: 변수분리

Ex 1. $(1+x)dy - ydx = 0$ 을 풀어라.

$(1+x)dy = ydx$

$\int \frac{1}{y}dy = \int \frac{1}{1+x}dx$ 양변 적분

$\ln y = \ln(1+x) + C_1$

$|y| = e^{\ln(1+x)+C_1}$

$y = \pm e^{C_1} \cdot e^{\ln(1+x)}$

$y = \pm C \cdot e^{\ln(1+x)}$ $C = \pm e^{C_1}$

$\therefore y = \pm C(1+x)$ $C \neq -1$

sep

Ex 2. 초깃값 문제 $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = -3$ 을 풀어라.

$y \cdot dy = -x \cdot dx$

$\int y \cdot dy = \int -x \cdot dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$

$y^2 = -x^2 + 2C_1$

$2C_1 = C$

$y^2 = -x^2 + C$

$x^2 + y^2 = C$

$25 = C$

$x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

$y = \pm\sqrt{25 - x^2}$

$\Rightarrow y = -\sqrt{25 - x^2}$

$$\frac{1}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\frac{1}{ab} = \frac{1}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Ex 3. (해 잃어버림)

$\frac{dy}{dx} = y^2 - 4$ 를 풀어라. 자율, sep

$$\frac{dy}{y^2-4} = dx \longrightarrow \frac{1}{y-2}\cdot\frac{1}{y+2} = \frac{1}{4}\left(\frac{1}{y-2} - \frac{1}{y+2}\right)$$

$$\int \frac{1}{y^2-4}\,dy = \int 1\,dx$$

$$\frac{1}{4}\int \frac{1}{y-2} - \frac{1}{y+2}\,dy = \int dx$$

$$\frac{1}{4}\left(\ln|y-2| - \ln|y+2|\right) = x + C_1$$

$$\ln|y-2| - \ln|y+2| = 4x + 4C_1$$

$$\ln\left|\frac{y-2}{y+2}\right| = 4x + 4C_1$$

$$\frac{y-2}{y+2} = \pm e^{4x+4C_1}$$

$$\frac{y-2}{y+2} = \pm e^{4x}\cdot e^{4C_1}$$

$$\because \pm e^{4C_1}$$

$$\frac{y-2}{y+2} = e^{4x}\cdot C$$

$$y-2 = Ce^{4x}(y+2)$$

$$y - Ce^{4x}\cdot y = 2Ce^{4x} + 2$$

$$\therefore y = \frac{2(Ce^{4x}+1)}{1-Ce^{4x}}$$

sep.

Ex 4. 초깃값 문제 $(e^{2y}-y)\cos x\dfrac{dy}{dx}=e^{y}\sin 2x,\ y(0)=0$ 을 풀어라.

$\int u\cdot v' = u\cdot v - \int u'v$

$$\frac{e^{2y}-y}{e^{y}}\,dy=\frac{\sin 2x}{\cos x}\,dx$$

$e^{y}\leftarrow e^{y}$

$$e^{y}-y\cdot e^{y}\,dy=\frac{2\sin x\cdot\cancel{\cos x}}{\cancel{\cos x}}\,dx$$

$$\int y\cdot e^{-y}dy = -ye^{-y}+\int e^{-y}dy = -ye^{-y}-e^{-y}$$

$$\int e^{y}dy-\int y\cdot e^{-y}dy=\int 2\sin x\,dx$$

$$e^{y}-(-ye^{-y}-e^{-y})=-2\cos x+C$$

$$e^{y}+ye^{-y}+e^{-y}=-2\cos x+C \quad \leftarrow y(0)=0$$

$$1+0+1=-2+C$$

$$C=4$$

$$\therefore\ e^{y}+ye^{-y}+e^{-y}=-2\cos x+4$$

sep.

Ex 5. 초깃값 문제 $\frac{dy}{dx} = e^{-x^2}$, $y(3) = 5$ 를 풀어라.

$$\int dy = \int e^{-x^2} \cdot dx$$

$$y = \int e^{-x^2} dx + C$$

$$F(x) = \int_3^x e^{-t^2} dt \quad F(3) = 0$$

$$\therefore y = \int_3^x e^{-t^2} dt + 5$$