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B. Tech. I Semester
ODD SEMESTER
MAJOR EXAMINATION 2017 - 2018
Subject Name: : Engineering Mathematics-I

Time: 3 Hrs. Max. Marks: 50

Note: Attempt all questions. Each question carry equal marks.

1. Attempt any four parts of the following: $(4\times 2.5 = 10)$

(a) If $y = x\log\frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$.

(b) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$.

(c) If u is the homogeneous function of degree n then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. Using this find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if $u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2+y^2}$.

(d) Find the rank of the matrix A by applying elementary transformation

$$A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}.$$

(e) Find the values of k for which the following system of equations has non-trivial solutions. Solve equations for such values of k

$$(k-1)x + (3k+1)y + 2kz = 0, \quad (k-1)x + (4k-2)y + (k+3)z = 0,$$
$$2x + (3k+1)y + 3(k-1)z = 0.$$

(f) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, use Cayley-Hamilton theorem to find A^{-1} and $B = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

2. Attempt any two parts of the following: $(2\times 5 = 10)$

(a) (i) Evaluate $\int_0^\infty \int_0^x xe^{-x^2/y}dxdy$ by changing the order of integration.

(ii) Evaluate $\iint xy\, dxdy$ over the positive quadrant of $x^2 + y^2 = a^2$.

(b) Find the volume and mass contained in the solid region of the positive octant of the surface $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r = 1$, where $p, q\ \&\ r > 0$, given that density at any point $\rho(x,y,z) = k\sqrt{xyz}$.

(c) Evaluate (i) $\int_0^2 x(8-x^3)^{1/3}\,dx$ (ii) $\int_{-\infty}^{\infty} \cos\frac{\pi}{2}x^2\,dx$.

3. Attempt any two parts of the following: $(2\times 5 = 10)$

(a) (i) Find the area enclosed between the parabolas $y^2 = 4a(a-x)$ and $y^2 = 4a(x+a)$.

(ii) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}dxdydz$.

(b) Let D be the region in the first quadrant bounded by $x = 0, y = 0$ and $x + y = 1$. Change the variables x, y to u, v, where $x + y = u, y = uv$ and evaluate $\iint_D xy(1-x-y)^{1/2}dxdy$.

(c) Prove that $\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}} dx = \beta(p+1,\ q) + \beta(p,\ q+1)$.

4. **Attempt any two parts of the following:** (2× 5 = 10)

(a) (i) Show that $div(curl\,\overline{V}) = 0$, for any vector point function $\overline{V}$.

(ii) Show that $div(grad\ f(r)) = \frac{d^2}{dr^2} f(r) + \frac{2}{r}\frac{d}{dr} f(r)$, where $r = \sqrt{x^2+y^2+z^2}$.

(b) Verify stokes theorem for the field for the vector field $\vec{F} = (2x-y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ over the upper half surface of $x^2+y^2+z^2 = 1$, bounded by its projection on the $x-y$ plane.

(c) Find the constant a, b and c so that vector function $\vec{A} = (x+10y+8az)\hat{\imath} + (bx-3y-5z)\hat{\jmath} + (4x+cy+2z)\hat{k}$ is irrotational. Then show that $\vec{A}$ can be expressed as the gradient of a scalar function ϕ and hence find ϕ.

5. **Attempt any two parts of the following:** (2× 5 = 10)

(a) (i) Find the direction in which the directional derivative of $\phi(x,y) = \frac{x^2+y^2}{xy}$ at (1, 1) is zero.

(ii) Find the constant a and b such that the surface $3x^2 - 2y^2 - 3z^2 + 8 = 0$ is orthogonal to $ax^2 + 9y^2 = bz$ at $P = (-1,\ 2, 1)$.

(b) Verify Gauss's divergence theorem for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b$ and $0 \le z \le c$.

(c) Apply Green's theorem to evaluate $\int_C (y - \sin x)\,dx + \cos x\,dy$ where C is the plane triangle enclosed lines $y = 0,\ x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$.