# Planar Languages

## Grammatical Inference beyond context free

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St Etienne GI Workshop

# Acknowledgements

This is joint work with Chris Watkins, Christophe Costa Florencio and Mariette Serayet.

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*Grammatical Inference with String Kernels*

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# Outline

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1. Motivation
   - First Language Acquisition
2. Planar Languages
   - Simple example
   - Formal definition
   - Learnability
3. Empirical results
   - Practical Issues
   - Results

# How do children learn language?

- Without explicit instruction
- Without correction (middle class Western families aside)
- Rapidly
  - after a small amount of data
  - after a small amount of time
- Some feedback on well-formedness of utterances
- All natural languages
  - Includes some languages that are not context free
  - Swiss German, Bambara

Caveat: no natural language in this talk!

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# Two possible research strategies

- The high road
  - Choose a sufficiently powerful class: CFGs, TAGs, . . . , that includes the natural languages.
  - Try to find an algorithm for learning some of them
- The low road
  - Choose a formalism that is inherently learnable
  - Try to make it powerful enough to represent natural languages.

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# Grammatical inference

- Formal languages
  Predicting language membership, not probability
- Positive data
- Unstructured examples
- No side information
- Polynomial bounds on data and computation
- Different assumptions about samples

# Problem with language theory

**Palindrome language**

$$L = \{ww^R | w \in \{a, b\}^*\}$$

**Copy language**

$$L = \{ww | w \in \{a, b\}^*\}$$

Question: why is the copy language much more complex than the palindrome language, when pre-theoretically it is simpler?

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# Simple example
## Parikh map

Consider the well known Parikh map from strings to a vector of counts of each of the letters.
If $|\Sigma| = n$ then $\phi_P : \Sigma^* \to \mathbb{R}^n$.

**Example: $\Sigma = \{a, b\}$**

$\phi_P(aaabab) = \binom{4}{2}$
$\phi_P(ab) = \binom{1}{1}$

**Parikh's lemma**

The image of a context free language under the Parikh map is semi-linear.

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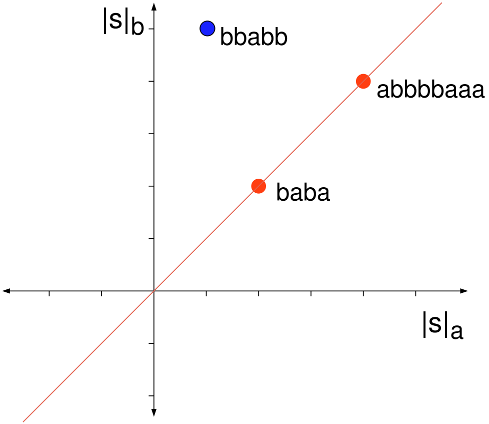
# Simple Example

## A context free language

- Let $\Sigma = \{a, b\}$
- Consider $L = \{s \in \Sigma^* : |s|_a = |s|_b\}$ where $|s|_a$ is the number of $a$'s in $s$
- $L$ consists of strings with equal numbers of $a$ and $b$

Examples $ab, ba, aabb, bababa, baab, \ldots$

# Image of this language under the Parikh map




String in the language if and only if its image is on the line.

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# Planar Languages

## Definition

For any feature map $\phi$ from $\Sigma^*$ to a Hilbert space $H$, for any finite subset $S = \{w_1, \ldots, w_n\} \subset \Sigma^*$. we define $L_\phi(S) = \{w \in \Sigma^* | \exists \alpha_i, \sum \alpha_i = 1 \sum_i \alpha_i \phi(w_i) = \phi(w)\}$

## Informally

Given a finite set of strings, a basis, we can define the language as the set of strings, whose images in feature space, that lie in the least hyperplane containing the images of the basis.

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# Comments on formal definition

- Finite basis; finite rank of hyperplane.
  $R = \{w_1, \ldots, w_n\}, \|R\| = \sum_i |w_i|$
- Affine combination.
  Rank of plane = $|R| - 1$, not necessarily through origin.
- Learnable using elementary linear algebra.
  Does a test point lie on the plane formed by the training points?
- Assume exact model of computation and neglect numerical issues.
  In practice we don't find accuracy a problem (using standard techniques).

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# Kernels

We can use the kernel trick.

- We need to use feature spaces with large or infinite dimension.
- Explicitly computing $\phi$ may be intractable or impossible.
- It is enough to compute $\kappa(u, v) = \langle \phi(u), \phi(v) \rangle$. Basic linear algebra becomes slightly less basic linear algebra.

# Outline

# Learnability 1

## Simple PAC algorithm

Given a polynomial kernel $\kappa$.

### Algorithm 1

Training data $S = \{w_1, \ldots, w_n\}$. Given a new string $w$, compute the distance to the hyperplane spanned by $S$. If this is large (non-zero), then this is not in the language, if it is small (close to zero) then it is in the language.

### Theorem

This algorithm PAC-learns the class of $\kappa$-planar languages with sample complexity $\frac{|R|}{\epsilon} \log \frac{|R|}{\delta}$

# Learnability 2

## Simple IIL algorithm

Given a polynomial kernel $\kappa$.

### Algorithm 1

Training data an infinite presentation of the language $S = \{w_1, \ldots, w_n, \ldots\}$. Start with $B = \{\}$. At each step $i$, if $w_i \in L(B)$, do nothing. Otherwise $B \leftarrow B \cup \{w_i\}$.

### Theorem

This algorithm polynomially identifies in the limit the class of $\kappa$-planar languages

- The number of errors at most $|R|$;
- The representation is itself a characteristic set.

# Formal properties

## Every language is planar

### Specific kernel

For any language $L$ define map
$\phi_L(w) = 1$ if $w \in L$ otherwise $\phi_L(w) = 0$.

### Fact

$L$ is $\phi_L$-planar

- One dimensional feature space
- Kernel represents prior knowledge; which can be very detailed.

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# General purpose kernels

Each kernel defines an implicit feature space.

- Parikh kernel
- Spectrum kernel
- Subsequence kernel
- Gap-weighted kernel
- Discrete kernel: $\kappa_D(u, v) = \delta(u, v)$.
- All subsequences kernel

Small feature spaces tend to give overgeneralisation; huge feature spaces give poor or no generalisation.

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# General program

## e.g. Discrete kernel

- Defines a feature space
  Infinite dimensional feature space with one feature for each string $\phi(cat) = (0, 0, \ldots 0, 1, 0, \ldots)$
- Identify the planar languages with respect to this kernel
- All finite languages
  $L(S) = S$
- No generalisation: basis is the whole language

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# $p$-subsequence kernel

**kernel hyperparameters**

$p$ length of subsequences

**$\Sigma = \{a, b\}$, $p = 2$**

Features are scattered substrings of length p
$(aa, ab, ba, bb)$
$\phi(aaba) = (3, 2, 1, 0)$
$\phi(abba) = \phi(baab) = (1, 2, 2, 1)$

Parikh kernel is the 1-subsequence kernel.

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Parikh kernel is the 1-subsequence kernel.

# Gap-weighted kernel

## kernel hyperparameters

$p$ length of subsequences
$\lambda$ gap penalty

## $\Sigma = \{a, b\}, p = 2, \lambda = 0.1$

Features are $(aa, ab, ba, bb)$
$\phi(aaba) = (1.11, 1.1, 1, 0)$

Values of features are polynomials in $\lambda$.

# Modularity of kernels

We can combine kernels freely. or kernels $\kappa_1$, $\kappa_2$ with feature spaces $H_1$, $H_2$.

- $\kappa_1 + \kappa_2$ has feature space $H_1 \oplus H_2$
- $\kappa_1 \times \kappa_2$ has feature space $H_1 \otimes H_2$

Most of our work is with the kernel $\kappa_{GW+} = \kappa_2^G + \kappa_P$.

# Injectivity

A key point is whether the feature map is injective.

**Definition**

A kernel $\kappa$ is injective if the feature map is injective i.e. if $\phi(u) = \phi(v) \Rightarrow u = v$.

$p$-subsequence kernel is not injective for any $p$:
$\phi_2(abba) = \phi_2(baab)$

**Theorem**

The gap-weighted kernel is injective if

- $\lambda$ is transcendental.
- $\lambda = \frac{p}{q}$, $p, q$ coprime, $q > 1$.

Open question: what about $\lambda = 2$?

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# Some planar languages

## 2-subsequence kernel

### Examples

$\{a^n b^n | n \geq 0\}$

### Not planar

$\{a^n b^n | n > 0\}$
$\{a^n b^m | n > m\}$

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# Some planar languages

$\kappa_{GW+}$

Swiss german.

- Set of verb classes $V = \{v_1, v_2, \ldots v_k\}$
- Set of noun classes $N = \{n_1, n_2, \ldots n_k\}$
- $L = \{uf(u) | u \in N^*\}$

Examples: $n_1 n_2 n_3 v_1 v_2 v_3$
This is not a context free language.

**$L$ is planar for $\kappa_{GW+}$**

$|u|_{v_i} = |u|_{n_i}$
$|u|_{v_i, v_j} = |u|_{n_i, n_j}, |u|_{v_i, n_j} = 0$

Not planar for 2-subsequence kernel, because of congruent pairs. $n_1 n_2 n_2 n_1 v_2 v_1 v_1 v_2$

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# Closure properties

Language theoretic properties of this class

## Concatenation

$L_1 = \{a^n b^n | n \geq 0\}, L_2 = \{b^*\}$.
$L_1 L_2 = \{a^n b^m | m > n\}$ not planar.

## Union

$L = \{a^n b^n\} \cup \{a^n b^{2n}\} \cup \{a^{2n} b^n\}$
generalises to $\{a^* b^*\}$

Intersection, reversal are the only closure properties.

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# Implementation

## Matlab sample code

### Centering the Gram matrix

```
D = sum(K)/n; E = sum(D)/n; J = ones(n,1) * D;
K2 = K - J - J' + E * ones(n,n);
```

### kernel PCA

```
k = rank(K2);
[V,L] = eigs(K2,k,'LM');
invL = diag(1./diag(L));
sqrtL = diag(sqrt(diag(L)));
invsqrtL = diag(1./diag(sqrtL));
Knew = K2' * V * invL * V' * K2;
```

# Experimental setup

Not really experiments: demonstrations

- Generate some random positive training data from example language
- Generate some random test data;
  - Negative data is challenging
  - Uniform samples are too easy
  - Added *ad hoc* approximation to the real samples to make the test harder.
- Induce model
- Test on the test data
  - False Positive rate = false positives / number of negatives
  - False Negative rate = false negatives / number of positives

# Languages

- Classic examples from language theory
- Various levels of Chomsky hierarchy
- Focussed particularly on natural languages
- Simple languages: short descriptions

# Baselines

Two baseline systems:

- Hidden Markov Model
  Non deterministic finite state automaton
- PCFG
  In CNF with every possible rule
- Trained to convergence with EM algorithm.
  Forward-backward algorithm/ inside outside algorithm
- Probability threshold for language membership

# Outline

# Results
## Overview

- Some simple languages can't be learned by GISK method but can be by baselines
- Some languages are impossibly hard
- (Most important) Some interesting CF and CS languages can be learned by GISK.

## Experiments: Even and Brackets

### GISK worse than baselines

| | | |
|---|---|---|
| Even (Regular) | Even number of symbols<br>Alphabet $\{a, b, c\}$ | abcb, ba, babacc, aaaa |
| Bracket (CF) | Balanced brackets<br>Alphabet $\{(, )\}$ | (), ()(), (()(())) |

| | PCFG | | HMM | | SUBS | | | GPWT | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | FP | FN | FP | FN | FP | FN | R | FP | FN | R |
| Even | 0 | 0 | 0 | 0 | 100 | 0 | 12 | 100 | 0 | 12 |
| Bracket | 0 | 0 | 3.4 | 1.3 | 10.8 | 0 | 3 | 10.8 | 0 | 5 |

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# Planar Languages not Learned by HMMs or PCFGs

$$A = \{a_1, \ldots, a_N\}, B = \{b_1, \ldots\}, \ldots$$

## Equality languages

$L_3 = \{A^n B^n C^n | n \geq 0\}$
$L_4 = \{A^n B^n C^n D^n | n \geq 0\}$
$L_5 = \{A^n B^n C^n D^n E^n | n \geq 0\}$

## Results

| | PCFG | | HMM | | 1+2-subseq | | | GapWeighted | | |
|---|---|---|---|---|---|---|---|---|---|---|
| L | FP | FN | FP | FN | FP | FN | R | FP | FN | R |
| $L_3$ | 8.8 | 0 | 20.4 | 0 | 0 | 0 | 17 | 0 | 0 | 25 |
| $L_4$ | 6.4 | 0 | 46.5 | 0 | 0 | 0 | 24 | 0 | 0 | 38 |
| $L_5$ | 38 | 0 | 37.5 | 0 | 0 | 0 | 32 | 0 | 0 | 54 |

# Copy languages
## Swiss german

Abstraction of Swiss German data (Shieber):

- Nouns with various cases $N_{acc}, N_{dat} \ldots$
- Verbs that require cases $V_{acc}, V_{dat} \ldots$
- Sentences consist of a sequence of nouns, followed by verbs, with cross serial dependencies.

$$L = \{N_{acc}N_{dat}N_{dat}V_{acc}V_{dat}V_{dat}, \ldots\}$$

# Copy languages
## Three variants

### Formal definition

$N = \{N_1, \dots N_n\}, V = \{V_1 \dots V_n\}, f : N \to V$, n= 4

$L_{copy} = \{wf(w) | w \in N^*\}$

$L_{copynd} = \{ww | w \in N^*\}$

$L_{copycs} = \{wxw | w \in N^*\}$

### Results

| | PCFG | | HMM | | 1+2-subseq | | | GapWeighted | | |
|---|---|---|---|---|---|---|---|---|---|---|
| L | FP | FN | FP | FN | FP | FN | R | FP | FN | R |
| $L_{copy}$ | 4.2 | 0 | 5.7 | 4.3 | 0 | 0 | 20 | 0 | 0 | 36 |
| $L_{copynd}$ | 64.2 | 5.0 | 76.3 | 6.3 | 70.0 | 6.7 | 71 | 100 | 0 | 72 |
| $L_{copycs}$ | 3.3 | 2.1 | 8.7 | 2.1 | 0 | 0 | 20 | 8.5 | 0 | 27 |

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# Palindromes

## Two variants

### Languages

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|---|---|---|---|---|---|---|---|---|---|---|
| L | FP | FN | FP | FN | FP | FN | R | FP | FN | R |
| $L_{palind}$ | 0.8 | 0 | 4 | 8.1 | 0 | 0 | 20 | 0 | 0 | 30 |
| $L_{palin}$ | 6.1 | 0 | 83.5 | 2.9 | 16.1 | 0 | 14 | 16.1 | 0 | 14 |

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# Results

## Very hard languages

*two thousand thousand one thousand*

### Chinese numbers

$$L = \{ab^{k_1} \ldots ab^{k_r} | k_1 > \cdots > k_r > 0\}$$

### Results

| PCFG | | HMM | | 1+2-subseq | | | GapWeighted | | |
|---|---|---|---|---|---|---|---|---|---|
| FP | FN | FP | FN | FP | FN | R | FP | FN | R |
| 100 | 0 | 99.2 | 1.2 | 100 | 0 | 6 | 100 | 0 | 6 |

# Distributional kernels
## Dealing with large alphabets

Assumption of a finite alphabet $\Sigma$ is too simplistic.

- Words have internal structure – sequence of phone(me)s, letters.
- Lexical structure – case, number, gender, conceptual structure
- Need some way of capturing this internal structure of the alphabet.
- This might be given *a priori*, or could be learned.
- Large alphabets are computationally intractable

**Subkernel**

Assume we have a kernel over $\Sigma$, $\kappa : \Sigma \times \Sigma \to R$

# Distributional kernels
## Dealing with large alphabets

Assumption of a finite alphabet $\Sigma$ is too simplistic.

- Words have internal structure – sequence of phone(me)s, letters.
- Lexical structure – case, number, gender, conceptual structure
- Need some way of capturing this internal structure of the alphabet.
- This might be given *a priori*, or could be learned.
- Large alphabets are computationally intractable

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# Distributional kernels

## Learning a kernel

Given two words *cat* and *dog* we can expect them to behave similarly based on their distribution. (Harris, Schuetze . . . )

- This can be learned by looking at the statistics of a large corpus.
- Normally, we derived distributional statistics (vectors), cluster them and then use the cluster labels
- Now, we can use the distributional statistics directly
- Kernel that uses similarity matrix between symbols dimensions represent combinations of dimensions in symbol feature space

# Distributional kernels

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# Experiments with distributional kernel

## Target language with large alphabet

- Target Language: $L = \{A^n B^n C^n | n \geq 0\}$,
  $A = \{a_1, \ldots, a_{30}\}, B = \{b_1, \ldots, b_{30}\}, C = \{c_1, \ldots, c_{30}\}$,
- Large alphabet $|\Sigma| = 90$, so language has rank $> 10^3$.
- Training data of 500 samples
- Three test sets of size 1000
  - Uniform
  - Positive
  - Hard $\{A^* B^* C^*\}$
- 2-subsequence kernel.

# Experiments with distributional kernel

## Basic approach

Target language is planar, but data is inadequate, so the generated language is a subclass of the target language

### Results

| Test Set | Positive | Uniform | Hard |
|---|---|---|---|
| FN | 890 | 4 | 0 |
| TN | 0 | 881 | 1000 |
| TP | 110 | 115 | 0 |
| FP | 0 | 0 | 0 |

Need at least $20,000$ data points to get good generalisation, but algorithms are cubic …

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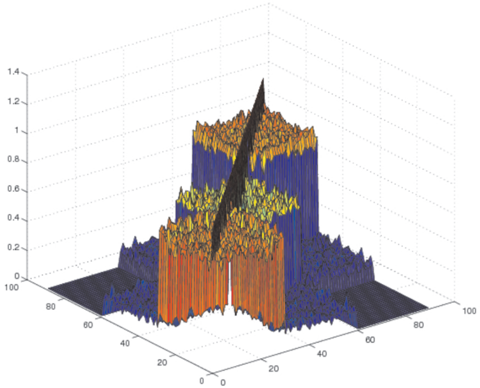
# Simple distributional kernel

- For a given symbol $a \in \Sigma$ consider the distribution of immediately adjacent symbols.
- Add a distinguished boundary symbol
- Gives a $2(|\Sigma| + 1)$-dimensional feature space.
- Count frequencies:
- $\kappa(a, b) = \sum_{\sigma \in \Sigma'} c(\sigma a)c(\sigma b) + c(a\sigma)c(b\sigma)$
- Normalise so $\kappa(a, a) = 1$.
- Efficient algorithm linear in size of data, so we can use as much data as we want.

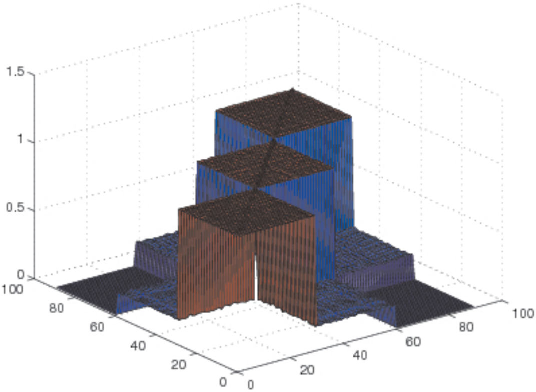
# Learned Gram matrix
## Distributional kernel

Learn a distributional kernel from 500 strings.

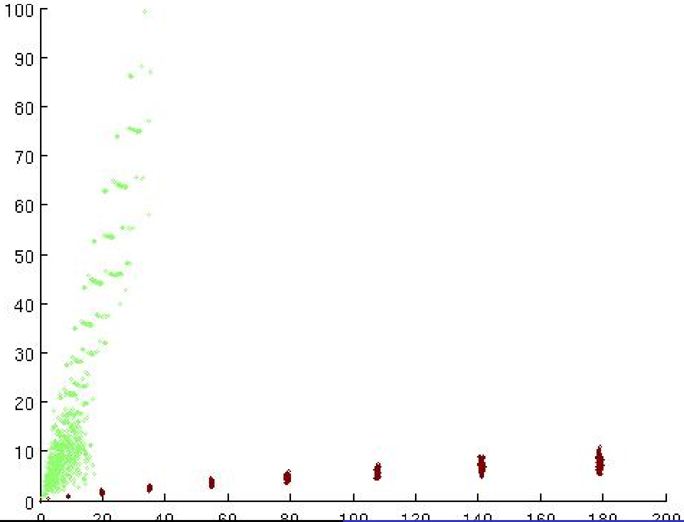


# Learned Gram matrix
## distributional kernel

Learn a distributional kernel from 10,000 strings.



# 20-dimensional approximation



# Approximate hyperplane

- Because of noise in the Gram matrix, strings will not lie exactly in hyperplane
- Longer strings have larger norm: measure angle to hyperplane, rather than perpendicular distance
- Threshold is set from training data (better – held out data)
- Perfect score: 0 FP, 0 FN

# Previous work

- Kontorovitch: learning linearly separable languages.
    - Learning from positive and negative examples
    - Locally testable languages (subclass of regular languages)
- Salomaa: defining languages by numerical equations. Purely theoretical; no learning, no discussion of language theoretic power, no kernels.

# Previous work

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# Interfaces

## Pure speculation

Linear representations of

- Semantics: LSA from bag of words
- Sound: Fourier kernels

Natural interface between a linear representation of syntax and linear models of the inputs and outputs.

# Critical review

## What are the weaknesses?

- Polynomial algorithms but cubic in number of sentences.
- Poor closure properties
- No experiments on real data (yet).
- Not a magic bullet; might need to be combined with another learning method.
- Useless – doesn't produce any structure.

# Summary

- We can define languages geometrically using hyperplanes in a feature space.
- These languages include classic examples of mildly context sensitive languages that occur in natural languages.
- These can be efficiently learned from positive data alone.

- Future work
  - Learning with manifolds, hyper-ellipsoids, half-spaces . . .
  - Learning with noise

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