Реализация метода Холецкого-Холесского
1.1.3 (а) и 1.1.3 (г) в матричном виде

Дано :

A - симметричная положительно - определенная матрица

f - вектор столбец

Реализация :

In[1]:=
```
Clear@sqrtMethod
```

In[2]:=
```
sqrtMethod[A_, F_] :=
 Module[
  {n = Length@A, L, g, x},
  L = ConstantArray[0, {n, n}];
  g = ConstantArray[0, {n}];
  x = ConstantArray[0, {n}];
  Do[
```

$$\text{If}\left[i == j,\ L[[i, j]] = \sqrt{A[[i, j]] - \sum_{p=1}^{i-1} \left(L[[i, p]] * L[[i, p]]\right)}\ ,\right.$$

$$\left.L[[j, i]] = \frac{1}{L[[i, i]]} * \left(A[[j, i]] - \sum_{k=1}^{i-1} L[[i, k]] * L[[j, k]]\right)\right],\ \{i, 1, n\},\ \{j, i, n\}];$$

$$g[[1]] = \left(1 / L[[1, 1]]\right) * F[[1]];$$

$$\text{Do}\left[g[[i]] = \left(1 / L[[i, i]]\right) * \left(F[[i]] - \sum_{k=1}^{i-1} L[[i, k]] * g[[k]]\right),\ \{i, 2, n\}\right];$$

$$x[[n]] = g[[n]] / L[[n, n]];$$

$$\text{Do}\left[x[[i]] = \left(1 / L[[i, i]]\right) * \left(g[[i]] - \sum_{j=i+1}^{n} L[[j, i]] * x[[j]]\right),\ \{i, n-1, 1, -1\}\right];$$

```
  x]
```

Результат работы алгоритма :

Пример 1

In[3]:=
```
A = {{81, -45, 45}, {-45, 50, -15}, {45, -15, 38}};
f = {531, -460, 193};
{A // MatrixForm, f // MatrixForm}
```

Out[5]=

$$\left\{\begin{pmatrix} 81 & -45 & 45 \\ -45 & 50 & -15 \\ 45 & -15 & 38 \end{pmatrix},\ \begin{pmatrix} 531 \\ -460 \\ 193 \end{pmatrix}\right\}$$

In[6]:=
```
sqrtMethod[A, f] // MatrixForm
```
Out[6]//MatrixForm=
$$\begin{pmatrix} 6 \\ -5 \\ -4 \end{pmatrix}$$

Проверка 1

In[7]:=
```
Inverse@A.f // MatrixForm
```
Out[7]//MatrixForm=
$$\begin{pmatrix} 6 \\ -5 \\ -4 \end{pmatrix}$$

Пример 2

In[8]:=
```
A = {{4.33, -1.12, -1.08, 1.14}, {-1.12, 4.33, 0.24, -1.22},
   {-1.08, 0.24, 7.21, -3.22}, {1.14, -1.22, -3.22, 5.43}};
f = {{0.3}, {0.5}, {0.7}, {0.9}};
{A // MatrixForm, f // MatrixForm}
```
Out[10]=
$$\left\{\begin{pmatrix} 4.33 & -1.12 & -1.08 & 1.14 \\ -1.12 & 4.33 & 0.24 & -1.22 \\ -1.08 & 0.24 & 7.21 & -3.22 \\ 1.14 & -1.22 & -3.22 & 5.43 \end{pmatrix}, \begin{pmatrix} 0.3 \\ 0.5 \\ 0.7 \\ 0.9 \end{pmatrix}\right\}$$

In[11]:=
```
sqrtMethod[A, f] // MatrixForm
```
Out[11]//MatrixForm=
$$\begin{pmatrix} 0.100579 \\ 0.225667 \\ 0.260999 \\ 0.350105 \end{pmatrix}$$

Проверка 2

In[12]:=
```
x = Inverse@A.f // MatrixForm
```
Out[12]//MatrixForm=
$$\begin{pmatrix} 0.100579 \\ 0.225667 \\ 0.260999 \\ 0.350105 \end{pmatrix}$$

Реализация метода Холецкого-Холесского 1.1.3 (г) в матричном виде

In[13]:=
```
Clear@sqrtMethodMatrix
```

In[14]:=
```
sqrtMethodMatrix[A_, F_] :=
 Module[
  {n = Length@A, L, g, x},
  L = ConstantArray[0, {n, n}];
  g = ConstantArray[0, {n, n}];
  x = ConstantArray[0, {n, n}];
  Do[
   If[i == j, L[[i, j]] = Sqrt[A[[i, j]] - Sum[(L[[i, p]] * L[[i, p]]), {p, 1, i-1}]],
    L[[j, i]] = 1/L[[i, i]] * (A[[j, i]] - Sum[L[[i, k]] * L[[j, k]], {k, 1, i-1}])], {i, 1, n}, {j, i, n}];
  (g[[1, #]] = (1/L[[1, 1]]) * F[[1, #]]) & /@ Range@n;
  Do[g[[i, j]] = (1./L[[i, i]]) * (F[[i, j]] - Sum[L[[i, k]] * g[[k, j]], {k, 1, i-1}]),
   {i, 2, n}, {j, 1, n}];
  (x[[n, #]] = (1/L[[n, n]]) * g[[n, #]]) & /@ Range@n;
  Do[x[[i, m]] = (1./L[[i, i]]) * (g[[i, m]] - Sum[L[[j, i]] * x[[j, m]], {j, i+1, n}]),
   {i, n - 1, 1, -1}, {m, 1, n}];
  x]
```

Результат работы алгоритма

Пример 1

In[21]:=
```
A = {{81., -45., 45.}, {-45., 50., -15.}, {45., -15., 38.}};
F = {{3, 5, 1 }, {5, 9, 1}, {5, 2, 8}};
{A // MatrixForm, F // MatrixForm}
```

Out[23]= $\left\{\begin{pmatrix} 81. & -45. & 45. \\ -45. & 50. & -15. \\ 45. & -15. & 38. \end{pmatrix}, \begin{pmatrix} 3 & 5 & 1 \\ 5 & 9 & 1 \\ 5 & 2 & 8 \end{pmatrix}\right\}$

In[24]:=
```
sqrtMethodMatrix[A, F] // MatrixForm
```

Out[24]//MatrixForm=

$$\begin{pmatrix} 0.127572 & 0.797805 & -0.542661 \\ 0.237037 & 0.715062 & -0.240988 \\ 0.0740741 & -0.609877 & 0.758025 \end{pmatrix}$$

Проверка 1

In[25]:=
```
x = Inverse@A.F // MatrixForm
```

Out[25]//MatrixForm=

$$\begin{pmatrix} 0.127572 & 0.797805 & -0.542661 \\ 0.237037 & 0.715062 & -0.240988 \\ 0.0740741 & -0.609877 & 0.758025 \end{pmatrix}$$

Пример 2

In[∘]:=
```
A = {{4.33, -1.12, -1.08, 1.14}, {-1.12, 4.33, 0.24, -1.22},
    {-1.08, 0.24, 7.21, -3.22}, {1.14, -1.22, -3.22, 5.43}};
F = {{3, 5, 1 , 6}, {5, 9, 1, 10}, {5, 2, 8, 8}, {1, 2, 3, 4}};
```

In[∘]:=
```
{A // MatrixForm, F // MatrixForm}
```

Out[∘]=

$$\left\{\begin{pmatrix} 4.33 & -1.12 & -1.08 & 1.14 \\ -1.12 & 4.33 & 0.24 & -1.22 \\ -1.08 & 0.24 & 7.21 & -3.22 \\ 1.14 & -1.22 & -3.22 & 5.43 \end{pmatrix}, \begin{pmatrix} 3 & 5 & 1 & 6 \\ 5 & 9 & 1 & 10 \\ 5 & 2 & 8 & 8 \\ 1 & 2 & 3 & 4 \end{pmatrix}\right\}$$

In[∘]:=
```
sqrtMethodMatrix[A, F] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1.16752 & 1.81709 & 0.438498 & 2.23194 \\ 1.69162 & 2.8358 & 0.738058 & 3.45359 \\ 1.29845 & 0.998214 & 1.94565 & 2.44189 \\ 1.0891 & 1.21592 & 1.78002 & 2.49205 \end{pmatrix}$$

Проверка 2

In[∘]:=
```
x =  Inverse@A.F // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1.16752 & 1.81709 & 0.438498 & 2.23194 \\ 1.69162 & 2.8358 & 0.738058 & 3.45359 \\ 1.29845 & 0.998214 & 1.94565 & 2.44189 \\ 1.0891 & 1.21592 & 1.78002 & 2.49205 \end{pmatrix}$$