KEB-45250
Numerical Techniques for Process Modeling
Exercise 5 - Initial Value Problem
10.02.2021

Antti Mikkonen
Niko Niemelä

Introduction

The example measurement in this exercise was originally performed live in the lectures. We now skip the live measurement demo but use the old measurements. The source code of the live measurement is, however, attached.

Problem 1

Today we model the cooling of a hot vertical aluminum plate with natural convection and radiation.

We will start with no radiation and a constant heat transfer coefficient and validate out model with an analytical solution.

Then we extend our model to include radiation heat transfer and solve the heat transfer coefficient from a correlation. Radiation makes the problem too complicated for analytical solution and we use measurement data for validation instead. The measurement data is included in file *temperature.log* and was measured by the lecturers using open source tools and cheap electronics (`http://wiki.seeedstudio.com/Grove-Digital_Infrared_Temperature_Sensor/`).

See lecture slides for hints. Use template. Geometry, mass, etc. is given in the template file.

(a) **Implement an explicit model for constant heat transfer coefficient, $h = 5\text{W/m}^2\text{K}$. Validate with analytical solution. Ignore radiation.**

(b) **With the same assumptions as in (a), implement an implicit model. You don't need iteration yet. Compare explicit and implicit model performance for high heat transfer coefficient, say $h = 50\text{W/m}^2\text{K}$.**

(c) **Include varying heat transfer coefficient to your explicit solution. The heat transfer coefficient is calculated in function *hVertical. Note that the analytical solution is no longer valid. Also, as we don't yet have implemented radiation, we can't compare the results to measurement data either.***

(d) **With the same assumptions as in (c), implement an implicit model. Now iteration is needed. Root is a good candidate for the iteration.**

(e) **Include radiation to the explicit model. Validate with measurement data. Take emissivity as $\epsilon = 0.96$ (Flir recommendation for electric tape).**

(f) **Include radiation to the implicit model. Validate with measurement data.**

P.S. There is also a symbolic math library (Sympy) available in Python. See *sympy_ ode.py* if you are interested.

Problem 2 - Extra

The explicit and implicit Euler schemes just discussed are the easiest methods to understand and implement. However, they are rarely the best ones. How would you improve accuracy with little or no additional computational cost? There are many methods, but for this case it would be easy to combine the analytical solution with numerical solution and get the best of both worlds.