SOLS optimum* tilt - quadratic estimate

In a SOLS microsope the light-sheet consumes some of the numerical aperture from the primary objective (NA_1). The trade of light-sheet NA_{ex} vs emmission NA_{eff} is governed by the tilt of the 3rd microscope (θ_{tilt}). Some builders have used $\theta_{tilt} = 30$ degrees. So the question is... what is the optimal* tilt?

Here's a simple analytical estimate for matching the light-sheet thickness to the axial PSF:

$$2\omega_0 \approx Z_{PSF_{eff}} \quad \textit{and} \quad \theta_{ex} = 2(\theta_1 - \theta_{eff})$$

$$\frac{\lambda_{ex}}{\pi NA_{ex}} \approx \frac{\lambda_{em}}{NA_{eff}^2} \quad \Rightarrow \quad \frac{n_1 \lambda_{ex}}{\pi \lambda_{em}} \sin^2\theta_{eff} \approx \sin 2(\theta_1 - \theta_{eff})$$

$$\left(\frac{n_1 \lambda_{ex}}{\pi \lambda_{em}} + \sin 2\theta_1\right)\tan^2\theta_{eff} + 2(1 - 2\sin^2\theta_1)\tan\theta_{eff} - \sin 2\theta_1 \approx 0$$

$$\Rightarrow \quad \tan\theta_{eff} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \textit{and} \quad \theta_{tilt} = \frac{\pi}{2} + \theta_1 - 2\theta_{eff}$$

(*depends on what you're trying to optimize!)

Worked example:

In [1]:

```
import numpy as np

NA1 = 1.35 # Nikon 100x1.35 Sil
n1 = 1.4 # Silicone refractive index
lambda_ex = 488 # typical laser
lambda_em = 510 # GFP like

theta1 = np.arcsin(NA1/n1)
a = (lambda_ex * n1)/(np.pi * lambda_em) + np.sin(2 * theta1)
b = 2 * (1 - 2 * (np.sin(theta1) ** 2))
c = - np.sin(2 * theta1)

tan_theta_eff = (- b + (b**2 - 4 * a * c)**0.5)/ (2 * a)
theta_eff = np.arctan(tan_theta_eff)
theta_tilt = np.pi/2 + theta1 - 2 * theta_eff

print("Optimum tilt = ", round(np.rad2deg(theta_tilt), 2), "deg")
```

```
Optimum tilt =  35.68 deg
```