Kinematic Analysis of the 4 Bar Recovery A-Frame
Exercise - 3, Modelling and Analysis Lab(Updated)

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In Exercise-1 of the course, displacement, velocity and energy analysis was done of the SpaceX Crew Dragon Endeavour, considering it as a simple pendulum. The next section of the course deals with the Kinematic and Dynamic Analysis of the Recovery A Frame installed in the Recovery ship, GO Navigator. This report is focused on the Kinematic Analysis of the A-frame, and the results obtained from this would be used for the Dynamic Analysis in the coming week.

I. AIM

The Fig. 1 shows the Recovery Crane in the A-Frame. The aim of this exercise is to dig into the basic design parameters of the crane, the dimensions and hence do a kinematic analysis and solving for the angular displacements, velocities and accelerations for the various links of the 4- Bar Mechanism.

FIG. 1. The A-Frame as seen from Actual Recovery images

II. INTRODUCTION

The Recovery crane from Fig. 1 is simplified for the analysis as shown in Fig. 2. It is further simplified into 2 separate mechanisms, as shown in Fig. 3 and Fig. 4. From the actual model of the recovery crane (Fig. 1), it is observed that the angle $\theta_2 = \theta_4$ and $\theta_3 = 0$ (angles as labelled in Fig. 3). These conditions would be used as per requirement and we have tried to develop the analysis without using these conditions.

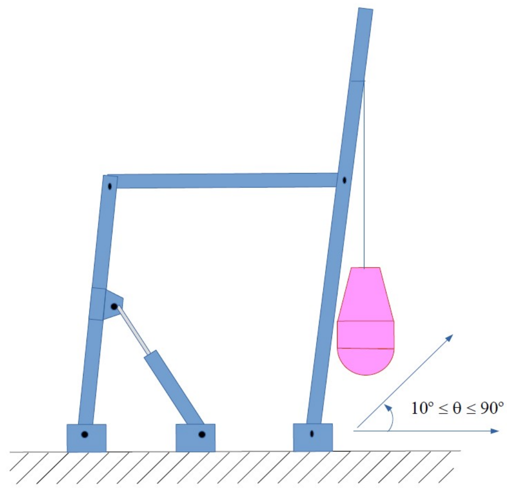

FIG. 2. The Side View of the Designed Recovery A-Frame

III. THE 4-BAR MECHANISM POSITION ANALYSIS

Let's begin with the Loop-Closure equation for the 4-Bar Mechanism as shown in Fig. 3.

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{0} \tag{1}$$

Resolving the cos and sin components from 1 and writing it with appropriate sign according to the Fig.3, we get:

$$l_2 \cos\theta_2 + l_3 \cos\theta_3 = l_4 \cos\theta_4 + l_1 \tag{2}$$

and,

$$l_2 \sin\theta_2 + l_3 \sin\theta_3 = l_4 \sin\theta_4 \tag{3}$$

We will now define the constants, variables and input parameters here. The definitions of lengths and angles as done in Table IV.1 is the most general definition possible. However, it would be very difficult practically to give 2 input parameters and control the system. Obviously it is not as difficult as a space-docking

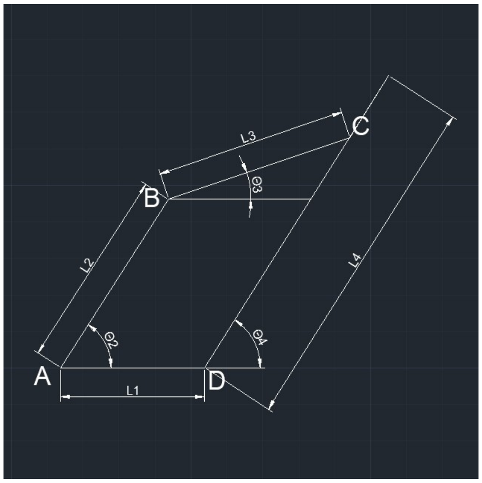

FIG. 3. Recovery Crane's Mechanism reduced to a 4-Bar Mechanism

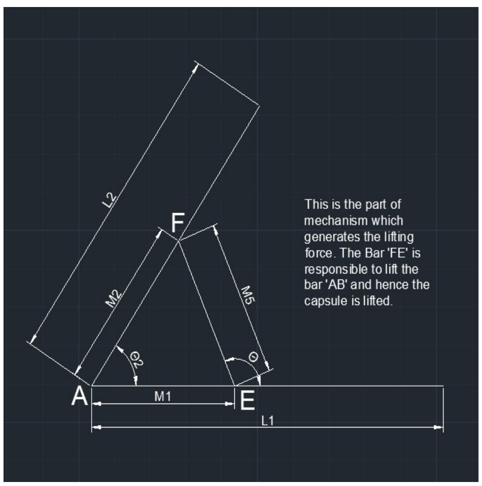

FIG. 4. The Designed Lifting Mechanism in the Recovery Crane

Variables	θ_3, l_4
Constants	l_1, l_2, l_3
Input Parameters	θ_2, θ_4

TABLE I. The Variables, Constants and Input Parameters according to the scenario

mission and a recovery process without getting the life of astronauts in risk, but bringing in 2 inputs would just make the system more complex. We shall give 2 inputs to the system only in case if it helps reduce the energy requirement or provides better stability to the system.
For, now let us fix to one input only. Let this input parameter be θ_2. And let the value of the input θ_4 be taken same as that of θ_2 at every instant.

The good thing about this fixing of θ_4 to θ_2 is that it should not an additional motor or an active system. Our intuition says that the angle θ_4 can be maintained equal to θ_2 just by mechanical parts which binds their motion together. More detailed analysis on this shall be carried out in the next report on Dynamic Analysis of the system.

Thus, for the model, we finally have an input parameter θ_2, whose value at any instant will help determine the values of unknown quantities or variables, θ_3 and l_4. And, the lengths of link 1, 2 and 3, ie. l_1, l_2, and l_3 respectively are constants. Substituting θ_4 as θ_2 in the equations, 2 and 3, they transform into:

$$l_2 \cos\theta_2 + l_3 \cos\theta_3 = l_4 \cos\theta_2 + l_1 \tag{4}$$

and,

$$l_2 \sin\theta_2 + l_3 \sin\theta_3 = l_4 \sin\theta_2 \tag{5}$$

Rearranging the equation 5 we get,

$$l_4 = l_2 + l_3 \frac{\sin\theta_3}{\sin\theta_2} \tag{6}$$

Hence, on substituting l_4 in equation 4 we get,

$$\sin(\theta_2 - \theta_3) = \frac{l_1}{l_3} \times \sin\theta_2$$

Hence, θ_3 can be expressed as,

$$\theta_3 = \theta_2 - (\arcsin\frac{l_1}{l_3} \times \sin\theta_2) \tag{7}$$

And, substituting θ_3 in equation 4 or 5 we get l_4. l_4 can be thus expressed as:

$$l_4 = l_2 + \frac{\sin\left(\theta_2 - (\arcsin\frac{l_1}{l_3} \times \sin\theta_2)\right)}{\sin\theta_2} \times l_3 \tag{8}$$

Hence, from equations 7 and 8, θ_3 and l_4 can be computed at any instant, provided we know the input parameter, that is θ_2 at that instant.
From the obtained expressions of θ_3 and l_4 we get a trivial solution as well. For,

$$\theta_3 = 0$$

We get,

$$l_1 = l_3 \tag{9}$$

And,

$$l_2 = l_4 \tag{10}$$

This is the case, which actually appears in the actual recovery crane, as seen in Fig. 1. In this case, the link 3, or the supporting link between link 2 and link 4 remains perfectly horizontal. In this case, we get a constraint on our selected constant link lengths, and the variable θ_3 vanishes as it's value is always zero.

IV. POSITION ANALYSIS, CONSIDERING THE EXTENDED MECHANISM

In section III, we solved for the positions of θ_3 and l_4 using θ_2 as the input parameter which was varied. 3 Test cases were performed taking different inputs of θ_2 and observing the variation in θ_3 and l_4.

Now, we will proceed with an analysis similar to the actual scenario. The basic idea of this analysis is what is seen in fig. 4. In the actual mechanism (as seen in fig. 1), θ_2 is varied with the help of some motor. The bar 'FE' is what is responsible to provide energy to the system and increase θ_2 to lift the module.
Thus, to increase θ_2, we can either change the length m_5 or the angle θ. So, for this analysis, we will assume, $_5$ or θ as the input parameter (one in each case, the other one will be treated as a constant), and θ_2 will be solved as a variable. Once we get θ_2, we will get θ_3 and l_4 the similar way (as we got in section III).

In the case of Variable angle θ, there will be a motor providing torque about the point E and hence adjusting the angle θ to increase θ_2.
And, for the case of variable bar length, there will be a piston cylinder arrangement, where the length is changed by expansion of a gas.

IV.1. Deriving Equations

$$\overrightarrow{AF} + \overrightarrow{FE} + \overrightarrow{EA} = \overrightarrow{0} \tag{11}$$

Resolving the cos and sin components from 11 and writing it with appropriate sign according to the Fig.4, we get:

$$m_2 \cos\theta_2 + = m_5 \cos\theta + m_1 \tag{12}$$

and,

$$m_2 \sin\theta_2 = m_5 \sin\theta \tag{13}$$

We will now define the constants, variables and input parameters here.

Variables	θ_2, m_2
Constants	m_5, m_1
Input Parameters	θ

TABLE II. The Variables, Constants and Input Parameters for Case of Variable Angle of Bar EF (θ)

IV.2. Position Analysis for some Sample Cases

Note: The values of 'k' and 'ϕ' are assumed arbitrarily.

Variables	θ_2, m_2
Constants	θ, m_1
Input Parameters	m_5

TABLE III. The Variables, Constants and Input Parameters for Case of Variable Length of Bar EF (m_5)

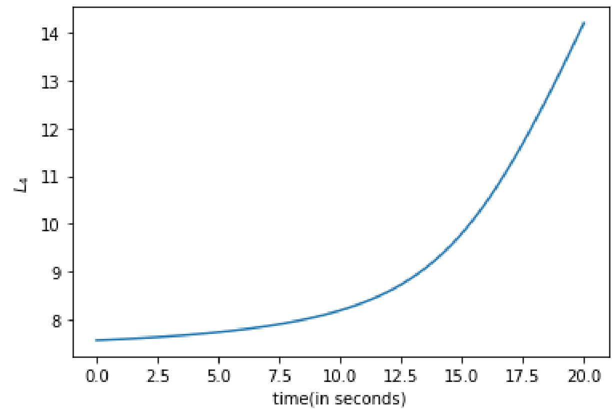

FIG. 5. Length of 4th link(l_4) for linear variation of θ_2 v/s time(minutes)(for $\theta_2 = kt + \phi$)

IV.2.1. Case-1

For a simple case, considering the variation of θ due to motor such that:

$$\theta_2 = kt + \phi \tag{14}$$

Here $\phi = 0.523599rad$ and k=0.067. The plot we got of the variation of θ_3 is shown. Also, we got the variation in length of 4th link w.r.t time.

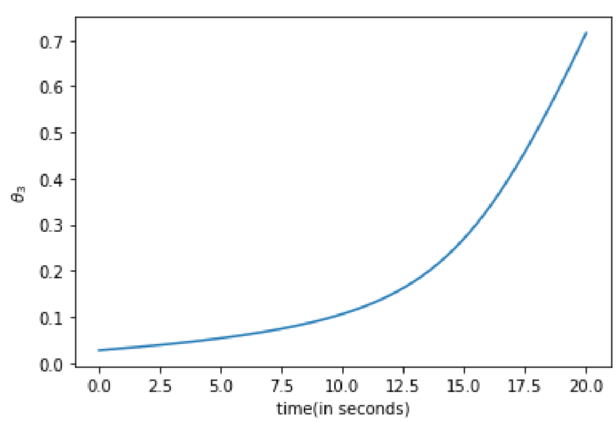

FIG. 6. θ_3 for linear variation of θ_2 v/s time(minutes)(for $\theta_2 = kt + \phi$)

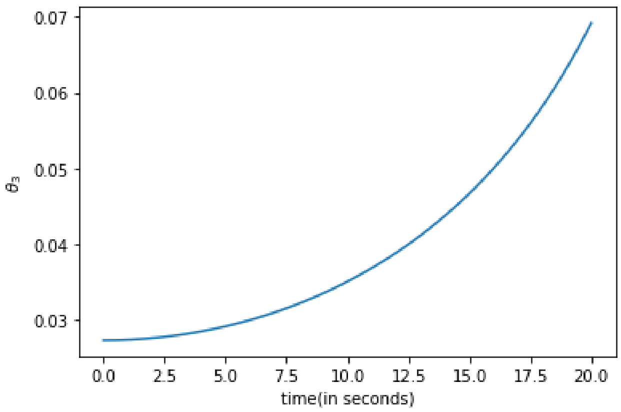

FIG. 7. θ_3 for quadratic variation of θ_2 with time(minutes)(for $\theta_2 = kt^2 + \phi$)

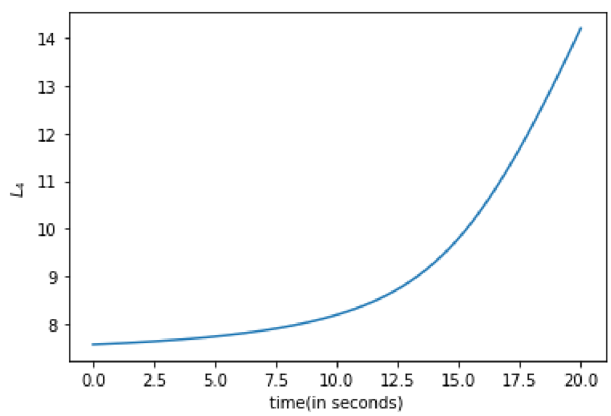

FIG. 8. Length of 4th link(l_4) for linear variation of θ_2 v/s time(minutes)(for $\theta_2 = kt + \phi$)

IV.2.2. Case-2

Now, taking another case where θ varies w.r.t such as-

$$\theta_2 = kt^2 + \phi \tag{15}$$

Here $\phi = 0.523599rad$ and k=0.0011693706. Similar to the previous case, we got plots of Angular displacement and 4th link length.

IV.2.3. Case-3

We took another case for the variation of θ such that-

$$\theta = kt^3 + \phi \tag{16}$$

Here $\phi = 0.523599rad$ and k=3.87463094e-5.

Thus, the position analysis of the Recovery A- Frame as a 4-Bar Mechanism is completed.

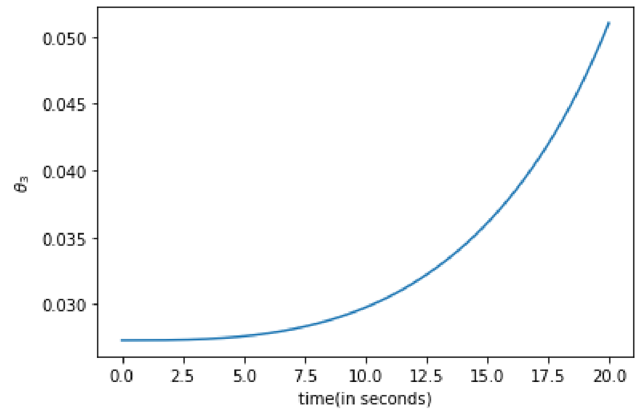

FIG. 9. θ_3 for cubic variation of θ v/s time(minutes)(for $\theta_2 = kt^3 + \phi$)

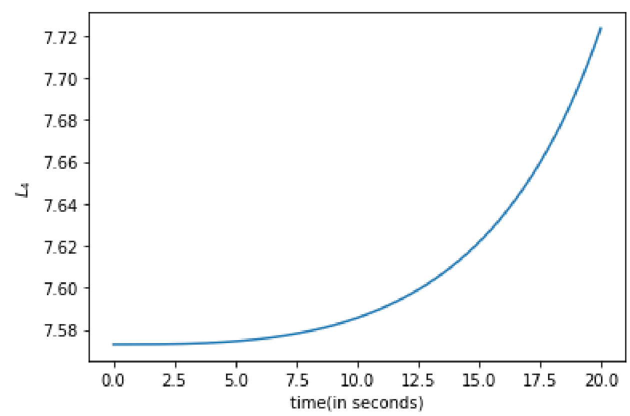

FIG. 10. Length of 4th link(L_4) for cubic variation of θ v/s time(minutes)(for $\theta_2 = kt^3 + \phi$)

V. THE 4-BAR MECHANISM VELOCITY ANALYSIS

For the velocity analysis, we begin by differentiating equations 4 and 5 with respect to time. Thus, we get:

$$-l_2 \sin\theta_2\dot{\theta}_2 - l_3 \sin\theta_3\dot{\theta}_3 = \dot{l}_4 \cos\theta_2 - l_4 \sin\theta_2\dot{\theta}_2 \tag{17}$$

And,

$$l_2 \cos\theta_2\dot{\theta}_2 + l_3 \cos\theta_3\dot{\theta}_3 = \dot{l}_4 \sin\theta_2 + l_4 \cos\theta_2\dot{\theta}_2 \tag{18}$$

Rearranging terms of equation 17, we get,

$$\dot{l}_4 = \frac{(l_4 - l_2)\sin\theta_2(\dot{\theta}_2) - l_3 \sin\theta_3(\dot{\theta}_3)}{\cos\theta} \tag{19}$$

Now substituting $\dot{l}_4$ in equation 17, we get:

$$\dot{\theta}_3 = -\frac{(l_4 - l_2)(\cos 2\theta_2)\cdot\dot{\theta}2}{l_3 \cos(\theta_3 - \theta_2)}$$

(20)

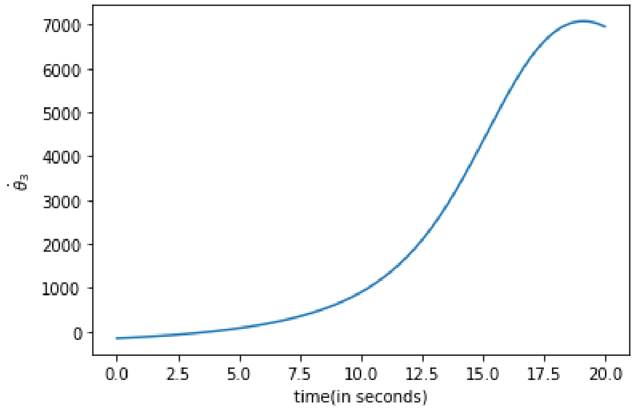

FIG. 11. Angular Velocity($\frac{d\theta_3}{dt}$) v/s time(minutes)(for $\theta_2 = kt + \phi$)

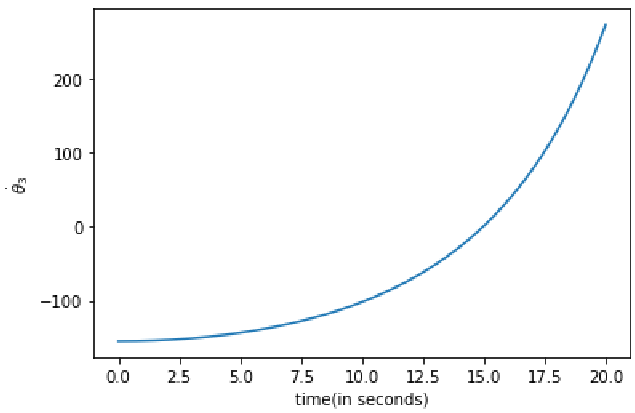

FIG. 12. Angular Velocity($\frac{d\theta_3}{dt}$) v/s time(minutes)(for $\theta_2 = kt^2 + \phi$)

Where, l_4 and θ_3 can be substituted from what we have obtained in equations 8 and 7.

V.1. Velocity Analysis for some Sample Cases

Using the above equations we computed the corresponding angular velocities and rate of change of length of 4th link. For the first case of $\theta_2 = kt + \phi$ we got the plots as shown in FIG.19 and 20. Similarly for the next case of-

$$\theta_2 = kt^2 \tag{21}$$

We got the plots as shown in FIG.21 and 22.

Considering the next case as we discussed-

$$\theta_2 = kt^3 \tag{22}$$

We got the plots as shown in FIG.23 and 24.

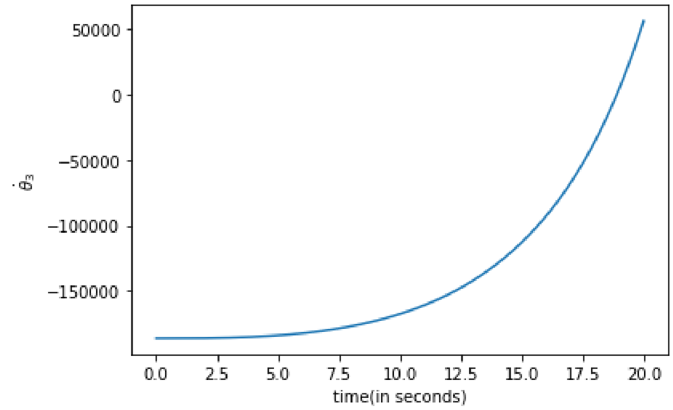

FIG. 13. Angular Velocity($\frac{d\theta_3}{dt}$) v/s time(minutes)(for $\theta_2 = kt^2 + \phi$)

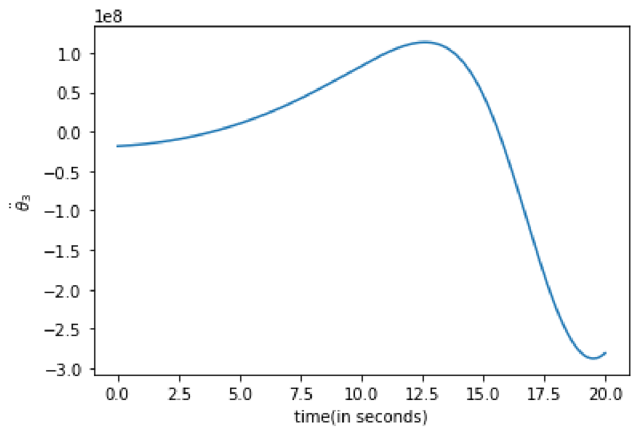

FIG. 14. Angular acceleration($\frac{d2\theta_3}{dt^2}$) v/s time(minutes)(for $\theta_2 = kt + \phi$)

VI. ACCELERATION ANALYSIS AND SOME SAMPLE CASES

The value of Acceleration found by double differentiation θ_3 w.r.t time came out as follows-

$$\frac{d^2\theta_3}{dt^2} = (l_2 - l_4)\cos 2\theta_2 l_3 \frac{d\theta_2}{dt}\sin\theta_3 - \theta_2\left(\frac{d\theta_3}{dt} - \frac{d\theta_2}{dt}\right) \tag{23}$$

Also we got another equation to find $\frac{d^2L_4}{dt^2}$ which is given as follows-

Eqn 11

$\frac{d^2L_4}{dt^2} sin\theta_1 + \frac{dL_4}{dt} cos\theta_1\dot{\theta}_1 - L_2 sin(\dot{\theta}_1)^2 + L_1 cos\theta_1\ddot{\theta}_1$
$+ bsin\theta_2(\dot{\theta}_2)^2 - bcos\theta_2\ddot{\theta}_2 = 0$

For the first case of $\theta_2 = kt$ we got the plot of acceleration as shown in FIG. 31. Now, considering another simple case of $\theta_2 = kt^2$. we got the plot of acceleration as shown in FIG. 32. Now, considering another simple

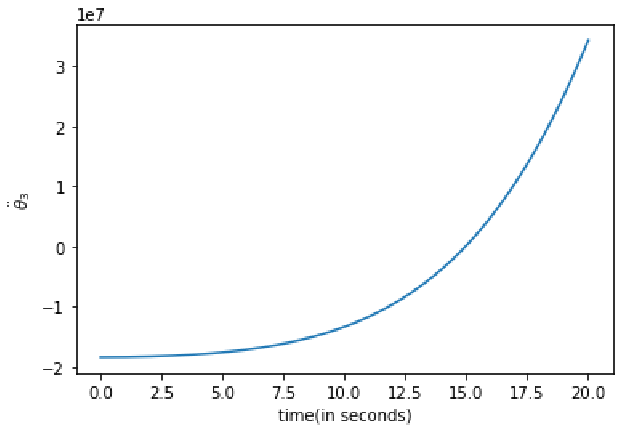

FIG. 15. Angular acceleration($\frac{d2\theta_3}{dt^2}$) v/s time(minutes)(for $\theta_2 = kt^2 + \phi$)

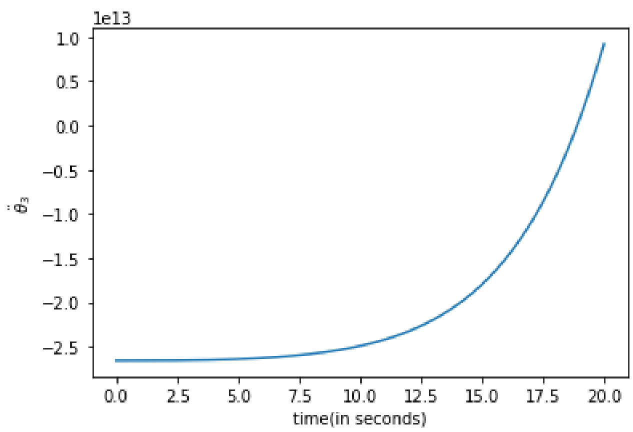

FIG. 16. Angular acceleration($\frac{d2\theta_3}{dt^2}$) v/s time(minutes)(for $\theta_2 = kt^3 + \phi$)

case of $\theta_2 = kt^3$. we got the plot of acceleration as shown in FIG. 33.

VII. CONCLUSION

The results obtained in this exercise will be very useful for the Dynamic Analysis. For the case when θ_3 is maintained at 0 degrees, the study will become simple. Thus, the Dynamic Analysis will focus on to find external torques or forces to maintain the angle θ_3.