Energy Signals

The **Average Energy** E_x of a signal $x(t)$ is defined as,

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

If $0 < E_x < \infty$, then $x(t)$ is an **Energy Signal.**

Power Signals

The **Average Power** P_x of a signal $x(t)$ is defined as,

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

If $0 < P_x < \infty$, then $x(t)$ is a **Power Signal.**

NOTE: a signal cannot be both an energy and a power signal. It can, however, be neither.

Energy Spectral Density

The **Energy Spectral Density** $\Psi_x(f)$ of a signal $x(t)$ is defined as,

$$\Psi_x(f) = |X(f)|^2$$

Power Spectral Density

The **Power Spectral Density** $S_x(f)$ of a signal $x(t)$ is defined as,

$$S_x(f) = \lim_{T \to \infty} \frac{1}{T} |X_T(f)|^2$$

where, $X_T(f)$ is the Fourier transform of $x_T(t)$ and,

$$x_T(t) = x(t)\Pi(\frac{t}{T})$$

In other words, $x_T(t)$ is the function that has value $x(t)$ for $-\frac{T}{2} < t < \frac{T}{2}$ and 0 for all other values of t:

$$x_T(t) = \begin{cases} x(t), & if \ -\frac{T}{2} < t < \frac{T}{2} \\ 0, & otherwise \end{cases}$$

Periodic Signals

In the special case where $x(t)$ is a periodic signal, the Average Power is defined as,

$$P_x = \sum_{n=-\infty}^{\infty} |X_n|^2$$

and the Power Spectral Density is defined as,

$$S_x(f) = \sum_{n=-\infty}^{\infty} |X_n|^2 \delta(f - \frac{n}{T})$$

Linear Time-Invariant System

For a linear time-invariant system $y(t)$ with input $x(t)$ and impulse response $h(t)$,

$$S_y(f) = S_x(f)|H(f)|^2$$

Autocorrelation Function

The **Autocorrelation** $R_x(T)$ of a signal $x(t)$ is defined as,

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^*(t)x(t+\tau)dt$$

Periodic Signals

In the special case where $x(t)$ is a periodic signal, the Autocorrelation is defined as,

$$R_x(\tau) = \sum_{n=-\infty}^{\infty} |X_n|^2 e^{j\frac{2n\pi}{T}\tau}$$

Wiener-Khinchin Theorem

The inverse Fourier transform of the Power Spectral Density of signal $x(t)$ is equal to the Autocorrelation of signal $x(t)$:

$$R_x(\tau) = \mathcal{F}^{-1}\{S_x(f)\}$$

Linear Time-Invariant System

For a linear time-invariant system $y(t)$ with input $x(t)$ and impulse response h(t),

$$R_y(\tau) = R_x(\tau) * h(\tau) * h^*(-\tau)$$

Properties

Symmetry

- $R_x^*(\tau) = R_x(-\tau)$

Mean-Squared Value

- $R_x(\tau)|_{\tau=0} = P_x$

Periodicity

- If $x(t)$ is periodic with time period T, then $R_x(\tau + T) = R_x(\tau)$

Maximum Value

- $|R_x(\tau)| \leq R_x(0)$

Crosscorrelation Function

The **Crosscorrelation Function** $R_{xy}(\tau)$ of two signals $x(t)$ and $y(t)$ is defined as,

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^*(t)y(t+\tau)dt$$

The Crosscorrelation Function measures the *similarity* between two signals. $R_{xy}(\tau) = 0$ implies $x(t)$ and $y(t)$ are uncorrelated.