VE216 Recitation Class 8

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UM-SJTU Joint Institute

VE216 SU20 TA Group

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Overview

Sampling Theorem

Sampling Theorem

- Equally Spaced Samples of $x(t)$

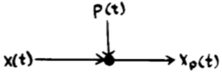
  $x(nT) \quad n = 0, \pm 1, \pm 2 \ldots$

- $x(t)$ Band limited

  $X(\omega) = 0 \quad |\omega| > \omega_M$

If $\frac{2\pi}{T} \triangleq \omega_s > 2\omega_M$

Then $x(t)$ uniquely recoverable



$$x_p(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

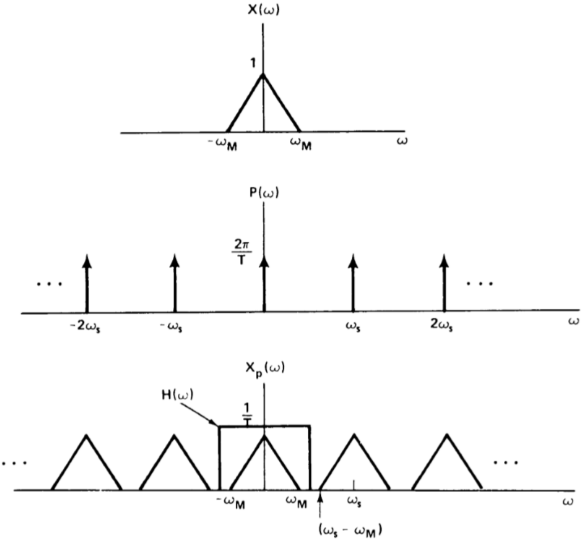
$$= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

$$X_p(\omega) = \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right]$$

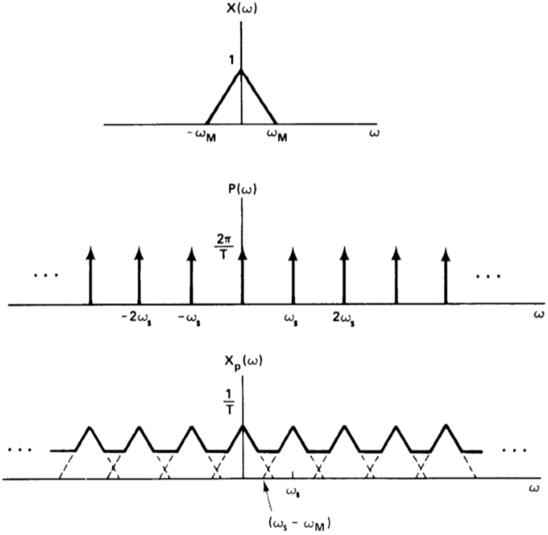
$$P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X\left(\omega - k\frac{2\pi}{T}\right)$$

Recoverable



Aliasing



Exercise - Quiz 4 Q2

A signal $x(t)$ with spectrum $X(\omega) = (1 - 4|\omega|)\,\mathrm{rect}(2\omega)$ is modulated by the following modified impulse train: $p(t) = \sum_{n=-\infty}^{\infty} 2\delta(t-5n) - \delta(t-5n-1) - \delta(t-5n+1)$. Determine and sketch the magnitude spectrum of the resulting signal.

Exercise - Quiz 6

Consider the input signal $x(t)$ below

$$x(t) = e^{-j45t} + e^{-j35t} + e^{-j25t} + e^{-j15t} + e^{-j5t} + e^{j5t} + e^{j15t} + e^{j25t} + e^{j35t} + e^{j45t}.$$

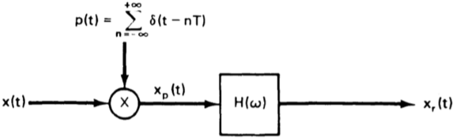
The signal $x(t)$ is first input to an analogy filter with impulse response

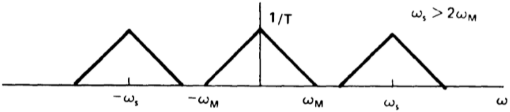
$$h_1(t) = \frac{\sin(10t)}{\pi t} + \frac{\sin(20t)}{\pi t} + \frac{\sin(30t)}{\pi t}$$

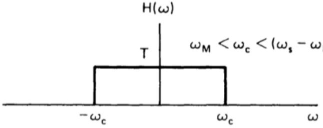
to form an output $x_1(t)$, and then $x_1(t)$ is sampled at a rate of $\omega_s = 40$ to form a sampled signal $x[n]$. The signal $x[n]$ thus obtained is then input to a lowpass filter with impluse response $h_2(t) = \frac{\sin(40t)}{\pi t}$ to reconstruct a single $z(t)$.

- [10 points] Find the Fourier Transform $x_1(t)$.
- [10 points] Write your expression for $z(t)$. Simply your result when possible.

Sampling & Reconstruction





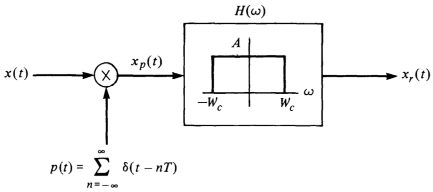


Exercise - HW5 Q6

6. [12]Consider the system in Figure 0506.

   If $X_1(\omega) = 0$ for $|\omega| > 2W$ and $X_2(\omega) = 0$ for $|\omega| > W$. For the following inputs $x(t)$, find the ranges for the cutoff frequency W_c in terms of T and W and find the maximum values of T and A, such that $x_r(t) = x(t)$.

   (a) $x(t) = x_1(t - \pi/2) + x_2(t)$

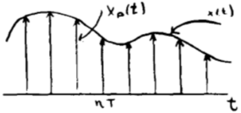


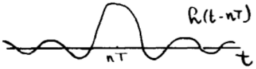
Sinc interplation (Ideal lowpass filter)

$$x_p(t) \quad x(t)p(t)$$

$$= x(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

$$= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t-nT)$$





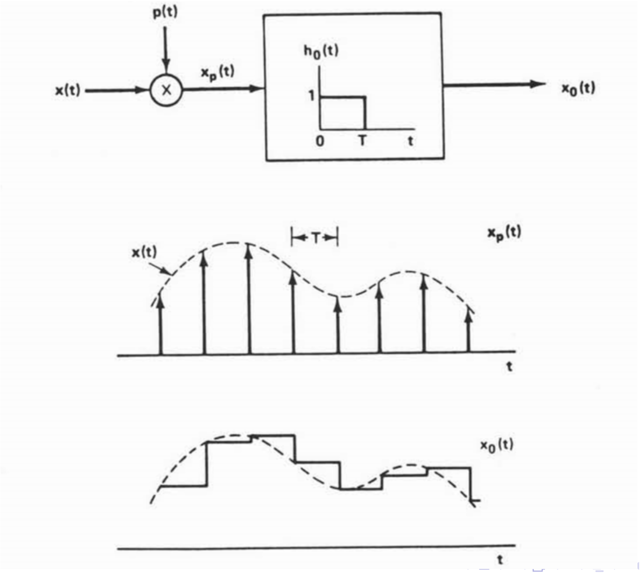
$$x_r(t) = x_p(t) * h(t)$$

For $H(\omega)$ an ideal lowpass filter with cutoff frequency ω_c,

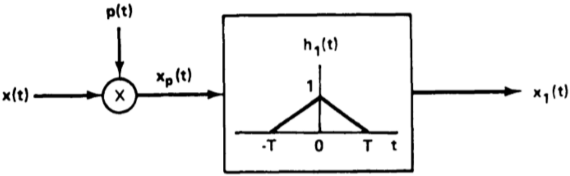
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)h(t-nT)$$

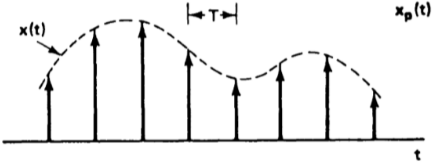
$$h(t) = T\frac{\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c t}{\pi}\right)$$

Nearest neighbor interplation (zero-order hold filter)

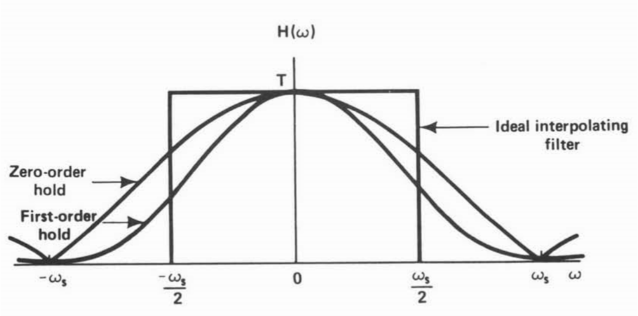


Linear interplation (first-order hold filter)





Comparison in Frequency Domain



Exercise - Interplation

7. [5]Suppose we have the system in Figure 0507(a) and 0507(b), in which $x(t)$ is sampled with an impulse train. Sketch $x_p(t)$, $y(t)$ and $w(t)$. State your reasoning.

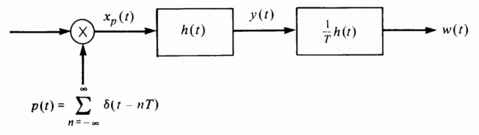
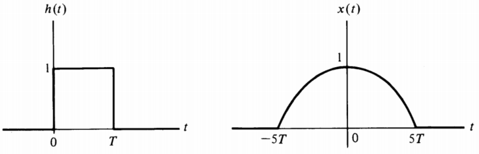


Figure 0507(a).



Conclusion

- Time domain and Frequency behavior are consistent
- Both sampling and reconstruction are indispensible
- Consider eye (watching the wheels) and ear (infrasonic, ultrasonic)
- For sampling & reconstruction-related problems, I prefer to view them graphically (often in freq. domain).
- Questions in HW5 are very interesting (or, tricky) - not much math involved, rather requires understanding

The End