HW4 李[illegible]

1. $G(f)=0$ for $|f|\ge 2$ $\quad g(-\frac{1}{4})=g(\frac{1}{4})=2$ $\quad g(0)=3$

$g(-\frac{2}{4})=g(\frac{2}{4})=1$ $\quad g(\frac{n}{4})=0$ if $n\in\mathbb{Z}$, $|n|\ge 3$

determine $g(x)$

Sol: $\Delta x=\frac{1}{4}$

$$g(x)=\sum g_n \operatorname{sinc}\left(\frac{x}{\Delta x}-n\right)$$

$$=2\operatorname{sinc}\left(4x+\frac{1}{4}\right)+2\operatorname{sinc}\left(4x-\frac{1}{4}\right)$$

$$+\operatorname{sinc}\left(4x-\frac{2}{4}\right)+\operatorname{sinc}\left(4x+\frac{2}{4}\right)+3\operatorname{sinc}4x$$

#

2. $A=\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) $A \otimes \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2&0&0&0&1&0&0&0 \\ 2&1&0&0&2&1&0&0 \\ 0&0&2&0&0&0&0&0 \\ 0&0&4&2&0&0&0&0 \\ 0&0&0&0&2&0&1&0 \\ 0&0&0&0&4&2&2&1 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{bmatrix}$ #

(b) $A \otimes A \otimes A \otimes A$ rank

$\because$ If $\text{rank } A = c_1$ $\text{rank } B = c_2$ $\quad \text{rank } A \otimes B = c_1 c_2$

$\therefore \text{rank}(A \otimes A \otimes A \otimes A) = (\text{rank } A)^4 = 3^4 = 81$ #

$\begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$ rank $= 3$

(c) $\det(A-\lambda I) = 0$

$\det\begin{pmatrix} 2-\lambda & 0 & 1 & 0 \\ 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 0 & -\lambda \end{pmatrix} = (2-\lambda)\begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & -\lambda \end{vmatrix} + 1\begin{vmatrix} 0 & 2-\lambda & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\lambda \end{vmatrix}$

$= (2-\lambda)\left[-(2-\lambda)^2\lambda\right] + 0 = 0$

$-(2-\lambda)^3\lambda = 0 \quad \therefore \lambda = 0, 2, 2, 2$

$\lambda=0 \quad (A-0)v=0 \quad \begin{pmatrix} 2&0&1&0\\ 0&2&0&0\\ 0&0&2&1\\ 0&0&0&0 \end{pmatrix}\begin{pmatrix} k_1\\k_2\\k_3\\k_4 \end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

$2k_1+k_3=0 \quad 2k_1=-k_3$

$2k_2=0 \quad k_2=0$

$2k_3+k_4=0 \quad 2k_3=-k_4$

取 $k_1=1$, $k_3=-2$, $k_4=4$

eigenvector $\begin{pmatrix}1\\0\\-2\\4\end{pmatrix}$

$\lambda=2 \quad A-2I \Rightarrow \begin{pmatrix} 0&0&1&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0 \end{pmatrix}\begin{pmatrix} k_1\\k_2\\k_3\\k_4 \end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

$k_3=0$

$k_4=0$

$\begin{pmatrix}k_1\\k_2\\0\\0\end{pmatrix}=k_1\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+k_2\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

eigen vector $\underset{e_1}{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}} \quad \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

$(A-2I)e_2=e_1$

$\begin{pmatrix} 0&0&1&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0 \end{pmatrix}\begin{pmatrix} k_1\\k_2\\k_3\\k_4 \end{pmatrix}=\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad k_3=1 \quad k_4=0 \quad \begin{pmatrix}k_1\\k_2\\1\\0\end{pmatrix}$ 取 $k_1=k_2=0$ $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$

$\begin{pmatrix} 0&0&1&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0 \end{pmatrix}\begin{pmatrix} k_1\\k_2\\k_3\\k_4 \end{pmatrix}=\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad k_3=0$

無解

$$A=E\begin{bmatrix} 0&0&0&0\\ 0&2&0&0\\ 0&0&2&1\\ 0&0&0&2 \end{bmatrix}E^{-1}$$

$$E=\begin{pmatrix} 1&0&1&0\\ 0&1&0&0\\ -2&0&0&1\\ 4&0&0&0 \end{pmatrix} \quad E^{-1}=\begin{pmatrix} 0&0&0&\frac{1}{4}\\ 0&1&0&0\\ 1&0&0&-\frac{1}{4}\\ 0&0&1&\frac{1}{2} \end{pmatrix}$$

(d) $\|A\|_1: \; 2+2+2+1+1=8$

$\|A\|_2: \; \sqrt{3\cdot 2^2+2\cdot 1^2}=\sqrt{14}$

$\|A\|_\infty = 2$

$\|A\|_0 = \lim_{\alpha\to 0}\left(\|A\|_\alpha\right)^\alpha = 5$ ~~不~~

(3) $A=\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix}$ $\det(A-\lambda I)=\det\begin{pmatrix}1-\lambda&1&1\\0&1-\lambda&0\\0&0&1-\lambda\end{pmatrix}$ $(1-\lambda)^3=0$ $\lambda=1,1,1$

$(A-I)v=0$

eigenvector

$\begin{pmatrix}0&1&1\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}k_1\\k_2\\k_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $0k_1+k_2+k_3=0$ $\Rightarrow k_2=-k_3$ $\begin{pmatrix}0\\k_2\\-k_2\end{pmatrix}=k_2\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\underset{e_1}{\begin{pmatrix}1\\0\\0\end{pmatrix}}$

或取 $k_1=1$ $k_2=k_3=0$

$(A-I)e_2=e_{1a}$

$\begin{pmatrix}0&1&1\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}k_1\\k_2\\k_3\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}$ $0+k_2+k_3=1$ 取 $k_2=1,\; k_3=0$ $\begin{pmatrix}0\\1\\0\end{pmatrix}$

$\therefore A=EDE^{-1}$

$\begin{pmatrix}e&e&\frac{e}{2}\\0&e&e\\0&0&e\end{pmatrix}$ //

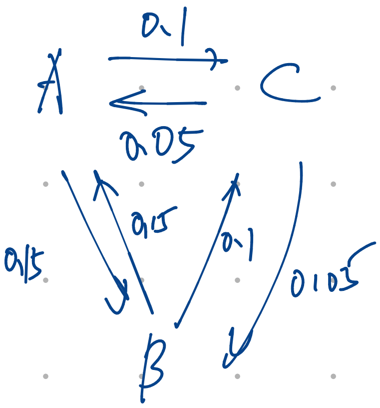
$D=\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix}$ $\therefore \exp A = E\exp(D)E^{-1}$

$E=\begin{pmatrix}0&1&0\\1&0&1\\-1&0&0\end{pmatrix}$ $E^{-1}=\begin{pmatrix}0&0&1\\1&0&0\\0&1&1\end{pmatrix}$

$$\therefore \exp(A) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} e & e & \frac{e}{2} \\ 0 & e & e \\ 0 & 0 & e \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} e & e & \frac{3}{2}e \\ e & e & \frac{3}{2}e \\ -e & -e & -\frac{1}{2}e \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} e & \frac{3}{2}e & \frac{5}{2}e \\ e & \frac{3}{2}e & \frac{5}{2}e \\ -e & -\frac{1}{2}e & -\frac{3}{2}e \end{pmatrix} \#$$

(4) $A = \begin{bmatrix} 0.75 & 0.15 & 0.05 \\ 0.15 & 0.75 & 0.05 \\ 0.1 & 0.1 & 0.9 \end{bmatrix}$ $\quad X = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$



(a) $$A^4 \begin{pmatrix} 400000 \\ 300000 \\ 300000 \end{pmatrix} = \begin{pmatrix} 297440 \\ 284480 \\ 418080 \end{pmatrix} \#$$

(b) $$\begin{pmatrix} 0.75 & 0.15 & 0.05 \\ 0.15 & 0.75 & 0.05 \\ 0.1 & 0.1 & 0.9 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{3}{5} & 0 & 0 \\ 0 & \frac{4}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\therefore t \to \infty \quad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore \lim_{t \to \infty} X(t) = \begin{pmatrix} -1 & -\frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 400000 \\ 300000 \\ 300000 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 4\times10^5 \\ 3\times10^5 \\ 3\times10^5 \end{pmatrix}$$

1.5

$$= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 4\times10^5 \\ 3\times10^5 \\ 3\times10^5 \end{pmatrix} = \begin{pmatrix} 10^5 + \frac{3}{4}\times10^5 + \frac{3}{4}\times10^5 \\ 10^5 + \frac{3}{4}\times10^5 + \frac{3}{4}\times10^5 \\ 2\times10^5 + 1.5\times10^5 + 1.5\times10^5 \end{pmatrix}$$

$$= \begin{pmatrix} 2.5\times10^5 \\ 2.5\times10^5 \\ 5\times10^5 \end{pmatrix} \#$$