1 Cooper-Frye integral for decay products

The spectrum of final decay products is given by

$$E_{\mathbf{p}}\frac{dN}{d^3\mathbf{p}} = \frac{\nu}{(2\pi)^3}\int_\sigma d\sigma_\mu g^\mu \tag{1}$$

where g^μ is the vector distribution function of decayed particles. g^μ is typically given as a sum of a couple Lorentz invariant weight functions and some explicit Lorentz vectors, which depends on what is the initial primary resonance distribution function. Below we document, which distribution components are calculated in `FastReso`. See the paper and code for further details of how the the decays are implemented.

2 Initial distribution functions

Code stores distribution components in an array `TParticle::fFj_arr`, which is of size `[grid_params:fN_f]` $\times$ `[grid_params:fN_pbar]`, where $N_f = 12$ is the total number of irreducible components and $N_{\bar{\mathbf{p}}} = 201$ is the number of (un-equally spaced) momentum grid points. Note that some momentum independent terms are factored out to make the stored values simple and dimensionless. As of time of writing (2018.09.27) `FastReso` calculates the decay components of the following initial distribution functions.

2.1 Equilibrium distribution

Initial distribution is given by Fermi-Dirac or Bose-Einstein distribution (there is also an option for Boltzmann distribution which is never used)

$$f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu) = \left(e^{(\bar{E}_{\mathbf{p}}-\mu)/T} \mp 1\right)^{-1}, \tag{2}$$

where $\mu = Q_B\mu_B + Q_{I_3}\mu_{I_3} + Q_S\mu_S + Q_C\mu_C$ where the charges and corresponding chemical potentials are that of baryon number, third component of the isospint, strangeness and charm. The decayed vector distribution function is given by

$$g_b^\mu(\bar{E}_{\mathbf{p}}) = f_1^{\mathrm{eq}}(\bar{E}_{\mathbf{p}})\left(p^\mu - \bar{E}_{\mathbf{p}}u^\mu\right) + f_2^{\mathrm{eq}}(\bar{E}_{\mathbf{p}})\bar{E}_{\mathbf{p}}u^\mu. \tag{3}$$

The actual data stored and printed is

$$f_1^{\mathrm{eq}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[0]}, \tag{4}$$
$$f_2^{\mathrm{eq}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[1]} \tag{5}$$

which is initialized by

$$\texttt{fFj\_arr[0]} = f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu), \tag{6}$$
$$\texttt{fFj\_arr[1]} = f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu) \tag{7}$$

2.2 Viscous bulk and shear distributions

Initial perturbations are given by (taken from [arXiv:1509.06738])

$$\delta f^{\text{bulk}}(x,p) = f_{\text{eq}}(1 \pm f_{\text{eq}}) \left[\frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right) - \frac{1}{3}\frac{m^2}{T\bar{E}_p} \right] \times \frac{\tau_\Pi \Pi}{\zeta}, \tag{8}$$

$$\delta f^{\text{shear}}(x,p) = f_{\text{eq}}(1 \pm f_{\text{eq}}) \times \pi_{\rho\nu} p^\rho p^\nu \times \frac{1}{2(e+p)T^2}. \tag{9}$$

where c_s is the speed of sound on the freeze-out surface (given as a parameter) and m is a primary resonance mass. The decayed vectro distribution function is given by

$$g^{\mu}_{\text{bulk}} \Pi = \left(p^\mu - \bar{E}_{\mathbf{p}} u^\mu\right) f_1^{\text{bulk}}(\bar{E}_{\mathbf{p}}) + \bar{E}_{\mathbf{p}} u^\mu f_2^{\text{bulk}}(\bar{E}_{\mathbf{p}}), \tag{10}$$

and

$$\begin{aligned} g^{\mu\nu\rho}_{\text{shear}} \pi_{\nu\rho} &= [p^\nu \pi_{\nu\rho} p^\rho (p^\mu - \bar{E}_{\mathbf{p}} u^\mu) - \frac{2}{5}|\bar{\mathbf{p}}|^2 p_\nu \pi^{\nu\mu}] f_1^{\text{shear}}(\bar{E}_{\mathbf{p}}) \\ &+ \frac{2}{5}|\bar{\mathbf{p}}|^2 p_\nu \pi^{\nu\mu} f_2^{\text{shear}}(\bar{E}_{\mathbf{p}}) + \pi_{\nu\rho} p^\nu p^\rho \bar{E}_{\mathbf{p}} u^\mu f_3^{\text{shear}}(\bar{E}_{\mathbf{p}}). \end{aligned} \tag{11}$$

The actual data stored and printed is

$$f_1^{\text{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[2]} \times \frac{1}{2(e+p)T^2}, \tag{12}$$

$$f_2^{\text{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[3]} \times \frac{1}{2(e+p)T^2}, \tag{13}$$

$$f_3^{\text{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[4]} \times \frac{1}{2(e+p)T^2}, \tag{14}$$

$$f_1^{\text{bulk}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[5]} \times \frac{-\tau_\Pi \Pi}{\zeta}, \tag{15}$$

$$f_2^{\text{bulk}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[6]} \times \frac{-\tau_\Pi \Pi}{\zeta}, \tag{16}$$

$$\tag{17}$$

which is initialized by

$$\texttt{fFj\_arr[2]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{18}$$

$$\texttt{fFj\_arr[3]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{19}$$

$$\texttt{fFj\_arr[4]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{20}$$

$$\texttt{fFj\_arr[5]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)) \left[\frac{1}{3}\frac{m^2}{T\bar{E}_p} - \frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right) \right], \tag{21}$$

$$\texttt{fFj\_arr[6]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)) \left[\frac{1}{3}\frac{m^2}{T\bar{E}_p} - \frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right) \right] \tag{22}$$

Note the sign in the bulk perturbation. Also, because of the explicit shear components carry $|\bar{p}|^2$, the irreducible weight function after decays are actually diverging at the origin, but no more than quadratically.

2.3 Temperature and velocity perturbations

Linear perturbations in temperature and velocity

$$\delta f^{\text{temp.}}(x,p) = f_{\text{eq}}(1 \pm f_{\text{eq}})\frac{\bar{E}_{\mathbf{p}}}{\bar{T}}\frac{\delta T}{\bar{T}}, \tag{23}$$

$$\delta f^{\text{velocity}}(x,p) = f_{\text{eq}}(1 \pm f_{\text{eq}})\frac{\delta u_\nu p^\nu}{\bar{T}}. \tag{24}$$

Vector distribution function is then given by

$$g^{\mu}_{\text{temp.}}\delta T = f_1^{\text{temp.}}(\bar{E}_{\mathbf{p}})\left(p^\mu - \bar{E}_{\mathbf{p}}u^\mu\right) + f_2^{\text{temp.}}(\bar{E}_{\mathbf{p}})\bar{E}_{\mathbf{p}}u^\mu. \tag{25}$$

and

$$\begin{aligned} g^{\mu\nu}_{\text{velocity}}\delta u_\nu &= [\delta u_\nu p^\nu (p^\mu - \bar{E}_{\mathbf{p}}u^\mu) - \frac{1}{3}|\bar{\mathbf{p}}|^2\delta u^\mu)] f_1^{\text{velocity}}(\bar{E}_{\mathbf{p}}) \\ &+ \frac{1}{3}|\bar{\mathbf{p}}|^2 \delta u^\mu f_2^{\text{velocity}}(\bar{E}_{\mathbf{p}}) + \delta u_\nu p^\nu \bar{E}_{\mathbf{p}} u^\mu f_3^{\text{velocity}}(\bar{E}_{\mathbf{p}}). \end{aligned} \tag{26}$$

The actual data stored and printed is

$$f_1^{\text{temp.}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[7]} \times \frac{\delta T}{T}, \tag{27}$$

$$f_2^{\text{temp.}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[8]} \times \frac{\delta T}{T}, \tag{28}$$

$$f_1^{\text{velocity}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[9]} \times \frac{1}{T}, \tag{29}$$

$$f_2^{\text{velocity}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[10]} \times \frac{1}{T}, \tag{30}$$

$$f_3^{\text{velocity}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[11]} \times \frac{1}{T}, \tag{31}$$

$$\tag{32}$$

Note that irreducible components for velocity perturbation can also be divergent, but the explicit momentum prefactors will make it finite.

Initialization is given by

$$\texttt{fFj\_arr[7]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu))\frac{\bar{E}_{\mathbf{p}}}{T}, \tag{33}$$

$$\texttt{fFj\_arr[8]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu))\frac{\bar{E}_{\mathbf{p}}}{T}, \tag{34}$$

$$\texttt{fFj\_arr[9]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{35}$$

$$\texttt{fFj\_arr[10]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{36}$$

$$\texttt{fFj\_arr[11]} = f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)) \tag{37}$$

3 Header of output files

```
#     Name      Mass     Gamma  Spin Isospin    I3 Nq Ns Naq Nas Nc
Nac        MC    Nyield
# pi0139plu  0.139570 0.00000e+00   0.0   1.0   1.0 1 0 1 0 00      211  0.000701
#   1:pbar [GeV] 2:m [GeV]         3:feq 1         4:feq 2      5:fshear 1      6:fshear 2
   7:fshear 3       8:fbulk 1       9:fbulk 2 10:ftemperature 1 11:ftemperature 2
12:fveloc
ity 1  13:fvelocity 2  14:fvelocity 3
```