Problem 14.1

Problem 14.1.2

Assuming priors of the form: $\pi \sim beta(\alpha_\psi, \beta_\pi), S \sim beta(\alpha_S, \beta_S)$ and $C \sim beta(\alpha_C, \beta_C)$, it is possible to code up a Gibbs sampler for this problem [8] of the form

$$Y_1|a, \pi, S, C \sim \mathcal{B}\left(a, \frac{\pi S}{\pi S + (1-\pi)(1-C)}\right) \tag{14.3}$$

$$Y_2|b, \pi, S, C \sim \mathcal{B}\left(b, \frac{\pi(1-S)}{\pi(1-S) + (1-\pi)C}\right) \tag{14.4}$$

$$\pi|a, b, Y_1, Y_2 \sim beta\left(Y_1 + Y_2 + \alpha_\pi, a + b - Y_1 - Y_2 + \beta_\pi\right) \tag{14.5}$$

$$S|Y_1, Y_2 \sim beta\left(Y_1 + \alpha_S, Y_2 + \beta_S\right) \tag{14.6}$$

$$C|a, b, Y_1, Y_2 \sim beta\left(b - Y_2 + \alpha_C, a - Y_1 + \beta_C\right) \tag{14.7}$$

Figure 1: equations

Priors

```
# Priors
alpha_pi = 1
beta_pi = 1
alpha_S = 1
beta_S = 1
alpha_C = 1
beta_C = 1
```

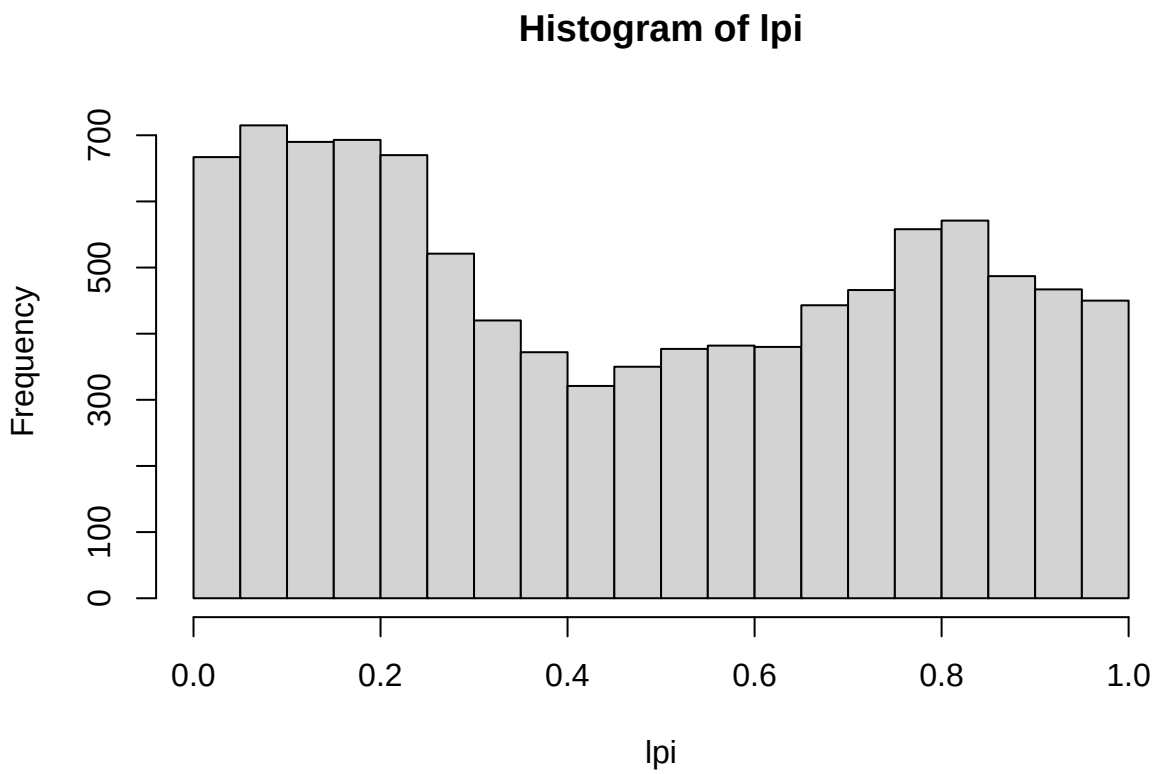
```
fGibbsSampling <- function(numSamples,a, b){

  Y1[1] <- as.integer(runif(1, 1, a))

  Y2[1] <- as.integer(runif(1, a, b+a))

  lpi[1] <- runif(1, 0, 1)

  S[1] <- runif(1, 0, 1)

  C[1] <- runif(1, 0, 1)

  # cat("\nBefore Loop: ")
  # cat("\nY1:",Y1, " Y2: ", Y1, " lp: ", lpi, " S: ", S, " C: ", C)
  for(t in 2:numSamples){

    Y1[t] <- rbinom(n = 1, size = a, prob =  lpi[t-1] * S[t-1] /
```

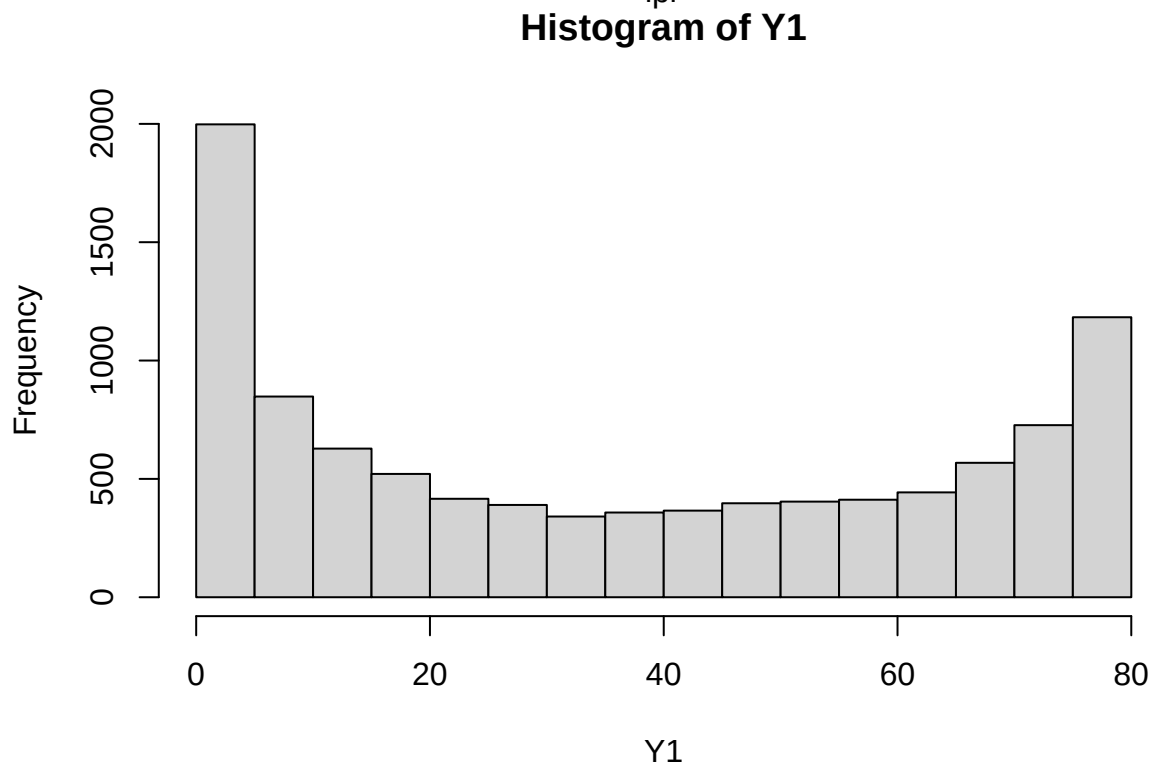
```
                    (lpi[t-1] * S[t-1] + (1 - lpi[t-1]) * (1 - C[t-1]) ))

    Y2[t] <- rbinom(n=1, size = b, prob = lpi[t-1] * (1 - S[t-1]) /
                    (lpi[t-1] * (1 - S[t-1]) + (1 - lpi[t-1]) * C[t-1]))

    lpi[t] <- rbeta(1, Y1[t] + Y2[t] + alpha_pi, a + b - Y1[t] -
                    Y2[t] + beta_pi)

    S[t] <- rbeta(1, Y1[t] + alpha_S, Y2[t] + beta_S)

    C[t] <- rbeta(1, b - Y2[t] + alpha_C, a - Y1[t] + beta_C)
    # cat("\n\nAfter t=", t)
    # cat("\nY1:",Y1, " Y2: ", Y1, " lp: ", lpi, " S: ", S, " C: ", C)

  }

  hist(lpi)
  hist(Y1)
  hist(Y2)
  hist(S)
  hist(C)

}
```

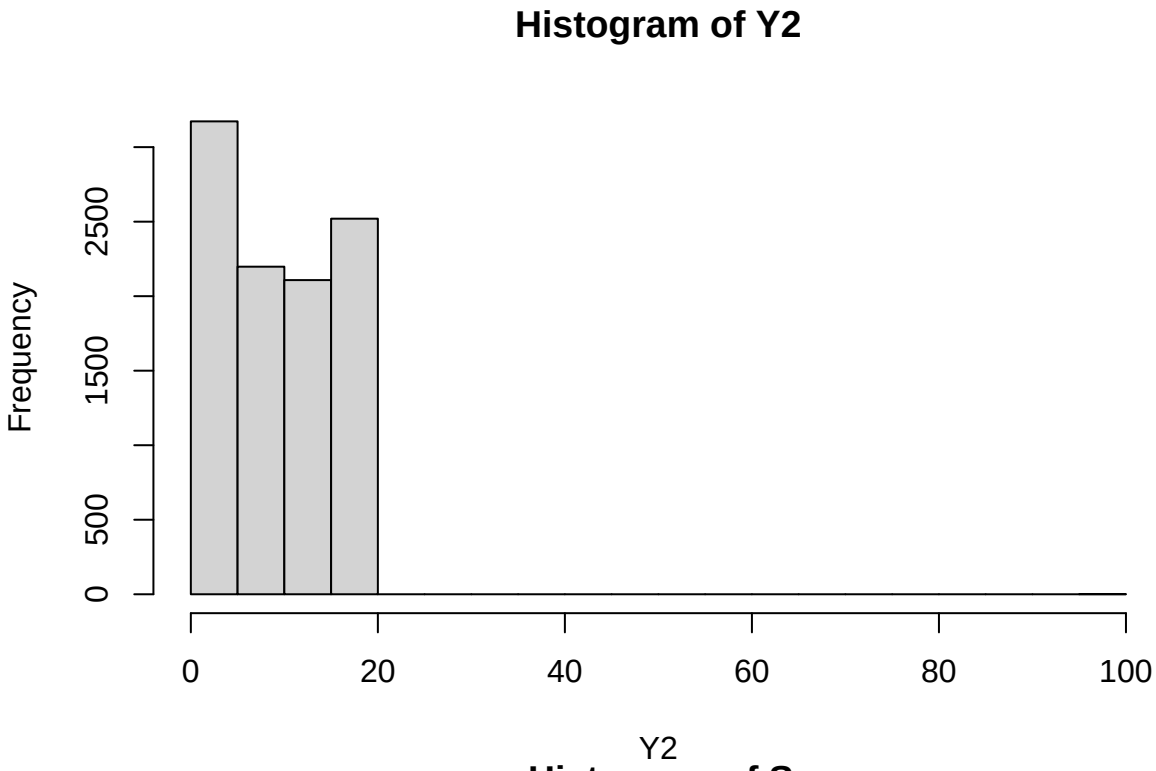
Problem 14.1.3

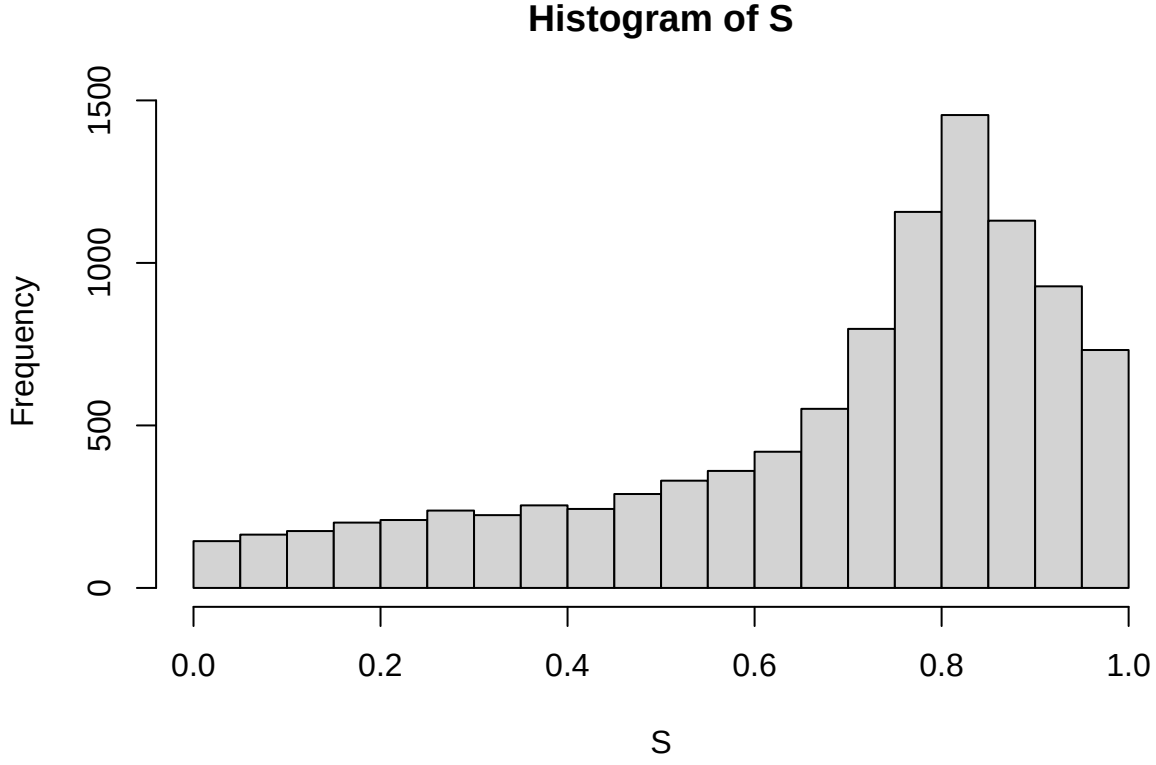
Suppose that out of a sample of 100 people, 20 of those tested negative and 80 positive. Assuming uniform priors on , S and C, use Gibbs sampling to generate posterior samples for . What do you conclude?

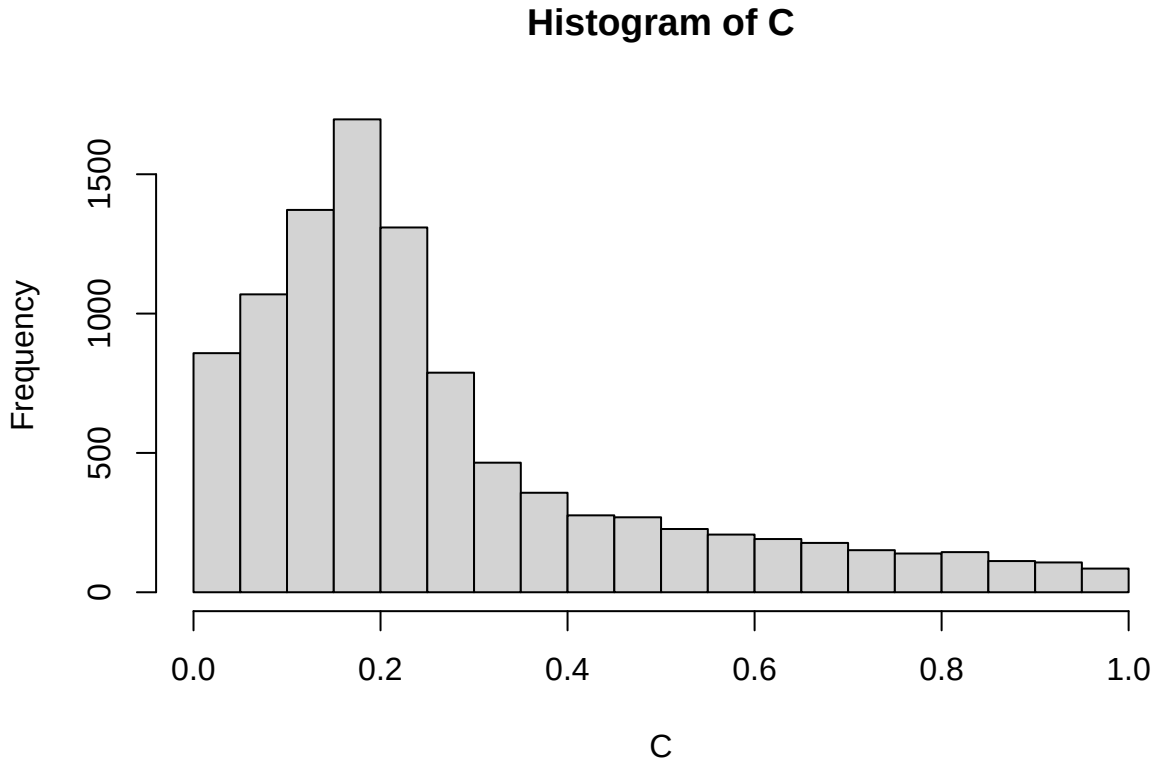
```
set.seed(41)
Y1 <- vector()
Y2 <- vector()
lpi <- vector()
S <- vector()
C <- vector()

s_lpi <- fGibbsSampling(10000, 80, 20)
```


Histogram of lpi
Frequency
0
100
300
500
700
0.0
0.2
0.4
0.6
0.8
1.0
lpi


Histogram of Y1
Frequency
0
500
1000
1500
2000
0
20
40
60
80
Y1


Histogram of Y2
Frequency
0
500
1500
2500
0
20
40
60
80
100
Y2


Histogram of S
Frequency
0
500
1000
1500
0.0
0.2
0.4
0.6
0.8
1.0
S


Histogram of C
Frequency
0
500
1000
1500
0.0
0.2
0.4
0.6
0.8
1.0
C