A simple **induction**:
$f(a^m b^n) = a^{min(n,m)} b^{|m-n|} c^{min(n,m)}$
$f(a^x b^y c^k) = b^{x+y} a^k$
$f(a^k) = a^k$
A simple **series**:
$U_0 = m, V_0 = n$
$U_n = |U_{n-1} - V_{n-1}|, V_n = min(U_{n-1}, V_{n-1})$
$\forall n \geq gcd(m,n), U_n = 0$

The **algorithm**:
Input: $a^m b^n$
1- Remove an 'a' an a 'b', otherwise (no a, no b or both) goto 3
2- Add 'c', goto 1
3- Switch all 'a' to 'b'
4- Switch all 'c' to 'a'
5- If there is 'b', goto 1, otherwise halt
Output: $a^x = a^{gcd(m,n)}$

An explanation:
The main idea is to continuously substruct n from m to get a 'remainder' $|m-n|$ 'b' and 'quotient' min(n, m) 'a' until we get no more 'b' (remainder is 0) at step 4, therfore $a^x = a^{gcd(n,m)}$

A sample running:
aaaaaabbbbbbbbbb
aaaaabbbbbbbbb
aaaaabbbbbbbbbc
aaaabbbbbbbbc
aaaabbbbbbbbcc
aaabbbbbbbcc
aaabbbbbbbccc
aabbbbbbccc
aabbbbbbcccc
abbbbbcccc
abbbbbccccc
bbbbccccc
bbbbcccccc
bbbbaaaaaa
bbbaaaaa
bbbaaaaac
bbaaaac
bbaaaacc
baaacc
baaaccc
aaccc
aacccc

bbcccc
bbaaaa
baaa
baaac
aac
aacc
bbaa
ba
bac
cc
aa
HALT
GCD(14, 6) = 2