VE216 Recitation Class 10

ZHU Yilun

UM-SJTU Joint Institute

VE216 SU20 TA Group

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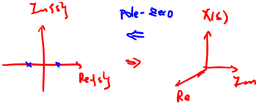
Overview

Laplace Transform

complex — decay/grow — mag=1

- $s = \sigma + j\omega$, $e^{st} = e^{\sigma t} \cdot e^{j\omega t}$: decaying/growing term and periodic term
- LT Definition:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$



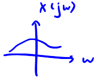
Notice: ROC - region that X(s) exist, or $\int_{-\infty}^{\infty} < \infty$. } detail next slide.

Study system behavior

- Compare with FT:

s=σ+jw, σ=0

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$



decompose signals; system as filters

w.r.t freq. components. $Y(w) = X(w) \cdot H(w)$

ROC

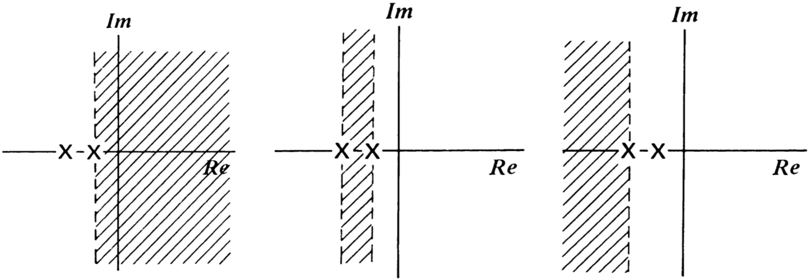
$e^{-at}\cdot u(t) \leftrightarrow \frac{1}{s+a}$. real{s} > real{-a}. e.g: $e^{-t}u(t)$

$-e^{-at}\cdot u(-t) \leftrightarrow \frac{1}{s+a}$. real{s} < real{-a} $\quad e^{-\sigma t}$ σ<0 σ>0

Definition: the subset of $\mathbb{C}$ that $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt < \infty$

Consider: - have an inverse problem $\quad x(t)\cdot e^{-\sigma t}\cdot |e^{-j\omega t}|$ 1

$$X(s) = \frac{1}{(s+1)(s+2)}$$



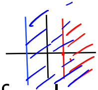
Different choice of ROC corresponds to different $x(t)$. - Quiz9 / Quite general here, next: focus on systems

(here, $x(t)$ can be input/output signal, or the impulse response)

LT - Study (rational) LTI System Behavior $H(s)$ $= \int_{-\infty}^{\infty} h(t)\cdot e^{-st}\,dt$

Recall: chap.2. $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

- stable $\Longleftrightarrow$ ROC includes $j\omega$ axis
- casual $\Longleftrightarrow$ ROC RHP $\quad h(t) = e^{t}\cdot u(t) \leftrightarrow \frac{1}{s-1}$



- casual and stable $\Longleftrightarrow$ all poles in the left half of s-plane $\quad h(t) = e^{-t}\cdot u(t) \leftrightarrow \frac{1}{s+1}$
- Not stable if: $H(s) = \frac{s^2+s+1}{s+1}$ $= \boxed{s} + \frac{1}{s+1} \Rightarrow \frac{Y(s)}{X(s)} = s + \dots \Rightarrow y(t) = \frac{d}{dt}x(t) + \dots$
- Differentiation: solve systems defined by diff. eqn.

$$\frac{d^n}{dt^n}x(t) \overset{\mathscr{L}}{\longleftrightarrow} s^n X(s)$$

$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) = \frac{d^2}{dt^2}x(t)$

$\Updownarrow$

$s^2\cdot Y(s) + 2\cdot Y(s) = s^2 X(s)$

- Convolution: get output y(t)

$$y(t) = h(t) * x(t) \overset{\mathscr{L}}{\longleftrightarrow} H(s)X(s) = Y(s)$$

- Block diagram: be able to read as well as draw - Quiz10

Exercise: HW6 Q5

5. [10] A causal LTI system S with impluse response h(t) has its input x(t) and output y(t) related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t)$$

,

how many poles does G(s) have?

$$\begin{array}{r} s^2+\alpha s+\alpha^2 \\ s+1 \,\big)\, \overline{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2} \\ s^3+s^2 \\ \hline \alpha s^2+\alpha(\alpha+1)s+\alpha^2 \\ \alpha s^2+\alpha s \\ \hline \alpha^2 s+\alpha^2 \\ \alpha^2 s+\alpha^2 \\ \hline 0 \end{array}$$

Hint: use long division

$s^3 \cdot Y(s) + (1+\alpha)\cdot s^2 \cdot Y(s) + \alpha(\alpha+1)\cdot s \cdot Y(s) + \alpha^2 \cdot Y(s) = X(s)$

$[s^3 + (1+\alpha)\cdot s^2 + \alpha(\alpha+1)\cdot s + \alpha^2]\cdot Y(s) = X(s)$

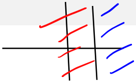
$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2}$ ← how to simplify? guess $s = \pm 1, 0, \cdots$ $= \frac{1}{(s+1)(s^2+\alpha s+\alpha^2)}$

$\Rightarrow G(s) = s\cdot H(s) + H(s) = (s+1)H(s). = \frac{1}{s^2+\alpha s+\alpha^2}$

Exercise: HW6 Q7

$= e^{2t} \cdot u(-t) + e^{2t} \cdot u(t)$

$-a=2 \qquad -a=2$



ROC ???

7. [10] A $\boxed{\text{causal LTI}}$ system with impulse response h(t) has the following properties:1.When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = \frac{1}{6}e^{2t}$ for all t. 2.The impulse respose h(t) satisfies the differential equation $\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t)$, where b is an unkhown constant.

   Determine the system function H(s) of the system, consistent with information above. There should be no unkhowm constants in your answer, that is, the constant b should not appear in the answer.

Hint: when input is an exponential signal

$x(t) = e^{st} \rightarrow \boxed{H} \rightarrow y(t) = x(t) * h(t)$

$= \cdots \leftarrow$ RC 4. P5

$= H(s) \cdot e^{st}$

$\Rightarrow H(2) = \frac{1}{6}$

Diff. eqn. : $s \cdot H(s) + 2 \cdot H(s) = \frac{1}{s+4} + \frac{b}{s}$

Plug in $H(2) = \frac{1}{6}$: $2 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} = \frac{1}{6} + \frac{b}{2} \Rightarrow b = 1 \Rightarrow H(s) = \frac{1}{s+2}\left(\frac{1}{s+4} + \frac{1}{s}\right)$

$= \frac{1}{s+2} \cdot \frac{2s+4}{s(s+4)}$

$= \frac{2}{s(s+4)}$

Conclusion - for Chap. 9

Vv256/286: use FS/FT/LT to solve diff. eqn.

periodic, both, system (causal, stable)

- FS vs. FT vs. LT
- Focus on system prospective
- Practice on PFE, block diagram, etc

read & draw

$$F(s) = \frac{1}{s^3+as^2+bs+1}$$

$$= \frac{1}{s+r_1} + \frac{1}{s+r_2} + \cdots$$

Conclusion - for the course

$\delta(t) \to h(t)$ $x(t) \to y(t) = x(t) * h(t)$

- LTI system, impulse response, convolution - Foundation, Time domain
- Fourier Analysis - signal, system (freq. comp.; filter) freq. domain
- Filtering, Sampling, Communication - most interesting topics to me
- Laplace Transform - ROC, system, block diagram
- This course is one of the most inspiring course I have ever took, as it provides a sense of the strong connection between mathematics and the real world.
- And I join research group then.
- If you are interested in signal processing, consider taking: VE351; VE401, VE501; VE455, VE489; VV214/417

DSP. after Chap. 7.

Random Signal $n(t)$ — Digital Comm. — Comm. Network — Linear Algebra – "most" important & inspiring. to me

The End