CMPS 2200

Introduction to Algorithms

Overview

Today's agenda:

- Introductions
- Motivation for course
- Formalisms used throughout the course
- Navigating the course

What is an algorithm?

> *an explicit, precise, unambiguous, mechanically-executable sequence of elementary instructions, usually intended to accomplish a specific purpose.*

-- **Jeff Erickson**

Examples?

BOB(n):

- for i n down to 1
    - Sing “i bottles of beer on the wall, i bottles of beer,”
    - Sing “Take one down, pass it around, $i - 1$ bottles of beer on the wall.”
- Sing “No bottles of beer on the wall, no bottles of beer,”
- Sing “Go to the store, buy some more, n bottles of beer on the wall.”

Examples of algorithm-like things that are not algorithms?

BeAMillionaireAndNeverPayTaxes():

- Get a million dollars.
- If the tax man comes to your door and says, “You have never paid taxes!”
    - Say “I forgot.”

What makes a good algorithm?

- correct
- user-friendly
- many features
- robust
- simple
- secure
- low programmer cost
- **efficient**
  - runs quickly
  - requires little memory

Then, why study efficiency?

- separates feasible from infeasible
- correlates with user-friendliness

> What if it took Google 2 minutes to return results?

Simple warmup: **What does this do?**

```
def my_function(a, b):
    for i,v in enumerate(a):
        if v == b:
            return i
    return -1
```

```python
def linear_search(mylist, key):
    """
    Args:
      mylist...a list
      key......a search key
    Returns:
      index of key in mylist; -1 if not present
    """
    for i,v in enumerate(mylist):
        if v == key:
            return i
    return -1

linear_search([5,1,10,7,12,4,2], 12)
```

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What factors affect the running time of this algorithm?

- Input **size**
- Input **values**: is key at start or end?
- Hardware!
    - TI-85 vs. Supercomputer

We need a way to compare the efficiency of algorithms that abstracts away details of hardware and input.

Analysis of Linear Search, the long way

- Assign a time cost c_i to each line i.
- Figure out how often each line is run n_i
- total cost is the cost of each line multiplied by the number of times it is run

$$\text{Cost}(\text{linear-search}, \text{mylist}, \text{key}) = \sum_i c_i * n_i$$

```
def linear_search(mylist, key):          #    cost          number of times run
    for i,v in enumerate(mylist):        #    c1                  ?
        if v == key:                     #    c2                  ?
            return i                     #    c3                  ?
    return -1                            #    c4                  ?
```

Best/Average/Worst case

To deal with the effects of the input values on performance, we can consider three types of analysis:

- **Worst-case**: maximum time for any input of size n

```
linear_search([5,1,10,7,12,4,2], 9999)
```

- **Best case**: minimum time of any input of size n

```
linear_search([5,1,10,7,12,4,2], 5)
```

- **Average case**: expected time over all inputs of size n
    - Need some probability distribution over inputs

```
for (mylist, key) in ???:
    linear_search(mylist, key)
```

Worst-case analysis of linear search

Assume $n \leftarrow$ `len(mylist)`

```
def linear_search(mylist, key):          #    cost          number of times run
    for i,v in enumerate(mylist):        #    c1                ?
        if v == key:                     #    c2                ?
            return i                     #    c3                ?
    return -1                            #    c4                ?
```

$\text{Cost}(\text{linear-search}, n) = c_1 n + c_2 n + c_4$

Cost is now just a function of:

- input size n
- constants c (depend on machine, compiler, etc)

How granular should we get?

Consider this slightly different implementation:

```
def new_linear_search(mylist, key):     #   cost          number of times run
    for i in range(len(mylist)):        #   c5                n
        if mylist[i] == key:            #   c6                n
            return i                    #   c3                0
    return -1                           #   c4                1
```

$$\text{Cost}(\text{new-linear-search}, n) = c_5 n + c_6 n + c_4 \tag{1}$$

$$\text{Cost}(\text{linear-search}, n) = c_1 n + c_2 n + c_4 \tag{2}$$

Which one is better?

Big Idea: Asymptotic Analysis

- Ignore machine-dependent constants
- Focus on **growth** of running time
  - What happens in the limit as $n \to \infty$

    https://en.wikipedia.org/wiki/Limit_(mathematics)

$c_1 n + c_2 n + c_4 \approx c_5 n + c_6 n + c_4$

e.g., consider two algorithms with running times:

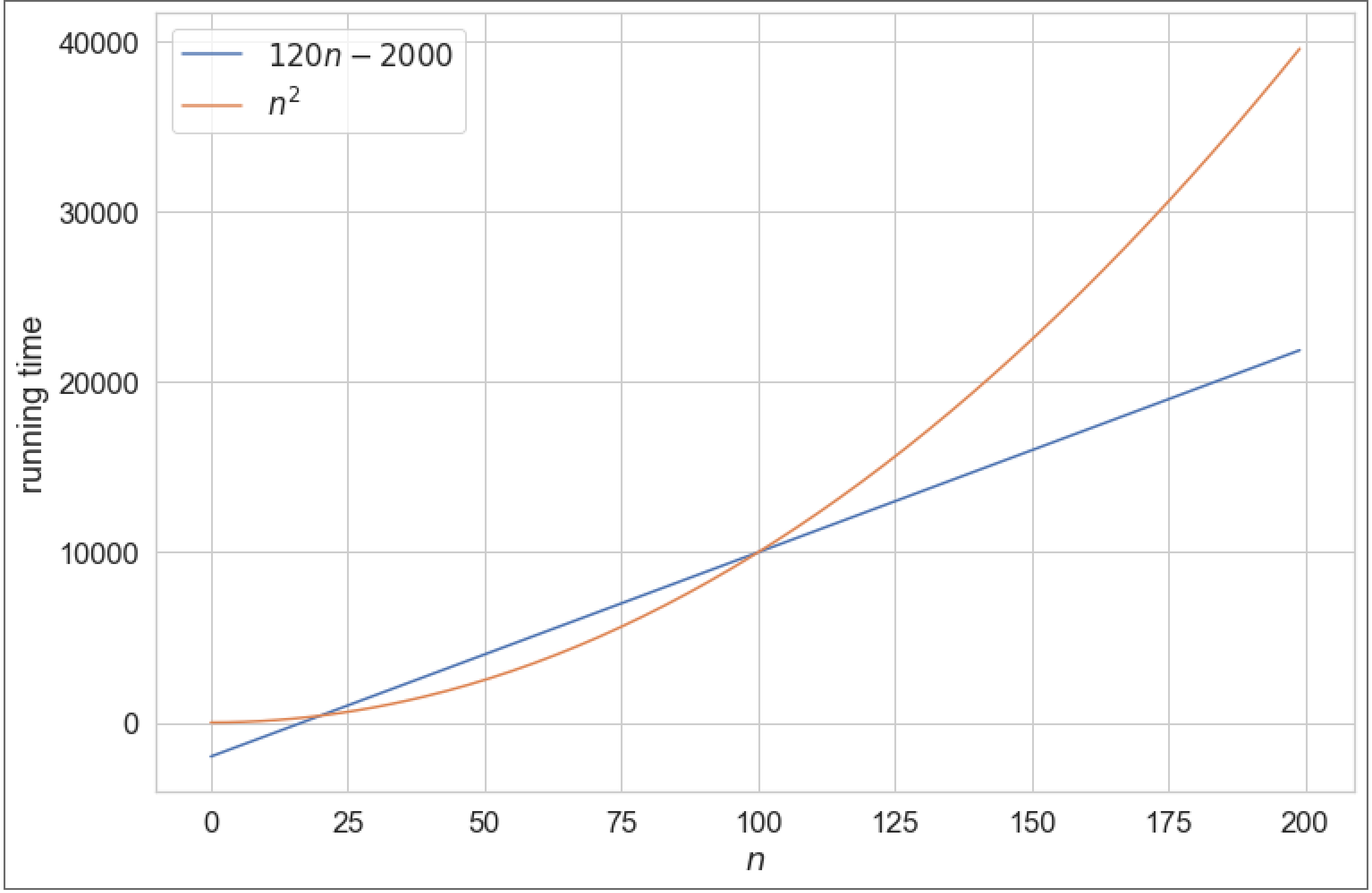
- algorithm 1: $c_1 n + c_2$
- algorithm 2: $c_3 n^2 + c_4 n + c_5$

Depending on the machine-dependent constants, algorithm 2 may sometimes be faster than algorithm 1:

- algorithm 1: $120n - 2000$
- algorithm 2: n^2

```
n = np.arange(200)
time1 = 120*n + - 2000
time2 = n*n

# plot
plt.figure()
plt.plot(n, time1, label='$120 n - 2000$')
plt.plot(n, time2, label='$n^2$')
plt.xlabel("$n$")
plt.ylabel('running time')
plt.legend()
plt.show()
```



But, as $n \to \infty$, there will be a point at which algorithm 2 will be slower, **no matter which machine it is run on**

Definition: Asymptotic dominance

Function $f(n)$ **asymptotically dominates** function $g(n)$ if **there exist** constants c and n_0 such that
$g(n) \leq c \cdot f(n)$ **for all** $n \geq n_0$

e.g., n^2 asymptotically dominates $120n - 2000$

Proof:
Find c and n_0 such that
$120n - 2000 \leq c * n^2$ for all $n > n_0$

Let $c = 1$. Find an n_0 such that
$120n - 2000 \leq n^2$

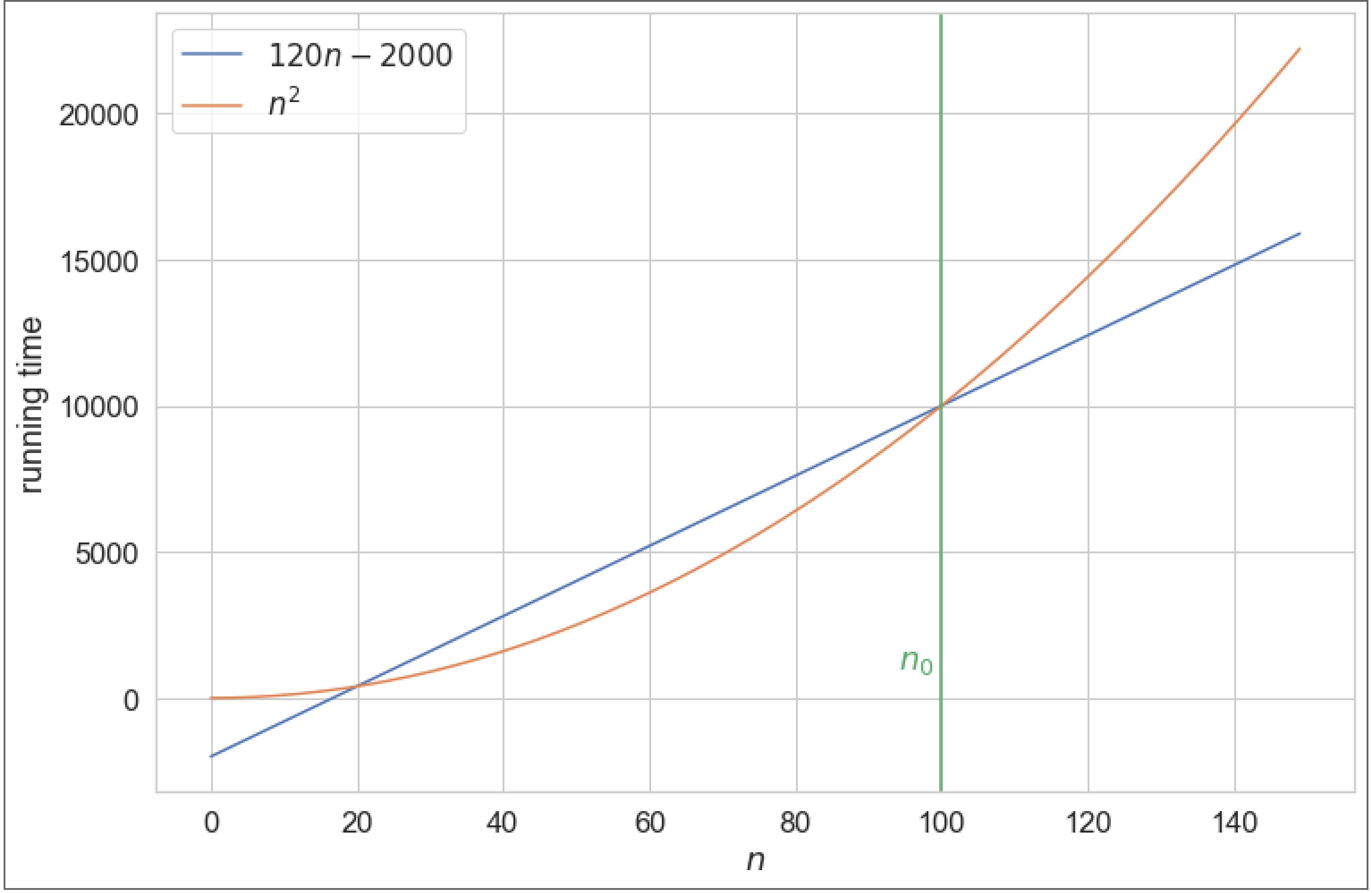
for all $n \geq n_0$

$$120n - 2000 \leq n^2 \tag{3}$$
$$0 \leq n^2 - 120n + 2000 \tag{4}$$
$$0 \leq (n - 100)(n - 20) \tag{5}$$

When $n = 100$, $120n - 2000 = n^2$
For all $n \geq 100$, $120n - 2000 \leq n^2$
So, $c = 1, n_0 = 100$ satisfies the definition of asymptotic dominance.

```
# show n_0
n = np.arange(150)
time1 = 120*n + - 2000
time2 = n*n

# plot
plt.figure()
plt.plot(n, time1, label='$120 n - 2000$')
plt.plot(n, time2, label='$n^2$')
plt.axvline(100, color='g')
plt.text(94,1000,'$n_0$', fontsize=18, color='g')
plt.xlabel("$n$")
plt.ylabel('running time')
plt.legend()
plt.show()
```



$120n - 2000$
n^2
running time
20000
15000
10000
5000
0
n_0
0
20
40
60
80
100
120
140
n

Asymptotic Notation

$$O(f(n)) = \{g(n) \mid f(n) \text{ asymptotically dominates } g(n)\} \tag{6}$$
$$\Omega(f(n)) = \{g(n) \mid g(n) \text{ asymptotically dominates } f(n)\} \tag{7}$$
$$\Theta(f(n)) = O(f(n)) \cap \Omega(f(n)) \tag{8}$$

e.g.

$120n - 2000 \in O(n^2)$

$10n^3 + 2n^2 - 100 \in \Omega(n^2)$

$14n^2 - 5n + 50 \in \Theta(n^2)$

We often abuse notation such as

$120n - 2000 = O(n^2)$

or

$120n - 2000$ is $O(n^2)$

Analogy:

O	Ω	Θ	o	ω
$\leq$	$\geq$	$=$	$<$	$>$

Course Overview

- Analyzing algorithms: methods to compute tight bounds on running time
- Designing algorithms: various approaches to designing efficient algorithms
    - lists, sequences, trees, graphs,…
- Distinct from typical courses like this, we will emphasize **parallel** algorithms from the start (next lecture)

Navigating the course

- Canvas: syllabus, dates, grades
- Diderot: interactive textbook
- Github: assignments, slides