

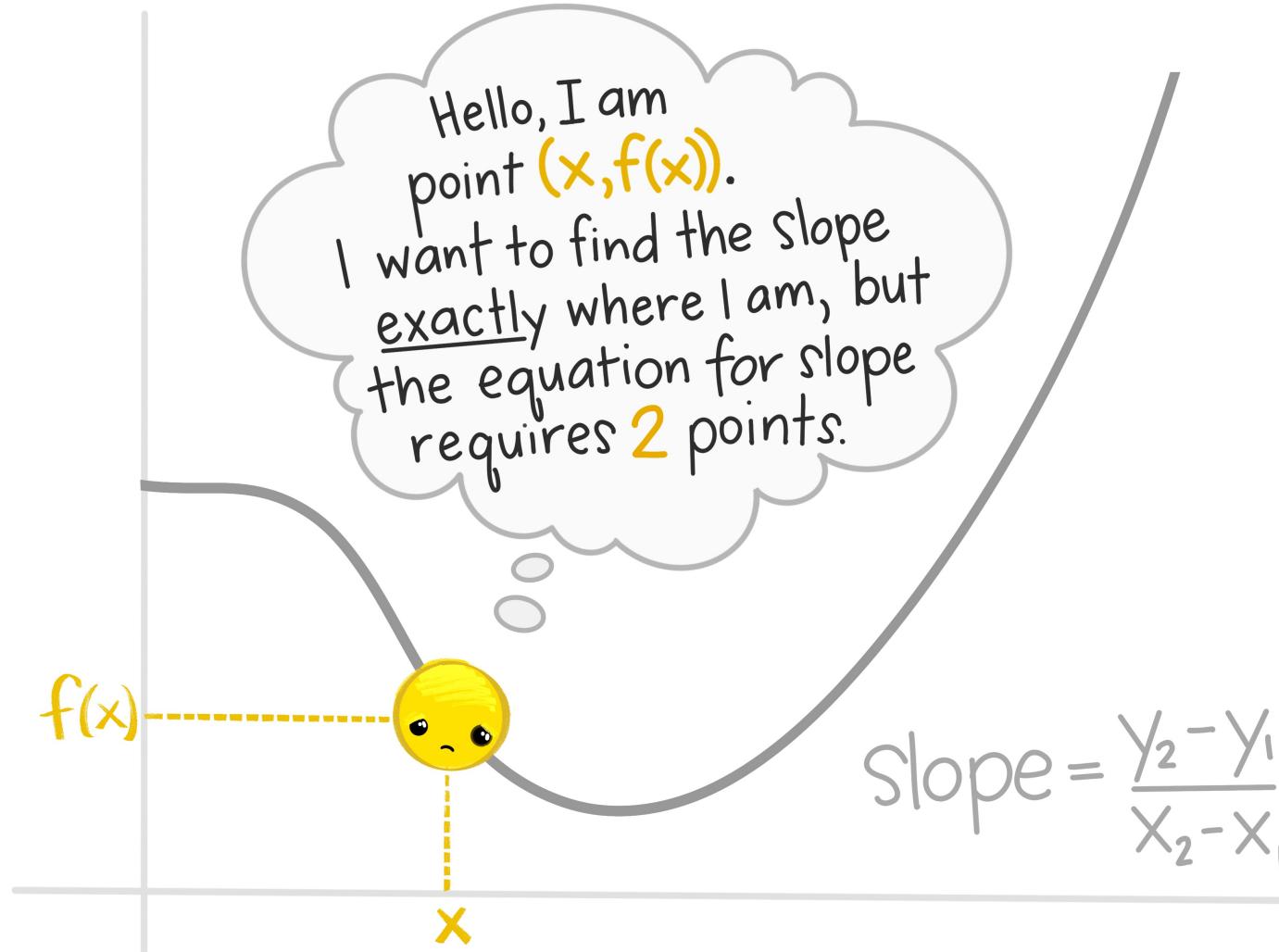
EDS 212

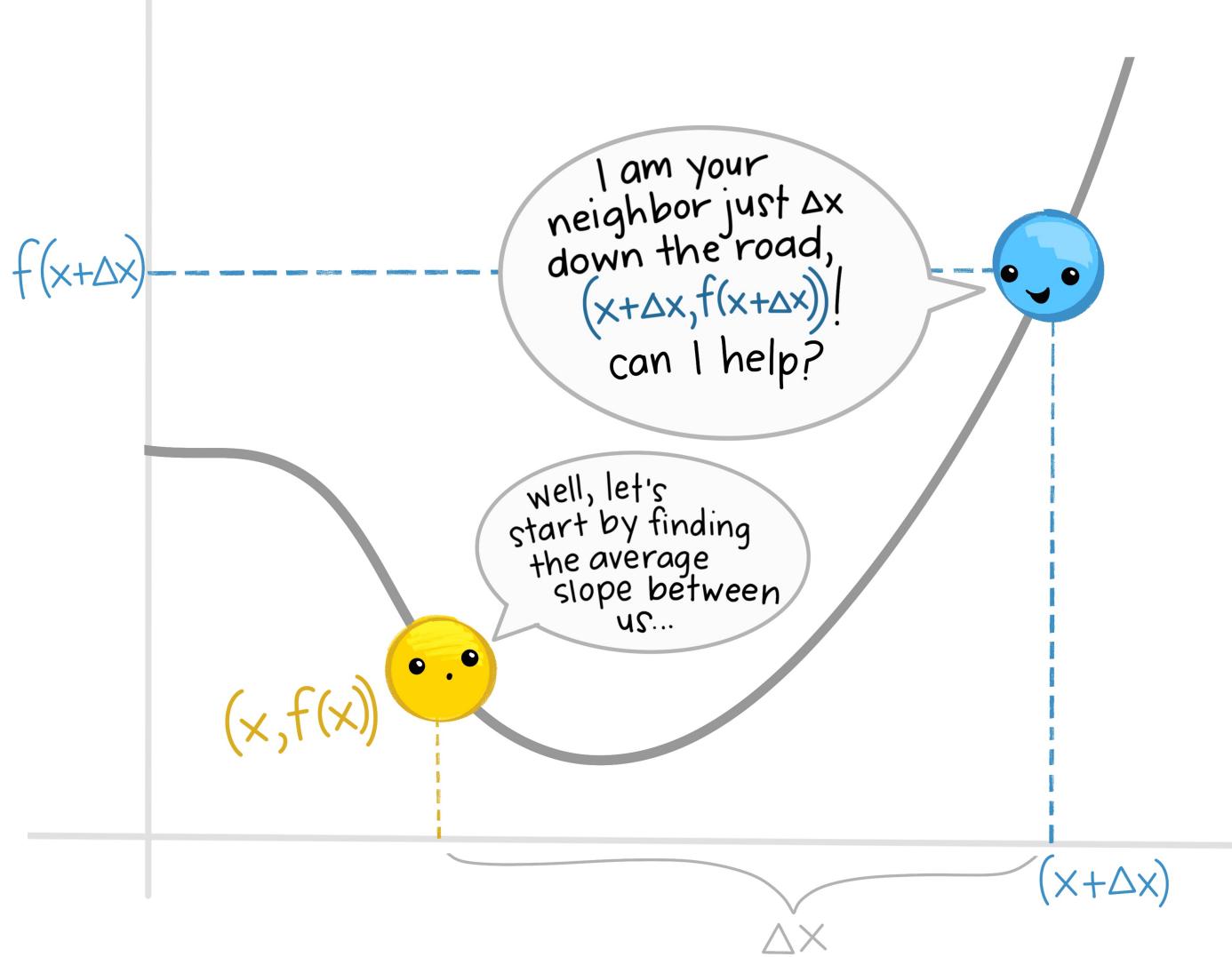
Day 2 Part 1: Definition of the derivative

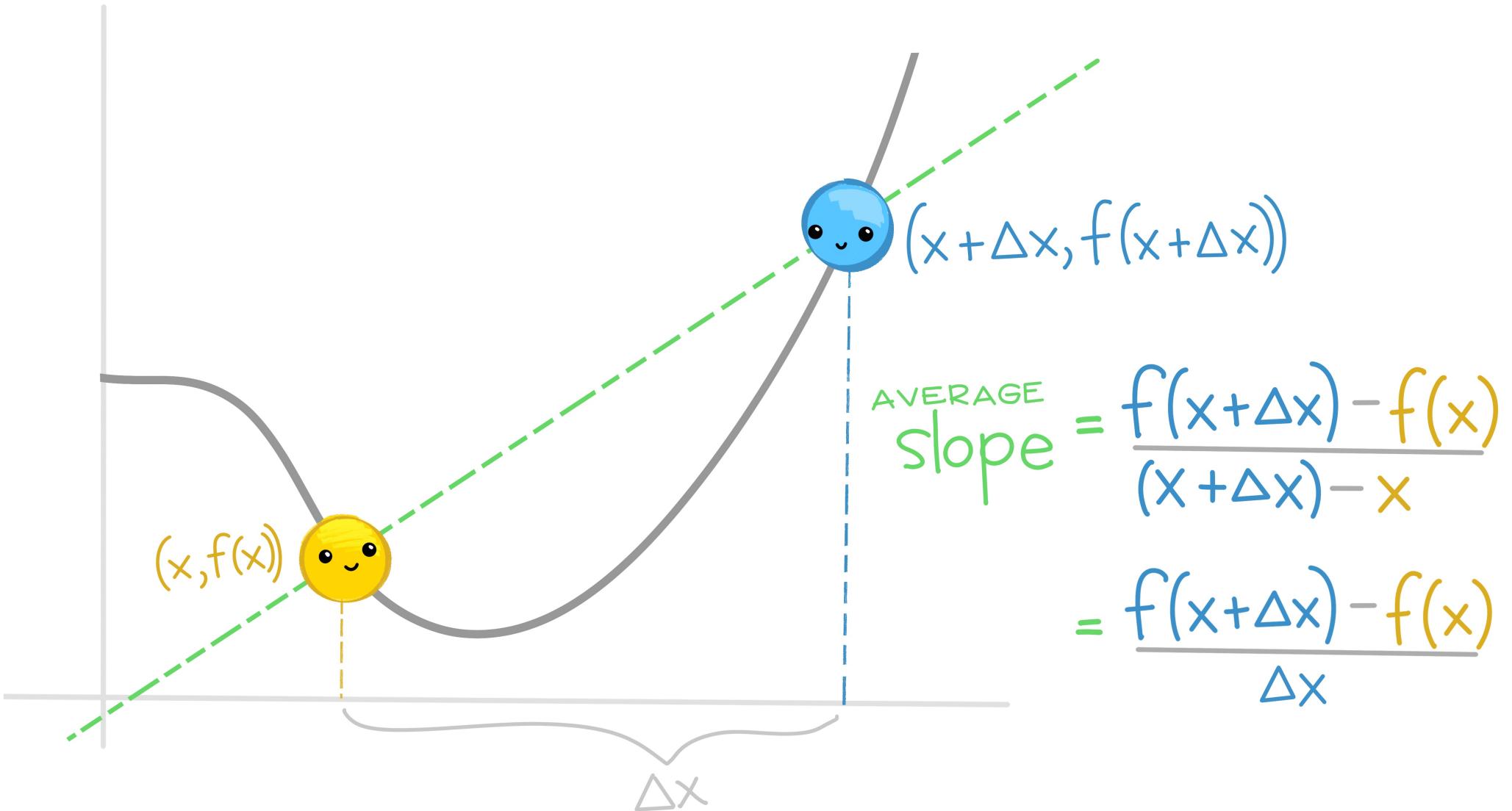
Overview of Day 2 topics

- Derivatives
- Higher order & partial derivatives
- Differential equations (reading, understanding, solving, using)

What if we want to find an expression that describes the rate of change (slope) at *any* point on a function?

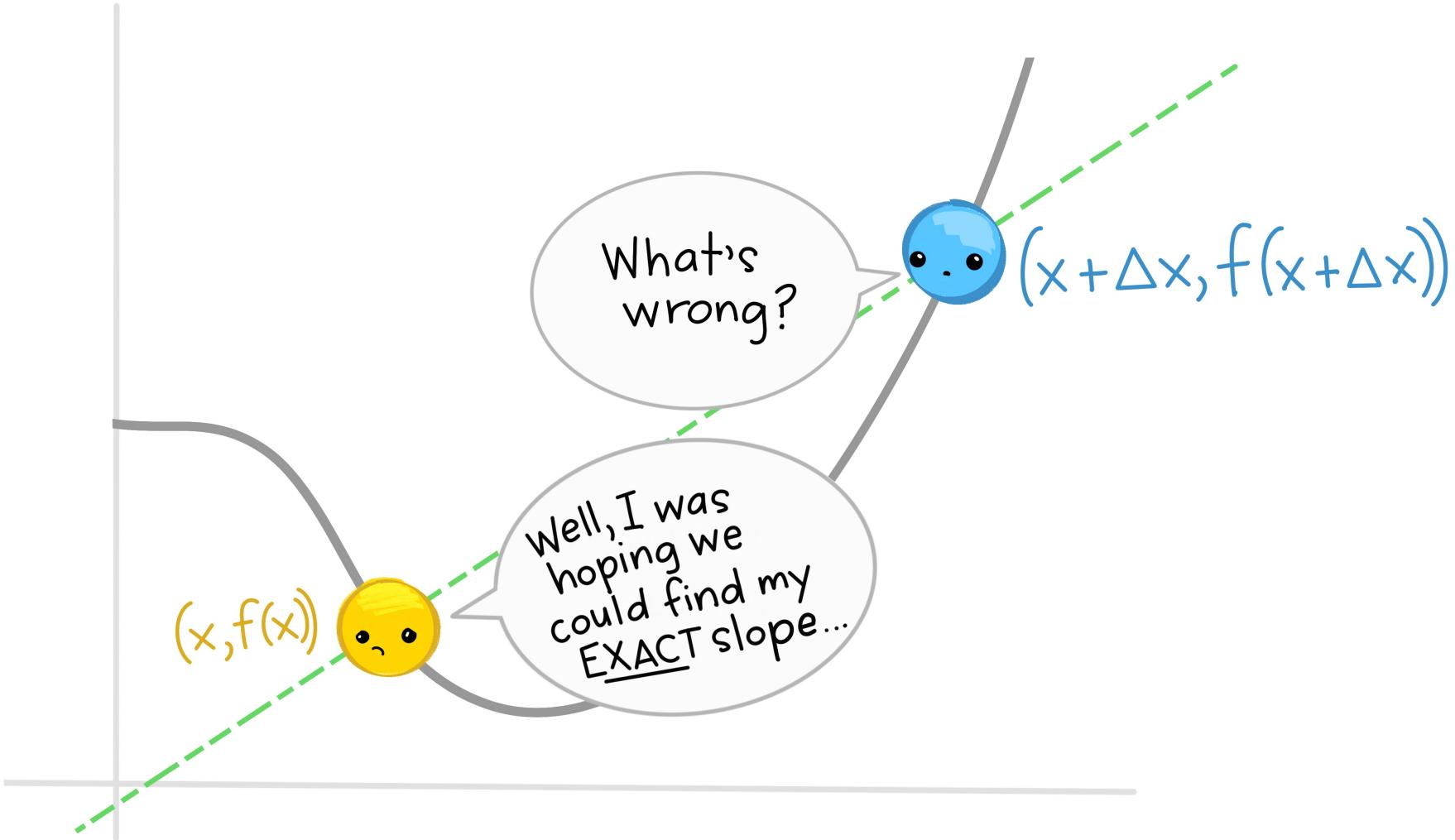




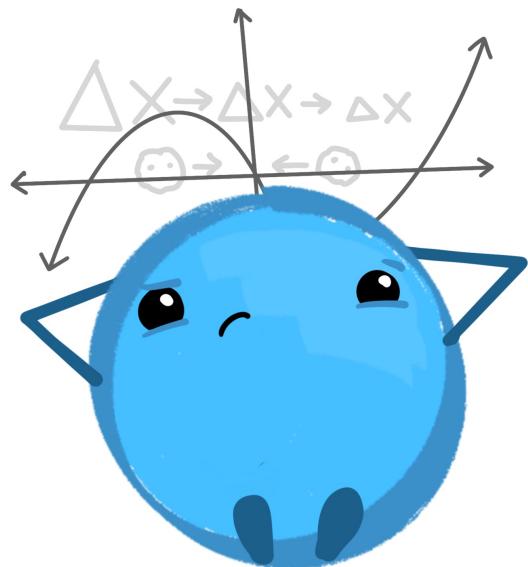


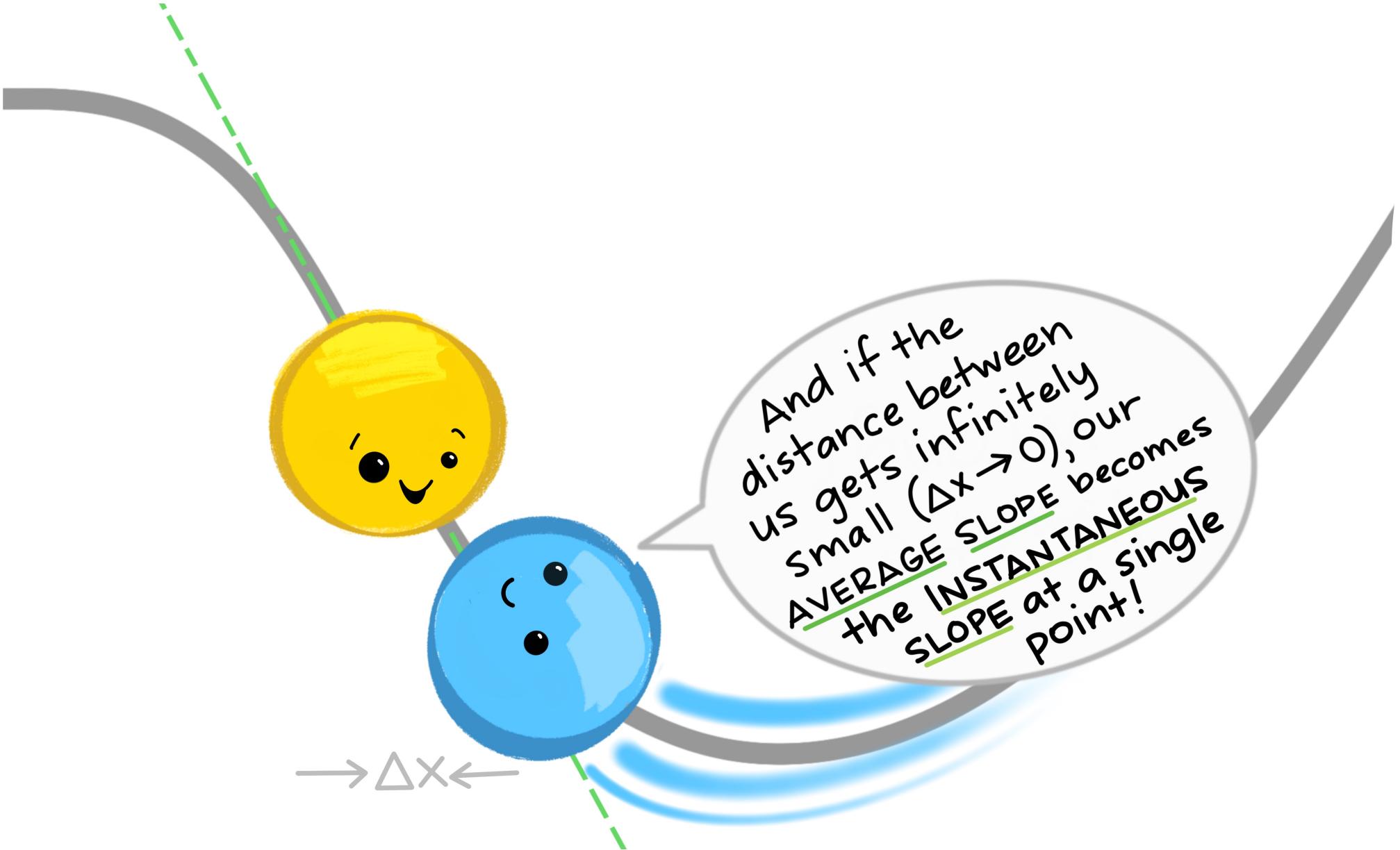
So: the average slope between ANY 2 POINTS on function $f(x)$ separated by Δx is

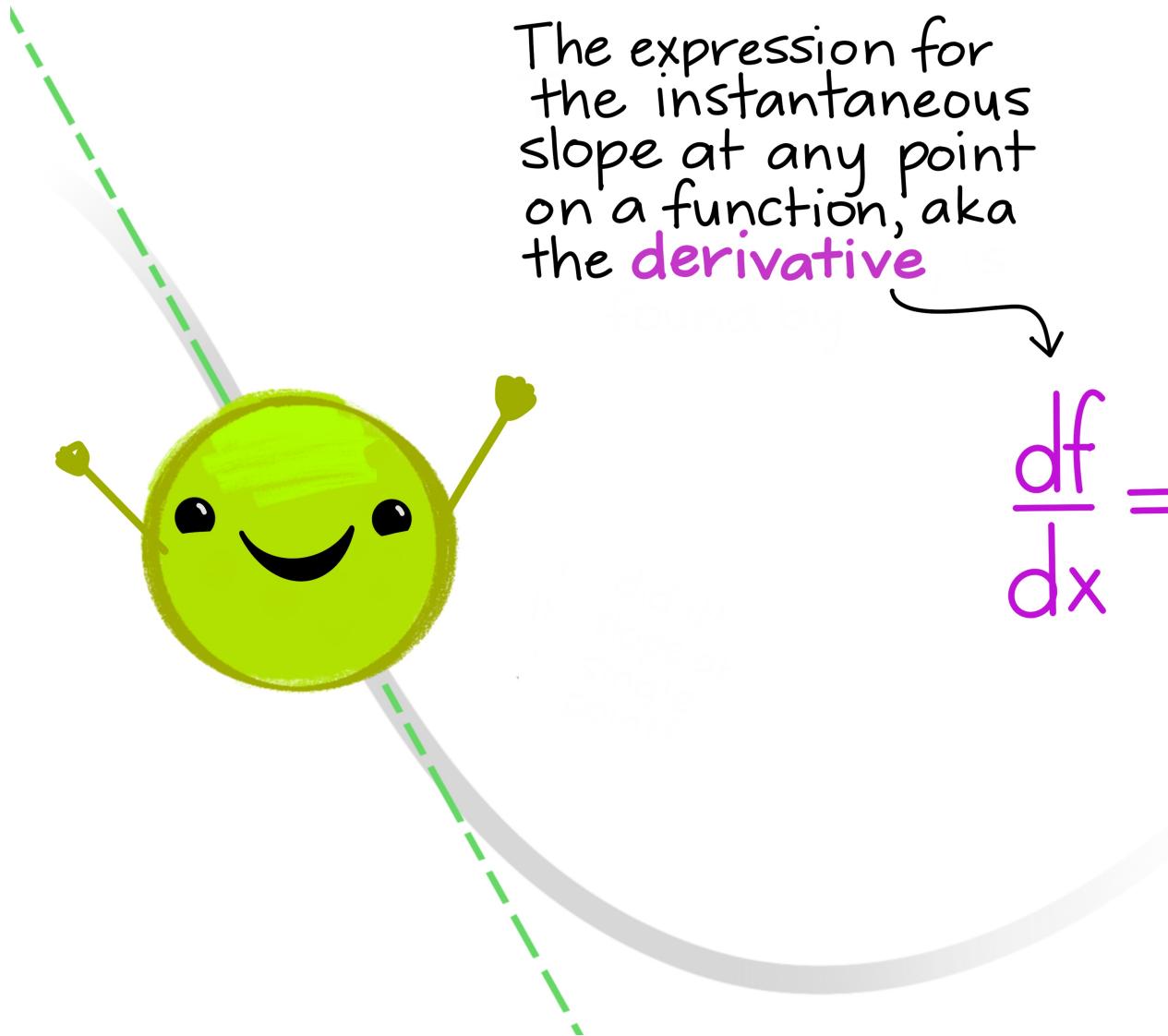
$$m = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



BRAINSTORM MONTAGE!







The expression for
the instantaneous
slope at any point
on a function, aka
the **derivative**

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

IS FOUND BY:

- ① Finding an expression for the **slope** between 2 points separated by Δx ...

- ② Evaluating that slope as the points get infinitely close together.

And that's how we find derivatives

Evaluate the slope between two generic points on any function (separated by Δx) as Δx becomes infinitely small.

Let's try one.

Find an expression that tells us the slope of the function $f(x) = x^2 - 18.2$ at any value of x .

$$\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - 18.2 - (x^2 - 18.2)}{\Delta x} = \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 18.2 - x^2 + 18.2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} 2x + \Delta x = 2x\end{aligned}$$

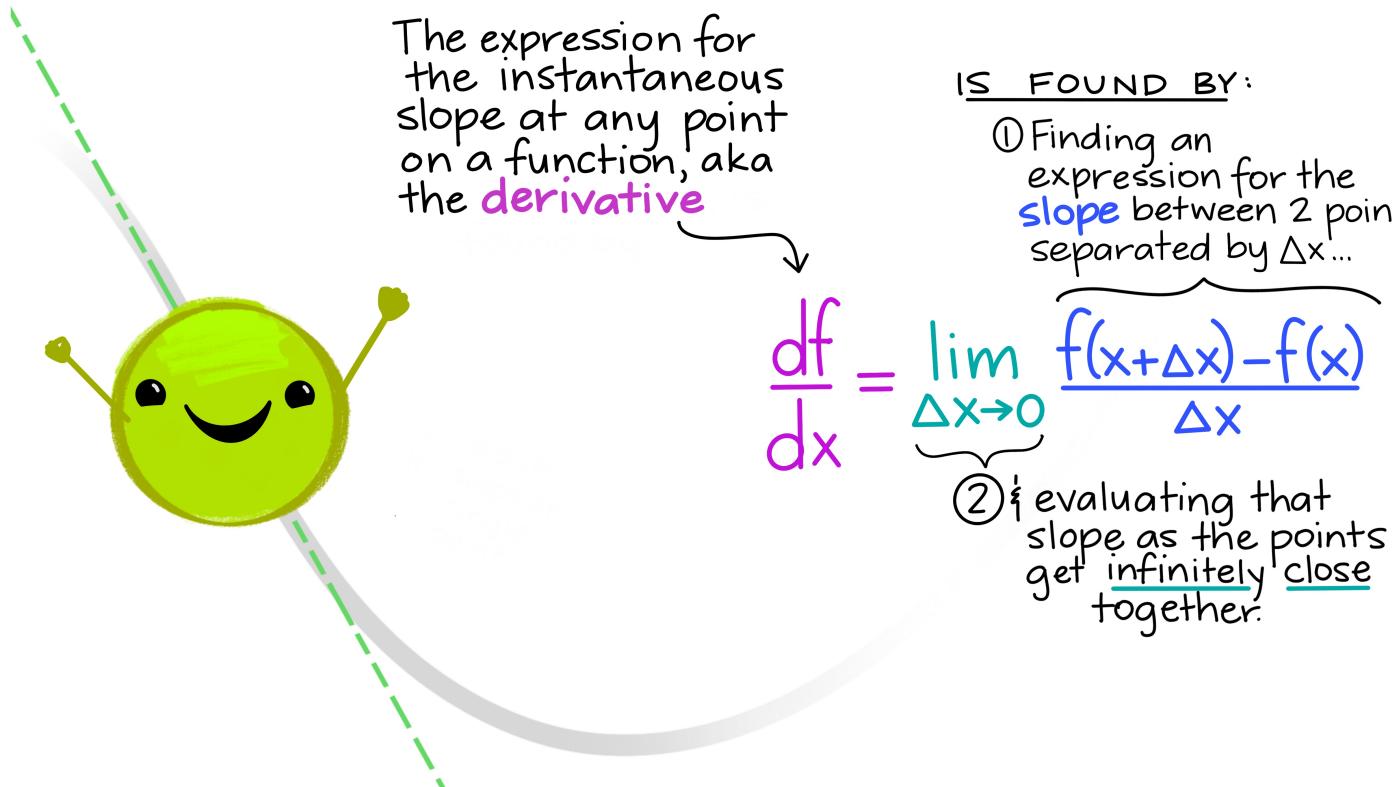
It's cool to do that a few times, but it gets really tedious really fast.

So we have some basic derivative shortcut rules to speed it up a bit:

- **Power rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Constant rule:** if k is a constant, $\frac{d}{dx}(k) = 0$
- **Constant multiple rule:** if k is a constant, $\frac{d}{dx}(kx) = k$
- **Sum and difference rule:** $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

Yeah there are more - we're not doing them.

ALL of those rules come from the definition of the derivative.



Reminder of what we're doing mathematically:

Finding the slope between two generic points on a function as the distance between them gets infinitely small. That will give us an expression for the slope at any point on the original function.

Derivative of logs & exponents

- $\frac{d}{dx}(e^x) = e^x$

- $\frac{d}{dx} \ln(x) = \frac{1}{x}$