Homework 8, Section 1.8; 4, 7, 8, 9, 15, 26, 30

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Homework

4.

RREF Matrix results in: $\begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$
So $x_1 = -17$
$x_2 = -7$
$x_3 = -1$
and $X = \begin{bmatrix} -17 \\ -7 \\ -1 \end{bmatrix}$
This makes it clear that X is unique.

7.

Let A be a 6x5 matrix. What must a and b be in order to define $T : \mathbb{R}^a \to \mathbb{R}^b$ by $T(X) = Ax$. The matrix A and the column vector x are multipliable. If A is a matrix of order p x q, and x is a matrix of order q x l, then the product matrix is of order p x l. This confirms that the dimension of range space is p, meaning the number of rows of A is the dimension of the range vector space. Since 6 is the dimension of the range space and 5 is the dimension of the domain vector space, then A is a matrix of order 6 x 5 and $T : \mathbb{R}^a \to \mathbb{R}^b$ by $T(X) = Ax$ that is A has $b = 6$ rows and $a = 5$ columns and so the co domain of T is $\mathbb{R}^6$ and the domain of T is $\mathbb{R}^5$. Therefore, $a = 5$ and $b = 6$.

8.

Since $X \in \mathbb{R}^5$, we have x has order $5 \cdot 1$
Since $Ax \in \mathbb{R}^7$, we have Ax with order $7 \cdot 1$. Thus, by the product rule of matrices the number of rows in Ax and the number of columns in X equal the number of rows and columns in A, so A must be a matrix with 7 rows and 5 columns.

9.

RREF Matrix results in: $\begin{bmatrix} 1 & 0 & -4 & 10 & 0 \\ 0 & 1 & -3 & 5 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix}$
Hence $x_4 = 0$

$x_3 = free$
$x_2 = 3x_3$
$x_1 = 4x_3$
and $X = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

15.

$T(u) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$
$T(5) = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$
The images of $T(u)$ and $T(v)$ are a reflection through the line $x_2 = x_1$

26.

30.

$T(x) = c_1, \cdot 0 + c_2 \cdot 0 + ... + c_p \cdot 0$
Therefore, $T(x) = 0$ and T is linear.