VE216 Recitation Class 7

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VE216 SP20 TA Group

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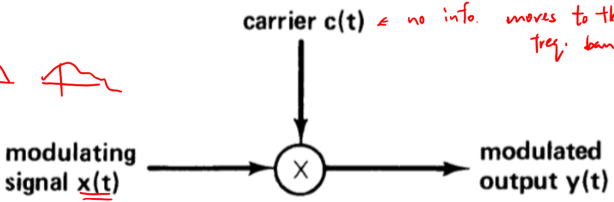
Overview

1. Chapter 8: Communications
   - Sinusoidal Amplitude Modulation (AM) - Synchronous
   - Sinusoidal Amplitude Modulation (AM) - Asynchronous
   - Frequency-division Multiplexing
2. Chapter 9: Laplace Transform
   - Definition
   - Study System Behavior
3. Conclusion

Modulation

- Modulation Property:



no info. moves to the wanted freq. band

carries info.

$$x(t)\,c(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\left[X(\omega) * C(\omega)\right]$$

- pulse carrier — sampling
- sinusoidal carrier : 1 st: $f(t)\cdot\cos(\omega_c t) \leftrightarrow \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$

$$c(t) = \cos(\omega_c t + \theta_c)$$

Sinusoidal Amplitude Modulation

- Block diagram of modulation system:
  $x(t)$ - information, $c(t)$ - carrier

$$x(t) \rightarrow \underset{\substack{\uparrow \\ \cos(\omega_c t + \theta_c)}}{\bigotimes} \rightarrow y(t) \rightarrow \text{ antenna}$$

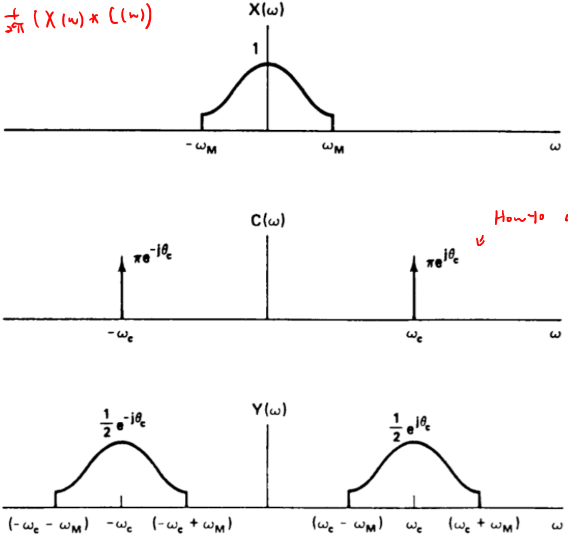
  Notice: here we multiply the carrier signal rather than do convolution
- Transmitted signal (i.e., modulated output $y(t)$): Here, consider strict case

$$y(t) = x(t)c(t) = x(t)\cos(\omega_c t + \theta_c)$$

$$\xleftrightarrow{\mathcal{F}} \quad Y(\omega) = \frac{1}{2}[e^{j\theta_c}X(\omega - \omega_c) + e^{-j\theta_c}X(\omega + \omega_c)]$$

Sinusoidal Amplitude Modulation - Synchronous

$x(t) \cdot c(t) \Leftrightarrow \frac{1}{2\pi}\left(X(\omega) * C(\omega)\right)$

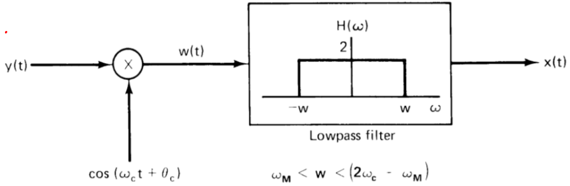


How to draw complex function:
- mag
- phase

> together for simplicity

Synchronous Demodulation

- Block diagram of demodulation system:



- First multiply $y(t)$ by another $cos(\omega_c t + \theta_c)$ signal:

$$w(t) = y(t)\cos(\omega_c t + \theta_c)$$

$$W(\omega) = \frac{1}{2}[e^{j\theta_c} Y(\omega - \omega_c) + e^{-j\theta_c} Y(\omega + \omega_c)]$$

$$= \boxed{\frac{1}{4}e^{2j\theta_c} X(\omega - 2\omega_c) + \frac{1}{2}X(\omega) + \frac{1}{4}e^{-2j\theta_c} X(\omega + 2\omega_c)}$$

- Then followed by lowpass filtering to extract $X(\omega)$

Synchronous Demodulation

$f(t)\cos(w_c t) \leftrightarrow \frac{F(w-w_c)+F(w+w_c)}{2}$

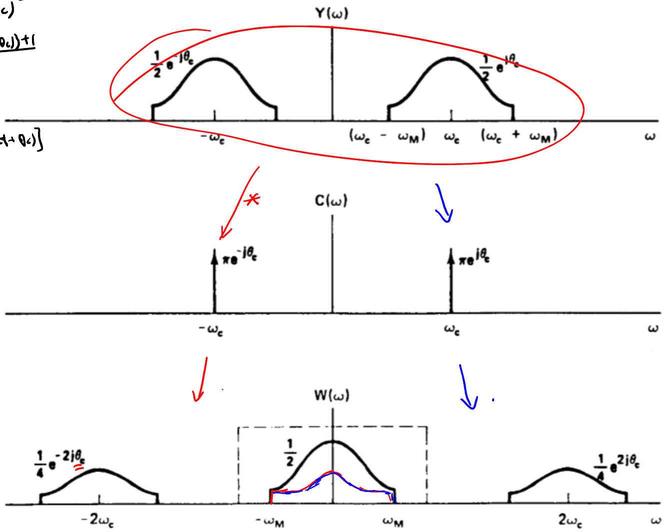
Time domain:

$x(t)\cdot\cos(w_c t+\theta_c)^2$

$= x(t)\cdot\frac{\cos(2(w_c t+\theta_c))+1}{2}$

$= \frac{1}{2}x(t) +$

$\frac{1}{2}x(t)\cdot\cos[2(w_c t+\theta_c)]$



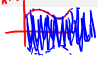
Asynchronous Demodulation: Motivation

consider $\theta_c = 0$.

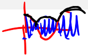
- It seems to be harmless to write the way synchronous Demodulation works on paper, but up to now we haven't considered how to implement it to hardware.
- The bad news is that in practice, the phase θ_c is not available, therefore a sophisticated phase-tracking receiver is needed. ⇒ difficult receiver.
- But for commercial products like AM radio, one would expect the receivers to be simple and inexpensive.
- Therefore a different demodulation scheme is needed, which uses a more complicated and power inefficient transmitter, but a simple receiver.

Asynchronous Demodulation: Modulated signal

$x(t) > 0$

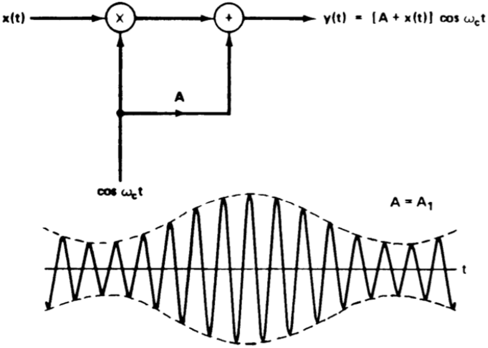


but if



$\emptyset$: $\hat{x}(t) \neq x(t)$

- Now the modulated signal is: $y(t) = (A + x(t))\cos(\omega_c t)$
- Often we choose A greater then the amplitude of $x(t)$
- The block diagram & how the output $y(t)$ looks like:

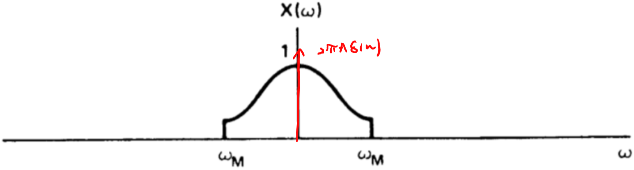


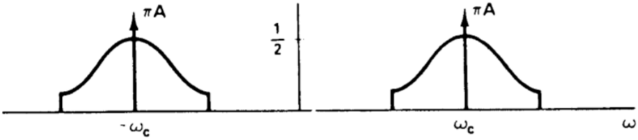
Asynchronous Demodulation: Frequency Domain

- In frequency domain:

$$\begin{cases} x(t)\cdot c(t) \leftrightarrow \dfrac{F(w-w_c)+F(w+w_c)}{2} \\ 1 \leftrightarrow 2\pi\delta(w) \end{cases}$$

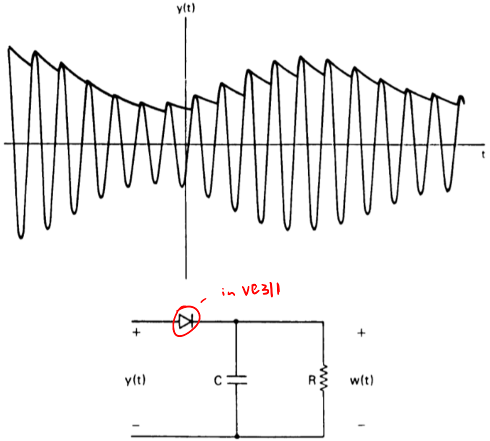
$$Y(\omega) = A\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \frac{1}{2}[X(\omega-\omega_c)+X(\omega+\omega_c)]$$





Asynchronous Demodulation

- Use a simple circuit to detect the envelop: $m(t) = A + \hat{x}(t)$
- It works because ω_c is much higher than frequency of $x(t)$



Asynchronous Demodulation

- The envolope detector gives us:

$$y(t) = (A + x(t))\cos(\omega_c t)$$

$$m(t) = A + \hat{x}(t)$$

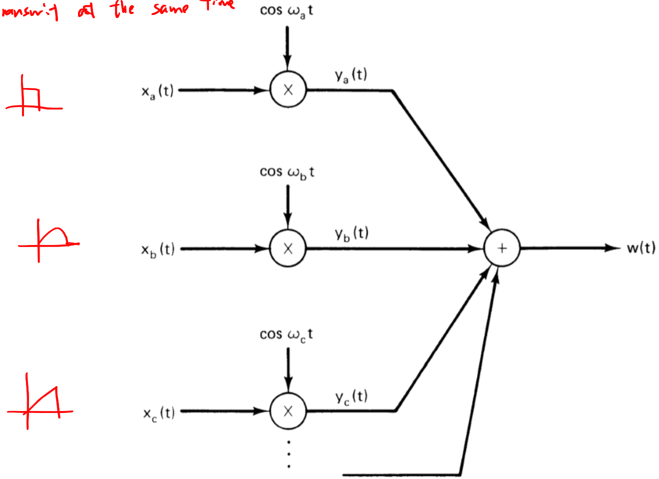
- Then eliminate the DC component (this is what we mean by "power inefficient") and you recover the orginal signal.
- The overall block diagram of demodulation:

$$y(t) \rightarrow \boxed{\text{Envelop detector}} \rightarrow m(t) \rightarrow \boxed{\text{DC blocking filter}} \rightarrow \hat{x}(t)$$

Frequency-division Multiplexing

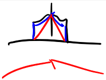
In time domain:

will transmit at the same time



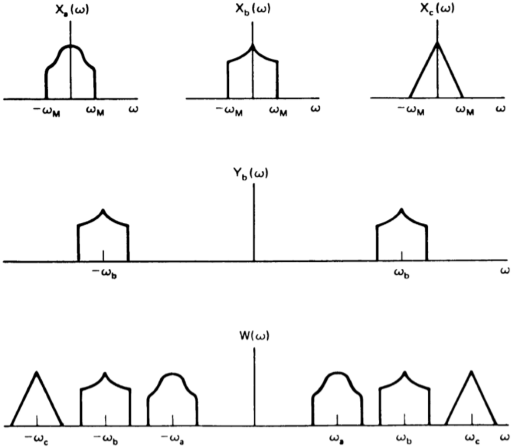
Frequency-division Multiplexing

In frequncy domain:

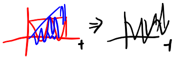


mixed together.

separated ⇒



time domain



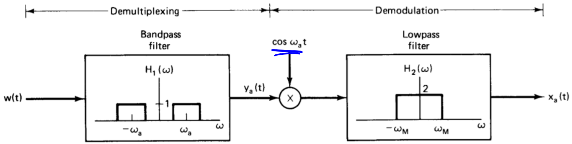
freq. gap large enough

A total mass in time

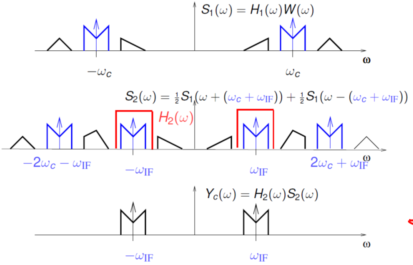
but separated in freq.

Demultiplexing and Demodulation

- synchronous demodulation



- asynchronous demodulation (using IF filter)



⇒ then envelope detector

Exercise

$x(t)\cdot\cos(w_0 t) \leftrightarrow \frac{F(w-w_0)+F(w+w_0)}{2}$

8. We discussed the effect of a loss of synchronization in phase between the carrier signals in the modulator and demodulator in sinusoidal amplitude modulation. We showed that the output of the demodulation is attenuated by the cosine of the phase difference, and in particular, when the modulator and demodulator

$y(t)=x(t)\cdot\cos w_c t$

$w(t)=y(t)\cos w_d t$

Let us denote the difference in frequency between the modulator and demodulator as $\Delta\omega$ (i.e., $\omega_d-\omega_c=\Delta\omega$). Also assume that $x(t)$ is band limited with $X(j\omega)=0$ for $|\omega|\geq\omega_M$, and assume that the cufoff frequency ω_{co} of the lowpass filter in the demodulator satisfies the inequality

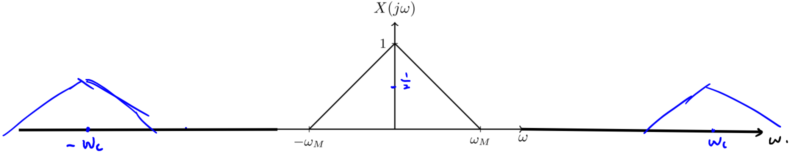
$$\omega_M+\Delta\omega<\omega_{co}<2\omega_c+\Delta\omega-\omega_M$$

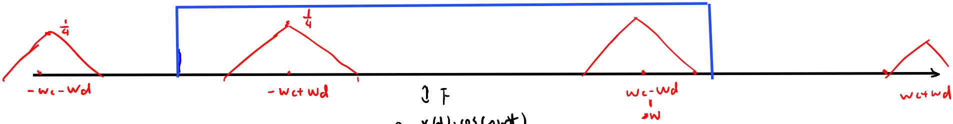
time varying.

(a) [5] Show that the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\underline{\Delta\omega} t)$.

multiply by another cos.

(b) [5] If the spectrum of $x(t)$ is that shown in figure below, sketch the spectrum of the output of the demodulator.





$\sim x(t)\cdot\cos(\Delta w t)$

Conclusion

$x(t) \rightarrow \otimes \rightarrow y(t)$
$c(t)$

$\otimes \rightarrow x(t)$
envelop $\rightarrow x(t)$

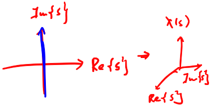
- Prelab2 provides a detailed discussion on Multiplexing
- Get the big picture of modulation; solve problems graphically
- I guess at one time you may complain about why do we have to go through such a painful way just to get $x(t)$.
  why not $x(t) \rightarrow x(t)$. ∵ impossible, transmit at 1 place, receive at another
- But in fact the task is not at all easy, given the constrain of physical laws and hardware implementation.
- To me, the outcomes of these issues are amazing, because Electrical Engineers not only managed to develop a brand new subject based on the fairly abstract mathematical property associated with the Fourier transform, but also turn the theory into real life applications.
  common.
  mod. property.
  all depends on one theorem.
- This course provides a sense of the strong connection between mathematics and the real world.

Laplace Transform

mag = 1

- $s = \sigma + j\omega$, $e^{st} = e^{\sigma t} \cdot e^{j\omega t}$: decaying/growing term and periodic term
- LT Definition:

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

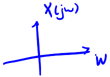


Notice: ROC
Study system behavior } later

- Compare with FT:

$s = \sigma + j\omega$, $\sigma = 0$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$



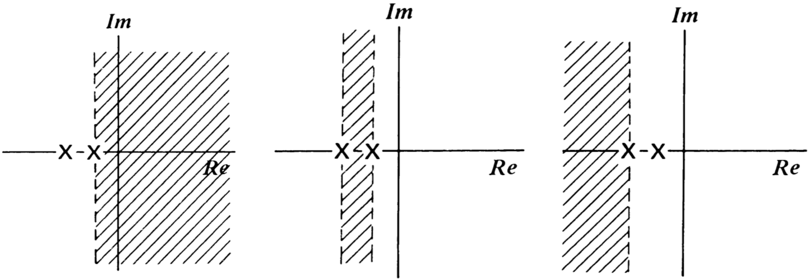
decompose signals; system as filters w.r.t. freq. components.

ROC

$e^{-at} \cdot u(t) \Leftrightarrow \frac{1}{s+a}$ real{s} > real{-a}

$-e^{-at} \cdot u(-t) \Leftrightarrow \frac{1}{s+a}$ real{s} < real{-a}

$$X(s) = \frac{1}{(s+1)(s+2)}$$



Different choice of ROC corresponds to different $x(t)$.

LT - Study System Behavior

$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$

- stable $\Longleftrightarrow$ ROC includes $j\omega$ axis
- casual $\Longleftrightarrow$ ROC RHP

  $h(t) = e^{-t} \cdot u(t) \leftrightarrow \frac{1}{s+1}$:
- (rational) casual and stable $\Longleftrightarrow$ all poles in the left half of s-plane
- Differentiation: solve systems defined by diff. eqn.

$$\frac{d^n}{dt^n}x(t) \overset{\mathscr{L}}{\longleftrightarrow} s^n X(s)$$

$$\frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) = \frac{d^2}{dt^2} x(t)$$

$$\updownarrow$$

$$s^2 Y(s) + 2 Y(s) = s^2 X(s)$$

- Convolution: get output y(t)

$$h(t) * x(t) \overset{\mathscr{F}}{\longleftrightarrow} H(s)X(s) = Y(s)$$

- Block diagram: be able to read as well as draw

Exercise

5. [10] A causal LTI system S with impluse response h(t) has its input x(t) and output y(t) related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t)$$

,

how many poles does G(s) have?

$$\begin{array}{r} s^2+\alpha s+\alpha^2 \\ s+1 \,\big)\, s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2 \\ \underline{s^3+s^2} \\ \alpha s^2+\alpha(\alpha+1)s+\alpha^2 \\ \underline{\alpha s^2+\alpha s} \\ \alpha^2 s+\alpha^2 \\ \underline{\alpha^2 s+\alpha^2} \\ 0 \end{array}$$

Hint: use long division

$s^3 \cdot Y(s) + (1+\alpha)\cdot s^2 \cdot Y(s) + \alpha(\alpha+1)\cdot s \cdot Y(s) + \alpha^2 \cdot Y(s) = X(s)$

$[s^3+(1+\alpha)\cdot s^2 + \alpha(\alpha+1)\cdot s+\alpha^2]\,Y(s) = X(s)$

$\Rightarrow \quad H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2}$

$G(s) = s\cdot H(s) + H(s) = (s+1)\cdot H(s) = \frac{1}{s^2+\alpha s+\alpha^2}$

how to simplify?

Conclusion - for Chap. 9

Vv256/286 : use FS/FT/LT to solve diff. eqn.

periodic — periodic & aperiodic — system (casual. stable)

- FS vs. FT vs. LT
- Focus on system prospective
- Practice on PFE, block diagram, etc

read & draw

$$H(s) = \frac{1}{s^3 + as^2 + bs + 1}$$

$$= \frac{1}{s + r_1} + \frac{1}{s + r_2} + \cdots$$

Conclusion - for the course

$\delta(t) \to h(t)$ $\quad$ $x(t) \to y(t) = x(t) * h(t)$

- LTI system, impulse response, convolution

freq. component $\quad$ filter

- Fourier Analysis - signal, system $\quad$ freq. domain
- Filtering, Sampling, Communication - most interesting topics to me
- Laplace Transform - ROC, system, block diagram

$H(s)$

- This course is one of the most inspiring course I have ever took, as it provides a sense of the strong connection between mathematics and the real world.
- If you are interested in signal processing, consider taking: VE351; VE401, VE501; VE455, VE489; VV214/417

dsp.

random signal $\quad$ Digital comm. $\quad$ comm. network $\quad$ Linear Algebra $\quad$ - "most" important & inspiring.

The End