

Thermodynamics of humid air

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1 The state variables of humid air, Mollier diagram and applications

1.1 Basic concepts

Humid air is a mixture of dry air and water vapor. Dry air means a mixture of all the gasses in the air (nitrogen, oxygen, carbon dioxide, and noble gasses) excluding water vapor.

The molar mass of dry air depends on its composition but for so called standard air is it $M_{da} = 0.028964$ kg/mol, for which in calculations we often use an approximation $M_{da} \approx 0.0290$ kg/mol. The molar mass of water vapor (and water) is $M_v = 0.0180153$ kg/mol, for which the approximation used in calculations is $M_v \approx 0.0180$ kg/mol.

According to the ideal gas law, the partial density of dry air in humid air is

$$\rho_{da} = \frac{p_{da} M_{da}}{RT}, \quad (1)$$

where p_{da} is the partial pressure of the dry air and R is the gas constant. To the best of our current knowledge, the gas constant is [1]

$$R = (8.31441 \pm 0.00026) \text{ J/molK}. \quad (2)$$

In practice, we use $R = 8.314 \text{ J/molK}$. The partial density of water vapor in humid air is

$$\rho_v = \frac{p_v M_v}{RT}, \quad (3)$$

where p_v is the partial pressure of the water vapor. The density of humid air is the sum of the partial densities of the dry air and the water vapor

$$\rho = \rho_{da} + \rho_v \quad (4)$$

and the pressure of the humid air is the sum of the partial pressures of the dry air and the water vapor

$$p = p_{da} + p_v. \quad (5)$$

Let us denote the amount of dry air in volume V with m_{da} ($\rho_{da} = m_{da}/V$) and the amount of water vapor with m_v ($\rho_v = m_v/V$). The absolute humidity of air is defined as the ratio between the mass of the vapor and the mass of dry air

$$x \equiv \frac{m_v}{m_{da}}. \quad (6)$$

We can also write this by using the partial densities as

$$x \equiv \frac{\rho_v}{\rho_{da}}. \quad (7)$$

The moisture content is a dimensionless number but usually a “dimension” from the definition in Eq. (6) is attached to it. We can write either $x = 0.05$ or $x = 0.05 \text{ kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{da}}$. Based on Eqs. (1), (3), and (5), we can also write the absolute humidity by using only the pressures

$$x = \frac{M_v}{M_{da}} \frac{p_v}{p_{da}} = 0.6220 \frac{p_v}{p_{da}} = 0.6220 \frac{p_v}{p - p_v}. \quad (8)$$

Solving this for the partial pressure of the water vapor, we obtain

$$p_v = \frac{x}{0.6220 + x} p. \quad (9)$$

Humid air can be seen as a mixture of dry air and water vapor. If the mixture is ideal, the enthalpy is¹

$$H = m_{da}h_{da} + m_vh_v, \quad (10)$$

where the h_{da} and h_v are the specific enthalpies (J/kg) of dry air and water vapor, respectively.

Technical calculations for humid air are easiest to be done by using the flow of dry air because it remains constant despite the changes in the amount of water vapor. It is useful to define enthalpy

$$h_k \equiv \frac{H}{m_{da}}, \quad (11)$$

where we have divided the enthalpy of the humid air by the mass of the dry air. The dimension of the enthalpy h_k is J/kg but it is often written as J/kg_{da} as a reminder that the total enthalpy of the mixture is given per one kilogram of dry air.

Combining Eqs. (10) and (11) yields

$$m_{da}h_k = m_{da}h_{da} + m_vh_v,$$

from which, by using Eq. (6), we get

$$h_k = h_{da} + xh_v. \quad (12)$$

In applications with a low enough pressure (total pressure at the most 1 bar), water vapor and dry air can be treated as ideal gasses like done in Eqs. (1) and (3). For ideal gasses the specific enthalpy is only a function of temperature

$$h_{da} = h_{da}(T)$$

and

$$h_v = h_v(T).$$

When the zero point of enthalpy for dry air is chosen to be 0 °C dry air and for water vapor 0 °C water, the enthalpies of dry air and water vapor can be obtained from the following equations

$$h_{da}(T) = \int_{273.15K}^T c_{pda}(T) dT \quad (13)$$

$$h_v(T) = l_{ho} + \int_{273.15K}^T c_{pv}(T) dT, \quad (14)$$

where $c_{pda}(T)$ and $c_{pv}(T)$ are the specific heats (J/kgK) of dry air and water vapor, respectively, and

¹Eq. (10) formally holds also for non-ideal mixtures. Then h_{da} and h_v are partial enthalpies, which depend, in addition to the temperature and the pressure, on the mixture ratio of the dry air and water vapor (i.e. the moisture content).

l_{ho} is the heat of evaporation of water at a temperature of 0 °C. The value of the evaporation heat is

$$l_{ho} = 2501 \text{ kJ/kg.}$$

The specific heats c_{pda} and c_{pv} depend somewhat on the temperature. In the temperature range of $-10 \text{ °C} \dots 40 \text{ °C}$, the average values of the specific heats are

$$\begin{aligned} c_{pda} &= 1.006 \text{ kJ/kg}^\circ\text{C} \\ c_{pv} &= 1.85 \text{ kJ/kg}^\circ\text{C}. \end{aligned}$$

At a temperature of 50°C, they are $c_{pda} = 1.008 \text{ kJ/kg}^\circ\text{C}$ and $c_{pv} = 1.87 \text{ kJ/kg}^\circ\text{C}$. Using the above average values, the enthalpy of humid air in Eq. (12) can be written in the form

$$h_k = 1.006t + x(2501 + 1.85t), \text{ kJ/kg} \quad (15a)$$

where t is the temperature in Celsius degrees. We can also of course write Eq. (15a) without substituting the average numbers as

$$h_k = c_{pda}t + x(l_{ho} + c_vt). \quad (15b)$$

We denoted the amount of dry air in volume V with m_{da} , that is $\rho_{da} = m_{da}/V$, and the amount of water vapor with m_v , that is $\rho_v = m_v/V$. In practice, we usually deal with the volume flow \dot{V} (m^3/s) instead of the volume V , because in general application the volume flow is usually a known parameter for example in an inlet of a fan. The volume flow \dot{V} expresses the total air flow including both the dry air and the water vapor and it might not remain constant throughout the system as the pressure and the temperature may vary. Therefore, calculations are usually done by using mass flow, and because the humidity might often change, it is most convenient to use the mass flow of dry air \dot{m}_{da} (kg_{da}/s). If for example we know the volume flow \dot{V} at a fan inlet, the mass flow of dry air going through the fan is

$$\dot{m}_{da} = \rho_{da}\dot{V}, \quad (16)$$

where ρ_{da} is the partial density of dry air at the inlet of the fan. Similarly, for the mass flow of water vapor we have that

$$\dot{m}_v = \rho_v\dot{V}, \quad (17)$$

where ρ_v is the partial density of the water vapor. Using the Eqs. (16) and (17), we can write based on the definition of the moisture content Eq. (6) that

$$\dot{m}_v = x\dot{m}_{da}. \quad (18)$$

When determining the energy balance of a system, we need the enthalpy flow of humid air \dot{H} , which based on Eqs. (10) and (18) can be written as

$$\dot{H} = \dot{m}_{da}h_{da} + \dot{m}_vh_v = \dot{m}_{da}(h_{da} + xh_v)$$

or in a shorter form by using the definition (11) as

$$\dot{H} = \dot{m}_{da}h_k. \quad (19)$$

In later examples, where we will be calculating energy balances, we will be using the Eq. (19).

Example 1. At an inlet of a fan, the temperature of air is 15°C and the volume flow is $0.5\text{ m}^3/\text{s}$. What is the mass flow of dry air \dot{m}_{da} at the inlet, when the moisture content of the air is $x = 0.009$ and the pressure is $p = 1.0\text{ bar}$?

The partial pressure of water vapor can be obtained from Eq. (3):

$$p_v = \frac{0.009}{0.6220 + 0.009} \cdot 1.0\text{ bar} = 0.01426\text{ bar}.$$

The partial pressure of dry air is $p_{da} = (1.0 - 0.01426)\text{ bar} = 0.986\text{ bar} = 0.986 \cdot 10^5\text{ Pa}$. The partial density of dry air is then solved from Eq. (1):

$$\rho_{da} = \frac{0.986 \cdot 10^5\text{ Pa} \cdot 0.0290\text{ kg/mol}}{8.314\text{ J/molK} \cdot (273.15015)\text{ K}} = 1.194\text{ kg}_{da}/\text{m}^3.$$

Thus the mass flow of dry air from Eq. (16) is

$$\dot{m}_{da} = 1.194\text{ kg}_{da}/\text{m}^3 \cdot 0.5\text{ m}^3/\text{s} = 0.597\text{ kg}_{da}/\text{s}.$$

The mass flow of water vapor through the fan is then

$$\dot{m}_v = x\dot{m}_{da} = 0.009 \cdot 0.597\text{ kg}_{da}/\text{s} = 0.005373\text{ kg}_{\text{H}_2\text{O}}/\text{s}.$$

1.2 The vapor pressure of water in the presence of dry air

The equilibrium between water and humid air is illustrated in Fig. 1.

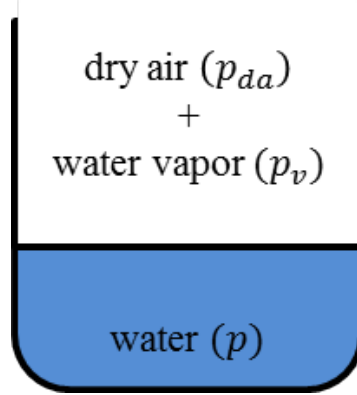


Figure 1: The equilibrium between water and water vapor in the presence of dry air.

The situation in Fig. 1 differs from an equilibrium between water and pure water vapor, as there is also inert gas, dry air, in the gas phase. The presence of dry air affects the situation because the water pressure equals the total pressure of the gas $p = p_{da} + p_v$, making it differ from the pressure of the water vapor p_v .

In equilibrium, the chemical potentials of water and water vapor are equal

$$\mu_w(T, p) = \mu_v(T, p_v), \tag{20}$$

where subscript w is for water and v for water vapor. Note that $p = p_{da} + p_v$. In principle, we can

solve the partial pressure of the water vapor p_h from Eq. (20). We notice that it depends on the temperature and the partial pressure of the dry air

$$p_v = p_v(T, p_{da}). \quad (21)$$

We will now proof that to a very good accuracy

$$p_v = p_v(T) \quad (22)$$

meaning that the dependency of the partial pressure of the water vapor on the partial pressure of the dry air is very small. We can proof this by differentiating Eq. (20)

$$\frac{\partial \mu_w}{\partial T} dT + \frac{\partial \mu_w}{\partial p} dp = \frac{\partial \mu_v}{\partial T} dT + \frac{\partial \mu_v}{\partial p_v} dp_v. \quad (23)$$

On the other hand it holds that

$$\begin{aligned} \frac{\partial \mu_w}{\partial T} &= -s_w, & \frac{\partial \mu_v}{\partial T} &= -s_v \\ \frac{\partial \mu_w}{\partial p} &= v_w, & \frac{\partial \mu_v}{\partial p} &= v_v, \end{aligned} \quad (24)$$

where s_w and s_v are the specific entropies of liquid water and water vapor, respectively, and v_w and v_v the specific volumes with the same labeling. Note that $v_v = 1/p_v$. Substituting the Eq. (24) into the Eq. (23) we get that

$$-s_w dT + v_w dp = -s_v dT + v_v dp_v,$$

from which follows that

$$dp_v = \frac{s_v - s_w}{v_v} dT + \frac{v_w}{v_v} dp. \quad (25)$$

On the other hand, because $\mu = h - Ts$, based on the equilibrium condition (20) it holds that

$$h_w - Ts_w = h_v - Ts_v,$$

from which

$$s_v - s_w = \frac{h_v - h_w}{T}.$$

Substituting this into Eq. (25) yields

$$dp_v = \frac{h_v - h_w}{T v_v} dT + \frac{v_w}{v_v} dp. \quad (26)$$

Then substituting the differential of the total pressure

$$dp = dp_{da} + dp_v$$

into Eq. (26), we finally get that

$$dp_v = \frac{h_v - h_w}{T(v_v - v_w)} dT + \frac{v_w}{v_v - v_w} dp_{da}. \quad (27)$$

The Eq. (27) differs from the Clapeyron equation by the second term, which is absent in the Clapeyron

equation. If $p_{da} = 0$, then $dp_{da} = 0$, and the Eq. (27) coincides with the Clapeyron equation as it should.

By taking into account that $v_v \gg v_w$ and $v_v = 1/p_v$, and by using the Eq. (3), we can write an approximative form for the Eq. (27)

$$\frac{dp_v}{p_v} = \frac{M_v(h_v - h_w)}{RT^2}dT + \frac{M_v v_v}{RT}dp_{da}. \quad (28)$$

On the other hand

$$\frac{dp_v}{p_v} = d(\ln p_v),$$

and thus by using the Eq. (28) we can write

$$\left(\frac{\partial (\ln p_v)}{\partial T} \right)_{p_{da}} = \frac{M_v(h_v - h_w)}{RT^2} \quad (29)$$

$$\left(\frac{\partial (\ln p_v)}{\partial p_{da}} \right)_T = \frac{M_v v_w}{RT} \quad (30)$$

The specific volume of water is approximately $v_w = 10^{-3} \text{ m}^3/\text{kg}$ and therefore at a temperature of 50 °C, we get from the Eq. (30) an estimate

$$\left(\frac{\partial (\ln p_v)}{\partial p_{da}} \right)_T = \frac{0.0180 \cdot 10^{-3}}{8.314 \cdot 323.15} \frac{1}{\text{Pa}} = 6.70 \cdot 10^{-9} \frac{1}{\text{Pa}}.$$

By integrating the Eq. (30) and using the above value, we can estimate the effect of the air pressure on the vapor pressure. When the partial pressure of dry air is $p_{da} = 0 \text{ Pa}$ and when it is $p_{da} = 10^5 \text{ Pa}$ at a temperature of 50 °C we get that

$$\ln \frac{p_v(50^\circ\text{C}, p_{da} = 10^5 \text{ Pa})}{p_v(50^\circ\text{C}, p_{da} = 0)} = 6.70 \cdot 10^{-9} \frac{1}{\text{Pa}} (10^5 - 0) \text{ Pa} = 6.70 \cdot 10^{-4}$$

thus

$$\frac{p_v(50^\circ\text{C}, p_{da} = 10^5 \text{ Pa})}{p_v(50^\circ\text{C}, p_{da} = 0)} = e^{6.70 \cdot 10^{-4}} = 1.0006702.$$

When $p_{da} = 0$, saturated water vapor is in equilibrium with water and there is no dry air. We can read the corresponding vapor pressure from the vapor pressure table

$$p_v(50^\circ\text{C}, p_{da} = 0) = p'_v(50^\circ\text{C}) = 0.12335 \text{ bar}.$$

We denote the vapor pressure of saturated water² at a temperature of T with $p'_v(T)$. The vapor pressure in the presence of dry air, when $p_{da} = 10^5 \text{ Pa}$, is

$$p_v(50^\circ\text{C}, p_{da} = 10^5 \text{ Pa}) = 1.0006702 \cdot 0.12335 \text{ bar} = 0.12343 \text{ bar}.$$

The difference between the vapor pressures is $p_v(50^\circ\text{C}, p_{da} = 10^5 \text{ Pa}) - p_v(50^\circ\text{C}, p_{da} = 0) = 8 \text{ Pa}$.

²The water at its boiling point is called saturated. The pressure of saturated water is the same as the vapor pressure and it thus depends on the temperature.

This shows that the vapor pressure of water depends only a little on the presence of dry air in the gas phase³. By repeating the above calculation for different temperatures, we can show that the vapor pressure of water can be taken with a very good precision from the pressure table of saturated water vapor, as if there is no dry air in the gas phase. To a good accuracy, the vapor pressure is only a function of temperature and the Eq. (22) holds. Therefore, no new vapor pressure tables are necessary for the calculations with humid air.

1.3 The vapor pressure of water and ice and the calculation of the state variables of humid air

The partial pressure of vapor in air cannot be greater than the vapor pressure of saturated water $p'_v(T)$ of the corresponding temperature T . If it would be greater, the water vapor would condensate until an equilibrium corresponding the saturated vapor pressure would be reached.

The pressure of saturated vapor can be obtained from the vapor pressure tables (see e.g. [2]). An approximation can be obtained from an equation [3]

$$\begin{aligned} \log \frac{p'_v(t)}{\text{bar}} = & 28.59051 - 8.2 \log \left(\frac{t}{^\circ\text{C}} + 273.16 \right) \\ & + 0.0024804 \left(\frac{t}{^\circ\text{C}} + 273.16 \right) - \frac{3142.31}{\frac{t}{^\circ\text{C}} + 273.16}. \end{aligned} \quad (31)$$

The logarithm is the 10 base logarithm, the pressure is in the units of bar, and the temperature in the units of Celsius.

Also a simpler approximative formula applies

$$p'_v(T) = p_0 \exp \left(11.78 \frac{T - 372.79}{T - 43.15} \right), \quad (32)$$

where $p_0 = 10^5$ Pa and the temperature is given in Kelvins.

When the temperature is below 0°C, the ice converts straight from solid to vapor i.e. sublimates. Then the saturation pressure p'_v is calculated based on the vapor pressure of air, for which an experimental formula holds

$$\log \frac{p'_v(t)}{\text{mbar}} = 10.5380997 - \frac{2663.91}{\frac{t}{^\circ\text{C}} + 273.16}, \quad (33)$$

where the temperature is in Celsius and the pressure in mbar.

Similarly to the water, we can write the Clayperon equation also for the vapor pressure of ice

$$dp_v = \frac{h_v - h_i}{T(v_v - v_i)} dT, \quad (34)$$

where h_i is the enthalpy and v_i the specific volume of ice.

³The amount of nitrogen, oxygen, and carbon dioxide in air also affects the vapor pressure. However, this can usually be neglected as the effect is extremely small.

The relative vapor pressure of air, that is the relative humidity, is defined as

$$\varphi = \frac{p_v}{p'_v(T)}, \quad (35)$$

where p_v is the partial pressure of water vapor in the air and $p'_v(T)$ is the saturated vapor pressure at a temperature of T . When $\varphi = 1$, that is 100 %, we say that the air is saturated.

Example 2. Let us calculate the properties of a) saturated air and b) air with a relative humidity of $\varphi = 50$ %, when the total space{10mm}pressure is $p = 1.0$ bar and the temperature is 20 °C.

a) Saturated air, $\varphi = 100$ %

We obtain the saturated vapor pressure from the Eq. (31)

$$\begin{aligned} \log \frac{p'_v(20^\circ\text{C})}{\text{bar}} &= 28.59051 - 8.2 \log(20 + 273.16) \\ &\quad + 0.0024804(20 + 273.16) - \frac{3142.31}{20 + 273.16} \\ &= -1.631 \\ \Rightarrow p'_v(20^\circ\text{C}) &= 10^{-1.631} \text{ bar} = 0.0234 \text{ bar} \end{aligned}$$

Moisture content from the Eq. (8)

$$x = 0.6220 \frac{0.0234}{1.0 - 0.0234} = 0.0149 \text{ kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{da}}$$

The densities ρ_{da} , ρ_v , and ρ from the Eqs. (1), (3), and (4), respectively

$$\rho_{da} = \frac{(1.0 - 0.0234) \cdot 10^5 \cdot 0.0290}{8.314 \cdot 293.15} = 1.162 \text{ kg/m}^3$$

$$\rho_v = \frac{0.0234 \cdot 10^5 \cdot 0.0180}{8.314 \cdot 293.15} = 0.0173 \text{ kg/m}^3$$

Note that when you know one of the partial densities and the moisture content, you can calculate the other one also by using the Eq. (7). The total density is then

$$\rho = (1.162 + 0.0173) \text{ kg/m}^3 = 1.179 \text{ kg/m}^3$$

The enthalpy of humid air is obtained from the Eq. (15a)

$$h_k = 1.006 \cdot 20 + 0.0149 \cdot (2501 + 1.85 \cdot 20) = 57.936 \text{ kJ/kg}_{\text{da}}$$

b) Humid air, $\varphi = 50$ %

From part a) we know that $p'_v(20^\circ\text{C}) = 0.0234$ bar. Thus we can get the partial pressure of the vapor from the definition of the relative humidity (35)

$$p_v = 0.50 \cdot 0.0234 \text{ bar} = 0.0117 \text{ bar}$$

The moisture content is

$$x = 0.6220 \frac{0.0117}{1 - 0.0117} = 0.00720 \text{ kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{da}}$$

The densities are obtained as in part a)

$$\rho_{da} = \frac{(1 - 0.0117) \cdot 10^5 \cdot 0.0290}{8.314 \cdot 293.15} = 1.176 \text{ kg/m}^3$$

$$\rho_v = x\rho_{da} = 0.00720 \cdot 1.176 \text{ kg/m}^3 = 0.00847 \text{ kg/m}^3$$

$$\rho = (1.176 + 0.00847) \text{ kg/m}^3 = 1.184 \text{ kg/m}^3$$

And finally the enthalpy

$$h_k = 1.006 \cdot 20 + 0.00720 \cdot (2501 + 1.85 \cdot 20) = 38.394 \text{ kJ/kg}_{\text{da}}$$

Example 3. Now let us calculate the same thing but with a total pressure of $p = 0.825$ bar.

Saturated air, $\varphi = 100$ %

$$p'_v(20^\circ\text{C}) = 0.0234 \text{ bar}$$

$$x = 0.6220 \frac{0.0234}{0.825 - 0.0234} = 0.0182 \text{ kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{da}}$$

$$\rho_{da} = \frac{(0.825 - 0.0234) \cdot 10^5 \cdot 0.0290}{8.314 \cdot 293.15} = 0.954 \text{ kg/m}^3$$

$$\rho_v = x\rho_{da} = 0.0174 \text{ kg/m}^3$$

$$\rho = (0.954 + 0.0174) \text{ kg/m}^3 = 0.971 \text{ kg/m}^3$$

The enthalpy of humid air is obtained from the Eq. (15a)

$$h_k = 1.006 \cdot 20 + 0.0182 \cdot (2501 + 1.85 \cdot 20) = 66.312 \text{ kJ/kg}_{\text{da}}$$

b) Humid air, $\varphi = 50$ %

From part a) we know that $p'_v(20^\circ\text{C}) = 0.0234$ bar. Thus we can get the partial pressure of the vapor from the definition of the relative humidity (35)

$$p_v = 0.50 \cdot 0.0234 \text{ bar} = 0.0117 \text{ bar}$$

The moisture content is

$$x = 0.6220 \frac{0.0117}{1 - 0.0117} = 0.00784 \text{ kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{da}}$$

The densities are obtained as in part a)

$$\rho_{da} = \frac{(1 - 0.0117) \cdot 10^5 \cdot 0.0290}{8.314 \cdot 293.15} = 1.176 \text{ kg/m}^3$$

$$\rho_v = x \rho_{da} = 0.00784 \cdot 1.176 \text{ kg/m}^3 = 0.00922 \text{ kg/m}^3$$

$$\rho = (1.176 + 0.00922) \text{ kg/m}^3 = 1.185 \text{ kg/m}^3$$

And finally the enthalpy

$$h_k = 1.006 \cdot 20 + 0.00784 \cdot (2501 + 1.85 \cdot 20) = 40.018 \text{ kJ/kg}_{da}.$$

By comparing the results of example 2 to the results of the example 3, we notice that the total pressure of the air affects the moisture content x , the partial density of dry air ρ_{da} , the total density ρ , and the enthalpy h_k . Knowing the total pressure is therefore relevant when calculating the thermodynamic properties of humid air.

The pressure and moisture content have also an effect on the mass flows. Let us continue the examples 2 and 3 by assuming that the volume flow of humid air at the fan inlet is $0.8 \text{ m}^3/\text{s}$. What is then the dry air flow and the vapor flow at the fan inlet in all the different cases?

Example 2 a), $\varphi = 100 \%$:

$$\begin{aligned} \dot{m}_{da} &= \rho_{da} \dot{V} = 1.162 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.930 \text{ kg}_{da}/\text{s} \\ \dot{m}_v &= \rho_v \dot{V} = 0.0173 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.014 \text{ kg/s} \end{aligned}$$

Example 2 b), $\varphi = 50 \%$:

$$\begin{aligned} \dot{m}_{da} &= 1.176 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.941 \text{ kg}_{da}/\text{s} \\ \dot{m}_v &= 0.00847 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.00678 \text{ kg/s} \end{aligned}$$

Example 3 a), $\varphi = 100 \%$:

$$\begin{aligned} \dot{m}_{da} &= 0.954 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.763 \text{ kg}_{da}/\text{s} \\ \dot{m}_v &= 0.0174 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.0139 \text{ kg/s} \end{aligned}$$

Example 3 b), $\varphi = 50 \%$:

$$\begin{aligned} \dot{m}_{da} &= 1.176 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.941 \text{ kg}_{da}/\text{s} \\ \dot{m}_v &= 0.00922 \text{ kg/m}^3 \cdot 0.8 \text{ m}^3/\text{s} = 0.00738 \text{ kg/s} \end{aligned}$$

We can see that the mass flows (also the total mass flow) differ in different cases. Note that you can also get the mass flow of vapor by multiplying the dry air mass flow with the moisture content (see Eq. (18)). When writing the energy balance, the quantity being used is the dry air mass flow \dot{m}_{da} . The effect of moisture is taken into account in the enthalpy h_k as shown in the Eq. (12).

1.4 Drawing of a Mollier diagram

The properties of humid air are usually illustrated with a Mollier diagram. A single air pressure is chosen for a single Mollier diagram. Usually the Mollier diagram is drawn for the standard atmospheric pressure

$$p = 1.013 \text{ bar} = 760 \text{ mm Hg} = 1 \text{ atm.}$$

As seen in the previous examples 2 and 3, we can only use the Mollier diagram when the pressure is the same (or approximately same) that was used in the drawing of the diagram.

The x-axis of a Mollier diagram is the moisture content x . Let us draw an equally spaced scale on the x-axis. Then draw an another axis at an 45° angle to the x-axis for the enthalpy $h_k = l_{ho} \cdot x$ (kJ/kg). Thus the h_k -axis is also equally spaced.⁴ The result is illustrated in Fig. 2.

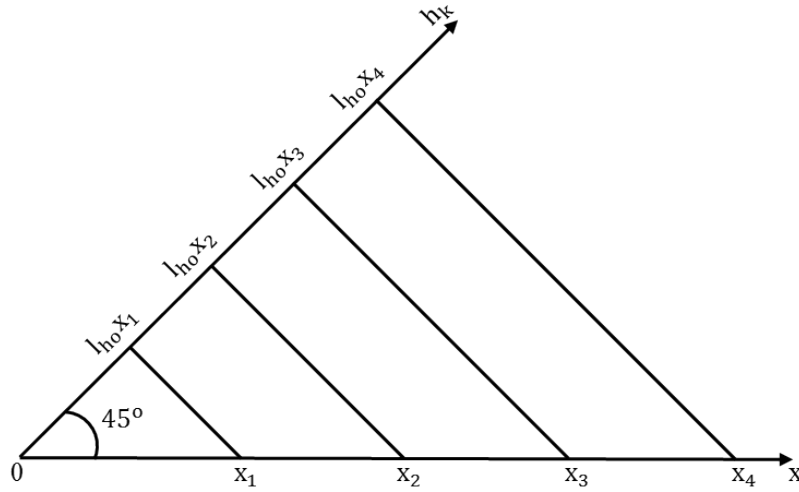


Figure 2: Example of a Mollier diagram.

Now let us draw the isotherms to the diagram

$$t = 0^\circ\text{C}, \quad h_k = c_{pda}t + x(c_{pv}t + l_{ho}) = xl_{ho} = 2501x, \quad \text{kJ/kg.}$$

Note that the h_k -scaling was constructed by using the equation $h_k = l_{ho}x = 2501x$. Therefore, the isotherm $t = 0^\circ\text{C}$ lies on top of the x-axis. The isotherms with $t = t_1 \neq 0$ are obtained from

$$t = t_1, \quad h_k = c_{pda}t_1 + x(c_{pv}t_1 + l_{ho}) = 1.006t_1 + x(1.85t_1 + 2501), \quad \text{kJ/kg.}$$

The isotherms t_1 are straight lines in the diagram. Due to the term $c_{pv}t_1$, they are not quite parallel. When $t_1 < 0^\circ\text{C}$, the isotherms are declining and when $t_1 > 0^\circ\text{C}$, the isotherms are ascending. The isotherms cross the y-axis at $x = 0$, that is $h_k = c_{pda}t$. When c_{pda} is a constant, the temperature scale on the y-axis is also equally spaced. The isotherms are illustrated in Fig. 3.

⁴Mollier diagrams can be drawn in many different ways. You can choose the direction of the h_k -axis and the scale differently. The illustration here is of a conventional way of drawing a Mollier diagram.

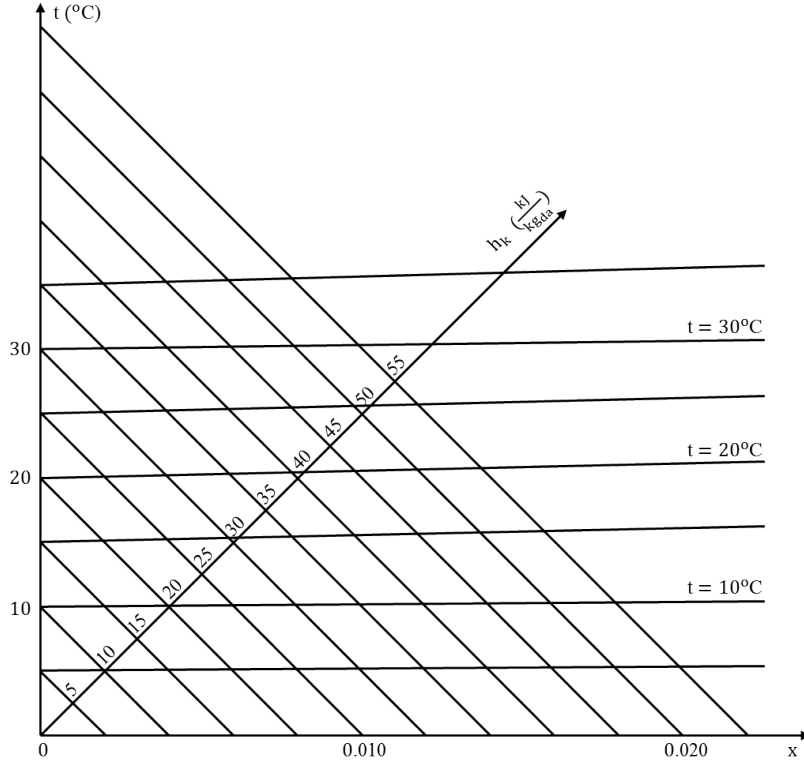


Figure 3: The basic scales of a Mollier diagram.

Adding a saturation curve ($\varphi = 100\%$) and other relative humidity (φ) curves completes the above Mollier diagram.

Example 4. Drawing of a Mollier diagram for air at a total pressure of 875 mbar.

First draw the h_k - x coordinate system as instructed above and add the temperatures to the y-axis by using the equation

$$h_k = c_{pda}t + x(c_{pv}t + l_{ho}) = c_{da}t = 1.006t, \quad \text{kJ/kg.}$$

For example when $t = -5^\circ\text{C}$, $h_k = -5.03$ kJ/kg and when $t = 5^\circ\text{C}$, $h_k = 5.03$ kJ/kg. The isotherms are equally spaced at the y-axis.

Then draw the saturation curve into the diagram. The vapor pressures can be calculated from the Eqs. (31) and (33) or they can be read from a vapor pressure table. The moisture content x' corresponding to the saturated vapor pressure $p'_v(t)$ can be obtained from the Eq. (8) (with $p = 0.875$ bar). The enthalpy of humid air from the Eq. (12) is

$$h'_k = 1.006t + x'(1.85t + 2501), \quad \text{kJ/kg.}$$

Now we can draw the saturation curves corresponding to different temperatures t by calculating the enthalpies h'_k with different moisture contents x' with the given t . Simultaneously the end points of the isotherms are determined. The curves corresponding to the different relative humidities $\varphi_1, \varphi_2, \dots$ can be drawn by calculating the corresponding moisture contents and using the already existing isotherms.

With high temperatures, the values for x' do not fit into the diagram unless we significantly extend the x-axis. Then it is useful to calculate the h_k -values for smaller values of x with a smaller relative

humidity to be able to draw the isotherms into the diagram. Values calculated with x' and $x = x_{50\%}$ are listed in the Table 1.

$t/^{\circ}\text{C}$	p'_v/bar	$\varphi = 100 \%$ x'	$\varphi = 100 \%$ $h'_k/\text{kJ/kg}$	$\varphi = 50 \%$ $x_{50\%}$	$\varphi = 50 \%$ $h_k/\text{kJ/kg}$
-10	0.00260	0.00185	-5.47	0.000925	
-5	0.00402	0.00287	2.12	0.001432	
0	0.00611	0.00437	10.93	0.00218	
5	0.00872	0.00626	20.7	0.00311	
10	0.01227	0.00885	32.4	0.00439	
15	0.01704	0.01235	46.3	0.00612	
20	0.0234	0.01709	63.5	0.00843	41.5
25	0.0317	0.0234	84.8	0.01147	54.4
30	0.0424	0.0317	111.2	0.01544	69.7
35	0.0562	0.0427	144.8	0.0206	88.1
40	0.0738	0.0573	187.8	0.0274	110.8

Table 1: Values for the drawing of the Mollier diagram with a total pressure of $p = 0.875$ bar.

By following the above instructions and using the values given in the Table 1 for the saturation curves we can now draw the full Mollier diagram. The result is illustrated in the Fig. 4.

Commonly used Mollier diagrams are illustrated in the Figs. 4-7. The diagram in the Fig. 5 holds for a total pressure of $p = 1$ bar and is used in problems concerning air conditioning technology. The Fig. 6 is the American version of the Fig. 5. It is the same diagram except with an inverted temperature scale. The diagram in the Fig. 7 covers a large range of temperatures and is therefore suitable for applications in process engineering. A diagram of the same type as in the Fig. 7 can be used for example in the designing of the drying part of a paper machine. In Fig. 7 the enthalpy is in the horizontal axis and the lines of constant enthalpy are vertical lines. The curves f are curves of moisture ratios defined as $f = x/x'(t)$, where $x'(t)$ is the moisture content of saturated air at a temperature of t . The moisture ratio f and the relative humidity φ are different things and should not be mixed up.

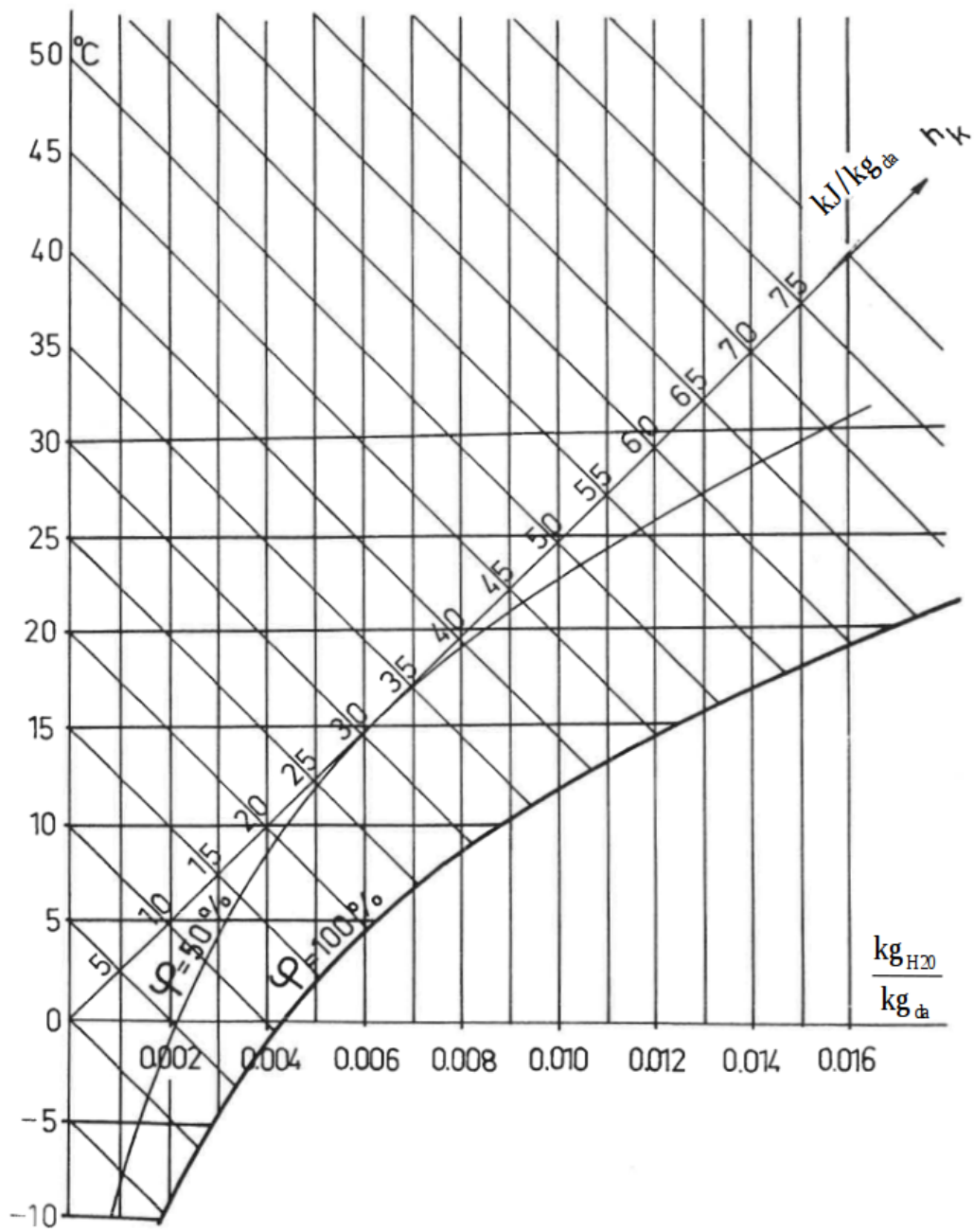


Figure 4: A Mollier diagram with a total pressure of $p = 0.875$ bar.

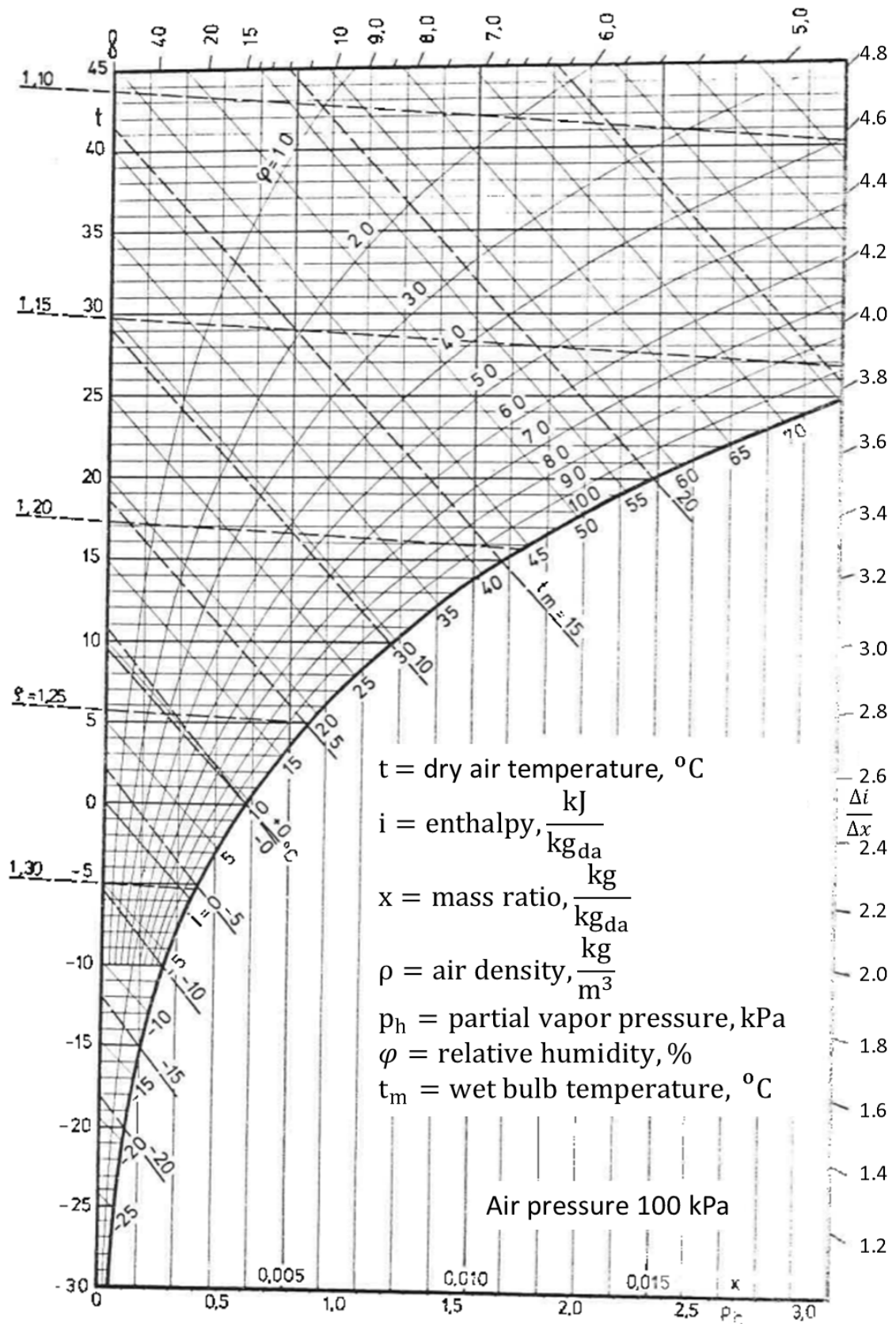


Figure 5: A Mollier diagram commonly used in problems concerning air conditioning technology. The total pressure in the diagram is $p = 1$ bar. Note that the notation in this figure does not completely coincide with the text. The used symbols are explained in the above figure.

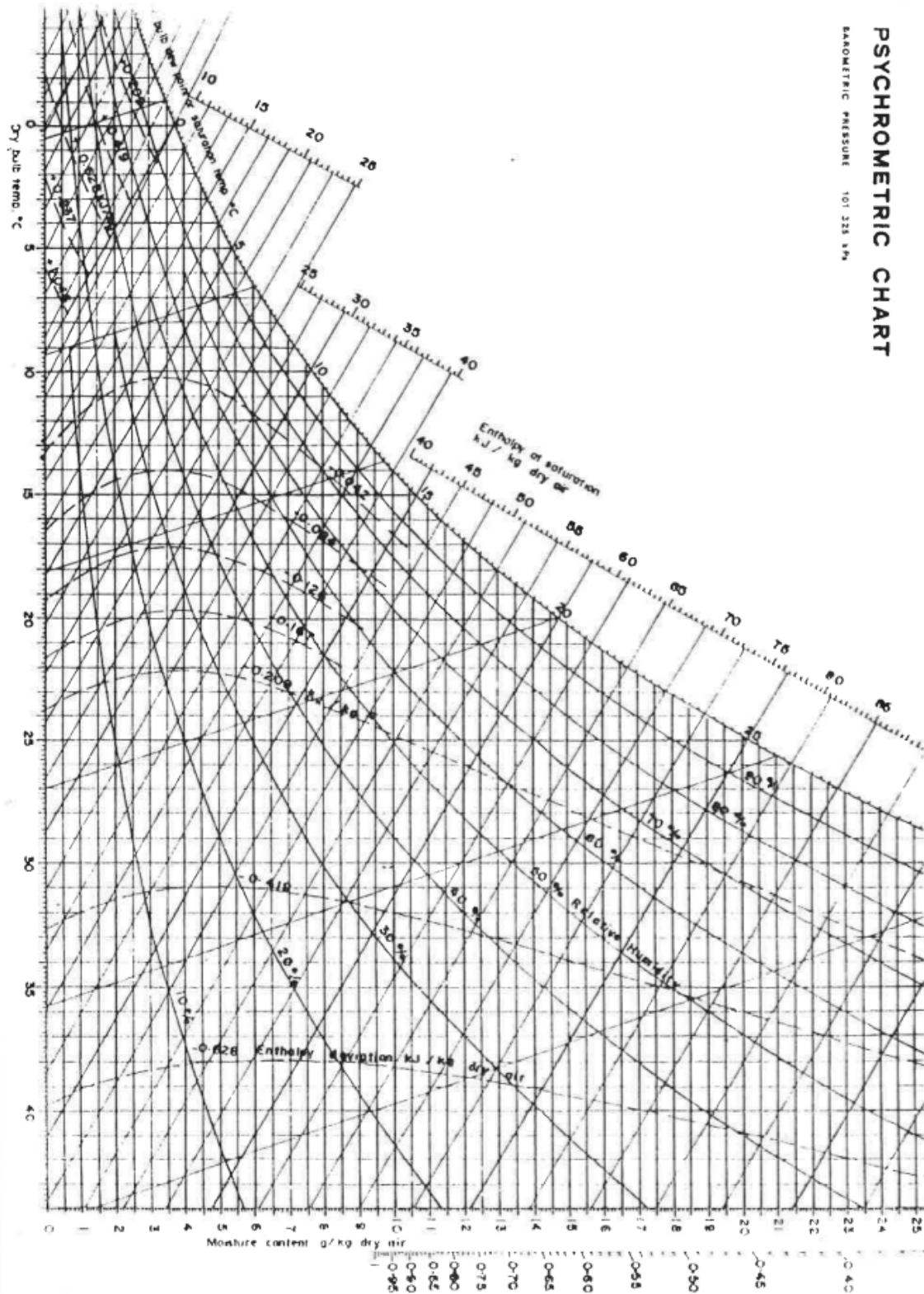


Figure 6: The American version of the diagram shown in the Fig. 5. The temperature scale is inverted.

1.5 Determining the air humidity

The humidity of air can be measured either by determining the dew point or the wet bulb temperature of the air.

The dew point is the temperature of the saturated vapor at which it has the same vapor pressure as the ambient humid air. When the total pressure is a constant, then the vapor pressure being a constant also means that the moisture content is a constant. In other words, the dew point is the temperature of the saturated vapor at which it has the same moisture content as the humid air. One can measure the dew point by cooling a surface down to a temperature at which water starts to condense on it. Then by measuring the temperature of the surface we can determine the dew point. Together with a measurement of the air temperature we can then determine the state of the air.

Example 5. The air temperature is 20°C and the dew point is 8°C . What is the relative humidity of the air?

$$\begin{aligned} p'_v(20^\circ\text{C}) &= 0.0234 \text{ bar} \\ p_v = p'_v(8^\circ\text{C}) &= 0.01072 \text{ bar} \end{aligned}$$

Thus

$$\varphi = \frac{p_v}{p'_v(20^\circ\text{C})} = 0.458 = \underline{45.8\%}.$$

Example 6. The air pressure is 950 mbar. The temperature of a room is 20°C and the relative humidity is $\varphi = 40\%$. What is the dew point of the room air?

$$\begin{aligned} p'_v(20^\circ\text{C}) &= 0.0234 \text{ bar} \\ p_v = 0.4 \cdot 0.0234 \text{ bar} &= 0.00936 \text{ bar} \end{aligned}$$

We can look up the result from the tables: $p'_v(t_1) = 0.00936 \text{ bar}$ corresponds to a temperature of $\underline{t_1 = 6.0^\circ\text{C}}$. The total pressure doesn't affect the result.

If we want to read the result from the Mollier diagram, we need to find the crossing of the line corresponding the to moisture content

$$x = 0.6620 \frac{0.00936}{0.950 - 0.00936} = 0.00659$$

and the saturation curve corresponding $\varphi = 40\%$. From there we can read the dew point temperature. To be precise, we should use a Mollier diagram drawn with a total pressure of $p = 950 \text{ mbar}$, but we can get a decent approximation from the diagram with the total pressure of $p = 1 \text{ bar}$.

When a moist cloth is placed into an air flow, it will after awhile reach an equilibrium temperature. This equilibrium temperature is known as the wet bulb temperature (t_{wb}) and it is determined by the heat and mass transfer of the system. If we neglect the heat flow radiating and conducting into the cloth, we can write the heat balance of the cloth in a stationary situation as

$$\alpha(t - t_{wb}) = \dot{m}_v l(t_{wb}), \quad (36)$$

where t is the temperature ($^\circ\text{C}$) of the air flow, \dot{m}_v the evaporation rate ($\text{kg/m}^2\text{s}$) from the cloth,

$l(t_{wb})$ the heat of evaporation of water (J/kg) at the temperature of t_{wb} ⁵ (°C), and α is the convective heat-transfer coefficient (W/m² °C).

For the evaporation rate of water from the cloth it holds that⁶ [4]

$$\dot{m}_v = M_v \frac{p}{RT} k \ln \frac{p - p_v}{p - p'_v(t_{wb})}, \quad (37)$$

where k is the mass transfer coefficient (m/s). The heat and mass transfer coefficients relate via [4]

$$k = \frac{\alpha}{\rho c_p} \text{Le}^{1-n}, \quad (38)$$

where the power n is between 0.33...0.5 and

$$\rho c_p = \rho_v c_{pv} + \rho_{da} c_{pda}, \quad \text{J/m}^3 \text{ °C} \quad (39)$$

is the heat capacity of humid air. The dimensionless number Le is the so called Lewis number (in Russian literature the Luikov number) and it is defined as

$$\text{Le} = \frac{\rho c_p D}{\lambda}, \quad (40)$$

where D is the diffusion coefficient of water vapor in air (m²/s) and λ is the thermal conductivity of humid air (W/m °C). The thermodynamic properties of saturated humid air are listed in the Table 2, where you can find e.g. the diffusion coefficient and the thermal conductivity.⁷

Substituting the Eqs. (37) and (38) into the Eq. (36) yields

$$t - t_{wb} = \frac{M_v}{\rho c_p} \frac{p}{RT} \text{Le}^{1-n} l(t_{wb}) \ln \frac{p - p_v}{p - p'_v(t_{wb})}. \quad (41)$$

Note that the heat-transfer coefficient α has disappeared from the Eq. (41) and the only term that depends on the air flow conditions of the system is the power n ⁸ in the Lewis number. Because the Lewis number is close to one, the dependence of the air flow conditions is very small.

When the state of the air, the temperature t , and the partial vapor pressure p_v are known, the wet bulb temperature can be obtained from the Eq. (41). Inversely, if the temperature t and the wet bulb temperature t_{wb} are known, we can solve the partial vapor pressure p_v from the Eq. (41) and thus the air humidity.

⁵The temperature dependence of the heat of evaporation for water is

$l(t_{wb}) = l_{ho} - (c_{pw} - c_{pv}) t_{wb} = 2501 - (4.186 - 1.85) t_{wb} = 2501 - 2.34 t_{wb}, \quad \text{kJ/kg}.$

⁶ T is the average absolute temperature at the boundary layer. In practical calculations we use $T = (t + t_{wb})/2 + 273.15 \text{ K}$.

⁷For comparison, there is a Table 3 on page 24 of thermodynamic properties of dry air. In the Table 2, the zero point of enthalpy is set at 0 °C whereas in the Table 3 the zero point is set at 0 K = -273.15 °C. For calculations with humid air, we always use the Table 2.

⁸The power n is the same as the power of the Prandtl (Pr) number in the heat-transfer equation $\text{Nu} = A \text{Re}^m \text{Pr}^n$, where Nu is the Nusselt number, A the cross-sectional area, and Re the Reynolds number. n depends on the air flow conditions of the system.

Temperature °C	Moisture content kg _{H₂O} /kg _{da}	Partial vapor pressure kN/m ²	Partial vapor density kg/m ³	Heat of evapo- ration of water kJ/kg	Enthalpy of the solution kJ/kg _{da}	Partial density of dry air kg _{da} /m ³	Kinematic viscosity 10 ⁴ m ² /s	Specific heat capacity of the solution kJ/kgK	Thermal conductivity W/mK	Diffusion coeffi- cient water-air 10 ⁶ m ² /s
0	0.003821	0.6108	0.004846	2500.8	9.55	1.285	13.25	1.0108	0.02380	22.2
2	0.004418	0.7054	0.005557	2495.9	13.06	1.275	13.43	1.0120	0.02413	22.4
4	0.005100	0.8129	0.006358	2491.3	16.39	1.264	13.61	1.0134	0.02427	22.6
6	0.005868	0.9346	0.007257	2486.6	20.77	1.254	13.79	1.0149	0.02440	22.8
8	0.006749	1.0721	0.008267	2481.9	25.00	1.243	13.97	1.0167	0.02454	23.1
10	0.007733	1.2271	0.009396	2477.2	29.52	1.232	14.15	1.0186	0.02466	23.3
12	0.008849	1.4015	0.01066	2472.5	34.37	1.221	14.34	1.0208	0.02478	23.6
14	0.010105	1.5974	0.01206	2467.8	39.57	1.211	14.52	1.0233	0.02490	23.9
16	0.011513	1.8168	0.01363	2463.1	45.18	1.199	14.71	1.0260	0.02500	24.2
18	0.013108	2.0622	0.01536	2458.4	51.29	1.188	14.89	1.0291	0.02511	24.5
20	0.014895	2.337	0.01729	2453.1	57.86	1.177	15.08	1.0325	0.02520	24.8
22	0.016892	2.642	0.01942	2449.0	65.02	1.175	15.27	1.0364	0.02529	25.2
24	0.019131	2.982	0.02177	2442.0	72.60	1.154	15.46	1.0407	0.02537	25.5
26	0.021635	3.360	0.02437	2439.5	81.22	1.141	15.65	1.0455	0.02544	25.9
28	0.024435	3.778	0.02723	2434.8	90.48	1.129	15.84	1.0509	0.02508	26.3
30	0.027558	4.241	0.03036	2430.0	100.57	1.116	16.03	1.0569	0.02556	26.6
32	0.031050	4.733	0.03380	2425.3	111.58	1.103	16.22	1.0635	0.02561	27.0
34	0.034950	5.318	0.03758	2420.5	123.72	1.090	16.41	1.0710	0.02565	27.4
36	0.039289	5.940	0.04171	2415.8	136.99	1.076	16.61	1.0793	0.02567	27.8
38	0.044136	6.624	0.04622	2411.0	151.60	1.061	16.80	1.0885	0.02569	28.3
40	0.049532	7.375	0.05114	2406.2	167.64	1.046	17.00	1.0989	0.02569	28.7
42	0.055560	8.198	0.05650	2401.4	185.40	1.030	17.20	1.1103	0.02568	29.1
44	0.062278	9.010	0.06233	2396.6	204.94	1.014	17.39	1.1232	0.02666	29.6
46	0.069778	10.085	0.06867	2391.8	226.55	0.9970	17.59	1.1375	0.02563	30.0
48	0.078146	11.161	0.07553	2387.0	250.45	0.9791	17.79	1.1534	0.02558	30.5
50	0.087516	12.335	0.08298	2382.1	277.04	0.9606	17.99	1.1713	0.02552	30.9
52	0.098018	13.613	0.09103	2377.3	306.64	0.9411	18.19	1.1913	0.02545	31.4
54	0.10976	15.002	0.09974	2372.4	339.51	0.9207	18.39	1.2137	0.02536	31.9
56	0.12297	16.509	0.1091	2367.6	373.31	0.8999	18.59	1.2389	0.02526	32.4
58	0.13790	18.146	0.1193	2362.7	417.73	0.8768	18.79	1.2673	0.02514	32.9
60	0.15472	19.92	0.1302	2357.9	464.11	0.8532	18.99	1.2994	0.02501	33.4
62	0.17380	21.84	0.1419	2353.0	516.57	0.8283	19.19	1.3357	0.02487	34.0
64	0.19541	23.91	0.1545	2348.1	575.77	0.8021	19.38	1.3770	0.02471	34.5
66	0.22021	26.14	0.1680	2343.1	643.51	0.7746	19.57	1.4241	0.02455	35.1
68	0.24866	28.55	0.1826	2338.2	721.01	0.7456	19.76	1.4782	0.02437	35.7
70	0.28154	31.16	0.1981	2333.3	810.36	0.7150	19.94	1.5418	0.02418	36.3
72	0.31966	33.96	0.2146	2328.3	915.57	0.6829	20.01	1.6132	0.02399	36.9
74	0.36468	36.96	0.2324	2323.3	1035.60	0.6489	20.28	1.6986	0.02379	37.6
76	0.41790	40.19	0.2514	2318.3	1179.42	0.6132	20.44	1.7994	0.02360	38.3
78	0.48048	43.65	0.2717	2313.3	1348.40	0.5755	20.58	1.9199	0.02341	39.0
80	0.55931	47.36	0.2933	2308.3	1560.80	0.5358	20.71	2.0664	0.02323	39.8
82	0.65573	51.33	0.3162	2303.2	1820.46	0.4939	20.81	2.2477	0.02307	40.7
84	0.77781	55.57	0.3406	2298.1	2148.92	0.4497	20.90	2.4767	0.02294	41.5
86	0.93768	60.50	0.3666	2293.0	2578.73	0.4031	20.96	2.7739	0.02285	42.5
88	1.15244	64.95	0.3942	2287.9	3155.67	0.3542	20.99	3.1708	0.02281	43.6
90	1.45873	70.11	0.4235	2282.8	3978.42	0.3026	20.99	3.7304	0.02293	44.7
92	1.92718	75.61	0.4545	2277.6	5236.61	0.2482	20.94	4.574	0.02295	46.0
94	2.73170	81.46	0.4873	2272.4	7395.49	0.1909	20.84	5.987	0.02318	47.4
96	4.42670	87.69	0.5221	2267.1	11944.39	0.1305	20.69	8.820	0.02355	49.0
98	10.30306	94.30	0.5588	2261.9	27711.34	0.06694	20.47	17.338	0.02409	50.8
100	∞	101.325	0.5977	2256.7	∞	0	20.08	∞	0.02486	52.5

Table 2: Thermodynamic properties of saturated humid air. [5] Values of the humidity and the humid enthalpy refer to a total pressure of 100 kN/m². The moisture diffusivity is estimated as the concentration-weighted mean value of th self-diffusion coefficient of water vapor and the limiting diffusivity of water vapor through air at dilute concentrations. Thermal conductivity is estimated from Mason and Saxena's rule for mixtures [6]. The kinematic viscosity is estimated from Wilke's method [7] of predicting the dynamic viscosity of mixtures.

°C	T		$\frac{* \rho}{\text{kg/m}^3}$ $= 1/v$	v m ³ /kg	η 10 ⁻⁶ kg/ms	$\frac{* \nu}{10^{-6} \text{m}^2/\text{s}}$ $= \eta/\nu$	c_p kJ/kgK	c_{pm} 1	h kJ/kg	λ 10 ⁻³ W/Km	$\frac{* a}{10^{-6} \text{m}^2/\text{s}}$ $= \lambda v / c_p$	$\frac{* \text{Pr}}{(\nu/a)}$ $= \eta c_p / \lambda$
	K	kJ/mol										
-180	93.15	0.7745	3.852	0.25962	6.45	1.67	1.073	3.5968	90.509	8.59	2.08	0.806
-160	113.15	0.9408	3.126	0.31991	7.85	2.51	1.037	3.5540	111.53	10.41	3.21	0.782
-140	133.15	1.1071	2.639	0.37891	9.20	3.49	1.021	3.5310	132.11	12.22	4.53	0.769
-120	153.15	1.2733	2.287	0.43730	10.48	4.58	1.014	3.5187	152.42	14.00	6.04	0.759
-100	173.15	1.4396	2.019	0.49540	11.71	5.80	1.011	3.5114	172.68	15.77	7.73	0.751
-80	193.15	1.6059	1.807	0.55331	12.89	7.13	1.009	3.5071	192.87	17.50	9.60	0.743
-60	213.15	1.7722	1.636	0.61109	14.02	8.57	1.008	3.5044	213.02	19.20	11.64	0.736
-40	233.15	1.9385	1.495	0.66879	15.11	10.11	1.007	3.5030	233.14	20.89	13.87	0.728
-20	253.15	2.1048	1.377	0.72646	16.15	11.73	1.006	3.5026	253.27	22.50	16.25	0.722
0	273.15	2.2711	1.275	0.78408	17.16	13.45	1.006	3.5032	273.41	24.08	18.77	0.717
20	293.15	2.4373	1.188	0.84168	18.14	15.27	1.006	3.5050	293.54	25.61	21.43	0.713
40	313.15	2.6036	1.112	0.89925	19.07	17.15	1.007	3.5078	313.68	27.08	24.18	0.709
60	333.15	2.7699	1.045	0.95680	19.99	19.13	1.009	3.5118	333.84	28.48	27.01	0.707
80	353.15	2.9362	0.9859	1.0143	20.88	21.18	1.010	3.5168	354.02	29.91	30.0	0.705
100	373.15	3.1025	0.9329	1.0719	21.73	23.29	1.012	3.5230	374.23	31.26	33.1	0.703
120	393.15	3.2688	0.8854	1.1294	22.58	25.50	1.014	3.5305	394.48	32.59	36.3	0.702
140	413.15	3.4351	0.8425	1.1869	23.39	27.76	1.017	3.5391	414.78	33.92	39.6	0.701
160	433.15	3.6014	0.8036	1.2444	24.18	30.09	1.020	3.5487	435.14	35.24	43.0	0.700
180	453.15	3.7676	0.7681	1.3019	24.57	32.51	1.023	3.5595	455.55	36.53	46.5	0.699
200	473.15	3.9339	0.7356	1.3594	25.70	34.94	1.026	3.5712	476.03	37.82	50.1	0.697
220	493.15	4.1002	0.7058	1.4169	26.44	37.46	1.029	3.5837	496.58	39.07	53.8	0.696
240	513.15	4.2665	0.6783	1.4743	27.17	40.06	1.033	3.5971	517.20	40.34	57.6	0.696
260	533.15	4.4329	0.6528	1.5318	27.90	42.74	1.037	3.6113	537.90	41.57	61.4	0.696
280	553.15	4.5991	0.6292	1.5893	28.59	45.44	1.041	3.6262	558.69	42.79	65.3	0.696
300	573.15	4.7654	0.6072	1.6468	29.28	48.22	1.046	3.6415	579.56	43.99	69.3	0.696
350	623.15	5.1811	0.5585	1.7905	30.92	55.4	1.057	3.6814	632.13	46.92	79.5	0.697
400	673.15	5.5968	0.5170	1.9341	32.50	62.9	1.069	3.7230	685.28	49.83	90.2	0.698
450	723.15	6.0125	0.4813	2.0778	34.02	70.7	1.081	3.7647	739.04	52.60	101.1	0.699
500	773.15	6.4282	0.4502	2.2214	35.49	78.8	1.093	3.8062	793.39	55.45	112.7	0.700
550	823.15	6.8439	0.4228	2.3651	36.89	87.2	1.105	3.8460	848.33	58.20	124.6	0.700
600	873.15	7.2597	0.3986	2.5087	38.26	96.0	1.116	3.886	903.83	60.93	137.0	0.701
650	923.15	7.6754	0.3770	2.6523	39.57	105.0	1.137	3.924	959.88	63.59	162.2	0.705
700	973.15	8.0911	0.3577	2.7960	40.87	114.3	1.155	3.960	1016.4	65.95	188.2	0.710
750	1023.15	8.5068	0.3396	2.9445	42.12	124.0	1.171	3.995	1073.5	68.24	215	0.715
800	1073.15	8.9225	0.3243	3.0832	43.32	133.6	1.185	4.028	1131.0	70.5	243	0.721
850	1123.15	9.3382	0.3099	3.2268	44.50	143.6	1.198	4.061	1188.9	72.7	272	0.726
900	1173.15	9.7540	0.2967	3.3705	45.65	153.7	1.198	4.092	1247.3	74.8	301	0.731
950	1223.15	10.1697	0.2846	3.5141	46.78	164.4	1.185	4.123	1305.9	76.8		
1000	1273.15	10.5854	0.2734	3.6578	47.88	175.1	1.185	4.154	1365.0	78.7		
1050	1323.15	11.0011	0.2631	3.8014	48.96	186.1	1.198	4.185	1424.4	80.6		
1100	1373.15	11.4168	0.2535	3.9450	50.02	197.3	1.198	4.214	1484.0	82.5		
1150	1423.15	11.8325	0.2446	4.0886	51.06	209	1.209	4.244	1543.9	84.3		
1200	1473.15	12.2483	0.2363	4.2323	52.07	220	1.209	4.273	1604.1	86.1		
1250	1523.15	12.6640	0.2285	4.3759	53.07	232	1.209	4.304	1664.5			

Table 3: The thermodynamic properties of dry air at a total pressure of 1 bar. The values of ν and c_p are taken from Ref. [8]. The values for viscosity (ν) are taken from Ref. [9] and for c_{pm} the values are interpolated from the same source. Due to the different sources, the values of c_p and c_{pm} do not exactly correspond to each other, since for the molar mass Ref. [8] uses 28.96 and Ref. [9] uses 28.966. The values of thermal conductivity are from [10] and are interpolated as with c_{pm} . The other variables, marked with *, are calculated from the values of the table using the corresponding equations below the variables. Many elements in the table are given with greater precision than the error limits would allow. The references have more insight on the reliability of the elements in question.

Example 7. The air temperature is $t = 20^\circ\text{C}$ and the wet bulb temperature is $t_{wb} = 10^\circ\text{C}$. What is the moisture content of the air when the air pressure is a) $p = 1$ bar and b) $p = 0.9$ bar?

Solving the vapor pressure p_v from the Eq. (41) gives

$$p - p_v = (p - p'_v(t_{wb})) \exp \left[(t - t_{wb}) \frac{\rho c_p}{M_v} \frac{RT}{p} \frac{1}{\text{Le}^{1-n}} \frac{1}{l(t_{wb})} \right]. \quad (42)$$

The diffusion coefficient D in the Lewis number is inversely proportional to the total pressure

$$D = \frac{1}{p} f(T). \quad (43)$$

At a temperature of 10°C and a pressure of $p = 1$ bar, the diffusion coefficient from the Table 2 is $D(10^\circ\text{C}, p = 1 \text{ bar}) = 23.3 \cdot 10^{-6} \text{m}^2/\text{s}$. With the same temperature and the pressure $p = 0.9$ bar, the diffusion coefficient can be obtained from the Eq. (43) $D(10^\circ\text{C}, p = 0.9 \text{ bar}) = 25.9 \cdot 10^{-6} \text{m}^2/\text{s}$.

On the other hand, an inverse proportionality like in Eq. (43) also holds for the diffusion coefficient with the heat-transfer coefficient. With a good accuracy $D/\lambda = g(T)$. From the Table 2, $\lambda(10^\circ\text{C}, p = 1 \text{ bar}) = 0.02466 \text{ W/mK}$ and thus $D/\lambda = 9.45 \cdot 10^{-4} \text{ m}^3\text{K/J}$.

The evaporation heat and the saturated vapor pressure can be obtained from the Table 2

$$\begin{aligned} l(t_{wb}) &= l(10^\circ\text{C}) = 2477 \cdot 10^3 \text{ J/K} \\ p'_v(t_{wb}) &= p'_v(t_{wb}) = 0.01227 \text{ bar.} \end{aligned}$$

The temperature T in the Eq. (42) is taken as the average temperature at the boundary layer (see Footnote 6)

$$T = \left(\frac{20 + 10}{2} + 273.15 \right) \text{ K} = 288.15 \text{ K}.$$

For the heat capacity

$$\rho c_p = \rho_{da} c_{pda} + \rho_v c_{pv} = \rho_{da} (c_{pda} + x c_{pv}) \quad (44)$$

we would have to find an iterative solution as the vapor pressure p_v and therefore the moisture content x are unknown at this point. When the moisture content is small enough, as in this example, we can approximate that $\rho c_p \cong \rho_{da} c_{da}$. For a more exact solution, we would have to then repeat the calculation with the vapor pressure from the Eq. (42) until a desired precision is reached. In this example, we will settle for the result obtained with the above mentioned approximation for the heat capacity.

When calculating the density and the heat capacity, we make an approximation $\rho \cong \rho_{da}$ or $p \cong p_{da}$ and $c_p \cong c_{pda}$. From this follows that

$$\frac{\rho c_p}{M_v} \frac{RT}{p} \cong \frac{M_{da}}{M_v} c_p \cong \frac{M_{da}}{M_v} c_{pda}, \quad (45)$$

which as a numerical value is

$$\frac{M_{da}}{M_v} c_{pda} = 1617 \text{ J/kg}^\circ\text{C}.$$

Now from the Eq. (1), the partial density of air in a) and b) is

a)

$$\rho_{da} = \frac{p_{da} M_{da}}{RT} \cong \frac{p M_{da}}{RT} = \frac{10^5 \cdot 0.029}{8.314 \cdot 288.15} = 1.211 \text{ kg/m}^3$$

b)

$$\rho_{da} = \frac{p_{da} M_{da}}{RT} \cong \frac{p M_{da}}{RT} = \frac{0.9 \cdot 10^5 \cdot 0.029}{8.314 \cdot 288.15} = 1.089 \text{ kg/m}^3$$

The Lewis number is

a)

$$\text{Le} = \frac{\rho c_p D}{\lambda} \cong \rho_{da} c_{pda} \frac{D}{\lambda} = 1.211 \cdot 1006 \cdot 9.45 \cdot 10^{-4} = 1.151$$

Assuming that $n = 0.5$, $\text{Le}^{1-n} = 1.073$.

b)

$$\text{Le} = \frac{\rho c_p D}{\lambda} \cong \rho_{da} c_{pda} \frac{D}{\lambda} = 1.089 \cdot 1006 \cdot 9.45 \cdot 10^{-4} = 1.035$$

and $\text{Le}^{1-n} = 1.017$.

Substituting all the obtained values into the Eq. (42) gives

a)

$$p - p_v = (1.0 - 0.012271) \exp \left[(20 - 10) \cdot 1617 \cdot \frac{1}{1.073} \cdot \frac{1}{2477.2 \cdot 10^3} \right] \text{ bar} = 0.9938 \text{ bar}$$

and thus

$$p_v = (1.0 - 0.9938) \text{ bar} = 0.0062 \text{ bar} = 620 \text{ Pa}$$

and

$$x = 0.662 \frac{p_v}{p - p_v} = 0.662 \frac{620}{10^5 - 620} = 0.00413.$$

b)

$$p - p_v = (0.9 - 0.012271) \exp \left[(20 - 10) \cdot 1617 \cdot \frac{1}{1.017} \cdot \frac{1}{2477.2 \cdot 10^3} \right] \text{ bar} = 0.8934 \text{ bar}$$

and thus

$$p_v = (0.9 - 0.8934) \text{ bar} = 0.0066 \text{ bar} = 660 \text{ Pa}$$

and finally

$$x = 0.662 \frac{p_v}{p - p_v} = 0.662 \frac{660}{0.9 \cdot 10^5 - 660} = 0.00489.$$

By comparing the a) and b) parts, we can notice that the pressure has a significant impact on the results. It is important that the effects of pressure are taken into account in industrial air conditioning and process measurements, where there can be a partial vacuum or considerable overpressure compared to the atmospheric pressure.

Next we derive an approximation for the Eq. (41) that can be used when the partial pressure of the vapor is small compared to the total pressure. It holds with a relatively good accuracy that⁹

$$\ln \frac{p - p_v}{p - p'_v(t_{wb})} = \ln \left[1 + \frac{p'_v(t_{wb}) - p_v}{p - p'_v(t_{wb})} \right] \cong \frac{p'_v(t_{wb}) - p_v}{p - p'_v(t_{wb})} \cong \frac{p'_v(t_{wb})}{p - p'_v(t_{wb})} - \frac{p_v}{p - p_v}.$$

Now using the Eq. (8), we get that

$$\ln \frac{p - p_v}{p - p'_v(t_{wb})} \cong \frac{M_{da}}{M_v} (x'(t_{wb}) - x), \quad (46)$$

where $x'(t_{wb})$ is the moisture content of saturated air corresponding to a temperature of t_{wb} and a total pressure of p . Substituting the approximations from the Eqs. (45) and (46) into the Eq. (41) yields

$$\frac{x'(t_{wb}) - x}{t - t_{wb}} = \frac{c_p}{l(t_{wb})} \frac{1}{\text{Le}^{1-n}}. \quad (47)$$

For air the Lewis number is approximately 1 (see example 7), so to a good accuracy $\text{Le}^{1-n} \cong 1$. Therefore the Eq. (47) simplifies to

$$\frac{x'(t_{wb}) - x}{t - t_{wb}} = \frac{c_p}{l(t_{wb})}. \quad (48)$$

Above we discussed to which temperature does a moist cloth set when it is considered to be thermally isolated except from an air flow. We also assumed that there is no radiant heat exchange between the cloth and the air flow. During this discussion, the state of the air flow remained unchanged.

If the moisture from the cloth humidifies the air adiabatically so much that the state of the air flow also changes, the moist cloth sets to a slightly different temperature. There is a wet bulb temperature t_{wb} for every state of air (t, x) , which can be calculated from the Eq. (41) or from the approximation (48), when the partial pressures are small compared to the total pressure. An interesting special case is when the state of the air reaches the saturation curve. Then the temperatures of the air flow and the moist cloth are the same. This equilibrium temperature is known as the thermodynamic wet bulb temperature or the adiabatic saturation temperature (t_{ad}).

When air is humidified with a water flow \dot{m}_w and when the ingoing and outgoing humid air flows are denoted as \dot{m}_1 and \dot{m}_2 , respectively, we can write the energy balance of the humidifying chamber as

$$\dot{m}_2 h_2 - \dot{m}_1 h_1 = \dot{m}_w h_w. \quad (49)$$

The Eq. (49) is illustrated in the Fig. 8.

⁹Natural logarithm can be expanded as a Taylor series

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots,$$

when $|x| \leq 1$ unless $x = -1$. If x is small, we can estimate that $\ln(1+x) \cong x$.

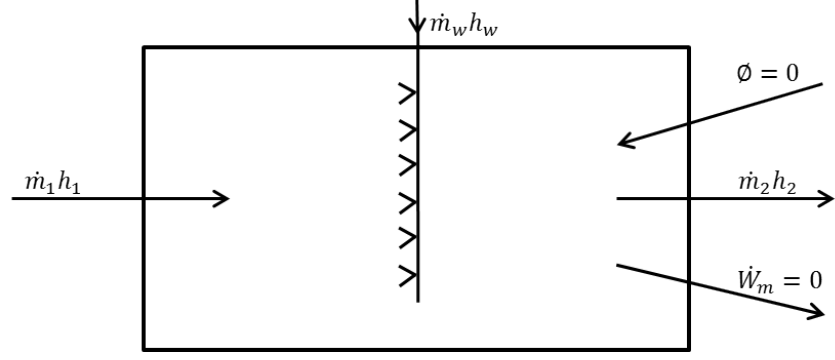


Figure 8: The energy balance of an adiabatic humidifying chamber. Here ϕ is net thermal power going into the system and \dot{W}_m is the net work done by the system into the environment.

In the Eq. (49) the incoming enthalpy flow of the humid air is

$$\dot{m}_1 h_1 = \dot{m}_{da,1} h_{da,1} + \dot{m}_{v,1} h_{v,1} \quad (50a)$$

and the outgoing is

$$\dot{m}_2 h_2 = \dot{m}_{da,2} h_{da,2} + \dot{m}_{v,2} h_{v,2}. \quad (50b)$$

Because the flow of dry air remains unchanged, it holds that

$$\dot{m}_{da} = \dot{m}_{da,1} = \dot{m}_{da,2}, \quad (51)$$

and thus we can rewrite the enthalpy flows in the Eqs. (50a) and (50b) as¹⁰

$$\dot{m}_1 h_1 = \dot{m}_{da} h_{k,1} \quad (52a)$$

$$\dot{m}_2 h_2 = \dot{m}_{da} h_{k,2}, \quad (52b)$$

where

$$h_{k,1} = h_{da,1} + x_1 h_{v,1} \quad (53a)$$

$$h_{k,2} = h_{da,2} + x_2 h_{v,2} \quad (53b)$$

as seen in the Eq. (12). When all the water supplied into the humidifying chamber is evaporated, the following humidity balance holds

$$\dot{m}_w = \dot{m}_{v,2} - \dot{m}_{v,1} = \dot{m}_{da} (x_2 - x_1). \quad (54)$$

Substituting the Eqs. (52a) and (52b) into the Eq. (54) yields

$$\frac{\Delta h_k}{\Delta x} = h_w, \quad (55)$$

where $\Delta h_k = h_{k,2} - h_{k,1}$ and $\Delta x = x_2 - x_1$.

When air is adiabatically humidified with water of a temperature of t_{ad} , the enthalpy of water is

$$h_w = c_{pw} t_{ad}. \quad (56)$$

¹⁰Note that the moisture content $x \equiv \frac{\dot{m}_v}{\dot{m}_{da}}$ expressed with the mass flows is $x = \frac{\dot{m}_v}{\dot{m}_{da}}$.

If the air is humidified such that it reaches a saturated state corresponding to the temperature of t_{ad} , we denote

$$\begin{aligned} h_{k,2} &= h_{k,ad} \\ x_2 &= x_{ad}. \end{aligned} \tag{57}$$

Using the notations (57), we can drop the subscript 1 and write that $x_1 = x$ and $h_{k,1} = h_k$. We can now rewrite the Eq. (55) by using the above notation and the Eq. (56) as

$$\frac{h_k - h_{k,ad}}{x - x_{ad}} = c_{pw} t_{ad}. \tag{58}$$

The state moves along a line in the Mollier diagram, where the enthalpy and the moisture content are determined by the Eq. (58). This is illustrated in the Fig. 9.

The Eq. (58) can be formally written in a form resembling the Eq. (48). To see this, let us first write the Eq. (58) as

$$h_k - h_{k,ad} - c_{pw} t_{ad} (x - x_{ad}) = 0.$$

On the other hand,

$$h_k = c_{pda} t + x (c_{pv} t + l_{ho})$$

and

$$h_{k,ad} = c_{pda} t_{ad} + x_{ad} (c_{pv} t_{ad} + l_{ho}).$$

Substituting these to the above equation we can rewrite it as

$$(c_{pda} + x c_{pv}) (t - t_{ad}) + (x - x_{ad}) (l_{ho} + c_{pv} t_{ad} - c_{pw} t_{ad}) = 0. \tag{59}$$

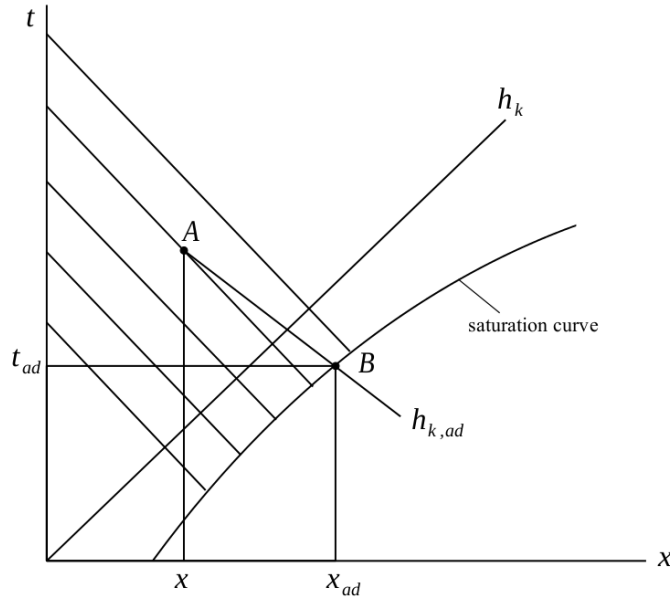


Figure 9: The change of the state of air in adiabatic humidifying. A denotes the initial state and B the final saturated state.

The heat of evaporation of water at a temperature of t_{ad} is

$$l(t_{ad}) = l_{ho} + c_{pv}t_{ad} - c_{pw}t_{ad} \quad (60)$$

and the specific heat of humid air per kg_{da} is

$$c_{pk} = c_{pda} + xc_{pv}. \quad (61)$$

Substituting the Eqs. (60) and (61) into the Eq. (59) yields

$$\frac{x_{ad} - x}{t - t_{ad}} = \frac{c_{pk}}{l(t_{ad})}, \quad (62)$$

which is equivalent with the Eq. (58).

The Eq. (62) is almost the same as the approximative equation (48) derived for the wet bulb temperature. When the partial vapor pressure is small compared to the air pressure, that is the moisture content is small, the specific heat of humid air per $\text{kg}_{\text{H}_2\text{O}}$ and per kg_{da} are approximately the same, that is $c_p \cong c_{pk}$. If the moisture content is small and $\text{Le} \cong 1$, the adiabatic saturation temperature (thermodynamic wet bulb temperature) t_{ad} is approximately the same as the “technical” wet bulb temperature t_{wb} .

Example 8. In the Mollier diagram, starting from the 14°C at the saturation curve, draw a) the line along which the state moves during adiabatic humidifying and b) the guideline associated with the wet bulb temperature measurement that can be used to define the state. The air pressure is 1 bar.

$$p'_v(14^\circ\text{C}) = 0.01597 \text{ bar (from the Table 2)}$$

$$x_{ad} = 0.6620 \frac{0.01597}{1.0 - 0.01597} = 0.01009 = x'(t_{wb})$$

$$c_{pk} = 1.006 + 0.01009 \cdot 1.85 = 1.025 \text{ KJ/}^\circ\text{Ckg}_{\text{da}}$$

$$l(14^\circ\text{C}) = 24678 \text{ kJ/kg (from the Table 2)}$$

$$\rho_{da} = 1.211 \text{ kg/m}^3 \text{ (from the Table 2)}$$

$$\rho_v = 0.01206 \text{ kg/m}^3 \text{ (from the Table 2)}$$

$$\rho = \rho_{da} + \rho_v = 1.223 \text{ kg/m}^3$$

$$\rho c_p = \rho_{da}c_{pda} + \rho_v c_{pv} = 1.211 \cdot 1.006 + 0.01206 \cdot 1.85 = 1.241 \text{ kJ/m}^3^\circ\text{C}$$

$$c_p = 1.015 \text{ kJ/kg}^\circ\text{C}$$

$$D = 23.9 \cdot 10^{-6} \text{ m}^2/\text{s} \text{ (from the Table 2)}$$

$$\lambda = 0.0249 \text{ W/m}^\circ\text{C} \text{ (from the Table 2)}$$

$$\text{Le} = \frac{D\rho c_p}{\lambda} = \frac{23.9 \cdot 10^{-6} \cdot 1.241}{0.0249} = 1.191$$

$$n \cong 0.5$$

a) Let us choose a point on the isotherm $t = 25\text{ }^{\circ}\text{C}$ to help with the drawing. From the Eq. (62)

$$x_{ad} - x = \frac{1.025}{2467.8} (25 - 14) = 0.00457$$

we get the location of the point $x = 0.01009 - 0.00457 = 0.00552$.

b) Similarly we get a point to help with the drawing of the line for the wet bulb temperature. From the Eq. (47)

$$x'(t_{wb}) - x = \frac{1.015}{2467.8} \frac{1}{1.191^{0.5}} (25 - 14) = 0.00415$$

and thus $x = 0.00594$. The results are illustrated in the Fig. 10

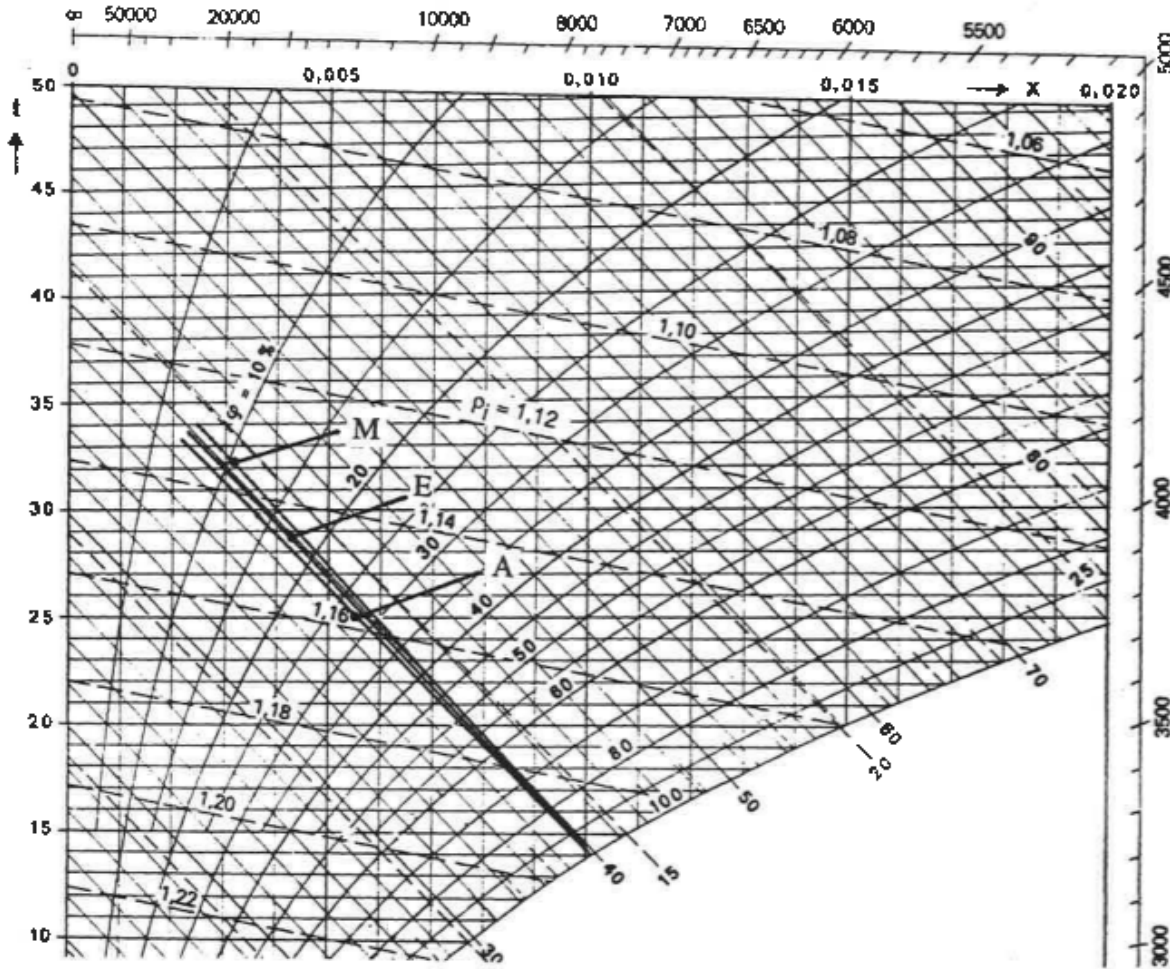


Figure 10: The lines used to define the state of air. M = the wet bulb temperature line, E = line of constant enthalpy, and A = line of adiabatic humidifying.

The enthalpy of humid air at a temperature of $14\text{ }^{\circ}\text{C}$ at the saturation curve is

$$h_{k,ad} = 1.0016 \cdot 14 + 0.01009 (1.85 \cdot 14 + 2501) = 39.58 \text{ kJ/kg.}$$

The moisture content corresponding to this enthalpy at a temperature of $25\text{ }^{\circ}\text{C}$ is determined by

$$39.58 = 1.005 \cdot 25 + x (1.85 \cdot 25 + 2501),$$

from which $x = 0.00567$.

Comparing the obtained value to the results of the Examples 6 and 7, we realize that the line of constant enthalpy lies in between the line corresponding to the wet bulb temperature and the line of adiabatic humidifying. The closer the Lewis number is to one the closer the wet bulb temperature and the adiabatic humidifying temperature are to each other.

In practical calculations you can use the line of constant enthalpy as the wet bulb temperature guideline. The crossing of the line of constant enthalpy and the isotherm, corresponding to the state of the air, gives the humidity of the air. For more precise calculations you must use the Eq. (41) or its approximative form (47) if the vapor pressures (p_v and p'_v) are small enough compared to the total pressure p .

As an example of using the Mollier diagram of the Fig. 7 to determine the state of air, we can take a typical measurement from the exhaust hood of a paper machine. The temperature and the wet bulb temperature of the extract air are 82 °C and 60 °C, respectively. In the Fig. 7, start from the saturation curve at a temperature of 60 °C and move up along the line of constant enthalpy ($h_k = 460 \text{ kJ/kg}_{\text{da}}$) until you reach the isotherm $t = 82 \text{ °C}$. The crossing point of the line of constant enthalpy and the isotherm represents the state of the air. To the accuracy of the Fig. 7, $x = 0.14$ and $f \equiv x/x'(82 \text{ °C}) = 0.20$. Using the obtained $x = 0.14$ and $p = 1.0 \text{ bar}$, we can calculate the corresponding relative humidity φ . From the Eq. (9) we get $p_v = 0.183 \text{ bar}$ and from the Table 2 $p'_v(82 \text{ °C}) = 0.5133 \text{ bar}$. Therefore, from the definition (35), $\varphi = p_v/p'_v(82 \text{ °C}) = 0.358 = 35.8 \%$. Notice that f and φ differ notably.

Let us write the Eq. (48) in a numerical form when $\text{Le} \cong 1$, $l(t_{wb}) \cong 2450 \text{ kJ/kg}$, and $c_v \cong 1.0 \text{ kJ/kg}^\circ\text{C}$. With the given values, the Eq. (48) can be written as¹¹

$$p_v = p'_v(t_{wb}) - 6.6 \cdot 10^{-4} \cdot p(t - t_{wb}) \frac{1}{^\circ\text{C}}, \quad (63)$$

from which the state of air can be quickly estimated. Usually the temperature of the air is referred to as dry temperature to clearly separate it from the wet bulb temperature of the air.

Finally we should emphasize that especially in process measurements, the radiation can have a significant effect on the wet bulb temperature. Therefore, the wet bulb temperature generally depends also on the measurement device and the way of measuring. If the air flow is really small, the radiation may have a major effect in addition to the convective heat transfer. In principle, for each wet bulb temperature measurement we can experimentally determine a formula analogous to the Eq. (63).

1.6 The changes of state of humid air

Let us study a balance boundary illustrated in the Fig. 11. The system can basically be any part of the air around process equipment where there are humid air flows. In air conditioning applications, the balance boundary can for example consist of the inside of a room or an office.

¹¹The Eq. (63) actually follows from the Eq. (47) and the above logarithm approximation (46) when we also notice that

$$6.6 \cdot 10^{-4} (t - t_{wb}) \frac{1}{^\circ\text{C}} \ll 1.$$

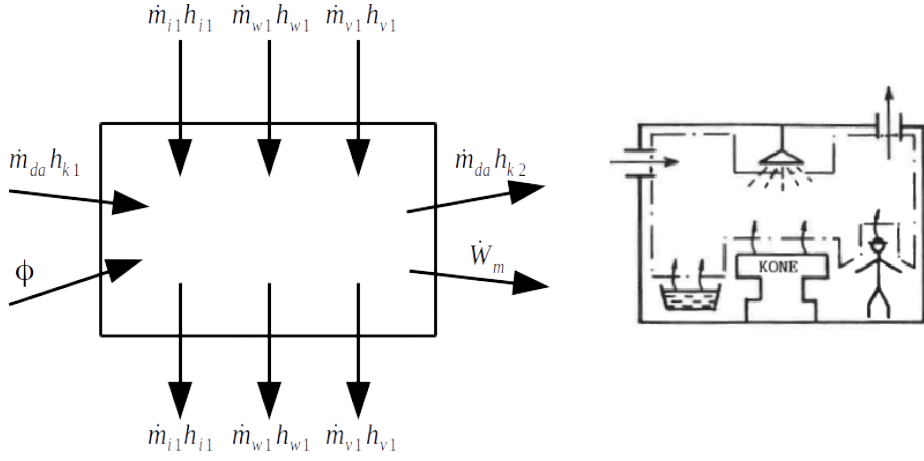


Figure 11: Energy and moisture balance boundaries.

In a stationary state, the energy balance of the area inside the balance boundary is

$$\begin{aligned} \phi - \dot{W}_m = & (\dot{m}_{da} h_{k2} - \dot{m}_{da} h_{k1}) + [(\dot{m}_{i2} h_{i2} - \dot{m}_{i1} h_{i1}) \\ & + (\dot{m}_{w2} h_{w2} - \dot{m}_{w1} h_{w1}) + (\dot{m}_{v2} h_{v2} - \dot{m}_{v1} h_{v1})], \end{aligned} \quad (64)$$

where ϕ is net thermal power going into the system, \dot{W}_m is the net work done by the system into the environment¹², \dot{m}_w is the water flow (1 = inflow, 2 = outflow), \dot{m}_i is the ice flow, and \dot{m}_v is the separate vapor flow not included in the air flows. The vapor flows within the humid air are $\dot{m}_{da} x_1$ and $\dot{m}_{da} x_2$ and they are taken into account in the terms h_{k1} and h_{k2} .

Similarly, the humidity or the water balance is

$$\dot{m}_{da} (x_2 - x_1) = (\dot{m}_{i1} - \dot{m}_{i2}) + (\dot{m}_{w1} - \dot{m}_{w2}) + (\dot{m}_{v1} - \dot{m}_{v2}). \quad (65)$$

In many cases, the inflow and outflow air can consist of multiple air flows in different states (temperature, moisture content) and these must be handled separately. This means that the enthalpy flows must also be separated accordingly.

¹²The expansion work is not included in this term, since it is already taken into account in the enthalpy terms.

Example 9. Mixing of two air flows.

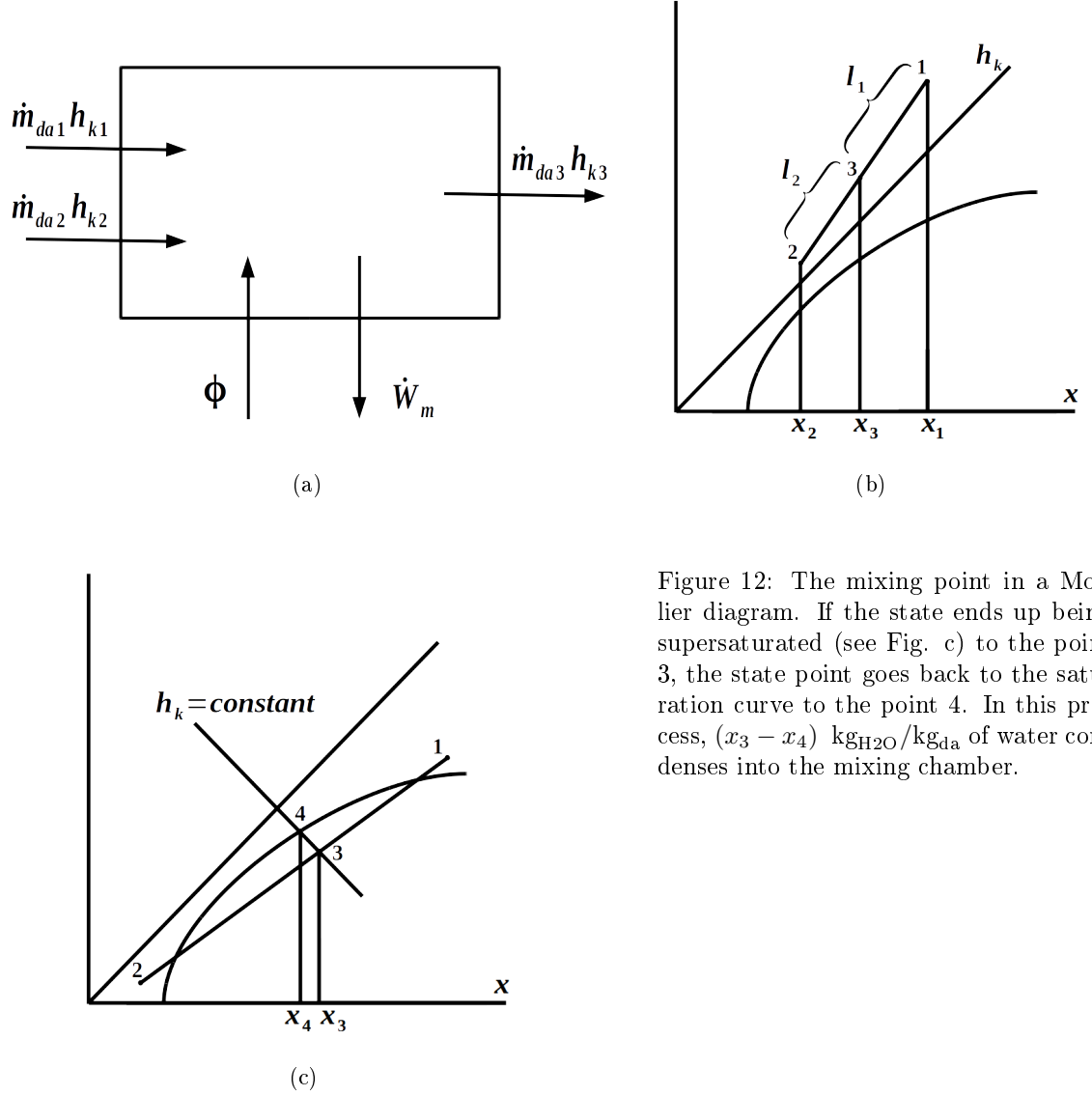


Figure 12: The mixing point in a Mollier diagram. If the state ends up being supersaturated (see Fig. c) to the point 3, the state point goes back to the saturation curve to the point 4. In this process, $(x_3 - x_4)$ kg_{H2O}/kg_{da} of water condenses into the mixing chamber.

In this case, the energy balance is (see Fig. 12a)

$$\dot{m}_{da1}h_{k1} + \dot{m}_{da2}h_{k2} = \dot{m}_{da3}h_{k3} \quad (66)$$

and the water balance is

$$\dot{m}_{da1}x_1 + \dot{m}_{da2}x_2 = \dot{m}_{da3}x_3. \quad (67)$$

For the dry air flows it holds that

$$\dot{m}_{da1} + \dot{m}_{da2} = \dot{m}_{da3}. \quad (68)$$

From the Eqs. (66)-(68) it follows that

$$\frac{h_{k3} - h_{k2}}{h_{k1} - h_{k3}} = \frac{x_3 - x_2}{x_1 - x_3} = \frac{\dot{m}_{da1}}{\dot{m}_{da2}}, \quad (69)$$

which shows that the mixing point 3 is on the line connecting the points 1 and 2, and that the position of the point 3 is obtained from the “lever rule” ($\dot{m}_{da1}l_1 = \dot{m}_{da2}l_2$, see Fig. 12b).

Example 10. Heating of an air flow. From the Eq. (64) follows that

$$\phi = \dot{m}_{da}(h_{k2} - h_{k1})$$

and from the Eq. (65) that

$$\dot{m}_{da}(x_2 - x_1) = 0$$

that is

$$x_2 = x_1.$$

During heating, the state of the air moves along a line of a constant moisture content.

Example 11. Cooling of an air flow. From the energy balance (64)

$$\phi = \dot{m}_{da}(h_{k2} - h_{k1}) + \dot{m}_{w2}h_{w2},$$

where $\phi < 0$. During the cooling, also water can condense and therefore the term $\dot{m}_{w2}h_{w2}$ is present in the above equation. Whether water condenses or not depends on the surface temperature of the cooling radiator. From the water balance (65)

$$\dot{m}_{da}(x_2 - x_1) = -\dot{m}_{w2}$$

so the final moisture content is less or equal the initial one $x_2 \leq x_1$. The cooling of an air flow is illustrated in Fig. 13.

When an air flow encounters a surface with a temperature below the dew point, water condenses on the surface. If all the air contacts the cold surface, the state of the air is at point 3 after the cooling. In practice, part of the air does not contact the cold surface and thus the air after the cooling radiator is a mixture of saturated air (3) and the air going past the cooling radiator (1). As shown in the Example 9, the mixing point 2 is on the line connecting the points 1 and 3. The closer the point 2 is to the point 3, the more effective the cooling radiator.

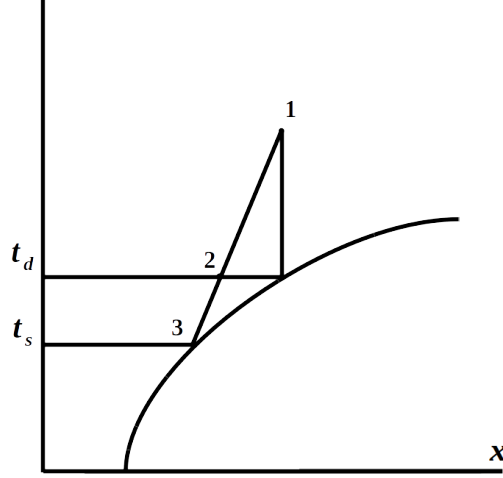


Figure 13: The change of the state of air in a cooling radiator. If the surface temperature t_s is below the dew point t_d , water condenses during cooling. If $t_s > t_d$, the cooling happens along the line of a constant moisture content $x_1 = x_2$.

Example 12. Adding vapor into air. From the Eqs. (64) and (65) follows that

$$\dot{m}_v h_v = \dot{m}_{da} (h_{k2} - h_{k1})$$

and

$$\dot{m}_v = \dot{m}_{da} (x_2 - x_1),$$

where \dot{m}_v is the added vapor flow and h_v its enthalpy. Combining the above equations yields

$$\frac{h_{k2} - h_{k1}}{x_2 - x_1} = h_v. \quad (70)$$

In differential form, the Eq. (70) is

$$\frac{dh_k}{dx} = h_v. \quad (71)$$

On the other hand, by differentiating Eq. (15b) with respect to t and x we get

$$dh_k = (c_{pda} + xc_{pv}) dt + (tc_{ph} + l_{ho}) dx,$$

which together with the Eq. (71) gives

$$\frac{dt}{dx} = \frac{h_v - (l_{ho} - tc_{pv})}{c_{pda} + xc_{pv}} = \frac{h_h - h_h(t)}{c_{pda} + xc_{pv}}, \quad (72)$$

where $h_h(t) = l_{ho} + c_{pv}t$ is the enthalpy of the vapor at a temperature of t .

We can see from the Eq. (72) that if the temperature of the vapor added into the air is lower than the air temperature, the air cools down $dt/dx < 0$. If the temperature of vapor is greater than the temperature of the air, the air warms up $dt/dx > 0$.

Example 13. The temperature of a room needs to stay at 20 °C and the relative humidity at $\varphi = 50$ %. The total thermal power of the room is 2.45 kW and the total vapor flow is $1.53 \cdot 10^{-3}$ kg/s. What needs to be the state (temperature and moisture content) of the inlet air when the inlet air flow is a) $\dot{m}_{da} = 0.3$ kg/s and b) $\dot{m}_{da} = 0.6$ kg/s?

$$p'_v(20^\circ\text{C}) = 0.02337 \text{ bar} \quad p_v = 0.5 \cdot 0.02337 \text{ bar} = 0.01169 \text{ bar}$$

Thus the moisture content in the room is

$$x = 0.6220 \frac{0.01169}{1.0 - 0.01169} = 0.00736.$$

When the inlet air mixes into the room air properly, the moisture content and the temperature of the outlet air are the same as the temperature $t_2 = 20^\circ\text{C}$ and moisture content $x_2 = 0.00736$ of the room. The enthalpy of the outlet air is then

$$h_{k2} = 1.006 \cdot 20 + 0.00736 (2501 + 1.85 \cdot 20) = 38.8 \text{ kJ/kg}.$$

The enthalpy of the inlet air h_{k1} and its moisture content x_1 are determined from the energy (64) and the water balances (65)

$$\begin{aligned} \dot{m}_{da} (h_{k2} - h_{k1}) &= \phi + \dot{m}_v h_v \\ \dot{m}_{da} (x_2 - x_1) &= \dot{m}_v. \end{aligned}$$

The total thermal power means that it also takes into account the enthalpy flow $\dot{m}_v h_v$. Thus

$$\phi + \dot{m}_v h_v = 2.45 \text{ kW}$$

and therefore

a)

$$\begin{aligned} h_{k2} - h_{k1} &= \frac{2.45}{0.3} \text{ kJ/kg} = 8.2 \text{ kJ/kg} \\ h_{k1} &= (38.8 - 8.2) \text{ kJ/kg} = 30.6 \text{ kJ/kg} \end{aligned}$$

b)

$$\begin{aligned} h_{k2} - h_{k1} &= \frac{2.45}{0.6} \text{ kJ/kg} = 4.1 \text{ kJ/kg} \\ h_{k1} &= (38.8 - 4.1) \text{ kJ/kg} = 34.7 \text{ kJ/kg}. \end{aligned}$$

The total vapor flow is $\dot{m}_v = 1.53 \cdot 10^{-3}$ kg/s and thus

a)

$$\begin{aligned} x_2 - x_1 &= \frac{1.53 \cdot 10^{-3}}{0.3} = 0.0051 \\ x_1 &= 0.00736 - 0.0051 = \underline{0.00226} \end{aligned}$$

b)

$$x_2 - x_1 = \frac{1.53 \cdot 10^{-3}}{0.6} = 0.00255$$

$$\underline{x_1} = 0.00736 - 0.00255 = \underline{0.00481}.$$

The corresponding temperatures are obtained from the following equation

$$t = \frac{h_k - xl_{ho}}{c_{pda} + xc_{pv}}.$$

a) $x = 0.00226$ and $h_k = 30.6$ kJ/kg

$$t = \frac{30.6 - 0.00226 \cdot 2501}{1.006 + 0.00226 \cdot 1.85} = 24.4^\circ\text{C}$$

b) $x = 0.00481$ and $h_k = 34.7$ kJ/kg

$$t = \frac{34.7 - 0.00481 \cdot 2501}{1.006 + 0.00481 \cdot 1.85} = 22.2^\circ\text{C}.$$

The results are illustrated in the Fig. 14.

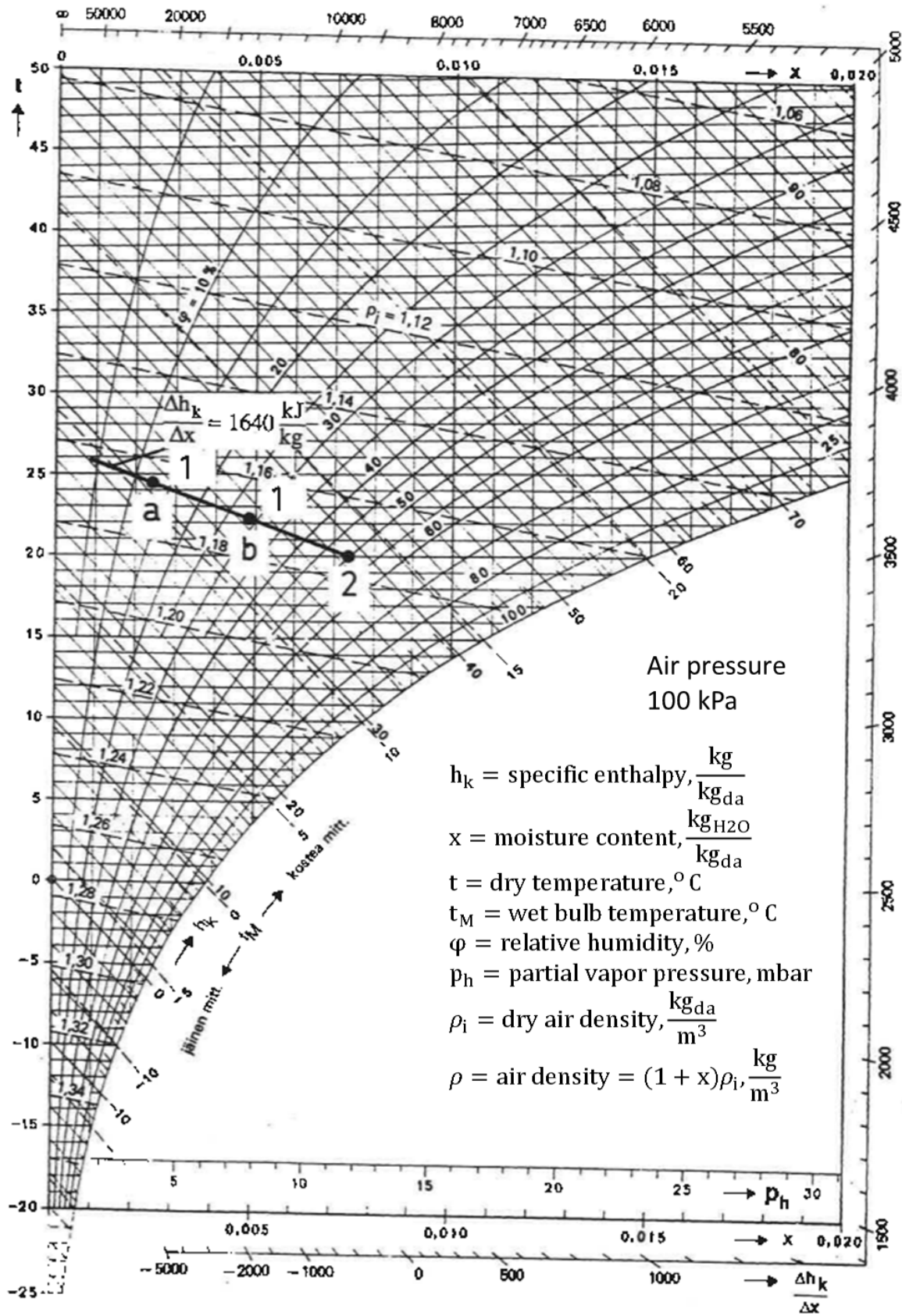


Figure 14: The state line of the inlet air. a and b denote the two different cases for the final state. Note that the notation in this figure does not completely coincide with the text. The used symbols are explained in the above figure.

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