# Aiyagari model with progressive taxes

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## 1 Model

Here I present a brief description of the model. Households face the following problem:

$$V(a,z) = \max_{c,a'} \left\{ \frac{c^{1-\sigma}-1}{1-\sigma} + \beta \sum_{z'} \Gamma_{z,z'} V(a',z') \right\}$$

subject to

$$y = wz + ra$$

$$c + a' = a + y - T(y)$$

$$c \geq 0, \quad a' \geq 0.$$

The individual state variables are asset holdings $a$ (endogenous state variable) and the idiosyncratic shock $z$ (exogenous state variable ). $\Gamma_{z,z'}$ denotes the transition matrix of the Markov chain over $z$, with $\sum_{z'} \Gamma_{z,z'} = 1$ for all $z$. Taxes are given by

$$T(y) = y - \lambda y^{1-\tau}.$$

The parameter $\tau \in [0,1)$ denotes tax progressivity. If $\tau = 0$, then taxes are proportional to income, with an average tax rate equal to $(1-\lambda)$. If instead $\tau \in (0,1)$, the average tax rate

$$\frac{T(y)}{y} = 1 - \lambda y^{-\tau}$$

is increasing with respect to income, i.e. the tax system is progressive. Households choose consumption $c$, and next-period assets $a'$. The policy function are denoted as $c = g_c(a,z)$, and $a' = g_a(a,z)$.

Factor prices $r$ and $w$ are pinned down by the first-order conditions of the representative firm

$$r = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha-1} - \delta$$

$$w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$$

and the aggregate production function is

$$Y = K^{\alpha} L^{1-\alpha}.$$

The market clearing conditions are

$$K = \int g_a (a, z)\, d\mu (a, z)$$

$$L = \int z d\mu (a, z),$$

where $\mu$ is the stationary distribution. The government budget constraint is simply

$$G = \int T(wz + ra) d\mu(a, z)$$

where $G$ denotes wasteful government spending. Then, by Walras' law, the aggregate resource constraint of the economy is automatically satisfied:

$$C + \delta K + G = K^{\alpha} L^{1-\alpha}.$$