

Let  $(R_1, R_2) \in \mathbb{R}^{\neq}$  two resistance values of two resistor  $r_1$  and  $r_2$ . Let  $(\epsilon_1, \epsilon_2) \in \mathbb{R}^{\neq}$  their tolerance values.

If  $r_1$  and  $r_2$  are in parallel:

- The first algebraic relation is:  $R_{par} = \frac{R_1 * R_2}{R_1 + R_2}$

$$\Rightarrow \epsilon_{par} = \frac{(R_1 + \epsilon_1)(R_2 + \epsilon_2)}{(R_1 + R_2) + (\epsilon_1 + \epsilon_2)} - \frac{R_1 * R_2}{R_1 + R_2}$$

- The second algebraic relation is:  $R_{par} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

$$\Rightarrow \epsilon_{par} = \frac{1}{\frac{1}{R_1 + \epsilon_1} + \frac{1}{R_2 + \epsilon_2}} - \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Given the two scheme functions *par1* and *par2* :

```
(define (div-interval x y)
  (cond ((= (width y) 0)
        (error "can't divide by an interval of width 0")))
; the old implementation inverts lower and upper bound
;
;      (mul-interval x
;      (make-interval (/ 1.0 (upper-bound y))
;                      (/ 1.0 (lower-bound y))))
;
;      (make-interval (/ (lower-bound x) (lower-bound y))
;                      (/ (upper-bound x) (upper-bound y))))

(define (par1 r1 r2)
  (div-interval (mul-interval r1 r2)
                (add-interval r1 r2)))

(define (par2 r1 r2)
  (let ((unit (make-interval 1.0 1.0)))
    (div-interval unit
                  (add-interval (div-interval unit r1)
                                (div-interval unit r2)))))
```

The algebraic relations both give the same error. Our calculations in **par1** and **par2** break from the algebraic notions of an interval as we will show.

Lets try to write the **lower-bound** and **upper-bound** in terms of the two forms of a parallel resistance (or how **par1** and **par2** calculate them).

Let  $u_r, l_r$  respectively be the upper and lower bounds of an interval  $r$  and let  $U_{r_1.r_2}$  be the upper bound for  $R1.R2$  and let  $L_{r_1.r_2}$  be the lower bound of  $R1.R2$ ;

$$l_{par1} = \frac{L_{r_1.r_2}}{U_{r_1+r_1}} < \frac{u_{r_1}u_{r_2}}{U_{r_1+r_2}}$$

$$l_{par2} = \frac{1}{\frac{1}{u_{r_1}} + \frac{1}{u_{r_2}}} > l_{par1}$$

By symmetry of the proof, this can be also proven that  $u_{par1} > u_{par2}$

$$\Rightarrow \forall (r_1, r_2) \in \mathbb{R}^{\neq}, e_{par1} > e_{par2} | e_{par1} = r_1 * r_2 - l_{par1} = u_{par1} - r_1 * r_2 \text{ AND } e_{par2} = r_1 * r_2 - l_{par2} = u_{par2} - r_1 * r_2$$

The **par2** procedure is more suitable for parallel resistances calculation because the division by the unit ( $1/r_1$  and  $1/r_2$ ) doesn't create an expansion of the result unreasonable. While the multiplication of  $R1 * R2$  has that effect. In a sense

par2 doesn't make the operation depend on an expansive multiplication of  $R_1$  and  $R_2$ . It seems this is inherit to arithmetic operations over intervals, it is not trivial to define an exact arithmetic of intervals.