Optimal Control for Autonomous Drone Racing

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***Abstract*— This paper studies the problem of optimal control of a quadrotor to minimize the time it takes it to pass trough several waypoints, that is, to finish a race.**

I. INTRODUCTION

Optimal control problems have been widely studied... The Red Bull Air Race, where airplanes cross gates to end a circuit as fast as possible is an example of a similar problem that has been studied... Nowadays, every team has the role of a 'tactician', which is in charge of...

MENTION PARKER AND AUTONOMOUS DRONE RACING COMPETITIONS...

The paper is structured as follows: Section II presents the formulation of this problem, Section...

II. PROBLEM FORMULATION

Several authors have studied quadrotor dynamics [1]. This section presents the assumptions presumed, the geometry of a quadrotor, its dynamics, the state-space model used and the mathematical formulation of the problem in hand.

A. Assumptions

The formulation that follows considers the following assumptions:

1) The maximum and minimum altitudes of the quadrotor are similar and thus the air density and gravity are constants.
2) The vehicle is flying in zero-wind conditions.
3) No ground effect is considered.
4) The angular velocities of the rotors are similar, and the rotors' inertia is small.
5) The quadrotor is symmetrical with its four arms aligned with the body x- and y-axes.
6) The propellers are rigid and thus blade flapping does not occur.

B. Quadrotor Geometry and Notation

The quadrotor's absolute linear position is defined in the inertial frame with the vector $\boldsymbol{\xi}$. Similarly, the attitude (angular position of the drone with respect to the inertial frame) is defined with the vector $\boldsymbol{\eta}$. Roll angle ϕ determines the rotation of the vehicle around the x-axis, pitch angle θ defines a rotation around the y-axis, and yaw angle ψ determines the quadrotor's rotation around the z-axis:

$$\boldsymbol{\xi} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \boldsymbol{\eta} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

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The origin of the body frame, indicated with a B, is the center of mass of the quadrotor. In this frame, the vehicle's linear velocities $\boldsymbol{V_B}$ and angular velocities $\boldsymbol{\nu}$ are:

$$\boldsymbol{V}_B = \begin{bmatrix} v_{x,B} \\ v_{y,B} \\ v_{z,B} \end{bmatrix}, \quad \boldsymbol{\nu} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Figure 1 shows the inertial and body frame, as well as the Euler and angular velocity angles defined previously.

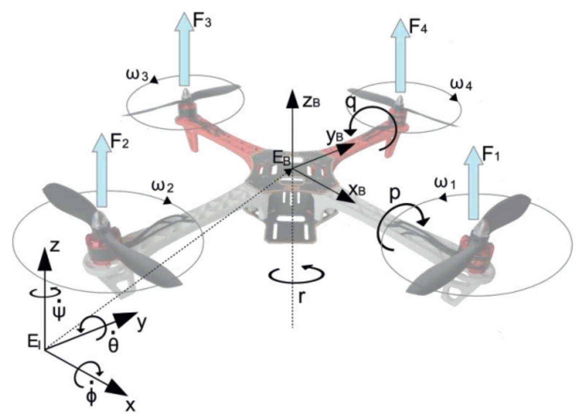

Fig. 1. Inertial and body-fixed frame of the quadrotor, showing the Euler and angular velocity angles [?]

The rotation from the body frame to the inertial frame can be defined with the matrix

$$\boldsymbol{R} = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}$$

in which $S_x = \sin(x)$ and $C_x = \cos(x)$. Note that this rotation matrix is orthogonal and thus the rotation matrix from the inertial frame to the body frame is $\boldsymbol{R^{-1}} = \boldsymbol{R^T}$.

To obtain the angular velocities in the body frame from the angular velocities in the inertial frame, the matrix $\boldsymbol{W_\eta}$ should be used:

$$\boldsymbol{\nu} = \boldsymbol{W}_\eta \dot{\boldsymbol{\eta}}, \qquad \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Its inverse is the transformation matrix from the body frame to the inertial frame, which will be used later to assemble the quadrotor dynamics:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{W}_\eta^{-1}\boldsymbol{\nu}, \quad \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi/C_\theta & C_\phi/C_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

where $T_x = \tan(x)$.

As shown in Figure 1 and stated in assumption 5, the drone is symmetric with the arms aligned with the body axes. Therefore, the inertia matrix $\mathbf{I}$ is diagonal, and $I_{xx} = I_{yy}$:

$$\boldsymbol{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

C. Quadrotor Dynamics

The force f_i created by the angular velocity of rotor i, denoted with ω_i, in the direction of the rotor axis is:

$$f_i = k\omega_i^2$$

Additionally, torque τ_{M_i} is created around the rotor axis:

$$\tau_{M_i} = b\omega_i^2 + I_r\dot\omega_i$$

where the lift constant is k, the drag constant is b and the inertia moment of the rotor is I_r. As assumption 4 considered, the rotor's inertia is small, and $\dot\omega_i$ is also usually small. Therefore, this term can be omitted.

The rotors create thrust T in the direction of the body z-axis, and torque $\boldsymbol{\tau_B}$ consists of torques $\tau_\phi, \tau_\theta, \tau_\psi$ in the direction of the corresponding body frame angles:

$$T = \sum_{i=1}^{4} f_i = k\sum_{i=1}^{4}\omega_i^2, \qquad \boldsymbol{T}^B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix}$$

$$\boldsymbol{\tau}_B = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} lk\left(-\omega_2^2 + \omega_4^2\right) \\ lk\left(-\omega_1^2 + \omega_3^2\right) \\ \sum_{i=1}^{4}\tau_{M_i} \end{bmatrix}$$

in which l is the distance between the rotor and the center of mass of the quadcopter.

The Newton-Euler equations for the quadrotor are:

$$m\ddot{\boldsymbol{\xi}} = \boldsymbol{G} + \boldsymbol{R}\boldsymbol{T}_B$$

$$\begin{bmatrix} \ddot x \\ \ddot y \\ \ddot z \end{bmatrix} = -g\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m}\begin{bmatrix} C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix}$$

where g is Earth's gravity, 9.81 m/s^2. Additionally:

$$\boldsymbol{I}\dot{\boldsymbol{\nu}} + \boldsymbol{\nu}\times(\boldsymbol{I}\boldsymbol{\nu}) + \boldsymbol{\Gamma} = \boldsymbol{\tau_B}$$

$$\dot{\boldsymbol{\nu}} = \boldsymbol{I}^{-1}\left(-\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} I_{xx}p \\ I_{yy}q \\ I_{zz}r \end{bmatrix} - I_r\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\omega_\Gamma + \boldsymbol{\tau_B}\right)$$

$$\begin{bmatrix} \dot p \\ \dot q \\ \dot r \end{bmatrix} = \begin{bmatrix} (I_{yy}-I_{zz})\,qr/I_{xx} \\ (I_{zz}-I_{xx})\,pr/I_{yy} \\ (I_{xx}-I_{yy})pq/I_{zz} \end{bmatrix} - I_r\begin{bmatrix} q/I_{xx} \\ -p/I_{yy} \\ 0 \end{bmatrix}\omega_\Gamma + \begin{bmatrix} \tau_\phi/I_{xx} \\ \tau_\theta/I_{yy} \\ \tau_\psi/I_{zz} \end{bmatrix}$$

where $\omega_\Gamma = \omega_1 - \omega_2 + \omega_3 - \omega_4$. By assumption 4, the term that includes I_r and ω_Γ can be omitted.

Finally, this work considers aerodynamic drag caused by the vehicle's translation. Therefore, the revised Netwon equation is:

$$\begin{bmatrix} \ddot x \\ \ddot y \\ \ddot z \end{bmatrix} = -g\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m}\begin{bmatrix} C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix} - \frac{1}{m}\begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix}\begin{bmatrix} \dot x \\ \dot y \\ \dot z \end{bmatrix}$$

where A_x, A_y, and A_z are the drag force coefficients for velocities in the corresponding directions of the inertial frame.

D. State-space Representation

The state of the quadrotor can be represented as follows:

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x} \\ \dot{\boldsymbol{x}} \\ \boldsymbol{\eta} \\ \boldsymbol{\nu} \end{bmatrix}$$

that is, a column vector of $4\cdot 3 = 12$ components that contains the inertial position, the inertial velocity, the Euler angles and the angular velocities in the body frame. The control inputs are:

$$\boldsymbol{U} = \begin{bmatrix} T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

that is, the total thrust T and the torques τ that cause a roll, pitch, and yaw angle change, respectively.

The dynamics previously derived can be assembled in vector $\dot{\boldsymbol{X}} = f(\boldsymbol{X}, \boldsymbol{U})$, which represents the change of the state as a function of the state and the inputs. That is:

$$
\dot{\boldsymbol{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \frac{T}{m}[C_\psi S_\theta C_\phi + S_\psi S_\phi] - \frac{A_x}{m}\dot{x} \\ \frac{T}{m}[S_\psi S_\theta C_\phi - C_\psi S_\phi] - \frac{A_y}{m}\dot{y} \\ -g + \frac{T}{m}[C_\theta C_\phi] - \frac{A_z}{m}\dot{z} \\ p + q[S_\phi T_\theta] + r[C_\phi T_\theta] \\ q[C_\phi] - r[S_\phi] \\ q\frac{S_\phi}{C_\theta} + r\frac{C_\phi}{C_\theta} \\ \frac{I_{yy}-I_{zz}}{I_{xx}}qr + \frac{\tau_\phi}{I_{xx}} \\ \frac{I_{zz}-I_{xx}}{I_{yy}}pr + \frac{\tau_\theta}{I_{yy}} \\ \frac{I_{xx}-I_{yy}}{I_{zz}}pq + \frac{\tau_\psi}{I_{zz}} \end{bmatrix}
$$

E. Mathematical Formulation

The problem in hand consists of minimizing the time a drone takes to complete a race, that is, to cross all the gates in a specific order as fast as possible. In this paper, the gates are considered point constraints only, and it does not matter the direction of the velocity vector. This can be formulated mathematically as an optimal control problem:

Minimize $\quad \int_{t_0}^{t_f} 1dt = \int_0^{t_f} 1dt = t_f$

subject to:

$$\dot{\boldsymbol{X}} = f(\boldsymbol{X}, \boldsymbol{U})$$

The dynamics, gates, starting point and constraints (zmin).

III. RESULTS

Explain solved using GPOPS.

IV. CONCLUSIONS

Conclusions

APPENDIX

Nothing...
Citation: [1].

REFERENCES

[1] S. Zhang, C. Zhu, J. K. O. Sin, and P. K. T. Mok. A novel ultrathin elevated channel low-temperature poly-Si TFT. 20:569–571, November 1999.