Homework 1 (Due: March 16th)

(1) Find the solutions of the following nonlinear DEs. (30 scores)

(a) $x^3 y''(x) - \left(y'(x)\right)^2 = 0 \qquad y'(1) = 2$

(b) $y''(x) = \left(y'(x)\right)^3 y(x) \qquad y(0) = 2, \quad y'(0) = -1/2$

(c) $y'(x) - xy + y''(x) = 0, \quad y(0) = 0, \quad y'(0) = 1$

(2) Solve the following PDEs. (30 scores)

(a) $x\dfrac{\partial u(x,y)}{\partial x} + y^2 \dfrac{\partial^2 u(x,y)}{\partial y^2} + y\dfrac{\partial u(x,y)}{\partial y} = 0$

(b) $\dfrac{\partial^2 u(x,t)}{\partial x^2} = 9\dfrac{\partial u(x,t)}{\partial t}, \qquad 0 < x < 2, \ t > 0, \qquad u(x,0) = \begin{cases} x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$

$u(0,t) = u(2,t) = 0$

(c) $\dfrac{\partial^2 u(x,y)}{\partial x^2} + \dfrac{\partial^2 u(x,y)}{\partial y^2} = 0, \qquad 0 < x < a, \quad 0 < y < b$

$\left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0, \quad \left.\dfrac{\partial u}{\partial x}\right|_{x=a} = 0, \quad u(x,0) = x, \quad u(x,b) = 0$

(3) Solve the following nonhomogeneous PDE. (10 scores)

$$u(x,t)+\frac{\partial u(x,t)}{\partial x}+x+t=\frac{\partial u(x,t)}{\partial t}$$

(4) Solve the following 1st order nonlinear DE numerically. The codes should be handed out by Ceiba. (30 scores)

$$\frac{\partial y(x)}{\partial x}=\sqrt{y}\exp(-0.1x^2),\quad y(0)=1,\quad 0\le x\le 10,\quad x_{n+1}-x_n=0.05$$

(a) By Euler's method.

(b) By modified Euler's method.

(c) By the RK4 method.