Modeling of the nonlinear stochastic dynamics of an orchard sprayer tower moving in an irregular terrain

Americo Cunha Jr

Universidade do Estado do Rio de Janeiro (UERJ)

NUMERICO – **Nu**cleus of **M**odeling and **E**xpe**ri**mentation with **Co**mputers

http://numerico.ime.uerj.br

In collaboration with: Jorge Luis Palacios Felix (UFFS)
José Manoel Balthazar (ITA)

WCCM - APCOM 2016
24 – 29 July, 2016
Seoul, Republic of Korea

Outline

Section 1

Introduction

Horticulture in Brazil

Economical and social aspects:

- Brazil is the world's third largest fruit producer
- annual production of about 40.8 million tonnes
- responsible for 27% of agribusiness workforce

Challenges:

- ensure fruit quality
- reduce crop losses
- reduce air and soil pollution
- increase productivity

2014 Balance and 2015 Perspectives for Brazilian Agribusiness.

Horticulture in Brazil

Economical and social aspects:

- Brazil is the world's third largest fruit producer
- annual production of about 40.8 million tonnes
- responsible for 27% of agribusiness workforce

Challenges:

- ensure fruit quality
- reduce crop losses
- reduce air and soil pollution
- increase productivity

careful pest control

2014 Balance and 2015 Perspectives for Brazilian Agribusiness.

Orchard spraying process

Pest control: done through dispersion of a pulverizing fluid.

UNIVERSIDADE DO ESTADO DO RIO DE JANEIRO UERJ

Orchard tower sprayer

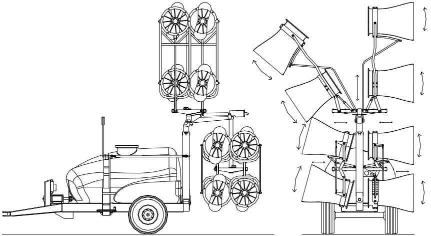


Figure: Schematic of an orchard tower sprayer.

Characteristics:

- an articulate tall structure
- subjected to soil irregularities induced vibrations

Orchard tower sprayer

Lateral vibrations can disturb the spraying process

Figure: Schematic of an orchard tower sprayer.

Characteristics:

- an articulate tall structure
- subjected to soil irregularities induced vibrations

Research objectives

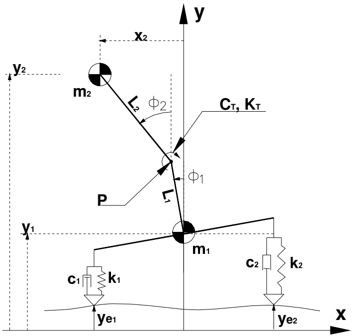
The objectives of this research are:

- construct a stochastic model for orchard sprayer dynamics
- compute the propagation of uncertainties through this model
- investigate the probability of large lateral vibrations

Section 2

Deterministic Modeling

Idealized mechanical system



Multibody system with 3 DoF
(trailer + tower + tires)

Degrees of Freedom:

- y_1 – trailer displacement
- ϕ_1 – trailer rotation
- ϕ_2 – tower rotation

External Excitation:

- y_{e1} – left tire displacement
- y_{e2} – right tire displacement

Quantity of Interest (QoI):

- $x_2 = -L_1 \sin\phi_1 - L_2 \sin\phi_2$

S. Sartori Junior, J. M. Balthazar, B. R. Pontes Junior, *Non-linear dynamics of a tower orchard sprayer based on an inverted pendulum model.* **Biosystems Engineering**, 103:417–426, 2009.

Model equations of motion

Lagrangian formalism:

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{T}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{T}}{\partial q} + \frac{\partial \mathcal{V}}{\partial q} + \frac{\partial \mathcal{D}}{\partial \dot{q}} = 0, \quad q = \{y_1, \phi_1, \phi_2\}$$

Functionals of energy and dissipation:

$$\mathcal{T} = \frac{1}{2} m_1 \, \dot{y}_1 + \frac{1}{2} m_2 \left(\dot{x}_2 + \dot{y}_2\right) + \frac{1}{2} I_1 \dot{\phi}_1 + \frac{1}{2} I_2 \dot{\phi}_2$$

$$\mathcal{V} = m_1 \, g \, y_1 + m_2 \, g \, y_2 + \frac{1}{2} k_1 \left(y_1 - B_1 \, \sin \phi_1 - y_{e1}\right)^2 + \frac{1}{2} k_2 \left(y_1 + B_2 \, \sin \phi_1 - y_{e2}\right)^2 + \frac{1}{2} k_T \left(\phi_2 - \phi_1\right)^2$$

$$\mathcal{D} = \frac{1}{2} c_1 \left(\dot{y}_1 - B_1 \, \dot{\phi}_1 \, \cos \phi_1 - y_{e1}\right)^2 + \frac{1}{2} c_2 \left(\dot{y}_1 + B_2 \, \dot{\phi}_1 \, \cos \phi_1 - y_{e1}\right)^2 + \frac{1}{2} c_T \left(\dot{\phi}_2 - \dot{\phi}_1\right)^2$$

S. Sartori Junior, J. M. Balthazar, B. R. Pontes Junior, *Non-linear dynamics of a tower orchard sprayer based on an inverted pendulum model.* **Biosystems Engineering**, 103:417–426, 2009.

Nonlinear dynamical system

$$[M]\begin{pmatrix}\ddot{y}_1(t)\\ \ddot{\phi}_1(t)\\ \ddot{\phi}_2(t)\end{pmatrix}+[N]\begin{pmatrix}\dot{y}_1^2(t)\\ \dot{\phi}_1^2(t)\\ \dot{\phi}_2^2(t)\end{pmatrix}+[C]\begin{pmatrix}\dot{y}_1(t)\\ \dot{\phi}_1(t)\\ \dot{\phi}_2(t)\end{pmatrix}+[K]\begin{pmatrix}y_1(t)\\ \phi_1(t)\\ \phi_2(t)\end{pmatrix}=\mathbf{g}-\mathbf{h},$$

$+$ initial conditions

- $[M]$, $[N]$, $[C]$, $[K]$, $\mathbf{g}$, and $\mathbf{h}$ are configuration dependent
- RKF45 method is used for numerical integration

S. Sartori Junior, J. M. Balthazar, B. R. Pontes Junior, *Non-linear dynamics of a tower orchard sprayer based on an inverted pendulum model.* **Biosystems Engineering**, 103:417–426, 2009.

Section 3

Stochastic Modeling

Aleatory nature of a tire displacement

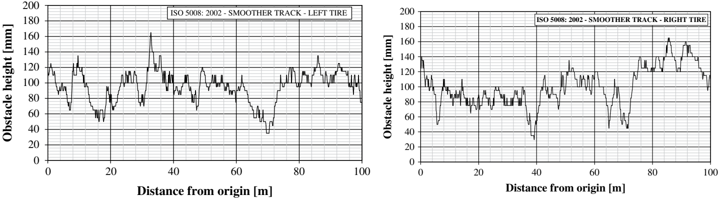


Figure: Typical paths followed by the tires of orchard sprayer.

Probabilistic model for tires displacement

External excitations are described as random processes:

- $\{\mathbb{y}_{e1}(t), t \in \mathbb{R}\}$
- $\{\mathbb{y}_{e2}(t), t \in \mathbb{R}\}$

Covariance functions are assumed to be:

$$\mathrm{cov}_{\mathbb{y}_{e1}}(t_1, t_2) = \mathrm{cov}_{\mathbb{y}_{e2}}(t_1, t_2) = \exp\left(-\frac{v\,(t_2 - t_1)}{a_{corr}}\right)$$

Random processes representation

Truncated Karhunen-Loève expansion:

$$\mathbb{y}(t) \approx \mu_{\mathbb{y}}(t) + \sum_{n=1}^{N_{KL}} \sqrt{\lambda_n}\, \varphi_n(t)\, \mathbb{Y}_n$$

$$\int_{\mathbb{R}} \text{cov}_{\mathbb{y}(t)}(t,\tau)\, \varphi_n(\tau)\, d\tau = \lambda_n\, \varphi_n(t), \qquad t \in \mathbb{R}$$

$$\mu_{\mathbb{Y}_n} = 0 \qquad \text{and} \qquad \mathbb{E}\left[\mathbb{Y}_n \mathbb{Y}_m\right] = \delta_{mn}$$

D. Xiu, **Numerical Methods for Stochastic Computations: A Spectral Method Approach**, Princeton University Press, 2010.

R. Ghanem, P. Spanos, **Stochastic Finite Elements: A Spectral Approach**, Dover Publications, 2003.

Random loading induced by irregular terrain

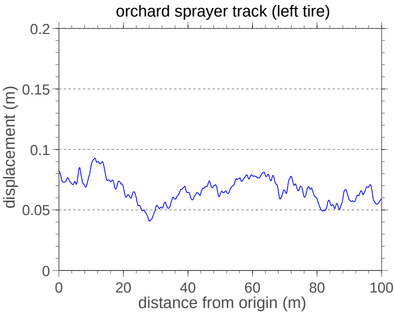


Figure: Random loadings generated by KL expansion.

Stochastic nonlinear dynamical system

$$[\mathbb{M}] \begin{pmatrix} \ddot{\mathbb{y}}_1(t) \\ \ddot{\mathbb{\phi}}_1(t) \\ \ddot{\mathbb{\phi}}_2(t) \end{pmatrix} + [\mathbb{N}] \begin{pmatrix} \dot{\mathbb{y}}_1^2(t) \\ \dot{\mathbb{\phi}}_1^2(t) \\ \dot{\mathbb{\phi}}_2^2(t) \end{pmatrix} + [\mathbb{C}] \begin{pmatrix} \dot{\mathbb{y}}_1(t) \\ \dot{\mathbb{\phi}}_1(t) \\ \dot{\mathbb{\phi}}_2(t) \end{pmatrix} + [\mathbb{K}] \begin{pmatrix} \mathbb{y}_1(t) \\ \mathbb{\phi}_1(t) \\ \mathbb{\phi}_2(t) \end{pmatrix} = \underline{\mathbb{g}} - \underline{\mathbb{h}}, \qquad a.s.$$

$+$ initial conditions

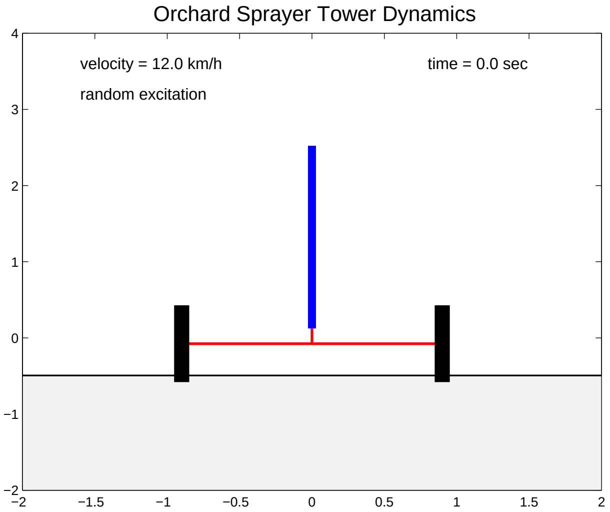
Propagation of uncertainties: Monte Carlo method

A. Cunha Jr, R. Nasser, R. Sampaio, H. Lopes, and K. Breitman, *Uncertainty quantification through Monte Carlo method in a cloud computing setting.* **Computer Physics Communications**, 185: 1355–1363, 2014.

Section 4

Numerical Experiments

Nonlinear dynamics animation



Lateral dynamics: typical realization time series

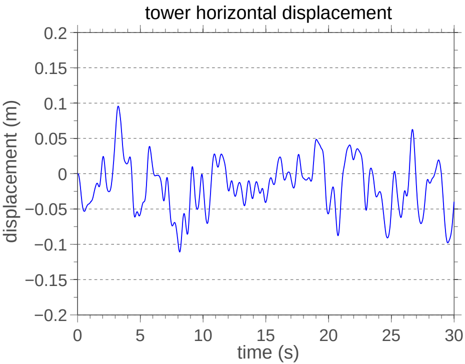


Figure: Time series of tower lateral displacement.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: typical realization time series

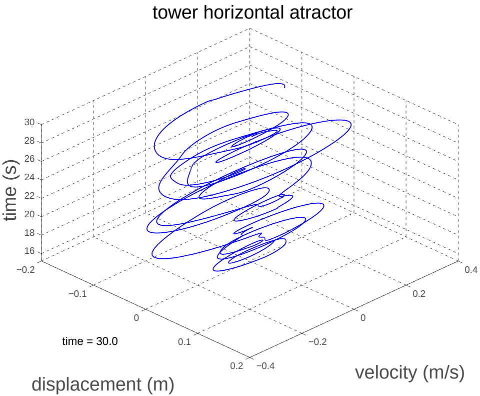


Figure: Projection of lateral dynamics phase space trajectory in $\mathbb{R}^3$.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: typical realization time series

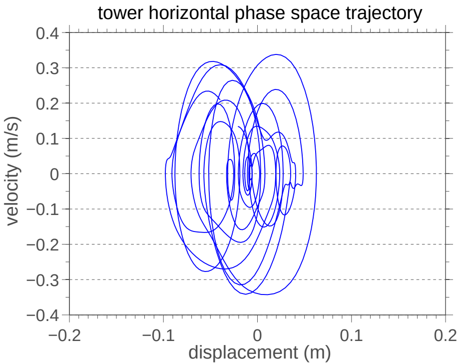


Figure: Projection of lateral dynamics phase space trajectory in $\mathbb{R}^2$.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: typical realization spectral analysis

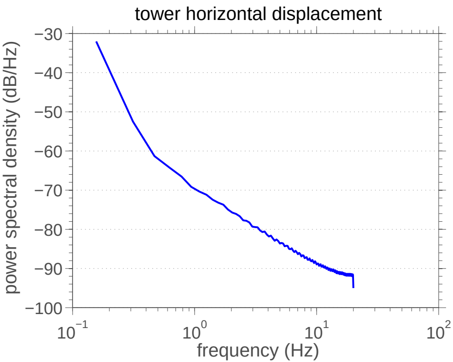


Figure: Power spectral density of tower lateral displacement.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: probabilistic confidence band

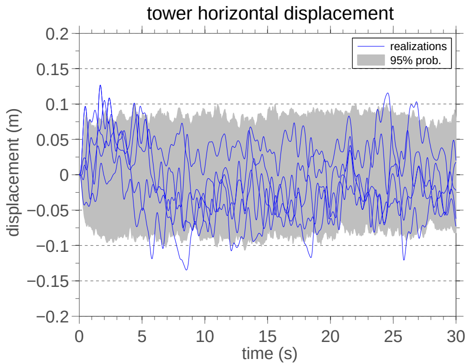


Figure: Confidence envelope for tower lateral displacement.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: low order statistics

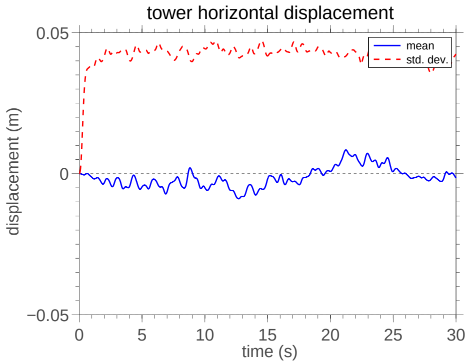


Figure: Sample mean/standard deviation for tower lateral displacement.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: PDF evolution

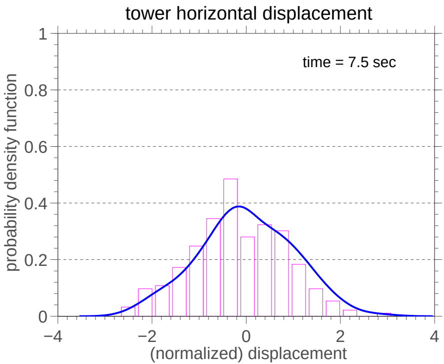


Figure: Evolution of tower lateral displacement PDF.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: PDF evolution

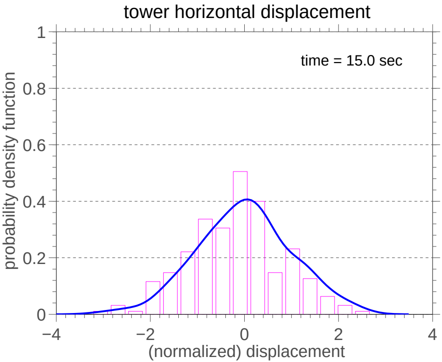


Figure: Evolution of tower lateral displacement PDF.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: PDF evolution

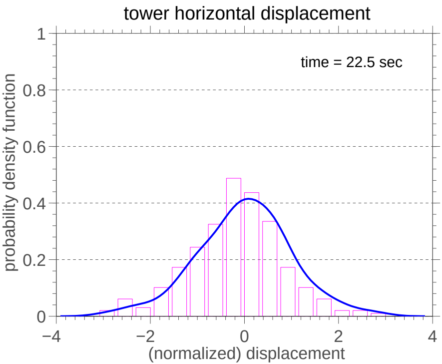


Figure: Evolution of tower lateral displacement PDF.

A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: PDF evolution

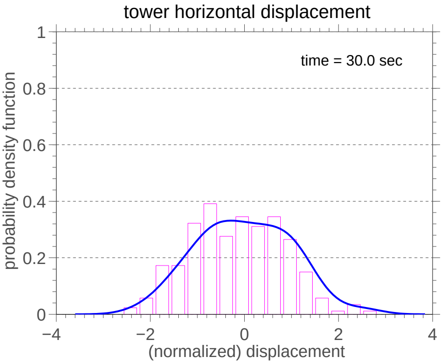
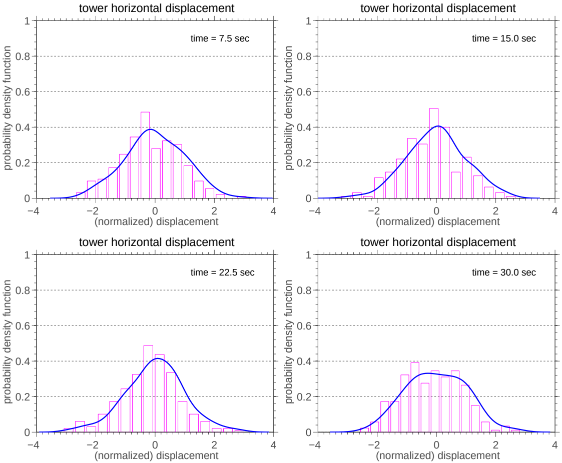


Figure: Evolution of tower lateral displacement PDF.

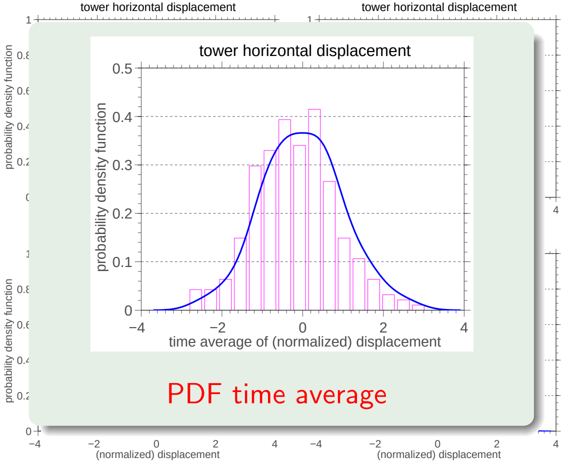
A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: PDF evolution



A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: PDF evolution



A. Cunha Jr, J. L. P. Felix, and J. M. Balthazar, **Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics** (in preparation).

Lateral dynamics: temporal average statistics

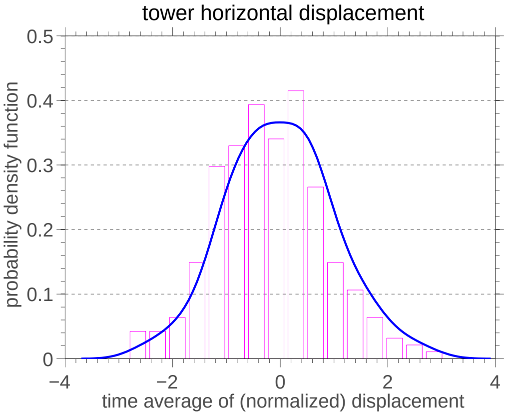


Figure: PDF time average.

Statistics of $\langle \mathbb{x}_2 \rangle$

$$
\begin{aligned}
\text{mean} &= 0.00\\
\text{std. dev.} &= 0.02\\
\text{skewness} &= 0.11\\
\text{kurtosis} &= 0.01
\end{aligned}
$$

$$\mathbb{P}\left\{ \underbrace{\langle \mathbb{x}_2 \rangle > 10\% \text{ of } B_1}_{\text{large vibration}} \right\} = 1\%$$

Section 5

Final Remarks

Concluding remarks

Contributions and conclusions:

- Construction of a parametric probabilistic model for orchard sprayer dynamics uncertainties
- Computation of propagation of random loading uncertainties through the nonlinear dynamics
- Numerical simulations shown the probability of sprayer presents large lateral vibrations is small, but not negligible

Next steps in this work:

- Solve a robust optimization problem seeking to reduce sprayer lateral vibrations

Acknowledgments

Important data supplied:

- Máquinas Agrícolas Jacto S/A

Financial support given to this research:

- CNPq
- CAPES
- FAPERJ
- FAPESP

Thank you for your attention!

americo@ime.uerj.br

www.americocunha.org

Simulation Parameters

Deterministic parameters

parameter	value	unit
m_1	6500	kg
m_2	800	kg
L_1	0.2	m
L_2	2.4	m
I_1	6850	$kg\ m^2$
I_2	6250	$kg\ m^2$
k_1	465×10^3	N/m
k_2	465×10^3	N/m
c_1	5.6×10^3	$N/m/s$
c_2	5.6×10^3	$N/m/s$
B_1	0.85	m
B_2	0.85	m
k_T	100×10^3	N/rad
c_T	50×10^3	$Nm/rad/s$

Stochastic parameters

parameter	value	unit
N_{KL}	173	
a_{corr}	0.2	m
$\sigma_{y_{e1}}$	0.0375	m
$\sigma_{y_{e2}}$	0.0375	m
$\mu_{y_{e1}}$	0.25	m
$\mu_{y_{e2}}$	0.25	m
v	12	km/h

Nonlinear Dynamics Operators

Mass Matrix (configuration dependent)

$$[M] = \begin{bmatrix} m_1 + m_2 & -m_2\, L_1\, \sin\phi_1 & -m_2\, L_2\, \sin\phi_1 \\ -m_2\, L_1\, \sin\phi_1 & I_1 + m_2\, L_1^2 & m_2\, L_1\, L_2\, \cos\left(\phi_2 - \phi_1\right) \\ -m_2\, L_2\, \sin\phi_1 & m_2\, L_1\, L_2\, \cos\left(\phi_2 - \phi_1\right) & I_2 + m_2\, L_2^2 \end{bmatrix},$$

Circulatory Matrix (configuration dependent)

$$[N] = \begin{bmatrix} 0 & -m_2\, L_1\, \cos\phi_1 & -m_2\, L_2\, \cos\phi_2 \\ 0 & 0 & -m_2\, L_1\, L_2\, \sin(\phi_2 - \phi_1) \\ 0 & -m_2\, L_1\, L_2\, \sin(\phi_2 - \phi_1) & 0 \end{bmatrix},$$

Damping Matrix (configuration dependent)

$$[C] = \begin{bmatrix} c_1 + c_2 & (c_2\,B_2 - c_1\,B_1)\cos\phi_1 & 0 \\ (c_2\,B_2 - c_1\,B1)\cos\phi_1 & c_T + (c_1\,B_1^2 + c_2\,B_2^2)\cos^2\phi_1 & -c_T \\ 0 & -c_T & c_T \end{bmatrix},$$

Stiffness Matrix (configuration dependent)

$$[K] = \begin{bmatrix} k_1 + k_2 & 0 & 0 \\ (k_2\, B_2 - k_1\, B1) \cos\phi_1 & k_T & -k_T \\ 0 & -k_T & k_T \end{bmatrix},$$

Weight Loading Vector (configuration dependent)

$$\mathbf{g} = \begin{pmatrix} (k_2\, B_2 - k_1\, B_1)\sin\phi_1 + (m_1 + m_2) g \\ (k_1\, B_1^2 + k_2\, B_2^2)\sin\phi_1\, \cos\phi_1 - m_2\, g\, L_1\, \sin\phi_1 \\ -m_2\, g\, L_2\, \sin\phi_2 \end{pmatrix},$$

External Loading Vector (configuration dependent)

$$\mathbf{h} = \begin{pmatrix} k_1\, y_{e1} + k_2\, y_{e2} + c_1\, \dot{y}_{e1} + c_2\, \dot{y}_{e2} \\ -k_1\, B_1\, \cos\phi_1\, y_{e1} + k_2\, B_2\, \cos\phi_1\, y_{e2} - c_1\, B_2\, \cos\phi_1\, \dot{y}_{e1} + c_2\, B_2\, \cos\phi_1\, \dot{y}_{e2} \\ 0 \end{pmatrix}.$$

Monte Carlo convergence

Study of convergence for MC simulation

The convergence of Monte Carlo simulation is measured with the following metric:

$$\mathrm{conv}(n_s) = \left(\frac{1}{n_s} \sum_{n=1}^{n_s} \int_{t=t_0}^{t_f} \left(\mathbb{y}_1(t,\theta_n)^2 + \mathbb{\phi}_1(t,\theta_n)^2 + \mathbb{\phi}_2(t,\theta_n)^2 \right) \, dt \right)^{1/2}$$

C. Soize, *A comprehensive overview of a non-parametric probabilistic approach of model uncertainties for predictive models in structural dynamics.* **Journal of Sound and Vibration**, 288: 623–652, 2005.

Study of convergence for MC simulation

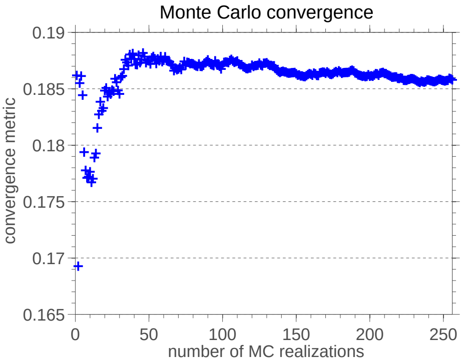


Figure: MC convergence metric as function of the number of realizations.

Other Numerical Results

Trailer vertical dynamics: time series analyses

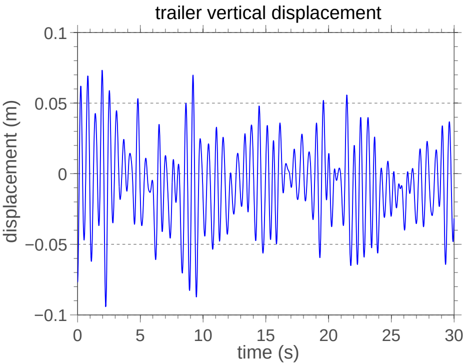


Figure: Time series of trailer vertical displacement.

Trailer vertical dynamics: time series analyses

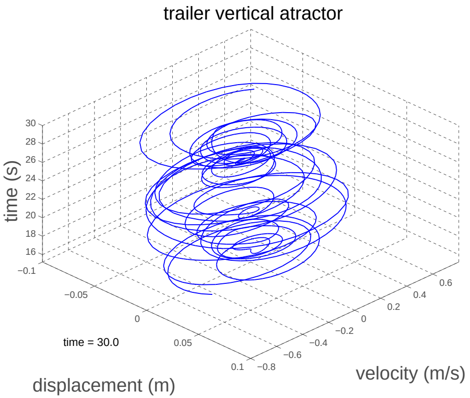


Figure: Projection of trailer vertical dynamics phase space trajectory in $\mathbb{R}^3$.

Trailer vertical dynamics: time series analyses

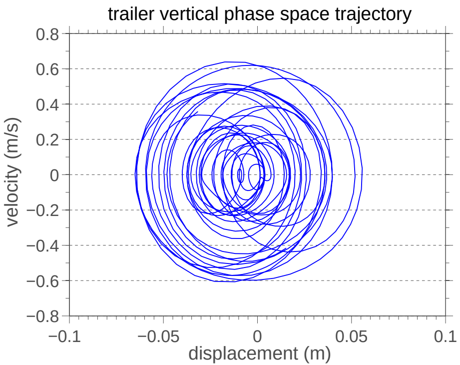


Figure: Projection of trailer vertical dynamics phase space trajectory in $\mathbb{R}^2$.

Trailer rotational dynamics: time series analyses

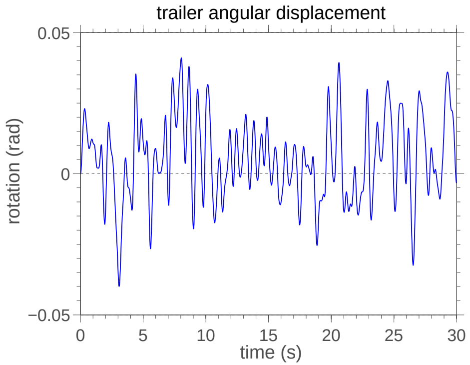


Figure: Time series of trailer rotational displacement.

Trailer rotational dynamics: time series analyses

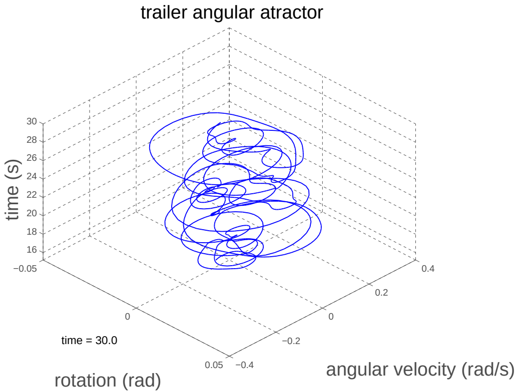


Figure: Projection of trailer rotational dynamics phase space trajectory in $\mathbb{R}^3$.

Trailer rotational dynamics: time series analyses

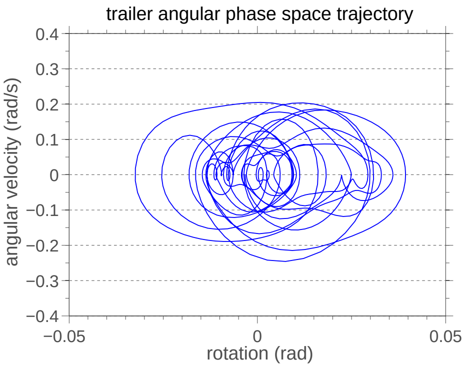


Figure: Projection of trailer rotational dynamics phase space trajectory in $\mathbb{R}^2$.

Tower rotational dynamics: time series analyses

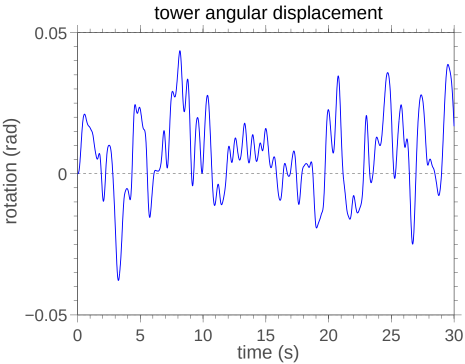


Figure: Time series of tower rotational displacement.

Tower rotational dynamics: time series analyses

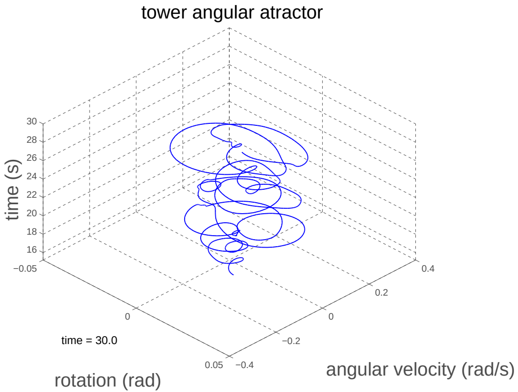


Figure: Projection of tower rotational dynamics phase space trajectory in $\mathbb{R}^3$.

Tower rotational dynamics: time series analyses

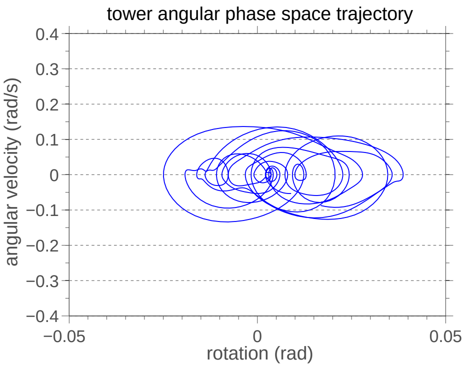


Figure: Projection of tower rotational dynamics phase space trajectory in $\mathbb{R}^2$.