Construction of large two-level nonregular designs of strength three

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Outline

1. Introduction
2. Evaluation of designs
3. Construction method
4. Results and discussion

Applications of large two-level orthogonal designs

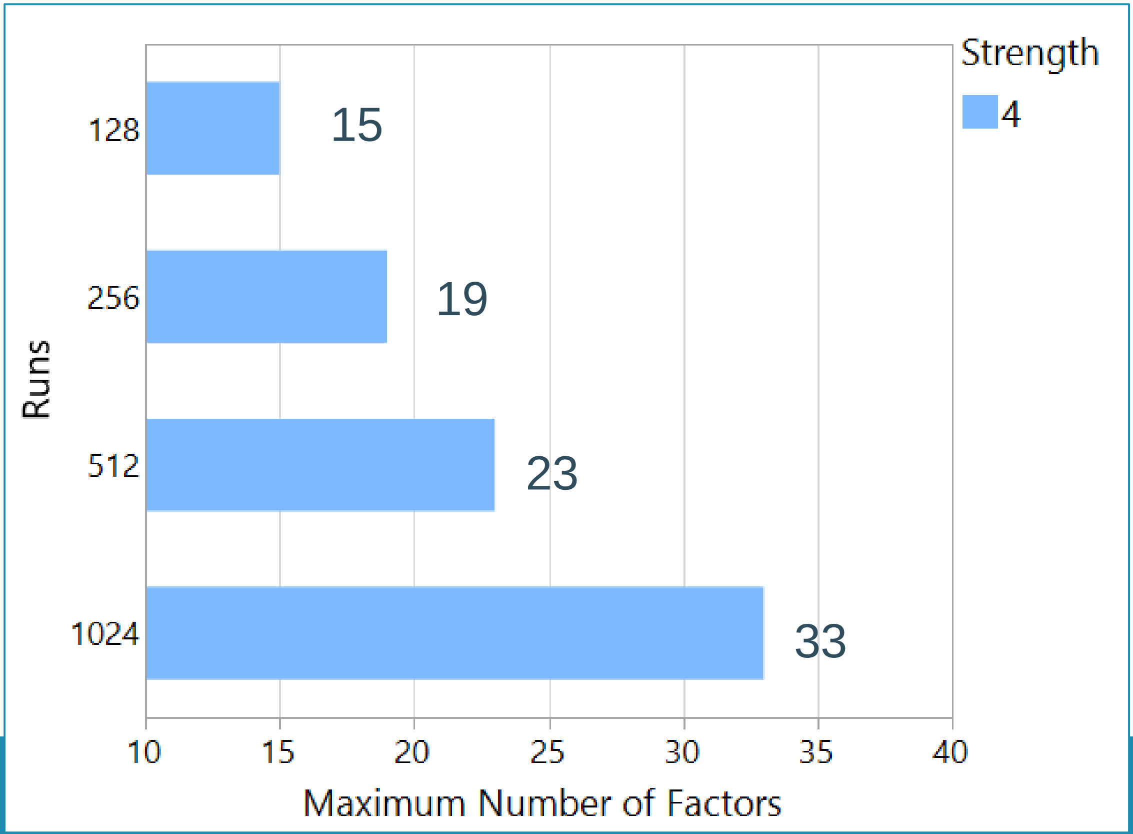
- Simulation of a software development process: 128 runs & 30 factors (Houston et al. 2001).
- Simulation of a combat aircraft: 1024 runs & 40 factors (Lefebvre et al. 2010).
- Investigate the regulation of specific cells: 128 runs & 13 factors (Barminko et al. 2014).
- Develop treatments to inhibit Tuberculosis: 128 runs & 14 factors (Silva et al. 2016).

Goal: Study the main effects and two-factor interactions of the factors.

Literature review

Strength-4 orthogonal designs. Properties:

- All main effects and all two-factor interactions are orthogonal to each other.

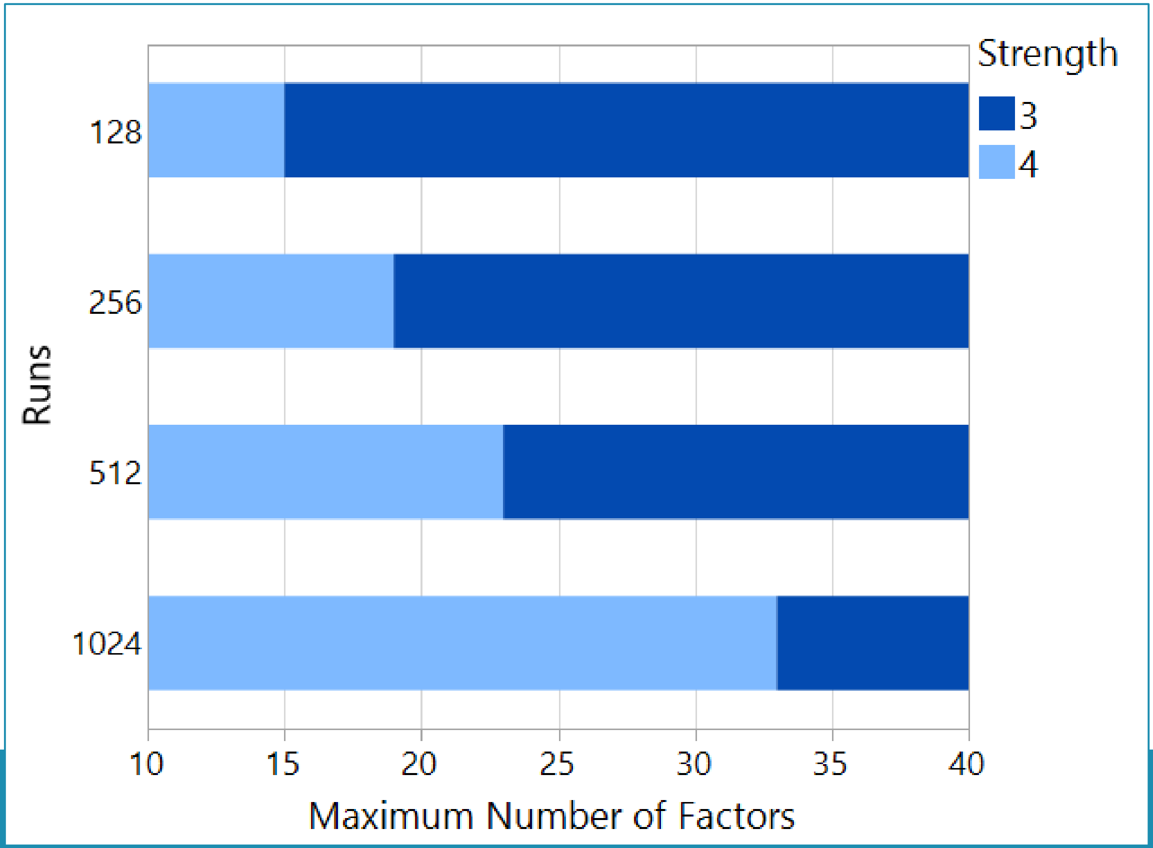


See Hedayat et al. (1999), Mee (2004), Sanchez & Sanchez (2005), Xu (2009) and Schoen et al. (2010).

Literature review

Strength-3 orthogonal designs. Properties:

1. Main effects are orthogonal to the two-factor interactions.
2. Pairs of two-factor interactions can be correlated.



See Block & Mee (2005), Xu & Wong (2007), Xu (2009), Ryan & Bulutoglu (2010).

Literature review

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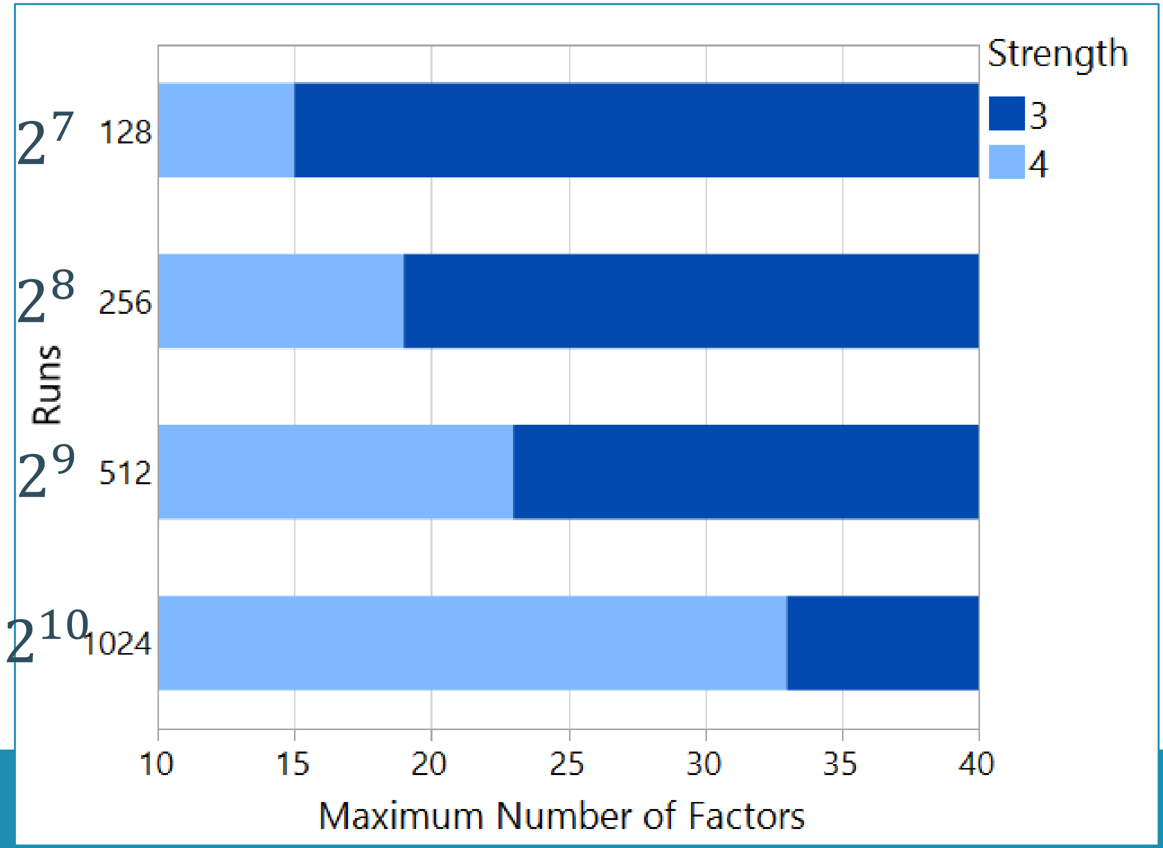
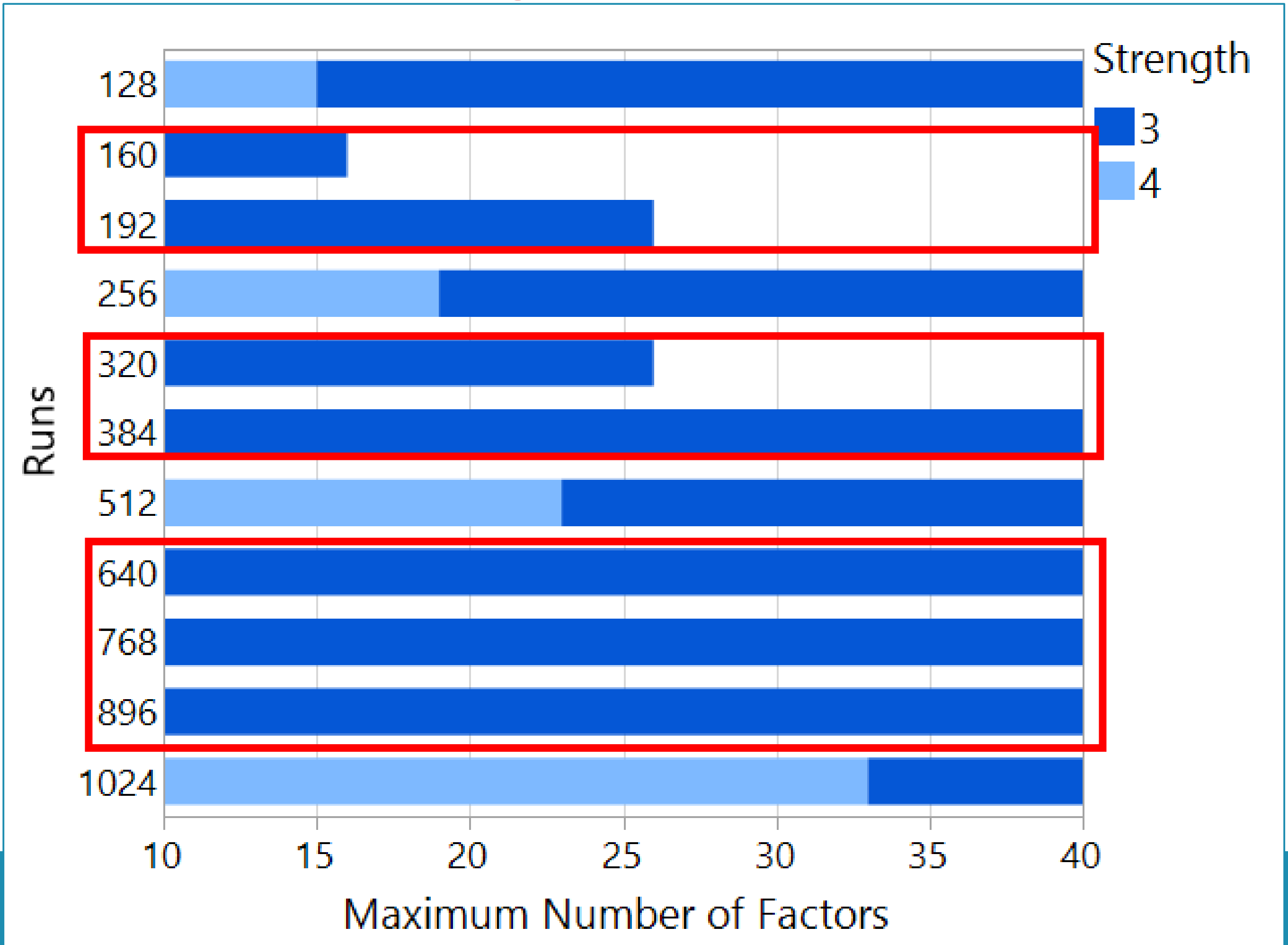


See Block & Mee (2005), Xu & Wong (2007), Xu (2009), Ryan & Bulutoglu (2010).

This talk

Alternative strength-3 orthogonal designs which fill the gaps between the available designs.



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Outline

1. Introduction

2. Evaluation of designs

3. Construction method

4. Results and discussion

Evaluating two-level strength-3 designs

<u>Example I</u>: Compare experimental designs with 32 runs and 10 factors.

Regular resolution-IV design

1	2	3	4	5	6	7	8	9	10
−1	−1	−1	−1	−1	−1	−1	−1	1	1
−1	−1	−1	−1	1	−1	−1	1	−1	−1
−1	−1	−1	1	−1	−1	1	−1	−1	−1
−1	−1	−1	1	1	−1	1	1	1	1
−1	−1	1	−1	−1	1	−1	−1	−1	−1
−1	−1	1	−1	1	1	−1	1	1	1
−1	−1	1	1	−1	1	1	−1	1	1
−1	−1	1	1	1	1	1	1	−1	−1
−1	1	−1	−1	−1	1	1	1	1	−1
−1	1	−1	−1	1	1	1	−1	−1	1
−1	1	−1	1	−1	1	−1	1	−1	1
−1	1	−1	1	1	1	−1	−1	1	−1
−1	1	1	−1	−1	−1	1	1	−1	1
−1	1	1	−1	1	−1	1	−1	1	−1
−1	1	1	1	−1	−1	−1	1	1	−1
−1	1	1	1	1	−1	−1	−1	−1	1
1	−1	−1	−1	−1	1	1	1	−1	1
1	−1	−1	−1	1	1	1	−1	1	−1
1	−1	−1	1	−1	1	−1	1	1	−1
1	−1	−1	1	1	1	−1	−1	−1	1
1	−1	1	−1	−1	−1	1	1	1	−1
1	−1	1	−1	1	−1	1	−1	−1	1
1	−1	1	1	−1	−1	−1	1	−1	1
1	−1	1	1	1	−1	−1	−1	1	−1
1	1	−1	−1	−1	−1	−1	−1	−1	−1
1	1	−1	−1	1	−1	−1	1	1	1
1	1	−1	1	−1	−1	1	−1	1	1
1	1	−1	1	1	−1	1	1	−1	−1
1	1	1	−1	−1	1	−1	−1	1	1
1	1	1	−1	1	1	−1	1	−1	−1
1	1	1	1	−1	1	1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1

Wu & Hamada (2009)

Nonregular strength-3 design

1	2	3	4	5	6	7	8	9	10
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	1	1	1	1
−1	−1	−1	−1	1	1	−1	−1	1	1
−1	−1	−1	−1	1	1	1	1	−1	−1
−1	−1	1	1	−1	−1	−1	−1	−1	−1
−1	−1	1	1	−1	1	−1	1	1	1
−1	−1	1	1	1	−1	1	1	−1	1
−1	−1	1	1	1	1	1	−1	1	−1
−1	1	−1	1	−1	−1	1	−1	1	1
−1	1	−1	1	−1	1	1	1	−1	−1
−1	1	−1	1	1	−1	−1	1	1	−1
−1	1	−1	1	1	1	−1	−1	−1	1
−1	1	1	−1	−1	1	−1	1	1	−1
−1	1	1	−1	−1	1	1	−1	−1	1
−1	1	1	−1	1	−1	−1	1	−1	1
−1	1	1	−1	1	−1	1	−1	1	−1
1	−1	−1	1	−1	1	−1	1	−1	1
1	−1	−1	1	−1	1	1	−1	1	−1
1	−1	−1	1	1	−1	−1	1	1	−1
1	−1	−1	1	1	−1	1	−1	−1	1
1	−1	1	−1	−1	−1	1	1	1	−1
1	−1	1	−1	−1	1	1	−1	−1	1
1	−1	1	−1	1	−1	−1	−1	1	1
1	−1	1	−1	1	1	−1	1	−1	−1
1	1	−1	−1	−1	−1	−1	1	−1	1
1	1	−1	−1	−1	1	−1	−1	1	−1
1	1	−1	−1	1	−1	1	−1	−1	−1
1	1	−1	−1	1	1	1	1	1	1
1	1	1	1	−1	−1	−1	−1	1	1
1	1	1	1	−1	−1	1	1	−1	−1
1	1	1	1	1	1	−1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1

Schoen & Mee (2010)

How can we measure the correlation between pairs of two-factor interactions in strength-3 designs?

Compute the J_4-characteristics (Deng & Tang, 1999) for all subsets of 4 factors.

Regular design

1	2	3	4	5	6	7	8	9	10
−1	−1	−1	−1	−1	−1	−1	−1	1	1
−1	−1	−1	−1	1	−1	−1	1	−1	−1
−1	−1	−1	1	−1	−1	1	−1	−1	−1
−1	−1	−1	1	1	−1	1	1	1	1
−1	−1	1	−1	−1	1	−1	−1	−1	−1
−1	−1	1	−1	1	1	−1	1	1	1
−1	−1	1	1	−1	1	1	−1	1	1
−1	−1	1	1	1	1	1	1	−1	−1
−1	1	−1	−1	−1	1	1	1	1	−1
−1	1	−1	−1	1	1	1	−1	−1	1
−1	1	−1	1	−1	1	−1	1	−1	1
−1	1	−1	1	1	1	−1	−1	1	−1
−1	1	1	−1	−1	−1	1	1	−1	1
−1	1	1	−1	1	−1	1	−1	1	−1
−1	1	1	1	−1	−1	−1	1	1	−1
−1	1	1	1	1	−1	−1	−1	−1	1
1	−1	−1	−1	−1	1	1	1	−1	1
1	−1	−1	−1	1	1	1	−1	1	−1
1	−1	−1	1	−1	1	−1	1	1	−1
1	−1	−1	1	1	1	−1	−1	−1	1
1	−1	1	−1	−1	−1	1	1	1	−1
1	−1	1	−1	1	−1	1	−1	−1	1
1	−1	1	1	−1	−1	−1	1	−1	1
1	−1	1	1	1	−1	−1	−1	1	−1
1	1	−1	−1	−1	−1	−1	−1	−1	−1
1	1	−1	−1	1	−1	−1	1	1	1
1	1	−1	1	−1	−1	1	−1	1	1
1	1	−1	1	1	−1	1	1	−1	−1
1	1	1	−1	−1	1	−1	−1	1	1
1	1	1	−1	1	1	−1	1	−1	−1
1	1	1	1	−1	1	1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1

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How can we measure the correlation between pairs of two-factor interactions in strength-3 designs?

Compute the J_4-characteristics (Deng & Tang, 1999) for all subsets of 4 factors.

1	2	3	6	1236
−1	−1	−1	−1	1
−1	−1	−1	−1	1
−1	−1	−1	1	1
−1	−1	−1	1	1
−1	−1	1	−1	1
−1	−1	1	1	1
−1	−1	1	−1	1
−1	−1	1	1	1
−1	1	−1	−1	1
−1	1	−1	1	1
−1	1	−1	−1	1
−1	1	−1	1	1
−1	1	1	1	1
−1	1	1	1	1
−1	1	1	−1	1
−1	1	1	−1	1
1	−1	−1	1	1
1	−1	−1	1	1
1	−1	−1	−1	1
1	−1	−1	−1	1
1	−1	1	−1	1
1	−1	1	1	1
1	−1	1	−1	1
1	−1	1	1	1
1	1	−1	−1	1
1	1	−1	1	1
1	1	−1	−1	1
1	1	−1	1	1
1	1	1	−1	1
1	1	1	−1	1
1	1	1	1	1
1	1	1	1	1

Sum of all elements in the column:

$$j_4(\{1,2,3,6\}) = 32.$$

$$J_4(\{1,2,3,6\}) = \frac{|j_4(\{1,2,3,6\})|}{N} = \frac{32}{32} = 1.$$

3 pairs of interactions which are fully correlated: *12 & 36*, *13 & 26,16 & 23.*

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For regular designs, all J_4-characteristics are either 0 or 1.

For nonregular designs, the J_4-characteristics can have values of 0, 0.5 or 1 (Deng & Tang, 1999).

Two criteria to compare the correlation between pairs of two-factor interactions in regular and nonregular designs:

- **The $\boldsymbol{F_4}$ vector.**
- **The $\boldsymbol{B_4}$ value.**

Alternative summaries of all the J_4-characteristics.

The F_4 vector

The F_4 vector has entries equal to the frequencies for the possible values of the J_4-characteristics (Deng & Tang, 1999).

Example I (cont.): 32-run 10-factor designs.

Regular design

$F_4(1, 0.5) = (10, 0)$
30 pairs of 2FIs

Nonregular design

$F_4(1, 0.5) = (1, 62)$
189 pairs of 2FIs

Sequentially minimizing F_4 is equivalent to minimizing: (1) the maximum absolute correlation between pairs of two-factor interactions; and, (2) the total number of pairs involved.

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The B_4 value

The B_4 value is the sum of squared J_4-characteristics (Tang & Deng, 1999).

Example I (cont.): 32-run 10-factor designs.

Regular design

$$B_4 = 10(1)^2 + 0(0.5)^2 = 10$$

Nonregular design

$$B_4 = 1(1)^2 + 62(0.5)^2 = 16.5$$

Minimizing B_4 is equivalent to minimizing the sum of squared correlations between pairs of two-factor interactions.

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Outline

Construction by example

Example II: Construct a two-level design with 96 runs and 14 factors.

Step 1: Consider the 32-run 14-factor, MA regular design, call it 2^{14-9}.

Step 2: Construct the concatenated design D.

Step 3: Consider column permutations and fold-overs of columns in the copies of 2^{14-9} to sequentially minimize F_4 or minimize the B_4 value of D.

$$D = \begin{pmatrix} 2^{14-9} \\ 2^{14-9} \\ 2^{14-9} \end{pmatrix}$$

See also Addelman (1961), Pajak & Addelman (1975), Mee (2004).

Construction by example

Example II: Construct a two-level design with 96 runs and 14 factors.

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$$D = \begin{pmatrix} 2^{14-9} \\ 2^{14-9} \\ 2^{14-9} \end{pmatrix}$$

Evaluating all possible concatenated designs would require $2 \times 14! \times 2^{14} = 2.8 \times 10^{15}$ evaluations.

Factor column permutations

$$2^{14-9} =$$

Basic factors / Generated factors

					123	124	134	234	125	135	235	145	245
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	1	−1	−1	−1	−1	1	1	1	1	1
−1	−1	−1	1	−1	−1	1	1	1	−1	−1	−1	1	1
−1	−1	−1	1	1	−1	1	1	1	1	1	1	−1	−1
−1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1
−1	−1	1	−1	1	1	−1	1	1	1	−1	−1	1	1
−1	−1	1	1	−1	1	1	−1	−1	−1	1	1	1	1
−1	−1	1	1	1	1	1	−1	−1	1	−1	−1	−1	−1
−1	1	−1	−1	−1	1	1	−1	1	1	−1	1	−1	1
−1	1	−1	−1	1	1	1	−1	1	−1	1	−1	1	−1
−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	−1
−1	1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
−1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	1
−1	1	1	−1	1	−1	1	1	−1	−1	−1	1	1	−1
−1	1	1	1	−1	−1	−1	−1	1	1	1	−1	1	−1
−1	1	1	1	1	−1	−1	−1	1	−1	−1	1	−1	1
1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	1	−1
1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	−1	1
1	−1	−1	1	−1	1	−1	−1	1	1	1	−1	−1	1
1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	1	−1
1	−1	1	−1	−1	−1	1	−1	1	1	−1	1	1	−1
1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	1
1	−1	1	1	−1	−1	−1	1	−1	1	−1	1	−1	1
1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1	−1
1	1	−1	−1	−1	−1	−1	1	1	−1	1	1	1	1
1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	−1	−1
1	1	−1	1	−1	−1	1	−1	−1	−1	1	1	−1	−1
1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	1
1	1	1	−1	−1	1	−1	−1	−1	−1	−1	−1	1	1
1	1	1	−1	1	1	−1	−1	−1	1	1	1	−1	−1
1	1	1	1	−1	1	1	1	1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1	1	1	1	1

Permute the basic factors using **linear permutations**.

Factor column permutations

$$2^{14-9} =$$

Basic factors | Generated factors

5	1	2	3	4	123 6	124 7	134 8	234 9	125 10	135 11	235 12	145 13	245 14
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
−1	−1	−1	−1	1	−1	1	1	1	−1	−1	−1	1	1
1	−1	−1	−1	1	−1	1	1	1	1	1	1	−1	−1
−1	−1	−1	1	−1	1	−1	1	1	−1	1	1	−1	−1
1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	1	1
−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	1	1
1	−1	−1	1	1	1	1	−1	−1	1	−1	−1	−1	−1
−1	−1	1	−1	−1	1	1	−1	1	1	−1	1	−1	1
1	−1	1	−1	−1	1	1	−1	1	−1	1	−1	1	−1
−1	−1	1	−1	1	1	−1	1	−1	1	−1	1	1	−1
1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1
−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	−1	1
1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1	−1
−1	−1	1	1	1	−1	−1	−1	1	1	1	−1	1	−1
1	−1	1	1	1	−1	−1	−1	1	−1	−1	1	−1	1
−1	1	−1	−1	−1	1	1	1	−1	1	1	−1	1	−1
1	1	−1	−1	−1	1	1	1	−1	−1	−1	1	−1	1
−1	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1
1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	1	−1
−1	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1
1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	−1	1
−1	1	−1	1	1	−1	−1	1	−1	1	−1	1	−1	1
1	1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	−1
−1	1	1	−1	−1	−1	−1	1	1	−1	1	1	1	1
1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	−1	−1
−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	−1	−1
1	1	1	−1	1	−1	1	−1	−1	1	−1	−1	1	1
−1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1
1	1	1	1	−1	1	−1	−1	−1	1	1	1	−1	−1
−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1	1	1	1	1

Permute the basic factors using **linear permutations**.

Blue copy 2^{10-5}

Column		New Position
1	→	2
2	→	3
3	→	4
4	→	5
5	→	1

Factor column permutations

$$2^{14-9} =$$

Basic factors | Generated factors

4	5	1	2	3	123 6	124 7	134 8	234 9	125 10	135 11	235 12	145 13	245 14
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
1	−1	−1	−1	−1	−1	1	1	1	−1	−1	−1	1	1
1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1
−1	−1	−1	−1	1	1	−1	1	1	−1	1	1	−1	−1
−1	1	−1	−1	1	1	−1	1	1	1	−1	−1	1	1
1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	1	1
1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	−1	−1
−1	−1	−1	1	−1	1	1	−1	1	1	−1	1	−1	1
−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1
1	−1	−1	1	−1	1	−1	1	−1	1	−1	1	1	−1
1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	−1	1
−1	−1	−1	1	1	−1	1	1	−1	1	1	−1	−1	1
−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	1	−1
1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	1	−1
1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	−1	1
−1	−1	1	−1	−1	1	1	1	−1	1	1	−1	1	−1
−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1	1
1	−1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1
1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1
−1	−1	1	−1	1	−1	1	−1	1	1	−1	1	1	−1
−1	1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	1
1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1
1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	−1
−1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	1
−1	1	1	1	−1	−1	−1	1	1	1	−1	−1	−1	−1
1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	−1	−1
1	1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1
−1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1
−1	1	1	1	1	1	−1	−1	−1	1	1	1	−1	−1
1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1	1	1	1	1

Permute the basic factors using **linear permutations**.

Red copy 2^{10-5}

Column		New Position
1	$\rightarrow$	3
2	$\rightarrow$	4
3	$\rightarrow$	5
4	$\rightarrow$	1
5	$\rightarrow$	2

Theorem: Let 2^{m-p} be a regular design with a prime number of basic factors $b = m - p$. We can calculate the J_4-characteristics of the concatenated design D from the ones of 2^{m-p}.

$$D = \begin{pmatrix} 2^{m-p} \\ 2^{m-p} \\ 2^{m-p} \end{pmatrix} \begin{array}{l} \\ \text{Shift 1 position} \\ \text{Shift 2 positions} \end{array}$$

Theorem: Let 2^{m-p} be a regular design with a prime number of basic factors $b = m - p$. We can calculate the J_4-characteristics of the concatenated design D from the ones of 2^{m-p}.

Let s be a 4-factor subset and $J_4\left(s; 2^{10-5}\right) = 1$.

- If $s = \{1, 2, 3, 6\}$ with a subset of basic factors,

  $\rightarrow J_4(\{1, 2, 3, 6\}; D) = J_4(\{2, 3, 4, 6\}; D) = J_4(\{3, 4, 5, 6\}; D) = 1/3.$

For each s of this type in 2^{m-p}

d J_4-characteristics equal to $1/d$ in D.

$d = \#$ concatenated copies of 2^{m-p}.

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Theorem: Let 2^{m-p} be a regular design with a prime number of basic factors $b = m - p$. We can calculate the J_4-characteristics of the concatenated design D from the ones of 2^{m-p}.

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  $\rightarrow J_4(\{1, 2, 3, 6\}; D) = J_4(\{2, 3, 4, 6\}; D) = J_4(\{3, 4, 5, 6\}; D) = 1/3.$

- If $s = \{6, 7, 8, 9\}$ with only generated factors,

  $\rightarrow J_4(\{6, 7, 8, 9\}; D) = 1.$

If $J_4\left(s; 2^{14-9}\right) = 0 \rightarrow J_4(s; D) = 0.$

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Example II (cont.):

32-run 14-factor design

Step 1: 2^{14-9} 5 basic factors

$$F_4(1, 0.5) = (77, 0)$$

- 68 include basic factors
- 9 include only generated factors

96-run 14-factor design

Steps 2 & 3: $D = \begin{pmatrix} 2^{14-9} \\ 2^{14-9} \\ 2^{14-9} \end{pmatrix}$ Shift 1 position / Shift 2 positions

$$F_4(1, 0.33) = (9, 204)$$

- 3 x 68 = 204 include basic factors
- 9 include only generated factors

How can we improve the 96-run concatenated design?

An algorithmic approach

Step 3': Variable Neighborhood Search (VNS).

- Framework to develop effective algorithms.
- One move: Fold-over a column in a copy of 2^{14-9}.

$F_4(1, 0.33) = (9, 204)$

$D =$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1	1	1	1	1	1
-1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	-1	1	1	-1	-1	-1	1	1	1	-1	-1
1	1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1
-1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1	-1
-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1
1	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	-1
1	-1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1

96 runs

An algorithmic approach

Step 3': Variable Neighborhood Search (VNS).

- Framework to develop effective algorithms.
- One move: Fold-over a column in a copy of 2^{14-9}.

$$F_4(1, 0.33) = (5, 208)$$

Fold-over column 8 in blue copy

$D =$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1	1	1	1	1	1
-1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	-1	1	1	-1	-1	-1	1	1	1	-1	-1
1	1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	-1	-1	1	-1	1	1	1	1	1
-1	-1	-1	-1	1	-1	1	-1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	-1	1	-1	1	-1	1	1	1	-1	-1
-1	1	1	1	1	1	1	-1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	-1	1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1
1	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	-1
1	-1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1

An algorithmic approach

Step 3': Variable Neighborhood Search (VNS).

- Framework to develop effective algorithms.
- One move: Fold-over a column in a copy of 2^{14-9}.

$$F_4(1, 0.33) = (3, 210)$$

$$D = \begin{array}{cccccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
\hline
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\
-1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}$$

Fold-over column 12 in blue copy

An algorithmic approach

Step 3': Variable Neighborhood Search (VNS).

- Framework to develop effective algorithms.
- One move: Fold-over a column in a copy of 2^{14-9}.

$$F_4(1, 0.33) = (1, 212)$$

$$D = \begin{array}{cccccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
\hline
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\
-1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}$$

Fold-over column 9 in red copy

An algorithmic approach

Step 3': Variable Neighborhood Search (VNS).

- Framework to develop effective algorithms.
- One move: Fold-over a column in a copy of 2^{14-9}.

$$F_4(1, 0.33) = (0, 213)$$

Fold-over column 13 in red copy

$D =$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1	1	1	1	1	1
-1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	-1	1	1	-1	-1	-1	1	1	1	-1	-1
1	1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	-1	-1
1	-1	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	1
-1	-1	-1	-1	1	-1	1	-1	1	-1	-1	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	-1	1	-1	1	-1	1	1	-1	-1	-1
-1	1	1	1	1	1	1	-1	1	-1	-1	1	-1	-1
1	1	1	1	1	1	1	-1	1	1	1	-1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	-1
-1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	1
1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-1	1	1	1	1	1	-1	-1	1	1	1	1	1	-1
1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	-1
1	1	1	1	1	1	1	1	-1	1	1	1	-1	1

An algorithmic approach

Step 3’: Variable Neighborhood Search (VNS).

- Framework to develop effective algorithms.
- One move: Fold-over a column in a copy of 2^{14-9}.

96-run 14-factor design

$$F_4(1, 0.33) = (0, 213)$$

$$B_4 = 23.67$$

$D =$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1	1	1	1	1	1
-1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	-1	1	1	-1	-1	-1	1	1	1	-1	-1
1	1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	-1	-1
1	-1	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	1
-1	-1	-1	-1	1	-1	1	-1	1	-1	-1	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	-1	1	-1	1	-1	1	1	-1	-1	-1
-1	1	1	1	1	1	1	-1	1	-1	-1	1	-1	-1
1	1	1	1	1	1	1	-1	1	1	1	-1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	-1
-1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	1
1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-1	1	1	1	1	1	-1	-1	1	1	1	1	1	-1
1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	-1
1	1	1	1	1	1	1	1	-1	1	1	1	-1	1

Outline

1. Introduction
2. Classification of designs
3. Construction method
4. Results and discussion

Results

We used our method to construct large two-level designs from 32- and 128-run resolution-IV MA designs.

	5 basic factors		7 basic factors
Number of concatenated copies	32-run MA design		128-run MA design
3	96		384
4	128		512
5	160		640
6	-		768
7	-		896
	Up to 16 factors		Up to 40 factors

Good strength-3 designs in terms of the F_4 vector.

Results

<u>Example III</u>: Construct large two-level designs for 28 factors.

Design	Runs	Maximum absolute correlation between pairs of 2FIs	B_4
Regular	512	1	13
Ours	512	0.5	43
Ours	640	0.6	32.5
Ours	768	0.33	26.9
Ours	896	0.14	22
Strength 4	1,024	0	0

Discussion

- Our method can be used to construct large concatenated designs using 2^{m-p} regular designs with $m-p$ not a prime number.
- We constructed two-level nonregular designs of strength 3 with 96, 128, 160, 192, 256, 320, 384, 512, 640, 768 and 896 runs, and up to 40 factors.
- An extra column can be added to block the designs.

Vazquez, A. R. and Xu, H. (2018) Construction of two-level nonregular designs of strength three with large run sizes. *Technometrics*. Published online.

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