6.0 Roof

6.1 Functional Framing

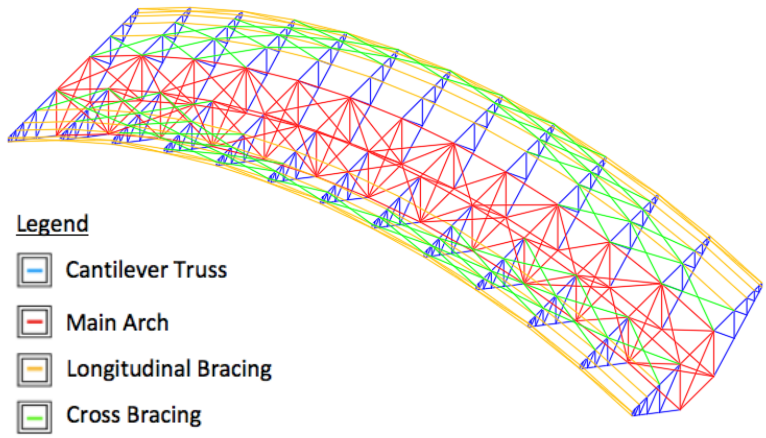


Figure 6.1 Functional framing of the roof

6.2 Study of Connection Conditions

The support conditions of the roof structure play a critical role in how the structure behaves and were a key decision in the final design. A study was carried out to determine which support conditions would be most suitable, with three scheme options considered:

- Near pin-pin supports utilizing stiff concrete cores at both end of the arch
- Pin-roller supports
- Use of a cable in tension to tie the arch

If the roof truss is pinned at one end and supported by a roller at the other, it will behave like a beam, with the critical members being those located at the centre of the roof where the maximum bending moment occurs. Alternatively, if the support conditions are pinned-pinned, the structure will behave as an arch acting entirely in compression. This results in the maximum axial force occurring in those members adjacent to the supports. Tying the arch will result in action somewhere between these extremes.

A fully pin-pin support would be unachievable given that the concrete cores to which the arch would attach would not be infinitely stiff, hence it was important to model the situation in-between the two extreme support conditions to assess each option. This was done by using the tied arch formulae shown in Equations 6.1 & 6.2 were used to provide a quick way to calculate the horizontal forces and bending moment in a tied arch. By varying the stiffness of the tie in the equation, the spectrum of support conditions from pin-roller to pin-pin could be simply modelled, and conclusions drawn about which configuration would be the most cost effective, considering the tonnage of the roof, core supports and foundations.

$$H_A = H_c = \frac{wL^2}{8f}\left(\frac{1}{1+\left(\frac{15}{8}\frac{\frac{EI}{E_T A_T}}{f^2}\right)}\right) \qquad M_B = \frac{wL^2}{8}\left(1-\frac{1}{1+\left(\frac{15}{8}\frac{\frac{EI}{E_T A_T}}{f^2}\right)}\right) \qquad (Eq\ 6.1\ \&\ 6.2)$$

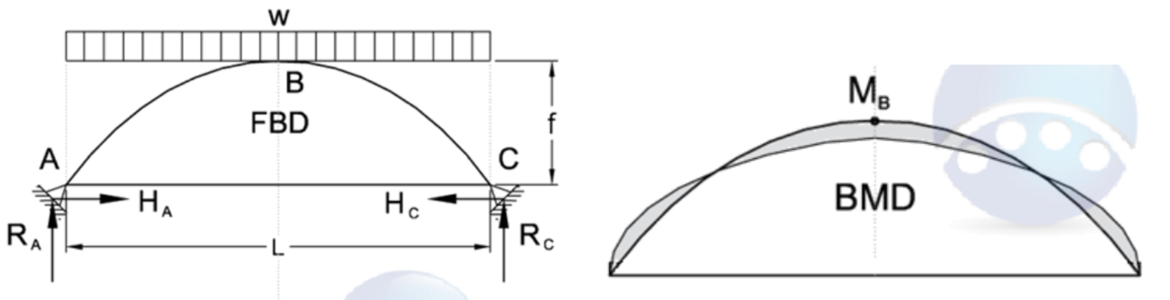


Figure 6.2 Bending moment diagram for a tied arch (StructX, 2015)

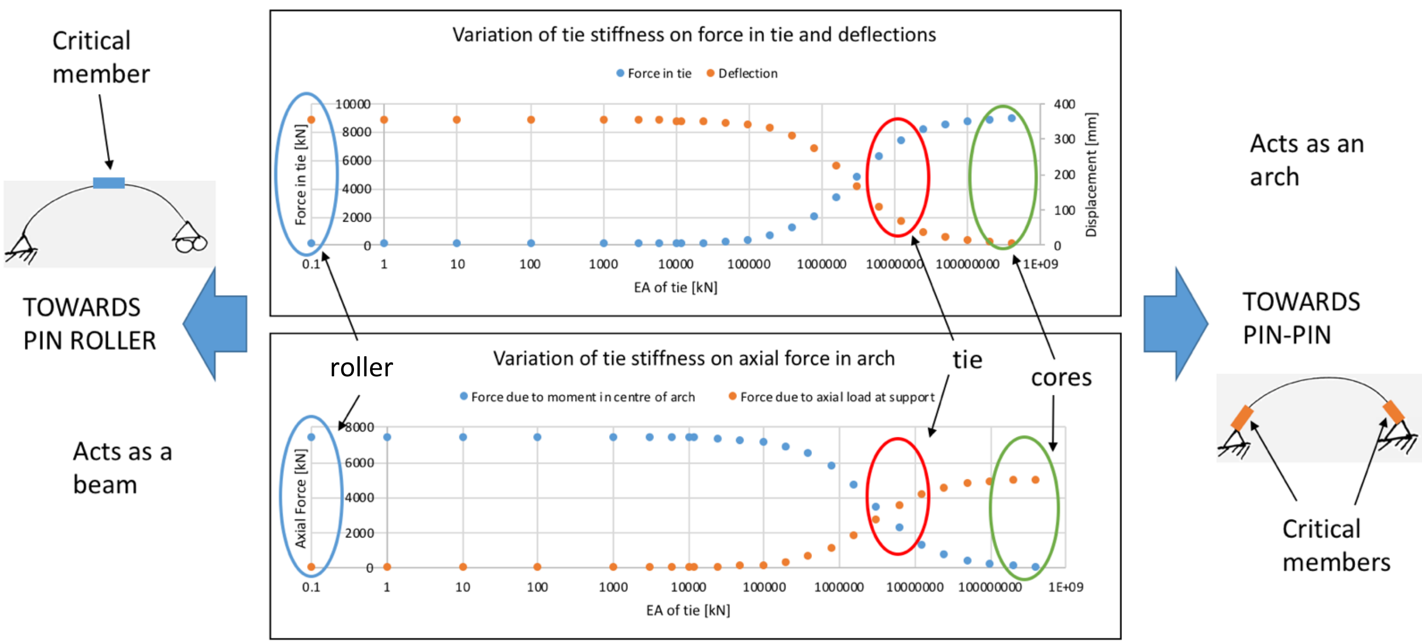


Figure 6.3 Top: Plot of tie force and lateral displacement at roller support against tie stiffness

Bottom: Plot of axial force in central member and end member with tie stiffness

The graph in Figure 6.3 indicates how varying the tie stiffness effects the lateral displacements and forces in the key members at the ends and centre of the support. An indication of where each scheme sits is also shown. Sections 6.2.1 to 6.2.3 summarise the calculation of the 2 extreme conditions and the compromise tied arch condition for a same chosen truss depth and configuration.

6.2.1 Pin-Pin Arch End Connections

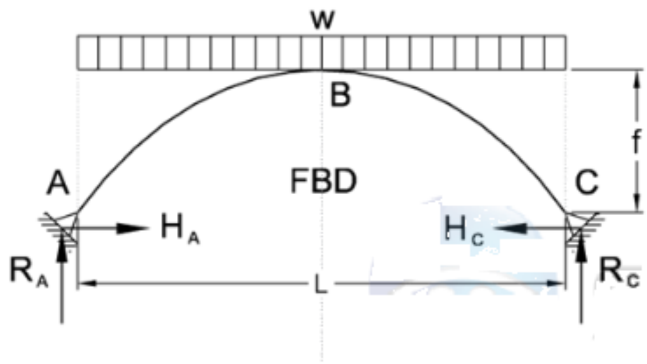


Figure 6.4 Pin-pin assumption (StructX, 2015)

The calculation details of the following results can be found in the Appendix F.

Tonnage of truss steel = 90.7 kg m^{-2}

Members:

CHS 406.4 x 16 on main arch

CHS 168.3 x 5.0 on cantilevers

Vertical reaction $R_A = \frac{wL}{2} = 4.594\ MN$

Horizontal reaction $H_A = \frac{wL^2}{8f} = 8.572\ MN$

The critical section in the pin-pin system is at end-span where the compressive forces are the largest. These were checked to be safe using equilibrium of the end-joint.

This scheme has the smallest steel tonnage because the required member sizes are the smallest and no ties are used. However, the arch produces large horizontal thrust on the core which leads to a large core with more reinforcement, as shown in Section X.

The core was also sized for this scheme, taking into account large moments at the base due to the significant thrust outwards from the arch. This gave a reinforcement 1.23 m^3 and concrete volume of 78.8 m^3 costing an approximate £33.8k.

6.2.2 Pin-Roller Arch End Connections

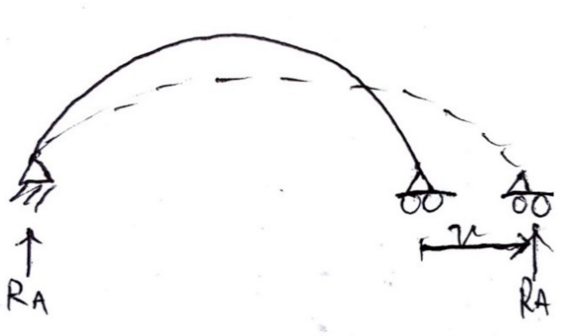


Figure 6.5 Pin-roller assumption

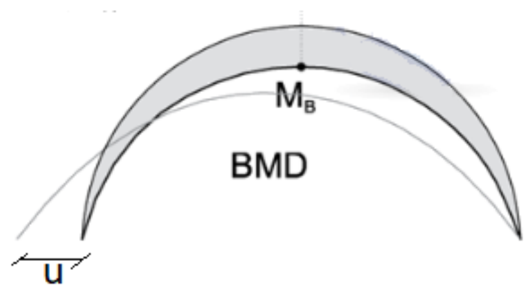


Figure 6.6 Displacement and associated moment

The calculation details of the following results can be found in the Appendix F.

Tonnage of truss steel = 283.8 kg m^{-2}

Members:

CHS 559 x 25 on main arch

CHS 168.3 x 5.0 on cantilevers

Vertical reaction $R_A = \frac{wL}{2} = 6.643\ MN$

Horizontal reaction $H_A = 0$

The displacement is found by considering the maximum bending moment M_b associated with an imposed u in the arch (Figure 4.3.6), and conservatively assume M_b to be that produced in a beam.

$$M_B = \frac{15}{8}\left(\frac{EIu}{f^2L}\right)f$$

$\Rightarrow u = \frac{8M_BfL}{15EI} = 0.23\ m$ where a beam's $M_b = \frac{wL^2}{8}$ is conservatively assumed

The critical section in the pin-roller system is at mid-span where the bending stresses are the largest. These were checked to be safe assuming that the bending moment is resisted by a couple of forces in the top and bottom chord.

The pin-roller condition provides the benefit of having no horizontal thrusts on the core but requires a greater tonnage because the truss members must be strong enough to sustain the concentrated bending moment at mid-span. Additionally, the end-span displacement of the pin-roller scheme is the greatest of the 3 schemes, and must be checked to be within limits.

6.2.3 Pin-Roller Arch End Connections with Tie

The calculation details of the following results can be found in the Appendix F.

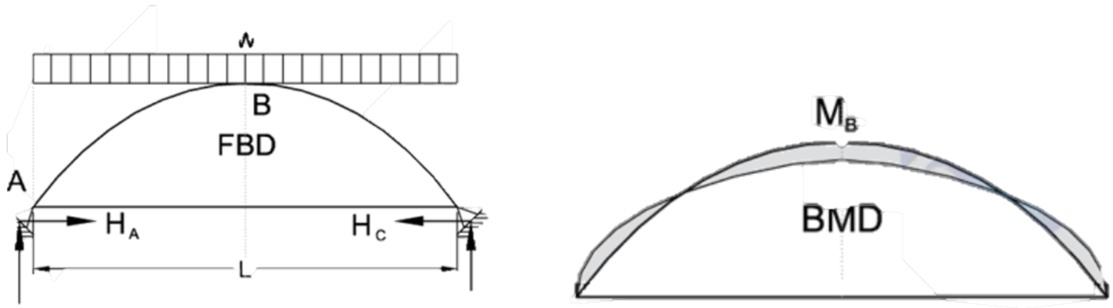


Figure 6.7 Bending moment diagram for a tied arch (StructX, 2015)

Tonnage of truss steel = 102.2 kg m^{-2}

Members:

CHS 406.4 x 16 on main arch

CHS 168.3 x 5.0 on cantilevers

$$R_A = \frac{wL}{2} = 4.794\ MN$$

$$H_A = H_c = \frac{wL^2}{8f}\left(\frac{1}{1+\left(\frac{15}{8}\frac{\frac{EI}{E_T A_T}}{f^2}\right)}\right) = 6.602\ MN$$

The displacement is found by considering the effect of the horizontal force found above acting as an axial force in one of the 2 parallel ties.

$$\frac{H_A}{2} = \frac{EA}{L}u$$

$$\Rightarrow u = \frac{H_a L}{2EA} = 0.092\ m$$

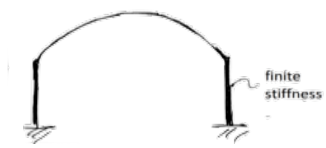
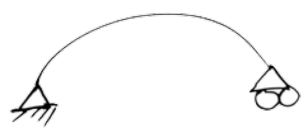
where one tie's $EA = 3780\ kN$ as discussed and chosen from Figure 6.3.

Since the tied arch solution is a compromise between the beam action of the pin-roller scheme and the arching action of the pin-pin scheme, both the members at end-span and at mid-span were checked to be safe, by respectively considering equilibrium of the end-span joint, and considering that the bending moment is resisted by a force couple in the top and bottom chord.

The core was also sized for this scheme, taking into account moments from uplift. This gave a reinforcement 0.41 m3 and concrete volume of 40.0 m^3 costing an approximate £12.5k.

6.2.4 Conclusion

Table 6.1 gives a summary of the conclusions of the study leading to the final decision.

	NEAR PIN-PIN CONNECTION (USING STIFF CORES) 	PIN-ROLLER CONNECTION 	PIN-ROLLER CONNECTION WITH PHYSICAL TIE 
PROS	Near pin-pin condition could be achieved using very large cores (9mx9m with 200mm walls, giving 10mm displacement) Reduction in arch forces and tonnage compared to pin-roller	No horizontal thrust to cores, much reduced size and cost of cores and foundation (8.6 MN horizontal force into core otherwise)	No horizontal thrust to cores, much reduced size and cost of cores and foundation EA of tie can be chosen such that the forces in the arch are smaller than for the pin-pin and pin-roller case Hence low arch tonnage, simple foundation and small core
CONS	Large moment in cores leads to high tensile forces in core, unfeasible Large moment and large horizontal force transferred to foundation, would require expensive and complicated inclined piles	Larger forces in centre of arch due to resisting a moment of $wL^2/8$ Significantly increased tonnage of steel in arch, approx. 2x that of near pin-pin Much larger displacements ~ 300mm	Thermal expansion of tie to consider Visibility and aesthetic issues with tie Tie constructability
CORE COST (SOUTH STAND)	£33.8k	£12.5k	£12.5k
ROOF COST	£874.3k	£2,735.5k	£985.1k

(SOUTH STAND)			

Table 6.1 Summary comparison of different support conditions

The tie captures the pin-roller scheme's benefit of producing no horizontal force on the core while decreasing the horizontal displacement at end-span. The member sizes and hence tonnage also do not need to be as large as in the pin-roller scheme because the horizontal restraint provided by the help ties provide some arching action to the truss. To make the pin-roller scheme viable in terms of tonnage of steel, the depth had to be increased to at least 14m, which was deemed aesthetically unacceptable. The cost of the tie and pinned-core schemes were comparable, but the complications to foundation design of the pinned-core scheme was the deciding factor in choosing a physical tie as the final scheme.

6.3 Roof Optimisation

6.3.1 Modelling Assumptions

As there are many interdependent factors in this roof framework analysis, several constraints have been established to restrain the boundaries of the problem and simplify the analysis. The constraints applied are listed below:

1. Arch spans = 105m.
   This was based on the superstructure stand geometry.
2. Arch cross section is square.
   This was to improve the torsional resistance of the main arch, allowing the structure to behave as a torsion tube.
3. Rise is fixed at 15m.
   This was not iterated as tonnage and lateral displacements decreases with reduced rise. Hence, the rise was determined based on the desired aesthetic outlook of the roof structure.
4. Diagonal members should have angles of approximately 45^{o}.
   Implications: For a particular depth, there will be a fixed number of cantilever sections and arch sections as illustrated below in Figure 6.8. An increase in depth will result in a decrease in number of sections and vice versa.

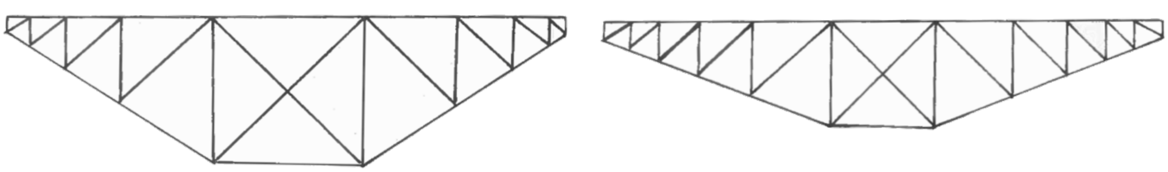


Figure 6.8 Influence of depth on number of cantilever sections and cantilever span

5. Upper chord members are taken to be continuous while vertical and diagonal members are taken to have pinned ends.
6. The selected pin-roller with tie arch end restraint scheme selected in Section 6.2 has been adopted.

6.3.2 Key Variables

Based on these constraints, we can now identify the key variables to optimise, which are:

1. Depth

   This variable influences

   - Cantilever span

Given that the arch is a square box section, the cantilever span = (stand width – arch depth)/2

- Number of cantilever and arch sections.
- Available nodes for longitudinal (out-of-plane) restraint.
- Tonnage.
- Tributary area supported by cantilever (illustrated in Figure 6.9)

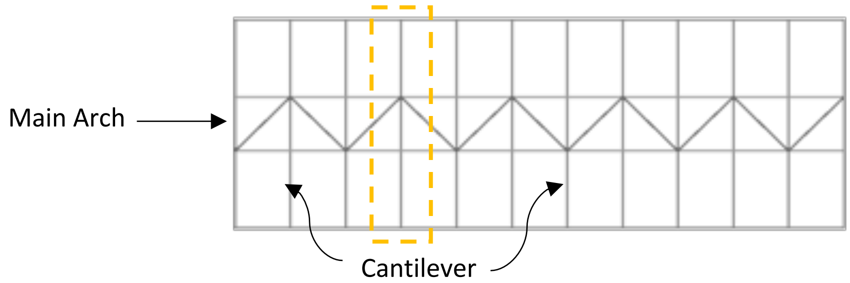


Figure 6.9 Effect of number of arch sections on tributary area supported by each cantilever

2. Longitudinal Bracing Configuration

- It is assumed that all nodes on the top chord will be restrained longitudinally for lateral stability (i.e.: bracing configuration is only explored for the bottom chords)
- For the bottom chords, the nodes circled in orange as illustrated in Figure 6.10 will always be restrained. These nodes are restrained by the main arch (on the long end) and the diaphragm (on the short end).
- Optimal configuration is found by analysing combinations of different longitudinal restraint on the remaining intermediate bottom chord nodes. An example is illustrated in Figure 6.10 for a combination resisting 2 out of 3 nodes.
- This variable influences:
    - Tonnage (weight per unit roof area).
    - Effective length for out-of-plane buckling check.

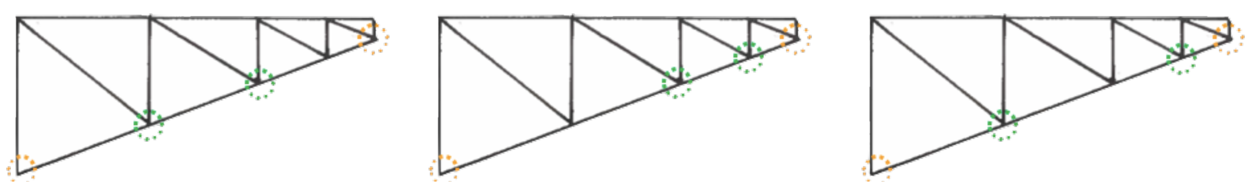


Figure 6.10 Example of restraining 2 out of 3 nodes

6.3.3 Procedural Flow Chart

From the procedural flow chart in Figure 6.11, the roof optimisation comprises of 2 separate processes. This was done first using MATLAB via a Finite Element Analysis to optimise the truss configuration. Once the configuration is determined, further optimisation of the cross-section dimensions using SAP2000 will be carried out.

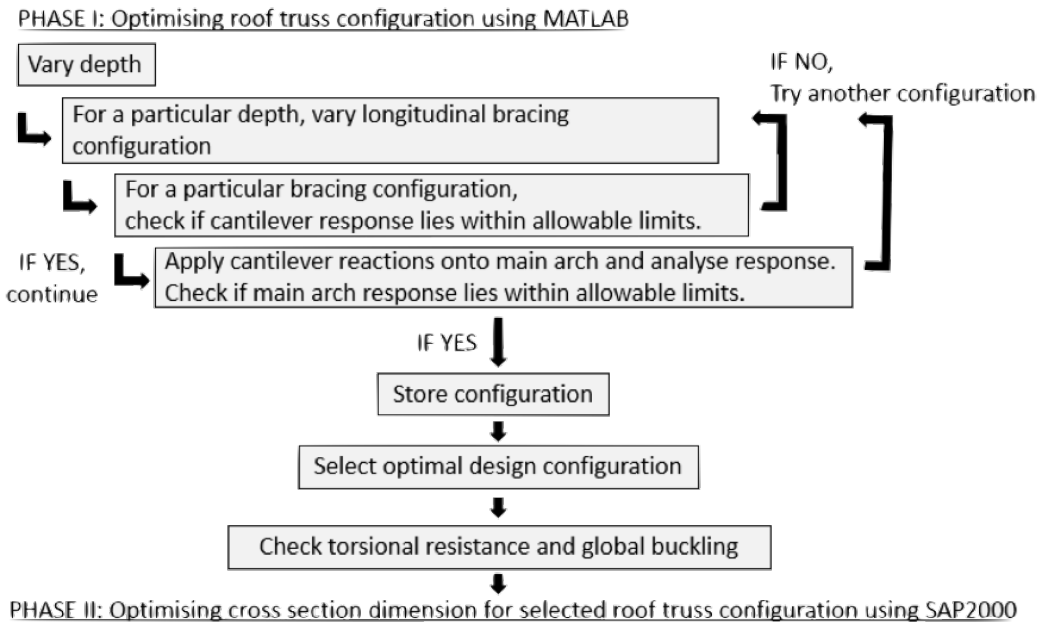


Figure 6.11 Procedural Flow Chart for Truss Arrangement Design

6.4 PHASE I: Steel Layout Optimisation

The key aim of this optimisation phase is to determine the depth and longitudinal restraint (i.e. key variables) that will provide the most optimised roof configuration.

6.4.1 Design Assumptions

1. All arch members are comprised of CHS 406.4 x 16.0 members
2. All cantilever members are comprised of CHS 168.3 x 5.0 members
3. Tie is comprised of LC 180 members.
4. All members have yield strength, f_y = 275 N/mm^2.
5. PHASE I will consider only the vertical downward load case of 1.35 Dead Load + 1.5 Imposed Load

6.4.2 Design Checks

As illustrated in the design flow chart (Figure 6.11), each configuration analysed will have to undergo a series of design checks for both the cantilever and arch sections. These include:

1. In-plane buckling
- The Perry Robertson formula as defined in Eurocode 3 was used. This takes into consideration of imperfections on the response of the members.
- For pinned diagonal and vertical members, leff = member length.
- For continuous chord members, leff = 0.5 x member length
- Axial forces of all the members within these restraints have to lie below the in-plane buckling resistance.

2. Out-of-plane buckling
- Similarly, the Perry Robertson formula as defined in Eurocode 3 was used.

- This was assessed only for the bottom chord members as only the effective lengths differ in these members from the in-plane buckling case.
- Bottom continuous chord's effective length, leff = 0.5 x distance between longitudinal restraints.
- Axial forces of all the members within these restraints have to lie below the out-of-plane buckling resistance.

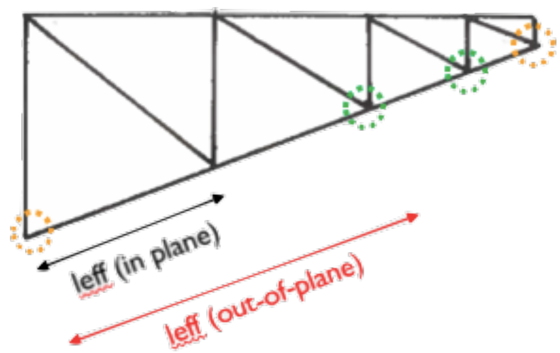


Figure 6.12 Illustration of effective lengths for different

3. Yielding
- Axial Forces of all members must lie below the yield stress x cross sectional area.

4. Deflection
   - Maximum vertical deflection of the configurations must lie below span/180.

6.4.3 Design Selection Process – Determining Factors

1. Adequate Factor of Safety
- Only when all the members within the cantilever and main arch of the configuration has passed through all design checks, will we consider them in the design selection process (i.e: the configuration must have a minimum factor of safety larger than 1).
- The design selection will then be based on this pool of safe configurations.

2. Depth limits
- These are governed by the spatial constraints of the site, whereby a limited core size of 10.5m and hence a corresponding maximum arch width of 10.5m is imposed on the design.
- This is illustrated below in Figure 6.13.

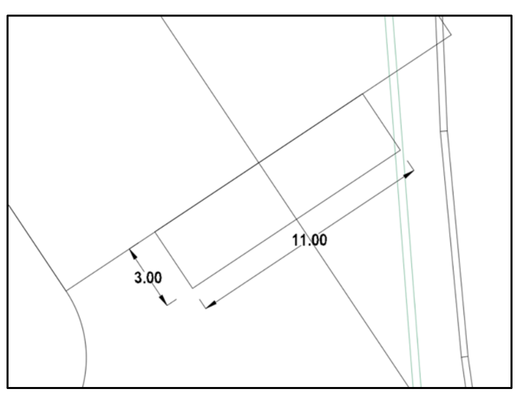


Figure 6.13 The maximum allowable core width for a central placement about the ends of the roof

3. Minimise tonnage.
- After sieving through various configurations and selecting from the allowable depth, the configuration selected is one that provides the minimum tonnage.

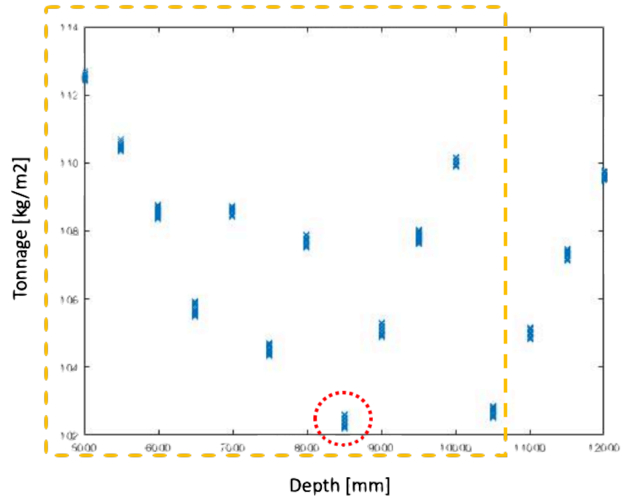


Figure 6.14 Orange indicates depth limits and red indicates the depth providing the configuration of least tonnage

6.4.4 Optimised Truss Configuration from PHASE I

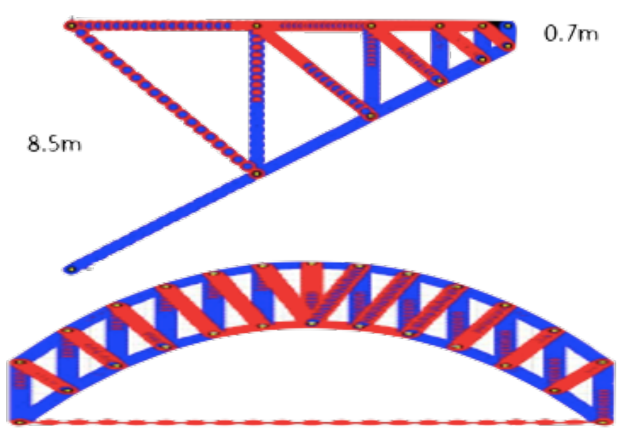


Figure 6.15 Final optimised configuration

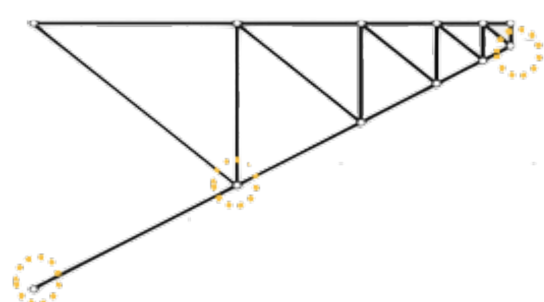


Figure 6.16 Longitudinally restrained nodes

Based on the procedure above, a truss depth of 8.5 m has been selected for the main arch. Similarly, the cantilever will have a depth of 8.5m at the deeper end and tapers to a depth of 0.7m. The number of sections along the main arch and along the cantilevers are 12 and 5 respectively (Figure 6.15). On the bottom chord of the cantilever, only the joints circled in orange are restrained out of plane as shown in Figure 6.16. This gives an overall minimum factor of safety of 1.5 against buckling in the main arch, for a 102.2 kg/m^2 tonnage which has been minimised within the constraints.