Notes on Psychrometry

- Psychrometric chart provides thermal and physical properties of a closed air-water system

air, T, P_{air}
water

open system
$T = 15 \sim 50°C$
$P = 1$ atm
(normal pressure)

air, T
$P > 1$ atm
water

closed system
$0\% < RH < 100\%$
$P = P_w + P_{air}$
$y = \frac{P_w}{P_w + P_{air}}$
$RH = \frac{P_w}{P_w^*}$

air, T
$RH = 100\%$
$P > 1$ atm
water

closed saturated system
$P_w^* \leftarrow$ Antoine equation
$P_T = P_w^* + P_{air}$
$y^* = \frac{P_w^*}{P_w^* + P_{air}}$
$RH = 100\%$

- T = Temperature of air-water mixture
  P = Total pressure of air-water system
  = partial pressure of water vapor + air pressure
  $\boxed{P = P_w + P_{air}} \approx$ atmospheric pressure $= RH \times P_w^* + P_{air}$
  y = mole fraction $= \frac{P_w}{P_w + P_{air}}$
  P_w^* = saturated water vapor pressure
  1 atm = 101325 Pa = sea level pressure at 15°C
  $= P_w^* + P_{air}$
  At 15°C, $P_w^* = 1703.762363$ Pa
  $P_{air} = 101325 - P_w^* = 99621.237637$ Pa

$PV = nRT$; $\therefore P \propto T \Rightarrow \frac{P_2}{P_1} = T_2/T_1$ $\boxed{\frac{P_2}{P_1} = \frac{z_2 T_2}{z_1 T_1}}$

$\therefore P_{air} = (T_2 + 273.15) \times \frac{99621.237637}{288.15}$ Pa

Dry Air Pressure, $P_{air} = 345.727009\,(T + 273.15)$ Pa
where T in °C
$P_{air} \approx 345.727(T + 273.15)$

Antoine Equation

$$\boxed{\lg(p^*) = A - \frac{B}{T+C}}$$

0–60°C, $A = 8.10765$, $B = 1750.286$, $C = 235.0$

60°C–150°C, $A = 7.96681$, $B = 1668.21$, $C = 228.0$

p^* in mmHg; T in °C

p^* = saturated water vapor pressure

Ferrel Equation

Ferrel constant, $A = \gamma(1 + 0.00115\, T_{wb})$

where γ ~~0~~ 0.000667 $\frac{\text{kPa}}{C}$ $\left(\frac{2}{3}\right)$

$$\boxed{P_w = p^*_{Twb} - A \cdot P_T \cdot (T - T_{wb})} = RH \cdot p^*_T \quad (2/3000)$$

where, p^*_{Twb} = saturated vapor pressure at the wet bulb temperature (°C)

p^*_T = saturated vapor pressure at the current temperature (°C)

T_{wb} = Wet bulb temperature

$P_T = 345.727(T + 273.15)\ \text{Pa} + RH \cdot p^*_T$

T, RH given; T_{wb} = ?

$$(T_{wb})_{new} = T - \frac{p^*_{wb,old} - P_w}{A \cdot P_T}$$

~~Not good for iteration~~

$$\boxed{T_{wb} = T - (p^*_{wb} - RH \cdot p^*_T)/A \cdot P}$$

Ideal gas; $PV = nRT$; $P \propto T$

Real gas: $PV = znRT$; $P \propto zT$

$$\boxed{P_2 = P_1(z_2 T_2 / z_1 T_1)}$$

$z = f(P, T,$

$z = z_0 + \omega z_1$; $P_r = \frac{P}{P_c}$, $T_r = T/T_c$

$z_0 = 1 + B_0 \frac{P_r}{T_r}$, $z_1 = B_1 \frac{P_r}{T_r}$

$B_0 = 0.083 - 0.422/T_r^{1.6}$

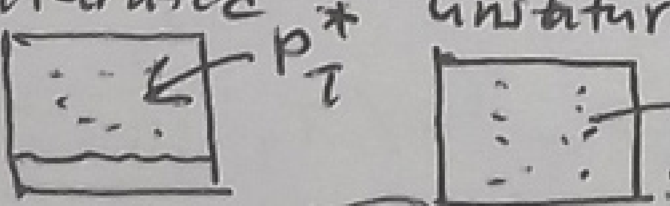
$B_1 = 0.139 - 0.172/T_r^{4.2}$

$P_c V_c = Z_c R T_c$

Dewpoint: Temperature at which current air-water vapor system becomes saturated

T, RH given, T_{dew} = ? $T \xrightarrow{\text{cooling}} p^*_T \xrightarrow{\times RH} P_w \xrightarrow[\text{cooling}]{\text{Antoine}} T_{dew}$

saturated unsaturates



p^*_T T T $P_w = RH \times p^*_w$

$T, p^*_w \xleftarrow{+H_2O} T, P_w \xleftarrow{\text{heating}} p^*_w$

$RH = \frac{P_w}{p^*_w}$

$\boxed{P_w \propto n_w}$

$\boxed{T_{dew}, p^*_w} \underset{\text{heating}}{\overset{\text{cooling}}{\rightleftarrows}} \boxed{T, P_w} \underset{-H_2O}{\overset{+H_2O,\ RH}{\rightleftarrows}} \boxed{T, p^*_w}$

Antoine eqn Antoine eqn

$p^*_{w,dew} \leftarrow P_{w,T}$

$y_{w,dew} = y_{w,T}$

Enthalpy Calculation

$$\Delta H = \int_{298.15}^{T} c_p\,dT + (H^r_{T} - H^r_{298.15}) + \Delta H_{298.15}$$

residual enthalpy

For pure elements, $\Delta H_{298.15} = 0$

Residual enthalpy, $H^r = P_r \cdot RT_c \cdot (B_0^r + wB_1^r)$

$B_0^r = 0.083 - \frac{1.097}{T_r^{1.6}}$; $B_1^r = 0.139 - \frac{0.894}{T_r^{4.2}}$

$Cp/R = A + BT + CT^2 + D/T^2$

Humid Enthalpy Correction

There is a difference between calculated value (theoretical value) and actual value

T/°C	Actual kJ/kgDA	soft value kJ/kgDA	Diff kJ/kgDA	ln(diff/kJ/kgDA)	
0	9.47	-16.8834	26.3534		
5	18.64	-7.5254	26.1654		
10	29.35	3.3856	25.9644		
15	42.12	16.3422	25.778		
20	57.56	31.9761	25.5839		
25	76.50	51.0985	25.4015		
30	100.01	74.7584	25.2516		
35	129.46	104.3256	25.1344		
40	166.69	141.6157	25.0743	3.2218	ln (diff) vs T
45	214.17	189.0836	25.0864	3.2223	A = 2.3571
50	275.35	250.1349	25.2151	3.2274	B = 0.01679
55	355.15	329.6457	25.5043	3.2388	r = 0.8426
60	460.89	434.8055	26.0845	3.2613	ln (diff) = A + B.T
65	604.00	577.0123	26.3867	3.2729	
70	803.48	774.8882	28.5918	3.3531	
75	1093.39	1061.9968	31.3922	3.4466	
80	1541.79	1505.3308	36.4592	3.5962	
85	2307.52	2261.0364	46.4836	3.8391	
90	3867.63	3797.3084	70.3216	4.2531	

Glossary

1) Saturated vapor: Air, water vapor mixture that cannot take any more water molecules. (Humidity=100%)
2) Dew point: Saturation temperature where saturation is made by forced cooling
3) Wet bulb temperature: saturation temperature where saturation is made by natural evaporation using by cotton wick
4) Absolute humidity = Moisture Content = Humidity Ratio = Specific Humidity = mass of water vapor per unit mass of dry air (kg/kgDA)
5) Relative Humidity = $\frac{\text{Partial pressure of vapor}}{\text{saturated vapor pressure}} \times 100\%$
6) Partial pressure = mole fraction x total pressure
7) Mole = Mass/Molar Mass
8) Volumetric Humidity = $\frac{\text{mass of water vapor}}{\text{volume of humid air}}$ $\left(\frac{kgV}{m^3}\right)$ = Vapor Density
9) Humid Density = $\frac{\text{mass of humid air}}{\text{volume of humid air}}$ $\left(\frac{kgHA}{m^3\,HA}\right)$
10) Humid Volume = $\frac{\text{volume of humid air}}{\text{mass of dry air}}$ $\left(\frac{m^3}{kgDA}\right)$

Equations

1) $PV = ZnRT = Z\frac{m}{M}RT \Rightarrow P_2/P_1 = Z_2T_2/Z_1T_1$
2) $\lg(P^*) = A - \frac{B}{T+C}$
3) $\Delta H_T^\circ = \Delta H_0 + \int_{T_0}^{T} C_p\,dT + H_2^R - H_1^R$

$\frac{C_p}{R} = A + BT + CT^2 + D/T^2$