EE/Ma/CS 126a: Information Theory

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Contents

0.1 Introduction and Course Information 1

1 Math Review **2**
1.1 Combinatorics & Probability 2
1.2 Logarithm Identities 3

2 Entropy Definitions **4**

0.1 Introduction and Course Information

This document offers an overview of EE/Ma/CS 126a at Caltech. They comprise my condensed course notes for the course. No promises are made relating to the correctness or completeness of the course notes. These notes are meant to highlight difficult concepts and explain them simply, not to comprehensively review the entire course.

Course Information

- Professor: Michelle Effros
- Term: 2022 Fall

Chapter 1

Math Review

1.1 Combinatorics & Probability

Binomial Distribution & Coefficient

- **Bernoulli Process**: Repeated trials, each with one binary outcome. The probability of a positive outcome is $p \in [0, 1]$. Each trial is independent.

- **Binomial Distribution**: Let x represent the number of successful trials in a Bernoulli process repeated n times with success probability p. The binomial distribution gives the probability distribution on x:

$$b(x; n, p) = \binom{n}{k} p^x (1-p)^{n-x} \tag{1.1}$$

  Which has $\mu = np$, $\sigma^2 = npq$.

- **Intuition for Binomial Distribution**: The probability of observing a sequence with x positive outcomes and $n - x$ negative outcomes is $p^x(1-p)^{n-x}$. There are $\binom{n}{k}$ different sequences (i.e., permutations) that have x positive cases and n negative cases. Thus the total probability of observing x positive cases is given by Eq 1.1.

- **Binomial Coefficient**:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{1.2}$$

1.2 Logarithm Identities

Entropy calculations and manipulations involve a lot of logarithms. They're not so bad once you get to know them, though:

- **Definition**:
$$a = b^{\log_b a}$$

- **Sum-Product**:
$$\log_c(ab) = \log_c a + \log_c b$$

- **Difference-Quotient**:
$$\log(a/c) = \log a - \log c$$
$$\log \frac{1}{a} = -\log a$$

- **Product-Exponent**:
$$\log_c(a^n) = n \log_c(a)$$

- **Swapping Base**:
$$\log_b(a) = \log_a(b)$$

- **Swapping Exponential**:
$$a^{\log n} = n^{\log a}$$

- **Change of Base**;
$$\log_b(a) = \frac{\log_x(a)}{\log_x(b)}$$

Chapter 2

Entropy Definitions

Chapter 2 of Elements of Information Theory.

Entropy Definition (Discrete):

$$H(X) = \sum_{x \in \mathcal{X}} p(x) \log(\frac{1}{p(x)}) \tag{2.1}$$

$$= -\sum_{x \in \mathcal{X}} p(x) \log p(x) \tag{2.2}$$

$$= \mathbb{E}[\log \frac{1}{p(x)}] \tag{2.3}$$