KEB-45251
Numerical Techniques for Process Modeling
Exercise 8 - Finite Volume Method and Heat Conduction
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Problem 1

Consider a source-free heat conduction in a insulated aluminum rod. The ends are maintained at constant temperatures, 100 °C and 500 °C, respectively. See Fig. 1.

Thermal conductivity is $k = 230\,\mathrm{W/mK}$, cross-sectional area is $A = 10 \times 10^{-3}\,\mathrm{m}^2$.

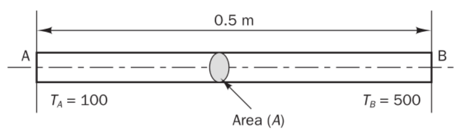

Figure 1: Problem sketch

Using Finite Volume Method with 5 control volumes

- derive the equations
- solve the problem
- compare the results with analytical solution

Feel free to use Example 4.1 from Versteeg and Malalasekera or course notes for reference.

Problem 2

After verifying your code in Problem 1, add uniform heat generation $q = 1 \times 10^6\,\mathrm{W/m}^3$.

Try to reuse you previous equations and code as much as possible.

Validate your code with analytical solution

$$T = \left[\frac{T_B - T_A}{L} + \frac{q}{2k}(L - x)\right] x + T_A \tag{1}$$

It may also be a good idea to change your input values to match those in Example 4.2 from Versteeg and Malalasekera to be able to check the matrix coefficients as you write the code.

Problem 3 - Extra

Similarly to Problem 1, we have an insulated 1D rod. This time the left boundary is insulated and right boundary has a constant temperature. We want to solve the transient heat conduction.

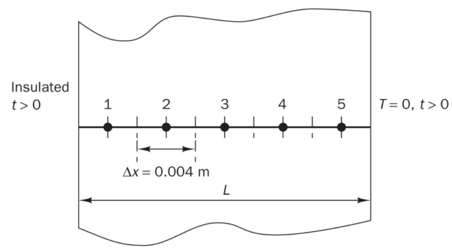

Figure 2: Problem sketch

Using a fully implicit scheme, derive the equations and solve the problem using FVM with 5 cells.

- Initial temperature $T_0 = 200\,°\mathrm{C}$.
- Right boundary temperature $T_{|x=L} = 0\,°\mathrm{C}$.
- Left boundary is insulated, $\frac{\mathrm{dT}}{\mathrm{d}x}_{|x=0} = 0$
- Rod length $L = 2\,\mathrm{cm}$.
- Thermal conductivity $k = 10\,\mathrm{W/mK}$.
- Product of density and heat capacity $\rho c = 10 \times 10^6\ \mathrm{J/m^3K}$
- Plot the results at times 40s, 80s, and 120s.

Feel free to use examples 8.1 and 8.2 from Versteeg and Malalasekera. Note that example 8.1 is for explicit time stepping.

Problem 4 - Extra

Modify your code to solve for a pin fin heat transfer. In other words, include heat transfer coefficient instead of volumetric heat power. See Verseeg and Malalasekera for hints or derive the equations from scratch.