Table 1. A brief description of the datasets

	Release	Description	Sparsity	Source
	Date		Level	
Film Trust	06/2011	1508 users with	98.863%	https://guoguibing.github.io/librec/datasets.ht
		35,497 ratings on		ml?fbclid=IwAR3E87ZfBAnjEwb1ykzY2_7
		2071 movies		Zwy_4jr4-
				BrR4u99A3oADaCjVHbmg9IKOMrg
Movielens 100-K	2/2003	1000 users with	94.118%	https://group.lens.org/datas.ets/movie.lens/
		100,000 ratings		
		on 1700 movies		
Movielens 1 M	2/2003	6040 users with	95.754%	https://group.lens.org/datas.ets/movie.lens/
		1000,209 ratings		
		on 3900 movies		

We followed the same procedure used in both sub-sections (6.1. and 6.2.) using the best nine similarity measures, which are obtained as a results of initial evaluation in sections 6.1 and 6.2. Three more datasets (see Table 10) with several K values (5, 20, 50 and 100) for neighborhood using KNN under two r values (0.7 and 0.9) in which the sparisty of each datasets is stressed. Experiments have been conducted on: (1) the Film Trust dataset, as one of the most sparse datasets, with K [5, 20, 50 and 100], (2) the recent version of Movielens-100k with K [5, 20 and 50], and (3) Movielens-1M with k [5, 20]. Tables (2-13) hold the results which are secured from these experiments. Our measures are still shown in the top particularly on MAE, MSE and R metrics. It is worth indicating that the top-performer list includes four similarity measures out of the best ninth measures. In next Tables, the AVG symbol stands for the averaged results of both r values (0.7 and 0.9). The detailed results of all datasets regarding all metrics are uploaded on Github¹.

Table 2- MAE - Movielens-100K

		r=	0.7			r=	0.9		AVG (0.7, 0.9)
	k=5	k=20	k=50	AVG	k=5	k=20	k=50	AVG	
Cosine	0.7736	0.7733	0.7737	0.7735	0.8473	0.8478	0.8477	0.8476	0.8106
CON	0.7889	0.7900	0.7899	0.7896	0.9069	0.9061	0.9056	0.9062	0.8479
Pearson	0.8091	0.8099	0.8094	0.8095	0.9161	0.9182	0.9175	0.9173	0.8634
MSDJ	0.7709	0.7707	0.7711	0.7709	0.8538	0.8535	0.8538	0.8537	0.8123
NHSM	0.7728	0.7726	0.7730	0.7728	0.8729	0.8720	0.8725	0.8725	0.8226
PIP	0.7726	0.7726	0.7728	0.7727	0.8723	0.8720	0.8723	0.8722	0.8224
TA	0.7726	0.7724	0.7727	0.7726	0.8487	0.8487	0.8488	0.8487	0.8107
TAJ	0.7704	0.7704	0.7707	0.7705	0.8552	0.8546	0.8551	0.8550	0.8127
SMD	0.7737	0.7735	0.7739	0.7737	0.8253	0.8241	0.8241	0.8245	0.7991

Table 3- MSE - Movielens-100K

		r=	0.7			r=		AVG (0.7, 0.9)	
	k=5	k=20	k=50	AVG	k=5	k=20	k=50	AVG	

¹ <u>https://github.com/aliamer/Enhancing-Recommendation-Systems-Performance-Using-Highly-Effective-Similarity-Measures</u>

Cosine	0.9681	0.9693	0.9698	0.9691	1.1801	1.1816	1.1818	1.1812	1.0751
CON	1.0019	1.0061	1.0059	1.0046	1.3392	1.3371	1.3358	1.3374	1.1710
Pearson	1.0438	1.0470	1.0468	1.0459	1.3464	1.3519	1.3492	1.3492	1.1975
MSDJ	0.9638	0.9651	0.9656	0.9648	1.1959	1.1957	1.1970	1.1962	1.0805
NHSM	0.9709	0.9718	0.9723	0.9717	1.2484	1.2470	1.2485	1.2480	1.1098
PIP	0.9740	0.9752	0.9754	0.9749	1.2526	1.2523	1.2534	1.2528	1.1138
TA	0.9657	0.9671	0.9675	0.9668	1.1833	1.1843	1.1846	1.1841	1.0754
TAJ	0.9627	0.9642	0.9645	0.9638	1.1993	1.1989	1.2003	1.1995	1.0817
SMD	0.9673	0.9682	0.9690	0.9682	1.1131	1.1084	1.1077	1.1097	1.0390

Table 4- R - Movielens-100K

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		r=	0.7			r=		AVG (0.7, 0.9)	
	k=5	k=20	k=50	AVG	k=5	k=20	k=50	AVG	
Cosine	0.3580	0.3578	0.3586	0.3581	0.2315	0.2269	0.2285	0.2290	0.2936
CON	0.2879	0.2843	0.2854	0.2859	0.1232	0.1230	0.1250	0.1237	0.2048
Pearson	0.1931	0.1887	0.1895	0.1904	0.0583	0.0603	0.0589	0.0592	0.1248
MSDJ	0.3539	0.3537	0.3546	0.3541	0.2230	0.2191	0.2199	0.2207	0.2874
NHSM	0.3449	0.3436	0.3450	0.3445	0.2102	0.2063	0.2074	0.2080	0.2762
PIP	0.3529	0.3517	0.3527	0.3524	0.2128	0.2102	0.2120	0.2117	0.2821
TA	0.3613	0.3606	0.3613	0.3611	0.2307	0.2270	0.2285	0.2287	0.2949
TAJ	0.3554	0.3550	0.3558	0.3554	0.2224	0.2186	0.2195	0.2202	0.2878
SMD	0.3619	0.3620	0.3619	0.3619	0.3008	0.3060	0.3045	0.3038	0.3329

Table 5- F1 - Movielens-100K

	r=0.7		
	precision -	Recall-	AVG
	AVG	AVG	
Cosine	0.0396	0.9071	0.075887
CON	0.0384	0.9079	0.073684
Pearson	0.0358	0.919067	0.068916
MSDJ	0.0399	0.907167	0.076438
NHSM	0.0402	0.904033	0.076977
PIP	0.0402	0.9071	0.076988
TA	0.0397	0.906567	0.076069
TAJ	0.04	0.9066	0.076619
SMD	0.037633	0.9082	0.072272

	r=0.9			AVG
	Precision -	Recall-	AVG	(0.7, 0.9)
	AVG	AVG		
Cosine	0.083733	0.801633	0.151629	0.113758
CON	0.0912	0.7329	0.162214	0.117949
Pearson	0.080433	0.7831	0.145883	0.107399
MSDJ	0.0837	0.7968	0.151487	0.113963
NHSM	0.083133	0.784267	0.150331	0.113654
PIP	0.0834	0.7876	0.150829	0.113908
TA	0.083767	0.800267	0.151659	0.113864
TAJ	0.083667	0.7955	0.151409	0.114014
SMD	0.055	0.8724	0.103476	0.087874

From Tables 2-5, the rank of best similarity measures on Movielens-100k has been as follows; TA, MSDJ, SMD, Cosine, NHSM and PIP. SMD and TA were the top performers with regard to the estimation process. On the other hand, TA, NHSM and MSDJ were the top performers in terms of the recommendation process. Tables 6-8 show the results of MAE, MSE and R metrics on Movielens-1 M dataset.

Table 6- MAE - Movielens-1 M

		r=0.7			r=0.9		AVG (0.7, 0.9)
	k=5	k=20	AVG	k=5	k=20	AVG	
Cosine	0.7400	0.7394	0.7397	0.7675	0.7672	0.7674	0.7951
CON	0.7452	0.7447	0.7450	0.8032	0.8034	0.8033	0.7516
Pearson	0.7610	0.7605	0.7608	0.8294	0.8293	0.8294	0.7548
MSDJ	0.7339	0.7332	0.7336	0.7698	0.7696	0.7697	0.7527
NHSM	0.7305	0.7298	0.7302	0.7794	0.7793	0.7794	0.7531
PIP	0.7311	0.7304	0.7308	0.7747	0.7746	0.7747	0.7516
TA	0.7392	0.7385	0.7389	0.7674	0.7672	0.7673	0.7531
TAJ	0.7334	0.7327	0.7331	0.7702	0.7700	0.7701	0.7951
SMD	0.7408	0.7401	0.7405	0.7658	0.7655	0.7657	0.7516

Table 7- MSE - Movielens-1 M

		r=0.7			r=0.9		AVG (0.7, 0.9)
	k=5	k=20	AVG	k=5	k=20	AVG	
Cosine	0.8848	0.8824	0.8836	0.9644	0.9642	0.9643	0.9240
CON	0.8891	0.8871	0.8881	1.0468	1.0474	1.0471	0.9676
Pearson	0.9191	0.9168	0.9180	1.0950	1.0941	1.0946	1.0063
MSDJ	0.8716	0.8692	0.8704	0.9674	0.9671	0.9673	0.9188
NHSM	0.8641	0.8617	0.8629	0.9895	0.9893	0.9894	0.9262
PIP	0.8729	0.8703	0.8716	0.9879	0.9880	0.9880	0.9298
TA	0.8827	0.8803	0.8815	0.9641	0.9639	0.9640	0.9228
TAJ	0.8702	0.8678	0.8690	0.9680	0.9677	0.9679	0.9184
SMD	0.8852	0.8827	0.8840	0.9511	0.9503	0.9507	0.9173

Table 8- R - Movielens-1 M

		r=0.7			r=0.9		AVG (0.7, 0.9)
	k=5	k=20	AVG	k=5	k=20	AVG	
Cosine	0.4276	0.4286	0.4281	0.3581	0.3578	0.3580	0.3930
CON	0.4138	0.4144	0.4141	0.2569	0.2563	0.2566	0.3354
Pearson	0.3310	0.3331	0.3321	0.1416	0.1419	0.1418	0.2369
MSDJ	0.4313	0.4323	0.4318	0.3447	0.3440	0.3444	0.3881
NHSM	0.4302	0.4315	0.4309	0.3239	0.3233	0.3236	0.3772
PIP	0.4337	0.4351	0.4344	0.3396	0.3387	0.3392	0.3868
TA	0.4299	0.4310	0.4305	0.3583	0.3578	0.3581	0.3943
TAJ	0.4326	0.4337	0.4332	0.3441	0.3434	0.3438	0.3885
SMD	0.4280	0.4288	0.4284	0.4061	0.4064	0.4063	0.4173

From Tables 6-8, the rank of best similarity measures on.Movielens-1 M has been as follows; SMD, TA, MSDJ, TAJ, Cosine, PIP. Tables 9-13 draw results of Film Trust evaluation in terms of MAE, MSE, R, Recall and F1.

Table 9- MAE - Film Trust

			r=0.7					r=0.9			AVG
	k=5	k=20	k=50	k=100	AVG	k=5	k=20	k=50	k=100	AVG	(0.7, 0.9)
Cosine	0.6364	0.6368	0.6370	0.6367	0.6367	0.6837	0.6845	0.6868	0.6846	0.6849	0.6608
CON	0.6457	0.6460	0.6466	0.6460	0.6461	0.6923	0.6939	0.6970	0.6945	0.6944	0.6703
Pearson	0.6591	0.6602	0.6611	0.6599	0.6601	0.7073	0.7087	0.7079	0.7064	0.7076	0.6838
MSDJ	0.6363	0.6366	0.6369	0.6365	0.6366	0.6835	0.6847	0.6868	0.6847	0.6849	0.6608
NHSM	0.6375	0.6378	0.6380	0.6378	0.6378	0.6872	0.6889	0.6909	0.6886	0.6889	0.6633
PIP	0.6365	0.6373	0.6374	0.6371	0.6371	0.6951	0.6944	0.6977	0.6953	0.6956	0.6664
TA	0.6362	0.6365	0.6368	0.6364	0.6365	0.6833	0.6843	0.6865	0.6841	0.6846	0.6605
TAJ	0.6364	0.6366	0.6369	0.6366	0.6366	0.6838	0.6853	0.6874	0.6851	0.6854	0.6610
SMD	0.6419	0.6405	0.6415	0.6412	0.6413	0.6803	0.6846	0.6868	0.6849	0.6842	0.6627

Table 10- MSE - Film Trust

			r=0	0.7				r=0	0.9		AVG
	k=5	k=20	k=50	k=100	AVG	k=5	k=20	k=50	k=100	AVG	(0.7, 0.9)
Cosine	0.7073	0.7095	0.7082	0.7074	0.7081	0.8179	0.8187	0.8229	0.8190	0.8196	0.7639
CON	0.7183	0.7194	0.7190	0.7173	0.7185	0.8207	0.8241	0.8297	0.8253	0.8250	0.7717
Pearson	0.7288	0.7310	0.7320	0.7290	0.7302	0.8240	0.8259	0.8250	0.8227	0.8244	0.7773
MSDJ	0.7068	0.7093	0.7078	0.7070	0.7077	0.8148	0.8160	0.8200	0.8161	0.8167	0.7622
NHSM	0.7091	0.7119	0.7103	0.7095	0.7102	0.8231	0.8251	0.8289	0.8244	0.8254	0.7678
PIP	0.7044	0.7076	0.7062	0.7051	0.7058	0.8327	0.8301	0.8372	0.8319	0.8330	0.7694
TA	0.7054	0.7074	0.7062	0.7053	0.7061	0.8131	0.8142	0.8180	0.8136	0.8147	0.7604
TAJ	0.7061	0.7084	0.7069	0.7061	0.7069	0.8138	0.8155	0.8193	0.8151	0.8159	0.7614
SMD	0.7203	0.7183	0.7190	0.7184	0.7190	0.8215	0.8283	0.8332	0.8299	0.8282	0.7736

Table 11- R - Film Trust

Table 11- N-1 IIII 11 USt												
				r=0.7				r=0.9			AVG	
	k=5	k=2	0	k=50	k=100	AVG	k=5	k=20	k=50	k=100	AVG	(0.7, 0.9)
Cosine	0.1938	0.19	959	0.1963	0.1976	0.1959	0.0962	0.1036	0.1030	0.1032	0.1015	0.1487
CON	0.1671	0.15	584	0.1545	0.1579	0.1595	0.0545	0.0551	0.0535	0.0537	0.0542	0.1068
Pearson	0.0904	0.08	328	0.0807	0.0834	0.0843	0.0253	0.0212	0.0187	0.0201	0.0213	0.0528
MSDJ	0.1998	0.19	88	0.1995	0.2012	0.1998	0.0948	0.1042	0.1024	0.1031	0.1011	0.1505
NHSM	0.1992	0.19	973	0.1981	0.2002	0.1987	0.0923	0.0974	0.0979	0.0988	0.0966	0.1477
PIP	0.2005	0.19	978	0.1980	0.1999	0.1991	0.0840	0.0919	0.0901	0.0916	0.0894	0.1442
TA	0.1986	0.20	001	0.2004	0.2022	0.2003	0.0979	0.1053	0.1043	0.1056	0.1033	0.1518
TAJ	0.2014	0.20	001	0.2013	0.2033	0.2015	0.0971	0.1047	0.1025	0.1036	0.1020	0.1518

SMD	0.2048	0.2087	0.2059	0.2064	0.2065	0.1621	0.1628	0.1656	0.1662	0.1642	0.1853	
				0.2004	0.2003				0.1002	0.1042	0.1033	

Table 12- Recall - Film Trust

		r=0.7				r=0.9				AVG	
	k=5	k=20	k=50	k=100	AVG	k=5	k=20	k=50	k=100	AVG	(0.7, 0.9)
Cosine	0.8512	0.8531	0.8525	0.8523	0.8523	0.7406	0.7378	0.7379	0.7383	0.7387	0.7955
CON	0.8502	0.8525	0.8515	0.8516	0.8515	0.7353	0.7328	0.7329	0.7330	0.7335	0.7925
Pearson	0.8494	0.8510	0.8504	0.8505	0.8503	0.7192	0.7171	0.7196	0.7186	0.7186	0.7845
MSDJ	0.8514	0.8529	0.8527	0.8524	0.8524	0.7406	0.7378	0.7380	0.7385	0.7387	0.7955
NHSM	0.8492	0.8506	0.8506	0.8503	0.8502	0.7360	0.7333	0.7333	0.7339	0.7341	0.7922
PIP	0.8539	0.8549	0.8544	0.8543	0.8544	0.7373	0.7359	0.7355	0.7362	0.7362	0.7953
TA	0.8513	0.8530	0.8526	0.8523	0.8523	0.7406	0.7374	0.7376	0.7382	0.7385	0.7954
TAJ	0.8511	0.8525	0.8524	0.8521	0.8520	0.7398	0.7367	0.7370	0.7374	0.7377	0.7949
SMD	0.8757	0.8784	0.8772	0.8772	0.8771	0.8065	0.7996	0.8008	0.8007	0.8019	0.8395

Table 13- F1 - Film Trust

	r=0.7				
	Precision - AVG	Recall - AVG	F1 - AVG		
Cosine	0.0256	0.8523	0.049707		
CON	0.026	0.8515	0.050459		
Pearson	0.0273	0.8503	0.052902		
MSDJ	0.0256	0.8524	0.049707		
NHSM	0.0256	0.8502	0.049703		
PIP	0.0258	0.8544	0.050087		
TA	0.0256	0.8523	0.049707		
TAJ	0.0256	0.8520	0.049707		
SMD	0.0142	0.8771	0.027948		

	r=0.9	AVG		
	Precision - AVG	Recall - AVG	F1 - AVG	(0.7, 0.9)
Cosine	0.1867	0.7387	0.298066	0.173887
CON	0.1895	0.7335	0.301188	0.175824
Pearson	0.2098	0.7186	0.324779	0.18884
MSDJ	0.1867	0.7387	0.298066	0.173887
NHSM	0.1861	0.7341	0.296927	0.173315
PIP	0.1865	0.7362	0.297608	0.173848
TA	0.1867	0.7385	0.29805	0.173879
TAJ	0.1866	0.7377	0.297857	0.173782
SMD	0.0398	0.8019	0.075836	0.051892

In conclusion of the drawn-above Tables, the final rank for the best sixth measures are: TA, SMD, MSDJ, PIP, NHSM and Cosine. Final Rank gives a priority for those measures whose values are the highest with both values of r in general, and r=0.9 in particular. That is because of that this value of r (r=0.9) reflects the highest level of sparsity of each datasets.