Homework 28, Section 4.5: 3, 9, 14, 20, 22, 25

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March 28, 2014

Homework

3.

This subspace is H =Span $\{v_1, v_2, v_3\}$, where $v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix}$. Theorem 4 in 4.3 shows that the columns of this set are linearly independent. $v_1 \neq 0, v_2$ is not a multiple of v_1 and since its first entry is not zero, v_3 is not a linear combination of v_1 or v_2 Thus, this set is a basis for H and the dimension of the subspace is 3.

9.

This subspace is $H = \begin{bmatrix} a \\ b \\ a \end{bmatrix} : a, b$ in $R =$ Span $\{v_1, v_2\}$ and where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Since v_1 and v_2 are not multiples of each other, they are linearly independent and are a basis for H. This means the dimension of H is 2.

14.

The matrix is in echelon form. There are two pivot columns so the dimension of Col A is 3. Since there are three columns without pivots, the equation $Ax = 0$ has three free variables thus the dimension of Nul A is 2.

20. A)

False. the set R^2 is not even a subset of R^3.

20. B)

False. The number of free variables is qual to the dimension of Null A.

20. C)

False. A basis could still have only finitely many elements, which would make the vector space finite.

20. D)

False. The set S must also have n elements.

20. E)

True. The subspaces can only be subsets of themselves.

22.

The matrix whose columns are the coordinate vectors of polynomials with the standard basis of P_3 is: $\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -1 & -4 & -18 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & -1 \end{bmatrix}$. This matrix has 4 pivots so it's columns are linearly independent. Since their coordinate vectors for a linearly independent set, the polynomials themselves are linearly independent in p_3. The Basis Theorem then states they form a basis for P_3.

25.

Suppose that S Spans V and that S contains fewer than n vectors. This means that by the spanning set theorem, some subset of S' of S is a basis for V. Since S contains fewer than n vectors, and S' is a subset of S, S' also contains fewer than n vectors. Thus there is a basis S' for V with fewer than n vectors. However, this is inpossible because of theorem 10, because dimV = n. Thus S cannot span V.