VE216 Recitation Class 1

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UM-SJTU Joint Institute

VE216 SU20 Teaching Group

2020 Summer

Overview

1. Introduction
   - RC Arrangement
   - General Advice
2. Chapter 1: Signals and Systems — use diff. order from prof.'s slide; today, study signal
   - Signal Characteristics — eg: energy, ...
   - Singularity Functions — $u(t)$, $\delta(t)$, ...
   - Transformation of Signals — connected with system
   - Preview: Systems — next time
3. Summary

RC Arrangement

Time often last to ~60min	Join Through
Monday 16:00 - 17:30	Zoom ID: 537 259 5052

For zoom RC:

- you may need to join twice, due to 40-min limit
- I will join only 5 min before class starts, due to 40-min limit

if you have questions.

- I prefer:
  Raise hand, then speak > public message > private message

For TA's job:

- ZHU Yilun - RC
- CHEN Ling, LI Zhipeng, YUAN Shuai - all the other staff, including Homework, Quiz, Lab, ...
- Please contact by email, rather than via Wechat

General Advice

- To me, this course seems like an (Applied) Math course, therefore:
  - Don't get lost in Math, think about physical meaning (of the concept)
  - Live with ambiguousness, don't treat it as a "Theoretical" Math course
- The course "Signals and Systems" on MIT Open Courseware by Prof. Alan V. Oppenheim's is highly recommended
  - Personally, recommend Video Lecture > Textbook
- This course is inspiring because it provides a different view
  - When I first study this course, the application to *Communication Systems* is really fascinating to me. chap. 8
  - After working as TA and reviewed all the contents again, I realized that this course is full of brilliant ideas.
  - To RC: I hope, at least, tell you the points that attracted me most
- Ever think of why this course is titled "Signals and Systems"?

system

$x(t) \rightarrow \boxed{T} \rightarrow y(t)$

input / output signal

Even and Odd

This RC: study signal

inspiring . ∵ decompose any signal into odd & even components

Theorem (Even and odd components)

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Energy and Power

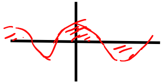
Imagine:

e.g.:

- Do not cram. Remember with the help of graph. (Eg.: power consumed by a resistor)
- Average value:

$\frac{\text{integral}}{\text{time}}$

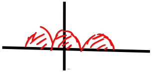
$$A = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt$$



- Energy (remember the square):

$\approx \int_{-\infty}^{\infty} \frac{v(t)^2}{R} dt$

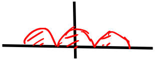
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



- Average power:

$\approx \frac{\text{Energy}}{\text{time}}$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$



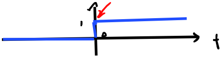
- Energy Signal, Power Signal

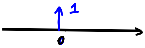
$E < \infty$ $E = \infty$, P finite

Singularity Functions

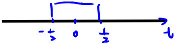
$\frac{d}{dt}u(t)|_{t=0} = \infty$ here, but for engineering purpose, denote $\delta(t) = \frac{d}{dt}u(t)$.

$= 0$ elsewhere





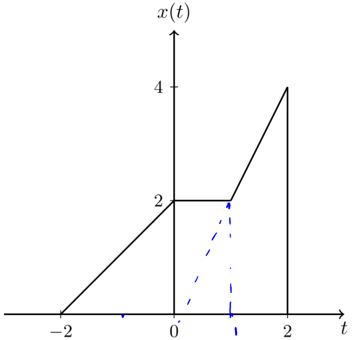
- Unit Step Function: $u(t)$: "turn on at $t=0$"
    - $\delta(t) = \frac{d}{dt}u(t)$
- Rectangle Function:
    - $rect(t)$
    - $rect(t) = rect(-t)$
    - $rect(\frac{t-t_0}{T})$ is centered at t_0 and with width T



; if $rect(\underbrace{-t-2}_{a}) = rect(a) = rect(-a) = rect(t+2)$

- Skill: Using these functions to represent piecewise functions — Q4,9

Exercise: Q4(a)

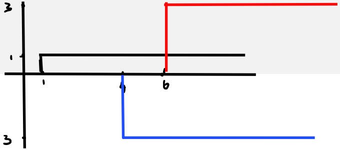


(a) Find a mathematical representation for $x(t)$.

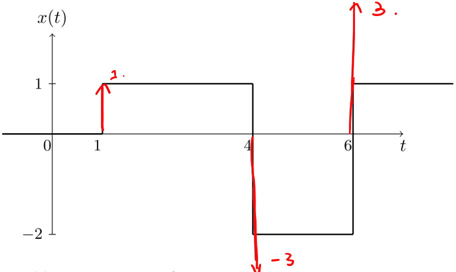
$$x(t) = (t+2)\cdot \text{rect}\left(\frac{t+1}{2}\right) + 2\cdot \text{rect}\left(t-\frac{1}{2}\right) + 2t\cdot \text{rect}\left(t-\frac{3}{2}\right)$$ — a more elegant representation

previous. you may consider $x(t) = \begin{cases} \vdots \end{cases}$

Exercise: Q9



9. [4!] Consider the signal illustrated below.



(a) Express the signal $x(t)$ using a sum of step functions.

(b) Find the derivative of the signal and carefully sketch it.

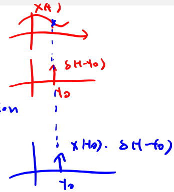
(a). $x(t) = u(t-1) - 3 \cdot u(t-4) + 3 \cdot u(t-6)$

(b) sol1: $\frac{d}{dt} x(t) = \delta(t-1) - 3\delta(t-4) + 3\delta(t-6)$

sol2: using graph

Singularity Functions

- Unit Impulse Function: $\delta(t)$, defined by properties.



  - sampling property — function

two functions — a number × a function

$$x(t)\delta(t-t_0) = \boxed{x(t_0)}\,\delta(t-t_0)$$

  - sifting property — number

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

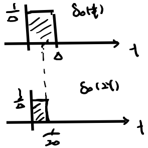
$= \int_{-\infty}^{\infty} x(t_0)\cdot\delta(t-t_0)\,dt = x(t_0)\cdot\int_{-\infty}^{\infty}\delta(t-t_0)\,dt = x(t_0)$

  - scaling property (prove : using area)

"prove by equivalence property". slides.

Here, prove: $\delta_\Delta(2t) = \frac{1}{2}\delta_\Delta(t)$

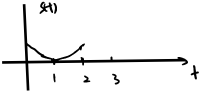
$$\delta(at) = \frac{1}{|a|}\delta(t)$$



$\Rightarrow$ Area: $\int \delta_\Delta(2t) = \int \frac{1}{2}\delta_\Delta(t)$ $\Rightarrow$ $\delta(2t) = \frac{1}{2}\delta(t)$

Exercise

$$\int_{-\infty}^{\infty} x(t)\cdot \delta(t-t_0)dt = x(t_0)$$



13. [6!] Let $s(t) = (\frac{t-1}{2})^2 rect(\frac{t-1}{2})$

(a) Make a sketch of s(t).

(b) Evaluate $\int_{-\infty}^{\infty} s(t)x(t)dt$, where $x(t) = \delta(t-\frac{1}{2}) + \delta(t-2) - \delta(3t-4)$. $= \frac{1}{3}\delta(t-\frac{4}{3})$

$$= \int_{-\infty}^{\infty} s(t)\cdot\left[\delta(t-\tfrac{1}{2}) + \delta(t-2) - \tfrac{1}{3}\delta(t-\tfrac{4}{3})\right] dt$$

$$= s(\tfrac{1}{2}) + s(2) - \tfrac{1}{3}s(\tfrac{4}{3})$$

$$= \ldots$$

Transformation of Signals

$s(t) = x(\frac{t}{w}) \rightarrow y(t) = s(t - t_0) = x(\frac{t-t_0}{w})$

Theorem (Time transformation)

1) $x(\frac{t-t_0}{w})$ 2) $x(at-b)$

For Graph:

1) First scale according to w, then shift according to t_0

2) First time-delay by b, then time-scale by a

Wait... The word "Transformation" sounds familiar?

Yes. Transformation of signals is performed by systems! ⇒ Now you see the connection!

talked in very beginning in class

talked quite late.

Think about the physical meaning: There are two systems, one can shift the time, another can scale the time. Different sequence of connection requires different specification of (w, t_0, a, b) to reach the same effect.

$y(t) = x(\frac{1}{2}t - 3) = x(\frac{t-6}{2})$

$x(t) \rightarrow \boxed{\frac{t}{2}} \rightarrow \boxed{t-6} \rightarrow y(t)$

$x(t) \rightarrow \boxed{t-3} \rightarrow \boxed{\frac{t}{2}} \rightarrow y(t)$

Transformation of Signals

skip. also performed by systems

Theorem (Amplitude transformation)

1) Reversal $y(t) = -x(t)$
2) Scaling $y(t) = ax(t)$
3) Shifting $y(t) = x(t) + b$

General Transformation

$y(t) = x(\frac{t}{2})$ — $y(t) = x(\sin(t))$

- "Time" transformation: $y(t) = x(g(t))$
- "Amplitude" transformation: $y(t) = h(x(t))$ $y(t) = 2x(t)$ — $y(t) = \sin(x(t))$

Consider:

- 1) $y(t) = x(t)$
- 2) $y(t) = x(\sin(t))$
- 3) $y(t) = \cos(x(t))$
- 4) $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

Question:

Think about whether the system that perform such transformation are: "linear, stable; time-invariant, causal, memoryless" in general.

We will come back to these after going through the system properties.

Preview: Systems

x(t) → $\boxed{T}$ → y(t)

- Transform the input signal to the output signal
- Understand the system in terms of input-output relation
- const. system $y(t) = 0$ vs. "non-causal" $y(t) = x(t+1) - x(t+1)$?

future !

∵ No matter what x(t), x(t+1) is, y(t) always 0

- Question from class: what if "x(t)" is non casual?

1st: a system is casual ...

2nd: Imagine: c(t) → $\boxed{T_1}$ → x(t) → $\boxed{T_2}$ → y(t).

non casual · casual

x(t) = c(t+1)

- Digression: physical meaning of knowing $x(t)$, e.g.: $x(t) = \sin(t)$?

x(t) = sin(t) need to know value of x(t) from −∞ to ∞

In real life, only sin(x) · rect($\frac{t-T_0}{w}$) is possible.

But for this course, simply assume x(t) is given. neglect the real-life issues.

Summary

- Singularity functions
- Connection between signals and systems
- 2nd RC will focus on system properties

The End