



1. **LT729** What is $16 \times 117 + 6 \times 8$?

Written by: Linus Tang

Answer: 1920

Solution 1:

$$\begin{aligned} &16 \times 117 + 6 \times 8 \\ &= 16 \times 125 - 16 \times 8 + 6 \times 8 \\ &= 16 \times 125 - 10 \times 8 \\ &= 2^4 \times 5^3 - 80 \\ &= 2 \times 10^3 - 80 \\ &= 2000 - 80 \\ &= \boxed{1920} \end{aligned}$$

Solution 2:

Computing directly,

$$\begin{aligned} &16 \times 117 + 6 \times 8 \\ &= 1872 + 48 \\ &= \boxed{1920} \end{aligned}$$

2. **LT731** Let m and n be positive integers. Albert calculates the quantity $1+2+3+\cdots+n$ and Betty calculates the quantity $5+6+7+\cdots+m$. Given that Albert and Betty get the same sum, what is this sum?

Written by: Linus Tang

Answer: 45

The sum of the first n positive integers is given by $\frac{n^2+n}{2}$.

So, $\frac{n^2+n}{2} = 5 + 6 + 7 + \cdots + m$.

$$1 + 2 + 3 + 4 + \frac{n^2+n}{2} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + \cdots + m$$

$$10 + \frac{n^2+n}{2} = \frac{m^2+m}{2}$$

$$20 = m^2 + m - n^2 - n$$

$$20 = m^2 - n^2 + m - n$$

$$20 = (m - n)(m + n) + m - n$$



$$20 = (m - n)(m + n + 1)$$

Note that $m - n$ and $m + n + 1$ have opposite parity, meaning that one of them is even and the other is odd.

The ways to factor 20 into numbers with opposite parity are $(4)(5)$ and $(1)(20)$.

The former gives $m = 4, n = 0$, which is not allowed since n is positive.

The latter gives $m = 10, n = 9$.

Thus, the answer is the common sum $1 + 2 + 3 + \cdots + 9 = 5 + 6 + 7 + \cdots + 10 = \boxed{45}$.

3. **TK919** Let $ABCD$ be a square. If the largest circle that can fit inside $ABCD$ has area 2023π , what is the area of $ABCD$?

Written by: Tristan Kay

Answer: $\boxed{8092}$

The area of a circle is given by πr^2 , where r is the radius. Thus, $\pi r^2 = 2023\pi$, so $r = \sqrt{2023}$.

The side length of the square is twice the radius of the circle, or $2\sqrt{2023}$. Its area is $(2\sqrt{2023})^2 = 2^2 \cdot \sqrt{2023}^2 = 4 \cdot 2023 = \boxed{8092}$.

4. **SR453** If $85,7A6$ divisible by both 3 and 8 for some digit A , what is A ?

Written by: Shak Ragoler

Answer: $\boxed{7}$

The sum of the digits of the number is $8 + 5 + 7 + A + 6 = 26 + A$. For the number to be divisible by 3, the sum $26 + A$ must also be divisible by 3, meaning A can be 1, 4, or 7. Note that A cannot be greater than 9 since A is a digit.

For the number to be divisible by 8, the last three digits combined must be divisible by 8. Using each of the possible values of A , we get 716, 746, and 776. 776 is divisible by 8, but 716 and 746 are not.

Since 776 is our only option, $A = \boxed{7}$.

5. **LT830** What is the smallest positive integer that cannot be expressed as the difference between two prime numbers?

Written by: Linus Tang

Answer: $\boxed{7}$

Note that $1 = 3 - 2$, $2 = 5 - 3$, $3 = 5 - 2$, $4 = 7 - 3$, $5 = 7 - 2$, and $6 = 11 - 5$. So, the positive integers up to 6 can all be expressed as the difference between two prime



numbers.

However, 7 cannot be expressed in this way: If $7 = p - q$ for primes p and q , then either p or q is even. The only even prime is 2, so either $p = 2$ or $q = 2$. The former case leads to $q = -5$, and the latter leads to $p = 9$. Neither is prime. Hence, $\boxed{7}$ is the smallest positive integer that cannot be expressed as the difference between two primes.

6. **TK921** There are 15 mustangs in a stable. There are 5 blue mustangs, 5 green mustangs, and 5 black mustangs. Yuuki and Anna each pick a different Mustang to ride on randomly. What is the probability they choose Mustangs of different colors? Express your answer as a common fraction.

Written by: Tristan Kay

Answer: $\boxed{\frac{5}{7}}$

Without loss of generality, assume Yuuki picked a blue Mustang. Note that Anna has 10 mustangs to choose from that are not blue, out of a total of 14 mustangs, since Yuuki has already picked one of them. Therefore, our answer is $\frac{10}{14} = \boxed{\frac{5}{7}}$.

7. **LT647** Let $\lfloor x \rfloor$ denote the greatest integer that is less than or equal to x . For example, $\lfloor 3.5 \rfloor = 3$ and $\lfloor -1.3 \rfloor = -2$. Evaluate the following expression:

$$\left\lfloor \frac{10}{10} \right\rfloor + \left\lfloor \frac{10}{9} \right\rfloor + \cdots + \left\lfloor \frac{10}{2} \right\rfloor + \left\lfloor \frac{10}{1} \right\rfloor + \left\lfloor \frac{10}{-1} \right\rfloor + \left\lfloor \frac{10}{-2} \right\rfloor + \cdots + \left\lfloor \frac{10}{-9} \right\rfloor + \left\lfloor \frac{10}{-10} \right\rfloor$$

Written by: Linus Tang

Answer: $\boxed{-6}$

Solution 1:

Regroup the terms as follows:

$$\left(\left\lfloor \frac{10}{10} \right\rfloor + \left\lfloor \frac{10}{-10} \right\rfloor \right) + \left(\left\lfloor \frac{10}{9} \right\rfloor + \left\lfloor \frac{10}{-9} \right\rfloor \right) + \cdots + \left(\left\lfloor \frac{10}{2} \right\rfloor + \left\lfloor \frac{10}{-2} \right\rfloor \right) + \left(\left\lfloor \frac{10}{1} \right\rfloor + \left\lfloor \frac{10}{-1} \right\rfloor \right)$$

Note that each pair is of the form $\lfloor x \rfloor + \lfloor -x \rfloor$.

This is helpful because $\lfloor x \rfloor + \lfloor -x \rfloor = 0$ when x is an integer and $\lfloor x \rfloor + \lfloor -x \rfloor = -1$ when x is not an integer.

It now suffices to count how many pairs above are equal to -1 .

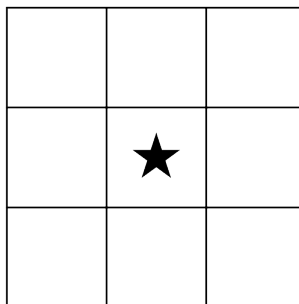
Note that $\frac{10}{10}$, $\frac{10}{5}$, $\frac{10}{2}$, and $\frac{10}{1}$ are integers and the other 6 positive fractions are not integers. Thus, 6 of the pairs are equal to -1 and 4 of them are equal to 0. The overall sum is $\boxed{-6}$.

Solution 2:

Evaluate each term individually and add them up:

$$\begin{aligned} & \left\lfloor \frac{10}{10} \right\rfloor + \left\lfloor \frac{10}{9} \right\rfloor + \cdots + \left\lfloor \frac{10}{2} \right\rfloor + \left\lfloor \frac{10}{1} \right\rfloor + \left\lfloor \frac{10}{-1} \right\rfloor + \left\lfloor \frac{10}{-2} \right\rfloor + \cdots + \left\lfloor \frac{10}{-9} \right\rfloor + \left\lfloor \frac{10}{-10} \right\rfloor \\ &= 1 + 1 + 1 + 1 + 1 + 2 + 2 + 3 + 5 + 10 + (-10) + (-5) + (-4) + (-3) + (-2) + (-2) + \\ & \quad (-2) + (-2) + (-2) + (-1) \\ &= \boxed{-6}. \end{aligned}$$

8. **LT831** In the 3 by 3 grid below, the center cell is marked with a star. How many rectangles created by the gridlines contain the star?



Written by: Linus Tang

Answer: $\boxed{16}$

Observe that a rectangle is bounded by a vertical line to its left, a vertical line to its right, a horizontal line above, and a horizontal line below. To choose a rectangle created by the gridlines that contains the star, there are two choices for each of these four bounding lines, so the number of such rectangles is $2^4 = 16$.

9. **ML804** There exist pairs of primes (p, q) that satisfy the equation $20^2 + p = 23^2 - q$. Compute the sum of all distinct possible values of pq .

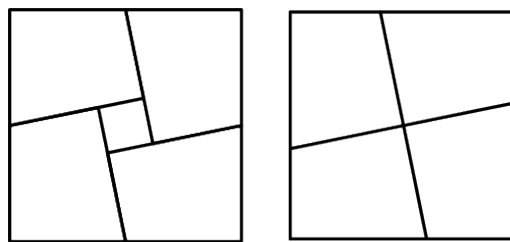
Written by: Michael Li

Answer: $\boxed{254}$

Rearranging terms, we get $23^2 - 20^2 = p + q$. Computing the left side gives $529 - 400 = 129 = p + q$.

Since $p + q$ is odd, one of p, q must be even. But p and q are prime, and 2 is the only even prime number. Thus, p or q is 2 and the other is $129 - 2 = 127$ (which is also prime). The answer is $2 \cdot 127 = \boxed{254}$.

10. **ARP564** A square of side length 5 is cut into four identical quadrilaterals and a unit square, such that the four quadrilaterals can be moved and rotated to form a new square, as shown in the diagram below. What is the sum of the smallest and largest side lengths of one of the four quadrilaterals? Express your answer in simplest radical form.



Before

After

Written by: Arpit Ranasaria

Answer: $2\sqrt{6}$

Notice that the smallest and largest sides of the quadrilateral together make a side of the smaller square. Hence the problem becomes finding the length of the smaller square.

Since the area of the larger square is $5 \cdot 5 = 25$ and we've removed a unit square to make the smaller one, so the area of the smaller square is $25 - 1 = 24$ and the side length is $\sqrt{24} = 2\sqrt{6}$.

11. **DL403** When 47 and 73 are divided by a positive integer n ($n > 1$), it results in the same remainder. How many possible values of n are there?

Written by: Daniel Z Lu

Answer: 3

The largest n that divides 47 and 73 with the same remainder is the difference between 47 and 73, which is 26. Any number greater than 26 will not be able to complete one cycle of remainders and thus will have different remainders for 47 and 73. The factors of 26 that are greater than 1 are 2, 13, and 26. Therefore, there are 3 possible values of n .

12. **TK922** Let $\triangle ABC$ and $\triangle ACD$ be similar right triangles with $\angle ABC = \angle ACD = 90^\circ$ and $\angle BAC = \angle CAD$. If $AB = 20$ and $AD = 23$, what is the ratio of the area of $\triangle ABC$ to $\triangle ACD$? Express your answer as a common fraction.

Written by: Tristan Kay

Answer: $\frac{20}{23}$

Let the ratio between the side lengths of $\triangle ACD$ to $\triangle ABC$ be r . Then we have $\frac{AD}{AC} = \frac{AC}{AB} = r$. Since, $r^2 = \frac{AD}{AB} = \frac{23}{20}$, the wanted ratio is $\frac{1}{r^2} = \frac{20}{23}$. Thus our answer is $23 + 20 = 43$.

Alternate slick solution: draw BD and let BD intersect AC at E . Then we know ABC



to ACD is BE to ED which, by angle bisector theorem, is $\frac{AB}{AD} = \frac{20}{23}$.

13. **LT421** A 6-digit number, N , has the property that each digit (except for the rightmost 2) is the product of the 2 digits to its right. For example, 1111 and 9331 are examples of 4-digit numbers with the given property. What is the largest possible value of N ?

Written by: Linus Tang

Answer: 842212

Since the problem asks for the largest possible value of 6-digit number N , we start by casework on the first digit of N , starting with the largest possible digit.

Case 1: $N = 9$

The next two digits have a product of 9, leaving 3 possibilities: 919—, 933—, and 991—. The first of these possibilities is a dead end, as 1 must be the product of 9 and the next missing digit. The second of these possibilities leads to 93313—, another dead end. Similarly, the third possibility leads to 9919—. Thus, there are no possible values of N with a first digit of 9.

Case 2: $N = 8$

By similar work to Case 1, we find 4 ways to assign the next two digits: 818—, 824—, 842—, and 881—. Only 842— leads to a 6-digit solution, 842212, whereas the other 3 ways result in dead ends similar to those in Case 1.

Checking other possible first digits than 9 and 8 will only lead to smaller solutions, so we have found the largest solution.

14. **YS934** Let O be the sum of all odd integers and E be the sum of all even integers between 0 and 9999, inclusive. Compute $\frac{O+E}{O-E}$.

Written by: Yuuki Sawanoi

Answer: 9999

Notice that $O+E = 0+1+2+\cdots+9999 = \frac{(9999)(10000)}{2} = 5000(9999)$, and $O-E = 9999-9998+9997+\cdots+1-0 = (9999-9998)+(9997-9996)+\cdots+(1-0) = 5000(1) = 5000$. Therefore, our answer is $\frac{(5000)(9999)}{5000} = \span style="border: 1px solid black; padding: 2px;">9999$

15. **SK721** In right triangle $\triangle ABC$ with hypotenuse \overline{AC} , D lies on \overline{AB} such that \overline{DC} bisects angle C . Given that $DB = 1$ and $BC = 2$, compute AC . Express your answer as a common fraction.

Written by: Sebastian Kumar

Answer: $\frac{10}{3}$



Using the the Angle Bisector Theorem, we get $AC = 2AD$.

Let $x = AD$. Then, by the Pythagorean Theorem, we have $(1 + x)^2 + 4 = 4x^2 \Rightarrow 3x^2 - 2x - 5 = 0 \Rightarrow (3x - 5)(x + 1) = 0$. Hence, $x = \frac{5}{3}$ (because x is positive).

Thus, $AC = 2x = \boxed{\frac{10}{3}}$.

16. **MLI927** 4 people each have a distinct number of coins from 1 to 4. If each person gives away all of their coins to someone else in the group at random, what is the probability no one ends up with the same number of coins they started with? Express your answer as a common fraction. Note that a person may receive coins from multiple people.

Written by: Michael Liu

Answer: $\boxed{\frac{23}{27}}$

For the sake of clarity, let's call the person who had one coin Person1, the person with two coins Person2, the person with three coins Person3, and the person with four coins Person4. We will complementary count this. There are two cases when somebody ends up with the same number of coins that they started with.

First, Person1 and Person2 both gives their coins to Person3, while Person4 gives it to somebody else. The probability of this happening is $\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$.

Second, Person1 and Person3 gives their coins to Person4 while Person2 gives it to somebody else. The probability of this happening is also $\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$.

Adding these two values together, we get $\frac{2}{27} + \frac{2}{27} = \frac{4}{27}$. Subtracting this from 1, we get $\frac{23}{27}$ favorable outcomes.

Thus, our answer is $\boxed{\frac{23}{27}}$

17. **AC883** Arpit needs to schedule a meeting with Anna. This week, both are available for 3 uniformly randomly selected days of the week (independent of each other). What is the probability that there is a day where both are available to meet?

Written by: Alex Chen

Answer: $\boxed{\frac{31}{35}}$

We solve using complementary counting.

Consider the four days during which Arpit is unavailable. The only scenario in which Arpit and Anna have no common availability is when all three of Anna's available days are among Arpit's four unavailalbe days. There are $\binom{4}{3} = 4$ possible triples of days that



fall under this scenario, out of $\binom{7}{3} = 35$ possible triples in total.

Thus, the probability that Arpit and Anna have no common availability is $\frac{4}{35}$.

The probability that they have common availability is $1 - \frac{4}{35} = \boxed{\frac{31}{35}}$.

18. **LT260** Let n be a positive integer with exactly 16 positive divisors. If n^2 has exactly 63 positive divisors, how many positive divisors does n^3 have?

Written by: Linus Tang

Answer: $\boxed{160}$

Recall the formula for the number of divisors of a number, which is the product of $e + 1$ over all exponents in the prime factorization.

Enumerating the ways that 16 can be written as such a product, namely $16 = 8 \cdot 2 = 4 \cdot 4 = 4 \cdot 2 \cdot 2 = 2 \cdot 2 \cdot 2 \cdot 2$, we find the corresponding possible prime factorizations p^{15} , p^7q , p^3q^3 , p^3qr , or $pqr s$, where p, q, r , and s are distinct primes.

With that, n^2 is equal to p^{30} , $p^{14}q^2$, p^6q^6 , $p^6q^2r^2$, or $p^2q^2r^2s^2$, which have 31, 45, 49, 63, and 81 positive divisors, respectively.

We are given that n^2 has 63 factors, so $n = p^3qr$. Thus, $n^3 = p^9q^3r^3$, which has $\boxed{160}$ positive divisors.

19. **YS949** Let r be the answer to the third question in this set. Compute the sum of the digits of r .

Written by: Yuuki Sawanoi

Answer: $\boxed{18}$

Since $10^n \equiv 1 \pmod{9}$ for any nonnegative integer n , the sum of the digits of a number has the same remainder as the original number when divided by 9. Thus, the only values we need to check are multiples of 9, and we get that $\boxed{18}$ is the only one that satisfies the cycle.

20. **MLI326** Let u be the answer to the first question in this set. Compute the sum of the two solutions to the quadratic $x^2 + (x - 1)^2 + (x - 2)^2 + \cdots + (x - u)^2 = 2023^2$.

Written by: Michael Liu

Answer: $\boxed{18}$

Solution 1: Let r be a root of the quadratic. If we plug in r to the quadratic, we get:

$$r^2 + (r - 1)^2 + (r - 2)^2 + \cdots + (r - u)^2 = 2023^2.$$



Note that by symmetry, we can also plug in $u - r$ into this quadratic to get:

$$(u - r)^2 + (u - r - 1)^2 + (u - r - 2)^2 + \cdots + (-r)^2 = 2023^2.$$

The left-hand sides of the two equations are the same, which means that the sum of the roots of the quadratic is simply u . After solving the first question in this set, we see that $u = \boxed{18}$.

Solution 2: First note that when expanded out, the equation we are trying to solve for is a quadratic, and the answer is the sum of the roots. So with Vieta's formulas, we only need to compute the coefficient of the x^2 term and the x term. After testing some small cases or using logic, we can see that the coefficient of the x^2 term is the amount of terms in the equation, which is $u + 1$. And that the coefficient of the x term is negative two times the sum $1 + 2 + \cdots + u$, or $-2(\frac{(u+1) \cdot (u)}{2}) = -(u + 1) \cdot (u)$. So using Vieta's, the sum of the roots to the equation is $-(-\frac{u(u+1)}{u+1}) = \boxed{u}$. After solving the cycle, we realize $u = \boxed{18}$.

21. **YS950** Let n be the answer to the second question in this set. Arpit has a collection of n standard 6-sided dice. Compute the total number of dots on Arpit's dice.

Written by: Yuuki Sawanoi

Answer: $\boxed{378}$

There are $1 + 2 + 3 + 4 + 5 + 6 = 21$ dots on each dice. Thus, the answer is $21n$, which, when solving for cycle gives $\boxed{378}$.

22. **CY882** For a positive integer n , the function $f(n)$ switches the units digit and tens digit of n . For how many positive integers $n < 1000$ is $f(n)$ strictly greater than n ? For example, $n = 156$ counts because $165 > 156$ but $n = 193$ does not because $139 < 193$. Note: If you have a single-digit number, assume the tens digit is 0. For example, 1 becomes 10 after switching digits.

Written by: Carsten Yeung

Answer: $\boxed{450}$

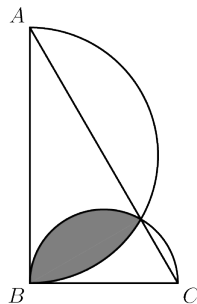
Notice that the digit in the hundreds digit will not be altered in any way by f , so there are 10 choices of digits in the hundreds place.

For $n < f(n)$, the units-digit must be strictly greater than the tens-digit. If we choose any two digits from $\{0, 1, 2, \dots, 9\}$, there is exactly one way to order those two digits so that the greater one comes first. For example, if we choose 0 and 1, the ten's digit must be 1 and the one's digits must be 0. Thus the amount of digit pairs that satisfies this will be the amount of ways to choose two numbers unordered from 0 to 9 which is $\binom{10}{2}$.

Thus, our answer is $10 \cdot \binom{10}{2} = 10 \cdot \frac{10 \cdot 9}{2} = \boxed{450}$.

23. **YS935** Right triangle ABC has $AB = 12\sqrt{3}$, $BC = 12$, and $AC = 24$. Let semicircles

C_1 and C_2 have diameters \overline{AB} and \overline{BC} , respectively. Compute the area of overlap between C_1 and C_2 . Express your answer in simplest radical form in terms of π .



Written by: Yuuki Sawanoi

Answer: $30\pi - 36\sqrt{3}$

The key observation in this problem is that $\overline{AC} = 2\overline{BC}$ and $\overline{AB} = \overline{BC}\sqrt{3}$, which tells us that ABC is a 30-60-90 triangle. Specifically, $\angle ACB = 60^\circ$ and $\angle BAC = 30^\circ$.

Next, define O_1 and O_2 to be the centers of C_1 and C_2 respectively, define D to be the point of intersection between the two circles, and draw segments O_1D , O_2D , and BD .

This gives us a few new shapes: Triangles O_1DB and O_2DB , and sectors O_1DB and O_2DB . Now, the answer we are looking for is (Area of Sector O_1DB - Area of Triangle O_1DB) + (Area of Sector O_2DB + Area of Triangle O_2DB).

To find these areas individually, we can use the fact that the four line segments involved are radii and say that $\overline{O_1D} = \overline{O_1B} = 6\sqrt{3}$ and $\overline{O_2D} = \overline{O_2B} = 6$. In addition, angles $\angle BO_1D$ and $\angle BO_2D$, by the Inscribed Angle Theorem, are 60° and 120° respectively.

Now, since we know the radii and angles, we can calculate the areas of sectors O_1DB and O_2DB to be 18π and 12π respectively. We can also calculate the areas of the triangles with the sin area formula and get $[\triangle O_1DB] = 27\sqrt{3}$ and $[\triangle O_2DB] = 9\sqrt{3}$. Plugging these values to the equation from earlier, we get our answer to be $12\pi + 18\pi - 27\sqrt{3} - 9\sqrt{3} = 30\pi - 36\sqrt{3}$.

24. **LT754** Amy chooses a random integer from 0 to 999, inclusive, and multiplies it by 11. What is the probability that the product is a palindrome with 4 digits? Express your answer as a common fraction. (A palindrome is a positive integer, such as 30303 or 5885, that is the same when read forward or backward)

Written by: Linus Tang

Answer: $\frac{9}{100}$

A 4-digit palindrome is a number of the form \overline{abba} , where a and b are digits and $a \neq 0$. There are 90 4-digit palindromes because there are 9 choices for a and 10 choices for b . By the divisibility by 11 rule, all 90 are multiples of 11. Furthermore, they are multiples



of 11 that lie between 11×0 and 11×999 . Thus, there are 90 integers within the range 0-999 that Amy could have chosen such that the product is a palindrome with 4 digits.

The probability is $\frac{90}{1000} = \boxed{\frac{9}{100}}$.

25. **TK951** Estimate the expression below to the nearest integer.

$$\frac{3}{2} \times \frac{5}{4} \times \frac{7}{6} \times \cdots \times \frac{2023}{2022}$$

Submit a positive integer N . If the correct answer is A , you will receive $\max(25 - |A - N|, 0)$ points (if you do not submit a positive integer, you will receive 0 points).

Written by: Tristan Kay

Answer: $\boxed{36}$

Let A be the exact value of the expression. Let

$$B = \frac{2}{1} \times \frac{4}{3} \times \frac{6}{5} \times \cdots \times \frac{2022}{2021}$$

Then notice that $AB = 2023$. If we assume $A \approx B$, then we get $B \approx \sqrt{2023} = 45$, giving us 16 points. However, we also have

$$\frac{A}{B} = \frac{1 \times 3^2 \times 5^2 \times 7^2 \times \cdots \times 2021^2 \times 2023}{2^2 \times 4^2 \times 6^2 \times \cdots \times 2022^2}$$

We can approximate $3^2 \times 5^2 \times \cdots \times 2021^2 \approx 2 \times 4^2 \times 6^2 \times \cdots \times 2020^2 \times 2022$ yielding

$$\frac{A}{B} \approx \frac{1 \times 2023}{2 \times 2022} \approx \frac{1}{2}$$

Hence, we have $AB \times \frac{A}{B} = A^2 \approx \frac{2023}{2}$, thus $A \approx \sqrt{\frac{2023}{2}} \approx \frac{45}{\sqrt{2}} \approx 32$.

Refining this estimation of $\frac{A}{B}$ can continue to give us increasingly close answers.

26. **TK953** Anna the Anaconda initially has 1 gold coin. Every second, she has a 50% chance of doubling the number of coins she currently has. Else, she has a 50% chance of losing one coin. This process terminates whenever Anna loses all of her coins. The probability that Anna still has coins after 10^{2023} seconds is p . Estimate $\lfloor 100p \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x .

Submit a positive integer N . If the correct answer is A , you will receive $\max(25 - 5|A - N|, 0)$ points (if you do not submit a positive integer, you will receive 0 points).

Written by: Tristan Kay

Answer: $\boxed{29}$

Let x_n denote the probability that Anna continues to have coins after 10^{2023} seconds



when she has n coins. Then, we have that $x_0 = 0$, $x_n = \frac{x_{n-1} + x_{2n}}{2}$, and as n approaches infinity, $x_n = 1$. Using this, we can write

$$2x_1 = x_2$$

$$2x_2 = x_1 + x_4$$

$$2x_4 = x_3 + x_8$$

$$4x_4 = x_2 + x_6 + 2x_8$$

$$3x_1 = x_4$$

$$4x_4 = 2x_1 + x_6 + 2x_8$$

$$10x_1 = x_6 + 2x_8$$

Logically, notice that it is quite difficult for Anna to get back to zero gold coins from 6 gold coins (one way to think about this is that for her to lose 6 coins in a row, there is a $\frac{1}{2^6} \approx 2\%$ of that happening. Therefore, we can approximate both x_6 and x_8 as very close to 1, to get

$$10x_1 \approx 1 + 2(1) = 3$$

$$x_1 \approx 0.3$$

But notice how this slightly over estimates, giving us an answer of $\lfloor 100x_1 \rfloor = \boxed{29}$.

Monte Carlo simulations of this problem yield an approximate value of 29.5%.

27. **TK954** For nonnegative integers M, A, T, H, F, U , and N , let X be the number of possible quadruplets (M, A, T, H) and Y be the number of possible triplets (F, U, N) that satisfy the equations below. Estimate $\frac{X}{Y}$ to the nearest integer.

$$17M + 19A + 21T + 23H = 2023$$

$$9F + 10U + 11N = 1011$$

Submit a positive integer N . If the correct answer is A , you will receive $\max(25 - (A - N)^2, 0)$ points for this question (if you do not submit a positive integer, you will receive 0 points).

Written by: Tristan Kay

Answer: $\boxed{18}$

If we first look at the number of solutions to

$$17M + 19A + 21T \leq 2023$$

assuming the number of lattice points are approximately equal to the total volume of the tetrahedron bounded by the plane and the xy -, yz -, and xz -axis, we get there are approximately $\frac{1}{6} \times \frac{2023}{17} \times \frac{2023}{19} \times \frac{2023}{21}$. Note that approximately $\frac{1}{23}$ of these lattice points will yield a solution $17M + 19A + 21T + 23H = 2023$. Thus, we have

$$X \approx \frac{2023^3}{6 \times 17 \times 19 \times 21 \times 23}$$



Now using similar logic for $9F + 10U + 11N = 1011$, there are

$$Y \approx \frac{1011^2}{2 \times 9 \times 10 \times 11}$$

Hence, our answer is around $\frac{2023^3 \times 2 \times 9 \times 10 \times 11}{1011^2 \times 6 \times 17 \times 19 \times 21 \times 23} \approx 2023 \times 2^2 \times \frac{1}{3} \times \frac{1}{2^3} \times \frac{1}{20} = \frac{2023}{6 \times 20} = 16.8$, close enough to give the vast majority of points.

More careful calculation of the above quantity yields 17.14. It can also be noted that the above is known to be an over estimate because the volume is strictly less than the number of lattice points.

The actual value of X and Y are 9379 and 532 respectively, so $\frac{X}{Y}$ is actually around 17.64 or 18.