Notes on `FastReso` implementation

July 24, 2019

1 Cooper-Frye integral for decay products

The spectrum of final decay products is given by

$$E_{\mathbf{p}} \frac{dN}{d^3\mathbf{p}} = \frac{\nu}{(2\pi)^3} \int_\sigma d\sigma_\mu g^\mu \tag{1}$$

where g^μ is the vector distribution function of decayed particles. g^μ is typically given as a sum of a couple Lorentz invariant weight functions and some explicit Lorentz vectors, which depends on what is the initial primary resonance distribution function. Below we document, which distribution components are calculated in `FastReso`. See the paper (Aleksas Mazeliauskas, Stefan Floerchinger, Eduardo Grossi, Derek Teaney , Eur. Phys. J. C79, (2019) https://arxiv.org/abs/1809.11049) and the code for further details of how the the decays are implemented.

2 Initial distribution functions

Code stores distribution components in an array `TParticle::fFj_arr`, which is of size `[grid_params:fN_f]` $\times$ `[grid_params:fN_pbar]`, where $N_f = 12$ is the total number of irreducible components and $N_{\bar{\mathbf{p}}} = 201$ is the number of (un-equally spaced) momentum grid points. Note that some momentum independent terms are factored out to make the stored values simple and dimensionless. As of time of writing (2018.09.27) `FastReso` calculates the decay components of the following initial distribution functions. As of time of writing (2019.07.24) irreducible components were multiplied by powers of $\bar{p}$ to make the integrals easier to evaluate numerically.

2.1 Equilibrium distribution

Initial distribution is given by Fermi-Dirac or Bose-Einstein distribution (there is also an option for Boltzmann distribution which is never used)

$$f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu) = \left(e^{(\bar{E}_{\mathbf{p}}-\mu)/T} \mp 1\right)^{-1}, \tag{2}$$

where $\mu = Q_B\mu_B + Q_{I_3}\mu_{I_3} + Q_S\mu_S + Q_C\mu_C$ where the charges and corresponding chemical potentials are that of baryon number, third component of the isospint, strangeness and charm.

The decayed vector distribution function is given by

$$g_b^\mu(\bar{E}_{\mathbf{p}}) = f_1^{\mathrm{eq}}(\bar{E}_{\mathbf{p}}) \left(p^\mu - \bar{E}_{\mathbf{p}} u^\mu\right) + f_2^{\mathrm{eq}}(\bar{E}_{\mathbf{p}}) \bar{E}_{\mathbf{p}} u^\mu. \tag{3}$$

The actual data stored and printed is

$$\bar{p} f_1^{\mathrm{eq}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[0]}, \tag{4}$$
$$\bar{p} f_2^{\mathrm{eq}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[1]} \tag{5}$$

which is initialized by

$$\texttt{fFj\_arr[0]} = \bar{p} f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu), \tag{6}$$
$$\texttt{fFj\_arr[1]} = \bar{p} f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu) \tag{7}$$

2.2 Viscous bulk and shear distributions

Initial perturbations are given by (taken from [arXiv:1509.06738])

$$\delta f^{\mathrm{bulk}}(x,p) = f_{\mathrm{eq}}(1 \pm f_{\mathrm{eq}}) \left[\frac{\bar{E}_p}{T} \left(\frac{1}{3} - c_s^2 \right) - \frac{1}{3} \frac{m^2}{T \bar{E}_p} \right] \times \frac{\tau_\Pi \Pi}{\zeta}, \tag{8}$$
$$\delta f^{\mathrm{shear}}(x,p) = f_{\mathrm{eq}}(1 \pm f_{\mathrm{eq}}) \times \pi_{\rho\nu} p^\rho p^\nu \times \frac{1}{2(e+p)T^2}. \tag{9}$$

where c_s is the speed of sound on the freeze-out surface (given as a parameter) and m is a primary resonance mass. The decayed vectro distribution function is given by

$$g_{\mathrm{bulk}}^\mu \Pi = \left(p^\mu - \bar{E}_{\mathbf{p}} u^\mu\right) f_1^{\mathrm{bulk}}(\bar{E}_{\mathbf{p}}) + \bar{E}_{\mathbf{p}} u^\mu f_2^{\mathrm{bulk}}(\bar{E}_{\mathbf{p}}), \tag{10}$$

and

$$\begin{aligned} g_{\mathrm{shear}}^{\mu\nu\rho} \pi_{\nu\rho} &= [p^\nu \pi_{\nu\rho} p^\rho (p^\mu - \bar{E}_{\mathbf{p}} u^\mu) - \frac{2}{5} |\bar{\mathbf{p}}|^2 p_\nu \pi^{\nu\mu}] f_1^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) \\ &+ \frac{2}{5} |\bar{\mathbf{p}}|^2 p_\nu \pi^{\nu\mu} f_2^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) + \pi_{\nu\rho} p^\nu p^\rho \bar{E}_{\mathbf{p}} u^\mu f_3^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}). \end{aligned} \tag{11}$$

The actual data stored and printed is

$$\bar{p}^3 f_1^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[2]} \times \frac{1}{2(e+p)T^2}, \tag{12}$$
$$\bar{p}^3 f_2^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[3]} \times \frac{1}{2(e+p)T^2}, \tag{13}$$
$$\bar{p}^3 f_3^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[4]} \times \frac{1}{2(e+p)T^2}, \tag{14}$$
$$\bar{p} f_1^{\mathrm{bulk}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[5]} \times \frac{-\tau_\Pi \Pi}{\zeta}, \tag{15}$$
$$\bar{p} f_2^{\mathrm{bulk}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[6]} \times \frac{-\tau_\Pi \Pi}{\zeta}, \tag{16}$$
$$\tag{17}$$

which is initialized by

$$\texttt{fFj\_arr[2]} = \bar{p}^3 f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{18}$$

$$\texttt{fFj\_arr[3]} = \bar{p}^3 f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{19}$$

$$\texttt{fFj\_arr[4]} = \bar{p}^3 f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{20}$$

$$\texttt{fFj\_arr[5]} = \bar{p} f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)) \left[\frac{1}{3}\frac{m^2}{T\bar{E}_p} - \frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right)\right], \tag{21}$$

$$\texttt{fFj\_arr[6]} = \bar{p} f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)) \left[\frac{1}{3}\frac{m^2}{T\bar{E}_p} - \frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right)\right] \tag{22}$$

Note the sign in the bulk perturbation. Also, because of the explicit shear components carry $|\bar{p}|^2$, the irreducible weight function after decays are actually diverging at the origin, but no more than quadratically.

2.3 Temperature and velocity perturbations

Linear perturbations in temperature and velocity

$$\delta f^{\rm temp.}(x, p) = f_{\rm eq}(1 \pm f_{\rm eq})\frac{\bar{E}_{\mathbf{p}}}{\bar{T}}\frac{\delta T}{\bar{T}}, \tag{23}$$

$$\delta f^{\rm velocity}(x, p) = f_{\rm eq}(1 \pm f_{\rm eq})\frac{\delta u_\nu p^\nu}{\bar{T}}. \tag{24}$$

Vector distribution function is then given by

$$g^{\mu}_{\rm temp.}\delta T = f_1^{\rm temp.}(\bar{E}_{\mathbf{p}})\left(p^\mu - \bar{E}_{\mathbf{p}} u^\mu\right) + f_2^{\rm temp.}(\bar{E}_{\mathbf{p}})\bar{E}_{\mathbf{p}} u^\mu. \tag{25}$$

and

$$\begin{aligned} g^{\mu\nu}_{\rm velocity}\delta u_\nu = {} & [\delta u_\nu p^\nu (p^\mu - \bar{E}_{\mathbf{p}} u^\mu) - \frac{1}{3}|\bar{\mathbf{p}}|^2 \delta u^\mu)] f_1^{\rm velocity}(\bar{E}_{\mathbf{p}}) \\ & + \frac{1}{3}|\bar{\mathbf{p}}|^2 \delta u^\mu f_2^{\rm velocity}(\bar{E}_{\mathbf{p}}) + \delta u_\nu p^\nu \bar{E}_{\mathbf{p}} u^\mu f_3^{\rm velocity}(\bar{E}_{\mathbf{p}}). \end{aligned} \tag{26}$$

The actual data stored and printed is

$$\bar{p} f_1^{\rm temp.}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[7]} \times \frac{\delta T}{T}, \tag{27}$$

$$\bar{p} f_2^{\rm temp.}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[8]} \times \frac{\delta T}{T}, \tag{28}$$

$$\bar{p}^2 f_1^{\rm velocity}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[9]} \times \frac{1}{T}, \tag{29}$$

$$\bar{p}^2 f_2^{\rm velocity}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[10]} \times \frac{1}{T}, \tag{30}$$

$$\bar{p}^2 f_3^{\rm velocity}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[11]} \times \frac{1}{T}, \tag{31}$$

$$\tag{32}$$

Note that irreducible components for velocity perturbation can also be divergent, but the explicit momentum prefactors will make it finite.

Initialization is given by

$$
\begin{aligned}
\texttt{fFj\_arr[7]} &= \bar{p} f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)) \frac{\bar{E}_{\mathbf{p}}}{T}, && (33)\\
\texttt{fFj\_arr[8]} &= \bar{p} f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)) \frac{\bar{E}_{\mathbf{p}}}{T}, && (34)\\
\texttt{fFj\_arr[9]} &= \bar{p}^2 f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)), && (35)\\
\texttt{fFj\_arr[10]} &= \bar{p}^2 f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)), && (36)\\
\texttt{fFj\_arr[11]} &= \bar{p}^2 f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\rm eq}(\bar{E}_{\mathbf{p}}, T, \mu)) && (37)
\end{aligned}
$$

3 New transformation rules for irreducible components

From appendix A, we have the following transformation rule for irreducible components f_i after a two body decay

$$
f_i^b(\bar{E}_{\mathbf{p}}) = B \frac{\nu_a}{\nu_b} \frac{m_a^2}{m_b^2} \frac{1}{2} \int_{-1}^{1} dw A_i(w) f_i^a(E(w)). \tag{38}
$$

The calculation is straightforward and the appropriate $A_i(w)$ function

$$
A_1^{\rm eq} = \frac{Q(w)}{|\bar{\mathbf{p}}|}, \qquad A_2^{\rm eq} = \frac{E(w)}{\bar{E}_{\mathbf{p}}} \tag{39}
$$

where we remind that $E(w)$ and $Q(w)$ were defined as

$$
E(w) \equiv \frac{m_a E^a_{b|c} \bar{E}_{\mathbf{p}}}{m_b^2} - w \frac{m_a p^a_{b|c} |\bar{\mathbf{p}}|}{m_b^2}, \tag{40a}
$$

$$
Q(w) = \frac{m_a E^a_{b|c} |\bar{\mathbf{p}}|}{m_b^2} - w \frac{m_a p^a_{b|c} \bar{E}_{\mathbf{p}}}{m_b^2}. \tag{40b}
$$

For the weight functions f_i for δu^μ perturbations we obtain

$$
A_1^{\rm velocity} = \frac{3}{2} \frac{Q(w)^2}{|\bar{\mathbf{p}}|^2} - \frac{1}{2} \frac{E(w)^2 - m_a^2}{|\bar{\mathbf{p}}|^2} \tag{41}
$$

$$
A_2^{\rm velocity} = \frac{E(w)^2 - m_a^2}{|\bar{\mathbf{p}}|^2}, \qquad A_3^{\rm velocity} = A_1^{\rm eq}(w) A_2^{\rm eq}(w) \tag{42}
$$

while for the shear-stress case

$$
A_1^{\rm shear} = \frac{5}{2} \frac{Q(w)^3}{|\bar{\mathbf{p}}|^3} - \frac{3}{2} \frac{Q(w)}{|\bar{\mathbf{p}}|} \frac{E(w)^2 - m_a^2}{|\bar{\mathbf{p}}|^2} \tag{43}
$$

$$
A_2^{\rm shear} = A_2^{\rm velocity}(w) A_1^{\rm eq}(w), \quad A_3 = A_1^{\rm velocity}(w) A_2^{\rm eq}(w). \tag{44}
$$

If instead we want to have components multiplied by powers of momenta, we have a new transformation rule

$$\bar{p}^n f_i^b(\bar{E}_{\mathbf{p}}) = B\frac{\nu_a}{\nu_b}\frac{m_a^2}{m_b^2}\frac{1}{2}\int_{-1}^{1} dw \frac{\bar{p}^n}{(E(w)^2 - m_a^2)^{n/2}} A_i(w)\,(\bar{p}^n f_i^a)(E(w)). \tag{45}$$

For the current choices of n we have

$$\text{new}\quad A_1^{\text{eq}} = \frac{Q(w)}{\sqrt{E(w)^2 - m_a^2}}, \qquad A_2^{\text{eq}} = \frac{E(w)}{\bar{E}_{\mathbf{p}}}\frac{|\bar{\mathbf{p}}|}{\sqrt{E(w)^2 - m_a^2}} \tag{46}$$

$$\text{new}\quad A_1^{\text{velocity}} = \frac{3}{2}\frac{Q(w)^2}{E(w)^2 - m_a^2} - \frac{1}{2} \tag{47}$$

$$\text{new}\quad A_2^{\text{velocity}} = 1, \qquad A_3^{\text{velocity}} = A_1^{\text{eq}}(w)A_2^{\text{eq}}(w) \tag{48}$$

$$\text{new}\quad A_1^{\text{shear}} = \frac{5}{2}\frac{Q(w)^3}{(E(w)^2 - m_a^2)^{3/2}} - \frac{3}{2}\frac{Q(w)}{\sqrt{E(w)^2 - m_a^2}} \tag{49}$$

$$\text{new}\quad A_2^{\text{shear}} = A_2^{\text{velocity}}(w)A_1^{\text{eq}}(w), \quad A_3 = A_1^{\text{velocity}}(w)A_2^{\text{eq}}(w). \tag{50}$$

Finally we note that for massless particles among the decay products we need to use the change of variables The limit of the massless final state particle $m_b = 0$ can be treated by a simple change of variables $u = (1-w)\frac{m_a^2}{m_b^2}\frac{|\bar{\mathbf{p}}|p_{b|c}^a}{m_a^2}$, so that

$$E(w) \equiv \frac{m_a(E_{b|c}^a \bar{E}_{\mathbf{p}} - p_{b|c}^a|\bar{\mathbf{p}}|)}{m_b^2} + (1-w)\frac{m_a p_{b|c}^a|\bar{\mathbf{p}}|}{m_b^2}, \tag{51}$$

$$= \frac{m_a}{2}\left(\frac{p_{b|c}^a}{\bar{\mathbf{p}}} + \frac{\bar{\mathbf{p}}}{p_{b|c}^a}\right) + u m_a \tag{52}$$

$$Q(w) = \frac{m_a(E_{b|c}^a|\bar{\mathbf{p}}| - p_{b|c}^a \bar{E}_{\mathbf{p}})}{m_b^2} + (1-w)\frac{m_a p_{b|c}^a \bar{E}_{\mathbf{p}}}{m_b^2} \tag{53}$$

$$= \frac{m_a}{2}\left(\frac{\bar{\mathbf{p}}}{p_{b|c}^a} - \frac{p_{b|c}^a}{\bar{\mathbf{p}}}\right) + u m_a \tag{54}$$

$$E(w)^2 - m_a^2 = Q^2(w) + (1-w^2)\frac{m_a^2}{m_b^2}\left(p_{b|c}^a\right)^2 \tag{55}$$

$$= Q^2(w) + (1+w)(1-w)\frac{m_a^2}{m_b^2}\left(p_{b|c}^a\right)^2 \tag{56}$$

$$= Q^2(w) + \left(2 + u\frac{m_b^2}{m_a^2}\frac{m_a^2}{p_{b|c}^a|\bar{\mathbf{p}}|}\right) u\frac{p_{b|c}^a}{\bar{\mathbf{p}}} m_a^2 \tag{57}$$

$$\lim_{m_b \to 0} = Q^2(w) + 2u\frac{p_{b|c}^a}{|\bar{\mathbf{p}}|} m_a^2. \tag{58}$$

and the integration variable goes from

$$\bar{p}^n f_i^b(\bar{E}_{\mathbf{p}}) = B\frac{\nu_a}{\nu_b}\frac{1}{2}\int_0^\infty \frac{m_a^2 du}{p^a_{b|c}|\bar{\mathbf{p}}|}\frac{\bar{p}^n}{(E(u)^2 - m_a^2)^{n/2}} A_i(u)\,(\bar{p}^n f_i^a)(E(u)). \tag{59}$$

4 Header of output files

```
#      Name      Mass     Gamma  Spin Isospin    I3 Nq Ns Naq Nas Nc Nac         MC
Nyield
# pi0139plu  0.139570 0.00000e+00   0.0    1.0    1.0 1 0 1 0 0 0       211 0.000816617
# 1:pbar [GeV] 2:m [GeV] 3:pbar*feq_1 4:pbar*feq_2 5:pbar^3*fshear_1 6:pbar^3*fshear_2
7:pbar^3*fshear_3 8:pbar*fbulk_1 9:pbar*fbulk_2 10:pbar*ftemperature_1
11:pbar*ftemperature_2 12:pbar^2*fvelocity 1 13:pbar^2*fvelocity_2 14:pbar^2*fvelocity_3
```