Cell Division Axis

scratch work

In[1510]:=
```
Clear[α, β];
DeleteDuplicates@Flatten@Fold[Insert[#1, #2[[2]], #2[[1]]] &,
   {{a, b}, {b, c}, {c, d}, {d, e}, {e, a}}, {{{4, 2}, β}, {{3, 2}, α}}]
```

Out[1511]=
```
{a, b, c, α, d, β, e}
```

In[1512]:=
```
Flatten@SequenceCases[{a, b, c, α, d, β, e}, {___, α} | {β, ___}]
```

Out[1512]=
```
{a, b, c, α, β, e}
```

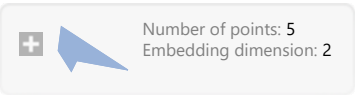
In[1513]:=
```
Flatten@SequenceCases[{a, b, c, α, d, β, e}, {α, __, β}]
```

Out[1513]=
```
{α, d, β}
```

finding axis of division

In[1514]:=
```
Clear[centroidPolygon];
centroidPolygon[vertices_] := Mean@vertices
```

In[1516]:=
```
(* allow cells to divide *)
```

In[1517]:=
```
me = Polygon[
```


```
];
```

In[1518]:=
```
p = MeshPrimitives[me, 0] /. Point → Sequence
```

Out[1518]=
```
{{0.190657, 0.259949}, {0.340267, 0.0238182},
 {0.40141, 0.146524}, {0.985985, 0.0397145}, {0.226276, 0.534713}}
```

In[1519]:=
```
Evaluate[Table[{xi, yi}, {i, 6}]] = Append[p, First@p];
```

In[1520]:=
$$I_{xx} = 1/12 \sum_{i=1}^{5} (x_i\, y_{i+1} - x_{i+1}\, y_i)\, \left(y_i^2 + y_i\, y_{i+1} + y_{i+1}^2\right)$$

Out[1520]=
```
0.0108197
```

In[1521]:=
$$I_{yy} = 1/12 \sum_{i=1}^{5} (x_i\, y_{i+1} - x_{i+1}\, y_i)\, \left(x_i^2 + x_i\, x_{i+1} + x_{i+1}^2\right)$$

Out[1521]=
```
0.036854
```

In[1522]:=
$$I_{xy} = 1/24 \sum_{i=1}^{5} (x_i\, y_{i+1} - x_{i+1}\, y_i)\, \left(x_i\, y_{i+1} + 2\, x_i\, y_i + 2\, x_{i+1}\, y_{i+1} + x_{i+1}\, y_i\right)$$

Out[1522]=
```
0.0152804
```

In[1523]:= mat = $\begin{pmatrix} I_{xx} & -I_{xy} \\ -I_{xy} & I_{yy} \end{pmatrix}$

Out[1523]= {{0.0108197, -0.0152804}, {-0.0152804, 0.036854}}

In[1524]:= Eigensystem[mat]

Out[1524]= {{0.0439101, 0.00376355}, {{-0.419236, 0.907877}, {-0.907877, -0.419236}}}

In[1525]:= eigvals = Eigenvalues[mat]

Out[1525]= {0.0439101, 0.00376355}

In[1526]:= eigvectors = Eigenvectors[mat]

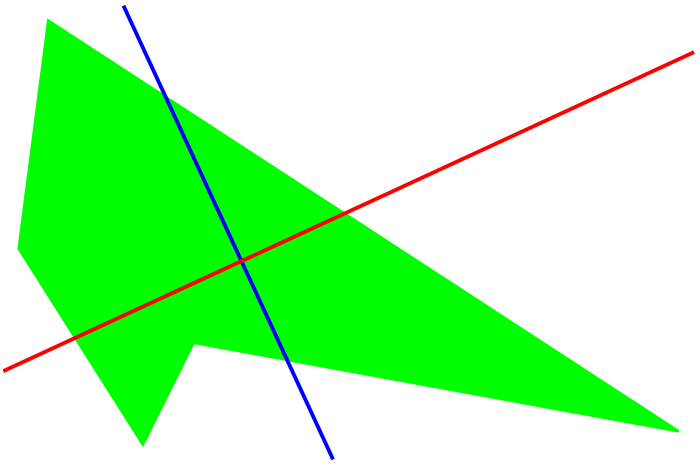
Out[1526]= {{-0.419236, 0.907877}, {-0.907877, -0.419236}}

In[1527]:= pos = Position[eigvals, Max[eigvals]];

In[1528]:= cent = RegionCentroid[me];

In[1529]:=
```
Graphics[
 {{Green, me}, Thick, Blue, InfiniteLine[{cent, cent + Extract[eigvectors, pos][[1]]}],
  Red, InfiniteLine[{cent, cent + Extract[eigvectors, {{2}}][[1]]}]}]
```

Out[1529]=



In[1530]:= edges = Partition[p, 2, 1, 1];

In[1531]:= edgePart = Line /@ Partition[p, 2, 1, 1];

In[1532]:=
```
intersects =
  RegionIntersection[InfiniteLine[{cent, cent + Extract[eigvectors, pos][[1]]}], #] & /@
   edgePart;
```

In[1533]:= intersectPts = Cases[intersects, {__Real}, {3}];

In[1534]:= posIntersects = Flatten@Position[intersects, _Point, {1}];

In[1535]:= repPart = Thread[{posIntersects, 2}];

In[1536]:= {α, β} = intersectPts;

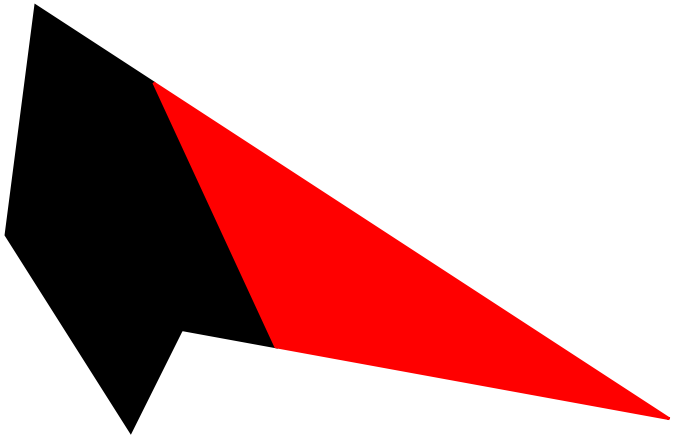
In[1537]:= inserts = Reverse@Thread[{repPart, intersectPts}];

In[1538]:=
```
withAdditions =
  DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, inserts], 1];
```

In[1539]:=
```
(* i=0;
Insert[p,"mark",{{4},{5}}]/."mark"⧴(++i;
    intersectPts[[i]])//Trace*)
```

In[1540]:=
```
Graphics[{Black, Polygon[Join @@ SequenceCases[withAdditions, {___, α} | {β, ___}]],
  Red, Polygon[Join @@ SequenceCases[withAdditions, {α, __, β}]]}]
```

Out[1540]=

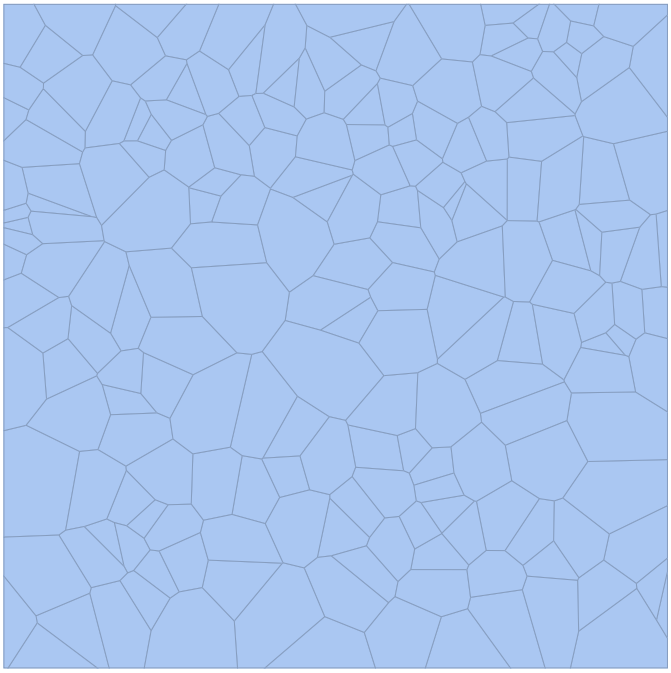


In[1541]:=
```
Table[
  {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, Length[p] + 1}];
```

Initialize mesh

In[1542]:=
```
SeedRandom[3]
mesh = VoronoiMesh[RandomReal[1, {200, 2}], {{0, 1}, {0, 1}}, ImageSize → Medium]
```

Out[1543]=



In[1544]:=
```
pts = MeshPrimitives[mesh, 0] /. Point → Sequence;
```

In[1545]:=
```
cornerpts = pts[[-4 ;;]];
pts = pts[[1 ;; -5]];
```

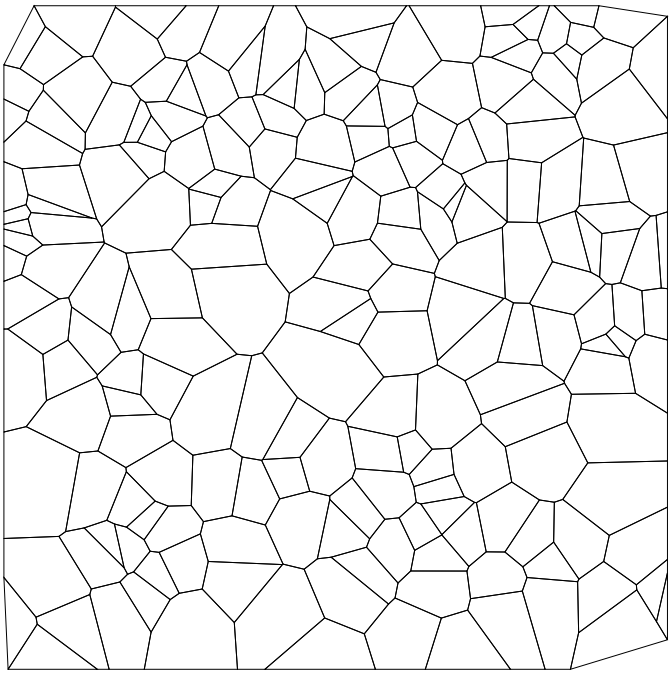
In[1547]:=
```
ptsToInd = AssociationThread[pts → Range@Length[pts]];
indToptsC = indTopts = AssociationMap[Reverse][ptsToInd];
```

In[1549]:=
```
cellmeshprim = MeshPrimitives[mesh, 2];
cells = (MeshPrimitives[#, 0] & /@ cellmeshprim) /. Point → Sequence /.
    Thread[cornerpts → Nothing];
```

In[1551]:=
```
cellToVertexG = AssociationThread[Range[Length@cells] → Map[ptsToInd, cells, {2}]];
vertexToCell =
  GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
```

In[1553]:=
```
plt = Graphics[Map[{FaceForm[None], EdgeForm[Black], Polygon@Lookup[indTopts, #]} &,
   Values@cellToVertexG], ImageSize → Medium]
```

Out[1553]=



In[1554]:=
```
Clear[areaOfPolygon];
areaOfPolygon[cells_ /; Head[cells] === Association] := Map[Area@*Polygon, cells];
```

In[1556]:=
```
Clear[areaPolygon];
areaPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  0.5 Abs@Total[(#[[1, 1]] * #[[2, 2]]) - (#[[2, 1]] * #[[1, 2]]) & /@ edges]
 ]
```

In[1558]:=
```
Clear[perimeterOfPolygon];
perimeterOfPolygon[cells_ /; Head[cells] === Association] :=
  (Perimeter@*Polygon) /@ cells;
```

In[1560]:=
```
Clear[perimeterPolygon];
perimeterPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  Total[Apply[EuclideanDistance] /@ edges]
 ]
```

In[1562]:=
```
Clear[cellDivision];
cellDivision[polygonind_, indToPoints_, areaAssoc_, perimAssoc_, cellToVertG_] :=
  Block[{x, y, num, matrix, xx, xy, yy, eigvals, eigVecs, maxeigpos, cent, edges,
    edgesL, intersects, intersectionPts, posIntersections, repPart, α, β,
    polygonPts, newkeys = Range[# + 1, # + 2] &[Max@Keys[indToPoints]], newPtToInds,
```

```
 indtoPtAssoc = indToPoints, ptToIndAssoc, edgeinds, contour, poly1, poly2, res, seq,
 newcells = Range[# + 1, # + 2] &[Max@Keys[areaAssoc]],
 CVG = cellToVertG, addcellsRule, polygonPtsInds, VCG},

VCG = GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
polygonPtsInds = CVG[polygonind];
num = Length@polygonPtsInds;
ptToIndAssoc = AssociationMap[Reverse, indToPoints];
polygonPts = Lookup[indToPoints, polygonPtsInds];
Evaluate[Table[{xi, yi}, {i, num + 1}]] = Append[polygonPts, First@polygonPts];
Ixx = (1/12) Sum_{i=1}^{num} (xi yi+1 - xi+1 yi) (yi^2 + yi yi+1 + yi+1^2);
Iyy = (1/12) Sum_{i=1}^{num} (xi yi+1 - xi+1 yi) (xi^2 + xi xi+1 + xi+1^2);
Ixy = (1/24) Sum_{i=1}^{num} (xi yi+1 - xi+1 yi) (xi yi+1 + 2 xi yi + 2 xi+1 yi+1 + xi+1 yi);
Table[
 {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, num + 1}];
matrix = ( Ixx   -Ixy
          -Ixy    Iyy );
{eigvals, eigVecs} = Eigensystem@matrix;
maxeigpos = Position[eigvals, Max@eigvals];
{edges, edgeinds} = Partition[#, 2, 1, 1] & /@ {polygonPts, polygonPtsInds};
edgesL = Line /@ edges;
cent = centroidPolygon[polygonPts];
intersects = RegionIntersection[
    InfiniteLine[{cent, cent + Extract[eigVecs, maxeigpos][[1]]}], #] & /@ edgesL;
intersectionPts = Cases[intersects, {(_Real | _Integer) ..}, {3}];
newPtToInds = Thread[intersectionPts → newkeys];
posIntersections = Flatten@Position[intersects, _Point, {1}];
MapThread[
 (res = Complement[Intersection @@ Lookup[VCG, #2], {polygonind}];
   If[res ≠ {},
    seq = Partition[CVG[First@res], 2, 1, 1];
    AppendTo[CVG,
     First@res → DeleteDuplicates@
       Flatten@SequenceSplit[seq, {x___, p : {OrderlessPatternSequence[
              #2[[1]], #2[[-1]]]}, y___} :↦ {x, Insert[p, #1, 2], y}]
    ];
   ]; ) &, {newkeys, edgeinds[[posIntersections]]}];

repPart =
 Thread[{Thread[{ReverseSort@posIntersections, 2}], Reverse[intersectionPts]}];
{α, β} = intersectionPts;
AppendTo[ptToIndAssoc, newPtToInds];
AppendTo[indtoPtAssoc, Reverse[newPtToInds, 2]];
contour =
 DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, repPart], 1];
poly1 = Join @@ SequenceCases[contour, {___, α} | {β, ___}];
```

```
    poly2 = Join @@ SequenceCases[contour, {α, __, β}];
    KeyDropFrom[CVG, polygonind];
    addcellsRule = Thread[newcells → {poly1, poly2}];
    AppendTo[CVG, addcellsRule /. ptToIndAssoc];
    {indtoPtAssoc, CVG, Append[KeyDrop[areaAssoc, polygonind],
      MapAt[Area@*Polygon, addcellsRule, {All, 2}]],
     Append[KeyDrop[perimAssoc, polygonind],
      MapAt[Perimeter@*Polygon, addcellsRule, {All, 2}]]}
   ];
```

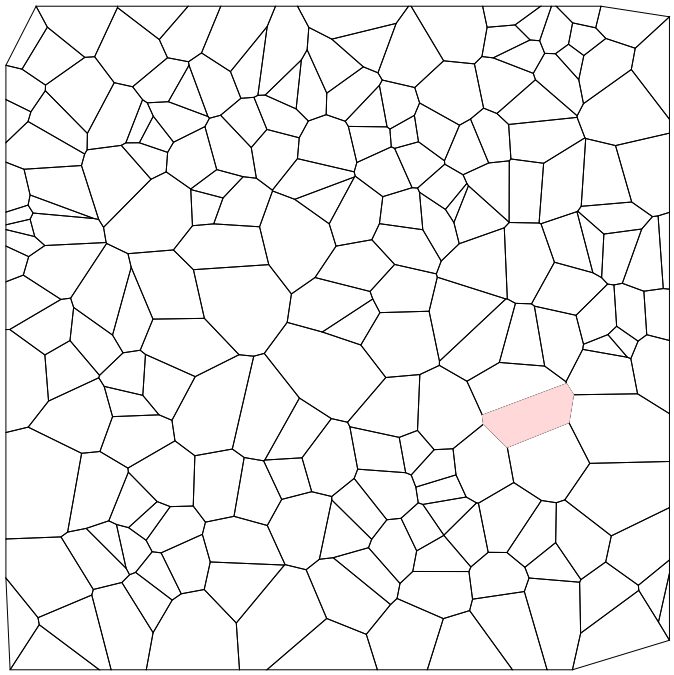
In[1564]:=
```
polys = Polygon /@ Map[Lookup[indTopts, #] &, cellToVertexG, {2}];
```

In[1565]:=
```
areaPolygonAssoc = Area /@ polys;
```

In[1566]:=
```
periPolygonAssoc = Perimeter /@ polys;
```

In[1567]:=
```
Show[plt, Graphics[{LightRed, Polygon@Lookup[indTopts, cellToVertexG[138]]}]]
```

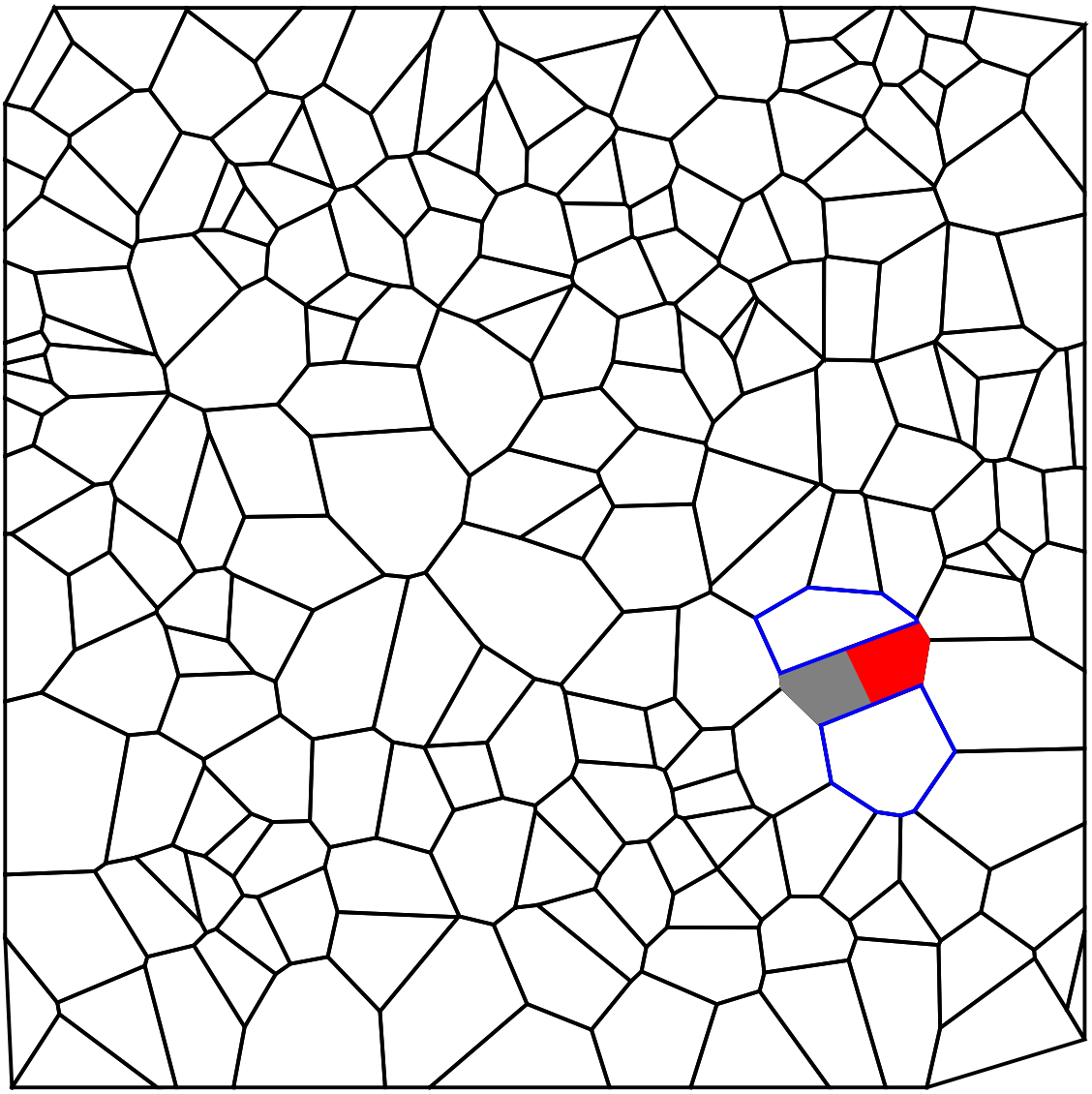
Out[1567]=



In[1568]:=
```
{indToptsCG, cellToVertexCG, areaPolygonAssocCG, periPolygonAssocCG} =
  cellDivision[138, indTopts, areaPolygonAssoc, periPolygonAssoc, cellToVertexG];
```

In[1569]:=
```
Graphics[{Black, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indToptsCG, #] &, cellToVertexCG],
  Gray, Polygon@Lookup[indToptsCG, cellToVertexCG[201]], Red,
  Polygon@Lookup[indToptsCG, cellToVertexCG[202]],
  Blue, Line[Join[##, {First@#}]] &@Lookup[indToptsCG, cellToVertexCG[#]] & /@ {94, 154}}
]
```

Out[1569]=



In[1570]:=
```
TakeLargest[areaPolygonAssoc, 5]
```

Out[1570]= ⟨| 190 → 0.0160202, 125 → 0.0158383, 187 → 0.0146222, 99 → 0.0138049, 176 → 0.0132217 |⟩

In[1571]:=
```
{indToptsCG, cellToVertexCG, areaPolygonAssocCG, periPolygonAssocCG} =
  cellDivision[190, indToptsCG, areaPolygonAssocCG, periPolygonAssocCG, cellToVertexCG];

Graphics[{Black, Thick,
  Values@Map[Line[Join[##, {First@#}]] &@Lookup[indToptsCG, #] &, cellToVertexCG],
  Gray, Polygon@Lookup[indToptsCG, cellToVertexCG[204]], Red,
  Polygon@Lookup[indToptsCG, cellToVertexCG[203]],
  Gray, Polygon@Lookup[indToptsCG, cellToVertexCG[202]], Red,
  Polygon@Lookup[indToptsCG, cellToVertexCG[201]]}]
```

Out[1572]=