In order to prove that $Fib(n)$ is the closest integer to $\frac{\phi^n}{\sqrt{5}}, \phi = \frac{1+\sqrt{5}}{2}$
We need to prove that $|Fib(n) - \frac{\phi^n}{\sqrt{5}}| \leq 1$

Let $\psi = \frac{(1-\sqrt{5})}{2}$, lets prove by induction that $Fib(n) = \frac{\phi^n - \psi^n}{\sqrt{5}}$
Let $H(n) : Fib(n) = \frac{\phi^n - \psi^n}{\sqrt{5}}$
$Fib(0) = 0 = \frac{\phi^0 - \psi^0}{\sqrt{5}} \therefore H(0)$ is true.
Suppose that $H(n)$ is true, lets prove that $H(n+1)$ is true.
$\Rightarrow Fib(n+1) = \frac{\phi^n - \psi^n}{\sqrt{5}} + \frac{\phi^{n-1} - \psi^{n-1}}{\sqrt{5}}$
$\Rightarrow Fib(n+1) = \frac{\phi^{n-1}(\phi+1) - \psi^{n-1}(\psi+1)}{\sqrt{5}}$
Having $\phi^2 = \phi + 1$ and $\psi^2 = \psi + 1$
$\Rightarrow Fib(n+1) = \frac{\phi^{(n+1)} - \psi^{(n+1)}}{\sqrt{5}}$
$\Rightarrow H(n+1)$ is true $\blacksquare$

Let $Fib(n) = \frac{\phi^n - \psi^n}{\sqrt{5}}$
$\Rightarrow |Fib(n) - \frac{\phi^n}{\sqrt{5}}| = |\frac{\psi^n}{\sqrt{5}}|$
Having $\sqrt{5} < 3 \Rightarrow |\psi^n| < 1$ and $\sqrt{5} > 1$
$\Rightarrow |Fib(n) - \frac{\phi^n}{\sqrt{5}}| \leq 1$ $\blacksquare$