KEB-45251
Numerical Techniques for Process Modeling
Exercise 2 - Nonlinear equations
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Problem 1

In this exercise we practice solving single variable nonlinear equations and plotting our results. Our target is to solve the implicit Darcy friction factor correlation by White-Colebrook

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{\mathrm{f}}}\right) \tag{1}$$

For validation we use Haaland explicit function

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\epsilon}{3.7d}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right) \tag{2}$$

giving near identical results. Haaland equation, Eq. 2, is given in template Python file, *template1.py*.

Solve

(a) First create your own iterative solution for Eq. 1 without using Scipy. Use *naive_white_colebrook* function in template. Test result with Eq. 2. $\mathrm{Re} = 10^4$ and $\epsilon/d = 0$.

(b) Solve case using *root* and *root_scalar*. See Scipy documentation for details.

(c) Repeat (b) for $4000 \leq \mathrm{Re} \leq 5 \times 10^6$, i.e. use a for-loop. Plot the results using logarithmic scale (see template).

Problem 2

In this exercise we practice solving a system of nonlinear equations. To save time, the used correlations are implemented in the template file *template2.py*. You need to implement the balance equation, Eq. 3, and solve it correctly. The balance equation is called *energy_ balance(T)* in the template.

We have hot gas (air for simplicity) in a chimney. The chimney is build from two layers of thin metal with an air cavity between them to provide insulation. We assume there to be no heat bridges between the two layers. **Calculate the thin metal plate temperatures.** We assume the plate to be isothermal. The nonlinear energy balance equations are given in Eq. 3. For convection correlations, see Eq. 6-10.

Inside the duct, heat is transfered with forced convection. We assume the gas to be transparent to heat radiation and therefore there is no radiative heat transfer inside the duct. Inside the cavity, heat is transfered with convection and radiation. Outside, heat is transfered with convection and radiation.

$$\begin{cases} 0 = \underbrace{h_{i0}\left(T_i - T_0\right)}_{\text{convection in duct}} - \underbrace{h_{01}\left(T_0 - T_1\right)}_{\text{convection in cavity}} - \underbrace{\epsilon\sigma\left(T_0^4 - T_1^4\right)}_{\text{radiation in cavity}} \\ 0 = \underbrace{h_{01}\left(T_0 - T_1\right)}_{\text{convection in cavity}} - \underbrace{h_{1\infty}\left(T_1 - T_\infty\right)}_{\text{convection outside}} + \underbrace{\epsilon\sigma\left(T_0^4 - T_1^4\right)}_{\text{radiation in cavity}} - \underbrace{\epsilon\sigma\left(T_1^4 - T_\infty^4\right)}_{\text{radiation outside}} \end{cases} \tag{3}$$

(a) **First, form the balance equations 3 in *energy_ balance* function. Try a few times to solve the equations by guessing the correct temperatures. How do you know when you have guessed correctly?**

(b) **Solve the system with *root* or similar**.

The chimney height is $L_v = 2\,\mathrm{m}$, hydraulic diameter is $D = 1\,\mathrm{cm}$. The mean velocity inside the duct is $V = 2\,\mathrm{m/s}$and the duct is smooth. The gas temperature is $T_{in} = 800\,°\mathrm{C}$and outside temperature is $T_\infty = 20\,°\mathrm{C}$. Use normal pressure as a reference. The metals are painted black and the_ emissivity is about $\epsilon = 0.9$.

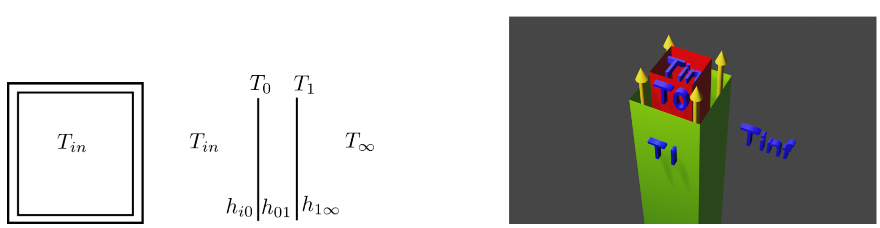

Figure 1: Sketch

Problem 3 - Extra

We have two parallel vertical plates facing each other. **Solve the plate temperatures.** Each plate is $L_{vertical}$ =10 cm high and $L_{horizontal}$ =5 cm wide. The back sides of the plates are insulated. Emissivity for both plates is $\epsilon = 0.8$. The view factor is $F = 0.3$ for the plates. Electrical input power for plate 1 is $Q_1 = 20\,\mathrm{W}$ and for plate 2 $Q_2 = 40\,\mathrm{W}$. The plates are in still air in room pressure and temperature. The resulting energy balance for the plates is

$$\begin{aligned} Q_1 &= A\epsilon\sigma T_1^4 - FA\epsilon\sigma T_2^4 - (1-F)\,A\epsilon\sigma T_\infty^4 + Ah_1\,(T_1 - T_\infty) \\ Q_2 &= A\epsilon\sigma T_2^4 - FA\epsilon\sigma T_1^4 - (1-F)\,A\epsilon\sigma T_\infty^4 + Ah_2\,(T_2 - T_\infty) \end{aligned} \tag{4}$$

where σ is Stefan-Boltman constant. Natural convection can be calculated from

$$\begin{aligned} \mathrm{Nu} &= 0.478\mathrm{Gr}^{1/4} \\ \mathrm{Gr} &= \frac{g\beta\,(T - T_\infty)\,L^3}{\nu^2} \end{aligned} \tag{5}$$

where Nu, Gr are Nusselt number and Grashof number, respectively. For details, see *for* example *Wikipedia.* NOTE! We did not check the primary source for this correlation. Use with caution!

Use template *template3.py.* To save time, natural convection is already written in the template.

Convection correlations for problem 2

For the forced convection inside the duct *(Gnielinski's correlation for turbulent flow in tubes, Haaland)*

$$\mathrm{Nu_s} = \frac{\frac{f}{8}(\mathrm{Re} - 1000)\mathrm{Pr}}{1 + 12.7\left(\frac{f}{8}\right)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)} \tag{6}$$

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\frac{6.9}{\mathrm{Re}}\right) \tag{7}$$

For the cap between plates 0 and 1 *(Building Physics - Heat, Air and Moisture, Hugo Hens, Eq.1.59, pp.57)*

$$\mathrm{Nu} = \max\left(1, 1 + \frac{0.024\mathrm{Ra}^{1.39}}{\mathrm{Ra} + 10100}\right) \tag{8}$$

$$\mathrm{Ra} = \frac{\beta g L^3 \left(T_0 - T_1\right)}{\nu^2}\mathrm{Pr} \tag{9}$$

For the natural convection on the outside (*https://www.sfu.ca/∼mbahrami/ENSC%20388/Notes/Natural%20Convection.pdf)* Primary sources not checked!.

$$\begin{aligned} &\mathrm{Nu} = 0.59\mathrm{Ra}^{1/4},\ \text{if Ra} < 10^4 \\ &\mathrm{Nu} = 0.1\mathrm{Ra}^{1/3},\ \text{ if Ra} > 10^4 \\ &\mathrm{Ra} = \mathrm{GrPr} \\ &\mathrm{Gr} = \frac{g\beta\left(T - T_\infty\right)L^3}{\nu^2} \end{aligned} \tag{10}$$