Laplace Transforms

Definition

Let $x(t)$ be a continuous-time signal and let s be a complex number such that $s = \sigma + j\omega$. Then,

$$\mathcal{L}\{x(t)\} = X(s) = \int_0^\infty x(\tau)e^{-s\tau}d\tau$$

where $\mathcal{L}\{x(t)\} = X(s)$ is the **Laplace Transform** of $x(t)$.

Properties

Linearity

- $\mathcal{L}\{c_1x_1(t) + c_2x_2(t)\} = c_1\mathcal{L}\{x_1(t)\} + c_2\mathcal{L}\{x_2(t)\}$

Exponential Shift

- $\mathcal{L}\{e^{\alpha t}x(t)\} = X(s - \alpha)$

Derivatives

- $\mathcal{L}\{\frac{d}{dx}(x(t))\} = sX(s) - x(0)$

Integrals

- $\mathcal{L}\{\int_0^t x(\tau)\,d\tau\} = \frac{X(s)}{s}$

Convolution

- $\mathcal{L}\{(x * y)(t)\} = X(s)Y(s)$

Initial Value Theorem

- $x(0) = \lim\limits_{s\to\infty} sX(s)$

Final Value Theorem

- $\lim\limits_{t\to\infty} x(t) = \lim\limits_{s\to 0} sX(s)$

Condition: all poles of $sX(s)$ must have strictly negative real parts.

Laplace Transform Table

Signal $x(t)$ **for** $t \geq 0$	**Laplace Transform** $X(s)$
$\delta(t)$	1
$\delta^{(n)}(t)$	s^n
$u(t)$	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{\alpha t} f(t)$	$F(s-\alpha)$
$e^{\alpha t}$	$\frac{n!}{(s-\alpha)^{n+1}}$
$sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
$cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$