# Astrometry of the Neptune-Triton System

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# Introduction

In this report I present the preliminary results of the astrometric reductions of the images from the Observatório do Pico dos Dias (OPD) in Brazil. The aim is to obtain precise positions for the Neptune - Triton system and to investigate the orbit of Triton around Neptune. Eventually, also investigate the orbit of Neptune around the Sun. The telescopes used were the Perkin-Elmer (160) with a diameter of 1.6m, the Boller & Chivens (IAG) with a diameter of 0.6m, and the Zeiss telescope with a diameter of 0.6m.

The observations were carried out since 1992 when a CCD big enough was installed in the OPD. The planet and satellite have been constantly observed, and still are, by our group. There were many CCDs (IKON, IXON, CCD101, CCD106, ...) and many filters (V, R, I, No Filter, ...) utilized.

There was more than 5000 images from June 1992 to September 2015. Many of the oldest images had no coordinates in header or they were wrong. Many nights had two exposure sets. The first one with low exposure times so Neptune was not saturated, but there were few reference stars in the field. The second one with higher exposure time so Triton was brighter and had more reference stars than with the the previous exposure, but the image of Neptune were saturated.

In Table 1 it is summarized the final number of (only non-saturated) images for Neptune (short-exposure observations) and Triton (all observations) for the 3 telescopes. It is also shown the number of positions where non-saturated-Neptune and Triton are in the same image (short-exposure observations for precision premium).

**Table 1:** Number of positions by object by telescope

| Telescope | Neptune | Triton | Matches |
|-----------|---------|--------|---------|
| 160 | 735 | 1251 | 682 |
| IAG | 2795 | 3341 | 2459 |
| Zeiss | 292 | 463 | 280 |
| Total | 3822 | 5055 | 3421 |

Number of positions identified of non-saturated-Neptune and Triton by telescope. Matches: Number of positions where non-saturaded-Neptune and Triton were identified automatically in the same image. This is the final number after all the process described in this report.

Fig. 1 shows the distribution of positions where Neptune and Triton are identified in the same image (short-exposure observations) over the years. Table 2 summarizes the distribution of positions with Neptune and Triton in the same image by filter for each telescope.

| Telescope | Clear | B | V | R | I | Metano |
|-----------|-------|---|---|---|---|--------|
| 160 | 569 | 4 | 5 | 5 | 86 | 13 |
| IAG | 919 | 21 | 126 | 243 | 1032 | 118 |
| Zeiss | 218 | - | - | - | 62 | - |

**Table 2:** Number of positions with Neptune and Triton in the same image by filter for each telescope.
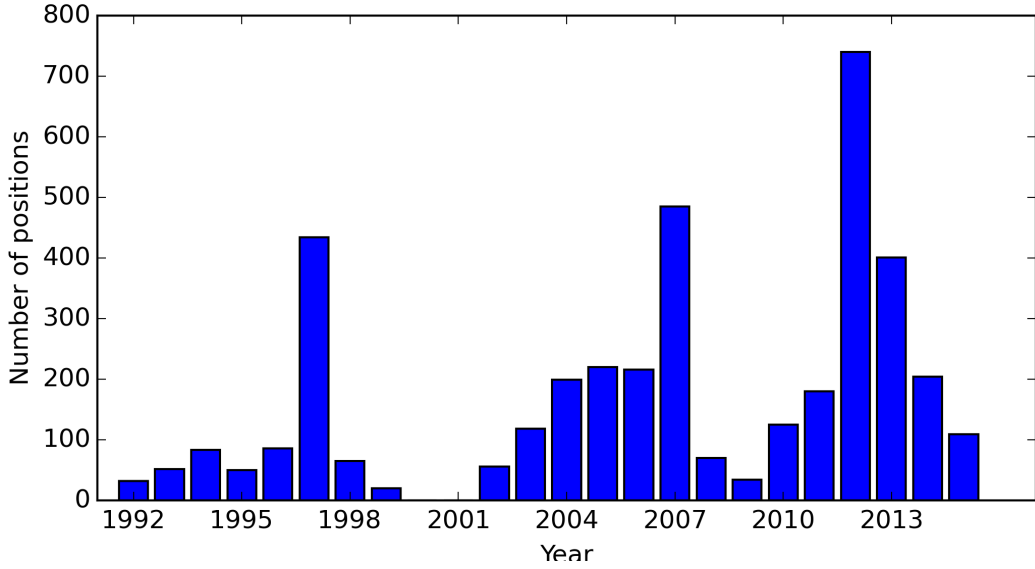
**Figure 1:** Distribution of positions with Neptune and Triton in the same image (short-exposure observations) by year.

# Reduction

The images were reduced using PRAIA, developed by Marcelo Assafin. To avoid the missing or wrong field-center coordinates I used the coordinates of the ephemeris as input. This way PRAIA could identify reference stars in the images. The reference catalogue used was UCAC4. The ephemeris used to identify Neptune and Triton in the images was DE430+NEP081. The positions where the image of Neptune was saturated and where there were less than 5 reference stars were removed of the results.

In Table 3 it is presented the mean errors in X and Y of the bidimensional circular Gaussian used to fit the PSF of the objects and the mean value of the dispersion of the offsets by night.

**Table 3:** Table of errors of the reduction. Gaussian error stands for the error in X and Y of the bidimensional circular Gaussian PSF used in the (x,y) fits. Mean offset errors is the average dispersion of the positions of each night.

| Telescope/Satellite | Gaussian error | Mean offet errors | |
|---|---|---|---|
| | (mas) | RA (mas) | DEC (mas) |
| 160/Neptune | 8±4 | 51 | 39 |
| 160/Triton | 14±8 | 35 | 38 |
| IAG/Neptune | 9±7 | 63 | 58 |
| IAG/Triton | 20±14 | 52 | 53 |
| Zeiss/Neptune | 9±6 | 49 | 57 |
| Zeiss/Triton | 25±13 | 40 | 51 |

We applied the digital coronagraphy technique to test if the scattered light of Neptune would influence in the Triton's photocenter. We found no variations larger than 1 mas. We thus concluded that we do not need digital coronagraphy to improve the positions, as was the case of Uranus and its satellite Miranda.

# Chromatic Refraction Test

Table 4 shows the colors for Triton (Pascu et al., 2006) and Neptune Schmude et al. (2016). Their colors are very different. Neptune is much bluer than Triton. So it is expected that their positions have influence of chromatic refraction (CR) with different intensities with respect to the reference stars. The apparent position of Neptune, which is more blue than Triton, would be more shifted towards the zenith than the Triton's position. There may also be noted that in 1992 Neptune had just exited the galactic plane, so the reference stars were redder due to dust.

| Object | U-B | B-V | V-R | R-I | V-I |
|--------|-----|-----|-----|-----|-----|
| Triton (leading side) | | +0.696±0.009 | | | +0.766±0.006 |
| Triton (trailing side) | | +0.699±0.006 | | | +0.776±0.007 |
| Neptune | +0.14 | +0.39 | -0.29 | -1.05 | -1.34* |

**Table 4:** Colors of Triton and Neptune. Leading side is the hemisphere of Triton that is in the direction of its movement. Trailing side is the opposite hemisphere.
*calculated from V-R and R-I colors.

(Pascu et al., 2006) data also support a secular "blueing" on Triton observed since 1954. They also evidence a reddening episode that happened in 1997 where the B-V color of Triton was bigger than 0.9. A similar reddening was also identified in 1979 (Fig. 2). The authors state that a possible cause of this event is an increase in the activity of geysers.
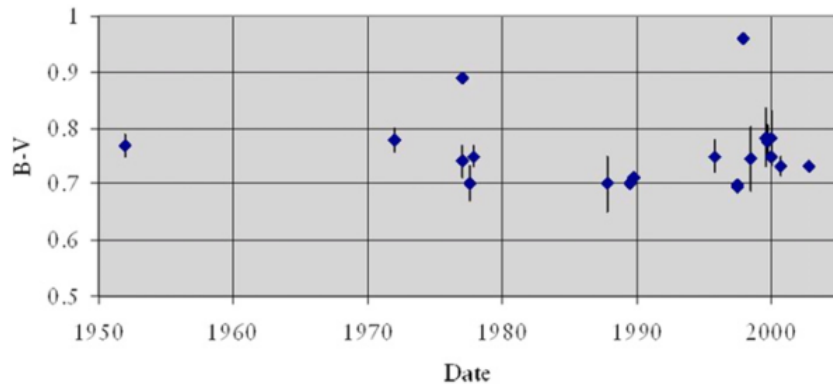


Summary of $B-V$ color observations of Triton. Redder colors result in higher values of $B-V$. Note the very red colors observed in 1977 and 1997—the latter occurrence only two months after the data reported here (which are among the bluest colors reported). Note that points at 1995.6 and 2002.8 are calculated from reflectance spectra published by Tryka and Bosh (1999) and by Marchi et al. (2004), respectively.

**Figure 2:** Figure extracted from Pascu et al. (2006).

For Neptune, Schmude et al. (2016) showed a secular brightening in the B-, V-, R-and I-bands (Fig. 3) from observations since 1954. They also identified, from Hubble observations, that Neptune has a variation of about 1 magnitude in the I-band over some hours caused by the presence of bright clouds on its atmosphere (Fig. 4). All these

circumstances can difficult the estimative of chromatic refraction parameters for Neptune and Triton.
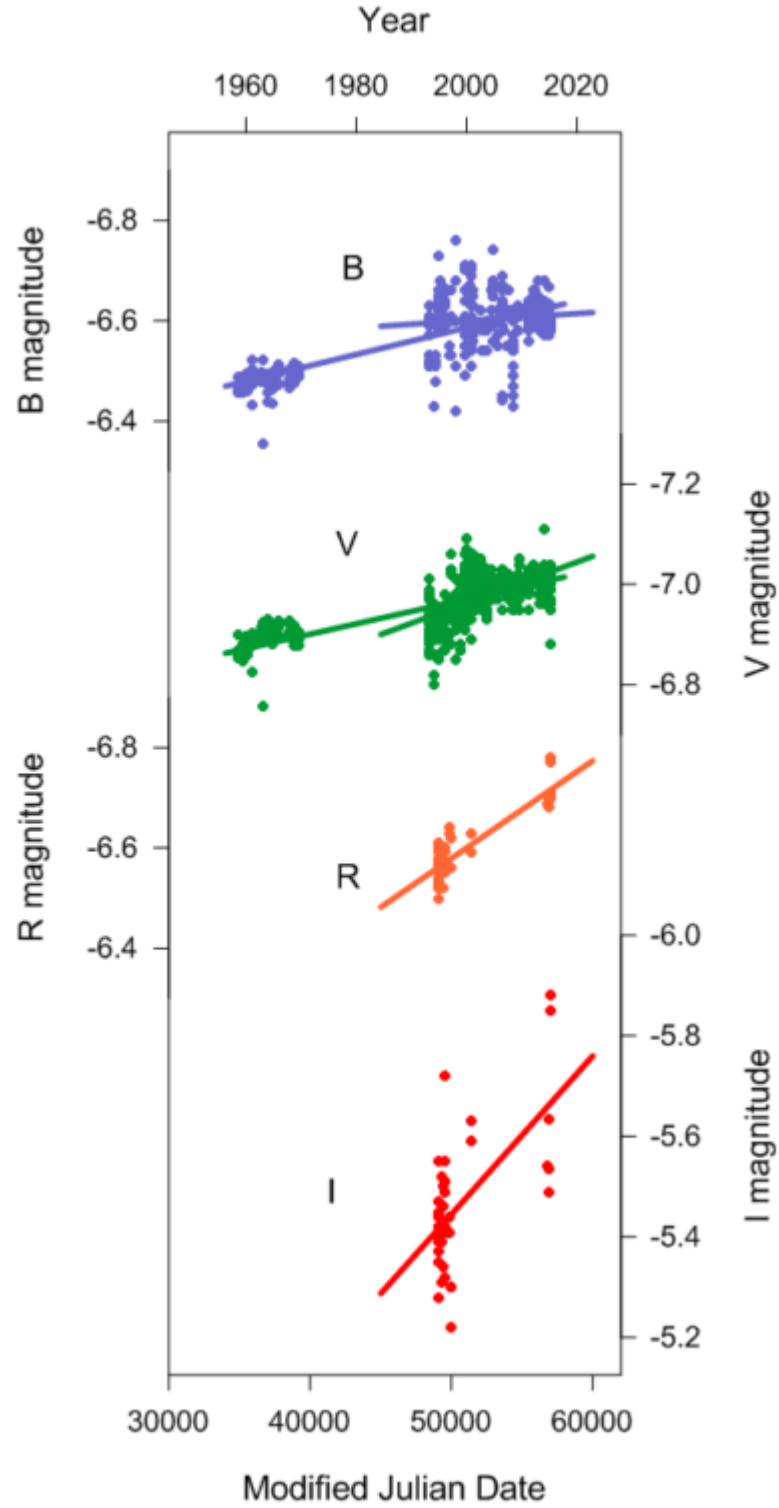


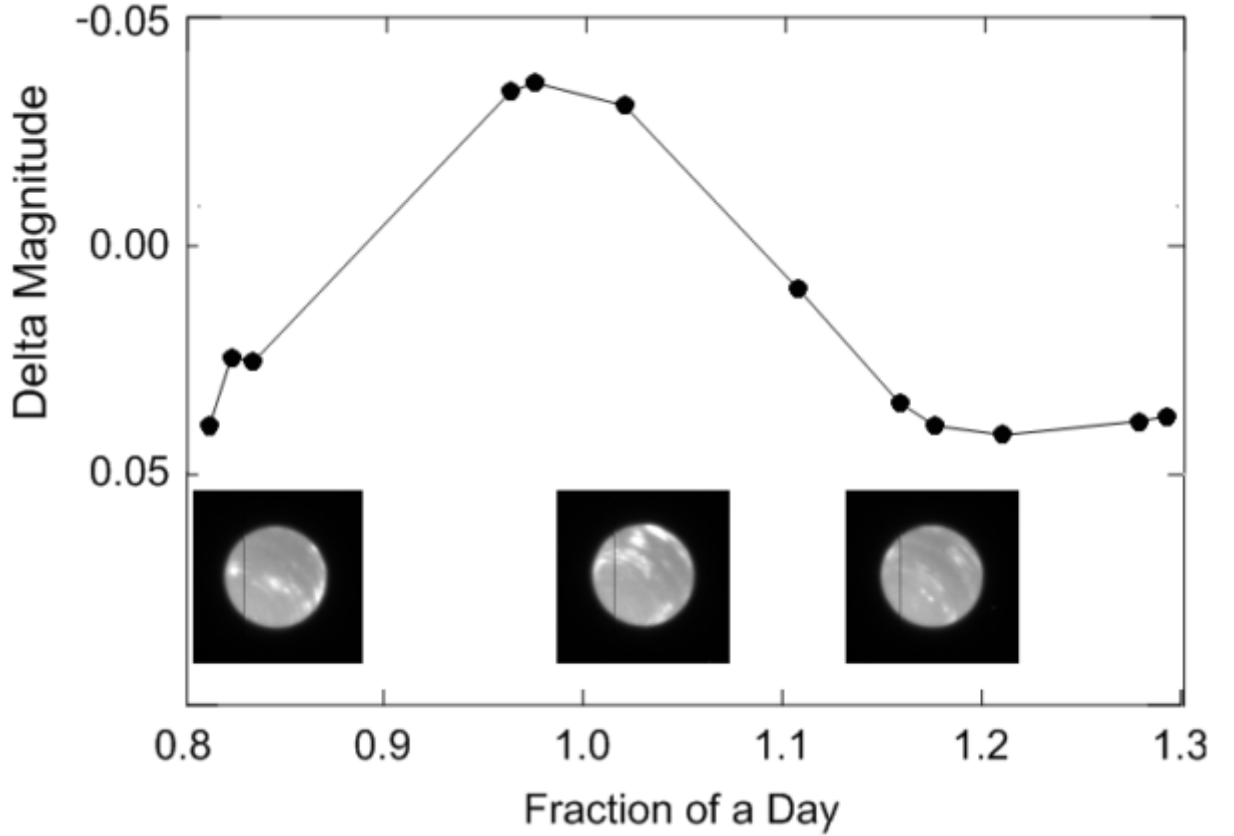**Figure 3:** Secular brightening of Neptune in B-, V-, R- and I-bands (Schmude et al., 2016).

**Figure 4:** HST images of Neptune at $\lambda = 8450$Å which show a variation of the I-magnitude caused by bright clouds on the atmosphere (Schmude et al., 2016).

To test the effects of chromatic refraction I used the method of Benedetti-Rossi et al. (2014) on all nights with observations distributed over more than 1.5h of hour angle. I used the equation

$$\Delta[\alpha, \delta] = V_{\alpha,\delta}(\phi, \delta, H) \cdot \Delta B, \tag{1}$$

where

$$V_\alpha(\phi, \delta, H) = \frac{\sec^2 \delta . \sin H}{\tan \delta . \tan \phi + \cos H} \quad and \quad V_\delta(\phi, \delta, H) = \frac{\tan \phi - \tan \delta . \cos H}{\tan \delta . \tan \phi + \cos H} \tag{2}$$

to model the chromatic refraction of the nights. $\Delta[\alpha, \delta]$ is the position offset for each coordinate $(\alpha, \delta)$, $V_{\alpha,\delta}(\phi, \delta, H)$ is the first part of refraction which is due to the position of the observed objects and is a function of the latitude of the site ($\phi$), of the object's declination ($\delta$), and of the hour angle ($H$) and $\Delta B$ is the the second part: the differential chromatic refraction which is due to the atmospheric conditions and the wavelength ($\lambda$) of the object and of the stars in the field. This equation is available in Benedetti-Rossi et al. (2014) where it was applied for observations of Pluto. Notice that positive values for $\Delta B$ indicates that the body's color is bluer than the average color of the reference stars used in the (RA, DEC) reductions.

The model is applied to the offsets in $\alpha$ and the chromatic parameter $\Delta B$ is obtained. It allows to obtain the zero point of the CR in right ascension since the CR in the meridian must be null. This parameter is then used to correct the offsets in $\delta$, where the effect of CR is much smaller because the declination of Neptune is close to the latitude of the observation site ($\delta = -21°$ in 1992 up to $\delta = -9°$ in 2015, OPD: $\phi = -22.5°$).

Fig. 5 shows the offsets for a sample night observed with the Perkin-Elmer telescope. In blue are the offsets before the correction and in green after correction. It is possible to see the increase in the offsets before the correction over time (blue).
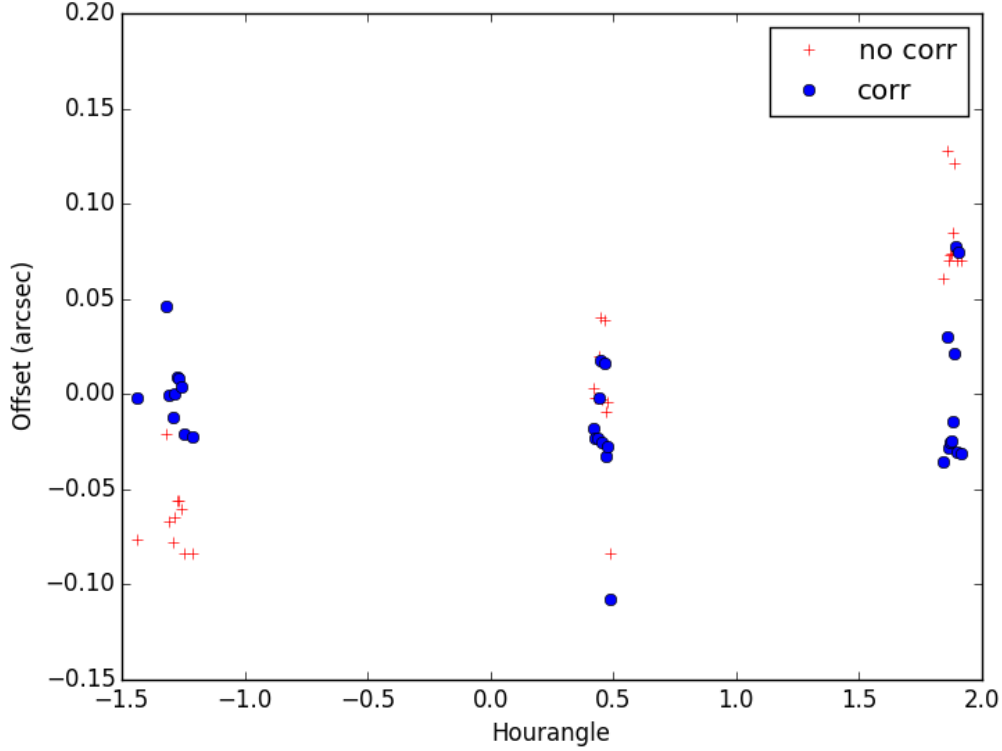


**Figure 5:** Offsets in right ascension before (red) and after (blue) CR correction for the night of August 20, 1993 observed with the Perkin-Elmer telescope.

For the nights with observations distributed in a smaller hour angle, the correction was made following the conditions:

- If the night only has observations between -1h and 1h of hour angle, no correction is made.

- If there is another night with $\Delta B$ calculated observed with the same filter and same telescope within at most 3 days apart, the $\Delta B$ from that night is used in the CR correction.

- If there is no close night with $\Delta B$ calculated, it is used the mean $\Delta B$ calculated for nights observed with the same filter and same telescope for the correction of CR.

- Other situations, no CR correction is made.

Figs. 6-9 show the distributions of the offsets in RA and DEC before and after the elimination of chromatic refraction (Neptune-160, Triton-160, Neptune-IAG, Triton-IAG, respectively) for all the nights. Figs. 10-13 shows the same, but using the mean values of the offsets of each night. Significant improvements were achieved in right ascension as expected, while in declination they are much smaller. As expected, more significant

improvements were achieved in right ascension than in declination, because the declination of Neptune is close to the latitude of the OPD site ($\delta = -21°$in 1992 up to $\delta = -9°$in 2015, OPD: $\phi = -22.5°$). We see that the Neptune's offsets are much more improved than for Triton's, as expected since Neptune is much bluer than the satellite.
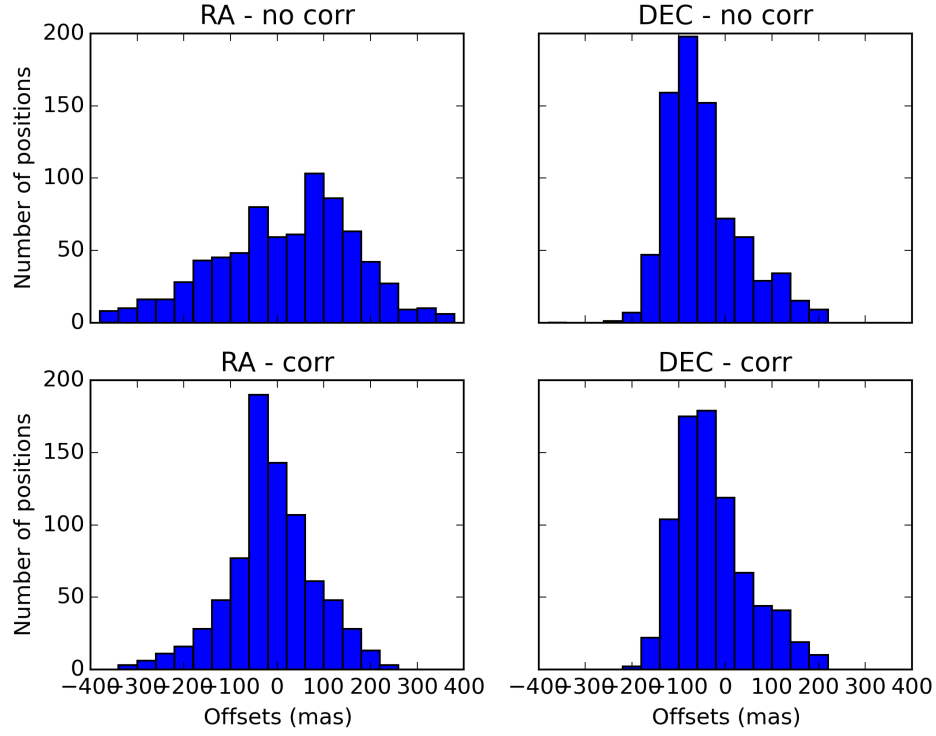


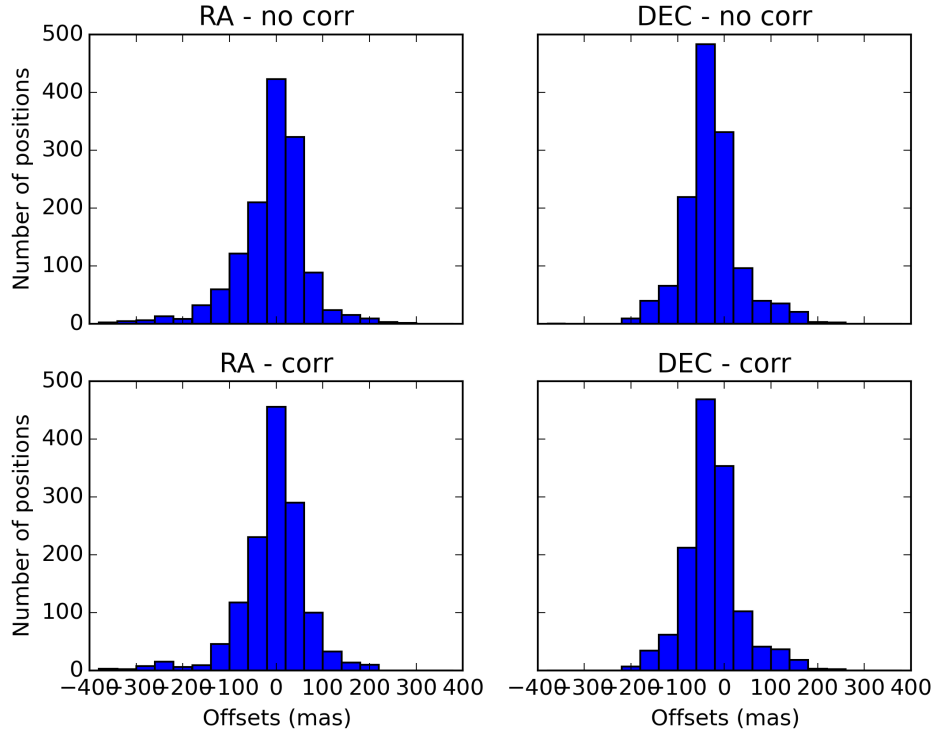**Figure 6:** Distribution of the offsets of Neptune observed in 160.

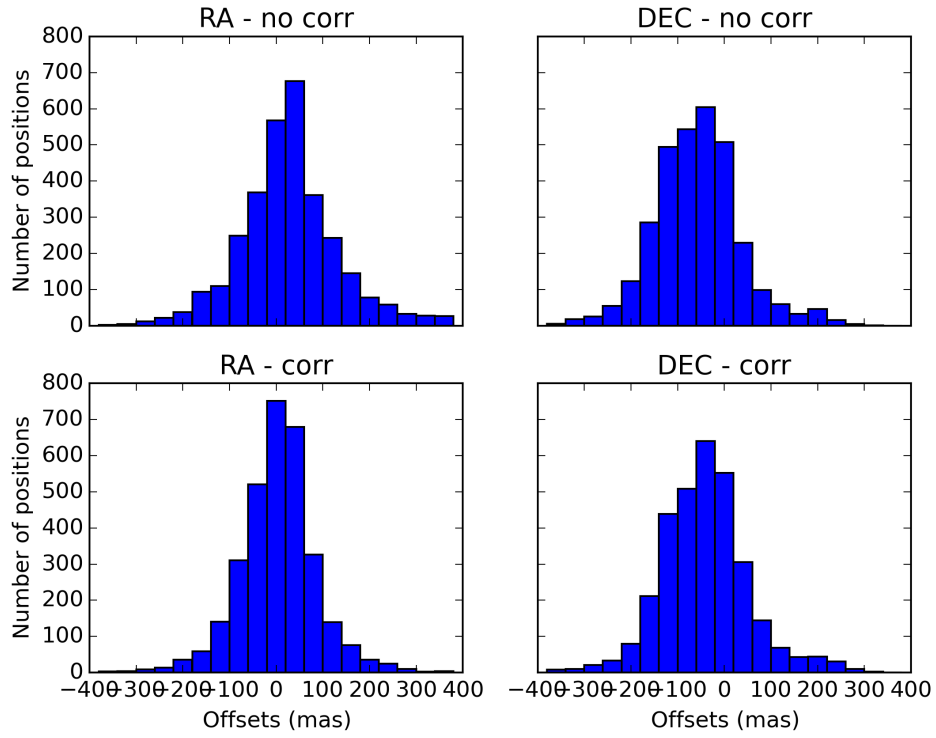**Figure 7:** Distribution of the offsets of Triton observed in 160.



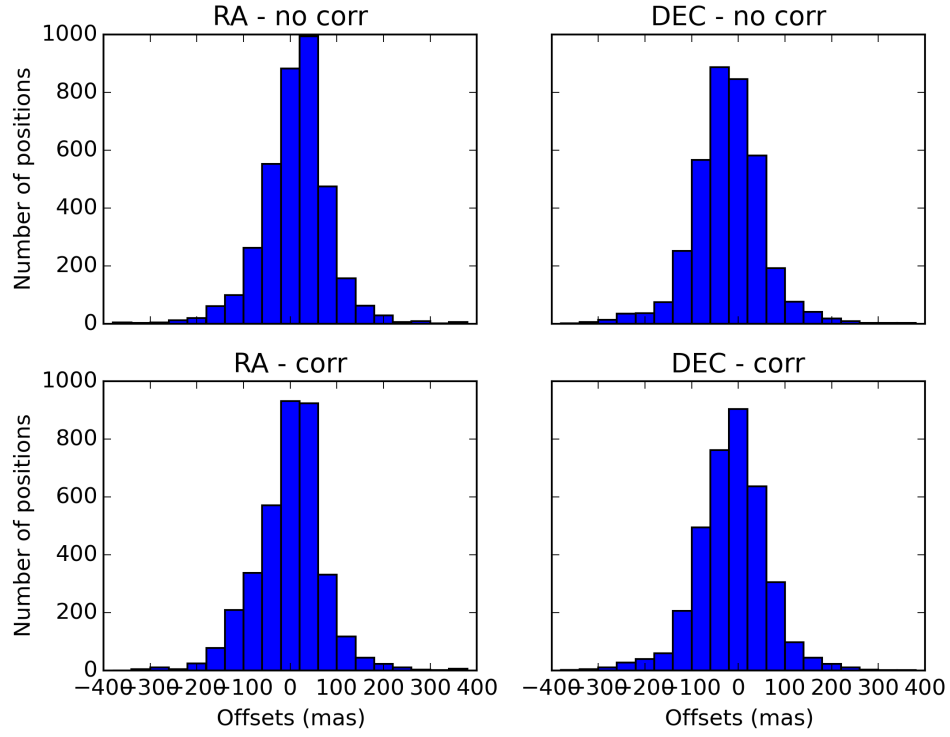**Figure 8:** Distribution of the offsets of Neptune observed in IAG.

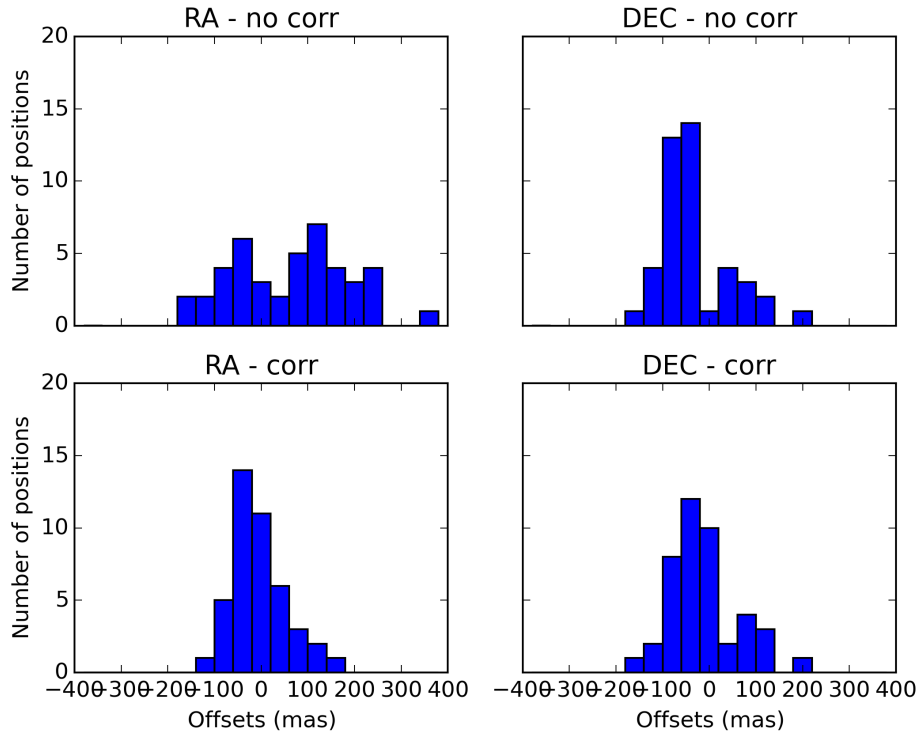**Figure 9:** Distribution of the offsets of Triton observed in IAG.



**Figure 10:** Distribution of the mean values night by night of the offsets of Neptune observed in 160.
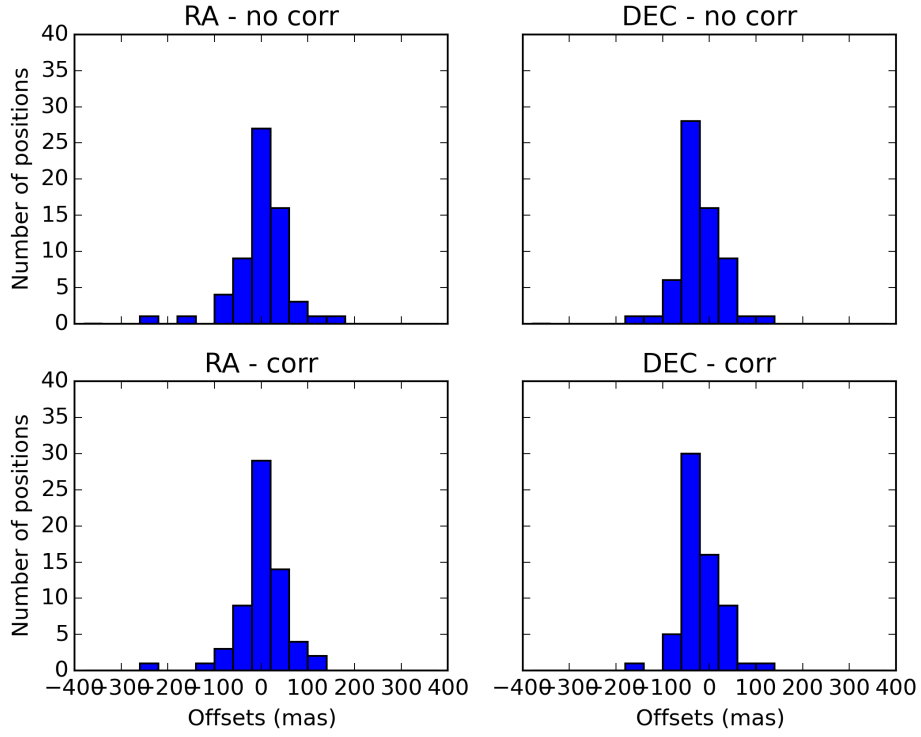
**Figure 11:** Distribution of the mean values night by night of the offsets of Triton observed in 160.
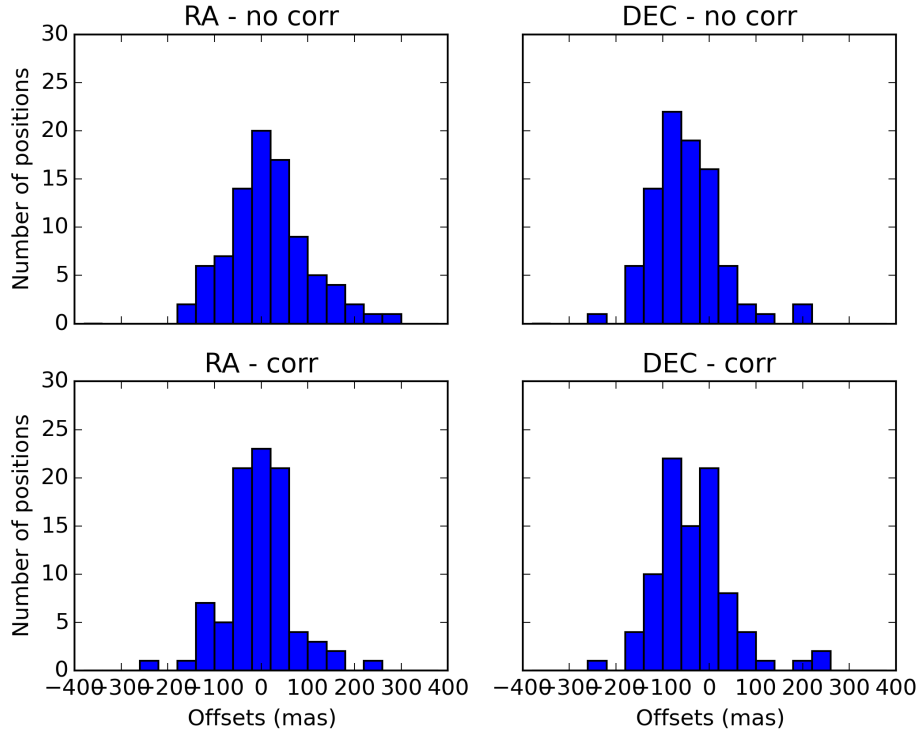


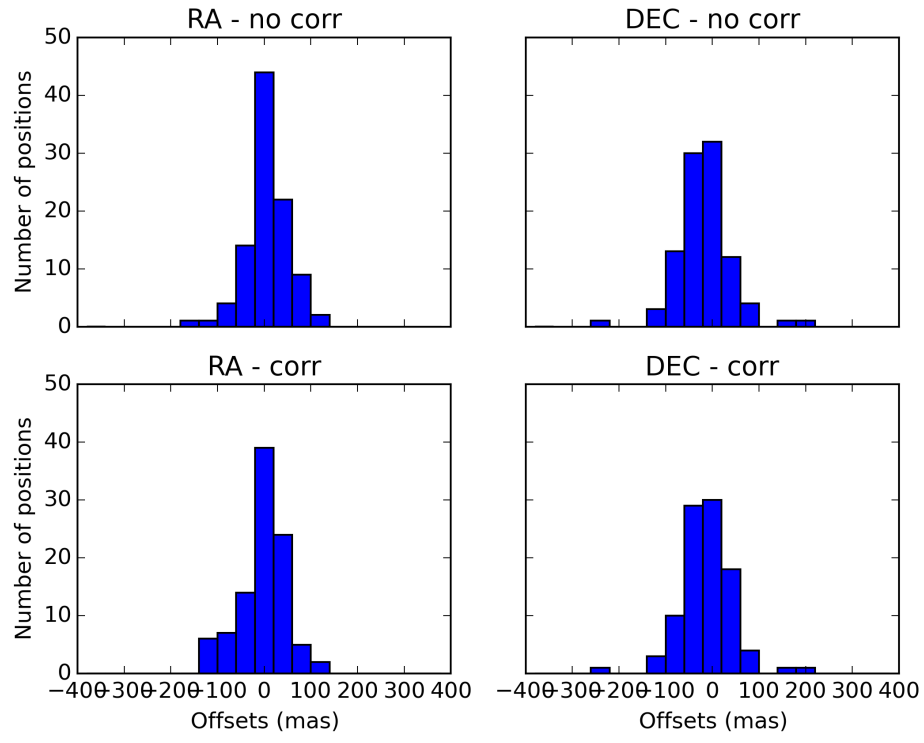**Figure 12:** Distribution of the mean values night by night of the offsets of Neptune observed in IAG.

**Figure 13:** Distribution of the mean values night by night of the offsets of Triton observed in IAG.

Table 5 shows the nights used to calculate the CR parameter (nights with $\Delta H > 1.5$). It is possible to see that $\Delta B$ has higher values for Neptune than for Triton, as expected from the much bluer color of the planet with regard to the satellite's color. Due to the low number of reference stars, the mean color of the stars may vary significantly among the nights. This causes the high variation in the $\Delta B$ parameter seen in column 4 of the table, both for Neptune and Triton.

**Table 5:** Obtained parameters and offsets from adjustments. Only nights with $\Delta H > 1.5h$. Also shown the filter, the variation in hour angle ($\Delta H$), the parameter obtained ($\Delta B$), number of images (Nimg), mean number of reference stars (Nstars), mean offsets before and after correction and mean difference between the non corrected and corrected offsets.

| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Neptune-160 | | | | | | | | | | | |
| 1992-06-09 | Clear | 1.57 | +0.29±0.03 | 13 | 11 | 190± 54 | 25± 76 | -43± 15 | 65± 68 | 233± 51 | -41± 12 |
| 1992-07-19 | Clear | 1.63 | +0.20±0.04 | 17 | 24 | 50± 46 | 117± 37 | 1± 26 | 125± 36 | 49± 37 | -7± 4 |
| 1993-06-24 | Clear | 1.61 | -0.07±0.04 | 10 | 17 | -17± 26 | -38± 48 | -21± 22 | -40± 48 | 4± 14 | 2± 0 |
| 1993-06-25 | Clear | 2.63 | +0.03±0.02 | 12 | 8 | -37± 26 | 70± 75 | -40± 24 | 72± 74 | 2± 11 | -1± 1 |
| 1993-08-20 | Clear | 3.35 | +0.20±0.02 | 31 | 29 | 15± 76 | -83± 43 | -7± 34 | -75± 44 | 21± 68 | -9± 4 |
| 1993-08-22 | Clear | 2.87 | +0.18±0.02 | 35 | 9 | -47± 63 | -74± 44 | -15± 38 | -67± 46 | -32± 50 | -7± 4 |
| 1996-06-22 | Clear | 1.68 | +0.24±0.02 | 19 | 16 | -25± 51 | -32± 35 | -48± 18 | -20± 35 | 23± 47 | -12± 2 |
| 1996-10-02 | Clear | 1.97 | +0.31±0.07 | 16 | 6 | 252±192 | -60± 73 | -59±126 | 10± 79 | 311±145 | -70± 46 |
| 1997-06-01 | Clear | 4.89 | +0.35±0.03 | 91 | 11 | -155±189 | -112± 36 | -120±107 | -81± 40 | -35±156 | -31± 11 |
| 1997-06-02 | Clear | 5.24 | +0.22±0.01 | 60 | 10 | -113±125 | -84± 36 | -73± 48 | -61± 33 | -40±115 | -23± 12 |
| 1998-06-06 | Clear | 2.51 | +0.38±0.04 | 35 | 11 | -140±101 | -66± 42 | -13± 56 | -33± 41 | -127± 83 | -33± 11 |
| 1998-09-03 | Clear | 1.52 | +0.30±0.03 | 20 | 10 | -86± 63 | -75± 49 | 20± 26 | -51± 47 | -106± 57 | -24± 8 |
| All nights above | | | | | | -63±164 | -64± 72 | -54± 83 | -41± 71 | -9±143 | -23± 20 |
| Triton-160 | | | | | | | | | | | |
| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
| 1992-06-09 | Clear | 1.57 | +0.01±0.03 | 16 | 16 | 12± 20 | 19± 79 | 3± 20 | 20± 79 | 10± 2 | -2± 1 |
| 1992-07-19 | Clear | 1.89 | +0.07±0.04 | 21 | 25 | -3± 33 | 15± 52 | -21± 30 | 17± 52 | 18± 14 | -3± 1 |
| 1993-06-24 | Clear | 1.61 | +0.01±0.04 | 15 | 13 | 8± 23 | 15± 55 | 9± 23 | 16± 55 | -1± 1 | -0± 0 |
| 1993-06-25 | Clear | 2.90 | -0.09±0.04 | 20 | 12 | -28± 60 | 36± 79 | -14± 52 | 32± 80 | -14± 28 | 4± 2 |
| 1993-08-20 | Clear | 3.35 | -0.01±0.02 | 30 | 27 | -8± 29 | -25± 62 | -6± 29 | -25± 62 | -2± 5 | 1± 0 |

Table 5 – *Continued from previous page*

| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1993-08-22 | Clear | 3.12 | +0.05±0.01 | 43 | 13 | -2± 28 | -9± 43 | 4± 24 | -7± 43 | -7± 14 | -2± 1 |
| 1994-09-22 | Clear | 1.94 | -0.12±0.08 | 13 | 12 | -33± 53 | 30± 75 | 48± 49 | 16± 71 | -81± 22 | 14± 6 |
| 1994-09-22 | Clear | 1.55 | +0.00±0.04 | 15 | 17 | -53± 21 | 24± 82 | -54± 21 | 24± 82 | 0± 1 | -0± 0 |
| 1995-08-07 | Clear | 3.02 | +0.02±0.02 | 11 | 21 | -10± 14 | -40± 30 | -10± 13 | -39± 30 | 0± 5 | -1± 0 |
| 1996-06-22 | Clear | 2.28 | -0.08±0.02 | 32 | 15 | -87± 24 | -9± 29 | -77± 19 | -13± 28 | -10± 15 | 4± 1 |
| 1996-08-22 | Clear | 1.56 | +0.01±0.02 | 29 | 13 | 43± 30 | -41± 44 | 37± 30 | -40± 44 | 5± 2 | -1± 0 |
| 1996-08-24 | Clear | 1.99 | -0.03±0.04 | 10 | 11 | 46± 15 | -66± 23 | 52± 15 | -67± 23 | -5± 4 | 1± 0 |
| 1996-10-02 | Clear | 1.97 | +0.04±0.07 | 16 | 6 | 73±121 | 1± 76 | 38±120 | 9± 77 | 35± 16 | -8± 5 |
| 1997-06-01 | Clear | 4.89 | +0.11±0.01 | 101 | 11 | -76± 71 | -107± 53 | -68± 51 | -98± 54 | -8± 49 | -10± 3 |
| 1997-06-02 | Clear | 5.41 | +0.02±0.02 | 81 | 10 | -18± 86 | -56± 30 | -17± 85 | -54± 30 | -1± 10 | -2± 1 |
| 1997-08-11 | Clear | 3.08 | +0.12±0.02 | 33 | 11 | -30± 46 | -23± 27 | 2± 28 | -14± 28 | -32± 37 | -9± 3 |
| 1997-08-13 | Clear | 1.67 | +0.04±0.03 | 19 | 6 | 44± 30 | -4± 16 | 60± 29 | -0± 16 | -16± 8 | -3± 1 |
| 1998-06-06 | Clear | 2.84 | +0.13±0.04 | 42 | 11 | -46± 62 | -55± 33 | -5± 53 | -44± 33 | -42± 31 | -11± 4 |
| 1998-09-03 | Clear | 1.52 | -0.04±0.04 | 20 | 10 | 3± 36 | -48± 54 | -11± 35 | -50± 54 | 13± 7 | 3± 1 |
| 1999-06-06 | Clear | 1.58 | +0.23±0.04 | 27 | 14 | 19± 47 | -63± 43 | 91± 32 | -43± 40 | -71± 34 | -20± 4 |
| 1999-08-22 | Clear | 3.06 | +0.03±0.02 | 30 | 15 | -32± 37 | -7± 25 | -45± 35 | -3± 26 | 13± 11 | -3± 2 |
| All nights above | | | | | | -22± 69 | -36± 64 | -12± 66 | -32± 61 | -10± 35 | -4± 7 |
| | | | | | | Neptune-IAG | | | | | |
| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
| 2001-08-26 | B | 2.11 | +0.15±0.03 | 44 | 15 | 89± 55 | -41± 57 | 14± 41 | -22± 57 | 75± 36 | -19± 6 |
| 2002-07-15 | Clear | 6.11 | +0.18±0.00 | 57 | 17 | 51±163 | 85± 90 | -36± 31 | 131± 76 | 87±160 | -46± 20 |
| 2002-07-18 | Clear | 3.86 | +0.21±0.02 | 30 | 13 | -100±115 | -140± 61 | -50± 61 | -111± 61 | -50± 98 | -29± 3 |
| 2003-07-22 | Clear | 2.38 | +0.33±0.04 | 20 | 15 | 174±137 | -75± 80 | -32± 57 | -13± 74 | 206±125 | -62± 27 |
| 2003-07-23 | Clear | 4.08 | +0.02±0.01 | 39 | 16 | -81± 36 | 13± 55 | -78± 34 | 17± 55 | -4± 12 | -3± 1 |
| 2003-07-25 | Clear | 6.19 | -0.01±0.02 | 21 | 9 | -107± 74 | 9± 62 | -106± 74 | 7± 62 | -1± 8 | 2± 1 |
| 2003-07-26 | Clear | 6.97 | -0.01±0.08 | 17 | 10 | -132±211 | 8±123 | -129±211 | 7±124 | -3± 4 | 1± 1 |
| 2003-07-27 | Clear | 1.54 | +0.06±0.06 | 26 | 14 | -21±104 | 4± 84 | -90±102 | 24± 84 | 70± 20 | -20± 6 |
| 2003-07-28 | Clear | 7.98 | +0.01±0.03 | 43 | 12 | -37±144 | 21±162 | -44±144 | 24±162 | 7± 4 | -2± 1 |

Table 5 – *Continued from previous page*

| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
|------|--------|-----------|-----------|------|--------|-----------|-------------|---------|----------|-----------|-------------|
| 2003-08-20 | Clear | 4.03 | +0.26±0.04 | 30 | 17 | 95±154 | -72± 66 | 18± 98 | -35± 60 | 76±118 | -37± 16 |
| 2003-10-14 | V | 2.33 | +0.02±0.03 | 8 | 30 | 20± 25 | 16± 31 | 16± 25 | 18± 31 | 5± 5 | -2± 1 |
| 2004-08-05 | V | 2.21 | -0.03±0.12 | 5 | 6 | -53± 67 | -61± 25 | -35± 67 | -66± 26 | -18± 9 | 5± 2 |
| 2004-08-07 | V | 4.31 | +0.21±0.33 | 6 | 11 | 128±333 | -40±210 | 110±317 | -9±212 | 18±102 | -31± 8 |
| 2004-08-21 | Clear | 4.09 | +0.08±0.02 | 30 | 21 | -32± 57 | -78± 55 | -56± 44 | -66± 57 | 24± 36 | -12± 4 |
| 2004-08-21 | Clear | 2.57 | +0.05±0.02 | 30 | 21 | -42± 28 | -86± 41 | -32± 26 | -81± 41 | -10± 12 | -6± 1 |
| 2004-08-23 | Clear | 5.20 | +0.06±0.01 | 70 | 18 | 10± 59 | -88± 52 | -29± 42 | -73± 50 | 40± 42 | -14± 10 |
| 2004-08-24 | Clear | 3.94 | +0.06±0.01 | 40 | 18 | -2± 34 | -83± 66 | -23± 26 | -74± 65 | 20± 23 | -8± 4 |
| 2004-09-24 | R | 3.08 | +0.27±0.05 | 35 | 13 | 157±183 | 12±136 | -0±133 | 63±126 | 157±126 | -51± 28 |
| 2004-09-25 | Clear | 3.75 | +0.26±0.03 | 40 | 14 | 201±164 | 2± 93 | 37±106 | 54± 86 | 165±126 | -52± 32 |
| 2005-09-24 | V | 2.78 | +0.05±0.01 | 88 | 14 | 2± 47 | -101± 61 | -52± 44 | -85± 60 | 54± 17 | -16± 4 |
| 2006-06-08 | Clear | 2.68 | +0.32±0.04 | 95 | 24 | -91±144 | -83± 93 | 14±115 | -30± 93 | -105± 87 | -53± 8 |
| 2011-09-26 | I | 4.02 | +0.05±0.00 | 250 | 18 | 76± 44 | -67± 48 | 38± 36 | -52± 45 | 38± 25 | -15± 7 |
| 2012-10-19 | R | 1.99 | +0.13±0.03 | 119 | 8 | 41±100 | -167±137 | -50± 92 | -130±138 | 90± 38 | -37± 8 |
| All nights above | | | | | | 25±130 | -62±105 | -16± 91 | -37±105 | 41± 93 | -24± 21 |
| | | | | | Triton-IAG | | | | | | |
| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
| 2001-08-26 | B | 2.11 | +0.03±0.03 | 24 | 15 | 1± 40 | -53± 46 | -13± 39 | -50± 46 | 14± 7 | -4± 1 |
| 2002-07-15 | Clear | 6.11 | +0.01±0.00 | 55 | 17 | 14± 27 | -29± 45 | 7± 24 | -26± 45 | 7± 12 | -3± 2 |
| 2002-07-18 | Clear | 3.86 | -0.01±0.03 | 31 | 13 | 25± 74 | -68±111 | 23± 74 | -69±111 | 2± 4 | 1± 0 |
| 2003-07-22 | Clear | 2.55 | +0.08±0.02 | 31 | 17 | -8± 57 | -7± 53 | -58± 49 | 8± 54 | 50± 29 | -15± 7 |
| 2003-07-23 | Clear | 4.08 | +0.04±0.02 | 38 | 16 | -44± 54 | -12± 45 | -37± 50 | -6± 45 | -7± 21 | -6± 1 |
| 2003-07-25 | Clear | 6.39 | -0.04±0.03 | 44 | 16 | -60±138 | 9± 50 | -64±136 | 2± 51 | 5± 27 | 7± 2 |
| 2003-07-26 | Clear | 6.97 | -0.04±0.04 | 33 | 14 | -133±167 | 21± 90 | -121±165 | 13± 91 | -12± 30 | 8± 5 |
| 2003-07-27 | Clear | 1.54 | +0.01±0.05 | 26 | 14 | -31± 76 | 31± 74 | -40± 76 | 34± 74 | 9± 3 | -3± 1 |
| 2003-07-28 | Clear | 7.98 | -0.01±0.02 | 60 | 14 | -51±130 | 46±133 | -44±130 | 43±133 | -6± 10 | 3± 2 |
| 2003-08-20 | Clear | 4.20 | +0.02±0.02 | 49 | 21 | 53± 69 | -8± 45 | 46± 68 | -5± 45 | 8± 9 | -3± 1 |
| 2003-10-14 | V | 3.63 | -0.04±0.03 | 20 | 33 | -4± 54 | -34± 72 | 13± 53 | -40± 74 | -17± 14 | 5± 2 |

Table 5 – *Continued from previous page*

| Date | Filter | $\Delta H$ | $\Delta B$ | Nimg | Nstars | RA no corr | DEC no corr | RA corr | DEC corr | $\Delta$RA | $\Delta$DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2003-10-15 | V | 2.27 | +0.01±0.05 | 18 | 33 | -22± 38 | -3± 78 | -23± 38 | -2± 77 | 1± 1 | -1± 0 |
| 2003-10-16 | V | 1.94 | -0.10±0.10 | 8 | 25 | -18± 64 | -15± 36 | 46± 58 | -32± 32 | -64± 25 | 17± 6 |
| 2003-10-17 | V | 2.09 | +0.02±0.04 | 10 | 29 | 1± 31 | 4± 92 | -7± 31 | 7± 92 | 8± 4 | -2± 1 |
| 2003-10-19 | V | 1.89 | -0.12±0.05 | 12 | 31 | -7± 40 | -45± 35 | 44± 32 | -60± 32 | -51± 23 | 15± 4 |
| 2004-08-05 | V | 2.38 | -0.06±0.02 | 21 | 15 | -13± 31 | -101± 35 | 21± 26 | -111± 35 | -34± 17 | 10± 4 |
| 2004-08-06 | V | 3.27 | +0.06±0.07 | 22 | 16 | 53± 99 | -110± 50 | 45± 97 | -102± 50 | 8± 21 | -8± 2 |
| 2004-08-07 | V | 4.50 | +0.00±0.03 | 23 | 19 | 54± 56 | -117± 61 | 54± 56 | -117± 61 | 0± 1 | -0± 0 |
| 2004-08-20 | Clear | 3.81 | +0.03±0.01 | 16 | 24 | -5± 26 | -40± 24 | -15± 23 | -35± 23 | 11± 13 | -5± 2 |
| 2004-08-21 | Clear | 4.09 | +0.02±0.02 | 27 | 22 | -13± 41 | -32± 52 | -22± 39 | -28± 53 | 9± 11 | -4± 1 |
| 2004-08-21 | Clear | 2.57 | +0.01±0.02 | 26 | 21 | -13± 28 | -51± 35 | -12± 27 | -51± 35 | -1± 2 | -1± 0 |
| 2004-08-23 | Clear | 5.20 | +0.02±0.01 | 43 | 18 | -2± 44 | -36± 49 | -11± 42 | -32± 49 | 10± 11 | -4± 3 |
| 2004-08-24 | Clear | 3.94 | +0.02±0.01 | 29 | 17 | -2± 29 | -34± 62 | -9± 28 | -31± 62 | 8± 9 | -3± 1 |
| 2004-09-24 | R | 3.08 | +0.04±0.04 | 37 | 14 | 26±111 | 83±131 | 2±109 | 91±130 | 24± 19 | -8± 4 |
| 2004-09-25 | Clear | 3.75 | +0.01±0.04 | 36 | 14 | 59±113 | 91± 77 | 51±113 | 93± 77 | 7± 5 | -2± 1 |
| 2005-09-24 | V | 2.78 | +0.01±0.01 | 155 | 16 | -14± 43 | -95± 56 | -26± 43 | -91± 56 | 11± 4 | -3± 1 |
| 2006-06-08 | Clear | 3.30 | +0.10±0.03 | 157 | 26 | -22±104 | -47± 69 | 2±100 | -32± 69 | -24± 27 | -15± 2 |
| 2009-07-22 | Clear | 2.19 | +0.08±0.07 | 17 | 15 | 10± 43 | 74± 96 | -1± 41 | 87± 95 | 10± 12 | -13± 0 |
| 2011-09-05 | I | 1.82 | +0.19±0.03 | 100 | 18 | 91± 47 | -1± 34 | 36± 38 | 38± 36 | 55± 28 | -39± 3 |
| 2011-09-26 | I | 4.02 | -0.00±0.00 | 250 | 18 | 43± 28 | 4± 41 | 43± 28 | 3± 41 | -1± 0 | 0± 0 |
| 2012-10-19 | R | 1.99 | +0.13±0.03 | 118 | 8 | 14±108 | -76±134 | -78±100 | -38±136 | 93± 39 | -38± 9 |
| All nights above | | | | | | 7± 88 | -25± 87 | -4± 85 | -17± 87 | 11± 36 | -8± 14 |

# Results

Finally, from the offsets in the sense "position minus ephemeris" it was made statistics night by night to eliminate discrepant positions with a sigma-clip procedure where offsets (modulus) larger than 80 mas or 2.5-sigma discrepant from the mean offset were removed.

Fig 14 shows the mean ephemeris offsets of each night and respective discrepancy (error bars) for Neptune in RA and DEC, respectively.
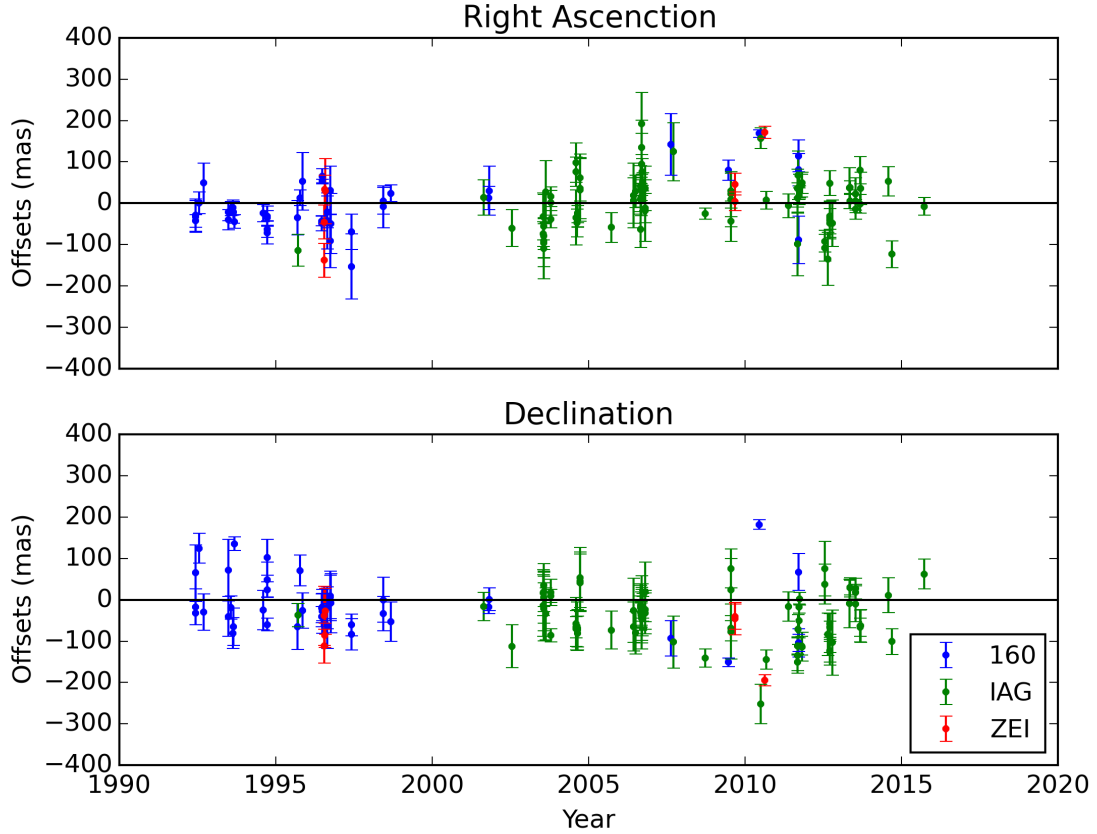


**Figure 14:** Neptune - Mean offsets by night. The plot shows the variation in the position of Neptune over time.

Fig. 23 shows the difference between the ephemeris offsets of Triton and Neptune over the Argument of Latitude, so that in fact we get the "observed minus ephemeris" offsets of the relative positions of Triton with respect to Neptune. As shown by Emelyanov and Samorodov (2015), the movement of Triton around Neptune has a peculiarity where the True Anomaly, or Mean Anomaly, since Triton has an almost circular orbit, oscillates around zero with an amplitude of about 17°. Emelyanov and Samorodov (2015) also shows that the Argument of Pericenter ($\omega$) of Triton's orbit rotates with an angular velocity approximately equal to the rotation of the satellite around the planet. These characteristics make the Argument of Latitude, sum of Mean Anomaly and Argument of Pericenter, more suitable to study the movement of Triton around Neptune.
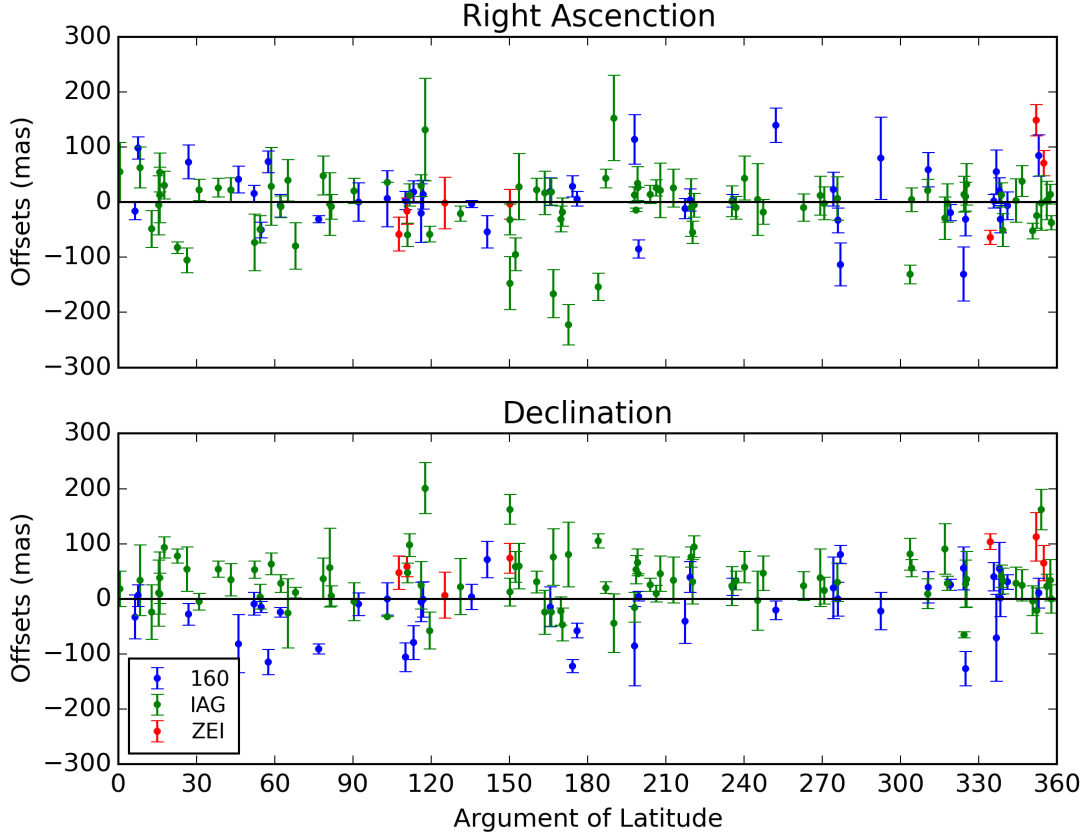
**Figure 15:** Difference between the offsets of Triton and Neptune by Argument of Latitude. It shows the variation in the position of Triton around Neptune.

In Appendix A it is shown the graphics of the difference in the offsets by Argument of Latitude separated for a time interval of 3 years. This is in order to see the variation of the position of Triton around Neptune in a smaller period, what can be masked in an overall graphic.

# Final tests

In order to consolidate the position data sets of Neptune and Triton prior to publication, we have two tests yet to perform.

a) Numerical PSF for Neptune:

Roberto Vieira Martins developed a numerical PSF for a circular extended object, based on the spread of a Gaussian PSF kernel over the apparent planet's disk. The effects of solar phase angle were not considered yet in this first numerical PSF model. I've already implemented this numerical PSF. I am testing it in a group of observations with negligible solar phase angle. The idea is to compare the results using this numerical PSF and the 2D Gaussian PSF, which was used in all observations.

If no significant improvements are achieved, we will stand with the 2D Gaussian PSF. This may be the case, as we are dealing with high S/N ratio images (Neptune) and PRAIA discards wing pixels in the (x,y) fits, so that in the end both PSFs may perform equally well. If the offsets of the observations tested are improved, the next step is to upgrade the
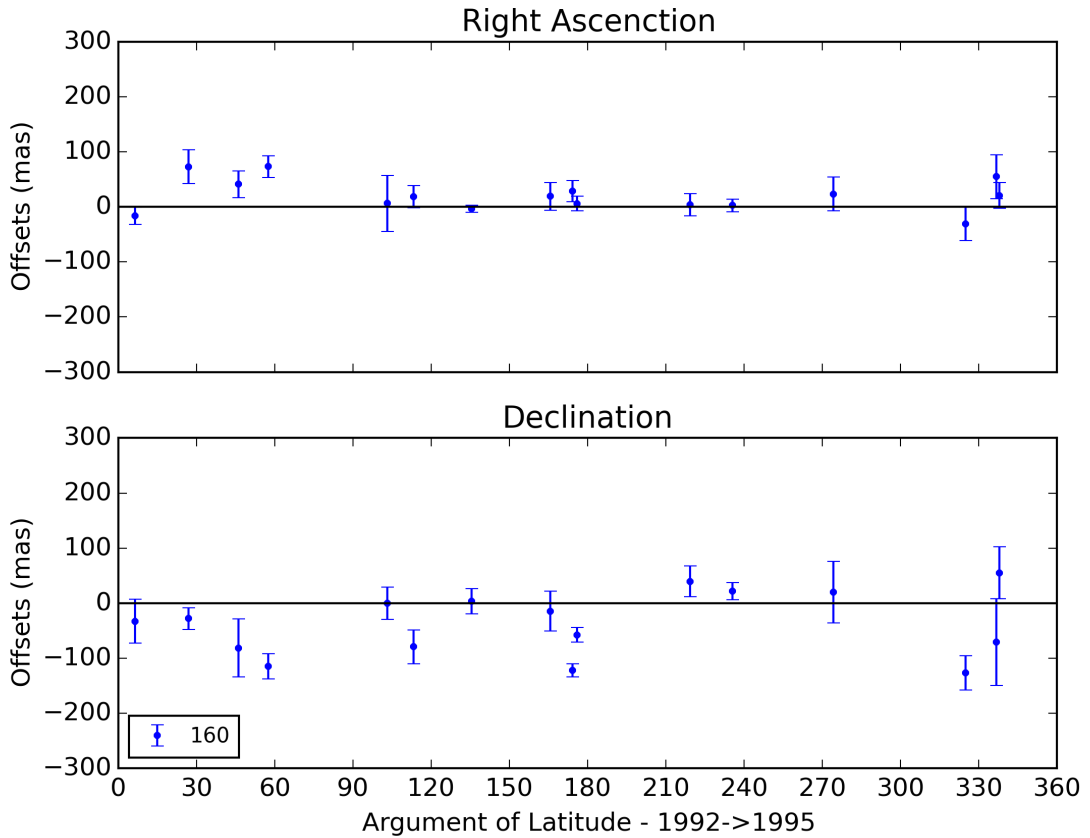
**Figure 17:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 1995 and 1998. It shows the variation in the position of Triton around Neptune.
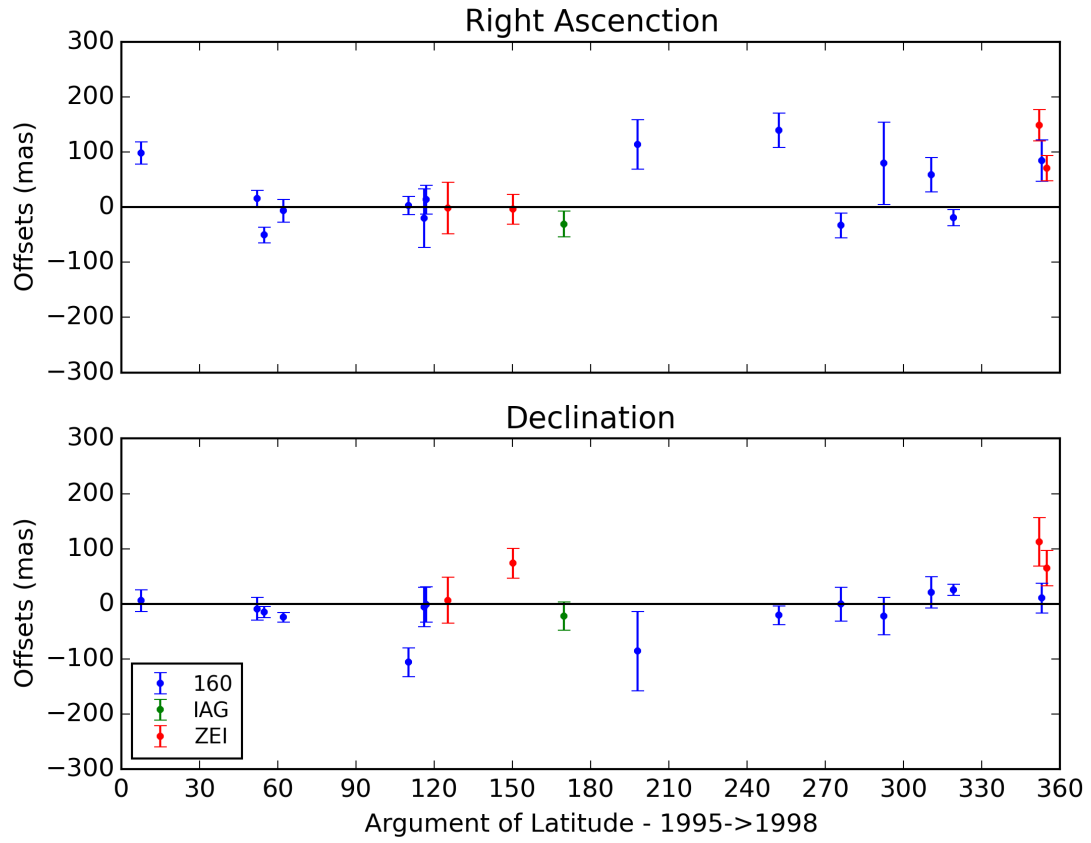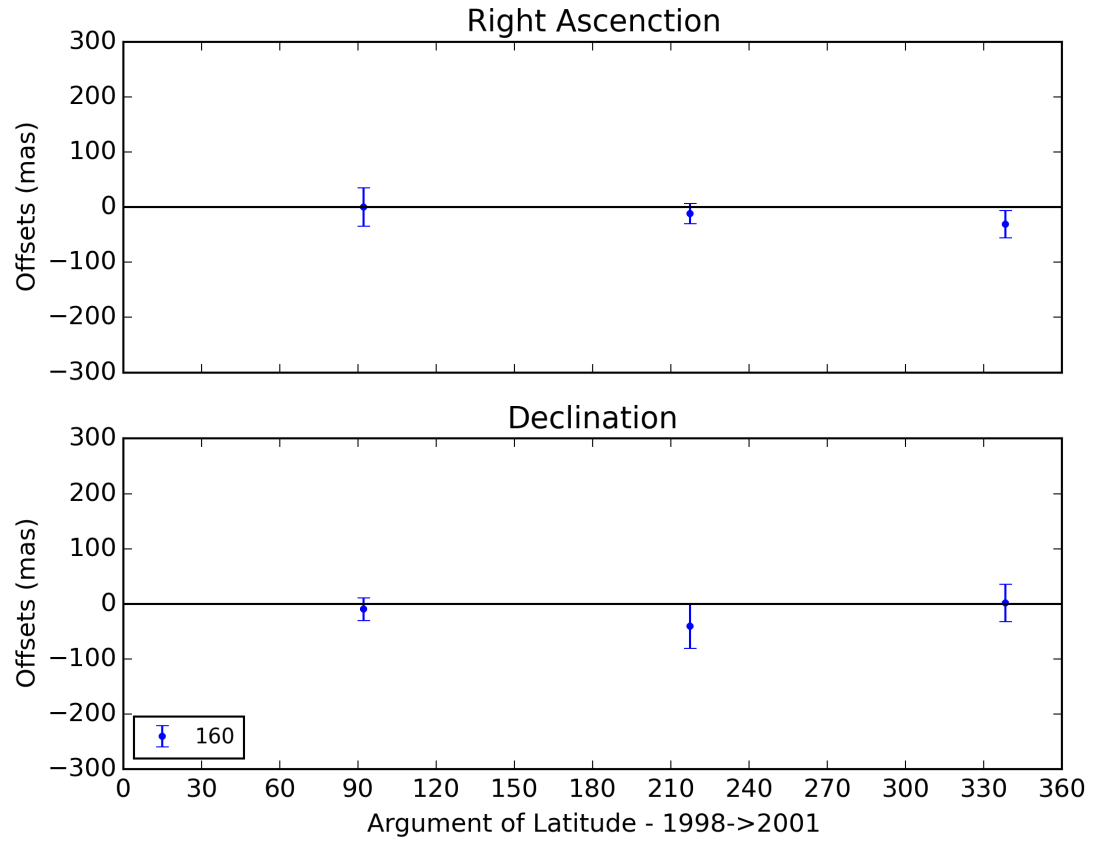
**Figure 18:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 1998 and 2001. It shows the variation in the position of Triton around Neptune.
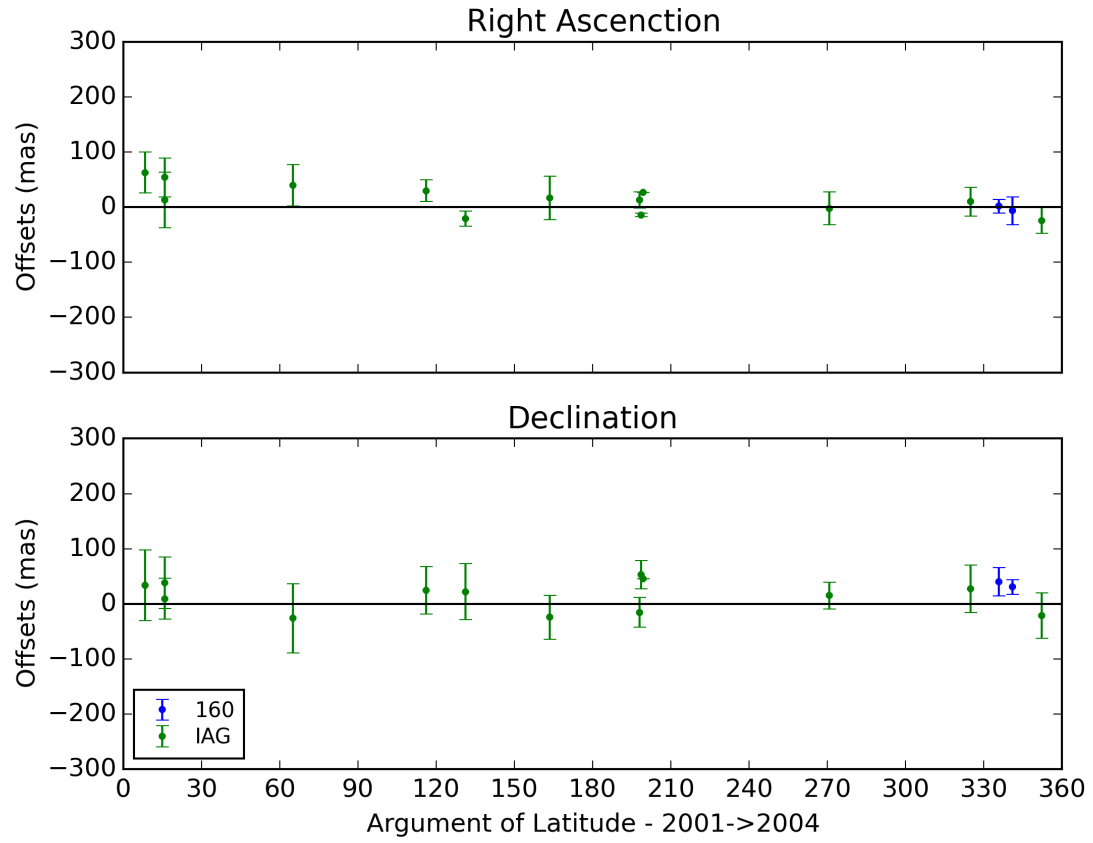
**Figure 19:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 2001 and 2004. It shows the variation in the position of Triton around Neptune.
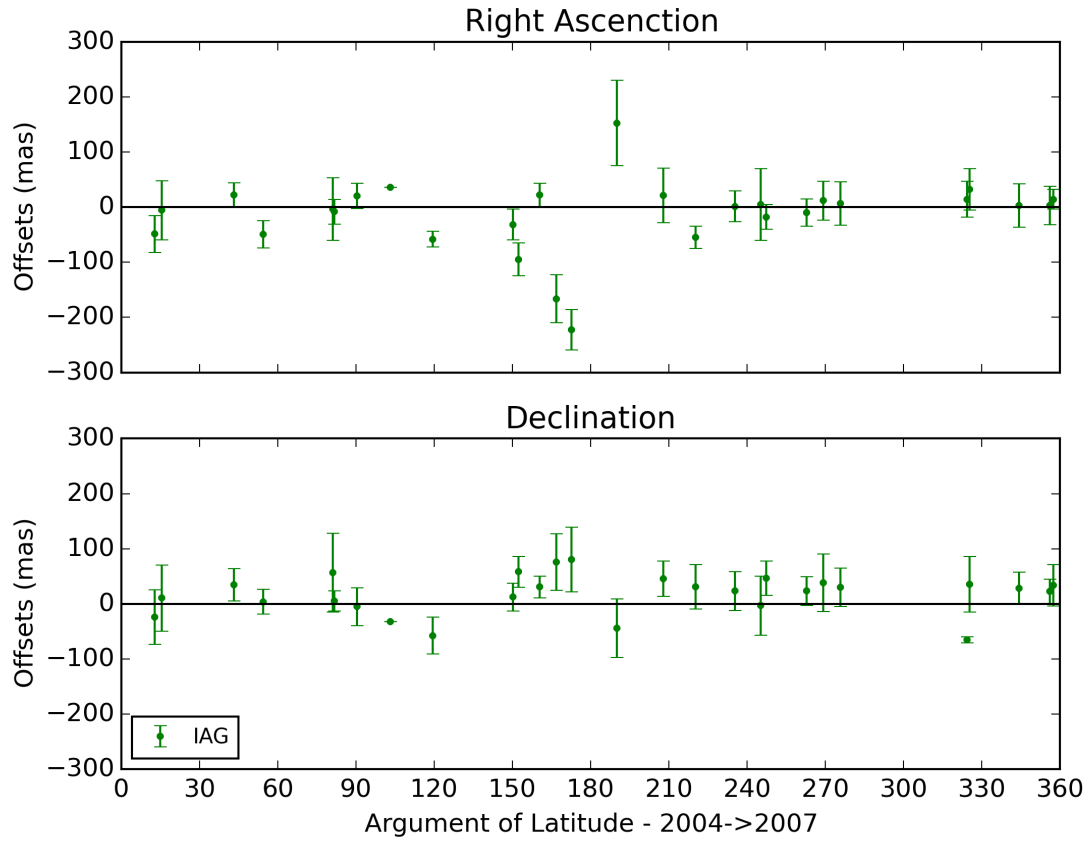
**Figure 20:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 2004 and 2007. It shows the variation in the position of Triton around Neptune.
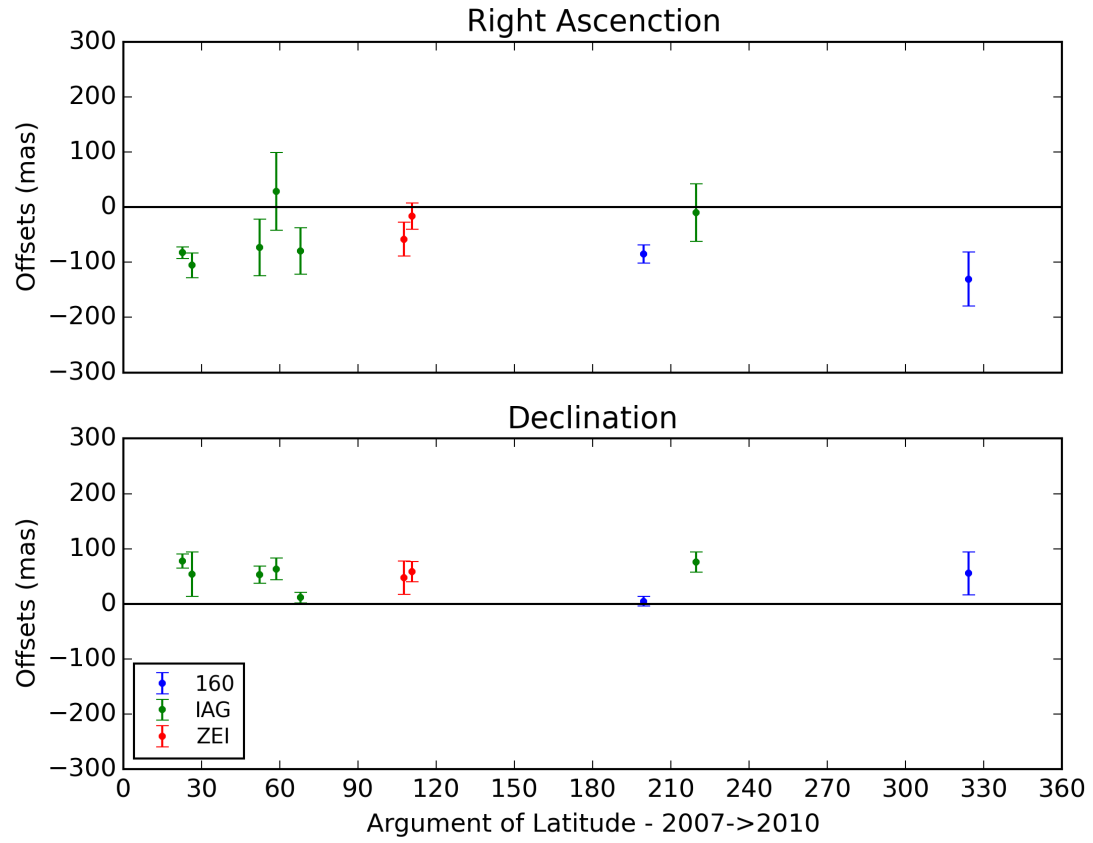
**Figure 21:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 2007 and 2010. It shows the variation in the position of Triton around Neptune.
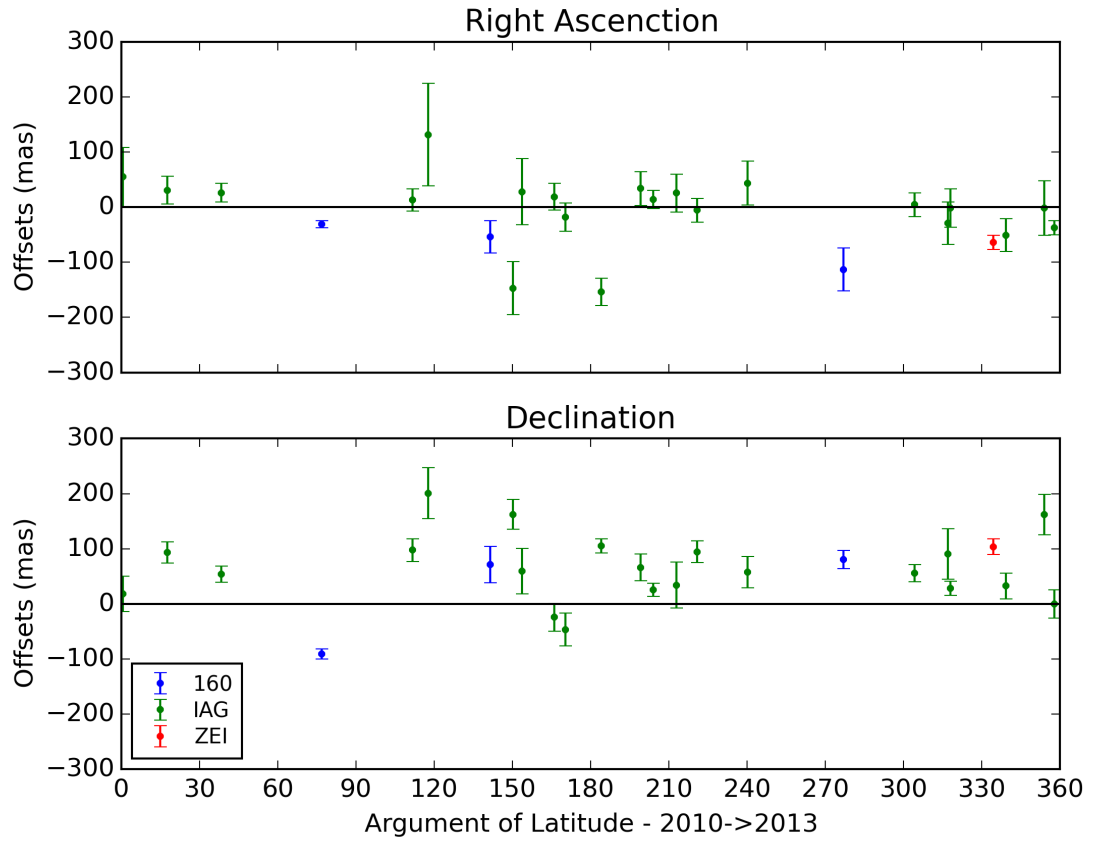
**Figure 22:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 2010 and 2013. It shows the variation in the position of Triton around Neptune.
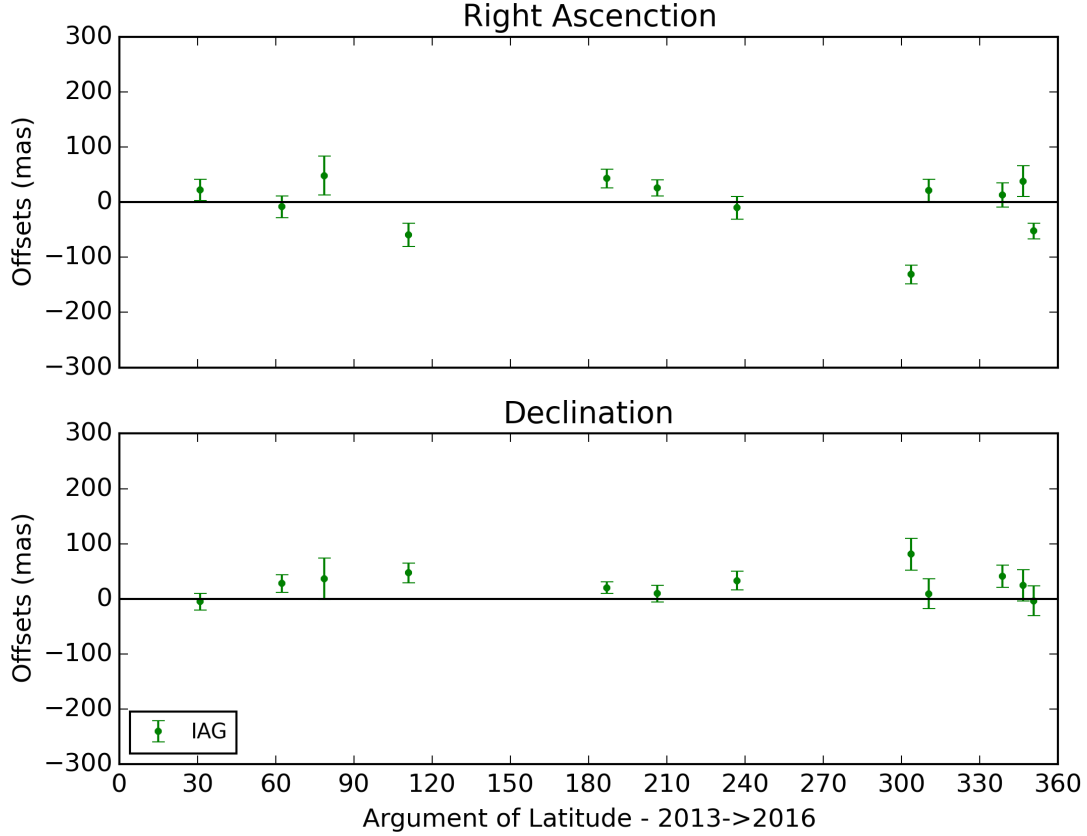
**Figure 23:** Difference between the offsets of Triton and Neptune by Argument of Latitude between 2013 and 2016. It shows the variation in the position of Triton around Neptune.

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