Exercise 1: Modelling and Analysis Lab

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This report aims to solve and simulate Ordinary Differential Equations using Python. Libraries such as 'numpy' and 'scipy' were used in solving the differential equations. The simple pendulum problem was analysed taking different assumptions and different conditions and the angular displacements were plotted using 'matplotlib.pyplot' library.

The frame of the Dragon module was considered as a simple pendulum system while keeping the following assumptions common in each case:

1. The module is considered as point mass and the length of string is measured up to the center of mass of the module.
2. The string is considered as unstretchable.
3. Initially, the module is at highest angular displacement.
4. Also the angular displacement is not small which is the case in simple pendulums.

CASE 1

- In this simple case, string's length is kept constant i.e, 8.5 m and any damping is ignored.
- The intial condition is:

$$\theta = 0.0821 radians \tag{1}$$

- The differential equation for the case can be found by balancing the net torque experienced by the point mass.

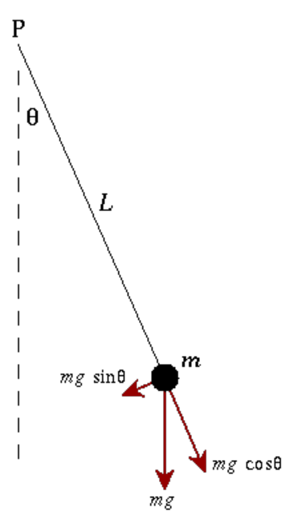

FIG. 1. A Simple Pendulum

- The differential equation for this case comes out as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \tag{2}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 2.

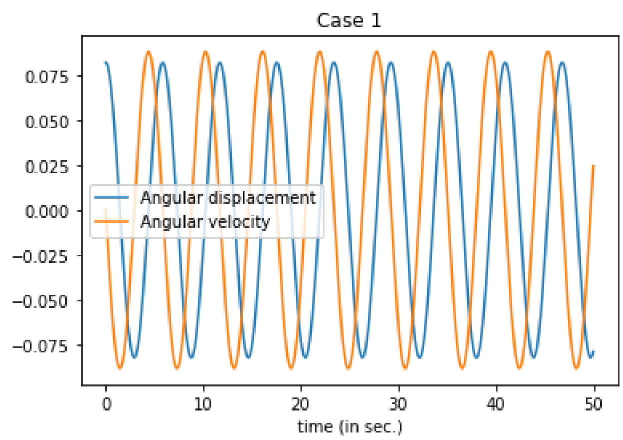

FIG. 2. Angular Displacement and Velocity as a function of time

CASE 2

- In this case, string's length is varying linearly w.r.t time any damping is ignored.

$$L(t) = 8.5 - 0.0857t \tag{3}$$

- The intial condition is:

$$\theta = 0.0821 radians \tag{4}$$

- The differential equation for this case comes out as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L(t)}\sin\theta = 0 \tag{5}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 3.

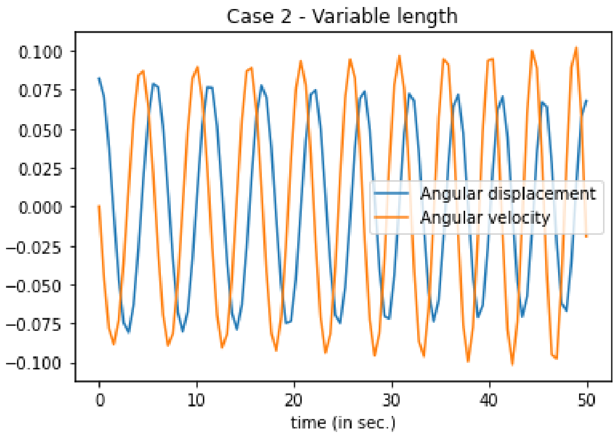

FIG. 3. Angular Displacement and Velocity as a function of time

CASE 3

- In this case, Air damping is considered as:

$$F_damp = -6.74 * 10^{-4} L \frac{d\theta}{dt} \tag{6}$$

- The intial condition is:

$$\theta = 0.0821 radians \tag{7}$$

- The length of string is constant in this case.

$$L = 8.5m \tag{8}$$

- The differential equation for this case comes out as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta + \frac{6.74 * 10^{-4}}{mL}\frac{d\theta}{dt} = 0 \tag{9}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 4.

CASE 4

- In this case, Air damping is ignored.
- The angular displacement is taken large is this case.
- The intial condition is:

$$\theta = 3.14 radians \tag{10}$$

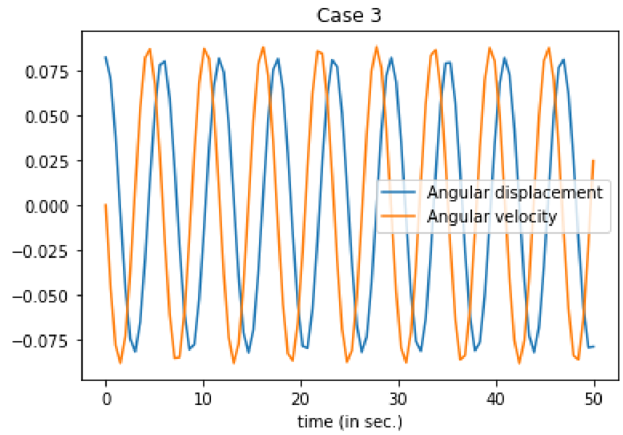

FIG. 4. Angular Displacement and Velocity as a function of time

- The length of string is constant in this case.

$$L = 8.5m \tag{11}$$

- The differential equation for this case comes out as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \tag{12}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 5.

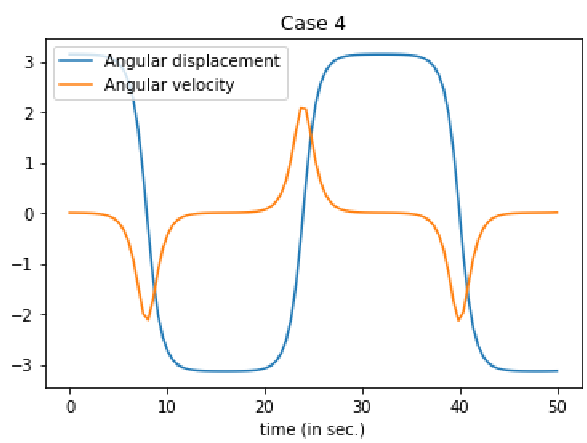

FIG. 5. Angular Displacement and Velocity as a function of time

CASE 5

- In this case, Air damping is ignored.
- Now, initial angular displacement is increased just above the last case.

- The intial condition is:

$$\theta = 3.15 radians \tag{13}$$

- The length of string is constant in this case.

$$L = 8.5m \tag{14}$$

- The differential equation for this case comes out as:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \tag{15}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 6.

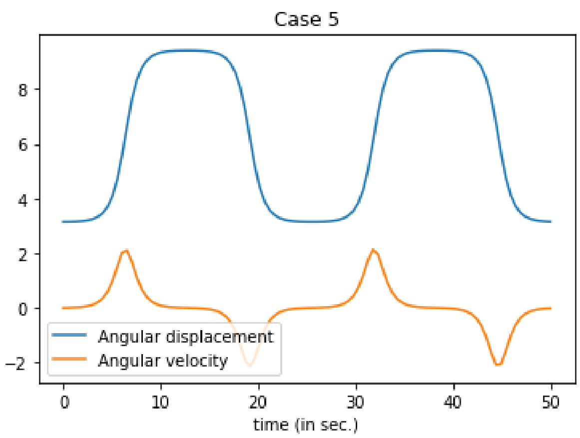

FIG. 6. Angular Displacement and Velocity as a function of time

CASE 6

- Now, we are discussing about the cases perpendicular to the frame.
- In this case, Air damping is ignored.
- The intial condition is:

$$\theta = 0.0821 radians \tag{16}$$

- The length of string is constant in this case.

$$L = 8.5m \tag{17}$$

- The differential equation for this case comes out as:

$$\frac{d^2\phi}{dt^2} + \frac{g}{L}\sin\phi = 0 \tag{18}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 7.

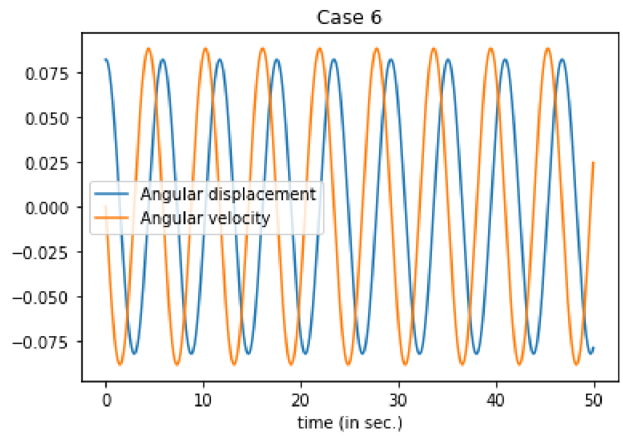

FIG. 7. Angular Displacement and Velocity as a function of time

CASE 7

- In this case, Air damping considered as a constant force of 12 N.
- The intial condition is:

$$\theta = 0.0821 radians \tag{19}$$

- The length of string is constant in this case.

$$L = 8.5m \tag{20}$$

- The differential equation for this case comes out as:

$$\frac{d^2\phi}{dt^2} + \frac{g}{L}\sin\phi + \frac{F_{air}}{mL}\cos\phi = 0 \tag{21}$$

Here we have plotted both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds as shown in FIG. 8.

CASE 8

- In this case, Air damping is considered a constant force of 12 N.
- The intial condition is:

$$\theta = 0.0821 radians \tag{22}$$

- The length of string is variable in this case.

$$L(t) = 8.5 - 0.0857t \tag{23}$$

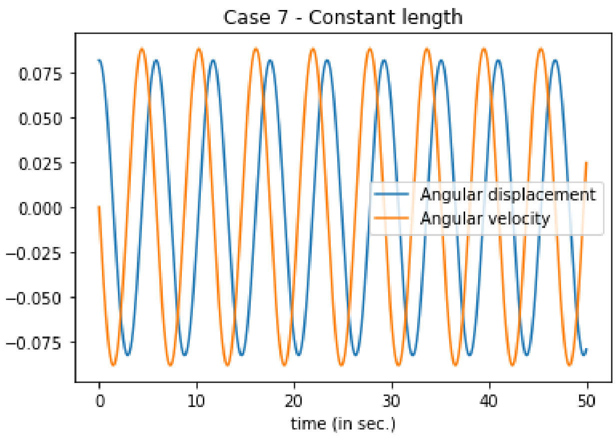

FIG. 8. Angular Displacement and Velocity as a function of time

- The differential equation for this case comes out as:

$$\frac{d^2\phi}{dt^2} + \frac{g}{L(t)} \sin\phi + \frac{F_{air}}{mL(t)} \cos\phi = 0 \tag{24}$$

The plot for both Angular velocity and Angular displacement of the module w.r.t time for first 50 seconds is shown in FIG. 9.

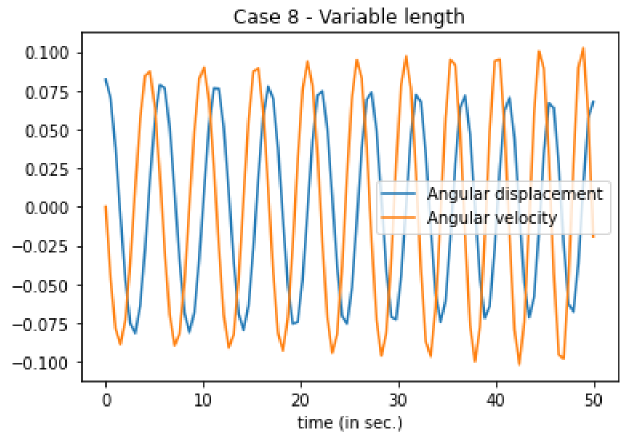

FIG. 9. Angular Displacement and Velocity as a function of time