Homework 7, Section 1.7: 6, 10, 12, 18, 21, 29, 36, 37

Alex Gordon

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Homework

6.

$$\begin{bmatrix} -4 & -3 & 0 & 0 & 0 \\ 0 & -1 & 5 & 0 & \\ 0 & 0 & -15 & 0 & \\ 0 & 0 & 0 & 0 & \end{bmatrix}$$

As we can see the three basic variables give the trivial solution and no free variable. Therefore the columns of A are linearly independent.

10. A)

v_3 in span $\{v_1, v_2\}$
The augmented matrix is:

$$\begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -5 \\ -5 & 15 & h \end{bmatrix}$$

The reduced matrix is:

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 10+h \end{bmatrix}$$

As we can see, $0 = 1$ is not possible, therefore the system has no solution to this equation.

10. B)

The reduced matrix is:

$$\begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 10+h & 0 \end{bmatrix}$$

If the system has a non trivial solution then $\{v_1, v_2, v_3\}$ is linearly dependent. For non-trivial solutions, $10 + h = 0$ means that $h = -10$. x_2 is a free variable, which means we have a non-trivial solution.

12.

The system is linearly dependent. Therefore the system has a non-trivial solution and $h = -18$

18.

Linearly dependent.

21. A)

Since the homogeneous system $Ax = 0$ always has the trivial solution, that means that regardless of A, to possess linearly independent columns, $Ax = 0$ always has a trivial solution. This means the statement is not true.

21. B)

If we consider that linear combinations of all vectors then set $S = 0$, then there exists at least one nonzero scalar to satisfy the equation.
Since that means every vector in the set S can be left on one side of the equation and all other vectors are on the other side that shows that S is a linear combination of the other vectors, meaning the statement is true.

21. C)

The columns of any 4x5 matrix are linearly dependent. The statement is true.

21. D)

If x and y are linearly independent then z is in the span of $\{x, y\}$.
Therefore any one of the conditions are true (they can't be true simultaneously)

36.

By theorem 7, it is true.

37.

The given set of vectors $\{v_1, v_2, v_3\}$ are linearly dependent and so the set of vectors $\{v_1, v_2, v_3, v_4\}$ are linearly dependent, hence the statement is true.