VE216 Recitation Class 7

ZHU Yilun

UM-SJTU Joint Institute

VE216 SU20 TA Group

2020 Summer

Overview

Before we start

- To me, concept of convolution, FS, FT are "theoretically inspiring"
- Now we turn to applications like
  - filtering (Chap. 6)
  - sampling (Chap. 7)
  - communication (Chap. 8)

  which are "practically inspiring"
- What's even more amazing is that all these applications depend on only two properties:
  - Convolution Property:

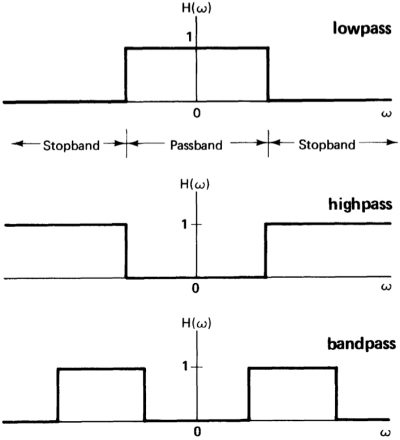
$$f_1(t) * f_2(t) \stackrel{\mathscr{F}}{\longleftrightarrow} F_1(\omega) \cdot F_2(\omega)$$

  - Time-domain Multiplication:

$$f_1(t) \cdot f_2(t) \stackrel{\mathscr{F}}{\longleftrightarrow} \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

Filtering

- Convolution Property: $h(t) * x(t) \overset{\mathscr{F}}{\longleftrightarrow} H(\omega)X(\omega)$



- LTI systems can be viewed as “filters” in frequency domain

Exercise - Filtering

Example

The signal $x(t) = \sum_{n=-\infty}^{\infty} \mathrm{rect}(t - 1/2 - 2n)$ is passed through a filter with frequency response $H(\omega) = 3\,\mathrm{rect}(\omega/\pi)$. Determine the output signal $y(t)$.
(Selected from Midterm Exam 2 of Summer 2014)

Overview of DSP

$$x(t) \rightarrow \boxed{\text{A/D}} \rightarrow \boxed{x[n] \rightarrow \boxed{\text{DSP}} \rightarrow y[n]} \rightarrow \boxed{\text{D/A}} \rightarrow y(t)$$

- Why DSP? Only way possible, Computers, low price, etc
- Here we only focus on the left and right part. (DSP will be discussed in VE351)
- We want to show that processing the discrete-time signal (the samples) is equivalent to processing the continuous-time signal (the initial signal).
- Time-domain Multiplication:

$$f_1(t) \cdot f_2(t) \overset{\mathscr{F}}{\longleftrightarrow} \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

Sampling Theorem

Sampling Theorem

- Equally Spaced Samples of $x(t)$

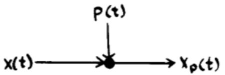
$$x(nT) \quad n = 0, \pm 1, \pm 2 \ldots$$

- $x(t)$ Band limited

$$X(\omega) = 0 \quad |\omega| > \omega_M$$

If $\frac{2\pi}{T} \triangleq \omega_s > 2\omega_M$

Then $x(t)$ uniquely recoverable



$$x_p(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

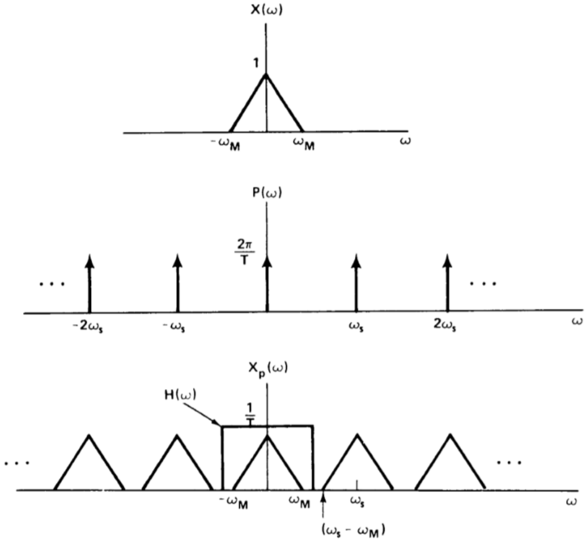
$$= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

$$X_p(\omega) = \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right]$$

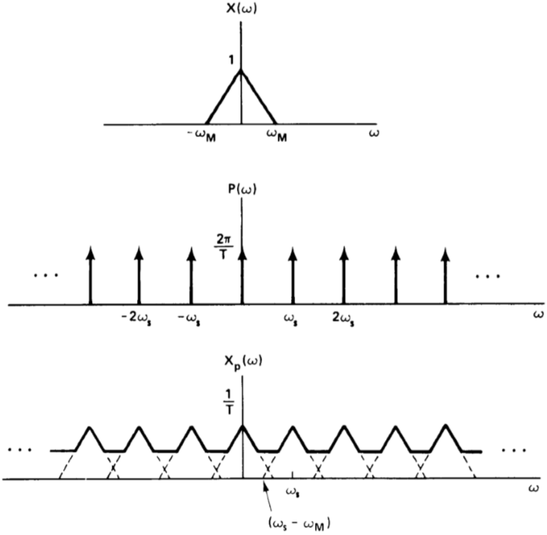
$$P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X\left(\omega - k\frac{2\pi}{T}\right)$$

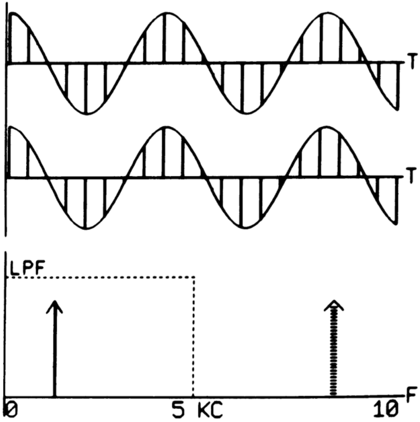
Recoverable



Aliasing

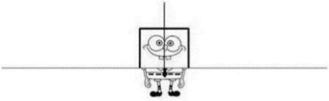


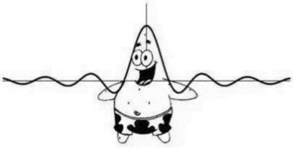
Nyquist rate - interpretation



- Interpretation: at least 2 samples in a period
- Information lost during sampling? Consider const. signal.

Another way to understand Sampling: relation with FS



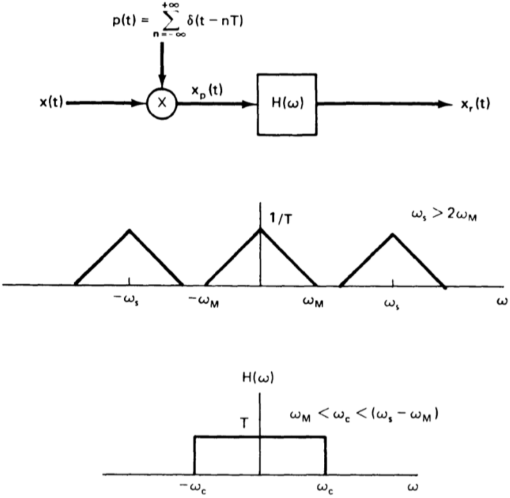


$$\text{Patrick Star} \overset{FT}{\longleftrightarrow} \text{SpongeBob SquarePants}$$

$$\text{Periodic SpongeBob SquarePants} \overset{FS/FT}{\longleftrightarrow} \text{Samples of Patrick Star}$$

$$\text{Samples of Patrick Star} \overset{FT}{\longleftrightarrow} \text{Periodic SpongeBob SquarePants}$$

Sampling & Reconstruction



Conclusion

- The concept of sampling itself is motivating - consider eye (watching the wheels) and ear (ultrasonic)
- Sampling is closely related with reconstruction, which will be the focus of next week.
- The place we are in the big picture.
- For sampling-related problems, I prefer to view them graphically (often in freq. domain).

The End