VE216 Recitation Class 6

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VE216 SU20 Teaching Group

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Overview

FS vs FT: for periodic signal

FOURIER TRANSFORM OF A PERIODIC SIGNAL $\tilde{x}(t)$

$\tilde{x}(t) \longleftrightarrow a_k$ — Fourier series coefficients

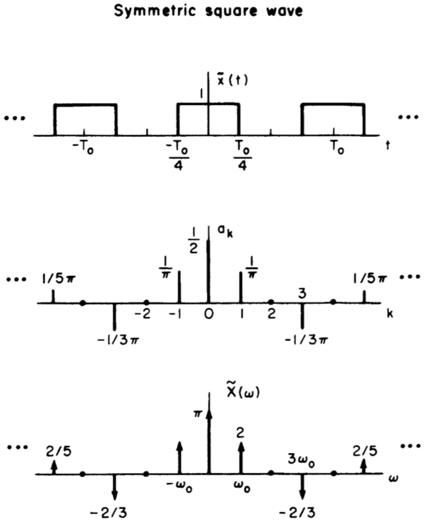
$\tilde{x}(t) \stackrel{\mathcal{F}}{\longleftrightarrow} \tilde{X}(\omega)$ — Fourier transform

$$\boxed{\tilde{X}(\omega) \triangleq \sum_{k=-\infty}^{+\infty} 2\pi a_k \; \delta(\omega - k\omega_o)}$$

$$\tilde{x}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{X}(\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} 2\pi a_k \underbrace{\int_{-\infty}^{+\infty} \delta(\omega - k\omega_o) e^{-j\omega t} d\omega}_{e^{-jk\omega_o}}$$

FS vs FT: for periodic signal - Example



FS vs FT: definition

Fourier Series: for periodic signals

$$\boxed{\begin{aligned} x(t) &= \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \\ a_k &= \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t} \end{aligned}}$$

synthesis

analysis

Fourier Transform: for "all" signals, often aperiodic

$$\boxed{\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega \\ X(\omega) &= \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \end{aligned}}$$

synthesis

analysis

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

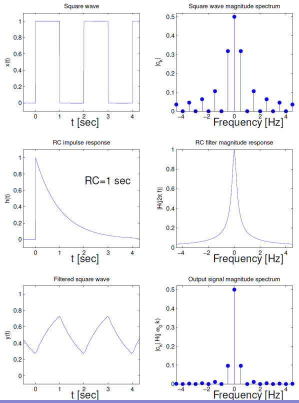
$$\begin{aligned} X(\omega) &= Re\{X(\omega)\} + j\, Im\{X(\omega)\} \\ &= |X(\omega)|\, e^{j\sphericalangle X(\omega)} \end{aligned}$$

FS: Filtering

Input-Output Relation:

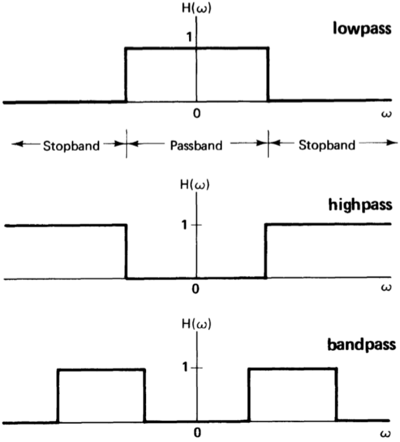
$$x(t) = \sum_k c_k e^{jk\omega_0 t} \rightarrow \boxed{\text{LTI } h(t)} \rightarrow y(t) = \sum_k c_k H(jk\omega_0) e^{jk\omega_0 t}$$

Example:



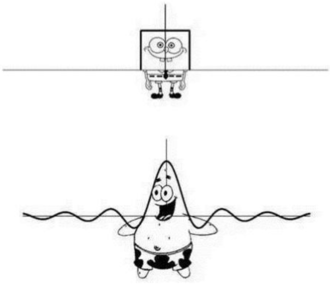
FT: Filtering

- Convolution Property: $h(t) * x(t) \overset{\mathscr{F}}{\longleftrightarrow} H(\omega)X(\omega)$



- LTI systems can be viewed as "filters" in frequency domain

Lowpass Filter



Patrick Star $\xleftrightarrow{FT}$ SpongeBob SquarePants

Exercise: FS Filtering - HW3 Q11

11. [5] Consider a continues-time ideal lowpass filter S whose frequency response is

$$H(j\omega) = \begin{cases} 1 & , |\omega| \leq 100 \\ 0 & , |\omega| > 100 \end{cases}$$

When the input to this filter is a signal $x(t)$ with fundamental period $T = \pi/6$ and Fourier series coefficients a_k, it is found that

$$x(t) \longrightarrow y(t) = x(t)$$

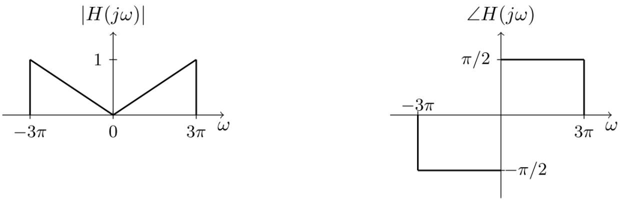
For what value of k it is guaranteed that a_k must be zero?

Exercise - FT: Filtering

12. [10] Shown in the figure 0403 is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals x(t) below, determine the filter output signal $y(t)$.

    (a) $x(t) = \cos(2\pi t + \theta)$

    (b) $x(t) = \cos(4\pi t + \theta)$



Hint:

$$x(t) = \cos(\omega t + \phi) \rightarrow \boxed{\text{LTI } h(t)} \rightarrow y(t) = |H(j\omega)| \cos(\omega t + \phi + \angle H(j\omega))$$

Exercise - FT: Filtering

13. [10] A power signal with the power spectral density shown in figure 0405 is the input of a linear system with the frequency response shown in figure 0406. Calculate and sketch the power spectral density of the system's output signal.

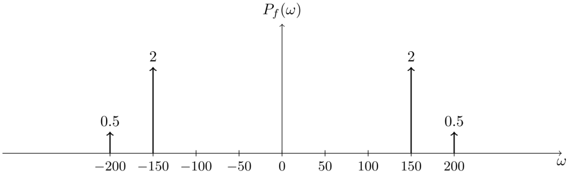


Figure: 0405

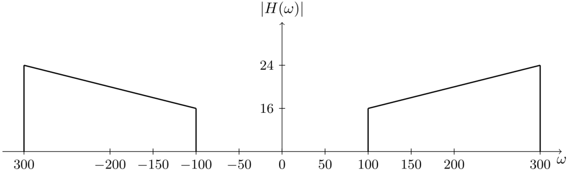


Figure: 0406

Summary

- FS vs FT
- Physical meaning of FT
- The place we are in the big picture
- For Filtering
  - if want y(t), then using FS is easier
  - for FT, it is more elegant: we are either in time domain or freq. domain

The End