MECCANICA
17 (1982), 211-221

A COMPARISON BETWEEN THE CRAIG-COX AND THE KACKER-OKAPUU METHODS OF TURBINE PERFORMANCE PREDICTION

Giovanni Lozza*

SOMMARIO. In questo articolo vengono considerati due metodi per la valutazione completa delle perdite fluidodinamiche in turbine assiali: il primo, ben noto, è quello proposto da Craig e Cox, mentre il secondo rappresenta il recente sviluppo del metodo di Ainley e Mathieson, operato da Kacker e Okapuu a conferma della validità dell'approccio originale.

Un primo confronto è stato effettuato valutando le perdite in alcune schiere tipiche, con diversi numeri di Reynolds e di Mach. Viene mostrato che i due metodi sono in buon accordo per schiere subsoniche aventi elevati coefficienti di flusso, mentre si rilevano differenze significative in pale ad alta deflessione, specialmente per ciò che riguarda le perdite secondarie.

Nella seconda parte si è voluto indagare come la scelta di una o dell'altra correlazione di perdite possa influenzare il progetto di stadi di turbine. Un certo numero di stadi tipici è stato ottimizzato mediante un codice automatico: vengono discusse le differenze tra le due soluzioni dello stesso problema progettuale risultate dall'uso di ambedue le correlazioni. I risultati sono presentati in funzione di parametri quali il numero di giri caratteristico, il rapporto di espansione e il coefficiente di «taglia», nell'intento di generalizzare i risultati.

SUMMARY. Two complete comprehensive loss correlations for estimating the efficienty of axial-flow turbines are examinated in this paper: the well-known Craig-Cox method, and the recent development of the Ainley-Mathieson method, presented by Kacker and Okapuu, which reproposes the validity of this approach.

Firstly, a comparison is done by evaluating the losses in a number representative cascades, having various solidities, aspect ratios, Reynolds and Mach numbers. It is shown that the two methods are in good agreement for subsonic cascades having high flow coefficients, while significant differences are found in high deflection blades, especially for the secondary losses.

In the second part, it is investigated how the choice of correlation affects the project of a turbine stage. A design procedure, automatically carried out by a computer program, was applied to a number of cases: the differences between the two solutions of the same project problem obtained by using the two correlations are discusses. Parameters like specific speed, expansion ratio and size parameter are used to generalize the results.

* Dept. of Energetics Politecnico di Milano, Italy.

NOMENCLATURE.

b	axial chord, m
bb	backbone lenght, m
c	blade chord, m
CR	contraction ratio, defined by C-C
D	mean diameter, m
e	blade back radius, m
Fl	lift coefficient, defined by C-C
FL	flaring angle, deg
h	blade height, m
Kis	head coefficient, $2(\Delta h_{is})/u^2$
ks	equivalent sand grain size, m
KP, KS	correction factors defined by K-O
$M1$	blade inlet Mach number
$M2$	blade isoentropic outlet Mach number
Ns	specific speed, $RPS \sqrt{Vout}/\Delta h_{is}{}^{3/4}$
o	throat opening, m
o_{min}	blade critical throat, m
Re	Reynolds number
RPS	speed of revolution, rps
r^*	isoentropic degree of reaction
s	pitch, m
t	trailing-edge thickness, m
u	peripheral speed at mean radius, m/s
v	absolute velocity, m/s
VH	size parameter, $\sqrt{V_{out}}/\Delta h_{is}{}^{1/4}$, m
Vin	volumetric flow rate, at turbine inlet total conditions, m^3/s
$Vout$	volumetric flow rate, at turbine isoentropic outlet static conditions, m^3/s
VR	volume ratio, V_{out}/V_{in}
w	relative velocity, m/s
Y	total pressure drop
z	number of blades
α	relative gas angles, deg
γ	specific heats ratio
δ_r	tip clearance, m
$\Delta\beta$	rotor deflection, deg
$\Delta\eta$	efficiency decay
Δh_{is}	total-to-static isoentropic enthalpy drop relative to the whole turbine, J/kg

ϕ_E	utilization factor of leaving losses
ηts	total-to-static efficiency
ηtt	total-to-total efficienty
ψz	loading coefficient by Zweifel
ρ	density, kg/m^3
ξ	energy drop coefficient, referred to the ideal kinetic outlet energy
ξ'	energy drop coefficient, referred to the real outlet kinetic energy

SUBSCRIPTS

1	blade inlet
2	blade outlet
S	relative to the stator
R	relative to the rotor
T	total conditions

NOTE:

The angles are expressed in european convention, starting from zero degrees in tangential direction.

INTRODUCTION

The first step in the design of a new turbine is the selection of a group of variables, able to completely describe the geometry and the aerodynamics of the machine. The second step is the inspection of a large number of variables combinations, each of them representing a turbine whose efficiency has to be found. Obviously, since it is not possible to resort to full scale tests, in order to obtain exact results, the performance must be predicted by using loss correlations, which yield useful, even if not exact, indications about the losses breakdown, and about the way to follow in the optimization. A little number of interesting solutions can be identified by repeating this procedure for many cases and, eventually, the turbine most important parameters, like velocity triangles and main blading dimensions, can be fixed by this analysis. Deeper studies starting from this point cannot significantly change the turbine performance, so that this first approach is of great importance. The above described design procedure has been carried out by a computer program able to perform a mean line calculation of a turbine stage. The input data are nine project parameters, and the code is able to change them up to the obtainment of the best efficiency solution. The program evaluates the losses using the two different methods proposed by Craig and Cox [1] and by Kacker and Okapuu [2] (indicated in the paper as C-C and K-O respectively).

The discussion of the differences in the optimization resulting from the use of the two loss correlations is the purpose of this paper. For a better understanding of the different losses distribution, a more conventional comparison, based on the analysis of some typical cascades, preceeds the description of the computer program and of the results concerning the whole turbine stage.

A SHORT REVIEW OF THE TWO METHODS

The choice of the two adopted methods, among the ones available in the literature, and reviewed in [3], is due to the following reasons. The Craig-Cox system is supported by a correct theoretical formulation and was derived from experience on both steam and gas turbines, in a wide range of aspect-rations; besides, it was satisfactorily used in previous papers [4, 5]. The Kacker-Okapuu system is the most recent development of the Ainley-Mathieson [6] method, after the contribution of Dunham-Came [7]: it represents a confirmation of the validity and longevity of this approach.

There are substantial differences between the two methods: in the first place, C-C calculate the losses by means of energy coefficients (ξ) (1), instead of total pressure drop coefficients (Y), as done by K-O. For a direct comparison, the two coefficients will be converted into a kinetic loss coefficient referred to the isentropic head (ξ), a parameter which is more directly correlated to the efficienty drops. (1)

For equal Y, the conversion formula yields lower ξ values for high pressure ratios than for nearly incompressible cases. Hence, a significant influence of subsonic Mach numbers is expected for the K-O predictions, which calculate the losses in terms of ξ, but not for the C-C ones.

The main variables which are used in the loss prediction are presented in table 1. The substantial differences in the determination of the *profile losses* basic term can be noticed: C-C use the lift coefficient and the contraction ratio, rather than flow angles. Besides, reference is made not to the chord but to the blade backbone-lenght, in a more correct – but less easy – way: an approximate relation has been used for its calculation (2). On the contrary, K-O use the same data and the same interpolation equation proposed by [6], decreasing the final value by a factor of 2/3, justified by the advances in aerodynamics of profile design, since 1951. Both methods consider an additional term due to the after-expansion; besides, K-O take into account leading-edge shocks, by adding another term, and the thinning of the boundary-layer, by multiplying the basic losses by a reduction coefficient named *KP*. In both systems, the *secondary losses* are depending on the aspect ratio in a non-linear way for its lowest values, as certainly correct. The basic term is a function of the flow angles for K-O, rather than of the relative velocity ratio, as more correctly suggested by C-C for highly compressible flows. Furthermore, it can be noticed that the *annulus losses* are not considered by K-O, while the *leakage losses* are evalutated by the two methods as a function of quite dissimilar geometric and

(1) See Appendix.

(2) For the K-O method, the blade chord is calculated from the axial chord using the approximate method suggested by [2]. The same procedure has been used to convert the s/b ratio given by the Zweifel [8] loading criterion in the s/c ratio plotted in fig. 1.

For the C-C method, the backbone-lenght is calculated by assuming that the blade mean-line is constituted by a circular arc from the inlet to the throat, and then by a straight line up to the outlet; for given relative angles and axial chord the lenght is found by using geometric relations.

Table 1. Review of the parameters involved in the two losses correlations.

Source of losses	Parameters involved by C-C	Parameters involved by K-O
Profile losses basic term	$Fl, CR, s/bb$	$\alpha_1, \alpha_2, s/c$
Trailing edge thickness	$t/s, \alpha_2$	$t/o, \alpha_1, \alpha_2$
Blade back radius	$s/e, M_2$	n.c.
After-expansion	M_{2is}	M_{2is}
Leading edge shocks	n.c.	M_1
Thinning of the boundary-layer	n.c.	M_1, M_2
Reynolds number correction	$Re = \rho W_2 o/_\mu$	$Re = \rho W_2 C/_\mu$
Blade surface roughness	ks/bb	n.c.
Secondary losses basic term	$Fl, s/bb, w_1/w_2$	α_1, α_2
Aspect ratio	h/bb	h/c
Reynolds number correction	$Re, ks/bb$	n.c.
Annulus losses	annulus geometry	n.c.

kinematic parameters.

Also the correction terms due to Reynolds number are substantially different: C-C calculate Re using the throat opening, instead of the blade chord, as used, as well as by K-O, by Ainley [6], Deich [10] and Traupel [11]. The reference done by C-C to the actual flow channel dimensions and roughness seems more appropriate. Besides, the corrective term is supplied by K-O only to the profile losses, and to both profile and secondary losses by C-C: there are not plain indications in the literature confirming one or the other hypothesis. In the author's opinion, the C-C hypothesis is more justified, because the secondary are connected to the boundary-layer evolution along the endwalls, with viscous effects.

INVESTIGATION ON SINGLE CASCADES.

Optimal solidity values resulting from the two correlations were searched for various combinations of inlet and outlet angles, by varying the s/c ratio up to the attainment of minimum profile losses. The results are shown in fig. 1, where also the Zweifel [8] loading criterion has been used for comparison, with $\psi_z = .9$, as suggested by Baljé, [9]. It can be noticed that correlations [1] and [8] give similar results and suggest completely a different trend from the one proposed by the K-O method, that recommends a reduction of s/c for increasing the deflection, for every kind of blading. Then the two correlations yield different optimum solidities especially for impulse bladings: a rather surprising discrepancy, which demonstrates that uncertainties exist in turbine design even on a simple matter like the solidity choice. For coherence, the profile losses were computed in the following analysis by using the respective optimal solidities, quite different between them.

Three typical cascades, with aspect-ratios varying from 0.5 to 3 have been analyzed operating at three isoentropic outlet Mach numbers: $M2 = .4$, for an almost incompressible flow; $M2 = 1$, for highly compressible flow and $M2 = 1.3$, where important supersonic effects (shocks and after-expansions)

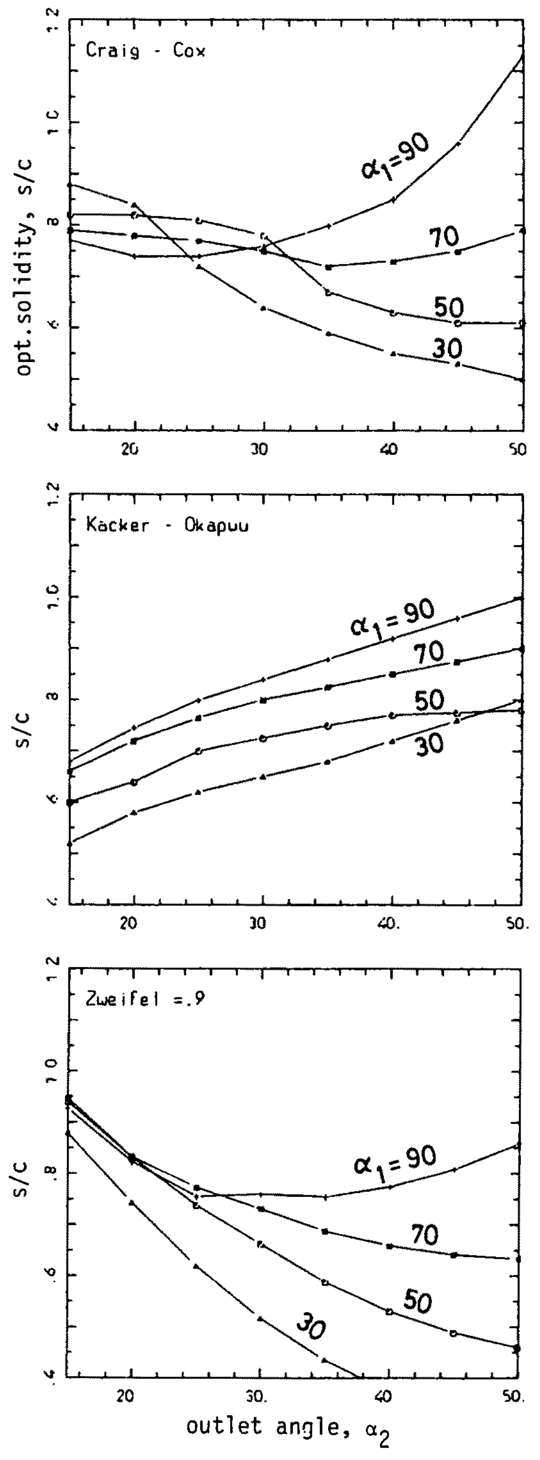

Fig. 1. Optimal solidities vs. outlet gas angles, for different inlet angles, by C-C, K-O and Zweifel methods.

take place. The calculations have been done for a perfect gas having $\gamma = 1.1$. (3)

The influence of fluid molecular complexity is not negligeable at the higher Mach number, but is does not change

(3) See later in the paper the reason for the choice of this value.

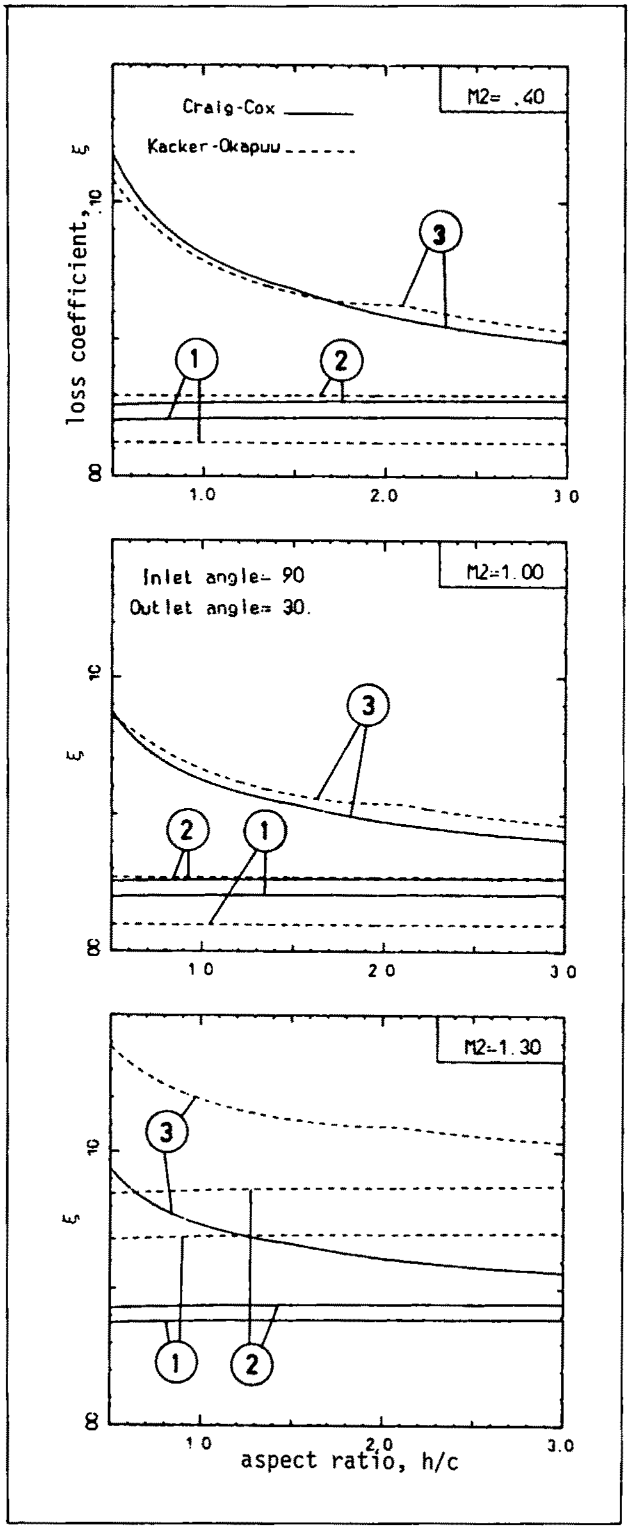

Fig. 2. Breakdown of losses vs. aspect rations for bladings having $\alpha 1 = 90°$, $\alpha 2 = 30°$, for three different outlet Mach number:
1) profile losses at optimal solidity
2) profile and trailing-edge (for $t/o = .10$) losses
3) profile, trailing-edge and secondary losses.

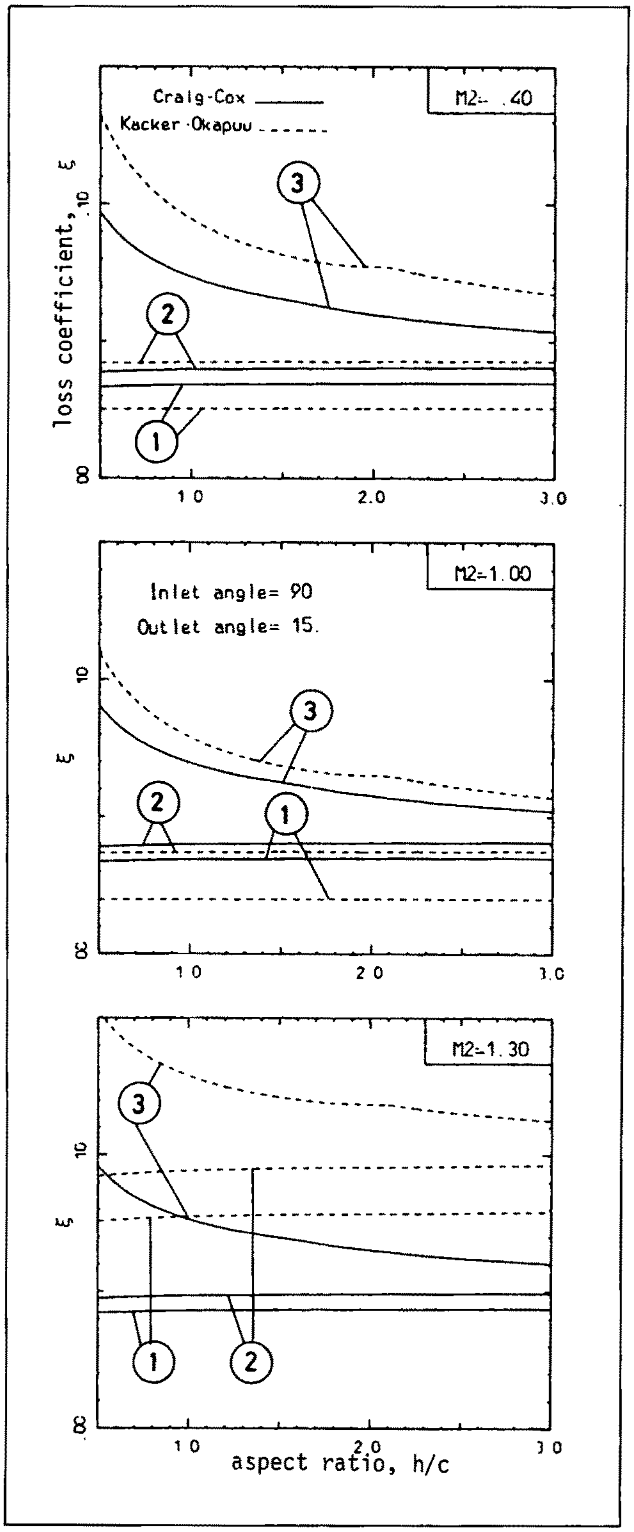

Fig. 3. Breakdown of losses vs. aspect-ratio, for bladings having $\alpha_1 = 90°$, $\alpha_2 = 15°$.

the qualitative trend of the results. No Reynolds number effects were accounted for in this first approach, by imposing correction coefficient equal to one. Let us discuss the results obtained for the three blades separately:

1) nozzle or reaction runner for turbines having high flow coefficients ($\alpha_1 = 90$, $\alpha_2 = 30$): referring to fig. 2, the two methods show a good agreement on overall losses, except for the case at $M2 = 1.3$ where a higher penalty for after-expansion is introduced by K-O. The trailing-edge effect (calculated for $t/o = 0.1$) is larger by K-O, with lower profile

losses, especially for $M2 = 1$, where the KP coefficient becomes important. The change is concavity of the secondary losses curves for $h/c > 2$ is due to the different aspect-ratio correction.

2) Nozzle or reaction runner for turbines having low flow coefficient ($\alpha_1 = 90$, $\alpha_2 = 15$): the fig. 3 shows that lower secondary losses are predicted by C-C than by K-O: in fact, in the C-C correlation, there is a strong reduction of the losses caused by the flow acceleration, either due to compressibility or to channel convergency. On the contrary, similar effects are not found in the K-O correlation, based only on geometric quantities. The profile losses show the same trend of the proceeding case: it has to be pointed out that there is no Mach number effect for subsonic flow in the C-C correlation, while the KP coefficient plays an important part in the K-O method, by reducing the losses. Besides, the losses are expressed in terms of Y, so that better performance of bladings are expected for increasing expansion ratios, up to the critical value.

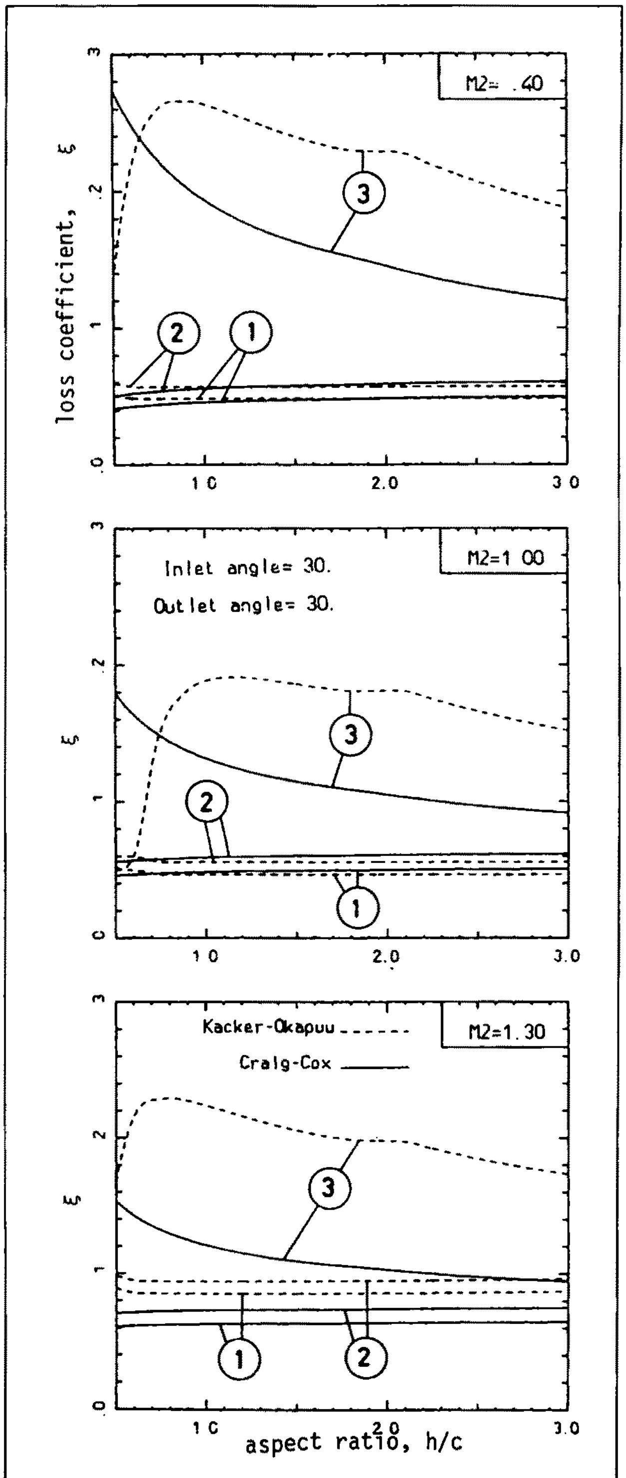

Fig. 4. Breakdown of losses vs. aspect-ratio, for bladings having $\alpha_1 = 30°$, $\alpha_2 = 30°$.

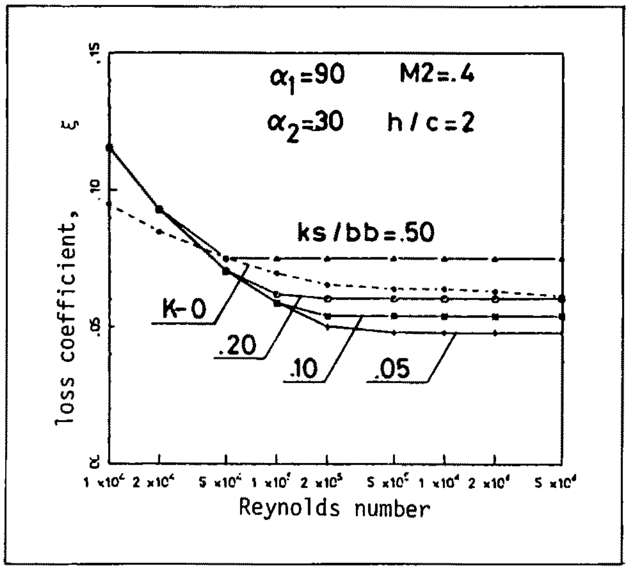

Fig. 5. Losses for one blading at different Reynolds number: four values of actual rugosity are used in the C-C lines.

3) Impulse runner with high deflection: ($\alpha_1 = 30$, $\alpha_2 = 30$) the fig. 4 shows a good agreement on profile losses between the two correlations: the influence of the after-expansion is not as strong as in the proceeding cases, and the repartition between profile and trailing-edge losses is almost equal. On the contrary, the secondary losses present significant anomalies. The values predicted by K-O are in fact two or three times larger than the ones proposed by C-C, for aspect-ratios greater than unity. For lower h/c, the K-O curves show a very strange trend, caused by the KS coefficient, which strongly reduces the secondary losses, because of its dependance from the square of the inverse aspect-ratios; the effect is present at high $M1/M2$ ratios only, so that is not noticed in the preceedings cases. This anomalous situation, clearly not occurring in reality, points out one limitation of the K-O method [2] unapplicable for impulse bladings with low aspect-ratio, as the ones often used in the first stages of steam turbines. Even if this situation was not present neither in [6] nor [7], the Ainley-Mathieson derived correlations are more pertinent to gas turbines, where such a kind of blading is not of interest.

A comparison about the Reynolds number influence in the two correlations is quite difficult because of all the quoted differences. However let's consider the example of fig. 5 where the losses of the first blading are plotted, as a function of Re, calculated using the throat lenght, for h/c

equal to 2, and with o/c fixed by the solidity. Four curves are represented for the C-C method, having various relative roughness (Ks/bb) the two lower values called «standard-finish» in [1]. For low Re, the K-O correlation gives significantly lower loss values: it is mainly due to the fact that the secondary losses are not multiplied by the corrective coefficient.

A comparison about the leakage losses has not been carried out, because they depend from different geometric ratios. Let us summarize the main differences which were found:

– The two predictions are in good agreement only in the case of cascades used in turbines having high flow coefficients, like aircraft gas turbines. As expected [3], the secondary losses are the major sources of unaccuracy, because they depend from a very complex problem of three-dimensional boundary-layers, whose behaviour is not well understood; the secondary losses are generally higher by K-O, especially in impulse bladings.

– The trailing-edge losses are a larger fraction of the profile losses, in the K-O correlation.

– The K-O method gives erroneous results for low aspect-ratio impulse bladings.

– The annulus losses are considered by C-C only.

A further limitation of the K-O method should be pointed out, although not seen in the considered bladings: the profile losses become negative for very low deflection bladings, with inlet angles greater than 90° ($\alpha_1 = 140°$, $\alpha_2 = 20°$, for example), as the ones used at the tip of the last stages of L.P. steam turbines. This demonstrates that the attempts done by K-O to extend the validity of the Ainley-Matheison correlation to these bladings is not successful.

DESCRIPTION OF THE COMPUTER PROGRAM.

The computer program optimizing the axial-flow turbine design has the same general formulation of the ones used in [4] and [5], but is can calculate the losses with the method [2], as well as [1]. Let us consider a short description of the program logics. The mean line calculation of a turbine stage requires three groups of input data, given in table 2: the optimizing variables (point 1), constant quantities that are imposed to the designer (point 2) and other quantities that describe the thermodynamics of the problem (point 3). The calculation procedure needs, in addition to the thermo-fluidynamic equations, a number of arbitrary correlations; clearly the most important is the loss prediction method, but the following ones have to be mentioned: the Ainley [6] or Vavra [12] methods for evaluating the gas outlet angles (respectively for the subsonic and supersonic flows); the Deich [10] method for the calculation of the throat lenghts in converging-diverging ducts. A more detailed description of these procedures can be found in [13].

Furthermore, the optimization is limited by a number of constraints (tab. 2, point 4), which keep the solution in the range of validity of the assumptions, and within reasonable technological limits for industrial production. Among the many constraints, let us pay a particular attention

Table 2. Computer program variables and constraints.

1. *Optimizing variables*

$r^*, k_{is}, (o_{\min})_s, (o/s)_s, b_s, o_R, (o/s)_R, b_R, r_R/r_S.$

2. *Fixed input variables*

axial entry nozzle
$\mathrm{Re} = 5 \cdot 10^5$
$k_s = 2 \cdot 10^{-3}$ mm
δ_r = max (0.2 mm or R/1000)
t_s = max (0.2 mm or $o_1/20$)
t_R = max (0.2 mm or $o_2/20$)

3. *Variable input data*

Fluid thermodynamic properties (molecular mass, specific heat ratio)
Inlet conditions (total pressure and temperature)
Outlet conditions (static pressure)
Mass flow rate
Speed of revolution

4. *Constraints*

$0 < M_{WS} < 0.8$
$0 < M_{WR} < 1.4$
$-20° < FL < +20°$ (for cases at $VR \geqslant 2$)
$-3° < FL < +3°$ (for cases at $VR = 1.05$)
$0.001 < (h/D)_s,\ (h/D)_R < 0.25$
$0 < (b/D)_s,\ (b/D)_R < 0.25$
$\sin 13° < (o/s)_s, (o/s)_R < \sin 60°$
$2\,o_S < b_S < 100$ mm
$2\,o_R < b_R < 100$ mm
1.5 mm $< o_{\min S} < 50$ mm
$1.5 < kis < 10$
$-.01 < r^* < 0.8$
$1 < r_R/r_S < 1.05$
$10 < z_S < 100$
$10 < z_R < 100$

to the Mach number limits: in the runner, a supersonic inlet is not allowed, and the outlet Mach number has not to exceed 1.4, a value compatible with blades having converging ducts only: the after-expansion losses are evaluated in accord to [1] and [2]. On the contrary, converging-diverging nozzles are allowed in the stator: in this case, no additional losses are computed, as the experience indicates for design-point conditions.

The calculation is repeated for various combinations of the independent variables: their variation logic is established by a numeric optimization procedure, up to the attainment of the maximum efficiency solution, within all the constraints. For single-stage turbines, the required number of iterations is generally about 5000. The optimized efficiency stipulates a recovery of fifty percent of the axial exhaust energy at the rotor exit, by using a coefficient ϕ_E (as proposed by Vavra in [12]) equal to 0.5. In the experience done in [4, 5], this assumption allows more reasonable flow coefficients than cases with $\phi_E = 0$.

PRESENTATION OF GENERAL RESULTS.

As shown in [4] and [5], an optimized turbine may be classified using three parameters: (1) the volume ratio (VR), which takes into account the compressibility, (2) the specific speed (Ns) and (3) the «size» parameter (VH), which takes

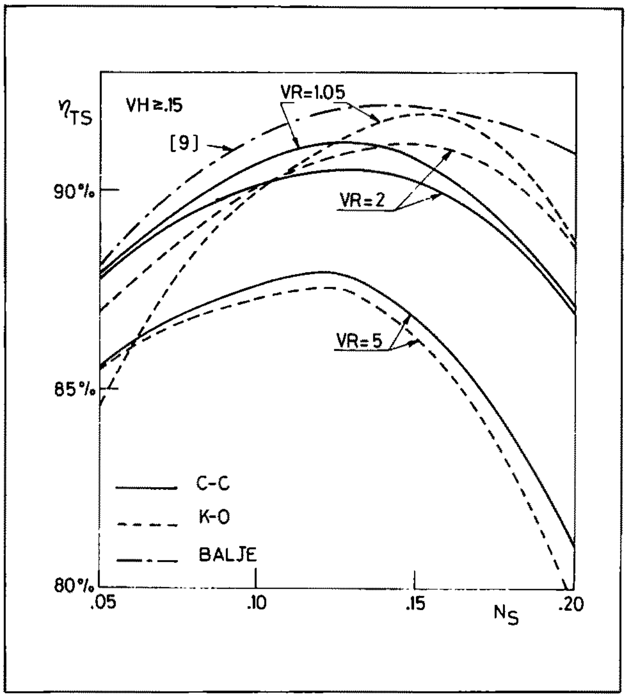

Fig. 6. Efficiency prediction for turbines having VR = 1.05, at different Ns and VH.

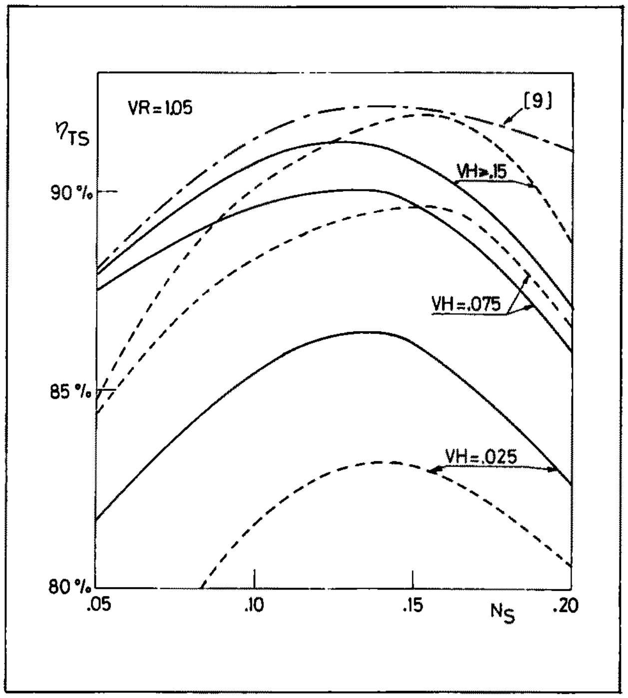

Fig. 7. Efficiency prediction for turbines having $VH \geqslant .15$, at different Ns and VR.

into account the non-respect of the similarity rules, due to the introduction of dimensional quantities (blade thickness, radial clearance, etc.). The Reynolds number influence was not considered, by limiting the analysis to high turbolent flow. The working fluid influence is limited by referring to VR instead of the pressure ratio, as shown by [4]. In this analysis, a fluid having $\gamma = 1.1$ has been supposed, because the proposed results are more useful for turbines operating with non-conventional fluids, having low values of γ, rather than air or steam, where a large experience exist to help the designer. The overall performance prediction of turbines optimized using the two methods can be seen in figg. 6 and 7, in the η_{TS}-Ns plane. The fig. 6 refers to VR = 1.05 (incompressible flow) and to different VH parameters. Let us consider the K-O curve for $VH > .15$ (for the highest values of VH, the similitude is exactly respected): the maximum efficiency is obtained for higher Ns and reaches a higher value. The runner profile losses by K-O become almost equal to zero, as it can be seen in the losses breakdown of table 3 (col. 1), due to the mentioned anomaly occurring for very low deflection blades (see the velocity trangles in fig. 8a). The leakage losses are less by C-C, but the annulus losses are also accounted for in this correlation: it is quite curious to notice that a kind of compensation is obtained in many cases. This consideration demonstrates that the addition of single portions of losses coming from different correlations is not warranted: generally, the accuracy of each correlation has been verified by its author by testing the overall performance, while the single contributions can be differently computed or included in a different group of

Table 3. Breakdown of losses for six significant turbines, optimized by the two methods.

	1		2		3		4		5		6	
VR - VH - Ns	1.05 - .15 - .15		1.05 - .15 - .05		1.05 - .025 - .125		5 - .15 - .125		1.05 - 15 - .1		1.05 - .15 - .15	
Loss system	C-C	K-O	C-C	K-O	C-C	K-O	C-C	K-O	C-C	K-O	C-C	K-O
guide profile	.0142	.0097	.0214	.0154	.0221	.0146	.0213	.0102	.0317	.0207	.0185	.0131
guide tr-edge	–	.0048	–	.0043	–	.0097	–	.0086	–	.0109	–	.0036
guide second.	.0038	.0057	.0136	.0229	.0121	.0202	.0080	.0096	.0085	.0133	.0112	.0079
guide annulus	.0105	–	.0141	–	.0114	–	.0133	–	.0202	–	.0103	–
runner profile	.0196	.0026	.0152	.0178	.0230	.0144	.0211	.0109	.0164	.0231	.0218	.0001
runner tr-edge	–	.0094	–	.0045	–	.0095	–	.0030	–	.0061	–	.0103
runner second.	.0047	.0098	.0122	.0244	.0118	.0209	.0123	.0243	.0082	.0200	.0167	.0103
runner tip leak.	.0075	.0117	.0202	.0414	.0334	.0584	.0055	.0093	.0033	.0061	.0073	.0119
exhaust energy	.0164	.0158	.0130	.0137	.0118	.0128	.0205	.0227	.0223	.0211	.0174	.0155
$\eta(\phi_E = .5)$	.9233	.9315	.8903	.8561	.8744	.8395	.8980	.8933	.8894	.8787	.8968	.9273

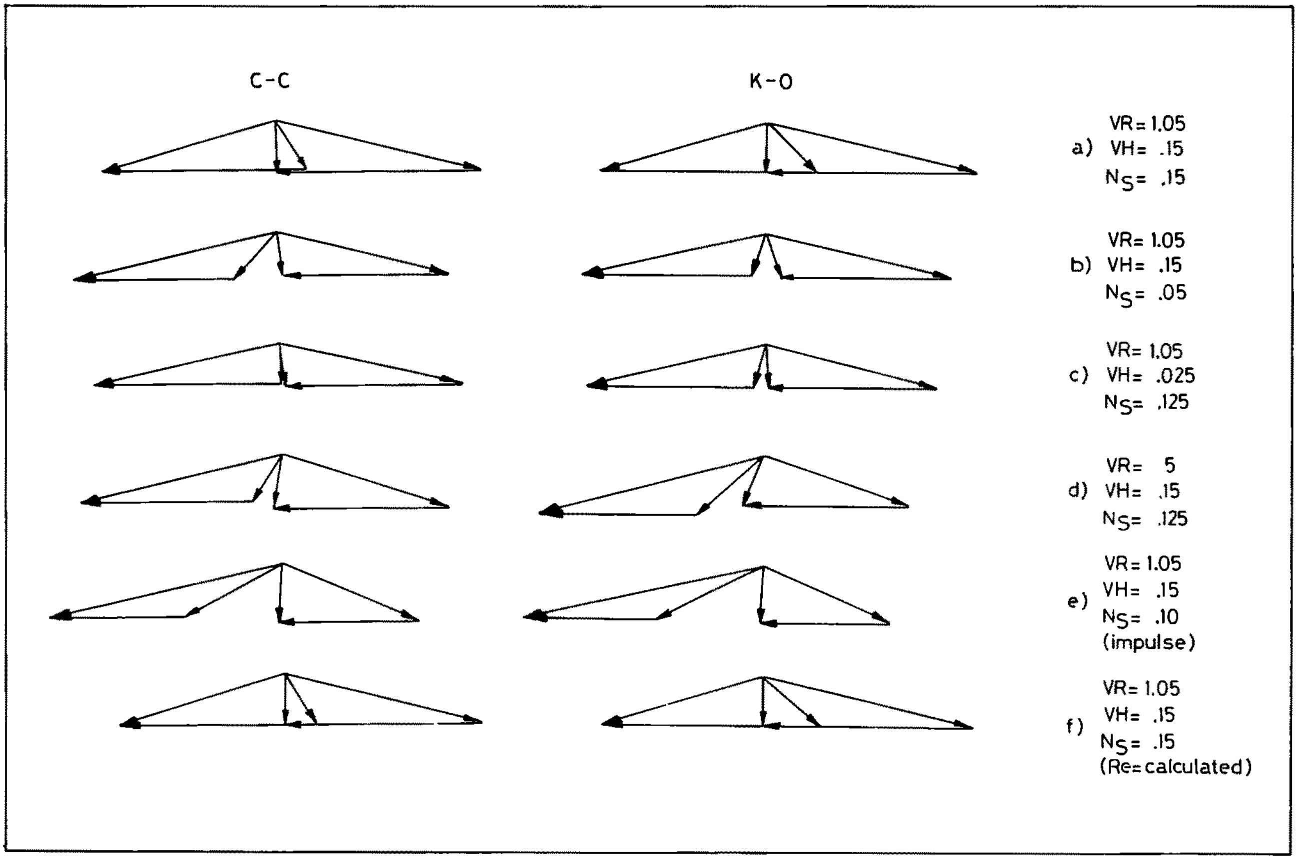

Fig. 8. Optimized velocity triangles for six significant cases: by the two methods.

losses.

The larger secondary and leakage losses predicted by K-O justify the remarkably lower efficiency of the turbines at low *Ns* (table 3 col. 2). The more rapid decrease of the K-O efficiency for lower *VH* can be noticed again from fig. 6: it is due to the greater influence of the trailing-edge and leakage losses predicted by this method (see table 3 col. 3). These curves also confirm [4] the large influence of *VH* in the overall efficiency prediction: the unaccuracy of a method which does not take into account *VH* would be greater that the one introduced by the use of different loss correlations. The trend already found in [4] of lower optimal *Ns* for lower *VH*, is well confirmed. The fig. 7 shows the efficiency variation due to the compressibility effects at constant *VH*. For $VR = 2$, there are only little efficiency decreases for optimal *Ns*. At low *Ns*, the K-O method predicts better performance than for the incompressible cases, owing to the use of Y as loss parameter, which yields lower ξ for higher pressure ratios below the critical value: this effect is more evident for low *Ns*, because is strongly reduces the secondary losses. This trend is not confirmed by the C-C correlation. The curves at high expansion ratio ($VR = 5$) are similar, but the adopted geometries are different: the K-O turbines have lower reaction degrees in order to avoid excessive after-expansion penalizations in the runner; this yields smaller mean diameter as it can be seen by the velocity traingles (fig. 8-d). Of course, the losses breakdown (tab. 3 col. 4) is in agreement with these choices: moreover, it can be noticed that the exhaust energy losses are highly reduced by the flaring angles, which cause large axial chords in the rotor blades, with a little increase of secondary losses.

An overall comparison among the various solutions given by the two methods on optimized turbines, can be seen in fig. 9, where seven important variables are represented. The figures relative to η_{TS} and η_{TT} pratically confirm the previous discussion: there is a quite good agreement between the two methods, but more optimistic values are predicted by C-C for the cases at lower *VH* and for the impulse turbines (see later in the paper). Also the head coefficient and the reaction degree do not show great differences; however, it can be noticed that the cases at $VR = 5$ generally have higher *Kis* and lower r^* by K-O. Both methods give optimized turbines at high reaction degrees ($0.5 \div 0.6$), as seen also in fig. 10; besides, the strong dependance between *Kis* and r^* is well documented in this figure, which gives results similar to the one presented by [4]. On the contrary, as seen in Fig. 9, the optimized runner aspect-ratios are strongly different: the greater importance given by K-O to the secondary losses yields shorter axial chords. The results about the runner turning angle are depending upon the particular case, and it is impossible to give a general trend, except for the cases at $VR = 5$ which show higher $\Delta\beta$ connected to lower r^*, as often noticed. Eventually, very different optimum values for the Zweifel coefficient ψz are found by the two correlations: a better understanding of the phenomena is possible looking at fig. 2, which shows how different are the optimal solidities for the two methods.

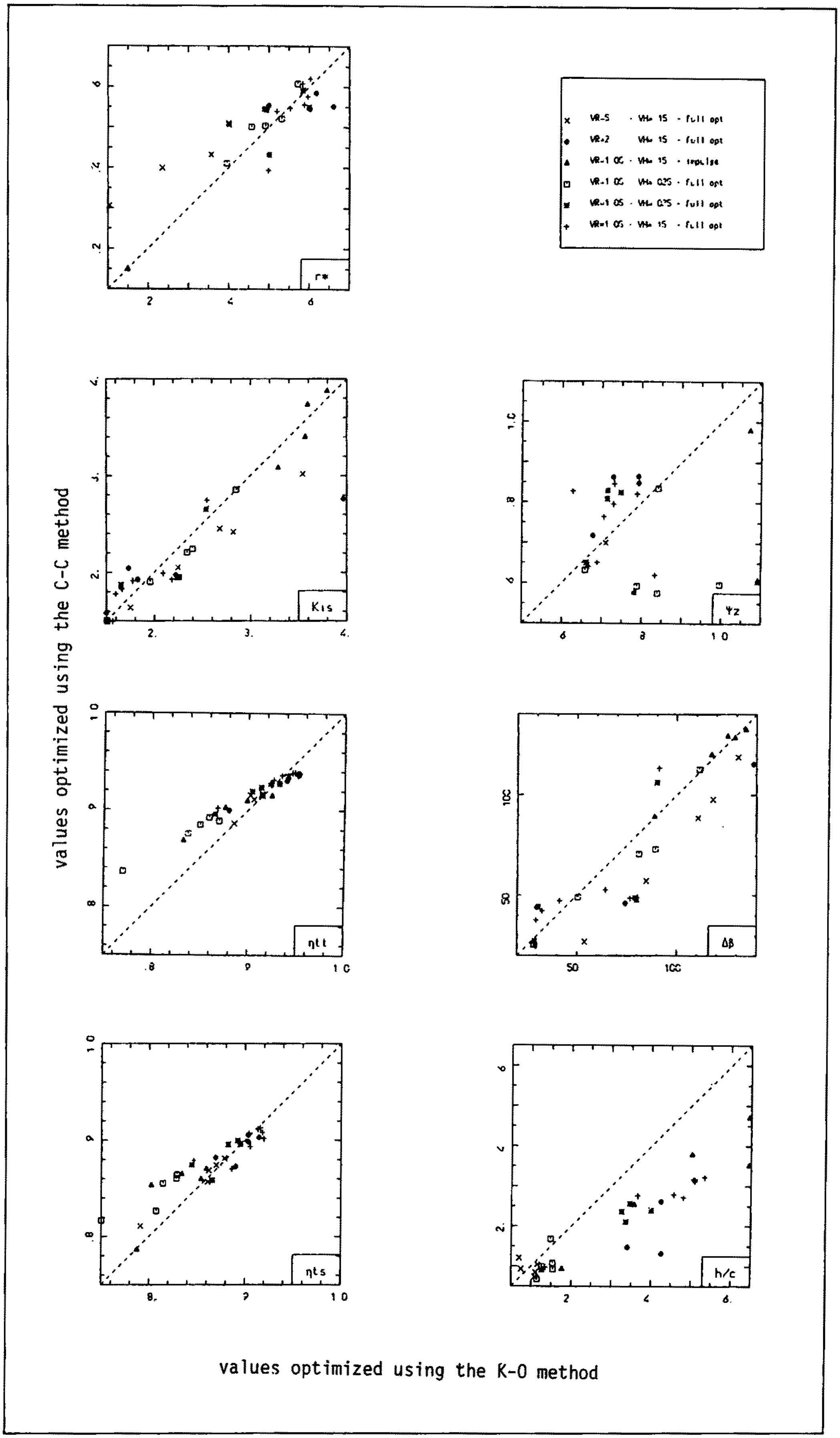

Fig. 9. Comparison between the solutions given by the two methods, for the same project problem. Each plot shows one significant optimized variable, for about 40 turbines.

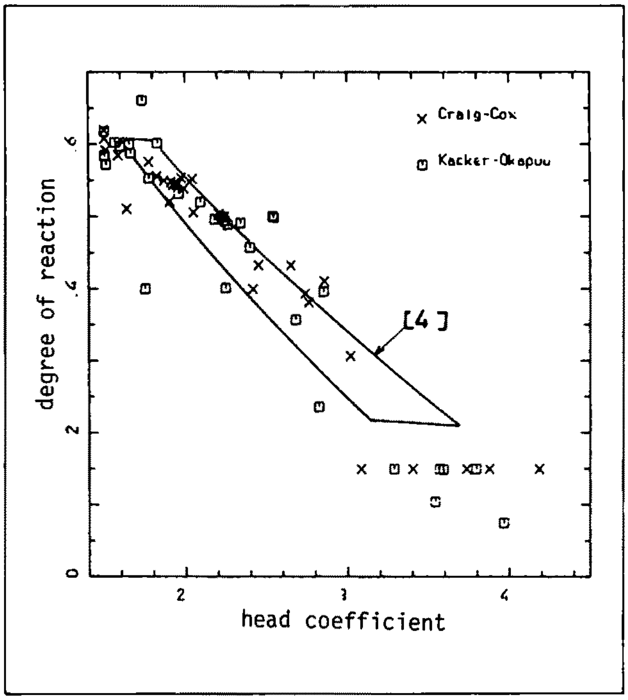

Fig. 10. Relation between *Kis* and r^* for optimized turbines.

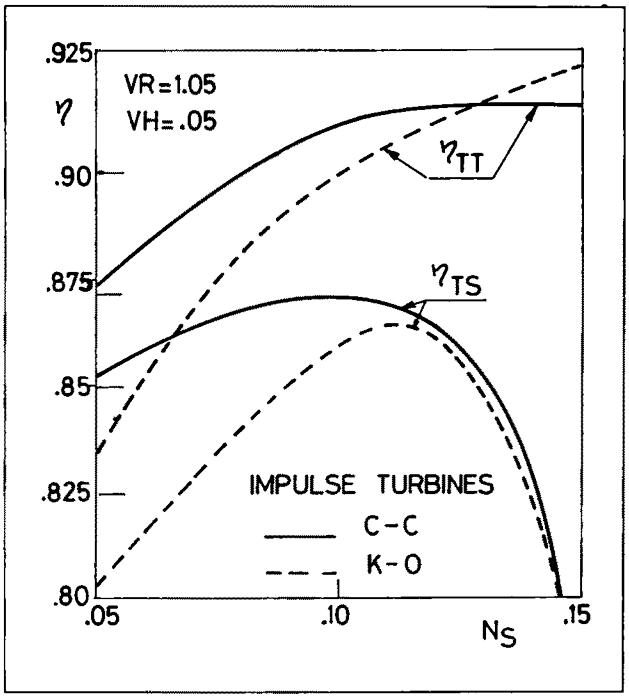

Fig. 11. Efficiency prediction for impulse turbines, at various *Ns*.

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Impulse turbines: since the optimized turbines present high reaction degrees, it would be of interest to investigate nearly impulse turbines, obtained imposing to r^* the maximum value of 0.15. The fig. 11 shows the obtained efficiency as a function of *Ns*: as expected, the *Ns* corresponding to maximum η_{ts} is lower than for reaction turbines; at high *Ns*, the losses due to exhaust energy become ingent; at low *Ns* the efficiency by K-O is considerably lower, because of larger secondary and leakage losses. Let us consider the cases at $Ns = .10$: while the velocity triangles for the two correlations are similar (fig. 8-e), the losses breakdown (table 3 col. 5) presents the expected differences: the runner secondary losses by K-O are about 2.5 times larger than the ones by C-C, while the guide annulus losses, considered only by C-C, perform an important part (2 points).

Reynolds number influence: all the cases so far considered have been computed for a Reynolds number equal to 5.10^5 (Re bases on the throat lenght). Let us remove this hypotesis for the incompressible case at optimal *Ns*: the influence of low Re values is introduced by getting viscosity value such that $Re = 3.10^4$ for the common initial case; Re will then experience some variations during the optimization, according to changes of geometric variables (*o* for C-C, *c* for K-O) and relative outlet velocities. The solution given by the C-C method presents larger backbone and throat length, with respect to one at Re = cost. In this way, the correction coefficient is strongly reduced, because of higher Re and lower *Ks/bb* ratios, but with an increase of secondary losses, as seen in table 3, col. 6. On the contrary, the velocity triangles are practically unchanged with respect to ones at Re = cost. The solution that uses the K-O method has a similar trend for the nozzle, while the computer optimization is able to modify the velocity triangles up to reach zero profile losses (caused by the quoted error in interpolation equation), so that in the runner there are not effects at all. For all these reasons, the Reynolds number effects are more important by C-C, as seen in Table 3.

CONCLUSIONS.

It was not the purpose of this paper to establish what is the better method: such a comparison can be done only having at one's disposal a very large number of tests results over real turbines operating in a wide range of conditions.

However, some points can be made according to the previous discussion:

– the two considered methods give similar results for optimized turbines operating with relatively low Mach numbers, high aspect-ratios, flow coefficients and Reynold numbers, as more commonly used in gas turbines. The differences become important in particular cases (low *Ns* or *VH* or Re), more frequently found in steam turbines or in turbines operating with non-conventional fluids;

– the choice of a loss correlation is a serious matter, because it yields different solutions also in apparently simple project problems. It is dangerous to mix up the single portions of losses coming from different methods, because every system has a proper logic and a proper range of validity;

– the great differences shown in some cases are a little discouraging: they are due to the poor knowledge of the turbo-machines flow, that is one of the most complex problems that fluiddynamic has dealed. A lot of work has to be done in this field;

– the K-O correlation cannot be applied for turbine stages with characteristics different from conventional gas turbines (in particular, for low aspect ractio, impulse stages and for blades with large inlet flow angles).

APPENDIX

– Losses coefficients used in the paper:

$$\xi = \frac{h_2 - h_{2is}}{W_{2is}^2/2} \ ; \ \xi' = \frac{h_2 - h_{2is}}{W_2^2/2} \ ; \ Y = \frac{p_{T1} - p_{T2}}{p_{T2} - p_2} .$$

– Conversion formulas used between the coefficients:

$$\xi = \left\{ \left(\frac{1 + Y}{1 + Y(p_2/p_{T1})} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} / \left\{ \frac{1}{(p_2/p_{T1})^{\frac{\gamma-1}{\gamma}}} - 1 \right\} ;$$

$\xi = \xi'/(1 + \xi')$.

– Efficiency drop as a function of ξ in a turbine stage:

$$\Delta\eta_x = \left(\frac{\xi_x}{1 - \xi_c} \right) * \left(\frac{W_2^2/2}{\Delta h_{is}} \right) * \left(\frac{p_{2R}}{p_{2C}} \right)^{\frac{\gamma-1}{\gamma}}$$

where the subscript x is referred to the single voice of loss, and the subscript c to the considered cascade.

Received: March 2, 1982

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