Efficacy of Super-Modeling in Climate Systems

Team Members: Alex Stoken, Jacob Warshawer, Xiaokun Tu, Lourdes Mota, Yuanjie Liu

Project Description

- Chaotic systems are very difficult to model via traditional techniques because of their high sensitivity to initial conditions and parameter values
- Super-models may offer improved accuracy for chaotic systems
- Determine the viability of the super-modeling approach by assessing Lyapunov exponents, model error, and qualitative trajectory agreement
- We will test the approach by applying it to the Lorenz 63 and Lorenz 84 climate systems

Lorenz 63

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

Nine Sub-Models (k=1,2,3)

$$\dot{x_k} = \sigma_k(y_k - x_k) + \sum_{j \neq k} C^x_{kj}(x_j - x_k)$$
$$\dot{y_k} = x_k(\rho_k - z_k) - y_k + \sum_{j \neq k} C^y_{kj}(y_j - y_k)$$
$$\dot{z_k} = x_k y_k - \beta_k z_k + \sum_{j \neq k} C^z_{kj}(z_j - z_k)$$

Super-Model [4]

$$x_s = \frac{1}{3}(x_1 + x_2 + x_3)$$
$$y_s = \frac{1}{3}(y_1 + y_2 + y_3)$$
$$z_s = \frac{1}{3}(z_1 + z_2 + z_3)$$

Figure 1: Standard Lorenz 63 model (left) and Lorenz 63 super-model (right). Process of building supermodel begins with adding connection coefficients and then averaging submodel results

Scientific Challenges

- Climate systems exhibit chaos which is inherently difficult to predict. [3]
- Accurate climate prediction is increasingly important as scientists attempt to forecast and prepare for global climate change
- Super-models are developed for high accuracy even with imperfect parameterizations, which can lead to new research into modeling techniques with high uncertainty in initial conditions or parameters

Glossary of Technical Terms

Super-model: A model that combines multiple sub-models with perturbed parameters and added connection coefficients
Sub-model: Once component of a super-model
Connection Coefficient: Additive, learned terms that "pull" sub-model trajectories together
Chaotic System: A system of differential equations that exhibit sensitivity to initial conditions
Cost Function: A measurement of error between the truth and the model as a function of the connection coefficients
Lyapunov Exponent: A quantitative measure of how chaotic a system of differential equations is

Methodology

1. Transform a model into a super-model by perturbing the standard parameters and adding connection coefficients
2. To optimize these connection coefficients, build a cost function that takes in a vector ***C*** containing all 18 connection coefficients and outputs cumulative model error.

$$F(\boldsymbol{C}) = \frac{1}{K\Delta} \sum_{i=1}^{K} \int_{t_i}^{t_i+\Delta} |\boldsymbol{x}_s(\boldsymbol{C}, t) - \boldsymbol{x}_o(t)|^2 \gamma^t \, dt$$

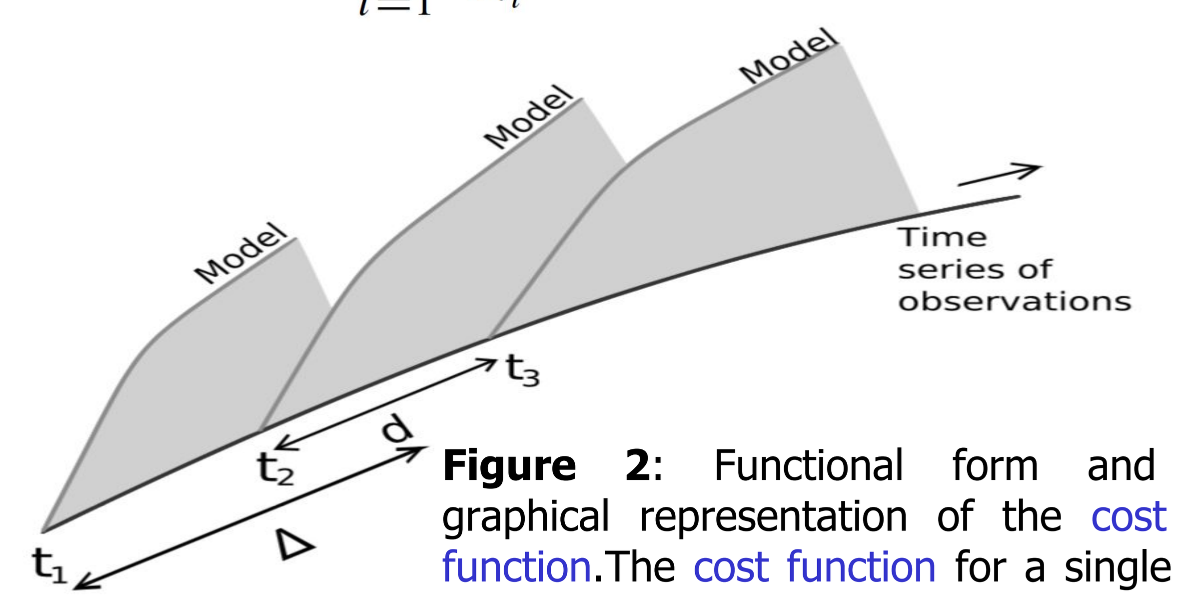


Figure 2: Functional form and graphical representation of the cost function.The cost function for a single time step represents the grey area inside Δ.[4]

3. Use the conjugate gradient method - a type of gradient descent minimization procedure [3] - to find the ***C*** that yields the super-model with the least error.
4. Compute model error and Lyapunov exponents for the super-model and compare it to the original system.

Results

- Super-model successfully produces qualitatively similar trajectories from sub-models that are qualitatively different
- Lyapunov exponent for our super-model is smaller than that of the Lorenz 63 system - the super-model is less chaotic than the Lorenz 63 system
  - Lorenz 63 Standard Model Lyapunov Exponent: 0.906
  - Lorenz 63 Super-Model Lyapunov Exponent: 0.690
- Minimal reduction in error with increased training points, but high increase in computation time - models can be build with relatively little data

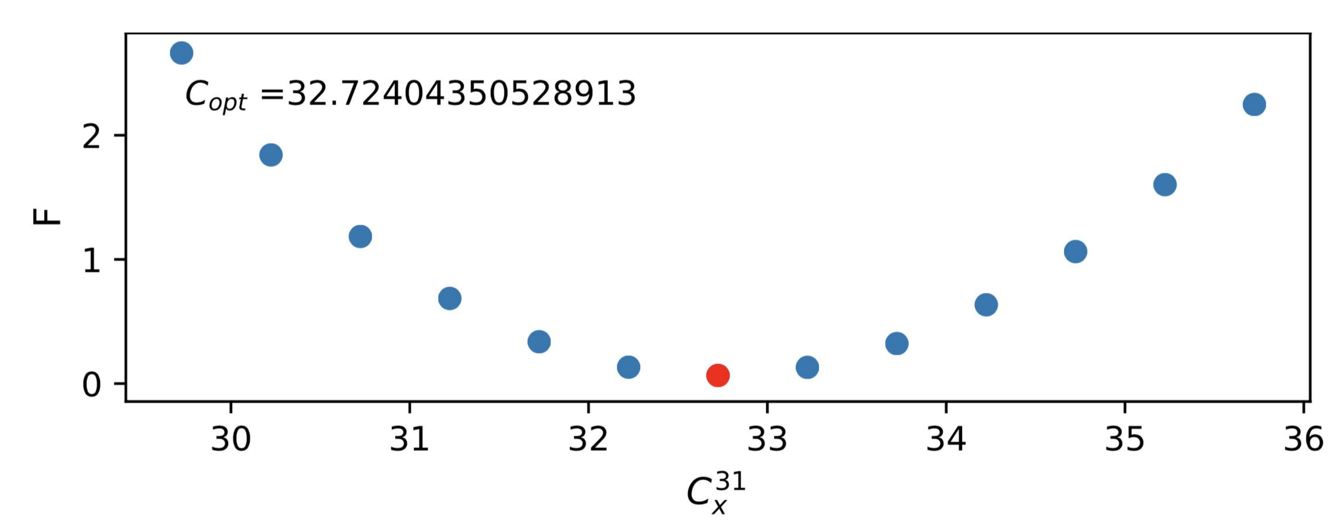
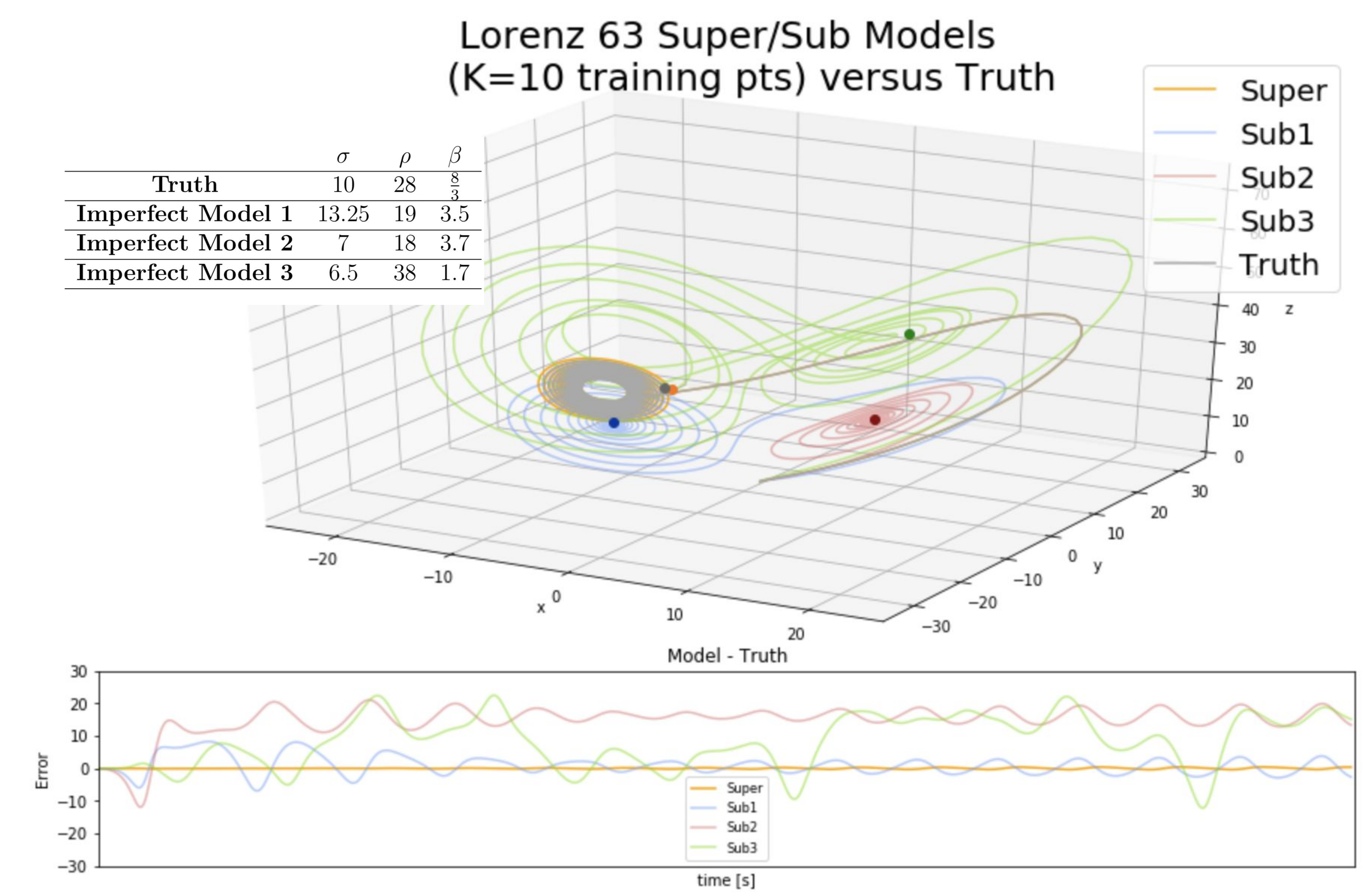


Figure 3: Minimization of Cost Function F with respect to a connection coefficient C_x^{31}. The cost function is minimized with respect to every connection coefficient C_k^{ij} using the Conjugate Gradient algorithm [3]



	σ	ρ	β
Truth	10	28	$\frac{8}{3}$
Imperfect Model 1	13.25	19	3.5
Imperfect Model 2	7	18	3.7
Imperfect Model 3	6.5	38	1.7

Figure 4: Time evolution of two Lorenz 63 super-models, the imperfect submodels, and the data values. Error between each model and data shown in the bottom of the figure

Conclusions and Potential Applications

- Super-models far outperform traditional models of imperfect parameters for chaotic systems
- Super-models can be built from relatively small datasets, and are less chaotic than the systems they model
- Super-modeling can be used in climate forecasting [5], as well as other chaotic systems including polymer manufacturing and asteroid orbits

References

1. R. Fletcher, C. M. Reeves, Function minimization by conjugate gradients, *The Computer Journal*, Vol 7, Iss. 2, 1964, P 149–154
2. F. M. Selten, F. Schevenhoven, and G. Duane. Simulating climate with a synchronization-based supermodel. *Chaos: An Inter disciplinary Journal of Nonlinear Science*, 27(12):126903, 2017.
3. M. Shen, N. Keenlyside, B. C. Bhatt, and G. Duane. Role of atmosphere-ocean interactions in supermodeling the tropical pacific climate. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 27(12):126704, 2017.
4. L. A. van den Berge, F. Selten, W. Wiegerinck, and G. Duane. A multi-model ensemble method that combines imperfect models through learning. *EarthSystem Dynamics*, 2(1):161–177, 2011.
5. W.Wiegerinck, SNN Radboudn University, *Super-modeling: combining imperfect models by synchronization,2017.*

Acknowledgments

This project was mentored by Ryan Coatney, whose help is acknowledged with great appreciation.