Design Exercise 2013-2014

Gas generator for an automobile

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1. Design characteristics

The next report presents the procedure and results of the design of a single stage turbine that provides work to a shaft in order to propel an automobile.

The constraints of design are the next ones:

- Compressor ratio $\pi_c = 4$
- Inlet total pressure $P_{01} = 101325\ Pa$
- Inlet total temperature $T_{01} = 288\ K$
- Net power to wheels $150\ kW$
- Exit combustor temperature $T_{03} = 1400\ K$

Fluid properties

- $\Upsilon_c = 1.4$
- $C_{p,c} = 1000 \frac{J}{kgK}$
- $\Upsilon_t = 1.3$
- $C_{p,t} = 1240 \frac{J}{kgK}$
- $R = 287 \frac{J}{kgK}$

2. Cycle analysis

First of all an analysis of the cycle is needed in order to compute the rest of needed flow quantities from the design constraints. Here there will be computed for instance parameters as the fuel or air massflow through the gas generator, the expansion ratio or the pressure at the exit of the combustor.

In order to solve the cycle the next assumptions are taken

- All the work extracted in a single stage
- Compressor efficiency $\eta_{compressor} = 0.88$
- Shaft efficiency $\eta_{shaft} = 0.96$
- Turbine efficiency $\eta_{turbine} = 0.91$
- Burner efficiency $\eta_{burner} = 0.88$
- Combustor pressure efficiency $\eta_{combustor,p} = 0.98$
- Transmission efficiency $\eta_{trans} = 0.89$

It will be assumed that the fuel that is feeding the combustor is gasoline, what seems a good approach since it is for an automobile. The gasoline energy density

$$L = 4440000 \left[\frac{J}{kg}\right]$$

The inlet conditions for the **compressor** are

$$P_{01} = 101325\ [Pa]$$

$$T_{01} = 288\ [K]$$

In the compressor the compression ratio is given, leading to the pressure ahead of the combustion chamber

$$P_{02} = P_{01} * \pi_c\ [Pa]$$

Following an isentropic evolution

$$T_{02is} = T_{01} * (\pi_c)^{\frac{\Upsilon-1}{\Upsilon}} [K]$$

And then considering the compressor efficiency, the real total temperature

$$T_{02} = T_{01} + \frac{T_{02s} - T_{01}}{\eta_{compressor}}\ [K]$$

Thus, the work required by the compressor can be computed as the difference of total temperature

$$\Delta H_{compresssor} = c_{p,c} * (T_{02} - T_{01}) \left[\frac{J}{kg}\right]$$

The next element that the flow faces is the **combustor**

In the combustor the flow is heated up till the outlet gas temperature, which was given as design constraint, T_{03} = 1400 K. Due to the combustion process and the losses that take place in the chamber the total pressure is slightly reduced. Considering the pressure losses that have been assumed, the pressure at the outlet of the combustion chamber can be computed

$$P_{03} = P_{02}.*\ \eta_{combustor}\ [Pa]$$

From the inlet and exit combustor temperature the temperature increase in the combustor can be obtained

$$\Delta T = T_{03} - T_{02} [K]$$

From the burner efficiency, the gasoline mass energy density and the temperature change the fuel to massflow ratio can be estimated just considering the energy balance.

$$(m_a + m_f) c_{p,t} * T_{03} = m_a c_{p,c} T_{02} + m_f L \eta_{burner}$$

$$f = \frac{m_f}{m_a}$$

$$f = \frac{c_{p,t} T_{03} - c_{p,c} T_{02}}{\eta_{burner} L - c_{p,t} T_{03}}$$

Then the flow is expanded on the **turbine**. Where the work to move the compressor and propel the shaft of the car should be extracted. In the turbine it will be assumed that the expansion is done till P_{04}, what will mean that the static pressure at the outlet will be lower than the atmospheric one.

$$P_{04} = P_{01} = 101325\ [Pa]$$

The expansion ratio in the turbine can be already obtained

$$\pi_{turbine} = \frac{P_{03}}{P_{04}}$$

Then following an isentropic process the turbine outlet temperature, in case no losses take place, can be obtained.

$$T_{04is} = T_{03} * \left(\frac{P_{04}}{P_{o3}}\right)^{\frac{(\Upsilon-1)}{\Upsilon}}\ [K]$$

Although, considering the turbine efficiency

$$T_{04} = T_{03} - \eta_{turbine} * (T_{03} - T_{04})\ [K]$$

So finally the real work available in the turbine is

$$\Delta H_{avaliable} = c_{p,t} * (T_{03} - T_{04}) \left[\frac{J}{kg}\right]$$

The turbine needs to provide the constrained net power to the wheels. So first of all it is needed to subtract from this power the compressor work, considering the shaft efficiency, and also the increase of flow (fuel addition)

$$\Delta H_{available,transmision} = \Delta H_{avaliable} - \frac{\Delta H_c}{\eta_{shaft}} * \frac{1}{1+f} \; \left[\frac{J}{kg}\right]$$

Finally the net power that will be provided is just function of the available specific power considering the car transmission efficiency times the massflow

$$Net\ power = (m_a + m_f)\, \Delta H_{available,transmision}\, \eta_{transmision} \; \left[\frac{J}{s}\right]$$

Extracting the massflow

$$(m_a + m_f) = m_a * (1+f) = \frac{Net\ power}{\Delta H * \eta_{transmision}}$$

$$m_a = \frac{Net\ power}{\Delta H * \eta_{transmision}\,(1+f)} \left[\frac{kg}{s}\right]$$

In the next graph the H-S diagram correspondent to the cycle that has been described is presented

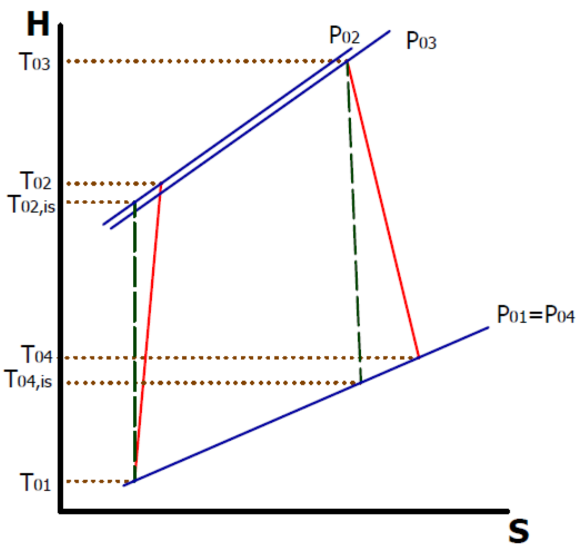

Figure 1: HS Diagram of the cycle

The value that was assumed for the turbine efficiency will be changed according to the losses estimation that will be computed in the subsequent parts of the design procedure. The rest of assumptions will be unchanged. This is the reason because of numerical values were not shown, since they will be changed in accordance with the losses modification. In the next tables the final assumptions and the global parameters that are the output of the cycle analysis will be presented.

Assumptions		
Compressor efficiency	0,82	[-]
Shaft efficiency	0,94	[-]
Turbine efficiency	0,836	[-]
Transmission efficiency	0,89	[-]
Fuel energy density (gasoline)	44400000	[J/kg]
Combustor P efficiency	0,98	[-]
Combustor burner efficiency	0,89	[-]

Global Parameters		
AHC	170690,68	[J/Kg]
π_c	4	[-]
Expansion ratio	3,92	[-]
P0exit	101325	[Pa]
T05	1083,52975	[K]
AHT available	392423,111	[J/Kg]
$\dot{m}_a + \dot{m}_f$	0,7775	[Kg/s]
$\dot{m}_f$	0,02542636	[Kg/s]
$\dot{m}_a$	0,75205594	[Kg/s]
AHT avaliable - AHC	216775,77	[J/Kg]
(AHT available-AHC)*η	192930,435	[J/Kg]

3. 1D Design

The next stage is the design based in a 1D analysis. The main constraints for the design are the expansion ratio, the massflow and the required specific work, all of them coming from the cycle analysis. First of a couple of figures with the description of the names of the sections and the angles and velocities criteria are presented.

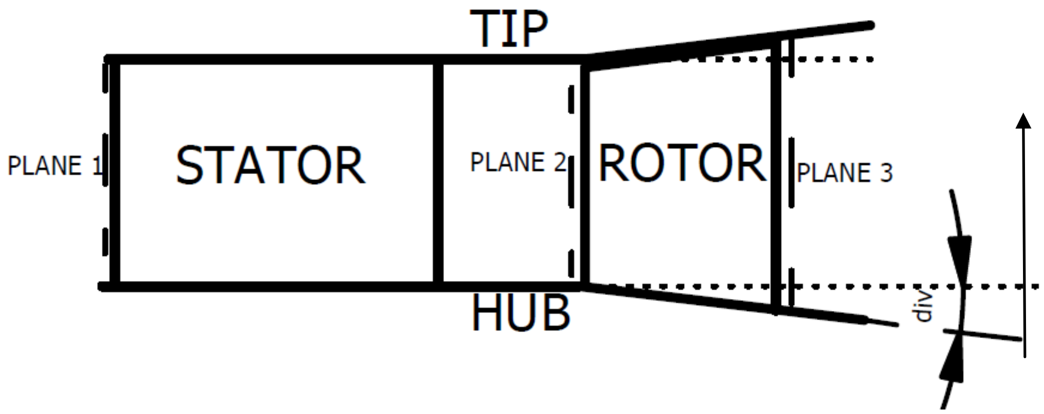

Figure 2: Sketch of the axial distribution of the stage

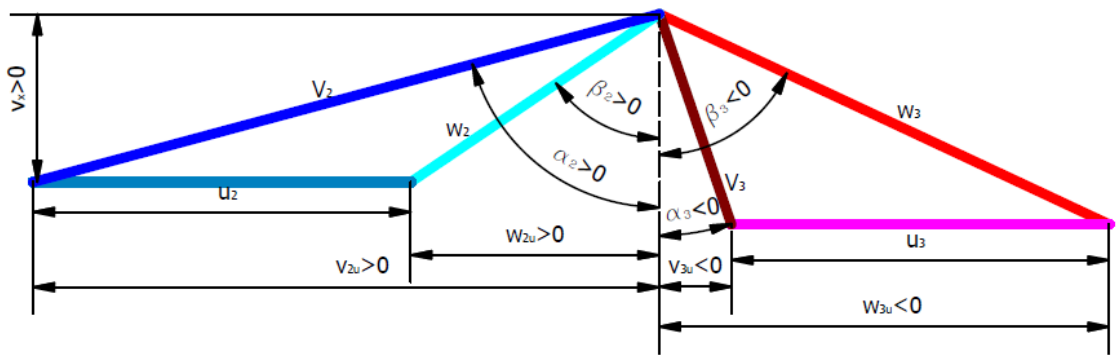

Figure 3: Velocity triangle criteria

As designers the process starts with the choice of a degree of reaction and the choice of a Mach number at the exit.

Assuming a Mach number in the exit, since the total pressure there is known; using the compressibility relations the static pressure can be computed. The final value of the exit Mach number (M_3) will be set via an iterative process, where it will be needed to match this imposition and the value that will be computed following the cycle. The designer task is to reduce as much as possible the outlet Mach number, thus reducing the ejection of kinetic energy.

$$P_3 = \frac{P_{03}}{\left(1 + \frac{\Upsilon - 1}{2} M_3^2\right)^{\frac{\Upsilon}{\Upsilon - 1}}}$$

Then from the definition of the degree of reaction the value of the static pressure in the plane 2 can be obtained. The degree of reaction will be one of the main relevant parameters defining the final performance of the machine. In this case the designer has decided to take a mid value, 0.32. The higher the degree of reaction the best the expected performance of the turbine, since the machine with higher degree of reaction came out with fewer losses. Although a degree of reaction extremely high will lead to a really high divergence in the rotor section and will also lead to higher values of turning. Since the huge amount of work extraction and the high expansion ration that is required push this machine to its limits, in terms of turning and peripheral speed, the selection of the degree of reaction has been based on the maximum possible that leads to an assumable turning and not excessive divergence in the rotor. Then from the selection of the degree of reaction and the values of the total pressure ahead of the stage and the exit static pressure, the intermediate static pressure can be obtained.

$$r_d = 1 - \frac{1-\left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left(\frac{P_3}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}}$$

Following an isentropic evolution it can be computed the value of the ideal temperature in the stator. In the stator no work is done, consequently the absolute total temperature is kept constant

$$T_{02} = T_{01}$$

$$T_{2is} = T_{02}\left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

Then assuming a certain stator efficiency

$$\eta_{st} = \frac{T_{02}-T_2}{T_{02}-T_{2is}}$$

The real static temperature

$$T_2 = T_{02} - \eta_{st}(T_{02}-T_{2is})$$

And knowing the static temperatures and the total one, the velocities can be obtained, both isentropic and real

$$V_2 = \sqrt{2c_p(T_{02}-T_2)}$$

$$V_{2is} = \sqrt{2c_p(T_{02}-T_{2is})}$$

Knowing all the stator static quantities the density can be computed

$$\rho_2 = \frac{P_2}{RT_2}$$

Also the speed of sound

$$a_2 = \sqrt{\Upsilon R T_2}$$

And the Mach number

$$M_2 = \frac{V_2}{a_2}$$

And following the isentropic evolution between the static and the total quantities, the total pressure in this plane

$$P_{02} = P_2 \left(\frac{T_{02}}{T_2}\right)^{\frac{\Upsilon}{\Upsilon-1}}$$

The stator kinetic loss coefficient can be obtained

$$\xi = \frac{V_{2is}^2 - V_2^2}{V_2^2}$$

no coincide?

And also the pressure loss coefficient in the stator

$$\omega = \frac{P_{01} - P_{02}}{P_{02} - P_2}$$

Assuming a value of the absolute outlet angle of the stator, the procedure will start with a first guess, and then it will modified in order to provide the power that is being required. In a similar way this value should also be corrected in order to avoid a huge increase of area in the rotor. Making it more tangential the area increase across the rotor is reduced. As common design limits, the absolute inlet flow angle α_2 should be in the range between 70 – 75º for HP turbines. Assuming a certain value the axial component in the absolute frame of reference

$$V_{2x} = V_2 \cos(\alpha_2)$$

And the tangential

$$V_{2u} = V_2 \sin(\alpha_2)$$

To select a peripheral speed the approximation based in the loading factor can be used, although it must be checked that the limit of 500 m/s is not exceeded. Consequently an assumption on the loading factor needs to be taken. The selection of the loading factor needs to be in agreement with the degree of reaction. A low reaction machine can have a loading factor around 2 and a high reaction one around 1.5. This machine is being designed for a degree of reaction of 0.32, so a value of 1.75 has been taken for this design.

$$U_2 = \sqrt{\frac{\Delta H}{\psi}}$$

Then solving the triangle

$$W_{2u} = V_{2u} - U_2$$

And together with the axial speed

$$W_2 = \sqrt{W_{2u}^2 + V_{2x}^2}$$

Thus the relative inlet angle and the relative Mach number

$$\beta_2 = \mathrm{atan}\left(\frac{W_{2u}}{V_{2x}}\right)$$

$$M_{2r} = w_2/a_2$$

The relative total quantities at the inlet of the rotor can be obtained using the compressible relationships and the isentropic evolution.

$$\frac{T}{T_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1}$$

$$T_{02r} = \left(1 + \frac{\gamma - 1}{2} M_{2r}^2\right) * T_2$$

$$\rightarrow T_{02r} = T_2\left(1 + \frac{\gamma-1}{2} M^2\right)$$

$$P_{02r} = \left(1 + \frac{\gamma - 1}{2} M_{2r}^2\right)^{\frac{\gamma}{\gamma-1}} * P_2$$

In the rotor the relative total temperature is kept constant, no work is extracted in the relative frame of reference

$$T_{02r} = T_{03r}$$

At the beginning a value of the exit Mach number was assumed leading to a value of the outlet static pressure. Following an isentropic evolution from the total relative conditions to the static the ideal outlet temperature can be estimated.

$$T_{3is} = T_{03r}\left(\frac{P_3}{P_{02r}}\right)^{\frac{\gamma-1}{\gamma}}$$

Then with the assumption of the rotor efficiency, the real static temperature value is obtained

$$\eta_{st} = \frac{T_{03r} - T_3}{T_{03r} - T_{3is}}$$

And knowing the temperatures the velocities can be calculated.

$$W_3 = \sqrt{2c_p(T_{03r} - T_3)}$$

$$W_{3is} = \sqrt{2c_p(T_{03r} - T_{3is})}$$

Once all the rotor static quantities are known, the density can be computed

$$\rho_3 = \frac{P_3}{RT_3}$$

Also the speed of sound and the Mach numbers

$$a_3 = \sqrt{\Upsilon R T_3}$$

$$M_{3r} = \frac{W_3}{a_3}$$

$$P_{03r} = P_3 \left(\frac{T_{03r}}{T_3}\right)^{\frac{\Upsilon}{\Upsilon-1}}$$

Then an estimation of the rotor losses can be computed, the rotor kinetic loss

$$\xi = (W_{3is}^2 - W_3^2)/W_3^2$$

And for the pressure loss coefficient

$$\omega = \frac{P_{02r} - P_{03r}}{P_{03r} - P_3}$$

Assuming a value of β_3 as was done for α_2, the components of the relative velocity can be extracted. Although this choice is not completely free, the selection must be based in the work that the turbine must provide. For high pressure turbines the value should be limited between 60º and 65º.

$$W_{3x} = W_3 \cos(\beta_3)$$

And the tangential

$$W_{3u} = W_3 \sin(\beta_3)$$

If the design of the machine is done in a way that the mean radius is kept constant the value of the peripheral speed does not change from the inlet to the outlet of the rotor

$$U_3 = U_2$$

And hence the tangential component of the speed at the outlet of the stator

$$V_{3u} = W_{3u} - U_3$$

Here is where it should be checked that the work that is being provided, and compare it with the work that is required. The difference should be less than 1 % of deviation. If the work provided by the turbine is not enough the values of β_3 and α_2 should be modified. It's recommended to set a constant value for α_2 and modify the relative exit angle. Making use of the Euler formulation for the work extraction a correction for β_3 is prescribed. Then the user should iterate with the design in order to match the work. And later on if it is needed modify the value of the absolute inlet angle to avoid an excessive increase of area in the rotor.

The work based on the Euler formulation can be obtained as

$$\Delta H = U_2(V_{2u} - V_{3u})$$

The procedure can continue extracting the value of the speed in the absolute frame o reference

$$V_{3x} = W_{3x}$$

$$V_3 = \sqrt{V_{3x}^2 + V_{3u}^2}$$

The velocity triangle of the 1D Analysis can be completed

$$\alpha_3 = asin\left(\frac{V_{3u}}{V_3}\right)$$

Then the value of the absolute Mach number at the exit can be extracted, which should be compared with the one that was assumed at the beginning. In case it's not matching a new input should be set and iterate till the convergence.

$$M_3 = V_3/a_3$$

$$T_{03} = \left(1 + \frac{\Upsilon - 1}{2} M_3^2\right)$$

$$P_{03} = \left(1 + \frac{\Upsilon - 1}{2} M_3^2\right)^{\frac{\Upsilon}{\Upsilon - 1}}$$

Now that the total temperatures ahead and after the turbine are known the work extraction can be obtained, and the comparison of this value with the one obtained with the Euler formula will give an indication of the global matching of the design.

$$\Delta H = c_p(T_{01} - T_{03})$$

The process will continue now obtaining the geometrical characteristics of the turbine. The massflow that pass through the stage was computed in the cycle analysis

$$\dot{m} = \dot{m}_{cycle}$$

Then assuming a value for the hub/tip radius factor

$$\frac{R_H}{R_T} = 0.9$$

$$R_T = \sqrt{\frac{\dot{m}}{\pi \rho_2 V_{2x}\left(1 - \left(\frac{R_H}{R_T}\right)^2\right)}}$$

The blade height can be computed and so the mean radius

$$h_2 = R_T - R_H$$

$$R_m = R_T - \frac{h_2}{2}$$

From the mean radius and the peripheral speed the angular speed of the rotor can be computed

$$\Omega = U_2/R_m$$

Taking into account the angular speed and the tip radius the maximum peripheral speed should be computed and check that its value doesn't exceed excessively the limit of 500 m/s.

The Area in the stage 2

$$A_2 = \pi(R_T^2 - R_H^2)$$

In order to know what are the conditions at the inlet of the stator some further assumptions need to be taken. It will be assumed that the flow enters completely axial to the stage, $\alpha_1 = 0$. The second assumption will be to consider that there is no change of area across the stator and then that the stator is a straight channel, as was shown in the **Figure 2**.

Then since the massflow is known an iterative procedure could be performed in order to compute the speed and the flow conditions at the inlet. If an assumption on the density is performed ρ_{1guess}

The global speed of the flow can be computed at the inlet

$$V_1 = \sqrt{2c_pT_{01}\left(1 - \left(\frac{\rho_{1guess}}{\rho_{01}}\right)^{\Upsilon-1}\right)}$$

And the area in the stage 1

$$A_1 = \left(\frac{\dot{m}}{\rho_{1guess}V_1}\right)$$

Then iterating till $A_1 = A_2$, the value of ρ_1 and V_1 will be known.

For the station 3 if the mean radius is kept constant, what means that both hub and tip radius will be modified as was shown in the **Figure 2.** The span of the blade at the outlet of the rotor

$$h_3 = \frac{\dot{m}}{\pi 2 R_m \rho_3 V_{x3}}$$

The values of the limiting radius and the area

$$R_{3T} = R_m + \frac{h_3}{2}$$

$$R_{3H} = R_m - \frac{h_3}{2}$$

$$A_3 = \pi(R_{3T}^2 - R_{3H}^2)$$

Some parameters should be checked before going further in the design, as the turning and the area increase through the rotor

$$\Delta\beta = \beta_2 - \beta_3$$

$$\Delta\alpha_r = \alpha_2 - \alpha_3$$

$$A_{ratio} = \frac{A_3}{A_2}$$

The turning in a common HP turbine should not exceed 120º and the area ratio should be lower than 1.2. Since the power requirement for this machine is on limit conditions, a huge specific power is being required, this values will reach their limiting thresholds. Particularly the turning will exceed this suggested limit.

The degree of reaction that was used to compute the value of the pressure in between rotor and stator was based in a pressure definition. However in the design descriptions is more common to refer to the enthalpy definition

$$R_d = (T_2 - T_3)/(T_1 - T_3)$$

The global efficiencies of the stage can be computed and their agreement with the efficiency that was assumed in the cycle analysis must be checked. In case there was no agreement, the value of the assumption in the cycle should be modified

$$\eta_{TT} = (T_{01} - T_{03})/(T_{01} - T_{03ss})$$

$$\eta_{TT} = (T_{01} - T_{03})/(T_{01} - T_{3ss})$$

Where the value of T_{3ss} comes from the isentropic process from the inlet pressure to the outlet one. Assuming isentropic expansion in both rotor and stator

$$T_{3ss} = T_{03}\left(\frac{P_3}{P_{03}}\right)^{\frac{\Upsilon-1}{\Upsilon}}$$

Although in order to check if the efficiencies that are assumed in the rotor and in the stator are realistic values, a comparison with the losses estimation based on the Soderberg correlation should be made. This will give the designer a first idea of the performance of the machine and provide a better prescription on the losses.

In case that the losses estimation coming from Soderberg correlation are not in agreement with the losses that have been assumed, an iterative procedure should be made. In which the cycle efficiency should be changed too in case variations of the global efficiency takes place. Just as a reminder, inside each one of the iterations the user should check that the design is matching the losses estimation, the work and that the divergence and turning are within the security range.

The Soderberg correlation provides the estimation the kinetic losses based in some geometrical and flow characteristics.

The correlation is applied for an optimum geometrical and flow conditions and further corrections for the real geometry and the real flow conditions should be applied.

The losses for optimum geometrical characteristics are based in the deflection of the flow, across the stator the change of the absolute angle $(\alpha_1 + \alpha_2)$, and across the rotor in the relative frame of reference $(\beta_2 - \beta_3)$

$$\xi^* = 0.04 + 0.06\left(\frac{(\Delta\beta, \Delta\alpha)}{100}\right)^2$$

And the corrections for the real geometrical conditions

For the stator

$$1 + \xi_s = (1 + \xi_s^*)(0.993 + 0.021c/h\,)$$

For the rotor

$$1 + \xi_r = (1 + \xi_r^*)(0.975 + 0.075c/h\,)$$

The aspect ratio h/c for the stator could be between 0.5-0.8, while for the rotor the chords are typically smaller and it should be set to a value between 1.1 – 1.5.

In this case the designer selection for the stator has been a slightly high value to reduce the stator chord.

$$\left(\frac{h}{c}\right)_{STATOR} = 0.7$$

And for the rotor a higher ratio has been chosen. The selection of these ratios is based in general values for turbine designs.

$$\left(\frac{h}{c}\right)_{ROTOR} = 1.4$$

Since the height of the stages was already known the chords can be computed

$$c_{STATOR} = 0.017\ [\mathrm{m}]$$

$$c_{ROTOR} = 0.01\ [\mathrm{m}]$$

Another correction should be made for the Reynolds number.

$$\xi = \xi\left(\frac{10^5}{Re}\right)^{\frac{1}{4}}$$

Where the Reynolds number should be based on the hydraulic diameter

$$D_h = 2\,g\,h\frac{cos(\alpha_2)}{g\cos(\alpha_2) + h}$$

The pitch can be estimated making use of the Zweifel criteria, where the pitch to chord ratio can be computed as function of a work parameter and some geometrical characteristics

$$\psi^* = \frac{2g}{c\cos\left(\Upsilon_{st}\right)}\cos^2(\alpha_2)\left(\frac{V_{x1}}{V_{x2}}\tan(\alpha_1) + \tan(\alpha_2)\right)\left(\frac{h_2}{h_1}\right)P_{02}/P_{01}$$

In this formula all the values are known for both stator and rotor. Except forΥ_{st}, which is the stagger angle, and the ratio between pitch and chord. The stagger angle can be estimated as function of the inlet and outlet angles based on a correlation of Kacker & Okapuu.

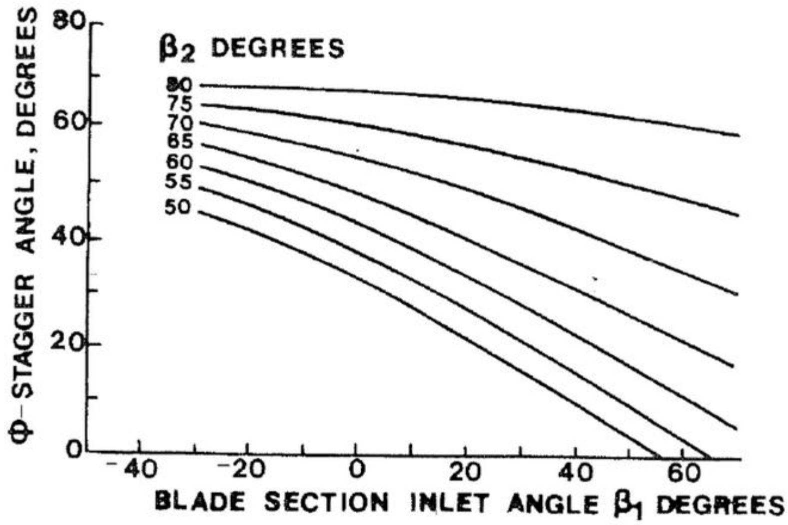

Figure 4: Stagger angle selection, form Kacker and Okapuu

The value for ψ^* should be between 0.6-0.8 depending on the characteristics of the design and the designer criteria. The value that has been selected for the work coefficient in the stator is of 0.6 and for the rotor a value of 0.8 has been prescribed. These values have been selected in order to limit the solidities between 0.6 and 0.75, so limiting the blade loading to a reasonable value.

Once the pitches are also known, the hydraulic diameters can be computed and the Reynolds number and the correction for the losses obtained.

The value found from the correlation is compared with the kinematic losses that were computed, and in case they do not match within a 1% of deviation, the assumption of the losses should be modified till agreement is reached. A change on the losses coefficients for stator and rotor will lead to a change in the global losses of the turbine so the iteration should include the modification of the cycle Analysis, which could lead to a variation on the massflow, the work extraction or the expansion ratio.

Apart from the area ratio, also the divergence angle in the hub and tip should be limited. This angle should not exceed the value of 15º in order to avoid the separation of the boundary layer.

The divergence in both end walls will be the same since in the design the mean radius has been kept constant

$$div = \operatorname{atan}\left(\frac{R_{t3} - R_{t2}}{c_{rotor}}\right)$$

More relevant parameters that can be computed are

The flow coefficient

$$\Phi = V_{3x}/U_3$$

The number of blades for both the stator and the rotor, that knowing the mean radius and the pitch

$$n_{blades} = \frac{2\,\pi\,r_m}{g}$$

Just as reminder the parameters that need to be assumed and modified according to the constraints, limitations or performance characteristics are the next ones

- M_3
- R_d
- α_2
- β_3
- ψ
- η_{stator}
- η_{rotot}

And the constraints that need to be satisfied

- $\dot{m}$
- ΔH
- Expansion ratio

Reasonable limits that should be taken into account

- M_3< 0.45
- α_2 < 75
- β_3 < 65
- $\Delta\beta < 120$
- $A_{ratio} < 1.2$

As mentioned before these limits are common thresholds for HP turbine designs. Although depending on the particular requirements of each machine some of them could be slightly exceeded. Anyway they set the security margins for the design and the designer should take them into account in order to keep a proper turbine efficiency.

Following the procedure that has been explained the 1D Design has taken place, a resume of the relevant quantities is listed in the next tables. Also a representation of the velocity triangle can be found.

Resume stage1		
T01	1400	[K]
P01	397194	Pa
V1	93,45861036	[m/s]
T1	1396,478019	[K]
P1	392882,1612	[Pa]
$\rho 1$	0,98317	[kg/m^3]
A1	0,008461407	m2
M1	0,129667398	[-]
$\boldsymbol{\alpha_1}$	0,000	[º]
$\dot{m}_1$	0,777482308	[kg/s]
Rt1	0,119061007	[m]
Rh1	0,107154906	[m]
h1 stator	0,011906101	[m]

Resume stator				Resume rotor		
T02	1400,000	[K]		**M3 (imposed)**	0,420	[-]
P02	353263,772	[Pa]		**P3**	90483,172	[Pa]
V2	778,667	[m/s]		**T03**	1085,120	[K]
T2	1155,515	[K]		**P03**	101372,147	[Pa]
P2 (chosen)	153781,890	[Pa]		**V3**	263,917	[m/s]
ρ2	0,465	[kg/m^3]		**T3**	1057,035	[K]
T2is	1124,679	[K]		**P3**	90483,172	[Pa]
M2	1,188	[-]		**ρ3**	0,299	[kg/m^3]
α_2	75,300	[º]		**T3is**	1022,400	[K]
V2x	197,593	[m/s]		**T3iss**	995,116	[K]
V2u	753,180	[m/s]		**M3 (achieved)**	0,421	[-]
W2	342,404	[m/s]		**α_3**	-15,686	[º]
T02r	1202,789	[K]		**V3x**	254,089	[m/s]
P02r	182964,074	[Pa]		**V3u**	-71,353	[m/s]
β_2	54,755	[º]		**W3**	601,225	[m/s]
W2x	197,593	[m/s]		**T03r**	1202,789	[K]
W2u	279,638	[m/s]		**P03r**	158368,671	[Pa]
Mw2	0,522	[-]		**β_3**	-65,000	[º]
V2is	826,315			**W3x**	254,089	[m/s]
U2	473,542	[m/s]		**W3u**	-544,895	[m/s]
ω_2	0,266	[-]		**Mw3**	0,959	[-]
A2	0,008460	[m2]		**U3**	473,542	[m/s]
rh2	0,107	[m]		**ω_3**	0,266	[-]
rt2	0,119	[m]		**A3**	0,01023	[m2]
rm2	0,113	[m]		**rh3**	0,106	[m]
h2	0,01191	[m]		**rt3**	0,120	[m]
Ω	4186,894	[rad/s]		**rm3**	0,113	[m]
H01	1736000,000	[J/kg]		**h3**	0,01439	[m]
kinetic loss stator	0,242	[-]		**Ω**	4186,894	[rad/s]
optimim pitch/chord	0,687	[-]		**H03**	1345549,219	[J/kg]
Chord	0,017	[-]		**H3ss**	1233944,201	[J/kg]
h/c	0,7	[-]		**kinetic loss rotor**	0,380	[-]
Re_{dh}	37520,102	[-]		**optimim pitch/chord**	0,637	[-]
Dh	0,005	[m]		**Chord**	0,010	[m]
Re_c	134386,8892	[-]		**h/c**	1,4	[-]
				Re_{dh}	19345,272	[-]
				dh	0,005	[m]
				Re_c	42838,65571	[-]

Global Parameters		
AHT euler based	390450,78	J/kg
AHT T0 based	390450,78	J/kg
GRp	0,320	[-]
GrH	0,328	[-]
Δβ	119,75	[º]
h3/h2	1,2	[-]
A3/A2	1,2	[-]
V2x/v1x	2,113	[-]
V3x/v2x	1,285	[-]
η_{TT}	0,836	[-]
η_{TS}	0,778	[-]
W3/w2	1,755	[-]
Loading factor	1,750	[-]
flow coefficient	0,536	[-]

Geometrical characteristics		
Stator chord	0,0170	[m]
Stator span	0,0119	[m]
Stator pitch	0,0117	[m]
α1	0,0000	[º]
α2	75,30	[º]
Stator blades	61,00	[n]
Rotor chord	0,0103	[m]
Rotor span	0,0144	[m]
Rotor pitch	0,0065	[m]
β2	54,7549	[º]
β3	-65,00	[º]
Rotor blades	109,00	[n]

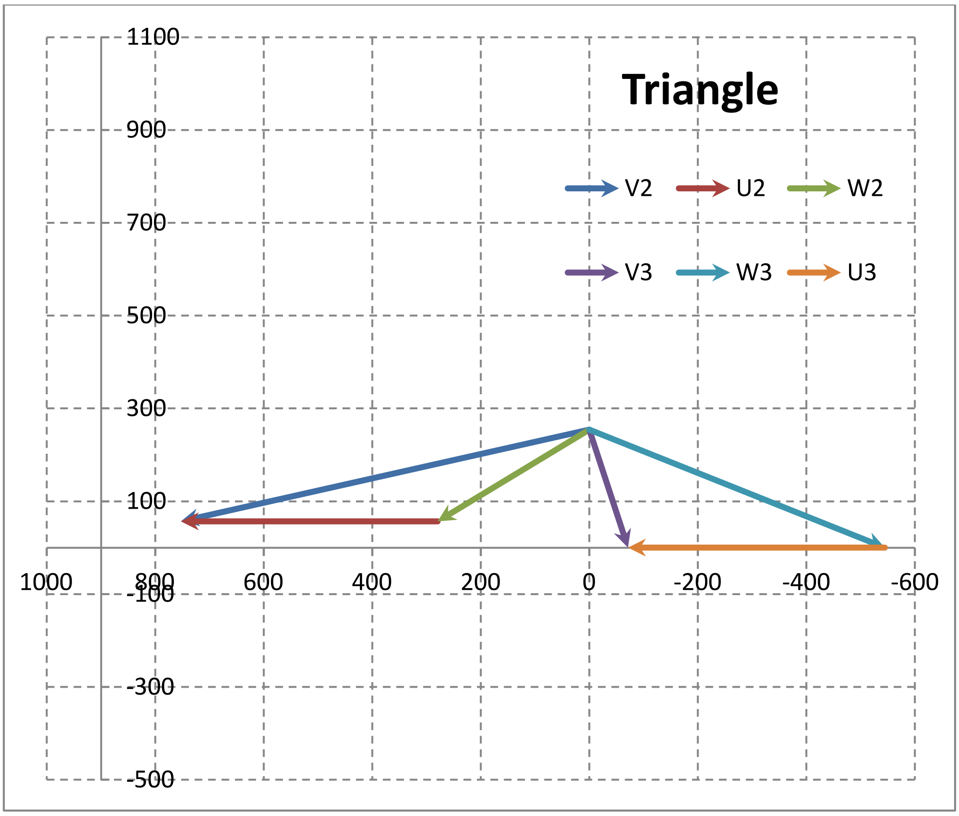

Figure 5: Velocity triangle for the 1D design

4. ISRE

The next stage in the design is the selection of a certain vortex distribution and the computation of the radial distribution of the flow and geometrical quantities along the radius in order to design the shape of the blades along the span.

The Non Isentropic Radial Equilibrium Equation (NISRE) equation is

$$\frac{dH}{dR} = \frac{TdS}{dR} + \frac{V_x dV_x}{dR} + \frac{V_u}{R}\frac{d(RV_u)}{dR}$$

In order to simplify and start the design with a simple procedure no radial distribution of losses will be considered, then solving the ISRE equation, the isentropic radial equilibrium

$$\frac{dH}{dR} = \frac{V_x dV_x}{dR} + \frac{V_u}{R}\frac{d(RV_u)}{dR}$$

All the values obtained from the 1D Analysis will serve as input for this stage of the design.

In order to solve the radial equilibrium the designer must select a certain distribution for the tangential component distribution, commonly referred as the vortex distribution. In this design the choice has been the selection of a 3 parameter distribution. The choice of this distribution will allow a wide range of designs in order to assess the best performance

$$V_u = AR^n \pm \frac{B}{R}$$

The minus is applied ahead the rotor and the plus after it.

As a result from this distribution the work is constant along the radius

$$\Delta H = \frac{2UB}{R} = 2\Omega B$$

The ISRE equation is simplified to

$$0 = \frac{V_x dV_x}{dR} + \frac{V_u}{R}\frac{d(RV_u)}{dR}$$

And solving it analytically

$$V_x^2 = -A(1+n)\left(\pm\frac{BR^{n-1}}{n-1} + \frac{AR^{2n}}{2\,n}\right)$$

Applying this last equation and the definition of work at the mean radius, assuming a value for n and taking as design choice the value of α_2 coming from the 1D phase the values of the constants, A and B, can be computed

In the mean radius everything is known from the 1D analysis. In order to know the radial distribution a space marching procedure from the mean radius will be performed.

And the axial speed in another radius can be obtained as

$$V_x = \sqrt{V_x(R_m) + 2 * \left(A(1+n)\left(\pm\frac{B(R^{n-1}-R_m{}^{n-1})}{n-1} + \frac{A(R^{2n}-R_m^{2n})}{2\,n}\right)\right)}$$ -ahead rotor + after rotor

The tangential speed will be also known from the evaluation of the free vortex distribution that has been selected

$$V_u = AR^n \pm \frac{B}{R}$$

Then the rest of flow characteristics can be computed.

4.1 Section 1, ahead of the stator

In the first plane of our stage, ahead the vane, it will be assumed constant flow properties and velocity distribution, everything will be constant along the radius: inlet angles and speeds. The flow will be assumed to be pure axial, $\alpha_1 = 0º$. The velocity V_1 will be the one obtained in the 1D design, where the massflow constraint was already satisfied.

4.2 Section 2, ahead of the rotor

Then for the second section behind the stator and ahead the rotor, the 3 parameters radial distribution will be applied.

The selection of the parameter n will be the result of an iterative procedure, the objective of the design is to minimize the difference in turning along the radius, avoiding excessive turning in both hub and tip. Since the turning in the mean radius was already high, the designer should avoid further increases imposed by the radial distribution selection. Once the losses distributions are considered, due to the effect of the clearance loses the turning will be increased in the neighborhood of the blade tip. For this reason the tangential distribution has been selected in order to reduce the turning close to the tip and increase it slightly in the rotor. Thanks to this modification the final distribution of turning is expected to be more uniform. After the evaluation of several n parameters, the choice has been a value of n = -0.5.

As was said before, the process starts in the mean radius, where the axial velocity is assumed, as first attempt, to be the one of the 1D analysis. In a similar way also the absolute flow angle α_{2m} is fixed, it was obtained from the 1D procedure after several iterations in order to match the work requirement without exceeding any design limit. Consequently the tangential component is also known

$$V_{2u} = \tan(\alpha_2)\, V_{2x}$$

Once these values are known, with selection of the coefficient n and applying the definition of the radial distribution and the work, the constants A and B are calculated.

Making use of the ISRE equation the values of the axial velocity along the radius will be computed. In a similar way applying the radial distribution that has been selected the tangential velocity is computed. Once the coefficients of the three parameters distribution have been set the tangential velocities ahead and behind the rotor can be obtained. As far as the radial distribution is not modified the values of the tangential components are kept constant through the rest of the design.

Then the rest of parameters in the section 2 can be obtained.

The inlet absolute speed and angle are obtained.

$$V_2 = \sqrt{V_{2x}^2 + V_{2u}^2}$$

$$\alpha_2 = \operatorname{acos} V_{2x}/V_2$$

For the peripheral speed the value of the angular velocity in the 1D design will be applied.

$$U_2 = r\,\Omega$$

Then the relative tangential component can be computed at every radius.

$$W_{2u} = V_{2u} - U_2$$

Once the tangential component is known, making use of the knowledge of the axial component the relative velocity can be define

$$W_2 = \sqrt{W_{2u}^2 + V_{2x}^2}$$

And the inlet relative angle

$$\beta_2 = \operatorname{acos} V_{2x}/W_2$$

Then the flow quantities will be computed. The absolute total temperature in this section will be constant since no work is performed in the stator and the radial distribution has been selected in order to avoid any radial variation. In a similar way since no total temperature variation are prescribed, the total pressure will be also constant along the radius. That will lead to the computation of the static pressure and temperature at this section. Since there is a certain velocity distribution the static quantities will be different along the radius

$$T_2 = T_{01} - \frac{V_2^2}{2\,c_p}$$

And with the isentropic evolution and assuming that P_{02} is constant and equal to the result found in the 1D procedure.

$$P_2 = P_{02}\left(\frac{T_2}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

So the value of the density

$$\rho_2 = \frac{P_2}{RT_2}$$

The value of V_z that was assumed at the beginning was the one of the 1D design and it will lead to certain massflow through the section. The massflow is one of the main constraints of the design, it should be ascertained that the massflow through each one of the sections is the one that was found in the cycle analysis.

The massflow in each one of the segments in which the section has been discretized need to be computed in order to verify the massflow limitation. In this case there have been computed 21 positions leading to 20 sectors. The massflow in each one of them will be computed with the average axial speed and density between both limits of the element considered.

In case the global massflow through the section is not matching the massflow constraint, the value of the axial speed in the mean radius should be changed V_{xm} till the agreement is reached.

The total relative flow quantities P_{02r} and T_{02r}, can be obtained from the static quantities and the relative velocity.

$$T_{02r} = T_2 + \frac{w_2^2}{2\, c_p}$$

Following an isentropic evolution, the relative total pressure

$$P_{02r} = P_2 \left(\frac{T_{02r}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

4.3 Section 3, after the rotor

For the section behind the rotor the procedure will be very similar to the previous one in the ISRE procedure. First of all a value of the axial velocity in the mean radius will be assumed, the tangential component will be given by the 3 parameter distribution and the rest of flow quantities will be computed as was done for the section 2.

$$V_3 = \sqrt{V_{3x}^2 + V_{3u}^2}$$

$$\alpha_3 = \operatorname{acos} V_{3x}/V_3$$

For the peripheral speed the value of the angular velocity in the 1D design will be applied.

$$U_3 = r\,\Omega$$

Then the relative tangential component can be computed at every radius.

$$W_{3u} = V_{3u} - U_3$$

$$W_3 = \sqrt{W_{3u}^2 + V_{3x}^2}$$

And the inlet relative angle

$$\beta_3 = \operatorname{acos} W_{3x}/W_3$$

The turning in each one of the sectors can be computed.

$$\Delta\beta = \beta_2 - \beta_3$$

This section has been also discretized in 21 segments. In order to compute the massflow and check again the constraint of the cycle analysis the values of the total temperature at the outlet of the rotor will be needed. The point is that due to the difference in height between the inlet and outlet of the rotor the segments in the section 3 are not aligned with the section 2.

To compute the density the value of the static temperature will be needed T_3. It can be obtained through the total temperature T_{03} and the velocity distribution. The temperature T_{03} can be assumed to be constant in the radial direction, but in order to check the proper resolution of the rotor; the total temperature can be obtained with the work based in the Euler definition

$$T_{03} = T_{02} - \ (u_2 V_{2u} - u_3 V_{3u})$$

From the total temperature and the absolute speed the static temperature is known in each segment

$$T_3 = T_{03} - \frac{V_3^2}{2\ c_p}$$

In order to know the static pressure this time the total pressure in the outlet cannot be assumed constant, but since the rothalpy is constant through the blade, no work in the relative frame of reference is done, the relative total temperature is constant.

At this point the relative total pressure after the rotor will be needed. In the stator the outlet pressure was considered uniform an equal to the value found in the 1D process. Consequently the consideration of this value implies the assumption the same losses through the stator and uniformly distributed in span-wise direction. In order to perform a similar procedure in the rotor and at the same time consider the effect of the radial distribution the distribution of the relative total pressure that was found ahead of the rotor will be multiplied by the factor between the relative pressures found in the 1D method.

$$P_3 = \ P_{03r} \left(\frac{T_3}{T_{03r}}\right)^{\frac{\Upsilon}{\Upsilon-1}} = P_{02r} \left(\frac{P_{03r,1D}}{P_{02r,1D}}\right) \left(\frac{T_3}{T_{03r}}\right)^{\frac{\Upsilon}{\Upsilon-1}}$$

The value of the relative total temperature T_{03r} is obtained directly from the static temperature and the relative velocity

$$T_{03r} = T_3 + \frac{W_3^2}{2c_p}$$

Despite the conservation of rothalpy in the rotor some deviations should be found from the results of the segments ahead and before the stator due to the fact that they are not at the same radial position. As was done for the second section an iterative procedure changing the axial speed in the mean radius takes place till the massflow constraint is satisfied.

Apart from the turning and the velocity triangles, also the degree of reaction variation along the radius will be of interest.

$$R_d = \frac{T_2 - T_3}{T_1 - T_3}$$

The static temperature at the inlet of the stator will be assumed to be constant along the entire radius, and computed with the combustion chamber exit temperature and the inlet speed coming from the 1D design.

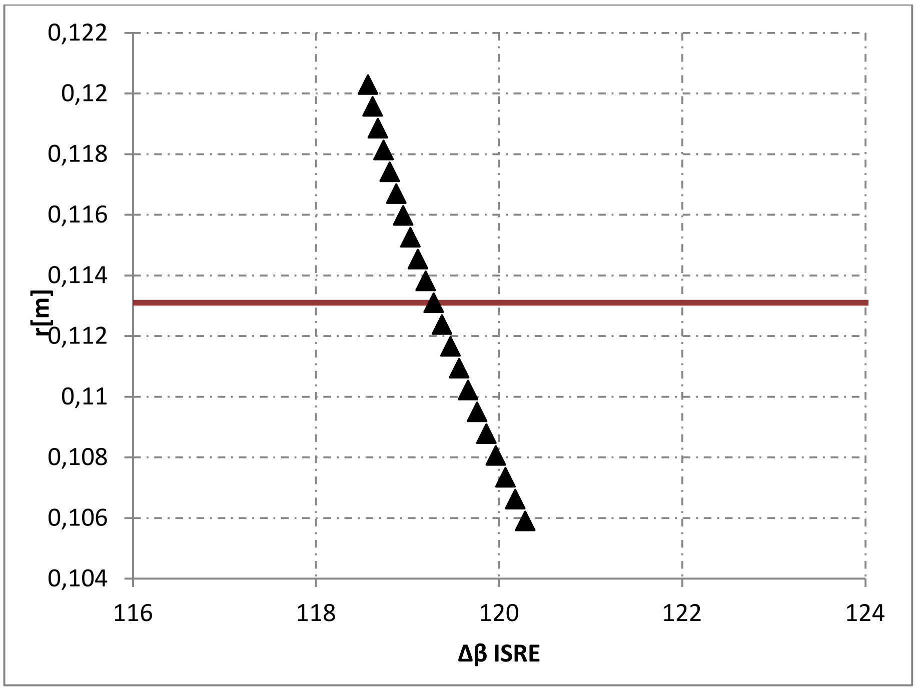

Figure 6: Radial distribution of turning, ISRE

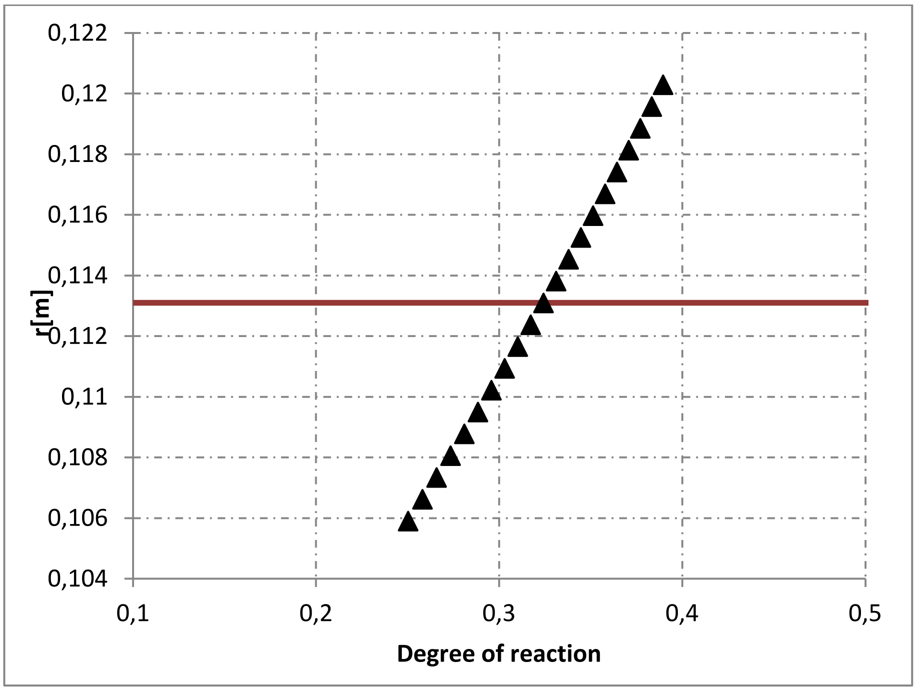

Figure 7: Radial distribution of degree of reaction, ISRE

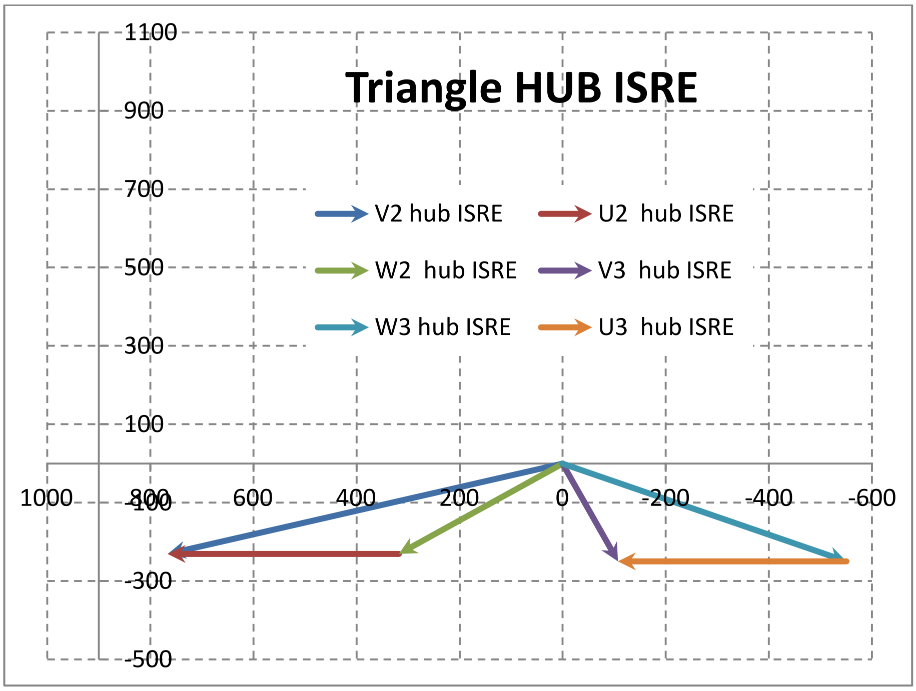

Figure 8: Triangle HUB ISRE

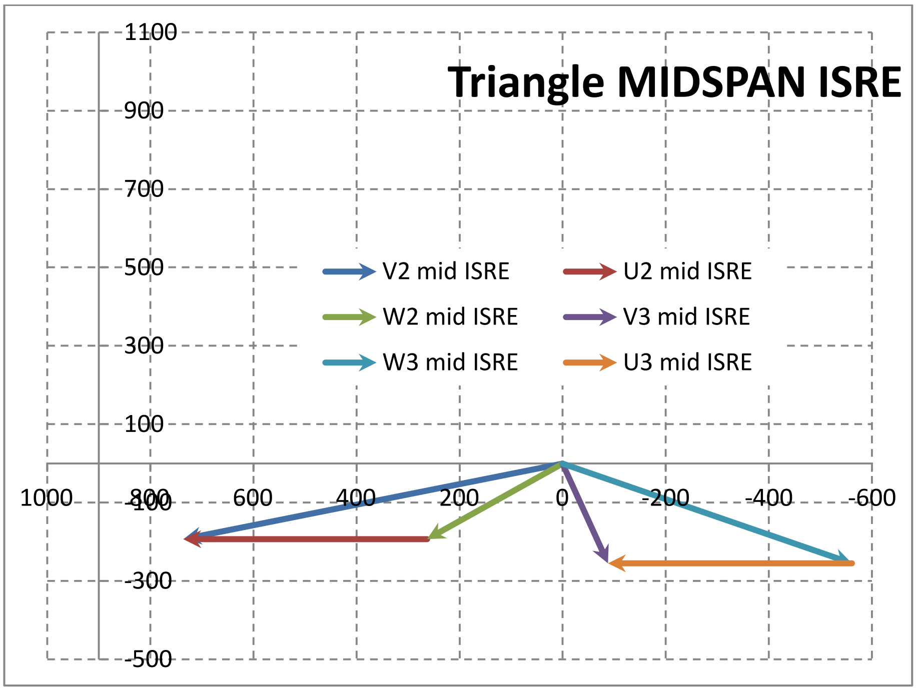

Figure 9: Triangle Midspan ISRE

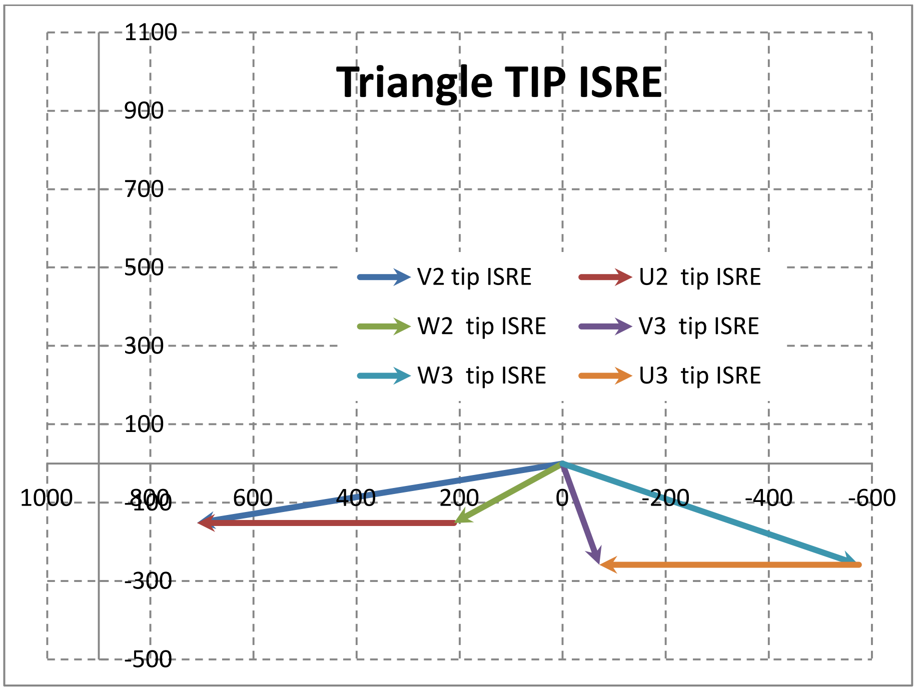

Figure 10: Triangle TIP, ISRE

5. NISRE

The previous procedure was done considering no gradient of losses in the radial direction, assuming the same losses for the entire blade span. To be more precise in the design the variation of the losses along the span should be considered. The losses increment through the stage will be estimated based on some literature correlations of turbine performance. Particularly in this case the correlations that have been selected are the ones published by Craig and Cox on Performance Estimation of axial flow turbines.

The losses that have been considered are the next ones

- Profile losses
- Secondary losses
- Tip clearance losses

In order to evaluate the correlations for the losses, the flow parameters coming from the ISRE analysis will be used

5.1 Section 1, ahead of stator

In this section everything will be considered to be uniform, so no losses gradient or velocity profile will be assumed, a flat profile will be assumed as inlet. Consequently the velocity (V_1), the inlet flow angle (α_1), the total enthalpy (H_1) and the pressure will be considered constant along the entire plane.

5.2 Section 2, ahead of rotor

In this section the stator losses should be considered and their effect in the radial distribution should be computed. The losses that are taken into account in the stator are the profile losses and the secondary losses.

The profile losses will be obtained in three different segments of the radial discretization (hub, mid and tip) and linear interpolation between these values will be done for the rest of radial positions.

For the profile losses Craig and Cox suggested the kinematic loss coefficient

$$\xi_{pl} = \xi_b N_r N_i N_t + \Delta\xi_t$$

Where the coefficients:

ξ_b, is function of the modified lift coefficient, some geometrical properties, and the contraction ratio. The contraction ratio and the lift coefficient can be obtained correlated with the flow angles.

N_r, is a correction for the Reynolds number based on the chord

N_t, is a correction based on the blade trailing thickness. In this case for the stator the trailing edge thickness has been assumed as 1 mm. Value in agreement with an uncooled trailing edge.

$\Delta\xi_t$, is a loss increment due to the trailing edge thickness

Finally the incidence effect is neglected at this stage of the design and N_i is assumed to be 1. As was mentioned before all the flow parameters that are needed as inputs will be taken from the ISRE procedure.

The distribution of the profile kinematic losses in the stator is shown in the next figure

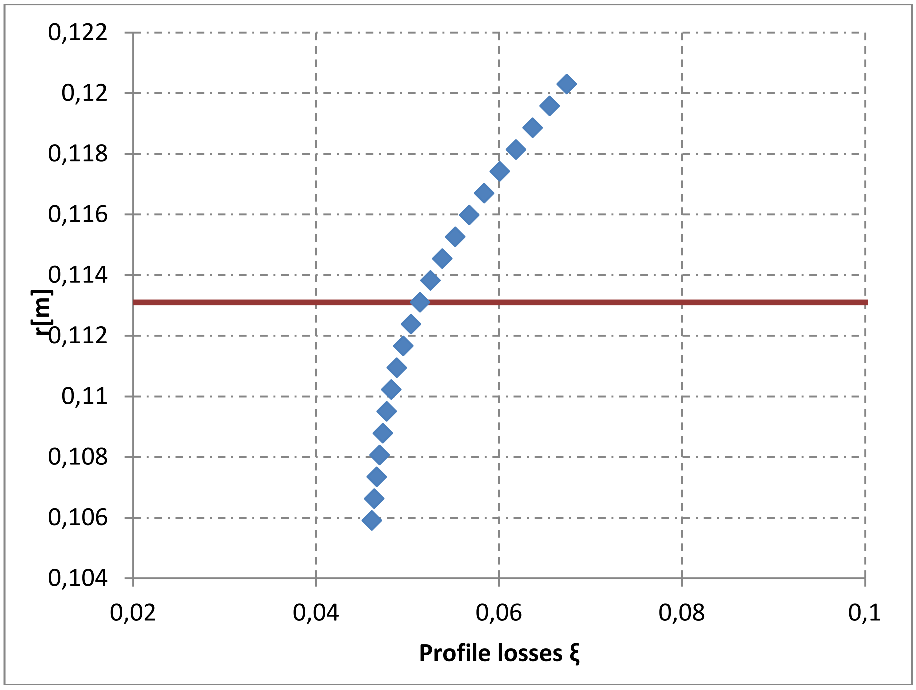

Figure 11: Profile kinematic losses stator

Regarding the secondary losses, these ones are only notorious up to certain spanwise percentage and the value that is obtained from the losses is an integral coefficient, so a distribution will need to be assumed at the hub and the tip of the vane. The distribution that has been chosen is a linear interpolation.

The integral coefficient that is found is divided by two, hub and tip, and then the linear distribution is prescribed. The maximum losses are just in the extreme of the blade. The penetration of the secondary losses can be obtained based on the turning, the contraction ratio and the boundary layer thickness. For the stator the penetration of the secondary losses is 40% of the blade span.

The secondary integral kinematic losses can be obtained as

$$\xi_{sl} = \xi_{s,0} N_r N_{h/b}$$

Where the coefficients

$\xi_{s,0}$, is function of the modified lift coefficient and the velocity ratio through the blade.

N_r, is again the correction for the Reynolds number

$N_{\frac{h}{b}}$, is a correction for the aspect ratio of the blade

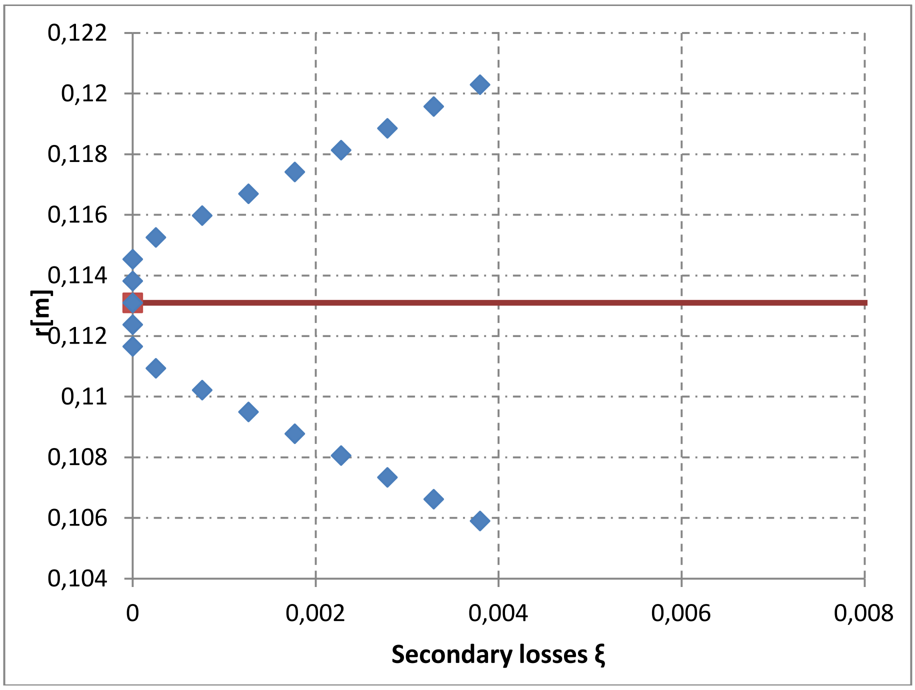

Figure 12: Kinematic secondary losses stator

The profile and secondary losses are obtained as kinematic losses. In order to convert these losses in a value of increment of entropy first a conversion to pressure losses will be performed.

$$\omega = \xi\left(1 + \frac{\Upsilon M_2^2}{2}\right)$$

Where the pressure loss coefficient was defined as

$$\omega = \frac{P_{01} - P_{02}}{P_{02} - P_2}$$

The increment of entropy will be computed as

$$\Delta S = c_p \ln\left(\frac{T_{02}}{T_{01}}\right) + R \ln\left(\frac{P_{01}}{P_{02}}\right)$$

Since in the stator no work is done, the first term disappears, and only the ratio of pressures is needed. The pressure ratio will be obtained from the pressure loss coefficient. The total pressure at the inlet of the stator (P_{01}) is considered constant along the radius and the distribution of static pressure at the outlet (P_2) will be taken from the ISRE results.

Once the losses increments are known along all the segments in the stator, the NISRE equation can be evaluated to solve the radial distribution of the axial velocity.

$$\frac{dH}{dR} = \frac{TdS}{dR} + \frac{V_x dV_x}{dR} + \frac{V_u}{R}\frac{d(RV_u)}{dR}$$

Again this equation should be particularized for the vortex distribution that the designer has chosen. Solving analytically the equation for the three parameter distribution the next expression for the stator is found

$$V_x = \sqrt{V_x(R_i) + 2 * \left(A(1+n)\left(-\frac{B(R^{n-1}-R_i{}^{n-1})}{n-1} + \frac{A(R^{2n}-R_i^{2n})}{2\,n}\right) - \frac{T\,(\Delta S-\Delta S_i)}{R-R_i}(R-R_i)\right)}$$

As was done for the ISRE the procedure will start from the mean radius where the axial velocity will be assumed to be the one coming from the ISRE procedure. For the tangential component the distribution will be exactly the same as in the isentropic radial equilibrium since the coefficients of the three parameters distributions are kept constant.

Then performing a finite difference radial marching process the axial speed is computed for the rest of segments. Although this time not all the process will be referred to the mean radius. The difference will be computed between two adjacent positions, in order to take into account in a more accurate way the losses gradient between the discrete points. The method will start from the mean radius and then the computation will advance both towards the hub and towards the tip.

Once the axial velocity at each one of the segments is known the process is similar to the one that took place in the ISRE stage. The main difference will be the way the static pressure is obtained. This time the total pressure P_{02} is not assumed any more constant along the radius; its value is taken from the results of the losses estimation. Computing the rest of flow quantities and iterating on the mean radius axial speed in order to match the constrained massflow, the non isentropic radial equilibrium is solved for the exit of the stator.

5.3 Section 3, behind the rotor

For the rotor the computation of the profile losses and the secondary losses will be exactly the same just changing the absolute flow quantities for the relative ones, taking them again from the ISRE results. Only the difference on two parameters should be mentioned. First the penetration of the secondary losses, that for the rotor will be assumed as 30 % of the span. Since the rotor chord is smaller than the stator one the penetration is reduced. The other parameter that is missing is the blade trailing edge thickness for the rotor, which has been considered as 0.6 mm. Both trailing edge thicknesses the rotor and the stator one are in agreement with the common values for uncooled trailing edges in small airfoils as the ones characteristics of this design.

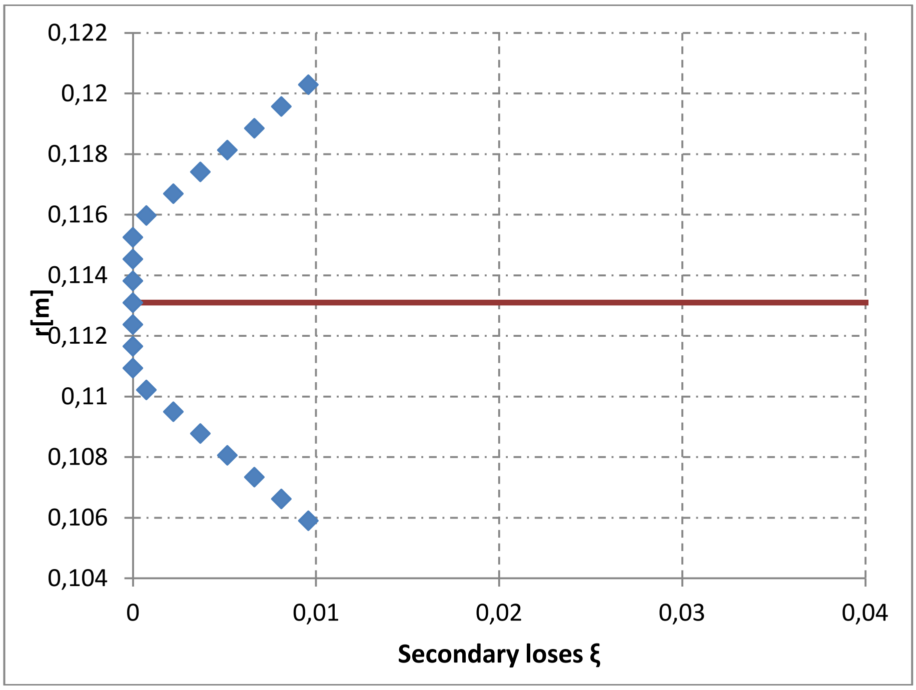

Figure 13: Kinematic secondary losses rotor

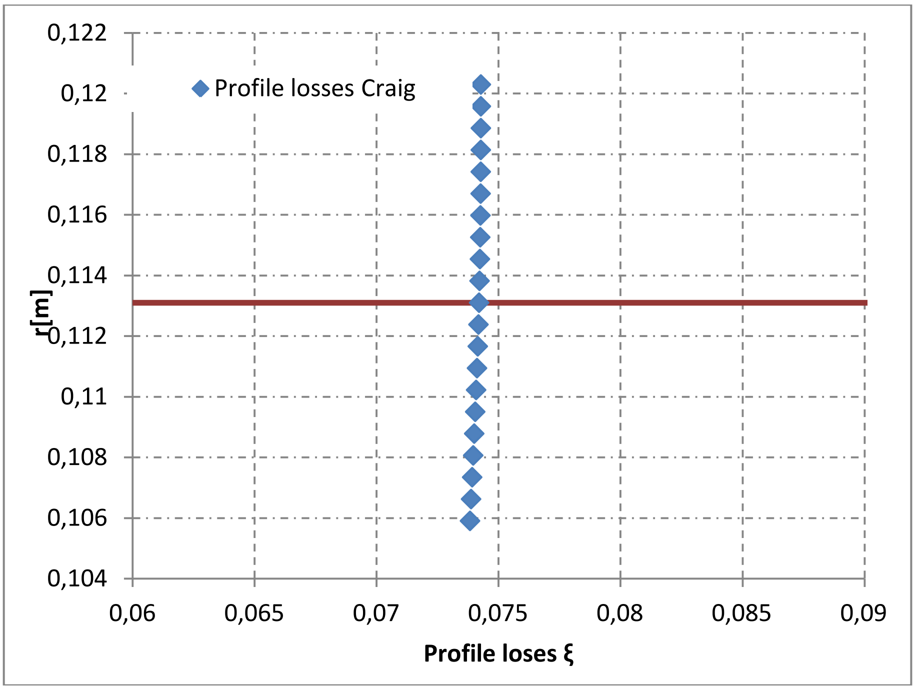

Figure 14: Kinematic profile losses rotor

Additionally the tip clearance losses will be computed. These losses are due to the gap existing between the rotor and the end wall of the engine. In this case a minimum value of the gap will be assumed and it will be translated in a certain percentage of the span of the blade. The smallest gap that could be considered for a safety operation of the machine is of 0.7 mm. If we translate this clearance in a percentage of the span in it will represent a 5 % of the blade span. This value is slightly higher than the common percentage for turbine designs but due to the small size of the blades it is reasonable. For the clearance losses Craig and Cox offered a correlation based on:

$$\omega_{cl} = \frac{kc}{h}\left(\frac{\delta}{c}\right)^{0.78} Z$$

Where δ is the tip clearance, k is a correction factor (0.5 for unshrouded blades) and Z is the Ainley's loading factor. That can be computed as function of the flow relative angles (β_2, β_3)

This correlation provides directly a value of pressure loss coefficient that will be translated in an increment of entropy. The loss coefficient that is found is an integral value that should be distributed in the surroundings of the tip of the blade. Again the profile that will be assumed will be a linear one with the higher losses in the tip and reducing them towards the hub. The penetration of the clearance losses will be assumed as 35 % of the blade span.

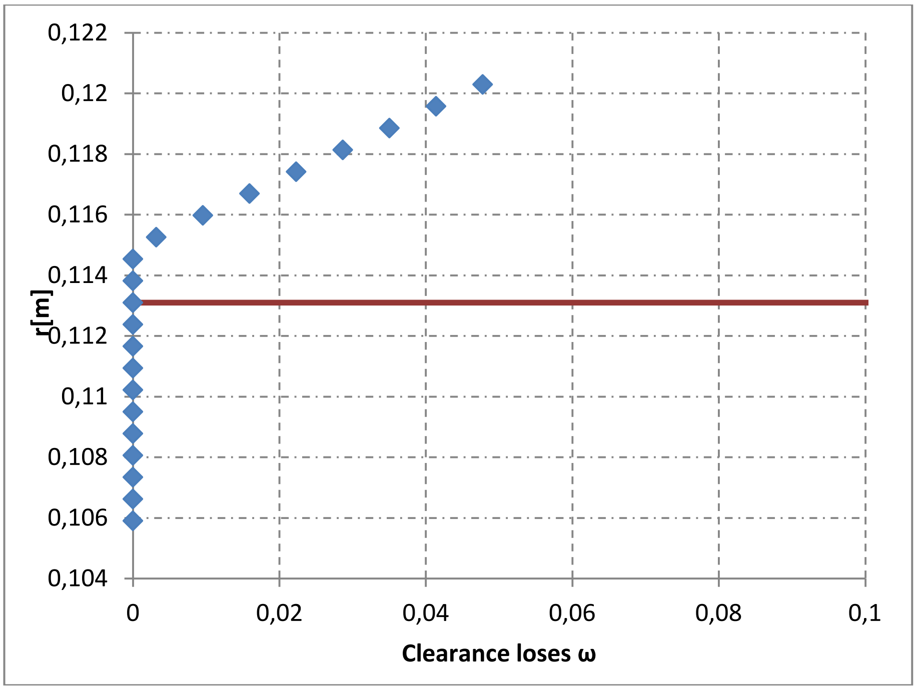

Figure 15: Clearance losses

To convert the profile and secondary losses into pressure losses and then into a pressure ratio in order to compute the entropy increment a slightly different procedure is needed. For the conversion into pressure losses the outlet relative Mach number is needed

$$\omega_r = \xi_r \left(1 + \frac{\Upsilon M_{3r}^2}{2}\right)$$

The pressure loss coefficient in the rotor

$$\omega = \frac{P_{02r} - P_{03r}}{P_{03r} - P_3}$$

$$\omega_r = \omega_{clearance} + \omega_{profile} + \omega_{secondary}$$

Then the clearance losses are added to the profile and secondary ones. The entropy increment through the rotor can be computed as

$$\Delta S = c_p \ln\left(\frac{T_{03r}}{T_{02r}}\right) + R \ln\left(\frac{P_{02r}}{P_{03r}}\right)$$

In the rotor there is no change of rothalpy and neglecting each variation due to the radial position variation, the only coefficient that is missing to know the entropy increase is the relative pressure ratio (P_{02r}/P_{03r}).

In order to compute the relative pressure ratio from the rotor pressure loss coefficient the values of P_3 and P_{02r} need to be considered. The outlet static pressure will be taken from the ISRE procedure. While the rotor inlet relative total pressure will be taken from the NISRE results of the section 2.

Finally the stator entropy increment is added to the rotor entropy increment in order to know the global losses distribution at the outlet of the rotor.

Then applying the 3 parameter distribution to the NISRE equation for the rotor

$$V_x = \sqrt{V_x(R_i) + 2 * \left(A(1+n)\left(+\frac{B(R^{n-1} - R_i{}^{n-1})}{n-1} + \frac{A(R^{2n} - R_i^{2n})}{2\,n}\right) - \frac{T\,(\Delta S - \Delta S_i)}{R - R_i}(R - R_i)\right)}$$

The temperature that is multiplying the losses increment will be taken as the mean one between the two segments considered.

The tangential component will be the result of the radial distribution, and the rest of the procedure is similar to the one that took place for the isentropic radial equilibrium. The only difference that will be found will be again in the way the density is computed. Taking as P_{03r} the value that was found once the losses estimation were assessed.

In the next graphs are represented some of the losses and flow distribution on both rotor and stator

First of all the stator entropy increment distribution is depicted

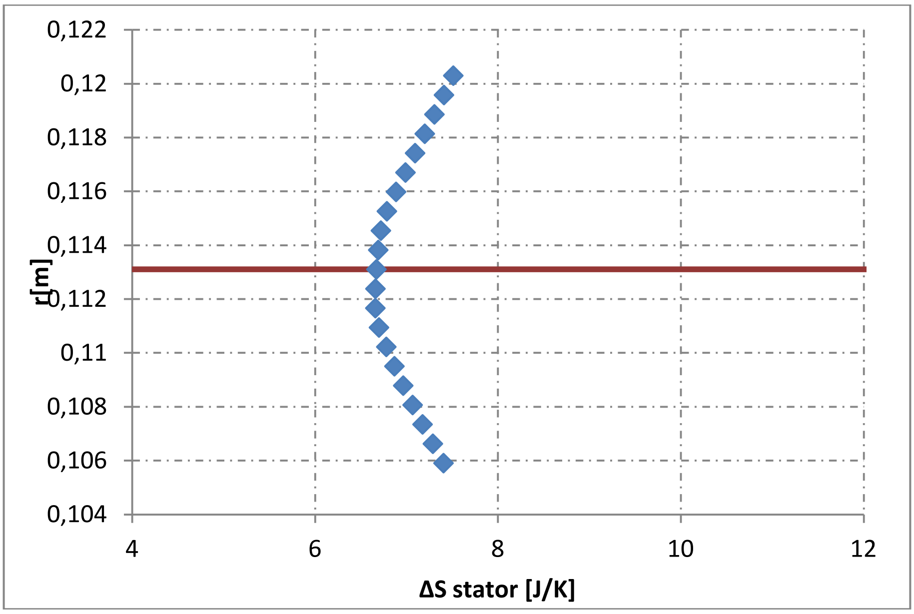

Figure 16: Entropy increment radial distribution stator

Comparing the massflow distribution for the ISRE and the NISRE design in the stator

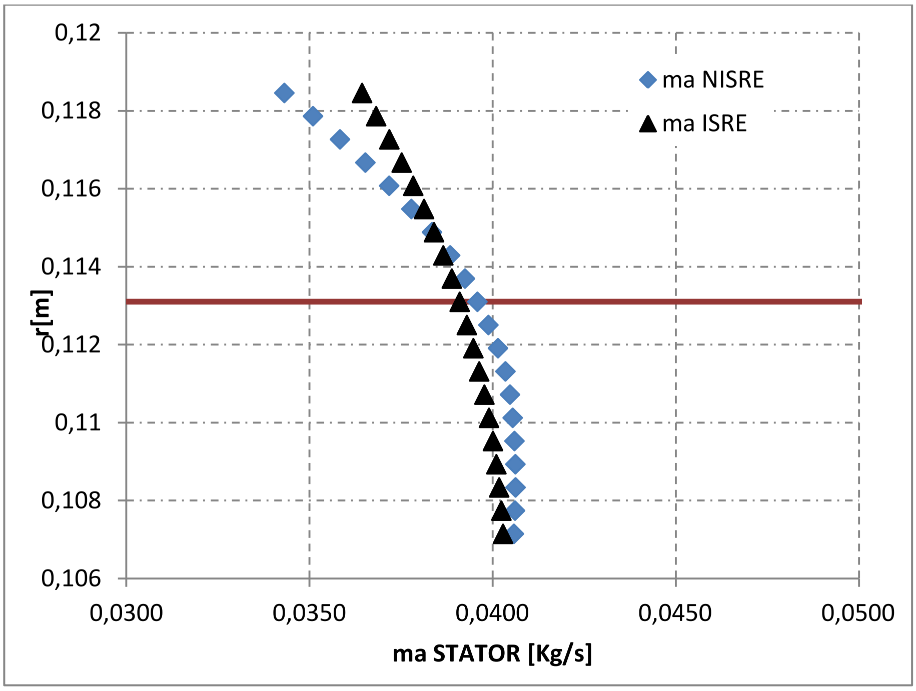

Figure 17: Massflow radial distribution stator

Both graphs lead to the same global massflow since is the design constraint but taking into account the losses distribution the massflow gradient between the TIP and the HUB of the blade is increased.

Representing the axial speed at the outlet of the stator.

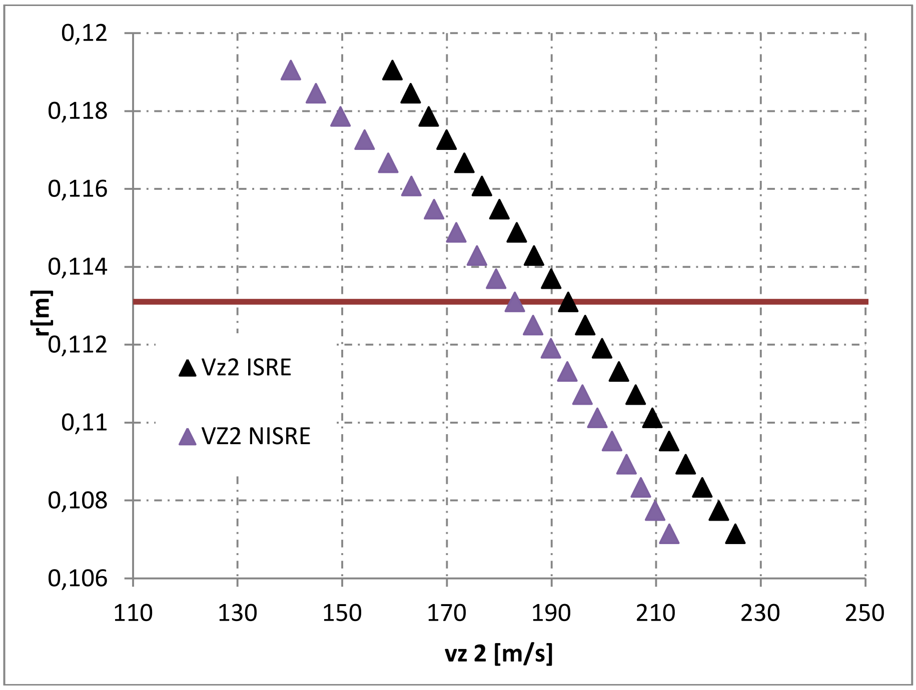

Figure 18: Axial speed radial distribution stator

Considering the performance estimation based on the Craig and Cox publication the axial speed has been decreased. This is due to the fact that applying the losses evaluation from the Craig and Cox performance higher efficiencies are reached. The estimation of the massflow and the specific work extraction was computed in the iterative process between the 1D method and the cycle analysis. The losses estimation with the Soderberg correlation leads to a worse performance of the machine. Following the procedure that has been explained a better performance in the non isentropic radial equilibrium will lead to slightly higher values of exit pressures since both the work and massflow will need to match the values that were found with a higher amount of losses. Because of the slightly increase of pressure the axial speed need to be reduced to match the constrained massflow.

For the rotor the global entropy distribution

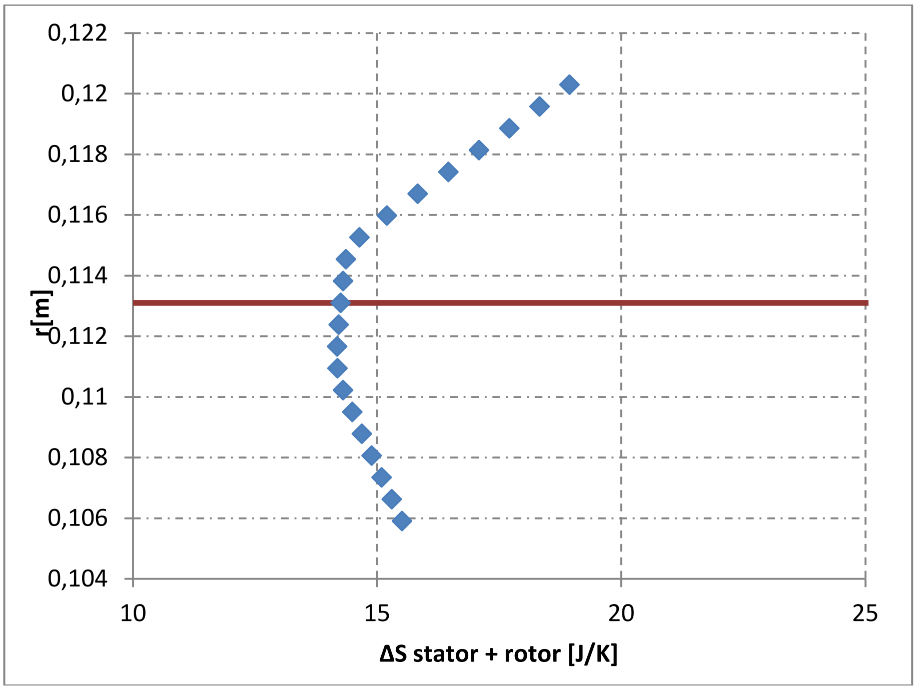

Figure 19:Entropy increment radial distribution rotor

Leading to the next axial speed profile

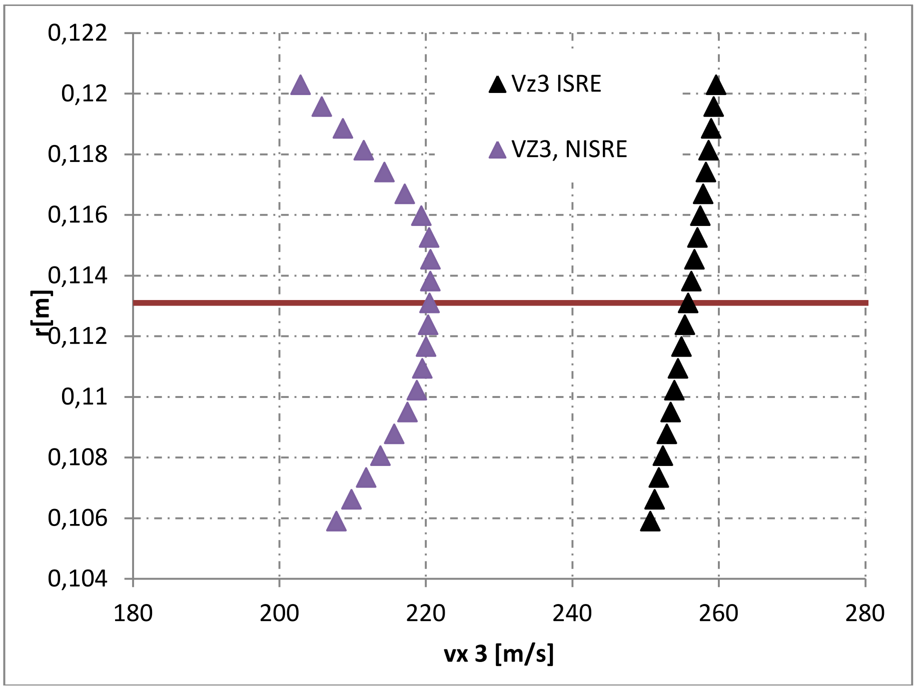

Figure 20: Axial speed radial distribution rotor

As was found in the stator the axial velocity from the NISRE is lower than the one found in the ISRE, again this is due to the variation of the losses. The losses estimation with the Craig and Cox correlation result in higher efficiencies, since the efficiency is higher and the work that is constrained as requirement, imposed by the tangential velocity distribution, is the same the exit static pressure and temperatures are modified. If the massflow that goes through the stage is constrained the axial velocity will be adapted to the new density.

Regarding the flow turning

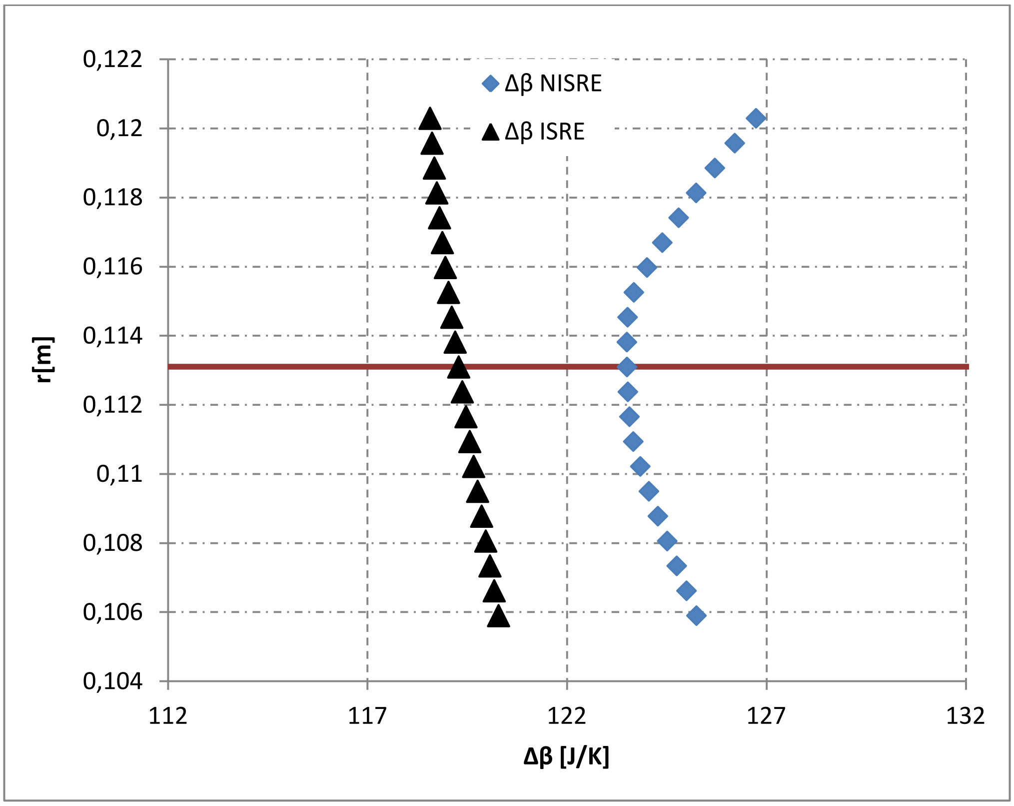

Figure 21: Turning radial distribution, NISRE vs. ISRE

The addition of the losses profiles results in a higher variation of the turning along the span. Particularly close the tip, where the higher losses take place, the turning is considerably modified, this was the reason that motivates the designer to select a distribution that actually reduces the turning at the tip. The final turning in the rotor tip is 127º. This value is slightly higher than the one for a safe performance operation and could eventually promote the boundary layer separation.

On the other hand it should be noticed that also in the midspan the turning is increased compared to the ISRE distribution, this bring the attention again to the disparity of losses estimation between both procedures. Due to the effect of the losses and the massflow limitation the axial speed was reduced, but the distribution of the tangential velocity is the same, then leading to higher values of turning.

If an iterative procedure in all the design exercise took place modifying the losses estimation in the cycle, 1D and ISRE stages the mean turning in the NISRE distribution will be reduced. Anyway due to the high expansion ratio and the high amount of power that is required a high value for the turning is expected.

The radial distribution of the different angles compared for the isentropic and the non isentropic radial equilibrium

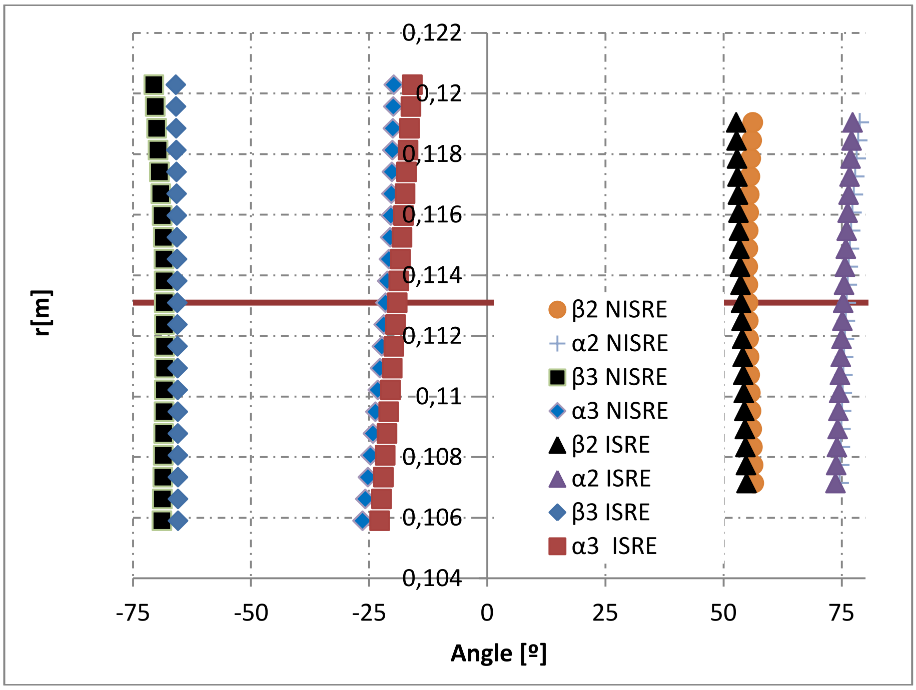

Figure 22: Radial angle distribution

Finally the variations for the massflow and the degree of reaction distribution are presented.

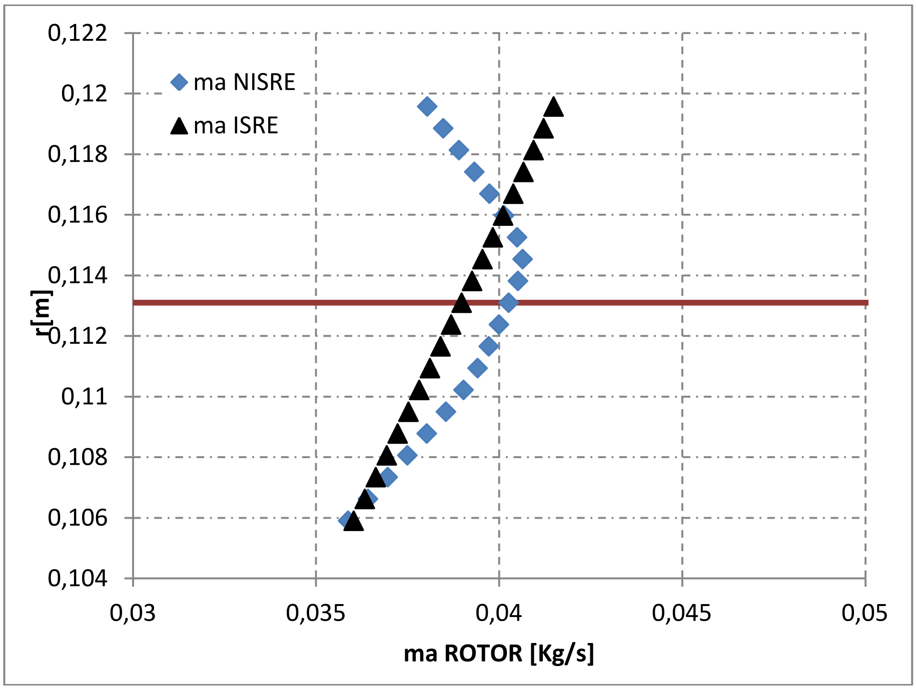

Figure 23: Massflow radial distribution rotor

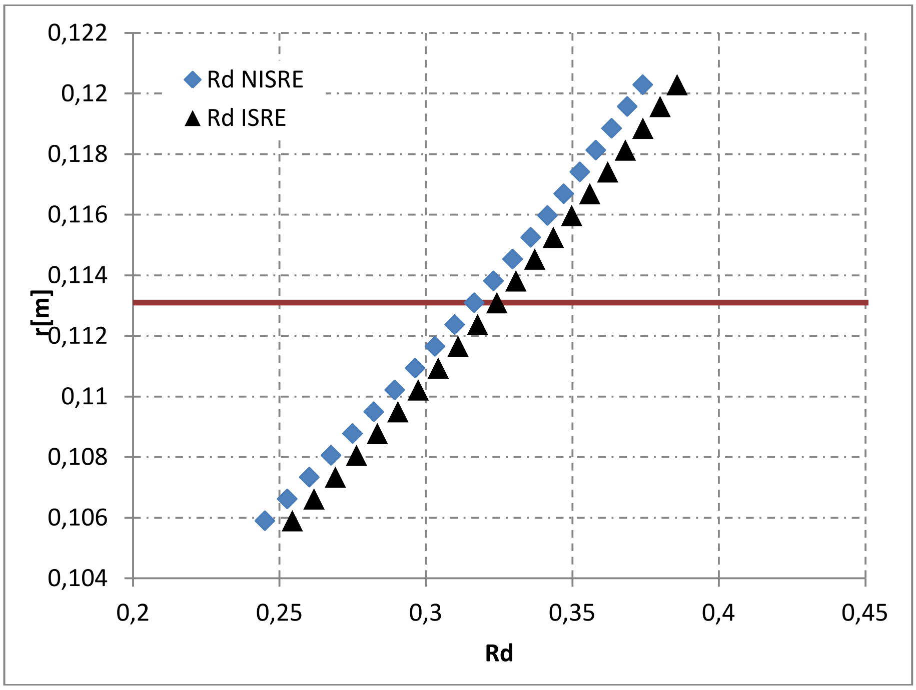

Figure 24: Degree of reaction radial distribution, ISRE vs. NISRE

To conclude the values of the velocity triangles at hub, mid span and tip are represented

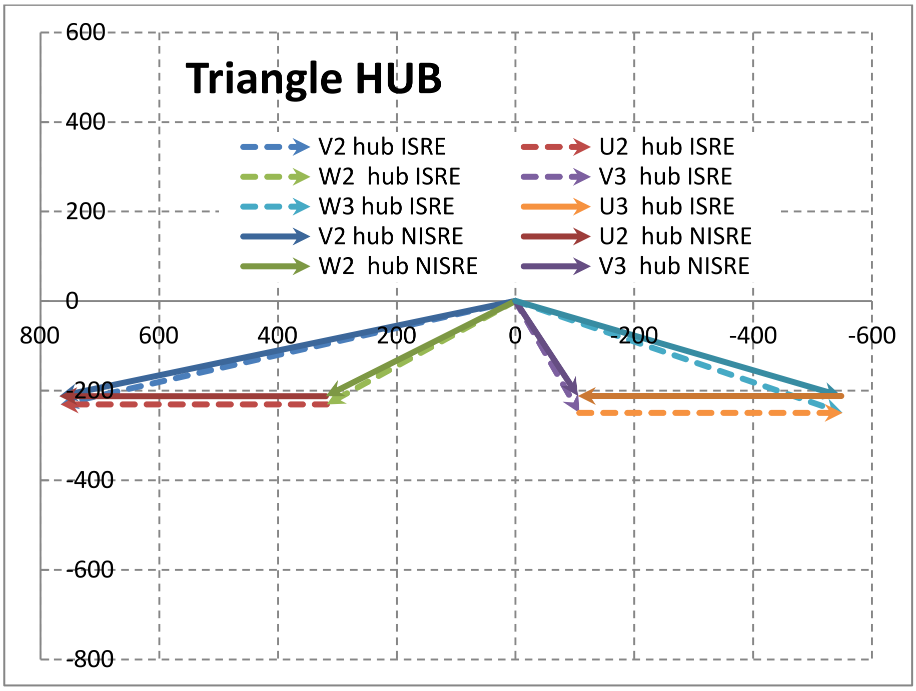

Figure 25: Velocity triangle HUB

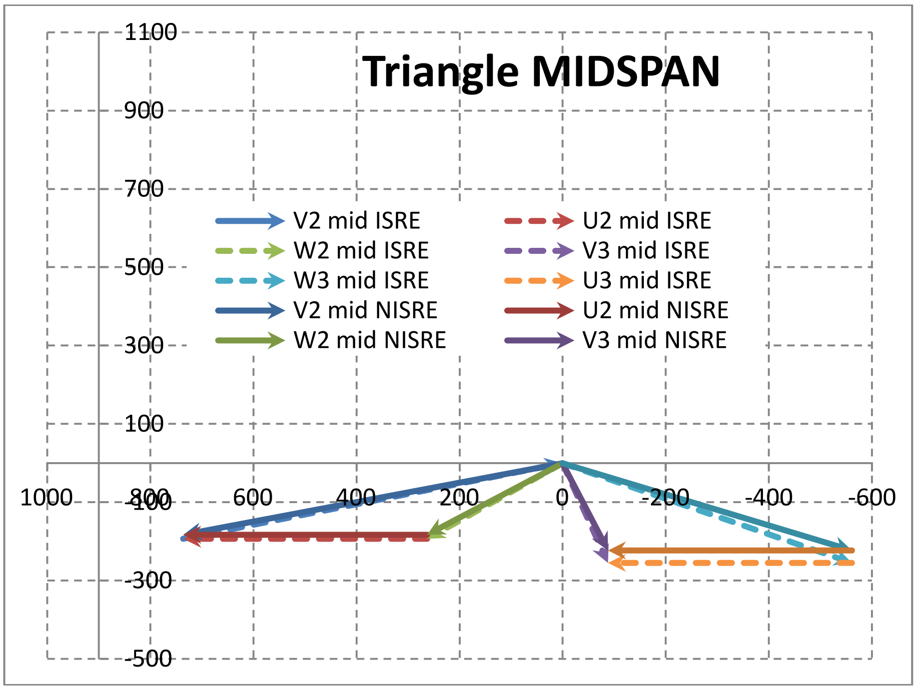

Figure 26: Velocity triangle MidSpan

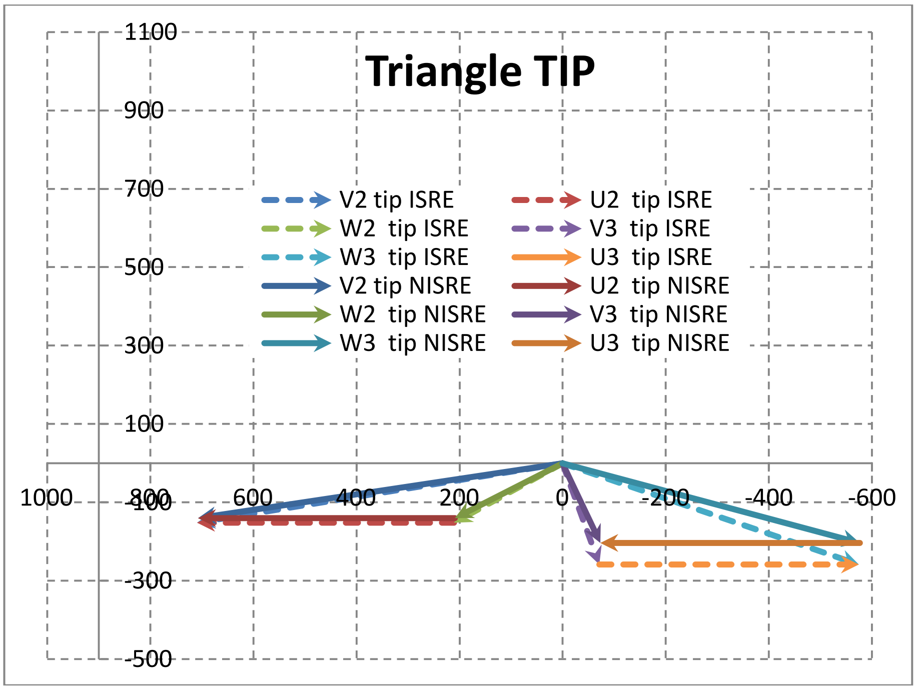

Figure 27: Velocity triangle TIP

The aim of the design was to keep the turning as uniform as possible along the entire blade. The mean turning, as was found in the 1D Analysis reaches a value around 120º and once the losses distribution are considered the turning exceeds 125º at some span locations close to the blade tip. These high turning angles are due to the expansion ratio that takes place in the stage 3.92. In order to reduce the turning the angular velocity of the machine has been raised as much as possible, 500 m/s at the blade tip. This high peripheral speed could increase the losses due to the presence of supersonic regions close to the tip gap. The maximum peripheral Mach number stays below 0.8 leading to relative Mach number values of 0.95 at the tip location. Although, it should be taken into account that the negative effect of the high angular velocity will be less severe than the effect caused by a lower of peripheral speed and the consequent increase of turning to provide the required power.

6. Conclusions

In this report is presented the design procedure for a gas generator to propel an automobile. The design has been based in a mid reaction machine in order to achieve the maximum efficiency possible avoiding excessive acceleration and excessive divergence in the rotor.

The first step that has been described is the cycle analysis where the main features of the machine are found in order to provide the required net power. Followed by a 1D design process where the geometrical and flow characteristics are obtained.

Among all this procedure the designer needs to perform certain assumptions and take some decisions that will guide to the final status of the design and will have its consequences on the turbine performance.

Finally the radial equilibrium is assessed. In first order assuming the same losses in all the radial direction (ISRE) and finally computing with the support of the Craig & Cox correlation the radial losses distribution in both rotor and stator, driving to the final shape and flow distribution of the design.

Both work and massflow are matched during all the phases since they have been selected as constrained parameters in all the procedures that have been explained. In the radial distribution the design choices that can be modified are the absolute inlet angle for the rotor and one parameter of the vortex distribution, the attempt of the design has been to limit the variation of turning along the span and limit its maximum value. It has been noticed the relevance of matching the losses estimation among all the procedure, consequently an iterative process in all the design stages is required.

In conclusion it has been reported the design procedure for a turbine stage inside a gas generator to propel an automobile. Due to the high expansion ratio and specific work that was required both high peripheral speed and blade turning angles are required.