First Order System

Transfer function

$$H(s) = \frac{b_0}{s + a_0}, \quad a_0 \neq 0$$

Step Response

Frequency response:

$$Y(s) = H(s)U(s)$$

$$= \frac{b_0}{s + a_0} \times \frac{1}{s}$$

$$= \frac{b_0}{a_0}\left(\frac{1}{s} - \frac{1}{s + a_0}\right)$$

Taking the Laplace transform:

$$y(t) = \frac{b_0}{a_0}\left(1 - e^{-a_0 t}\right), \quad t \geq 0$$

Performance Measures

Rise Time, t_r

Rise time is the time required to go from 10% to 90% of the final value.

$$t_r = \frac{ln(9)}{a_0}$$

Settling Time, t_s

Settling time is the time required to get within 2% of the final value and stay there.

$$t_s \approx \frac{3.91}{a_0}$$

Time Constant, τ

Time constant is the time at which output becomes $\frac{b_0}{a_0}(1 - e^{-1})$, i.e. roughly 63% of the final value.

$$\tau = \frac{1}{a_0}$$

Bandwidth, ω_{BW}

Bandwidth is the frequency where,

$$|H(j\omega_{BW})| = \frac{1}{\sqrt{2}} H(0)$$

For a first order system,

$$\omega_{BW} = a_0$$