Homework 1, Section 2.3: 3, 5, 8(f), 10

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Homework

3. A)

First we must start with an outline of the proof we are about to perform. The table below includes the verification of the n = 1 through 4. It also shows the last row checked (n = m - 1) and the next row to be checked (n = m).

n	a_n (recursive formula)	closed formula	equal?
1	1	$4 \times 1 - 3 = 1$	yes
2	$1 + 4 = 5$	$4 \times 2 - 3 = 5$	yes
3	$5 + 4 = 9$	$4 \times 3 - 3 = 9$	yes
4	$9 + 4 = 13$	$4 \times 4 - 3 = 13$	yes
...	...	...	...
$m - 1$	$a_{m-2} + 4 = 4m - 7$	$4(m - 1) - 3 = 4m - 7$	yes
m	$a_{m-1} + 4$	$4m - 3$	???

All let is left is to simplify the recursive formula for the closed formula. We can do this by some simple substitution.

$$\begin{aligned} a_m &= a_{m-1} + 4 \\ a_{m-1} &= (4m - 7) \\ a_m &= (4m - 7) + 4 \\ &= 4m - 3 \end{aligned}$$

This now shows that the recursive formula is equal to the closed formula.

3. B)

First we must start with an outline of the proof we are about to perform. The table below includes the verification of the n = 1 through 4. It also shows the last row checked (n = m - 1) and the next row to be checked (n = m).

n	a_n (recursive formula)	closed formula	equal?
1	5	$\frac{1(1+9)}{2} = 5$	yes
2	$5+2+4=11$	$\frac{2(2+9)}{2} = 11$	yes
3	$11+3+4=18$	$\frac{3(3+9)}{2} = 18$	yes
4	$18+4+4=26$	$\frac{4(4+9)}{2} = 26$	yes
...	...	...	...
$m-1$	$a_{m-2}+a_{m-1}+4 = \frac{(m-1)((m)+9)}{2}$	$\frac{(m-1)(m-1)+9}{2} = \frac{m(m+9)}{2}$	yes
m	$a_{m-1}+a_m+4$	$\frac{4(m)}{2}$	???

All let is left is to simplify the recursive formula for the closed formula. We can do this by some simple substitution.

$$\begin{aligned} a_m &= \frac{(m-1)((m)+9)}{2} \\ a_{m-1} &= \frac{m(m+9)}{2} \\ &= \frac{m^2+9m}{2} \end{aligned}$$

This now shows that the recursive formula is equal to the closed formula.

3. C)

First we must start with an outline of the proof we are about to perform. The table below includes the verification of the n = 1 through 4. It also shows the last row checked (n = m - 1) and the next row to be checked (n = m).

n	a_n (recursive formula)	closed formula	equal?
1	1	$\frac{1(2)(3)}{6} = 1$	yes
2	$1+2^2=5$	$\frac{2(3)(5)}{6} = 5$	yes
3	$5+3^2=14$	$\frac{3(4)(7)}{2} = 14$	yes
4	$14+4^2=30$	$\frac{4(5)(9)}{2} = 30$	yes
...	...	...	...
$m-1$	$a_{m-2}+(m-1)^2 = \frac{(m-1)(m)(2m-1)}{6}$	$\frac{(m-1)(m)(2m-1)}{6}$	yes
m	$a_{m-1}+m^2$	$\frac{(m)(m+1)(2m+1)}{6}$	???

All let is left is to simplify the recursive formula for the closed formula. We can do this by some simple substitution.

$$\begin{aligned} a_m &= a_{m-1}+m^2 \\ &= \frac{(m-1)(m)(2m-1)}{6} + m^2 \\ &= \frac{(m-1)(m)(2m-1)+6m^2}{6} \\ &= \frac{m[(m-1)(2m-1)+6m]}{6} \\ &= \frac{m(m+1)(2m+1)}{6} \end{aligned}$$

This now shows that the recursive formula is equal to the closed formula.

3. D)

First we must start with an outline of the proof we are about to perform. The table below includes the verification of the n = 1 through 4. It also shows the last row checked (n = m - 1) and the next row to be checked (n = m).

n	a_n (recursive formula)	closed formula	equal?
1	1	$2^1 = 1$	yes
2	$1 + 2^2 = 5$	$2^2 - 1 = 3$	yes
3	$5 + 3^2 = 14$	$2^3 - 1 = 7$	yes
4	$14 + 4^2 = 30$	$2^4 - 1 = 15$	yes
...	...	...	...
$m - 1$	$2(a_{m-2}) + 1 = 2^{m-1} - 1$	$2^{m-1} - 1$	yes
m	$2(a_{m-1}) + 1$	$2^m - 1$	???

3. E)

3. F)

5. A)

10. A)

$\sum_{i=1}^{n} = \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$