# Gravitational-Wave (GW) standard siren measurement of the Hubble constant

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2018.01.09

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# Background

1.1 History

1.2 Significance

1.3 Measurement

# 1.1 History

1922 Friedmann: universe might expand

1927 Lemaître: expanding universe &

estimated rate

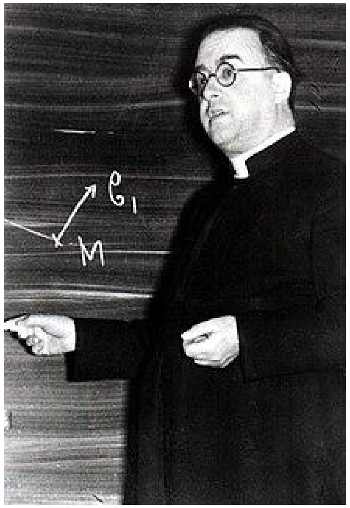
1929 Edwin Hubble

@ Mount Wilson Observetory

Galaxies nearby (d<50 Mpc)

Hubble: 24 Nebulae(galaxy) distances

Vesto Slipher & Milton Humason: redshifts



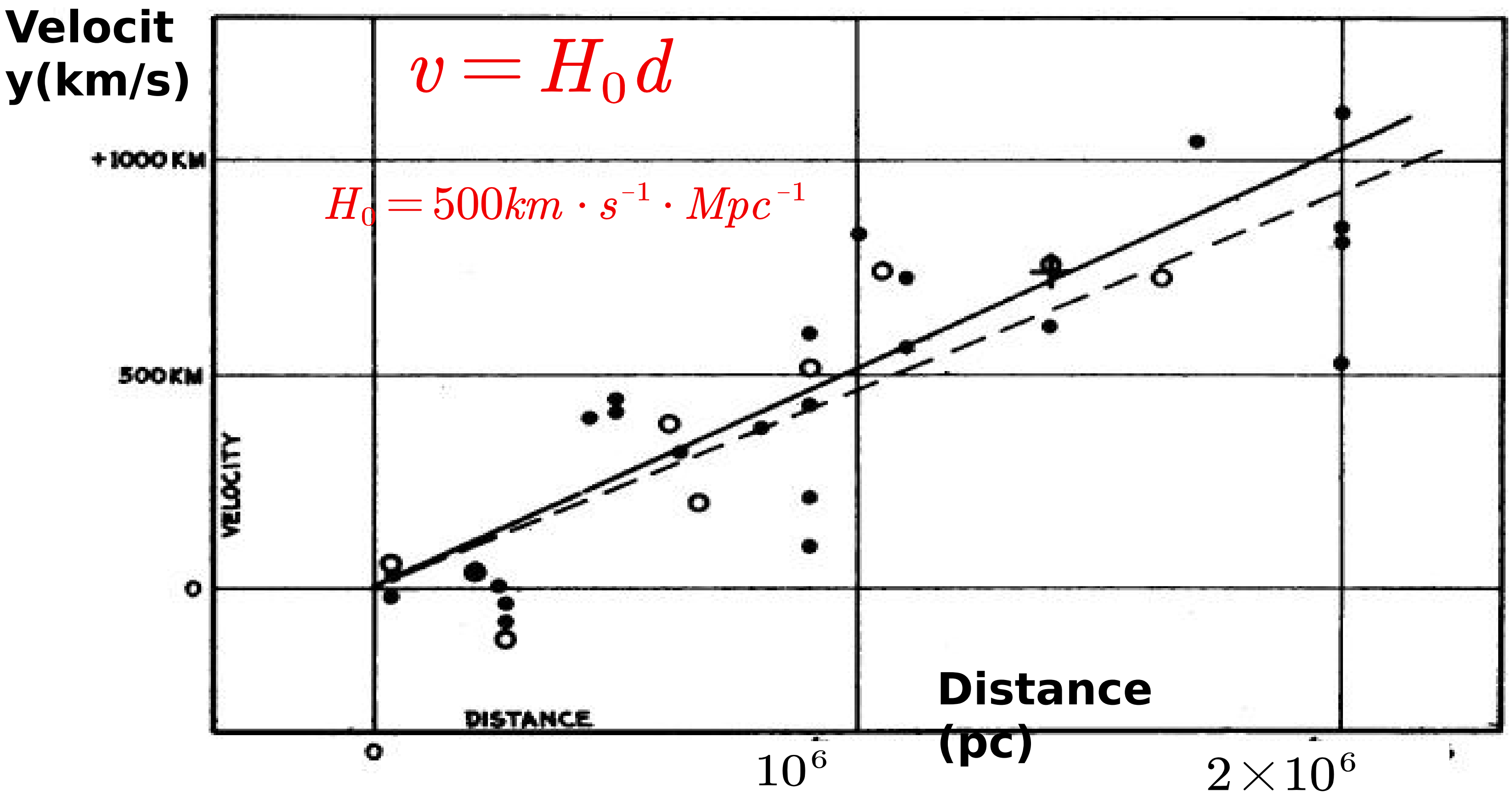
Alexander Friedmann (1888-1925)

Georges Lemaître (1894-1966)

Edwin Hubble (1889-1953)

pictures's credit: wiki

# 1.1 History



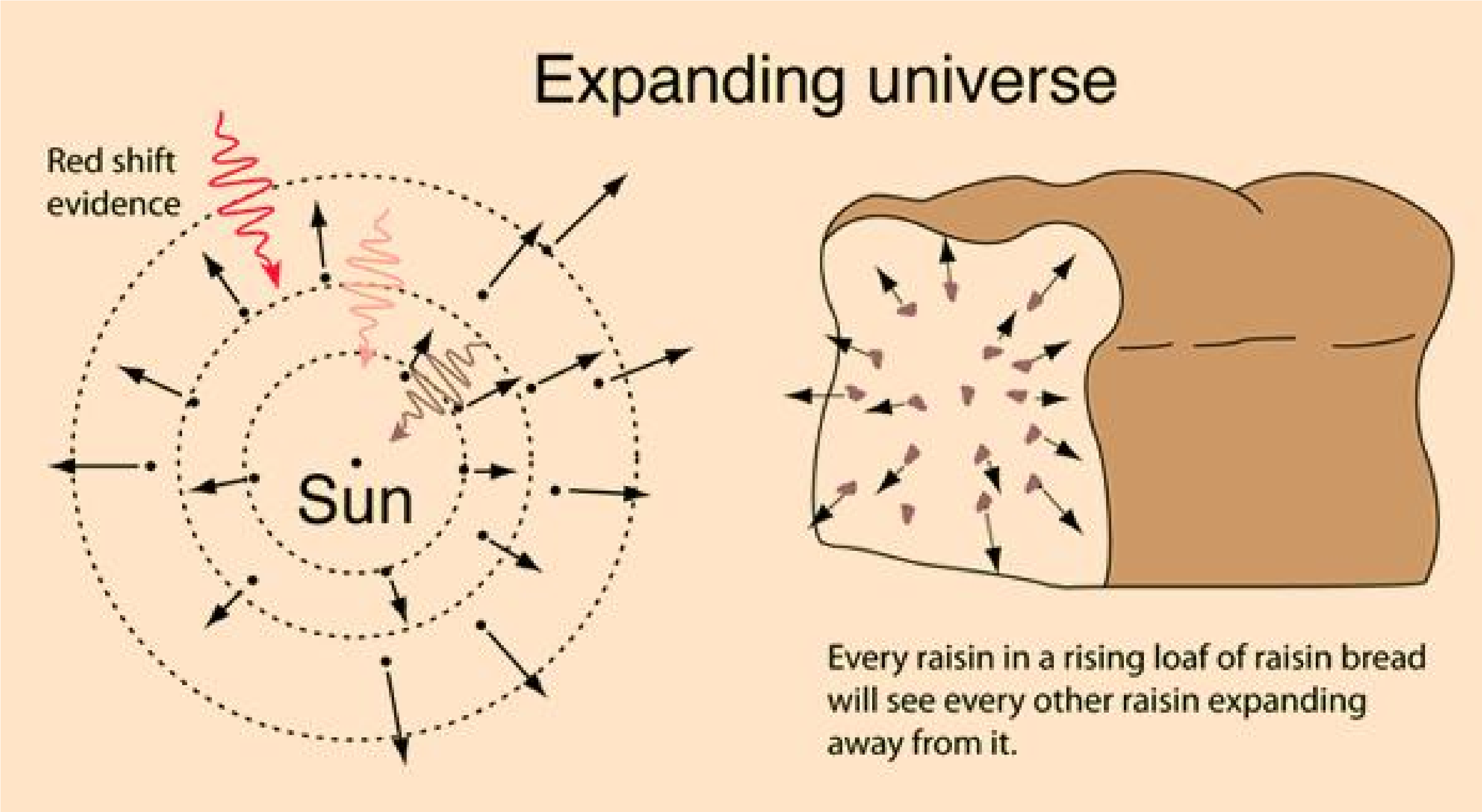

**Velocity-Distance Relation among Extra-Galactic Nebulae.**

E. Hubble A Relation Between Distance and Radial Velocity Among Extra-galactic Nebulae *Astronomy 15,* 168-173 (1929)

# 1.2 Significance

Hubble constant $H_0$ :

expansion rate of the universe



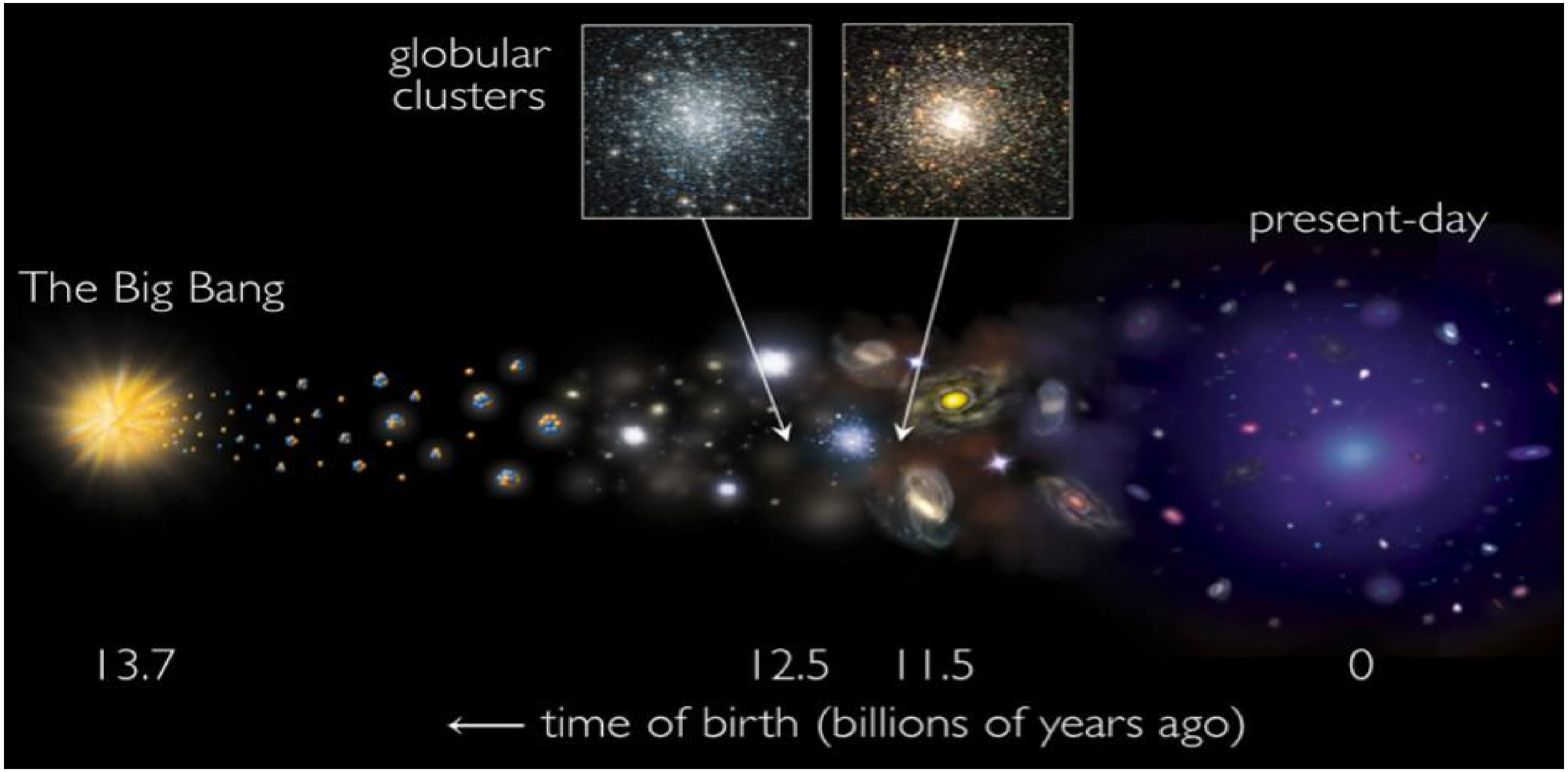

http://hyperphysics.phy-astr.gsu.edu/hbase/Astro/hubble.html

# 1.2 Significance

Hubble time $t_H$ : $t_H = \frac{1}{H_0} = \frac{1}{67.8km \cdot s^{-1} \cdot Mpc^{-1}} = 4.55 \times 10^{7} s \approx 14.4\ billion\ years$

A dimensionless factor

**Age of the universe ~** *13.7 billion years*




credit: astronomynow.com

# 1.2 Significance

Hubble length:

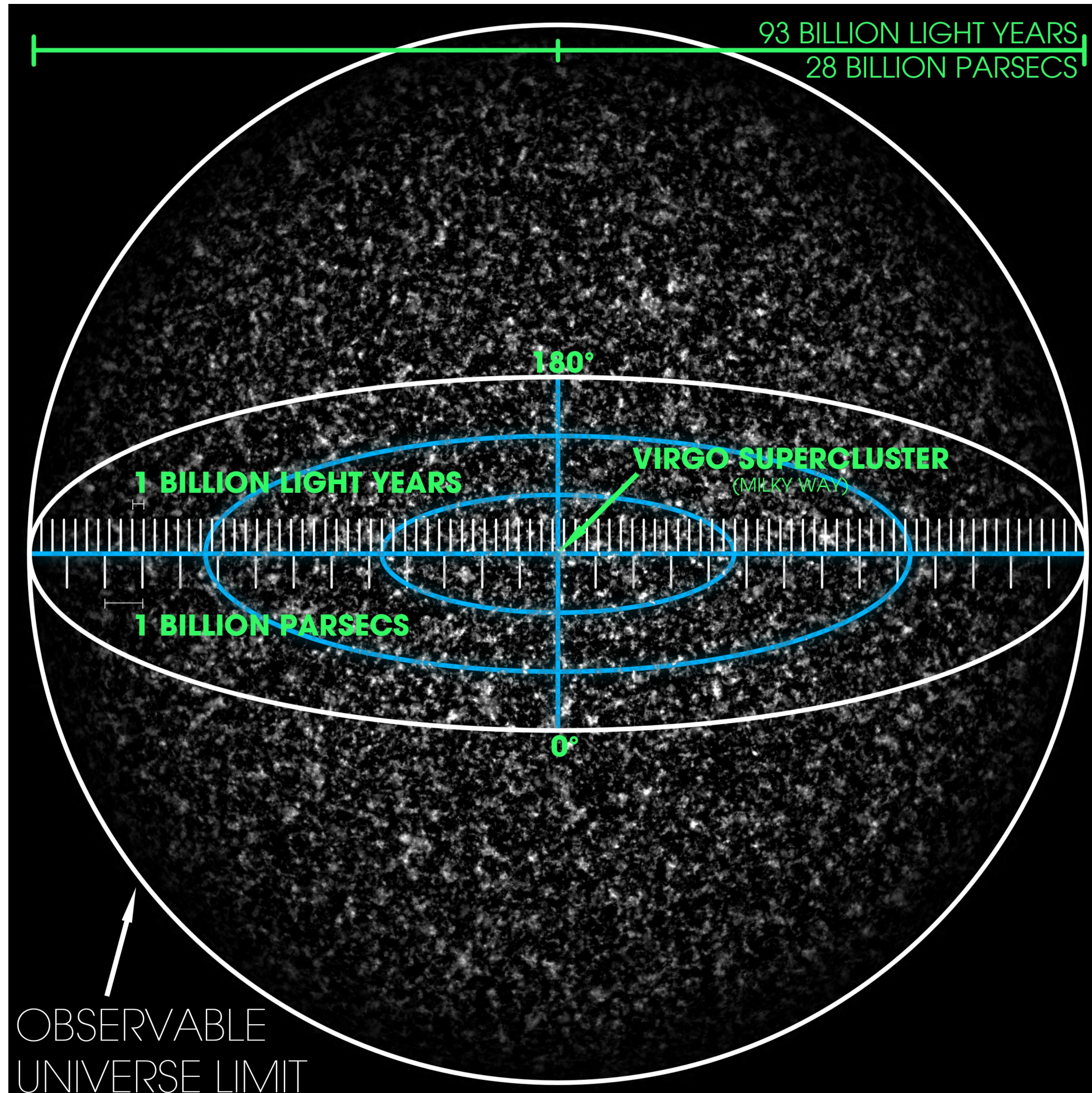
$$D = \frac{c}{H_0}$$

Hubble volume:

Sphere ($R = D$) or

Cube ($d = D$)

Observable universe




en.wikipedia.org/wiki/Hubble_v

# 1.3 Measurement

| **Measurement methods** | **Project** | **Shortcomings** |
| --- | --- | --- |
| CMB | Planck | |
| high-angular-multipole CMB | SPT | |
| type Ia supernova | SHoES | |
| Cepheid | HST key project | |
| Baryon Acoustic Oscillations | SDSS | not independent |
| Strong lensing | H0LiCOW | |

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# GW standard siren measurement

2.1 Basics

2.2 Measurement example

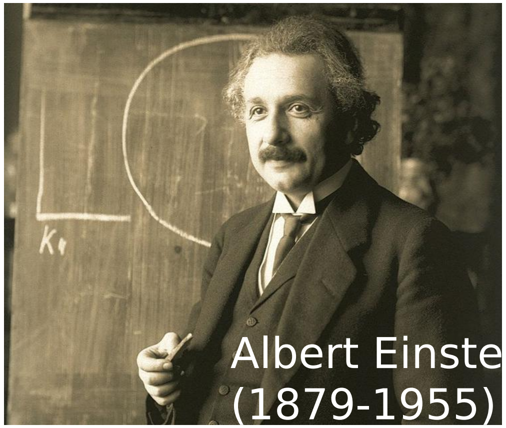
2.3 Results & discussion

# 2.1 Basics

1915 Albert Einstein

Einstein field equation:

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$



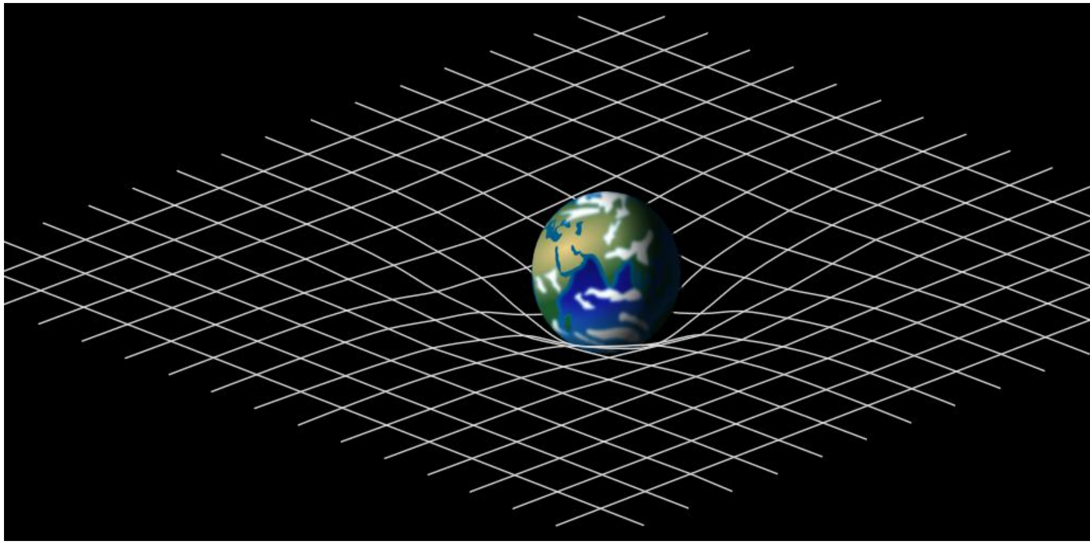

$G_{\mu\nu}$: *Einstein tensor*

$\lambda$: *cosmological constant*

$g_{\mu\nu}$: *metric tensor*

$G$: *Newton's gravitational constant*

$T_{\mu\nu}$: *stress-energy tensor*



credit:en.wikipedia.org/wiki/einstein_field_equations

# 2.1 Basics

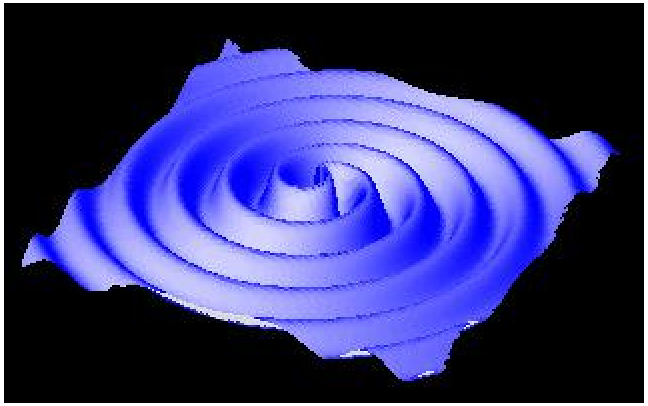
1916 Albert Einstein

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1$$

linerized weak-field equations: $h_{\mu\nu,\alpha}{}^{,\alpha} = -16\pi G T_{\mu\nu}$

wave solutions: $h_{\mu\nu} = C_{\mu\nu} \exp(i k_\sigma x^\sigma)$

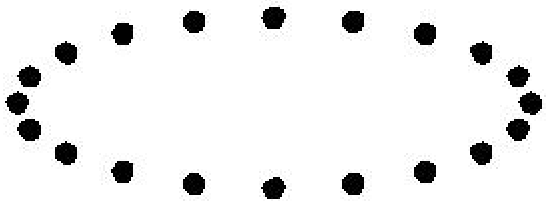


credit:wiki

# 2.1 Basics

GW sources: 1) Cosmological

2) Astrophical : isolated system



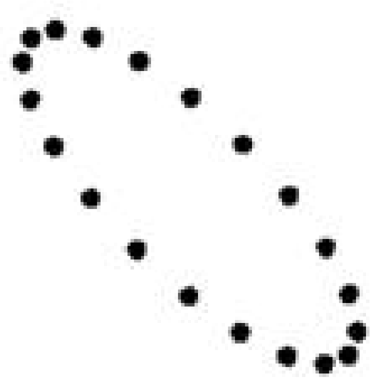
Effects of plus-polarized GW on particles

$$plus-polarized:$$

$$h_+(t) = \frac{2M_z}{D_L}\left[\pi M_z f(t)\right]^{2/3}\left[1+\left(\hat{L}\cdot\hat{n}\right)^2\right]\cos\left[\Phi(t)\right]$$

$$cross-polarized:$$

$$h_\times(t) = -\frac{4M_z}{D_L}\left[\pi M_z f(t)\right]^{2/3}\left(\hat{L}\cdot\hat{n}\right)\sin\left[\Phi(t)\right]$$

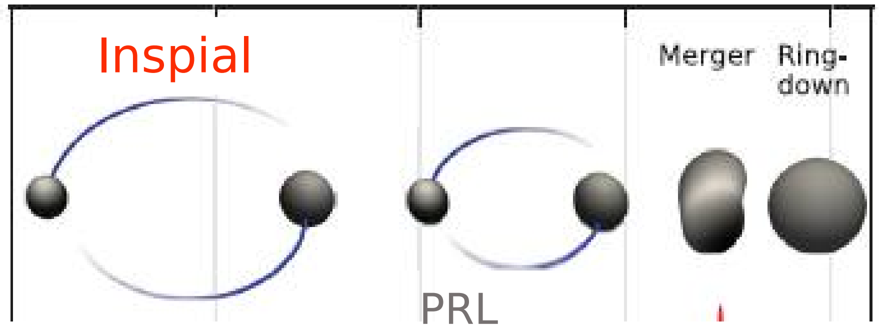


Effects of cross-polarized GW on particles

credit:wiki

# 2.1 Basics

Binary's merging stages




PRL 116,061102(2016)

$$GWs\ emitted\ by\ the\ source:$$

$$h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$$

$$response\ tensor\ for\ dector\ a:$$

$$D_a^{ij} = \frac{1}{2}\left[(\hat{x}_a)^i(\hat{x}_a)^j - (\hat{y}_a)^i(\hat{y}_a)^j\right]$$

# 2.1 Basics

Binary's merging stages:

$$response\ of\ dector\ a\ to\ a\ GW:$$

$$h_a = D_a^{ij} h_{ij} \equiv e^{-2\pi i (n \cdot x) f} \left( F_{a,+} h_+ + F_{a,\times} h_\times \right)$$

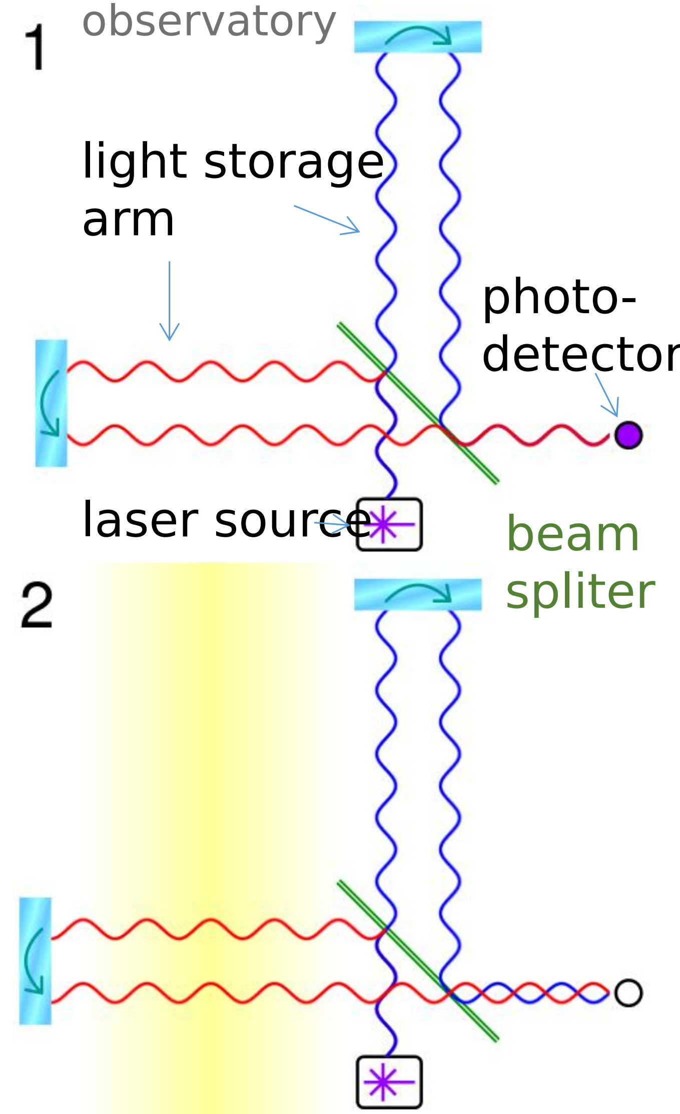
$$datastrem\ of\ detecor\ a:$$

$$s_a(t) = h_a\left(t; \hat{\theta}\right) + n_a(t)$$

data reduction

$$joint\ posterior\ of\ D_L, \cos\iota,$$

Simple operation of laser interferometer GW observatory




credit:wiki

# 2.1 Basics

for multiple dectors: $\vec{s} = (s_a(t), s_b(t), ....)$

Estimating f and ḟ from the data

$$M = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

$M_z$: *chirp mass*

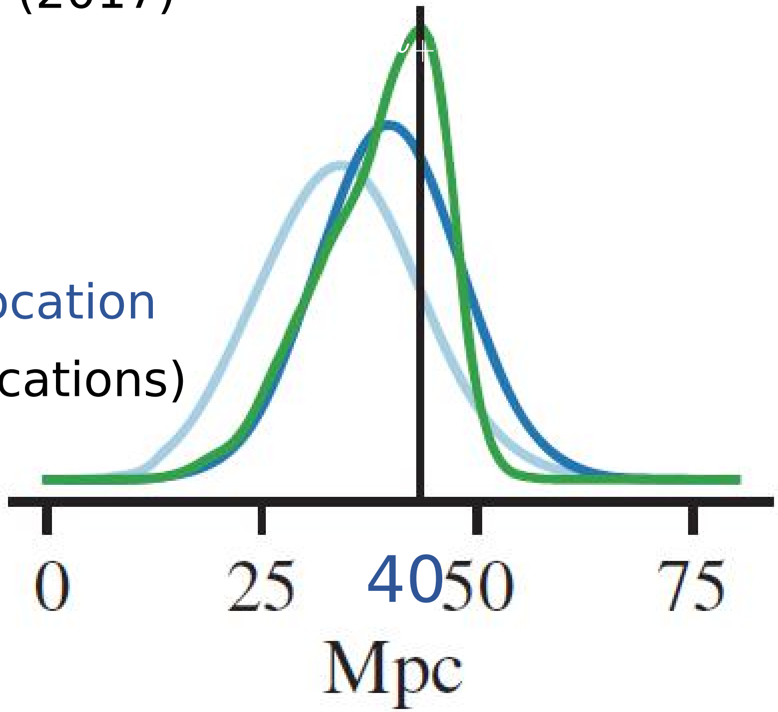
$f$: *frequency of GW*

$\dot{f}$: *time derivative of the frequency*

# 2.2 Measurement example

In Abbott, B.P. *et al*. *Nature* 551, 85-88 (2017)

$$D_L = 40^{+8}_{-14}\,Mpc$$




$$d = 43.8^{+2.9}_{-6.9}\,Mpc$$

(optical counterpart represents true location not marginalizing over potential sky locations)

consistent :

$$d_{TF} = 41.1 \pm 5.8\;\,Mpc$$

(Tully-Fisher relationship)

Posterior distributioon of 3 detector's localization analysis
credit: PRL 119,161101(2017)

# 2.2 Measurement example

NGC4993(GW170817's host galaxy):

group of galaxies ESO-508

$v_{ESO-508} = 3327 \pm 72 \ km \cdot s^{-1}$ (in frame of CMB)

NGC4993 = 40Mpc

correcting its measured recessional velocity for local peculiar velocity

# 2.2 Measurement example

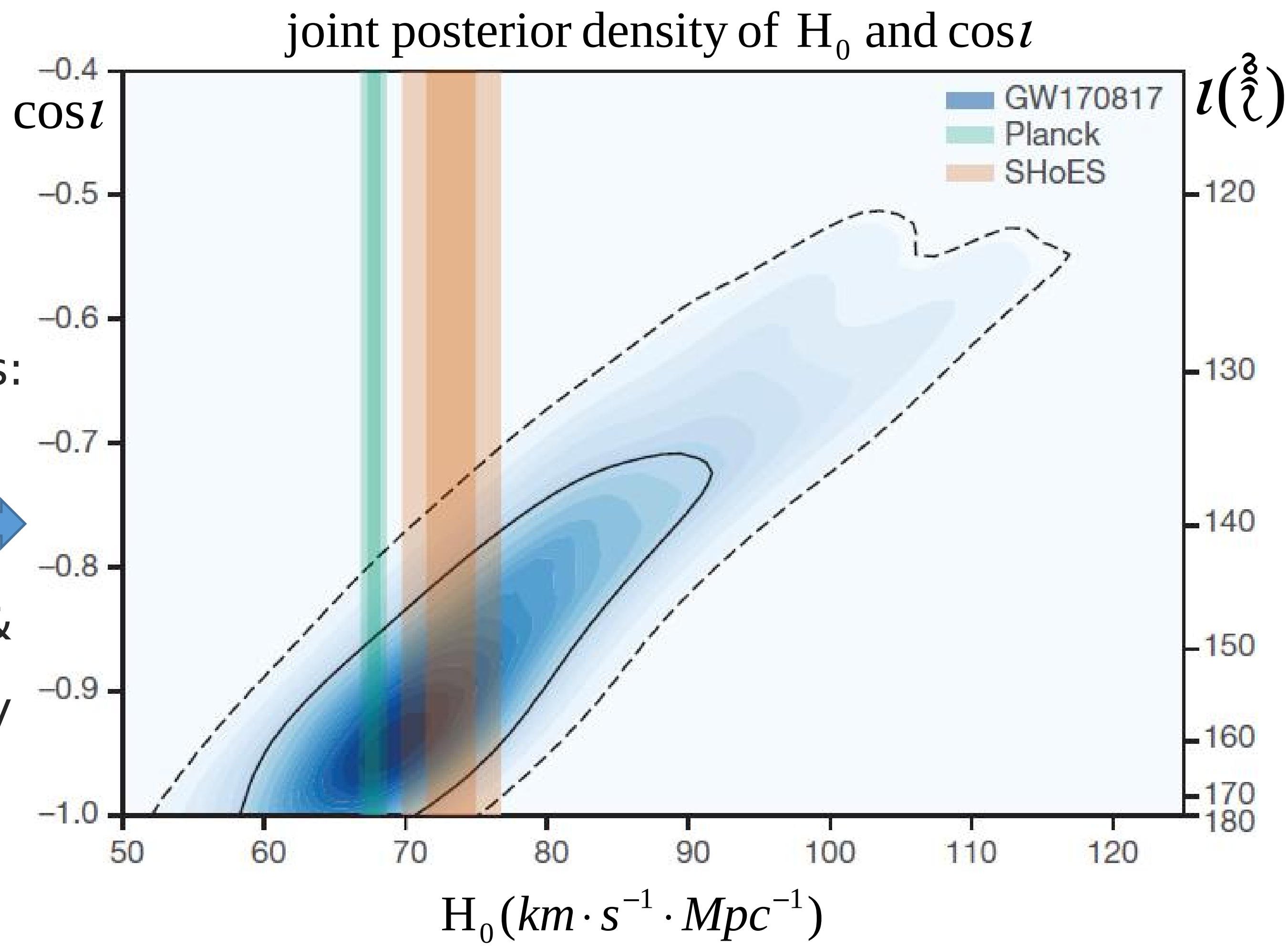
Bayesian analysis:

maginalized over

uncertainties of

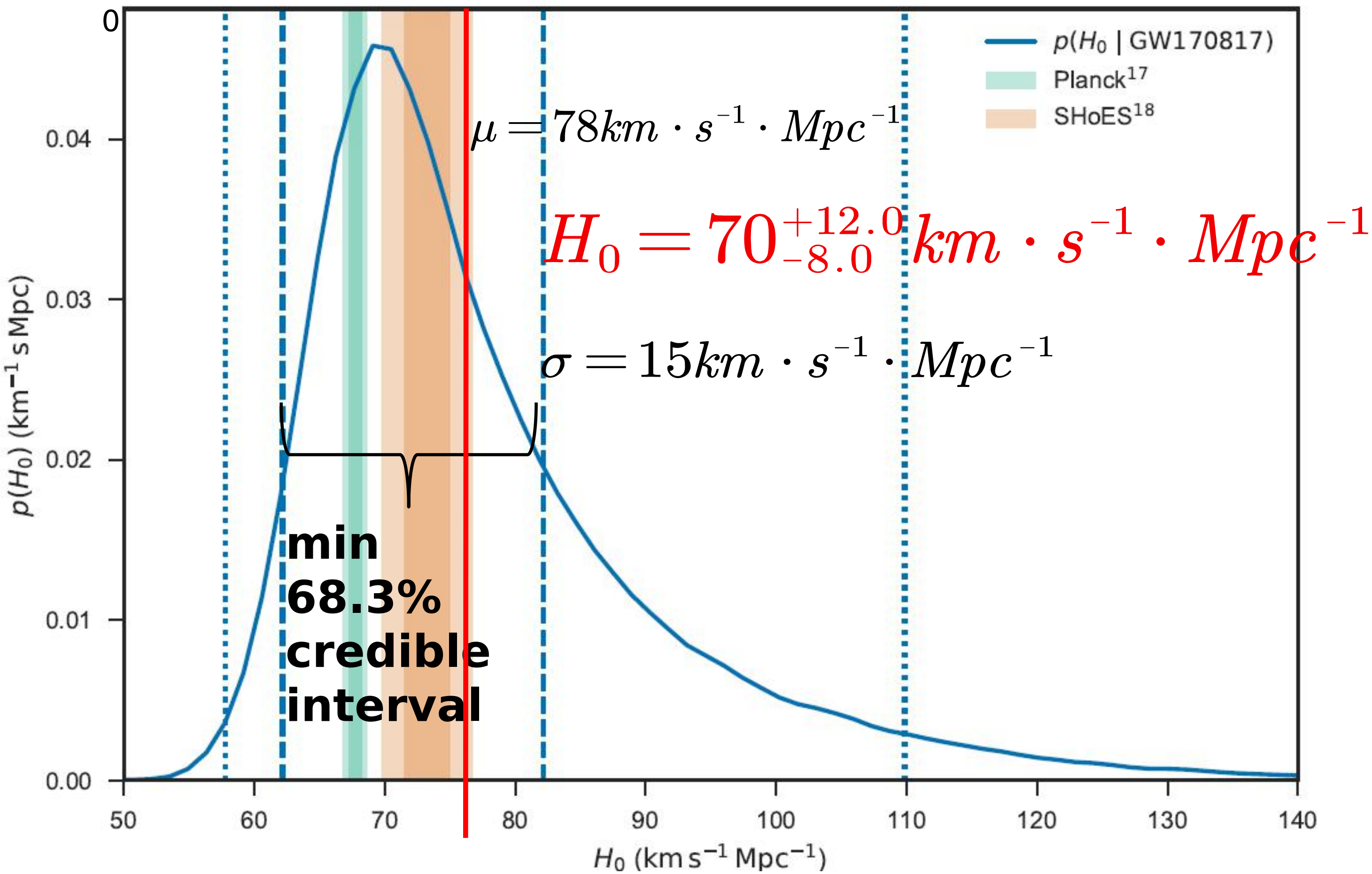
peculiar velocity &

recesional velocity



# 2.3 Results & discussion

The marginalized posterior density for $H_0$
$p(H_0|GW170817)$




credit: Abbott, B.P. et al. *Nature* 551, 85-88 (2017)

# 2.3 Results & discussion

| Measurement methods | Project | Date | $H_0$(km·s$^{-1}$·Mpc$^{-1}$) |
|---|---|---|---|
| GW standard siren | LIGO & Virgo | 2017.10 | $70^{+12.0}_{-8.0}$ |
| CMB | Planck | 2015.02 | $67.74 \pm 0.46$ |
| high-angular-multipole CMB | SPT | 2017.08 | $71.2 \pm 2.1$ |
| type Ia supernova | SHoES | 2016 | $73.24 \pm 1.74$ |
| Cepheid | HST key project | 2001. | $72 \pm 8$ |
| Baryon Acoustic Oscillations | SDSS | | $67.3 \pm 1.1$ |
| Strong lensing | H0LiCOW | 2017.01 | $71.9^{+2.4}_{-3.0}$ |

$\sigma$

# 2.3 Results & discussion

1. Agree well with popular measurement methods

2. No systematic differences between GW-based estimates & EM-based estimates

3. Planck and ShoES: inconsistent at a level greater than 3$\sigma$
This GW measurement: not solving this problem (broadly consistent with both)

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# Summary

3.1 Comments

3.2 Outlook

# 3.1 Comments

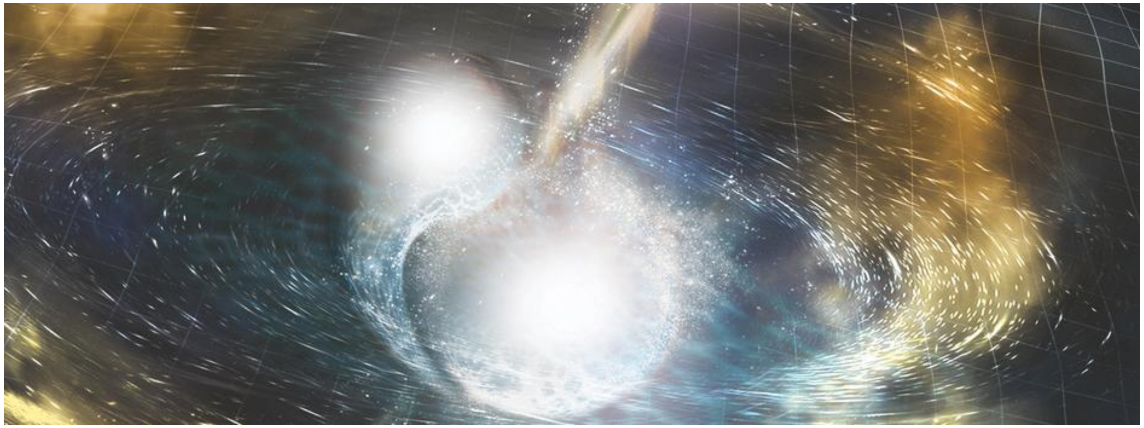
1. First GW standard siren measurement of $H_0$
2. An independent determination of $H_0$
3. Not using a distance ladder
4. Demonstrating the potential for cosmological inference from GW standard sirens

# 3.2 Outlook

1.More multi-messaenger binary neutron-star events be detected

10 events(100Mpc): Hubble constant to 3% accuracy

2.Leading to an era of precision gravitational-wave cosmology



# *Thank you !*

End

References:

[1] Abbott, B.P. et al. A gravitational-wave standard siren measurement of the Hubble constant *Nature* 551(2017):85-88
[2] Nissanke, S. *et al*. Determining the Hubble constant from gravitatioal wave obervations of merging compact binaries arxiv.org/abs/1307.2638 (2013)
[3] Nissanke, Samaya. *et al.* Exploring short Gamma-ray bursts as gravitational- wave standard sirens *ApJ* 725.1(2010):496-514
[4] Holz, D. E. & Hughes, S. A. Using gravitational wave standard sirens *ApJ.* 629(2005):15-22
[5] Schutz, B. F. Determining the Hubble constant from gravitational wave obserbations *Nature* 323 (1986):310-311