Homework 29, Section 5.3: 5, 21, 22, 25

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Homework

5.

By the Diagonalization Theorem, eigenvectors form the columns of the left factor and they correspond respectively to the eigenvalues on the diagonal of the middle factor. $\lambda = 5 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \lambda = 5 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

21. A)

False. The symbol D does not automatically denote a diagonal matrix.

21. B)

True. A is diagonalizable if and only if there are enough eigenvectors to form a basis of R^n. WE call such a basis an eigenvector basis of R^n

21. C)

False. The 3x3 matrix in Example 4 has 3 eigenvalues but is not diagonalizable

21. D)

False. Invertibility depends on - not being an eigenvalue. A Diagonalizable matrix may or may not have 0 as an eigenvalue.

22. A)

False. The n eigenvectors must be linearly independent by the Diagonalization theorem.

22. B)

False. The matrix in example 3 is Diagonalizable but it only has 2 different eigenvalues.

22. C)

True. This follows from AP = PD and the first two formulas given in the section 5.3

22. D)

False. In example 4 the matrix is invertible because 0 is not an eigenvalue, but the matrix is not Diagonalizable.

25.

Let $\{v_1\}$be a basis for the one-dimensional eigenspace. Let v_2 and v_3 form a basis for the two dimensional eigenspace and let v_4 be any eigenvector in the eigenspace. By theorem 7, $v_1, ..., v_4$ has to be linearly independent. It then follows that since A is 4x4, the Diagonalization theorem shows that A is diagonalizable.