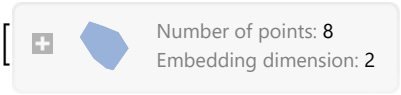
Cell Division Axis

scratch work

finding axis of division

In[460]:=
```
Clear[centroidPolygon];
centroidPolygon[vertices_] := Mean@vertices
```

In[462]:=
```
me = Polygon[ 

];
```

In[463]:=
```
p = MeshPrimitives[me, 0] /. Point → Sequence;
```

In[464]:=
```
q = With[{mean = Mean[p]},
      # - mean] & /@ p;
```

In[465]:=
```
Evaluate[Table[{xi, yi}, {i, 9}]] = Append[q, First@q];
```

In[466]:=
$$I_{xx} = 1 / 12 \sum_{i=1}^{8} (x_i\, y_{i+1} - x_{i+1}\, y_i)\, \left(y_i^2 + y_i\, y_{i+1} + y_{i+1}^2\right);$$

$$I_{yy} = 1 / 12 \sum_{i=1}^{8} (x_i\, y_{i+1} - x_{i+1}\, y_i)\, \left(x_i^2 + x_i\, x_{i+1} + x_{i+1}^2\right);$$

$$I_{xy} = 1 / 24 \sum_{i=1}^{8} (x_i\, y_{i+1} - x_{i+1}\, y_i)\, (x_i\, y_{i+1} + 2\, x_i\, y_i + 2\, x_{i+1}\, y_{i+1} + x_{i+1}\, y_i);$$

In[469]:=
$$mat = \begin{pmatrix} I_{xx} & -I_{xy} \\ -I_{xy} & I_{yy} \end{pmatrix}$$

Out[469]=
$$\left\{\left\{0.0000185152,\ 8.34941 \times 10^{-6}\right\},\ \left\{8.34941 \times 10^{-6},\ 0.0000274721\right\}\right\}$$

In[470]:=
```
Eigensystem[mat]
```

Out[470]=
```
{{0.0000324683, 0.000013519}, {{0.51348, 0.858102}, {-0.858102, 0.51348}}}
```

In[471]:=
```
eigvals = Eigenvalues[mat]
```

Out[471]=
```
{0.0000324683, 0.000013519}
```

In[472]:=
```
eigvectors = Eigenvectors[mat]
```

Out[472]=
```
{{0.51348, 0.858102}, {-0.858102, 0.51348}}
```

In[473]:=
```
Norm /@ eigvectors
```

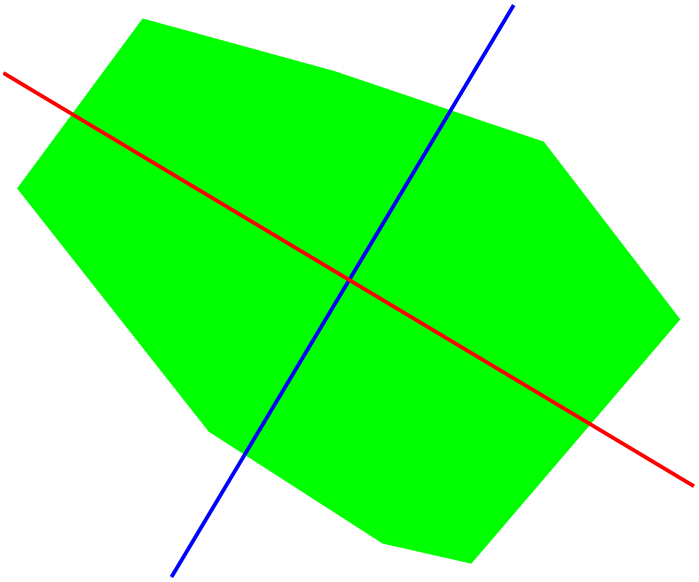
Out[473]=
```
{1., 1.}
```

In[474]:=
```
pos = Position[eigvals, Max[eigvals]];
```

In[475]:=
```
cent = RegionCentroid[me];
```

In[476]:=
```
Graphics[
 {{Green, me}, Thick, Blue, InfiniteLine[{cent, cent + Extract[eigvectors, pos][[1]]}],
  Red, InfiniteLine[{cent, cent + Extract[eigvectors, {{2}}][[1]]}]}]
```

Out[476]=



In[477]:=
```
edges = Partition[p, 2, 1, 1];
```

In[478]:=
```
edgePart = Line /@ Partition[p, 2, 1, 1];
```

In[479]:=
```
intersects =
  RegionIntersection[InfiniteLine[{cent, cent + Extract[eigvectors, pos][[1]]}], #] & /@
   edgePart;
```

In[480]:=
```
intersectPts = Cases[intersects, {__Real}, {3}];
```

In[481]:=
```
posIntersects = Flatten@Position[intersects, _Point, {1}];
```

In[482]:=
```
repPart = Thread[{posIntersects, 2}];
```

In[483]:=
```
{α, β} = intersectPts;
```

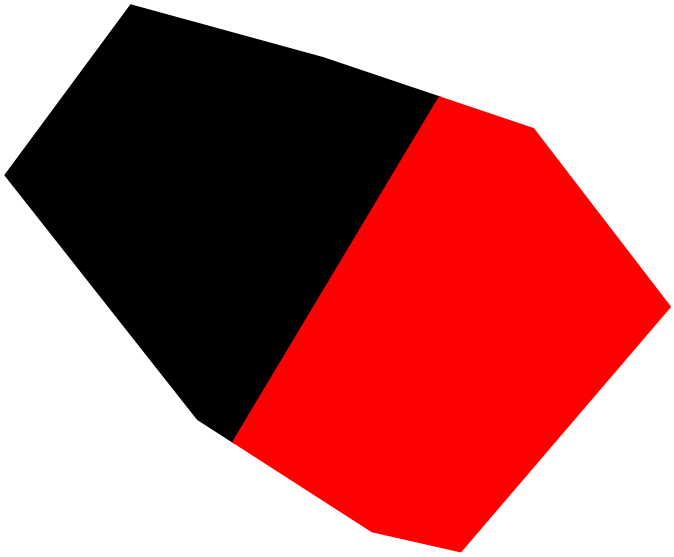
In[484]:=
```
inserts = Reverse@Thread[{repPart, intersectPts}];
```

In[485]:=
```
withAdditions =
  DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, inserts], 1];
```

In[486]:=
```
(* i=0;
Insert[p,"mark",{{4},{5}}]/."mark":>(++i;
    intersectPts[[i]])//Trace*)
```

In[487]:=
```
Graphics[{Black, Polygon[Join @@ SequenceCases[withAdditions, {___, α} | {β, ___}]],
  Red, Polygon[Join @@ SequenceCases[withAdditions, {α, __, β}]]}]
```

Out[487]=



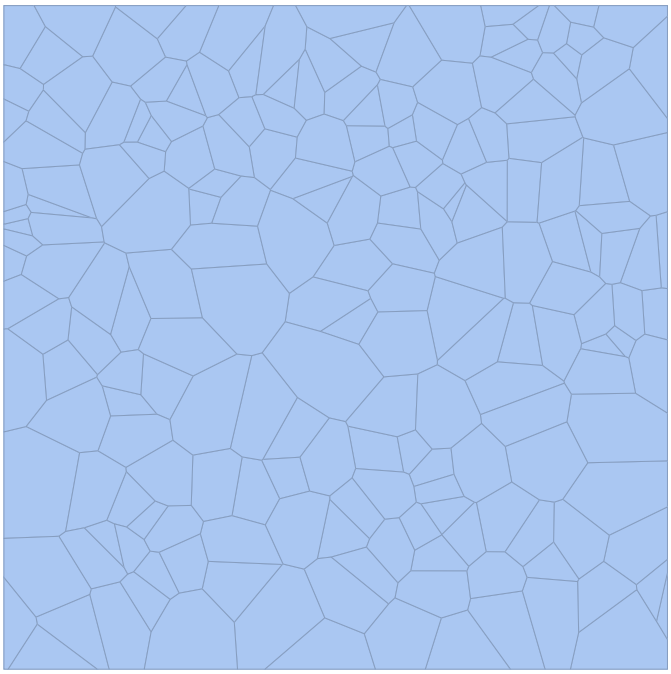
In[488]:=
```
Table[
  {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, Length[p] + 1}];
```

Initialize mesh

In[489]:=
```
SeedRandom[3]
mesh = VoronoiMesh[RandomReal[1, {200, 2}], {{0, 1}, {0, 1}}, ImageSize → Medium]
```

Out[489]=
```
RandomGeneratorState[ Method: ExtendedCA
                      State hash: -2 772 983 392 497 998 393 ]
```

Out[490]=



In[491]:=
```
pts = MeshPrimitives[mesh, 0] /. Point → Sequence;
```

In[492]:=
```
cornerpts = pts[[-4 ;;]];
pts = pts[[1 ;; -5]];
```

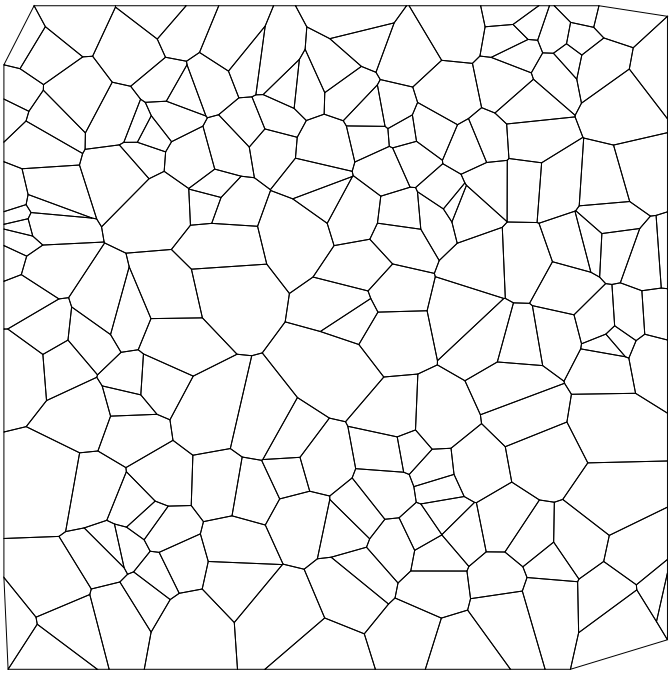
In[494]:=
```
ptsToInd = AssociationThread[pts → Range@Length[pts]];
indToptsC = indTopts = AssociationMap[Reverse][ptsToInd];
```

In[496]:=
```
cellmeshprim = MeshPrimitives[mesh, 2];
cells = (MeshPrimitives[#, 0] & /@ cellmeshprim) /. Point → Sequence /.
    Thread[cornerpts → Nothing];
```

In[498]:=
```
cellToVertexG = AssociationThread[Range[Length@cells] → Map[ptsToInd, cells, {2}]];
vertexToCell =
  GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
```

In[500]:=
```
plt = Graphics[Map[{FaceForm[None], EdgeForm[Black], Polygon@Lookup[indTopts, #]} &,
   Values@cellToVertexG], ImageSize → Medium]
```

Out[500]=



cell division line

In[501]:=
```
Clear[areaOfPolygon];
areaOfPolygon[cells_ /; Head[cells] === Association] := Map[Area@*Polygon, cells];
```

In[503]:=
```
Clear[areaPolygon];
areaPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  0.5 Abs@Total[(#[[1, 1]] * #[[2, 2]]) - (#[[2, 1]] * #[[1, 2]]) & /@ edges]
 ]
```

In[505]:=
```
Clear[perimeterOfPolygon];
perimeterOfPolygon[cells_ /; Head[cells] === Association] :=
  (Perimeter@*Polygon) /@ cells;
```

In[507]:=
```
Clear[perimeterPolygon];
perimeterPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  Total[Apply[EuclideanDistance] /@ edges]
 ]
```

In[509]:=
```
Clear[cellDivision];
cellDivision[polygonind_, indToPoints_, areaAssoc_, perimAssoc_, cellToVertG_] :=
```

```
Block[{x, y, num, matrix, xx, xy, yy, eigvals, eigVecs, maxeigpos, cent, edges,
  edgesL, intersects, intersectionPts, posIntersections, repPart, α, β,
  polygonPts, newkeys = Range[# + 1, # + 2] &[Max@Keys[indToPoints]], newPtToInds,
  indtoPtAssoc = indToPoints, ptToIndAssoc, edgeinds, contour, poly1, poly2, res, seq,
  newcells = Range[# + 1, # + 2] &[Max@Keys[areaAssoc]], CVG = cellToVertG,
  addcellsRule, polygonPtsInds, VCG, polygonptsTrans},

 VCG = GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
 polygonPtsInds = CVG[polygonind];
 num = Length@polygonPtsInds;
 ptToIndAssoc = AssociationMap[Reverse, indToPoints];
 polygonPts = Lookup[indToPoints, polygonPtsInds];
 polygonptsTrans = With[{mean = Mean@polygonPts},
   # - mean & /@ polygonPts
  ];
 Evaluate[Table[{x_i, y_i}, {i, num + 1}]] =
  Append[polygonptsTrans, First@polygonptsTrans];
 I_xx = (1/12) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (y_i^2 + y_i y_{i+1} + y_{i+1}^2);
 I_yy = (1/12) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (x_i^2 + x_i x_{i+1} + x_{i+1}^2);
 I_xy = (1/24) Sum_{i=1}^{num} (x_i y_{i+1} - x_{i+1} y_i) (x_i y_{i+1} + 2 x_i y_i + 2 x_{i+1} y_{i+1} + x_{i+1} y_i);
 Table[
  {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, num + 1}];
 matrix = {{I_xx, -I_xy}, {-I_xy, I_yy}};
 {eigvals, eigVecs} = Eigensystem@matrix;
 maxeigpos = Position[eigvals, Max@eigvals];
 {edges, edgeinds} = Partition[#, 2, 1, 1] & /@ {polygonPts, polygonPtsInds};
 edgesL = Line /@ edges;
 cent = centroidPolygon[polygonPts];
 intersects = RegionIntersection[
     InfiniteLine[{cent, cent + Extract[eigVecs, maxeigpos][[1]]}], #] & /@ edgesL;
 intersectionPts = Cases[intersects, {(_Real | _Integer) ..}, {3}];
 newPtToInds = Thread[intersectionPts → newkeys];
 posIntersections = Flatten@Position[intersects, _Point, {1}];
 MapThread[
  (res = Complement[Intersection @@ Lookup[VCG, #2], {polygonind}];
    If[res ≠ {},
     seq = Partition[CVG[First@res], 2, 1, 1];
     AppendTo[CVG,
      First@res → DeleteDuplicates@
        Flatten@SequenceSplit[seq, {x___, p : {OrderlessPatternSequence[
                #2[[1]], #2[[-1]]]}, y___} :→ {x, Insert[p, #1, 2], y}]
     ];
    ];) & , {newkeys, edgeinds[[posIntersections]]}];
```

```
  repPart =
   Thread[{Thread[{ReverseSort@posIntersections, 2}], Reverse[intersectionPts]}];
  {α, β} = intersectionPts;
  AppendTo[ptToIndAssoc, newPtToInds];
  AppendTo[indtoPtAssoc, Reverse[newPtToInds, 2]];
  contour =
   DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, repPart], 1];
  poly1 = Join @@ SequenceCases[contour, {___, α} | {β, ___}];
  poly2 = Join @@ SequenceCases[contour, {α, __, β}];
  KeyDropFrom[CVG, polygonind];
  addcellsRule = Thread[newcells → {poly1, poly2}];
  AppendTo[CVG, addcellsRule /. ptToIndAssoc];
  {indtoPtAssoc, CVG, Append[KeyDrop[areaAssoc, polygonind],
     MapAt[Area@*Polygon, addcellsRule, {All, 2}]],
    Append[KeyDrop[perimAssoc, polygonind],
     MapAt[Perimeter@*Polygon, addcellsRule, {All, 2}]]}
 ];
```

Apply division algorithm

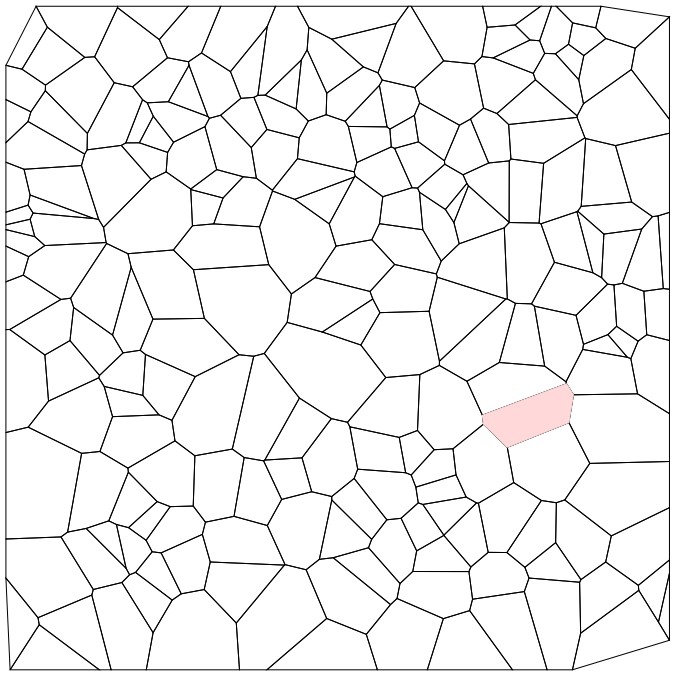
In[511]:=
```
polys = Polygon /@ Map[Lookup[indTopts, #] &, cellToVertexG, {2}];
```

In[512]:=
```
areaPolygonAssoc = Area /@ polys;
```

In[513]:=
```
periPolygonAssoc = Perimeter /@ polys;
```

In[514]:=
```
Show[plt, Graphics[{LightRed, Polygon@Lookup[indTopts, cellToVertexG[138]]}]]
```

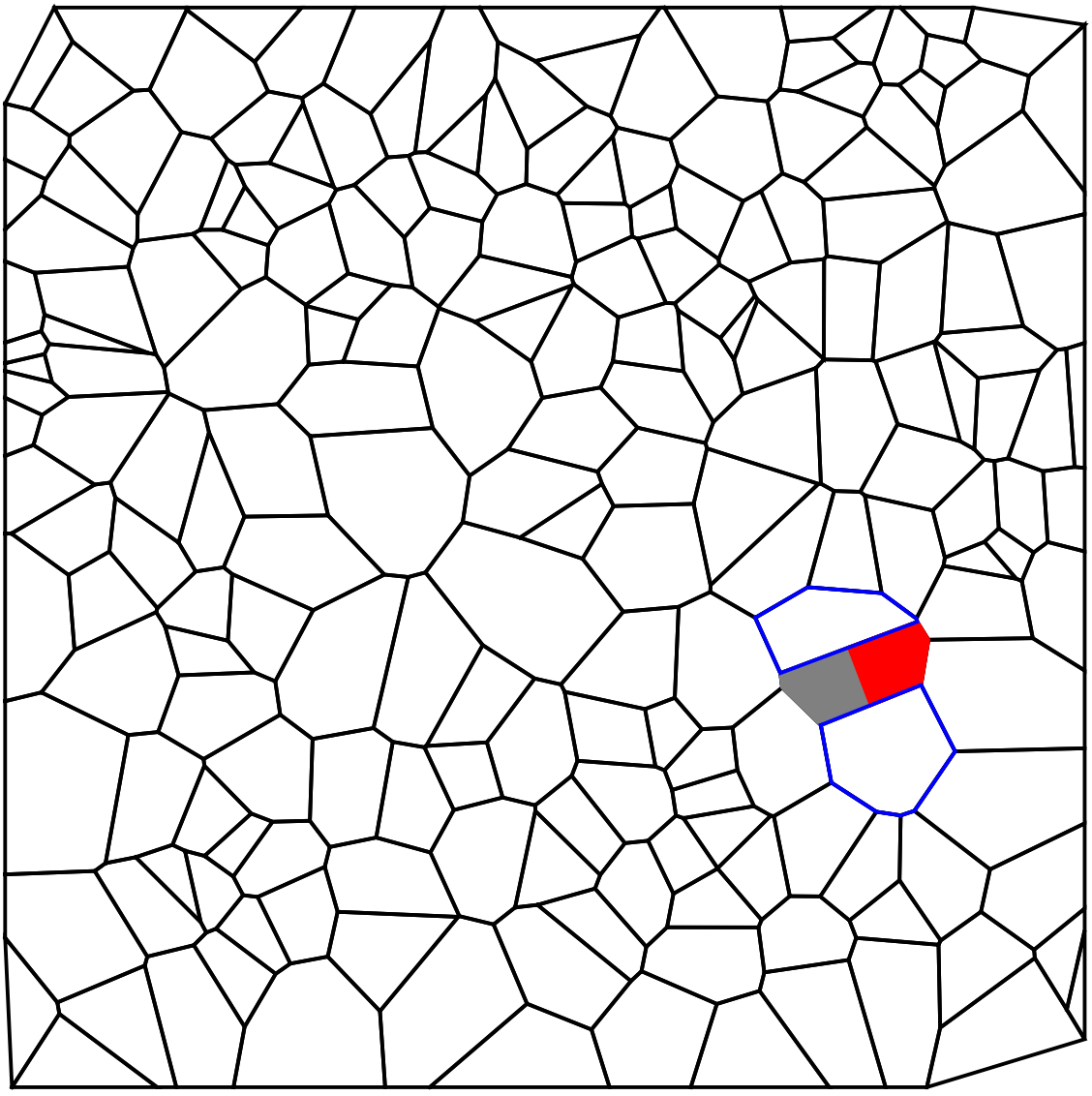
Out[514]=



In[515]:=
```
{indToptsCG, cellToVertexCG, areaPolygonAssocCG, periPolygonAssocCG} =
  cellDivision[138, indTopts, areaPolygonAssoc, periPolygonAssoc, cellToVertexG];
```

In[516]:=
```
Graphics[{Black, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indToptsCG, #] &, cellToVertexCG],
  Gray, Polygon@Lookup[indToptsCG, cellToVertexCG[201]], Red,
  Polygon@Lookup[indToptsCG, cellToVertexCG[202]],
  Blue, Line[Join[##, {First@#}]] &@Lookup[indToptsCG, cellToVertexCG[#]] & /@ {94, 154}}
]
```

Out[516]=



In[517]:=
```
TakeLargest[areaPolygonAssoc, 5]
```

Out[517]=
```
⟨| 190 → 0.0160202, 125 → 0.0158383, 187 → 0.0146222, 99 → 0.0138049, 176 → 0.0132217 |⟩
```

In[518]:=
```
{indToptsCG, cellToVertexCG, areaPolygonAssocCG, periPolygonAssocCG} =
  cellDivision[190, indToptsCG, areaPolygonAssocCG, periPolygonAssocCG, cellToVertexCG];

Graphics[{Black, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indToptsCG, #] &, cellToVertexCG],
  Gray, Polygon@Lookup[indToptsCG, cellToVertexCG[204]], Red,
  Polygon@Lookup[indToptsCG, cellToVertexCG[203]],
  Gray, Polygon@Lookup[indToptsCG, cellToVertexCG[202]], Red,
  Polygon@Lookup[indToptsCG, cellToVertexCG[201]]}]
```

Out[519]=