ASSIGNMENT #1

worked with: Keegan Blain

1 a)

Least squares

$$f[x] \longrightarrow \boxed{T = \{\alpha x + \beta\}} \longrightarrow v[x]$$

1b)

$$\begin{cases} \alpha x_1 + \beta = V_1 \\ \alpha x_2 + \beta = V_2 \\ \vdots \qquad\qquad \vdots \\ \alpha x_n + \beta = V_n \end{cases} \qquad \underbrace{\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}}_{\vec{f}} \underbrace{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}}_{\vec{V}}$$

3)

$$\frac{\partial Q}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left(\sum_i v[x_k] - (\alpha x_k + \beta)^2\right)$$

$$= \frac{1}{2}\sum_i -2x_k\left(v[x_k] - (\alpha x_k + \beta)\right)$$

$$= -\sum_i x_k\left(v[x_k] - (\alpha x_k + \beta)\right)$$

$$\frac{\partial Q}{\partial \beta} = \frac{\partial}{\partial \beta}\left(\sum_i v[x_k] - (\alpha x_k + \beta)^2\right)$$

$$= \frac{1}{2}\sum_i -2\left(v[x_k] - (\alpha x_k + \beta)\right)$$

$$= -\sum_i v[x_k] - (\alpha x_k + \beta)$$

$$Q[x] = (A\theta - b)^T(A\theta - b)$$

if $\frac{\partial}{\partial \theta}(A\theta - b) = A$

$$Q'[x] = A^T(A\theta - b) + A^T(A\theta - b)$$

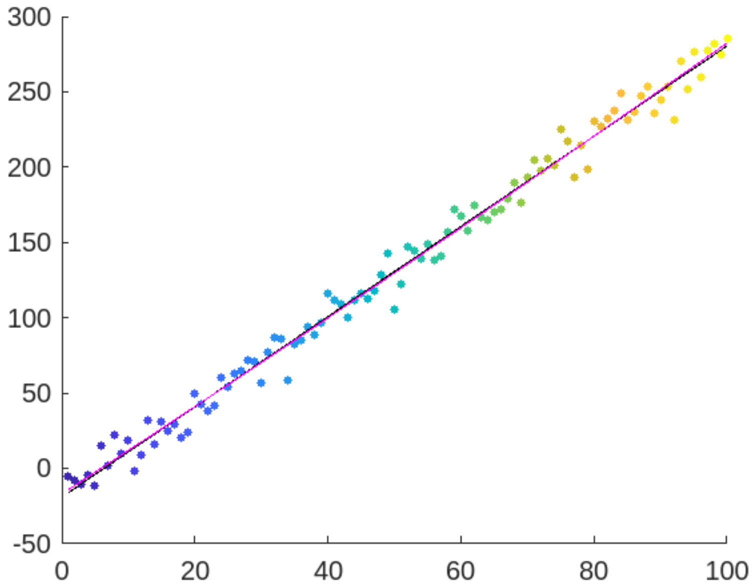
$$2A^T(A\theta - b) = 0$$

$$A^TA\theta = A^Tb$$

where $A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_k & 1 \end{bmatrix}$ $\quad \theta = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ $\quad b = v\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}$

$$\left(\begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_k & 1 \end{bmatrix}\right) \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ 1 & 1 & \cdots & 1 \end{bmatrix} v \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}$$

$$\begin{bmatrix} \sum_i x_k^2 & \sum_i x_k \\ \sum_i x_k & \sum_i 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \sum_i x_k v[x_k] \\ \sum_i v[x_k] \end{bmatrix}$$



Quadratic Fit: Magenta
Linear Fit: Black