Modelling urban traffic configuration with the influence of human factors

The proposed mathematical model receives parameters from map, weather conditions, vehicles, people and the queuing system. These parameters are presented in Table 1. The possible values shown are specific to the data sources of this research then, they may be different if other sources are used.

Table 1: Model input parameters

Group	Name	Symbol	Possible values
Queuing system	Arrival rate	(λ)	-
	Service rate	(μ)	-
Map	Street type	(T_s)	Primary, Secondary, Motorway
	Pavement conditions	(U_s)	Good, Regular, Bad, Repair
	Maximum allowed speed	(V_s)	> 0
Vehicle	Type	(T_v)	Motorcycle, Auto, Bus, Articulated
	Technical conditions	(E_v)	Good, Slight problems, Severe problems
	Size	(l_a)	> 0
People	Age	(G_p)	> 0
	Sex	(S_p)	Male, Female
	Zone Knowledge	(K_p)	[0,1]
	Experience	(X_p)	≥ 0
Weather	Temperature	$(Temp)$	-
	Humidity	(Hum)	[0,100]
	Rain	(ll)	[1,5]
Probabilities	Red light violation	(R_{prob})	[0,1]
	Stop violation	(S_{prob})	[0,1]
	Obstacles	(O_{prob})	[0,1]
Traffic conditions	Traffic light cycle minimum	(Min_c)	> 0
	Traffic light cycle maximum	(Max_c)	> 0
	Standard speed for people	(V_{ep})	> 0
	Speed by Street and vehicle	(V_{ev})	V for each T_v y T_s
	Distance between vehicles	(l_o)	> 0

From the parameters shown in Table 1, there are others obtained using equations described below.

$$a = \frac{d}{dt} V \quad (1)$$

Eq. (1) corresponds to the instantaneous acceleration of vehicles at the start of the green light [1].

$$\mu_0(t) = \begin{cases} {}^{a\tau}\!/_{2(l_0 + l_a)}, \tau \in [0;\tau^*] \\ {}^{{}^{V^2}\!/_{2a} + V(\tau - \tau^*)}\!\Big/_{\tau(l_0 + l_a)}, \tau \in [\tau^*;\tau^{**}] \\ {}^{{}^{{}^{V^2}\!/_{2a} + V(\tau^{**} - \tau^*)}\!\Big/_{(l_o + l_a)} + \lambda(\tau - \tau^{**})}\!\Big/_{\tau}, \tau \in [\tau^{**};\tau^{***}] \end{cases} \tag{2}$$

Eq. (2) is used to calculate the average service rate of traffic light stations using the model presented in [1].

The service of traffic light is divided into three phases during the green light time of a traffic light [1]

- from the movement beginning of the first car in the queue until the flow of the recommended speed is reached;
- from the moment the recommended speed is reached until the last car in the queue has passed;
- from the moment of passing the queue to the end of the green traffic light.

In this equation τ is the current time of the model and $\tau^*, \tau^{**}, \tau^{***}$ the duration times of each subphase.

$$\rho = \frac{\lambda}{s\mu} \tag{3}$$

Eq. (3) calculates the factor of utilization of the queue [2].

Service provision occurs at two types of stations: traffic lights and stop signs. Stop sign stations also have exponential distribution because the service time is short and not very variable so it will be directly proportional to the arrival distribution. So the model for these stations will be $(M/M/s)$. Traffic light stations will follow a general distribution [3] and the model will be $(M/G/s)$. This type of model accepts any data distribution for service times. It has been shown to fit the behavior of traffic lights [2].

$$L_q = \begin{cases} \frac{\lambda^2\sigma_s^2 + (\frac{\lambda}{s\mu})^2}{2(1 - \frac{\lambda}{s\mu})} \rightarrow (M/G/s) \\ \frac{\left(\frac{\lambda}{\mu}\right)^s \rho * P_0}{s!(1-\rho)^2} \rightarrow (M/M/s) \end{cases} \tag{4}$$

$$L = \begin{cases} \lambda W \rightarrow (M/G/s) \\ \mathrm{L}_q + \left(\frac{\lambda}{\mu}\right) \rightarrow (M/M/s) \end{cases} \tag{5}$$

Eq. (4) and (5) corresponds with number of vehicles in every traffic signal (queues) and in the system respectively [2, 4]. These measurements are calculated differently for each model used, so you will get one value for stop signs and another for traffic lights.

$$W_{q=}\frac{L_q}{\lambda} \quad (6)$$

$$W = W_q + \frac{1}{\mu} \quad (7)$$

Eq. (6) and (7) get the waiting time in queue and system respectively [2, 4]. These measures are calculated in the same way for both models used.

Eq. (8) obtains the humidity temperature index which is used to determine the level of comfort driver experience with a given combination of these values [5].

$$ITH = 0.8 \times Temp + Hum \times (Temp - 14.4) + 46.4 \quad (8)$$

Normal ITH levels are between [60 - 80]. When, in addition to temperature and humidity, there is rain in the environment, this factor is multiplied by the level of rainfall. The increase in the basic ITH corresponds to a higher unconformity, which also occurs when it starts to rain. This transformation is shown in equation (9) and only applies if the rainfall level is greater than 0. Otherwise $CF = ITH$.

$$CF = ITH \times ll \quad (9)$$

The following equations model the variations that occur in vehicle and pedestrian speeds when evaluating their characteristics. Equation (10) uses the comfort already calculated and the sex of the person.

$$Var_{CF-S_p} = 0.01 \times (CF - 70)^2 + (1.5 - S_p) \quad (10)$$

The difference between CF and 70 gives the variation between actual and ideal comfort. This variation is squared to maintain its positivity. The result is multiplied by 0.01 to ensure that the speed remains in range. For example, if a vehicle traveling at 50km/h, obtains a variation of 90, the speed will be added to 9km/h and not 90, because speed changes must be gradual and can be reached in a short period of time.

The second summand of the variation considers a variation of [0.5 - 1] km/h. The allowed values for sex will be 0 (male) or 1 (female), so that for males a larger variation will be obtained according to the assumption that males drive and walk faster than females.

The second variation expressed in equation (11) uses age and knowledge of the area. Two constants are recognized for this equation: 35 years as the balance age for road users, and level 3 (medium) in knowledge of the area they travel. For the knowledge of the zone, level 1 is qualified as Expert. The ages and levels of knowledge above the identified average will return smaller variations, due to the fact that the older people move slower, and because they do not know the region where they travel, they go slower in search of help or directions. The ages and levels below, on the contrary, will obtain a greater variation and will move faster.

$$Var_{G_p-K_p} = \frac{35}{G_p} \times \frac{3}{K_p} \quad (11)$$

The last variation is applicable only to vehicle drivers, is expressed in equation (12) and takes into account the technical condition of the vehicle, the condition of the road pavement and the driving experience of the person. Values of [1 - 3] are considered in these parameters, where level 1 is the desired value for all. Thus, if all parameters are evaluated at 1, there is no variation, otherwise the speed will decrease due to the different combinations expressed by the sum of the three factors.

$$Var_{U_s-E_v-X_p} = \frac{3}{U_s+E_v+X_p} \tag{12}$$

Equation (13) calculates the new speed for each pedestrian based on the standard speed set as a parameter and the variations that depend on the characteristics of each pedestrian. The variation $CF - Xp$ is added because it obtains a value in the range of speeds, while the variation for age and knowledge is a ratio that will tell, according to the current speed, how many times faster or slower the pedestrian will now travel.

$$V_p = [V_{ep} + Var_{CF-S_p}] \times Var_{G_p-K_p} \tag{13}$$

The same analysis is performed in equation (14), also applying the variation due to pavement, technical condition and experience.

$$V_v = [\frac{V_{ev}+V_c}{2} + Var_{CF-S_p}] \times Var_{G_p-K_p} \times Var_{U_s-E_v-X_p} \tag{14}$$

In addition to the variation of speeds, the human factor is present in the reckless actions on the road. For the model, the number of violations that occur is calculated in order to apply it as a penalty to the objective function. In equation (15) the violation probabilities obtained in the parameters are used and multiplied by the number of vehicles in each of the system queues, either at traffic lights and stop signs.

$$viol = R_{prob} \times S_{prob} \times \sum L_i \tag{15}$$

Pedestrian obstructions in the path of vehicles are calculated in equation (16), using the width of the road and the speed of the pedestrian to know how long it will take to complete the crossing. The probability of occurrence of this situation is also a parameter of the model.

$$E_p = \frac{A_s}{V_p} \times O_{prob} \tag{16}$$

The variables that determine the performance of the model will be S and Times.

$$S = [s_1 \quad s_2 \quad \cdots \quad s_n]$$

$$Times = \begin{bmatrix} TV_1 & TR_1 & A_1 \\ TV_2 & TR_2 & A_2 \\ \cdots & \cdots & \cdots \\ TV_k & TR_k & A_k \end{bmatrix}$$

The variable S represents the location of the signals where the values taken by the matrix positions are 0 if no signal, 1 if traffic light and 2 if stop sign. For each traffic light corner there will be a Times matrix where the times of the lights (GT – green time, RT - red time) of the k phases of the traffic light are determined.

Times also stores the active phase in each traffic light. To determine the active phase, the Eq. (17) is applied. The active phase will have value 1 in the matrix, while the remaining phases will have value 0.

This equation assumes a uniform distribution among the phases of the traffic light, so that the subtraction of k and the division between the total green times, and the current time of the cycle gives the active phase.

$$Active = k - \left\lfloor \frac{\sum TV_k}{\tau} \right\rfloor \quad (17)$$

$$Min \quad Z = W_s + W_p + E_{ag} + inf \quad (18)$$

s.t:

$$TV_n + TR_n + TA_n \leq Max_c \quad (19)$$

$$TV_n + TR_n + TA_n \geq Min_c \quad (20)$$

$$\sum_{i=1}^{k} A_i = 1 \quad (21)$$

The objective function of the model (18) gets the total waiting time in the system, where W_t is the waiting time obtained for the traffic lights of the queuing system: $W \rightarrow (M/G/s)$ and W_s is the waiting time at Stop signs of the queuing system: $W \rightarrow (M/M/s)$. A penalization is applied for vehicles violations. Constraints 19 and 20 correspond to obtaining light cycles that are in the range set in the parameters. Constraint 21 ensures that each traffic light has only one active phase at each time τ.

References

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