2D VERTEX MODEL

geometrical f(x)

In[1]:=
```
Clear[getCounterClockwise];
getCounterClockwise[vertex_, vertices_] := Block[{pos, v},
   pos = First @@ Position[vertices, vertex];
   If[pos == Length[vertices], pos = 1, pos += 1];
   vertices[[pos]]
  ];
```

In[3]:=
```
Clear[getClockwise];
getClockwise[vertex_, vertices_] := Block[{ls, pos},
   pos = First @@ Position[vertices, vertex];
   If[pos == 1, pos = Length[vertices], pos -= 1];
   vertices[[pos]]
  ];
```

In[5]:=
```
getCounterClockwise[{xi, yi}, {{xi-1, yi-1}, {xi, yi}, {xi+1, yi+1}, d, e}]
```

Out[5]=
$$\{x_{1+i},\ y_{1+i}\}$$

In[6]:=
```
getClockwise[{xi, yi}, {{xi-1, yi-1}, {xi, yi}, {xi+1, yi+1}, d, e}]
```

Out[6]=
$$\{x_{-1+i},\ y_{-1+i}\}$$

In[7]:=
```
Clear[areaOfPolygon];
areaOfPolygon[cells_ /; Head[cells] === Association] := Map[Area@*Polygon, cells];
```

In[9]:=
```
Clear[areaPolygon];
areaPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  0.5 Abs@Total[(#[[1, 1]] * #[[2, 2]]) - (#[[2, 1]] * #[[1, 2]]) & /@ edges]
 ]
```

In[11]:=
```
Clear[perimeterOfPolygon];
perimeterOfPolygon[cells_ /; Head[cells] === Association] :=
  (Perimeter@*Polygon) /@ cells;
```

In[13]:=
```
Clear[perimeterPolygon];
perimeterPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  Total[Apply[EuclideanDistance] /@ edges]
 ]
```

In[15]:=
```
Clear[centroidPolygon];
centroidPolygon[vertices_] := Mean@vertices
```

In[17]:=
```
Clear[polyCounterClockwiseQ];
polyCounterClockwiseQ[poly_] := Block[{area = 0, j, vertLength = Length[poly]},
  Do[
   j = Mod[i, vertLength] + 1;
   area += poly[[i, 1]] * poly[[j, 2]];
   area -= poly[[j, 1]] * poly[[i, 2]],
   {i, vertLength}];
  (area / 2) > 0
 ]
```

In[19]:=
```
Clear[sortCC];
sortCC[polyinds_, indTopts_, ptsToInds_] := Block[{cent, poly},
   poly = Lookup[indTopts, polyinds];
   Lookup[ptsToInds,
    DeleteDuplicates@
     Flatten[MeshPrimitives[ConvexHullMesh[poly], 1] /. Line → Sequence, 1]
   ]
  ];
```

In[21]:=
```
Clear[sortPointsCC];
sortPointsCC[polyinds_, indTopts_, ptsToInds_] := Block[{cent, ordering, polyPoints},
  polyPoints = Lookup[indTopts, polyinds];
  cent = Mean@polyPoints;
  ordering = Ordering[ArcTan[#[[1]], #[[2]]] &@ (# - cent) & /@ polyPoints];
  Lookup[ptsToInds, Part[polyPoints, ordering]]
 ]
```

mesh restructuring operations

T1 transition (neighbour swapping)

In[23]:=
```
(* T1 transition: neighbour switching *)
```

In[24]:=
```
Clear@edgesforT1;
edgesforT1[edgeLs_, indToPts_, threshLength_ : 0.0015] := Block[{edges, dist},
   edges = Lookup[indToPts, #] & /@ edgeLs;
   dist = EuclideanDistance @@ # & /@ edges;
   {Pick[edges, Thread[dist ≤ threshLength], True],
    Pick[edgeLs, Thread[dist ≤ threshLength], True]}
  ];
```

In[26]:=
```
Clear@T1transitionFn;
T1transitionFn[edges_, indToPtsAssoc_, vertexToCellG_, cellToVertexG_, dSep_ : 0.01] :=
```

```
Block[{findEdges, edgeind, connectedcellKeys, edge,
  newpts, cellvertIndices, cellvertices, pos, cellpolys, memF, keyscellP,
  selcellKeys, ptToCell, newptsindices, indToPts = indToPtsAssoc, ptsToInds,
  PtIndToCell, keysToMap, cellindicesAssoc, f1, otherkeys, f2, polysharingEdge,
  bag = CreateDataStructure["DynamicArray"], vertToCellG = vertexToCellG,
  cellToVertG = cellToVertexG, testpts, edgechanged},
 {edgechanged, findEdges} = edgesforT1[edges, indToPts];
 (* finding all possible edges for T1 transition *)
 If[findEdges ≠ {},
  Scan[
   (edgeind = #;
    If[ContainsAll[Keys[indToPts], edgeind],
     (* should be an edge not
      connected to an edge that has already undergone a T1 *)
     connectedcellKeys = DeleteDuplicates[Flatten@Lookup[vertToCellG, edgeind]];
     cellvertIndices = Lookup[cellToVertG, connectedcellKeys];
     edge = Lookup[indToPts, edgeind];
     newpts = With[{midPt = Mean[edge]},
       midPt + dSep Normalize[(# - midPt)] & /@
        Flatten[RotationTransform[-π/2, midPt] /@ {edge}, 1]
      ];
     testpts = With[{midPt = Mean[edge]},
       midPt + 0.000001 Normalize[(# - midPt)] & /@ newpts
      ];
     pos = Position[cellvertIndices, {OrderlessPatternSequence[
         ___, First@edgeind, ___, Last@edgeind, ___]}, {1}];
     polysharingEdge = Extract[cellvertIndices, pos];
     (* the edge should not be part of any Δ *)
     If[(AllTrue[polysharingEdge, Length[#] ≠ 3 &]) &&
       ContainsNone[edgeind, Union@*Flatten@*Normal@bag],
      cellvertices = Map[Lookup[indToPts, #] &, cellvertIndices];
      cellpolys = Polygon /@ cellvertices;
      memF = Function[x, RegionMember@x, Listable][Extract[cellpolys, pos]];
      If[k == 17, Print["conn cells:", connectedcellKeys]];
      keyscellP = Extract[connectedcellKeys, pos];
      selcellKeys = Thread[keyscellP → memF];
      ptToCell = Quiet[# → First @@ Select[selcellKeys, Function[x, Last[x][#]]] & /@
          testpts /. HoldPattern[_ → First[]] → Nothing] ;
      (* pt to cell *)
      ptToCell = ptToCell /. Thread[testpts → newpts];
      newptsindices = Range[# + 1, # + 2] &[Max[Keys@indToPts]];
      KeyDropFrom[indToPts, edgeind];
      AppendTo[indToPts, Thread[newptsindices → newpts]];
      ptsToInds = AssociationMap[Reverse, indToPts];
      bag["Append", edgeind];
      PtIndToCell = MapAt[ptsToInds, ptToCell, {All, 1}] /. Rule → List ;
      (*index to cell*)
      keysToMap = MapAt[Key, PtIndToCell, {All, 2}];
```

```
        cellindicesAssoc = AssociationThread[connectedcellKeys, cellvertIndices];

        f1 = Fold[MapAt[Function[x, DeleteDuplicates[x /. Thread[ edgeind → #2[[1]] ]]],
            #1, #2[[2]]] &, cellindicesAssoc, keysToMap];
        otherkeys = List@* Key /@ Complement[connectedcellKeys, keyscellP];
        f2 = MapAt[ (# /. (Alternatives @@ edgeind) → Splice[newptsindices] //
             sortPointsCC[#, indToPts, ptsToInds] &) &, f1, otherkeys];
        AppendTo[cellToVertG, f2];
        vertToCellG = GroupBy[
          Flatten[ (Reverse[#, 2] &) @* Thread /@ Normal@cellToVertG], First → Last];
       ]
      ]) &,
    findEdges]
  ];
  {edgechanged, indToPts, cellToVertG, vertToCellG}
 ];
```

T2 transition

In[28]:=
```
(* T2 transition: removal of cell *)
```

In[29]:=
```
Clear@cellsforT2;
cellsforT2[areaAssoc_, cellVertexG_, thresh_ : 10^-5] := Block[{keys, ls, inds},
   keys = Keys@Select[areaAssoc, # < thresh &];
   ls = Lookup[cellVertexG, keys];
   inds = Flatten@Position[ls, x_ /; (3 ≤ Length[x] ≤ 6), {1}];
   (* 3 ≤ detect ≤ 6 *)
   If[inds ≠ {}, {keys[[inds]], ls[[inds]]}, {{}, {}}] (*cell inds, vertices*)
  ];
```

In[31]:=
```
Clear@T2TransitionFn;
T2TransitionFn[{cellsToRemove_, vertindsRemove_}, indTopts_, cellToVertexG_,
   areaPolygonAssoc_, periPolygonAssoc_] := Block[{newVertices, maxkey, newindices,
     newentries, indToPts = indTopts, ruleDisp, removeentries,
     CVG = cellToVertexG, notaCell, VertCellGrouping},
    newVertices = Mean@Lookup[indTopts, #] & /@ vertindsRemove;
    maxkey = Max@*Keys@indTopts;
    newindices = Range[maxkey + 1, maxkey + Length[newVertices]];
    newentries = Thread[newindices → newVertices];
    KeyDropFrom[indToPts, Union@Flatten[vertindsRemove]];
    AppendTo[indToPts, newentries];
    ruleDisp =
     Dispatch@Flatten[MapThread[Thread[#1 → #2] &, {vertindsRemove, newindices}]];
    removeentries = Union@Flatten@cellsToRemove;
    KeyDropFrom[CVG, removeentries];
    CVG = DeleteDuplicates /@ Replace[CVG, ruleDisp, {2}];
    notaCell = Keys@Select[Length /@ CVG, # < 3 &];
    KeyDropFrom[CVG, notaCell];
    VertCellGrouping =
     GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
    {indToPts, CVG, VertCellGrouping, KeyDrop[areaPolygonAssoc,
      removeentries ~ Join ~ notaCell],
     KeyDrop[periPolygonAssoc, removeentries ~ Join ~ notaCell]}
   ] /; vertindsRemove ≠ {};
```

cell division

In[33]:=
```
(* probability of division based on the cell area *)
```

In[34]:=
```
Clear[selectDivCells];
selectDivCells[areaPolygon_, areathresh_ : 2.2, thresh_ : 0.0025] :=
  Block[{candidates, pos},
   candidates = Normal@Select[areaPolygon / Mean[areaPolygon], # > areathresh &];
   pos = Position[0.1 RandomReal[1, Length@candidates], x_ /; x < thresh];
   Keys@Extract[candidates, pos]
  ];
```

In[36]:=
```
(* division more random *)
```

In[37]:=
```
Clear[pickcellsDiv];
pickcellsDiv[cellToVertG_, areaAssoc_] := Block[{pickcells, selcells, pos},
   pickcells = Keys@Select[Pick[areaAssoc,
       Thread[RandomReal[{0, 1}, Length[areaAssoc]] < 0.001], True], # > 0.005 &];
   pos = Position[Lookup[cellToVertG, pickcells], x_ /; Length[x] > 3, {1}];
   Extract[pickcells, pos]
  ];
```

In[39]:=
```
Clear[cellDivision];
cellDivision[polygonind_, indToPoints_, areaAssoc_, perimAssoc_, cellToVertG_] :=
```

```
Block[{x, y, num, matrix, xx, xy, yy, eigvals, eigVecs, maxeigpos, cent, edges,
  edgesL, intersects, intersectionPts, posIntersections, repPart, α, β,
  polygonPts, newkeys = Range[# + 1, # + 2] &[Max@Keys[indToPoints]], newPtToInds,
  indtoPtAssoc = indToPoints, ptToIndAssoc, edgeinds, contour, poly1, poly2, res, seq,
  newcells = Range[# + 1, # + 2] &[Max@Keys[areaAssoc]],
  CVG = cellToVertG, addcellsRule, polygonPtsInds, VCG},
 VCG = GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
 polygonPtsInds = CVG[polygonind];
 num = Length@polygonPtsInds;
 ptToIndAssoc = AssociationMap[Reverse, indToPoints];
 polygonPts = Lookup[indToPoints, polygonPtsInds];
 Evaluate[Table[{xi, yi}, {i, num}]] = polygonPts;
 Ixx = (1/12) Sum_{i=1}^{num-1} (xi yi+1 - xi+1 yi) (yi^2 + yi yi+1 + yi+1^2);
 Iyy = (1/12) Sum_{i=1}^{num-1} (xi yi+1 - xi+1 yi) (xi^2 + xi xi+1 + xi+1^2);
 Ixy = (1/24) Sum_{i=1}^{num-1} (xi yi+1 - xi+1 yi) (xi yi+1 + 2 xi yi + 2 xi+1 yi+1 + xi+1 yi);
 Table[
  {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, num}];
 matrix = {{Ixx, -Ixy}, {-Ixy, Iyy}};
 {eigvals, eigVecs} = Eigensystem@matrix;
 maxeigpos = Position[eigvals, Max@eigvals];
 {edges, edgeinds} = Partition[#, 2, 1, 1] & /@ {polygonPts, polygonPtsInds};
 edgesL = Line /@ edges;
 cent = centroidPolygon[polygonPts];
 intersects = RegionIntersection[
     InfiniteLine[{cent, cent + Extract[eigVecs, maxeigpos][[1]]}], #] & /@ edgesL;
 intersectionPts = Cases[intersects, {(_Real | _Integer) ..}, {3}];
 newPtToInds = Thread[intersectionPts → newkeys];
 posIntersections = Flatten@Position[intersects, _Point, {1}];
 MapThread[
  (res = Complement[Intersection @@ Lookup[VCG, #2], {polygonind}];
    If[res ≠ {},
     seq = Partition[CVG[First@res], 2, 1, 1];
     AppendTo[CVG,
      First@res → DeleteDuplicates@
        Flatten@SequenceSplit[seq, {x___, p : {OrderlessPatternSequence[
                #2[[1]], #2[[-1]]]}, y___} :→ {x, Insert[p, #1, 2], y}]
     ];
    ]) &, {newkeys, edgeinds[[posIntersections]]}];

 repPart =
  Thread[{Thread[{ReverseSort@posIntersections, 2}], Reverse[intersectionPts]}];
 {α, β} = intersectionPts;
 AppendTo[ptToIndAssoc, newPtToInds];
 AppendTo[indtoPtAssoc, Reverse[newPtToInds, 2]];
 contour =
```

```
    DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, repPart], 1];
   poly1 = Join @@ SequenceCases[contour, {___, α} | {β, ___}];
   poly2 = Join @@ SequenceCases[contour, {α, __, β}];
   KeyDropFrom[CVG, polygonind];
   addcellsRule = Thread[newcells → {poly1, poly2}];
   AppendTo[CVG, addcellsRule /. ptToIndAssoc];
   {indtoPtAssoc, CVG, Append[KeyDrop[areaAssoc, polygonind],
     MapAt[Area@*Polygon, addcellsRule, {All, 2}]],
    Append[KeyDrop[perimAssoc, polygonind],
     MapAt[Perimeter@*Polygon, addcellsRule, {All, 2}]]}
  ];
```

force computation

In[41]:=
```
ka = 1; A0 = 1; γ = 0.04 * ka * A0; δt = 0.02; P0 = 0; κ = 0.025;
```

In[42]:=
```
F_AreaElasticity[indTopts_, vertexToCellG_, cellToVertexG_, areaPolygonAssoc_] :=
 Block[{cellinds, temp, vertKeys = Keys[indTopts],
   vertLs, vertex, gc, gcc, diffVec, grad, coeff},
  First@*Last@Reap@Do[
     cellinds = Lookup[vertexToCellG, i];
     temp = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = Lookup[indTopts, Lookup[cellToVertexG, j]];
      gcc = getCounterClockwise[vertex, vertLs];
      gc = getClockwise[vertex, vertLs];
      diffVec = gcc - gc;
      grad = 0.5 * {{0, 1}, {-1, 0}}.diffVec;
      coeff = ka (areaPolygonAssoc[j] - A0);
      temp += grad * coeff, {j, cellinds}
     ];
     Sow@temp, {i, vertKeys}]
 ]
```

In[43]:=
```
MatrixForm[{{0, 1}, {-1, 0}}.({x_{i+1}, y_{i+1}} - {x_{i-1}, y_{i-1}})]
```

Out[43]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

In[44]:=
```
FPerimeterElasticity[indTopts_, vertexToCellG_, cellToVertexG_, periPolygonAssoc_] :=
 Block[{cellinds, temp, vertKeys = Keys[indTopts], vertLs,
   vertex, gc, gcc, v1, v2, coeff, grad},
  First@*Last@Reap@Do[
     cellinds = Lookup[vertexToCellG, i];
     temp = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = Lookup[indTopts, Lookup[cellToVertexG, j]];
      gc = getClockwise[vertex, vertLs];
      gcc = getCounterClockwise[vertex, vertLs];
      v1 = Normalize[vertex - gc];
      v2 = Normalize[vertex - gcc];
      grad = v1 + v2;
      coeff = γ (periPolygonAssoc[j] - P0);
      temp += grad * coeff, {j, cellinds}];
     Sow@temp, {i, vertKeys}]
 ]
```

In[45]:=
```
MatrixForm@Normalize[{xi, yi} - {xj, yj}]
```

Out[45]//MatrixForm=

$$\begin{pmatrix} \frac{x_i - x_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \\ \frac{y_i - y_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \end{pmatrix}$$

In[46]:=
```
FLineTension[indTopts_, vertexToCellG_, cellToVertexG_] :=
 Block[{cellinds, temp, vertKeys = Keys@indTopts, vertLs,
   vertex, gc, gcc, v1, v2},
  First@*Last@Reap@Do[
     cellinds = Lookup[vertexToCellG, i];
     temp = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = Lookup[indTopts, Lookup[cellToVertexG, j]];
      gc = getClockwise[vertex, vertLs];
      gcc = getCounterClockwise[vertex, vertLs];
      v1 = Normalize[vertex - gc];
      v2 = Normalize[vertex - gcc];
      temp += κ v1 + κ v2, {j, cellinds}];
     Sow@temp, {i, vertKeys}]
 ]
```

In[47]:=
```
FActiveContraction[indTopts_, vertexToCellG_, cellToVertexG_, areaPolygonAssoc_] :=
 Block[{cellinds, temp, vertKeys = Keys@indTopts, vertLs,
   vertex, gc, gcc, diffVec, grad, coeff},
  First@*Last@Reap@Do[
     cellinds = Lookup[vertexToCellG, i];
     temp = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = Lookup[indTopts, Lookup[cellToVertexG, j]];
      gcc = getCounterClockwise[vertex, vertLs];
      gc = getClockwise[vertex, vertLs];
      diffVec = gcc - gc;
      grad = 0.5 * {{0, 1}, {-1, 0}}.diffVec;
      coeff = 0.1 ka * (areaPolygonAssoc[j]);
      temp += grad * coeff, {j, cellinds}];
     Sow@temp, {i, vertKeys}]
 ]
```

In[48]:=
```
FT[indTopts_, vertexToCellG_,
   cellToVertexG_, areaPolygonAssoc_, periPolygonAssoc_] := -(
    FAreaElasticity[indTopts, vertexToCellG, cellToVertexG, areaPolygonAssoc] +
     FPerimeterElasticity[indTopts, vertexToCellG, cellToVertexG, periPolygonAssoc] +
     FLineTension[indTopts, vertexToCellG, cellToVertexG] +
     FActiveContraction[indTopts, vertexToCellG, cellToVertexG, areaPolygonAssoc]
   );
```

create mesh and run simulation

In[49]:=
```
SeedRandom[3];
mesh = VoronoiMesh[RandomReal[1, {200, 2}], {{0, 1}, {0, 1}}, ImageSize → Medium];
```

In[51]:=
```
pts = MeshPrimitives[mesh, 0] /. Point → Sequence;
```

In[52]:=
```
cornerpts = pts[[-4 ;;]];
pts = pts[[1 ;; -5]];
```

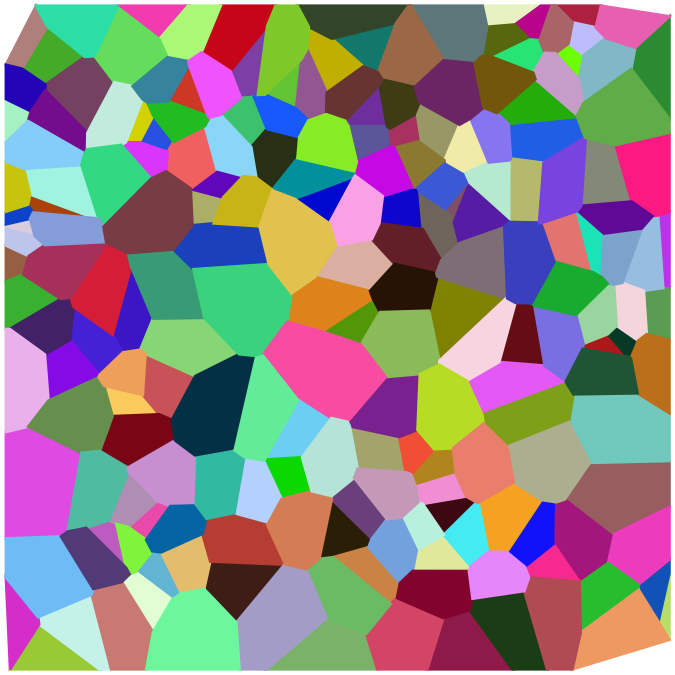
In[54]:=
```
$ptsToInd = ptsToInd = AssociationThread[pts → Range@Length@pts];
$indTopts = indTopts = AssociationMap[Reverse][ptsToInd];
```

In[56]:=
```
cellmeshprim = MeshPrimitives[mesh, 2];
cells = (MeshPrimitives[#, 0] & /@ cellmeshprim) /. Point → Sequence /.
   Thread[cornerpts → Nothing];
```

In[58]:=
```
$cellToVertexG =
  cellToVertexG = AssociationThread[Range[Length@cells] → Map[ptsToInd, cells, {2}]];
$vertexToCell = vertexToCell =
   GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
```

In[60]:=
```
Graphics[Map[{RandomColor[], Polygon@Lookup[indTopts, #]} &, Values@cellToVertexG],
 ImageSize → Medium]
```

Out[60]=

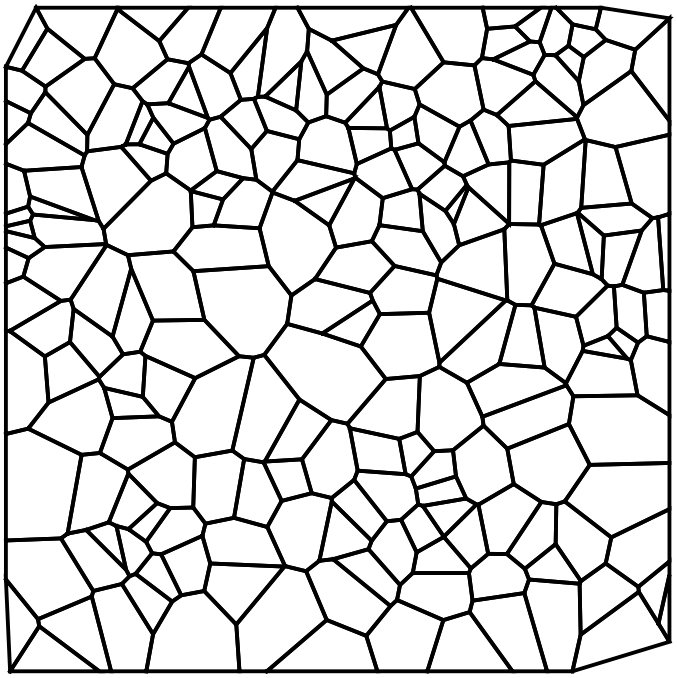


In[61]:=
```
(*edges=Flatten[Map[Partition[#,2,1,1]&,Values[cellToVertexG]],1];*)
$cellToPts = cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
$periPolygonAssoc = periPolygonAssoc = perimeterPolygon /@ cellToPts;
$areaPolygonAssoc = areaPolygonAssoc = areaPolygon /@ cellToPts;
```

In[64]:=
```
Clear[plt, indTopts, ptsToInd, vertexToCell,
  cellToVertexG, periPolygonAssoc, areaPolygonAssoc, cellToPts, edges];
```

In[65]:=
```
pltOriginal = Graphics[{Black, Thick,
   Values@Map[Line[Join[##, {First@#}]] &@Lookup[$indTopts, #] &, $cellToVertexG]}]
```

Out[65]=

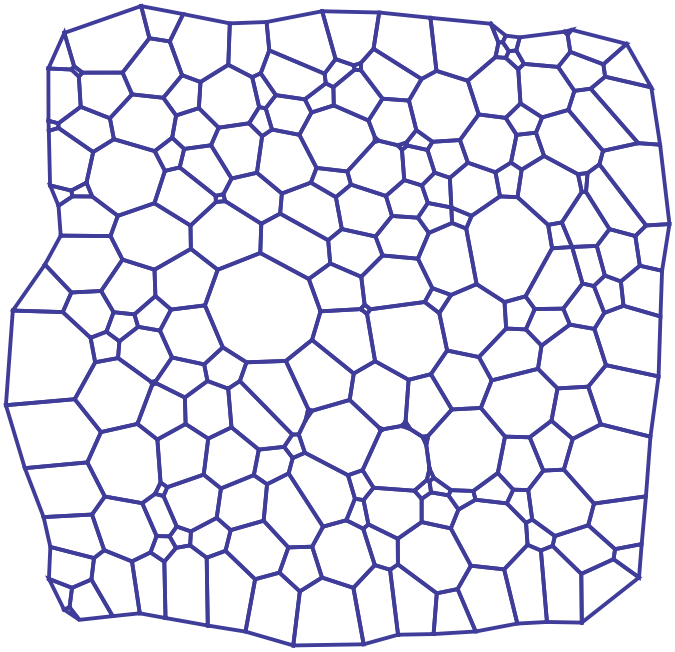


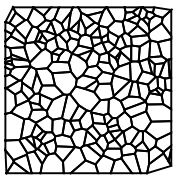
In[66]:=
```
t = δt;
indTopts = $indTopts;
ptsToInd = $ptsToInd;
vertexToCell = $vertexToCell;
cellToVertexG = $cellToVertexG;
periPolygonAssoc = $periPolygonAssoc;
areaPolygonAssoc = $areaPolygonAssoc;
cellToPts = $cellToPts;
edges = DeleteDuplicatesBy[
   Flatten[Map[Partition[#, 2, 1, 1] &, Values@$cellToVertexG], 1], Sort];
```

In[75]:=
```
Module[{cellsToRemove, vertsToRemove, edgechanged, polydiv},
  saveres = First@Last@Reap@Monitor[
       While[t ≤ 80 δt,
        (* T2 transitions *)
        {cellsToRemove, vertsToRemove} = cellsforT2[areaPolygonAssoc, cellToVertexG];
        If[cellsToRemove ≠ {},
         {indTopts, cellToVertexG, vertexToCell, areaPolygonAssoc, periPolygonAssoc} =
          T2TransitionFn[{cellsToRemove, vertsToRemove}, indTopts,
           cellToVertexG, areaPolygonAssoc, periPolygonAssoc]
        ];
        (* T1 transitions *)
        edges = DeleteDuplicatesBy[
          Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1], Sort];
        {edgechanged, indTopts, cellToVertexG, vertexToCell} =
         T1transitionFn[edges, indTopts, vertexToCell, cellToVertexG];
        cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ cellToPts;
        periPolygonAssoc = perimeterPolygon /@ cellToPts;
        (* Divisions *)
        polydiv = selectDivCells[areaPolygonAssoc];
        (*polydiv=pickcellsDiv[cellToVertexG,areaPolygonAssoc];*)
        If[polydiv ≠ {},
         Scan[
           ({indTopts, cellToVertexG, areaPolygonAssoc, periPolygonAssoc} = cellDivision[
               #, indTopts, areaPolygonAssoc, periPolygonAssoc, cellToVertexG]) &,
          polydiv];
         vertexToCell = GroupBy[
           Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
        ];
        (* update positions *)
        indTopts = AssociationThread[Keys[indTopts] → (Values[indTopts] + F_T[indTopts,
              vertexToCell, cellToVertexG, areaPolygonAssoc, periPolygonAssoc] δt)];
        cellToPts = Lookup[indTopts, #] & /@ cellToVertexG;
        areaPolygonAssoc = areaPolygon /@ cellToPts;
        periPolygonAssoc = perimeterPolygon /@ cellToPts;
        (*plt=Graphics[{ColorData[1][1],Thick,Values@Map[Line[Join[##,{First@#}]]&@
                 Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
        plt = Graphics[{FaceForm[LightBlue], EdgeForm[Black],
           Values[Polygon@Lookup[indTopts, #] & /@ cellToVertexG]}];
        Sow[plt];
        t += δt;
       ], plt
      ]
 ];
```

In[76]:=
```
Graphics[{■, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indTopts, #] &, cellToVertexG]},
 ImageSize → Medium]
```

Out[76]=



In[78]:=
```
Show[  /. Black → Red, Graphics[{ , Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indTopts, #] &, cellToVertexG]}]]
```

Out[78]=