Paper Code: BAS 07

Roll No. 2011703103 3

B. Tech.
Year: 1st Semester: 2nd
Major Examination: 2017-18
Engineering Mathematics: II

Note: Attempt all questions. Each question carries equal marks.

Q. 1 Attempt any five parts of the following: 5*2=10

(a) Solve, $(D^2+3D+2)y = xe^x \sin x$.

(b) Solve, by the variation of parameters $(D^2-1)y = e^{-2x}\sin e^{-x}$.

(c) Solve, $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 4y = 4x^2 - 6x^3, y(2) = 4, y'(2) = -1$.

(d) Solve, $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (x^2+2)y = e^{\frac{1}{2}(x^2+2x)}$.

(e) Show that $\int_1^1 (x^2-1)P_{n+1}P_n' dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$.

(f) Show that $\int J_3(x)dx = -J_2 - \frac{2}{x}J_1$.

(g) Solve, $xy' - 3y = k$ where k is a constant using power series method.

Q. 2 Attempt any two parts of the following: 2*5=10

(a)

i. Find the Laplace transform of
$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ where $f(t+2a) = f(t)$.

ii. Find the Laplace transform $f(t) = \frac{\sin at}{t}$. Does the Laplace transform of $\frac{\cos at}{t}$ exist?

(b)

i. Find the Laplace of the following function by representing it in terms of unit step function
$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$.

ii. Evaluate the following integrals using Laplace transform
$\int_0^\infty \frac{e^{-t}\sin^2 t}{t^2} dt, \int_0^\infty e^{-2t}\, t \sin 3t\, dt$.

(c) Find the inverse Laplace transform of

i. $\frac{As+B}{Cs^2+Ds+E}$.

ii. $\frac{e^{-s}}{s^4+4a^4}$.

Printed Pages: 2

Q. 3 Attempt any two parts of the following. 2*5=1[0]

(a) Solve the following differential equation using Laplace transform
$x'' + 9x = \cos 2t$, if $x' = \frac{dx}{dt}$, $x(0) = 1$ and $x\left(\frac{\pi}{2}\right) = -1$.

(b) Solve, $t\frac{d^2x}{dt^2} + \frac{dx}{dt} + tx = 0$, $x(0) = 1, x'(0) = 0$ using Laplace transform.

(c) (i) Solve the following simultaneous differential equations using Laplace transform
$\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = sint$, given $x(0) = 0 = y(0)$.
(ii) Solve the integral equation
$y'(t) = t + \int_0^t y(t-u)\cos u\, du$, $y(0) = 4$.

Q. 4 Attempt any two parts of the following. 2*5=10

(a) Find the Fourier series of the function $f(x) = x^2$. ., $-5 \le x \le 5$

Hence find the value of

i. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$,

ii. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and

iii. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

(b) Obtain the Fourier series for
$f(x) = \begin{cases} x, & -1 < x \le 0 \\ x+2, & 0 < x < 1 \end{cases}$
Hence, deduce the sum of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots\dots\dots$.

(c) (i). Solve: $(D^2 + DD' - 6D'^2)y = x^2 sin(x+y)$.
(ii). Find a real function V of x and y, reducing to zero when $y = 0$ and satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + 4\pi(x^2 + y^2) = 0$.

Q. 5 Attempt any two parts of the following. 2*5=

(a) (i) Solve, $(D^2 + 4DD' - 5D'^2)z = 2cosy - xsiny$.
(ii) Solve, $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+2y}$.

(b) (i) Solve: $(r + 2s + t + 2p + 2q + 1)z = cosmx\ cosnx + \sqrt{x+y}$.
(ii) Solve, $(D^2 - D'^2)z = e^{x-y}\sin(2x+3y)$.

(c) Find the half range sine series for $f(x) = x(\pi - x)$ in the interval $(0,\pi)$ and deduce $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots\dots = \frac{\pi}{32}$.