KEB-45251
Numerical Techniques for Process Modeling
Exercise 9 - Advection
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Problem 1

Consider a one dimensional source-free heat conduction and advection. The ends are maintained at constant temperatures of $L_{x=0} = 10\,°\mathrm{C}$ and $L_{x=L} = 5\,°\mathrm{C}$.

Thermal conductivity is $k = 0.1\,\mathrm{W/mK}$, product of density and heat capacity is $c\rho = 1\,\mathrm{J/m^3K}$, and length $L = 1\,\mathrm{m}$.

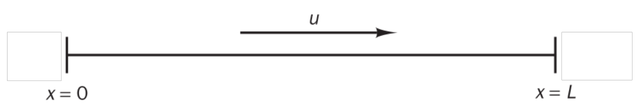

Figure 1: Problem sketch

Using Finite Volume Method and central differencing (linear interpolation) for advection term

- solve the problem using

  a) $n = 5$, $u = 0.1\,\mathrm{m/s}$

  b) $n = 5$, $u = 2.5\,\mathrm{m/s}$

  c) $n = 5$, $u = -0.1\,\mathrm{m/s}$

  d) $n = 5$, $u = -2.5\,\mathrm{m/s}$

  where n is the number of cells and u is velocity.

- Calculate Peclet number for each case. How does Peclet number and your results correlate? Why?

- Compare the results with analytical solution

$$\frac{T - T_0}{T_L - T_0} = \frac{\exp(c\rho ux/k) - 1}{\exp(c\rho uL/k) - 1} \tag{1}$$

Feel free to use Examples 5.1 and 5.2 from Versteeg and Malalasekera or course notes, for reference. Note that we use non-zero boundary values here. Zero boundary values may easily mask a error in code as the contribution often becomes zero.

Problem 2

Repeat problem 1 using upwind scheme for advection.

Extras

- Add a source term to you solver.
- Add heat transfer coefficient to your solver. Maybe pipe flow.
- How would you implement a zero gradient boundary condition?
- Make a transient case solver.