VE216 Recitation Class 7

ZHU Yilun

UM-SJTU Joint Institute

VE216 SU20 TA Group

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Overview

Before we start

uniquely characterize LTI systems

$h(t)$

Decompose signals: LTI. Freq. domain

- To me, concept of convolution, FS, FT are “theoretically inspiring”
- Now we turn to applications like
  - filtering (Chap. 6)
  - sampling (Chap. 7)
  - communication (Chap. 8)

  which are “practically inspiring”
- What’s even more amazing is that all these applications depend on only $\boxed{\text{two}}$ properties:
  - Convolution Property:

    $x(t) \to \boxed{h(t)} \to *$

    Filtering:

    $$f_1(t) * f_2(t) \overset{\mathscr{F}}{\longleftrightarrow} F_1(\omega) \cdot F_2(\omega)$$

  - Time-domain Multiplication:

    sampling

    $$f_1(t) \cdot f_2(t) \overset{\mathscr{F}}{\longleftrightarrow} \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

    $x(t) \to \otimes \to$

    $p(t)$

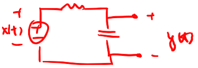
Filtering

x(t) → LTI h(t) → y(t) = h(t) * x(t)

Y(ω) = H(ω) · X(ω)

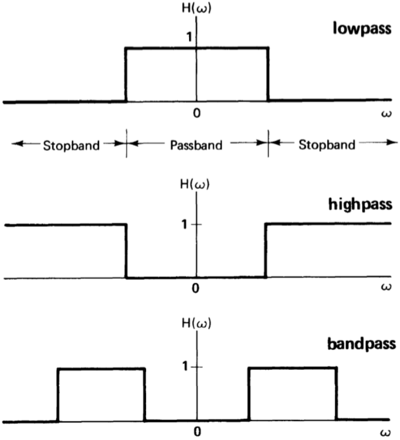
- Convolution Property: $h(t) * x(t) \overset{\mathscr{F}}{\longleftrightarrow} H(\omega)X(\omega)$

Lab 1:



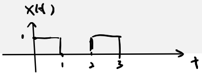
lose "edge"

low pass



- LTI systems can be viewed as "filters" in frequency domain

Exercise - Filtering



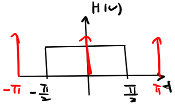
$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \pi.$

Example

The signal $x(t) = \sum_{n=-\infty}^{\infty} \operatorname{rect}(t - 1/2 - 2n)$ is passed through a filter with frequency response $H(\omega) = 3\,\operatorname{rect}(\omega/\pi)$. Determine the output signal $y(t)$.
(Selected from Midterm Exam 2 of Summer 2014)

FS for Periodic signal: $X(\omega) = \sum_k 2\pi \cdot C_k \cdot \delta(\omega - k\omega_0)$

$= \sum_k 2\pi \cdot C_k \cdot \delta(\omega - k \cdot \pi).$

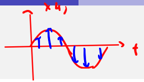


$\Rightarrow$ Only DC term left.

FS table: $c_0 = \frac{1}{2}$

$\Rightarrow Y(\omega) = 2\pi \times \frac{1}{2} \times \delta(\omega) \times 3 = 3\pi\,\delta(\omega) \overset{\mathcal{F}}{\longleftrightarrow} y(t) = \frac{3}{2}$

Overview of DSP



conti: infinite values

VE351

$$x(t) \rightarrow \boxed{\text{A/D}} \rightarrow \boxed{x[n] \rightarrow \boxed{\text{DSP}} \rightarrow y[n]} \rightarrow \boxed{\text{D/A}} \rightarrow y(t)$$

┌ handle finite values

c.f.: analog-circuit

┌ write program

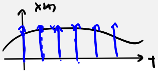
- Why DSP? Only way possible, Computers, low price, etc
- Here we only focus on the left and right part. (DSP will be discussed in VE351)
- We want to show that processing the discrete-time signal (the samples) is equivalent to processing the continuous-time signal (the initial signal). - i.e.: we don't need the all the infinite values ⇒ therefore DSP with confident
- Time-domain Multiplication:

$$f_1(t) \cdot f_2(t) \overset{\mathscr{F}}{\longleftrightarrow} \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

$x(t) \rightarrow \otimes \rightarrow x_p(t)$

$p(t)$

Sampling Theorem



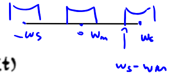
later, when see $\sum \delta(t-nT)$, think of sampling

Sampling Theorem

- Equally Spaced Samples of $x(t)$

  $x(nT) \quad n = 0, \pm 1, \pm 2 \ldots$

periodic



$$x(t) \longrightarrow \otimes \longrightarrow x_p(t), \quad p(t)$$

$$x_p(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT) \quad \Big| \quad x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$$

$$= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

- $X(t)$ Band limited ← composed of only low freq.

  $X(\omega) = 0 \quad |\omega| > \omega_M$

$$X_p(\omega) = \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right] \quad \Big| \quad x(t) * \delta(t - t_0) = x(t - t_0)$$

If $\frac{2\pi}{T} \triangleq \underline{\omega_s} > 2\omega_M$ ← sample fast enough

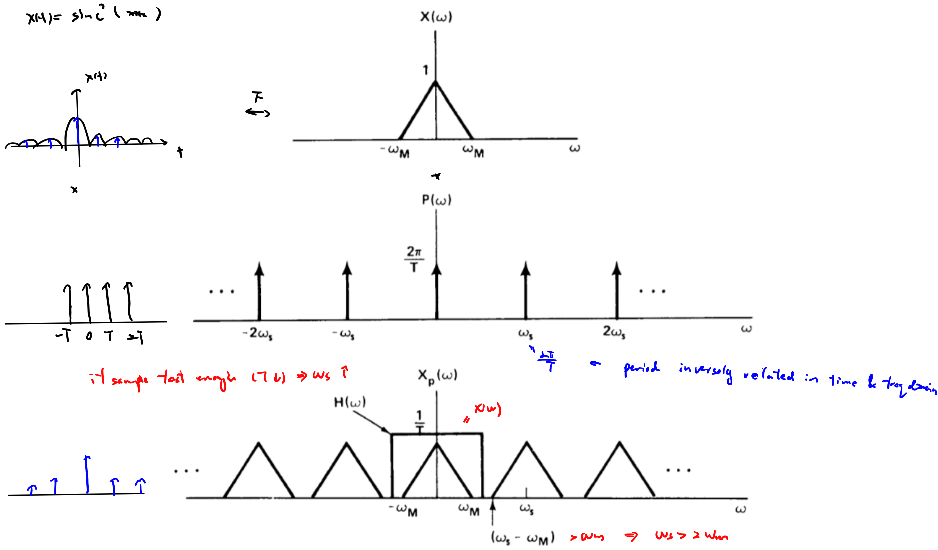
$$p(t) \xleftrightarrow{F} P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$ ← see previous or check 7-1 table

Then $x(t)$ uniquely recoverable

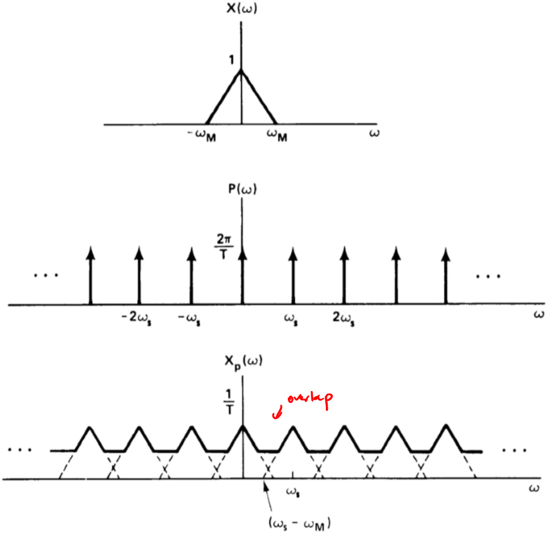
↑ recover from samples

$$X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X\left(\omega - k\underbrace{\boxed{\frac{2\pi}{T}}}_{\omega_s}\right) = \frac{1}{T}\left(X(\omega) + X(\omega - k\omega_s) + \cdots\right)$$

Recoverable

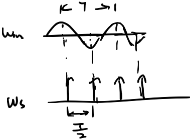


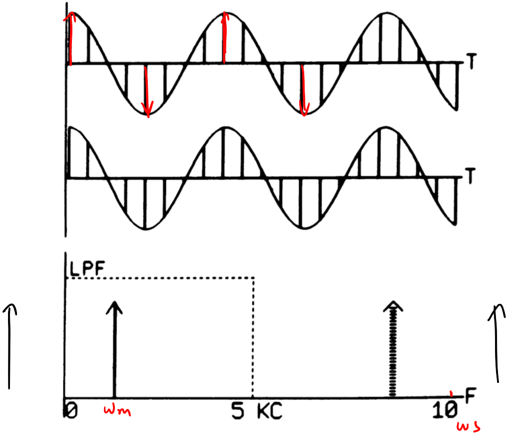
Aliasing

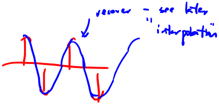


Nyquist rate - interpretation

$w_s \geq 2 w_m$: $x_s(t) = \sum_n x(n \cdot T_s) \cdot \delta(t - nT_s)$



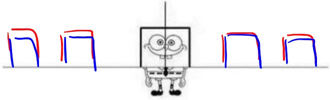




- Interpretation: at least 2 samples in a period — No, as long as $x(t)$ band limited
- Information lost during sampling? Consider const. signal. — 1 sample enough

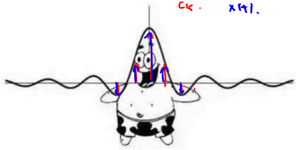
Another way to understand Sampling: relation with FS

x(t) X(ω).



t ω.

ck. x(t).



t

Time domain Freq. domain.

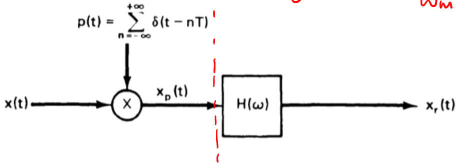
chap.4. Patrick Star $\overset{FT}{\longleftrightarrow}$ SpongeBob SquarePants ← lowpass filter.

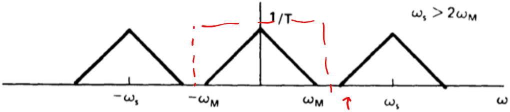
chap.3 Periodic SpongeBob SquarePants $\overset{FS/FT}{\longleftrightarrow}$ Samples of Patrick Star

chap.7 Samples of Patrick Star $\overset{FT}{\longleftrightarrow}$ Periodic SpongeBob SquarePants

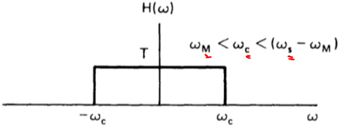
Sampling & Reconstruction

ω_m vs ω_s vs ω_c





$\omega_s - \omega_m$.



cut off freq.

Conclusion

$\omega_s \geq 2\omega_m$?

- The concept of sampling itself is motivating - consider eye (watching the wheels) and ear (ultrasonic)
- Sampling is closely related with reconstruction, which will be the focus of next week.
- For sampling-related problems, I prefer to view them graphically (often in freq. domain).
- Questions in HW5 are very interesting (or, tricky) - not much math involved, rather requires understanding

The End