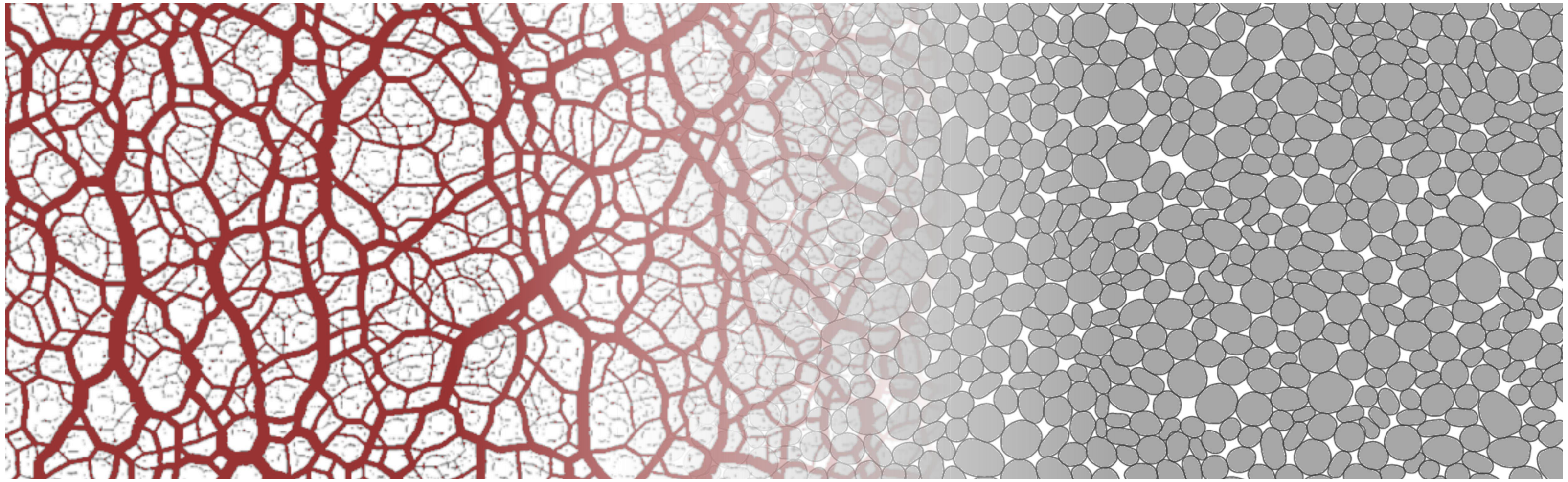
Data-Driven Modeling of Geomaterials



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Alert Doctoral School, Aussois, France
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Introduction

Conventional (+ ML-based surrogate)

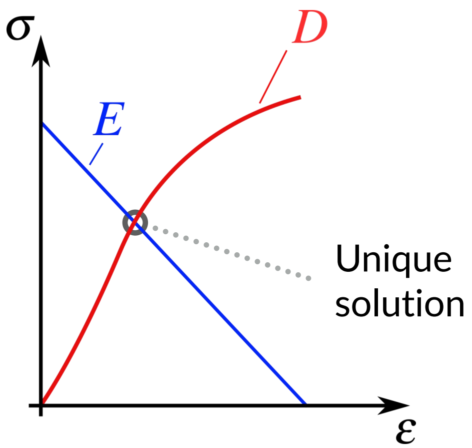
Mozzafar et al., 2019

Masi et al., 2021

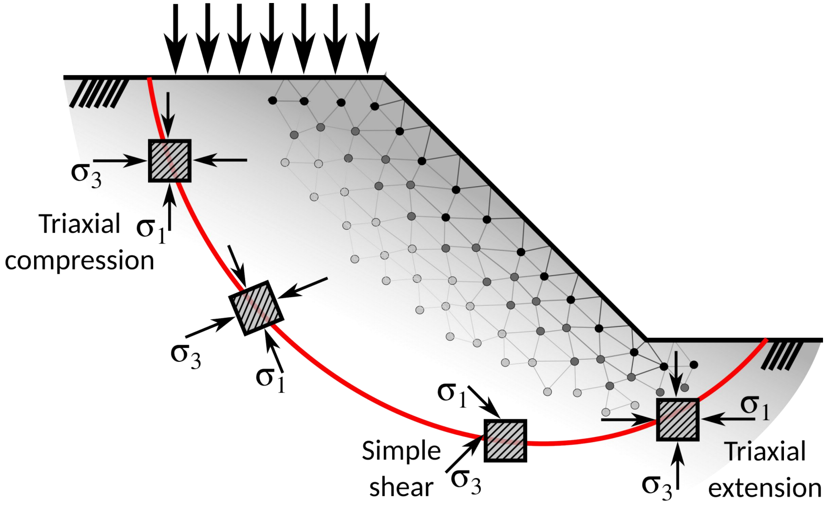
Fung et al., 2022

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{b} = \boldsymbol{0} & (1) \\ \boldsymbol{\epsilon} = \mathrm{sym}(\nabla \boldsymbol{u}) & (2) \\ \boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\epsilon}) \end{cases}$$

e.g. learned constitutive relation (LSTM/TANN/..)



- Uncertainty in functional form
- Uncontrolled extrapolation
- Laborious training/Fast inference



Data-Driven Computing (DD)

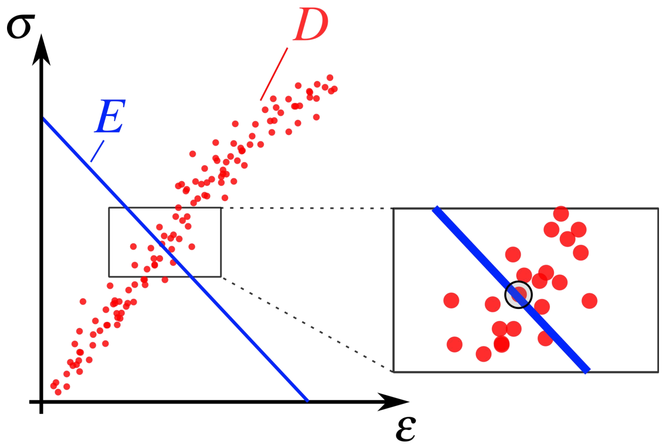
Kirchdoerfer et al., 2016

Phase Space: $Z = \{(\boldsymbol{\epsilon}, \boldsymbol{\sigma})\}$

Equilibrium Set: $E = \{\boldsymbol{z} \in Z \,|\, (1), (2)\}$

Material Data Set: $D \subset Z$

$$\min_{\boldsymbol{z} \in E} \min_{\boldsymbol{y} \in D} d(\boldsymbol{z}, \boldsymbol{y})$$



- Pure data/No fitting/Explicit conservation laws
- Eminent physical meaning (distance)

Overview

I) Data-Driven Computational Mechanics

a) Formulation
b) Hands-off example
c) Improving convergence

II) Data Mining

a) Experiments (Identification)
b) Simulations (Multiscale)
c) Adaptive data sampling

III) History-dependent Formulation

IV) Multi-field Formulation

V) Advanced Applications

a) Shear banding
b) Breakage mechanics

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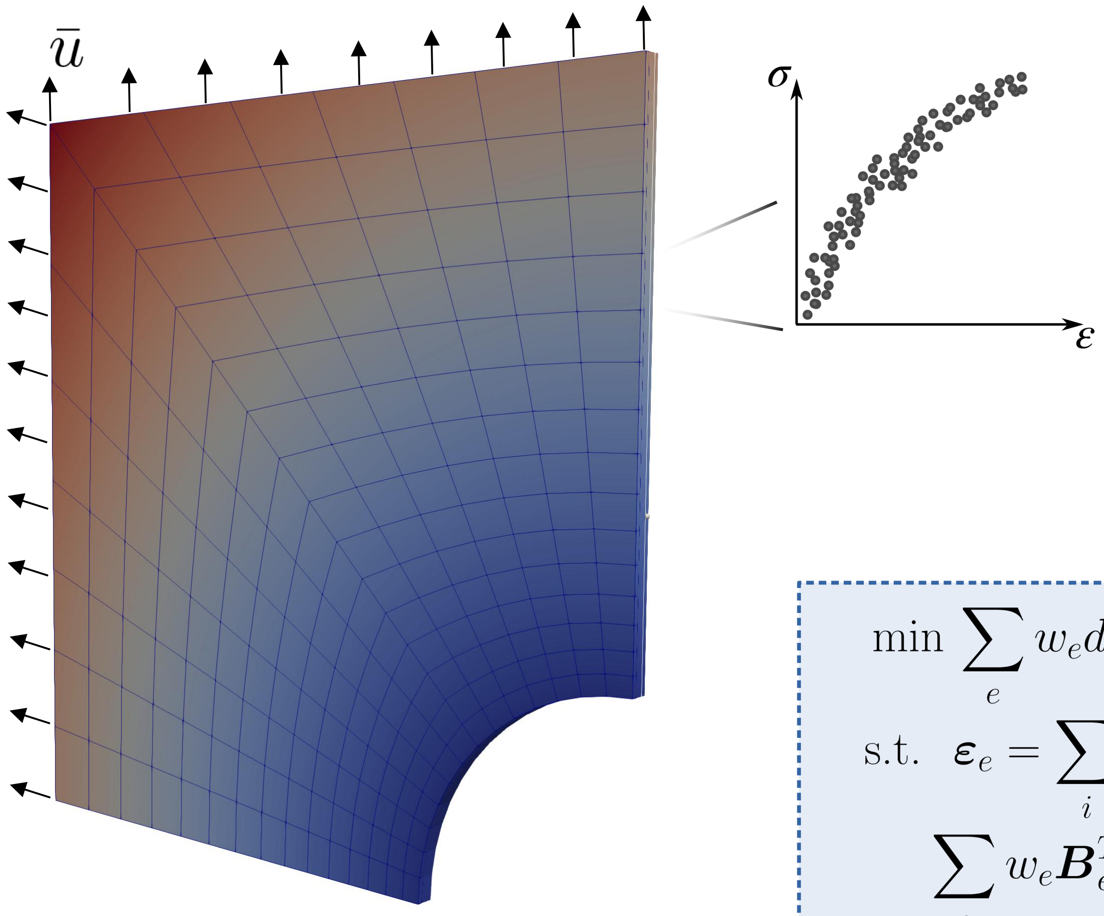
Data-Driven Computational Mechanics

Kirchdoerfer et al, 2016



Perform double minimization in staggered fashion

$$\min_{\boldsymbol{z}\in E} \min_{\boldsymbol{y}\in D} d(\boldsymbol{z}, \boldsymbol{y})$$

$$\min \sum_e w_e d_e(\boldsymbol{\varepsilon}_e, \boldsymbol{\sigma}_e)$$
$$\text{s.t.} \quad \boldsymbol{\varepsilon}_e = \sum_i \boldsymbol{B}_{ei} u_i,$$
$$\sum_e w_e \boldsymbol{B}_{ei}^T \boldsymbol{\sigma}_e = F_i$$

→ Mechanical states

$$\mathbf{z} = \{(\boldsymbol{\varepsilon}^e, \boldsymbol{\sigma}^e)\}_{e=1,\ldots,N}$$

$$d_e(\boldsymbol{\varepsilon}_e, \boldsymbol{\sigma}_e) = \min_{\boldsymbol{\varepsilon}_e^*, \boldsymbol{\sigma}_e^* \in D_e} (\boldsymbol{\varepsilon}_e - \boldsymbol{\varepsilon}_e^*)^T \mathbb{C}_e (\boldsymbol{\varepsilon}_e - \boldsymbol{\varepsilon}_e^*) + (\boldsymbol{\sigma}_e - \boldsymbol{\sigma}_e^*)^T \mathbb{C}_e^{-1} (\boldsymbol{\sigma}_e - \boldsymbol{\sigma}_e^*)$$

→ Material states

$$\mathbf{y} = \{(\boldsymbol{\varepsilon}^{e*}, \boldsymbol{\sigma}^{e*})\}_{e=1,\ldots,N}$$

The stiffness $\mathbb{C}_e$ is simply a distance-inducing numerical parameter

Fixed-point algorithm

Iterative scheme, involving:

i) Solution of two modified 'elasticity' problems

ii) Database search

$$\left(\sum_{e=1}^{M} w_e \mathbf{B}_e^T \mathbb{C}_e \mathbf{B}_e\right)\mathbf{u}^{(i)} = \sum_{e=1}^{M} w_e \mathbf{B}_e^T \mathbb{C}_e \varepsilon_e^{*(i)} \quad (1)$$

$$\left(\sum_{e=1}^{M} w_e \mathbf{B}_e^T \mathbb{C}_e \mathbf{B}_e\right)\boldsymbol{\eta}^{(i)} = \mathbf{f} - \sum_{e=1}^{M} w_e \mathbf{B}_e^T \sigma_e^{*(i)} \quad (2)$$

$$\sigma_e^{(i)} = \sigma_e^{*(i)} + \mathbb{C}_e \sum_{\alpha=1}^{N} \mathbf{B}_{e\alpha} \eta_\alpha^{(i)}$$

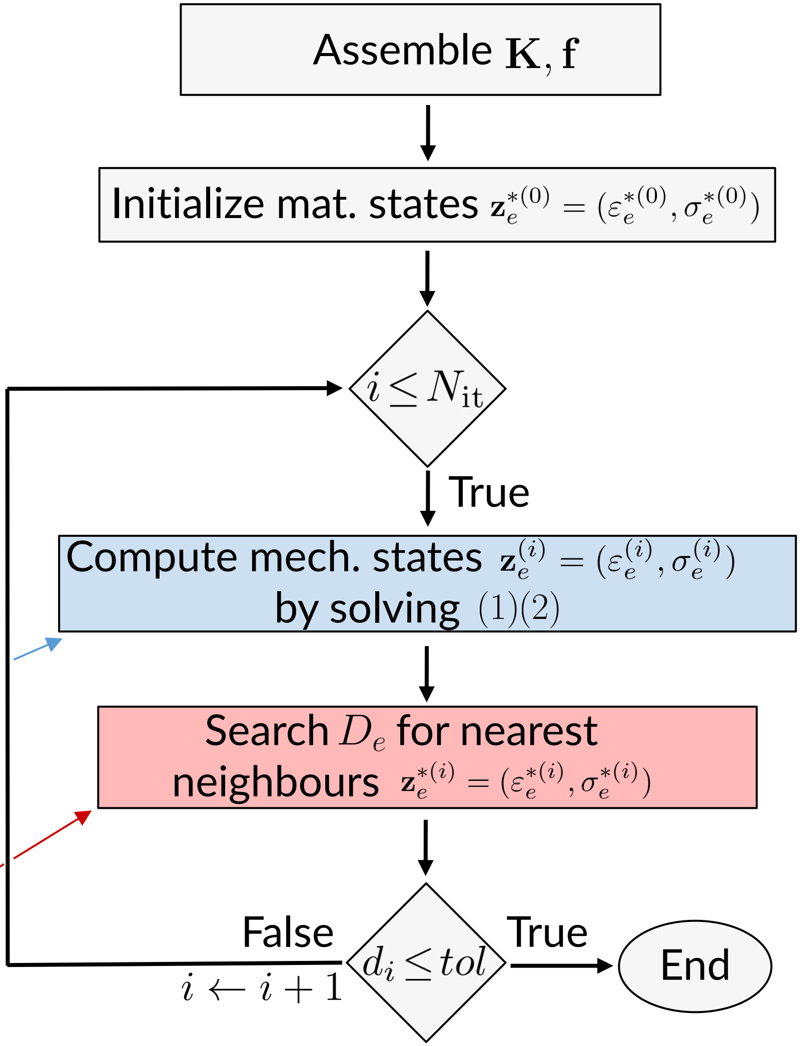
Positive-definite distance-inducing tensor

$$|\boldsymbol{z}^e| = \mathbb{C}^e \boldsymbol{\varepsilon}^e \cdot \boldsymbol{\varepsilon}^e + \mathbb{C}^{e^{-1}} \boldsymbol{\sigma}^e \cdot \boldsymbol{\sigma}^e$$

$$\mathbf{K}\,\mathbf{u} = \mathbf{F}_\varepsilon$$

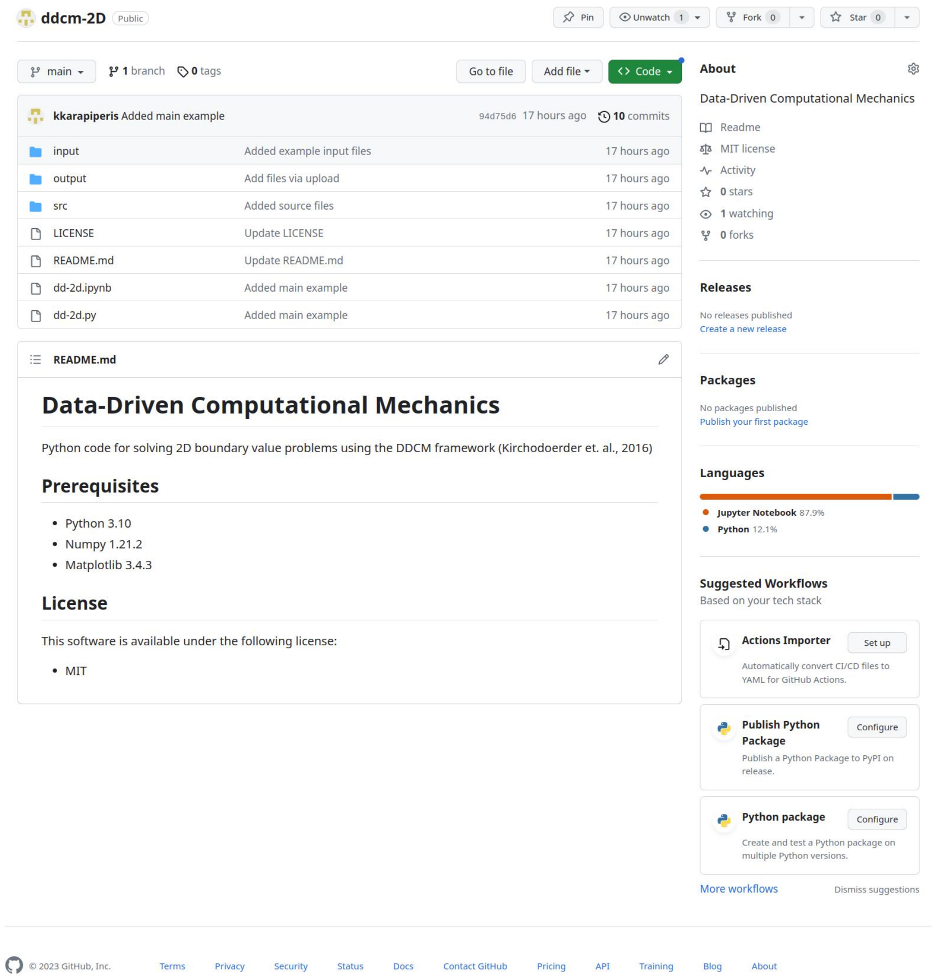
$$\mathbf{K}\,\eta = \mathbf{F}_\sigma$$

Major computational burden (tree-search)

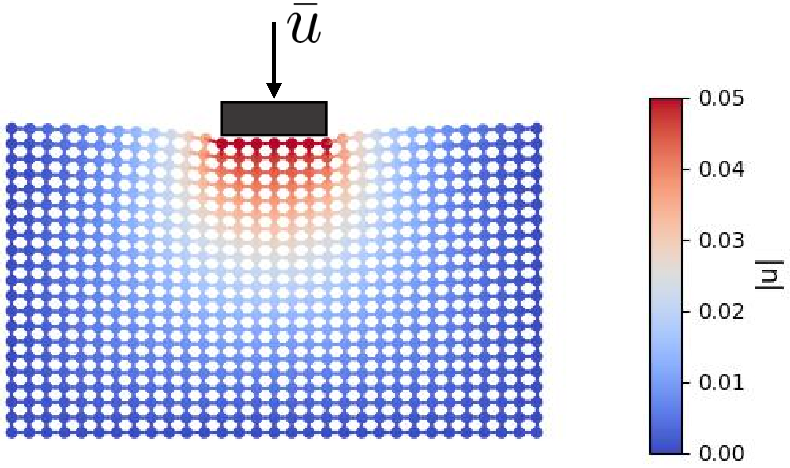


An open-source Python library

github.com/kkarapiperis/ddcm-2D



A simple flat-punch boundary value problem

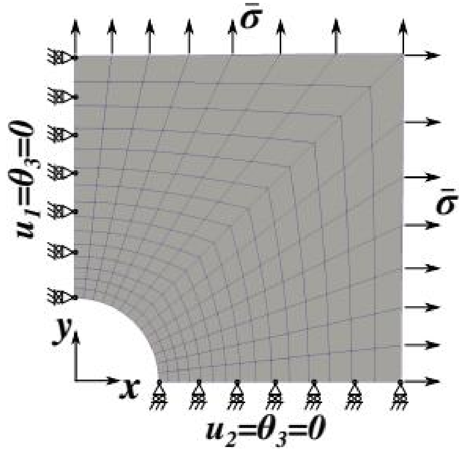


Improving accuracy

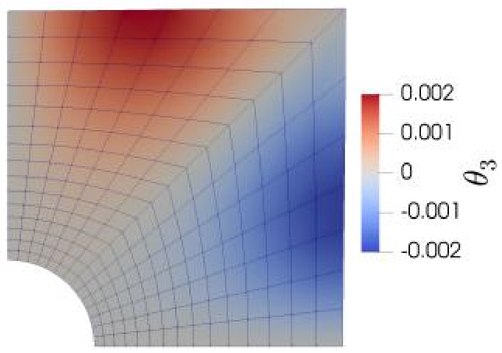
Karapiperis et. al., 2021

Can we solve for the metric-inducing tensor?

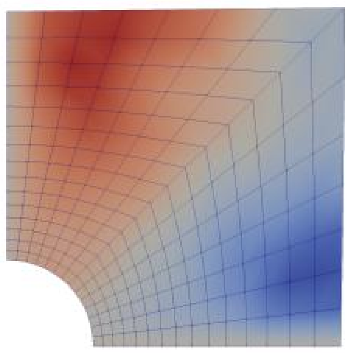
$$|\boldsymbol{z}^e| = \boxed{\min_{\mathbb{C}^e \succeq 0}} \mathbb{C}^e \boldsymbol{\varepsilon}^e \cdot \boldsymbol{\varepsilon}^e + \mathbb{C}^{e^{-1}} \boldsymbol{\sigma}^e \cdot \boldsymbol{\sigma}^e$$



Classical



Data-Driven

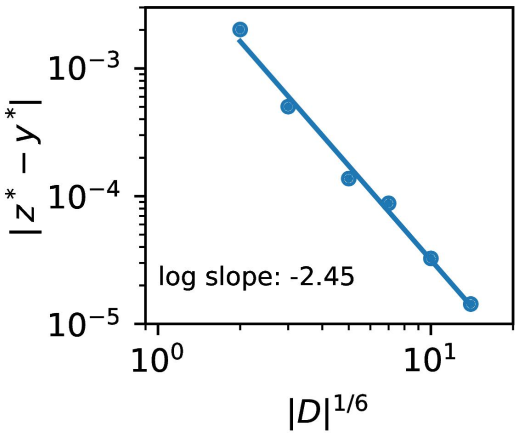


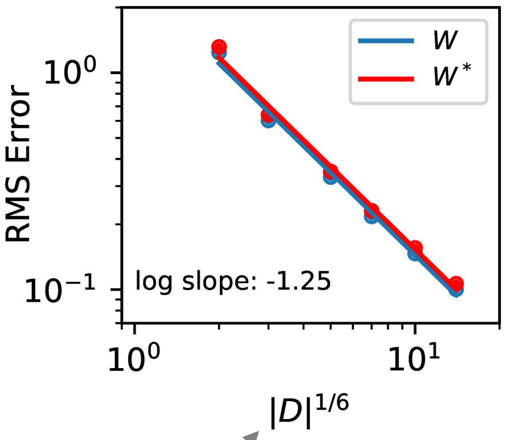
Karapiperis et al., 2021

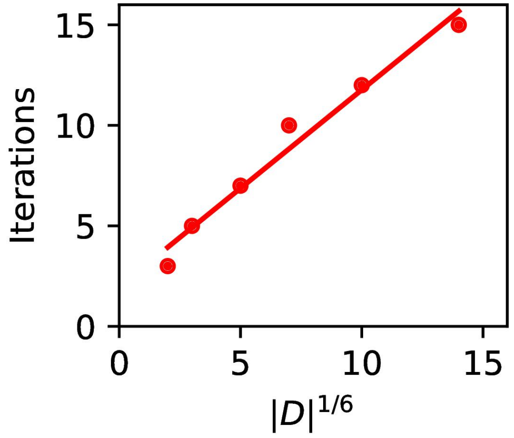
$$RMS_W = \left(\frac{\sum_e w_e W(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{ref}})}{\sum_e w_e W(\boldsymbol{\varepsilon}^{\text{ref}})} \right)^{1/2}$$

$$RMS_{W^*} = \left(\frac{\sum_e w_e W^*(\boldsymbol{\sigma} - \boldsymbol{\sigma}^{\text{ref}})}{\sum_e w_e W^*(\boldsymbol{\sigma}^{\text{ref}})} \right)^{1/2}$$

- Truly parameter-free scheme
- Balanced error in compatibility (W^*) and equilibium (W)







Data-set size

Tackling noise and improving convergence

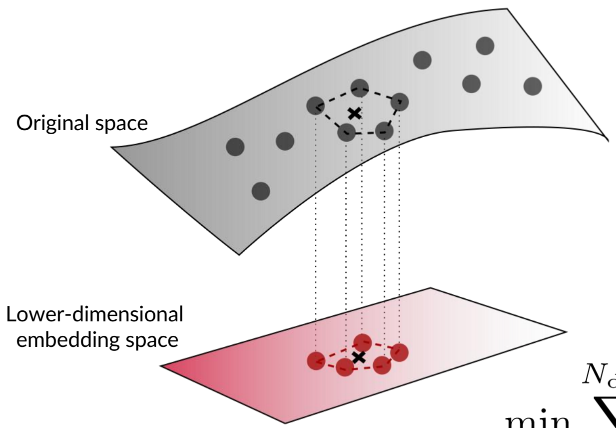
Locally convex/linear embedding

Kirchdoerfer and Ortiz, 2017

Ibanez, 2018

He and Chen, 2021

Eggersmann et al, 2021



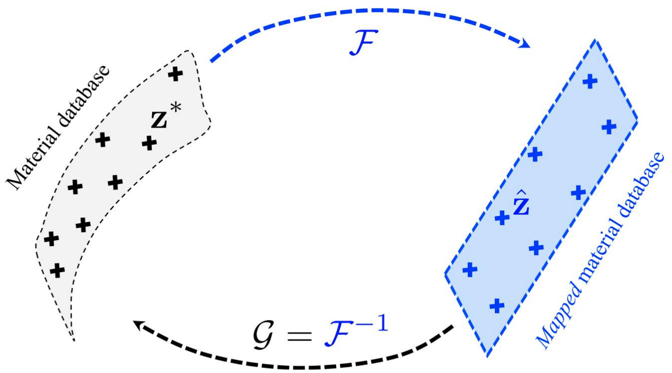
$$\min_{\boldsymbol{w}} \sum_{i}^{N_{\text{data}}} \|\boldsymbol{z}_i - \sum_{j \in \mathcal{N}_i}^{N_{\text{data}}} w_{ij} \boldsymbol{z}_j\|^2$$

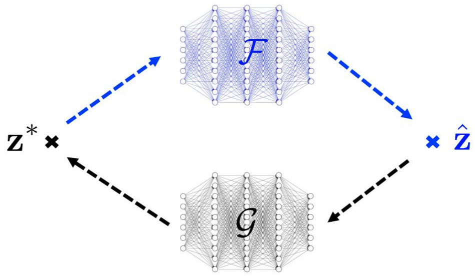
$$\text{s.t.} \sum_{j \in \mathcal{N}_i}^{N_{\text{data}}} w_{ij} = 1$$

$$w_{ij} = 0 \text{ if } j \notin \mathcal{N}_i$$

Global embedding

Bahmani and Sun, 2021





$$\min_{\boldsymbol{\theta},\boldsymbol{\beta},\hat{\mathbb{C}}} \sum_{i}^{N_{\text{data}}} \|\hat{\boldsymbol{\sigma}}_i - \hat{\mathbb{C}}\, \hat{\boldsymbol{\epsilon}}_i\|^2 + \|\boldsymbol{z}_i^* - \mathcal{G}(\mathcal{F}(\boldsymbol{z}_i^*, \boldsymbol{\theta}), \boldsymbol{\beta})\|^2$$

- Often noisy/high dimensional data. Live in manifolds with structure.
- Potential to improve convergence

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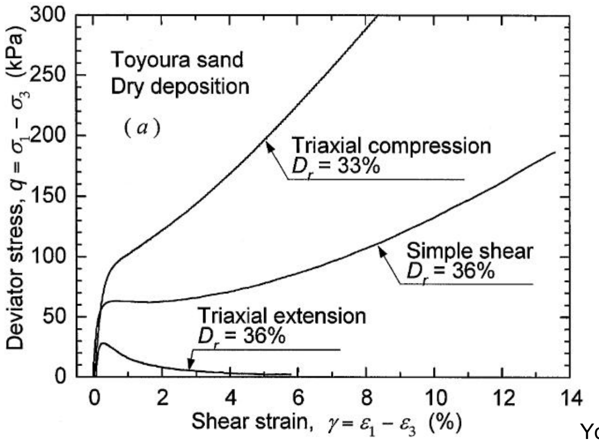
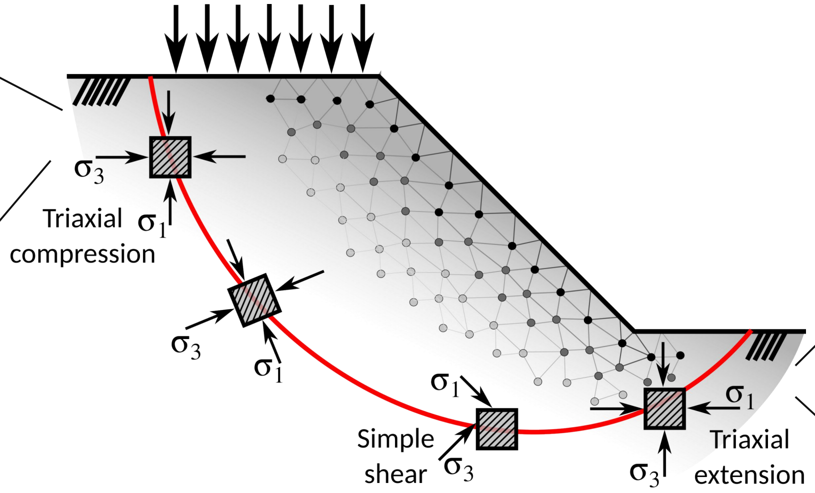
Data mining

Physical Experiments

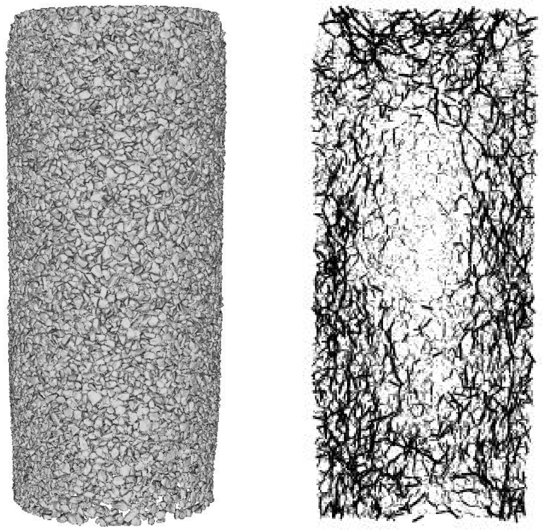
wikipedia.com

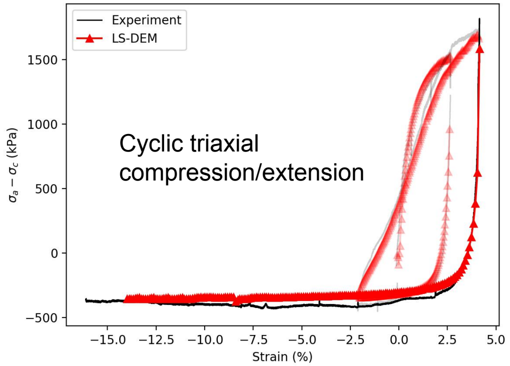


Yoshimine et al., 1999



DEM simulations (Multiscale DD)

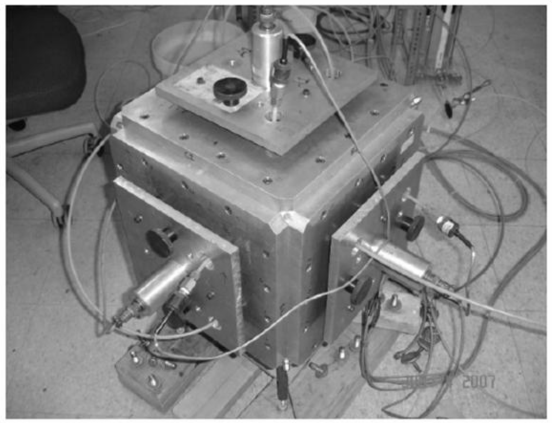


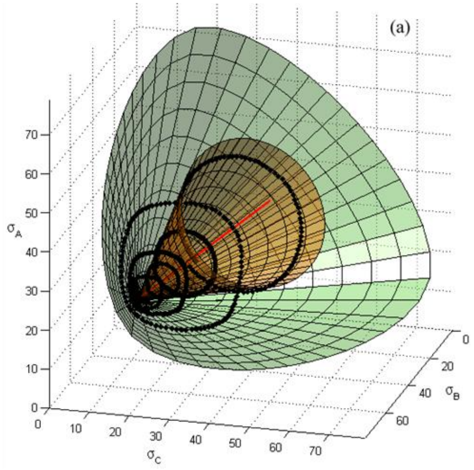


Experimental data mining

Element tests

- Boundary measurements
  (e.g. from true triaxial test)

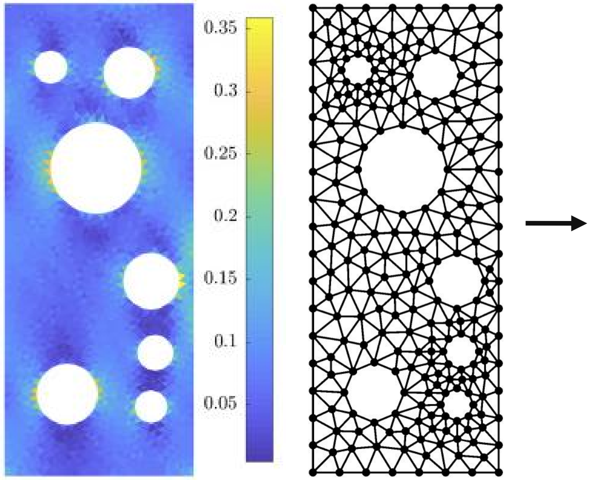


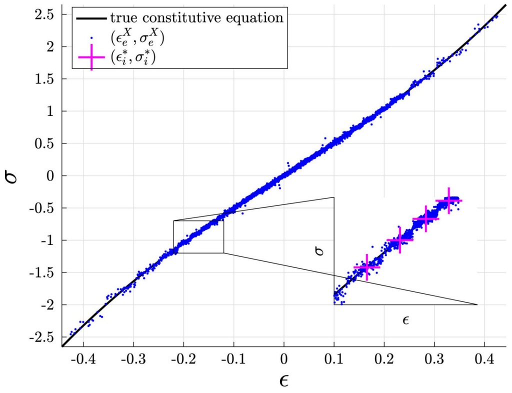


http://faculty.washington.edu/parduino

Data-Driven identification from full-field measurements

- Displacement field e.g. from DIC
- Measured reaction forces





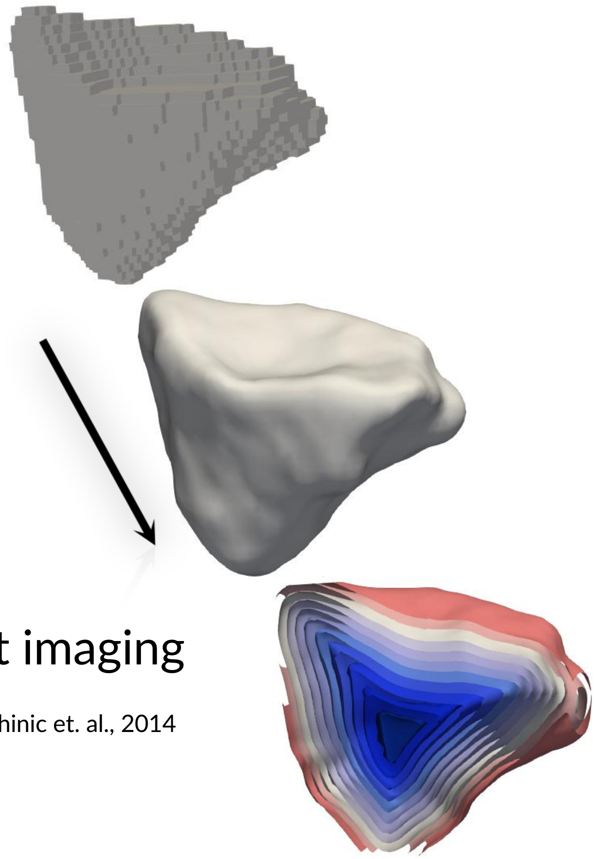
Leygue et al., 2017
Stainier et al., 2019

Find the stress-strain pairs that best satisfy equilibrium/compatibility

Level-Set DEM for granular simulations

Kawamoto et al, 2018, Karapiperis et al, 2020

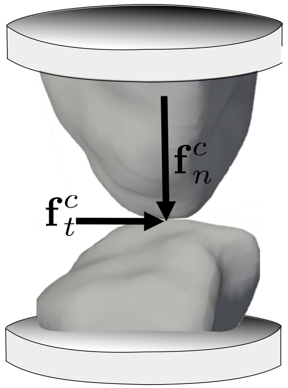
Morphology

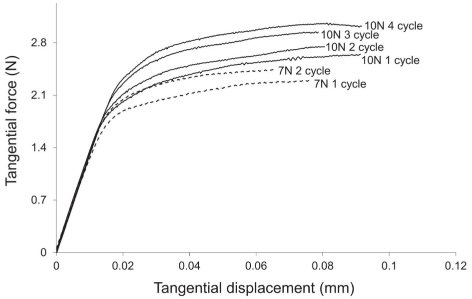


Level set imaging

Vlahinic et. al., 2014

Interaction





Sandeep et al, *Tribology Int.* (2018)

$$\mathbf{f}^c = \mathbf{f}_n^c + \mathbf{f}_t^c$$

$$\mathbf{f}_n^c = k_n \delta_n \mathbf{n}$$

$$\mathbf{f}_t^c = -\frac{\Delta \mathbf{s}}{\|\Delta \mathbf{s}\|} \min(k_t \|\Delta \mathbf{s}\|, \mu \|\mathbf{f}_n^c\|)$$

Specimen generation

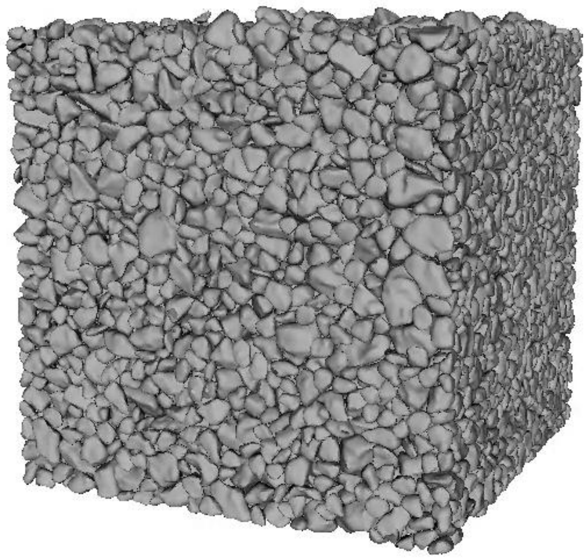
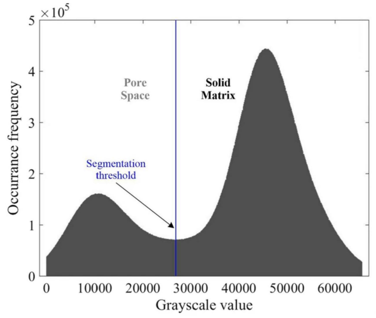


Image-based reconstruction of porous rocks/conglomerates

X-ray CT

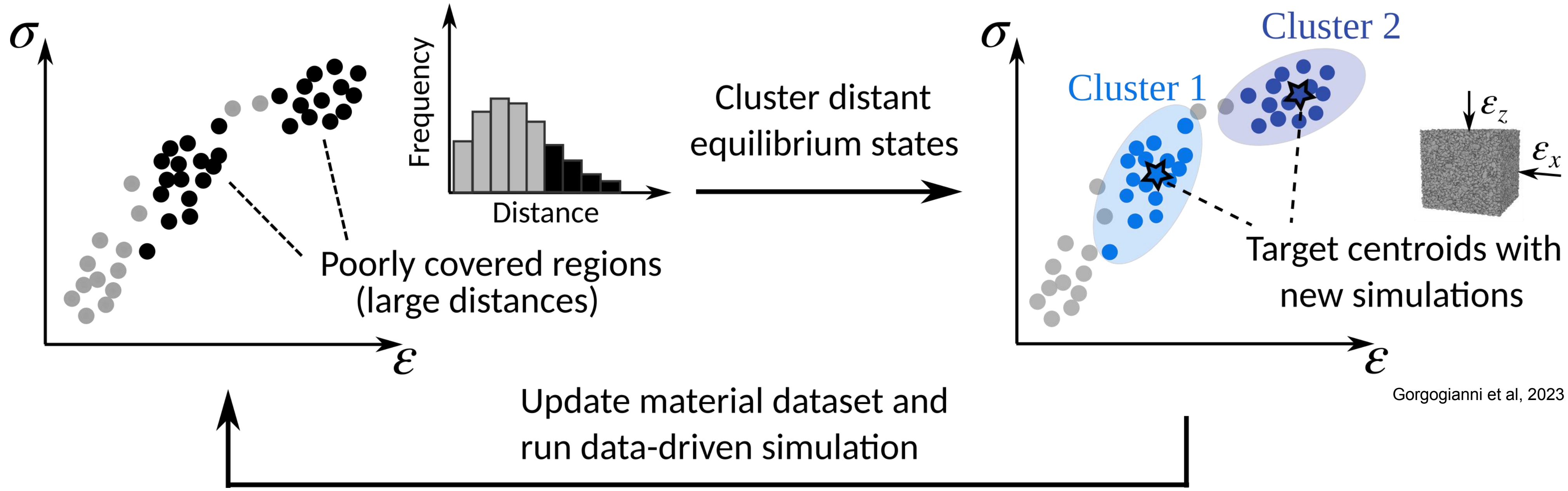
Fu et. al., 2022



Computational specimen

What if data are still scarce?

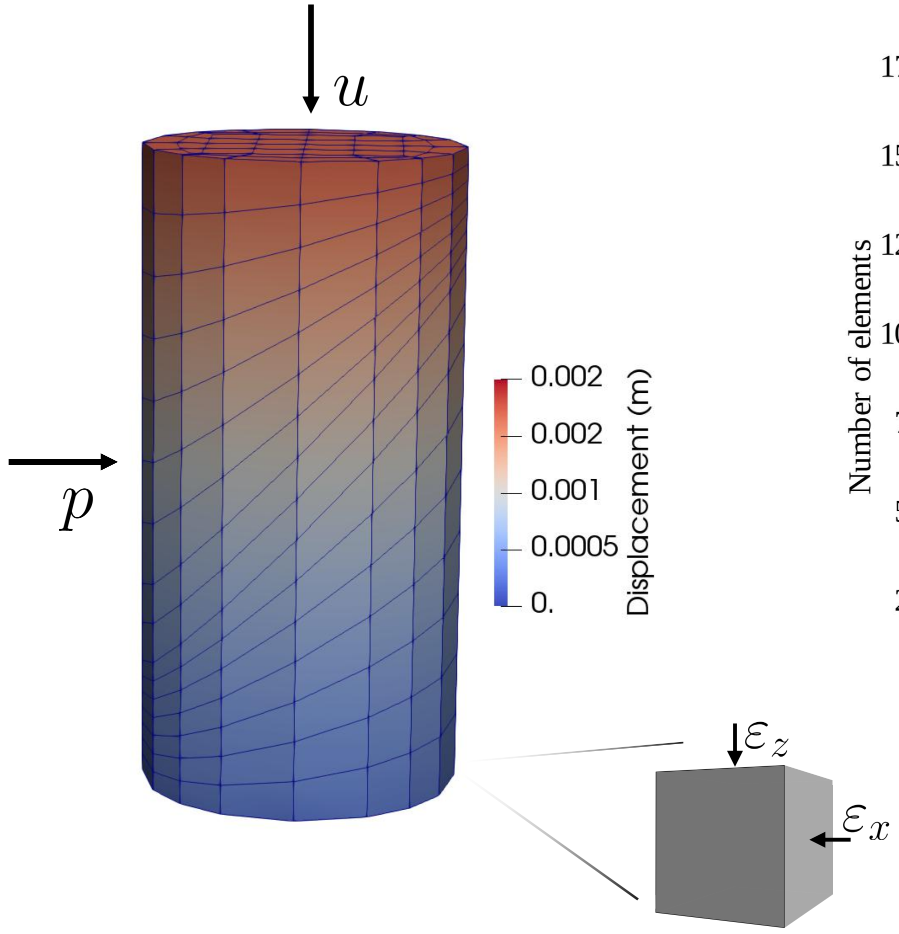
Iteratively sample poorly populated regions of the phase space



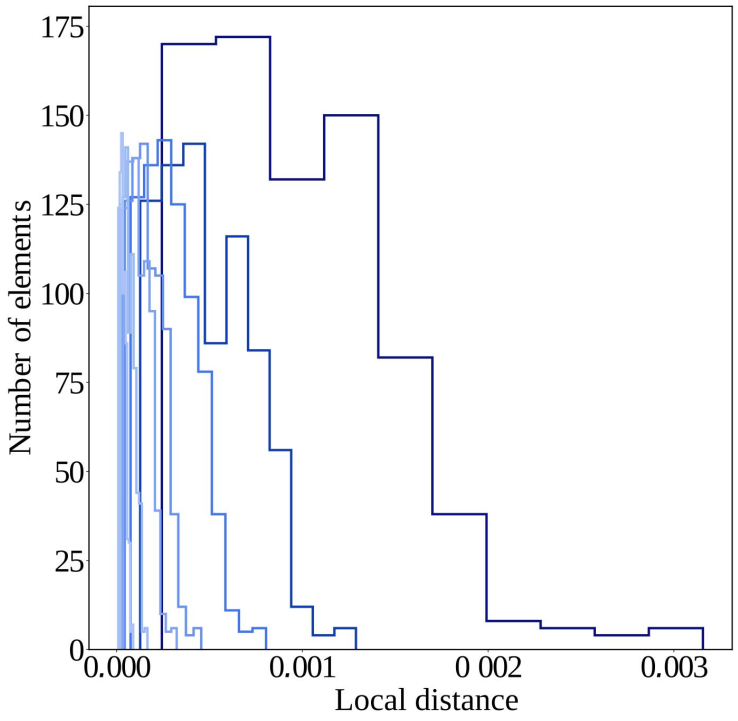
Adaptive sampling via *unsupervised learning*
Repeat until distance is small enough

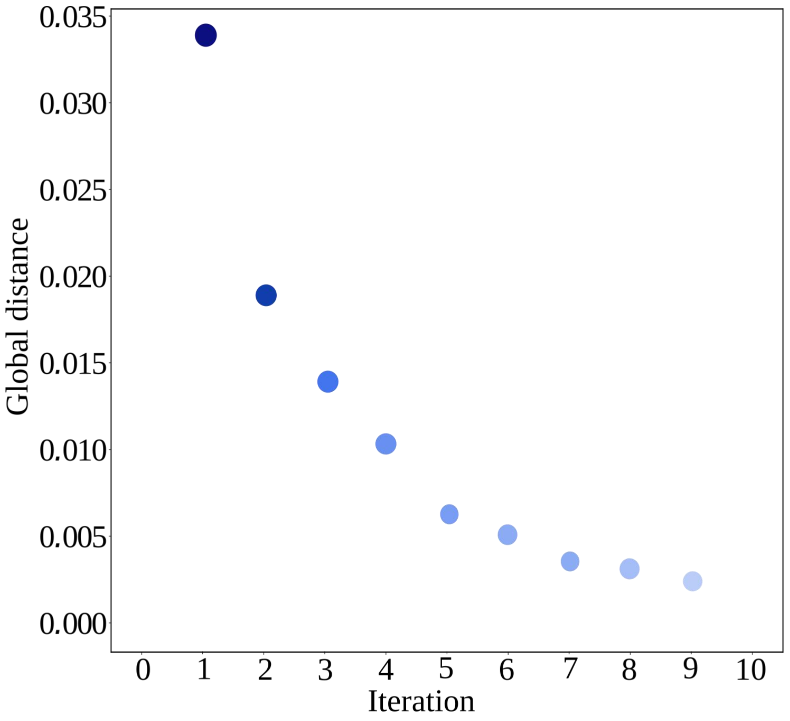
Convergence study

Triaxially compressed elastic cylinder



Error decays with each iteration





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History-dependent formulation

Data-Driven Computing (DD)

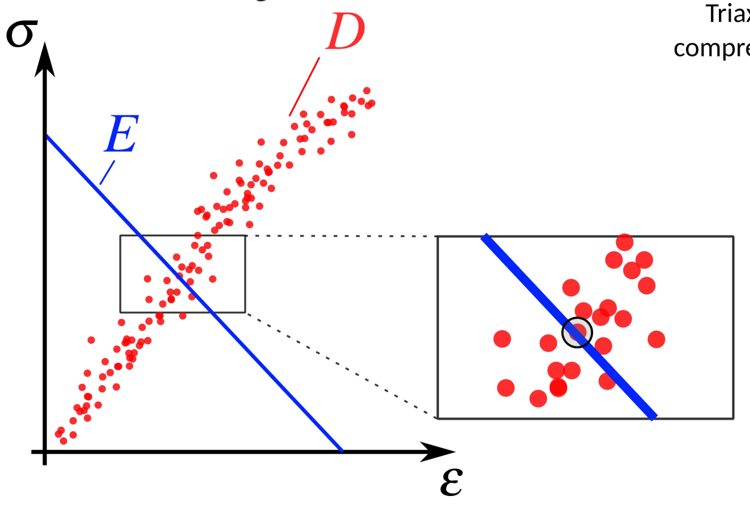
Kirchdoerfer et al., 2016

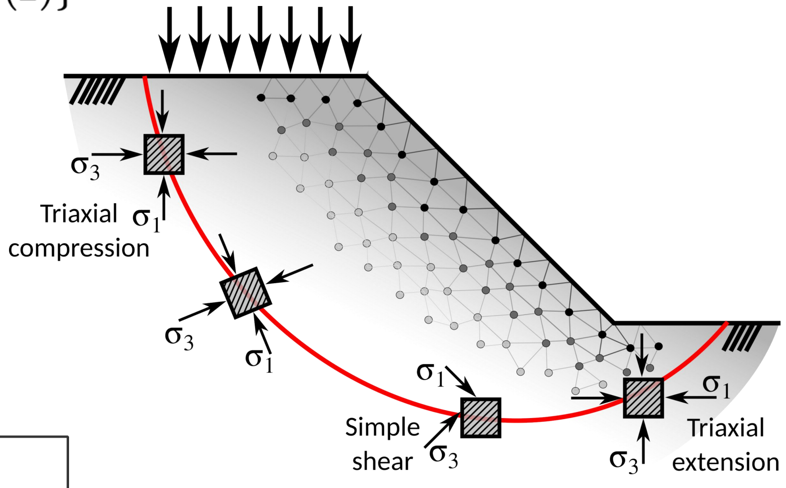
Phase Space: $Z = \{(\boldsymbol{\epsilon}, \boldsymbol{\sigma})\}$

Equilibrium Set: $E = \{\boldsymbol{z} \in Z \,|\, (1), (2)\}$

Material Data Set: $D \subset Z$

$$\min_{\boldsymbol{z} \in E} \min_{\boldsymbol{y} \in D} d(\boldsymbol{z}, \boldsymbol{y})$$





History-dependent DD

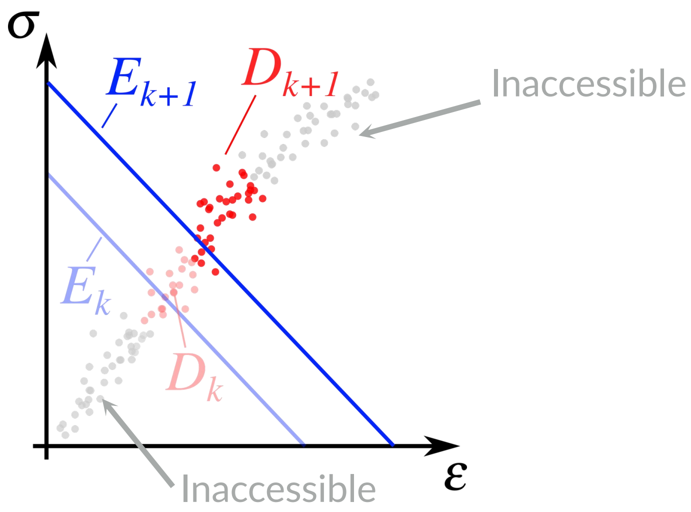
Karapiperis et al., 2020
Eggersmann et al., 2019

History-dependence $\longrightarrow$ Notion of time

Time-dependent Equilibrium Set: $E_k \subset Z$

Time-dependent Material Data Set: $D_k \subset Z$

$$\min_{\mathbf{z}_k \in E_k} \min_{\mathbf{y}_k \in D_k} d(\mathbf{z}_k, \mathbf{y}_k)$$



Modified solution algorithm

Iterative scheme, involving:

i) Solution of two modified ‘elasticity’ problems

ii) Database search

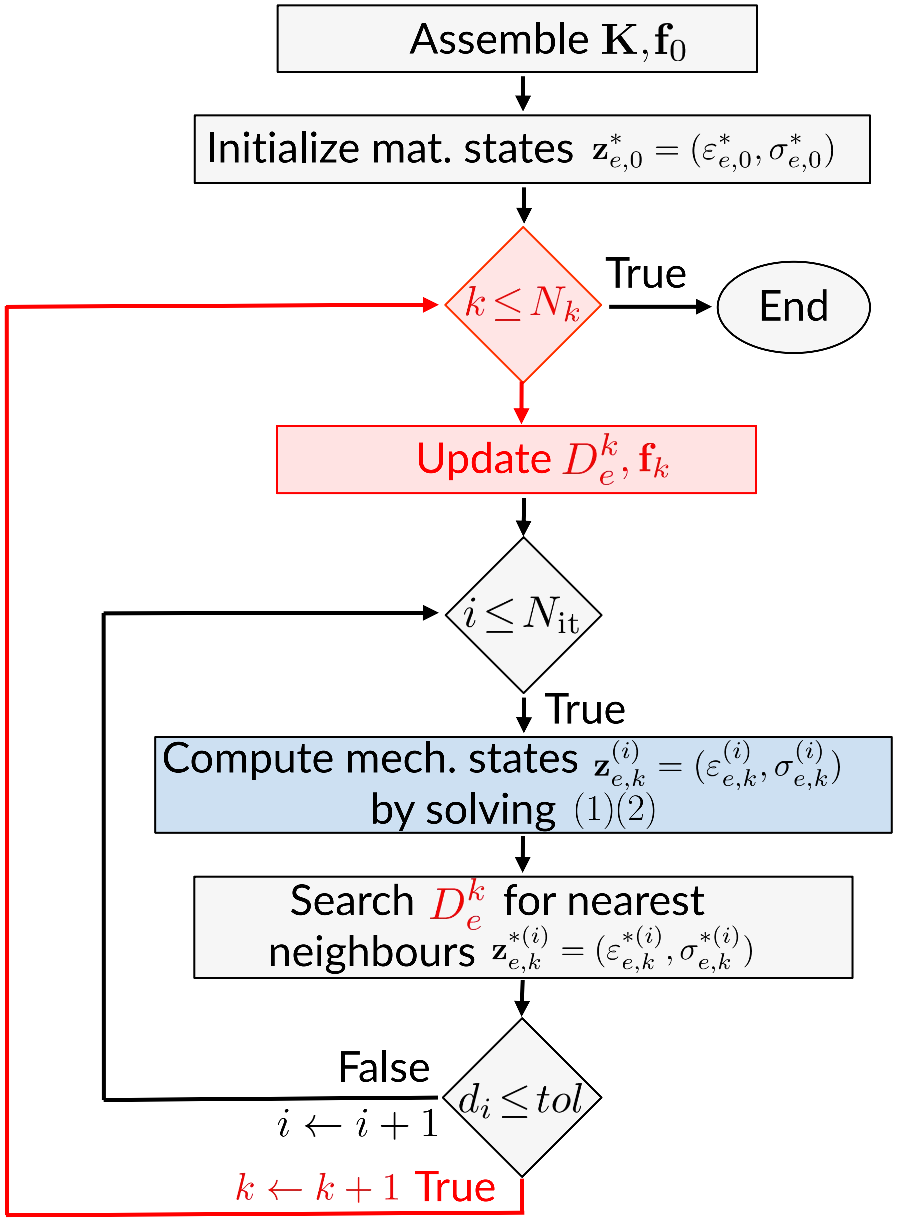
$$\left(\sum_{e=1}^{M} w_e \mathbf{B}_e^T \mathbb{C}_e \mathbf{B}_e\right)\mathbf{u}_k^{(i)} = \sum_{e=1}^{M} w_e \mathbf{B}_e^T \mathbb{C}_e \varepsilon_{e,k}^{*(i)} \quad (1)$$

$$\left(\sum_{e=1}^{M} w_e \mathbf{B}_e^T \mathbb{C}_e \mathbf{B}_e\right)\boldsymbol{\eta}_k^{(i)} = \mathbf{f}_k - \sum_{e=1}^{M} w_e \mathbf{B}_e^T \sigma_{e,k}^{*(i)} \quad (2)$$

$$\sigma_{e,k}^{(i)} = \sigma_{e,k}^{*(i)} + \mathbb{C}_e \sum_{\alpha=1}^{N} \mathbf{B}_{e\alpha} \eta_{\alpha,k}^{(i)}$$

$$\mathbf{K}\,\mathbf{u} = \mathbf{F}_\varepsilon$$

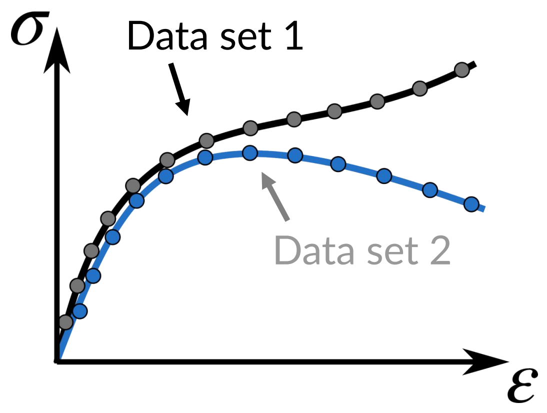
$$\mathbf{K}\,\eta = \mathbf{F}_\sigma$$



History parametrization

History-matching
Eggersmann et al., 2019

$$D_{k+1} = \{(\boldsymbol{\epsilon}_{k+1}, \boldsymbol{\sigma}_{k+1}) \,|\, \{\boldsymbol{\epsilon}_l\}_{l \leq k}\}$$

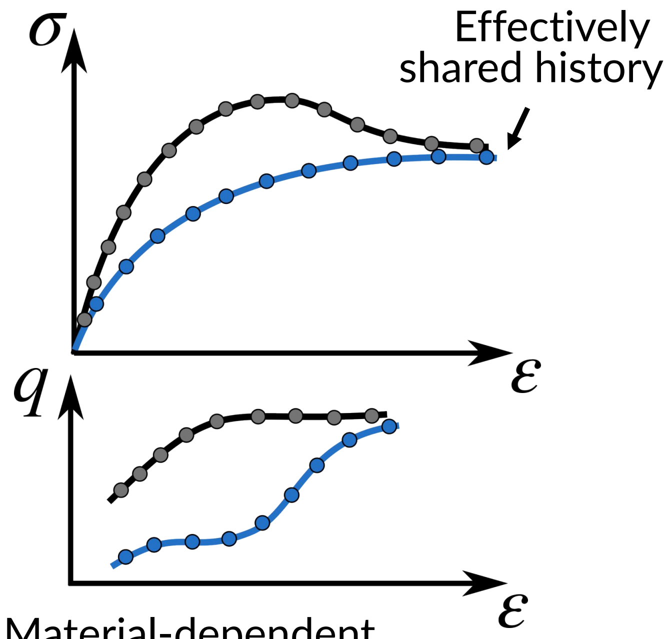


Material-independent
Inefficient

Internal variable
Eggersmann et al., 2019
Karapiperis et al., 2020

$$D_{k+1} = \{(\boldsymbol{\epsilon}_{k+1}, \boldsymbol{\sigma}_{k+1}) \,|\, (\boldsymbol{\epsilon}_k, \boldsymbol{\sigma}_k, \mathbf{q}_k)\}$$

Internal variable: $\mathbf{q} = \{\mathbf{F}, \ldots\}$



Material-dependent
Requires access to micromechanics
Augments space by set of int. variables

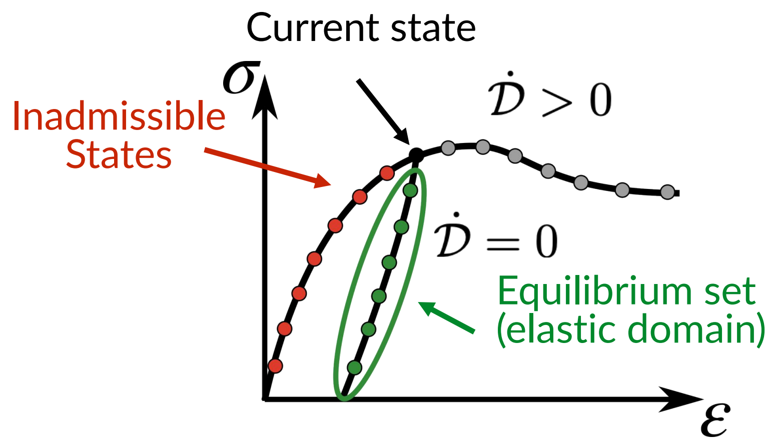
Thermodynamics
Karapiperis et al., 2020

$$D_{k+1} = \{(\boldsymbol{\epsilon}_{k+1}, \boldsymbol{\sigma}_{k+1}) \,|\, (\boldsymbol{\epsilon}_k, \boldsymbol{\sigma}_k), (1)\}$$

$$\mathcal{D}_{k+1} - \mathcal{D}_k = \frac{\boldsymbol{\sigma}_k + \boldsymbol{\sigma}_{k+1}}{2} : (\boldsymbol{\epsilon}_{k+1} - \boldsymbol{\epsilon}_k) - (\mathcal{A}_{k+1} - \mathcal{A}_k) \geq 0 \quad (1)$$

Dissipation

Free energy



Material-independent

Application to geomaterials

Free energy/Dissipation

$$\boldsymbol{\varepsilon} = \frac{1}{2V}\text{sym}\Big(\sum_{p\in\partial\mathcal{P}} \mathbf{u}^p \otimes \mathbf{n}^p\Big)$$

$$\boldsymbol{\sigma} = \frac{1}{V}\text{sym}\Big(\sum_{c\in\mathcal{C}} \mathbf{f}^c \otimes \mathbf{l}^c\Big)$$

$$\mathcal{A} = \sum_c \mathcal{A}^c = \frac{1}{2V}\sum_c \Big(\frac{\|\mathbf{f_n^c}\|^2}{k_n} + \frac{\|\mathbf{f_t^c}\|^2}{k_t}\Big)$$

$$d\mathcal{D} = \sum_c d\mathcal{D}^c = \frac{1}{V}\sum_c \mathbf{f}_t^c \cdot d\mathbf{u}^{c,slip}$$

Internal variables

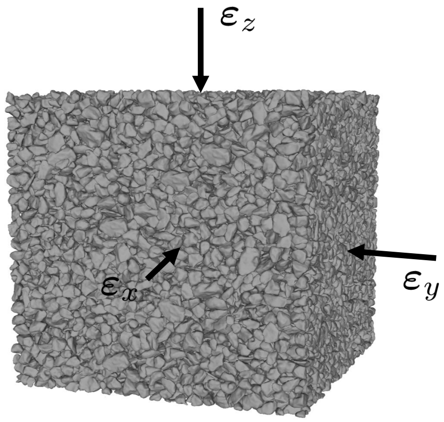
$$\mathbf{q} = P_{\mathbf{nfl}}(\mathbf{n},\mathbf{f},\mathbf{l})$$

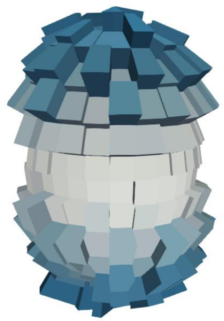
$$= \underbrace{P_{\mathbf{n}}(\mathbf{n})}_{\text{Contact structure}}\underbrace{P_{\mathbf{f}|\mathbf{n}}(\mathbf{f}|\mathbf{n})}_{\text{Frictional history}}\underbrace{P_{\mathbf{l}|\mathbf{f},\mathbf{n}}(\mathbf{l}|\mathbf{f},\mathbf{n})}_{\text{Grain size/shape}}$$

Second-order statistics

Fabric tensors

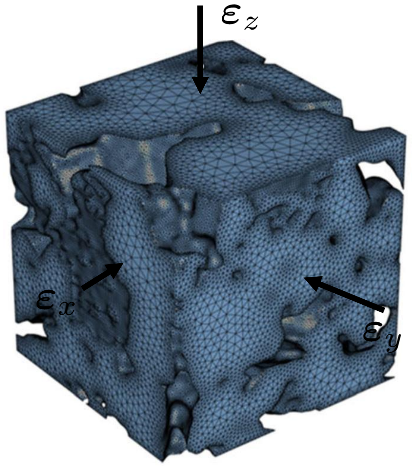
Granular RVE



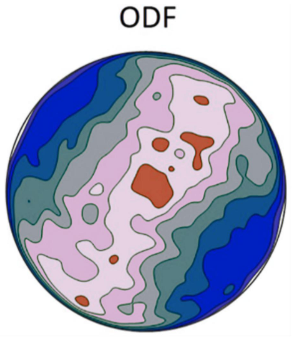


Contact Fabric

Continuum RVE



(e.g. porous rock)



Pore texture

Free energy/Dissipation

$$\bar{\boldsymbol{\varepsilon}} = \frac{1}{V}\int_{\partial V} \text{sym}(\boldsymbol{u}\otimes\boldsymbol{n})dS$$

$$\bar{\boldsymbol{\sigma}} = \frac{1}{V}\int_{\partial V} \boldsymbol{t}\otimes\boldsymbol{x}dS$$

$$\bar{\mathcal{A}} = \frac{1}{V}\int_{V} \mathcal{A}(\mathbf{x})dS$$

$$d\bar{\mathcal{D}} = \bar{\boldsymbol{\sigma}} : d\bar{\boldsymbol{\varepsilon}} - d\bar{\mathcal{A}}$$

Internal variables

$$\mathbf{q} = \underbrace{P(s,\mathbf{n})}$$

Pore size   Pore orientation

Second-order statistics

Pore texture tensor

Application to granular materials - Material point simulations

A simplified problem:

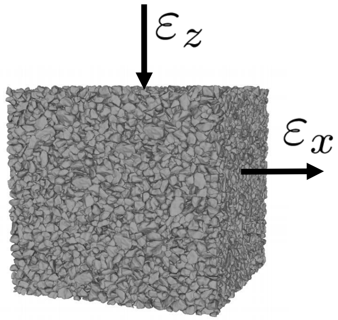
$$\min_{\mathbf{z}\in D_k} d(\mathbf{z}_k, \mathbf{y}_k)$$

Periodic unit cells

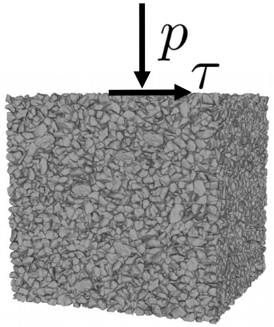
Quasistatic conditions

$I \leq 10^{-4}$

Biaxial Compression

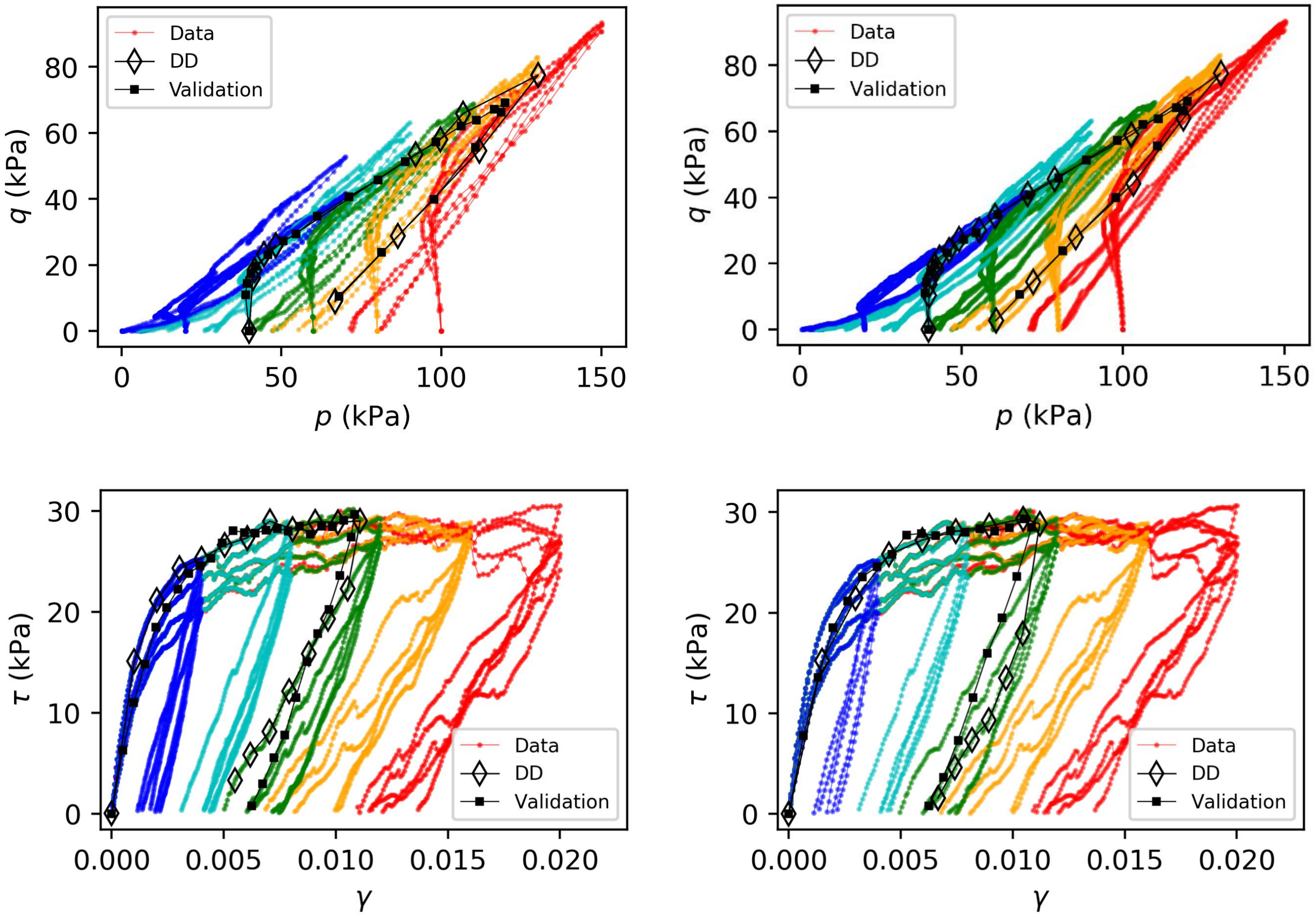


Simple Shear



Thermodynamics-informed

IV-based (Anisotropy tensors)



Overview

I) Data-Driven Computational Mechanics
- a) Formulation
- b) Live example
- c) Improving convergence

II) Data Mining
- a) Experiments (Identification)
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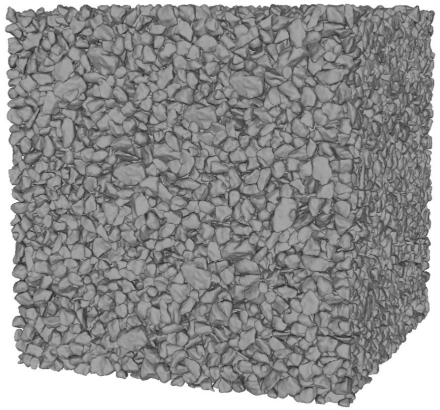
III) History-dependent Formulation

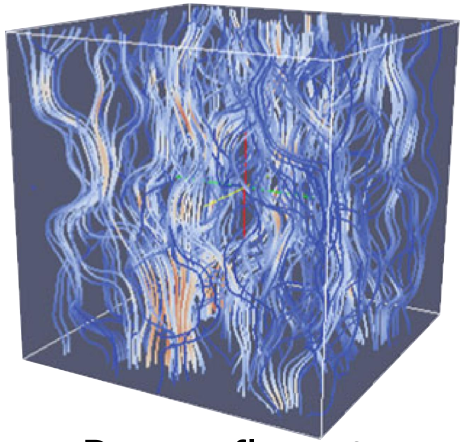
IV) Multi-field Formulation

V) Advanced Applications
- a) Shear banding
- b) Breakage mechanics

Multi-field problems

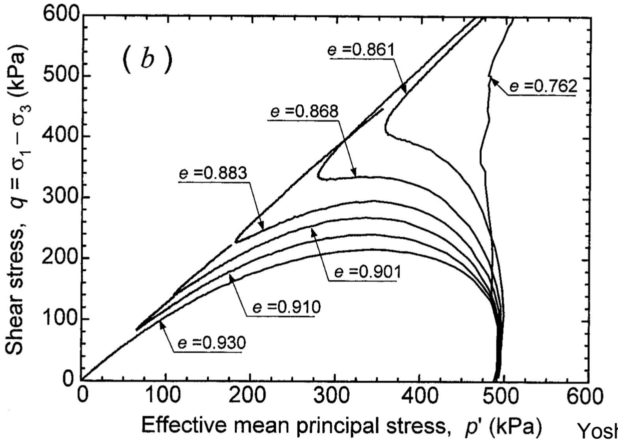
Coupled problems





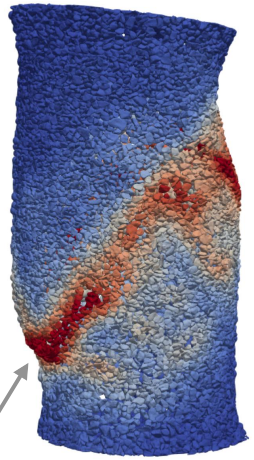
Porous flow

Sun et al., 2013



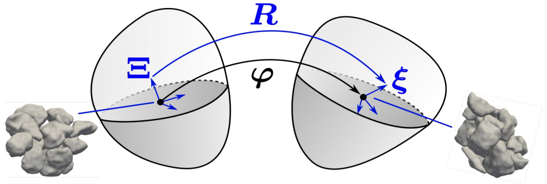
Yoshimine et al., 1998

Gradient/nonlocal theories



Characteristic width ~ 10d

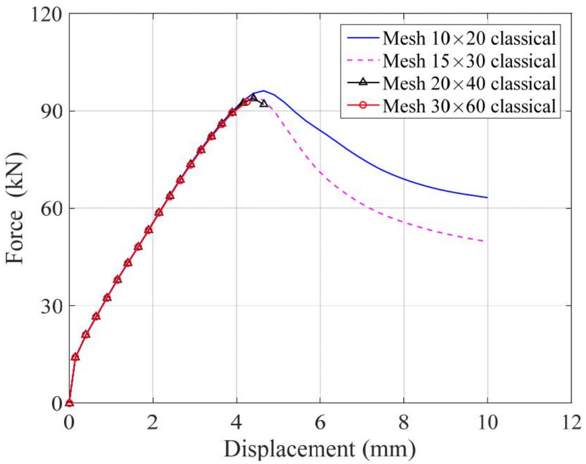
Locally nonaffine deformation



$$\mathcal{A} = \mathcal{A}(\nabla \mathbf{u}, \boldsymbol{\theta}, \nabla^2 \mathbf{u}, \nabla \theta, \dots) \to \ell$$

Eringen, 1964

Muhlhaus and Vardoulakis, 1989



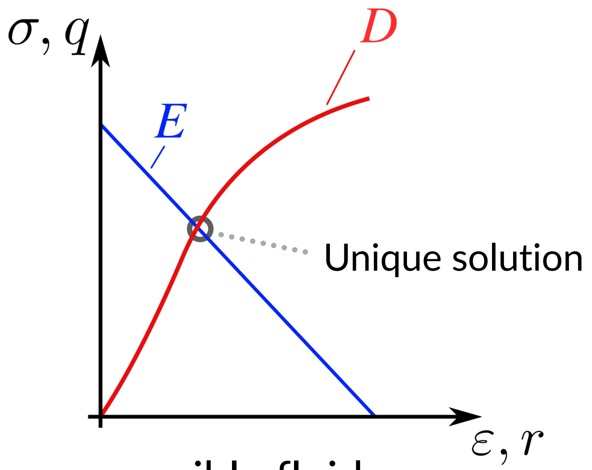
Liu et al., 2015

Poromechanical problem

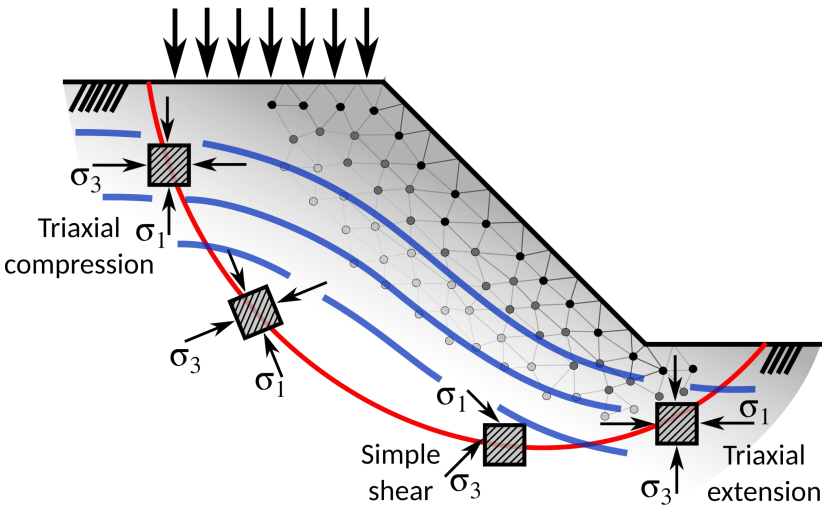
Conventional poromechanics

$$
\begin{cases}
\nabla\cdot\boldsymbol{\sigma} + \rho\,\boldsymbol{b} = 0 & (1)\\
\nabla\cdot\boldsymbol{q} + \dot{\varepsilon}_{\mathrm{vol}} + s = 0 & (2)\\
\boldsymbol{\varepsilon} = \mathrm{sym}(\nabla\boldsymbol{u}) & (3)\\
\boldsymbol{r} = \nabla p & (4)\\
\boldsymbol{\sigma} = \boldsymbol{\sigma}'(\boldsymbol{\varepsilon}) - p\boldsymbol{I} & (5)\\
\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{r}) & (6)
\end{cases}
\quad + \text{B.Cs}
$$

Constitutive relations



- Incompressible fluid
- Compressible solid skeleton



Data-Driven poromechanics

Bahmani et al., 2021

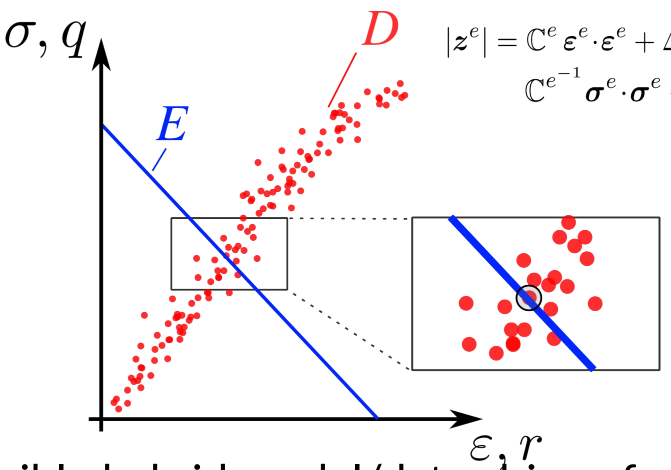
Phase Space $Z = \{(\boldsymbol{\epsilon}, \boldsymbol{r}, \boldsymbol{\sigma}, \boldsymbol{q})\}$

Equilibrium/Mass Balance set $E = \{\boldsymbol{z} \in Z \,|\, (1)-(4)\}$

Material Data set $D \subset Z$

$$\min_{\boldsymbol{z}\in E}\min_{\boldsymbol{y}\in D} d(\boldsymbol{z},\boldsymbol{y})$$

$$|z^e| = \mathbb{C}^e\,\boldsymbol{\varepsilon}^e\cdot\boldsymbol{\varepsilon}^e + \Delta t\boldsymbol{D}^e\,\boldsymbol{r}^e\cdot\boldsymbol{r}^e + \mathbb{C}^{e^{-1}}\boldsymbol{\sigma}^e\cdot\boldsymbol{\sigma}^e + \Delta t\boldsymbol{D}^{e^{-1}}\boldsymbol{q}^e\cdot\boldsymbol{q}^e$$



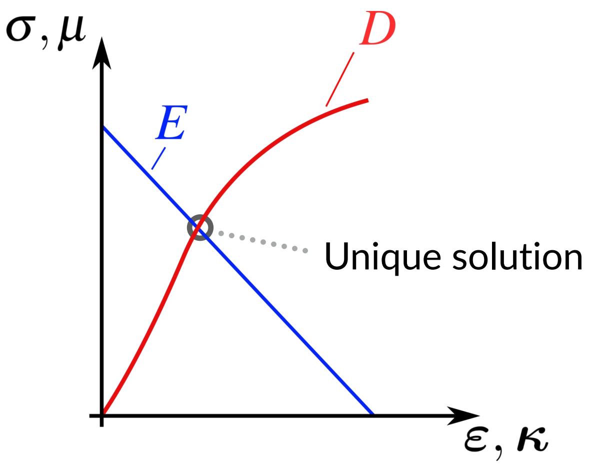
- Possible hybrid model/data-driven formulations
- Modified distance definition

Micropolar formulation

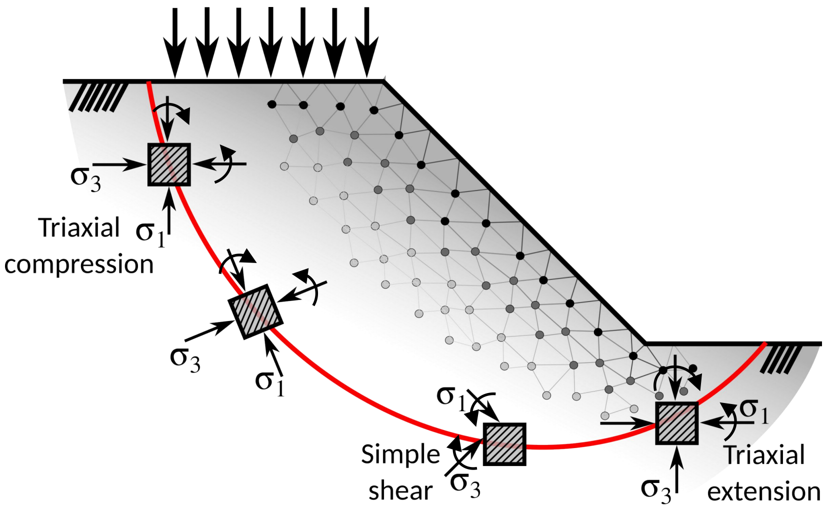
Conventional micropolar

$$
\begin{cases}
\nabla \cdot \boldsymbol{\sigma} = 0 & (1) \\
\nabla \cdot \boldsymbol{\mu} - \boldsymbol{\epsilon} : \boldsymbol{\sigma} = 0 & (2) \\
\boldsymbol{\varepsilon} = \nabla \boldsymbol{u} - \boldsymbol{\epsilon} : \boldsymbol{\theta} & (3) \\
\boldsymbol{\kappa} = \nabla \boldsymbol{\theta} & (4) \\
\{\boldsymbol{\sigma}, \boldsymbol{\mu}\} = \{\boldsymbol{\sigma}, \boldsymbol{\mu}\}(\boldsymbol{\varepsilon}, \boldsymbol{\kappa}, \ell)
\end{cases}
\quad + \text{ B.Cs}
$$

Constitutive relation



- Complicated constitutive models
- Hard to define the internal length scale



Data-Driven micropolar

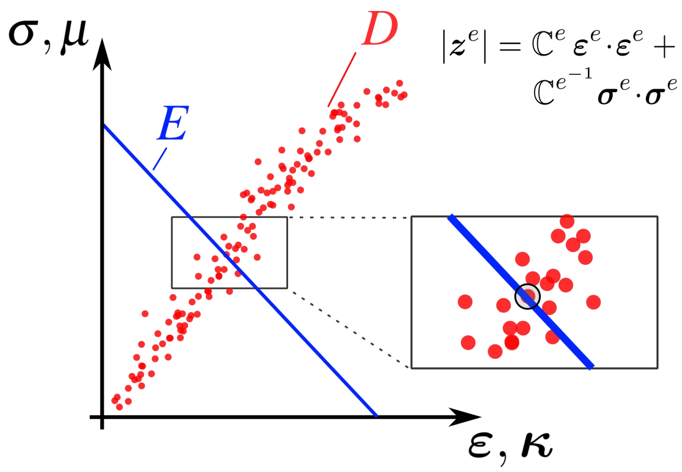
Karapiperis et al., 2021

Phase Space $Z = \{(\boldsymbol{\varepsilon}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \boldsymbol{\mu})\}$

Equilibrium set $E = \{\boldsymbol{z} \in Z | (1) - (4)\}$

Material Data set $D \subset Z$

$$\min_{\boldsymbol{z} \in E} \min_{\boldsymbol{y} \in D} d(\boldsymbol{z}, \boldsymbol{y})$$



$$|\boldsymbol{z}^e| = \mathbb{C}^e \boldsymbol{\varepsilon}^e \cdot \boldsymbol{\varepsilon}^e + \mathbb{D}^e \boldsymbol{\kappa}^e \cdot \boldsymbol{\kappa}^e + \mathbb{C}^{e^{-1}} \boldsymbol{\sigma}^e \cdot \boldsymbol{\sigma}^e + \mathbb{D}^{e^{-1}} \boldsymbol{\mu}^e \cdot \boldsymbol{\mu}^e$$

- Discover internal length scale from the data
- Modified distance definition

Multiscale formulation and thermodynamics

Free energy/Dissipation

$$\boldsymbol{\varepsilon} = \frac{1}{V}\Big(\sum_{p\in\partial\mathcal{P}} \boldsymbol{u}^p \otimes \mathbf{n}^p + \epsilon \cdot \sum_{p\in\mathcal{P}} \boldsymbol{\theta}^p V^p\Big)$$

$$\boldsymbol{\kappa} = \frac{1}{V}\sum_{p\in\partial\mathcal{P}} \boldsymbol{\theta}^p \otimes \mathbf{n}^p$$

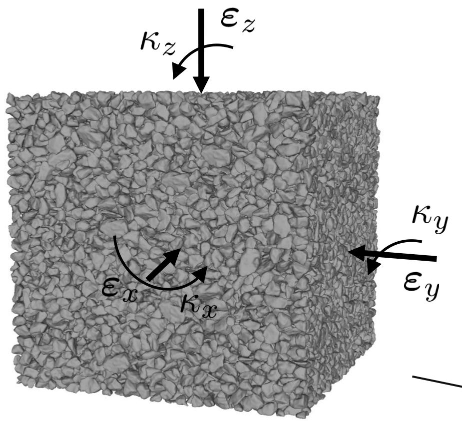
$$\bar{\boldsymbol{\sigma}} = \frac{1}{V}\sum_{c\in\mathcal{C}} \boldsymbol{f}^c \otimes \boldsymbol{l}^c$$

$$\boldsymbol{\mu} = \frac{1}{V}\sum_{c\in\partial\mathcal{C}} (\boldsymbol{l}^c \times \mathbf{f}^c) \otimes \mathbf{x}^{p(c)}$$

$$\mathcal{A} = \sum_c \mathcal{A}^c = \frac{1}{2V}\sum_c \Big(\frac{\|\mathbf{f}^{\mathbf{c}}_{\mathbf{n}}\|^2}{k_n} + \frac{\|\mathbf{f}^{\mathbf{c}}_{\mathbf{t}}\|^2}{k_t}\Big)$$

$$d\mathcal{D} = \sum_c d\mathcal{D}^c = \frac{1}{V}\sum_c \mathbf{f}^c_t \cdot d\mathbf{u}^{c,slip}$$

Granular RVE

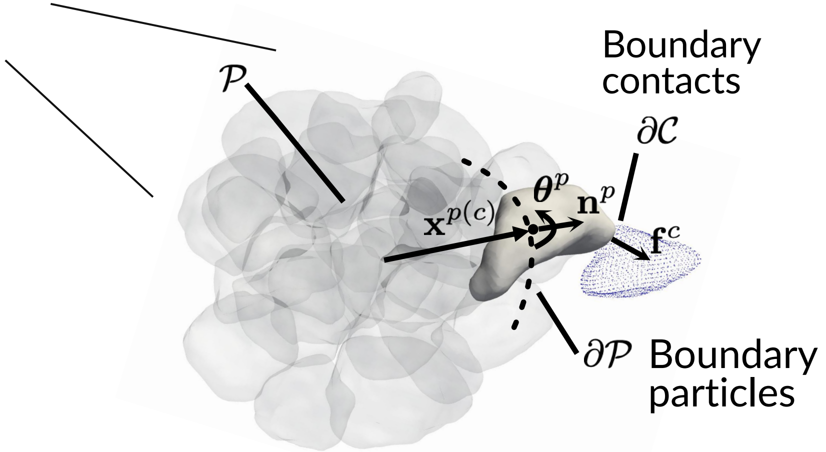


Augmented Dissipation Inequality

$$\mathcal{D}_{k+1} - \mathcal{D}_k = \frac{\boldsymbol{\sigma}_k + \boldsymbol{\sigma}_{k+1}}{2} : (\boldsymbol{\epsilon}_{k+1} - \boldsymbol{\epsilon}_k) + \frac{\boldsymbol{\mu}_k + \boldsymbol{\mu}_{k+1}}{2} : (\boldsymbol{\kappa}_{k+1} - \boldsymbol{\kappa}_k) - (\mathcal{A}_{k+1} - \mathcal{A}_k) \geq 0$$

Dissipation

Free energy



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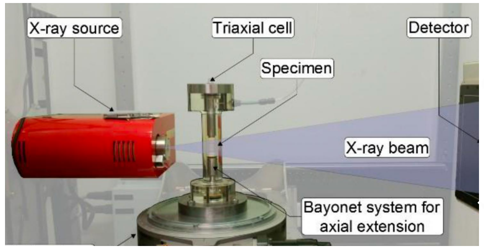
Application I: Shear banding

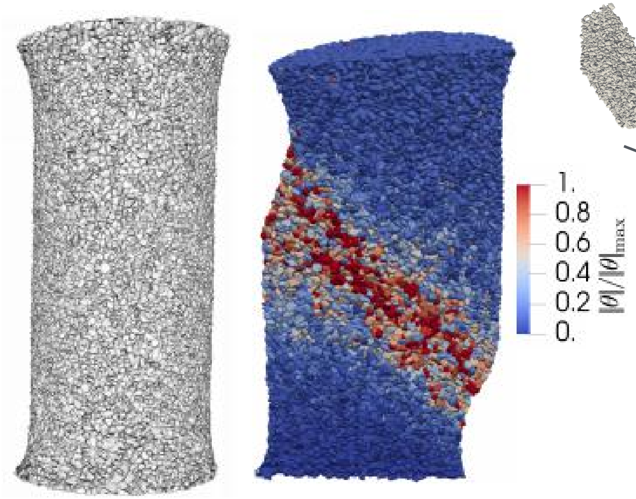
Karapiperis et al, 2021

Experiment

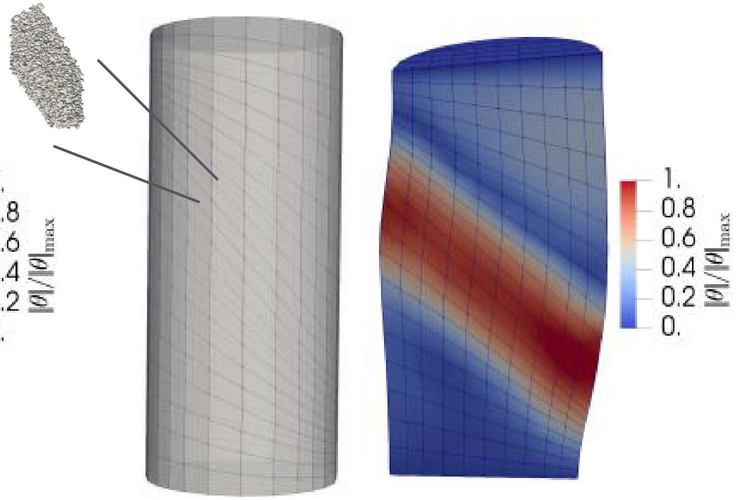


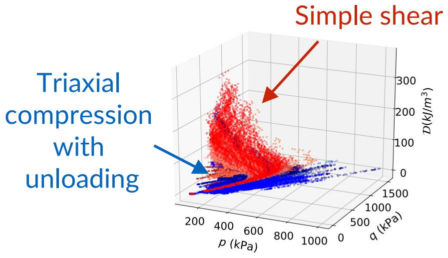
Ando et al, 2016

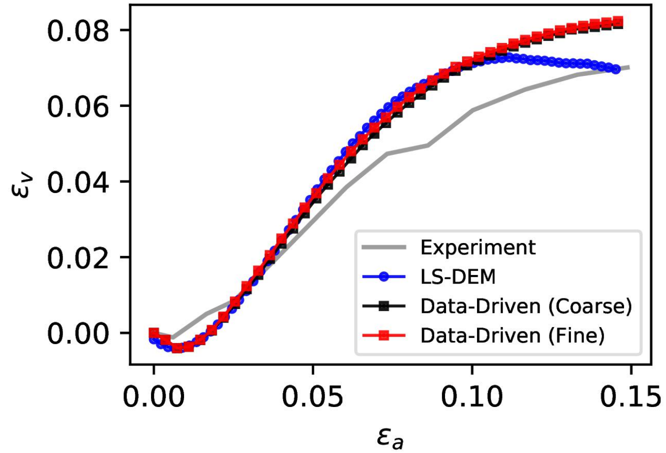
LS-DEM

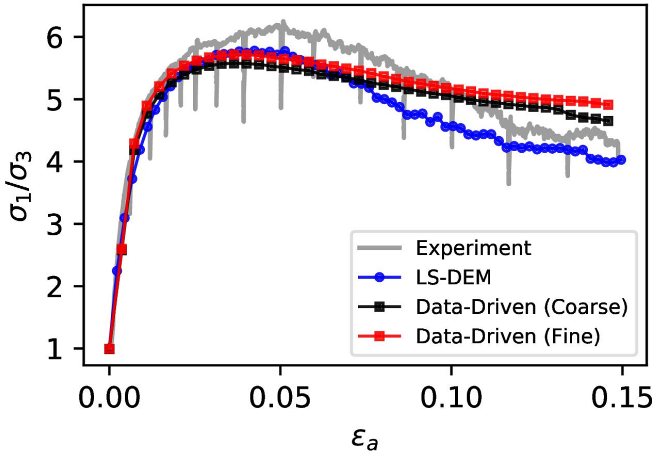


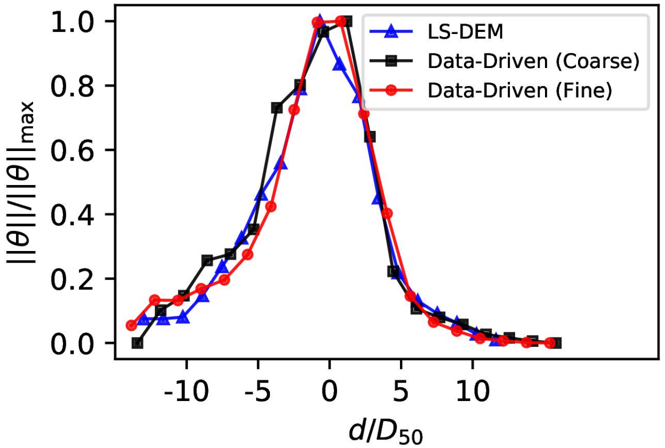
Data-Driven











Application II: Breakage mechanics

Gorgogianni et al, 2023 (CMAME)

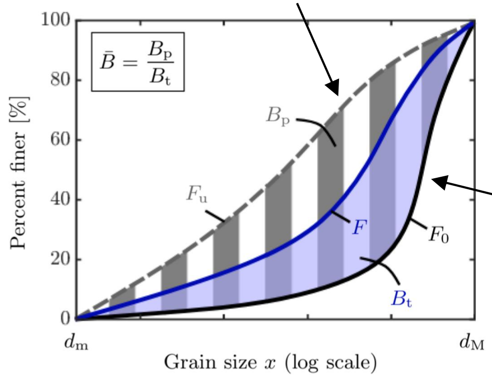
Model-based Breakage Mechanics → Data-Driven Breakage Mechanics

Einav, *J. Mech. Phys. Solids* (2007)

Breakage (internal) variable:

$$B(t) = \frac{p(t) - p_0}{p_u - p_0} = \frac{F(t) - F_0}{F_u - F_0}$$

Ultimate GSD



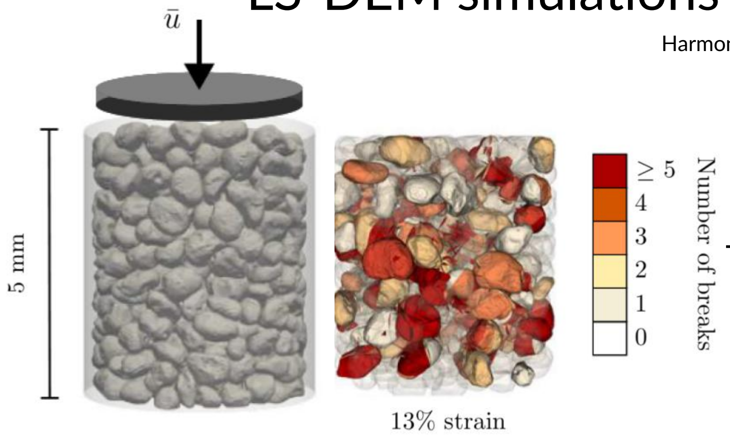
Initial GSD

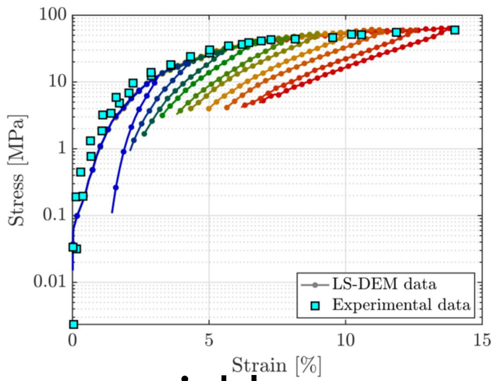
Free energy $\psi := \psi(\boldsymbol{\varepsilon}, B)$

Dissipation potential $\phi := \phi(\dot{B})$

LS-DEM simulations of breakable particles

Harmon et al, *Comp. Meth. Appl. Mech. Eng.* (2020)

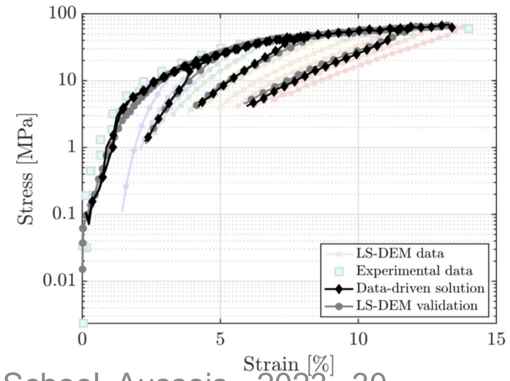




State space augmented by breakage variable

$$D_{k+1} = \{(\boldsymbol{\varepsilon}_{k+1}, \boldsymbol{\sigma}_{k+1}) \mid B_{k+1} = \hat{B}(\boldsymbol{\varepsilon}_{k+1}, \boldsymbol{\sigma}_{k+1}) \geq B_k\}$$

Data-driven oedometric compression

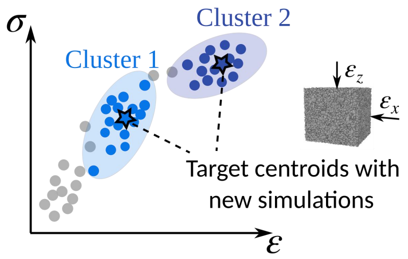


Summary

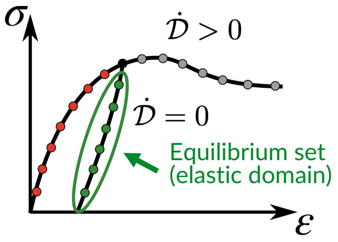
- **Interpretable** distance-based framework

- Data from various provenances (**experiments**, high-fidelity **simulations**)

- Requires a lot of data ⟶ Resolved via unsupervised **adaptive sampling**, **embeddings** into lower dimensional spaces



- History dependence (internal variables + **thermodynamic** constraints)



- Extension to **coupled/gradient problems**