

$$\begin{aligned}
&\text{Let } (a, b) \in \mathbb{N}^2 \text{ such that } \gcd(a, b) = 1 \\
&\implies \forall (p, q) \in \mathbb{N}^2, \gcd(a^p, b^q) = 1 \\
&\implies \forall (p, q) \in \mathbb{N}^2, \frac{\log\left(\frac{a^p \cdot b^q}{a^p}\right)}{\log(b)} = q \text{ AND } \frac{\log\left(\frac{a^p \cdot b^q}{b^q}\right)}{\log(a)} = p
\end{aligned}$$