2-dimensional vertex model :→ simulating epithelial tissue

In[1]:=
```
LaunchKernels[]
ParallelTable[$KernelID, {i, $KernelCount}]
```

Out[1]= {KernelObject[Name: Local kernel KernelID: 1], KernelObject[Name: Local kernel KernelID: 2],
KernelObject[Name: Local kernel KernelID: 3], KernelObject[Name: Local kernel KernelID: 4],
KernelObject[Name: Local kernel KernelID: 5], KernelObject[Name: Local kernel KernelID: 6]}

Out[2]= {6, 5, 4, 3, 2, 1}

geometrical f(x)s

In[5]:=
```
(*get vertex in the list by ccw order*)
Clear[getCounterClockwise];
getCounterClockwise[vertex_, vertices_] := Block[{pos, v},
   pos = First @@ Position[vertices, vertex];
   If[pos == Length[vertices], pos = 1, pos += 1];
   vertices[[pos]]
  ];
```

In[7]:=
```
(*get vertex in the list by cw order*)
Clear[getClockwise];
getClockwise[vertex_, vertices_] := Block[{ls, pos},
   pos = First @@ Position[vertices, vertex];
   If[pos == 1, pos = Length[vertices], pos -= 1];
   vertices[[pos]]
  ];
```

In[9]:= $\texttt{getCounterClockwise}[\{x_i, y_i\}, \{\{x_{i-1}, y_{i-1}\}, \{x_i, y_i\}, \{x_{i+1}, y_{i+1}\}, d, e\}]$

Out[9]= $\{x_{1+i}, y_{1+i}\}$

In[10]:= $\texttt{getClockwise}[\{x_i, y_i\}, \{\{x_{i-1}, y_{i-1}\}, \{x_i, y_i\}, \{x_{i+1}, y_{i+1}\}, d, e\}]$

Out[10]= $\{x_{-1+i}, y_{-1+i}\}$

In[11]:=
```
Clear[areaOfPolygon];
areaOfPolygon[cells_ /; Head[cells] === Association] :=
  Parallelize[Map[Area@*Polygon, cells]];
```

In[13]:=
```
Clear[areaPolygon];
areaPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  0.5 Abs@Total[(#[[1, 1]] * #[[2, 2]]) - (#[[2, 1]] * #[[1, 2]]) & /@ edges]
 ]
```

In[15]:=
```
Clear[perimeterOfPolygon];
perimeterOfPolygon[cells_ /; Head[cells] === Association] :=
  Parallelize[(Perimeter@*Polygon) /@ cells];
```

In[17]:=
```
Clear[perimeterPolygon];
perimeterPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  Total[Apply[EuclideanDistance] /@ edges]
 ]
```

In[19]:=
```
Clear[centroidPolygon];
centroidPolygon[vertices_] := Mean[vertices]
```

In[21]:=
```
(*counterclockwise polygonQ*)
Block[{signedarea = 0, j, vertlen = 5},
  Do[
   j = Mod[i, vertlen] + 1;
   signedarea += (x_i y_j - x_j y_i),
   {i, vertlen}];
  Echo[1/2 (signedarea)]
 ];
```

» $\frac{1}{2}\left(-x_2 y_1 + x_5 y_1 + x_1 y_2 - x_3 y_2 + x_2 y_3 - x_4 y_3 + x_3 y_4 - x_5 y_4 - x_1 y_5 + x_4 y_5\right)$

In[22]:=
```
Clear[polyCounterClockwiseQ];
polyCounterClockwiseQ[poly_] := Block[{area = 0, j, vertLength = Length[poly]},
  Do[
   j = Mod[i, vertLength] + 1;
   area += poly[[i, 1]] * poly[[j, 2]];
   area -= poly[[j, 1]] * poly[[i, 2]],
   {i, vertLength}
  ];
  (area / 2.) > 0
 ]
```

In[24]:=
```
(*Clear[sortCC];
sortCC[polyinds_,indTopts_,ptsToInds_]:=Block[{cent,poly},
  poly=Lookup[indTopts,polyinds];
  Lookup[ptsToInds,
   DeleteDuplicates@Flatten[MeshPrimitives[ConvexHullMesh[poly],1]/.Line→Sequence,1]
  ]
 ];*)
```

In[25]:=
```
(*sort points for a convex polygon in counter-clockwise direction*)
```

In[26]:=
```
Clear[sortPointsCC];
sortPointsCC[polyinds_, indTopts_, ptsToInds_] := Block[{cent, ordering, polyPoints},
  polyPoints = Lookup[indTopts, polyinds];
  cent = Mean[polyPoints];
  ordering = Ordering[ArcTan[#[[1]], #[[2]]] &@(# - cent) & /@ polyPoints];
  Lookup[ptsToInds, Part[polyPoints, ordering]]
 ]
```

In[28]:=
```
outeredges[indToPtsAssoc_, localtopology_] := Block[{k, temp, tcells, assoc},
   Reap[
      Scan[(k = #; temp = localtopology[k];
         tcells = temp[[2]];
         If[tcells ≠ {},
          MapAt[Sow@Cases[Partition[#, 2, 1,
                 1], OrderlessPatternSequence[{x : indToPtsAssoc[k], y : _}] :>
                 {x, y}] &, temp[[1]], {Key[#]} & /@ tcells];
         ]) &, Keys@indToPtsAssoc]
     ][[2]] // Flatten[#, 2] &
  ];
```

mesh restructuring operations

simulation domain

In[29]:=
```
xLim = yLim = {-1, 1};
𝒟 = Rectangle[{First@xLim, First@yLim}, {Last@xLim, Last@yLim}];
```

local topology

In[31]:=
```
Clear@outerCellsFn;
outerCellsFn[faceListCoords_, vertexToCell_, ptsToIndAssoc_] :=
  With[{xlim1 = xLim[[1]], xlim2 = xLim[[2]], ylim1 = yLim[[1]], ylim2 = yLim[[2]]},
   Block[{boundaryCells, bcells, temp, res},
    temp = faceListCoords;
    boundaryCells =
     Union[First /@ Position[temp, {x_ /; x ≥ xlim2, _} | {x_ /; x ≤ xlim1, _} |
           {_, y_ /; y ≥ ylim2} | {_, y_ /; y ≤ ylim1}] /. Key[x_] :→ x];
    bcells = KeyTake[faceListCoords, boundaryCells];
    res = Union@(Flatten@Lookup[vertexToCell, Lookup[ptsToIndAssoc, DeleteDuplicates@
             Cases[bcells, {x_ /; x ≥ xlim2, _} {x_ /; x ≤ xlim1, _} {_, y_ /; y ≥ ylim2}
               {_, y_ /; y ≤ ylim1}, {3}] /. periodicRules]] ~Join~ boundaryCells);
    res
   ]
  ];
```

In[33]:=
```
ClearAll[periodicRules, transformRules];
periodicRules::Information =
  "shift the points outside the simulation domain to inside the domain";
transformRules::Information =
  "vector that shifts the point outside the simulation domain back inside";
With[{xlim1 = xLim[[1]], xlim2 = xLim[[2]], ylim1 = yLim[[1]], ylim2 = yLim[[2]], dstep = 2},
  periodicRules = Dispatch[{
     {x_ /; x ≥ xlim2, y_ /; y ≤ ylim1} :> SetPrecision[{x - dstep, y + dstep}, 10],
     {x_ /; x ≥ xlim2, y_ /; ylim1 < y < ylim2} :> SetPrecision[{x - dstep, y}, 10],
     {x_ /; xlim1 < x < xlim2, y_ /; y ≤ ylim1} :> SetPrecision[{x, y + dstep}, 10],
     {x_ /; x ≤ xlim1, y_ /; y ≤ ylim1} :> SetPrecision[{x + dstep, y + dstep}, 10],
     {x_ /; x ≤ xlim1, y_ /; ylim1 < y < ylim2} :> SetPrecision[{x + dstep, y}, 10],
     {x_ /; x ≤ xlim1, y_ /; y ≥ ylim2} :> SetPrecision[{x + dstep, y - dstep}, 10],
     {x_ /; xlim1 < x < xlim2, y_ /; y ≥ ylim2} :> SetPrecision[{x, y - dstep}, 10],
     {x_ /; x ≥ xlim2, y_ /; y ≥ ylim2} :> SetPrecision[{x - dstep, y - dstep}, 10]
    }];

  transformRules = Dispatch[{
     {x_ /; x ≥ xlim2, y_ /; y ≤ ylim1} :> {-dstep, dstep} ~ SetPrecision ~ 10,
     {x_ /; x ≥ xlim2, y_ /; ylim1 < y < ylim2} :> {-dstep, 0} ~ SetPrecision ~ 10,
     {x_ /; xlim1 < x < xlim2, y_ /; y ≤ ylim1} :> {0, dstep} ~ SetPrecision ~ 10,
     {x_ /; x ≤ xlim1, y_ /; y ≤ ylim1} :> {dstep, dstep} ~ SetPrecision ~ 10,
     {x_ /; x ≤ xlim1, y_ /; ylim1 < y < ylim2} :> {dstep, 0} ~ SetPrecision ~ 10,
     {x_ /; x ≤ xlim1, y_ /; y ≥ ylim2} :> {dstep, -dstep} ~ SetPrecision ~ 10,
     {x_ /; xlim1 < x < xlim2, y_ /; y ≥ ylim2} :> {0, -dstep} ~ SetPrecision ~ 10,
     {x_ /; x ≥ xlim2, y_ /; y ≥ ylim2} :> {-dstep, -dstep} ~ SetPrecision ~ 10,
     {___Real} :> {0, 0} ~ SetPrecision ~ 10}];
 ];
```

In[37]:=
```
Clear@getLocalTopology;
getLocalTopology[ptsToIndAssoc_, indToPtsAssoc_, vertexToCell_,
    cellVertexGrouping_, wrappedMat_, faceListCoords_][vertices_] :=
  Block[{localtopology = <||>, wrappedcellList = {}, vertcellconns,
    localcellunion, v, wrappedcellpos, vertcs = vertices, rl1, rl2,
    transVector, wrappedcellCoords, wrappedcells, vertOutofBounds,
    shiftedPt, transvecList = {}, $faceListCoords = faceListCoords,
    vertexQ, boundsCheck, rules, extractcellkeys, vertind,
    cellsconnected, wrappedcellsrem},
   vertexQ = MatchQ[vertices, {__?NumberQ}];
   If[vertexQ,
    (vertcellconns =
      AssociationThread[{#} , {vertexToCell[ptsToIndAssoc[#]]}] &@vertices;
     vertcs = {vertices};
     localcellunion = Flatten[Values@vertcellconns]),
    (vertcellconns = AssociationThread[# ,
         Lookup[vertexToCell, Lookup[ptsToIndAssoc, #]]] &@vertices;
```

```
    localcellunion = Union@Flatten[Values@vertcellconns])
  ];
  If[localcellunion ≠ {},
   AppendTo[localtopology,
    Thread[localcellunion →
      Map[Lookup[indToPtsAssoc, #] &, cellVertexGrouping /@ localcellunion, {2}]]
   ]
  ];
  (* condition to be an internal edge: both vertices should have 3 neighbours *)
  (* if a vertex has 3 cells in its local neighbourhood then the entire
    network topology about the vertex is known → no wrapping required *)
  (* else we need to wrap around the vertex because other cells
    are connected to it → periodic boundary conditions *)
  With[{vert = #},
     vertind = ptsToIndAssoc[vert];
     cellsconnected = vertexToCell[vertind];
     If[Length[cellsconnected] ≠ 3,
      If[(𝒟~RegionMember~vert),
        v = vert;
        With[{x = v[[1]], y = v[[2]]},
         boundsCheck = (x == xLim[[1]] || x == xLim[[2]] || y == yLim[[1]] || y == yLim[[2]])];
        extractcellkeys = If[boundsCheck,
          {rl1, rl2} = {v, v /. periodicRules};
          rules = Block[{x$},
            With[{r = rl1, s = rl2},
             DeleteDuplicates[HoldPattern[SameQ[x$, r]] || HoldPattern[SameQ[x$, s]]]
            ]
           ];
          Position@@
           With[{rule = rules}, Hold[wrappedMat, x_ /; ReleaseHold@rule, {2}]],
          Position[wrappedMat, x_ /; SameQ[x, v], {2}]
         ];
        (* find cell indices that are attached to the vertex in wrappedMat *)
        wrappedcellpos = DeleteDuplicatesBy[
          Cases[extractcellkeys,
           {Key[p : Except[Alternatives@@
                  Join[localcellunion, Flatten@wrappedcellList]]], y__} :> {p, y}],
          First];
        (*wrappedcellpos = wrappedcellpos/.
           {Alternatives@@Flatten[wrappedcellList],__} :> Sequence[];*)
        (* if a wrapped cell has not been considered earlier (i.e. is new)
         then we translate it to the position of the vertex *)
        If[wrappedcellpos ≠ {},
         If[vertexQ,
          transVector = SetPrecision[(v - Extract[$faceListCoords,
                  Replace[#, {p_, q__} :> {Key[p], q}]]) & /@ wrappedcellpos, 10],
          (* call to function is enquiring an edge and not a vertex*)
```

```
  transVector = SetPrecision[(v - Extract[$faceListCoords,
          Replace[#, {p_, q__} :> {Key[p], q}]]) & /@ wrappedcellpos, 10]
 ];
 wrappedcellCoords = MapThread[#1 -> Map[Function[x,
       SetPrecision[x + #2, 10]], $faceListCoords[[Key@#1]], {1}] &,
   {First /@ wrappedcellpos, transVector}];
 wrappedcells = Keys@wrappedcellCoords;
 AppendTo[wrappedcellList, Flatten@wrappedcells];
 AppendTo[transvecList, transVector];
 AppendTo[localtopology, wrappedcellCoords];
],
(* the else clause: vertex is out of bounds *)
vertOutofBounds = vert;
(* translate the vertex back into mesh *)
transVector = vertOutofBounds /. transformRules;
shiftedPt = SetPrecision[vertOutofBounds + transVector, 10];
(* ------------- CORE B ------------- *)
(* find which cells the
 shifted vertex is a part of in the wrapped matrix *)
wrappedcells = Complement[
  Union@Cases[Position[wrappedMat, x_ /; SameQ[x, shiftedPt], {2}],
     x_Key :> Sequence @@ x, {2}] /.
   Alternatives @@ localcellunion -> Sequence[],
  Flatten@wrappedcellList];
(*forming local topology now that we know the wrapped cells *)
If[wrappedcells ≠ {},
 AppendTo[wrappedcellList, Flatten@wrappedcells];
 wrappedcellCoords = AssociationThread[wrappedcells,
   Map[Lookup[indToPtsAssoc, #] &,
    cellVertexGrouping[#] & /@ wrappedcells, {1}]];
 With[{opt = (vertOutofBounds /. periodicRules)},
  Block[{pos, vertref, transvec},
    Do[
     With[{cellcoords = wrappedcellCoords[cell]},
      pos = FirstPosition[cellcoords /. periodicRules, opt];
      vertref = Extract[cellcoords, pos];
      transvec = SetPrecision[vertOutofBounds - vertref, 10];
      AppendTo[transvecList, transvec];
      AppendTo[localtopology,
       cell -> Map[SetPrecision[# + transvec, 10] &, cellcoords]];
     ], {cell, wrappedcells}]
   ];
 ];
];
(* to detect wrapped cells not detected by CORE B*)
(* ------------- CORE C ------------- *)
Block[{pos, celllocs, ls, transvec, assoc, tvecLs = {}, ckey},
```

```
          ls = Union@Flatten@Join[cellsconnected, wrappedcells];
          If[Length[ls] ≠ 3,
           pos = Position[$faceListCoords, x_ /; SameQ[x, shiftedPt], {2}];
           celllocs = DeleteDuplicatesBy[Cases[pos, Except[{Key[Alternatives @@ ls],
                 __}]], First] /. {Key[x_], z__} :> {Key[x], {z}};
           If[celllocs ≠ {},
            celllocs = Transpose@celllocs;
            assoc = <|
              MapThread[
               (transvec = SetPrecision[vertOutofBounds -
                    Extract[$faceListCoords[Sequence @@ #1], #2], 10];
                 ckey = Identity @@ #1;
                 AppendTo[tvecLs, transvec];
                 ckey → Map[SetPrecision[Lookup[indToPtsAssoc, #] + transvec, 10] &,
                   cellVertexGrouping[Sequence @@ #1], {1}]
                ) &, celllocs]
              |>;
            AppendTo[localtopology, assoc];
            AppendTo[wrappedcellList, Keys@assoc];
            AppendTo[transvecList, tvecLs];
           ];
          ];
         ];
        ];
       ];
      ] & /@ vertcs;
     transvecList = Which[
       MatchQ[transvecList, {{{__?NumberQ}}}], First[transvecList],
       MatchQ[transvecList, {{__?NumberQ} ..}], transvecList,
       True, transvecList //. {x___, {p : {__?NumberQ} ..}, y___} :> {x, p, y}
      ];
     {localtopology, Flatten@wrappedcellList, transvecList}
    ];
```

T1 transition

In[49]:=
```
(*find edge function*)
```

In[50]:=
```
bagged = CreateDataStructure["DynamicArray"];
```

find edges

In[51]:=
```
Clear@edgesforT1;
edgesforT1[edgeLs_, indToPts_, threshLength_ : 0.0015] :=
  Block[{edges, dist, sel, filt, b, cand, sameedg2Q, del},
   edges = Lookup[indToPts, #] & /@ edgeLs;
   dist = EuclideanDistance @@ # & /@ edges;
   sel = Pick[edges, Thread[dist ≤ threshLength], True];
   If[Length[sel] == 1,
    Nothing,
    filt = sel /. periodicRules;
    b = MapThread[SameQ, {filt, sel}];
    cand = Pick[filt, b, False];
    sameedg2Q = MemberQ[sel, {OrderlessPatternSequence @@ #}] & /@ cand;
    del = (OrderlessPatternSequence @@@ Pick[cand, sameedg2Q]);
    If[Length[cand] > 1, del = Alternatives @@ del];
    sel = DeleteCases[sel, {del}]
   ];
   sel
  ];
```

T1

In[53]:=
```
Clear@T1transitionFn;
T1transitionFn[edges_, indToPtsAssoc_, ptsToIndAssoc_,
   vertexToCellG_, cellToVertexG_, wrappedMat_, faceListCoords_, dSep_ : 0.01] :=
  Block[{edgeind, connectedcellKeys, edge, newpts, cellvertices, pos,
    cellpolys, memF, keyscellP, selcellKeys, ptToCell, newptsindices,
    indToPts = indToPtsAssoc, ptsToInds = ptsToIndAssoc, PtIndToCell,
    keysToMap, f1, otherkeys, f2, bag = bagged, vertToCellG = vertexToCellG,
    cellToVertG = cellToVertexG, testpts, localtopology, translCells, transVector,
    edgepts, celltopo, polysharingEdge, ls, cs, cc, ccw, trimmedtopo, ordering,
    edgeordered, topotransl, newtransvert, findEdges, $wrappedMat = wrappedMat,
    $faceListCoords = faceListCoords, cellToVertGC, dropinds, bagopen, maxlab},
   findEdges = edgesforT1[edges, indToPts];
   (* finding all possible edges for T1 transition *)
   If[findEdges ≠ {},
    (*run if there are any edges for T1*)
    Scan[
     (edge = #;
      cellToVertGC = cellToVertG;
      edgeind = Lookup[ptsToInds, edge];
```

```
If[ContainsAll[Keys[indToPts], edgeind],
 (* should be an edge not
  connected to an edge that has already undergone a T1 *)
 (* let us get the local network topology *)
 {localtopology, translCells, transVector} =
  getLocalTopology[ptsToInds, indToPts, vertToCellG,
      cellToVertG, $wrappedMat, $faceListCoords][#] &[edge];
 If[! DuplicateFreeQ[translCells],
  {translCells, transVector} =
   DeleteDuplicates[Transpose[{translCells, transVector}]]^T
 ];
 connectedcellKeys = Keys@localtopology;
 celltopo = Values@localtopology;
 (*Print[Show[pvor,Graphics[{Polygon/@celltopo,Red,Line@edge}]]];*)
 newpts = With[{midPt = Mean@edge},
   SetPrecision[midPt + dSep Normalize[(# - midPt)], 10] & /@
    Flatten[RotationTransform[-π/2, midPt] /@ {edge}, 1]
  ];
 testpts =
  With[{midPt = Mean@edge}, midPt + 0.00001 Normalize[(# - midPt)] & /@ newpts];
 (*plt1=Graphics[{{FaceForm[LightGray],EdgeForm[{Thick,Black}],
     Polygon/@celltopo},{Red,Line@edge,Orange,
     Point@newpts,Green,Point@testpts}},ImageSize→Tiny];
 Print@plt1;*)
 (*which cells contain both vertices*)
 pos = Position[celltopo,
   {OrderlessPatternSequence[___, First[edge], ___, Last[edge], ___]}, {1}];
 polysharingEdge = Extract[celltopo, pos];
 (* the edge should not be part of any Δ *)
 bagopen = Union@*Flatten@*Normal@bag;
 (*Print[Show[Graphics[{Polygon/@Values@$faceListCoords}],plt1]];*)
 If[(AllTrue[polysharingEdge, Length[#] ≠ 3 &]) && ContainsNone[
    edgeind, bagopen] && ContainsNone[edge /. periodicRules, bagopen],
  cellvertices = celltopo;
  cellpolys = Polygon /@ cellvertices;
  memF = Function[x, RegionMember@x, Listable][Extract[cellpolys, pos]];
  keyscellP = Extract[connectedcellKeys, pos];
  selcellKeys = Thread[keyscellP → memF];
  ptToCell = Quiet[# → First @@ Select[selcellKeys, Function[x, Last[x][#]]] & /@
      testpts /. HoldPattern[_ → First[]] → Nothing] ;
  (* testpt to cell *)
  ptToCell = ptToCell /. Thread[testpts → newpts];
  newptsindices = Range[# + 1, # + 2] &[Max@Keys@indToPts];
  KeyDropFrom[indToPts, edgeind];
  AppendTo[indToPts, Thread[newptsindices → newpts]];
```

```
 bag["Append", Lookup[ptsToInds, edge /. periodicRules]];
 ptsToInds = AssociationMap[Reverse, indToPts];
 bag["Append", edgeind];
 PtIndToCell = MapAt[ptsToInds, ptToCell, {All, 1}] /. Rule → List ;
 (*index to cell*)
 keysToMap = Map[{Lookup[indToPts, #[[1]]], Key@#[[2]]} &, PtIndToCell];
 f1 = Fold[MapAt[Function[x, DeleteDuplicates@Replace[x, (Alternatives @@
           edge ) :→ #2[[1]], {1}]], #1, #2[[-1]]] &, localtopology, keysToMap];
 otherkeys = List@* Key /@ Complement[connectedcellKeys, keyscellP];
 ls = Lookup[localtopology, First @@ otherkeys];
 cs = FirstCase[ls, Alternatives @@ edge];
 cc = getClockwise[cs, ls];
 ccw = getCounterClockwise[cs, ls];
 trimmedtopo =
  KeyDrop[localtopology, connectedcellKeys ~ Complement ~ keyscellP];
 ordering = {FirstCase[Position[trimmedtopo, cc],
    x : {Key[First@keyscellP], _} | {Key[Last@keyscellP], _} :→ First @@ x],
   FirstCase[Position[trimmedtopo, ccw], x : {Key[First@keyscellP],
         _} | {Key[Last@keyscellP], _} :→ First @@ x ]};
 edgeordered = ordering /. Reverse[ptToCell, 2];
 f2 = MapAt[Replace[#, Alternatives @@ edge :→ Splice[edgeordered], {1}] &,
   f1, First@otherkeys];
 f2 = MapAt[Replace[#, Alternatives @@ edge :→ Splice[Reverse[edgeordered]],
     {1}] &, f2, Last@otherkeys];
 (*plt2=Values@f2//Map[Polygon]//Graphics[{FaceForm[LightGray],
       EdgeForm[{Thin,Red}],#},ImageSize→Tiny]&;*)
 (* once we make the transition, we translate the cell back and
  add the shifted vertex into the indtopts and ptstoinds *)
 If[translCells ≠ {},
  f2 = Fold[MapAt[Function[x, Function[y, SetPrecision[y - #2[[2]], 10]] /@ x], #1,
      {#2[[1]]}] &, f2, Thread[{Key /@ translCells, transVector}]];
  newtransvert = Complement[Flatten[Values@f2, 1],
    Values@indToPts, SameTest → (#1 == #2 &)];
  If[newtransvert ≠ {},
   newptsindices =
    Range[# + 1, # + Length@newtransvert] &[Max[Keys@indToPts]];
   AppendTo[indToPts, Thread[newptsindices → newtransvert ]];
   AppendTo[ptsToInds, Thread[newtransvert → newptsindices]];
  ];
 ];
 (*plt3=Values@f2//Map[Polygon]//Graphics[
      {FaceForm[LightGray],EdgeForm[{Thin,Red}],#},ImageSize→Tiny]&;*)
 f2 = Lookup[ptsToInds, #] & /@ f2;
 AppendTo[cellToVertG, f2];
];
cellToVertG = KeySort@cellToVertG;
dropinds =
```

```
        Flatten@Map[Complement[cellToVertGC[#], cellToVertG[#]] &, translCells];
       KeyDropFrom[indToPts, dropinds];
       ptsToInds = AssociationMap[Reverse, indToPts];
       vertToCellG = KeySort@GroupBy[
          Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertG], First → Last];
       $faceListCoords = Lookup[indToPts, #] & /@ cellToVertG;
       $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
       (*Print@{plt1,plt2};*)
      ])) &, findEdges]
  ];
  bagged["DropAll"];
  {indToPts, vertToCellG, cellToVertG, $wrappedMat, $faceListCoords}
 ];
```

T2 transition

find cells for T2

T2

In[57]:=

```
Clear@T2TransitionFn;
T2TransitionFn[removeelem_, indToPtsAssoc_, ptsToIndAssoc_,
   vertexCellAssoc_, cellVertexGrouping_, wrappedMat_, faceListCoords_] :=
  Block[{assoc, translC, transVec, ptsToInds = ptsToIndAssoc, indToPts = indToPtsAssoc,
    vertCellAssoc = vertexCellAssoc, cvG = cellVertexGrouping,
    $wrappedMat = wrappedMat, $faceListCoords = faceListCoords, cellkey,
    res, mergedtopo, vertinds, vertpts, mean, ruletrans, changedtopo,
    transtopo, ckeys, pos, newpt, maxlab, newind, vertinddrop, oldptind, re},
   re = Replace[removeelem, x_Integer :→ {x}];
   If[removeelem =!= {},
    Scan[
      (cellkey = #;
       {assoc, translC, transVec} =
        getLocalTopology[ptsToInds, indToPts, vertCellAssoc, cvG, $wrappedMat,
             $faceListCoords][#] & /@ Lookup[indToPts, cvG[cellkey]] // Transpose;
       res = DeleteDuplicates[Flatten[{translC, transVec}, {3, 2}] //.
          {p___, x : {}, q___} :→ {p, q}];
       mergedtopo = KeySort[Join @@ assoc];
       vertinds = cvG[cellkey];
       vertpts = Lookup[indToPts, vertinds];
       mean = SetPrecision[Mean@vertpts, 10];
       ruletrans = Flatten[Thread[{# → mean}] & /@ vertpts];
       (*Print[Values@mergedtopo//Graphics[{FaceForm[LightBlue],
```

```
            EdgeForm[Black],Map[Polygon][#]},ImageSize→Small]&];*)
      If[res == {}, KeyDropFrom[mergedtopo, cellkey]];
      changedtopo = (mergedtopo /. ruletrans);
      (*Print[Values@changedtopo//Graphics[{FaceForm[LightBlue],
            EdgeForm[Black],Map[Polygon][#]},ImageSize→Small]&];*)
      If[res ≠ {},
       transtopo = Fold[MapAt[Function[x, Function[y, SetPrecision[y - #2[[-1]], 10]] /@
             x], #1, Key@#2[[1]]] &, changedtopo, res];
       ckeys = res[[All, 1]];
       pos = Cases[Position[<|Thread[ckeys → Lookup[mergedtopo, ckeys]]|>,
          Alternatives @@ vertpts], {Key[Alternatives @@ ckeys], _}];
       newpt = DeleteDuplicates@Extract[transtopo, pos];
       oldptind = Extract[<|Thread[ckeys → Lookup[cvG, ckeys]]|>, pos];
       newpt = Join[{mean}, newpt];
       maxlab = Max[Keys@indToPts];
       newind = Range[maxlab + 1, maxlab + Length[newpt]];
       vertinddrop = Join[oldptind, vertinds];
       AppendTo[indToPts, Thread[newind → newpt]],
       newind = Max[Keys@indToPts] + 1;
       newpt = mean;
       vertinddrop = vertinds;
       AppendTo[indToPts, newind → newpt];
      ];
      ptsToInds = AssociationMap[Reverse, indToPts];
      If[res ≠ {},
       AppendTo[cvG, DeleteDuplicates@Lookup[ptsToInds, #] & /@ transtopo],
       AppendTo[cvG, DeleteDuplicates@Lookup[ptsToInds, #] & /@ changedtopo]
      ];
      cvG = KeySort@cvG;
      KeyDropFrom[indToPts, vertinddrop];
      ptsToInds = AssociationMap[Reverse, indToPts];
      KeyDropFrom[cvG, cellkey];
      vertCellAssoc = GroupBy[
         Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cvG], First → Last] // KeySort;
      $wrappedMat = AssociationThread[Keys[cvG] →
         Map[Lookup[indToPts, #] /. periodicRules &, Lookup[cvG, Keys[cvG]], {2}]];
      $faceListCoords = Map[Lookup[indToPts, #] &, cvG, {2}]
      (*$wrappedMat= Map[#/.periodicRules&,$faceListCoords]*)) &, re];
 ];
 (*Print@Graphics[{EdgeForm[Black],FaceForm[LightBlue],
     Polygon/@Values@Map[Lookup[indToPts,#]&,cvG,{2}]},ImageSize→Medium];*)
 {indToPts, ptsToInds, vertCellAssoc, cvG, $wrappedMat, $faceListCoords}
];
```

cell division

Difference from Farahdifar's paper: in his article he dilates the cells before dividing them. Here I randomly select a cell and divide it into two. This does not really matter

In[59]:=
```
Clear[selectDivCells];
selectDivCells[areaPolygon_, areathresh_ : 2.2, thresh_ : 0.0025] :=
  Block[{candidates, pos},
   candidates = Normal@Select[areaPolygon / Mean[areaPolygon], # > areathresh &];
   pos = Position[0.1 RandomReal[1, Length@candidates], x_ /; x < thresh];
   Keys@Extract[candidates, pos]
  ];
```

In[61]:=
```
(* division events more random *)
```

In[62]:=
```
Clear[pickcellsDiv];
pickcellsDiv[cellToVertG_, areaAssoc_] := Block[{pickcells, selcells, pos},
   pickcells = Keys@Select[Pick[areaAssoc,
       Thread[RandomReal[{0, 1}, Length[areaAssoc]] < 0.001], True], # > 0.005 &];
   pos = Position[Lookup[cellToVertG, pickcells], x_ /; Length[x] > 3, {1}];
   Extract[pickcells, pos]
  ];
```

In[64]:=
```
Clear[cellDivision, i];
cellDivision[polygonind_, indToPoints_, areaAssoc_, perimAssoc_, cellToVertG_] :=
  Block[{x, y, num, matrix, xx, xy, yy, eigvals, eigVecs, maxeigpos, cent, edges,
    edgesL, intersects, intersectionPts, posIntersections, repPart, α, β,
    polygonPts, newkeys = Range[# + 1, # + 2] &[Max@Keys[indToPoints]], newPtToInds,
    indtoPtAssoc = indToPoints, ptToIndAssoc, edgeinds, contour, poly1, poly2, res, seq,
    newcells = Range[# + 1, # + 2] &[Max@Keys[areaAssoc]], CVG = cellToVertG,
    addcellsRule, polygonPtsInds, VCG, polygonptsTrans},
   VCG = GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
   polygonPtsInds = CVG[polygonind];
   num = Length@polygonPtsInds;
   ptToIndAssoc = AssociationMap[Reverse, indToPoints];
   polygonPts = Lookup[indToPoints, polygonPtsInds];
   polygonptsTrans = TranslationTransform[-Mean[polygonPts]]@polygonPts;
   Evaluate[Table[{x_i, y_i}, {i, num + 1}]] =
    Append[polygonptsTrans, First@polygonptsTrans];
   I_xx = (1/12) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (y_i^2 + y_i y_{i+1} + y_{i+1}^2);
   I_yy = (1/12) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (x_i^2 + x_i x_{i+1} + x_{i+1}^2);
   I_xy = (1/24) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (x_i y_{i+1} + 2 x_i y_i + 2 x_{i+1} y_{i+1} + x_{i+1} y_i);
```

```
Table[
 {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, num + 1}];
matrix = ( Ixx   -Ixy
          -Ixy    Iyy );
{eigvals, eigVecs} = Eigensystem@matrix;
maxeigpos = Position[eigvals, Max@eigvals];
{edges, edgeinds} = Partition[#, 2, 1, 1] & /@ {polygonPts, polygonPtsInds};
edgesL = Line /@ edges;
cent = centroidPolygon[polygonPts];
intersects = RegionIntersection[
    InfiniteLine[{cent, cent + Extract[eigVecs, maxeigpos][[1]]}], #] & /@ edgesL;
intersectionPts = Cases[intersects, {(_Real | _Integer) ..}, {3}];
newPtToInds = Thread[intersectionPts → newkeys];
posIntersections = Flatten@Position[intersects, _Point, {1}];
MapThread[
 (res = Complement[Intersection @@ Lookup[VCG, #2], {polygonind}];
   If[res ≠ {},
    seq = Partition[CVG[First@res], 2, 1, 1];
    AppendTo[CVG,
     First@res → DeleteDuplicates@
       Flatten@SequenceSplit[seq, {x___, p : {OrderlessPatternSequence[
              #2[[1]], #2[[-1]]]}, y___} :→ {x, Insert[p, #1, 2], y}]
    ];
   ]) & , {newkeys, edgeinds[[posIntersections]]}];

repPart =
 Thread[{Thread[{ReverseSort@posIntersections, 2}], Reverse[intersectionPts]}];
{α, β} = intersectionPts;
AppendTo[ptToIndAssoc, newPtToInds];
AppendTo[indtoPtAssoc, Reverse[newPtToInds, 2]];
contour =
 DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, repPart], 1];
poly1 = Join @@ SequenceCases[contour, {___, α} | {β, ___}];
poly2 = Join @@ SequenceCases[contour, {α, __, β}];
KeyDropFrom[CVG, polygonind];
addcellsRule = Thread[newcells → {poly1, poly2}];
AppendTo[CVG, addcellsRule /. ptToIndAssoc];
{indtoPtAssoc, CVG, Append[KeyDrop[areaAssoc, polygonind],
   MapAt[Area@*Polygon, addcellsRule, {All, 2}]],
 Append[KeyDrop[perimAssoc, polygonind],
   MapAt[Perimeter@*Polygon, addcellsRule, {All, 2}]]}
];
```

computing forces

In[80]:=
```
ka = 1000;
A0 = 0.01;
γ = 0.04 * ka * A0;
P0 = 0;
κ = 0.025;
δt = 0.001;
```

In[86]:=
```
{γ / (ka * A0), κ / (ka * A0^(3/2))}
```

Out[86]=
```
{0.04, 0.025}
```

area elasticity

In[87]:=
```
F_AreaElasticity[indTopts_, localtopo_, areaPolygonAssoc_] :=
 Block[{assoc, cellinds, force, vertKeys = Keys[indTopts],
   vertLs, vertex, gc, gcc, diffVec, grad, coeff},
  First@*Last@Reap@Do[
     assoc = First@Lookup[localtopo, i];
     cellinds = Keys[assoc];
     force = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = assoc[j];
      gcc = getCounterClockwise[vertex, vertLs];
      gc = getClockwise[vertex, vertLs];
      diffVec = SetPrecision[gcc - gc, 10];
      grad = SetPrecision[1/2 {{0, 1}, {-1, 0}}.diffVec, 10];
      coeff = SetPrecision[2 ka (areaPolygonAssoc[j] - A0), 10];
      force += SetPrecision[coeff * grad, 10], {j, cellinds}
     ];
     Sow@force, {i, vertKeys}]
 ]
```

In[88]:=
```
{{0, 1}, {-1, 0}}.(getCounterClockwise[{x_i, y_i}, {{x_(i-1), y_(i-1)}, {x_i, y_i}, {x_(i+1), y_(i+1)}}] -
   getClockwise[{x_i, y_i}, {{x_(i-1), y_(i-1)}, {x_i, y_i}, {x_(i+1), y_(i+1)}}]) // MatrixForm
```

Out[88]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

In[89]:=
```
MatrixForm[{{0, 1}, {-1, 0}} . ({x_{i+1}, y_{i+1}} - {x_{i-1}, y_{i-1}})]
```

Out[89]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

perimeter elasticity

In[90]:=
```
F_PerimeterElasticity[indTopts_, localtopo_, periPolygonAssoc_] :=
 Block[{assoc, cellinds, force, vertKeys = Keys[indTopts],
   vertLs, vertex, gc, gcc, v1, v2, coeff, grad},
  First@*Last@Reap@Do[
     assoc = First@Lookup[localtopo, i];
     cellinds = Keys@assoc;
     force = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = assoc[j];
      gc = getClockwise[vertex, vertLs];
      v1 = Normalize[vertex - gc] ~SetPrecision~ 10;
      gcc = getCounterClockwise[vertex, vertLs];
      v2 = Normalize[gcc - vertex] ~SetPrecision~ 10;
      grad = SetPrecision[v1 - v2, 10];
      coeff = SetPrecision[2 γ (periPolygonAssoc[j] - P0), 10];
      force += SetPrecision[coeff * grad, 10], {j, cellinds}
     ];
     Sow@force, {i, vertKeys}]
 ]
```

In[91]:=
```
MatrixForm@Normalize[{x_i, y_i} - {x_j, y_j}]
```

Out[91]//MatrixForm=

$$\begin{pmatrix} \frac{x_i - x_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \\ \frac{y_i - y_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \end{pmatrix}$$

line tension

In[92]:=
```
F_LineTension[indTopts_, ptsToInd_, edges_] :=
 Block[{vertKeys = Keys@indTopts, $v1, $v2, v1, force, uv},
  force = AssociationThread[vertKeys → 0.];
  Do[
   {$v1, $v2} = i;
   uv = Normalize[$v1 - $v2] ~ SetPrecision ~ 10;
   v1 = ptsToInd[$v1];
   force[v1] += SetPrecision[κ * uv, 10],
   {i, edges}];
  Values[force]
 ]
```

ΣF

In[93]:=
```
F_T[indTopts_, ptsToInds_, localtopology_,
    areaPolygonAssoc_, periPolygonAssoc_, edges_] :=
  - (F_AreaElasticity[indTopts, localtopology, areaPolygonAssoc]
      + F_PerimeterElasticity[indTopts, localtopology, periPolygonAssoc] +
      F_LineTension[indTopts, ptsToInds, edges]);
```

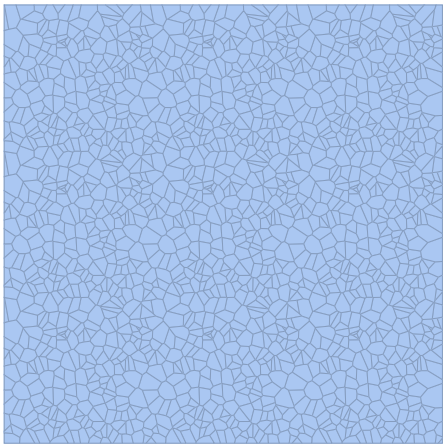
generating mesh

In[•]:=
```
(* ensure PBC logically integrates with the rest of the code *)
```

In[94]:=
```
SeedRandom[1];
pts = RandomReal[{-1, 1}, {200, 2}];
```

In[96]:=
```
pts2 = Flatten[Table[TranslationTransform[{2 i, 2 j}][pts], {i, -1, 1}, {j, -1, 1}], 2];
vor = VoronoiMesh[pts2, {{-3, 3}, {-3, 3}}]
```

Out[97]=



In[98]:=
```
vcells = Catenate[NearestMeshCells[{vor, 2}, #] & /@ pts];
pvor = MeshRegion[MeshCoordinates[vor], MeshCells[vor, vcells]];
```

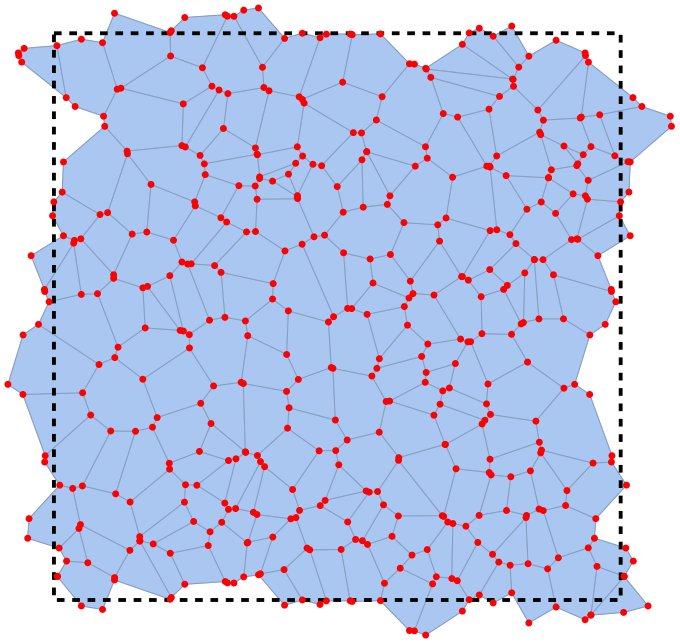
In[100]:=
```
Show[Table[MeshRegion[TransformedRegion[pvor, TranslationTransform[{2 i, 2 j}]],
   MeshCellStyle → {1 → Black, 2 → ColorData[2, 7 i + j + 25]}], {i, -3, 3}, {j, -3, 3}]]
```

Out[100]=

In[101]:=
```
vpts = Extract[MeshCoordinates[vor],
   Union@Flatten[MeshCells[vor, vcells] /. Polygon[x_] :> x] ~Partition~ 1];
```

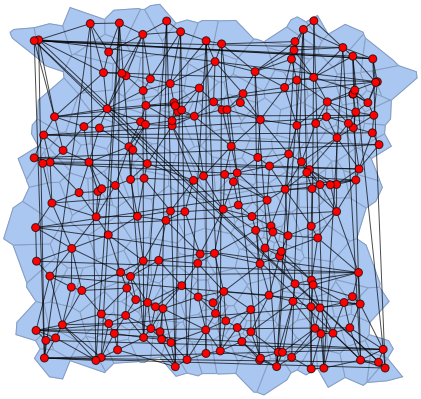
In[102]:=
```
Show[pvor, Graphics[
  {{EdgeForm[{Thick, Dashed, Black}], FaceForm[None], Rectangle[{-1, -1}, {1, 1}]},
   Red, PointSize[0.01], Point@vpts}]]
```

Out[102]=



In[103]:=
```
len = Length[pts];
C22 = #.Transpose[#] &@vor["ConnectivityMatrix"[2, 1]];
cells = Region`Mesh`MeshMemberCellIndex[vor, pts2][[All, 2]];
C22perm = C22[[cells, cells]];
pC22 = SparseArray[Unitize[Total[Partition[Unitize[C22perm], {len, len}], 2]]];
pC22 -= IdentityMatrix[len, SparseArray];
Show[pvor,
 Graph[AdjacencyGraph[pC22, VertexCoordinates → pts], VertexStyle → Red, EdgeStyle → Black]]
```

Out[109]=



In[111]:=
```
indToPtsAssoc = AssociationThread[
   Replace[First@MeshCells[pvor], Point[x_] :> x, {1}] ->
    Replace[MeshPrimitives[pvor, 0], Point -> Sequence, {2}, Heads -> True]
  ];
```

In[112]:=
```
ptsToIndAssoc = <|Reverse[Normal@indToPtsAssoc, 2]|>;
```

In[113]:=
```
cellVertexGrouping = AssociationThread[Range@Length[#] -> #] &@
   Replace[MeshCells[pvor, 2], x_Polygon :> Sequence @@ x, {1}, Heads -> True];
```

In[114]:=
```
vertexCellAssoc = KeySort@
   GroupBy[Flatten[Thread[#] & /@ Reverse[Normal@cellVertexGrouping, 2]], First -> Last];
```

In[115]:=
```
wrappedMat = AssociationThread[
   Keys[cellVertexGrouping] -> Map[Lookup[indToPtsAssoc, #] /. periodicRules &,
     Lookup[cellVertexGrouping, Keys[cellVertexGrouping]], {2}]];
```

In[116]:=
```
faceListCoords = AssociationThread[Keys[cellVertexGrouping] ->
    Map[Lookup[indToPtsAssoc, #] &, Values@Normal@cellVertexGrouping]];
```

In[117]:=
```
ptsToIndAssoc = KeyMap[SetPrecision[#, 10] &, ptsToIndAssoc];
indToPtsAssoc = SetPrecision[#, 10] & /@ indToPtsAssoc;
wrappedMat = SetPrecision[#, 10] & /@ wrappedMat;
faceListCoords = SetPrecision[#, 10] & /@ faceListCoords;
```

Main( )

In[121]:=
```
indTopts = indToPtsAssoc;
ptsToInd = ptsToIndAssoc;
cellToVertexG = cellVertexGrouping;
vertexToCell = vertexCellAssoc;
$wrappedMat = wrappedMat;
$wrappedMatTrim =
  KeyTake[wrappedMat, outerCellsFn[faceListCoords, vertexCellAssoc, ptsToIndAssoc]];
$faceListCoords = faceListCoords;
areaPolygonAssoc = areaOfPolygon@faceListCoords;
periPolygonAssoc = perimeterOfPolygon@faceListCoords;

SeedRandom[1];
cj = 0;
Tmax = 4000;
t = 0;
```

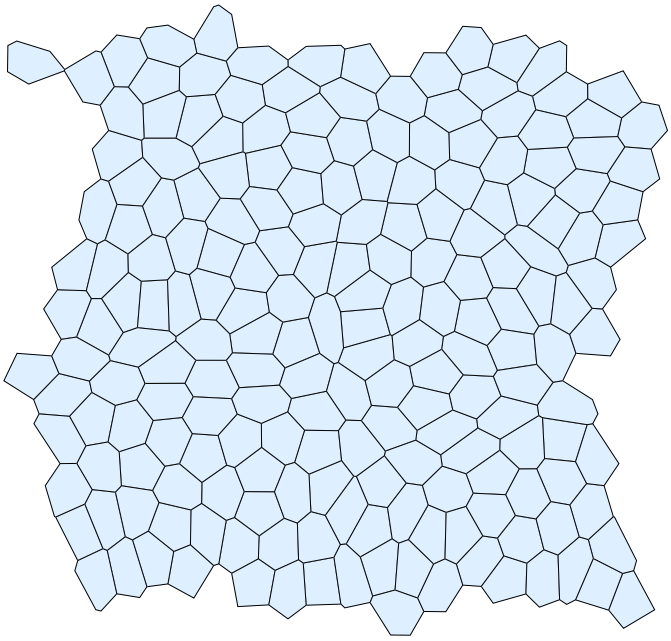
```
tt = {};

Module[{cellsToRemove, edgechanged, polydiv, findEdges},
   saveres = First@Last@Reap@Monitor[
        While[t ≤ Tmax * δt,
         cj = Round[t / δt];

         (* T2 transitions *)
         cellsToRemove = cellsforT2[areaPolygonAssoc, cellToVertexG];
         If[cellsToRemove ≠ {},
          {indTopts, ptsToInd, vertexToCell, cellToVertexG, $wrappedMat,
            $faceListCoords} = T2TransitionFn[cellsToRemove, indTopts, ptsToInd,
            vertexToCell, cellToVertexG, $wrappedMatTrim, $faceListCoords];
          bcells = outerCellsFn[$faceListCoords, vertexToCell, ptsToInd];
          $wrappedMatTrim = KeyTake[$wrappedMat, bcells];
         ];

         (* T1 transitions *)
         edges = DeleteDuplicatesBy[
           Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1], Sort];
         {indTopts, vertexToCell, cellToVertexG, $wrappedMat, $faceListCoords} =
          T1transitionFn[edges, indTopts, ptsToInd, vertexToCell,
           cellToVertexG, $wrappedMatTrim, $faceListCoords];
         bcells = outerCellsFn[$faceListCoords, vertexToCell, ptsToInd];
         $wrappedMatTrim = KeyTake[$wrappedMat, bcells];
         ptsToInd = AssociationMap[Reverse, indTopts];
         areaPolygonAssoc = areaPolygon /@ $faceListCoords;
         periPolygonAssoc = perimeterPolygon /@ $faceListCoords;

         (* cell divisions *)
         polydiv = selectDivCells[areaPolygonAssoc];
         If[polydiv ≠ {},
          Scan[({indTopts, cellToVertexG, areaPolygonAssoc, periPolygonAssoc} =
               cellDivision[#, indTopts, areaPolygonAssoc,
                periPolygonAssoc, cellToVertexG]) &, polydiv];
          vertexToCell = KeySort@GroupBy[Flatten[(Reverse[#, 2] &) @*
                 Thread /@ Normal@cellToVertexG], First → Last];
          $faceListCoords = Lookup[indTopts, #] & /@ cellToVertexG;
          $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
          bcells = outerCellsFn[$faceListCoords, vertexToCell, ptsToInd];
          $wrappedMatTrim = KeyTake[$wrappedMat, bcells];
         ];
         ptsToInd = AssociationMap[Reverse, indTopts];


         (* update positions *)
         localtopo = getLocalTopology[ptsToInd, indTopts, vertexToCell,
```

```
            cellToVertexG, $wrappedMatTrim, $faceListCoords][#] & /@ indTopts;
        edgeLst = SortBy[Flatten[Map[Partition[#, 2, 1, 1] &,
            Values[$faceListCoords]], 1], First];
        externedges = outeredges[indTopts, localtopo];
        edgeLst = Join[edgeLst, externedges];
        (*capture the first and the last output for the magnitude of force*)
        If[cj == 3 || (cj + 1) == Tmax, AppendTo[tt, F_T[indTopts, ptsToInd,
            localtopo, areaPolygonAssoc, periPolygonAssoc, edgeLst] δt]];

        indTopts = AssociationThread[
          Keys[indTopts] → SetPrecision[(Values[indTopts] + F_T[indTopts, ptsToInd,
                localtopo, areaPolygonAssoc, periPolygonAssoc, edgeLst] δt), 10]];

        ptsToInd = AssociationMap[Reverse, indTopts];
        $faceListCoords = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ $faceListCoords;
        periPolygonAssoc = perimeterPolygon /@ $faceListCoords;
        vertexToCell = GroupBy[
          Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
        $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
        bcells = outerCellsFn[$faceListCoords, vertexToCell, ptsToInd];
        $wrappedMatTrim = KeyTake[$wrappedMat, bcells];

        (*plt=Graphics[{ColorData[1][1],Thick,Values@Map[Line[Join[##,{First@#}]]&@
                Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
        plt = Graphics[{FaceForm[LightBlue], EdgeForm[{Black}], Values[
            Polygon@Lookup[indTopts, #] & /@ cellToVertexG]}, ImageSize → Medium];
        (*plt=Graphics[{■,Thick,Values@Map[Line[Join[##,{First@#}]]&@
                Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
        If[Mod[cj, 2] == 0, Sow[plt]];
        t += δt;
       ], {cj, plt}
      ]
]; // AbsoluteTiming
```

Out[135]= {1606.05, Null}

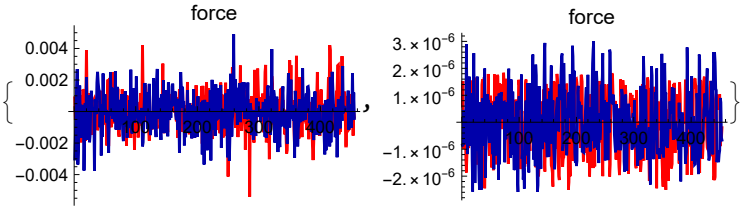
In[136]:=
```
plt
```

Out[136]=



In[137]:=
```
(ListLinePlot[#, PlotStyle → {{Thickness[0.01], Red}, {Thickness[0.01], Darker@Blue}},
    PlotLabel → "force"] &) @* Transpose /@ tt
```

Out[137]=



miscellaneous

In[138]:=
```
localtopotemp =
  getLocalTopology[ptsToIndAssoc, indToPtsAssoc, vertexCellAssoc, cellVertexGrouping,
      wrappedMat, faceListCoords][#] & /@ indToPtsAssoc;
edgeLsttemp = SortBy[Flatten[Map[Partition[#, 2, 1, 1] &, Values[faceListCoords]], 1],
   First];
externedgestemp = outeredges[indToPtsAssoc, localtopotemp];
Show[pvor, Graphics[{{Thin, Black, Arrowheads[Small], Arrow /@ edgeLsttemp},
   {Thin, Red, Arrowheads[Small], Arrow /@ externedgestemp}}], ImageSize → Medium]
```

Out[141]=