

Let  $(x, y) \in \mathbb{N}^2$ , let  $(\epsilon', \epsilon) \in [0, 1]^2$   
 $\implies (x + \epsilon') * (y + \epsilon) = xy + y\epsilon' + x\epsilon + \epsilon'\epsilon$   
 Given  $\epsilon' = x\alpha'$  and  $\epsilon = y\alpha$ ,  $(\alpha, \alpha') \in [0, 1]^2$   
 $\implies (x + \epsilon') * (y + \epsilon) = xy + xy\alpha' + xy\alpha + xy\alpha\alpha'$   
 Given  $\alpha\alpha' = o(xy + xy\alpha' + xy\alpha)$   
 $\implies (x + \epsilon') * (y + \epsilon) \sim xy + xy(\alpha + \alpha') \blacksquare$