Let $(R_1, R_2) \in \mathbb{R}^{\nvDash}$ two resistance values of two resistor r_1 and r_2. Let $(\epsilon_1, \epsilon 2) \in \mathbb{R}^{\nvDash}$ their tolerance values.
If r_1 and r_2 are in parallel:
- The first algebraic relation is: $R_{par} = \frac{R_1 * R_2}{R_1 + R_2}$
$\implies \epsilon_{par} = \frac{(R_1+\epsilon_1)(R_2+\epsilon_2)}{(R_1+R_2)+(\epsilon_1+\epsilon_2)} - \frac{R_1 * R_2}{R_1+R_2}$
- The second algebraic relation is: $R_{par} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$
$\implies \epsilon_{par} = \frac{1}{\frac{1}{R_1+\epsilon_1} + \frac{1}{R_2+\epsilon_2}} - \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$
Given the two scheme functions $par1$ and $par2$:

```
  (define (div-interval x y)
    (cond ((= (width y) 0)
          (error "can't divide by an interval of width 0")))
; the old implementation inverts lower and upper bound
;          (mul-interval x
;                 (make-interval (/ 1.0 (upper-bound y))
;                                (/ 1.0 (lower-bound y)))))
      (make-interval (/ (lower-bound x) (lower-bound y))
                     (/ (upper-bound x) (upper-bound y))))

  (define (par1 r1 r2)
    (div-interval (mul-interval r1 r2)
                  (add-interval r1 r2)))

  (define (par2 r1 r2)
    (let ((unit (make-interval 1.0 1.0)))
      (div-interval unit
                    (add-interval (div-interval unit r1)
                                  (div-interval unit r2)))))
```

The algebraic relations both give the same error. Our calculations in `par1` and `par2` break from the algebraic notions of an interval as we will show.
Lets try to write the `lower-bound` and `upper-bound` in terms of the two forms of a parallel resistance (or how `par1` and `par2` calculate them).

Let u_r, l_r respectively be the upper and lower bounds of an interval r and let $U_{r_1.r_2}$ be the upper bound for $R1.R2$ and let $L_{r_1.r_2}$ be the lower bound of $R1.R2$;
$l_{par1} = \frac{L_{r_1 r_2}}{U_{r_1+r_1}} < \frac{u_{r1} u_{r_2}}{U_{r_1+r_2}}$
$l_{par2} = \frac{1}{\frac{1}{u_{r_1}} + \frac{1}{u_{r_2}}} > l_{par1}$
By symmetry of the proof, this can be also proven that $u_{par1} > u_{par2}$
$\implies \forall (r1, r2) \in \mathbb{N}^{\nvDash}, e_{par1} > e_{par_2} | e_{par1} = r1 * r2 - l_{par1} = u_{par1} - r1 * r2$ AND $e_{par2} = r1 * r2 - l_{par2} = u_{par2} - r1 * r2$