Homework 5, Section 1.5: 1, 4, 6, 15, 18, 24, 25, 26, 28-31

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Homework

1.

RREF Matrix $\begin{bmatrix} 1 & 0 & 17/8 & 0 \\ 0 & 1 & -3/4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -17/8, x_2 = 3/4$ and x_3 is a free variable.

This means that $Ax = 0$ has the form: $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -17/8 \\ 3/4 \\ 1 \end{bmatrix}$

4.

RREF Matrix $\begin{bmatrix} 1 & -3/5 & 2/5 & 0 \\ 0 & 1 & -16/29 & 0 \end{bmatrix}$

$x_2 = 16/29(x_3), x_1 = 3/5(x_2) - 2/5(x_3)$. Clearly x_3 is a free variable which means that the given system $Ax = 0$ has a non-trivial solution

6.

RREF Matrix $\begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_2 = x_3, x_1 = x_3$. The general solution of $Ax = 0$ has the form: $X = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

15.

The general solution takes the form: $X = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ Now, solving for x_1 in terms of the free variables, the general solution of x_1 is $x_1 = -2 - 5x_2 + 3x_3$.

This means the general solution is: $X = x_1 \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

18.

RREF Matrix $\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = 5 - 2x_2 + 3x_3, x_2 = -1 + x_3, x_1 = 7 + x_3$. The general solution of $Ax = 0$ has the form: $X = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

24. A)

The statement is false. A system of linear equation is said to be homogeneous if it can be written in the form $Ax = 0$.

24. B)

The statement is false. If x is a non-trivial solution of $Ax = 0$, then at least one entry in x is non-zero.

24. C)

The statement is true. The effect of adding p to a vector v is to move v in a direction parallel to the line through p and o.

24. D)

The statement is true. If x = 0 and b = 0 then $Ax = 0$.

24. E)

The statement is true. If $Ax = b$ is consistent, then the solution set of $Ax = b$ is obtained by translating the solution set of $Ax = 0$

25. A)

25. B)

25. C)

26.

Given A, in which it is a 3x3 Zero Matrix, let $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \in R^3$

now, since $0 + 0 = 0$, the the matrix equals zero. Therefore, the solution set is all vectors in R^3

28. A)

28. B)

29. A)

A has three pivot positions. A does not have pivot positions in all four rows. One row not having pivot positions means that the equation $Ax = b$ does not have a solution for every possible b in R^3.

29. B)

A has three pivot positions. A does not have pivot positions in all four rows. One row not having pivot positions means that the equation $Ax = b$ does not have a solution for every possible b in R^3.

30. A)

A is a 2x5 matrix and has two pivot positions. Hence it is non-trivial.

30. B)

A has 2 pivot positions and A has a pivot in every row, so the equation $Ax = b$ has a solution for every b.

31. A)

Non-trivial

31. B)

31. C)

31. D)