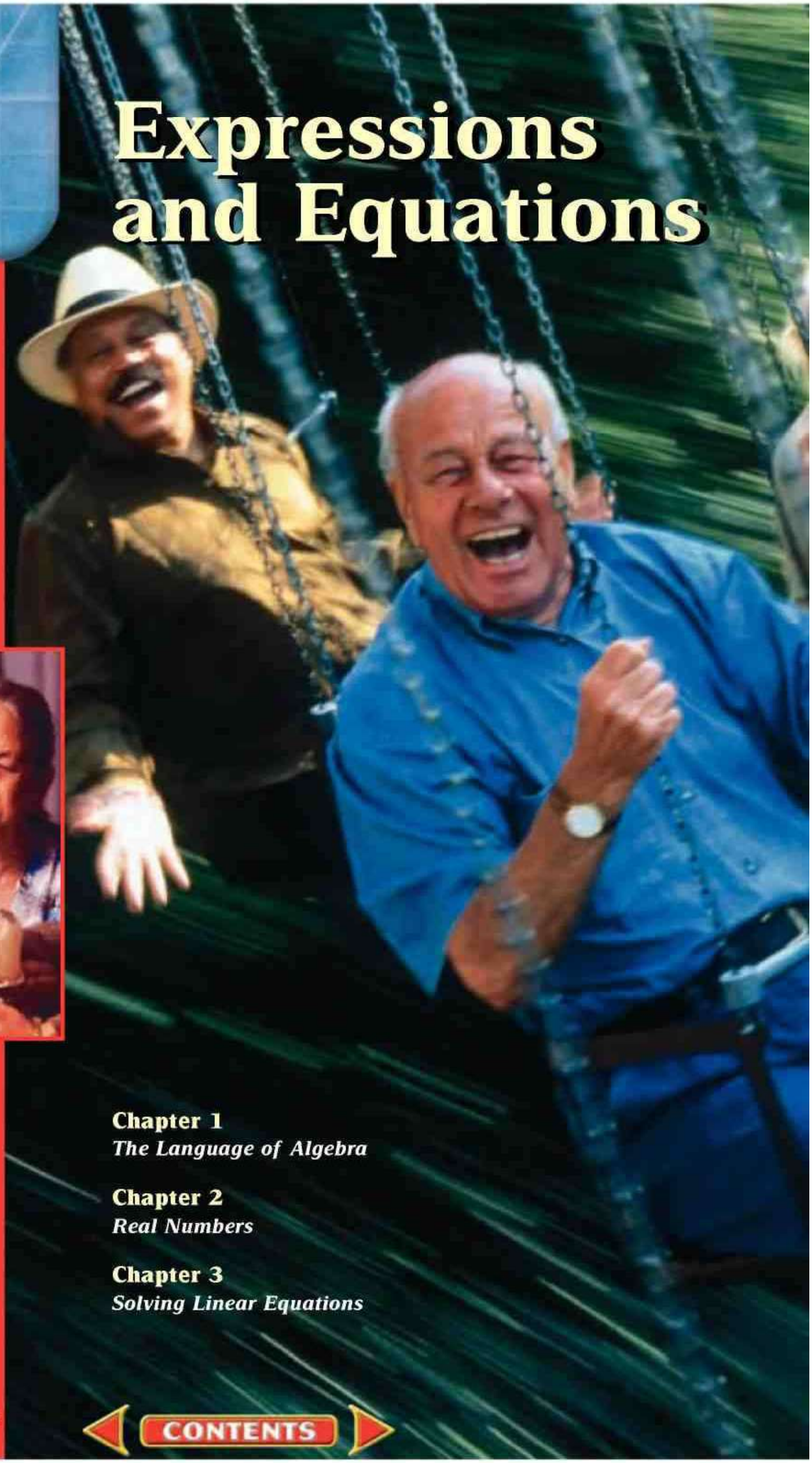
UNIT 1

Expressions and Equations

You can use algebraic expressions and equations to model and analyze real-world situations. In this unit, you will learn about expressions, equations, and graphs.



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WebQuest Internet Project

Can You Fit 100 Candles on a Cake?

Source: *USA TODAY,* January, 2001

"The mystique of living to be 100 will be lost by the year 2020 as 100th birthdays become commonplace, predicts Mike Parker, assistant professor of social work, University of Alabama, Tuscaloosa, and a gerontologist specializing in successful aging. He says that, in the 21st century, the fastest growing age group in the country will be centenarians—those who live 100 years or longer." In this project, you will explore how equations, functions, and graphs can help represent aging and population growth.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.

Lesson	1-9	2-6	3-6
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Chapter 1

The Language of Algebra

What You'll Learn

- **Lesson 1-1** Write algebraic expressions.
- **Lessons 1-2 and 1-3** Evaluate expressions and solve open sentences.
- **Lessons 1-4 through 1-6** Use algebraic properties of identity and equality.
- **Lesson 1-7** Use conditional statements and counterexamples.
- **Lessons 1-8 and 1-9** Interpret graphs of functions and analyze data in statistical graphs.

Key Vocabulary

- variable (p. 6)
- order of operations (p. 11)
- identity (p. 21)
- like terms (p. 28)
- counterexample (p. 38)

Why It's Important

In every state and in every country, you find unique and inspiring architecture. Architects can use algebraic expressions to describe the volume of the structures they design. A few of the shapes these buildings can resemble are a rectangle, a pentagon, or even a pyramid. *You will find the amount of space occupied by a pyramid in Lesson 1-2.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lessons 1-1, 1-2, and 1-3 — Multiply and Divide Whole Numbers

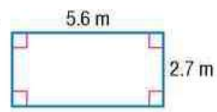
Find each product or quotient.

1. $8 \cdot 8$ **2.** $4 \cdot 16$ **3.** $18 \cdot 9$ **4.** $23 \cdot 6$

5. $57 \div 3$ **6.** $68 \div 4$ **7.** $\frac{72}{3}$ **8.** $\frac{90}{6}$

For Lessons 1-1, 1-2, 1-5, and 1-6 — Find Perimeter

Find the perimeter of each figure. *(For review, see pages 820 and 821.)*

9.


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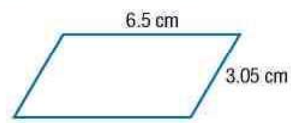

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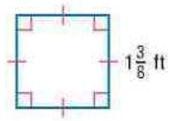

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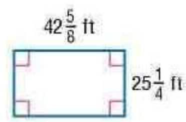


For Lessons 1-5 and 1-6 — Multiply and Divide Decimals and Fractions

Find each product or quotient. *(For review, see page 821.)*

13. $6 \cdot 1.2$ **14.** $0.5 \cdot 3.9$ **15.** $3.24 \div 1.8$ **16.** $10.64 \div 1.4$

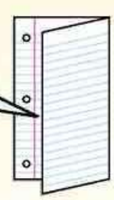
17. $\frac{3}{4} \cdot 12$ **18.** $1\frac{2}{3} \cdot \frac{3}{4}$ **19.** $\frac{5}{16} \div \frac{9}{12}$ **20.** $\frac{5}{6} \div \frac{2}{3}$

FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about algebraic properties. Begin with a sheet of notebook paper.

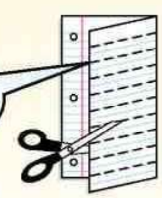
Step 1 Fold

Fold lengthwise to the holes.



Step 2 Cut

Cut along the top line and then cut 9 tabs.



Step 3 Label

Label the tabs using the lesson numbers and concepts.

Reading and Writing Store the Foldable in a 3-ring binder. As you read and study the chapter, write notes and examples under the tabs.

1-1 Variables and Expressions

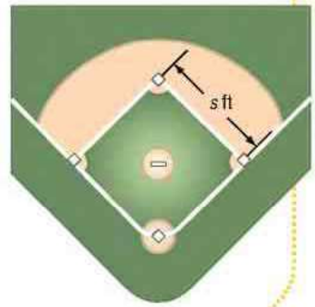
What You'll Learn

- Write mathematical expressions for verbal expressions.
- Write verbal expressions for mathematical expressions.

Vocabulary

- variables
- algebraic expression
- factors
- product
- power
- base
- exponent
- evaluate

What expression can be used to find the perimeter of a baseball diamond?



A baseball infield is a square with a base at each corner. Each base lies the same distance from the next one. Suppose s represents the length of each side of the square. Since the infield is a square, you can use the expression 4 times s, or $4s$ to find the perimeter of the square.

WRITE MATHEMATICAL EXPRESSIONS In the algebraic expression $4s$, the letter s is called a variable. In algebra, **variables** are symbols used to represent unspecified numbers or values. Any letter may be used as a variable. *The letter s was used above because it is the first letter of the word side.*

An **algebraic expression** consists of one or more numbers and variables along with one or more arithmetic operations. Here are some examples of algebraic expressions.

$$5x \qquad 3x - 7 \qquad 4 + \frac{p}{q} \qquad m \times 5n \qquad 3ab \div 5cd$$

In algebraic expressions, a raised dot or parentheses are often used to indicate multiplication as the symbol $\times$ can be easily mistaken for the letter x. Here are several ways to represent the product of x and y.

$$xy \qquad x \cdot y \qquad x(y) \qquad (x)y \qquad (x)(y)$$

In each expression, the quantities being multiplied are called **factors**, and the result is called the **product**.

It is often necessary to translate verbal expressions into algebraic expressions.

Example 1 Write Algebraic Expressions

Write an algebraic expression for each verbal expression.

a. eight more than a number n

The words *more than* suggest addition.

$$\underbrace{8}_{\text{eight}} \quad \underbrace{+}_{\text{more than}} \quad \underbrace{n}_{\text{a number } n}$$

Thus, the algebraic expression is $8 + n$.

b. the difference of 7 and 4 times a number x

Difference implies subtract, and *times* implies multiply. So the expression can be written as $7 - 4x$.

c. one third of the size of the original area a

The word *of* implies multiply, so the expression can be written as $\frac{1}{3}a$ or $\frac{a}{3}$.

An expression like x^n is called a **power** and is read "x to the nth power." The variable x is called the **base**, and n is called the **exponent**. The exponent indicates the number of times the base is used as a factor.

Symbols	Words	Meaning
3^1	3 to the first power	3
3^2	3 to the second power or 3 squared	$3 \cdot 3$
3^3	3 to the third power or 3 cubed	$3 \cdot 3 \cdot 3$
3^4	3 to the fourth power	$3 \cdot 3 \cdot 3 \cdot 3$
$2b^6$	2 times b to the sixth power	$2 \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$
x^n	x to the nth power	$\underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$

By definition, for any nonzero number x, $x^0 = 1$.

Study Tip

Reading Math

When no exponent is shown, it is understood to be 1. For example, $a = a^1$.

Example 2 Write Algebraic Expressions with Powers

Write each expression algebraically.

a. the product of 7 and m to the fifth power

$7m^5$

b. the difference of 4 and x squared

$4 - x^2$

To **evaluate** an expression means to find its value.

Example 3 Evaluate Powers

Evaluate each expression.

a. 2^6

$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ Use 2 as a factor 6 times.

$= 64$ Multiply.

b. 4^3

$4^3 = 4 \cdot 4 \cdot 4$ Use 4 as a factor 3 times.

$= 64$ Multiply.

WRITE VERBAL EXPRESSIONS Another important skill is translating algebraic expressions into verbal expressions.

Example 4 Write Verbal Expressions

Write a verbal expression for each algebraic expression.

a. $4m^3$

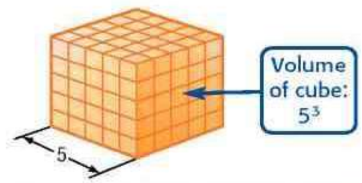
the product of 4 and m to the third power

b. $c^2 + 21d$

the sum of c squared and 21 times d

c. 5^3

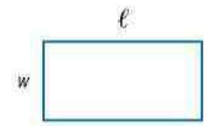
five to the third power or five cubed



Check for Understanding

Concept Check

1. **Explain** the difference between an algebraic expression and a verbal expression.
2. **Write** an expression that represents the perimeter of the rectangle.
3. **OPEN ENDED** Give an example of a variable to the fifth power.



Guided Practice

Write an algebraic expression for each verbal expression.

4. the sum of j and 13
5. 24 less than three times a number

Evaluate each expression.

6. 9^2
7. 4^4

Write a verbal expression for each algebraic expression.

8. $4m^4$
9. $\frac{1}{2}n^3$

Application

10. **MONEY** Lorenzo bought several pounds of chocolate-covered peanuts and gave the cashier a \$20 bill. Write an expression for the amount of change he will receive if p represents the cost of the peanuts.

Practice and Apply

Homework Help

For Exercises	See Examples
11–18	1, 2
21–28	3
31–42	4

Extra Practice

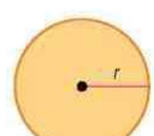
See page 820.

Write an algebraic expression for each verbal expression.

11. the sum of 35 and z
12. the sum of a number and 7
13. the product of 16 and p
14. the product of 5 and a number
15. 49 increased by twice a number
16. 18 and three times d
17. two-thirds the square of a number
18. one-half the cube of n

19. **SAVINGS** Kendra is saving to buy a new computer. Write an expression to represent the amount of money she will have if she has s dollars saved and she adds d dollars per week for the next 12 weeks.

20. **GEOMETRY** The area of a circle can be found by multiplying the number π by the square of the radius. If the radius of a circle is r, write an expression that represents the area of the circle.



Evaluate each expression.

21. 6^2
22. 8^2
23. 3^4
24. 6^3
25. 3^5
26. 15^3
27. 10^6
28. 100^3

29. **FOOD** A bakery sells a dozen bagels for \$8.50 and a dozen donuts for \$3.99. Write an expression for the cost of buying b dozen bagels and d dozen donuts.

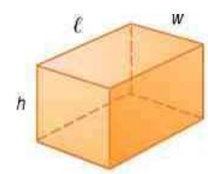
30. **TRAVEL** Before starting her vacation, Sari's car had 23,500 miles on the odometer. She drives an average of m miles each day for two weeks. Write an expression that represents the mileage on Sari's odometer after her trip.

Write a verbal expression for each algebraic expression.

31. $7p$	**32.** $15r$	**33.** 3^3	**34.** 5^4
35. $3x^2 + 4$	**36.** $2n^3 + 12$	**37.** $a^4 \cdot b^2$	**38.** $n^3 \cdot p^5$
39. $\frac{12z^2}{5}$	**40.** $\frac{8g^3}{4}$	**41.** $3x^2 - 2x$	**42.** $4f^5 - 9k^3$

43. **PHYSICAL SCIENCE** When water freezes, its volume is increased by one-eleventh. In other words, the volume of ice equals the sum of the volume of the water and the product of one-eleventh and the volume of the water. If x cubic centimeters of water is frozen, write an expression for the volume of the ice that is formed.

44. **GEOMETRY** The surface area of a rectangular prism is the sum of:



- the product of twice the length ℓ and the width w,
- the product of twice the length and the height h, and
- the product of twice the width and the height.

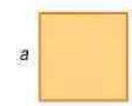
Write an expression that represents the surface area of a prism.

More About...

Recycling
In 2000, about 30% of all waste was recycled.
Source: U.S. Environmental Protection Agency

45. **RECYCLING** Each person in the United States produces approximately 3.5 pounds of trash each day. Write an expression representing the pounds of trash produced in a day by a family that has m members. **Source:** *Vitality*

46. **CRITICAL THINKING** In the square, the variable a represents a positive whole number. Find the value of a such that the area and the perimeter of the square are the same.



47. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

What expression can be used to find the perimeter of a baseball diamond?

Include the following in your answer:
- two different verbal expressions that you can use to describe the perimeter of a square, and
- an algebraic expression other than 4s that you can use to represent the perimeter of a square.

Standardized Test Practice

48. What is 6 more than 2 times a certain number x?

Ⓐ $2x - 6$ Ⓑ $2x$ Ⓒ $6x - 2$ Ⓓ $2x + 6$

49. Write $4 \cdot 4 \cdot 4 \cdot c \cdot c \cdot c \cdot c$ using exponents.

Ⓐ $3^4 4^c$ Ⓑ $4^3 c^4$ Ⓒ $(4c)^7$ Ⓓ $4c$

Maintain Your Skills

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
*(To review **operations with fractions**, see pages 798–801.)*

50. $14.3 + 1.8$	**51.** $10 - 3.24$	**52.** 1.04×4.3	**53.** $15.36 \div 4.8$
54. $\frac{1}{3} + \frac{2}{5}$	**55.** $\frac{3}{4} - \frac{1}{6}$	**56.** $\frac{3}{8} \times \frac{4}{9}$	**57.** $\frac{7}{10} \div \frac{3}{5}$

Reading Mathematics

Translating from English to Algebra

You learned in Lesson 1-1 that it is often necessary to translate words into algebraic expressions. Generally, there are "clue" words such as *more than*, *times*, *less than*, and so on, which indicate the operation to use. These words also help to connect numerical data. The table shows a few examples.

Words	Algebraic Expression
four times x plus y	$4x + y$
four times the sum of x and y	$4(x + y)$
four times the quantity x plus y	$4(x + y)$

Notice that all three expressions are worded differently, but the first expression is the only one that is different algebraically. In the second expression, parentheses indicate that the *sum*, $x + y$, is multiplied by four. In algebraic expressions, terms grouped by parentheses are treated as one quantity. So, $4(x + y)$ can also be read as *four times the quantity x plus y*.

Words that may indicate parentheses are *sum*, *difference*, *product*, and *quantity*.

Reading to Learn

Read each verbal expression aloud. Then match it with the correct algebraic expression.

1. nine divided by 2 plus n

2. four divided by the difference of n and six

3. n plus five squared

4. three times the quantity eight plus n

5. nine divided by the quantity 2 plus n

6. three times eight plus n

7. the quantity n plus five squared

8. four divided by n minus six

a. $(n + 5)^2$
b. $4 \div (n - 6)$
c. $9 \div 2 + n$
d. $3(8) + n$
e. $4 \div n - 6$
f. $n + 5^2$
g. $9 \div (2 + n)$
h. $3(8 + n)$

Write each algebraic expression in words.

9. $5x + 1$

10. $5(x + 1)$

11. $3 + 7x$

12. $(3 + x) \cdot 7$

13. $(6 + b) \div y$

14. $6 + (b \div y)$

1-2 Order of Operations

What You'll Learn

- Evaluate numerical expressions by using the order of operations.
- Evaluate algebraic expressions by using the order of operations.

Vocabulary

- order of operations

How is the monthly cost of internet service determined?

Nicole is signing up with a new internet service provider. The service costs \$4.95 a month, which includes 100 hours of access. If she is online for more than 100 hours, she must pay an additional \$0.99 per hour. Suppose Nicole is online for 117 hours the first month. The expression $4.95 + 0.99(117 - 100)$ represents what Nicole must pay for the month.

EVALUATE RATIONAL EXPRESSIONS Numerical expressions often contain more than one operation. A rule is needed to let you know which operation to perform first. This rule is called the **order of operations**.

Key Concept — *Order of Operations*

Step 1 Evaluate expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

Example 1 *Evaluate Expressions*

Evaluate each expression.

a. $3 + 2 \cdot 3 + 5$

$3 + 2 \cdot 3 + 5 = 3 + 6 + 5$ Multiply 2 and 3.

$= 9 + 5$ Add 3 and 6.

$= 14$ Add 9 and 5.

b. $15 \div 3 \cdot 5 - 4^2$

$15 \div 3 \cdot 5 - 4^2 = 15 \div 3 \cdot 5 - 16$ Evaluate powers.

$= 5 \cdot 5 - 16$ Divide 15 by 3.

$= 25 - 16$ Multiply 5 by 5.

$= 9$ Subtract 16 from 25.

Grouping symbols such as parentheses ( ), brackets [ ], and braces { } are used to clarify or change the order of operations. They indicate that the expression within the grouping symbol is to be evaluated first.

Study Tip

Grouping Symbols
When more than one grouping symbol is used, start evaluating within the innermost grouping symbols.

Example 2 Grouping Symbols

Evaluate each expression.

a. $2(5) + 3(4 + 3)$

$$\begin{aligned} 2(5) + 3(4 + 3) &= 2(5) + 3(7) && \text{Evaluate inside grouping symbols.} \\ &= 10 + 21 && \text{Multiply expressions left to right.} \\ &= 31 && \text{Add 10 and 21.} \end{aligned}$$

b. $2[5 + (30 \div 6)^2]$

$$\begin{aligned} 2[5 + (30 \div 6)^2] &= 2[5 + (5)^2] && \text{Evaluate innermost expression first.} \\ &= 2[5 + 25] && \text{Evaluate power inside grouping symbol.} \\ &= 2[30] && \text{Evaluate expression in grouping symbol.} \\ &= 60 && \text{Multiply.} \end{aligned}$$

A fraction bar is another type of grouping symbol. It indicates that the numerator and denominator should each be treated as a single value.

Example 3 Fraction Bar

Evaluate $\frac{6 + 4^2}{3^2 \cdot 4}$.

$\frac{6 + 4^2}{3^2 \cdot 4}$ means $(6 + 4^2) \div (3^2 \cdot 4)$.

$$\begin{aligned} \frac{6 + 4^2}{3^2 \cdot 4} &= \frac{6 + 16}{3^2 \cdot 4} && \text{Evaluate the power in the numerator.} \\ &= \frac{22}{3^2 \cdot 4} && \text{Add 6 and 16 in the numerator.} \\ &= \frac{22}{9 \cdot 4} && \text{Evaluate the power in the denominator.} \\ &= \frac{22}{36} \text{ or } \frac{11}{18} && \text{Multiply 9 and 4 in the denominator. Then simplify.} \end{aligned}$$

EVALUATE ALGEBRAIC EXPRESSIONS Like numerical expressions, algebraic expressions often contain more than one operation. Algebraic expressions can be evaluated when the values of the variables are known. First, replace the variables with their values. Then, find the value of the numerical expression using the order of operations.

Example 4 Evaluate an Algebraic Expression

Evaluate $a^2 - (b^3 - 4c)$ if $a = 7$, $b = 3$, and $c = 5$.

$$\begin{aligned} a^2 - (b^3 - 4c) &= 7^2 - (3^3 - 4 \cdot 5) && \text{Replace } a \text{ with 7, } b \text{ with 3, and } c \text{ with 5.} \\ &= 7^2 - (27 - 4 \cdot 5) && \text{Evaluate } 3^3. \\ &= 7^2 - (27 - 20) && \text{Multiply 4 and 5.} \\ &= 7^2 - 7 && \text{Subtract 20 from 27.} \\ &= 49 - 7 && \text{Evaluate } 7^2. \\ &= 42 && \text{Subtract.} \end{aligned}$$

Career Choices

Architect

Architects must consider the function, safety, and needs of people, as well as appearance when they design buildings.

Online Research For more information about a career as an architect, visit: www.algebra1.com/careers

Example 5 Use Algebraic Expressions

ARCHITECTURE **The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. The area of its base is 360,000 square feet, and it is 321 feet high. The volume of any pyramid is one third of the product of the area of the base B and its height h.**

a. Write an expression that represents the volume of a pyramid.

$$\underbrace{\frac{1}{3}}_{\text{one third}} \quad \underbrace{\times}_{\text{of}} \quad \underbrace{(B \cdot h)}_{\text{the product of area of base and height}} \quad \text{or } \frac{1}{3}Bh$$

b. Find the volume of the Pyramid Arena.

Evaluate $\frac{1}{3}(Bh)$ for $B = 360{,}000$ and $h = 321$.

$\frac{1}{3}(Bh) = \frac{1}{3}(360{,}000 \cdot 321)$ $B = 360{,}000$ and $h = 321$

$= \frac{1}{3}(115{,}560{,}000)$ Multiply 360,000 by 321.

$= \frac{115{,}560{,}000}{3}$ Multiply $\frac{1}{3}$ by 115,560,000.

$= 38{,}520{,}000$ Divide 115,560,000 by 3.

The volume of the Pyramid Arena is 38,520,000 cubic feet.

Check for Understanding

Concept Check

1. **Describe** how to evaluate $8[6^2 - 3(2 + 5)] \div 8 + 3$.
2. **OPEN ENDED** Write an expression involving division in which the first step in evaluating the expression is addition.
3. **FIND THE ERROR** Laurie and Chase are evaluating $3[4 + (27 \div 3)]^2$.

Laurie

$3[4 + (27 \div 3)]^2 = 3(4 + 9^2)$
$= 3(4 + 81)$
$= 3(85)$
$= 255$

Chase

$3[4 + (27 \div 3)]^2 = 3(4 + 9)^2$
$= 3(13)^2$
$= 3(169)$
$= 507$

Who is correct? Explain your reasoning.

Guided Practice

Evaluate each expression.

4. $(4 + 6)7$
5. $50 - (15 + 9)$
6. $29 - 3(9 - 4)$
7. $[7(2) - 4] + [9 + 8(4)]$
8. $\frac{(4 \cdot 3)^2 \cdot 5}{9 + 3}$
9. $\frac{3 + 2^3}{5^2(4)}$

Evaluate each expression if $g = 4$, $h = 6$, $j = 8$, and $k = 12$.

10. $hk - gj$
11. $2k + gh^2 - j$
12. $\frac{2g(h - g)}{gh - j}$

Application

SHOPPING **For Exercises 13 and 14, use the following information.**
A computer store has certain software on sale at 3 for \$20.00, with a limit of 3 at the sale price. Additional software is available at the regular price of \$9.95 each.

13. Write an expression you could use to find the cost of 5 software packages.
14. How much would 5 software packages cost?

Practice and Apply

Homework Help

For Exercises	See Examples
15–28	1–3
29–31	5
32–39	4, 5

Extra Practice

See page 820.

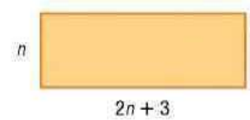
Evaluate each expression.

15. $(12 - 6) \cdot 2$
16. $(16 - 3) \cdot 4$
17. $15 + 3 \cdot 2$
18. $22 + 3 \cdot 7$
19. $4(11 + 7) - 9 \cdot 8$
20. $12(9 + 5) - 6 \cdot 3$
21. $12 \div 3 \cdot 5 - 4^2$
22. $15 \div 3 \cdot 5 - 4^2$
23. $288 \div [3(9 + 3)]$

Evaluate each expression.

24. $390 \div [5(7 + 6)]$
25. $\dfrac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}$
26. $\dfrac{4 \cdot 6^2 - 4^2 \cdot 6}{4 \cdot 6}$
27. $\dfrac{[(8 + 5)(6 - 2)^2] - (4 \cdot 17 \div 2)}{[(24 \div 2) \div 3]}$
28. $6 - \left[\dfrac{2 + 7}{3} - (2 \cdot 3 - 5)\right]$

29. **GEOMETRY** Find the area of the rectangle when $n = 4$ centimeters.



ENTERTAINMENT **For Exercises 30 and 31, use the following information.**
Derrick and Samantha are selling tickets for their school musical. Floor seats cost \$7.50 and balcony seats cost \$5.00. Samantha sells 60 floor seats and 70 balcony seats, Derrick sells 50 floor seats and 90 balcony seats.

30. Write an expression to show how much money Samantha and Derrick have collected for tickets.

31. Evaluate the expression to determine how much they collected.

Evaluate each expression if $x = 12$, $y = 8$, and $z = 3$.

32. $x + y^2 + z^2$
33. $x^3 + y + z^3$
34. $3xy - z$
35. $4x - yz$
36. $\dfrac{2xy - z^3}{z}$
37. $\dfrac{xy^2 - 3z}{3}$
38. $\left(\dfrac{x}{y}\right)^2 - \dfrac{3y - z}{(x - y)^2}$
39. $\dfrac{x - z^2}{y \div x} + \dfrac{2y - x}{y^2 \div 2}$

40. **BIOLOGY** Most bacteria reproduce by dividing into identical cells. This process is called *binary fission*. A certain type of bacteria can double its numbers every 20 minutes. Suppose 100 of these cells are in one culture dish and 250 of the cells are in another culture dish. Write and evaluate an expression that shows the total number of bacteria cells in both dishes after 20 minutes.

BUSINESS **For Exercises 41–43, use the following information.**
Mr. Martinez is a sales representative for an agricultural supply company. He receives a salary and monthly commission. He also receives a bonus each time he reaches a sales goal.

41. Write a verbal expression that describes how much Mr. Martinez earns in a year if he receives four equal bonuses.

42. Let e represent earnings, s represent his salary, c represent his commission, and b represent his bonus. Write an algebraic expression to represent his earnings if he receives four equal bonuses.

43. Suppose Mr. Martinez's annual salary is \$42,000 and his average commission is \$825 each month. If he receives four bonuses of \$750 each, how much does he earn in a year?

44. **CRITICAL THINKING** Choose three numbers from 1 to 6. Write as many expressions as possible that have different results when they are evaluated. You must use all three numbers in each expression, and each can only be used once.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

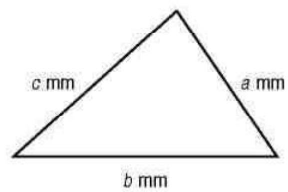
How is the monthly cost of internet service determined?

Include the following in your answer:

- an expression for the cost of service if Nicole has a coupon for $25 off her base rate for her first six months, and
- an explanation of the advantage of using an algebraic expression over making a table of possible monthly charges.

Standardized Test Practice

46. Find the perimeter of the triangle using the formula $P = a + b + c$ if $a = 10$, $b = 12$, and $c = 17$.



Ⓐ 39 mm   Ⓑ 19.5 mm
Ⓒ 60 mm   Ⓓ 78 mm

47. Evaluate $(5 - 1)^3 + (11 - 2)^2 + (7 - 4)^3$.

Ⓐ 586   Ⓑ 172   Ⓒ 106   Ⓓ 39

Graphing Calculator

EVALUATING EXPRESSIONS Use a calculator to evaluate each expression.

48. $\frac{0.25x^2}{7x^3}$ if $x = 0.75$ **49.** $\frac{2x^2}{x^2 - x}$ if $x = 27.89$ **50.** $\frac{x^3 + x^2}{x^3 - x^2}$ if $x = 12.75$

Maintain Your Skills

Mixed Review

Write an algebraic expression for each verbal expression. *(Lesson 1-1)*

51. the product of the third power of a and the fourth power of b

52. six less than three times the square of y

53. the sum of a and b increased by the quotient of b and a

54. four times the sum of r and s increased by twice the difference of r and s

55. triple the difference of 55 and the cube of w

Evaluate each expression. *(Lesson 1-1)*

56. 2^4 **57.** 12^1 **58.** 8^2 **59.** 4^4

Write a verbal expression for each algebraic expression. *(Lesson 1-1)*

60. $5n + \frac{n}{2}$ **61.** $q^2 - 12$ **62.** $\frac{(x + 3)}{(x - 2)^2}$ **63.** $\frac{x^3}{9}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the value of each expression.**
*(To review **operations with decimals and fractions**, see pages 798–801.)*

64. $0.5 - 0.0075$ **65.** $5.6 + 1.612$ **66.** $14.9968 \div 5.2$ **67.** $2.3(6.425)$

68. $4\frac{1}{8} - 1\frac{1}{2}$ **69.** $\frac{3}{5} + 2\frac{5}{7}$ **70.** $\frac{5}{6} \cdot \frac{4}{5}$ **71.** $8 \div \frac{2}{9}$

1-3 Open Sentences

What You'll Learn

- Solve open sentence equations.
- Solve open sentence inequalities.

Vocabulary

- open sentence
- solving an open sentence
- solution
- equation
- replacement set
- set
- element
- solution set
- inequality

How can you use open sentences to stay within a budget?

The Daily News sells garage sale kits. The Spring Creek Homeowners Association is planning a community garage sale, and their budget for advertising is \$135. The expression $15.50 + 5n$ can be used to represent the cost of purchasing $n + 1$ kits. The open sentence $15.50 + 5n \leq 135$ can be used to ensure that the budget is met.

SOLVE EQUATIONS A mathematical statement with one or more variables is called an **open sentence**. An open sentence is neither true nor false until the variables have been replaced by specific values. The process of finding a value for a variable that results in a true sentence is called **solving the open sentence**. This replacement value is called a **solution** of the open sentence. A sentence that contains an equals sign, =, is called an **equation**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A **set** is a collection of objects or numbers. It is often shown using braces, { }, and is usually named by a capital letter. Each object or number in the set is called an **element**, or member. The **solution set** of an open sentence is the set of elements from the replacement set that make an open sentence true.

Example 1 Use a Replacement Set to Solve an Equation

Find the solution set for each equation if the replacement set is {3, 4, 5, 6, 7}.

a. $6n + 7 = 37$

Replace n in $6n + 7 = 37$ with each value in the replacement set.

n	$6n + 7 = 37$	True or False?
3	$6(3) + 7 \stackrel{?}{=} 37 \rightarrow 25 \neq 37$	false
4	$6(4) + 7 \stackrel{?}{=} 37 \rightarrow 31 \neq 37$	false
5	$6(5) + 7 \stackrel{?}{=} 37 \rightarrow 37 = 37$	true ✓
6	$6(6) + 7 \stackrel{?}{=} 37 \rightarrow 43 \neq 37$	false
7	$6(7) + 7 \stackrel{?}{=} 37 \rightarrow 49 \neq 37$	false

Since $n = 5$ makes the equation true, the solution of $6n + 7 = 37$ is 5.

The solution set is {5}.

b. $5(x + 2) = 40$

Replace x in $5(x + 2) = 40$ with each value in the replacement set.

x	$5(x + 2) = 40$	True or False?
3	$5(3 + 2) \stackrel{?}{=} 40 \rightarrow 25 \neq 40$	false
4	$5(4 + 2) \stackrel{?}{=} 40 \rightarrow 30 \neq 40$	false
5	$5(5 + 2) \stackrel{?}{=} 40 \rightarrow 35 \neq 40$	false
6	$5(6 + 2) \stackrel{?}{=} 40 \rightarrow 40 = 40$	true ✓
7	$5(7 + 2) \stackrel{?}{=} 40 \rightarrow 45 \neq 40$	false

The solution of $5(x + 2) = 40$ is 6. The solution set is $\{6\}$.

You can often solve an equation by applying the order of operations.

Example 2 Use Order of Operations to Solve an Equation

Solve $\frac{13 + 2(4)}{3(5 - 4)} = q$.

$\frac{13 + 2(4)}{3(5 - 4)} = q$ Original equation

$\frac{13 + 8}{3(1)} = q$ Multiply 2 and 4 in the numerator. Subtract 4 from 5 in the denominator.

$\frac{21}{3} = q$ Simplify.

$7 = q$ Divide.

The solution is 7.

Study Tip

Reading Math

Inequality symbols are read as follows.

$<$ *is less than*

$\leq$ *is less than or equal to*

$>$ *is greater than*

$\geq$ *is greater than or equal to*

SOLVE INEQUALITIES An open sentence that contains the symbol $<$, $\leq$, $>$, or $\geq$ is called an **inequality**. Inequalities can be solved in the same way as equations.

Example 3 Find the Solution Set of an Inequality

Find the solution set for $18 - y < 10$ if the replacement set is $\{7, 8, 9, 10, 11, 12\}$.

Replace y in $18 - y < 10$ with each value in the replacement set.

y	$18 - y < 10$	True or False?
7	$18 - 7 \stackrel{?}{<} 10 \rightarrow 11 \nless 10$	false
8	$18 - 8 \stackrel{?}{<} 10 \rightarrow 10 \nless 10$	false
9	$18 - 9 \stackrel{?}{<} 10 \rightarrow 9 < 10$	true ✓
10	$18 - 10 \stackrel{?}{<} 10 \rightarrow 8 < 10$	true ✓
11	$18 - 11 \stackrel{?}{<} 10 \rightarrow 7 < 10$	true ✓
12	$18 - 12 \stackrel{?}{<} 10 \rightarrow 6 < 10$	true ✓

The solution set for $18 - y < 10$ is $\{9, 10, 11, 12\}$.

Example 4 Solve an Inequality

FUND-RAISING **Refer to the application at the beginning of the lesson. How many garage sale kits can the association buy and stay within their budget?**

Explore The association can spend no more than \$135. So the situation can be represented by the inequality $15.50 + 5n \leq 135$.

(continued on the next page)

Plan Since no replacement set is given, estimate to find reasonable values for the replacement set.

Solve Start by letting $n = 10$ and then adjust values up or down as needed.

$15.50 + 5n \leq 135$ Original inequality

$15.50 + 5(10) \leq 135$ $n = 10$

$15.50 + 50 \leq 135$ Multiply 5 and 10.

$65.50 \leq 135$ Add 15.50 and 50.

The estimate is too low. Increase the value of n.

n	$15.50 + 5n \leq 135$	Reasonable?
20	$15.50 + 5(20) \overset{?}{\leq} 135 \rightarrow 115.50 \leq 135$	too low
25	$15.50 + 5(25) \overset{?}{\leq} 135 \rightarrow 140.50 \not\leq 135$	too high
23	$15.50 + 5(23) \overset{?}{\leq} 135 \rightarrow 130.50 \leq 135$	almost
24	$15.50 + 5(24) \overset{?}{\leq} 135 \rightarrow 135.50 \leq 135$	too high

Study Tip

Reading Math

In {1, 2, 3, 4, ...}, the three dots are an *ellipsis*. In math, an ellipsis is used to indicate that numbers continue in the same pattern.

Examine The solution set is {0, 1, 2, 3, ..., 21, 22, 23}. In addition to the first kit, the association can buy as many as 23 additional kits. So, the association can buy as many as 1 + 23 or 24 garage sale kits and stay within their budget.

Check for Understanding

Concept Check

1. **Describe** the difference between an expression and an open sentence.
2. **OPEN ENDED** Write an inequality that has a solution set of {8, 9, 10, 11, ...}.
3. **Explain** why an open sentence always has at least one variable.

Guided Practice

Find the solution of each equation if the replacement set is {10, 11, 12, 13, 14, 15}.

4. $3x - 7 = 29$
5. $12(x - 8) = 84$

Find the solution of each equation using the given replacement set.

6. $x + \frac{2}{5} = 1\frac{3}{20}; \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}\right\}$
7. $7.2(x + 2) = 25.92$; {1.2, 1.4, 1.6, 1.8}

Solve each equation.

8. $4(6) + 3 = x$
9. $w = \frac{14 - 8}{2}$

Find the solution set for each inequality using the given replacement set.

10. $24 - 2x \geq 13$; {0, 1, 2, 3, 4, 5, 6}
11. $3(12 - x) - 2 \leq 28$; {1.5, 2, 2.5, 3}

Application

NUTRITION **For Exercises 12 and 13, use the following information.**
A person must burn 3500 Calories to lose one pound of weight.

12. Write an equation that represents the number of Calories a person would have to burn a day to lose four pounds in two weeks.
13. How many Calories would the person have to burn each day?

Practice and Apply

Homework Help

For Exercises	See Examples
14–25	1
26–28	4
29–36	2
37–44	3

Extra Practice

See page 820.

Find the solution of each equation if the replacement sets are $A = \{0, 3, 5, 8, 10\}$ and $B = \{12, 17, 18, 21, 25\}$.

14. $b - 12 = 9$ **15.** $34 - b = 22$ **16.** $3a + 7 = 31$

17. $4a + 5 = 17$ **18.** $\frac{40}{a} - 4 = 0$ **19.** $\frac{b}{3} - 2 = 4$

Find the solution of each equation using the given replacement set.

20. $x + \frac{7}{4} = \frac{17}{8}; \left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$ **21.** $x + \frac{7}{12} = \frac{25}{12}; \left\{\frac{1}{2}, 1, 1\frac{1}{2}, 2\right\}$

22. $\frac{2}{5}(x + 1) = \frac{8}{15}; \left\{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}\right\}$ **23.** $2.7(x + 5) = 17.28; \{1.2, 1.3, 1.4, 1.5\}$

24. $16(x + 2) = 70.4; \{2.2, 2.4, 2.6, 2.8\}$ **25.** $21(x + 5) = 216.3; \{3.1, 4.2, 5.3, 6.4\}$

MOVIES **For Exercises 26–28, use the table and the following information.** The Conkle family is planning to see a movie. There are two adults, a daughter in high school, and two sons in middle school. They do not want to spend more than \$30.

26. The movie theater charges the same price for high school and middle school students. Write an inequality to show the cost for the family to go to the movies.

27. How much will it cost for the family to see a matinee?

28. How much will it cost to see an evening show?

Admission Prices		
	Evening	Matinee
Adult	\$7.50	All Seats \$4.50
Student	\$4.50	
Child	\$4.50	
Senior	\$3.50	

Solve each equation.

29. $14.8 - 3.75 = t$ **30.** $a = 32.4 - 18.95$ **31.** $y = \frac{12 \cdot 5}{15 - 3}$

32. $g = \frac{15 \cdot 6}{16 - 7}$ **33.** $d = \frac{7(3) + 3}{4(3 - 1)} + 6$ **34.** $a = \frac{4(14 - 1)}{3(6) - 5} + 7$

35. $p = \frac{1}{4}[7(2^3) + 4(5^2) - 6(2)]$ **36.** $n = \frac{1}{8}[6(3^2) + 2(4^3) - 2(7)]$

Find the solution set for each inequality using the given replacement set.

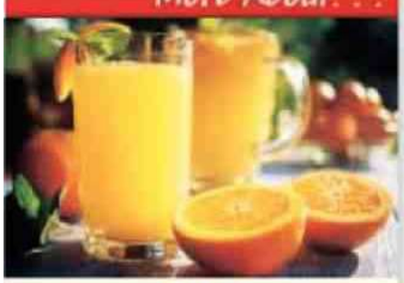
37. $a - 2 < 6; \{6, 7, 8, 9, 10, 11\}$ **38.** $a + 7 < 22; \{13, 14, 15, 16, 17\}$

39. $\frac{a}{5} \geq 2; \{5, 10, 15, 20, 25\}$ **40.** $\frac{2a}{4} \leq 8; \{12, 14, 16, 18, 20, 22\}$

41. $4a - 3 \geq 10.6; \{3.2, 3.4, 3.6, 3.8, 4\}$ **42.** $6a - 5 \geq 23.8; \{4.2, 4.5, 4.8, 5.1, 5.4\}$

43. $3a \leq 4; \left\{0, \frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}\right\}$ **44.** $2b < 5; \left\{1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3\right\}$

More About...



Food

During a lifetime, the average American drinks 15,579 glasses of milk, 6220 glasses of juice, and 18,995 glasses of soda.

Source: *USA TODAY*

FOOD **For Exercises 45 and 46, use the information about food at the left.**

45. Write an equation to find the total number of glasses of milk, juice, and soda the average American drinks in a lifetime.

46. How much milk, juice, and soda does the average American drink in a lifetime?

MAIL ORDER **For Exercises 47 and 48, use the following information.** Suppose you want to order several sweaters that cost \$39.00 each from an online catalog. There is a \$10.95 charge for shipping. You have \$102.50 to spend.

47. Write an inequality you could use to determine the maximum number of sweaters you can purchase.

48. What is the maximum number of sweaters you can buy?

www.algebra1.com/self_check_quiz

49. **CRITICAL THINKING** Describe the solution set for x if $3x \leq 1$.

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use open sentences to stay within a budget?

Include the following in your answer:

- an explanation of how to use open sentences to stay within a budget, and
- examples of real-world situations in which you would use an inequality and examples where you would use an equation.

Standardized Test Practice

51. Find the solution set for $\frac{(5 \cdot n)^2 + 5}{(9 \cdot 3^2) - n} < 28$ if the replacement set is {5, 7, 9, 11, 13}.

Ⓐ {5} Ⓑ {5, 7} Ⓒ {7} Ⓓ {7, 9}

52. Which expression has a value of 17?

Ⓐ $(9 \times 3) - 63 \div 7$ Ⓑ $6(3 + 2) \div (9 - 7)$

Ⓒ $27 \div 3 + (12 - 4)$ Ⓓ $2[2(6 - 3)] - 5$

Maintain Your Skills

Mixed Review

Write an algebraic expression for each verbal expression. Then evaluate each expression if $r = 2$, $s = 5$, and $t = \frac{1}{2}$. *(Lesson 1-2)*

53. r squared increased by 3 times s

54. t times the sum of four times s and r

55. the sum of r and s times the square of t

56. r to the fifth power decreased by t

Evaluate each expression. *(Lesson 1-2)*

57. $5^3 + 3(4^2)$ 58. $\frac{38 - 12}{2 \cdot 13}$ 59. $[5(2 + 1)]^4 + 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product. Express in simplest form.**
*(To review **multiplying fractions**, see pages 800 and 801.)*

60. $\frac{1}{6} \cdot \frac{2}{5}$ 61. $\frac{4}{9} \cdot \frac{3}{7}$ 62. $\frac{5}{6} \cdot \frac{15}{16}$ 63. $\frac{6}{14} \cdot \frac{12}{18}$

64. $\frac{8}{13} \cdot \frac{2}{11}$ 65. $\frac{4}{7} \cdot \frac{4}{9}$ 66. $\frac{3}{11} \cdot \frac{7}{16}$ 67. $\frac{2}{9} \cdot \frac{24}{25}$

Practice Quiz 1

Lessons 1-1 through 1-3

Write a verbal expression for each algebraic expression. *(Lesson 1-1)*

1. $x - 20$ 2. $5n + 2$ 3. a^3 4. $n^4 - 1$

Evaluate each expression. *(Lesson 1-2)*

5. $6(9) - 2(8 + 5)$ 6. $4[2 + (18 \div 9)^3]$ 7. $9(3) - 4^2 + 6^2 \div 2$ 8. $\frac{(5 - 2)^2}{3(4 \cdot 2 - 7)}$

9. Evaluate $\frac{5a^2 + c - 2}{6 + b}$ if $a = 4$, $b = 5$, and $c = 10$. *(Lesson 1-2)*

10. Find the solution set for $2n^2 + 3 \leq 75$ if the replacement set is {4, 5, 6, 7, 8, 9}. *(Lesson 1-3)*

1-4 Identity and Equality Properties

What You'll Learn

- Recognize the properties of identity and equality.
- Use the properties of identity and equality.

Vocabulary

- additive identity
- multiplicative identity
- multiplicative inverses
- reciprocal

How are identity and equality properties used to compare data?

During the college football season, teams are ranked weekly. The table shows the last three rankings of the top five teams for the 2000 football season. The open sentence below represents the change in rank of Oregon State from December 11 to the final rank.

	Dec. 4	Dec. 11	Final Rank
University of Oklahoma	1	1	1
University of Miami	2	2	2
University of Washington	4	3	3
Oregon State University	5	4	4
Florida State University	3	5	5

$$\underbrace{4}_{\text{Rank on December 11, 2000}} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{r}_{\text{increase in rank}} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{4}_{\text{final rank for 2000 season.}}$$

The solution of this equation is 0. Oregon State's rank changed by 0 from December 11 to the final rank. In other words, $4 + 0 = 4$.

IDENTITY AND EQUALITY PROPERTIES The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

Key Concept *Additive Identity*

- **Words** For any number a, the sum of a and 0 is a.
- **Symbols** $a + 0 = 0 + a = a$
- **Examples** $5 + 0 = 5$, $0 + 5 = 5$

There are also special properties associated with multiplication. Consider the following equations.

$$7 \cdot n = 7$$

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

$$9 \cdot m = 0$$

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

$$\frac{1}{3} \cdot 3 = 1$$

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

Key Concept Multiplication Properties

Property	Words	Symbols	Examples
Multiplicative Identity	For any number a, the product of a and 1 is a.	$a \cdot 1 = 1 \cdot a = a$	$12 \cdot 1 = 12$, $1 \cdot 12 = 12$
Multiplicative Property of Zero	For any number a, the product of a and 0 is 0.	$a \cdot 0 = 0 \cdot a = 0$	$8 \cdot 0 = 0$, $0 \cdot 8 = 0$
Multiplicative Inverse	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.	$\frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1$	$\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$, $\frac{3}{2} \cdot \frac{2}{3} = \frac{6}{6} = 1$

Example 1 Identify Properties

Name the property used in each equation. Then find the value of n.

a. $42 \cdot n = 42$

Multiplicative Identity Property

$n = 1$, since $42 \cdot 1 = 42$.

b. $n + 0 = 15$

Additive Identity Property

$n = 15$, since $15 + 0 = 15$.

c. $n \cdot 9 = 1$

Multiplicative Inverse Property

$n = \frac{1}{9}$, since $\frac{1}{9} \cdot 9 = 1$.

There are several properties of equality that apply to addition and multiplication. These are summarized below.

Key Concept Properties of Equality

Property	Words	Symbols	Examples
Reflexive	Any quantity is equal to itself.	For any number a, $a = a$.	$7 = 7$, $2 + 3 = 2 + 3$
Symmetric	If one quantity equals a second quantity, then the second quantity equals the first.	For any numbers a and b, if $a = b$, then $b = a$.	If $9 = 6 + 3$, then $6 + 3 = 9$.
Transitive	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $5 + 7 = 8 + 4$ and $8 + 4 = 12$, then $5 + 7 = 12$.
Substitution	A quantity may be substituted for its equal in any expression.	If $a = b$, then a may be replaced by b in any expression.	If $n = 15$, then $3n = 3 \cdot 15$.

USE IDENTITY AND EQUALITY PROPERTIES The properties of identity and equality can be used to justify each step when evaluating an expression.

Example 2 Evaluate Using Properties

Evaluate $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$. Name the property used in each step.

$2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3} = 2(6 - 5) + 3 \cdot \frac{1}{3}$	Substitution; $3 \cdot 2 = 6$
$= 2(1) + 3 \cdot \frac{1}{3}$	Substitution; $6 - 5 = 1$
$= 2 + 3 \cdot \frac{1}{3}$	Multiplicative Identity; $2 \cdot 1 = 2$
$= 2 + 1$	Multiplicative Inverse; $3 \cdot \frac{1}{3} = 1$
$= 3$	Substitution; $2 + 1 = 3$

Check for Understanding

Concept Check

1. **Explain** whether 1 can be an additive identity.
2. **OPEN ENDED** Write two equations demonstrating the Transitive Property of Equality.
3. **Explain** why 0 has no multiplicative inverse.

Guided Practice

Name the property used in each equation. Then find the value of n.

4. $13n = 0$
5. $17 + 0 = n$
6. $\frac{1}{6}n = 1$
7. Evaluate $6(12 - 48 \div 4)$. Name the property used in each step.
8. Evaluate $\left(15 \cdot \frac{1}{15} + 8 \cdot 0\right) \cdot 12$. Name the property used in each step.

Application

HISTORY **For Exercises 9–11, use the following information.**
On November 19, 1863, Abraham Lincoln delivered the famous Gettysburg Address. The speech began "Four score and seven years ago, . . ."

9. Write an expression to represent four score and seven. (*Hint*: A score is 20.)
10. Evaluate the expression. Name the property used in each step.
11. How many years is four score and seven?

Practice and Apply

Homework Help

For Exercises	See Examples
12–19	1
20–23	1, 2
24–29	2
30–35	1, 2

Extra Practice
See page 821.

Name the property used in each equation. Then find the value of n.

12. $12n = 12$
13. $n \cdot 1 = 5$
14. $8 \cdot n = 8 \cdot 5$
15. $0.25 + 1.5 = n + 1.5$
16. $8 = n + 8$
17. $n + 0 = \frac{1}{3}$
18. $1 = 2n$
19. $4 \cdot \frac{1}{4} = n$
20. $(9 - 7)(5) = 2(n)$
21. $3 + (2 + 8) = n + 10$
22. $n\left(5^2 \cdot \frac{1}{25}\right) = 3$
23. $6\left(\frac{1}{2} \cdot n\right) = 6$

Evaluate each expression. Name the property used in each step.

24. $\frac{3}{4}[4 \div (7 - 4)]$
25. $\frac{2}{3}[3 \div (2 \cdot 1)]$
26. $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$
27. $6 \cdot \frac{1}{6} + 5(12 \div 4 - 3)$
28. $3 + 5(4 - 2^2) - 1$
29. $7 - 8(9 - 3^2)$

FUND-RAISING **For Exercises 30 and 31, use the following information.**

The spirit club at Central High School is selling items to raise money. The profit the club earns on each item is the difference between what an item sells for and what it costs the club to buy.

School Spirit Items		
Item	Cost	Selling Price
Pennant	\$3.00	\$5.00
Button	\$1.00	\$2.50
Cap	\$6.00	\$10.00

30. Write an expression that represents the profit for 25 pennants, 80 buttons, and 40 caps.
31. Evaluate the expression, indicating the property used in each step.

MILITARY PAY **For Exercises 32 and 33, use the table that shows the monthly base pay rates for the first five ranks of enlisted personnel.**

Monthly Basic Pay Rates by Grade, Effective July 1, 2001								
	Years of Service							
Grade	<2	>2	>3	>4	>6	>8	>10	>12
E-5	1381.80	1549.20	1623.90	1701.00	1779.30	1888.50	1962.90	2040.30
E-4	1288.80	1423.80	1500.60	1576.20	1653.00	1653.00	1653.00	1653.00
E-3	1214.70	1307.10	1383.60	1385.40	1385.40	1385.40	1385.40	1385.40
E-2	1169.10	1169.10	1169.10	1169.10	1169.10	1169.10	1169.10	1169.10
E-1	1042.80	1042.80	1042.80	1042.80	1042.80	1042.80	1042.80	1042.80

Source: U.S. Department of Defense

32. Write an equation using addition that shows the change in pay for an enlisted member at grade E-2 from 3 years of service to 12 years.
33. Write an equation using multiplication that shows the change in pay for someone at grade E-4 from 6 years of service to 10 years.

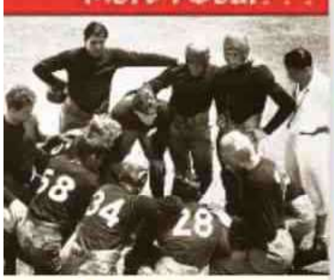
FOOTBALL **For Exercises 34–36, use the table that shows the base salary and various bonus plans for the NFL from 2002–2005.**

34. Suppose a player rushed for 12 touchdowns in 2002 and another player scored 76 points that same year. Write an equation that compares the two salaries and bonuses.
35. Write an expression that could be used to determine what a team owner would pay in base salaries and bonuses in 2004 for the following:
    - eight players who keep their weight under 240 pounds and are involved in at least 35% of the offensive plays,
    - three players who score 12 rushing touchdowns and score 76 points, and
    - four players who run 1601 yards of total offense and average 4.5 yards per carry.
36. Evaluate the expression you wrote in Exercise 35. Name the property used in each step.

NFL Salaries and Bonuses	
Year	Base Salary
2002	\$350,000
2003	375,000
2004	400,000
2005	400,000
Goal	Bonus
Involved in 35% of offensive plays	\$50,000
Average 4.5 yards per carry	50,000
12 rushing touchdowns	50,000
12 receiving touchdowns	50,000
76 points scored	50,000
1601 yards of total offense	50,000
Keep weight below 240 lb	100,000
Goal—Rushing Yards	Bonus
1600 yards	\$1 million
1800 yards	1.5 million
2000 yards	2 million
2100 yards	2.5 million

Source: *ESPN Sports Almanac*

More About. . .



Football

Nationally organized football began in 1920 and originally included five teams. In 2002, there were 32 teams.

Source: www.infoplease.com

Online Research **Data Update** Find the most recent statistics for a professional football player. What was his base salary and bonuses? Visit www.algebra1.com/data_update to learn more.

37. **CRITICAL THINKING** The Transitive Property of Inequality states that if $a < b$ and $b < c$, then $a < c$. Use this property to determine whether the following statement is *sometimes*, *always*, or *never* true.

If $x > y$ and $z > w$, then $xz > yw$.

Give examples to support your answer.

38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are identity and equality properties used to compare data?

Include the following in your answer:

- a description of how you could use the Reflexive or Symmetric Property to compare a team's rank for any two time periods, and
- a demonstration of the Transitive Property using one of the team's three rankings as an example.

Standardized Test Practice

39. Which equation illustrates the Symmetric Property of Equality?

Ⓐ If $a = b$, then $b = a$.
Ⓑ If $a = b$, $b = c$, then $a = c$.
Ⓒ If $a = b$, then $b = c$.
Ⓓ If $a = a$, then $a + 0 = a$.

40. The equation $(10 - 8)(5) = (2)(5)$ is an example of which property of equality?

Ⓐ Reflexive
Ⓑ Substitution
Ⓒ Symmetric
Ⓓ Transitive

Extending the Lesson

The sum of any two whole numbers is always a whole number. So, the set of whole numbers {0, 1, 2, 3, …} is said to be closed under addition. This is an example of the **Closure Property**. State whether each of the following statements is *true* or *false*. If false, justify your reasoning.

41. The set of whole numbers is closed under subtraction.

42. The set of whole numbers is closed under multiplication.

43. The set of whole numbers is closed under division.

Maintain Your Skills

Mixed Review

Find the solution set for each inequality using the given replacement set. *(Lesson 1-3)*

44. $10 - x > 6$; {3, 5, 6, 8}

45. $4x + 2 < 58$; {11, 12, 13, 14, 15}

46. $\frac{x}{2} \geq 3$; {5.8, 5.9, 6, 6.1, 6.2, 6.3}

47. $8x \leq 32$; {3, 3.25, 3.5, 3.75, 4}

48. $\frac{7}{10} - 2x < \frac{3}{10}$; $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}$

49. $2x - 1 \leq 2$; $\left\{1\frac{1}{4}, 2, 3, 3\frac{1}{2}\right\}$

Evaluate each expression. *(Lesson 1-2)*

50. $(3 + 6) \div 3^2$

51. $6(12 - 7.5) - 7$

52. $20 \div 4 \cdot 8 \div 10$

53. $\frac{(6 + 2)^2}{16} + 3(9)$

54. $[6^2 - (2 + 4)2]3$

55. $9(3) - 4^2 + 6^2 \div 2$

56. Write an algebraic expression for the sum of twice a number squared and 7. *(Lesson 1-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
*(To review **order of operations**, see Lesson 1-2.)*

57. $10(6) + 10(2)$

58. $(15 - 6) \cdot 8$

59. $12(4) - 5(4)$

60. $3(4 + 2)$

61. $5(6 - 4)$

62. $8(14 + 2)$

1-5 The Distributive Property

What You'll Learn

- Use the Distributive Property to evaluate expressions.
- Use the Distributive Property to simplify expressions.

Vocabulary

- term
- like terms
- equivalent expressions
- simplest form
- coefficient

How can the Distributive Property be used to calculate quickly?

Instant Replay Video Games sells new and used games. During a Saturday morning sale, the first 8 customers each bought a bargain game and a new release. To calculate the total sales for these customers, you can use the Distributive Property.

Sale Prices	
Used Games	\$9.95
Bargain Games	\$14.95
Regular Games	\$24.95
New Releases	\$34.95

EVALUATE EXPRESSIONS There are two methods you could use to calculate the video game sales.

Method 1

$$\underbrace{8(14.95)}_{\text{sales of bargain games}} \underbrace{+}_{\text{plus}} \underbrace{8(34.95)}_{\text{sales of new releases}}$$

$= 119.60 + 279.60$

$= 399.20$

Method 2

$$\underbrace{8}_{\text{number of customers}} \underbrace{\times}_{\text{times}} \underbrace{(14.95 + 34.95)}_{\text{each customer's purchase price}}$$

$= 8(49.90)$

$= 399.20$

Either method gives total sales of \$399.20 because the following is true.

$$8(14.95) + 8(34.95) = 8(14.95 + 34.95)$$

This is an example of the **Distributive Property**.

Key Concept — Distributive Property

- **Symbols** For any numbers a, b, and c,
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and
  $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.
- **Examples**
  $3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$ $4(9 - 7) = 4 \cdot 9 - 4 \cdot 7$
  $3(7) = 6 + 15$ $4(2) = 36 - 28$
  $21 = 21$ ✓ $8 = 8$ ✓

Notice that it does not matter whether a is placed on the right or the left of the expression in the parentheses.

The Symmetric Property of Equality allows the Distributive Property to be written as follows.

If $a(b + c) = ab + ac$, then $ab + ac = a(b + c)$.

Example 1 Distribute Over Addition

Rewrite 8(10 + 4) using the Distributive Property. Then evaluate.

$8(10 + 4) = 8(10) + 8(4)$ Distributive Property

$= 80 + 32$ Multiply.

$= 112$ Add.

Example 2 Distribute Over Subtraction

Rewrite (12 − 3)6 using the Distributive Property. Then evaluate.

$(12 - 3)6 = 12 \cdot 6 - 3 \cdot 6$ Distributive Property

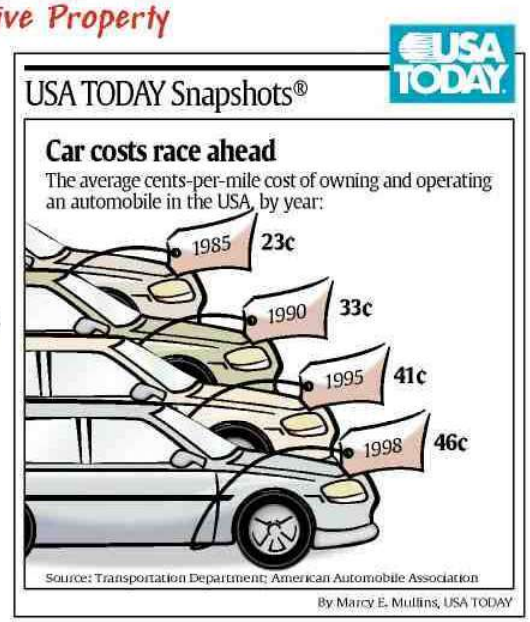
$= 72 - 18$ Multiply.

$= 54$ Subtract.

Log on for:
- Updated data
- More activities on the Distributive Property www.algebra1.com/usa_today

Example 3 Use the Distributive Property

CARS The Morris family owns two cars. In 1998, they drove the first car 18,000 miles and the second car 16,000 miles. Use the graph to find the total cost of operating both cars.



Use the Distributive Property to write and evaluate an expression.

$0.46(18{,}000 + 16{,}000)$ Distributive Prop.

$= 8280 + 7360$ Multiply.

$= 15{,}640$ Add.

It cost the Morris family \$15,640 to operate their cars.

The Distributive Property can be used to simplify mental calculations.

Example 4 Use the Distributive Property

Use the Distributive Property to find each product.

a. $15 \cdot 99$

$15 \cdot 99 = 15(100 - 1)$ Think: $99 = 100 - 1$

$= 15(100) - 15(1)$ Distributive Property

$= 1500 - 15$ Multiply.

$= 1485$ Subtract.

b. $35\left(2\frac{1}{5}\right)$

$35\left(2\frac{1}{5}\right) = 35\left(2 + \frac{1}{5}\right)$ Think: $2\frac{1}{5} = 2 + \frac{1}{5}$

$= 35(2) + 35\left(\frac{1}{5}\right)$ Distributive Property

$= 70 + 7$ Multiply.

$= 77$ Add.

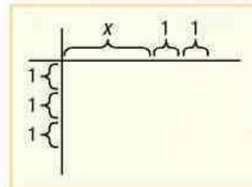
SIMPLIFY EXPRESSIONS You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.

Algebra Activity

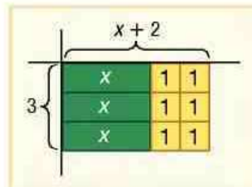
The Distributive Property

Consider the product $3(x + 2)$. Use a product mat and algebra tiles to model $3(x + 2)$ as the area of a rectangle whose dimensions are 3 and $(x + 2)$.

Step 1 Use algebra tiles to mark the dimensions of the rectangle on a product mat.



Step 2 Using the marks as a guide, make the rectangle with the algebra tiles. The rectangle has 3 x-tiles and 6 1-tiles. The area of the rectangle is $x + 1 + 1 + x + 1 + 1 + x + 1 + 1$ or $3x + 6$. Therefore, $3(x + 2) = 3x + 6$.



Model and Analyze

Find each product by using algebra tiles.

1. $2(x + 1)$ **2.** $5(x + 2)$ **3.** $2(2x + 1)$

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing.

4. $3(x + 3) = 3x + 3$ **5.** $x(3 + 2) = 3x + 2x$

Make a Conjecture

6. Rachel says that $3(x + 4) = 3x + 12$, but José says that $3(x + 4) = 3x + 4$. Use words and models to explain who is correct and why.

You can apply the Distributive Property to algebraic expressions.

Study Tip

Reading Math
The expression $5(g - 9)$ is read *5 times the quantity g minus 9* or *5 times the difference of g and 9.*

Example 5 Algebraic Expressions

Rewrite each product using the Distributive Property. Then simplify.

a. $5(g - 9)$

$$5(g - 9) = 5 \cdot g - 5 \cdot 9 \quad \text{Distributive Property}$$
$$= 5g - 45 \quad \text{Multiply.}$$

b. $-3(2x^2 + 4x - 1)$

$$-3(2x^2 + 4x - 1) = (-3)(2x^2) + (-3)(4x) - (-3)(1) \quad \text{Distributive Property}$$
$$= -6x^2 + (-12x) - (-3) \quad \text{Multiply.}$$
$$= -6x^2 - 12x + 3 \quad \text{Simplify.}$$

A **term** is a number, a variable, or a product or quotient of numbers and variables. For example, y, p^3, $4a$, and $5g^2h$ are all terms. **Like terms** are terms that contain the same variables, with corresponding variables having the same power.

$2x^2 + 6x + 5$ — three terms

$3a^2 + 5a^2 + 2a$ — like terms, unlike terms

CONTENTS

The Distributive Property and the properties of equality can be used to show that $5n + 7n = 12n$. In this expression, $5n$ and $7n$ are like terms.

$$\begin{aligned} 5n + 7n &= (5 + 7)n && \text{Distributive Property} \\ &= 12n && \text{Substitution} \end{aligned}$$

The expressions $5n + 7n$ and $12n$ are called **equivalent expressions** because they denote the same number. An expression is in **simplest form** when it is replaced by an equivalent expression having no like terms or parentheses.

Example 6 Combine Like Terms

Simplify each expression.

a. $15x + 18x$

$$\begin{aligned} 15x + 18x &= (15 + 18)x && \text{Distributive Property} \\ &= 33x && \text{Substitution} \end{aligned}$$

b. $10n + 3n^2 + 9n^2$

$$\begin{aligned} 10n + 3n^2 + 9n^2 &= 10n + (3 + 9)n^2 && \text{Distributive Property} \\ &= 10n + 12n^2 && \text{Substitution} \end{aligned}$$

Study Tip

Like Terms
Like terms may be defined as terms that are the same or vary only by the coefficient.

The **coefficient** of a term is the numerical factor. For example, in $17xy$, the coefficient is 17, and in $\frac{3y^2}{4}$, the coefficient is $\frac{3}{4}$. In the term m, the coefficient is 1 since $1 \cdot m = m$ by the Multiplicative Identity Property.

Check for Understanding

Concept Check

1. **Explain** why the Distributive Property is sometimes called The Distributive Property of Multiplication Over Addition.
2. **OPEN ENDED** Write an expression that has five terms, three of which are like terms and one term with a coefficient of 1.
3. **FIND THE ERROR** Courtney and Ben are simplifying $4w^4 + w^4 + 3w^2 - 2w^2$.

Courtney	Ben
$4w^4 + w^4 + 3w^2 - 2w^2$	$4w^4 + w^4 + 3w^2 - 2w^2$
$= (4 + 1)w^4 + (3 - 2)w^2$	$= (4)w^4 + (3 - 2)w^2$
$= 5w^4 + 1w^2$	$= 4w^4 + 1w^2$
$= 5w^4 + w^2$	$= 4w^4 + w^2$

Who is correct? Explain your reasoning.

Guided Practice

Rewrite each expression using the Distributive Property. Then simplify.

4. $6(12 - 2)$
5. $2(4 + t)$
6. $(g - 9)5$

Use the Distributive Property to find each product.

7. $16(102)$
8. $\left(3\frac{1}{17}\right)(17)$

Simplify each expression. If not possible, write *simplified*.

9. $13m + m$
10. $3(x + 2x)$
11. $14a^2 + 13b^2 + 27$
12. $4(3g + 2)$

CONTENTS

Application

COSMETOLOGY For Exercises 13 and 14, use the following information.
Ms. Curry owns a hair salon. One day, she gave 12 haircuts. She earned \$19.95 for each and received an average tip of \$2 for each haircut.

13. Write an expression to determine the total amount she earned.

14. How much did Ms. Curry earn?

Practice and Apply

Homework Help

For Exercises	See Examples
15–18	1, 2
19–28	5
29, 30, 37–41	3
31–36	4
42–53	6

Extra Practice

See page 821.

Rewrite each expression using the Distributive Property. Then simplify.

15. $8(5 + 7)$ **16.** $7(13 + 12)$ **17.** $12(9 - 5)$

18. $13(10 - 7)$ **19.** $3(2x + 6)$ **20.** $8(3m + 4)$

21. $(4 + x)2$ **22.** $(5 + n)3$ **23.** $28\left(y - \frac{1}{7}\right)$

24. $27\left(2b - \frac{1}{3}\right)$ **25.** $a(b - 6)$ **26.** $x(z + 3)$

27. $2(a - 3b + 2c)$ **28.** $4(8p + 4q - 7r)$

OLYMPICS For Exercises 29 and 30, use the following information.
At the 2000 Summer Olympics in Australia, about 110,000 people attended events at Olympic Stadium each day while another 17,500 fans were at the aquatics center.

29. Write an expression you could use to determine the total number of people at Olympic Stadium and the Aquatic Center over 4 days.

30. What was the attendance for the 4-day period?

Use the Distributive Property to find each product.

31. $5 \cdot 97$ **32.** $8 \cdot 990$ **33.** $17 \cdot 6$

34. $24 \cdot 7$ **35.** $18\left(2\frac{1}{9}\right)$ **36.** $48\left(3\frac{1}{6}\right)$

More About. . .

Olympics

The first modern Olympics were held in Athens, Greece, in 1896. The games featured 43 events and included 14 nations. The 2000 Olympics featured 300 events and included 199 nations.

Source: www.olympic.org

COMMUNICATIONS For Exercises 37 and 38, use the following information.
A public relations consultant keeps a log of all contacts made by e-mail, telephone, and in person. In a typical week, she averages 5 hours using e-mail, 12 hours of meeting in person, and 18 hours on the telephone.

37. Write an expression that could be used to predict how many hours she will spend on these activities over the next 12 weeks.

38. How many hours should she plan for contacting people for the next 12 weeks?

INSURANCE For Exercises 39–41, use the table that shows the monthly cost of a company health plan.

Available Insurance Plans—Monthly Charge			
Coverage	Medical	Dental	Vision
Employee	\$78	\$20	\$12
Family (additional coverage)	\$50	\$15	\$7

39. Write an expression that could be used to calculate the cost of medical, dental, and vision insurance for an employee for 6 months.

40. How much does it cost an employee to get all three types of insurance for 6 months?

41. How much would an employee expect to pay for individual and family medical and dental coverage per year?

Simplify each expression. If not possible, write *simplified*.

42. $2x + 9x$ **43.** $4b + 5b$ **44.** $5n^2 + 7n$

45. $3a^2 + 14a^2$ **46.** $12(3c + 4)$ **47.** $15(3x - 5)$

48. $6x^2 + 14x - 9x$ **49.** $4y^3 + 3y^3 + y^4$ **50.** $6(5a + 3b - 2b)$

51. $5(6m + 4n - 3n)$ **52.** $x^2 + \frac{7}{8}x - \frac{x}{8}$ **53.** $a + \frac{a}{5} + \frac{2}{5}a$

54. **CRITICAL THINKING** The expression $2(\ell + w)$ may be used to find the perimeter of a rectangle. What are the length and width of a rectangle if the area is $13\frac{1}{2}$ square units and the length of one side is $\frac{1}{5}$ the measure of the perimeter?

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can the Distributive Property be used to calculate quickly?

Include the following in your answer:

- a comparison of the two methods of finding the total video game sales.

Standardized Test Practice

56. Simplify $3(x + y) + 2(x + y) - 4x$.

Ⓐ $5x + y$ Ⓑ $9x + 5y$ Ⓒ $5x + 9y$ Ⓓ $x + 5y$

57. If $a = 2.8$ and $b = 4.2$, find the value of c in the equation $c = 7(2a + 3b)$.

Ⓐ 18.2 Ⓑ 238.0 Ⓒ 127.4 Ⓓ 51.8

Maintain Your Skills

Mixed Review **Name the property illustrated by each statement or equation.** *(Lesson 1-4)*

58. If $7 \cdot 2 = 14$, then $14 = 7 \cdot 2$. **59.** $8 + (3 + 9) = 8 + 12$

60. $mnp = 1mnp$ **61.** $3\left(5^2 \cdot \frac{1}{25}\right) = 3 \cdot 1$

62. $\left(\frac{3}{4}\right)\left(\frac{4}{3}\right) = 1$ **63.** $32 + 21 = 32 + 21$

PHYSICAL SCIENCE **For Exercises 64 and 65, use the following information.**
Sound travels 1129 feet per second through air. *(Lesson 1-3)*

64. Write an equation that represents how many feet sound can travel in 2 seconds when it is traveling through air.

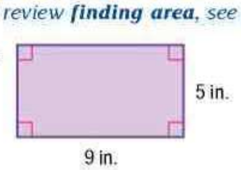
65. How far can sound travel in 2 seconds when traveling through air?

Evaluate each expression if $a = 4$, $b = 6$, and $c = 3$. *(Lesson 1–2)*

66. $3ab - c^2$ **67.** $8(a - c)^2 + 3$ **68.** $\frac{6ab}{c(a + 2)}$ **69.** $(a + c)\left(\frac{a + b}{2}\right)$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Find the area of each figure.**
*(To review **finding area**, see pages 813 and 814.)*

70.



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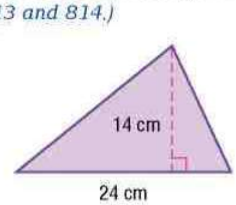

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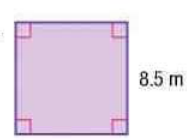

1-6 Commutative and Associative Properties

What You'll Learn

- Recognize the Commutative and Associative Properties.
- Use the Commutative and Associative Properties to simplify expressions.

How can properties help you determine distances?

The South Line of the Atlanta subway leaves Five Points and heads for Garnett, 0.4 mile away. From Garnett, West End is 1.5 miles. The distance from Five Points to West End can be found by evaluating the expression $0.4 + 1.5$. Likewise, the distance from West End to Five Points can be found by evaluating the expression $1.5 + 0.4$.

COMMUTATIVE AND ASSOCIATIVE PROPERTIES In the situation above, the distance from Five Points to West End is the same as the distance from West End to Five Points. This distance can be represented by the following equation.

$$\overbrace{0.4 + 1.5}^{\text{The distance from Five Points to West End}} \overbrace{=}^{\text{equals}} \overbrace{1.5 + 0.4}^{\text{the distance from West End to Five Points.}}$$

This is an example of the **Commutative Property**.

Key Concept — *Commutative Property*

- **Words** The order in which you add or multiply numbers does not change their sum or product.
- **Symbols** For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
- **Examples** $5 + 6 = 6 + 5$, $3 \cdot 2 = 2 \cdot 3$

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

Key Concept — *Associative Property*

- **Words** The way you group three or more numbers when adding or multiplying does not change their sum or product.
- **Symbols** For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.
- **Examples** $(2 + 4) + 6 = 2 + (4 + 6)$, $(3 \cdot 5) \cdot 4 = 3 \cdot (5 \cdot 4)$

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Example 1 Multiplication Properties

Evaluate $8 \cdot 2 \cdot 3 \cdot 5$.

You can rearrange and group the factors to make mental calculations easier.

$$\begin{aligned} 8 \cdot 2 \cdot 3 \cdot 5 &= 8 \cdot 3 \cdot 2 \cdot 5 && \text{Commutative } (\times) \\ &= (8 \cdot 3) \cdot (2 \cdot 5) && \text{Associative } (\times) \\ &= 24 \cdot 10 && \text{Multiply.} \\ &= 240 && \text{Multiply.} \end{aligned}$$

Example 2 Use Addition Properties

TRANSPORTATION **Refer to the application at the beginning of the lesson. Find the distance between Five Points and Lakewood/Ft. McPherson.**

$$\underbrace{0.4}_{\text{Five Points to Garnett}} + \underbrace{1.5}_{\text{Garnett to West End}} + \underbrace{1.5}_{\text{West End to Oakland City}} + \underbrace{1.1}_{\text{Oakland City to Lakewood/Ft. McPherson}}$$

$$\begin{aligned} 0.4 + 1.5 + 1.5 + 1.1 &= 0.4 + 1.1 + 1.5 + 1.5 && \text{Commutative } (+) \\ &= (0.4 + 1.1) + (1.5 + 1.5) && \text{Associative } (+) \\ &= 1.5 + 3.0 && \text{Add.} \\ &= 4.5 && \text{Add.} \end{aligned}$$

Lakewood/Ft. McPherson is 4.5 miles from Five Points.

More About...

Transportation

New York City has the most extensive subway system, covering 842 miles of track and serving about 4.3 million passengers per day.

Source: *The Guinness Book of Records*

SIMPLIFY EXPRESSIONS The Commutative and Associative Properties can be used with other properties when evaluating and simplifying expressions.

Concept Summary — Properties of Numbers

The following properties are true for any numbers *a*, *b*, and *c*.

Properties	Addition	Multiplication
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	0 is the identity. $a + 0 = 0 + a = a$	1 is the identity. $a \cdot 1 = 1 \cdot a = a$
Zero	—	$a \cdot 0 = 0 \cdot a = 0$
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Substitution	If $a = b$, then a may be substituted for b.	

Example 3 Simplify an Expression

Simplify $3c + 5(2 + c)$.

$$\begin{aligned} 3c + 5(2 + c) &= 3c + 5(2) + 5(c) && \text{Distributive Property} \\ &= 3c + 10 + 5c && \text{Multiply.} \\ &= 3c + 5c + 10 && \text{Commutative } (+) \\ &= (3c + 5c) + 10 && \text{Associative } (+) \\ &= (3 + 5)c + 10 && \text{Distributive Property} \\ &= 8c + 10 && \text{Substitution} \end{aligned}$$

Example 4 Write and Simplify an Expression

Use the expression ***four times the sum of a and b increased by twice the sum of a and 2b.***

a. Write an algebraic expression for the verbal expression.

$$\underbrace{4(a+b)}_{\text{four times the sum of } a \text{ and } b} \quad \underbrace{+}_{\text{increased by}} \quad \underbrace{2(a+2b)}_{\text{twice the sum of } a \text{ and } 2b}$$

b. Simplify the expression and indicate the properties used.

$4(a+b)+2(a+2b) = 4(a)+4(b)+2(a)+2(2b)$	Distributive Property
$= 4a+4b+2a+4b$	Multiply.
$= 4a+2a+4b+4b$	Commutative (+)
$= (4a+2a)+(4b+4b)$	Associative (+)
$= (4+2)a+(4+4)b$	Distributive Property
$= 6a+8b$	Substitution

Check for Understanding

Concept Check

1. **Define** the Associative Property in your own words.
2. **Write** a short explanation as to whether there is a Commutative Property of Division.
3. **OPEN ENDED** Write examples of the Commutative Property of Addition and the Associative Property of Multiplication using 1, 5, and 8 in each.

Guided Practice

Evaluate each expression.

4. $14+18+26$ **5.** $3\frac{1}{2}+4+2\frac{1}{2}$ **6.** $5 \cdot 3 \cdot 6 \cdot 4$ **7.** $\frac{5}{6} \cdot 16 \cdot 9\frac{3}{4}$

Simplify each expression.

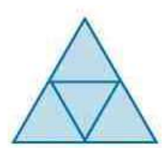
8. $4x+5y+6x$ **9.** $5a+3b+2a+7b$ **10.** $\frac{1}{4}q+2q+2\frac{3}{4}q$

11. $3(4x+2)+2x$ **12.** $7(ac+2b)+2ac$ **13.** $3(x+2y)+4(3x+y)$

14. Write an algebraic expression for *half the sum of p and 2q increased by three-fourths q*. Then simplify, indicating the properties used.

Application

15. GEOMETRY Find the area of the large triangle if each smaller triangle has a base measuring 5.2 centimeters and a height of 7.86 centimeters.



Practice and Apply

Evaluate each expression.

16. $17+6+13+24$ **17.** $8+14+22+9$ **18.** $4.25+3.50+8.25$

19. $6.2+4.2+4.3+5.8$ **20.** $6\frac{1}{2}+3+\frac{1}{2}+2$ **21.** $2\frac{3}{8}+4+3\frac{3}{8}$

22. $5 \cdot 11 \cdot 4 \cdot 2$ **23.** $3 \cdot 10 \cdot 6 \cdot 3$ **24.** $0.5 \cdot 2.4 \cdot 4$

25. $8 \cdot 1.6 \cdot 2.5$ **26.** $3\frac{3}{7} \cdot 14 \cdot 1\frac{1}{4}$ **27.** $2\frac{5}{8} \cdot 24 \cdot 6\frac{2}{3}$

Homework Help

For Exercises	See Examples
16–29	1, 2
30, 31	2
32–43	3
44–47	4

Extra Practice

See page 821.

TRAVEL **For Exercises 28 and 29, use the following information.**
Hotels often have different rates for weeknights and weekends. The rates of one hotel are listed in the table.

Hotel Rates	
Weeknights (M–F)	\$72
Weekends	\$63
Weekly (5 weeknights)	\$325

28. If a traveler checks into the hotel on Friday and checks out the following Tuesday morning, what is the total cost of the room?

29. Suppose there is a sales tax of \$5.40 for weeknights and \$5.10 for weekends. What is the total cost of the room including tax?

ENTERTAINMENT **For Exercises 30 and 31, use the following information.**
A video store rents new release videos for \$4.49, older videos for \$2.99, and DVDs for \$3.99. The store also sells its used videos for \$9.99.

30. Write two expressions to represent the total sales of a clerk after renting 2 DVDs, 3 new releases, 2 older videos, and selling 5 used videos.

31. What are the total sales of the clerk?

Simplify each expression.

32. $4a + 2b + a$

33. $2y + 2x + 8y$

34. $x^2 + 3x + 2x + 5x^2$

35. $4a^3 + 6a + 3a^3 + 8a$

36. $6x + 2(2x + 7)$

37. $5n + 4(3n + 9)$

38. $3(x + 2y) + 4(3x + y)$

39. $3.2(x + y) + 2.3(x + y) + 4x$

40. $3(4m + n) + 2m$

41. $6(0.4f + 0.2g) + 0.5f$

42. $\frac{3}{4} + \frac{2}{3}(s + 2t) + s$

43. $2p + \frac{3}{5}\left(\frac{1}{2}p + 2q\right) + \frac{2}{3}$

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

44. twice the sum of s and t decreased by s

45. five times the product of x and y increased by $3xy$

46. the product of six and the square of z, increased by the sum of seven, z^2, and 6

47. six times the sum of x and y squared decreased by three times the sum of x and half of y squared

48. **CRITICAL THINKING** Tell whether the Commutative Property *always*, *sometimes*, or *never* holds for subtraction. Explain your reasoning.

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can properties help you determine distances?

Include the following in your answer:
- an expression using the Commutative and Associative Properties that you could use to easily determine the distance from the airport to Five Points, and
- an explanation of how the Commutative and Associative Properties are useful in performing calculations.

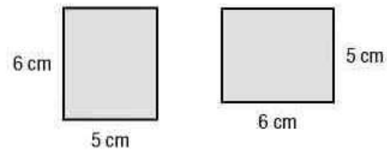
Stop	Distance from Previous Stop
Five Points	0
Garnett	0.4
West End	1.5
Oakland City	1.5
Lakewood/ Ft. McPherson	1.1
East Point	1.9
College Park	1.8
Airport	0.8

Standardized Test Practice

50. Simplify $6(ac + 2b) + 2ac$.

Ⓐ $10ab + 2ac$ Ⓑ $12ac + 20b$ Ⓒ $8ac + 12b$ Ⓓ $12abc + 2ac$

51. Which property can be used to show that the areas of the two rectangles are equal?



Ⓐ Associative
Ⓑ Commutative
Ⓒ Distributive
Ⓓ Reflexive

Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 1-5)*

52. $5(2 + x) + 7x$
53. $3(5 + 2p)$
54. $3(a + 2b) - 3a$
55. $7m + 6(n + m)$
56. $(d + 5)f + 2f$
57. $t^2 + 2t^2 + 4t$

58. Name the property used in each step. *(Lesson 1-4)*

$$\begin{aligned} 3(10 - 5 \cdot 2) + 21 \div 7 &= 3(10 - 10) + 21 \div 7 \\ &= 3(0) + 21 \div 7 \\ &= 0 + 21 \div 7 \\ &= 0 + 3 \\ &= 3 \end{aligned}$$

Evaluate each expression. *(Lesson 1-2)*

59. $12(5) - 6(4)$
60. $7(0.2 + 0.5) - 0.6$
61. $8[6^2 - 3(2 + 5)] \div 8 + 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression for the given value of the variable.**
(To review ***evaluating expressions****, see Lesson 1-2.)*

62. If $x = 4$, then $2x + 7 =$ __?__.
63. If $x = 8$, then $6x + 12 =$ __?__.
64. If $n = 6$, then $5n - 14 =$ __?__.
65. If $n = 7$, then $3n - 8 =$ __?__.
66. If $a = 2$, and $b = 5$, then $4a + 3b =$ __?__.

Practice Quiz 2

Lessons 1-4 through 1-6

Write the letters of the properties given in the right-hand column that match the examples in the left-hand column.

1. $28 + 0 = 28$
2. $(18 - 7)6 = 11(6)$
3. $24 + 15 = 15 + 24$
4. $8 \cdot 5 = 8 \cdot 5$
5. $(9 + 3) + 8 = 9 + (3 + 8)$
6. $1(57) = 57$
7. $14 \cdot 0 = 0$
8. $3(13 + 10) = 3(13) + 3(10)$
9. If $12 + 4 = 16$, then $16 = 12 + 4$.
10. $\frac{2}{5} \cdot \frac{5}{2} = 1$

a. Distributive Property
b. Multiplicative Property of 0
c. Substitution Property of Equality
d. Multiplicative Identity Property
e. Multiplicative Inverse Property
f. Reflexive Property of Equality
g. Associative Property
h. Symmetric Property of Equality
i. Commutative Property
j. Additive Identity Property

1-7 Logical Reasoning

What You'll Learn

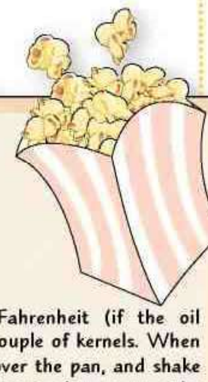
- Identify the hypothesis and conclusion in a conditional statement.
- Use a counterexample to show that an assertion is false.

Vocabulary

- conditional statement
- if-then statement
- hypothesis
- conclusion
- deductive reasoning
- counterexample

How is logical reasoning helpful in cooking?

Popcorn is a popular snack with 16 billion quarts consumed in the United States each year. The directions at the right can help you make perfect popcorn. If the popcorn burns, then the heat was too high or the kernels heated unevenly.



Stovetop Popping

To pop popcorn on a stovetop, you need:

- A 3- to 4-quart pan with a loose lid that allows steam to escape
- Enough popcorn to cover the bottom of the pan, one kernel deep
- 1/4 cup of oil for every cup of kernels (Don't use butter!)

Heat the oil to 400–460 degrees Fahrenheit (if the oil smokes, it is too hot). Test the oil on a couple of kernels. When they pop, add the rest of the popcorn, cover the pan, and shake to spread the oil. When the popping begins to slow, remove the pan from the stovetop. The heated oil will pop the remaining kernels.

Source: Popcorn Board

Study Tip

Reading Math
Note that "if" is not part of the hypothesis and "then" is not part of the conclusion.

CONDITIONAL STATEMENTS The statement *If the popcorn burns, then the heat was too high or the kernels heated unevenly* is called a conditional statement. **Conditional statements** can be written in the form *If A, then B*. Statements in this form are called **if-then statements**.

If *A*, then *B*.

If the popcorn burns, then the heat was too high or the kernels heated unevenly.

The part of the statement immediately following the word *if* is called the **hypothesis**.

The part of the statement immediately following the word *then* is called the **conclusion**.

Example 1 Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

a. If it is Friday, then Madison and Miguel are going to the movies.

Recall that the hypothesis is the part of the conditional following the word *if* and the conclusion is the part of the conditional following the word *then*.

Hypothesis: it is Friday

Conclusion: Madison and Miguel are going to the movies

b. If $4x + 3 > 27$, then $x > 6$.

Hypothesis: $4x + 3 > 27$

Conclusion: $x > 6$

Sometimes a conditional statement is written without using the words *if* and *then*. But a conditional statement can always be rewritten as an if-then statement. For example, the statement *When it is not raining, I ride my bike* can be written as *If it is not raining, then I ride my bike.*

Example 2 Write a Conditional in If-Then Form

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a. I will go to the ball game with you on Saturday.

Hypothesis: it is Saturday

Conclusion: I will go to the ball game with you

If it is Saturday, then I will go to the ball game with you.

b. For a number x such that $6x - 8 = 16$, $x = 4$.

Hypothesis: $6x - 8 = 16$

Conclusion: $x = 4$

If $6x - 8 = 16$, then $x = 4$.

DEDUCTIVE REASONING AND COUNTEREXAMPLES

Deductive reasoning is the process of using facts, rules, definitions, or properties to reach a valid conclusion. Suppose you have a true conditional and you know that the hypothesis is true for a given case. Deductive reasoning allows you to say that the conclusion is true for that case.

Study Tip

Common Misconception

Suppose the conclusion of a conditional is true. This does not mean that the hypothesis is true. Consider the conditional "If it rains, Annie will stay home." If Annie stays home, it does not necessarily mean that it is raining.

Example 3 Deductive Reasoning

Determine a valid conclusion that follows from the statement "If two numbers are odd, then their sum is even" for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

a. The two numbers are 7 and 3.

7 and 3 are odd, so the hypothesis is true.

Conclusion: The sum of 7 and 3 is even.

CHECK $7 + 3 = 10$ ✓ The sum, 10, is even.

b. The sum of two numbers is 14.

The conclusion is true. If the numbers are 11 and 3, the hypothesis is true also. However, if the numbers are 8 and 6, the hypothesis is false. There is no way to determine the two numbers. Therefore, there is no valid conclusion.

Not all if-then statements are always true or always false. Consider the statement "If Luke is listening to CDs, then he is using his portable CD player." Luke may be using his portable CD player. However, he could also be using a computer, a car CD player, or a home CD player.

To show that a conditional is false, we can use a counterexample. A **counterexample** is a specific case in which a statement is false. It takes only one counterexample to show that a statement is false.

Example 4 Find Counterexamples

Find a counterexample for each conditional statement.

a. If you are using the Internet, then you own a computer.

You could be using the Internet on a computer at a library.

b. If the Commutative Property holds for multiplication, then it holds for division.

$2 \div 1 \stackrel{?}{=} 1 \div 2$

$2 \neq 0.5$

Standardized Test Practice

Example 5 Find a Counterexample

Multiple-Choice Test Item

Which numbers are counterexamples for the statement below?

If $x \div y = 1$, then x and y are whole numbers.

Ⓐ $x = 2, y = 2$	Ⓑ $x = 0.25, y = 0.25$
Ⓒ $x = 1.2, y = 0.6$	Ⓓ $x = 6, y = 3$

Read the Test Item

Find the values of x and y that make the statement false.

The Princeton Review

Test-Taking Tip

Since choice B is the correct answer, you can check your result by testing the other values.

Solve the Test Item

Replace x and y in the equation $x \div y = 1$ with the given values.

Ⓐ $x = 2, y = 2$

$2 \div 2 \stackrel{?}{=} 1$

$1 = 1$ ✓

The hypothesis is true and both values are whole numbers. The statement is true.

Ⓑ $x = 0.25, y = 0.25$

$0.25 \div 0.25 \stackrel{?}{=} 1$

$1 = 1$ ✓

The hypothesis is true, but 0.25 is not a whole number. Thus, the statement is false.

Ⓒ $x = 1.2, y = 0.6$

$1.2 \div 0.6 \stackrel{?}{=} 1$

$2 \neq 1$

The hypothesis is false, and the conclusion is false. However, this is not a counterexample. A counterexample is a case where the hypothesis is true and the conclusion is false.

Ⓓ $x = 6, y = 3$

$6 \div 3 \stackrel{?}{=} 1$

$2 \neq 1$

The hypothesis is false. Therefore, there is no valid conclusion.

The only values that prove the statement false are $x = 0.25$ and $y = 0.25$. So these numbers are counterexamples. The answer is B.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a conditional statement and label its hypothesis and conclusion.
2. **Explain** why counterexamples are used.
3. **Explain** how deductive reasoning is used to show that a conditional is true or false.

Guided Practice **Identify the hypothesis and conclusion of each statement.**

4. If it is January, then it might snow.
5. If you play tennis, then you run fast.
6. If $34 - 3x = 16$, then $x = 6$.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

7. Lance watches television when he does not have homework.
8. A number that is divisible by 10 is also divisible by 5.
9. A rectangle is a quadrilateral with four right angles.

Determine a valid conclusion that follows from the statement *If the last digit of a number is 2, then the number is divisible by 2* for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

10. The number is 10,452.
11. The number is divisible by 2.
12. The number is 946.

Find a counterexample for each statement.

13. If Anna is in school, then she has a science class.
14. If you can read 8 pages in 30 minutes, then you can read a book in a day.
15. If a number x is squared, then $x^2 > x$.
16. If $3x + 7 \geq 52$, then $x > 15$.

Standardized Test Practice

17. Which number is a counterexample for the statement $x^2 > x$?

Ⓐ 1 Ⓑ 4 Ⓒ 5 Ⓓ 8

Practice and Apply

Homework Help

For Exercises	See Examples
18–23	1
24–29	2
30–35	3
36–43	4

Extra Practice

See page 822.

Identify the hypothesis and conclusion of each statement.

18. If both parents have red hair, then their children have red hair.
19. If you are in Hawaii, then you are in the tropics.
20. If $2n - 7 > 25$, then $n > 16$.
21. If $4(b + 9) \leq 68$, then $b \leq 8$.
22. If $a = b$, then $b = a$.
23. If $a = b$, and $b = c$, then $a = c$.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

24. The trash is picked up on Monday.
25. Greg will call after school.
26. A triangle with all sides congruent is an equilateral triangle.
27. The sum of the digits of a number is a multiple of 9 when the number is divisible by 9.
28. For $x = 8$, $x^2 - 3x = 40$.
29. $4s + 6 > 42$ when $s > 9$.

Determine whether a valid conclusion follows from the statement *If a VCR costs less than \$150, then Ian will buy one* for the given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

30. A VCR costs \$139.

31. A VCR costs \$99.

32. Ian did not buy a VCR.

33. The price of a VCR is \$199.

34. A DVD player costs \$229.

35. Ian bought 2 VCRs.

Find a counterexample for each statement.

36. If you were born in Texas, then you live in Texas.

37. If you are a professional basketball player, then you play in the United States.

38. If a baby is wearing blue clothes, then the baby is a boy.

39. If a person is left-handed, then each member of that person's family is left-handed.

40. If the product of two numbers is even, then both numbers must be even.

41. If two times a number is greater than 16, then the number must be greater than 7.

42. If $4n - 8 \geq 52$, then $n > 15$.

43. If $x \cdot y = 1$, then x or y must equal 1.

GEOMETRY **For Exercises 44 and 45, use the following information.**
If points P, Q, and R lie on the same line, then Q is between P and R.

44. Copy the graph. Label the points so that the conditional is true.

45. Copy the graph. Provide a counterexample for the conditional.

More About. . .

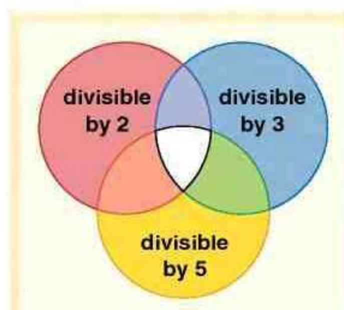
Groundhog Day

Groundhog Day has been celebrated in the United States since 1897. The most famous groundhog, Punxsutawney Phil, has seen his shadow about 85% of the time.

Source: www.infoplease.com

46. RESEARCH On Groundhog Day (February 2) of each year, some people say that if a groundhog comes out of its hole and sees its shadow, then there will be six more weeks of winter weather. However, if it does not see its shadow, then there will be an early spring. Use the Internet or another resource to research the weather on Groundhog Day for your city for the past 10 years. Summarize your data as examples or counterexamples for this belief.

NUMBER THEORY **For Exercises 47–49, use the following information.**
Copy the Venn diagram and place the numbers 1 to 25 in the appropriate places on the diagram.



47. What conclusions can you make about the numbers and where they appear on the diagram?

48. What conclusions can you form about numbers that are divisible by 2 and 3?

49. Find a counterexample for the data you have collected if possible.

www.algebra1.com/self_check_quiz

50. **CRITICAL THINKING** Determine whether the following statement is always true. If it is not, provide a counterexample.

*If the mathematical operation * is defined for all numbers a and b as $a + 2b$, then the operation * is commutative.*

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is logical reasoning helpful in cooking?

Include the following in your answer:

- the hypothesis and conclusion of the statement *If you have small, underpopped kernels, then you have not used enough oil in your pan*, and
- examples of conditional statements used in cooking food other than popcorn.

Standardized Test Practice

52. **GRID IN** What value of n makes the following statement true?
If $14n - 12 \geq 100$, *then* $n \geq$ _?_.

53. If # is defined as $\#x = \frac{x^3}{2}$, what is the value of #4?

(A) 8 (B) 16 (C) 32 (D) 64

Maintain Your Skills

Mixed Review

Simplify each expression. (Lesson 1-6)

54. $2x + 5y + 9x$
55. $a + 9b + 6b$
56. $\frac{3}{4}g + \frac{2}{5}f + \frac{5}{8}g$
57. $4(5mn + 6) + 3mn$
58. $2(3a + b) + 3b + 4$
59. $6x^2 + 5x + 3(2x^2) + 7x$

60. **ENVIRONMENT** According to the U.S. Environmental Protection Agency, a typical family of four uses 100 gallons of water flushing the toilet each day, 80 gallons of water showering and bathing, and 8 gallons of water using the bathroom sink. Write two expressions that represent the amount of water a typical family of four uses for these purposes in d days. (Lesson 1-5)

Name the property used in each expression. Then find the value of n. (Lesson 1-4)

61. $1(n) = 64$
62. $12 + 7 = n + 12$
63. $(9 - 7)5 = 2n$
64. $\frac{1}{4}n = 1$
65. $n + 18 = 18$
66. $36n = 0$

Solve each equation. (Lesson 1-3)

67. $5(7) + 6 = x$
68. $7(4^2) - 6^2 = m$
69. $p = \frac{22 - (13 - 5)}{28 \div 2^2}$

Write an algebraic expression for each verbal expression. (Lesson 1-1)

70. the product of 8 and a number x raised to the fourth power
71. three times a number n decreased by 10
72. twelve more than the quotient of a number a and 5

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. Round to the nearest tenth.
(To review **percents**, see pages 802 and 803.)

73. 40% of 90
74. 23% of 2500
75. 18% of 950
76. 38% of 345
77. 42.7% of 528
78. 67.4% of 388

1-8 Graphs and Functions

What You'll Learn

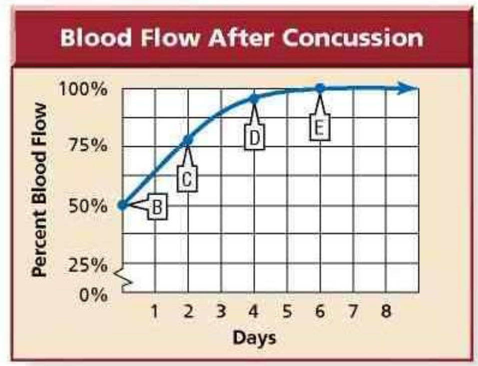
- Interpret graphs of functions.
- Draw graphs of functions.

Vocabulary

- function
- coordinate system
- *x*-axis
- *y*-axis
- origin
- ordered pair
- *x*-coordinate
- *y*-coordinate
- independent variable
- dependent variable
- relation
- domain
- range

How can real-world situations be modeled using graphs and functions?

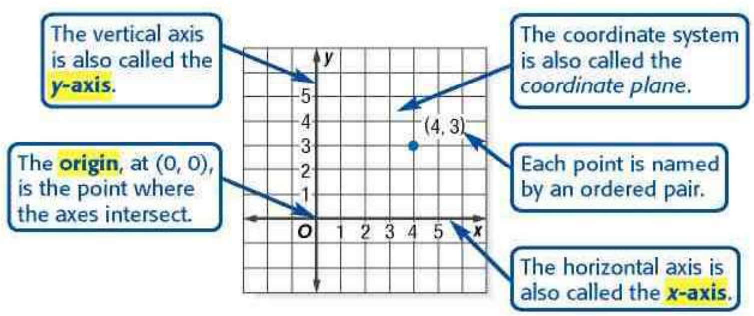
Many athletes suffer concussions as a result of sports injuries. The graph shows the relationship between blood flow to the brain and the number of days after the concussion. The graph shows that as the number of days increases, the percent of blood flow increases.



Source: *Scientific American*

INTERPRET GRAPHS The return of normal blood flow to the brain is said to be a function of the number of days since the concussion. A **function** is a relationship between input and output. In a function, the output depends on the input. There is exactly one output for each input.

A function is graphed using a **coordinate system**. It is formed by the intersection of two number lines, the *horizontal axis* and the *vertical axis*.



Each input x and its corresponding output y can be represented on a graph using ordered pairs. An **ordered pair** is a set of numbers, or *coordinates*, written in the form (x, y). The x value, called the **x-coordinate**, corresponds to the x-axis and the y value, or **y-coordinate**, corresponds to the y-axis.

Example 1 Identify Coordinates

SPORTS MEDICINE Refer to the application above. Name the ordered pair at point *C* and explain what it represents.

Point C is at 2 along the x-axis and about 80 along the y-axis. So, its ordered pair is (2, 80). This represents 80% normal blood flow 2 days after the injury.

In Example 1, the percent of normal blood flow depends on the number of days from the injury. Therefore, the number of days from the injury is called the **independent variable** or *quantity*, and the percent of normal blood flow is called the **dependent variable** or *quantity*. Usually the independent variable is graphed on the horizontal axis and the dependent variable is graphed on the vertical axis.

Example 2 Independent and Dependent Variables

Identify the independent and dependent variables for each function.

a. In general, the average price of gasoline slowly and steadily increases throughout the year.

Time is the independent variable as it is unaffected by the price of gasoline, and the price is the dependent quantity as it is affected by time.

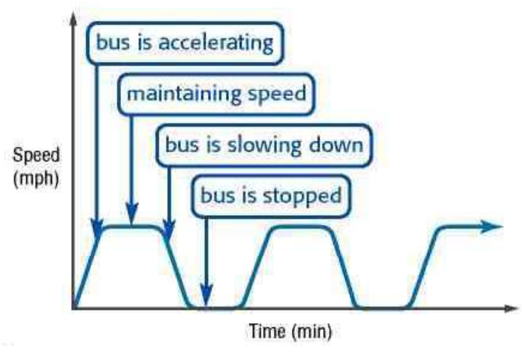
b. The profit that a business makes generally increases as the price of their product increases.

In this case, price is the independent quantity. Profit is the dependent quantity as it is affected by the price.

Functions can be graphed without using a scale on either axis to show the general shape of the graph that represents a function.

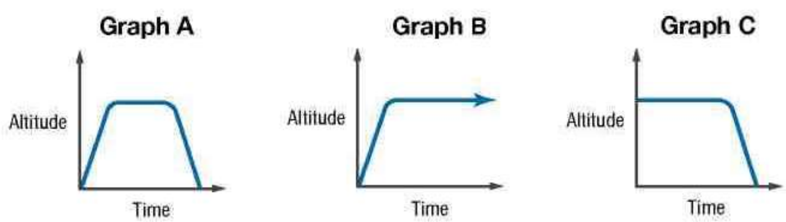
Example 3 Analyze Graphs

a. The graph at the right represents the speed of a school bus traveling along its morning route. Describe what is happening in the graph.



At the origin, the bus is stopped. It accelerates and maintains a constant speed. Then it begins to slow down, eventually stopping. After being stopped for a short time, the bus accelerates again. The starting and stopping process repeats continually.

b. Identify the graph that represents the altitude of a space shuttle above Earth, from the moment it is launched until the moment it lands.



Before it takes off, the space shuttle is on the ground. It blasts off, gaining altitude until it reaches space where it orbits Earth at a constant height until it comes back to Earth. Graph A shows this situation.

DRAW GRAPHS

Graphs can be used to represent many real-world situations.

Example 4 Draw Graphs

An electronics store is having a special sale. For every two DVDs you buy at the regular price of $29 each, you get a third DVD free.

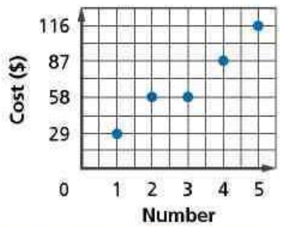
a. Make a table showing the cost of buying 1 to 5 DVDs.

Number of CDs	1	2	3	4	5
Total Cost ($)	29	58	58	87	116

b. Write the data as a set of ordered pairs.

The ordered pairs can be determined from the table. The number of DVDs is the independent variable, and the total cost is the dependent variable. So, the ordered pairs are (1, 29), (2, 58), (3, 58), (4, 87), and (5, 116).

c. Draw a graph that shows the relationship between the number of DVDs and the total cost.



Study Tip

Different Representations
Example 4 illustrates several of the ways data can be represented—tables, ordered pairs, and graphs.

A set of ordered pairs, like those in Example 4, is called a **relation**. The set of the first numbers of the ordered pairs is the **domain**. The domain contains values of the independent variable. The set of second numbers of the ordered pairs is the **range** of the relation. The range contains the values of the dependent variable.

Example 5 Domain and Range

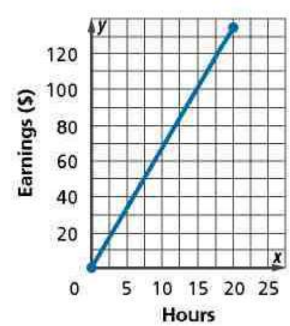
JOBS Rasha earns $6.75 per hour working up to 4 hours each day after school. Her weekly earnings are a function of the number of hours she works.

a. Identify a reasonable domain and range for this situation.

The domain contains the number of hours Rasha works each week. Since she works up to 4 hours each weekday, she works up to 5×4 or 20 hours a week. Therefore, a reasonable domain would be values from 0 to 20 hours. The range contains her weekly earnings from $0 to $20 \times \$6.75$ or $135. Thus, a reasonable range is $0 to $135.

b. Draw a graph that shows the relationship between the number of hours Rasha works and the amount she earns each week.

Graph the ordered pairs (0, 0) and (20, 135). Since she can work any amount of time up to 20 hours, connect the two points with a line to include those points.



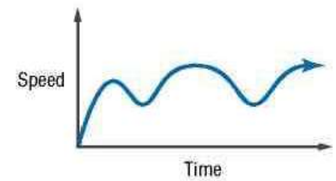
Check for Understanding

Concept Check

1. **Explain** why the order of the numbers in an ordered pair is important.
2. **Describe** the difference between dependent and independent variables.
3. **OPEN ENDED** Give an example of a relation. Identify the domain and range.

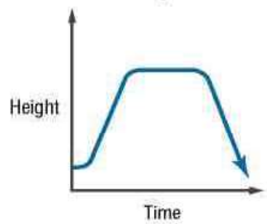
Guided Practice

4. The graph at the right represents Alexi's speed as he rides his bike. Give a description of what is happening in the graph.

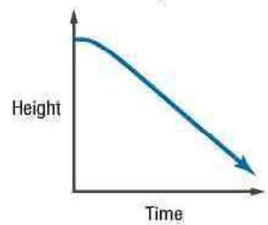


5. Identify the graph that represents the height of a skydiver just before she jumps from a plane until she lands.

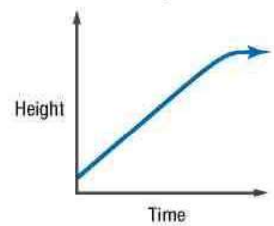
Graph A



Graph B



Graph C



Applications

PHYSICAL SCIENCE **For Exercises 6–8, use the table and the information.** During an experiment, the students of Ms. Roswell's class recorded the height of an object above the ground at several intervals after it was dropped from a height of 5 meters. Their results are in the table below.

Time (s)	0	0.2	0.4	0.6	0.8	1
Height (cm)	500	480	422	324	186	10

6. Identify the independent and dependent variables.
7. Write a set of ordered pairs representing the data in the table.
8. Draw a graph showing the relationship between the height of the falling object and time.
9. **BASEBALL** Paul is a pitcher for his school baseball team. Draw a reasonable graph that shows the height of the baseball from the ground from the time he releases the ball until the time the catcher catches the ball. Let the horizontal axis show the time and the vertical axis show the height of the ball.

Practice and Apply

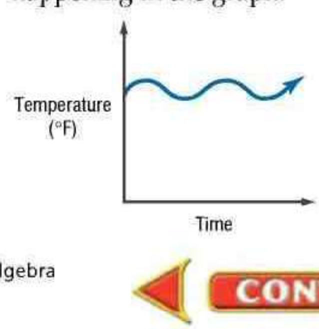
Homework Help

For Exercises	See Examples
10, 11	2
12, 13	3
14–21	4, 5

Extra Practice

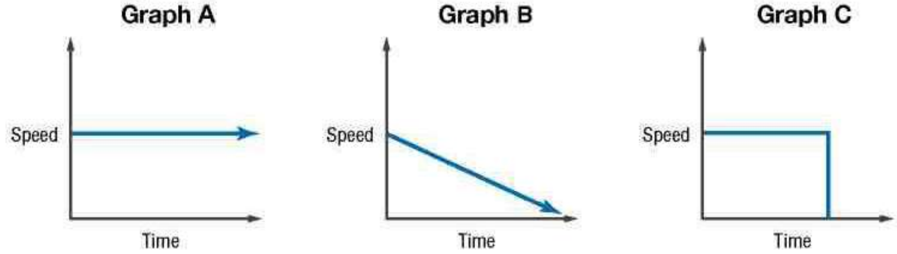
See page 822.

10. The graph below represents Michelle's temperature when she was sick. Describe what is happening in the graph.

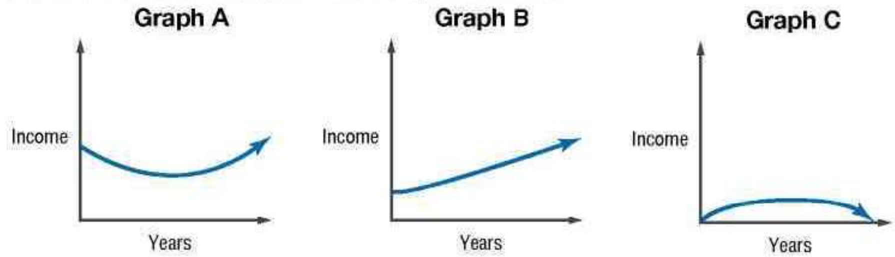


11. The graph below represents the balance in Rashaad's checking account. Describe what is happening in the graph.

12. **TOYS** Identify the graph that displays the speed of a radio-controlled car as it moves along and then hits a wall.



13. **INCOME** In general, as a person gets older, their income increases until they retire. Which of the graphs below represents this?



TRAVEL **For Exercises 14–16, use the table that shows the charges for parking a car in the hourly garage at an airport.**

Time Parked (h)	0–2	2–4	4–6	6–12	12–24
Cost ($)	1	2	4	5	30
	After 24 hours: $15 per each 24-hour period				

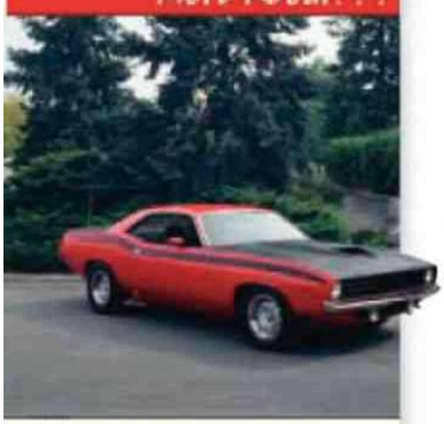
14. Write the ordered pairs that represent the cost of parking for up to 36 hours.
15. Draw a graph to show the cost of parking for up to 36 hours.
16. What is the cost of parking if you arrive on Monday at 7:00 A.M. and depart on Tuesday at 9:00 P.M.?

GEOMETRY **For Exercises 17–19, use the table that shows the relationship between the sum of the measures of the interior angles of convex polygons and the number of sides of the polygons.**

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Number of Sides	3	4	5	6	7
Interior Angle Sum	180	360	540	720	900

17. Identify the independent and dependent variables.
18. Draw a graph of the data.
19. Use the data to predict the sum of the interior angles for an octagon, nonagon, and decagon.
20. **CARS** A car was purchased new in 1970. The owner has taken excellent care of the car, and it has relatively low mileage. Draw a reasonable graph to show the value of the car from the time it was purchased to the present.
21. **CHEMISTRY** When ice is exposed to temperatures above 32°F, it begins to melt. Draw a reasonable graph showing the relationship between the temperature of a block of ice as it is removed from a freezer and placed on a counter at room temperature. (*Hint*: The temperature of the water will not exceed the temperature of its surroundings.)

More About. . .



Cars

Most new cars lose 15 to 30 percent of their value in the first year. After about 12 years, more popular cars tend to increase in value.

Source: *Consumer Guide*

22. **CRITICAL THINKING** Mallory is 23 years older than Lisa.

   a. Draw a graph showing Mallory's age as a function of Lisa's age for the first 40 years of Lisa's life.

   b. Find the point on the graph when Mallory is twice as old as Lisa.

23. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

   How can real-world situations be modeled using graphs and functions?

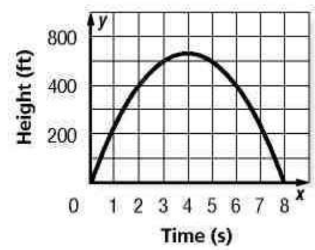
   Include the following in your answer:

   - an explanation of how the graph helps you analyze the situation,
   - a summary of what happens during the first 24 hours from the time of a concussion, and
   - an explanation of the time in which significant improvement occurs.

Standardized Test Practice

24. The graph shows the height of a model rocket shot straight up. How many seconds did it take for the rocket to reach its maximum height?

   (A) 3 (B) 4 (C) 5 (D) 6



25. Andre owns a computer backup service. He charges his customers \$2.50 for each backup CD. His expenses include \$875 for the CD recording equipment and \$0.35 for each blank CD. Which equation could Andre use to calculate his profit p for the recording of n CDs?

   (A) $p = 2.15n - 875$ (B) $p = 2.85 + 875$

   (C) $p = 2.50 - 875.65$ (D) $p = 875 - 2.15n$

Maintain Your Skills

Mixed Review **Identify the hypothesis and conclusion of each statement.** *(Lesson 1-7)*

26. You can send e-mail with a computer.

27. The express lane is for shoppers who have 9 or fewer items.

28. Name the property used in each step. *(Lesson 1-6)*

$$\begin{aligned} ab(a + b) &= (ab)a + (ab)b \\ &= a(ab) + (ab)b \\ &= (a \cdot a)b + a(b \cdot b) \\ &= a^2b + ab^2 \end{aligned}$$

Name the property used in each statement. Then find the value of n. *(Lesson 1-4)*

29. $(12 - 9)(4) = n(4)$ 30. $7(n) = 0$ 31. $n(87) = 87$

Getting Ready for the Next Lesson

32. **PREREQUISITE SKILL** Use the information in the table to construct a bar graph.
*(To review **making bar graphs**, see pages 806 and 807.)*

U.S. Commercial Radio Stations by Format, 2000					
Format	country	adult contemporary	news/ talk	oldies	rock
Number	2249	1557	1426	1135	827

Source: *The World Almanac*

Algebra Activity

A Follow-Up of Lesson 1-8

Investigating Real-World Functions

The table shows the number of students enrolled in elementary and secondary schools in the United States for the given years.

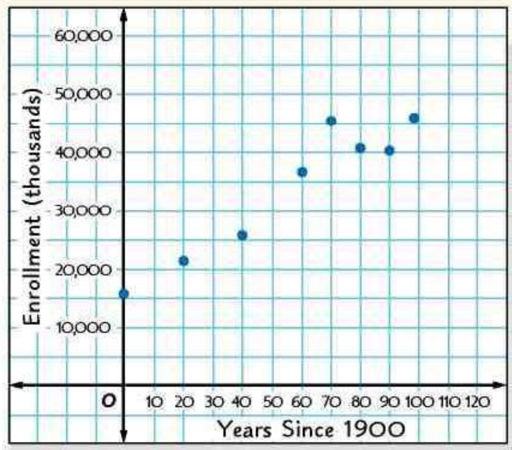
Year	Enrollment (thousands)	Year	Enrollment (thousands)
1900	15,503	1970	45,550
1920	21,578	1980	41,651
1940	25,434	1990	40,543
1960	36,807	1998	46,327

Source: *The World Almanac*

Step 1 On grid paper, draw a vertical and horizontal axis as shown. Make your graph large enough to fill most of the sheet. Label the horizontal axis 0 to 120 and the vertical axis 0 to 60,000.

Step 2 To make graphing easier, let x represent the number of years since 1900. Write the eight ordered pairs using this method. The first will be (0, 15,503).

Step 3 Graph the ordered pairs on your grid paper.



Analyze

1. Use your graph to estimate the number of students in elementary and secondary school in 1910 and in 1975.
2. Use your graph to estimate the number of students in elementary and secondary school in 2020.

Make a Conjecture

3. Describe the methods you used to make your estimates for Exercises 1 and 2.
4. Do you think your prediction for 2020 will be accurate? Explain your reasoning.
5. Graph this set of data, which shows the number of students per computer in U.S. schools. Predict the number of students per computer in 2010. Explain how you made your prediction.

Year	Students per Computer	Year	Students per Computer	Year	Students per Computer	Year	Students per Computer
1984	125	1988	32	1992	18	1996	10
1985	75	1989	25	1993	16	1997	7.8
1986	50	1990	22	1994	14	1998	6.1
1987	37	1991	20	1995	10.5	1999	5.7

Source: *The World Almanac*

1-9 Statistics: Analyzing Data by Using Tables and Graphs

What You'll Learn

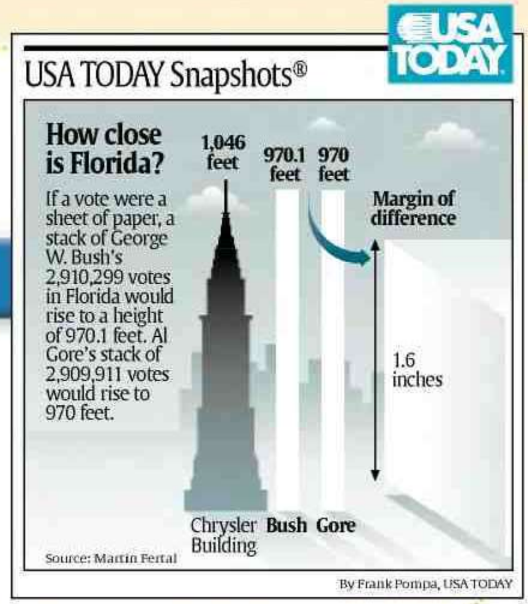
- Analyze data given in tables and graphs (bar, line, and circle).
- Determine whether graphs are misleading.

Vocabulary

- bar graph
- data
- circle graph
- line graph

Why are graphs and tables used to display data?

For several weeks after Election Day in 2000, data regarding the presidential vote counts changed on a daily basis. The bar graph at the right illustrates just how close the election was at one point and the importance of each vote in the election. The graph allows you to compare the data visually.



ANALYZE DATA A **bar graph** compares different categories of numerical information, or **data**, by showing each category as a bar whose length is related to the frequency. Bar graphs can also be used to display multiple sets of data in different categories at the same time. Graphs with multiple sets of data always have a key to denote which bars represent each set of data.

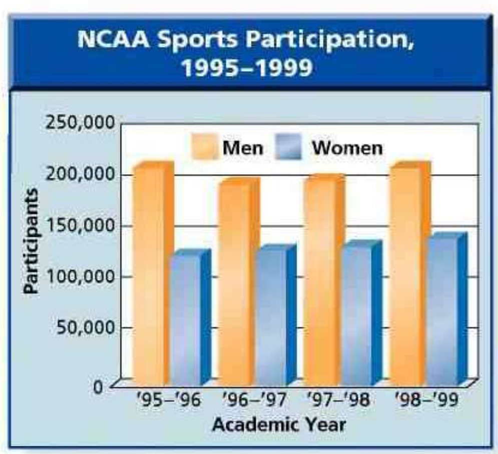
Example 1 Analyze a Bar Graph

The table shows the number of men and women participating in NCAA championship sports programs from 1995 to 1999.

NCAA Championship Sports Participation 1995–1999				
Year	'95–'96	'96–'97	'97–'98	'98–'99
Men	206,366	199,375	200,031	207,592
Women	125,268	129,295	133,376	145,832

Source: NCAA

This same data is displayed in a bar graph.



a. Describe the general trend shown in the graph.

The graph shows that the number of men has remained fairly constant while the number of women has been increasing.

Study Tip

Graphs and Tables

Graphs are useful for visualizing data and for estimations. Tables are used when you need precise data for computation.

CONTENTS

b. Approximately how many more men than women participated in sports during the 1997–1998 school year?

The bar for the number of men shows about 200,000 and the bar for the women shows about 130,000. So, there were approximately 200,000–130,000 or 70,000 more men than women participating in the 1997–1998 school year.

c. What was the total participation among men and women in the 1998–1999 academic year?

Since the table shows the exact numbers, use the data in it.

$$\underbrace{207{,}592}_{\text{Number of men}} \underbrace{+}_{\text{plus}} \underbrace{145{,}832}_{\text{number of women}} \underbrace{=}_{\text{equals}} \underbrace{353{,}424}_{\text{total participation.}}$$

There was a total of 353,424 men and women participating in sports in the 1998–1999 academic year.

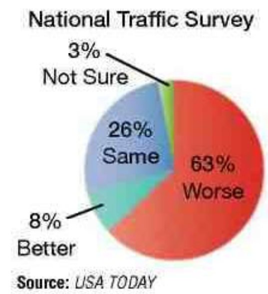
Study Tip

Reading Math
In everyday life, circle graphs are sometimes called *pie graphs* or *pie charts*.

Another type of graph used to display data is a circle graph. A **circle graph** compares parts of a set of data as a percent of the whole set. The percents in a circle graph should always have a sum of 100%.

Example 2 Analyze a Circle Graph

A recent survey asked drivers in several cities across the United States if traffic in their area had gotten better, worse, or had not changed in the past five years. The results of the survey are displayed in the circle graph.



a. If 4500 people were surveyed, how many felt that traffic had improved in their area?

The section of the graph representing people who said traffic is better is 8% of the circle, so find 8% of 4500.

$$\underbrace{0.08}_{8\%} \underbrace{\times}_{\text{of}} \underbrace{4500}_{4500} \underbrace{=}_{\text{equals}} \underbrace{360}_{360.}$$

360 people said that traffic was better.

b. If a city with a population of 647,000 is representative of those surveyed, how many people could be expected to think that traffic conditions are worse?

63% of those surveyed said that traffic is worse, so find 63% of 647,000.

$0.63 \times 647{,}000 = 407{,}610$

Thus, 407,610 people in the city could be expected to say that traffic conditions are worse.

A third type of graph used to display data is a line graph. **Line graphs** are useful when showing how a set of data changes over time. They can also be helpful when making predictions.

Career Choices

Professor

A college professor may teach by lecturing to several hundred students at a time or by supervising students in small groups in a laboratory. Often they also do their own research to expand knowledge in their field.

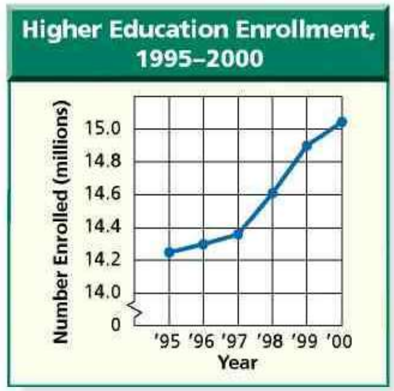
Online Research
For information about a career as a professor, visit: www.algebra1.com/careers

Example 3 Analyze a Line Graph

EDUCATION **Refer to the line graph below.**

a. Estimate the change in enrollment between 1995 and 1999.

The enrollment for 1995 is about 14.25 million, and the enrollment for 1999 is about 14.9 million. So, the change in enrollment is 14.9 − 14.25 or 0.65 million.



Source: U.S. National Center for Educational Statistics

b. If the rate of growth between 1998 and 1999 continues, predict the number of people who will be enrolled in higher education in the year 2005.

Based on the graph, the increase in enrollment from 1998 to 1999 is 0.3 million. So, the enrollment should increase by 0.3 million per year.

$14.9 + 0.3(6) = 14.9 + 1.8$ Multiply the annual increase, 0.3, by the number of years, 6.

$= 16.7$ Enrollment in 2005 should be about 16.7 million.

Concept Summary — Statistical Graphs

Type of Graph	bar graph	circle graph	line graph
When to Use	to compare different categories of data	to show data as parts of a whole set of data	to show the change in data over time

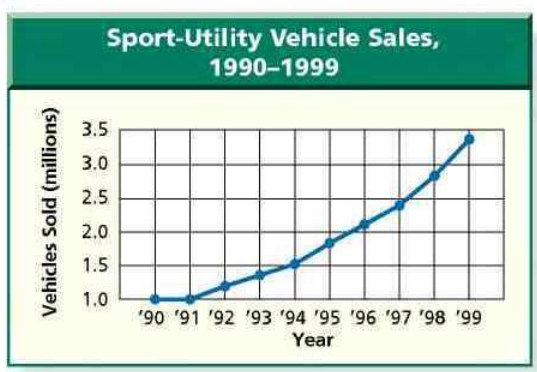
MISLEADING GRAPHS Graphs are very useful for displaying data. However, graphs that have been constructed incorrectly can be confusing and can lead to false assumptions. Many times these types of graphs are mislabeled, incorrect data is compared, or the graphs are constructed to make one set of data appear greater than another set. Here are some common ways that a graph may be misleading.

- Numbers are omitted on an axis, but no break is shown.
- The tick marks on an axis are not the same distance apart or do not have the same-sized intervals.
- The percents on a circle graph do not have a sum of 100.

Example 4 Misleading Graphs

AUTOMOBILES **The graph shows the number of sport-utility vehicle (SUV) sales in the United States from 1990 to 1999. Explain how the graph misrepresents the data.**

The vertical axis scale begins at 1 million. This causes the appearance of no vehicles sold in 1990 and 1991, and very few vehicles sold through 1994.



Source: *The World Almanac*

Check for Understanding

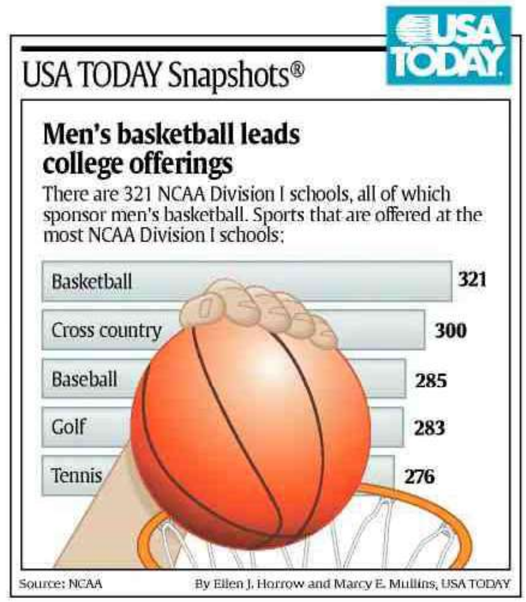
Concept Check

1. **Explain** the appropriate use of each type of graph.
   - circle graph
   - bar graph
   - line graph
2. **OPEN ENDED** Find a real-world example of a graph in a newspaper or magazine. Write a description of what the graph displays.
3. **Describe** ways in which a circle graph could be drawn so that it is misleading.

Guided Practice

SPORTS **For Exercises 4 and 5, use the following information.** There are 321 NCAA Division I schools. The graph at the right shows the sports that are offered at the most Division I schools.

4. How many more schools participate in basketball than in golf?
5. What sport is offered at the fewest schools?



EDUCATION **For Exercises 6–9, use the table that shows the number of foreign students as a percent of the total college enrollment in the United States.**

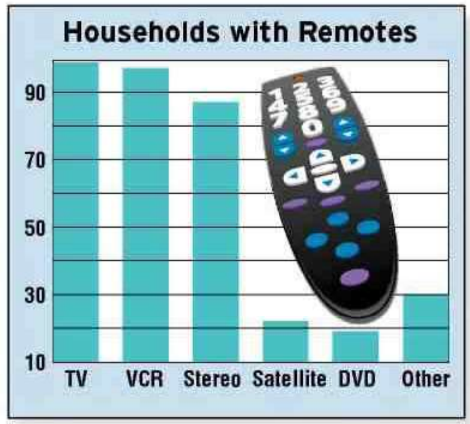
Country of Origin	Total Student Enrollment (%)
Australia	0.02
Canada	0.15
France	0.04
Germany	0.06
Italy	0.22
Spain	0.03
United Kingdom	0.05

Source: *Statistical Abstract of the United States*

6. There were about 14.9 million students enrolled in colleges in 1999. How many of these students were from Germany?
7. How many more students were from Canada than from the United Kingdom in 1999?
8. Would it be appropriate to display this data in a circle graph? Explain.
9. Would a bar or a line graph be more appropriate to display these data? Explain.

HOME ENTERTAINMENT For Exercises 10 and 11, refer to the graph.

10. Describe why the graph is misleading.
11. What should be done so that the graph displays the data more accurately?



Practice and Apply

Homework Help

For Exercises	See Examples
12, 13	1
14, 15	2
16	3, 4
17	2–4

Extra Practice

See page 822.

VIDEOGRAPHY For Exercises 12 and 13, use the table that shows the average cost of preparing one hour of 35-millimeter film versus one hour of digital video.

12. What is the total cost of using 35-millimeter film?
13. Estimate how many times as great the cost of using 35-millimeter film is as using digital video.

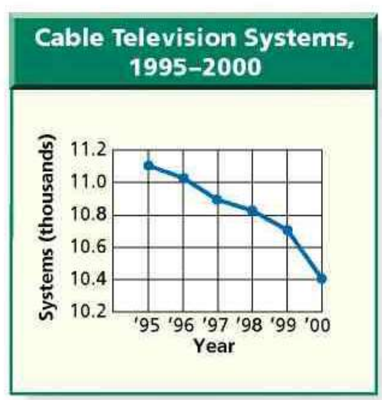
35 mm, editing video	
Film stock	$3110.40
Processing	621.00
Prep for telecine	60.00
Telecine	1000.00
Tape stock	73.20
Digital, editing on video	
Tape stock (original)	$10.00
Tape stock (back up)	10.00

Source: *USA TODAY*

BOOKS For Exercises 14 and 15, use the graph that shows the time of year people prefer to buy books.

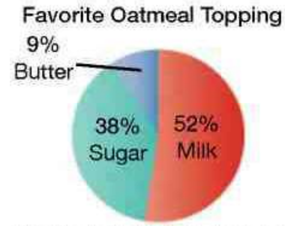
14. Suppose the total number of books purchased for the year was 25 million. Estimate the number of books purchased in the spring.
15. Suppose the manager of a bookstore has determined that she sells about 15,000 books a year. Approximately how many books should she expect to sell during the summer?

16. **ENTERTAINMENT** The line graph shows the number of cable television systems in the United States from 1995 to 2000. Explain how the graph misrepresents the data.



Data Source: *The World Almanac*

17. **FOOD** Oatmeal can be found in 80% of the homes in the United States. The circle graph shows favorite oatmeal toppings. Is the graph misleading? If so, explain why and tell how the graph can be fixed so that it is not misleading.



Data Source: NPD Group for Quaker Oats

WebQuest

A graph of the number of people over 65 in the U.S. for the years since 1900 will help you predict trends. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

18. **CRITICAL THINKING** The table shows the percent of United States households owning a color television for the years 1980 to 2000.
    a. Display the data in a line graph that shows little increase in ownership.
    b. Draw a line graph that shows a rapid increase in the number of households owning a color television.
    c. Are either of your graphs misleading? Explain.

Households with Color Televisions

Year	Percent
1980	83
1985	91
1990	98
1995	99
2000	99

Source: *The World Almanac*

19. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

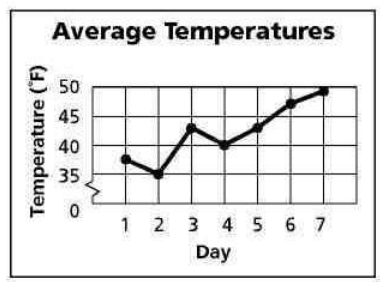
Why are graphs and tables used to display data?

Include the following in your answer:

- a description of how to use graphs to make predictions, and
- an explanation of how to analyze a graph to determine whether the graph is misleading.

Standardized Test Practice A B C D

20. According to the graph, the greatest increase in temperature occurred between which two days?
    (A) 1 and 2 (B) 6 and 7
    (C) 2 and 3 (D) 5 and 6



21. A graph that is primarily used to show the change in data over time is called a
    (A) circle graph. (B) bar graph.
    (C) line graph. (D) data graph.

Maintain Your Skills

Mixed Review

22. **PHYSICAL FITNESS** Pedro likes to exercise regularly. On Mondays, he walks two miles, runs three miles, sprints one-half of a mile, and then walks for another mile. Sketch a graph that represents Mitchell's heart rate during his Monday workouts. *(Lesson 1-8)*

Find a counterexample for each statement. *(Lesson 1-7)*

23. If $4x - 5 \leq 42$, then $x \leq 12$.
24. If $x > 1$, then $x < \frac{1}{x}$.
25. If the perimeter of a rectangle is 16 inches, then each side is 4 inches long.

Simplify each expression. *(Lesson 1-6)*

26. $7a + 5b + 3b + 3a$
27. $4x^2 + 9x + 2x^2 + x$
28. $\frac{1}{2}n + \frac{2}{3}m + \frac{1}{2}m + \frac{1}{3}n$

Statistical Graphs

You can use a computer spreadsheet program to display data in different ways. The data is entered into a table and then displayed in your chosen type of graph.

Example

Use a spreadsheet to make a line graph of the data on sports equipment sales.

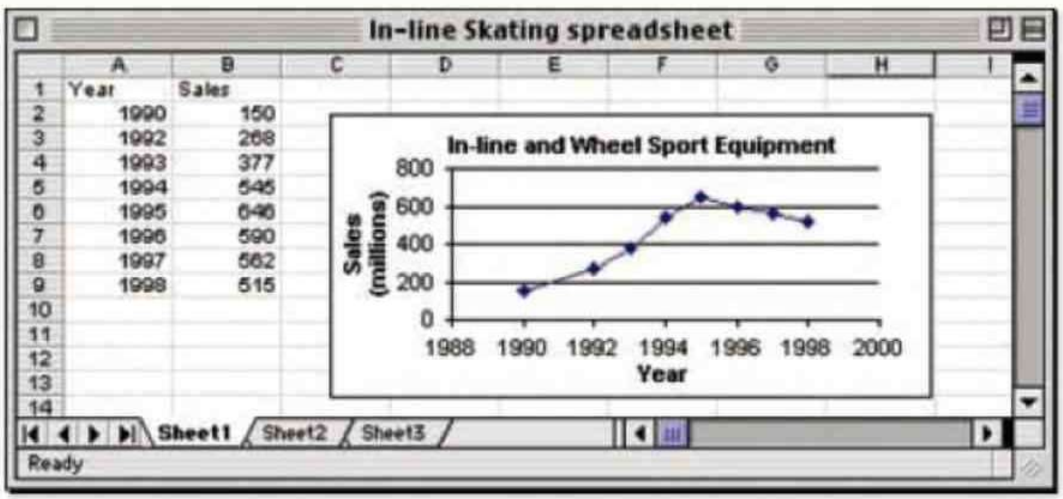
In-line Skating and Wheel Sports Equipment Sales								
Year	1990	1992	1993	1994	1995	1996	1997	1998
Sales (millions)	150	268	377	545	646	590	562	515

Source: National Sporting Goods Association

Step 1 Enter the data in a spreadsheet. Use Column A for the years and Column B for the sales.

Step 2 Select the data to be included in your graph. Then use the graph tool to create the graph.

The spreadsheet will allow you to change the appearance of the graph by adding titles and axis labels, adjusting the scales on the axes, changing colors, and so on.



Exercises

For Exercises 1–3, use the data on snowmobile sales in the table below.

Snowmobile Sales								
Year	1990	1992	1993	1994	1995	1996	1997	1998
Sales (millions)	322	391	515	715	910	974	975	957

Source: National Sporting Goods Association

1. Use a spreadsheet program to create a line graph of the data.

2. Use a spreadsheet program to create a bar graph of the data.

3. Adjust the scales on each of the graphs that you created. Is it possible to create a misleading graph using a spreadsheet program? Explain.

Chapter 1

Study Guide and Review

Vocabulary and Concept Check

additive identity (p. 21)
algebraic expression (p. 6)
Associative Property (p. 32)
bar graph (p. 50)
base (p. 7)
circle graph (p. 51)
Closure Property (p. 25)
coefficient (p. 29)
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conditional statement (p. 37)
coordinate system (p. 43)
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domain (p. 45)
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ordered pair (p. 43)
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vertical axis (p. 43)
x-axis (p. 43)
x-coordinate (p. 43)
y-axis (p. 43)
y-coordinate (p. 43)

Choose the letter of the property that best matches each statement.

1. For any number a, $a + 0 = 0 + a = a$.
2. For any number a, $a \cdot 1 = 1 \cdot a = a$.
3. For any number, a, $a \cdot 0 = 0 \cdot a = 0$.
4. For any nonzero number a, there is exactly one number $\frac{1}{a}$ such that $\frac{1}{a} \cdot a = a \cdot \frac{1}{a} = 1$.
5. For any number a, $a = a$.
6. For any numbers a and b, if $a = b$, then $b = a$.
7. For any numbers a and b, if $a = b$, then a may be replaced by b in any expression.
8. For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
9. For any numbers a, b, and c, $a(b + c) = ab + ac$.
10. For any numbers a, b, and c, $a + (b + c) = (a + b) + c$.

a. Additive Identity Property
b. Distributive Property
c. Commutative Property
d. Associative Property
e. Multiplicative Identity Property
f. Multiplicative Inverse Property
g. Multiplicative Property of Zero
h. Reflexive Property
i. Substitution Property
j. Symmetric Property
k. Transitive Property

Lesson-by-Lesson Review

1-1 Variables and Expressions

See pages 6–9.

Concept Summary

- Variables are used to represent unspecified numbers or values.
- An algebraic expression contains letters and variables with an arithmetic operation.

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Examples

1 **Write an algebraic expression for *the sum of twice a number x and fifteen.***

$\underbrace{2x}_{\text{twice a number } x} \quad \underbrace{+}_{\text{sum of}} \quad \underbrace{15}_{\text{fifteen}}$ The algebraic expression is $2x + 15$.

2 **Write a verbal expression for $4x^2 - 13$.**

Four times a number x squared minus thirteen.

Exercises **Write an algebraic expression for each verbal expression.**
See Examples 1 and 2 on pages 6 and 7.

11. a number x to the fifth power
12. five times a number x squared
13. the sum of a number x and twenty-one
14. the difference of twice a number x and 8

Evaluate each expression. *See Example 3 on page 7.*

15. 3^3
16. 2^5
17. 5^4

Write a verbal expression for each algebraic expression. *See Example 4 on page 7.*

18. $2p^2$
19. $3m^5$
20. $\frac{1}{2} + 2$

1-2 Order of Operations

See pages 11–15.

Concept Summary

- Expressions must be simplified using the order of operations.
  - **Step 1** Evaluate expressions inside grouping symbols.
  - **Step 2** Evaluate all powers.
  - **Step 3** Do all multiplications and/or divisions from left to right.
  - **Step 4** Do all additions and/or subtractions from left to right.

Example

Evaluate $x^2 - (y + 2)$ if $x = 4$ and $y = 3$.

$x^2 - (y + 2) = 4^2 - (3 + 2)$ Replace x with 4 and y with 3.
$= 4^2 - 5$ Add 3 and 2.
$= 16 - 5$ Evaluate power.
$= 11$ Subtract 5 from 16.

Exercises **Evaluate each expression.** *See Examples 1–3 on pages 11 and 12.*

21. $3 + 2 \cdot 4$
22. $\frac{(10 - 6)}{8}$
23. $18 - 4^2 + 7$
24. $8(2 + 5) - 6$
25. $4(11 + 7) - 9 \cdot 8$
26. $288 \div [3(9 + 3)]$
27. $16 \div 2 \cdot 5 \cdot 3 \div 6$
28. $6(4^3 + 2^2)$
29. $(3 \cdot 1)^3 - \frac{(4 + 6)}{(5 \cdot 2)}$

Evaluate each expression if $x = 3$, $t = 4$, and $y = 2$. *See Example 4 on page 12.*

30. $t^2 + 3y$
31. xty^3
32. $\frac{ty}{x}$
33. $x + t^2 + y^2$
34. $3ty - x^2$
35. $8(x - y)^2 + 2t$

1-3 Open Sentences

See pages 16–20.

Concept Summary

- Open sentences are solved by replacing the variables in an equation with numerical values.
- Inequalities like $x + 2 \geq 7$ are solved the same way that equations are solved.

Example **Solve $5^2 - 3 = y$.**

$5^2 - 3 = y$ Original equation

$25 - 3 = y$ Evaluate the power.

$22 = y$ Subtract 3 from 25.

The solution is 22.

Exercises **Solve each equation.** *See Example 2 on page 17.*

36. $x = 22 - 13$ **37.** $y = 4 + 3^2$ **38.** $m = \frac{64 + 4}{17}$

39. $x = \frac{21 - 3}{12 - 3}$ **40.** $a = \frac{14 + 28}{4 + 3}$ **41.** $n = \frac{96 \div 6}{8 \div 2}$

42. $b = \frac{7(4 \cdot 3)}{18 \div 3}$ **43.** $\frac{6(7) - 2(3)}{4^2 - 6(2)}$ **44.** $y = 5[2(4) - 1^3]$

Find the solution set for each inequality if the replacement set is {4, 5, 6, 7, 8}.
See Example 3 on page 17.

45. $x + 2 > 7$ **46.** $10 - x < 7$ **47.** $2x + 5 \geq 15$

1-4 Identity and Equality Properties

See pages 21–25.

Concept Summary

- Adding zero to a quantity or multiplying a quantity by one does not change the quantity.
- Using the Reflexive, Symmetric, Transitive, and Substitution Properties along with the order of operations helps in simplifying expressions.

Example **Evaluate $36 + 7 \cdot 1 + 5(2 - 2)$. Name the property used in each step.**

$36 + 7 \cdot 1 + 5(2 - 2) = 36 + 7 \cdot 1 + 5(0)$ Substitution (=)

$= 36 + 7 + 5(0)$ Multiplicative Identity

$= 36 + 7$ Multiplicative Prop. of Zero

$= 43$ Substitution

Exercises **Evaluate each expression. Name the property used in each step.**
See Example 2 on page 23.

48. $2[3 \div (19 - 4^2)]$ **49.** $\frac{1}{2} \cdot 2 + 2[2 \cdot 3 - 1]$ **50.** $4^2 - 2^2 - (4 - 2)$

51. $1.2 - 0.05 + 2^3$ **52.** $(7 - 2)(5) - 5^2$ **53.** $3(4 \div 4)^2 - \frac{1}{4}(8)$

1-5 The Distributive Property

See pages 26–31.

Concept Summary

- For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$.
- For any numbers a, b, and c, $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Examples

1 Rewrite $5(t + 3)$ using the Distributive Property. Then simplify.

$5(t + 3) = 5(t) + 5(3)$ Distributive Property

$= 5t + 15$ Multiply.

2 Simplify $2x^2 + 4x^2 + 7x$.

$2x^2 + 4x^2 + 7x = (2 + 4)x^2 + 7x$ Distributive Property

$= 6x^2 + 7x$ Substitution

Exercises **Rewrite each product using the Distributive Property. Then simplify.**
See Examples 1 and 2 on page 27.

54. $2(4 + 7)$ **55.** $8(15 - 6)$ **56.** $4(x + 1)$

57. $3\left(\frac{1}{3} - p\right)$ **58.** $6(a + b)$ **59.** $8(3x - 7y)$

Simplify each expression. If not possible, write *simplified*. *See Example 6 on page 29.*

60. $4a + 9a$ **61.** $4np + 7mp$ **62.** $3w - w + 4v - 3v$

63. $3m + 5m + 12n - 4n$ **64.** $2p(1 + 16r)$ **65.** $9y^2 + -5y + 3y^2$

1-6 Commutative and Associative Properties

See pages 32–36.

Concept Summary

- For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
- For any numbers a, b and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Example

Simplify $3x + 7xy + 9x$.

$3x + 7xy + 9x = 3x + 9x + 7xy$ Commutative (+)

$= (3 + 9)x + 7xy$ Distributive Property

$= 12x + 7xy$ Substitution

Exercises **Simplify each expression.** *See Example 3 on page 33.*

66. $3x + 4y + 2x$ **67.** $7w^2 + w + 2w^2$ **68.** $3\frac{1}{2}m + \frac{1}{2}m + n$

69. $6a + 5b + 2c + 8b$ **70.** $3(2 + 3x) + 21x$ **71.** $6(2n - 4) + 5n$

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used. *See Example 4 on page 34.*

72. five times the sum of x and y decreased by $2x$

73. twice the product of p and q increased by the product of p and q

74. six times a plus the sum of eight times b and twice a

75. three times the square of x plus the sum of x squared and seven times x

1-7 Logical Reasoning

See pages 37–42.

Concept Summary

- Conditional statements can be written in the form *If A, then B.* where *A* is the hypothesis and *B* is the conclusion.
- One counterexample can be used to show that a statement is false.

Example **Identify the hypothesis and conclusion of the statement *The trumpet player must audition to be in the band.* Then write the statement in if-then form.**

Hypothesis: a person is a trumpet player

Conclusion: the person must audition to be in the band

If a person is a trumpet player, then the person must audition to be in the band.

Exercises **Identify the hypothesis and conclusion of each statement. Then, write each statement in if-then form.** *See Example 2 on page 38.*

76. School begins at 7:30 A.M.

77. Triangles have three sides.

Find a counterexample for each statement. *See Example 4 on page 39.*

78. If $x > y$, then $2x > 3y$.

79. If $a > b$ and $a > c$, then $b > c$.

1-8 Graphs and Functions

See pages 43–48.

Concept Summary

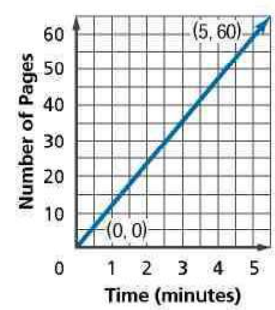
- Graphs can be used to represent a function and to visualize data.

Example **A computer printer can print 12 pages of text per minute.**

a. Make a table showing the number of pages printed in 1 to 5 minutes.

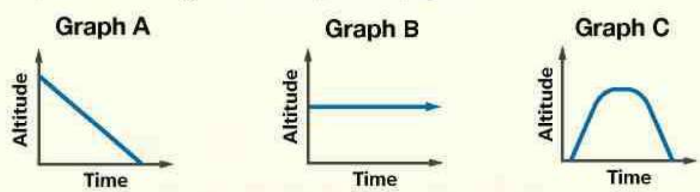
Time (min)	1	2	3	4	5
Pages	12	24	36	48	60

b. Sketch a graph that shows the relationship between time and the number of pages printed.



Exercises

80. Identify the graph that represents the altitude of an airplane taking off, flying for a while, then landing. *See Example 3 on page 44.*



• Extra Practice, see pages 820–822.
• Mixed Problem Solving, see page 853.

81. Sketch a reasonable graph that represents the amount of helium in a balloon if it is filled until it bursts. *See Examples 3–5 on pages 44 and 45.*

For Exercises 82 and 83, use the following information.
The planet Mars takes longer to orbit the sun than does Earth. One year on Earth is about 0.54 year on Mars. *See Examples 4 and 5 on page 45.*

82. Construct a table showing the relationship between years on Earth and years on Mars.

83. Draw a graph showing the relationship between Earth years and Mars years.

1-9 Statistics: Analyzing Data by Using Tables and Graphs

See pages 50–55.

Concept Summary

- Bar graphs are used to compare different categories of data.
- Circle graphs are used to show data as parts of a whole set of data.
- Line graphs are used to show the change in data over time.

Example **The bar graph shows ways people communicate with their friends.**

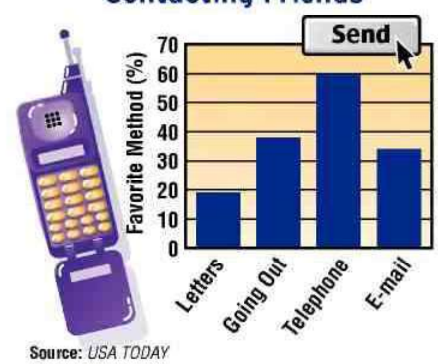
a. About what percent of those surveyed chose e-mail as their favorite way to talk to friends?

The bar for e-mail is about halfway between 30% and 40%. Thus, about 35% favor e-mail.

b. What is the difference in the percent of people favoring letters and those favoring the telephone?

The bar for those favoring the telephone is at 60%, and the bar for letters is about 20%. So, the difference is 60 − 20 or 40%.

Favorite Method of Contacting Friends



Source: *USA TODAY*

Exercises

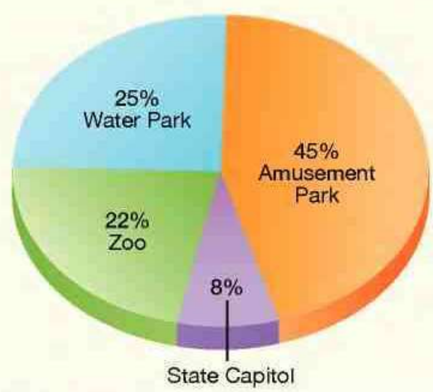
CLASS TRIP **For Exercises 84 and 85, use the circle graph and the following information.**

A survey of the ninth grade class asked members to indicate their choice of locations for their class trip. The results of the survey are displayed in the circle graph. *See Example 2 on page 51.*

84. If 120 students were surveyed, how many chose the amusement park?

85. If 180 students were surveyed, how many more chose the amusement park than the water park?

9th Grade Class Survey



Chapter 1 Practice Test

Vocabulary and Concepts

Choose the letter of the property that best matches each statement.

1. For any number a, $a = a$.
2. For any numbers a and b, if $a = b$, then b may be replaced by a in any expression or equation.
3. For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

a. Substitution Property of Equality
b. Symmetric Property of Equality
c. Transitive Property of Equality
d. Reflexive Property of Equality

Skills and Applications

Write an algebraic expression for each verbal expression.

4. the sum of a number x and 13
5. the difference of 7 and number x squared

Simplify each expression.

6. $5(9 + 3) - 3 \cdot 4$
7. $12 \cdot 6 \div 3 \cdot 2 \div 8$

Evaluate each expression if $a = 2$, $b = 5$, $c = 3$, and $d = 1$.

8. $a^2b + c$
9. $(cd)^3$
10. $(a + d)c$

Solve each equation.

11. $y = (4.5 + 0.8) - 3.2$
12. $4^2 - 3(4 - 2) = x$
13. $\frac{2^3 - 1^3}{2 + 1} = n$

Evaluate each expression. Name the property used in each step.

14. $3^2 - 2 + (2 - 2)$
15. $(2 \cdot 2 - 3) + 2^2 + 3^2$

Rewrite each expression in simplest form.

16. $2m + 3m$
17. $4x + 2y - 2x + y$
18. $3(2a + b) - 5a + 4b$

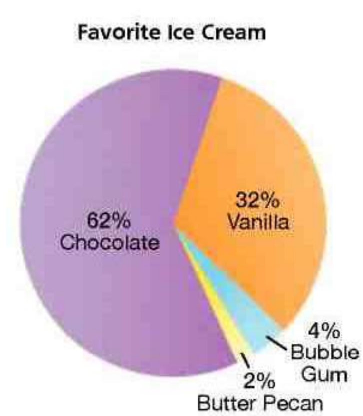
Find a counterexample for each conditional statement.

19. If you run fifteen minutes today, then you will be able to run a marathon tomorrow.
20. If $2x - 3 < 9$, then $x \leq 6$.

Sketch a reasonable graph for each situation.

21. A basketball is shot from the free throw line and falls through the net.
22. A nickel is dropped on a stack of pennies and bounces off.

ICE CREAM **For Exercises 23 and 24, use the following information.** A school survey at West High School determined the favorite flavors of ice cream are chocolate, vanilla, butter pecan, and bubble gum. The results of the survey are displayed in the circle graph.



23. If 200 students were surveyed, how many more chose chocolate than vanilla?
24. What was the total percent of students who chose either chocolate or vanilla?
25. **STANDARDIZED TEST PRACTICE** Which number is a counterexample for the statement below?

If a is a prime number, then a is odd.

(A) 5 (B) 4 (C) 3 (D) 2

Chapter 1 Standardized Test Practice

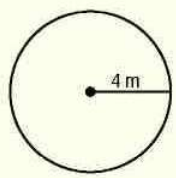
Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The Maple Grove Warehouse measures 800 feet by 200 feet. If $\frac{3}{4}$ of the floor space is covered, how many square feet are *not* covered? (Prerequisite Skill)

Ⓐ 4000 Ⓑ 40,000
Ⓒ 120,000 Ⓓ 160,000

2. The radius of a circular flower garden is 4 meters. How many meters of edging will be needed to surround the garden? (Prerequisite Skill)



Ⓐ 7.14 m Ⓑ 12.56 m
Ⓒ 25.12 m Ⓓ 20.24 m

3. The Johnson family spends about \$80 per week on groceries. Approximately how much do they spend on groceries per year? (Prerequisite Skill)

Ⓐ \$400 Ⓑ \$4000
Ⓒ \$8000 Ⓓ \$40,000

4. Daria is making 12 party favors for her sister's birthday party. She has 50 stickers, and she wants to use as many of them as possible. If she puts the same number of stickers in each bag, how many stickers will she have left over? (Prerequisite Skill)

Ⓐ 2 Ⓑ 4 Ⓒ 6 Ⓓ 8

The Princeton Review Test-Taking Tip

Questions 1, 3, and 8 Read each question carefully. Be sure you understand what the question asks. Look for words like *not*, *estimate*, and *approximately*.

5. An auto repair shop charges \$36 per hour, plus the cost of replaced parts. Which of the following expressions can be used to calculate the total cost of repairing a car, where h represents the number of hours of work and the cost of replaced parts is \$85? (Lesson 1-1)

Ⓐ $36 + h + 85$ Ⓑ $(85 \times h) + 36$
Ⓒ $36 + 85 \times h$ Ⓓ $(36 \times h) + 85$

6. Which expression is equivalent to $3(2x + 3) + 2(x + 1)$? (Lessons 1-5 and 1-6)

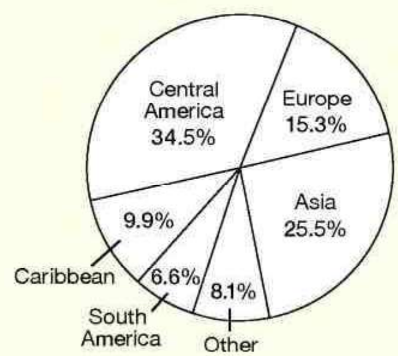
Ⓐ $7x + 8$ Ⓑ $8x + 4$
Ⓒ $8x + 9$ Ⓓ $8x + 11$

7. Find a counterexample for the following statement. (Lesson 1-7)
If x is a positive integer, then x^2 is divisible by 2.

Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 6

8. The circle graph shows the regions of birth of foreign-born persons in the United States in 2000. According to the graph, which statement is *not* true? (Lesson 1-9)

Regions of Birth



Ⓐ More than $\frac{1}{3}$ of the foreign-born population is from Central America.

Ⓑ More foreign-born people are from Asia than Central America.

Ⓒ About half of the foreign-born population comes from Central America or Europe.

Ⓓ About half of the foreign-born population comes from Central America, South America, or the Caribbean.

CONTENTS

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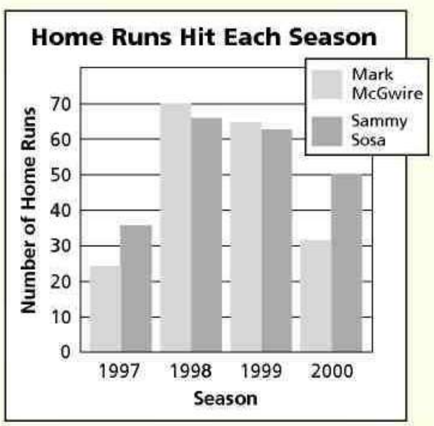
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. There are 32 students in the class. Five eighths of the students are girls. How many boys are in the class? (Prerequisite Skill)

10. Tonya bought two paperback books. One book cost $8.99 and the other $13.99. Sales tax on her purchase was 6%. How much change should she receive if she gives the clerk $25? (Prerequisite Skill)

11. According to the bar graph of the home runs hit by two baseball players, in which year was the difference between the numbers of home runs hit by the two players the least? (Prerequisite Skill)



Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the two quantities are equal, or**

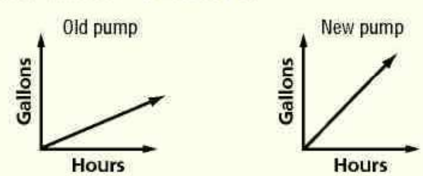
Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
12.	15% of 80	25% of 50
		(Prerequisite Skill)
13.	$\frac{10}{\frac{2}{3}}$	1.5
		(Prerequisite Skill)
14.	$2x - 1$	$2x + 1$
		(Lesson 1-3)
15.	$\frac{1}{4}(a + b)c$	$\frac{ac + bc}{4}$
		(Lesson 1-5)
16.	$(26 \times 39) + (39 \times 13)$	$(39)^2$
		(Lesson 1-5)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

17. Workers are draining water from a pond. They have an old pump and a new pump. The graphs below show how each pump drains water. (Lesson 1-8)



a. Describe how the old and new pumps are different in the amount of water they pump per hour.

b. Draw a graph that shows the gallons pumped per hour by both pumps at the same time.

c. Explain what the graph below tells about how the water is pumped out.

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Chapter

2 Real Numbers

What You'll Learn

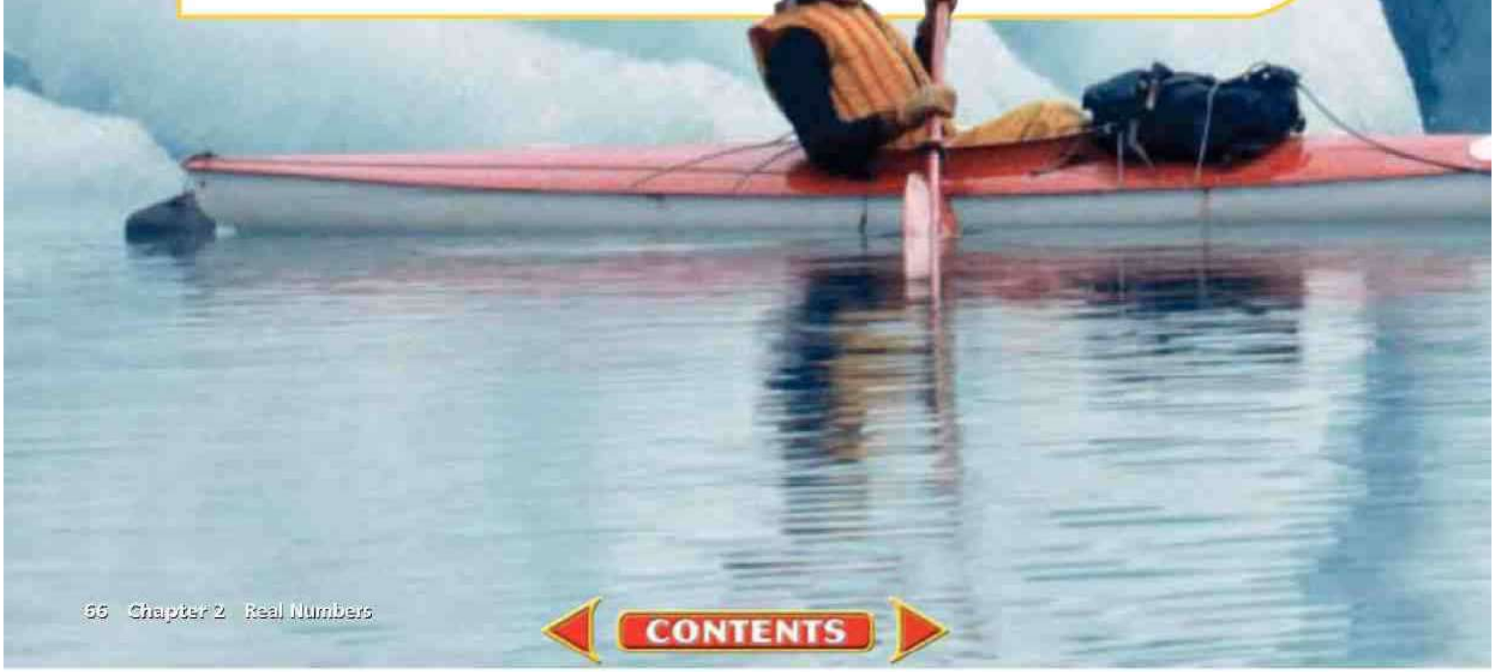
- **Lesson 2-1** Classify and graph rational numbers.
- **Lessons 2-2 through 2-4** Add, subtract, multiply, and divide rational numbers.
- **Lesson 2-5** Display and interpret statistical data on line graphs and stem-and-leaf plots.
- **Lesson 2-6** Determine simple probability and odds.
- **Lesson 2-7** Find square roots and compare real numbers.

Key Vocabulary

- **rational number** (p. 68)
- **absolute value** (p. 69)
- **probability** (p. 96)
- **square root** (p.103)
- **real number** (p. 104)

Why It's Important

The ability to work with real numbers lays the foundation for further study in mathematics and allows you to solve a variety of real-world problems. For example, temperatures in the United States vary greatly from cold arctic regions to warm tropical regions. You can use real numbers and absolute value to compare these temperature extremes. *You will use absolute value and real numbers to compare temperatures in Lessons 2-1 and 2-2.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

For Lessons 2-1 through 2-5 — Operations with Decimals and Fractions

Perform the indicated operation. *(For review, see pages 798 and 799.)*

1. $2.2 + 0.16$ **2.** $13.4 - 4.5$ **3.** $6.4 \cdot 8.8$ **4.** $76.5 \div 4.25$

5. $\frac{1}{4} + \frac{2}{3}$ **6.** $\frac{1}{2} - \frac{1}{3}$ **7.** $\frac{5}{4} \cdot \frac{3}{10}$ **8.** $\frac{4}{9} \div \frac{1}{3}$

For Lessons 2-1 through 2-5 — Evaluate Expressions

Evaluate each expression if $a = 2$, $b = \frac{1}{4}$, $x = 7$, and $y = 0.3$. *(For review, see Lesson 1-2.)*

9. $3a - 2$ **10.** $2x + 5$ **11.** $8(y + 2.4)$ **12.** $4(b + 2)$

13. $a - \frac{1}{2}$ **14.** $b + 3$ **15.** xy **16.** $y(a \div b)$

For Lesson 2-5 — Find Mean, Median, and Mode

Find the mean, median, and mode for each set of data. *(For review, see pages 818 and 819.)*

17. 2, 4, 7, 9, 12, 15 **18.** 23, 23, 23, 12, 12, 14 **19.** 7, 19, 2, 7, 4, 9

For Lesson 2-7 — Square Numbers

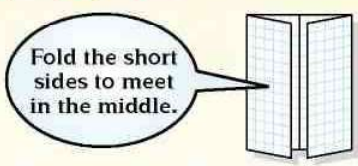
Simplify. *(For review, see Lesson 1-1.)*

20. 11^2 **21.** 0.9^2 **22.** $\left(\frac{2}{3}\right)^2$ **23.** $\left(\frac{4}{5}\right)^2$

FOLDABLES™ Study Organizer

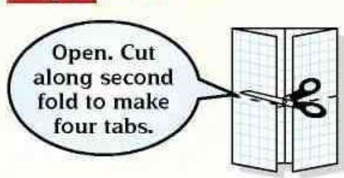
Make this Foldable to collect examples and notes about operations with real numbers. Begin with a sheet of grid paper.

Step 1 Fold

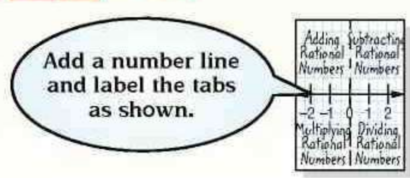


Step 2 Fold Again

Step 3 Cut



Step 4 Label



Reading and Writing As you read and study the chapter, use the number line to help you solve problems. Write examples and notes under each tab.

2-1 Rational Numbers on the Number Line

What You'll Learn

- Graph rational numbers on a number line.
- Find absolute values of rational numbers.

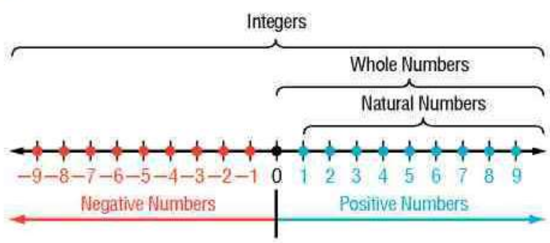
How can you use a number line to show data?

A river's level rises and falls depending on rainfall and other conditions. The table shows the percent of change in river depths for various rivers in Texas over a 24-hour period. You can use a number line to graph these values and compare the changes in each river.

Vocabulary

- natural number
- whole number
- integers
- positive number
- negative number
- rational number
- infinity
- graph
- coordinate
- absolute value

GRAPH RATIONAL NUMBERS A number line can be used to show the sets of **natural numbers**, **whole numbers**, and **integers**. Values greater than 0, or **positive numbers**, are listed to the right of 0, and values less than 0, or **negative numbers**, are listed to the left of 0.



Study Tip

Number Line
Although only a portion of the number line is shown, the arrowheads indicate that the line and the set continue to **infinity**, which means that they never end.

Another set of numbers you can display on a number line is the set of rational numbers. A **rational number** is any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Some examples of rational numbers are shown below.

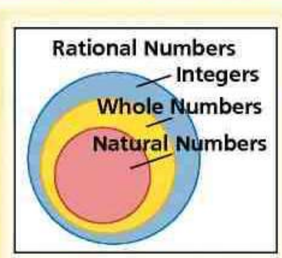
$$\frac{1}{2} \qquad \frac{-2}{3} \qquad \frac{17}{5} \qquad \frac{15}{-3} \qquad \frac{-14}{-11} \qquad \frac{3}{1}$$

A rational number can also be expressed as a decimal that terminates, or as a decimal that repeats indefinitely.

$$0.5 \qquad -0.\overline{3} \qquad 3.4 \qquad 2.6767\ldots \qquad -5 \qquad 1.\overline{27} \qquad -1.23568994141\ldots$$

Concept Summary — Rational Numbers

Natural Numbers	{1, 2, 3, ...}
Whole Numbers	{0, 1, 2, 3, ...}
Integers	{..., −2, −1, 0, 1, 2, ...}
Rational Numbers	numbers expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$



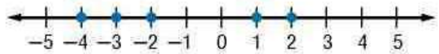
Later in this chapter, you will be introduced to numbers that are not rational.

To **graph** a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the **coordinate** of that point.

Example 1 Identify Coordinates on a Number Line

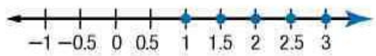
Name the coordinates of the points graphed on each number line.

a.



The dots indicate each point on the graph.
The coordinates are $\{-4, -3, -2, 1, 2\}$.

b.

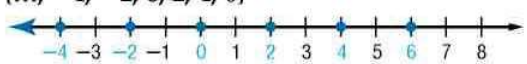


The bold arrow on the right means that the graph continues indefinitely in that direction. The coordinates are $\{1, 1.5, 2, 2.5, 3, \ldots\}$.

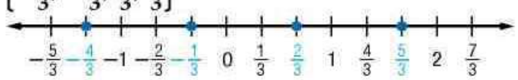
Example 2 Graph Numbers on a Number Line

Graph each set of numbers.

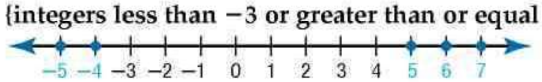
a. $\{\ldots, -4, -2, 0, 2, 4, 6\}$



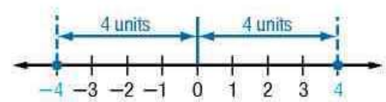
b. $\left\{-\frac{4}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{5}{3}\right\}$



c. {integers less than −3 or greater than or equal to 5}



ABSOLUTE VALUE On a number line, 4 is four units from zero in the positive direction, and −4 is four units from zero in the negative direction. This number line illustrates the meaning of **absolute value**.



Key Concept — Absolute Value

- **Words** The absolute value of any number n is its distance from zero on a number line and is written as $|n|$.
- **Examples** $|-4| = 4$ $|4| = 4$

Since distance cannot be less than zero, absolute values are always greater than or equal to zero.

Study Tip

Reading Math
$|-7| = 7$ is read *the absolute value of negative 7 equals 7.*

Example 3 Absolute Value of Rational Numbers

Find each absolute value.

a. $|-7|$

−7 is seven units from zero in the negative direction.

$|-7| = 7$

b. $\left|\frac{7}{9}\right|$

$\frac{7}{9}$ is seven-ninths unit from zero in the positive direction.

$\left|\frac{7}{9}\right| = \frac{7}{9}$

You can also evaluate expressions involving absolute value. The absolute value bars serve as grouping symbols.

Example 4 Expressions with Absolute Value

Evaluate $15 - |x + 4|$ if $x = 8$.

$15 - \lvert x + 4\rvert$	$= 15 - \lvert 8 + 4\rvert$	Replace x with 8.
	$= 15 - \lvert 12\rvert$	$8 + 4 = 12$
	$= 15 - 12$	$\lvert 12\rvert = 12$
	$= 3$	Simplify.

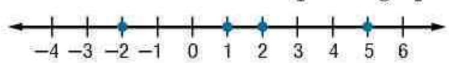
Check for Understanding

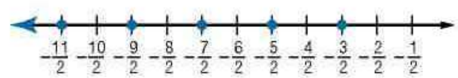
Concept Check

1. **Tell** whether the statement is *sometimes*, *always*, or *never* true.
   An integer is a rational number.
2. **Explain** the meaning of absolute value.
3. **OPEN ENDED** Give an example where absolute values are used in a real-life situation.

Guided Practice

Name the coordinates of the points graphed on each number line.

4. 

5. 

Graph each set of numbers.

6. $\{-4, -2, 1, 5, 7\}$
7. $\{-2.8, -1.5, 0.2, 3.4\}$
8. $\left\{-\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{5}{3}\right\}$
9. {integers less than or equal to -4}

Find each absolute value.

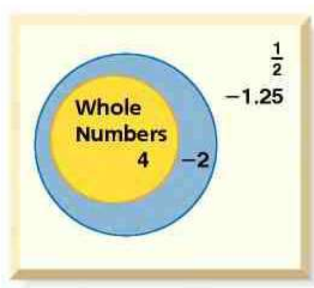
10. $|-2|$
11. $|18|$
12. $|2.5|$
13. $\left|-\frac{5}{6}\right|$

Evaluate each expression if $x = 18$, $y = 4$, and $z = -0.76$.

14. $57 - |x + 34|$
15. $19 + |21 - y|$
16. $|z| - 0.26$

Application

17. **NUMBER THEORY** Copy the Venn diagram at the right. Label the remaining sets of numbers. Then place the numbers -3, -13, 0, 53, $\frac{2}{3}$, $-\frac{1}{5}$, 0.33, 40, 2.98, -49.98, and $-\frac{5}{2}$ in the most specific categories.



CONTENTS

Practice and Apply

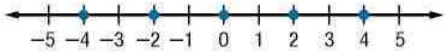
Homework Help

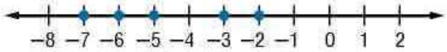
For Exercises	See Examples
18–23	1
24–33	2
34–41	3
42–44, 58, 59	2, 3
45–56	4

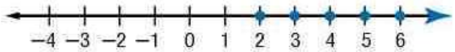
Extra Practice

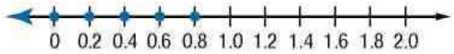
See page 823.

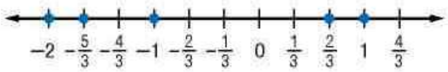
Name the coordinates of the points graphed on each number line.

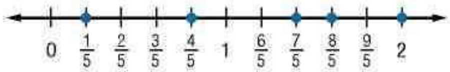
18. 

19. 

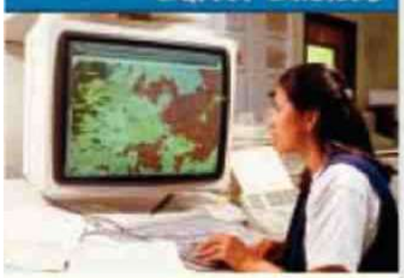
20. 

21. 

22. 

23. 

Career Choices



Demographer

A demographer analyses the size, nature, and movement of human populations. Many demographers specialize in one area such as health, housing, education, agriculture, or economics.

Online Research For information about a career as a demographer, visit www.algebra1.com/careers

Graph each set of numbers.

24. $\{-4, -2, -1, 1, 3\}$

25. $\{0, 2, 5, 6, 9\}$

26. $\{-5, -4, -3, -2, \ldots\}$

27. $\{\ldots, -2, 0, 2, 4, 6\}$

28. $\{-8.4, -7.2, -6.0, -4.8\}$

29. $\{-2.4, -1.6, -0.8, 3.2, \ldots\}$

30. $\left\{\ldots, -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots\right\}$

31. $\left\{-3\frac{2}{5}, -2\frac{1}{5}, -1\frac{4}{5}, -\frac{4}{5}, 1\right\}$

32. {integers less than -7 or greater than -1}

33. {integers greater than -5 and less than 9}

Find each absolute value.

34. $|-38|$

35. $|10|$

36. $|97|$

37. $|-61|$

38. $|3.9|$

39. $|-6.8|$

40. $\left|-\frac{23}{56}\right|$

41. $\left|\frac{35}{80}\right|$

POPULATION For Exercises 42–44, refer to the table below.

Population of Various Counties, 1990–1999

County	Percent Change	County	Percent Change
Kings, NY	−1.4	Wayne, MI	−0.2
Los Angeles, CA	5.3	Philadelphia, PA	−10.6
Cuyahoga, OH	−2.9	Suffolk, NY	4.7
Santa Clara, CA	10.0	Alameda, CA	8.5
Cook, IL	1.7	New York, NY	4.3

Source: *The World Almanac*

42. Use a number line to order the percent of change from least to greatest.

43. Which population had the greatest percent increase or decrease? Explain.

44. Which population had the least percent increase or decrease? Explain.

Evaluate each expression if $a = 6$, $b = \frac{2}{3}$, $c = \frac{5}{4}$, $x = 12$, $y = 3.2$, and $z = -5$.

45. $48 + |x - 5|$

46. $25 + |17 + x|$

47. $|17 - a| + 23$

48. $|43 - 4a| + 51$

49. $|z| + 13 - 4$

50. $28 - 13 + |z|$

51. $6.5 - |8.4 - y|$

52. $7.4 + |y - 2.6|$

53. $\frac{1}{6} + \left|b - \frac{7}{12}\right|$

54. $\left(b + \frac{1}{2}\right) - \left|-\frac{5}{6}\right|$

55. $|c - 1| + \frac{2}{5}$

56. $|-c| + \left(2 + \frac{1}{2}\right)$

57. CRITICAL THINKING Find all values for x if $|x| = -|x|$.

More About. . .

Weather

The lowest temperature ever recorded in the world was −129°F at the Soviet Antarctica station of Vostok.

Source: *The World Almanac*

WEATHER For Exercises 58 and 59, use the table at the right and the information at the left.

Same Day Low Temperatures for Certain U.S. Cities

City	Low Temperature (°F)
Bismarck, ND	−11
Caribou, ME	−5
Chicago, IL	−4
Fairbanks, AK	−9
International Falls, MN	−13
Kansas City, MO	7
Sacramento, CA	34
Shreveport, LA	33

Source: *The World Almanac*

58. Draw a number line and graph the set of numbers that represents the low temperatures for these cities.

59. Write the absolute value of the low temperature for each city.

60. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you use a number line to show data?

Include the following in your answer:

- an explanation of how to choose the range for a number line, and
- an explanation of how to tell which river had the greatest increase or decrease.

Standardized Test Practice

61. Which number is a natural number?

Ⓐ -2.5 Ⓑ $5 - |5|$ Ⓒ $-|3 + 5|$ Ⓓ $|-8| - 2$

62. Which sentence is *not* true?

Ⓐ All natural numbers are whole numbers.

Ⓑ Natural numbers are positive numbers.

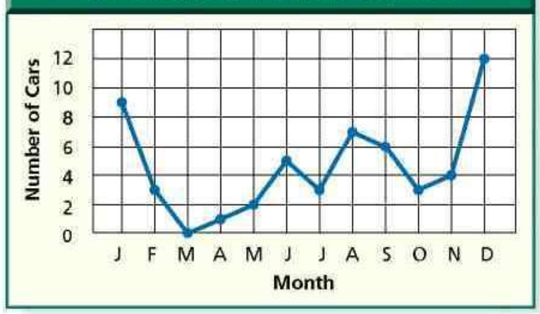
Ⓒ Every whole number is a natural number.

Ⓓ Zero is neither positive nor negative.

Maintain Your Skills

Mixed Review

SALES For Exercises 63–65, refer to the graph. *(Lesson 1-9)*



63. In which month did Mr. Michaels have the greatest sales?

64. Between which two consecutive months did the greatest change in sales occur?

65. In which months were sales equal?

66. **ENTERTAINMENT** Juanita has the volume on her stereo turned up. When her telephone rings, she turns the volume down. After she gets off the phone, she returns the volume to its previous level. Sketch a reasonable graph to show the volume of Juanita's stereo during this time. *(Lesson 1-8)*

Simplify each expression. *(Lesson 1-6)*

67. $8x + 2y + x$ **68.** $7(5a + 3b) - 4a$ **69.** $4[1 + 4(5x + 2y)]$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each sum or difference.

*(To review **addition and subtraction of fractions**, see pages 798 and 799.)*

70. $\frac{3}{8} + \frac{1}{8}$ **71.** $\frac{7}{12} - \frac{3}{12}$ **72.** $\frac{7}{10} + \frac{1}{5}$ **73.** $\frac{3}{8} + \frac{2}{3}$

74. $\frac{5}{6} + \frac{1}{2}$ **75.** $\frac{3}{4} - \frac{1}{3}$ **76.** $\frac{9}{15} - \frac{1}{2}$ **77.** $\frac{7}{9} - \frac{7}{18}$

2-2 Adding and Subtracting Rational Numbers

What You'll Learn

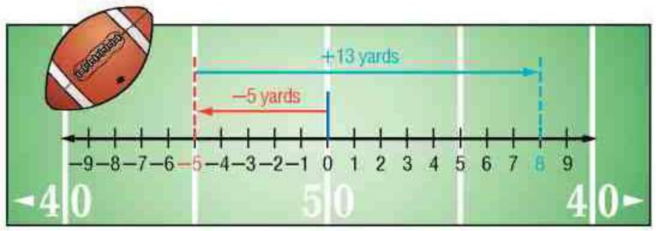
- Add integers and rational numbers.
- Subtract integers and rational numbers.

Vocabulary

- opposites
- additive inverses

How can a number line be used to show a football team's progress?

In one series of plays during Super Bowl XXXV, the New York Giants received a five-yard penalty before completing a 13-yard pass.



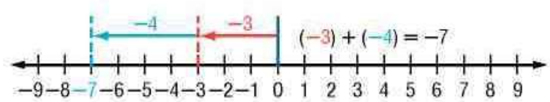
The number line shows the yards gained during this series of plays. The total yards gained was 8 yards.

ADD RATIONAL NUMBERS The number line above illustrates how to add integers on a number line. You can use a number line to add any rational numbers.

Example 1 Use a Number Line to Add Rational Numbers

Use a number line to find each sum.

a. $-3 + (-4)$

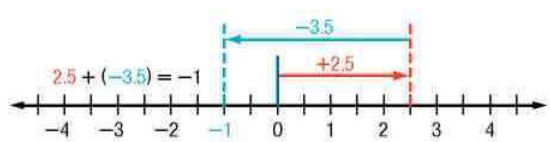


Step 1 Draw an arrow from 0 to -3.

Step 2 Then draw a second arrow 4 units to the left to represent adding -4.

Step 3 The second arrow ends at the sum -7. So, $-3 + (-4) = -7$.

b. $2.5 + (-3.5)$



Step 1 Draw an arrow from 0 to 2.5.

Step 2 Then draw a second arrow 3.5 units to the left.

Step 3 The second arrow ends at the sum -1. So, $2.5 + (-3.5) = -1$.

You can use absolute value to add rational numbers.

Same Signs		Different Signs	
$+\ +$	$-\ -$	$+\ -$	$-\ +$
$3 + 5 = 8$	$-3 + (-5) = -8$	$3 + (-5) = -2$	$-3 + 5 = 2$
3 and 5 are positive, so the sum is positive.	-3 and -5 are negative, so the sum is negative.	Since -5 has the greater absolute value, the sum is negative.	Since 5 has the greater absolute value, the sum is positive.

The examples above suggest the following rules for adding rational numbers.

Key Concept *Addition of Rational Numbers*

- To add rational numbers with the *same sign*, add their absolute values. The sum has the same sign as the addends.
- To add rational numbers with *different signs*, subtract the lesser absolute value from the greater absolute value. The sum has the same sign as the number with the greater absolute value.

Example 2 Add Rational Numbers

Find each sum.

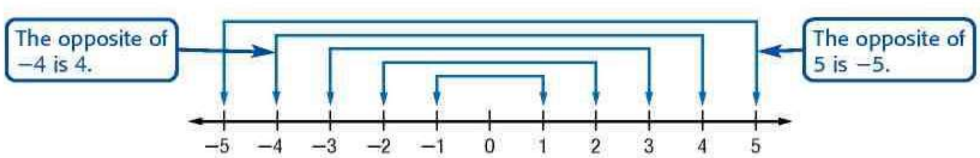
a. $-11 + (-7)$

$$\begin{aligned} -11 + (-7) &= -(|-11| + |-7|) && \text{Both numbers are negative, so the sum is negative.} \\ &= -(11 + 7) \\ &= -18 \end{aligned}$$

b. $\frac{7}{16} + \left(-\frac{3}{8}\right)$

$$\begin{aligned} \frac{7}{16} + \left(-\frac{3}{8}\right) &= \frac{7}{16} + \left(-\frac{6}{16}\right) && \text{The LCD is 16. Replace } -\tfrac{3}{8} \text{ with } -\tfrac{6}{16}. \\ &= +\left(\left|\frac{7}{16}\right| - \left|-\frac{6}{16}\right|\right) && \text{Subtract the absolute values.} \\ &= +\left(\frac{7}{16} - \frac{6}{16}\right) && \text{Since the number with the greater absolute value is } \tfrac{7}{16}, \text{ the sum is positive.} \\ &= \frac{1}{16} \end{aligned}$$

SUBTRACT RATIONAL NUMBERS Every positive rational number can be paired with a negative rational number. These pairs are called **opposites**.



Study Tip

Additive Inverse
Since $0 + 0 = 0$, zero is its own additive inverse.

A number and its opposite are **additive inverses** of each other. When you add two opposites, the sum is always 0.

Key Concept — Additive Inverse Property

- **Words** The sum of a number and its additive inverse is 0.
- **Symbols** For every number a, $a + (-a) = 0$.
- **Examples** $2 + (-2) = 0$ $\quad -4.25 + 4.25 = 0$ $\quad \frac{1}{3} + \left(-\frac{1}{3}\right) = 0$

Additive inverses can be used when you subtract rational numbers.

Subtraction	Addition
$9 - 4 = 5$	$9 + (-4) = 5$

additive inverses

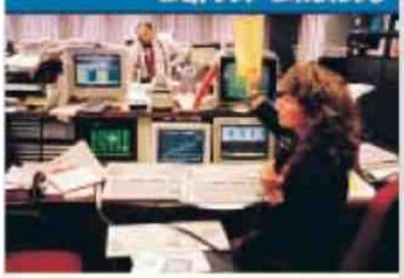
same result

This example suggests that subtracting a number is equivalent to adding its inverse.

Key Concept — Subtraction of Rational Numbers

- **Words** To subtract a rational number, add its additive inverse.
- **Symbols** For any numbers a and b, $a - b = a + (-b)$.
- **Examples** $8 - 15 = 8 + (-15)$ or -7
  $-7.6 - 12.3 = -7.6 + (-12.3)$ or -19.9

Career Choices



Stockbroker

Stockbrokers perform various duties, including buying or selling stocks, bonds, mutual funds, or other financial products for an investor.

Online Research
For information about a career as a stockbroker, visit: www.algebra1.com/careers

Example 3 Subtract Rational Numbers to Solve a Problem

STOCKS **During a five-day period, a telecommunications company's stock price went from \$17.82 to \$15.36 per share. Find the change in the price of the stock.**

Explore The stock price began at \$17.82 and ended at \$15.36. You need to determine the change in price for the week.

Plan Subtract to find the change in price.

$$\underbrace{15.36}_{\text{ending price}} \quad \underbrace{-}_{\text{minus}} \quad \underbrace{17.82}_{\text{beginning price}}$$

Solve

$15.36 - 17.82 = 15.36 + (-17.82)$ To subtract 17.82, add its inverse.

$= -(|-17.82| - |15.36|)$ Subtract the absolute values.

$= -(17.82 - 15.36)$ The absolute value of -17.82 is greater, so the result is negative.

$= -2.46$

The price of the stock changed by $-\$2.46$.

Examine The problem asks for the change in a stock's price from the beginning of a week to the end. Since the change was negative, the price dropped. This makes sense since the ending price is less than the beginning price.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a subtraction expression using rational numbers that has a difference of $-\frac{2}{5}$.
2. **Describe** how to subtract real numbers.
3. **FIND THE ERROR** Gabriella and Nick are subtracting fractions.

Gabriella

$$\left(-\frac{4}{9}\right) - \left(-\frac{2}{3}\right) = \left(-\frac{4}{9}\right) - \left(-\frac{6}{9}\right)$$
$$= \left(-\frac{4}{9}\right) + \left(\frac{6}{9}\right)$$
$$= \left(\frac{6}{9} - \frac{4}{9}\right)$$
$$= \frac{2}{9}$$

Nick

$$\left(-\frac{4}{9}\right) - \left(-\frac{2}{3}\right) = \left(-\frac{4}{9}\right) - \left(-\frac{6}{9}\right)$$
$$= \left(-\frac{4}{9}\right) + \left(-\frac{6}{9}\right)$$
$$= -\left(\frac{6}{9} + \frac{4}{9}\right)$$
$$= -\frac{10}{9}$$

Who is correct? Explain your reasoning.

Guided Practice

Find each sum.

4. $-15 + (-12)$
5. $-24 + (-45)$
6. $38.7 + (-52.6)$
7. $-4.62 + (-12.81)$
8. $\frac{4}{7} + \left(-\frac{1}{2}\right)$
9. $-\frac{5}{12} + \frac{8}{15}$

Find each difference.

10. $18 - 23$
11. $12.7 - (-18.4)$
12. $(-3.86) - 1.75$
13. $-32.25 - (-42.5)$
14. $-\frac{2}{9} - \frac{3}{10}$
15. $\left(-\frac{7}{10}\right) - \left(-\frac{11}{12}\right)$

Application

16. **WEATHER** The highest recorded temperature in the United States was in Death Valley, California, while the lowest temperature was recorded at Prospect Creek, Alaska. What is the difference between these two temperatures?

Practice and Apply

Homework Help

For Exercises	See Examples
17–38	1, 2
39–62	3

Extra Practice

See page 823.

Find each sum.

17. $-8 + 13$
18. $-11 + 19$
19. $41 + (-63)$
20. $80 + (-102)$
21. $-77 + (-46)$
22. $-92 + (-64)$
23. $-1.6 + (-3.8)$
24. $-32.4 + (-4.5)$
25. $-38.9 + 24.2$
26. $-7.007 + 4.8$
27. $43.2 + (-57.9)$
28. $38.7 + (-61.1)$
29. $\frac{6}{7} + \frac{2}{3}$
30. $\frac{3}{18} + \frac{6}{17}$
31. $-\frac{4}{11} + \frac{3}{5}$
32. $-\frac{2}{5} + \frac{17}{20}$
33. $-\frac{4}{15} + \left(-\frac{9}{16}\right)$
34. $-\frac{16}{40} + \left(-\frac{13}{20}\right)$
35. Find the sum of $4\frac{1}{8}$ and $-1\frac{1}{2}$.
36. Find the sum of $1\frac{17}{50}$ and $-3\frac{17}{25}$.

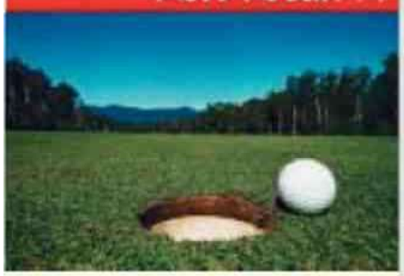
37. **GAMES** Sarah was playing a computer trivia game. Her scores for round one were $+100$, $+200$, $+500$, -300, $+400$, and -500. What was her total score at the end of round one?

38. **FOOTBALL** The Northland Vikings' offense began a drive from their 20-yard line. They gained 6 yards on the first down, lost 8 yards on the second down, then gained 3 yards on third down. What yard line were they on at fourth down?

Find each difference.

39. $-19 - 8$	40. $16 - (-23)$	41. $9 - (-24)$
42. $12 - 34$	43. $22 - 41$	44. $-9 - (-33)$
45. $-58 - (-42)$	46. $79.3 - (-14.1)$	47. $1.34 - (-0.458)$
48. $-9.16 - 10.17$	49. $67.1 - (-38.2)$	50. $72.5 - (-81.3)$
51. $-\frac{1}{6} - \frac{2}{3}$	52. $\frac{1}{2} - \frac{4}{5}$	53. $-\frac{7}{8} - \left(-\frac{3}{16}\right)$
54. $-\frac{1}{12} - \left(-\frac{3}{4}\right)$	55. $2\frac{1}{4} - 6\frac{1}{3}$	56. $5\frac{3}{10} - 1\frac{31}{50}$

More About...



Golf

In the United States, there are more than 16,000 golf courses played by 26 million people each year.

Source: Encarta Online

GOLF **For Exercises 57–59, use the following information.**
In golf, scores are based on *par*. Par 72 means that a golfer should hit the ball 72 times to complete 18 holes of golf. A score of 67, or 5 under par, is written as -5. A score of 3 over par is written as $+3$. At the Masters Tournament (par 72) in April, 2001, Tiger Woods shot 70, 66, 68, and 68 during four rounds of golf.

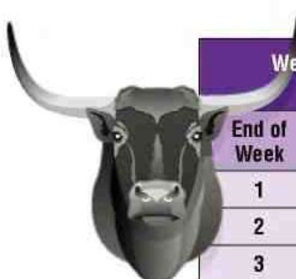
57. Use integers to write his score for each round as over or under par.

58. Add the integers to find his overall score.

59. Was his score under or over par? Would you want to have his score? Explain.

Online Research **Data Update** Find the most recent winner of the Masters Tournament. What integer represents the winner's score for each round as over or under par? What integer represents the winner's overall score? Visit www.algebra1.com/data_update to learn more.

STOCKS **For Exercises 60–62, refer to the table that shows the weekly closing values of the stock market for an eight-week period.**



Weekly Dow Jones Industrial Average (April – May 2000)

End of Week	Closing Value	End of Week	Closing Value
1	9791.09	5	10,951.24
2	10,126.94	6	10,821.31
3	10,579.85	7	11,301.74
4	10,810.05	8	11,257.24

Source: *The Wall Street Journal*

60. Find the change in value from week 1 to week 8.

61. Which week had the greatest change from the previous week?

62. Which week had the least change from the previous week?

63. **CRITICAL THINKING** Tell whether the equation $x + |x| = 0$ is *always*, *sometimes*, or *never* true. Explain.

64. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a number line be used to show a football team's progress?

Include the following in your answer:

- an explanation of how you could use a number line to determine the yards gained or lost by the Giants on their next three plays, and
- a description of how to determine the total yards gained or lost without using a number line.

Standardized Test Practice

65. What is the value of n in $-57 - n = -144$?

Ⓐ -201 Ⓑ 201 Ⓒ -87 Ⓓ 87

66. Which expression is equivalent to $5 - (-8)$?

Ⓐ $(-5) + 8$ Ⓑ $8 + 5$ Ⓒ $8 - 5$ Ⓓ $5 - 8$

Maintain Your Skills

Mixed Review

Evaluate each expression if $x = 4.8$, $y = -7.4$, and $z = 10$. *(Lesson 2-1)*

67. $12.2 + |8 - x|$

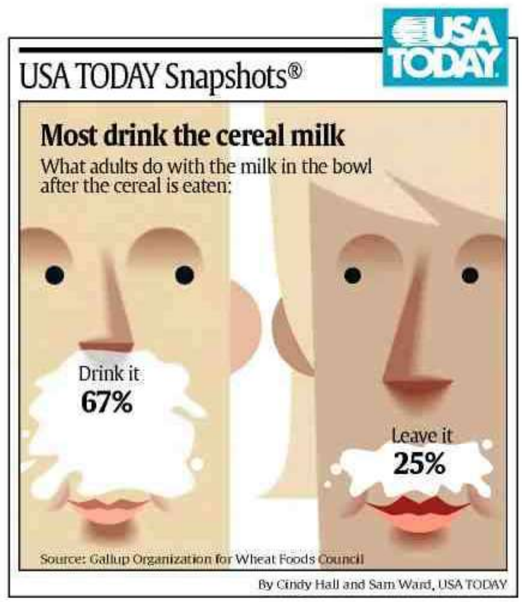
68. $|y| + 9.4 - 3$

69. $24.2 - |18.3 - z|$

For Exercises 70 and 71, refer to the graph. *(Lesson 1-9)*

70. If you wanted to make a circle graph of the data, what additional category would you have to include so that the circle graph would not be misleading?

71. Construct a circle graph that displays the data accurately.



Find the solution sets for each inequality if the replacement sets are $A = \{2, 3, 4, 5, 6\}$, $B = \{0.3, 0.4, 0.5, 0.6, 0.7\}$, and $C = \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}\right\}$. *(Lesson 1-3)*

72. $b + 1.3 \geq 1.8$

73. $3a - 5 > 7$

74. $c + \frac{1}{2} < 2\frac{1}{4}$

Write an algebraic expression for each verbal phrase. *(Lesson 1-1)*

75. eight less than the square of q

76. 37 less than 2 times a number k

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**

*(To review **multiplication of fractions**, see pages 800 and 801.)*

77. $\frac{1}{2} \cdot \frac{2}{3}$

78. $\frac{1}{4} \cdot \frac{2}{5}$

79. $\frac{3}{4} \cdot \frac{5}{6}$

80. $4 \cdot \frac{3}{5}$

81. $8 \cdot \frac{5}{8}$

82. $\frac{7}{9} \cdot 12$

2-3 Multiplying Rational Numbers

What You'll Learn

- Multiply integers.
- Multiply rational numbers.

How do consumers use multiplication of rational numbers?

Stores often offer coupons to encourage people to shop in their stores. The receipt shows a purchase of four CDs along with four coupons for \$1.00 off each CD. How could you determine the amount saved by using the coupons?

MULTIPLY INTEGERS One way to find the savings from the coupons is to use repeated addition.

$$-\$1.00 + (-\$1.00) + (-\$1.00) + (-\$1.00) = -\$4.00$$

An easier way to find the savings would be to multiply $-\$1.00$ by 4.

$$4(-\$1.00) = -\$4.00$$

Suppose the coupons were expired and had to be removed from the total. You can represent this by multiplying $-\$1.00$ by -4.

$$(-4)(-\$1.00) = \$4.00$$

In other words, \$4.00 would be added back to the total.

These examples suggest the following rules for multiplying integers.

Key Concept — Multiplication of Integers

- **Words** The product of two numbers having the same sign is positive. The product of two numbers having *different signs* is negative.
- **Examples** $(-12)(-7) = 84$ same signs → positive product
  $15(-8) = -120$ different signs → negative product

Study Tip

Multiplying Integers
When multiplying integers, if there are an even number of negative integers, the product is positive. If there are an odd number of negative integers, the product is negative.

Example 1 Multiply Integers

Find each product.

a. $4(-5)$

$4(-5) = -20$ different signs → negative product

b. $(-12)(-14)$

$(-12)(-14) = 168$ same signs → positive product

You can simplify expressions by applying the rules of multiplication.

Example 2 Simplify Expressions

Simplify the expression $4(-3y) - 15y$.

$$\begin{aligned} 4(-3y) - 15y &= 4(-3)y - 15y && \text{Associative Property } (\times) \\ &= -12y - 15y && \text{Substitution} \\ &= (-12 - 15)y && \text{Distributive Property} \\ &= -27y && \text{Simplify.} \end{aligned}$$

MULTIPLY RATIONAL NUMBERS Multiplying rational numbers is similar to multiplying integers.

Example 3 Multiply Rational Numbers

Find $\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right)$.

$\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right) = -\frac{9}{32}$ different signs → negative product

Example 4 Multiply Rational Numbers to Solve a Problem

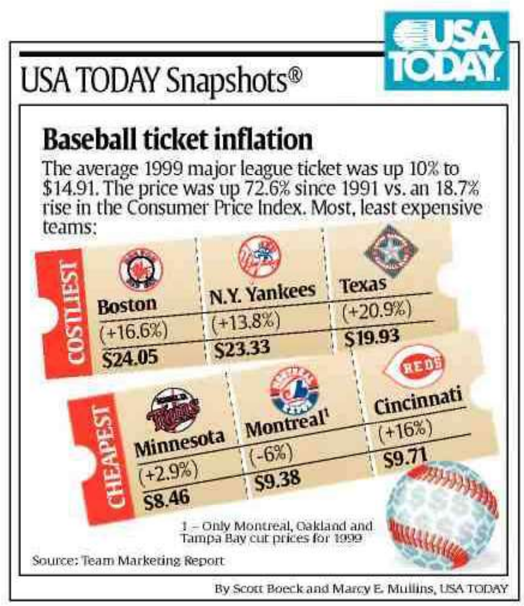
BASEBALL **Fenway Park, home of the Boston Red Sox, is the oldest ball park in professional baseball. It has a seating capacity of about 34,000. Determine the approximate total ticket sales for a sold-out game.**

To find the approximate total ticket sales, multiply the number of tickets sold by the average price.

$34{,}000 \cdot 24.05 = 817{,}770$

same signs → positive product

The total ticket sales for a sold-out game are about $817,770.



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www.algebra1.com/usa_today

You can evaluate expressions that contain rational numbers.

Example 5 Evaluate Expressions

Evaluate $n^2\left(-\frac{5}{8}\right)$ if $n = -\frac{2}{5}$.

$$\begin{aligned} n^2\left(-\frac{5}{8}\right) &= \left(-\frac{2}{5}\right)^2\left(-\frac{5}{8}\right) && \text{Substitution} \\ &= \left(\frac{4}{25}\right)\left(-\frac{5}{8}\right) && \left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) \text{ or } \frac{4}{25} \\ &= -\frac{20}{200} \text{ or } -\frac{1}{10} && \text{different signs} \rightarrow \text{negative product} \end{aligned}$$

In Lesson 1-4, you learned about the Multiplicative Identity Property, which states that any number multiplied by 1 is equal to the number. Another important property is the Multiplicative Property of -1.

Key Concept — *Multiplicative Property of -1*

- **Words** The product of any number and -1 is its additive inverse.
- **Symbols** For any number a, $-1(a) = a(-1) = -a$.
- **Examples** $(-1)(4) = (4)(-1) = -4$ $\quad$ $(-1)(-2.3) = (-2.3)(-1) = 2.3$

Check for Understanding

Concept Check

1. **List** the conditions under which the product ab is negative. Give examples to support your answer.
2. **OPEN ENDED** Describe a real-life situation in which you would multiply a positive rational number by a negative rational number. Write a corresponding multiplication expression.
3. **Explain** why the product of two negative numbers is positive.

Guided Practice

Find each product.

4. $(-6)(3)$
5. $(5)(-8)$
6. $(4.5)(2.3)$
7. $(-8.7)(-10.4)$
8. $\left(\frac{5}{3}\right)\left(-\frac{2}{7}\right)$
9. $\left(-\frac{4}{9}\right)\left(\frac{7}{15}\right)$

Simplify each expression.

10. $5s(-6t)$
11. $6x(-7y) + (-15xy)$

Evaluate each expression if $m = -\frac{2}{3}$, $n = \frac{1}{2}$, and $p = -3\frac{3}{4}$.

12. $6m$
13. np
14. $n^2(m + 2)$

Application

15. **NATURE** The average worker honeybee makes about $\frac{1}{12}$ teaspoon of honey in its lifetime. How much honey do 675 honeybees make?

Practice and Apply

Homework Help

For Exercises	See Examples
16–33	1, 3
34–39	2
40, 41	4
42–49	5
50–54	4

Extra Practice

See page 823.

Find each product.

16. $5(18)$
17. $8(22)$
18. $-12(15)$
19. $-24(8)$
20. $-47(-29)$
21. $-81(-48)$
22. $\left(\frac{4}{5}\right)\left(\frac{3}{8}\right)$
23. $\left(\frac{5}{12}\right)\left(\frac{4}{9}\right)$
24. $\left(-\frac{3}{5}\right)\left(\frac{5}{6}\right)$
25. $\left(-\frac{2}{5}\right)\left(\frac{6}{7}\right)$
26. $\left(-3\frac{1}{5}\right)\left(-7\frac{1}{2}\right)$
27. $\left(-1\frac{4}{5}\right)\left(-2\frac{1}{2}\right)$
28. $7.2(0.2)$
29. $6.5(0.13)$
30. $(-5.8)(2.3)$
31. $(-0.075)(6.4)$
32. $\frac{3}{5}(-5)(-2)$
33. $\frac{2}{11}(-11)(-4)$

Simplify each expression.

34. $6(-2x) - 14x$
35. $5(-4n) - 25n$
36. $5(2x - x)$
37. $-7(3d + d)$
38. $-2a(-3c) + (-6y)(6r)$
39. $7m(-3n) + 3s(-4t)$

STOCK PRICES **For Exercises 40 and 41, use the table that lists the closing prices of a company's stock over a one-week period.**

Closing Stock Price ($)	
Day	Price
1	64.38
2	63.66
3	61.66
4	61.69
5	62.34

40. What was the change in price of 35 shares of this stock from day 2 to day 3?

41. If you bought 100 shares of this stock on day 1 and sold half of them on day 4, how much money did you gain or lose on those shares?

Evaluate each expression if $a = -2.7$, $b = 3.9$, $c = 4.5$, and $d = -0.2$.

42. $-5c^2$

43. $-2b^2$

44. $-4ab$

45. $-5cd$

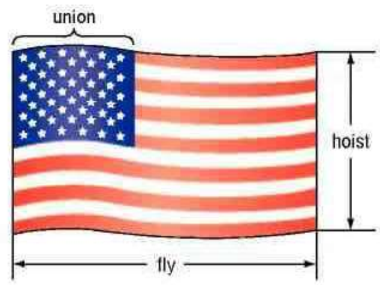
46. $ad - 8$

47. $ab - 3$

48. $d^2(b - 2a)$

49. $b^2(d - 3c)$

50. **CIVICS** In a United States flag, the length of the union is $\frac{2}{5}$ of the fly, and the width is $\frac{7}{13}$ of the hoist. If the fly is 6 feet, how long is the union?



51. **COMPUTERS** The price of a computer dropped \$34.95 each month for 7 months. If the starting price was \$1450, what was the price after 7 months?

More About. . .

Civics

The Marine Corps War Memorial in Washington, D.C., is dedicated to all Marines who have defended the United States since 1775. It is the most famous memorial that is centered around the flag.

Source: The United States National Park Service

52. **BALLOONING** The temperature drops about 2°F for every rise of 530 feet in altitude. Per Lindstrand achieved the altitude record of 64,997 feet in a hot-air balloon over Laredo, Texas, on June 6, 1988. About how many degrees difference was there between the ground temperature and the air temperature at that altitude? **Source:** *The Guinness Book of Records*

ECOLOGY **For Exercises 53 and 54, use the following information.**
Americans use about 2.5 million plastic bottles every hour.
Source: www.savethewater.com

53. About how many plastic bottles are used in one day?

54. About how many bottles are used in one week?

55. **CRITICAL THINKING** An even number of negative numbers is multiplied. What is the sign of the product? Explain your reasoning.

56. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How do consumers use multiplication of rational numbers?

Include the following in your answer:

- an explanation of why the amount of a coupon is expressed as a negative value, and
- an explanation of how you could use multiplication to find your total discount if you bought 3 CDs for \$13.99 each and there was a discount of \$1.50 on each CD.

Standardized Test Practice

57. Which expression can be simplified as $-8xy$?

(A) $2y - 4x$ (B) $-2x(4y)$ (C) $(-4)^2xy$ (D) $-4x(-2y)$

58. Find the value of m if $m = -2ab$, $a = -4$, and $b = 6$.

(A) 8 (B) 48 (C) 12 (D) -48

Maintain Your Skills

Mixed Review

Find each sum or difference. *(Lesson 2-2)*

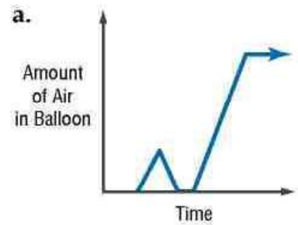
59. $-6.5 + (-5.6)$ **60.** $\frac{4}{5} + \left(-\frac{3}{4}\right)$ **61.** $42 - (-14)$ **62.** $-14.2 - 6.7$

Graph each set of numbers on a number line. *(Lesson 2-1)*

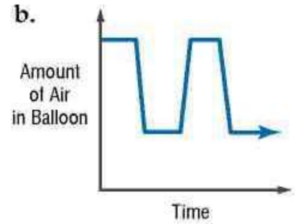
63. $\{\ldots, -3, -1, 1, 3, 5\}$ **64.** $\{-2.5, -1.5, 0.5, 4.5\}$ **65.** $\{-1, -\frac{1}{3}, \frac{2}{3}, 2\}$

66. Identify the graph below that best represents the following situation. Brandon has a deflated balloon. He slowly fills the balloon up with air. Without tying the balloon, he lets it go. *(Lesson 1-8)*

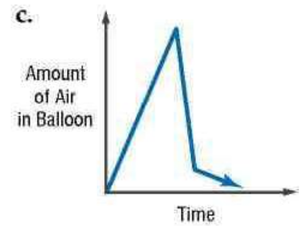
a.



b.



c.



Write a counterexample for each statement. *(Lesson 1-7)*

67. If $2x - 4 \geq 6$, then $x > 5$. **68.** If $|a| > 3$, then $a > 3$.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each quotient.**

*(To review **division of fractions**, see pages 800 and 801.)*

69. $\frac{5}{8} \div 2$ **70.** $\frac{2}{3} \div 4$ **71.** $5 \div \frac{3}{4}$ **72.** $1 \div \frac{2}{5}$

73. $\frac{1}{2} \div \frac{3}{8}$ **74.** $\frac{7}{9} \div \frac{5}{6}$ **75.** $\frac{4}{5} \div \frac{6}{5}$ **76.** $\frac{7}{8} \div \frac{2}{3}$

Practice Quiz 1

Lessons 2-1 through 2-3

1. Name the set of points graphed on the number line. *(Lesson 2-1)*

2. Evaluate $32 - |x + 8|$ if $x = 15$. *(Lesson 2-1)*

Find each sum or difference. *(Lesson 2-2)*

3. $-15 + 7$ **4.** $27 - (-12)$ **5.** $-6.05 + (-2.1)$ **6.** $-\frac{3}{4} - \left(-\frac{2}{5}\right)$

Find each product. *(Lesson 2-3)*

7. $-9(-12)$ **8.** $(3.8)(-4.1)$

9. Simplify $(-8x)(-2y) + (-3y)(z)$. *(Lesson 2-3)*

10. Evaluate $mn + 5$ if $m = 2.5$ and $n = -3.2$. *(Lesson 2-3)*

2-4 Dividing Rational Numbers

What You'll Learn

- Divide integers.
- Divide rational numbers.

How can you use division of rational numbers to describe data?

Each year, many sea turtles are stranded on the Texas Gulf Coast. The number of sea turtles stranded from 1997 to 2000 and the changes in number from the previous years are shown in the table. The following expression can be used to find the *mean* change per year of the number of stranded turtles.

$$\text{mean} = \frac{(-127) + 54 + (-65)}{3}$$

Stranded Sea Turtles Texas Gulf Coast

Year	Number of Turtles	Change
1997	523	——
1998	396	−127
1999	450	+54
2000	385	−65

Source: www.ndleyturtles.org

DIVIDE INTEGERS Since multiplication and division are inverse operations, the rule for finding the sign of the quotient of two numbers is similar to the rule for finding the sign of a product of two numbers.

Key Concept — *Division of Integers*

- **Words** The quotient of two numbers having the *same sign* is positive. The quotient of two numbers having *different signs* is negative.
- **Examples** $(-60) \div (-5) = 12$ same signs → positive quotient
  $32 \div (-8) = -4$ different signs → negative quotient

Example 1 *Divide Integers*

Find each quotient.

a. $-77 \div 11$

$-77 \div 11 = -7$ negative quotient

b. $\frac{-51}{-3}$

$\frac{-51}{-3} = -51 \div (-3)$ Divide.

$= 17$ positive quotient

When simplifying fractions, recall that the fraction bar is a grouping symbol.

Example 2 *Simplify Before Dividing*

Simplify $\frac{-3(-12 + 8)}{7 + (-5)}$.

$\frac{-3(-12 + 8)}{7 + (-5)} = \frac{-3(-4)}{7 + (-5)}$ Simplify the numerator first.

$= \frac{12}{7 + (-5)}$ Multiply.

$= \frac{12}{2}$ or 6 same signs → positive quotient

DIVIDE RATIONAL NUMBERS The rules for dividing positive and negative integers also apply to division with rational numbers. Remember that to divide by any nonzero number, multiply by the reciprocal of that number.

Example 3 Divide Rational Numbers

Find each quotient.

a. $245.66 \div (-14.2)$

$245.66 \div (-14.2) = -17.3$ Use a calculator. different signs → negative quotient

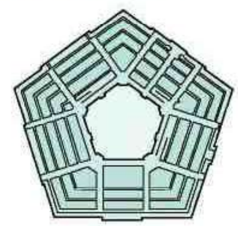
b. $-\frac{2}{5} \div \frac{1}{4}$

$-\frac{2}{5} \div \frac{1}{4} = -\frac{2}{5} \cdot \frac{4}{1}$ Multiply by $\frac{4}{1}$, the reciprocal of $\frac{1}{4}$.

$= -\frac{8}{5}$ or $-1\frac{3}{5}$ different signs → negative quotient

Example 4 Divide Rational Numbers to Solve a Problem

ARCHITECTURE The Pentagon in Washington, D.C., has an outside perimeter of 4608 feet. Find the length of each outside wall.



The Pentagon

To find the length of each wall, divide the perimeter by the number of sides.

$4608 \div 5 = 921.6$ same signs → positive quotient

The length of each outside wall is 921.6 feet.

More About...

Architecture

The Pentagon is one of the world's largest office buildings. It contains 131 stairways, 19 escalators, 13 elevators, 284 restrooms, and 691 drinking fountains.

Source: www.infoplease.com

You can use the Distributive Property to simplify fractional expressions.

Example 5 Simplify Algebraic Expressions

Simplify $\frac{24 - 6a}{3}$.

$\frac{24 - 6a}{3} = (24 - 6a) \div 3$ The fraction bar indicates division.

$= (24 - 6a)\left(\frac{1}{3}\right)$ Multiply by $\frac{1}{3}$, the reciprocal of 3.

$= 24\left(\frac{1}{3}\right) - 6a\left(\frac{1}{3}\right)$ Distributive Property

$= 8 - 2a$ Simplify.

Example 6 Evaluate Algebraic Expressions

Evaluate $\frac{ab}{c^2}$ if $a = -7.8$, $b = 5.2$, and $c = -3$. Round to the nearest hundredth.

$\frac{ab}{c^2} = \frac{(-7.8)(5.2)}{(-3)^2}$ Replace a with -7.8, b with 5.2, and c with -3.

$= \frac{-40.56}{9}$ Find the numerator and denominator separately.

≈ 4.51 Use a calculator. same signs → positive quotient

Check for Understanding

Concept Check

1. **Compare and contrast** multiplying and dividing rational numbers.
2. **OPEN ENDED** Find a value for x if $\frac{1}{x} > x$.
3. **Explain** how to divide any rational number by another rational number.

Guided Practice

Find each quotient.

4. $96 \div (-6)$
5. $-36 \div 4$
6. $-64 \div 5$
7. $64.4 \div 2.5$
8. $-\frac{2}{3} \div 12$
9. $-\frac{2}{3} \div \frac{4}{5}$

Simplify each expression.

10. $\frac{25 + 3}{-4}$
11. $\frac{-650a}{10}$
12. $\frac{6b + 18}{-2}$

Evaluate each expression if $a = 3$, $b = -4.5$, and $c = 7.5$. Round to the nearest hundredth.

13. $\frac{2ab}{-ac}$
14. $\frac{cb}{4a}$
15. $-\frac{a}{b} \div \frac{a}{c}$

Application

16. **ONLINE SHOPPING** During the 2000 holiday season, the sixth most visited online shopping site recorded 419,000 visitors. This is eight times as many visitors as in 1999. About how many visitors did the site have in 1999?

Practice and Apply

Homework Help

For Exercises	See Examples
17–36	1, 3
37–44	5
45, 46, 55–57	4
47–54	6

Extra Practice

See page 824.

Find each quotient.

17. $-64 \div (-8)$
18. $-78 \div (-4)$
19. $-78 \div (-1.3)$
20. $108 \div (-0.9)$
21. $42.3 \div (-6)$
22. $68.4 \div (-12)$
23. $-23.94 \div 10.5$
24. $-60.97 \div 13.4$
25. $-32.25 \div (-2.5)$
26. $-98.44 \div (-4.6)$
27. $-\frac{1}{3} \div 4$
28. $-\frac{3}{4} \div 12$
29. $-7 \div \frac{3}{5}$
30. $-5 \div \frac{2}{7}$
31. $\frac{16}{36} \div \frac{24}{60}$
32. $-\frac{24}{56} \div \frac{31}{63}$
33. $\frac{14}{32} \div \left(-\frac{12}{25}\right)$
34. $\frac{80}{25} \div \left(-\frac{2}{3}\right)$
35. Find the quotient of -74 and $-\frac{5}{3}$.
36. Find the quotient of -156 and $-\frac{3}{8}$.

Simplify each expression.

37. $\frac{81c}{9}$
38. $\frac{105g}{5}$
39. $\frac{8r + 24}{-8}$
40. $\frac{7h + 35}{-7}$
41. $\frac{40a - 50b}{2}$
42. $\frac{42c - 18d}{3}$
43. $\frac{-8f + (-16g)}{8}$
44. $\frac{-5x + (-10y)}{5}$
45. **CRAFTS** Hannah is making pillows. The pattern states that she needs $1\frac{3}{4}$ yards of fabric for each pillow. If she has $4\frac{1}{2}$ yards of fabric, how many pillows can she make?
46. **BOWLING** Bowling centers in the United States made \$2,800,000,000 in 1990. Their receipts in 1998 were \$2,764,000,000. What was the average change in revenue for each of these 8 years? **Source:** U.S. Census Bureau

Evaluate each expression if $m = -8$, $n = 6.5$, $p = 3.2$, and $q = -5.4$. Round to the nearest hundredth.

47. $\frac{mn}{p}$ **48.** $\frac{np}{m}$ **49.** $mq \div np$ **50.** $pq \div mn$

51. $\frac{n+p}{m}$ **52.** $\frac{m+p}{q}$ **53.** $\frac{m-2n}{-n+q}$ **54.** $\frac{p-3q}{-q-m}$

55. BUSINESS The president of a small business is looking at her profit/loss statement for the past year. The loss in income for the last year was $23,985. On average, what was the loss per month last year?

More About. . .

Jewelry

The discovery of gold at Sutter's Mill early in 1848 brought more than 40,000 prospectors to California within two years.

Source: www.infoplease.com

JEWELRY **For Exercises 56 and 57, use the following information.**
The gold content of jewelry is given in karats. For example, 24-karat gold is pure gold, and 18-karat gold is $\frac{18}{24}$ or 0.75 gold.

56. What fraction of 10-karat gold is pure gold? What fraction is not gold?

57. If a piece of jewelry is $\frac{2}{3}$ gold, how would you describe it using karats?

58. CRITICAL THINKING What is the least positive integer that is divisible by all whole numbers from 1 to 9?

59. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

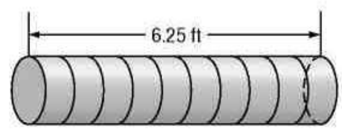
How can you use division of rational numbers to describe data?

Include the following in your answer:

- an explanation of how you could use the mean of a set of data to describe changes in the data over time, and
- reasons why you think the change from year to year is not consistent.

Standardized Test Practice

60. If the rod is cut as shown, how many inches long will each piece be?



Ⓐ 0.625 in. Ⓑ 1.875 in.
Ⓒ 5.2 in. Ⓓ 7.5 in.

61. If $\frac{17}{3} = x$, then what is the value of $6x + 1$?

Ⓐ 32 Ⓑ 33 Ⓒ 44 Ⓓ 35

Maintain Your Skills

Mixed Review

Find each product. *(Lesson 2-3)*

62. $-4(11)$ **63.** $-2.5(-1.2)$ **64.** $\frac{1}{4}(-5)$ **65.** $1.6(0.3)$

Find each difference. *(Lesson 2-2)*

66. $8 - (-6)$ **67.** $15 - 21$ **68.** $-7.5 - 4.8$ **69.** $-\frac{5}{8} - \left(-\frac{1}{6}\right)$

70. Name the property illustrated by $2(1.2 + 3.8) = 2 \cdot 5$.

Simplify each expression. If not possible, write *simplified*. *(Lesson 1-5)*

71. $8b + 12(b + 2)$ **72.** $6(5a + 3b - 2b)$ **73.** $3(x + 2y) - 2y$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the mean, median, and mode for each set of data.**
*(To review **mean**, **median**, and **mode**, see pages 818 and 819.)*

74. 40, 34, 40, 28, 38 **75.** 3, 9, 0, 2, 11, 8, 14, 3

76. 1.2, 1.7, 1.9, 1.8, 1.2, 1.0, 1.5 **77.** 79, 84, 81, 84, 75, 73, 80, 78

2-5 Statistics: Displaying and Analyzing Data

What You'll Learn

- Interpret and create line plots and stem-and-leaf plots.
- Analyze data using mean, median, and mode.

Vocabulary

- line plot
- frequency
- stem-and-leaf plot
- back-to-back stem-and-leaf plot
- measures of central tendency

How are line plots and averages used to make decisions?

How many people do you know with the same first name? Some names are more popular than others. The table below lists the top five most popular names for boys and girls born in each decade from 1950 to 1999.

Top Five First Names of America

1950-59	Boys	Michael	James	Robert	John	David
	Girls	Deborah	Mary	Linda	Patricia	Susan
1960-69	Boys	Michael	John	David	James	Robert
	Girls	Lisa	Deborah	Mary	Karen	Michelle
1970-79	Boys	Michael	Christopher	Jason	David	James
	Girls	Jennifer	Michelle	Amy	Melissa	Kimberly
1980-89	Boys	Michael	Christopher	Matthew	Joshua	David
	Girls	Jessica	Jennifer	Ashley	Sarah	Amanda
1990-99	Boys	Michael	Christopher	Matthew	Joshua	Nicholas
	Girls	Ashley	Jessica	Sarah	Brittany	Emily

Source: *The World Almanac*

To help determine which names appear most frequently, these data could be displayed graphically.

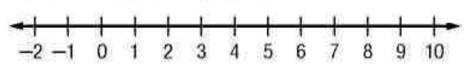
CREATE LINE PLOTS AND STEM-AND-LEAF PLOTS In some cases, data can be presented using a **line plot**. Most line plots have a number line labeled with a scale to include all the data. Then an × is placed above a data point each time it occurs to represent the **frequency** of the data.

Example 1 Create a Line Plot

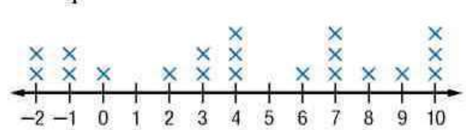
Draw a line plot for the data.

−2 4 3 2 6 10 7 4 −2 0 10 8 7 10 7 4 −1 9 −1 3

Step 1 The value of the data ranges from −2 to 10, so construct a number line containing those points.



Step 2 Then place an × above a number each time it occurs.



CONTENTS

Line plots are a convenient way to organize data for comparison.

Example 2 Use a Line Plot to Solve a Problem

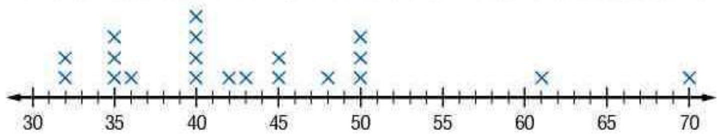
ANIMALS **The speeds (mph) of 20 of the fastest land animals are listed below.**

45	70	43	45	32	42	40	40	35	50
40	35	61	48	35	32	50	36	50	40

Source: *The World Almanac*

a. Make a line plot of the data.

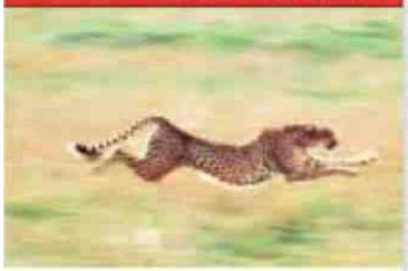
The lowest value is 30, and the highest value is 70, so use a scale that includes those values. Place an × above each value for each occurrence.



b. Which speed occurs most frequently?

Looking at the line plot, we can easily see that 40 miles per hour occurs most frequently.

More About...



Animals

Whereas the cheetah is the fastest land animal, the fastest marine animal is the sailfish. It is capable of swimming 68 miles per hour.

Source: *The Top 10 of Everything*

Another way to organize and display data is by using a **stem-and-leaf plot**. In a stem-and-leaf plot, the greatest common place value is used for the *stems*. The numbers in the next greatest place value are used to form the *leaves*. In Example 2, the greatest place value is tens. Thus, 32 miles per hour would have a stem of 3 and a leaf of 2. A complete stem-and-leaf plot for the data in Example 2 is shown below.

Stem	Leaf
3	2 2 5 5 5 6
4	0 0 0 0 2 3 5 5 8
5	0 0 0
6	1
7	0 *3 | 2 = 32*

key

Study Tip

Stem-and-Leaf Plots

A key is included on stem-and-leaf plots to indicate what the stems and leaves represent when read.

Example 3 Create a Stem-and-Leaf Plot

Use the data below to make a stem-and-leaf plot.

108	104	86	82	80	72	70	62	64	68	84	64	98	96	98
103	87	65	83	79	97	96	112	62	80	62	83	76	66	97

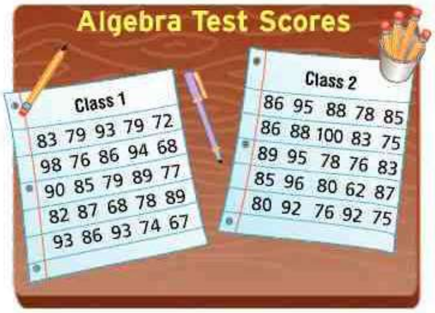
The greatest common place value is tens, so the digits in the tens place are the stems.

Stem	Leaf
6	2 2 2 4 4 5 6 8
7	0 2 6 9
8	0 0 2 3 3 4 6 7
9	6 6 7 7 8 8
10	3 4 8
11	2 *10 | 3 = 103*

A **back-to-back stem-and-leaf plot** can be used to compare two related sets of data.

Example 4 Back-to-Back Stem-and-Leaf Plot

Mrs. Evans wants to compare recent test scores from her two algebra classes. The table shows the scores for both classes.



a. Make a stem-and-leaf plot to compare the data.
To compare the data, we can use a back-to-back stem-and-leaf plot. Since the data represent similar measurements, the plot will share a common stem.

Class 1	Stem	Class 2
8 8 7	6	2
9 9 9 8 7 6 4 2	7	5 5 6 6 8 8
9 9 7 6 6 5 3 2	8	0 0 3 3 5 5 6 6 7 8 8 9
8 4 3 3 3 0	9	2 2 5 5 6
7|6 = 67	10	0 6|2 = 62

b. What is the difference between the highest score in each class?
100 − 98 or 2 points

c. Which class scored higher overall on the test?
Looking at the scores of 80 and above, we see that class 2 has a greater number of scores at or above 80 than class 1.

ANALYZE DATA When analyzing data, it is helpful to have one number that describes the set of data. Numbers known as **measures of central tendency** are often used to describe sets of data because they represent a centralized, or middle, value. Three of the most commonly used measures of central tendency are the mean, median, and mode.

When you use a measure of central tendency to describe a set of data, it is important that the measure you use best represents all of the data.

- Extremely high or low values can affect the mean, while not affecting the median or mode.
- A value with a high frequency can cause the mode to be misleading.
- Data that is clustered with a few values separate from the cluster can cause the median to be too low or too high.

Study Tip

Look Back
To review **finding mean**, **median**, and **mode**, see pages 818 and 819.

Example 5 Analyze Data

Which measure of central tendency best represents the data?

Stem	Leaf
7	7 8 9
8	2 2 2 2 2 3 4 4 6
9	
10	8
11	6 8 7|9 = 0.79

Determine the mean, median, and mode.

The mean is about 0.88. Add the data and divide by 15.
The median is 0.82. The middle value is 0.82.
The mode is 0.82. The most frequent value is 0.82.

Either the median or the mode best represent the set of data since both measures are located in the center of the majority of the data. In this instance, the mean is too high.

Example 6 Determine the Best Measure of Central Tendency

PRESIDENTS The numbers below show the ages of the U.S. Presidents since 1900 at the time they were inaugurated. Which measure of central tendency best represents the data?

42	51	56	55	51	54	51	60	62
43	55	56	61	52	69	64	46	54

The mean is about 54.6. Add the data and divide by 18.

The median is 54.5. The middle value is 54.5.

The mode is 51. The most frequent value is 51.

The mean or the median can be used to best represent the data. The mode for the data is too low.

Check for Understanding

Concept Check

1. **Explain** why it is useful to find the mean, median, and mode of a set of data.
2. Mitchell says that a line plot and a line graph are the same thing. **Find a counterexample** to show that he is incorrect.
3. **OPEN ENDED** Write a set of data for which the median is a better representation than the mean.

Guided Practice

4. Use the data to make a line plot.

22 19 14 15 14 21 19 16 22 19 10 15 19 14 19

For Exercises 5–7, use the list that shows the number of hours students in Mr. Ricardo's class spent online last week.

7	4	7	11	3	1	5	10	10	0	9	4	0	14	13	4
11	3	1	12	0	9	13	14	7	6	10	5	12	0	6	5

5. Make a line plot of the data.
6. Which value occurs most frequently?
7. Does the mean, median, or mode best represent the data? Explain.
8. Use the data to make a stem-and-leaf plot.

68 66 68 88 76 71 88 93 86 64 73 80 81 72 68

For Exercises 9 and 10, use the data in the stem-and-leaf plot.

Stem	Leaf
9	3 5 5
10	2 2 5 8
11	5 8 8 9 9 9
12	0 1 7 8 9 $9\|3 = 9.3$

9. What is the difference between the least and greatest values?
10. Which measure of central tendency best describes the data? Explain.

Application

BUILDINGS For Exercises 11–13, use the data below that represents the number of stories in the 25 tallest buildings in the world.

88	88	110	88	80	69	102	78	70	54	80	85	
83	100	60	90	77	55	73	55	56	61	75	64	105

11. Make a stem-and-leaf plot of the data.
12. Which value occurs most frequently?
13. Does the mode best describe the set of data? Explain.

Practice and Apply

Homework Help

For Exercises	See Examples
14–18	1, 2
20–22, 28, 29, 32, 33, 35, 36	3, 4
19, 23–27, 30, 31, 34, 37	5, 6

Extra Practice

See page 824.

Use each set of data to make a line plot.

14. 43 36 48 52 41 54 45 48 49 52 35 44 53 46 38 41 53

15. 1.0 −1.5 1.5 2.0 −1.5 2.1 −2.0 2.4 1.5 −1.4
2.5 1.4 −1.2 1.3 1.0 2.2 2.3 −1.2 −1.5 2.1

BASKETBALL **For Exercises 16–19, use the table that shows the seeds, or rank, of the NCAA men's basketball Final Four from 1991 to 2001.**

Year	Seeds
1991	1 1 2 3
1992	1 2 4 6
1993	1 1 1 2
1994	1 1 2 3
1995	1 2 2 4
1996	1 1 4 5
1997	1 1 1 4
1998	1 2 3 3
1999	1 1 1 4
2000	1 5 8 8
2001	1 1 2 3

Source: www.espn.com

16. Make a line plot of the data.

17. How many of the teams in the Final Four were *not* number 1 seeds?

18. How many teams were seeded higher than third? (*Hint*: Higher seeds have lesser numerical value.)

19. Which measure of central tendency best describes the data? Explain.

Use each set of data to make a stem-and-leaf plot.

20. 6.5 6.3 6.9 7.1 7.3 5.9 6.0 7.0 7.2 6.6 7.1 5.8

21. 31 30 28 26 22 34 26 31 47 32 18 33 26 23 18 29

WEATHER **For Exercises 22–24, use the list of the highest recorded temperatures in each of the 50 states.**

112	100	128	120	134	118	106	110	109	112
100	118	117	116	118	121	114	114	105	109
107	112	114	115	118	117	118	125	106	110
122	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: *The World Almanac*

22. Make a stem-and-leaf plot of the data.

23. Which temperature occurs most frequently?

24. Does the mode best represent the data? Explain.

25. **RESEARCH** Use the Internet or another source to find the total number of each CD sold over the past six months to reach number one. Which measure of central tendency best describes the average number of top selling CDs sold? Explain.

GEOLOGY **For Exercises 26 and 27, refer to the stem-and-leaf plot that shows the magnitudes of earthquakes occurring in 2000 that measured at least 5.0 on the Richter scale.**

Stem	Leaf
5	1 2 2 3 4 8 8 9 9
6	1 1 2 3 4 5 6 7 7 8
7	0 1 1 2 2 3 5 5 5 5 6 8 8
8	0 0 2 $5 \mid 1 = 5.1$

Source: National Geophysical Data Center

26. What was the most frequent magnitude of these earthquakes?

27. Which measure of central tendency best describes this set of data? Explain.

OLYMPICS **For Exercises 28–31, use the information in the table that shows the number of medals won by the top ten countries during the 2000 Summer Olympics in Sydney, Australia.**

Sydney Olympics Total Medals by Country

Country	Gold	Silver	Bronze	Total
United States	40	24	33	97
Russia	32	28	28	88
China	28	16	15	59
Australia	16	25	17	58
Germany	13	17	26	56
France	13	14	11	38
Italy	14	8	13	35
Cuba	11	11	7	29
Britain	11	10	7	28
Korea	8	10	10	28

Source: www.espn.com

28. Make a line plot showing the number of gold medals won by the countries.

29. How many countries won fewer than 25 gold medals?

30. What was the median number of gold medals won by a country?

31. Is the median the best measure to describe this set of data? Explain.

CARS **For Exercises 32–34, use the list of the fuel economy of various vehicles in miles per gallon.**

25	28	29	30	24	28	29	31	34	30
33	47	34	43	33	36	37	29	30	30
29	26	29	22	23	19	18	20	23	21
20	20	19	16	18	21	20	19	28	20

Source: United States Environmental Protection Agency

32. Make a stem-and-leaf plot of the data.

33. How many of the vehicles get more than 25 miles per gallon?

34. Which measure of central tendency would you use to describe the fuel economy of the vehicles? Explain your reasoning.

EDUCATION **For Exercises 35–37, use the table that shows the top ten public libraries in the United States by population served.**

Top Libraries

Location	Number of Branches	Location	Number of Branches
Brooklyn, NY	59	Los Angeles County, CA	84
Broward County, FL	34	Miami, FL	30
Chicago, IL	77	New York, NY	85
Houston, TX	37	Philadelphia P	52
Los Angeles, CA	67	Queens Borough, NY	62

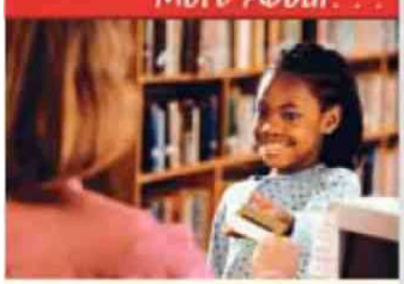
35. Make a stem-and-leaf plot to show the number of library branches.

36. Which interval has the most values?

37. What is the mode of the data?

38. **CRITICAL THINKING** Construct a set of twelve numbers with a mean of 7, a median of 6, and a mode of 8.

More About. . .

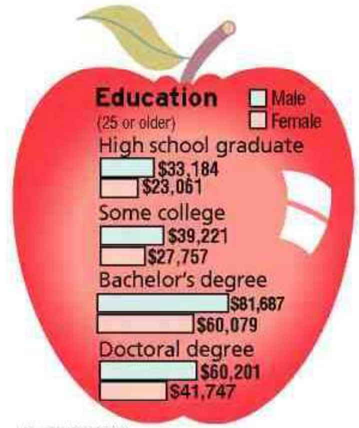


Education

In 1848, the Boston Public Library became the first public library to allow users to borrow books and materials.

Source: The Boston Public Library

SALARIES **For Exercises 39–41, refer to the bar graph that shows the median income of males and females based on education levels.**



Source: *USA TODAY*

39. What are the differences between men's and women's salaries at each level of education?

40. What do these graphs say about the difference between salaries and education levels?

41. Why do you think that salaries are usually represented by the median rather than the mean?

42. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are line plots and averages used to make decisions?

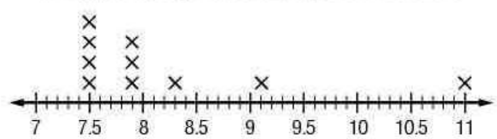
Include the following in your answer:

- a line plot to show how many male students in your class have the most popular names for the decade in which they were born, and
- a convincing argument that explains how you would use this information to sell personalized T-shirts.

Standardized Test Practice

For Exercises 43 and 44, refer to the line plot.

Wingspan (in.) of Ten Largest Butterflies



43. What is the average wingspan for these types of butterflies?

Ⓐ 7.6 in. Ⓑ 7.9 in.

Ⓒ 8.2 in. Ⓓ 9.1 in.

44. Which sentence is *not* true?

Ⓐ The difference between the greatest and least wingspan is 3.5 inches.

Ⓑ Most of the wingspans are in the 7.5 inch to 8.5 inch interval.

Ⓒ Most of the wingspans are greater than 8 inches.

Ⓓ The mode of the data is 7.5 inches.

Maintain Your Skills

Mixed Review

Find each quotient. *(Lesson 2-4)*

45. $56 \div (-14)$ **46.** $-72 \div (-12)$ **47.** $-40.5 \div 3$ **48.** $102 \div 6.8$

Simplify each expression. *(Lesson 2-3)*

49. $-2(6x) - 5x$ **50.** $3x(-7y) - 4x(5y)$ **51.** $5(3t - 2t) - 2(4t)$

52. Write an algebraic expression to represent the amount of money in Kara's savings account if she has d dollars and adds x dollars per week for 12 weeks. *(Lesson 1-1)*

Evaluate each expression if $x = 5$, $y = 16$, and $z = 9$. *(Lesson 1-2)*

53. $y - 3x$ **54.** $xz \div 3$ **55.** $2x - x + (y \div 4)$ **56.** $\frac{x^2 - z}{2y}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write each fraction in simplest form.**
*(To review **simplifying fractions**, see pages 798 and 799.)*

57. $\frac{12}{18}$ **58.** $\frac{54}{60}$ **59.** $\frac{21}{30}$ **60.** $\frac{42}{48}$

61. $\frac{32}{64}$ **62.** $\frac{28}{52}$ **63.** $\frac{16}{36}$ **64.** $\frac{84}{90}$

Reading Mathematics

Interpreting Statistics

The word *statistics* is associated with the collection, analysis, interpretation, and presentation of numerical data. Sometimes, when presenting data, *notes* and *unit indicators* are included to help you interpret the data.

Headnotes give information about the table as a whole.

If the numerical data are too large, *unit indicators* are used to save space.

Public Elementary and Secondary School Enrollment, 1994–1998
[(in thousands) 44,111 represents 44,111,000.]
As of fall year. Kindergarten includes nursery schools.

Grade	1994	1995	1996	1997	1998, prel.
Pupils enrolled	**44,111**	**44,840**	**45,611**	**46,127**	**46,535**
Kindergarten and grades 1 to 8	31,898	32,341	32,764	33,073	33,344
Kindergarten	4047	4173	4202	4198	4171
First	3593	3671	3770	3755	3727
Second	3440	3507	3600	3689	3682
Third	3439	3445	3524	3597	3696
Fourth	3426	3431	3454	3507	3592
Fifth	3372	3438	3453	3458	3520
Sixth	3381	3395	3494	3492	3497
Seventh	3404	3422	3464	3520	3530
Eighth	3302	3356	3403	3415	3480
Unclassified[1]	494	502	401	442	460
Grades 9 to 12	12,213	12,500	12,847	13,054	13,191
Ninth	3604	3704	3801	3819	3856
Tenth	3131	3237	3323	3376	3382
Eleventh	2748	2826	2930	2972	3018
Twelfth	2488	2487	2586	2673	2724
Unclassified[1]	242	245	206	214	211

[1] Includes ungraded and special education.

Source: U.S. Census Bureau

Footnotes give information about specific items within the table.

Suppose you need to find the number of students enrolled in the 9th grade in 1997. The following steps can be used to determine this information.

Step 1 Locate the number in the table. The number that corresponds to 1997 and 9th grade is 3819.

Step 2 Determine the unit indicator. The *unit indicator* is thousands.

Step 3 If the unit indicator is not 1 unit, multiply to find the data value. In this case, multiply 3819 by 1000.

Step 4 State the data value. The number of students enrolled in the 9th grade in 1997 was 3,819,000.

Reading to Learn

Use the information in the table to answer each question.

1. Describe the data.
2. What information is given by the footnote?
3. How current is the data?
4. What is the unit indicator?
5. How many acres of state parks and recreation areas does New York have?
6. Which of the states shown had the greatest number of visitors? How many people visited that state's parks and recreation areas in 1999?

State Parks and Recreation Areas for Selected States, 1999

State	Acreage (1000)	Visitors (1000)[1]
United States	**12,916**	**766,842**
Alaska	3291	3855
California	1376	76,736
Florida	513	14,645
Indiana	178	18,652
New York	1016	61,960
North Carolina	158	13,269
Oregon	94	38,752
South Carolina	82	9563
Texas	628	21,446

Source: U.S. Census Bureau

[1] Includes overnight visitors.

2-6 Probability: Simple Probability and Odds

What You'll Learn

- Find the probability of a simple event.
- Find the odds of a simple event.

Why is probability important in sports?

A basketball player is at the free throw line. Her team is down by one point. If she makes an average of 75% of her free throws, what is the probability that she will tie the game with her first shot?

Vocabulary

- probability
- simple event
- sample space
- equally likely
- odds

PROBABILITY One way to describe the likelihood of an event occurring is with probability. The **probability** of a **simple event**, like a coin landing heads up when it is tossed, is a ratio of the number of favorable outcomes for the event to the total number of possible outcomes of the event. The probability of an event can be expressed as a fraction, a decimal, or a percent.

Suppose you wanted to find the probability of rolling a 4 on a die. When you roll a die, there are six possible outcomes, 1, 2, 3, 4, 5, or 6. This list of all possible outcomes is called the **sample space**. Of these outcomes, only one, a 4, is favorable. So, the probability of rolling a 4 is $\frac{1}{6}$, $0.1\overline{6}$, or about 16.7%.

Key Concept — Probability

The probability of an event a can be expressed as

$$P(a) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}.$$

Study Tip

Reading Math
$P(a)$ is read *the probability of a*.

Example 1 Find Probabilities of Simple Events

a. Find the probability of rolling an even number on a die.

There are six possible outcomes. Three of the outcomes are favorable. That is, three of the six outcomes are even numbers.

3 even numbers

Sample space: 1, 2, 3, 4, 5, 6 $\quad \frac{3}{6}$

6 total possible outcomes

So, $P(\text{even number}) = \frac{3}{6}$ or $\frac{1}{2}$.

b. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find P(blue).

There are 7 blue chips and 28 total chips.

$P(\text{blue chip}) = \frac{7}{28}$ ← number of favorable outcomes

$= \frac{1}{4}$ or 0.25 Simplify.

The probability of selecting a blue chip is $\frac{1}{4}$ or 25%.

c. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find P(red or yellow).

There are 5 ways to pick a red chip and 6 ways to pick a yellow chip. So there are $5 + 6$ or 11 ways to pick a red or a yellow chip.

$$P(\text{red or yellow}) = \frac{11}{28} \quad \leftarrow \text{number of favorable outcomes}$$

$$\approx 0.39 \quad \text{Divide.}$$

The probability of selecting a red chip or a yellow chip is $\frac{11}{28}$ or about 39%.

d. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find P(not green).

There are $5 + 7 + 6$ or 18 chips that are not green.

$$P(\text{not green}) = \frac{18}{28} \quad \leftarrow \text{number of favorable outcomes}$$

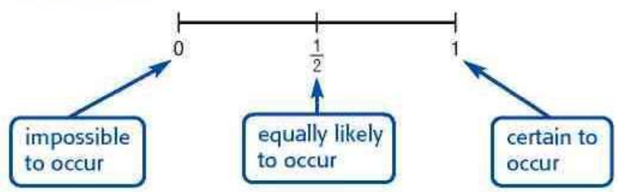
$$\approx 0.64 \quad \text{Divide.}$$

The probability of selecting a chip that is not green is $\frac{9}{14}$ or about 64%.

Study Tip

Reading Math
Inclusive means that the end values are included.

Notice that the probability that an event will occur is somewhere between 0 and 1 inclusive. If the probability of an event is 0, that means that it is impossible for the event to occur. A probability equal to 1 means that the event is certain to occur. There are outcomes for which the probability is $\frac{1}{2}$. When this happens, the outcomes are **equally likely** to occur or not to occur.



ODDS

Another way to express the chance of an event occurring is with **odds**.

Key Concept — *Odds*

The odds of an event occurring is the ratio that compares the number of ways an event can occur (successes) to the number of ways it cannot occur (failures).

Study Tip

Odds
Odds are usually written in the form *number of successes : number of failures.*

Example 2 Odds of an Event

Find the odds of rolling a number less than 3.

There are 6 possible outcomes, 2 are successes and 4 are failures.

2 numbers less than 3

Sample space: 1, 2, 3, 4, 5, 6 $\qquad \frac{2}{4}$ or $\frac{1}{2}$

4 numbers not less than 3

So, the odds of rolling a number less than three are $\frac{1}{2}$ or 1:2.

The odds *against* an event occurring are the odds that the event will *not* occur.

Study Tip

In this text, a *standard deck of cards* always indicates 52 cards in 4 suits of 13 cards each.

Example 3 Odds Against an Event

A card is selected at random from a standard deck of 52 cards. What are the odds against selecting a 3?

There are four 3s in a deck of cards, and there are $52 - 4$ or 48 cards that are not a 3.

odds against a 3 $= \frac{48}{4}$ ← number of ways to *not* pick a 3

The odds against selecting a 3 from a deck of cards are 12:1.

Example 4 Probability and Odds

WEATHER **A weather forecast states that the probability of rain the next day is 40%. What are the odds that it will rain?**

The probability that it will rain is 40%, so the probability that it will not rain is 60%.

odds of rain = 40:60 or 2:3

The odds that it will rain tomorrow are 2:3.

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an impossible event, a certain event, and an equally likely event when a die is rolled.
2. **Describe** how to find the odds of an event occurring if the probability that the event will occur is $\frac{3}{5}$.
3. **FIND THE ERROR** Mark and Doug are finding the probability of picking a red card from a standard deck of cards.

Mark	Doug
$P(\text{red card}) = \frac{26}{26}$ or $\frac{1}{1}$	$P(\text{red card}) = \frac{26}{52}$ or $\frac{1}{2}$

Who is correct? Explain your reasoning.

Guided Practice

A card is selected at random from a standard deck of cards. Determine each probability.

4. $P(5)$
5. P(red 10)
6. P(odd number)
7. P(queen of hearts or jack of diamonds)

Find the odds of each outcome if the spinner is spun once.

8. multiple of 3
9. even number less than 8
10. odd number or blue
11. red or yellow

Application

NUMBER THEORY **One of the factors of 48 is chosen at random.**

12. What is the probability that the chosen factor is not a multiple of 4?
13. What is the probability that the number chosen has 4 and 6 as two of its factors?

Practice and Apply

Homework Help

For Exercises	See Examples
14–35, 51, 54, 56	1
36–47, 52, 53, 55	2, 3
48, 49	4

Extra Practice

See page 824.

One coin is randomly selected from a jar containing 70 nickels, 100 dimes, 80 quarters, and 50 1-dollar coins. Find each probability.

14. P(quarter)

15. P(dime)

16. P(nickel or dollar)

17. P(quarter or nickel)

18. P(value less than \$1.00)

19. P(value greater than \$0.10)

20. P(value at least \$0.25)

21. P(value at most \$1.00)

Two dice are rolled, and their sum is recorded. Find each probability.

22. P(sum less than 7)

23. P(sum less than 8)

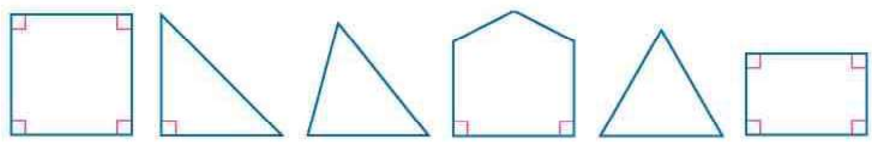
24. P(sum is greater than 12)

25. P(sum is greater than 1)

26. P(sum is between 5 and 10)

27. P(sum is between 2 and 9)

One of the polygons is chosen at random. Find each probability.



28. P(triangle)

29. P(pentagon)

30. P(not a triangle)

31. P(not a quadrilateral)

32. P(more than three sides)

33. P(more than one right angle)

34. If a person's birthday is in April, what is the probability that it is the 29th?

35. If a person's birthday is in July, what is the probability that it is after the 16th?

Find the odds of each outcome if a computer randomly picks a letter in the name *The United States of America*.

36. the letter a

37. the letter t

38. a vowel

39. a consonant

40. an uppercase letter

41. a lowercase vowel

More About. . .

Stamp Collecting

Stamp collecting can be a very inexpensive hobby. Most stamp collectors start by saving stamps from letters, packages, and postcards.

Source: United States Postal Service

STAMP COLLECTING **Lanette collects stamps from different countries. She has 12 from Mexico, 5 from Canada, 3 from France, 8 from Great Britain, 1 from Russia, and 3 from Germany. Find the odds of each of the following if she accidentally loses one stamp.**

42. the stamp is from Canada

43. the stamp is from Mexico

44. the stamp is not from France

45. the stamp is not from a North American country

46. the stamp is from Germany or Russia

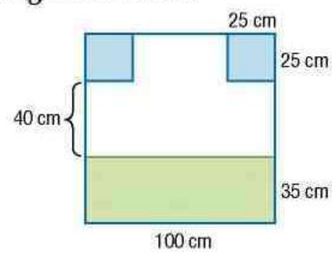
47. the stamp is from Canada or Great Britain

48. If the probability that an event will occur is $\frac{3}{7}$, what are the odds that it will occur?

49. If the probability that an event will occur is $\frac{2}{3}$, what are the odds against it occurring?

50. **CONTESTS** Every Tuesday, Mike's Submarine Shop has a business card drawing for a free lunch. Four coworkers from InvoAccounting put their business cards in the bowl for the drawing. If there are 80 cards in the bowl, what are the odds that one of the coworkers will win a free lunch?

GAMES **For Exercises 51–53, use the following information.**
A game piece is randomly placed on the board shown at the right by blindfolded players.



51. What is the probability that a game piece is placed on a shaded region?

52. What are the odds against placing a game piece on a shaded region?

53. What are the odds that a game piece will be placed within the green rectangle?

More About. . .

Baseball

The record for the most home runs in a single season is 84. It was set by Joshua Gibson of the Homestead Grays in 1934.

Source: National Baseball Hall of Fame

BASEBALL **For Exercises 54–56, use the following information.**
The stem-and-leaf plot shows the number of home runs hit by the top major league baseball players in the 2000 season. **Source:** www.espn.com

Stem	Leaf
3	0 0 0 0 1 1 1 1 1 1 1 2 2 2 3
	3 4 4 4 5 5 5 6 6 6 7 7 8 8 9
4	0 1 1 1 1 2 2 3 3 3 4 4 7 7 9
5	0 *3|0 = 30*

54. What is the probability that one of these players picked at random hit more than 35 home runs?

55. What are the odds that a randomly selected player hit less than 45 home runs?

56. If a player batted 439 times and hit 38 home runs, what is the probability that the next time the player bats he will hit a home run?

CONTESTS **For Exercises 57 and 58, use the following information.**
A fast-food restaurant is holding a contest in which the grand prize is a new sports car. Each customer is given a game card with their order. The contest rules state that the odds of winning the grand prize are 1:1,000,000.

57. For any randomly-selected game card, what is the probability that it is the winning game card for the grand prize?

58. Do your odds of winning the grand prize increase significantly if you have several game cards? Explain.

WebQuest

You can use real-world data to find the probability that a person will live to be 100. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

59. **CRITICAL THINKING** Three coins are tossed, and a tail appears on at least one of them. What is the probability that at least one head appears?

60. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is probability important in sports?

Include the following in your answer:

- examples of two sports in which probability is used and an explanation of each sport's importance, and
- examples of methods other than probability used to show chance.

Standardized Test Practice

61. If the probability that an event will occur is $\frac{12}{25}$, what are the odds that the event will *not* occur?

(A) 12:13 (B) 13:12 (C) 13:25 (D) 25:12

62. What is the probability that a number chosen at random from the domain $\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$ will satisfy the inequality $3x + 2 \leq 17$?

(A) 20% (B) 27% (C) 73% (D) 80%

Maintain Your Skills

Mixed Review

63. **WEATHER** The following data represents the average daily temperature in Fahrenheit for Sacramento, California, for two weeks during the month of May. Organize the data using a stem-and-leaf plot. *(Lesson 2-5)*

58.3	64.3	66.7	65.1	68.7	67.0	69.3
70.0	72.8	77.4	77.4	73.2	75.8	65.5

Evaluate each expression if $a = -\frac{1}{3}$, $b = \frac{2}{5}$, and $c = \frac{1}{2}$. *(Lesson 2-4)*

64. $b \div c$
65. $2a \div b$
66. $\frac{ab}{c}$

Find each sum. *(Lesson 2-2)*

67. $4.3 + (-8.2)$
68. $-12.2 + 7.8$
69. $-\frac{1}{4} + \left(-\frac{3}{8}\right)$
70. $\frac{7}{12} + \left(-\frac{5}{6}\right)$

Find each absolute value. *(Lesson 2-1)*

71. $|4.25|$
72. $|-8.4|$
73. $\left|-\frac{2}{3}\right|$
74. $\left|\frac{1}{6}\right|$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
*(To review **evaluating expressions**, see Lesson 1-2.)*

75. 6^2
76. 17^2
77. $(-8)^2$
78. $(-11.5)^2$
79. 1.6^2
80. $\left(\frac{5}{12}\right)^2$
81. $\left(-\frac{4}{9}\right)^2$
82. $\left(-\frac{16}{15}\right)^2$

Practice Quiz 2 — Lessons 2-4 through 2-6

Find each quotient. *(Lesson 2-4)*

1. $-136 \div (-8)$
2. $15 \div \left(-\frac{3}{8}\right)$
3. $(-46.8) \div 4$

Simplify each expression. *(Lesson 2-4)*

4. $\frac{3a + 9}{3}$
5. $\frac{4x + 32}{4}$
6. $\frac{15n - 20}{-5}$

7. State the scale you would use to make a line plot for the following data. Then draw the line plot. *(Lesson 2-5)*

1.9	1.1	3.2	5.0	4.3	2.7	2.5	1.1	1.4	1.8	1.8	1.6
4.3	2.9	1.4	1.7	3.6	2.9	1.9	0.4	1.3	0.9	0.7	1.9

Determine each probability if two dice are rolled. *(Lesson 2-6)*

8. $P(\text{sum of } 10)$
9. $P(\text{sum} \geq 6)$
10. $P(\text{sum} < 10)$

Algebra Activity

A Follow-Up of Lesson 2-6

Investigating Probability and Pascal's Triangle

Collect the Data

- If a family has one child, you know that the child is either a boy or a girl. You can make a simple table to show this type of family.

1 boy	1 girl
B	G

You can see that there are 2 possibilities for a one-child family.

- If a family has two children, the table below shows the possibilities for two children, including the order of birth. For example, BG means that a boy is born first and a girl second.

2 boys, 0 girls	1 boy, 1 girl	0 boys, 2 girls
BB	BG	GG
	GB	

There are 4 possibilities for the two-child family: BB, BG, GB, or GG.

Analyze the Data

3 boys	2 boys, 1 girl	1 boy, 2 girls	3 girls
BBB	BBG	BGG	GGG

1. Copy and complete the table that shows the possibilities for a three-child family.

2. Make your own table to show the possibilities for a four-child family.

3. List the total number of possibilities for a one-child, two-child, three-child, and four-child family. How many possibilities do you think there are for a five-child family? a six-child family? Describe the pattern of the numbers you listed.

4. Find the probability that a three-child family has 2 boys and 1 girl.

5. Find the probability that a four-child family has 2 boys and 2 girls.

Make a Conjecture

6. Blaise Pascal was a French mathematician who lived in the 1600s. He is known for this triangle of numbers, called Pascal's triangle, although the pattern was known by other mathematicians before Pascal's time.

				1					Row 0
			1		1				Row 1
		1		2		1			Row 2
	1		3		3		1		Row 3
1		4		6		4		1	Row 4

Explain how Pascal's triangle relates to the possibilities for the make-up of families. (*Hint*: The first row indicates that there is 1 way to have 0 children.)

7. Use Pascal's triangle to find the probability that a four-child family has 1 boy.

2-7 Square Roots and Real Numbers

What You'll Learn

- Find square roots.
- Classify and order real numbers.

Vocabulary

- square root
- perfect square
- radical sign
- principal square root
- irrational numbers
- real numbers
- rational approximations

How can using square roots determine the surface area of the human body?

In the 2000 Summer Olympics, Australian sprinter Cathy Freeman wore a special running suit that covered most of her body. The surface area of the human body may be found using the formula below, where height is measured in centimeters and weight is in kilograms.

Surface Area $= \sqrt{\dfrac{\text{height} \times \text{weight}}{3600}}$ square meters

The symbol $\sqrt{}$ designates a square root.

SQUARE ROOTS A **square root** is one of two equal factors of a number. For example, one square root of 64 is 8 since $8 \cdot 8$ or 8^2 is 64. Another square root of 64 is -8 since $(-8) \cdot (-8)$ or $(-8)^2$ is also 64. A number like 64, whose square root is a rational number is called a **perfect square**.

The symbol $\sqrt{}$, called a **radical sign**, is used to indicate a nonnegative or **principal square root** of the expression under the radical sign.

> **Study Tip**
>
> *Reading Math*
>
> $\pm\sqrt{64}$ is read *plus or minus the square root of 64.*

$\sqrt{64} = 8$ ← $\sqrt{64}$ indicates the *principal* square root of 64.

$-\sqrt{64} = -8$ ← $-\sqrt{64}$ indicates the *negative* square root of 64.

$\pm\sqrt{64} = \pm 8$ ← $\pm\sqrt{64}$ indicates *both* square roots of 64.

Note that $-\sqrt{64}$ is not the same as $\sqrt{-64}$. The notation $-\sqrt{64}$ represents the negative square root of 64. The notation $\sqrt{-64}$ represents the square root of -64, which is not a real number since no real number multiplied by itself is negative.

Example 1 Find Square Roots

Find each square root.

a. $-\sqrt{\dfrac{49}{256}}$

$-\sqrt{\dfrac{49}{256}}$ represents the negative square root of $\dfrac{49}{256}$.

$\dfrac{49}{256} = \left(\dfrac{7}{16}\right)^2 \rightarrow -\sqrt{\dfrac{49}{256}} = -\dfrac{7}{16}$

b. $\pm\sqrt{0.81}$

$\pm\sqrt{0.81}$ represents the positive and negative square roots of 0.81.

$0.81 = 0.9^2$ and $0.81 = (-0.9)^2$

$\pm\sqrt{0.81} = \pm 0.9$

CLASSIFY AND ORDER NUMBERS Recall that rational numbers are numbers that can be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

As you have seen, the square roots of perfect squares are rational numbers. However, numbers such as $\sqrt{3}$ and $\sqrt{24}$ are the square roots of numbers that are not perfect squares. Numbers like these cannot be expressed as a terminating or repeating decimal.

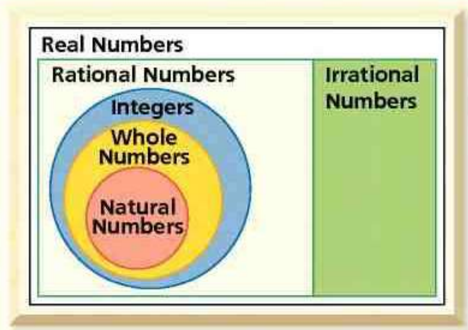
$$\sqrt{3} = 1.73205080\ldots$$

$$\sqrt{24} = 4.89897948\ldots$$

Numbers that cannot be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers**. Irrational numbers and rational numbers together form the set of **real numbers**.

Concept Summary — Real Numbers

The set of real numbers consists of the set of rational numbers and the set of irrational numbers.



Example 2 Classify Real Numbers

Name the set or sets of numbers to which each real number belongs.

a. $\frac{5}{22}$

Because 5 and 22 are integers and $5 \div 22 = 0.2272727\ldots$ is a repeating decimal, this number is a rational number.

b. $\sqrt{121}$

Because $\sqrt{121} = 11$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{56}$

Because $\sqrt{56} = 7.48331477\ldots$, which is not a repeating or terminating decimal, this number is irrational.

d. $-\frac{36}{4}$

Because $-\frac{36}{4} = -9$, this number is an integer and a rational number.

Study Tip

Common Misconception

Pay close attention to the placement of a negative sign when working with square roots. $\sqrt{-121}$ is undefined for real numbers since no real number multiplied by itself can result in a negative product.

CONTENTS

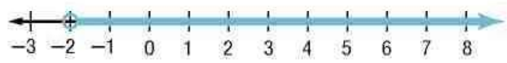
In Lesson 2-1 you graphed rational numbers on a number line. However, the rational numbers alone do not complete the number line. By including irrational numbers, the number line is complete. This is illustrated by the **Completeness Property** which states that each point on the number line corresponds to exactly one real number.

Recall that inequalities like $x < 7$ are open sentences. To solve the inequality, determine what replacement values for x make the sentence true. This can be shown by the solution set {all real numbers less than 7}. Not only does this set include integers like 5 and -2, but it also includes rational numbers like $\frac{3}{8}$ and $-\frac{12}{13}$ and irrational numbers like $\sqrt{40}$ and π.

Example 3 Graph Real Numbers

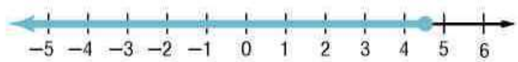
Graph each solution set.

a. $x > -2$



The heavy arrow indicates that all numbers to the right of -2 are included in the graph. The circle at -2 indicates -2 is *not* included in the graph.

b. $a \leq 4.5$



The heavy arrow indicates that all points to the left of 4.5 are included in the graph. The dot at 4.5 indicates that 4.5 *is* included in the graph.

To express irrational numbers as decimals, you need to use a rational approximation. A **rational approximation** of an irrational number is a rational number that is close to, but not equal to, the value of the irrational number. For example, a rational approximation of $\sqrt{2}$ is 1.41 when rounded to the nearest hundredth.

Example 4 Compare Real Numbers

Replace each ● with <, >, or = to make each sentence true.

a. $\sqrt{19}$ ● $3.\overline{8}$

Find two perfect squares closest to $\sqrt{19}$ and write an inequality.

$16 < 19 < 25$ 19 is between 16 and 25.

$\sqrt{16} < \sqrt{19} < \sqrt{25}$ Find the square root of each number.

$4 < \sqrt{19} < 5$ $\sqrt{19}$ is between 4 and 5.

Since $\sqrt{19}$ is between 4 and 5, it must be greater than $3.\overline{8}$.

So, $\sqrt{19} > 3.\overline{8}$.

b. $7.\overline{2}$ ● $\sqrt{52}$

You can use a calculator to find an approximation for $\sqrt{52}$.

$\sqrt{52} = 7.211102551\ldots$

$7.\overline{2} = 7.222\ldots$

Therefore, $7.\overline{2} > \sqrt{52}$.

You can write a set of real numbers in order from greatest to least or from least to greatest. To do so, find a decimal approximation for each number in the set and compare.

Example 5 Order Real Numbers

Write $2.\overline{63}$, $-\sqrt{7}$, $\frac{8}{3}$, $\frac{53}{-20}$ in order from least to greatest.

Write each number as a decimal.

$2.\overline{63} = 2.6363636\ldots$ or about 2.636.

$-\sqrt{7} = -2.64575131\ldots$ or about -2.646.

$\frac{8}{3} = 2.66666666\ldots$ or about 2.667.

$\frac{53}{-20} = -2.65$

$-2.65 < -2.646 < 2.636 < 2.667$

The numbers arranged in order from least to greatest are $\frac{53}{-20}$, $-\sqrt{7}$, $2.\overline{63}$, $\frac{8}{3}$.

You can use rational approximations to test the validity of some algebraic statements involving real numbers.

Standardized Test Practice

Example 6 Rational Approximation

Multiple-Choice Test Item

For what value of x is $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ true?

Ⓐ $\frac{1}{2}$ Ⓑ 0 Ⓒ -2 Ⓓ 3

Read the Test Item

The expression $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ is an open sentence, and the set of choices $\left\{\frac{1}{2}, 0, -2, 3\right\}$ is the replacement set.

Solve the Test Item

Replace x in $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ with each given value.

Ⓐ $x = \frac{1}{2}$

$\frac{1}{\sqrt{\frac{1}{2}}} \stackrel{?}{>} \sqrt{\frac{1}{2}} \stackrel{?}{>} \frac{1}{2}$ Use a calculator.

$1.41 > 0.71 > 0.5$ ✓ True

Ⓑ $x = 0$

$\frac{1}{\sqrt{0}} \stackrel{?}{>} \sqrt{0} \stackrel{?}{>} 0$

False; $\frac{1}{\sqrt{0}}$ is not a real number.

Ⓒ $x = -2$

$\frac{1}{\sqrt{-2}} \stackrel{?}{>} \sqrt{-2} \stackrel{?}{>} -2$

False; $\frac{1}{\sqrt{-2}}$ and $\sqrt{-2}$ are not real numbers.

Ⓓ $x = 3$

$\frac{1}{\sqrt{3}} \stackrel{?}{>} \sqrt{3} \stackrel{?}{>} 3$ Use a calculator.

$\cancel{0.58 > 1.73 > 3}$ False

The inequality is true for $x = \frac{1}{2}$, so the correct answer is A.

The Princeton Review

Test-Taking Tip

You could stop when you find that A is a solution. But testing the other values is a good check.

Check for Understanding

Concept Check

1. **Tell** whether the square root of any real number is *always*, *sometimes* or *never* positive. Explain your answer.
2. **OPEN ENDED** Describe the difference between rational numbers and irrational numbers. Give examples of both.
3. **Explain** why you cannot evaluate $\sqrt{-25}$ using real numbers.

Guided Practice

Find each square root. If necessary, round to the nearest hundredth.

4. $-\sqrt{25}$
5. $\sqrt{1.44}$
6. $\pm\sqrt{\frac{16}{49}}$
7. $\sqrt{32}$

Name the set or sets of numbers to which each real number belongs.

8. $-\sqrt{64}$
9. $\frac{8}{3}$
10. $\sqrt{28}$
11. $\frac{56}{7}$

Graph each solution set.

12. $x < -3.5$
13. $x \geq -7$

Replace each ● with <, >, or = to make each sentence true.

14. $0.3 \bullet \frac{1}{3}$
15. $\frac{2}{9} \bullet 0.\overline{2}$
16. $\frac{1}{6} \bullet \sqrt{6}$

Write each set of numbers in order from least to greatest.

17. $\frac{1}{8}, \sqrt{\frac{1}{8}}, 0.\overline{15}, -15$
18. $\sqrt{30}, 5\frac{4}{9}, 13, \frac{1}{\sqrt{30}}$

Standardized Test Practice

19. For what value of a is $-\sqrt{a} < -\frac{1}{\sqrt{a}}$ true?

(A) $\frac{1}{3}$ (B) -4 (C) 2 (D) 1

Practice and Apply

Homework Help

For Exercises	See Examples
20–31, 50, 51	1
32–49	2
52–57	3
58–63	4
64–69	5

Extra Practice

See page 825.

Find each square root. If necessary, round to the nearest hundredth.

20. $\sqrt{49}$
21. $\sqrt{81}$
22. $\sqrt{5.29}$
23. $\sqrt{6.25}$
24. $-\sqrt{78}$
25. $-\sqrt{94}$
26. $\pm\sqrt{\frac{36}{81}}$
27. $\pm\sqrt{\frac{100}{196}}$
28. $\sqrt{\frac{9}{14}}$
29. $\sqrt{\frac{25}{42}}$
30. $\pm\sqrt{820}$
31. $\pm\sqrt{513}$

Name the set or sets of numbers to which each real number belongs.

32. $-\sqrt{22}$
33. $\frac{36}{6}$
34. $\frac{1}{3}$
35. $-\frac{5}{12}$
36. $\sqrt{\frac{82}{20}}$
37. $-\sqrt{46}$
38. $\sqrt{10.24}$
39. $\frac{-54}{19}$
40. $-\frac{3}{4}$
41. $\sqrt{20.25}$
42. $\frac{18}{3}$
43. $\sqrt{2.4025}$
44. $\frac{-68}{35}$
45. $\frac{6}{11}$
46. $\sqrt{5.5696}$
47. $\sqrt{\frac{78}{42}}$
48. $-\sqrt{9.16}$
49. π

50. **PHYSICAL SCIENCE** The time it takes for a falling object to travel a certain distance d is given by the equation $t = \sqrt{\frac{d}{16}}$, where t is in seconds and d is in feet. If Krista dropped a ball from a window 28 feet above the ground, how long would it take for the ball to reach the ground?

More About...

Tourism

Built in 1758, the Sambro Island Lighthouse at Halifax Harbor is the oldest operational lighthouse in North America.

Source: Canadian Coast Guard

51. **LAW ENFORCEMENT** Police can use the formula $s = \sqrt{24d}$ to estimate the speed s of a car in miles per hour by measuring the distance d in feet a car skids on a dry road. On his way to work, Jerome skidded trying to stop for a red light and was involved in a minor accident. He told the police officer that he was driving within the speed limit of 35 miles per hour. The police officer measured his skid marks and found them to be $43\frac{3}{4}$ feet long. Should the officer give Jerome a ticket for speeding? Explain.

Graph each solution set.

52. $x > -12$
53. $x \leq 8$
54. $x \geq -10.2$
55. $x < -0.25$
56. $x \neq -2$
57. $x \neq \pm\sqrt{36}$

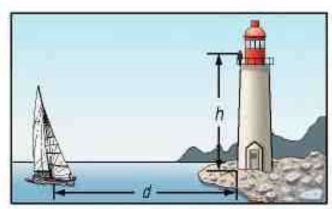
Replace each ● with <, >, or = to make each sentence true.

58. $5.\overline{72}$ ● $\sqrt{5}$
59. $2.\overline{63}$ ● $\sqrt{8}$
60. $\frac{1}{7}$ ● $\frac{1}{\sqrt{7}}$
61. $\frac{2}{3}$ ● $\frac{2}{\sqrt{3}}$
62. $\frac{1}{\sqrt{31}}$ ● $\frac{\sqrt{31}}{31}$
63. $\frac{\sqrt{2}}{2}$ ● $\frac{1}{2}$

Write each set of numbers in order from least to greatest.

64. $\sqrt{0.42}, 0.\overline{63}, \frac{\sqrt{4}}{3}$
65. $\sqrt{0.06}, 0.\overline{24}, \frac{\sqrt{9}}{12}$
66. $-1.\overline{46}, 0.2, \sqrt{2}, -\frac{1}{6}$
67. $-4.\overline{83}, 0.4, \sqrt{8}, -\frac{3}{8}$
68. $-\sqrt{65}, -6\frac{2}{5}, -\sqrt{27}$
69. $\sqrt{122}, 7\frac{4}{9}, \sqrt{200}$

TOURISM **For Exercises 70–72, use the following information.**
The formula to determine the distance d in miles that an object can be seen on a clear day on the surface of a body of water is $d = 1.4\sqrt{h}$, where h is the height in feet of the viewer's eyes above the surface of the water.



70. A charter plane is used to fly tourists on a sightseeing trip along the coast of North Carolina. If the plane flies at an altitude of 1500 feet, how far can the tourists see?

71. Dillan and Marissa are parasailing while on vacation. Marissa is 135 feet above the ocean while Dillan is 85 feet above the ocean. How much farther can Marissa see than Dillan?

72. The observation deck of a lighthouse stands 120 feet above the ocean surface. Can the lighthouse keeper see a boat that is 17 miles from the lighthouse? Explain.

73. **CRITICAL THINKING** Determine when the following statements are all true for real numbers q and r.

a. $q^2 > r^2$
b. $\frac{1}{q} < \frac{1}{r}$
c. $\sqrt{q} > \sqrt{r}$
d. $\frac{1}{\sqrt{q}} < \frac{1}{\sqrt{r}}$

GEOMETRY **For Exercises 74–76, use the table.**

Squares		
Area (units^2)	Side Length	Perimeter
1		
4		
9		
16		
25		

74. Copy and complete the table. Determine the length of each side of each square described. Then determine the perimeter of each square.

75. Describe the relationship between the lengths of the sides and the area.

76. Write an expression you can use to find the perimeter of a square whose area is a units^2.

77. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

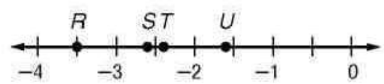
How can using square roots determine the surface area of the human body?

Include the following in your answer:

- an explanation of the order of operations that must be followed to calculate the surface area of the human body,
- a description of other situations in which you might need to calculate the surface area of the human body, and
- examples of real-world situations involving square roots.

Standardized Test Practice

78. Which point on the number line is closest to $-\sqrt{7}$?



Ⓐ R Ⓑ S
Ⓒ T Ⓓ U

79. Which of the following is a true statement?

Ⓐ $-\frac{6}{3} > \frac{3}{6}$ Ⓑ $-\frac{3}{6} > -\frac{6}{3}$ Ⓒ $-\frac{3}{6} < -\frac{6}{3}$ Ⓓ $\frac{6}{3} < \frac{3}{6}$

Maintain Your Skills

Mixed Review **Find the odds of each outcome if a card is randomly selected from a standard deck of cards.** *(Lesson 2-6)*

80. red 4 **81.** even number

82. against a face card **83.** against an ace

84. **AUTO RACING** Jeff Gordon's finishing places in the 2000 season races are listed below. Which measure of central tendency best represents the data? Explain. *(Lesson 2-5)*

34 10 28 9 8 8 25 4 1 11 14 10 32 14 8 1 4
10 5 3 33 23 36 23 4 1 6 9 5 39 4 2 7 7

Simplify each expression. *(Lesson 2-3)*

85. $4(-7) - 3(11)$ **86.** $3(-4) + 2(-7)$

87. $1.2(4x - 5y) - 0.2(-1.5x + 8y)$ **88.** $-4x(y - 2z) + x(6z - 3y)$

Chapter 2 Study Guide and Review

Vocabulary and Concept Check

absolute value (p. 69)
additive inverses (p. 74)
back-to-back stem-and-leaf plot (p. 89)
Completeness Property (p. 105)
coordinate (p. 69)
equally likely (p. 97)
frequency (p. 88)
graph (p. 69)
infinity (p. 68)
integers (p. 68)
irrational number (p. 104)
line plot (p. 88)
measures of central tendency (p. 90)
natural number (p. 68)
negative number (p.68)
odds (p. 97)
opposites (p. 74)
perfect square (p. 103)
positive number (p. 68)
principal square root (p. 103)
probability (p. 96)
radical sign (p. 103)
rational approximation (p. 105)
rational number (p. 68)
real number (p. 104)
sample space (p. 96)
simple event (p. 96)
square root (p. 103)
stem-and-leaf plot (p. 89)
whole number (p. 68)

State whether each sentence is *true* or *false*. If false, replace the underlined term or number to make a true sentence.

1. The absolute value of -26 is 26.
2. Terminating decimals are rational numbers.
3. The principal square root of 144 is 12.
4. $-\sqrt{576}$ is an irrational number.
5. 225 is a perfect square.
6. -3.1 is an integer.
7. 0.666 is a repeating decimal.
8. The product of two numbers with different signs is negative.

Lesson-by-Lesson Review

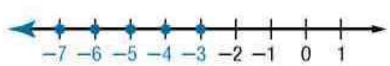
2-1 Rational Numbers on the Number Line

See pages 68–72.

Concept Summary

- A set of numbers can be graphed on a number line by drawing points.
- To evaluate expressions with absolute value, treat the absolute value symbols as grouping symbols.

Example Graph $\{\ldots, -5, -4, -3\}$.



The bold arrow means that the graph continues indefinitely in that direction.

Exercises **Graph each set of numbers.** *See Example 2 on page 69.*

9. $\{5, 3, -1, -3\}$
10. $\left\{-1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}, \ldots\right\}$
11. {integers less than -4 and greater than or equal to 2}

Evaluate each expression if $x = -4$, $y = 8$, and $z = -9$. *See Example 4 on page 70.*

12. $32 - |y - 3|$
13. $3|x| - 7$
14. $4 + |z|$
15. $46 - y|x|$

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2-2 Adding and Subtracting Rational Numbers

See pages 73–78.

Concept Summary

- To add rational numbers with the *same* sign, add their absolute values. The sum has the same sign as the addends.
- To add rational numbers with *different* signs, subtract the lesser absolute value from the greater absolute value. The sum has the same sign as the number with the greater absolute value.
- To subtract a rational number, add its additive inverse.

Examples

1 **Find $-4 + (-3)$.**

$-4 + (-3)$
$= -(|-4| + |-3|)$ Both numbers are negative, so the sum is negative.
$= -(4 + 3)$
$= -7$

2 **Find $12 - 18$.**

$12 - 18$
$= 12 + (-18)$ To subtract 18, add its inverse.
$= -(|-18| - |12|)$ The absolute value of 18 is greater, so the result is negative.
$= -(18 - 12)$
$= -6$

Exercises **Find each sum or difference.** *See Examples 1–3 on pages 73–75.*

16. $4 + (-4)$ **17.** $2 + (-7)$ **18.** $-0.8 + (-1.2)$

19. $-3.9 + 2.5$ **20.** $-\frac{1}{4} + \left(-\frac{1}{8}\right)$ **21.** $\frac{5}{6} + \left(-\frac{1}{3}\right)$

22. $-2 - 10$ **23.** $9 - (-7)$ **24.** $1.25 - 0.18$

25. $-7.7 - (-5.2)$ **26.** $\frac{9}{2} - \left(-\frac{1}{2}\right)$ **27.** $-\frac{1}{8} - \left(-\frac{2}{3}\right)$

2-3 Multiplying Rational Numbers

See pages 79–83.

Concept Summary

- The product of two numbers having the same sign is positive.
- The product of two numbers having different signs is negative.

Example

Multiply $\left(-2\frac{1}{7}\right)\left(3\frac{2}{3}\right)$.

$\left(-2\frac{1}{7}\right)\left(3\frac{2}{3}\right) = \frac{-15}{7} \cdot \frac{11}{3}$ Write as improper fractions.

$= \frac{-55}{7}$ or $-7\frac{6}{7}$ Simplify.

Exercises **Find each product.** *See Examples 1 and 3 on pages 79 and 80.*

28. $(-11)(9)$ **29.** $12(-3)$ **30.** $-8.2(4.5)$

31. $-2.4(-3.6)$ **32.** $\frac{3}{4} \cdot \frac{7}{12}$ **33.** $\left(-\frac{1}{3}\right)\left(-\frac{9}{10}\right)$

Simplify each expression. *See Example 2 on page 80.*

34. $8(-3x) + 12x$ **35.** $-5(-2n) - 9n$ **36.** $-4(6a) - (-3)(-7a)$

2-4 Dividing Rational Numbers

See pages 84–87.

Concept Summary

- The quotient of two positive numbers is positive.
- The quotient of two negative numbers is positive.
- The quotient of a positive number and a negative number is negative.

Example **Simplify $\frac{-3(4)}{-2-3}$.**

$\frac{-3(4)}{-2-3} = \frac{-12}{-2-3}$ Simplify the numerator.

$= \frac{-12}{-5}$ Simplify the denominator.

$= 2\frac{2}{5}$ same signs → positive quotient

Exercises **Find each quotient.** *See Examples 1–3 on pages 84 and 85.*

37. $\frac{-54}{6}$ **38.** $-\frac{74}{8}$ **39.** $21.8 \div (-2)$

40. $-7.8 \div (-6)$ **41.** $-15 \div \left(\frac{3}{4}\right)$ **42.** $\frac{21}{24} \div \frac{1}{3}$

Simplify each expression. *See Example 5 on page 85.*

43. $\frac{14-28x}{-7}$ **44.** $\frac{-5+25x}{5}$ **45.** $\frac{-4x+24y}{4}$

Evaluate each expression if $x = -4$, $y = 2.4$, and $z = 3$. *See Example 6 on page 85.*

46. $xz - 2y$ **47.** $-2\left(\frac{2y}{z}\right)$ **48.** $\frac{2x-z}{4} + 3y$

2-5 Statistics: Displaying and Analyzing Data

See pages 88–94.

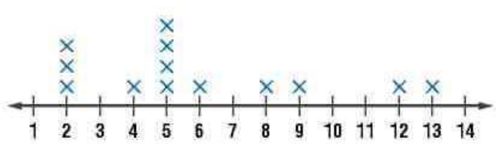
Concept Summary

- A set of numerical data can be displayed in a line plot or stem-and-leaf plot.
- A measure of central tendency represents a centralized value of a set of data. Examine each measure of central tendency to choose the one most representative of the data.

Examples **1** **Draw a line plot for the data.**

2 8 6 4 5 9 13 12 5 2 5 5 2

The value of the data ranges from 2 to 13. Construct a number line containing those points. Then place an × above a number each time it occurs.



2 **SCHOOL** **Melinda's scores on the 25-point quizzes in her English class are 20, 21, 12, 21, 22, 22, 22, 21, 20, 20, and 21. Which measure of central tendency best represents her grade?**

mean:	20.2	Add the data and divide by 11.
median:	21	The middle value is 21.
mode:	21	The most frequent value is 21.

The median and mode are both representative of the data. The mean is less than most of the data.

Exercises

49. Draw a line plot for the data. Then make a stem-and-leaf plot.
See Examples 1–3 on pages 88 and 89.

28	17	16	18	19	21	26	15
19	19	16	14	21	12	26	17
30	17	13	18	14	22	20	12
19	19	15	12	15	21	15	17

50. **BUSINESS** Of the 42 employees at Pirate Printing, four make \$6.50 an hour, sixteen make \$6.75 an hour, six make \$6.85 an hour, thirteen make \$7.25 an hour, and three make \$8.85 an hour. Which measure best describes the average wage? Explain. *See Examples 5 and 6 on pages 90 and 91.*

51. **HOCKEY** Professional hockey uses a point system based on wins, losses and ties, to determine teams' rank. The stem-and-leaf plot shows the number of points earned by each of the 30 teams in the National Hockey League during the 2000–2001 season. Which measure of central tendency best describes the average number of points earned? Explain.
See Example 5 on page 90.

Stem	Leaf
11	1 1 8
10	0 3 6 9
9	0 0 0 2 3 5 6 6 8
8	0 8 8
7	0 1 1 2 3
6	0 6 6 8
5	2 9 *11|1 = 111*

2-6 Probability: Simple Probability and Odds

See pages 96–101.

Concept Summary

- The probability of an event a can be expressed as

$$P(a) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}.$$

- The odds of an event can be expressed as the ratio of the number of successful outcomes to the number of unsuccessful outcomes.

Examples

1 **Find the probability of randomly choosing the letter *I* in the word *MISSISSIPPI*.**

$$P(\text{letter I}) = \frac{4}{11} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$
$$\approx 0.36$$

The probability of choosing an I is $\frac{4}{11}$ or about 36%.

Chapter 2 For More ...
- Extra Practice, see pages 823–825.
- Mixed Problem Solving, see page 854.

2 **Find the odds that you will randomly select a letter that is *not* S in the word *MISSISSIPPI*.**

number of successes : number of failures = 7:4

The odds of not selecting an S are 7:4.

Exercises **Find the probability of each outcome if a computer randomly chooses a letter in the word *REPRESENTING*.** *See Example 1 on pages 96 and 97.*

52. $P(\text{S})$ **53.** $P(\text{E})$ **54.** $P(\text{not N})$ **55.** $P(\text{R or P})$

Find the odds of each outcome if you randomly select a coin from a jar containing 90 pennies, 75 nickels, 50 dimes, and 30 quarters.
See Examples 2 and 3 on pages 97 and 98.

56. a dime **57.** a penny **58.** *not* a nickel **59.** a nickel or a dime

2-7 Square Roots and Real Numbers

See pages 103–109.

Concept Summary

- A square root is one of two equal factors of a number.
- The symbol $\sqrt{}$ is used to indicate the nonnegative square root of a number.

Example

Find $\sqrt{169}$.

$\sqrt{169}$ represents the square root of 169.

$169 = 13^2 \rightarrow \sqrt{169} = 13$

Exercises **Find each square root. If necessary, round to the nearest hundredth.**
See Example 1 on page 103.

60. $\sqrt{196}$ **61.** $\pm\sqrt{1.21}$ **62.** $-\sqrt{160}$ **63.** $\pm\sqrt{\frac{4}{225}}$

Name the set or sets of numbers to which each real number belongs.
See Example 2 on page 104.

64. $\frac{16}{25}$ **65.** $\frac{\sqrt{64}}{2}$ **66.** $-\sqrt{48.5}$

Replace each ● with <, >, or = to make each sentence true. *See Example 4 on page 105.*

67. $\frac{1}{8}$ ● $\frac{1}{\sqrt{49}}$ **68.** $\sqrt{\frac{2}{3}}$ ● $\frac{4}{9}$ **69.** $\sqrt{\frac{3}{4}}$ ● $\sqrt{\frac{1}{3}}$

70. WEATHER Meteorologists can use the formula $t = \sqrt{\frac{d^3}{216}}$ to estimate the amount of time t in hours a storm of diameter d will last. Suppose the eye of a hurricane, which causes the greatest amount of destruction, is 9 miles in diameter. To the nearest tenth of an hour, how long will the worst part of the hurricane last?
See Example 1 on pages 103 and 104.

Chapter 2

Practice Test

Vocabulary and Concepts

Choose the correct term to complete each sentence.

1. The *(absolute value, square)* of a number is its distance from zero on a number line.
2. A number that can be written as a fraction where the numerator and denominator are integers and the denominator does not equal zero is a *(repeating, rational)* number.
3. The list of all possible outcomes is called the *(simple event, sample space)*.

Skills and Applications

Evaluate each expression.

4. $-|x| - 38$ if $x = -2$
5. $34 - |x + 21|$ if $x = -7$
6. $-12 + |x - 8|$ if $x = 1.5$

Find each sum or difference.

7. $-19 + 12$
8. $-21 - (-34)$
9. $16.4 + (-23.7)$
10. $6.32 - (-7.41)$
11. $-\frac{7}{16} + \frac{3}{8}$
12. $-\frac{7}{12} - \left(-\frac{5}{9}\right)$

Find each quotient or product.

13. $-5(19)$
14. $-56 \div (-7)$
15. $96 \div (-0.8)$
16. $(-7.8)(5.6)$
17. $-\frac{1}{8} \div -5$
18. $-\frac{15}{32} \div \frac{3}{4}$

Simplify each expression.

19. $5(-3x) - 12x$
20. $7(6h - h)$
21. $-4m(-7n) + (3d)(-4c)$
22. $\frac{36k}{4}$
23. $\frac{9a + 27}{-3}$
24. $\frac{70x - 30y}{-5}$

Find each square root. If necessary, round to the nearest hundredth.

25. $-\sqrt{64}$
26. $\sqrt{3.61}$
27. $\pm\sqrt{\frac{16}{81}}$

Replace each ● with <, >, or = to make each sentence true.

28. $\frac{1}{\sqrt{3}}$ ● $\frac{1}{3}$
29. $\sqrt{\frac{1}{2}}$ ● $\frac{8}{11}$
30. $\sqrt{0.56}$ ● $\frac{\sqrt{3}}{2}$

STATISTICS **For Exercises 31 and 32, use the following information.**
The height, in inches, of the students in a health class are 65, 63, 68, 66, 72, 61, 62, 63, 59, 58, 61, 74, 65, 63, 71, 60, 62, 63, 71, 70, 59, 66, 61, 62, 68, 69, 64, 63, 70, 61, 68, and 67.

31. Make a line plot of the data.
32. Which measure of central tendency best describes the data? Explain.

33. **STANDARDIZED TEST PRACTICE** During a 20-song sequence on a radio station, 8 soft-rock, 7 hard-rock, and 5 rap songs are played at random. Assume that all of the songs are the same length. What is the probability that when you turn on the radio, a hard-rock song will be playing?

(A) $\frac{1}{4}$ (B) $\frac{7}{20}$ (C) $\frac{2}{5}$ (D) $\frac{13}{20}$ (E) $\frac{7}{10}$

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Chapter 2 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

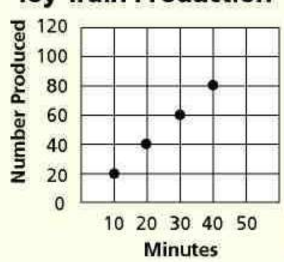
1. Darryl works 9 days at the State Fair and earns \$518.40. If he works 8 hours each day, what is his hourly pay? (Prerequisite Skill)

Ⓐ \$6.48 Ⓑ \$7.20
Ⓒ \$30.50 Ⓓ \$57.60

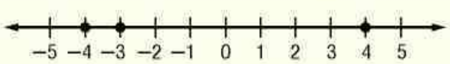
2. The graph below shows how many toy trains are assembled at a factory at the end of 10-minute intervals. What is the best prediction for the number of products assembled per hour? (Prerequisite Skill)

Ⓐ 80
Ⓑ 100
Ⓒ 120
Ⓓ 130

Toy Train Production



3. Which graph shows the integers greater than -2 and less than or equal to 3? (Lesson 2-1)

Ⓐ


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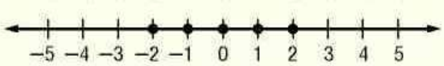

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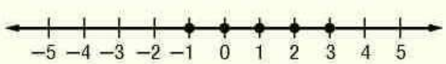
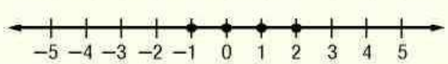


Ⓓ


The Princeton Review Test-Taking Tip

Question 1 If you don't know how to solve a problem, eliminate the answer choices you know are incorrect and then guess from the remaining choices. Even eliminating only one answer choice greatly increases your chance of guessing the correct answer.

4. Which number is the greatest? (Lesson 2-1)

Ⓐ $|-4|$ Ⓑ $|4|$
Ⓒ $|7|$ Ⓓ $|-9|$

5. What is $-3.8 + 4.7$? (Lesson 2-2)

Ⓐ 0.9 Ⓑ -0.9
Ⓒ 8.5 Ⓓ -8.5

6. Simplify $3(-2m) - 7m$. (Lesson 2-3)

Ⓐ $-12m$ Ⓑ $-m$
Ⓒ $-2m$ Ⓓ $-13m$

7. Which statement about the stem-and-leaf plot is *not* true? (Lesson 2-5)

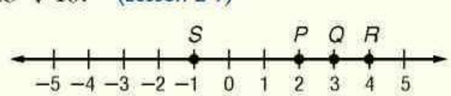
Stem	Leaf
3	1 1 5 6 8 8
4	2 2 2 4
5	0 0
6	0 3 7 8 9 9
7	4 $7\|4 = 74$

Ⓐ The greatest value is 74.
Ⓑ The mode is 42.
Ⓒ Seven of the values are greater than 50.
Ⓓ The least value is 38.

8. There are 4 boxes. If you choose a box at random, what are the odds that you will choose the one box with a prize? (Lesson 2-6)

Ⓐ 1:3 Ⓑ 1:4
Ⓒ 3:1 Ⓓ 3:4

9. Which point on the number line is closest to $\sqrt{10}$? (Lesson 2-7)



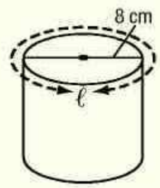
Ⓐ point P Ⓑ point Q
Ⓒ point R Ⓓ point S

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Ethan needs to wrap a label around a jar of homemade jelly so that there is no overlap. Find the length of the label. (Prerequisite Skill)



11. Evaluate $\frac{5-1}{4+12 \div 3 \times 2}$. (Lesson 1-2)

12. Find the solution of $4m - 3 = 9$ if the replacement set is {0, 2, 3, 5}. (Lesson 1-3)

13. Write an algebraic expression for *2p plus three times the difference of m and n*. (Lesson 1-6)

14. State the hypothesis in the statement *If $3x + 3 > 24$, then $x > 7$.* (Lesson 1-7)

Part 3 Quantitative Comparison

Compare the quantity in Column A to the quantity in Column B. Then determine whether:

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the quantities are equal, or**

Ⓓ **the relationship cannot be determined from the information given.**

15. $x > 0$

Column A	Column B
$\lvert x \rvert$	$\lvert -x \rvert$

(Lesson 2-1)

16. $x > y > 0$

Column A	Column B
$\frac{1}{x}$	$\frac{1}{y}$

(Lesson 2-5)

17. $\frac{1}{n} > 1$

Column A	Column B
1	n

(Lesson 2-6)

18. $a^2 = 49$

Column A	Column B
a	7

(Lesson 2-7)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

19. Mia has created the chart below to compare the three cellular phone plans she is considering. (Lessons 2-2 and 2-3)

Plan	Monthly Fee	Cost/Minute
A	\$5.95	\$0.30
B	\$12.95	\$0.10
C	\$19.99	\$0.08

a. Write an algebraic expression that Mia can use to figure the monthly cost of each plan. Use C for the total monthly cost, m for the cost per minute, x for the monthly fee, and y for the minutes used per month.

b. If Mia uses 150 minutes of calls each month, which plan will be least expensive? Explain.

20. The stem-and-leaf plot lists the annual profit for seven small businesses. (Lesson 2-5)

Stem	Leaf
3	2 9
4	1 1 3 5
5	0

a. Explain how the absence of a key could lead to misinterpreting the data.

b. How do the keys below affect how the data should be interpreted?

$3 | 2 = 3.2$ $\quad$ $3 | 2 = 0.32$

Chapter 3

Solving Linear Equations

What You'll Learn

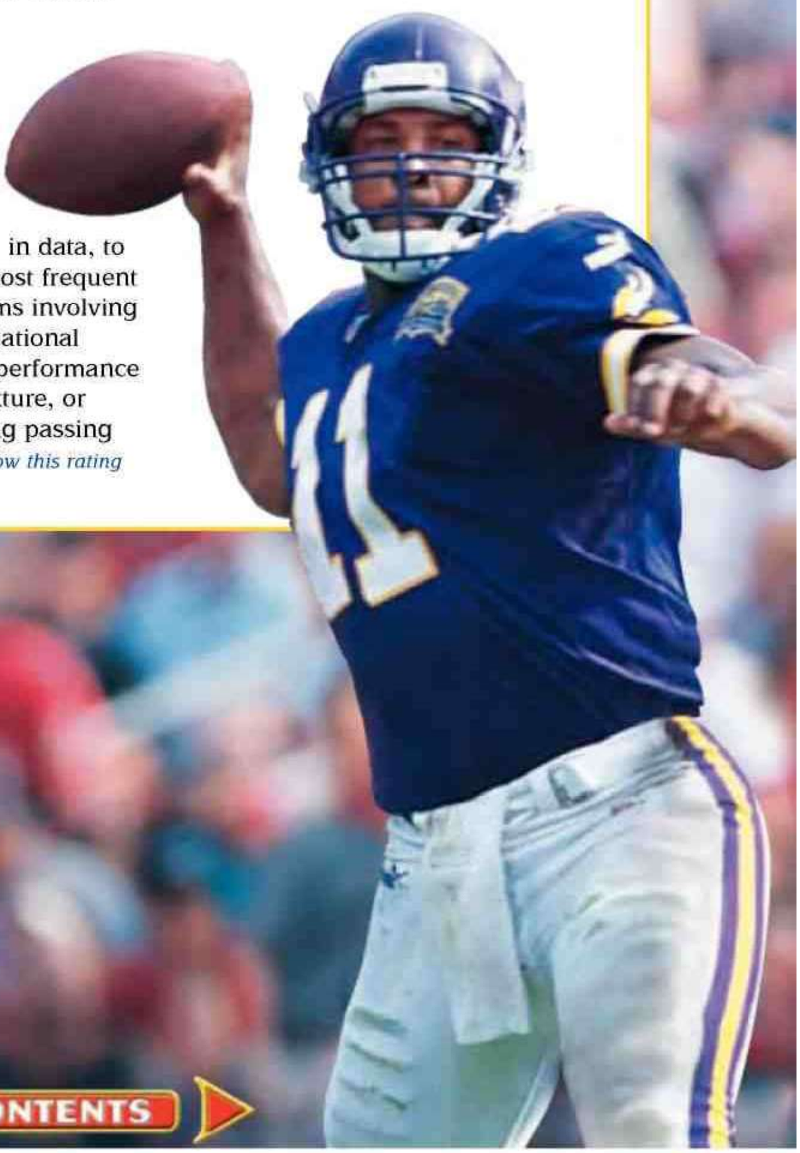
- **Lesson 3-1** Translate verbal sentences into equations and equations into verbal sentences.
- **Lessons 3-2 through 3-6** Solve equations and proportions.
- **Lesson 3-7** Find percents of change.
- **Lesson 3-8** Solve equations for given variables.
- **Lesson 3-9** Solve mixture and uniform motion problems.

Key Vocabulary

- equivalent equations (p. 129)
- identity (p. 150)
- proportion (p. 155)
- percent of change (p. 160)
- mixture problem (p. 171)

Why It's Important

Linear equations can be used to solve problems in every facet of life from planning a garden, to investigating trends in data, to making wise career choices. One of the most frequent uses of linear equations is solving problems involving mixtures or motion. For example, in the National Football League, a quarterback's passing performance is rated using an equation based on a mixture, or weighted average, of five factors, including passing attempts and completions. *You will learn how this rating system works in Lesson 3-9.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.

For Lesson 3-1 — Write Mathematical Expressions

Write an algebraic expression for each verbal expression. *(For review, see Lesson 1-1.)*

1. five greater than half of a number t

2. the product of seven and s divided by the product of eight and y

3. the sum of three times a and the square of b

4. w to the fifth power decreased by 37

5. nine times y subtracted from 95

6. the sum of r and six divided by twelve

For Lesson 3-4 — Use the Order of Operations

Evaluate each expression. *(For review, see Lesson 1-2.)*

7. $3 \cdot 6 - \frac{12}{4}$ **8.** $5(13 - 7) - 22$ **9.** $5(7 - 2) - 3^2$ **10.** $\frac{2 \cdot 6 - 4}{2}$

11. $(25 - 4) \div (2^2 - 1)$ **12.** $36 \div 4 - 2 + 3$ **13.** $\frac{19 - 5}{7} + 3$ **14.** $\frac{1}{4}(24) - \frac{1}{2}(12)$

For Lesson 3-7 — Find the Percent

Find each percent. *(For review, see pages 802 and 803.)*

15. Five is what percent of 20?

16. What percent of 300 is 21?

17. What percent of 5 is 15?

18. Twelve is what percent of 60?

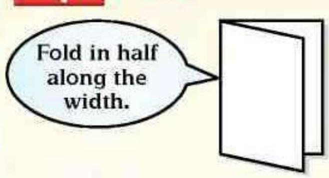
19. Sixteen is what percent of 10?

20. What percent of 50 is 37.5?

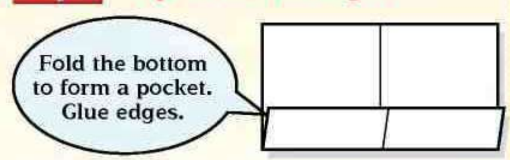
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about solving linear equations. Begin with 4 sheets of plain $8\frac{1}{2}$" by 11" paper.

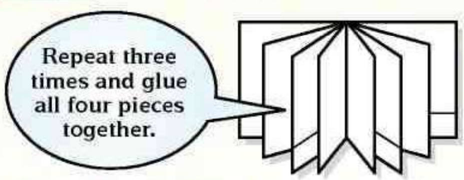
Step 1 Fold



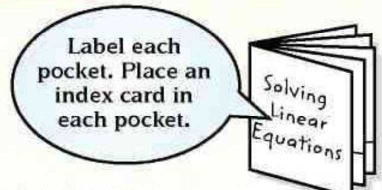
Step 2 Open and Fold Again



Step 3 Repeat Steps 1 and 2



Step 4 Label



Reading and Writing As you read and study the chapter, you can write notes and examples on each index card.

3-1 Writing Equations

What You'll Learn

- Translate verbal sentences into equations.
- Translate equations into verbal sentences.

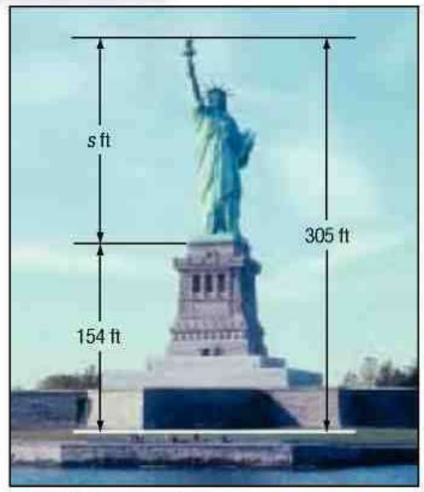
Vocabulary

- four-step problem-solving plan
- defining a variable
- formula

How are equations used to describe heights?

The Statue of Liberty sits on a pedestal that is 154 feet high. The height of the pedestal and the statue is 305 feet. If s represents the height of the statue, then the following equation represents the situation.

$$154 + s = 305$$



Source: *World Book Encyclopedia*

WRITE EQUATIONS When writing equations, use variables to represent the unspecified numbers or measures referred to in the sentence or problem. Then write the verbal expressions as algebraic expressions. Some verbal expressions that suggest the *equals sign* are listed below.

- is
- equals
- is equal to
- is the same as
- is as much as
- is identical to

Study Tip

Look Back

To review **translating verbal expressions to algebraic expressions**, see Lesson 1-1.

Example 1 Translate Sentences into Equations

Translate each sentence into an equation.

a. Five times the number a is equal to three times the sum of b and c.

Five	times	a	is equal to	three	times	the sum of b and c.
5	$\times$	a	$=$	3	$\times$	$(b + c)$

The equation is $5a = 3(b + c)$.

b. Nine times y subtracted from 95 equals 37.

Rewrite the sentence so it is easier to translate.

95	less	nine times y	equals	37.
95	$-$	$9y$	$=$	37

The equation is $95 - 9y = 37$.

Using the **four-step problem-solving plan** can help you solve any word problem.

Key Concept *Four-Step Problem-Solving Plan*

Step 1 Explore the problem.

Step 2 Plan the solution.

Step 3 Solve the problem.

Step 4 Examine the solution.

Study Tip

Reading Math
In a verbal problem, the sentence that tells what you are asked to find usually contains *find*, *what*, *when*, or *how*.

Each step of the plan is important.

Step 1 **Explore the Problem**
To solve a verbal problem, first read the problem carefully and explore what the problem is about.

- Identify what information is given.
- Identify what you are asked to find.

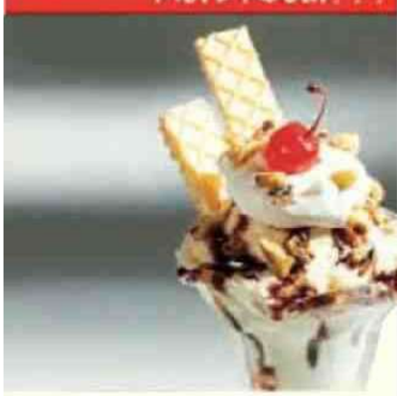
Step 2 **Plan the Solution**
One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecific numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers in the problem. *You will learn to use other strategies throughout this book.*

Step 3 **Solve the Problem**
Use the strategy you chose in Step 2 to solve the problem.

Step 4 **Examine the Solution**
Check your answer in the context of the original problem.

- Does your answer make sense?
- Does it fit the information in the problem?

More About...



Ice Cream

The first ice cream plant was established in 1851 by Jacob Fussell. Today, 2,000,000 gallons of ice cream are produced in the United States each day.

Source: *World Book Encyclopedia*

Example 2 Use the Four-Step Plan

ICE CREAM **Use the information at the left. In how many days can 40,000,000 gallons of ice cream be produced in the United States?**

Explore You know that 2,000,000 gallons of ice cream are produced in the United States each day. You want to know how many days it will take to produce 40,000,000 gallons of ice cream.

Plan Write an equation to represent the situation. Let d represent the number of days needed to produce the ice cream.

$$\overbrace{2{,}000{,}000}^{\text{2,000,000}} \quad \overbrace{\times}^{\text{times}} \quad \overbrace{d}^{\text{the number of days}} \quad \overbrace{=}^{\text{equals}} \quad \overbrace{40{,}000{,}000}^{\text{40,000,000}}$$

Solve $2{,}000{,}000d = 40{,}000{,}000$ Find d mentally by asking, "What number times 2,000,000 equals 40,000,000?"

$d = 20$

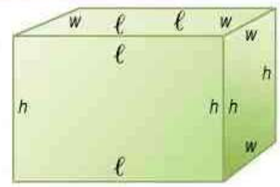
It will take 20 days to produce 40,000,000 gallons of ice cream.

Examine If 2,000,000 gallons of ice cream are produced in one day, $2{,}000{,}000 \times 20$ or 40,000,000 gallons are produced in 20 days. The answer makes sense.

A **formula** is an equation that states a rule for the relationship between certain quantities. Sometimes you can develop a formula by making a model.

Algebra Activity

Surface Area



- Mark each side of a rectangular box as the length ℓ, the width w, or the height h.
- Use scissors to cut the box so that each surface or face of the box is a separate piece.

Analyze

1. Write an expression for the area of the front of the box.
2. Write an expression for the area of the back of the box.
3. Write an expression for the area of one side of the box.
4. Write an expression for the area of the other side of the box.
5. Write an expression for the area of the top of the box.
6. Write an expression for the area of the bottom of the box.
7. The surface area of a rectangular box is the sum of all the areas of the faces of the box. If S represents surface area, write a formula for the surface area of a rectangular box.

Make a Conjecture

8. If s represents the length of the side of a cube, write a formula for the surface area of a cube.

Example 3 Write a Formula

Translate the sentence into a formula.

The perimeter of a rectangle equals two times the length plus two times the width.

Words Perimeter equals two times the length plus two times the width.

Variables Let P = perimeter, ℓ = length, and w = width.

Formula $\underbrace{P}_{\text{Perimeter}} \underbrace{=}_{\text{equals}} \underbrace{2\ell}_{\text{two times the length}} \underbrace{+}_{\text{plus}} \underbrace{2w}_{\text{two times the width.}}$

The formula for the perimeter of a rectangle is $P = 2\ell + 2w$.

WRITE VERBAL SENTENCES You can also translate equations into verbal sentences or make up your own verbal problem if you are given an equation.

Study Tip

Look Back
To review **translating algebraic expressions to verbal expressions**, see Lesson 1-1.

Example 4 Translate Equations into Sentences

Translate each equation into a verbal sentence.

a. $3m + 5 = 14$

$\underbrace{3m}_{\text{Three times } m} \underbrace{+}_{\text{plus}} \underbrace{5}_{\text{five}} \underbrace{=}_{\text{equals}} \underbrace{14}_{\text{fourteen.}}$

b. $w + v = y^2$

$\underbrace{\text{The sum of } w \text{ and } v}_{w+v}$ $\underbrace{\text{equals}}_{=}$ $\underbrace{\text{the square of } y.}_{y^2}$

Example 5 Write a Problem

Write a problem based on the given information.

a = Rafael's age **$a + 5$ = Tierra's age** **$a + 2(a + 5) = 46$**

You know that a represents Rafael's age and $a + 5$ represents Tierra's age. The equation adds a plus twice $(a + 5)$ to get 46. A sample problem is given below.

Tierra is 5 years older than Rafael. The sum of Rafael's age and twice Tierra's age equals 46. How old is Rafael?

Check for Understanding

Concept Check

1. **List** the four steps used in solving problems.
2. **Analyze** the following problem.

   Misae has \$1900 in the bank. She wishes to increase her account to a total of \$3500 by depositing \$30 per week from her paycheck. Will she reach her savings goal in one year?

   a. How much money did Misae have in her account at the beginning?

   b. How much money will Misae add to her account in 10 weeks? in 20 weeks?

   c. Write an expression representing the amount added to the account after w weeks have passed.

   d. What is the answer to the question? Explain.
3. **OPEN ENDED** Write a problem that can be answered by solving $x + 16 = 30$.

Guided Practice

Translate each sentence into an equation.

4. Two times a number t decreased by eight equals seventy.
5. Five times the sum of m and n is the same as seven times n.

Translate each sentence into a formula.

6. The area A of a triangle equals one half times the base b times the height h.
7. The circumference C of a circle equals the product of two, pi, and the radius r.

Translate each equation into a verbal sentence.

8. $14 + d = 6d$
9. $\frac{1}{3}b - \frac{3}{4} = 2a$

10. Write a problem based on the given information.

    c = cost of a suit $c - 25 = 150$

Application

WRESTLING **For Exercises 11 and 12, use the following information.**
Darius is training to prepare for wrestling season. He weighs 155 pounds now. He wants to gain weight so that he starts the season weighing 160 pounds.

11. If g represents the number of pounds he wants to gain, write an equation to represent the situation.
12. How many pounds does Darius need to gain to reach his goal?

Practice and Apply

Homework Help

For Exercises	See Examples
13–22	1
23–28	3
29–38	4
39, 40	5
41–51	2

Extra Practice

See page 825.

Translate each sentence into an equation.

13. Two hundred minus three times x is equal to nine.

14. The sum of twice r and three times s is identical to thirteen.

15. The sum of one-third q and 25 is as much as twice q.

16. The square of m minus the cube of n is sixteen.

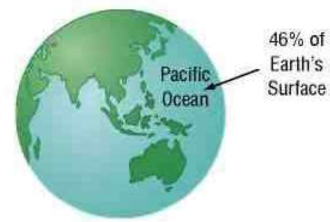
17. Two times the sum of v and w is equal to two times z.

18. Half of the sum of nine and p is the same as p minus three.

19. The number g divided by the number h is the same as seven more than twice the sum of g and h.

20. Five-ninths the square of the sum of a, b, and c equals the sum of the square of a and the square of c.

21. GEOGRAPHY The Pacific Ocean covers about 46% of Earth. If P represents the area of the Pacific Ocean and E represents the area of Earth, write an equation for this situation.

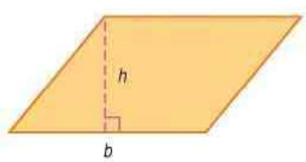


Source: *World Book Encyclopedia*

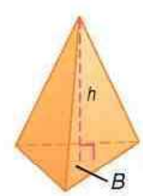
22. GARDENING Mrs. Patton is planning to place a fence around her vegetable garden. The fencing costs $1.75 per yard. She buys f yards of fencing and pays $3.50 in tax. If the total cost of the fencing is $73.50, write an equation to represent the situation.

Translate each sentence into a formula.

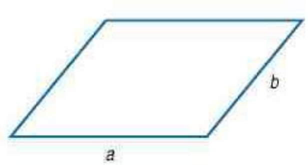
23. The area A of a parallelogram is the base b times the height h.



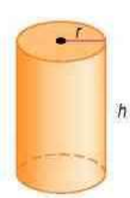
24. The volume V of a pyramid is one-third times the product of the area of the base B and its height h.



25. The perimeter P of a parallelogram is twice the sum of the lengths of the two adjacent sides, a and b.



26. The volume V of a cylinder equals the product of π, the square of the radius r of the base, and the height.



27. In a right triangle, the square of the measure of the hypotenuse c is equal to the sum of the squares of the measures of the legs, a and b.

28. The temperature in degrees Fahrenheit F is the same as nine-fifths of the degrees Celsius C plus thirty-two.

Translate each equation into a verbal sentence.

29. $d - 14 = 5$

30. $2f + 6 = 19$

31. $k^2 + 17 = 53 - j$

32. $2a = 7a - b$

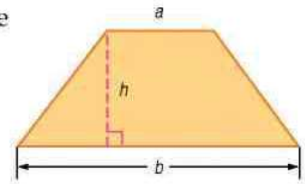
33. $\frac{3}{4}p + \frac{1}{2} = p$

34. $\frac{2}{5}w = \frac{1}{2}w + 3$

35. $7(m + n) = 10n + 17$

36. $4(t - s) = 5s + 12$

37. **GEOMETRY** If a and b represent the lengths of the bases of a trapezoid and h represents its height, then the formula for the area A of the trapezoid is $A = \frac{1}{2}h(a + b)$. Write the formula in words.



38. **SCIENCE** If r represents rate, t represents time, and d represents distance, then $rt = d$. Write the formula in words.

Write a problem based on the given information.

39. y = Yolanda's height in inches
$y + 7$ = Lindsey's height in inches
$2y + (y + 7) = 193$

40. p = price of a new backpack
$0.055p$ = tax
$p + 0.055p = 31.65$

GEOMETRY **For Exercises 41 and 42, use the following information.**
The volume V of a cone equals one-third times the product of π, the square of the radius r of the base, and the height h.

41. Write the formula for the volume of a cone.

42. Find the volume of a cone if r is 10 centimeters and h is 30 centimeters.

GEOMETRY **For Exercises 43 and 44, use the following information.**
The volume V of a sphere is four-thirds times π times the radius r of the sphere cubed.

43. Write a formula for the volume of a sphere.

44. Find the volume of a sphere if r is 4 inches.

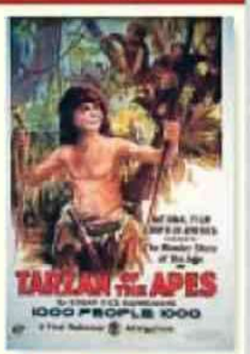
LITERATURE **For Exercises 45–47, use the following information.**
Edgar Rice Burroughs is the author of the *Tarzan of the Apes* stories. He published his first Tarzan story in 1912. Some years later, the town in southern California where he lived was named Tarzana.

45. Let y represent the number of years after 1912 that the town was named Tarzana. Write an expression for the year the town was named.

46. The town was named in 1928. Write an equation to represent the situation.

47. Use what you know about numbers to determine the number of years between the first Tarzan story and the naming of the town.

More About. . .



Literature

More than 50 movies featuring Tarzan have been made. The first, *Tarzan of the Apes*, in 1918, was among the first movies to gross over $1 million.

Source: www.tarzan.org

TELEVISION **For Exercises 48–51, use the following information.**
During a highly rated one-hour television program, the entertainment portion lasted 15 minutes longer than 4 times the advertising portion.

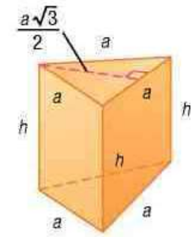
48. If a represents the time spent on advertising, write an expression for the entertainment portion.

49. Write an equation to represent the situation.

50. Use your equation and the guess-and-check strategy to determine the number of minutes spent on advertising. Choose different values of a and evaluate to find the solution.

51. Time the entertainment and advertising portions of a one-hour television program you like to watch. Describe what you found. Are the results of this problem similar to your findings?

52. **CRITICAL THINKING** The surface area of a prism is the sum of the areas of the faces of the prism. Write a formula for the surface area of the triangular prism at the right.



53. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are equations used to describe heights?

Include the following in your answer:

- an equation relating the Sears Tower, which is 1454 feet tall; the twin antenna towers on top of the building, which are a feet tall; and a total height, which is 1707 feet, and
- an equation representing the height of a building of your choice.

Standardized Test Practice

54. Which equation represents the following sentence?
One fourth of a number plus five equals the number minus seven.

(A) $\frac{1}{4}n + 7 = n - 5$ (B) $\frac{1}{4}n + 5 = n - 7$

(C) $4n + 7 = n - 5$ (D) $4n + 5 = n - 7$

55. Which sentence can be represented by $7(x + y) = 35$?

(A) Seven times x plus y equals 35.

(B) One seventh of the sum of x and y equals 35.

(C) Seven plus x and y equals 35.

(D) Seven times the sum of x and y equals 35.

Maintain Your Skills

Mixed Review

Find each square root. Use a calculator if necessary. Round to the nearest hundredth if the result is not a whole number or a simple fraction. *(Lesson 2-7)*

56. $\sqrt{8100}$ 57. $-\sqrt{\frac{25}{36}}$ 58. $\sqrt{90}$ 59. $-\sqrt{55}$

Find the probability of each outcome if a die is rolled. *(Lesson 2-6)*

60. a 6 61. an even number 62. a number greater than 2

Simplify each expression. *(Lesson 1-5)*

63. $12d + 3 - 4d$ 64. $7t^2 + t + 8t$ 65. $3(a + 2b) + 5a$

Evaluate each expression. *(Lesson 1-2)*

66. $5(8 - 3) + 7 \cdot 2$ 67. $6(4^3 + 2^2)$ 68. $7(0.2 + 0.5) - 0.6$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each sum or difference.**
(To review ***operations with fractions****, see pages 798 and 799.)*

69. $5.67 + 3.7$ 70. $0.57 + 2.8$ 71. $5.28 - 3.4$ 72. $9 - 7.35$

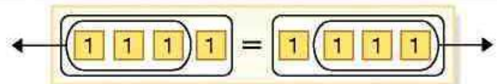
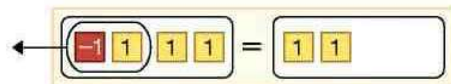
73. $\frac{2}{3} + \frac{1}{5}$ 74. $\frac{1}{6} + \frac{2}{3}$ 75. $\frac{7}{9} - \frac{2}{3}$ 76. $\frac{3}{4} - \frac{1}{6}$

Algebra Activity

A Preview of Lesson 3-2

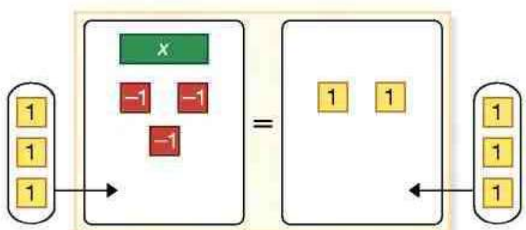
Solving Addition and Subtraction Equations

You can use algebra tiles to solve equations. To solve an equation means to find the value of the variable that makes the equation true. After you model the equation, the goal is to get the x tile by itself on one side of the mat using the rules stated below.

Rules for Equation Models	
You can remove or add the same number of identical algebra tiles to each side of the mat without changing the equation.	
One positive tile and one negative tile of the same unit are a **zero pair**. Since $1 + (-1) = 0$, you can remove or add zero pairs to the equation mat without changing the equation.	

Use an equation model to solve $x - 3 = 2$.

Step 1 *Model the equation.*

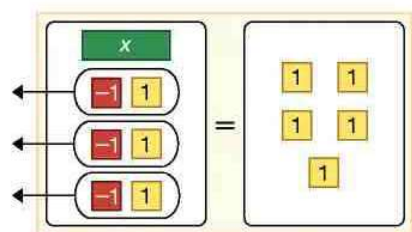


$$x - 3 = 2$$

$$x - 3 + 3 = 2 + 3$$

Place 1 x tile and 3 negative 1 tiles on one side of the mat. Place 2 positive 1 tiles on the other side of the mat. Then add 3 positive 1 tiles to each side.

Step 2 *Isolate the x term.*



$$x = 5$$

Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is $x = 5$.

Model and Analyze

Use algebra tiles to solve each equation.

1. $x + 5 = 7$

2. $x + (-2) = 28$

3. $x + 4 = 27$

4. $x + (-3) = 4$

5. $x + 3 = -4$

6. $x + 7 = 2$

Make a Conjecture

7. If $a = b$, what can you say about $a + c$ and $b + c$?

8. If $a = b$, what can you say about $a - c$ and $b - c$?

3-2 Solving Equations by Using Addition and Subtraction

What You'll Learn

- Solve equations by using addition.
- Solve equations by using subtraction.

Vocabulary

- equivalent equation
- solve an equation

How can equations be used to compare data?

The graph shows some of the fastest-growing occupations from 1992 to 2005.

Source: Bureau of Labor Statistics

The difference between the percent of growth for medical assistants and the percent of growth for travel agents in these years is 5%. An equation can be used to find the percent of growth expected for medical assistants. If m is the percent of growth for medical assistants, then $m - 66 = 5$. You can use a property of equality to find the value of m.

SOLVE USING ADDITION Suppose your school's boys' soccer team has 15 members and the girls' soccer team has 15 members. If each team adds 3 new players, the number of members on the boys' and girls' teams would still be equal.

$15 = 15$	Each team has 15 members before adding the new players.
$15 + 3 = 15 + 3$	Each team adds 3 new members.
$18 = 18$	Each team has 18 members after adding the new members.

This example illustrates the **Addition Property of Equality**.

Key Concept — Addition Property of Equality

- **Words** If the same number is added to each side of an equation, the resulting equation is true.
- **Symbols** For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.
- **Examples**

$$7 = 7 \qquad\qquad 14 = 14$$
$$7 + 3 = 7 + 3 \qquad\qquad 14 + (-6) = 14 + (-6)$$
$$10 = 10 \qquad\qquad 8 = 8$$

If the same number is added to each side of an equation, then the result is an equivalent equation. **Equivalent equations** have the same solution.

$t + 3 = 5$	The solution of this equation is 2.
$t + 3 + 2 = 5 + 2$	Using the Addition Property of Equality, add 2 to each side.
$t + 5 = 7$	The solution of this equation is also 2.

Study Tip

Reading Math
Remember that x means $1 \cdot x$. The coefficient of x is 1.

To **solve an equation** means to find all values of the variable that make the equation a true statement. One way to do this is to isolate the variable having a coefficient of 1 on one side of the equation. You can sometimes do this by using the Addition Property of Equality.

Example 1 Solve by Adding a Positive Number

Solve $m - 48 = 29$. Then check your solution.

$m - 48 = 29$	Original equation
$m - 48 + 48 = 29 + 48$	Add 48 to each side.
$m = 77$	$-48 + 48 = 0$ and $29 + 48 = 77$

To check that 77 is the solution, substitute 77 for m in the original equation.

CHECK	$m - 48 = 29$	Original equation
	$77 - 48 \stackrel{?}{=} 29$	Substitute 77 for m.
	$29 = 29$ ✓	Subtract.

The solution is 77.

Example 2 Solve by Adding a Negative Number

Solve $21 + q = -18$. Then check your solution.

$21 + q = -18$	Original equation
$21 + q + (-21) = -18 + (-21)$	Add -21 to each side.
$q = -39$	$21 + (-21) = 0$ and $-18 + (-21) = -39$

CHECK	$21 + q = -18$	Original equation
	$21 + (-39) \stackrel{?}{=} -18$	Substitute -39 for q.
	$-18 = -18$ ✓	Add.

The solution is -39.

SOLVE USING SUBTRACTION Similar to the Addition Property of Equality, there is a **Subtraction Property of Equality** that may be used to solve equations.

Key Concept — Subtraction Property of Equality

- **Words** If the same number is subtracted from each side of an equation, the resulting equation is true.
- **Symbols** For any numbers a, b, and c, if $a = b$, then $a - c = b - c$.
- **Examples**

$$17 = 17 \qquad\qquad 3 = 3$$
$$17 - 9 = 17 - 9 \qquad\qquad 3 - 8 = 3 - 8$$
$$8 = 8 \qquad\qquad -5 = -5$$

Example 3 Solve by Subtracting

Solve $142 + d = 97$. Then check your solution.

$$142 + d = 97 \quad \text{Original equation}$$

$$142 + d - 142 = 97 - 142 \quad \text{Subtract 142 from each side.}$$

$$d = -45 \quad 142 - 142 = 0 \text{ and } 97 - 142 = -45$$

CHECK

$$142 + d = 97 \quad \text{Original equation}$$

$$142 + (-45) \stackrel{?}{=} 97 \quad \text{Substitute } -45 \text{ for } d.$$

$$97 = 97 \checkmark \quad \text{Add.}$$

The solution is -45.

Remember that subtracting a number is the same as adding its inverse.

Example 4 Solve by Adding or Subtracting

Solve $g + \frac{3}{4} = -\frac{1}{8}$ in two ways.

Method 1 Use the Subtraction Property of Equality.

$$g + \frac{3}{4} = -\frac{1}{8} \quad \text{Original equation}$$

$$g + \frac{3}{4} - \frac{3}{4} = -\frac{1}{8} - \frac{3}{4} \quad \text{Subtract } \frac{3}{4} \text{ from each side.}$$

$$g = -\frac{7}{8} \quad \frac{3}{4} - \frac{3}{4} = 0 \text{ and } -\frac{1}{8} - \frac{3}{4} = -\frac{1}{8} - \frac{6}{8} \text{ or } -\frac{7}{8}$$

The solution is $-\frac{7}{8}$.

Method 2 Use the Addition Property of Equality.

$$g + \frac{3}{4} = -\frac{1}{8} \quad \text{Original equation}$$

$$g + \frac{3}{4} + \left(-\frac{3}{4}\right) = -\frac{1}{8} + \left(-\frac{3}{4}\right) \quad \text{Add } -\frac{3}{4} \text{ to each side.}$$

$$g = -\frac{7}{8} \quad \frac{3}{4} + \left(-\frac{3}{4}\right) = 0 \text{ and } -\frac{1}{8} + \left(-\frac{3}{4}\right) = -\frac{1}{8} + \left(-\frac{6}{8}\right) \text{ or } -\frac{7}{8}$$

The solution is $-\frac{7}{8}$.

Example 5 Write and Solve an Equation

Write an equation for the problem. Then solve the equation and check your solution.

A number increased by 5 is equal to 42. Find the number.

$$\underbrace{n}_{\text{A number}} \quad \underbrace{+}_{\text{increased by}} \quad \underbrace{5}_{5} \quad \underbrace{=}_{\text{is equal to}} \quad \underbrace{42}_{42.}$$

$$n + 5 = 42 \quad \text{Original equation}$$

$$n + 5 - 5 = 42 - 5 \quad \text{Subtract 5 from each side.}$$

$$n = 37 \quad 5 - 5 = 0 \text{ and } 42 - 5 = 37$$

CHECK

$$n + 5 = 42 \quad \text{Original equation}$$

$$37 + 5 \stackrel{?}{=} 42 \quad \text{Substitute 37 for } n.$$

$$42 = 42 \checkmark$$

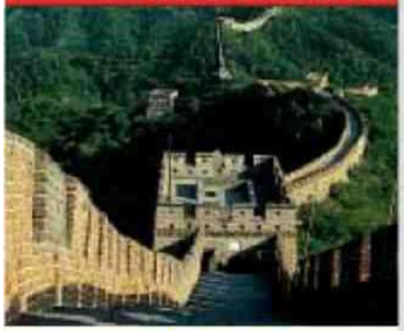
The solution is 37.

Study Tip

Checking Solutions

You should always check your solution in the context of the original problem. For instance, in Example 5, is 37 increased by 5 equal to 42? The solution checks.

More About. . .



History

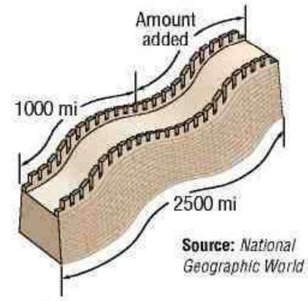
The first emperor of China, Qui Shi Huangdi, ordered the building of the Great Wall of China to protect his people from nomadic tribes that attacked and looted villages. By 204 B.C., this wall guarded 1000 miles of China's border.

Source: *National Geographic World*

Example 6 Write an Equation to Solve a Problem

HISTORY Refer to the information at the right.

In the fourteenth century, the part of the Great Wall of China that was built during Qui Shi Huangdi's time was repaired, and the wall was extended. When the wall was completed, it was 2500 miles long. How much of the wall was added during the 1300s?



Source: *National Geographic World*

Words The original length plus the additional length equals 2500.

Variable Let a = the additional length.

Equation $\underbrace{1000}_{\text{The original length}} \underbrace{+}_{\text{plus}} \underbrace{a}_{\text{the additional length}} \underbrace{=}_{\text{equals}} \underbrace{2500}_{2500.}$

$1000 + a = 2500$ Original equation

$1000 + a - 1000 = 2500 - 1000$ Subtract 1000 from each side.

$a = 1500$ $1000 - 1000 = 0$ and $2500 - 1000 = 1500$.

The Great Wall of China was extended 1500 miles in the 1300s.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write three equations that are equivalent to $n + 14 = 27$.
2. **Compare and contrast** the Addition Property of Equality and the Subtraction Property of Equality.
3. **Show** two ways to solve $g + 94 = 75$.

Guided Practice

Solve each equation. Then check your solution.

4. $t - 4 = -7$
5. $p + 19 = 6$
6. $15 + r = 71$
7. $104 = y - 67$
8. $h - 0.78 = 2.65$
9. $\frac{2}{3} + w = 1\frac{1}{2}$

Write an equation for each problem. Then solve the equation and check your solution.

10. Twenty-one subtracted from a number is -8. Find the number.
11. A number increased by -37 is -91. Find the number.

Application

CARS For Exercises 12–14, use the following information.

The average time it takes to manufacture a car in the United States is equal to the average time it takes to manufacture a car in Japan plus 8.1 hours. The average time it takes to manufacture a car in the United States is 24.9 hours.

12. Write an addition equation to represent the situation.
13. What is the average time to manufacture a car in Japan?
14. The average time it takes to manufacture a car in Europe is 35.5 hours. What is the difference between the average time it takes to manufacture a car in Europe and the average time it takes to manufacture a car in Japan?

Practice and Apply

Homework Help

For Exercises	See Examples
15–40	1–4
41–48	5
51–64	6

Extra Practice
See page 825.

Solve each equation. Then check your solution.

15. $v - 9 = 14$ **16.** $s - 19 = -34$ **17.** $g + 5 = 33$

18. $18 + z = 44$ **19.** $a - 55 = -17$ **20.** $t - 72 = -44$

21. $-18 = -61 + d$ **22.** $-25 = -150 + q$ **23.** $r - (-19) = -77$

24. $b - (-65) = 15$ **25.** $18 - (-f) = 91$ **26.** $125 - (-u) = 88$

27. $-2.56 + c = 0.89$ **28.** $k + 0.6 = -3.84$ **29.** $-6 = m + (-3.42)$

30. $6.2 = -4.83 + y$ **31.** $t - 8.5 = 7.15$ **32.** $q - 2.78 = 4.2$

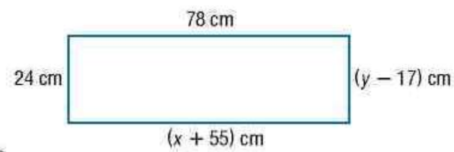
33. $x - \frac{3}{4} = \frac{5}{6}$ **34.** $a - \frac{3}{5} = -\frac{7}{10}$ **35.** $-\frac{1}{2} + p = \frac{5}{8}$

36. $\frac{2}{3} + r = -\frac{4}{9}$ **37.** $\frac{2}{3} = v + \frac{4}{5}$ **38.** $\frac{2}{5} = w + \frac{3}{4}$

39. If $x - 7 = 14$, what is the value of $x - 2$?

40. If $t + 8 = -12$, what is the value of $t + 1$?

GEOMETRY **For Exercises 41 and 42, use the rectangle at the right.**



41. Write an equation you could use to solve for x and then solve for x.

42. Write an equation you could use to solve for y and then solve for y.

Write an equation for each problem. Then solve the equation and check your solution.

43. Eighteen subtracted from a number equals 31. Find the number.

44. What number decreased by 77 equals -18?

45. A number increased by -16 is -21. Find the number.

46. The sum of a number and -43 is 102. What is the number?

47. What number minus one-half is equal to negative three-fourths?

48. The sum of 19 and 42 and a number is equal to 87. What is the number?

49. Determine whether $x + x = x$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

50. Determine whether $x + 0 = x$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

GAS MILEAGE **For Exercises 51–55, use the following information.**
A midsize car with a 4-cylinder engine goes 10 miles more on a gallon of gasoline than a luxury car with an 8-cylinder engine. A midsize car consumes one gallon of gas for every 34 miles driven.

51. Write an addition equation to represent the situation.

52. How many miles does a luxury car travel on a gallon of gasoline?

53. A subcompact car with a 3-cylinder engine goes 13 miles more than a luxury car on one gallon of gas. How far does a subcompact car travel on a gallon of gasoline?

54. How many more miles does a subcompact travel on a gallon of gasoline than a midsize car?

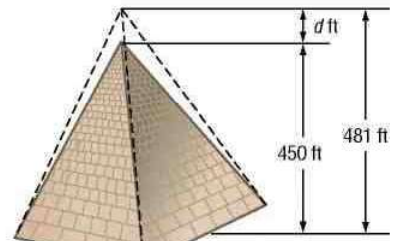
55. Estimate how many miles a full-size car with a 6-cylinder engine goes on one gallon of gasoline. Explain your reasoning.

CONTENTS

HISTORY **For Exercises 56 and 57, use the following information.**
Over the years, the height of the Great Pyramid at Giza, Egypt, has decreased.

56. Write an addition equation to represent the situation.

57. What was the decrease in the height of the pyramid?



Source: *World Book Encyclopedia*

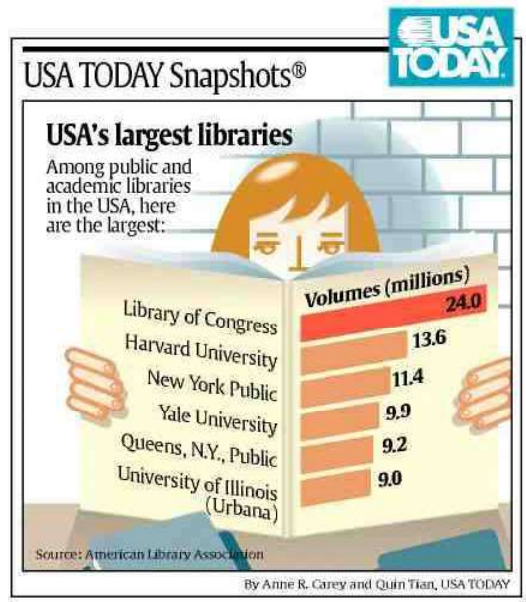
LIBRARIES **For Exercises 58–61, use the graph at the right to write an equation for each situation. Then solve the equation.**

58. How many more volumes does the Library of Congress have than the Harvard University Library?

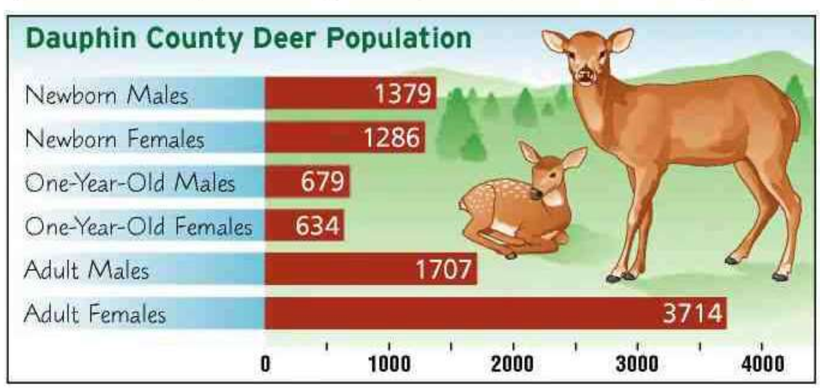
59. How many more volumes does the Harvard University Library have than the New York Public Library?

60. How many more volumes does the Library of Congress have than the New York Public Library?

61. What is the total number of volumes in the three largest U.S. libraries?



ANIMALS **For Exercises 62–64, use the information below to write an equation for each situation. Then solve the equation.**
Wildlife authorities monitor the population of animals in various regions. One years' deer population in Dauphin County, Pennsylvania, is shown in the graph below.



Source: www.visi.com

62. How many more newborns are there than one-year-olds?

63. How many more females are there than males?

64. What is the total deer population?

65. **CRITICAL THINKING** If $a - b = x$, what values of a, b, and x would make the equation $a + x = b + x$ true?

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to compare data?

Include the following in your answer:

- an explanation of how to solve the equation to find the growth rate for medical assistants, and
- a sample problem and related equation using the information in the graph.

Standardized Test Practice

67. Which equation is *not* equivalent to $b - 15 = 32$?

Ⓐ $b + 5 = 52$ Ⓑ $b - 20 = 27$

Ⓒ $b - 13 = 30$ Ⓓ $b = 47$

68. What is the solution of $x - 167 = -52$?

Ⓐ 115 Ⓑ -115

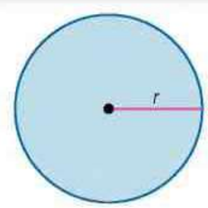
Ⓒ 219 Ⓓ -219

Maintain Your Skills

Mixed Review

GEOMETRY For Exercises 69 and 70, use the following information.

The area of a circle is the product of π times the radius r squared. *(Lesson 3-1)*



69. Write the formula for the area of the circle.

70. If a circle has a radius of 16 inches, find its area.

Replace each ● with >, <, or = to make the sentence true. *(Lesson 2-7)*

71. $\frac{1}{2}$ ● $\sqrt{2}$ 72. $\frac{3}{4}$ ● $\frac{2}{3}$ 73. 0.375 ● $\frac{3}{8}$

Use each set of data to make a stem-and-leaf plot. *(Lesson 2-5)*

74. 54, 52, 43, 41, 40, 36, 35, 31, 32, 34, 42, 56

75. 2.3, 1.4, 1.7, 1.2, 2.6, 0.8, 0.5, 2.8, 4.1, 2.9, 4.5, 1.1

Identify the hypothesis and conclusion of each statement. *(Lesson 1-7)*

76. For $y = 2$, $4y - 6 = 2$.

77. There is a science quiz every Friday.

Evaluate each expression. Name the property used in each step. *(Lesson 1-4)*

78. $4(16 \div 4^2)$ 79. $(2^5 - 5^2) + (4^2 - 2^4)$

Find the solution set for each inequality, given the replacement set. *(Lesson 1-3)*

80. $3x + 2 > 2$; $\{0, 1, 2\}$ 81. $2y^2 - 1 > 0$; $\{1, 3, 5\}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each product or quotient.

*(To review **operations with fractions**, see pages 800 and 801.)*

82. 6.5×2.8 83. 70.3×0.15 84. $17.8 \div 2.5$ 85. $0.33 \div 1.5$

86. $\frac{2}{3} \times \frac{5}{8}$ 87. $\frac{5}{9} \times \frac{3}{10}$ 88. $\frac{1}{2} \div \frac{2}{5}$ 89. $\frac{8}{9} \div \frac{4}{15}$

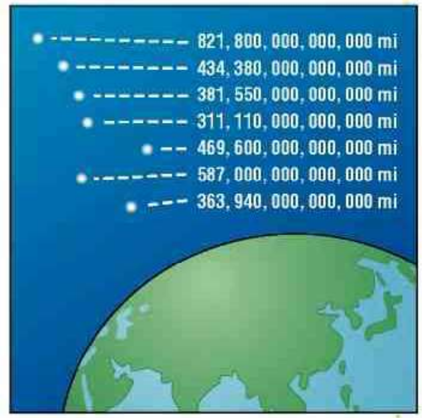
3-3 Solving Equations by Using Multiplication and Division

What You'll Learn

- Solve equations by using multiplication.
- Solve equations by using division.

How can equations be used to find how long it takes light to reach Earth?

It may look like all seven stars in the Big Dipper are the same distance from Earth, but in fact, they are not. The diagram shows the distance between each star and Earth.



Source: *National Geographic World*

Light travels at a rate of about 5,870,000,000,000 miles per year. In general, the rate at which something travels times the time equals the distance ($rt = d$). The following equation can be used to find the time it takes light to reach Earth from the closest star in the Big Dipper.

$$rt = d$$
$$5{,}870{,}000{,}000{,}000t = 311{,}110{,}000{,}000{,}000$$

SOLVE USING MULTIPLICATION To solve equations such as the one above, you can use the **Multiplication Property of Equality**.

Key Concept — Multiplication Property of Equality

- **Words** If each side of an equation is multiplied by the same number, the resulting equation is true.
- **Symbols** For any numbers a, b, and c, if $a = b$, then $ac = bc$.
- **Examples**

$6 = 6$	$9 = 9$	$10 = 10$
$6 \times 2 = 6 \times 2$	$9 \times (-3) = 9 \times (-3)$	$10 \times \frac{1}{2} = 10 \times \frac{1}{2}$
$12 = 12$	$-27 = -27$	$5 = 5$

Example 1 Solve Using Multiplication by a Positive Number

Solve $\frac{t}{30} = \frac{7}{10}$. Then check your solution.

$\frac{t}{30} = \frac{7}{10}$ Original equation

$30\left(\frac{t}{30}\right) = 30\left(\frac{7}{10}\right)$ Multiply each side by 30.

$t = 21$ $\frac{t}{30}(30) = t$ and $\frac{7}{10}(30) = 21$

(continued on the next page)

CHECK $\frac{t}{30} = \frac{7}{10}$ Original equation

$\frac{21}{30} \stackrel{?}{=} \frac{7}{10}$ Substitute 21 for t.

$\frac{7}{10} = \frac{7}{10}$ ✓ The solution is 21.

Example 2 Solve Using Multiplication by a Fraction

Solve $\left(2\frac{1}{4}\right)g = 1\frac{1}{2}$.

$\left(2\frac{1}{4}\right)g = 1\frac{1}{2}$ Original equation

$\left(\frac{9}{4}\right)g = \frac{3}{2}$ Rewrite each mixed number as an improper fraction.

$\frac{4}{9}\left(\frac{9}{4}\right)g = \frac{4}{9}\left(\frac{3}{2}\right)$ Multiply each side by $\frac{4}{9}$, the reciprocal of $\frac{9}{4}$.

$g = \frac{12}{18}$ or $\frac{2}{3}$ Check this result.

The solution is $\frac{2}{3}$.

Example 3 Solve Using Multiplication by a Negative Number

Solve $42 = -6m$.

$42 = -6m$ Original equation

$-\frac{1}{6}(42) = -\frac{1}{6}(-6m)$ Multiply each side by $-\frac{1}{6}$, the reciprocal of -6.

$-7 = m$ Check this result.

The solution is -7.

You can write an equation to represent a real-world problem. Then use the equation to solve the problem.

More About. . .

Space Travel

On July 20, 1969, Neil Armstrong stepped on the surface of the moon. On the moon, his suit and life-support backpacks weighed about 33 pounds.

Source: NASA

Example 4 Write and Solve an Equation Using Multiplication

SPACE TRAVEL Refer to the information about space travel at the left. The weight of anything on the moon is about one-sixth its weight on Earth. What was the weight of Neil Armstrong's suit and life-support backpacks on Earth?

Words One sixth times the weight on Earth equals the weight on the moon.

Variable Let w = the weight on Earth.

Equation $\underbrace{\frac{1}{6}}_{\text{One sixth}} \underbrace{\cdot}_{\text{times}} \underbrace{w}_{\text{the weight on Earth}} \underbrace{=}_{\text{equals}} \underbrace{33}_{\text{the weight on the moon}}$

$\frac{1}{6}w = 33$ Original equation

$6\left(\frac{1}{6}w\right) = 6(33)$ Multiply each side by 6.

$w = 198$ $\frac{1}{6}(6) = 1$ and $33(6) = 198$

The weight of Neil Armstrong's suit and life-support backpacks on Earth was about 198 pounds.

SOLVE USING DIVISION The equation in Example 3, $42 = -6m$, was solved by multiplying each side by $-\frac{1}{6}$. The same result could have been obtained by dividing each side by -6. This method uses the **Division Property of Equality**.

Key Concept *Division Property of Equality*

- **Words** If each side of an equation is divided by the same nonzero number, the resulting equation is true.
- **Symbols** For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
- **Examples**
$$15 = 15 \qquad 28 = 28$$
$$\frac{15}{3} = \frac{15}{3} \qquad \frac{28}{-7} = \frac{28}{-7}$$
$$5 = 5 \qquad -4 = -4$$

Example 5 Solve Using Division by a Positive Number

Solve $13s = 195$. Then check your solution.

$13s = 195$ Original equation

$\frac{13s}{13} = \frac{195}{13}$ Divide each side by 13.

$s = 15$ $\frac{13s}{13} = s$ and $\frac{195}{13} = 15$

CHECK $13s = 195$ Original equation

$13(15) \stackrel{?}{=} 195$ Substitute 15 for s.

$195 = 195$ ✓

The solution is 15.

Study Tip

Alternative Method

You can also solve equations like those in Examples 5, 6, and 7 by using the Multiplication Property of Equality. For instance, in Example 6, you could multiply each side by $-\frac{1}{3}$.

Example 6 Solve Using Division by a Negative Number

Solve $-3x = 12$.

$-3x = 12$ Original equation

$\frac{-3x}{-3} = \frac{12}{-3}$ Divide each side by -3.

$x = -4$ $\frac{-3x}{-3} = x$ and $\frac{12}{-3} = -4$

The solution is -4.

Example 7 Write and Solve an Equation Using Division

Write an equation for the problem below. Then solve the equation.
Negative eighteen times a number equals −198.

$$\underbrace{\text{Negative eighteen}}_{-18}\quad \underbrace{\text{times}}_{\times}\quad \underbrace{\text{a number}}_{n}\quad \underbrace{\text{equals}}_{=}\quad \underbrace{-198.}_{-198}$$

$-18n = -198$ Original equation

$\frac{-18n}{-18} = \frac{-198}{-18}$ Divide each side by -18.

$n = 11$ Check this result.

The solution is 11.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a multiplication equation that has a solution of -3.
2. **Explain** why the Multiplication Property of Equality and the Division Property of Equality can be considered the same property.
3. **FIND THE ERROR** Casey and Juanita are solving $8n = -72$.

Casey	Juanita
$8n = -72$	$8n = -72$
$8n(8) = -72(8)$	$\frac{8n}{8} = \frac{-72}{8}$
$n = -576$	$n = -9$

Who is correct? Explain your reasoning.

Guided Practice **Solve each equation. Then check your solution.**

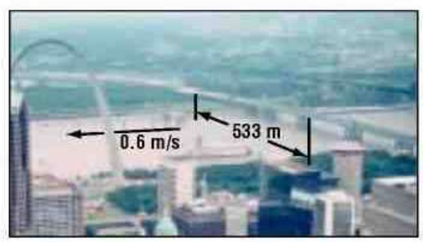
4. $-2g = -84$
5. $\frac{t}{7} = -5$
6. $\frac{a}{36} = \frac{4}{9}$
7. $\frac{4}{5}k = \frac{8}{9}$
8. $3.15 = 1.5y$
9. $\left(3\frac{1}{4}\right)p = 2\frac{1}{2}$

Write an equation for each problem. Then solve the equation.

10. Five times a number is 120. What is the number?
11. Two fifths of a number equals -24. Find the number.

Application

12. **GEOGRAPHY** The discharge of a river is defined as the width of the river times the average depth of the river times the speed of the river. At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second. How deep is the Mississippi River at this location?



Practice and Apply

Homework Help

For Exercises	See Examples
13–32	1–3, 5, 6
33–38	7
39–49	4

Extra Practice
See page 826.

Solve each equation. Then check your solution.

13. $-5r = 55$
14. $8d = 48$
15. $-910 = -26a$
16. $-1634 = 86s$
17. $\frac{b}{7} = -11$
18. $-\frac{v}{5} = -45$
19. $\frac{2}{3}n = 14$
20. $\frac{2}{5}g = -14$
21. $\frac{g}{24} = \frac{5}{12}$
22. $\frac{z}{45} = \frac{2}{5}$
23. $1.9f = -11.78$
24. $0.49k = 6.272$
25. $-2.8m = 9.8$
26. $-5.73q = 97.41$
27. $\left(-2\frac{3}{5}\right)t = -22$
28. $\left(3\frac{2}{3}\right)x = -5\frac{1}{2}$
29. $-5h = -3\frac{2}{3}$
30. $3p = 4\frac{1}{5}$

31. If $4m = 10$, what is the value of $12m$?
32. If $15b = 55$, what is the value of $3b$?

Write an equation for each problem. Then solve the equation.

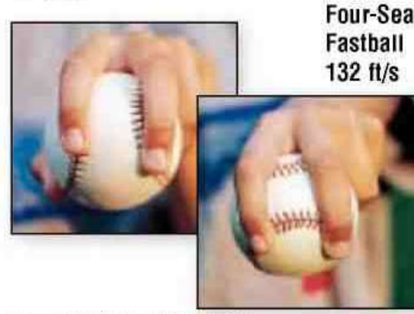
33. Seven times a number equals -84. What is the number?
34. Negative nine times a number is -117. Find the number.
35. One fifth of a number is 12. Find the number.
36. Negative three eighths times a number equals 12. What is the number?
37. Two and one half times a number equals one and one fifth. Find the number.
38. One and one third times a number is -4.82. What is the number?

GENETICS **For Exercises 39–41, use the following information.**
Research conducted by a daily U.S. newspaper has shown that about one seventh of people in the world are left-handed.

39. Write a multiplication equation relating the number of left-handed people ℓ and the total number of people p.
40. About how many left-handed people are there in a group of 350 people?
41. If there are 65 left-handed people in a group, about how many people are in that group?

42. **WORLD RECORDS** In 1993, a group of people in Utica, New York, made a very large round jelly doughnut which broke the world record for doughnut size. It weighed 1.5 tons and had a circumference of 50 feet. What was the diameter of the doughnut? (*Hint*: $C = \pi d$)

BASEBALL **For Exercises 43–45, use the following information.**
In baseball, if all other factors are the same, the speed of a four-seam fastball is faster than a two-seam fastball. The distance from the pitcher's mound to home plate is 60.5 feet.



Source: *Baseball and Mathematics*

43. How long does it take a two-seam fastball to go from the pitcher's mound to home plate? Round to the nearest hundredth. (*Hint*: $rt = d$)
44. How long does it take a four-seam fastball to go from the pitcher's mound to home plate? Round to the nearest hundredth.
45. How much longer does it take for a two-seam fastball to reach home plate than a four-seam fastball?

PHYSICAL SCIENCE **For Exercises 46–49, use the following information.**
In science lab, Devin and his classmates are asked to determine how many grams of hydrogen and how many grams of oxygen are in 477 grams of water. Devin used what he learned in class to determine that for every 8 grams of oxygen in water, there is 1 gram of hydrogen.

46. If x represents the number of grams of hydrogen, write an expression to represent the number of grams of oxygen.
47. Write an equation to represent the situation.
48. How many grams of hydrogen are in 477 grams of water?
49. How many grams of oxygen are in 477 grams of water?

50. **CRITICAL THINKING** If $6y - 7 = 4$, what is the value of $18y - 21$?

51. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

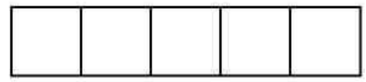
How can equations be used to find how long it takes light to reach Earth?

Include the following in your answer:

- an explanation of how to find the length of time it takes light to reach Earth from the closest star in the Big Dipper, and
- an equation describing the situation for the furthest star in the Big Dipper.

Standardized Test Practice

52. The rectangle at the right is divided into 5 identical squares. If the perimeter of the rectangle is 48 inches, what is the area of each square?



Ⓐ 4 in^2 Ⓑ 9.8 in^2 Ⓒ 16 in^2 Ⓓ 23.04 in^2

53. Which equation is equivalent to $4t = 20$?

Ⓐ $-2t = -10$ Ⓑ $t = 80$ Ⓒ $2t = 5$ Ⓓ $-8t = 40$

Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. *(Lesson 3-2)*

54. $m + 14 = 81$ 55. $d - 27 = -14$ 56. $17 - (-w) = -55$

57. Translate the following sentence into an equation. *(Lesson 3-1)*
Ten times a number a is equal to 5 times the sum of b and c.

Find each product. *(Lesson 2-3)*

58. $(-5)(12)$ 59. $(-2.93)(-0.003)$ 60. $(-4)(0)(-2)(-3)$

Graph each set of numbers on a number line. *(Lesson 2-1)*

61. $\{-4, -3, -1, 3\}$ 62. {integers between -6 and 10}

63. {integers less than -4} 64. {integers less than 0 and greater than -6}

Name the property illustrated by each statement. *(Lesson 1-6)*

65. $67 + 3 = 3 + 67$ 66. $(5 \cdot m) \cdot n = 5 \cdot (m \cdot n)$

Getting Ready for the Next Lesson

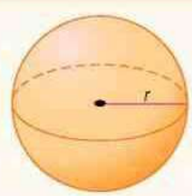
PREREQUISITE SKILL **Use the order of operations to find each value.**
(To review the ***order of operations****, see Lesson 1-2.)*

67. $2 \times 8 + 9$ 68. $24 \div 3 - 8$ 69. $\frac{3}{8}(17 + 7)$ 70. $\frac{15 - 9}{26 + 12}$

Practice Quiz 1

Lessons 3-1 through 3-3

GEOMETRY **For Exercises 1 and 2, use the following information.**
The surface area S of a sphere equals four times π times the square of the radius r. *(Lesson 3-1)*



1. Write the formula for the surface area of a sphere.
2. What is the surface area of a sphere if the radius is 7 centimeters?

Solve each equation. Then check your solution. *(Lessons 3-2 and 3-3)*

3. $d + 18 = -27$ 4. $m - 77 = -61$ 5. $-12 + a = -36$ 6. $t - (-16) = 9$

7. $\frac{2}{3}p = 18$ 8. $-17y = 391$ 9. $5x = -45$ 10. $-\frac{2}{5}d = -10$

Algebra Activity

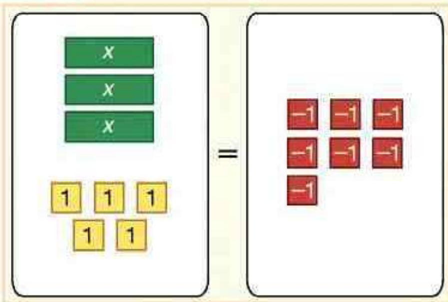
A Preview of Lesson 3-4

Solving Multi-Step Equations

You can use an equation model to solve multi-step equations.

Solve $3x + 5 = -7$.

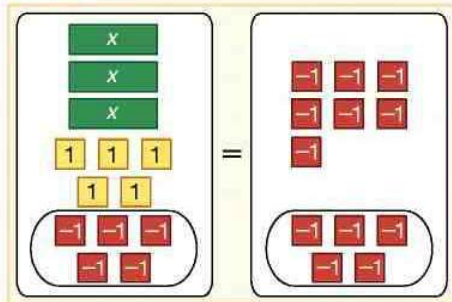
Step 1 *Model the equation.*



$$3x + 5 = -7$$

Place 3 x tiles and 5 positive 1 tiles on one side of the mat. Place 7 negative 1 tiles on the other side of the mat.

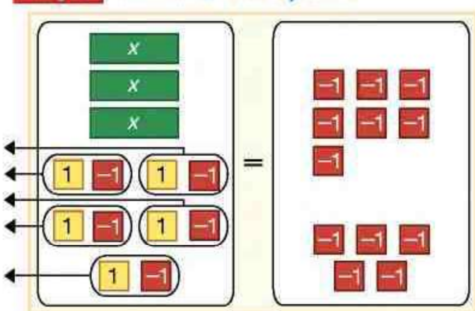
Step 2 *Isolate the x term.*



$$3x + 5 - 5 = -7 - 5$$

Since there are 5 positive 1 tiles with the x tiles, add 5 negative 1 tiles to each side to form zero pairs.

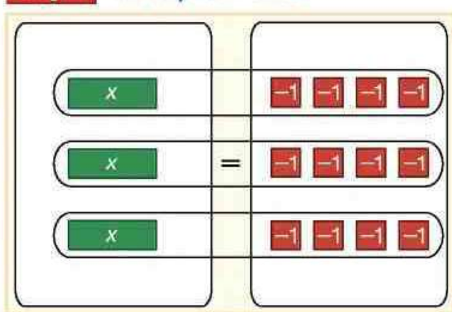
Step 3 *Remove zero pairs.*



$$3x = -12$$

Group the tiles to form zero pairs and remove the zero pairs.

Step 4 *Group the tiles.*



$$\frac{3x}{3} = \frac{-12}{3}$$

Separate the tiles into 3 equal groups to match the 3 x tiles. Each x tile is paired with 4 negative 1 tiles. Thus, $x = -4$.

Model **Use algebra tiles to solve each equation.**

1. $2x - 3 = -9$ **2.** $3x + 5 = 14$ **3.** $3x - 2 = 10$ **4.** $-8 = 2x + 4$

5. $3 + 4x = 11$ **6.** $2x + 7 = 1$ **7.** $9 = 4x - 7$ **8.** $7 + 3x = -8$

9. MAKE A CONJECTURE What steps would you use to solve $7x - 12 = -61$?

CONTENTS

3-4 Solving Multi-Step Equations

What You'll Learn

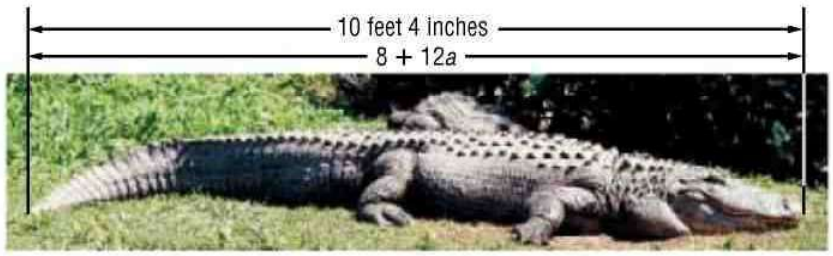
- Solve problems by working backward.
- Solve equations involving more than one operation.

Vocabulary

- work backward
- multi-step equations
- consecutive integers
- number theory

How can equations be used to estimate the age of an animal?

An American alligator hatchling is about 8 inches long. These alligators grow about 12 inches per year. Therefore, the expression $8 + 12a$ represents the length in inches of an alligator that is a years old.



Since 10 feet 4 inches equals $10(12) + 4$ or 124 inches, the equation $8 + 12a = 124$ can be used to estimate the age of the alligator in the photograph. Notice that this equation involves more than one operation.

WORK BACKWARD **Work backward** is one of many *problem-solving strategies* that you can use. Here are some other problem-solving strategies.

Problem-Solving Strategies	
draw a diagram	solve a simpler (or similar) problem
make a table or chart	eliminate the possibilities
make a model	look for a pattern
guess and check	act it out
check for hidden assumptions	list the possibilities
use a graph	identify the subgoals

Example 1 Work Backward to Solve a Problem

Solve the following problem by working backward.

After cashing her paycheck, Tara paid her father the \$20 she had borrowed. She then spent half of the remaining money on a concert ticket. She bought lunch for \$4.35 and had \$10.55 left. What was the amount of the paycheck?

Start at the end of the problem and undo each step.

Statement	Undo the Statement
She had \$10.55 left.	\$10.55
She bought lunch for \$4.35.	\$10.55 + \$4.35 = \$14.90
She spent half of the money on a concert ticket.	\$14.90 × 2 = \$29.80
She paid her father \$20.	\$29.80 + \$20.00 = \$49.80

The paycheck was for \$49.80. *Check this answer in the context of the problem.*

SOLVE MULTI-STEP EQUATIONS

To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward.

Example 2 Solve Using Addition and Division

Solve $7m - 17 = 60$. Then check your solution.

$$7m - 17 = 60 \quad \text{Original equation}$$
$$7m - 17 + 17 = 60 + 17 \quad \text{Add 17 to each side.}$$
$$7m = 77 \quad \text{Simplify.}$$
$$\frac{7m}{7} = \frac{77}{7} \quad \text{Divide each side by 7.}$$
$$m = 11 \quad \text{Simplify.}$$

CHECK
$$7m - 17 = 60 \quad \text{Original equation}$$
$$7(11) - 17 \stackrel{?}{=} 60 \quad \text{Substitute 11 for } m.$$
$$77 - 17 \stackrel{?}{=} 60 \quad \text{Multiply.}$$
$$60 = 60 \checkmark \quad \text{The solution is 11.}$$

Study Tip

Solving Multi-Step Equations
When solving a multi-step equation, "undo" the operations in reverse of the order of operations.

You have seen a multi-step equation in which the first, or *leading*, coefficient is an integer. You can use the same steps if the leading coefficient is a fraction.

Example 3 Solve Using Subtraction and Multiplication

Solve $\frac{t}{8} + 21 = 14$. Then check your solution.

$$\frac{t}{8} + 21 = 14 \quad \text{Original equation}$$
$$\frac{t}{8} + 21 - 21 = 14 - 21 \quad \text{Subtract 21 from each side.}$$
$$\frac{t}{8} = -7 \quad \text{Simplify.}$$
$$8\left(\frac{t}{8}\right) = 8(-7) \quad \text{Multiply each side by 8.}$$
$$t = -56 \quad \text{Simplify.}$$

CHECK
$$\frac{t}{8} + 21 = 14 \quad \text{Original equation}$$
$$\frac{-56}{8} + 21 \stackrel{?}{=} 14 \quad \text{Substitute } -56 \text{ for } t.$$
$$-7 + 21 \stackrel{?}{=} 14 \quad \text{Divide.}$$
$$14 = 14 \checkmark \quad \text{The solution is } -56.$$

Example 4 Solve Using Multiplication and Addition

Solve $\frac{p - 15}{9} = -6$.

$$\frac{p - 15}{9} = -6 \quad \text{Original equation}$$
$$9\left(\frac{p - 15}{9}\right) = 9(-6) \quad \text{Multiply each side by 9.}$$
$$p - 15 = -54 \quad \text{Simplify.}$$
$$p - 15 + 15 = -54 + 15 \quad \text{Add 15 to each side.}$$
$$p = -39 \quad \text{The solution is } -39.$$

www.algebra1.com/extra_examples

Example 5 Write and Solve a Multi-Step Equation

Write an equation for the problem below. Then solve the equation.
Two-thirds of a number minus six is −10.

$$\underbrace{\text{Two-thirds}}_{\frac{2}{3}} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{\text{a number}}_{n} \quad \underbrace{\text{minus}}_{-} \quad \underbrace{\text{six}}_{6} \quad \underbrace{\text{is}}_{=} \quad \underbrace{-10.}_{-10}$$

$\frac{2}{3}n - 6 = -10$	Original equation
$\frac{2}{3}n - 6 + 6 = -10 + 6$	Add 6 to each side.
$\frac{2}{3}n = -4$	Simplify.
$\frac{3}{2}\left(\frac{2}{3}n\right) = \frac{3}{2}(-4)$	Multiply each side by $\frac{3}{2}$.
$n = -6$	Simplify.

The solution is −6.

Consecutive integers are integers in counting order, such as 7, 8, and 9. Beginning with an even integer and counting by two will result in *consecutive even integers*. For example, −4, −2, 0, and 2 are consecutive even integers. Beginning with an odd integer and counting by two will result in *consecutive odd integers*. For example, −3, −1, 1, 3 and 5 are consecutive odd integers. The study of numbers and the relationships between them is called **number theory**.

Example 6 Solve a Consecutive Integer Problem

Study Tip

Representing Consecutive Integers

You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of *n*—odd or even—that differs between the two expressions.

NUMBER THEORY **Write an equation for the problem below. Then solve the equation and answer the problem.**

Find three consecutive even integers whose sum is −42.

Let n = the least even integer.

Then $n + 2$ = the next greater even integer, and

$n + 4$ = the greatest of the three even integers.

$$\underbrace{\text{The sum of three consecutive even integers}}_{n + (n + 2) + (n + 4)} \quad \underbrace{\text{is}}_{=} \quad \underbrace{-42.}_{-42}$$

$n + (n + 2) + (n + 4) = -42$	Original equation
$3n + 6 = -42$	Simplify.
$3n + 6 - 6 = -42 - 6$	Subtract 6 from each side.
$3n = -48$	Simplify.
$\frac{3n}{3} = \frac{-48}{3}$	Divide each side by 3
$n = -16$	Simplify.

$n + 2 = -16 + 2$ or -14 $\qquad$ $n + 4 = -16 + 4$ or -12

The consecutive even integers are −16, −14, and −12.

CHECK −16, −14, and −12 are consecutive even integers.
$-16 + (-14) + (-12) = -42$ ✓

Check for Understanding

Concept Check

1. **OPEN ENDED** Give two examples of multi-step equations that have a solution of -2.
2. **List** the steps used to solve $\frac{w+3}{5} - 4 = 6$.
3. **Write** an expression for the odd integer before odd integer n.
4. Justify each step.

$\frac{4-2d}{5} + 3 = 9$	
$\frac{4-2d}{5} + 3 - 3 = 9 - 3$	**a.** ?
$\frac{4-2d}{5} = 6$	**b.** ?
$\frac{4-2d}{5}(5) = 6(5)$	**c.** ?
$4 - 2d = 30$	**d.** ?
$4 - 2d - 4 = 30 - 4$	**e.** ?
$-2d = 26$	**f.** ?
$\frac{-2d}{-2} = \frac{26}{-2}$	**g.** ?
$d = -13$	**h.** ?

Guided Practice

Solve each problem by working backward.

5. A number is multiplied by seven, and then the product is added to 13. The result is 55. What is the number?
6. **LIFE SCIENCE** A bacteria population triples in number each day. If there are 2,187,000 bacteria on the seventh day, how many bacteria were there on the first day?

Solve each equation. Then check your solution.

7. $4g - 2 = -6$
8. $18 = 5p + 3$
9. $\frac{3}{2}a - 8 = 11$
10. $\frac{b+4}{-2} = -17$
11. $0.2n + 3 = 8.6$
12. $3.1y - 1.5 = 5.32$

Write an equation and solve each problem.

13. Twelve decreased by a twice a number equals –34. Find the number.
14. Find three consecutive integers whose sum is 42.

Application

15. **WORLD CULTURES** The English alphabet contains 2 more than twice as many letters as the Hawaiian alphabet. How many letters are there in the Hawaiian alphabet?

Practice and Apply

Solve each problem by working backward.

16. A number is divided by 4, and then the quotient is added to 17. The result is 25. Find the number.
17. Nine is subtracted from a number, and then the difference is multiplied by 5. The result is 75. What is the number?

Homework Help

For Exercises	See Examples
16–21	1
22–41	2–4
42–54	5, 6

Extra Practice

See page 826.

Solve each problem by working backward.

18. **GAMES** In the Trivia Bowl, each finalist must answer four questions correctly. Each question is worth twice as much as the question before it. The fourth question is worth \$6000. How much is the first question worth?

19. **ICE SCULPTING** Due to melting, an ice sculpture loses one-half its weight every hour. After 8 hours, it weighs $\frac{5}{16}$ of a pound. How much did it weigh in the beginning?

20. **FIREFIGHTING** A firefighter spraying water on a fire stood on the middle rung of a ladder. The smoke lessened, so she moved up 3 rungs. It got too hot, so she backed down 5 rungs. Later, she went up 7 rungs and stayed until the fire was out. Then, she climbed the remaining 4 rungs and went into the building. How many rungs does the ladder have?

21. **MONEY** Hugo withdrew some money from his bank account. He spent one third of the money for gasoline. Then he spent half of what was left for a haircut. He bought lunch for \$6.55. When he got home, he had \$13.45 left. How much did he withdraw from the bank?

Solve each equation. Then check your solution.

22. $5n + 6 = -4$
23. $7 + 3c = -11$
24. $15 = 4a - 5$
25. $-63 = 7g - 14$
26. $\frac{c}{-3} + 5 = 7$
27. $\frac{y}{5} + 9 = 6$
28. $3 - \frac{a}{7} = -2$
29. $-9 - \frac{p}{4} = 5$
30. $\frac{t}{8} - 6 = -12$
31. $\frac{m}{-5} + 6 = 31$
32. $\frac{17 - s}{4} = -10$
33. $\frac{-3j - (-4)}{-6} = 12$
34. $-3d - 1.2 = 0.9$
35. $-2.5r - 32.7 = 74.1$
36. $-0.6 + (-4a) = -1.4$
37. $\frac{p}{-7} - 0.5 = 1.3$
38. $3.5x + 5 - 1.5x = 8$
39. $\frac{9z + 4}{5} - 8 = 5.4$

40. If $3a - 9 = 6$, what is the value of $5a + 2$?
41. If $2x + 1 = 5$, what is the value of $3x - 4$?

Write an equation and solve each problem.

42. Six less than two thirds of a number is negative ten. Find the number.
43. Twenty-nine is thirteen added to four times a number. What is the number?
44. Find three consecutive odd integers whose sum is 51.
45. Find three consecutive even integers whose sum is -30.
46. Find four consecutive integers whose sum is 94.
47. Find four consecutive odd integers whose sum is 8.

48. **BUSINESS** Adele Jones is on a business trip and plans to rent a subcompact car from Speedy Rent-A-Car. Her company has given her a budget of \$60 per day for car rental. What is the maximum distance Ms. Jones can drive in one day and still stay within her budget?

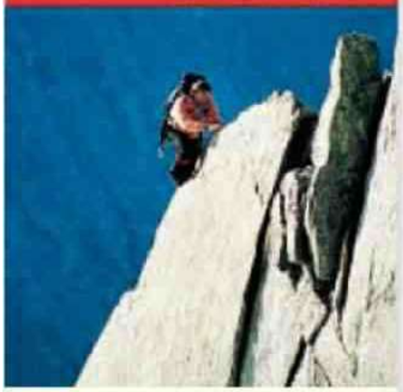
Speedy Rent-A-Car Price List

Subcompact
\$14.95 per day plus \$0.10 per mile
Compact
\$19.95 per day plus \$0.12 per mile
Full Size
\$22.95 per day plus \$0.15 per mile

49. **GEOMETRY** The measures of the three sides of a triangle are consecutive even integers. The perimeter of the triangle is 54 centimeters. What are the lengths of the sides of the triangle?

50. **MOUNTAIN CLIMBING** A general rule for those climbing more than 7000 feet above sea level is to allow a total of $\left(\frac{a - 7000}{2000} + 2\right)$ weeks of camping during the ascension. In this expression, a represents the altitude in feet. If a group of mountain climbers have allowed for 9 weeks of camping in their schedule, how high can they climb without worrying about altitude sickness?

More About. . .



Mountain Climbing

Many mountain climbers experience altitude sickness caused by a decrease in oxygen. Climbers can acclimate themselves to these higher altitudes by camping for one or two weeks at various altitudes as they ascend the mountain.

Source: *Shape*

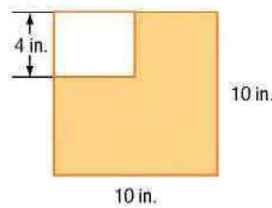
SHOE SIZE **For Exercises 51 and 52, use the following information.**
If ℓ represents the length of a person's foot in inches, the expression $2\ell - 12$ can be used to estimate his or her shoe size.

51. What is the approximate length of the foot of a person who wears size 8?

52. Measure your foot and use the expression to determine your shoe size. How does this number compare to the size of shoe you are wearing?

53. **SALES** Trever Goetz is a salesperson who is paid a monthly salary of \$500 plus a 2% commission on sales. How much must Mr. Goetz sell to earn \$2000 this month?

54. **GEOMETRY** A rectangle is cut from the corner of a 10-inch by 10-inch of paper. The area of the remaining piece of paper is $\frac{4}{5}$ of the area of the original piece of paper. If the width of the rectangle removed from the paper is 4 inches, what is the length of the rectangle?



55. **CRITICAL THINKING** Determine whether the following statement is *sometimes*, *always*, or *never* true.

The sum of two consecutive even numbers equals the sum of two consecutive odd numbers.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to estimate the age of an animal?

Include the following in your answer:
- an explanation of how to solve the equation representing the age of the alligator, and
- an estimate of the age of the alligator.

Standardized Test Practice

57. Which equation represents the following problem?

Fifteen minus three times a number equals negative twenty-two. Find the number.

Ⓐ $3n - 15 = -22$ Ⓑ $15 - 3n = -22$
Ⓒ $3(15 - n) = -22$ Ⓓ $3(n - 15) = -22$

58. Which equation has a solution of -5?

Ⓐ $2a - 6 = 4$ Ⓑ $3a + 7 = 8$
Ⓒ $\frac{3a - 7}{4} = 2$ Ⓓ $\frac{3}{5}a + 19 = 16$

Graphing Calculator

EQUATION SOLVER **You can use a graphing calculator to solve equations that are rewritten as expressions that equal zero.**

Step 1 Write the equation so that one side is equal to 0.

Step 2 On a TI-83 Plus, press [MATH] and choose 0, for solve.

Step 3 Enter the equation after 0=. Use [ALPHA] to enter the variables. Press [ENTER].

Step 4 Press [ALPHA] [SOLVE] to reveal the solution. Use the [▲] key to begin entering a new equation.

Use a graphing calculator to solve each equation.

59. $0 = 11y + 33$ **60.** $\frac{w+2}{5} - 4 = 0$ **61.** $6 = -12 + \frac{h}{-7}$

62. $\frac{p-(-5)}{-2} = 6$ **63.** $0.7 = \frac{r-0.8}{6}$ **64.** $4.91 + 7.2t = 38.75$

Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. *(Lesson 3-3)*

65. $-7t = 91$ **66.** $\frac{r}{15} = -8$ **67.** $-\frac{2}{3}b = -1\frac{1}{2}$

TRANSPORTATION **For Exercises 68 and 69, use the following information.** In the year 2000, there were 18 more models of sport utility vehicles than there were in the year 1990. There were 47 models of sport utility vehicles in 2000. *(Lesson 3-2)*

68. Write an addition equation to represent the situation.

69. How many models of sport utility vehicles were there in 1990?

Find the odds of each outcome if you spin the spinner at the right. *(Lesson 2-6)*

70. spinning a number divisible by 3

71. spinning a number equal to or greater than 5

72. spinning a number less than 7

Find each quotient. *(Lesson 2-4)*

73. $-\frac{6}{7} \div 3$ **74.** $\frac{\frac{2}{3}}{8}$ **75.** $\frac{-3a+16}{4}$ **76.** $\frac{15t-25}{-5}$

Use the Distributive Property to find each product. *(Lesson 1-5)*

77. $17 \cdot 9$ **78.** $13(101)$ **79.** $16\left(1\frac{1}{4}\right)$ **80.** $18\left(2\frac{1}{9}\right)$

Write an algebraic expression for each verbal expression. *(Lesson 1-1)*

81. the product of 5 and m plus half of n

82. the sum of 3 and b divided by y

83. the sum of 3 times a and the square of b

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression.**

*(To review **simplifying expressions**, see Lesson 1-5.)*

84. $5d - 2d$ **85.** $11m - 5m$

86. $8t + 6t$ **87.** $7g - 15g$

88. $-9f + 6f$ **89.** $-3m + (-7m)$

3-5 Solving Equations with the Variable on Each Side

What You'll Learn

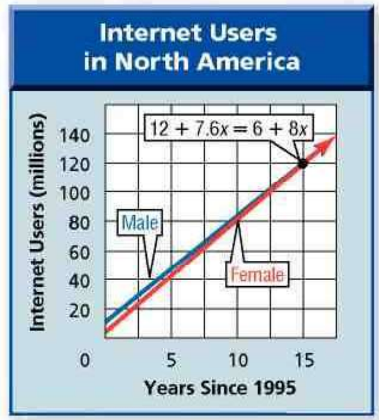
- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

Vocabulary

- identity

How can an equation be used to determine when two populations are equal?

In 1995, there were 18 million Internet users in North America. Of this total, 12 million were male, and 6 million were female. During the next five years, the number of male Internet users on average increased 7.6 million per year, and the number of female Internet users increased 8 million per year. If this trend continues, the following expressions represent the number of male and female Internet users x years after 1995.



Male Internet Users: $12 + 7.6x$

Female Internet Users: $6 + 8x$

The equation $12 + 7.6x = 6 + 8x$ represents the time at which the number of male and female Internet users are equal. Notice that this equation has the variable x on each side.

VARIABLES ON EACH SIDE Many equations contain variables on each side. To solve these types of equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side.

Example 1 Solve an Equation with Variables on Each Side

Solve $-2 + 10p = 8p - 1$. Then check your solution.

$-2 + 10p = 8p - 1$	Original equation
$-2 + 10p - 8p = 8p - 1 - 8p$	Subtract $8p$ from each side.
$-2 + 2p = -1$	Simplify.
$-2 + 2p + 2 = -1 + 2$	Add 2 to each side.
$2p = 1$	Simplify.
$\frac{2p}{2} = \frac{1}{2}$	Divide each side by 2.
$p = \frac{1}{2}$ or 0.5	Simplify.

CHECK

$-2 + 10p = 8p - 1$	Original equation
$-2 + 10(0.5) \stackrel{?}{=} 8(0.5) - 1$	Substitute 0.5 for p.
$-2 + 5 \stackrel{?}{=} 4 - 1$	Multiply.
$3 = 3$ ✓	The solution is $\frac{1}{2}$ or 0.5.

GROUPING SYMBOLS When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 2 Solve an Equation with Grouping Symbols

Solve $4(2r - 8) = \frac{1}{7}(49r + 70)$. Then check your solution.

$4(2r - 8) = \frac{1}{7}(49r + 70)$	Original equation
$8r - 32 = 7r + 10$	Distributive Property
$8r - 32 - 7r = 7r + 10 - 7r$	Subtract $7r$ from each side.
$r - 32 = 10$	Simplify.
$r - 32 + 32 = 10 + 32$	Add 32 to each side.
$r = 42$	Simplify.

CHECK

$4(2r - 8) = \frac{1}{7}(49r + 70)$	Original equation
$4[2(42) - 8] \stackrel{?}{=} \frac{1}{7}[49(42) + 70]$	Substitute 42 for r.
$4(84 - 8) \stackrel{?}{=} \frac{1}{7}(2058 + 70)$	Multiply.
$4(76) \stackrel{?}{=} \frac{1}{7}(2128)$	Add and subtract.
$304 = 304$ ✓	

The solution is 42.

Study Tip

Look Back
To review the **Distributive Property**, see Lesson 1-5.

Some equations with the variable on each side may have no solution. That is, there is no value of the variable that will result in a true equation.

Example 3 No Solutions

Solve $2m + 5 = 5(m - 7) - 3m$.

$2m + 5 = 5(m - 7) - 3m$	Original equation
$2m + 5 = 5m - 35 - 3m$	Distributive Property
$2m + 5 = 2m - 35$	Simplify.
$2m + 5 - 2m = 2m - 35 - 2m$	Subtract $2m$ from each side.
$5 = -35$	This statement is false.

Since $5 = -35$ is a false statement, this equation has no solution.

An equation that is true for every value of the variable is called an **identity**.

Example 4 An Identity

Solve $3(r + 1) - 5 = 3r - 2$.

$3(r + 1) - 5 = 3r - 2$	Original equation
$3r + 3 - 5 = 3r - 2$	Distributive Property
$3r - 2 = 3r - 2$	Reflexive Property of Equality

Since the expressions on each side of the equation are the same, this equation is an identity. The statement $3(r + 1) - 5 = 3r - 2$ is true for all values of r.

Concept Summary — Steps for Solving Equations

Step 1 Use the Distributive Property to remove the grouping symbols.

Step 2 Simplify the expressions on each side of the equals sign.

Step 3 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.

Step 4 Simplify the expressions on each side of the equals sign.

Step 5 Use the Multiplication or Division Property of Equality to solve.
- If the solution results in a false statement, there is no solution of the equation.
- If the solution results in an identity, the solution is all numbers.

Standardized Test Practice

Example 5 Use Substitution to Solve an Equation

Multiple-Choice Test Item

Solve $2(b - 3) + 5 = 3(b - 1)$.

(A) -2 (B) 2 (C) -3 (D) 3

Read the Test Item

You are asked to solve an equation.

Solve the Test Item

You can solve the equation or substitute each value into the equation and see if it makes the equation true. We will solve by substitution.

A

$$\begin{aligned} 2(b - 3) + 5 &= 3(b - 1) \\ 2(-2 - 3) + 5 &\stackrel{?}{=} 3(-2 - 1) \\ 2(-5) + 5 &\stackrel{?}{=} 3(-3) \\ -10 + 5 &\stackrel{?}{=} -9 \\ -5 &\neq -9 \end{aligned}$$

B

$$\begin{aligned} 2(b - 3) + 5 &= 3(b - 1) \\ 2(2 - 3) + 5 &\stackrel{?}{=} 3(2 - 1) \\ 2(-1) + 5 &\stackrel{?}{=} 3(1) \\ -2 + 5 &\stackrel{?}{=} 3 \\ 3 &= 3 \quad \checkmark \end{aligned}$$

Since the value 2 results in a true statement, you do not need to check -3 and 3. The answer is B.

The Princeton Review

Test-Taking Tip

If you are asked to solve a complicated equation, it sometimes takes less time to check each possible answer rather than to actually solve the equation.

Check for Understanding

Concept Check

1. **Determine** whether each solution is correct. If the solution is not correct, find the error and give the correct solution.

a.

$$\begin{aligned} 2(g + 5) &= 22 \\ 2g + 5 &= 22 \\ 2g + 5 - 5 &= 22 - 5 \\ 2g &= 17 \\ \frac{2g}{2} &= \frac{17}{2} \\ g &= 8.5 \end{aligned}$$

b.

$$\begin{aligned} 5d &= 2d - 18 \\ 5d - 2d &= 2d - 18 - 2d \\ 3d &= -18 \\ \frac{3d}{3} &= \frac{-18}{3} \\ d &= -6 \end{aligned}$$

c.

$$\begin{aligned} -6z + 13 &= 7z \\ -6z + 13 - 6z &= 7z - 6z \\ 13 &= z \end{aligned}$$

2. **Explain** how to determine whether an equation is an identity.

3. **OPEN ENDED** Find a counterexample to the statement *all equations have a solution.*

Guided Practice

4. Justify each step.

$$6n + 7 = 8n - 13$$
$$6n + 7 - 6n = 8n - 13 - 6n \quad \textbf{a.}\ \underline{\ ?\ }$$
$$7 = 2n - 13 \quad \textbf{b.}\ \underline{\ ?\ }$$
$$7 + 13 = 2n - 13 + 13 \quad \textbf{c.}\ \underline{\ ?\ }$$
$$20 = 2n \quad \textbf{d.}\ \underline{\ ?\ }$$
$$\frac{20}{2} = \frac{2n}{2} \quad \textbf{e.}\ \underline{\ ?\ }$$
$$10 = n \quad \textbf{f.}\ \underline{\ ?\ }$$

Solve each equation. Then check your solution.

5. $20c + 5 = 5c + 65$
6. $\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$
7. $3(a - 5) = -6$
8. $7 - 3r = r - 4(2 + r)$
9. $6 = 3 + 5(d - 2)$
10. $\frac{c + 1}{8} = \frac{c}{4}$
11. $5h - 7 = 5(h - 2) + 3$
12. $5.4w + 8.2 = 9.8w - 2.8$

Standardized Test Practice

13. Solve $75 - 9t = 5(-4 + 2t)$.

Ⓐ -5 Ⓑ -4 Ⓒ 4 Ⓓ 5

Practice and Apply

Homework Help

For Exercises	See Examples
14–48	1–4
51, 52	5

Extra Practice

See page 826.

Justify each step.

14. $\frac{3m - 2}{5} = \frac{7}{10}$

$$\frac{3m - 2}{5}(10) = \frac{7}{10}(10) \quad \textbf{a.}\ \underline{\ ?\ }$$
$$(3m - 2)2 = 7 \quad \textbf{b.}\ \underline{\ ?\ }$$
$$6m - 4 = 7 \quad \textbf{c.}\ \underline{\ ?\ }$$
$$6m - 4 + 4 = 7 + 4 \quad \textbf{d.}\ \underline{\ ?\ }$$
$$6m = 11 \quad \textbf{e.}\ \underline{\ ?\ }$$
$$\frac{6m}{6} = \frac{11}{6} \quad \textbf{f.}\ \underline{\ ?\ }$$
$$m = 1\frac{5}{6} \quad \textbf{g.}\ \underline{\ ?\ }$$

15. $v + 9 = 7v + 9$

$$v + 9 - v = 7v + 9 - v \quad \textbf{a.}\ \underline{\ ?\ }$$
$$9 = 6v + 9 \quad \textbf{b.}\ \underline{\ ?\ }$$
$$9 - 9 = 6v + 9 - 9 \quad \textbf{c.}\ \underline{\ ?\ }$$
$$0 = 6v \quad \textbf{d.}\ \underline{\ ?\ }$$
$$\frac{0}{6} = \frac{6v}{6} \quad \textbf{e.}\ \underline{\ ?\ }$$
$$0 = v \quad \textbf{f.}\ \underline{\ ?\ }$$

Solve each equation. Then check your solution.

16. $3 - 4q = 10q + 10$
17. $3k - 5 = 7k - 21$
18. $5t - 9 = -3t + 7$
19. $8s + 9 = 7s + 6$
20. $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$
21. $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$
22. $8 = 4(3c + 5)$
23. $7(m - 3) = 7$
24. $6(r + 2) - 4 = -10$
25. $5 - \frac{1}{2}(x - 6) = 4$
26. $4(2a - 1) = -10(a - 5)$
27. $4(f - 2) = 4f$
28. $3(1 + d) - 5 = 3d - 2$
29. $2(w - 3) + 5 = 3(w - 1)$

30. $\frac{3}{2}y - y = 4 + \frac{1}{2}y$

31. $3 + \frac{2}{5}b = 11 - \frac{2}{5}b$

32. $\frac{1}{4}(7 + 3g) = -\frac{g}{8}$

33. $\frac{1}{6}(a - 4) = \frac{1}{3}(2a + 4)$

34. $28 - 2.2x = 11.6x + 262.6$

35. $1.03p - 4 = -2.15p + 8.72$

36. $18 - 3.8t = 7.36 - 1.9t$

37. $13.7v - 6.5 = -2.3v + 8.3$

38. $2[s + 3(s - 1)] = 18$

39. $-3(2n - 5) = 0.5(-12n + 30)$

40. One half of a number increased by 16 is four less than two thirds of the number. Find the number.

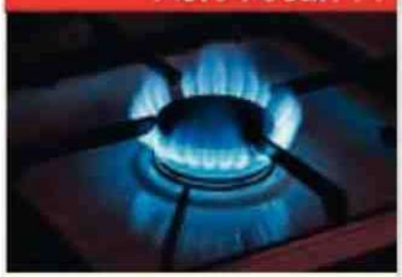
41. The sum of one half of a number and 6 equals one third of the number. What is the number?

42. **NUMBER THEORY** Twice the greater of two consecutive odd integers is 13 less than three times the lesser number. Find the integers.

43. **NUMBER THEORY** Three times the greatest of three consecutive even integers exceeds twice the least by 38. What are the integers?

44. **HEALTH** When exercising, a person's pulse rate should not exceed a certain limit, which depends on his or her age. This maximum rate is represented by the expression $0.8(220 - a)$, where a is age in years. Find the age of a person whose maximum pulse is 152.

More About. . .



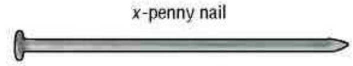
Energy

One British Thermal Unit (BTU) is the amount of energy needed to raise the temperature of one pound of water 1°F. If a heating system is 100% efficient, one cubic foot of natural gas provides 1000 BTU.

Source: *World Book Encyclopedia*

45. **HARDWARE** Traditionally, nails are given names such as 2-penny, 3-penny, and so on. These names describe the lengths of the nails. What is the name of a nail that is $2\frac{1}{2}$ inches long?

x-penny nail

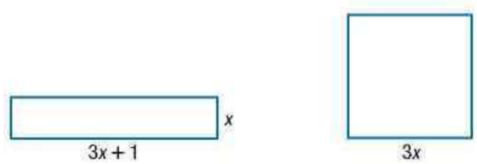


nail length $= 1 + \frac{1}{4}(x - 2)$

Source: *World Book Encyclopedia*

46. **TECHNOLOGY** About 4.9 million households had one brand of personal computers in 2001. The use of these computers grew at an average rate of 0.275 million households a year. In 2001, about 2.5 million households used another type of computer. The use of these computers grew at an average rate of 0.7 million households a year. How long will it take for the two types of computers to be in the same number of households?

47. **GEOMETRY** The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.



48. **ENERGY** Use the information on energy at the left. The amount of energy E in BTUs needed to raise the temperature of water is represented by the equation $E = w(t_f - t_O)$. In this equation, w represents the weight of the water in pounds, t_f represents the final temperature in degrees Fahrenheit, and t_O represents the original temperature in degrees Fahrenheit. A 50-gallon water heater is 60% efficient. If 10 cubic feet of natural gas are used to raise the temperature of water with the original temperature of 50°F, what is the final temperature of the water? (One gallon of water weighs about 8 pounds.)

49. **CRITICAL THINKING** Write an equation that has one or more grouping symbols, the variable on each side of the equals sign, and a solution of -2.

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can an equation be used to determine when two populations are equal?

Include the following in your answer:

- a list of the steps needed to solve the equation,
- the year when the number of female Internet users will equal the number of male Internet users according to the model, and
- an explanation of why this method can be used to predict future events.

Standardized Test Practice

51. Solve $8x - 3 = 5(2x + 1)$.

Ⓐ 4 Ⓑ 2 Ⓒ -2 Ⓓ -4

52. Solve $5n + 4 = 7(n + 1) - 2n$.

Ⓐ 0 Ⓑ -1 Ⓒ no solution Ⓓ all numbers

Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. *(Lesson 3-4)*

53. $\frac{2}{9}v - 6 = 14$ 54. $\frac{x-3}{7} = -2$ 55. $5 - 9w = 23$

HEALTH For Exercises 56 and 57, use the following information.
Ebony burns 4.5 Calories per minute pushing a lawn mower. *(Lesson 3-3)*

56. Write a multiplication equation representing the number of Calories C burned if Ebony pushes the lawn mower for m minutes.

57. How long will it take Ebony to burn 150 Calories mowing the lawn?

Use each set of data to make a line plot. *(Lesson 2-5)*

58. 13, 15, 11, 15, 16, 17, 12, 12, 13, 15, 16, 15

59. 22, 25, 19, 21, 22, 24, 22, 25, 28, 21, 24, 22

Find each sum or difference. *(Lesson 2-2)*

60. $-10 + (-17)$ 61. $-12 - (-8)$ 62. $6 - 14$

Write a counterexample for each statement. *(Lesson 1-7)*

63. If the sum of two numbers is even, then both addends are even.

64. If you are baking cookies, you will need chocolate chips.

Evaluate each expression when $a = 5$, $b = 8$, $c = 7$, $x = 2$, and $y = 1$. *(Lesson 1-2)*

65. $\frac{3a^2}{b+c}$ 66. $x(a + 2b) - y$ 67. $5(x + 2y) - 4a$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each fraction.
*(To review **simplifying fractions**, see pages 798 and 799.)*

68. $\frac{12}{15}$ 69. $\frac{28}{49}$ 70. $\frac{36}{60}$ 71. $\frac{8}{120}$

72. $\frac{108}{9}$ 73. $\frac{28}{42}$ 74. $\frac{16}{40}$ 75. $\frac{19}{57}$

3-6 Ratios and Proportions

What You'll Learn

- Determine whether two ratios form a proportion.
- Solve proportions.

How are ratios used in recipes?

The ingredients in the recipe will make 4 servings of honey frozen yogurt. Keri can use ratios and equations to find the amount of each ingredient needed to make enough yogurt for her club meeting.

Honey Frozen Yogurt	
2 cups 2% milk	2 eggs, beaten
$\frac{3}{4}$ cup honey	2 cups plain low-fat
1 dash salt	yogurt
	1 tablespoon vanilla

Vocabulary

- ratio
- proportion
- extremes
- means
- rate
- scale

RATIOS AND PROPORTIONS A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

$$x \text{ to } y \qquad x{:}y \qquad \frac{x}{y}$$

Ratios are often expressed in simplest form. For example, the recipe above states that for 4 servings you need 2 cups of milk. The ratio of servings to milk may be written as 4 to 2, 4:2, or $\frac{4}{2}$. Written in simplest form, the ratio of servings to milk can be written as 2 to 1, 2:1, or $\frac{2}{1}$.

Study Tip

Reading Math
A ratio that is equivalent to a whole number is written with a denominator of 1.

Suppose you wanted to double the recipe to have 8 servings. The amount of milk required would be 4 cups. The ratio of servings to milk is $\frac{8}{4}$. When this ratio is simplified, the ratio is $\frac{2}{1}$. Notice that this ratio is equal to the original ratio.

$$\underset{\div 2}{\overset{\div 2}{\frac{4}{2} = \frac{2}{1}}} \qquad\qquad \underset{\div 4}{\overset{\div 4}{\frac{8}{4} = \frac{2}{1}}}$$

An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{4}{2} = \frac{8}{4}$ is a proportion.

Example 1 Determine Whether Ratios Form a Proportion

Determine whether the ratios $\frac{4}{5}$ and $\frac{24}{30}$ form a proportion.

$$\underset{\div 1}{\overset{\div 1}{\frac{4}{5} = \frac{4}{5}}} \qquad\qquad \underset{\div 6}{\overset{\div 6}{\frac{24}{30} = \frac{4}{5}}}$$

The ratios are equal. Therefore, they form a proportion.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

Example 2 Use Cross Products

Use cross products to determine whether each pair of ratios form a proportion.

a. $\frac{0.4}{0.8}, \frac{0.7}{1.4}$

$\frac{0.4}{0.8} \stackrel{?}{=} \frac{0.7}{1.4}$ Write the equation.

$0.4(1.4) \stackrel{?}{=} 0.8(0.7)$ Find the cross products.

$0.56 = 0.56$ Simplify.

The cross products are equal, so $\frac{0.4}{0.8} = \frac{0.7}{1.4}$. Since the ratios are equal, they form a proportion.

b. $\frac{6}{8}, \frac{24}{28}$

$\frac{6}{8} \stackrel{?}{=} \frac{24}{28}$ Write the equation.

$6(28) \stackrel{?}{=} 8(24)$ Find the cross products.

$168 \neq 192$ Simplify.

The cross products are not equal, so $\frac{6}{8} \neq \frac{24}{28}$. The ratios do not form a proportion.

Study Tip

Cross Products
When you find cross products, you are said to be *cross multiplying*.

In the proportion $\frac{0.4}{0.8} = \frac{0.7}{1.4}$ above, 0.4 and 1.4 are called the **extremes**, and 0.8 and 0.7 are called the **means**.

Key Concept — Means-Extremes Property of Proportion

- **Words** In a proportion, the product of the extremes is equal to the product of the means.
- **Symbols** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
- **Examples** Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

SOLVE PROPORTIONS You can write proportions that involve a variable. To solve the proportion, use cross products and the techniques used to solve other equations.

Example 3 Solve a Proportion

Solve the proportion $\frac{n}{15} = \frac{24}{16}$.

$\frac{n}{15} = \frac{24}{16}$ Original equation

$16(n) = 15(24)$ Find the cross products.

$16n = 360$ Simplify.

$\frac{16n}{16} = \frac{360}{16}$ Divide each side by 16.

$n = 22.5$ Simplify.

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of \$1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of \$30,000 per year are all rates. Proportions are often used to solve problems involving rates.

Example 4 Use Rates

BICYCLING Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

Explore Let m represent the number of miles Trent can ride in 6 hours.

Plan Write a proportion for the problem.

$$\begin{matrix}\text{miles} \rightarrow \\ \text{hours} \rightarrow\end{matrix} \quad \frac{30}{4} = \frac{m}{6} \quad \begin{matrix}\leftarrow \text{miles} \\ \leftarrow \text{hours}\end{matrix}$$

Solve

$\frac{30}{4} = \frac{m}{6}$ Original proportion

$30(6) = 4(m)$ Find the cross products.

$180 = 4m$ Simplify.

$\frac{180}{4} = \frac{4m}{4}$ Divide each side by 4.

$45 = m$ Simplify.

Examine If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6×7.5 or 45 miles. The answer is correct.

Since the rates are equal, they form a proportion. So, Trent can ride 45 miles in 6 hours.

A ratio or rate called a **scale** is used when making a model or drawing of something that is too large or too small to be conveniently drawn at actual size. The scale compares the model to the actual size of the object using a proportion. Maps and blueprints are two commonly used scale drawings.

More About. . .

Crater Lake

Crater Lake is a volcanic crater in Oregon that was formed by an explosion 42 times the blast of Mount St. Helens.

Source: travel.excite.com

Example 5 Use a Scale Drawing

CRATER LAKE The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance between these two places?

Let d represent the actual distance.

$$\begin{matrix}\text{scale} \rightarrow \\ \text{actual} \rightarrow\end{matrix} \quad \frac{2}{9} = \frac{1\frac{3}{4}}{d} \quad \begin{matrix}\leftarrow \text{scale} \\ \leftarrow \text{actual}\end{matrix}$$

$2(d) = 9\left(1\frac{3}{4}\right)$ Find the cross products.

$2d = \frac{63}{4}$ Simplify.

$2d \div 2 = \frac{63}{4} \div 2$ Divide each side by 2.

$d = \frac{63}{8}$ or $7\frac{7}{8}$ Simplify.

The actual distance is about $7\frac{7}{8}$ miles.

Check for Understanding

Concept Check

1. **OPEN ENDED** Find an example of ratios used in advertisements.
2. **Explain** the difference between a ratio and a proportion.
3. **Describe** how to solve a proportion if one of the ratios contains a variable.

Guided Practice

Use cross products to determine whether each pair of ratios form a proportion. Write *yes* or *no*.

4. $\frac{4}{11}, \frac{12}{33}$ **5.** $\frac{16}{17}, \frac{8}{9}$ **6.** $\frac{2.1}{3.5}, \frac{0.5}{0.7}$

Solve each proportion. If necessary, round to the nearest hundredth.

7. $\frac{3}{4} = \frac{6}{x}$ **8.** $\frac{a}{45} = \frac{5}{15}$ **9.** $\frac{0.6}{1.1} = \frac{n}{8.47}$

Application

10. TRAVEL The Lehmans' minivan requires 5 gallons of gasoline to travel 120 miles. How much gasoline will they need for a 350-mile trip?

Practice and Apply

Homework Help

For Exercises	See Examples
11–18	1, 2
19–30	3
31, 32	4
33, 34	5

Extra Practice
See page 827.

Use cross products to determine whether each pair of ratios form a proportion. Write *yes* or *no*.

11. $\frac{3}{2}, \frac{21}{14}$ **12.** $\frac{8}{9}, \frac{12}{18}$ **13.** $\frac{2.3}{3.4}, \frac{3.0}{3.6}$

14. $\frac{4.2}{5.6}, \frac{1.68}{2.24}$ **15.** $\frac{21.1}{14.4}, \frac{1.1}{1.2}$ **16.** $\frac{5}{2}, \frac{4}{1.6}$

SPORTS For Exercises 17 and 18, use the graph at the right.

17. Write a ratio of the number of gold medals won to the total number of medals won for each country.

18. Do any two of the ratios you wrote for Exercise 17 form a proportion? If so, explain the real-world meaning of the proportion.

USA TODAY Snapshots®

USA stands atop all-time medals table

The USA, which led the 2000 Summer Olympics with 97 medals, has dominated the medal standings over the years. The all-time Summer Olympics medal standings:

	Gold	Silver	Bronze	Total
USA	**871**	**659**	**586**	**2,116**
USSR/Russia[1]	498	409	371	1,278
Germany[2]	374	392	416	1,182
Great Britain	180	233	225	638
France	188	193	217	598
Italy	179	143	157	479
Sweden	136	156	177	469

1 – Competed as the Unified Team in 1992 after the breakup of the Soviet Union
2 – Totals include medals won by both East and West Germany.
Source: *The Ultimate Book of Sports Lists*

By Ellen J. Horrow and Marcy E. Mullins, USA TODAY

Solve each proportion. If necessary, round to the nearest hundredth.

19. $\frac{4}{x} = \frac{2}{10}$ **20.** $\frac{1}{y} = \frac{3}{15}$ **21.** $\frac{6}{5} = \frac{x}{15}$

22. $\frac{20}{28} = \frac{n}{21}$ **23.** $\frac{6}{8} = \frac{7}{a}$ **24.** $\frac{16}{7} = \frac{9}{b}$

25. $\frac{1}{0.19} = \frac{12}{n}$ **26.** $\frac{2}{0.21} = \frac{8}{n}$ **27.** $\frac{2.405}{3.67} = \frac{s}{1.88}$

28. $\frac{7}{1.066} = \frac{z}{9.65}$ **29.** $\frac{6}{14} = \frac{7}{x-3}$ **30.** $\frac{5}{3} = \frac{6}{x+2}$

WebQuest

A percent of increase or decrease can be used to describe trends in populations. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

31. **WORK** Seth earns \$152 in 4 days. At that rate, how many days will it take him to earn \$532?

32. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?

33. **BLUEPRINTS** A blueprint for a house states that 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall in the blueprint?

34. **MODELS** A collector's model racecar is scaled so that 1 inch on the model equals $6\frac{1}{4}$ feet on the actual car. If the model is $\frac{2}{3}$ inch high, how high is the actual car?

35. **PETS** A research study shows that three out of every twenty pet owners got their pet from a breeder. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from a breeder?

36. **CRITICAL THINKING** Consider the proportion $a{:}b{:}c = 3{:}1{:}5$. What is the value of $\frac{2a + 3b}{4b + 3c}$? (*Hint*: Choose different values of a, b, and c for which the proportion is true and evaluate the expression.)

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are ratios used in recipes?

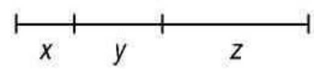
Include the following in your answer:
- an explanation of how to use a proportion to determine how much honey is needed if you use 3 eggs, and
- a description of how to alter the recipe to get 5 servings.

Standardized Test Practice

38. Which ratio is *not* equal to $\frac{9}{12}$?

Ⓐ $\frac{18}{24}$ Ⓑ $\frac{3}{4}$ Ⓒ $\frac{15}{20}$ Ⓓ $\frac{18}{27}$

39. In the figure at the right, $x{:}y = 2{:}3$ and $y{:}z = 3{:}5$. If $x = 10$, find the value of z.



Ⓐ 15 Ⓑ 20 Ⓒ 25 Ⓓ 30

Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. *(Lessons 3-4 and 3-5)*

40. $8y - 10 = -3y + 2$
41. $17 + 2n = 21 + 2n$
42. $-7(d - 3) = -4$
43. $5 - 9w = 23$
44. $\frac{m}{-5} + 6 = 31$
45. $\frac{z - 7}{5} = -3$

Find each product. *(Lesson 2-3)*

46. $(-7)(-6)$
47. $\left(-\frac{8}{9}\right)\left(\frac{9}{8}\right)$
48. $\left(\frac{3}{7}\right)\left(\frac{3}{7}\right)$
49. $(-0.075)(-5.5)$

Find each absolute value. *(Lesson 2-1)*

50. $|-33|$
51. $|77|$
52. $|2.5|$
53. $|-0.85|$

54. Sketch a reasonable graph for the temperature in the following statement. *In August, you enter a hot house and turn on the air conditioner.* *(Lesson 1-9)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each percent.** *(To review* ***percents****, see pages 802 and 803.)*

55. Eighteen is what percent of 60?
56. What percent of 14 is 4.34?
57. Six is what percent of 15?
58. What percent of 2 is 8?

3-7 Percent of Change

What You'll Learn

- Find percents of increase and decrease.
- Solve problems involving percents of change.

Vocabulary

- percent of change
- percent of increase
- percent of decrease

How can percents describe growth over time?

Phone companies began using area codes in 1947. The graph shows the number of area codes in use in different years. The growth in the number of area codes can be described by using a percent of change.

PERCENT OF CHANGE When an increase or decrease is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original, the percent of change is a **percent of decrease**.

Example 1 Find Percent of Change

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change.

a. original: 25
new: 28

Find the *amount* of change. Since the new amount is greater than the original, the percent of change is a percent of increase.

$28 - 25 = 3$

Find the percent using the original number, 25, as the base.

$$\begin{array}{rl} \text{change} \rightarrow & \dfrac{3}{25} = \dfrac{r}{100} \\ \text{original amount} \rightarrow & \end{array}$$

$$3(100) = 25(r)$$
$$300 = 25r$$
$$\frac{300}{25} = \frac{25r}{25}$$
$$12 = r$$

The percent of increase is 12%.

b. original: 30
new: 12

The percent of change is a percent of decrease because the new amount is less than the original. Find the change.

$30 - 12 = 18$

Find the percent using the original number, 30, as the base.

$$\begin{array}{rl} \text{change} \rightarrow & \dfrac{18}{30} = \dfrac{r}{100} \\ \text{original amount} \rightarrow & \end{array}$$

$$18(100) = 30(r)$$
$$1800 = 30r$$
$$\frac{1800}{30} = \frac{30r}{30}$$
$$60 = r$$

The percent of decrease is 60%.

Study Tip

Look Back
To review the **percent proportion**, see page 834.

More About . . .

Football

On November 12, 1892, the Allegheny Athletic Association paid William "Pudge" Heffelfinger \$500 to play football. This game is considered the start of professional football.

Source: *World Book Encyclopedia*

Example 2 Find the Missing Value

FOOTBALL **The field used by the National Football League (NFL) is 120 yards long. The length of the field used by the Canadian Football League (CFL) is 25% longer than the one used by the NFL. What is the length of the field used by the CFL?**

Let ℓ = the length of the CFL field. Since 25% is a percent of increase, the length of the NFL field is less than the length of the CFL field. Therefore, $\ell - 120$ represents the amount of change.

change $\rightarrow$ $\dfrac{\ell - 120}{120} = \dfrac{25}{100}$	Percent proportion
$(\ell - 120)(100) = 120(25)$	Find the cross products.
$100\ell - 12{,}000 = 3000$	Distributive Property
$100\ell - 12{,}000 + 12{,}000 = 3000 + 12{,}000$	Add 12,000 to each side.
$100\ell = 15{,}000$	Simplify.
$\dfrac{100\ell}{100} = \dfrac{15{,}000}{100}$	Divide each side by 100.
$\ell = 50$	Simplify.

The length of the field used by the CFL is 150 yards.

SOLVE PROBLEMS Two applications of percent of change are sales tax and discounts. Sales tax is a tax that is added to the cost of the item. It is an example of a percent of increase. Discount is the amount by which the regular price of an item is reduced. It is an example of a percent of decrease.

Example 3 Find Amount After Sales Tax

SALES TAX **A concert ticket costs \$45. If the sales tax is 6.25%, what is the total price of the ticket?**

The tax is 6.25% of the price of the ticket.

6.25% of \$45 = 0.0625×45	6.25% = 0.0625
= 2.8125	Use a calculator.

Round \$2.8125 to \$2.82 since tax is always rounded up to the nearest cent. Add this amount to the original price.

\$45.00 + \$2.82 = \$47.82

The total price of the ticket is \$47.82.

Example 4 Find Amount After Discount

DISCOUNT **A sweater is on sale for 35% off the original price. If the original price of the sweater is \$38, what is the discounted price?**

The discount is 35% of the original price.

35% of \$38 = 0.35×38	35% = 0.35
= 13.30	Use a calculator.

Subtract \$13.30 from the original price.

\$38.00 − \$13.30 = \$24.70

The discounted price of the sweater is \$24.70.

www.algebra1.com/extra_examples

CONTENTS

Check for Understanding

Concept Check

1. **Compare and contrast** percent of increase and percent of decrease.
2. **OPEN ENDED** Give a counterexample to the statement *The percent of change must always be less than 100%.*
3. **FIND THE ERROR** Laura and Cory are writing proportions to find the percent of change if the original number is 20 and the new number is 30.

Laura	Cory
Amount of change: 30 − 20 = 10	Amount of change: 30 − 20 = 10
$\frac{10}{20} = \frac{r}{100}$	$\frac{10}{30} = \frac{r}{100}$

Who is correct? Explain your reasoning.

Guided Practice

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

4. original: 72
new: 36
5. original: 45
new: 50
6. original: 14
new: 16
7. original: 150
new: 120

Find the total price of each item.

8. software: $39.50
sales tax: 6.5%
9. compact disc: $15.99
sales tax: 5.75%

Find the discounted price of each item.

10. jeans: $45.00
discount: 25%
11. book: $19.95
discount: 33%

Application

EDUCATION **For Exercises 12 and 13, use the following information.**
According to the Census Bureau, the average income of a person with a bachelor's degree is $40,478, and the average income of a person with a high school diploma is $22,895.

12. Write an equation that could be used to find the percent of increase in average income for a person with a high school diploma to average income for a person with a bachelor's degree.
13. What is the percent of increase?

Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1
28–30, 46, 47	2
31–36	3
37–42	4
43–45	3, 4

Extra Practice
See page 827.

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

14. original: 50
new: 70
15. original: 25
new: 18
16. original: 66
new: 30
17. original: 58
new: 152
18. original: 13.7
new: 40.2
19. original: 15.6
new: 11.4
20. original: 132
new: 150
21. original: 85
new: 90
22. original: 32.5
new: 30
23. original: 9.8
new: 12.1
24. original: 40
new: 32.5
25. original: 25
new: 21.5

26. **THEME PARKS** In 1990, 253 million people visited theme parks in the United States. In 2000, the number of visitors increased to 317 million people. What was the percent of increase?

27. **MILITARY** In 1987, the United States had 2 million active-duty military personnel. By 2000, there were only 1.4 million active-duty military personnel. What was the percent of decrease?

28. The percent of increase is 16%. If the new number is 522, find the original number.

29. **FOOD** In order for a food to be marked "reduced fat," it must have at least 25% less fat than the same full-fat food. If one ounce of reduced fat chips has 6 grams of fat, what is the least amount of fat in one ounce of regular chips?

30. **TECHNOLOGY** From January, 1996, to January, 2001, the number of internet hosts increased by 1054%. There were 109.6 million internet hosts in January, 2001. Find the number of internet hosts in January, 1996.

Career Choices

Military

A military career can involve many different duties like working in a hospital, programming computers, or repairing helicopters. The military provides training and work in these fields and others for the Army, Navy, Marine Corps, Air Force, Coast Guard, and the Air and Army National Guard.

Online Research
For information about a career in the military, visit: www.algebra1.com/careers

Find the total price of each item.

31. umbrella: $14.00
tax: 5.5%

32. backpack: $35.00
tax: 7%

33. candle: $7.50
tax: 5.75%

34. hat: $18.50
tax: 6.25%

35. clock radio: $39.99
tax: 6.75%

36. sandals: $29.99
tax: 5.75%

Find the discounted price of each item.

37. shirt: $45.00
discount: 40%

38. socks: $6.00
discount: 20%

39. watch: $37.55
discount: 35%

40. gloves: $24.25
discount: 33%

41. suit: $175.95
discount: 45%

42. coat: $79.99
discount: 30%

Find the final price of each item.

43. lamp: $120.00
discount: 20%
tax: 6%

44. dress: $70.00
discount: 30%
tax: 7%

45. camera: $58.00
discount: 25%
tax: 6.5%

POPULATION For Exercises 46 and 47, use the following table.

Country	1997 Population (billions)	Projected Percent of Increase for 2050
China	1.24	22.6%
India	0.97	57.8%
United States	0.27	44.4%

Source: *USA TODAY*

46. What are the projected 2050 populations for each country in the table?

47. Which of these three countries is projected to be the most populous in 2050?

48. **RESEARCH** Use the Internet or other reference to find the tuition for the last several years at a college of your choice. Find the percent of change for the tuition during these years. Predict the tuition for the year you plan to graduate from high school.

49. **CRITICAL THINKING** Are the following expressions *sometimes*, *always*, or *never* equal? Explain your reasoning.

$x\%$ of y $\qquad\qquad$ $y\%$ of x

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can percents describe growth over time?

Include the following in your answer:

- the percent of increase in the number of area codes from 1996 to 1999, and
- an explanation of why knowing a percent of change can be more informative than knowing how much the quantity changed.

Standardized Test Practice

51. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. Which proportion represents the percent of change?

(A) $\frac{70}{910} = \frac{r}{100}$ (B) $\frac{70}{840} = \frac{r}{100}$ (C) $\frac{r}{910} = \frac{70}{100}$ (D) $\frac{r}{840} = \frac{70}{100}$

52. The list price of a television is \$249.00. If it is on sale for 30% off the list price, what is the sale price of the television?

(A) \$74.70 (B) \$149.40 (C) \$174.30 (D) \$219.00

Maintain Your Skills

Mixed Review

Solve each proportion. *(Lesson 3-6)*

53. $\frac{a}{45} = \frac{3}{15}$ 54. $\frac{2}{3} = \frac{8}{d}$ 55. $\frac{5.22}{13.92} = \frac{t}{48}$

Solve each equation. Then check your solution. *(Lesson 3-5)*

56. $6n + 3 = -3$ 57. $7 + 5c = -23$ 58. $18 = 4a - 2$

Find each quotient. *(Lesson 2-4)*

59. $\frac{2}{5} \div 4$ 60. $-\frac{4}{5} \div \frac{2}{3}$ 61. $-\frac{1}{9} \div \left(-\frac{3}{4}\right)$

State whether each equation is *true* or *false* for the value of the variable given. *(Lesson 1-3)*

62. $a^2 + 5 = 17 - a, a = 3$ 63. $2v^2 + v = 65, v = 5$

64. $8y - y^2 = y + 10, y = 4$ 65. $16p - p = 15p, p = 2.5$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Then check your solution.**
*(To review **solving equations**, see Lesson 3-5.)*

66. $-43 - 3t = 2 - 6t$ 67. $7y + 7 = 3y - 5$ 68. $7(d - 3) - 2 = 5$

69. $6(p + 3) = 4(p - 1)$ 70. $-5 = 4 - 2(a - 5)$ 71. $8x - 4 = -10x + 50$

Practice Quiz 2 — Lessons 3-4 through 3-7

Solve each equation. Then check your solution. *(Lessons 3-4 and 3-5)*

1. $-3x - 7 = 18$ 2. $5 = \frac{m - 5}{4}$ 3. $4h + 5 = 11$

4. $5d - 6 = 3d + 9$ 5. $7 + 2(w + 1) = 2w + 9$ 6. $-8(4 + 9r) = 7(-2 - 11r)$

Solve each proportion. *(Lesson 3-6)*

7. $\frac{2}{10} = \frac{1}{a}$ 8. $\frac{3}{5} = \frac{24}{x}$ 9. $\frac{y}{4} = \frac{y + 5}{8}$

10. **POSTAGE** In 1975, the cost of a first-class stamp was 10¢. In 2001, the cost of a first-class stamp became 34¢. What is the percent of increase in the price of a stamp? *(Lesson 3-7)*

Reading Mathematics

Sentence Method and Proportion Method

Recall that you can solve percent problems using two different methods. With either method, it is helpful to use "clue" words such as *is* and *of*. In the sentence method, *is* means equals and *of* means multiply. With the proportion method, the "clue" words indicate where to place the numbers in the proportion.

Sentence Method

15% of 40 is what number?

$0.15 \cdot 40 = ?$

Proportion Method

15% of 40 is what number?

$\frac{\text{(is) } P}{\text{(of) } B} = \frac{R\text{(percent)}}{100} \rightarrow \frac{P}{40} = \frac{15}{100}$

You can use the proportion method to solve percent of change problems. In this case, use the proportion $\frac{\text{difference}}{\text{original}} = \frac{\%}{100}$. When reading a percent of change problem, or any other word problem, look for the important numerical information.

Example In chemistry class, Kishi heated 20 milliliters of water. She let the water boil for 10 minutes. Afterward, only 17 milliliters of water remained, due to evaporation. What is the percent of decrease in the amount of water?

$\frac{\text{difference}}{\text{original}} = \frac{\%}{100} \rightarrow \frac{20 - 17}{20} = \frac{r}{100}$	Percent proportion
$\frac{3}{20} = \frac{r}{100}$	Simplify.
$3(100) = 20(r)$	Find the cross products.
$300 = 20r$	Simplify.
$\frac{300}{20} = \frac{20r}{20}$	Divide each side by 20.
$15 = r$	Simplify.

There was a 15% decrease in the amount of water.

Reading to Learn

Give the original number and the amount of change. Then write and solve a percent proportion.

1. Monsa needed to lose weight for wrestling. At the start of the season, he weighed 166 pounds. By the end of the season, he weighed 158 pounds. What is the percent of decrease in Monsa's weight?

2. On Carla's last Algebra test, she scored 94 points out of 100. On her first Algebra test, she scored 75 points out of 100. What is the percent of increase in her score?

3. In a catalog distribution center, workers processed an average of 12 orders per hour. After a reward incentive was offered, workers averaged 18 orders per hour. What is the percent of increase in production?

3-8 Solving Equations and Formulas

What You'll Learn

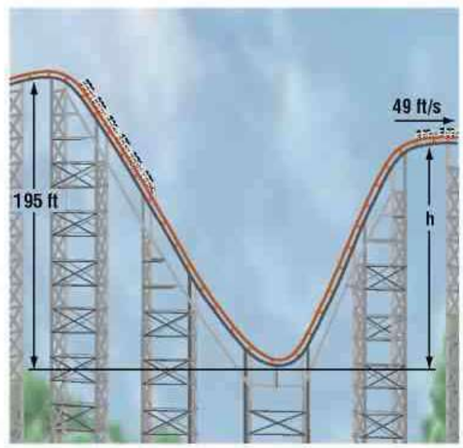
- Solve equations for given variables.
- Use formulas to solve real-world problems.

Vocabulary

- dimensional analysis

How are equations used to design roller coasters?

Ron Toomer designs roller coasters, including the Magnum XL-200. This roller coaster starts with a vertical drop of 195 feet and then ascends a second shorter hill. Suppose when designing this coaster, Mr. Toomer decided he wanted to adjust the height of the second hill so that the coaster would have a speed of 49 feet per second when it reached its top.



If we ignore friction, the equation $g(195 - h) = \frac{1}{2}v^2$ can be used to find the height of the second hill. In this equation, g represents the force of gravity (32 feet per second squared), h is the height of the second hill, and v is the velocity of the coaster when it reaches the top of the second hill.

SOLVE FOR VARIABLES Some equations such as the one above contain more than one variable. At times, you will need to solve these equations for one of the variables.

Example 1 Solve an Equation for a Specific Variable

Solve $3x - 4y = 7$ for y.

$3x - 4y = 7$	Original equation
$3x - 4y - 3x = 7 - 3x$	Subtract $3x$ from each side.
$-4y = 7 - 3x$	Simplify.
$\frac{-4y}{-4} = \frac{7 - 3x}{-4}$	Divide each side by -4.
$y = \frac{7 - 3x}{-4}$ or $\frac{3x - 7}{4}$	Simplify.

The value of y is $\frac{3x - 7}{4}$.

It is sometimes helpful to use the Distributive Property to isolate the variable for which you are solving an equation or formula.

Example 2 Solve an Equation for a Specific Variable

Solve $2m - t = sm + 5$ for m.

$2m - t = sm + 5$	Original equation
$2m - t - sm = sm + 5 - sm$	Subtract sm from each side.
$2m - t - sm = 5$	Simplify.
$2m - t - sm + t = 5 + t$	Add t to each side.
$2m - sm = 5 + t$	Simplify.
$m(2 - s) = 5 + t$	Use the Distributive Property.
$\frac{m(2-s)}{2-s} = \frac{5+t}{2-s}$	Divide each side by $2 - s$.
$m = \frac{5+t}{2-s}$	Simplify.

The value of m is $\frac{5+t}{2-s}$. Since division by 0 is undefined, $2 - s \neq 0$ or $s \neq 2$.

USE FORMULAS Many real-world problems require the use of formulas. Sometimes solving a formula for a specific variable will help you solve the problem.

Example 3 Use a Formula to Solve Problems

WEATHER **Use the information about the Kansas City hailstorm at the left. The formula for the circumference of a circle is $C = 2\pi r$, where C represents circumference and r represent radius.**

a. Solve the formula for r.

$C = 2\pi r$	Formula for circumference
$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$	Divide each side by 2π.
$\frac{C}{2\pi} = r$	Simplify.

b. Find the radius of one of the large hailstones that fell on Kansas City in 1898.

$\frac{C}{2\pi} = r$	Formula for radius
$\frac{9.5}{2\pi} = r$	$C = 9.5$
$1.5 \approx r$	The largest hailstones had a radius of about 1.5 inches.

More About. . .

Weather

On May 14, 1898, a severe hailstorm hit Kansas City. The largest hailstones were 9.5 inches in circumference. Windows were broken in nearly every house in the area.

Source: National Weather Service

When using formulas, you may want to use dimensional analysis. **Dimensional analysis** is the process of carrying units throughout a computation.

Example 4 Use Dimensional Analysis

PHYSICAL SCIENCE **The formula $s = \frac{1}{2}at^2$ represents the distance s that a free-falling object will fall near a planet or the moon in a given time t. In the formula, a represents the acceleration due to gravity.**

a. Solve the formula for a.

$s = \frac{1}{2}at^2$	Original formula
$\frac{2}{t^2}(s) = \frac{2}{t^2}\left(\frac{1}{2}at^2\right)$	Multiply each side by $\frac{2}{t^2}$.
$\frac{2s}{t^2} = a$	Simplify.

b. **A free-falling object near the moon drops 20.5 meters in 5 seconds. What is the value of a for the moon?**

$a = \frac{2s}{t^2}$ Formula for a

$a = \frac{2(20.5m)}{(5s)^2}$ $s = 20.5m$ and $t = 5s$.

$a = \frac{1.64m}{s^2}$ or $1.64\ m/s^2$ Use a calculator.

The acceleration due to gravity on the moon is 1.64 meters per second squared.

Check for Understanding

Concept Check

1. **List** the steps you would use to solve $ax - y = az + w$ for a.
2. **Describe** the possible values of t if $s = \frac{r}{t-2}$.
3. **OPEN ENDED** Write a formula for A, the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than A.

Guided Practice

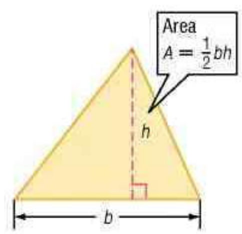
Solve each equation or formula for the variable specified.

4. $-3x + b = 6x$, for x
5. $-5a + y = -54$, for a
6. $4z + b = 2z + c$, for z
7. $\frac{y+a}{3} = c$, for y
8. $p = a(b + c)$, for a
9. $mw - t = 2w + 5$, for w

Application

GEOMETRY **For Exercises 10–12, use the formula for the area of a triangle.**

10. Find the area of a triangle with a base of 18 feet and a height of 7 feet.
11. Solve the formula for h.
12. What is the height of a triangle with area of 28 square feet and base of 8 feet?



Practice and Apply

Homework Help

For Exercises	See Examples
13–30	1, 2
31–41	3, 4

Extra Practice

See page 827.

Solve each equation or formula for the variable specified.

13. $5g + h = g$, for g
14. $8t - r = 12t$, for t
15. $y = mx + b$, for m
16. $v = r + at$, for a
17. $3y + z = am - 4y$, for y
18. $9a - 2b = c + 4a$, for a
19. $km + 5x = 6y$, for m
20. $4b - 5 = -t$, for b
21. $\frac{3ax - n}{5} = -4$, for x
22. $\frac{5x + y}{a} = 2$, for a
23. $\frac{by + 2}{3} = c$, for y
24. $\frac{6c - t}{7} = b$, for c
25. $c = \frac{3}{4}y + b$, for y
26. $\frac{3}{5}m + a = b$, for m
27. $S = \frac{n}{2}(A + t)$, for A
28. $p(t + 1) = -2$, for t
29. $at + b = ar - c$, for a
30. $2g - m = 5 - gh$, for g

Write an equation and solve for the variable specified.

31. Five less than a number t equals another number r plus six. Solve for t.

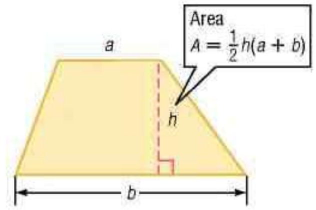
32. Five minus twice a number p equals six times another number q plus one. Solve for p.

33. Five eighths of a number x is three more than one half of another number y. Solve for y.

GEOMETRY **For Exercises 34 and 35, use the formula for the area of a trapezoid.**

34. Solve the formula for h.

35. What is the height of a trapezoid with an area of 60 square meters and bases of 8 meters and 12 meters?



WORK **For Exercises 36 and 37, use the following information.**

The formula $s = \frac{w - 10e}{m}$ is often used by placement services to find keyboarding speeds. In the formula, s represents the speed in words per minute, w represents the number of words typed, e represents the number of errors, and m represents the number of minutes typed.

36. Solve the formula for e.

37. If Miguel typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make?

FLOORING **For Exercises 38 and 39, use the following information.**

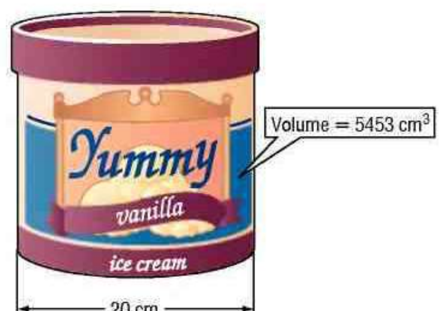
The formula $P = \frac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by the heel of a shoe. In this formula, P represents the pressure in pounds per square inch, W represents the weight of a person wearing the shoe in pounds, and H is the width of the heel of the shoe in inches.

38. Solve the formula for W.

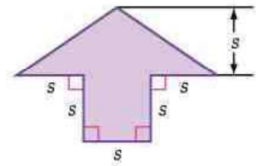
39. Find the weight of the person if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

40. **ROCKETRY** In the book *October Sky,* high school students were experimenting with different rocket designs. One formula they used was $R = \frac{S + F + P}{S + P}$, which relates the mass ratio R of a rocket to the mass of the structure S, the mass of the fuel F, and the mass of the payload P. The students needed to determine how much fuel to load in the rocket. How much fuel should be loaded in a rocket whose basic structure and payload each have a mass of 900 grams, if the mass ratio is to be 6?

41. **PACKAGING** The Yummy Ice Cream Company wants to package ice cream in cylindrical containers that have a volume of 5453 cubic centimeters. The marketing department decides the diameter of the base of the containers should be 20 centimeters. How tall should the containers be? (*Hint*: $V = \pi r^2 h$)



42. **CRITICAL THINKING** Write a formula for the area of the arrow.



43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are equations used to design roller coasters?

Include the following in your answer:

- a list of steps you could use to solve the equation for h, and
- the height of the second hill of the roller coaster.

Standardized Test Practice

44. If $2x + y = 5$, what is the value of $4x$?

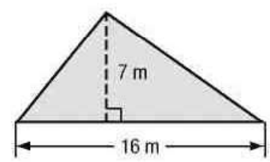
Ⓐ $10 - y$ Ⓑ $10 - 2y$

Ⓒ $\frac{5 - y}{2}$ Ⓓ $\frac{10 - y}{2}$

45. What is the area of the triangle?

Ⓐ 23 m^2 Ⓑ 28 m^2

Ⓒ 56 m^2 Ⓓ 112 m^2



Maintain Your Skills

Mixed Review

Find the discounted price of each item. *(Lesson 3-7)*

46. camera: \$85.00
discount: 20%

47. scarf: \$15.00
discount: 35%

48. television: \$299.00
discount: 15%

Solve each proportion. *(Lesson 3-6)*

49. $\frac{2}{9} = \frac{5}{a}$

50. $\frac{15}{32} = \frac{t}{8}$

51. $\frac{x + 1}{8} = \frac{3}{4}$

Write the numbers in each set in order from least to greatest. *(Lesson 2-7)*

52. $\frac{1}{4}, \sqrt{\frac{1}{4}}, 0.\overline{5}, 0.2$

53. $\sqrt{5}, 3, \frac{2}{3}, 1.1$

Find each sum or difference. *(Lesson 2-2)*

54. $2.18 + (-5.62)$

55. $-\frac{1}{2} - \left(-\frac{3}{4}\right)$

56. $-\frac{2}{3} - \frac{2}{5}$

Name the property illustrated by each statement. *(Lesson 1-4)*

57. $mnp = 1mnp$

58. If $6 = 9 - 3$, then $9 - 3 = 6$.

59. $32 + 21 = 32 + 21$

60. $8 + (3 + 9) = 8 + 12$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Use the Distributive Property to rewrite each expression without parentheses.** *(To review the **Distributive Property**, see Lesson 1-5.)*

61. $6(2 - t)$

62. $(5 + 2m)3$

63. $-7(3a + b)$

64. $\frac{2}{3}(6h - 9)$

65. $-\frac{3}{5}(15 - 5t)$

66. $0.25(6p + 12)$

3-9 Weighted Averages

What You'll Learn

- Solve mixture problems.
- Solve uniform motion problems.

Vocabulary

- weighted average
- mixture problem
- uniform motion problem

How are scores calculated in a figure skating competition?

In individual figure skating competitions, the score for the long program is worth twice the score for the short program. Suppose Olympic gold medal winner Ilia Kulik scores 5.5 in the short program and 5.8 in the long program at a competition. His final score is determined using a weighted average.

$$\frac{5.5(1) + 5.8(2)}{1 + 2} = \frac{5.5 + 11.6}{3}$$

$$= \frac{17.1}{3} \text{ or } 5.7 \quad \text{His final score would be 5.7.}$$

MIXTURE PROBLEMS Ilia Kulik's average score is an example of a weighted average. The **weighted average** M of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages.

Example 1 Solve a Mixture Problem with Prices

TRAIL MIX **Assorted dried fruit sells for \$5.50 per pound. How many pounds of mixed nuts selling for \$4.75 per pound should be mixed with 10 pounds of dried fruit to obtain a trail mix that sells for \$4.95 per pound?**

Let w = the number of pounds of mixed nuts in the mixture. Make a table.

	Units (lb)	Price per Unit (lb)	Total Price
Dried Fruit	10	\$5.50	5.50(10)
Mixed Nuts	w	\$4.75	$4.75w$
Trail Mix	$10 + w$	\$4.95	$4.95(10 + w)$

$$\underbrace{5.50(10)}_{\text{Price of dried fruit}} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{4.75w}_{\text{price of nuts}} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{4.95(10 + w)}_{\text{price of trail mix.}}$$

$5.50(10) + 4.75w = 4.95(10 + w)$ — Original equation

$55.00 + 4.75w = 49.50 + 4.95w$ — Distributive Property

$55.00 + 4.75w - 4.75w = 49.50 + 4.95w - 4.75w$ — Subtract $4.75w$ from each side.

$55.00 = 49.50 + 0.20w$ — Simplify.

$55.00 - 49.50 = 49.50 + 0.20w - 49.50$ — Subtract 49.50 from each side.

$5.50 = 0.20w$ — Simplify.

$\frac{5.50}{0.20} = \frac{0.20w}{0.20}$ — Divide each side by 0.20.

$27.5 = w$ — Simplify.

27.5 pounds of nuts should be mixed with 10 pounds of dried fruit.

Sometimes mixture problems are expressed in terms of percents.

Example 2 Solve a Mixture Problem with Percents

SCIENCE A chemistry experiment calls for a 30% solution of copper sulfate. Kendra has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?

Let x = the amount of 60% solution to be added. Make a table.

	Amount of Solution (mL)	Amount of Copper Sulfate
25% Solution	40	0.25(40)
60% Solution	x	$0.60x$
30% Solution	$40 + x$	$0.30(40 + x)$

Study Tip

Mixture Problems
When you organize the information in mixture problems, remember that the final mixture must contain the sum of the parts in the correct quantities and at the correct percents.

Write and solve an equation using the information in the table.

$$\underbrace{0.25(40)}_{\text{Amount of copper sulfate in 25\% solution}} \underbrace{+}_{\text{plus}} \underbrace{0.60x}_{\text{amount of copper sulfate in 60\% solution}} \underbrace{=}_{\text{equals}} \underbrace{0.30(40 + x)}_{\text{amount of copper sulfate in 30\% solution.}}$$

$0.25(40) + 0.60x = 0.30(40 + x)$	Original equation
$10 + 0.60x = 12 + 0.30x$	Distributive Property
$10 + 0.60x - 0.30x = 12 + 0.30x - 0.30x$	Subtract 0.30x from each side.
$10 + 0.30x = 12$	Simplify.
$10 + 0.30x - 10 = 12 - 10$	Subtract 10 from each side.
$0.30x = 2$	Simplify.
$\frac{0.30x}{0.30} = \frac{2}{0.30}$	Divide each side by 0.30.
$x \approx 6.67$	Simplify.

Kendra should add 6.67 milliliters of the 60% solution to the 40 milliliters of the 25% solution.

UNIFORM MOTION PROBLEMS Motion problems are another application of weighted averages. **Uniform motion problems** are problems where an object moves at a certain speed, or rate. The formula $d = rt$ is used to solve these problems. In the formula, d represents distance, r represents rate, and t represents time.

Example 3 Solve for Average Speed

TRAVEL On Alberto's drive to his aunt's house, the traffic was light, and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

To find the average speed for each leg of the trip, rewrite $d = rt$ as $r = \frac{d}{t}$.

Going

$r = \frac{d}{t}$

$= \frac{45 \text{ miles}}{1 \text{ hour}}$ or 45 miles per hour

Returning

$r = \frac{d}{t}$

$= \frac{45 \text{ miles}}{2 \text{ hours}}$ or 22.5 miles per hour

You may think that the average speed of the trip would be $\frac{45 + 22.5}{2}$ or 33.75 miles per hour. However, Alberto did not drive at these speeds for equal amounts of time. You must find the weighted average for the trip.

Round Trip

$M = \frac{45(1) + 22.5(2)}{1 + 2}$ Definition of weighted average

$= \frac{90}{3}$ or 30 Simplify.

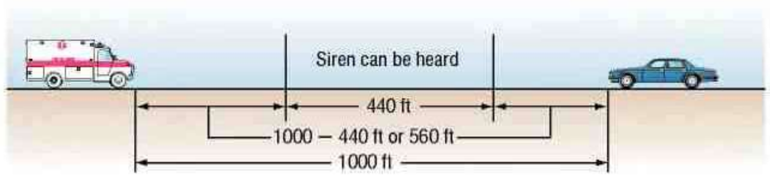
Alberto's average speed was 30 miles per hour.

Sometimes a table is useful in solving uniform motion problems.

Example 4 Solve a Problem Involving Speeds of Two Vehicles

SAFETY Use the information about sirens at the left. A car and an emergency vehicle are heading toward each other. The car is traveling at a speed of 30 miles per hour or about 44 feet per second. The emergency vehicle is traveling at a speed of 50 miles per hour or about 74 feet per second. If the vehicles are 1000 feet apart and the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

Draw a diagram. The driver can hear the siren when the total distance traveled by the two vehicles equals 1000 − 440 or 560 feet.



Let t = the number of seconds until the driver can hear the siren. Make a table of the information.

	r	t	$d = rt$
Car	44	t	$44t$
Emergency Squad	74	t	$74t$

Write an equation.

$$\underbrace{44t}_{\text{Distance traveled by car}} \underbrace{+}_{\text{plus}} \underbrace{74t}_{\text{distance traveled by emergency vehicle}} \underbrace{=}_{\text{equals}} \underbrace{560}_{\text{560 feet.}}$$

Solve the equation.

$44t + 74t = 560$ Original equation

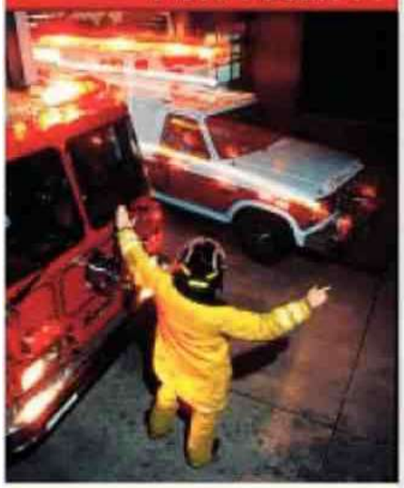
$118t = 560$ Simplify.

$\frac{118t}{118} = \frac{560}{118}$ Divide each side by 118.

$t \approx 4.75$ Round to the nearest hundredth.

The driver of the car will hear the siren in about 4.75 seconds.

More About...



Safety

Under ideal conditions, a siren can be heard from up to 440 feet. However, under normal conditions, a siren can be heard from only 125 feet.

Source: U.S. Department of Transportation

Check for Understanding

Concept Check

1. **OPEN ENDED** Give a real-world example of a weighted average.
2. **Write** the formula used to solve uniform motion problems and tell what each letter represents.
3. **Make a table** that can be used to solve the following problem. *Lakeisha has $2.55 in dimes and quarters. She has 8 more dimes than quarters. How many quarters does she have?*

Guided Practice

FOOD For Exercises 4–7, use the following information.
How many quarts of pure orange juice should Michael add to a 10% orange drink to create 6 quarts of a 40% orange juice mixture? Let p represent the number of quarts of pure orange juice he should add to the orange drink.

4. Copy and complete the table representing the problem.

	Quarts	Amount of Orange Juice
10% Juice	$6 - p$	
100% Juice	p	
40% Juice		

5. Write an equation to represent the problem.
6. How much pure orange juice should Michael use?
7. How much 10% juice should Michael use?
8. **BUSINESS** The Nut Shoppe sells walnuts for $4.00 a pound and cashews for $7.00 a pound. How many pounds of cashews should be mixed with 10 pounds of walnuts to obtain a mixture that sells for $5.50 a pound?
9. **GRADES** Many schools base a student's grade point average, or GPA, on the student's grade and the class credit rating. Brittany's grade card for this semester is shown. Find Brittany's GPA if a grade of A equals 4 and a B equals 3.

Grade Card

Class	Credit Rating	Grade
Algebra 1	1	A
Science	1	B
English	1	A
Spanish	1	B
Phys. Ed.	$\frac{1}{2}$	A

10. **CYCLING** Two cyclists begin traveling in the same direction on the same bike path. One travels at 20 miles per hour, and the other travels at 14 miles per hour. When will the cyclists be 15 miles apart?

Practice and Apply

Homework Help

For Exercises	See Examples
11–18, 22–25, 27–29, 33	1, 2
19–21, 26, 30–32, 34	3, 4

Extra Practice

See page 828.

BUSINESS For Exercises 11–14, use the following information.
Cookies Inc. sells peanut butter cookies for $6.50 per dozen and chocolate chip cookies for $9.00 per dozen. Yesterday, they sold 85 dozen more peanut butter cookies than chocolate chip cookies. The total sales for both types of cookies were $4055.50. Let p represent the number of dozens of peanut butter cookies sold.

11. Copy and complete the table representing the problem.

	Number of Dozens	Price per Dozen	Total Price
Peanut Butter Cookies	p		
Chocolate Chip Cookies	$p - 85$		

12. Write an equation to represent the problem.
13. How many dozen peanut butter cookies were sold?
14. How many dozen chocolate chip cookies were sold?

METALS **For Exercises 15–18, use the following information.**
In 2000, the international price of gold was \$270 per ounce, and the international price of silver was \$5 per ounce. Suppose gold and silver were mixed to obtain 15 ounces of an alloy worth \$164 per ounce. Let g represent the amount of gold used in the alloy.

15. Copy and complete the table representing the problem.

	Number of Ounces	Price per Ounce	Value
Gold	g		
Silver	$15 - g$		
Alloy			

16. Write an equation to represent the problem.

17. How much gold was used in the alloy?

18. How much silver was used in the alloy?

TRAVEL **For Exercises 19–21, use the following information.**
Two trains leave Pittsburgh at the same time, one traveling east and the other traveling west. The eastbound train travels at 40 miles per hour, and the westbound train travels at 30 miles per hour. Let t represent the amount of time since their departure.

19. Copy and complete the table representing the situation.

	r	t	$d = rt$
Eastbound Train			
Westbound Train			

20. Write an equation that could be used to determine when the trains will be 245 miles apart.

21. In how many hours will the trains be 245 miles apart?

22. **FUND-RAISING** The Madison High School marching band sold gift wrap. The gift wrap in solid colors sold for \$4.00 per roll, and the print gift wrap sold for \$6.00 per roll. The total number of rolls sold was 480, and the total amount of money collected was \$2340. How many rolls of each kind of gift wrap were sold?

23. **COFFEE** Charley Baroni owns a specialty coffee store. He wants to create a special mix using two coffees, one priced at \$6.40 per pound and the other priced at \$7.28 per pound. How many pounds of the \$7.28 coffee should he mix with 9 pounds of the \$6.40 coffee to sell the mixture for \$6.95 per pound?

24. **FOOD** Refer to the graphic at the right. How much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat?

25. **METALS** An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each alloy should be used to make 1000 grams of an alloy that is 45% copper?

26. **TRAVEL** An airplane flies 1000 miles due east in 2 hours and 1000 miles due south in 3 hours. What is the average speed of the airplane?

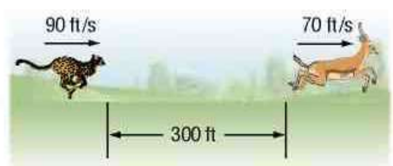
27. **SCIENCE** Hector is performing a chemistry experiment that requires 140 milliliters of a 30% copper sulfate solution. He has a 25% copper sulfate solution and a 60% copper sulfate solution. How many milliliters of each solution should he mix to obtain the needed solution?

28. **CAR MAINTENANCE** One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

29. **GRADES** In Ms. Martinez's science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student's average grade?

30. **RESCUE** A fishing trawler has radioed the Coast Guard for a helicopter to pick up an injured crew member. At the time of the emergency message, the trawler is 660 kilometers from the helicopter and heading toward it. The average speed of the trawler is 30 kilometers per hour, and the average speed of the helicopter is 300 kilometers per hour. How long will it take the helicopter to reach the trawler?

31. **ANIMALS** A cheetah is 300 feet from its prey. It starts to sprint toward its prey at 90 feet per second. At the same time, the prey starts to sprint at 70 feet per second. When will the cheetah catch its prey?



32. **TRACK AND FIELD** A sprinter has a bad start, and his opponent is able to start 1 second before him. If the sprinter averages 8.2 meters per second and his opponent averages 8 meters per second, will he be able to catch his opponent before the end of the 200-meter race? Explain.

33. **CAR MAINTENANCE** A car radiator has a capacity of 16 quarts and is filled with a 25% antifreeze solution. How much must be drained off and replaced with pure antifreeze to obtain a 40% antifreeze solution?

34. **TRAVEL** An express train travels 80 kilometers per hour from Ironton to Wildwood. A local train, traveling at 48 kilometers per hour, takes 2 hours longer for the same trip. How far apart are Ironton and Wildwood?

35. **FOOTBALL** NFL quarterbacks are rated for their passing performance by a type of weighted average as described in the formula below.
$R = [50 + 2000(C \div A) + 8000(T \div A) - 10{,}000(I \div A) + 100(Y \div A)] \div 24$
In this formula,
- R represents the rating,
- C represents number of completions,
- A represents the number of passing attempts,
- T represents the number to touchdown passes,
- I represents the number of interceptions, and
- Y represents the number of yards gained by passing.

In the 2000 season, Daunte Culpepper had 297 completions, 474 passing attempts, 33 touchdown passes, 16 interceptions, and 3937 passing yards. What was his rating for that year?

Online Research **Data Update** What is the current passing rating for your favorite quarterback? Visit www.algebra1.com/data_update to get statistics on quarterbacks.

36. **CRITICAL THINKING** Write a mixture problem for the equation $1.00x + 0.28(40) = 0.40(x + 40)$.

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are scores calculated in a figure skating competition?

Include the following in your answer:

- an explanation of how a weighted average can be used to find a skating score, and
- a demonstration of how to find the weighted average of a skater who received a 4.9 in the short program and a 5.2 in the long program.

Standardized Test Practice

38. Eula Jones is investing \$6000 in two accounts, part at 4.5% and the remainder at 6%. If d represents the number of dollars invested at 4.5%, which expression represents the amount of interest earned in the account paying 6%?

(A) $0.06d$ (B) $0.06(d - 6000)$

(C) $0.06(d + 6000)$ (D) $0.06(6000 - d)$

39. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was his average speed?

(A) 38.5 mph (B) 40 mph (C) 44 mph (D) 47.5 mph

Maintain Your Skills

Mixed Review

Solve each equation for the variable specified. *(Lesson 3-8)*

40. $3t - 4 = 6t - s$, for t

41. $a + 6 = \frac{b-1}{4}$, for b

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent. *(Lesson 3-7)*

42. original: 25
new: 14

43. original: 35
new: 42

44. original: 244
new: 300

45. If the probability that an event will occur is $\frac{2}{3}$, what are the odds that the event will occur? *(Lesson 2-6)*

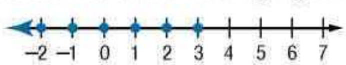
Simplify each expression. *(Lesson 2-3)*

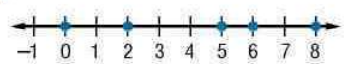
46. $(2b)(-3a)$

47. $3x(-3y) + (-6x)(-2y)$

48. $5s(-6t) + 2s(-8t)$

Name the set of numbers graphed. *(Lesson 2-1)*

49. 

50. 

WebQuest Internet Project

Can You Fit 100 Candles on a Cake?

It's time to complete your project. Use the information and data you have gathered about living to be 100 to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

www.algebra1.com/webquest

Spreadsheet Investigation

A Follow-Up of Lesson 3-9

Finding a Weighted Average

You can use a computer spreadsheet program to calculate weighted averages. A spreadsheet allows you to make calculations and print almost anything that can be organized in a table.

The basic unit in a spreadsheet is called a **cell**. A cell may contain numbers, words, or a formula. Each cell is named by the column and row that describe its location. For example, cell B4 is in column B, row 4.

Example

Greta Norris manages the Java Roaster Coffee Shop. She has entered the price per pound and the number of pounds sold in October for each type of coffee in a spreadsheet. What was the average price per pound of coffee sold?

October Sales

	A	B	C	D
1	Product	Price per Pound	Pounds Sold	Income
2	Hawaiian Cafe	16.95	59	=B2*C2
3	Mocha Java	12.59	85	=B3*C3
4	House Blend	10.75	114	=B4*C4
5	Decaf Espresso	10.15	75	=B5*C5
6	Breakfast Blend	11.25	93	=B6*C6
7	Italian Roast	9.95	55	=B7*C7
8	Total		=SUM(C2:C7)	=SUM(D2:D7)
9	Weighted Average	=D8 / C8		
10				

Sheet1 Sheet2 Sheet3

The spreadsheet shows the formula that will calculate the weighted average. The formula multiplies the price of each product by its volume and calculates its sum for all the products. Then it divides that value by the sum of the volume for all products together. To the nearest cent, the weighted average of a pound of coffee is $11.75.

Exercises

For Exercises 1–4, use the spreadsheet of coffee prices.

1. What is the average price of a pound of coffee for the November sales shown in the table at the right?

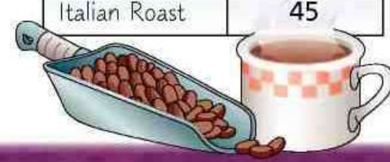
2. How does the November weighted average change if all of the coffee prices are increased by $1.00?

3. How does the November weighted average change if all of the coffee prices are increased by 10%?

4. Find the weighted average of a pound of coffee if the shop sold 50 pounds of each type of coffee. How does the weighted average compare to the average of the per-pound coffee prices? Explain.

November Sales

Product	Pounds Sold
Hawaiian Cafe	56
Mocha Java	97
House Blend	124
Decaf Espresso	71
Breakfast Blend	69
Italian Roast	45



Chapter 3 Study Guide and Review

Vocabulary and Concept Check

Addition Property of Equality (p. 128)
consecutive integers (p. 144)
defining a variable (p. 121)
dimensional analysis (p. 167)
Division Property of Equality (p. 137)
equivalent equation (p. 129)
extremes (p. 156)
formula (p. 122)
four-step problem-solving plan (p. 121)
identity (p. 150)
means (p. 156)
mixture problem (p. 171)
Multiplication Property of Equality (p. 135)
multi-step equations (p. 143)
number theory (p. 144)
percent of change (p. 160)
percent of decrease (p. 160)
percent of increase (p. 160)
proportion (p. 155)
rate (p. 157)
ratio (p. 155)
scale (p. 157)
solve an equation (p. 129)
Subtraction Property of Equality (p. 129)
uniform motion problem (p. 172)
weighted average (p. 171)
work backward (p. 142)

Choose the correct term to complete each sentence.

1. According to the *(Addition, Multiplication)* Property of Equality, if $a = b$, then $a + c = b + c$.
2. A *(means, ratio)* is a comparison of two numbers by division.
3. A rate is the ratio of two measurements with *(the same, different)* units of measure.
4. The first step in the four-step problem-solving plan is to *(explore, solve)* the problem.
5. $2x + 1 = 2x + 1$ is an example of a(n) *(identity, formula)*.
6. An equivalent equation for $3x + 5 = 7$ is $(3x = 2, 3x = 12)$.
7. If the original amount was 80 and the new amount is 90, then the percent of *(decrease, increase)* is 12.5%.
8. *(Defining the variable, Dimensional analysis)* is the process of carrying units throughout a computation.
9. The *(weighted average, rate)* of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
10. An example of consecutive integers is *(8 and 9, 8 and 10)*.

Lesson-by-Lesson Review

3-1 Writing Equations

See pages 120–126.

Concept Summary

- Variables are used to represent unknowns when writing equations.
- Formulas given in sentence form can be written as algebraic equations.

Example **Translate the following sentence into an equation.**

The sum of x and y equals 2 plus two times the product of x and y.

$$\underbrace{x + y}_{\text{The sum of } x \text{ and } y} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{2}_{2} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{2xy}_{\text{two times the product of } x \text{ and } y.}$$

The equation is $x + y = 2 + 2xy$.

Exercises **Translate each sentence into an equation.** *See Example 1 on page 120.*

11. Three times a number n decreased by 21 is 57.

12. Four minus three times z is equal to z decreased by 2.

13. The sum of the square of a and the cube of b is 16.

14. Translate the equation $16 - 9r = r$ into a verbal sentence. *See Example 4 on pages 122 and 123.*

3-2 Solving Equations by Using Addition and Subtraction

See pages 128–134.

Concept Summary

- **Addition Property of Equality** For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.
- **Subtraction Property of Equality** For any numbers a, b, and c, if $a = b$, then $a - c = b - c$.

Example **Solve $x - 13 = 45$. Then check your solution.**

$x - 13 = 45$ Original equation

$x - 13 + 13 = 45 + 13$ Add 13 to each side.

$x = 58$ Simplify.

CHECK $x - 13 = 45$ Original equation

$58 - 13 \stackrel{?}{=} 45$ Substitute 58 for x.

$45 = 45$ ✓ Simplify.

The solution is 58.

Exercises **Solve each equation. Then check your solution.**
See Examples 1–4 on pages 129 and 130.

15. $r - 21 = -37$ **16.** $14 + c = -5$ **17.** $27 = 6 + p$

18. $b + (-14) = 6$ **19.** $d - (-1.2) = -7.3$ **20.** $r + \left(-\frac{1}{2}\right) = -\frac{3}{4}$

3-3 Solving Equations by Using Multiplication and Division

See pages 135–140.

Concept Summary

- **Multiplication Property of Equality** For any numbers a, b, and c, if $a = b$, then $ac = bc$.
- **Division Property of Equality** For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example **Solve $\frac{4}{9}t = -72$.**

$\frac{4}{9}t = -72$ Original equation

$\frac{9}{4}\left(\frac{4}{9}t\right) = \frac{9}{4}(-72)$ Multiply each side by $\frac{9}{4}$.

$t = -162$ Simplify.

CHECK $\frac{4}{9}t = -72$ Original equation

$\frac{4}{9}(-162) \stackrel{?}{=} -72$ Substitute -162 for t.

$-72 = -72$ ✓ Simplify.

The solution is -162.

Exercises **Solve each equation. Then check your solution.**
See Examples 1–3 on pages 135 and 136.

21. $6x = -42$ **22.** $-7w = -49$ **23.** $\frac{3}{4}n = 30$

24. $-\frac{3}{5}y = -50$ **25.** $\frac{5}{2}a = -25$ **26.** $5 = \frac{r}{2}$

3-4 Solving Multi-Step Equations

See pages 142–148.

Concept Summary

- Multi-step equations can be solved by undoing the operations in reverse of the order of operations.

Example **Solve $34 = 8 - 2t$. Then check your solution.**

$34 = 8 - 2t$	Original equation
$34 - 8 = 8 - 2t - 8$	Subtract 8 from each side.
$26 = -2t$	Simplify.
$\frac{26}{-2} = \frac{-2t}{-2}$	Divide each side by −2.
$-13 = t$	Simplify.

CHECK

$34 = 8 - 2t$	Original equation
$34 \stackrel{?}{=} 8 - 2(-13)$	Substitute −13 for *t*.
$34 = 34$ ✓	The solution is −13.

Exercises **Solve each equation. Then check your solution.**
See Examples 2–4 on page 143.

27. $4p - 7 = 5$ **28.** $6 = 4v + 2$ **29.** $\frac{y}{3} + 6 = -45$

30. $\frac{c}{-4} - 8 = -42$ **31.** $\frac{4d + 5}{7} = 7$ **32.** $\frac{7n + (-1)}{8} = 8$

3-5 Solving Equations with the Variable on Each Side

See pages 149–154.

Concept Summary

Steps for Solving Equations

Step 1 Use the Distributive Property to remove the grouping symbols.

Step 2 Simplify the expressions on each side of the equals sign.

Step 3 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.

Step 4 Simplify the expressions on each side of the equals sign.

Step 5 Use the Multiplication and/or Division Properties of Equalities to solve.

Example **Solve $\frac{3}{4}q - 8 = \frac{1}{4}q + 9$.**

$$\frac{3}{4}q - 8 = \frac{1}{4}q + 9 \quad \text{Original equation}$$

$$\frac{3}{4}q - 8 - \frac{1}{4}q = \frac{1}{4}q + 9 - \frac{1}{4}q \quad \text{Subtract } \tfrac{1}{4}q \text{ from each side.}$$

$$\frac{1}{2}q - 8 = 9 \quad \text{Simplify.}$$

$$\frac{1}{2}q - 8 + 8 = 9 + 8 \quad \text{Add 8 to each side.}$$

$$\frac{1}{2}q = 17 \quad \text{Simplify.}$$

$$2\left(\frac{1}{2}q\right) = 2(17) \quad \text{Multiply each side by 2.}$$

$$q = 34 \quad \text{Simplify.}$$

The solution is 34.

Exercises **Solve each equation. Then check your solution.**
See Examples 1–4 on pages 149 and 150.

33. $n - 2 = 4 - 2n$ **34.** $3t - 2(t + 3) = t$ **35.** $3 - \frac{5}{6}y = 2 + \frac{1}{6}y$

36. $\frac{x - 2}{6} = \frac{x}{2}$ **37.** $2(b - 3) = 3(b - 1)$ **38.** $8.3h - 2.2 = 6.1h - 8.8$

3-6 Ratios and Proportions

See pages 155–159.

Concept Summary

- A ratio is a comparison of two numbers by division.
- A proportion is an equation stating that two ratios are equal.
- A proportion can be solved by finding the cross products.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example **Solve the proportion $\frac{8}{7} = \frac{a}{1.75}$.**

$$\frac{8}{7} = \frac{a}{1.75} \quad \text{Original equation}$$

$$8(1.75) = 7(a) \quad \text{Find the cross products.}$$

$$14 = 7a \quad \text{Simplify.}$$

$$\frac{14}{7} = \frac{7a}{7} \quad \text{Divide each side by 7.}$$

$$2 = a \quad \text{Simplify.}$$

Exercises **Solve each proportion.** *See Example 3 on page 156.*

39. $\frac{6}{15} = \frac{n}{45}$ **40.** $\frac{x}{11} = \frac{35}{55}$ **41.** $\frac{12}{d} = \frac{20}{15}$

42. $\frac{14}{20} = \frac{21}{m}$ **43.** $\frac{2}{3} = \frac{b + 5}{9}$ **44.** $\frac{6}{8} = \frac{9}{s - 4}$

3-7 Percent of Change

See pages 160–164.

Concept Summary

- The proportion $\frac{\text{amount of change}}{\text{original amount}} = \frac{r}{100}$ is used to find percents of change.

Example **Find the percent of change. original: \$120**
new: \$114

First, subtract to find the amount of change.

$\$120 - \$114 = \$6$ Note that since the new amount is less than the original, the percent of change will be a percent of decrease.

Then find the percent using the original number, 120, as the base.

change → original amount →	$\frac{6}{120} = \frac{r}{100}$	Percent proportion
	$6(100) = 120(r)$	Find the cross products.
	$600 = 120r$	Simplify.
	$\frac{600}{120} = \frac{120r}{120}$	Divide each side by 120.
	$5 = r$	Simplify.

The percent of decrease is 5%.

Exercises **State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.** *See Example 1 on page 160.*

45. original: 40
new: 32

46. original: 50
new: 88

47. original: 35
new: 37.1

48. Find the total price of a book that costs \$14.95 plus 6.25% sales tax.
See Example 3 on page 161.

49. A T-shirt priced at \$12.99 is on sale for 20% off. What is the discounted price?
See Example 4 on page 161.

3-8 Solving Equations and Formulas

See pages 166–170.

Concept Summary

- For equations with more than one variable, you can solve for one of the variables by using the same steps as solving equations with one variable.

Example **Solve $\frac{x + y}{b} = c$ for x.**

$\frac{x + y}{b} = c$	Original equation
$b\left(\frac{x + y}{b}\right) = b(c)$	Multiply each side by b.
$x + y = bc$	Simplify.
$x + y - y = bc - y$	Subtract y from each side.
$x = bc - y$	Simplify.

Chapter 3 For More ...
• Extra Practice, see pages 825–828.
• Mixed Problem Solving, see page 855.

Exercises **Solve each equation or formula for the variable specified.**
See Examples 1 and 2 on pages 166 and 167.

50. $5x = y$, for x

51. $ay - b = c$, for y

52. $yx - a = cx$, for x

53. $\frac{2y - a}{3} = \frac{a + 3b}{4}$, for y

3-9 Weighted Averages

See pages 171–177.

Concept Summary

- The weighted average of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
- The formula $d = rt$ is used to solve uniform motion problems.

Example

SCIENCE **Mai Lin has a 35 milliliters of 30% solution of copper sulfate. How much of a 20% solution of copper sulfate should she add to obtain a 22% solution?**

Let x = amount of 20% solution to be added. Make a table.

	Amount of Solution (mL)	Amount of Copper Sulfate
30% Solution	35	0.30(35)
20% Solution	x	$0.20x$
22% Solution	$35 + x$	$0.22(35 + x)$

$$0.30(35) + 0.20x = 0.22(35 + x) \quad \text{Write and solve an equation.}$$
$$10.5 + 0.20x = 7.7 + 0.22x \quad \text{Distributive Property}$$
$$10.5 + 0.20x - 0.20x = 7.7 + 0.22x - 0.20x \quad \text{Subtract } 0.20x \text{ fom each side.}$$
$$10.5 = 7.7 + 0.02x \quad \text{Simplify.}$$
$$10.5 - 7.7 = 7.7 + 0.02x - 7.7 \quad \text{Subtract 7.7 from each side.}$$
$$2.8 = 0.02x \quad \text{Simplify.}$$
$$\frac{2.8}{0.02} = \frac{0.02x}{0.02} \quad \text{Divide each side by 0.02.}$$
$$140 = x \quad \text{Simplify.}$$

Mai Lin should add 140 milliliters of the 20% solution.

Exercises

54. **COFFEE** Ms. Anthony wants to create a special blend using two coffees, one priced at \$8.40 per pound and the other at \$7.28 per pound. How many pounds of the \$7.28 coffee should she mix with 9 pounds of the \$8.40 coffee to sell the mixture for \$7.95 per pound? *See Example 1 on page 171.*

55. **TRAVEL** Two airplanes leave Dallas at the same time and fly in opposite directions. One airplane travels 80 miles per hour faster than the other. After three hours, they are 2940 miles apart. What is the speed of each airplane? *See Example 3 on pages 172 and 173.*

Chapter 3

Practice Test

Vocabulary and Concepts

Choose the correct term to complete each sentence.

1. The study of numbers and the relationships between them is called *(consecutive, number)* theory.
2. An equation that is true for *(every, only one)* value of the variable is called an identity.
3. When a new number is *(greater than, less than)* the original number, the percent of change is called a percent of increase.

Skills and Applications

Translate each sentence into an equation.

4. The sum of twice x and three times y is equal to thirteen.
5. Two thirds of a number is negative eight fifths.

Solve each equation. Then check your solution.

6. $-15 + k = 8$
7. $-1.2x = 7.2$
8. $k - 16 = -21$
9. $\frac{t-7}{4} = 11$
10. $\frac{3}{4}y = -27$
11. $-12 = 7 - \frac{y}{3}$
12. $t - (-3.4) = -5.3$
13. $-3(x + 5) = 8x + 18$
14. $5a = 125$
15. $\frac{r}{5} - 3 = \frac{2r}{5} + 16$
16. $0.1r = 19$
17. $-\frac{2}{3}z = -\frac{4}{9}$
18. $-w + 11 = 4.6$
19. $2p + 1 = 5p - 11$
20. $25 - 7w = 46$

Solve each proportion.

21. $\frac{36}{t} = \frac{9}{11}$
22. $\frac{n}{4} = \frac{3.25}{52}$
23. $\frac{5}{12} = \frac{10}{x-1}$

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

24. original: 45
    new: 9
25. original: 12
    new: 20

Solve each equation or formula for the variable specified.

26. $h = at - 0.25vt^2$, for a
27. $a(y + 1) = b$, for y

28. **SALES** Suppose the Central Perk coffee shop sells a cup of espresso for \$2.00 and a cup of cappuccino for \$2.50. On Friday, Destiny sold 30 more cups of cappuccino than espresso for a total of \$178.50 worth of espresso and cappuccino. How many cups of each were sold?

29. **BOATING** *The Yankee Clipper* leaves the pier at 9:00 A.M. at 8 knots (nautical miles per hour). A half hour later, *The River Rover* leaves the same pier in the same direction traveling at 10 knots. At what time will *The River Rover* overtake *The Yankee Clipper*?

30. **STANDARDIZED TEST PRACTICE** If $\frac{4}{5}$ of $\frac{3}{4} = \frac{2}{5}$ of $\frac{x}{4}$, find the value of x.

    (A) 12 (B) 6 (C) 3 (D) $\frac{3}{2}$

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Chapter 3 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

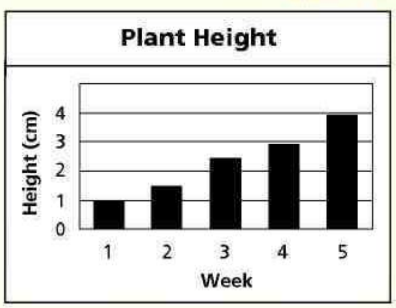
1. Bailey planted a rectangular garden that is 6 feet wide by 15 feet long. What is the perimeter of the garden? (Prerequisite Skill)

(A) 21 ft (B) 27 ft
(C) 42 ft (D) 90 ft

2. Which of the following is true about 65 percent of 20? (Prerequisite Skill)

(A) It is greater than 20.
(B) It is less than 10.
(C) It is less than 20.
(D) Can't tell from the information given

3. For a science project, Kelsey measured the height of a plant grown from seed. She made the bar graph below to show the height of the plant at the end of each week. Which is the most reasonable estimate of the plant's height at the end of the sixth week? (Lesson 1-8)



(A) 2 to 3.5 cm (B) 4 to 5.5 cm
(C) 6 to 7 cm (D) 8 to 8.5 cm

4. WEAT predicted a 25% chance of snow. WFOR said the chance was 1 in 4. Myweather.com showed the chance of snow as $\frac{1}{5}$, and Allweather.com listed the chance as 0.3. Which forecast predicted the greatest chance of snow? (Lesson 2-7)

(A) WEAT (B) WFOR
(C) Myweather.com (D) Allweather.com

5. Amber owns a business that transfers photos to CD-ROMs. She charges her customers \$24.95 for each CD-ROM. Her expenses include \$575 for equipment and \$0.80 for each blank CD-ROM. Which of these equations could be used to calculate her profit p for creating n CD-ROMs? (Lesson 3-1)

(A) $p = (24.95 - 0.8)n - 575$
(B) $p = (24.95 + 0.8)n + 575$
(C) $p = 24.95n - 574.2$
(D) $p = 24.95n + 575$

6. Which of the following equations has the same solution as $8(x + 2) = 12$? (Lesson 3-4)

(A) $8x + 2 = 12$
(B) $x + 2 = 4$
(C) $8x = 10$
(D) $2x + 4 = 3$

7. Eduardo is buying pizza toppings for a birthday party. His recipe uses 8 ounces of shredded cheese for 6 servings. How many ounces of cheese are needed for 27 servings? (Lesson 3-6)

(A) 27 (B) 32
(C) 36 (D) 162

8. The sum of x and $\frac{1}{y}$ is 0, and y does not equal 0. Which of the following is true? (Lesson 3-8)

(A) $x = -y$ (B) $\frac{x}{y} = 0$
(C) $x = 1 - y$ (D) $x = -\frac{1}{y}$

The Princeton Review Test-Taking Tip

Questions 2, 6, 8 Always read every answer choice, particularly in questions that ask, "Which of the following is true?"

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Let $x = 2$ and $y = -3$. Find the value of $\frac{x(xy + 5)}{4}$. (Lesson 1-2)

10. Use the formula $F = \frac{9}{5}C + 32$ to convert temperatures from Celsius (C) to Fahrenheit (F). If it is $-5°$ Celsius, what is the temperature in degrees Fahrenheit? (Lesson 2-3)

11. Darnell keeps his cotton socks folded in pairs in his drawer. Five pairs are black, 2 pairs are navy, and 1 pair is brown. In the dark, he pulls out one pair at random. What are the odds that it is black? (Lesson 2-6)

12. The sum of the ages of the Kruger sisters is 39. Their ages can be represented as three consecutive integers. What is the age of the middle sister? (Lesson 3-4)

13. On a car trip, Tyson drove 65 miles more than half the number of miles Pete drove. Together they drove 500 miles. How many miles did Tyson drive? (Lesson 3-4)

14. Solve $7(x + 2) + 4(2x - 3) = 47$ for x. (Lesson 3-5)

15. A bookshop sells used hardcover books with a 45% discount. The price of a book was $22.95 when it was new. What is the discounted price for that book? (Lesson 3-7)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
16.	$\lvert a \rvert$	$\lvert -a \rvert$

(Lesson 2-1)

	Column A	Column B
17.	solution of $3x + 7 = 10$	solution of $4y - 2 = 6$

(Lesson 3-4)

	Column A	Column B
18.	the percent of increase from \$75 to \$100	the percent of increase from \$150 to \$200

(Lesson 3-7)

Part 4 Open Ended

Record your answers on a sheet of paper.

19. Kirby's pickup truck travels at a rate of 6 miles every 10 minutes. Nola's SUV travels at a rate of 15 miles every 25 minutes. The speed limit on this street is 40 mph. Is either vehicle or are both vehicles exceeding the speed limit? Explain. (Lesson 3-6)

20. A chemist has one solution of citric acid that is 20% acid and another solution of citric acid that is 80% acid. She plans to mix these solutions together to make 200 liters of a solution that is 50% acid. (Lesson 3-9)

a. Complete the table to show the liters of 20% and 80% solutions that will be used to make the 50% solution. Use x to represent the number of liters of the 80% solution that will be used to make the 50% solution.

	Liters of Solution	Liters of Acid
20% Solution		
80% Solution	x	
50% Solution	200	0.50(200)

b. Write an equation that represents the number of liters of acid in the solution.

c. How many liters of the 20% solution and how many of the 80% solution will the chemist need to mix together to make 200 liters of a 50% solution?

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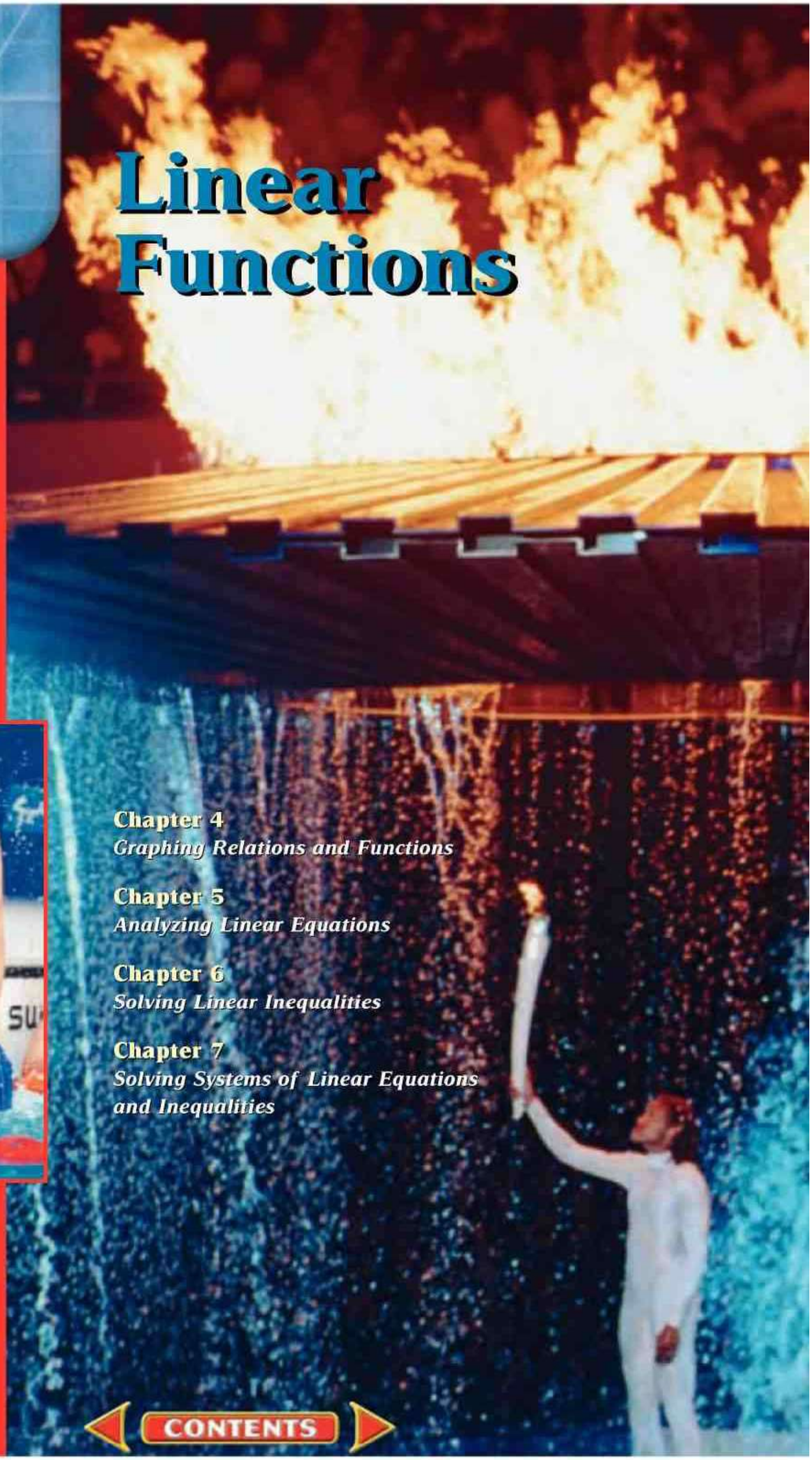
UNIT 2

Linear Functions

Many real-world situations such as Olympic race times can be represented using functions. In this unit, you will learn about linear functions and equations.



CONTENTS

WebQuest Internet Project

The Spirit of the Games

The first Olympic Games featured only one event— a foot race. The 2004 Games will include thousands of competitors in about 300 events. In this project, you will explore how linear functions can be illustrated by the Olympics.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 2.

Lesson	4-6	5-7	6-6	7-1
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