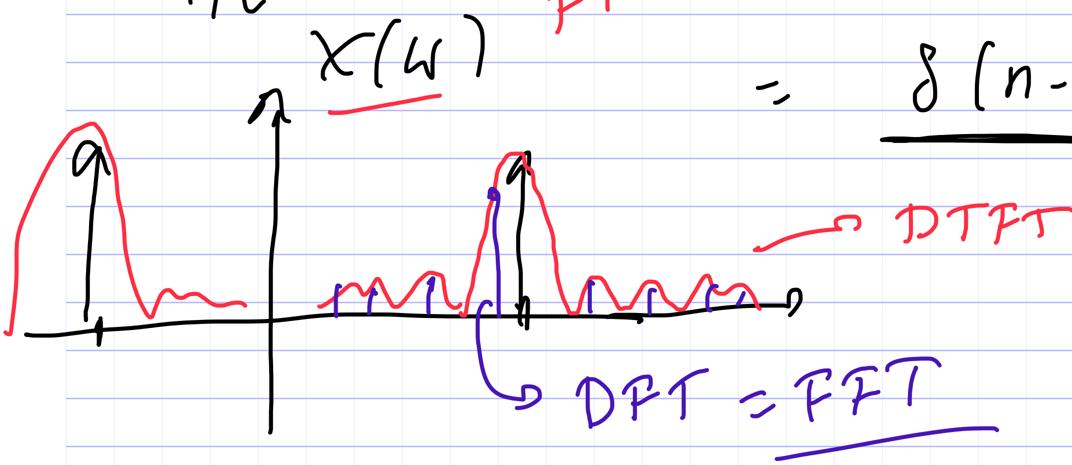
Efecto de fuga. DFT, DTFT.

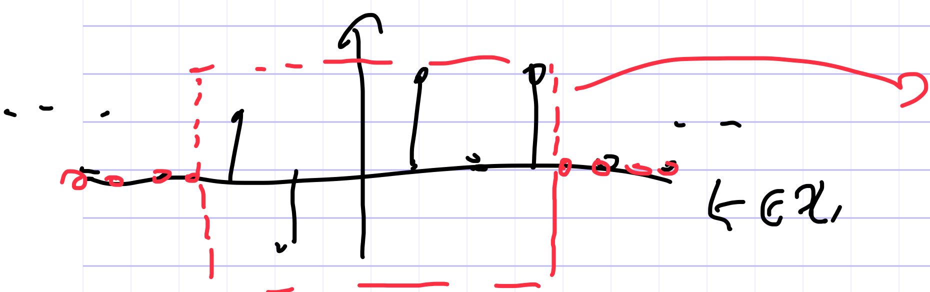
$x(t) = e^{jn\omega_0 t}$; $\varphi_n(t) = \{ e^{jm\omega_0 t} \}_{m \in \mathbb{Z}}$.

$$\int_{-T/2}^{T/2} \underline{\varphi_n(t)\varphi_m^*(t)}\, dt = \begin{cases} 0 & n \neq m \\ \varepsilon_n & n = m \end{cases} = \underline{\delta(n-m)}\ \varepsilon_n$$

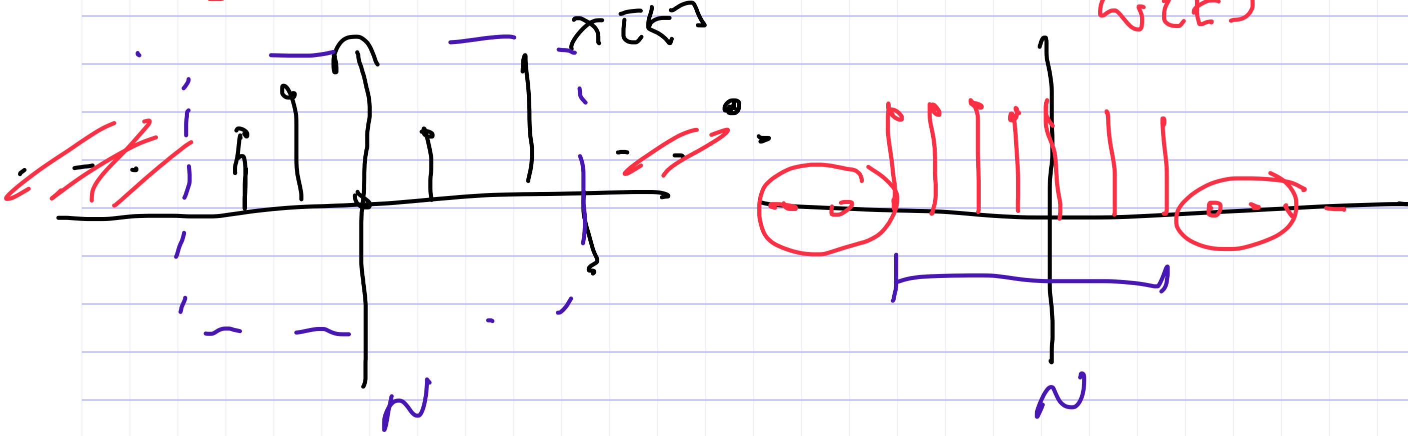


DTFT: $x(t) \rightarrow$ discretizar.

$x[k] = e^{j\omega_0 k}$; $k \in \mathbb{Z}$. PC ?



$X_N[k] = x[k] \cdot w[k]$



$\underline{x[k] = e^{j\omega_0 k}}$ → armónico simple

$X(\omega) = DTFT(x[k])$; $\omega \in [-\pi, +\pi)$

$\omega \in [0, 2\pi)$

$$X(\omega) = \sum_{k \in \mathbb{Z}} x[k] e^{-j\omega k}$$

$$X(\omega) = \sum_{k \in \mathbb{Z}} e^{j\omega_0 k} e^{-j\omega k} = \sum_{k \in \mathbb{Z}} e^{jk(\omega_0 - \omega)}$$

Relación de $e^{j\theta}$ con función delta

$$F\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} \partial t = 1.$$

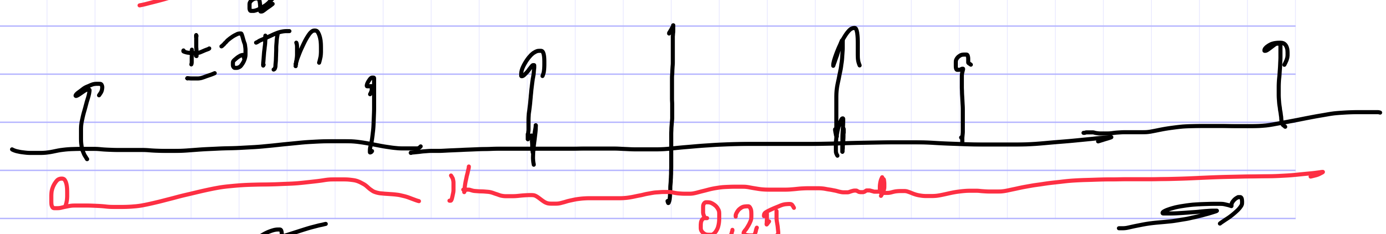
$$F\{1\} = 2\pi\delta(-\omega) = 2\pi\delta(\omega) = \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t} \partial t$$

$$\int_{-\infty}^{\infty} e^{-j\omega t} \partial t = 2\pi\delta(\omega)$$

$$\int_{-\infty}^{\infty} e^{-j(\omega - \omega_0)t} \partial t = 2\pi\delta(\omega - \omega_0)$$

$$X(\omega) = \sum_{k \in \mathbb{Z}} e^{jk(\omega_0 - \omega)} = \sum_{n \in \mathbb{Z}} 2\pi\delta((\omega - \omega_0) - 2\pi n)$$

$\pm 2\pi n$



ventana. (seleccionar N muestras)

$$X_N[k] = x[k]\cdot s[k]$$

$$X_N[\omega] = DTFT\{x_N[k]\} = DTFT\{x[k]s[k]\}$$

$$F\{x(t)s(t)\} = \frac{1}{2\pi} X(\omega) * Y(\omega)$$

$$DTFT\{x[k]\cdot s[k]\} = \frac{1}{2\pi} X(\omega) \circledast S(\omega)$$

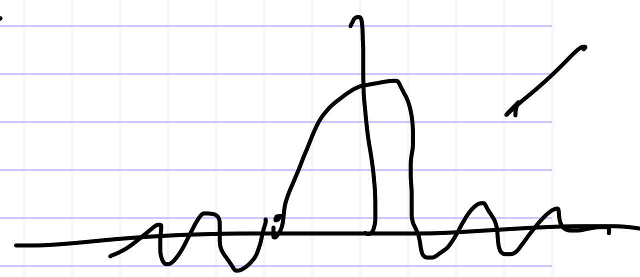
$\circledast$ → convolución cíclica → k discreto

$\omega \in [-\pi, +\pi)$

$$X(\omega) = \sum_{n\in\mathbb{Z}} 2\pi\,\delta(\omega-\omega_0-2\pi n)$$

$$s[k] = rect_N[k]$$

$$F\{A\,rect_\tau(t)\} = A\tau\,\frac{\sin(\theta)}{\theta} = A\tau\,\mathrm{Sinc}(\theta)$$



$$S(\omega) = DTFT\{s[k]\}$$

$$= \sum_{k\in\mathbb{Z}} rect_N[k]\, e^{-j\omega k} = \sum_{k=0}^{N-1} 1\cdot e^{-j\omega k}$$

$$S(\omega) = \sum_{k=0}^{N-1} e^{-j\omega k} =$$

$$\sum_{n=1}^{N} a r^{n-1} = a \frac{1-r^N}{1-r}$$

$$\sum_{n=0}^{N-1} a r^n = a \frac{1-r^N}{1-r}.$$

Serie geométrica.

$|r| < 1.$

$$\sum_{k=0}^{N-1} e^{-j\omega k}$$

$$\sum_{k=0}^{N-1} \left(e^{-j\omega}\right)^k$$

$a = 1, \; r = e^{-j\omega}.$

$$S(\omega) = \sum_{k=0}^{N-1} \left(e^{-j\omega}\right)^k = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}$$

$$\operatorname{Sen}(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j},$$

$$S(\omega) = \frac{\underbrace{e^{-j\omega\frac{N}{2}}}\,\cancel{2j}\,\underbrace{\left[e^{j\omega\frac{N}{2}} - e^{-j\omega\frac{N}{2}}\right]/2j}}{e^{-j\frac{\omega}{2}}\,\cancel{2j}\,\underbrace{\left[e^{+j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right]}_{2j}},$$

$$S(\omega) = e^{-j\omega\frac{N}{2}}\, e^{+j\frac{\omega}{2}}\, \frac{\operatorname{Sen}\left(\omega\frac{N}{2}\right)}{\operatorname{Sen}\left(\frac{\omega}{2}\right)}$$

$$S(\omega) = e^{-j\omega\left[\frac{N}{2} - \frac{1}{2}\right]} \operatorname{Sen}(\omega N/2) / \operatorname{Sen}(\omega/2)$$

$$\rightarrow S(\omega) = e^{-j\omega\left[\frac{N-1}{2}\right]} \operatorname{Sen}(\omega N/2) / \operatorname{Sen}(\omega/2)$$

$$X_N[\omega] = DTFT\{x[k]\cdot \xi[k]\}$$
$$= \frac{1}{2\pi} X(\omega) \circledast \xi(\omega)$$

$$X(\omega) = \sum_{n\in\mathbb{Z}} 2\pi\,\delta(\omega-\omega_0-2\pi n)$$

$$\xi(\omega) = e^{-j\omega\left[\frac{N-1}{2}\right]} \sin(\omega N/2)\,/\,\sin(\omega/2)$$

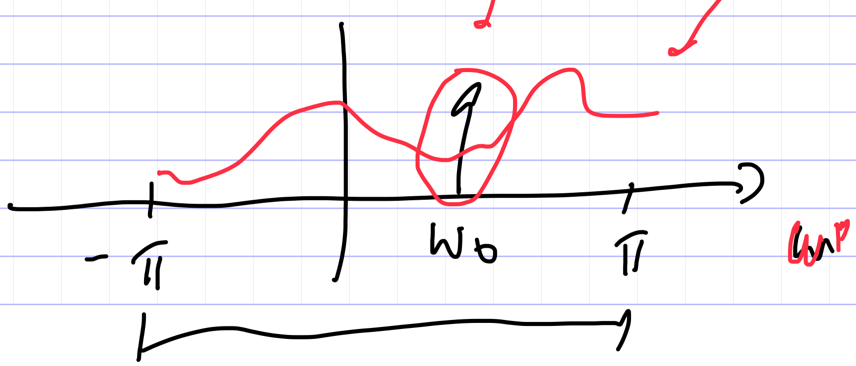
$$X_N(\omega) = \frac{1}{2\pi}\int_{\omega\in\mathbb{R}} X(\omega-\omega')\,\xi(\omega')\,\partial\omega'$$
$$= \frac{1}{2\pi}\int_{\omega\in\mathbb{R}} \xi(\omega-\omega')\,X(\omega')\,\partial\omega'$$

Resolver para 1 ciclo $\rightarrow$ $\omega\in[-\pi,\pi)$

$[0,2\pi]$ $\quad n=0$

$$X_N(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega')\,\xi(\omega-\omega')\,\partial\omega'.$$

$$= \frac{1}{\cancel{2\pi}}\int_{-\pi}^{\pi} \cancel{2\pi}\,\delta(\omega'-\omega_0)\,\xi(\omega-\omega')\,\partial\omega' = \xi(\omega-\omega_0)$$



$$X_N(\omega) = e^{-j\left(\frac{N-1}{2}\right)(\omega-\omega_0)} \frac{\operatorname{Sen}\left((\omega-\omega_0)\frac{N}{2}\right)}{\operatorname{Sen}\left(\frac{\omega-\omega_0}{2}\right)}$$

$$DTFT\{x_N[k]\} = DTFT\{x[k] \cdot g[k]\}$$

DFT → muestreo en frecuencia de la DTFT

$$DFT\{x[k]\} = X[m] = \sum_{k=0}^{N-1} x[k] e^{-j\frac{2\pi m}{N}k}$$

→ w en discreto.

discretizar en tiempo → $\boxed{t = kT_s = k/F_s}$.

$$\cos(\omega t) = \cos(2\pi F_c t) \Rightarrow \cos\left[2\pi F_c \frac{k}{F_s}\right]$$

$f = \frac{F_c}{F_s}$ → Nyquist $F_s \geq 2F_{c\,(MAX)}$

$-\frac{1}{2} \leq f \leq \frac{1}{2}$ → RACIONAL

$$f = \frac{\left[\frac{1}{s} \to \text{ciclo}\right]}{\left[\frac{\text{muest}}{s} \to \text{ciclo}\right]}$$

$\frac{\text{ciclos}}{\text{muestras}}$ → # entero / # entero

$\theta = 2\pi f k = \omega_{dis} k$; $\omega_{dis} = 2\pi f = 2\pi \frac{F_c}{F_s} = \frac{2\pi}{N}$

$f = \frac{1}{N}$ → ciclo / → muestras

$$\omega_m = m\omega_0 = m\frac{2\pi}{N} = m 2\pi f = 2\pi \underline{m f} = 2\pi f_m$$

$f = \frac{F_0}{F_s}$

$\omega_m = 2\pi f_m = 2\pi m f$

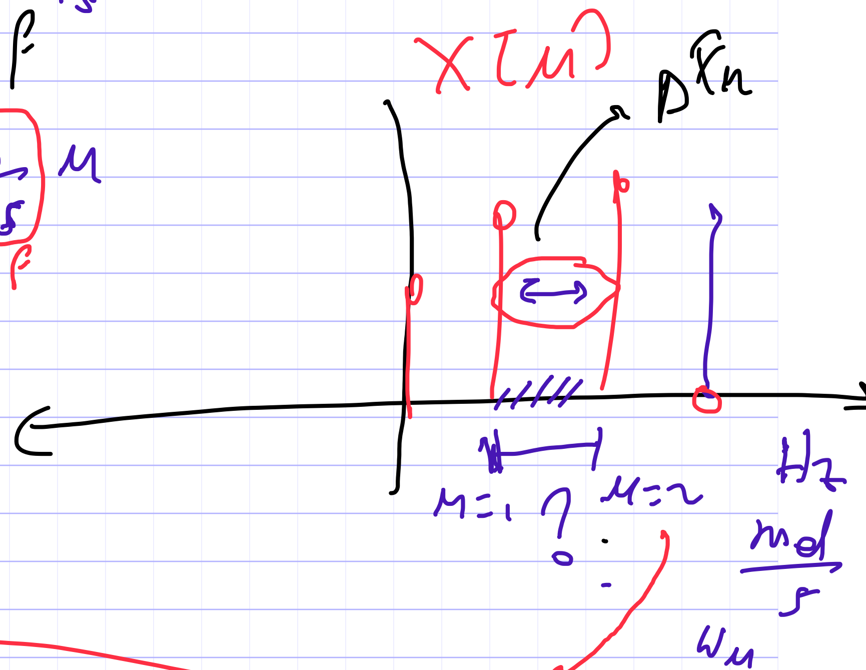
$= \frac{2\pi}{N} m = 2\pi \boxed{\frac{F_0}{F_s}} m$

$f \qquad f$

$F_m = F_0 m$

$\cancel{2\pi} \frac{F_m}{F_s} = \frac{\cancel{2\pi}}{N} m$

$$\boxed{F_m = F_s \frac{m}{N}} \quad [Hz]$$



$m = 2$

$Hz = ?$

$\Delta F_m = ?$ [Hz].

$$\Delta F_m = F_m - F_{m-1} = \frac{F_s m}{N} - \frac{F_s (m-1)}{N} = \cancel{\frac{F_s m}{N}} - \cancel{\frac{F_s m}{N}} + \frac{F_s}{N}$$

$$\boxed{\Delta F_m = \frac{F_s}{N}}$$

Resolución del muestreo en frecuencia.

$N \to \infty$

mejor resolución en frec.

DTFT → DFT

incertidumbre

$\frac{F_s}{N}$.

$F_s \to 0$

PERO resolución de tiempo empeora.