FastReso documentation

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January 1, 2021

1 Cooper-Frye integral for decay products

The spectrum of final decay products is given by

$$E_{\mathbf{p}}\frac{dN}{d^3\mathbf{p}} = \frac{\nu}{(2\pi)^3}\int_\sigma d\sigma_\mu g^\mu \tag{1}$$

where g^μ is the vector distribution function of decayed particles. g^μ is typically given as a sum of a couple Lorentz invariant weight functions and some explicit Lorentz vectors, which depends on what is the initial primary resonance distribution function. Below we document, which distribution components are calculated in `FastReso`. See the paper, code and `README` for further details of how the decays are implemented [1].

2 Initial distribution functions

Code stores distribution components in an array `TParticle::fFj_arr`, which is of size `[grid_params:fN_f]` $\times$ `[grid_params:fN_pbar]`, where $N_f = 12$ is the total number of irreducible components and $N_{\bar{\mathbf{p}}} = 401$ is the number of (un-equally spaced) momentum grid points. Note that some momentum independent terms are factored out to make the stored values simple and dimensionless. One can speed up computations by selecting only specific components. However, re-initialization of `TParticle` object from the output file is only supported if all 12 components are computed. As of time of writing (2021.01.01) `FastReso` calculates the decay components of the following initial distribution functions.

2.1 Equilibrium distribution

Initial distribution is given by Fermi-Dirac or Bose-Einstein distribution (there is also an option for Boltzmann distribution, which is never used)

$$f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu) = \left(e^{(\bar{E}_{\mathbf{p}}-\mu)/T} \mp 1\right)^{-1}, \tag{2}$$

[1] Mazeliauskas, S. Floerchinger, E. Grossi, D. Teaney, *Fast resonance decays in nuclear collisions*, **Eur. Phys. J. C** (2019), arXiv:1809.11049

where $\mu = Q_B\mu_B + Q_{I_3}\mu_{I_3} + Q_S\mu_S + Q_C\mu_C$ where the charges and corresponding chemical potentials are that of baryon number, third component of the isospin, strangeness and charm. The decayed vector distribution function is given by

$$g_b^\mu(\bar{E}_{\mathbf{p}}) = f_1^{\text{eq}}(\bar{E}_{\mathbf{p}})\left(p^\mu - \bar{E}_{\mathbf{p}}u^\mu\right) + f_2^{\text{eq}}(\bar{E}_{\mathbf{p}})\bar{E}_{\mathbf{p}}u^\mu. \tag{3}$$

The actual data stored and printed is (note that we multiply by additional fluid frame momentum $\bar{p}$ relative to what is written in the paper to make the $\bar{p} \to 0$ limit regular on numerical grid)

$$\bar{p}f_1^{\text{eq}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[0]}, \tag{4}$$
$$\bar{p}f_2^{\text{eq}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[1]} \tag{5}$$

which is initialized by `get_initial_Fj` in `TFastReso_formulas.h`

$$\texttt{fFj\_arr[0]} = \bar{p}f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu), \tag{6}$$
$$\texttt{fFj\_arr[1]} = \bar{p}f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu) \tag{7}$$

The momentum distribution of a product from the two-body decay is calculated by doing a one-dimensional integral

$$F_j^b(\bar{E}_{\mathbf{p}}) = B\frac{\nu_a}{\nu_b}\frac{m_a^2}{m_b^2}\frac{1}{2}\int_{-1}^{1} dw A_i(w) F_j^a(E(w)). \tag{8}$$

The calculation is straightforward and the appropriate $A_i(w)$ functions, which are given by `get_factor_Aj` in `TFastReso_formulas.h`, are

$$A_1^{\text{eq}} = \frac{Q(w)}{p(w)}, \qquad A_2^{\text{eq}} = \frac{E(w)\bar{p}}{\bar{E}_{\mathbf{p}}p(w)} \tag{9}$$

where we remind that $E(w)$ and $Q(w)$ were defined as

$$E(w) \equiv \frac{m_a E_{b|c}^a \bar{E}_{\mathbf{p}}}{m_b^2} - w\frac{m_a p_{b|c}^a|\bar{\mathbf{p}}|}{m_b^2}, \tag{10a}$$
$$Q(w) = \frac{m_a E_{b|c}^a|\bar{\mathbf{p}}|}{m_b^2} - w\frac{m_a p_{b|c}^a \bar{E}_{\mathbf{p}}}{m_b^2}. \tag{10b}$$
$$p(w)^2 \equiv E(w)^2 - m_a^2 = Q(w)^2 + \left(1 - w^2\right)\frac{m_a^2 (p_{b|c}^a)^2}{m_b^2}, \tag{10c}$$

Note that in the special case of massless limit $m_b = 0$ a change of integration variables is made

$$u = (1 - w)\frac{m_a^2}{m_b^2}\frac{p_{b|c}^a|\bar{\mathbf{p}}|}{m_a^2}. \tag{11}$$

$$F_j^b(\bar{E}_{\mathbf{p}}) = B\frac{\nu_a}{\nu_b}\frac{1}{2}\int_0^\infty du A_i(u)\frac{m_a^2}{p_{b|c}^a|\bar{\mathbf{p}}|}F_j^a(E(u)). \tag{12}$$

$$E(u) \equiv \frac{m_a}{2}\left(\frac{|\bar{\mathbf{p}}|}{p^a_{b|c}} + \frac{p^a_{b|c}}{|\bar{\mathbf{p}}|}\right) + um_a \tag{13a}$$

$$Q(u) \equiv \frac{m_a}{2|\bar{\mathbf{p}}|}\left(\frac{|\bar{\mathbf{p}}|^2}{p^a_{b|c}} - p^a_{b|c}\right) + um_a \tag{13b}$$

$$p(u)^2 \equiv E(u)^2 - m_a^2 = Q(w)^2 + 2u\frac{p^a_{b|c}}{|\bar{\mathbf{p}}|}m_a^2 \tag{13c}$$

2.2 Viscous bulk and shear distributions

Initial perturbations are given by (taken from [arXiv:1509.06738])

$$\delta f^{\text{bulk}}(x,p) = f_{\text{eq}}(1 \pm f_{\text{eq}})\left[\frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right) - \frac{1}{3}\frac{m^2}{T\bar{E}_p}\right] \times \frac{\tau_\Pi \Pi}{\zeta}, \tag{14}$$

$$\delta f^{\text{shear}}(x,p) = f_{\text{eq}}(1 \pm f_{\text{eq}}) \times \pi_{\rho\nu}p^\rho p^\nu \times \frac{1}{2(e+p)T^2}. \tag{15}$$

where c_s is the speed of sound on the freeze-out surface (given as a parameter) and m is a primary resonance mass. The speed of sound should be adjusted to match the underlying hydrodynamic evolution. Current implementation uses the parametrization motivated by fits to lattice and HRG data from [arxiv:1811.01870][2] and given by numerical function in `TFastReso.h`

$$c_s^2\,(x = T\ (\text{GeV})) = \texttt{cs2FluiduM}(x). \tag{16}$$

User is responsible for supplying the right value of the speed of sound (especially if $\mu \neq 0$).

The decayed vector distribution function for bulk perturbation is given by

$$g^\mu_{\text{bulk}}\Pi = \Big[\big(p^\mu - \bar{E}_{\mathbf{p}}u^\mu\big)f_1^{\text{bulk}}(\bar{E}_{\mathbf{p}}) + \bar{E}_{\mathbf{p}}u^\mu f_2^{\text{bulk}}(\bar{E}_{\mathbf{p}})\Big] \times \frac{-\tau_\pi\Pi}{\zeta}, \tag{17}$$

and shear perturbations

$$\begin{aligned} g^{\mu\nu\rho}_{\text{shear}}\pi_{\nu\rho} = \Big\{&[\eta^{\rho\sigma}(p^\mu - \bar{E}_{\mathbf{p}}u^\mu) - \frac{2}{5}\eta^{\rho\mu}\Delta^{\sigma\alpha}p_\alpha]f_1^{\text{shear}}(\bar{E}_{\mathbf{p}}) \\ &+ \frac{2}{5}\eta^{\rho\mu}\Delta^{\sigma\alpha}p_\alpha f_2^{\text{shear}}(\bar{E}_{\mathbf{p}}) + \eta^{\rho\sigma}\bar{E}_{\mathbf{p}}u^\mu f_3^{\text{shear}}(\bar{E}_{\mathbf{p}})\Big\} \times \frac{p^\nu\pi_{\nu\rho}p_\sigma}{2(e+p)T^2}. \end{aligned} \tag{18}$$

[2]Stefan Floerchinger, Eduardo Grossi and Jorrit Lion, Phys. Rev. C100 (2019)

The actual data stored and printed is

$$\bar{p}^3 f_1^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[2]} \tag{19}$$
$$\bar{p}^3 f_2^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[3]} \tag{20}$$
$$\bar{p}^3 f_3^{\mathrm{shear}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[4]} \tag{21}$$
$$\bar{p} f_1^{\mathrm{bulk}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[5]} \tag{22}$$
$$\bar{p} f_2^{\mathrm{bulk}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[6]} \tag{23}$$
$$\tag{24}$$

which is initialized by

$$\texttt{fFj\_arr[2]} = \bar{p}^3 f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{25}$$
$$\texttt{fFj\_arr[3]} = \bar{p}^3 f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{26}$$
$$\texttt{fFj\_arr[4]} = \bar{p}^3 f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{27}$$
$$\texttt{fFj\_arr[5]} = \bar{p} f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)) \left[\frac{1}{3}\frac{m^2}{T\bar{E}_p} - \frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right)\right], \tag{28}$$
$$\texttt{fFj\_arr[6]} = \bar{p} f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\mathrm{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)) \left[\frac{1}{3}\frac{m^2}{T\bar{E}_p} - \frac{\bar{E}_p}{T}\left(\frac{1}{3} - c_s^2\right)\right] \tag{29}$$

Note the sign in the bulk perturbation. Also, because of the explicit shear components carry $|\bar{p}|^2$, the irreducible weight function after decays are actually diverging at the origin, but no more than quadratically, so we multiply by cubic and linear $\bar{p}$ weights.

The same procedure of finding the transformation rules also apply for the irreducible decomposition of tensor distribution function $g^{\mu\nu}_{\mathrm{velocity}}$ and $g^{\mu\nu\rho}_{\mathrm{shear}}$. For the weight functions f_i for δu^μ perturbations (with appropriate momentum power) we obtain

$$A_1^{\mathrm{shear}} = \frac{Q(w)}{p(w)}\left(\frac{5}{2}\frac{Q(w)^2}{p(w)^2} - \frac{3}{2}\right) \tag{30}$$
$$A_2^{\mathrm{shear}} = A_2^{\mathrm{velocity}}(w) A_1^{\mathrm{eq}}(w), \quad A_3 = A_1^{\mathrm{velocity}}(w) A_2^{\mathrm{eq}}(w). \tag{31}$$

$$A_1^{\mathrm{velocity}} = \frac{3}{2}\frac{Q(w)^2}{p(w)^2} - \frac{1}{2} \quad A_2^{\mathrm{velocity}} = 1, \qquad A_3^{\mathrm{velocity}} = A_1^{\mathrm{eq}}(w) A_2^{\mathrm{eq}}(w) \tag{32}$$

2.3 Temperature and velocity perturbations

Linear perturbations in temperature and velocity

$$\delta f^{\mathrm{temp.}}(x,p) = f_{\mathrm{eq}}(1 \pm f_{\mathrm{eq}})\frac{\bar{E}_{\mathbf{p}}}{\bar{T}}\frac{\delta T}{\bar{T}}, \tag{33}$$
$$\delta f^{\mathrm{velocity}}(x,p) = f_{\mathrm{eq}}(1 \pm f_{\mathrm{eq}})\frac{\delta u_\nu p^\nu}{\bar{T}}. \tag{34}$$

$$g^{\mu}_{\text{temp.}}\delta T = \Big[\left(p^\mu - \bar{E}_{\mathbf{p}} u^\mu\right) f_1^{\text{temp.}}(\bar{E}_{\mathbf{p}}) + \bar{E}_{\mathbf{p}} u^\mu f_2^{\text{temp.}}(\bar{E}_{\mathbf{p}})\Big] \times \frac{\delta T}{\bar{T}}. \tag{35}$$

$$\begin{aligned} g^{\mu\nu}_{\text{vel.}}\delta u_\nu = \Big\{ & [\eta^{\nu\rho}(p^\mu - \bar{E}_{\mathbf{p}} u^\mu) - \frac{1}{3}\eta^{\nu\mu} p_\sigma \Delta^{\sigma\rho})] f_1^{\text{velocity}}(\bar{E}_{\mathbf{p}}) \\ & + \frac{1}{3}\eta^{\nu\mu} p_\sigma \Delta^{\sigma\rho} f_2^{\text{velocity}}(\bar{E}_{\mathbf{p}}) + \eta^{\nu\rho}\bar{E}_{\mathbf{p}} u^\mu f_3^{\text{velocity}}(\bar{E}_{\mathbf{p}}) \Big\} \times \frac{\delta u_\nu p_\rho}{\bar{T}}. \end{aligned} \tag{36}$$

Vector distribution function is then given by The actual data stored and printed is

$$\bar{p} f_1^{\text{temp.}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[7]} \tag{37}$$
$$\bar{p} f_2^{\text{temp.}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[8]} \tag{38}$$
$$\bar{p}^2 f_1^{\text{velocity}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[9]} \tag{39}$$
$$\bar{p}^2 f_2^{\text{velocity}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[10]} \tag{40}$$
$$\bar{p}^2 f_3^{\text{velocity}}(\bar{E}_{\mathbf{p}}) = \texttt{fFj\_arr[11]} \tag{41}$$

Note that irreducible components for velocity perturbation can also be divergent, but the explicit momentum prefactors will make it finite.

Initialization is given by

$$\texttt{fFj\_arr[7]} = \bar{p} f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu))\frac{\bar{E}_{\mathbf{p}}}{T}, \tag{42}$$
$$\texttt{fFj\_arr[8]} = \bar{p} f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu))\frac{\bar{E}_{\mathbf{p}}}{T}, \tag{43}$$
$$\texttt{fFj\_arr[9]} = \bar{p}^2 f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{44}$$
$$\texttt{fFj\_arr[10]} = \bar{p}^2 f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)), \tag{45}$$
$$\texttt{fFj\_arr[11]} = \bar{p}^2 f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)(1 \pm f_{\text{eq}}(\bar{E}_{\mathbf{p}}, T, \mu)) \tag{46}$$

$A_j(w)$ functions for the scalar temperature perturbation are the same as for the equilibrium distribution. The velocity $A_j(w)$ functions were defined above.

3 Resonance lists

The particle data and resonance decay lists are *input files* and is the sole responsibility of the user and not part of FastReso publication. Consequently, any work using FastReso must document (and appropriately cite) the resonance lists used. For convenience, two types of input files are supported. One is based on the input for `THERMINATOR 2` Monte-Carlo decay code[3,4] (also used in SHARE[5]) and is called THERMINATOR. The other common format is used in

[3]Mikolaj Chojnacki, Adam Kisiel, Wojciech Florkowski, Wojciech Broniowski , Comput.Phys.Commun. 183 (2012) 746-773, arXiv:1102.0273

[4]`https://therminator2.ifj.edu.pl/`

[5]Giorgio Torrieri, Steve Steinke, Wojciech Broniowski, Wojciech Florkowski Jean Letessier, Johann Rafelski, , Comput.Phys.Commun.167:229-251,2005, arXiv:nucl-th/0404083

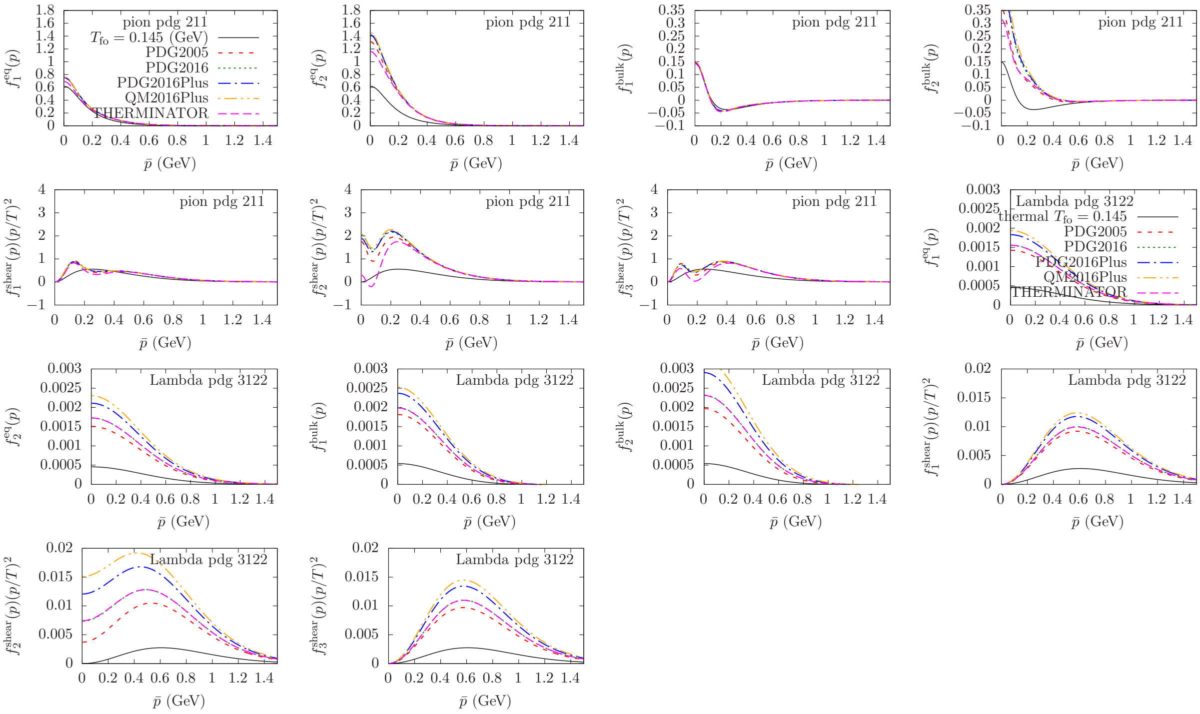

Figure 1: Pion and Lambda irreducible components calculated for different resonance decay lists. Freeze-out temperature $T_{\rm fo} = 145\,\mathrm{MeV}$ and zero chemical potentials. Weak decays are excluded (this is different from 1809.11049).

some hydrodynamic models. Here we call this format AZYHYDRO. The most recent public compilation of different PDG versions in this format were composed by Paolo Parotto[6] and can be downloaded from this link[7]

It is customary for experimentalists to present identified particle spectra with weak decays subtracted. Therefore, weak decays should not be included when making comparison to data. In Fig. 1 we show different components for pion and Lambda for different resonance decay lists. Note that here therminator list is modified compared to original publication, namely, that weak decays are excluded. Table 3 lists the number of particles and resonances in each list. Note that QM2016Plus is includes a large number of predicted particles/decays and takes substantially longer to compute. Note that `FastReso` does not support currently 4-particle decays. All decays are also assumed to be zero-width.

[6]Please refer to the following works: Phys.Rev. D96 (2017) no.3, 034517 `https://inspirehep.net/record/1512119`, Phys.Rev. C98 (2018) no.3, 034909 `https://inspirehep.net/record/1636208`, Phys.Rev. C101 (2020) no.5, 054905 `https://inspirehep.net/literature/1782970`

[7]`http://nsmn1.uh.edu/cratti/decays.html`

List	Number of particles	number of 2-body decays	number of 3-body decays
PDG2005	143	395	82
PDG2016	609	2677	489
PDG2016Plus	739	3291	513
QM2016Plus	1518	16043	4065
THERMINATOR	381	1739	105

Table 1: Summary of number of particles and decays in the resonance lists. Note that FastReso currently supports only 2-body and 3-body decays, so any 4-body decays are ignored.

3.1 Header of output files

`TParticle` class objects have `print()` method which prints out the stored grid of irreducible components and some information about the particle. These files can be used to load the class using creation method (global grid parameters should be the same).

For AZYHYDRO particle format

```
# mc-number                  name            mass           width       deg
Qb      Qs      Qc      Qbot    isospin charge  decays     Nyield
#       211                 pi+          0.14000       0.00000e+00     1
0         0     0       0       1          1        1     0.000350054
# 1:pbar [GeV]  2:m [GeV]       3:feq 1 4:feq 2 5:fshear 1      6:fshear 2
7:fshear 3      8:fbulk 1       9:fbulk 2       10:ftemperature 1
11:ftemperature 2       12:fvelocity 1  13:fvelocity 2  14:fvelocity 3
```

For THERMINATOR particle format

```
#     Name      Mass     Gamma  Spin Isospin    I3 Nq Ns Naq Nas Nc Nac        MC
Nyield
# pi0139plu  0.139570 0.00000e+00   0.0   1.0   1.0 1 0 1 0 00      211  0.000701
#    1:pbar [GeV] 2:m [GeV]         3:feq 1         4:feq 2      5:fshear 1
6:fshear 2   7:fshear 3       8:fbulk 1       9:fbulk 2 10:ftemperature 1
11:ftemperature 2  12:fvelocity 1  13:fvelocity 2  14:fvelocity 3
```

4 Kernels

`TKernel` class performs the rapidity and azimuthal angle integrals of the irreducible components, which generates kernels for calculating spectra on boost invariant and azimuthally symmetric freeze-out surface.

$$d\sigma_\mu = \tau(\alpha) r(\alpha) \left(\frac{\partial r}{\partial \alpha}, -\frac{\partial \tau}{\partial \alpha}, 0, 0 \right) d\alpha d\phi d\eta \tag{47}$$

Namely, it performs the following numerical integrals. In practise rapidity is restricted to $|\eta| < 5$

$$
\begin{aligned}
&\frac{dN}{2\pi p_T dp_T dy} = \frac{\nu}{(2\pi)^3}\int_0^1 d\alpha\ \tau(\alpha) r(\alpha) \\
&\bigg\{ \frac{\partial r}{\partial \alpha}\Big[K_1^{\rm eq} + \frac{\bar{\pi}_\eta^\eta}{2(e+p)T^2}\ K_1^{\rm shear} + \frac{\bar{\pi}_\phi^\phi}{2(e+p)T^2}\ K_3^{\rm shear} + \frac{-\tau_\pi \bar{\Pi}}{\zeta}\ K_1^{\rm bulk}\Big] \\
&-\frac{\partial \tau}{\partial \alpha}\Big[K_2^{\rm eq} + \frac{\bar{\pi}_\eta^\eta}{2(e+p)T^2}\ K_2^{\rm shear} + \frac{\bar{\pi}_\phi^\phi}{2(e+p)T^2}\ K_4^{\rm shear} + \frac{-\tau_\pi \bar{\Pi}}{\zeta}\ K_2^{\rm bulk}\Big]\bigg\},
\end{aligned} \tag{48}
$$

$$
K_1^{\rm eq}(p_T, u^r) = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \big\{ f_1^{\rm eq}(\bar{E}_{\mathbf{p}}) m_T \cosh(\eta) + \big(f_2^{\rm eq}(\bar{E}_{\mathbf{p}}) - f_1^{\rm eq}(\bar{E}_{\mathbf{p}})\big)\ \bar{E}_{\mathbf{p}} u^\tau \big\}, \tag{49}
$$

$$
K_2^{\rm eq}(p_T, u^r) = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \big\{ f_1^{\rm eq}(\bar{E}_{\mathbf{p}}) p_T \cos(\phi) + \big(f_2^{\rm eq}(\bar{E}_{\mathbf{p}}) - f_1^{\rm eq}(\bar{E}_{\mathbf{p}})\big)\ \bar{E}_{\mathbf{p}} u^r \big\}, \tag{50}
$$

$$
\begin{aligned}
K_1^{\rm shear}(p_T, u^r) = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \Big\{ &\big[f_1^{\rm shear}(\bar{E}_{\mathbf{p}}) m_T \cosh(\eta) + \Big(f_3^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \bar{E}_{\mathbf{p}} u^\tau \big] \\
&\times \big[m_T^2 \sinh(\eta)^2 - \{m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)\}^2 \big] \\
&+ \Big(f_2^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \frac{2}{5}|\bar{\mathbf{p}}|^2 u^r\ [m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)]\ \Big\},
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
K_2^{\rm shear}(p_T, u^r) = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \Big\{ &\big[f_1^{\rm shear}(\bar{E}_{\mathbf{p}}) p_T \cos(\phi) + \Big(f_3^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \bar{E}_{\mathbf{p}} u^r \big] \\
&\times \big[m_T^2 \sinh(\eta)^2 - \{m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)\}^2 \big] \\
&+ \Big(f_2^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \frac{2}{5}|\bar{\mathbf{p}}|^2 u^\tau\ [m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)]\ \Big\},
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
K_3^{\rm shear}(p_T, u^r) = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \Big\{ &\big[f_1^{\rm shear}(\bar{E}_{\mathbf{p}}) m_T \cosh(\eta) + \Big(f_3^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \bar{E}_{\mathbf{p}} u^\tau \big] \\
&\times \big[p_T^2 \sin(\phi)^2 - \{m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)\}^2 \big] \\
&+ \Big(f_2^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \frac{2}{5}|\bar{\mathbf{p}}|^2 u^r\ [m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)]\ \Big\},
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
K_4^{\rm shear}(p_T, u^r) = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \Big\{ &\big[f_1^{\rm shear}(\bar{E}_{\mathbf{p}}) p_T \cos(\phi) + \Big(f_3^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \bar{E}_{\mathbf{p}} u^r \big] \\
&\times \big[p_T^2 \sin(\phi)^2 - \{m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)\}^2 \big] \\
&+ \Big(f_2^{\rm shear}(\bar{E}_{\mathbf{p}}) - f_1^{\rm shear}(\bar{E}_{\mathbf{p}})\Big)\ \frac{2}{5}|\bar{\mathbf{p}}|^2 u^\tau\ [m_T u^r \cosh(\eta) - p_T u^\tau \cos(\phi)]\ \Big\}.
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
K_1^{\rm bulk}(p_T,u^r) &= \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \{ f_1^{\rm bulk}(\bar{E}_{\mathbf{p}}) m_T \cosh(\eta) + \left(f_2^{\rm bulk}(\bar{E}_{\mathbf{p}}) - f_1^{\rm bulk}(\bar{E}_{\mathbf{p}}) \right) \bar{E}_{\mathbf{p}} u^\tau \}, \\
K_2^{\rm bulk}(p_T,u^r) &= \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \{ f_1^{\rm bulk}(\bar{E}_{\mathbf{p}}) p_T \cos(\phi) + \left(f_2^{\rm bulk}(\bar{E}_{\mathbf{p}}) - f_1^{\rm bulk}(\bar{E}_{\mathbf{p}}) \right) \bar{E}_{\mathbf{p}} u^r \},
\end{aligned} \tag{55}
$$

4.1 Blast-wave spectra

Fig. 2 show the equilibrium long-lived particle spectra after the decays from a simple constant time blast-wave surface with constant velocity.

$$
\frac{dN}{2\pi p_T dp_T dy} = \frac{\nu}{(2\pi)^3} \frac{\tau_{\rm fo} R^2}{2} K_1^{\rm eq}(p_T,u^r). \tag{56}
$$

Note that `FastReso` uses GeV as a default units, so $\hbar c$ conversion is needed if time and radius is given in fm. Here ν is the spin degeneracy of particles (note that Lambda spectra is usually given as a sum of particle and anti-partcile spectra, so additional factor of 2 might be needed in comparison with data).

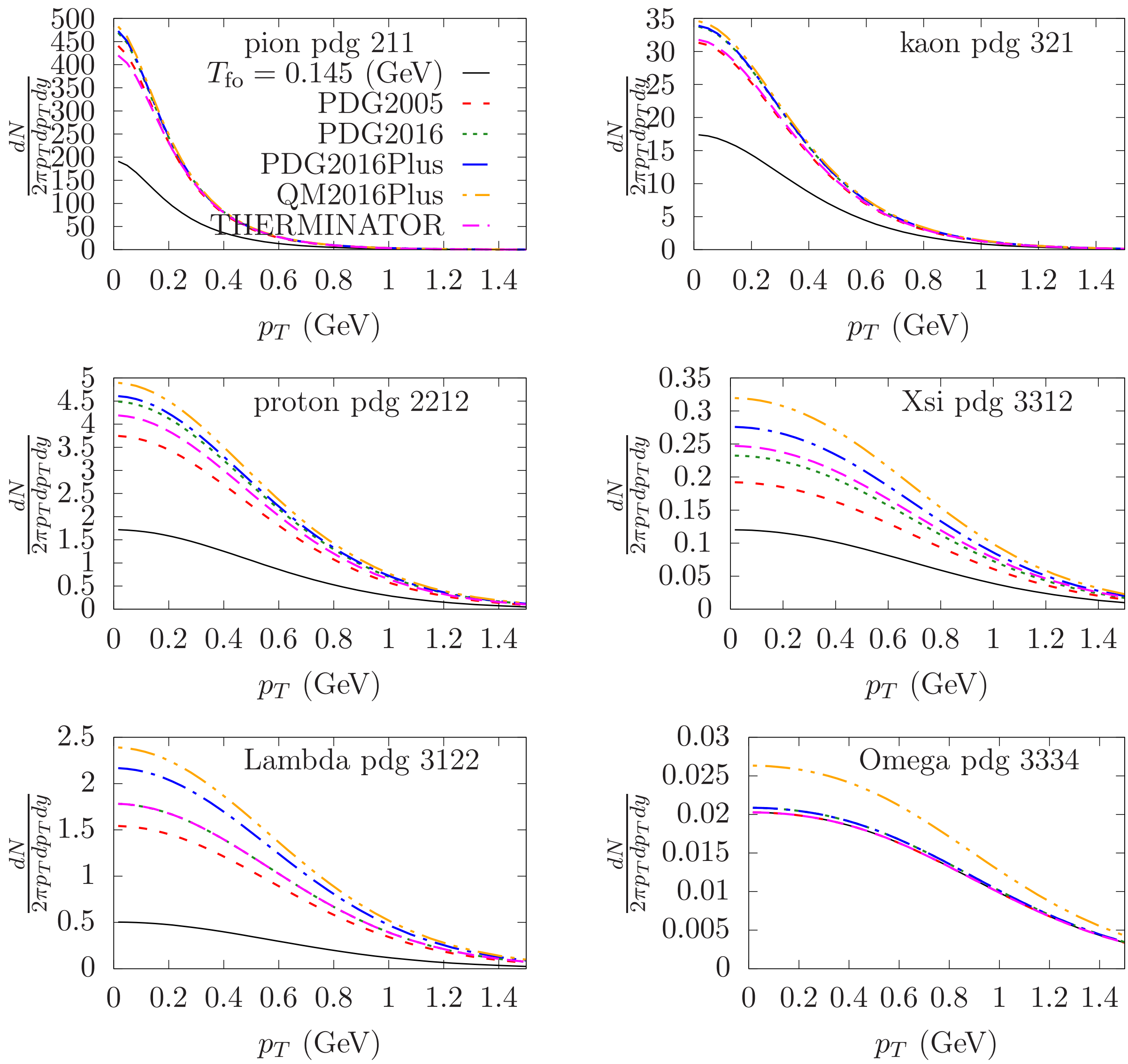

Figure 2: Decay spectra for different resonance decay lists. Freeze-out temperature $T_{\text{fo}} = 145\,\text{MeV}$ and zero chemical potentials. $R = 8.21\,\text{fm}$, $\tau_{\text{fo}} = 8.17\,\text{fm}$ and $u^r = 0.3712$. Weak decays are excluded (this is different from 1809.11049).