Homework 7, 2.2: 1(b,d,f), 3(a,f), 4(a), 7(b,c,d), 8, 12, 15, 21(extra credit)

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Homework

1. B)

$187 = 17 \times 11 + 0$

1. D)

$-24 = -6 \times 4 + 0$

1. F)

$(9k^2 + 5) = (3k^2 + 5)3 + 1$

3. A)

$55 = 9 \times 6 + 1$

3. F)

$(3k^4 - k^2 - 10k + 3)3 + 2$

4. A)

If a is 12, b is 6 and c is 3, then $a \mod b\ and\ b \mod c\ but\ b \neq$ a

7. B)

If b/a and c/a then $\frac{a^2/}{b \times c}$. First, if a divides b and a divides c, then by the closure properties b and c must be multiples of a. Again, by the closure properties, since a(a) $= a^2$ then that result is a multiple of a. If b and c are multiples of a, then by the closure properties, b $\times$ c is a multiple of a. If b(c) is a multiple of a, then it naturally follows that a divides b(c).

7. C)

If a divides b and c divides d then ac divides bd. If a divides b then b must be a multiple of a. If c divides d then d must be a multiple of c. It then naturally follows that $a \times c$ is a multiple of c and and it also naturally follows that $btimesd$ is a multiple of d. Because of that, in the equation $\frac{ac}{bd}$ a/b leaves a remainder of 0 and c/d leaves a remainder of 0 and by the closure properties that means that there is a remainder of 0 meaning bd divides ac.

7. D)

If c divides a then a is a multiple of c. Therefore, for any integer x, and the problem $\frac{ax}{cx}$ the x's simply cancel out to $\frac{a(1)}{c}$

8.

1) a is rational 2) b is rational 3) xw + yz 4)xw + yz 5) w $\neq$ 0 6) y $\neq$ 0

12.

If x is any integer, then $x = x(1)$, and so $x = \frac{x}{1}$. If x = $\frac{x}{1}$, then x and 1 are both integers and $1 \neq 0$. Thus, x can be written as a quotient of integers with a nonzero denominator, meaning x is rational.

15.

x is divisible by 3, thus $x = 3a$ for some integer a. Similarly, $x = 4b$ for some integer b. It then follows that if you multiply $x = 3a$ by 4 and get $4x = 12a$ and you multiply $x = 4b$ by 3 and get $3x = 12b$ then

$$x = 4x = 3x = 12a - 12b = 12(a - b)$$
$$c = (a - b)$$
$$x = 12c$$

Therefore, x is divisible by 12.

21.

Prove that no perfect square ends in the digit 2. Let x be equal to any integer. Let a "perfect square" $= x^2$. Since x^2 equals $x(x)$ then if $x(x) = y$ then the $\sqrt{y}$ must equal an integer. Therefore because the $\sqrt{2}$ is not an integer, and $x(\sqrt{y})$ is not an integer, then no perfect square root can ever end in 2.