Optimal Control for Autonomous Drone Racing

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***Abstract*— This paper studies the problem of optimal control of a quadrotor to minimize the time it takes it to pass trough several waypoints, that is, to finish a race.**

I. INTRODUCTION

Optimal control problems have been widely studied... The Red Bull Air Race, where airplanes cross gates to end a circuit as fast as possible is an example of a similar problem that has been studied... Nowadays, every team has the role of a ‘tactician’, which is in charge of...

MENTION PARKER AND AUTONOMOUS DRONE RACING COMPETITIONS...

The paper is structured as follows: Section II presents the formulation of this problem, Section...

II. PROBLEM FORMULATION

Several authors have studied quadrotor dynamics [1]. This section presents the geometry of a quadrotor, its dynamics and the mathematical formulation of the problem in hand.

A. Assumptions

The formulation that follows considers the following assumptions:

1) The maximum and minimum altitudes of the quadrotor are similar and thus the air density and gravity are constants.
2) The vehicle is flying in zero-wind conditions.
3) No ground effect is considered.
4) The angular velocities of the rotors are similar, and the rotors’ inertia I_r is small.
5) The quadrotor is symmetrical with its four arms aligned with the body x- and y-axes.
6) The propellers are rigid and thus blade flapping does not occur.

B. Quadrotor Geometry and Notation

The quadrotor’s absolute linear position is defined in the inertial frame with the vector $\boldsymbol{\xi}$. Similarly, the attitude (angular position of the drone with respect to the inertial frame) is defined with the vector $\boldsymbol{\eta}$. Roll angle ϕ determines the rotation of the vehicle around the x-axis, pitch angle θ defines a rotation around the y-axis, and yaw angle ψ determines the quadrotor’s rotation around the z-axis:

$$\boldsymbol{\xi} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \boldsymbol{\eta} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

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The origin of the body frame, indicated with a B, is the center of mass of the quadrotor. In this frame, the vehicle’s linear velocities $\boldsymbol{V_B}$ and angular velocities $\boldsymbol{\nu}$ are:

$$\boldsymbol{V}_B = \begin{bmatrix} v_{x,B} \\ v_{y,B} \\ v_{z,B} \end{bmatrix}, \quad \boldsymbol{\nu} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Figure 1 shows the inertial and body frame, as well as the Euler and angular velocity angles defined previously.

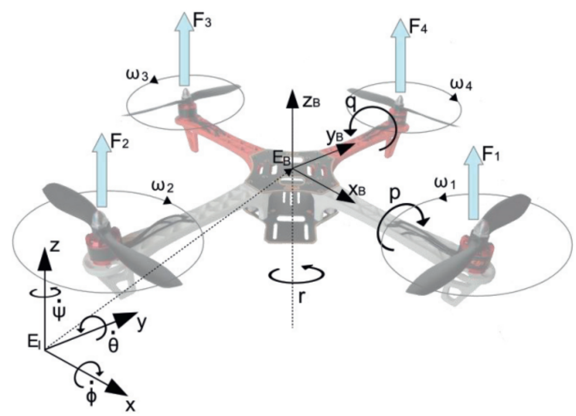

Fig. 1. Inertial and body-fixed frame of the quadrotor, showing the Euler and angular velocity angles [?]

The rotation from the body frame to the inertial frame can be defined with the matrix

$$\boldsymbol{R} = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}$$

in which $S_x = \sin(x)$ and $C_x = \cos(x)$. Note that this rotation matrix is orthogonal and thus the rotation matrix from the inertial frame to the body frame is $\boldsymbol{R^{-1}} = \boldsymbol{R^T}$.

To obtain the angular velocities in the body frame from the angular velocities in the inertial frame, the matrix $\boldsymbol{W_\eta}$ should be used:

$$\boldsymbol{\nu} = \boldsymbol{W}_\eta \dot{\boldsymbol{\eta}}, \qquad \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Its inverse is the transformation matrix from the body frame to the inertial frame, which will be used later to assemble the quadrotor dynamics:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{W}_\eta^{-1}\boldsymbol{\nu}, \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & C_\phi / C_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

where $T_x = \tan(x)$.

As shown in Figure 1 and stated in assumption 5, the drone is symmetric with the arms aligned with the body axes. Therefore, the inertia matrix **I** is diagonal, and $I_{xx} = I_{yy}$:

$$\boldsymbol{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

C. Quadrotor Dynamics

III. RESULTS

Explain solved using GPOPS.

IV. CONCLUSIONS

Conclusions

APPENDIX

Nothing...

Citation: [1].

REFERENCES

[1] S. Zhang, C. Zhu, J. K. O. Sin, and P. K. T. Mok. A novel ultrathin elevated channel low-temperature poly-Si TFT. 20:569–571, November 1999.