

$$\text{Let } \forall (a,b), (d,e) \in (\mathbb{N}^2)^2, \text{ let } w = \frac{\max(a,b) - \min(a,b)}{2}, w' = \frac{\max(e,d) - \min(e,d)}{2} \\ \implies \frac{\max(a,b) + \max(e,d) - (\min(a,b) + \min(e,d))}{2} = w + w' \blacksquare$$

$$\text{In the case of } I_1 = (a,b), I_2 = (d,e), w_{I_1 * I_2} = \frac{\max((ae, ad, be, bd)) - \min((ae, ad, be, bd))}{2}, \\ \text{let } b = 0, d = 0 \implies w_{I_1 * I_2} = ae \neq f(w_{I_1}, w_{I_2})$$