Introduction to Structured Prediction (with Python)

Andreas C. Müller

Columbia University

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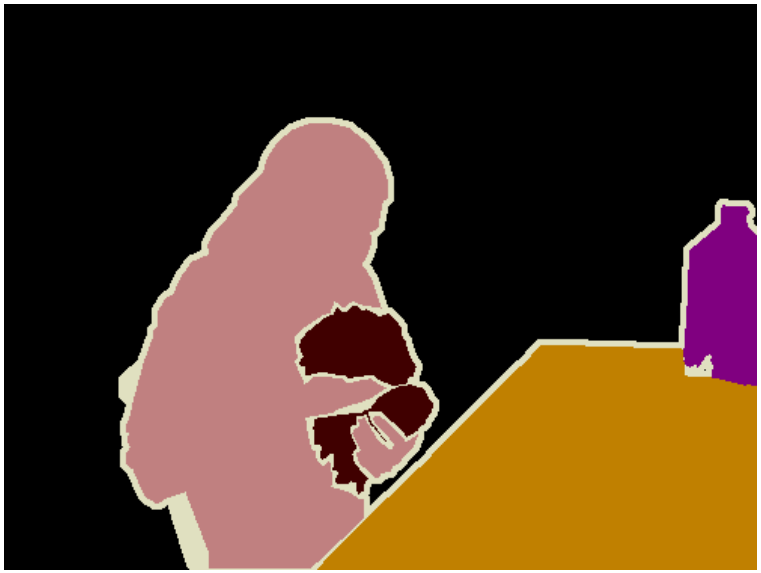
1. Inference Algorithms

2. Learning Algorithms
   - Optimizing Structural SVMs

3. Structured Prediction with PyStruct

Semantic Segmentation

Semantic Segmentation



Multi-Label Classification

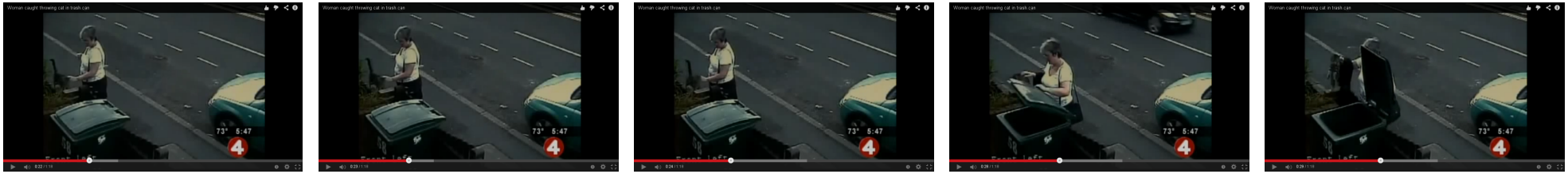
	Politics	Sports	Finance	Domestic	Religion
News Story1	1	0	0	1	1
News Story2	0	1	0	1	0
News Story3	0	0	1	0	0

Multi-Label Classification

	Politics	Sports	Finance	Domestic	Religion
News Story1	1	0	0	1	1
News Story2	0	1	0	1	0
News Story3	0	0	1	0	0

	Owns Car	Smokes	Married	Self-Employed	Has Kids
Customer1	1	0	1	0	1
Customer2	1	1	0	1	0
Customer3	0	1	1	0	0

Sequence Tagging



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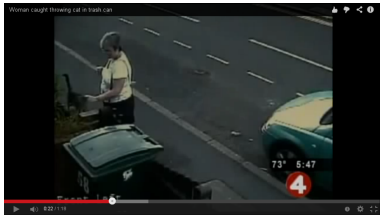
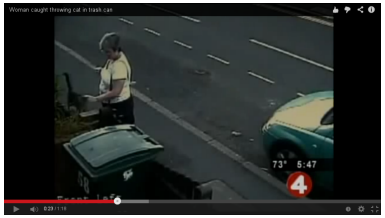


Stroke cat.



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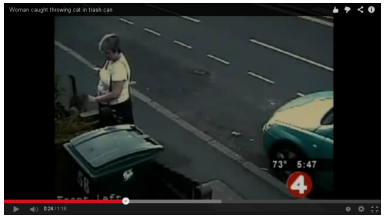
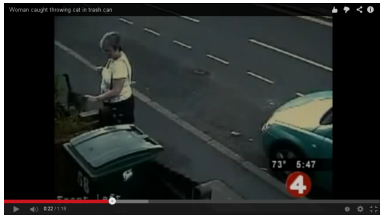


Stroke cat.

Open trash can.

Put cat in trash can.

Sequence Tagging



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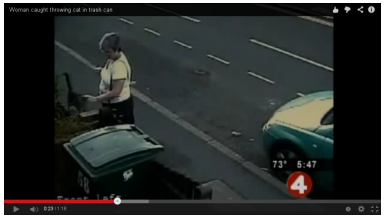

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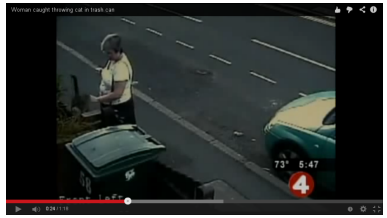

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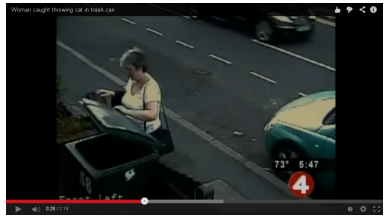

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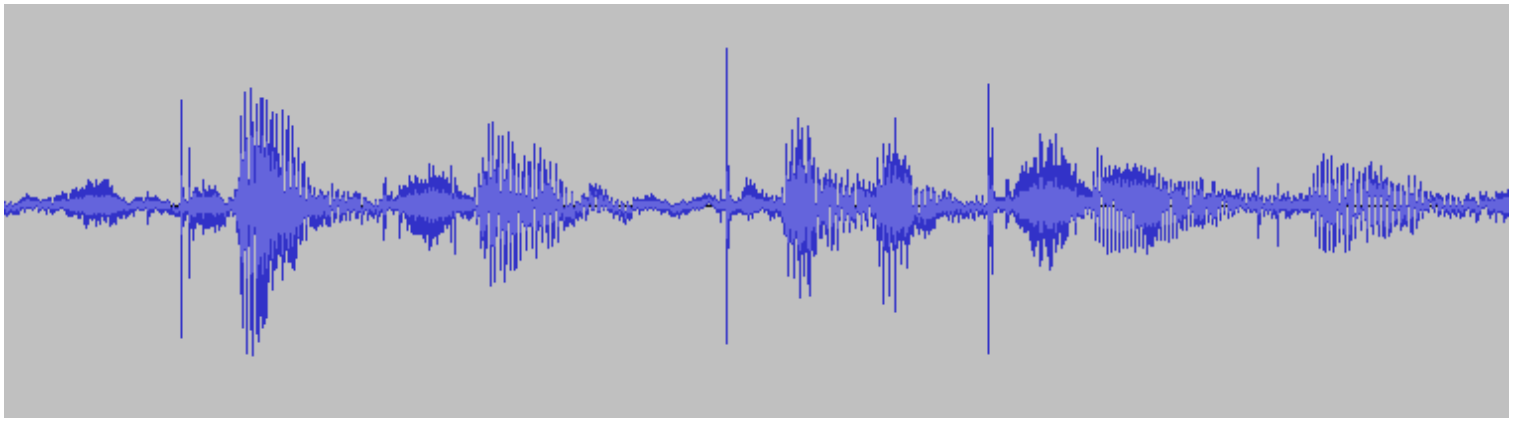

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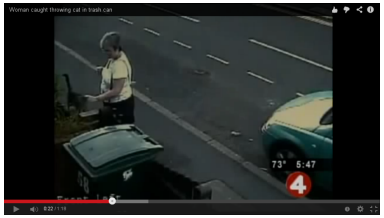

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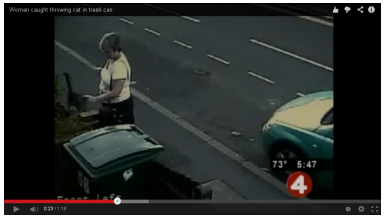

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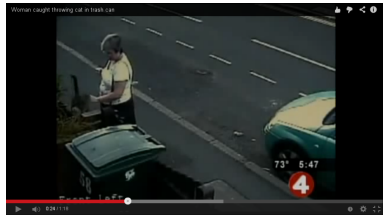

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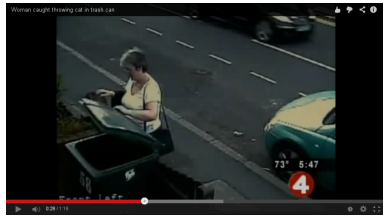

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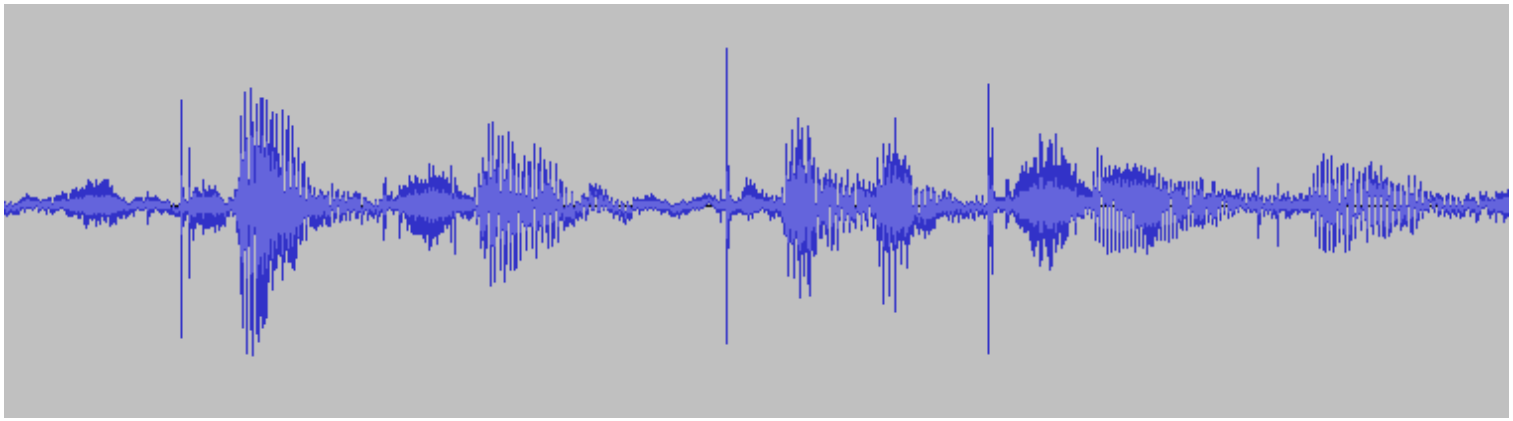

Struc-tured pre-dic-tion in Py-thon

Predicting Structured Objects

$$f(x, w) := \arg\max_{y \in \mathcal{Y}} g(x, y, w)$$

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If you like:

$$\arg\max_{y \in \mathcal{Y}} p(y|x, w)$$

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$$f(x, w) := \arg\max_{y \in \mathcal{Y}} w^T \psi(x, y)$$

Predicting discrete vectors

$$y = (y_1, y_2, \ldots, y_{n_i})$$

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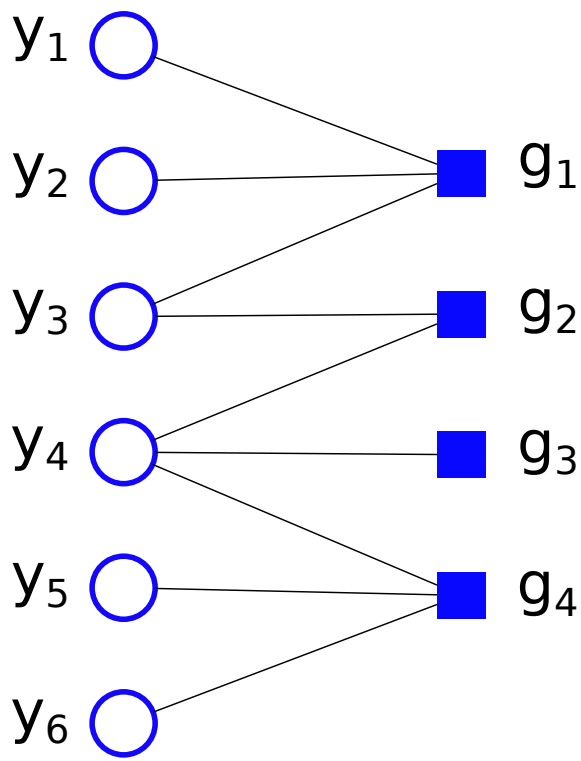
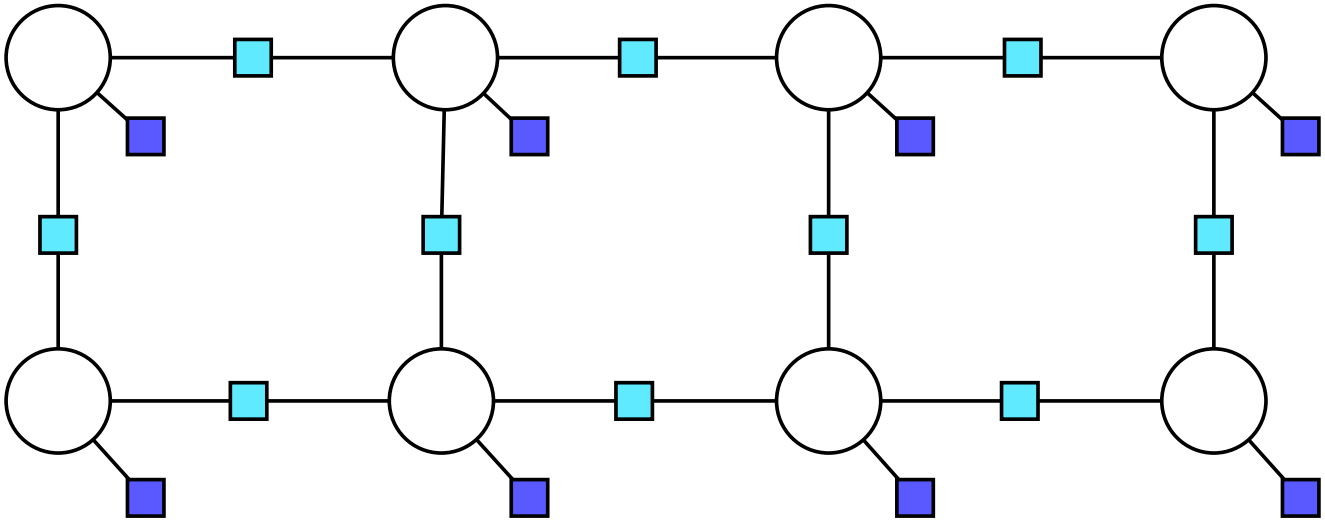
$$\begin{aligned} f(x, w) &= \arg\max_{y \in \mathcal{Y}} w^T \psi(x, y) \\ &= \arg\max_{y_1, y_2, \ldots, y_{n_i}} w^T \psi(x, y) \end{aligned}$$

Factor Graphs

$$g(x, y) = g_1(x, y_1, y_2, y_3) + g_2(x, y_3, y_4) + g_3(x, y_4) + g_4(x, y_4, y_5, y_6)$$

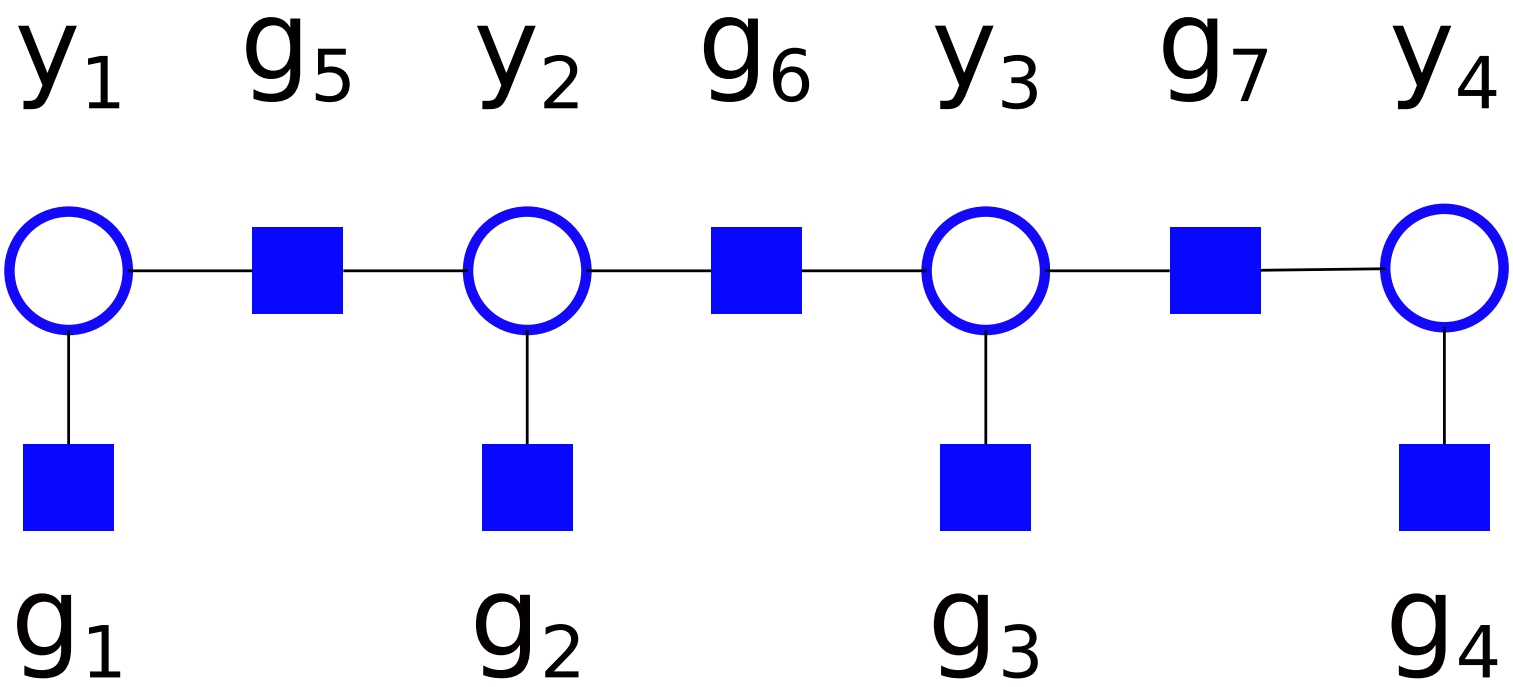


Pairwise Models



$$w^T \psi(x, y) = \sum_{(i,j) \in E} w_{i,j} \psi_{i,j}(x, y_i, y_j) + \sum_{i \in I} w_i \psi_i(x, y_i)$$

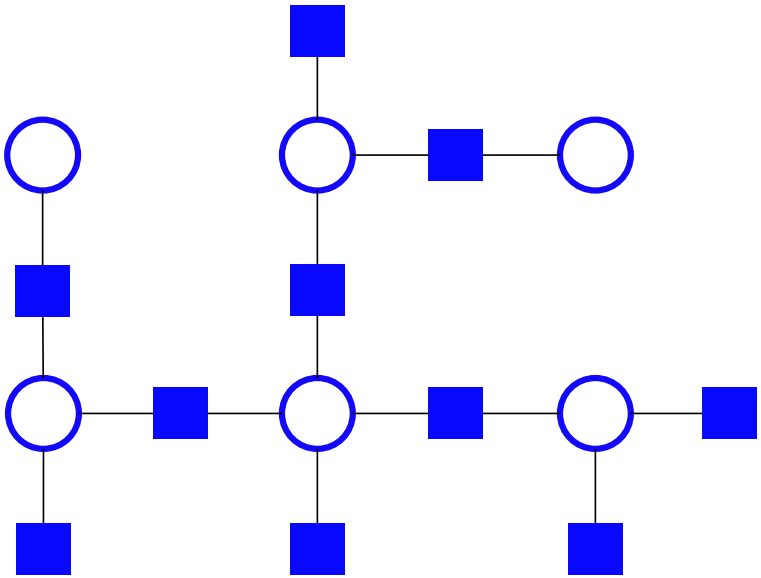
Factor Graph for HMM



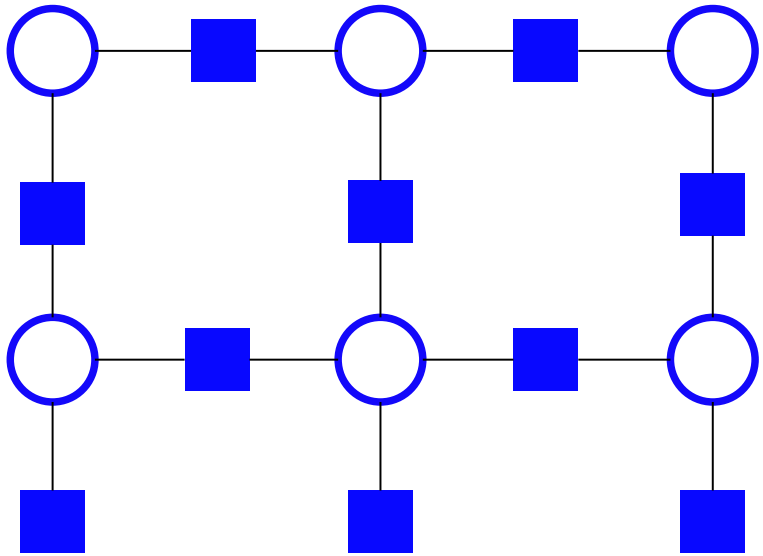
Factor Graphs Multi-Label Classification

Inference

Easy



Tricky



Inference Algorithms

Brute Force

Sum-Product (Viterbi)

Sampling

Relaxations

Move-making

Learning Algorithms

Probabilistic Learning

$$p(y|x, w) = \frac{1}{Z} exp(w^T \psi(x, y))$$

$$Z = \sum_{y' \in \mathcal{Y}} exp(w^T \psi(x, y'))$$

Probabilistic Learning

$$p(y|x, w) = \frac{1}{Z} exp(w^T \psi(x, y))$$

$$Z = \sum_{y' \in \mathcal{Y}} exp(w^T \psi(x, y'))$$

Objective

$$\max_w \sum_i \log(p(y^i|x^i, w))$$

$$= \max_w \sum_i w^T \psi(x^i, y^i) - \log(Z)$$

Max-Margin Learning

Learn prediction function of the form

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$$g(x, w) := \arg\max_{y \in \mathcal{Y}} w^T \psi(x, y)$$

Objective

$$\min_w \frac{1}{2}\|w\|^2 + C \sum_i \ell(x^i, y^i, w)$$

$$\ell(x^i, y^i, w) = [\max_{y \in \mathcal{Y}} \Delta(y^i, y) + w^T \psi(x^i, y) - w^T \psi(x^i, y^i)]_+.$$

Joachims, Finley, and Yu 2009

Compare to Binary SVM

Binary

$$\min_w \frac{1}{2}\|w\|^2 + C\sum_i \max(0, 1 - y^i w^T x^i)$$

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Crammer-Singer Multi-Class

$$\min_w \frac{1}{2}\|w\|^2 + C\sum_i \ell(x^i, y^i, w)$$

$$\ell(x^i, y^i, w) = [1 + \max_{\hat{y}!=y_i} w_{\hat{y}}^T x^i - w_{y^i}^T x^i]_+.$$

Compare to Binary SVM

Binary

$$\min_w \frac{1}{2}\|w\|^2 + C\sum_i \max(0, 1 - y^i w^T x^i)$$

Crammer-Singer Multi-Class

$$\min_w \frac{1}{2}\|w\|^2 + C\sum_i \ell(x^i, y^i, w)$$

$$\ell(x^i, y^i, w) = [1 + \max_{\hat{y}!=y_i} w_{\hat{y}}^T x^i - w_{y^i}^T x^i]_+.$$

Structured

$$\min_w \frac{1}{2}\|w\|^2 + C\sum_i \ell(x^i, y^i, w)$$

$$\ell(x^i, y^i, w) = [\max_{y \in \mathcal{Y}} \Delta(y^i, y) + w^T\psi(x^i, y) - w^T\psi(x^i, y^i)]_+.$$

Subgradient learning

Require: training samples $\{(x^1, y^1), \ldots, (x^k, y^k)\}$, regularization parameter C, learning rates η_t.

Ensure: parameters θ

$w \leftarrow 0$

repeat

  for $i = 1, \ldots, k$ **do** $\hat{y} \leftarrow \operatorname{argmax}_{\hat{y} \in \mathcal{Y}} \delta(y^i, \hat{y}) - \theta^T [\Phi(x^i, y^i) - \Phi(x^i, \hat{y})]$

    $w \leftarrow (1 - \eta_t) w - \eta_t C [\Phi(x^i, y^i) - \Phi(x^i, \hat{y})]$

until stopping tolerance reached.

n-Slack Cutting Plane Training of Structural SVMs

Require: training samples $\{(x^1, y^1), \ldots, (x^k, y^k)\}$, regularization parameter C, stopping tolerance ϵ.

Ensure: parameters θ, slack $(\xi_1, \ldots, \xi_k)$

$\mathcal{W}_i \leftarrow \emptyset, \xi_i \leftarrow 0$ for $i = 1, \ldots, k$

repeat

  for i=1, . . . , k **do**

    $\hat{y} \leftarrow \arg\max_{\hat{y} \in \mathcal{Y}} \delta(y^i, \hat{y}) - \theta^T [\Phi(x^i, y^i) - \Phi(x^i, \hat{y})]$

    if $\delta(y^i, \hat{y}) - \theta^T [\Phi(x^i, y^i) - \Phi(x^i, \hat{y})] \geq \xi_i + \epsilon$ **then**

    $\mathcal{W}_i \leftarrow \mathcal{W}_i \cup \{\hat{y}\}$

$$(\theta, \xi_1, \ldots, \xi_k) \leftarrow \arg\min_{\theta, \xi_1, \ldots, \xi_k} \frac{\|\theta\|^2}{2} + C \sum_{i=1}^{k} \xi_i$$

$$\text{s.t. for } i = 1, \ldots, k \; \forall \hat{y} \in \mathcal{W}_i :$$

$$\theta^T [\Phi(x^i, y^i) - \Phi(x^i, \hat{y}^i)] \geq \delta(y^i, \hat{y}^i) - \xi_i$$

until no $\mathcal{W}_i$ changes anymore.

1-Slack Cutting Plane Training of Structural SVMs

Require: training samples $\{(x^i, y^i), \ldots, (x^i, y^i)\}$, regularization parameter C, stopping tolerance ϵ.

Ensure: parameters θ, slack ξ

$\mathcal{W} \leftarrow \emptyset$

repeat

$$(\theta, \xi) \leftarrow \arg\min_{\theta,\xi} \frac{\|\theta\|^2}{2} + C\xi$$

$$\text{s.t. } \forall \hat{\mathbf{y}} = (\hat{y}^1, \ldots, \hat{y}^k) \in \mathcal{W} :$$

$$\theta^T \sum_{i=1}^{k} [\Phi(x^i, y^i) - \Phi(x^i, \hat{y}^i)] \geq \sum_{i=1}^{k} \delta(y^i, \hat{y}^i) - \xi$$

for i=1, . . . , k **do**

$$\hat{y}^i \leftarrow \arg\max_{\hat{y} \in \mathcal{Y}} \sum_{i=1}^{k} \delta(y^i, \hat{y}) - \theta^T \sum_{i=1}^{k} [\Phi(x^i, y^i) - \Phi(x^i, \hat{y})]$$

$\mathcal{W} \leftarrow \mathcal{W} \cup \{(\hat{y}^i, \ldots, \hat{y}^i)\}$

$$\xi' \leftarrow \sum_{i=1}^{k} \delta(y^i, \hat{y}^i) - \theta^T \sum_{i=1}^{k} [\Phi(x^i, y^i) - \Phi(x^i, \hat{y}^i)]$$

until $\xi' - \xi < \epsilon$

Structured Prediction with PyStruct

Simple structured prediction

Estimator = Learner + Model + Inference

Simple structured prediction

Estimator = Learner + Model + Inference

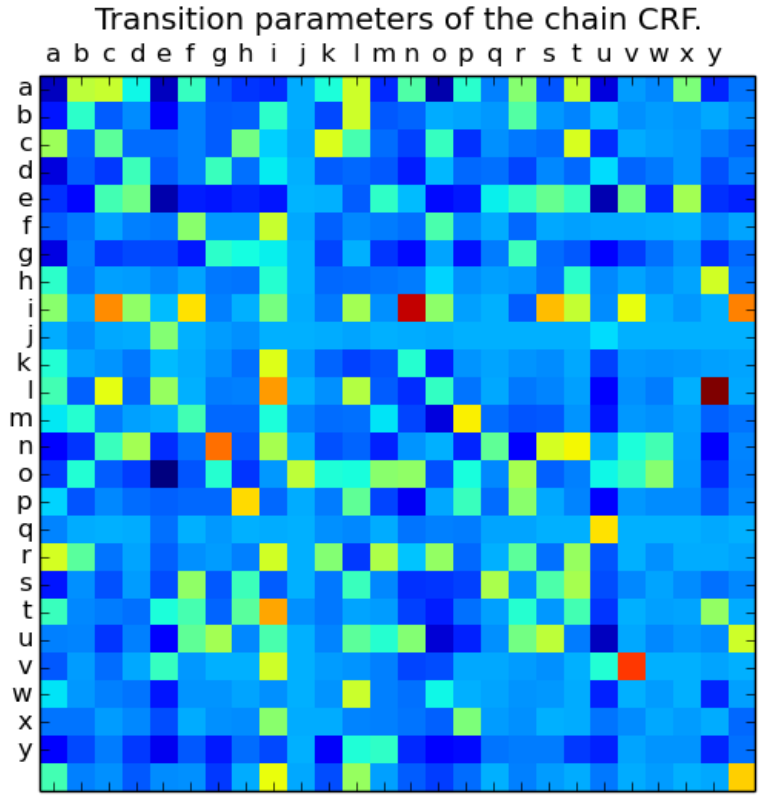
- Learner: SubgradientSSVM, StructuredPerceptron, OneSlackSSVM, LatentSSVM
- Model: BinaryClf, MultiLabelClf, ChainCRF, GraphCRF, EdgeFeatureGraphCRF
- Inference: Linear Programming, QPBO (PyQPBO), Dual Decomposition (AD3), Message Passing, Everything (OpenGM)

Example OCR

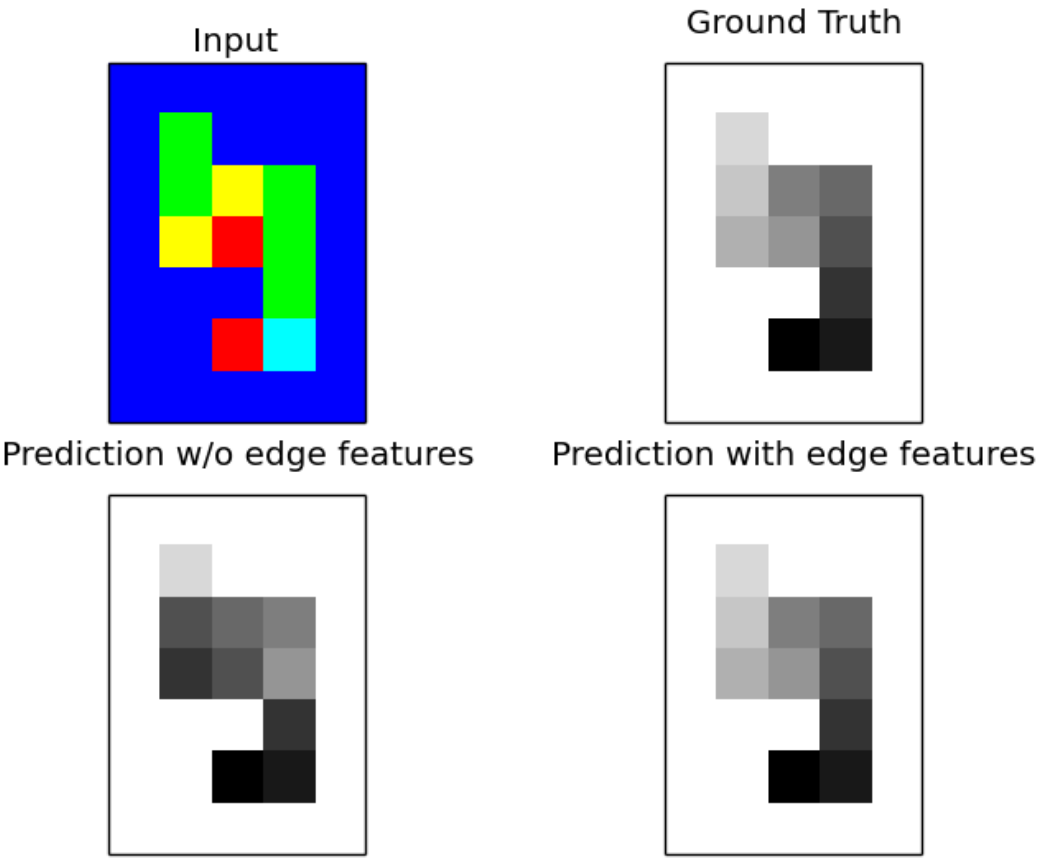
```
from pystruct.datasets import load_letters
from pystruct.models import ChainCRF
from pystruct.learners import OneSlackSSVM

abc = "abcdefghijklmnopqrstuvwxyz"

letters = load_letters()
X, y, folds = letters['data'], letters['labels'], letters['folds']
# we convert the lists to object arrays, as that makes slicing much more
# convenient
X, y = np.array(X), np.array(y)
X_train, X_test = X[folds == 1], X[folds != 1]
y_train, y_test = y[folds == 1], y[folds != 1]

# Train linear SVM
svm = LinearSVC(dual=False, C=.1)
# flatten input
svm.fit(np.vstack(X_train), np.hstack(y_train))

# Train linear chain CRF
model = ChainCRF()
ssvm = OneSlackSSVM(model=model, C=.1, inference_cache=50, tol=0.1, verbose=3)
ssvm.fit(X_train, y_train)
```

Example OCR

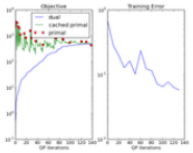

a f e t e r i a
a a f t a a f t m a r r i i a a
m b r a c e s
m m b b r r a a c c e e s s
p o i l f n g
p p a a i i c c i i m n d g
a f e t e r i a
a a f f o o d d o o l l z i n n


Transition parameters of the chain CRF.
a b c d e f g h i j k l m n o p q r s t u v w x y

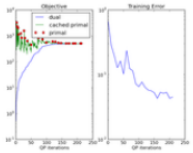
Example Snake



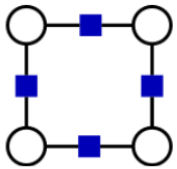
Examples



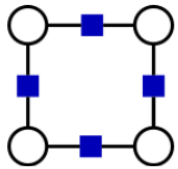
Plotting the objective and constraint caching in 1-slack SSVM



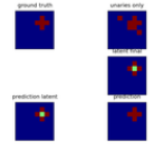
Efficient exact learning of 1-slack SSVMs



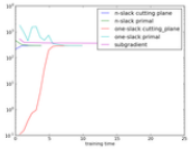
SVM as CRF



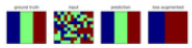
Semantic Image Segmentation on Pascal VOC



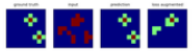
Latent Dynamics CRF



SVM objective values

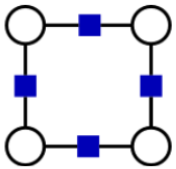


Learning directed interactions on a 2d grid



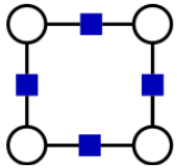
Learning interactions on a 2d grid

OCR Letter sequence recognition



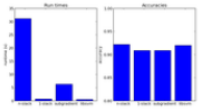
Crammer-Singer Multi-Class SVM

Latent SVM for odd vs. even digit classification

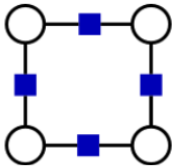


Mult-label classification

Latent Variable Hierarchical CRF



Binary SVM as SSVM



Comparing PyStruct and SVM-Struct

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http://pystruct.github.io

Thank you

References


Thorsten Joachims, Thomas Finley, and Chun-Nam John Yu. “Cutting-plane training of structural SVMs”. In: *Machine Learning* 77.1 (2009).