Contents

List of Abbreviations

Geometric parameters

c	Chord (distance between leading edge and trailing edge)	m
D_m	Mean diameter of blade	m
h	Height (blade height between hub and tip)	m
R	Total radius of rotor (axis to tip)	m
R_h	Radius of hub	m
R_m	Mean radius of blade	m
R_t	Radius of tip	m
s	Pitch or spacing (distance between two adjacent blades)	m

Other Symbols

η	Isentropic efficiency	
ψ	Flow coefficient or flow function	
GR	***TRADUCIR*** Grado de reacción, degree of reaction??	
$h_t = h_0$	Specific total enthalpy	$\mathrm{J\,kg^{-1}}$

Subindices and stages

1	Start of turbine stage, entry to vane/stator	
2	Stator exit, entry to rotor	
3	Rotor exit, end of turbine stage	

Thermodynamic variables

ΔH	Enthalpy change	$\mathrm{J\,kg^{-1}}$
γ	Heat capacity ratio	

ρ	Static density	kg m^{-3}
ρ_0	Total density	kg m^{-3}
C_p	Heat capacity	
P	Static pressure	Pa
P_0	Total pressure	Pa
R	Specific gas constant	J kg^{-1} K^{-1}
T	Static temperature	K
T_0	Total temperature	K
T_s	Ideal static temperature	K

Flow velocities and angles

α_1	Entry angle in vane (between v_1 and axis)	rad
α_2	Exit angle in vane (between v_2 and axis)	rad
β_2	Entry angle in rotor (between w_2 and axis)	rad
β_3	Exit angle in rotor (between w_3 and axis)	rad
ω	Angular velocity	rad s^{-1}
a	Speed of sound	m s^{-1}
M	Mach number	
RPM	Revolutions per minute	rev min^{-1}
u	Peripheral velocity	m s^{-1}
v_u	Tangential component of velocity in fixed axes	m s^{-1}
v_x	Axial component of velocity in fixed axes	m s^{-1}
w_u	Tangential component of velocity in fixed axes	m s^{-1}
w_x	Axial component of velocity in rotating axes	m s^{-1}

Chapter 1

Introduction

This document contains a background, method, and results of the project completed as the *Trabajo Fin de Grado* (final project for undergraduate degree) in Aerospace Engineering.

In particular, the project involves the creation of a simulation and prediction tool for high-pressure axial turbines, including the most relevant loss phenomena present in expansive turbine machinery. This prediction and design model will then be integrated with an optimization tool. The final results is a software tool which allows the user to define the required work and geometric constraints, and which provides in turn an optimized turbine model which minimizes energy losses.

This model is created using a variety of reference works and a wide variety of programming resources, so that both the study of thermodynamic theory and loss models (as included from the reference studies) and the technical implementation of the design tools provided challenges in the design process, thus increasing the author's grasp of applied methods and theory.

1.1 Objectives

1.2 Methodology

The methodology followed in the creation of the design tool contains 8 distinct steps, all of which were essential to create the final tool. These were not followed sequentially in every case, overlapping and feeding back when needed.

1. **Preliminary research** on high-pressure turbines and similar design projects, as well as various loss models which will be included in the tool. During this research, a large variety of sources are considered. Notes are taken of each work, building a reference log, and a summary of references is made which is be useful later on in the project.

2. **Initial turbine model** with no pressure losses, with given entry conditions and thrust requirements. This model follows the usual thermodynamic calculations, assuming fixed isentropic efficiencies to model the losses in compressor and turbine. Some numerical tools are implemented to automate the process for a variety of conditions.

3. Iteration on the initial model by including **pressure losses** as a function of the blade angles.

4. Development/adaptation of a **global optimization strategy**, after preliminary research of various alternatives and existing algorithms.

5. Integration of the turbine design tool with the optimization strategy selected in the previous step, to achieve a complete tool which, given the requirements, provides the **optimal design configuration**.

6. Comparison of the **results provided by the design tool** to existing turbines, experimental results, and other models.

7. Possible inclusion of **additional losses** for more accurate results.

8. **Design tool analysis and conclusion**.

1.3 Topic selection and relevance

This subject was selected because

Chapter 2

Reference studies

2.1 Thermodynamic model of turbine

- Apuntes jorge y valencia: modelo general de turbina, introducción a triangulos de velocidades [1] [2]
- Slides VKI: turbine design process, efficiency correlations, swindell graphs, etc. [3]
- Fluid Mechanics and Thermodynamics of Turbomachinery (Dixon): basic thermodynamics turbine, two-dimensional flow, un poco de todo [4]

2.2 Isentropic equations

*** Explain these since they are referenced in chapter 6 ***

$$\frac{P}{P_0} = \left(1 + \frac{\gamma + 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}} \tag{2.1}$$

$$\frac{T}{T_0} = \left(1 + \frac{\gamma + 1}{2} M^2\right)^{-1} \tag{2.2}$$

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma - 1}} \tag{2.3}$$

$$T_0 = T_{dynamic} + T_{static} = \frac{V^2}{2C_p} + T \tag{2.4}$$

$$\Delta H = u \Delta v_u = u\left(v_{au} - v_{bu}\right) \tag{2.5}$$

2.3 ***notes***

(see if any of these notes from propulsión are useful in the final report) (cuidado con plagios)

2.3.1 Intro turbines

[1] [2] Una turbina es una maquina fluido térmica motora:

- Máquina de **fluido**: conjunto de elementos mecánicos que permiten intercambiar energía con el exterior, generalmente a través de un eje, por variación de la energía disponible en el fluido que la atraviesa.
- Máquina **térmica**: trabaja con fluido compresible, y al aumentar la presión aumenta la temperatura.
- Máquina **motora**: se produce la transformación de energía de fluido en energía mecánica del eje. La presión y entalpía (temperatura) del fluido disminuyen, el eje gira para comprimir el compresor o un fan.

A turbine stage is composed of two elements: the **guide vanes** and **rotor blades**, in that order.

The **vane** adapts the flow to a desired entry angle for the following rotor stage, as well as the first stage of expansion.

$$\frac{\Delta c^2}{2} > 0 \qquad \Delta P < 0 \tag{2.6}$$

The **rotor** extracts work from the flow energy, as the flow rotates the axis as it expands through the rotor blades. This axis transfers the extracted work to a compressor or fan stage/s.

$$\frac{\Delta c^2}{2} < 0 \qquad \frac{\Delta \omega^2}{2} > 0 \qquad \Delta P < 0 \tag{2.7}$$

2.3.2 Velocity triangles

The flow can be measured according to the fixed axes (where the z-axis follows the engine's rotating axis), and this velocity is called c. On the other hand, the velocity relative to the rotating blades is w. These velocities are easily converted using the blade velocity $u = \omega R$, where ω is the angular velocity of the turbine's rotor and R is the radius of the blade section under analysis–or the mean radius, if the entire blade is under analysis.

$$\vec{c} = \vec{w} + \vec{u} = \vec{w} + \vec{\omega} \times \vec{R_m} \tag{2.8}$$

At the stage entry points (0), the flow has a velocity of c and an angle α_0 when entering the turbine vanes. If the flow is purely axial, then $\alpha_0 = 0$. Since the vane is by definition fixed to the engine, the rotational velocity and blade velocity are 0. (All angles are measured to the engine's rotational axis.)

diagram entry to vane

Exiting the vane, the flow has been expanded, accelerated, and turned to a velocity c_1 and angle $\angle_1$. Entering the rotor, the relative velocity is found by subtracting the blade velocity from the flow's absolute velocity:

$$\vec{w_1} = \vec{c_1} - \vec{u} \tag{2.9}$$

This velocity has an angle β_1. Since the blade velocity is purely tangential, the axial velocity of the flow does not vary: $c_{1a} = w_{1a}$.

At the rotor exit, the blade velocity is added to the relative velocity to find the absolute velocity, and the relative exit angle β_2 becomes the absolute exit angle, α_2:

$$\vec{c_2} = \vec{w_2} + \vec{u} \tag{2.10}$$

2.3.3 Reaction ratio (grado de reacción translation? and change "GR")

The reaction ratio compares the expansion which takes place in the rotor to the total expansion of the stage. This can be done by comparing enthalpies, velocities, or pressures from the entry and exit stages of the turbine.

*** some of these are referenced, don't delete ***

$$GR = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{h_1 - h_2}{h_0 - h_2} \tag{2.11}$$

$$GR = \frac{w_2^2 - w_1^2}{w_2^2 - w_1^2 + v_0^2 + v_1^2} \tag{2.12}$$

$$GR = \frac{V_a}{u} \frac{\tan\beta_2 - \tan\beta_1}{2} \tag{2.13}$$

$$GR = 1 - \frac{1 - \left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{P_3}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{2.14}$$

En turbinas de **acción o impulso**: toda la expansión del fluido se produce en el estator $(GR = 0)$. En estas turbinas se tiene mayor trabajo útil por escalonamiento (triángulo de velocidad), y menos fugas por no haber diferencia de presiones y posibilidad de inyección parcial, ausencia de esfuerzos axiales en el rotor.

In **reaction** turbines, the expansion is produced in both the vane and rotor, an the expansion ratio is $0 < GR < 1$. These turbines have higher efficiencies and a larger operating range.

In a symmetrical turbine stage, the ratio is equal to $GR = \frac{1}{2}$, and the velocities are such that stages can be concatenated with no additional modifications, since the rotor exit conditions are appropriate for the vane entrance:

$$c_1 = w_2, \quad w_1 = c2, \quad \alpha_1, \quad \beta_2, \quad \beta_1 = \alpha_2 \tag{2.15}$$

figure of cross sections vanes/blades for various turbine types with differing reaction ratios

[3]: Since the 2-D loss is approximately proportional to the square of the Mach number, the minimum loss in this sense will occur when the velocity triangles are symmetrical, $GR = 0.5$. (This is assuming that the efficiency in the estator is the same as the efficiency in the rotor.

Common practice in high-pressure turbine design is to use $GR = 0.45$, since this leads to reduced relative total temperature and pressure in the rotor:

$$T_{OR} = T_O + \frac{1}{2c_p}\left(w^2 - v_c^2\right) \tag{2.16}$$

It reduces the inlet angles for the estator of the next stage (lowering the GR lowers the turning of the fluid). Bearing loads are also reduced.

This solution, however, is a compromise, and will not necessarily lead to the optimal design.

2.3.4 Isentropic efficiency

The turbine efficiency is measures in the ratio of enthalpy to the ideal adiabatic and isentropic expansion:

$$\eta_t = \frac{\text{Real work extracted form turbine}}{\text{Ideal work extracted from turbine}} = \frac{W_{real}}{W_s} = \frac{h_{4t} - h_{5t}}{h_{4t} - h_{5ts}} \tag{2.17}$$

Under the ideal gas hypothesis, the isentropic efficiency can be expressed as a function of pressure and temperature ratios:

$$\eta_t = \frac{h_{4t} - h_{5t}}{h_{4t} - h_{5ts}} = \frac{c_P\left(T_{4t} - T_{5t}\right)}{c_P\left(T_{4t} - T_{5ts}\right)} = \frac{1 - \frac{T_{5t}}{T_{4t}}}{1 - \frac{T_{5ts}}{T_{4t}}} = \frac{1 - \frac{T_{5t}}{T_{4t}}}{1 - \left(\frac{P_{5ts}}{P_{4t}}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1 - \frac{T_{5t}}{T_{4t}}}{1 - {\pi_t}^{\frac{\gamma-1}{\gamma}}} \tag{2.18}$$

2.3.5 Ecuación fundamental de las turbomáquinas

[2] Tenemos una turbina con eje de giro z y velocidad angular ω. Aplicamos el teorema de la cantidad de movimiento a un tubo de corriente entre los puntos 1 y 2, separados por una cierta distancia axial.

$$\sum \vec{M} = \frac{d\left(\vec{r} \times m \cdot \vec{c}\right)}{dt} \tag{2.19}$$

Si se considera que las magnitudes son estacionarias, se puede hallar el momento en función del gasto másico:

$$\sum \vec{M} = \left(\vec{r} \times \vec{c}\right)\frac{dm}{dt} \tag{2.20}$$

Realizando un producto escalar para hallar el momento según el eje axial z:

$$M_z = \left(r_2 \cdot c_{2u} - r_1 \cdot c_{1u}\right)\dot{m} \tag{2.21}$$

Las fuerzas que ejerce el fluido contenido en el volumen de control sobre las paredes de éste crean un **momento** $-M_z$, y con la ecuación de Euler para máquinas motoras de $(M_z > 0)$:

$$M_z = \left(r_1 \cdot c_{1u} - r_2 \cdot c_{2u}\right)\dot{m} \tag{2.22}$$

Y la **potencia**:

$$N_u = \omega M_z = \omega \cdot (r_1 \cdot c_{1u} - r_2 \cdot c_{2u})\, \dot{m} \tag{2.23}$$
$$= (\omega \cdot r_1 \cdot c_{1u} - \omega \cdot r_2 \cdot c_{2u})\, \dot{m} \tag{2.24}$$
$$= (u_1 \cdot c_{1u} - u_2 \cdot c_{2u})\, \dot{m} \tag{2.25}$$

El **trabajo específico** o trabajo periférico se obtiene a partir de la potencia, y esto se denomina la **1ª forma de la ecuación de Euler**.

$$w_u = \frac{N_u}{\dot{m}} = (u_1 \cdot c_{1u} - u_2 \cdot c_{2u}) \tag{2.26}$$

Esta deducción es válida tanto para turbomáquinas motoras $(Wu, Mz > 0)$ como generadoras $(Wu, Mz < 0)$.

Para desarrollar esta ecuación, miramos el triangulo de velocidades:

$$\vec{w} = \vec{c} - \vec{u} \tag{2.27}$$

Tomando la dirección u, perpendicular al eje de giro z:

$$c_u = c \cdot \cos(90° - \alpha) = c \cdot \sin\alpha \tag{2.28}$$

Aplicando el teorema del coseno:

$$w^2 = c^2 + u^2 - 2u \cdot c \cdot \cos(90° - \alpha) \tag{2.29}$$

Sustituyendo:

$$u \cdot c \cdot \cos\alpha = u \cdot c_u = \frac{c^2 + u^2 - w^2}{2} \tag{2.30}$$

La **2ª forma de la ecuación de Euler** (para referencia fija).

$$w_u = \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2} + \frac{u_1^2 - u_2^2}{2} \tag{2.31}$$

Aplicando el primer principio para el observador fijo, suponiendo un evolución adiabática entre 1 y 2:

$$w_u = h_{01} - h_{02} = h_1 - h_2 + \frac{c_1^2 - c_2^2}{2} \tag{2.32}$$

Con un sistema de referencia móvil:

$$w_u = \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2} + \frac{u_1^2 - u_2^2}{2} = h_1 - h_2 + \frac{c_1^2 - c_2^2}{2} \tag{2.33}$$

Reorganizando:

$$h_1 + \frac{w_1^2}{2} + \frac{u_2^2 - u_1^2}{2} = h_2 + \frac{w_2^2}{2} \tag{2.34}$$

Al igual que en el sistema fijo, se puede definir una entalpía de parada relativa (rotalpía) a partir de la siguiente expresión:

$$h_{0r} = h + \frac{w^2}{2} \tag{2.35}$$

Por lo que:

$$h_{01r} + \frac{u_2^2 - u_1^2}{2} = h_{02r} \tag{2.36}$$

El trabajo específico generado o consumido por la turbomáquina se obtiene a partir de la ecuación de Euler y se expresa como:

$$w_u = u_1 c_{1u} - u_2 c_{2u} = \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2} + \frac{u_1^2 - u_2^2}{2} = h_1 - h_2 + \frac{c_1^2 - c_2^2}{2} \tag{2.37}$$

En una maquina turboaxial pura, la velocidad angular y periférica son idénticas, por lo que $u_1 = u_2 = u$:

$$w_u = u\left(c_{1u} - c_{2u}\right) \tag{2.38}$$

2.3.6 Turbine design parameters

[3] (change variable names in this table)

Specific work	$\frac{C_p \Delta T_0}{T_0}$
Engine non-dimensional speed	$\frac{\omega}{\sqrt{T_0}}$
Turbine non-dimensional speed	$\frac{U}{\sqrt{T_0}}$
Stage loading (factor de carga)	$\psi = \frac{\Delta H}{U^2}$
Flow coefficient (flow function, factor de flujo)	$\frac{c_{1a}}{u} \approx \frac{\dot{m}_a}{rpm}$
Inlet capacity	$\frac{\dot{m}\sqrt{T_0}}{P_0}$
Stage Reaction	$\frac{t_2 - t_3}{T_{01} - t_3}$

Stage loading (factor de carga):

$$\psi = \frac{\Delta H}{U^2} \tag{2.39}$$

The **flow coefficient**, sometimes called φ, is a representation of how large the motor is, relative to how rapidly the axis rotates.

The **stage loading** represents the expansion, or work extraction, occurring at each stage. A large stage loading factor is one in which each stage produces more expansion: the engine has fewer total stages, is more compact, and represents more advanced technology, since the pressure drop in a single stage is larger.

2.3.7 Factors improving turbine efficiency: Engine thermodynamic cycle

[3]

- Increase Inlet Total Pressure and Temperature (PR),
- Reduced Core Inlet Capacity (Increased Bypass Ratio),
- Increased Specific Work on Core Turbines,
- Cooling and Leakage Flows Have Greater Effect,
- Mechanical Features Do Not Scale - Seals, Bearings, Shafts, etc.

The energy extracted from the fluid is:

$$E = \dot{m}\Delta H = \dot{m}C_p\Delta T_0 \tag{2.40}$$

Since the energy extracted from the fluid must be equal to the work done by the turbine, these equations bust be equal: The energy extracted from the fluid is:

$$\dot{m}u\Delta c_u = \dot{m}\Delta H \tag{2.41}$$

Which brings us to equation:

$$\frac{\Delta H}{u^2} = \frac{\frac{C_p\Delta T_0}{T_0}}{\left(\frac{u}{\sqrt{T_0}}\right)^2} \tag{2.42}$$

Hence, the stage loading is equal to the specific work divided by the square of turbine non-dimensional blade speed.

At this point, the diameter is set with the stage loading, $\frac{\Delta H}{u^2}$, where typical values for high-pressure turbines are between 1-5-2.

High stage loading leads to higher turning and an increase in Mach Number, however there is more work per stage, which can lead to fewer stages. (the velocity triangle is more "pointy".

Low stage loading leads to lower turning and a decrease in Mach Number, however you are not getting the best out of the turbine.

Since the 2-D loss is approximately proportional to the square of the Mach number, the minimum loss in this sense will occur when the velocity triangles are symmetrical, $GR = 0.5$. (This is assuming that the efficiency in the estator is the same as the efficiency in the rotor.

Common practice in high-pressure turbine design is to use $GR = 0.45$, since this leads to reduced relative total temperature and pressure in the rotor:

$$T_0R = T_0 + \frac{1}{2c_p}\left(w^2 - v_c^2\right) \tag{2.43}$$

It reduces the inlet angles for the estator of the next stage (lowering the GR lowers the turning of the fluid). Bearing loads are also reduced.

This solution, however, is a compromise, and will not necessarily lead to the optimal design.

Once the flow coefficient $\varphi = \frac{c_{1a}}{u}$ has been fixed, the velocity triangles are locked. Then, variations in $V_A = c_{1a}$ varies the annulus height, hub and case diameters.

Increasing the flow coefficient φ leads to increased Mach numbers, reduced exit angles and turning, in both vane and rotor, and a smaller annulus height. This results in increased airfoil chord, increased trailing edge loss, and increased cost. In addition the aspect ratio of the airfoils will be reduced, resulting in increased secondary loss. However, the turbine is smaller and lighter and the blade stress will be reduced.

As the hub diameter will increase, there is the potential for more leakage loss due to the increased area of the seals. At the casing the overall result depends on two opposing effects, as the area of the seals is reduced there is the potential for reduced leakage, however, assuming the tip gap is fixed, the tip gap to height ratio of the rotor will increase, providing the potential for increased tip leakage flow per unit area.

Typical values for flow coefficients φ in high-pressure turbines are 0.4-0.6.

2.4 Loss models

2.5 Other turbine analyses / state of the art

Chapter 3

Hypotheses

Chapter 4

Software and code resources

4.1 Implementation

The initial models of the program have been implemented in Python 3.8.5. This high-level programming language was selected due to its simplicity and relevance in the current aerospace industry, in addition to the facilities provided by existing libraries and packages for computational algorithms, optimization, and visual depiction of data and results. A brief summary of the libraries and functions used, as well as the appropriate citations, is included in this section.

The optimization was done by implementing / using the existing package / ... ***complete when optimization method is selected***

4.2 Python libraries

Several open-source python libraries have been used in the design tool, containing both mathematical tools and visualization resources. Most of these are commonly-used python libraries, although more specific tools have been applied in some cases. All the code implemented in this project has been written by the author except the use of these libraries.

Numpy [5] is the fundamental Python library for all the high-level mathematics in this design process. Based in C, it brings more powerful computational power to Python code. It is also the foundation for the additional libraries used.

Scipy [6] is based on Nympy, and contains many algorithms and functions for numerical data processing and solving. The optimization libraries have been particularly useful–some specific functions are described in the next section.

Matplotlib [7] is a comprehensive library with all the necessary tools for visual representation of data: all figures, animations and data plots included in this document have been generated using this library. *** confirm this when finished ***

Pandas *** rellenar ***

4.3 Specific functions

Within the above-mentioned libraries, some functions are worth detailing, especially when the mathematical algorithms applied in these may have an effect on the results and/or computational speed.

`fsolve` part of the SciPy optimization package, this function is a Python wrapper for the `hybrd` function from the FORTRAN library MINPACK. In this algorithm, Powell's dog leg method is used to find the roots of a system of N non-linear functions [8]. This function is used for convergence cases within the turbine calculations.

`least_squares` with the `trf` or Trust Region Reflective algorithm method. Also included in the SciPy optimization package, this function applies the least-squares method to find the minimum in a non-linear problem. In contrast to `fsolve`, this function allows the user to specify bounds for each of the variables under optimization.

4.4 Optimization strategies

4.5 Front-end visualization

Chapter 5

Tool overview and applications

Chapter 6

Turbine model

6.1 Overview and definitions

The basis of the design tool will be the turbine model, a one-dimensional calculation tool which completes the necessary iterations to solve for all the interior variables. Given the basic constraints (as discussed in section **??**), the model can run through the thermodynamic and geometric variables for the two turbine components: stator and rotor

Note: the analysis in a one-dimensional single-stage turbine model is reduced to three points (por precisely, planes) along the axial direction of the turbine: **inlet (1)**, **middle** or **stator (2)**, and **outlet** or **rotor (3)**. For a visual depiction, see Figure 6.1.

6.1.1 Inputs

The design tool needs only a limited number of input values: inlet and outlet temperatures and pressures, as well as the enthalpy required of the turbine. These can be deduced from a global cycle analysis, or from the constraints imposed directly on the turbine by manufacturer.

Although this design tool is prepared for use of these parameters, it may be modified with only slight alterations to the upper layers for application using different input variables, or else these parameters could be calculated directly from available values.

Table 6.1: Input constraints

Input parameter		Unit
Inlet: total temperature	$T01$	K
Inlet: total pressure	$P01$	Pa
Outlet: total temperature	$T03$	K
Outlet: total pressure	$P03$	Pa
Turbine enthalpy	ΔH_{prod}	$\mathrm{J\,kg^{-1}}$

It should be noted that ΔH_{prod} represents the energy which must be produced by the turbine–this can be found by analyzing the compressor of the given generator, or through similar turbine designs.

6.1.2 Turbine geometry

The basic form of the turbine must be pre-defined within the design tool, since the geometric calculations are included in the thermodynamic equations, as well as the loss calculations.

In this case, a traditional configuration is selected, where the radius of both tip and hub is constant in the stator, whereas the rotor has a diverging shape: the radius of the tip is larger at the outlet than at the middle stage. The planes under study, as can be seen in figure 6.1, mark the three key points in the turbine.

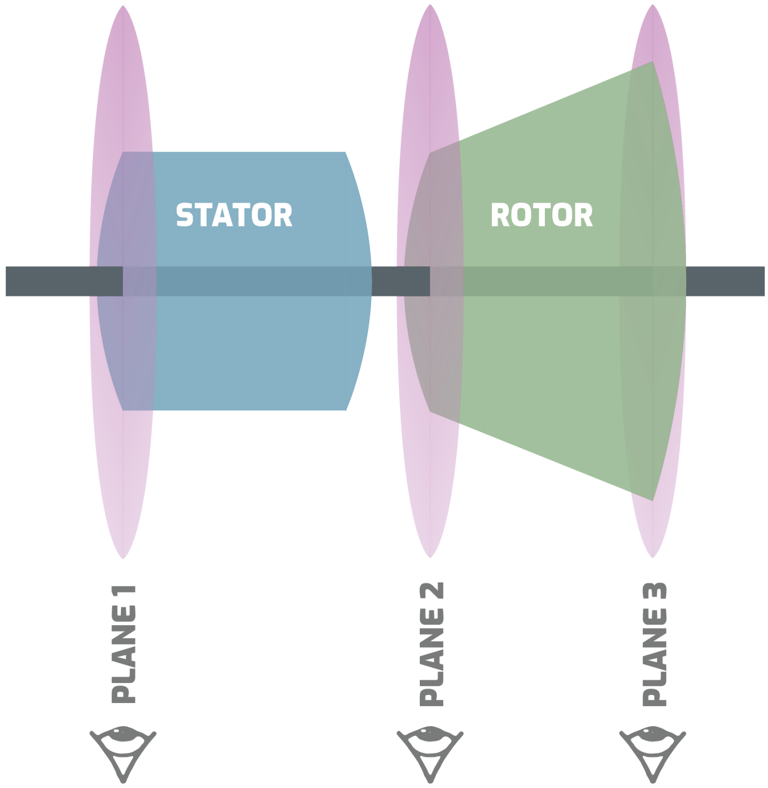

Figure 6.1: Representation of the three analysis planes on a turbine stage

Moving in closer, the relevant radii can be defined: R_t marks the tip (the outer edge of the blades), R_h the hub (where the blades meet the shaft), R_m the mean radius for that section. h represents the blade height.

6.1.3 Velocity triangles

The flow can be measured according to the fixed axes (where the z-axis follows the engine's shaft), and this velocity is called v. On the other hand, the velocity relative to the rotating

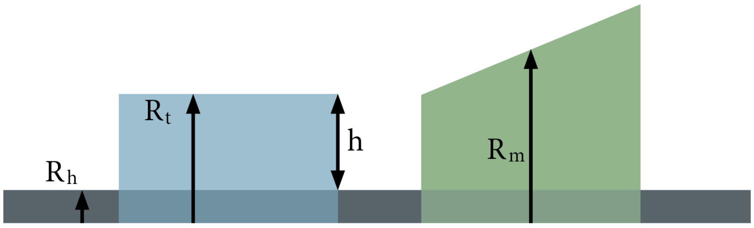

Figure 6.2: Geometry of turbine blades

blades is w. These velocities are easily converted using the blade velocity $u = \omega R$, where ω is the angular velocity of the turbine's rotor and R is the radius of the blade section under analysis–or the mean radius $\vec{R}_m$, if the entire blade is under analysis.

$$\vec{v} = \vec{w} + \vec{u} = \vec{w} + \vec{\omega} \times \vec{R}_m \tag{6.1}$$

On the other hand, the velocities can be projected along the shaft axis (v_x) or perpendicular to this (v_u). In the same way, the relative velocities in the rotor can also be projected: w_x, w_u.

At the inlet point (1), the flow has a velocity of v_1 and an angle α_1 when entering the turbine vanes. If the flow is purely axial, then $\alpha_1 = 0$. Since the stator vane is by definition fixed to the engine, the rotational velocity and blade velocity are 0. (All angles are measured to the engine's rotational axis.)

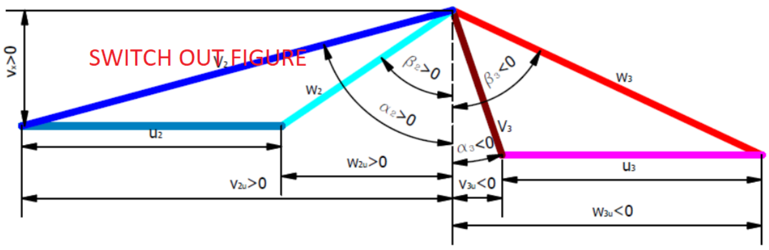

Figure 6.3: SWITCH OUT FIGURE Diagram of velocity triangles in turbine stage

Exiting the vane, the flow has been expanded and accelerated to v_2 and angle α_2. Entering the rotor, the relative velocity is found by subtracting the blade velocity from the flow's absolute velocity:

$$\vec{w_2} = \vec{c_2} - \vec{u} \tag{6.2}$$

This velocity has an angle β_2. Since the blade velocity is purely tangential, the axial velocity of the flow does not vary: $v_{2x} = w_{2x}$.

At the rotor exit, the blade velocity is added to the relative velocity to find the absolute velocity, and the relative exit angle β_3 becomes the absolute exit angle, α_3:

$$\vec{v_3} = \vec{w_3} + \vec{u} \tag{6.3}$$

6.1.4 Outputs

*** completar *** A number of variables are calculated within the design tool: both thermodynamic quantities, used in the models and calculations, and geometric values, translating the numerical solutions to a visual model.

All of these values could be extracted form the design tool as output, if desired. In this case, the selected outputs are shown in **??**. In addition, graphs for the velocity triangles, pressure and temperatures, and models of the turbine geometry are generated. *** complete when finished ***

6.2 Assumptions and constants

A small number of assumptions are made in addition to the hypotheses and models discussed previously. This is done to limit certain variables to reasonable ranges, or to fix parameters which will be included in the optimization in the next chapter of the design tool. In addition, thermodynamic constants and fuel characteristics must be defined.

First, a degree of reaction GR is selected. This value, as discussed in **??**, represents the expansion in the rotor compared to the total expansion in the turbine stage. A higher value for GR will, in general, reduce losses and lead to better performance. However, as GR increases, so necessarily does the rotational speed in the rotor–this may lead to divergence and lower overall performance [].

The degree of reaction will be optimized in the next phase of the project, but for the initial no-loss model, a value for GR must be selected. The recommended range is between 0.3 and 0.5 [], with the default value set to *** completar ***.

The stator inlet angle α_1 represents the angle between the flow velocity v_1 and the shaft axis. This is assumed to be $\alpha_1 = 0$, that is, the flow is purely axial. If the inlet flow has a tangential component (for example, if the stage under design is the second stage in a multi-stage turbine immediately following a non-symmetric stage), then this angle must be increased and the tangential component taken into account.

In addition, the thermodynamic efficiencies of both stator and rotor must be assumed at this stage. *** find reasonable numbers and references ***

Table 6.2: Turbine design assumptions

Parameter		Default value	Unit
Degree of reaction	GR	0.32	-
Loading factor	ϕ	1.75	-
Inlet angle	α_1	0	rad
Stator efficiency	η_{stator}	0.888	-
Rotor efficiency	η_{rotor}	0.808	-

The thermodynamic constants needed in the design tool can be seen in Table 6.3. These can also be modified when the design tool is run, if the fuel mix used is different to the standard–these values refer to the air-and-fuel mix which exits the combustion chamber (see **??** for a discussion on the combustion chamber exit conditions).

Table 6.3: Thermodynamic constants

Parameter		Default value	Unit
Heat capacity ratio	γ	1.3	-
Heat capacity	C_p	1240	$\mathrm{J\,K^{-1}}$
Specific gas constant	R	286.15	$\mathrm{J\,kg^{-1}\,K^{-1}}$

6.3 Convergence functions

The design tool, with the inputs described in subsection 6.1.1, will find one single solution for turbine stage–however this cannot be solved for directly. There are a number of parameters which can only be found using an initial guess and subsequent iterations until convergence is found, with the acceptable ranges for each value defined. Rather than complete these processes by hand, as has been done in the past, these parameters have been fixed by use of non-linear solvers (as specified in section 4.2).

Using these, an initial guess is made for the needed value (although, as will be seen later on, the results are insensitive to guesses within a large range of reasonable values). Next, the non-linear solver is applied to these functions, a convergence point is applied (where the final calculated value meets the initial guess), and the results extracted.

6.3.1 Outlet Mach

The first such parameter, on which all the subsequent calculations depend, is for the **outlet Mach number** M_3. Once this value is calculated, the outlet conditions are given, and using the reaction degree the middle stage can be solved for as well. An initial guess is needed for this convergence–however nearly any value for subsonic flow, M_3, leads to the same convergence point in all tested cases.

Given a guess for $M_{3,init}$, the static pressure at the outlet is defined by using the isentropic equation which relates total pressure and static pressure by means of the Mach number (Equation 2.1), solving for P_3:

$$P_3 = P_{03}\left(1 + \frac{\gamma+1}{2}M_{3,init}^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{6.4}$$

An the static temperature T_3 is computed through the isentropic relation (Equation 2.3):

$$\frac{T_3}{T_{03}} = \left(\frac{P_3}{P_{03}}\right)^{\frac{\gamma-1}{\gamma}} \tag{6.5}$$

Intermediate plane

Since a value for GR has been assumed, the pressure ratio definition (Equation 2.14) is applied to solve for the static pressure in the intermediate plane:

$$P_2 = P_{01}\left(1 + (GR - 1)\left(1 - \left(\frac{P_3}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right)\right)^{\frac{\gamma}{\gamma-1}} \tag{6.6}$$

Once this pressure is found, the pressure in each of the planes has been determined. Next, the remaining variables in plane 2 are found. For this, it is necessary to assume an isentropic evolution in the stator. Since there is no rotation in the stator, no work is applied by the flow, and therefore the total temperature is constant, and because the total temperature at inlet is a design input:

$$T_{02} = T_{01} \tag{6.7}$$

The ideal static pressure T_{2s} can be found by applying the same isentropic flow relation as previously (Equation 2.3) between planes 1 and 2:

$$T_{2s} = T_{02}\left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \tag{6.8}$$

Using the assumed stator efficiency, the real static pressure T_2 is easily found:

$$\eta_{stator} = \frac{T_{02} - T_2}{T_{02} - T_{2s}} \qquad \longrightarrow \qquad T_2 = T_{02} - \eta_{stator}\left(T_{02} - T_{2s}\right) \tag{6.9}$$

Next, the flow velocity is calculated by using the dynamic pressure (Equation 2.4) and the temperature calculated in the middle plane:

$$T_{02} = \frac{v_2^2}{2C_p} + T_2 \qquad \longrightarrow \qquad v_2 = \sqrt{2C_p\left(T_{02} - T_2\right)} \tag{6.10}$$

The same equation can be used to calculate v_{2s} by substituting the ideal temperature:

$$v_{2s} = \sqrt{2C_p\left(T_{02} - T_{2s}\right)} \tag{6.11}$$

Once the static quantities have been calculated, the density ρ_2, speed of sound a_2, and Mach number M_2 of the stator exit are easily found:

$$\rho_2 = \frac{P_2}{RT_2} \qquad a_2 = \sqrt{\gamma R T_2} \qquad M_2 = \frac{v_2}{a_2} \tag{6.12}$$

6.3.2 Rotor inlet and outlet angles

At this point in the program, it is necessary to have the stator outlet α_2 and rotor outlet β_3 angles, so that the velocities can be projected along these axes, and the relative conditions calculated.

For a given set of angles, the program will perform a series of calculations (as follows) until the work produced can be calculated with the change in tangential velocity (**??**). This value is compared to the required ΔH produced by the turbine, and a set of angles selected which meets this criteria. Using the least squares method, convergence is found and the best angle combination selected.

*** the problem here is that we have two variables and only one condition to meet... so there could be several answers. Try 1. plotting results to see trends. or 2. Coming up with another condition. or 3. ? ***

The initial guess for α_2 and β_2, as well as the upper and lower bounds for each, are specified within the program. Because the results are sensitive to these parameters, the selection must be made with care. In addition, a larger entry angle (more tangential flow) will also increase the frontal area of the rotor, which is not desired. Applying limits from reference studies [], the limits can be set to $\alpha_2 \in [70°, 75°]$ for high pressure turbines. Equivalently, the rotor outlet angle can be fixed in this range: $\beta_3 \in [-65°, -60°]$.

Once an angle has been selected, the calculations are straightforward. First, the velocity at the intermediate plane v_2 is projected in axial and tangential components (respectively), using the angle α_2 as seen in Figure 6.3:

$$v_{2x} = v_2 \cos \alpha_2 \tag{6.13}$$

$$v_{2u} = v_2 \sin \alpha_2 \tag{6.14}$$

Next, using the selected loading factor ψ, the peripheral speed in the rotor can be determined, applying the total change in enthalpy produced by the turbine (Equation 2.39):

$$\psi = \frac{\Delta H_{prod}}{u_2^2} \qquad \longrightarrow \qquad u_2 = \sqrt{\frac{\Delta H_{prod}}{\psi}} \tag{6.15}$$

Relative conditions

Using this peripheral speed, the absolute velocities at plane 2 can be converted to relative velocities, taking into account the rotor's rotation. This is done by components, tangential and axial, since the peripheral velocity u Thus, according to Equation 6.2, the relative velocity in the tangential plane is a simple subtraction:

$$w_{2u} = v_{2u} - u \tag{6.16}$$

whereas in the axial direction, the magnitude remains constant (perpendicular to $\vec{u}$):

$$w_{2x} = v_{2x} \tag{6.17}$$

and the total speed is easily found as the magnitude:

$$w_2 = \sqrt{w_{2u}^2 + w_{2x}^2} \tag{6.18}$$

The speed of sound at 2 is still the same as was computed in Equation 6.12–therefore the relative Mach number M_{2r} is calculated. Necessarily, it will be much lower than the absolute Mach.

$$M_{2r} = \frac{w_2}{a_2} \tag{6.19}$$

The relative inlet angle is given by a simple trigonometric relation (see Figure 6.3):

$$\beta_2 = \arctan \frac{w_{2u}}{w_{2x}} \tag{6.20}$$

Finally, to complete all values for the intermediate plane, the relative total properties are calculated using the relative Mach conditions (Equations 2.1 and 2.2):

$$T_{02r} = T_2 \left(1 + \frac{\gamma - 1}{2} M_{2r}^2\right) \tag{6.21}$$

$$P_{02r} = P_2 \left(1 + \frac{\gamma - 1}{2} M_{2r}^2\right)^{\frac{\gamma}{\gamma-1}} \tag{6.22}$$

Rotor outlet

In its passage through the rotor, the flow maintains a constant rothalpy–therefore the relative total temperature s the same in the outlet as in the intermediate plane:

$$T_{03r} = T_{02r} \tag{6.23}$$

The isentropic evolution is calculated (Equation 2.3), and using the rotor efficiency assumed at the outset, the static temperature at the outlet is calculated:

$$T_{3s} = T_{03r} \left(\frac{P_3}{P_{02r}}\right)^{\frac{\gamma-1}{\gamma}} \tag{6.24}$$

$$\eta_{rotor} = \frac{T_{03r - T_3}}{T_{03r} - T_{3s}} \qquad \longrightarrow \qquad T_3 = T_{03r} - \eta_{rotor} \left(T_{03r} - T_{3s}\right) \tag{6.25}$$

Finally, just as was done for the intermediate plane, the relative velocities are calculated from the relative dynamic pressure:

$$T_{03} = \frac{w_3^2}{2C_p} + T_3 \qquad \longrightarrow \qquad w_3 = \sqrt{2C_p \left(T_{03r} - T_3\right)} \tag{6.26}$$

With the current guess value for β_3, the relative velocity w_3 can be projected into axial and tangential directions, respectively:

$$w_{3x} = w_3 \cos \beta_3 \tag{6.27}$$

$$w_{3u} = w_3 \sin \beta_3 \tag{6.28}$$

Since the peripheral speed is a function of the rotor's angular velocity ω and the mean radius R_m, so long as the mean radius is constant, the peripheral speed will be constant:

$$u_3 = u_2 \tag{6.29}$$

Following the inverse method as was completed for plane 2, equation 6.2 can be applied to find the absolute tangential speed at the stator outlet:

$$v_{3u} = w_{3u} + u_3 \tag{6.30}$$

Once again, the axial component of the peripheral speed is 0, and therefore the relative axial speed is equal to the absolute axial speed:

$$v_{3x} = w_{3x} \tag{6.31}$$

which leads to the magnitude of the absolute velocity:

$$v_3 = \sqrt{v_{3x}^2 + v_{3u}^2} \tag{6.32}$$

Now all the outlet velocities have been determined, and the enthalpy produced can be calculated. This will be given by the work applied by the flow on the rotor:

$$\Delta H_{calc} = u_2 \left(v_{2u} - v_{3u} \right) \tag{6.33}$$

This value, referred to as *calculated enthalpy*, is the final value in this convergence loop, where the cost function is set to the difference between this value and the defined turbine enthalpy, where this value is set to be minimized by the convergence loop:

Rotor angle convergence: find α_2 and β_3 such that $(\Delta H_{prod} - \Delta H_{calc}) \rightarrow 0$

6.3.3 Outlet Mach (continued)

Once the angles and velocities for this iteration have been completed, and the rotor angles α_2, β_3 established, the convergence loop for M_3 can be completed.

First, the remaining outlet quantities are found. Once again, the density ρ_3, speed of sound a_3, and relative Mach number M_{3r} at the rotor outlet are easily found:

$$\rho_3 = \frac{P_3}{RT_3} \qquad a_3 = \sqrt{\gamma R T_3} \qquad M_{3r} = \frac{w_3}{a_3} \tag{6.34}$$

The absolute angle at outlet is also easily found though a trigonometric relationship:

$$\alpha_3 = \arcsin \frac{v_{3u}}{v_3} \tag{6.35}$$

Finally, the absolute Mach number is found using the absolute velocity:

$$M_3 = \frac{v_3}{a_3} \tag{6.36}$$

This value for outlet Mach is compared to the initial guess. The non-linear solver is integrated to find a solution for M_3 which, when set as the initial guess, provides a calculated value identical to the first.

Outlet Mach convergence: find $M_{3,init}$ such that $(M_{3,init} - M_3) \rightarrow 0$

Chapter 7

Loss models

Chapter 8

Optimization methodology *

(maybe move up in report)

8.1 Selected method

8.2 Implementation/adaptation

Chapter 9

Integration of turbine model with optimization tool

Chapter 10

Results

Chapter 11

Conclusion

(incluyendo, en su caso, líneas de trabajos futuros, y mencionando obligatoriamente las competencias –conocimientos y/o capacidades– del grado que el alumno ha aplicado al TFG y las nuevas competencias –conocimientos y/o capacidades– que el alumno ha adquirido con la realización del TFG)

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List of Figures

List of Tables