VE216 Recitation Class 2

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VE216 SU20 Teaching Group

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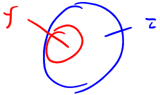
Overview

1. Chapter 1: Signals and Systems
   - Properties of Systems
   - Transformation of Signals

2. Summary

Systems



$x(t) \rightarrow \boxed{T} \rightarrow y(t)$

- Transform the input signal to the output signal
- Transformation is more general than function composition (which is pointwise)

$g \circ f(x) = g(f(x))$

Consider: 1) $y(t) = 2x(t)$ $\Rightarrow y(x(t)) = 2x(t)$ 2) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ $\Rightarrow y(x(t)) = ?$

- invertible system vs. bijective (ve203) : similar
- Understand the system in terms of input-output relation
- const. system $y(t) = 0$ vs. "future" $y(t) = x(t+1) - x(t+1)$?

$x(t) \rightarrow \boxed{T} \rightarrow y(t)$

$x(t) = t \rightarrow y(t) = 0$

$x(t) = \sin t \rightarrow y(t) = 0$

Amplitude properties

$\sum_k a_k \cdot x_k(t) \rightarrow \sum_k a_k \cdot y_k(t)$

- linearity: zero in $\rightarrow$ zero out
  - $x(t) = 0, \forall t$ $\Rightarrow$ $y(t) = 0 \quad \forall t.$ not $x(t) = 7 \quad \forall t = T_0 \Rightarrow \cdots$ shown later
  - $y(t) = 2x(t) + 1$
- stability: bounded in $\rightarrow$ bounded out
  - should assume $x(t)$ bounded
  - $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$ $\leftarrow$ let $x(t) = u(t) \Rightarrow y(t) = t \cdot u(t) \rightarrow \infty.$ as $t \rightarrow \infty.$
- invertibility: each output signal correspond to only one input signal
  - $y(t) = x^2(t)$

  $y(t)$: $-x(t)$, $x(t)$

Time properties: if no given formula

1st consider: casual & TI

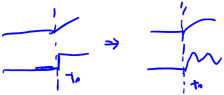
understand generally

- causality: output depends only on the "present" or "past" inputs
- time invariant: if $x(t) \to y(t)$, then $\underset{\text{delayed input}}{x(t-t_0)} \to \underset{\text{delayed output}}{y(t-t_0)}$

14. [2!] Show that causality for a continuous-time linear system is equivalent to the following statement:

For any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

$\Rightarrow$: if casual & Linear, then ...

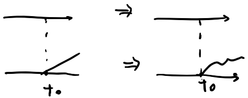


- casual (by def): $y(t_0)$ depends on $x(t)$, $t \le t_0$

$\begin{cases} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \end{cases}$. If $x_1(t) = x_2(t)$ for $t \le t_0$ | then $y_1(t_0) = y_2(t_0)$ $\xrightarrow{\text{for } \forall t = t_0}$ $y_1(t) = y_2(t)$ for $t \le t_0$

- casual + Linear :

linear :



← zero in, zero out

← ∵ depend on "past"

$\Rightarrow$ $x(t) = 0 \ \forall t \le t_0 \Rightarrow y(t) = 0 \ \forall t \le t_0$

$\Leftarrow$: skip : linear + ... $\Rightarrow$ casual .

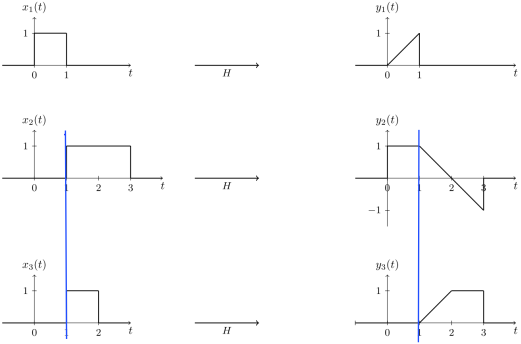
Exercise: Type 1 - Given Input-Ouput Pairs

in general, 2 types

can only show not xxx. ∵ no formular

only a few pairs

Different from HW1 Q11

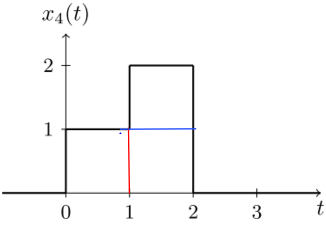


- Is it casual? No, compare $x_2(t)$ and $x_3(t)$.
- Is it linear? Maybe, may need zero in pair, or other pairs to know
- If linear, is there other ways to determine casuality? 逆否. if linear + casual, then $x_2(t)$...

Exercise

Conti.

- If linear, what is the output of $x_4(t)$?



$x_4(t) = x_1(t) + 2 \cdot x_3(t) \longrightarrow y_4(t) = y_1(t) + 2 \cdot y_3(t)$

cannot : $x_4(t) = x_2(t+1) + x_3(t) \longrightarrow$? $\because$ Not necessarily TI

Time properties

- causality: output depends only on the "present" or "past" inputs
- memoryless: output depends only on the "present" input $x(t)$
- time invariant: if $x(t) \to y(t)$, then $x(t - t_0) \to y(t - t_0)$
- Notice that the input refers to "$x(t)$", while there can be other terms relating to "t"

$y(t) = \dfrac{e^{x(t)}}{\sqrt{|t+1|}}$ current input

future time

- casual : ✓ $x(t)$
- memoryless : ✓ $x(t)$
- TI : ✗ , ∵ $|t+1|$

$x_d(t) = x(t+1) \to y_d(t) = \dfrac{e^{x(t+1)}}{\sqrt{|t+1|}}$

but. $y(t+1) = \dfrac{e^{x(t+1)}}{\sqrt{|t+2|}}$

Transformation of Signals

Theorem (Time transformation)

1) $x(\frac{t-t_0}{w})$ 2) $x(at-b)$

For Graph:
1) First scale according to w, then shift according to t_0
2) First time-delay by b, then time-scale by a

Think about the physical meaning: There are two systems, one can shift the time, another can scale the time. Different sequence of connection requires different specification of (w, t_0, a, b) to reach the same effect.

$y(t) = x(\frac{1}{2}t-3) = x(\frac{t-6}{2})$

$x(t) \rightarrow \boxed{\frac{1}{2}t} \rightarrow \boxed{t-6} \rightarrow y(t)$

$x(t) \rightarrow \boxed{t-3} \rightarrow \boxed{\frac{1}{2}t} \rightarrow y(t)$.

Question: for what system, the order doesn't matter? — LTI

if: $x(t) \rightarrow \boxed{\frac{1}{2}t} \rightarrow \boxed{t-3} \rightarrow \neq y(t)$ $\because$ $\boxed{\frac{1}{2}t}$ not TI

Transformation of Signals

Theorem (Amplitude transformation)

1) Reversal $y(t) = -x(t)$
2) Scaling $y(t) = ax(t)$
3) Shifting $y(t) = x(t) + b$

General Transformation

$y(t) = x(\frac{t}{3})$, $y(t) = x(\sin(t))$

- "Time" transformation: $y(t) = x(g(t))$
- "Amplitude" transformation: $y(t) = h(x(t))$ $y(t) = 2x(t)$. $y(t) = \sin(x(t))$

Consider:

- 1) $y(t) = x(t)$
- 2) $y(t) = x(\sin(t))$
- 3) $y(t) = \cos(x(t))$
- 4) $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

Question:

Think about whether the system that perform such transformation are: "linear, stable; time-invariant, causal, memoryless" in general.

We will come back to these after going through the system properties.

Exercise: Type 2 - Given Formula

General Transform - Revisited

- "Time" transformation: $y(t) = x(g(t))$
  — often lead to the violation of Time properties (TI, Casual)
- "Amplitude" transformation: $y(t) = h(x(t))$
  — often lead to the violation of Amplitude properties (linear)

Guess the result

System	**Time-Invariant**	**Casual**	**Linear**	**Stable**
$y(t) = x(2-t)$	✗	✗	✓	✓
$y(t) = x(t/3)$	✗	✗	✓	✓
$y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$	✗	✗	✓	✗
$y(t) = cos(x(t))$	✓	✓	✗	✓

Summary

- Maybe you were confused about so many concepts and feel boring.
- But at least I hope you could tell signal properties from system properties
- I would be glad if you can see the connection between signals and systems
- The first fascinating result: impulse response will be covered next week.

The End