{-7} \quad \text{Divide each side by } -7 \text{ and change } < \text{ to } >.$$

$$b > 5 \quad \text{Simplify.}$$

CHECK To check this solution, substitute 5, a number less than 5, and a number greater than 5.

Let $b = 5$.	Let $b = 4$.	Let $b = 6$.
$-7b + 19 < -16$	$-7b + 19 < -16$	$-7b + 19 < -16$
$-7(5) + 19 \overset{?}{<} -16$	$-7(4) + 19 \overset{?}{<} -16$	$-7(6) + 19 \overset{?}{<} -16$
$-35 + 19 \overset{?}{<} -16$	$-28 + 19 \overset{?}{<} -16$	$-42 + 19 \overset{?}{<} -16$
$-16 \not< -16$	$-9 \not< -16$	$-23 < -16$ ✓

The solution set is $\{b \mid b > 5\}$.

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.
Three times a number minus eighteen is at least five times the number plus twenty-one.

$$\underbrace{\text{Three times a number}}_{3n}\quad \underbrace{\text{minus}}_{-}\quad \underbrace{\text{eighteen}}_{18}\quad \underbrace{\text{is at least}}_{\geq}\quad \underbrace{\text{five times the number}}_{5n}\quad \underbrace{\text{plus}}_{+}\quad \underbrace{\text{twenty one.}}_{21}$$

$3n - 18 \geq 5n + 21$	Original inequality
$3n - 18 - 5n \geq 5n + 21 - 5n$	Subtract $5n$ from each side.
$-2n - 18 \geq 21$	Simplify.
$-2n - 18 + 18 \geq 21 + 18$	Add 18 to each side.
$-2n \geq 39$	Simplify.
$\frac{-2n}{-2} \leq \frac{39}{-2}$	Divide each side by -2 and change $\geq$ to $\leq$.
$n \leq -19.5$	Simplify.

The solution set is $\{n \mid n \leq -19.5\}$.

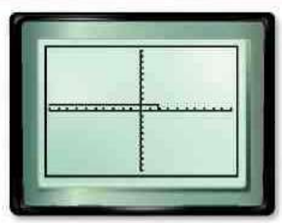
A graphing calculator can be used to solve inequalities.

Graphing Calculator Investigation

Solving Inequalities

You can find the solution of an inequality in one variable by using a graphing calculator. On a TI-83 Plus, clear the Y= list. Enter $6x + 9 < -4x + 29$ as Y1. (The symbol $<$ is item 5 on the TEST menu.)

Press GRAPH.



[10, 10] scl: 1 by [10, 10] scl: 1

Think and Discuss

1. Describe what is shown on the screen.
2. Use the TRACE function to scan the values along the graph. What do you notice about the values of y on the graph?
3. Solve the inequality algebraically. How does your solution compare to the pattern you noticed in Exercise 2?

SOLVE INEQUALITIES INVOLVING THE DISTRIBUTIVE PROPERTY

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 4 Distributive Property

Solve $3d - 2(8d - 9) > 3 - (2d + 7)$.

$3d - 2(8d - 9) > 3 - (2d + 7)$	Original inequality
$3d - 16d + 18 > 3 - 2d - 7$	Distributive Property
$-13d + 18 > -2d - 4$	Combine like terms.
$-13d + 18 + 13d > -2d - 4 + 13d$	Add 13d to each side.
$18 > 11d - 4$	Simplify.
$18 + 4 > 11d - 4 + 4$	Add 4 to each side.
$22 > 11d$	Simplify.
$\frac{22}{11} > \frac{11d}{11}$	Divide each side by 11.
$2 > d$	Simplify.

Since $2 > d$ is the same as $d < 2$, the solution set is $\{d \mid d < 2\}$.

If solving an inequality results in a statement that is always true, the solution is all real numbers. If solving an inequality results in a statement that is never true, the solution is the empty set $\varnothing$. The empty set has no members.

Example 5 Empty Set

Solve $8(t + 2) - 3(t - 4) < 5(t - 7) + 8$.

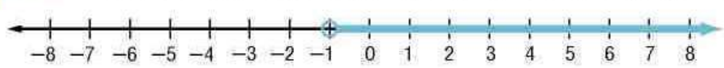
$8(t + 2) - 3(t - 4) < 5(t - 7) + 8$	Original inequality
$8t + 16 - 3t + 12 < 5t - 35 + 8$	Distributive Property
$5t + 28 < 5t - 27$	Combine like terms.
$5t + 28 - 5t < 5t - 27 - 5t$	Subtract 5t from each side.
$28 < -27$	This statement is false.

Since the inequality results in a false statement, the solution set is the empty set $\varnothing$.

Check for Understanding

Concept Check

1. **Compare and contrast** the method used to solve $-5h + 6 = -7$ and the method used to solve $-5h + 6 \le -7$.
2. **OPEN ENDED** Write a multi-step inequality with the solution graphed below.



Guided Practice

3. Justify each indicated step.

$3(a - 7) + 9 \le 21$	
$3a - 21 + 9 \le 21$	a. ___?___
$3a - 12 \le 21$	
$3a - 12 + 12 \le 21 + 12$	b. ___?___
$3a \le 33$	
$\frac{3a}{3} \le \frac{33}{3}$	c. ___?___
$a \le 11$	

Solve each inequality. Then check your solution.

4. $-4y - 23 < 19$
5. $\frac{2}{3}r + 9 \geq -3$
6. $7b + 11 > 9b - 13$
7. $-5(g + 4) > 3(g - 4)$
8. $3 + 5t \leq 3(t + 1) - 4(2 - t)$

9. Define a variable, write an inequality, and solve the problem below. Then check your solution.
Seven minus two times a number is less than three times the number plus thirty-two.

Application

10. **SALES** A salesperson is paid \$22,000 a year plus 5% of the amount of sales made. What is the amount of sales needed to have an annual income greater than \$35,000?

Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1–5
15–34	2, 4, 5
35–38	3
39–52	1

Extra Practice

See page 834.

Justify each indicated step.

11.
$$\frac{2}{5}w + 7 \leq -9$$
$$\frac{2}{5}w + 7 - 7 \leq -9 - 7 \quad \textbf{a.}\ \underline{\ ?\ }$$
$$\frac{2}{5}w \leq -16$$
$$\left(\frac{5}{2}\right)\frac{2}{5}w \leq \left(\frac{5}{2}\right)(-16) \quad \textbf{b.}\ \underline{\ ?\ }$$
$$w \leq -40$$

12.
$$m > \frac{15 - 2m}{-3}$$
$$(-3)m < (-3)\frac{15 - 2m}{-3} \quad \textbf{a.}\ \underline{\ ?\ }$$
$$-3m < 15 - 2m$$
$$-3m + 2m < 15 - 2m + 2m \quad \textbf{b.}\ \underline{\ ?\ }$$
$$-m < 15$$
$$(-1)(-m) > (-1)15 \quad \textbf{c.}\ \underline{\ ?\ }$$
$$m > -15$$

13. Solve $4(t - 7) \leq 2(t + 9)$. Show each step and justify your work.
14. Solve $-5(k + 4) > 3(k - 4)$. Show each step and justify your work.

Solve each inequality. Then check your solution.

15. $-3t + 6 \leq -3$
16. $-5 - 8f > 59$
17. $-2 - \frac{d}{5} < 23$
18. $\frac{w}{8} - 13 > -6$
19. $7q - 1 + 2q \leq 29$
20. $8a + 2 - 10a \leq 20$
21. $9r + 15 \leq 24 + 10r$
22. $13k - 11 > 7k + 37$
23. $\frac{2v - 3}{5} \geq 7$
24. $\frac{3a + 8}{2} < 10$
25. $\frac{3w + 5}{4} \geq 2w$
26. $\frac{5b + 8}{3} < 3b$
27. $7 + 3t \leq 2(t + 3) - 2(-1 - t)$
28. $5(2h - 6) - 7(h + 7) > 4h$
29. $3y + 4 > 2(y + 3) + y$
30. $3 - 3(b - 2) < 13 - 3(b - 6)$
31. $3.1v - 1.4 \geq 1.3v + 6.7$
32. $0.3(d - 2) - 0.8d > 4.4$

33. Solve $4(y + 1) - 3(y - 5) \geq 3(y - 1)$. Then graph the solution.

34. Solve $5(x + 4) - 2(x + 6) \geq 5(x + 1) - 1$. Then graph the solution.

Define a variable, write an inequality, and solve each problem. Then check your solution.

35. One eighth of a number decreased by five is at least thirty.
36. Two thirds of a number plus eight is greater than twelve.
37. Negative four times a number plus nine is no more than the number minus twenty-one.
38. Three times the sum of a number and seven is greater than five times the number less thirteen.

GEOMETRY **For Exercises 39 and 40, use the following information.**
By definition, the measure of any acute angle is less than 90 degrees. Suppose the measure of an acute angle is $3a - 15$.

39. Write an inequality to represent the situation.

40. Solve the inequality.

SCHOOL **For Exercises 41 and 42, use the following information.**
Carmen's scores on three math tests were 91, 95, and 88. The fourth and final test of the grading period is tomorrow. She needs an average (mean) of at least 92 to receive an A for the grading period.

41. If s is her score on the fourth test, write an inequality to represent the situation.

42. If Carmen wants an A in math, what must she score on the test?

PHYSICAL SCIENCE **For Exercises 43 and 44, use the information at the left and the information below.**
The melting point for an element is the temperature where the element changes from a solid to a liquid. If C represents degrees Celsius and F represents degrees Fahrenheit, then $C = \frac{5(F - 32)}{9}$.

43. Write an inequality that can be used to find the temperatures in degrees Fahrenheit for which mercury is a solid.

44. For what temperatures will mercury be a solid?

More About. . .

Physical Science

Mercury is a metal that is a liquid at room temperature. In fact, its melting point is −38°C. Mercury is used in thermometers because it expands evenly as it is heated.

Source: *World Book Encyclopedia*

45. HEALTH Keith weighs 200 pounds. He wants to weigh less than 175 pounds. If he can lose an average of 2 pounds per week on a certain diet, how long should he stay on his diet to reach his goal weight?

46. CRITICAL THINKING Write a multi-step inequality that has no solution and one that has infinitely many solutions.

47. PERSONAL FINANCES Nicholas wants to order a pizza. He has a total of \$13.00 to pay the delivery person. The pizza costs \$7.50 plus \$1.25 per topping. If he plans to tip 15% of the total cost of the pizza, how many toppings can he order?

LABOR **For Exercises 48–50, use the following information.**
A union worker made \$500 per week. His union sought a one-year contract and went on strike. Once the new contract was approved, it provided for a 4% raise.

48. Assume that the worker was not paid during the strike. Given his raise in salary, how many weeks could he strike and still make at least as much for the next 52 weeks as he would have made without a strike?

49. How would your answer to Exercise 48 change if the worker had been making \$600 per week?

50. How would your answer to Exercise 48 change if the worker's union provided him with \$150 per week during the strike?

51. NUMBER THEORY Find all sets of two consecutive positive odd integers whose sum is no greater than 18.

52. NUMBER THEORY Find all sets of three consecutive positive even integers whose sum is less than 40.

53. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are linear inequalities used in science?

Include the following in your answer:

- an inequality for the temperatures in degrees Celsius for which bromine is a gas, and
- a description of a situation in which a scientist might use an inequality.

Standardized Test Practice

54. What is the first step in solving $\frac{y-5}{9} \geq 13$?

Ⓐ Add 5 to each side. Ⓑ Subtract 5 from each side.

Ⓒ Divide each side by 9. Ⓓ Multiply each side by 9.

55. Solve $4t + 2 < 8t - (6t - 10)$.

Ⓐ $\{t \mid t < -6\}$ Ⓑ $\{t \mid t > -6\}$ Ⓒ $\{t \mid t < 4\}$ Ⓓ $\{t \mid t > 4\}$

Graphing Calculator

Use a graphing calculator to solve each inequality.

56. $3x + 7 > 4x + 9$ 57. $13x - 11 \leq 7x + 37$ 58. $2(x - 3) < 3(2x + 2)$

Maintain Your Skills

Mixed Review

59. **BUSINESS** The charge per mile for a compact rental car at Great Deal Rentals is \$0.12. Mrs. Ludlow must rent a car for a business trip. She has a budget of \$50 for mileage charges. How many miles can she travel without going over her budget? *(Lesson 6-2)*

Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

60. $d + 13 \geq 22$ 61. $t - 5 < 3$ 62. $4 > y + 7$

Write the standard form of an equation of the line that passes through the given point and has the given slope. *(Lesson 5-5)*

63. $(1, -3), m = 2$ 64. $(-2, -1), m = -\frac{2}{3}$ 65. $(3, 6), m = 0$

Determine the slope of the line that passes through each pair of points. *(Lesson 5-1)*

66. $(3, -1), (4, -6)$ 67. $(-2, -4), (1, 3)$ 68. $(0, 3), (-2, -5)$

Determine whether each equation is a linear equation. If an equation is linear, rewrite it in the form $Ax + By = C$. *(Lesson 4-5)*

69. $4x = 7 + 2y$ 70. $2x^2 - y = 7$ 71. $x = 12$

Solve each equation. Then check your solution. *(Lesson 3-5)*

72. $2(x - 2) = 3x - (4x - 5)$ 73. $5t - 7 = t + 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Graph each set of numbers on a number line.**

*(To review **graphing integers on a number line**, see Lesson 2-1.)*

74. $\{-2, 3, 5\}$ 75. $\{-1, 0, 3, 4\}$ 76. $\{-5, -4, -1, 1\}$

77. {integers less than 5} 78. {integers greater than -2}

79. {integers between 1 and 6} 80. {integers between -4 and 2}

81. {integers greater than or equal to -4}

82. {integers less than 6 but greater than -1}

Reading Mathematics

Compound Statements

Two simple statements connected by the words *and* or *or* form a compound statement. Before you can determine whether a compound statement is true or false, you must understand what the words *and* and *or* mean. Consider the statement below.

A triangle has three sides, *and* a hexagon has five sides.
For a compound statement connected by the word *and* to be true, both simple statements must be true. In this case, it is true that a triangle has three sides. However, it is false that a hexagon has five sides; it has six. Thus, the compound statement is false.

A compound statement connected by the word *or* may be *exclusive* or *inclusive*. For example, the statement "With your dinner, you may have soup *or* salad," is exclusive. In everyday language, *or* means one or the other, but not both. However, in mathematics, *or* is inclusive. It means one or the other or both. Consider the statement below.

A triangle has three sides, *or* a hexagon has five sides.
For a compound statement connected by the word *or* to be true, at least one of the simple statements must be true. Since it is true that a triangle has three sides, the compound statement is true.

Reading to Learn

Determine whether each compound statement is *true* or *false*. Explain your answer.

1. A hexagon has six sides, *or* an octagon has seven sides.

2. An octagon has eight sides, *and* a pentagon has six sides.

3. A pentagon has five sides, *and* a hexagon has six sides.

4. A triangle has four sides, *or* an octagon does *not* have seven sides.

5. A pentagon has three sides, *or* an octagon has ten sides.

6. A square has four sides, *or* a hexagon has six sides.

7. $5 < 4$ or $8 < 6$

8. $-1 > 0$ and $1 < 5$

9. $4 > 0$ and $-4 < 0$

10. $0 = 0$ or $-2 > -3$

11. $5 \neq 5$ or $-1 > -4$

12. $0 > 3$ and $2 > -2$

CONTENTS

6-4 Solving Compound Inequalities

What You'll Learn

- Solve compound inequalities containing the word *and* and graph their solution sets.
- Solve compound inequalities containing the word *or* and graph their solution sets.

Vocabulary

- compound inequality
- intersection
- union

How are compound inequalities used in tax tables?

Richard Kelley is completing his income tax return. He uses the table to determine the amount he owes in federal income tax.

2000 Tax Tables

If taxable income is—		Single	Married filing jointly	Married filing separately	Head of a household
At least	Less than				
41,000	41,050	8140	6154	8689	6996
41,050	41,100	8154	6161	8703	7010
41,100	41,150	8168	6169	8717	7024
41,150	41,200	8182	6176	8731	7038
41,200	41,250	8196	6184	8754	7052
41,250	41,300	8210	6191	8759	7066
41,300	41,350	8224	6199	8773	7080
41,350	41,400	8238	6206	8787	7094
41,400	41,450	8252	6214	8801	7108
41,450	41,500	8266	6221	8815	7122
41,500	41,550	8280	6229	8829	7136
41,550	41,600	8294	6236	8843	7150

Source: IRS

Let c represent the amount of Mr. Kelley's income. His income is at least \$41,350 and it is less than \$41,400. This can be written as $c \geq 41{,}350$ and $c < 41{,}400$. When considered together, these two inequalities form a **compound inequality**. This compound inequality can be written without using *and* in two ways.

$$41{,}350 \leq c < 41{,}400 \text{ or } 41{,}400 > c \geq 41{,}350$$

Study Tip

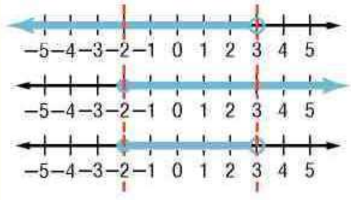
Reading Math The statement $41{,}350 \leq c < 41{,}400$ can be read *41,350 is less than or equal to c, which is less than 41,400.*

INEQUALITIES CONTAINING *AND* A compound inequality containing *and* is true only if both inequalities are true. Thus, the graph of a compound inequality containing *and* is the **intersection** of the graphs of the two inequalities. In other words, the solution must be a solution of *both* inequalities.

The intersection can be found by graphing each inequality and then determining where the graphs overlap.

Example 1 Graph an Intersection

Graph the solution set of $x < 3$ and $x \geq -2$.



Graph $x < 3$.

Graph $x \geq -2$.

Find the intersection.

The solution set is $\{x \mid -2 \leq x < 3\}$. Note that the graph of $x \geq -2$ includes the point -2. The graph of $x < 3$ does *not* include 3.

Study Tip

Reading Math

When solving problems involving inequalities,

- *within* is meant to be inclusive. Use $\leq$ or $\geq$.
- *between* is meant to be exclusive. Use $<$ or $>$.

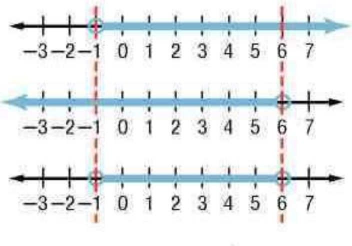
Example 2 Solve and Graph an Intersection

Solve $-5 < x - 4 < 2$. Then graph the solution set.

First express $-5 < x - 4 < 2$ using *and*. Then solve each inequality.

$$-5 < x - 4 \quad \text{and} \quad x - 4 < 2$$
$$-5 + 4 < x - 4 + 4 \qquad x - 4 + 4 < 2 + 4$$
$$-1 < x \qquad\qquad x < 6$$

The solution set is the intersection of the two graphs.



Graph $-1 < x$ or $x > -1$.

Graph $x < 6$.

Find the intersection.

The solution set is $\{x \mid -1 < x < 6\}$.

Career Choices

Pilot

Pilots check aviation weather forecasts to choose a route and altitude that will provide the smoothest flight.

Online Research For information about a career as a pilot, visit: www.algebra1.com/careers

INEQUALITIES CONTAINING *OR* Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if one or more of the inequalities is true. The graph of a compound inequality containing *or* is the **union** of the graphs of the two inequalities. In other words, the solution of the compound inequality is a solution of *either* inequality, not necessarily both.

The union can be found by graphing each inequality.

Example 3 Write and Graph a Compound Inequality

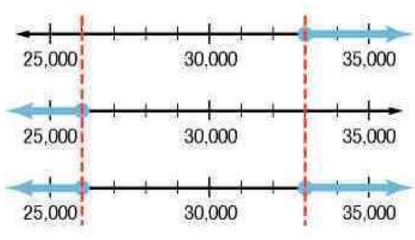
AVIATION An airplane is experiencing heavy turbulence while flying at 30,000 feet. The control tower tells the pilot that he should increase his altitude to at least 33,000 feet or decrease his altitude to no more than 26,000 feet to avoid the turbulence. Write and graph a compound inequality that describes the altitude at which the airplane should fly.

Words The pilot has been told to fly at an altitude of at least 33,000 feet or no more than 26,000 feet.

Variables Let a be the plane's altitude.

	The plane's altitude	is at least	33,000 feet	or	the altitude	is no more than	26,000 feet.
Inequality	a	$\geq$	33,000	or	a	$\leq$	26,000

Now, graph the solution set.



Graph $a \geq 33{,}000$.

Graph $a \leq 26{,}000$.

Find the union.

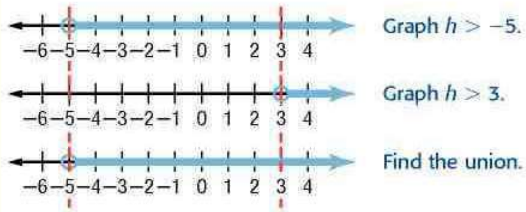
$a \geq 33{,}000$ or $a \leq 26{,}000$

Example 4 Solve and Graph a Union

Solve $-3h + 4 < 19$ or $7h - 3 > 18$. Then graph the solution set.

$$
\begin{array}{rcl}
-3h + 4 < 19 & \text{or} & 7h - 3 > 18 \\
-3h + 4 - 4 < 19 - 4 & & 7h - 3 + 3 > 18 + 3 \\
-3h < 15 & & 7h > 21 \\
\frac{-3h}{-3} > \frac{15}{-3} & & \frac{7h}{7} > \frac{21}{7} \\
h > -5 & & h > 3
\end{array}
$$

The solution set is the union of the two graphs.



Notice that the graph of $h > -5$ contains every point in the graph of $h > 3$. So, the union is the graph of $h > -5$. The solution set is $\{h \mid h > -5\}$.

Check for Understanding

Concept Check

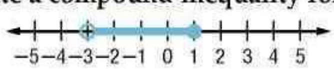
1. **Describe** the difference between a compound inequality containing *and* and a compound inequality containing *or*.
2. **Write** 7 *is less than t, which is less than* 12 as a compound inequality.
3. **OPEN ENDED** Give an example of a compound inequality containing *and* that has no solution.

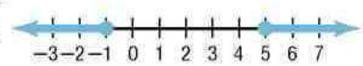
Guided Practice

Graph the solution set of each compound inequality.

4. $a \le 6$ and $a \ge -2$
5. $y > 12$ or $y < 9$

Write a compound inequality for each graph.

6. 

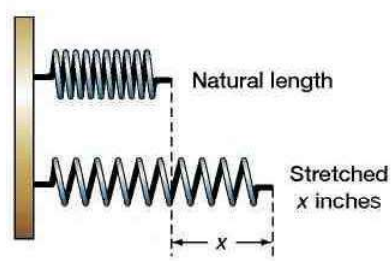
7. 

Solve each compound inequality. Then graph the solution set.

8. $6 < w + 3$ and $w + 3 < 11$
9. $n - 7 \le -5$ or $n - 7 \ge 1$
10. $3z + 1 < 13$ or $z \le 1$
11. $-8 < x - 4 \le -3$
12. Define a variable, write a compound inequality, and solve the following problem. Then check your solution.
Three times a number minus 7 is less than 17 and greater than 5.

Application

13. **PHYSICAL SCIENCE** According to Hooke's Law, the force F in pounds required to stretch a certain spring x inches beyond its natural length is given by $F = 4.5x$. If forces between 20 and 30 pounds, inclusive, are applied to the spring, what will be the range of the increased lengths of the stretched spring?



Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1
28–45	2, 4
46–48	3

Extra Practice

See page 834.

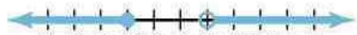
Graph the solution set of each compound inequality.

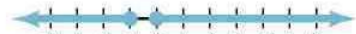
14. $x > 5$ and $x \le 9$
15. $s < -7$ and $s \le 0$
16. $r < 6$ or $r > 6$
17. $m \ge -4$ or $m > 6$
18. $7 < d < 11$
19. $-1 \le g < 3$

Write a compound inequality for each graph.

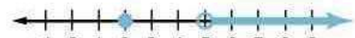
20. 

21. 

22. 

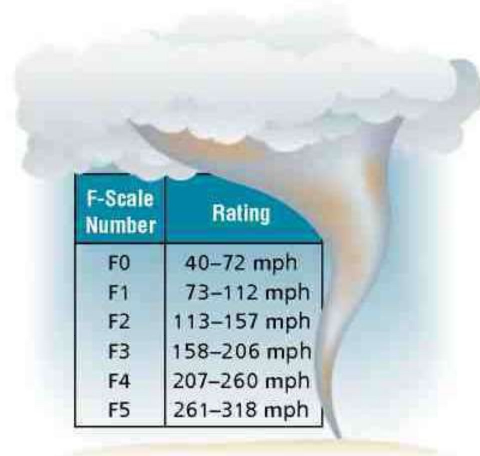
23. 

24. 

25. 

26. WEATHER The Fujita Scale (F-scale) is the official classification system for tornado damage. One factor used to classify a tornado is wind speed. Use the information in the table to write an inequality for the range of wind speeds of an F3 tornado.

27. BIOLOGY Each type of fish thrives in a specific range of temperatures. The optimum temperatures for sharks range from 18°C to 22°C, inclusive. Write an inequality to represent temperatures where sharks will *not* thrive.



F-Scale Number	Rating
F0	40–72 mph
F1	73–112 mph
F2	113–157 mph
F3	158–206 mph
F4	207–260 mph
F5	261–318 mph

Solve each compound inequality. Then graph the solution set.

28. $k + 2 > 12$ and $k + 2 \le 18$
29. $f + 8 \le 3$ and $f + 9 \ge -4$
30. $d - 4 > 3$ or $d - 4 \le 1$
31. $h - 10 < -21$ or $h + 3 < 2$
32. $3 < 2x - 3 < 15$
33. $4 < 2y - 2 < 10$
34. $3t - 7 \ge 5$ and $2t + 6 \le 12$
35. $8 > 5 - 3q$ and $5 - 3q > -13$
36. $-1 + x \le 3$ or $-x \le -4$
37. $3n + 11 \le 13$ or $-3n \ge -12$
38. $2p - 2 \le 4p - 8 \le 3p - 3$
39. $3g + 12 \le 6 + g \le 3g - 18$
40. $4c < 2c - 10$ or $-3c < -12$
41. $0.5b > -6$ or $3b + 16 < -8 + b$

Define a variable, write an inequality, and solve each problem. Then check your solution.

42. Eight less than a number is no more than 14 and no less than 5.

43. The sum of 3 times a number and 4 is between −8 and 10.

44. The product of −5 and a number is greater than 35 or less than 10.

45. One half a number is greater than 0 and less than or equal to 1.

46. HEALTH About 20% of the time you sleep is spent in rapid eye movement (REM) sleep, which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep?

47. SHOPPING A store is offering a $30 mail-in rebate on all color printers. Luisana is looking at different color printers that range in price from $175 to $260. How much can she expect to spend after the mail-in rebate?

48. **FUND-RAISING** Rashid is selling chocolates for his school's fund-raiser. He can earn prizes depending on how much he sells. So far, he has sold \$70 worth of chocolates. How much more does he need to sell to earn a prize in category D?

Sales (\$)	Prize
0–25	A
26–60	B
61–120	C
121–180	D
180+	E

49. **CRITICAL THINKING** Write a compound inequality that represents the values of x which make the following expressions *false*.

a. $x < 5$ or $x > 8$ **b.** $x \leq 6$ and $x \geq 1$

HEARING **For Exercises 50–52, use the following information.**
Humans hear sounds with sound waves within the 20 to 20,000 hertz range. Dogs hear sounds in the 15 to 50,000 hertz range.

50. Write a compound inequality for the hearing range of humans and one for the hearing range of dogs.

51. What is the union of the two graphs? the intersection?

52. Write an inequality or inequalities for the range of sounds that dogs can hear, but humans cannot.

53. **RESEARCH** Use the Internet or other resource to find the altitudes in miles of the layers of Earth's atmosphere, troposphere, stratosphere, mesosphere, thermosphere, and exosphere. Write inequalities for the range of altitudes for each layer.

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are compound inequalities used in tax tables?

Include the following in your answer:
- a description of the intervals used in the tax table shown at the beginning of the lesson, and
- a compound inequality describing the income of a head of a household paying \$7024 in taxes.

Standardized Test Practice

55. Ten pounds of fresh tomatoes make between 10 and 15 cups of cooked tomatoes. How many cups does one pound of tomatoes make?

Ⓐ between 1 and $1\frac{1}{2}$ cups Ⓑ between 1 and 5 cups

Ⓒ between 2 and 3 cups Ⓓ between 2 and 4 cups

56. Solve $-7 < x + 2 < 4$.

Ⓐ $-5 < x < 6$ Ⓑ $-9 < x < 2$

Ⓒ $-5 < x < 2$ Ⓓ $-9 < x < 6$

Graphing Calculator

57. **SOLVE COMPOUND INEQUALITIES** In Lesson 6-3, you learned how to use a graphing calculator to find the values of x that make a given inequality true. You can also use this method to test compound inequalities. The words *and* and *or* can be found in the LOGIC submenu of the TEST menu of a TI-83 Plus. Use this method to solve each of the following compound inequalities using your graphing calculator.

a. $x + 4 < -2$ or $x + 4 > 3$ **b.** $x - 3 \leq 5$ and $x + 6 \geq 4$

Maintain Your Skills

Mixed Review

58. **FUND-RAISING** A university is running a drive to raise money. A corporation has promised to match 40% of whatever the university can raise from other sources. How much must the school raise from other sources to have a total of at least $800,000 after the corporation's donation? *(Lesson 6-3)*

Solve each inequality. Then check your solution. *(Lesson 6-2)*

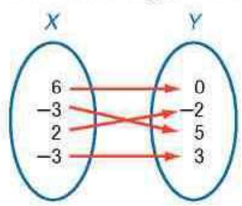
59. $18d \geq 90$ 60. $-7v < 91$ 61. $\frac{t}{13} < 13$ 62. $-\frac{3}{8}b > 9$

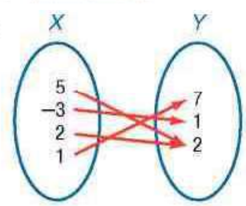
Solve. Assume that y varies directly as x. *(Lesson 5-2)*

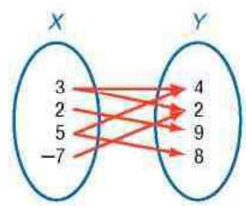
63. If $y = -8$ when $x = -3$, find x when $y = 6$.

64. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

Express the relation shown in each mapping as a set of ordered pairs. Then state the domain, range, and inverse. *(Lesson 4-3)*

65.


66.


67.


Find the odds of each outcome if a die is rolled. *(Lesson 2-6)*

68. a number greater than 2 69. not a 3

Find each product. *(Lesson 2-3)*

70. $-\frac{5}{6}\left(-\frac{2}{5}\right)$ 71. $-100(4.7)$ 72. $-\frac{7}{12}\left(\frac{6}{7}\right)\left(-\frac{3}{4}\right)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each value.** *(To review **absolute value**, see Lesson 2-1.)*

73. $|-7|$ 74. $|10|$ 75. $|-1|$ 76. $|-3.5|$

77. $|12 - 6|$ 78. $|5 - 9|$ 79. $|20 - 21|$ 80. $|3 - 18|$

Practice Quiz 2

Lessons 6-3 and 6-4

Solve each inequality. Then check your solution. *(Lesson 6-3)*

1. $5 - 4b > -23$
2. $\frac{1}{2}n + 3 \geq -5$
3. $3(t + 6) < 9$
4. $9x + 2 > 20$
5. $2m + 5 \leq 4m - 1$
6. $a < \frac{2a - 15}{3}$

Solve each compound inequality. Then graph the solution set. *(Lesson 6-4)*

7. $x - 2 < 7$ and $x + 2 > 5$
8. $2b + 5 \leq -1$ or $b - 4 \geq -4$
9. $4m - 5 > 7$ or $4m - 5 < -9$
10. $a - 4 < 1$ and $a + 2 > 1$

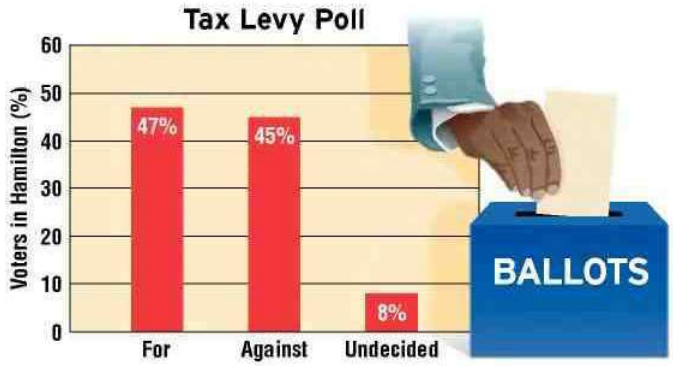
6-5 Solving Open Sentences Involving Absolute Value

What You'll Learn

- Solve absolute value equations.
- Solve absolute value inequalities.

How is absolute value used in election polls?

Voters in Hamilton will vote on a new tax levy in the next election. A poll conducted before the election found that 47% of the voters surveyed were for the tax levy, 45% were against the tax levy, and 8% were undecided. The poll has a 3-point margin of error.



The margin of error means that the result may be 3 percentage points higher or lower. So, the number of people in favor of the tax levy may be as high as 50% or as low as 44%. This can be written as an inequality using absolute value.

$|x - 47| \leq 3$ The difference between the actual number and 47 is within 3 points.

Study Tip

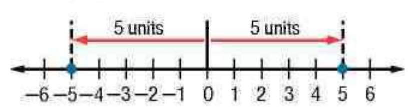
Look Back
To review **absolute value**, see Lesson 2-1.

ABSOLUTE VALUE EQUATIONS

There are three types of open sentences that can involve absolute value.

$$|x| = n \qquad |x| < n \qquad |x| > n$$

Consider the case of $|x| = n$. $|x| = 5$ means the distance between 0 and x is 5 units.



If $|x| = 5$, then $x = -5$ or $x = 5$. The solution set is $\{-5, 5\}$.

When solving equations that involve absolute value, there are two cases to consider.

Case 1 The value inside the absolute value symbols is positive.

Case 2 The value inside the absolute value symbols is negative.

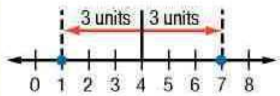
Equations involving absolute value can be solved by graphing them on a number line or by writing them as a compound sentence and solving it.

Example 1 Solve an Absolute Value Equation

Solve $|a - 4| = 3$.

Method 1 Graphing

$|a - 4| = 3$ means that the distance between a and 4 is 3 units. To find a on the number line, start at 4 and move 3 units in either direction.



The distance from 4 to 1 is 3 units.
The distance from 4 to 7 is 3 units.

The solution set is $\{1, 7\}$.

Study Tip

Absolute Value
Recall that $|a| = 3$ means $a = 3$ or $-a = 3$. The second equation can be written as $a = -3$. So, $|a - 4| = 3$ means $a - 4 = 3$ or $-(a - 4) = 3$. These can be written as $a - 4 = 3$ or $a - 4 = -3$.

Method 2 Compound Sentence

Write $|a - 4| = 3$ as $a - 4 = 3$ or $a - 4 = -3$.

Case 1

$a - 4 = 3$

$a - 4 + 4 = 3 + 4$ Add 4 to each side.

$a = 7$ Simplify.

Case 2

$a - 4 = -3$

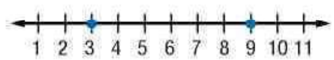
$a - 4 + 4 = -3 + 4$ Add 4 to each side.

$a = 1$ Simplify.

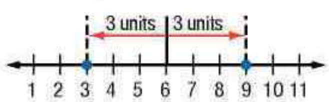
The solution set is $\{1, 7\}$.

Example 2 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.



Find the point that is the same distance from 3 as the distance from 9. The midpoint between 3 and 9 is 6.



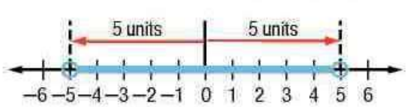
The distance from 6 to 3 is 3 units.
The distance from 6 to 9 is 3 units.

So, an equation is $|x - 6| = 3$.

CHECK Substitute 3 and 9 into $|x - 6| = 3$.

$$|x - 6| = 3 \qquad |x - 6| = 3$$
$$|3 - 6| \stackrel{?}{=} 3 \qquad |9 - 6| \stackrel{?}{=} 3$$
$$|-3| \stackrel{?}{=} 3 \qquad |3| \stackrel{?}{=} 3$$
$$3 = 3 \ \checkmark \qquad 3 = 3 \ \checkmark$$

ABSOLUTE VALUE INEQUALITIES Consider the inequality $|x| < n$. $|x| < 5$ means that the distance from 0 to x is less than 5 units.



Therefore, $x > -5$ and $x < 5$. The solution set is $\{x \mid -5 < x < 5\}$.

The Algebra Activity explores an inequality of the form $|x| < n$.

Algebra Activity

Absolute Value

Collect the Data

- Work in pairs. One person is the timekeeper.
- Start timing. The other person tells the timekeeper to stop timing after he or she thinks that one minute has elapsed.
- Write down the time in seconds.
- Switch places. Make a table that includes the results of the entire class.

Analyze the Data

1. Determine the error by subtracting 60 seconds from each student's time.
2. What does a negative error represent? a positive error?
3. The *absolute error* is the absolute value of the error. Since absolute value cannot be negative, the absolute error is positive. If the absolute error is 6 seconds, write two possibilities for a student's estimated time of one minute.
4. What estimates would have an absolute error less than 6 seconds?
5. Graph the responses and highlight all values such that $|60 - x| < 6$. How many guesses were within 6 seconds?

When solving inequalities of the form $|x| < n$, find the intersection of these two cases.

Case 1 The value inside the absolute value symbols is less than the positive value of n.

Case 2 The value inside the absolute value symbols is greater than the negative value of n.

Study Tip

Less Than

When an absolute value is on the left and the inequality symbol is $<$ or $\le$, the compound sentence uses *and*.

Example 3 Solve an Absolute Value Inequality (<)

Solve $|t + 5| < 9$. Then graph the solution set.

Write $|t + 5| < 9$ as $t + 5 < 9$ and $t + 5 > -9$.

Case 1

$t + 5 < 9$

$t + 5 - 5 < 9 - 5$ Subtract 5 from each side.

$t < 4$ Simplify.

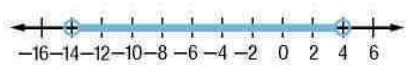
Case 2

$t + 5 > -9$

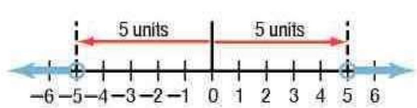
$t + 5 - 5 > -9 - 5$ Subtract 5 from each side.

$t > -14$ Simplify.

The solution set is $\{t \mid -14 < t < 4\}$.



Consider the inequality $|x| > n$. $|x| > 5$ means that the distance from 0 to x is greater than 5 units.



Therefore, $x < -5$ or $x > 5$. The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$.

When solving inequalities of the form $|x| > n$, find the union of these two cases.

Case 1 The value inside the absolute value symbols is greater than the positive value of n.

Case 2 The value inside the absolute value symbols is less than the negative value of n.

Study Tip

Greater Than
When the absolute value is on the left and the inequality symbol is $>$ or $\geq$, the compound sentence uses *or*.

Example 4 Solve an Absolute Value Inequality (>)

Solve $|2x + 8| \geq 6$. Then graph the solution set.

Write $|2x + 8| \geq 6$ as $2x + 8 \geq 6$ or $2x + 8 \leq -6$.

Case 1

$$2x + 8 \geq 6$$

$$2x + 8 - 8 \geq 6 - 8 \quad \text{Subtract 8 from each side.}$$

$$2x \geq -2 \quad \text{Simplify.}$$

$$\frac{2x}{2} \geq \frac{-2}{2} \quad \text{Divide each side by 2.}$$

$$x \geq -1 \quad \text{Simplify.}$$

Case 2

$$2x + 8 \leq -6$$

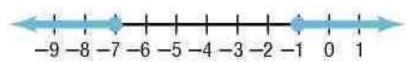
$$2x + 8 - 8 \leq -6 - 8 \quad \text{Subtract 8 from each side.}$$

$$2x \leq -14 \quad \text{Simplify.}$$

$$\frac{2x}{2} \leq \frac{-14}{2} \quad \text{Divide each side by 2.}$$

$$x \leq -7 \quad \text{Simplify.}$$

The solution set is $\{x \mid x \leq -7 \text{ or } x \geq -1\}$.



In general, there are three rules to remember when solving equations and inequalities involving absolute value.

Concept Summary *Absolute Value Equations and Inequalities*

If $|x| = n$, then $x = -n$ or $x = n$.

If $|x| < n$, then $x < n$ and $x > -n$.

If $|x| > n$, then $x > n$ or $x < -n$.

These properties are also true when $>$ or $<$ is replaced with $\geq$ or $\leq$.

Check for Understanding

Concept Check

1. **Compare and contrast** the solution of $|x - 2| > 6$ and the solution of $|x - 2| < 6$.
2. **OPEN ENDED** Write an absolute value inequality and graph its solution set.
3. **FIND THE ERROR** Leslie and Holly are solving $|x + 3| = 2$.

Leslie

$x + 3 = 2$ or $x + 3 = -2$

$x + 3 - 3 = 2 - 3$ $\quad$ $x + 3 - 3 = -2 - 3$

$x = -1$ $\quad$ $x = -5$

Holly

$x + 3 = 2$ or $x - 3 = 2$

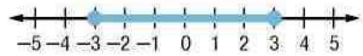
$x + 3 - 3 = 2 - 3$ $\quad$ $x - 3 + 3 = 2 + 3$

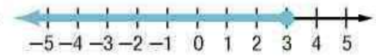
$x = -1$ $\quad$ $x = 5$

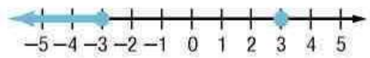
Who is correct? Explain your reasoning.

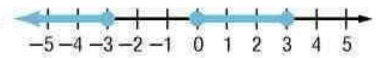
Guided Practice

4. Which graph represents the solution of $|k| \leq 3$?

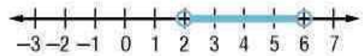
a. 

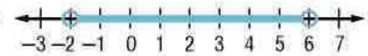
b. 

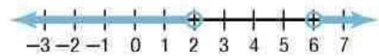
c. 

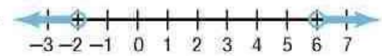
d. 

5. Which graph represents the solution of $|x - 4| > 2$?

a. 

b. 

c. 

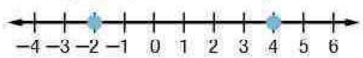
d. 

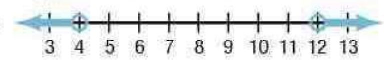
6. Express the statement in terms of an inequality involving absolute value. Do not solve.
A jar contains 832 gumballs. Amanda's guess was within 46 pieces.

Solve each open sentence. Then graph the solution set.

7. $|r + 3| = 10$
8. $|c - 2| < 6$
9. $|10 - w| > 15$
10. $|2g + 5| \geq 7$

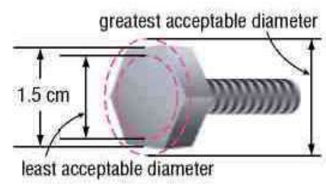
For each graph, write an open sentence involving absolute value.

11. 

12. 

Application

13. **MANUFACTURING** A manufacturer produces bolts which must have a diameter within 0.001 centimeter of 1.5 centimeters. What are the acceptable measurements for the diameter of the bolts?



Practice and Apply

Homework Help

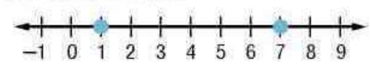
For Exercises	See Examples
14–19, 24–39, 46–51	1, 3, 4
20–23	3
40–45	2

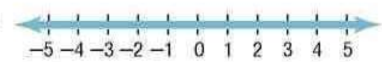
Extra Practice

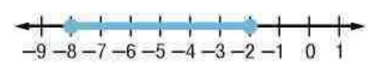
See page 834.

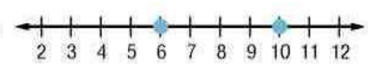
Match each open sentence with the graph of its solution set.

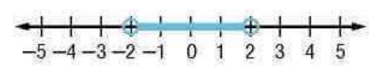
14. $|x + 5| \leq 3$
15. $|x - 4| > 4$
16. $|2x - 8| = 6$
17. $|x + 3| \geq -1$
18. $|x| < 2$
19. $|8 - x| = 2$

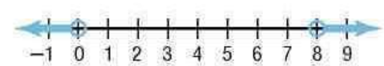
a. 

b. 

c. 

d. 

e. 

f. 

Express each statement using an inequality involving absolute value. Do *not* solve.

20. The pH of a buffered eye solution must be within 0.002 of a pH of 7.3.
21. The temperature inside a refrigerator should be within 1.5 degrees of 38°F.
22. Ramona's bowling score was within 6 points of her average score of 98.
23. The cruise control of a car set at 55 miles per hour should keep the speed within 3 miles per hour of 55.

Solve each open sentence. Then graph the solution set.

24. $|x - 5| = 8$

25. $|b + 9| = 2$

26. $|2p - 3| = 17$

27. $|5c - 8| = 12$

28. $|z - 2| \leq 5$

29. $|t + 8| < 2$

30. $|v + 3| > 1$

31. $|w - 6| \geq 3$

32. $|3s + 2| > -7$

33. $|3k + 4| \geq 8$

34. $|2n + 1| < 9$

35. $|6r + 8| < -4$

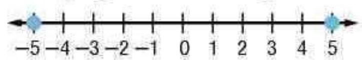
36. $|6 - (3d - 5)| \leq 14$

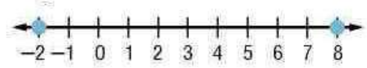
37. $|8 - (w - 1)| \leq 9$

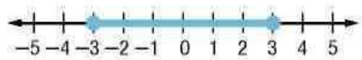
38. $\left|\frac{5h + 2}{6}\right| = 7$

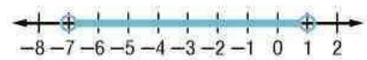
39. $\left|\frac{2 - 3x}{5}\right| \geq 2$

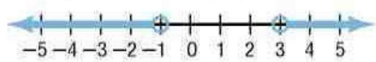
For each graph, write an open sentence involving absolute value.

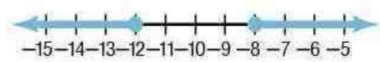
40. 

41. 

42. 

43. 

44. 

45. 

HEALTH For Exercises 46 and 47, use the following information.
The *average* length of a human pregnancy is 280 days. However, a healthy, full-term pregnancy can be 14 days longer or shorter.

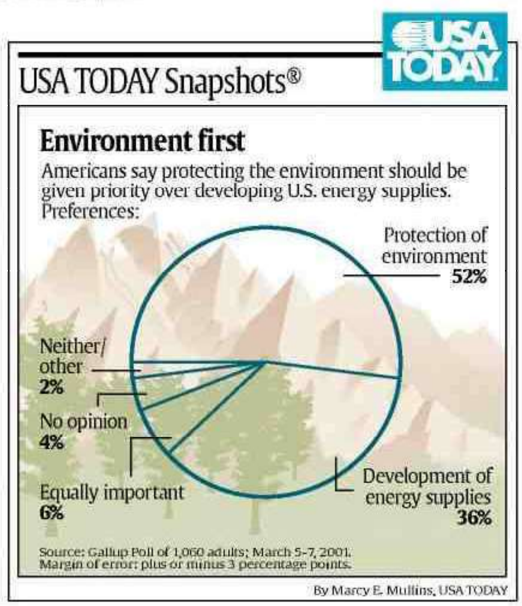
46. Write an absolute value inequality for the length of a full-term pregnancy.

47. Solve the inequality for the length of a full-term pregnancy.

48. **FIRE SAFETY** The pressure of a typical fire extinguisher should be within 25 pounds per square inch (psi) of 195 psi. Write the range of pressures for safe fire extinguishers.

49. **HEATING** A thermostat with a 2-degree differential will keep the temperature within 2 degrees Fahrenheit of the temperature set point. Suppose your home has a thermostat with a 3-degree differential. If you set the thermostat at 68°F, what is the range of temperatures in the house?

50. **ENERGY** Use the margin of error indicated in the graph at the right to find the range of the percent of people who say protection of the environment should have priority over developing energy supplies.



51. **TIRE PRESSURE** Tire pressure is measured in pounds per square inch (psi). Tires should be kept within 2 psi of the manufacturer's recommended tire pressure. If the recommended inflation pressure for a tire is 30 psi, what is the range of acceptable pressures?

52. **CRITICAL THINKING** State whether each open sentence is *always*, *sometimes*, or *never* true.
   a. $|x + 3| < -5$
   b. $|x - 6| > -1$
   c. $|x + 2| = 0$

More About...

Tire Pressure

Always inflate your tires to the pressure that is recommended by the manufacturer. The pressure stamped on the tire is the *maximum* pressure and should only be used under certain circumstances.

Source: www.etires.com

CONTENTS

More About. . .

Physical Science

The common name for sodium chloride is salt. Seawater is about 2.5% salt, and salt obtained by evaporating seawater is 95% to 98% pure.

Source: *World Book Encyclopedia*

53. **PHYSICAL SCIENCE** During an experiment, Li-Cheng must add 3.0 milliliters of sodium chloride to a solution. To get accurate results, the amount of sodium chloride must be within 0.5 milliliter of the required amount. How much sodium chloride can she add and still obtain the correct results?

54. **ENTERTAINMENT** Luis Gomez is a contestant on a television game show. He must guess within \$1500 of the actual price of the car without going over in order to win the car. The actual price of the car is \$18,000. What is the range of guesses in which Luis can win the vehicle?

55. **CRITICAL THINKING** The symbol $\pm$ means *plus* or *minus*.
   a. If $x = 3 \pm 1.2$, what are the values of x?
   b. Write $x = 3 \pm 1.2$ as an expression involving absolute value.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

   How is absolute value used in election polls?

   Include the following in your answer:
   - an explanation of how to solve the inequality describing the percent of people who are against the tax levy, and
   - a prediction of whether you think the tax levy will pass and why.

Standardized Test Practice

57. Choose the replacement set that makes $|x + 5| = 2$ true.
   (A) $\{-3, 3\}$ (B) $\{-3, -7\}$ (C) $\{2, -2\}$ (D) $\{3, -7\}$

58. What can you conclude about x if $-6 < |x| < 6$?
   (A) $-x \geq 0$ (B) $x \leq 0$ (C) $-x < 6$ (D) $-x > 6$

Maintain Your Skills

Mixed Review

59. **FITNESS** To achieve the maximum benefits from aerobic activity, your heart rate should be in your target zone. Your target zone is the range between 60% and 80% of your maximum heart rate. If Rafael's maximum heart rate is 190 beats per minute, what is his target zone? *(Lesson 6-4)*

Solve each inequality. Then check your solution. *(Lesson 6-3)*

60. $2m + 7 > 17$
61. $-2 - 3x \geq 2$
62. $\frac{2}{3}w - 3 \leq 7$

Find the slope and y-intercept of each equation. *(Lesson 5-4)*

63. $2x + y = 4$
64. $2y - 3x = 4$
65. $\frac{1}{2}x + \frac{3}{4}y = 0$

Solve each equation or formula for the variable specified. *(Lesson 3-8)*

66. $I = prt$, for r
67. $ex - 2y = 3z$, for x
68. $\frac{a + 5}{3} = 7x$, for x

Find each sum or difference. *(Lesson 2-2)*

69. $-13 + 8$
70. $-13.2 - 6.1$
71. $-4.7 - (-8.9)$

Name the property illustrated by each statement. *(Lesson 1-6)*

72. $10x + 10y = 10(x + y)$
73. $(2 + 3)a + 7 = 5a + 7$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Graph each equation.**
*(To review **graphing linear equations**, see Lesson 4-5.)*

74. $y = 3x + 4$
75. $y = -2$
76. $x + y = 3$
77. $y - 2x = -1$
78. $2y - x = -6$
79. $2(x + y) = 10$

6-6 Graphing Inequalities in Two Variables

What You'll Learn

- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.

Vocabulary

- half-plane
- boundary

How are inequalities used in budgets?

Hannah allots up to \$30 a month for lunch on school days. On most days, she brings her lunch. She can also buy lunch at the cafeteria or at a fast-food restaurant. She spends an average of \$3 a day at the cafeteria and an average of \$4 a day at a restaurant. How many times a month can Hannah buy her lunch and remain within her budget?

Let x represent the number of days she buys lunch at the cafeteria, and let y represent the number of days she buys lunch at a restaurant. Then the following inequality can be used to represent the situation.

$$\underbrace{3x}_{\text{The cost of eating in the cafeteria}} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{4y}_{\text{the cost of eating in a restaurant}} \quad \underbrace{\leq}_{\text{is less than or equal to}} \quad \underbrace{30}_{\$30.}$$

There are many solutions of this inequality.

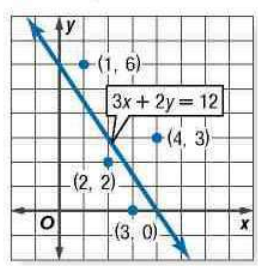
GRAPH LINEAR INEQUALITIES Like a linear equation in two variables, the solution set of an inequality in two variables is graphed on a coordinate plane. The solution set of an inequality in two variables is the set of all ordered pairs that satisfy the inequality.

Example 1 Ordered Pairs that Satisfy an Inequality

From the set {(1, 6), (3, 0), (2, 2), (4, 3)}, which ordered pairs are part of the solution set for $3x + 2y < 12$?

Use a table to substitute the x and y values of each ordered pair into the inequality.

x	y	$3x + 2y < 12$	True or False
1	6	$3(1) + 2(6) < 12$ $15 < 12$	false
3	0	$3(3) + 2(0) < 12$ $9 < 12$	true
2	2	$3(2) + 2(2) < 12$ $10 < 12$	true
4	3	$3(4) + 2(3) < 12$ $18 < 12$	false

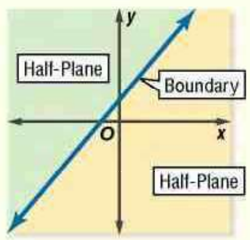


The ordered pairs {(3, 0), (2, 2)} are part of the solution set of $3x + 2y < 12$. In the graph, notice the location of the two ordered pairs that are solutions for $3x + 2y < 12$ in relation to the line.

The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a **half-plane**. An equation defines the **boundary** or edge for each half-plane.

Key Concept — Half-Planes and Boundaries

- **Words** Any line in the plane divides the plane into two regions called half-planes. The line is called the boundary of each of the two half-planes.
- **Model**



Study Tip

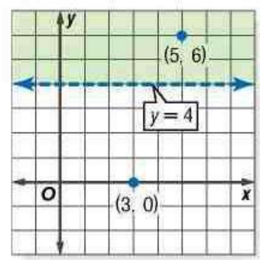
Dashed Line

- Like a circle on a number line, a dashed line on a coordinate plane indicates that the boundary is *not* part of the solution set.

Solid Line

- Like a dot on a number line, a solid line on a coordinate plane indicates that the boundary *is* included.

Consider the graph of $y > 4$. First determine the boundary by graphing $y = 4$, the equation you obtain by replacing the inequality sign with an equals sign. Since the inequality involves y-values greater than 4, but not equal to 4, the line should be dashed. The boundary divides the coordinate plane into two half-planes.



To determine which half-plane contains the solution, choose a point from each half-plane and test it in the inequality.

Try (3, 0).	Try (5, 6).
$y > 4$ $\quad y = 0$	$y > 4$ $\quad y = 6$
$0 > 4$ false	$6 > 4$ true

The half-plane that contains (5, 6) contains the solution. Shade that half-plane.

Example 2 Graph an Inequality

Graph $y - 2x \leq -4$.

Step 1 Solve for y in terms of x.

$$y - 2x \leq -4 \quad \text{Original inequality}$$

$$y - 2x + 2x \leq -4 + 2x \quad \text{Add } 2x \text{ to each side.}$$

$$y \leq 2x - 4 \quad \text{Simplify.}$$

Step 2 Graph $y = 2x - 4$. Since $y \leq 2x - 4$ means $y < 2x - 4$ or $y = 2x - 4$, the boundary is included in the solution set. The boundary should be drawn as a solid line.

(continued on the next page)

Study Tip

Origin as the Test Point

Use the origin as a standard test point because the values are easy to substitute into the inequality.

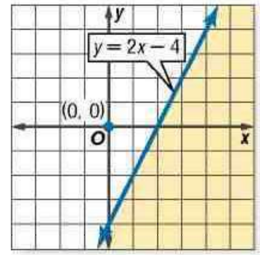
Step 3 Select a point in one of the half-planes and test it. Let's use (0, 0).

$y \leq 2x - 4$ Original inequality

$0 \leq 2(0) - 4$ $x = 0, y = 0$

$0 \leq -4$ false

Since the statement is false, the half-plane containing the origin is not part of the solution. Shade the other half-plane.



CHECK Test a point in the other half plane, for example, (3, −3).

$y \leq 2x - 4$ Original inequality

$-3 \leq 2(3) - 4$ $x = 3, y = -3$

$-3 \leq 2$ ✓

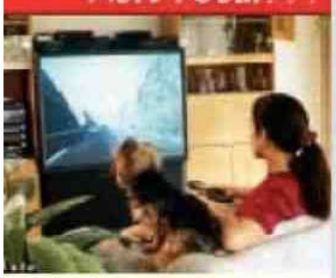
Since the statement is true, the half-plane containing (3, −3) should be shaded. The graph of the solution is correct.

SOLVE REAL-WORLD PROBLEMS When solving real-world inequalities, the domain and range of the inequality are often restricted to nonnegative numbers or whole numbers.

Example 3 Write and Solve an Inequality

ADVERTISING Rosa Padilla sells radio advertising in 30-second and 60-second time slots. During every hour, there are up to 15 minutes available for commercials. How many commercial slots can she sell for one hour of broadcasting?

More About...



Advertising

A typical one-hour program on television contains 40 minutes of the program and 20 minutes of commercials. During peak periods, a 30-second commercial can cost an average of $2.3 million.

Source: www.superbowl-ads.com

Step 1 Let x equal the number of 30-second commercials. Let y equal the number of 60-second or 1-minute commercials. Write an open sentence representing this situation.

$$\underbrace{\tfrac{1}{2}\text{ min}}\quad\underbrace{\text{times}}\quad\underbrace{\text{the number of 30-s commercials}}\quad\underbrace{\text{plus}}\quad\underbrace{\text{the number of 1-min commercials}}\quad\underbrace{\text{is up to}}\quad\underbrace{15\text{ min.}}$$

$$\frac{1}{2} \quad\cdot\quad x \quad+\quad y \quad\leq\quad 15$$

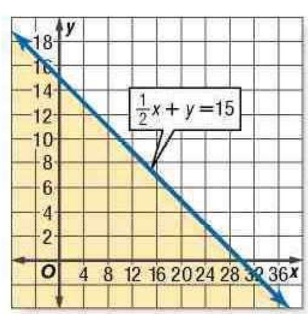
Step 2 Solve for y in terms of x.

$\frac{1}{2}x + y \leq 15$ Original inequality

$\frac{1}{2}x + y - \frac{1}{2}x \leq 15 - \frac{1}{2}x$ Subtract $\frac{1}{2}x$ from each side.

$y \leq 15 - \frac{1}{2}x$ Simplify.

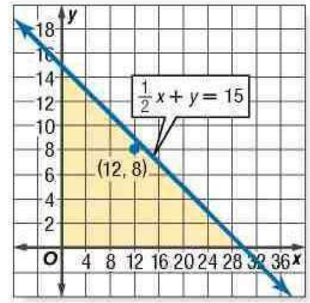
Step 3 Since the open sentence includes the equation, graph $y = 15 - \frac{1}{2}x$ as a solid line. Test a point in one of the half-planes, for example (0, 0). Shade the half-plane containing (0, 0) since $0 \leq 15 - \frac{1}{2}(0)$ is true.



CONTENTS

Step 4 Examine the solution.

- Rosa cannot sell a negative number of commercials. Therefore, the domain and range contain only nonnegative numbers.
- She also cannot sell half of a commercial. Thus, only points in the shaded half-plane whose x- and y-coordinates are whole numbers are possible solutions.



One solution is (12, 8). This represents twelve 30-second commercials and eight 60-second commercials in a one hour period.

Check for Understanding

Concept Check

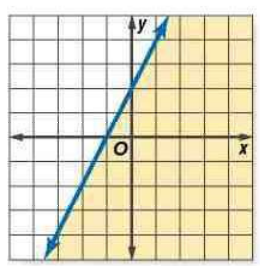
1. **Compare and contrast** the graph of $y = x + 2$ and the graph of $y < x + 2$.
2. **OPEN ENDED** Write an inequality in two variables and graph it.
3. **Explain** why it is usually only necessary to test one point when graphing an inequality.

Guided Practice

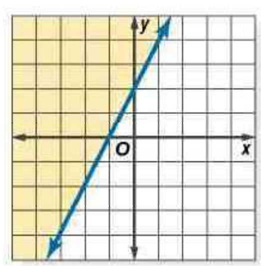
Determine which ordered pairs are part of the solution set for each inequality.

4. $y \leq x + 1$, $\{(-1, 0), (3, 2), (2, 5), (-2, 1)\}$
5. $y > 2x$, $\{(2, 6), (0, -1), (3, 5), (-1, -2)\}$
6. Which graph represents $y - 2x \geq 2$?

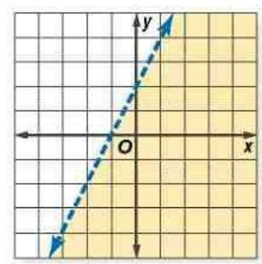
a.



b.



c.



Graph each inequality.

7. $y \geq 4$
8. $y \leq 2x - 3$
9. $4 - 2x < -2$
10. $1 - y > x$

Application

11. **ENTERTAINMENT** Coach Riley wants to take her softball team out for pizza and soft drinks after the last game of the season. She doesn't want to spend more than \$60. Write an inequality that represents this situation and graph the solution set.

Practice and Apply

Homework Help

For Exercises	See Examples
12–19	1
20–37	2
38–44	3

Extra Practice

See page 835.

Determine which ordered pairs are part of the solution set for each inequality.

12. $y \leq 3 - 2x$, $\{(0, 4), (-1, 3), (6, -8), (-4, 5)\}$

13. $y < 3x$, $\{(-3, 1), (-3, 2), (1, 1), (1, 2)\}$

14. $x + y < 11$, $\{(5, 7), (-13, 10), (4, 4), (-6, -2)\}$

15. $2x - 3y > 6$, $\{(3, 2), (-2, -4), (6, 2), (5, 1)\}$

16. $4y - 8 \geq 0$, $\{(5, -1), (0, 2), (2, 5), (-2, 0)\}$

17. $3x + 4y < 7$, $\{(1, 1), (2, -1), (-1, 1), (-2, 4)\}$

18. $|x - 3| \geq y$, $\{(6, 4), (-1, 8), (-3, 2), (5, 7)\}$

19. $|y + 2| < x$, $\{(2, -4), (-1, -5), (6, -7), (0, 0)\}$

Match each inequality with its graph.

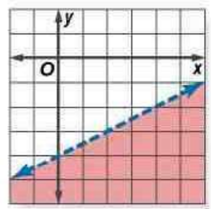
20. $2y + x \leq 6$

21. $\frac{1}{2}x - y > 4$

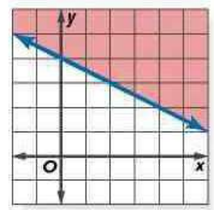
22. $y > 3 + \frac{1}{2}x$

23. $4y + 2x \geq 16$

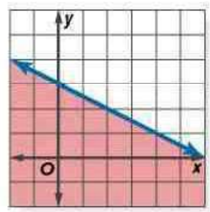
a.



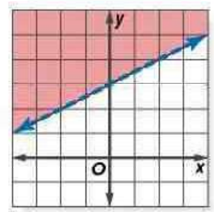
b.



c.



d.



24. Is the point $A(2, 3)$ on, above, or below the graph of $-2x + 3y = 5$?

25. Is the point $B(0, 1)$ on, above, or below the graph of $4x - 3y = 4$?

Graph each inequality.

26. $y < -3$

27. $x \geq 2$

28. $5x + 10y > 0$

29. $y < x$

30. $2y - x \leq 6$

31. $6x + 3y > 9$

32. $3y - 4x \geq 12$

33. $y \leq -2x - 4$

34. $8x - 6y < 10$

35. $3x - 1 \geq y$

36. $3(x + 2y) > -18$

37. $\frac{1}{2}(2x + y) < 2$

POSTAGE **For Exercises 38 and 39, use the following information.**
The U.S. Postal Service defines a large package as having the length of its longest side plus the distance around its thickest part less than or equal to 108 inches.

38. Write an inequality that represents this situation.

39. Are there any restrictions on the domain or range?

Online Research **Data Update** What are the current postage rates and regulations? Visit www.algebra1.com/data_update to learn more.

SHIPPING **For Exercises 40 and 41, use the following information.**
A delivery truck is transporting televisions and microwaves to an appliance store. The weight limit for the truck is 4000 pounds. The televisions weigh 77 pounds, and the microwaves weigh 55 pounds.

40. Write an inequality for this situation.

41. Will the truck be able to deliver 35 televisions and 25 microwaves at once?

FALL DANCE **For Exercises 42–44, use the following information.**
Tickets for the fall dance are \$5 per person or \$8 for couples. In order to cover expenses, at least \$1200 worth of tickets must be sold.

42. Write an inequality that represents this situation.

43. Graph the inequality.

44. If 100 single tickets and 125 couple tickets are sold, will the committee cover its expenses?

WebQuest

A linear inequality can be used to represent trends in Olympic times. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

45. **CRITICAL THINKING** Graph the intersection of the graphs of $y \le x - 1$ and $y \ge -x$.

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are inequalities used in budgets?

Include the following in your answer:

- an explanation of the restrictions placed on the domain and range of the inequality used to describe the number of times Hannah can buy her lunch, and
- three possible solutions of the inequality.

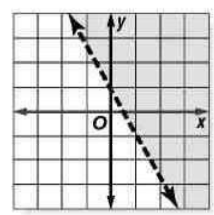
Standardized Test Practice

47. Which ordered pair is *not* a solution of $y - 2x < -5$?

Ⓐ $(2, -2)$ Ⓑ $(-1, -8)$ Ⓒ $(4, 1)$ Ⓓ $(5, 6)$

48. Which inequality is represented by the graph at the right?

Ⓐ $2x + y < 1$ Ⓑ $2x + y > 1$
Ⓒ $2x + y \le 1$ Ⓓ $2x + y \ge 1$



Maintain Your Skills

Mixed Review **Solve each open sentence. Then graph the solution set.** *(Lesson 6-5)*

49. $|3 + 2t| = 11$ **50.** $|x + 8| < 6$ **51.** $|2y + 5| \ge 3$

Solve each compound inequality. Then graph the solution. *(Lesson 6-4)*

52. $y + 6 > -1$ and $y - 2 < 4$ **53.** $m + 4 < 2$ or $m - 2 > 1$

State whether each percent of change is a percent of *increase* or *decrease*. Then find the percent of change. Round to the nearest whole percent. *(Lesson 3-7)*

54. original: 200
new: 172

55. original: 100
new: 142

56. original: 53
new: 75

Solve each equation. *(Lesson 3-4)*

57. $\frac{d - 2}{3} = 7$ **58.** $3n + 6 = -15$ **59.** $35 + 20h = 100$

Simplify. *(Lesson 2-4)*

60. $\frac{-64}{4}$ **61.** $\frac{27c}{-9}$ **62.** $\frac{12a - 14b}{-2}$ **63.** $\frac{18y - 9}{3}$

www.algebra1.com/self_check_quiz

Graphing Calculator Investigation

A Follow-Up of Lesson 6-6

Graphing Inequalities

You can use a TI-83 Plus graphing calculator to investigate the graphs of inequalities. Since graphing calculators only shade between two functions, enter a lower boundary as well as an upper boundary for each inequality.

Graph two different inequalities on your graphing calculator.

Step 1 *Graph $y \leq 3x + 1$.*

- Clear all functions from the Y= list.

KEYSTROKES: [Y=] [CLEAR]

- Graph $y \leq 3x + 1$ in the standard window.

KEYSTROKES: [2nd] [DRAW] 7 [(–)] 10 [,] 3 [X,T,θ,n] [+] 1 [)] [ENTER]

The lower boundary is Ymin or -10. The upper boundary is $y = 3x + 1$. All ordered pairs for which y is *less than or equal to* $3x + 1$ lie *below or on* the line and are solutions.

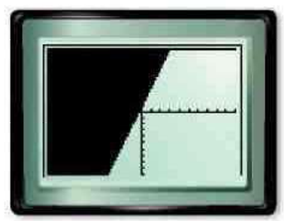
Step 2 *Graph $y - 3x \geq 1$.*

- Clear the drawing that is currently displayed.

KEYSTROKES: [2nd] [DRAW] 1

- Rewrite $y - 3x \geq 1$ as $y \geq 3x + 1$ and graph it.

KEYSTROKES: [2nd] [DRAW] 7 3 [X,T,θ,n] [+] 1 [,] 10 [)] [ENTER]



This time, the lower boundary is $y = 3x + 1$. The upper boundary is Ymax or 10. All ordered pairs for which y is *greater than or equal to* $3x + 1$ lie *above or on* the line and are solutions.

Exercises

1. Compare and contrast the two graphs shown above.

2. Graph the inequality $y \geq -2x + 4$ in the standard viewing window.

- **a.** What functions do you enter as the lower and upper boundaries?
- **b.** Using your graph, name four solutions of the inequality.

3. Suppose student movie tickets cost \$4 and adult movie tickets cost \$8. You would like to buy at least 10 tickets, but spend no more than \$80.

- **a.** Let x = number of student tickets and y = number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets.
- **b.** Which inequalities would you use as the lower and upper bounds?
- **c.** Graph the inequalities. Use the viewing window [0, 20] scl: 1 by [0, 20] scl: 1.
- **d.** Name four possible combinations of student and adult tickets.

www.algebra1.com/other_calculator_keystrokes

Chapter 6 Study Guide and Review

Vocabulary and Concept Check

Addition Property of Inequalities (p. 318)
boundary (p. 353)
compound inequality (p. 339)
Division Property of Inequalities (p. 327)
half-plane (p. 353)
intersection (p. 339)
Multiplication Property of Inequalities (p. 325)
set-builder notation (p. 319)
Subtraction Property of Inequalities (p. 319)
union (p. 340)

Choose the letter of the term that best matches each statement, algebraic expression, or algebraic sentence.

1. $\{w \mid w \geq -14\}$
2. If $x \leq y$, then $-5x \geq -5y$.
3. $p > -5$ and $p \leq 0$
4. If $a < b$, then $a + 2 < b + 2$.
5. the graph on one side of a boundary
6. If $s \geq t$, then $s - 7 \geq t - 7$.
7. $g \geq 7$ or $g < 2$
8. If $m > n$, then $\frac{m}{7} > \frac{n}{7}$.

a. Addition Property of Inequalities
b. Division Property of Inequalities
c. half-plane
d. intersection
e. Multiplication Property of Inequalities
f. set-builder notation
g. Subtraction Property of Inequalities
h. union

Lesson-by-Lesson Review

6-1 Solving Inequalities by Addition and Subtraction

See pages 318–323.

Concept Summary

- If any number is added to each side of a true inequality, the resulting inequality is also true.
- If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

Examples **Solve each inequality.**

1 $f + 9 \leq -23$

$f + 9 \leq -23$ Original inequality
$f + 9 - 9 \leq -23 - 9$ Subtract.
$f \leq -32$ Simplify.

The solution set is $\{f \mid f \leq -32\}$.

2 $v - 19 > -16$

$v - 19 > -16$ Original inequality
$v - 19 + 19 > -16 + 19$ Add.
$v > 3$ Simplify.

The solution set is $\{v \mid v > 3\}$.

Exercises **Solve each inequality. Then check your solution, and graph it on a number line.** *See Examples 1–5 on pages 318–320.*

9. $c + 51 > 32$
10. $r + 7 > -5$
11. $w - 14 \leq 23$
12. $a - 6 > -10$
13. $-0.11 \geq n - (-0.04)$
14. $2.3 < g - (-2.1)$
15. $7h \leq 6h - 1$
16. $5b > 4b + 5$

17. Define a variable, write an inequality, and solve the problem. Then check your solution. *Twenty-one is no less than the sum of a number and negative two.*

6-2 Solving Inequalities by Multiplication and Division

See pages 325–331.

Concept Summary

- If each side of a true inequality is multiplied or divided by the same positive number, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same negative number, the direction of the inequality must be *reversed*.

Examples **Solve each inequality.**

1 $-14g \geq 126$

$-14g \geq 126$	Original inequality
$\frac{-14g}{-14} \leq \frac{126}{-14}$	Divide and change $\geq$ to $\leq$.
$g \leq -9$	Simplify.

The solution set is $\{g \mid g \leq -9\}$.

2 $\frac{3}{4}d < 15$

$\frac{3}{4}d < 15$	Original inequality
$\left(\frac{4}{3}\right)\frac{3}{4}d < \left(\frac{4}{3}\right)15$	Multiply each side by $\frac{4}{3}$.
$d < 20$	Simplify.

The solution set is $\{d \mid d < 20\}$.

Exercises **Solve each inequality. Then check your solution.**

See Examples 1–5 on pages 326–328.

18. $15v > 60$ **19.** $12r \leq 72$ **20.** $-15z \geq -75$ **21.** $-9m < 99$

22. $\frac{b}{-12} \leq 3$ **23.** $\frac{d}{-13} > -5$ **24.** $\frac{2}{3}w > -22$ **25.** $\frac{3}{5}p \leq -15$

26. Define a variable, write an inequality, and solve the problem. Then check your solution. *Eighty percent of a number is greater than or equal to 24.*

6-3 Solving Multi-Step Inequalities

See pages 332–337.

Concept Summary

- Multi-step inequalities can be solved by undoing the operations.
- Remember to reverse the inequality sign when multiplying or dividing each side by a negative number.
- When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example **Solve $4(n - 1) < 7n + 8$.**

$4(n-1) < 7n + 8$	Original inequality
$4n - 4 < 7n + 8$	Distributive Property
$4n - 4 - 7n < 7n + 8 - 7n$	Subtract $7n$ from each side.
$-3n - 4 < 8$	Simplify.
$-3n - 4 + 4 < 8 + 4$	Add 4 to each side.
$-3n < 12$	Simplify.
$\frac{-3n}{-3} > \frac{12}{-3}$	Divide each side by -3 and change $<$ to $>$.
$n > -4$	Simplify.

The solution set is $\{n \mid n > -4\}$.

Exercises **Solve each inequality. Then check your solution.**
See Examples 1–5 on pages 332–334.

27. $-4h + 7 > 15$ **28.** $5 - 6n > -19$ **29.** $-5x + 3 < 3x + 19$

30. $15b - 12 > 7b + 60$ **31.** $-5(q + 12) < 3q - 4$ **32.** $7(g + 8) < 3(g + 2) + 4g$

33. $\frac{2(x + 2)}{3} \geq 4$ **34.** $\frac{1 - 7n}{5} > 10$

35. Define a variable, write an inequality, and solve the problem. Then check your solution. *Two thirds of a number decreased by 27 is at least 9.*

6-4 Solving Compound Inequalities

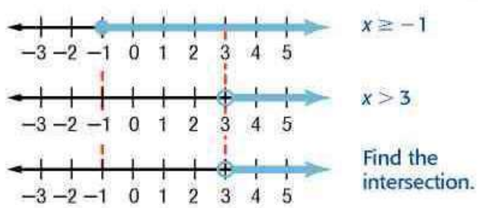
See pages 339–344.

Concept Summary

- The solution of a compound inequality containing *and* is the intersection of the graphs of the two inequalities.
- The solution of a compound inequality containing *or* is the union of the graphs of the two inequalities.

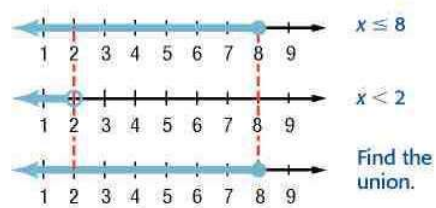
Examples **Graph the solution set of each compound inequality.**

1 $x \geq -1$ **and** $x > 3$



The solution set is $\{x \mid x > 3\}$.

2 $x \leq 8$ **or** $x < 2$



The solution set is $\{x \mid x \leq 8\}$.

Exercises **Solve each compound inequality. Then graph the solution set.**
See Examples 1–4 on pages 339–341.

36. $-1 < p + 3 < 5$ **37.** $-3 < 2k - 1 < 5$ **38.** $3w + 8 < 2$ or $w + 12 > 2 - w$

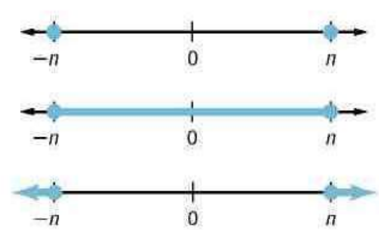
39. $a - 3 \leq 8$ or $a + 5 \geq 21$ **40.** $m + 8 < 4$ and $3 - m < 5$ **41.** $10 - 2y > 12$ and $7y < 4y + 9$

6-5 Solving Open Sentences Involving Absolute Value

See pages 345–351.

Concept Summary

- If $|x| = n$, then $x = -n$ or $x = n$.
- If $|x| < n$, then $x > -n$ and $x < n$.
- If $|x| > n$, then $x < -n$ or $x > n$.



CONTENTS

Chapter 6 For More ...
- Extra Practice, see pages 833–835.
- Mixed Problem Solving, see page 858.

Example **Solve $|x + 6| = 15$.**

$$|x + 6| = 15$$

$$\begin{aligned} x + 6 &= 15 & \text{or} \quad x + 6 &= -15 \\ x + 6 - 6 &= 15 - 6 & x + 6 - 6 &= -15 - 6 \\ x &= 9 & x &= -21 \end{aligned}$$

The solution set is $\{-21, 9\}$.

Exercises **Solve each open sentence. Then graph the solution set.**
See Examples 1, 3, and 4 on pages 346–348.

42. $|w - 8| = 12$ **43.** $|q + 5| = 2$ **44.** $|h + 5| > 7$ **45.** $|w + 8| \geq 1$
46. $|r + 10| < 3$ **47.** $|t + 4| \leq 3$ **48.** $|2x + 5| < 4$ **49.** $|3d + 4| < 8$

6-6 Graphing Inequalities in Two Variables

See pages 352–357.

Concept Summary

- To graph an inequality in two variables:
  - **Step 1** Determine the boundary and draw a dashed or solid line.
  - **Step 2** Select a test point. Test that point.
  - **Step 3** Shade the half-plane that contains the solution.

Example **Graph $y \geq x - 2$.**

Since the boundary is included in the solution, draw a solid line.

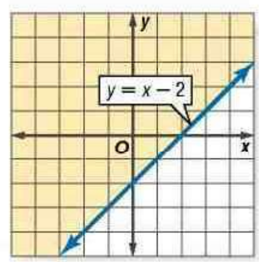
Test the point (0, 0).

$y \geq x - 2$ Original inequality

$0 \geq 0 - 2$ $x = 0, y = 0$

$0 \geq -2$ true

The half plane that contains (0, 0) should be shaded.



Exercises **Determine which ordered pairs are part of the solution set for each inequality.** *See Example 1 on page 352.*

50. $3x + 2y < 9$, $\{(1, 3), (3, 2), (-2, 7), (-4, 11)\}$

51. $5 - y \geq 4x$, $\left\{(2, -5), \left(\frac{1}{2}, 7\right), (-1, 6), (-3, 20)\right\}$

52. $\frac{1}{2}y \leq 6 - x$, $\{(-4, 15), (5, 1), (3, 8), (-2, 25)\}$

53. $-2x < 8 - y$, $\{(5, 10), (3, 6), (-4, 0), (-3, 6)\}$

Graph each inequality. *See Example 2 on pages 353 and 354.*

54. $y - 2x < -3$ **55.** $x + 2y \geq 4$ **56.** $y \leq 5x + 1$ **57.** $2x - 3y > 6$

Chapter 6 Practice Test

Vocabulary and Concepts

1. **Write** *the set of all numbers t such that t is greater than or equal to 17* in set-builder notation.
2. **Show** how to solve $6(a + 5) < 2a + 8$. Justify your work.
3. **OPEN ENDED** Give an example of a compound inequality that is an intersection and an example of a compound inequality that is a union.
4. **Compare and contrast** the graphs of $|x| \leq 3$ and $|x| \geq 3$.

Skills and Applications

Solve each inequality. Then check your solution.

5. $-23 \geq g - 6$
6. $9p < 8p - 18$
7. $d - 5 < 2d - 14$
8. $\frac{7}{8}w \geq -21$
9. $-22b \leq 99$
10. $4m - 11 \geq 8m + 7$
11. $-3(k - 2) > 12$
12. $\frac{f-5}{3} > -3$
13. $0.3(y - 4) \leq 0.8(0.2y + 2)$

14. **REAL ESTATE** A homeowner is selling her house. She must pay 7% of the selling price to her real estate agent after the house is sold. To the nearest dollar, what must be the selling price of her house to have at least $110,000 after the agent is paid?

15. Solve $6 + |r| = 3$.
16. Solve $|d| > -2$.

Solve each compound inequality. Then graph the solution set.

17. $r + 3 > 2$ and $4r < 12$
18. $3n + 2 \geq 17$ or $3n + 2 \leq -1$
19. $9 + 2p > 3$ and $-13 > 8p + 3$
20. $|2a - 5| < 7$
21. $|7 - 3s| \geq 2$
22. $|7 - 5z| > 3$

Define a variable, write an inequality, and solve each problem. Then check your solution.

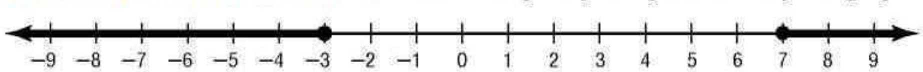
23. One fourth of a number is no less than -3.
24. Three times a number subtracted from 14 is less than two.
25. Five less than twice a number is between 13 and 21.

26. **TRAVEL** Megan's car gets between 18 and 21 miles per gallon of gasoline. If her car's tank holds 15 gallons, what is the range of distance that Megan can drive her car on one tank of gasoline?

Graph each inequality.

27. $y \geq 3x - 2$
28. $2x + 3y < 6$
29. $x - 2y > 4$

30. **STANDARDIZED TEST PRACTICE** Which inequality is represented by the graph?



Ⓐ $|x - 2| \leq 5$ Ⓑ $|x - 2| \geq 5$ Ⓒ $|x + 2| \leq 5$ Ⓓ $|x + 2| \geq 5$

Chapter 6 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the following is a correct statement? (Lesson 2-1)

   (A) $-\frac{9}{3} > \frac{3}{9}$ (B) $-\frac{3}{9} > -\frac{9}{3}$

   (C) $-\frac{3}{9} < -\frac{9}{3}$ (D) $\frac{9}{3} < \frac{3}{9}$

2. $(-6)(-7) =$ (Lesson 2-3)

   (A) -42 (B) -13

   (C) 13 (D) 42

3. A cylindrical can has a volume of 5625π cubic centimeters. Its height is 25 centimeters. What is the radius of the can? Use the formula $V = \pi r^2 h$. (Lessons 2-8 and 3-8)

   (A) 4.8 cm (B) 7.5 cm

   (C) 15 cm (D) 47.1 cm

4. A furnace repair service charged a customer \$80 for parts and \$65 per hour worked. The bill totaled \$177.50. About how long did the repair technician work on the furnace? (Lessons 3-1 and 3-4)

   (A) 0.5 hour (B) 1.5 hours

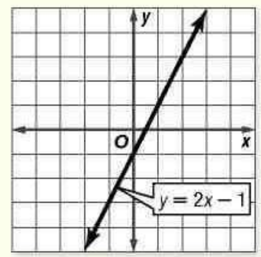
   (C) 2 hours (D) 4 hours

5. The formula $P = \frac{4(220 - A)}{5}$ determines the recommended maximum pulse rate P during exercise for a person who is A years old. Cameron is 15 years old. What is his recommended maximum pulse rate during exercise? (Lesson 3-8)

   (A) 162 (B) 164

   (C) 173 (D) 263

6. The graph of the function $y = 2x - 1$ is shown. If the graph is translated 3 units up, which equation will best represent the new line? (Lesson 4-2)



   (A) $y = 2x + 2$ (B) $y = 2x - 3$

   (C) $y = 2x + 3$ (D) $y = 2x - 4$

7. The table shows a set of values for x and y. Which equation best represents this set of data? (Lesson 4-8)

x	−4	−1	2	5	8
y	−16	−4	8	20	32

   (A) $y = 3x - 4$ (B) $y = 3x + 2$

   (C) $y = 2x - 10$ (D) $y = 4x$

8. Ali's grade depends on 4 test scores. On the first 3 tests, she earned scores of 78, 82, and 75. She wants to average at least 80. Which inequality can she use to find the score x that she needs on the fourth test in order to earn a final grade of at least 80? (Lesson 6-3)

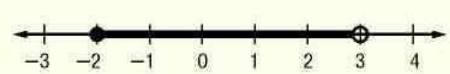
   (A) $\frac{78 + 82 + 75 + x}{3} \geq 80$

   (B) $\frac{78 + 82 + 75 + x}{4} \geq 80$

   (C) $\frac{78 + 82 + 75 - x}{4} \geq 80$

   (D) $\frac{78 + 82 + 75 + x}{4} \leq 80$

9. Which inequality is represented by the graph? (Lesson 6-4)



   (A) $-2 < x < 3$ (B) $-2 < x \leq 3$

   (C) $-2 \leq x < 3$ (D) $-2 \leq x \leq 3$

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

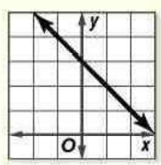
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A die is rolled. What are the odds of rolling a number less than 5? (Lesson 2-6)

11. A car is traveling at an average speed of 54 miles per hour. How many minutes will it take the car to travel 117 miles? (Lesson 2-4)

12. The price of a tape player was cut from \$48 to \$36. What was the percent of decrease? (Lesson 3-7)

13. Write an equation in slope-intercept form that describes the graph. (Lesson 5-4)



14. A line is parallel to the graph of the equation $\frac{1}{3}y = \frac{2}{3}x - 1$. What is the slope of the parallel line? (Lessons 5-4 and 5-6)

15. Solve $\frac{1}{2}(10x - 8) - 3(x - 1) \geq 15$ for x. (Lesson 6-3)

16. Find all values of x that make the inequality $|x - 3| > 5$ true. (Lesson 6-5)

The Princeton Review Test-Taking Tip

Questions 13 and 14

- Know the slope-intercept form of linear equations: $y = mx + b$.
- Understand the definition of slope.
- Recognize the relationships between the slopes of parallel lines and between the slopes of perpendicular lines.

17. Graph the equation $y = -2x + 4$ and indicate which region represents $y < -2x + 4$. (Lesson 6-6)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
18.	$\sqrt{68}$	9

(Lesson 2-7)

19. $x > 5$ or $x < -7$

$-3 < y < 4$

Column A	Column B
$\lvert x \rvert$	$\lvert y \rvert$

(Lesson 6-4)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

20. The Carlson family is building a house on a lot that is 91 feet long and 158 feet wide. (Lessons 6-1, 6-2, and 6-4)

a. Town law states that the sides of a house cannot be closer than 10 feet to the edges of a lot. Write an inequality for the possible lengths of the Carlson family's house, and solve the inequality.

b. The Carlson family wants their house to be at least 2800 square feet and no more than 3200 square feet. They also want their house to have the maximum possible length. Write an inequality for the possible widths of their house, and solve the inequality. Round your answer to the nearest whole number of feet.

Chapter 7 Solving Systems of Linear Equations and Inequalities

What You'll Learn

- **Lesson 7-1** Solve systems of linear equations by graphing.
- **Lessons 7-2 through 7-4** Solve systems of linear equations algebraically.
- **Lesson 7-5** Solve systems of linear inequalities by graphing.

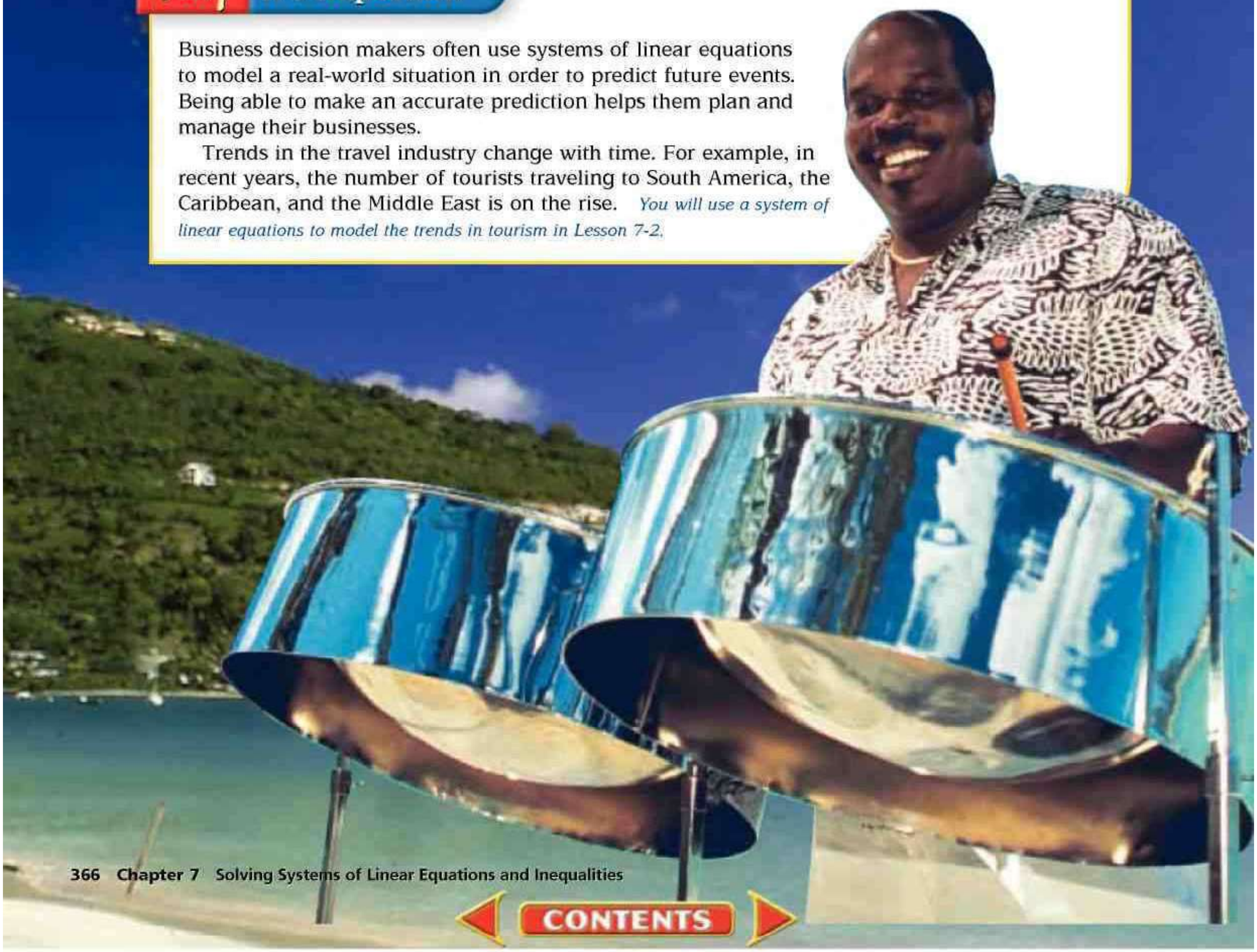
Key Vocabulary

- system of equations (p. 369)
- substitution (p. 376)
- elimination (p. 382)
- system of inequalities (p. 394)

Why It's Important

Business decision makers often use systems of linear equations to model a real-world situation in order to predict future events. Being able to make an accurate prediction helps them plan and manage their businesses.

Trends in the travel industry change with time. For example, in recent years, the number of tourists traveling to South America, the Caribbean, and the Middle East is on the rise. *You will use a system of linear equations to model the trends in tourism in Lesson 7-2.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1 — Graph Linear Equations

Graph each equation. *(For review, see Lesson 4-5.)*

1. $y = 1$ **2.** $y = -2x$ **3.** $y = 4 - x$

4. $y = 2x + 3$ **5.** $y = 5 - 2x$ **6.** $y = \frac{1}{2}x + 2$

For Lesson 7-2 — Solve for a Given Variable

Solve each equation or formula for the variable specified. *(For review, see Lesson 3-8.)*

7. $4x + a = 6x$, for x **8.** $8a + y = 16$, for a

9. $\frac{7bc - d}{10} = 12$, for b **10.** $\frac{7m + n}{q} = 2m$, for q

For Lessons 7-3 and 7-4 — Simplify Expressions

Simplify each expression. If not possible, write *simplified.* *(For review, see Lesson 1-5.)*

11. $(3x + y) - (2x + y)$ **12.** $(7x - 2y) - (7x + 4y)$ **13.** $(16x - 3y) + (11x + 3y)$

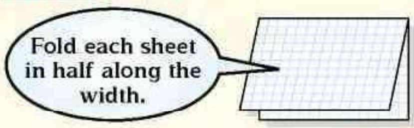
14. $(8x - 4y) + (-8x + 5y)$ **15.** $4(2x + 3y) - (8x - y)$ **16.** $3(x - 4y) + (x + 12y)$

17. $2(x - 2y) + (3x + 4y)$ **18.** $5(2x - y) - 2(5x + 3y)$ **19.** $3(x + 4y) + 2(2x - 6y)$

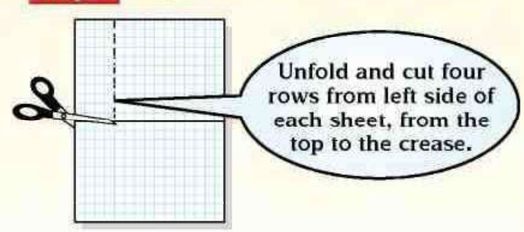
FOLDABLES™ Study Organizer

Make this Foldable to record information about solving systems of equations and inequalities. Begin with five sheets of grid paper.

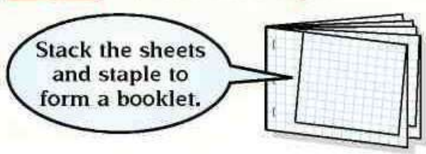
Step 1 Fold



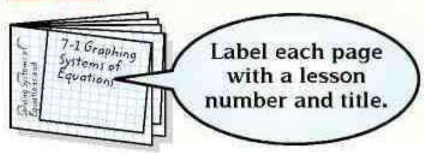
Step 2 Cut



Step 3 Stack and Staple



Step 4 Label



Reading and Writing As you read and study the chapter, unfold each page and fill the journal with notes, graphs, and examples for systems of equations and inequalities.

Spreadsheet Investigation

A Preview of Lesson 7-1

Systems of Equations

You can use a spreadsheet to investigate when two quantities will be equal. Enter each formula into the spreadsheet and look for the time when both formulas have the same result.

Example

Bill Winters is considering two job offers in telemarketing departments. The salary at the first job is $400 per week plus 10% commission on Mr. Winters' sales. At the second job, the salary is $375 per week plus 15% commission. For what amount of sales would the weekly salary be the same at either job?

Enter different amounts for Mr. Winters' weekly sales in column A. Then enter the formula for the salary at the first job in each cell in column B. In each cell of column C, enter the formula for the salary at the second job.

Job Salaries

	A	B	C
1	Sales	Salary 1	Salary 2
2	0	400	375
3	100	410	390
4	200	420	405
5	300	430	420
6	400	440	435
7	500	450	450
8	600	460	465
9	700	470	480
10	800	480	495
11	900	490	510
12	1000	500	525
13			

Sheet1

The spreadsheet shows that for sales of $500 the total weekly salary for each job is $450.

Exercises

For Exercises 1–4, use the spreadsheet of weekly salaries above.

1. If x is the amount of Mr. Winters' weekly sales and y is his total weekly salary, write a linear equation for the salary at the first job.

2. Write a linear equation for the salary at the second job.

3. Which ordered pair is a solution for both of the equations you wrote for Exercises 1 and 2?

a. (100, 410) **b.** (300, 420) **c.** (500, 450) **d.** (900, 510)

4. Use the graphing capability of the spreadsheet program to graph the salary data using a line graph. At what point do the two lines intersect? What is the significance of that point in the real-world situation?

5. How could you find the sales for which Mr. Winters' salary will be equal without using a spreadsheet?

7-1 Graphing Systems of Equations

What You'll Learn

- Determine whether a system of linear equations has 0, 1, or infinitely many solutions.
- Solve systems of equations by graphing.

Vocabulary

- system of equations
- consistent
- inconsistent
- independent
- dependent

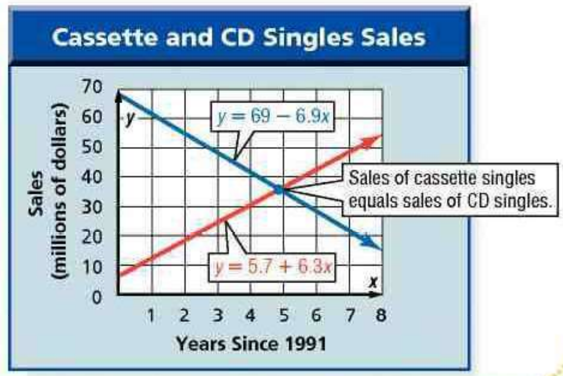
How can you use graphs to compare the sales of two products?

During the 1990s, sales of cassette singles decreased, and sales of CD singles increased. Assume that the sales of these singles were linear functions. If x represents the years since 1991 and y represents the sales in millions of dollars, the following equations represent the sales of these singles.

Cassette singles: $y = 69 - 6.9x$
CD singles: $y = 5.7 + 6.3x$

These equations are graphed at the right.

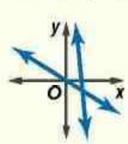
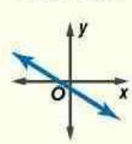
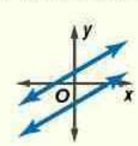
The point at which the two graphs intersect represents the time when the sales of cassette singles equaled the sales of CD singles. The ordered pair of this point is a solution of both equations.



NUMBER OF SOLUTIONS Two equations, such as $y = 69 - 6.9x$ and $y = 5.7 + 6.3x$, together are called a **system of equations**. A solution of a system of equations is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have 0, 1, or an infinite number of solutions.

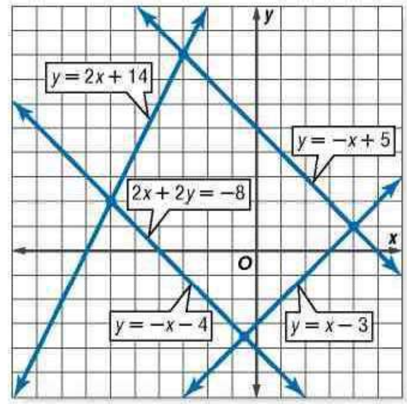
- If the graphs intersect or coincide, the system of equations is said to be **consistent**. That is, it has at least one ordered pair that satisfies both equations.
- If the graphs are parallel, the system of equations is said to be **inconsistent**. There are *no* ordered pairs that satisfy both equations.
- Consistent equations can be **independent** or **dependent**. If a system has exactly one solution, it is independent. If the system has an infinite number of solutions, it is dependent.

Concept Summary — Systems of Equations

	Intersecting Lines	Same Line	Parallel Lines
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

Example 1 Number of Solutions

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.



a. $y = -x + 5$
$y = x - 3$

Since the graphs of $y = -x + 5$ and $y = x - 3$ intersect, there is one solution.

b. $y = -x + 5$
$2x + 2y = -8$

Since the graphs of $y = -x + 5$ and $2x + 2y = -8$ are parallel, there are no solutions.

c. $2x + 2y = -8$
$y = -x - 4$

Since the graphs of $2x + 2y = -8$ and $y = -x - 4$ coincide, there are infinitely many solutions.

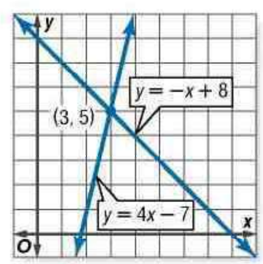
SOLVE BY GRAPHING One method of solving systems of equations is to carefully graph the equations on the same coordinate plane.

Example 2 Solve a System of Equations

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

a. $y = -x + 8$
$y = 4x - 7$

The graphs appear to intersect at the point with coordinates (3, 5). Check this estimate by replacing x with 3 and y with 5 in each equation.



CHECK

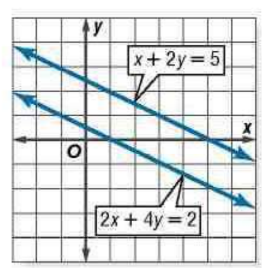
$y = -x + 8$
$5 \stackrel{?}{=} -3 + 8$
$5 = 5$ ✓

$y = 4x - 7$
$5 \stackrel{?}{=} 4(3) - 7$
$5 \stackrel{?}{=} 12 - 7$
$5 = 5$ ✓

The solution is (3, 5).

b. $x + 2y = 5$
$2x + 4y = 2$

The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Notice that the lines have the same slope but different y-intercepts. *Recall that a system of equations that has no solution is said to be inconsistent.*



Study Tip

Look Back

To review **graphing linear equations**, see Lesson 4-5.

Example 3 Write and Solve a System of Equations

WORLD RECORDS **Use the information on Guy Delage's swim at the left. If Guy can swim 3 miles per hour for an extended period and the raft drifts about 1 mile per hour, how many hours did he spend swimming each day?**

Words You have information about the amount of time spent swimming and floating. You also know the rates and the total distance traveled.

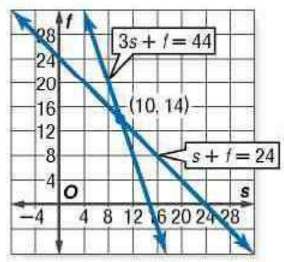
Variables Let s = the number of hours Guy swam, and let f = the number of hours he floated each day. Write a system of equations to represent the situation.

Equations

$$\underbrace{s}_{\text{The number of hours swimming}} \underbrace{+}_{\text{plus}} \underbrace{f}_{\text{the number of hours floating}} \underbrace{=}_{\text{equals}} \underbrace{24}_{\text{the total number of hours in a day.}}$$

$$\underbrace{3s}_{\text{The daily miles traveled swimming}} \underbrace{+}_{\text{plus}} \underbrace{1f}_{\text{the daily miles traveled floating}} \underbrace{=}_{\text{equals}} \underbrace{44}_{\text{the total miles traveled in a day.}}$$

Graph the equations $s + f = 24$ and $3s + f = 44$. The graphs appear to intersect at the point with coordinates (10, 14). Check this estimate by replacing s with 10 and f with 14 in each equation.



CHECK

$s + f = 24$
$10 + 14 \stackrel{?}{=} 24$
$24 = 24$ ✓

$3s + f = 44$
$3(10) + 14 \stackrel{?}{=} 44$
$30 + 14 \stackrel{?}{=} 44$
$44 = 44$ ✓

Guy Delage spent about 10 hours swimming each day.

More About. . .

World Records

In 1994, Guy Delage swam 2400 miles across the Atlantic Ocean from Cape Verde to Barbados. Everyday he would swim awhile and then rest while floating with the current on a huge raft. He averaged 44 miles per day.

Source: Banner Aerospace, Inc.

Check for Understanding

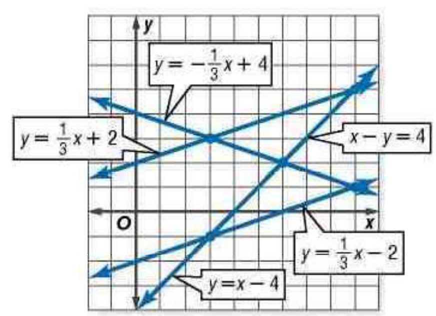
Concept Check

1. **OPEN ENDED** Draw the graph of a system of equations that has one solution at (−2, 3).
2. **Determine** whether a system of equations with (0, 0) and (2, 2) as solutions *sometimes*, *always*, or *never* has other solutions. Explain.
3. **Find a counterexample** for the following statement.
If the graphs of two linear equations have the same slope, then the system of equations has no solution.

Guided Practice

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

4. $y = x - 4$
$y = \frac{1}{3}x - 2$
5. $y = \frac{1}{3}x + 2$
$y = \frac{1}{3}x - 2$
6. $x - y = 4$
$y = x - 4$
7. $x - y = 4$
$y = -\frac{1}{3}x + 4$



Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

8. $y = -x$
   $y = 2x$
9. $x + y = 8$
   $x - y = 2$
10. $2x + 4y = 2$
    $3x + 6y = 3$
11. $x + y = 4$
    $x + y = 1$
12. $x - y = 2$
    $3y + 2x = 9$
13. $x + y = 2$
    $y = 4x + 7$

Application

14. **RESTAURANTS** The Rodriguez family and the Wong family went to a brunch buffet. The restaurant charges one price for adults and another price for children. The Rodriguez family has two adults and three children, and their bill was \$40.50. The Wong family has three adults and one child, and their bill was \$38.00. Determine the price of the buffet for an adult and the price for a child.

Practice and Apply

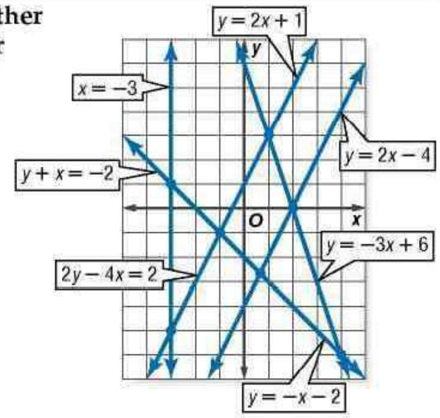
Homework Help

For Exercises	See Examples
15–22	1
23–40	2
41–54	3

Extra Practice

See page 835.

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

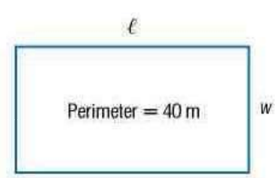


15. $x = -3$
    $y = 2x + 1$
16. $y = -x - 2$
    $y = 2x - 4$
17. $y + x = -2$
    $y = -x - 2$
18. $y = 2x + 1$
    $y = 2x - 4$
19. $y = -3x + 6$
    $y = 2x - 4$
20. $2y - 4x = 2$
    $y = 2x - 4$
21. $2y - 4x = 2$
    $y = -3x + 6$
22. $2y - 4x = 2$
    $y = 2x + 1$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

23. $y = -6$
    $4x + y = 2$
24. $x = 2$
    $3x - y = 8$
25. $y = \frac{1}{2}x$
    $2x + y = 10$
26. $y = -x$
    $y = 2x - 6$
27. $y = 3x - 4$
    $y = -3x - 4$
28. $y = 2x + 6$
    $y = -x - 3$
29. $x - 2y = 2$
    $3x + y = 6$
30. $x + y = 2$
    $2y - x = 10$
31. $3x + 2y = 12$
    $3x + 2y = 6$
32. $2x + 3y = 4$
    $-4x - 6y = -8$
33. $2x + y = -4$
    $5x + 3y = -6$
34. $4x + 3y = 24$
    $5x - 8y = -17$
35. $3x + y = 3$
    $2y = -6x + 6$
36. $y = x + 3$
    $3y + x = 5$
37. $2x + 3y = -17$
    $y = x - 4$
38. $y = \frac{2}{3}x - 5$
    $3y = 2x$
39. $6 - \frac{3}{8}y = x$
    $\frac{2}{3}x + \frac{1}{4}y = 4$
40. $\frac{1}{2}x + \frac{1}{3}y = 6$
    $y = \frac{1}{2}x + 2$

41. **GEOMETRY** The length of the rectangle at the right is 1 meter less than twice its width. What are the dimensions of the rectangle?



GEOMETRY For Exercises 42 and 43, use the graphs of $y = 2x + 6$, $3x + 2y = 19$, and $y = 2$, which contain the sides of a triangle.

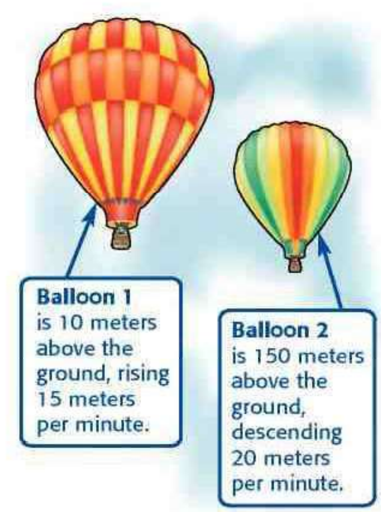
42. Find the coordinates of the vertices of the triangle.
43. Find the area of the triangle.

WebQuest

You can graph a system of equations to predict when men's and women's Olympic times will be the same. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

BALLOONING For Exercises 44 and 45, use the information in the graphic at the right.

44. In how many minutes will the balloons be at the same height?
45. How high will the balloons be at that time?

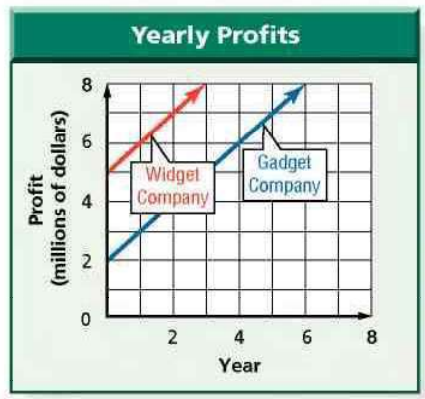


SAVINGS For Exercises 46 and 47, use the following information.
Monica and Michael Gordon both want to buy a scooter. Monica has already saved \$25 and plans to save \$5 per week until she can buy the scooter. Michael has \$16 and plans to save \$8 per week.

46. In how many weeks will Monica and Michael have saved the same amount of money?
47. How much will each person have saved at that time?

BUSINESS For Exercises 48–50, use the graph at the right.

48. Which company had the greater profit during the ten years?
49. Which company had a greater rate of growth?
50. If the profit patterns continue, will the profits of the two companies ever be equal? Explain.



POPULATION For Exercises 51–54, use the following information.
The U.S. Census Bureau divides the country into four sections. They are the Northeast, the Midwest, the South, and the West.

51. In 1990, the population of the Midwest was about 60 million. During the 1990s, the population of this area increased an average of about 0.4 million per year. Write an equation to represent the population of the Midwest for the years since 1990.
52. The population of the West was about 53 million in 1990. The population of this area increased an average of about 1 million per year during the 1990s. Write an equation to represent the population of the West for the years since 1990.
53. Graph the population equations.
54. Assume that the rate of growth of each of these areas remains the same. Estimate when the population of the West would be equal to the population of the Midwest.

55. **CRITICAL THINKING** The solution of the system of equations $Ax + y = 5$ and $Ax + By = 20$ is $(2, -3)$. What are the values of A and B?

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

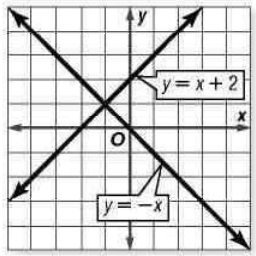
How can you use graphs to compare the sales of two products?

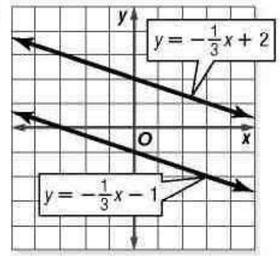
Include the following in your answer:

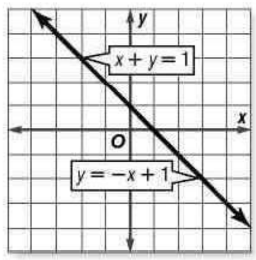
- an estimate of the year in which the sales of cassette singles equaled the sales of CD singles, and
- an explanation of why graphing works.

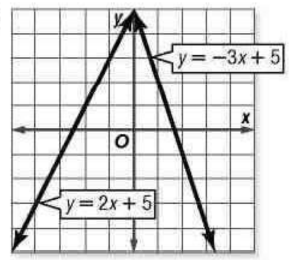
Standardized Test Practice

57. Which graph represents a system of equations with no solution?

(A)


(B)


(C)


(D)


58. How many solutions exist for the system of equations below?

$4x + y = 7$
$3x - y = 0$

(A) no solution
(B) one solution
(C) infinitely many solutions
(D) cannot be determined

Maintain Your Skills

Mixed Review

Determine which ordered pairs are part of the solution set for each inequality. *(Lesson 6-6)*

59. $y \leq 2x$, $\{(1, 4), (-1, 5), (5, -6), (-7, 0)\}$

60. $y < 8 - 3x$, $\{(-4, 2), (-3, 0), (1, 4), (1, 8)\}$

61. **MANUFACTURING** The inspector at a perfume manufacturer accepts a bottle if it is less than 0.05 ounce above or below 2 ounces. What are the acceptable numbers of ounces for a perfume bottle? *(Lesson 6-5)*

Write each equation in standard form. *(Lesson 5-5)*

62. $y - 1 = 4(x - 5)$ 63. $y + 2 = \frac{1}{3}(x + 3)$ 64. $y - 4 = -6(x + 2)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation for the variable specified.**
*(To review **solving equations for a specified variable**, see Lesson 3-8.)*

65. $12x - y = 10x$, for y
66. $6a + b = 2a$, for a
67. $\frac{7m - n}{q} = 10$, for q
68. $\frac{5tz - s}{2} = 6$, for z

Graphing Calculator Investigation

A Follow-Up of Lesson 7-1

Systems of Equations

You can use a TI-83 Plus graphing calculator to solve a system of equations.

Example

Solve the system of equations. State the decimal solution to the nearest hundredth.

$\mathbf{2.93x + y = 6.08}$

$\mathbf{8.32x - y = 4.11}$

Step 1 Solve each equation for y to enter them into the calculator.

$2.93x + y = 6.08$	First equation
$2.93x + y - 2.93x = 6.08 - 2.93x$	Subtract $2.93x$ from each side.
$y = 6.08 - 2.93x$	Simplify.
$8.32x - y = 4.11$	Second equation
$8.32x - y - 8.32x = 4.11 - 8.32x$	Subtract $8.32x$ from each side.
$-y = 4.11 - 8.32x$	Simplify.
$(-1) - y = (-1)(4.11 - 8.32x)$	Multiply each side by -1.
$y = -4.11 + 8.32x$	Simplify.

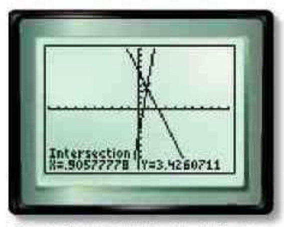
Step 2 Enter these equations in the Y= list and graph.

KEYSTROKES: *Review on pages 224–225.*

Step 3 Use the CALC menu to find the point of intersection.

KEYSTROKES: 2nd [CALC] 5 ENTER ENTER ENTER

The solution is approximately (0.91, 3.43).



[10, 10] scl: 1 by [−10, 10] scl: 1

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 3x - 4$
$y = -0.5x + 6$

2. $y = 2x + 5$
$y = -0.2x - 4$

3. $x + y = 5.35$
$3x - y = 3.75$

4. $0.35x - y = 1.12$
$2.25x + y = -4.05$

5. $1.5x + y = 6.7$
$5.2x - y = 4.1$

6. $5.4x - y = 1.8$
$6.2x + y = -3.8$

7. $5x - 4y = 26$
$4x + 2y = 53.3$

8. $2x + 3y = 11$
$4x + y = -6$

9. $0.22x + 0.15y = 0.30$
$-0.33x + y = 6.22$

10. $125x - 200y = 800$
$65x - 20y = 140$

www.algebra1.com/other_calculator_keystrokes

7-2 Substitution

What You'll Learn

- Solve systems of equations by using substitution.
- Solve real-world problems involving systems of equations.

Vocabulary

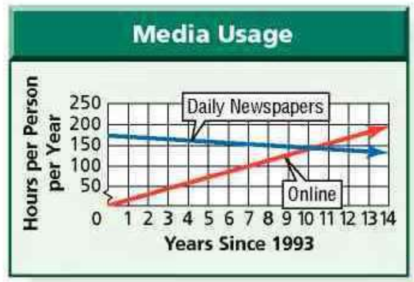
- substitution

How can a system of equations be used to predict media use?

Americans spend more time online than they spend reading daily newspapers. If x represents the number of years since 1993 and y represents the average number of hours per person per year, the following system represents the situation.

reading daily newspapers: $y = -2.8x + 170$
online: $y = 14.4x + 2$

The solution of the system represents the year that the number of hours spent on each activity will be the same. To solve this system, you could graph the equations and find the point of intersection. However, the exact coordinates of the point would be very difficult to determine from the graph. You could find a more accurate solution by using algebraic methods.



SUBSTITUTION The exact solution of a system of equations can be found by using algebraic methods. One such method is called **substitution**.

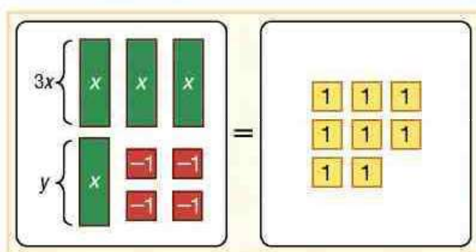
Algebra Activity

Using Substitution

Use algebra tiles and an equation mat to solve the system of equations.
$3x + y = 8$ and $y = x - 4$

Model and Analyze

Since $y = x - 4$, use 1 positive x tile and 4 negative 1 tiles to represent y. Use algebra tiles to represent $3x + y = 8$.



1. Use what you know about equation mats to solve for x. What is the value of x?
2. Use the $y = x - 4$ to solve for y.
3. What is the solution of the system of equations?

Make a Conjecture

4. Explain how to solve the following system of equations using algebra tiles.
   $4x + 3y = 10$ and $y = x + 1$
5. Why do you think this method is called substitution?

Example 1 Solve Using Substitution

Use substitution to solve the system of equations.

$y = 3x$
$x + 2y = -21$

Since $y = 3x$, substitute $3x$ for y in the second equation.

$x + 2y = -21$ Second equation

$x + 2(3x) = -21$ $y = 3x$

$x + 6x = -21$ Simplify.

$7x = -21$ Combine like terms.

$\frac{7x}{7} = \frac{-21}{7}$ Divide each side by 7.

$x = -3$ Simplify.

Use $y = 3x$ to find the value of y.

$y = 3x$ First equation

$y = 3(-3)$ $x = -3$

$y = -9$ The solution is $(-3, -9)$.

Study Tip

Look Back
To review **solving linear equations**, see Lesson 3-5.

Example 2 Solve for One Variable, Then Substitute

Use substitution to solve the system of equations.

$x + 5y = -3$
$3x - 2y = 8$

Solve the first equation for x since the coefficient of x is 1.

$x + 5y = -3$ First equation

$x + 5y - 5y = -3 - 5y$ Subtract $5y$ from each side.

$x = -3 - 5y$ Simplify.

Find the value of y by substituting $-3 - 5y$ for x in the second equation.

$3x - 2y = 8$ Second equation

$3(-3 - 5y) - 2y = 8$ $x = -3 - 5y$

$-9 - 15y - 2y = 8$ Distributive Property

$-9 - 17y = 8$ Combine like terms.

$-9 - 17y + 9 = 8 + 9$ Add 9 to each side.

$-17y = 17$ Simplify.

$\frac{-17y}{-17} = \frac{17}{-17}$ Divide each side by -17.

$y = -1$ Simplify.

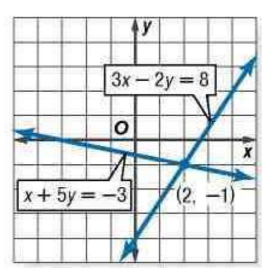
Substitute -1 for y in either equation to find the value of x.
Choose the equation that is easier to solve.

$x + 5y = -3$ First equation

$x + 5(-1) = -3$ $y = -1$

$x - 5 = -3$ Simplify.

$x = 2$ Add 5 to each side.



The solution is $(2, -1)$. The graph verifies the solution.

www.algebra1.com/extra_examples

Example 3 Dependent System

Use substitution to solve the system of equations.

$6x - 2y = -4$
$y = 3x + 2$

Since $y = 3x + 2$, substitute $3x + 2$ for y in the first equation.

$$6x - 2y = -4 \quad \text{First equation}$$
$$6x - 2(3x + 2) = -4 \quad y = 3x + 2$$
$$6x - 6x - 4 = -4 \quad \text{Distributive Property}$$
$$-4 = -4 \quad \text{Simplify.}$$

The statement $-4 = -4$ is true. This means that there are infinitely many solutions of the system of equations. This is true because the slope-intercept form of both equations is $y = 3x + 2$. That is, the equations are equivalent, and they have the same graph.

In general, if you solve a system of linear equations and the result is a true statement (an identity such as $-4 = -4$), the system has an infinite number of solutions. However, if the result is a false statement (for example, $-4 = 5$), the system has no solution.

Study Tip

Alternative Method

Using a system of equations is an alternative method for solving the weighted average problems that you studied in Lesson 3-9.

REAL-WORLD PROBLEMS Sometimes it is helpful to organize data before solving a problem. Some ways to organize data are to use tables, charts, different types of graphs, or diagrams.

Example 4 Write and Solve a System of Equations

METAL ALLOYS **A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?**

Let a = the number of grams of the 25% copper alloy and b = the number of grams of the 50% copper alloy. Use a table to organize the information.

	25% Copper	50% Copper	45% Copper
Total Grams	a	b	1000
Grams of Copper	$0.25a$	$0.50b$	0.45(1000)

The system of equations is $a + b = 1000$ and $0.25a + 0.50b = 0.45(1000)$. Use substitution to solve this system.

$$a + b = 1000 \quad \text{First equation}$$
$$a + b - b = 1000 - b \quad \text{Subtract } b \text{ from each side.}$$
$$a = 1000 - b \quad \text{Simplify.}$$

$$0.25a + 0.50b = 0.45(1000) \quad \text{Second equation}$$
$$0.25(1000 - b) + 0.50b = 0.45(1000) \quad a = 1000 - b$$
$$250 - 0.25b + 0.50b = 450 \quad \text{Distributive Property}$$
$$250 + 0.25b = 450 \quad \text{Combine like terms.}$$
$$250 + 0.25b - 250 = 450 - 250 \quad \text{Subtract 250 from each side.}$$
$$0.25b = 200 \quad \text{Simplify.}$$
$$\frac{0.25b}{0.25} = \frac{200}{0.25} \quad \text{Divide each side by 0.25.}$$
$$b = 800 \quad \text{Simplify.}$$

$a + b = 1000$	First equation
$a + 800 = 1000$	$b = 800$
$a + 800 - 800 = 1000 - 800$	Subtract 800 from each side.
$a = 200$	Simplify.

200 grams of the 25% copper alloy and 800 grams of the 50% copper alloy should be used.

Check for Understanding

Concept Check

1. **Explain** why you might choose to use substitution rather than graphing to solve a system of equations.
2. **Describe** the graphs of two equations if the solution of the system of equations yields the equation $4 = 2$.
3. **OPEN-ENDED** Write a system of equations that has infinitely many solutions.

Guided Practice

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

4. $x = 2y$
   $4x + 2y = 15$
5. $y = 3x - 8$
   $y = 4 - x$
6. $2x + 7y = 3$
   $x = 1 - 4y$
7. $6x - 2y = -4$
   $y = 3x + 2$
8. $x + 3y = 12$
   $x - y = 8$
9. $y = \frac{3}{5}x$
   $3x - 5y = 15$

Application

10. **TRANSPORTATION** The Thrust SSC is the world's fastest land vehicle. Suppose the driver of a car whose top speed is 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car?

Practice and Apply

Homework Help

For Exercises	See Examples
11–28	1–3
29–37	4

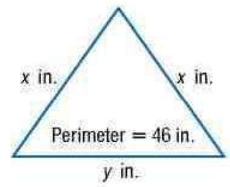
Extra Practice

See page 835.

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

11. $y = 5x$
    $2x + 3y = 34$
12. $x = 4y$
    $2x + 3y = 44$
13. $x = 4y + 5$
    $x = 3y - 2$
14. $y = 2x + 3$
    $y = 4x - 1$
15. $4c = 3d + 3$
    $c = d - 1$
16. $4x + 5y = 11$
    $y = 3x - 13$
17. $8x + 2y = 13$
    $4x + y = 11$
18. $2x - y = -4$
    $-3x + y = -9$
19. $3x - 5y = 11$
    $x - 3y = 1$
20. $2x + 3y = 1$
    $-3x + y = 15$
21. $c - 5d = 2$
    $2c + d = 4$
22. $5r - s = 5$
    $-4r + 5s = 17$
23. $3x - 2y = 12$
    $x + 2y = 6$
24. $x - 3y = 0$
    $3x + y = 7$
25. $-0.3x + y = 0.5$
    $0.5x - 0.3y = 1.9$
26. $0.5x - 2y = 17$
    $2x + y = 104$
27. $y = \frac{1}{2}x + 3$
    $y = 2x - 1$
28. $x = \frac{1}{2}y + 3$
    $2x - y = 6$

29. **GEOMETRY** The base of the triangle is 4 inches longer than the length of one of the other sides. Use a system of equations to find the length of each side of the triangle.



30. **FUND-RAISING** The Future Teachers of America Club at Paint Branch High School is making a healthy trail mix to sell to students during lunch. The mix will have three times the number of pounds of raisins as sunflower seeds. Sunflower seeds cost $4.00 per pound, and raisins cost $1.50 per pound. If the group has $34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy?

31. **CHEMISTRY** MX Labs needs to make 500 gallons of a 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. How many gallons of each solution should be mixed to make the 34% solution?

32. **GEOMETRY** Supplementary angles are two angles whose measures have the sum of 180 degrees. Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the measures of angles X and Y.

33. **SPORTS** At the end of the 2000 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win?

JOBS For Exercises 34 and 35, use the following information.
Shantel Jones has two job offers as a car salesperson. At one dealership, she will receive $600 per month plus a commission of 2% of the total price of the automobiles she sells. At the other dealership, she will receive $1000 per month plus a commission of 1.5% of her total sales.

34. What is the total price of the automobiles that Ms. Jones must sell each month to make the same income from either dealership?
35. Explain which job offer is better.

36. **LANDSCAPING** A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, when would you expect the trees to be the same height?

More About . . .

Tourism

Every year, multitudes of visitors make their way to South America to stand in awe of Machu Picchu, the spectacular ruins of the Lost City of the Incas.

Source: www.about.com

37. **TOURISM** In 2000, approximately 40.3 million tourists visited South America and the Caribbean. The number of tourists to that area had been increasing at an average rate of 0.8 million tourists per year. In the same year, 17.0 million tourists visited the Middle East. The number of tourists to the Middle East had been increasing at an average rate of 1.8 million tourists per year. If the trend continues, when would you expect the number of tourists to South America and the Caribbean to equal the number of tourists to the Middle East?

38. **RESEARCH** Use the Internet or other resources to find the pricing plans for various cell phones. Determine the number of minutes you would need to use the phone for two plans to cost the same amount of money. Support your answer with a table, a graph, and/or an equation.

39. **CRITICAL THINKING** Solve the system of equations. Write the solution as an ordered triple of the form (x, y, z).

$2x + 3y - z = 17$
$y = -3z - 7$
$2x = z + 2$

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a system of equations be used to predict media use?

Include the following in your answer:
- an explanation of solving a system of equations by using substitution, and
- the year when the number of hours spent reading daily newspapers is the same as the hours spent online.

Standardized Test Practice

41. When solving the following system, which expression could be substituted for x?

$x + 4y = 1$
$2x - 3y = -9$

(A) $4y - 1$ (B) $1 - 4y$ (C) $3y - 9$ (D) $-9 - 3y$

42. If $x - 3y = -9$ and $5x - 2y = 7$, what is the value of x?

(A) 1 (B) 2 (C) 3 (D) 4

Maintain Your Skills

Mixed Review

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

43. $x + y = 3$
$x + y = 4$

44. $x + 2y = 1$
$2x + y = 5$

45. $2x + y = 3$
$4x + 2y = 6$

Graph each inequality. *(Lesson 6-6)*

46. $y < -5$ 47. $x \geq 4$ 48. $2x + y > 6$

49. **RECYCLING** When a pair of blue jeans is made, the leftover denim scraps can be recycled. One pound of denim is left after making every five pair of jeans. How many pounds of denim would be left from 250 pairs of jeans? *(Lesson 3-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression.**
*(To review **simplifying expressions**, see Lesson 1-5.)*

50. $6a - 9a$ 51. $8t + 4t$ 52. $-7g - 8g$ 53. $7d - (2d + b)$

Practice Quiz 1

Lessons 7-1 and 7-2

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

1. $x + y = 3$
$x - y = 1$

2. $3x - 2y = -6$
$3x - 2y = 6$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. *(Lesson 7-2)*

3. $x + y = 0$
$3x + y = -8$

4. $x - 2y = 5$
$3x - 5y = 8$

5. $x + y = 2$
$y = 2 - x$

7-3 Elimination Using Addition and Subtraction

What You'll Learn

- Solve systems of equations by using elimination with addition.
- Solve systems of equations by using elimination with subtraction.

Vocabulary

- elimination

How can you use a system of equations to solve problems about weather?

On the winter solstice, there are fewer hours of daylight in the Northern Hemisphere than on any other day. On that day in Seward, Alaska, the difference between the number of hours of darkness n and the number of hours of daylight d is 12. The following system of equations represents the situation.

$$n + d = 24$$
$$n - d = 12$$

Notice that if you add these equations, the variable d is eliminated.

$$\begin{array}{r} n + d = 24 \\ \underline{(+)\,n - d = 12} \\ 2n \quad\;\; = 36 \end{array}$$

ELIMINATION USING ADDITION Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

Example 1 Elimination Using Addition

Use elimination to solve each system of equations.

$3x - 5y = -16$
$2x + 5y = 31$

Since the coefficients of the y terms, -5 and 5, are additive inverses, you can eliminate the y terms by adding the equations.

$3x - 5y = -16$	Write the equations in column form and add.
$(+)\,2x + 5y = 31$	
$5x \quad = 15$	Notice that the y variable is eliminated.
$\frac{5x}{5} = \frac{15}{5}$	Divide each side by 5.
$x = 3$	Simplify.

Now substitute 3 for x in either equation to find the value of y.

$3x - 5y = -16$	First equation
$3(3) - 5y = -16$	Replace x with 3.
$9 - 5y = -16$	Simplify.
$9 - 5y - 9 = -16 - 9$	Subtract 9 from each side.
$-5y = -25$	Simplify.
$\frac{-5y}{-5} = \frac{-25}{-5}$	Divide each side by -5.
$y = 5$	Simplify.

The solution is (3, 5).

Example 2 Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let x represent the first number and y represent the second number.

$$\underbrace{2x}_{\text{Twice one number}} \quad \underbrace{+}_{\text{added to}} \quad \underbrace{y}_{\text{another number}} \quad \underbrace{=}_{\text{is}} \quad \underbrace{18}_{18.}$$

$$\underbrace{4x}_{\text{Four times the first number}} \quad \underbrace{-}_{\text{minus}} \quad \underbrace{y}_{\text{the other number}} \quad \underbrace{=}_{\text{is}} \quad \underbrace{12}_{12.}$$

Use elimination to solve the system.

$2x + y = 18$ $(+)\ 4x - y = 12$	Write the equations in column form and add.
$6x = 30$	Notice that the variable y is eliminated.
$\frac{6x}{6} = \frac{30}{6}$	Divide each side by 6.
$x = 5$	Simplify.

Now substitute 5 for x in either equation to find the value of y.

$4x - y = 12$	Second equation
$4(5) - y = 12$	Replace x with 5.
$20 - y = 12$	Simplify.
$20 - y - 20 = 12 - 20$	Subtract 20 from each side.
$-y = -8$	Simplify.
$\frac{-y}{-1} = \frac{-8}{-1}$	Divide each side by -1.
$y = 8$	The numbers are 5 and 8.

Study Tip

Look Back

To review **translating verbal sentences into equations**, see Lesson 3-1.

ELIMINATION USING SUBTRACTION Sometimes subtracting one equation from another will eliminate one variable.

Example 3 Elimination Using Subtraction

Use elimination to solve the system of equations.

$5s + 2t = 6$
$9s + 2t = 22$

Since the coefficients of the t terms, 2 and 2, are the same, you can eliminate the t terms by subtracting the equations.

$5s + 2t = 6$ $(-)\ 9s + 2t = 22$	Write the equations in column form and subtract.
$-4s = -16$	Notice that the variable t is eliminated.
$\frac{-4s}{-4} = \frac{-16}{-4}$	Divide each side by -4.
$s = 4$	Simplify.

Now substitute 4 for s in either equation to find the value of t.

$5s + 2t = 6$	First equation
$5(4) + 2t = 6$	$s = 4$
$20 + 2t = 6$	Simplify.
$20 + 2t - 20 = 6 - 20$	Subtract 20 from each side.
$2t = -14$	Simplify.
$\frac{2t}{2} = \frac{-14}{2}$	Divide each side by 2.
$t = -7$	The solution is $(4, -7)$.

Standardized Test Practice

Example 4 Elimination Using Subtraction

Multiple-Choice Test Item

If $x - 3y = 7$ and $x + 2y = 2$, what is the value of x?			
(A) 4	(B) -1	(C) $(-1, 4)$	(D) $(4, -1)$

The Princeton Review

Test-Taking Tip

Always read the question carefully. Ask yourself, "What does the question ask?" Then answer that question.

Read the Test Item

You are given a system of equations, and you are asked to find the value of x.

Solve the Test Item

You can eliminate the x terms by subtracting one equation from the other.

$$\begin{array}{rll} x - 3y &= 7 & \text{Write the equations in column form and subtract.} \\ (-)\; x + 2y &= 2 & \\ \hline -5y &= 5 & \text{Notice the } x \text{ variable is eliminated.} \\ \frac{-5y}{-5} &= \frac{5}{-5} & \text{Divide each side by } -5. \\ y &= -1 & \text{Simplify.} \end{array}$$

Now substitute -1 for y in either equation to find the value of x.

$$\begin{array}{rll} x + 2y &= 2 & \text{Second equation} \\ x + 2(-1) &= 2 & y = -1 \\ x - 2 &= 2 & \text{Simplify.} \\ x - 2 + 2 &= 2 + 2 & \text{Add 2 to each side.} \\ x &= 4 & \text{Simplify.} \end{array}$$

Notice that B is the value of y and D is the solution of the system of equations. However, the question asks for the value of x. The answer is A.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a system of equations that can be solved by using addition to eliminate one variable.
2. **Describe** a system of equations that can be solved by using subtraction to eliminate one variable.
3. **FIND THE ERROR** Michael and Yoomee are solving a system of equations.

Michael

$$\begin{array}{rl} 2r + s &= 5 \\ (+)\; r - s &= 1 \\ \hline 3r \quad &= 6 \\ r &= 2 \\ 2r + s &= 5 \\ 2(2) + s &= 5 \\ 4 + s &= 5 \\ s &= 1 \end{array}$$

The solution is (2, 1).

Yoomee

$$\begin{array}{rl} 2r + s &= 5 \\ (-)\; r - s &= 1 \\ \hline r \quad &= 4 \\ r - s &= 1 \\ 4 - s &= 1 \\ -s &= -3 \\ s &= 3 \end{array}$$

The solution is (4, 3).

Who is correct? Explain your reasoning.

Guided Practice

Use elimination to solve each system of equations.

4. $x - y = 14$
$x + y = 20$

5. $2a - 3b = -11$
$a + 3b = 8$

6. $4x + y = -9$
$4x + 2y = -10$

7. $6x + 2y = -10$
$2x + 2y = -10$

8. $2a + 4b = 30$
$-2a - 2b = -21.5$

9. $-4m + 2n = 6$
$-4m + n = 8$

10. The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

Standardized Test Practice

11. If $2x + 7y = 17$ and $2x + 5y = 11$, what is the value of $2y$?

Ⓐ -4 Ⓑ -2 Ⓒ 3 Ⓓ 6

Practice and Apply

Homework Help

For Exercises	See Examples
12–29	1, 3
30–39	2
42, 43	4

Extra Practice

See page 836.

Use elimination to solve each system of equations.

12. $x + y = -3$
$x - y = 1$

13. $s - t = 4$
$s + t = 2$

14. $3m - 2n = 13$
$m + 2n = 7$

15. $-4x + 2y = 8$
$4x - 3y = -10$

16. $3a + b = 5$
$2a + b = 10$

17. $2m - 5n = -6$
$2m - 7n = -14$

18. $3r - 5s = -35$
$2r - 5s = -30$

19. $13a + 5b = -11$
$13a + 11b = 7$

20. $3x - 5y = 16$
$-3x + 2y = -10$

21. $6s + 5t = 1$
$6s - 5t = 11$

22. $4x - 3y = 12$
$4x + 3y = 24$

23. $a - 2b = 5$
$3a - 2b = 9$

24. $4x + 5y = 7$
$8x + 5y = 9$

25. $8a + b = 1$
$8a - 3b = 3$

26. $1.44x - 3.24y = -5.58$
$1.08x + 3.24y = 9.99$

27. $7.2m + 4.5n = 129.06$
$7.2m + 6.7n = 136.54$

28. $\frac{3}{5}c - \frac{1}{5}d = 9$
$\frac{7}{5}c + \frac{1}{5}d = 11$

29. $\frac{2}{3}x - \frac{1}{2}y = 14$
$\frac{5}{6}x - \frac{1}{2}y = 18$

More About . . .

Parks

Mammoth Cave in Kentucky was declared a national park in 1941. It has more than 336 miles of explored caves, making it the longest recorded cave system in the world.

Source: National Park Service

30. The sum of two numbers is 48, and their difference is 24. What are the numbers?

31. Find the two numbers whose sum is 51 and whose difference is 13.

32. Three times one number added to another number is 18. Twice the first number minus the other number is 12. Find the numbers.

33. One number added to twice another number is 23. Four times the first number added to twice the other number is 38. What are the numbers?

34. BUSINESS In 1999, the United States produced about 2 million more motor vehicles than Japan. Together, the two countries produced about 22 million motor vehicles. How many vehicles were produced in each country?

35. PARKS A youth group and their leaders visited Mammoth Cave. Two adults and 5 students in one van paid \$77 for the Grand Avenue Tour of the cave. Two adults and 7 students in a second van paid \$95 for the same tour. Find the adult price and the student price of the tour.

36. FOOTBALL During the National Football League's 1999 season, Troy Aikman, the quarterback for the Dallas Cowboys, earned \$0.467 million more than Deion Sanders, the Cowboys cornerback. Together they cost the Cowboys \$12.867 million. How much did each player make?

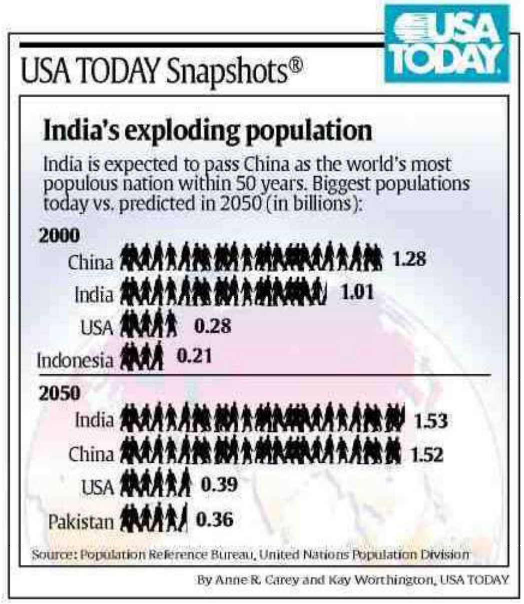
www.algebra1.com/self_check_quiz

POPULATIONS For Exercises 37–39, use the information in the graph at the right.

37. Let x represent the number of years since 2000 and y represent population in billions. Write an equation to represent the population of China.

38. Write an equation to represent the population of India.

39. Use elimination to find the year when the populations of China and India are predicted to be the same. What is the predicted population at that time?



40. **CRITICAL THINKING** The graphs of $Ax + By = 15$ and $Ax - By = 9$ intersect at (2, 1). Find A and B.

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use a system of equations to solve problems about weather?

Include the following in your answer:
- an explanation of how to use elimination to solve a system of equations, and
- a step-by-step solution of the Seward daylight problem.

Standardized Test Practice

42. If $2x - 3y = -9$ and $3x - 3y = -12$, what is the value of y?
Ⓐ -3 Ⓑ 1 Ⓒ $(-3, 1)$ Ⓓ $(1, -3)$

43. What is the solution of $4x + 2y = 8$ and $2x + 2y = 2$?
Ⓐ $(-2, 3)$ Ⓑ $(3, 2)$ Ⓒ $(3, -2)$ Ⓓ $(12, -3)$

Maintain Your Skills

Mixed Review

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. *(Lesson 7-2)*

44. $y = 5x$
$x + 2y = 22$

45. $x = 2y + 3$
$3x + 4y = -1$

46. $2y - x = -5$
$4y - 3x = -1$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

47. $x - y = 3$
$3x + y = 1$

48. $2x - 3y = 7$
$3y = 7 + 2x$

49. $4x + y = 12$
$x = 3 - \frac{1}{4}y$

50. Write an equation of a line that is parallel to the graph of $y = \frac{5}{4}x - 3$ and passes through the origin. *(Lesson 5-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Use the Distributive Property to rewrite each expression without parentheses.** *(To review the **Distributive Property**, see Lesson 1-5.)*

51. $2(3x + 4y)$
52. $6(2a - 5b)$
53. $-3(-2m + 3n)$
54. $-5(4t - 2s)$

7-4 Elimination Using Multiplication

What You'll Learn

- Solve systems of equations by using elimination with multiplication.
- Determine the best method for solving systems of equations.

How can a manager use a system of equations to plan employee time?

The Finneytown Bakery is making peanut butter cookies and loaves of quick bread. The preparation and baking times for each are given in the table below.

For these two items, the management has allotted 800 minutes of employee time and 900 minutes of oven time. If c represents the number of batches of cookies and b represents the number of loaves of bread, the following system of equations can be used to determine how many of each to bake.

$$20c + 10b = 800$$
$$10c + 30b = 900$$

	Cookies (per batch)	Bread (per loaf)
Preparation	20 min	10 min
Baking	10 min	30 min

ELIMINATION USING MULTIPLICATION Neither variable in the system above can be eliminated by simply adding or subtracting the equations. However, you can use the Multiplication Property of Equality so that adding or subtracting eliminates one of the variables.

Example 1 Multiply One Equation to Eliminate

Use elimination to solve the system of equations.

$\mathbf{3x + 4y = 6}$
$\mathbf{5x + 2y = -4}$

Multiply the second equation by -2 so the coefficients of the y terms are additive inverses. Then add the equations.

$$\begin{aligned} 3x + 4y &= 6 \\ 5x + 2y &= -4 \end{aligned} \quad \text{Multiply by } -2. \quad \begin{aligned} 3x + 4y &= 6 \\ (+)\ -10x - 4y &= 8 \\ \hline -7x \qquad &= 14 && \text{Add the equations.} \\ \frac{-7x}{-7} &= \frac{14}{-7} && \text{Divide each side by } -7. \\ x &= -2 && \text{Simplify.} \end{aligned}$$

Now substitute -2 for x in either equation to find the value of y.

$$\begin{aligned} 3x + 4y &= 6 && \text{First equation} \\ 3(-2) + 4y &= 6 && x = -2 \\ -6 + 4y &= 6 && \text{Simplify.} \\ -6 + 4y + 6 &= 6 + 6 && \text{Add 6 to each side.} \\ 4y &= 12 && \text{Simplify.} \\ \frac{4y}{4} &= \frac{12}{4} && \text{Divide each side by 4.} \\ y &= 3 && \text{The solution is } (-2, 3). \end{aligned}$$

For some systems of equations, it is necessary to multiply each equation by a different number in order to solve the system by elimination. You can choose to eliminate either variable.

Example 2 Multiply Both Equations to Eliminate

Use elimination to solve the system of equations.

$$3x + 4y = -25$$
$$2x - 3y = 6$$

Method 1 Eliminate x.

$$\begin{array}{ll} 3x + 4y = -25 & \text{Multiply by 2.} \\ 2x - 3y = 6 & \text{Multiply by } -3. \end{array} \quad \begin{array}{rl} 6x + 8y &= -50 \\ (+)\ -6x + 9y &= -18 \\ \hline 17y &= -68 \end{array}$$

$17y = -68$ Add the equations.

$\frac{17y}{17} = \frac{-68}{17}$ Divide each side by 17.

$y = -4$ Simplify.

Now substitute -4 for y in either equation to find the value of x.

$2x - 3y = 6$	Second equation
$2x - 3(-4) = 6$	$y = -4$
$2x + 12 = 6$	Simplify.
$2x + 12 - 12 = 6 - 12$	Subtract 12 from each side.
$2x = -6$	Simplify.
$\frac{2x}{2} = \frac{-6}{2}$	Divide each side by 2.
$x = -3$	Simplify.

The solution is $(-3, -4)$.

Method 2 Eliminate y.

$$\begin{array}{ll} 3x + 4y = -25 & \text{Multiply by 3.} \\ 2x - 3y = 6 & \text{Multiply by 4.} \end{array} \quad \begin{array}{rl} 9x + 12y &= -75 \\ (+)\ 8x - 12y &= 24 \\ \hline 17x &= -51 \end{array}$$

$17x = -51$ Add the equations.

$\frac{17x}{17} = \frac{-51}{17}$ Divide each side by 17.

$x = -3$ Simplify.

Now substitute -3 for x in either equation to find the value of y.

$2x - 3y = 6$	Second equation
$2(-3) - 3y = 6$	$x = -3$
$-6 - 3y = 6$	Simplify.
$-6 - 3y + 6 = 6 + 6$	Add 6 to each side.
$-3y = 12$	Simplify.
$\frac{-3y}{-3} = \frac{12}{-3}$	Divide each side by -3.
$y = -4$	Simplify.

The solution is $(-3, -4)$, which matches the result obtained with Method 1.

Study Tip

Using Multiplication

There are many other combinations of multipliers that could be used to solve the system in Example 2. For instance, the first equation could be multiplied by -2 and the second by 3.

DETERMINE THE BEST METHOD You have learned five methods for solving systems of linear equations.

Concept Summary — *Solving Systems of Equations*

Method	The Best Time to Use
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or −1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations

Example 3 *Determine the Best Method*

Determine the best method to solve the system of equations. Then solve the system.

$\mathbf{4x - 3y = 12}$
$\mathbf{x + 2y = 14}$

- For an exact solution, an algebraic method is best.
- Since neither the coefficients of x nor the coefficients of y are the same or additive inverses, you cannot use elimination using addition or subtraction.
- Since the coefficient of x in the second equation is 1, you can use the substitution method. You could also use elimination using multiplication.

The following solution uses substitution. *Which method would you prefer?*

$x + 2y = 14$ Second equation
$x + 2y - 2y = 14 - 2y$ Subtract $2y$ from each side.
$x = 14 - 2y$ Simplify.

$4x - 3y = 12$ First equation
$4(14 - 2y) - 3y = 12$ $x = 14 - 2y$
$56 - 8y - 3y = 12$ Distributive Property
$56 - 11y = 12$ Combine like terms.
$56 - 11y - 56 = 12 - 56$ Subtract 56 from each side.
$-11y = -44$ Simplify.
$\frac{-11y}{-11} = \frac{-44}{-11}$ Divide each side by −11.
$y = 4$ Simplify.

$x + 2y = 14$ Second equation
$x + 2(4) = 14$ $y = 4$
$x + 8 = 14$ Simplify.
$x + 8 - 8 = 14 - 8$ Subtract 8 from each side.
$x = 6$ Simplify.

The solution is (6, 4).

Study Tip

Alternative Method
This system could also be solved easily by multiplying the second equation by 4 and then subtracting the equations.

www.algebra1.com/extra_examples

More About...

Transportation

About 203 million tons of freight are transported on the Ohio River each year making it the second most used commercial river in the United States.

Source: *World Book Encyclopedia*

Example 4 Write and Solve a System of Equations

TRANSPORTATION **A coal barge on the Ohio River travels 24 miles upstream in 3 hours. The return trip takes the barge only 2 hours. Find the rate of the barge in still water.**

Let b = the rate of the barge in still water and c = the rate of the current. Use the formula rate × time = distance, or $rt = d$.

	r	t	d	$rt = d$
Downstream	$b + c$	2	24	$2b + 2c = 24$
Upstream	$b - c$	3	24	$3b - 3c = 24$

This system cannot easily be solved using substitution. It cannot be solved by just adding or subtracting the equations.

The best way to solve this system is to use elimination using multiplication. Since the problem asks for b, eliminate c.

$2b + 2c = 24$ Multiply by 3. → $6b + 6c = 72$

$3b - 3c = 24$ Multiply by 2. → $(+)\ 6b - 6c = 48$

$$12b = 120 \quad \text{Add the equations.}$$

$$\frac{12b}{12} = \frac{120}{12} \quad \text{Divide each side by 12.}$$

$$b = 10 \quad \text{Simplify.}$$

The rate of the barge in still water is 10 miles per hour.

Check for Understanding

Concept Check

1. **Explain** why multiplication is sometimes needed to solve a system of equations by elimination.

2. **OPEN ENDED** Write a system of equations that could be solved by multiplying one equation by 5 and then adding the two equations together to eliminate one variable.

3. **Describe** two methods that could be used to solve the following system of equations. Which method do you prefer? Explain.
   $a - b = 5$
   $2a + 3b = 15$

Guided Practice

Use elimination to solve each system of equations.

4. $2x - y = 6$
   $3x + 4y = -2$

5. $x + 5y = 4$
   $3x - 7y = -10$

6. $4x + 7y = 6$
   $6x + 5y = 20$

7. $4x + 2y = 10.5$
   $2x + 3y = 10.75$

Determine the best method to solve each system of equations. Then solve the system.

8. $4x + 3y = 19$
   $3x - 4y = 8$

9. $3x - 7y = 6$
   $2x + 7y = 4$

10. $y = 4x + 11$
    $3x - 2y = -7$

11. $5x - 2y = 12$
    $3x - 2y = -2$

Application **12. BUSINESS** The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners purchase?

Practice and Apply

Homework Help

For Exercises	See Examples
13–26	1, 2
27–38	3
39–43	4

Extra Practice

See page 836.

Use elimination to solve each system of equations.

13. $-5x + 3y = 6$, $x - y = 4$

14. $x + y = 3$, $2x - 3y = 16$

15. $2x + y = 5$, $3x - 2y = 4$

16. $4x - 3y = 12$, $x + 2y = 14$

17. $5x - 2y = -15$, $3x + 8y = 37$

18. $8x - 3y = -11$, $2x - 5y = 27$

19. $4x - 7y = 10$, $3x + 2y = -7$

20. $2x - 3y = 2$, $5x + 4y = 28$

21. $1.8x - 0.3y = 14.4$, $x - 0.6y = 2.8$

22. $0.4x + 0.5y = 2.5$, $1.2x - 3.5y = 2.5$

23. $3x - \frac{1}{2}y = 10$, $5x + \frac{1}{4}y = 8$

24. $2x + \frac{2}{3}y = 4$, $x - \frac{1}{2}y = 7$

25. Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

26. Five times a number minus twice another number equals twenty-two. The sum of the numbers is three. Find the numbers.

Determine the best method to solve each system of equations. Then solve the system.

27. $3x - 4y = -10$, $5x + 8y = -2$

28. $9x - 8y = 42$, $4x + 8y = -16$

29. $y = 3x$, $3x + 4y = 30$

30. $x = 4y + 8$, $2x - 8y = -3$

31. $2x - 3y = 12$, $x + 3y = 12$

32. $4x - 2y = 14$, $y = x$

33. $x - y = 2$, $5x + 3y = 18$

34. $y = 2x + 9$, $2x - y = -9$

35. $6x - y = 9$, $6x - y = 11$

36. $x = 8y$, $2x + 3y = 38$

37. $\frac{2}{3}x - \frac{1}{2}y = 14$, $\frac{5}{6}x - \frac{1}{2}y = 18$

38. $\frac{1}{2}x - \frac{2}{3}y = \frac{7}{3}$, $\frac{3}{2}x + 2y = -25$

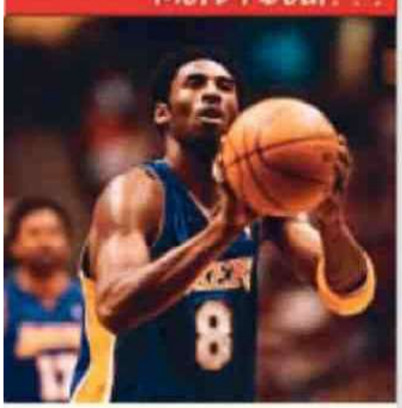
39. BASKETBALL In basketball, a free throw is 1 point and a field goal is either 2 points or 3 points. In the 2000–2001 season, Kobe Bryant scored a total of 1938 points. The total number of 2-point field goals and 3-point field goals was 701. Use the information at the left to find the number of Kobe Bryant's 2-point field goals and 3-point field goals that season.

Online Research **Data Update** What are the current statistics for Kobe Bryant and other players? Visit www.algebra1.com/data_update to learn more.

40. CRITICAL THINKING The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b.

41. CAREERS Mrs. Henderson discovered that she had accidentally reversed the digits of a test and shorted a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score without looking at his test. What was his actual score on the test?

More About. . .



Basketball

In the 2000–2001 season, Kobe Bryant ranked 18th in the NBA in free-throw percentage. He made 475 of the 557 free throws that he attempted.

Source: NBA

42. **NUMBER THEORY** The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 less than the original number. Find the original number.

43. **TRANSPORTATION** Traveling against the wind, a plane flies 2100 miles from Chicago to San Diego in 4 hours and 40 minutes. The return trip, traveling with a wind that is twice as fast, takes 4 hours. Find the rate of the plane in still air.

44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a manager use a system of equations to plan employee time?

Include the following in your answer:

- a demonstration of how to solve the system of equations concerning the cookies and bread, and
- an explanation of how a restaurant manager would schedule oven and employee time.

Standardized Test Practice

45. If $5x + 3y = 12$ and $4x - 5y = 17$, what is the value of y?

(A) -1 (B) 3 (C) $(-1, 3)$ (D) $(3, -1)$

46. Determine the number of solutions of the system $x + 2y = -1$ and $2x + 4y = -2$.

(A) 0 (B) 1 (C) 2 (D) infinitely many

Maintain Your Skills

Mixed Review

Use elimination to solve each system of equations. *(Lesson 7-3)*

47. $x + y = 8$
$x - y = 4$

48. $2r + s = 5$
$r - s = 1$

49. $x + y = 18$
$x + 2y = 25$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. *(Lesson 7-2)*

50. $2x + 3y = 3$
$x = -3y$

51. $x + y = 0$
$3x + y = -8$

52. $x - 2y = 7$
$-3x + 6y = -21$

53. **CAREERS** A store manager is paid \$32,000 a year plus 4% of the revenue the store makes above quota. What is the amount of revenue above quota needed for the manager to have an annual income greater than \$45,000? *(Lesson 6-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL Graph each inequality.
*(To review **graphing inequalities**, see Lesson 6-6.)*

54. $y \geq x - 7$ 55. $x + 3y \geq 9$ 56. $-y \leq x$ 57. $-3x + y \geq -1$

Practice Quiz 2

Lessons 7-3 and 7-4

Use elimination to solve each system of equations. *(Lessons 7-3 and 7-4)*

1. $5x + 4y = 2$
$3x - 4y = 14$

2. $2x - 3y = 13$
$2x + 2y = -2$

3. $6x - 2y = 24$
$3x + 4y = 27$

4. $5x + 2y = 4$
$10x + 4y = 9$

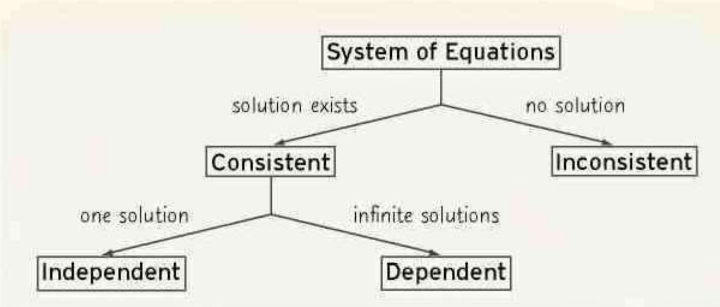
5. The price of a cellular telephone plan is based on peak and nonpeak service. Kelsey used 45 peak minutes and 50 nonpeak minutes and was charged \$27.75. That same month, Mitch used 70 peak minutes and 30 nonpeak minutes for a total charge of \$36. What are the rates per minute for peak and nonpeak time? *(Lesson 7-4)*

Reading Mathematics

Making Concept Maps

After completing a chapter, it is wise to review each lesson's main topics and vocabulary. In Lesson 7-1, the new vocabulary words were *system of equations*, *consistent*, *inconsistent*, *independent*, and *dependent*. They are all related in that they explain how many and what kind of solutions a system of equations has.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the vocabulary words for Lesson 7-1. The main ideas are placed in boxes. Any information that describes how to move from one box to the next is placed along the arrows.

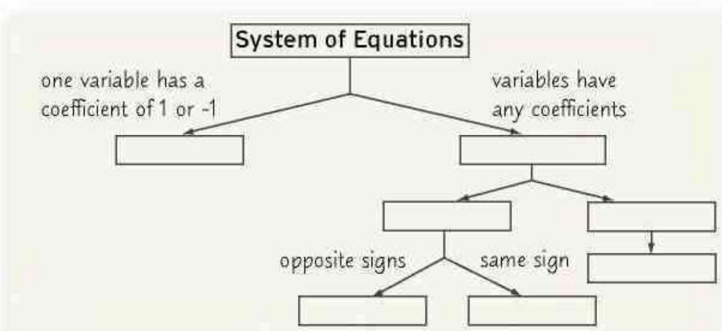


Concept maps are used to organize information. They clearly show how ideas are related to one another. They also show the flow of mental processes needed to solve problems.

Reading to Learn

Review Lessons 7-2, 7-3, and 7-4.

1. Write a couple of sentences describing the information in the concept map above.
2. How do you decide whether to use substitution or elimination? Give an example of a system that you would solve using each method.
3. How do you decide whether to multiply an equation by a factor?
4. How do you decide whether to add or subtract two equations?
5. Copy and complete the concept map below for solving systems of equations by using either substitution or elimination.



7-5 Graphing Systems of Inequalities

What You'll Learn

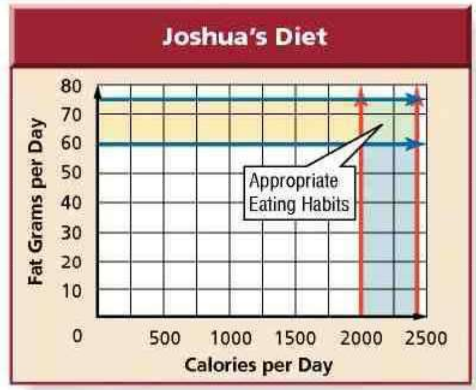
- Solve systems of inequalities by graphing.
- Solve real-world problems involving systems of inequalities.

Vocabulary

- system of inequalities

How can you use a system of inequalities to plan a sensible diet?

Joshua watches what he eats. His doctor told him to eat between 2000 and 2400 Calories per day. The doctor also wants him to keep his daily fat intake between 60 and 75 grams. The graph indicates the appropriate amounts of Calories and fat for Joshua. The graph is of a system of inequalities. He should try to keep his Calorie and fat intake to amounts represented in the green section.

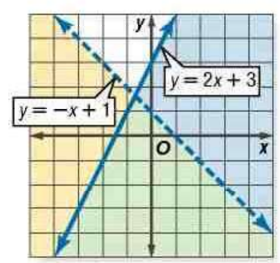


SYSTEMS OF INEQUALITIES To solve a **system of inequalities**, you need to find the ordered pairs that satisfy all the inequalities involved. One way to do this is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection, or overlap, of the graphs.

Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

$y < -x + 1$
$y \leq 2x + 3$



The solution includes the ordered pairs in the intersection of the graphs of $y < -x + 1$ and $y \leq 2x + 3$. This region is shaded in green at the right. The graphs of $y = -x + 1$ and $y = 2x + 3$ are boundaries of this region. The graph of $y = -x + 1$ is dashed and is *not* included in the graph of $y < -x + 1$. The graph of $y = 2x + 3$ is included in the graph of $y \leq 2x + 3$.

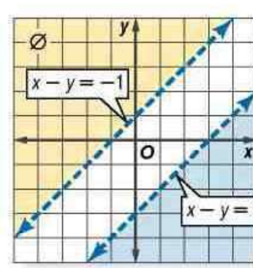
Study Tip

Look Back
To review **graphing linear inequalities**, see Lesson 6-6.

Example 2 No Solution

Solve the system of inequalities by graphing.

$x - y < -1$
$x - y > 3$



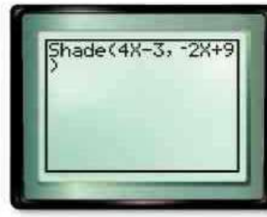
The graphs of $x - y = -1$ and $x - y = 3$ are parallel lines. Because the two regions have no points in common, the system of inequalities has no solution.

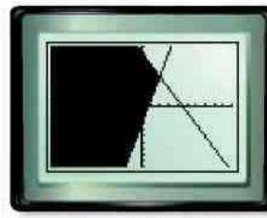
You can use a TI-83 Plus to solve systems of inequalities.

Graphing Calculator Investigation

Graphing Systems of Inequalities

To graph the system $y \geq 4x - 3$ and $y \leq -2x + 9$ on a TI-83 Plus, select the SHADE feature in the DRAW menu. Enter the function that is the lower boundary of the region to be shaded, followed by the upper boundary. (Note that inequalities that have $>$ or $\geq$ are lower boundaries and inequalities that have $<$ or $\leq$ are upper boundaries.)





[−10, 10] scl: 1 by [−10, 10] scl: 1

Think and Discuss

1. To graph the system $y \leq 3x + 1$ and $y \geq -2x - 5$ on a graphing calculator, which function should you enter first?
2. Use a graphing calculator to graph the system $y \leq 3x + 1$ and $y \geq -2x - 5$.
3. Explain how you could use a graphing calculator to graph the system $2x + y \geq 7$ and $x - 2y \geq 5$.
4. Use a graphing calculator to graph the system $2x + y \geq 7$ and $x - 2y \geq 5$.

REAL-WORLD PROBLEMS In real-life problems involving systems of inequalities, sometimes only whole-number solutions make sense.

Example 3 Use a System of Inequalities to Solve a Problem

COLLEGE The middle 50% of first-year students attending Florida State University score between 520 and 620, inclusive, on the verbal portion of the SAT and between 530 and 630, inclusive, on the math portion. Graph the scores that a student would need to be in the middle 50% of FSU freshmen.

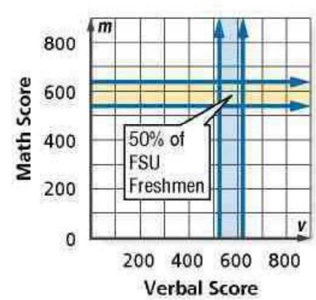
Words The verbal score is between 520 and 620, inclusive. The math score is between 530 and 630, inclusive.

Variables If v = the verbal score and m = the math score, the following inequalities represent the middle 50% of Florida State University freshmen.

Inequalities

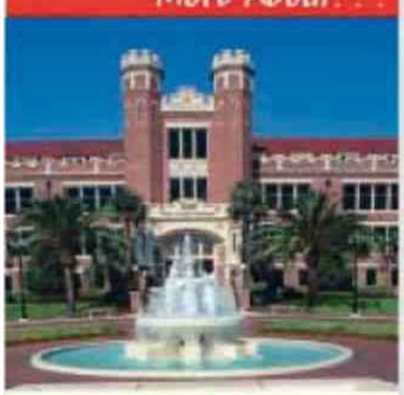
$$\underbrace{520 \leq v \leq 620}_{\text{The verbal score is between 520 and 620, inclusive.}}$$

$$\underbrace{530 \leq m \leq 630}_{\text{The math score is between 530 and 630, inclusive.}}$$



The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. However, since SAT scores are whole numbers, only whole-number solutions make sense in this problem.

More About...



College

FSU is the most wired campus in Florida and has been recently ranked the 18th most technologically connected university in the nation.

Source: www.fsu.edu

www.algebra1.com/extra_examples

Example 4 Use a System of Inequalities

AGRICULTURE **To ensure a growing season of sufficient length, Mr. Hobson has at most 16 days left to plant his corn and soybean crops. He can plant corn at a rate of 250 acres per day and soybeans at a rate of 200 acres per day. If he has at most 3500 acres available, how many acres of each type of crop can he plant?**

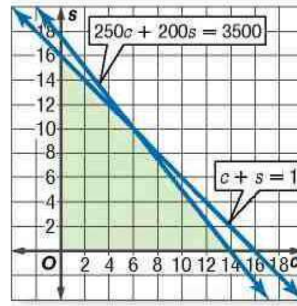
Let c = the number of days that corn will be planted and s = the number of days that soybeans will be planted. Since both c and s represent a number of days, neither can be a negative number. The following system of inequalities can be used to represent the conditions of this problem.

$c \geq 0$

$s \geq 0$

$c + s \leq 16$

$250c + 200s \leq 3500$



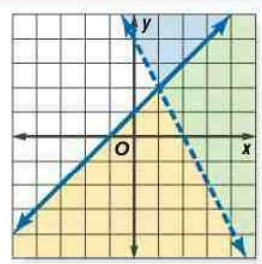
The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. This region is shown in green at the right. Only the portion of the region in the first quadrant is used since $c \geq 0$ and $s \geq 0$.

Any point in this region is a possible solution. For example, since (7, 8) is a point in the region, Mr. Hobson could plant corn for 7 days and soybeans for 8 days. In this case, he would use 15 days to plant 250(7) or 1750 acres of corn and 200(8) or 1600 acres of soybeans.

Check for Understanding

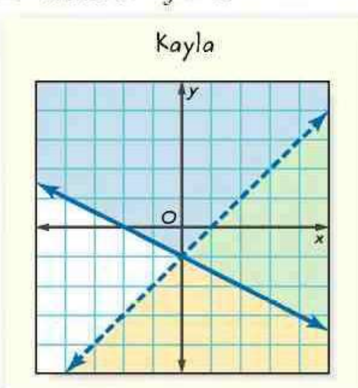
Concept Check

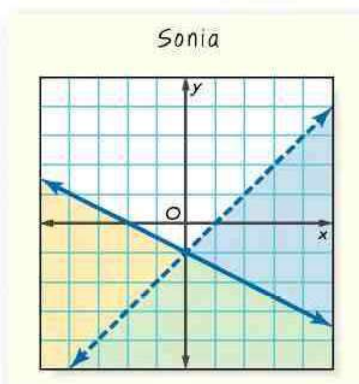
1. **OPEN ENDED** Draw the graph of a system of inequalities that has no solution.
2. **Determine** which of the following ordered pairs represent a solution of the system of inequalities graphed at the right.



a. (3, 1) **b.** $(-1, -3)$
c. (2, 3) **d.** $(4, -2)$
e. $(3, -2)$ **f.** (1, 4)

3. **FIND THE ERROR** Kayla and Sonia are solving the system of inequalities $x + 2y \geq -2$ and $x - y > 1$.





Who is correct? Explain your reasoning.

CONTENTS

Guided Practice

Solve each system of inequalities by graphing.

4. $x > 5$
   $y \leq 4$
5. $y > 3$
   $y > -x + 4$
6. $y \leq -x + 3$
   $y \leq x + 3$
7. $2x + y \geq 4$
   $y \leq -2x - 1$
8. $2y + x < 6$
   $3x - y > 4$
9. $x - 2y \leq 2$
   $3x + 4y \leq 12$
   $x \geq 0$

Application

HEALTH For Exercises 10 and 11, use the following information.
Natasha walks and jogs at least 3 miles every day. Natasha walks 4 miles per hour and jogs 8 miles per hour. She only has a half-hour to exercise.

10. Draw a graph of the possible amounts of time she can spend walking and jogging.
11. List three possible solutions.

Practice and Apply

Homework Help

For Exercises	See Examples
12–28	1–2
29–31, 33–35	3–4

Extra Practice

See page 836.

Career Choices

Visual Artist

Visual artists create art to communicate ideas. The work of fine artists is made for display. Illustrators and graphic designers produce art for clients, such as advertising and publishing companies.

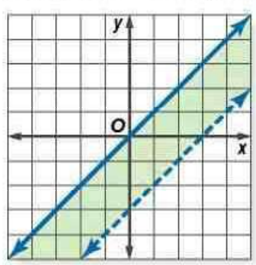
Online Research
For information about a career as a visual artist, visit: www.algebra1.com/careers

Solve each system of inequalities by graphing.

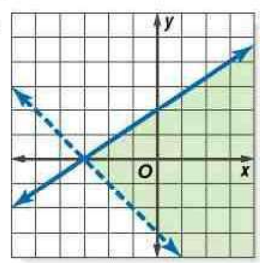
12. $y < 0$
    $x \geq 0$
13. $x > -4$
    $y \leq -1$
14. $y \geq -2$
    $y - x < 1$
15. $x \geq 2$
    $y + x \leq 5$
16. $x \leq 3$
    $x + y > 2$
17. $y \geq 2x + 1$
    $y \leq -x + 1$
18. $y < 2x + 1$
    $y \geq -x + 3$
19. $y - x < 1$
    $y - x > 3$
20. $y - x < 3$
    $y - x \geq 2$
21. $2x + y \leq 4$
    $3x - y \geq 6$
22. $3x - 4y < 1$
    $x + 2y \leq 7$
23. $x + y > 4$
    $-2x + 3y < -12$
24. $2x + y \geq -4$
    $-5x + 2y < 1$
25. $y \leq x + 3$
    $2x - 7y \leq 4$
    $3x + 2y \leq 6$
26. $x < 2$
    $4y > x$
    $2x - y > -9$
    $x + 3y < 9$

Write a system of inequalities for each graph.

27.



28.



ART For Exercises 29 and 30, use the following information.
A painter has exactly 32 units of yellow dye and 54 units of blue dye. She plans to mix the dyes to make two shades of green. Each gallon of the lighter shade of green requires 4 units of yellow dye and 1 unit of blue dye. Each gallon of the darker shade of green requires 1 unit of yellow dye and 6 units of blue dye.

29. Make a graph showing the numbers of gallons of the two greens she can make.
30. List three possible solutions.

31. **HEALTH** The LDL or "bad" cholesterol of a teenager should be less than 110. The HDL or "good" cholesterol of a teenager should be between 35 and 59. Make a graph showing appropriate levels of cholesterol for a teenager.

32. **CRITICAL THINKING** Write a system of inequalities that is equivalent to $|x| \leq 4$.

MANUFACTURING **For Exercises 33 and 34, use the following information.**
The Natural Wood Company has machines that sand and varnish desks and tables. The table gives the time requirements of the machines.

Machine	Hours per Desk	Hours per Table	Total Hours Available Each Week
Sanding	2	1.5	31
Varnishing	1.5	1	22

33. Make a graph showing the number of desks and the number of tables that can be made in a week.

34. List three possible solutions.

35. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you use a system of inequalities to plan a sensible diet?

Include the following in your answer:
- two appropriate Calorie and fat intakes for a day, and
- the system of inequalities that is represented by the graph.

Graphing Calculator

GRAPHING SYSTEMS OF INEQUALITIES **Use a graphing calculator to solve each system of inequalities. Sketch the results.**

36. $y \le x + 9$
$y \ge -x - 4$

37. $y \le 2x + 10$
$y \ge 7x + 15$

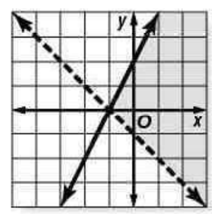
38. $3x - y \le 6$
$x - y \ge -1$

Standardized Test Practice

39. Which ordered pair does *not* satisfy the system $x + 2y > 5$ and $3x - y < -2$?

Ⓐ $(-3, 7)$ Ⓑ $(0, 5)$ Ⓒ $(-1, 4)$ Ⓓ $(0, 2.5)$

40. Which system of inequalities is represented by the graph?



Ⓐ $y \le 2x + 2$
$y > -x - 1$

Ⓑ $y \ge 2x + 2$
$y < -x - 1$

Ⓒ $y < 2x + 2$
$y \le -x - 1$

Ⓓ $y > 2x + 2$
$y \le -x - 1$

Maintain Your Skills

Mixed Review

Use elimination to solve each system of equations. *(Lessons 7-3 and 7-4)*

41. $2x + 3y = 1$
$4x - 5y = 13$

42. $5x - 2y = -3$
$3x + 6y = -9$

43. $-3x + 2y = 12$
$2x - 3y = -13$

44. $6x - 2y = 4$
$5x - 3y = -2$

45. $2x + 5y = 13$
$3x - 5y = -18$

46. $3x - y = 6$
$3x + 2y = 15$

Write an equation of the line that passes through each point with the given slope. *(Lesson 5-4)*

47. $(4, -1)$, $m = 2$

48. $(1, 0)$, $m = -6$

49. $(5, -2)$, $m = \frac{1}{3}$

WebQuest Internet Project

The Spirit of the Games

It's time to complete your project. Use the information and data you have gathered about the Olympics to prepare a portfolio or Web page. Be sure to include graphs and/or tables in your project.

www.algebra1.com/webquest

Chapter 7

Study Guide and Review

Vocabulary and Concept Check

consistent (p. 369)	elimination (p. 382)	independent (p. 369)	system of equations (p. 369)
dependent (p. 369)	inconsistent (p. 369)	substitution (p. 376)	system of inequalities (p. 394)

Choose the correct term to complete each statement.

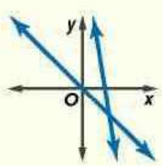
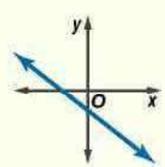
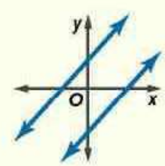
1. If a system of equations has exactly one solution, it is *(dependent, independent)*.
2. If the graph of a system of equations is parallel lines, the system is *(consistent, inconsistent)*.
3. A system of equations that has infinitely many solutions is *(dependent, independent)*.
4. If the equations in a system have the same slope and different intercepts, the graph of the system is *(intersecting lines, parallel lines)*.
5. If a system of equations has the same slope and intercepts, the system has *(exactly one, infinitely many)* solution(s).
6. The solution of a system of equations is $(3, -5)$. The system is *(consistent, inconsistent)*.

Lesson-by-Lesson Review

7-1 Graphing Systems of Inequalities

See pages 369–374.

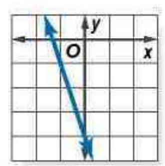
Concept Summary

	Intersecting Lines	Same Line	Parallel Lines
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

Example

Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

$3x + y = -4$
$6x + 2y = -8$



When the lines are graphed, they coincide. There are infinitely many solutions.

Exercises **Graph each system of equations. Then determine whether the system of equations has *one* solution, *no* solution, or *infinitely many* solutions. If the system has one solution, name it.** *See Example 2 on page 370.*

7. $x - y = 9$; $x + y = 11$
8. $9x + 2 = 3y$; $y - 3x = 8$
9. $2x - 3y = 4$; $6y = 4x - 8$
10. $3x - y = 8$; $3x = 4 - y$

7-2 Substitution

See pages 376–381.

Concept Summary

- In a system of equations, solve one equation for a variable, and then substitute that expression into the second equation to solve.

Example **Use substitution to solve the system of equations.**

$y = x - 1$
$4x - y = 19$

Since $y = x - 1$, substitute $x - 1$ for y in the second equation.

$4x - y = 19$ Second equation

$4x - (x - 1) = 19$ $y = x - 1$

$4x - x + 1 = 19$ Distributive Property

$3x + 1 = 19$ Combine like terms.

$3x = 18$ Subtract 1 from each side.

$x = 6$ Divide each side by 3.

Use $y = x - 1$ to find the value of y.

$y = x - 1$ First equation

$y = 6 - 1$ $x = 6$

$y = 5$ The solution is (6, 5).

Exercises **Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solutions or *infinitely many* solutions.** *See Examples 1–3 on pages 377 and 378.*

11. $2m + n = 1$, $m - n = 8$
12. $x = 3 - 2y$, $2x + 4y = 6$
13. $3x - y = 1$, $2x + 4y = 3$
14. $0.6m - 0.2n = 0.9$, $n = 4.5 - 3m$

7-3 Elimination Using Addition and Subtraction

See pages 382–386.

Concept Summary

- Sometimes adding or subtracting two equations will eliminate one variable.

Example **Use elimination to solve the system of equations.**

$2m - n = 4$
$m + n = 2$

You can eliminate the n terms by adding the equations.

$2m - n = 4$ Write the equations in column form and add.

$\underline{(+)\ m + n = 2}$

$3m \quad = 6$ Notice the variable n is eliminated.

$m = 2$ Divide each side by 3.

Now substitute 2 for m in either equation to find n.

$m + n = 2$	Second equation
$2 + n = 2$	$m = 2$
$2 + n - 2 = 2 - 2$	Subtract 2 from each side.
$n = 0$	Simplify.

The solution is (2, 0).

Exercises **Use elimination to solve each system of equations.**
See Examples 1–3 on pages 382 and 383.

15. $x + 2y = 6$, $x - 3y = -4$ **16.** $2m - n = 5$, $2m + n = 3$ **17.** $3x - y = 11$, $x + y = 5$ **18.** $3x + 1 = -7y$, $6x + 7y = 0$

7-4 Elimination Using Multiplication

See pages 387–392.

Concept Summary

- Multiplying one equation by a number or multiplying each equation by a different number is a strategy that can be used to solve a system of equations by elimination.
- There are five methods for solving systems of equations.

Method	The Best Time to Use
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or −1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations

Example **Use elimination to solve the system of equations.**

$x + 2y = 8$
$3x + y = 1.5$

Multiply the second equation by −2 so the coefficients of the y terms are additive inverses. Then add the equations.

$x + 2y = 8$ → $x + 2y = 8$
$3x + y = 1.5$ Multiply by −2. → $(+)\ -6x - 2y = -3$

$-5x = 5$ Add the equations.

$\frac{-5x}{-5} = \frac{5}{-5}$ Divide each side by −5.

$x = -1$ Simplify.

(continued on the next page)

• Extra Practice, see pages 835–836.
• Mixed Problem Solving, see page 859.

$$x + 2y = 8 \quad \text{First equation}$$
$$-1 + 2y = 8 \quad x = -1$$
$$-1 + 2y + 1 = 8 + 1 \quad \text{Add 1 to each side.}$$
$$2y = 9 \quad \text{Simplify.}$$
$$\frac{2y}{2} = \frac{9}{2} \quad \text{Divide each side by 2.}$$
$$y = 4.5 \quad \text{Simplify.}$$

The solution is $(-1, 4.5)$.

Exercises **Use elimination to solve each system of equations.**
See Examples 1 and 2 on pages 387 and 388.

19. $x - 5y = 0$
$2x - 3y = 7$

20. $x - 2y = 5$
$3x - 5y = 8$

21. $2x + 3y = 8$
$x - y = 2$

22. $-5x + 8y = 21$
$10x + 3y = 15$

Determine the best method to solve each system of equations. Then solve the system. *See Example 3 on page 389.*

23. $y = 2x$
$x + 2y = 8$

24. $9x + 8y = 7$
$18x - 15y = 14$

25. $3x + 5y = 2x$
$x + 3y = y$

26. $2x + y = 3x - 15$
$x + 5 = 4y + 2x$

7-5 Graphing Systems of Inequalities

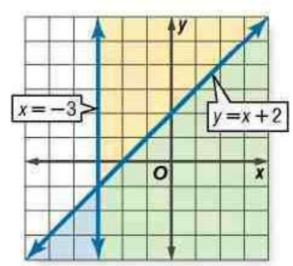
See pages 394–398.

Concept Summary

- Graph each inequality on a coordinate plane to determine the intersection of the graphs.

Example **Solve the system of inequalities.**

$x \geq -3$
$y \leq x + 2$



The solution includes the ordered pairs in the intersection of the graphs $x \geq -3$ and $y \leq x + 2$. This region is shaded in green. The graphs of $x \geq -3$ and $y \leq x + 2$ are boundaries of this region.

Exercises **Solve each system of inequalities by graphing.**
See Examples 1 and 2 on page 394.

27. $y < 3x$
$x + 2y \geq -21$

28. $y > -x - 1$
$y \leq 2x + 1$

29. $2x + y < 9$
$x + 11y < -6$

30. $x \geq 1$
$y + x \leq 3$

Chapter 7

Practice Test

Vocabulary and Concepts

Choose the letter that best matches each description.

1. a system of equations with two parallel lines
2. a system of equations with at least one ordered pair that satisfies both equations
3. a system of equations may be solved using this method

a. consistent
b. elimination
c. inconsistent

Skills and Applications

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

4. $y = x + 2$
$y = 2x + 7$

5. $x + 2y = 11$
$x = 14 - 2y$

6. $3x + y = 5$
$2y - 10 = -6x$

Use substitution or elimination to solve each system of equations.

7. $2x + 5y = 16$
$5x - 2y = 11$

8. $y + 2x = -1$
$y - 4 = -2x$

9. $2x + y = -4$
$5x + 3y = -6$

10. $y = 7 - x$
$x - y = -3$

11. $x = 2y - 7$
$y - 3x = -9$

12. $x + y = 10$
$x - y = 2$

13. $3x - y = 11$
$x + 2y = -36$

14. $3x + y = 10$
$3x - 2y = 16$

15. $5x - 3y = 12$
$-2x + 3y = -3$

16. $2x + 5y = 12$
$x - 6y = -11$

17. $x + y = 6$
$3x - 3y = 13$

18. $3x + \frac{1}{3}y = 10$
$2x - \frac{5}{3}y = 35$

19. **NUMBER THEORY** The units digit of a two-digit number exceeds twice the tens digit by 1. Find the number if the sum of its digits is 10.

20. **GEOMETRY** The difference between the length and width of a rectangle is 7 centimeters. Find the dimensions of the rectangle if its perimeter is 50 centimeters.

Solve each system of inequalities by graphing.

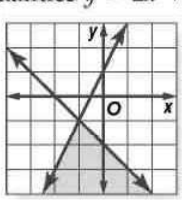
21. $y > -4$
$y < -1$

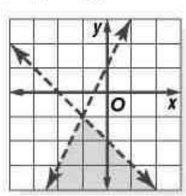
22. $y \leq 3$
$y > -x + 2$

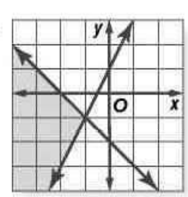
23. $x \leq 2y$
$2x + 3y \leq 7$

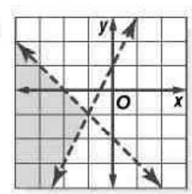
24. **FINANCE** Last year, Jodi invested $10,000, part at 6% annual interest and the rest at 8% annual interest. If she received $760 in interest at the end of the year, how much did she invest at each rate?

25. **STANDARDIZED TEST PRACTICE** Which graph represents the system of inequalities $y > 2x + 1$ and $y < -x - 2$?

(A)


(B)


(C)


(D)


Chapter 7 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the value of x in $4x - 2(x - 2) - 8 = 0$? (Lesson 3-4)

Ⓐ -2 Ⓑ 2
Ⓒ 5 Ⓓ 6

2. Noah paid \$17.11 for a CD, including tax. If the tax rate is 7%, then what was the price of the CD before tax? (Lesson 3-5)

Ⓐ \$10.06 Ⓑ \$11.98
Ⓒ \$15.99 Ⓓ \$17.04

3. What is the range of $f(x) = 2x - 3$ when the domain is $\{3, 4, 5\}$? (Lesson 4-3)

Ⓐ $\{0, 1, 2\}$ Ⓑ $\{3, 5, 7\}$
Ⓒ $\{6, 8, 10\}$ Ⓓ $\{9, 11, 13\}$

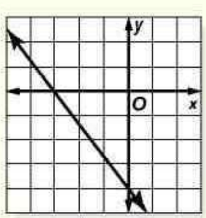
4. Jolene kept a log of the numbers of birds that visited a birdfeeder over periods of several hours. On the table below, she recorded the number of hours she watched and the cumulative number of birds that she saw each session. Which equation best represents this data set shown in the table? (Lesson 4-8)

Number of hours, x	1	3	4	6
Number of birds, y	6	14	18	26

Ⓐ $y = x + 5$ Ⓑ $y = 3x + 3$
Ⓒ $y = 3x + 5$ Ⓓ $y = 4x + 2$

5. Which equation describes the graph? (Lesson 5-3)

Ⓐ $3y - 4x = -12$
Ⓑ $4y + 3x = -16$
Ⓒ $3y + 4x = -12$
Ⓓ $3y + 4x = -9$



6. Which equation represents a line parallel to the line given by $y - 3x = 6$? (Lesson 5-6)

Ⓐ $y = -3x + 4$ Ⓑ $y = 3x - 2$
Ⓒ $y = \frac{1}{3}x + 6$ Ⓓ $y = -\frac{1}{3}x + 4$

7. Tamika has \$185 in her bank account. She needs to deposit enough money so that she can withdraw \$230 for her car payment and still have at least \$200 left in the account. Which inequality describes d, the amount she needs to deposit? (Lesson 6-1)

Ⓐ $d(185 - 230) \geq 200$
Ⓑ $185 - 230d \geq 200$
Ⓒ $185 + 230 + d \geq 200$
Ⓓ $185 + d - 230 \geq 200$

8. The perimeter of a rectangular garden is 68 feet. The length of the garden is 4 more than twice the width. Which system of equations will determine the length ℓ and the width w of the garden? (Lesson 7-2)

Ⓐ $2\ell + 2w = 68$, $\ell = 4 - 2w$ Ⓑ $2\ell + 2w = 68$, $\ell = 2w + 4$
Ⓒ $2 + 2w = 68$, $2\ell - w = 4$ Ⓓ $2\ell + 2w = 68$, $w = 2\ell + 4$

9. Ernesto spent a total of \$64 for a pair of jeans and a shirt. The jeans cost \$6 more than the shirt. What was the cost of the jeans? (Lesson 7-2)

Ⓐ \$26 Ⓑ \$29
Ⓒ \$35 Ⓓ \$58

10. What is the value of y in the following system of equations? (Lesson 7-3)

$3x + 4y = 8$
$3x + 2y = -2$

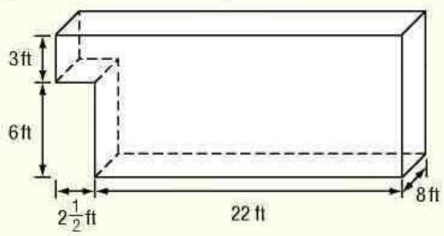
Ⓐ -2 Ⓑ 4
Ⓒ 5 Ⓓ 6

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

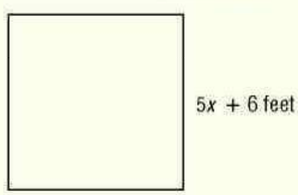
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. The diagram shows the dimensions of the cargo area of a delivery truck.



What is the maximum volume of cargo, in cubic feet, that can fit in the truck? (Prerequisite Skill)

12. The perimeter of the square below is 204 feet. What is the value of x? (Lesson 3-4)



13. What is the x-intercept of the graph of $4x + 3y = 12$? (Lesson 4-5)

14. What are the slope and the y-intercept of the graph of the equation $4x - 2y = 5$? (Lesson 5-4)

15. Solve the following system of equations. (Lesson 7-2)

$$5x - y = 10$$
$$7x - 2y = 11$$

The Princeton Review Test-Taking Tip

Questions 11 and 12 To prepare for a standardized test, make flash cards of key mathematical terms, such as "perimeter" and "volume." Use the glossary of your textbook to determine the important terms and their correct definitions.

www.algebra1.com/standardized_test

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- **Ⓐ the quantity in Column A is greater,**
- **Ⓑ the quantity in Column B is greater,**
- **Ⓒ the two quantities are equal, or**
- **Ⓓ the relationship cannot be determined from the information given.**

	Column A	Column B
16.	3^4	9^2
		(Lesson 1-1)
17.	the slope of the line that contains $A(2, 4)$ and $B(-1, 3)$	the slope of the line that contains $C(-2, 1)$ and $D(5, 3)$
		(Lesson 5-1)
18.	$x - 3y = 11$, $3x + y = 13$	
	y	0
		(Lesson 7-4)
19.	$3x - 2y = 19$, $5x + 4y = 17$	
	x	y
		(Lesson 7-4)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

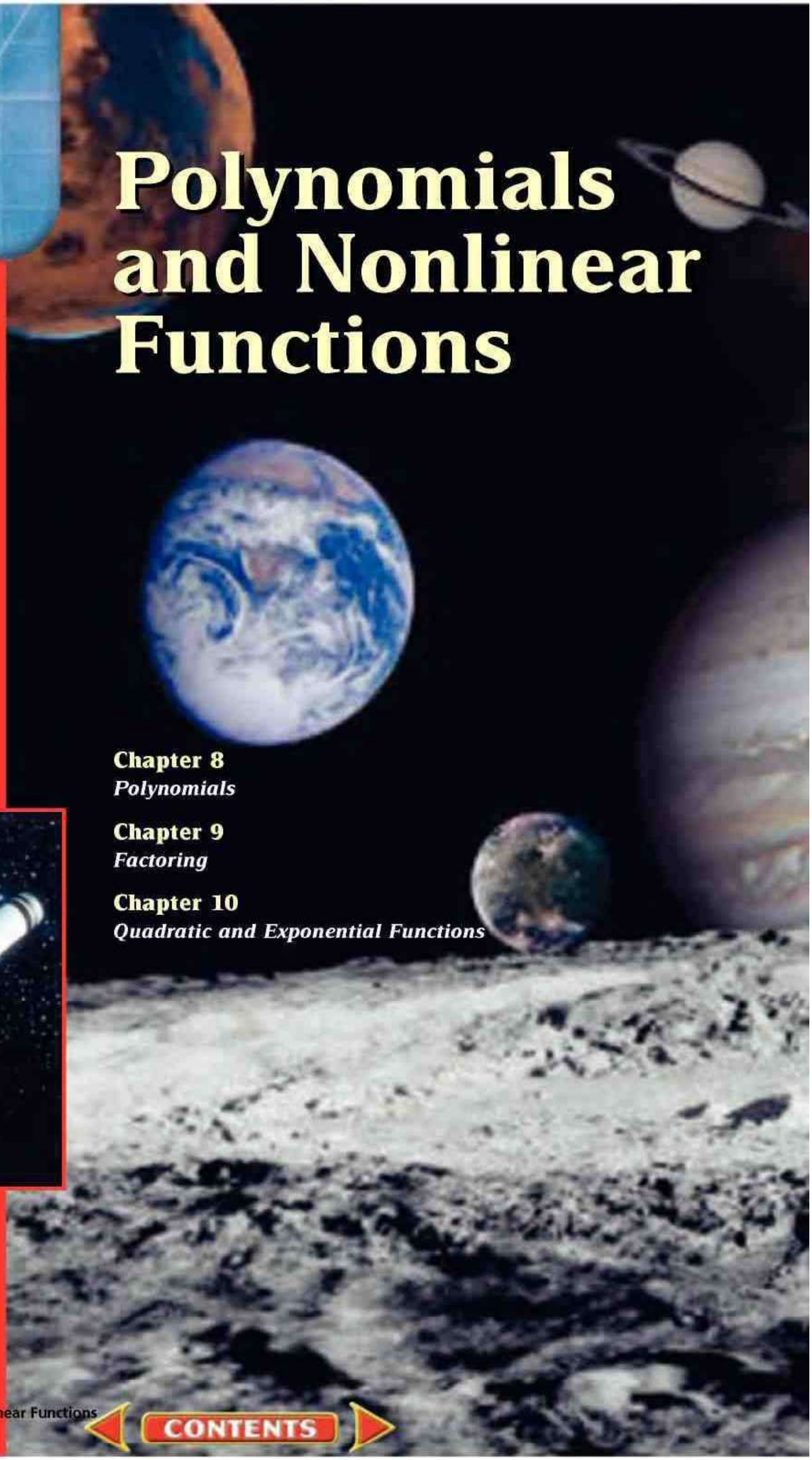
20. The manager of a movie theater found that Saturday's sales were \$3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost \$7.50, and children's tickets cost \$4.50. (Lesson 7-2)

a. Write equations to represent the number of tickets sold and the amount of money collected.

b. How many of each kind of ticket were sold? Show your work. Include all steps.

UNIT 3

Polynomials and Nonlinear Functions



Not all real-world situations can be modeled using a linear function. In this unit, you will learn about polynomials and nonlinear functions.

WebQuest Internet Project

Pluto Is Falling From Status as Distant Planet

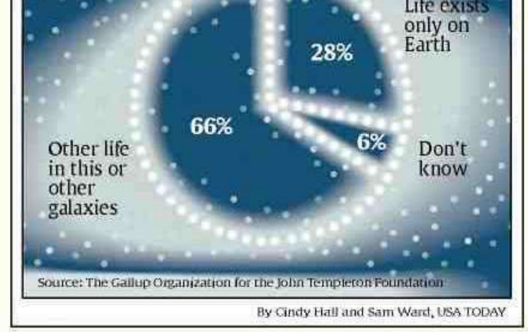
Source: *USA TODAY*, March 28, 2001

"Like any former third-grader, Catherine Beyhl knows that the solar system has nine planets, and she knows a phrase to help remember their order: 'My Very Educated Mother Just Served Us Nine Pizzas.' But she recently visited the American Museum of Natural History's glittering new astronomy hall at the Hayden Planetarium and found only eight scale models of the planets. No Pizza—no Pluto." In this project, you will examine how scientific notation, factors, and graphs are useful in presenting information about the planets.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 3.

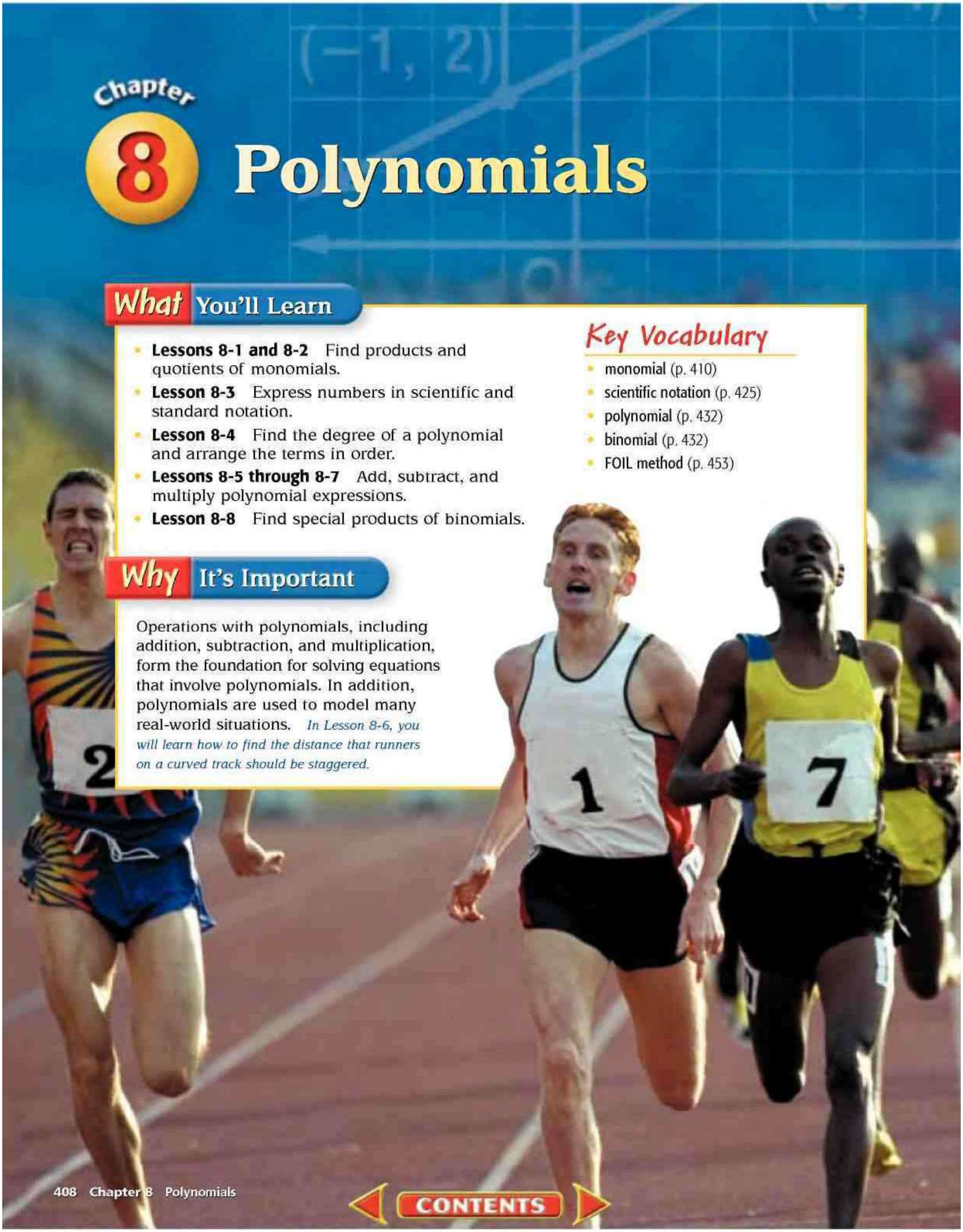
Lesson	8-3	9-1	10-2
Page	429	479	537



CONTENTS

Chapter

8 Polynomials



What You'll Learn

- **Lessons 8-1 and 8-2** Find products and quotients of monomials.
- **Lesson 8-3** Express numbers in scientific and standard notation.
- **Lesson 8-4** Find the degree of a polynomial and arrange the terms in order.
- **Lessons 8-5 through 8-7** Add, subtract, and multiply polynomial expressions.
- **Lesson 8-8** Find special products of binomials.

Key Vocabulary

- **monomial** (p. 410)
- **scientific notation** (p. 425)
- **polynomial** (p. 432)
- **binomial** (p. 432)
- **FOIL method** (p. 453)

Why It's Important

Operations with polynomials, including addition, subtraction, and multiplication, form the foundation for solving equations that involve polynomials. In addition, polynomials are used to model many real-world situations. *In Lesson 8-6, you will learn how to find the distance that runners on a curved track should be staggered.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 8.

For Lessons 8-1 and 8-2 — Exponential Notation

Write each expression using exponents. *(For review, see Lesson 1-1.)*

1. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ **2.** $3 \cdot 3 \cdot 3 \cdot 3$ **3.** $5 \cdot 5$ **4.** $x \cdot x \cdot x$

5. $a \cdot a \cdot a \cdot a \cdot a \cdot a$ **6.** $x \cdot x \cdot y \cdot y \cdot y$ **7.** $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ **8.** $\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{c}{d} \cdot \frac{c}{d} \cdot \frac{c}{d}$

For Lessons 8-1 and 8-2 — Evaluating Powers

Evaluate each expression. *(For review, see Lesson 1-1.)*

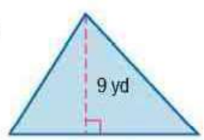
9. 3^2 **10.** 4^3 **11.** 5^2 **12.** 10^4

13. $(-6)^2$ **14.** $(-3)^3$ **15.** $\left(\frac{2}{3}\right)^4$ **16.** $\left(-\frac{7}{8}\right)^2$

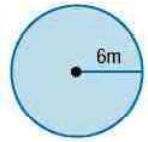
For Lessons 8-1, 8-2, and 8-5 through 8-8 — Area and Volume

Find the area or volume of each figure shown below. *(For review, see pages 813–817.)*

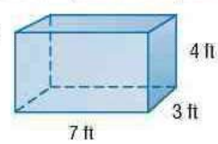
17.



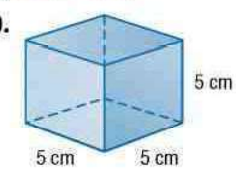
18.



19.



20.

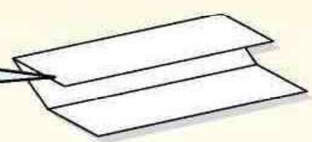


FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about polynomials. Begin with a sheet of 11" by 17" paper.

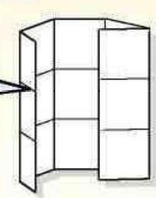
Step 1 Fold

Fold in thirds lengthwise.



Step 2 Open and Fold

Fold a 2" tab along the width. Then fold the rest in fourths.



Step 3 Label

Draw lines along folds and label as shown.

	+	−	×	÷
Mon.				
Poly.				

Reading and Writing As you read and study the chapter, write examples and notes for each operation.

8-1 Multiplying Monomials

What You'll Learn

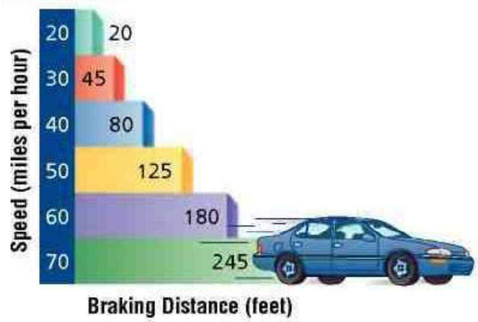
- Multiply monomials.
- Simplify expressions involving powers of monomials.

Vocabulary

- monomial
- constant

Why does doubling speed quadruple braking distance?

The table shows the braking distance for a vehicle at certain speeds. If s represents the speed in miles per hour, then the approximate number of feet that the driver must apply the brakes is $\frac{1}{20}s^2$. Notice that when speed is doubled, the braking distance is quadrupled.



Source: *British Highway Code*

MULTIPLY MONOMIALS An expression like $\frac{1}{20}s^2$ is called a monomial. A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression involving the division of variables is not a monomial. Monomials that are real numbers are called **constants**.

Example 1 Identify Monomials

Determine whether each expression is a monomial. Explain your reasoning.

	Expression	Monomial?	Reason
a.	-5	yes	-5 is a real number and an example of a constant.
b.	$p + q$	no	The expression involves the addition, not the product, of two variables.
c.	x	yes	Single variables are monomials.
d.	$\frac{c}{d}$	no	The expression is the quotient, not the product, of two variables.
e.	$\frac{abc^8}{5}$	yes	$\frac{abc^8}{5} = \frac{1}{5}abc^8$. The expression is the product of a number, $\frac{1}{5}$, and three variables.

Study Tip

Reading Math
The expression x^n is read *x to the nth power*.

Recall that an expression of the form x^n is called a *power* and represents the product you obtain when x is used as a factor n times. The number x is the *base*, and the number n is the *exponent*.

exponent

$$2^5 = \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5 \text{ factors}} \text{ or } 32$$

base

In the following examples, the definition of a power is used to find the products of powers. Look for a pattern in the exponents.

$$2^3 \cdot 2^5 = \underbrace{\overbrace{2 \cdot 2 \cdot 2}^{3 \text{ factors}} \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5 \text{ factors}}}_{3 + 5 \text{ or } 8 \text{ factors}} \text{ or } 2^8 \qquad 3^2 \cdot 3^4 = \underbrace{\overbrace{3 \cdot 3}^{2 \text{ factors}} \cdot \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{4 \text{ factors}}}_{2 + 4 \text{ or } 6 \text{ factors}} \text{ or } 3^6$$

These and other similar examples suggest the property for multiplying powers.

Key Concept *Product of Powers*

- **Words** To multiply two powers that have the same base, add the exponents.
- **Symbols** For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.
- **Example** $a^4 \cdot a^{12} = a^{4+12}$ or a^{16}

Example 2 Product of Powers

Simplify each expression.

a. $(5x^7)(x^6)$

$$\begin{aligned} (5x^7)(x^6) &= (5)(1)(x^7 \cdot x^6) && \text{Commutative and Associative Properties} \\ &= (5 \cdot 1)(x^{7+6}) && \text{Product of Powers} \\ &= 5x^{13} && \text{Simplify.} \end{aligned}$$

b. $(4ab^6)(-7a^2b^3)$

$$\begin{aligned} (4ab^6)(-7a^2b^3) &= (4)(-7)(a \cdot a^2)(b^6 \cdot b^3) && \text{Commutative and Associative Properties} \\ &= -28(a^{1+2})(b^{6+3}) && \text{Product of Powers} \\ &= -28a^3b^9 && \text{Simplify.} \end{aligned}$$

Study Tip

Power of 1
Recall that a variable with no exponent indicated can be written as a power of 1. For example, $x = x^1$ and $ab = a^1b^1$.

POWERS OF MONOMIALS You can also look for a pattern to discover the property for finding the power of a power.

$$\begin{aligned} (4^2)^5 &= \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5 \text{ factors}} \\ &= 4^{2+2+2+2+2} \\ &= 4^{10} \end{aligned} \qquad \longleftarrow \text{Apply rule for Product of Powers.} \longrightarrow \qquad \begin{aligned} (z^8)^3 &= \overbrace{(z^8)(z^8)(z^8)}^{3 \text{ factors}} \\ &= z^{8+8+8} \\ &= z^{24} \end{aligned}$$

Therefore, $(4^2)^5 = 4^{10}$ and $(z^8)^3 = z^{24}$. These and other similar examples suggest the property for finding the power of a power.

Key Concept *Power of a Power*

- **Words** To find the power of a power, multiply the exponents.
- **Symbols** For any number a and all integers m and n, $(a^m)^n = a^{m \cdot n}$.
- **Example** $(k^5)^9 = k^{5 \cdot 9}$ or k^{45}

Study Tip

Look Back
To review **using a calculator to find a power of a number**, see Lesson 1-1.

Example 3 Power of a Power

Simplify $((3^2)^3)^2$.

$$\begin{aligned} ((3^2)^3)^2 &= (3^{2 \cdot 3})^2 && \text{Power of a Power} \\ &= (3^6)^2 && \text{Simplify.} \\ &= 3^{6 \cdot 2} && \text{Power of a Power} \\ &= 3^{12} \text{ or } 531{,}441 && \text{Simplify.} \end{aligned}$$

Look for a pattern in the examples below.

$$(xy)^4 = (xy)(xy)(xy)(xy) = (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y) = x^4y^4$$

$$(6ab)^3 = (6ab)(6ab)(6ab) = (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b) = 6^3a^3b^3 \text{ or } 216a^3b^3$$

These and other similar examples suggest the following property for finding the power of a product.

Study Tip

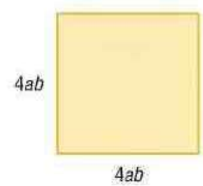
Powers of Monomials
Sometimes the rules for the Power of a Power and the Power of a Product are combined into one rule.
$(a^mb^n)^p = a^{mp}b^{np}$

Key Concept — *Power of a Product*

- **Words** To find the power of a product, find the power of each factor and multiply.
- **Symbols** For all numbers a and b and any integer m, $(ab)^m = a^mb^m$.
- **Example** $(-2xy)^3 = (-2)^3x^3y^3$ or $-8x^3y^3$

Example 4 Power of a Product

GEOMETRY **Express the area of the square as a monomial.**



Area $= s^2$	Formula for the area of a square
$= (4ab)^2$	$s = 4ab$
$= 4^2a^2b^2$	Power of a Product
$= 16a^2b^2$	Simplify.

The area of the square is $16a^2b^2$ square units.

The properties can be used in combination to simplify more complex expressions involving exponents.

Concept Summary — *Simplifying Monomial Expressions*

To *simplify* an expression involving monomials, write an equivalent expression in which:

- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

Example 5 Simplify Expressions

Simplify $\left(\frac{1}{3}xy^4\right)^2[(-6y)^2]^3$.

$\left(\frac{1}{3}xy^4\right)^2[(-6y)^2]^3 = \left(\frac{1}{3}xy^4\right)^2(-6y)^6$	Power of a Power
$= \left(\frac{1}{3}\right)^2x^2(y^4)^2(-6)^6y^6$	Power of a Product
$= \frac{1}{9}x^2y^8(46{,}656)y^6$	Power of a Power
$= \frac{1}{9}(46{,}656)x^2 \cdot y^8 \cdot y^6$	Commutative Property
$= 5184x^2y^{14}$	Product of Powers

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an expression that can be simplified using each property. Then simplify each expression.
   - **a.** Product of Powers
   - **b.** Power of a Power
   - **c.** Power of a Product
2. **Determine** whether each pair of monomials is equivalent. Explain.
   - **a.** $5m^2$ and $(5m)^2$
   - **b.** $(yz)^4$ and y^4z^4
   - **c.** $-3a^2$ and $(-3a)^2$
   - **d.** $2(c^7)^3$ and $8c^{21}$
3. **FIND THE ERROR** Nathan and Poloma are simplifying $(5^2)(5^9)$.

Nathan	Poloma
$(5^2)(5^9) = (5 \cdot 5)^{2+9}$	$(5^2)(5^9) = 5^{2+9}$
$= 25^{11}$	$= 5^{11}$

Who is correct? Explain your reasoning.

Guided Practice **Determine whether each expression is a monomial. Write *yes* or *no*. Explain.**

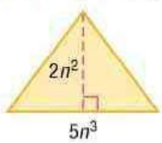
4. $5 - 7d$
5. $\frac{4a}{3b}$
6. n

Simplify.

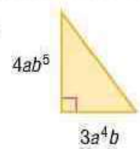
7. $x(x^4)(x^6)$
8. $(4a^4b)(9a^2b^3)$
9. $[(2^3)^2]^3$
10. $(3y^5z)^2$
11. $(-4mn^2)(12m^2n)$
12. $(-2v^3w^4)^3(-3vw^3)^2$

Application **GEOMETRY** **Express the area of each triangle as a monomial.**

13.



14.



Practice and Apply

Homework Help

For Exercises	See Examples
15–20	1
21–48	2, 3, 5
49–54	4

Extra Practice
See page 837.

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

15. 12
16. $4x^3$
17. $a - 2b$
18. $4n + 5m$
19. $\frac{x}{y^2}$
20. $\frac{1}{5}abc^{14}$

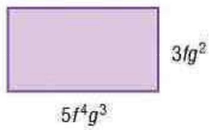
Simplify.

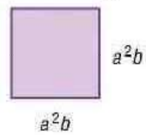
21. $(ab^4)(ab^2)$
22. $(p^5q^4)(p^2q)$
23. $(-7c^3d^4)(4cd^3)$
24. $(-3j^7k^5)(-8jk^8)$
25. $(5a^2b^3c^4)(6a^3b^4c^2)$
26. $(10xy^5z^3)(3x^4y^6z^3)$
27. $(9pq^7)^2$
28. $(7b^3c^6)^3$
29. $[(3^2)^4]^2$
30. $[(4^2)^3]^2$
31. $(0.5x^3)^2$
32. $(0.4h^5)^3$
33. $\left(-\frac{3}{4}c\right)^3$
34. $\left(\frac{4}{5}a^2\right)^2$
35. $(4cd)^2(-3d^2)^3$
36. $(-2x^5)^3(-5xy^6)^2$
37. $(2ag^2)^4(3a^2g^3)^2$
38. $(2m^2n^3)^3(3m^3n)^4$
39. $(8y^3)(-3x^2y^2)\left(\frac{3}{8}xy^4\right)$
40. $\left(\frac{4}{7}m\right)^2(49m)(17p)\left(\frac{1}{34}p^5\right)$

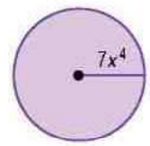
41. Simplify the expression $(-2b^3)^4 - 3(-2b^4)^3$.

42. Simplify the expression $2(-5y^3)^2 + (-3y^3)^3$.

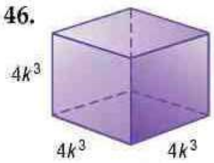
GEOMETRY **Express the area of each figure as a monomial.**

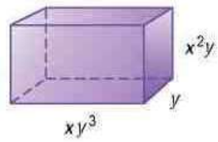
43. 

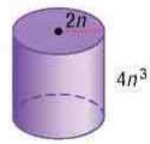
44. 

45. 

GEOMETRY **Express the volume of each solid as a monomial.**

46. 

47. 

48. 

TELEPHONES **For Exercises 49 and 50, use the following information.**
The first transatlantic telephone cable has 51 amplifiers along its length. Each amplifier strengthens the signal on the cable 10^6 times.

49. After it passes through the second amplifier, the signal has been boosted $10^6 \cdot 10^6$ times. Simplify this expression.

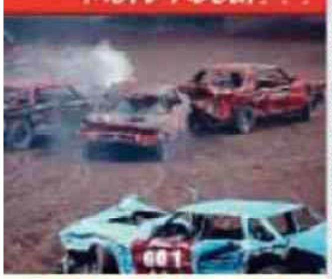
50. Represent the number of times the signal has been boosted after it has passed through the first four amplifiers as a power of 10^6. Then simplify the expression.

DEMOLITION DERBY **For Exercises 51 and 52, use the following information.**
When a car hits an object, the damage is measured by the collision impact. For a certain car, the collision impact I is given by $I = 2s^2$, where s represents the speed in kilometers per minute.

51. What is the collision impact if the speed of the car is 1 kilometer per minute? 2 kilometers per minute? 4 kilometers per minute?

52. As the speed doubles, explain what happens to the collision impact.

More About...



Demolition Derby

In a demolition derby, the winner is not the car that finishes first but the last car still moving under its own power.

Source: *Smithsonian Magazine*

TEST TAKING **For Exercises 53 and 54, use the following information.**
A history test covers two chapters. There are 2^{12} ways to answer the 12 true-false questions on the first chapter and 2^{10} ways to answer the 10 true-false questions on the second chapter.

53. How many ways are there to answer all 22 questions on the test? (*Hint*: Find the product of 2^{12} and 2^{10}.)

54. If a student guesses on each question, what is the probability of answering all questions correctly?

CRITICAL THINKING **Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, give a counterexample.**

55. For any real number a, $(-a)^2 = -a^2$.

56. For all real numbers a and b, and all integers m, n, and p, $(a^m b^n)^p = a^{mp} b^{np}$.

57. For all real numbers a, b, and all integers n, $(a + b)^n = a^n + b^n$.

CONTENTS

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why does doubling speed quadruple braking distance?

Include the following in your answer:

- the ratio of the braking distance required for a speed of 40 miles per hour and the braking distance required for a speed of 80 miles per hour, and
- a comparison of the expressions $\frac{1}{20}s^2$ and $\frac{1}{20}(2s)^2$.

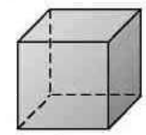
Standardized Test Practice

59. $4^2 \cdot 4^5 = ?$

Ⓐ 16^7 Ⓑ 8^7 Ⓒ 4^{10} Ⓓ 4^7

60. Which of the following expressions represents the volume of the cube?

Ⓐ $15x^3$ Ⓑ $25x^2$
Ⓒ $25x^3$ Ⓓ $125x^3$



$5x$

Maintain Your Skills

Mixed Review

Solve each system of inequalities by graphing. *(Lesson 7-5)*

61. $y \le 2x + 2$
$y \ge -x - 1$

62. $y \ge x - 2$
$y < 2x - 1$

63. $x > -2$
$y < x + 3$

Use elimination to solve each system of equations. *(Lesson 7-4)*

64. $-4x + 5y = 2$
$x + 2y = 6$

65. $3x + 4y = -25$
$2x - 3y = 6$

66. $x + y = 20$
$0.4x + 0.15y = 4$

Solve each compound inequality. Then graph the solution set. *(Lesson 6-4)*

67. $4 + h \le -3$ or $4 + h \ge 5$

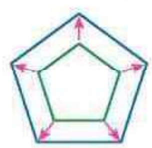
68. $4 < 4a + 12 < 24$

69. $14 < 3h + 2 < 2$

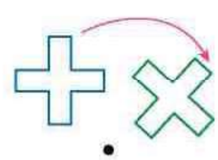
70. $2m - 3 > 7$ or $2m + 7 > 9$

Determine whether each transformation is a *reflection, translation, dilation,* or *rotation*. *(Lesson 4-2)*

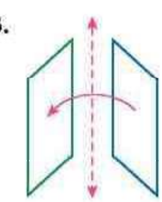
71.



72.



73.



74. **TRANSPORTATION** Two trains leave York at the same time, one traveling north, the other south. The northbound train travels at 40 miles per hour and the southbound at 30 miles per hour. In how many hours will the trains be 245 miles apart? *(Lesson 3-7)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **simplifying fractions**, see pages 798 and 799.)*

75. $\frac{2}{6}$

76. $\frac{3}{15}$

77. $\frac{10}{5}$

78. $\frac{27}{9}$

79. $\frac{14}{36}$

80. $\frac{9}{48}$

81. $\frac{44}{32}$

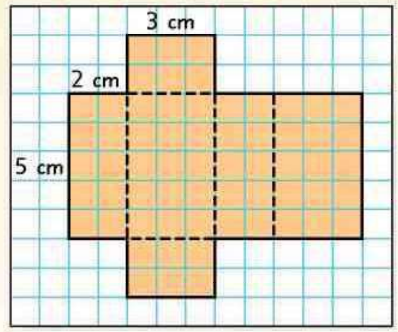
82. $\frac{45}{18}$

Algebra Activity

A Follow-Up of Lesson 8-1

Investigating Surface Area and Volume

Collect the Data



- Cut out the pattern shown from a sheet of centimeter grid paper. Fold along the dashed lines and tape the edges together to form a rectangular prism with dimensions 2 centimeters by 5 centimeters by 3 centimeters.
- Find the surface area SA of the prism by counting the squares on all the faces of the prism or by using the formula $SA = 2w\ell + 2wh + 2\ell h$, where w is the width, ℓ is the length, and h is the height of the prism.
- Find the volume V of the prism by using the formula $V = \ell wh$.
- Now construct another prism with dimensions that are 2 times each of the dimensions of the first prism, or 4 centimeters by 10 centimeters by 6 centimeters.
- Finally, construct a third prism with dimensions that are 3 times each of the dimensions of the first prism, or 6 centimeters by 15 centimeters by 9 centimeters.

Analyze the Data

1. Copy and complete the table using the prisms you made.

Prism	Dimensions	Surface Area (cm^2)	Volume (cm^3)	Surface Area Ratio $\left(\frac{SA \text{ of New}}{SA \text{ of Original}}\right)$	Volume Ratio $\left(\frac{V \text{ of New}}{V \text{ of Original}}\right)$
Original	2 by 5 by 3	62	30	——	——
A	4 by 10 by 6				
B	6 by 15 by 9				

2. Make a prism with different dimensions from any in this activity. Repeat the steps in **Collect the Data**, and make a table similar to the one in Exercise 1.

Make a Conjecture

3. Suppose you multiply each dimension of a prism by 2. What is the ratio of the surface area of the new prism to the surface area of the original prism? What is the ratio of the volumes?

4. If you multiply each dimension of a prism by 3, what is the ratio of the surface area of the new prism to the surface area of the original? What is the ratio of the volumes?

5. Suppose you multiply each dimension of a prism by a. Make a conjecture about the ratios of surface areas and volumes.

Extend the Activity

6. Repeat the steps in **Collect the Data** and **Analyze the Data** using cylinders. To start, make a cylinder with radius 4 centimeters and height 5 centimeters. To compute surface area SA and volume V, use the formulas $SA = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Do the conjectures you made in Exercise 5 hold true for cylinders? Explain.

8-2 Dividing Monomials

What You'll Learn

- Simplify expressions involving the quotient of monomials.
- Simplify expressions containing negative exponents.

Vocabulary

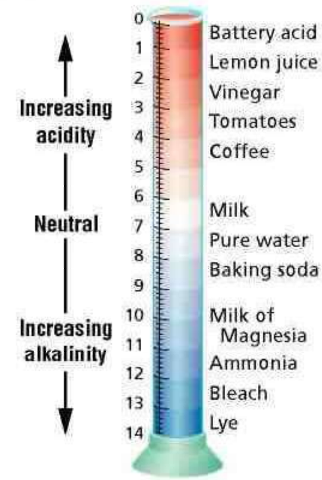
- zero exponent
- negative exponent

How can you compare pH levels?

To test whether a solution is a *base* or an *acid*, chemists use a pH test. This test measures the concentration c of hydrogen ions (in moles per liter) in the solution.

$$c = \left(\frac{1}{10}\right)^{\text{pH}}$$

The table gives examples of solutions with various pH levels. You can find the quotient of powers and use negative exponents to compare measures on the pH scale.



Source: U.S. Geological Survey

QUOTIENTS OF MONOMIALS In the following examples, the definition of a power is used to find quotients of powers. Look for a pattern in the exponents.

$$\frac{4^5}{4^3} = \frac{\overbrace{\cancel{4}^{1} \cdot \cancel{4}^{1} \cdot \cancel{4}^{1} \cdot 4 \cdot 4}^{5 \text{ factors}}}{\underbrace{\cancel{4}_{1} \cdot \cancel{4}_{1} \cdot \cancel{4}_{1}}_{3 \text{ factors}}} = \underbrace{4 \cdot 4}_{5 - 3 \text{ or } 2 \text{ factors}} \text{ or } 4^2$$

$$\frac{3^6}{3^2} = \frac{\overbrace{\cancel{3}^{1} \cdot \cancel{3}^{1} \cdot 3 \cdot 3 \cdot 3 \cdot 3}^{6 \text{ factors}}}{\underbrace{\cancel{3}_{1} \cdot \cancel{3}_{1}}_{2 \text{ factors}}} = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{6 - 2 \text{ or } 4 \text{ factors}} \text{ or } 3^4$$

These and other similar examples suggest the following property for dividing powers.

Key Concept — Quotient of Powers

- **Words** To divide two powers that have the same base, subtract the exponents.
- **Symbols** For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
- **Example** $\frac{b^{15}}{b^7} = b^{15-7}$ or b^8

Example 1 Quotient of Powers

Simplify $\frac{a^5b^8}{ab^3}$. Assume that a and b are not equal to zero.

$\frac{a^5b^8}{ab^3} = \left(\frac{a^5}{a}\right)\left(\frac{b^8}{b^3}\right)$	Group powers that have the same base.
$= (a^{5-1})(b^{8-3})$	Quotient of Powers
$= a^4b^5$	Simplify.

In the following example, the definition of a power is used to compute the power of a quotient. Look for a pattern in the exponents.

$$\left(\frac{2}{5}\right)^3 = \underbrace{\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)}_{\text{3 factors}} = \frac{\overbrace{2 \cdot 2 \cdot 2}^{\text{3 factors}}}{\underbrace{5 \cdot 5 \cdot 5}_{\text{3 factors}}} \text{ or } \frac{2^3}{5^3}$$

This and other similar examples suggest the following property.

Key Concept — Power of a Quotient

- **Words** To find the power of a quotient, find the power of the numerator and the power of the denominator.
- **Symbols** For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.
- **Example** $\left(\frac{c}{d}\right)^5 = \frac{c^5}{d^5}$

Example 2 Power of a Quotient

Simplify $\left(\frac{2p^2}{3}\right)^4$.

$\left(\frac{2p^2}{3}\right)^4 = \frac{(2p^2)^4}{3^4}$ Power of a Quotient

$= \frac{2^4(p^2)^4}{3^4}$ Power of a Product

$= \frac{16p^8}{81}$ Power of a Power

NEGATIVE EXPONENTS A graphing calculator can be used to investigate expressions with 0 as an exponent as well as expressions with negative exponents.

Graphing Calculator Investigation

Zero Exponent and Negative Exponents

Use the [^] key on a TI-83 Plus to evaluate expressions with exponents.

Think and Discuss

1. Copy and complete the table below.

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Value									

2. Describe the relationship between each pair of values.
   a. 2^4 and 2^{-4} **b.** 2^3 and 2^{-3} **c.** 2^2 and 2^{-2} **d.** 2^1 and 2^{-1}
3. **Make a Conjecture** as to the fractional value of 5^{-1}. Verify your conjecture using a calculator.
4. What is the value of 5^0?
5. What happens when you evaluate 0^0?

Study Tip

Graphing Calculator

To express a value as a fraction, press

[MATH] [ENTER] [ENTER].

To understand why a calculator gives a value of 1 for 2^0, study the two methods used to simplify $\frac{2^4}{2^4}$.

Study Tip

Alternative Method

Another way to look at the problem of simplifying $\frac{2^4}{2^4}$ is to recall that any nonzero number divided by itself is 1: $\frac{2^4}{2^4} = \frac{16}{16}$ or 1.

Method 1

$$\frac{2^4}{2^4} = 2^{4-4} \quad \text{Quotient of Powers}$$

$$= 2^0 \quad \text{Subtract.}$$

Method 2

$$\frac{2^4}{2^4} = \frac{\cancel{2}^1 \cdot \cancel{2}^1 \cdot \cancel{2}^1 \cdot \cancel{2}^1}{\cancel{2}_1 \cdot \cancel{2}_1 \cdot \cancel{2}_1 \cdot \cancel{2}_1} \quad \text{Definition of powers}$$

$$= 1 \quad \text{Simplify.}$$

Since $\frac{2^4}{2^4}$ cannot have two different values, we can conclude that $2^0 = 1$.

Key Concept — *Zero Exponent*

- **Words** Any nonzero number raised to the zero power is 1.
- **Symbols** For any nonzero number a, $a^0 = 1$.
- **Example** $(-0.25)^0 = 1$

Example 3 *Zero Exponent*

Simplify each expression. Assume that x and y are not equal to zero.

a. $\left(-\frac{3x^5y}{8xy^7}\right)^0$

$$\left(-\frac{3x^5y}{8xy^7}\right)^0 = 1 \quad a^0 = 1$$

b. $\frac{t^3s^0}{t}$

$$\frac{t^3s^0}{t} = \frac{t^3(1)}{t} \quad a^0 = 1$$

$$= \frac{t^3}{t} \quad \text{Simplify.}$$

$$= t^2 \quad \text{Quotient of Powers}$$

To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{8^2}{8^5}$ in two ways.

Method 1

$$\frac{8^2}{8^5} = 8^{2-5} \quad \text{Quotient of Powers}$$

$$= 8^{-3} \quad \text{Subtract.}$$

Method 2

$$\frac{8^2}{8^5} = \frac{\cancel{8}^1 \cdot \cancel{8}^1}{\cancel{8}_1 \cdot \cancel{8}_1 \cdot 8 \cdot 8 \cdot 8} \quad \text{Definition of powers}$$

$$= \frac{1}{8^3} \quad \text{Simplify.}$$

Since $\frac{8^2}{8^5}$ cannot have two different values, we can conclude that $8^{-3} = \frac{1}{8^3}$.

Key Concept — *Negative Exponent*

- **Words** For any nonzero number a and any integer n, a^{-n} is the reciprocal of a^n. In addition, the reciprocal of a^{-n} is a^n.
- **Symbols** For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.
- **Examples** $5^{-2} = \frac{1}{5^2}$ or $\frac{1}{25}$ $\qquad \frac{1}{m^{-3}} = m^3$

An expression involving exponents is not considered simplified if the expression contains negative exponents.

Example 4 Negative Exponents

Simplify each expression. Assume that no denominator is equal to zero.

a. $\dfrac{b^{-3}c^2}{d^{-5}}$

$\dfrac{b^{-3}c^2}{d^{-5}} = \left(\dfrac{b^{-3}}{1}\right)\left(\dfrac{c^2}{1}\right)\left(\dfrac{1}{d^{-5}}\right)$ Write as a product of fractions.

$= \left(\dfrac{1}{b^3}\right)\left(\dfrac{c^2}{1}\right)\left(\dfrac{d^5}{1}\right)$ $a^{-n} = \dfrac{1}{a^n}$

$= \dfrac{c^2d^5}{b^3}$ Multiply fractions.

b. $\dfrac{-3a^{-4}b^7}{21a^2b^7c^{-5}}$

$\dfrac{-3a^{-4}b^7}{21a^2b^7c^{-5}} = \left(\dfrac{-3}{21}\right)\left(\dfrac{a^{-4}}{a^2}\right)\left(\dfrac{b^7}{b^7}\right)\left(\dfrac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \dfrac{-1}{7}(a^{-4-2})(b^{7-7})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \dfrac{-1}{7}a^{-6}b^0c^5$ Simplify.

$= \dfrac{-1}{7}\left(\dfrac{1}{a^6}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= -\dfrac{c^5}{7a^6}$ Multiply fractions.

Study Tip

Common Misconception

Do not confuse a negative number with a number raised to a negative power.

$3^{-1} = \frac{1}{3}$ $\quad -3 \neq \frac{1}{3}$

Standardized Test Practice

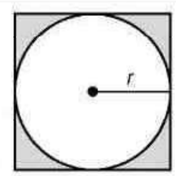
A B C D

Example 5 Apply Properties of Exponents

Multiple-Choice Test Item

Write the ratio of the area of the circle to the area of the square in simplest form.

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{2\pi}{1}$ (D) $\dfrac{\pi}{3}$



Read the Test Item

A ratio is a comparison of two quantities. It can be written in fraction form.

Solve the Test Item

- area of circle $= \pi r^2$
  length of square = diameter of circle or $2r$
  area of square $= (2r)^2$
- $\dfrac{\text{area of circle}}{\text{area of square}} = \dfrac{\pi r^2}{(2r)^2}$ Substitute.

  $= \dfrac{\pi}{4}r^{2-2}$ Quotient of Powers

  $= \dfrac{\pi}{4}r^0$ or $\dfrac{\pi}{4}$ $\quad r^0 = 1$

The answer is B.

The Princeton Review Test-Taking Tip

Some problems can be solved using estimation. The area of the circle is less than the area of the square. Therefore, the ratio of the two areas must be less than 1. Use 3 as an approximate value for π to determine which of the choices is less than 1.

Check for Understanding

Concept Check

1. **OPEN ENDED** Name two monomials whose product is $54x^2y^3$.
2. **Show** a method of simplifying $\frac{a^3b^5}{ab^2}$ using negative exponents instead of the Quotient of Powers Property.
3. **FIND THE ERROR** Jamal and Emily are simplifying $\frac{-4x^3}{x^5}$.

Jamal	Emily
$\frac{-4x^3}{x^5} = -4x^{3-5}$	$\frac{-4x^3}{x^5} = \frac{x^{3-5}}{4}$
$= -4x^{-2}$	$= \frac{x^{-2}}{4}$
$= \frac{-4}{x^2}$	$= \frac{1}{4x^2}$

Who is correct? Explain your reasoning.

Guided Practice

Simplify. Assume that no denominator is equal to zero.

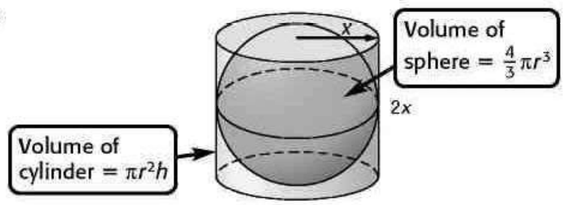
4. $\frac{7^8}{7^2}$
5. $\frac{x^8y^{12}}{x^2y^7}$
6. $\left(\frac{2c^3d}{7z^2}\right)^3$
7. $y^0(y^5)(y^{-9})$
8. 13^{-2}
9. $\frac{c^{-5}}{d^3g^{-8}}$
10. $\frac{-5pq^7}{10p^6q^3}$
11. $\frac{(cd^{-2})^3}{(c^4d^9)^{-2}}$
12. $\frac{(4m^{-3}n^5)^0}{mn}$

Standardized Test Practice

13. Find the ratio of the volume of the cylinder to the volume of the sphere.

Ⓐ $\frac{1}{2}$ Ⓑ 1

Ⓒ $\frac{3}{2}$ Ⓓ $\frac{3\pi}{2}$



Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1, 2
22–37	1–4

Extra Practice

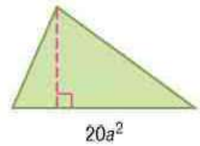
See page 837.

Simplify. Assume that no denominator is equal to zero.

14. $\frac{4^{12}}{4^2}$
15. $\frac{3^{13}}{3^7}$
16. $\frac{p^7n^3}{p^4n^2}$
17. $\frac{y^3z^9}{yz^2}$
18. $\left(\frac{5b^4n}{2a^6}\right)^2$
19. $\left(\frac{3m^7}{4x^5y^3}\right)^4$
20. $\frac{-2a^3}{10a^8}$
21. $\frac{15b}{45b^5}$
22. $x^3y^0x^{-7}$
23. $n^2(p^{-4})(n^{-5})$
24. 6^{-2}
25. 5^{-3}
26. $\left(\frac{4}{5}\right)^{-2}$
27. $\left(\frac{3}{2}\right)^{-3}$
28. $\frac{28a^7c^{-4}}{7a^3b^0c^{-8}}$
29. $\frac{30h^{-2}k^{14}}{5hk^{-3}}$
30. $\frac{18x^3y^4z^7}{-2x^2yz}$
31. $\frac{-19y^0z^4}{-3z^{16}}$
32. $\frac{(5r^{-2})^{-2}}{(2r^3)^2}$
33. $\frac{p^{-4}q^{-3}}{(p^5q^2)^{-1}}$
34. $\left(\frac{r^{-2}t^5}{t^{-1}}\right)^0$
35. $\left(\frac{4c^{-2}d}{b^{-2}c^3d^{-1}}\right)^0$
36. $\left(\frac{5b^{-2}n^4}{n^2z^{-3}}\right)^{-1}$
37. $\left(\frac{2a^{-2}bc^{-1}}{3ab^{-2}}\right)^{-3}$

38. The area of the rectangle is $24x^5y^3$ square units. Find the length of the rectangle.

39. The area of the triangle is $100a^3b$ square units. Find the height of the triangle.



SOUND **For Exercises 40–42, use the following information.**
The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds.

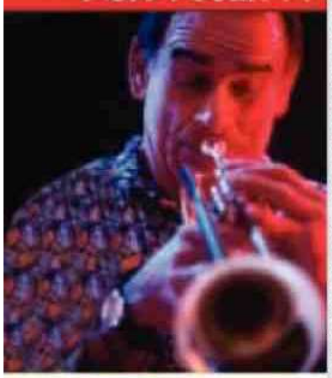
Watts/Square Meter	Common Sounds
10^2	jet plane (30 m away)
10^1	pain level
10^0	amplified music (2 m away)
10^{-2}	noisy kitchen
10^{-3}	heavy traffic
10^{-6}	normal conversation
10^{-7}	average home
10^{-9}	soft whisper
10^{-12}	barely audible

40. How many times more intense is the sound from heavy traffic than the sound from normal conversation?

41. What sound is 10,000 times as loud as a noisy kitchen?

42. How does the intensity of a whisper compare to that of normal conversation?

More About. . .



Sound

Timbre is the quality of the sound produced by a musical instrument. Sound quality is what distinguishes the sound of a note played on a flute from the sound of the same note played on a trumpet with the same frequency and intensity.

Source: www.school.discovery.com

PROBABILITY **For Exercises 43 and 44, use the following information.**
If you toss a coin, the probability of getting heads is $\frac{1}{2}$. If you toss a coin 2 times, the probability of getting heads each time is $\frac{1}{2} \cdot \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2$.

43. Write an expression to represent the probability of tossing a coin n times and getting n heads.

44. Express your answer to Exercise 43 as a power of 2.

LIGHT **For Exercises 45 and 46, use the table below.**

45. Express the range of the wavelengths of visible light using positive exponents. Then evaluate each expression.

46. Express the range of the wavelengths of X-rays using positive exponents. Then evaluate each expression.

Spectrum of Electromagnetic Radiation

Region	Wavelength (cm)
Radio	greater than 10
Microwave	10^1 to 10^{-2}
Infrared	10^{-2} to 10^{-5}
Visible	10^{-5} to 10^{-4}
Ultraviolet	10^{-4} to 10^{-7}
X-rays	10^{-7} to 10^{-9}
Gamma Rays	less than 10^{-9}

CRITICAL THINKING **Simplify. Assume that no denominator is equal to zero.**

47. $a^n(a^3)$

48. $(5^{4x-3})(5^{2x+1})$

49. $\frac{c^{x+7}}{c^{x-4}}$

50. $\frac{3b^{2n-9}}{b^{3(n-3)}}$

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you compare pH levels?

Include the following in your answer:

- an example comparing two pH levels using the properties of exponents.

Standardized Test Practice

52. What is the value of $\frac{2^2 \cdot 2^3}{2^{-2} \cdot 2^{-3}}$?

Ⓐ 2^{10} Ⓑ 2^{12} Ⓒ -1 Ⓓ $\frac{1}{2}$

53. **EXTENDED RESPONSE** Write a convincing argument to show why $3^0 = 1$ using the following pattern.
$3^5 = 243, 3^4 = 81, 3^3 = 27, 3^2 = 9, \ldots$

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 8-1)*

54. $(m^3n)(mn^2)$

55. $(3x^4y^3)(4x^4y)$

56. $(a^3x^2)^4$

57. $(3cd^5)^2$

58. $[(2^3)^2]^2$

59. $(-3ab)^3(2b^3)^2$

NUTRITION **For Exercises 60 and 61, use the following information.**
Between the ages of 11 and 18, you should get at least 1200 milligrams of calcium each day. One ounce of mozzarella cheese has 147 milligrams of calcium, and one ounce of Swiss cheese has 219 milligrams. Suppose you wanted to eat no more than 8 ounces of cheese. *(Lesson 7-5)*

60. Draw a graph showing the possible amounts of each type of cheese you can eat and still get your daily requirement of calcium. Let x be the amount of mozzarella cheese and y be the amount of Swiss cheese.

61. List three possible solutions.

Write an equation of the line with the given slope and y-intercept. *(Lesson 5-3)*

62. slope: 1, y-intercept: -4

63. slope: -2, y-intercept: 3

64. slope: $-\frac{1}{3}$, y-intercept: -1

65. slope: $\frac{3}{2}$, y-intercept: 2

Graph each equation by finding the x- and y-intercepts. *(Lesson 4-5)*

66. $2y = x + 10$

67. $4x - y = 12$

68. $2x = 7 - 3y$

Find each square root. If necessary, round to the nearest hundredth. *(Lesson 2-7)*

69. $\pm\sqrt{121}$

70. $\sqrt{3.24}$

71. $-\sqrt{52}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **Products of Powers**, see Lesson 8-1.)*

72. $10^2 \times 10^3$

73. $10^{-8} \times 10^{-5}$

74. $10^{-6} \times 10^9$

75. $10^8 \times 10^{-1}$

76. $10^4 \times 10^{-4}$

77. $10^{-12} \times 10$

Reading Mathematics

Mathematical Prefixes and Everyday Prefixes

You may have noticed that many prefixes used in mathematics are also used in everyday language. You can use the everyday meaning of these prefixes to better understand their mathematical meaning. The table shows two mathematical prefixes along with their meaning and an example of an everyday word using that prefix.

Prefix	Everyday Meaning	Example
mono-	**1.** one; single; alone	**monologue** A continuous series of jokes or comic stories delivered by one comedian.
bi-	**1.** two **2.** both **3.** both sides, parts, or directions	**bicycle** A vehicle consisting of a light frame mounted on two wire-spoked wheels one behind the other and having a seat, handlebars for steering, brakes, and two pedals or a small motor by which it is driven.
tri-	**1.** three **2.** occurring at intervals of three **3.** occurring three times during	**trilogy** A group of three dramatic or literary works related in subject or theme.
poly-	**1.** more than one; many; much	**polygon** A closed plane figure bounded by three or more line segments.

Source: *The American Heritage Dictionary of the English Language*

You can use your everyday understanding of prefixes to help you understand mathematical terms that use those prefixes.

Reading to Learn

1. Give an example of a geometry term that uses one of these prefixes. Then define that term.

2. **MAKE A CONJECTURE** Given your knowledge of the meaning of the word monomial, make a conjecture as to the meaning of each of the following mathematical terms.

a. binomial **b.** trinomial **c.** polynomial

3. Research the following prefixes and their meanings.

a. semi- **b.** hexa- **c.** octa-

8-3 Scientific Notation

What You'll Learn

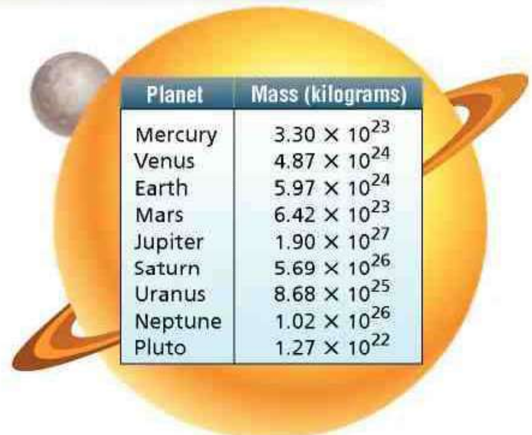
- Express numbers in scientific notation and standard notation.
- Find products and quotients of numbers expressed in scientific notation.

Vocabulary

- scientific notation

Why is scientific notation important in astronomy?

Astronomers often work with very large numbers, such as the masses of planets. The mass of each planet in our solar system is given in the table. Notice that each value is written as the product of a number and a power of 10. These values are written in scientific notation.



Planet	Mass (kilograms)
Mercury	3.30×10^{23}
Venus	4.87×10^{24}
Earth	5.97×10^{24}
Mars	6.42×10^{23}
Jupiter	1.90×10^{27}
Saturn	5.69×10^{26}
Uranus	8.68×10^{25}
Neptune	1.02×10^{26}
Pluto	1.27×10^{22}

Source: NASA

SCIENTIFIC NOTATION When dealing with very large or very small numbers, keeping track of place value can be difficult. For this reason, numbers such as these are often expressed in **scientific notation**.

Key Concept — *Scientific Notation*

- **Words** A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.
- **Symbols** A number in scientific notation is written as $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Study Tip

Reading Math
Standard notation is the way in which you are used to seeing a number written, where the decimal point determines the place value for each digit of the number.

The following examples show one way of expressing a number that is written in scientific notation in its decimal or standard notation. Look for a relationship between the power of 10 and the position of the decimal point in the standard notation of the number.

$$6.59 \times 10^4 = 6.59 \times 10{,}000 = 65{,}900$$

The decimal point moved 4 places to the right.

$$4.81 \times 10^{-6} = 4.81 \times \frac{1}{10^6} = 4.81 \times 0.000001 = 0.00000481$$

The decimal point moved 6 places to the left.

These examples suggest the following rule for expressing a number written in scientific notation in standard notation.

Concept Summary — Scientific to Standard Notation

Use these steps to express a number of the form $a \times 10^n$ in standard notation.

1. Determine whether $n > 0$ or $n < 0$.
2. If $n > 0$, move the decimal point in a to the right n places.
   If $n < 0$, move the decimal point in a to the left n places.
3. Add zeros, decimal point, and/or commas as needed to indicate place value.

Example 1 Scientific to Standard Notation

Express each number in standard notation.

a. $\mathbf{2.45 \times 10^8}$

$2.45 \times 10^8 = 245{,}000{,}000$ — $n = 8$; move decimal point 8 places to the right.

b. $\mathbf{3 \times 10^{-5}}$

$3 \times 10^{-5} = 0.00003$ — $n = -5$; move decimal point 5 places to the left.

To express a number in scientific notation, reverse the process used above.

Concept Summary — Standard to Scientific Notation

Use these steps to express a number in scientific notation.

1. Move the decimal point so that it is to the right of the first nonzero digit. The result is a decimal number a.
2. Observe the number of places n and the direction in which you moved the decimal point.
3. If the decimal point moved to the left, write as $a \times 10^n$.
   If the decimal point moved to the right, write as $a \times 10^{-n}$.

Example 2 Standard to Scientific Notation

Express each number in scientific notation.

a. 30,500,000

$30{,}500{,}000 \rightarrow 3.0500000 \times 10^n$ — Move decimal point 7 places to the left.

$30{,}500{,}000 = 3.05 \times 10^7$ — $a = 3.05$ and $n = 7$

b. 0.000781

$0.000781 \rightarrow 00007.81 \times 10^n$ — Move decimal point 4 places to the right.

$0.000781 = 7.81 \times 10^{-4}$ — $a = 7.81$ and $n = -4$

Study Tip

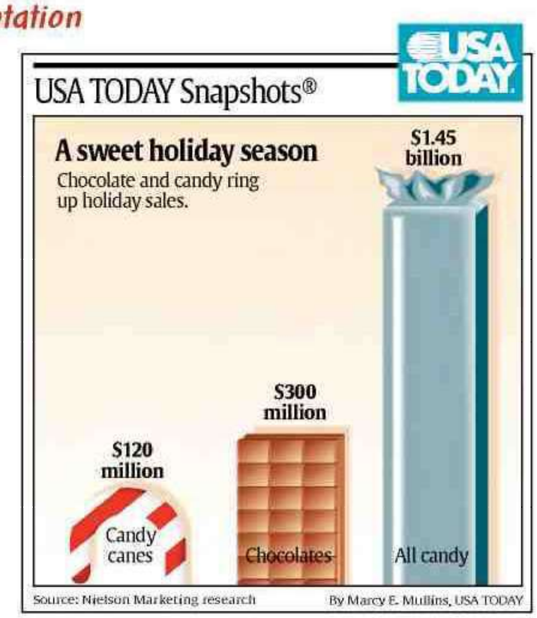
Scientific Notation
Notice that when a number is in scientific notation, no more than one digit is to the left of the decimal point.

You will often see large numbers in the media written using a combination of a number and a word, such as 3.2 million. To write this number in standard notation, rewrite the word *million* as 10^6. The exponent 6 indicates that the decimal point should be moved 6 places to the right.

$$3.2 \text{ million} = 3{,}200{,}000$$

Example 3 Use Scientific Notation

The graph shows chocolate and candy sales during a recent holiday season.



a. Express the sales of candy canes, chocolates, and all candy in standard notation.

Candy canes:
$120 million = $120,000,000

Chocolates:
$300 million = $300,000,000

All candy:
$1.45 billion = $1,450,000,000

b. Write each of these sales figures in scientific notation.

Candy canes:
$\$120{,}000{,}000 = \$1.2 \times 10^8$

Chocolates:
$\$300{,}000{,}000 = \$3.0 \times 10^8$

All candy: $\$1{,}450{,}000{,}000 = \$1.45 \times 10^9$

Log on for:
- Updated data
- More activities on scientific notation www.algebra1.com/usa_today

PRODUCTS AND QUOTIENTS WITH SCIENTIFIC NOTATION

You can use scientific notation to simplify computation with very large and/or very small numbers.

Example 4 Multiplication with Scientific Notation

Evaluate $(5 \times 10^{-8})(2.9 \times 10^2)$. Express the result in scientific and standard notation.

$(5 \times 10^{-8})(2.9 \times 10^2)$

$= (5 \times 2.9)(10^{-8} \times 10^2)$ Commutative and Associative Properties

$= 14.5 \times 10^{-6}$ Product of Powers

$= (1.45 \times 10^1) \times 10^{-6}$ $14.5 = 1.45 \times 10^1$

$= 1.45 \times (10^1 \times 10^{-6})$ Associative Property

$= 1.45 \times 10^{-5}$ or 0.0000145 Product of Powers

Example 5 Division with Scientific Notation

Evaluate $\frac{1.2789 \times 10^9}{5.22 \times 10^5}$. Express the result in scientific and standard notation.

$\frac{1.2789 \times 10^9}{5.22 \times 10^5} = \left(\frac{1.2789}{5.22}\right)\left(\frac{10^9}{10^5}\right)$ Associative Property

$= 0.245 \times 10^4$ Quotient of Powers

$= (2.45 \times 10^{-1}) \times 10^4$ $0.245 = 2.45 \times 10^{-1}$

$= 2.45 \times (10^{-1} \times 10^4)$ Associative Property

$= 2.45 \times 10^3$ or 2450 Product of Powers

Check for Understanding

Concept Check

1. **Explain** how you know to use a positive or a negative exponent when writing a number in scientific notation.
2. **State** whether 65.2×10^3 is in scientific notation. Explain your reasoning.
3. **OPEN ENDED** Give an example of a large number written using a decimal number and a word. Write this number in standard and then in scientific notation.

Guided Practice

Express each number in standard notation.

4. 2×10^{-8}
5. 4.59×10^3
6. 7.183×10^{14}
7. 3.6×10^{-5}

Express each number in scientific notation.

8. 56,700,000
9. 0.00567
10. 0.00000000004
11. 3,002,000,000,000,000

Evaluate. Express each result in scientific and standard notation.

12. $(5.3 \times 10^2)(4.1 \times 10^5)$
13. $(2 \times 10^{-5})(9.4 \times 10^{-3})$
14. $\frac{1.5 \times 10^2}{2.5 \times 10^{12}}$
15. $\frac{1.25 \times 10^4}{2.5 \times 10^{-6}}$

Application

CREDIT CARDS **For Exercises 16 and 17, use the following information.** During the year 2000, 1.65 billion credit cards were in use in the United States. During that same year, $1.54 trillion was charged to these cards. (*Hint*: 1 trillion $= 1 \times 10^{12}$) **Source:** U.S. Department of Commerce

16. Express each of these values in standard and then in scientific notation.
17. Find the average amount charged per credit card.

Practice and Apply

Homework Help

For Exercises	See Examples
18–29	1
30–43	2
44–55	3, 4
56–59	5

Extra Practice

See page 837.

Express each number in standard notation.

18. 5×10^{-6}
19. 6.1×10^{-9}
20. 7.9×10^4
21. 8×10^7
22. 1.243×10^{-7}
23. 2.99×10^{-1}
24. 4.782×10^{13}
25. 6.89×10^0

PHYSICS **Express the number in each statement in standard notation.**

26. There are 2×10^{11} stars in the Andromeda Galaxy.
27. The center of the moon is 2.389×10^5 miles away from the center of Earth.
28. The mass of a proton is 1.67265×10^{-27} kilograms.
29. The mass of an electron is 9.1095×10^{-31} kilograms.

Express each number in scientific notation.

30. 50,400,000,000
31. 34,402,000
32. 0.000002
33. 0.00090465
34. 25.8
35. 380.7
36. 622×10^6
37. 87.3×10^{11}
38. 0.5×10^{-4}
39. 0.0081×10^{-3}
40. 94×10^{-7}
41. 0.001×10^{12}

The distances of the planets from the Sun can be written in scientific notation. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

42. **STARS** In the 1930s, the Indian physicist Subrahmanyan Chandrasekhar and others predicted the existence of neutron stars. These stars can have a density of 10 billion tons per teaspoonful. Express this density in scientific notation.

43. **PHYSICAL SCIENCE** The unit of measure for counting molecules is a *mole*. One mole of a substance is the amount that contains about 602,214,299,000,000,000,000,000 molecules. Write this number in scientific notation.

Evaluate. Express each result in scientific and standard notation.

44. $(8.9 \times 10^4)(4 \times 10^3)$
45. $(3 \times 10^6)(5.7 \times 10^2)$
46. $(5 \times 10^{-2})(8.6 \times 10^{-3})$
47. $(1.2 \times 10^{-5})(1.2 \times 10^{-3})$
48. $(3.5 \times 10^7)(6.1 \times 10^{-8})$
49. $(2.8 \times 10^{-2})(9.1 \times 10^6)$
50. $\dfrac{7.2 \times 10^9}{4.8 \times 10^4}$
51. $\dfrac{7.2 \times 10^3}{1.8 \times 10^7}$
52. $\dfrac{3.162 \times 10^{-4}}{5.1 \times 10^2}$
53. $\dfrac{1.035 \times 10^{-2}}{4.5 \times 10^3}$
54. $\dfrac{2.795 \times 10^{-8}}{4.3 \times 10^{-4}}$
55. $\dfrac{4.65 \times 10^{-1}}{5 \times 10^5}$

56. **HAIR GROWTH** The usual growth rate of human hair is 3.3×10^{-4} meter per day. If an individual hair grew for 10 years, how long would it be in meters? (Assume 365 days in a year.)

57. **NATIONAL DEBT** In April 2001, the national debt was about \$5.745 trillion, and the estimated U.S. population was 283.9 million. About how much was each U.S. citizen's share of the national debt at that time?

Online Research **Data Update** What is the current U.S. population and amount of national debt? Visit www.algebra1.com/data_update to learn more.

More About. . .

Baseball

The contract Alex Rodriguez signed with the Texas Rangers on December 11, 2000, guarantees him \$25.2 million a year for 10 seasons.

Source: Associated Press

58. **BASEBALL** The table below lists the greatest yearly salary for a major league baseball player for selected years.

Baseball Salary Milestones

Year	Player	Yearly Salary
1979	Nolan Ryan	\$1 million
1982	George Foster	\$2.04 million
1990	Jose Canseco	\$4.7 million
1992	Ryne Sandberg	\$7.1 million
1996	Ken Griffey, Jr.	\$8.5 million
1997	Pedro Martinez	\$12.5 million
2000	Alex Rodriguez	\$25.2 million

Source: *USA TODAY*

About how many times as great was the yearly salary of Alex Rodriguez in 2000 as that of George Foster in 1982?

59. **ASTRONOMY** The Sun burns about 4.4×10^6 tons of hydrogen per second. How much hydrogen does the Sun burn in one year? (*Hint*: First, find the number of seconds in a year and write this number in scientific notation.)

60. **CRITICAL THINKING** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

   a. If $1 \leq a < 10$ and n and p are integers, then $(a \times 10^n)^p = a^p \times 10^{np}$.

   b. The expression $a^p \times 10^{np}$ in part **a** is in scientific notation.

61. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

Why is scientific notation important in astronomy?

Include the following in your answer:
- the mass of each of the planets in standard notation, and
- an explanation of how scientific notation makes presenting and computing with large numbers easier.

Standardized Test Practice

62. Which of the following is equivalent to 360×10^{-4}?

Ⓐ 3.6×10^3 Ⓑ 3.6×10^2 Ⓒ 3.6×10^{-2} Ⓓ 3.6×10^{-3}

63. **SHORT RESPONSE** There are an average of 25 billion red blood cells in the human body and about 270 million hemoglobin molecules in each red blood cell. Find the average number of hemoglobin molecules in the human body.

Graphing Calculator

SCIENTIFIC NOTATION You can use a graphing calculator to solve problems involving scientific notation. First, put your calculator in scientific mode. To enter 4.5×10^9, enter 4.5 [×] 10 [^] 9.

64. $(4.5 \times 10^9)(1.74 \times 10^{-2})$
65. $(7.1 \times 10^{-11})(1.2 \times 10^5)$
66. $(4.095 \times 10^5) \div (3.15 \times 10^8)$
67. $(6 \times 10^{-4}) \div (5.5 \times 10^{-7})$

Maintain Your Skills

Mixed Review

Simplify. Assume no denominator is equal to zero. *(Lesson 8-2)*

68. $\frac{49a^4b^7c^2}{7ab^4c^3}$
69. $\frac{-4n^3p^{-5}}{n^{-2}}$
70. $\frac{(8n^7)^2}{(3n^2)^{-3}}$

Determine whether each expression is a monomial. Write *yes* or *no*. *(Lesson 8-1)*

71. $3a + 4b$
72. $\frac{6}{n}$
73. $\frac{v^2}{3}$

Solve each inequality. Then check your solution and graph it on a number line. *(Lesson 6-1)*

74. $m - 3 < -17$
75. $-9 + d > 9$
76. $-x - 11 \geq 23$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression when $a = 5$, $b = -2$, and $c = 3$.** *(To review **evaluating expressions**, see Lesson 1-2.)*

77. $5b^2$
78. $c^2 - 9$
79. $b^3 + 3ac$
80. $a^2 + 2a - 1$
81. $-2b^4 - 5b^3 - b$
82. $3.2c^3 + 0.5c^2 - 5.2c$

Practice Quiz 1

Lessons 8-1 through 8-3

Simplify. *(Lesson 8-1)*

1. $n^3(n^4)(n)$
2. $4ad(3a^3d)$
3. $(-2w^3z^4)^3(-4wz^3)^2$

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

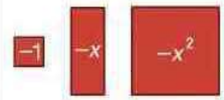
4. $\frac{25p^{10}}{15p^3}$
5. $\left(\frac{6k^3}{7np^4}\right)^2$
6. $\frac{4x^0y^2}{(3y^{-3}z^5)^{-2}}$

Evaluate. Express each result in scientific and standard notation. *(Lesson 8-3)*

7. $(6.4 \times 10^3)(7 \times 10^2)$
8. $(4 \times 10^2)(15 \times 10^{-6})$
9. $\frac{9.2 \times 10^3}{2.3 \times 10^5}$
10. $\frac{3.6 \times 10^7}{1.2 \times 10^{-2}}$

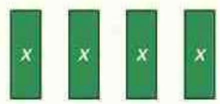
Polynomials

Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram at the right shows the models.

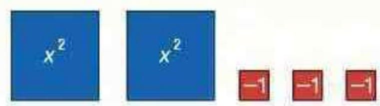
Polynomial Models	
Polynomials are modeled using three types of tiles.	
Each tile has an opposite.	

Use algebra tiles to model each polynomial.

- **$4x$**
  To model this polynomial, you will need 4 green x tiles.



- **$2x^2 - 3$**
  To model this polynomial, you will need 2 blue x^2 tiles and 3 red -1 tiles.



- **$-x^2 + 3x + 2$**
  To model this polynomial, you will need 1 red $-x^2$ tile, 3 green x tiles, and 2 yellow 1 tiles.

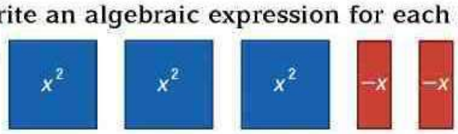
Model and Analyze

Use algebra tiles to model each polynomial. Then draw a diagram of your model.

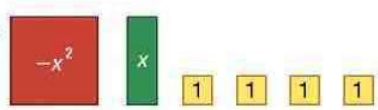
1. $-2x^2$ **2.** $5x - 4$ **3.** $3x^2 - x$ **4.** $x^2 + 4x + 3$

Write an algebraic expression for each model.

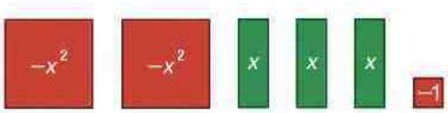
5.



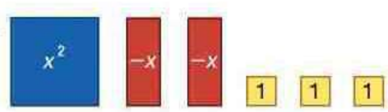
6.



7.



8.



9. MAKE A CONJECTURE Write a sentence or two explaining why algebra tiles are sometimes called *area tiles*.

8-4 Polynomials

What You'll Learn

- Find the degree of a polynomial.
- Arrange the terms of a polynomial in ascending or descending order.

Vocabulary

- polynomial
- binomial
- trinomial
- degree of a monomial
- degree of a polynomial

How are polynomials useful in modeling data?

The number of hours H spent per person per year playing video games from 1992 through 1997 is shown in the table. These data can be modeled by the equation

$$H = \frac{1}{4}(t^4 - 9t^3 + 26t^2 - 18t + 76),$$

where t is the number of years since 1992. The expression $t^4 - 9t^3 + 26t^2 - 18t + 76$ is an example of a polynomial.

Year	Hours spent per person
1992	19
1993	19
1994	22
1995	24
1996	26
1997	36

Source: U.S. Census Bureau

Study Tip

Common Misconception
Before deciding if an expression is a polynomial, write each term of the expression so that there are no variables in the denominator. Then look for negative exponents. Recall that the exponents of a monomial must be nonnegative integers.

DEGREE OF A POLYNOMIAL A **polynomial** is a monomial or a sum of monomials. Some polynomials have special names. A **binomial** is the sum of *two* monomials, and a **trinomial** is the sum of *three* monomials. Polynomials with more than three terms have no special names.

Monomial	Binomial	Trinomial
7	$3 + 4y$	$x + y + z$
$13n$	$2a + 3c$	$p^2 + 5p + 4$
$-5z^3$	$6x^2 + 3xy$	$a^2 - 2ab - b^2$
$4ab^3c^2$	$7pqr + pq^2$	$3v^2 - 2w + ab^3$

Example 1 Identify Polynomials

State whether each expression is a polynomial. If it is a polynomial, identify it as a *monomial, binomial,* or *trinomial.*

	Expression	Polynomial?	Monomial, Binomial, or Trinomial?
a.	$2x - 3yz$	Yes, $2x - 3yz = 2x + (-3yz)$. The expression is the sum of two monomials.	binomial
b.	$8n^3 + 5n^{-2}$	No. $5n^{-2} = \frac{5}{n^2}$, which is not a monomial.	none of these
c.	-8	Yes. -8 is a real number.	monomial
d.	$4a^2 + 5a + a + 9$	Yes. The expression simplifies to $4a^2 + 6a + 9$, so it is the sum of three monomials.	trinomial

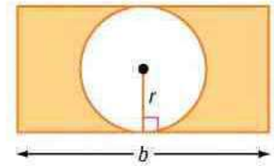
Study Tip

Like Terms
Be sure to combine any like terms before deciding if a polynomial is a monomial, binomial, or trinomial.

Polynomials can be used to express geometric relationships.

Example 2 Write a Polynomial

GEOMETRY **Write a polynomial to represent the area of the shaded region.**



Words The area of the shaded region is the area of the rectangle minus the area of the circle.

Variables area of shaded region = A
width of rectangle = $2r$
rectangle area = $b(2r)$
circle area = πr^2

Equation

area of shaded region	=	rectangle area	−	circle area
A	=	$b(2r)$	−	πr^2
A	=	$2br$	−	πr^2

The polynomial representing the area of the shaded region is $2br - \pi r^2$.

The **degree of a monomial** is the sum of the exponents of all its variables.

The **degree of a polynomial** is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term.

Monomial	Degree
$8y^4$	4
$3a$	1
$-2xy^2z^3$	$1 + 2 + 3$ or 6
7	0

Study Tip

Degrees of 1 and 0

- Since $a = a^1$, the monomial $3a$ can be rewritten as $3a^1$. Thus $3a$ has degree 1.
- Since $x^0 = 1$, the monomial 7 can be rewritten as $7x^0$. Thus 7 has degree 0.

Example 3 Degree of a Polynomial

Find the degree of each polynomial.

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	$5mn^2$	$5mn^2$	1, 2	3
b.	$-4x^2y^2 + 3x^2 + 5$	$-4x^2y^2$, $3x^2$, 5	4, 2, 0	4
c.	$3a + 7ab - 2a^2b + 16$	$3a$, $7ab$, $2a^2b$, 16	1, 2, 3, 0	3

WRITE POLYNOMIALS IN ORDER The terms of a polynomial are usually arranged so that the powers of one variable are in *ascending* (increasing) order or *descending* (decreasing) order.

Example 4 Arrange Polynomials in Ascending Order

Arrange the terms of each polynomial so that the powers of x are in ascending order.

a. $7x^2 + 2x^4 - 11$

$7x^2 + 2x^4 - 11 = 7x^2 + 2x^4 - 11x^0$ $\quad x^0 = 1$

$= -11 + 7x^2 + 2x^4$ $\quad$ Compare powers of x: $0 < 2 < 4$.

b. $2xy^3 + y^2 + 5x^3 - 3x^2y$

$2xy^3 + y^2 + 5x^3 - 3x^2y$

$= 2x^1y^3 + y^2 + 5x^3 - 3x^2y^1$ $\quad x = x^1$

$= y^2 + 2xy^3 - 3x^2y + 5x^3$ $\quad$ Compare powers of x: $0 < 1 < 2 < 3$.

Example 5 Arrange Polynomials in Descending Order

Arrange the terms of each polynomial so that the powers of x are in descending order.

a. $\mathbf{6x^2 + 5 - 8x - 2x^3}$

$$6x^2 + 5 - 8x - 2x^3 = 6x^2 + 5x^0 - 8x^1 - 2x^3 \quad x^0 = 1 \text{ and } x = x^1$$
$$= -2x^3 + 6x^2 - 8x + 5 \quad 3 > 2 > 1 > 0$$

b. $\mathbf{3a^3x^2 - a^4 + 4ax^5 + 9a^2x}$

$$3a^3x^2 - a^4 + 4ax^5 + 9a^2x = 3a^3x^2 - a^4x^0 + 4a^1x^5 + 9a^2x^1 \quad a = a^1,\ x^0 = 1, \text{ and } x = x^1$$
$$= 4ax^5 + 3a^3x^2 + 9a^2x - a^4 \quad 5 > 2 > 1 > 0$$

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of a monomial of degree zero.
2. **Explain** why a polynomial cannot contain a variable raised to a negative power.
3. **Determine** whether each statement is *true* or *false*. If false, give a counterexample.
   a. All binomials are polynomials.
   b. All polynomials are monomials.
   c. All monomials are polynomials.

Guided Practice

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

4. $5x - 3xy + 2x$
5. $\frac{2z}{5}$
6. $9a^2 + 7a - 5$

Find the degree of each polynomial.

7. 1
8. $3x + 2$
9. $2x^2y^3 + 6x^4$

Arrange the terms of each polynomial so that the powers of x are in ascending order.

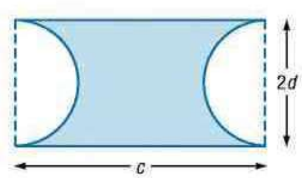
10. $6x^3 - 12 + 5x$
11. $-7a^2x^3 + 4x^2 - 2ax^5 + 2a$

Arrange the terms of each polynomial so that the powers of x are in descending order.

12. $2c^5 + 9cx^2 + 3x$
13. $y^3 + x^3 + 3x^2y + 3xy^2$

Application

14. **GEOMETRY** Write a polynomial to represent the area of the shaded region.



Practice and Apply

Homework Help

For Exercises	See Examples
15–20	1
21–24	2
25–36	3
37–52	4, 5

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

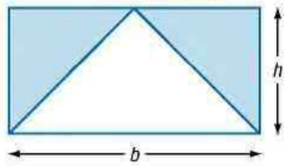
15. 14
16. $\frac{6m^2}{p} + p^3$
17. $7b - 3.2c + 8b$
18. $\frac{1}{3}x^2 + x - 2$
19. $6gh^2 - 4g^2h + g$
20. $-4 + 2a + \frac{5}{a^2}$

Extra Practice

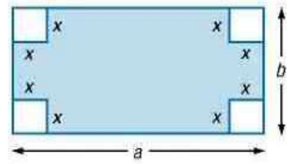
See page 838.

GEOMETRY **Write a polynomial to represent the area of each shaded region.**

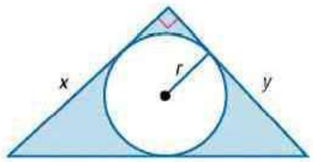
21.



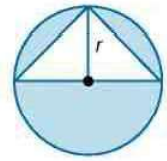
22.



23.



24.



Find the degree of each polynomial.

25. $5x^3$
26. $9y$
27. $4ab$
28. -13
29. $c^4 + 7c^2$
30. $6n^3 - n^2p^2$
31. $15 - 8ag$
32. $3a^2b^3c^4 - 18a^5c$
33. $2x^3 - 4y + 7xy$
34. $3z^5 - 2x^2y^3z - 4x^2z$
35. $7 + d^5 - b^2c^2d^3 + b^6$
36. $11r^2t^4 - 2s^4t^5 + 24$

Arrange the terms of each polynomial so that the powers of *x* are in ascending order.

37. $2x + 3x^2 - 1$
38. $9x^3 + 7 - 3x^5$
39. $c^2x^3 - c^3x^2 + 8c$
40. $x^3 + 4a + 5a^2x^6$
41. $4 + 3ax^5 + 2ax^2 - 5a^7$
42. $10x^3y^2 - 3x^9y + 5y^4 + 2x^2$
43. $3xy^2 - 4x^3 + x^2y + 6y$
44. $-8a^5x + 2ax^4 - 5 - a^2x^2$

Arrange the terms of each polynomial so that the powers of *x* are in descending order.

45. $5 + x^5 + 3x^3$
46. $2x - 1 + 6x^2$
47. $4a^3x^2 - 5a + 2a^2x^3$
48. $b^2 + x^2 - 2xb$
49. $c^2 + cx^3 - 5c^3x^2 + 11x$
50. $9x^2 + 3 + 4ax^3 - 2a^2x$
51. $8x - 9x^2y + 7y^2 - 2x^4$
52. $4x^3y + 3xy^4 - x^2y^3 + y^4$

53. **MONEY** Write a polynomial to represent the value of q quarters, d dimes, and n nickels.

54. **MULTIPLE BIRTHS** The number of quadruplet births Q in the United States from 1989 to 1998 can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where t represents the number of years since 1989. For what values of t does this model no longer give realistic data? Explain your reasoning.

More About. . .

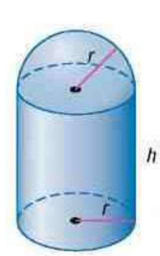
Multiple Births

From 1980 to 1997, the number of triplet and higher births rose 404% (from 1377 to 6737 births). This steep climb in multiple births coincides with the increased use of fertility drugs.

Source: National Center for Health and Statistics

PACKAGING **For Exercises 55 and 56, use the following information.**

A convenience store sells milkshakes in cups with semispherical lids. The volume of a cylinder is the product of π, the square of the radius r, and the height h. The volume of a sphere is the product of $\frac{4}{3}$, π, and the cube of the radius.



55. Write a polynomial that represents the volume of the container.

56. If the height of the container is 6 inches and the radius is 2 inches, find the volume of the container.

57. **CRITICAL THINKING** Tell whether the following statement is *true* or *false*. Explain your reasoning.

The degree of a binomial can never be zero.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are polynomials useful in modeling data?

Include the following in your answer:

- a discussion of the accuracy of the equation by evaluating the polynomial for $t = \{0, 1, 2, 3, 4, 5\}$, and
- an example of how and why someone might use this equation.

Standardized Test Practice

59. If $x = -1$, then $3x^3 + 2x^2 + x + 1 =$

Ⓐ -5. Ⓑ -1. Ⓒ 1. Ⓓ 2.

60. **QUANTITATIVE COMPARISON** **Compare the quantity in Column A and the quantity in Column B. Then determine whether:**

Ⓐ **the quantity in Column A is greater,**
Ⓑ **the quantity in Column B is greater,**
Ⓒ **the two quantities are equal, or**
Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B
the degree of $5x^2y^3$	the degree of $3x^3y^2$

Maintain Your Skills

Mixed Review **Express each number in scientific notation.** *(Lesson 8-3)*

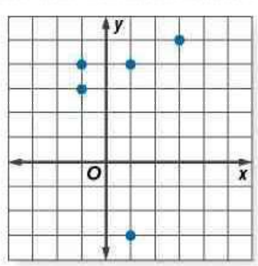
61. 12,300,000 62. 0.00345 63. 12×10^6 64. 0.77×10^{-10}

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

65. $a^0b^{-2}c^{-1}$ 66. $\frac{-5n^5}{n^8}$ 67. $\left(\frac{4x^3y^2}{3z}\right)^2$ 68. $\frac{(-y)^5m^8}{y^3m^{-7}}$

Determine whether each relation is a function. *(Lesson 4-6)*

69.



70.

x	y
−2	−2
0	1
3	4
5	−2

71. **PROBABILITY** A card is selected at random from a standard deck of 52 cards. What is the probability of selecting a black card? *(Lesson 2-6)*

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Simplify each expression. If not possible, write *simplified*.**

*(To review **evaluating expressions**, see Lesson 1-5.)*

72. $3n + 5n$ 73. $9a^2 + 3a - 2a^2$ 74. $12x^2 + 8x - 6$

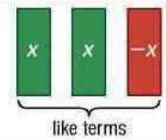
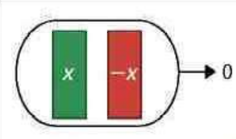
75. $-3a + 5b + 4a - 7b$ 76. $4x + 3y - 6 + 7x + 8 - 10y$

Algebra Activity

A Preview of Lesson 8-5

Adding and Subtracting Polynomials

Monomials such as $5x$ and $-3x$ are called *like terms* because they have the same variable to the same power. When you use algebra tiles, you can recognize like terms because the individual tiles have the same size and shape.

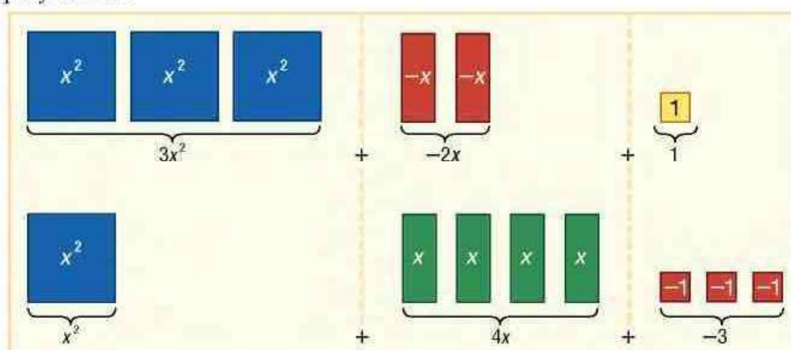
Polynomial Models	
Like terms are represented by tiles that have the same shape and size.	
A *zero pair* may be formed by pairing one tile with its opposite. You can remove or add zero pairs without changing the polynomial.	

Activity 1 Use algebra tiles to find $(3x^2 - 2x + 1) + (x^2 + 4x - 3)$.

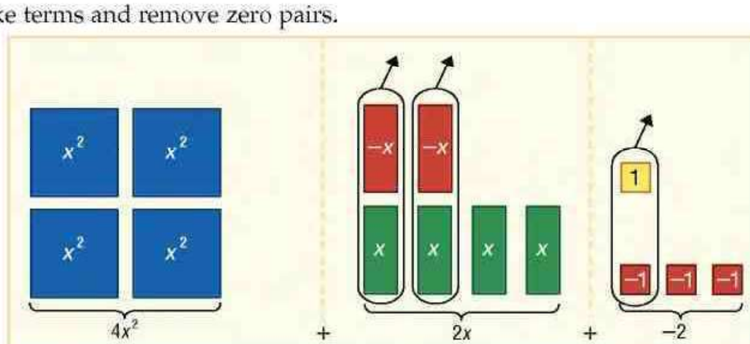
Step 1 Model each polynomial.

$3x^2 - 2x + 1 \rightarrow$

$x^2 + 4x - 3 \rightarrow$



Step 2 Combine like terms and remove zero pairs.



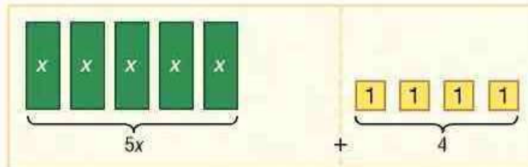
Step 3 Write the polynomial for the tiles that remain.

$(3x^2 - 2x + 1) + (x^2 + 4x - 3) = 4x^2 + 2x - 2$

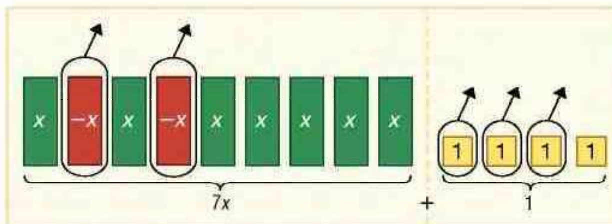
Algebra Activity

Activity 2 **Use algebra tiles to find $(5x + 4) - (-2x + 3)$.**

Step 1 Model the polynomial $5x + 4$.



Step 2 To subtract $-2x + 3$, you must remove 2 red $-x$ tiles and 3 yellow 1 tiles. You can remove the yellow 1 tiles, but there are no red $-x$ tiles. Add 2 zero pairs of x tiles. Then remove the 2 red $-x$ tiles.



Step 3 Write the polynomial for the tiles that remain.
$(5x + 4) - (-2x + 3) = 7x + 1$

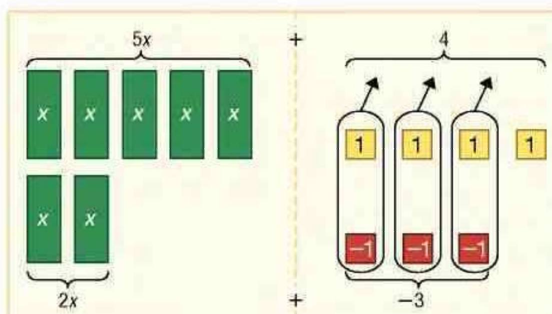
Recall that you can subtract a number by adding its additive inverse or opposite. Similarly, you can subtract a polynomial by adding its opposite.

Activity 3 **Use algebra tiles and the additive inverse, or opposite, to find $(5x + 4) - (-2x + 3)$.**

Step 1 To find the difference of $5x + 4$ and $-2x + 3$, add $5x + 4$ and the opposite of $-2x + 3$.

$5x + 4 \rightarrow$

The opposite of $-2x + 3$ is $2x - 3$. $\rightarrow$



Step 2 Write the polynomial for the tiles that remain.
$(5x + 4) - (-2x + 3) = 7x + 1$ Notice that this is the same answer as in Activity 2.

Model and Analyze

Use algebra tiles to find each sum or difference.

1. $(5x^2 + 3x - 4) + (2x^2 - 4x + 1)$
2. $(2x^2 + 5) + (3x^2 - 2x + 6)$
3. $(-4x^2 + x) + (5x - 2)$
4. $(3x^2 + 4x + 2) - (x^2 - 5x - 5)$
5. $(-x^2 + 7x) - (2x^2 + 3x)$
6. $(8x + 4) - (6x^2 + x - 3)$
7. Find $(2x^2 - 3x + 1) - (2x + 3)$ using each method from Activity 2 and Activity 3. Illustrate with drawings and explain in writing how zero pairs are used in each case.

8-5 Adding and Subtracting Polynomials

What You'll Learn

- Add polynomials.
- Subtract polynomials.

How can adding polynomials help you model sales?

From 1996 to 1999, the amount of sales (in billions of dollars) of video games V and traditional toys R in the United States can be modeled by the following equations, where t is the number of years since 1996.

Source: *Toy Industry Fact Book*

$$V = -0.05t^3 + 0.05t^2 + 1.4t + 3.6$$
$$R = 0.5t^3 - 1.9t^2 + 3t + 19$$

The total toy sales T is the sum of the video game sales V and traditional toy sales R.

ADD POLYNOMIALS To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms.

Example 1 Add Polynomials

Find $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$.

Method 1 Horizontal

Group like terms together.

$(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$

$= [3x^2 + (-7x^2)] + (-4x + 2x) + [8 + (-5)]$ Associative and Commutative Properties

$= -4x^2 - 2x + 3$ Add like terms.

Method 2 Vertical

Align the like terms in columns and add.

$$\begin{array}{r} 3x^2 - 4x + 8 \\ (+)\ -7x^2 + 2x - 5 \\ \hline -4x^2 - 2x + 3 \end{array}$$

Notice that terms are in descending order with like terms aligned.

Study Tip

Adding Columns

When adding like terms in column form, remember that you are adding integers. Rewrite each monomial to eliminate subtractions. For example, you could rewrite $3x^2 - 4x + 8$ as $3x^2 + (-4x) + 8$.

SUBTRACT POLYNOMIALS Recall that you can subtract a rational number by adding its opposite or additive inverse. Similarly, you can subtract a polynomial by adding its additive inverse.

Polynomial	Additive Inverse
$-5m + 3n$	$5m - 3n$
$2y^2 - 6y + 11$	$-2y^2 + 6y - 11$
$7a + 9b - 4$	$-7a - 9b + 4$

To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

www.algebra1.com/extra_examples

Example 2 Subtract Polynomials

Find $(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$.

Method 1 Horizontal

Subtract $7n + 4n^3$ by adding its additive inverse.

$(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$

$= (3n^2 + 13n^3 + 5n) + (-7n - 4n^3)$ — The additive inverse of $7n + 4n^3$ is $-7n - 4n^3$.

$= 3n^2 + [13n^3 + (-4n^3)] + [5n + (-7n)]$ — Group like terms.

$= 3n^2 + 9n^3 - 2n$ — Add like terms.

Method 2 Vertical

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 3n^2 + 13n^3 + 5n \\ \underline{(-)} & \underline{4n^3 + 7n} \end{array} \quad \text{Add the opposite.} \quad \begin{array}{rr} & 3n^2 + 13n^3 + 5n \\ \underline{(+)} & \underline{-4n^3 - 7n} \\ & 3n^2 + 9n^3 - 2n \end{array}$$

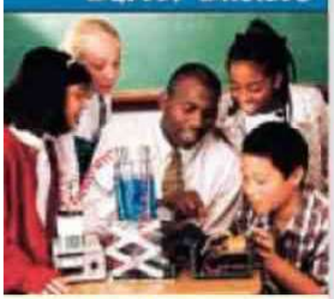
Thus, $(3n^2 + 13n^3 + 5n) - (7n + 4n^3) = 3n^2 + 9n^3 - 2n$ or, arranged in descending order, $9n^3 + 3n^2 - 2n$.

Study Tip

Inverse of a Polynomial
When finding the additive inverse of a polynomial, remember to find the additive inverse of *every* term.

When polynomials are used to model real-world data, their sums and differences can have real-world meaning too.

Career Choices



Teacher

The educational requirements for a teaching license vary by state. In 1999, the average public K–12 teacher salary was $40,582.

Online Research For information about a career as a teacher, visit: www.algebra1.com/careers

Example 3 Subtract Polynomials

EDUCATION The total number of public school teachers T consists of two groups, elementary E and secondary S. From 1985 through 1998, the number (in thousands) of secondary teachers and total teachers in the United States could be modeled by the following equations, where n is the number of years since 1985.

$$S = 11n + 942$$
$$T = 44n + 2216$$

a. Find an equation that models the number of elementary teachers E for this time period.

You can find a model for E by subtracting the polynomial for S from the polynomial for T.

Total	$44n + 2216$		$44n + 2216$
$-$ Secondary	$(-)\ 11n + 942$	Add the opposite.	$(+)\ -11n - 942$
Elementary			$33n + 1274$

An equation is $E = 33n + 1274$.

b. Use the equation to predict the number of elementary teachers in the year 2010.

The year 2010 is $2010 - 1985$ or 25 years after the year 1985.

If this trend continues, the number of elementary teachers in 2010 would be $33(25) + 1274$ thousand or about 2,099,000.

Check for Understanding

Concept Check

1. **Explain** why $5xy^2$ and $3x^2y$ are *not* like terms.
2. **OPEN ENDED** Write two polynomials whose difference is $2x^2 + x + 3$.
3. **FIND THE ERROR** Esteban and Kendra are finding $(5a - 6b) - (2a + 5b)$.

Esteban

$(5a - 6b) - (2a + 5b)$
$= (-5a + 6b) + (-2a - 5b)$
$= -7a + b$

Kendra

$(5a - 6b) - (2a + 5b)$
$= (5a - 6b) + (-2a - 5b)$
$= 3a - 11b$

Who is correct? Explain your reasoning.

Guided Practice

Find each sum or difference.

4. $(4p^2 + 5p) + (-2p^2 + p)$
5. $(5y^2 - 3y + 8) + (4y^2 - 9)$
6. $(8cd - 3d + 4c) + (-6 + 2cd)$
7. $(6a^2 + 7a - 9) - (-5a^2 + a - 10)$
8. $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$
9. $(3ax^2 - 5x - 3a) - (6a - 8a^2x + 4x)$

Application

POPULATION **For Exercises 10 and 11, use the following information.**
From 1990 through 1999, the female population F and the male population M of the United States (in thousands) is modeled by the following equations, where n is the number of years since 1990. **Source:** U.S. Census Bureau

$$F = 1247n + 126{,}971 \qquad M = 1252n + 120{,}741$$

10. Find an equation that models the total population T in thousands of the United States for this time period.
11. If this trend continues, what will the population of the United States be in 2010?

Practice and Apply

Homework Help

For Exercises	See Examples
12–31	1, 2
32, 33	3

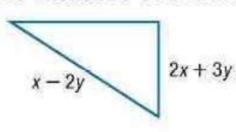
Extra Practice
See page 838.

Find each sum or difference.

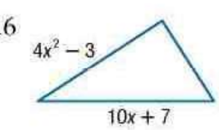
12. $(6n^2 - 4) + (-2n^2 + 9)$
13. $(9z - 3z^2) + (4z - 7z^2)$
14. $(3 + a^2 + 2a) + (a^2 - 8a + 5)$
15. $(-3n^2 - 8 + 2n) + (5n + 13 + n^2)$
16. $(x + 5) + (2y + 4x - 2)$
17. $(2b^3 - 4b + b^2) + (-9b^2 + 3b^3)$
18. $(11 + 4d^2) - (3 - 6d^2)$
19. $(4g^3 - 5g) - (2g^3 + 4g)$
20. $(-4y^3 - y + 10) - (4y^3 + 3y^2 - 7)$
21. $(4x + 5xy + 3y) - (3y + 6x + 8xy)$
22. $(3x^2 + 8x + 4) - (5x^2 - 4)$
23. $(5ab^2 + 3ab) - (2ab^2 + 4 - 8ab)$
24. $(x^3 - 7x + 4x^2 - 2) - (2x^2 - 9x + 4)$
25. $(5x^2 + 3a^2 - 5x) - (2x^2 - 5ax + 7x)$
26. $(3a + 2b - 7c) + (6b - 4a + 9c) + (-7c - 3a - 2b)$
27. $(5x^2 - 3) + (x^2 - x + 11) + (2x^2 - 5x + 7)$
28. $(3y^2 - 8) + (5y + 9) - (y^2 + 6y - 4)$
29. $(9x^3 + 3x - 13) - (6x^2 - 5x) + (2x^3 - x^2 - 8x + 4)$

GEOMETRY **The measures of two sides of a triangle are given. If P is the perimeter, find the measure of the third side.**

30. $P = 7x + 3y$



31. $P = 10x^2 - 5x + 16$



More About. . .

Movies

In 1998, attendance at movie theaters was at its highest point in 40 years with 1.48 billion tickets sold for a record \$6.95 billion in gross income.

Source: The National Association of Theatre Owners

MOVIES **For Exercises 32 and 33, use the following information.**
From 1990 to 1999, the number of indoor movie screens I and total movie screens T in the U.S. could be modeled by the following equations, where n is the number of years since 1990.

$$I = 161.6n^2 - 20n + 23{,}326 \qquad T = 160.3n^2 - 26n + 24{,}226$$

32. Find an equation that models the number of outdoor movie screens D in the U.S. for this time period.

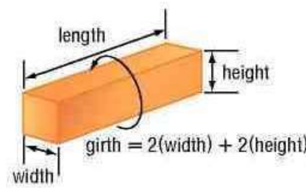
33. If this trend continues, how many outdoor movie screens will there be in the year 2010?

NUMBER TRICK **For Exercises 34 and 35, use the following information.**
Think of a two-digit number whose ones digit is greater than its tens digit. Multiply the difference of the two digits by 9 and add the result to your original number. Repeat this process for several other such numbers.

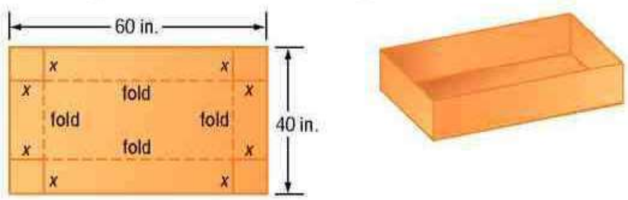
34. What observation can you make about your results?

35. Justify that your observation holds for all such two-digit numbers by letting x equal the tens digit and y equal the ones digit of the original number. (*Hint*: The original number is then represented by $10x + y$.)

POSTAL SERVICE **For Exercises 36–40, use the information below and in the figure at the right.**
The U.S. Postal Service restricts the sizes of boxes shipped by parcel post. The sum of the length and the girth of the box must not exceed 108 inches.



Suppose you want to make an open box using a 60-by-40 inch piece of cardboard by cutting squares out of each corner and folding up the flaps. The lid will be made from another piece of cardboard. You do not know how big the squares should be, so for now call the length of the side of each square x.



36. Write a polynomial to represent the length of the box formed.

37. Write a polynomial to represent the width of the box formed.

38. Write a polynomial to represent the girth of the box formed.

39. Write and solve an inequality to find the least possible value of x you could use in designing this box so it meets postal regulations.

40. What is the greatest integral value of x you could use to design this box if it does not have to meet regulations?

CRITICAL THINKING **For Exercises 41–43, suppose x is an integer.**

41. Write an expression for the next integer greater than x.

42. Show that the sum of two consecutive integers, x and the next integer after x, is always odd. (*Hint*: A number is considered even if it is divisible by 2.)

43. What is the least number of consecutive integers that must be added together to always arrive at an even integer?

44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can adding polynomials help you model sales?

Include the following in your answer:

- an equation that models total toy sales, and
- an example of how and why someone might use this equation.

Standardized Test Practice

45. The perimeter of the rectangle shown at the right is $16a + 2b$. Which of the following expressions represents the length of the rectangle?

Ⓐ $3a + 2b$ Ⓑ $10a + 2b$

Ⓒ $2a - 3b$ Ⓓ $6a + 4b$

46. If $a^2 - 2ab + b^2 = 36$ and $a^2 - 3ab + b^2 = 22$, find ab.

Ⓐ 6 Ⓑ 8 Ⓒ 12 Ⓓ 14

Maintain Your Skills

Mixed Review

Find the degree of each polynomial. *(Lesson 8-4)*

47. $15t^3y^2$ **48.** 24 **49.** $m^2 + n^3$ **50.** $4x^2y^3z - 5x^3z$

Express each number in standard notation. *(Lesson 8-3)*

51. 8×10^6 **52.** 2.9×10^5 **53.** 5×10^{-4} **54.** 4.8×10^{-7}

KEYBOARDING **For Exercises 55–59, use the table below that shows the keyboarding speeds and experience of 12 students.** *(Lesson 5-2)*

Experience (weeks)	4	7	8	1	6	3	5	2	9	6	7	10
Keyboarding Speed (wpm)	33	45	46	20	40	30	38	22	52	44	42	55

55. Make a scatter plot of these data.

56. Draw a best-fit line for the data.

57. Find the equation of the line.

58. Use the equation to predict the keyboarding speed of a student after a 12-week course.

59. Can this equation be used to predict the speed for any number of weeks of experience? Explain.

State the domain and range of each relation. *(Lesson 4-3)*

60. $\{(-2, 5), (0, -2), (-6, 3)\}$ **61.** $\{(-4, 2), (-1, -3), (5, 0), (-4, 1)\}$

62. **MODEL TRAINS** One of the most popular sizes of model trains is called the HO. Every dimension of the HO model measures $\frac{1}{87}$ times that of a real engine. The HO model of a modern diesel locomotive is about 8 inches long. About how many feet long is the real locomotive? *(Lesson 3-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review the **Distributive Property**, see Lesson 1-7.)*

63. $6(3x - 8)$ **64.** $-2(b + 9)$ **65.** $-7(-5p + 4q)$

66. $9(3a + 5b - c)$ **67.** $8(x^2 + 3x - 4)$ **68.** $-3(2a^2 - 5a + 7)$

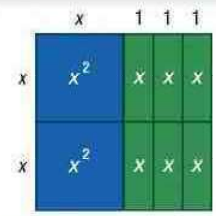
8-6 Multiplying a Polynomial by a Monomial

What You'll Learn

- Find the product of a monomial and a polynomial.
- Solve equations involving polynomials.

How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?

The algebra tiles shown are grouped together to form a rectangle with a width of $2x$ and a length of $x + 3$. Notice that the rectangle consists of 2 blue x^2 tiles and 6 green x tiles. The area of the rectangle is the sum of these algebra tiles or $2x^2 + 6x$.



PRODUCT OF MONOMIAL AND POLYNOMIAL The Distributive Property can be used to multiply a polynomial by a monomial.

Study Tip

Look Back
To review the **Distributive Property**, see Lesson 1-5.

Example 1 Multiply a Polynomial by a Monomial

Find $-2x^2(3x^2 - 7x + 10)$.

Method 1 Horizontal

$-2x^2(3x^2 - 7x + 10)$

$= -2x^2(3x^2) - (-2x^2)(7x) + (-2x^2)(10)$ Distributive Property

$= -6x^4 - (-14x^3) + (-20x^2)$ Multiply.

$= -6x^4 + 14x^3 - 20x^2$ Simplify.

Method 2 Vertical

$$\begin{array}{r} 3x^2 - 7x + 10 \\ (\times) \qquad\qquad -2x^2 \\ \hline -6x^4 + 14x^3 - 20x^2 \end{array}$$

Distributive Property

Multiply.

When expressions contain like terms, simplify by combining the like terms.

Example 2 Simplify Expressions

Simplify $4(3d^2 + 5d) - d(d^2 - 7d + 12)$.

$4(3d^2 + 5d) - d(d^2 - 7d + 12)$

$= 4(3d^2) + 4(5d) + (-d)(d^2) - (-d)(7d) + (-d)(12)$ Distributive Property

$= 12d^2 + 20d + (-d^3) - (-7d^2) + (-12d)$ Product of Powers

$= 12d^2 + 20d - d^3 + 7d^2 - 12d$ Simplify.

$= -d^3 + (12d^2 + 7d^2) + (20d - 12d)$ Commutative and Associative Properties

$= -d^3 + 19d^2 + 8d$ Combine like terms.

Example 3 Use Polynomial Models

PHONE SERVICE **Greg pays a fee of \$20 a month for local calls. Long-distance rates are 6¢ per minute for in-state calls and 5¢ per minute for out-of-state calls. Suppose Greg makes 300 minutes of long-distance phone calls in January and m of those minutes are for in-state calls.**

a. Find an expression for Greg's phone bill for January.

Words The bill is the sum of the monthly fee, in-state charges, and the out-of-state charges.

Variables If m = number of minutes of in-state calls, then $300 - m$ = number of minutes of out-of-state calls. Let B = phone bill for the month of January.

Equation

$$\overbrace{B}^{\text{bill}} \overbrace{=}^{=} \overbrace{20}^{\text{service fee}} \overbrace{+}^{+} \overbrace{m}^{\text{in-state minutes}} \overbrace{\cdot}^{\cdot} \overbrace{0.06}^{6¢\text{ per minute}} \overbrace{+}^{+} \overbrace{(300 - m)}^{\text{out-of-state minutes}} \overbrace{\cdot}^{\cdot} \overbrace{0.05}^{5¢\text{ per minute}}$$

$= 20 + 0.06m + 300(0.05) - m(0.05)$ Distributive Property

$= 20 + 0.06m + 15 - 0.05m$ Simplify.

$= 35 + 0.01m$ Simplify.

An expression for Greg's phone bill for January is $35 + 0.01m$, where m is the number of minutes of in-state calls.

b. Evaluate the expression to find the cost if Greg had 37 minutes of in-state calls in January.

$35 + 0.01m = 35 + 0.01(37)$ $m = 37$

$= 35 + 0.37$ Multiply.

$= \$35.37$ Add.

Greg's bill was \$35.37.

More About...

Phone Service

About 98% of long-distance companies service their calls using the network of one of three companies. Since the quality of phone service is basically the same, a company's rates are the primary factor in choosing a long-distance provider.

Source: Chamberland Enterprises

SOLVE EQUATIONS WITH POLYNOMIAL EXPRESSIONS Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example 4 Polynomials on Both Sides

Solve $y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$.

$y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$	Original equation
$y^2 - 12y + y^2 + 2y + 25 = 2y^2 + 10y - 15$	Distributive Property
$2y^2 - 10y + 25 = 2y^2 + 10y - 15$	Combine like terms.
$-10y + 25 = 10y - 15$	Subtract $2y^2$ from each side.
$-20y + 25 = -15$	Subtract $10y$ from each side.
$-20y = -40$	Subtract 25 from each side.
$y = 2$	Divide each side by -20.

The solution is 2.

CHECK

$y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$	Original equation
$2(2 - 12) + 2(2 + 2) + 25 \stackrel{?}{=} 2(2)(2 + 5) - 15$	$y = 2$
$2(-10) + 2(4) + 25 \stackrel{?}{=} 4(7) - 15$	Simplify.
$-20 + 8 + 25 \stackrel{?}{=} 28 - 15$	Multiply.
$13 = 13$ ✓	Add and subtract.

Check for Understanding

Concept Check

1. **State** the property used in each step to multiply $2x(4x^2 + 3x - 5)$.

$$\begin{aligned} 2x(4x^2 + 3x - 5) &= 2x(4x^2) + 2x(3x) - 2x(5) && \underline{\ ?\ } \\ &= 8x^{1+2} + 6x^{1+1} - 10x && \underline{\ ?\ } \\ &= 8x^3 + 6x^2 - 10x && \text{Simplify.} \end{aligned}$$

2. **Compare and contrast** the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.
3. **OPEN ENDED** Write a monomial and a trinomial involving a single variable. Then find their product.

Guided Practice

Find each product.

4. $-3y(5y + 2)$
5. $9b^2(2b^3 - 3b^2 + b - 8)$
6. $2x(4a^4 - 3ax + 6x^2)$
7. $-4xy(5x^2 - 12xy + 7y^2)$

Simplify.

8. $t(5t - 9) - 2t$
9. $5n(4n^3 + 6n^2 - 2n + 3) - 4(n^2 + 7n)$

Solve each equation.

10. $-2(w + 1) + w = 7 - 4w$
11. $x(x + 2) - 3x = x(x - 4) + 5$

Application

SAVINGS For Exercises 12–14, use the following information.
Kenzie's grandmother left her $10,000 for college. Kenzie puts some of the money into a savings account earning 4% per year, and with the rest, she buys a certificate of deposit (CD) earning 7% per year.

12. If Kenzie puts x dollars into the savings account, write an expression to represent the amount of the CD.
13. Write an equation for the total amount of money T Kenzie will have saved for college after one year.
14. If Kenzie puts $3000 in savings, how much money will she have after one year?

Practice and Apply

Homework Help

For Exercises	See Examples
15–28	1
29–38	2
39–48	4
49–54, 58–62	3

Extra Practice
See page 838.

Find each product.

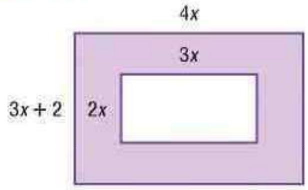
15. $r(5r + r^2)$
16. $w(2w^3 - 9w^2)$
17. $-4x(8 + 3x)$
18. $5y(-2y^2 - 7y)$
19. $7ag(g^3 + 2ag)$
20. $-3np(n^2 - 2p)$
21. $-2b^2(3b^2 - 4b + 9)$
22. $6x^3(5 + 3x - 11x^2)$
23. $8x^2y(5x + 2y^2 - 3)$
24. $-cd^2(3d + 2c^2d - 4c)$
25. $-\frac{3}{4}hk^2(20k^2 + 5h - 8)$
26. $\frac{2}{3}a^2b(6a^3 - 4ab + 9b^2)$
27. $-5a^3b(2b + 5ab - b^2 + a^3)$
28. $4p^2q^2(2p^2 - q^2 + 9p^3 + 3q)$

Simplify.

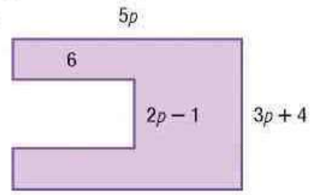
29. $d(-2d + 4) + 15d$
30. $-x(4x^2 - 2x) - 5x^3$
31. $3w(6w - 4) + 2(w^2 - 3w + 5)$
32. $5n(2n^3 + n^2 + 8) + n(4 - n)$
33. $10(4m^3 - 3m + 2) - 2m(-3m^2 - 7m + 1)$
34. $4y(y^2 - 8y + 6) - 3(2y^3 - 5y^2 + 2)$
35. $-3c^2(2c + 7) + 4c(3c^2 - c + 5) + 2(c^2 - 4)$
36. $4x^2(x + 2) + 3x(5x^2 + 2x - 6) - 5(3x^2 - 4x)$

GEOMETRY **Find the area of each shaded region in simplest form.**

37.



38.



Solve each equation.

39. $2(4x - 7) = 5(-2x - 9) - 5$

40. $2(5a - 12) = -6(2a - 3) + 2$

41. $4(3p + 9) - 5 = -3(12p - 5)$

42. $7(8w - 3) + 13 = 2(6w + 7)$

43. $d(d - 1) + 4d = d(d - 8)$

44. $c(c + 3) - c(c - 4) = 9c - 16$

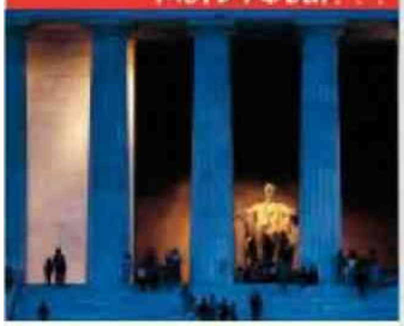
45. $y(y + 12) - 8y = 14 + y(y - 4)$

46. $k(k - 7) + 10 = 2k + k(k + 6)$

47. $2n(n + 4) + 18 = n(n + 5) + n(n - 2) - 7$

48. $3g(g - 4) - 2g(g - 7) = g(g + 6) - 28$

More About. . .



Class Trip

Inside the Lincoln Memorial is a 19-foot marble statue of the United States' 16th president. The statue is flanked on either side by the inscriptions of Lincoln's Second Inaugural Address and Gettysburg Address.

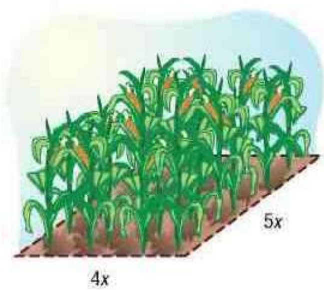
Source: www.washington.org

SAVINGS **For Exercises 49 and 50, use the following information.**
Marta has \$6000 to invest. She puts x dollars of this money into a savings account that earns 3% per year, and with the rest, she buys a certificate of deposit that earns 6% per year.

49. Write an equation for the total amount of money T Marta will have after one year.

50. Suppose at the end of one year, Marta has a total of \$6315. How much money did Marta invest in each account?

51. **GARDENING** A gardener plants corn in a garden with a length-to-width ratio of 5:4. Next year, he plans to increase the garden's area by increasing its length by 12 feet. Write an expression for this new area.



52. **CLASS TRIP** Mr. Smith's American History class will take taxis from their hotel in Washington, D.C., to the Lincoln Memorial. The fare is \$2.75 for the first mile and \$1.25 for each additional mile. If the distance is m miles and t taxis are needed, write an expression for the cost to transport the group.

NUMBER THEORY **For Exercises 53 and 54, let x be an odd integer.**

53. Write an expression for the next odd integer.

54. Find the product of x and the next odd integer.

CRITICAL THINKING **For Exercises 55–57, use the following information.**
An even number can be represented by $2x$, where x is any integer.

55. Show that the product of two even integers is always even.

56. Write a representation for an odd integer.

57. Show that the product of an even and an odd integer is always even.

VOLUNTEERING **For Exercises 58 and 59, use the following information.**
Laura is making baskets of apples and oranges for homeless shelters. She wants to place a total of 10 pieces of fruit in each basket. Apples cost 25¢ each, and oranges cost 20¢ each.

58. If a represents the number of apples Laura uses, write a polynomial model in simplest form for the total amount of money T Laura will spend on the fruit for each basket.

59. If Laura uses 4 apples in each basket, find the total cost for fruit.

SALES **For Exercises 60 and 61, use the following information.**
A store advertises that all sports equipment is 30% off the retail price. In addition, the store asks customers to select and pop a balloon to receive a coupon for an additional n percent off the already marked down price of one of their purchases.

60. Write an expression for the cost of a pair of inline skates with retail price p after receiving both discounts.

61. Use this expression to calculate the cost, not including sales tax, of a $200 pair of inline skates for an additional 10 percent off.

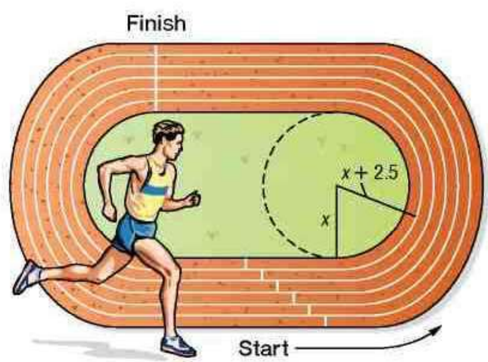
More About. . .

Volunteering

Approximately one third of young people in grades 7–12 suggested that "working for the good of my community and country" and "helping others or volunteering" were important future goals.

Source: Primedia/Roper National Youth Opinion Survey

62. **SPORTS** You may have noticed that when runners race around a curved track, their starting points are staggered. This is so each contestant runs the same distance to the finish line.



If the radius of the inside lane is x and each lane is 2.5 feet wide, how far apart should the officials start the runners in the two inside lanes? (*Hint*: Circumference of a circle: $C = 2\pi r$, where r is the radius of the circle)

63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?

Include the following in your answer:
- the product of $2x$ and $x + 3$ derived algebraically, and
- a representation of another product of a monomial and a polynomial using algebra tiles and multiplication.

Standardized Test Practice

64. Simplify $[(3x^2 - 2x + 4) - (x^2 + 5x - 2)](x + 2)$.

Ⓐ $2x^3 + 7x^2 + 8x + 4$ Ⓑ $2x^3 - 3x^2 - 8x + 12$
Ⓒ $4x^3 + 11x^2 + 8x + 4$ Ⓓ $-4x^3 - 11x^2 - 8x - 4$

65. A plumber charges $70 for the first thirty minutes of each house call plus $4 for each additional minute that she works. The plumber charges Ke-Min $122 for her time. What amount of time, in minutes, did the plumber work?

Ⓐ 43 Ⓑ 48 Ⓒ 58 Ⓓ 64

CONTENTS

Maintain Your Skills

Mixed Review

Find each sum or difference. *(Lesson 8-5)*

66. $(4x^2 + 5x) + (-7x^2 + x)$
67. $(3y^2 + 5y - 6) - (7y^2 - 9)$
68. $(5b - 7ab + 8a) - (5ab - 4a)$
69. $(6p^3 + 3p^2 - 7) + (p^3 - 6p^2 - 2p)$

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*. *(Lesson 8-4)*

70. $4x^2 - 10ab + 6$
71. $4c + ab - c$
72. $\frac{7}{y} + y^2$
73. $\frac{n^2}{3}$

Define a variable, write an inequality, and solve each problem. Then check your solution. *(Lesson 6-3)*

74. Six increased by ten times a number is less than nine times the number.
75. Nine times a number increased by four is no less than seven decreased by thirteen times the number.

Write an equation of the line that passes through each pair of points. *(Lesson 5-4)*

76. $(-3, -8), (1, 4)$
77. $(-4, 5), (2, -7)$
78. $(3, -1), (-3, 2)$

79. **EXPENSES** Kristen spent one fifth of her money on gasoline to fill up her car. Then she spent half of what was left for a haircut. She bought lunch for \$7. When she got home, she had \$13 left. How much money did Kristen have originally? *(Lesson 3-4)*

For Exercises 80 and 81, use each set of data to make a stem-and-leaf plot. *(Lesson 2-5)*

80. 49 51 55 62 47 32 56 57 48 47 33 68 53 45 30
81. 21 18 34 30 20 15 14 10 22 21 18 43 44 20 18

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **products of polynomials**, see Lesson 8-1.)*

82. $(a)(a)$
83. $2x(3x^2)$
84. $-3y^2(8y^2)$
85. $4y(3y) - 4y(6)$
86. $-5n(2n^2) - (-5n)(8n) + (-5n)(4)$
87. $3p^2(6p^2) - 3p^2(8p) + 3p^2(12)$

Practice Quiz 2

Lessons 8-4 through 8-6

Find the degree of each polynomial. *(Lesson 8-4)*

1. $5x^4$
2. $-9n^3p^4$
3. $7a^2 - 2ab^2$
4. $-6 - 8x^2y^2 + 5y^3$

Arrange the terms of each polynomial so that the powers of x are in ascending order. *(Lesson 8-4)*

5. $4x^2 + 9x - 12 + 5x^3$
6. $2xy^4 + x^3y^5 + 5x^5y - 13x^2$

Find each sum or difference. *(Lesson 8-5)*

7. $(7n^2 - 4n + 10) + (3n^2 - 8)$
8. $(3g^3 - 5g) - (2g^3 + 5g^2 - 3g + 1)$

Find each product. *(Lesson 8-6)*

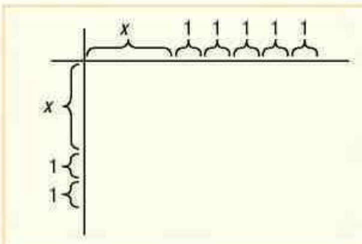
9. $5a^2(3a^3b - 2a^2b^2 + 6ab^3)$
10. $7x^2y(5x^2 - 3xy + y)$

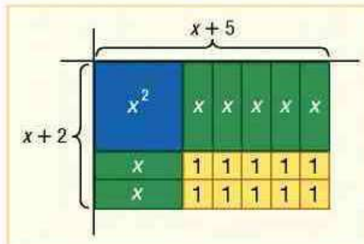
Multiplying Polynomials

You can use algebra tiles to find the product of two binomials.

Activity 1 Use algebra tiles to find $(x + 2)(x + 5)$.

The rectangle will have a width of $x + 2$ and a length of $x + 5$. Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.

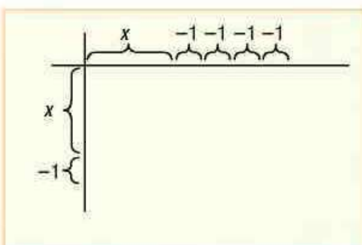


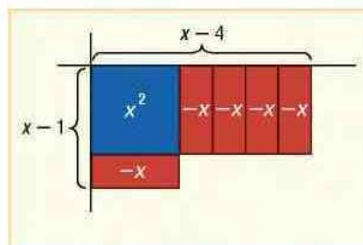


The rectangle consists of 1 blue x^2 tile, 7 green x tiles, and 10 yellow 1 tiles. The area of the rectangle is $x^2 + 7x + 10$. Therefore, $(x + 2)(x + 5) = x^2 + 7x + 10$.

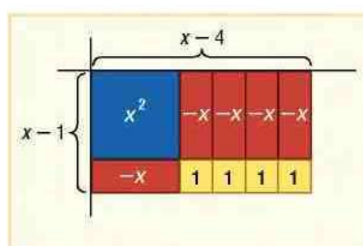
Activity 2 Use algebra tiles to find $(x - 1)(x - 4)$.

Step 1 The rectangle will have a width of $x - 1$ and a length of $x - 4$. Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.





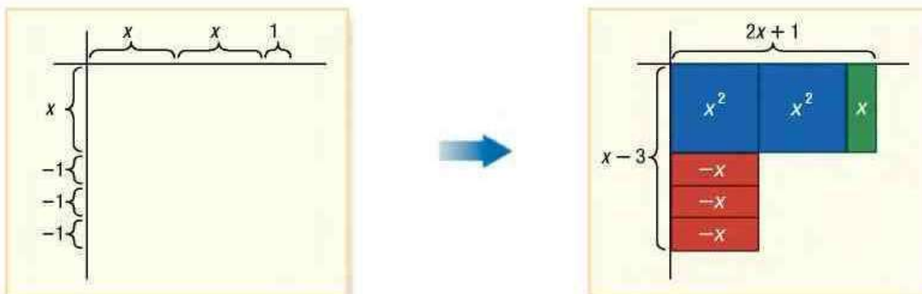
Step 2 Determine whether to use 4 yellow 1 tiles or 4 red -1 tiles to complete the rectangle. Remember that the numbers at the top and side give the dimensions of the tile needed. The area of each tile is the product of -1 and -1 or 1. This is represented by a yellow 1 tile. Fill in the space with 4 yellow 1 tiles to complete the rectangle.



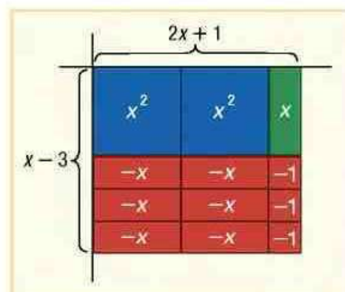
The rectangle consists of 1 blue x^2 tile, 5 red $-x$ tiles, and 4 yellow 1 tiles. The area of the rectangle is $x^2 - 5x + 4$. Therefore, $(x - 1)(x - 4) = x^2 - 5x + 4$.

Activity 3 Use algebra tiles to find $(x - 3)(2x + 1)$.

Step 1 The rectangle will have a width of $x - 3$ and a length of $2x + 1$. Mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.

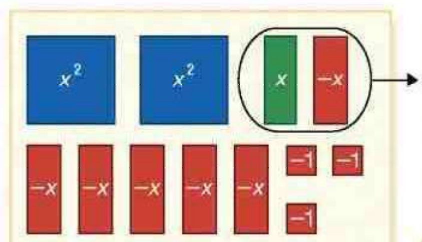


Step 2 Determine what color x tiles and what color 1 tiles to use to complete the rectangle. The area of each x tile is the product of x and -1. This is represented by a red $-x$ tile. The area of the 1 tile is represented by the product of 1 and -1 or -1. This is represented by a red -1 tile. Complete the rectangle with 3 red $-x$ tiles and 3 red -1 tiles.



Step 3 Rearrange the tiles to simplify the polynomial you have formed. Notice that a zero pair is formed by one positive and one negative x tile.

There are 2 blue x^2 tiles, 5 red $-x$ tiles, and 3 red -1 tiles left. In simplest form, $(x - 3)(2x + 1) = 2x^2 - 5x - 3$.



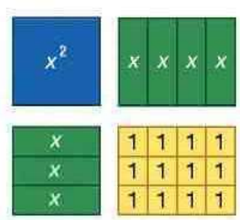
Model and Analyze

Use algebra tiles to find each product.

1. $(x + 2)(x + 3)$ **2.** $(x - 1)(x - 3)$ **3.** $(x + 1)(x - 2)$

4. $(x + 1)(2x + 1)$ **5.** $(x - 2)(2x - 3)$ **6.** $(x + 3)(2x - 4)$

7. You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for $(x + 3)(x + 4)$ separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.



8-7 Multiplying Polynomials

What You'll Learn

- Multiply two binomials by using the FOIL method.
- Multiply two polynomials by using the Distributive Property.

Vocabulary

- FOIL method

How is multiplying binomials similar to multiplying two-digit numbers?

To compute 24×36, we multiply each digit in 24 by each digit in 36, paying close attention to the place value of each digit.

Step 1 Multiply by the ones.	Step 2 Multiply by the tens.	Step 3 Add like place values.
$\begin{array}{r} 24 \\ \times\ 36 \\ \hline 144 \end{array}$	$\begin{array}{r} 24 \\ \times\ 36 \\ \hline 144 \\ 720 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ \times\ 36 \\ \hline 144 \\ +\ 720 \\ \hline 864 \end{array}$
$6 \times 24 = 6(20 + 4)$ $= 120 + 24$ or 144	$30 \times 24 = 30(20 + 4)$ $= 600 + 120$ or 720	

You can multiply two binomials in a similar way.

MULTIPLY BINOMIALS To multiply two binomials, apply the Distributive Property twice as you do when multiplying two-digit numbers.

Example 1 The Distributive Property

Find $(x + 3)(x + 2)$.

Method 1 Vertical

Multiply by 2.	Multiply by x.	Add like terms.
$\begin{array}{r} x + 3 \\ (\times)\ x + 2 \\ \hline 2x + 6 \end{array}$	$\begin{array}{r} x + 3 \\ (\times)\ x + 2 \\ \hline 2x + 6 \\ x^2 + 3x \quad \\ \hline \end{array}$	$\begin{array}{r} x + 3 \\ (\times)\ x + 2 \\ \hline 2x + 6 \\ x^2 + 3x \quad \\ \hline x^2 + 5x + 6 \end{array}$
$2(x + 3) = 2x + 6$	$x(x + 3) = x^2 + 3x$	

Method 2 Horizontal

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$ Distributive Property

$= x(x) + x(2) + 3(x) + 3(2)$ Distributive Property

$= x^2 + 2x + 3x + 6$ Multiply.

$= x^2 + 5x + 6$ Combine like terms.

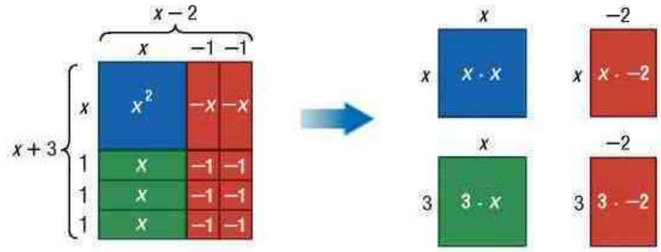
Study Tip

Look Back

To review the **Distributive Property**, see Lesson 1-7.

An alternative method for finding the product of two binomials can be shown using algebra tiles.

Consider the product of $x + 3$ and $x - 2$. The rectangle shown below has a length of $x + 3$ and a width of $x - 2$. Notice that this rectangle can be broken up into four smaller rectangles.



The product of $(x - 2)$ and $(x + 3)$ is the sum of these four areas.

$$(x + 3)(x - 2) = (x \cdot x) + (x \cdot -2) + (3 \cdot x) + (3 \cdot -2) \quad \text{Sum of the four areas}$$
$$= x^2 + (-2x) + 3x + (-6) \quad \text{Multiply.}$$
$$= x^2 + x - 6 \quad \text{Combine like terms.}$$

This example illustrates a shortcut of the Distributive Property called the **FOIL method**. You can use the FOIL method to multiply two binomials.

Key Concept — FOIL Method for Multiplying Binomials

- **Words** To multiply two binomials, find the sum of the products of
  - F the *First* terms,
  - O the *Outer* terms,
  - I the *Inner* terms, and
  - L the *Last* terms.
- **Example**

		Product of First terms		Product of Outer terms		Product of Inner terms		Product of Last terms
$(x + 3)(x - 2)$	$=$	$(x)(x)$	$+$	$(-2)(x)$	$+$	$(3)(x)$	$+$	$(3)(-2)$
	$=$	$x^2 - 2x + 3x - 6$						
	$=$	$x^2 + x - 6$						

Example 2 FOIL Method

Find each product.

a. $(x - 5)(x + 7)$

$$(x - 5)(x + 7) = \overset{F}{(x)(x)} + \overset{O}{(x)(7)} + \overset{I}{(-5)(x)} + \overset{L}{(-5)(7)} \quad \text{FOIL method}$$
$$= x^2 + 7x - 5x - 35 \quad \text{Multiply.}$$
$$= x^2 + 2x - 35 \quad \text{Combine like terms.}$$

b. $(2y + 3)(6y - 7)$

$(2y + 3)(6y - 7)$

$$= \overset{F}{(2y)(6y)} + \overset{O}{(2y)(-7)} + \overset{I}{(3)(6y)} + \overset{L}{(3)(-7)} \quad \text{FOIL method}$$
$$= 12y^2 - 14y + 18y - 21 \quad \text{Multiply.}$$
$$= 12y^2 + 4y - 21 \quad \text{Combine like terms.}$$

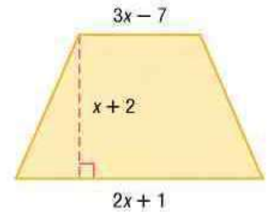
Study Tip

Checking Your Work

You can check your products in Examples 2a and 2b by reworking each problem using the Distributive Property.

Example 3 FOIL Method

GEOMETRY **The area A of a trapezoid is one-half the height h times the sum of the bases, b_1 and b_2. Write an expression for the area of the trapezoid.**



Identify the height and bases.

$h = x + 2$

$b_1 = 3x - 7$

$b_2 = 2x + 1$

Now write and apply the formula.

$$\underbrace{A}_{\text{Area}} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{\frac{1}{2}}_{\text{one-half}} \cdot \underbrace{h}_{\text{height}} \quad \underbrace{\cdot}_{\text{times}} \quad \underbrace{(b_1 + b_2)}_{\text{sum of bases}}$$

$A = \frac{1}{2}h(b_1 + b_2)$	Original formula
$= \frac{1}{2}(x + 2)[(3x - 7) + (2x + 1)]$	Substitution
$= \frac{1}{2}(x + 2)(5x - 6)$	Add polynomials in the brackets.
$= \frac{1}{2}[x(5x) + x(-6) + 2(5x) + 2(-6)]$	FOIL method
$= \frac{1}{2}(5x^2 - 6x + 10x - 12)$	Multiply.
$= \frac{1}{2}(5x^2 + 4x - 12)$	Combine like terms.
$= \frac{5}{2}x^2 + 2x - 6$	Distributive Property

The area of the trapezoid is $\frac{5}{2}x^2 + 2x - 6$ square units.

MULTIPLY POLYNOMIALS The Distributive Property can be used to multiply any two polynomials.

Study Tip

Common Misconception

A common mistake when multiplying polynomials horizontally is to combine terms that are not alike. For this reason, you may prefer to multiply polynomials in column form, aligning like terms.

Example 4 The Distributive Property

Find each product.

a. $(4x + 9)(2x^2 - 5x + 3)$

$(4x + 9)(2x^2 - 5x + 3)$	
$= 4x(2x^2 - 5x + 3) + 9(2x^2 - 5x + 3)$	Distributive Property
$= 8x^3 - 20x^2 + 12x + 18x^2 - 45x + 27$	Distributive Property
$= 8x^3 - 2x^2 - 33x + 27$	Combine like terms.

b. $(y^2 - 2y + 5)(6y^2 - 3y + 1)$

$(y^2 - 2y + 5)(6y^2 - 3y + 1)$	
$= y^2(6y^2 - 3y + 1) - 2y(6y^2 - 3y + 1) + 5(6y^2 - 3y + 1)$	Distributive Property
$= 6y^4 - 3y^3 + y^2 - 12y^3 + 6y^2 - 2y + 30y^2 - 15y + 5$	Distributive Property
$= 6y^4 - 15y^3 + 37y^2 - 17y + 5$	Combine like terms.

Check for Understanding

Concept Check

1. **Draw a diagram** to show how you would use algebra tiles to find the product of $2x - 1$ and $x + 3$.
2. **Show** how to find $(3x + 4)(2x - 5)$ using each method.
   a. Distributive Property
   b. FOIL method
   c. vertical or column method
   d. algebra tiles
3. **OPEN ENDED** State which method of multiplying binomials you prefer and why.

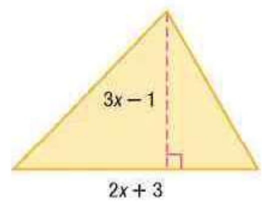
Guided Practice

Find each product.

4. $(y + 4)(y + 3)$
5. $(x - 2)(x + 6)$
6. $(a - 8)(a + 5)$
7. $(4h + 5)(h + 7)$
8. $(9p - 1)(3p - 2)$
9. $(2g + 7)(5g - 8)$
10. $(3b - 2c)(6b + 5c)$
11. $(3k - 5)(2k^2 + 4k - 3)$

Application

12. **GEOMETRY** The area A of a triangle is half the product of the base b times the height h. Write a polynomial expression that represents the area of the triangle at the right.



Practice and Apply

Homework Help

For Exercises	See Examples
13–38	1, 2, 4
39–42	3

Extra Practice

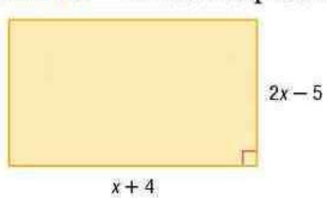
See page 839.

Find each product.

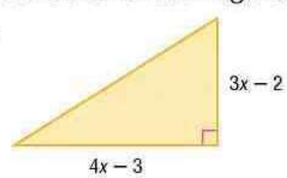
13. $(b + 8)(b + 2)$
14. $(n + 6)(n + 7)$
15. $(x - 4)(x - 9)$
16. $(a - 3)(a - 5)$
17. $(y + 4)(y - 8)$
18. $(p + 2)(p - 10)$
19. $(2w - 5)(w + 7)$
20. $(k + 12)(3k - 2)$
21. $(8d + 3)(5d + 2)$
22. $(4g + 3)(9g + 6)$
23. $(7x - 4)(5x - 1)$
24. $(6a - 5)(3a - 8)$
25. $(2n + 3)(2n + 3)$
26. $(5m - 6)(5m - 6)$
27. $(10r - 4)(10r + 4)$
28. $(7t + 5)(7t - 5)$
29. $(8x + 2y)(5x - 4y)$
30. $(11a - 6b)(2a + 3b)$
31. $(p + 4)(p^2 + 2p - 7)$
32. $(a - 3)(a^2 - 8a + 5)$
33. $(2x - 5)(3x^2 - 4x + 1)$
34. $(3k + 4)(7k^2 + 2k - 9)$
35. $(n^2 - 3n + 2)(n^2 + 5n - 4)$
36. $(y^2 + 7y - 1)(y^2 - 6y + 5)$
37. $(4a^2 + 3a - 7)(2a^2 - a + 8)$
38. $(6x^2 - 5x + 2)(3x^2 + 2x + 4)$

GEOMETRY **Write an expression to represent the area of each figure.**

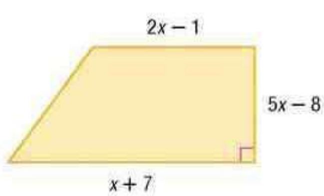
39.



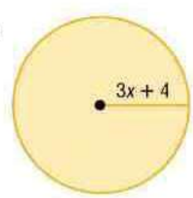
40.



41.

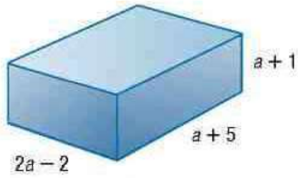


42.

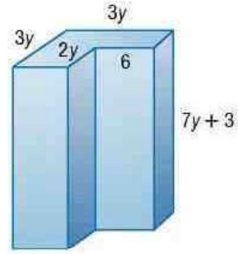


GEOMETRY **The volume V of a prism equals the area of the base B times the height h. Write an expression to represent the volume of each prism.**

43.



44.



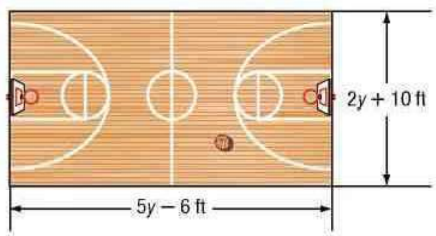
NUMBER THEORY **For Exercises 45–47, consider three consecutive integers. Let the least of these integers be a.**

45. Write a polynomial representing the product of these three integers.

46. Choose an integer for a. Find their product.

47. Evaluate the polynomial in Exercise 45 for the value of a you chose in Exercise 46. Describe the result.

48. BASKETBALL The dimensions of a professional basketball court are represented by a width of $2y + 10$ feet and a length of $5y - 6$ feet. Find an expression for the area of the court.



More About. . .

Basketball

More than 200 million people a year pay to see basketball games. That is more admissions than for any other American sport.

Source: *Compton's Encyclopedia*

OFFICE SPACE **For Exercises 49–51, use the following information.**
Latanya's modular office is square. Her office in the company's new building will be 2 feet shorter in one direction and 4 feet longer in the other.

49. Write expressions for the dimensions of Latanya's new office.

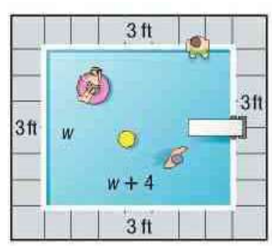
50. Write a polynomial expression for the area of her new office.

51. Suppose her office is presently 9 feet by 9 feet. Will her new office be bigger or smaller than her old office and by how much?

52. MENTAL MATH One way to mentally multiply 25 and 18 is to find $(20 + 5)(20 - 2)$. Show how the FOIL method can be used to find each product.

a. $35(19)$ **b.** $67(102)$ **c.** $8\frac{1}{2} \cdot 6\frac{3}{4}$ **d.** $12\frac{3}{5} \cdot 10\frac{2}{3}$

53. POOL CONSTRUCTION A homeowner is installing a swimming pool in his backyard. He wants its length to be 4 feet longer than its width. Then he wants to surround it with a concrete walkway 3 feet wide. If he can only afford 300 square feet of concrete for the walkway, what should the dimensions of the pool be?



54. CRITICAL THINKING Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The product of a binomial and a trinomial is a polynomial with four terms.

55. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is multiplying binomials similar to multiplying two-digit numbers?

Include the following in your answer:

- a demonstration of a horizontal method for multiplying 24×36, and
- an explanation of the meaning of "like terms" in the context of vertical two-digit multiplication.

Standardized Test Practice

56. $(x + 2)(x - 4) - (x + 4)(x - 2) =$

(A) 0 (B) $2x^2 + 4x - 16$ (C) $-4x$ (D) $4x$

57. The expression $(x - y)(x^2 + xy + y^2)$ is equivalent to which of the following?

(A) $x^2 - y^2$ (B) $x^3 - y^3$ (C) $x^3 - xy^2$ (D) $x^3 - x^2y + y^2$

Maintain Your Skills

Mixed Review

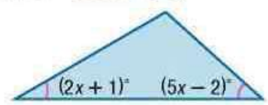
Find each product. *(Lesson 8-6)*

58. $3d(4d^2 - 8d - 15)$ 59. $-4y(7y^2 - 4y + 3)$ 60. $2m^2(5m^2 - 7m + 8)$

Simplify. *(Lesson 8-6)*

61. $3x(2x - 4) + 6(5x^2 + 2x - 7)$ 62. $4a(5a^2 + 2a - 7) - 3(2a^2 - 6a - 9)$

GEOMETRY For Exercises 63 and 64, use the following information.
The sum of the degree measures of the angles of a triangle is 180. *(Lesson 8-5)*

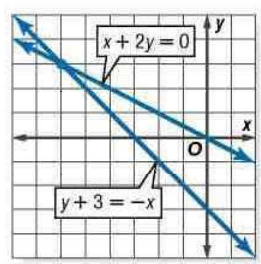


63. Write an expression to represent the measure of the third angle of the triangle.

64. If $x = 15$, find the measures of the three angles of the triangle.

65. Use the graph at the right to determine whether the system below has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

$x + 2y = 0$
$y + 3 = -x$



If $f(x) = 2x - 5$ and $g(x) = x^2 + 3x$, find each value. *(Lesson 4-6)*

66. $f(-4)$ 67. $g(-2) + 7$ 68. $f(a + 3)$

Solve each equation or formula for the variable specified. *(Lesson 3-8)*

69. $a = \frac{v}{t}$ for t 70. $ax - by = 2cz$ for y 71. $4x + 3y = 7$ for y

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify.
*(To review **Power of a Power** and **Power of a Product** Properties, see Lesson 8-1.)*

72. $(6a)^2$ 73. $(7x)^2$ 74. $(9b)^2$

75. $(4y^2)^2$ 76. $(2v^3)^2$ 77. $(3g^4)^2$

8-8 Special Products

What You'll Learn

- Find squares of sums and differences.
- Find the product of a sum and a difference.

Vocabulary

- difference of squares

When is the product of two binomials also a binomial?

In the previous lesson, you learned how to multiply two binomials using the FOIL method. You may have noticed that the *Outer* and *Inner* terms often combine to produce a trinomial product.

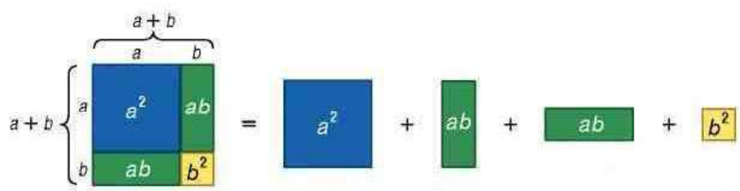
$$\begin{aligned}(x+5)(x-3) &= \overset{F}{x^2} \overset{O}{-\,3x} \overset{I}{+\,5x} \overset{L}{-\,15} \\ &= x^2 + 2x - 15 \qquad \text{Combine like terms.}\end{aligned}$$

This is not always the case, however. Examine the product below.

$$\begin{aligned}(x+3)(x-3) &= \overset{F}{x^2} \overset{O}{-\,3x} \overset{I}{+\,3x} \overset{L}{-\,9} \\ &= x^2 + 0x - 9 \qquad \text{Combine like terms.} \\ &= x^2 - 9 \qquad \text{Simplify.}\end{aligned}$$

Notice that the product of $x + 3$ and $x - 3$ is a binomial.

SQUARES OF SUMS AND DIFFERENCES While you can always use the FOIL method to find the product of two binomials, some pairs of binomials have products that follow a specific pattern. One such pattern is the *square of a sum*, $(a + b)^2$ or $(a + b)(a + b)$. You can use the diagram below to derive the pattern for this special product.



$$\begin{aligned}(a+b)^2 &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Key Concept *Square of a Sum*

- **Words** The square of $a + b$ is the square of a plus twice the product of a and b plus the square of b.
- **Symbols** $(a + b)^2 = (a + b)(a + b)$
  $= a^2 + 2ab + b^2$
- **Example** $(x + 7)^2 = x^2 + 2(x)(7) + 7^2$
  $= x^2 + 14x + 49$

Example 1 Square of a Sum

Find each product.

a. $(4y + 5)^2$

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{Square of a Sum}$$
$$(4y + 5)^2 = (4y)^2 + 2(4y)(5) + 5^2 \quad a = 4y \text{ and } b = 5$$
$$= 16y^2 + 40y + 25 \quad \text{Simplify.}$$

CHECK Check your work by using the FOIL method.

$$(4y + 5)^2 = (4y + 5)(4y + 5)$$
$$= \overset{F}{(4y)(4y)} + \overset{O}{(4y)(5)} + \overset{I}{5(4y)} + \overset{L}{5(5)}$$
$$= 16y^2 + 20y + 20y + 25$$
$$= 16y^2 + 40y + 25 \checkmark$$

b. $(8c + 3d)^2$

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{Square of a Sum}$$
$$(8c + 3d)^2 = (8c)^2 + 2(8c)(3d) + (3d)^2 \quad a = 8c \text{ and } b = 3d$$
$$= 64c^2 + 48cd + 9d^2 \quad \text{Simplify.}$$

Study Tip

$(a + b)^2$
In the pattern for $(a + b)^2$, a and b can be numbers, variables, or expressions with numbers and variables.

To find the pattern for the *square of a difference*, $(a - b)^2$, write $a - b$ as $a + (-b)$ and square it using the square of a sum pattern.

$$(a - b)^2 = [a + (-b)]^2$$
$$= a^2 + 2(a)(-b) + (-b)^2 \quad \text{Square of a Sum}$$
$$= a^2 - 2ab + b^2 \quad \text{Simplify. Note that } (-b)^2 = (-b)(-b) \text{ or } b^2.$$

The square of a difference can be found by using the following pattern.

Key Concept — Square of a Difference

- **Words** The square of $a - b$ is the square of a minus twice the product of a and b plus the square of b.
- **Symbols** $(a - b)^2 = (a - b)(a - b)$
  $= a^2 - 2ab + b^2$
- **Example** $(x - 4)^2 = x^2 - 2(x)(4) + 4^2$
  $= x^2 - 8x + 16$

Example 2 Square of a Difference

Find each product.

a. $(6p - 1)^2$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Square of a Difference}$$
$$(6p - 1)^2 = (6p)^2 - 2(6p)(1) + 1^2 \quad a = 6p \text{ and } b = 1$$
$$= 36p^2 - 12p + 1 \quad \text{Simplify.}$$

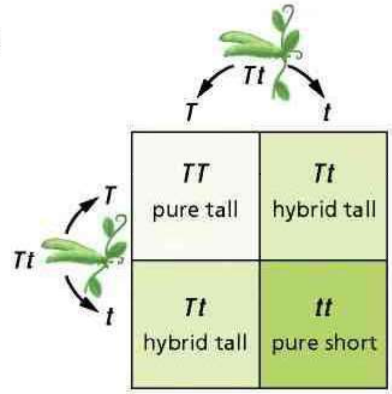
b. $(5m^3 - 2n)^2$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Square of a Difference}$$
$$(5m^3 - 2n)^2 = (5m^3)^2 - 2(5m^3)(2n) + (2n)^2 \quad a = 5m^3 \text{ and } b = 2n$$
$$= 25m^6 - 20m^3n + 4n^2 \quad \text{Simplify.}$$

www.algebra1.com/extra_examples

Example 3 Apply the Sum of a Square

GENETICS **The Punnett square shows the possible gene combinations of a cross between two pea plants. Each plant passes along one *dominant* gene T for tallness and one *recessive* gene t for shortness.**



Show how combinations can be modeled by the square of a binomial. Then determine what percent of the offspring will be pure tall, hybrid tall, and pure short.

Each parent has half the genes necessary for tallness and half the genes necessary for shortness. The makeup of each parent can be modeled by $0.5T + 0.5t$. Their offspring can be modeled by the product of $0.5T + 0.5t$ and $0.5T + 0.5t$ or $(0.5T + 0.5t)^2$.

If we expand this product, we can determine the possible heights of the offspring.

$(a + b)^2 = a^2 + 2ab + b^2$	Square of a Sum
$(0.5T + 0.5t)^2 = (0.5T)^2 + 2(0.5T)(0.5t) + (0.5t)^2$	$a = 0.5T$ and $b = 0.5t$
$= 0.25T^2 + 0.5Tt + 0.25t^2$	Simplify.
$= 0.25TT + 0.5Tt + 0.25tt$	$T^2 = TT$ and $t^2 = tt$

Thus, 25% of the offspring are TT or pure tall, 50% are Tt or hybrid tall, and 25% are tt or pure short.

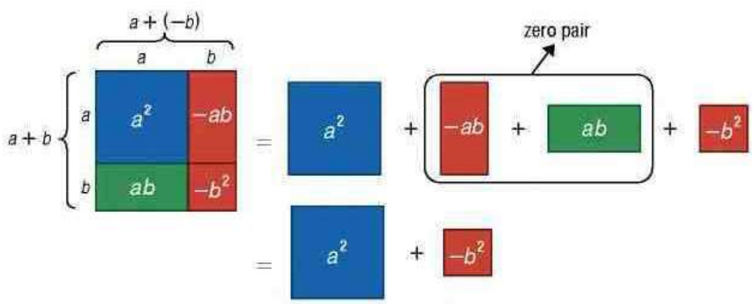
Career Choices

Geneticist

Laboratory geneticists work in medicine to find cures for disease, in agriculture to breed new crops and livestock, and in police work to identify criminals.

Online Research For information about a career as a geneticist, visit: www.algebra1.com/careers

PRODUCT OF A SUM AND A DIFFERENCE You can use the diagram below to find the pattern for the product of a sum and a difference of the *same two terms*, $(a + b)(a - b)$. Recall that $a - b$ can be rewritten as $a + (-b)$.



$= a^2 + (-b^2)$

$= a^2 - b^2$

The resulting product, $a^2 - b^2$, has a special name. It is called a **difference of squares**. Notice that this product has no middle term.

Key Concept — Product of a Sum and a Difference

- **Words** The product of $a + b$ and $a - b$ is the square of a minus the square of b.
- **Symbols** $(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$
- **Example** $(x + 9)(x - 9) = x^2 - 9^2 = x^2 - 81$

Example 4 Product of a Sum and a Difference

Find each product.

a. $(3n + 2)(3n - 2)$

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and a Difference

$(3n + 2)(3n - 2) = (3n)^2 - 2^2$ $a = 3n$ and $b = 2$

$= 9n^2 - 4$ Simplify.

b. $(11v - 8w^2)(11v + 8w^2)$

$(a - b)(a + b) = a^2 - b^2$ Product of a Sum and a Difference

$(11v - 8w^2)(11v + 8w^2) = (11v)^2 - (8w^2)^2$ $a = 11v$ and $b = 8w^2$

$= 121v^2 - 64w^4$ Simplify.

The following list summarizes the special products you have studied.

Key Concept — Special Products

- **Square of a Sum** $(a + b)^2 = a^2 + 2ab + b^2$
- **Square of a Difference** $(a - b)^2 = a^2 - 2ab + b^2$
- **Product of a Sum and a Difference** $(a - b)(a + b) = a^2 - b^2$

Check for Understanding

Concept Check

1. **Compare and contrast** the pattern for the square of a sum with the pattern for the square of a difference.
2. **Explain** how the square of a difference and the difference of squares differ.
3. **Draw a diagram** to show how you would use algebra tiles to model the product of $x - 3$ and $x - 3$, or $(x - 3)^2$.
4. **OPEN ENDED** Write two binomials whose product is a difference of squares.

Guided Practice

Find each product.

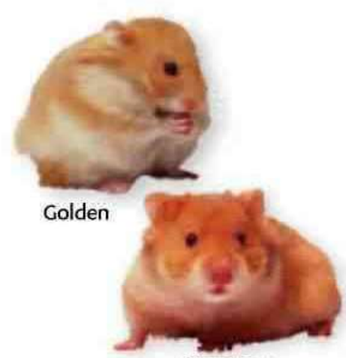
5. $(a + 6)^2$
6. $(4n - 3)(4n - 3)$
7. $(8x - 5)(8x + 5)$
8. $(3a + 7b)(3a - 7b)$
9. $(x^2 - 6y)^2$
10. $(9 - p)^2$

Application

GENETICS For Exercises 11 and 12, use the following information.

In hamsters, golden coloring G is dominant over cinnamon coloring g. Suppose a purebred cinnamon male is mated with a purebred golden female.

11. Write an expression for the genetic makeup of the hamster pups.
12. What is the probability that the pups will have cinnamon coloring? Explain your reasoning.



Golden

Cinnamon

Practice and Apply

Homework Help

For Exercises	See Examples
13–38	1, 2, 4
39, 40	3

Extra Practice

See page 839.

Find each product.

13. $(y + 4)^2$
14. $(k + 8)(k + 8)$
15. $(a - 5)(a - 5)$
16. $(n - 12)^2$
17. $(b + 7)(b - 7)$
18. $(c - 2)(c + 2)$
19. $(2g + 5)^2$
20. $(9x + 3)^2$
21. $(7 - 4y)^2$
22. $(4 - 6h)^2$
23. $(11r + 8)(11r - 8)$
24. $(12p - 3)(12p + 3)$
25. $(a + 5b)^2$
26. $(m + 7n)^2$
27. $(2x - 9y)^2$
28. $(3n - 10p)^2$
29. $(5w + 14)(5w - 14)$
30. $(4d - 13)(4d + 13)$
31. $(x^3 + 4y)^2$
32. $(3a^2 - b^2)^2$
33. $(8a^2 - 9b^3)(8a^2 + 9b^3)$
34. $(5x^4 - y)(5x^4 + y)$
35. $\left(\frac{2}{3}x - 6\right)^2$
36. $\left(\frac{4}{5}x + 10\right)^2$
37. $(2n + 1)(2n - 1)(n + 5)$
38. $(p + 3)(p - 4)(p - 3)(p + 4)$

GENETICS **For Exercises 39 and 40, use the following information.**
Pam has brown eyes and Bob has blue eyes. Brown genes B are dominant over blue genes b. A person with genes BB or Bb has brown eyes. Someone with genes bb has blue eyes. Suppose Pam's genes for eye color are Bb.

39. Write an expression for the possible eye coloring of Pam and Bob's children.

40. What is the probability that a child of Pam and Bob would have blue eyes?

MAGIC TRICK **For Exercises 41–44, use the following information.**
Julie says that she can perform a magic trick with numbers. She asks you to pick a whole number, any whole number. Square that number. Then, add twice your original number. Next add 1. Take the square root of the result. Finally, subtract your original number. Then Julie exclaims with authority, "Your answer is 1!"

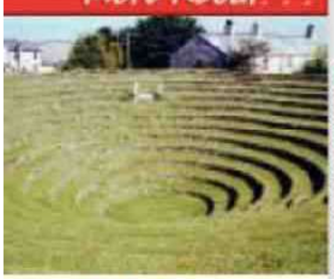
41. Pick a whole number and follow Julie's directions. Is your result 1?

42. Let a represent the whole number you chose. Then, find a polynomial representation for the first three steps of Julie's directions.

43. The polynomial you wrote in Exercise 42 is the square of what binomial sum?

44. Take the square root of the perfect square you wrote in Exercise 43, then subtract a, your original number. What is the result?

More About . . .

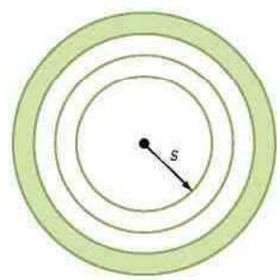


Architecture

The historical Gwennap Pit, an outdoor amphitheater in southern England, consists of a circular stage surrounded by circular levels used for seating. Each seating level is about 1 meter wide.

Source: *Christian Guide to Britain*

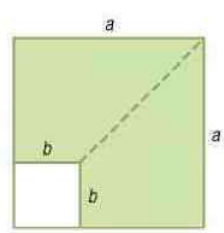
ARCHITECTURE **For Exercises 45 and 46, use the following information.**
A diagram of a portion of the Gwennap Pit is shown at the right. Suppose the radius of the stage is s meters.



45. Use the information at the left to find binomial representations for the radii of the second and third seating levels.

46. Find the area of the shaded region representing the third seating level.

47. **GEOMETRY** The area of the shaded region models the difference of two squares, $a^2 - b^2$. Show that the area of the shaded region is also equal to $(a - b)(a + b)$. (*Hint*: Divide the shaded region into two trapezoids as shown.)



48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

When is the product of two binomials also a binomial?

Include the following in your answer:

- an example of two binomials whose product is a binomial, and
- an example of two binomials whose product is not a binomial.

Standardized Test Practice

49. If $a^2 + b^2 = 40$ and $ab = 12$, find the value of $(a - b)^2$.

Ⓐ 1 Ⓑ 121 Ⓒ 16 Ⓓ 28

50. If $x - y = 10$ and $x + y = 20$, find the value of $x^2 - y^2$.

Ⓐ 400 Ⓑ 200 Ⓒ 100 Ⓓ 30

Extending the Lesson

51. Does a pattern exist for the cube of a sum, $(a + b)^3$?

a. Investigate this question by finding the product of $(a + b)(a + b)(a + b)$.

b. Use the pattern you discovered in part **a** to find $(x + 2)^3$.

c. Draw a diagram of a geometric model for the cube of a sum.

Maintain Your Skills

Mixed Review

Find each product. *(Lesson 8-7)*

52. $(x + 2)(x + 7)$ **53.** $(c - 9)(c + 3)$ **54.** $(4y - 1)(5y - 6)$

55. $(3n - 5)(8n + 5)$ **56.** $(x - 2)(3x^2 - 5x + 4)$ **57.** $(2k + 5)(2k^2 - 8k + 7)$

Solve. *(Lesson 8-6)*

58. $6(x + 2) + 4 = 5(3x - 4)$ **59.** $-3(3a - 8) + 2a = 4(2a + 1)$

60. $p(p + 2) + 3p = p(p - 3)$ **61.** $y(y - 4) + 2y = y(y + 12) - 7$

Use elimination to solve each system of equations. *(Lessons 7-3 and 7-4)*

62. $\frac{3}{4}x + \frac{1}{5}y = 5$, $\frac{3}{4}x - \frac{1}{5}y = -5$ **63.** $2x - y = 10$, $5x + 3y = 3$ **64.** $2x = 4 - 3y$, $3y - x = -11$

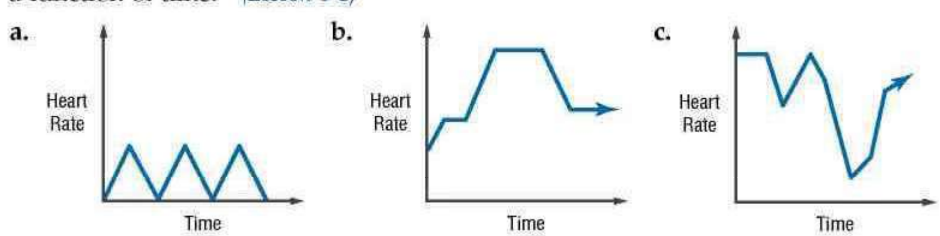
Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation. *(Lesson 5-6)*

65. $5x + 5y = 35, (-3, 2)$ **66.** $2x - 5y = 3, (-2, 7)$ **67.** $5x + y = 2, (0, 6)$

Find the nth term of each arithmetic sequence described. *(Lesson 4-7)*

68. $a_1 = 3, d = 4, n = 18$ **69.** $-5, 1, 7, 13, \dots$ for $n = 12$

70. PHYSICAL FITNESS Mitchell likes to exercise regularly. He likes to warm up by walking two miles. Then he runs five miles. Finally, he cools down by walking for another mile. Identify the graph that best represents Mitchell's heart rate as a function of time. *(Lesson 1-8)*



Chapter 8

Study Guide and Review

Vocabulary and Concept Check

binomial (p. 432)	monomial (p. 410)	Product of Powers (p. 411)
constant (p. 410)	negative exponent (p. 419)	Quotient of Powers (p. 417)
degree of a monomial (p. 433)	polynomial (p. 432)	scientific notation (p. 425)
degree of a polynomial (p. 433)	Power of a Power (p. 411)	trinomial (p. 432)
difference of squares (p. 460)	Power of a Product (p. 412)	zero exponent (p. 419)
FOIL method (p. 453)	Power of a Quotient (p. 418)	

Choose a term from the vocabulary list that best matches each example.

1. $4^{-3} = \frac{1}{4^3}$

2. $(n^3)^5 = n^{15}$

3. $\frac{4x^2y}{8xy^3} = \frac{x}{2y^2}$

4. $4x^2$

5. $x^2 - 3x + 1$

6. $2^0 = 1$

7. $x^4 - 3x^3 + 2x^2 - 1$

8. $(x + 3)(x - 4) = x^2 - 4x + 3x - 12$

9. $x^2 + 2$

10. $(a^3b)(2ab^2) = 2a^4b^3$

Lesson-by-Lesson Review

8-1 Multiplying Monomials

See pages 410–415.

Concept Summary

	Examples
• A monomial is a number, a variable, or a product of a number and one or more variables.	$6x^2, -5, \frac{2c}{3}$
• To multiply two powers that have the same base, add exponents.	$a^2 \cdot a^3 = a^5$
• To find the power of a power, multiply exponents.	$(a^2)^3 = a^6$
• The power of a product is the product of the powers.	$(ab^2)^3 = a^3b^6$

Examples

1 **Simplify $(2ab^2)(3a^2b^3)$.**

$(2ab^2)(3a^2b^3) = (2 \cdot 3)(a \cdot a^2)(b^2 \cdot b^3)$ Commutative Property

$= 6a^3b^5$ Product of Powers

2 **Simplify $(2x^2y^3)^3$.**

$(2x^2y^3)^3 = 2^3(x^2)^3(y^3)^3$ Power of a Product

$= 8x^6y^9$ Power of a Power

Exercises **Simplify.** *See Examples 2, 3, and 5 on pages 411 and 412.*

11. $y^3 \cdot y^3 \cdot y$

12. $(3ab)(-4a^2b^3)$

13. $(-4a^2x)(-5a^3x^4)$

14. $(4a^2b)^3$

15. $(-3xy)^2(4x)^3$

16. $(-2c^2d)^4(-3c^2)^3$

17. $-\frac{1}{2}(m^2n^4)^2$

18. $(5a^2)^3 + 7(a^6)$

19. $[(3^2)^2]^3$

8-2 Dividing Monomials

See pages 417–423.

Concept Summary

- To divide two powers that have the same base, subtract the exponents.
- To find the power of a quotient, find the power of the numerator and the power of the denominator.
- Any nonzero number raised to the zero power is 1.
- For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Examples

$\frac{a^5}{a^3} = a^2$

$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

$(3a^3b^2)^0 = 1$

$a^{-3} = \frac{1}{a^3}$

Example Simplify $\frac{2x^6y}{8x^2y^2}$. Assume that x and y are not equal to zero.

$\frac{2x^6y}{8x^2y^2} = \left(\frac{2}{8}\right)\left(\frac{x^6}{x^2}\right)\left(\frac{y}{y^2}\right)$ Group the powers with the same base.

$= \left(\frac{1}{4}\right)(x^{6-2})(y^{1-2})$ Quotient of Powers

$= \frac{x^4}{4y}$ Simplify.

Exercises Simplify. Assume that no denominator is equal to zero.

See Examples 1–4 on pages 417–420.

20. $\frac{(3y)^0}{6a}$
21. $\left(\frac{3bc^2}{4d}\right)^3$
22. $x^{-2}y^0z^3$
23. $\frac{27b^{-2}}{14b^{-3}}$
24. $\frac{(3a^3bc^2)^2}{18a^2b^3c^4}$
25. $\frac{-16a^3b^2x^4y}{-48a^4bxy^3}$
26. $\frac{(-a)^5b^8}{a^5b^2}$
27. $\frac{(4a^{-1})^{-2}}{(2a^4)^2}$
28. $\left(\frac{5xy^{-2}}{35x^{-2}y^{-6}}\right)^0$

8-3 Scientific Notation

See pages 425–430.

Concept Summary

- A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

$a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Examples

1 **Express 5.2×10^7 in standard notation.**

$5.2 \times 10^7 = 52{,}000{,}000$ $n = 7$; move decimal point 7 places to the right.

2 **Express 0.0021 in scientific notation.**

$0.0021 \rightarrow 0002.1 \times 10^n$ Move decimal point 3 places to the right.

$0.0021 = 2.1 \times 10^{-3}$ $a = 2.1$ and $n = -3$

3 **Evaluate $(2 \times 10^2)(5.2 \times 10^6)$. Express the result in scientific and standard notation.**

$(2 \times 10^2)(5.2 \times 10^6) = (2 \times 5.2)(10^2 \times 10^6)$ Associative Property

$= 10.4 \times 10^8$ Product of Powers

$= (1.04 \times 10^1) \times 10^8$ $10.4 = 1.04 \times 10^1$

$= 1.04 \times (10^1 \times 10^8)$ Associative Property

$= 1.04 \times 10^9$ or 1,040,000,000 Product of Powers

Exercises **Express each number in standard notation.** *See Example 1 on page 426.*

29. 2.4×10^5 **30.** 3.14×10^{-4} **31.** 4.88×10^9

Express each number in scientific notation. *See Example 2 on page 426.*

32. 0.00000187 **33.** 796×10^3 **34.** 0.0343×10^{-2}

Evaluate. Express each result in scientific and standard notation.
See Examples 3 and 4 on page 427.

35. $(2 \times 10^5)(3 \times 10^6)$ **36.** $\frac{8.4 \times 10^{-6}}{1.4 \times 10^{-9}}$ **37.** $(3 \times 10^2)(5.6 \times 10^{-8})$

8-4 Polynomials

See pages 432–436.

Concept Summary

- A polynomial is a monomial or a sum of monomials.
- A binomial is the sum of *two* monomials, and a trinomial is the sum of *three* monomials.
- The degree of a monomial is the sum of the exponents of all its variables.
- The degree of the polynomial is the greatest degree of any term. To find the degree of a polynomial, you must find the degree of each term.

Examples

1 **Find the degree of $2xy^3 + x^2y$.**

Polynomial	Terms	Degree of Each Term	Degree of Polynomial
$2xy^3 + x^2y$	$2xy^3, x^2y$	4, 3	4

2 **Arrange the terms of $4x^2 + 9x^3 - 2 - x$ so that the powers of x are in descending order.**

$4x^2 + 9x^3 - 2 - x = 4x^2 + 9x^3 - 2x^0 - x^1$ $x^0 = 1$ and $x = x^1$

$= 9x^3 + 4x^2 - x - 2$ $3 > 2 > 1 > 0$

Exercises **Find the degree of each polynomial.** *See Example 3 on page 433.*

38. $n - 2p^2$ **39.** $29n^2 + 17n^2t^2$ **40.** $4xy + 9x^3z^2 + 17rs^3$

41. $-6x^5y - 2y^4 + 4 - 8y^2$ **42.** $3ab^3 - 5a^2b^2 + 4ab$ **43.** $19m^3n^4 + 21m^5n$

Arrange the terms of each polynomial so that the powers of x are in descending order. *See Example 5 on page 433.*

44. $3x^4 - x + x^2 - 5$ **45.** $-2x^2y^3 - 27 - 4x^4 + xy + 5x^3y^2$

8-5 Adding and Subtracting Polynomials

See pages 439–443.

Concept Summary

- To add polynomials, group like terms horizontally or write them in column form, aligning like terms vertically.
- Subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse.

Example **Find $(7r^2 + 9r) - (12r^2 - 4)$.**

$$\begin{aligned} (7r^2 + 9r) - (12r^2 - 4) &= 7r^2 + 9r + (-12r^2 + 4) && \text{The additive inverse of } 12r^2 - 4 \text{ is } -12r^2 + 4. \\ &= (7r^2 - 12r^2) + 9r + 4 && \text{Group like terms.} \\ &= -5r^2 + 9r + 4 && \text{Add like terms.} \end{aligned}$$

Exercises **Find each sum or difference.** *See Examples 1 and 2 on pages 439 and 440.*

46. $(2x^2 - 5x + 7) - (3x^3 + x^2 + 2)$
47. $(x^2 - 6xy + 7y^2) + (3x^2 + xy - y^2)$
48. $(7z^2 + 4) - (3z^2 + 2z - 6)$
49. $(13m^4 - 7m - 10) + (8m^4 - 3m + 9)$
50. $(11m^2n^2 + 4mn - 6) + (5m^2n^2 + 6mn + 17)$
51. $(-5p^2 + 3p + 49) - (2p^2 + 5p + 24)$

8-6 Multiplying a Polynomial by a Monomial

See pages 444–449.

Concept Summary

- The Distributive Property can be used to multiply a polynomial by a monomial.

Examples

1 **Simplify $x^2(x + 2) + 3(x^3 + 4x^2)$.**

$$\begin{aligned} x^2(x + 2) + 3(x^3 + 4x^2) &= x^2(x) + x^2(2) + 3(x^3) + 3(4x^2) && \text{Distributive Property} \\ &= x^3 + 2x^2 + 3x^3 + 12x^2 && \text{Multiply.} \\ &= 4x^3 + 14x^2 && \text{Combine like terms.} \end{aligned}$$

2 **Solve $x(x - 10) + x(x + 2) + 3 = 2x(x + 1) - 7$.**

$$\begin{aligned} x(x - 10) + x(x + 2) + 3 &= 2x(x + 1) - 7 && \text{Original equation} \\ x^2 - 10x + x^2 + 2x + 3 &= 2x^2 + 2x - 7 && \text{Distributive Property} \\ 2x^2 - 8x + 3 &= 2x^2 + 2x - 7 && \text{Combine like terms.} \\ -8x + 3 &= 2x - 7 && \text{Subtract } 2x^2 \text{ from each side.} \\ -10x + 3 &= -7 && \text{Subtract } 2x \text{ from each side.} \\ -10x &= -10 && \text{Subtract 3 from each side.} \\ x &= 1 && \text{Divide each side by } -10. \end{aligned}$$

Exercises **Simplify.** *See Example 2 on page 444.*

52. $b(4b - 1) + 10b$
53. $x(3x - 5) + 7(x^2 - 2x + 9)$
54. $8y(11y^2 - 2y + 13) - 9(3y^3 - 7y + 2)$
55. $2x(x - y^2 + 5) - 5y^2(3x - 2)$

Solve each equation. *See Example 4 on page 445.*

56. $m(2m - 5) + m = 2m(m - 6) + 16$
57. $2(3w + w^2) - 6 = 2w(w - 4) + 10$

Chapter 8 For More ...
• Extra Practice, see pages 837–839.
• Mixed Problem Solving, see page 860.

8-7 Multiplying Polynomials

See pages 452–457.

Concept Summary

- The FOIL method is the sum of the products of the first terms F, the outer terms O, the inner terms I, and the last terms L.
- The Distributive Property can be used to multiply any two polynomials.

Examples

1 Find $(3x + 2)(x - 2)$.

$$\begin{aligned} (3x + 2)(x - 2) &= \overset{F}{(3x)(x)} + \overset{O}{(3x)(-2)} + \overset{I}{(2)(x)} + \overset{L}{(2)(-2)} && \text{FOIL Method} \\ &= 3x^2 - 6x + 2x - 4 && \text{Multiply.} \\ &= 3x^2 - 4x - 4 && \text{Combine like terms.} \end{aligned}$$

2 Find $(2y - 5)(4y^2 + 3y - 7)$.

$$\begin{aligned} &(2y - 5)(4y^2 + 3y - 7) \\ &= 2y(4y^2 + 3y - 7) - 5(4y^2 + 3y - 7) && \text{Distributive Property} \\ &= 8y^3 + 6y^2 - 14y - 20y^2 - 15y + 35 && \text{Distributive Property} \\ &= 8y^3 - 14y^2 - 29y + 35 && \text{Combine like terms.} \end{aligned}$$

Exercises **Find each product.** *See Examples 1, 2, and 4 on pages 452–454.*

58. $(r - 3)(r + 7)$ **59.** $(4a - 3)(a + 4)$ **60.** $(3x + 0.25)(6x - 0.5)$

61. $(5r - 7s)(4r + 3s)$ **62.** $(2k + 1)(k^2 + 7k - 9)$ **63.** $(4p - 3)(3p^2 - p + 2)$

8-8 Special Products

See pages 458–463.

Concept Summary

- Square of a Sum: $(a + b)^2 = a^2 + 2ab + b^2$
- Square of a Difference: $(a - b)^2 = a^2 - 2ab + b^2$
- Product of a Sum and a Difference: $(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$

Examples

1 Find $(r - 5)^2$.

$$\begin{aligned} (a - b)^2 &= a^2 - 2ab + b^2 && \text{Square of a Difference} \\ (r - 5)^2 &= r^2 - 2(r)(5) + 5^2 && a = r \text{ and } b = 5 \\ &= r^2 - 10r + 25 && \text{Simplify.} \end{aligned}$$

2 Find $(2c + 9)(2c - 9)$.

$$\begin{aligned} (a + b)(a - b) &= a^2 - b^2 && \text{Product of a Sum and a Difference} \\ (2c + 9)(2c - 9) &= 2c^2 - 9^2 && a = 2c \text{ and } b = 9 \\ &= 4c^2 - 81 && \text{Simplify.} \end{aligned}$$

Exercises **Find each product.** *See Examples 1, 2, and 4 on pages 459 and 461.*

64. $(x - 6)(x + 6)$ **65.** $(4x + 7)^2$ **66.** $(8x - 5)^2$

67. $(5x - 3y)(5x + 3y)$ **68.** $(6a - 5b)^2$ **69.** $(3m + 4n)^2$

CONTENTS

Chapter 8

Practice Test

Vocabulary and Concepts

1. **Explain** why $(4^2)(4^3) \neq 16^5$.
2. **Write** $\frac{1}{5}$ using a negative exponent.
3. **Define and give an example** of a monomial.

Skills and Applications

Simplify. Assume that no denominator is equal to zero.

4. $(a^2b^4)(a^3b^5)$
5. $(-12abc)(4a^2b^4)$
6. $\left(\frac{3}{5}m\right)^2$
7. $(-3a)^4(a^5b)^2$
8. $(-5a^2)(-6b^3)^2$
9. $\frac{mn^4}{m^3n^2}$
10. $\frac{9a^2bc^2}{63a^4bc}$
11. $\frac{48a^2bc^5}{(3ab^3c^2)^2}$

Express each number in scientific notation.

12. 46,300
13. 0.003892
14. 284×10^3
15. 52.8×10^{-9}

Evaluate. Express each result in scientific notation and standard notation.

16. $(3 \times 10^3)(2 \times 10^4)$
17. $\frac{14.72 \times 10^{-4}}{3.2 \times 10^{-3}}$
18. $(15 \times 10^{-7})(3.1 \times 10^4)$

19. **SPACE EXPLORATION** A space probe that is 2.85×10^9 miles away from Earth sends radio signals to NASA. If the radio signals travel at the speed of light (1.86×10^5 miles per second), how long will it take the signals to reach NASA?

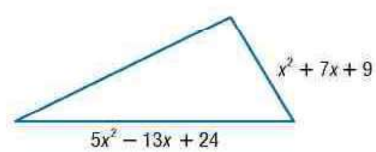
Find the degree of each polynomial. Then arrange the terms so that the powers of y are in descending order.

20. $2y^2 + 8y^4 + 9y$
21. $5xy - 7 + 2y^4 - x^2y^3$

Find each sum or difference.

22. $(5a + 3a^2 - 7a^3) + (2a - 8a^2 + 4)$
23. $(x^3 - 3x^2y + 4xy^2 + y^3) - (7x^3 + x^2y - 9xy^2 + y^3)$

24. **GEOMETRY** The measures of two sides of a triangle are given. If the perimeter is represented by $11x^2 - 29x + 10$, find the measure of the third side.



Simplify.

25. $(h - 5)^2$
26. $(4x - y)(4x + y)$
27. $3x^2y^3(2x - xy^2)$
28. $(2a^2b + b^2)^2$
29. $(4m + 3n)(2m - 5n)$
30. $(2c + 5)(3c^2 - 4c + 2)$

Solve each equation.

31. $2x(x - 3) = 2(x^2 - 7) + 2$
32. $3a(a^2 + 5) - 11 = a(3a^2 + 4)$

33. **STANDARDIZED TEST PRACTICE** If $x^2 + 2xy + y^2 = 8$, find $3(x + y)^2$.

Ⓐ 2 Ⓑ 4 Ⓒ 24 Ⓓ cannot be determined

Chapter 8 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A basketball team scored the following points during the first five games of the season: 70, 65, 75, 70, 80. During the sixth game, they scored only 30 points. Which of these measures changed the most as a result of the sixth game? (Lessons 2-2 and 2-5)

Ⓐ mean

Ⓑ median

Ⓒ mode

Ⓓ They all changed the same amount.

2. A machine produces metal bottle caps. The number of caps it produces is proportional to the number of minutes the machine operates. The machine produces 2100 caps in 60 minutes. How many minutes would it take the machine to produce 5600 caps? (Lesson 2-6)

Ⓐ 35 Ⓑ 58.3 Ⓒ 93.3 Ⓓ 160

3. The odometer on Juliana's car read 20,542 miles when she started a trip. After 4 hours of driving, the odometer read 20,750 miles. Which equation can be used to find r, her average rate of speed for the 4 hours? (Lesson 3-1)

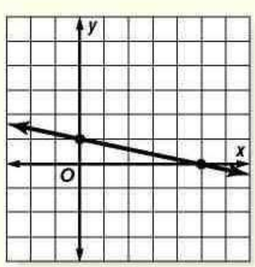
Ⓐ $r = 20{,}750 - 20{,}542$

Ⓑ $r = 4(20{,}750 - 20{,}542)$

Ⓒ $r = \frac{20{,}750}{4}$

Ⓓ $r = \frac{20{,}750 - 20{,}542}{4}$

4. Which equation best describes the graph? (Lesson 5-4)



Ⓐ $y = -\frac{1}{5}x + 1$

Ⓑ $y = -5x + 1$

Ⓒ $y = \frac{1}{5}x + 5$

Ⓓ $y = -5x - 5$

5. Which equation represents the line that passes through the point at $(-1, 4)$ and has a slope of -2? (Lesson 5-5)

Ⓐ $y = -2x - 2$ Ⓑ $y = -2x + 2$

Ⓒ $y = -2x + 6$ Ⓓ $y = -2x + 7$

6. Mr. Puram is planning an addition to the school library. The budget is \$7500. Each bookcase costs \$125, and each set of table and chairs costs \$550. If he buys 4 sets of tables and chairs, which inequality shows the number of bookcases b he can buy? (Lesson 6-6)

Ⓐ $4(550) + 125b \leq 7500$

Ⓑ $125b \leq 7500$

Ⓒ $4(550 + 125)b \leq 7500$

Ⓓ $4(125) + 550b \leq 7500$

7. Sophia and Allie went shopping and spent \$122 altogether. Sophia spent \$25 less than twice as much as Allie. How much did Allie spend? (Lesson 7-2)

Ⓐ \$39 Ⓑ \$49 Ⓒ \$53 Ⓓ \$73

8. The product of $2x^3$ and $4x^4$ is (Lesson 8-1)

Ⓐ $8x^{12}$. Ⓑ $6x^{12}$. Ⓒ $6x^7$. Ⓓ $8x^7$.

9. If 0.00037 is expressed as 3.7×10^n, what is the value of n? (Lesson 8-3)

Ⓐ -5 Ⓑ -4 Ⓒ 4 Ⓓ 5

10. When $x^2 - 2x + 1$ is subtracted from $3x^2 - 4x + 5$, the result will be (Lesson 8-5)

Ⓐ $2x^2 - 2x + 4$. Ⓑ $2x^2 - 6x + 4$.

Ⓒ $3x^2 - 6x + 6$. Ⓓ $4x^2 - 6x + 6$.

The Princeton Review Test-Taking Tip

Question 5 When you write an equation, check that the given values make a true statement. For example, in Question 5, substitute the values of the coordinates $(-1, 4)$ into your equation to check.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Find the 15th term in the arithmetic sequence $-20, -11, -2, 7, \ldots$. (Lesson 4-7)

12. Write a function that includes all of the ordered pairs in the table. (Lesson 4-8)

x	-3	-1	1	3	4
y	12	4	-4	-12	-16

13. Find the y-intercept of the line represented by $3x - 2y + 8 = 0$. (Lesson 5-4)

14. Graph the solution of the linear inequality $3x - y \leq 2$. (Lesson 6-6)

15. Let $P = 3x^2 - 2x - 1$ and $Q = -x^2 + 2x - 2$. Find $P + Q$. (Lesson 8-5)

16. Find $(x^2 + 1)(x - 3)$. (Lesson 8-7)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

(A) the quantity in Column A is greater,

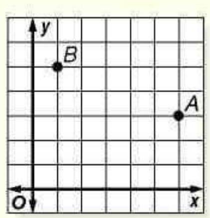
(B) the quantity in Column B is greater,

(C) the two quantities are equal, or

(D) the relationship cannot be determined from the information given.

Column A	Column B

17.



the domain of point A	the range of point B

(Lesson 4-3)

18.

$4x - 10 \geq 20$	$\frac{-6(x-1)}{8} \geq 3$

(Lesson 6-3)

19.

the x value in the solution of $x - 3y = 2$ and $x + 3y = 0$	the x value in the solution of $3x + 8y = 6$ and $x - 8y = 2$

(Lesson 7-3)

20.

$\frac{2b^{-3}c^2}{4bc}$	$\frac{10b^4}{20b^8c^{-1}}$

(Lesson 8-2)

21.

5.01×10^{-2}	50.1×10^{-4}

(Lesson 8-3)

22.

the degree of $x^2 + 5 - 6x + 13x^3$	the degree of $10 - y - 2y^2 - 4y^3$

(Lesson 8-4)

23. $m^2 + n^2 = 10$ and $mn = -6$

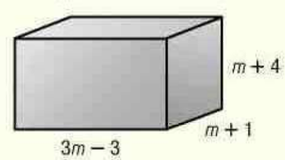
$(m+n)^2$	$(m-n)^2$

(Lesson 8-8)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

24. Use the rectangular prism below to solve the following problems. (Lessons 8-1 and 8-7)



a. Write a polynomial expression that represents the surface area of the top of the prism.

b. Write a polynomial expression that represents the surface area of the front of the prism.

c. Write a polynomial expression that represents the volume of the prism.

d. If $m = 2$ centimeters, then what is the volume of the prism?

www.algebra1.com/standardized_test

CONTENTS