2-dimensional vertex model :→ simulating epithelial tissue

In[1]:=

```
LaunchKernels[]
ParallelTable[$KernelID, {i, $KernelCount}]
```

Out[1]= {KernelObject[Name: Local kernel KernelID: 1], KernelObject[Name: Local kernel KernelID: 2],
KernelObject[Name: Local kernel KernelID: 3], KernelObject[Name: Local kernel KernelID: 4],
KernelObject[Name: Local kernel KernelID: 5], KernelObject[Name: Local kernel KernelID: 6]}

Out[2]= {6, 5, 4, 3, 2, 1}

geometrical f(x)s

In[3]:=

```
(*get vertex in the list by ccw order*)
Clear[getCounterClockwise];
getCounterClockwise[vertex_, vertices_] := Block[{pos, v},
    pos = First @@ Position[vertices, vertex];
    If[pos == Length[vertices], pos = 1, pos += 1];
    vertices[[pos]]
   ];
```

In[5]:=

```
(*get vertex in the list by cw order*)
Clear[getClockwise];
getClockwise[vertex_, vertices_] := Block[{ls, pos},
    pos = First @@ Position[vertices, vertex];
    If[pos == 1, pos = Length[vertices], pos -= 1];
    vertices[[pos]]
   ];
```

In[7]:= getCounterClockwise[$\{x_i, y_i\}$, $\{\{x_{i-1}, y_{i-1}\}, \{x_i, y_i\}, \{x_{i+1}, y_{i+1}\}$, d, e}]

Out[7]= $\{x_{1+i}, y_{1+i}\}$

In[8]:= getClockwise[$\{x_i, y_i\}$, $\{\{x_{i-1}, y_{i-1}\}, \{x_i, y_i\}, \{x_{i+1}, y_{i+1}\}$, d, e}]

Out[8]= $\{x_{-1+i}, y_{-1+i}\}$

In[9]:=
```
Clear[areaOfPolygon];
areaOfPolygon[cells_ /; Head[cells] === Association] :=
  Parallelize[Map[Area@*Polygon, cells]];
```

In[11]:=
```
Clear[areaPolygon];
areaPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  0.5 Abs@Total[(#[[1, 1]] * #[[2, 2]]) - (#[[2, 1]] * #[[1, 2]]) & /@ edges]
 ]
```

In[13]:=
```
Clear[perimeterOfPolygon];
perimeterOfPolygon[cells_ /; Head[cells] === Association] :=
  Parallelize[(Perimeter@*Polygon) /@ cells];
```

In[15]:=
```
Clear[perimeterPolygon];
perimeterPolygon[vertices_] := Block[{edges},
  edges = Partition[vertices, 2, 1, 1];
  Total[Apply[EuclideanDistance] /@ edges]
 ]
```

In[17]:=
```
Clear[centroidPolygon];
centroidPolygon[vertices_] := Mean[vertices]
```

In[19]:=
```
(*counterclockwise polygonQ*)
Block[{signedarea = 0, j, vertlen = 5},
  Do[
   j = Mod[i, vertlen] + 1;
   signedarea += (x_i y_j - x_j y_i),
   {i, vertlen}];
  Echo[1/2 (signedarea)]
 ];
```

» $\frac{1}{2}(-x_2 y_1 + x_5 y_1 + x_1 y_2 - x_3 y_2 + x_2 y_3 - x_4 y_3 + x_3 y_4 - x_5 y_4 - x_1 y_5 + x_4 y_5)$

In[20]:=
```
Clear[polyCounterClockwiseQ];
polyCounterClockwiseQ[poly_] := Block[{area = 0, j, vertLength = Length[poly]},
  Do[
   j = Mod[i, vertLength] + 1;
   area += poly⟦i, 1⟧ * poly⟦j, 2⟧;
   area -= poly⟦j, 1⟧ * poly⟦i, 2⟧,
   {i, vertLength}
  ];
  (area / 2.) > 0
 ]
```

In[22]:=
```
(*Clear[sortCC];
sortCC[polyinds_,indTopts_,ptsToInds_]:=Block[{cent,poly},
  poly=Lookup[indTopts,polyinds];
  Lookup[ptsToInds,
   DeleteDuplicates@Flatten[MeshPrimitives[ConvexHullMesh[poly],1]/.Line→Sequence,1]
  ]
 ];*)
```

In[23]:=
```
(*sort points for a convex polygon in counter-clockwise direction*)
```

In[24]:=
```
Clear[sortPointsCC];
sortPointsCC[polyinds_, indTopts_, ptsToInds_] := Block[{cent, ordering, polyPoints},
  polyPoints = Lookup[indTopts, polyinds];
  cent = Mean[polyPoints];
  ordering = Ordering[ArcTan[#⟦1⟧, #⟦2⟧] &@ (# - cent) & /@ polyPoints];
  Lookup[ptsToInds, Part[polyPoints, ordering]]
 ]
```

In[26]:=
```
outeredges[indToPtsAssoc_, localtopology_] := Block[{k, temp, tcells, assoc},
   Reap[
      Scan[(k = #; temp = localtopology[k];
         tcells = temp⟦2⟧;
         If[tcells ≠ {},
          MapAt[Sow@Cases[Partition[#, 2, 1,
                1], OrderlessPatternSequence[{x : indToPtsAssoc[k], y : _}] :→
                {x, y}] &, temp⟦1⟧, {Key[#]} & /@ tcells];
         ]) &, Keys@indToPtsAssoc]
     ]⟦2⟧ // Flatten[#, 2] &
  ];
```

mesh restructuring operations

simulation domain

In[27]:=
```
xLim = yLim = {-1, 1};
𝒟 = Rectangle[{First@xLim, First@yLim}, {Last@xLim, Last@yLim}];
```

local topology

In[29]:=
```
ClearAll[periodicRules, transformRules];
periodicRules::Information =
  "shift the points outside the simulation domain to inside the domain";
transformRules::Information =
  "vector that shifts the point outside the simulation domain back inside";
With[{xlim1 = xLim⟦1⟧, xlim2 = xLim⟦2⟧, ylim1 = yLim⟦1⟧, ylim2 = yLim⟦2⟧, dstep = 2},
  periodicRules = Dispatch[{
     {x_ /; x ≥ xlim2, y_ /; y ≤ ylim1} :→ SetPrecision[{x - dstep, y + dstep}, 10],
     {x_ /; x ≥ xlim2, y_ /; ylim1 < y < ylim2} :→ SetPrecision[{x - dstep, y}, 10],
     {x_ /; xlim1 < x < xlim2, y_ /; y ≤ ylim1} :→ SetPrecision[{x, y + dstep}, 10],
     {x_ /; x ≤ xlim1, y_ /; y ≤ ylim1} :→ SetPrecision[{x + dstep, y + dstep}, 10],
     {x_ /; x ≤ xlim1, y_ /; ylim1 < y < ylim2} :→ SetPrecision[{x + dstep, y}, 10],
     {x_ /; x ≤ xlim1, y_ /; y ≥ ylim2} :→ SetPrecision[{x + dstep, y - dstep}, 10],
     {x_ /; xlim1 < x < xlim2, y_ /; y ≥ ylim2} :→ SetPrecision[{x, y - dstep}, 10],
     {x_ /; x ≥ xlim2, y_ /; y ≥ ylim2} :→ SetPrecision[{x - dstep, y - dstep}, 10]
    }];

  transformRules = Dispatch[{
     {x_ /; x ≥ xlim2, y_ /; y ≤ ylim1} :→ {-dstep, dstep}~SetPrecision~10,
     {x_ /; x ≥ xlim2, y_ /; ylim1 < y < ylim2} :→ {-dstep, 0}~SetPrecision~10,
     {x_ /; xlim1 < x < xlim2, y_ /; y ≤ ylim1} :→ {0, dstep}~SetPrecision~10,
     {x_ /; x ≤ xlim1, y_ /; y ≤ ylim1} :→ {dstep, dstep}~SetPrecision~10,
     {x_ /; x ≤ xlim1, y_ /; ylim1 < y < ylim2} :→ {dstep, 0}~SetPrecision~10,
     {x_ /; x ≤ xlim1, y_ /; y ≥ ylim2} :→ {dstep, -dstep}~SetPrecision~10,
     {x_ /; xlim1 < x < xlim2, y_ /; y ≥ ylim2} :→ {0, -dstep}~SetPrecision~10,
     {x_ /; x ≥ xlim2, y_ /; y ≥ ylim2} :→ {-dstep, -dstep}~SetPrecision~10,
     {___Real} :→ {0, 0}~SetPrecision~10}];
 ];
```

In[33]:=
```
Clear@getLocalTopology;
getLocalTopology[ptsToIndAssoc_, indToPtsAssoc_, vertexToCell_,
    cellVertexGrouping_, wrappedMat_, faceListCoords_][vertices_] :=
  Block[{localtopology = <||>, wrappedcellList = {}, vertcellconns,
    localcellunion, v, wrappedcellpos, vertcs = vertices, rl1, rl2,
    transVector, wrappedcellCoords, wrappedcells, vertOutofBounds,
    shiftedPt, transvecList = {}, $faceListCoords = faceListCoords,
    vertexQ, boundsCheck, rules, extractcellkeys, vertind,
    cellsconnected, wrappedcellsrem},
```

```
    vertexQ = MatchQ[vertices, {__?NumberQ}];
    If[vertexQ,
      (vertcellconns =
        AssociationThread[{#} , {vertexToCell[ptsToIndAssoc[#]]}] &@vertices;
       vertcs = {vertices};
       localcellunion = Flatten[Values@vertcellconns]),
      (vertcellconns = AssociationThread[# ,
            Lookup[vertexToCell, Lookup[ptsToIndAssoc, #]]] &@vertices;
       localcellunion = Union@Flatten[Values@vertcellconns])
    ];
    If[localcellunion ≠ {},
     AppendTo[localtopology,
       Thread[localcellunion →
         Map[Lookup[indToPtsAssoc, #] &, cellVertexGrouping /@ localcellunion, {2}]]
     ]
    ];
    (* condition to be an internal edge: both vertices should have 3 neighbours *)
    (* if a vertex has 3 cells in its local neighbourhood then the entire
       network topology about the vertex is known → no wrapping required *)
    (* else we need to wrap around the vertex because other cells
       are connected to it → periodic boundary conditions *)
    With[{vert = #},
        vertind = ptsToIndAssoc[vert];
        cellsconnected = vertexToCell[vertind];
        If[Length[cellsconnected] ≠ 3,
         If[(𝒟~RegionMember~vert),
           v = vert;
           With[{x = v⟦1⟧, y = v⟦2⟧},
            boundsCheck = (x == xLim⟦1⟧ || x == xLim⟦2⟧ || y == yLim⟦1⟧ || y == yLim⟦2⟧)];
           extractcellkeys = If[boundsCheck,
             {rl1, rl2} = {v, v /. periodicRules};
             rules = Block[{x$},
               With[{r = rl1, s = rl2},
                DeleteDuplicates[HoldPattern[SameQ[x$, r]] || HoldPattern[SameQ[x$, s]]]
               ]
              ];
             Position @@
              With[{rule = rules}, Hold[wrappedMat, x_ /; ReleaseHold@rule, {2}]],
             Position[wrappedMat, x_ /; SameQ[x, v], {2}]
            ];
           (* find cell indices that are attached to the vertex in wrappedMat *)
           wrappedcellpos = DeleteDuplicatesBy[
             Cases[extractcellkeys,
              {Key[p : Except[Alternatives @@
                    Join[localcellunion, Flatten@wrappedcellList]]], y__} ⧴ {p, y}],
             First];
           (*wrappedcellpos = wrappedcellpos/.
```

```
      {Alternatives@@Flatten[wrappedcellList],__} :> Sequence[];*)
  (* if a wrapped cell has not been considered earlier (i.e. is new)
   then we translate it to the position of the vertex *)
  If[wrappedcellpos ≠ {},
   If[vertexQ,
    transVector = SetPrecision[(v - Extract[$faceListCoords,
           Replace[#, {p_, q__} :> {Key[p], q}]]) & /@ wrappedcellpos, 10],
    (* call to function is enquiring an edge and not a vertex*)
    transVector = SetPrecision[(v - Extract[$faceListCoords,
           Replace[#, {p_, q__} :> {Key[p], q}]]) & /@ wrappedcellpos, 10]
   ];
   wrappedcellCoords = MapThread[#1 -> Map[Function[x,
         SetPrecision[x + #2, 10]], $faceListCoords[[Key@#1]], {1}] &,
      {First /@ wrappedcellpos, transVector}];
   wrappedcells = Keys@wrappedcellCoords;
   AppendTo[wrappedcellList, Flatten@wrappedcells];
   AppendTo[transvecList, transVector];
   AppendTo[localtopology, wrappedcellCoords];
  ],
  (* the else clause: vertex is out of bounds *)
  vertOutofBounds = vert;
  (* translate the vertex back into mesh *)
  transVector = vertOutofBounds /. transformRules;
  shiftedPt = SetPrecision[vertOutofBounds + transVector, 10];
  (* ------------- CORE B ------------- *)
  (* find which cells the
   shifted vertex is a part of in the wrapped matrix *)
  wrappedcells = Complement[
    Union@Cases[Position[wrappedMat, x_ /; SameQ[x, shiftedPt], {2}],
       x_Key :> Sequence @@ x, {2}] /.
     Alternatives @@ localcellunion -> Sequence[],
    Flatten@wrappedcellList];
  (*forming local topology now that we know the wrapped cells *)
  If[wrappedcells ≠ {},
   AppendTo[wrappedcellList, Flatten@wrappedcells];
   wrappedcellCoords = AssociationThread[wrappedcells,
     Map[Lookup[indToPtsAssoc, #] &,
      cellVertexGrouping[#] & /@ wrappedcells, {1}]];
   With[{opt = (vertOutofBounds /. periodicRules)},
    Block[{pos, vertref, transvec},
      Do[
       With[{cellcoords = wrappedcellCoords[cell]},
        pos = FirstPosition[cellcoords /. periodicRules, opt];
        vertref = Extract[cellcoords, pos];
        transvec = SetPrecision[vertOutofBounds - vertref, 10];
        AppendTo[transvecList, transvec];
        AppendTo[localtopology,
```

```
                 cell → Map[SetPrecision[# + transvec, 10] &, cellcoords]];
               ], {cell, wrappedcells}]
             ];
           ];
         ];
         (* to detect wrapped cells not detected by CORE B*)
         (* ------------- CORE C ------------- *)
         Block[{pos, celllocs, ls, transvec, assoc, tvecLs = {}, ckey},
          ls = Union@Flatten@Join[cellsconnected, wrappedcells];
          If[Length[ls] ≠ 3,
           pos = Position[$faceListCoords, x_ /; SameQ[x, shiftedPt], {2}];
           celllocs = DeleteDuplicatesBy[Cases[pos, Except[{Key[Alternatives @@ ls],
                 __}]], First] /. {Key[x_], z__} :→ {Key[x], {z}};
           If[celllocs ≠ {},
            celllocs = Transpose@celllocs;
            assoc = <|
              MapThread[
               (transvec = SetPrecision[vertOutofBounds -
                    Extract[$faceListCoords[Sequence @@ #1], #2], 10];
                 ckey = Identity @@ #1;
                 AppendTo[tvecLs, transvec];
                 ckey → Map[SetPrecision[Lookup[indToPtsAssoc, #] + transvec, 10] &,
                   cellVertexGrouping[Sequence @@ #1], {1}]
                ) &, celllocs]
              |>;
            AppendTo[localtopology, assoc];
            AppendTo[wrappedcellList, Keys@assoc];
            AppendTo[transvecList, tvecLs];
           ];
          ];
         ];
        ];
      ];
    ] & /@ vertcs;
  transvecList = Which[
    MatchQ[transvecList, {{{__?NumberQ}}}], First[transvecList],
    MatchQ[transvecList, {{__?NumberQ} ..}], transvecList,
    True, transvecList //. {x___, {p : {__?NumberQ} ..}, y___} :→ {x, p, y}
   ];
  {localtopology, Flatten@wrappedcellList, transvecList}
 ];
```

T1 transition

In[35]:=
```
(*find edge function*)
```

find edges

In[36]:=
```
Clear@edgesforT1;
edgesforT1[edgeLs_, indToPts_, threshLength_ : 0.0015] :=
  Block[{edges, dist, sel, filt, b, cand, sameedg2Q, del},
   edges = Lookup[indToPts, #] & /@ edgeLs;
   dist = EuclideanDistance @@ # & /@ edges;
   sel = Pick[edges, Thread[dist ≤ threshLength], True];
   If[Length[sel] == 1,
    Nothing,
    filt = sel /. periodicRules;
    b = MapThread[SameQ, {filt, sel}];
    cand = Pick[filt, b, False];
    sameedg2Q = MemberQ[sel, {OrderlessPatternSequence @@ #}] & /@ cand;
    del = (OrderlessPatternSequence @@@ Pick[cand, sameedg2Q]);
    If[Length[cand] > 1, del = Alternatives @@ del];
    sel = DeleteCases[sel, {del}]
   ];
   sel
  ];
```

T1

In[38]:=
```
bagged = CreateDataStructure["DynamicArray"]
```

Out[38]= DataStructure[Type:DynamicArray Length:0]

In[39]:=
```
Clear@T1transitionFn;
T1transitionFn[edges_, indToPtsAssoc_, ptsToIndAssoc_, vertexToCellG_,
   cellToVertexG_, wrappedMat_, faceListCoords_, dSep_ : 0.004] :=
  Block[{edgeind, connectedcellKeys, edge, newpts, cellvertices, pos,
    cellpolys, memF, keyscellP, selcellKeys, ptToCell, newptsindices,
    indToPts = indToPtsAssoc, ptsToInds = ptsToIndAssoc, PtIndToCell,
    keysToMap, f1, otherkeys, f2, bag = bagged, vertToCellG = vertexToCellG,
    cellToVertG = cellToVertexG, testpts, localtopology, translCells, transVector,
    edgepts, celltopo, polysharingEdge, ls, cs, cc, ccw, trimmedtopo, ordering,
    edgeordered, topotransl, newtransvert, findEdges, $wrappedMat = wrappedMat,
    $faceListCoords = faceListCoords, cellToVertGC, dropinds, bagopen, maxlab, mm, ll},
   findEdges = edgesforT1[edges, indToPts];
   (* finding all possible edges for T1 transition *)
   If[findEdges ≠ {},
    (*run if there are any edges for T1*)
    Scan[
```

```
(edge = #;
  cellToVertGC = cellToVertG;
  edgeind = Lookup[ptsToInds, edge];
  If[ContainsAll[Keys[indToPts], edgeind],
   (* should be an edge not
    connected to an edge that has already undergone a T1 *)
   (* let us get the local network topology *)
   {localtopology, translCells, transVector} =
    getLocalTopology[ptsToInds, indToPts, vertToCellG,
        cellToVertG, $wrappedMat, $faceListCoords][#] &[edge];
   If[! DuplicateFreeQ[translCells],
    {translCells, transVector} =
     DeleteDuplicates[Transpose[{translCells, transVector}]]ᵀ
   ];
   connectedcellKeys = Keys@localtopology;
   celltopo = Values@localtopology;
   newpts = With[{midPt = Mean@edge},
     SetPrecision[midPt + dSep Normalize[(# - midPt)], 10] & /@
      Flatten[RotationTransform[-π/2, midPt] /@ {edge}, 1]
    ];
   testpts =
    With[{midPt = Mean@edge}, midPt + 0.00001 Normalize[(# - midPt)] & /@ newpts];
   (*plt1=Graphics[{{FaceForm[LightGray],EdgeForm[{Thick,Black}],
       Polygon/@celltopo},{Red,Line@edge,Orange,
       Point@newpts,Green,Point@testpts}},ImageSize→Tiny];
   Print@plt1;*)
   (*which cells contain both vertices*)
   pos = Position[celltopo,
     {OrderlessPatternSequence[___, First[edge], ___, Last[edge], ___]}, {1}];
   polysharingEdge = Extract[celltopo, pos];
   (* the edge should not be part of any Δ *)
   bagopen = Union@*Flatten@*Normal@bag;
   (*Print[Show[Graphics[{Polygon/@Values@$faceListCoords}],plt1]];*)
   mm = edge /. periodicRules;
   If[(AllTrue[polysharingEdge, Length[#] ≠ 3 &]) &&
     ContainsNone[edgeind, bagopen] && ContainsNone[mm, bagopen],
    cellvertices = celltopo;
    cellpolys = Polygon /@ cellvertices;
    memF = Function[x, RegionMember@*DiscretizeRegion@x, Listable][
      Extract[cellpolys, pos]];
    keyscellP = Extract[connectedcellKeys, pos];
    selcellKeys = Thread[keyscellP → memF];
    ptToCell = Quiet[# → First @@ Select[selcellKeys, Function[x, Last[x][#]]] & /@
        testpts /. HoldPattern[_ → First[]] → Nothing] ;
```

```
(* testpt to cell *)
ptToCell = ptToCell /. Thread[testpts → newpts];
newptsindices = Range[# + 1, # + 2] &[Max@Keys@indToPts];
KeyDropFrom[indToPts, edgeind];
AppendTo[indToPts, Thread[newptsindices → newpts]];

ll = Lookup[ptsToInds, mm];
If[ll ≠ edge, bag["Append", ll]];

ptsToInds = AssociationMap[Reverse, indToPts];
bag["Append", edgeind];
PtIndToCell = MapAt[ptsToInds, ptToCell, {All, 1}] /. Rule → List ;
(*index to cell*)
keysToMap = Map[{Lookup[indToPts, #[[1]]], Key@#[[2]]} &, PtIndToCell];
f1 = Fold[MapAt[Function[x, DeleteDuplicates@Replace[x, (Alternatives @@
          edge ) :> #2[[1]], {1}]], #1, #2[[-1]]] &, localtopology, keysToMap];
otherkeys = List@*Key /@ Complement[connectedcellKeys, keyscellP];

ls = Lookup[localtopology, First @@ otherkeys];
cs = FirstCase[ls, Alternatives @@ edge];
cc = getClockwise[cs, ls];
ccw = getCounterClockwise[cs, ls];
trimmedtopo =
 KeyDrop[localtopology, connectedcellKeys ~ Complement ~ keyscellP];
ordering = {
  FirstCase[Position[trimmedtopo, cc],
   x : {Key[First@keyscellP], _} | {Key[Last@keyscellP], _} :> First @@ x],
  FirstCase[Position[trimmedtopo, ccw], x : {Key[First@keyscellP],
      _} | {Key[Last@keyscellP], _} :> First @@ x ]
 };
edgeordered = ordering /. Reverse[ptToCell, 2];
f2 = MapAt[Replace[#, Alternatives @@ edge :> Splice[edgeordered], {1}] &,
  f1, First@otherkeys];
f2 = MapAt[Replace[#, Alternatives @@ edge :> Splice[Reverse[edgeordered]],
    {1}] &, f2, Last@otherkeys];

(*plt2=Values@f2//Map[Polygon]//Graphics[
     {FaceForm[LightGray],EdgeForm[{Thin,Red}],#},ImageSize→Tiny]&;*)
(* once we make the transition, we translate the cell back and
 add the shifted vertex into the indtopts and ptstoinds *)
If[translCells ≠ {},
 f2 = Fold[MapAt[Function[x, Function[y, SetPrecision[y - #2[[2]], 10]] /@ x], #1,
     {#2[[1]]}] &, f2, Thread[{Key /@ translCells, transVector}]];
 newtransvert = Complement[Flatten[Values@f2, 1],
   Values@indToPts, SameTest → (#1 == #2 &)];
 If[newtransvert ≠ {},
  newptsindices =
```

```
              Range[# + 1, # + Length@newtransvert] &[Max[Keys@indToPts]];
             AppendTo[indToPts, Thread[newptsindices → newtransvert ]];
             AppendTo[ptsToInds, Thread[newtransvert → newptsindices]];
            ];
           ];
           (*plt3=Values@f2//Map[Polygon]//Graphics[
                {FaceForm[LightGray],EdgeForm[{Thin,Red}],#},ImageSize→Tiny]&;*)
           f2 = Lookup[ptsToInds, #] & /@ f2;
           AppendTo[cellToVertG, f2];
          ];

          cellToVertG = KeySort@cellToVertG;
          dropinds =
           Flatten@Map[Complement[cellToVertGC[#], cellToVertG[#]] &, translCells];
          KeyDropFrom[indToPts, dropinds];
          ptsToInds = AssociationMap[Reverse, indToPts];
          vertToCellG = KeySort@GroupBy[
             Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertG], First → Last];
          $faceListCoords = Lookup[indToPts, #] & /@ cellToVertG;
          $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
          (*Print@{plt1,plt2};*)
         ]) &, findEdges]
    ];
    bagged["DropAll"];
    {indToPts, vertToCellG, cellToVertG, $wrappedMat, $faceListCoords}
   ];
```

T2 transition

find cells for T2

In[41]:=

```
Clear@cellsforT2;
cellsforT2[areaAssoc_, cellVertexG_, thresh_ : 0.1] := Block[{keys, ls, inds},
   keys = Keys@Select[areaAssoc, # < thresh &];
   ls = Lookup[cellVertexG, keys];
   inds = Flatten@Position[ls, x_ /; (Length[x] == 3), {1}];
   (* cell_edges == 3 *)
   If[inds ≠ {}, keys〚inds〛, {}] (*cell inds*)
  ];
```

T2

In[43]:=

```
Clear@T2TransitionFn;
T2TransitionFn[removeelem_, indToPtsAssoc_, ptsToIndAssoc_,
```

```
 vertexCellAssoc_, cellVertexGrouping_, wrappedMat_, faceListCoords_] :=
Block[{assoc, translC, transVec, ptsToInds = ptsToIndAssoc, indToPts = indToPtsAssoc,
  vertCellAssoc = vertexCellAssoc, cvG = cellVertexGrouping,
  $wrappedMat = wrappedMat, $faceListCoords = faceListCoords, cellkey,
  res, mergedtopo, vertinds, vertpts, mean, ruletrans, changedtopo,
  transtopo, ckeys, pos, newpt, maxlab, newind, vertinddrop, oldptind, re},
 re = Replace[removeelem, x_Integer :→ {x}];
 If[removeelem =!= {},
  Scan[
    (cellkey = #;
      {assoc, translC, transVec} =
       getLocalTopology[ptsToInds, indToPts, vertCellAssoc, cvG, $wrappedMat,
            $faceListCoords][#] & /@ Lookup[indToPts, cvG[cellkey]] // Transpose;
      res = DeleteDuplicates[Flatten[{translC, transVec}, {3, 2}] //.
         {p___, x : {}, q___} :→ {p, q}];
      mergedtopo = KeySort[Join @@ assoc];
      vertinds = cvG[cellkey];
      vertpts = Lookup[indToPts, vertinds];
      mean = SetPrecision[Mean@vertpts, 10];
      ruletrans = Flatten[Thread[{# → mean}] & /@ vertpts];
      (*Print[Values@mergedtopo//Graphics[{FaceForm[LightBlue],
            EdgeForm[Black],Map[Polygon][#]},ImageSize→Small]&];*)
      If[res == {}, KeyDropFrom[mergedtopo, cellkey]];
      changedtopo = (mergedtopo /. ruletrans);
      (*Print[Values@changedtopo//Graphics[{FaceForm[LightBlue],
            EdgeForm[Black],Map[Polygon][#]},ImageSize→Small]&];*)
      If[res ≠ {},
       transtopo = Fold[MapAt[Function[x, Function[y, SetPrecision[y - #2〚-1〛, 10]] /@
              x], #1, Key@#2〚1〛] &, changedtopo, res];
       ckeys = res〚All, 1〛;
       pos = Cases[Position[<|Thread[ckeys → Lookup[mergedtopo, ckeys]]|>,
          Alternatives @@ vertpts], {Key[Alternatives @@ ckeys], _}];
       newpt = DeleteDuplicates@Extract[transtopo, pos];
       oldptind = Extract[<|Thread[ckeys → Lookup[cvG, ckeys]]|>, pos];
       newpt = Join[{mean}, newpt];
       maxlab = Max[Keys@indToPts];
       newind = Range[maxlab + 1, maxlab + Length[newpt]];
       vertinddrop = Join[oldptind, vertinds];
       AppendTo[indToPts, Thread[newind → newpt]],
       newind = Max[Keys@indToPts] + 1;
       newpt = mean;
       vertinddrop = vertinds;
       AppendTo[indToPts, newind → newpt];
      ];
      ptsToInds = AssociationMap[Reverse, indToPts];
      If[res ≠ {},
       AppendTo[cvG, DeleteDuplicates@Lookup[ptsToInds, #] & /@ transtopo],
```

```
          AppendTo[cvG, DeleteDuplicates@Lookup[ptsToInds, #] & /@ changedtopo]
         ];
         cvG = KeySort@cvG;
         KeyDropFrom[indToPts, vertinddrop];
         ptsToInds = AssociationMap[Reverse, indToPts];
         KeyDropFrom[cvG, cellkey];
         vertCellAssoc = GroupBy[
            Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cvG], First → Last] // KeySort;
         $wrappedMat = AssociationThread[Keys[cvG] →
            Map[Lookup[indToPts, #] /. periodicRules &, Lookup[cvG, Keys[cvG]], {2}]];
         $faceListCoords = Map[Lookup[indToPts, #] &, cvG, {2}]
         (*$wrappedMat= Map[#/.periodicRules&,$faceListCoords]*)) &, re];
    ];
    (*Print@Graphics[{EdgeForm[Black],FaceForm[LightBlue],
        Polygon/@Values@Map[Lookup[indToPts,#]&,cvG,{2}]},ImageSize→Medium];*)
    {indToPts, ptsToInds, vertCellAssoc, cvG, $wrappedMat, $faceListCoords}
   ];
```

cell division

Difference from Farahdifar's paper: in his article he dilates the cells before dividing them. Here I randomly select a cell and divide it into two. This does not really matter

In[45]:=
```
Clear[selectDivCells];
selectDivCells[areaPolygon_, areathresh_ : 2.2, thresh_ : 0.0025] :=
  Block[{candidates, pos},
   candidates = Normal@Select[areaPolygon / Mean[areaPolygon], # > areathresh &];
   pos = Position[0.1 RandomReal[1, Length@candidates], x_ /; x < thresh];
   Keys@Extract[candidates, pos]
  ];
```

In[47]:=
```
(* division events more random *)
```

In[48]:=
```
Clear[pickcellsDiv];
pickcellsDiv[cellToVertG_, areaAssoc_] := Block[{pickcells, selcells, pos},
   pickcells = Keys@Select[Pick[areaAssoc,
       Thread[RandomReal[{0, 1}, Length[areaAssoc]] < 0.001], True], # > 0.005 &];
   pos = Position[Lookup[cellToVertG, pickcells], x_ /; Length[x] > 3, {1}];
   Extract[pickcells, pos]
  ];
```

In[50]:=
```
Clear[cellDivision, i];
cellDivision[polygonind_, indToPoints_, areaAssoc_, perimAssoc_, cellToVertG_] :=
  Block[{x, y, num, matrix, xx, xy, yy, eigvals, eigVecs, maxeigpos, cent, edges,
```

```
  edgesL, intersects, intersectionPts, posIntersections, repPart, α, β,
  polygonPts, newkeys = Range[# + 1, # + 2] &[Max@Keys[indToPoints]], newPtToInds,
  indtoPtAssoc = indToPoints, ptToIndAssoc, edgeinds, contour, poly1, poly2, res, seq,
  newcells = Range[# + 1, # + 2] &[Max@Keys[areaAssoc]], CVG = cellToVertG,
  addcellsRule, polygonPtsInds, VCG, polygonptsTrans},
 VCG = GroupBy[Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@CVG], First → Last];
 polygonPtsInds = CVG[polygonind];
 num = Length@polygonPtsInds;
 ptToIndAssoc = AssociationMap[Reverse, indToPoints];
 polygonPts = Lookup[indToPoints, polygonPtsInds];
 polygonptsTrans = TranslationTransform[-Mean[polygonPts]]@polygonPts;
 Evaluate[Table[{x_i, y_i}, {i, num + 1}]] =
  Append[polygonptsTrans, First@polygonptsTrans];
 I_xx = (1/12) Sum[(x_i y_{i+1} - x_{i+1} y_i) (y_i^2 + y_i y_{i+1} + y_{i+1}^2), {i, 1, num}];
 I_yy = (1/12) Sum[(x_i y_{i+1} - x_{i+1} y_i) (x_i^2 + x_i x_{i+1} + x_{i+1}^2), {i, 1, num}];
 I_xy = (1/24) Sum[(x_i y_{i+1} - x_{i+1} y_i) (x_i y_{i+1} + 2 x_i y_i + 2 x_{i+1} y_{i+1} + x_{i+1} y_i), {i, 1, num}];
 Table[
  {Unevaluated[Subscript[x, j]] =., Unevaluated[Subscript[y, j]] =.}, {j, num + 1}];
 matrix = ( I_xx   -I_xy
           -I_xy    I_yy );
 {eigvals, eigVecs} = Eigensystem@matrix;
 maxeigpos = Position[eigvals, Max@eigvals];
 {edges, edgeinds} = Partition[#, 2, 1, 1] & /@ {polygonPts, polygonPtsInds};
 edgesL = Line /@ edges;
 cent = centroidPolygon[polygonPts];
 intersects = RegionIntersection[
     InfiniteLine[{cent, cent + Extract[eigVecs, maxeigpos][[1]]}], #] & /@ edgesL;
 intersectionPts = Cases[intersects, {(_Real | _Integer) ..}, {3}];
 newPtToInds = Thread[intersectionPts → newkeys];
 posIntersections = Flatten@Position[intersects, _Point, {1}];
 MapThread[
  (res = Complement[Intersection @@ Lookup[VCG, #2], {polygonind}];
    If[res ≠ {},
     seq = Partition[CVG[First@res], 2, 1, 1];
     AppendTo[CVG,
      First@res → DeleteDuplicates@
        Flatten@SequenceSplit[seq, {x___, p : {OrderlessPatternSequence[
               #2[[1]], #2[[-1]]]}, y___} :→ {x, Insert[p, #1, 2], y}]
     ];
    ]) &, {newkeys, edgeinds[[posIntersections]]}];
```

```
  repPart =
   Thread[{Thread[{ReverseSort@posIntersections, 2}], Reverse[intersectionPts]}];
  {α, β} = intersectionPts;
  AppendTo[ptToIndAssoc, newPtToInds];
  AppendTo[indtoPtAssoc, Reverse[newPtToInds, 2]];
  contour =
   DeleteDuplicates@Flatten[Fold[Insert[#1, #2[[2]], #2[[1]]] &, edges, repPart], 1];
  poly1 = Join @@ SequenceCases[contour, {___, α} | {β, ___}];
  poly2 = Join @@ SequenceCases[contour, {α, __, β}];
  KeyDropFrom[CVG, polygonind];
  addcellsRule = Thread[newcells → {poly1, poly2}];
  AppendTo[CVG, addcellsRule /. ptToIndAssoc];
  {indtoPtAssoc, CVG, Append[KeyDrop[areaAssoc, polygonind],
    MapAt[Area@*Polygon, addcellsRule, {All, 2}]],
   Append[KeyDrop[perimAssoc, polygonind],
    MapAt[Perimeter@*Polygon, addcellsRule, {All, 2}]]}
 ];
```

computing forces

area elasticity

In[53]:=

```
F_AreaElasticity[indTopts_, localtopo_, areaPolygonAssoc_] :=
 Block[{assoc, cellinds, force, vertKeys = Keys[indTopts],
   vertLs, vertex, gc, gcc, diffVec, grad, coeff},
  First@*Last@Reap@Do[
     assoc = First@Lookup[localtopo, i];
     cellinds = Keys[assoc];
     force = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = assoc[j];
      gcc = getCounterClockwise[vertex, vertLs];
      gc = getClockwise[vertex, vertLs];
      diffVec = SetPrecision[gcc - gc, 10];
      grad = SetPrecision[1/2 {{0, 1}, {-1, 0}}.diffVec, 10];
      coeff = SetPrecision[2 ka (areaPolygonAssoc[j] - A0), 10];
      force += SetPrecision[coeff * grad, 10], {j, cellinds}
      ];
     Sow@force, {i, vertKeys}]
  ]
```

In[54]:=

```
{{0, 1}, {-1, 0}}.(getCounterClockwise[{x_i, y_i}, {{x_(i-1), y_(i-1)}, {x_i, y_i}, {x_(i+1), y_(i+1)}}] -
    getClockwise[{x_i, y_i}, {{x_(i-1), y_(i-1)}, {x_i, y_i}, {x_(i+1), y_(i+1)}}]) // MatrixForm
```

Out[54]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

In[55]:=

```
MatrixForm[({{0, 1}, {-1, 0}}).({x_(i+1), y_(i+1)} - {x_(i-1), y_(i-1)})]
```

Out[55]//MatrixForm=

$$\begin{pmatrix} -y_{-1+i} + y_{1+i} \\ x_{-1+i} - x_{1+i} \end{pmatrix}$$

perimeter elasticity

In[56]:=
```
F_PerimeterElasticity[indTopts_, localtopo_, periPolygonAssoc_] :=
 Block[{assoc, cellinds, force, vertKeys = Keys[indTopts],
   vertLs, vertex, gc, gcc, v1, v2, coeff, grad},
  First@*Last@Reap@Do[
     assoc = First@Lookup[localtopo, i];
     cellinds = Keys@assoc;
     force = {0, 0};
     vertex = indTopts[i];
     Do[
      vertLs = assoc[j];
      gc = getClockwise[vertex, vertLs];
      v1 = Normalize[vertex - gc] ~SetPrecision~ 10;
      gcc = getCounterClockwise[vertex, vertLs];
      v2 = Normalize[gcc - vertex] ~SetPrecision~ 10;
      grad = SetPrecision[v1 - v2, 10];
      coeff = SetPrecision[2 γ (periPolygonAssoc[j] - P0), 10];
      force += SetPrecision[coeff * grad, 10], {j, cellinds}
     ];
     Sow@force, {i, vertKeys}]
 ]
```

In[57]:= `MatrixForm@Normalize[{x_i, y_i} - {x_j, y_j}]`

Out[57]//MatrixForm=

$$\begin{pmatrix} \frac{x_i - x_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \\ \frac{y_i - y_j}{\sqrt{\text{Abs}[x_i - x_j]^2 + \text{Abs}[y_i - y_j]^2}} \end{pmatrix}$$

line tension

In[58]:=
```
F_LineTension[indTopts_, ptsToInd_, edges_] :=
 Block[{vertKeys = Keys[indTopts], $v1, $v2, v1, force, uv},
  force = AssociationThread[vertKeys → 0.];
  Do[
   {$v1, $v2} = i;
   uv = Normalize[$v1 - $v2] ~SetPrecision~ 10;
   v1 = ptsToInd[$v1];
   force[v1] += SetPrecision[κ * uv, 10],
   {i, edges}];
  Values[force]
 ]
```

ΣF

In[59]:=
```
F_T[indTopts_, ptsToInds_, localtopology_,
   areaPolygonAssoc_, periPolygonAssoc_, edges_] :=
  - (F_AreaElasticity[indTopts, localtopology, areaPolygonAssoc]
     + F_PerimeterElasticity[indTopts, localtopology, periPolygonAssoc] +
     F_LineTension[indTopts, ptsToInds, edges]);
```

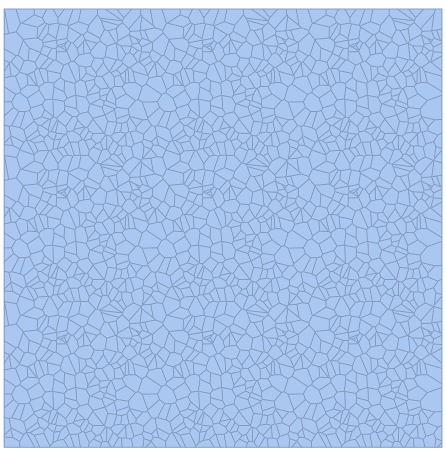
generating mesh

In[ ]:=
```
(* ensure PBC logically integrates with the rest of the code *)
```

In[60]:=
```
SeedRandom[1];
pts = RandomReal[{-1, 1}, {200, 2}];
```

In[62]:=
```
pts2 = Flatten[Table[TranslationTransform[{2 i, 2 j}][pts], {i, -1, 1}, {j, -1, 1}], 2];
vor = VoronoiMesh[pts2, {{-3, 3}, {-3, 3}}]
```

Out[63]=



In[64]:=
```
vcells = Catenate[NearestMeshCells[{vor, 2}, #] & /@ pts];
pvor = MeshRegion[MeshCoordinates[vor], MeshCells[vor, vcells]];
```

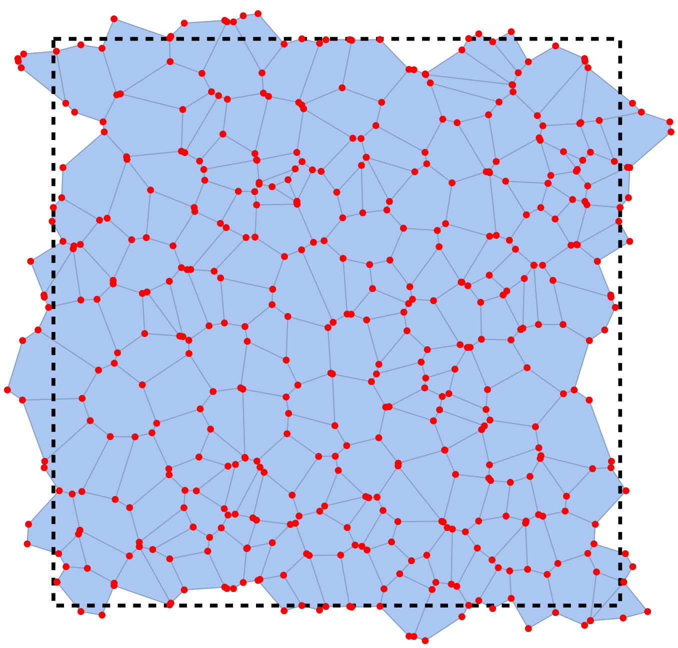
In[66]:=
```
Show[Table[MeshRegion[TransformedRegion[pvor, TranslationTransform[{2 i, 2 j}]],
   MeshCellStyle → {1 → Black, 2 → ColorData[2, 7 i + j + 25]}], {i, -3, 3}, {j, -3, 3}]]
```

Out[66]=

In[67]:=
```
vpts = Extract[MeshCoordinates[vor],
   Union@Flatten[MeshCells[vor, vcells] /. Polygon[x_] :→ x] ~Partition~ 1];
```

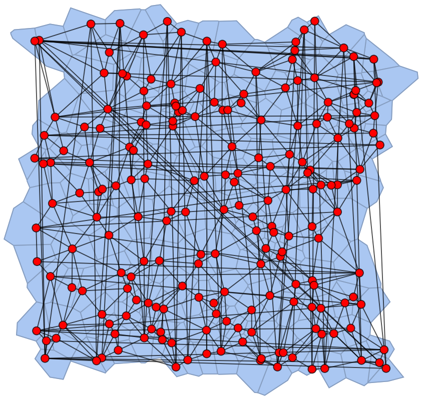
In[68]:=
```
Show[pvor, Graphics[
  {{EdgeForm[{Thick, Dashed, Black}], FaceForm[None], Rectangle[{-1, -1}, {1, 1}]},
   Red, PointSize[0.01], Point@vpts}]]
```

Out[68]=



In[69]:=
```
len = Length[pts];
C22 = #.Transpose[#] &@vor["ConnectivityMatrix"[2, 1]];
cells = Region`Mesh`MeshMemberCellIndex[vor, pts2]〚All, 2〛;
C22perm = C22〚cells, cells〛;
pC22 = SparseArray[Unitize[Total[Partition[Unitize[C22perm], {len, len}], 2]]];
pC22 -= IdentityMatrix[len, SparseArray];
Show[pvor,
 Graph[AdjacencyGraph[pC22, VertexCoordinates → pts], VertexStyle → Red, EdgeStyle → Black]]
```

Out[75]=



In[76]:=
```
indToPtsAssoc = AssociationThread[
   Replace[First@MeshCells[pvor], Point[x_] :→ x, {1}] →
    Replace[MeshPrimitives[pvor, 0], Point → Sequence, {2}, Heads → True]
  ];
```

In[77]:=
```
ptsToIndAssoc = <|Reverse[Normal@indToPtsAssoc, 2]|>;
```

In[78]:=
```
cellVertexGrouping = AssociationThread[Range@Length[#] → #] &@
   Replace[MeshCells[pvor, 2], x_Polygon :→ Sequence @@ x, {1}, Heads → True];
```

In[79]:=
```
vertexCellAssoc = KeySort@
   GroupBy[Flatten[Thread[#] & /@ Reverse[Normal@cellVertexGrouping, 2]], First → Last];
```

In[80]:=
```
wrappedMat = AssociationThread[
   Keys[cellVertexGrouping] → Map[Lookup[indToPtsAssoc, #] /. periodicRules &,
     Lookup[cellVertexGrouping, Keys[cellVertexGrouping]], {2}]];
```

In[81]:=
```
faceListCoords = AssociationThread[Keys[cellVertexGrouping] →
    Map[Lookup[indToPtsAssoc, #] &, Values@Normal@cellVertexGrouping]];
```

In[82]:=
```
ptsToIndAssoc = KeyMap[SetPrecision[#, 10] &, ptsToIndAssoc];
indToPtsAssoc = SetPrecision[#, 10] & /@ indToPtsAssoc;
wrappedMat = SetPrecision[#, 10] & /@ wrappedMat;
faceListCoords = SetPrecision[#, 10] & /@ faceListCoords;
```

Main( )

```
(* rigid network parameter choices *)
```

In[86]:=
```
ka = 1000;
A0 = 0.01;
γ = 0.04 * ka * A0;
δt = 0.001;
P0 = 0;
κ = 0.025;
```

In[92]:= $\{\gamma / (ka * A0), \kappa / (ka * A0^{3/2})\}$

Out[92]= {0.04, 0.025}

```
indTopts = indToPtsAssoc;
ptsToInd = ptsToIndAssoc;
cellToVertexG = cellVertexGrouping;
vertexToCell = vertexCellAssoc;
$wrappedMat = wrappedMat;
$faceListCoords = faceListCoords;
areaPolygonAssoc = areaOfPolygon@faceListCoords;
periPolygonAssoc = perimeterOfPolygon@faceListCoords;
SeedRandom[1];
cj = 0;
Tmax = 4000;
t = 0;
tt = {};
Module[{cellsToRemove, edgechanged, polydiv, findEdges},
   saveres = First@Last@Reap@Monitor[
        While[t ≤ Tmax * δt,
         cj = Round[t / δt];

         (* T2 transitions *)
         cellsToRemove = cellsforT2[areaPolygonAssoc, cellToVertexG];
         If[cellsToRemove ≠ {},
          {indTopts, ptsToInd, vertexToCell, cellToVertexG, $wrappedMat,
             $faceListCoords} = T2TransitionFn[cellsToRemove, indTopts, ptsToInd,
             vertexToCell, cellToVertexG, $wrappedMat, $faceListCoords];
         ];
```

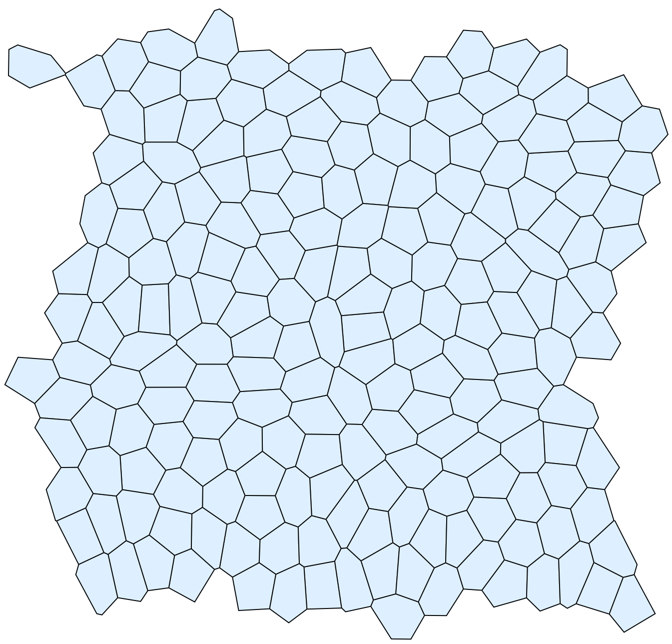
```
(* T1 transitions *)
edges = DeleteDuplicatesBy[
   Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1], Sort];
{indTopts, vertexToCell, cellToVertexG, $wrappedMat, $faceListCoords} =
  T1transitionFn[edges, indTopts, ptsToInd, vertexToCell,
   cellToVertexG, $wrappedMat, $faceListCoords];
ptsToInd = AssociationMap[Reverse, indTopts];
areaPolygonAssoc = areaPolygon /@ $faceListCoords;
periPolygonAssoc = perimeterPolygon /@ $faceListCoords;

(* cell divisions *)
polydiv = selectDivCells[areaPolygonAssoc];
If[polydiv ≠ {},
  Scan[({indTopts, cellToVertexG, areaPolygonAssoc, periPolygonAssoc} =
      cellDivision[#, indTopts, areaPolygonAssoc,
       periPolygonAssoc, cellToVertexG]) &, polydiv];
  vertexToCell = KeySort@GroupBy[Flatten[(Reverse[#, 2] &) @*
        Thread /@ Normal@cellToVertexG], First → Last];
  $faceListCoords = Lookup[indTopts, #] & /@ cellToVertexG;
  $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
];
ptsToInd = AssociationMap[Reverse, indTopts];

(* update positions *)
localtopo = getLocalTopology[ptsToInd, indTopts, vertexToCell,
      cellToVertexG, $wrappedMat, $faceListCoords][#] & /@ indTopts;
edgeLst = SortBy[Flatten[Map[Partition[#, 2, 1, 1] &,
     Values[$faceListCoords]], 1], First];
externedges = outeredges[indTopts, localtopo];
edgeLst = Join[edgeLst, externedges];
(*capture the first and the last output for the magnitude of force*)
If[cj == 3 || (cj + 1) == Tmax, AppendTo[tt, F_T[indTopts, ptsToInd,
     localtopo, areaPolygonAssoc, periPolygonAssoc, edgeLst] δt]];

indTopts = AssociationThread[
   Keys[indTopts] → SetPrecision[(Values[indTopts] + F_T[indTopts, ptsToInd,
         localtopo, areaPolygonAssoc, periPolygonAssoc, edgeLst] δt), 10]];

ptsToInd = AssociationMap[Reverse, indTopts];
$faceListCoords = Lookup[indTopts, #] & /@ cellToVertexG;
areaPolygonAssoc = areaPolygon /@ $faceListCoords;
periPolygonAssoc = perimeterPolygon /@ $faceListCoords;
vertexToCell = GroupBy[
   Flatten[(Reverse[#, 2] &) @* Thread /@ Normal@cellToVertexG], First → Last];
$wrappedMat = Map[# /. periodicRules &, $faceListCoords];
(*plt=Graphics[{ColorData[1][1],Thick,Values@Map[Line[Join[##,{First@#}]]&@
```

```
                    Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
          plt = Graphics[{FaceForm[LightBlue], EdgeForm[{Black}], Values[
               Polygon@Lookup[indTopts, #] & /@ cellToVertexG]}, ImageSize → Medium];
          (*plt=Graphics[{■,Thick,Values@Map[Line[Join[##,{First@#}]]&@
                    Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
          If[Mod[cj, 2] == 0, Sow[plt]];
          t += δt;
         ], {cj, plt}
        ]
   ]; // AbsoluteTiming
```

Out[107]= {1555.28, Null}

In[108]:= `plt`

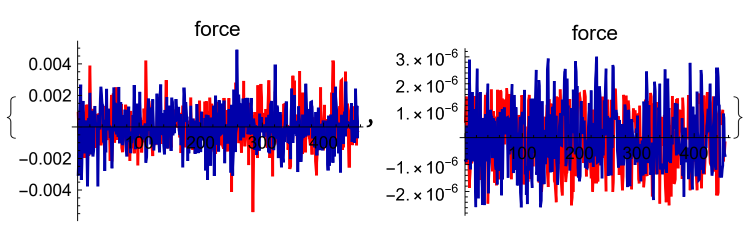
Out[108]=



In[109]:=

```
(ListLinePlot[#, PlotStyle → {{Thickness[0.01], Red}, {Thickness[0.01], Darker@Blue}},
      PlotLabel → "force"] &) @* Transpose /@ tt
```

Out[109]=



In[•]:= `(* different parameter choices *)`

In[110]:=
```
ka = 1000;
A0 = 0.01;
γ = 0.015 * ka * A0;
P0 = 0;
κ = -0.025;
δt = 0.00008;
```

In[116]:=
```
{γ / (ka * A0), κ / (ka * A0^(3/2))}
```

Out[116]=
```
{0.015, -0.025}
```

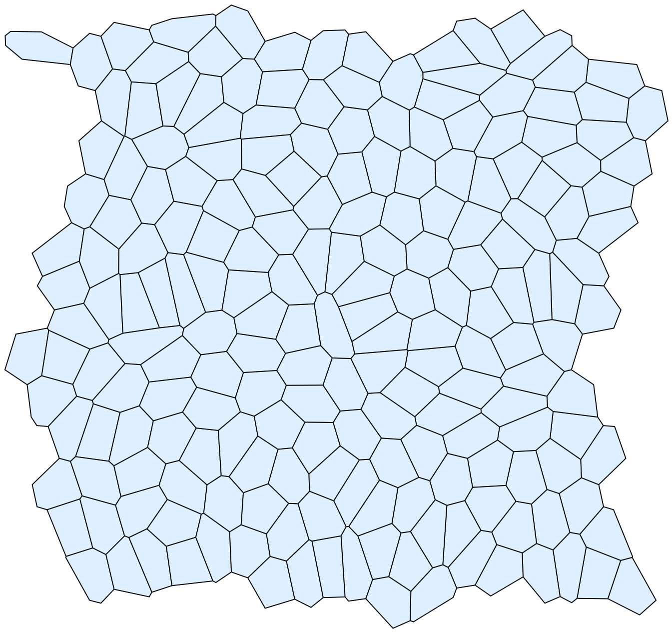
In[117]:=
```
indTopts = indToPtsAssoc;
ptsToInd = ptsToIndAssoc;
cellToVertexG = cellVertexGrouping;
vertexToCell = vertexCellAssoc;
$wrappedMat = wrappedMat;
$faceListCoords = faceListCoords;
areaPolygonAssoc = areaOfPolygon@faceListCoords;
periPolygonAssoc = perimeterOfPolygon@faceListCoords;
SeedRandom[1];
cj = 0;
Tmax = 4000;
t = 0;
tt = {};
Module[{cellsToRemove, edgechanged, polydiv, findEdges},
   saveres = First@Last@Reap@Monitor[
        While[t ≤ Tmax * δt,
         cj = Round[t / δt];
         (* T2 transitions *)
         cellsToRemove = cellsforT2[areaPolygonAssoc, cellToVertexG];
         If[cellsToRemove ≠ {},
          {indTopts, ptsToInd, vertexToCell, cellToVertexG, $wrappedMat,
             $faceListCoords} = T2TransitionFn[cellsToRemove, indTopts, ptsToInd,
             vertexToCell, cellToVertexG, $wrappedMat, $faceListCoords];
         ];

         (* T1 transitions *)
         edges = DeleteDuplicatesBy[
           Flatten[Map[Partition[#, 2, 1, 1] &, Values[cellToVertexG]], 1], Sort];
         {indTopts, vertexToCell, cellToVertexG, $wrappedMat, $faceListCoords} =
          T1transitionFn[edges, indTopts, ptsToInd, vertexToCell,
           cellToVertexG, $wrappedMat, $faceListCoords];
         ptsToInd = AssociationMap[Reverse, indTopts];
         areaPolygonAssoc = areaPolygon /@ $faceListCoords;
         periPolygonAssoc = perimeterPolygon /@ $faceListCoords;
```

```
          (* cell divisions *)
          polydiv = selectDivCells[areaPolygonAssoc];
          If[polydiv ≠ {},
           Scan[({indTopts, cellToVertexG, areaPolygonAssoc, periPolygonAssoc} =
               cellDivision[#, indTopts, areaPolygonAssoc,
                periPolygonAssoc, cellToVertexG]) &, polydiv];
           vertexToCell = KeySort@GroupBy[Flatten[(Reverse[#, 2] &)@*
                 Thread /@ Normal@cellToVertexG], First → Last];
           $faceListCoords = Lookup[indTopts, #] & /@ cellToVertexG;
           $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
          ];
          ptsToInd = AssociationMap[Reverse, indTopts];

          (* update positions *)
          localtopo = getLocalTopology[ptsToInd, indTopts, vertexToCell,
               cellToVertexG, $wrappedMat, $faceListCoords][#] & /@ indTopts;
          edgeLst = SortBy[Flatten[Map[Partition[#, 2, 1, 1] &,
              Values[$faceListCoords]], 1], First];
          externedges = outeredges[indTopts, localtopo];
          edgeLst = Join[edgeLst, externedges];
          (*capture the first and the last output for the magnitude of force*)
          If[cj == 3 || (cj + 1) == Tmax, AppendTo[tt, F_T[indTopts, ptsToInd,
              localtopo, areaPolygonAssoc, periPolygonAssoc, edgeLst] δt]];

          indTopts = AssociationThread[
            Keys[indTopts] → SetPrecision[(Values[indTopts] + F_T[indTopts, ptsToInd,
                  localtopo, areaPolygonAssoc, periPolygonAssoc, edgeLst] δt), 10]];

          ptsToInd = AssociationMap[Reverse, indTopts];
          $faceListCoords = Lookup[indTopts, #] & /@ cellToVertexG;
          areaPolygonAssoc = areaPolygon /@ $faceListCoords;
          periPolygonAssoc = perimeterPolygon /@ $faceListCoords;
          vertexToCell = GroupBy[
            Flatten[(Reverse[#, 2] &)@*Thread /@ Normal@cellToVertexG], First → Last];
          $wrappedMat = Map[# /. periodicRules &, $faceListCoords];
          (*plt=Graphics[{ColorData[1][1],Thick,Values@Map[Line[Join[##,{First@#}]]&@
                  Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
          plt = Graphics[{FaceForm[LightBlue], EdgeForm[{Black}], Values[
              Polygon@Lookup[indTopts, #] & /@ cellToVertexG]}, ImageSize → Medium];
          (*plt=Graphics[{,Thick,Values@Map[Line[Join[##,{First@#}]]&@
                  Lookup[indTopts,#]&,cellToVertexG]},ImageSize→Medium];*)
          If[Mod[cj, 2] == 0, Sow[plt]];
          t += δt;
        ], {cj, plt}
       ]
 ]; // AbsoluteTiming
```

Out[130]= {1716.86, Null}

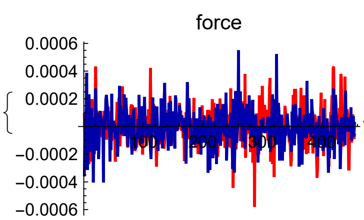
In[132]:= **plt**

Out[132]=

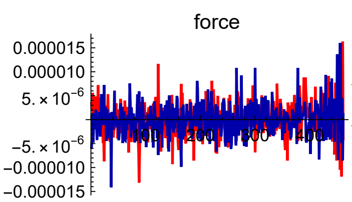


In[133]:=
```
(ListLinePlot[#, PlotStyle → {{Thickness[0.01], Red}, {Thickness[0.01], Darker@Blue}},
    PlotLabel → "force"] &) @* Transpose /@ tt
```

Out[133]= {



,



}

miscellaneous

In[134]:=
```
localtopotemp =
  getLocalTopology[ptsToIndAssoc, indToPtsAssoc, vertexCellAssoc, cellVertexGrouping,
      wrappedMat, faceListCoords][#] & /@ indToPtsAssoc;
edgeLsttemp = SortBy[Flatten[Map[Partition[#, 2, 1, 1] &, Values[faceListCoords]], 1],
   First];
externedgestemp = outeredges[indToPtsAssoc, localtopotemp];
Show[pvor, Graphics[{{Thin, Black, Arrowheads[Small], Arrow /@ edgeLsttemp},
   {Thin, Red, Arrowheads[Small], Arrow /@ externedgestemp}}], ImageSize → Medium]
```

Out[137]=