

Astrometry of the Neptune-Triton System

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Introduction

In this report I present the preliminary results of the astrometric reductions of the images from the Observatório do Pico dos Dias (OPD) in Brazil. The aim is to obtain precise positions for the Neptune - Triton system and to investigate the orbit of Neptune alone around the Sun. The telescopes used were the Perkin-Elmer (160) with a diameter of 1.6m, the Boller & Chivens (IAG) with a diameter of 0.6m, and the Zeiss telescope with a diameter of 0.6m.

The observations were carried out since 1992 when a CCD big enough was installed in the OPD. The planet and satellite have been constantly observed, and still are, by our group. There were many CCDs (IKON, IXON, CCD101, CCD106, ...) and many filters (V, R, I, No Filter, ...) utilized.

There was more than 5000 images from June 1992 to September 2015. Many of the oldest images had no coordinates in header or they were wrong. Sometimes the filter was missing. Many nights had two exposure sets. The first one with low exposure times so Neptune was not saturated, but there were few reference stars in the field. The second one with higher exposure time so Triton was brighter and had more reference stars than with the the previous exposure, but the image of Neptune were saturated.

In Table 1 it is summarized the final number of images for Neptune (short-exposure observations) and Triton (all observations) for the 3 telescopes. It is also shown the number of positions where Neptune and Triton were identified automatically in the same image (short-exposure observations for precision premium).

Table 1: Number of positions by object by telescope

Telescope	Neptune	Triton	Matches
160	735	1251	682
IAG	2795	3341	2459
Zeiss	292	463	280
Total	3822	5055	3421

Number of positions identified of Neptune and Triton by telescope. Matches: Number of positions where Neptune and Triton were identified automatically in the same image.

This is the final number after all the process described in this report.

Fig. 1 shows the distribution of positions where Neptune and Triton are identified in the same image (short-exposure observations) over the years. Figs 2-3 summarizes the distribution of positions with Neptune and Triton in the same image by filter obtained in the Perkin-Elmer and Boller & Chivens telescopes, respectively. Zeiss only has images observed in Clear and I filters.

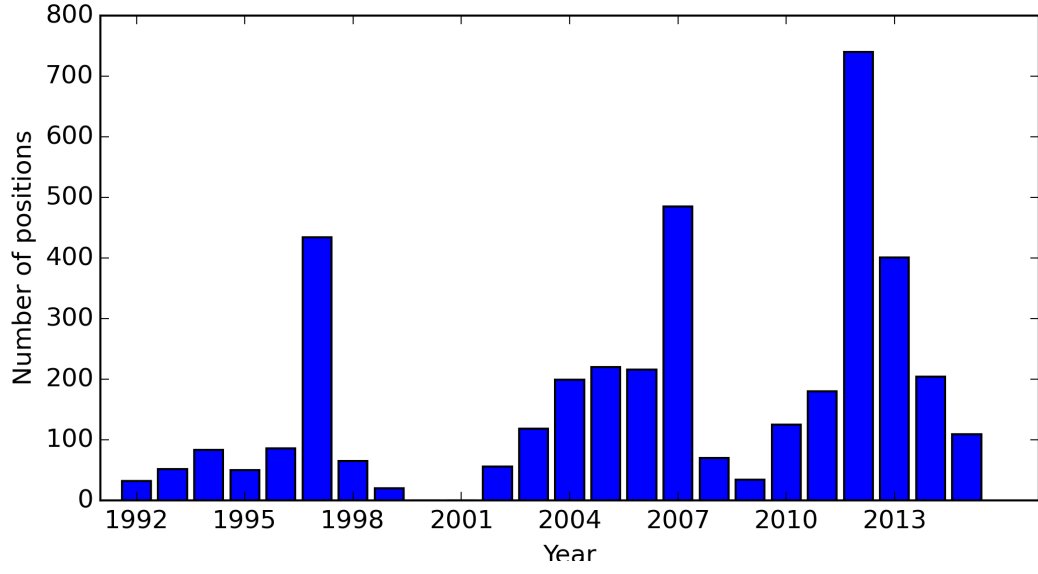


Figure 1: Distribution of positions with Neptune and Triton in the same image (short-exposure observations) by year.

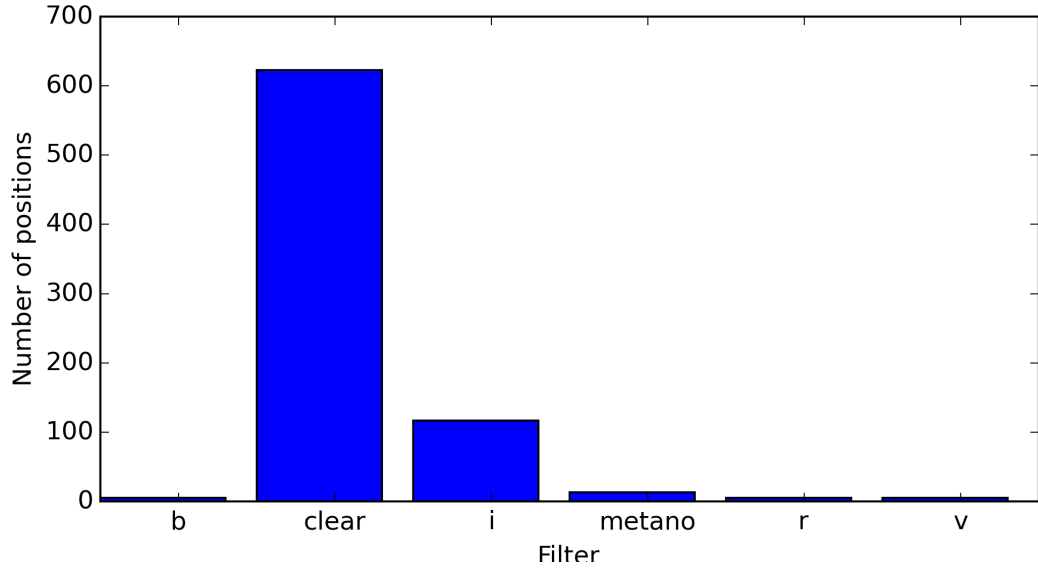


Figure 2: Distribution of positions with Neptune and Triton in the same image (short-exposure observations) by filter for the Perkin-Elmer telescope.

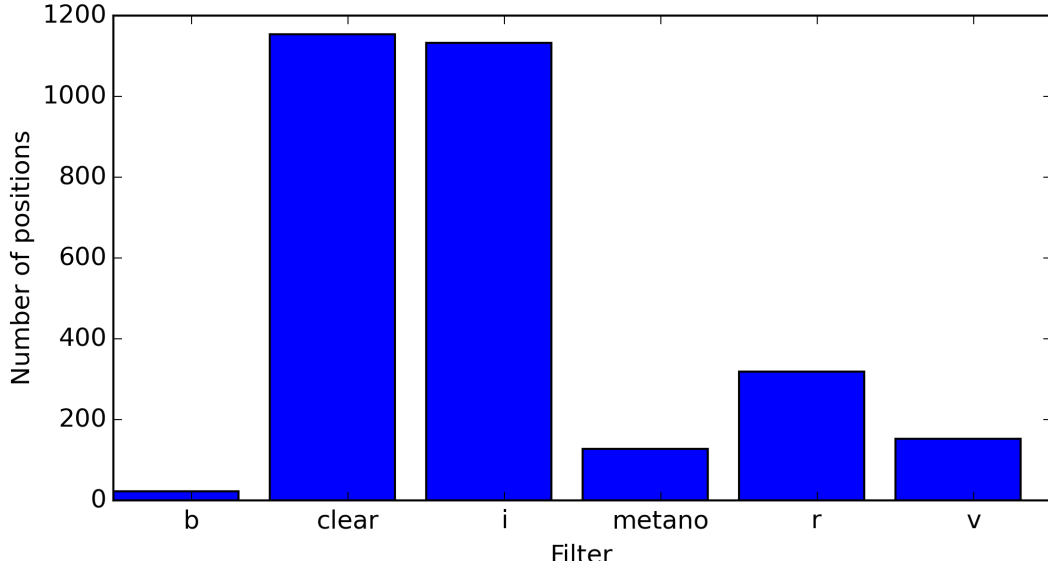


Figure 3: Same as in Fig 2 for the Boller & Chivens telescope.

Reduction

The images were reduced using PRAIA, developed by Marcelo Assafin. To avoid the missing or wrong field-center coordinates I used the coordinates of the ephemeris as input. This way PRAIA could identify reference stars in the images. The reference catalogue used was UCAC4. The ephemeris used to identify Neptune and Triton in the images was DE430+NEP081. The positions where the image of Neptune were saturated and where there were less than 5 reference stars were removed of the results.

In Table 2 it is presented the mean errors in X and Y of the bidimensional Gaussian used to fit the PSF of the objects and the mean value of the dispersion of the offsets by night.

Table 2: Table of errors of the reduction. Gaussian error stands for the error in X and Y of the bidimensional Gaussian PSF used in the (x,y) fits. Mean offset errors is the average dispersion of the positions of each night.

Telescope/Satellite	Gaussian error		Mean offset errors	
	X (mas)	Y (mas)	RA (mas)	DEC (mas)
160/Neptune	8±4	8±4	51	39
160/Triton	14±8	14±8	35	38
IAG/Neptune	9±7	9±7	63	58
IAG/Triton	20±14	20±14	52	53
Zeiss/Neptune	9±6	9±6	49	57
Zeiss/Triton	25±13	25±13	40	51

We applied the digital coronagraphy technique to test if the scattered light of Neptune would influence in the Triton's photocenter. We found no variations larger than 1 mas. We thus concluded that we do not need digital coronagraphy to improve the positions, as was the case of Uranus and its satellite Miranda..

Chromatic Refraction Test

Table 3 shows the colors for Triton (Pascu et al., 2006) and Neptune Schmude et al. (2016). Their colors are very different. Neptune is much bluer than Triton. So it is expected that their positions have influence of chromatic refraction (CR) with different intensities with respect to the reference stars. The apparent position of Neptune, which is more blue than Triton, would be more shifted towards the zenith than the Triton's position. There may also be noted that in 1992 Neptune had just exited the galactic plane, so the reference stars were redder due to dust.

Object	U-B	B-V	V-R	R-I	V-I
Triton (leading side)		$+0.696 \pm 0.009$			$+0.766 \pm 0.006$
Triton (trailing side)		$+0.699 \pm 0.006$			$+0.776 \pm 0.007$
Neptune	+0.14	+0.39	-0.29	-1.05	-1.34*

Table 3: Colors of Triton and Neptune. Leading side is the hemisphere of Triton that is in the direction of its movement. Trailing side is the opposite hemisphere.

*calculated from V-R and R-I colors.

(Pascu et al., 2006) data also support a secular "blueing" on Triton observed since 1954. They also evidence a reddening episode that happened in 1997 where the B-V color of Triton was bigger than 0.9. A similar reddening was also identified in 1979 (Fig. 4). The authors state that a possible cause of this event is an increase in the activity of geysers.

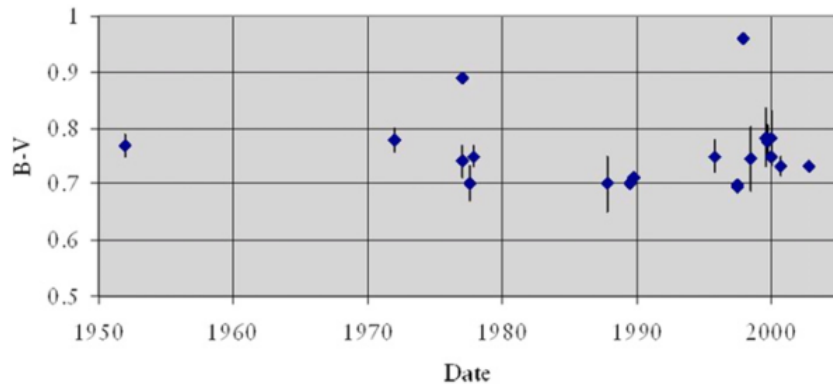


Fig. 1. Summary of $B-V$ color observations of Triton. Redder colors result in higher values of $B-V$. Note the very red colors observed in 1977 and 1997—the latter occurrence only two months after the data reported here (which are among the bluest colors reported). Note that points at 1995.6 and 2002.8 are calculated from reflectance spectra published by Tryka and Bosh (1999) and by Marchi et al. (2004), respectively.

Figure 4: Figure extracted from Pascu et al. (2006).

For Neptune, Schmude et al. (2016) showed a secular brightening in the B-, V-, R- and I-bands (Fig. 5) from observations since 1954. They also identified, from Hubble observations, that Neptune has a variation of about 1 magnitude in the I-band over some hours caused by the presence of bright clouds on its atmosphere (Fig. 6). All these

circumstances can difficult the estimative of chromatic refraction parameters for Neptune and Triton.

To test the effects of chromatic refraction I used the method of Benedetti-Rossi et al. (2014) on all nights with observations distributed over more than 1.5h of hour angle. I used the equation

$$\Delta[\alpha, \delta] = V_{\alpha, \delta}(\phi, \delta, H) \cdot \Delta B, \quad (1)$$

to model the chromatic refraction of the nights. $\Delta[\alpha, \delta]$ is the position offset for each coordinate (α, δ) , $V_{\alpha, \delta}(\phi, \delta, H)$ is the first part of refraction which is due to the position of the observed objects and is a function of the latitude of the site (ϕ), of the object's declination (δ), and of the hour angle (H) and ΔB is the the second part: the differential chromatic refraction which is due to the atmospheric conditions and the wavelength (λ) of the object and of the stars in the field. This equation is available in Benedetti-Rossi et al. (2014) where it was applied for observations of Pluto.

The model is applied to the offsets in α and the chromatic parameter ΔB is obtained. This parameter is then used to correct the offsets in δ .

Fig. 7 shows the offsets for a sample night observed with the Perkin-Elmer telescope. In blue are the offsets before the correction and in green after correction. It is possible to see the increase in the offsets before the correction over time (blue).

For the nights with observations distributed in a smaller hour angle, the correction was made following the conditions:

- If the night only has observations between -1h and 1h of hour angle, no correction is made.
- If there is another night with ΔB calculated observed with the same filter and same telescope within at most 3 days apart, the ΔB from that night is used in the CR correction.
- If there is no close night with ΔB calculated, it is used the mean ΔB calculated for nights observed with the same filter and same telescope for the correction of CR.
- Other situations, no CR correction is made.

Figs. 8-11 show the distributions of the offsets in RA and DEC before and after the elimination of chromatic refraction (Neptune-160, Triton-160, Neptune-IAG, Triton-IAG, respectively) for all the nights. Significant improvements were only achieved in right ascension. That was expected, since between 1992-2016 Neptune/Triton were always observed with declinations very close to the OPD's zenith ($\delta = -21^\circ$ in 1992 up to $\delta = -9^\circ$ in 2015, OPD: $\phi = -22.5^\circ$). We see a marginal improvement for Triton, as also expected since its red color is not far from the average color of the reference stars. And we see a significant improvement for Neptune positions, also expected because of its much bluer color.

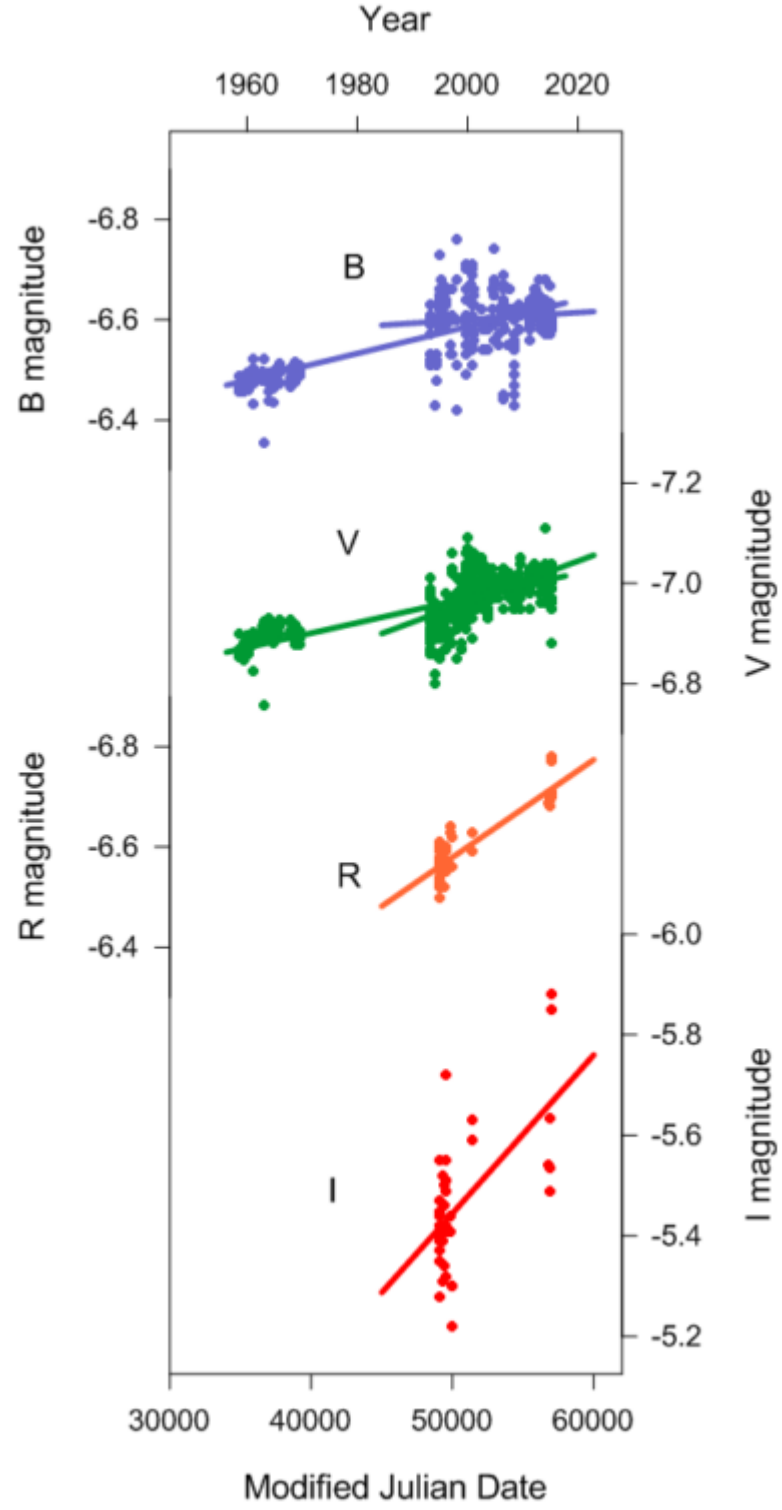


Figure 5: Secular brightening of Neptune in B-, V-, R- and I-bands (Schmude et al., 2016).

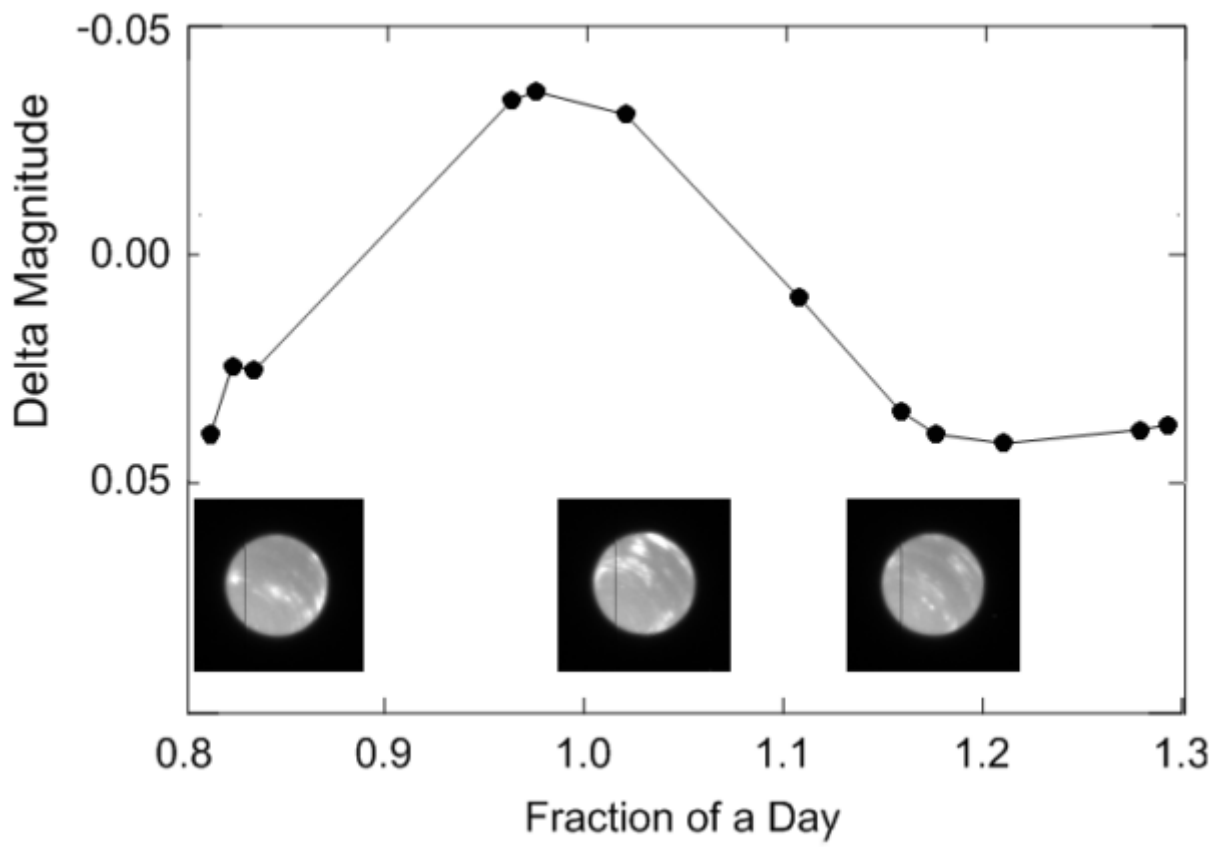


Figure 6: HST images of Neptune at $\lambda = 8450\text{\AA}$ which show a variation of the I-magnitude caused by bright clouds on the atmosphere (Schmude et al., 2016).

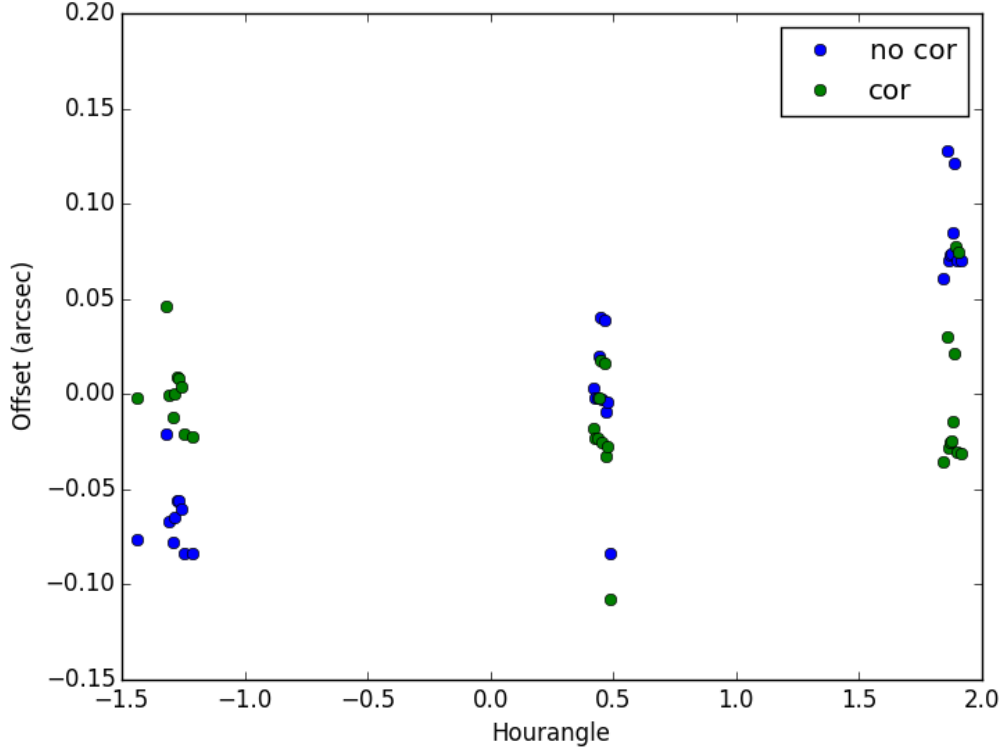


Figure 7: Offsets before (blue) and after (green) CR correction for the night of August 20, 1993 observed with the Perkin-Elmer telescope.

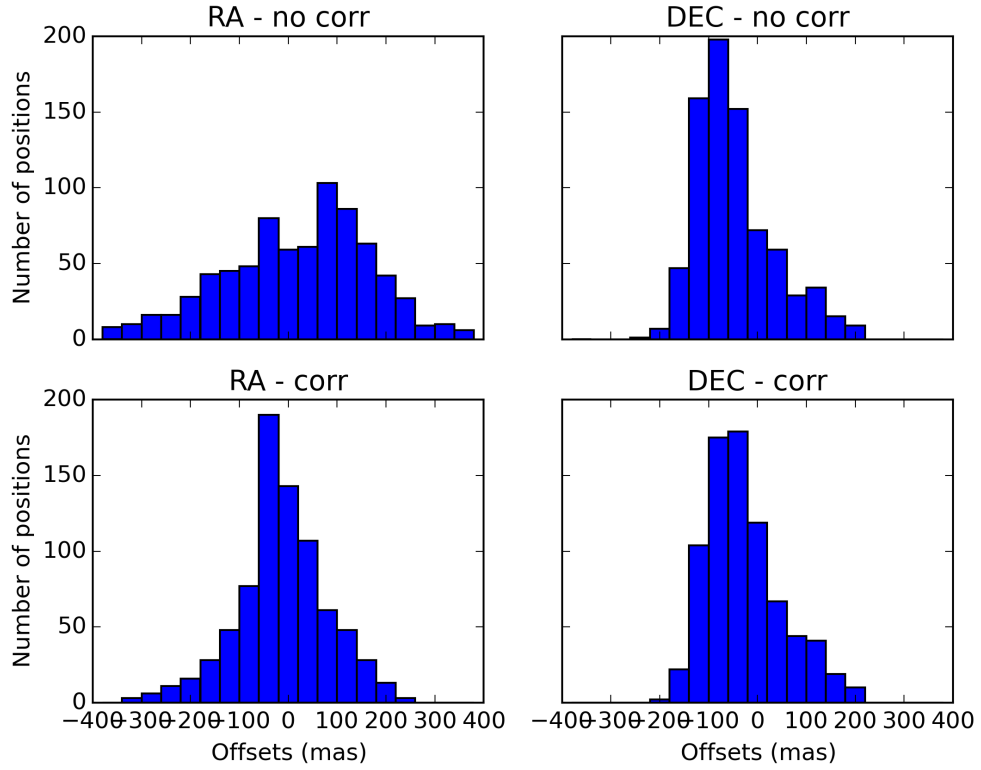


Figure 8: Distribution of the offsets of Neptune observed in 160.

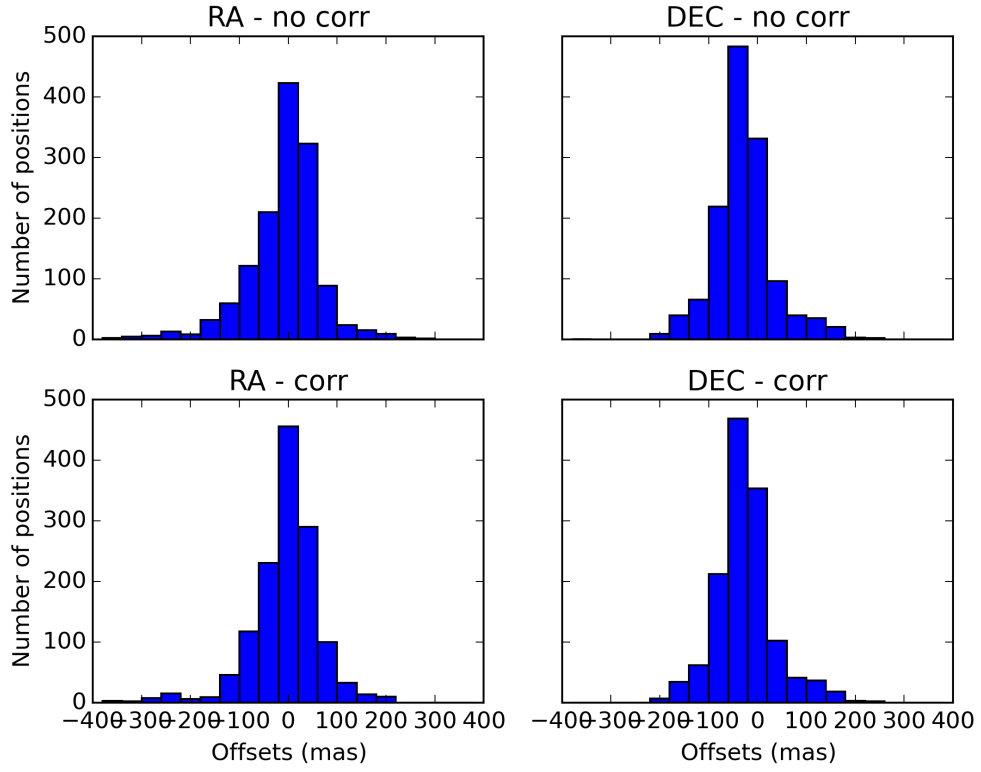


Figure 9: Distribution of the offsets of Triton observed in 160.

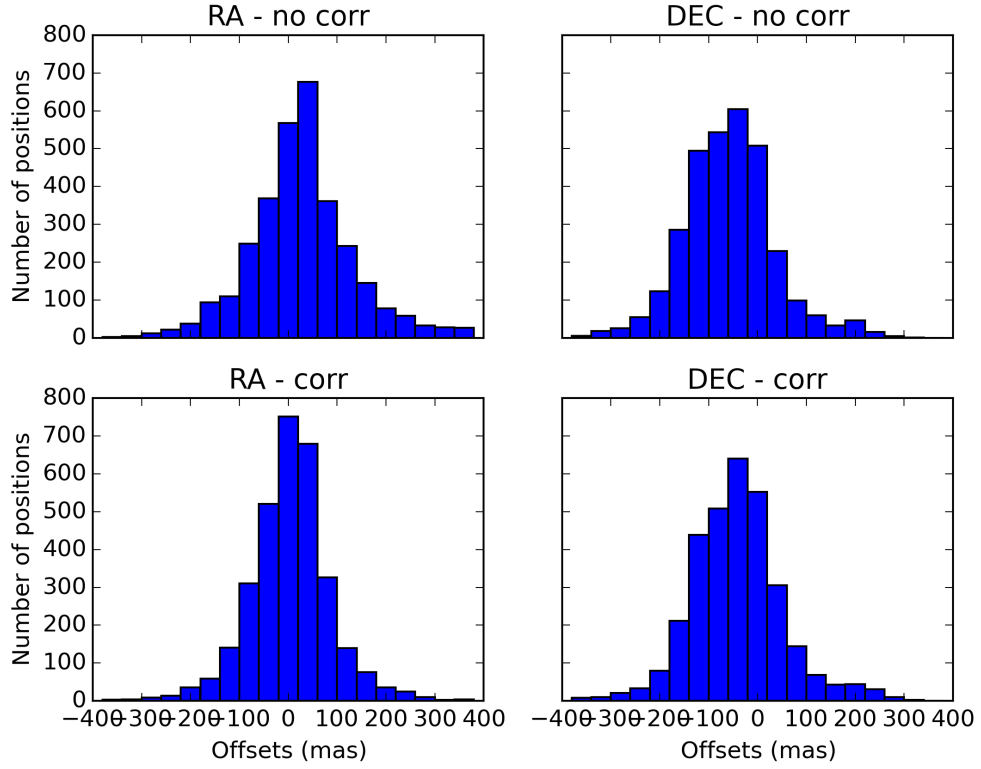


Figure 10: Distribution of the offsets of Neptune observed in IAG.

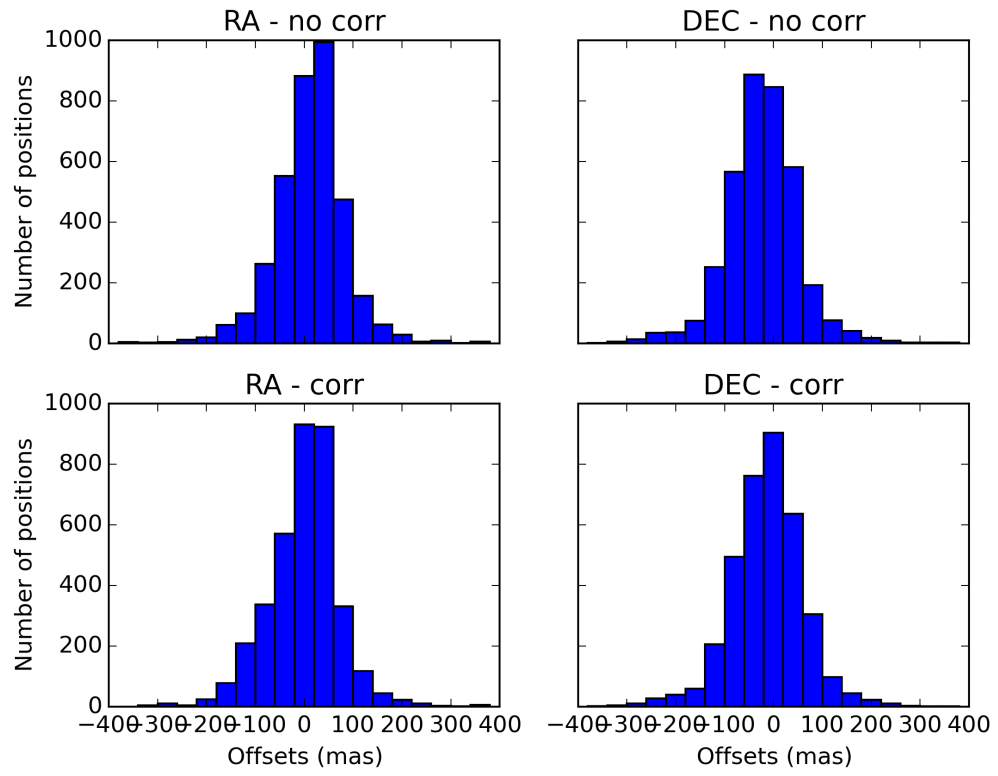


Figure 11: Distribution of the offsets of Triton observed in IAG.

Table 4 show the nights used to calculate CR parameter (nights with $\Delta H > 1.5$), the filter, the variation in hour angle (ΔH), the parameter obtained (ΔB), number of images (Nimg), mean number of reference stars (Nstars) and mean offsets before and after correction. It is possible to see that the ΔB calculated has high values for Neptune, while for Triton they are smaller.

Table 4: Obtained parameters and offsets from adjustments. Only nights with $\Delta H > 1.5h$.

Neptune-160									
Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
1992-06-09	Clear	1.57	+0.29±0.03	13	11	190± 54	25± 76	-43± 15	65± 68
1992-07-19	Clear	1.63	+0.20±0.04	17	24	50± 46	117± 37	1± 26	125± 36
1993-06-24	Clear	1.61	-0.07±0.04	10	17	-17± 26	-38± 48	-21± 22	-40± 48
1993-06-25	Clear	2.63	+0.03±0.02	12	8	-37± 26	70± 75	-40± 24	72± 74
1993-08-20	Clear	3.35	+0.20±0.02	31	29	15± 76	-83± 43	-7± 34	-75± 44
1993-08-22	Clear	2.87	+0.18±0.02	35	9	-47± 63	-74± 44	-15± 38	-67± 46
1996-06-22	Clear	1.68	+0.24±0.02	19	16	-25± 51	-32± 35	-48± 18	-20± 35
1996-10-02	Clear	1.97	+0.31±0.07	16	6	252±192	-60± 73	-59±126	10± 79
1997-06-01	Clear	4.89	+0.35±0.03	91	11	-155±189	-112± 36	-120±107	-81± 40
1997-06-02	Clear	5.24	+0.22±0.01	60	10	-113±125	-84± 36	-73± 48	-61± 33
1998-06-06	Clear	2.51	+0.38±0.04	35	11	-140±101	-66± 42	-13± 56	-33± 41
1998-09-03	Clear	1.52	+0.30±0.03	20	10	-86± 63	-75± 49	20± 26	-51± 47
Triton-160									
Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
1992-06-09	Clear	1.57	+0.01±0.03	16	16	12± 20	19± 79	3± 20	20± 79
1992-07-19	Clear	1.89	+0.07±0.04	21	25	-3± 33	15± 52	-21± 30	17± 52
1993-06-24	Clear	1.61	+0.01±0.04	15	13	8± 23	15± 55	9± 23	16± 55
1993-06-25	Clear	2.90	-0.09±0.04	20	12	-28± 60	36± 79	-14± 52	32± 80
1993-08-20	Clear	3.35	-0.01±0.02	30	27	-8± 29	-25± 62	-6± 29	-25± 62
1993-08-22	Clear	3.12	+0.05±0.01	43	13	-2± 28	-9± 43	4± 24	-7± 43
1994-09-22	Clear	1.94	-0.12±0.08	13	12	-33± 53	30± 75	48± 49	16± 71
1994-09-22	Clear	1.55	+0.00±0.04	15	17	-53± 21	24± 82	-54± 21	24± 82
1995-08-07	Clear	3.02	+0.02±0.02	11	21	-10± 14	-40± 30	-10± 13	-39± 30

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Table 4 – *Continued from previous page*

Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
1996-06-22	Clear	2.28	-0.08±0.02	32	15	-87± 24	-9± 29	-77± 19	-13± 28
1996-08-22	Clear	1.56	+0.01±0.02	29	13	43± 30	-41± 44	37± 30	-40± 44
1996-08-24	Clear	1.99	-0.03±0.04	10	11	46± 15	-66± 23	52± 15	-67± 23
1996-10-02	Clear	1.97	+0.04±0.07	16	6	73±121	1± 76	38±120	9± 77
1997-06-01	Clear	4.89	+0.11±0.01	101	11	-76± 71	-107± 53	-68± 51	-98± 54
1997-06-02	Clear	5.41	+0.02±0.02	81	10	-18± 86	-56± 30	-17± 85	-54± 30
1997-08-11	Clear	3.08	+0.12±0.02	33	11	-30± 46	-23± 27	2± 28	-14± 28
1997-08-13	Clear	1.67	+0.04±0.03	19	6	44± 30	-4± 16	60± 29	-0± 16
1998-06-06	Clear	2.84	+0.13±0.04	42	11	-46± 62	-55± 33	-5± 53	-44± 33
1998-09-03	Clear	1.52	-0.04±0.04	20	10	3± 36	-48± 54	-11± 35	-50± 54
1999-06-06	Clear	1.58	+0.23±0.04	27	14	19± 47	-63± 43	91± 32	-43± 40
1999-08-22	Clear	3.06	+0.03±0.02	30	15	-32± 37	-7± 25	-45± 35	-3± 26
Neptune-IAG									
Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
2001-08-26	B	2.11	+0.15±0.03	44	15	89± 55	-41± 57	14± 41	-22± 57
2002-07-15	Clear	6.11	+0.18±0.00	57	17	51±163	85± 90	-36± 31	131± 76
2002-07-18	Clear	3.86	+0.21±0.02	30	13	-100±115	-140± 61	-50± 61	-111± 61
2003-07-22	Clear	2.38	+0.33±0.04	20	15	174±137	-75± 80	-32± 57	-13± 74
2003-07-23	Clear	4.08	+0.02±0.01	39	16	-81± 36	13± 55	-78± 34	17± 55
2003-07-25	Clear	6.19	-0.01±0.02	21	9	-107± 74	9± 62	-106± 74	7± 62
2003-07-26	Clear	6.97	-0.01±0.08	17	10	-132±211	8±123	-129±211	7±124
2003-07-27	Clear	1.54	+0.06±0.06	26	14	-21±104	4± 84	-90±102	24± 84
2003-07-28	Clear	7.98	+0.01±0.03	43	12	-37±144	21±162	-44±144	24±162
2003-08-20	Clear	4.03	+0.26±0.04	30	17	95±154	-72± 66	18± 98	-35± 60
2003-10-14	V	2.33	+0.02±0.03	8	30	20± 25	16± 31	16± 25	18± 31
2004-08-05	V	2.21	-0.03±0.12	5	6	-53± 67	-61± 25	-35± 67	-66± 26
2004-08-06	V	3.10	+0.18±0.03	4	6	-160± 61	-173± 60	-165± 14	-151± 59
2004-08-07	V	4.31	+0.21±0.33	6	11	128±333	-40±210	110±317	-9±212

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Table 4 – *Continued from previous page*

Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
2004-08-21	Clear	4.09	+0.08 \pm 0.02	30	21	-32 \pm 57	-78 \pm 55	-56 \pm 44	-66 \pm 57
2004-08-21	Clear	2.57	+0.05 \pm 0.02	30	21	-42 \pm 28	-86 \pm 41	-32 \pm 26	-81 \pm 41
2004-08-23	Clear	5.20	+0.06 \pm 0.01	70	18	10 \pm 59	-88 \pm 52	-29 \pm 42	-73 \pm 50
2004-08-24	Clear	3.94	+0.06 \pm 0.01	40	18	-2 \pm 34	-83 \pm 66	-23 \pm 26	-74 \pm 65
2004-09-24	R	3.08	+0.27 \pm 0.05	35	13	157 \pm 183	12 \pm 136	-0 \pm 133	63 \pm 126
2004-09-25	Clear	3.75	+0.26 \pm 0.03	40	14	201 \pm 164	2 \pm 93	37 \pm 106	54 \pm 86
2005-09-24	V	2.78	+0.05 \pm 0.01	88	14	2 \pm 47	-101 \pm 61	-52 \pm 44	-85 \pm 60
2006-06-08	Clear	2.68	+0.32 \pm 0.04	95	24	-91 \pm 144	-83 \pm 93	14 \pm 115	-30 \pm 93
2011-09-26	I	4.02	+0.05 \pm 0.00	250	18	76 \pm 44	-67 \pm 48	38 \pm 36	-52 \pm 45
2012-10-19	R	1.99	+0.13 \pm 0.03	119	8	41 \pm 100	-167 \pm 137	-50 \pm 92	-130 \pm 138
Triton-IAG									
Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
2001-08-26	B	2.11	+0.03 \pm 0.03	24	15	1 \pm 40	-53 \pm 46	-13 \pm 39	-50 \pm 46
2002-07-15	Clear	6.11	+0.01 \pm 0.00	55	17	14 \pm 27	-29 \pm 45	7 \pm 24	-26 \pm 45
2002-07-18	Clear	3.86	-0.01 \pm 0.03	31	13	25 \pm 74	-68 \pm 111	23 \pm 74	-69 \pm 111
2003-07-22	Clear	2.55	+0.08 \pm 0.02	31	17	-8 \pm 57	-7 \pm 53	-58 \pm 49	8 \pm 54
2003-07-23	Clear	4.08	+0.04 \pm 0.02	38	16	-44 \pm 54	-12 \pm 45	-37 \pm 50	-6 \pm 45
2003-07-25	Clear	6.39	-0.04 \pm 0.03	44	16	-60 \pm 138	9 \pm 50	-64 \pm 136	2 \pm 51
2003-07-26	Clear	6.97	-0.04 \pm 0.04	33	14	-133 \pm 167	21 \pm 90	-121 \pm 165	13 \pm 91
2003-07-27	Clear	1.54	+0.01 \pm 0.05	26	14	-31 \pm 76	31 \pm 74	-40 \pm 76	34 \pm 74
2003-07-28	Clear	7.98	-0.01 \pm 0.02	60	14	-51 \pm 130	46 \pm 133	-44 \pm 130	43 \pm 133
2003-08-20	Clear	4.20	+0.02 \pm 0.02	49	21	53 \pm 69	-8 \pm 45	46 \pm 68	-5 \pm 45
2003-10-14	V	3.63	-0.04 \pm 0.03	20	33	-4 \pm 54	-34 \pm 72	13 \pm 53	-40 \pm 74
2003-10-15	V	2.27	+0.01 \pm 0.05	18	33	-22 \pm 38	-3 \pm 78	-23 \pm 38	-2 \pm 77
2003-10-16	V	1.94	-0.10 \pm 0.10	8	25	-18 \pm 64	-15 \pm 36	46 \pm 58	-32 \pm 32
2003-10-17	V	2.09	+0.02 \pm 0.04	10	29	1 \pm 31	4 \pm 92	-7 \pm 31	7 \pm 92
2003-10-19	V	1.89	-0.12 \pm 0.05	12	31	-7 \pm 40	-45 \pm 35	44 \pm 32	-60 \pm 32
2004-08-05	V	2.38	-0.06 \pm 0.02	21	15	-13 \pm 31	-101 \pm 35	21 \pm 26	-111 \pm 35

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Date	Filter	ΔH	ΔB	Nimg	Nstars	RA no corr	DEC no corr	RA corr	DEC corr
2004-08-06	V	3.27	+0.06±0.07	22	16	53± 99	-110± 50	45± 97	-102± 50
2004-08-07	V	4.50	+0.00±0.03	23	19	54± 56	-117± 61	54± 56	-117± 61
2004-08-20	Clear	3.81	+0.03±0.01	16	24	-5± 26	-40± 24	-15± 23	-35± 23
2004-08-21	Clear	4.09	+0.02±0.02	27	22	-13± 41	-32± 52	-22± 39	-28± 53
2004-08-21	Clear	2.57	+0.01±0.02	26	21	-13± 28	-51± 35	-12± 27	-51± 35
2004-08-23	Clear	5.20	+0.02±0.01	43	18	-2± 44	-36± 49	-11± 42	-32± 49
2004-08-24	Clear	3.94	+0.02±0.01	29	17	-2± 29	-34± 62	-9± 28	-31± 62
2004-09-24	R	3.08	+0.04±0.04	37	14	26±111	83±131	2±109	91±130
2004-09-25	Clear	3.75	+0.01±0.04	36	14	59±113	91± 77	51±113	93± 77
2005-09-24	V	2.78	+0.01±0.01	155	16	-14± 43	-95± 56	-26± 43	-91± 56
2006-06-08	Clear	3.30	+0.10±0.03	157	26	-22±104	-47± 69	2±100	-32± 69
2009-07-22	Clear	2.19	+0.08±0.07	17	15	10± 43	74± 96	-1± 41	87± 95
2011-09-05	I	1.82	+0.19±0.03	100	18	91± 47	-1± 34	36± 38	38± 36
2011-09-26	I	4.02	-0.00±0.00	250	18	43± 28	4± 41	43± 28	3± 41
2012-10-19	R	1.99	+0.13±0.03	118	8	14±108	-76±134	-78±100	-38±136

Results

Finally, from the offsets in the sense "position minus ephemeris" identified it was made statistics night by night to eliminate discrepant positions with a sigma-clip procedure where offsets (modulus) larger than 80 mas or 2.5-sigma discrepant from the mean offset were removed.

Fig 12 shows the mean offsets of each night and respective discrepancy (error bars) for Neptune in RA and DEC, respectively.

Fig. 13 shows the difference between the offsets of Triton and Neptune over True Anomaly.

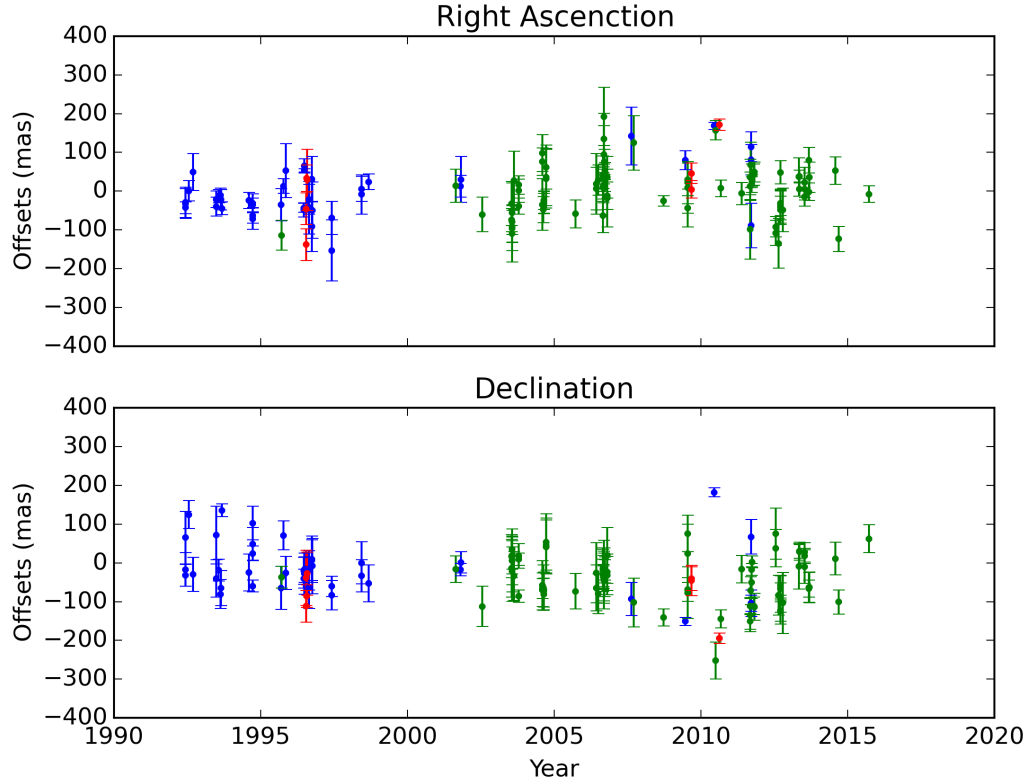


Figure 12: Neptune - Mean offsets by night

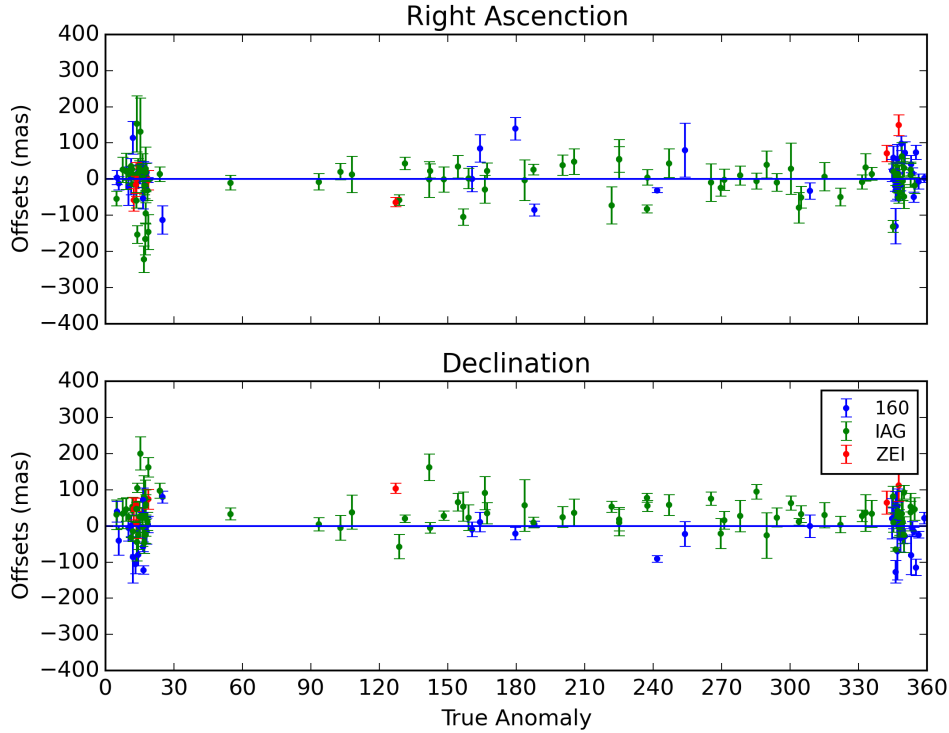


Figure 13: Difference between the offsets of Triton and Neptune by True Anomaly

Final tests

In order to consolidate the position data sets of Neptune and Triton prior to publication, we have two tests yet to perform.

a) Numerical PSF for Neptune

Roberto Vieira Martins developed a numerical PSF for a circular extended object, based on the spread of a Gaussian PSF kernel over the apparent planet's disk. I've already implemented this numerical PSF. I am testing it in a group of observations with negligible solar phase angle. The idea is to compare the results using this numerical PSF and the 2D Gaussian PSF, which was used in all observations. If no significant improvements are achieved, we will stand with the 2D Gaussian PSF. This may be the case, as we are dealing with high S/N ratio images (Neptune) and PRAIA discards wing pixels in the (x,y) fits, so that in the end both PSFs may perform equally well.

b) Uniform reduction and global reduction

We will make tests on selected nights, using rigorously the same reference stars in the (RA,Dec) reduction of each individual field of that night (uniform reduction), or using the global reduction technique with the same underlying objective, in order to see if we get a better chromatic refraction correction.

If we do not find any position improvements in tests (a) and (b), then the current sets of positions will be the final ones. If improvements can be made from the tests, then we

will apply the procedures in the reductions in order to have the final sets of positions.

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