Homework 15, Section 3.3: 3(a,c), 10(b), 11(b), 19(b)

Alex Gordon

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Homework

3. A)

If a is divisible by b, then$\{a \cdot m : m \in \mathbb{Z}\} \subseteq \{b \cdot n : n \in \mathbb{Z}\}$.
Let us suppose that $a = 4$, $b = 2$ and $m = 3$, $n = 5$
$a \cdot m = 12$
$b \cdot n = 10$
If this is the case then 12 is in the domain of integers, and 10 is a subset of the domain of integers showing the assertion valid.

3. C)

Let a prime number be equal to any number that is only divisible by 1 or itself.
Let k be equal to some integer.
Let us assume that x is prime and $x = (k + 1)(k - 1)$
If $x = 3$, then $x = (2 + 1)(2 - 1) = 3$, then x is prime for the value of $k = 2$
However, for all other values, x would be equal to $(k + 1)(k - 1)$ which is equal to (an integer +1)(an integer -1). Assuming that (an integer +1)(an integer -1) equals (a)(b) then we know that a or b only equals 1 when $k = 2$ and $k = 0$ (however, when $k = 0$, $x = 0$).
Since a prime number is only divisible by 1 and itself, we know that a prime number can not be equal to a multiple of any number two numbers other than 1 and itself, which means that since a or b is only equal to 1 when $k = 2$, then our assertion that the intersection of prime numbers and $k^2 - 1$ is only equal to the set of $\{3\}$

10. B)

(Distributive property) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$
Let sets A, B, C be given. Let $x \in A \cup (B \cap C)$. Then

$$\begin{aligned} X \in A \cup (B \cap C) &= X \in A \ \vee \ X \in B \cap C \\ &= X \in A \ \vee \ (X \in B \ \wedge \ X \in C) \\ &= (X \in A \ \vee \ X \in B) \wedge (X \in A \ \vee \ X \in C) \\ &= (X \in A \cup B) \wedge (X \in A \cup C) \\ &= X \in (A \cup B) \cap (A \cup C) \end{aligned}$$

We can now see that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

11. B)

If $A \cap B = B$, then $A \cup B = A$
Let $B = \{x\}$
If x is $\in B$ and $B \cup A = A$ then $x \in$ A.
If x is $\in A$ then the set of $A \cup B$ is the set of $\{x\} \cup \{x\}$ which is the same as $A \cup B$, which means $A \cup B = A$

19. B)

If $A \cup B = B$, then $A \cup (B \cap A') = B$
Let us begin by simplifying $A \cup (B \cap A')$

$$\begin{aligned} &= A \cup (B \cap A') \quad Absorbtion \\ &= A \cap (B \cup A') \quad Demorgan's \\ &= (A \cap A') \cup (A \cap B) \quad Distributive \\ &= \emptyset \cup (A \cap B) \quad negation \\ &= (A \cap B) \quad identity \\ &= B \quad hypothesis \end{aligned}$$

Therefore: $A \cup (B \cap A') = B$