THERMODYNAMICS AND FLUID MECHANICS GROUP

PERFORMANCE ESTIMATION OF AXIAL FLOW TURBINES

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A comprehensive method of estimating the performance of axial flow steam and gas turbines is presented, based on analysis of linear cascade tests on blading, on a number of turbine test results, and on air tests of model casings. The validity of the use of such data is briefly considered. Data are presented to allow performance estimation of actual machines over a wide range of Reynolds number, Mach number, aspect ratio and other relevant variables. The use of the method in connection with three-dimensional methods of flow estimation is considered, and data presented showing encouraging agreement between estimates and available test results. Finally 'carpets' are presented showing the trends in efficiencies that are attainable in turbines designed over a wide range of loading, axial velocity/blade speed ratio, Reynolds number and aspect ratio.

INTRODUCTION

TURBINE PERFORMANCE can only be satisfactorily determined by tests on full scale machines. Such tests, however, reflect the aggregate effect of a large number of features influencing efficiency, and for a basic understanding of turbine performance it is necessary to analyse such features. The procedure presented in this paper can be used in the preliminary performance estimation of new designs and will supplement estimation methods based on tests of previous turbines of similar design.

With an accurate theoretical method of performance prediction the designer can assess in the early design phase the performance merit or penalty that will result from any proposed departure from standard tested machines. While it is not suggested that the method could ever become a substitute for absolute test data, it should prove to be of direct help in assessing the changes in performance which occur at conditions other than those for which the basic tests were originally carried out. Finally, this method can also be used to indicate the lines along which future aerodynamic development should proceed.

A number of methods of performance estimation have been published in the past, such as that of Ainley and Mathieson (1)‡ and Traupel (2). The former found wide acceptance in the aircraft and industrial gas turbine fields, but in the manner in which it was often used—prior at least to modifications suggested by Dunham and Came (3)—little allowance was made for aspect ratio and blade height effects. Thus, use of the method produced unconvincing answers for typical steam turbine designs. Here a method is set out which has been developed for use with steam and gas turbines. It attempts to take into account the full range of Reynolds number and aspect ratio encountered in such machines, and in addition to deal with most of the auxiliary sources of loss which are sometimes omitted from published methods. It is a development of the method published by Craig and Janota at the 1965 CIMAC conference (4), extended to cover off-design conditions and other effects.

Examined against turbine test data, this analysis has generally given results correct to an accuracy of $\pm 1\frac{1}{4}$ per cent.

Notation

A	Fluid relative inlet angle.
Aa	Annulus area.
Ak	Total effective area of clearance.
At	Total blade throat area.
B	Fluid relative outlet angle.
b	Blade backbone length.
C_{cr}	Critical throat velocity.
C_{w1}	Guide outlet absolute tangential velocity.
C_{w2}	Runner outlet absolute tangential velocity.
C_z	Axial velocity.
C_1	Guide outlet absolute velocity.
Co	Guide inlet absolute velocity.
CR	Contraction ratio.
D	Diameter: D_h = hub diameter.

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e	Back surface radius.
F	Parameter used in evaluating losses, with various subscripts.
G	Mass flow through blades.
g	Gravitational constant.
h	Height.
i	Incidence.
$i_{\min}$	Incidence for minimum loss.
$i+$stall	Positive stalling incidence.
$i-$stall	Negative stalling incidence.
$(i+\text{stall})_{\text{basic}}$	Value for standard value of s/b and CR.
$(i-\text{stall})_{\text{basic}}$	Value for standard value of s/b.
J	Mechanical equivalent of heat.
l	Length.
Ma	Mach number.
N	Loss ratio.
o	Blade throat opening.
p	Perimeter length.
Re	Reynolds number.
Re_o	Reynolds number based on blade opening.
s	Blade pitch.
T_w	Torque coefficient.
te	Trailing edge thickness.
U	Blade speed.
v	Specific volume.
W_1	Relative velocity at inlet to runner.
W_2	Relative velocity at outlet to runner.
X	Total loss factor.
x	Basic loss factor.
α	Blade inlet angle.
ΔC_w	$C_{w1}+C_{w2}$.
Δg	Leakage flow bypassing blades.
$\Delta i+$stall	Correction added to basic value with suffix denoting effect of s/b or CR.
$\Delta i-$stall	Correction added to basic value.
ΔL	Lap defined in Fig. 21.
Δx	Additional loss factor added to basic loss factor.
$\Delta\eta$	Efficiency debit, with subscript denoting component.
η_b	Blading efficiency.
η_p	Profile section efficiency.
η_t	Total stage efficiency.
λ	Loss coefficient.
ϕ	Guide blade velocity coefficient.
ψ	Runner blade velocity coefficient.

Subscripts for loss factors and ratios

a	Annulus.
b	Basic.
h/b	Aspect ratio.
i	Incidence.
m	Mach number.
p	Profile.
r	Reynolds number.
s	Secondary.
s/e	Back curvature.
t	Trailing edge thickness.

RELEVANCE OF CASCADE DATA

The method is based on a correlation of profile and secondary losses obtained from linear cascade tests, supplemented by information on losses (such as clearance losses) derived from specific turbine tests, and by data on casing losses, derived for the most part from air tests. It is relevant therefore to start by considering how far cascade tests provide a satisfactory basis for estimating the losses in an actual turbine.

A linear cascade differs from blading in a real turbine in two ways. First, differences occur when the cascade tests are carried out:

(1) with a different working fluid;
(2) with a different Reynolds number;
(3) with a different scale of blade;
(4) with a different surface roughness;
(5) with a different Mach number.

Differences of this sort, if they occur, are capable of being corrected, with the major proviso that the information is available as to how the correction should be made. Second, the following, more fundamental, differences also exist:

Cascade flow	*Turbine flow*
Uniform inlet conditions.	Inlet flow containing wakes, disturbances due to preceding secondary flow, and stage leakage.
Linear cascade.	Annular flow.
Walls stationary relative to blades.	For unshrouded stages walls may be moving relative to blades.

Differences of this type are an inherent limitation in the use of stationary linear cascade data, and no measurements, however extensive, obtained from such cascades can completely allow for such differences. The only check that can be made upon them is to compare the results of carefully interpreted cascade data with actual turbine performance, and to deduce from the overall result the magnitude and importance of the errors involved.

If it be thought that the effects of such differences are so extensive as to swamp the points of similarity, then clearly no useful purpose can be served by cascade testing or its analysis, and turbine development has to proceed by trial and error. If it be thought that the effects are negligible, then turbines can be designed wholly round cascade data, and the results predicted with some degree of precision. It is the experience of the writers that the truth lies between these two extremes, and that carefully used cascade data serve as a guide to trends in performance which will occur in actual machines; but that some care is needed in the interpretation of cascade data to avoid errors caused by the differences between the cascade and the turbine.

The model turbine occupies an intermediate position between that of the linear cascade analogy and a real turbine. Model turbines are generally designed to an exact scale of the actual machine and differences of the second type listed above are usually eliminated. On the other hand, differences of the first type may be present, especially those given by conditions (2)–(4). Extrapolation from such model turbine data to the real fluid condition is therefore subject to some of the same uncertainties as occur with cascade data, and indeed can only be properly carried out if the measured loss is broken down into its constituent parts and each constituent individually corrected.

Other methods of performance prediction based essentially on calculations of the potential flow round aerofoils are coming increasingly into use in the industry. Such methods are inherently more discriminating than the type of analysis described in this paper (taking into account the precise blade profile), but it should be remembered that such methods deal only with the profile loss in turbines, which contribute rather less than half the total losses in many real machines. Until the more difficult problem of theoretical secondary flow prediction has been resolved, cascade and turbine test data must remain the basic sources of information on secondary loss.

BREAKDOWN OF STAGE LOSS

Before proceeding to deal with the losses in detail, it is convenient to consider first the nature of the losses in a stage. The work done on the rotor blades is indicated by the change in tangential momentum, and the overall integrated value can be calculated from the velocity conditions for the mass actually passing through the rotor blades. The energy given up by the gas is more than this, owing to the friction on the blade profiles, and loss in blade wakes (profile loss); the friction on the walls at root and tip, and other end effects (secondary loss); and any losses due to sudden enlargements in the fluid path, or wall cavities (annulus loss).

However, not all the fluid passes through the rotor blades, because of leakage through diaphragm glands, balance holes, and over the rotor blade tips; so the actual work per unit total mass flow is less than the work done on the blades per unit blade mass flow as evaluated above. Further, windage and bearing losses reduce the coupling power below that produced at the blades. Losses resulting from partial admission are also (in part) similar to windage loss, and can conveniently all be treated as a difference between blade and coupling work, as can lacing wire and wetness losses with rather less theoretical justification.

The total breakdown in losses can therefore be subdivided into the following constituent parts, assumed to be non-interacting:

Group 1	*Group 2*
Guide profile loss.	Guide gland leakage loss.
Runner profile loss.	Balance hole loss.
Guide secondary loss.	Rotor tip leakage loss.
Runner secondary loss.	Lacing wire loss.
Guide annulus loss (lap and cavity).	Wetness loss (where two-phase flow occurs).
Runner annulus loss (lap, cavity and annulus).	Disc windage loss.
	Losses due to partial admission.

A blading efficiency can then be defined as

$$\eta_b = \frac{\text{Work done in blading}}{\text{Work done in blading} + \text{Group 1 losses}} \quad (1)$$

this being essentially

$$\frac{\Delta H \text{ useful at blades}}{\Delta H \text{ isentropic}}$$

referred to the mass flow passing through the rotor blade.

The overall stage efficiency is then defined in a similar form as

$$\eta_t = \frac{\text{Work done in blading} - \text{Group 2 losses}}{\text{Work done in blading} + \text{Group 1 losses}} \quad (2)$$

this being essentially

$$\frac{\Delta H \text{ useful at coupling}}{\Delta H \text{ isentropic}}$$

referred to the total mass flow at the stage.

It is convenient to evaluate the Group 1 losses as loss factors based on the relative blade outlet velocities in the case of profile, secondary and lap losses, and on the inlet velocity in the case of the annulus loss. The Group 2 losses are evaluated as a net deficit in stage efficiency, this being the simplest way in which they are derived from test data. In this form the overall stage total head efficiency becomes

$$\eta_t = \frac{\text{Work done in blading}}{\text{Work done in blading} + \text{Group 1 losses}} - \sum(\text{Group 2 efficiency debits}) \quad (3)$$

in which the Group 1 loss term may be written approximately as

$$\text{Group 1 losses} = (X_p+X_s+X_a)_g \frac{C_1^2}{200gJ} + \left(X_p+X_s+X_a\frac{C_2^2}{W_2^2}\right)_r \frac{W_2^2}{200gJ} \quad (4)$$

where the first term of the Group 1 loss equation refers to the guide blade losses and the second term to the runner blade.

It should be noted that the definition of stage efficiency is based directly on inlet and outlet total head conditions specified at stations corresponding to guide entry of the particular stage assessed and the downstream stage respectively. Total head conditions applicable to the downstream stage are thus specified by the net work done and the overall stage efficiency defined above, as shown typically in the enthalpy–entropy diagram in Fig. 2. This procedure modifies that normally used in steam turbine practice which is based on static conditions where the implied total head at guide entry is obtained through an estimated

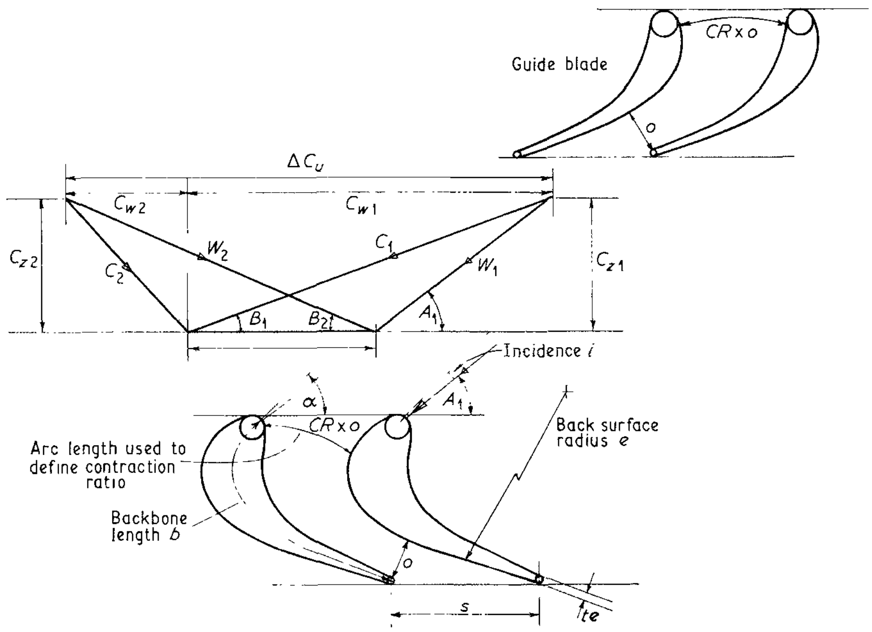

Fig. 1. Turbine blade and velocity triangle notation

'carry-in' kinetic energy. The latter is calculated as a fraction of the kinetic energy at runner outlet (stage leaving loss). This procedure is clumsy to apply where there are variations in velocity between runner outlet and guide entry, whereas the total head method is always direct.

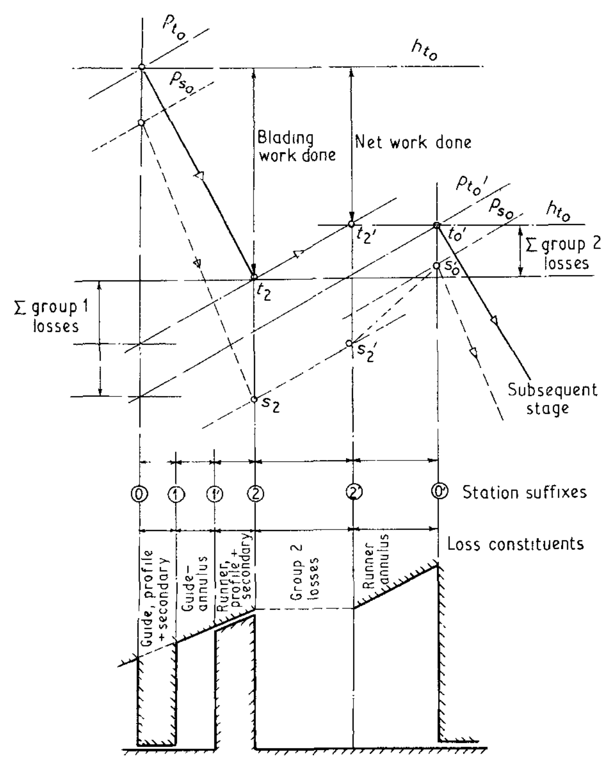

Fig. 2. Typical enthalpy–entropy diagram of stage

Once total head conditions at entry to the downstream stage are fixed the static conditions follow simply from mass flow continuity.

The four subsequent sections are concerned with the estimation of the loss coefficients and efficiency debits required to evaluate the overall stage total head efficiency defined by equation (3). These losses have to be evaluated consistently in conjunction with a three-dimensional flow solution, and to make this solution as realistic as possible the correct losses must be introduced into the flow analysis between specific axial stations within the stage with which each loss can be directly associated. Profile loss is essentially variable and calculable along the blade height, while secondary loss can be evaluated separately for root and tip conditions. Lap, annulus, and cavity losses occur between blades but these losses are essentially one-dimensional and should not be applied as a variable along the blade height.

The precise manner in which Group 2 losses are absorbed into the main flow field is uncertain. For convenience they—and any unshrouded rotor tip loss—are incorporated in an assumed mixing zone just downstream of the runner outlet station.

CORRELATION OF CASCADE DATA

The correlation of profile and secondary loss is based on an analysis of linear cascade data. One major problem in such correlation is the choice of independent variables, since cascade tests are not normally carried out with a variation of one parameter only. For instance, if the effect of Reynolds number is being measured, almost invariably the Mach number or the aspect ratio of the cascade is being simultaneously altered unless—and this is rarely so —the test is done on a variable density rig. If the wrong choice of independent variable is made, satisfactory correlation can still frequently be achieved within the range covered by the tests. However, errors, which may possibly be serious, will then arise when such a correlation is applied to values outside that range. Only by checking the final correlation against the range of variables encountered in practice can one ascertain whether the right choice of independent variable has been made.

The correlation evolved and presented in this paper is based on the analysis of over 100 specific cascade tests and on comparisons with a wide variety of published information. All losses in this section are related on a basis of velocity coefficients and are dependent on the following parameters:

(1) Reynolds number (Re) (based on outlet velocity and blade opening);
(2) aspect ratio (blade height/backbone length ratio);
(3) blade angles and passage geometry;
(4) pitch to backbone length ratio;
(5) Mach number (Ma);
(6) incidence.

The profile loss correlation is presented in the form of a basic loss correlation for incompressible flow conditions involving a variation in (3) and (4) only. To this basic value multiplying correction factors are presented which are to be applied where values of the other parameters differ from the standard values assumed in the basic condition. The basic correlation itself was derived originally from low speed tests where it could be assumed that the Ma effects could be ignored and that only items (1)–(4) were truly independent.

Effect of Reynolds number

The loss effect of Re on blade cascade performance is pronounced; typically in the range of Re between 2×10^4 and 2×10^5 the loss will be halved. A general prediction method for use in steam turbine analysis requires that the effect of Re should be predictable up to values of Re_o equal to about 4×10^6 which are now obtained at the inlet of modern high pressure (h.p.) cylinders. Thus any correlation of cascade data which neglects the Reynolds number of test is of little value.

In the method presented, the Re has been based on the blade opening, rather than on the chord or axial width, because it gives better correlation: and this correlation, derived from an analysis of Re effects in cascade, is shown by the standard finish curve of Fig. 3. At high Re values the surface roughness of the blade (or of the annulus walls with secondary loss) becomes important in conditions where it effectively controls the boundary layer thickness. To allow for this effect, data derived from Speidel (5) have been superimposed on the cascade Re correction to form the composite plot given in Fig. 3 covering a range of relative surface roughness.

When the Re effect defined in Fig. 3 was taken into account, no consistent effect of subsonic Ma on velocity coefficient was found for blade profiles designed with little profile curvature on the suction surface downstream of the throat. Thus the proposed correlation contains no Mach number effect on this type of profile at subsonic conditions.

Estimation of profile loss

The correlation of the profile loss for subsonic flow at or near the incidence at which the loss is a minimum (basic loss) is given in Figs 4 and 5. Fig. 4 defines a lift parameter F_L and Fig. 5 gives the basic loss parameter $x_p(s/b) \sin B$ as a function of two variables: $F_L s/b$ and the blade passage contraction ratio (CR). The losses on

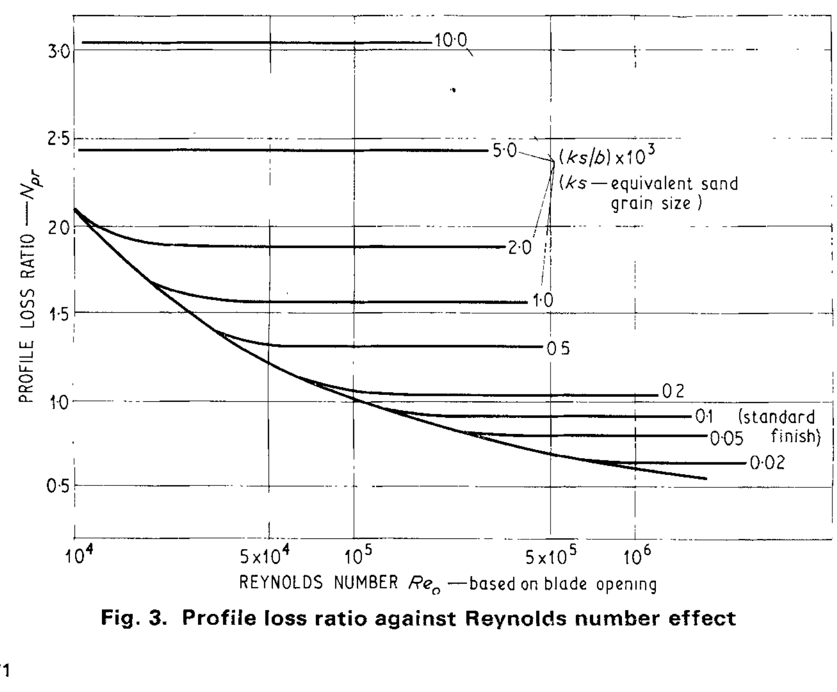

Fig. 3. Profile loss ratio against Reynolds number effect

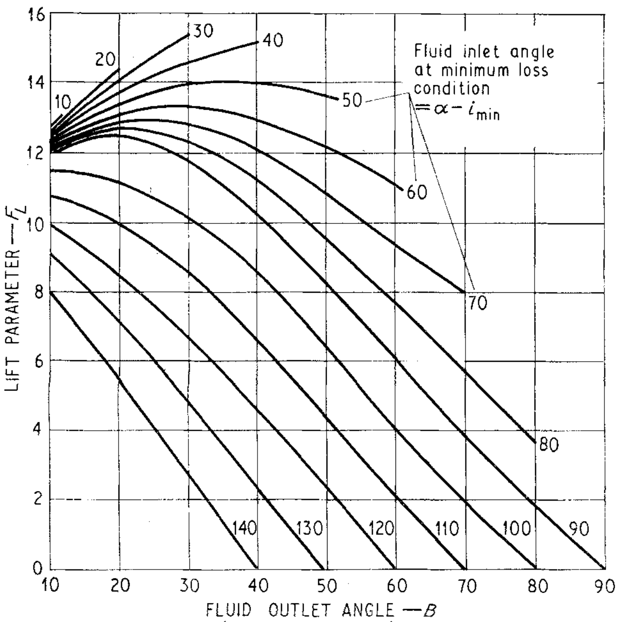

Fig. 4. Lift parameter, F_L

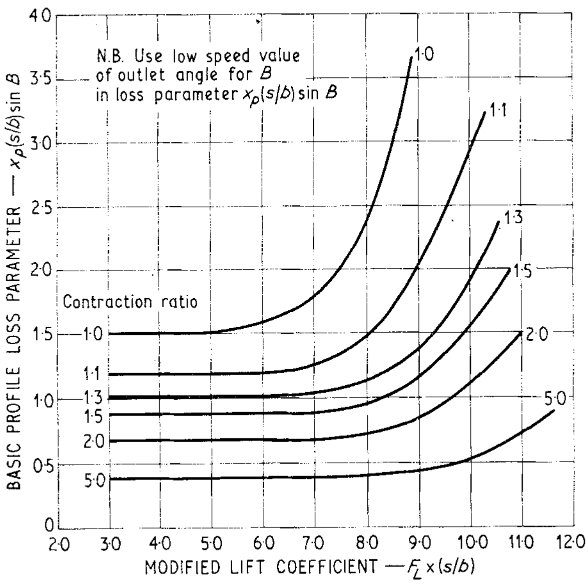

Fig. 5. Basic profile loss

which Fig. 5 is based have been adjusted to a zero trailing edge thickness condition, using the correction given in Fig. 6, this being derived by using a theoretical approach similar to that of Stewart (**6**). This correlation satisfactorily predicts the increase of losses associated with separation from the blade surface as the pitch to chord ratio is increased.

The basic profile loss correlation defined above relies upon a geometric cascade property referred to as the *CR*. This term denotes the internal blade passage width ratio

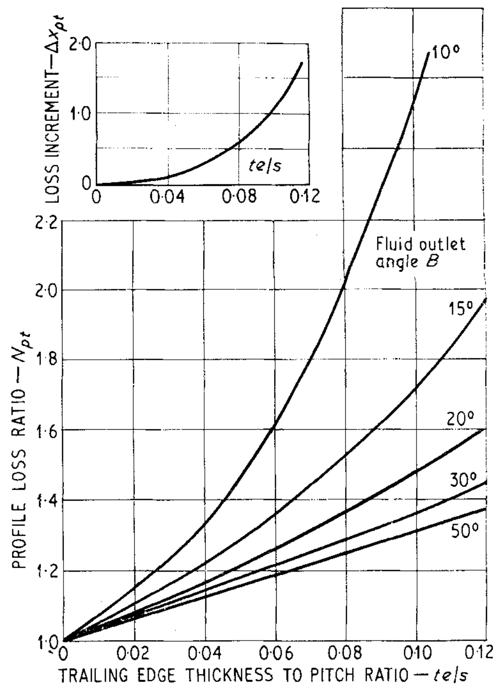

Fig. 6. Trailing edge thickness losses

of the particular cascade considered based on inlet to throat. The inlet internal passage width is not easy to define uniquely for any arbitrary cascade, but may be taken as the length of the maximum circular arc which can be drawn wholly within the passage at inlet and which is normal to both profile surfaces. For application to design analysis where the particular blade profiles have not been specified, a typical value of contraction ratio may be obtained from the data in Fig. 7 which cover a range of profile geometry.

Where the blade outlet *Ma* exceeds unity, a fundamental additive correction is made to the basic profile loss defined above. The correlation of this additive loss, referred specifically to convergent profiles designed with a straight suction surface downstream of the throat, is given in Fig. 8. For profiles designed with a pronounced convex suction surface curvature downstream of the throat, a further additive loss is required over and above that given for straight backed blades. The correlation of this second correction is shown in Fig. 9 where the additive loss factor is given as a function of outlet *Ma* and the ratio of the blade pitch to the mean suction surface radius s/e. As can be seen from the data in Fig. 9, *Ma* effects on curved suction surface profiles can exist at subsonic Mach numbers as low as 0·7 and occur when sonic conditions are attained locally on the suction surface.

While the above loss factor analysis is adequate for design calculations where the incidences are held close to optimum values, it is also necessary to consider off-design application where incidence losses may become appreciable. A correction for incidence is given in Fig. 10 in the

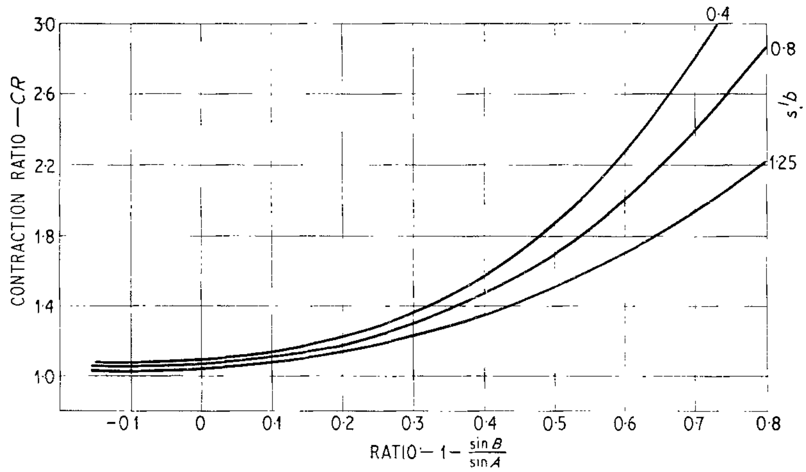

Fig. 7. Contraction ratio for average profiles

form of a loss ratio plotted against the incidence parameter $i-i_{\text{min}}/i_{\text{stall}}-i_{\text{min}}$. The form of correction is similar to that given by Ainley and Mathieson (1) except that in the present correlation the negative stalling incidence and minimum loss incidence have been correlated independently of the positive stalling value. Throughout the incidence analysis the stalling incidence is defined arbitrarily, as in (1), as the incidence at which the profile loss is twice the minimum value.

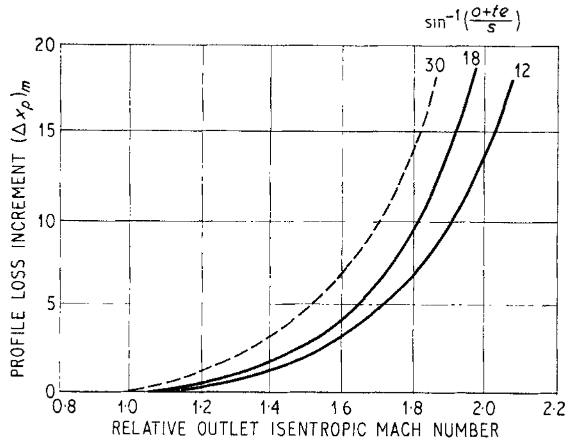

Fig. 8. Mach number loss for convergent blading

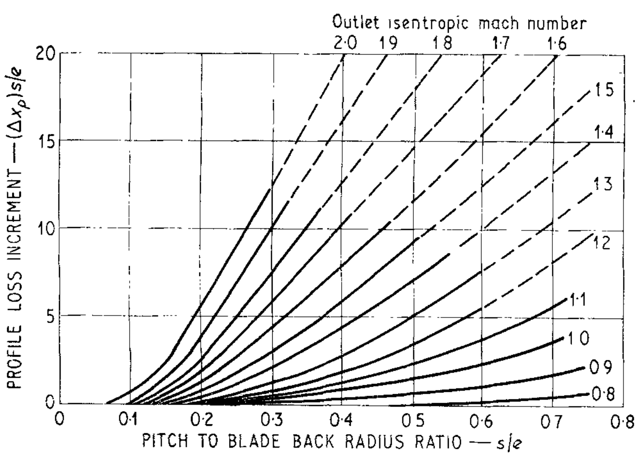

Fig. 9. Blade back radius losses

Figs 11–14 present data from which the positive and negative stalling incidences can be estimated. The minimum loss incidence is calculated from the data in Fig. 15 and equation (9) using the estimated values of the stalling incidences. A loss curve for each blade section can then be evaluated, using these values and the basic curve in Fig. 10. For initial design studies it is of course generally permissible to ignore incidence effects in the $-10°$ to $+5°$ range.

The incidence parameters are then evaluated from the equations, when $\alpha \leqslant 90°$:

$$i+\text{stall} = (i+\text{stall})_{\text{basic}}+(\Delta i+\text{stall})_{s/b}+(\Delta i+\text{stall})_{CR} \quad . \quad . \quad . \quad (5)$$

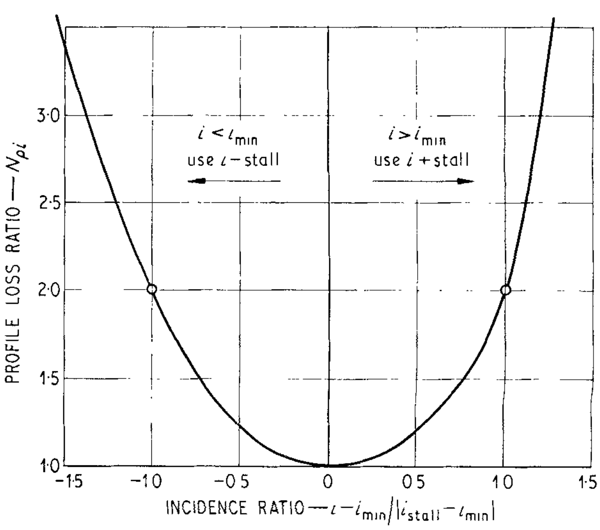

Fig. 10. Incidence losses

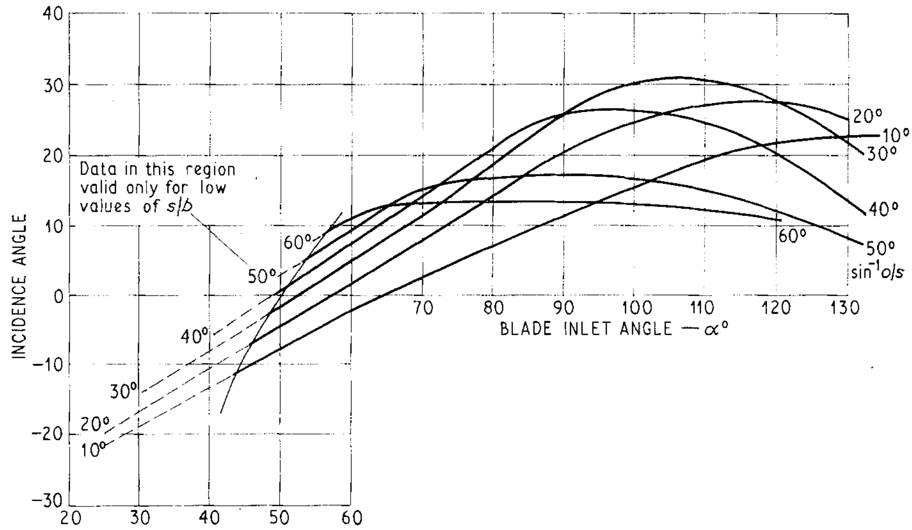

Fig. 11. Basic positive stalling incidence $(i+\text{stall})_{\text{basic}}$

where $(i+\text{stall})_{\text{basic}}$ is given by Fig. 11, $(\Delta i+\text{stall})_{s/b}$ and $(\Delta i+\text{stall})_{CR}$ are given in Fig. 12.

$$i-\text{stall} = (i-\text{stall})_{\text{basic}}+(\Delta i-\text{stall})_{s/b} \quad . \quad (6)$$

where $(i-\text{stall})_{\text{basic}}$ and $(\Delta-i_{\text{stall}})_{s/b}$ are given in Fig. 13.

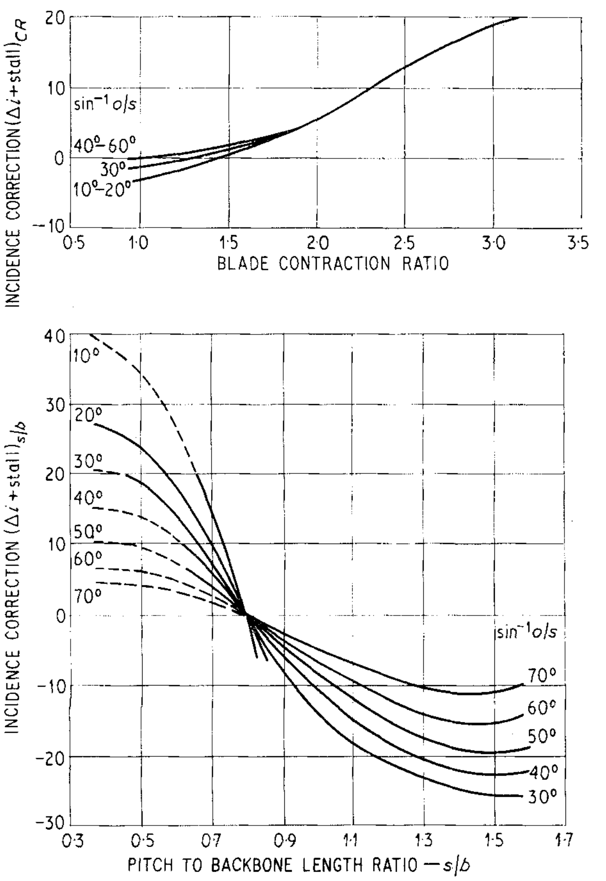

Fig. 12. Incidence corrections for positive stalling incidence

When $\alpha > 90°$:

$$i+\text{stall} = (i+\text{stall})_{\text{basic}}+\left(1-\frac{\alpha-90}{90-\sin^{-1} o/s}\right) \times[(\Delta i+\text{stall})_{s/b}+(\Delta i+\text{stall})_{CR}] \quad (7)$$

where $(i+\text{stall})_{\text{basic}}$ is given in Fig. 14, $(\Delta i+\text{stall})_{s/b}$ and $(\Delta i+\text{stall})_{CR}$ are given in Fig. 12.

$$i-\text{stall} = (i-\text{stall})_{\text{basic}}+\left(1-\frac{\alpha-90}{90-\sin^{-1} o/s}\right) \times(\Delta i-\text{stall})_{s/b} \quad (8)$$

where $(i-\text{stall})_{\text{basic}}$ is given in Fig. 14, $(\Delta i-\text{stall})_{s/b}$ is given in Fig. 13.

The minimum loss incidence may then be evaluated from the equation

$$i_{\min} = \frac{(i+\text{stall})+F_i(i-\text{stall})}{1+F_i} \quad . \quad . \quad (9)$$

where the incidence parameter F_i is given in Fig. 15.

To summarize, the profile loss factor in the proposed correlation is obtained using the equation

$$X_p = x_{pb}N_{pr}N_{pi}N_{pt}+(\Delta x_p)_t+(\Delta x_p)_{s/e}+(\Delta x_p)_m \quad (10)$$

A further correction is required to allow for three-dimensional flow effects where the meridional streamlines are not parallel, and this is discussed in a later section.

When the relative isentropic outlet Ma exceeds about 1·4, the losses can be reduced if a blade profile giving a convergent–divergent passage is used. The profile loss of such a blade is best derived from a cascade test, but may be estimated from theoretical considerations for the purposes of a generalized correlation. Detailed specification of the design and performance prediction of such blades is outside the scope of this paper, but briefly the loss can be calculated at the limit loading condition where the blade is just fully loaded. The trailing shock system at this condition is estimated from the theoretical surface velocity

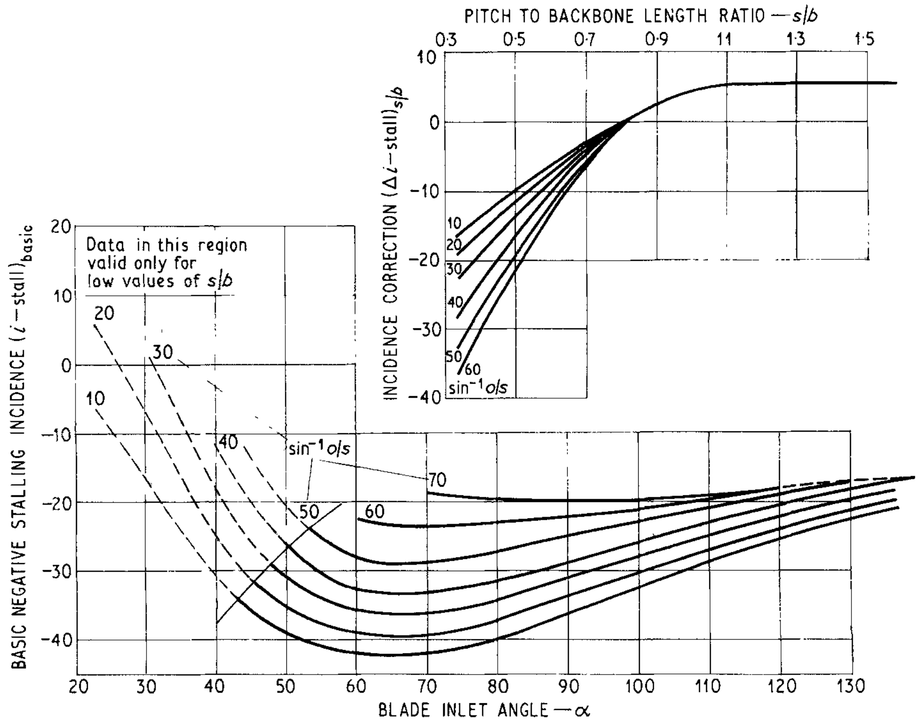

Fig. 13. Negative stalling incidence

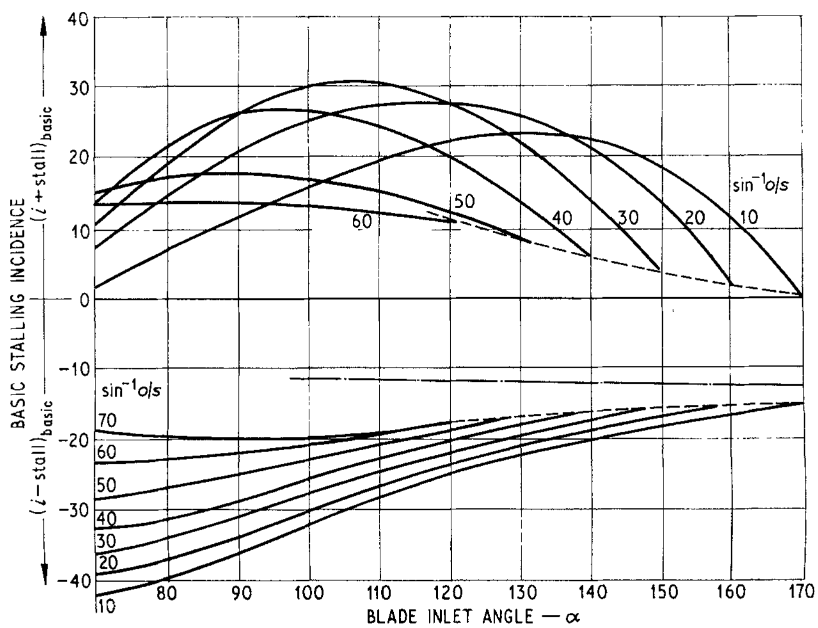

Fig. 14. Basic stalling incidences—for values of blade angle greater than 90°

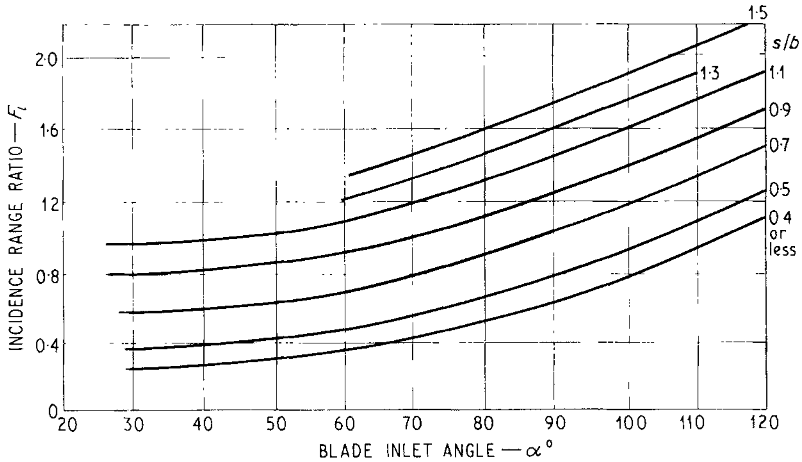

Fig. 15. Minimum loss incidence—range ratio F_i

condition just upstream of the trailing edge, by using shock reattachment criteria derived from data given by Nash (7); the wake thickness is estimated from the surface boundary layer properties. At conditions other than at limit load the variation of profile loss with *Ma* can be obtained from a generalized correlation of test data. An example of this procedure is illustrated in Fig. 16 where this general method has been applied to a particular blade profile for which test data are available for comparison.

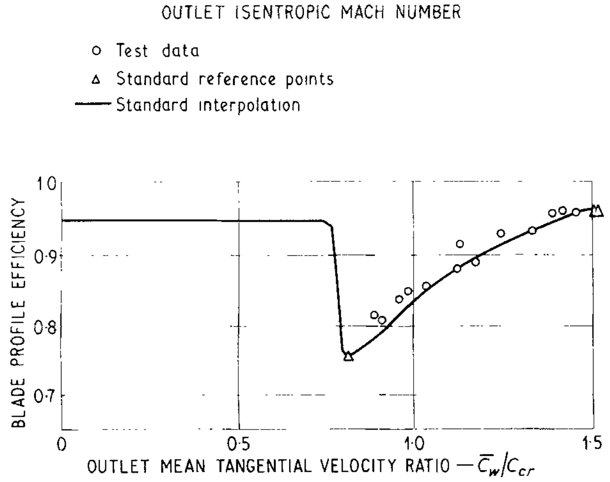

Fig. 16. Predicted and measured profile efficiencies for a typical convergent–divergent tip profile

Estimation of secondary loss

Secondary loss in a turbine consists partly of a true aerodynamic secondary loss and partly also of wall friction, both effects being complicated by any irregularities in the wall shape that may exist and by interaction with clearance flows. Precise prediction cannot therefore be expected and since the relative velocity between fluid and wall is of some importance, distinction must be made between shrouded and unshrouded stages. The correlation proposed here refers specifically to shrouded blade rows, but it is capable of adaptation for application to unshrouded blade rows.

The correlation is based on the assumption that the secondary loss is approximately inversely proportional to the aspect ratio of the blading and in addition shows a Reynolds number effect similar to that exhibited by the basic profile loss. Consideration of the Reynolds number effect serves to explain, qualitatively at least, the anomaly existing in other correlations where differing aspect ratio effects are quoted, depending on whether the blade height or the chord is being varied. For a change in aspect ratio obtained by varying the chord only, the *Re* effect will partly offset the fundamental aspect ratio effect, resulting in a reduced change of loss compared with that predicted for the same change in aspect ratio obtained by varying the height. In this correlation the effect depends strongly upon the absolute level of Reynolds number; at very high values the *Re* effect will become negligible and aspect ratio losses of blades of equivalent roughness will be equally affected by height and chord changes.

The proposed correlation is given in Figs 17 and 18 where a blade loading parameter and the relative velocity ratio are used as the independent variables. As the aspect ratio decreases the tip and root secondary loss concentrations tend to merge together and the rate of increase in loss is less than would be anticipated from a strict inverse law. The overall secondary loss factor is then estimated from the equation

$$X_s = (N_s)_r (N_s)_{h/b} (x_s)_b \quad . \quad . \quad . \quad (11)$$

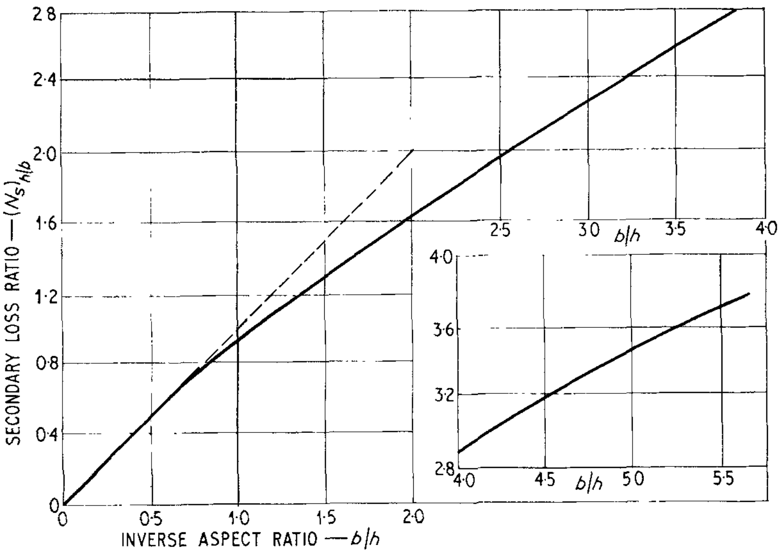

Fig. 17. Secondary loss–aspect ratio factor

and the correlation approximately holds over a range of blade incidences provided that the correct velocities are used in evaluating $(x_s)_b$.

DIFFUSING PASSAGES BETWEEN STAGES AND WALL CAVITIES

Additional losses occur where there is an appreciable amount of diffusion between two adjacent stages or where wall cavities occur between the guide and runner blades, and a further loss will be incurred where lap is introduced. The effect of lap on clearance losses is treated separately later in the paper.

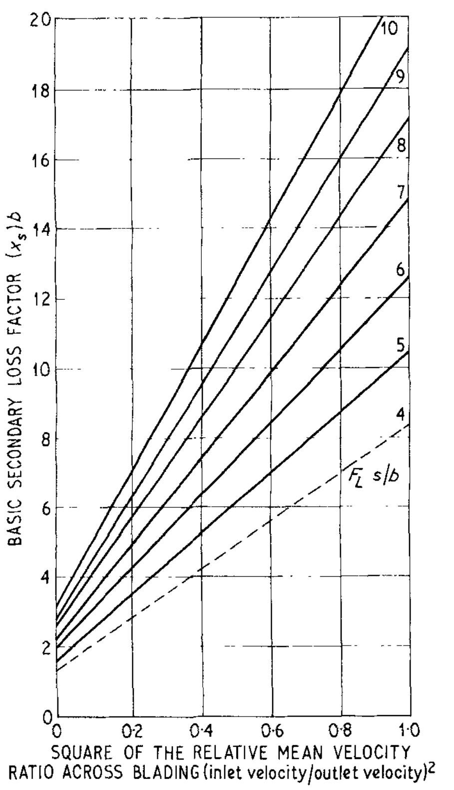

Fig. 18. Secondary loss–basic loss factor

The annulus loss factor X_a is given by the sum of the following three individual loss factors:

annulus loss factor (X_{a1})—given in Fig. 19;
cavity loss factor (X_{a2})—given in Fig. 20;
cavity loss factor (X_{a3})—calculated as a sudden expansion loss.

In all cases the equivalent non-dimensional loss factor is based on the inlet dynamic head. Distinction in the application of the data in Fig. 19 has to be made, depending upon whether the expansion is controlled or uncontrolled. The controlled expansion data are given by the full lines as a function of the equivalent diffuser cone angle, while the uncontrolled expansion data are represented by the broken lines as a function of distance ratio. Both sets of data are dependent upon the overall area ratio (inlet to outlet) while the effect of flow extraction is best simulated by considering the area ratio defined by the outer streamline of the flow passing into the downstream stage.

Typical data on cavity losses have been taken directly from Yablonik, Markovich and Al'tshuler (8) and presented in Fig. 20 in the form of a loss parameter dependent upon two variables largely defined by the geometric dimensions of the cavity and the stage. Differences between the loss through cavities situated between the guide and runner blades or between the runner and the downstream guide blade are said to be largely due to blade interaction effects which are much greater in the former instance. The data presented here are given mainly in order to emphasize that a comprehensive method of performance evaluation must allow for such losses. However, it should be noted that losses arising from cavities fundamentally

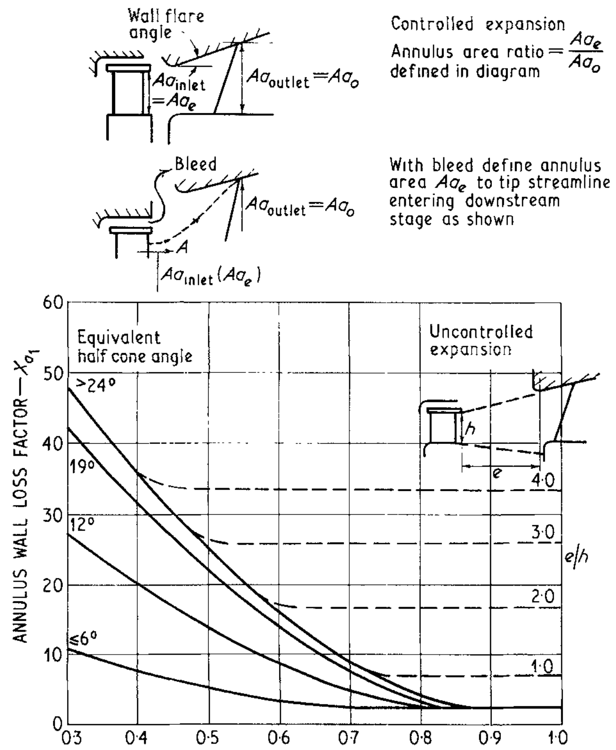

Fig. 19. Annulus wall loss

different in shape from that considered in (8) should be obtained from actual test data.

LAP, CLEARANCE, BALANCE HOLES AND GLANDS

The third main source of loss in a turbine is that from leakage, either over blade tips, around shrouding, or—with disc and diaphragm designs of steam turbine—through

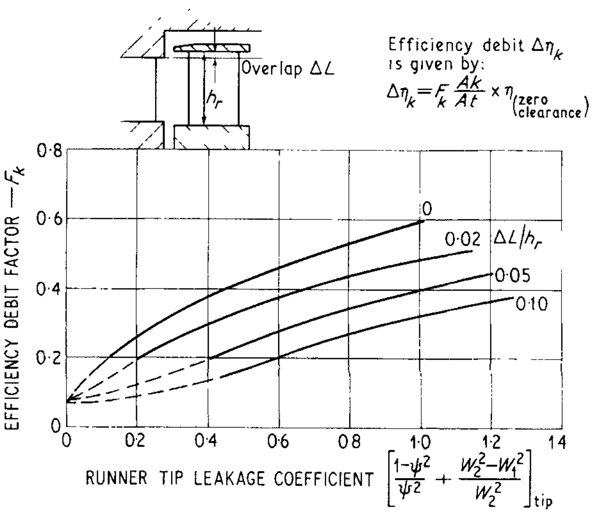

Fig. 21. Shrouded efficiency loss

disc balance holes and diaphragm glands. Such flows may be modified by the presence of lap (a sudden enlargement between stationary and moving blade rows frequently found in steam turbine designs).

Positive lap appears to have two effects: to give a loss which closely approximates to a sudden enlargement calculated by standard formula; and to influence the static pressure immediately after the lap in a manner which will reduce leakage flow.

This static pressure reduction may be the equivalent of perhaps a 10 per cent reduction in reaction. The existence of positive lap therefore tends to reduce leakage effects, and for any given clearance an optimum lap exists.

Typical clearance loss correlation is given in Fig. 21 for shrouded blading. At zero reaction this loss does not become zero, as some windage loss from the shroud band remains, and this strictly requires separate calculation. Where

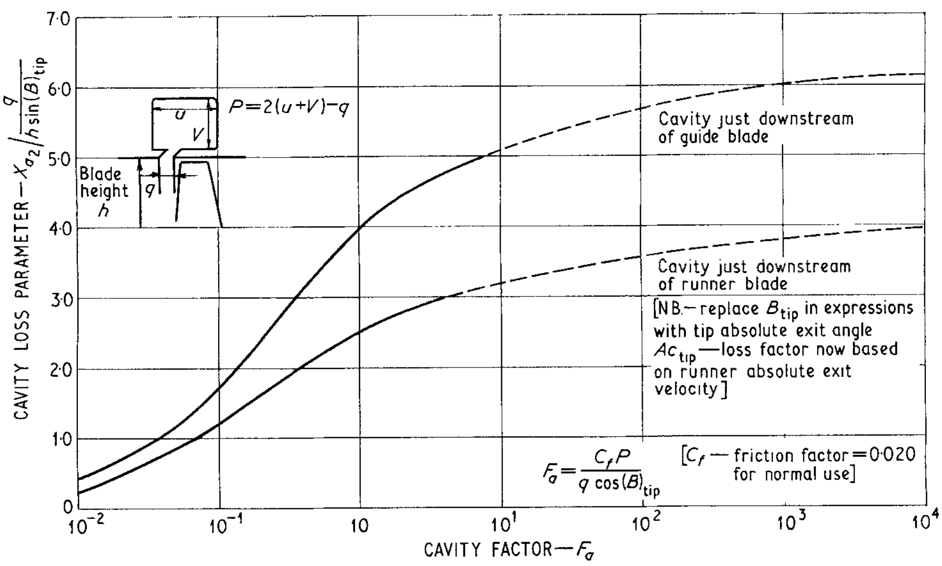

Fig. 20. Cavity loss

multiple seals are used to control leakage, an effective leakage area can be calculated by using an inverse square law. The data presented here refer specifically to seals operating just downstream of an abrupt flow disturbance. In cases where more uniform entry conditions to a seal are likely, the area of that seal should be multiplied by 1·5, to allow for the increase in its effective discharge coefficient, before using it in conjunction with Fig. 21. Additional correction may be made to take into account the detail of shroud overhang design. Lap is given in Fig. 21 as a function of blade height, which is a reasonable approximation in high and intermediate pressure cylinders, but requires modification for stages with long blades.

For unshrouded blades, the authors believe that the data in (**1**) are reasonably representative, provided that axial velocity remains approximately constant across the blade row and provided that the relative velocities are well below the sonic value. In steam turbine designs, unshrouded blading normally occurs only at the rear of the low pressure (l.p.) cylinder where neither of the above conditions will apply. For these conditions, it is suggested that a fair approximation to the tip loss is given by a value equal to 1·5 times that derived from use of the data in Fig. 21.

The leakage flow through glands can be calculated by standard formulae. It is convenient to represent the effective loss and other flow bypass losses, including that caused by balance holes, by an equivalent efficiency debit $\Delta\eta$, where

$$(\Delta\eta)\text{ leakage} = \left(\frac{\Delta g}{G+\Delta g}\right)\eta_b \quad . \quad . \quad (12)$$

and $\Delta g/(G+\Delta g)$ denotes the leakage fraction.

Balance holes are fitted, mainly in high and intermediate pressure steam turbine stages, to reduce or control the axial thrust by allowing a small radial flow down the face of the disc. The effect of this flow is to maintain an inwardly decreasing pressure across the upstream face of the disc which materially reduces the axial thrust, accurate estimates of which can then be made from the predictable pressure gradients. Calculation of the leakage flow can be carried out using the data on flow coefficients for balance holes given in (**9**), by considering a net flow balance across the rotor disc, though our own tests do not wholly confirm the values in (**9**).

Correctly designed balance holes may provide some benefit to the overall stage performance since they will automatically swallow the gland leakage flow. In this event the leakage flow will not pass up the upstream face of the disc, and spillage effects into the main flow which could induce an early separation of the flow along the inner casing wall will be avoided.

MISCELLANEOUS LOSSES

Lacing wire

For wires of circular cross section the mean blade loss is increased by approximately 1 per cent of the local relative velocity head at the wire section for each 1 per cent of passage area blocked by the wire. Then, in terms of an equivalent efficiency debit required to calculate the overall stage efficiency, the wire loss is given by

$$\Delta\eta_l = \frac{\left\{\dfrac{\text{Wire area} \times C_d}{\text{Passage area}}\dfrac{W^2_{\text{local}}}{2gJ}\right\}\eta_b}{\text{W.D. blading}} \quad . \quad (13)$$

where a wire drag coefficient is added to account for non-circular wires. For wires of elliptical section of fineness ratio of $\frac{1}{4}$ the loss will be decreased by 70 per cent compared with a circular wire ($C_d = 1{\cdot}0$). Efficiency analysis on particular stages shows considerable scatter because of a reactive effect of the wire on the flow through the blading, as distinct from the wire drag loss itself. It should be noted that in the form given above the wire efficiency debit will depend quite strongly on the stage reaction.

Wetness loss

Various corrections have been proposed to allow for the additional losses suffered in stages operating with wet steam and these have been reviewed by Wood (**10**). No absolute evidence has been published which completely substantiates one method in preference to the others, most of which in any case give somewhat similar predictions of loss. In order to complete the turbine efficiency correlations given in this paper, it is suggested that (at present) the simplest procedure, given by Baumann (**11**), be used. This proposes a loss of 1 per cent in stage efficiency per 1 per cent of mean stage wetness.

However, it should be pointed out that most of the data on which such approximations are based are obtained from tests on low pressure wet steam. Where the machine operates on high pressure wet steam, as for instance on turbines for water cooled nuclear reactors, there is evidence that the loss may be appreciably less. High pressure wet steam has its liquid phase in droplets whose maximum size (determined on stability grounds) is much smaller. The droplets thus tend, for a given wetness, to be more numerous and better dispersed. They may be typically only about seven diameters apart. It is not difficult, therefore, to believe that they may behave less independently of the vapour phase than at low pressure.

Disc windage

The windage efficiency debit is derived from a power loss analysis, the actual debit being calculated by the equation

$$\Delta\eta_{\text{ disc windage}} = \frac{\Delta P_w}{\text{W.D. blading}} - \eta_b \quad . \quad (14)$$

where the power loss term (in W.D. units) is given by

$$\Delta P_w = T_w \frac{\left(\dfrac{\text{rev/min}}{100}\right)^3 (D_h)^5}{Gv} \frac{10}{3{\cdot}471^{-10}}\text{ (B.t.u./lb)} \quad . \quad . \quad . \quad (15)$$

It is recommended that the values of torque coefficient, T_w, be taken from the results given by Daily and Nece

(12). The torque coefficient varies with disc Re and disc-casing spacing ratio.

Partial admission

No new correlation is proposed here, the best available being that given by Suter and Traupel (13).

CASINGS

Any performance method must make allowance for losses in inlet and exhaust casings as these are very important, particularly on cylinders with few stages or where high axial velocities are employed. A comprehensive treatment of the subject, however, is outside the scope of a paper of this length.

In general it is always desirable, and sometimes essential, that casings should be tested on models using the correct Reynolds number and Mach number, and closely simulating the flow distribution at the boundary planes. Interactions between blading and casing, and in certain instances between casing and condenser, can be of considerable significance.

For general performance prediction, where air model tests are not available, one can deduce approximate casing losses based on model tests. Comprehensive data for exhaust diffusers are given by Sovran and Klomp (14); for casings with restricted space, typical of some steam turbine low pressure exhausts, correlations of the type given in Fig. 22 can be used. Where casing geometry is restricted to a particular design concept, more accurate but less general correlations can be produced.

Using the data given in Fig. 22 the exhaust loss coefficient can be estimated from the equation

$$\lambda = \frac{F_{m0}}{4}\left[1-\left(\frac{Aa_0}{Aa_1}\right)^2\right]+F_fF_{m1}\left[\left(\frac{Aa_0}{Aa_1}\right)^2+\left(\frac{Aa_0}{Aa_2}\right)^2+\left(\frac{Aa_0}{Aa_3}\right)^2\right] \quad (16)$$

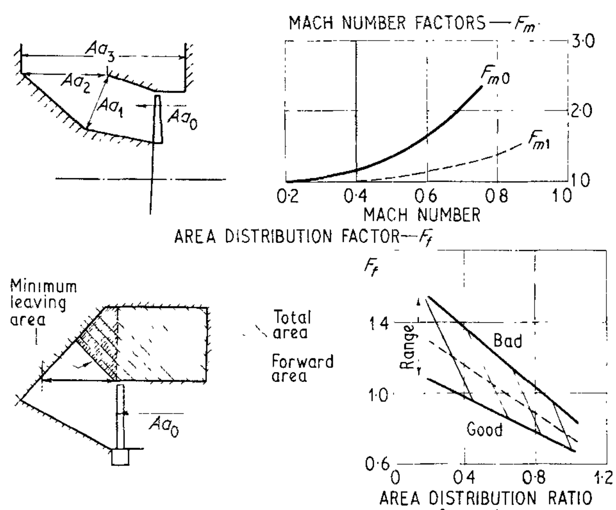

Fig. 22. Exhaust casing loss

where parameters F_{m0}, F_f and F_m are given in Fig. 22 and the area terms defined in the diagram.

APPLICATION TO TURBINE EFFICIENCY EVALUATION

In order to evaluate the performance of a complete machine, the radial velocity distributions at blade inlet and outlet stations are required in addition to the component loss data. One-dimensional mean diameter methods, i.e. using average velocity values at each station together with mean section loss data, may not introduce significant errors in very short blading, but this procedure should be discouraged since it fails to discriminate between good and bad root or tip conditions. For shorter gas turbine blading and steam turbine blading used in the h.p. and intermediate pressure (i.p.) cylinders, it is usually adequate to derive the radial velocity distributions from calculations using only the simple radial equilibrium condition. For longer blading, used typically in the rear stages of the l.p. cylinders, the radial flow solution must take into account both the streamline curvature and radial velocity effects, together with the influence of the upstream and downstream stages.

The general procedure used to evaluate stage efficiency follows simply from the subdivision of loss constituents given in the earlier part of the paper, where equation (3) defines the total head stage efficiency in terms of Group 1 losses and Group 2 efficiency debits. Whereas the Group 2 constituents are simply overall stage efficiency debits which each have a unique value for the particular stage considered, the losses included in Group 1 are essentially variable along the blade length. Thus if the efficiency in equation (3) is taken to represent the overall mean stage efficiency, the losses of Group 1 in that equation must be interpreted as the integrated value over the blade length. For short blades it is recommended that the Group 1 loss term should be evaluated from at least three separate sections along each blade length corresponding to root, mean and tip radii. The local value of the sum of Group 1 losses for each blade can then be obtained by applying the respective part of equation (4) individually to each section, using local computed values of the loss factors and velocity conditions. An overall value of loss can then be established by a simple averaging process which, assuming a parabolic distribution of loss between the three section values for each blade, is given by

$$(\textstyle\sum \text{losses}_1)_{\text{average}} = \tfrac{1}{6}[(\sum \text{losses}_1)_{\text{root}} + (\sum \text{losses}_1)_{\text{tip}} + 4(\sum \text{losses}_1)_{\text{mean}}] \quad (17)$$

where $(\sum \text{losses}_1)$ refers to Group 1 losses evaluated from equation (4) for both guide and runner blade, at local conditions indicated by the subscripts. For stage designs showing some variation of stage work from root to tip the value used in equation (3) must similarly be derived from averaging techniques.

For very long blades the accuracy of the above procedure will be improved by weighting the individual section losses and work done by the local mass flow. For stages in which streamline curvature procedures are used to establish the velocity conditions, the Group 1 losses can be introduced locally along each streamline within the calculation procedure. Thus the work done by the blading and the isentropic heat drop are evaluated locally across the stage; overall mean values can then be obtained by integration. Substitution of the identical calculation for the sum of the efficiency debits will give the net work done, and the overall total head stage efficiency can be obtained from the basic definition of total head stage efficiency as

$$\eta_T = \frac{\text{integrated net work done}}{\text{integrated overall isentropic total heat drop}}$$

This process is exact and, for very large heat drops, can show up to 0·5 per cent discrepancy from that derived from the approximate relation given in equations (3) and (4). It should be noted that the annulus loss has been included in Group 1 losses for convenience, but it should, in fact, be evaluated using the average exit velocity and held constant over the blade length.

While a detailed consideration of streamline curvature techniques is outside the scope of this paper, the following relevant comments can be made. There is substantial evidence that the multi-stage streamline curvature programmes do in fact compute a flow solution similar to that actually occurring within a model turbine stage, but it is also clear that considerable care must be exercised over the assumptions which have to be made in order to effect a solution. For example, the form of the tip streamline has a considerable influence on the overall flow solution, and in wide flared steam turbine stages it is not immediately obvious where the effective inviscid flow boundary should be positioned. Local reaction values are considerably influenced by the values of loss coefficient used in the solution; thus correct allowance for the radial variation of profile and secondary loss should be introduced automatically within the calculation procedure. Care must be taken, however, to ensure that the secondary flow loss concentrations near the tip and root are not allowed to accumulate stage by stage, as these losses are redistributed by internal shear action in a real fluid.

In the rear stages of a l.p. cylinder, considerable streamline displacements occur in the regions where there is a large annulus flare. Under certain conditions this effect can result in a difference of specific mass flow between inlet and outlet stations across a blade row. This flow condition, of course, will considerably modify the loss of each section compared with its equivalent cascade value, which has been evaluated from conditions where the specific mass flow is constant across the section. A simple procedure to employ in these circumstances is to modify the one-dimensional contraction ratio definition used in Fig. 5 to a two-dimensional value obtained by multiplying the basic section value by the streamline contraction ratio from inlet to outlet. Partial confirmation of this procedure is provided by the tests reported by Deich *et al.* on flared guide blade losses (**15**). Finally, it should be noted that the geometric data on which the section loss is based should correspond to the cross section defined by the intersection of each meridional stream surface with the blade. In conditions where the velocity has a strong radial flow component this procedure will greatly modify the backbone length of the blade section.

In diverging streamline flow situations, the method proposed above is adequate only for blade sections which have a reasonably high value of the basic one-dimensional contraction ratio. Where this is not so, it is suggested that the following procedure be used. From some specific relation between total surface diffusion and loss, a value of the diffusion parameter can be evaluated which will correspond to the predicted (cascade equivalent) value of loss coefficient. This value of surface diffusion can then be increased in proportion to the relative decrease in specific mass flow across the blade section; a corresponding increased loss can be obtained from the loss–surface diffusion correlation. It is further suggested that an adequate loss–diffusion correlation may be derived from the correlation of blade outlet momentum thickness with total blade diffusion, described by Stewart, Whitney and Wong (**16**) in conjunction with the standard relation between total outlet momentum thickness and blade loss.

It should be noted that this correction is only strictly applicable to convergent blades operating with subsonic outlet velocities. If diverging streamline flow occurs across a blade section of small contraction ratio operating with supersonic outlet velocities, it is quite possible that the physical throat is effectively at the blade inlet and that the blade passage is of a convergent–divergent form. If this effect is pronounced, severe losses can result in the outlet *Ma* range 0·8–1·3.

ACCURACY OF THE METHOD

The overall validity of the method has been checked against a large number of turbines for which adequate test information was available. The purpose of the assessment, while obviously checking the reliability and accuracy of the overall method, was to establish whether consistent errors

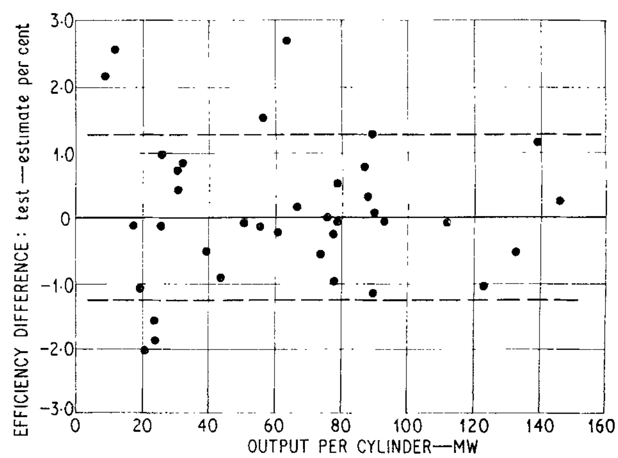

Fig. 23. Comparison of predicted and measured test efficiencies

could be detected from the comparison of calculated to measured performance. For instance, if the method involved optimistic or pessimistic assumptions about secondary loss, then it might be expected that this would have appeared as a consistent error in the comparison with data obtained from measurements on turbines operating at h.p. levels. In fact no systematic or major discrepancies have been found in an analysis of over fifty machines, most calculated values of overall efficiency being substantially within $\pm 1\frac{1}{4}$ per cent of the measured values. A plot of the data obtained is given in Fig. 23 where the efficiency error defined as the difference between the measured and calculated values of efficiency is shown, the output of each machine examined being used as a reference parameter. The tests with greater errors at small output (Fig. 23) refer largely to certain l.p. cylinder data where the measured values have been deduced from overall heat balances rather than from direct measurements.

It is claimed that these results give some grounds for confidence in the accuracy of the method and suggest that cascade test data, which form the foundation of the method, are a more reliable guide than some would have us believe

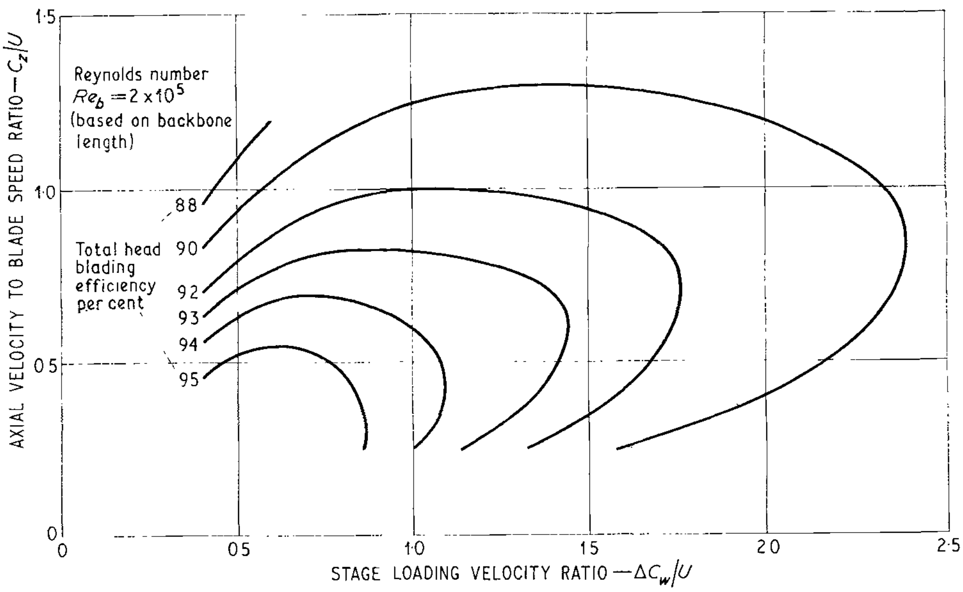

Fig. 24. Calculated total head efficiency for symmetric velocity triangle designs with guide and runner aspect ratios equal to 4·0

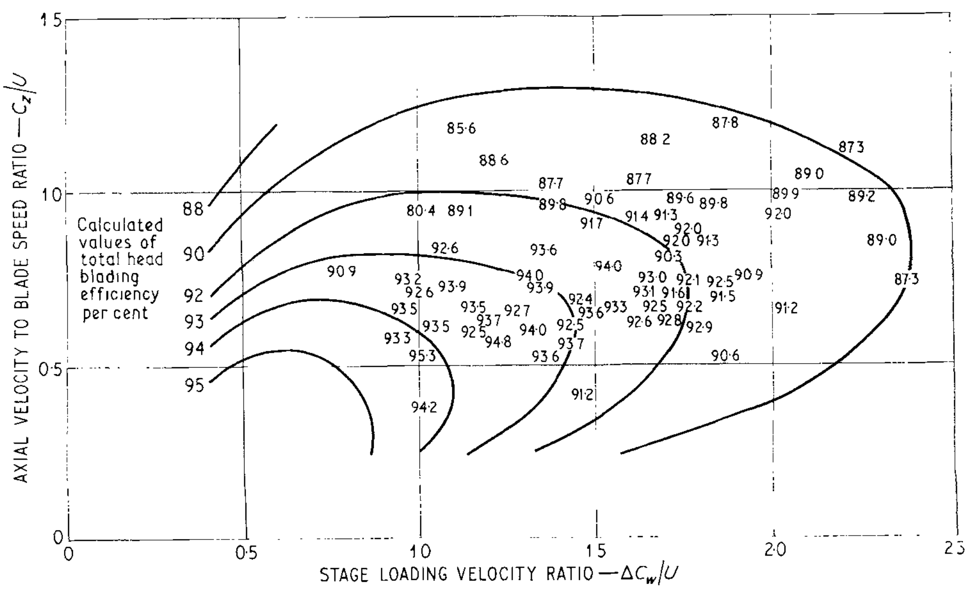

Fig. 25. Overlay of calculated total head efficiency contours given in Fig. 24 with test data published in (18)

when they are carefully interpreted. It is recognized that the efficiency comparison shown in Fig. 23 does not necessarily imply that the error on each loss constituent is small, but simply that the algebraic sum of the errors is small, i.e. some errors could be self-cancelling. However, the comparison of measured to calculated efficiencies has been extended over such a wide variety of designs that if any large constituent error existed in practice, one could expect it to have shown up as a major discrepancy in the overall efficiency.

Application to general turbine design

A theoretical method of performance prediction has the advantage that it gives the designer a method by which the relative performance merits of varying arrangements can be assessed in the early design phase. For preliminary design studies it is useful to construct generalized performance carpets which predict the stage efficiency changes that are implied by the variation of certain basic aerodynamic parameters. Three such carpets are presented in Figs 24, 26 and 27. In each instance blade data representing good design practice is used to compute the performance.

The data in Fig. 24 have been based on conditions typical of normal gas turbine practice, assuming a stage design based on symmetrical velocity triangles with a blade aspect ratio $h/b = 4{\cdot}0$ and $Re_b = 2\times10^5$. In Fig. 25 this carpet has been superimposed on the test results for gas turbines published by Smith (17), and the agreement is good. Data in Figs 26 and 27 are based on velocity triangles more typical of h.p. impulse steam turbine practice assuming a zero outlet swirl. The data in Fig. 26

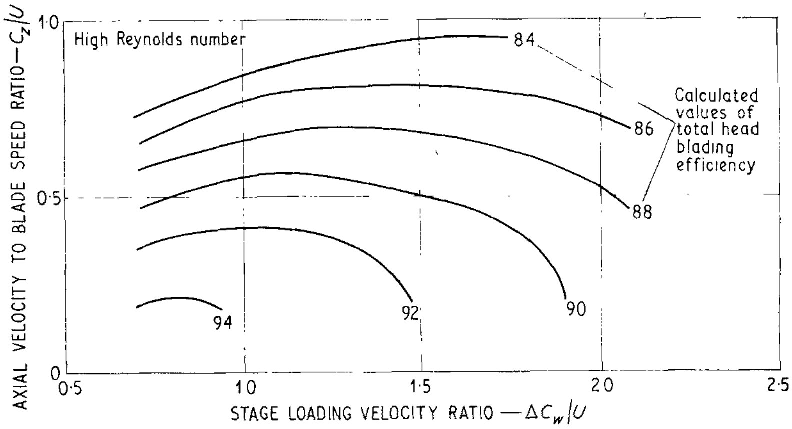

Fig. 26. Calculated total head blading efficiency for zero outlet swirl designs with guide and runner aspect ratios equal to 0·5 and 1·0 respectively

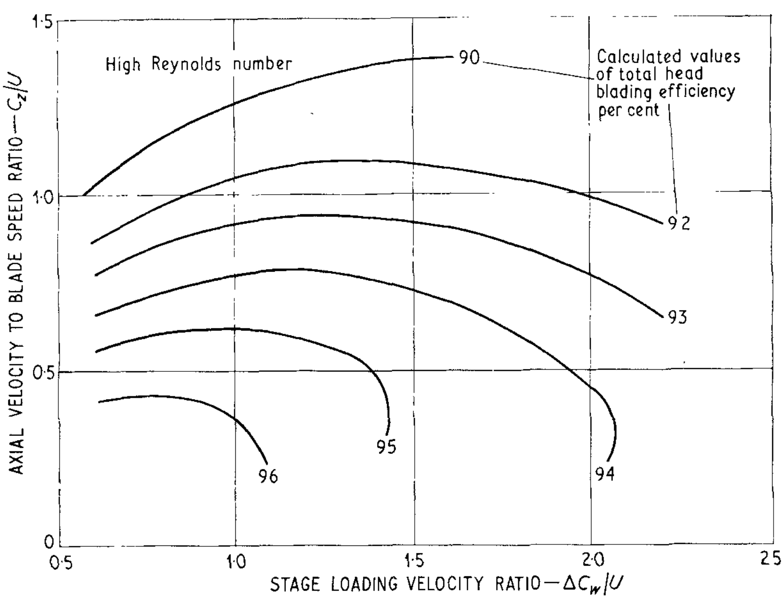

Fig. 27. Calculated total head blading effiency for zero outlet swirl designs with guide and runner aspect ratios equal to 4·0

assume aspect ratios of 0·5 and 1·0 on the guide and runner blade respectively, in contrast to the data in Fig. 27 which assume the uniformly high value of 4·0 for both blades. Comparison of Figs 26 and 27 indicates the magnitude of the aspect ratio effect predicted by the method at high *Re*. The *Re* effect can be deduced from a comparison of Figs 24 and 27 at the value of the stage loading parameter $\Delta C_w/U = 1{\cdot}0$, a condition where both velocity triangle configurations are identical.

In all calculations used to construct the carpet diagrams in Figs 24, 26 and 27 the method presented has been used in conjunction with the following simplifying assumptions:

(1) efficiency is based on the sum of profile and secondary losses only, i.e. excluding the tip clearance losses, etc.;

(2) the value of efficiency applies to a single section only, i.e. that corresponding to the specified velocity triangles;

(3) subsonic flow.

ACKNOWLEDGEMENTS

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APPENDIX

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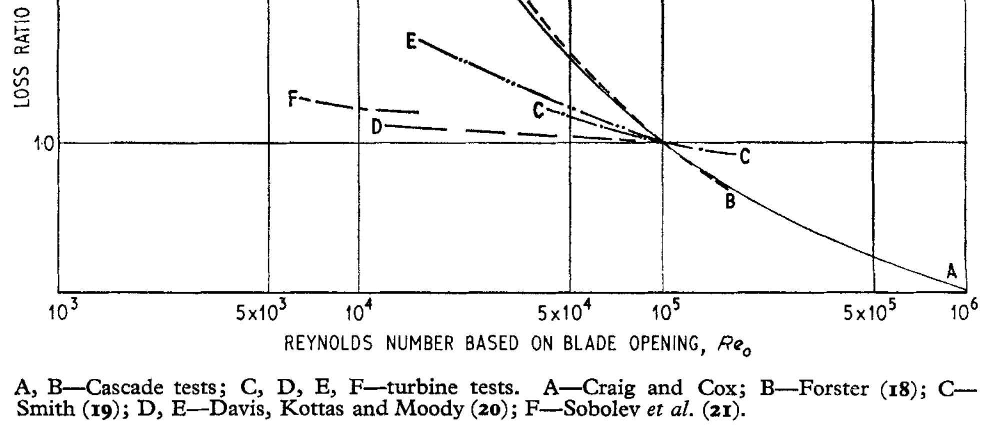
Discussion

V. J. Andrews Graduate

My comment concerns Fig. 3 which shows the variation of profile loss ratio with Reynolds number (*Re*) and which, according to the authors, can also be applied to secondary loss. Fig. 28 shows the authors' curve superimposed on the loss ratio–*Re* curves from a number of alternative sources, all of which have been reduced to the common basis of *Re* based on opening and loss ratio based on the loss at $Re_0 = 10^5$. Curve B from cascade tests (**18**) suggests, by its close proximity to the authors' curve, that all such curves show a similarly marked reduction in loss with increasing *Re*. Curve C is from tests on the Rolls-Royce Conway gas turbine (**19**). Curves D and E show the range of a number of tests on single stage impulse turbines (**20**), and curve F is from tests on a three stage low pressure model steam turbine working into the wet region (**21**). The loss ratios for this last curve, which does not pass through the datum point, have been found by extrapolating the loss curve to $Re_0 = 10^5$ assuming a power law relationship between *Re* and loss which has been found to hold approximately for all the turbines. Curves C to F cover a wide range of axial flow turbines, but they are all notably less steep than those based on cascade tests.

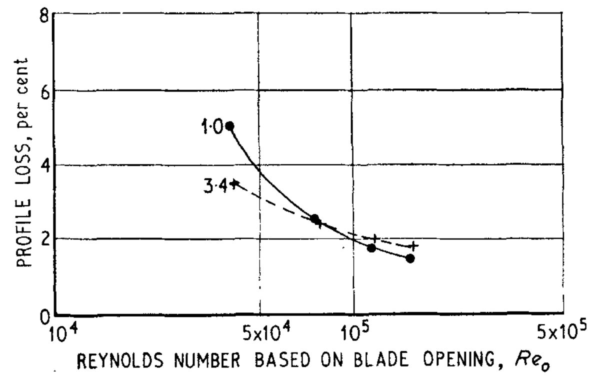
An examination of the effect of guide and runner blade losses on turbine losses shows that, assuming conditions at the cascade exactly to simulate turbine conditions, the only factor which will cause the slopes of the turbine curves in Fig. 28 to differ significantly from the slopes of the cascade curves is the presence of other turbine losses, such as tip leakage, which do not change with *Re*.

Since we are looking at curves of relative loss, the presence of a bulk of *Re* insensitive losses reduces the change of relative loss, i.e. it reduces the steepness of the curves. Without a detailed knowledge of the design of the turbines represented here, it may be reasonably argued that the ratio of *Re* sensitive losses to total losses is of the order of 0·5 for curve C, 0·4 for curve F and something rather less for curves D and E. If we divide the slopes of the turbine curves by the corresponding ratios to bring them to the same basis as the cascade curves, we find that the slopes



A, B—Cascade tests; C, D, E, F—turbine tests. A—Craig and Cox; B—Forster (**18**); C—Smith (**19**); D, E—Davis, Kottas and Moody (**20**); F—Sobolev *et al.* (**21**).

Fig. 28. Loss ratio against Reynolds number: comparison of cascade tests with turbine tests



Figures against curves are values of turbulence intensity per cent.

Fig. 29. Effect of turbulence and Reynolds number on profile loss for a low camber blade (22)

are still considerably less than the slopes of the cascade curves, and we are led to suspect that more factors are present in the effect of *Re* on turbines than are suggested by simple cascade tests.

Fig. 29 shows the result of some cascade tests by Sawyer (22). The full curve shows the variation of profile loss with *Re* for a 90° inlet cascade with an inlet turbulence intensity of 1 per cent which is fairly typical of most cascade wind tunnels. The broken curve shows the same tests with a turbulence of 3·4 per cent. Most turbines operate with a turbulence intensity of over 10 per cent in all but the first stage. Therefore Sawyer's curves suggest that the omission of turbulence from cascade tests sensibly affects their applicability to turbines. Perhaps it is also worth remembering that the *Re* curve for a flat plate with laminar flow obeys a $\frac{1}{2}$ power law but with turbulent flow obeys a $\frac{1}{5}$ power law giving a much smaller slope. Unfortunately, the blade profile used in Sawyer's test is not typical of a modern turbine, having a low camber and severe laminar separation at the trailing edge. Nevertheless, since the effect of turbulence on both Sawyer's blade and the flat plate is to reduce the slope of the *Re* curve, it suggests that turbulence may be the missing factor between the cascade and turbine tests described here and that its inclusion should give better agreement.

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A. V. Cooke Graduate

I should like to comment on the relative importance of changes in aspect ratio and *Re* on the velocity loss coefficient.

Last year the G.E.C. Air Flow Laboratory (AFL) at Rugby tested our design of rotor blade for a highly loaded stage in their low-speed wind tunnel (see Table 1 for blade details). The AFL had previously tested a large number of different cascades, from both gas and steam turbines, and had produced a set of generalized curves, plotting total loss coefficient against gas angles into and leaving the blade. These tests had been performed on the same wind tunnel as for our blade, using the same blade height and inlet boundary layer conditions but different aspect ratios.

The results of the particular and general AFL tests are listed in Table 2 and compared with the results obtained from applying the Craig and Cox analysis. There is some doubt about the correct equivalent sand roughness, so two analytic values are given corresponding to a smooth blade and one with 0·09 per cent roughness.

It will be seen that for the smooth blade the Craig and Cox analysis gives lower absolute loss levels and the difference between high and low aspect ratio blades is 2·3 per cent while the cascade tests give only 0·3 per cent difference.

For the 'rough' blades, the measured absolute loss level for the low aspect ratio blade agrees well with the analytic level, but the difference in losses for the different aspect ratio blades has risen to 3·8 per cent.

It would appear therefore that either the aspect ratio effect is overestimated in the Craig and Cox treatment; or the *Re* effect is underestimated; or that our design of rotor blade is considerably better than the blades previously tested by AFL.

Table 1. Rotor blade details

A	B	i	b/h	s/b	te/s	s/e	$(W1/W2)^2$	Ma
42·7	29·05	0	1·17	0·57	0·0251	0·263	0·341	1

Table 2

Basis for results	Blade	b/L	Re	Velocity loss coefficient, per cent	
AFL cascade test	Rotor blade	1·17	6×10^5	10·4	
AFL cascade test	General series	0·468	$2{\cdot}4\times10^5$	10·1	
				Smooth blade	Rough blade
Craig and Cox .	Rotor blade	1·17	6×10^5	8·2	10·6
Craig and Cox .	General series	0·468	$2{\cdot}4\times10^5$	5·9	6·8

J. D. Denton Marchwood, Southampton

This paper is a valuable addition to the very limited number of turbine performance estimation methods which have been published. It is especially welcome since it is the only recent method in which the emphasis is on steam turbines rather than on gas turbines.

It is not generally realized that most loss correlations are based on highly scattered experimental data and are of very limited accuracy. As a result different loss prediction methods can give widely different results when applied to the same blading. For example, I have compared the profile loss of 76 different cascades, obtained from a survey of the literature (**23**), with the loss predicted by the present method. The average value of predicted loss/experimental loss was 1·27 and the standard deviation was 0·49. Some other methods of profile loss estimation show better agreement than this, but none has a standard deviation less than 33 per cent of the mean. Even wider scatter is shown by comparison of predicted and measured secondary losses (**24**) and estimates of these may differ by an order of magnitude. Results of model turbine tests relating efficiency debit to leakage area ratio are similarly scattered.

Any performance estimation method which is based on cascade and model turbine tests alone is, therefore, likely to be very inaccurate and in practice it is often necessary to scale one or more of the loss components, usually the secondary loss, to obtain the correct level of efficiency for real machines. In view of the very good agreement between predicted and experimental efficiencies shown in Fig. 23 and particularly since the average value of the error appears to be zero, it seems probable that the loss components of the present method have been scaled in this way. If so, it would be interesting to know which of the loss components has been scaled and by how much. Since such a scaling is only likely to be accurate over a limited range of designs it would also be helpful to have more information on the range of designs covered by Fig. 23.

I should also like to point out that the blade opening is not a logical length on which to base the *Re* since it is in no way associated with the boundary layer growth on the blade surface. The fact that a better correlation of profile loss was obtained by using the opening rather than the blade chord or backbone length merely serves to illustrate the extreme variability of *Re* effects.

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J. Dunham Member

I congratulate the authors on this important work. Their conclusions are in many ways similar in nature to N.G.T.E's (**3**), especially in respect of secondary losses (**24**), and the overall accuracy appears similar. In a single paper, it was clearly impracticable to present supporting evidence in detail. May I urge the authors to do so in a second paper?

In particular, comparison with other methods is lacking, so I have prepared a few comparisons. Fig. 30 shows Smith's experimental curves (**17**) in comparison with Fig. 24 and also with curves generated by the original Ainley–Mathieson method (**1**) taken from (**25**) and assuming optimum swirl. It will be seen that the curves are similar, but the authors' curves are closer to Smith's.

Fig. 31 shows recent low speed cascade measurements of minimum profile loss of the root mean and tip sections of the turbine rotor blade given in (**26**), each at three pitch/chord ratios s/b, in comparison with predictions by the three methods of (1) the authors; (2) Ainley–Mathieson (**1**); and (3) direct boundary layer calculations using calculated velocity distributions. It will be seen that the authors' predictions are closer than (2); indeed (2) is unsuitable for inlet angles $>90°$ (such blading does not occur in mean sections for which the method was intended). Outstandingly better agreement is obtained by (3).

Turning to secondary losses, Fig. 32 shows one of Wolf's results (**27**) in comparison with predictions by the three methods of: (1) the authors; (2) Dunham (**24**) for a cascade; and (3) Dunham and Came (**3**) for a turbine. Methods (1) and (3) agree but are double the measured values. Since (2) was based on (**27**), they agree well. The difference between the cascade and the turbine lies in the upstream wall boundary layer thickness, which is a major factor, only broadly accounted for in the authors' method by the *Re* correction. Fig. 33 shows other data from (**27**), showing an incorrect trend in the authors' formula for gas

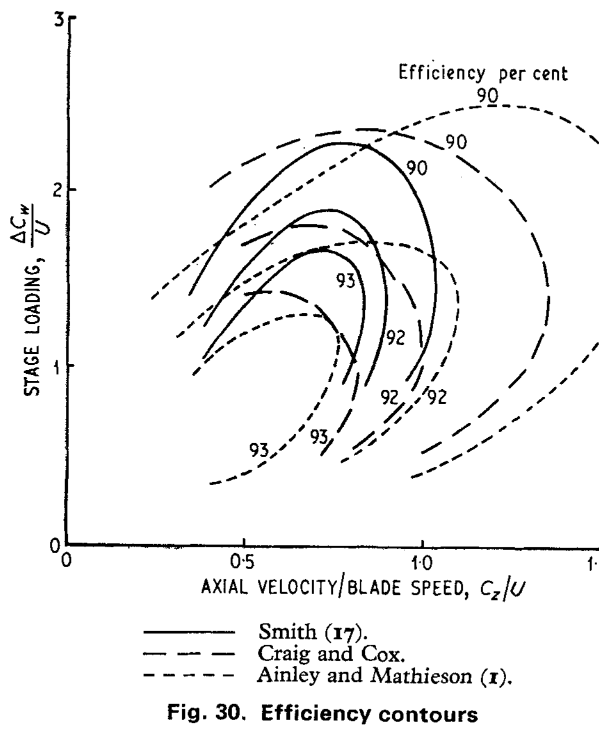
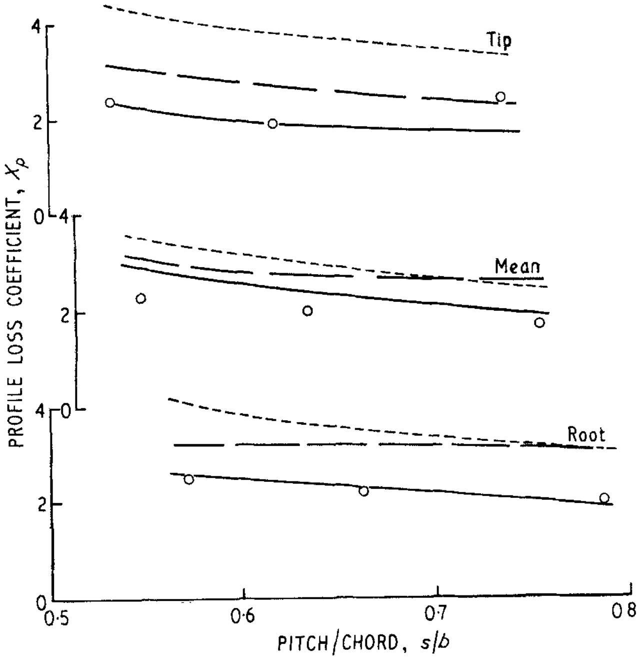


Fig. 30. Efficiency contours



— — — Craig and Cox.
- - - - - Ainley and Mathieson (1).
———— Direct boundary layer calculations.
○ Experiment.

Fig. 31. Profile losses

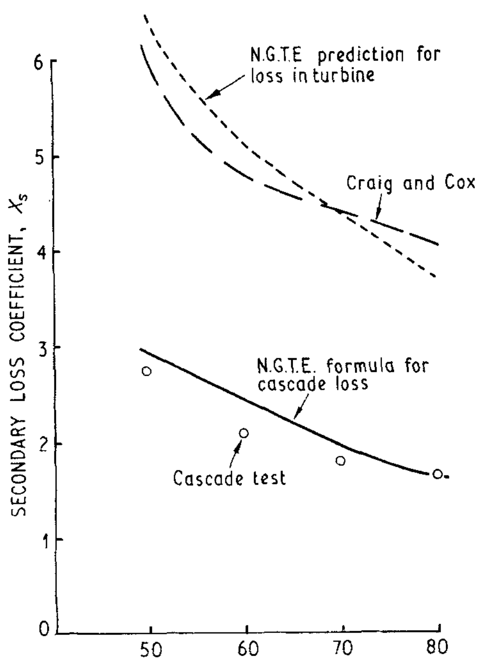


Fig. 32. Secondary losses (Wolf, blade 102, 25° stagger)

inlet angles exceeding 90°. Was the authors' secondary loss correlation deduced from cascade data or turbine data?

I would be interested to hear the authors' comments on

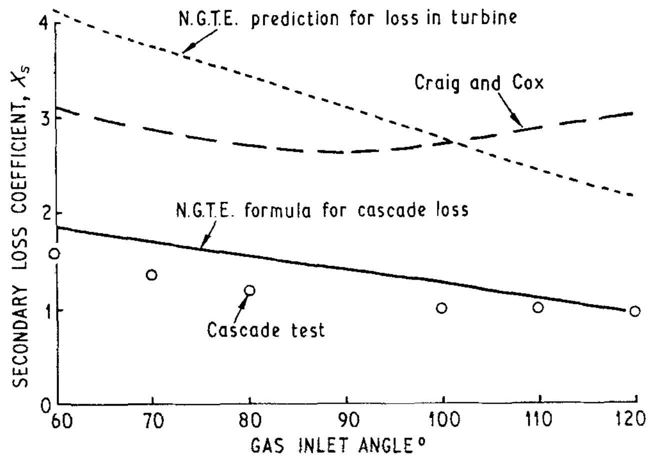


Fig. 33. Secondary losses (Wolf, blade 102, 40° stagger)

two other minor points also: why should not lap, annulus and cavity losses be treated (like secondary losses) as end losses instead of 'essentially one-dimensional'; and are the various curves stored in the authors' computer program as tables or as equations? Do the authors regard this paper as the last word in one-dimensional methods, or do they foresee further improvements? How do they believe better predictions can be obtained in the future, if indeed they are needed?

I am pleased to be given this rare opportunity of discussing mutual problems with steam turbine engineers. I urge more discussion between gas turbine and steam turbine industries because I believe such contact could afford mutual benefits.

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V. T. Forster Member

During the last 20 years, in fact since Ainley and Mathieson (1), no method of assessing axial turbine performance in a comprehensive manner has been published in this country, and therefore this paper must be especially welcomed by turbine engineers as a new contribution to the field. In particular, the ability to segregate the loss items makes an important contribution to the design and analysis process.

In recent years two other methods of rather similar pattern have appeared abroad, the one by Traupel mentioned by the authors (2) and also that of Deich (28) representing the Russian steam turbine practice, which is available as a C.E.G.B. translation. I would like to ask the authors whether they have in fact utilized any of this additional evidence in their presented curve data.

I have recently been analysing some previous cascade results from the Rugby AFL with reference to the important and difficult item of secondary flow losses, and have taken the opportunity of comparing these with the data of the authors and also of Traupel, Deich and Dunham. The study involved three turbine profiles which cover quite well the range needed, i.e. a 30/20°, a 54/18° and a 90/13°, and the results are shown in Figs 34, 35, and 36.

Briefly:

(1) One can learn that if we are to use cascade data we must learn always to measure our inlet boundary layer profiles in order to correlate our secondary losses, and also try and establish the correct inlet conditions from model turbines, as a theoretical approach seems rather intractable at the moment. The one exception appears to be the strongly accelerating nozzle where at H/W ratios above 1·0 the secondary loss is independent of the inlet boundary layer (see Fig. 36).

(2) Our own evidence suggests that the Traupel, Deich and Dunham approach, in assuming the inverse law for secondary loss, is incorrect and that the authors' proposed curve is more realistic.

(3) Again using our own data it appears that the authors' secondary losses are rather too low and profile losses too high, but the combined total loss agrees quite well.

In the final section the authors deal with application to general turbine design, which of course is the important end point of the process. Would the authors like to comment on how many of their Group 1 and Group 2 losses they would be prepared to leave out in arriving at an efficiency carpet if it is not to be misleading, particularly in design optimization? It seems to me that independent variation of reaction and stage velocity ratio is perhaps required, and there is also the question of whether stage velocity ratio is arrived at by varying the number of stages or varying the mean diameter. And if C_z/u is changed, the effect of both inlet and exhaust branches would be involved in considering a practical steam or gas turbine cylinder. Again the introduction of tip leakage losses would probably have a large influence on the curves of Fig. 26 because of the very short blade heights.

Finally, regarding losses caused by trailing edge thickness shown in Fig. 6, I have always felt that the operative parameter should be te/o rather than te/s since this gives a measure of the total flow area taken up by the wake. One finds in fact that if the authors' loss ratio in Fig. 6 is plotted against te/o rather than te/s then all their curves fall on a single line, at least within the 10–30° outlet angle range. Would the authors agree that this simplification is a reasonable approach?

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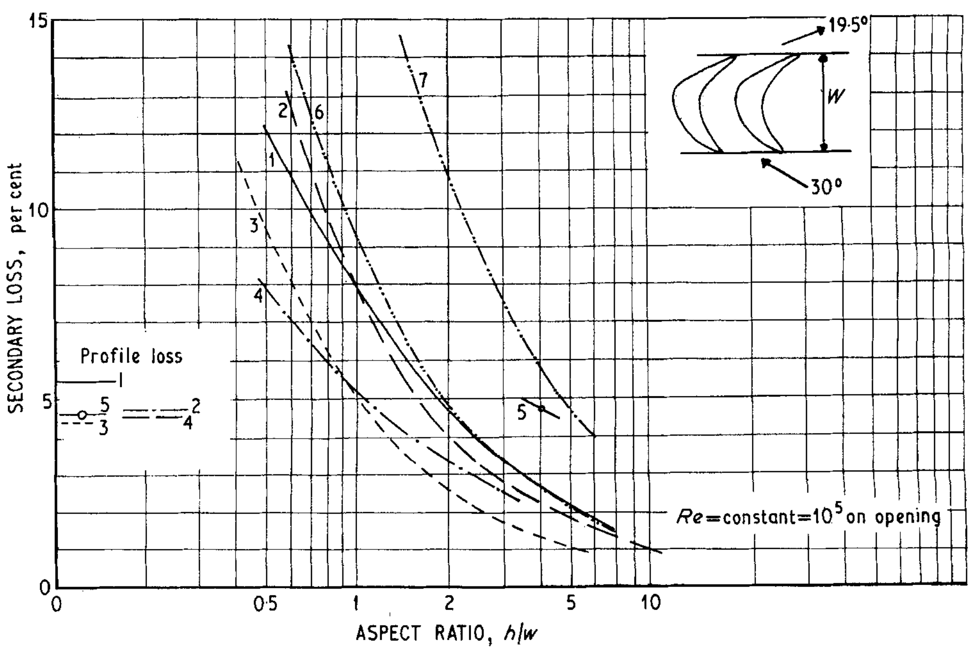
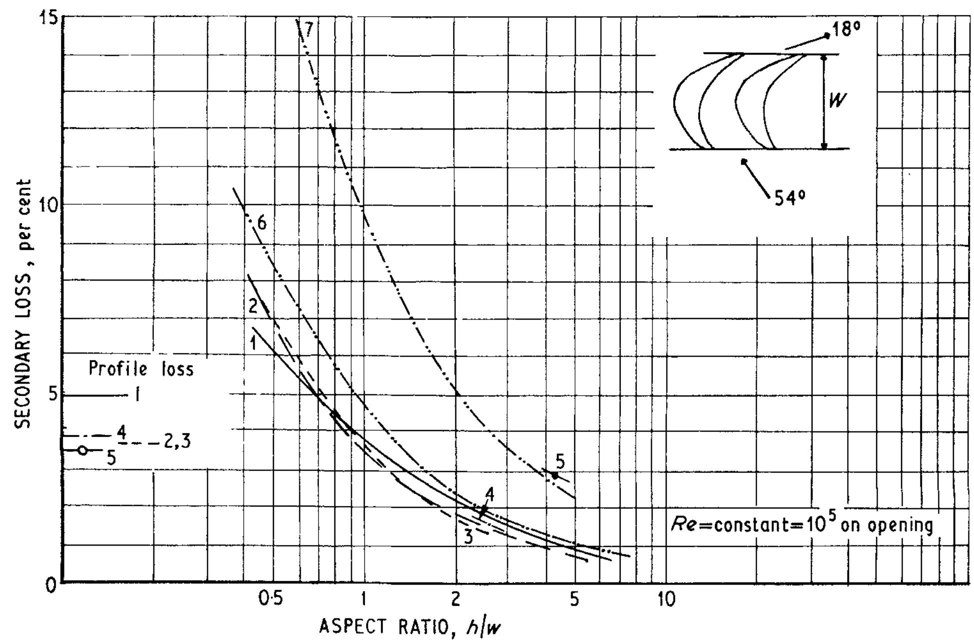


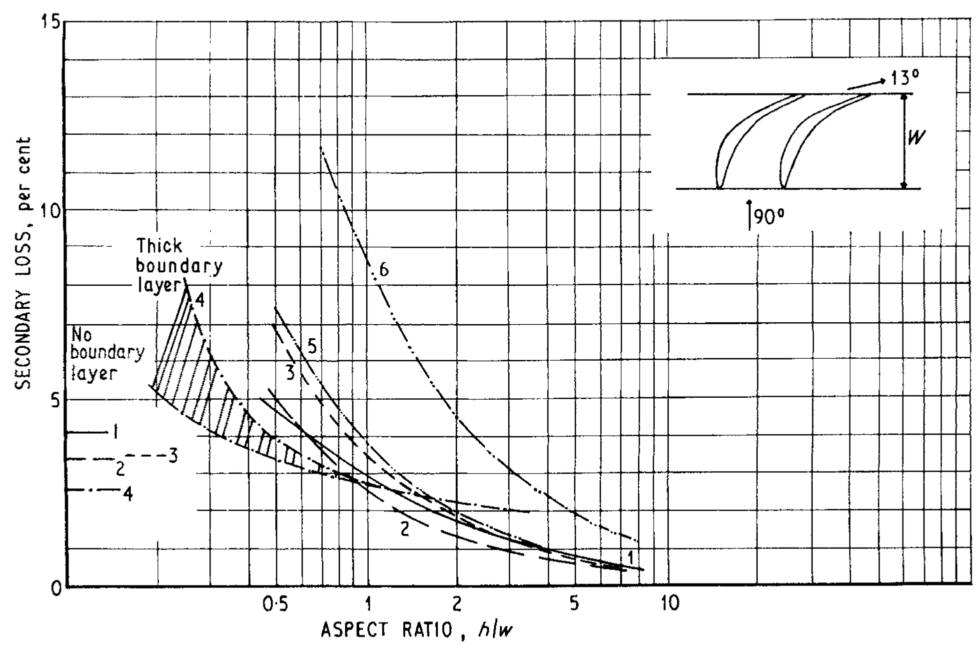
1 Craig and Cox; 2 Traupel (2); 3 Deich (28); 4 Rugby high speed tunnel; 5 Rugby low speed tunnel; 6 Dunham (24); 7 Dunham (3).

Fig. 34. Secondary losses in cascades (30/20° turbine profile)



1 Craig and Cox; 2 Traupel (2); 3 Deich (28); 4 Rugby high speed tunnel; 5 Rugby low speed tunnel; 6 Dunham (24); 7 Dunham (3).

Fig. 35. Secondary losses in cascades (54/18° turbine profile)



1 Craig and Cox; 2 Traupel (2); 3 Deich (28); 4 Rugby high speed tunnel; 5 Rugby low speed tunnel; 6 Dunham (24); 7 Dunham (3).

Fig. 36. Secondary losses in cascades (90/13° turbine profile)

A. A. Garson Fellow

The authors' excellent paper provides much food for thought. The method they describe has been shown to be very good for performance estimation, but in their extension of the use of the method to the optimization of stage design they may be on less firm ground.

As the authors have said when describing their comparison with commercial turbine test results, the algebraic sum of the errors is small, but there may be errors in the individual loss components.

On page 408 they list a number of significant differences between conditions inside a turbine stage compared with a cascade test. Incidentally, two further differences may be added:

(1) fine-grained turbulence arising from the gradual dissipation of the periodic wakes and other disturbances, described by the authors, but from stages higher up the turbine; and

(2) possible effects of centrifugal force on the boundary layers on the moving blades.

Because of all these differences, it would have perhaps been desirable if the authors' method had been compared with the results of systematic tests of model or experimental turbines, in addition to the test results of commercial turbines. They have mentioned some disadvantages of model turbine tests but these may not be very significant, as it seems that, for example, the *Re* may not be very important once the latter exceeds the transition value.

Advantages of correlating performance methods with model or experimental turbine tests, in addition to tests on commercial turbines, include: (*a*) quicker testing of up-to-date stage parameters; (*b*) tests can be carried out over a wide range of velocity ratio; (*c*) stage parameters can be varied systematically; and (*d*) parameters can be varied outside the 'commercial' range in order to emphasize their effect.

In Fig. 25, the authors have in fact compared predictions by their method with experimental test results for gas turbines reported in (**17**). However, study of (**17**) shows that degree of reaction, incidence and effects of clearances and leakages for various reasons do not appear in these particular test results.

The value of such a comparison would obviously have been greater had it been possible to include these significant parameters.

W. H. Gibson Member

The authors are to be congratulated on the tremendous effort undoubtedly involved in developing and checking this method of performance prediction.

My experience with a series of six experimental multi-stage turbines to the specifications shown in Table 3 with average blade heights of 0·93, 1·4 and 2·8 in tested at 1 in chord Reynolds numbers up to 3×10^5 confirms the accuracy of the method to within ± 1 per cent.

Table 3

Turbine blading	Low reaction, twisted, root	50 per cent reaction, untwisted, mean
Design, $\Delta C_w/u$	1·7	1·0
Design, C_z/u	0·4	0·4
Guide blade chord, in	2·0	1·08
Runner blade chord, in	0·625	1·08
Number of stages	4	7
Outlet swirl	zero	zero
Rotor	disc	drum

In dealing with clearance losses, the data are mainly concerned with turbines using disc and diaphragm construction rather than drum construction which is also widely used in high pressure (h.p.) steam turbines. For the reader concerned with the clearance losses of turbines using a drum construction, a paper by Jefferson (**29**) will be of interest. The runner tip leakage coefficient given in Fig. 21 is about 0·86 for 50 per cent reaction blading at $o/s = 0{\cdot}37$ with zero swirl. At 2 per cent overlap the value of $F_k = 0{\cdot}5$ per row is in good agreement with Jefferson's results. Jefferson also gives clearance losses for unshrouded blading which confirms the use of $1{\cdot}5F_k$. While unshrouded blading is more sensitive to clearance, its zero clearance efficiency is higher since there is no shroud windage loss. Hence, for certain values of clearance the efficiencies of the two types of blading are equal.

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W. S. Hall Newcastle upon Tyne

Some checks have been made with the cascade data available in my firm which show that profile loss is over-estimated by the authors, particularly for some of the newer profiles obtained with the controlled boundary layer designs of LeFoll. Secondary loss is also over-estimated and we find that the correlation proposed by Dunham (**24**), which takes into account upstream wall boundary layer thickness, is rather more accurate.

On the other hand, as W. H. Gibson has stated in his contribution, the prediction of model turbine performance is accurate. There must thus be some element of self-cancelling of errors, as the authors themselves mention.

J. H. Horlock Fellow

The authors' excellent summary paper will repay careful study by any turbine designer. A great deal of experimental information has gone into the correlations presented, and the basis of the performance estimation is logical and evidently accurate. I shall list a number of minor comments and questions first, and raise a few more general matters subsequently.

Minor comments and questions

(1) What is the basis of the lift parameter presented in Fig. 4? It is clearly not a normal lift coefficient (or the tangential lift coefficient of Zweifel).

(2) The presentation of the *Re* effect on profile loss on the basis of *Re* based on blade opening may give a better correlation, but can have no analytical basis unless the flow between the blade suraces is so developed that boundary layers merge together, which is surely not so except in very closely spaced impulse blades. This is probably not too important a point since for a given surface finish the effect is independent of *Re* above *Re* of about 10^5.

(3) I still have difficulty in accepting that secondary loss is inversely proportional to the aspect ratio. It would be expected that the secondary loss coefficient integrated across the wall boundary layer region (L_{s_w}) might be written as a function of the following parameters:

$$L_{s_w} = \int_0^\delta L_s \, \mathrm{d}r$$

$$= f(\delta_1, H_1, b, h, \rho, C, \mu, a, s, \text{blade geometry})$$

i.e.

$$L_{s_w} = f\left\{\left(\frac{\delta_1}{h}\right), H_1, \left(\rho\frac{Cb}{\mu}\right), \left(\frac{C}{a}\right), \left(\frac{h}{b}\right), \left(\frac{s}{b}\right), \begin{matrix}\text{blade}\\ \text{geometry}\end{matrix}\right\}$$

where δ_1 is the inlet boundary layer thickness and H_1 is the shape parameter. We would expect such a loss coefficient to be almost independent of h/b unless the sum of the two boundary layers, 2δ, approaches the blade height h. Thus the averaged loss along the blade ($L_{s_{ave}}$) would be given by

$$L_{s_{ave}} = \frac{2\delta}{h} L_{s_w}$$

$$= \frac{\delta}{h} f\left(\frac{\delta_1}{h}, H_1, R_e, M_a, \frac{h}{b}, \frac{s}{b}, \text{blade geometry}\right)$$

The inverse dependence of $L_{s_{ave}}$ on h/b is not apparent unless δ is proportional to b.

(4) The tabulation of the data on lap and cavity losses and the Group 2 losses is most valuable.

(5) On the basis of experience with compressor design, I commend to turbine designers the warning given by the authors to allow for divergence of the streamlines in the meridional flow calculations.

General comments

(1) The authors have taken a very balanced view of the relevance of cascade data. Clearly cascade data have been of some assistance to them in their performance estimation, but it is up to the designer to make a judgement on whether such data are of value. I would add the point that a cascade test may be of assistance in understanding particular phenomena.

(2) The discussion of the use of the loss data in streamline curvature calculations (page 42) is very important. Doyle, at Liverpool, and Treaster, at Pennsylvania State University, have followed similar procedures in assessing compressor and pump performance, by distributing the losses radially and using such distributed losses in a matrix through-flow or streamline curvature calculation with mixed success. The basic question is whether loss correlations which work when put into a crude 'one-dimensional' mean radius calculation will also work when the identical losses are distributed radially and used in a three-dimensional calculation.

(3) The alternative approach which we are following at Cambridge is to develop annulus wall boundary layer calculations. We have had some success with lightly loaded compressor blading, but the heavily loaded turbine blade presents a formidable problem.

What is extremely important in attempts to calculate the flow near the wall is to get both the losses and the angle variation right. There is a delicate balance between work transfers, dependent on the latter, and the losses.

I should like to have further explanation from the authors on their warning not to accumulate secondary losses row by row. If the angle and loss distributions are correct in any blade row, then surely the velocity distribution calculated should be carried forward to the next row: this must be so in any boundary layer calculation which allows for dissipation.

J. A. C. Hyde Member

Strong current trends in industrial gas turbines are forcing the design of turbine stages with loading and flow coefficients well away from the region where over 93 per cent basic aerodynamic efficiency is straightforward. The trends one may particularly note are: (1) the rapid growth of the 3000 rev/min single-shaft gas turbine from 25 MW a few years ago to current development programmes in the 80–100 MW range, with no end in sight; and (2) single stage power turbines being supplied for aerojet gas producers.

The authors' method has already proved to give more realistic predictions in these areas than previous methods.

For such turbines it is necessary to design aerofoils with lower profile losses than traditional good standards, and which incorporate measures to control secondary losses, which are often the predominant loss component. The appreciable influence of axial velocity ratio on the profile loss for given gas and blade angles, particularly with the large deflections and low reactions of rotor blade profiles, can now be understood with potential flow analysis, and advantage can be taken of this knowledge in designing new aerofoils. It is surprising that the authors' lift coefficient and loss coefficient correlation neglects this. Is this because axial velocity variations in multi-stage turbines have conventionally been small in the past, as on the bulk of cascades tested? Certain secondary loss aspects have been dealt with by my colleague, A. V. Cooke, in relation to a highly loaded stage by gas turbine standards, $\Delta H/U^2 = 3{\cdot}1$ at hub.

Disappointingly, the authors did not 'indicate the lines on which future aerodynamic development should proceed', to use the words of their introduction.

H. A. Kirby Fellow

The character of steam turbine blading design has changed markedly in the last 20 years; previously scant recognition was given to the work and attitudes of the aerodynamicist. Designers trained in those days have to face the fact that we have from the authors a major paper on turbine performance which goes the full distance with only two brief references to stage reaction. The influence of reaction is, of course, implicit in many places in the paper.

The comparison between 50 per cent reaction turbines and impulse reaction machines has been going on since the earliest days. In this connection, the authors' reference on page 422 to the fact that an analysis of over 50 machines revealed 'no systematic or major discrepancy' is interesting. Would it be true that all or most of these machines were of impulse design? In the majority of machines produced by the authors' company, reaction varies from a low value in the high pressure (h.p.) cylinder to a mean value approaching 50 per cent in the last stage in the low pressure (l.p.) cylinder. In other designs produced by the authors' company, particularly large single-cylinder back-pressure turbines in the chemical industry, reaction remains at a low value from inlet to exhaust. It would be useful to know whether any of the latter machines are represented in Fig. 23. Has a systematic study based upon stage reaction been made?

As far as 50 per cent reaction designs are concerned, it will be of interest to apply the analysis and results which have now been made available by the authors, for comparison with existing test results and other analytical methods.

In multi-cylinder condensing turbines, power output is not equally divided between cylinders. Normally the intermediate pressure (i.p.) cylinder in a turbine containing three or more cylinders has a significantly greater output than the others. This would suggest that the 'output per cylinder' criterion used in Fig. 23 is a somewhat blunt instrument. Turbine total output, with an indication of the type of machine represented by each point, would have been more convincing.

However, in whatever light Fig. 23 is viewed, there is no doubt that it justifies the authors' method of performance estimation and the work which they have done.

R. I. Lewis Member

When future generations of bio-technologists reconstruct the tree of life, we shall no doubt see rising through the shrouded mists of primeval technology the strong and steady shoot of steam turbinology, evolving exponentially with classical symmetry and dignity towards its omega. Somewhere along the stem bursting forth rudely and busily will appear the gas turbine shoot budding vigorously in a multitude of directions towards its less certain multitude of omega points. On that day the mystique which surrounds the steam turbine and its midwifery will vanish and perhaps we shall be surprised to discover that these cousins are not so distant after all. The authors are to be congratulated for demonstrating this fact in advance by presenting a paper on axial turbine performance with such equanimity to both branches of the family.

In particular, the efficiency contours presented in Fig. 25 show an encouraging measure of agreement with the gas turbine correlation published by Smith (17) in 1965 for machines operating at their peak efficiency points. As a fairly obvious design aim one would wish peak efficiency to be achieved at the design duty point. Could the authors comment on their degree of success in achieving this objective and tell me if the curves shown in Fig. 25 represent design duty or peak efficiencies?

Smith in his 1965 paper (17) made what I thought was a splendid contribution to the general understanding of turbine stage performance in his attempt to interpret the form of the efficiency contours in terms of velocity triangles. Having found his approach so helpful I have attempted to rationalize the arguments one step further in order to develop a logical and coherent approach to stage performance for presentation to students. My ultimate aim in the following analysis is to demonstrate that efficiency is dependent at least as much upon (ϕ, ψ) duty point selection as upon profile design. With this objective in mind we may start with a broad statement of dependency of η_T upon a range of variables of which a selection of only 15 is given below:

$$\eta_T = f(\Delta h_0, h_1, h_2, h_3, \Omega, D, C_x, C_2, w_3, a_2, a_3, \mu, \rho, \Delta p_{0S}, \Delta p_{0R})$$

where Δh_0, h_1, h_2, h_3 are thermodynamic design parameters; Ω, D are plant design variables; C_x, C_2, w_3 are velocity triangles; a_2, a_3, μ, ρ are physical properties of fluid; and Δp_{0S}, Δp_{0R} are frictional losses.

By formation of dimensionless groups these can be reduced to eight relevant parameters:

$$f(\psi, R, \phi, M_2, M_3, R_{eD}, \zeta_{S2}, \zeta_{R3})$$

where

$$\psi = \frac{\Delta h_0}{\Omega^2(D/2)^2}, \quad R = \frac{h_2-h_3}{h_1-h_3}, \quad \phi = \frac{C_x}{\Omega D/2}$$

$$M_2 = \frac{C_2}{a_2}, \quad M_3 = \frac{w_3}{a_3}, \quad R_{eD} = \frac{\rho\Omega D^2}{2\mu}$$

$$\zeta_{S2} = \frac{\Delta p_{0S}}{\frac{1}{2}\rho C_2{}^2}, \quad \zeta_{R3} = \frac{\Delta p_{0R}}{\frac{1}{2}\rho w_3{}^2}$$

Gas turbine stages with interstage whirl

For 50 per cent reaction stages, remembering also that ζ is a function of Ma and Re, this expression reduces to

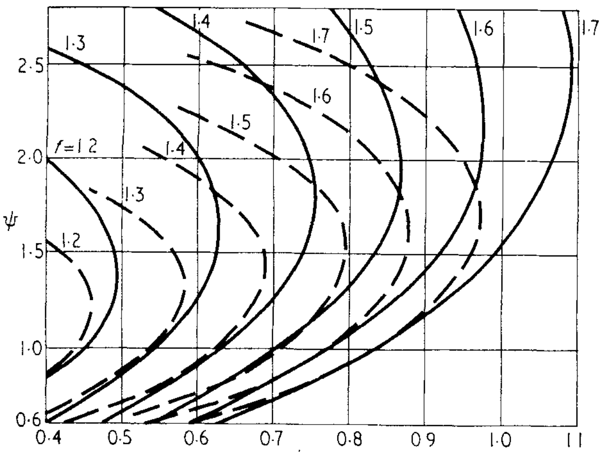
$$\eta_T = F(\phi, \psi, \zeta)$$

It can be shown that the function F is not simply a matter for empiricism but is reducible to the analytical form

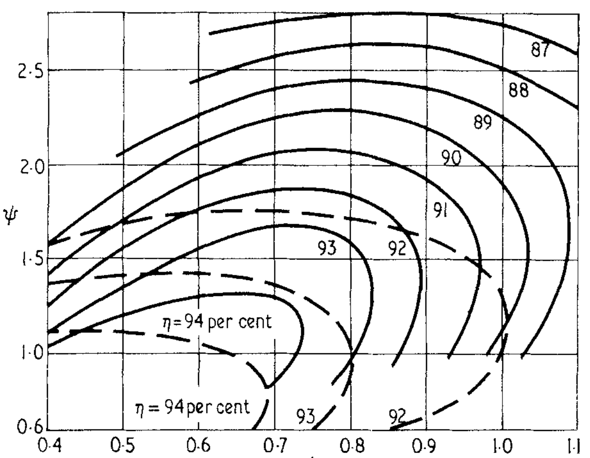
$$\eta_T = \frac{1}{1+f\zeta}$$

where
$$f = \frac{1}{\psi}\{\phi^2+\tfrac{1}{4}(\psi+1)^2\}$$

In effect f is a weighting coefficient, depending only upon the selected duty (ϕ, ψ) and thus velocity triangles, which scales the losses ζ. The magnitude of this coefficient (Fig. 37) reflects the fluid dynamic environment in which the blades are located. This weighting coefficient is directly related to the parameter $\Delta H/(C_1{}^2+C_2{}^2)$ referred to by Smith resulting in similar contours. It is striking that contours of f closely resemble efficiency contours, an



a $f(\phi, \psi)$ contours.
— — — Stages of varying reaction with zero interstage whirl.



b Efficiency contours.
— — — Craig and Cox.

Fig. 37. Performance charts for 50 per cent reaction axial turbine stages (17)

observation which demonstrates the dominating effect of duty point selection upon efficiency.

The loss coefficient ζ is also a function of ϕ and ψ but a much weaker one. ζ is obviously more closely related to fluid deflection, but this in turn can be expressed as a function of (ϕ, ψ)

$$\epsilon = \tan^{-1}\left\{\frac{4\phi\psi}{4\phi^2-\psi^2+1}\right\}$$

as can all the angles of the velocity triangles. Velocity triangle design can thus be related very conveniently to duty point selection.

When considering the relative merits of several alternative duties the weighting coefficient $f(\phi, \psi)$ would thus seem to offer a quick means for comparison and interpretation.

Similar analytic expressions may be derived for other reactions if the additional variable R is introduced.

$$\eta_T = F(\phi, \psi, \zeta, R) = \frac{1}{1+f_S\zeta_S+f_R\zeta_R}$$

where separate stator and rotor weighting coefficients are given by

$$f_S = -\frac{1}{2\psi}\left\{\phi^2+\left(\frac{\psi^2}{2}+1-R\right)^2\right\}$$

$$f_R = -\frac{1}{2\psi}\left\{\phi^2+\left(\frac{\psi}{2}+R\right)^2\right\}$$

These are shown for zero reaction stages in Fig. 38 which indicates without reference to profile losses that the contributions made by the rotor will in general be less than those made by the stator for zero reaction.

This method of presenting stage performance demonstrates in a very clear way the separate influences of the aerodynamic losses ζ and the overall design parameters ϕ, ψ and R. It should be understood of course that ϕ and ψ are here based on local blade speed at any given radius under consideration.

Steam turbine stages: arbitrary reaction and zero interstage whirl

The preceding analysis refers to stages of approximately 50 per cent reaction and finite interstage whirl ($\alpha_1 = \alpha_3$) defined by Smith. With zero interstage whirl and varying reaction the efficiency reduces to

$$\eta_T = \frac{1}{1+\frac{1}{2\psi}\{(\phi^2+\psi^2)\zeta_{S2}+(\phi^2+1)\zeta_{R3}\}}$$

If one assumes

$$\zeta_{S2} \approx \zeta_{R3} = \zeta$$

this reduces to

$$\zeta_T \approx \frac{1}{1+f_0\zeta}$$

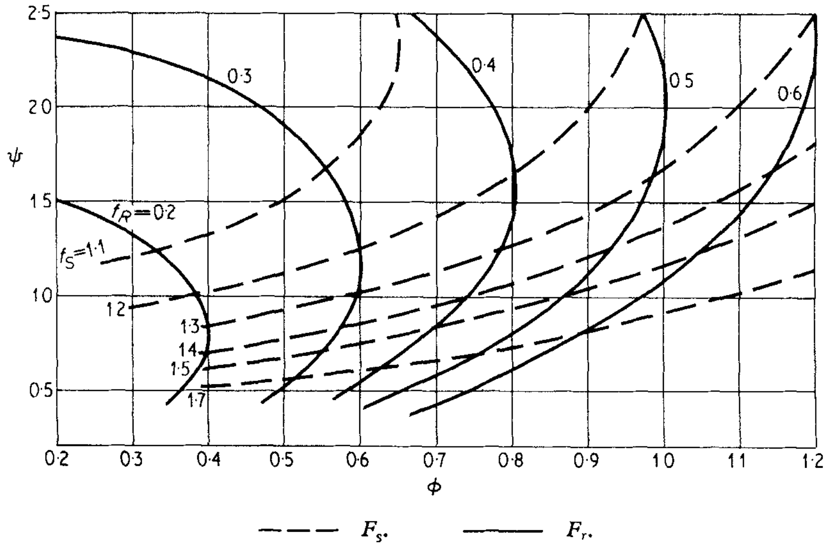


Fig. 38. Weighting coefficients for zero reaction axial turbine stages

where

$$f_0 = \frac{1}{2\psi}\{2\phi^2+\psi^2+1\}$$

Contours of f_0 are superimposed on the gas turbine f contours in Fig. 37*a* with revealing results. At low head coefficients f and f_0 tend to coincide. However, at values of ψ exceeding 1·0 the zero interstage whirl curves fall away rapidly. Precisely the same trend is exhibited by the efficiency contours published by the authors when overlaid on the gas turbine curves of S. F. Smith. The immediate conclusion is that more highly loaded stages are possible for the same efficiency if interstage whirl is introduced. Have the authors found this to be so with their machines?

In view of the considerations listed above, would the authors be prepared to show on Fig. 24 actual machine duty points obtained from steam turbines and models and to comment on aerodynamic or other constraints which led to their selection?

I would contend also that blade/speed ratio is not a very useful design parameter since it is really only a dependent variable of η_T and ψ

$$\sigma_{is} = \sqrt{\frac{\eta_T}{2\psi}}$$

For zero interstage whirl designs reaction R is uniquely related to ψ by

$$R = 1-\frac{\psi}{2}$$

and blade/speed ratio then becomes

$$\sigma_{is} = \sqrt{\frac{\eta_T}{4(1-R)}}$$

The more important parameters for selection of stages with one eye on efficiency are ϕ and ψ or $\Delta C_w/U$.

A. N. MacDonald London

The authors have pointed out that an analysis of losses shows the designer where to concentrate his efforts when looking for increased efficiency, both in absolute terms and in order to offset other losses, such as those which might be incurred by having to increase clearances of tip seals and diaphragm glands for mechanical reasons.

A reliable method of predicting efficiency levels is also of considerable value in the comparative assessment of different types of plant some five years before commissioning, and even longer before comprehensive testing. Tests on turbines in service, which follow similar design principles, can give a useful guide, but extrapolation is almost always necessary.

An analytical method of predicting cylinder efficiencies requires both a prediction of the flow lines through the cylinder and an estimation of the losses which must be applied, both locally and generally, to the flow prediction calculation.

The authors present a comprehensive description of a method for estimating losses which is a most valuable addition to the published literature. I shall first ask them three specific questions on the loss estimation method:

(1) Fig. 20 shows the losses occasioned by wall cavities. Surely this loss will depend on the amount of steam extracted, which will affect the position and shape of the neighbouring streamlines?

(2) Equation (15) is dimensional and to apply it, it is necessary to know the units of the quantities on the right-hand side.

(3) Why does one in effect add the kinetic energies at

the three sections 1, 2, 3 in the second term in equation (16)?

I should also like to relate the loss predictions to the methods used for predicting the flow lines in the cylinder. The authors have stated the generally accepted principle that a three-diameter calculation with radial equilibrium is adequate for h.p. and i.p. cylinders, but that for l.p. cylinders streamline curvature and radial velocity effects must be considered. I feel, however, that the evidence for this has never been clearly seen, for two reasons:

(1) the difficulty of obtaining accurate l.p. cylinder efficiencies, which involve either measurements of the wetness or enthalpy of wet steam, or deductions from a comprehensive test. Even the latter method involves several assumptions such as the level of mechanical losses; and

(2) the several different types of loss almost entirely confined to the l.p. cylinder, e.g. *Ma* loss, cavity loss, wetness loss and annulus wall loss.

It is therefore difficult to apportion any discrepancy between test and calculation between (i) poor measurements, (ii) inadequate loss correlations and (iii) inadequate flow line calculations. The authors have stated that they found no systematic or major discrepancy in their comparison of test and calculated values, but it would be interesting to know if the inclusion of streamline curvature and radial velocities gives a significant improvement in the accuracy of prediction for large l.p. cylinders when this comprehensive method of loss analysis is used, and what proportion of the points in Fig. 23 represent such cylinders, and whether these effects were considered.

L. Ptáček Brno, Czechoslovakia

The calculation described in this paper is relatively complicated in comparison with the Ainley and Mathieson method (1) which, as the authors state, could well be counterbalanced by a higher accuracy.

It would be useful to compare this method with our own test results. Since the time I have had in which to prepare my contribution was short, I shall confine my comments to an evaluation of some partial losses.

Effect of Reynolds number

The *Re* effect was measured on our multi-stage test steam turbine with reaction blading (50 per cent reaction). The value $ks/b \times 10^3$ was approximatley 0·25. The efficiency drop started at about $Re = 5 \times 10^4$ (based on blade opening and outlet velocity), which is substantially in accordance with the curves in Fig. 3.

We verified the results of the turbine tests in a wind tunnel. Even though the results on the cascade in the tunnel agreed essentially with the turbine tests, some other problems presented themselves, e.g. the effect of the turbulence of the tunnel flow, which was modified by means of wire grids.

It would surely be possible to discuss the calculation of the *Re* from blade opening as well as the dependence of *Re* on the reaction degree and on the characteristic blading ratio u/c_{ad}, where u denotes circumferential velocity and c_{ad} the velocity corresponding to the isentropic heat drop in the stage.

On the other hand it is a fact that in most cases of turbine blading the *Re* is high enough to make the *Re* effect on the efficiency relatively small.

Profile loss

The basic profile loss is given by the authors in Figs 4 and 5. In spite of the fact that the various methods for calculating profile loss generally are not at any great variance with actual values, we considered it useful to measure the velocity coefficient ψ of a given profile for various cascade adjustments and for varying incidence (Fig. 39). The values of ψ measured in the wind tunnel were the initial data for further calculation. As to the absolute value of the ψ coefficient, it is naturally affected by the tunnel flow turbulence.

In any case, the method presented by the authors is more general and opens wider aspects of profile geometry evaluation.

Radial clearance loss

The size of the radial clearance for reaction blading (without shrouding) greatly affects the turbine efficiency. From our steam turbine test results (50 per cent reaction blading) the radial clearance loss for a given blade profile and for various outlet angles was determined. These results were compared with the calculation methods of various authors, Fig. 40, including the Ainley–Mathieson method recommended by the authors.

Comparing the Ainley–Mathieson method in general with our turbine measurements, we obtained an acceptable agreement with the actual values. But radial clearance loss calculation according to this method is rather less accurate. Even if at the optimal point ($u/c_{ad} = 0{\cdot}65-0{\cdot}70$) there are no major discrepancies; at non-optimal conditions the difference increases. Moreover, the character of the loss curve in Fig. 40 according to Ainley does not agree with other authors.

The values in Fig. 40 are calculated for reaction blading (50 per cent reaction), for a fluid relative outlet angle of 32° (for smaller angles the accuracy of the Ainley method even decreases), and for a radial clearance change of 1 per cent of the blade length.

Carpet diagrams

The diagrams in Figs 24–27 are very instructive and enable one to make a choice of the optimal parameters, but also show the operating conditions at non-optimal parameters. We elaborated diagrams analogous to Fig. 39, where instead of the velocity coefficient we plotted the curves of constant efficiencies and took as co-ordinates, for instance, the blading ratio u/c_{ad}, outlet angles, etc.

The conclusion of the authors, that the method they

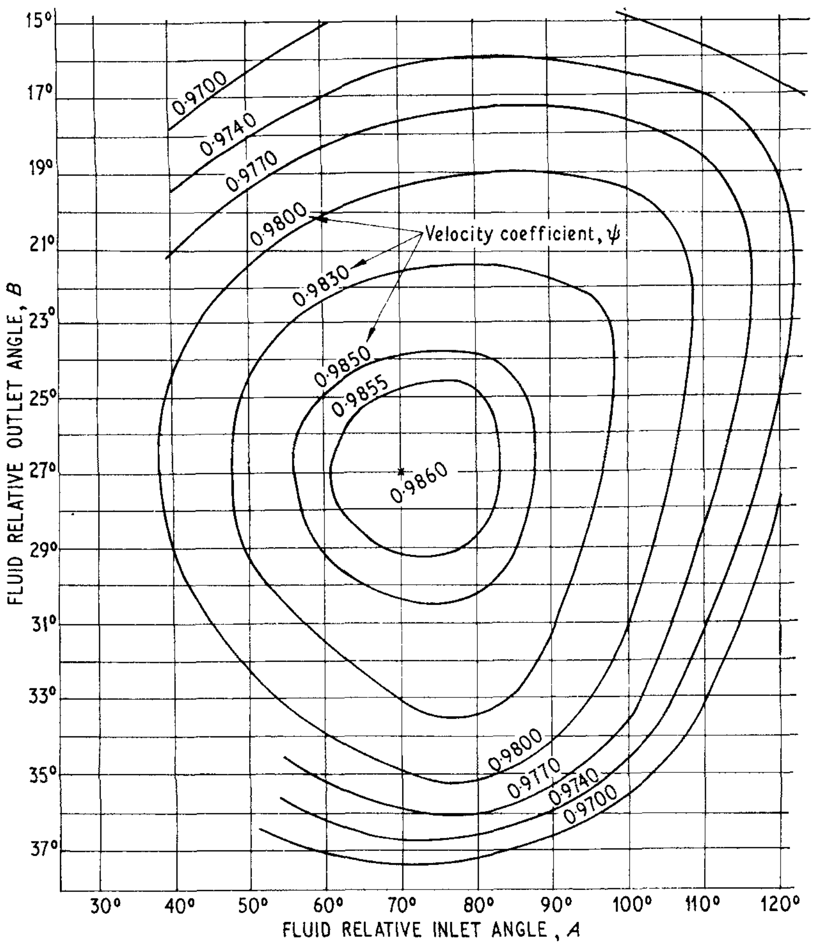


Fig. 39

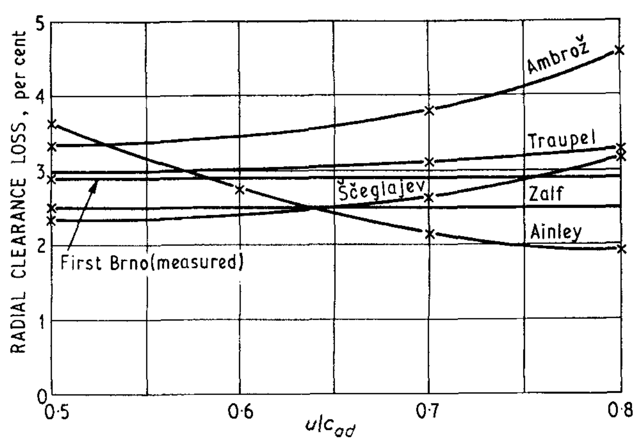


Fig. 40

presented was verified on more than 50 turbines, the discrepancy being within $\pm 1\frac{1}{4}$ per cent, testifies to the high accuracy of this method. I presume that this method was used specifically for the recalculation of the English-Electric turbines, i.e. most for turbines of the impulse type.

It would be most interesting to know the results of the application of this method to reaction blading calculation.

A. Smith Fellow

I should like to make three comments, the first relating to profile losses at high Mach numbers, the second on lacing wire losses, and the third on exhaust losses.

Figs 8 and 9 would suggest that the profile losses of convergent blading invariably decrease with *Ma* above 0·7 for blades with convex curvature at the run-out towards the trailing edge on the suction surface; and unity for straight run-outs. Whereas it is generally true that convex run-outs are inferior to straight run-outs above 0·8*Ma*, wind tunnel results indicate that in straight-tailed blading, having made due allowance for the rise in *Re*, the efficiency normally decreases locally in the transonic zone and subsequently recovers to exceed the subsonic efficiency in the *Ma* range 1·1–1·3. Above an *Ma* of 1·3, however, the efficiency would again fall, making convergent–divergent nozzles preferable.

Turbine tests would suggest that use of a drag coefficient of unity would result in a considerable underestimate of the losses caused by lacing wires. Our own 50 per cent reaction blading results would suggest that if the loss is to be introduced in the form of a drag coefficient, a value of from 2·5 to 3·3 would be necessary, dependent upon radial position. Lower reaction results published by Kirillov and Kuzmichev (30) would also give higher figures ranging from 1·5 to 3·4. The latter results, however, were obtained on a single stage whereas the reaction values refer to seven-stage tests in which the deterioration in axial velocity profile can result in additional losses.

The reason for the equivalent drag coefficient exceeding unity can be explained by additional losses being caused by separation on the blade surfaces behind the wires.

The authors intimate that the exhaust casing loss expressions given in equation (16) and Fig. 22 refer to a given basic type and I would like to endorse this point. Considerable reductions in the losses predicted by this method are possible at low speeds by the use of annular cascades; moreover, the necessity of providing adequate downstream axial length to avoid premature choking at high speeds is not apparent in the form of the equation. It is also worth noting that the disposition of supports in the lower half of the casing can seriously modify both low and high speed performance.

REFERENCE

(30) Kirillov, I. I. and Kuzmichev, R. B. 'Power losses in a turbine stage due to the lashing wires', *Elekt. Stantsii* 1962, No. 7 (July), 38 (in Russian).

K. W. Todd Fellow

While it is accepted that the authors have deliberately chosen to define their basic parameters in terms which fit most agreeably with empirical correlation, I would welcome their comment on the following points.

Reynolds number: as the non-dimensional parameter which compares the inertia forces with the viscous forces, and which thus provides a means of judging the nature of the boundary layer conditions, the *Re* should contain a term closely allied to the flow path length. The use of blade opening may be criticized in that it does not fully accord with this requirement. Its use may suppress the very flow feature which is most relevant. Is it possible that this explains the apparently good correlation which is claimed for its adoption?

Aspect ratio: mainly of use in assessing the relevance of secondary loss, this parameter should attempt to take account of the pressure gradients developed normal to the flow direction as a result of both span-wise velocity profiles at inlet and passage curvature. Such gradients will not be fully developed until the passage throat has been reached, and it would seem therefore that, to use the authors' definitions, height/opening is preferable to height/backbone length.

Contraction ratio: the definition used by the authors does not take account of the convergence generated by the blade nose, nor does it allow for the diminution in effective convergence resulting from positive incidence. If instead of using inlet internal passage width, which it is admitted is difficult to define, the length of the normal between the incoming 'stagnation' stream lines to adjacent blades is used, both these features are embraced, and the complete loss of advantage of convergence within the passage at high positive incidences is allowed for. It is even possible by these means to deduce a 'safe' working incidence for a given cascade set-up.

Turbulence: little is said about the influence of this feature in the analyses so ably put forward in this paper. I feel it would add considerably to its value if the authors permitted themselves some comment, be it only in parentheses.

M. Trounce Member

This paper, although comprehensive, prompts a number of questions.

First, what is the definition of, and theoretical background to, the lift parameter F_L? Fig. 4 is said to define it, but in fact does no more than indicate that F_L is dependent only on the fluid inlet and outlet angles.

Second, it would be interesting to be given the background to the use of the blade backbone length b as a significant length, rather than the more usual blade chord or axial width.

In the discussion on secondary losses it is stated that the *Re* effect is 'similar' to that exhibited by the basic profile loss. Does this in fact mean that $N_{sr} \equiv N_{pr}$?

Finally, there is no mention of fluid outlet angles in relation to blade outlet angles under differing outlet flow conditions. Surely a knowledge of flow angles is of fundamental importance in performance estimation, indeed of comparable importance to that of blade losses?

H. R. M. Craig Fellow and H. J. A. Cox Member (Authors)

We are grateful to the contributors to the discussion for the kind way they have received our paper, and for the numerous helpful comments that have materially added to its value. In view of the large number of points raised, it may assist if we answer under subject headings.

Reynolds number

Various contributors have questioned our use of *Re* based on opening. Considerable variations in *Re* effects are found with differing blade shapes. We do not attempt to defend our choice on theoretical grounds. We merely observe, as stated in the paper, that we found closer correspondence between differing blades when opening was used rather than backbone length or chord.

Profile loss

The blade-loading coefficient started life as the tangential change of whirl divided by the vector mean velocity, the latter being defined such that the value of the vector mean velocity approximated to the arithmetic mean of the inlet and outlet relative velocity (**4**). To improve the correlation, this value was subsequently multiplied by a parameter dependent only on fluid outlet angle. The values given in the paper assume a constant axial velocity across the blade; an estimation of the effect of a varying axial velocity is provided on page 421.

Backbone length has been used, instead of chord, primarily because it gives more consistent correlation between high deflection impulse sections and low cambered blades. It is in any case theoretically preferable.

As stated in the paper (page 409), calculations of loss by potential flow and boundary layer methods ought to be more discriminating than the method given in the paper; they are, however, much less convenient in the early stages of design. The results quoted by J. Dunham therefore do not surprise us. We would expect that mathematically designed profiles, of the type referred to by W. S. Hall, would have a profile loss up to 25 per cent less than that assessed in the paper for average profiles. It would be interesting to know if the model tests, to which he and W. H. Gibson refer, used mathematically designed profiles.

We agree with V. T. Forster that a trailing edge correlation using te/o gives a useful simplification. We used a theoretical approach (**6**) consistently.

K. W. Todd wishes us to use $\sin A/\sin B$ instead of contraction ratio for profile loss and incidence correlation. We did try it and found it markedly inferior to the definition we used. We did not find that a satisfactory correlation of incidence losses, of general application, could be found along these lines.

With respect to A. Smith's remarks about behaviour of blades in the transonic region, it is true that some variation from the mean given by our predictions occurs with particular profiles. We would wonder, however, from his description of a particular blade, if it possibly has a nearly parallel final portion to the passage and has therefore the performance of a slightly convergent–divergent form.

Secondary loss

J. H. Horlock questions our use of a secondary loss inversely proportional to aspect ratio. Where aspect ratio is altered by changes in blade height, it is self-evident that, except at low values of aspect ratio, the loss per unit mass flow will be inversely proportional to the actual aspect ratio. Where the change results from changes in chord, the issue is less simple. The secondary loss itself will change, while the mass flow will remain unaltered. The estimated secondary loss by the method disclosed will vary both with aspect ratio and Re, each parameter in opposite sense, and it is by no means clear that this is inconsistent with theoretical considerations of the type to which J. H. Horlock refers.

Our correlation is based on cascade, not turbine, tests; data where the boundary layer had been artificially reduced by suction were excluded. V. T. Forster may well be right when he argues that cascade tests should be carried out with the right boundary layer. He does not, however, tell us how he proposes to simulate correctly skewed boundary layers modified by flow up and down disc faces, across extraction slots, and through clearance spaces. If one must be wrong, there is merit in being wrong the simple way. K. W. Todd tells us that he would prefer aspect ratio correlation on h/o, but quotes no test data to support this statement. We can offer him no encouragement.

Clearance loss

We agree with W. H. Gibson that the clearance loss data presented agree reasonably closely with those given by Jefferson. The presence of a shroud does not only provide a shroud windage loss, it may considerably modify the secondary loss (page 416). L. Ptáček's interesting comparison of various methods of estimating clearance loss draws attention to the fact that such correlations depend on the estimation of tip reaction and the actual value of running clearance. In addition it should be remembered that clearance and secondary flow effects are often interacting. Precision of estimation is therefore unlikely. We ourselves pointed out (page 419) some limitations in Ainley's method. It is not clear whether L. Ptáček's results are in this area.

Other losses

A. N. MacDonald rightly questions the universality of the data in Fig. 20; we would refer to our own last sentence on page 417. The losses will also be modified by the quantity of steam extracted, though some unpublished work suggests that in certain cases, at least, the effect may be small. The dimensions of equation (15) are given in (**12**). Losses in exhaust casings are a function of the kinetic energies; Fig. 22 was included as an example of the sort of correlation that can be produced, rather than as being of universal application.

Lap and annulus losses may be treated as end distributed losses rather than one-dimensional values, provided that adequate information is available to define their distribution.

We note A. Smith's comments on lacing wire loss, and refer him to our own penultimate sentence on the subject on page 419. We endorse his remarks about annular cascades and axial length in connection with exhaust casings.

Three-dimensional flow

J. H. Horlock asks for further explanation of our warning about accumulating secondary losses. The '3-D' programmes which we have used with this method do not allow for the local variations in efflux angle occasioned by secondary flow, nor for the distribution of such losses by shear forces in the flow. If care is not taken, the secondary losses build up in a manner unrepresentative of real flow.

This would not occur in programmes which allowed for local angle variation and for shear forces.

A. N. MacDonald is right in stating that we accept a simple radial equilibrium calculation for h.p. and i.p. cylinders, but we would regard a three-diameter calculation as a minimum requirement (page 420). Evidence on the effect of streamline divergence and radial flow is clearly reported in (15) (p. 421). We would not consider design or analysis of l.p. cylinders without the use of '3-D' programmes.

General

A number of contributors refer to our Fig. 23. This figure contains a comparison of test and predicted efficiencies mainly for steam turbines of AEI design. Similar comparison has been made on a number of English-Electric machines, for model and experimental turbines and also for gas turbines, with similar results. Both reaction and impulse designs have been examined and of these about one-third of the cylinders are l.p.; these exhibit rather greater scatter than the others. Whether this is because of less accurate prediction, or difficulties of obtaining the accurate test data to which A. N. MacDonald refers is not clear, but certainly no consistent error has been observed in l.p. cylinder analysis.

We have not observed any difference in accuracy between impulse and reaction designs, and think, in any case, that reaction as an independent variable is rather overworked. The range of machines examined extend from small model turbines to 500 MW installations.

J. D. Denton attributes an average error of 27 per cent to our profile loss correlation, and a standard deviation of 0·49. We have not as yet had an opportunity of studying his report which we are informed is not yet available. The figures he quotes are, however, surprising. We were selective in the data we used, rejecting any results for which we did not have full details of the method and conditions of testing, and also any alleged profile loss measurements in which it appeared possible that insufficient aspect ratio had been used to give a true profile loss. No results were rejected because they referred to unconventional blade shapes. All the test results accepted were correlated to an accuracy of about ± 25 per cent. On other data subsequently examined, the error is sometimes higher. In support of his contention that cascade data cannot give accurate predictions, J. D. Denton then states that it seems probable that we have scaled the loss components. This we emphatically deny. It is far more probable that the fact our cascade correlation gives a reasonable answer for full-scale turbines demonstrates that he either overestimates the errors inherent in the cascade approach, or has experience based on inferior methods of correlation.

A. A. Garson's advocacy of model turbine tests ignores the fact that, unless they are multi-stage, they also suffer from the first of the additional limitations of cascade tests which he lists.

R. I. Lewis refers to our carpets, and asks if one can get peak efficiency at turbine design point? This depends upon the design point. The values quoted in Fig. 25 are design point efficiencies. Use of our method will confirm that with small clearances, increased loading is possible for a given efficiency by using inter-stage whirl, but we must warn against generalizations. The true answer will depend on clearance and on detail design of the inter-stage space. At the *Re* and aspect ratio of Fig. 24, typical steam turbine designs might be in the range of 1·3 to 1·5 for $\Delta Cw/U$ and 0·4 to 0·7 for Cz/U based on mean diameter conditions. We agree with R. I. Lewis that the choice of velocity triangle is highly relevant to the final stage efficiency, but a full analysis must allow adequately for secondary loss, *Re* and clearance losses. The carpets obtained vary appreciably as these variables are changed.

This is a very important point, and should be appreciated in comparison of carpets such as that shown by J. Dunham in Fig. 30, since the other carpets quoted made no such allowance. Direct comparison is therefore misleading.

For initial design purposes, we would advise V. T. Forster that carpets must include *Re*, axial velocity/blade speed ratio, stage loading and aspect ratio at least and, very desirably, clearance.

A. V. Cooke quotes some tests on a gas turbine blade. We do not accept his conclusions. We do not accept fully the generalized curves from which he quotes which are based on data averaged over an appreciable range of values of *Re* and aspect ratio; this in our opinion is insufficiently discriminating. To establish a secondary loss effect by comparison of a specific test with generalized data is, in any case, always dangerous. The specific test that he quotes was carried out at *Re* based on opening of $3{\cdot}5 \times 10^5$, and not as stated. If allowance is made for this, and the likely roughness of the blade, the tests and predictions are within 10 per cent.

K. W. Todd and V. J. Andrews refer to turbulence. We fully expected that a correction for turbulence would have to be added, but initially used the method without it. When we found that it gave substantially the right answer without the turbulence correction, we thought it right to publish in that form. The number of tests where turbulence would materially affect the answer is rather limited; these are the tests which are subject to most other unknowns. We therefore strongly suspect that a turbulence correction ought to be added, at least in respect of profile loss. However, there are grounds for thinking that the simplified ways of doing this, which have been proposed in the past, are inadequate.

M. Trounce asks about blade efflux angle. Our method has been used with a consistent prediction of blade efflux angle, but this has not been extended to cover local variations of efflux angle in the secondary loss zone.

We are particularly grateful to those who have added data for a comparison with our predictions. The fact that we appear to be under fire equally for estimates which are too low and too high causes us some quiet satisfaction.