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(1) (a) Determine generalized inverse

$$A=\begin{bmatrix}0&0&1\\-1&1&1\\1&-1&1\end{bmatrix} \quad \det(A-\lambda I)=\begin{vmatrix}-\lambda&0&1\\-1&1-\lambda&1\\1&-1&1-\lambda\end{vmatrix}$$

eigenvalue $\lambda=0, 1, 1$

det A $=1-1=0$

$$=-\lambda(1-\lambda)^2+1-[(1-\lambda)+\lambda]$$
$$=-\lambda(1-\lambda)^2+\cancel{1}-\cancel{\lambda}-\cancel{1}+\cancel{\lambda}=0$$

$\lambda=0$

$$\begin{pmatrix}0&0&1\\-1&1&1\\1&-1&1\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \quad \begin{cases}a_3=0\\-a_1+a_2+a_3=0\\a_1-a_2+a_3=0\end{cases}$$

$\therefore a_1=a_2$ 取 $\begin{pmatrix}1\\1\\0\end{pmatrix}$

$\lambda=1$

$$\begin{pmatrix}0&0&1\\-1&1&1\\1&-1&1\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \quad \begin{cases}a_3=a_1\\-a_1+\cancel{a_2}+a_3=\cancel{a_2}\\a_1-a_2+\cancel{a_3}=\cancel{a_3}\end{cases}$$

$a_1=a_2=a_3$ 取 $\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\begin{pmatrix}-1&0&1\\-1&0&1\\1&-1&0\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix} \quad \begin{cases}-a_1+a_3=1\\-a_1+a_3=1\\a_1-a_2=1\end{cases}$$

取 $\begin{pmatrix}-1\\-2\\0\end{pmatrix}$

$$\therefore A=E\begin{pmatrix}0&0&0\\0&1&1\\0&0&1\end{pmatrix}E^{-1} \quad E=\begin{pmatrix}1&1&-1\\1&1&-2\\0&1&0\end{pmatrix} \quad E^{-1}=\begin{pmatrix}2&-1&-1\\0&0&1\\1&-1&0\end{pmatrix}$$

$$D^+=\begin{pmatrix}D_1^+&0&0\\0&D_2^+&0\\0&0&D_3^+\end{pmatrix}$$

$\lambda_1=0$

$D_1=0 \therefore D_1^+=0$

$\lambda=1$

$$D_2=\begin{pmatrix}1&1\\0&1\end{pmatrix} \quad D_k^+=\begin{pmatrix}1&-1\\0&1\end{pmatrix}$$

$$\therefore D^{+} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore A^{+} = E D^{+} E^{-1}$$

$$= \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & -2 \\ 0 & 1 & -3 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 2 & 1 \\ -3 & 3 & 1 \\ -1 & 1 & 1 \end{pmatrix} \#$$

(b) $B=\begin{bmatrix} -2 & 1 \\ -1 & -1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$

高斯消去法

$$\begin{pmatrix} -2 & 1 \\ -1 & -1 \\ 1 & 1 \\ 2 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 0 & 3 \\ 0 & -3 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\therefore \operatorname{rank}(B)=2$

$\therefore$ Col are indep.

$\therefore$ Case 2

$$A^{+}=(A^TA)^{-1}A^T$$

$-2+1+1-2$ $\quad$ $1+1+1+1$

$-2+1+1-2$

$$\left(\begin{bmatrix} -2 & -1 & 1 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -1 & -1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 10 & -2 \\ -2 & 4 \end{bmatrix}^{-1} = \frac{1}{36}\begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix}$$

$$= \begin{pmatrix} \frac{1}{9} & \frac{1}{18} \\ \frac{1}{18} & \frac{5}{18} \end{pmatrix}$$

$$\therefore B^{+} = \begin{pmatrix} \frac{1}{9} & \frac{1}{18} \\ \frac{1}{18} & \frac{5}{18} \end{pmatrix}\begin{bmatrix} -2 & -1 & 1 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{pmatrix} \frac{-2}{9}+\frac{1}{18} & \frac{-1}{9}+\frac{-1}{18} & \frac{1}{9}+\frac{1}{18} & \frac{2}{9}+\frac{-1}{18} \\ \frac{-2}{18}+\frac{5}{18} & \frac{-1}{18}+\frac{-5}{18} & \frac{1}{18}+\frac{5}{18} & \frac{2}{18}+\frac{-5}{18} \end{pmatrix} = \begin{pmatrix} -\frac{1}{6} & -\frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$$

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2.(a) Find SVD

$$A=\begin{bmatrix}1&1\\0&1\\1&0\end{bmatrix} \qquad C=\begin{bmatrix}1&1\\0&1\\1&0\end{bmatrix}\begin{bmatrix}1&0&1\\1&1&0\end{bmatrix}$$

Sol: $B=\begin{bmatrix}1&0&1\\1&1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\\1&0\end{bmatrix}=\begin{bmatrix}2&1\\1&2\end{bmatrix}$ $\qquad =\begin{bmatrix}2&1&1\\1&1&0\\1&0&1\end{bmatrix}$

$B=VDV^H$

$\left|\begin{pmatrix}2-\lambda&1\\1&2-\lambda\end{pmatrix}\right|=0 \quad (2-\lambda)^2-1=0 \Rightarrow (2-\lambda)^2=1 \quad 2-\lambda=\pm1 \quad \lambda=2\pm1$

$=3 \vee 1$

$Av=\lambda v$

$\begin{pmatrix}2&1\\1&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}=3\begin{pmatrix}a_1\\a_2\end{pmatrix}$ $\quad \begin{cases}2a_1+a_2=3a_1\\a_1+2a_2=3a_2\end{cases}$ $\quad \begin{matrix}a_1=a_2\\a_1=a_2\end{matrix}$ 取 $\begin{pmatrix}\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}\end{pmatrix}$

$\begin{pmatrix}2&1\\1&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}=1\begin{pmatrix}a_1\\a_2\end{pmatrix}$ $\quad \begin{cases}2a_1+a_2=a_1\\a_1+2a_2=a_2\end{cases}$ $\quad \begin{matrix}a_1=-a_2\\a_1=a_2\end{matrix}$ 取 $\begin{pmatrix}\frac{1}{\sqrt2}\\-\frac{1}{\sqrt2}\end{pmatrix}$

$\therefore V=\begin{pmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{pmatrix} \qquad D=\begin{bmatrix}3&0\\0&1\end{bmatrix}$

$\begin{vmatrix}2-\lambda&1&1\\1&1-\lambda&0\\1&0&1-\lambda\end{vmatrix}=(2-\lambda)(1-\lambda)^2-\left[(1-\lambda)+(1-\lambda)\right]=(2-\lambda)(1-\lambda)^2-2(1-\lambda)$

$=(1-\lambda)\left[(2-\lambda)(1-\lambda)-2\right]$

$\therefore \lambda=1 \qquad (2-\lambda)(1-\lambda)-2 \qquad \lambda=0 \vee 3$

$=2-3\lambda+\lambda^2-2$

$=\lambda^2-3\lambda=0$

$\lambda=3$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 3\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$2a_1+a_2+a_3=3a_1 \qquad a_1=a_2+a_3$$
$$a_1+a_2=3a_2 \qquad a_1=2a_2$$
$$a_1+a_3=3a_3 \qquad a_1=2a_3$$

$$\Rightarrow \begin{cases} a_2=a_3 \\ a_1=2a_2 \\ a_1=2a_3 \end{cases} \qquad \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \sqrt{6}\begin{pmatrix} \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$$

$\lambda=1$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \begin{cases} 2a_1+a_2+a_3=a_1 \\ a_1+\cancel{a_2}=\cancel{a_2} \\ a_1+\cancel{a_3}=\cancel{a_3} \end{cases}$$

$$a_1+a_2+a_3=0$$
$$a_2+a_3=-a_1$$

取 $a_1=0$

$$a_2=-a_3$$

取 $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \sqrt{2}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

$\lambda=0$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{cases} 2a_1+a_2+a_3=0 \\ a_1+a_2=0 \\ a_1+a_3=0 \end{cases}$$

$$a_1=-a_2 \qquad a_1=-a_3$$
$$-2a_2+a_2+a_3=0 \qquad a_2=a_3$$

取 $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = \sqrt{3}\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \end{pmatrix}$

$$\therefore \tilde{U} = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \end{pmatrix}$$

$$\Omega = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$S_1 = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}}_{\begin{pmatrix} \sqrt{2} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}}$$

$$= \begin{pmatrix} \frac{2}{\sqrt{3}} + \frac{1}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} \\ \frac{1}{2} - \frac{1}{2} & -\frac{1}{2} - \frac{1}{2} \\ \frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} \end{pmatrix} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\therefore S = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \because S_1[2,2] < 0$$

$$\therefore U = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \end{bmatrix} \qquad A = USV^H$$

$$S = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

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$$V^H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

(b) $B=\begin{pmatrix}1&1&1\\1&0&-1\end{pmatrix}$

$$B'=\begin{pmatrix}1&1\\1&0\\1&-1\end{pmatrix}\begin{pmatrix}1&1&1\\1&0&-1\end{pmatrix}=\begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}$$

$$\begin{vmatrix}2-\lambda&1&0\\1&1-\lambda&1\\0&1&2-\lambda\end{vmatrix}=(1-\lambda)(2-\lambda)^2-2(2-\lambda)=(2-\lambda)\Big[\overbrace{(1-\lambda)(2-\lambda)-2}^{\lambda^2-3\lambda}\Big]$$

$\lambda=2 \vee 3 \vee 0$

$$C=\begin{pmatrix}1&1&1\\1&0&-1\end{pmatrix}\begin{pmatrix}1&1\\1&0\\1&-1\end{pmatrix}=\begin{pmatrix}3&0\\0&2\end{pmatrix}\quad \begin{vmatrix}3-\lambda&0\\0&2-\lambda\end{vmatrix}=(3-\lambda)(2-\lambda)=0\quad \lambda=2\vee 3$$

$B'$:

$$\begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=3\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\quad \begin{cases}2a_1+a_2=3a_1\\a_1+a_2+a_3=3a_2\\a_2+2a_3=3a_3\end{cases}\quad \begin{matrix}a_1=a_2\\a_1+a_3=2a_2\Rightarrow a_1=a_3\\a_2=a_3\end{matrix}$$

取 $\sqrt{3}\begin{pmatrix}\frac{1}{\sqrt3}\\\frac{1}{\sqrt3}\\\frac{1}{\sqrt3}\end{pmatrix}$

$$\begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=2\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\quad \begin{cases}2a_1+a_2=2a_1\\a_1+a_2+a_3=2a_2\\a_2+2a_3=2a_3\end{cases}\quad \begin{matrix}a_2=0\\a_1=-a_3\\a_2=0\end{matrix}$$

取 $\sqrt{2}\begin{pmatrix}\frac{1}{\sqrt2}\\0\\-\frac{1}{\sqrt2}\end{pmatrix}$

$$\begin{pmatrix}2&1&0\\1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\quad \begin{cases}2a_1+a_2=0\\a_1+a_2+a_3=0\\a_2+2a_3=0\end{cases}\quad \begin{matrix}2a_1=-a_2\\a_1=-a_3\\a_2=-2a_3\end{matrix}$$

$$\begin{pmatrix}1\\-2\\1\end{pmatrix}=\sqrt{6}\begin{pmatrix}\frac{1}{\sqrt6}\\\frac{-2}{\sqrt6}\\\frac{1}{\sqrt6}\end{pmatrix}$$

$B'=VDV^H \quad B'=\hat{U}\Omega\hat{U}^H$

$$\underset{\tilde{U}}{V}=\begin{pmatrix}\frac{1}{\sqrt3}&\frac{1}{\sqrt2}&\frac{1}{\sqrt6}\\\frac{1}{\sqrt3}&0&-\frac{2}{\sqrt6}\\\frac{1}{\sqrt3}&-\frac{1}{\sqrt2}&\frac{1}{\sqrt6}\end{pmatrix}\quad \underset{\Omega}{D}=\begin{pmatrix}3&0&0\\0&2&0\\0&0&0\end{pmatrix}$$

C:

$$\begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}=3\begin{pmatrix}a_1\\a_2\end{pmatrix}\quad \begin{cases}3a_1=3a_1\\2a_2=3a_2\end{cases}\quad 取\begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}=2\begin{pmatrix}a_1\\a_2\end{pmatrix}\quad \begin{cases}3a_1=2a_1\\2a_2=2a_2\end{cases}\quad 取\begin{pmatrix}0\\1\end{pmatrix}$$

$\therefore C=\tilde{U}\Omega\tilde{U}^H \quad C=VDV^H$

$$\underset{U}{\tilde{U}}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\quad \underset{D}{\Omega}=\begin{pmatrix}3&0\\0&2\end{pmatrix}$$

$$\therefore \underset{2\times3}{S_1}=\tilde{U}^HBV=\underset{2\times2}{\begin{pmatrix}1&0\\0&1\end{pmatrix}}\underset{2\times3}{\begin{pmatrix}1&1&1\\1&0&-1\end{pmatrix}}\underset{3\times3}{\begin{pmatrix}\frac{1}{\sqrt3}&\frac{1}{\sqrt2}&\frac{1}{\sqrt6}\\\frac{1}{\sqrt3}&0&\frac{2}{\sqrt6}\\\frac{1}{\sqrt3}&-\frac{1}{\sqrt2}&\frac{1}{\sqrt6}\end{pmatrix}}$$

$$=\begin{pmatrix}\sqrt3&0&0\\0&\sqrt2&0\end{pmatrix}$$

$\therefore U = \tilde{U} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\qquad S = S_1 = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$

$\therefore A = USV^H$

$$V = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix} \#$$

(3) $A = USV^H \qquad A^+ = VS^+U^H$

Prove $(AA^+)^H = AA^+$

pf: $(USV^H VS^+U^H)^H = (USS^+U^H)^H = U(SS^+)^H U^H$

$= USS^+U^H$

$= USV^H V S^+ U^H$

$= AA^+ \#$

prove

$(A^+A)^H = A^+A$

pf. $(VS^+U^HUSV^H)^H = (VS^+SV^H)^H = V(S^+S)^H V^H$

$= VS^+SV^H$

$= VS^+U^HUSV^H$

$= A^+A \#$

(4) (a) Prove that $X+c$ has same var / ~~skewness~~ / kurtosis of $X$ if $c$ is const

sol for $X$

$$m_k = E[(X-\mu)^k]$$

for $X+c$ $\quad \mu' = \mu + c, \ \sigma' = \sigma$

$$\therefore Var = m_2 = E[(X-\mu)^2]$$

$$m_2' = E[(X+c-(\mu+c))^2] = E[(X-\mu)^2] = m_2$$

$\therefore$ same var #

$$\text{Skewness} = \frac{m_3}{\sigma^3} = \frac{E[(X-\mu)^3]}{\sigma^3}$$

$$\frac{m_3'}{\sigma'^3} = \frac{E[(X+c-(\mu+c))^3]}{\sigma^3} = \frac{E[(X-\mu)^3]}{\sigma^3} = \frac{m_3}{\sigma^3}$$

$\therefore$ skewness same #

$$\text{kurtosis} = \frac{m_4}{\sigma^4} = \frac{E[(X-\mu)^4]}{\sigma^4}$$

$$\frac{m_4'}{\sigma'^4} = \frac{E[(X+c-(\mu+c))^4]}{\sigma^4} = \frac{E[(X-\mu)^4]}{\sigma^4} = \frac{m_4}{\sigma^4}$$

$\therefore$ same kurtosis #

Central Moment

$$m_k = \int_{48}^{50} (x-50)^k \frac{x-48}{4}\,dx + \int_{50}^{52} (x-50)^k \frac{52-x}{4}\,dx$$

令 $t = x-50 \quad dt = dx$

(b) $\begin{cases} f_X(x) = \dfrac{2-|50-x|}{4} & 48<x<52 \\ f_X(x) = 0 & \text{ow} \end{cases}$

$$= \int_{-2}^{0} t^k \frac{t+2}{4}\,dt + \int_0^2 t^k \frac{2-t}{4}\,dt$$

$$= \int_{-2}^{0} \frac{1}{4} t^{k+1} + \frac{1}{2} t^k\,dt + \int_0^2 \frac{1}{2} t^k - \frac{1}{4} t^{k+1}\,dt$$

$$= \frac{1}{4(k+2)} t^{k+2} + \frac{1}{2(k+1)} t^{k+1} \Big|_{-2}^{0} + \frac{1}{2(k+1)} t^{k+1} - \frac{1}{4(k+2)} t^{k+2} \Big|_{0}^{2}$$

$$= -\left(\frac{(-2)^{k+2}}{4(k+2)} + \frac{(-2)^{k+1}}{2(k+1)}\right) + \frac{2^{k+1}}{2(k+1)} - \frac{2^{k+2}}{4(k+2)} \qquad (-1)^3$$

$$= \frac{2^{k+1}-(-2)^{k+1}}{2(k+1)} + \frac{-2^{k+2}-(-2)^{k+2}}{4(k+2)}$$

if $k \in$ odd

$$m_k = 2^k\left(0+0\right) = 0$$

$$= \frac{2^{k+1}\left(1-(-1)^{k+1}\right)}{2(k+1)} + \frac{2^{k+2}\left(-1-(-1)^{k+2}\right)}{4(k+2)}$$

if $k \in$ even

$$m_k = 2^k\left(\frac{2}{k+1} + \frac{-2}{k+2}\right)$$

$$= 2^{k+1}\left(\frac{1}{k+1} - \frac{1}{k+2}\right)$$

$$= 2^{k+1} \frac{1}{(k+1)(k+2)}$$

$$= \frac{2^{k+1}}{(k+1)(k+2)}$$

$$= \frac{2^k\left(1-(-1)^{k+1}\right)}{k+1} + \frac{2^k\left(-1-(-1)^{k+2}\right)}{k+2}$$

$$= 2^k\left[\frac{1-(-1)^{k+1}}{k+1} + \frac{-1-(-1)^{k+2}}{k+2}\right]$$

Moment

$\therefore V_k = \frac{m_k}{\sigma^k}$

$\therefore$ Variance $\quad m_2 = \frac{2^3}{3\cdot 4} = \frac{2}{3}$ # $\quad \therefore \sigma = \sqrt{\frac{2}{3}}$

Skewness

$$V_3 = \frac{m_3}{\sigma^3} = \frac{\frac{2^4}{4\times 5}}{\sqrt{\frac{2}{3}}^{\,3}} = \frac{\frac{16}{20}}{\frac{2\sqrt{2}}{3\sqrt{3}}} = \frac{\overset{6}{\cancel{12}}\,\cancel{48}\sqrt{3}}{\underset{1}{\cancel{5}}\,\cancel{40}\sqrt{2}}$$

$$= \frac{6\sqrt{6}}{10} = \frac{3}{5}\sqrt{6} \ \#$$

Kurtosis

$$V_4 = \frac{m_4}{\sigma^4} = \frac{\frac{2^5}{5\times 6}}{\frac{4}{9}} = \frac{\overset{8\ 4}{\cancel{32}}\times \overset{3}{\cancel{9}}}{\cancel{4}\times 5\times \cancel{6}\,\cancel{2}} = \frac{12}{5} \ \#$$