Initial Proposal for AMATH 574

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In this project, we will mainly focus on models of traffic flow which are based on the Lighthill-Whitham-Richards model (LWR). The LWR nonlinear model satisfies one-dimensional conservation law,

$$\rho_t + f(\rho)_x = 0,$$

where $\rho(x,t)$ is the traffic density and $f(\rho)$ is the flux function,

$$f(\rho) = \rho U(\rho).$$

Here $U(\rho)$ is a special velocity function of density.

In this project, we have two goals.

- Based on the LWR model, assuming that we have a discontinuous, piecewise-linear flux function, we would like to explore the behavior and the physical background of solution, after smoothing out the discontinuity in the flux function over a tiny correction $\epsilon \ll 1$. The Riemann problem can be solved by limiting $\epsilon \to 0$.

  From [2], we will explore using a Godunov-type numerical scheme to implement this method.

- The second goal of this project is to study non-convex flux for the night-time traffic model. For this model we use car-following model: the local density $\rho_k(t)$ is observed by the kth driver at time t

  $$\rho_k(t) = \frac{1}{X_{k+1}(t) - X_k(t)},$$

  where $X_k(t)$ is the position of each individual car.

  From a non-convex flux, we can see an unusual feature of clustering. This is resulted from the unstability of uniformly spaced traffic flow. Our goal is to study the behavior with different perturbations, especially the situation where the random effect is considered.

References

[1] LeVeque, R. J. (2001). Some traffic flow models illustrating interesting hyperbolic behavior. Minisymposium on traffic flow. Chicago.

[2] Wiens, J. K., Stockie, J. M., & Williams, J. F. (2013). Riemann solver for a kinematic wave traffic model with discontinuous flux. Journal of Computational Physics, 242, 1-23.

[3] Jiang, R., & Wu, Q. S. (2007). The night driving behavior in a car-following model. Physica A: Statistical Mechanics and its Applications, 375(1), 297-306.