Assignment 2 Report

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1 QUESTION-1

Given a cube C centered at the origin $O(0,0,0)$ in a coordinate system described by unit vectors along the x,y and z directions, we would like to rotate it by 30° anticlockwise along axis described by vector $V(1,2,2)$.

To do so the following will be done:

- Compute M_{uvw} to convert the coordinate system from xyz to uvw such that w is the unit vector in the direction of the rotation axis V.
- Rotate anticlockwise by 30° around w using rotation matrix R. This is the same as rotating about z in the xyz coordinate system.
- Revert back to the original xyz coordinate system by using the transformation $M_{xyz} = M_{uvw}^{-1}$.

Thus the overall transformation M_V^{rot} is as follows -

$$M_V^{rot} = M_{uvw}^{-1}\ R\ M_{uvw}$$

To compute M_{uvw} we first declare some terminology. Let the xyz coordinate system be defined by basis vectors $\hat{\mathbf{x}}$,$\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ and the uvw coordinate system be defined by basis vectors - $\hat{\mathbf{u}}$,$\hat{\mathbf{v}}$ and $\hat{\mathbf{w}}$.

In the xyz coordinate system, vectors $\hat{\mathbf{u}}$,$\hat{\mathbf{v}}$ and $\hat{\mathbf{w}}$ are as follows -

$$\hat{\mathbf{u}} = \alpha_u\hat{\mathbf{x}} + \beta_u\hat{\mathbf{y}} + \gamma_u\hat{\mathbf{z}}$$

$$\hat{\mathbf{v}} = \alpha_v\hat{\mathbf{x}} + \beta_v\hat{\mathbf{y}} + \gamma_v\hat{\mathbf{z}}$$

$$\hat{\mathbf{w}} = \alpha_w\hat{\mathbf{x}} + \beta_w\hat{\mathbf{y}} + \gamma_w\hat{\mathbf{z}}$$

For a point P described by $P_{uvw}(P_u, P_v, P_w)$ in the uvw coordinate system and $P_{xyz}(P_x, P_y, P_z)$ in the xyz coordinate system, the inter-conversion can be done as follows-

$$\begin{aligned}
P_{uvw} &= P_u\hat{\mathbf{u}} + P_v\hat{\mathbf{v}} + P_w\hat{\mathbf{w}} \\
&= P_u(\alpha_u\hat{\mathbf{x}} + \beta_u\hat{\mathbf{y}} + \gamma_u\hat{\mathbf{z}}) + P_v(\alpha_v\hat{\mathbf{x}} + \beta_v\hat{\mathbf{y}} + \gamma_v\hat{\mathbf{z}}) + P_w(\alpha_w\hat{\mathbf{x}} + \beta_w\hat{\mathbf{y}} + \gamma_w\hat{\mathbf{z}}) \\
&= (P_u\alpha_u + P_v\alpha_v + P_w\alpha_w)\hat{\mathbf{x}} + (P_u\beta_u + P_v\beta_v + P_w\beta_w)\hat{\mathbf{y}} + (P_u\gamma_u + P_v\gamma_v + P_w\gamma_w)\hat{\mathbf{z}}) \\
&= P_x\hat{\mathbf{x}} + P_y\hat{\mathbf{y}} + P_y\hat{\mathbf{y}} \\
&= P_{xyz}
\end{aligned}$$

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Thus we can write M_{uvw} as,

$$M_{uvw} = \begin{bmatrix} \alpha_u & \alpha_v & \alpha_w & 0 \\ \beta_u & \beta_v & \beta_w & 0 \\ \gamma_u & \gamma_v & \gamma_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here the matrix has been expanded to 4×4 to make it compatible with the rotation matrix used subsequently. For the points will be of the form $P(x, y, z, 0)$ to make them compatible for matrix multiplication.

In general if the origins of the two coordinates do not coincide, i.e., the origin of the uvw system is $E(x_e, y_e, z_e)$ in the xyz coordinate system, then all points are first left multiplied by the translation matrix,

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

However, in this question both the **origins coincide**, hence T **is the identity matrix and can be omitted** from the calculations.

Now that we have computed the transformation matrix, we need to rotate the transformed points about the z axis (after transformation it is the w axis) by $\theta = +30°$ (+ve sign means anticlockwise). To do so we simply use the rotation matrix R about the z axis. This matrix is given by,

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now the overall transformation is,

$$M_V^{rot} = M_{uvw}^{-1}\, R\, M_{uvw}$$

Notice that the columns of M_{uvw} are mutually orthogonal unit vectors, spanning $\mathbb{R}^4$. Thus M_{uvw} is an orthonormal matrix which has the property,

$$M_{uvw}^{-1} = M_{uvw}^{T}$$

M_{uvw}^{T} being the transpose. Thus the overall transformation can be written as,

$$M_V^{rot} = \left(\begin{bmatrix} \alpha_u & \beta_u & \gamma_u & 0 \\ \alpha_v & \beta_v & \gamma_v & 0 \\ \alpha_w & \beta_w & \gamma_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} \alpha_u & \alpha_v & \alpha_w & 0 \\ \beta_u & \beta_v & \beta_w & 0 \\ \gamma_u & \gamma_v & \gamma_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)$$

All that remains now is computing the vectors $\hat{\mathbf{u}}$,$\hat{\mathbf{v}}$ and $\hat{\mathbf{w}}$. For this we use two vectors - the axis vector $V(1, 2, 2)$ and the up vector $t(0, 0, 1)$. The unit vectors are described as follows,

$$\hat{\mathbf{w}} = \frac{V}{\|V\|} \quad \hat{\mathbf{u}} = \frac{t \times \hat{\mathbf{w}}}{\|t \times \hat{\mathbf{w}}\|} \quad \hat{\mathbf{v}} = \frac{\hat{\mathbf{w}} \times \hat{\mathbf{u}}}{\|\hat{\mathbf{w}} \times \hat{\mathbf{u}}\|}$$

Note that here $\times$ refers to the cross product of two vectors, while $\|.\|$ refers to the vector norm.

The values computed for the above are,

$$\hat{\mathbf{w}} = (\frac{1}{3}, \frac{2}{3}, \frac{2}{3}) \quad \hat{\mathbf{u}} = (\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{0}{\sqrt{5}}) \quad \hat{\mathbf{v}} = (\frac{-2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{5}{\sqrt{45}})$$

$$M_V^{rot} = \left(\begin{bmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{0}{\sqrt{5}} & 0 \\ \frac{-2}{\sqrt{45}} & \frac{4}{\sqrt{45}} & \frac{5}{\sqrt{45}} & 0 \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-\sqrt{1}}{2} & 0 & 0 \\ \frac{\sqrt{1}}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} \frac{-2}{\sqrt{5}} & \frac{-2}{\sqrt{45}} & \frac{1}{3} & 0 \\ \frac{1}{\sqrt{5}} & \frac{4}{\sqrt{45}} & \frac{2}{3} & 0 \\ \frac{0}{\sqrt{5}} & \frac{5}{\sqrt{45}} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)$$

2 QUESTION-2

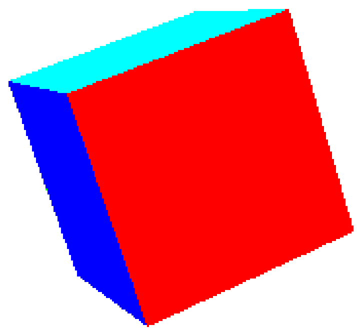

Fig. 1. Screenshot of cube transformed with matrices derived in Q1.

3 QUESTION-3

There is no difference in the output of my code as compared to the lookAt function.

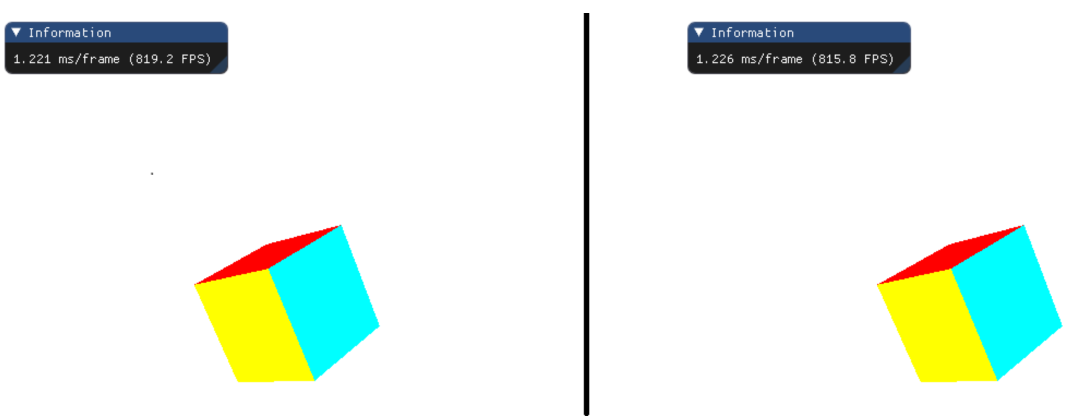

Fig. 2. Screenshots of cube transformed with my code (**left**) and lookAt function (**right**)

4 QUESTION-4

Renderings from different perspectives.

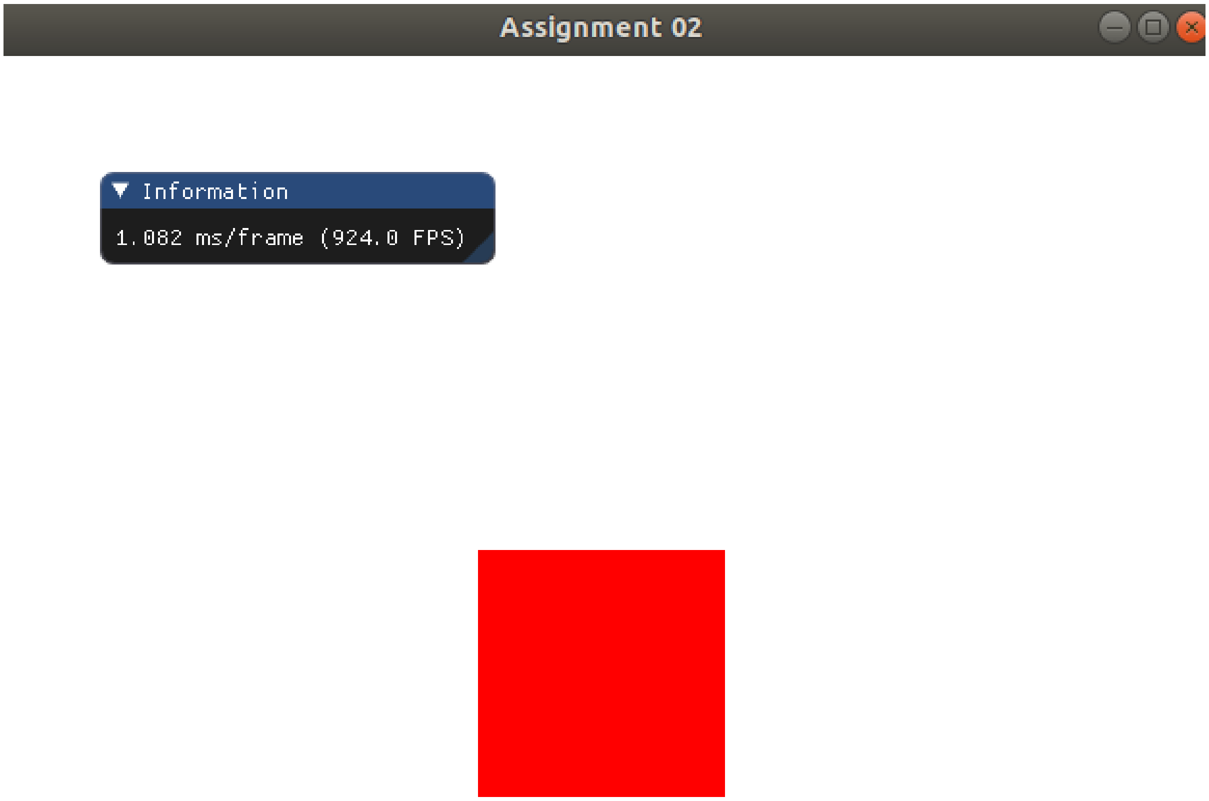

Fig. 3. One point Perspective. Gaze is along on -ve z direction.

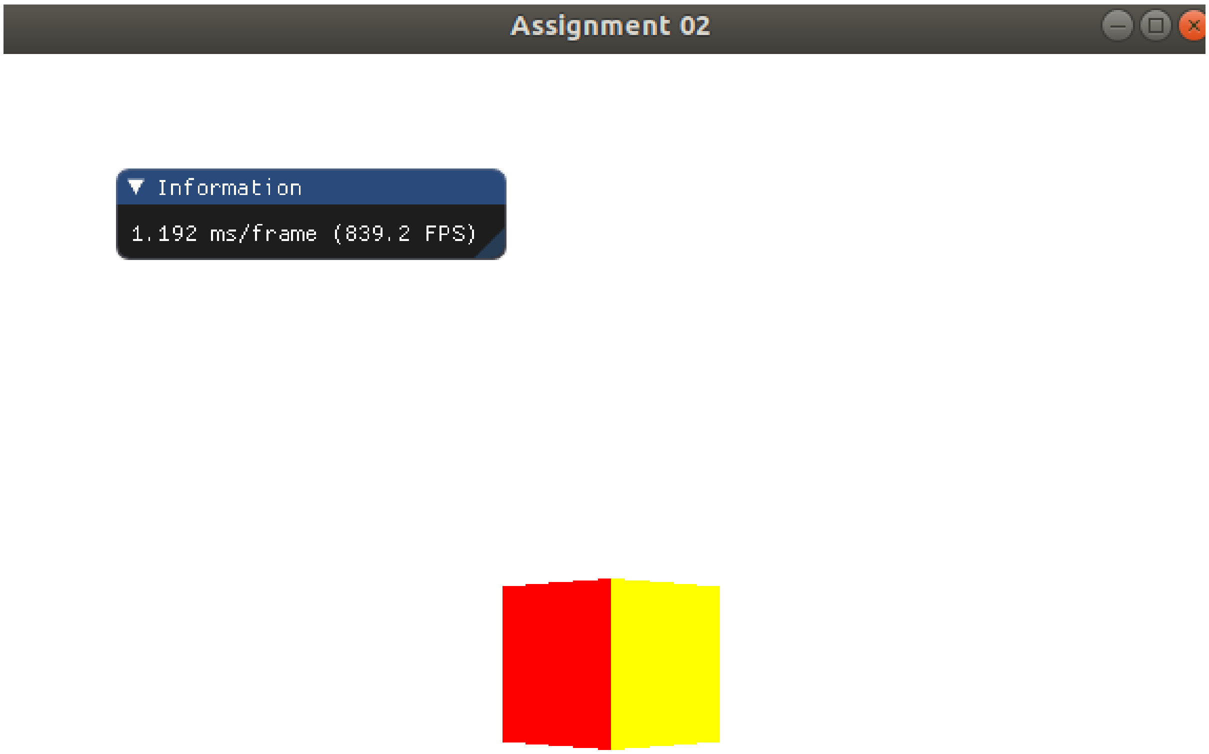

Fig. 4. Two point Perspective.

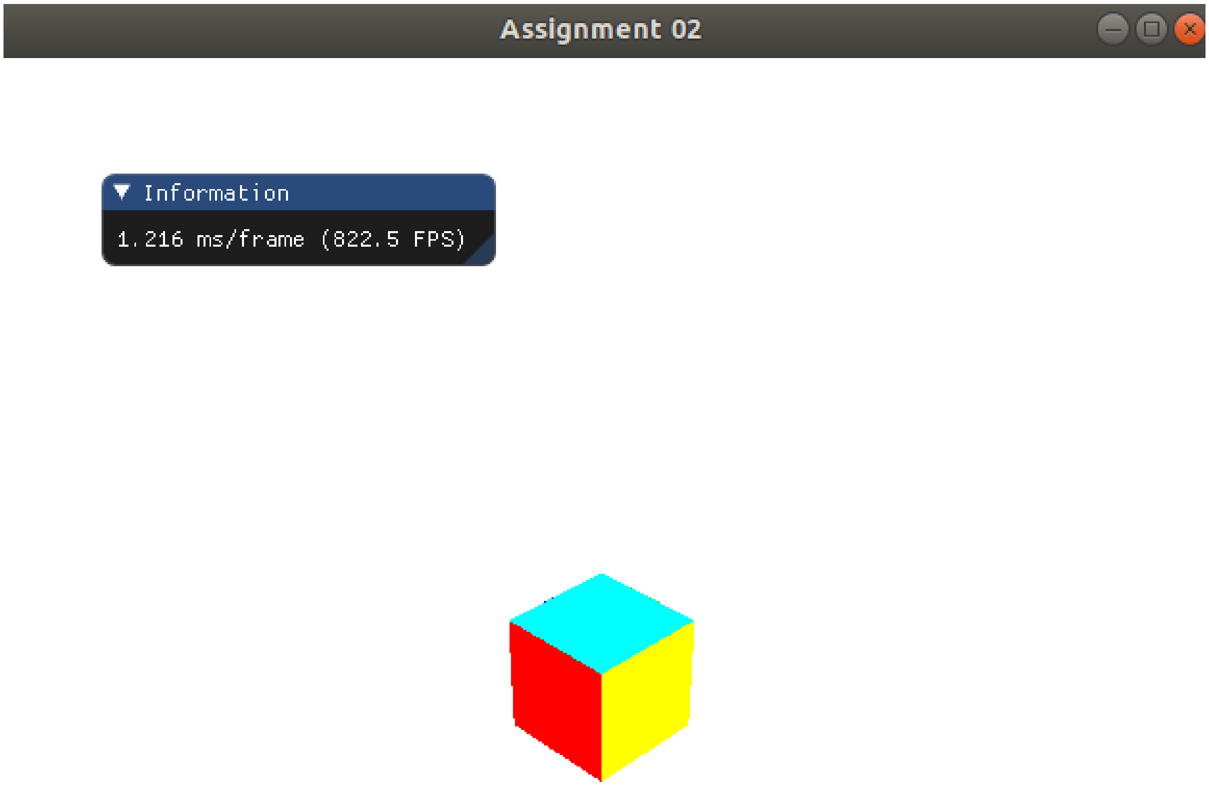

Fig. 5. Three point Perspective.Bird's Eye View

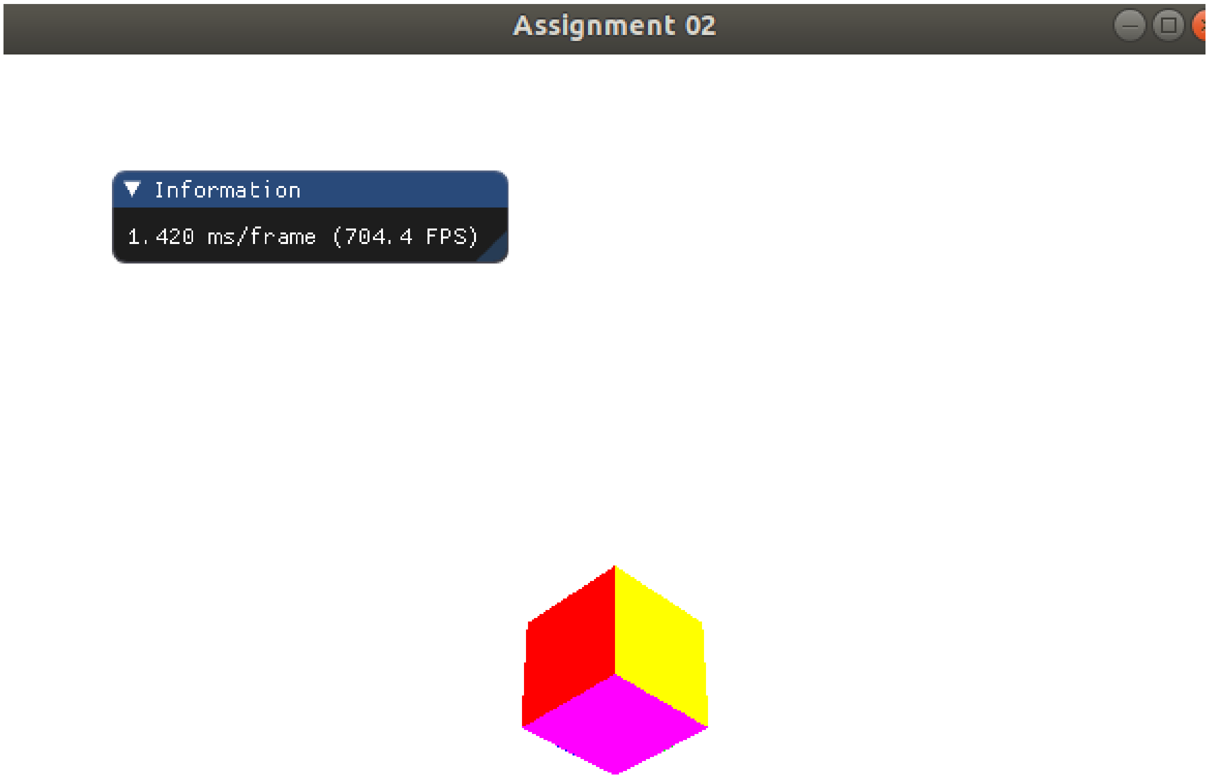

Fig. 6. Three point Perspective.Rat's Eye View