2D Vertex Model

geometrical f(x)

In[91]:=
```
(*get vertex in the list by ccw order*)
Clear[getCounterClockwise];
getCounterClockwise[vertex_, vertices_] := Block[{pos, v},
   pos = First @@ Position[vertices, vertex];
   If[pos == Length[vertices], pos = 1, pos += 1];
   vertices[[pos]]
  ];
```

In[93]:=
```
(*get vertex in the list by cw order*)
Clear[getClockwise];
getClockwise[vertex_, vertices_] := Block[{ls, pos},
   pos = First @@ Position[vertices, vertex];
   If[pos == 1, pos = Length[vertices], pos -= 1];
   vertices[[pos]]
  ];
```

In[95]:=
```
getCounterClockwise[{xi, yi}, {{xi-1, yi-1}, {xi, yi}, {xi+1, yi+1}, d, e}]
```

Out[95]=
$$\{x_{1+i},\ y_{1+i}\}$$

In[96]:=
```
getClockwise[{xi, yi}, {{xi-1, yi-1}, {xi, yi}, {xi+1, yi+1}, d, e}]
```

Out[96]=
$$\{x_{-1+i},\ y_{-1+i}\}$$

In[97]:=
```
Clear[areaOfPolygon];
areaOfPolygon[cells_ /; Head[cells] === Association] := Map[Area@*Polygon, cells];
```

In[99]:=
```
Clear[areaPolygon];
areaPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  0.5 Abs@Total[(#[[1, 1]] * #[[2, 2]]) - (#[[2, 1]] * #[[1, 2]]) & /@ edges]
 ]
```

In[101]:=
```
randpoly = Polygon[
```

```
];
```

Number of points: 5
Embedding dimension: 2
Type: simple polygon
Bounds: {{0.05, 0.98}, {0.27, 0.92}}
Area: 0.29

In[102]:=
```
Area[randpoly]
```
Out[102]= 0.287052

In[103]:=
```
areaPolygon[MeshPrimitives[randpoly, 0] /. Point → Sequence]
```
Out[103]= 0.287052

In[104]:=
```
Clear[perimeterOfPolygon];
perimeterOfPolygon[cells_ /; Head[cells] === Association] :=
  (Perimeter@*Polygon) /@ cells;
```

In[106]:=
```
Clear[perimeterPolygon];
perimeterPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  Total[Apply[EuclideanDistance] /@ edges]
 ]
```

In[108]:=
```
Perimeter@randpoly
```
Out[108]= 2.4033

In[109]:=
```
perimeterPolygon[MeshPrimitives[randpoly, 0] /. Point → Sequence]
```
Out[109]= 2.4033

In[110]:=
```
Clear[centroidPolygon];
centroidPolygon[vertices_] := Mean[vertices]
```

In[112]:=
```
(*counterclockwise polygonQ*)
Block[{signedarea = 0, j, vertlen = 5},
  Do[
   j = Mod[i, vertlen] + 1;
   signedarea += (x_i y_j - x_j y_i),
   {i, vertlen}];
  Echo[1/2 (signedarea)]
 ];
```
» $\frac{1}{2}\left(-x_2 y_1 + x_5 y_1 + x_1 y_2 - x_3 y_2 + x_2 y_3 - x_4 y_3 + x_3 y_4 - x_5 y_4 - x_1 y_5 + x_4 y_5\right)$

In[113]:=
```
Clear[polyCounterClockwiseQ];
polyCounterClockwiseQ[poly_] := Block[{area = 0, j, vertLength = Length[poly]},
  Do[
   j = Mod[i, vertLength] + 1;
   area += poly[[i, 1]] * poly[[j, 2]];
   area -= poly[[j, 1]] * poly[[i, 2]],
   {i, vertLength}
  ];
  (area / 2.) > 0
 ]
```

In[115]:=
```
(*Clear[sortCC];
sortCC[polyinds_,indTopts_,ptsToInds_]:=Block[{cent,poly},
  poly=Lookup[indTopts,polyinds];
  Lookup[ptsToInds,
   DeleteDuplicates@Flatten[MeshPrimitives[ConvexHullMesh[poly],1]/.Line→Sequence,1]
  ]
 ];*)
```

In[116]:=
```
(*sort points for a convex polygon in counter-clockwise direction*)
```

In[117]:=
```
Clear[sortPointsCC];
sortPointsCC[polyinds_, indTopts_, ptsToInds_] := Block[{cent, ordering, polyPoints},
  polyPoints = Lookup[indTopts, polyinds];
  cent = Mean[polyPoints];
  ordering = Ordering[ArcTan[#[[1]], #[[2]]] &@(# - cent) & /@ polyPoints];
  Lookup[ptsToInds, Part[polyPoints, ordering]]
 ]
```

mesh restructuring operations

In[119]:=
```
bagged = CreateDataStructure["DynamicArray"]
```

Out[119]=
```
DataStructure[ Type:DynamicArray
               Length:0 ]
```

T1 transition

In[120]:=
```
(* T1 transition: neighbour switching *)
```

In[121]:=
```
Clear@edgesforT1;
edgesforT1[edgeLs_, indToPts_, threshLength_ : 0.002] := Block[{edges, dist, bool},
   edges = Lookup[indToPts, #] & /@ edgeLs;
   dist = EuclideanDistance @@ # & /@ edges;
   bool = Thread[dist ≤ threshLength];
   {Pick[edges, bool, True], Pick[edgeLs, bool, True]}
  ];
```

In[123]:=
```
Clear@T1transitionFn;
T1transitionFn[edges_, indToPtsAssoc_,
   vertexToCellG_, cellToVertexG_, dSep_ : 0.0075] :=
  Block[{findEdges, edgeind, connectedcellKeys, edge, newpts, cellvertIndices,
    cellvertices, pos, cellpolys, memF, keyscellP, selcellKeys, ptToCell,
    newptsindices, indToPts = indToPtsAssoc, ptsToInds, PtIndToCell, keysToMap,
    cellindicesAssoc, f1, otherkeys, f2, polysharingEdge, bag = bagged,
    vertToCellG = vertexToCellG, cellToVertG = cellToVertexG, testpts, edgechanged,
    localtopology, ls, cs, cc, ccw, trimmedtopo, ordering, edgeordered},
   {edgechanged, findEdges} = edgesforT1[edges, indToPts];
   (* finding all possible edges for T1 transition *)

   If[findEdges ≠ {},
    Scan[
     (edgeind = #;
      If[ContainsAll[Keys[indToPts], edgeind],
       (* should be an edge not
        connected to an edge that has already undergone a T1 *)
       connectedcellKeys = DeleteDuplicates[Flatten@Lookup[vertToCellG, edgeind]];
       cellvertIndices = Lookup[cellToVertG, connectedcellKeys];
       edge = Lookup[indToPts, edgeind];
       If[Length[connectedcellKeys] == 1,
        (*edge that is exposed to the void to be merged as a single vertex*)
        newpts = Mean[edge];
        newptsindices = Max[Keys@indToPts] + 1;
        KeyDropFrom[indToPts, edgeind];
        AppendTo[indToPts, newptsindices → newpts];
        bag["Append", edgeind];
        ptsToInds = AssociationMap[Reverse, indToPts];
        cellToVertG = MapAt[
          DeleteDuplicates[# /. (Alternatives @@ edgeind) → newptsindices] &,
          cellToVertG, Key[connectedcellKeys /. {z_Integer} :> z]
         ],
        (*else proceed with T1 transition*)
```

```
newpts = With[{midPt = Mean[edge]},
  midPt + dSep Normalize[(# - midPt)] & /@
   Flatten[RotationTransform[-π/2, midPt] /@ {edge}, 1]
 ];
testpts = With[{midPt = Mean[edge]},
  midPt + 0.000001 Normalize[(# - midPt)] & /@ newpts
 ];
pos = Position[cellvertIndices, {OrderlessPatternSequence[
    ___, First@edgeind, ___, Last@edgeind, ___]}, {1}];
polysharingEdge = Extract[cellvertIndices, pos];
(* the edge should not be part of any Δ *)
If[(AllTrue[polysharingEdge, Length[#] ≠ 3 &]) &&
  ContainsNone[edgeind, Union@*Flatten@*Normal@bag],
 cellvertices = Map[Lookup[indToPts, #] &, cellvertIndices];
 cellpolys = Polygon /@ cellvertices;
 memF = Function[x, RegionMember@*DiscretizeRegion@x, Listable][
   Extract[cellpolys, pos]];
 keyscellP = Extract[connectedcellKeys, pos];
 selcellKeys = Thread[keyscellP → memF];
 ptToCell = Quiet[# → First @@ Select[selcellKeys, Function[x,
          Last[x][#]]] & /@ testpts /. HoldPattern[_ → First[]] → Nothing] ;
 (* pt to cell *)
 ptToCell = ptToCell /. Thread[testpts → newpts];
 newptsindices = Range[# + 1, # + 2] &[Max[Keys@indToPts]];

 AppendTo[indToPts, Thread[newptsindices → newpts]];
 ptsToInds = AssociationMap[Reverse, indToPts];
 bag["Append", edgeind];
 PtIndToCell = MapAt[ptsToInds, ptToCell, {All, 1}] /. Rule → List ;
 (*index to cell*)
 keysToMap = MapAt[Key, PtIndToCell, {All, 2}];
 cellindicesAssoc = AssociationThread[connectedcellKeys, cellvertIndices];

 f1 = Fold[MapAt[Function[x, DeleteDuplicates[x /. Thread[ edgeind → #2[[1]] ]]],
     #1, #2[[2]] ] &, cellindicesAssoc, keysToMap];
 f1 = Lookup[indToPts, #] & /@ f1;

 KeyDropFrom[indToPts, edgeind];

 otherkeys = List@*Key /@ Complement[connectedcellKeys, keyscellP];
 (*Print@Graphics[{EdgeForm[Black],
     FaceForm[Red],Polygon/@cellvertices},ImageSize→Tiny];*)

 localtopology = AssociationThread[connectedcellKeys → cellvertices];
 ls = Lookup[localtopology, First @@ otherkeys];
```

```
            cs = FirstCase[ls, Alternatives @@ edge];
            cc = getClockwise[cs, ls];
            ccw = getCounterClockwise[cs, ls];
            trimmedtopo =
             KeyDrop[localtopology, connectedcellKeys ~ Complement ~ keyscellP];
            ordering = {FirstCase[Position[trimmedtopo, cc],
                x : {Key[First@keyscellP], _} | {Key[Last@keyscellP], _} :> First @@ x],
              FirstCase[Position[trimmedtopo, ccw], x : {Key[First@keyscellP], _} |
                  {Key[Last@keyscellP], _} :> First @@ x ]};

            If[Length[otherkeys] == 1,
             edgeordered = ordering /. Reverse[ptToCell, 2];
             f2 = MapAt[Replace[#, Alternatives @@ edge :>
                   Splice[edgeordered], {1}] &, f1, First@otherkeys];
             (*Print@Graphics[{EdgeForm[Black],FaceForm[Red],Polygon/@
                   Values@f2},ImageSize→Tiny];*)
             ,
             If[MatchQ[ordering, {_Missing, _} | {_, _Missing}],
              ordering =
               Flatten[ordering /. _Missing :> Complement[newpts, Keys@ptToCell], 1];
              edgeordered = ordering,
              edgeordered = ordering /. Reverse[ptToCell, 2];
             ];
             f2 = MapAt[Replace[#, Alternatives @@ edge :> Splice[edgeordered], {1}] &,
               f1, First@otherkeys];
             f2 = MapAt[Replace[#, Alternatives @@ edge :> Splice[
                    Reverse[edgeordered]], {1}] &, f2, Last@otherkeys];
             (*Print@Graphics[{EdgeForm[Black],FaceForm[Red],Polygon/@
                   Values@f2},ImageSize→Tiny];*)
            ];
            AppendTo[cellToVertG, Lookup[ptsToInds, #] & /@ f2];
           ];
          ];
          vertToCellG = GroupBy[
            Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertG], First → Last];
         ]) &, findEdges]
     ];
     bagged["DropAll"];
     {edgechanged, indToPts, cellToVertG, vertToCellG}
    ];
```

T2 transition

In[125]:=
```
(* T2 transition: removal of cell *)
```

In[126]:=
```
Clear@cellsforT2;
cellsforT2[areaAssoc_, cellVertexG_, thresh_ : 10^-4] := Block[{keys, ls, inds},
   keys = Keys@Select[areaAssoc, # < thresh &];
   ls = Lookup[cellVertexG, keys];
   (*inds=Flatten@Position[ls,x_/;(3≤Length[x]≤10),{1}];
   (* 3 ≤ cell edges ≤ 6 *) *)
   inds = Flatten@Position[ls, x_ /; (Length[x] == 3), {1}];
   (* cell edges == 3 *)
   If[inds ≠ {}, {keys[[inds]], ls[[inds]]}, {{}, {}}] (*cell inds, vertices*)
  ];
```

In[128]:=
```
Clear@T2TransitionFn;
T2TransitionFn[{cellsToRemove_, vertindsRemove_}, indTopts_, cellToVertexG_,
   areaPolygonAssoc_, periPolygonAssoc_] := Block[{newVertices, maxkey, newindices,
     newentries, indToPts = indTopts, ruleDisp, removeentries,
     CVG = cellToVertexG, notaCell, VertCellGrouping},
    newVertices = Mean@Lookup[indTopts, #] & /@ vertindsRemove;
    maxkey = Max@*Keys@indTopts;
    newindices = Range[maxkey + 1, maxkey + Length[newVertices]];
    newentries = Thread[newindices → newVertices];
    KeyDropFrom[indToPts, Union@Flatten[vertindsRemove]];
    AppendTo[indToPts, newentries];
    ruleDisp =
     Dispatch@Flatten[MapThread[Thread[#1 → #2] &, {vertindsRemove, newindices}]];
    removeentries = Union@Flatten@cellsToRemove;
    KeyDropFrom[CVG, removeentries];
    CVG = DeleteDuplicates /@ Replace[CVG, ruleDisp, {2}];
    notaCell = Keys@Select[Length /@ CVG, # < 3 &];
    KeyDropFrom[CVG, notaCell];
    VertCellGrouping =
     GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
    {indToPts, CVG, VertCellGrouping, KeyDrop[areaPolygonAssoc,
      removeentries ~ Join ~ notaCell],
     KeyDrop[periPolygonAssoc, removeentries ~ Join ~ notaCell]}
   ] /; vertindsRemove ≠ {};
```

T3 transition

In[130]:=
```
T3candidates[vertexToCell_, indTopts_, cellToVertexG_] :=
  Block[{outervertindices, outercellsinds, outercells, regmem, outerverticespts},
   {outervertindices, outercellsinds} = Through[{Keys, Union@*Flatten@*Values}[#]] &@
     Select[vertexToCell, Length[#] < 3 &];
   outercells = Lookup[indTopts, cellToVertexG@#] & /@ outercellsinds;
   regmem = SignedRegionDistance@*Polygon /@ outercells;
   outerverticespts = Lookup[indTopts, outervertindices];
   {Position[(Thread[(#[outerverticespts] < 0)] & /@ regmem), True],
    outercellsinds, outercells, outervertindices}
  ];
```

In[131]:=
```
T3Transition[markers_, outercellsinds_,
   outercells_, outervertindices_, vertToCell_, pToI_, ItoP_, CVG_] :=
  Block[{ci, vi, minorcellind, vert, vertexToCell = vertToCell, numcells,
    majorcellind, intersectcell, ptsToInd = pToI, majorcell, cellToVertexG = CVG,
    commonvertexQ, edgespartof, indTopts = ItoP, edgecoords, edgesminorcell,
    lines, intersects, fpos, intersectpts, newptsindices, ls},
   If[markers ≠ {},
    Do[
     (*take the marker and handle cases *)
     {ci, vi} = marker;
     minorcellind = outercellsinds[[ci]];
     vert = outervertindices[[vi]];
     majorcellind = If[Head[#] === Integer, numcells = 1;
         #, numcells = Length[#];
         #] &@Replace[Lookup[vertexToCell, vert], {z_Integer} :> z];
     intersectcell = Lookup[ptsToInd, outercells[[ci]]];
     majorcell = Lookup[cellToVertexG, majorcellind];
     Print[Flatten@{majorcellind, minorcellind}];
     commonvertexQ = If[numcells == 1,
        (Union[Flatten@Cases[Partition[majorcell, 2, 1, 1],
             {OrderlessPatternSequence[vert, _]}]] ⋂ intersectcell),
       Function[(Union[Flatten@Cases[Partition[#, 2, 1, 1],
            {OrderlessPatternSequence[vert, _]}]] ⋂ intersectcell)] /@ majorcell
      ] // (If[# ≠ {}, First@#, {}] &) @* Flatten;
     Which[
      (*Case A*)
      numcells == 1 && commonvertexQ === {},
      edgespartof = Cases[Partition[cellToVertexG[majorcellind], 2, 1, 1],
        {OrderlessPatternSequence[vert, _]}];
      edgecoords = Lookup[indTopts, #] & /@ edgespartof;
      edgesminorcell = Partition[cellToVertexG[minorcellind], 2, 1, 1];
```

```
lines = Map[Line@Lookup[indTopts, #] &, edgesminorcell];
intersects =
 Map[Function[x, Map[RegionIntersection[Line[x], #] &, lines]], edgecoords];
fpos = Last@FirstPosition[intersects, _Point, {2}];
intersectpts = Cases[intersects, {__?NumberQ}, {-2}];
newptsindices = Range[Max[ptsToInd] + 1, Max[ptsToInd] + 2];
AppendTo[indTopts, Thread[newptsindices → intersectpts]];
KeyDropFrom[indTopts, vert];
ptsToInd = AssociationMap[Reverse, indTopts];
cellToVertexG = MapAt[sortPointsCC[
    Flatten[# /. Thread[vert → {newptsindices}]], indTopts, ptsToInd] &,
  cellToVertexG, Key[majorcellind]];
cellToVertexG = MapAt[
  Block[{y},
    y = Partition[#, 2, 1, 1];
    sortPointsCC[DeleteDuplicates@
      Flatten@Replace[y, {x : OrderlessPatternSequence @@ y⟦fpos⟧} :↦ Flatten@
          Insert[{x}, newptsindices, 2], {1}], indTopts, ptsToInd]
   ] &, cellToVertexG, Key[minorcellind]];
,
(*Case B*)
numcells == 2 && commonvertexQ === {},
edgesminorcell = Partition[cellToVertexG[minorcellind], 2, 1, 1];
lines = Map[Line@Lookup[indTopts, #] &, edgesminorcell];
Do[
 edgespartof = Cases[Partition[cellToVertexG[majcelliter], 2, 1, 1],
   {OrderlessPatternSequence[vert, _]}];
 edgecoords = Lookup[indTopts, #] & /@ edgespartof;
 intersects =
  Map[Function[x, Map[RegionIntersection[Line[x], #] &, lines]], edgecoords];
 fpos = Last@FirstPosition[intersects, _Point, {2}];
 intersectpts = Cases[intersects, {__?NumberQ}, {-2}];
 Scan[(If[KeyFreeQ[ptsToInd, #],
     newptsindices = Max[ptsToInd] + 1;
     ptsToInd[#] = newptsindices;
     indTopts[newptsindices] = #]) &, intersectpts];
 newptsindices = Lookup[ptsToInd, intersectpts];
 cellToVertexG = MapAt[sortPointsCC[
     Flatten[# /. Thread[vert → {newptsindices}]], indTopts, ptsToInd] &,
   cellToVertexG, Key[majcelliter]]
 , {majcelliter, majorcellind}
];
KeyDropFrom[indTopts, vert];
ptsToInd = AssociationMap[Reverse, indTopts];
cellToVertexG = MapAt[
  Block[{y},
    y = Partition[#, 2, 1, 1];
```

```
    sortPointsCC[DeleteDuplicates@
      Flatten@Replace[y, {x : OrderlessPatternSequence @@ y[[fpos]]} :> Flatten@
          Insert[{x}, newptsindices, 2], {1}], indTopts, ptsToInd]
   ] &, cellToVertexG, Key[minorcellind]],
(*Case C*)
numcells == 1 && commonvertexQ =!= {},
edgespartof = Cases[Partition[cellToVertexG[majorcellind], 2, 1, 1],
  {OrderlessPatternSequence[vert, _]}];
edgecoords = Lookup[indTopts, #] & /@ edgespartof;
edgesminorcell = Partition[cellToVertexG[minorcellind], 2, 1, 1];
lines = Map[Line@Lookup[indTopts, #] &, edgesminorcell];
intersects =
 Map[Function[x, Map[RegionIntersection[Line[x], #] &, lines]], edgecoords];
fpos = Last@FirstPosition[intersects, _Point, {2}];
intersectpts = First@
  DeleteCases[Cases[intersects, {__?NumberQ}, {-2}], indTopts[commonvertexQ]];
newptsindices = Max[ptsToInd] + 1;
AppendTo[indTopts, newptsindices -> intersectpts];
KeyDropFrom[indTopts, {vert, commonvertexQ}];
ptsToInd = AssociationMap[Reverse, indTopts];
cellToVertexG =
 MapAt[DeleteDuplicates[# /. (commonvertexQ | vert) -> newptsindices] &,
  cellToVertexG, Key[majorcellind]];
cellToVertexG = MapAt[# /. commonvertexQ -> newptsindices &,
  cellToVertexG, Key[minorcellind]],
(*Case D*)
numcells == 2 && commonvertexQ =!= {},
ls = {};
edgesminorcell = Partition[cellToVertexG[minorcellind], 2, 1, 1];
lines = Map[Line@Lookup[indTopts, #] &, edgesminorcell];
Do[
 edgespartof = Cases[Partition[cellToVertexG[majcelliter], 2, 1, 1],
   {OrderlessPatternSequence[vert, _]}];
 edgecoords = Lookup[indTopts, #] & /@ edgespartof;
 intersects =
  Map[Function[x, Map[RegionIntersection[Line[x], #] &, lines]], edgecoords];
 fpos = Last@FirstPosition[intersects, _Point, {2}];
 intersectpts =
  DeleteCases[Cases[intersects, {__?NumberQ}, {-2}], indTopts[commonvertexQ]];
 If[Length[intersectpts] == 2, AppendTo[ls, intersectpts]];
 Scan[(If[KeyFreeQ[ptsToInd, #],
     newptsindices = Max[ptsToInd] + 1;
     ptsToInd[#] = newptsindices;
     indTopts[newptsindices] = #]) &, intersectpts];
 newptsindices = Lookup[ptsToInd, intersectpts];
 cellToVertexG = MapAt[
   sortPointsCC[DeleteDuplicates@
```

```
            Flatten[# /. (commonvertexQ | vert) → newptsindices], indTopts, ptsToInd] &,
          cellToVertexG, Key[majcelliter]];
        ,
        {majcelliter, majorcellind}
       ];
       cellToVertexG = MapAt[
         sortPointsCC[DeleteDuplicates@Flatten[# /.
              commonvertexQ → Lookup[ptsToInd, intersectpts]], indTopts, ptsToInd] &,
         cellToVertexG, Key[minorcellind]];
      ];
      KeyDropFrom[indTopts, {vert, commonvertexQ}];
      ptsToInd = AssociationMap[Reverse, indTopts];
      vertexToCell = GroupBy[
        Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
      , {marker, markers}]
    ];
    {indTopts, ptsToInd, vertexToCell, cellToVertexG}
   ];
```

cell division

Difference from Farahdifar's paper: in his article he dilates the cells before dividing them. Here I randomly select a cell and divide it into two.

In[132]:=
```
(* probability of division based on the cell area *)
```

In[133]:=
```
Clear[selectDivCells];
selectDivCells[areaPolygon_, areathresh_ : 2.2, thresh_ : 0.0025] :=
  Block[{candidates, pos},
   candidates = Normal@Select[areaPolygon / Mean[areaPolygon], # > areathresh &];
   pos = Position[0.1 RandomReal[1, Length@candidates], x_ /; x < thresh];
   Keys@Extract[candidates, pos]
  ];
```

In[135]:=
```
(* division events more random *)
```

In[136]:=
```
Clear[pickcellsDiv];
pickcellsDiv[cellToVertG_, areaAssoc_] := Block[{pickcells, selcells, pos},
   pickcells = Keys@Select[Pick[areaAssoc,
       Thread[RandomReal[{0, 1}, Length[areaAssoc]] < 0.001], True], # > 0.005 &];
   pos = Position[Lookup[cellToVertG, pickcells], x_ /; Length[x] > 3, {1}];
   Extract[pickcells, pos]
  ];
```

In[138]:=
```
Clear[cellDivision];
cellDivision[polygonind_, indToPoints_, areaAssoc_, perimAssoc_, cellToVertG_] :=
```

```
Block[{x, y, num, matrix, xx, xy, yy, eigvals, eigVecs, maxeigpos, cent, edges,
  edgesL, intersects, intersectionPts, posIntersections, repPart, α, β,
  polygonPts, newkeys = Range[# + 1, # + 2] &[Max@Keys[indToPoints]], newPtToInds,
  indtoPtAssoc = indToPoints, ptToIndAssoc, edgeinds, contour, poly1, poly2, res, seq,
  newcells = Range[# + 1, # + 2] &[Max@Keys[areaAssoc]], CVG = cellToVertG,
  addcellsRule, polygonPtsInds, VCG, polygonptsTrans},
 VCG = GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
 polygonPtsInds = CVG[polygonind];
 num = Length@polygonPtsInds;
 ptToIndAssoc = AssociationMap[Reverse, indToPoints];
 polygonPts = Lookup[indToPoints, polygonPtsInds];
 polygonptsTrans = TranslationTransform[-Mean[polygonPts]]@polygonPts;
 Evaluate[Table[{x_i, y_i}, {i, num + 1}]] =
  Append[polygonptsTrans, First@polygonptsTrans];
 I_xx = (1/12) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (y_i^2 + y_i y_{i+1} + y_{i+1}^2);
 I_yy = (1/12) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (x_i^2 + x_i x_{i+1} + x_{i+1}^2);
 I_xy = (1/24) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (x_i y_{i+1} + 2 x_i y_i + 2 x_{i+1} y_{i+1} + x_{i+1} y_i);
 Table[
  {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, num + 1}];
 matrix = {{I_xx, -I_xy}, {-I_xy, I_yy}};
 {eigvals, eigVecs} = Eigensystem@matrix;
 maxeigpos = Position[eigvals, Max@eigvals];
 {edges, edgeinds} = Partition[#, 2, 1, 1] & /@ {polygonPts, polygonPtsInds};
 edgesL = Line /@ edges;
 cent = centroidPolygon[polygonPts];
 intersects = RegionIntersection[
     InfiniteLine[{cent, cent + Extract[eigVecs, maxeigpos][[1]]}], #] & /@ edgesL;
 intersectionPts = Cases[intersects, {(_Real | _Integer) ..}, {3}];
 newPtToInds = Thread[intersectionPts → newkeys];
 posIntersections = Flatten@Position[intersects, _Point, {1}];
 MapThread[
  (res = Complement[Intersection @@ Lookup[VCG, #2], {polygonind}];
    If[res ≠ {},
     seq = Partition[CVG[First@res], 2, 1, 1];
     AppendTo[CVG,
      First@res → DeleteDuplicates@
        Flatten@SequenceSplit[seq, {x___, p : {OrderlessPatternSequence[
                #2[[1]], #2[[-1]]]}, y___} :→ {x, Insert[p, #1, 2], y}]
     ];
    ]) &, {newkeys, edgeinds[[posIntersections]]}];
```

```
  repPart =
   Thread[{Thread[{ReverseSort@posIntersections, 2}], Reverse[intersectionPts]}];
  {α, β} = intersectionPts;
  AppendTo[ptToIndAssoc, newPtToInds];
  AppendTo[indtoPtAssoc, Reverse[newPtToInds, 2]];
  contour =
   DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, repPart], 1];
  poly1 = Join @@ SequenceCases[contour, {___, α} | {β, ___}];
  poly2 = Join @@ SequenceCases[contour, {α, __, β}];
  KeyDropFrom[CVG, polygonind];
  addcellsRule = Thread[newcells → {poly1, poly2}];
  AppendTo[CVG, addcellsRule /. ptToIndAssoc];
  {indtoPtAssoc, CVG, Append[KeyDrop[areaAssoc, polygonind],
    MapAt[Area@*Polygon, addcellsRule, {All, 2}]],
   Append[KeyDrop[perimAssoc, polygonind],
    MapAt[Perimeter@*Polygon, addcellsRule, {All, 2}]]}
 ];
```

force computation

In[140]:=

```
F_AreaElasticity[indTopts_, vertexToCellG_, cellToVertexG_, areaPolygonAssoc_] :=
 Block[{cellinds, force, vertKeys = Keys[indTopts],
   vertLs, vertex, gc, gcc, diffVec, grad, coeff},
  First@*Last@Reap@Do[
     cellinds = Lookup[vertexToCellG, i];
     force = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = Lookup[indTopts, Lookup[cellToVertexG, j]];
      gcc = getCounterClockwise[vertex, vertLs];
      gc = getClockwise[vertex, vertLs];
      diffVec = gcc - gc;
      grad = 1/2 {{0, 1}, {-1, 0}}.diffVec;
      coeff = 2 ka (areaPolygonAssoc[j] - A0);
      force += coeff * grad, {j, cellinds}
     ];
     Sow@force, {i, vertKeys}]
 ]
```

In[141]:=
```
{{0, 1}, {-1, 0}}.(getCounterClockwise[{x_i, y_i}, {{x_(i-1), y_(i-1)}, {x_i, y_i}, {x_(i+1), y_(i+1)}}] -
    getClockwise[{x_i, y_i}, {{x_(i-1), y_(i-1)}, {x_i, y_i}, {x_(i+1), y_(i+1)}}]) // MatrixForm
```

Out[141]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

In[142]:=
```
MatrixForm[{{0, 1}, {-1, 0}}.({x_(i+1), y_(i+1)} - {x_(i-1), y_(i-1)})]
```

Out[142]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

In[143]:=
```
F_PerimeterElasticity[indTopts_, vertexToCellG_, cellToVertexG_, periPolygonAssoc_] :=
 Block[{cellinds, force, vertKeys = Keys[indTopts], vertLs,
   vertex, gc, gcc, v1, v2, coeff, grad},
  First@*Last@Reap@Do[
     cellinds = Lookup[vertexToCellG, i];
     force = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = Lookup[indTopts, Lookup[cellToVertexG, j]];
      gc = getClockwise[vertex, vertLs];
      v1 = Normalize[vertex - gc];
      gcc = getCounterClockwise[vertex, vertLs];
      v2 = Normalize[gcc - vertex];
      grad = v1 - v2;
      coeff = 2 γ (periPolygonAssoc[j] - P0);
      force += coeff * grad, {j, cellinds}
     ];
     Sow@force, {i, vertKeys}]
 ]
```

In[144]:=
```
MatrixForm@Normalize[{x_i, y_i} - {x_j, y_j}]
```

Out[144]//MatrixForm=

$$\begin{pmatrix} \frac{x_i - x_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \\ \frac{y_i - y_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \end{pmatrix}$$

In[145]:=
```
F_LineTension[indTopts_, ptsToInd_, edges_] :=
 Block[{vertKeys = Keys@indTopts, $v1, $v2, v1, force, uv},
  force = AssociationThread[vertKeys → 0.];
  Do[
   {$v1, $v2} = Lookup[indTopts, i];
   uv = Normalize[$v1 - $v2];
   v1 = ptsToInd[$v1];
   force[v1] += κ * uv, {i, edges}];
  Values[force]
 ]
```

In[146]:=
```
F_T[indTopts_, ptsToInds_, vertexToCellG_,
   cellToVertexG_, areaPolygonAssoc_, periPolygonAssoc_, edges_] := -(
    F_AreaElasticity[indTopts, vertexToCellG, cellToVertexG, areaPolygonAssoc] +
     F_PerimeterElasticity[indTopts, vertexToCellG, cellToVertexG, periPolygonAssoc] +
     F_LineTension[indTopts, ptsToInds, edges]);
```

create mesh

In[147]:=
```
SeedRandom[3];
mesh = VoronoiMesh[RandomReal[1, {200, 2}], {{0, 1}, {0, 1}}, ImageSize → Medium];
```

In[149]:=
```
pts = MeshPrimitives[mesh, 0] /. Point → Sequence;
```

In[150]:=
```
cornerpts = pts[[-4 ;;]];
pts = pts[[1 ;; -5]];
```

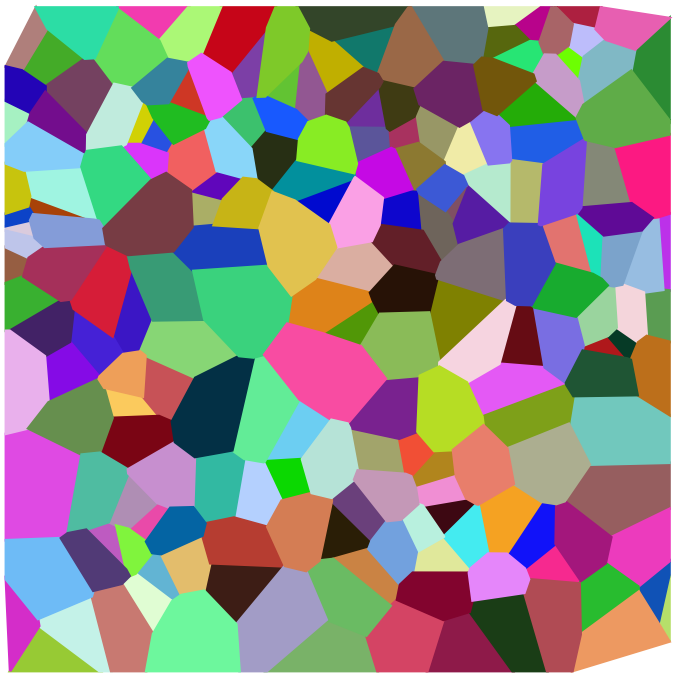
In[152]:=
```
$ptsToInd = ptsToInd = AssociationThread[pts → Range@Length@pts];
$indTopts = indTopts = AssociationMap[Reverse][ptsToInd];
```

In[154]:=
```
cellmeshprim = MeshPrimitives[mesh, 2];
cells = (MeshPrimitives[#, 0] & /@ cellmeshprim) /. Point → Sequence /.
   Thread[cornerpts → Nothing];
```

In[156]:=
```
$cellToVertexG =
  cellToVertexG = AssociationThread[Range[Length@cells] → Map[ptsToInd, cells, {2}]];
$vertexToCell = vertexToCell =
   GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
```

In[158]:=
```
Graphics[Map[{RandomColor[], Polygon@Lookup[indTopts, #]} &, Values@cellToVertexG],
 ImageSize → Medium]
```

Out[158]=

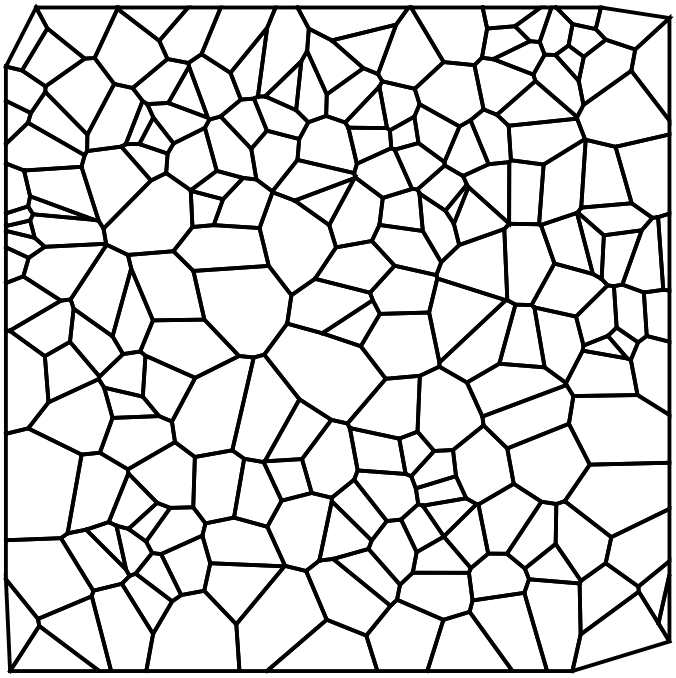


In[159]:=
```
(*edges=Flatten[Map[Partition[#,2,1,1]&,Values[cellToVertexG]],1];*)
$cellToPts = cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
$periPolygonAssoc = periPolygonAssoc = perimeterPolygon /@ cellToPts;
$areaPolygonAssoc = areaPolygonAssoc = areaPolygon /@ cellToPts;
```

In[162]:=
```
Clear[plt, indTopts, ptsToInd, vertexToCell,
  cellToVertexG, periPolygonAssoc, areaPolygonAssoc, cellToPts, edges];
```

In[163]:=
```
pltOriginal = Graphics[{Black, Thick,
   Values@Map[Line[Join[##, {First@#}]] &@Lookup[$indTopts, #] &, $cellToVertexG]}]
```

Out[163]=



run simulation 1

In[ ]:=
```
(* PARAMETERS*)
```

In[ ]:=
```
ka = 1; A0 = 1; γ = 0.04 * ka * A0; δt = 0.0013; P0 = 0; κ = 0.025;
```

In[ ]:=
```
ka = 1000; A0 = 0.01; γ = 0.04 * ka * A0; δt = 0.0008; P0 = 0; κ = 0.025;
```

In[ ]:=
```
{γ / (ka A0), κ / (ka A0^(3/2))}
```

Out[ ]=
```
{0.04, 0.025}
```

In[ ]:=
```
Tmax = 4000;
t = δt;
indTopts = $indTopts;
ptsToInd = $ptsToInd;
vertexToCell = $vertexToCell;
cellToVertexG = $cellToVertexG;
periPolygonAssoc = $periPolygonAssoc;
areaPolygonAssoc = $areaPolygonAssoc;
cellToPts = $cellToPts;
edges = DeleteDuplicatesBy[
   Flatten[Map[Partition[#, 2, 1, 1] &, Values@$cellToVertexG], 1], Sort];
```

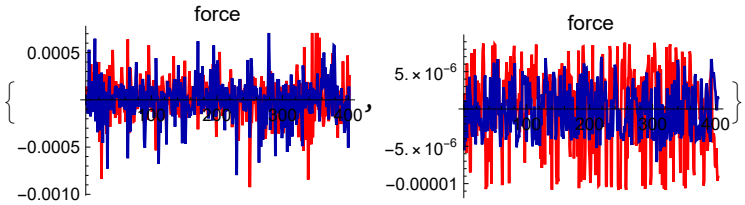
In[ ]:=
```
tt = {};
SeedRandom[1];
```

```
Module[{cellsToRemove, vertsToRemove, edgechanged, polydiv},
  saveres = First@Last@Reap@Monitor[
       While[t ≤ Tmax δt,
        (* T2 transitions *)
        cj = Round[t / δt];
        {cellsToRemove, vertsToRemove} = cellsforT2[areaPolygonAssoc, cellToVertexG];
        If[cellsToRemove ≠ {},
         {indTopts, cellToVertexG, vertexToCell, areaPolygonAssoc, periPolygonAssoc} =
          T2TransitionFn[{cellsToRemove, vertsToRemove}, indTopts,
           cellToVertexG, areaPolygonAssoc, periPolygonAssoc]
        ];

        (* T1 transitions *)
        edges = DeleteDuplicatesBy[
          Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1], Sort];
        {edgechanged, indTopts, cellToVertexG, vertexToCell} =
         T1transitionFn[edges, indTopts, vertexToCell, cellToVertexG];
        cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ cellToPts;
        periPolygonAssoc = perimeterPolygon /@ cellToPts;

        (* Divisions *)
        polydiv = selectDivCells[areaPolygonAssoc];
        (*polydiv=pickcellsDiv[cellToVertexG,areaPolygonAssoc];*)
        If[polydiv ≠ {},
         Scan[
          ({indTopts, cellToVertexG, areaPolygonAssoc, periPolygonAssoc} = cellDivision[
              #, indTopts, areaPolygonAssoc, periPolygonAssoc, cellToVertexG]) &,
          polydiv];
         vertexToCell = GroupBy[
           Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
        ];
        ptsToInd = AssociationMap[Reverse, indTopts];
        edges = Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1];
        (* update positions *)
        If[cj == 5 || (cj + 1) == Tmax, AppendTo[tt, F_T[indTopts, ptsToInd, vertexToCell,
            cellToVertexG, areaPolygonAssoc, periPolygonAssoc, edges] δt]];

        indTopts = AssociationThread[
          Keys[indTopts] → (Values[indTopts] + F_T[indTopts, ptsToInd, vertexToCell,
               cellToVertexG, areaPolygonAssoc, periPolygonAssoc, edges] δt)];
        cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ cellToPts;
        periPolygonAssoc = perimeterPolygon /@ cellToPts;
        (*plt=Graphics[{ColorData[1][1],Thick,Values@Map[Line[Join[##,{First@#}]]&@
                Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
        (*plt=Graphics[{FaceForm[LightBlue],EdgeForm[{Black}],
```

```
            Values[Polygon@Lookup[indTopts,#]&/@cellToVertexG]},ImageSize→Large];*)
        plt = Graphics[{■, Thick, Values@Map[Line[Join[##, {First@#}]] &@
              Lookup[indTopts, #] &, cellToVertexG]}, ImageSize → Medium];
        If[Mod[cj, 2] == 0, Sow[plt]];
        t += δt;
       ], {cj, plt}
      ]
];
```

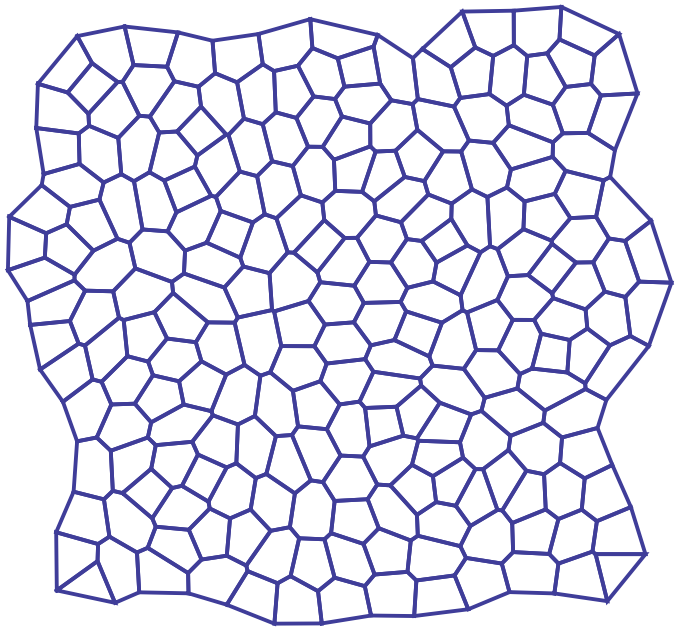
In[ ]:=
```
(ListLinePlot[#, PlotStyle → {{Thickness[0.01], Red}, {Thickness[0.01], Darker@Blue}},
    PlotLabel → "force"] &) @* Transpose /@ tt
```

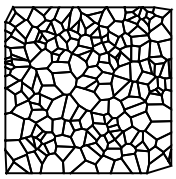
Out[ ]=



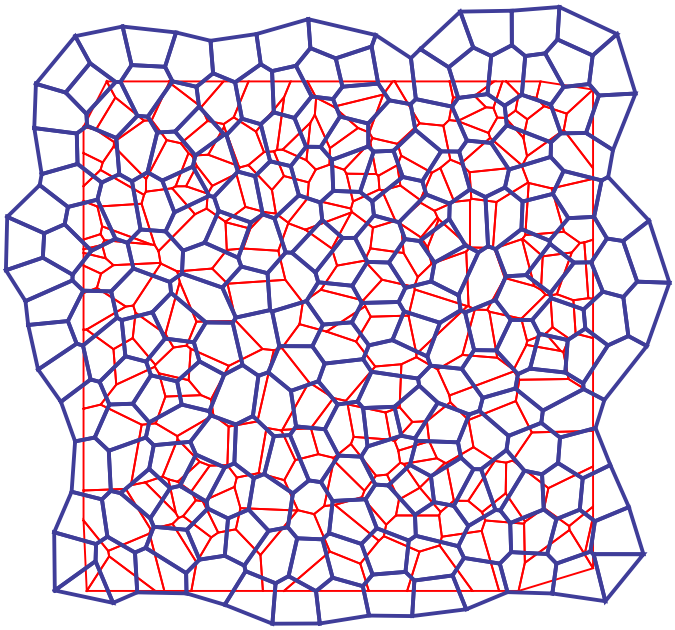
In[ ]:=
```
Graphics[{■, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indTopts, #] &, cellToVertexG]},
 ImageSize → Medium]
```

Out[ ]=



In[ ]:=
```
Show[  /. Black → Red, Graphics[{ , Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indTopts, #] &, cellToVertexG]}]]
```

Out[ ]=



run simulation 2

In[164]:=
```
ka = 1000; A0 = 0.01; γ = 0.015 * ka * A0; P0 = 0; κ = -0.025; δt = 0.0001;
```

In[165]:=
```
Tmax = 4000;
t = δt;
indTopts = $indTopts;
ptsToInd = $ptsToInd;
vertexToCell = $vertexToCell;
cellToVertexG = $cellToVertexG;
periPolygonAssoc = $periPolygonAssoc;
areaPolygonAssoc = $areaPolygonAssoc;
cellToPts = $cellToPts;
edges = DeleteDuplicatesBy[
   Flatten[Map[Partition[#, 2, 1, 1] &, Values@$cellToVertexG], 1], Sort];
```

In[175]:=
```
tt = {};
SeedRandom[1];
```

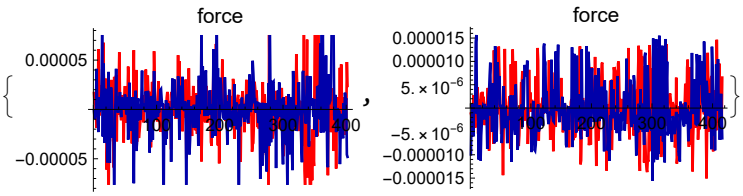
```
Module[{cellsToRemove, vertsToRemove, edgechanged, polydiv},
  saveres = First@Last@Reap@Monitor[
       While[t ≤ Tmax δt,
        (* T2 transitions *)
        cj = Round[t / δt];
        {cellsToRemove, vertsToRemove} = cellsforT2[areaPolygonAssoc, cellToVertexG];
        If[cellsToRemove ≠ {},
         {indTopts, cellToVertexG, vertexToCell, areaPolygonAssoc, periPolygonAssoc} =
          T2TransitionFn[{cellsToRemove, vertsToRemove}, indTopts,
           cellToVertexG, areaPolygonAssoc, periPolygonAssoc]
        ];

        (* T1 transitions *)
        edges = DeleteDuplicatesBy[
          Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1], Sort];
        {edgechanged, indTopts, cellToVertexG, vertexToCell} =
         T1transitionFn[edges, indTopts, vertexToCell, cellToVertexG];
        cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ cellToPts;
        periPolygonAssoc = perimeterPolygon /@ cellToPts;

        (* Divisions *)
        polydiv = selectDivCells[areaPolygonAssoc];
        (*polydiv=pickcellsDiv[cellToVertexG,areaPolygonAssoc];*)
        If[polydiv ≠ {},
         Scan[
          ({indTopts, cellToVertexG, areaPolygonAssoc, periPolygonAssoc} = cellDivision[
              #, indTopts, areaPolygonAssoc, periPolygonAssoc, cellToVertexG]) &,
          polydiv];
         vertexToCell = GroupBy[
           Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
        ];
        ptsToInd = AssociationMap[Reverse, indTopts];
        edges = Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1];
        (* update positions *)
        If[cj == 5 || (cj + 1) == Tmax, AppendTo[tt, F_T[indTopts, ptsToInd, vertexToCell,
            cellToVertexG, areaPolygonAssoc, periPolygonAssoc, edges] δt]];

        indTopts = AssociationThread[
          Keys[indTopts] → (Values[indTopts] + F_T[indTopts, ptsToInd, vertexToCell,
               cellToVertexG, areaPolygonAssoc, periPolygonAssoc, edges] δt)];
        cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ cellToPts;
        periPolygonAssoc = perimeterPolygon /@ cellToPts;
        (*plt=Graphics[{ColorData[1][1],Thick,Values@Map[Line[Join[##,{First@#}]]&@
                Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
        (*plt=Graphics[{FaceForm[LightBlue],EdgeForm[{Black}],
```

```
        Values[Polygon@Lookup[indTopts,#]&/@cellToVertexG]},ImageSize→Large];*)
      plt = Graphics[{■, Thick, Values@Map[Line[Join[##, {First@#}]] &@
            Lookup[indTopts, #] &, cellToVertexG]}, ImageSize → Medium];
      If[Mod[cj, 2] == 0, Sow[plt]];
      t += δt;
     ], {cj, plt}
    ]
];
```

```
(*  with soft network parameter *)
```

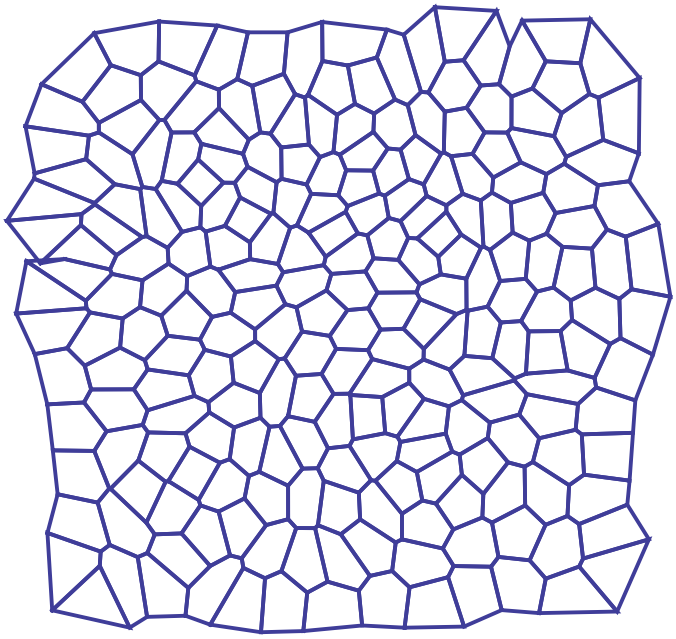
In[178]:=
```
(ListLinePlot[#, PlotStyle → {{Thickness[0.01], Red}, {Thickness[0.01], Darker@Blue}},
    PlotLabel → "force"] &) @* Transpose /@ tt
```

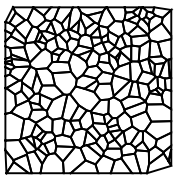
Out[178]=



In[180]:=
```
Graphics[{■, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indTopts, #] &, cellToVertexG]},
 ImageSize → Medium]
```

Out[180]=



In[183]:=
```
Show[  /. Black → Red, Graphics[{ , Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indTopts, #] &, cellToVertexG]}]]
```

Out[183]=