Efficacy of Super-Modeling in Climate Systems

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Project Description

- Chaotic systems are very difficult to model via traditional techniques because of their high sensitivity to initial conditions and parameter values
- Super-models may offer improved accuracy for chaotic systems
- Determine the viability of the super-modeling approach by assessing Lyapunov exponents, model error, and qualitative trajectory agreement
- We will test the approach by applying it to the Lorenz 63 and Lorenz 84 climate systems

Lorenz 63	Nine Sub-Models (k=1,2,3)	Super-Model [4]
$\dot{x} = \sigma(y - x)$	$\dot{x_k} = \sigma_k(y_k - x_k) + \sum_{j \neq k} C^x_{kj}(x_j - x_k)$	$x_s = \frac{1}{3}(x_1 + x_2 + x_3)$
$\dot{y} = x(\rho - z) - y$ →	$\dot{y_k} = x_k(\rho_k - z_k) - y_k + \sum_{j \neq k} C^y_{kj}(y_j - y_k)$ →	$y_s = \frac{1}{3}(y_1 + y_2 + y_3)$
$\dot{z} = xy - \beta z$	$\dot{z_k} = x_k y_k - \beta_k z_k + \sum_{j \neq k} C^z_{kj}(z_j - z_k)$	$z_s = \frac{1}{3}(z_1 + z_2 + z_3)$

Figure 1: Standard Lorenz 63 model (left) and Lorenz 63 super-model (right). Process of building supermodel begins with adding connection coefficients and then averaging submodel results

Scientific Challenges

- Climate systems exhibit chaos which is inherently difficult to predict. [3]
- Accurate climate prediction is increasingly important as scientists attempt to forecast and prepare for global climate change
- Super-models are developed for high accuracy even with imperfect parameterizations, which can lead to new research into modeling techniques with high uncertainty in initial conditions or parameters

Glossary of Technical Terms

Super-model: A model that combines multiple sub-models with perturbed parameters and added connection coefficients
Sub-model: Once component of a super-model
Connection Coefficient: Additive, learned terms that "pull" sub-model trajectories together
Chaotic System: A system of differential equations that exhibit sensitivity to initial conditions
Cost Function: A measurement of error between the truth and the model as a function of the connection coefficients
Lyapunov Exponent: A quantitative measure of how chaotic a system of differential equations is

Methodology

1. Transform a model into a super-model by perturbing the standard parameters and adding connection coefficients
2. To optimize these connection coefficients, build a cost function that takes in a vector $\boldsymbol{C}$ containing all 18 connection coefficients and outputs cumulative model error.

$$F(\boldsymbol{C}) = \frac{1}{K\Delta}\sum_{i=1}^{K}\int_{t_i}^{t_i+\Delta} |\boldsymbol{x}_s(\boldsymbol{C}, t) - \boldsymbol{x}_o(t)|^2 \gamma^t \, dt$$

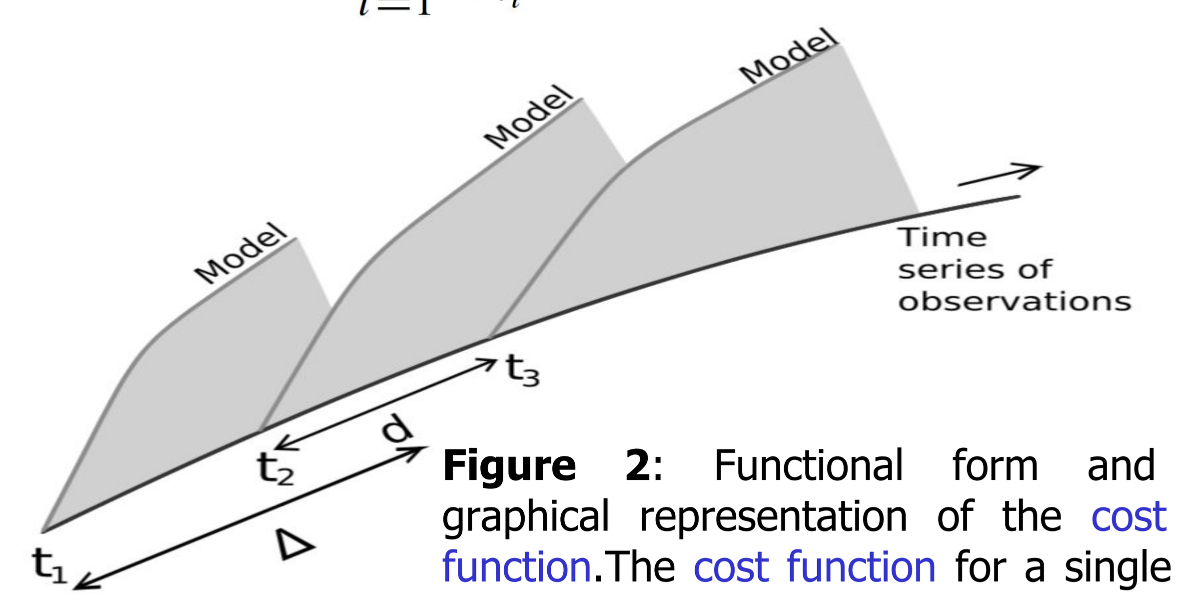


Figure 2: Functional form and graphical representation of the cost function.The cost function for a single time step represents the grey area inside Δ.[4]

3. Use the conjugate gradient method - a type of gradient descent minimization procedure [3] - to find the $\boldsymbol{C}$ that yields the super-model with the least error.
4. Compute model error and Lyapunov exponents for the super-model and compare it to the original system.

Results

- Super-model successfully produces qualitatively similar trajectories from sub-models that are qualitatively different
- Lyapunov exponent for our super-model is smaller than that of the Lorenz 63 system - the super-model is less chaotic than the Lorenz 63 system
  - Lorenz 63 Standard Model   Lyapunov Exponent: 0.906
  - Lorenz 63 Super-Model   Lyapunov Exponent: 0.690
- Minimal reduction in error with increased training points, but high increase in computation time - models can be build with relatively little data

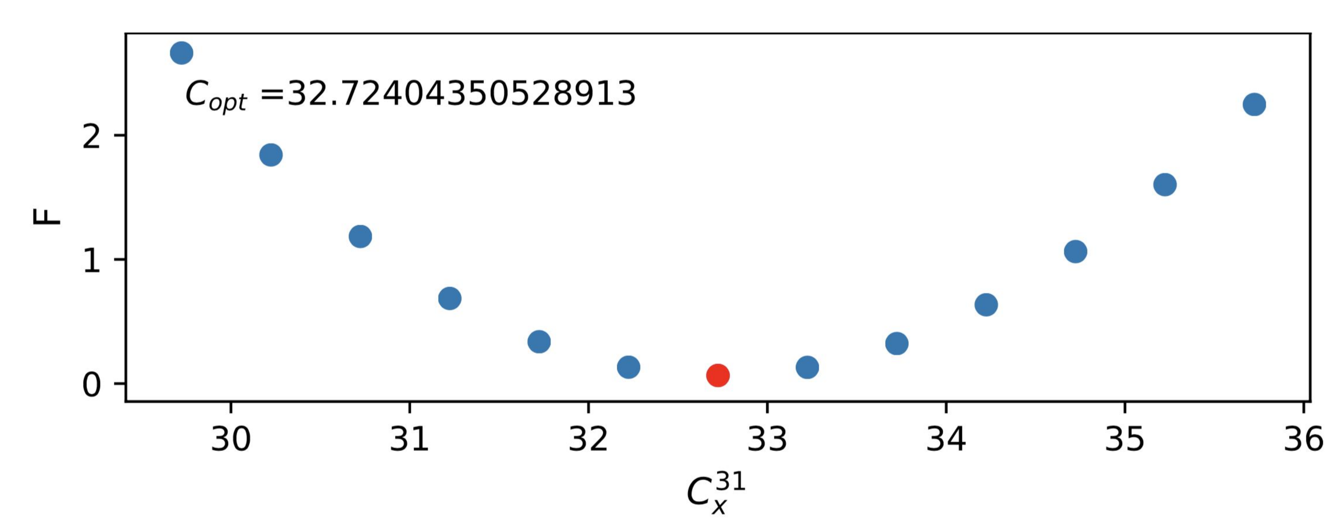
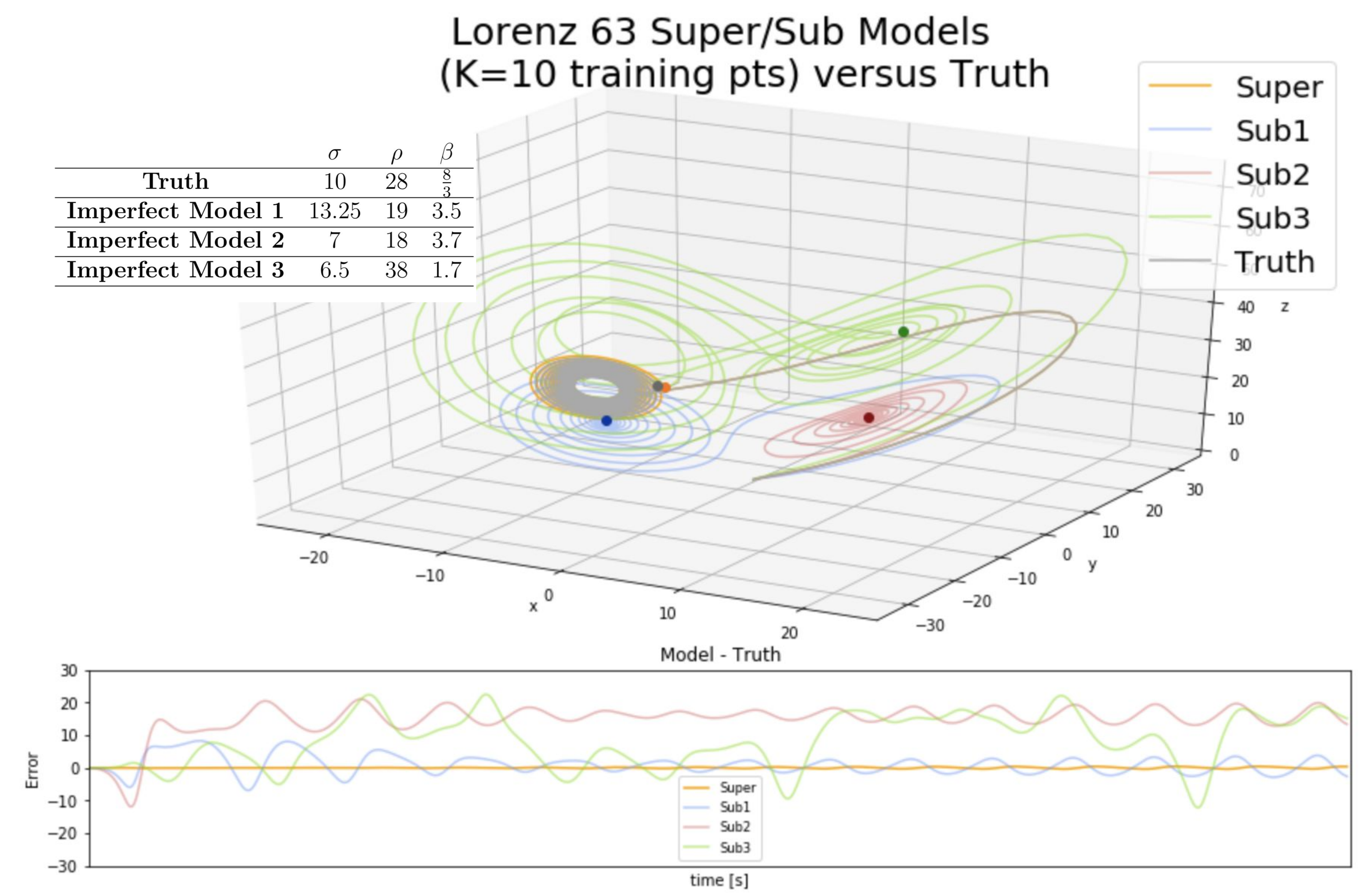


Figure 3: Minimization of Cost Function F with respect to a connection coefficient C_x^{31}. The cost function is minimized with respect to every connection coefficient C_k^{ij} using the Conjugate Gradient algorithm [3]



	σ	ρ	β
Truth	10	28	$\frac{8}{3}$
Imperfect Model 1	13.25	19	3.5
Imperfect Model 2	7	18	3.7
Imperfect Model 3	6.5	38	1.7

Figure 4: Time evolution of two Lorenz 63 super-models, the imperfect submodels, and the data values. Error between each model and data shown in the bottom of the figure

Conclusions and Potential Applications

- Super-models far outperform traditional models of imperfect parameters for chaotic systems
- Super-models can be built from relatively small datasets, and are less chaotic than the systems they model
- Super-modeling can be used in climate forecasting [5], as well as other chaotic systems including polymer manufacturing and asteroid orbits

References

1. R. Fletcher, C. M. Reeves, Function minimization by conjugate gradients, *The Computer Journal*, Vol 7, Iss. 2, 1964, P 149–154
2. F. M. Selten, C. Schevenhoven, and G. Duane. Simulating climate with a synchronization-based supermodel. *Chaos: An Inter disciplinary Journal of Nonlinear Science*, 27(12):126903, 2017.
3. M. Shen, N. Keenlyside, B. C. Bhatt, and G. Duane. Role of atmosphere-ocean interactions in supermodeling the tropical pacific climate. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 27(12):126704, 2017.
4. L. A. van den Berge, F. Selten, W. Wiegerinck, and G. Duane. A multi-model ensemble method that combines imperfect models through learning. *EarthSystem Dynamics*, 2(1):161–177, 2011.
5. W.Wiegerinck, SNN Radboudn University, *Super-modeling: combining imperfect models by synchronization,2017.*

Acknowledgments

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Efficacy of Super-modeling in Climate Systems

Project Description

- Describe the motivations of your project.
- Summarize what is known in the literature in terms of relevant experiments or models (use brackets, e.g. [1], to refer to references).
- Explain the need for a model.
- Describe the goals of your project.
- Use short sentences.
- Each bullet should correspond to only one idea, concept or statement.

Scientific Challenges

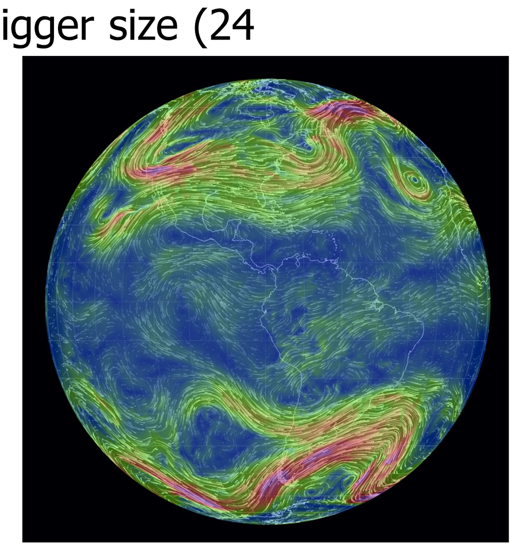
- Explain why it is important or interesting to develop such a model.

Potential Applications

- describe climate stuff

show the L63 model, and then the L63 super model ,
show the L63 time evolution that lourdes made

Use this font (Tahoma) and size (20 pts) for figure, table and diagram captions. The rest of the poster uses the same font, but with bigger size (24 pts).



Team Members:

List your team members here.
Use one line per name.
Use alphabetical order, unless you want to indicate that the amount of effort put into the project decreases as one goes down the list of names.

Methodology

1. Describe the steps taken to solve the problem. -> minimize cost fn w/ CG algo
2. For instance, you can say that you collected experimental data and analyzed it, or that you developed numerical simulations, or you can explain how you wrote down model equations. -> this is basically what we did
3. Use blue font color to single out terms that are defined in the Glossary of Technical Terms, or to indicate that more information is to be found elsewhere on the poster.
4. Use arrows to point to diagrams, tables, figures, or other sections of the poster.
5. Make sure to include all relevant references to the literature or the internet.

Results

1. Summarize your results. -> Show supermodel vs submodels and truth
2. quantize error, show supermodel is smaller, consider making ensemble model
3. Use arrows to point to figures, tables, diagrams, etc. -> put in FvsC plots to show minimizing conditions
4. Make sure proper credit is given to other people's work. In particular, if you use a figure from a research paper, clearly indicate the source in the figure caption.

Glossary of Technical Terms

Super-model Corresponding definition.
Submodel: Corresponding definition.
Connection Coefficient: Corresponding definition.

we can put out Fvs C or the time evolution of the system here

Dynamics of the Oregonator model [2], plotted with the software PPLANE [3].

References

1. Name of authors, *Title*, Journal **Volume**, first page number – last page number (year).
2. R.J. Field and R.M. Noyes, *Oscillations in chemical systems. IV. Limit cycle behavior in a model of a real chemical reaction*, J. Chem. Phys. **60**, 1877-1884 (1974).
3. PPLANE (http://math.rice.edu/~dfield/) is developed by John C. Polking, Department of Mathematics, Rice University.

Acknowledgments

This project was mentored by your mentor's name, whose help is acknowledged with great appreciation.