Biomedical Imaging Exercise – Week 4

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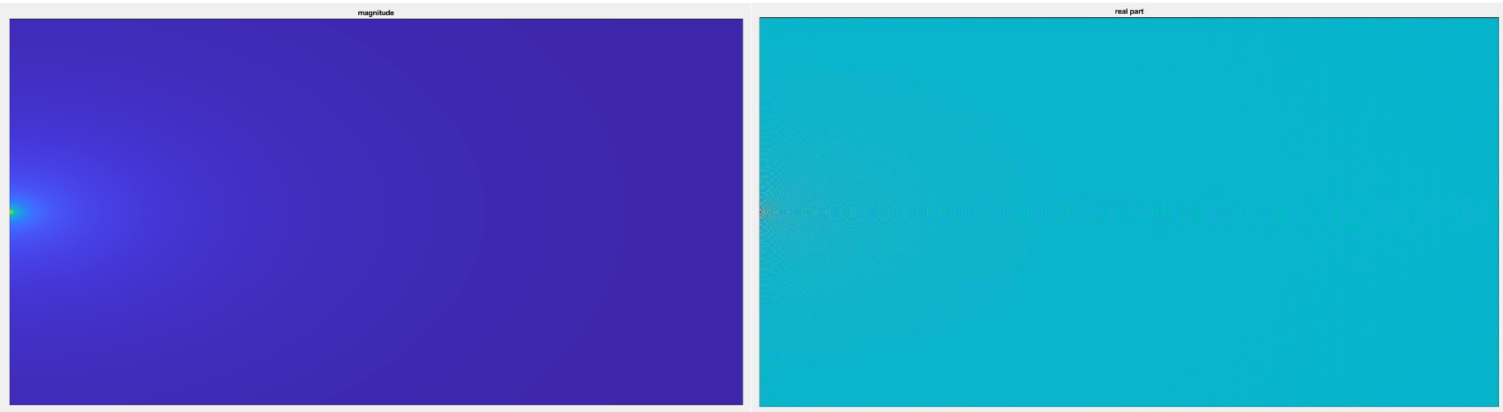
1. The resulting waveforms for the real part and the magnitude are the following:

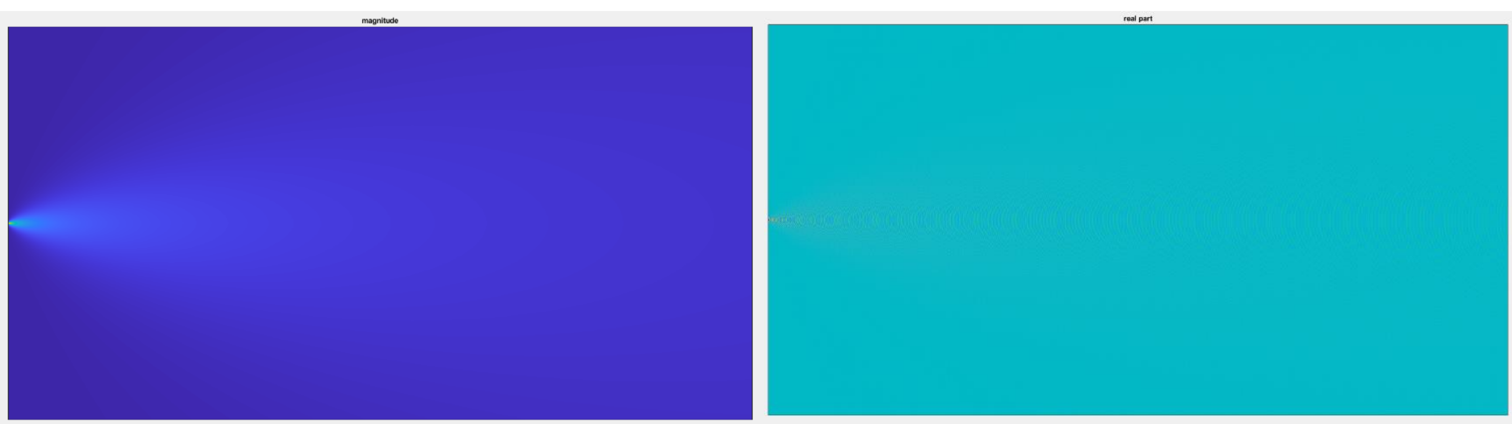


There is only one array element emitting an acoustic wave, which has negligible width in the y-direction as mentioned in the question. Therefore, it is considered a point source and its diameter and radius r tend to 0. From this observation the following conclusions are extracted:

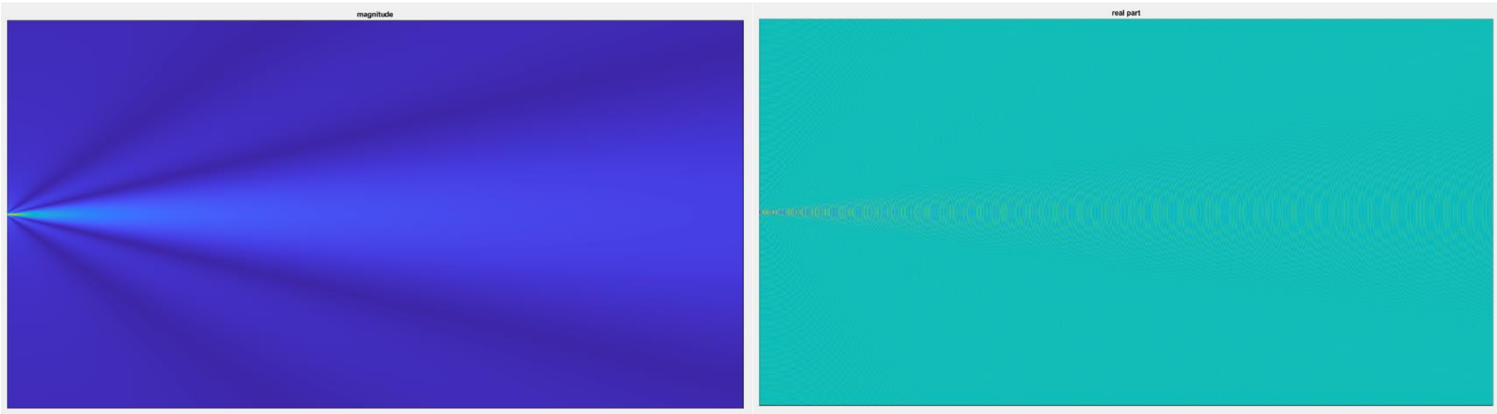
- <u>Near-field Boundary (NFB):</u> Because the NFB is given by: $Z_{NFB} = \frac{r^2}{\lambda}$ and $r \rightarrow 0$, then $Z_{NFB} \rightarrow 0$ as well. This can be interpreted as the near-field boundary being at the border of the array element.
- <u>Lateral Resolution:</u> $lateral\ resolution \rightarrow 0$ until the NFB, then the lateral resolution deteriorates in proportion to the broadening angle θ.
- <u>Broadening Angle θ:</u> The angle is given by: $\theta = \sin^{-1}(\frac{0.61 \cdot \lambda}{r})$, but because $r \rightarrow 0$, then θ diverges.

2. The resulting waves from increasing the number of transducer elements are the following:

- **n = 2:**



- **n = 5:**



- **n = 10:**

- **n = 20:**

As seen from the above illustrations, as more transducer elements are added then r (radius) is increased as it is true that: $r = \frac{(n-1) \cdot pitch}{2}$. Also, the NFB is now further away from the source elements and the lateral resolution increases. Furthermore, it is obvious that the broadening angle θ, becomes smaller as more elements are added.

The above behaviour is the same as for a single flat transducer, therefore Huygens' Principle is proved. More specifically, the individual waves contribute to the net wave by following the principle of superposition.

3. For the wavelength of the wave in water: $\lambda = \frac{c}{f} = \frac{1480}{1.5 \cdot 10^6} \approx 1\ mm$

For 40 transducer elements the following formula is true:

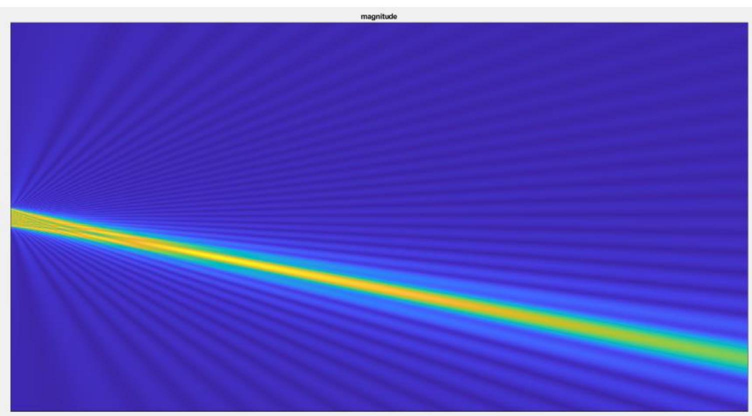
$$\Delta\Phi = k \cdot \Delta s = k \cdot pitch \cdot \sin(a), \text{ where } a \text{ is the deflection angle of the beam}$$

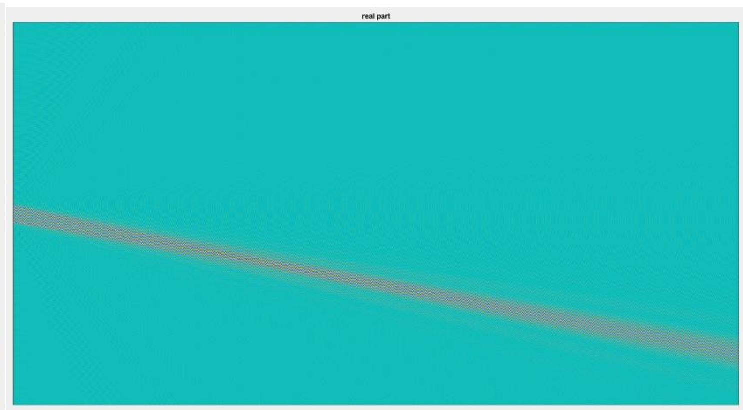
$$\Delta\Phi = \frac{2\pi}{\lambda} \cdot pitch \cdot \sin(a) = \frac{2\pi}{1} \cdot 0.5 \cdot sin(20°) = 1.075\, rad = 0.342\pi\, rad$$

The exponential part of the code is changed from *exp(i*phase)* to *exp(i*(t-1)*phase)* and the code snippet looks like the following:

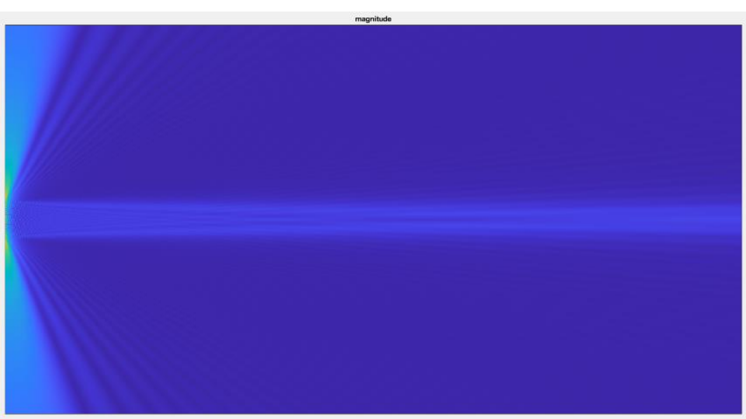
```
for t = 1:nt

    x0 = -0.001;                        % x coordinate of transducer, 1 mm outside the container
    y0 = pitch*(t-(nt+1)/2);            % y coordinate of transducer

    r = sqrt((x-x0).^2+(y-y0).^2);      % distance from transducer, on the grid

    % gaussian_amp = a*exp(-((t-b)^2)/(2*c^2)); % gaussian amplitude calculation

    phase = 0.342*pi;                     % phase of transducer input [rad]
    amplitude = 1;                      % relative amplitude of transducer input

    wave = wave + exp(1i*(k*r))./sqrt(r) .* amplitude .*exp(1i*(t-1)*phase);  % add partial wave to net wave

end
```

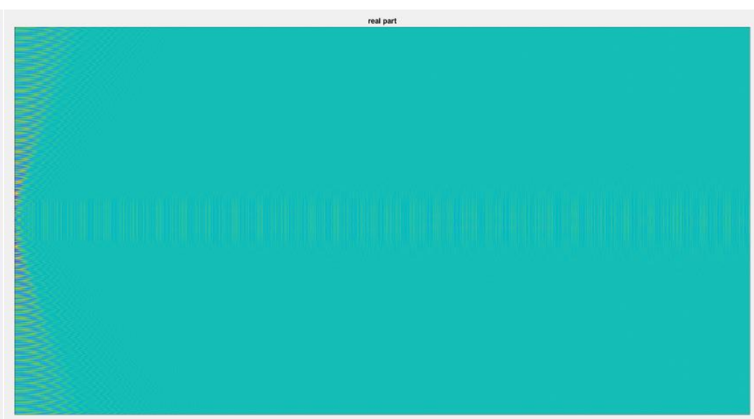
Therefore, by changing the *phase* variable to 0.342π rad outputs the following graphs:





4. A grating lobe can be introduced by violating the necessary condition of $d \leq \frac{\lambda}{2}$. Therefore, if we change the pitch to be equal to the wavelength $(pitch = \lambda)$, the grating lobes are visible from the following waveforms:



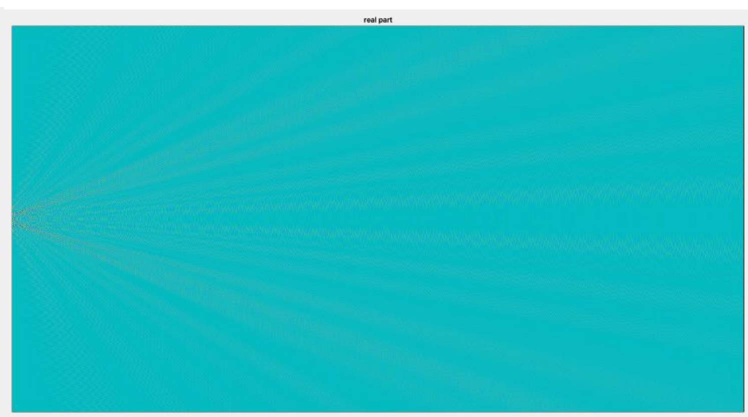


5. In order to focus at a depth of 5 cm we need to change the phase of the individual transducer elements. The phase needs to be 0 for the outer elements and it needs to be gradually increasing as we move towards the center of the transducer

array. The phase is changed according to $\Delta\Phi = k \cdot \Delta s$, where $\Delta s = depth\ of\ focus \cdot (\frac{1}{sin\theta} - 1)$. For the angle θ, it is true that: $\theta = \tan^{-1}(\frac{depth\ of\ focus}{distance\ of\ transducer\ from\ center\ of\ array})$. This algorithm is used to write the following MATLAB snippet:

```
for t = 1:(nt/2)

    x0 = -0.001;                        % x coordinate of transducer, 1 mm outside the container
    y0 = -pitch*(t-(nt+1)/2);           % y coordinate of transducer

    r = sqrt((x-x0).^2+(y-y0).^2);      % distance from transducer, on the grid

    % gaussian_amp = a*exp(-((t-b)^2)/(2*c^2)); % gaussian amplitude calculation

    dist_cent = (pitch/2) + pitch*abs(t-floor((nt+1)/2));
    angle = atand(depth/dist_cent);
    delta_s = depth*((1/sind(angle))-1);
    phase = k*delta_s;                  % phase of transducer input [rad]
    amplitude = 1;                      % relative amplitude of transducer input
    wave = wave + exp(1i*(k*r))./sqrt(r) .* amplitude .* exp(1i*(t-1)*phase);   % add partial wave to net wave

end

for t = (nt/2)+1:nt

    x0 = -0.001;                        % x coordinate of transducer, 1 mm outside the container
    y0 = -pitch*(t-(nt+1)/2);           % y coordinate of transducer

    r = sqrt((x-x0).^2+(y-y0).^2);      % distance from transducer, on the grid

    % gaussian_amp = a*exp(-((t-b)^2)/(2*c^2)); % gaussian amplitude calculation

    dist_cent = (pitch/2) + pitch*abs(t-ceil((nt+1)/2));
    angle = atand(depth/dist_cent);
    delta_s = depth*((1/sind(angle))-1);
    phase = k*delta_s;                  % phase of transducer input [rad]
    amplitude = 1;                      % relative amplitude of transducer input

    wave = wave + exp(1i*(k*r))./sqrt(r) .* amplitude .* exp(1i*(nt-t)*phase);   % add partial wave to net wave

end
```

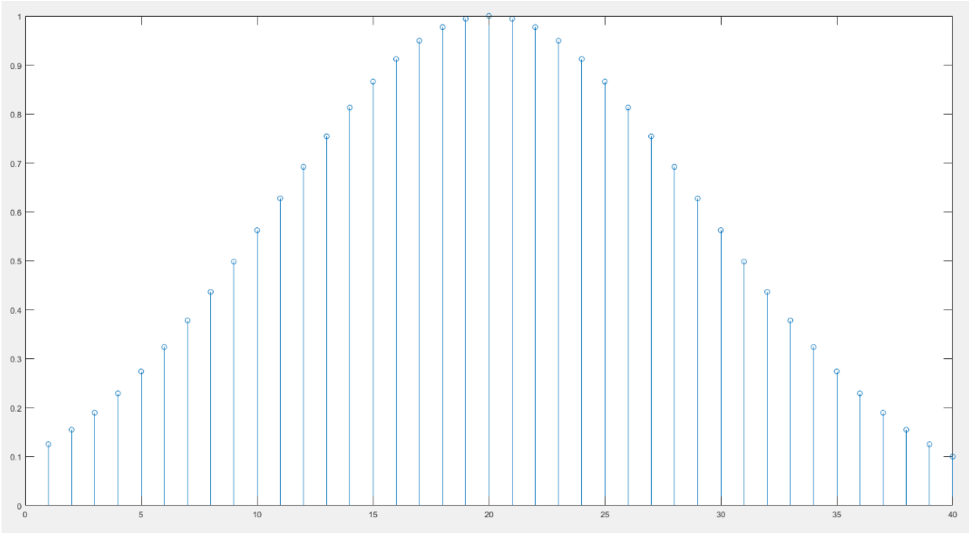
Running the program with the aforementioned changes outputs the following graphs:



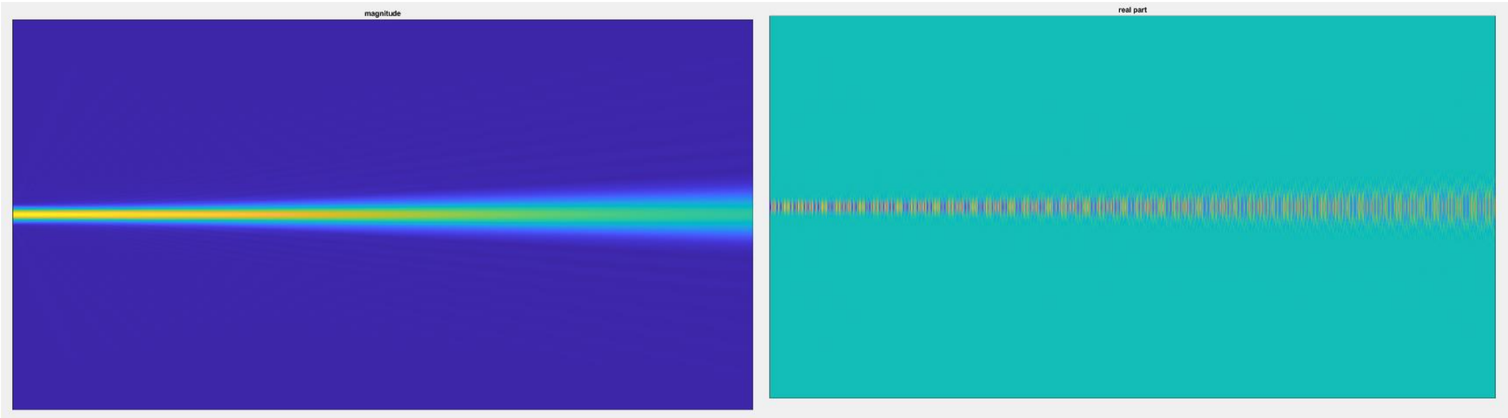
6. The following Gaussian bell function is defined:

```
% Gaussian bell function

a = 1;
b = 20;
c = 9.32;

gaussian_amp = a*exp(-((t-b)^2)/(2*c^2)); % gaussian amplitude calculation
```

If this function is plotted, it can be seen that the width is chosen so that the amplitude of the outer elements is dropped to 10% of the maximum value:



Therefore, if the above function is applied as the amplitude of the transducer elements of the phased array the following waveforms are extracted:



It is obvious that the beam is smoother than when the amplitude was the same for all the transducer elements, because the amplitude gradually increases as we are moving towards the center of the transducer array.