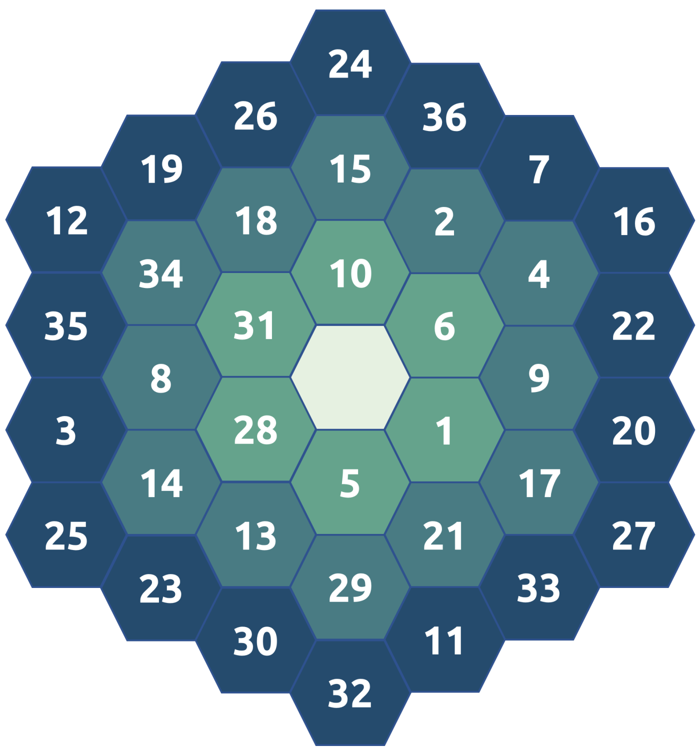
Mustang Math Tournament

HERDING HEXES EXAMPLE RULES

1. **This round serves as AN EXAMPLE. It reuses questions from MMT 2021's Bingo Round, and the grid layout, number of questions, and scoring rules may ALL be different on MMT 2023 competition day!**
2. The "Hexes" Round will consist of 36 questions to be solved in 60 minutes.
3. The questions will vary in difficulty, but not necessarily in increasing order - rather the difficult problems will be randomly distributed.
4. The Hexes board is the hexagonal board shown below. The numbers refer to question numbers

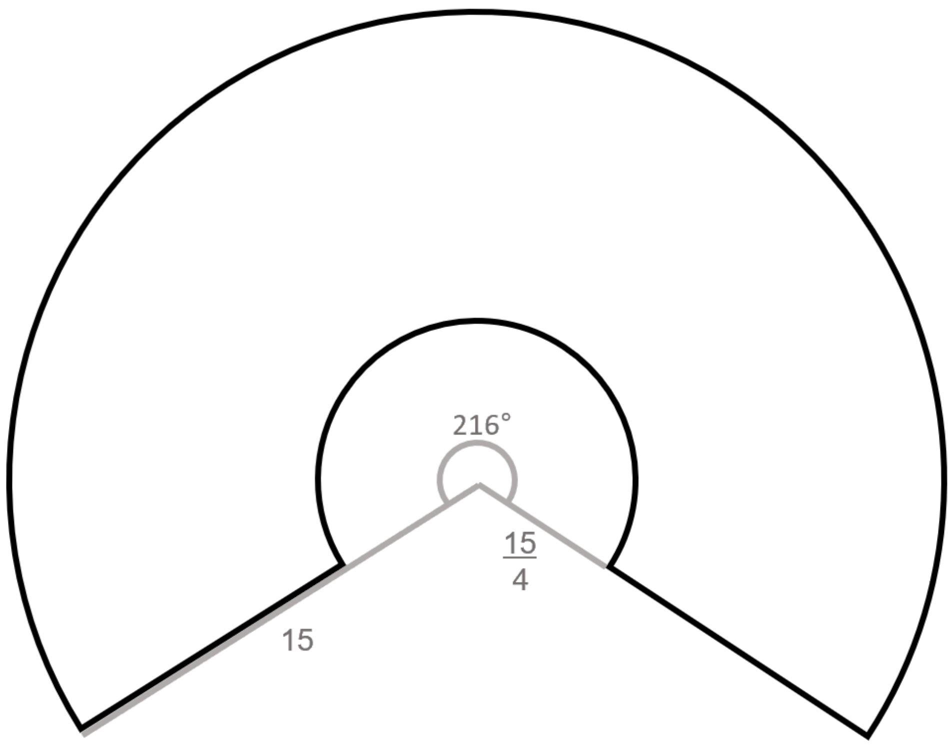


5. In this round, each question in the inner ring is worth 2 points, each question in the middle ring is worth 3 points, and each question in the outer ring is worth 4 points. Additionally, each correct answer that is connected with other correct answers to the center hexagon is worth double points!
6. You are not expected to solve all the questions in this round within the allotted time. Instead, we encourage you to strategize how you might use your time to maximize your points!

HERDING HEXES EXAMPLE ROUND

1. Find the sum of real numbers x and y that satisfy the equations $6x + 5y = 5$ and $4x + 10y = 6$. Express your answer in decimal form.
2. The trainers are having a potluck to celebrate their victory in a stakes race! Medha made a 13 cm by 12 cm pan of brownies, and needs to divide them into brownies with dimensions 2 cm by 3 cm. What is the maximum number of whole brownies she can have?
3. The price of sugar cubes was originally at 15 dollars at the end of the first day. Each morning, a merchant increases the price of sugar cubes by 15% multiplicatively, and each afternoon, he decreases the current price by 15% multiplicatively. This increase and decrease in prices goes on forever. What is the sum of the prices of sugar cubes at the end of each day over all the days?
4. The 3D irregular shape formed by a square pyramid with its base as one of the faces of a cube. All edges equal to 6. The volume of the entire shape can be expressed as $x\sqrt{y} + z$. Find $\frac{x \cdot y}{z}$.
5. A triangle has vertices $(3, 18)$, $(3, 3)$, and $(16, 3)$. Find the area of the triangle.
6. A jar contains black, gold, green, and white horse toys. $\frac{1}{2}$ of the toys are black, $\frac{1}{4}$ are gold, 5 are green, and 4 are white. Find the fraction of total toys that are green.
7. A function f is defined as $f(a, b) = \frac{1}{a} + \frac{2}{ab} + \frac{3}{ab^2} + \cdots$. Evaluate $f(16, 15)$.
8. A trainer wants to build a regular sixteen-sided pen. What is the measure of each interior angle?
9. Find the larger root of the quadratic $2x^2 - x - 36$.
10. Evaluate $(3^2 + 2^3)^2 \cdot 3$.
11. Stallions Daniel and Fiona are racing. Daniel can run 480 meters per minute, and Fiona can run 320 meters per minute. However, every 30 seconds, Daniel needs a 30 second rest (when he completely stops), while Fiona doesn't need any breaks. They begin running at the same time from the starting line on a 3200 meter circular track. After they start racing, how long will it take, in minutes, until the next time the two stallions are side-by-side again? Express your answer in decimal form.
12. A parabola has equation $y = x^2 - 6x + 8$. The radius of the circle with center at $(3, 1)$ that has two points of tangency with this parabola has length r. Find r^2.
13. Horse trainer Evan wanted to distribute sugar cubes to his stallions evenly without any remaining. Splitting the sugar cubes into 11 equal piles left 3 cubes leftover. After dividing the sugar cubes into 15 equal piles, there was still 1 cube left. He finally succeeded when he divided the sugar cubes into 13 equal piles. What is the minimum amount of sugar cubes that he has?

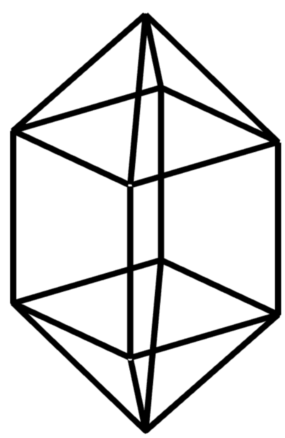
14. The least common multiple of 24 and m is 360. Find the smallest value of m, where m is an integer.
15. A rhombus with side length 9 has one diagonal of length 10. If the length of the other diagonal is x, find x^2.
16. A frustum of a cone or pyramid is defined as the portion of the object which remains after its upper part has been cut off by a plane parallel to its base. A frustum of a cone is formed by folding the net in the diagram such that the straight segments meet perfectly. The radius of the outer circle is 15, the inner circle is $\frac{15}{4}$, and the central angle is $216°$. The frustum of a square pyramid of the same height is then perfectly inscribed within the cone frustum such that all 8 vertices and the 4 lateral edges of the pyramid frustum are on the cone. The volume outside of the pyramid frustum but inside the cone frustum can be expressed as $\frac{x}{y}\pi - \frac{w}{z}$. Find $\frac{x}{y} + \frac{w}{z}$ expressed as a common fraction.



17. The trainers need to put 5 foals in a row for a race. However, the foals Emily and Seabiscuit need to be together or otherwise both will throw a tantrum. How many lineups are there such that no tantrums are thrown?
18. In the stable, there is a drawer of red and blue blankets where the ratio of red blankets to blue blankets is $24 : 7$. A horse has figured out how to open the drawer and has taken a blanket at random. What is the probability the horse has a blue blanket?

19. Two foals Abby and Becky are searching for a treat hidden in the Primes horse barn, which has infinitely many stalls. Foal Abby searches the first stall, and then the two foals take turns going through each stall until one of them finds the treat. Each stall has a $\frac{1}{10}$ chance of containing the treat. What is the probability that the foal Becky will find the treat?
20. Triangle ABC has area 60. D, E, and F are the midpoints of $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively. G and H are the midpoints of $\overline{EF}$ and $\overline{DF}$, respectively. I is the midpoint of $\overline{GD}$. Find the area of triangle HID in decimal form.
21. Find the sum of the distinct prime factors of 2021.
22. Two colts are trying to meet up to race. Each can arrive at any time between $1-5$ pm, and will stay for 30 minutes before leaving. What is the probability that these horses will meet and race that day?
23. How many integers from 1 to 1000 (inclusive) are relatively prime to 1001?
24. Wilson is leaving to go to Krish's house. Wilson's clock gains 3 minutes every hour, and Krish's clock loses 5 minutes every hour. (i.e Wilson's clock progresses 63 minutes for every hour of real time and Krish's clock progresses 55 minutes for every hour of real time). Besides the gain or loss of time, the clocks advance normally at a constant rate. Both set their clocks to the right time at 12 am, and when Wilson leaves his house the same day, his clock shows that the time is 9:00 am. When he arrives at Krish's house, he notices that on Krish's clock, the time is 10:30 am. How long did Wilson travel to get to Krish's house? Express your answer in hours as a common fraction.
25. A five-term mountain sequence is defined as a_1, a_2, a_3, a_4, a_5, where $a_1 < a_2$, $a_2 < a_3$, $a_3 > a_4$, and $a_4 > a_5$. 5 distinct random numbers are selected from the first 20 positive integers. What is the probability that these numbers form a mountain sequence in the order that they were selected?
26. There are 10 rings to be distributed among Abbie, Betty, Charlie and Daniel. Abbie insists on receiving more rings than any other siblings. Betty and Charlie are twins, and want to receive the same amount of rings. Find the number of ways the rings can be distributed amongst the siblings Abbie, Betty, Charlie, and Daniel if siblings can receive 0 candies.
27. A sequence is constructed by interweaving two geometric sequences, 2^n and 3^n, like this: $2, 3, 4, 9, 8, 27, 16, 81, 32, 243 \ldots 2^n, 3^n \ldots$. When the number of terms in the sequence is 120, the sum of the sequence is divisible by 10. Find the total number of terms the next time the sum of the sequence is divisible by 10
28. Find the perimeter of an equilateral triangle with side length 4.

29. In a math camp, there are algebra and geometry classes. Everyone in the camp is taking at least one class. $\frac{1}{2}$ of the total people chose the algebra class, and $\frac{9}{10}$ of the total people chose the geometry class. If 32 people chose both classes, find the number of people who only chose algebra.
30. Stallions Charlie and Charlotte are fighting over a circular-cone shaped carrot with base radius 4 and height 6. As a trainer, Arpit must make a circular cut parallel to the base such that both stallions get an equal share. If the radius of the cut is r, find r^3.
31. In Horse Paradise, 4 saddles can be traded for 6 horseshoes. 9 horseshoes can be traded for 3 reins and 5 bales of hay can be obtained for 2 reins. How many bales of hay can I get from 12 saddles?
32. Consider the toy below constructed by placing two regular pyramids above and below a cube. A trainer paints each face of the toy with one of the colors black, gold, green, and white such that the faces completely opposite (parallel) to each other are painted the same color. What is the number of unique ways she can paint it such that no two adjacent faces (ones that share an edge) share the same color? Note that two ways are not considered unique if they can be rotated to look like another.



33. How many ways can 10 foals split into 5 pairs (unordered) for the relay race?
34. A fraction $\frac{x}{y}$ can expressed as $0.11\overline{36}$. Find $\frac{y}{x}$ in decimal form.
35. Grace is saving change from different shopping trips. She has pennies, nickels, dimes, and quarters in her piggy bank. She has a total of 56 coins, and her coins are worth 3.27 dollars. Grace notices that the number of dimes she has is one more than twice the number of quarters and the number of pennies is also one more than twice the number of nickels. How many pennies does she have?
36. The reciprocal of every prime greater than or equal to 7 results can be expressed as a repeating decimal in the form $0.\overline{a_1a_2a_3\cdots a_n}$. We can define the n-digit number to be $a_1a_2a_3...a_n$, which is the part that repeats. Find the greatest common divisor of all such repeating parts for all primes greater than or equal to 7.