

Let $P(n) = \sum_{k=1}^n 2k - 1$. Prove the $P(n) = n^2$

Proof: we proceed by induction

For $n = 1$, $P(1) = 1^2$

Let $P(n) = n^2$, we demonstrate that $P(n + 1) = (n + 1)^2$

$$P(n + 1) = P(n) + 2(n + 1) - 1$$

$$\Rightarrow P(n + 1) = n^2 + 2n + 1$$

$$\Rightarrow P(n + 1) = (n + 1)^2$$

Then : $\forall n > 0, P(n) = n^2$