Selected Topics in Engineering Mathematics Finals

(2 pages)

1. Solve the following nonlinear DE: (8 scores)

$$y''(x) = \left(y'(x)\right)^2$$

2. Solve the following PDEs (24 scores)

(a) $\frac{\partial^2}{\partial x^2}u(x,y) + \frac{\partial^2}{\partial y^2}u(x,y) = 0$, $0 < x < 5$, $0 < y < 5$

$u(0,y) = u(5,y) = 0$, $\left.\frac{\partial}{\partial y}u(x,y)\right|_{y=0} = 0$, $u(x,0) = \sin\left(\frac{2\pi}{5}x\right)\cos\left(\frac{\pi}{5}x\right)$.

(b) $\frac{\partial^2 u(x,y,z)}{\partial x^2} + \frac{\partial^2 u(x,y,z)}{\partial y^2} + \frac{\partial^2 u(x,y,z)}{\partial z^2} = 0$, $0 < x < 1$, $0 < y < 1$, $0 < z < \infty$,

$u(0,y,z) = u(1,y,z) = u(x,0,z) = u(x,1,z) = u(x,y,0) = 0$

(just find the general solution)

(c) $\frac{\partial^2 u(x,y)}{\partial x^2} = \frac{\partial u(x,y)}{\partial y} + y$, $0 < x < 1$, $0 < y < \infty$

$\left.\frac{\partial}{\partial x}u(x,y)\right|_{x=0} = \left.\frac{\partial}{\partial x}u(x,y)\right|_{x=1} = 0$ (just find the general solution)

3. (a) Try to approximate (14 scores)

$$y(x) = \begin{cases} 1-|x| & -2 < x < 2 \\ 0 & otherwise \end{cases}$$

by $c_0 + c_1x + c_2x^2$ such that $\left\|y(x) - c_0 - c_1x - c_2x^2\right\| = \sqrt{\int_{-2}^{2}\left(y(x) - c_0 - c_1x - c_2x^2\right)^2 dx}$ is minimal.

(b) Try to approximate $\mathbf{z}$ = [3, 1, 3, 5, 3, 5] by $d_0\mathbf{b_0} + d_1\mathbf{b_1} + d_2\mathbf{b_2}$ where

$\mathbf{b_0}$ = [1, 1, 1, 1, 1, 1], $\mathbf{b_1}$ = [1, -1, 1, -1, 1, -1], $\mathbf{b_2}$ = [1, 2, 3, 4, 5, 6],

such that $\left\|\mathbf{z} - d_0\mathbf{b_0} - d_1\mathbf{b_1} - d_2\mathbf{b_2}\right\|$ is minimal.

4. Determine (15 scores)

(a) $\mathcal{F}\left[\exp\left(-2\pi x^2\right)\cos(6\pi x)\right]$ where $\mathcal{F}$ means the Fourier transform.

(b) $\exp\left(-\pi x^2\right) * \exp\left(-2\pi x^2\right) * \exp\left(-3\pi x^2\right)$ where * means the convolution. (*Cont.*)

(c) $\exp\left(-x^2\right)*\delta\left(x+1\right)*\delta\left(2x\right)*\delta\left(3x\right)$ where * means the convolution

5. Suppose that $G(f)$ is the Fourier transform of $g(t)$ and $G(f) = 0$ for $|f| \geq 1$. If

$g(-1) = g(-1/2) = 1$, $g(0) = 2$, $g(1/2) = g(1) = 3$,

and $g(n/2) = 0$ if n is an integer and $|n| > 2$,

try to determine $g(t)$. (6 scores)

6. Suppose that (15 scores)

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) Determine the Jordan-canonical form of $\mathbf{A}$.

(b) Determine $\mathbf{A}^{100}$.

(c) Determine $\lim_{\alpha\to 0}\left(\|\mathbf{A}\|_\alpha\right)^\alpha$, $\|\mathbf{A}\|_1$, $\|\mathbf{A}\|_2$, and $\|\mathbf{A}\|_\infty$.

7. Determine the SVD of (6 scores)

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

8. Suppose that (12 scores)

$P_X\left(1\right) = 2/5$, $P_X\left(2\right) = P_X\left(3\right) = P_X\left(4\right) = 1/5$, $P_X\left(n\right) = 0$ otherwise,

$P_Y\left(1\right) = P_Y\left(2\right) = P_Y\left(3\right) = P_Y\left(4\right) = P_Y\left(5\right) = 1/5$, $P_Y\left(n\right) = 0$ otherwise.

(a) Determine the entropy of X and the entropy of Y (express the solution in terms of ln).

(b) Determine the KL divergence from $P_Y(n)$ to $P_X(n)$ (express the solution in terms of ln).

答題完成後，請將答案的電子檔 (打字，手寫後掃描，或手寫後拍照皆可，但是要清楚) 繳交至 ceiba。

雖計算過程不需一步一步寫，但不能完全沒有計算過程

繳交期限：6 月 22 日下午 9 點