# EDS 212 - Essential Math in Environmental Data Science

Day 4 Part 1 - Linear algebra continued

## Part 1: Linear algebra continued

- Refresher: vectors and working with them
- Matrices: notation, language, basic algebra
- Representing systems of equations with matrices
- Linear algebra in environmental science

#### **Matrices**

A matrix is a table of values (multiple vectors in combination). A vector, therefore, can be thought of as a matrix with a single column.

- Dimensions: the size of the matrix, in rows x columns (m x n)
- Elements: values in a matrix, often denoted symbolically with a subscript where the first number is the row and the second number is the column (e.g.  $a_{34}$  indicates the element in row 3, column 4)

## Matrix algebra

#### Addition & subtraction

Add or subtract the corresponding elements (by matrix position) to create a new matrix of the same dimensions.

$$\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & 8 \end{bmatrix}$$

#### Scalar multiplication

To multiply a matrix by a scalar, multiply each element in the matrix by the scalar to get a scaled matrix of the same dimensions.

For example:

$$6 \times \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -18 \\ 0 & 12 \end{bmatrix}$$

### Recall: dot product

The dot product of two vectors is the sum of their elements multiplied:

For  $\vec{a} = [1, 5]$  and  $\vec{b} = [2, -3]$ :

$$\vec{a} \cdot \vec{b} = (1)(2) + (5)(-3) = -13$$

#### Matrix multiplication

We find the dot product of row · column vectors:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

#### Practice problem:

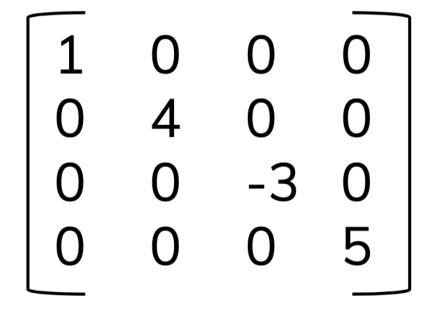
$$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 5 \\ 6 & -3 \end{bmatrix} = ?$$

## Critical thinking: Matrices with unequal dimensions

What do you think the output matrix would contain if you were multiplying the following?

#### Diagonal matrix

A diagonal matrix is (almost always) a square matrix (\$m\$ = n) where only elements on the diagonal are non-zero values.



## What happens when we multiply a matrix by a diagonal matrix?

A diagonal matrix is also called a scaling matrix because it scales rows proportionally, but not by the same value:

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 8 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 16 \\ 8 & 4 \end{bmatrix}$$

### Matrices as systems of equations

Often in environmental data science, we have multiple equations representing processes. Matrices give us a way to express these systems of equations in data structures that are easy to store and work with in data science. For example, let's say we have a system:

$$3x - 8y = 5$$

$$x + 2y = 10$$

How can we write this using matrices?

#### Rewriting in matrix form:

$$3x - 8y = 5$$

$$x + 2y = 10$$

The matrix form of this system of equations looks as follows:

$$\begin{bmatrix} 3 & -8 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

## Example: matrices and linear algebra in environmental science

#### Leslie Matrix: Population ecology

A matrix model that accounts for survival / fecundity rates at different life stages for a species.

#### Overview:

- Define life stages
- Estimate probability of survival / reproduction at different life stages to create a matrix over time

### Writing estimates as equations:

• For our species, each adult female will lay  $\sim$ 600 eggs during each cycle (let's say that's a year). Which means that the eggs at time  $_{t+1}$  can be estimated by the number of adult females \* 600:

$$E_{t+1} = 600 * F_t$$

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We also estimate that 20% of eggs survive to reach larval stage:

$$L_{t+1}=0.2st E_t$$

$$E_{t+1} = 600 * F_t$$

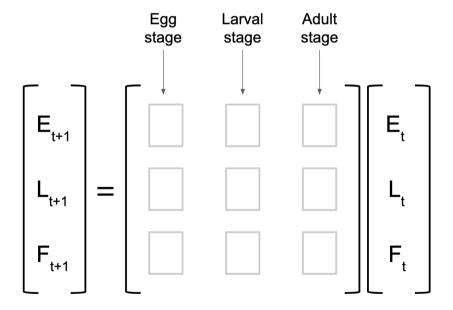
$$L_{t+1} = 0.2 * E_t$$

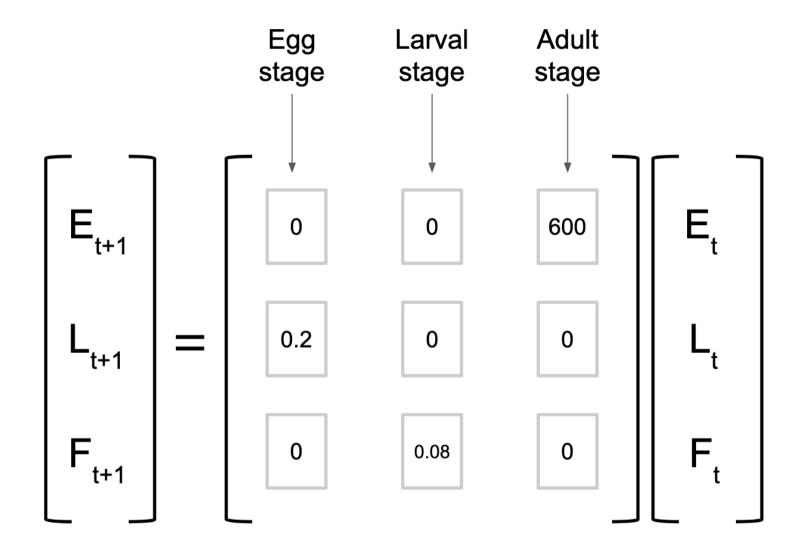
We also estimate that 8% of those that reach larval stage will survive to become reproducing female adults:

$$F_{t+1} = 0.08 * L_t$$

$$E_{t+1} = 600 * F_t$$
  $L_{t+1} = 0.2 * E_t$   $F_{t+1} = 0.08 * L_t$ 

How can we write this in matrix form?





See also: Bison model

https://www.google.com/url?

sa=t&rct=j&q=&esrc=s&source=web&cd=&ved=2ahUKEwjy2ZDKuOPxAhUJ6J4K