Biomedical Imaging Exercise – Week 6

Name: Alkinoos Sarioglou

Student ID: 20-947-743

1. Task 1.1

a) The density values and the mass attenuation coefficients of all the structures at 50 and 150 keV are entered in the code as following:

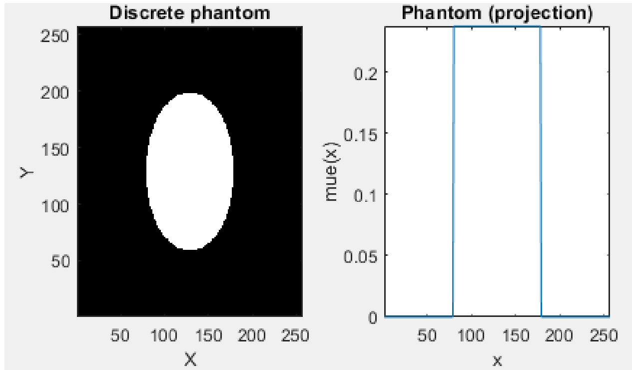
```
% ----------------------------------------------------------------------
% TASK 1.1 (begin)
% ----------------------------------------------------------------------
% Set density
% (see: Table 2 in www.nist.gov/pml/data/xraycoef/)
% ----------------------------------------------------------------------

% TASK 1.1 EDIT HERE
rho_blood       = 1.060E+00;                % density blood    [g/cm3]
rho_bone        = 1.920E+00;                % density bone     [g/cm3]
rho_lung        = 1.050E+00;                % density lung/air [g/cm3]
rho_muscle      = 1.050E+00;                % density muscle   [g/cm3]

% ----------------------------------------------------------------------
% Set X-ray mass attenuation coefficients for 50 and 150 keV
% (see: Table 4 in www.nist.gov/pml/data/xraycoef/)
% ----------------------------------------------------------------------

% TASK 1.1 EDIT HERE
mac_blood(1)    = 2.278E-01;                % blood @  50 keV    [cm2/g]
mac_blood(2)    = 1.492E-01;                % blood @ 150 keV    [cm2/g]

mac_bone(1)     = 4.242E-01;                % bone @  50 keV     [cm2/g]
mac_bone(2)     = 1.480E-01;                % bone @ 150 keV     [cm2/g]

mac_lung(1)     = 2.270E-01;                % lung @  50 keV     [cm2/g]
mac_lung(2)     = 1.493E-01;                % lung @ 150 keV     [cm2/g]

mac_muscle(1)   = 2.262E-01;                % muscle @  50 keV  [cm2/g]
mac_muscle(2)   = 1.492E-01;                % muscle @ 150 keV  [cm2/g]
```

b) The linear attenuation coefficients are calculated by multiplication of the density and the mass attenuation coefficient. This is implemented in the code as following:

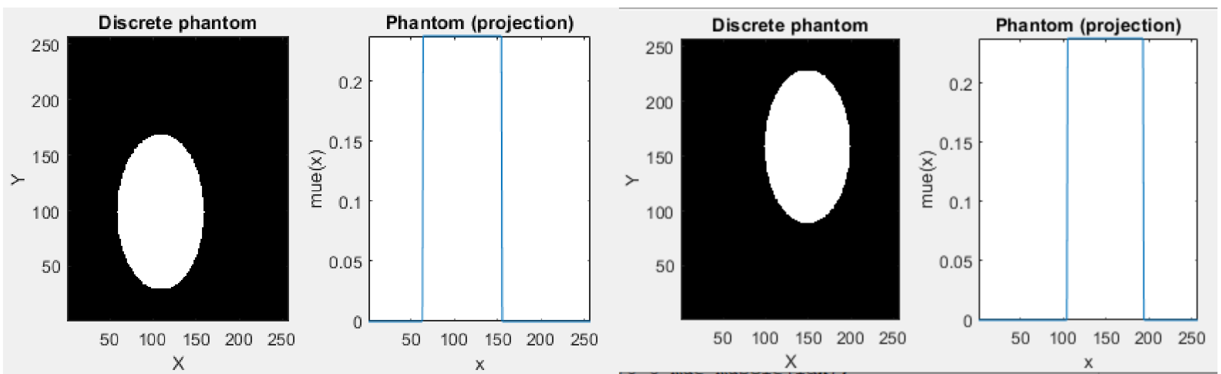
```
% ----------------------------------------------------------------------
% Calculate linear attenuation coefficients
% ----------------------------------------------------------------------

% TASK 1.1 EDIT HERE
mue_blood       = mac_blood.*rho_blood;
mue_bone        = mac_bone.*rho_bone;
mue_lung        = mac_lung.*rho_lung;
mue_muscle      = mac_muscle.*rho_muscle;

idx = 1; if ua==150 idx = 2; end
```

c) With higher anode voltage U_A, the energy of the photons E will be higher as they will have higher acceleration due to the more intense electric field. For higher photon energies, the cross-section σ of the area that could result in either photo effect or Compton effect gets smaller. Therefore, it is less likely that these events will occur and as a result less attenuation is imposed on the photons. This results in the mass attenuation coefficient to get smaller. Due to the fact that the linear attenuation coefficient is proportional to the mass attenuation coefficient, then the linear attenuation coefficient μ will be inversely proportional to U_A. As a result, when U_A increases, μ decreases, because less attenuation occurs for high-energy photons and when U_A decreases, μ increases, because higher attenuation occurs for low-energy photons.

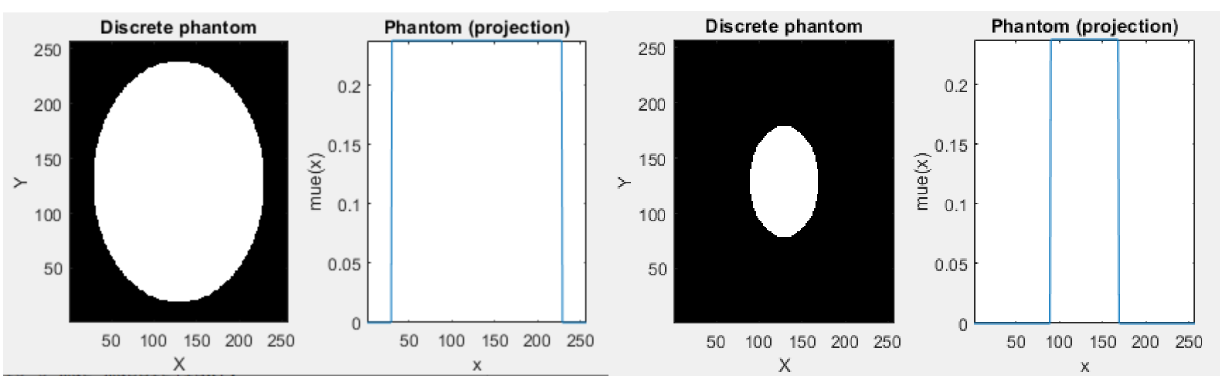
d) Initially, one muscle object is created. When its center is at (0,0), then the following graph is extracted:



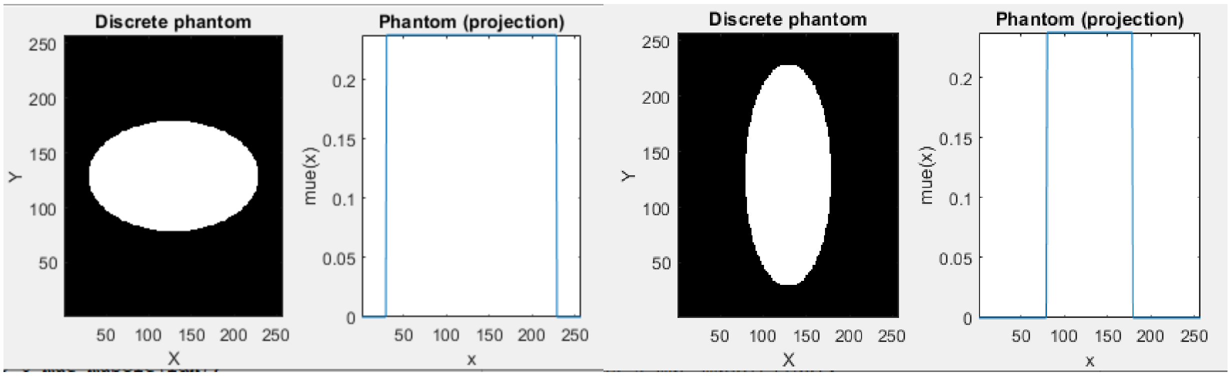
Then its position is modified as illustrated in the following graphs:



The size is then adjusted:



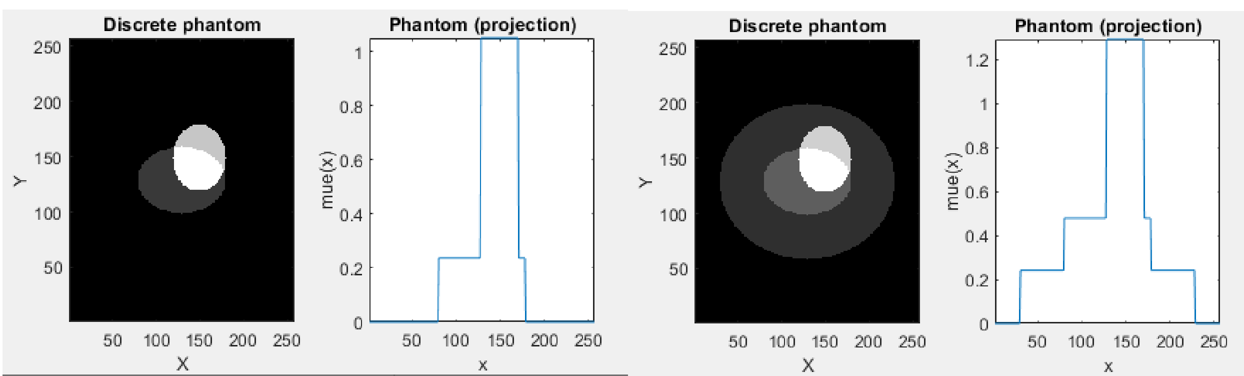
Finally, the shape is altered:



Then, other structures of bone and blood can be also added:

```
% ------------------------------------------------------------------
% Define test phantom using ellipses with [x0 y0 a b phi mue]
%
%       x0,y0    - center point [cm] (+x -> left-right, +y -> bottom-up)
%       a,b      - half axes [cm]
%       theta    - rotation angle relative to x-axis [deg]
%       mue      - linear attenuation coefficient [1/cm]
% ------------------------------------------------------------------

% TASK 1.1 EDIT HERE
phantom.ellipse = [50 60 20 20 0 mue_muscle(idx);
                   0 0 20 20 0 mue_bone(idx);
                   -50 -60 20 20 0 mue_blood(idx);

];
```

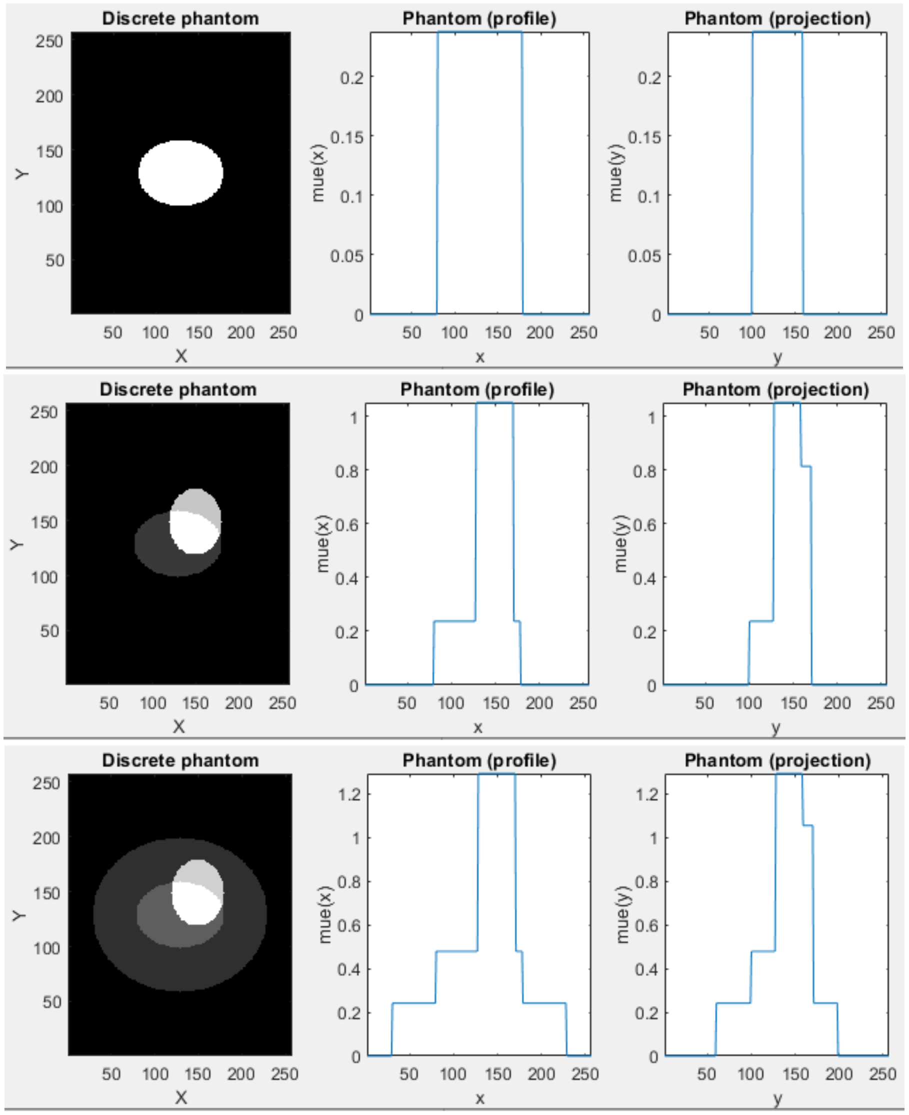
The following graphs are extracted:



e) Furthermore, the sum along the vertical direction can be computed with the following snippet of code:

```
% ------------------------------------------------------------------
% Display column-wise sum of mue(y)
% ------------------------------------------------------------------

% TASK 1.1 FILL IN HERE
DisplayData(phantom.discrete(:,fix(matrix/2)),[1,4,3]);
title('Phantom (projection)'); xlabel('y'); ylabel('mue(y)');
```

The resulting vertical projections are shown in the following diagrams for all the combinations of structures:



The graph of the horizontal projection (phantom(profile)) shows how the linear attenuation coefficient varies as a function of the horizontal distance. Its values are affected by all the body structures that are present in a specific horizontal distance.

Similarly, the graph of the vertical projection (phantom(projection)) shows the column-wise sum of μ, which is changed as a function of the vertical distance. Here, the resulting intensity is also affected by the values of μ of all the body structures present at that vertical distance. The resulting graphs above demonstrate these conclusions.

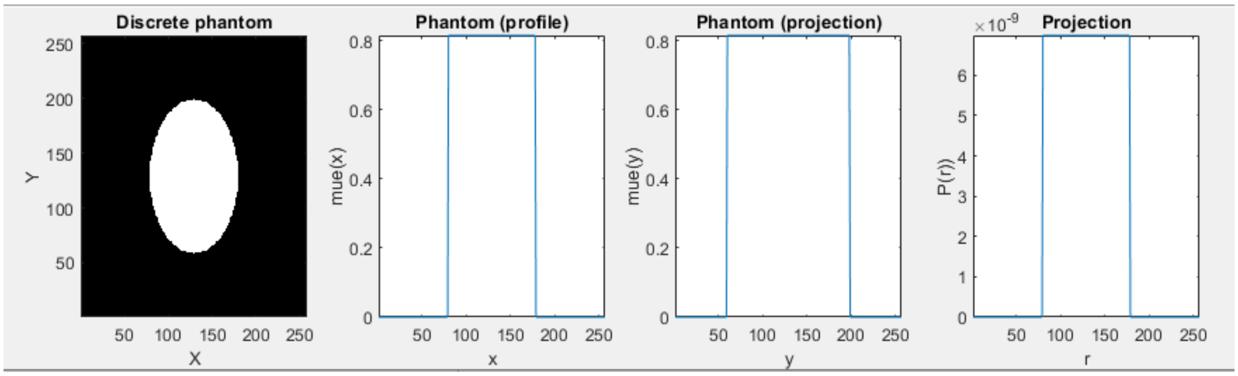
2. Task 1.2

a) To compute the line integral the following code snippet was used:

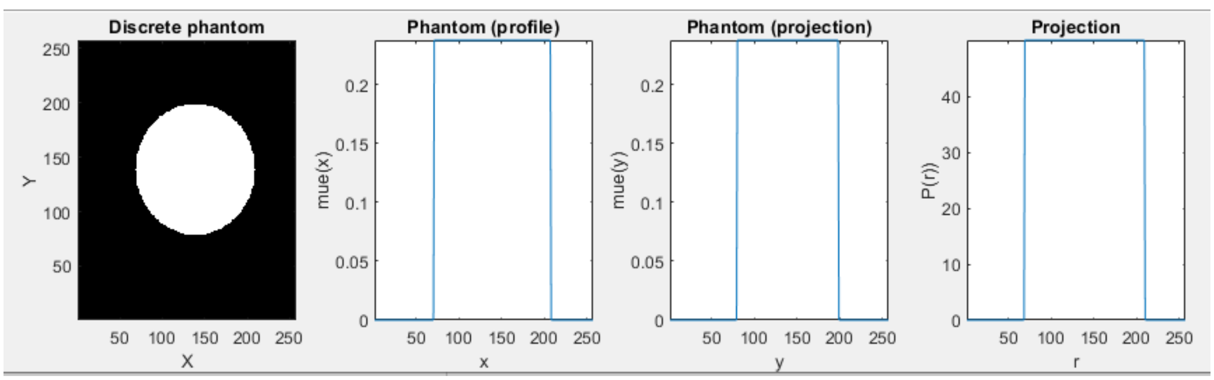
```
% ----------------------------------------------------------------
% TASK 1.2 (begin)
% ----------------------------------------------------------------

% TASK 1.2 FILL IN/EDIT HERE

lsf     = @(y) mue+0*y;

for m=1:length(i)
    proj    = ua*exp(-integral(lsf,sq(m),sp(m)));
end

% ----------------------------------------------------------------
% TASK 1.2 (end)
% ----------------------------------------------------------------
```

The intensity of the photon beam is decreased exponentially according to the Beer-Lambert's Law.

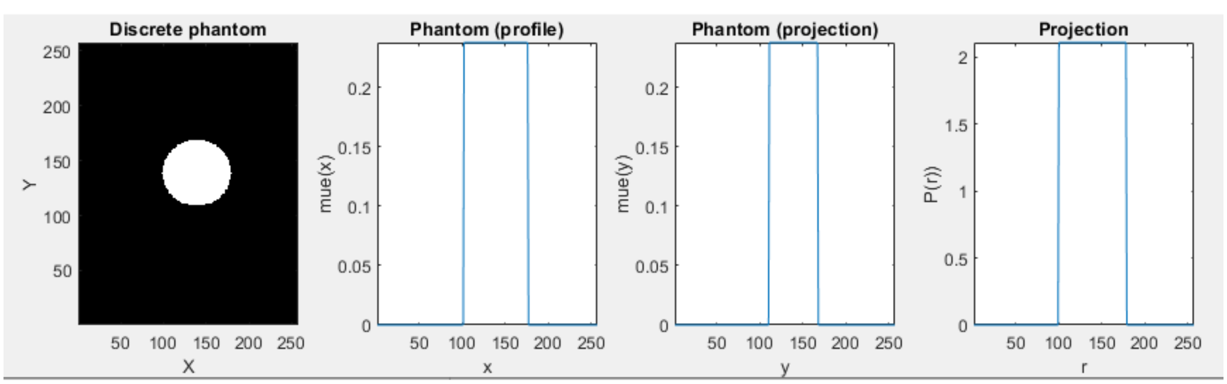
b) The resulting graphs are the following for a bone structure:



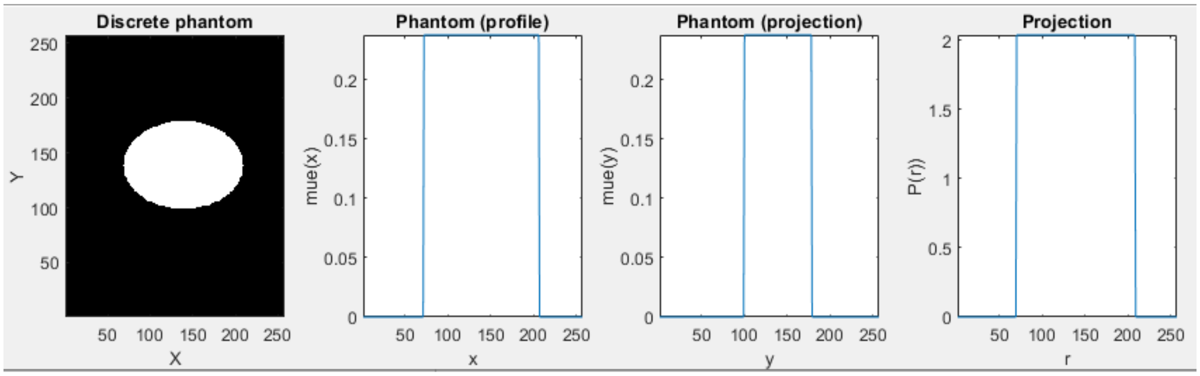
If the position of the object is changed then:



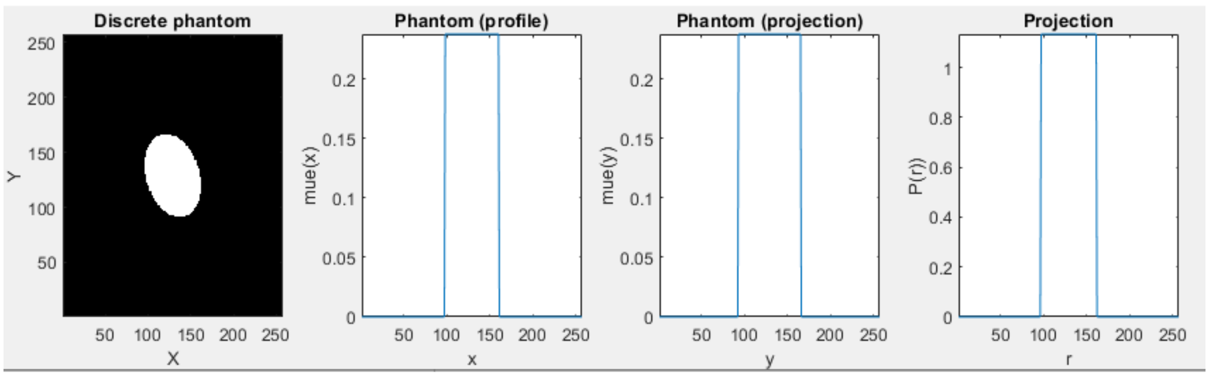
With a change in the size:



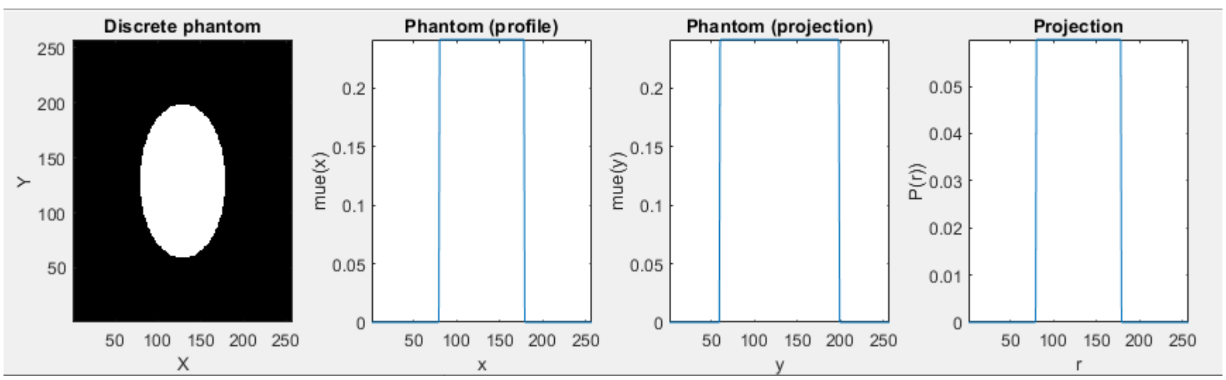
And with a change in the shape:



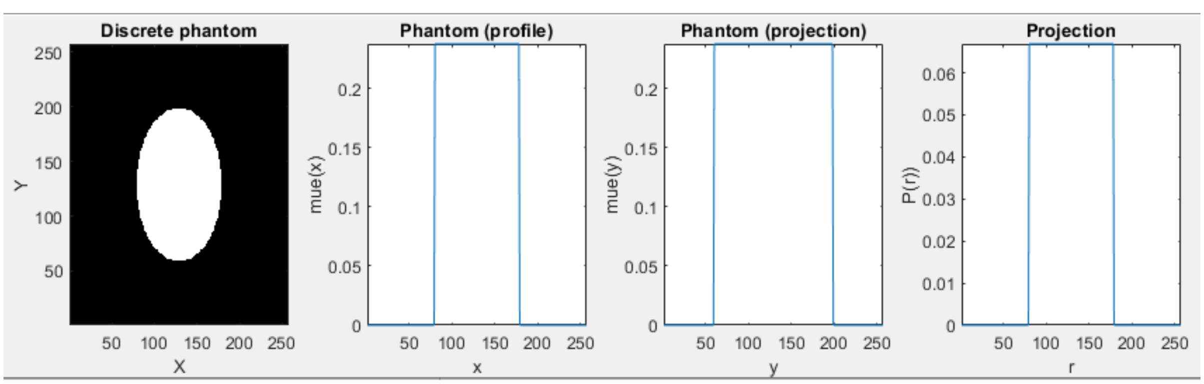
A change in the angle θ results in the following graphs:



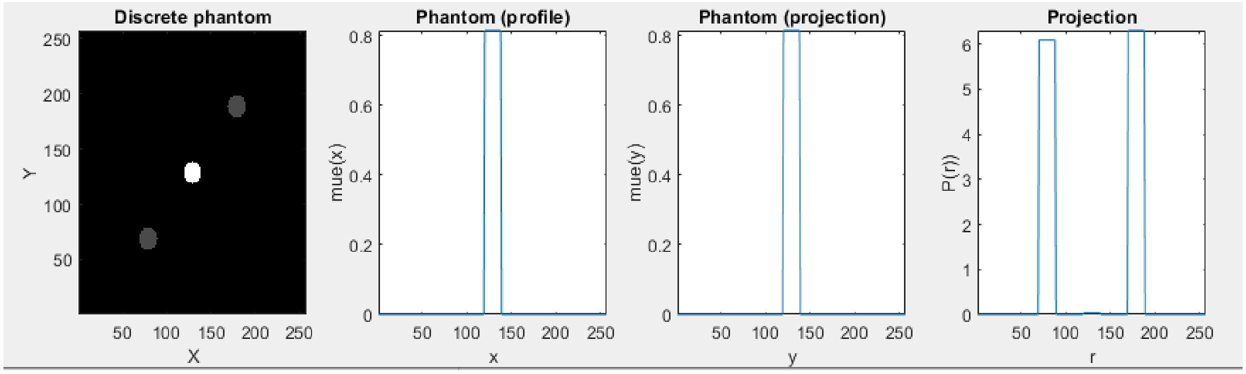
Additionally, for a blood object we get:



Then, the graphs for a muscle structure are the following:



A combination of these structures outputs the following:



The graph of projection now shows how the photon beam is attenuated when passed inside different structures, whereas the other two graphs (of Task 1.1) demonstrate how the linear attenuation coefficient changes according to the structures present.

In the new graph of projection, when $P(r)$ is low then that means that at this part of the image the whole energy of the photon beam is absorbed by the structures (such as here in the middle where there is a bone) and therefore there are no photons to be detected at this part by the detector. That means that this structure has high μ. Therefore, when the graph of μ is high, then the detected projection will be of low power, because most photon energy will have been attenuated by the structure. On the other hand, when the graph of μ has a low value for a structure, then that means that the graph of projection will be high because most photons will not have been attenuated enough and therefore they will be detected.

The differences can be seen in the above graphs of bone, blood and muscle. Due to the fact that the bone has high μ, the value of projection is very low (on the order of 10^{-9}) when compared to blood and muscle.

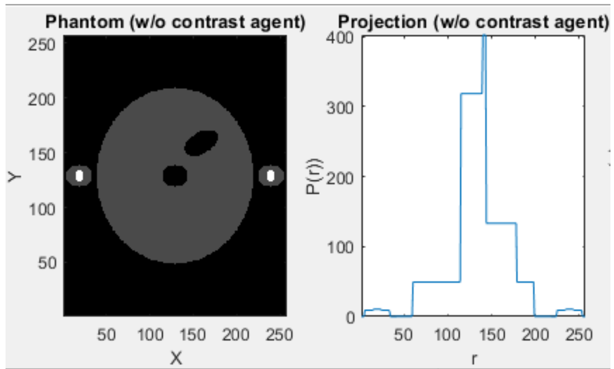
The other graphs also demonstrate that the structures are detected even with changes in position, angle, shape and size.

3. Task 1.3

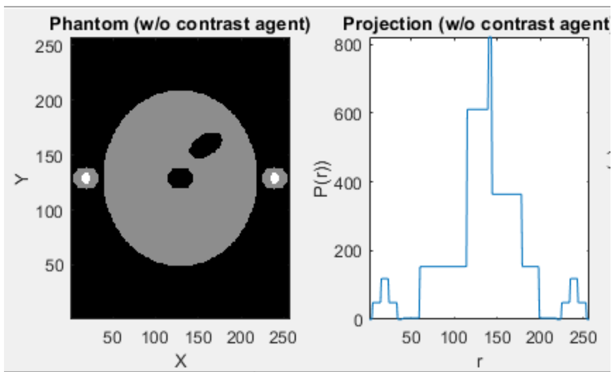
a) The code used to construct the objects is the following:

```
% ---------------------------------------------------------------------
% TASK 1.3 (begin)
% ---------------------------------------------------------------------
% Define thorax phantom using ellipses with [x0 y0 a b phi mue]
%
%       x0,y0    - center point [cm] (+x -> left-right, +y -> bottom-up)
%       a,b      - half axes [cm]
%       theta    - rotation angle relative to x-axis [deg]
%       mue      - linear attenuation coefficient [1/cm]
% ---------------------------------------------------------------------
mue_lung_new = mue_lung(idx)-mue_muscle(idx);
mue_armbone_new = mue_bone(idx)-mue_muscle(idx);
mue_aorta_new = mue_blood(idx)-mue_muscle(idx)-mue_lung(idx);
mue_heart_new = mue_muscle(idx)-mue_muscle(idx)-mue_lung(idx);

% TASK 1.3 EDIT HERE
phantom.ellipse = [    0    0    90   80   0    mue_muscle(idx);                              % thorax
                       0    0    70   60   0    mue_lung_new;                                 % lung
                     -110   0    15   10   0    mue_muscle(idx);                              % left arm muscle
                     -110   0    5    5    0    mue_armbone_new;                              % left arm bone
                      110   0    15   10   0    mue_muscle(idx);                              % right arm muscle
                      110   0    5    5    0    mue_armbone_new;                              % right arm bone
                       0    0    15   10   0    mue_aorta_new;                                % aorta
                       30   30   20   10   20   mue_heart_new];                               % heart
```

For anode voltage of 50 keV:



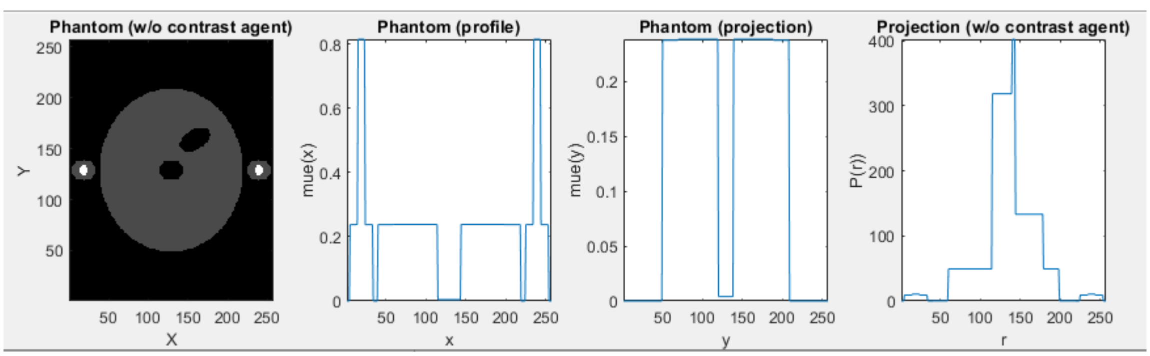
Also, for anode voltage of 150 keV:



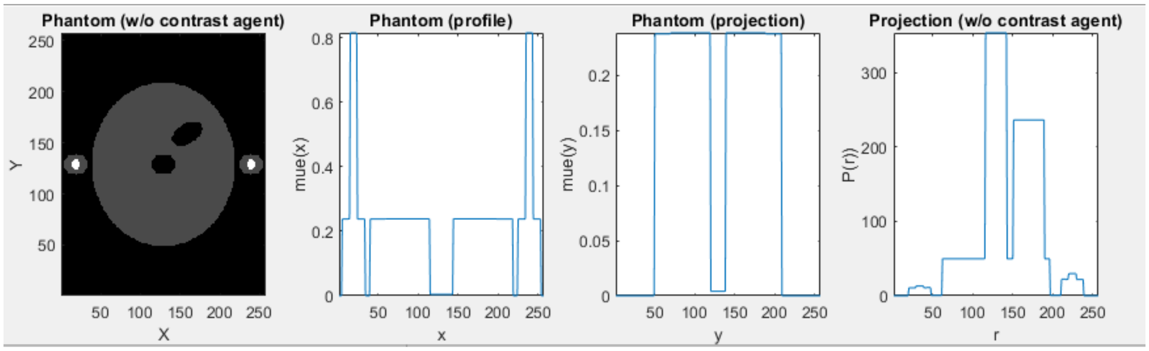
The contrast between the bone and the muscle is better at 50 keV of photon energy. That occurs because the linear attenuation coefficients for the two structures have a bigger difference at lower photon energies. More specifically, :

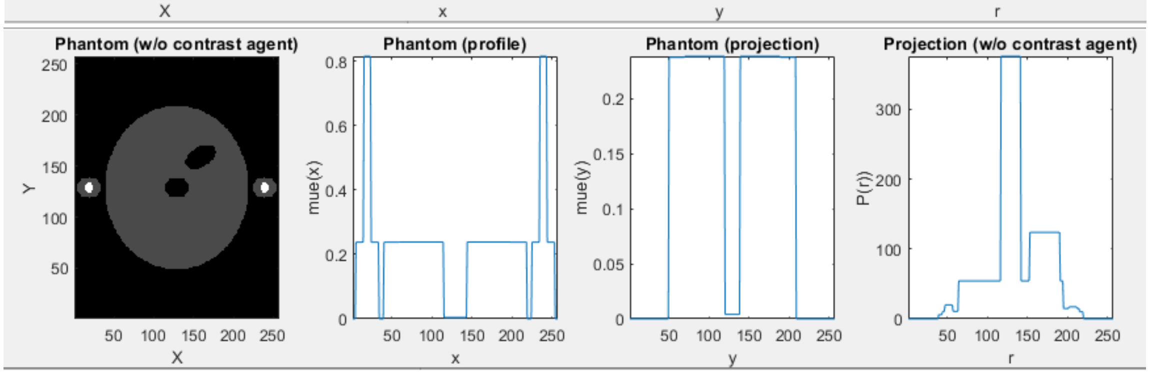
$\mu_{bone-50\ keV} = 0.814464\ \frac{1}{cm}$ and $\mu_{muscle-50\ keV} = 0.23751\ \frac{1}{cm}$, whereas $\mu_{bone-150\ keV} = 0.28416\ \frac{1}{cm}$ and $\mu_{muscle-50\ keV} = 0.15666\ \frac{1}{cm}$.

b) The horizontal and vertical projections are shown in the following graphs. For 50 keV:

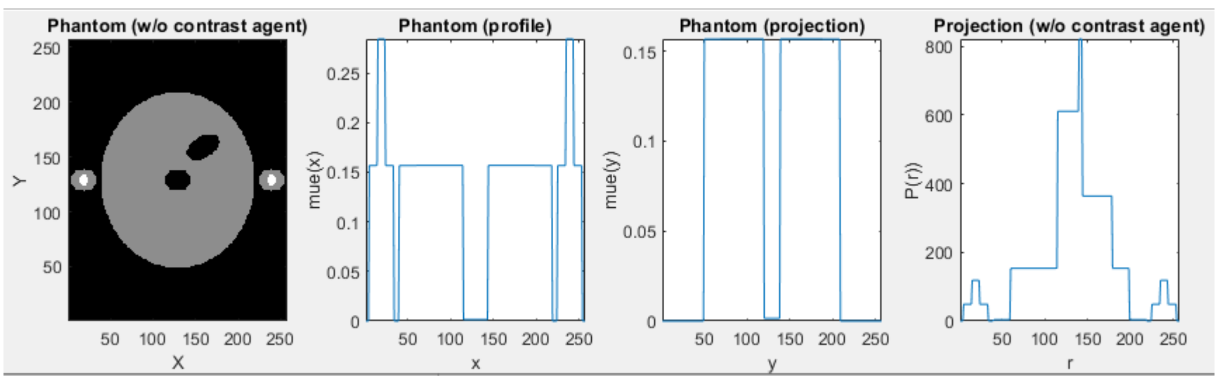


The angular projection for angles 30 (a) and 45 (b) degrees are shown below:





On the other hand, for 150 keV:



The aorta is not seen well because the neighbouring structures (lungs, thorax) have similar values of linear attenuation coefficients. Therefore, the contrast between the structures is poor and as a result the aorta cannot be clearly recognized in the X-ray graph. That is also because the aorta (blood structure) has a low value of μ at 50 and 150 keV:

$\mu_{50\ keV} = 0.241468\ \frac{1}{cm}$ and $\mu_{150\ keV} = 0.158152\ \frac{1}{cm}$.

c) According to the Beer-Lambert's Law:

$I(x+dx) - I(x) = -\mu I dx \leftrightarrow |I| = e^{-\mu x} + e^{c}$, where c is a constant. It can be seen that the contrast in intensity will be bigger for higher μ. Also:

$I(x+dx) = I(x) - \mu I dx$ for the intensity without a contrast agent and also:

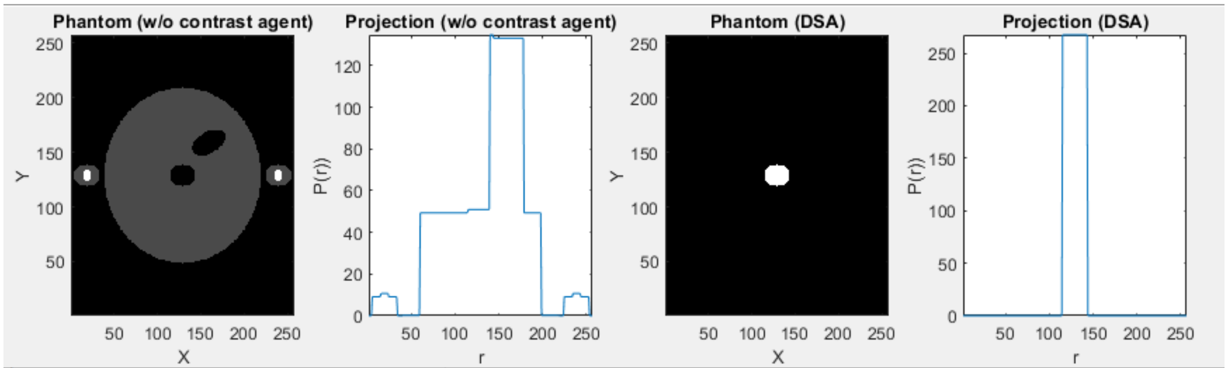
$I'(x+dx) = I(x) - 2\mu I dx$ for the intensity with the intravascular administration of iodine contrast agent which doubles the mass attenuation coefficient

Therefore: $I'(x+dx) - I(x+dx) = -\mu I dx$. As a result, the contrast will be bigger if a subtraction of the two images is performed.

d) The implementation of the DSA algorithm is performed with the following code snippet:

```
% -------------------------------------------------------------------
% Recompute phantom and projections (blood with contrast agent)
% -------------------------------------------------------------------

% TASK 1.3 EDIT HERE
phantom.ellipse(7,6) = 2*mue_blood(idx);   % mue of iodine contrast agent in blood

phantom.discrete    = CalcDiscretePhantom(x,y,phantom,ua);
phantom.projection  = CalcLineIntegrals(r,phi,phantom,ua);


% -------------------------------------------------------------------
% Subtract images, projections and display (with contrast agent)
% -------------------------------------------------------------------

% TASK 1.3 FILL IN HERE
phantom_dsa = phantom.discrete - phantom_no_contrast;
project_dsa = phantom.projection - project_no_contrast;

% TASK 1.3 UNCOMMENT HERE
DisplayData(phantom.projection',[1,4,2]);
title('Projection (w/o contrast agent)'); xlabel('r'); ylabel('P(r))');

DisplayData(phantom_dsa,[1,4,3]);
title('Phantom (DSA)');

DisplayData(project_dsa',[1,4,4]);
title('Projection (DSA)'); xlabel('r'); ylabel('P(r))');
```

e) The resulting graph from the DSA algorithm is the following:



The graphs demonstrate the difference in contrast in the two implementations. The vessel contrast with DSA has been improved significantly as due to the intravascular administration of the iodine contrast agent, the mass attenuation coefficient of blood doubled. That means that the photons will experience bigger attenuation in the aorta than in the neighbouring structures. Furthermore, by subtracting the image with no contrast from the image with the high contrast will only keep the difference in contrast in the aorta as the other intensity values for the other structures are the same. Therefore, the only intensity left in the final image is the difference in contrast in the aorta as seen in the resulting DSA graph.