

A simple **induction**:

$f(a^m b^n) = f(a^{\min(n,m)} b^{|m-n|})$ A simple **series**:

$U_0 = m, V_0 = m$

$U_n = |U_{n-1} - V_{n-1}|, V_n = \min(U_{n-1}, V_{n-1})$

The **algorithm**:

Input: $a^m b^n$

1- Remove an 'a' and a 'b', otherwise (no a, no b or both) goto 3

2- Add 'c', goto 1

3- Switch all 'a' to 'b'

4- Switch all 'c' to 'a'

5- If there is 'b', goto 1, otherwise halt

Output: $a^x = a^{\gcd(m,n)}$

An explanation:

The main idea is to continuously subtract n from m to get a 'remainder' $|m-n|$ 'b' and 'quotient' $\min(n, m)$ 'a' until we get no more 'b' (remainder is 0) at step 4, therefore $a^x = a^{\gcd(n,m)}$

A sample running:

aaaaaabbbbbbbbbb

aaaaabbbbbbbbbb

aaaaabbbbbbbbbc

aaaabbbbbbbbbc

aaaabbbbbbbbcc

aaabbbbbbbbcc

aaabbbbbbbcccc

aabbbbbbbccc

aabbbbbbbcccc

abbbbbbbccc

abbbbbbbcccc

bbbbbbccc

bbbbbbccc

bbbbaaaaaa

bbbbaaaa

bbbbaaaac

bbbaaac

bbbaaac

baaac

baaac

aacc

aacc

bbccc

bbaaaa

baaa

baaac

aac
aacc
bbaa
ba
bac
cc
aa
HALT
 $\text{GCD}(14, 6) = 2$