Homework 30, Section 5.4: 7, 12, 14, 24, 25

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Homework

7.

Since $T(b_1) = t(1) = 3 + 5t, (T(b_1))_B = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$. also, since $T(b_2) = T(t) = -2t + 4t^2, (T(b_2))_B = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$, and $T(b_3) = T(t^2) = t^2, (T(b_3))_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Thus the matrix representation of T relative to the basis b is $\begin{bmatrix} 3 & 0 & 0 \\ 5 & -2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

12.

The eigenvalues of the matrix are 1 and 3.
For $\lambda = 1, A - I = \begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix}$. The basis vector for the eigenspace, with x_2 being free, is $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
For $\lambda = 3, A - I = \begin{bmatrix} -3 & 1 \\ -3 & 1 \end{bmatrix}$. The basis vector for the eigenspace, with x_2 being free, is $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

14.

24.

If $A = PBP^{-1}$ then rank A = rank $P(BP^{-1}$ = rank BP^{-1}. Also, Rank BP^{-1} = rank B, since p^{-1} is invertible. Thus, rank$A$ =rank$B$.

25.

IF $A = PBP^{-1}$ then by the trace property $tr(A) = tr(P^{-1}PB) = tr(IB) = tr(B)$. if B is diagonal, then the diagonal entries of B must be the eigenvalues of A, by the diagonalization theorem. So, tr$a$ = tr b = the sum of the eigenvalues of A.