VE216 Recitation Class 2

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Overview

Systems

- Transform the input signal to the output signal
- Transformation is more general than function composition (which is pointwise)

- invertible system vs. bijective (ve203)
- Understand the system in terms of input-output relation
- const. system $y(t) = 0$ vs. "future" $y(t) = x(t+1) - x(t+1)$?

Amplitude properties

- linearity: zero in $\rightarrow$ zero out
  - $x(t) = 0, \forall t$
  - $y(t) = 2x(t) + 1$
- stability: bounded in $\rightarrow$ bounded out
  - should assume $x(t)$ bounded
  - $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$
- invertibility: each output signal correspond to only one input signal
  - $y(t) = x^2(t)$

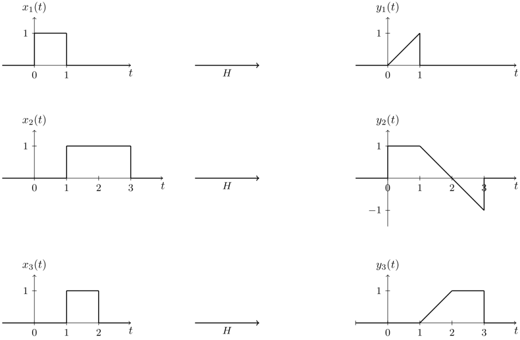
Time properties: if no given formula

- causality: output depends only on the "present" or "past" inputs
- time invariant: if $x(t) \to y(t)$, then $x(t - t_0) \to y(t - t_0)$

14. [2!] Show that causality for a continuous-time linear system is equivalent to the following statement:
For any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

Exercise: Type 1 - Given Input-Ouput Pairs

Different from HW1 Q11

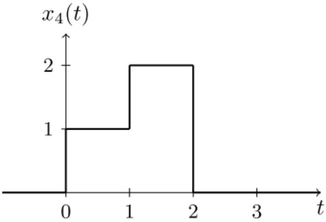


- Is it casual?
- Is it linear?
- If linear, is there other ways to determine casuality?

Exercise

Conti.

- If linear, what is the output of $x_4(t)$?



Time properties

- causality: output depends only on the "present" or "past" inputs
- memoryless: output depends only on the "present" input $x(t)$
- time invariant: if $x(t) \to y(t)$, then $x(t-t_0) \to y(t-t_0)$
- Notice that the input refers to "$x(t)$", while there can be other terms relating to "t"

Consider:

$$y(t) = \frac{e^{x(t)}}{|t+1|}$$

- casual:
- memoryless:
- Time Invariant:

Transformation of Signals

Theorem (Time transformation)

1) $x(\frac{t-t_0}{w})$ 2) $x(at-b)$

For Graph:
1) First scale according to w, then shift according to t_0
2) First time-delay by b, then time-scale by a

Think about the physical meaning: There are two systems, one can shift the time, another can scale the time. Different sequence of connection requires different specification of (w, t_0, a, b) to reach the same effect.

Question: for what system, the order doesn't matter?

Transformation of Signals

Theorem (Amplitude transformation)

1) Reversal $y(t) = -x(t)$
2) Scaling $y(t) = ax(t)$
3) Shifting $y(t) = x(t) + b$

General Transformation

- "Time" transformation: $y(t) = x(g(t))$
- "Amplitude" transformation: $y(t) = h(x(t))$

Consider:

- 1) $y(t) = x(t)$
- 2) $y(t) = x(\sin(t))$
- 3) $y(t) = \cos(x(t))$
- 4) $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$

Question:

Think about whether the system that perform such transformation are: "linear, stable; time-invariant, causal, memoryless" in general.

Exercise: Type 2 - Given Formula

General Transform - Revisited

- "Time" transformation: $y(t) = x(g(t))$
  — often lead to the violation of Time properties (TI, Casual)
- "Amplitude" transformation: $y(t) = h(x(t))$
  — often lead to the violation of Amplitude properties (linear)

Guess the result

System	**Time-Invariant**	**Casual**	**Linear**	**Stable**
$y(t) = x(2-t)$				
$y(t) = x(t/3)$				
$y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$				
$y(t) = cos(x(t))$				

Summary

- Maybe you were confused about so many concepts and feel boring.
- But at least I hope you could tell signal properties from system properties
- I would be glad if you can see the connection between signals and systems
- The first fascinating result: impulse response will be covered next week.

The End