Homework 4, Section 1.4: 4, 5, 14, 17, 19, 21, 22, 24, 31, 35

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Homework

4. A)

$\begin{bmatrix} 3 \\ 8 \end{bmatrix}$

4. B)

$\begin{bmatrix} 3 \\ 8 \end{bmatrix}$

5.

$\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

14. A)

$x_1 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$ This, this is a matrix equation.

Converting this to REF gives

$$\begin{bmatrix} 2 & 0 & 4 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Once this is converted to a linear system we can see that the final row results in $0 = 3$, which is not possible so the system is inconsistent, meaning u does not span the columns of A.

17.

Since there is no pivot position in the 4th row of A, then by theorem 4, the equation $Ax = b$ does not have a solution for each b in the domain of R^4.

This goes on to show that statement D has to be false

19.

Since statement D is false, then all 4 statements have to be false. Therefore, since not all vectors in R^4 can be written as a linear combination of the columns of A.

21.

The matrix $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$ does not have a pivot in every row, thus by Theorem 4, $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$ does not span R^4.

22.

Since the system is consistent $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$ does span R^3.

24. A)

$Ax = b$ Since this is a vector equation and $Ax = b$, then the matrix equation and the vector equation have the same x values. Therefore, they have the same solution set.

24. B)

Since the system is consistent, then the x values exists. Therefore, any b can be expressed by $b = a_1x_1 + a_2x_2$...... This means that b is spanned by the columns of A.

24. C)

False

24. D)

False

24. E)

True

24. F)

False

31.

Since a does not have a pivot position in every row, the equation $Ax = b$ cannot be consistent for all b in R^M

35.

Suppose that y and z satisfy $Ay = z$. This means that $4z = 4Ay$. Since that fits the definition of a scalar in an mn matrix, then if we replace the constant scalar we get $4(Ay) = A(4y)$. This shows that 4y is a solution, thus making it consistent.