VE216 Recitation Class 5

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VE216 SU20 Teaching Group

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Overview

1. Chapter 3: Fourier Series
   - Properties
   - Common FS Pairs
2. Chapter 4: Fourier Transform
   - FS vs FT
   - Common FT Pairs
   - Properties
3. Summary

Properties

- Understand $e^{j\theta}$
- Time shift:

$$x(t - t_0) \longleftrightarrow c_k e^{-jk\omega_0 t_0}$$

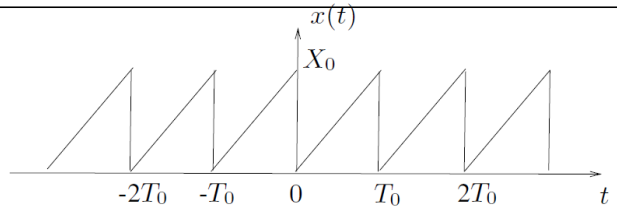
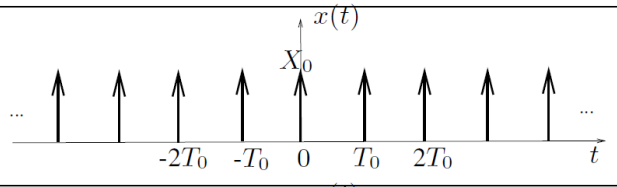
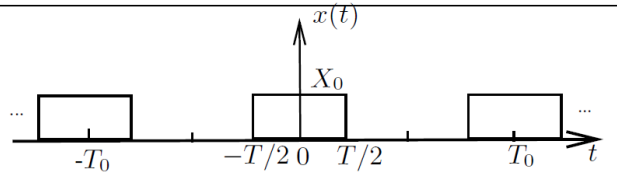
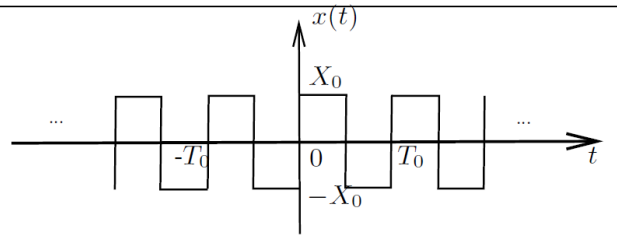
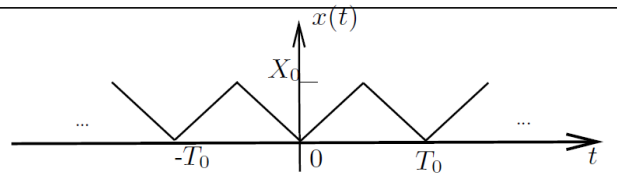
- Differentiation:

$$y(t) = \frac{d}{dt}x(t) \longleftrightarrow jk\omega_0 c_k$$

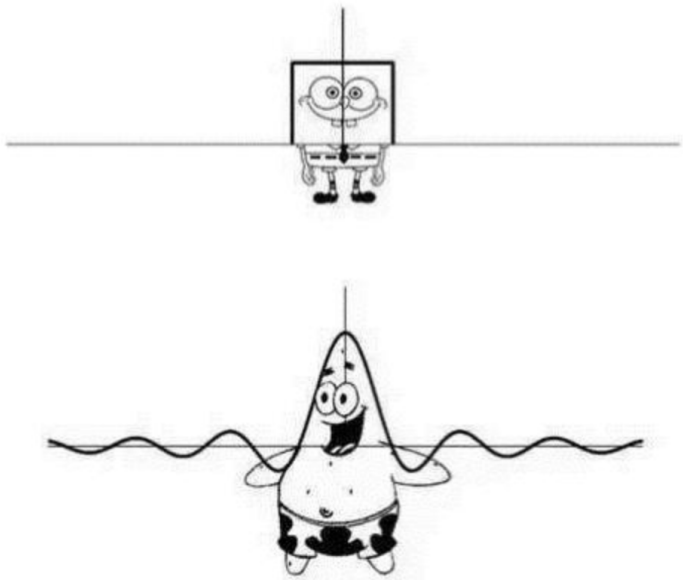
- Etc ...

Common FS Pairs

Table of Fourier Series for Common Signals

Name	Waveform	c_0	$c_k, k \neq 0$	Comments
Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi k}$	
Impulse train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	
Rectangular wave		$\frac{TX_0}{T_0}$	$\frac{TX_0}{T_0}\,\mathrm{sinc}\left(\frac{Tk\omega_0}{2\pi}\right)$	$\frac{Tk\omega_0}{2\pi} = \frac{Tk}{T_0}$
Square wave		0	$-j\frac{2X_0}{\pi k}$	$c_k = 0$, k even
Triangular wave sine		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi k)^2}$	$c_k = 0$, k even

To help remember

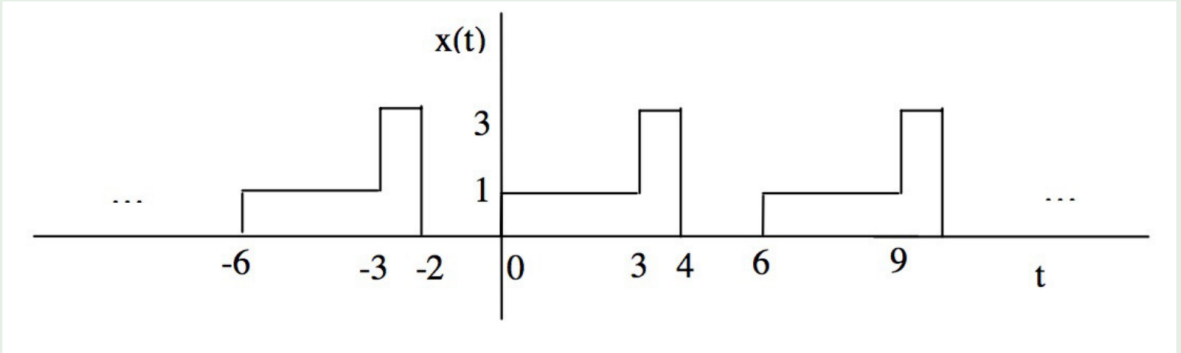


SpongeBob SquarePants $\overset{FT}{\longleftrightarrow}$ Patrick Star

Periodic SpongeBob SquarePants $\overset{FS}{\longleftrightarrow}$ Samples of Patrick Star

Exercise - Use Table Lookup: Quiz3

Find the Fourier series for the following signal [10]. Also, sketch the approximation if a large number of terms are kept in the series (say $N = 40$) [10].



FS vs FT: definition

Fourier Series: for periodic signals

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_o t} \quad \text{synthesis}$$

$$a_k = \frac{1}{T_o} \int_{T_o} x(t) e^{-jk\omega_o t} \quad \text{analysis}$$

Fourier Transform: for "all" signals, often aperiodic

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega \quad \text{synthesis}$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \text{analysis}$$

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

$$X(\omega) = Re\{X(\omega)\} + j\, Im\{X(\omega)\}$$
$$= |X(\omega)| e^{j\measuredangle X(\omega)}$$

FS vs FT: for periodic signal

FOURIER TRANSFORM OF A PERIODIC SIGNAL $\tilde{x}(t)$

$\tilde{x}(t) \longleftrightarrow a_k$ — Fourier series coefficients

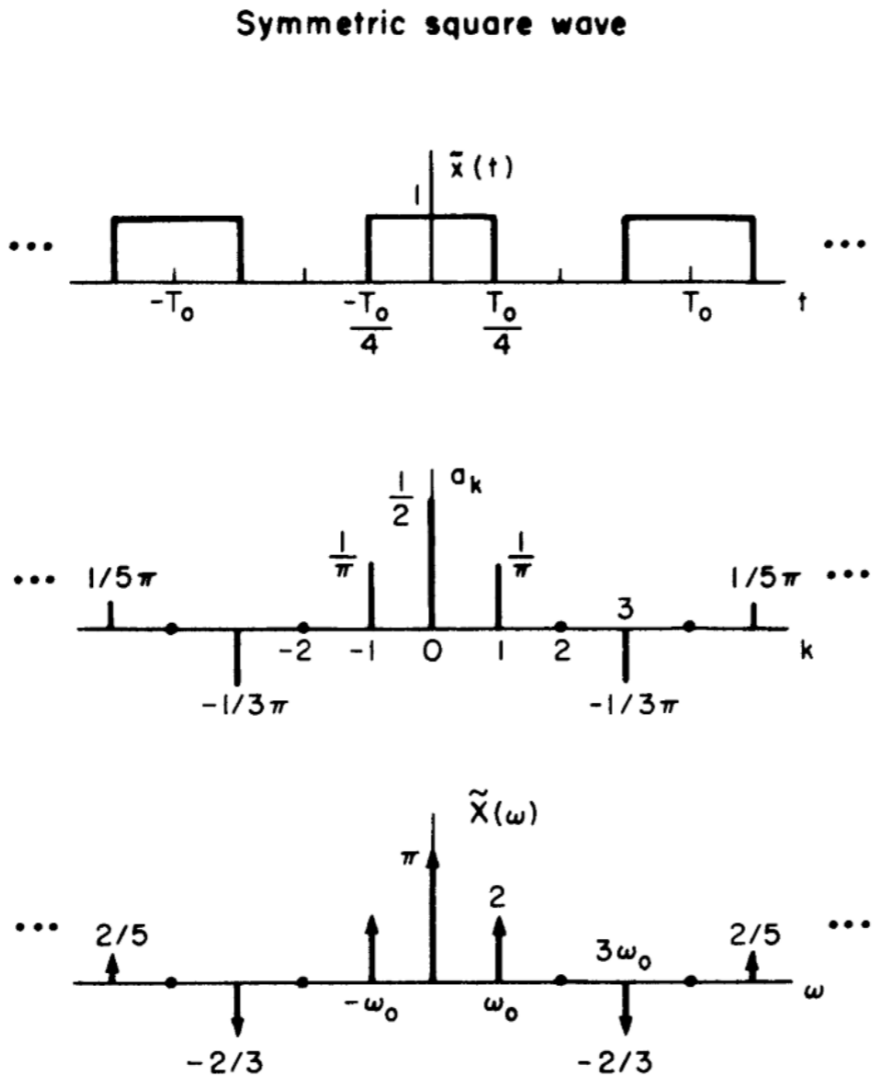
$\tilde{x}(t) \overset{\mathcal{F}}{\longleftrightarrow} \tilde{X}(\omega)$ — Fourier transform

$$\tilde{X}(\omega) \triangleq \sum_{k=-\infty}^{+\infty} 2\pi a_k \, \delta(\omega - k\omega_o)$$

$$\tilde{x}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{X}(\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} 2\pi a_k \underbrace{\int_{-\infty}^{+\infty} \delta(\omega - k\omega_o) e^{-j\omega t} d\omega}_{e^{-jk\omega_o}}$$

FS vs FT: for periodic signal - Example



FS vs FT: for periodic signal - Quiz4 Q1

[10] Find the FT of the following signal: $x(t) = \sum_{n=-\infty}^{\infty} 2\delta(t-6n) - \delta(t-6n-2) - \delta(t-6n+2)$. sketch the magnitude of the spectrum.

Common FT Pairs

$$\delta(t) \overset{\mathscr{F}}{\longleftrightarrow} 1$$

$$\delta(t - t_0) \overset{\mathscr{F}}{\longleftrightarrow} e^{-jk\omega t_0}$$

$$1 \overset{\mathscr{F}}{\longleftrightarrow} 2\pi\delta(\omega)$$

$$e^{j\omega_0 t} \overset{\mathscr{F}}{\longleftrightarrow} 2\pi\delta(\omega - \omega_0)$$

$$\cos(\omega_0 t) \overset{\mathscr{F}}{\longleftrightarrow} \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

Properties

- Time shift:
$$f(t-\tau) \overset{\mathscr{F}}{\longleftrightarrow} F(\omega)e^{-j\omega\tau}$$
- Time reversal:
$$f(-t) \overset{\mathscr{F}}{\longleftrightarrow} F(-\omega)$$
- Time Differentiation:
$$\frac{d^n}{dt^n}f(t) \overset{\mathscr{F}}{\longleftrightarrow} (j\omega)^n F(\omega)$$
- Freq. Differentiation:
$$(-jt)^n f(t) \overset{\mathscr{F}}{\longleftrightarrow} \frac{d^n}{d\omega^n}F(\omega)$$
- Hermitian: If f(t) real, then
$$F(-\omega) = F^*(\omega)$$

Exercise - Use FT Table

HW4 Q3

Find the FT of $t^2 e^{-(t/2)^2}$

Hint:

- $e^{-bt^2} \overset{\mathscr{F}}{\longleftrightarrow} \sqrt{\pi/b} e^{-\omega^2/4b}$
- $(-jt)^n f(t) \overset{\mathscr{F}}{\longleftrightarrow} \frac{d^n}{d\omega^n} F(\omega)$

Summary

- FS vs FT
- FS Table, FT Table
- Next week:
  - Closure of FT
  - Focus on Filtering (problem-solving)
- The place we are in the big picture
- Actually, we will spend a whole month on the applications related to FS and FT

The End