VE216 Recitation Class 7

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UM-SJTU Joint Institute

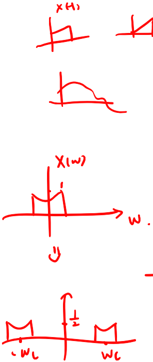
VE216 SP20 TA Group

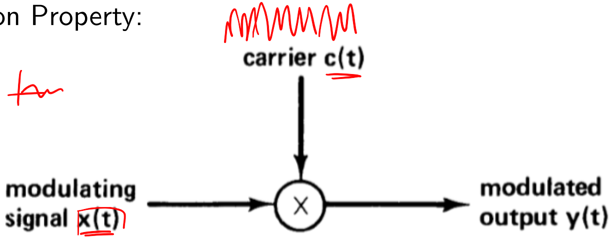
2020 Spring

Overview

Modulation

- Modulation Property:





carries info.

$$\rightarrow : \quad x(t) \cdot c(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\left[X(\omega) * C(\omega)\right]$$

- pulse carrier   chap. 7.   — sampling.
- → sinusoidal carrier : 1st $x(t)\cdot\cos(w_c t) \overset{F}{\longleftrightarrow} \frac{X(w-w_c)+X(w+w_c)}{2}$

$$c(t) = \cos(\omega_c t + \theta_c)$$

Sinusoidal Amplitude Modulation

$x(t) \to \boxed{h(t)} \to x(t) * h(t)$

- Block diagram of modulation system:
  $x(t)$ - information, $c(t)$ - carrier

$$x(t) \to \otimes \to y(t) \to \text{ antenna}$$
$$\uparrow$$
$$\cos(\omega_c t + \theta_c)$$

Notice: here we multiply the carrier signal rather than do convolution

- Transmitted signal (i.e., modulated output $y(t)$):

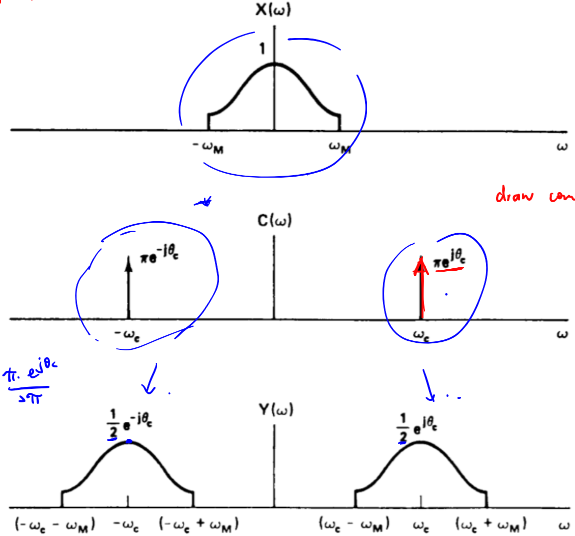
$$y(t) = x(t)c(t) = x(t)\cos(\omega_c t + \theta_c)$$
$$\overset{\mathcal{F}}{\longleftrightarrow} \quad Y(\omega) = \frac{1}{2}[\boxed{e^{j\theta_c}}X(\omega - \omega_c) + \boxed{e^{-j\theta_c}}X(\omega + \omega_c)]$$

Sinusoidal Amplitude Modulation - Synchronous

$x(t)\cdot c(t) \xleftrightarrow{F} \frac{1}{2\pi}\left(X(\omega) * C(\omega)\right)$

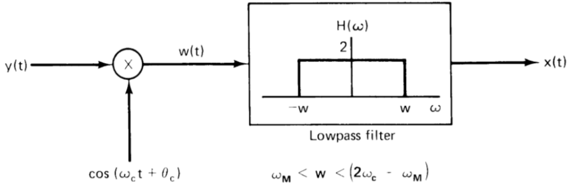
draw complex function: $\begin{cases} |C(\omega)| \\ \angle C(\omega) \end{cases}$

$\frac{\pi \cdot e^{j\theta_c}}{2\pi}$



Synchronous Demodulation

- Block diagram of demodulation system:



$\omega_M < w < (2\omega_c - \omega_M)$

- First multiply $y(t)$ by another $cos(\omega_c t + \theta_c)$ signal:

$$w(t) = y(t)\cos(\omega_c t + \theta_c)$$

$$W(\omega) = \frac{1}{2}[e^{j\theta_c} Y(\omega - \omega_c) + e^{-j\theta_c} Y(\omega + \omega_c)]$$

$$= \boxed{\frac{1}{4}e^{2j\theta_c}X(\omega - 2\omega_c) + \frac{1}{2}X(\omega) + \frac{1}{4}e^{-2j\theta_c}X(\omega + 2\omega_c)}$$

- Then followed by lowpass filtering to extract $X(\omega)$

Synchronous Demodulation

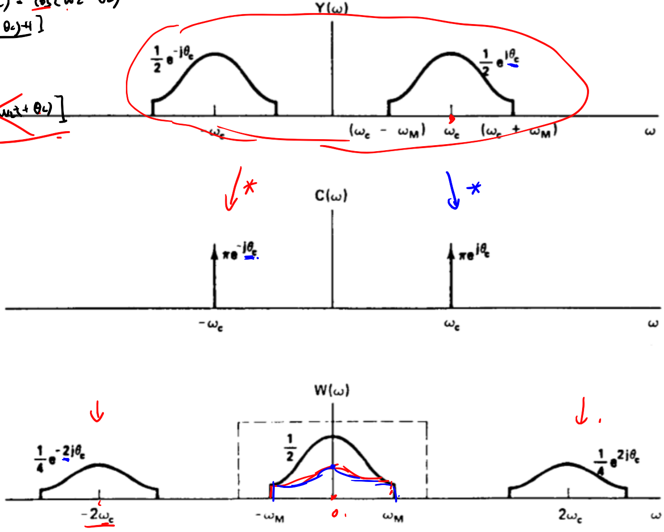
$y(t)\cdot c(t)$

Time domain:

$x(t)\cdot\cos(\omega_c t+\theta_c)\cdot\cos(\omega_c+\theta_c)$

$= x(t)\cdot\frac{\cos[2(\omega_c t+\theta_c)-t]}{2}$

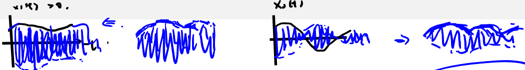
$= \boxed{\frac{1}{2}x(t)} + \frac{1}{2}x(t)\cdot\cancel{\cos[2(\omega_c t+\theta_c)]}$



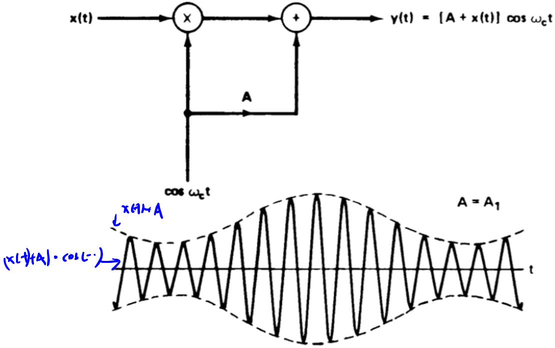
Asynchronous Demodulation: Motivation

- It seems to be harmless to write the way synchronous Demodulation works on paper, but up to now we haven't considered how to implement it to hardware.
- The bad news is that in practice, the phase θ_c is not available, therefore a sophisticated phase-tracking receiver is needed.
- But for commercial products like AM radio, one would expect the receivers to be simple and inexpensive.
- Therefore a different demodulation scheme is needed, which uses a more complicated and power inefficient transmitter, but a simple receiver.

Asynchronous Demodulation: Modulated signal



- Now the modulated signal is: $y(t) = (A + x(t))\cos(\omega_c t)$ >0. $+\theta_c$
- Often we choose A greater then the amplitude of $x(t)$
- The block diagram & how the output $y(t)$ looks like:



Asynchronous Demodulation: Frequency Domain

$x(t)\cdot c(t) + A\cdot c(t).$

$c(t) = \cos(\omega_c t)$

- In frequency domain:

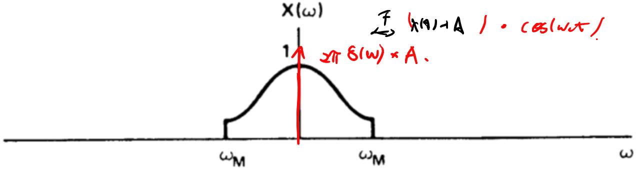
$$\begin{cases} x(t)\cdot c(t) \leftrightarrow \dfrac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2} \\ 1 \leftrightarrow 2\pi\delta(\omega) \end{cases}$$

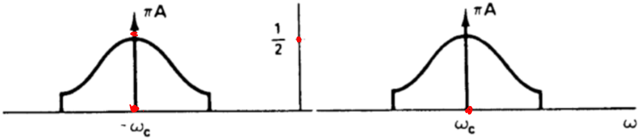
$$Y(\omega) = A\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \frac{1}{2}[X(\omega-\omega_c)+X(\omega+\omega_c)]$$

$\overset{F}{\leftrightarrow} (x(t)+A)\cdot\cos(\omega_c t)$

$2\pi\delta(\omega) * A.$

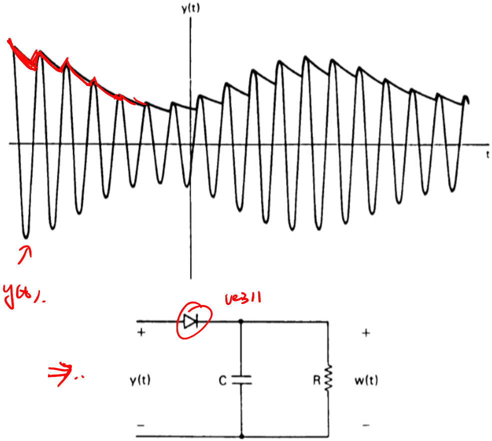
$y(t)\cdot$





Asynchronous Demodulation

- Use a simple circuit to detect the envelop: $m(t) = A + \hat{x}(t)$
- It works because ω_c is much higher than frequency of $x(t)$



Asynchronous Demodulation

yes

envelop detector

- The envolope detector gives us:

$$y(t) = (A + x(t))\cos(\omega_c t)$$

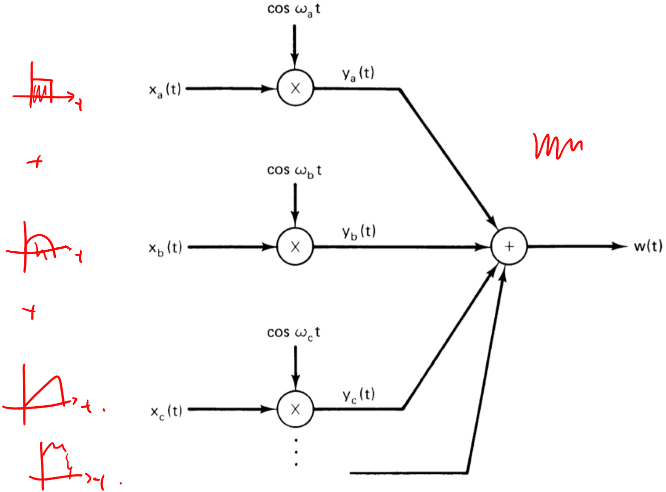
$$\to m(t) = \cancel{A} + x(t)$$

- Then eliminate the DC component (this is what we mean by "power inefficient") and you recover the orginal signal.
- The overall block diagram of demodulation:

$$y(t) \to \boxed{\text{Envelop detector}} \to m(t) \to \boxed{\text{DC blocking filter}} \to x(t)$$

Frequency-division Multiplexing

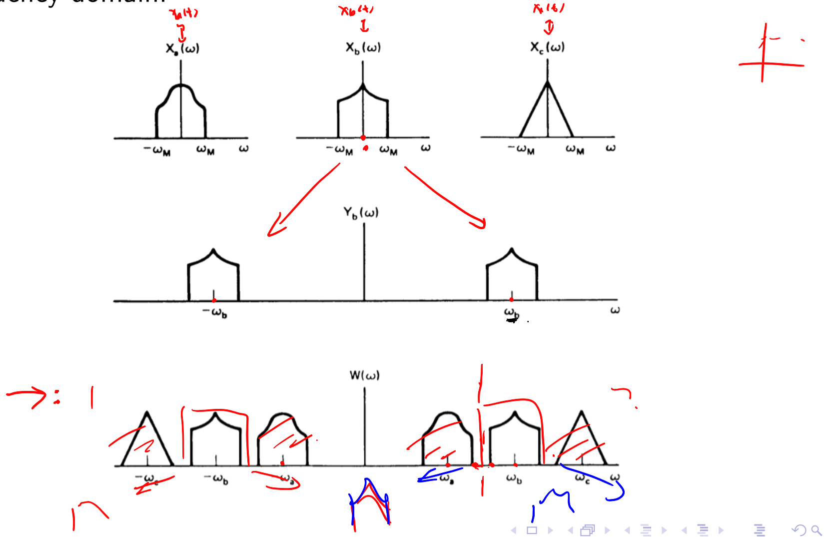
In time domain: want to transmit multiple signals at the same time



Frequency-division Multiplexing

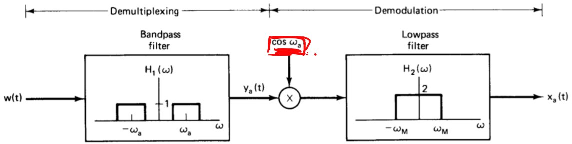
time domain:

In frequency domain:

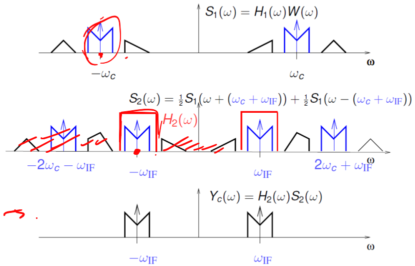


Demultiplexing and Demodulation

- synchronous demodulation



- asynchronous demodulation (using IF filter) – prob 2.



Exercise

Consider the amplitude modulation and demodulation systems with $\theta_c = 0$ and with a change in the frequency of the modulator carrier so that

$$w(t) = y(t)\cos\omega_d t$$

where

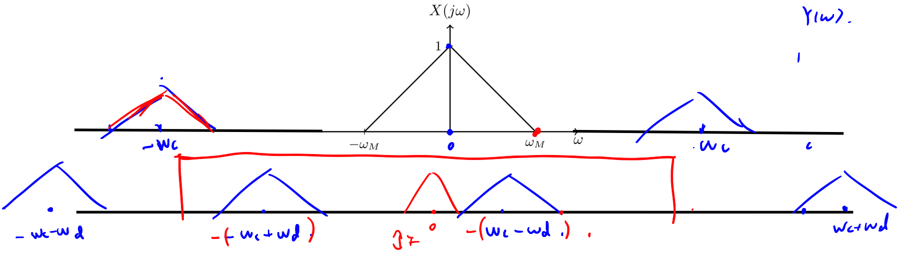
$$y(t) = x(t)\cos\omega_c t$$

Let us denote the difference in frequency between the modulator and demodulator as $\Delta\omega$ (i.e., $\omega_d - \omega_c = \Delta\omega$). Also assume that $x(t)$ is band limited with $X(j\omega) = 0$ for $|\omega| \geq \omega_M$, and assume that the cutoff frequency ω_{co} of the lowpass filter in the demodulator satisfies the inequality

$$\omega_M + \Delta\omega < \omega_{co} < 2\omega_c + \Delta\omega - \omega_M$$

(a) [5] Show that the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\Delta\omega t)$.

(b) [5] If the spectrum of $x(t)$ is that shown in figure below, sketch the spectrum of the output of the demodulator.



Conclusion

x(t)

- Prelab2 provides a detailed discussion on Multiplexing
- Get the big picture of modulation; solve problems graphically
- I guess at one time you may complain about why do we have to go through such a painful way just to get $x(t)$. x(t) → x(t)
- But in fact the task is not at all easy, given the constrain of physical laws and hardware implementation. syn. asyn.
- To me, the outcomes of these issues are amazing, because Electrical Engineers not only managed to develop a brand new subject based on the fairly abstract mathematical property associated with the Fourier transform, but also turn the theory into real life applications.

mod.

Laplace Transform

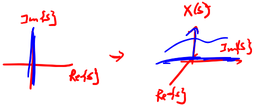
$\cos(\omega t)+j\sin(\omega t)$ → stability.

- $s = \sigma + j\omega$, $\boxed{e^{st}} = \boxed{e^{\sigma t}} \cdot \boxed{e^{j\omega t}}$: decaying/growing term and periodic term

  $|e^{j\omega t}|=1.$

- LT Definition:

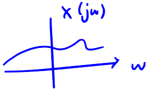
$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$



- Notice: ROC ~ later.
- Study system behavior
- Compare with FT:

$s=\sigma+j\omega.$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$



decompose signals; system as filters

$\cos(\omega_0 t)$ → ← ω_0 ×2.

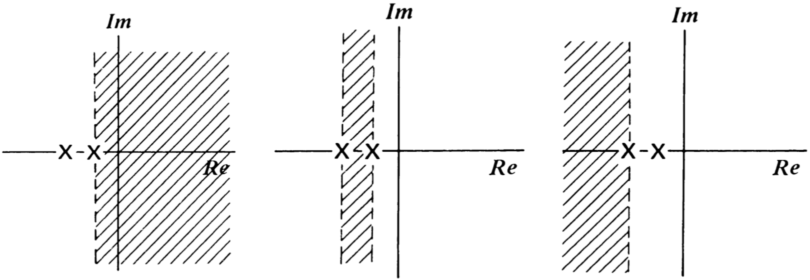
← $2\omega_0$ × 0.01

← $\frac{1}{2}\omega_0$

ROC

$$\begin{cases} e^{-at}\cdot u(t) \leftrightarrow \frac{1}{s+a}, & real\{s\} > real\{-a\} \\ -e^{-at}\, u(-t) \leftrightarrow \frac{1}{s+a}, & real\{s\} < real\{-a\} \end{cases}$$

$$x(t). \xleftrightarrow{L} X(s) = \frac{1}{(s+1)(s+2)}$$



Different choice of ROC corresponds to different $x(t)$.

LT - Study System Behavior

- stable $\Longleftrightarrow$ ROC includes $j\omega$ axis
- casual $\Longleftrightarrow$ ROC RHP
- (rational) casual and stable $\Longleftrightarrow$ all poles in the left half of s-plane
- Differentiation: solve systems defined by diff. eqn.

$$\frac{d^n}{dt^n}x(t) \overset{\mathscr{L}}{\longleftrightarrow} s^n X(s)$$

- Convolution: get output y(t)

$$h(t) * x(t) \overset{\mathscr{L}}{\longleftrightarrow} H(s)X(s)$$

- Block diagram: be able to read as well as draw

$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$

$X(\sigma + j\omega)$

$X(j\omega)$

right-hand

$h(t) = e^{-t} \cdot u(t) \leftrightarrow \frac{1}{s+1}$

$s = -1$

$\mathrm{Re}\{s\} > -1$

$\frac{s+1}{s^2+s+1}$

$\frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) = \frac{d}{dt} x(t)$

$s^2 Y(s) + 2 \cdot s \cdot Y(s) = s \cdot X(s)$

$Y(s) = \boxed{H(s)} \cdot X(s)$

$y(t)$

e.g. $H(s) = \frac{s+1}{s^2(s+1)}$

Exercise

5. [10] A causal LTI system S with impluse response h(t) has its input x(t) and output y(t) related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$ $\Rightarrow H(s)$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t)$$

,

how many poles does G(s) have?

Hint: use long division

$\Leftrightarrow\ s^3\cdot Y(s) + (1+\alpha)\cdot s^2\cdot Y(s) + \alpha(\alpha+1)\cdot s\cdot Y(s) + \alpha^2\cdot Y(s) = X(s)$

$[s^3 + (1+\alpha)\cdot s^2 + \alpha(\alpha+1)\cdot s + \alpha^2]\cdot Y(s) = X(s)$

$\Rightarrow\ H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3 + (1+\alpha)s^2 + \alpha(\alpha+1)s + \alpha^2}$

$\Leftrightarrow\ G(s) = s\cdot H(s) + H(s) = (s+1)\cdot H(s) = \frac{1}{s^2 + \alpha\cdot s + \alpha^2}$

$$\begin{array}{r} s^2 + \alpha\cdot s + \alpha^2 \\ s+1 \enclose{longdiv}{s^3 + (\alpha+1)s^2 + \alpha(\alpha+1)s + \alpha^2} \\ s^3 + s^2 \\ \hline \alpha\cdot s^2 + \alpha(\alpha+1)s + \alpha^2 \\ \alpha s^2 + \alpha s \\ \hline \alpha^2 s + \alpha^2 \\ \alpha^2 s + \alpha^2 \\ \hline 0 \end{array}$$

$\swarrow\ \frac{c}{s+1} + \frac{1}{s+\frac{1}{2}} + \cdots + \cdots$

Conclusion - for Chap. 9

Vv 256/286: use FS/FT/LT to solve diff. eqn.

periodic | periodic & aperiodic | system.

- FS vs. FT vs. LT ?
- Focus on system prospective
- Practice on PFE, block diagram, etc

Conclusion - for the course

$\delta(t) \to h(t)$ $x(t) \to y(t) = x(t) * h(t)$.

- LTI system, impulse response, convolution
- Fourier Analysis - signal, system FS FT.
- Filtering, Sampling, Communication - most interesting topics to me
- Laplace Transform - ROC, system, block diagram
- This course is one of the most inspiring course I have ever took, as it provides a sense of the strong connection between mathematics and the real world. sampling / interpolation : e.g. lears. application
- If you are interested in signal processing, consider taking: VE351; VE401, VE501; VE455, VE489; VV214/417 dsp.

  $\underbrace{\text{VE401, VE501}}_{\text{random signal.}}$ VE455: digital comm. VE489: comm. network.

The End