KEB-45250
Numerical Techniques for Process Modeling
Exercise 1 - Correlations and a primer to linear algebra
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Problem 1

We have a smooth, straight pipe with diameter D =10 mm filled with water. The volumetric flow rate is $Q = 15$ l/min. The inlet temperature is $T_{in} = 30$ °C and wall temperature is $T_w = 120$ °C. Outlet pressure is $p_{out} = 2$ bar and outlet temperature is $T_{out} = 90$ °C. **How long is the pipe? What is the pressure at inlet? Plot the temperature in the pipe. Repeat the calculations for air.** Assume fully developed flow.

A template is provided. It is recommended to follow the structure in the template as it makes teaching a large group a lot easier.

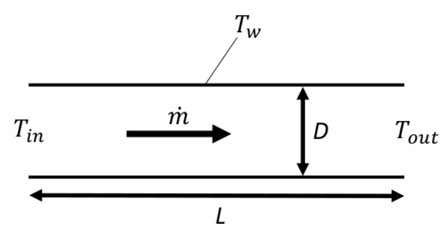

Figure 1: Pipe

The relevant equations are given below. Eqs. 2 and 3 are valid for turbulent flow only. For definitions of dimensionless numbers etc. see lecture slides. For fluid properties, use CoolProps, Google, or what ever method you prefer. For simplicity, you may take the fluid properties at the mean temperature and at the outlet pressure.

How could you improve accuracy? How reliable are the results? What would happed if we dropped the flow rate to $Q = 5$ l/min?

$$\frac{T_w - T_{out}}{T_w - T_{in}} = \exp(\frac{-hA_{wall}}{\dot{m}c_p}) \tag{1}$$

$$\mathrm{Nu} = \frac{\frac{f}{8}(\mathrm{Re} - 1000)\mathrm{Pr}}{1 + 12.7\left(\frac{f}{8}\right)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)} \tag{2}$$

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\frac{6.9}{\mathrm{Re}}\right) \tag{3}$$

$$\Delta p = \frac{1}{2}\rho V^{2}\frac{L}{D}\mathrm{f} \tag{4}$$

Where A_{wall} is the pipe wall area. Example CoolProps command for density "*rho = PropsSI("D", "T", T_m, "P", 1e5, fluid)*". Substitute D with "*V*" for dynamic viscosity, "Prandtl" for Prandtl number, "C" for heat capacity, and "conductivity" for heat conductivity.

Some useful Python: "sp.log" for natural logarithm, "sp.log10" for 10-based logarithm, "sp.pi" for π, "sp.exp(x)" for e^x.

Problem 2

Later on the course, we will be using matrices to solve practical problems. In this primer, we will learn a sufficient level of math/programming skills to solve them.

In a template you'll find an example solution for the following linear algebra problem

$$\underbrace{\begin{bmatrix} 3 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 5 & 1 \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} 2 \\ -2 \\ 9 \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}}_{\boldsymbol{b}} \tag{5}$$

Using the methods learned from the template, transform the following linear system in to standard form (Ax=b) and solve it.

$$\begin{aligned} x + 3y + 4z &= 3 \\ 7x + 2y + 3z &= 5 \\ 7x + 9y + 2z &= 6 \end{aligned}$$

Problem 3 - Extra

From the literature, we can find multiple correlations for the stagnation point heat transfer in impinging jets. See schematics in Fig. 2. For stagnation point $r = 0$. One such correlation is

$$\mathrm{Nu}_0 = a\mathrm{Re}^b\mathrm{Pr}^{0.4}\left(H/d\right)^{0.064} \tag{6}$$

where $a = 0.56$ and $b = 0.53$ and it works well for the larger nozzle diameters $d = 6\,\mathrm{mm}$ and $d = 8\,\mathrm{mm}$ shown in Fig. 2. However, from Fig. 2 we can readily see that for the smallest diameter nozzle $d = 1.1\,\mathrm{mm}$, the correlation fails.

In the literature, there is are large differences between the reported values of a and b. The differences mirror the differences in experimental setups.

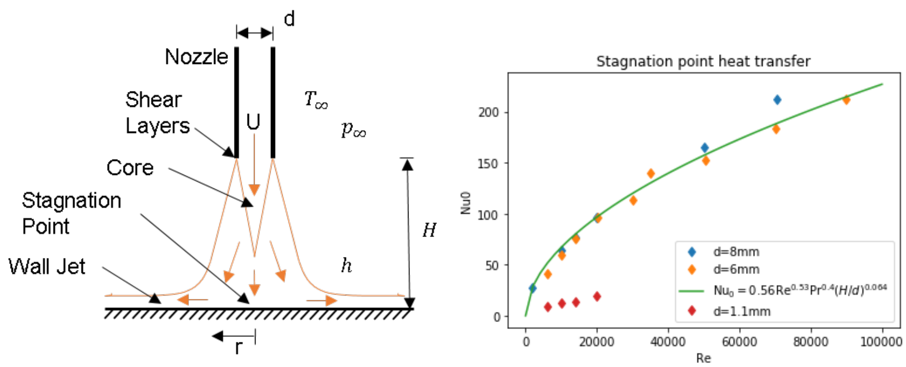

Figure 2: Impinging jet heat transfer. Unpublished data measured by Mikkonen and Pirttiniemi.

1. **Modify a and b in Eq. 6 to match $d = 1.1\,\mathrm{mm}$ in Fig. 2. How reliable do you consider this correlation?**

2. **Solve heat flux density at stagnation point** for $d = 6\,\mathrm{mm}$ and $d = 1.1\,\mathrm{mm}$, $H/d = 2$, $V = 50\,\mathrm{m/s}$ air jet. Wall temperature is $T_w = 50\,°\mathrm{C}$ and jet temperature is $T_n = 20\,°\mathrm{C}$. Take fluid properties at jet temperature.