Homework 1, Section 1.2: 2, 5, 9, 21, 29

Alex Gordon

September 4, 2013

Homework

2. A)

$$\begin{aligned} a_n &= 5n - 2 \\ a_{k-1} &= 5(k-1) - 2 \\ &= 5k - 5 - 2 \\ &\boxed{= 5k - 7} \end{aligned}$$

$$\begin{aligned} a_n &= 5n - 2 \\ a_{k+1} &= 5(k+1) - 2 \\ &= 5k + 5 - 2 \\ &\boxed{= 5k + 3} \end{aligned}$$

2. B)

$$\begin{aligned} a_n &= 3n^2 + n \\ a_{k-1} &= 3(k-1)^2 + (k-1) \\ &= 3(k-1)(k-1) + (k-1) \\ &\boxed{= 3k^2 - 5k + 2} \end{aligned}$$

$$\begin{aligned} a_n &= 3n^2 + n \\ a_{k+1} &= 3(k+1)^2 + (k+1) \\ &= 3(k-1)(k+1) + (k+1) \\ &\boxed{= 3k^2 + 7k + 4} \end{aligned}$$

2. C)

$$\begin{aligned} a_n &= 2n + 7 \\ a_{k-1} &= 2(k-1) + 7 \\ &= 2k - 2 + 7 \\ &\boxed{= 2k + 5} \end{aligned}$$

$$\begin{aligned} a_n &= 2n + 7 \\ a_{k+1} &= 2(k+1) + 7 \\ &= 2k + 2 + 7 \\ &\boxed{= 2a + 9} \end{aligned}$$

2. D)

$$a_n = 3^n + 4$$
$$a_{k-1} = 3^{k-1} + 4$$
$$\boxed{= 3^{k-1} + 4}$$

$$a_n = 3^n + 4$$
$$a_{k+1} = 3^{k+1} + 4$$
$$\boxed{= 3^{k+1} + 4}$$

2. E)

$$a_n = n^{3n+1} - 1$$
$$a_{k-1} = n^{3(k-1)+1} - 1$$
$$= 2^{3k-3+1} - 1$$
$$\boxed{= 2^{3k-2} - 1}$$

$$a_n = n^{3n+1} - 1$$
$$a_{k+1} = n^{3(k+1)+1} - 1$$
$$= n^{3k+3+1} - 1$$
$$\boxed{= 2^{3k+4} - 1}$$

2. F)

$$a_n = \frac{n(2n-1)(n+2)}{6}$$
$$a_{k-1} = \frac{k-1(2(k-1)-1)((k-1)+2)}{6}$$
$$= \frac{k-1(2k-2-1)(k+2)}{6}$$
$$\boxed{= \frac{-2k^2-2k-3}{6}}$$

$$a_n = \frac{n(2n-1)(n+2)}{6}$$
$$a_{k+1} = \frac{k+1(2(k+1)-1)(k+1+2)}{6}$$
$$= \frac{k+1(2k+2)-1)(k+3)}{6}$$
$$\boxed{= \frac{2k^2+8k+3}{6}}$$

5. A)

2, 4, 6, 8, 10
The pattern here is fairly obvious. It works out to $\boxed{2n}$.

5. B)

1, 7, 13, 19, 25
There are a few answers to this pattern, however, the easiest one that I could find through experimenting is $\boxed{6n-5}$

5. C)

2, 5, 11, 23, 47
The pattern for this was a little harder to figure out. So to figure it out, I used the differences method. The differences are 3, 6, 12, 24.

5. D)

2, 4, 16, 256, 65536
2^{2^n}

9. A)

5, 11, 18, 26, 35
$a_n = \frac{1}{2}$

9. B)

1, 4, 9, 16, 25 $a_n = n^2$

9. C)

-3, 5, 17, 33, 53 $a_n = 2n(n+1) - 7$

21. A)

$$\sum_{k=1}^{7} = 3k$$
$$\sum_{k=1}^{7} = 3(1+2+3+4+5+6+7)$$
$$\boxed{\sum_{k=1}^{7} = 84}$$

21. B)

$$\sum_{k=1}^{3} = 3k$$
$$\sum_{k=1}^{3} = 3(1+2+3)$$
$$\boxed{\sum_{k=1}^{3} = 42}$$

21. C)

$$\sum_{k=1}^{9} = 4$$
$$\sum_{k=1}^{9} = 4(9)$$
$$\boxed{\sum_{k=1}^{9} = 36}$$

21. D)

$$\sum_{k=0}^{4} = \frac{1}{2^k}$$
$$\boxed{\sum_{k=0}^{4} = \frac{31}{16}}$$

21. E)

$$\sum_{k=-1}^{3} = 3 - 2k$$
$$\boxed{\sum_{k=-1}^{3} = 5}$$

29. A)

$$a_3 = A + ABBA + BA$$
$$\boxed{= ABBABA}$$

$$a_4 = A + a_{4-1} + BA$$
$$\boxed{= AAABBABABA}$$

$$a_5 = A + a_{5-1} + BA$$
$$\boxed{= AAAABBABABABA}$$

$$a_6 = A + a_{6-1} + BA$$
$$\boxed{= AAAAABBABABABABA}$$