Optimization of an energy harvester via the cross-entropy method

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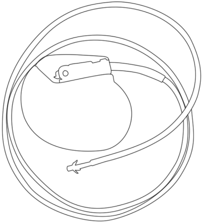
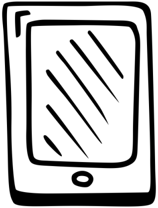
NUMERICO – **Nu**cleus of **M**odeling and **E**xperi**m**entation with **Co**mputers

ICoEV 2020

Virtual Congress, December 14-16, 2020

What these devices have in common?





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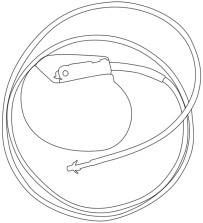
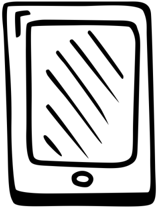
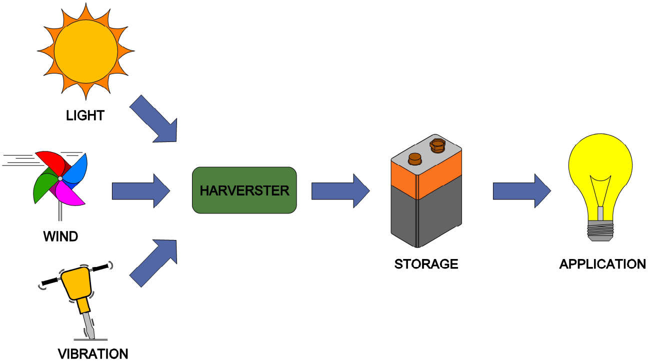




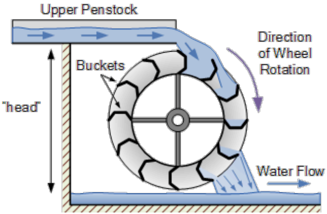
Both demand an autonomous power source to operate!

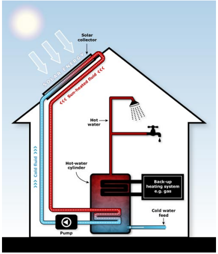
Energy Harvesting concept



- Capture wasted energy from external sources
- Store this wasted energy for future use
- Use the stored energy to supply other devices

Classical Technologies in Energy Harvesting





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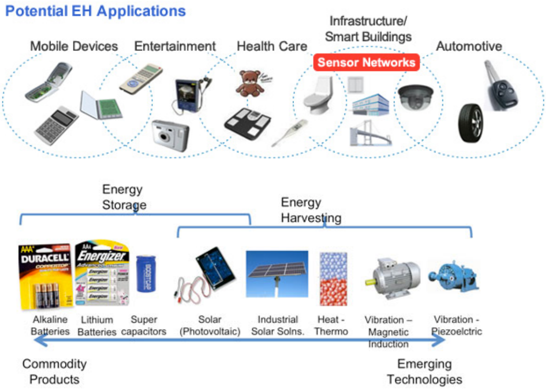
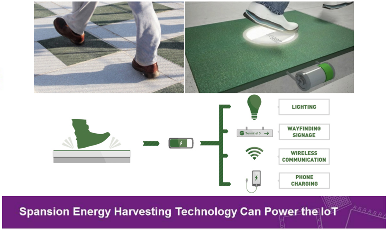
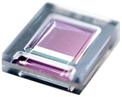
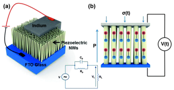
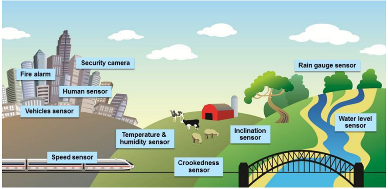
Emergent Technologies in Energy Harvesting











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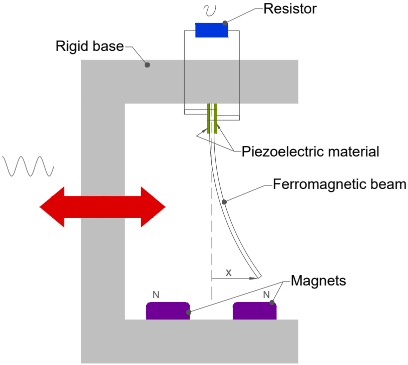
Research objectives

- Propose a strategy of design to enhance the recovered energy
  - Formulate a nonlinear non-convex optimization problem
  - Use the cross-entropy method to obtain an efficient solution



$$\ddot{x} + 2\,\xi\,\dot{x} - \frac{1}{2}x\left(1 - x^2\right) - \chi\,\upsilon = f\,\cos\left(\Omega t\right)$$

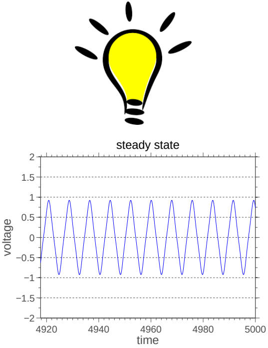
$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0$$

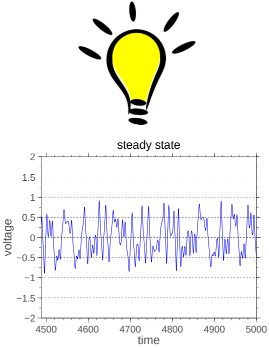
+ initial conditions

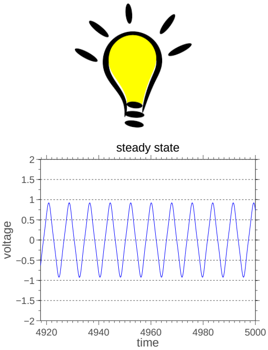
Mean output power:

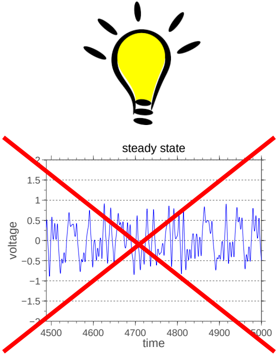
$$P = \frac{1}{T}\int_{t}^{t+T} \lambda\, v^2(\tau)\, d\tau$$

A. Erturk, J. Hoffmann and D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting.* **Applied Physics Letters**, 94: 254102, 2009.

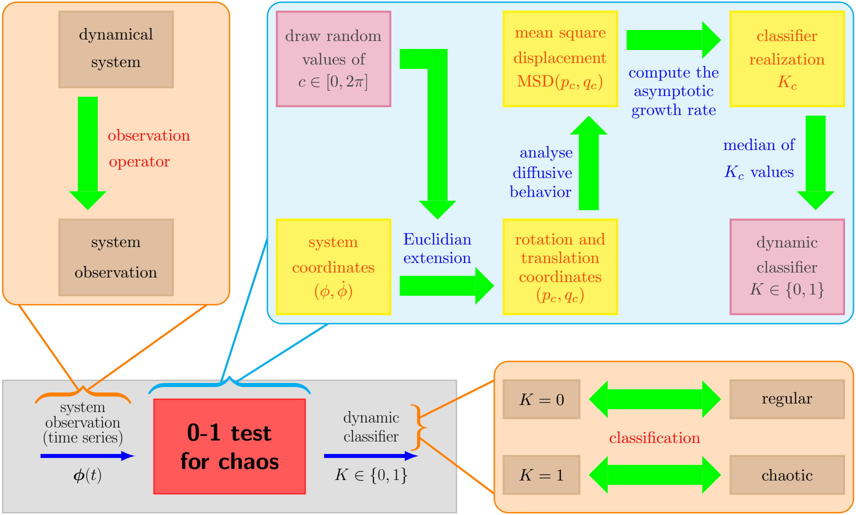

steady state
voltage
time
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
4920
4940
4960
4980
5000


steady state
voltage
time
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
4500
4600
4700
4800
4900
5000

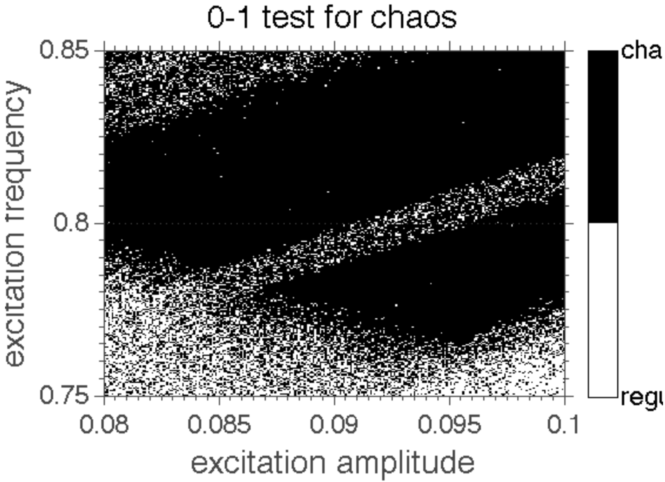




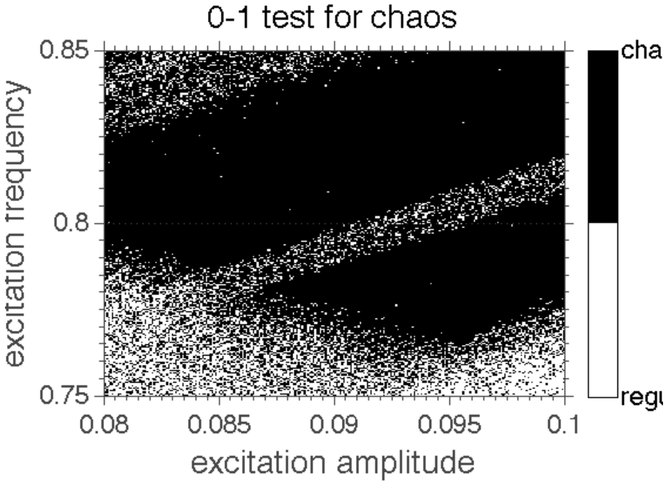
... irregular voltage is undesirable!


dynamical system
observation operator
system observation
draw random values of $c \in [0, 2\pi]$
mean square displacement $\mathrm{MSD}(p_c, q_c)$
compute the asymptotic growth rate
classifier realization K_c
analyse diffusive behavior
median of K_c values
system coordinates $(\phi, \dot{\phi})$
Euclidian extension
rotation and translation coordinates (p_c, q_c)
dynamic classifier $K \in \{0, 1\}$
system observation (time series)
$\phi(t)$
0-1 test for chaos
dynamic classifier
$K \in \{0, 1\}$
$K = 0$
regular
classification
$K = 1$
chaotic

Characterization of chaos



Characterization of chaos



There is a large number of high-energy periodic orbits embedded into the chaotic region

Optimization framework

- $\boldsymbol{x}$ — design variables vector
- $\mathcal{S}(\boldsymbol{x})$ — mean power
- $\mathcal{G}(\boldsymbol{x})$ — 0-1 test for chaos classifier

Constrained formulation:

$$\max \mathcal{S}(\boldsymbol{x}) \quad \text{s.t.} \quad \mathcal{G}(\boldsymbol{x}) = 0 \quad \text{and} \quad \boldsymbol{x}_{min} \leq \boldsymbol{x} \leq \boldsymbol{x}_{max}$$

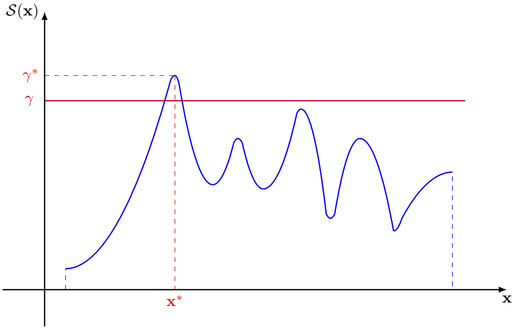
Penalized formulation:

$$\boldsymbol{x}^{\star} = \arg\max \left\{ \mathcal{S}(\boldsymbol{x}) + H \max\left(0, \mathcal{G}(\boldsymbol{x})\right) \right\}$$

Peculiarities:

- Test 0-1 for chaos constraint is a discontinuous function of $\boldsymbol{x}$
- Gradient-based methods are not applicable
- Evolutionary algorithms can be used (but we prefer not!)

Transform the optimization problem into a rare-event estimation problem



$$\mathcal{P}\left\{\mathcal{S}(\boldsymbol{X}) \geq \gamma\right\} \approx 0 \text{ for } \gamma \approx \gamma^\star$$

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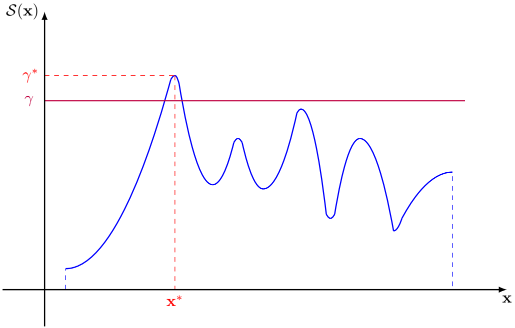


$$\mathcal{P}\left\{\mathcal{S}(\boldsymbol{X}) \geq \gamma\right\} \approx 0 \text{ for } \gamma \approx \gamma^{\star}$$

$\mathcal{S}(\boldsymbol{X}) \geq \gamma$ is a rare-event

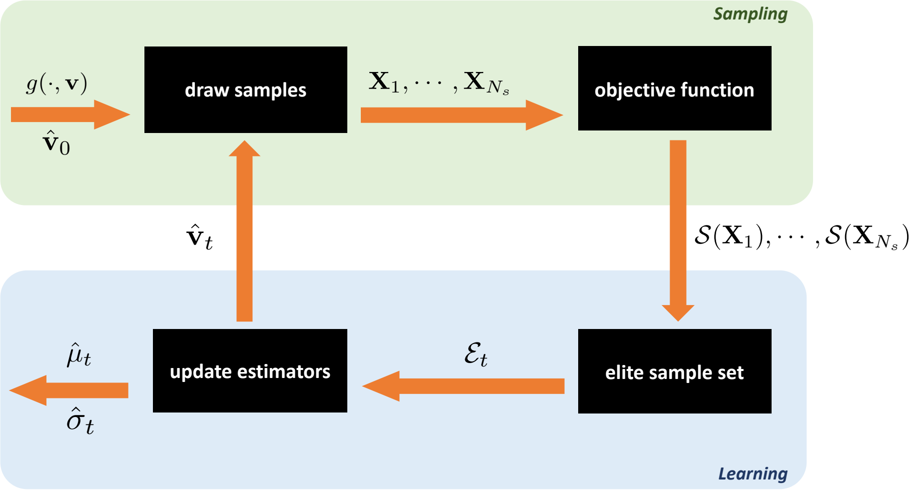
1. **Sampling:** Generate an iid sample of objects in the search space according to a specified probability distribution $g\left(\cdot;\boldsymbol{v}\right)$
2. **Learning:** Update the distribution parameters, based on the best performing samples (elite samples), using cross-entropy minimization

CE method generates an "optimal sequence" of estimators $\left(\widehat{\gamma}_t, \widehat{\boldsymbol{v}}_t\right)$
such that $\widehat{\gamma}_t \to \gamma^\star$ and $g\left(\boldsymbol{x}, \widehat{\boldsymbol{v}}_t\right) \to \delta\left(\boldsymbol{x} - \boldsymbol{x}^\star\right)$

- $\widehat{\boldsymbol{v}}_t = \arg\max_{\boldsymbol{v}} \sum_{\boldsymbol{X}_k \in \mathcal{E}_t} \ln g\left(\boldsymbol{X}_k; \boldsymbol{v}\right)$
  (maximum likelihood estimator)
- minimize KL divergence between $\mathbb{1}_{\{\mathcal{S}(\boldsymbol{x}) \geq \gamma\}}$ and $g(\cdot\,,\ \boldsymbol{v})$

R. Y. Rubinstein, and Dirk P. Kroese, **Simulation and the Monte Carlo Method**, Wiley, 3rd Edition, 2017.

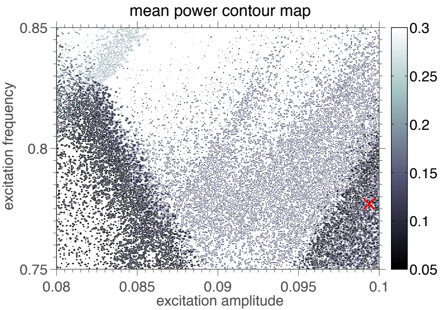
Cross-entropy algorithm

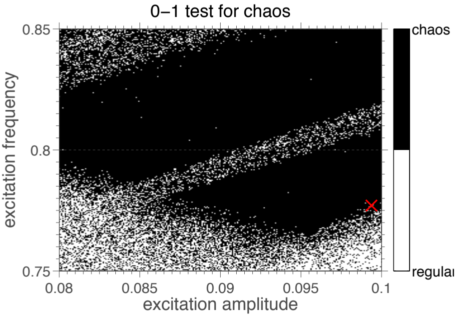


- Design variables: f and Ω
- Feasible domain: $\mathcal{D} = \left\{0.08 \leq f \leq 0.1 \;\text{ and }\; 0.75 \leq \Omega \leq 0.85\right\}$
- Grid resolution: 256×256 points
- Function evaluations: 65 536
- CPU time:[1] $\approx$ 4 hours

[1] Dell Inspiron Core i7-3632QM 2.20 GHz RAM 12GB

Reference solution: mean power





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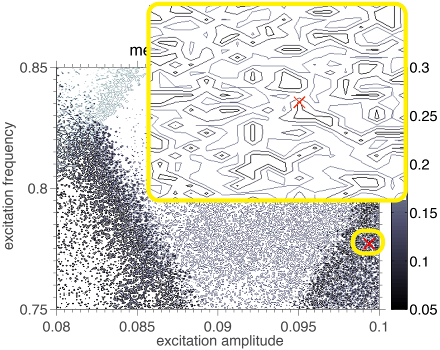

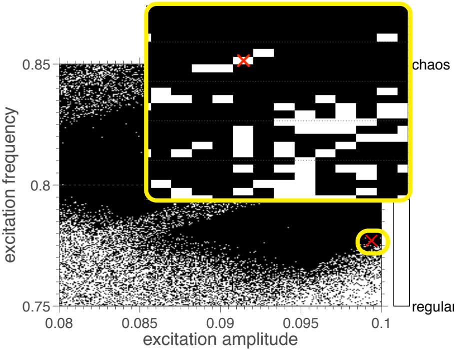

- Design variables: f and Ω
- Feasible domain: $\mathcal{D} = \left\{0.08 \leq f \leq 0.1 \text{ and } 0.75 \leq \Omega \leq 0.85\right\}$
- Number of CE samples: 50
- Percentage of elite samples: 10%
- CE samples distribution: Truncated Gaussian
- Convergence criterium: $||\boldsymbol{\sigma}||_{\infty} < 1 \times 10^{-3}$
- Function evaluations: 1 300
- CPU time:[2] $\approx 5\,\text{minutes}$

[2] Dell Inspiron Core i7-3632QM 2.20 GHz RAM 12GB

Cross-entropy animation (50 samples)

Cross-entropy performance

samples	levels	CPU time[3]	speed-up	function evaluation
reference	—	$\sim$ 3.6h	—	65 536
25	19	$\sim$ 2 min	$\sim$ 120	475
50	26	$\sim$ 5 min	$\sim$ 45	1 300
75	30	$\sim$ 8 min	$\sim$ 25	2 250
100	28	$\sim$ 10 min	$\sim$ 20	2 800

[3] Dell Inspiron Core i7-3632QM 2.20 GHz RAM 12GB

Noisily and high-dimensional cases

Noisily external forcing: $\boldsymbol{x} = (f, \Omega)$

Direct search:

$P_{max} = 0.0173$

$\boldsymbol{x}^* = (0.0998, 0.7763)$

$\approx$ 4 hours

Cross-entropy:

$P_{max} = 0.0170$

$\boldsymbol{x}^* = (0.0991, 0.7675)$

$\approx$ 4 minutes

- robustness to noise

Moderate high-dimensional case: $\boldsymbol{x} = (\xi, \chi, \lambda, \kappa)$

Direct search:

$P_{max} = 0.1761$

$\boldsymbol{x}^* =$

$(0.0340, 0.0600, 0.2000, 1.5000)$

$\approx$ 4 hours

Cross-entropy:

$P_{max} = 0.1612$

$\boldsymbol{x}^* =$

$(0.0237, 0.1053, 0.1953, 1.4923)$

$\approx$ 35 minutes

- good performance

Concluding remarks

Contributions:

- Formulation of a nonlinear non-convex optimization problem to enhance power recovered by a bistable energy harvesting system
- Efficient solution of this optimization problem by means of cross-entropy method

Conclusions:

- The CE method is a power technique to deal with non-convex optimization problems in dynamical systems, in particular, for energy harvesting systems
- It is simple, robust, efficient, generalizable and extensible.

Future direction:

- Parallelization of the CE optimization algorithm

Acknowledgments

Invitation for the scientific committee:

- Profa. Ekatherina Pavlovskaia
- Prof. Marian Wiercigroch
- ICoEV 2020 Organizing Committee

Financial support:

FAPERJ
Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

Thank you for your attention!

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A. Cunha Jr, *Enhancing the performance of a bistable energy harvesting device via the cross-entropy method.* **Nonlinear Dynamics**, (in press) 2020.