VE216 Recitation Class 10

ZHU Yilun

UM-SJTU Joint Institute

VE216 SU20 TA Group

2020 Summer

Overview

1. Chapter 9: Laplace Transform
   - Definition
   - Study System Behavior
   - Exercise

2. Conclusion

Laplace Transform

- $s = \sigma + j\omega, e^{st} = e^{\sigma t} \cdot e^{j\omega t}$: decaying/growing term and periodic term
- LT Definition:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

  Notice: ROC
  Study system behavior
- Compare with FT:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

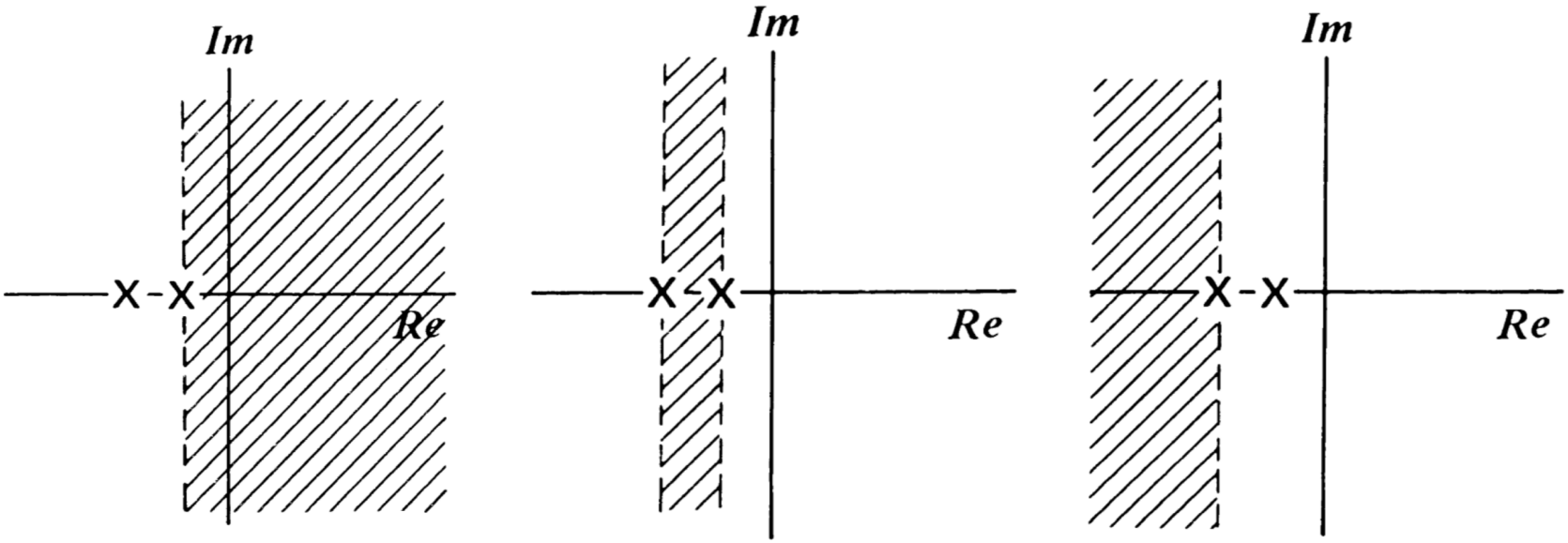
  decompose signals; system as filters

ROC

Definition: the subset of $\mathbb{C}$ that $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt < \infty$

Consider:

$$X(s) = \frac{1}{(s+1)(s+2)}$$



Different choice of ROC corresponds to different $x(t)$. - Quiz9

(here, $x(t)$ can be input/output signal, or the impulse response)

LT - Study (rational) LTI System Behavior $H(s)$

- stable $\Longleftrightarrow$ ROC includes $j\omega$ axis
- casual $\Longleftrightarrow$ ROC RHP
- casual and stable $\Longleftrightarrow$ all poles in the left half of s-plane
- Not stable if: $H(s) = \frac{s^2+s+1}{s+1}$
- Differentiation: solve systems defined by diff. eqn.

$$\frac{d^n}{dt^n}x(t) \overset{\mathscr{L}}{\longleftrightarrow} s^n X(s)$$

- Convolution: get output y(t)

$$h(t) * x(t) \overset{\mathscr{L}}{\longleftrightarrow} H(s)X(s)$$

- Block diagram: be able to read as well as draw - Quiz10

Exercise: HW6 Q5

5. [10] A causal LTI system S with impluse response h(t) has its input x(t) and output y(t) related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t)$$

,
how many poles does G(s) have?

Hint: use long division

Exercise: HW6 Q7

7. [10] A causal LTI system with impulse response h(t) has the following properties:1.When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = \frac{1}{6}e^{2t}$ for all t. 2.The impulse respose h(t) satisfies the differential equation $\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t)$, where b is an unkhown constant.

   Determine the system function H(s) of the system, consistent with information above. There should be no unkhowm constants in your answer, that is, the constant b should not appear in the answer.

Hint: when input is an exponential signal

Conclusion - for Chap. 9

- FS vs. FT vs. LT
- Focus on system prospective
- Practice on PFE, block diagram, etc

Conclusion - for the course

- LTI system, impulse response, convolution - Foundation, Time domain
- Fourier Analysis - signal, system
- Filtering, Sampling, Communication - most interesting topics to me
- Laplace Transform - ROC, system, block diagram
- This course is one of the most inspiring course I have ever took, as it provides a sense of the strong connection between mathematics and the real world.
- And I join research group then.
- If you are interested in signal processing, consider taking: VE351; VE401, VE501; VE455, VE489; VV214/417

The End