1. **LT751** 10 numbers, $a_1, a_2, \cdots, a_{10}$ are written in a row on a blackboard. For any integer s in the range $1 \leq s \leq 10$, the average of the first s numbers is equal to s^2. What is $a_5 + a_{10}$?

Written by: Linus Tang

Answer: $\boxed{332}$

The sum of the first n numbers would be $n \cdot n^2 = n^3$, so we have

$$a_5 = (a_5 + a_4 + ... + a_1) - (a_4 + a_3 + a_2 + a_1) = 5^3 - 4^3 = 61$$

and

$$a_{10} = (a_{10} + a_9 + ... + a_1) - (a_9 + a_8 + ... + a_1) = 10^3 - 9^3 = 271$$

The final answer is hence $61 + 271 = \boxed{332}$.

2. **YS940** Find the smallest integer n such that $0.1 > \left(\frac{4}{5}\right)^n$.

Written by: Yuuki Sawanoi

Answer: $\boxed{11}$

Notice that $\left(\frac{4}{5}\right)^3 = 0.8^3 = 0.512 \approx 0.5$.

Hence, $\left(\frac{4}{5}\right)^9 = \left[\left(\frac{4}{5}\right)^3\right]^3 \approx 0.5^3 \approx 0.125$.

Notice that $0.125 \times \frac{4}{5} = 0.1$. Thus, we have that $\left(\frac{4}{5}\right)^{10} \approx 0.1$. However, note that because 0.512 is larger than 0.5, $\left(\frac{4}{5}\right)^{10}$ is slightly larger than 10. Thus, our answer becomes $10 + 1 = \boxed{11}$.

Alternatively, it is also possible to remember the fact that $\log(2) \approx 0.301$ and thus:

$$\log\left(\left(\frac{4}{5}\right)^n\right) = n\log(\frac{4}{5}) = n(\log 8 - 1) = n(3\log 2 - 1) \approx n(0.903 - 1) \approx -0.97n$$

Thus, we are trying to find the minimum n such that $-1 > -0.97n$ which happens to be $\boxed{11}$.

3. **TK889** Tristan is trying to estimate $(1 + x)^2$, and ends up approximating it as $1 + 2x$. For what positive value of x is Tristan's approximation half the real value? Round x to the nearest tenth.

Written by: Tristan Kay

Answer: $\boxed{2.4}$

Interpreting the problem as $\frac{\text{real value}}{2} = \text{estimated value}$, we have:

$$\frac{(1 + x)^2}{2} = (1 + 2x)$$

$$
\begin{aligned}
1+2x+x^2 &= 2+4x \\
x^2-2x-1 &= 0 \\
(x-1)^2-2 &= 0 \\
x &= 1 \pm \sqrt{2}
\end{aligned}
$$

As x is positive, our answer is $1+\sqrt{2} \approx \boxed{2.4}$

4. **TK890** Let $7a+2b=54$ and let $4a+7b=43$. Evaluate $a+12b$.

   Written by: Tristan Kay

   Answer: $\boxed{32}$

$$2(4a+7b)-(7a+2b) = 2 \times 43 - 54 = \boxed{32}$$

5. **LT728** A fair six-sided die has faces labelled $1, 1, 2, 3, 5$, and 8. Davy rolls the die twice and takes the sum of the numbers rolled. What is the probability that this sum is even?

   Written by: Linus Tang

   Answer: $\boxed{\dfrac{5}{9}}$

   Observe the probability that the sum is even is the probability that his dice rolls are both odd or both even. There are 4 odds and 2 evens, thus the probability of the former is $\left(\frac{4}{6}\right)^2 = \frac{4}{9}$ and the probability of the latter is $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$. Altogether, our answer is $\frac{4}{9} + \frac{1}{9} = \boxed{\dfrac{5}{9}}$

6. **LT272** A standard deck of 52 cards is shuffled into a random order. Given that the top card is a king, what is the probability that the bottom card is the king of diamonds?

   Written by: Linus Tang

   Answer: $\boxed{\dfrac{1}{68}}$

   We are given that the top card is a king. With probability $\frac{1}{4}$, the top card is the king of diamonds, so the bottom card cannot possibly be the king of diamonds. With probability $\frac{3}{4}$, the top card is one of the other kings. In this case, the king of diamonds is in one of the 51 positions other than the top. It is at the bottom with probability $\frac{1}{51}$. Therefore, the overall probability that the king of diamonds is at the bottom is $\frac{3}{4} \cdot \frac{1}{51} = \boxed{\dfrac{1}{68}}$.

7. **CY592** At a party, $\frac{7}{17}$ of the people are wearing green jackets. An additional 300 people arrive on a bus. Now, $\frac{7}{12}$ of the people are wearing green jackets. What is the smallest possible number of people that were wearing green jackets on the bus?

   Written by: Carsten Yeung

   Answer: $\boxed{210}$

   If there is x people initially in the party, then

   $$\frac{7}{12}(x+300) - \frac{7}{17}x = x\frac{35}{12 \cdot 17} + 175$$

   people wearing green jackets on the bus. For this expression to be an integer and minimized, $x = 12 \cdot 17$, leading our answer to be $35 + 175 = \boxed{210}$.

8. **DG943** How many ways are there to rearrange the letters in the word $MOONSHINE$ such that the word $MOON$ appears in that order continuously? ($SEMOONIHN$ is valid, but $MOOSNHINE$ and $NOOMNHSIE$ are not)

   Written by: Maggie Shen

   Answer: $\boxed{720}$

   We can consider the letters $MOON$ together as 1 "letter" or object. Then, there are 6 objects we will need to rearrange - $MOON$, S, H, I, N, and E. Because order matters, there are $6! = \boxed{720}$ ways to arrange the letters.

9. **MLI317** In square $ABCD$, midpoints E and F are drawn on $\overline{BC}$ and $\overline{CD}$ respectively. Compute the ratio between the area of $\triangle AEF$ to $ABCD$.

   Written by: Michael Liu

   Answer: $\boxed{\frac{3}{8}}$

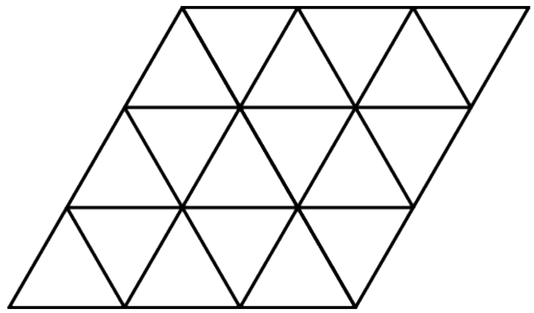
   Let side length $AB = x$. Therefore, we know the area of $ABCD = x^2$. Note that the area of $\triangle AEF$ is equal to $[ABCD] - [ADF] - [ABE] - [EFC]$. Since E and F are midpoints of sides of the square, we know that $DF = FC = CE = BE = \frac{x}{2}$. With all this information, we can now determine the areas of triangles in terms of x. We can now find that $[ADF] = [ABE] = x \cdot \frac{x}{2} \cdot \frac{1}{2} = \frac{x^2}{4}$ and $[EFC] = \frac{x}{2} \cdot \frac{x}{2} \cdot \frac{1}{2} = \frac{x^2}{8}$. Tberefore, $[AEF] = x^2 - 2 \cdot \frac{x^2}{4} - \frac{x^2}{8} = \frac{3x^2}{8}$, so our final answer is $\frac{\frac{3x^2}{8}}{x^2} = \boxed{\frac{3}{8}}$.

10. **YS941** Tristan constructs a shape by gluing together 18 equilateral triangles of side length 1 with no overlap. What is the smallest possible perimeter of Tristan's shape?

    Written by: Yuuki Sawanoi

    Answer: $\boxed{12}$

There are multiple configurations of perimeter 12 that work. One is shown below:



11. **YS948** Which of the following is not a possible area for a triangle with perimeter 60?

Written by: Yuuki Sawanoi

Answer: $\boxed{225}$

Let's say there is a triangle with area A. Notice that every other area below A can be achieved by squishing the triangle without changing the perimeter. For example, a $20-20-20$ triangle can be squished to a $29-29-2$ triangle, which preserves a perimeter of 60 while decreasing the area. Thus, the answer must be the largest answer choice, else all the areas are achievable.

To solve this problem properly, the maximum area of a triangle with perimeter 60 is when the triangle is equilateral. Thus, the side length of the triangle is 20, and the area is $\frac{(20)^2\sqrt{3}}{4} = 100\sqrt{3}$, which is less than the largest answer choice of $\boxed{225}$.

12. **SG297** Equilateral triangle $\triangle ABC$ with side length 60 is cut into 3600 smaller equilateral triangles with side length 1. Point D is chosen on segment $\overline{AC}$ such that $AD = 20$. If a bug starts at point B and travels in a straight line path to point D, find the total number of triangles the bug passes through the interior of.

Written by: Sambhu Ganesan

Answer: $\boxed{80}$

We can place triangle $\triangle ABC$ on the coordinate plane with vertex B at the origin. This means that the coordinates of D are $(40, 20\sqrt{3})$.

We see that the line passes through points $(2i, i\sqrt{3})$, where $i \leq 30$. Note that between $(0,0)$ and $(2,\sqrt{3})$, the bug passes through 4 triangles and between $(2,\sqrt{3})$ and $(4, 2\sqrt{3})$ the bug passes through 4 triangles. Hence, the bug crosses 4 triangles between $(2(i-1),(i-1)\sqrt{3})$ and $(2i, i\sqrt{3})$.

Since the minimum value is $i = 1$ and the maximum value is $i = 20$, we have 20 such intervals. Thus our answer is $4 \cdot 20 = \boxed{80}$.

13. **DG399** How many digits does the base-16 number 3421_{16} have in base-4?

Written by: *Daniel Ge*

Answer: $\boxed{7}$

Sice $4^2 = 16$, two digits in base-4 make up one digit in base-16. Thus the conversions from base-16 to base-4 would be:

$$\begin{gathered} 1 \to 01, \\ 2 \to 02, \\ 4 \to 10, \\ \text{and } 3 \to 03. \end{gathered}$$

Together, our base-4 conversion would be 03100201. If we drop the leading zero, we have $\boxed{7}$ digits.

14. **ARP254** How many triples of positive integers (a, b, c) are there that satisfy $a \leq b \leq c$ and $abc = 2023$?

Written by: *Arpit Ranasaria*

Answer: $\boxed{4}$

Since the prime factorization of 2023 is $7 \cdot 17^2$, we can count the possibilities starting with the smallest values of a. For $a = 1$ we have $(1, 1, 2023)$, $(1, 7, 289)$, $(1, 17, 119)$. For $a = 7$ we have $(7, 17, 17)$. Since we cannot have any other values of a, satisfying the inequality, these are all our possibilities, giving us an answer of $\boxed{4}$.

15. **YS936** What digit O makes the 5-digit number $2O23O$ divisible by every answer choice except O?

Written by: *Yuuki Sawanoi*

Answer: $\boxed{4}$

One can simply try each possibility to get $\boxed{4}$. Note that 7 is clearly false as $7|2023$ and 3 and 6 are impossible due to logic as both are multiples of 3.

16. **TK892** A *meaningful* number is a number whose prime factors sum to 42. What is the sum of the two smallest meaningful numbers?

Written by: *Tristan Kay*

Answer: $\boxed{407}$

If there are only two prime factors of a meaningful number, the meaningful number is minimized when they are as far apart as possible. The pair of primes that are most far apart that sum to 42 is 5 and 37. The next pair of primes after that is 11 and 31.

If there are three prime factors, notice one prime factor must be even. In other words, 2. Then we need a pair of primes that sum to 40, for which the most furthest apart they can be is with 3 and 37.

Looking at the above numbers, the sum of the two smallest is $5 \times 37 + 2 \times 3 \times 37 = 11 \times 37 = \boxed{407}$.