Innovations on Teaching Design of Experiments: The Computer-Generated Approach

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Outline

1. Motivation
2. Undergraduate course
3. Discussion

Motivating example

Suppose that we have the following experimental scenario:

- Eight factors under study at two levels.
- Initial budget allows for 32 test combinations or runs.
- One continuous response.
- **Goal**: Identify the active main effects and two-factor interactions.

Design problem:

Propose an efficient experimental design

First design option

- The first option is an 8-factor 32-run fractional factorial design of resolution IV.
- The design can estimate all main effects **but not all** two-factor interactions.
- Therefore, the design may not satisfactory if we would like to identify the important interactions.

Number of Factors, k	Fraction	Number of Runs	Design Generators
3	2_{III}^{3-1}	4	$C = \pm AB$
4	2_{IV}^{4-1}	8	$D = \pm ABC$
5	2_{V}^{5-1}	16	$E = \pm ABCD$
	2_{III}^{5-2}	8	$D = \pm AB$
			$E = \pm AC$
6	2_{VI}^{6-1}	32	$F = \pm ABCDE$
	2_{IV}^{6-2}	16	$E = \pm ABC$
			$F = \pm BCD$
	2_{III}^{6-3}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
7	2_{VII}^{7-1}	64	$G = \pm ABCDEF$
	2_{IV}^{7-2}	32	$F = \pm ABCD$
			$G = \pm ABDE$
	2_{IV}^{7-3}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
	2_{III}^{7-4}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
			$G = \pm ABC$
8	2_{V}^{8-2}	64	$G = \pm ABCD$
			$H = \pm ABEF$
	2_{IV}^{8-3}	32	$F = \pm ABC$
			$G = \pm ABD$
			$H = \pm BCDE$
	2_{IV}^{8-4}	16	$E = \pm BCD$
			$F = \pm ACD$
			$G = \pm ABC$
			$H = \pm ABD$

Montgomery (2012, Ch. 8)

Second design option

- Another option is an 8-factor 64-run fractional factorial design of resolution V.
- The design can estimate all main effects and all two-factor interactions.
- However, the second design is twice as expensive as the first one.

Number of Factors, k	Fraction	Number of Runs	Design Generators
3	2_{III}^{3-1}	4	$C = \pm AB$
4	2_{IV}^{4-1}	8	$D = \pm ABC$
5	2_{V}^{5-1}	16	$E = \pm ABCD$
	2_{III}^{5-2}	8	$D = \pm AB$
			$E = \pm AC$
6	2_{VI}^{6-1}	32	$F = \pm ABCDE$
	2_{IV}^{6-2}	16	$E = \pm ABC$
			$F = \pm BCD$
	2_{III}^{6-3}	8	$D = \pm AB$
			$E = \pm AC$
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	2_{IV}^{7-2}	32	$F = \pm ABCD$
			$G = \pm ABDE$
	2_{IV}^{7-3}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
	2_{III}^{7-4}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
			$G = \pm ABC$
8	2_{V}^{8-2}	64	$G = \pm ABCD$
			$H = \pm ABEF$
	2_{IV}^{8-3}	32	$F = \pm ABC$
			$G = \pm ABD$
			$H = \pm BCDE$
	2_{IV}^{8-4}	16	$E = \pm BCD$
			$F = \pm ACD$
			$G = \pm ABC$
			$H = \pm ABD$

Montgomery (2012, Ch. 8)

Is there anything between 32 and 64 runs?

Answer: Yes! We have computer-generated optimal designs (Goos and Jones, 2011).

Construction:

1. Define a linear model with one intercept, the 8 main effects and the 28 two-factor interactions.
2. To estimate all coefficients in the model, we need at least 37 observations.
3. A D-optimal design minimizes the determinant of the variance-covariance matrix of the least squares estimates of the coefficients.
4. We construct a D-optimal design, we use the R software package called AlgDesign (Wheeler, 2022).

R Demo!

More flexible experimental designs

There are other scenarios where computer-generated optimal designs are better than standard fractional factorial designs:

- Constraints on the experimental domain.
- Economical follow-up experiments to resolve ambiguities left by the initial design.
- Experiments with a number of blocks (groups of observations) that is not a power of 2.

Therefore, optimal designs must be included in undergraduate and graduate courses in Design of Experiments!

Outline

1. Motivation
2. Undergraduate course
3. Future evolution

STATS 101B: Introduction to Design and Analysis of Experiments

- Upper division course which is required for the statistics major at UCLA.
- Offered in Spring quarter (10 weeks).
- Typically, four sessions with around 60 students each.
- Requirement: “STATS 101A: Introduction to Data Analysis and Regression.”

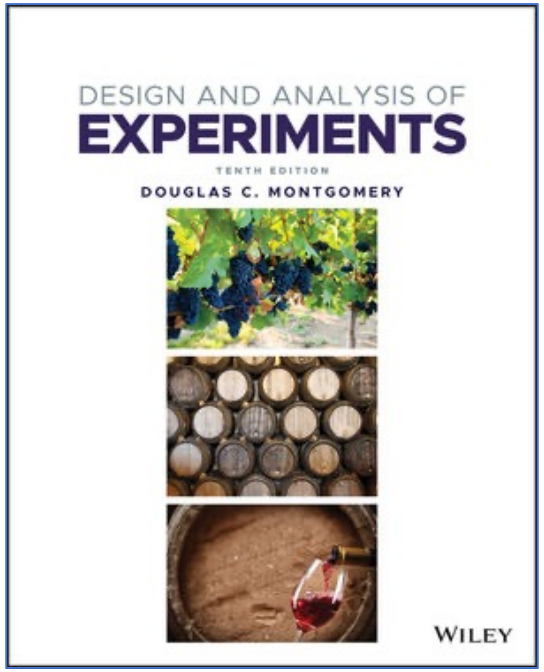
Course contents

Fundamental topics

- Randomization, blocking and replication
- Simple comparative experiments
- ANOVA
- Randomized complete block designs
- Factorial designs
- Full and fractional factorial designs with two levels

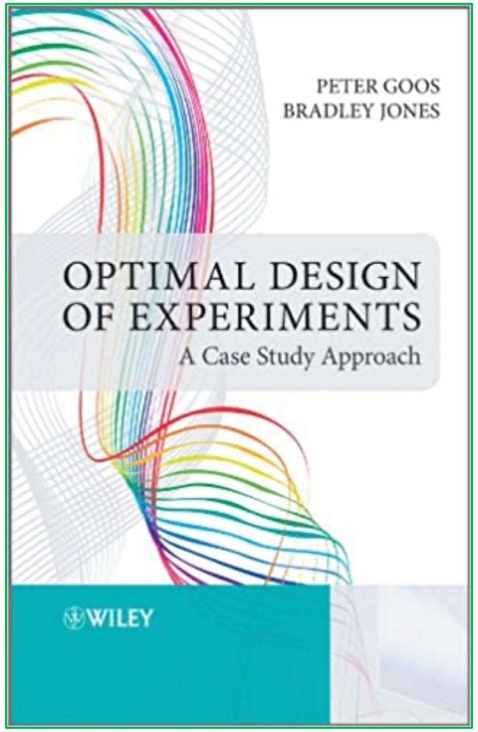
Optimal design

- D- and I-optimal designs for completely randomized experiments
- Follow-up experiments
- Blocked experiments



R software:
FrF2 (Gromping, 2014)

R software:
AlgDesign
(Wheeler, 2022)



Lectures

- Theory, examples and R demos (very popular!).

Evaluation

- Weekly assignments with problems from Montgomery's textbook and other sources.

- A final team project that requires to design and analyze an experiment.
  - Optimize the hyperparameters of machine learning algorithms.

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Discussion

- My DoE class provides students with traditional experimental designs as well as computer-generated optimal designs, which are more flexible.
- Optimal designs improve the estimation and prediction efficiency by optimizing specific measures of multi-collinearity. Therefore, their discussion is natural after a course on linear regression.
- In the future, I plan to remove ANOVA from my DoE course, and adopt the linear regression model as the main data analysis tool.