VE216 Recitation Class

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UM-SJTU Joint Institute

VE216 SP20 Teaching Group

2020 Spring

Overview

Recitation Class Arrangement

10:00 ~ 11:40

No.	Coverage	Date	Join Through
1	Chap.1	Mar.13, Fri.	Canvas (In-class)
2	Chap.2	Mar.19, Thur.	Zoom ID: 869-301-2154
3	Chap.3	Mar.26, Thur.	Zoom ID: 869-301-2154
4	Chap.4	Apr.2, Thur.	Zoom ID: 869-301-2154
5	Chap.6,7	Apr.9, Thur.	Zoom ID: 869-301-2154
6	Chap.8	Apr.16, Thur.	Zoom ID: 869-301-2154
7	Chap.9	Apr.24, Fri.	Canvas (In-class)

Table: RC Arrangement

- CHEN Ling, LI Zhipeng - Homework, Quiz
- ZHU Yilun - RC
- For zoom RC, I prefer:
  Raise hand, then speak > public message > private message

General Advice

- To me, this course seems like an (Applied) Math course, therefore:
  - Don't get lost in Math, think about physical meaning
  - Live with ambiguousness, don't treat it as a "Theoretical" Math course
- The course "Signals and Systems" on MIT Open Courseware by Prof. Alan V. Oppenheim's is highly recommended
  - Personally, recommend Video Lecture > Textbook
- This course is inspiring because it provides a different view
  - When I first study this course, the application to *Communication Systems* is really fascinating to me. Chap. 8 FT ?
  - After working as TA and reviewed all the contents again, I realized that this course is full of brilliant ideas.
  - RL: I hope, at least, tell you the points that attracted me most
- Ever think of why this course is titled "Signals and Systems"?

Input signal System Output signal

$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$

Study signal 1st.

Transformation of Signals

$y(t) = x(\frac{1}{2}t - 3) = x(\frac{t-6}{2})$

1): $x(t) \rightarrow \boxed{\frac{t}{2}} \rightarrow \boxed{t-6} \rightarrow y(t) = x(\frac{t-6}{2})$

2): $x(t) \rightarrow \boxed{t-3} \rightarrow \boxed{\frac{t}{2}} \rightarrow y(t) = x(\frac{t-6}{2})$

$s(t) = x(\frac{t}{w}) \rightarrow y(t) = s(t - t_0) = x(\frac{t-t_0}{w})$

Theorem (Time transformation)

1) $x(\frac{t-t_0}{w})$ 2) $x(at-b)$

For Graph:

1) First scale according to w, then shift according to t_0
2) First time-delay by b, then time-scale by a

Wait... The word "Transformation" sounds familiar?
Yes. Transformation of signals is performed by systems!

Think about the physical meaning: There are two systems, one can shift the time, another can scale the time. Different sequence of connection requires different specification of (w, t_0, a, b) to reach the same effect.

Transformation of Signals

Theorem (Amplitude transformation)

1) Reversal $y(t) = -x(t)$
2) Scaling $y(t) = ax(t)$
3) Shifting $y(t) = x(t) + b$

General Transformation

$y(t) = x(\frac{t}{2})$

- "Time" transformation: $y(t) = x(g(t))$ — $y(t) = x(\cos(t))$
- "Amplitude" transformation: $y(t) = h(x(t))$ — $y(t) = \sin(x(t))$

$y(t) = a \cdot x(t)$

Consider:

- 1) $y(t) = x(t)$
- 2) $y(t) = x(\sin(t))$
- 3) $y(t) = \cos(x(t))$
- 4) $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

Question:

Think about whether the system that perform such transformation are: "linear, stable; time-invariant, causal, memoryless" in general.

We will come back to these after going through the system properties.

Even and Odd

Any signal → Odd, Even

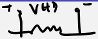
Inspiring:

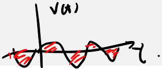
Theorem (Even and odd components)

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Energy and Power





- Do not cram. Remember with the help of graph. (Eg.: power consumed by a resistor)
- Average value:

$$A = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)dt \quad \approx \frac{\text{area/integral}}{\text{time}}$$

- Energy (remember the square): $\int_{-\infty}^{\infty} \frac{\boxed{V(t)^2}}{R} dt$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$\downarrow \frac{\text{Energy}}{\text{time}}.$

- Average power:

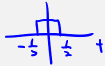
$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

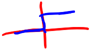
- Energy Signal, Power Signal

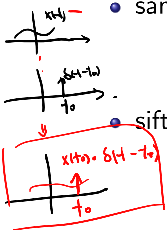
$E < \infty.$ $E = \infty.$ P finite.

Singularity Functions

rect(t) =



- Unit Step Function: $u(t)$  "turn on" at $t=0$
- Rectangle Function: $rect(\frac{t-t_0}{T})$ is centered at t_0 and with width T
- Unit Impulse Function: $\delta(t)$ - defined by property.
  - sampling property — function



$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

  - sifting property — number

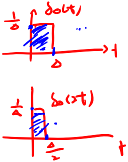
$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

$= \int_{-\infty}^{\infty} x(t_0)\delta(t-t_0)dt = x(t_0)\cdot\int_{-\infty}^{\infty}\delta(t-t_0)dt$ $= x(t_0)$

  - scaling property (prove) $\delta(2t) = \frac{1}{2}\delta(t)$

"prove by equi. property"

$$\delta(at) = \frac{1}{|a|}\delta(t)$$



$\delta_0(t)$, $\delta_0(2t)$

$\int \delta(2t)dt$, $\int \frac{1}{2}\delta(t)dt$

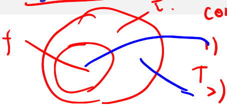
$\approx \delta(2t) = \frac{1}{2}\delta(t)$

- Skill: Using these functions to represent piecewise functions — Q4,9

Systems

$x(t) \to \boxed{T} \to y(t)$

conside:



1) $y(t) = 2x(t) \Rightarrow y(x(t)) = 2x(t)$

2) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau \neq y(x(t))$?

- Transform the input signal to the output signal
- Transformation is more general than function composition (which is pointwise) $g \circ f$
- invertible system vs. bijective (ve203) system function
- Understand the system in terms of input-output relation
- const. system $y(t) = 0$ vs. "non-causal" $y(t) = x(t+1) - x(t+1)$?

$y(t) = 0$

$\boxed{= x(t+1) - x(t+1)}$

future

$t=0$: $y(0) = x(1) - x(1)$

$x(t) \to \boxed{0}$ $y(t) = 0$.

Amplitude properties

$x(t)=0$ $\forall t$ $\qquad$ $y(t)=0$ $\forall t$ $\qquad$ X: $x(t)=0$ $\forall t\le t_0 \Rightarrow y(t)=0$ $\forall t\le 0$.

- linearity: zero in $\to$ zero out
- stability: bounded in $\to$ bounded out
- invertibility: each output signal correspond to only one input signal
- consider: $y(t) = 2x(t) + 1$, $\quad y(t) = x^2(t)$ $\quad$ - X inv.

non-linear.

$y(t)$: $-x(t)$, $+x(t)$

Time properties

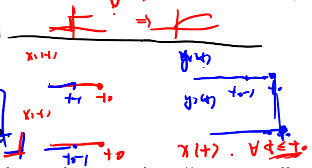
Tough:

1) If the formula is given: $y(t) = 2x(t)$

2) If only some input-output pairs are given:

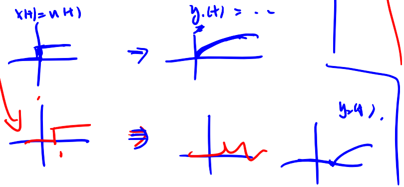
$x_1(t) \to y_1(t)$, $x_2(t) \to y_2(t)$

If $x_1(t) = x_2(t)$ $\forall t \le t_0$, then $y_1(t_0) = y_2(t_0)$ $\Rightarrow$ $y_1(t) = y_2(t)$, $\forall t \le t_0$



- causality: output depends only on the "present" or "past" inputs
- memoryless: output depends only on the "present" input $x(t)$
- time invariant: if $x(t) \to y(t)$, then $x(t - t_0) \to y(t - t_0)$
- Notice that the input refers to "$x(t)$", while there can be other terms relating to "t"

$x(t+1)$, $t \in \mathbb{R}$



$y(t) = e^{x(t)} (t+1)$ — memoryless.

$x(t+1)$ future.

System: memoryless

$x_1(t_1) = x_2(t_2)$ $\Downarrow$ $y_1(t_1) = y_2(t_2)$

Exercise

y(t) = x(-t).
delay input by 1 ⇒ output delay is reversed × TI
t = -1 y(-1) = x(1)

General Transform - Revisited

y(t) = x(sin(t))

- "Time" transformation: $y(t) = x(g(t))$
  — often lead to the violation of Time properties (TI, Casual)
- "Amplitude" transformation: $y(t) = h(x(t))$ y(t) = sin(x(t))
  — often lead to the violation of Amplitude properties (linear)

1 - Guess the result

time AM.

System	Time-Invariant	Casual	Linear	Stable
$y(t) = x(2-t)$ (x(g(t)))	✗	✗	✓	✓
$y(t) = x(t/3)$ (t/3)	✗	✗	✓	✓
$y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$ (t/2)	✗	✗	✓	✗
$y(t) = \cos(x(t))$	✓	✓	✗	✓

$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

y(t) = h(x(t)) -1 ~ 1,

Exercise

If a system is L & Casual ⇒ Then if $x(t)=0$ $\forall t \le t_0$, then $y(t)=0$ $\forall t \le t_0$.

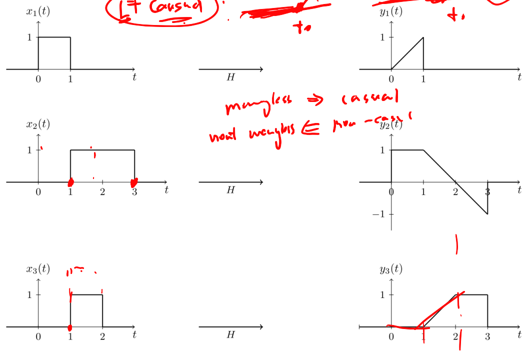
Linear: x $\forall t$ y: $\forall t$

2 - Different from HW1 Q11

zero in → "

LT Causal:

t_0



memoryless ⇒ casual

need weights ∈ non-casual

x(t)

- Is it casual? $x_2(t)$ vs $x_3(t)$.
- Is it linear? Maybe. "zero in → zero out" only $x_2(t)$
- If linear, is there other ways to determine casuality? Most casual ways.

Linear:

Exercise

$x_4(t) = x_1(t) + 2x_3(t)$

$\Downarrow$ L.

$y_4(t) = y_1(t) + 2\cdot y_3(t)$

Conti.

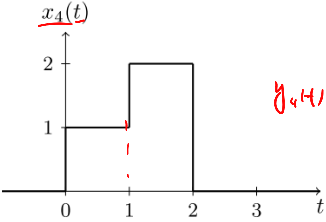
- If linear, what is the output of $x_4(t)$?

$x_4(t) = x_1(t) + 2x_3(t)$

$= x_2(t+1) + x_3(t)$

$\Downarrow$

$y_4(t) = y_2(t+1) + y_3(t)$?



Summary

$x_1(t) \to \boxed{T} \to y_1(t)$

$x_2(t) \to \boxed{T} \to y_2(t)$

L TI, S.

- Maybe you were confused about so many concepts and feel boring.
- But at least I hope you could tell signal properties from system properties

  Linear, TI.. periodic ...

- I would be glad if you can see the connection between signals and systems

  time trans. $\Leftrightarrow$ system ~ properties.

- The first fascinating result will come soon, i.e., impulse response. $h(t)$. LTI
- 2nd RC will be held next Thursday (via zoom, rather than canvas)

  $h(t)$ *

$y(t) = T[x(t)] = t \cdot \boxed{x(t)}$

$x(t)$.

" memory " w.r.t. input $x(t)$

x t.

The End

x

$(t, x(t))$

$y(t)$