Bode Plots

Bode Form

Consider a transfer function $H(s)$ with real poles and zeros:

$$H(s) = K\frac{(s+z_1)(s+z_2)\dots(s+z_m)}{(s+p_1)(s+p_2)\dots(s+p_n)}$$

This can be written in **Bode form**:

$$H(s) = K_0\frac{(\frac{s}{z_1}+1)(\frac{s}{z_2}+1)\dots(\frac{s}{z_m}+1)}{(\frac{s}{p_1}+1)(\frac{s}{p_2}+1)\dots(\frac{s}{p_n}+1)}$$

where K_0 is the DC gain,

$$K_0 = K\frac{z_1 z_2 \dots z_m}{p_1 p_2 \dots p_n}$$

Bode Plots

Constant

Let $H(s) = K_0$. Then,

$$20\ log|H(j\omega)| = 20\ log|K_0|$$

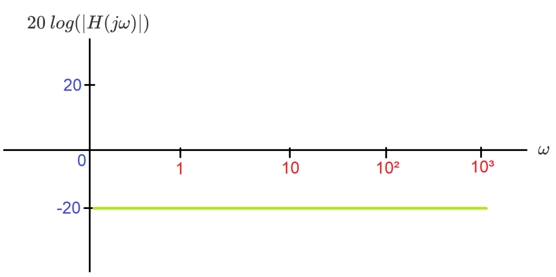
$$\angle H(j\omega) = \begin{cases} 0 & if\ K_0 > 0 \\ \pi & if\ K_0 < 0 \end{cases}$$

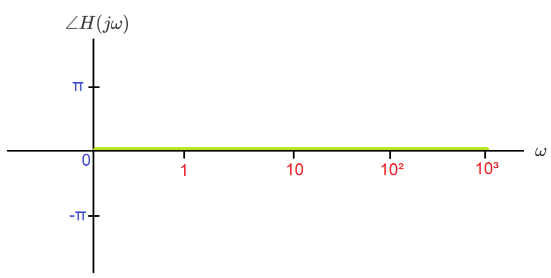
Example: Bode Plot for $\frac{1}{10}$:

$$20\ log(\frac{1}{10}) = -20$$

$$\angle\frac{1}{10} = 0$$

Bode plot:





Zeros & Poles at origin

Let $H(s) = s^q$. Then,

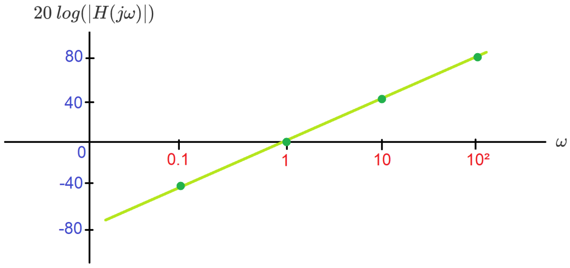
$$20\ log|H(j\omega)| = 20q\ log|j\omega| = 20q\ log|\omega|$$

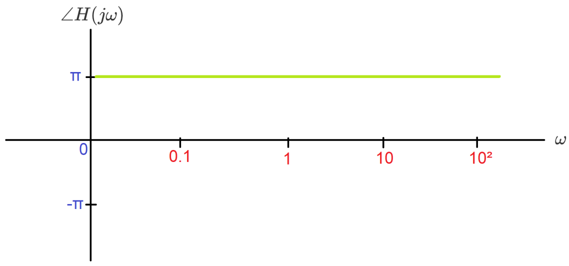
$$\angle(j\omega)^q = q\angle j\omega = q\frac{\pi}{2}$$

Example: Bode Plot for s^2:

$$20\ log|(j\omega)^2| = 40\ log|\omega|$$

$$\angle(j\omega)^2 = \pi$$





Real Zeros & Poles

Let $H(s) = (\frac{s}{z} + 1)^{\pm 1}$. Then,

$$20\ log|(j\frac{\omega}{z} + 1)^{\pm 1}| = \pm 20\ log|j\frac{\omega}{z} + 1| = \pm 20\ log\sqrt{1 + (\frac{\omega}{z})^2}$$

$$\approx \begin{cases} 0 & if\ \omega \ll z \\ \pm 20\ log(\omega) \pm 20\ log(\frac{1}{z}) & if\ \omega \gg z \end{cases}$$

For convenience, we draw the magnitude as a linear transition of slope ± 20 dB/decade starting at $\omega = z$.

$$\angle(j\frac{\omega}{z} + 1)^{\pm 1} = \pm\, tan^{-1}(\frac{\omega}{z})$$

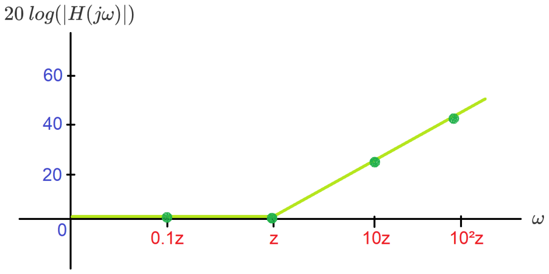
$$\approx \begin{cases} 0 & if\ \omega \ll z \\ \pm\frac{\pi}{4} & if\ \omega = z \\ \pm\frac{\pi}{2} & if\ \omega \gg z \end{cases}$$

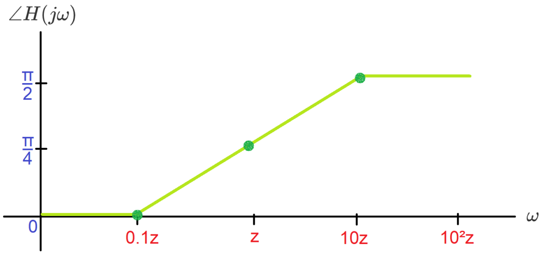
For convenience, we draw the phase as a linear transition starting at $\omega = 0.1z$ and ending at $\omega = 10z$.

Example: Bode Plot for $(\frac{s}{z} + 1)$:

$$20\ log\sqrt{1 + (\frac{\omega}{z})^2} \approx \begin{cases} 0 & if\ \omega \ll z \\ 20q\ log(\omega) + 20q\ log(\frac{1}{z}) & if\ \omega \gg z \end{cases}$$

$$\angle(j\frac{\omega}{z} + 1) \approx \begin{cases} 0 & if\ \omega \ll z \\ \frac{\pi}{4} & if\ \omega = z \\ \frac{\pi}{2} & if\ \omega \gg z \end{cases}$$





Complex Zeros & Poles

Let $H(s) = ((\frac{s}{\omega_n})^2 + 2\zeta(\frac{s}{\omega_n}) + 1)^{\pm 1}$. Then,

$$20\ log|((\frac{j\omega}{\omega_n})^2 + 2\zeta(\frac{\omega}{\omega_n})j + 1)^{\pm 1}| = \pm 20\ log\sqrt{(1 - (\frac{\omega}{\omega_n}^2)^2) + 4\zeta^2(\frac{\omega}{\omega_n})^2}$$

$$\approx \begin{cases} 0 & if\ \omega \ll \omega_n \\ \pm 40\ log(\omega) \mp 40\ log(\omega_n) & if\ \omega \gg \omega_n \end{cases}$$

For convenience, we draw the magnitude as a linear transition of slope ± 40 dB/decade starting at $\omega =$\omega_n.

$$\angle((\frac{j\omega}{\omega_n})^2 + 2\zeta(\frac{\omega}{\omega_n})j + 1)^{\pm 1} = \pm tan^{-1}(\frac{2\zeta\frac{\omega}{\omega_n}}{1 - (\frac{\omega}{\omega_n})^2})$$

$$\angle(j\frac{\omega}{z} + 1) \approx \begin{cases} 0 & if\ \omega \ll \omega_n \\ \pm\frac{\pi}{2} & if\ \omega = \omega_n \\ \pm\pi & if\ \omega \gg \omega_n \end{cases}$$

Example: Bode Plot for $((\frac{s}{\omega_n})^2 + 2\zeta(\frac{s}{\omega_n}) + 1)$

$$20\ log|(\frac{j\omega}{\omega_n})^2 + 2\zeta(\frac{\omega}{\omega_n})j + 1| \approx \begin{cases} 0 & if\ \omega \ll \omega_n \\ 40\ log(\omega) - 40\ log(\omega_n) & if\ \omega \gg \omega_n \end{cases}$$

$$\angle(\frac{j\omega}{\omega_n})^2 + 2\zeta(\frac{\omega}{\omega_n})j + 1 = tan^{-1}(\frac{2\zeta\frac{\omega}{\omega_n}}{1 - (\frac{\omega}{\omega_n})^2})$$

$$\angle(j\frac{\omega}{z} + 1) \approx \begin{cases} 0 & if\ \omega \ll \omega_n \\ \frac{\pi}{2} & if\ \omega = \omega_n \\ \pi & if\ \omega \gg \omega_n \end{cases}$$