

A simple **induction**:

$$f(a^m b^n) = f(a^{\min(n,m)} b^{|m-n|})$$

A simple **series**:

$$U_0 = m, V_0 = m$$

$$U_n = |U_{n-1} - V_{n-1}|, V_n = \min(U_{n-1}, V_{n-1})$$

The **algorithm**:

Input: $a^m b^n$

1- Remove an 'a' and a 'b', otherwise (no a, no b or both) goto 3

2- Add 'c', goto 1

3- Switch all 'a' to 'b'

4- Switch all 'c' to 'a'

5- If there is 'b', goto 1, otherwise halt

Output: $a^x = a^{\gcd(m,n)}$

An explanation:

The main idea is to continuously subtract n from m to get a 'remainder' $|m-n|$ 'b' and 'quotient' $\min(n, m)$ 'a' until we get no more 'b' (remainder is 0) at step 4, therefore $a^x = a^{\gcd(n,m)}$

A sample running:

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bbbaaac

baaac

baaac

aacc

aacc

bbcc

bbbaaa

baaa

baaac
aac
aacc
bbaa
ba
bac
cc
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HALT
 $\text{GCD}(14, 6) = 2$