Feature Learning

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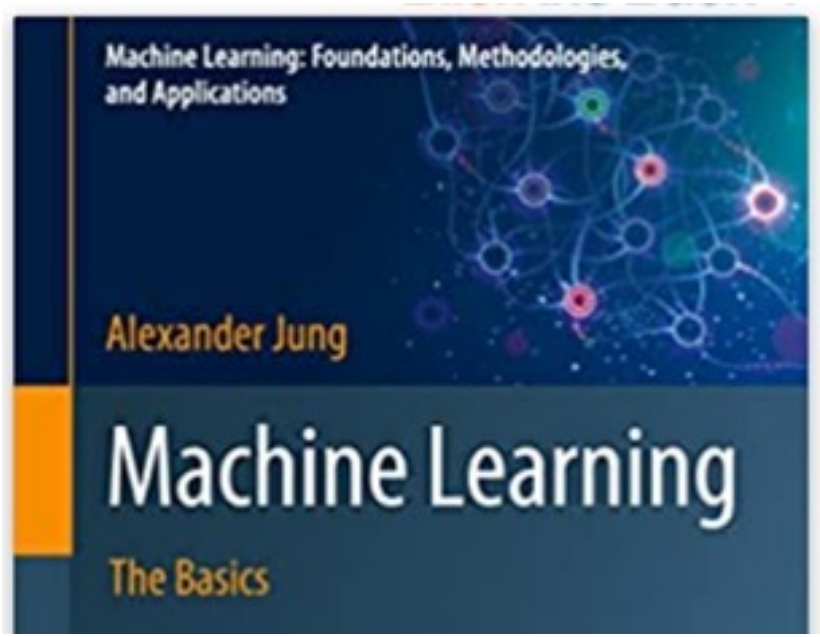
Reading.

Ch. 9 of AJ, ”Machine Learning: The Basics,” Springer, Singapore, 2022.

preprint: https://mlbook.cs.aalto.fi



https://scikit-learn.org/stable/modules/feature_selection.html

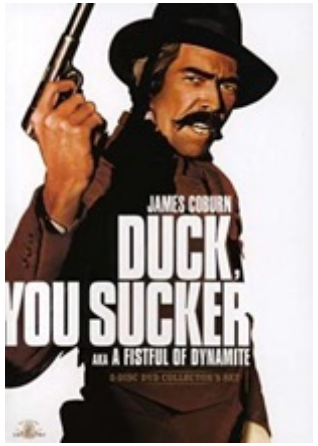
Learning Goals

- understand challenges with long raw feature vectors
- basic idea of feature learning
- feature learning for visualization and privacy-protection
- principal component analysis
- random projections

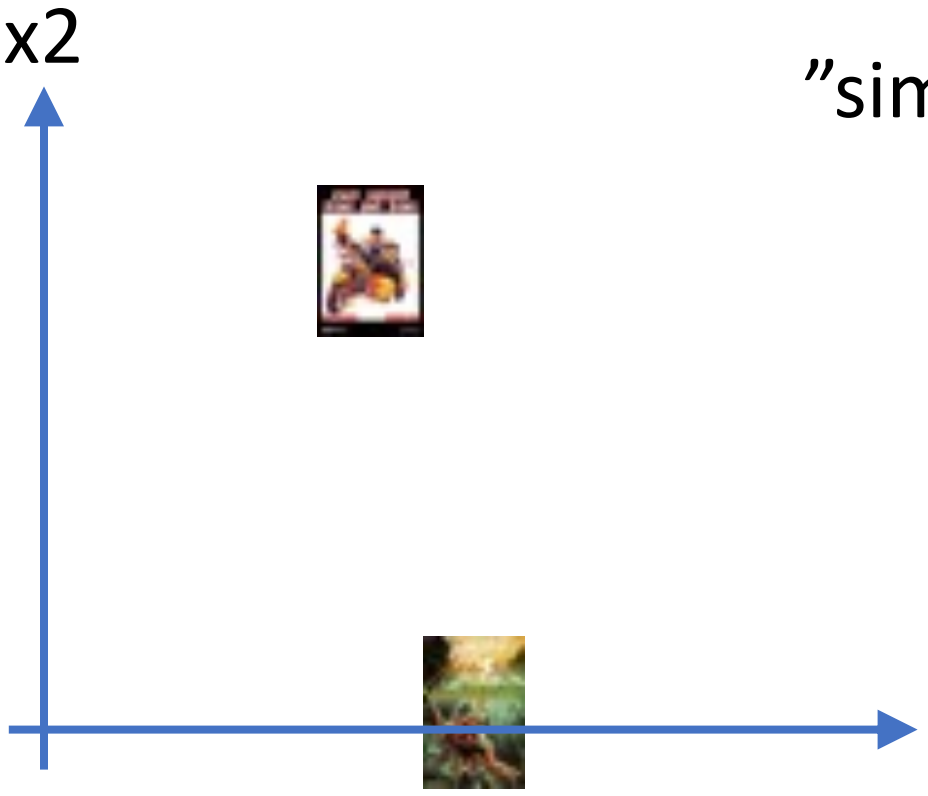
Data Point = “Some Movie”

several Gigabytes of raw feature bits!

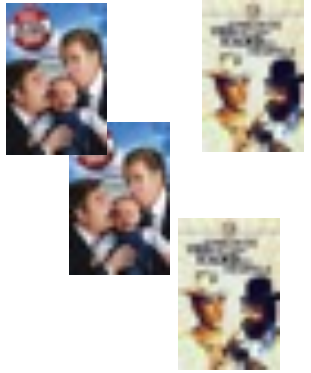
Dataset = ”Bunch of Movies”



Scatterplot of Movies?



”similar” movies should be placed nearby!



movie represented by two features x1,x2 !

Curse of Big Data

Overwhelmed by Tons of Features!

- consider data point representing a person
- digital footprint can be used for constructing features
- health-records (including genetic fingerprint)
- credit-card transactions
- social media posts
- media collections
- travelling profile over last 20 years
- ….

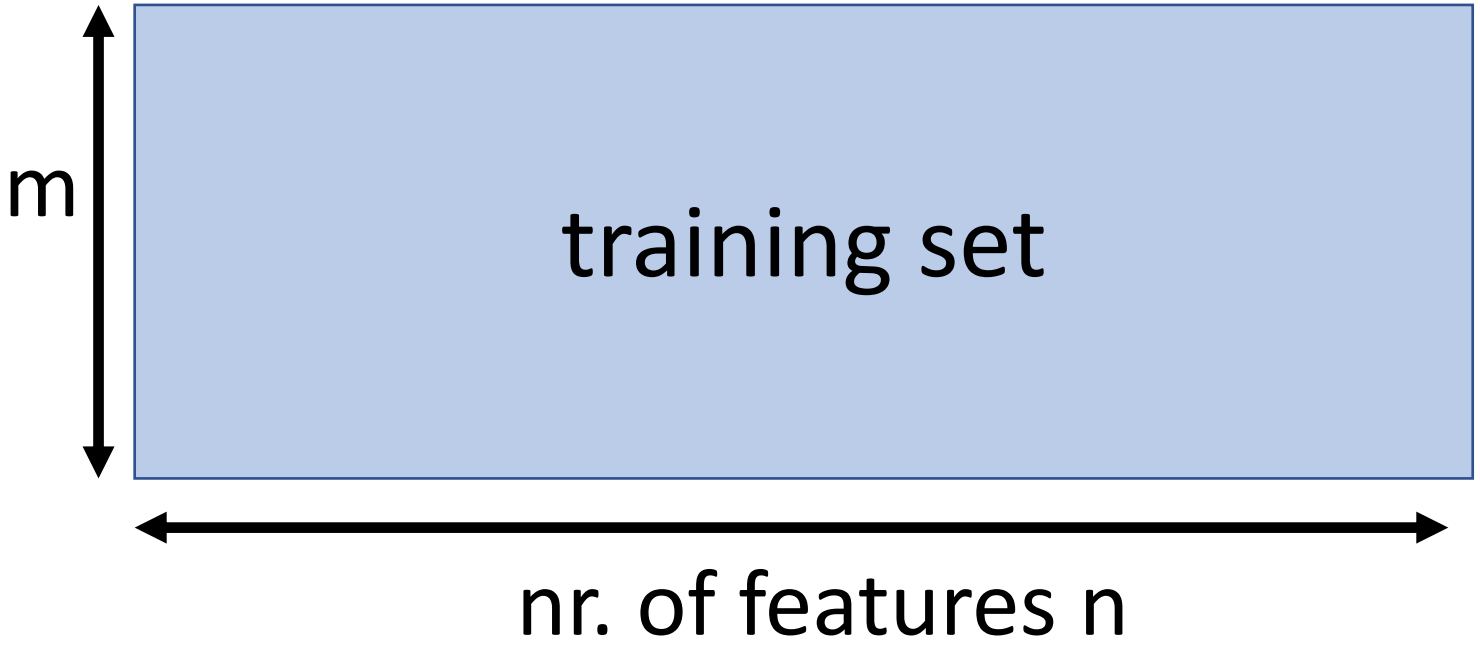
Statistical Challenge

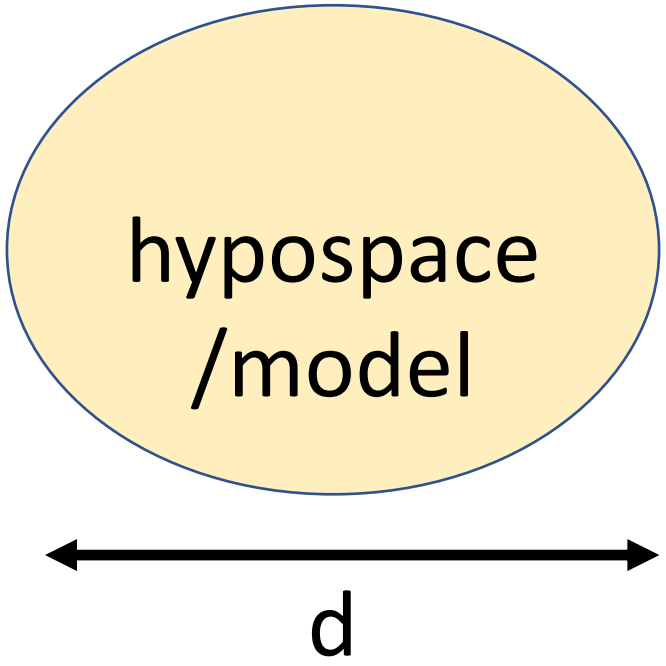
- effective dimension d of hypothesis space
- d typically increases with larger nr. of features
- overfitting is likely when d > sample size m

Linear Regression

- consider data points with n features
- have m labeled data points
- for m < n, plain linear regression will overfit
- n might be billions (e.g., HD-movies)
- would need billions of labeled data points

Data and Model Size





overfitting as soon as

$d/m > 1$!

Computational Challenge

linear regression on data points with d features amounts to inverting a matrix size d x d

GDPR-Compliant Feature Selection

Data minimisation: The use of personal data has to be limited to what is necessary to fulfil the purpose it was collected for …

Proportionality…The amount and nature of the data used has to be proportionate to the purpose and the least invasive for the data subject…

source: https://www.auditingalgorithms.net/

Feature Selection for Trustworthy AI

"Privacy and data governance: besides ensuring full respect for privacy and data protection, …into account the quality, integrity …and ensuring legitimised access to data…

Diversity, non-discrimination and fairness: Unfair bias must be avoided, as it could could have multiple negative implications, from the marginalization of vulnerable groups, to the exacerbation of prejudice and discrimination…."

https://digital-strategy.ec.europa.eu/en/library/ethics-guidelines-trustworthy-ai

The Basic Idea of Feature Learning.

Basic Idea of Feature Learning

raw feature vector

reconstruction

n $\mathbf{z}$

tunable map W

short feature vector

$\mathbf{x}$ n'

tunable map R

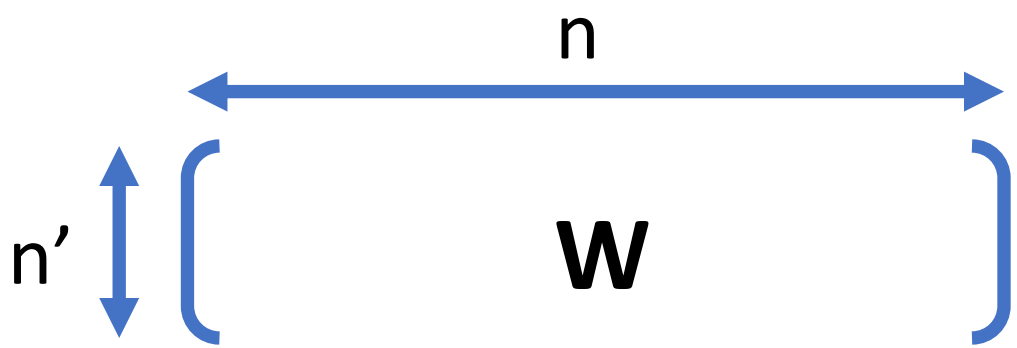
$\hat{\mathbf{z}}$ n

choose (learn) W and R to minimize reconstruction error $\mathbf{z} - \hat{\mathbf{z}}$

Linear Feature Learning

- use linear maps for compression and reconstruction

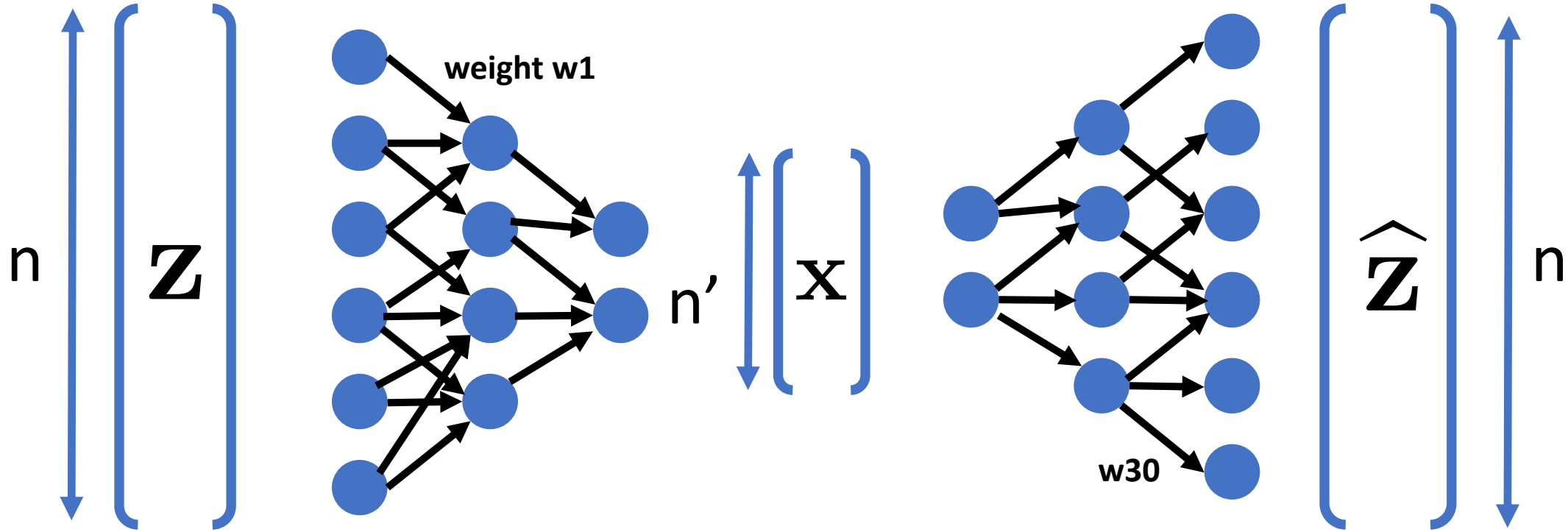
$$\mathbf{x} = \mathbf{W}\mathbf{z} \qquad \widehat{\mathbf{z}} = \mathbf{R}\mathbf{x}$$



choose matrices W and R to minimize $\mathbf{z} - \widehat{\mathbf{z}} = \big(\mathbf{I} - \mathbf{R}\mathbf{W}\big)\mathbf{z}$

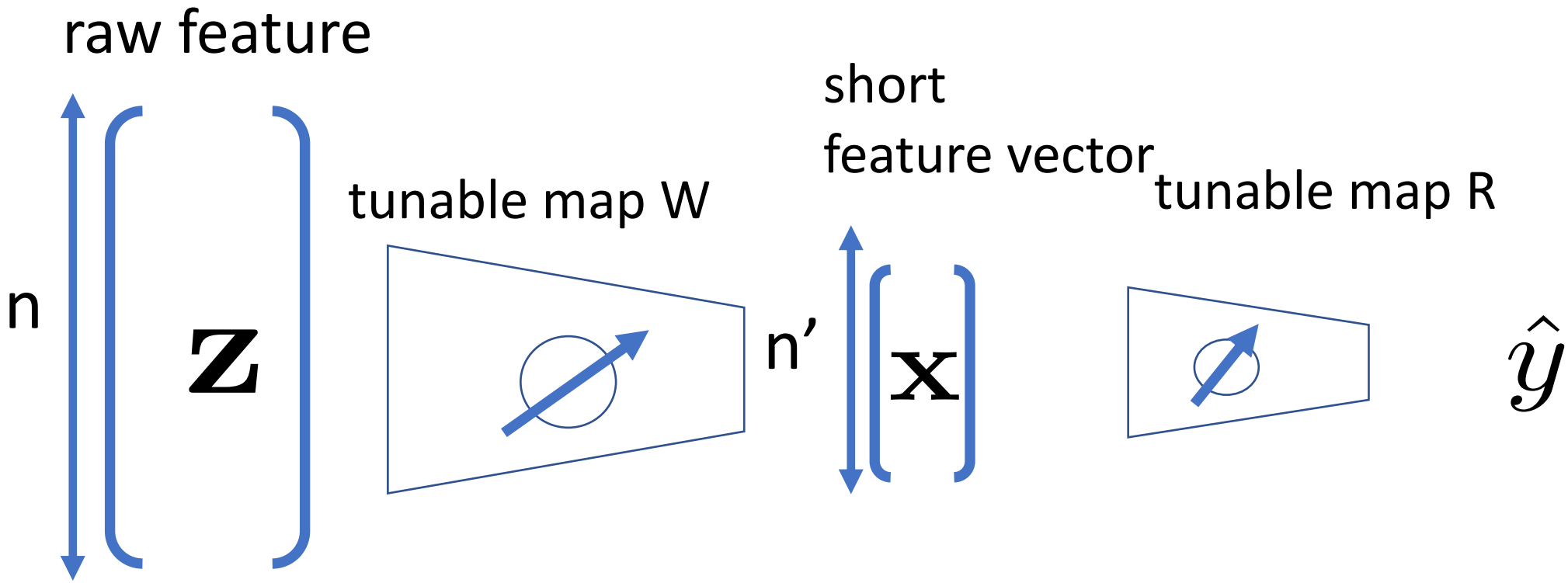
Non-Linear Feature Learning (“Autoencoder”)

use artificial neural networks for compression and reconstruction



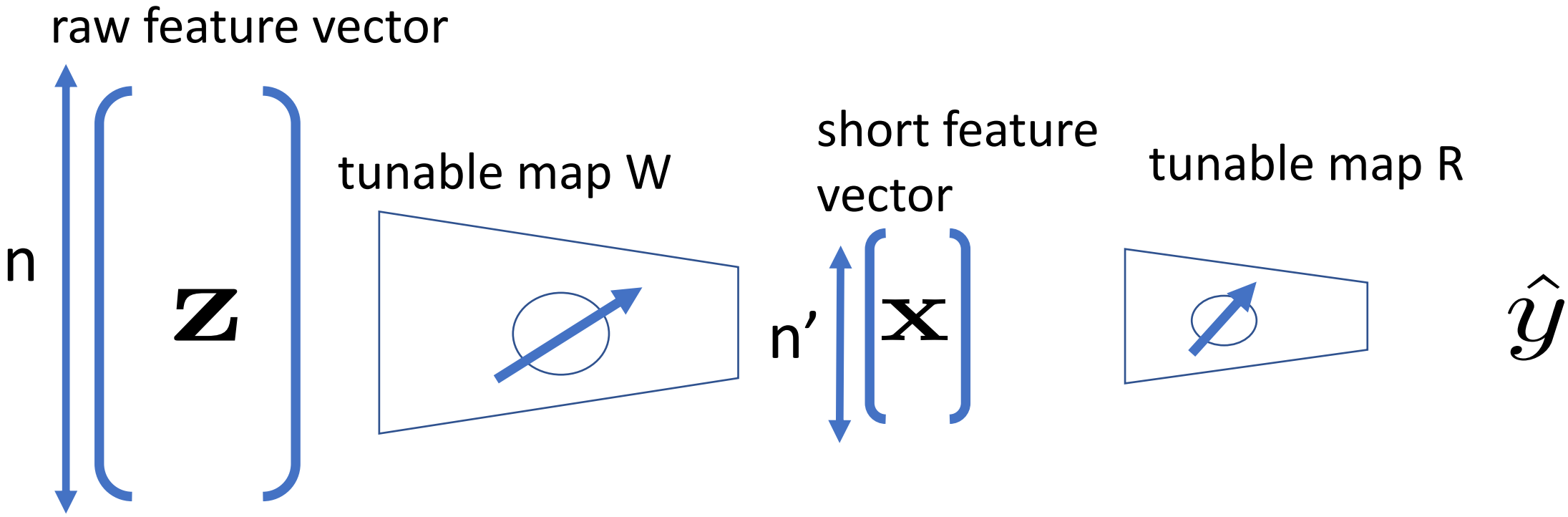
much like linear maps, ANNs are just **parametrized maps**!

Feature Learning for Labeled Data



choose W such that we can predict (using some map R) the label y from z with maximum accuracy

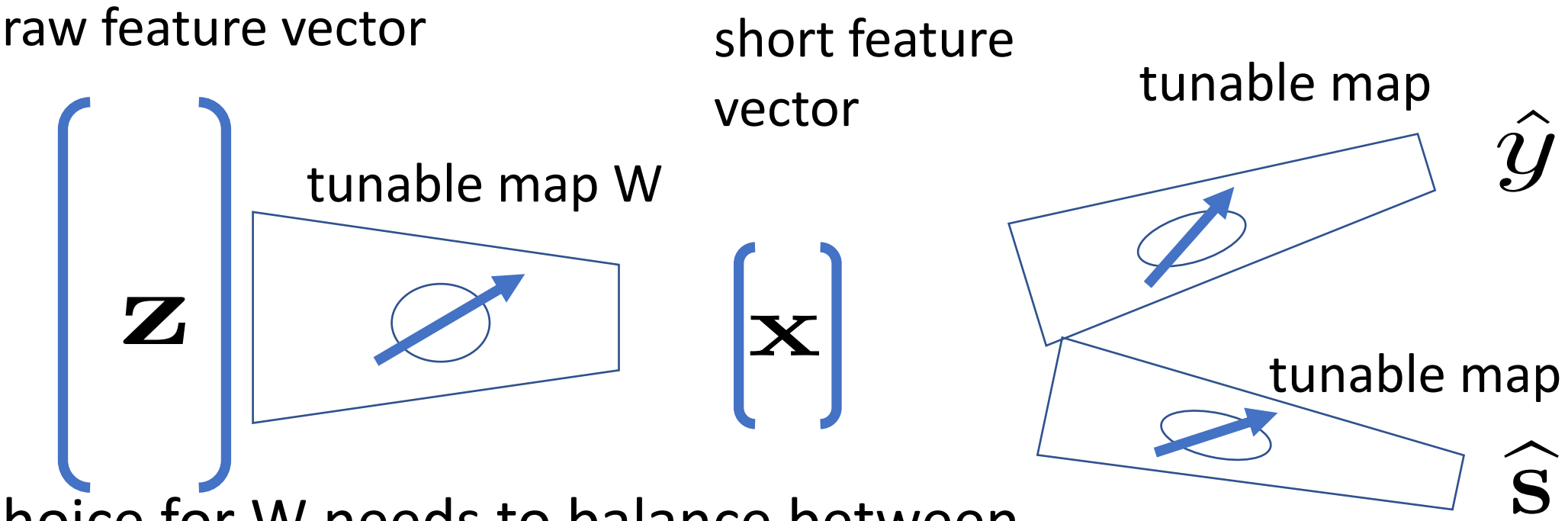
Feature Learning for Labeled Data



choice for W needs to balance between

- compressing raw feature vector as much as possible
- keep parts of raw features that are relevant for predicting y

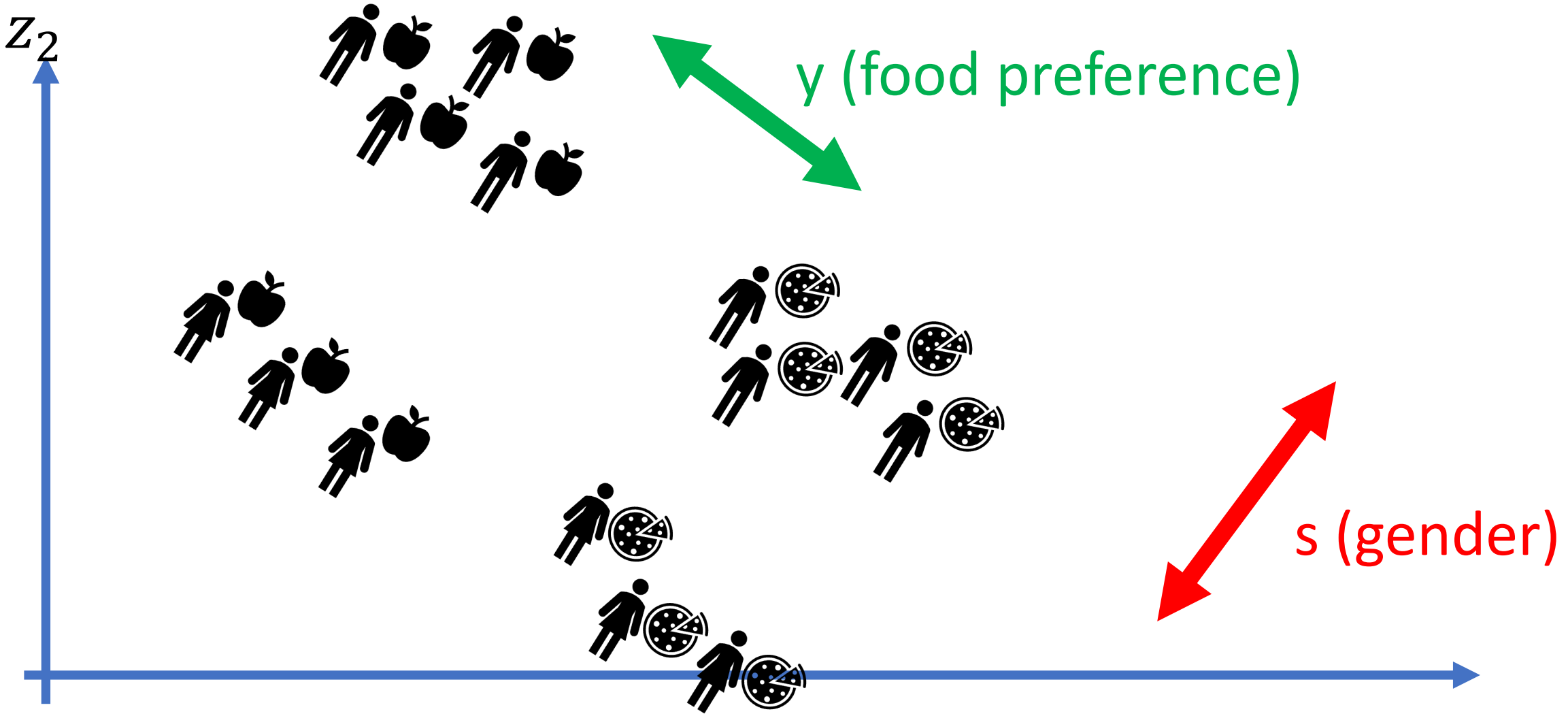
Privacy-Preserving Feature Learning



choice for W needs to balance between

- compressing raw feature vector as much as possible
- keep parts of raw features that are relevant for predicting y
- predicting private variable ”s” is not predictable from x

Privacy-Preserving Feature Learning

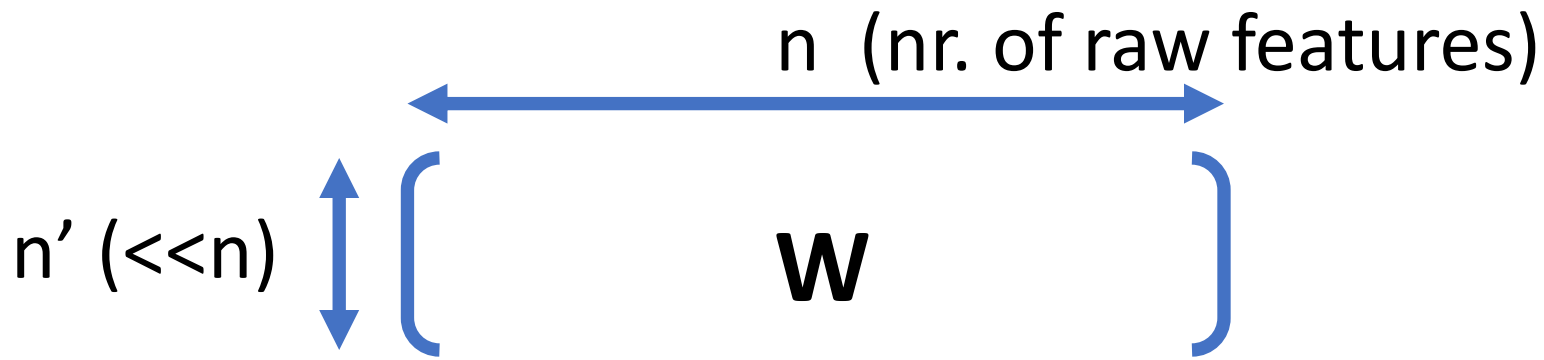


Principal Component Analysis (PCA)

Linear Feature Learning

- use linear maps for compression and reconstruction

$$\mathbf{x} = \mathbf{W}\mathbf{z} \qquad \widehat{\mathbf{z}} = \mathbf{R}\mathbf{x}$$



choose matrices W and R to minimize $\mathbf{z} - \widehat{\mathbf{z}} = \big(\mathbf{I} - \mathbf{R}\mathbf{W}\big)\mathbf{z}$

PCA as Risk Minimization

- m datapoints with raw feature vecs $\mathbf{z}^{(1)}, \cdots, \mathbf{z}^{(m)}$

  choose matrices **W, R** to minimize reconstruction error

$$\mathrm{L}(\mathbf{W}, \mathbf{R}) = \frac{\mathbf{1}}{\mathbf{m}} \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{m}} \left\| \mathbf{z}^{(\mathrm{i})} - \mathbf{R}\mathbf{W}\mathbf{z}^{(\mathrm{i})} \right\|^{\mathbf{2}}$$

Principal Component Analysis (PCA)

- optimal compression matrix $\mathbf{W} = \big(\mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(\mathrm{n}')}\big)$

- using "top" eigenvectors $\boldsymbol{u}^{(i)}$ of sample covariance matrix

$$\widehat{\mathbf{C}} = (1/m) \sum_{i=1}^{m} \mathbf{z}^{(i)} \big(\mathbf{z}^{(i)}\big)^{T}$$

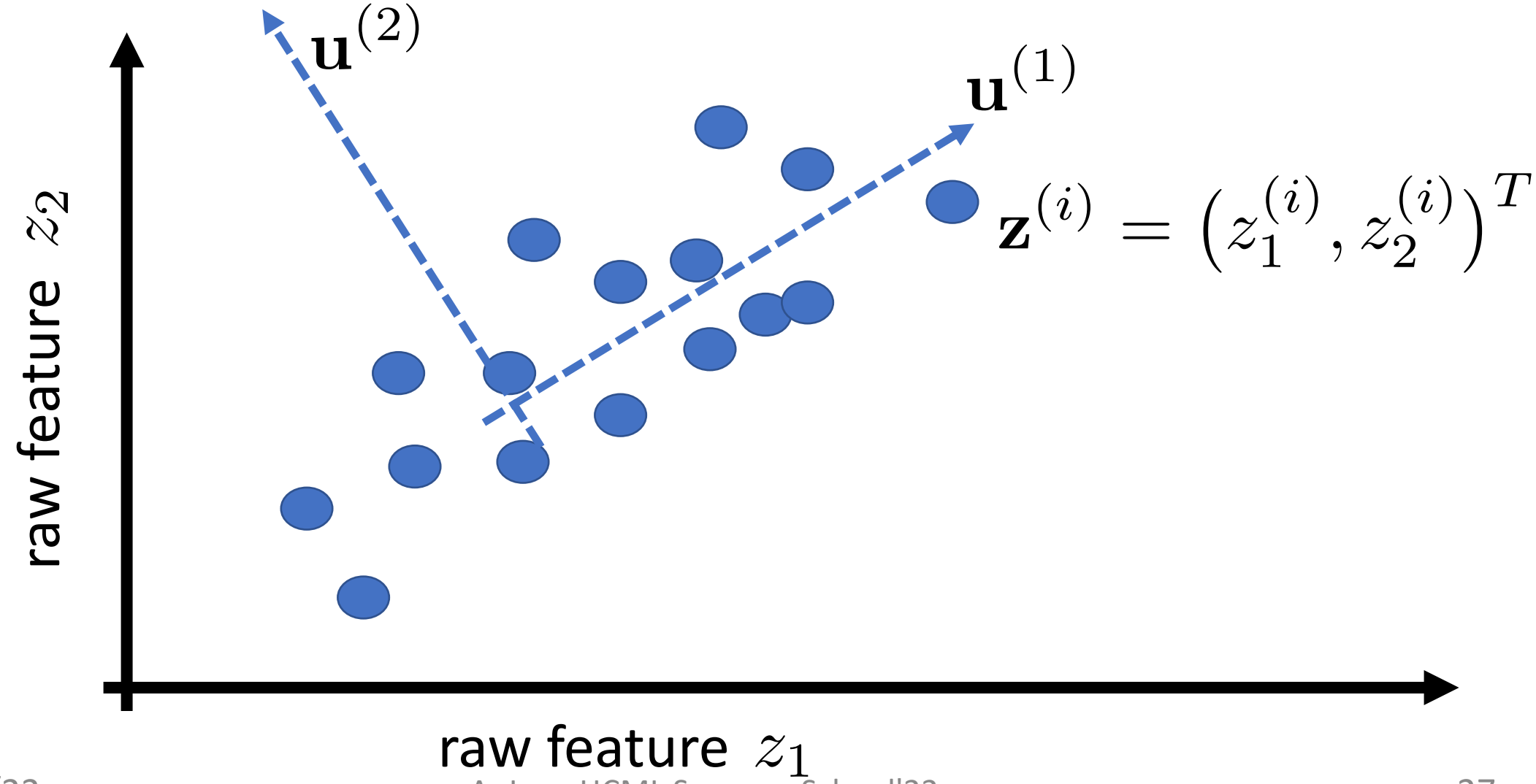
- eigenvalue decomposition of psd sample cov. matrix:

$$\widehat{\mathbf{C}} = \big(\mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(n)}\big) \mathrm{diag}\big(\lambda_1, \ldots, \lambda_n\big) \big(\mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(n)}\big)^{T}$$

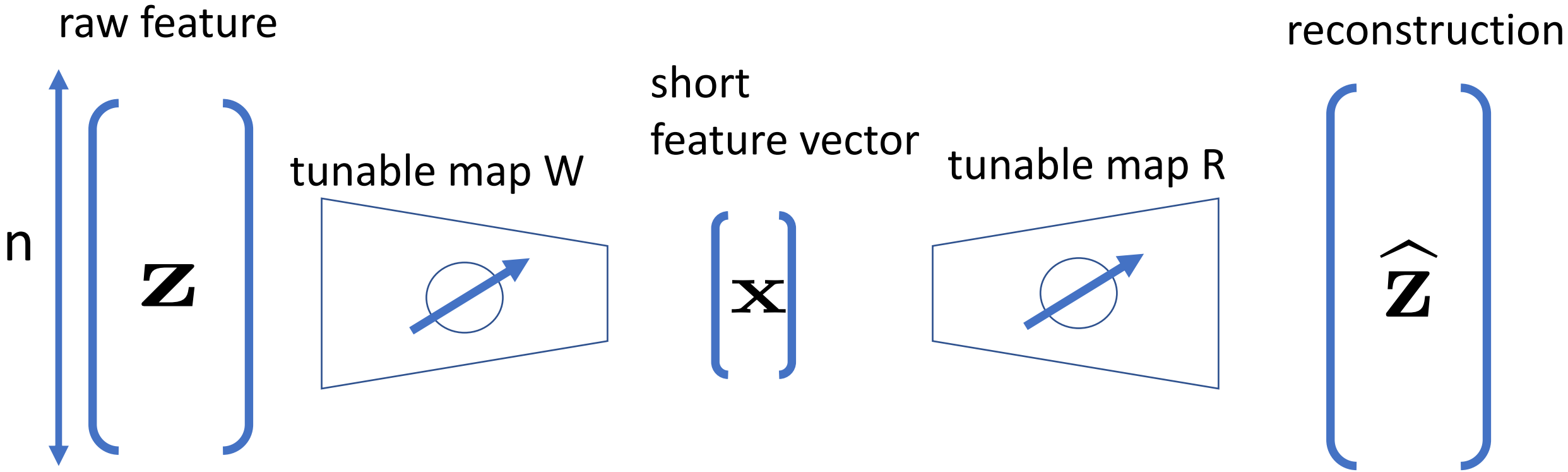
non-negative eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0.$$

Geometry of PCA



Computational Complexity



choose (learn) W and R to minimize reconstruction error $\mathbf{z} - \widehat{\mathbf{z}}$

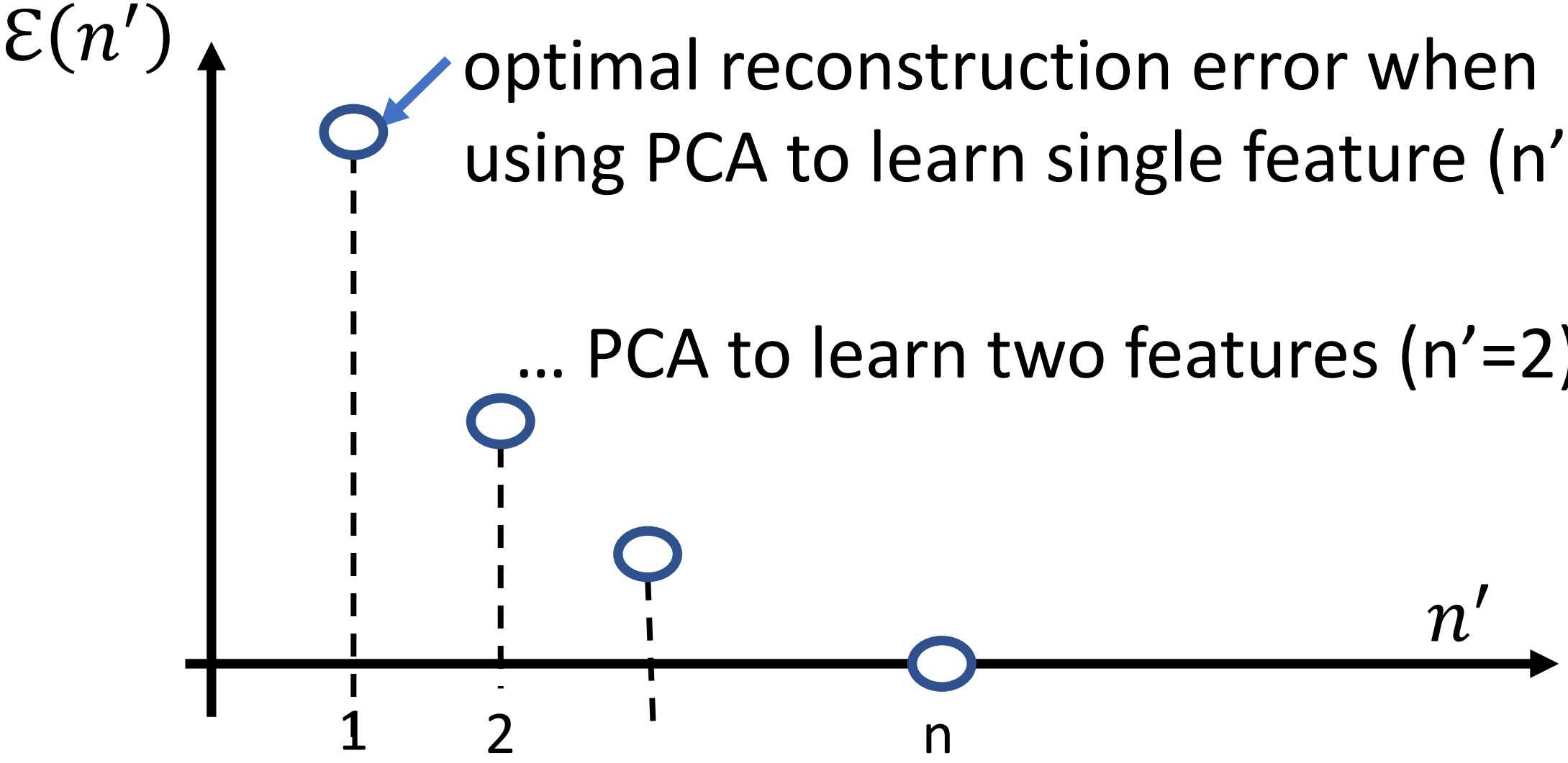
PCA requires eigenvalue decomposition of “n x n” matrix!

How to choose n’ ?

- PCA requires number n’ of learned features as input
- n’=2 for visualization (scatter plot)
- chose n’ to balance compression with optimal reconstruction error

$$\mathrm{L}(n') = \min_{\substack{\mathbf{W}\in\mathbb{R}^{\mathrm{n}'\times\mathrm{n}} \\ \mathbf{R}\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}'}}} L(\mathbf{W},\mathbf{R})$$

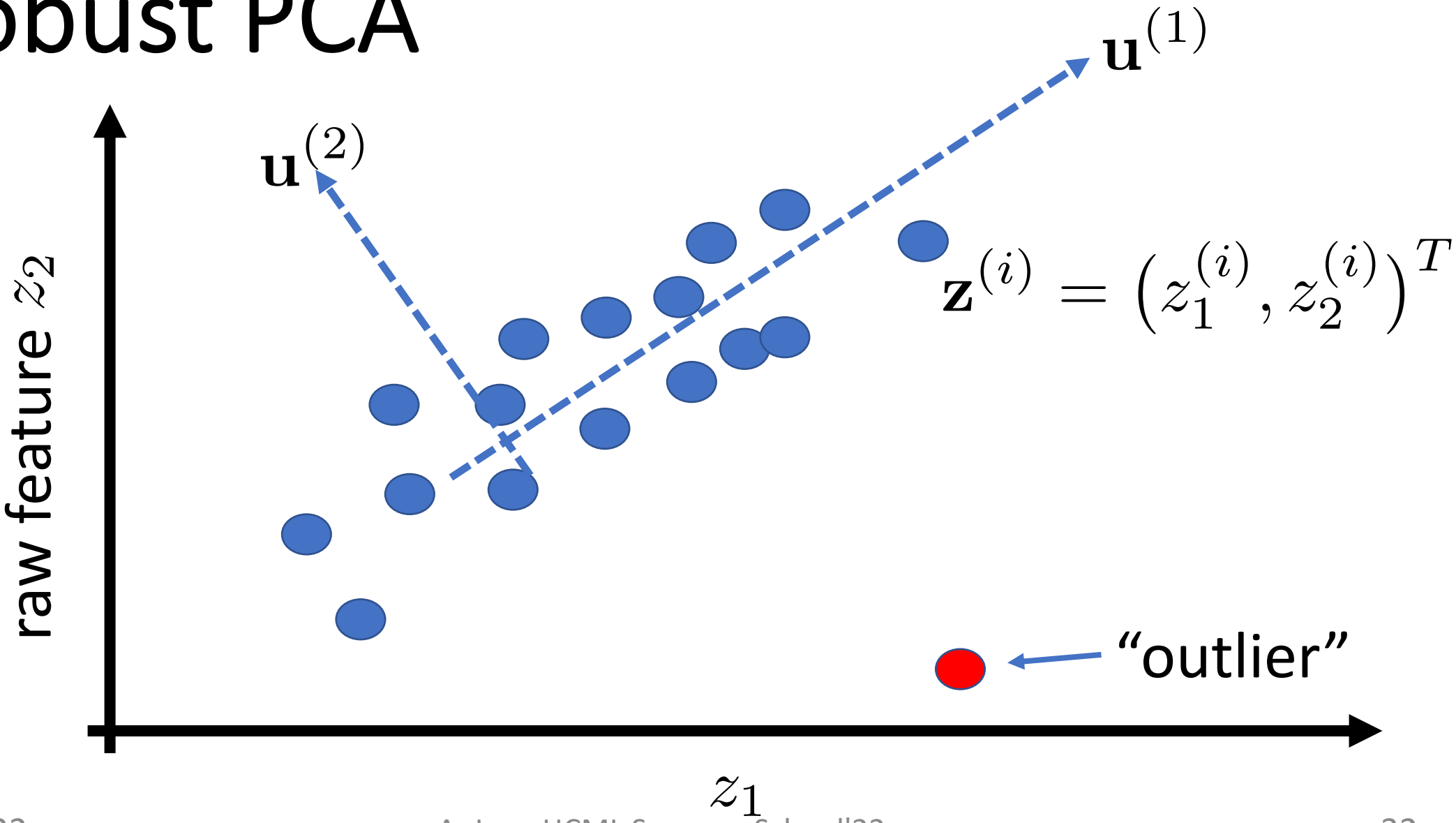
Elbow Method



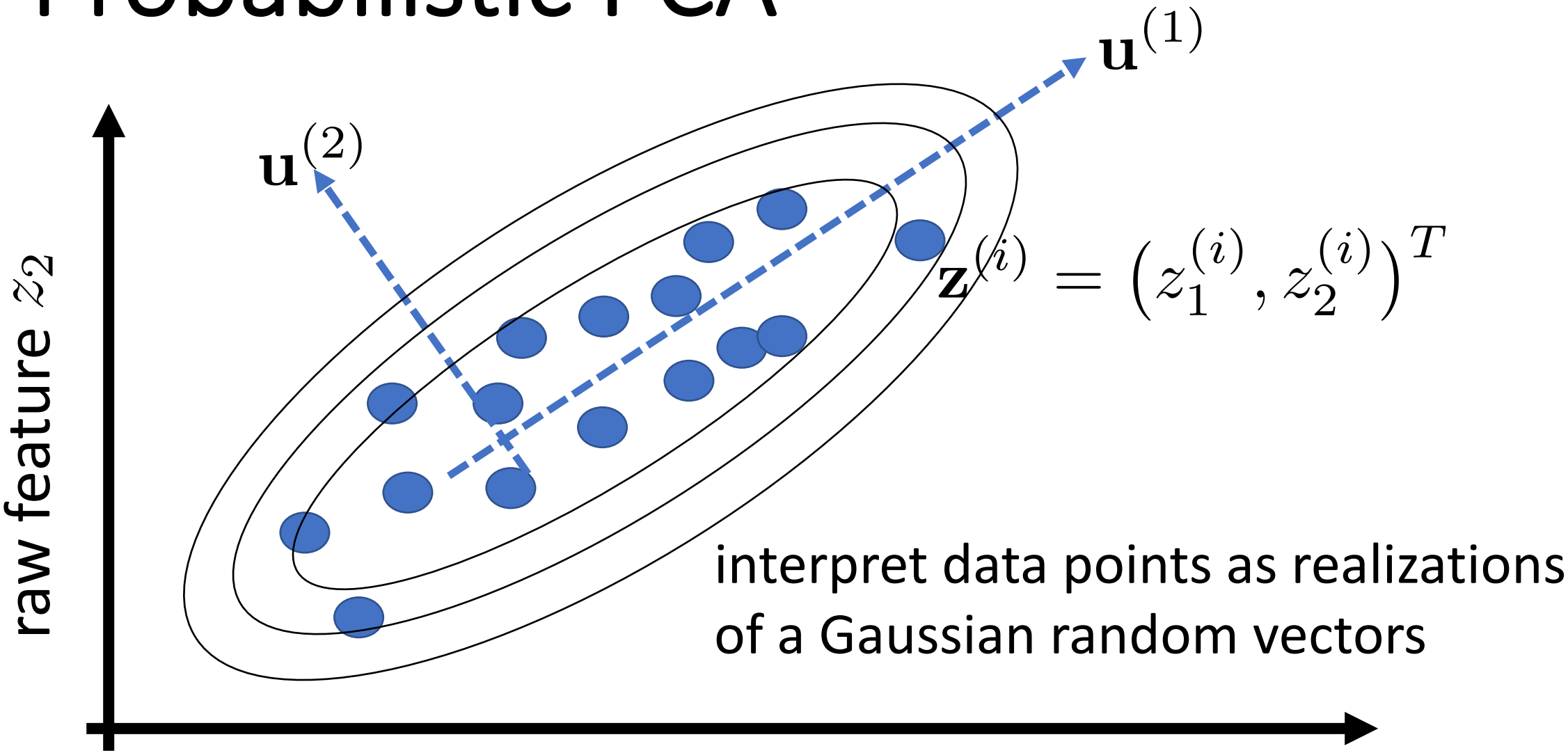
Variations of PCA

- robust PCA: uses a different measure of reconstruction error
- probabilistic PCA: uses a statistical model for data points
- sparse PCA: new features depend on few raw features

Robust PCA


$\mathbf{u}^{(1)}$
$\mathbf{u}^{(2)}$
$\mathbf{z}^{(i)} = \left(z_1^{(i)}, z_2^{(i)}\right)^T$
raw feature z_2
"outlier"
z_1

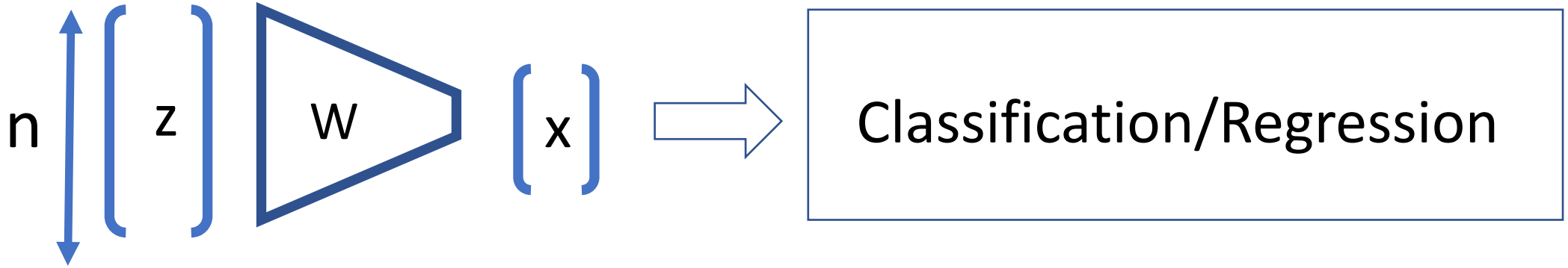
Probabilistic PCA



$$\mathbf{z}^{(i)} = \left(z_1^{(i)}, z_2^{(i)}\right)^T$$

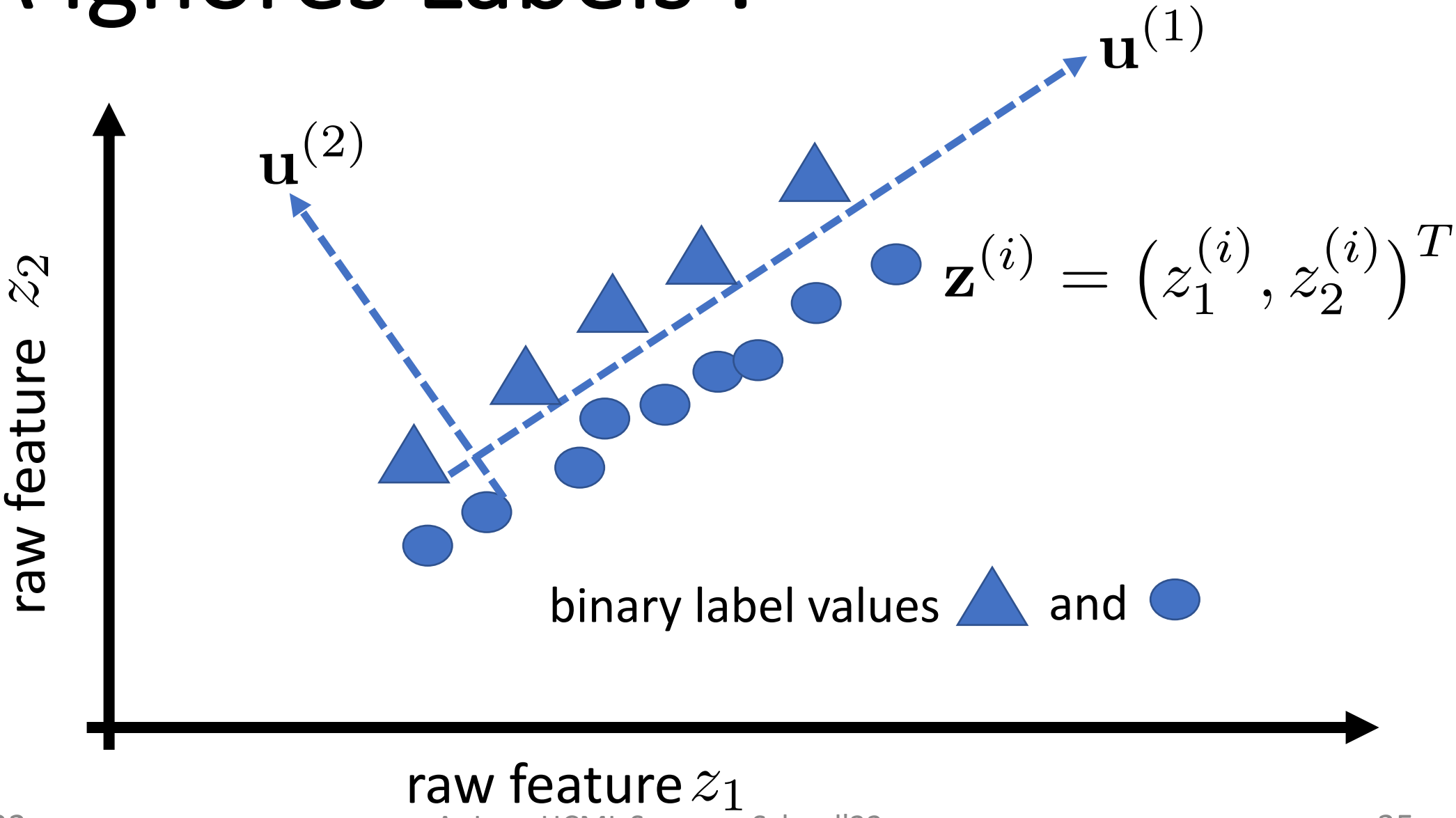
interpret data points as realizations of a Gaussian random vectors

PCA as Pre-Processing

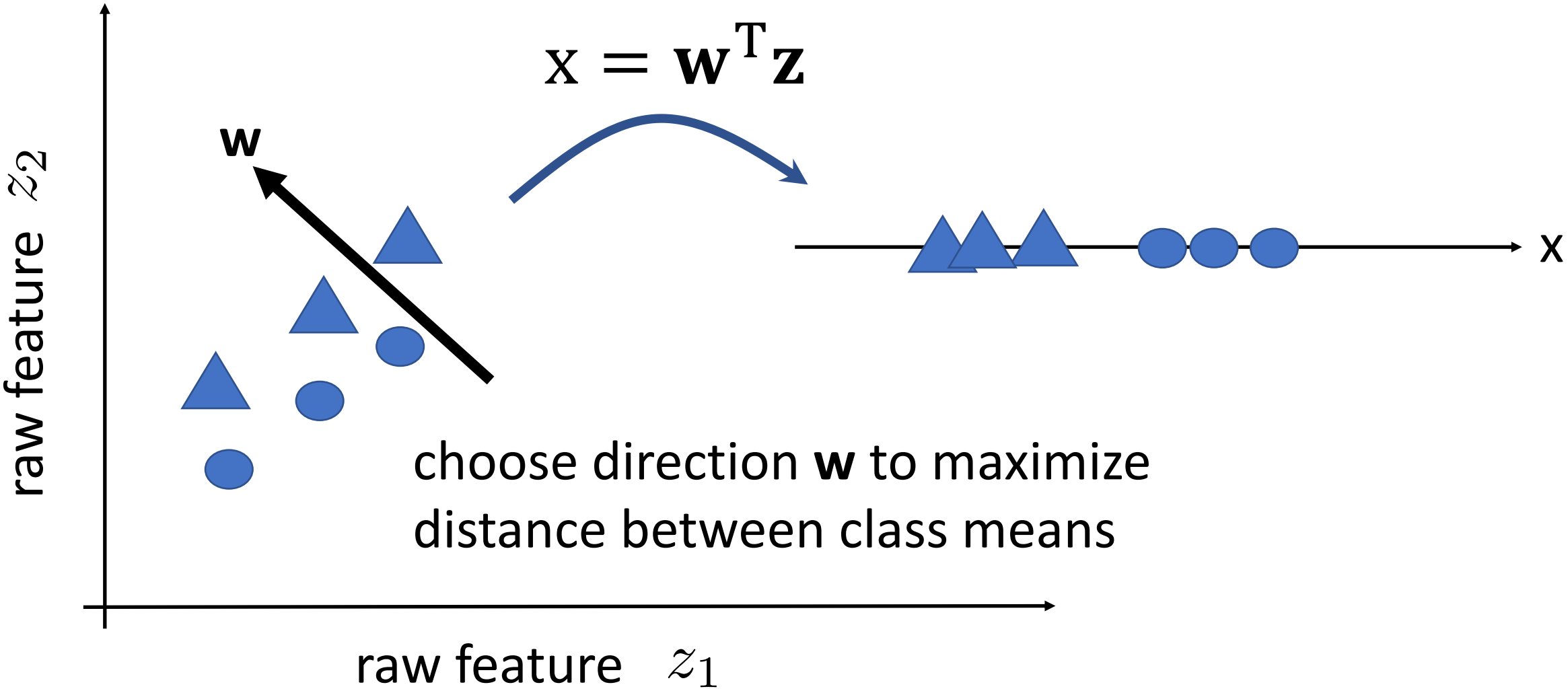


- PCA delivers a compression matrix W
- replace (long) raw features z with shorter features x =Wz
- apply regression/classification methods to new features x
- CAUTION: PCA ignores label information!

PCA Ignores Labels !



Fisher's Linear Discriminant



Random Projections

- consider random projection $\mathbf{x} = \mathbf{W}\,\mathbf{z}$
- entries of matrix $\mathbf{W}$ are randomly chosen
- no learning/tuning of $\mathbf{W}$ required!
- in many settings, works surprisingly well
- known as compressed sensing

So What?

- feature learning methods determine relevant features
- learning two features allows to scatter plot !
- optimal linear feature learning = PCA
- PCA ignores label information!
- random projections as computationally light alternative

Thank You !