CHAPTER 9

Design of Beams for Moments

9.1 INTRODUCTION

If gravity loads are applied to a fairly long, simply supported beam, the beam will bend downward, and its upper part will be placed in compression and will act as a compression member. The cross section of this "column" will consist of the portion of the beam cross section above the neutral axis. For the usual beam, the "column" will have a much smaller moment of inertia about its y or vertical axis than about its x axis. If nothing is done to brace it perpendicular to the y axis, it will buckle laterally at a much smaller load than would otherwise have been required to produce a vertical failure. (You can verify these statements by trying to bend vertically a magazine held in a vertical position. The magazine will, just as a steel beam, always tend to buckle laterally unless it is braced in that direction.)

Lateral buckling will not occur if the compression flange of a member is braced laterally or if twisting of the beam is prevented at frequent intervals. In this chapter, the buckling moments of a series of compact ductile steel beams with different lateral or torsional bracing situations are considered. (As previously defined, a *compact section* is one that has a sufficiently stocky profile so that it is capable of developing a fully plastic stress distribution before buckling.)

In this chapter, we will look at beams as follows:

1. First, the beams will be assumed to have continuous lateral bracing for their compression flanges.
2. Next, the beams will be assumed to be braced laterally at short intervals.
3. Finally, the beams will be assumed to be braced laterally at larger and larger intervals.

In Fig. 9.1, a typical curve showing the nominal resisting or buckling moments of one of these beams with varying unbraced lengths is presented.

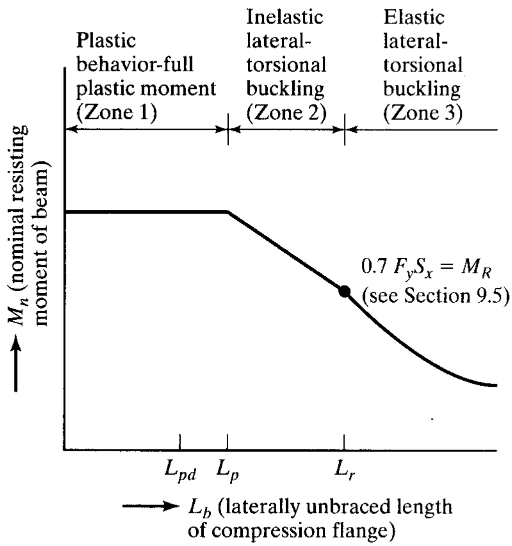

FIGURE 9.1

Nominal moment as function of unbraced length of compression flange.

An examination of Fig. 9.1 will show that beams have three distinct ranges, or zones, of behavior, depending on their lateral bracing situation. If we have continuous or closely spaced lateral bracing, the beams will experience yielding of the entire cross section and fall into what is classified as Zone 1. As the distance between lateral bracing is increased further, the beams will begin to fail inelastically at smaller moments and fall into Zone 2. Finally, with even larger unbraced lengths, the beams will fail elastically and fall into Zone 3. A brief discussion of these three types of behavior is presented in this section, while the remainder of the chapter is devoted to a detailed discussion of each type, together with a series of numerical examples.

9.1.1 Plastic Behavior (Zone 1)

If we were to take a compact beam whose compression flange is continuously braced laterally, we would find that we could load it until its full plastic moment M_p is reached at some point or points; further loading then produces a redistribution of moments, as was described in Chapter 8. In other words, the moments in these beams can reach M_p and then develop a rotation capacity sufficient for moment redistribution.

If we now take one of these compact beams and provide closely spaced intermittent lateral bracing for its compression flanges, we will find that we can still load it until the plastic moment plus moment redistribution is achieved if the spacing between the bracing does not exceed a certain value, called L_p herein. (The value of L_p is dependent on the dimensions of the beam cross section and on its yield stress.) *Most beams fall in Zone 1.*

9.1.2 Inelastic Buckling (Zone 2)

If we now further increase the spacing between points of lateral or torsional bracing, the section may be loaded until some, but not all, of the compression fibers are stressed to F_y. The section will have insufficient rotation capacity to permit full moment

Bridge over Allegheny River at Kittanning, PA. (Courtesy of the American Bridge Company.)

redistribution and thus will not permit plastic analysis. In other words, in this zone we can bend the member until the yield strain is reached in some, but not all, of its compression elements before lateral buckling occurs. This is referred to as *inelastic buckling*.

As we increase the unbraced length, we will find that the moment the section resists will decrease, until finally it will buckle before the yield stress is reached anywhere in the cross section. The maximum unbraced length at which we can still reach F_y at one point is the end of the inelastic range. It's shown as L_r in Fig. 9.1; its value is dependent upon the properties of the beam cross section, as well as on the yield and residual stresses of the beam. At this point, as soon as we have a moment that theoretically causes the yield stress to be reached at some point in the cross section (actually, it's less than F_y because of residual stresses), the section will buckle.

9.1.3 Elastic Buckling (Zone 3)

If the unbraced length is greater than L_r, the section will buckle elastically before the yield stress is reached anywhere. As the unbraced length is further increased, the buckling moment becomes smaller and smaller. As the moment is increased in such a beam, the beam will deflect more and more transversely until a critical moment value M_{cr} is reached. At this time, the beam cross section will twist and the compression flange will move laterally. The moment M_{cr} is provided by the torsional resistance and the warping resistance of the beam, as will be discussed in Section 9.7.

150 Federal Street, Boston, MA. (Courtesy Owen Steel Company, Inc.)

9.2 YIELDING BEHAVIOR—FULL PLASTIC MOMENT, ZONE 1

In this section and the next two, beam formulas for yielding behavior (Zone 1) are presented, while in Sections 9.5 through 9.7, formulas are presented for inelastic buckling (Zone 2) and elastic buckling (Zone 3). After seeing some of these latter expressions, the reader may become quite concerned that he or she is going to spend an enormous amount of time in formula substitution. This is not generally true, however, as the values needed are tabulated and graphed in simple form in Part 3 of the AISC Manual.

If the unbraced length L_b of the compression flange of a compact I- or C-shaped section, including hybrid members, does not exceed L_p (if elastic analysis is being used) or L_{pd} (if plastic analysis is being used), then the member's bending strength about its major axis may be determined as follows:

$$M_n = M_p = F_y Z \qquad \text{(LRFD Equation F2-1)}$$

$$\phi_b M_n = \phi_b F_y Z \ (\phi_b = 0.90)$$

$$\frac{M_n}{\Omega_b} = \frac{F_y Z}{\Omega_b} \ (\Omega_b = 1.67)$$

When a conventional elastic analysis approach is used to establish member forces, L_b may not exceed the value L_p to follow if M_n is to equal $F_y Z$.

$$L_p = 1.76\, r_y \sqrt{\frac{E}{F_y}} \qquad \text{(AISC Equation F2-5)}$$

When a plastic analysis approach is used to establish member forces for doubly symmetric and singly symmetric I-shaped members, with the compression flanges larger than their tension flanges (including hybrid members) loaded in the plane of the web, L_b (which is defined as the laterally unbraced length of the compression flange at plastic hinge locations associated with failure mechanisms) may not exceed the value L_{pd} to follow if M_n is to equal $F_y Z$.

$$L_{pd} = \left[0.12 - 0.076\left(\frac{M_1'}{M_2'}\right)\right]\left(\frac{E}{F_y}\right) r_y \qquad \text{(AISC Appendix Equation A-1-5)}$$

In this expression, M_1 is the smaller moment at the end of the unbraced length of the beam and M_2 is the larger moment at the end of the unbraced length, and the ratio M_1/M_2 is positive when the moments cause the member to be bent in double curvature  and negative if they bend it in single curvature . Only steels with F_y values (F_y is the specified minimum yield stress of the compression flange) of 65 ksi or less may be considered. Higher-strength steels may not be ductile.

There is no limit of the unbraced length for circular or square cross sections or for I-shaped beams bent about their minor axes. (If I-shaped sections are bent about their minor or y axes, they will not buckle before the full plastic moment M_p about the y axis is developed, as long as the flange element is compact.) Appendix Equation A1-8 of the AISC Specification also provides a value of L_{pd} for solid rectangular bars and symmetrical box beams.

9.3 DESIGN OF BEAMS, ZONE 1

Included in the items that need to be considered in beam design are moments, shears, deflections, crippling, lateral bracing for the compression flanges, fatigue, and others. Beams will probably be selected that provide sufficient design moment capacities

($\phi_b M_n$) and then checked to see if any of the other items are critical. The factored moment will be computed, and a section having that much design moment capacity will be initially selected from the AISC Manual, Part 3, Table 3-2, entitled "W Shapes Selection by Z_x." From this table, steel shapes having sufficient plastic moduli to resist certain moments can quickly be selected. Two important items should be remembered in selecting shapes:

1. These steel sections cost so many cents per pound, and it is therefore desirable to select the lightest possible shape having the required plastic modulus (assuming that the resulting section is one that will reasonably fit into the structure). The table has the sections arranged in various groups having certain ranges of plastic moduli. The heavily typed section at the top of each group is the lightest section in that group, and the others are arranged successively in the order of their plastic moduli. Normally, the deeper sections will have the lightest weights giving the required plastic moduli, and they will be generally selected, unless their depth causes a problem in obtaining the desired headroom, in which case a shallower but heavier section will be selected.
2. The plastic moduli values in the table are given about the horizontal axes for beams in their upright positions. If a beam is to be turned on its side, the proper plastic modulus about the y axis can be found in Table 3-4 of the Manual or in the tables giving dimensions and properties of shapes in Part 1 of the AISC Manual. A W shape turned on its side may be only from 10 to 30 percent as strong as one in the upright position when subjected to gravity loads. In the same manner, the strength of a wood joist with the actual dimensions 2×10 in turned on its side would be only 20 percent as strong as in the upright position.

The examples to follow illustrate the analysis and design of compact steel beams whose compression flanges have full lateral support or bracing, thus permitting plastic analysis. For the selection of such sections, the designer may enter the tables either with the required plastic modulus or with the factored design moment (if $F_y = 50$ ksi).

Example 9-1

Is the compact and laterally braced section shown in Fig. 9.2 sufficiently strong to support the given loads if $F_y = 50$ ksi? Check the beam with both the LRFD and ASD methods.

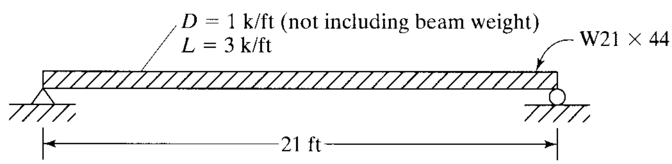

FIGURE 9.2

Solution. Using a W21 × 44 (Z_x = 95.4 in^3)

LRFD $\phi_b = 0.9$	ASD $\Omega_b = 1.67$
Given beam wt = 0.044 k/ft	Given beam wt = 0.044 k/ft
$w_u = (1.2)(1 + 0.044) + (1.6)(3) = 6.05$ k/ft	$w_a = (1 + 0.044) + 3 = 4.044$ k/ft
$M_u = \dfrac{(6.05)(21)^2}{8} = 333.5$ ft-k	$M_a = \dfrac{(4.044)(21)^2}{8} = 222.9$ ft-k
M_n of section $= \dfrac{F_y Z}{12} = M_{px}$	$M_n = 397.5$ ft-k from LRFD solution
$= \dfrac{(50 \text{ ksi})(95.4 \text{ in}^3)}{12 \text{ in/ft}} = 397.5$ ft-k	$\dfrac{M_n}{\Omega_2} = \dfrac{397.5}{1.67} = 238$ ft-k >222.9 ft-k **OK**
$M_u = \phi_b M_{px} = (0.9)(397.5)$	
$= 358$ ft-k > 333.5 ft-k **OK**	

Note: Instead of using Z_x and $F_y Z_x$, we will usually find it easier to use the moment columns $\phi_b M_{px}$ and M_{px}/Ω_b in AISC Table 3-2. There, the term M_{px} represents the plastic moment of a section about its x axis. Following this procedure for a W21 × 44, we find the values $\phi_b M_{px}$ = 358 ft-k and M_{px}/Ω_b = 238 ft-k. These values agree with the preceding calculations.

9.3.1 Beam Weight Estimates

In each of the examples to follow, the weight of the beam is included in the calculation of the bending moment to be resisted, as the beam must support itself as well as the external loads. The estimates of beam weight are very close here, because the authors were able to perform a little preliminary paperwork before making his estimates. The beginning student is not expected to be able to glance at a problem and estimate exactly the weight of the beam required. A very simple method is available, however, with which the student can quickly and accurately estimate beam weights. He or she can calculate the maximum bending moment, not counting the effect of the beam weight, and pick a section from AISC Table 3-2. Then the weight of that section or a little bit more (since the beam's weight will increase the moment somewhat) can be used as the estimated beam weight. The resulting beam weights will almost always be very close to the weights of the member selected for the final designs. For future example problems in this text, the author does not show his calculations for estimated beam weights. The weight estimates used for those problems, however, were obtained in exactly the same manner as they are in Example 9-2, which follows.

Example 9-2

Select a beam section by using both the LRFD and ASD methods for the span and loading shown in Fig. 9.3, assuming full lateral support is provided for the compression flange by the floor slab above (that is, $L_b = 0$) and $F_y = 50$ ksi.

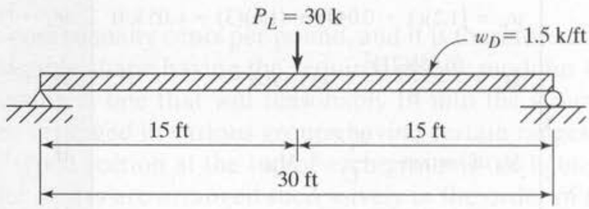

FIGURE 9.3

Solution

Estimate beam weights.

LRFD	ASD
w_u not including beam weight	w_a not including beam weight
$= (1.2)(1.5) = 1.8$ k/ft	$= 1.5$ k/ft
$P_u = (1.6)(30) = 48$ k	$P_a = 30$ k
$M_u = \frac{(1.8)(30)^2}{8} + \frac{(48)(30)}{4}$	$M_a = \frac{(1.5)(30)^2}{8} + \frac{(30)(30)}{4}$
$= 562.5$ ft-k	$= 393.8$ ft-k
From AISC Table 3-2 and the LRFD moment column ($\phi_b M_{px}$), a W24 × 62 is required.	From AISC Table 3-2 and the ASD moment column (M_{px}/Ω_b), a W21 × 68 is required.
$\phi_b M_{px} = 574$ ft-k	$\frac{M_{px}}{\Omega_b} = 399$ ft-k
Assume beam wt = 62 lb/ft.	**Assume beam wt = 68 lb/ft.**

Select beam section.

LRFD	ASD
$w_u = (1.2)(1.5 + 0.062) = 1.874$ k/ft	$w_a = 1.5 + 0.068 = 1.568$ k/ft
$P_u = (1.6)(30) = 48$ k	$P_a = 30$ k
$M_u = \frac{(1.874)(30)^2}{8} + \frac{(48)(30)}{4}$	$M_a = \frac{(1.568)(30)^2}{8} + \frac{(30)(30)}{4}$
$= 570.8$ ft-k	$= 401.4$ ft-k

(Continued)

LRFD	ASD
From AISC Table 3-2	From AISC Table 3-2
Use W24 × 62.	**Use W24 × 68.**
($\phi_b M_{px}$ = 574 ft-k > 570.8 ft-k)	(M_{px}/Ω_b = 442 ft-k > 401.4 ft k)
OK	**OK**

Example 9-3

The 5-in reinforced-concrete slab shown in Fig. 9.4 is to be supported with steel W sections 8 ft 0 in on centers. The beams, which will span 20 ft, are assumed to be simply supported. If the concrete slab is designed to support a live load of 100 psf, determine the lightest steel sections required to support the slab by the LRFD and ASD procedures. It is assumed that the compression flange of the beam will be fully supported laterally by the concrete slab. The concrete weighs 150 lb/ft^3. F_y = 50 ksi.

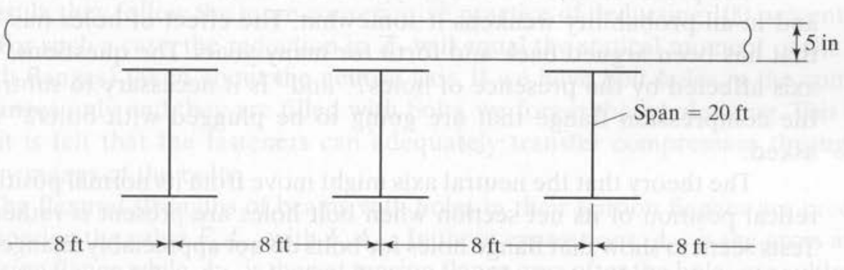

FIGURE 9.4

Solution

LRFD	ASD
Assume beam wt = 22 lb/ft	Assume beam wt = 22 lb/ft
Slab wt = $\left(\frac{5}{12}\right)(150)(8)$ = 500 lb/ft	Slab wt = 500 lb/ft
w_D = 522 lb/ft	w_D = 522 lb/ft
w_L = (8)(100) = 800 lb/ft	w_L = 800 lb/ft
w_u = (1.2)(522) + (1.6)(800) = 1906 lb/ft = 1.906 k/ft	w_a = 522 + 800 = 1322 lb/ft = 1.322 k/ft
$M_u = \frac{(1.906)(20)^2}{8}$ = 95.3 ft-k	$M_a = \frac{(1.322)(20)^2}{8}$ = 66.1 ft-k
From AISC Table 3-2	From AISC Table 3-2
Use W10 × 22.	**Use W12 × 22.**
($\phi_b M_{px}$ = 97.5 ft-k > 95.3 ft-k)	$\left(\frac{M_{px}}{\Omega_b}\right.$ = 73.1 ft-k > 66.1 ft-k$\left.\right)$

9.3.2 Holes in Beams

It is often necessary to have holes in steel beams. They are obviously required for the installation of bolts and sometimes for pipes, conduits, ducts, etc. If at all possible, these latter types of holes should be completely avoided. When absolutely necessary, they should be placed through the web if the shear is small and through the flange if the moment is small and the shear is large. Cutting a hole through the web of a beam does not reduce its section modulus greatly or its resisting moment; but, as will be described in Section 10.2, a large hole in the web tremendously reduces the shearing strength of a steel section. When large holes are put in beam webs, extra plates are sometimes connected to the webs around the holes to serve as reinforcing against possible web buckling.

When large holes are placed in beam webs, the strength limit states of the beams—such as local buckling of the compression flange, the web, or the tee-shaped compression zone above or below the opening—or the moment–shear interaction or serviceability limit states may control the size of the member. A general procedure for estimating these effects and the design of any required reinforcement is available for both steel and composite beams.[1,2]

The presence of holes of any type in a beam certainly does not make it stronger and in all probability weakens it somewhat. The effect of holes has been a subject that has been argued back and forth for many years. The questions, "Is the neutral axis affected by the presence of holes?" and "Is it necessary to subtract holes from the compression flange that are going to be plugged with bolts?" are frequently asked.

The theory that the neutral axis might move from its normal position to the theoretical position of its net section when bolt holes are present is rather questionable. Tests seem to show that flange holes for bolts do not appreciably change the location of the neutral axis. It is logical to assume that the location of the neutral axis will not follow the exact theoretical variation with its abrupt changes in position at bolt holes, as shown in part (b) of Fig. 9.5. A more reasonable change in neutral axis location is shown in part (c) of this figure, where it is assumed to have a more gradual variation in position.

It is interesting to note that flexure tests of steel beams seem to show that their failure is based on the strength of the compression flange, even though there may be bolt holes in the tension flange. The presence of these holes does not seem to be as serious as might be thought, particularly when compared with holes in a pure tension member. These tests show little difference in the strengths of beams with no holes and in beams with an appreciable number of bolt holes in either flange.

Bolt holes in the webs of beams are generally considered to be insignificant, as they have almost no effect on Z calculations.

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[1] D. Darwin, "*Steel and Composite Beams with Web Openings*," AISC Design Guide Series No. 2 (Chicago: American Institute of Steel Construction, 1990).

[2] ASCE Task Committee on Design Criteria for Composite Structures in Steel and Concrete, "Proposed Specification for Structural Steel Beams with Web Openings," D. Darwin, Chairman, *Journal of Structural Engineering*, ASCE, vol. 118 (New York: ASCE, December, 1992).

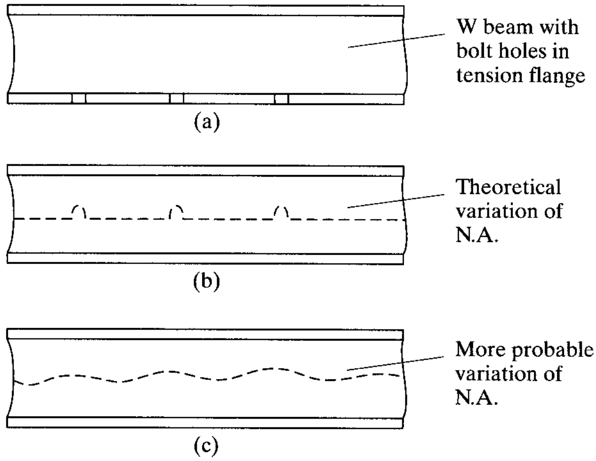


FIGURE 9.5

Some specifications, notably the bridge ones, and some designers have not adopted the idea of ignoring the presence of all or part of the holes in tension flanges. As a result, they follow the more conservative practice of deducting 100 percent of all holes. For such a case, the reduction in Z_x will equal the statical moment of the holes (in both flanges) taken about the neutral axis. If we have bolt holes in the compression flanges only and they are filled with bolts, we forget the whole thing. This is because it is felt that the fasteners can adequately transfer compression through the holes by means of the bolts.

The flexural strengths of beams with holes in their tension flanges are predicted by comparing the value F_yA_{fg} with F_uA_{fn}. In these expressions, A_{fg} is the gross area of the tension flange while A_{fn} is the net tension flange area after the holes are subtracted. In the expressions given herein for computing M_n, there is a term Y_t, which is called the hole reduction coefficient. Its value is taken as 1.0 if $F_y/F_u \leq 0.8$. For cases when the ratio of F_y/F_u is >0.8, Y_t is taken as 1.1.[3,4]

a. If $F_uA_{fn} \geq Y_tF_yA_{fg}$, the limit state of tensile rupture does not apply and there is no reduction in M_n because of the holes.

b. If $F_uA_{fn} < Y_tF_yA_{fg}$, the nominal flexural strength of the member at the holes is to be determined by the following expression, in which S_x is the section modulus of the member:

$$M_n = \frac{F_uA_{fn}}{A_{fg}}S_x \qquad \text{(AISC Equation F13-1)}$$

[3]R.J. Dexter and S.A. Altstadt, "Strength and Ductility of Tension Flanges in Girders," Proceedings of the Second New York City Bridge Conference (New York, 2003).

[4]Q. Yuan, J. Swanson and G.A. Rassati, "An Investigation of Hole Making Practices in the Fabrication of Structural Steel" (University of Cincinnati, Cincinnati, OH, 2004).

Example 9-4

Determine $\phi_b M_n$ and $\dfrac{M_n}{\Omega_b}$ for the W24 × 176 ($F_y = 50$ ksi, $F_u = 65$ ksi) beam shown in Fig. 9.6 for the following situations:

a. Using the AISC Specification and assuming two lines of 1-in bolts in standard holes in each flange (as shown in Fig. 9.6).
b. Using the AISC Specification and assuming four lines of 1-in bolts in standard holes in each flange.

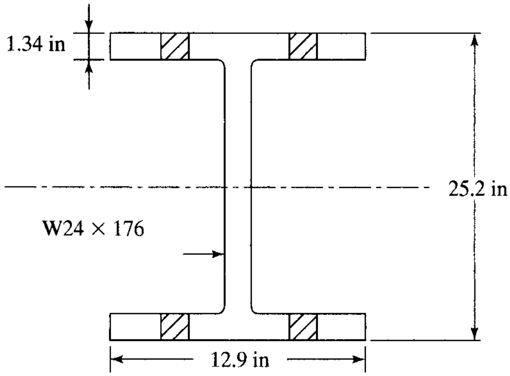


FIGURE 9.6

Solution. Using a W24 × 176 ($b_f = 12.9$ in, $t_f = 1.34$ in and $S_x = 450$ in^3)

a.

$$A_{fg} = b_f t_f = (12.9\text{ in})(1.34\text{ in}) = 17.29\text{ in}^2$$

$$A_{fn} = 17.29\text{ in}^2 - (2)\left(1\tfrac{1}{8}\text{ in}\right)(1.34\text{ in}) = 14.27\text{ in}^2$$

$$F_u A_{fn} = (65\text{ ksi})(14.27\text{ in}^2) = 927.6\text{ k}$$

$$\frac{F_y}{F_u} = \frac{50}{65} = 0.77 < 0.8 \quad \therefore\ Y_t = 1.0$$

$$927.6\text{ k} > Y_t F_y A_{fg} = (1.0)(50\text{ ksi})(17.29\text{ in}^2) = 864.5\text{ k}$$

$\therefore$ **Tensile rupture does not apply and $\boldsymbol{\phi_b M_{px} = 1920}$ ft-k and $\boldsymbol{\dfrac{M_{px}}{\Omega_b} = 1270}$ ft-k from AISC Table 3-2.**

b.

$$A_{fn} = 17.29\text{ in}^2 - (4)\left(1\tfrac{1}{8}\text{ in}\right)(1.34\text{ in}) = 11.26\text{ in}^2$$

$$\frac{F_y}{F_u} = \frac{50}{65} = 0.77 \quad \therefore\ Y_t = 1.0$$

$$F_u A_{fn} = (65\text{ ksi})(11.26\text{ in}^2) = 731.9\text{ k}$$
$$< Y_t F_y A_{fg} = (1.0)(50\text{ ksi})(17.29\text{ in}^2) = 864.5\text{ k}$$

$\therefore$ Tensile rupture expression does apply.

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{(65\text{ ksi})(11.26\text{ in}^2)(450\text{ in}^3)}{(17.29\text{ in}^2)} = 19{,}048\text{ in-k} = 1587.4\text{ ft-k}$$

LRFD $\phi_b = 0.9$	ASD $\Omega_b = 1.67$
$\phi_b M_n = (0.9)(1587.4)$	$\frac{M_n}{\Omega_b} = \frac{1587.4}{1.67}$
= 1429 ft-k	**= 951 ft-k**

Should a hole be present in only one side of a flange of a W section, there will be no axis of symmetry for the net section of the shape. A correct theoretical solution of the problem would be very complex. Rather than going through such a lengthy process over a fairly minor point, it seems logical to assume that there are holes in both sides of the flange. The results obtained will probably be just as satisfactory as those obtained by a laborious theoretical method.

9.4 LATERAL SUPPORT OF BEAMS

Probably most steel beams are used in such a manner that their compression flanges are restrained against lateral buckling. (Unfortunately, however, the percentage has not been quite as high as the design profession has assumed.) The upper flanges of

An ironworker awaits the lifting of a steel beam that will be field bolted to the column. (Courtesy of CMC South Carolina Steel.)

beams used to support concrete building and bridge floors are often incorporated in these concrete floors. For situations of this type, where the compression flanges are restrained against lateral buckling, the beams will fall into Zone 1.

Should the compression flange of a beam be without lateral support for some distance, it will have a stress situation similar to that existing in columns. As is well known, the longer and slenderer a column becomes, the greater becomes the danger of its buckling for the same loading condition. When the compression flange of a beam is long enough and slender enough, it may quite possibly buckle, unless lateral support is provided.

There are many factors affecting the amount of stress that will cause buckling in the compression flange of a beam. Some of these factors are properties of the material, the spacing and types of lateral support provided, residual stresses in the sections, the types of end support or restraints, the loading conditions, etc.

The tension in the other flange of a beam tends to keep that flange straight and restrain the compression flange from buckling; but as the bending moment is increased, the tendency of the compression flange to buckle may become large enough to overcome the tensile restraint. When the compression flange does begin to buckle, twisting or torsion will occur, and the smaller the torsional strength of the beam the more rapid will be the failure. The W, S, and channel shapes so frequently used for beam sections do not have a great deal of resistance to lateral buckling and the resulting torsion. Some other shapes—notably, the built-up box shapes—are tremendously stronger. These types of members have a great deal more torsional resistance than the W, S, and plate girder sections. Tests have shown that they will not buckle laterally until the strains developed are well in the plastic range.

Some judgment needs to be used in deciding what does and what does not constitute satisfactory lateral support for a steel beam. Perhaps the most common question asked by practicing steel designers is, "What is lateral support?" A beam that is wholly encased in concrete or that has its compression flange incorporated in a concrete slab is certainly well supported laterally. When a concrete slab rests on the top flange of a beam, the engineer must study the situation carefully before he or she counts on friction to provide full lateral support. Perhaps if the loads on the slab are fairly well fixed in position, they will contribute to the friction and it may be reasonable to assume full lateral support. If, on the other hand, there is much movement of the loads and appreciable vibration, the friction may well be reduced and full lateral support cannot be assumed. Such situations occur in bridges due to traffic, and in buildings with vibrating machinery such as printing presses.

Should lateral support of the compression flange not be provided by a floor slab, it is possible that such support may be provided with connecting beams or with special members inserted for that purpose. Beams that frame into the sides of the beam or girder in question and are connected to the compression flange can usually be counted on to provide full lateral support at the connection. If the connection is made primarily to the tensile flange, little lateral support is provided to the compression flange. Before support is assumed from these beams, the designer should note whether the beams themselves are prevented from moving. The beams represented with horizontal dashed lines in Fig. 9.7 provide questionable lateral support for the main beams between columns. For a situation of this type, some system of x-bracing may be desirable in one of the bays. Such a system is shown in Fig. 9.7. This one system will provide sufficient lateral support for the beams for several bays.

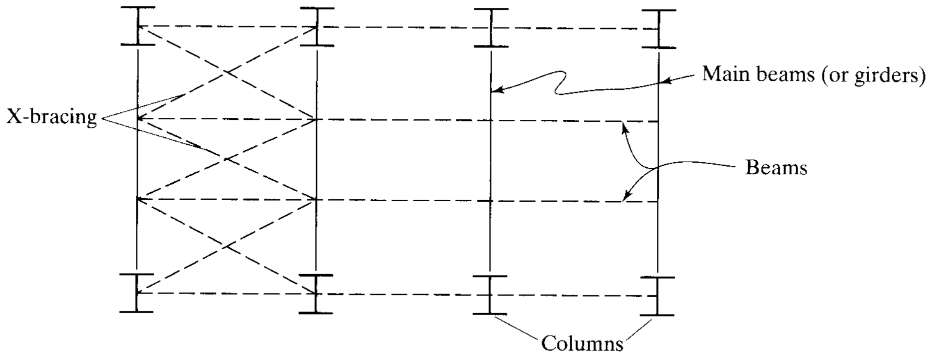


FIGURE 9.7

X-bracing for floor system.

The intermittent welding of metal roof or floor decks to the compression flanges of beams will probably provide sufficient lateral bracing. The corrugated sheet-metal roofs that are usually connected to the purlins with metal straps probably furnish only partial lateral support. A similar situation exists when wood flooring is bolted to supporting steel beams. At this time, the student quite naturally asks, "If only partial support is available, what am I to consider to be the distance between points of lateral support?" The answer to this question is for the student to use his or her judgment. As an illustration, a wood floor is assumed to be bolted every 4 ft to the supporting steel beams in such a manner that, it is thought, only partial lateral support is provided at those points. After studying the situation, the engineer might well decide that the equivalent of full lateral support at 8-ft intervals is provided. Such a decision seems to be within the spirit of the Specification.

Should there be doubt in the designer's mind as to the degree of lateral support provided, he or she should probably be advised to assume that there is none.

The reader should carefully study the provisions of Section C1 and Appendix 6 of the AISC Specification regarding stability bracing for beams and columns. In this Appendix, values are provided for calculating necessary bracing strengths and stiffnesses, and design formulas are given for obtaining those values. Included are various types of bracing for columns as well as torsional bracing for flexural members.

Two categories of bracing are considered in the Appendix: relative bracing and nodal bracing. With relative bracing a particular point is restrained in relation to another point or points. In other words, relative bracing is connected not only to the member to be braced but to other members as well (diagonal cross bracing, for example). Nodal bracing is used to prevent lateral movement or twisting of a member independently of other braces.

In Section 10.9 of this text, lateral bracing for the ends of beams supported on bearing plates is discussed.

9.5 INTRODUCTION TO INELASTIC BUCKLING, ZONE 2

If intermittent lateral bracing is supplied for the compression flange of a beam section, or if intermittent torsional bracing is supplied to prevent twisting of the cross section at the bracing points such that the member can be bent until the yield strain is reached in some (but not all) of its compression elements before lateral buckling occurs, we have inelastic buckling. In other words, the bracing is insufficient to permit the member to reach a full plastic strain distribution before buckling occurs.

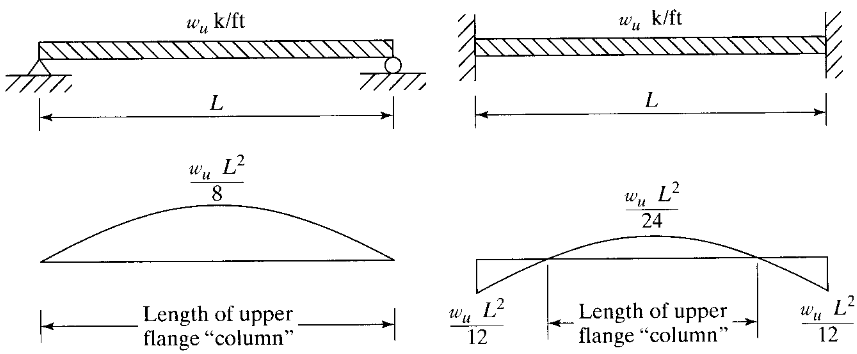
Because of the presence of residual stresses (discussed in Section 5.2), yielding will begin in a section at applied stresses equal to $F_y - F_r$, where F_y is the yield stress of the web and F_r equals the compressive residual stress. The AISC Specification estimates this value ($F_y - F_r$) to be equal to about $0.7F_y$, and we will see that value in the AISC equations. It should be noted that the definition of plastic moment F_yZ in Zone 1 is not affected by residual stresses, because the sum of the compressive residual stresses equals the sum of the tensile residual stresses in the section and the net effect is, theoretically, zero.

When a constant moment occurs along the unbraced length, L_b, of a compact I- or C-shaped section and L_b is larger than L_p, the beam will fail inelastically, unless L_b is greater than a distance L_r (to be discussed) beyond which the beam will fail elastically before F_y is reached (thus falling into Zone 3).

9.5.1 Bending Coefficients

In the formulas presented in the next few sections for inelastic and elastic buckling, we will use a term C_b, called the *lateral-torsional buckling modification factor* for nonuniform moment diagrams, when both ends of the unsupported segment are braced. This is a moment coefficient that is included in the formulas to account for the effect of different moment gradients on lateral-torsional buckling. In other words, lateral buckling may be appreciably affected by the end restraint and loading conditions of the member.

As an illustration, the reader can see that the moment in the unbraced beam of part (a) of Fig. 9.8 causes a worse compression flange situation than does the moment in the unbraced beam of part (b). For one reason, the upper flange of the beam in part (a) is in compression for its entire length, while in (b) the length of the "column"—that is, the length of the upper flange that is in compression—is much less (thus, in effect, a much shorter "column").



(a) Single curvature

(b) Reverse curvature

FIGURE 9.8

For the simply supported beam of part (a) of the figure, we will find that C_b is 1.14, while for the beam of part (b) it is 2.38 (see Example 9-5). The basic moment capacity equations for Zones 2 and 3 were developed for laterally unbraced beams subject to single curvature, with $C_b = 1.0$. Frequently, beams are not bent in single curvature, with the result that they can resist more moment. We have seen this in Fig. 9.8. To handle this situation, the AISC Specification provides moment or C_b coefficients larger than 1.0 that are to be multiplied by the computed M_n values. The results are higher moment capacities. The designer who conservatively says, "I'll always use $C_b = 1.0$ in is missing out on the possibility of significant savings in steel weight for some situations. *When using C_b values, the designer should clearly understand that the moment capacity obtained by multiplying M_n by C_b may not be larger than the plastic M_n of Zone 1, which is M_p* and is *equal to $F_y Z$*. This situation is illustrated in Fig. 9.9.

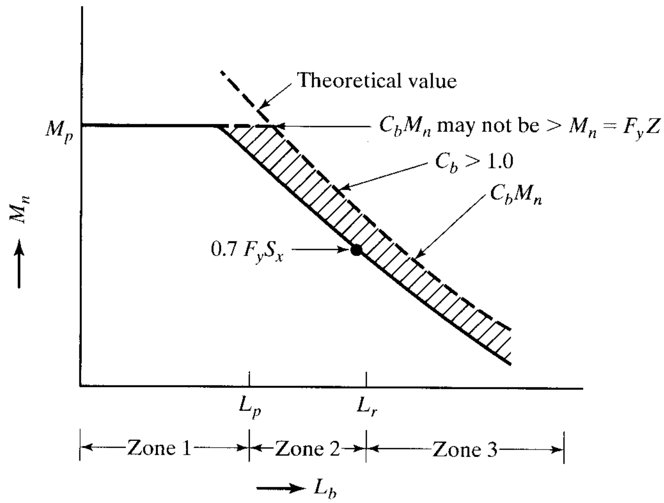


FIGURE 9.9

The value of C_b for singly symmetric members in single curvature and all doubly symmetric members is determined from the expression to follow, in which M_{max} is the largest moment in an unbraced segment of a beam, while M_A, M_B, and M_C are, respectively, the moments at the 1/4 point, 1/2 point, and 3/4 point in the segment:

$$C_b = \frac{12.5\, M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad \text{(AISC Equation F1-1)}$$

In singly symmetric members subject to reverse curvature bending, the lateral-torsional buckling strength shall be checked for both top and bottom flanges. A more detailed analysis for C_b of singly symmetric members is presented in the AISC Commentary, F1. General Provisions.

C_b is equal to 1.0 for cantilevers or overhangs where the free end is unbraced. Some typical values of C_b, calculated with the previous AISC Equation F1-1, are shown in Fig. 9.10 for various beam and moment situations. Some of these values are also given in Table 3-1 of the AISC Manual.

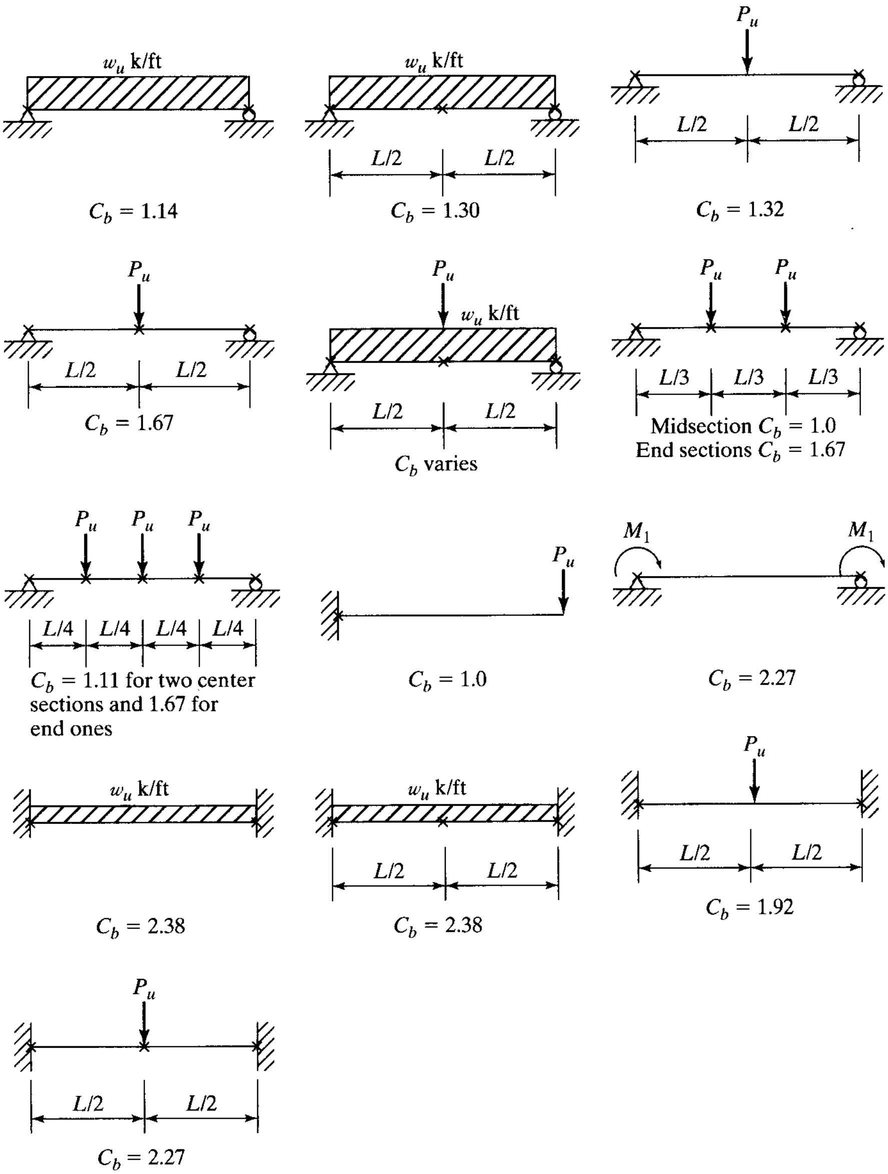


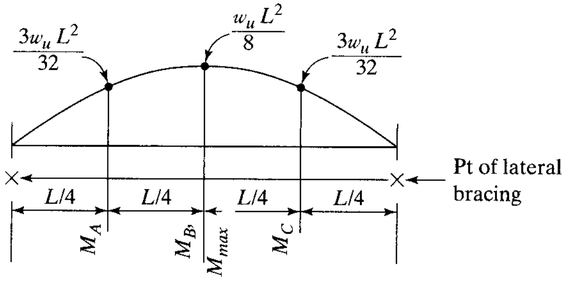
FIGURE 9.10

Sample C_b values for doubly symmetric members. (The X marks represent points of lateral bracing of the compression flange.)

Example 9-5

Determine C_b for the beam shown in Fig. 9.8 parts (a) and (b). Assume the beam is a doubly symmetric member.

a.



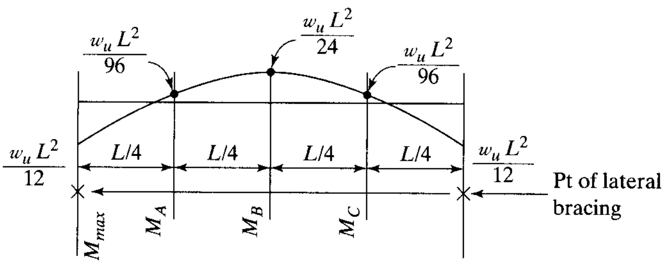
$$USE: \frac{w_u L^2}{8} = \frac{1}{8}$$

$$\frac{3w_u L^2}{32} = \frac{3}{32}$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5\left(\frac{1}{8}\right)}{2.5\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right) + 4\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right)} = 1.14$$

b.



$$USE: \frac{w_u L^2}{12} = \frac{1}{12}$$

$$\frac{w_u L^2}{96} = \frac{1}{96}$$

$$\frac{w_u L^2}{24} = \frac{1}{24}$$

$$C_b = \frac{12.5\left(\frac{1}{12}\right)}{2.5\left(\frac{1}{12}\right) + 3\left(\frac{1}{96}\right) + 4\left(\frac{1}{24}\right) + 3\left(\frac{1}{96}\right)} = 2.38$$

9.6 MOMENT CAPACITIES, ZONE 2

When constant moment occurs along the unbraced length, or as the unbraced length of the compression flange of a beam or the distance between points of torsional bracing is increased beyond L_p, the moment capacity of the section will become smaller and smaller. Finally, at an unbraced length L_r, the section will buckle elastically as soon as the yield stress is reached. Owing to the rolling operation, however, there is a residual stress in the section equal to F_r. Thus, the elastically computed stress caused by bending can reach only $F_y - F_r = 0.7F_y$. The nominal moment strengths for unbraced lengths between L_p and L_r are calculated with the equation to follow:

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(AISC Equation F2-2)}$$

L_r is a function of several of the section's properties, such as its cross-sectional area, modulus of elasticity, yield stress, and warping and torsional properties. The very complex formulas needed for its computation are given in the AISC Specification (F1), and space is not taken to show them here. Fortunately, numerical values have been determined for sections normally used as beams and are given in AISC Manual Table 3-2, entitled "W Shapes Selected by Z_x."

Going backward from an unbraced length of L_r toward an unbraced length L_p, we can see that buckling does not occur when the yield stress is first reached. We are in the inelastic range (Zone 2), where there is some penetration of the yield stress into the section from the extreme fibers. For these cases when the unbraced length falls between L_p and L_r, the nominal moment strength will fall approximately on a straight line between $M_{nx} = F_yZ_x$ at L_p and $0.7F_yS_x$ at L_r. For intermediate values of the unbraced length between L_p and L_r, we may interpolate between the end values that fall on a straight line. It probably is simpler, however, to use the expressions given at the end of this paragraph to perform the interpolation. Should C_b be larger than 1.0, the nominal moment strength will be larger, but not more than, $M_p = F_yZ_x$.

The interpolation expressions that follow are presented on page 3-8 of the AISC Manual. The bending factors (*BF*s) represent part of AISC Equation F2-2, as can be seen by comparing the equations to follow with that equation. Their numerical values in kips are given in Manual Table 3-2 for W shapes.

$$\textbf{For LRFD} \quad \phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p)] \leq \phi_b M_{px}$$

$$\textbf{For ASD} \quad \frac{M_n}{\Omega_b} = C_b\left[\frac{M_{px}}{\Omega_b} - BF(L_b - L_p)\right] \leq \frac{M_{px}}{\Omega_b}$$

Example 9-6

Determine the LRFD design moment capacity and the ASD allowable moment capacity of a W24 × 62 with $F_y = 50$ ksi, $L_b = 8.0$ ft, and $C_b = 1.0$.

Solution

Using a W24 × 62 (from AISC Table 3-2: $\phi_bM_{px} = 574$ ft-k, $M_{px}/\Omega_b = 382$ ft-k, $\phi_bM_{rx} = 344$ ft-k, $M_{rx}/\Omega_b = 229$ ft-k, $L_p = 4.87$ ft, $L_r = 14.4$ ft, *BF* for LRFD = 24.1 k, and *BF* for ASD = 16.1 k)

Noting $L_b > L_p < L_r \therefore$ falls in Zone 2, Fig. 9.1 in text.

LRFD	ASD
$\phi_b M_{nx} = C_b[\phi_b M_{px} - BF(L_b - L_p)] \leq \phi_b M_{px}$	$\dfrac{M_{nx}}{\Omega_b} = C_b\left[\dfrac{M_{px}}{\Omega_b} - \text{BF}(L_b - L_p)\right] \leq \dfrac{M_{px}}{\Omega_b}$
$\phi_b M_{nx} = 1.0[574 - 24.1(8.0 - 4.87)]$ $= \mathbf{499\ ft\text{-}k} < 574\text{ ft-k}$	$\dfrac{M_{nx}}{\Omega_b} = 1.0[382 - 16.1(8.0 - 4.87)]$ $= \mathbf{332\ ft\text{-}k} < 382\text{ ft-k}$
$\therefore \phi_b M_{nx} = 499\text{ ft-k}$	$\therefore \dfrac{M_{nx}}{\Omega} = 332\text{ ft-k}$

9.7 ELASTIC BUCKLING, ZONE 3

When the unbraced length of a beam is greater than L_r, the beam will fall in Zone 3. Such a member may fail due to buckling of the compression portion of the cross section laterally about the weaker axis, with twisting of the entire cross section about the beam's longitudinal axis between the points of lateral bracing. This will occur even though the beam is loaded so that it supposedly will bend about the stronger axis. The beam will bend initially about the stronger axis until a certain critical moment M_{cr} is reached. At that time, it will buckle laterally about its weaker axis. As it bends laterally, the tension in the other flange will try to keep the beam straight. As a result, the buckling of the beam will be a combination of lateral bending and a twisting (or torsion) of the beam cross section. A sketch of this situation is shown in Fig. 9.11.

The critical moment, or flexural-torsional moment M_{cr} in a beam will be made up of the torsional resistance (commonly called *St-Venant torsion*) and the warping resistance of the section.

If the unbraced length of the compression flange of a beam section or the distance between points that prevent twisting of the entire cross section is greater than L_r,

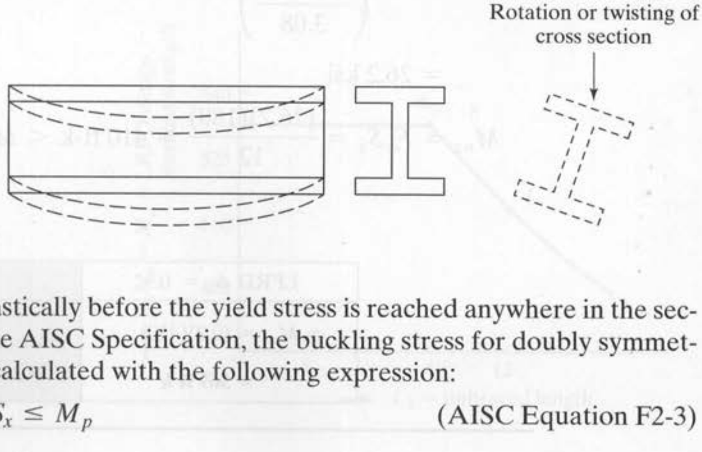


FIGURE 9.11

Lateral-torsional buckling of a simply supported beam.

the section will buckle elastically before the yield stress is reached anywhere in the section. In Section F2.2 of the AISC Specification, the buckling stress for doubly symmetric I-shaped members is calculated with the following expression:

$$M_n = F_{cr}S_x \leq M_p \qquad \text{(AISC Equation F2-3)}$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{(AISC Equation F2-4)}$$

In this calculation,

r_{ts} = effective radius of gyration, in (provided in AISC Table 1-1)

J = torsional constant, in^4 (AISC Table 1-1)

c = 1.0 for doubly symmetric I-shapes

h_o = distance between flange centroids, in (AISC Table 1-1)

It is not possible for lateral-torsional buckling to occur if the moment of inertia of the section about the bending axis is equal to or less than the moment of inertia out of plane. For this reason the limit state of lateral-torsional buckling is not applicable for shapes bent about their minor axes, for shapes with $I_x \leq I_y$, or for circular or square shapes. Furthermore, yielding controls if the section is noncompact.

Example 9-7

Using AISC Equation F2-4, determine the values of $\phi_b M_{nx}$ and M_{nx}/Ω_b for a W18 × 97 with F_y = 50 ksi and an unbraced length L_b = 38 ft. Assume that C_b = 1.0.

Solution

Using a W18 × 97 (L_r = 30.4 ft, r_{ts} = 3.08 in, J = 5.86 in^4, c = 1.0 for doubly symmetric I section, S_x = 188 in^3, h_o = 17.7 in and Z_x = 211 in^3)

Noting L_b = 38 ft > L_r = 30.4 ft (from AISC Table 3-2), section is in Zone 3.

$$F_{cr} = \frac{(1.0)(\pi)^2(29 \times 10^3)}{\left(\frac{12 \times 38}{3.08}\right)^2} \sqrt{1 + (0.078)\frac{(5.86)(1.0)}{(188)(17.7)}\left(\frac{12 \times 38}{3.08}\right)^2}$$

$$= 26.2 \text{ ksi}$$

$$M_{nx} = F_{cr} S_x = \frac{(26.2)(188)}{12} = 410 \text{ ft-k} < M_p = \frac{(50)(211)}{12} = 879 \text{ ft-k}$$

LFRD $\phi_b = 0.9$	ASD $\Omega_b = 1.67$
$\phi_b M_{nx} = (0.9)(410)$	$\frac{M_{nx}}{\Omega_b} = \frac{410}{1.67}$
= 369 ft-k	**= 246 ft-k**

9.8 DESIGN CHARTS

Fortunately, the values of $\phi_b M_n$ and M_n/Ω_b for sections normally used as beams have been computed by the AISC, plotted for a wide range of unbraced lengths, and shown as Table 3-10 in the AISC Manual. These diagrams enable us to solve any of the problems previously considered in this chapter in just a few seconds.

The values provided cover unbraced lengths in the plastic range, in the inelastic range, and on into the elastic buckling range (Zones 1–3). They are plotted for $F_y = 50$ ksi and $C_b = 1.0$.

The LRFD curve for a typical W section is shown in Fig. 9.12. For each of the shapes, L_p is indicated with a solid circle (•), while L_r is shown with a hollow circle (○).

The charts were developed without regard to such things as shear, deflection, etc.—items that may occasionally control the design, as described in Chapter 10. They cover almost all of the unbraced lengths encountered in practice. If C_b is greater than 1.0, the values given will be magnified somewhat, as illustrated in Fig. 9.9.

To select a member, it is necessary to enter the chart only with the unbraced length L_b and the factored design moment M_u or the ASD moment M_a. For an illustration, let's assume that $C_b = 1.0$, $F_y = 50$ ksi and that we wish to select a beam with $L_b = 18$ ft, $M_u = 544$ ft-k (or $M_a = 362.7$ ft-k). For this problem, the appropriate page from AISC Table 3-10 is shown in Fig. 9.13, with the permission of the AISC.

First, for the LRFD solution, we proceed up from the bottom of the chart for an unbraced length $L_b = 18$ ft until we intersect a horizontal line from the ϕM_n column for $M_u = 544$ ft-k. Any section to the right and above this intersection point (↗) will have a greater unbraced length and a greater design moment capacity.

Moving up and to the right, we first encounter the W16 × 89 and W14 × 90 sections. In this area of the charts, these sections are shown with dashed lines. The dashèd lines indicate that the sections will provide the necessary moment capacities, but are in an uneconomical range. If we proceed further upward and to the right, the first solid line encountered will represent the lightest satisfactory section. In this case, it is a W24 × 84. For an ASD solution of the same problem, we enter the chart with $L_b = 18$ ft and $M_a = 362.7$ ft-k and use the left column entitled M_n/Ω. The result, again, is a W24 × 84. Other illustrations of these charts are presented in Examples 9-8 to 9-10.

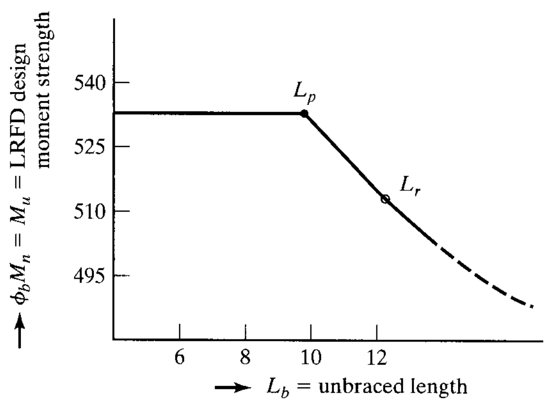
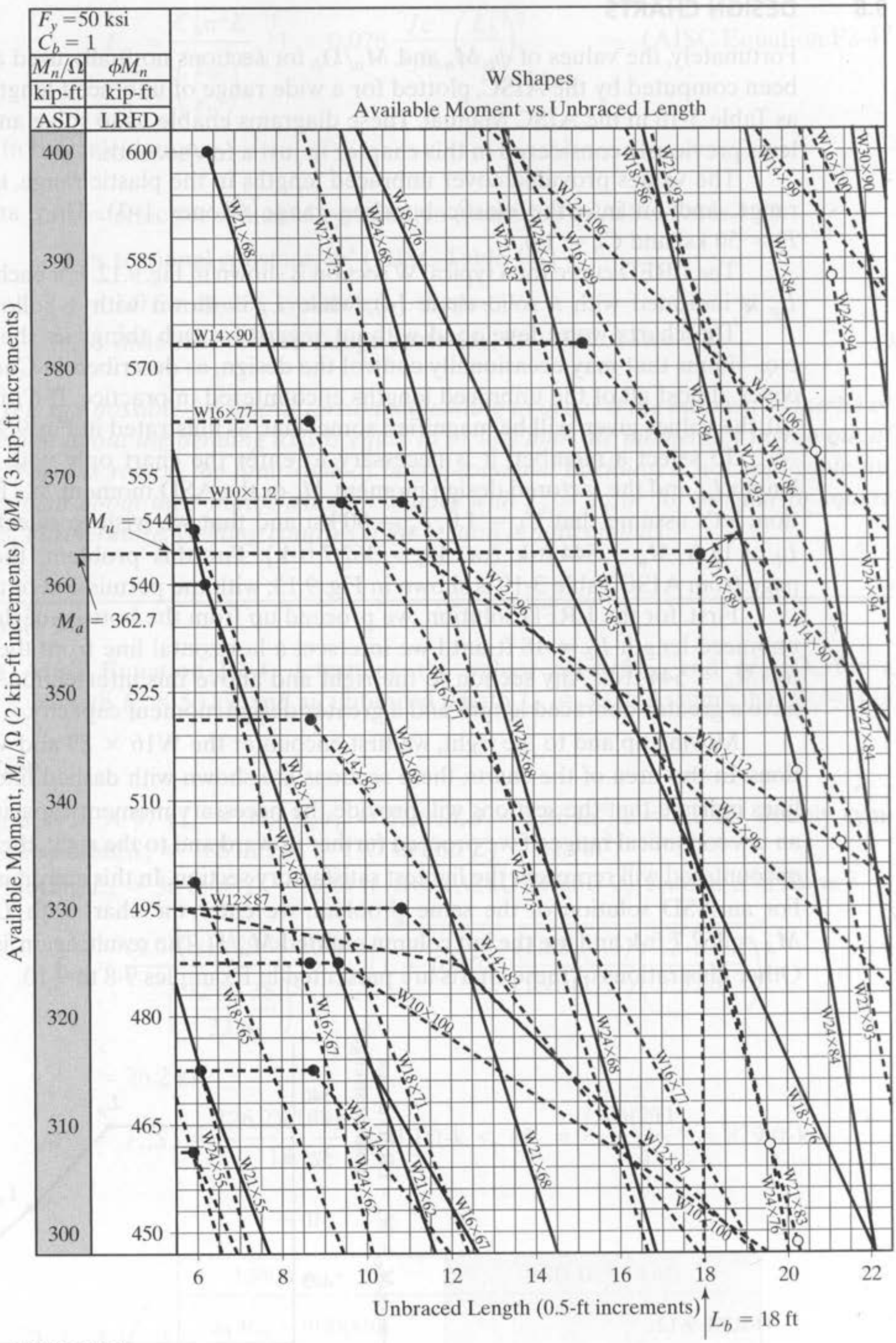


FIGURE 9.12

LRFD design moment for a beam plotted versus unbraced length, L_b.



F_y = 50 ksi
C_b = 1
M_n/Ω kip-ft ASD
ϕM_n kip-ft LRFD
W Shapes
Available Moment vs. Unbraced Length
Available Moment, M_n/Ω (2 kip-ft increments) ϕM_n (3 kip-ft increments)
Unbraced Length (0.5-ft increments)
M_u = 544
M_a = 362.7
L_b = 18 ft
400 600
390 585
380 570
370 555
360 540
350 525
340 510
330 495
320 480
310 465
300 450
6
8
10
12
14
16
18
20
22
W14×90
W16×77
W10×112
W12×87
W21×68
W21×73
W24×68
W18×76
W21×83
W24×76
W12×106
W16×89
W18×86
W21×93
W14×99
W16×100
W30×90
W27×84
W24×94
W24×84
W12×96
W14×90
W10×112
W16×77
W21×68
W14×82
W18×71
W24×62
W21×62
W16×67
W18×65
W24×55
W21×55
W14×74
W21×75
W10×100
W24×68
W12×87
W21×83
W24×76
W27×84
W21×93
W18×76

FIGURE 9.13

Example 9-8

Using 50 ksi steel, select the lightest available section for the beam of Fig. 9.14, which has lateral bracing provided for its compression flange, only at its ends. Assume that $C_b = 1.00$ for this example. (It's actually 1.14.) Use both LRFD and ASD methods.

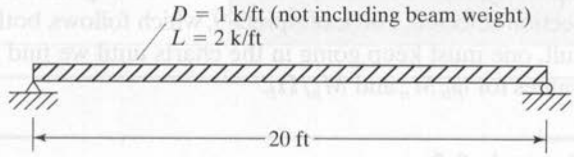


FIGURE 9.14

Solution

LRFD	ASD
Neglect beam wt initially then check after member selection is made	
$w_u = 1.2\,(1.0\text{ k/ft}) + 1.6\,(2.0\text{ k/ft}) = 4.4\text{ k/ft}$	$w_a = 1.0\text{ k/ft} + 2.0\text{ k/ft} = 3.0\text{ k/ft}$
$M_u = \dfrac{(4.4\text{ k/ft})\,(20\text{ ft})^2}{8} = 220\text{ ft-k}$	$M_a = \dfrac{(3.0\text{ k/ft})\,(20\text{ ft})^2}{8} = 150\text{ ft-k}$
Enter AISC Table 3-10 with $L_b = 20$ ft and $M_u = 220$ ft-k	Enter AISC Table 3-10 with $L_b = 20$ ft and $M_a = 150$ ft-k
Try W12 × 53	Try W12 × 53
Add self wt of 53 lb/ft	Add self wt of 53 lb/ft
$w_u = 1.2\,(1.053\text{ k/ft}) + 1.6\,(2.0\text{ k/ft}) = 4.46\text{ k/ft}$	$w_a = 1.053\text{ k/ft} + 2.0\text{ k/ft} = 3.05\text{ k/ft}$
$M_u = \dfrac{(4.46\text{ k/ft})\,(20\text{ ft})^2}{8} = 223\text{ ft-k}$	$M_a = \dfrac{(3.05\text{ k/ft})\,(20\text{ ft})^2}{8} = 153\text{ ft-k}$
Re-enter AISC Table 3-10	Re-enter AISC Table 3-10
Use W12 × 53.	**Use W12 × 53.**
$\phi M_n = 230.5\text{ ft-k} \geq M_u = 223\text{ ft-k}$ **OK**	$\dfrac{M_n}{\Omega} = 153.6\text{ ft-k} \geq M_a = 153\text{ ft-k}$ **OK**

Note: ϕM_n and M_n/Ω may be calculated from AISC Equations or more conveniently read from Table 3-10. To obtain the value of ϕM_n or M_n/Ω, proceed up from the bottom of the chart for an $L_b = 20$ ft until we intersect the line for the W12 × 53 member. Turn left and proceed with a horizontal line and read the value of either ϕM_n or M_n/Ω from the vertical axis.

For the example problem that follows, C_b is greater than 1.0. For such a situation, the reader should look back to Fig. 9.9. There, he or she will see that the design moment strength of a section can go to $\phi_b C_b M_n$ when $C_b > 1.0$, **but may under no circumstances exceed $\phi_b M_p = \phi_b F_y Z$, nor may M_a exceed M_p/Ω_b for the section.**

To handle such a problem, we calculate an effective moment, as shown next. (The numbers are taken from Example 9-9, which follows. Notice $C_b = 1.67$.

$$M_{u\,\text{effective}} = \frac{850}{1.67} = 509 \text{ ft-k and } M_{a\,\text{effective}} = \frac{595}{1.67} = 356 \text{ ft-k}$$

Then we enter the charts with $L_b = 17$ ft, and with M_u effective = 509 ft-k or M_a effective = 356 ft-k, and select a section. We must, however, make sure that, for LRFD design, the calculated M_u, does not exceed $\phi_b M_n = \phi F_y Z$ for the section selected. Similarly, for ASD design, the calculated M_a must not exceed $M_n/\Omega_b = F_y Z/\Omega_b$ for the section selected. For Example 9-9, which follows, both of the values are exceeded. As a result, one must keep going in the charts until we find a section that provides the necessary values for $\phi_b M_n$ and M_n/Ω_b.

Example 9-9

Using 50 ksi steel and both the LRFD and ASD methods, select the lightest available section for the situation shown in Fig. 9.15. Bracing is provided only at the ends and center line of the member, and thus, $L_b = 17$ ft.

Using Fig. 9.10, C_b is 1.67 if the only uniform load is the member self-weight and it is neglected. If the self-weight is considered then C_b will be between 1.67 and 1.30. Since the self-weight is a small portion of the design moment, C_b is near the value of 1.67 and using it would be a reasonable assumption.

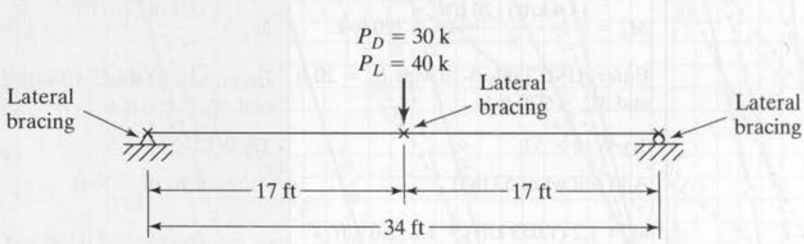


FIGURE 9.15

Solution

LRFD	ASD
Neglect beam wt initially then check after member selection is made	
$P_u = 1.2\,(30\text{ k}) + 1.6\,(40\text{ k}) = 100\text{ k}$	$P_a = 30\text{ k} + 40\text{ k} = 70\text{ k}$
$M_u = \dfrac{100\text{ k}\,(34\text{ ft})}{4} = 850\text{ ft-k}$	$M_a = \dfrac{70\text{ k}\,(34\text{ ft})}{4} = 595\text{ ft-k}$
Enter AISC Table 3-10 with $L_b = 17$ ft and M_u effective $= \dfrac{850}{1.67} = 509$ ft-k	Enter AISC Table 3-10 with $L_b = 17$ ft and M_a effective $= \dfrac{595}{1.67} = 356$ ft-k
Try W24 × 76 ($\phi_b M_p$ from AISC Table 3-2 = 750 ft-k < M_u = 850 ft-k **N.G.**)	Try W24 × 84 $\left(\dfrac{M_p}{\Omega_b} = 559\text{ ft-k from AISC Table 3-2} < M_a = 595\text{ ft-k } \textbf{N.G.}\right)$
Try W27 × 84 ($\phi_b M_p$ = 915 ft-k) Add self-weight of 84 lb/ft	Try W27 × 84 $\left(\dfrac{M_p}{\Omega_b} = 609\text{ ft-k}\right)$ Add self-weight of 84 lb/ft

$w_u = 1.2\ (0.084\ \text{k/ft}) = 0.101\ \text{k/ft}$	$w_a = 0.084\ \text{k/ft}$
$M_u = \dfrac{(0.101\ \text{k/ft})\ (34\ \text{ft})^2}{8} + \dfrac{100\ \text{k}\ (34\ \text{ft})}{4}$	$M_a = \dfrac{(0.084\ \text{k/ft})\ (34\ \text{ft})^2}{8} + \dfrac{70\ \text{k}\ (34\ \text{ft})}{4}$
$M_u = 865\ \text{ft-k} < \phi_b M_p = 915\ \text{ft-k}$ **OK**	$M_a = 607\ \text{ft-k} < \dfrac{M_p}{\Omega_b} = 609\ \text{ft-k}$ **OK**
Use W27 × 84.	**Use W27 × 84.**

Example 9-10

Using 50 ksi steel and both the LRFD and ASD methods, select the lightest available section for the situation shown in Fig. 9.16. Bracing is provided only at the ends and at midspan.

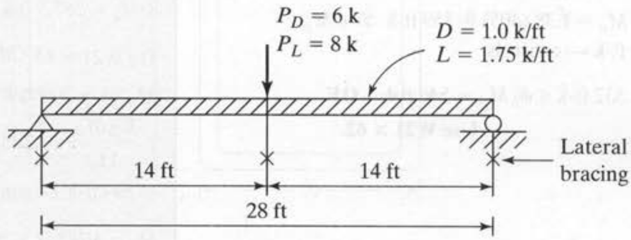
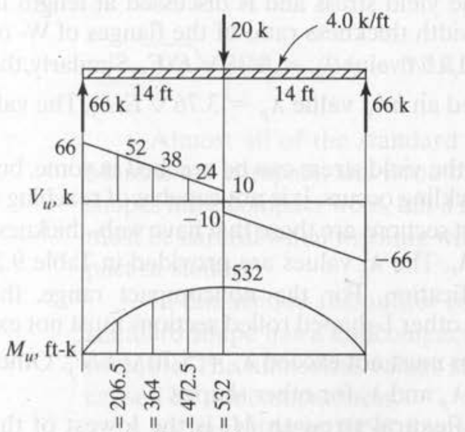
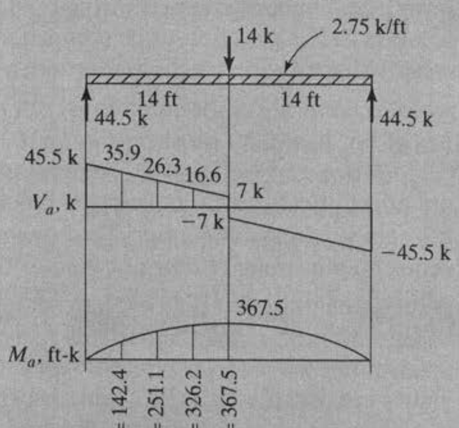


FIGURE 9.16

Solution

LRFD	ASD
$P_u = 1.2\ (6\ \text{k}) + 1.6\ (8\ \text{k}) = 20\ \text{k}$	$P_a = 6\ \text{k} + 8\ \text{k} = 14\ \text{k}$
$W_u = 1.2\ (1.0\ \text{k/ft}) + 1.6\ (1.75\ \text{k/ft}) = 4.0\ \text{k/ft}$	$W_a = 1.0\ \text{k/ft} + 1.75\ \text{k/ft} = 2.75\ \text{k/ft}$
	

$$C_b = \frac{12.5\ M_{max}}{2.5\ M_{max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5(532)}{2.5(532) + 3(206.5) + 4(364) + 3(472.5)}$$
$$= 1.38$$

Enter AISC Table 3-10 with $L_b = 14$ ft and

$$M_u \text{ effective} = \frac{532}{1.38} = 386 \text{ ft-k}$$

Try W21 × 62 ($\phi_b M_n = 405$ ft-k from Table 3-10, $\phi_b M_p$ = 540 ft-k from Table 3-2)

$C_b\ \phi_b\ M_n = 1.38\ (405) = 559$ ft-k $\not\leq \phi M_p$
= 540 ft-k ← controls

$M_u = 532$ ft-k $\leq \phi_b M_p = 540$ ft-k **OK**

Use W21 × 62.

$$C_b = \frac{12.5\ M_{max}}{2.5\ M_{max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5(367.5)}{2.5\ (367.5) + 3(142.4) + 4(251.1) + 3(326.2)}$$
$$= 1.38$$

Enter AISC Table 3-10 with $L_b = 14$ ft and

$$M_a \text{ effective} = \frac{367.5}{1.38} = 267 \text{ ft-k}$$

Try W21 × 62 ($M_n/\Omega_b = 270$ ft-k from Table 3.10,

$M_p/\Omega_b = 359$ ft-k $< M_u = 367.5$ ft-k **N.G.**

Try W24 × 62 ($M_p/\Omega_b = 382$ ft-k from Table 3-2)

$M_n/\Omega_b = 236$ ft-k from Table 3-10 with $C_b = 1.0$

$$\therefore \frac{C_b M_n}{\Omega_b} = 1.38(236) = 326 \text{ ft-k}$$

$< M_a = 367.5$ ft-k **N.G.**

Try W21 × 68 ($M_p/\Omega_b = 399$ ft-k from Table 3-2)

$M_n/\Omega = 304$ ft-k from Table 3-10 with $C_b = 1.0$

$$\therefore \frac{C_b M_n}{\Omega_b} = 1.38\ (304) = 420 \text{ ft-k} > M_p/\Omega_b$$

= 399 ft-k ← controls

$$M_a = 367.5 \text{ ft-k} \leq \frac{M_p}{\Omega_b} = 399 \text{ ft-k} \quad \textbf{OK}$$

Use W21 × 68.

9.9 NONCOMPACT SECTIONS

A compact section is a section that has a sufficiently stocky profile so that it is capable of developing a fully plastic stress distribution before buckling locally (web or flange). The term *plastic* means stressed throughout to the yield stress and is discussed at length in Chapter 8. For a section to be compact, the width thickness ratio of the flanges of W- or other I-shaped rolled sections must not exceed a b/t value $\lambda_p = 0.38\sqrt{E/F_y}$. Similarly, the webs in flexural compression must not exceed an h/t_w value $\lambda_p = 3.76\sqrt{E/F_y}$. The values of b, t, h, and t_w are shown in Fig. 9.17.

A noncompact section is one for which the yield stress can be reached in some, but not all, of its compression elements before buckling occurs. It is not capable of reaching a fully plastic stress distribution. The noncompact sections are those that have web–thickness ratios greater than λ_p, but not greater than λ_r. The λ_r values are provided in Table 9.2, which is Table B4.1b of the AISC Specification. For the noncompact range, the width–thickness ratios of the flanges or W- or other I-shaped rolled sections must not exceed $\lambda_r = 1.0\sqrt{E/F_y}$, while those for the webs must not exceed $\lambda_r = 5.70\sqrt{E/F_y}$. Other values are provided in AISC Table B4.1b for λ_p and λ_r for other shapes.

For noncompact beams, the nominal flexural strength M_n is the lowest of the lateral-torsional buckling strength, the compression flange local buckling strength, or the web local buckling strength.

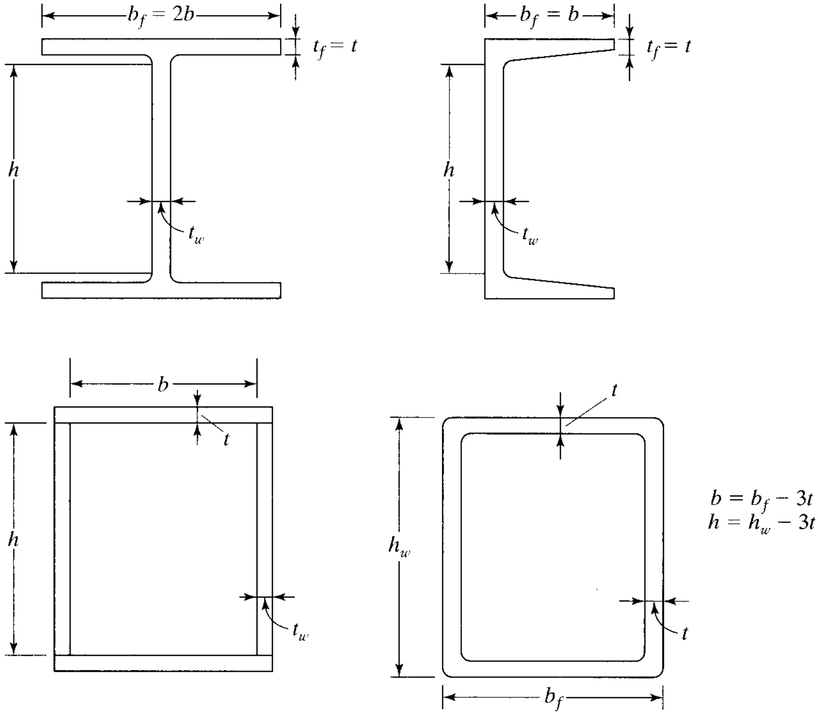


FIGURE 9.17
Values of h, b, t, and t_w to be used for computing λ = width–thickness ratios.

If we have a section with noncompact flanges—that is, one where $\lambda_p < \lambda \leq \lambda_r$ the value of M_n is given by the equation to follow, in which $k_c = 4/\sqrt{h/t_w} \geq 0.35 \leq 0.76$:

$$M_n = \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \qquad \text{(AISC Equation F3-1)}$$

Almost all of the standard hot-rolled W, M, S, and C shapes listed in the AISC Manual are compact, and none of them fall into the slender classification. All of these shapes have compact webs, but a few of them have noncompact flanges. We particularly must be careful when working with built-up sections as they may very well be noncompact or slender.

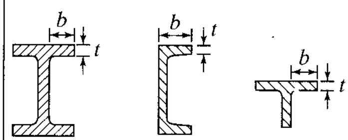
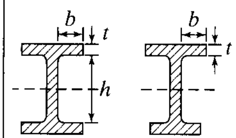
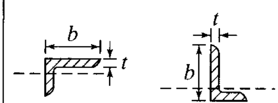
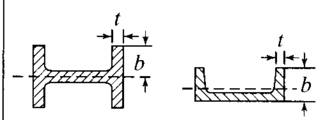
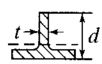
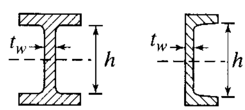
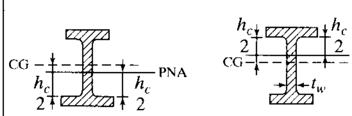
In this section the author considers a shape which has a noncompact flange. If a standard shape has a noncompact flange, it will be indicated in the Manual with an "f" footnote. The numerical values shown in the tables are based on the reduced stresses caused by noncompactness.

As indicated in AISC Specification F3 the flange of a member is noncompact if $\lambda_p < \lambda \leq \lambda_r$ and the member will buckle inelastically. These values are given for different shapes in AISC Specification Table B4.1b.

For built-up sections with slender flanges (that is, where $\lambda > \lambda_r$),

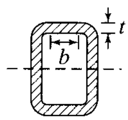
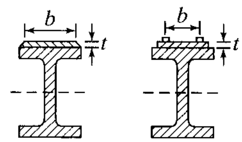
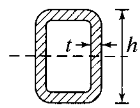
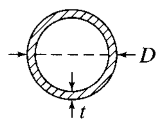
$$M_n = \frac{0.9Ek_c S_x}{\lambda^2}. \qquad \text{(AISC Equation F3-2)}$$

TABLE 9.1 Width-to-Thickness Ratios: Compression Elements in Members Subject to Flexure

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratios λ_r compact / noncompact)	Limiting Width-to-Thickness Ratios λ_r noncompact / slender)	Example
Unstiffened Elements	10	Flanges of rolled I-shaped sections, channels, and tees	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
	11	Flanges of doubly and singly symmetric I-shaped built-up sections	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$0.95\sqrt{\frac{K_cE}{F_L}}$ [a][b]	
	12	Legs of single angles	b/t	$0.54\sqrt{\frac{E}{F_y}}$	$0.91\sqrt{\frac{E}{F_y}}$	
	13	Flanges of all I-shaped sections and channels in flexure about the weak axis	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
	14	Stems of tees	d/t	$0.84\sqrt{\frac{E}{F_y}}$	$1.03\sqrt{\frac{E}{F_y}}$	
Stiffened Elements	15	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	
	16	Webs of singly-symmetric I-shaped sections	h_c/t_w	$\dfrac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y} - 0.09\right)^2} \le \lambda_r$ [c]	$5.70\sqrt{\frac{E}{F_y}}$	

(*Continued*)

TABLE 9.1 (Continued)

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratios: λ_r compact/noncompact)	λ_r noncompact/slender)	Example
Stiffened Elements	17	Flanges of rectangular HSS and boxes of uniform thickness	b/t	$1.12\sqrt{\frac{E}{F_y}}$	$1.40\sqrt{\frac{E}{F_y}}$	
Stiffened Elements	18	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.12\sqrt{\frac{E}{F_y}}$	$1.40\sqrt{\frac{E}{F_y}}$	
Stiffened Elements	19	Webs of rectangular HSS and boxes	h/t	$2.42\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	
Stiffened Elements	20	Round HSS	D/t	$0.07\frac{E}{F_y}$	$0.31\frac{E}{F_y}$	

[a] $K_c = \frac{4}{\sqrt{h/t_w}}$ but shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

[b] $F_L = 0.7F_y$ for major axis bending of compact and noncompact web built-up I-shaped members with $S_{xy}/S_{xc} \geq 0.7$, $F_L = F_yS_{xy}/S_{xc} > 0.5F_y$ for major-axis bending of compact and noncompact web built-up I-shaped members with $S_{xy}/S_{xc} < 0.7$.

[c] M_y is the moment at yielding of the extreme fiber. M_p = plastic bending moment, kip-in. (N-mm)

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

F_y = specified minimum yield stress, ksi (MPa)

Source: AISC Specification, Table B4.1b, p. 16.1-17. June 22, 2010.

Example 9-11

Determine the LRFD flexural design stress and the ASD allowable flexural stress for a 50 ksi W12 × 65 section which has full lateral bracing.

Solution

Using a W12 × 65 (b_f = 12.00 in, t_f = 0.605 in, S_x = 87.9 in³, Z_x = 96.8 in³)

Is the flange noncompact?

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29 \times 10^3}{50}} = 9.15$$

$$\lambda = \frac{b_f}{2t_f} = \frac{12.00}{(2)(0.605)} = 9.92$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29 \times 10^3}{50}} = 24.08$$

$$\lambda_p = 9.15 < \lambda = 9.92 < \lambda_r = 24.08$$

$\therefore$ The flange is noncompact.

Calculate the nominal flexural stress.

$$M_p = F_y Z = (50)(96.8) = 4840 \text{ in-k}$$

$$M_n = \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \quad \text{(AISC Eq F3-1)}$$

$$M_n = \left[4840 - (4840 - 0.7 \times 50 \times 87.9)\left(\frac{9.92 - 9.15}{24.08 - 9.15}\right)\right]$$

$$= 4749 \text{ in-k} = 395.7 \text{ ft-k}$$

Determine $\phi_b M_n$ and M_n/Ω.

LRFD $\phi_b = 0.9$	ASD $\Omega_b = 1.67$
$\phi_b M_n = (0.9)(395.7) = 356$ ft-k	$\frac{M_n}{\Omega_b} = \frac{395.7}{1.67} = 237$ ft-k

Note: These values correspond to the values given in AISC Table 3-2.

The equations mentioned here were used, where applicable, to obtain the values used for the charts plotted in AISC Table 3-10. The designer will have little trouble with noncompact sections when F_y is no more than 50 ksi. He or she, however, will have to use the formulas presented in this section for shapes with larger F_y values.

9.10 PROBLEMS FOR SOLUTION

9-1 to 9-8. *Using both LRFD and ASD, select the most economical sections, with F_y = 50 ksi, unless otherwise specified, and assuming full lateral bracing for the compression flanges. Working or service loads are given for each case, and beam weights are not included.*

9-1.

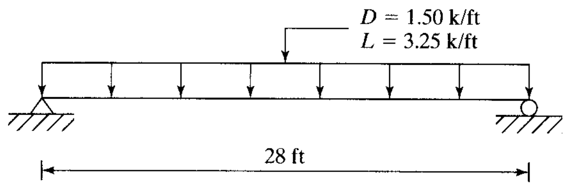


FIGURE P9-1 (*Ans.* W24 × 76 LRFD and ASD)

9-2.

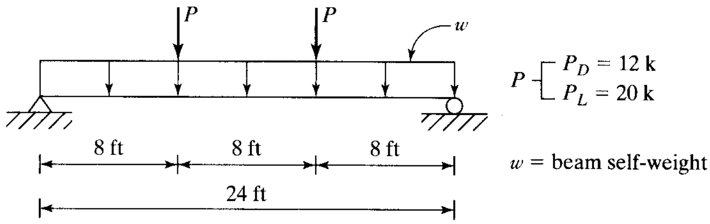


FIGURE P9-2

9-3.

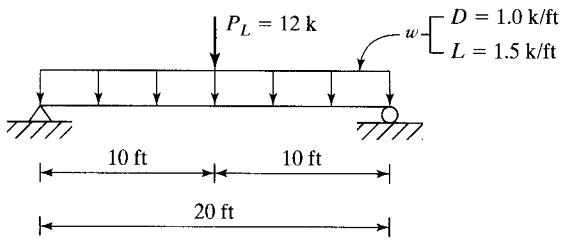


FIGURE P9-3 (*Ans.* W18 × 40 LRFD and ASD)

9-4. Repeat Prob. 9-3, using P_L = 20 k.

9-5.

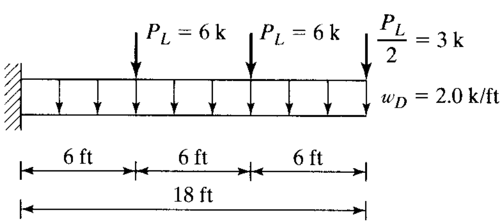


FIGURE P9-5 (*Ans.* W24 × 68 LRFD, W24 × 76 ASD)

9-6.

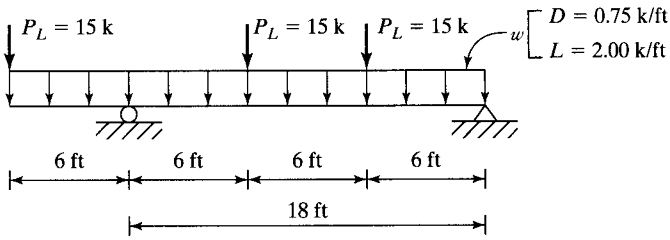


FIGURE P9-6

9-7.

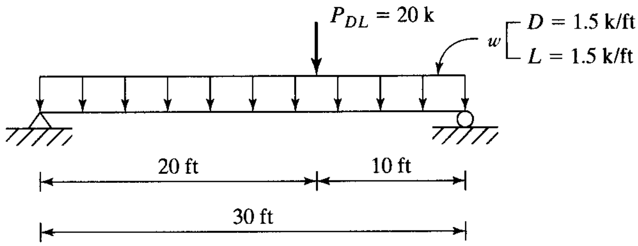


FIGURE P9-7 (*Ans.* W24 × 68 LRFD, W24 × 76 ASD)

9-8. The accompanying figure shows the arrangement of beams and girders that are used to support a 5 in reinforced concrete floor for a small industrial building. Design the beams and girders assuming that they are simply supported. Assume full lateral support of the compression flange and a live load of 80 psf. Concrete weight is 150 lb/ft^3.

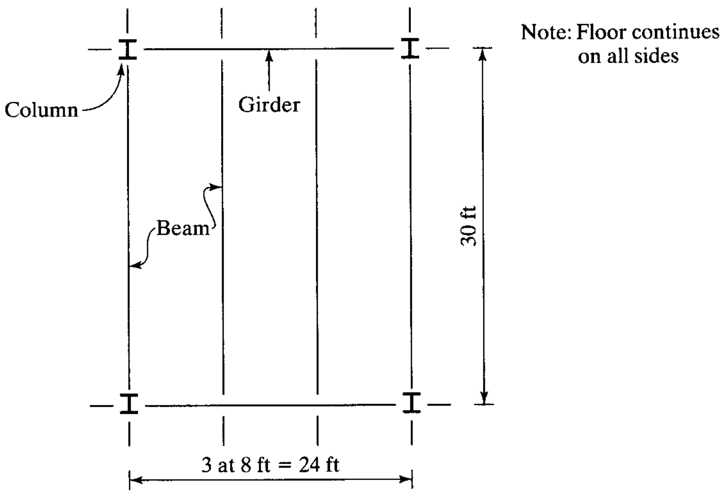


FIGURE P9-8

9-9. A beam consists of a W18 × 35 with 3/8 in × 8 in cover plates welded to each flange. Determine the LRFD design uniform load, w_u, and the ASD allowable uniform load,

w_a, that the member can support in addition to its own weight for a 28 ft simple span. (*Ans.* 2.85 k/ft LRFD, 3.02 k/ft ASD)

9-10. The member shown is made with 36 ksi steel. Determine the maximum service live load that can be placed on the beam if, in addition to its own weight, it is supporting a service dead load of 0.80 klf. The member is used for a 20 ft simple span. Use both LRFD and ASD methods.

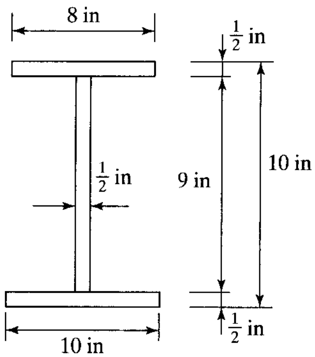


FIGURE P9-10

9-11 to 9-14. *Use both LRFD and ASD methods for these beams for which full lateral bracing of the compression flange is provided.*

9-11. Select a W section for a 24 ft simple span to support a service dead uniform load of 1.5 k/ft and a live service load of 1.0 k/ft if two holes for 3/4-in ϕ bolts are assumed present in each flange at the section of maximum moment. Use AISC Specification and A36 steel. Use both LRFD and ASD methods. (*Ans.* W21 × 44 LRFD, W21 × 48 ASD)

9-12. Rework Prob. 9-11, assuming that four holes for 3/4-in ϕ bolts pass through each flange at the point of maximum moment. Use A992 steel.

9-13. The section shown in Fig. P9-13 has two 3/4-in ϕ bolts passing though each flange and cover plate. Find the design load, w_a, and factored load, w_u, that the section can support, in addition to its own weight, for a 22 ft simple span if it consists of a steel with $F_y = 50$ ksi. Deduct all holes for calculating section properties. (*Ans.* Net w_u = 4.74 k/ft, Net w_a = 3.14 k/ft)

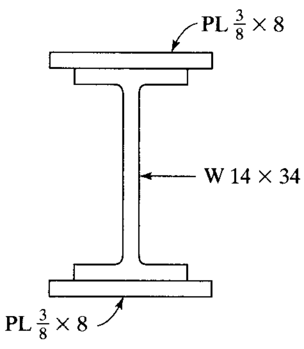


FIGURE P9-13

9-14. A 36 ft simple span beam is to support two movable service 20 kip loads a distance of 12 ft apart. Assuming a dead load of 1.0 k/ft including the beam self-weight, select a 50 ksi steel section to resist the largest possible moment. Use LRFD method only.

9-15 to 9-28. *For these problems, different values of L_b are given. Dead loads do not include beam weights. Use both LRFD and ASD methods.*

9-15. Determine ΦM_n and M_n/Ω for a W18 × 46 used as a beam with an unbraced length of the compression flange of 4 ft and 12 ft. Use A992 steel and $C_b = 1.0$.

(*Ans.* L_b = 4 ft, 340 ft-k LRFD; 226 ft-k ASD)
(*Ans.* L_b = 12 ft, 231.4 ft-k LRFD; 154.3 ft-k ASD)

9-16. Determine the lightest satisfactory W shape to carry a uniform dead load of 4.0 k/ft plus the beam self-weight and a uniform live load of 2.75 k/ft on a simple span of 12 ft. Assume bracing is provided at the ends only. Obtain C_b from Fig. 9.10 in text.

9-17. Select the lightest satisfactory W-shape section if F_y = 50 ksi. Lateral bracing is provided at the ends only. Determine C_b. (*Ans.* W14 × 61 LRFD, W12 × 65 ASD)

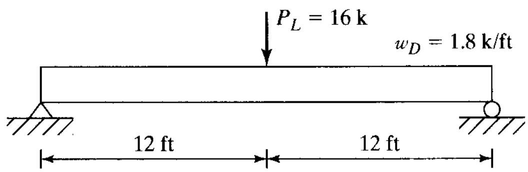


FIGURE P9-17

9-18. Repeat Prob. 9-17 if lateral bracing is provided at the concentrated load as well as at the ends of the span. Determine C_b.

9-19. A W18 × 55 of A992 steel is used on a simple span of 15 ft and has lateral support of compression flange at its ends only. If the only dead load present is the beam self-weight, what is the largest service concentrated live load that can be placed at the 1/3 points of the beam? Determine C_b. (*Ans.* 41.7 k LRFD, 44.3 k ASD)

9-20. Repeat Prob. 9-19 if lateral bracing is supplied at the beam ends and at the concentrated loads. Determine C_b.

9-21. The cantilever beam shown in Fig. P9-21 is a W18 × 55 of A992 steel. Lateral bracing is supplied at the fixed end only. The uniform load is a service dead load and includes the beam self-weight. The concentrated loads are service live loads. Determine whether the beam is adequate using LRFD and ASD methods. Assume C_b = 1.0. (*Ans.* LRFD OK, 363 ft-k > 335 ft-k; ASD OK, 241 ft-k > 212.5 ft-k)

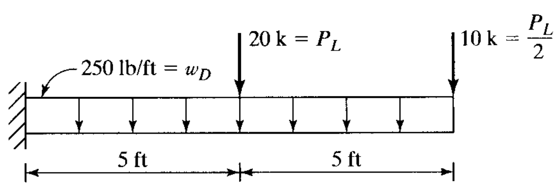


FIGURE P9-21

9-22. The given beam in Fig. P9-22 is A992 steel. If the live load is twice the dead load, what is the maximum total service load in k/ft that can be supported when (a) the compression flange is braced laterally for its full length, and (b) lateral bracing is supplied at the ends and centerline only?

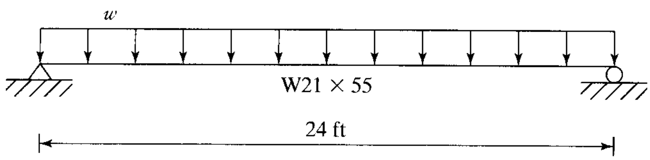
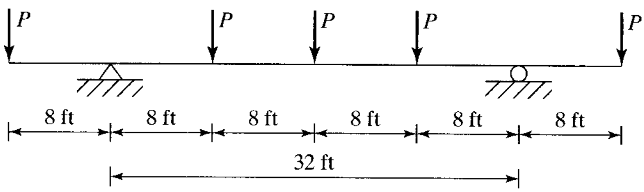


FIGURE P9-22

9-23. A W21 × 68 beam of A992 steel carries a uniformly distributed service dead load of 1.75 k/ft plus its self-weight and two concentrated service live loads at the third points of a 33 ft simply supported span. If lateral bracing is provided at the ends and the concentrated loads, determine the maximum service live load, P_L. Assume the concentrated loads are equal in value, determine C_b. (*Ans.* 12.26 k LRFD, 8.50 k ASD)

9-24. A beam of $F_y = 50$ ksi steel is used to support the loads shown in Fig. P9-24. Neglecting the beam self-weight, determine the lightest W shape to carry the loads if full lateral bracing is provided.



P: $P_D = 8.5$ k, $P_L = 6.0$ k

FIGURE P9-24

9-25. Redesign the beam of Prob. 9-24 if lateral bracing is only provided at the supports and at the concentrated loads. Determine C_b. (*Ans.* W16 × 26 LRFD, W14 × 30 ASD)

9-26. Design the lightest W shape beam of 50 ksi steel to support the loads shown in Fig. P9-26. Neglect the beam self-weight. The beam has continuous lateral bracing between A and B, but is laterally unbraced between B and C. Determine C_b.

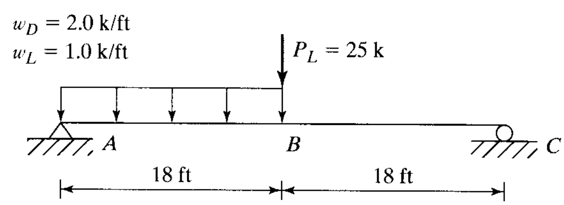


FIGURE P9-26

9-27. A W16 × 36 beam of A992 steel is fixed at one support and simply supported at the other end. A concentrated load of dead load of 9.25 k and live load of 6.50 k is applied at the center of the 32 ft span. Assume lateral bracing of the compression flange is provided at the pinned support, the load point, and the fixed support. You may neglect the beam self-weight and assume that $C_b = 1.0$. Is the W16 adequate? (*Ans.* LRFD OK, 129.0 ft-k ≤ 136.6 ft-k; ASD N.G., 94.5 ft-k ≥ 90.9 ft-k)

9-28. A W24 × 104 beam is used to support the loads shown in Fig. P9-28. Lateral bracing of the compression flange is supplied only at the ends. Determine C_b. If $F_y = 50$ ksi, determine if the W24 is adequate to support these loads.

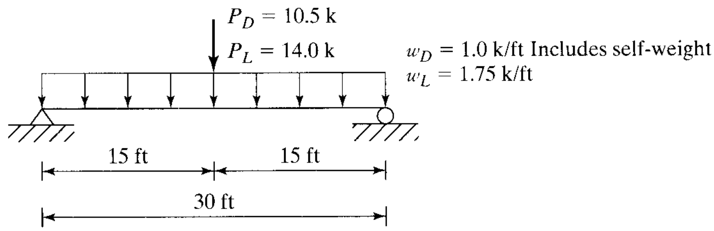
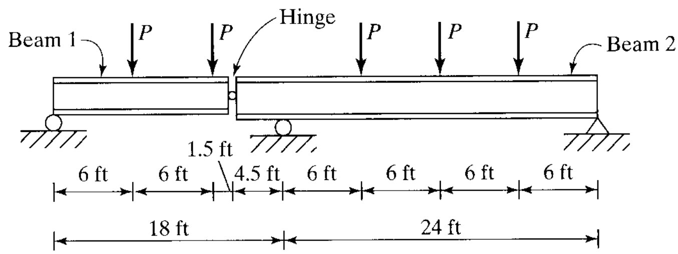


FIGURE P9-28

9-29. An A992, W18 × 60 steel beam is used on a 36 ft simple span to carry a uniformly distributed load. Determine the location of the lateral support, L_b, in order to provide just enough strength to carry a design moment. Use $M_u = 416.8$ ft-k for LRFD method and $M_a = 277.5$ ft-k for ASD method. Assume $C_b = 1.0$. (*Ans.* 9 ft LRFD and ASD)

9-30. The two steel beams shown in Fig. P9-30 are part of a two-span beam framing system with a pin (hinge) located 4.5 ft left of the interior support, making the system statically determinate. Determine the sizes (lightest) of the two W shape beams. Assume A992 steel and continuous lateral support of the compression flanges. The beam self-weight may be neglected. Use LRFD and ASD methods.



P: $P_D = 5.0$ k, $P_L = 7.5$ k

FIGURE P9-30

9-31. A built-up shape steel beam consists of a ¼ in × 12 in web, and ⅜ in × 4 in top and bottom flanges. The member has the compression flange fully braced, therefore the moment capacity, ΦM_n, using LRFD method was calculated to be 103.4 ft-k using $F_y = 50$ ksi steel. During design it was thought that the factored moment, M_u, was 100 ft-k, but after the member was fabricated it was found that the actual design moment, M_u, should have been 130 ft-k. A brilliant and resourceful young engineer suggested adding a ¼ in × 6 in cover plate to the bottom flange of the member to increase its capacity. Compute the new moment capacity, ΦM_n, and state whether or not it will safely support the design moment, $M_u = 130$ ft-k. (*Ans.* Yes, $\Phi M_n = 131.5$ ft-k $> M_u = 130$ ft-k)

9-32. A W21 × 93 has been specified for use on your design project. By mistake, a W21 × 73 was shipped to the field. This beam must be erected today. Assuming that ½ in thick plates are obtainable immediately, select cover plates to be welded to the top and bottom flanges to obtain the necessary section capacity. Use $F_y = 50$ ksi steel for all materials and assume that full bracing is supplied for the compression flange. Use LRFD and ASD methods.

CHAPTER 10

Design of Beams—Miscellaneous Topics (Shear, Deflection, etc.)

10.1 DESIGN OF CONTINUOUS BEAMS

Section B3 of the AISC Specification states that beams may be designed according to the provisions of the LRFD or ASD methods. Analysis of the members to determine their required strengths may be made by elastic, inelastic, or plastic analysis procedures. Design based on plastic analysis is permitted only for sections with yield stresses no greater than 65 ksi and is subject to some special requirements in Appendix 1 of the Specification Commentary.

Both theory and tests show clearly that continuous ductile steel members meeting the requirements for compact sections with sufficient lateral bracing supplied for their compression flanges have the desirable ability of being able to redistribute moments caused by overloads. If plastic analysis is used, this advantage is automatically included in the analysis.

If elastic analysis is used, the AISC handles the redistribution by a rule of thumb that approximates the real plastic behavior. AISC Appendix 1 Commentary, Section A1.3, states that for continuous compact sections, the design *may* be made on the basis of nine-tenths of the maximum negative moments caused by gravity loads that are maximum at points of support if the positive moments are increased by one-tenth of the average negative moments at the adjacent supports. *(The 0.9 factor is applicable only to gravity loads and not to lateral loads such as those caused by wind and earthquake.)* The factor can also be applied to columns that have axial forces not exceeding $0.15\phi_c F_y A_g$ for LRFD or

$0.15F_y A_g/\Omega_c$ for ASD. This moment reduction does not apply to moments produced by loading on cantilevers nor for designs made according to Sections 1.4 through 1.8 of Appendix 1 of the AISC Specification Commentary.

Example 10-1

The beam shown in Fig. 10.1 is assumed to consist of 50 ksi steel. (a) Select the lightest W section available, using plastic analysis and assuming that full lateral support is provided for its compression flanges. (b) Design the beam, using elastic analysis with the service loads and the 0.9 rule, and assuming that full lateral support is provided for both the flanges.

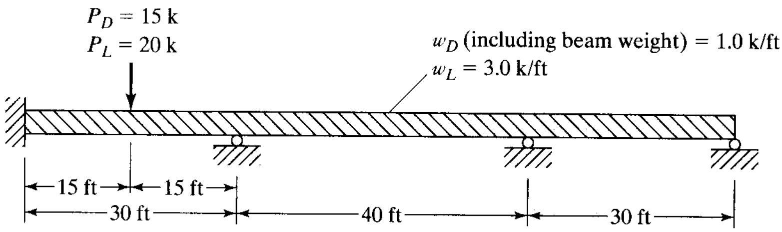


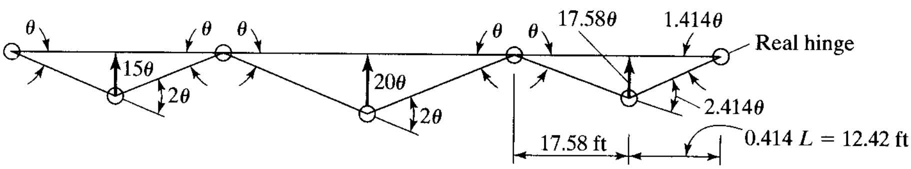
FIGURE 10.1

Solution

a. Plastic analysis and LRFD design

$$w_u = (1.2)(1.0) + (1.6)(3) = 6.0 \text{ klf}$$

$$P_u = (1.2)(15) + (1.6)(20) = 50 \text{ k}$$



$$\text{Span 1}\begin{cases} M_u 4\theta = (30w_u)\left(\dfrac{1}{2}\right)(15\theta) + (P_u)(15\theta) \\ M_u = 56.25w_u + 3.75P_u \\ M_u = (56.25)(6.0) + (3.75)(50) \\ M_u = 525 \text{ ft-k} \end{cases}$$

$$\text{Span 2}\begin{cases} M_u(4\theta) = (40w_u)\left(\dfrac{1}{2}\right)(20\theta) \\ M_u = 100w_u = (100)(6.0) \\ M_u = 600 \text{ ft-k} \leftarrow \text{controls} \end{cases}$$

$$\text{Span 3}\begin{cases} M_u(3.414\theta) = (30w_u)\left(\dfrac{1}{2}\right)(17.58\theta) \\ M_u = 77.24w_u = (77.24)(6.0) \\ \quad = 463.4 \text{ ft-k} \end{cases}$$

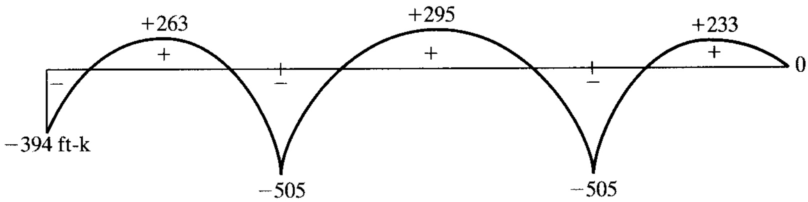
Use W21 × 68 (AISC Table 3-2). $\phi_b M_p = 600 \text{ ft-k} = M_u = 600 \text{ ft-k}$

b. Analyzing beam of Fig. 10.1 for service loads

$$w_a = 1.0 + 3.0 = 4 \text{ k/ft}$$

$$P_a = 15 + 20 = 35 \text{ k}$$

Drawing moment diagram, ft-k



Maximum negative moment for design

$$= (0.9)(-505) = -454.5 \text{ ft-k} \leftarrow \text{controls}$$

Maximum positive moment for design

$$= +295 + \left(\frac{1}{10}\right)\left(\frac{505 + 505}{2}\right) = +345.5 \text{ ft-k}$$

Use W24 × 76 (AISC Table 3-2). $M_p/\Omega = 499 \text{ ft-k} > M_a = 454.5 \text{ ft-k}$

Important Note: Should the lower flange of this W24 × 76 not be braced laterally, we must check the lengths of the span where negative moments are present, because L_b values may very well exceed L_p for the section and the design may have to be revised.

10.2 SHEAR

For this discussion, the beam of Fig. 10.2(a) is considered. As the member bends, shear stresses occur because of the changes in length of its longitudinal fibers. For positive bending the lower fibers are stretched and the upper fibers are shortened, while somewhere in between there is a neutral axis where the fibers do not change in length. Owing to these varying deformations, a particular fiber has a tendency to slip on the fiber above or below it.

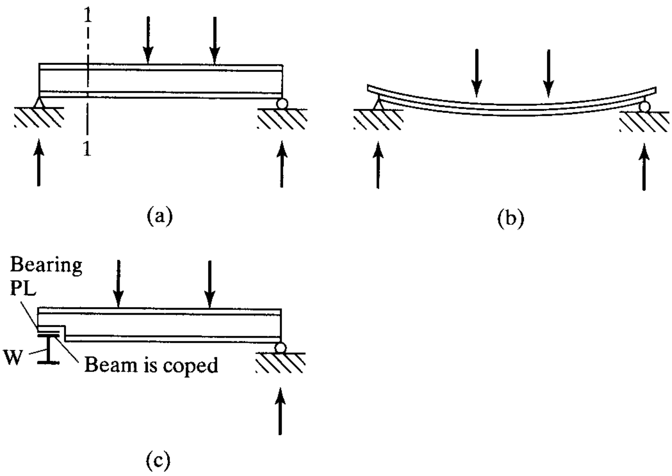


FIGURE 10.2

If a wooden beam was made by stacking boards on top of each other and not connecting them, they obviously would tend to take the shape shown in part (b) of the figure. The student may have observed short, heavily loaded timber beams with large transverse shears that split along horizontal planes.

This presentation may be entirely misleading in seeming to completely separate horizontal and vertical shears. In reality, horizontal and vertical shears at any point are the same, as long as the critical section at which the shear stress is evaluated is taken parallel to the axis of symmetry. Furthermore, one cannot occur without the other.

Generally, shear is not a problem in steel beams, because the webs of rolled shapes are capable of resisting rather large shearing forces. Perhaps it is well, however, to list here the most common situations where shear might be excessive:

1. Should large concentrated loads be placed near beam supports, they will cause large internal forces without corresponding increases in bending moments. A fairly common example of this type of loading occurs in tall buildings where, on a particular floor, the upper columns are offset with respect to the columns below. The loads from the upper columns applied to the beams on the floor level in question will be quite large if there are many stories above.
2. Probably the most common shear problem occurs where two members (as a beam and a column) are rigidly connected together so that their webs lie in a common plane. This situation frequently occurs at the junction of columns and beams (or rafters) in rigid frame structures.
3. Where beams are notched or coped, as shown in Fig. 10.2(c), shear can be a problem. For this case, shear forces must be calculated for the remaining beam depth. A similar discussion can be made where holes are cut in beam webs for ductwork or other items.
4. Theoretically, very heavily loaded short beams can have excessive shears, but practically, this does not occur too often unless it is like Case 1.

5. Shear may very well be a problem even for ordinary loadings when very thin webs are used, as in plate girders or in light-gage cold-formed steel members.

From his or her study of mechanics of materials, the student is familiar with the horizontal shear stress formula $f_v = VQ/Ib$, where V is the external shear; Q is the statical moment of that portion of the section lying outside (either above or below) the line on which f_v is desired, taken about the neutral axis; and b is the width of the section where the unit shearing stress is desired.

Figure 10.3(a) shows the variation in shear stresses across the cross section of an I-shaped member, while part (b) of the same figure shows the shear stress variation in a member with a rectangular cross section. It can be seen in part (a) of the figure that the shear in I-shaped sections is primarily resisted by the web.

If the load is increased on an I-shaped section until the bending yield stress is reached in the flange, the flange will be unable to resist shear stress and it will be carried in the web. If the moment is further increased, the bending yield stress will penetrate farther down into the web and the area of web that can resist shear will be further reduced. Rather than assuming the nominal shear stress is resisted by part of the web, the AISC Specification assumes that a reduced shear stress is resisted by the entire web area. This web area, A_w, is equal to the overall depth of the member, d, times the web thickness, t_w.

Shear strength expressions are given in AISC Specification G2. In these expressions, h is the clear distance between the web toes of the fillets for rolled shapes, while for built-up welded sections it is the clear distance between flanges. For bolted built-up sections, h is the distance between adjacent lines of bolts in the web. Different expressions are given for different h/t_w ratios, depending on whether shear failures would be plastic, inelastic, or elastic.

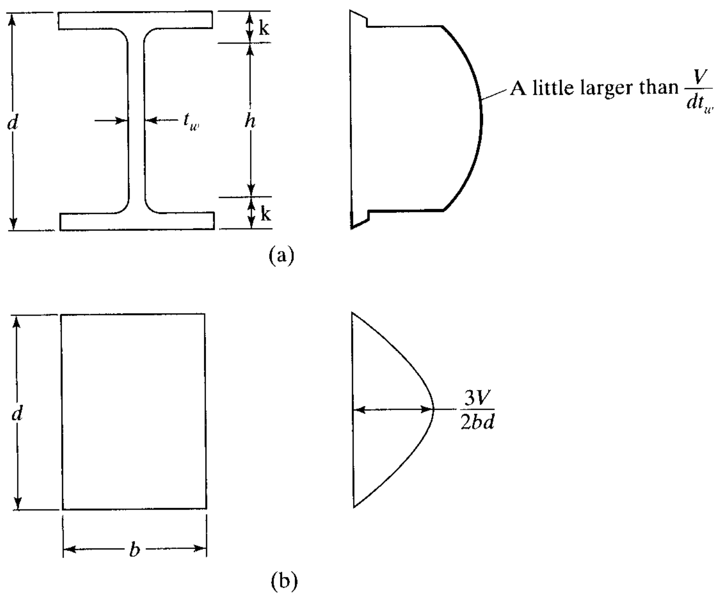


FIGURE 10.3

Combined welded and bolted joint, Transamerica Pyramid, San Francisco, CA.
(Courtesy of Kaiser Steel Corporation.)

The nominal shear strength of unstiffered or stiffened webs is specified as

$$V_n = 0.6F_yA_wC_v \qquad \text{(AISC Equation G2-1)}$$

Using this equation for the webs of I-shaped members when $h/t_w \leq 2.24\sqrt{E/F_y}$, we find that $C_v = 1.0$, $\phi_v = 1.00$, and $\Omega_v = 1.50$. (Almost all current W, S, and HP shapes fall into this class. The exceptions are listed in Section G2 of the AISC Specification.)

For the webs of all doubly symmetric shapes, singly symmetric shapes, and channels, except round HSS, $\phi_v = 0.90$ and $\Omega_v = 1.67$ are used to determine the design shear strength, ϕ_vV_n, and the allowable shear strength V_n/Ω. C_v, the web shear coefficient, is determined from the following situations and is substituted into AISC Equation G2-1:

a. For $\dfrac{h}{t_w} \leq 1.10\sqrt{\dfrac{k_vE}{F_y}}$

$$C_v = 1.0 \qquad \text{(AISC Equation G2-3)}$$

b. For $1.10\sqrt{\dfrac{k_vE}{F_y}} < \dfrac{h}{t_w} \leq 1.37\sqrt{\dfrac{k_vE}{F_y}}$

$$C_v = \frac{1.10\sqrt{\dfrac{k_vE}{F_y}}}{\dfrac{h}{t_w}} \qquad \text{(AISC Equation G2-4)}$$

c. For $\dfrac{h}{t_w} > 1.37\sqrt{\dfrac{k_v E}{F_y}}$

$$C_v = \frac{1.51 E k_v}{\left(\dfrac{h}{t_w}\right)^2 F_y} \qquad \text{(AISC Equation G2-5)}$$

The web plate shear buckling coefficient, k_v, is specified in the AISC Specification G2.1b, parts (i) and (ii). For webs without transverse stiffeners and with $h/t_w < 260$: $k_v = 5$. This is the case for most rolled I-shaped members designed by engineers.

Example 10-2

A W21 × 55 with $F_y = 50$ ksi is used for the beam and loads of Fig. 10.4. Check its adequacy in shear.

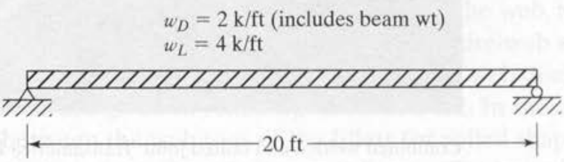


FIGURE 10.4

Solution

Using a W21 × 55 ($A = 16.2\text{ in}^2$, $d = 20.8$ in, $t_w = 0.375$ in, and $k_{des} = 1.02$ in)

$$h = 20.8 - 2k_{des} = 20.8 - (2)(1.02) = 18.76\text{ in}$$

$$\frac{h}{t_w} = \frac{18.76}{0.375} = 50.03 < 2.24\sqrt{\frac{29{,}000}{50}} = 53.95$$

$$\therefore C_v = 1.0,\ \phi_v = 1.0 \text{ and } \Omega_v = 1.50$$

$$A_w = d\,t_w = (20.8\text{ in})(0.375\text{ in}) = 7.80\text{ in}^2$$

$$\therefore V_n = 0.6\,F_y\,A_w\,C_v = 0.6\,(50\text{ ksi})(7.80\text{ in}^2)(1.0) = 234\text{ k}$$

LRFD $\phi_v = 1.00$	ASD $\Omega_v = 1.50$
$w_u = (1.2)(2) + (1.6)(4) = 8.8$ k/ft	$w_a = 2 + 4 = 6$ k/ft
$V_u = \dfrac{8.8\text{ k/ft (20 ft)}}{2} = 88$ k	$V_a = \dfrac{6.0\text{ k/ft (20 ft)}}{2} = 60$ k
$\phi_v V_n = (1.00)(234) = 234$ k	$\dfrac{V_n}{\Omega_v} = \dfrac{234}{1.50} = 156$ k
> 88 k **OK**	> 60 k **OK**

Notes

1. The values of $\phi_v V_{nx}$ and V_{nx}/Ω_v with $F_y = 50$ ksi are given for W shapes in the manual, Table 3-2.
2. Two values are given in the AISC Manual for k. One is given in decimal form and is to be used for design calculations, while the other one is given in fractions and is to be used for detailing. These two values are based, respectively, on the minimum and maximum radii of the fillets and will usually be quite different from each other.
3. A very useful table (3-6) is provided in Part 3 of the AISC Manual for determining the maximum uniform load each W shape can support for various spans. The values given are for $F_y = 50$ ksi and are controlled by maximum moments or shears, as specified by LRFD or ASD.

Should V_u for a particular beam exceed the AISC specified shear strengths of the member, the usual procedure will be to select a slightly heavier section. If it is necessary, however, to use a much heavier section than required for moment, doubler plates (Fig. 10.5) may be welded to the beam web, or stiffeners may be connected to the webs in zones of high shear. Doubler plates must meet the width–thickness requirements for compact stiffened elements, as prescribed in Section B4 of the AISC Specification. In addition, they must be welded sufficiently to the member webs to develop their proportionate share of the load.

The AISC specified shear strengths of a beam or girder are based on the entire area of the web. Sometimes, however, a connection is made to only a small portion or depth of the web. For such a case, the designer may decide to assume that the shear is spread over only part of the web depth, for purposes of computing shear strength. Thus, he or she may compute A_w as being equal to t_w times the smaller depth, for use in the shear strength expression.

When beams that have their top flanges at the same elevations (the usual situation) are connected to each other, it is frequently necessary to cope one of them, as shown in Fig. 10.6. For such cases, there is a distinct possibility of a block shear failure along the broken lines shown. This subject was previously discussed in Section 3.7 and is continued in Chapter 15.

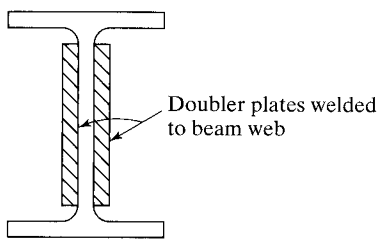


FIGURE 10.5

Increasing shear strength of beam by using doubler plates.

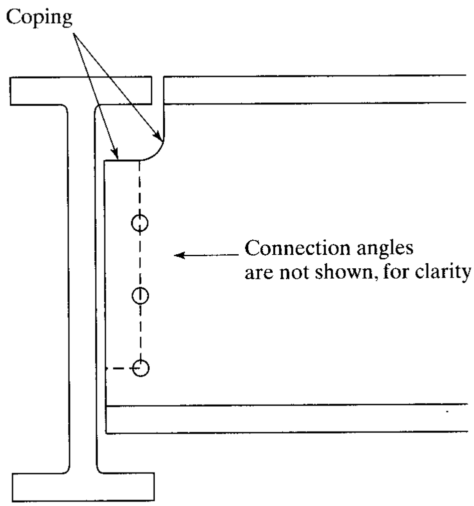


FIGURE 10.6

Block shear failure possible along dashed line.

10.3 DEFLECTIONS

The deflections of steel beams are usually limited to certain maximum values. Among the several excellent reasons for deflection limitations are the following:

1. Excessive deflections may damage other materials attached to or supported by the beam in question. Plaster cracks caused by large ceiling joist deflections are one example.
2. The appearance of structures is often damaged by excessive deflections.
3. Extreme deflections do not inspire confidence in the persons using a structure, although the structure may be completely safe from a strength standpoint.
4. It may be necessary for several different beams supporting the same loads to deflect equal amounts.

Standard American practice for buildings has been to limit service live-load deflections to approximately 1/360 of the span length. This deflection is supposedly the largest value that ceiling joists can deflect without causing cracks in underlying plaster. The 1/360 deflection is only one of many maximum deflection values in use because of different loading situations, different engineers, and different specifications. For situations where precise and delicate machinery is supported, maximum deflections may be limited to 1/1500 or 1/2000 of the span lengths. The 2010 AASHTO Specifications limit deflections in steel beams and girders due to live load and impact to 1/800 of the span. (For bridges in urban areas that are shared by pedestrians, the AASHTO *recommends* a maximum value equal to 1/1000 of the span lengths.)

The AISC Specification does not specify exact maximum permissible deflections. There are so many different materials, types of structures, and loadings that no one single set of deflection limitations is acceptable for all cases. Thus, limitations must be set by the individual designer on the basis of his or her experience and judgment.

The reader should note that deflection limitations fall in the serviceability area. Therefore, deflections are determined for service loads, and thus the calculations are identical for both LRFD and ASD designs.

Before substituting blindly into a formula that will give the deflection of a beam for a certain loading condition, the student should thoroughly understand the theoretical methods of calculating deflections. These methods include the moment area, conjugate beam, and virtual-work procedures. From these methods, various expressions can be determined, such as the following common one for the center line deflection of a uniformly loaded simple beam:

$$\Delta_{¢} = \frac{5wL^4}{384EI}$$

To use deflection expressions such as this one, the reader must be very careful to apply consistent units. Example 10-3 illustrates the application of the preceding expression. The author has changed all units to pounds and inches. Thus, the uniform load given in the problem as so many kips per foot is changed to so many lb/in.

Example 10-3

A W24 × 55 (I_x = 1350 in) has been selected for a 21-ft simple span to support a total service live load of 3 k/ft (including beam weight). Is the center line deflection of this section satisfactory for the service live load if the maximum permissible value is 1/360 of the span?

Solution. Use $E = 29 \times 10^6\,\text{lb/in}^2$

$$\Delta_{¢} = \frac{5wL^4}{384EI} = \frac{(5)(3000/12)(12 \times 21)^4}{(384)(29 \times 10^6)(1350)} = 0.335 \text{ in total load deflection}$$

$$< \left(\frac{1}{360}\right)(12 \times 21) = 0.70 \text{ in} \qquad \text{OK}$$

Another way many engineers use to account for units in the deflection calculation is to keep the uniform load, *w*, in units of k/ft and span, *L*, in units of ft, and then convert units from ft to in by multiplying by 1728 (i.e. 12 × 12 × 12). In this method, *E* has units of k/in^2 (i.e. 29,000).

On page 3-7 in the AISC Manual, the following simple formula for determining maximum beam deflections for W, M, HP, S, C, and MC sections for several different loading conditions is presented:

$$\Delta = \frac{ML^2}{C_1 I_x}$$

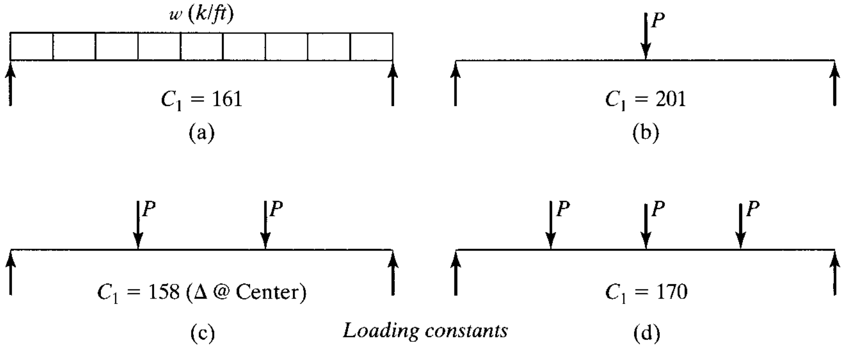


FIGURE 10.7

Values of constant C_1 for use in deflection expression. (Figure 3-2 in AISC Manual.)

In this expression, M is the maximum service load moment in ft-k, and it can be based on the 4 different loading conditions from Fig. 10.7; C_1 is a constant whose value can be determined from Fig. 10.7; L is the span length (ft); and I_x is the moment of inertia (in^4).

If we want to use this expression for the beam of Example 10-3, C_1 from part (a) of Fig. 10.7 would be 161, the center line bending moment would be $wL^2/8 = (3)(21)^2/8 = 165.375$ ft-k, and the center line deflection for the live service load would be

$$\Delta_{\text{¢}} = \frac{(165.375)(21)^2}{(161)(1350)} = 0.336 \text{ in}$$

Table 1604.3 of IBC 2009 presents maximum permissible deflections for quite a few types of members and loading conditions. Several of these are presented in Table 10.1. These values are not applicable to ponding situations.

Some specifications handle the deflection problem by requiring certain minimum depth–span ratios. For example, the AASHTO suggests the depth–span ratio be limited to a minimum value of 1/25. A shallower section is permitted, but it should have sufficient stiffness to prevent a deflection greater than would have occurred if the 1/25 ratio had been used.

A steel beam can be cold-bent, or cambered, an amount equal to the deflection caused by dead load, or the deflection caused by dead load plus some percentage of the live load. Approximately 25 percent of the camber so produced is elastic and will disappear when the cambering operation is completed. It should be remembered that a beam which is bent upward looks much stronger and safer than one that sags downward (even a very small distance).

A camber requirement is quite common for longer steel beams. In fact, a rather large percentage of the beams used in composite construction today (see Chapter 16) are cambered. On many occasions, however, it is more economical to select heavier

TABLE 10.1 Deflection Limits from IBC 2009

Members	Loading conditions		
	L	D + L	S or W
For floor members	$\frac{L}{360}$	$\frac{L}{240}$	—
For roof members supporting plaster ceiling*	$\frac{L}{360}$	$\frac{L}{240}$	$\frac{L}{360}$
For roof members supporting nonplaster ceilings*	$\frac{L}{240}$	$\frac{L}{180}$	$\frac{L}{240}$
For roof members not supporting ceilings*	$\frac{L}{180}$	$\frac{L}{120}$	$\frac{L}{180}$

*All roof members should be investigated for ponding.

beams with their larger moments of inertia so as to reduce deflections and avoid the labor costs involved in cambering. One commonly used rule of thumb is that it takes about one extra man-work-hour to camber each beam.

Cambering is something of a nuisance to many fabricators, and it may very well introduce some additional problems. For instance, when beams are cambered, it may be necessary to adjust the connection detail so that proper fitting of members is achieved. The end of a cambered beam will be rotated, and thus it may be necessary to rotate the connection details by the same amount to insure proper fitting.

If we can move up one or two sections in weight, thus reducing deflections so that cambering is not needed, we may have a very desirable solution. Similarly, if a higher-strength steel is being used, it may be be desirable to switch the beams that need cambering to a lower yield stress steel. The results will be larger beams, but smaller deflections, and perhaps some economy will be achieved.

The most economical way to camber beams is with a mechanical press. However, beams less than about 24 ft in length may not fit into a standard press. Such a situation may require the use of heat to carry out the cambering, but expenses will increase by two or three or more times. For this reason, it is almost always wise for the sizes of such short members to be increased until cambering is not needed.

If necessary for a particular project, a member may be bent or cambered about its horizontal axis. This is called *sweep*.

Deflections may very well control the sizes of beams for longer spans, or for short ones where deflection limitations are severe. To assist the designer in selecting sections where deflections may control, the AISC Manual includes a set of tables numbered 3-3 and entitled "W-shapes Selection by I_x," in which the I_x values are given in numerically descending order for the sections normally used as beams. In this table, the sections are arranged in groups, with the lightest section in each group printed in roman type. Example 10-4 presents the design of a beam where deflections control the design.

Example 10-4

Using the LRFD and ASD methods, select the lightest available section with $F_y = 50$ ksi to support a service dead load of 1.2 k/ft and a service live load of 3 k/ft for a 30-ft simple span. The section is to have full lateral bracing for its compression flange, and the maximum total service load deflection is not to exceed 1/1500 the span length.

Solution. After some scratch work, assume that beam wt = 167 lb/ft

LRFD	ASD
$w_u = 1.2(1.2 + 0.167) + (1.6)(3) = 6.44$ klf	$w_a = (1.2 + 0.167) + 3 = 4.37$ k/ft
$M_u = \dfrac{(6.44 \text{ k/ft})(30 \text{ ft})^2}{8} = 724.5$ ft-k	$M_a = \dfrac{(4.37 \text{ k/ft})(30 \text{ ft})^2}{8} = 491.6$ ft-k
From AISC Table 3-2, try W24 × 76 ($I_x = 2100$ in^4)	From AISC Table 3-2, try W24 × 76
Maximum permissible $\Delta = \left(\dfrac{1}{1500}\right)(12 \times 30) = 0.24$ in Actual $\Delta = \dfrac{ML^2}{C_1 I_x}$ $M = M_a = M_{\text{service}} = \dfrac{(4.37 \text{ k/ft})(30 \text{ ft})^2}{8}$ $= 491.6$ ft-k $\Delta = \dfrac{(491.6)(30)^2}{(161)(2100)} = 1.31$ in > 0.24 in **N.G.** Min I_x required to limit Δ to 0.24 in $= \left(\dfrac{1.31}{0.24}\right)(2100) = 11{,}463$ in^4 From AISC Table 3-3	All other calculations same as LRFD
Use W40 × 167. ($I_x = 11{,}600$ in^4)	**Use W40 × 167.**

10.3.1 Vibrations

Though steel members may be selected that are satisfactory as to moment, shear, deflections, and so on, some very annoying floor vibrations may still occur. This is perhaps the most common serviceability problem faced by designers. Objectionable vibrations will frequently occur where long spans and large open floors without partitions or other items that might provide suitable damping are used. The reader may often have noticed this situation in the floors of large malls.

Damping of vibrations may be achieved by using framed-in-place partitions, each attached to the floor system in at least three places, or by installing "false" sheetrock partitions between ceilings and the underside of floor slabs. Further damping may be achieved by the thickness of the floor slabs, the stability of partitions, and by the weight of office furniture and perhaps the equipment used in the building. A better procedure is to control the stiffness of the structural system.[1,2] (A rather common practice in the past has been to try to limit vibrations by selecting beams no shallower than 1/20 times span lengths.)

If the occupants of a building feel uneasy about or are annoyed by vibrations, the design is unsuccessful. It is rather difficult to correct a situation of this type in an existing structure. On the other hand, the situation can be easily predicted and corrected in the design stage. Several good procedures have been developed that enable the structural designer to estimate the acceptability of a given system by its users.

F. J. Hatfield[3] has prepared a useful chart for estimating the perceptibility of vibrations of steel beams and concrete slabs for both office and residential buildings.

10.3.2 Ponding

If water on a flat roof accumulates faster than it runs off, the increased load causes the roof to deflect into a dish shape that can hold more water, which causes greater deflections, and so on. This process of *ponding* continues until equilibrium is reached or until collapse occurs. Ponding is a serious matter, as illustrated by the large annual number of flat-roof failures in the United States.

Ponding will occur on almost any flat roof to a certain degree, even though roof drains are present. Drains may be inadequate during severe storms, or they may become stopped up. Furthermore, they are often placed along the beam lines, which are actually the high points of the roof. The best method of preventing ponding is to have an appreciable slope on the roof (1/4 in/ft or more), together with good drainage facilities. It has been estimated that probably two-thirds of the flat roofs in the United States have slopes less than this value, which is the minimum recommended by the National Roofing Contractors Association (NRCA). It costs approximately 3 to 6 percent more to construct a roof with this desired slope than to build with no slope.[4]

When a very large flat roof (perhaps an acre or more) is being considered, the effect of wind on water depth may be quite important. A heavy rainstorm will frequently be accompanied by heavy winds. When a large quantity of water is present on the roof, a strong wind may very well push a great deal of water to one end, creating a dangerous depth of water in terms of the load in pounds per square foot applied to the roof.

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[1]T.M. Murray, "Controlling Floor Movement." *Modern Steel Construction* (AISC, Chicago, IL, June 1991) pp. 17–19.

[2]T.M. Murray, "Acceptability Criterion for Occupant-Induced Floor Vibrations," *Engineering Journal*, AISC, 18, 2 (2nd Quarter, 1981), pp. 62–70.

[3]F. J. Hatfield, "Design Chart for Vibration of Office and Residential Floors," *Engineering Journal*, AISC, 29, 4 (4th Quarter, 1992), pp. 141–144.

[4]Gary Van Ryzin, "Roof Design: Avoid Ponding by Sloping to Drain," *Civil Engineering* (New York: ASCE, January 1980), pp. 77–81.

For such situations, *scuppers* are sometimes used. These are large holes, or tubes, in the walls or parapets that enable water above a certain depth to quickly drain off the roof.

Ponding failures will be prevented if the roof system (consisting of the roof deck and supporting beams and girders) has sufficient stiffness. The AISC Specification (Appendix 2) describes a minimum stiffness to be achieved if ponding failures are to be prevented. If this minimum stiffness is not provided, it is necessary to make other investigations to be sure that a ponding failure is not possible.

Theoretical calculations for ponding are very complicated. The AISC requirements are based on work by F. J. Marino,[5] in which he considered the interaction of a two-way system of secondary members, or submembers, supported by a primary system of main members, or girders. A numerical ponding example is presented in Appendix E of this book.

10.4 WEBS AND FLANGES WITH CONCENTRATED LOADS

When steel members have concentrated loads applied that are perpendicular to one flange and symmetric to the web, their flanges and webs must have sufficient flange and web design strength in the areas of flange bending, web yielding, web crippling, and sidesway web buckling. Should a member have concentrated loads applied to both flanges, it must have a sufficient web design strength in the areas of web yielding, web crippling, and column web buckling. In this section, formulas for determining strengths in these areas are presented.

If flange and web strengths do not satisfy the requirements of AISC Specification Section J.10, it will be necessary to use transverse stiffeners at the concentrated loads. Should web design strengths not satisfy the requirements of AISC Specification J.10 it will be necessary to use doubler plates or diagonal stiffeners, as described herein. These situations are discussed in the paragraphs to follow.

10.4.1 Local Flange Bending

The flange must be sufficiently rigid so that it will not deform and cause a zone of high stress concentrated in the weld in line with the web. The nominal tensile load that may be applied through a plate welded to the flange of a W section is to be determined by the expression to follow, in which F_{yf} is the specified minimum yield stress of the flange (ksi) and t_f is the flange thickness (in):

$$R_n = 6.25 t_f^2 F_{yf} \qquad \text{(AISC Equation J10-1)}$$

$$\phi = 0.90\,(\text{LRFD}) \quad \Omega = 1.67\,(\text{ASD})$$

It is not necessary to check this formula if the length of loading across the beam flange is less than 0.15 times the flange width b_f or if a pair of half-depth or deeper web stiffeners are provided. Fig. 10.9(a) shows a beam with local flange bending.

[5]F. J. Marino, "Ponding of Two-Way Roof System," *Engineering Journal* (New York: AISC, July 1966), pp. 93–100.

10.4.2 Local Web Yielding

The subject of local web yielding applies to all concentrated forces, tensile or compressive. Here we will try to limit the stress in the web of a member in which a force is being transmitted. Local web yielding is illustrated in part (b) of Fig. 10.9.

The nominal strength of the web of a beam at the web toe of the fillet when a concentrated load or reaction is applied is to be determined by one of the following two expressions, in which k is the distance from the outer edge of the flange to the web toe of the fillet, l_b is the length of bearing (in) of the force parallel to the plane of the web, F_{yw} is the specified minimum yield stress (ksi) of the web, and t_w is the thickness of the web:

If the force is a concentrated load or reaction that causes tension or compression and is applied at a distance greater than the member depth, d, from the end of the member, then

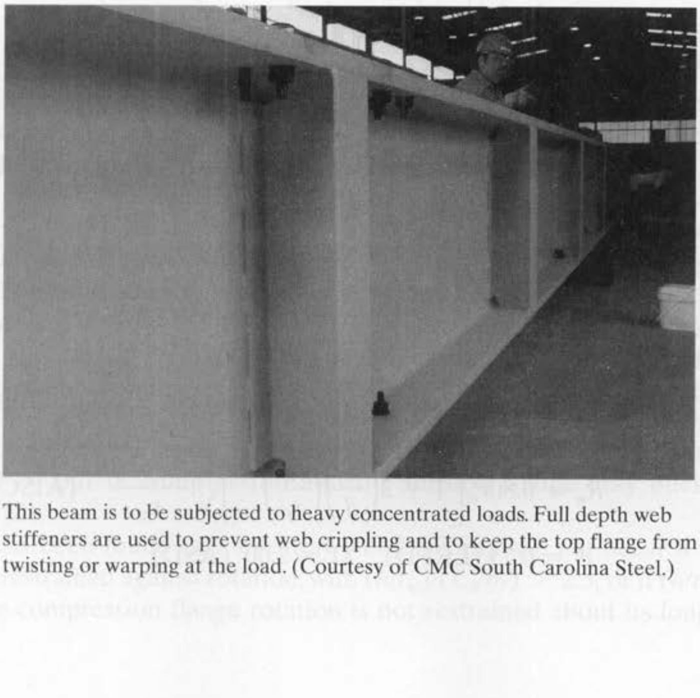
$$R_n = (5k + l_b)F_{yw}t_w \qquad \text{(AISC Equation J10-2)}$$

$$\phi = 1.00 \text{ (LRFD)} \quad \Omega = 1.50 \text{ (ASD)}$$

If the force is a concentrated load or reaction applied at a distance d or less from the member end, then

$$R_n = (2.5k + l_b)F_{yw}t_w \qquad \text{(AISC Equation J10-3)}$$

$$\phi = 1.00 \text{ (LRFD)} \quad \Omega = 1.50 \text{ (ASD)}$$



This beam is to be subjected to heavy concentrated loads. Full depth web stiffeners are used to prevent web crippling and to keep the top flange from twisting or warping at the load. (Courtesy of CMC South Carolina Steel.)

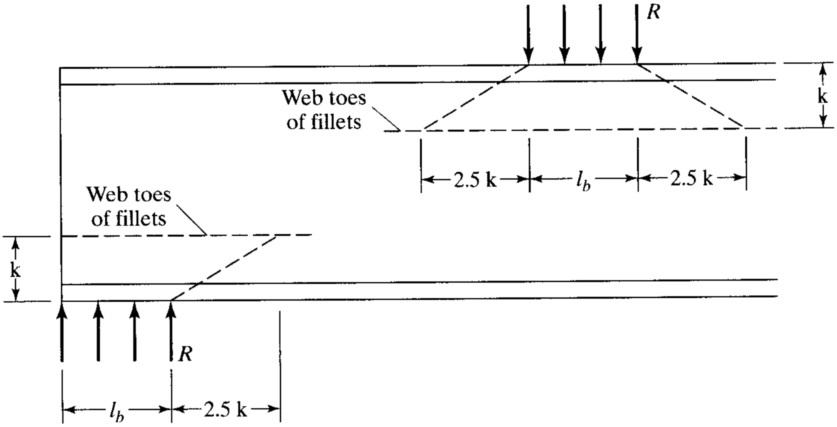


FIGURE 10.8

Local web yielding.

Reference to Fig. 10.8 clearly shows where these expressions were obtained. The nominal strength R_n equals the length over which the force is assumed to be spread when it reaches the web toe of the fillet times the web thickness times the yield stress of the web. Should a stiffener extending for at least half the member depth or a doubler plate be provided on each side of the web at the concentrated force, it is not necessary to check for web yielding.

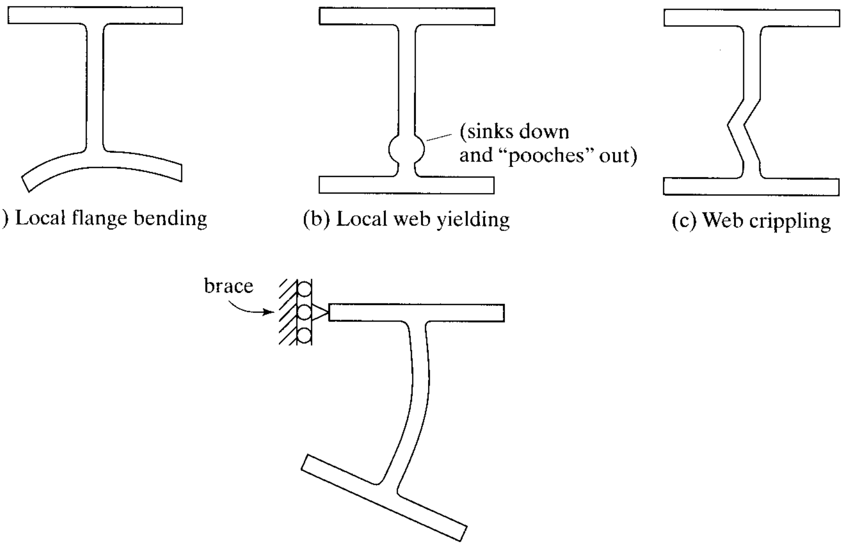
10.4.3 Web Crippling

Should concentrated compressive loads be applied to a member with an unstiffened web (the load being applied in the plane of the web), the nominal web crippling strength of the web is to be determined by the appropriate equation of the two that follow (in which d is the overall depth of the member). If one or two web stiffeners or one or two doubler plates are provided and extend for at least half of the web depth, web crippling will not have to be checked. Research has shown that when web crippling occurs, it is located in the part of the web adjacent to the loaded flange. Thus, it is thought that stiffening the web in this area for half its depth will prevent the problem. Web crippling is illustrated in part (c) of Fig. 10.9.

If the concentrated load is applied at a distance greater than or equal to $d/2$ from the end of the member, then

$$R_n = 0.80t_w^2\left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}} \qquad \text{(AISC Equation J10-4)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$



(a) Local flange bending (b) Local web yielding (c) Web crippling

(d) Sidesway web buckling

FIGURE 10.9

If the concentrated load is applied at a distance less than $d/2$ from the end of the member, then

for $\quad \dfrac{l_b}{d} \le 0.2,$

$$R_n = 0.40t_w^2\left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}} \qquad \text{(AISC Equation J10-5a)}$$

$$\phi = 0.75 \text{ (LRFD)} \quad \Omega = 2.00 \text{ (ASD)}$$

For $\dfrac{l_b}{d} > 0.2,$

$$R_n = 0.40t_w^2\left[1 + \left(4\frac{l_b}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}} \qquad \text{(AISC Equation J10-5b)}$$

$$\phi = 0.75 \text{ (LRFD)} \quad \Omega = 2.00 \text{ (ASD)}$$

10.4.4 Sidesway Web Buckling

Should compressive loads be applied to laterally braced compression flanges, the web will be put in compression and the tension flange may buckle, as shown in Fig. 10-9(d).

It has been found that sidesway web buckling will not occur if the compression flange is restrained against rotation, with $(h/t_w)/(L_b/b_f) > 2.3$, or if $(h/t_w)/(L_b/b_f) > 1.7$ when the compression flange rotation is not restrained about its longitudinal axis. In these expressions, h is the web depth between the web toes of the fillets—that is, $d - 2k$—and L_b is the largest laterally unbraced length along either flange at the point of the load.

It is also possible to prevent sidesway web buckling with properly designed lateral bracing or stiffeners at the load point. The AISC Commentary suggests that local bracing for *both* flanges be designed for 1 percent of the magnitude of the concentrated load applied at the point. If stiffeners are used, they must extend from the point of load for at least one-half of the member depth and should be designed to carry the full load. Flange rotation must be prevented if the stiffeners are to be effective.

Should members not be restrained against relative movement by stiffeners or lateral bracing and be subject to concentrated compressive loads, their strength may be determined as follows:

When the loaded flange is braced against rotation and $(h/t_w)/(L_b/b_f)$ is ≤ 2.3,

$$R_n = \frac{C_r t_w^3 t_f}{h^2}\left[1 + 0.4\left(\frac{h/t_w}{L_b/b_f}\right)^3\right] \qquad \text{(AISC Equation J10-6)}$$

$$\phi = 0.85 \text{ (LRFD)} \quad \Omega = 1.76 \text{ (ASD)}$$

C&W Warehouse, Spartanburg, SC. (Courtesy of Britt, Peters and Associates.)

When the loaded flange is not restrained against rotation and $\dfrac{h/t_w}{L_b/b_f} \leq 1.7$,

$$R_n = \frac{C_r t_w^3 t_f}{h^2}\left[0.4\left(\frac{h/t_w}{L_b/b_f}\right)^3\right] \qquad \text{(AISC Equation J10-7)}$$

$$\phi = 0.85 \text{ (LRFD)} \quad \Omega = 1.76 \text{ (ASD)}$$

It is not necessary to check Equations J10-6 and J10-7 if the webs are subject to distributed load. Furthermore, these equations were developed for bearing connections and *do not apply to moment connections*. In these expressions,

C_r = 960,000 ksi when $M_u < M_y$ (LRFD) or $1.5M_a < M_y$ (ASD) at the location of the force, ksi.

C_r = 480,000 ksi when $M_u \geq M_y$ (LRFD) or $1.5M_a \geq M_y$ (ASD) at the location of the force, ksi.

10.4.5 Compression Buckling of the Web

This limit state relates to concentrated compression loads applied to both flanges of a member, such as, moment connections applied to both ends of a column. For such a situation, it is necessary to limit the slenderness ratio of the web to avoid the possibility of buckling. Should the concentrated loads be larger than the value of ϕR_n given in the next equation, it will be necessary to provide either one stiffener, a pair of stiffeners, or a doubler plate, extending for the full depth of the web and meeting the requirements of AISC Specification J10.8. (The equation to follow is applicable to moment connections, but not to bearing ones.)

$$R_n = \frac{24\, t_w^3 \sqrt{EF_{yw}}}{h} \qquad \text{(AISC Equation J10-8)}$$

$$\phi = 0.90 \text{ (LRFD)} \quad \Omega = 1.67 \text{ (ASD)}$$

If the concentrated forces to be resisted are applied at a distance from the member end that is less than $d/2$, then the value of R_n shall be reduced by 50 percent.

Example 10-5, which follows, presents the review of a beam for the applicable items discussed in this section.

Example 10-5

A W21 × 44 has been selected for moment in the beam shown in Fig. 10.10. Lateral bracing is provided for both flanges at beam ends and at concentrated loads. If the end bearing length is 3.50 in and the concentrated load bearing lengths are each 3.00 in, check the beam for web yielding, web crippling, and sidesway web buckling.

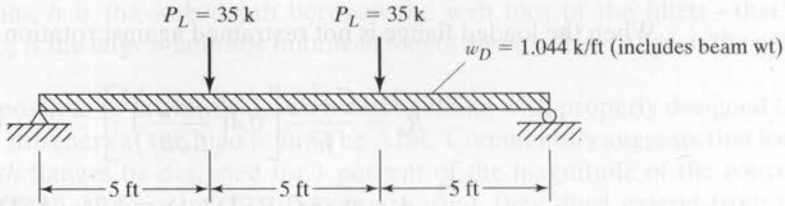


FIGURE 10.10

Solution

Using a W21 × 44 (d = 20.7 in, b_f = 6.50 in, t_w = 0.350 in, t_f = 0.450 in, k = 0.950 in)

LRFD	ASD
End reaction	End reaction
$R_u = (1.2)(1.044 \text{ k/ft})\left(\frac{15 \text{ ft}}{2}\right) + (1.6)(35 \text{ k})$	$R_a = (1.044 \text{ k/ft})\left(\frac{15 \text{ ft}}{2}\right) + 35 \text{ k}$
$= 65.4$ k	$= 42.83$ k
Concentrated load	Concentrated load
$P_u = (1.6)(35 \text{ k}) = 56$ k	$P_a = 35$ k

Local web yielding

(l_b = bearing length of reactions = 3.50 in, for concentrated loads l_b = 3.00 in)

At end reactions (AISC Equation J10-3)

$$R_n = (2.5\text{ k} + l_b)F_{yw}t_w = (2.5 \times 0.950 \text{ in} + 3.50 \text{ in})(50 \text{ ksi})(0.350 \text{ in}) = 102.8 \text{ k}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$\phi R_n = (1.00)(102.8) = 102.8$ k	$\frac{R_n}{\Omega} = \frac{102.8}{1.50} = 68.5$ k
> 65.4 k **OK**	> 42.83 k **OK**

At concentrated loads (AISC Equation J10-2)

$$R_n = (5\,k + l_b)F_{yw}t_w = (5 \times 0.950\text{ in} + 3.00\text{ in})(50\text{ ksi})(0.350\text{ in}) = 135.6\text{ k}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$\phi R_n = (1.00)(135.6) = 135.6\text{ k}$ $> 56\text{ k}$ **OK**	$\dfrac{R_n}{\Omega} = \dfrac{135.6}{1.50} = 90.4\text{ k}$ $> 35\text{ k}$ **OK**

Web crippling

At end reactions (AISC Equation J10-5a) since $\dfrac{l_b}{d} \leq 0.20$

$$\frac{l_b}{d} = \frac{3.5}{20.7} = 0.169 < 0.20$$

$$R_n = 0.40 t_w^2\left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

$$= (0.40)(0.350\text{ in})^2\left[1 + 3\left(\frac{3.5\text{ in}}{20.7\text{ in}}\right)\left(\frac{0.350\text{ in}}{0.450\text{ in}}\right)^{1.5}\right]\sqrt{\frac{(29 \times 10^3\text{ ksi})(50\text{ ksi})(0.450\text{ in})}{0.350\text{ in}}}$$

$$= 90.3\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(90.3) = 67.7\text{ k}$ $> 65.4\text{ k}$ **OK**	$\dfrac{R_n}{\Omega} = \dfrac{90.3}{2.00} = 45.1\text{ k}$ $> 42.83\text{ k}$ **OK**

At concentrated loads (AISC Equation J10-4)

$$R_n = 0.80\, t_w^2\left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

$$= (0.80)(0.350)^2\left[1 + 3\left(\frac{3.0}{20.7}\right)\left(\frac{0.350}{0.450}\right)^{1.5}\right]\sqrt{\frac{(29 \times 10^3)(50)(0.450)}{0.350}}$$

$$= 173.7\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(173.7)$	$\frac{R_n}{\Omega} = \frac{173.7}{2.00} = 86.8$ k
$= 130.3$ k > 56 k **OK**	> 35 k **OK**

Sidesway web buckling

The compression flange is restrained against rotation.

$$\frac{h}{t_w}\Big/\frac{L_b}{b_f} = \frac{20.7\text{ in} - 2 \times 0.950\text{ in}}{0.350\text{ in}}\Big/\left(\frac{12\text{ in/ft} \times 5\text{ ft}}{6.50\text{ in}}\right) = 5.82 > 2.3$$

$\therefore$ **Sidesway web buckling does not have to be checked.**

The preceding calculations can be appreciably shortened if use is made of the Manual tables numbered 9-4 and entitled "Beam Bearing Constants." In those tables, values are shown for ϕR_1, ϕR_2, ϕR_3, R_1/Ω, R_2/Ω, R_3/Ω, and so on. The values given represent parts of the equations used for checking web yielding and web crippling and are defined on page 9-19 in the Manual.

Instructions for use of the tables are provided on pages 9-19 and 9-20 of the Manual. The expressions for local web yielding at beam ends of a W21 × 44 are written next making use of the table values. Then, those expressions and the table values are used to check the previous calculations.

LRFD	ASD
$\phi R_n = \phi R_1 + \phi l_b R_2$	$\frac{R_n}{\Omega} = \left(\frac{R_1}{\Omega}\right) + l_b\left(\frac{R_2}{\Omega}\right)$
$= 41.6 + (3.5)(17.5)$	$= 27.7 + (3.5)(11.7)$
$= 102.8$ k **OK**	$= 68.6$ k **OK**

10.5 UNSYMMETRICAL BENDING

From mechanics of materials, it should be remembered that each beam cross section has a pair of mutually perpendicular axes known as the principal axes for which the product of inertia is zero. Bending that occurs about any axis other than one of the principal axes is said to be unsymmetrical bending. When the external loads are not in a plane with either of the principal axes, or when loads are simultaneously applied to the beam from two or more directions, unsymmetrical bending results.

If a load is not perpendicular to one of the principal axes, it may be broken into components that are perpendicular to those axes and to the moments about each axis, M_{ux} and M_{uy}, or M_{ax} and M_{ay}, determined as shown in Fig. 10.11.

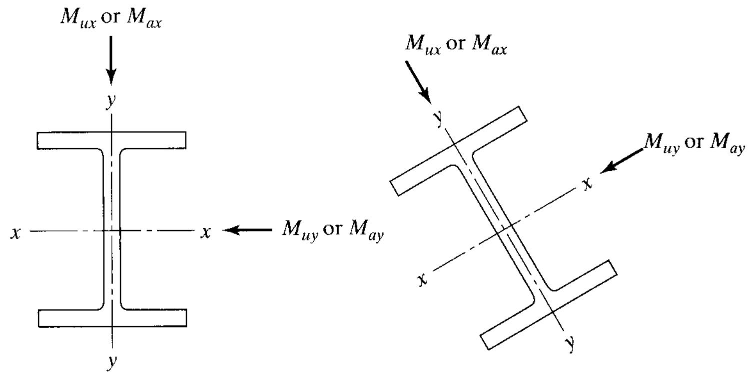


FIGURE 10.11

When a section has one axis of symmetry, that axis is one of the principal axes, and the calculations necessary for determining the moments are quite simple. For this reason, unsymmetrical bending is not difficult to handle in the usual beam section, which is probably a W, S, M, or C. Each of these sections has at least one axis of symmetry, and the calculations are appreciably reduced. A further simplifying factor is that the loads are usually gravity loads and probably perpendicular to the x axis.

Among the beams that must resist unsymmetrical bending are crane girders in industrial buildings and purlins for ordinary roof trusses. The x axes of purlins are parallel to the sloping roof surface, while the large percentage of their loads (roofing, snow, etc.) are gravity loads. These loads do not lie in a plane with either of the principal axes of the inclined purlins, and the result is unsymmetrical bending. Wind loads are generally considered to act perpendicular to the roof surface and thus perpendicular to the x axes of the purlins, with the result that they are not considered to cause unsymmetrical bending. The x axes of crane girders are usually horizontal, but the girders are subjected to lateral thrust loads from the moving cranes, as well as to gravity loads.

To check the adequacy of members bent about both axes simultaneously, the AISC provides an equation in Section H1 of their Specification. The equation that follows is for combined bending and axial force, if $P_r/P_c < 0.2$:

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1b)}$$

Here, the values are defined as follows:

1. P_r is the required axial strength under LRFD or the required allowable axial strength with ASD.
2. P_c is the available axial strength with LRFD or the available allowable axial strength with ASD.
3. M_{rx} and M_{ry} are the required design flexural strengths about the x and y axes under LRFD and the required allowable flexural strengths with ASD.

4. M_{cx} and M_{cy} are the design flexural strengths about the x and y axes using LRFD and the available allowable flexural strengths about those axes using ASD.

Since, for the problem considered here, P_r is equal to zero, the equation becomes

$$\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \leq 1.0$$

This is an interaction, or percentage, equation. If M_{rx} is 75 percent of M_{cx}, then M_{ry} can be no greater than 25 percent of M_{cy}.

Examples 10-6 and 10-7 illustrate the design of beams subjected to unsymmetrical bending. To illustrate the trial-and-error nature of the problem, the author did not do quite as much scratch work as in some of his earlier examples. The first design problems of this type that the student attempts may quite well take several trials. Consideration needs to be given to the question of lateral support for the compression flange. Should the lateral support be of questionable nature, the engineer should reduce the design moment resistance by means of one of the expressions previously given for that purpose.

Example 10-6

A steel beam in its upright position must resist the following service moments: $M_{Dx} = 60$ ft-k, $M_{Lx} = 100$ ft-k, $M_{Dy} = 15$ ft-k, and $M_{Ly} = 25$ ft-k. These moments include the effects of the estimated beam weight. The loads are assumed to pass through the centroid of the section. Select a W24 shape of 50 ksi steel that can resist these moments, assuming full lateral support for the compression flange.

Solution

Try a W24 × 62 ($\phi_b M_{px} = 574$ ft-k, $\dfrac{M_{px}}{\Omega_b} = 382$ ft-k, $Z_y = 15.7$ in^3)

$$\phi_b M_{py} = \phi_b F_y Z_y = \frac{(0.9)(50\text{ ksi})(15.7\text{ in}^3)}{12\text{ in/ft}} = 58.8\text{ ft-k},\ \frac{M_{py}}{\Omega_b} = \frac{F_y Z_y}{\Omega_b}$$

$$= \frac{(50\text{ ksi})(15.7\text{ in}^3)}{(12\text{ in/ft})(1.67)} = 39.1\text{ ft-k}$$

LRFD	ASD
$M_{ux} = (1.2)(60) + (1.6)(100) = 232$ ft-k	$M_{ax} = 60 + 100 = 160$ ft-k
$M_{uy} = (1.2)(15) + (1.6)(25) = 58$ ft-k	$M_{ay} = 15 + 25 = 40$ ft-k
$\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}} \leq 1.0$	$\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}} \leq 1.0$
$\dfrac{232}{574} + \dfrac{58}{58.8} = 1.39 > 1.0$ **N.G.**	$\dfrac{160}{382} + \dfrac{40}{39.1} = 1.44 > 1.0$ **N.G.**

Try a W24 × 68 ($\phi_b M_{px}$ = 664 ft-k, $\dfrac{M_{px}}{\Omega_b}$ = 442 ft-k, Z_y = 24.5 in^3)

$$\phi_b M_{py} = \frac{(0.9)(50 \text{ ksi})(24.5 \text{ in}^3)}{12 \text{ in/ft}} = 91.9 \text{ ft-k}, \frac{M_{py}}{\Omega_b} = \frac{(50 \text{ ksi})(24.5 \text{ in}^3)}{(12 \text{ in/ft})(1.67)} = 61.1 \text{ ft-k}$$

LRFD	ASD
$\dfrac{232}{664} + \dfrac{58}{91.9} = 0.98$	$\dfrac{160}{442} + \dfrac{40}{61.1} = 1.02$
< 1.0 **OK**	> 1.00 **N.G.**
Use W24 × 68.	**Try a larger section, W24 × 76.**

It should be noted in the solution for Example 10-6 that, although the procedure used will yield a section that will adequately support the moments given, the selection of the absolutely lightest section listed in the AISC Manual could be quite lengthy because of the two variables Z_x and Z_y, which affect the size. If we have a large M_{ux} and a small M_{uy}, the most economical section will probably be quite deep and rather narrow, whereas if we have a large M_{uy} in proportion to M_{ux}, the most economical section may be rather wide and shallow.

10.6 DESIGN OF PURLINS

To avoid bending in the top chords of roof trusses, it is theoretically desirable to place purlins only at panel points. For large trusses, however, it is more economical to space them at closer intervals. If this practice is not followed for large trusses, the purlin sizes may become so large as to be impractical. When intermediate purlins are used, the top chords of the truss should be designed for bending as well as for axial stress, as described in Chapter 11. Purlins are usually spaced from 2 to 6 ft apart, depending on loading conditions, while their most desirable depth-to-span ratios are probably in the neighborhood of 1/24. Channels or S sections are the most frequently used sections, but on some occasions other shapes may be convenient.

As previously mentioned, the channel and S sections are very weak about their web axes, and sag rods may be necessary to reduce the span lengths for bending about those axes. Sag rods, in effect, make the purlins continuous sections for their y axes, and the moments about these axes are greatly reduced, as shown in Fig. 10.12. These moment diagrams were developed on the assumption that the changes in length of the sag rods are negligible. It is further assumed that the purlins are simply supported at the trusses. This assumption is on the conservative side, since they are often continuous over two or more trusses and appreciable continuity may be achieved at their splices. The student can easily reproduce these diagrams from his or her knowledge of moment distribution or by other analysis methods. In the diagrams, L is the distance between

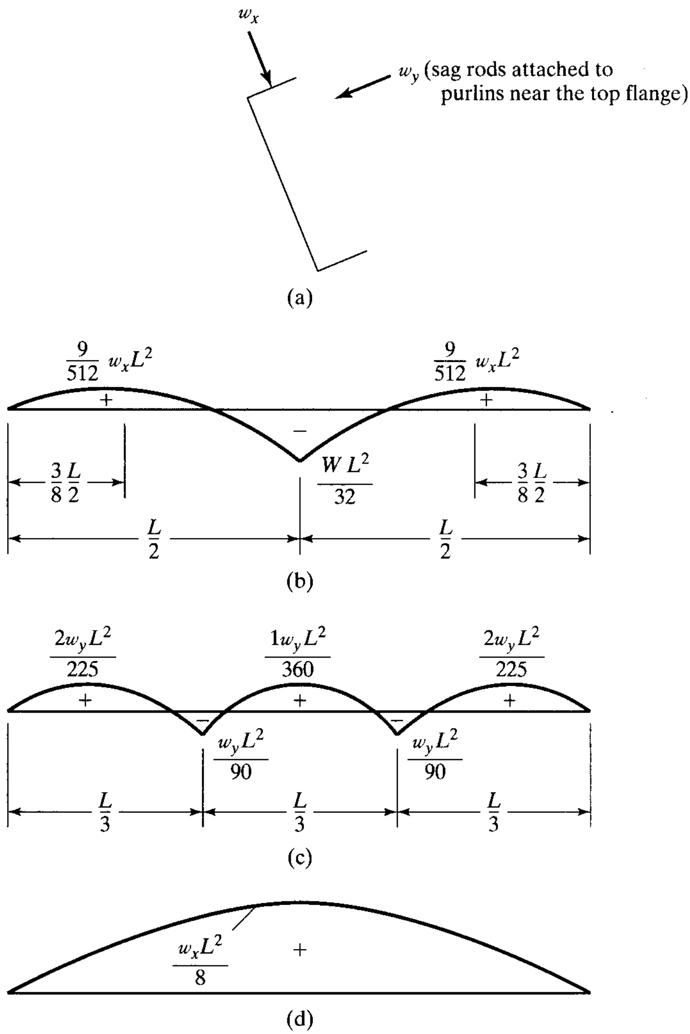


FIGURE 10.12

(a) Channel purlin. (b) Moment about web axis of purlins acting on the upper half of purlin—sag rods at midspan. (c) Moment about web axis of purlins acting on the upper half of purlin—sag rods at one-third points. (d) Moment about x axis of purlin.

trusses, w_{uy} is the load component *perpendicular* to the web axis of the purlin, and w_{ux} is the load component *parallel* to the web axis.

If sag rods were not used, the maximum moment about the web axis of a purlin would be $w_y L^2/8$. When sag rods are used at midspan, this moment is reduced to a maximum of $w_y L^2/32$ (a 75 percent reduction), and when used at one-third points is reduced to a maximum of $w_y L^2/90$ (a 91 percent reduction). In Example 10-7, sag rods are used at the midpoints, and the purlins are designed for a moment of $w_x L^2/8$ parallel to the web axis and $w_y L^2/32$ perpendicular to the web axis.

In addition to being advantageous in reducing moments about the web axes of purlins, sag rods can serve other useful purposes. First, they can provide lateral support for the purlins; second, they are useful in keeping the purlins in proper alignment during erection until the roof deck is installed and connected to the purlins.

Example 10-7

Using both the LRFD and ASD methods, select a W6 purlin for the roof shown in Fig. 10.13. The trusses are 18 ft 6 in on center, and sag rods are used at the midpoints between trusses. Full lateral support is assumed to be supplied from the roof above. Use 50 ksi steel and the AISC Specification. Loads are as follows in terms of pounds per square foot of roof surface:

$$\text{Snow} = 30 \text{ psf}$$
$$\text{Roofing} = 6 \text{ psf}$$
$$\text{Estimated purlin weight} = 3 \text{ psf}$$
$$\text{Wind pressure} = 15 \text{ psf} \perp \text{ to roof surface}$$

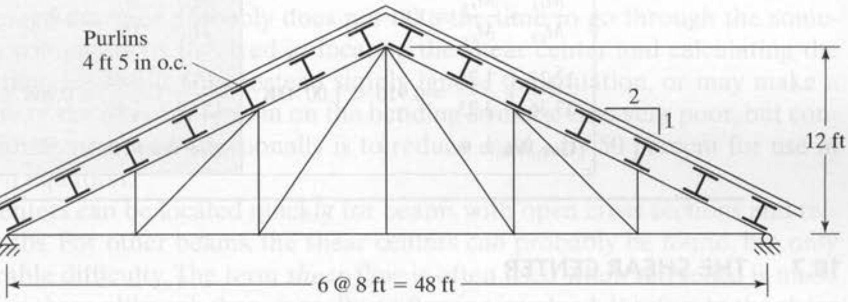


FIGURE 10.13

LRFD	ASD
$w_{ux} = 1.2D + 1.6S + 0.8W$	$w_{ax} = D + S + W$
$= (1.2)(6+3)(4.42)\left(\frac{2}{\sqrt{5}}\right) + (1.6)(30)(4.42)\left(\frac{2}{\sqrt{5}}\right)$	$= (6+3)(4.42)\left(\frac{2}{\sqrt{5}}\right) + (30)(4.42)\left(\frac{2}{\sqrt{5}}\right)$
$+(0.8)(15)(4.42) = 285.5$ lb/ft	$+ 15(4.42) = 220.5$ lb/ft
$M_{ux} = \frac{(0.2855)(18.5)^2}{8} = 12.21$ ft-k	$M_{ax} = \frac{(0.2205)(18.5)^2}{8} = 9.43$ ft-k
$w_{uy} = [1.2D + 1.6S]\left(\frac{1}{\sqrt{5}}\right)$	$w_{ay} = (6+3+30)\left(\frac{1}{\sqrt{5}}\right)(4.42)$
$= [(1.2)(6+3)(4.42) + (1.6)(30)(4.42)]\left(\frac{1}{\sqrt{5}}\right)$	
$= 116.5$ lb/ft	$= 77.1$ lb/ft
$M_{uy} = \frac{(0.1165)(18.5)^2}{32} = 1.25$ ft-k	$M_{ay} = \frac{(0.0771)(18.5)^2}{32} = 0.82$ ft-k

Solution

$$\text{Try W6} \times 9 \left(Z_x = 6.23 \text{ in}^3, \phi_b M_{px} = \frac{(0.9)(50)(6.23)}{12} = 23.36 \text{ ft-k},\right.$$

$$Z_y = 1.72 \text{ in}^3, \phi_b M_{py} = \frac{(0.9)(50)(1.72)}{12}\left(\frac{1}{2}\right) = 3.23 \text{ ft-k},$$

$$\frac{M_{px}}{\Omega_b} = \frac{(50)(6.23)}{(12)(1.67)} = 15.54 \text{ ft-k}, \frac{M_{py}}{\Omega_b} = \frac{(50)(1.72)}{(12)(1.67)}\left(\frac{1}{2}\right) = 2.15 \text{ ft-k}$$

(the $\frac{1}{2}$ used on $\phi_b M_{py}$ and $\frac{M_{py}}{\Omega_b}$ since sag rod attached to top of purlin).

LRFD	ASD
$\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \leq 1.0$	$\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \leq 1.0$
$\frac{12.21}{23.36} + \frac{1.25}{3.23} = 0.910 < 1.00$ **OK**	$\frac{9.43}{15.54} + \frac{0.82}{2.15} = 0.988 < 1.00$ **OK**
Use W6 × 9.	**Use W6 × 9.**

10.7 THE SHEAR CENTER

The shear center is defined as the point on the cross section of a beam through which the resultant of the transverse loads must pass so that the stresses in the beam may be calculated only from the theories of pure bending and transverse shear. Should the resultant pass through this point, it is unnecessary to analyze the beam for torsional moments. For a beam with two axes of symmetry, the shear center will fall at the intersection of the two axes, thus coinciding with the centroid of the section. For a beam with one axis of symmetry, the shear center will fall somewhere on that axis, but not necessarily at the centroid of the section. This surprising statement means that to avoid torsion in some beams the lines of action of the applied loads and beam reactions should not pass through the centroids of the sections.

The shear center is of particular importance for beams whose cross sections are composed of thin parts that provide considerable bending resistance, but little resistance to torsion. Many common structural members, such as the W, S, and C sections, angles, and various beams made up of thin plates (as in aircraft construction) fall into this class, and the problem has wide application.

The location of shear centers for several open sections are shown with the solid dots (•) in Fig. 10.14. Sections such as these are relatively weak in torsion, and for them and similar shapes, the location of the resultant of the external loads can be a very serious matter. A previous discussion has indicated that the addition of one or more webs to these sections—so that they are changed into box shapes—greatly increases their torsional resistance.

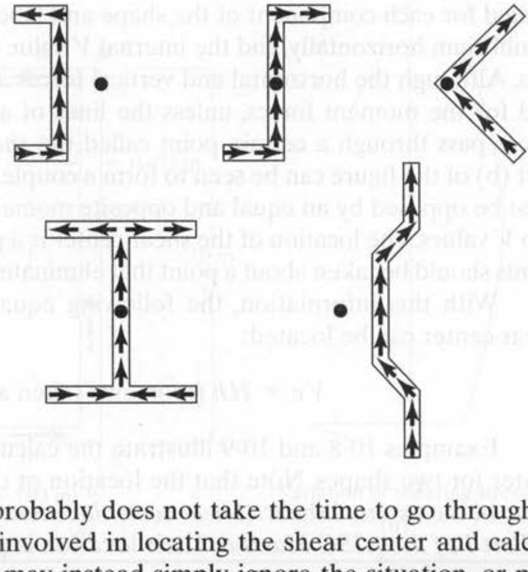


FIGURE 10.14

The average designer probably does not take the time to go through the sometimes tedious computations involved in locating the shear center and calculating the effect of twisting. He or she may instead simply ignore the situation, or may make a rough estimate of the effect of torsion on the bending stresses. One very poor, but conservative, estimate practiced occasionally is to reduce $\phi_b M_{ny}$ by 50 percent for use in the interaction equation.

Shear centers can be located quickly for beams with open cross sections and relatively thin webs. For other beams, the shear centers can probably be found, but only with considerable difficulty. The term *shear flow* is often used when reference is made to thin-wall members, although there is really no flowing involved. It refers to the shear per inch of the cross section and equals the unit shearing stress times the thickness of the member. (The unit shearing stress F_v has been determined by the expression VQ/bI, and the shear flow q_v can be determined by VQ/I if the shearing stress is assumed to be constant across the thickness of the section.) The shear flow acts parallel to the sides of each element of a member.

The channel section of Fig. 10.15(a) will be considered for this discussion. In this figure, the shear flow is shown with the small arrows, and in part (b) the values are

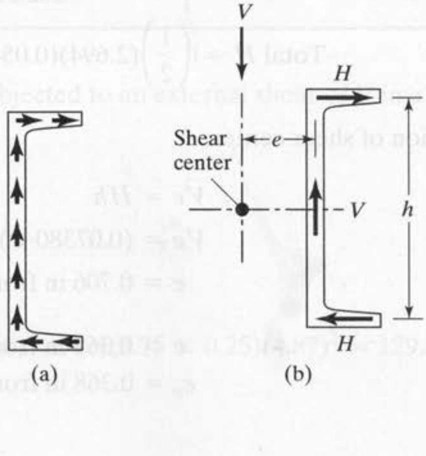


FIGURE 10.15

totaled for each component of the shape and labeled H and V. The two H values are in equilibrium horizontally, and the internal V value balances the external shear at the section. Although the horizontal and vertical forces are in equilibrium, the same cannot be said for the moment forces, unless the lines of action of the resultant of the external forces pass through a certain point called the *shear center.* The horizontal H forces in part (b) of the figure can be seen to form a couple. The moment produced by this couple must be opposed by an equal and opposite moment, which can be produced only by the two V values. The location of the shear center is a problem in equilibrium; therefore, moments should be taken about a point that eliminates the largest number of forces possible.

With this information, the following equation can be written from which the shear center can be located:

$$Ve = Hh \text{ (moments taken about c.g. of web)}$$

Examples 10-8 and 10-9 illustrate the calculations involved in locating the shear center for two shapes. Note that the location of the shear center is independent of the value of the external shear. (*Shear center locations are provided for channels in* Table 1-5 *of Part I of the AISC Manual.* Their location is given by e_o in those tables.)

Example 10-8

The C10 × 30 channel section shown in Fig. 10.16(a) is subjected to an external shear of V in the vertical plane. Locate the shear center.

Solution

Properties of section:

For this channel, the flanges are idealized as rectangular elements with a thickness equal to the average flange thickness.

$$I_x = \left(\frac{1}{12}\right)(3)(10)^3 - \left(\frac{1}{12}\right)(2.327)(9.128)^3 = 102.52 \text{ in}^4$$

$$q_v \text{ at } B = \frac{(V)(2.694 \times 0.436 \times 4.782)}{102.52} = 0.05479\ V/\text{in}$$

$$\text{Total } H = \left(\frac{1}{2}\right)(2.694)(0.05479\ V) = 0.07380\ V$$

Location of shear center:

$$Ve = Hh$$
$$Ve = (0.07380\ V)(9.564)$$
$$e = 0.706 \text{ in from } \mathbb{C}\!\!\!\!L \text{ of web}$$
$$\text{or}$$
$$e = 0.369 \text{ in from back of web}$$
$$e_o = 0.368 \text{ in from Table 1-5 (C10} \times 30)$$

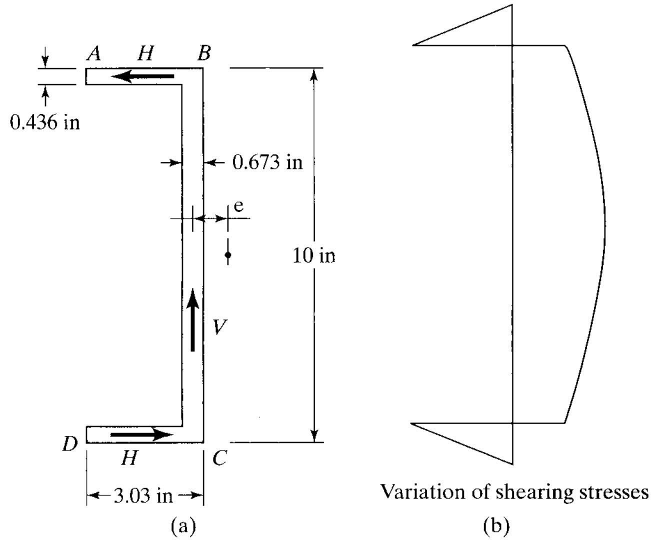


FIGURE 10.16

The student should clearly understand that shear stress variation across the corners, where the webs and flanges join, cannot be determined correctly with the mechanics of materials expression (VQ/bI or VQ/I for shear flow) and cannot be determined too well, even after a complicated study with the theory of elasticity. As an approximation, in the two examples presented here, we have assumed that shear flow continues up to the middle of the corners on the same pattern (straight-line variation for horizontal members and parabolic for others). The values of Q are computed for the corresponding dimensions. Other assumptions could have been made, such as assuming that a shear flow variation continues vertically for the full depth of webs, and only for the protruding parts of flanges horizontally, or vice versa. It is rather disturbing to find that, whichever assumption is made, the values do not check out perfectly. For instance, in Example 10-9, which follows, the sum of the vertical shear flow values does not check out very well with the external shear.

Example 10-9

The open section of Fig. 10.17 is subjected to an external shear of V in a vertical plane. Locate the shear center.

Solution

Properties of section:

$$I_x = \left(\frac{1}{12}\right)(0.25)(16)^3 + (2)(3.75 \times 0.25)(4.87)^2 = 129.8 \text{ in}^4$$

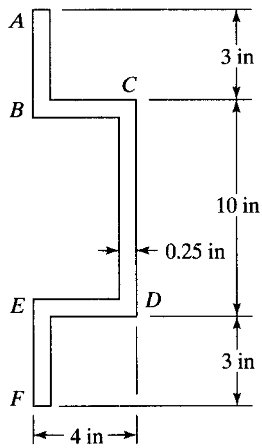


FIGURE 10.17

Values of shear flow (labeled q):

$$q_A = 0$$

$$q_B = \frac{(V)(3.12 \times 0.25 \times 6.44)}{129.8} = 0.0387\ V/\text{in}$$

$$q_C = q_B + \frac{(V)(3.75 \times 0.25 \times 4.87)}{129.8} = 0.0739\ V/\text{in}$$

$$q_{\mathfrak{L}} = q_C + \frac{(V)(4.87 \times 0.25 \times 2.44)}{129.8} = 0.0968\ V/\text{in}$$

These shear flow values are shown in Fig. 10.18(a), and the summation for each part of the member is given in part (b) of the figure.

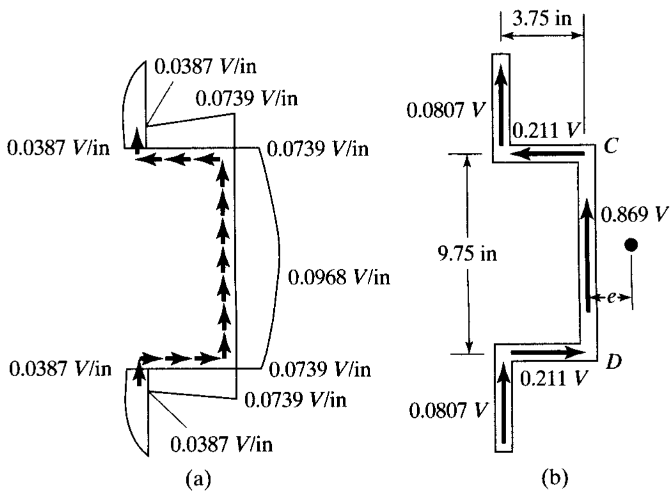


FIGURE 10.18

Taking moments about the center line of CD

$$-(0.211\,V)(9.75) + (2)(0.0807\,V)(3.75) + Ve = 0$$
$$e = 1.45 \text{ in}$$

The theory of the shear center is a very useful one in design, but it has certain limitations that should be clearly understood. For instance, the approximate analysis given in this section is valid only for thin sections. In addition, steel beams often have variable cross sections along their spans, with the result that the loci of the shear centers are not straight lines along those spans. Thus, if the resultant of the loads passes through the shear center at one cross section, it might not do so at other cross sections along the beam.

When designers are faced with the application of loads to thin-walled sections, such that twisting of those sections will be a problem, they will usually provide some means by which the twisting can be constrained. They may specify special bracing at close intervals, or attachments to flooring or roofing, or other similar devices. Should such solutions not be feasible, designers will probably consider selecting sections with greater torsional stiffnesses. Two references on this topic are given here.[6,7]

10.8 BEAM-BEARING PLATES

When the ends of beams are supported by direct bearing on concrete or other masonry construction, it is frequently necessary to distribute the beam reactions over the masonry by means of beam-bearing plates. The reaction is assumed to be spread uniformly through the bearing plate to the masonry, and the masonry is assumed to push up against the plate with a uniform pressure equal to the reaction R_u or R_a over the area of the plate A_1. This pressure tends to curl up the plate and the bottom flange of the beam. The AISC Manual recommends that the bearing plate be considered to take the entire bending moment produced and that the critical section for moment be assumed to be a distance k from the center line of the beam (see Fig. 10.19). The distance k is the same as the distance from the outer face of the flange to the web toe of the fillet, given in the tables for each section (or it equals the flange thickness plus the fillet radius).

The determination of the true pressure distribution in a beam-bearing plate is a very formidable task, and the uniform pressure distribution assumption is usually made. This assumption is probably on the conservative side, as the pressure is typically larger at the center of the beam than at the edges. The outer edges of the plate and flange tend to bend upward, and the center of the beam tends to go down, concentrating the pressure there.

The required thickness of a 1-in-wide strip of plate can be determined as follows, with reference being made to Fig. 10.19:

$$Z \text{ of a 1-in-wide piece of plate of } t \text{ thickness} = (1)\left(\frac{t}{2}\right)\left(\frac{t}{4}\right)(2) = \frac{t^2}{4}$$

---

[6]C. G. Salmon and J. E. Johnson, *Steel Structures, Design and Behavior*, 4th ed. (New York: Harper & Row, 1996), pp. 430–443.

[7]AISC Engineering Staff, "Torsional Analysis of Steel Members" (Chicago: AISC, 1983).

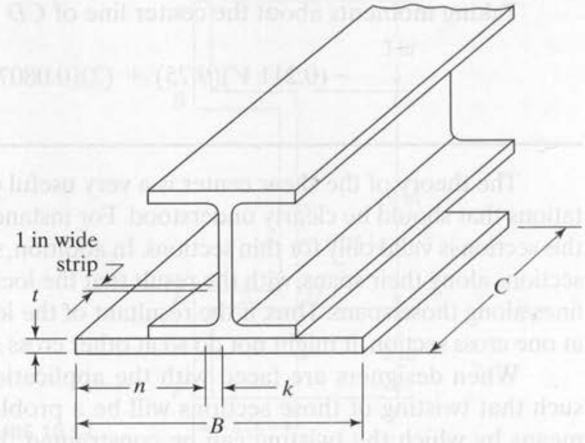


FIGURE 10.19

The moments M_u and M_a are computed at a distance k from the web center line and are equated, respectively, to $\phi_b F_y Z$ and $F_y Z/\Omega_b$; the resulting equations are then solved for the required plate thickness.

LRFD $\phi_b = 0.90$	ASD $\Omega_b = 1.67$
$M_u = \frac{R_u}{A_1} n \left(\frac{n}{2}\right) = \frac{R_u n^2}{2A_1}$	$M_a = \frac{R_a}{A_1} n \left(\frac{n}{2}\right) = \frac{R_a n^2}{2A_1}$
$\frac{R_u n^2}{2A_1} = \phi_b F_y \frac{t^2}{4}$	$\frac{R_a n^2}{2A_1} = \frac{F_y \frac{t^2}{4}}{\Omega_b}$
From which $t_{reqd} = \sqrt{\frac{2R_u n^2}{\phi_b A_1 F_y}}$	From which $t_{reqd} = \sqrt{\frac{2R_a n^2 \Omega_b}{A_1 F_y}}$

In the absence of code regulations specifying different values, the design strength for bearing on concrete is to be taken equal to $\phi_c P_p$ or P_p/Ω_a according to AISC Specification J8. This specification states that when a bearing plate extends for the full area of a concrete support, the bearing strength of the concrete can be determined as follows:

$$P_p = 0.85 f'_c A_1 \qquad \text{(LRFD Equation J8-1)}$$

(The bearing values provided are the same as those given in Section 10.17 of the 2005 *ACI Building Code Requirements for Structural Concrete*.)

Should the bearing load be applied to an area less than the full area of the concrete support, $\phi_c P_p$ is to be determined with the following equation, in which A_2 is the maximum area of the supporting surface that is geometrically similar to and concentric with the loaded area, with $\sqrt{A_2/A_1}$ having a maximum value of 2:

$$P_p = 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7 f'_c A_1 \qquad \text{(AISC Equation J8-2)}$$

In this expression, f'_c is the compression strength of the concrete in psi and A_1 is the area of the plate (in^2) bearing concentrically on the concrete. For the design of such a plate, its required area A_1 can be determined by dividing the factored reaction R_u by $\phi_c 0.85 f'_c$ for LRFD, or by dividing R_a by $0.85 f'_c/\Omega_c$ for ASD.

$$A_1 = \frac{R_u}{\phi_c 0.85 f'_c} \text{ with } \phi_c = 0.65 \quad \text{or } A_1 = \frac{\Omega_c R_a}{0.85 f'_c} \text{ with } \Omega_c = 2.31$$

After A_1 is determined, its length (parallel to the beam) and its width are selected. The length may not be less than the N required to prevent web yielding or web crippling of the beam, nor may it be less than about 3 1/2 or 4 in for practical construction reasons. It may not be greater than the thickness of the wall or other support, and actually, it may have to be less than that thickness, particularly at exterior walls, to prevent the steel from being exposed.

Example 10-10 illustrates the calculations involved in designing a beam-bearing plate. Notice that the width and the length of the plate are desirably taken to the nearest full inch.

Example 10-10

A W18 × 71 beam ($d = 18.5$ in, $t_w = 0.495$ in, $b_f = 7.64$ in, $t_f = 0.810$ in, $k = 1.21$ in) has one of its ends supported by a reinforced-concrete wall with $f'_c = 3$ ksi. Design a bearing plate for the beam with A36 steel, for the service loads $R_D = 30$ k and $R_L = 50$ k. The maximum length of end bearing ⊥ to the wall is the full wall thickness = 8.0 in.

Solution

Compute plate area A_1.

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$R_u = (1.2)(30) + (1.6)(50) = 116$ k	$R_a = 30 + 50 = 80$ k
$A_1 = \frac{R_u}{\phi_c 0.85 f'_c} = \frac{116}{(0.65)(0.85)(3)}$	$A_1 = \frac{\Omega_c R_a}{0.85 f'_c} = \frac{(2.31)(80)}{(0.85)(3)}$
$= 70.0$ in^2	$= 72.5$ in^2
Try PL 8 × 10 (80 in^2).	Try PL 8 × 10 (80 in^2).

Check web local yielding.

$$R_n = (2.5k + l_b)F_{yw}t_w \qquad \text{(AISC Equation J10-3)}$$
$$= (2.5 \times 1.21 + 8)(36)(0.495) = 196.5 \text{ k}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$R_u = \phi R_n = (1.00)(196.5)$	$R_a = \dfrac{R_n}{\Omega} = \dfrac{196.5}{1.50}$
$= 196.5\text{ k} > 116\text{ k}$ **OK**	$= 131\text{ k} > 80\text{ k}$ **OK**

Check web crippling.

$$\frac{l_b}{d} = \frac{8}{18.5} = 0.432 > 0.2 \quad \therefore \text{ Must use AISC Equation (J10-5b)}$$

$$R_n = 0.40t_w^{\,2}\left[1 + \left(\frac{4l_b}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

$$= (0.40)(0.495)^2\left[1 + \left(\frac{4 \times 8}{18.5} - 0.2\right)\left(\frac{0.495}{0.810}\right)^{1.5}\right]\sqrt{\frac{(29 \times 10^3)(36)(0.810)}{0.495}}$$

$$= 221.7\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$R_u = \phi R_n = (0.75)(221.7)$	$R_a = \dfrac{R_n}{\Omega} = \dfrac{221.7}{2.00} = 111\text{ k}$
$= 166\text{ k} > 116\text{ k}$ **OK**	$>80\text{ k}$ **OK**

Determine plate thickness.

$$n = \frac{10}{2} - 1.21 = 3.79\text{ in}$$

LRFD $\phi_b = 0.90$	ASD $\Omega_b = 1.67$
$t = \sqrt{\dfrac{2R_u n^2}{\phi_b A_1 F_y}} = \sqrt{\dfrac{(2)(116)(3.79)^2}{(0.9)(80)(36)}}$	$t = \sqrt{\dfrac{2R_a n^2 \Omega_b}{A_1 F_y}} = \sqrt{\dfrac{(2)(80)(3.79)^2(1.67)}{(80)(36)}}$
$= 1.13$ in	$= 1.15$ in
Use PL $1\frac{1}{4} \times 8 \times 10$ (A36).	**Use PL $1\frac{1}{4} \times 8 \times 10$ (A36).**

On some occasions, the beam flanges alone probably provide sufficient bearing area, but bearing plates are nevertheless recommended, as they are useful in erection and ensure an even bearing surface for the beam. They can be placed separately from the beams and carefully leveled to the proper elevations. When the ends of steel beams are enclosed by the concrete or masonry walls, it is considered desirable to use some type of wall anchor to prevent the beam from moving longitudinally with respect to the wall. The usual anchor consists of a bent steel bar called a government anchor passing through the web of the beam and running parallel to the wall. Occasionally, clip angles attached to the web are used instead of government anchors. Should longitudinal loads of considerable size be anticipated, regular vertical anchor bolts may be used at the beam ends.

If we were to check to see if the flange thickness alone is sufficient, we would have $\left(\text{with } n = \dfrac{b_f}{2} - k\right) = \dfrac{7.64}{2} - 1.21 = 2.61$ in.

LRFD $\phi_b = 0.90$	ASD $\Omega_b = 1.67$
$t = \sqrt{\dfrac{(2)(116)(2.61)^2}{(0.9)(8 \times 7.64)(36)}}$	$t = \sqrt{\dfrac{(2)(80)(2.61)^2(1.67)}{(8)(7.64)(36)}}$
$= 0.893$ in $> t_f = 0.810$ in for W18 × 71 **N.G.**	$= 0.910 > t_f = 0.810$ in for W18 × 71 **N.G.**

$\therefore$ Flange t_f is not sufficient alone for either LRFD or ASD designs.

10.9 LATERAL BRACING AT MEMBER ENDS SUPPORTED ON BASE PLATES

The ends of beams and girders (and trusses) supported on base plates must be restrained against rotation about their longitudinal axes. This is clearly stated in Section F1 (2) of the AISC Specification. Stability at these locations may be obtained in several ways. Regardless of the method selected, the beams and bearing plates must be anchored to the supports.

Beam flanges may be connected to the floor or roof system which themselves must be anchored to prevent translation; the beam ends may be built into solid masonry or concrete walls; or transverse bearing stiffeners or end plates may be used as illustrated in Fig. 10.20.

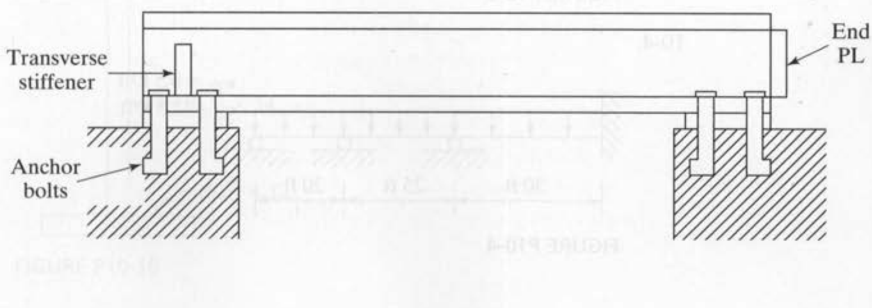


FIGURE 10.20

Bracing at beam ends

10.10 PROBLEMS FOR SOLUTION

Use LRFD for Probs. 10-1 to 10-9 except as indicated. Use both methods for all others.

10-1 to 10-5. *Considering moment only and assuming full lateral support for the compression flanges, select the lightest sections available, using 50 ksi steel and the LRFD method. The loads shown include the effect of the beam weights. Use elastic analysis, factored loads, and the 0.9 rule.*

10-1. (*Ans.* W24 × 62)

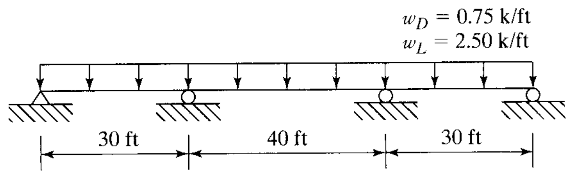


FIGURE P10-1

10-2.

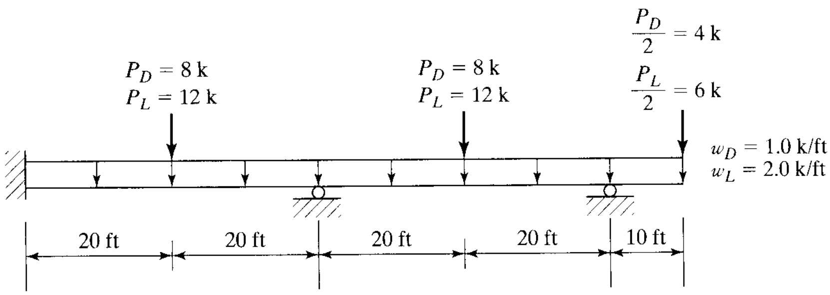


FIGURE P10-2

10-3. (*Ans.* W21 × 50)

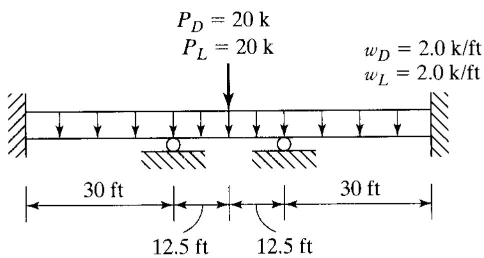


FIGURE P10-3

10-4.

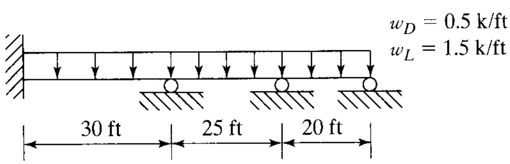


FIGURE P10-4

10-5. (*Ans.* W21 × 44)

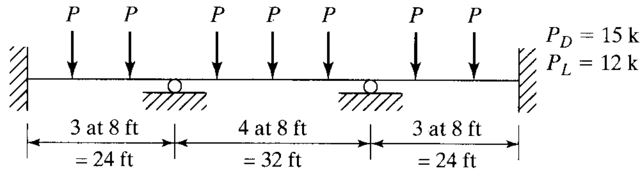


FIGURE P10-5

10-6. Repeat Prob. 10-1, using plastic analysis.
10-7. Repeat Prob. 10-3, using plastic analysis. (*Ans.* W21 × 48)
10-8. Repeat Prob. 10-5, using plastic analysis.
10-9. Three methods of supporting a roof are shown in Fig. P10-9. Using an elastic analysis with factored loads, $F_y = 50$ ksi, and assuming full lateral support in each case, select the lightest section if a dead uniform service load (including the beam self-weight) of 1.5 k/ft and a live uniform service load of 2.0 k/ft is to be supported. Consider moment only.

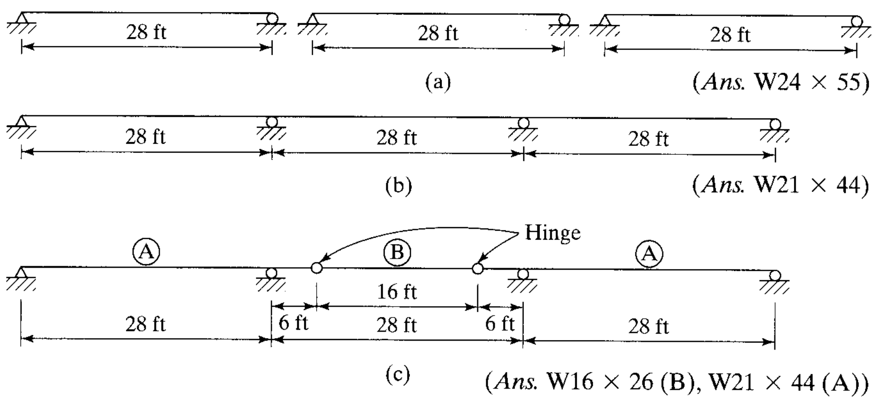


FIGURE P10-9

10-10. The welded plate girder section shown, made from 50 ksi steel, has full lateral support for its compression flange and is bent about its major axis. If $C_b = 1.0$, determine its design and allowable moments and shear strengths.

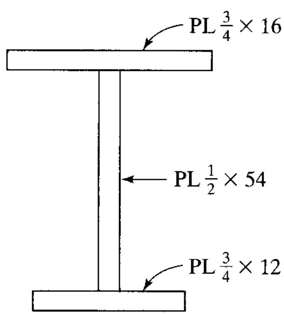


FIGURE P10-10

10-11 to 10-13. *Using F_y = 50 ksi, select the lightest available W-shape section for the span and loading shown. Make your initial member selection based on moment and check for shear. Neglect the beam self-weight in your calculations. The members are assumed to have full lateral bracing of their compression flanges.*

10-11. (*Ans.* W21 × 62 LRFD, W24 × 62 ASD)

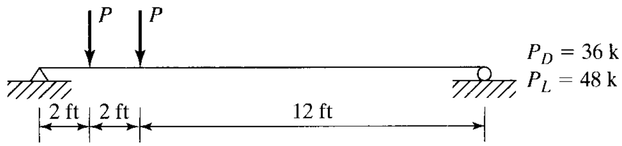


FIGURE P10-11

10-12.

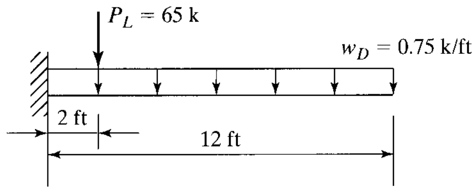


FIGURE P10-12

10-13. Repeat Prob. 10-11, using F_y = 36 ksi. (*Ans.* W24 × 76 LRFD, W24 × 84 ASD)

10-14. A W14 × 26 is to be used as a simply supported beam on a span of 12 ft with a single concentrated load located at 2 ft from the left support. Check for moment and shear to determine the maximum P_D and P_L permitted on the beam, using 50 ksi steel and the LRFD and the ASD methods. Assume P is 25 percent dead load and 75 percent live load. Neglect beam weight. Full lateral support of the compression flange exists.

10-15. A W10 × 17 is to be used as a simply supported beam on a span of 8 ft with a single moving concentrated service live load of 56 k. Use A992 steel and neglect the beam self-weight. Where along the span of the beam can the load be placed and not exceed the shear strength capacity using the LRFD method? (*Ans.* 1.5 ft ≤ x ≤ 6.5 ft)

10-16. A W12 × 40 consisting of 50 ksi steel is used as a simple beam for a span of 7.25 ft. If it has full lateral support, determine the maximum uniform loads w_u and w_a that it can support in addition to its own weight. Use LRFD and ASD methods and consider shear and moment only.

10-17. A 24-ft, simply supported beam must support a moving concentrated service live load of 50 k in addition to a uniform service dead load of 2.5 k/ft. Using 50 ksi steel, select the lightest section considering moments and shear only. Use LRFD and ASD methods and neglect the beam self-weight. (*Ans.* W24 × 76 LRFD and ASD)

10-18. A 36-ft simple beam that supports a service uniform dead load of 1.0 k/ft and a service concentrated live load of 25 k at midspan is laterally unbraced except at its ends and at the concentrated load at midspan. If the maximum permissible centerline deflection under service loads equal $L/360$ total load and $L/1000$ live load, select the most economical W section of 50 ksi steel, considering moment, shear and deflection. The beam self-weight is included in the uniform dead load. Use C_b = 1.0.

10-19. Design a beam for a 30-ft simple span to support the working uniform loads of w_D = 1.25 k/ft (includes beam self-weight) and w_L = 1.75 k/ft. The maximum permissible total load deflection under working loads is 1/360 of the span. Use 50 ksi steel and consider moment, shear and deflection. The beam is to be braced laterally at its ends and the midspan only. (*Ans.* W24 × 76 LRFD and ASD)

10-20. Select the lightest W shape of A992 steel for uniform service dead load and the concentrated service live load shown in Fig. P10-20. The dead load includes the beam self-weight. The beam has lateral support for its compression flange and the ends and at the concentrated load. The maximum service live load deflection may not exceed 1/1000 of the span. Consider moment, shear and deflection.

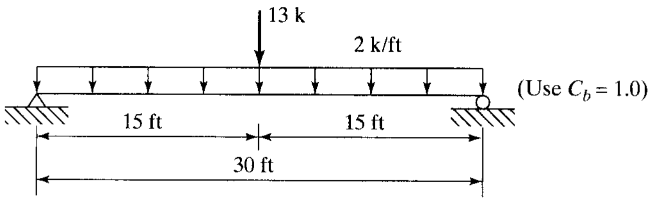


FIGURE P10-20

10-21. The cantilever beam shown in Fig. P10-21 is a W14 × 34 of A992 steel. There is no lateral support other than at the fixed end. Use an unbraced length equal to the span length. The uniform load is a service dead load that includes the beam weight, and the concentrated load is a service live load. Determine if the beam is adequate for moment, shear and an allowable total load deflection of 0.25 in. (*Ans.* OK for moment and deflection, N.G. for shear)

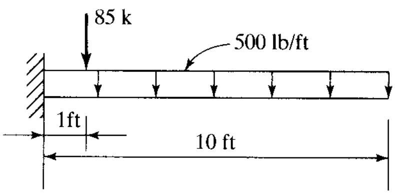


FIGURE P10-21

10-22. A W24 × 68 consisting of 50 ksi steel is used as a simply supported beam shown in Fig. P10-22 Include the beam weight and determine if the member has sufficient shear capacity using the equations from Chapter G of the AISC Specification (you may check your answer using the values from the tables), and determine whether or not the beam will meet the following deflection criteria: max LL = L/360 and max TL = L/240.

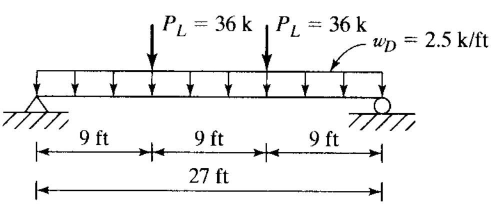


FIGURE P10-22

10-23. Select the lightest available W sections (F_y = 50 ksi) for the beams and girders shown in Fig. P10-23. The floor slab is 6 in reinforced concrete (weight = 145 lb/ft^3) and supports a 125 psf uniform live load. Assume that continuous lateral bracing of the compression flange is provided. The maximum permissible TL deflection is L/240. (*Ans.* Beam = W21 × 44 LRFD and ASD, Girder = W24 × 62 LRFD and ASD)

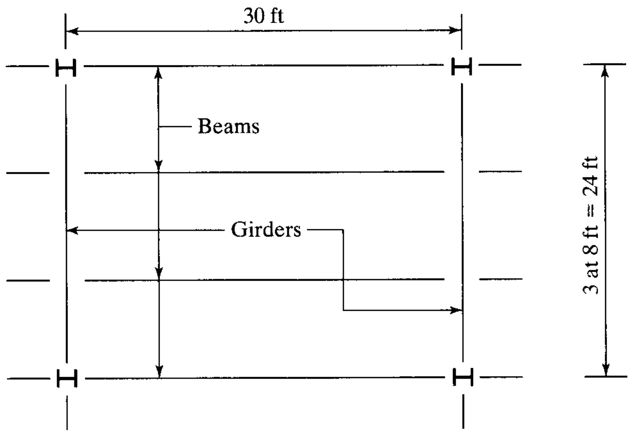


FIGURE P10-23

10-24. Repeat Prob. 10-23 if the live load is 250 psf.

10-25. Select the lightest available W section of 50 ksi steel for a beam that is simply supported on the left end and a fixed support on the right end of a 36-ft span. The member supports a service dead load of 2.4 k/ft, including its self-weight and a service live load of 3.0 k/ft. Assume full lateral support of the compression flange and maximum TL deflection of L/600. Consider moment, shear and deflection. (*Ans.* W30 × 108 LRFD, W30 × 116 ASD)

10-26. Repeat Prob. 10-25 if the nominal depth of the beam is limited to 27 in and lateral support of the compression flange is provided at the ends and the 1/3 points of the span. Use C_b = 1.0.

10-27. The beam shown in Fig. P10-27 is a W14 × 34 of A992 steel and has lateral support of the compression flange at the ends and at the points of the concentrated loads. The two concentrated loads are service live loads. Check the beam for shear and for Web Local Yielding and Web Crippling at the concentrated load if l_b = 6 in. Neglect the self-weight of the beam. (*Ans.* Shear and web crippling N.G., web local yielding OK)

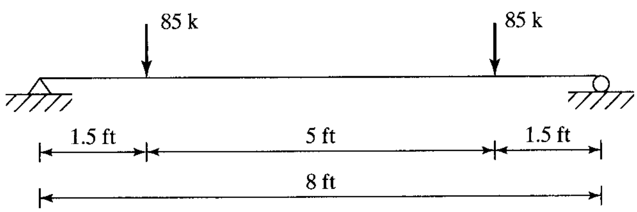


FIGURE P10-27

10-28. A 7-ft beam with full lateral support for its compression flange is supporting a moving concentrated live load of 58 k. Using 50 ksi steel, select the lightest W section. Assume the moving load can be placed anywhere in the middle 5 ft of the beam span. Choose a member based on moment then check if it is satisfactory for shear, and compute the minimum length of bearing required at the supports from the standpoint of web local yielding and web crippling. Neglect self-weight.

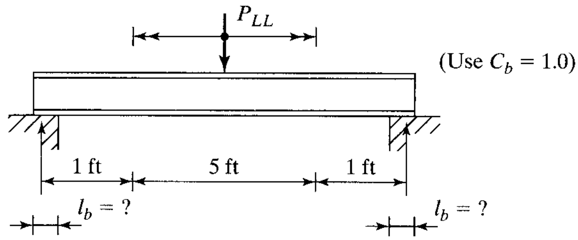


FIGURE P10-28

10-29. A 12 ft beam with full lateral support of the compression flange is used on a simple span. A concentrated service live load of 64 k is applied 3 ft from the left support. Use A992 steel and neglect the member's self-weight. Select the lightest W section based on moment. Check the beam for Local Flange Bending at the concentrated load and for Web Local Yielding at the concentrated load if $l_b = 4$ in. Neglect beam self-weight. Resize beam if necessary. (*Ans.* W18 × 46 LRFD and ASD)

10-30. A W21 × 68 member is used as a simply supported beam with a span length of 12 ft. Determine C_b, since the lateral support of the compression flange is provided only at the ends. The member is uniformly loaded. The loads will produce factored moments of $M_{Dx} = 75$ ft-k, $M_{Lx} = 90$ ft-k and $M_{Dy} = 15$ ft-k, $M_{Ly} = 18$ ft-k. Is this member satisfactory for bending strength based on the interaction equation in Chapter H of the AISC Specification?

10-31. The 30-ft, simply supported beam shown in Fig. P10-31 has full support of its compression flange and is A992 steel. The beam supports a gravity service dead load of 132 lb/ft (includes beam weight) and gravity live load of 165 lb/ft. The loads are assumed to act through the c.g. of the section. Select the lightest available W10 section. (*Ans.* W10 × 22 LRFD, W10 × 26 ASD)

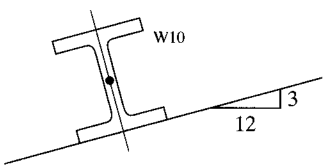


FIGURE P10-31

10-32. Design a steel bearing plate from A572 (Grade 50) steel for a W18 × 35 beam, with end reactions of $R_D = 12$ k and $R_L = 16$ k. The beam will bear on a reinforced concrete wall with $f'_c = 3$ ksi. In the direction perpendicular to wall, the bearing plate maximum length of end bearing may not be longer than 6 in. W18 is A992 steel.

10-33. Design a steel bearing plate from A36 steel for a W24 × 55 beam supported by a reinforced concrete wall with $f'_c = 3$ ksi. The maximum beam reaction is $R_D =$ 30 k and $R_L = 40$ k. The maximum length of end bearing perpendicular to the wall is the full wall thickness of 10 in. W24 is A992 steel. (*Ans.* Use PL11/8 × 9 3 0 ft to 8 in LRFD and ASD)

CHAPTER 11

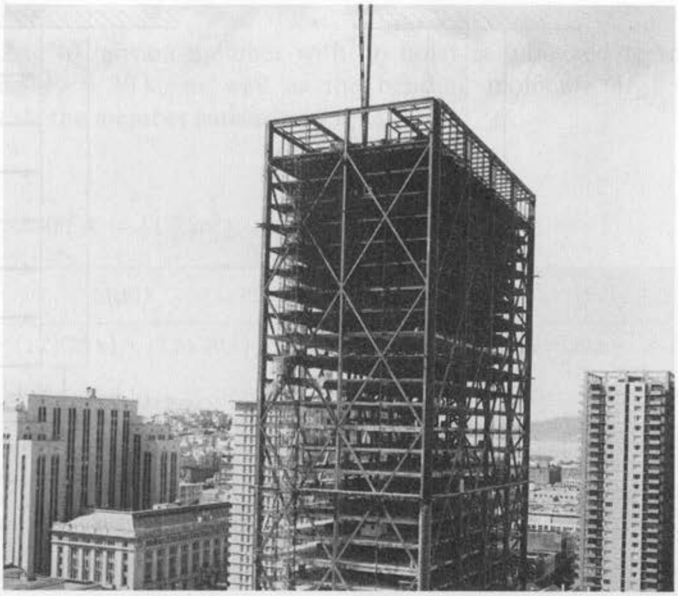
Bending and Axial Force

11.1 OCCURRENCE

Structural members that are subjected to a combination of bending and axial force are far more common than the student may realize. This section is devoted to listing some of the more obvious cases. Columns that are part of a steel building frame must nearly always resist sizable bending moments in addition to the usual compressive loads. It is almost impossible to erect and center loads exactly on columns, even in a testing lab; in an actual building, one can see that it is even more difficult. Even if building loads could be perfectly centered at one time, they would not stay in one place. Furthermore, columns may be initially crooked or have other flaws resulting in lateral bending. The beams framing into columns are commonly supported with framing angles or brackets on the sides of the columns. These eccentrically applied loads produce moments. Wind and other lateral loads cause columns to bend laterally, and the columns in rigid frame buildings are subjected to moments, even when the frame is supporting gravity loads alone. The members of bridge portals must resist combined forces, as do building columns. Among the causes of the combined forces are heavy lateral wind loads or seismic loads, vertical traffic loads—whether symmetrical or not—and the centrifugal effect of traffic on curved bridges.

The previous practice of the student has probably been to assume that truss members are only axially loaded. Purlins for roof trusses, however, are frequently placed between truss joints, causing the top chords to bend. Similarly, the bottom chords may be bent by the hanging of light fixtures, ductwork, and other items between the truss joints. All horizontal and inclined truss members have moments caused by their own weights, while all truss members—whether vertical or not—are subjected to secondary bending forces. Secondary forces are developed because the members are not connected with frictionless pins; as assumed in the usual analysis, the members' centers of gravity or those of their connectors do not exactly coincide at the joints, etc.

Moments in tension members are not as serious as those in compression members, because tension tends to reduce lateral deflections while compression increases them. Increased lateral deflections in turn result in larger moments, which cause larger



The Alcoa Building under construction in San Francisco, CA. (Courtesy of Bethlehem Steel Corporation.)

lateral deflections, and so on. It is hoped that members in such situations are stiff enough to prevent the additional lateral deflections from becoming excessive.

11.2 MEMBERS SUBJECT TO BENDING AND AXIAL TENSION

A few types of members subject to both bending and axial tension are shown in Fig. 11.1.

In Section H1 of the AISC Specification, the interaction equations that follow are given for symmetric shapes subjected simultaneously to bending and axial tensile forces. These equations are also applicable to members subjected to bending and compression forces, as will be described in Sections 11.3 to 11.9.

For $\frac{P_r}{P_c} \geq 0.2$,

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1a)}$$

and for $\frac{P_r}{P_c} < 0.2$,

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1b)}$$

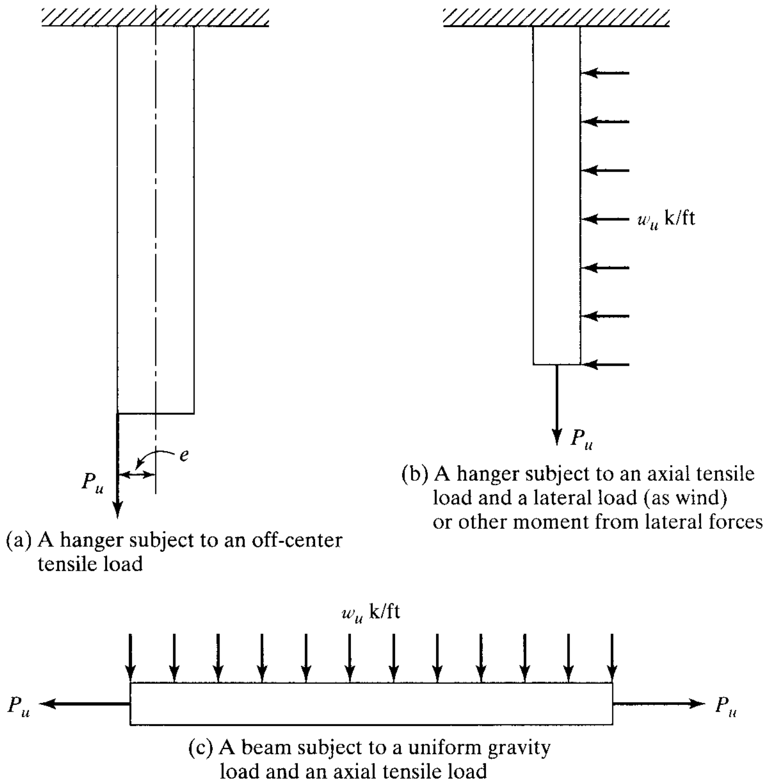


FIGURE 11.1

Some members subject to bending and axial tension.

in which

P_r = required axial tensile strength, P_u (LRFD) or P_a (ASD), kips

P_c = design axial tensile strength ($\phi_c P_n$) or allowable axial tensile strength $\left(\dfrac{P_n}{\Omega_c}\right)$, kips

M_r = required flexural strength, M_u (LRFD) or M_a (ASD), ft-k

M_c = design flexural strength ($\phi_b M_n$) or allowable flexural strength $\left(\dfrac{M_n}{\Omega_b}\right)$, ft-k

Usually, only a first-order analysis (that is, not including any secondary forces as described in the next section) is made for members subject to bending and axial tension. It is conservative to neglect the effects of tension forces acting with bending moments. However, the analyst may—in fact, is encouraged to—make second-order analyses for these members and use the results in his or her designs. Examples 11-1 and 11-2 illustrate the use of the interaction equations to review members subjected simultaneously to bending and axial tension.

Example 11-1

A 50 ksi W12 × 40 tension member with no holes is subjected to the axial loads $P_D = 25$ k and $P_L = 30$ k, as well as the bending moments $M_{Dy} = 10$ ft-k and $M_{Ly} = 25$ ft-k. Is the member satisfactory if $L_b < L_p$?

Solution

Using a W12 × 40 ($A = 11.7$ in^2)

LRFD	ASD
$P_r = P_u = (1.2)(25\text{ k}) + (1.6)(30\text{ k}) = 78\text{ k}$	$P_r = P_a = 25\text{ k} + 30\text{ k} = 55\text{ k}$
$M_{ry} = M_{uy} = (1.2)(10\text{ ft-k}) + (1.6)(25\text{ ft-k})$	$M_{ry} = M_{ay} = 10\text{ ft-k} + 25\text{ ft-k} = 35\text{ ft-k}$
$= 52\text{ ft-k}$	
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50\text{ ksi})(11.7\text{ in}^2)$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_c} = \dfrac{(50\text{ ksi})(11.7\text{ in}^2)}{1.67}$
$= 526.5\text{ k}$	$= 350.3\text{ k}$
$M_{cy} = \phi_b M_{py} = 63.0$ ft-k (AISC Table 3-4)	$M_{cy} = \dfrac{M_{cy}}{\Omega_b} = 41.9$ ft-k (AISC Table 3-4)
$\dfrac{P_r}{P_c} = \dfrac{78\text{ k}}{526.5\text{ k}} = 0.148 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{55\text{ k}}{350.3\text{ k}} = 0.157 < 0.2$
$\therefore$ **Must use AISC Eq. H1-1b**	$\therefore$ **Must use AISC Eq. H1-1b**
$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$
$\dfrac{78}{(2)(526.5)} + \left(0 + \dfrac{52}{63}\right)$	$\dfrac{55}{(2)(350.3)} + \left(0 + \dfrac{35}{41.9}\right)$
$= 0.899 < 1.0$ **OK**	$= 0.914 < 1.0$ **OK**

Example 11-2

A W10 × 30 tensile member with no holes, consisting of 50 ksi steel and with $L_b = 12.0$ ft, is subjected to the axial service loads $P_D = 30$ k and $P_L = 50$ k and to the service moments $M_{Dx} = 20$ ft-k and $M_{Lx} = 40$ ft-k. If $C_b = 1.0$, is the member satisfactory?

Solution

Using a W10 × 30 ($A = 8.84$ in^2, $L_p = 4.84$ ft and $L_r = 16.1$ ft, $\phi_b M_{px} = 137$ ft-k, BF for LRFD = 4.61, BF for ASD = 3.08 and $M_{px}/\Omega_b = 91.3$ ft-k from AISC Table 3-2)

LRFD	ASD
$P_r = P_u = (1.2)(30\text{k}) + (1.6)(50\text{k}) = 116 \text{ k}$	$P_r = P_a = 30 \text{ k} + 50 \text{ k} = 80 \text{ k}$
$M_{rx} = M_{ux} = (1.2)(20 \text{ ft-k}) + (1.6)(40 \text{ ft-k})$	$M_{rx} = M_{ax} = 20 \text{ ft-k} + 40 \text{ ft-k}$
$= 88 \text{ ft-k}$	$= 60 \text{ ft-k}$
$P_c = \phi P_n = \phi_t F_y A_g = (0.9)(50 \text{ ksi})(8.84 \text{ in}^2)$	$P_c = \dfrac{P_n}{\Omega_c} = \dfrac{F_y A_g}{\Omega_c} = \dfrac{(50 \text{ ksi})(8.84 \text{ in}^2)}{1.67}$
$= 397.8 \text{ k}$	$= 264.7 \text{ k}$
$M_{cx} = \phi_b M_{nx} = C_b[\phi_b M_{px} - BF(L_b - L_p)]$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b} = C_b\left[\dfrac{M_{px}}{\Omega_b} - BF(L_b - L_p)\right]$
$= 1.0[137 - 4.61(12.0 - 4.84)]$	$= 1.0[91.3 - (3.08)(12 - 4.84)]$
$= 104.0 \text{ ft-k}$	$= 69.2 \text{ ft-k}$
$\dfrac{P_r}{P_c} = \dfrac{116}{397.8} = 0.292 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{80}{264.7} = 0.302 > 0.2$
$\therefore$ **Must use AISC Eq. H1-1a**	$\therefore$ **Must use AISC Eq. H1-1a**
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{rx}}{M_{cy}}\right) \leq 1.0$
$\dfrac{116}{397.8} + \dfrac{8}{9}\left(\dfrac{88}{104.0} + 0\right)$	$\dfrac{80}{264.7} + \dfrac{8}{9}\left(\dfrac{60}{69.2} + 0\right)$
$= 1.044 > 1.0$ **N.G.**	$= 1.073 > 1.0$ **N.G.**

11.3 FIRST-ORDER AND SECOND-ORDER MOMENTS FOR MEMBERS SUBJECT TO AXIAL COMPRESSION AND BENDING

When a beam column is subjected to moment along its unbraced length, it will be displaced laterally in the plane of bending. The result will be an increased or secondary moment equal to the axial compression load times the lateral displacement or eccentricity. In Fig. 11.2, we can see that the member moment is increased by an amount $P_{nt}\delta$, where P_{nt} is the axial compression force determined by a first-order analysis. This moment will cause additional lateral deflection, which will in turn cause a larger column moment, which will cause a larger lateral deflection, and so on until equilibrium is reached. M_r is the required moment strength of the member. M_{nt} is the first-order moment, assuming no lateral translation of the frame.

If a frame is subject to sidesway where the ends of the columns can move laterally with respect to each other, additional secondary moments will result. In Fig. 11.3, the secondary moment produced due to sidesway is equal to $P_{nt}\ \Delta$.

The moment M_r is assumed by the AISC Specification to equal M_{lt} (which is the moment due to the lateral loads) plus the moment due to $P_u\ \Delta$.

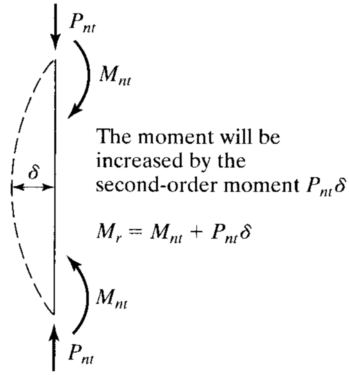


FIGURE 11.2

Moment amplification of a column that is braced against sidesway.

The required total flexural strength of a member must at least equal the sum of the first-order and second-order moments. Several methods are available for determining this required strength, ranging from very simple approximations to very rigorous procedures.

A rigorous second-order inelastic analysis of the structure computes and takes into account the anticipated deformations in calculating the required maximum compressive strength, P_r, and the maximum required flexural strength, M_r. This method is usually more complex than is necessary for the structural design of typical structures. Should the designer make a second-order analysis, he or she should realize that it must account for the interaction of the various load effects. That is, one must consider combinations of loads acting at the same time. We cannot correctly make separate analyses and superimpose the results because it is inherently a non-linear problem.

The AISC Specification Chapter C.1 states that any rational method of design for stability that considers all of the effects list below is permitted.

1. flexural, shear, and axial member deformation, and all other deformations that contribute to displacement of the structure;
2. second-order effect (both P-Δ and P-δ effects);
3. geometric imperfections;

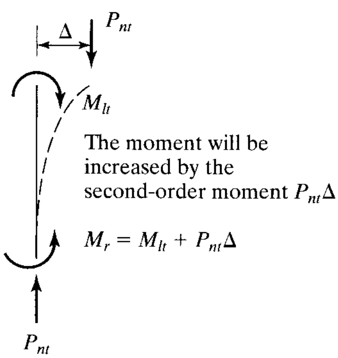


FIGURE 11.3

Column in an unbraced frame.

4. stiffness reductions due to inelasticity;
5. uncertainty in stiffness and strength.

Three methods are presented in the AISC Specification. The *Direct Analysis Method* of design in Chapter C, the *Effective Length Method*, and the *First-Order Analysis Method* in Appendix 7 of the Specification are permitted alternatives. In this chapter, the authors present the Direct Analysis Method and the Effective Length Method.

11.4 DIRECT ANALYSIS METHOD (DM)

This method is applicable to all types of structures. It does not distinguish between building structural systems such as braced frames, moment frames, shear wall, or any combination of systems. It has the additional advantage of not having to calculate the effective length factor, K. This means that in determining the available axial compressive strength, P_c, $K = 1.0$ is used.

11.4.1 Second-Order Effects (C2.1 (2) – AISC Specification)

The required strength, P_r, can be determined using a rigorous second-order analysis that requires an iterative computer analysis of the model or by the approximate technique of utilizing an amplified first-order analysis using magnification factors, B_1 and B_2, that is specified in Appendix 8 and discussed in Section 11.5 of the text.

11.4.2 Stiffness Reduction (C2.3 – AISC Specification)

The Direct Analysis Method uses a reduced flexural and axial stiffness to account for the influence of inelasticity and uncertainty in strength and stiffness on second-order effects. In the analysis, reduced stiffness EI^*, is replaced with $0.8\tau_b EI$ and EA^*, is replaced with $0.8EA$.

The τ_b factor depends on the level of axial stress in the member which implies that the modulus of elasticity reduces as the material goes inelastic.

When: $\dfrac{\alpha P_r}{P_y} \le 0.5$

$$\tau_b = 1.0 \qquad \text{(AISC Equation C2-2a)}$$

When: $\dfrac{\alpha P_r}{P_y} > 0.5$

$$\tau_b = 4\left(\frac{\alpha P_r}{P_y}\right)\left[1 - \frac{\alpha P_r}{P_y}\right] \qquad \text{(AISC Equation C2-2b)}$$

Where: $\alpha = 1.0$ (LRFD) and $\alpha = 1.6$ (ASD)
P_r = required axial compressive strength, kips
P_y = axial yield strength, kips

11.4.3 Notional Loads (C2.2b – AISC Specification)

To account for initial out-of-plumbness of columns (geometric imperfections), the Direct Analysis provisions require the application of notional loads. Notional loads are applied as lateral loads to a model of the structure that is based on its nominal geometry. The magnitude of the notional loads shall be:

$$N_i = 0.002\alpha Y_i \qquad \text{(AISC Equation C2-1)}$$

Where: $\alpha = 1.0$ (LRFD) and $\alpha = 1.6$ (ASD)
N_i = notional load applied at level i, kips
Y_i = gravity load applied at level i from load combinations, kips

Note: The 0.002 term represents an out-of-plumbness of 1/500 which is the maximum tolerance on column plumbness specified in the *AISC Code of Standard Practice for Steel Buildings and Bridges.*

The notional load shall be additive to other lateral loads and shall be applied in all load combinations, except as noted in AISC Specification section C2.2b(4). This section states that for structures where the ratio of maximum second-order drift to maximum first-order drift in all stories is equal to or less than 1.7,

$$B_2 = \frac{\Delta_{second\text{-}order}}{\Delta_{first\text{-}order}} \leq 1.7$$

it is permissible to apply the notional load, N_i, only in gravity-only load combinations and not in combinations that include other lateral loads.

11.5 EFFECTIVE LENGTH METHOD (ELM)

This method, found in Appendix 7 of the AISC Specification, is applicable where the ratio of maximum second-order drift to maximum first-order drift in all stories is equal to or less than 1.5.

That is:
$$B_2 = \frac{\Delta_{second\text{-}order}}{\Delta_{first\text{-}order}} \leq 1.5$$

The required strength, P_r, is calculated from an analysis conforming to the requirements of AISC Specification C2.1, except that the stiffness reduction indicated in C2.1 (2) need not be applied. The nominal stiffness of structural members is used.

Notional loads need only be applied in gravity-only load cases.

The K factor must be determined from a sidesway buckling analysis or the alignment charts as shown in Chapter 7 of this text. It is permitted to use a $K = 1.0$ in the design of all braced systems and in moment frames where the ratio of maximum second-order drift to maximum first-order drift in all stories is equal to or less than 1.1.

That is:
$$B_2 = \frac{\Delta_{second\text{-}order}}{\Delta_{first\text{-}order}} \leq 1.1$$

TABLE 11.1 Comparison of Basic Stability Requirements with Specific Provisions

Basic Requirement in Section C1		Provision in Direct Analysis Method (DM)	Provision in Effective Length Method (ELM)
(1) Consider all deformations		C2.1(1). Consider all deformations	Same as DM (by reference to C2.1)
(2) Consider second-order effects (both P-Δ and P-δ)		C2.1(2). Consider second-order effects (P-Δ and P-δ)**	Same as DM (by reference to C2.1)
(3) Consider geometric imperfections *This includes joint-position imperfections*(which affect structure response) and member imperfections (which affect structure response and member strength)*	Effect of joint-position imperfections* on structure response	C2.2a. Direct modeling or C2.2b. Notional loads	Same as DM, second option only (by reference to C2.2b)
	Effect of member imperfections on structure response	Included in the stiffness reduction specified in C2.3	All these effects are considered by using KL from a sidesway buckling analysis in the member strength check. Note that the only difference between DM and ELM is that: • DM uses reduced stiffness in the analysis; $KL = L$ in the member strength check • ELM uses full stiffness in the analysis; KL from sidesway buckling analysis in the member strength check for frame members
	Effect of member imperfections on member strength	Included in member strength formulas, with $KL = L$	
(4) Consider stiffness reduction due to inelasticity *This affects structure response and member strength*	Effect of stiffness reduction on structure response	Included in the stiffness reduction specified in C2.3	
	Effect of stiffness reduction on member strength	Included in member strength formulas, with $KL = L$	
(5) Consider uncertainty in strength and stiffness *This affects structure response and member strength*	Effect of stiffness/strength uncertainty on structure response	Included in the stiffness reduction specified in C2.3	
	Effect of stiffness/strength uncertainty on member strength	Included in member strength formulas, with $KL = L$	

*In typical building structures, the "joint-position imperfections" refers to column out-of-plumbness.
**Second-order effects may be considered either by rigorous second-order analysis or by the approximate technique (using B_1 and B_2) specified in Appendix 8.

Table C-C1.1 of the commentary of Chapter C of the AISC Specification, which is reproduced in Table 11.1, presents a comparison of the basic stability requirements of the Direct Analysis Method (DM) and the Effective Length Method (ELM).

11.6 APPROXIMATE SECOND-ORDER ANALYSIS

In this chapter, the authors present the approximate second-order analysis given in Appendix 8 of the AISC Specification. We will make two first-order elastic analyses—one an analysis where the frame is assumed to be braced so that it cannot sway. We will call these moments M_{nt} and will multiply them by a magnification factor called B_1 to account for the P-δ effect (see Fig. 11.2). Then we will analyze the frame again, allowing it to sway. We will call these moments M_{lt} and will multiply them by a magnification factor called B_2 to account for the P-Δ effect (see Fig. 11.3). The final moment in a particular member will equal

$$M_r = B_1 M_{nt} + B_2 M_{lt}. \qquad \text{(AISC Equation A-8-1)}$$

The final axial strength P_r must equal

$$P_r = P_{nt} + B_2 P_{lt}. \qquad \text{(AISC Equation A-8-2)}$$

Instead of using the AISC empirical procedure described here, the designer may—and is encouraged to—use a theoretical second-order elastic analysis, provided that he or she meets the requirements of Chapter C of the Specification.

11.6.1 Magnification Factors

The magnification factors are B_1 and B_2. With B_1, the analyst attempts to estimate the $P_{nt}\delta$ effect for a column, whether the frame is braced or unbraced against sidesway. With B_2, he or she attempts to estimate the $P_{lt}\,\Delta$ effect in unbraced frames.

These factors are theoretically applicable when the connections are fully restrained or when they are completely unrestrained. The AISC Manual indicates that the determination of secondary moments in between these two classifications for partially restrained moment connections is beyond the scope of their specification. The terms *fully restrained* and *partially restrained* are discussed at length in Chapter 15 of this text.

The horizontal deflection of a multistory building due to wind or seismic load is called *drift*. It is represented by Δ in Fig. 11.3. Drift is measured with the so-called *drift index* Δ_H/L, where Δ_H is the first-order lateral inter-story deflection and L is the story height. For the comfort of the occupants of a building, the index usually is limited at working or service loads to a value between 0.0015 and 0.0030, and at factored loads to about 0.0040.

The expression to follow for B_1 was derived for a member braced against sidesway. It will be used only to magnify the M_{nt} moments (*those moments computed, assuming that there is no lateral translation of the frame*).

$$B_1 = \frac{C_m}{1 - \alpha \dfrac{P_r}{P_{e1}}} \geq 1.0 \qquad \text{(AISC Equation A-8-3)}$$

In this expression, C_m is a term that is defined in the next section of this chapter; α is a factor equal to 1.00 for LRFD and 1.60 for ASD; P_r is the required axial strength of the member; and P_{e1} is the member's Euler buckling strength calculated on the

$$P_{e1} = \frac{EI^*}{(K_1 L)^2} \qquad \text{(AISC Equation A-8-5)}$$

basis of zero sidesway. One is permitted to use the first-order estimate of P_r (that is, $P_r = P_{nt} + P_{lt}$) when calculating magnification factor, B_1. Also, K is the effective length factor in the plane of bending, determined based on the assumption of no lateral translation, set equal to 1.0, unless analysis justifies a smaller value. EI^* is 0.8 $\tau_b EI$ when the direct analysis method is used, and EI when the effective length method is used.

In a similar fashion, P_{e2} is the elastic critical buckling resistance for the story in question, determined by a sidesway buckling analysis. For this analysis, K_2L is the effective length in the plane of bending, based on the sidesway buckling analysis. For this case, the sidesway buckling resistance may be calculated with the following expression, in which Σ is used to include all of the columns on that level or story:

$$P_{e\ story} = \Sigma \frac{\pi^2 EI}{(K_2L)^2}$$

Furthermore, the AISC permits the use of the following alternative expression for calculating $P_{e\ story}$

$$P_{e\ story} = R_M \frac{HL}{\Delta_H} \qquad \text{(AISC Equation A-8-7)}$$

Here, the factors are defined as follows:

$$R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{story}}\right) \qquad \text{(AISC Equation A-8-8)}$$

H = story shear produced by the lateral loads used to compute Δ_H, kips

L = story height, in

Δ_H = first-order interstory drift due to the lateral loads computed using stiffness required for the analysis method used, in

P_{mf} = total vertical load in columns in the story that are part of moment frame, kips ($P_{mf} = 0$ for braced frame systems)

The values shown for P_{story} and $P_{e\ story}$ are for all of the columns on the floor in question. This is considered to be necessary because the B_2 term is used to magnify column moments for sidesway. For sidesway to occur in a particular column, it is necessary for all of the columns on the floor to sway simultaneously.

$$B_2 = \frac{1}{1 - \dfrac{P_{story}}{P_{e\ story}}} \qquad \text{(AISC Equation A-8-6)}$$

We must remember that the amplification factor B_2 is only applicable to moments caused by forces that cause sidesway and is to be computed for an entire story. (Of course, if you want to be conservative, you can multiply B_2 times the sum of the no-sway and the sway moments—that is, M_{nt} and M_{lt}—but that's probably overdoing it.) To use the B_2 value given by AISC Equation A-8-6, we must select initial member sizes (that is, so we can compute a value for $P_{e\ story}$ or Δ_H).

To calculate the values of P_{story} and $P_{e\ story}$ some designers will calculate the values for the columns in the one frame under consideration. This, however, is a rather bad practice, unless all the other frames on that level are exactly the same as the one under study.

11.6.2 Moment Modification or C_m Factors

In Section 11.6.1, the subject of moment magnification due to lateral deflections was introduced, and the factors B_1 and B_2 were presented with which the moment increases could be estimated. In the expression for B_1, a term C_m, called the *modification factor*, was included. The magnification factor B_1 was developed for the largest possible lateral displacement. On many occasions, the displacement is not that large, and B_1 overmagnifies the column moment. As a result, the moment may need to be reduced or modified with the C_m factor. In Fig. 11.4, we have a column bent in single curvature, with equal end moments such that the column bends laterally by an amount δ at mid-depth. The maximum total moment occurring in the column clearly will equal M plus the increased moment $P_{nt}\delta$. As a result, no modification is required and $C_m = 1.0$.

An entirely different situation is considered in Fig. 11.5, where the end moments tend to bend the member in reverse curvature. The initial maximum moment occurs at one of the ends, and we shouldn't increase it by a value $P_{nt}\delta$ that occurs some distance out in the column, because we will be overdoing the moment magnification. The purpose of the modification factor is to modify or reduce the magnified moment when the variation of the moments in the column is such that B_1 is made too large. If we didn't use a modification factor, we would end up with the same total moments in the columns of both Figs. 11.4 and 11.5, assuming the same dimensions and initial moments and load.

Modification factors are based on the rotational restraint at the member ends and on the moment gradients in the members. The AISC Specification (Appendix 8) includes two categories of C_m, as described in the next few paragraphs.

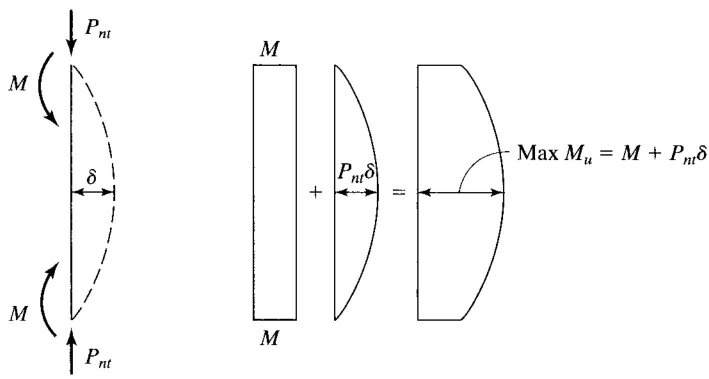


FIGURE 11.4

Moment magnification for column bent in single curvature.

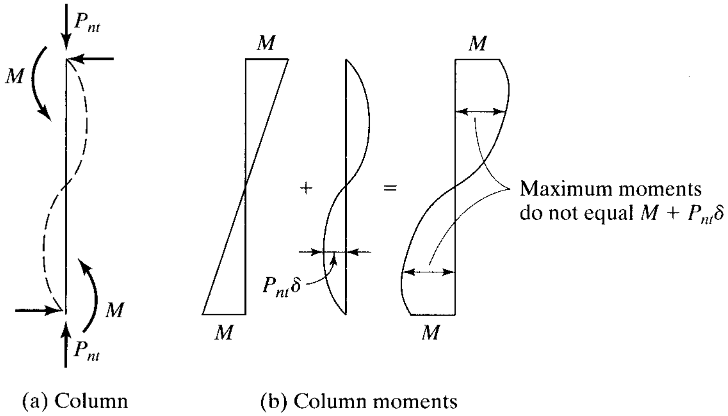


FIGURE 11.5

Moment magnification for column bent in double curvature.

In Category 1, the members are prevented from joint translation or sidesway, and they are not subject to transverse loading between supports. For such members, the modification factor is based on an elastic first-order analysis.

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \qquad \text{(AISC Equation A-8-4)}$$

In this expression, M_1/M_2 is the ratio of the smaller moment to the larger moment at the ends of the unbraced length in the plane of bending under consideration. The ratio is negative if the moments cause the member to bend in single curvature, and positive if they bend the members in reverse or double curvature. As previously described, a member in single curvature has larger lateral deflections than a member bent in reverse curvature. With larger lateral deflections, the moments due to the axial loads will be larger.

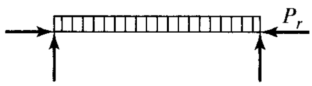
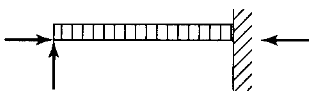
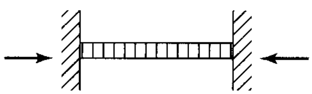
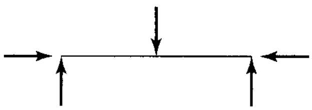
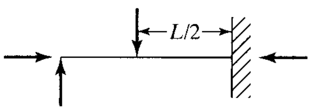
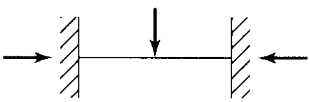
Category 2 applies to members that are subjected to transverse loading between supports. The compression chord of a truss with a purlin load between its joints is a typical example of this category. The AISC Specification states that the value of C_m for this situation may be determined by rational analysis or by setting it conservatively equal to 1.0.

Instead of using these values for transversely loaded members, the values of C_m for Category 2 may be determined for various end conditions and loads by the values given in Table 11.2, which is a reproduction of Table C-A-8.1 of the Commentary on Appendix 8 of the AISC Specification. In the expressions given in the table, P_r is the required column axial load and P_{e1} is the elastic buckling load for a braced column for the axis about which bending is being considered.

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2} \qquad \text{(AISC Equation A-8-5)}$$

In Table 11.2, note that some members have rotationally restrained ends and some do not. Sample values of C_m are calculated for four beam columns and shown in Fig. 11.6.

TABLE 11.2 Amplification Factors (ψ) and Modification Factors (C_m) for Beam Columns Subject to Transverse Loads between Joints.

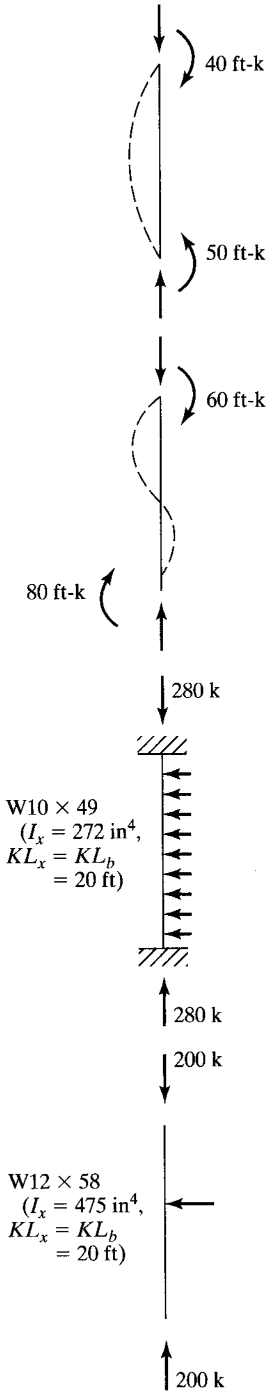
Case	ψ	C_m
	0	1.0
	−0.4	$1 - 0.4\dfrac{\alpha P_r}{P_{e1}}$
	−0.4	$1 - 0.4\dfrac{\alpha P_r}{P_{e1}}$
	−0.2	$1 - 0.2\dfrac{\alpha P_r}{P_{e1}}$
	−0.3	$1 - 0.3\dfrac{\alpha P_r}{P_{e1}}$
	−0.2	$1 - 0.2\dfrac{\alpha P_r}{P_{e1}}$

Source: Commentary on the Specification, Appendix 8–Table C–A–8.1, p16.1–525. June 22, 2010.

11.7 BEAM–COLUMNS IN BRACED FRAMES

The same interaction equations are used for members subject to axial compression and bending as were used for members subject to axial tension and bending. However, some of the terms involved in the equations are defined somewhat differently. For instance, P_a and P_u refer to compressive forces rather than tensile forces.

To analyze a particular beam column or a member subject to both bending and axial compression, we need to perform both a first-order and a second-order analysis to obtain the bending moments. The first-order moment is usually obtained by making an elastic analysis and consists of the moments M_{nt} (these are the moments in



(a) No sidesway and no transverse loading.
Moments bend member in single curvature.

$$C_m = 0.6 - (0.4)\left(-\frac{40}{50}\right) = 0.92$$

(b) No sidesway and no transverse loading.
Moments bend member in reverse curvature.

$$C_m = 0.6 - 0.4\left(+\frac{60}{80}\right) = 0.30$$

(c) Member has restrained ends and transverse loading and is bent about *x* axis.
C_m can be determined from Table 11.1 (AISC Table C-A-8.1) as follows:

$$\alpha P_r = 280 \text{ k}$$

$$P_{e1} = \frac{\pi^2 EI}{(KL)_x^2} = \frac{(\pi^2)(29 \times 10^3)(272)}{(12 \times 20)^2}$$

$$= 1351 \text{ k}$$

$$C_m = 1 - 0.4\left(+\frac{280}{1351}\right) = 0.92$$

(d) Member has unrestrained ends and transverse loading and is bent about *x* axis.
C_m can be determined from Table 11.1 (AISC Table C-A-8.1).

$$\alpha P_r = 200 \text{ k}$$

$$P_{e1} = \frac{(\pi^2)(29 \times 10^3)(475)}{(12 \times 20)^2} = 2360 \text{ k}$$

$$C_m = 1 - 0.2\left(+\frac{200}{2360}\right) = 0.98$$

FIGURE 11.6

Example modification or C_m factors.

beam-columns caused by gravity loads) and the moments M_{lt} (these are the moments in beam-columns due to the lateral loads).

Theoretically, if both the loads and frame are symmetrical, M_{lt} will be zero. Similarly, if the frame is braced, M_{lt} will be zero. For practical purposes, you can have lateral deflections in taller buildings with symmetrical dimensions and loads.

Examples 11-3 to 11-5 illustrate the application of the interaction equations to beam-columns that are members of braced frames. In these examples, the approximate second-order analysis is used, only B_1 will be computed, as B_2 is not applicable. *It is to be remembered that C_m was developed for braced frames and thus must be used in these three examples for calculating B_1.* Also, the effective length method is used. This means that the values of axial loads and moments were determined in a first-order analysis using unreduced member stiffness and notional loads were added to the gravity-only load cases.

Example 11-3

A 12-ft W12 × 96 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments and is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 175$ k, $P_L = 300$ k, and first-order $M_{Dx} = 60$ ft-k and $M_{Lx} = 60$ ft-k?

Solution. Using a W12 × 96 ($A = 28.2\text{ in}^2$, $I_x = 833\text{ in}^4$, $\phi_b M_{px} = 551$ ft-k, $\dfrac{M_{px}}{\Omega_b} = 367$ ft-k, $L_p = 10.9$ ft, $L_r = 46.7$ ft, $BF = 5.78$ k for LRFD and 3.85 k for ASD).

LRFD	ASD
$P_{nt} = P_u = (1.2)(175) + (1.6)(300) = 690\text{ k}$	$P_{nt} = P_a = 175 + 300 = 475\text{ k}$
$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(60) = 168\text{ ft-k}$	$M_{ntx} = M_{ax} = 60 + 60 = 120\text{ ft-k}$
For a braced frame, let $K = 1.0$	For a braced frame, let $K = 1.0$
$\therefore (KL)_x = (KL)_y = (1.0)(12) = 12\text{ ft}$	$\therefore (KL)_x = (KL)_y = (1.0)(12) = 12\text{ ft}$
$P_c = \phi_c P_n = 1080\text{ k}$ (AISC Table 4-1)	$P_c = \dfrac{P_n}{\Omega_c} = 720\text{ k}$ (AISC Table 4-1)
$P_r = P_{nt} + \beta_2 P_{lt} = 690 + 0 = 690\text{ k}$	$P_r = P_{nt} + \beta_2 P_{lt} = 475 + 0 = 475\text{ k}$
$\dfrac{P_r}{P_c} = \dfrac{690}{1080} = 0.639 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{475}{720} = 0.660 > 0.2$
$\therefore$ Must use AISC Eq. H1-1a	$\therefore$ Must use AISC Eq. H1-1a
$C_{mx} = 0.6 - 0.4\dfrac{M_1}{M_2}$	$C_{mx} = 0.6 - 0.4\dfrac{M_1}{M_2}$
$C_{mx} = 0.6 - 0.4\left(-\dfrac{168}{168}\right) = 1.0$	$C_{mx} = 0.6 - 0.4\left(-\dfrac{120}{120}\right) = 1.0$

(Continued)

LRFD	ASD
$P_{e1x} = \dfrac{\pi^2 EI_x}{(K_1L_x)^2} = \dfrac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$	$P_{e1x} = \dfrac{\pi^2 EI_x}{(K_1L_x)^2} = \dfrac{(\pi^2)(29{,}000)(833)}{(1.0 \times 12 \times 12)^2}$
$= 11{,}498 \text{ k}$	$= 11{,}498 \text{ k}$
$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{(1.0)(690)}{11{,}498}} = 1.064$	$B_{1x} = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{(1.6)(475)}{11{,}498}} = 1.071$
$M_{rx} = B_{1x}M_{ntx} = (1.064)(168) = 178.8 \text{ ft-k}$	$M_{rx} = (1.071)(120) = 128.5 \text{ ft-k}$
Since $L_b = 12 \text{ ft} > L_p = 10.9 \text{ ft} < L_r = 46.6 \text{ ft}$	Since $L_b = 12 \text{ ft} > L_p = 10.9 \text{ ft} < L_r = 46.6 \text{ ft}$
$\therefore$ **Zone 2**	$\therefore$ **Zone 2**
$\phi_b M_{px} = 1.0[551 - (5.78)(12 - 10.9)] = 544.6 \text{ ft-k}$	$\dfrac{M_{px}}{\Omega_b} = 1.0[367 - 3.85\,(12 - 10.9)] = 362.7 \text{ ft-k}$
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right)$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) = \dfrac{475}{720} + \dfrac{8}{9}\left(\dfrac{128.5}{362.7} + 0\right)$
$= \dfrac{690}{1080} + \dfrac{8}{9}\left(\dfrac{178.8}{544.6} + 0\right) = 0.931 < 1.0 \text{ OK}$	$= 0.975 < 1.0 \text{ OK}$
$\therefore$ **Section is satisfactory.**	$\therefore$ **Section is satisfactory.**

In Part 6 of the AISC Manual, a somewhat simplified procedure for solving AISC Equations H1-1a and H1-1b is presented. The student, after struggling through Example 11-3, will surely be delighted to see these expressions, which are used for some of the remaining examples in this chapter.

Various parts of the equations are taken out, and numerical values substituted into them for W sections and recorded in Table 6-1 of the Manual. Each of these terms, such as p, b_x, and b_y, are shown on page 6-3 of the Manual. The revised forms of the equations, which follow, are given in the Manual on page 6-4:

$$pP_r + b_xM_{rx} + b_yM_{ry} \le 1.0 \quad \text{(Modified AISC Equation H1-1a)}$$

$$\frac{1}{2}pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) \le 1.0 \quad \text{(Modified AISC Equation H1-1b)}$$

The value of p is based on the larger of $(KL)_y$ and equivalent $(KL)_y = (KL)_x/(r_x/r_y)$, and b_x is based on the unbraced length L_b. A single value of b_y applies for any W shape member because unbraced length is not a factor in weak-axis bending.

Example 11-3 is repeated as Example 11-4 with these simplified expressions. You must be sure to use the magnified values of M_{rx} and P_r in these equations.

Example 11-4

Repeat Example 11-3, using the AISC simplified method of Part 6 of the Manual and the values for K, L, P_r and M_{rx} determined in that earlier example.

Solution

LRFD	ASD
From Example 11-3 (LRFD)	From Example 11-3 (ASD)
$P_r = 690$ k	$P_r = 475$ k
$M_{rx} = 178.8$ ft-k	$M_{rx} = 128.5$ ft-k
From AISC Table 6-1 for a W12 × 96 with $KL = 12$ ft and $L_b = 12$ ft	From AISC Table 6-1 for a W12 × 96 with $KL = 12$ ft and $L_b = 12$ ft
$p = 0.924 \times 10^{-3}$	$p = 1.39 \times 10^{-3}$
$b_x = 1.63 \times 10^{-3}$	$b_x = 2.45 \times 10^{-3}$
$b_y = 3.51 \times 10^{-3}$ (from bottom of table)	$b_y = 5.28 \times 10^{-3}$ (from bottom of table)
Then with the modified equation $(0.924 \times 10^{-3})(690) + (1.63 \times 10^{-3})(178.8) + (3.51 \times 10^{-3})(0) = 0.929 < 1.0$	Then with the modified equation $(1.39 \times 10^{-3})(475) + (2.45 \times 10^{-3})(128.5) + (5.28 \times 10^{-3})(0) = 0.975 < 1.0$
Section is satisfactory.	**Section is satisfactory.**

In the examples of this chapter, the student may find that when it becomes necessary to compute a value for P_e, it is slightly easier to use a number given in AISC Table 4-1 for the appropriate steel section than to substitute into the equation $P_e = \dfrac{\pi^2 EI}{(K_1L)^2}$. For instance, for a W12 × 96, we read at the bottom of the table a value given for P_{ex}, $(KL)^2/10^4 = 23{,}800$. If $KL = 12$ ft, P_{ex} will equal $\dfrac{(23{,}800)(10^4)}{(1.0 \times 12 \times 12)^2} = 11{,}478$ k.

Example 11-5

A 14-ft W14 × 120 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite moments. Its ends are rotationally restrained, and it is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 70$ k, and $P_L = 100$ k and if it has the first-order moments $M_{Dx} = 60$ ft-k, $M_{Lx} = 80$ ft-k, $M_{Dy} = 40$ ft-k, and $M_{Ly} = 60$ ft-k?

Solution. Using a W14 × 120 ($A = 35.3$ in^2, $I_x = 1380$ in^4, $I_y = 495$ in^4, $Z_x = 212$ in^3, $Z_y = 102$ in^3, $L_p = 13.2$ ft, $L_r = 51.9$ ft, BF for LRFD = 7.65 k, and BF for ASD = 5.09 k).

LRFD	ASD
$P_{nt} = P_u = (1.2)(70) + (1.6)(100) = 244$ k	$P_{nt} = P_a = 70 + 100 = 170$ k
$M_{ntx} = M_{ux} = (1.2)(60) + (1.6)(80) = 200$ ft-k	$M_{ntx} = M_{ax} = 60 + 80 = 140$ ft-k
$M_{nty} = M_{uy} = (1.2)(40) + (1.6)(60) = 144$ ft-k	$M_{nty} = M_{ay} = 40 + 60 = 100$ ft-k
For a braced frame $K = 1.0$	For a braced frame $K = 1.0$
$KL = (1.0)(14) = 14$ ft	$KL = (1.0)(14) = 14$ ft
$P_c = \phi_c P_n = 1370$ k (AISC Table 4-1)	$P_c = \dfrac{P_n}{\Omega_c} = 912$ k (AISC Table 4-1)
$P_r = P_{nt} + \beta_2 P_{lt} = 244 + 0 = 244$ k	$P_r = P_{nt} + \beta_2 P_{lt} = 170 + 0 = 170$ k
$\dfrac{P_r}{P_c} = \dfrac{244}{1370} = 0.178 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{170}{912} = 0.186 < 0.2$
$\therefore$ Must use AISC Equation H1-1b	$\therefore$ Must use AISC Equation H1-1b
$C_{mx} = 0.6 - 0.4\left(-\dfrac{200}{200}\right) = 1.0$	$C_{mx} = 0.6 - 0.4\left(-\dfrac{140}{140}\right) = 1.0$
$P_{e1x} = \dfrac{(\pi^2)(29{,}000)(1380)}{(1.0 \times 12 \times 14)^2} = 13{,}995$ k	$P_{e1x} = \dfrac{(\pi^2)(29{,}000)(1380)}{(1.0 \times 12 \times 14)^2} = 13{,}995$ k
$B_{1x} = \dfrac{1.0}{1 - \dfrac{(1.0)(244)}{13{,}995}} = 1.018$	$B_{1x} = \dfrac{1.0}{1 - \dfrac{(1.6)(170)}{13{,}995}} = 1.020$
$M_{rx} = (1.018)(200) = 203.6$ ft-k	$M_{rx} = (1.020)(140) = 142.8$ ft-k
$C_{my} = 0.6 - 0.4\left(-\dfrac{144}{144}\right) = 1.0$	$C_{my} = 0.6 - 0.4\left(-\dfrac{100}{100}\right) = 1.0$
$P_{e1y} = \dfrac{(\pi^2)(29{,}000)(495)}{(1.0 \times 12 \times 14)^2} = 5020$ k	$P_{e1y} = \dfrac{(\pi^2)(29{,}000)(495)}{(1.0 \times 12 \times 14)^2} = 5020$ k
$B_{1y} = \dfrac{1.0}{1 - \dfrac{(1.0)(244)}{5020}} = 1.051$	$B_{1y} = \dfrac{1.0}{1 - \dfrac{(1.6)(170)}{5020}} = 1.057$
$M_{ry} = (1.051)(144) = 151.3$ ft-k	$M_{ry} = (1.057)(100) = 105.7$ ft-k
From AISC Table 6-1, for $KL = 14$ ft and $L_b = 14$ ft	From AISC Table 6-1, for $KL = 14$ ft and $L_b = 14$ ft
$p = 0.730 \times 10^{-3}$, $b_x = 1.13 \times 10^{-3}$, $b_y = 2.32 \times 10^{-3}$	$p = 1.10 \times 10^{-3}$, $b_x = 1.69 \times 10^{-3}$, $b_y = 3.49 \times 10^{-3}$

(*Continued*)

LRFD	ASD
$\frac{1}{2}p\,P_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \leq 1.0$	$\frac{1}{2}p\,P_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \leq 1.0$
$= \frac{1}{2}(0.730 \times 10^{-3})(244)$	$= \frac{1}{2}(1.10 \times 10^{-3})(170)$
$+ \frac{9}{8}(1.13 \times 10^{-3})(203.6)$	$+ \frac{9}{8}(1.69 \times 10^{-3})(142.8)$
$+ \frac{9}{8}(2.32 \times 10^{-3})(151.3)$	$+ \frac{9}{8}(3.49 \times 10^{-3})(105.7)$
$= 0.743 \leq 1.0$ OK	$= 0.780 \leq 1.0$ OK
Section is satisfactory but perhaps overdesigned.	**Section is satisfactory but perhaps overdesigned.**

Examples 11–3 and 11–5 are reworked using the Direct Analysis Method in Examples 11–6 and 11–7. The values of axial loads and moments given were determined in a first-order analysis using *reduced* member stiffness and application of the notional loads. The analysis yielded values that were essentially equal to the values from the Effective Length Method.

Example 11-6

A 12 ft long W12 × 96 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments and is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 175$ k, $P_L = 300$ k, and first-order $M_{Dx} = 60$ ft-k and $M_{Lx} = 60$ ft-k?

Solution. Using a $W12 \times 96$ ($A = 28.2$ in^2, $I_x = 833$ in^4, $\phi_b M_{px} = 551$ ft-k, $M_{px}/\Omega_b = 367$ ft-k, $L_p = 10.9$ ft, $L_r = 46.7$ ft, $BF = 5.78$ k for LRFD and 3.85 k for ASD).

LRFD	ASD
$P_{nt} = P_u = 1.2\,(175) + 1.6\,(300) = 690$ kips	$P_{nt} = P_a = 175 + 300 = 475$ kips
$M_{ntx} = M_{ux} = 1.2\,(60) + 1.6\,(60) = 168$ ft-k	$M_{ntx} = M_{ax} = 60 + 60 = 120$ ft-k
For Direct Analysis Method, $K = 1.0$	For Direct Analysis Method, $K = 1.0$
$\therefore\ (KL)_x = (KL)_y = 1.0(12) = 12$ ft	$\therefore\ (KL)_x = (KL)_y = 1.0(12) = 12$ ft
$P_c = \Phi_c P_n = 1080$ k (AISC Table 4-1)	$P_c = P_n / \Omega_c = 720$ k (AISC Table 4-1)
B_2 is not required, since braced frame, therefore	B_2 is not required, since braced frame, therefore
$P_r = P_{nt} + B_2 P_{lt} = 690 + 0 = 690$ k	$P_r = P_{nt} + B_2 P_{lt} = 475 + 0 = 475$ k
$\frac{P_r}{P_c} = \frac{690}{1080} = 0.639 > 0.2$	$\frac{P_r}{P_c} = \frac{475}{720} = 0.660 > 0.2$

(Continued)

LRFD	ASD
∴ Must use AISC Equation H1-1a	∴ Must use AISC Equation H1-1a
$C_m = 0.6 - 0.4\dfrac{M_1}{M_2}$	$C_m = 0.6 - 0.4\dfrac{M_1}{M_2}$
$= 0.6 - 0.4\left(-\dfrac{168}{168}\right) = 1.0$	$= 0.6 - 0.4\left(-\dfrac{120}{120}\right) = 1.0$
Determine τ_b:	Determine τ_b:
$\dfrac{\alpha P_r}{P_y} = \dfrac{1.0(690\text{ k})}{(28.2\text{ in}^2)(50\text{ ksi})} = 0.49 < 0.5$	$\dfrac{\alpha P_r}{P_y} = \dfrac{1.6(475\text{ k})}{(28.2\text{ in}^2)(50\text{ ksi})} = 0.539 > 0.5$
$\therefore\ \tau_b = 1.0$	$\therefore\ \tau_b = 4(\alpha P_r/P_y)[1-(\alpha P_r/P_y)]$
	$\tau_b = 4(0.539)[1-(0.539)] = 0.994$
$P_{e1x} = \dfrac{\pi^2 0.8\tau_b EI^*}{(K_1L_x)^2} = \dfrac{\pi^2 0.8(1.0)(29{,}000)(833)}{(1.0\times 12\times 12)^2} = 9198\text{ k}$	$P_{e1x} = \dfrac{\pi^2 0.8\tau_b EI^*}{(K_1L_x)^2}$ $= \dfrac{\pi^2 0.8(0.994)(29{,}000)(833)}{(1.0\times 12\times 12)^2} = 9143\text{ k}$
$B_{1x} = \dfrac{C_{mx}}{1-\dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1-\dfrac{1.0(690)}{9198}} = 1.081$	$B_{1x} = \dfrac{C_{mx}}{1-\dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1-\dfrac{1.6(475)}{9143}} = 1.091$
$M_{rx} = B_{1x}M_{ntx} = 1.081(168) = 181.6$ ft-k	$M_{rx} = B_{1x}M_{ntx} = 1.091(120) = 130.9$ ft-k
Since $L_b = 12\text{ ft} > L_p = 10.9\text{ ft}, < L_r = 46.6\text{ ft}$	Since $L_b = 12\text{ ft} > L_p = 10.9\text{ ft}, < L_r = 46.6\text{ ft}$
∴ Zone 2	∴ Zone 2
$M_{cx} = \Phi_b M_{px} = 1.0[551-(5.78)(12-10.9)] = 544.6$ ft-k	$M_c = M_{px}/\Omega_b = 1.0[367-(3.85)(12-10.9)] = 362.7$ ft-k
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$
$\dfrac{690}{1080} + \dfrac{8}{9}\left(\dfrac{181.6}{544.6} + 0\right) = 0.935 \le 1.0$	$\dfrac{475}{720} + \dfrac{8}{9}\left(\dfrac{130.9}{362.7} + 0\right) = 0.981 \le 1.0$
∴ Section is satisfactory.	**∴ Section is satisfactory.**

Example 11-7

A 14 ft long W14 × 120 (50 ksi steel) is used as a beam-column in a braced frame. It is bent in single curvature with equal and opposite end moments. Its ends are rotationally restrained and it is not subjected to intermediate transverse loads. Is the section satisfactory if $P_D = 70$ k, $P_L = 100$ k, and if it has first-order moments $M_{Dx} = 60$ ft-k, $M_{Lx} =$ 80 ft-k, $M_{Dy} = 40$ ft-k and $M_{Ly} = 60$ ft-k?

Solution. Using a W14 × 120 ($A = 35.3\text{ in}^2$, $I_x = 1380\text{ in}^4$, $I_y = 495\text{ in}^4$, $Z_x = 212\text{ in}^3$, $Z_y = 102\text{ in}^3$, $L_p = 13.2$ ft, $L_r = 51.9$ ft, $BF = 7.65$ k for LRFD and 5.09 k for ASD).

LRFD	ASD
$P_{nt} = P_u = 1.2\ (70\text{ k}) + 1.6\ (100\text{ k}) = 244\text{ kips}$	$P_{nt} = P_a = 70\text{ k} + 100\text{ k} = 170\text{ kips}$
$M_{ntx} = M_{ux} = 1.2\ (60\text{ ft-k}) + 1.6\ (80\text{ ft-k}) = 200\text{ ft-k}$	$M_{ntx} = M_{ax} = 60\text{ ft-k} + 80\text{ ft-k} = 140\text{ ft-k}$
$M_{nty} = M_{uy} = 1.2\ (40\text{ ft-k}) + 1.6\ (60\text{ ft-k}) = 144\text{ ft-k}$	$M_{nty} = M_{ay} = 40\text{ ft-k} + 60\text{ ft-k} = 100\text{ ft-k}$
For Direct Analysis Method, $K = 1.0$	For Direct Analysis Method, $K = 1.0$
$\therefore\ (KL)_x = (KL)_y = 1.0(14) = 14\text{ ft}$	$\therefore\ (KL)_x = (KL)_y = 1.0(14) = 14\text{ ft}$
$P_c = \Phi_c P_n = 1370\text{ k}$ (AISC Table 4-1)	$P_c = P_n/\Omega_c = 912\text{ k}$ (AISC Table 4-1)
B_2 is not required, since braced frame, therefore	B_2 is not required, since braced frame, therefore
$P_r = P_{nt} + B_2 P_{lt} = 244 + 0 = 244\text{ k}$	$P_r = P_{nt} + B_2 P_{lt} = 170 + 0 = 170\text{ k}$
$\dfrac{P_r}{P_c} = \dfrac{244}{1370} = 0.178 < 0.2$	$\dfrac{P_r}{P_c} = \dfrac{170}{912} = 0.186 < 0.2$
$\therefore$ Must use AISC Equation H1-1b	$\therefore$ Must use AISC Equation H1-1b
$C_{mx} = 0.6 - 0.4\left(-\dfrac{200}{200}\right) = 1.0$	$C_{mx} = 0.6 - 0.4\left(-\dfrac{140}{140}\right) = 1.0$
Determine τ_b:	Determine τ_b:
$\dfrac{\alpha P_r}{P_y} = \dfrac{1.0(244\text{ k})}{(35.3\text{ in}^2)(50\text{ ksi})} = 0.138 < 0.5$	$\dfrac{\alpha P_r}{P_y} = \dfrac{1.0(170\text{ k})}{(35.3\text{ in}^2)(50\text{ ksi})} = 0.154 > 0.5$
$\therefore\ \tau_b = 1.0$	$\therefore\ \tau_b = 1.0$
$P_{e1x} = \dfrac{\pi^2 0.8\tau_b EI^*}{(K_1L_x)^2} = \dfrac{\pi^2 0.8(1.0)(29{,}000)(1380)}{(1.0 \times 12 \times 14)^2}$ $= 11{,}196\text{ k}$	$P_{e1x} = \dfrac{\pi^2 0.8\tau_b EI^*}{(K_1L_x)^2} = \dfrac{\pi^2 0.8(1.0)(29{,}000)(1380)}{(1.0 \times 12 \times 14)^2} = 11{,}196\text{ k}$
$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{1.0(244)}{11{,}196}} = 1.022$	$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{1.0}{1 - \dfrac{1.6(170)}{11{,}196}} = 1.025$
$M_{rx} = B_{1x}M_{ntx} = 1.022(200) = 204.5\text{ ft-k}$	$M_{rx} = B_{1x}M_{ntx} = 1.025(140) = 143.5\text{ ft-k}$
$C_{my} = 0.6 - 0.4\left(-\dfrac{144}{144}\right) = 1.0$	$C_{my} = 0.6 - 0.4\left(-\dfrac{100}{100}\right) = 1.0$
$P_{e1y} = \dfrac{\pi^2 0.8\tau_b EI^*}{(K_1L_y)^2} = \dfrac{\pi^2 0.8(1.0)(29{,}000)(495)}{(1.0 \times 12 \times 14)^2}$ $= 4016\text{ k}$	$P_{e1y} = \dfrac{\pi^2 0.8\tau_b EI^*}{(K_1L_y)^2} = \dfrac{\pi^2 0.8(1.0)(29{,}000)(495)}{(1.0 \times 12 \times 14)^2} = 4016\text{ k}$
$B_{1y} = \dfrac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \dfrac{1.0}{1 - \dfrac{1.0(244)}{4016}} = 1.065$	$B_{1y} = \dfrac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \dfrac{1.0}{1 - \dfrac{1.6(170)}{4016}} = 1.073$
$M_{ry} = B_{1y}M_{nty} = 1.065(144) = 153.3\text{ ft-k}$	$M_{ry} = B_{1y}M_{nty} = 1.073(100) = 107.3\text{ ft-k}$

(Continued)

LRFD	ASD
From AISC Table 6-1, for $KL = 14$ ft and $L_b = 14$ ft	From AISC Table 6-1, for $KL = 14$ ft and $L_b = 14$ ft
$p = 0.730 \times 10^{-3}, b_x = 1.13 \times 10^{-3}, b_y = 2.32 \times 10^{-3}$	$p = 1.10 \times 10^{-3}, b_x = 1.69 \times 10^{-3}, b_y = 3.49 \times 10^{-3}$
$\frac{1}{2}pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) \le 1.0$	$\frac{1}{2}pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) \le 1.0$
$\frac{1}{2}(0.730 \times 10^{-3})(244) + \frac{9}{8}(1.13 \times 10^{-3})(204.5)$	$\frac{1}{2}(1.10 \times 10^{-3})(170) + \frac{9}{8}(1.69 \times 10^{-3})(143.5)$
$+\frac{9}{8}(2.32 \times 10^{-3})(153.3) = 0.749 \le 1.0$ OK	$+\frac{9}{8}(3.49 \times 10^{-3})(107.3) = 0.788 \le 1.0$ OK
Section is satisfactory but perhaps overdesigned.	**Section is satisfactory but perhaps overdesigned.**

Example 11-8

For the truss shown in Fig. 11.7(a), a W8 × 35 is used as a continuous top chord member from joint L_0 to joint U_3. If the member consists of 50 ksi steel, does it have sufficient strength to resist the loads shown in parts (b) and (c) of the figure? The factored or LRFD loads are shown in part (b), while the service or ASD loads are shown in part (c). The 17.6 k and 12 k loads represent the reaction from a purlin. The compression flange of the W8 is braced only at the ends about the *x-x* axis, $L_x = 13$ ft, and at the ends and the concentrated load about the *y-y* axis, $L_y = 6.5$ ft and $L_b = 6.5$ ft.

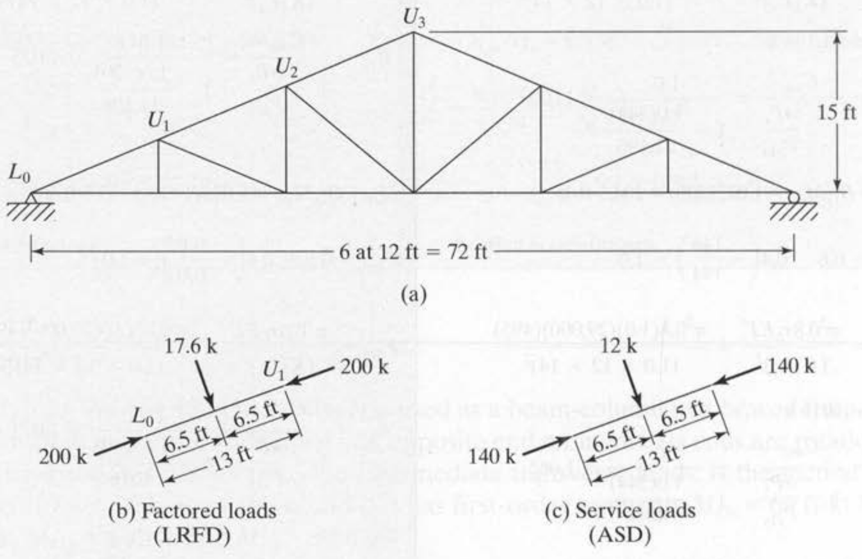
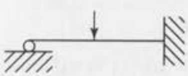


FIGURE 11.7
A truss whose top chord is subject to intermediate loads.

Solution. The Effective Length Method is used in this problem.

Using a W8 × 35 ($A = 10.3\text{ in}^2$, $I_x = 127\text{ in}^4$, $r_x = 3.51$ in, $r_y = 2.03$ in, $L_P = 7.17$ ft, $\phi_b M_{Px} = 130$ ft-k, $\dfrac{M_{Px}}{\Omega_b} = 86.6$ ft-k, $r_x/r_y = 1.73$).

LRFD	ASD
$P_{nt} = P_u$ from figure $= 200\text{ k} = P_r$	$P_{nt} = P_a$ from figure $= 140\text{ k} = P_r$
Conservatively assume $K_x = K_y = 1.0$. In truth, the K-factor is somewhere between $K = 1.0$ (pinned–pinned end condition) and $K = 0.8$ (pinned–fixed end condition) for segment L_oU_i	Conservatively assume $K_x = K_y = 1.0$. In truth, the K-factor is somewhere between $K = 1.0$ (pinned–pinned end condition) and $K = 0.8$ (pinned–fixed end condition) for segment L_oU_i
$\left(\dfrac{KL}{r}\right)_x = \dfrac{(1.0)(12 \times 13)}{3.51} = 44.44 \leftarrow$	$\left(\dfrac{KL}{r}\right)_x = \dfrac{(1.0)(12 \times 13)}{3.51} = 44.44 \leftarrow$
$\left(\dfrac{KL}{r}\right)_y = \dfrac{(1.0)(12 \times 6.5)}{2.03} = 38.42$	$\left(\dfrac{KL}{r}\right)_y = \dfrac{(1.0)(12 \times 6.5)}{2.03} = 38.42$
From AISC Table 4-22, $F_y = 50$ ksi	From AISC Table 4-22, $F_y = 50$ ksi
$\phi_c F_{cr} = 38.97$ ksi	$\dfrac{F_{cr}}{\Omega_c} = 25.91$ ksi
$\phi_c P_n = (38.97)(10.3) = 401.4\text{ k} = P_c$	$\dfrac{P_n}{\Omega_c} = (25.91)(10.3) = 266.9\text{ k} = P_c$
$\dfrac{P_r}{P_c} = \dfrac{200}{401.4} = 0.498 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{140}{266.9} = 0.525 > 0.2$
∴ Must use AISC Eq. H1-1a	∴ Must use AISC Eq. H1-1a
Computing P_{e1x} and C_{mx}	Computing P_{e1x} and C_{mx}
$P_{e1x} = \dfrac{(\pi^2)(29{,}000)(127)}{(1.0 \times 12 \times 13)^2} = 1494\text{ k}$	$P_{e1x} = \dfrac{(\pi^2)(29{,}000)(127)}{(1.0 \times 12 \times 13)^2} = 1494\text{ k}$
	Computing C_m as in LRFD
From Table 11.1 For 	From Table 11.1 For 
$C_{mx} = 1 - 0.2\left(\dfrac{1.0\,(200)}{1494}\right) = 0.973$	$C_{mx} = 1 - 0.2\left(\dfrac{1.6\,(140)}{1494}\right) = 0.970$
For 	For 

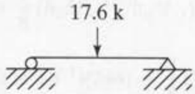
(*Continued*)

$$C_{mx} = 1 - 0.3\left(\frac{1.0\,(200)}{1494}\right) = 0.960$$

Avg $C_{mx} = 0.967$

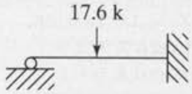
Computing M_{ux}

For



$$M_{ux} = \frac{PL}{4} = \frac{(17.6)(13)}{4} = 57.2 \text{ ft-k}$$

For



$$M_{ux} = \frac{3\,PL}{16} = \frac{(3)(17.6)(13)}{16} = 42.9 \text{ ft-k}$$

Avg $M_{ux} = 50.05 \text{ ft-k} = M_{rx}$

$$B_{1x} = \frac{0.967}{1 - \dfrac{(1)(200)}{1494}} = 1.116$$

$M_r = (1.116)(50.05) = 55.86$ ft-k

Since $L_b = 6.5 \text{ ft} < L_p = 7.17 \text{ ft}$

$\therefore$ Zone ①

$\phi_b M_{nx} = 130 \text{ ft-k} = M_{cx}$

Using Equation H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{200}{401.4} + \frac{8}{9}\left(\frac{55.86}{130} + 0\right) \le 1.0$$

$0.880 \le 1.0$ Section OK

From AISC Table 6-1

$(KL)_y = 6.5$ ft

$$(KL)_{yEQUIV} = \frac{(KL)_x}{r_x/r_y} = \frac{13}{1.73} = 7.51 \text{ ft} \leftarrow$$

$P = 2.50 \times 10^{-3}$, for $KL = 7.51$ ft

$b_x = 6.83 \times 10^{-3}$, for $L_b = 6.5$ ft

$p\,P_r + b_x\,M_{rx} + b_y\,M_{ry} \le 1.0$

$= (2.50 \times 10^{-3})\,(200) + (6.83 \times 10^{-3})\,(55.86) + 0$

$= 0.882 \le 1.0$ Section OK

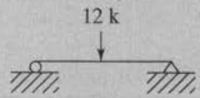
Section is Satisfactory.

$$C_{mx} = 1 - 0.3\left(\frac{1.6\,(140)}{1494}\right) = 0.955$$

Avg $C_{mx} = 0.963$

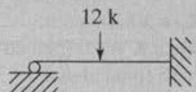
Computing M_{ax}

For



$$M_{ax} = \frac{(12)(13)}{4} = 39 \text{ ft-k}$$

For



$$M_{ax} = \frac{(3)(12)(13)}{16} = 29.25 \text{ ft-k}$$

Avg $M_{ax} = 34.13 \text{ ft-k} = M_{rx}$

$$B_{1x} = \frac{0.967}{1 - \dfrac{(1.6)(140)}{1494}} = 1.138$$

$M_r = (1.138)(34.13) = 38.84$ ft-k

Since $L_b = 6.5 \text{ ft} < L_P = 7.17 \text{ ft}$

$\therefore$ Zone ①

$$\frac{M_{nx}}{\Omega_b} = 86.6 \text{ ft-k} = M_{cx}$$

Using Equation H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{140}{266.9} + \frac{8}{9}\left(\frac{38.84}{88.6} + 0\right) \le 1.0$$

$0.914 \le 1.0$ Section OK

From AISC Table 6-1

$(KL)_y = 6.5$ ft

$$(KL)_{yEQUIV} = \frac{(KL)_x}{r_x/r_y} = \frac{13}{1.73} = 7.51 \text{ ft} \leftarrow$$

$P = 3.75 \times 10^{-3}$, for $KL = 7.51$ ft

$b_x = 10.3 \times 10^{-3}$, for $L_b = 6.5$ ft

$p\,P_r + b_x\,M_{rx} + b_y\,M_{ry} \le 1.0$

$= (3.75 \times 10^{-3})\,(140) + (10.3 \times 10^{-3})\,(38.84) + 0$

$= 0.925 \le 1.0$ Section OK

Section is Satisfactory.

11.8 BEAM-COLUMNS IN UNBRACED FRAMES

The maximum primary moments in unbraced frames almost always occur at the column ends. As you can see in Fig. 11.3, the maximum sidesway moments always occur at the member ends and the total moment for a particular column is determined by adding its primary end moment, M_{lt}, to its sidesway moment, $P_{nt}\,\Delta$. As described in Section 11.3, B_2 is the multiplier used in the Approximate Second-Order Analysis to account for P-Δ effect.

In Examples 11-9 and 11-10, beam-column sections will be analyzed using both the Direct Analysis Method and the Effective Length Method. In both examples, the approximate analysis method is used to account for the second-order effects. The magnification factors, B_1 and B_2, are determined for each beam-column in each direction of lateral translation. These second-order loads and moments are substituted into the appropriate interaction equation to determine if the section is satisfactory.

Example 11-9

Part (a) – Direct Analysis Method

A W10 × 39 of $F_y = 50$ ksi steel, is used for a 14 ft long beam-column in an unbraced frame about the x-x axis but is braced about the y-y axis. Based on a first-order analysis using the requirements of the Direct Analysis Method, the member supports the following factored loads: $P_{nt} = 130$ k, $P_{lt} = 25$ ft-k, $M_{ntx} = 45$ ft-k, and $M_{ltx} = 15$ ft-k. C_{mx} was determined to be 0.85. P_{story} is 1604 k and the ratio of P_{mf}/P_{story} is 0.333. H, the story shear, is equal to 33.4 k and the drift index (Δ_H/L) is 0.0025. Using the LRFD procedure, is the member satisfactory?

Solution

W10 × 39 ($A = 11.5$ in^2, $I_x = 209$ in^4)

$C_{mx} = 0.85$ (given) $\quad \alpha = 1.0$ (LRFD)

$P_r = P_{nt} + P_{lt} = 130\text{ k} + 25\text{ k} = 155\text{ k}$

Determine τ_b:

$$\frac{\alpha P_r}{P_y} = \frac{1.0(155\text{ k})}{(11.5\text{ in}^2)(50\text{ ksi})} = 0.27 < 0.5$$

$\therefore\ \tau_b = 1.0$

$$P_{e1x} = \frac{\pi^2 0.8\tau_b EI^*}{(K_1L_x)^2} = \frac{\pi^2 0.8(1.0)(29{,}000)(209)}{(1.0 \times 12 \times 14)^2} = 1{,}696\text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(155)}{1{,}696}} = 0.94 < 1.0 \qquad \therefore\ B_{1x} = 1.0$$

$P_{story} = 1604$ k (given) $\quad \Delta_H/L = 0.0025$ (given)

$H = 33.4$ k (given) $\quad \alpha = 1.0$ (LRFD)

$$R_m = 1 - 0.15(P_{mf}/P_{story}) = 1 - 0.15(0.333) = 0.95$$

$$P_{e\ story\ x} = R_m \frac{(H)}{\frac{\Delta_H}{L}} = 0.95\left(\frac{33.4}{0.0025}\right) = 12{,}692\text{ k}$$

$$B_{2x} = \frac{1}{1 - \frac{\alpha(P_{story})}{P_{e\ story\ x}}} = \frac{1}{1 - \frac{1.0(1604)}{12{,}692}} = 1.15$$

$\therefore\ P_r = P_{nt} + B_2\,P_{lt} = 130\text{ k} + 1.15(25\text{ k}) = 158.8\text{ k}$

$M_{rx} = B_{1x}\,M_{ntx} + B_{2x}\,M_{ltx} = 1.0(45\text{ ft-k}) + 1.15(15\text{ ft-k}) = 62.3\text{ ft-k}$

For Direct Analysis Method, $K = 1.0$

$\therefore\ (KL)_x = (KL)_y = 1.0(14) = 14\text{ ft}$

$P_c = \Phi_c P_n = 306\text{ k}$ (AISC Table 4-1)

$$\frac{P_r}{P_c} = \frac{158.8}{306} = 0.52 > 0.2$$

$\therefore$ Must use AISC Equation H1-1a.

For W10 × 39, $\Phi M_{px} = 176$ ft-k, $L_p = 6.99$ ft, $L_r = 24.2$ ft

$BF = 3.78$ k, $L_b = 14$ ft, Zone 2, $C_b = 1.0$

$\Phi M_{nx} = C_b\,[\Phi M_{px} - BF(L_b - L_p)]\ \Phi M_{px}$

$\Phi M_{nx} = 1.0[176 - 3.78(14 - 6.99)] = 149.5\text{ ft-k}$

Equation H1-1a:

$$\frac{158.8}{306} + \frac{8}{9}\left(\frac{62.3}{149.5} + 0\right) = 0.889 < 1.0 \quad \text{OK}$$

Additional check:

From Table 6-1 for $KL = 14$ ft and $L_b = 14$ ft

$p = 3.27 \times 10^{-3}, b_x = 5.96 \times 10^{-3}$

$3.27 \times 10^{-3}\,(158.8) + 5.96 \times 10^{-3}\,(62.3) = 0.891 < 1.0$ OK

$\therefore$ **Section is satisfactory.**

Part (b) – Effective Length Method

Repeat using the same W10 × 39, 14 ft long section. Based on a first-order analysis using the requirement of the Effective Length Method, the member essentially has the same loads and moments. C_{mx} is still 0.85. P_{story} is 1604 k, the ratio of P_{mf}/P_{story} is 0.333 and H, the story shear, is equal to 33.4 k. The drift index (Δ_H/L) is reduced to 0.0020 due to the increased member stiffness in the analysis when compared the direct analysis method. K_x was determined to be 1.2 and the K_y is equal 1.0. Using the LRFD procedure, is the member satisfactory?

Solution

W10 × 39 ($A = 11.5\text{ in}^2$, $I_x = 209\text{ in}^4$)

$C_{mx} = 0.85$ (given) $\qquad \alpha = 1.0$ (LRFD)

$P_r = P_{nt} + P_{lt} = 130\text{ k} + 25\text{ k} = 155\text{ k}$

$$P_{e1x} = \frac{\pi^2 EI^*}{(K_1 L_x)^2} = \frac{\pi^2(29{,}000)(209)}{(1.0 \times 12 \times 14)^2} = 2120\text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(155)}{2120}} = 0.92 < 1.0$$

$\therefore \quad B_{1x} = 1.0$

$P_{story} = 1604\text{ k}$ (given) $\qquad \Delta_H/L = 0.0020$ (given)

$H = 33.4\text{ k}$ (given) $\qquad \alpha = 1.0$ (LRFD)

$R_m = 1 - 0.15(P_{mf}/P_{story}) = 1 - 0.15(0.333) = 0.95$

$$P_{e\,story\,x} = R_m\left(\frac{H}{\dfrac{\Delta_H}{L}}\right) = 0.95\left(\frac{33.4}{0.0020}\right) = 15{,}865\text{ k}$$

$$B_{2x} = \frac{1}{1 - \dfrac{\alpha(P_{story})}{P_{e\,story\,x}}} = \frac{1}{1 - \dfrac{1.0(1604)}{15{,}865}} = 1.11$$

$\therefore \quad P_r = P_{nt} + B_2 P_{lt} = 130\text{ k} + 1.11(25\text{ k}) = 157.8\text{ k}$

$M_{rx} = B_{1x} M_{ntx} + B_{2x} M_{ltx} = 1.0(45\text{ ft-k}) + 1.11(15\text{ ft-k}) = 61.7\text{ ft-k}$

Effective Length Method: $K_y = 1.0$ and $K_x = 1.2$

$\therefore \quad (KL)_y = 1.0(14) = 14\text{ ft} \leftarrow$ controls

$$\text{Equivalent } (KL)_y = \frac{(KL)_x}{\dfrac{r_x}{r_y}} = \frac{(1.2)(14)}{2.16} = 7.78\text{ ft}$$

$P_c = \Phi_c P_n = 306\text{ k}$ (AISC Table 4-1)

$$\frac{P_r}{P_c} = \frac{157.8}{306} = 0.52 > 0.2$$

$\therefore$ Must use AISC Equation H1-1a.

For W10 × 39, $\Phi M_{px} = 176$ ft-k, $L_p = 6.99$ ft, $L_r = 24.2$ ft

$BF = 3.78$ k, $L_b = 14$ ft, Zone 2, $C_b = 1.0$

$\Phi M_{nx} = C_b\,[\Phi M_{px} - BF(L_b - L_p)] \leq \Phi M_{px}$

$\Phi M_{nx} = 1.0[176 - 3.78(14 - 6.99)] = 149.5$ ft-k

Equation H1-1a:

$$\frac{157.8}{306} + \frac{8}{9}\left(\frac{61.7}{149.5} + 0\right) = 0.883 < 1.0 \quad \text{OK}$$

Additional check:

From Table 6-1 for $KL = 14$ ft and $L_b = 14$ ft

$p = 3.27 \times 10^{-3}$, $b_x = 5.96 \times 10^{-3}$

$3.27 \times 10^{-3}\,(157.8) + 5.96 \times 10^{-3}\,(61.7) = 0.884 < 1.0$ OK

$\therefore$ **Section is satisfactory.**

Example 11-10

Part (a) – Direct Analysis Method

A 14 ft W10 × 45 of $F_y = 50$ ksi steel, is used in the same unbraced frame building as given in Example 11-9. The major difference is that this beam-column is bent about both the x-x axis and y-y axis. Based on a first-order analysis using the requirements of the Direct Analysis Method, the member supports the following factored loads: $P_{nt} = 65$ k, $P_{lt} = 30$ ft-k, $M_{ntx} = 50$ ft-k, $M_{ltx} = 20$ ft-k, $M_{nty} = 16$ ft-k, and $M_{lty} = 8$ ft-k. C_{mx} and C_{my} were determined to be 0.85. P_{story} is 1604 k and the ratio of P_{mf}/P_{story} is 0.333. H, the story shear, is equal to 33.4 k and the drift indices are $(\Delta_H/L)_x = 0.0025$ and $(\Delta_H/L)_y = 0.0043$. Using the LRFD procedure, is the member satisfactory?

Solution

W10 × 45 ($A = 13.3$ in^2, $I_x = 248$ in^4, $I_y = 53.4$ in^4)

$C_{mx} = C_{my} = 0.85$ (given) $\qquad \alpha = 1.0$ (LRFD)

$P_r = P_{nt} + P_{lt} = 65\text{ k} + 30\text{ k} = 95\text{ k}$

Determine τ_b:

$$\frac{\alpha P_r}{P_y} = \frac{1.0(95\text{ k})}{(13.3\text{ in}^2)(50\text{ ksi})} = 0.14 < 0.5$$

$\therefore\ \tau_b = 1.0$

$$P_{e1x} = \frac{\pi^2 0.8\tau_b EI^*}{(K_1L_x)^2} = \frac{\pi^2 0.8(1.0)(29{,}000)(248)}{(1.0 \times 12 \times 14)^2} = 2012 \text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(95)}{2012}} = 0.89 < 1.0 \qquad \therefore\ B_{1x} = 1.0$$

$$P_{e1y} = \frac{\pi^2 0.8\tau_b EI^*}{(K_1L_y)^2} = \frac{\pi^2 0.8(1.0)(29{,}000)(53.4)}{(1.0 \times 12 \times 14)^2} = 433 \text{ k}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \frac{0.85}{1 - \dfrac{1.0(95)}{433}} = 1.09 \qquad \therefore\ B_{1y} = 1.09$$

$P_{story} = 1604$ k (given) $\quad (\Delta_H/L)_x = 0.0025$ and $(\Delta_H/L)_y = 0.0043$ (given)

$H = 33.4$ k (given) $\quad \alpha = 1.0$ (LRFD)

$R_m = 1 - 0.15(P_{mf}/P_{story}) = 1 - 0.15(0.333) = 0.95$

$$P_{e\,story\,x} = R_m\left(\frac{H}{\dfrac{\Delta_H}{L}}\right) = 0.95\left(\frac{33.4}{0.0025}\right) = 12{,}692 \text{ k}$$

$$B_{2x} = \frac{1}{1 - \dfrac{\alpha(P_{story})}{P_{e\,story\,x}}} = \frac{1}{1 - \dfrac{1.0(1604)}{12{,}692}} = 1.15$$

$$P_{e\,story\,y} = R_m\left(\frac{H}{\dfrac{\Delta_H}{L}}\right) = 0.95\left(\frac{33.4}{0.0043}\right) = 7379 \text{ k}$$

$$B_{2y} = \frac{1}{1 - \dfrac{\alpha(P_{story})}{P_{e\,story\,y}}} = \frac{1}{1 - \dfrac{1.0(1604)}{7379}} = 1.28$$

$\therefore\ P_r = P_{nt} + B_2P_{lt} = 65 \text{ k} + 1.28(30 \text{ k}) = 103.4 \text{ k}$

$M_{rx} = B_{1x}M_{ntx} + B_{2x}M_{ltx} = 1.0(50 \text{ ft-k}) + 1.15(20 \text{ ft-k}) = 73.0 \text{ ft-k}$

$M_{ry} = B_{1y}M_{nty} + B_{2y}M_{lty} = 1.09(16 \text{ ft-k}) + 1.28(8 \text{ ft-k}) = 27.7 \text{ ft-k}$

For Direct Analysis Method, $K = 1.0$

$\therefore\ (KL)_x = (KL)_y = 1.0(14) = 14$ ft

$P_c = \Phi_c P_n = 359$ k (AISC Table 4-1)

$$\frac{P_r}{P_c} = \frac{103.4}{359} = 0.29 > 0.2$$

$\therefore$ Must use AISC Equation H1-1a

For W10 $\times$ 45, $\Phi M_{px} = 206$ ft-k, $L_p = 7.10$ ft, $L_r = 26.9$ ft

$BF = 3.89$ k, $L_b = 14$ ft, Zone 2, $C_b = 1.0$

$\Phi M_{nx} = C_b\,[\Phi M_{px} - BF(L_b - L_p)] \leq \Phi M_{px}$

$\Phi M_{nx} = 1.0[206 - 3.89(14 - 7.10)] = 179.2$ ft-k

$\Phi M_{ny} = \Phi M_{py} = 76.1$ ft-k (AISC Table 3-4)

Equation H1-1a:

$$\frac{103.4}{359} + \frac{8}{9}\left(\frac{73.0}{179.2} + \frac{27.7}{76.1}\right) = 0.974 < 1.0 \quad \text{OK}$$

Additional check:

From Table 6-1 for $KL = 14$ ft and $L_b = 14$ ft

$p = 2.78 \times 10^{-3}, b_x = 4.96 \times 10^{-3}, b_y = 11.7 \times 10^{-3}$

$2.78 \times 10^{-3}\,(103.4) + 4.96 \times 10^{-3}\,(73.0) + 11.7 \times 10^{-3}(27.7) = 0.974 < 1.0$ OK

$\therefore$ **Section is satisfactory.**

Part (b) – Effective Length Method
Repeat using the same W10 $\times$ 45, 14 ft long section. Based on a first-order analysis using the requirement of the Effective Length Method, the member essentially has the same loads and moments. C_{mx} is still 0.85. P_{story} is 1604 k, the ratio of P_{mf}/P_{story} is 0.333 and *H*, the story shear, is equal to 33.4 k. The drift indices are reduced to $(\Delta_H/L)_x = 0.0020$ and $(\Delta_H/L)_y = 0.0034$ due to the increased member stiffness in the analysis when compared with the direct analysis method. K_x was determined to be 1.31 and the K_y equal to 1.25. Using the LRFD procedure, is the member satisfactory?

Solution

W10 $\times$ 45 ($A = 13.3$ in^2, $I_x = 248$ in^4, $I_y = 53.4$ in^4)

$C_{mx} = C_{my} = 0.85$ (given) $\qquad \alpha = 1.0$ (LRFD)

$P_r = P_{nt} + P_{lt} = 65\text{ k} + 30\text{ k} = 95\text{ k}$

$$P_{e1x} = \frac{\pi^2 EI^*}{(K_1 L_x)^2} = \frac{\pi^2(29{,}000)(248)}{(1.0 \times 12 \times 14)^2} = 2515\text{ k}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(95)}{2515}} = 0.88 < 1.0 \qquad \therefore B_{1x} = 1.0$$

$$P_{e1y} = \frac{\pi^2 EI^*}{(K_1 L_x)^2} = \frac{\pi^2(29{,}000)(53.4)}{(1.0 \times 12 \times 14)^2} = 542 \text{ k}$$

$$B_{1y} = \frac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \frac{0.85}{1 - \dfrac{1.0(95)}{542}} = 1.03 \quad \therefore \; B_{1y} = 1.03$$

$P_{story} = 1604$ k (given) $\quad (\Delta_H/L)_x = 0.0020$ and $(\Delta_H/L)_y = 0.0034$ (given)

$H = 33.4$ k (given) $\quad \alpha = 1.0$ (LRFD)

$R_m = 1 - 0.15(P_{mf}/P_{story}) = 1 - 0.15(0.333) = 0.95$

$$P_{e\ story\ x} = R_m\left(\frac{H}{\dfrac{\Delta_H}{L}}\right) = 0.95\left(\frac{33.4}{0.0020}\right) = 15{,}865 \text{ k}$$

$$B_{2x} = \frac{1}{1 - \dfrac{\alpha(P_{story})}{P_{e\ story\ x}}} = \frac{1}{1 - \dfrac{1.0(1604)}{15{,}865}} = 1.11$$

$$P_{e\ story\ y} = R_m\left(\frac{H}{\dfrac{\Delta_H}{L}}\right) = 0.95\left(\frac{33.4}{0.0034}\right) = 9332 \text{ k}$$

$$B_{2y} = \frac{1}{1 - \dfrac{\alpha(P_{story})}{P_{e\ story\ y}}} = \frac{1}{1 - \dfrac{1.0(1604)}{9332}} = 1.21$$

$\therefore \; P_r = P_{nt} + B_2 P_{lt} = 65 \text{ k} + 1.21(30 \text{ k}) = 101.3 \text{ k}$

$M_{rx} = B_{1x} M_{ntx} + B_{2x} M_{ltx} = 1.0(50 \text{ ft-k}) + 1.11\ (20 \text{ ft-k}) = 72.2 \text{ ft-k}$

$M_{ry} = B_{1y} M_{nty} + B_{2y} M_{lty} = 1.03(16 \text{ ft-k}) + 1.21(8 \text{ ft-k}) = 26.2 \text{ ft-k}$

Effective Length Method: $K_y = 1.25$ and $K_x = 1.31$

$\therefore \; (KL)_y = 1.25(14) = 17.5 \text{ ft} \leftarrow$ controls

$$\text{Equivalent } (KL)_y = \frac{(KL)_x}{r_x/r_y} = \frac{(1.31)(14)}{2.15} = 8.53 \text{ ft}$$

$P_c = \Phi_c P_n = 269.5$ k (AISC Table 4-1)

$$\frac{P_r}{P_c} = \frac{101.3}{269.5} = 0.38 > 0.2$$

$\therefore$ Must use AISC Equation H1-1a

For W10 $\times$ 45, $\Phi M_{px} = 206$ ft-k, $L_p = 7.10$ ft, $L_r = 26.9$ ft

$BF = 3.89$ k, $L_b = 14$ ft, Zone 2, $C_b = 1.0$

$\Phi M_{nx} = C_b\,[\Phi M_{px} - BF(L_b - L_p)] \le \Phi M_{px}$

$\Phi M_{nx} = 1.0[206 - 3.89(14 - 7.10)] = 179.2$ ft-k

$\Phi M_{ny} = \Phi M_{py} = 76.1$ ft-k (AISC Table 3-4)

Equation H1-1a:

$$\frac{101.3}{268.5} + \frac{8}{9}\left(\frac{72.2}{179.2} + \frac{26.2}{76.1}\right) = 1.041 > 1.0 \quad \text{N.G.}$$

Additional check:

From Table 6-1 for $KL = 17.5$ ft and $L_b = 14$ ft

$p = 3.72 \times 10^{-3}$, $b_x = 4.96 \times 10^{-3}$, $b_y = 11.7 \times 10^{-3}$

$3.72 \times 10^{-3}\,(101.3) + 4.96 \times 10^{-3}\,(72.2) + 11.7 \times 10^{-3}(26.2) = 1.041 > 1.0$ N.G.

$\therefore$ **Section is NOT satisfactory.**

11.9 DESIGN OF BEAM–COLUMNS—BRACED OR UNBRACED

The design of beam–columns involves a trial-and-error procedure. A trial section is selected by some process and is then checked with the appropriate interaction equation. If the section does not satisfy the equation, or if it's too much on the safe side (that is, if it's overdesigned), another section is selected, and the interaction equation is applied again. Probably, the first thought of the reader is, "I sure hope that we can select a good section the first time and not have to go through all that rigamarole more than once or twice." We certainly can make a good estimate, and that's the topic of the remainder of this section.

A common method used for selecting sections to resist both moments and axial loads is the **equivalent axial load**, or **effective axial load, method.** With this method, the axial load (P_u or P_a) and the bending moment or bending moments M_{ux}, M_{uy}, or M_{ax} and M_{ay}) are replaced with a fictitious concentric load P_{ueq} or P_{aeq}, equivalent approximately to the actual axial load plus the moment effect.

It is assumed for this discussion that it is desired to select the most economical section to resist both a moment and an axial load. By a trial-and-error procedure, it is possible eventually to select the lightest section. Somewhere, however, there is a fictitious axial load that will require the same section as the one required for the actual moment and the actual axial load. This fictitious load is called the equivalent axial load, or the effective axial load P_{ueq} or P_{aeq}.

Equations are used to convert the bending moment into an estimated equivalent axial load P'_u or P'_a, which is added to the design axial load P_u, or to P_a for ASD. The total of $P_u + P'_u$ or $P_a + P'_a$ is the equivalent or effective axial load P_{ueq} or P_{aeq}, and it is used to enter the concentric column tables of Part 4 of the AISC Manual for the choice of a trial section. In the approximate formula for P_{ueq} (or P_{aeq}) that follows, m is a factor given in Table 11.3 of this chapter. This table is taken from the second edition of the *Manual of Steel Construction Load and Resistance Factor Design* published in 1994 (where it was Table 3-2). The equivalent loads are estimated with the following expressions:

$$P_{ueq} = P_u + M_{ux}m + M_{uy}mu \qquad \text{(LRFD)}$$

or

$$P_{aeq} = P_a + M_{ax}m + M_{ay}mu \qquad \text{(ASD)}$$

To apply these expressions, a value of m is taken from the first approximation section of Table 11.3, and u is assumed equal to 2. *In applying the equation, the* moments M_{ux} and M_{uy} (or M_{ax} and M_{ay}) must be used in ft-k. The equations are solved for P_{ueq} or P_{aeq}. A column is selected from the concentrically loaded column tables for each load. Then the equation for P_{ueq} (or P_{aeq}) is solved again with a revised value of m from the subsequent approximations part of Table 11.3, and the value of u is kept equal to 2.0. (Actually, a more precise value of u for each column section was provided in the 1994 Manual.)

Limitations of P_{ueq} Formula

The application of the equivalent axial load formula and Table 11.3 results in economical beam–column designs, unless the moment becomes quite large in comparison with the axial load. For such cases, the members selected will be capable of supporting the loads and moments, but may very well be rather uneconomical. The tables for

TABLE 11.3 Preliminary Beam–Column Design F_y = 36 ksi, F_y = 50 ksi

Values of m														
F_y	36 ksi							50 ksi						
KL(ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
1st Approximation														
All Shapes	2.0	1.9	1.8	1.7	1.6	1.5	1.3	1.9	1.8	1.7	1.6	1.4	1.3	1.2
Subsequent Approximation														
W4	3.1	2.3	1.7	1.4	1.1	1.0	0.8	2.4	1.8	1.4	1.1	1.0	0.9	0.8
W5	3.2	2.7	2.1	1.7	1.4	1.2	1.0	2.8	2.2	1.7	1.4	1.1	1.0	0.9
W6	2.8	2.5	2.1	1.8	1.5	1.3	1.1	2.5	2.2	1.8	1.5	1.3	1.2	1.1
W8	2.5	2.3	2.2	2.0	1.8	1.6	1.4	2.4	2.2	2.0	1.7	1.5	1.3	1.2
W10	2.1	2.0	1.9	1.8	1.7	1.6	1.4	2.0	1.9	1.8	1.7	1.5	1.4	1.3
W12	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.7	1.6	1.5	1.5	1.4	1.3	1.2
W14	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.5	1.4	1.4	1.3	1.3	1.2	1.2

Source: This table is from a paper in AISC *Engineering Journal* by Uang, Wattar, and Leet (1990).

concentrically loaded columns of Part 4 of the Manual are limited to the W14s, W12s, and shallower sections, but when the moment is large in proportion to the axial load, there will often be a much deeper and appreciably lighter section, such as a W27 or W30, that will satisfy the appropriate interaction equation.

Examples of the equivalent axial load method applied with the values given in Table 11.3 are presented in Examples 11-11 and 11-12. After a section is selected by the approximate P_{ueq} or P_{aeq} formula, it is necessary to check it with the appropriate interaction equation.

Example 11-11

Select a trial W section for both LRFD and ASD for the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 12$ ft, $P_{nt} = 690$ k and $M_{ntx} = 168$ ft-k for LRFD, and $P_{nt} = 475$ k and $M_{ntx} = 120$ ft-k for ASD. These were the values used in Example 11-3.

Solution

LRFD	ASD
Assume B_1 and $B_2 = 1.0$	Assume B_1 and $B_2 = 1.0$
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 690 + 0 = 690$ k	$P_a = 475 + 0 = 475$ k
and, $M_{rx} = M_{ux} = B_1(M_{ntx}) + B_2(M_{ltx})$	and, $M_{rx} = M_{ax} = B_1(M_{ntx}) + B_2(M_{ltx})$
$M_{ux} = 1.0(168) + 0 = 168$ ft-k	$M_{ax} = 1.0(120) + 0 = 120$ ft-k
$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$	$P_{aeq} = P_a + M_{ax}m + M_{ay}mu$
From "1st Approximation" part of Table 11.3	From "1st Approximation" part of Table 11.3
$m = 1.8$ for $KL = 12$ ft, $F_y = 50$ ksi	$m = 1.8$ for $KL = 12$ ft, $F_y = 50$ ksi
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 690 + 168(1.8) + 0 = 992.4$ k	$P_{aeq} = 475 + 120(1.8) + 0 = 691.0$ k
1st trial section: W12 × 96 ($\Phi_c P_n = 1080$ k) from AISC Table 4-1	1st trial section: W12 × 96 ($P_n/\Omega_c = 720$ k) from AISC Table 4-1
From "Subsequent Approximation" part of Table 11.3, W12's	From "Subsequent Approximation" part of Table 11.3, W12's
$m = 1.6$	$m = 1.6$
$P_{ueq} = 690 + 168(1.6) + 0 = 958.8$ k	$P_{aeq} = 475 + 120(1.6) + 0 = 667.0$ k
Try W12 × 87, ($\Phi_c P_n = 981$ k > 958.8 k)	**Try W12 × 96,** ($P_n/\Omega_c = 720$ k > 667.0 k)

Note: These are trial sizes. B_1 and B_2, which were assumed, must be calculated and these W12 sections checked with the appropriate interaction equations.

Example 11-12

Select a trial W section for both LRFD and ASD for an unbraced frame and the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 10$ ft.

For LRFD: $P_{nt} = 175$ k and $P_{lt} = 115$ k, $M_{ntx} = 102$ ft-k and $M_{ltx} = 68$ ft-k, $M_{nty} = 84$ ft-k and $M_{lty} = 56$ ft-k

For ASD: $P_{nt} = 117$ k and $P_{lt} = 78$ k, $M_{ntx} = 72$ ft-k and $M_{ltx} = 48$ ft-k, $M_{nty} = 60$ ft-k and $M_{lty} = 40$ ft-k

Solution

LRFD	ASD
Assume B_{1x}, B_{1y}, B_{2x} and $B_{2y} = 1.0$	Assume B_{1x}, B_{1y}, B_{2x} and $B_{2y} = 1.0$
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 175 + 1.0(115) = 290$ k	$P_a = 117 + 1.0(78) = 195$ k
and, $M_{rx} = M_{ux} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$	and, $M_{rx} = M_{ax} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$
$M_{ux} = 1.0(102) + 1.0(68) = 170$ ft-k	$M_{ax} = 1.0(72) + 1.0(48) = 120$ ft-k
and, $M_{ry} = M_{uy} = B_{1y}(M_{nty}) + B_{2x}(M_{lty})$	and, $M_{ry} = M_{ay} = B_{1y}(M_{nty}) + B_{2x}(M_{lty})$
$M_{uy} = 1.0(84) + 1.0(56) = 140$ ft-k	$M_{ay} = 1.0(60) + 1.0(40) = 100$ ft-k
$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$	$P_{aeq} = P_a + M_{ax}\,m + M_{ay}mu$
From "1st Approximation" part of Table 11.3	From "1st Approximation" part of Table 11.3
$m = 1.9$ for $KL = 10$ ft, $F_y = 50$ ksi	$m = 1.9$ for $KL = 10$ ft, $F_y = 50$ ksi
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 290 + 170(1.9) + 140(1.9)(2.0) = 1145$ k	$P_{aeq} = 195 + 120(1.9) + 100(1.9)(2.0) = 803$ k
1st trial section from Table 4.1:	1st trial section from Table 4.1:
W14 → W14 × 99 ($\Phi_c P_n = 1210$ k)	W14 → W14 × 99 ($P_n/\Omega_c = 807$ k)
W12 → W12 × 106 ($\Phi_c P_n = 1260$ k)	W12 → W12 × 106 ($P_n/\Omega_c = 838$ k)
W10 → W10 × 112 ($\Phi_c P_n = 1280$ k)	W10 → W10 × 112 ($P_n/\Omega_c = 851$ k)
Suppose we decide to use a W14 section:	Suppose we decide to use a W14 section:
From "Subsequent Approximation" part of Table 11.3, W14's	From "Subsequent Approximation" part of Table 11.3, W14's
$m = 1.5$	$m = 1.5$
$P_{ueq} = 290 + 170(1.5) + 140(1.5)(2.0) = 965$ k	$P_{aeq} = 195 + 120(1.5) + 100(1.5)(2.0) = 675$ k
Try W14 × 90, ($\Phi_c P_n = 1100$ k > 965 k)	**Try W14 × 90,** ($P_n/\Omega_c = 735$ k > 675 k)

Note: These are trial sizes. B_{1x}, B_{1y}, B_{2x} and B_{2y}, which were assumed, must be calculated and these W14 sections checked with the appropriate interaction equations.

Examples of complete beam-column design are presented in Examples 11-13 and 11-14. In the examples, a trial section is first determined using the equivalent load method as shown in Examples 11-11 and 11-12. The trial section is then checked with the appropriate interaction equation. Both examples use the Effective Length Method. Example 11-13 is a beam-column in a braced frame and Example 11-14 is a beam-column in an unbraced frame.

Example 11-13

Select the lightest W12 section for both LRFD and ASD for the following data: $F_y = 50$ ksi, $(KL)_x = (KL)_y = 12$ ft, $P_{nt} = 250$ k, $M_{ntx} = 180$ ft-k and $M_{nty} = 70$ ft-k for LRFD, and $P_{nt} = 175$ k, $M_{ntx} = 125$ ft-k and $M_{nty} = 45$ ft-k for ASD. $C_b = 1.0$, $C_{mx} = C_{my} = 0.85$.

Solution

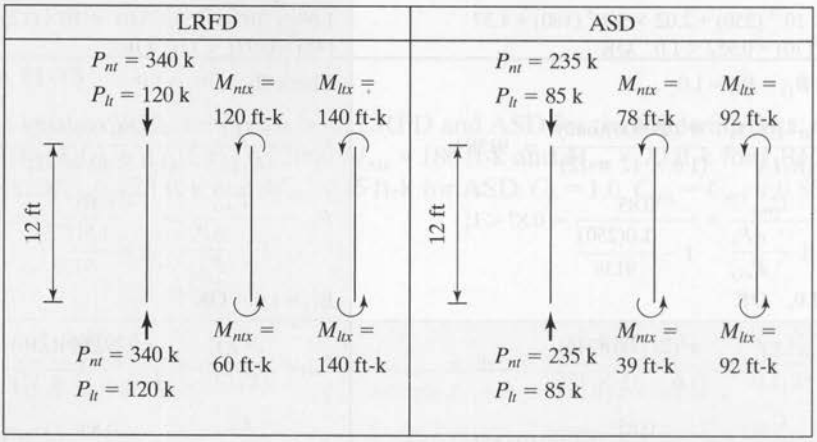
LRFD	ASD
Assume $B_{1x} = B_{1y} = 1.0$, B_2 not required	Assume $B_{1x} = B_{1y} = 1.0$, B_2 not required
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 250 + 0 = 250$ k	$P_a = 175 + 0 = 175$ k
and, $M_{rx} = M_{ux} = B_1(M_{ntx}) + B_2(M_{ltx})$	and, $M_{rx} = M_{ax} = B_1(M_{ntx}) + B_2(M_{ltx})$
$M_{ux} = 1.0(180) + 0 = 180$ ft-k	$M_{ax} = 1.0(120) + 0 = 125$ ft-k
and, $M_{ry} = M_{uy} = B_1(M_{nty}) + B_2(M_{lty})$	and, $M_{ry} = M_{ay} = B_1(M_{nty}) + B_2(M_{lty})$
$M_{uy} = 1.0(70) + 0 = 70$ ft-k	$M_{ay} = 1.0(45) + 0 = 45$ ft-k
$P_{ueq} = P_u + M_{ux}m + M_{uy}mu$	$P_{aeq} = P_a + M_{ax}\,m + M_{ay}\,mu$
From "Subsequent Approximation" part of	From "Subsequent Approximation" part of
Table 11.3, W12's	Table 11.3, W12's
$m = 1.6$	$m = 1.6$
$u = 2.0$ (assumed)	$u = 2.0$ (assumed)
$P_{ueq} = 250 + 180(1.6) + 70(1.6)(2.0) = 762$ k	$P_{aeq} = 175 + 125(1.6) + 45(1.6)(2.0) = 519$ k
Try W12 × 72, ($\Phi_c P_n = 806$ k > 762 k) from Table 4.1	**Try W12 × 72,**($P_n/\Omega_c = 536$ k > 519 k) from Table 4.1
From Table 6.1 for $KL = 12$ ft and $L_b = 12$ ft	From Table 6.1 for $KL = 12$ ft and $L_b = 12$ ft
$p = 1.24 \times 10^{-3}$, $b_x = 2.23 \times 10^{-3}$, $b_y = 4.82 \times 10^{-3}$	$p = 1.87 \times 10^{-3}$, $b_x = 3.36 \times 10^{-3}$, $b_y = 7.24 \times 10^{-3}$
$P_r/\Phi_c P_n = 250/806 = 0.310 > 0.2$ Use modified Equation H1-1a.	$P_r/P_n/\Omega_c = 175/536 = 0.326 > 0.2$ Use modified Equation H1-1a.
$1.24 \times 10^{-3}(250) + 2.23 \times 10^{-3}(180) + 4.82 \times 10^{-3}(70) = 1.049 > 1.0$ N.G.	$1.87 \times 10^{-3}(175) + 3.36 \times 10^{-3}(125) + 7.24 \times 10^{-3}(45) = 1.073 > 1.0$ N.G.

(*Continued*)

LRFD	ASD
Try W12 × 79, ($\Phi_c P_n$ = 887 k > 762 k) from Table 4.1	Try W12 × 79, (P_n/Ω_c = 590 k > 519 k) from Table 4.1
From Table 6.1 for KL = 12 ft and L_b = 12 ft	From Table 6.1 for KL = 12 ft and L_b = 12 ft
$p = 1.13 \times 10^{-3}, b_x = 2.02 \times 10^{-3}, b_y = 4.37 \times 10^{-3}$	$p = 1.69 \times 10^{-3}, b_x = 3.04 \times 10^{-3}, b_y = 6.56 \times 10^{-3}$
$1.13 \times 10^{-3}(250) + 2.02 \times 10^{-3}(180) + 4.37 \times 10^{-3}(70) = 0.952 < 1.0$ OK	$1.69 \times 10^{-3}(175) + 3.04 \times 10^{-3}(125) + 6.56 \times 10^{-3}(45) = 0.971 < 1.0$ OK
Check $B_{1x} = B_{1y} = 1.0$	Check $B_{1x} = B_{1y} = 1.0$
$P_{e1x} = \dfrac{\pi^2 EI^*}{(K_1L)^2} = \dfrac{\pi^2(29{,}000)(662)}{(1.0 \times 12 \times 12)^2} = 9138$ k	$P_{e1x} = \dfrac{\pi^2 EI^*}{(K_1L)^2} = \dfrac{\pi^2(29{,}000)(662)}{(1.0 \times 12 \times 12)^2} = 9138$ k
$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{0.85}{1 - \dfrac{1.0(250)}{9138}} = 0.87 < 1;$	$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{0.85}{1 - \dfrac{1.6(175)}{9138}} = 0.88 < 1;$
$B_{1x} = 1.0$, OK	$B_{1x} = 1.0$, OK
$P_{e1y} = \dfrac{\pi^2 EI^*}{(K_1L)^2} = \dfrac{\pi^2(29{,}000)(216)}{(1.0 \times 12 \times 12)^2} = 2981$ k	$P_{e1y} = \dfrac{\pi^2 EI^*}{(K_1L)^2} = \dfrac{\pi^2(29{,}000)(216)}{(1.0 \times 12 \times 12)^2} = 2981$ k
$B_{1y} = \dfrac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \dfrac{0.85}{1 - \dfrac{1.0(250)}{2981}} = 0.93 < 1;$	$B_{1y} = \dfrac{C_{my}}{1 - \dfrac{\alpha P_r}{P_{e1y}}} = \dfrac{0.85}{1 - \dfrac{1.6(175)}{2981}} = 0.94 < 1;$
$B_{1y} = 1.0$, OK	$B_{1y} = 1.0$, OK
With $B_{1x} = B_{1y} = 1.0$, section is sufficient based on previous check using modified Equation H1-1a.	With $B_{1x} = B_{1y} = 1.0$, section is sufficient based on previous check using modified Equation H1-1a.
Will perform additional check using Equation H1-1a:	Will perform additional check using Equation H1-1a:
For W12 × 79, ΦM_{px} = 446 ft-k, L_p = 10.8 ft, L_r = 39.9 ft	For W12 × 79, M_{px}/Ω_b = 297 ft-k, L_p = 10.8 ft, L_r = 39.9 ft
BF = 5.67, L_b = 12 ft, Zone 2, C_b = 1.0, ΦM_{py} = 204 ft-k	BF = 3.78, L_b = 12 ft, Zone 2, C_b = 1.0, M_{py}/Ω_b = 135 ft-k
$\Phi M_{nx} = C_b\,[\Phi M_{px} - BF(L_b - L_p)]\ \Phi M_{px}$	$M_{nx}/\Omega_b = C_b\,[M_{px}/\Omega_b - BF(L_b - L_p)] \le M_{px}/\Omega_b$
$\Phi M_{nx} = 1.0\,[446 - 5.67(12 - 10.8)] = 439.2$ ft-k	$M_{nx}/\Omega_b = 1.0\,[297 - 3.78\,(12 - 10.8)] = 292.4$ ft-k
$\Phi M_{ny} = \Phi M_{py} = 204$ ft-k	$M_{ny}/\Omega_b = M_{py}/\Omega_b = 135$ ft-k
Equation H1-1a:	Equation H1-1a:
$\dfrac{250}{887} + \dfrac{8}{9}\left(\dfrac{180}{439.2} + \dfrac{70}{204}\right) = 0.951 < 1.0$ OK	$\dfrac{175}{590} + \dfrac{8}{9}\left(\dfrac{125}{292.4} + \dfrac{45}{135}\right) = 0.973 < 1.0$ OK
Use W12 × 79, LRFD.	**Use W12 × 79, ASD.**

Example 11-14

Select a trial W12 section for both LRFD and ASD for an unbraced frame and the following data: $F_y = 50$ ksi, $L_x = L_y = 12$ ft, $K_x = 1.72$, $K_y = 1.0$, $P_{story} = 2400$ k (LRFD) = 1655k (ASD), $P_{e\,story} = 50{,}000$ k.



Solution

LRFD	ASD
Assume $B_{1x} = 1.0$	Assume $B_{1x} = 1.0$
$B_{2x} = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \dfrac{1}{1 - \dfrac{1.0(2400)}{50{,}000}} = 1.05$	$B_{2x} = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \dfrac{1}{1 - \dfrac{1.6(1655)}{50{,}000}} = 1.056$
Assume $(KL)_y = 1.0(12) = 12$ ft	Assume $(KL)_y = 1.0(12) = 12$ ft
$\therefore\ P_r = P_u = P_{nt} + B_2(P_{lt})$	$\therefore\ P_r = P_a = P_{nt} + B_2(P_{lt})$
$P_u = 340 + 1.05(120) = 466$ k	$P_a = 235 + 1.056(85) = 325$ k
and, $M_{rx} = M_{ux} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$	and, $M_{rx} = M_{ax} = B_{1x}(M_{ntx}) + B_{2x}(M_{ltx})$
$M_{ux} = 1.0(120) + 1.05(140) = 267$ ft-k	$M_{ax} = 1.0(78) + 1.056(92) = 176$ ft-k
$P_{ueq} = P_u + M_{ux}\,m + M_{uy}\,mu$	$P_{aeq} = P_a + M_{ax}m + M_{ay}mu$
From "Subsequent Approximation" part of Table 11.3, W12's	From "Subsequent Approximation" part of Table 11.3, W12's
$m = 1.6$	$m = 1.6$
$u = 2.0$ (assumed) not required	$u = 2.0$ (assumed) not required
$P_{ueq} = 466 + 267(1.6) + 0 = 894$ k	$P_{aeq} = 325 + 176(1.6) + 0 = 607$ k

(Continued)

LRFD	ASD
1st trial section from Table 4.1:	1st trial section from Table 4.1:
Try W12 × 87, ($\Phi_c P_n = 981 \text{ k} > 894 \text{ k}$)	Try W12x87, ($P_n/\Omega_c = 653 \text{ k} > 607 \text{ k}$)
Check $\Phi_c P_n$:	Check P_n/Ω_c:
$(KL)_y = 12$ ft	$(KL)_y = 12$ ft
Equivalent $(KL)_y = \dfrac{(KL)_x}{\dfrac{r_x}{r_y}} = \dfrac{(1.72)(12)}{1.75} = 11.79$ ft	Equivalent $(KL)_y = \dfrac{(KL)_x}{\dfrac{r_x}{r_y}} = \dfrac{(1.72)(12)}{1.75} = 11.79$ ft
$(KL)_y = 12$ ft controls so $\Phi_c P_n = 981$ k	$(KL)_y = 12$ ft controls so $P_n/\Omega_c = 653$ k
From Table 6-1 for $KL = 12$ ft and $L_b = 12$ ft	From Table 6-1 for $KL = 12$ ft and $L_b = 12$ ft
$p = 1.02 \times 10^{-3}, b_x = 1.82 \times 10^{-3}$	$p = 1.53 \times 10^{-3}, b_x = 2.74 \times 10^{-3}$
$P_r/\Phi_c P_n = 466/981 = 0.475 > 0.2$ Use modified H1-1a	$P_r/P_n/\Omega_c = 325/653 = 0.498 > 0.2$ Use modified H1-1a
$1.02 \times 10^{-3}(466) + 1.82 \times 10^{-3}(267) = 0.961 < 1.0$ OK	$1.53 \times 10^{-3}(325) + 2.74 \times 10^{-3}(176) = 0.979 < 1.0$ OK
Check $B_{1x} = 1.0$	Check $B_{1x} = 1.0$
$P_{e1x} = \dfrac{\pi^2 EI^*}{(K_1L)^2} = \dfrac{\pi^2(29{,}000)(740)}{(1.0 \times 12 \times 12)^2} = 10{,}214$ k	$P_{e1x} = \dfrac{\pi^2 EI^*}{(K_1L)^2} = \dfrac{\pi^2(29{,}000)(740)}{(1.0 \times 12 \times 12)^2} = 10{,}214$ k
Find C_{mx}: $\dfrac{M_1}{M_2} = \dfrac{60}{120} = +0.50$	Find C_{mx}: $\dfrac{M_1}{M_2} = \dfrac{39}{78} = +0.50$
$C_{mx} = 0.6 - 0.4(M_1/M_2) = 0.6 - 0.4(0.50) = 0.40$	$C_{mx} = 0.6 - 0.4(M_1/M_2) = 0.6 - 0.4(0.50) = 0.40$
$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{0.40}{1 - \dfrac{1.0(466)}{10214}} = 0.42 < 1;$	$B_{1x} = \dfrac{C_{mx}}{1 - \dfrac{\alpha P_r}{P_{e1x}}} = \dfrac{0.40}{1 - \dfrac{1.6(325)}{10214}} = 0.42 < 1;$
$B_{1x} = 1.0$, OK	$B_{1x} = 1.0$, OK
With $B_{1x} = 1.0$, section is sufficient based on previous check using modified Equation H1-1a.	With $B_{1x} = 1.0$, section is sufficient based on previous check using modified Equation H1-1a.
Will perform additional check using Equation H1-1a:	Will perform additional check using Equation H1-1a:
For W12 × 87, $\Phi M_{px} = 495$ ft-k, $L_p = 10.8$ ft, $L_r = 43.1$ ft	For W12 × 87, $M_{px}/\Omega_b = 329$ ft-k, $L_p = 10.8$ ft, $L_r = 43.1$ ft
$BF = 5.73$, $L_b = 12$ ft, Zone 2, $C_b = 1.0$	$BF = 3.81$, $L_b = 12$ ft, Zone 2, $C_b = 1.0$
$\Phi M_{nx} = C_b(\Phi M_{px} - BF(L_b - L_p) \le \Phi M_{px}$	$M_{nx}/\Omega_b = C_b[M_{px}/\Omega_b - BF(L_b - L_p)]\, M_{px}/\Omega_b$
$\Phi M_{nx} = 1.0[495 - 5.73(12\text{–}10.8)] = 488.1$ ft-k	$M_{nx}/\Omega_b = 1.0[329 - 3.81(12 - 10.8)] = 324.4$ ft-k
Equation H1-1a:	Equation H1-1a:
$\dfrac{466}{981} + \dfrac{8}{9}\left(\dfrac{267}{488.1} + 0\right) = 0.961 < 1.0$ OK	$\dfrac{325}{653} + \dfrac{8}{9}\left(\dfrac{176}{324.4} + 0\right) = 0.980 < 1.0$ OK
Use W12 × 87, LRFD.	**Use W12 × 87, ASD.**

11.10 PROBLEMS FOR SOLUTION

All problems are to be solved with both LRFD and ASD methods, unless noted otherwise.

Bending and Axial Tension

11-1 to 11-6. *Analysis Problems*

11-1. A W10 × 54 tension member with no holes and $F_y = 50$ ksi ($F_u = 65$ ksi) is subjected to service loads $P_D = 90$ k and $P_L = 120$ k and to service moments $M_{Dx} = 32$ ft-k and $M_{Lx} = 50$ ft-k. Is the member satisfactory if $L_b = 12$ ft and if $C_b = 1.0$? (*Ans.* OK 0.863– LRFD, OK 0.903 ASD)

11-2. A W8 × 35 tension member with no holes, consisting of $F_y = 50$ ksi steel ($F_u = 65$ ksi), is subjected to service loads $P_D = 50$ k and $P_L = 30$ k and to service moments $M_{Dx} = 35$ ft-k and $M_{Lx} = 25$ ft-k. Is the member satisfactory if $L_b = 12$ ft and if $C_b = 1.0$?

11-3. Repeat Prob. 11-2 if the member has 2 – 3/4 in diameter bolts in each flange and $U = 0.85$. (*Ans.* OK 0.920 LRFD, N.G. 1.015 ASD)

11-4. A W10 × 39 tension member with no holes and $F_y = 50$ ksi ($F_u = 65$ ksi) is subjected to service loads $P_D = 56$ k and $P_L = 73$ k that are placed with an eccentricity of 7 in with respect to the x axis. The member is to be 16 ft long and is braced laterally only at its supports. Is the member satisfactory if $C_b = 1.0$?

11-5. A W12 × 30 tension member with no holes is subjected to an axial load, P, which is 40 percent dead load and 60 percent live load and a uniform service wind load of 2.40 k/ft. The member is 14 ft long, laterally braced at its ends only and bending is about the x axis. Assume $C_b = 1.0$, $F_y = 50$ ksi and $F_u = 65$ ksi. What is the maximum value of P for this member to be satisfactory? (*Ans.* $P = 66.9$ k, LRFD; $P = 76.5$ k, ASD)

11-6. A W10 × 45 tension member with no holes and $F_y = 50$ ksi ($F_u = 65$ ksi) is subjected to service loads $P_D = 60$ k and $P_L = 40$ k and to service moments $M_{Dx} = 40$ ft-k, $M_{Lx} = 20$ ft-k, $M_{Dy} = 15$ ft-k and $M_{Ly} = 10$ ft-k. Is the member satisfactory if $L_b = 10.5$ ft and if $C_b = 1.0$?

11-7 to 11-9. *Design Problems*

11-7. Select the lightest available W12 section ($F_y = 50$ ksi, $F_u = 65$ ksi) to support service loads $P_D = 50$ k and $P_L = 90$ k and to service moments $M_{Dx} = 20$ ft-k and $M_{Lx} = 35$ ft-k. The member is 14 ft long and is laterally braced at its ends only. Assume $C_b = 1.0$. (*Ans.* W12 × 35, LRFD and ASD)

11-8. Select the lightest available W8 section ($F_y = 50$ ksi, $F_u = 65$ ksi) to support service loads $P_D = 55$ k and $P_L = 30$ k that are placed with an eccentricity of 2.5 in with respect to the y-axis. The member is 12 ft long and is braced laterally only at its supports. Assume $C_b = 1.0$.

11-9. Select the lightest available W10 section of A992 steel ($F_u = 65$ ksi) to support service loads $P_D = 30$ k and $P_L = 40$ k and to service moments $M_{Dx} = 40$ ft-k, $M_{Lx} = 55$ ft-k, $M_{Dy} = 8$ ft-k and $M_{Ly} = 14$ ft-k. The member is 12 ft long and is laterally braced at its ends only. Assume $C_b = 1.0$. (*Ans.* W10 × 54, LRFD and ASD)

Bending and Axial Compression

All problems (11-10 to 11-22) have loads and moments that were obtained from first-order analyses, and the approximate analysis method (Appendix 8 – AISC Specification) should be used to account for the second-order effects.

11-10 and 11-11. *Analysis problems in braced frames – using loads and moments obtained using the requirements of the Direct Analysis Method.*

11-10. A 14 ft long W12 × 65 beam-column is part of a braced frame and supports service loads of $P_D = 180$ k and $P_L = 110$ k. These loads are applied to the member at its upper end with an eccentricity of 3 in so as to cause bending about the major axis of the section. Check the adequacy of the member if it consists of A992 steel. Assume $C_b = 1.0$ and $C_{mx} = 1.0$.

11-11. A W10 × 60 beam-column member is in a braced frame and must support service loads of $P_D = 60$ k and $P_L = 120$ k and service moments $M_{Dx} = 30$ ft-k and $M_{Lx} = 60$ ft-k. These moments occur at one end while the opposite end is pinned. The member is 15 ft long, and $C_b = 1.0$. Is the member satisfactory if it consists of 50 ksi steel? (*Ans.* OK 0.931 LRFD, OK 0.954 ASD)

11-12 and 11-13. *Analysis problems in braced frames - using loads and moments obtained using the requirements of the Effective Length Method.*

11-12. A W 12 × 58 beam-column member in a braced frame must support the service loads in the Fig. P11-12. The member is A992 steel, C_b may be assumed to be 1.0, and bending is about the strong axis. The member is laterally braced at its ends only and $L_x = L_y = 16$ ft. Is the member adequate?

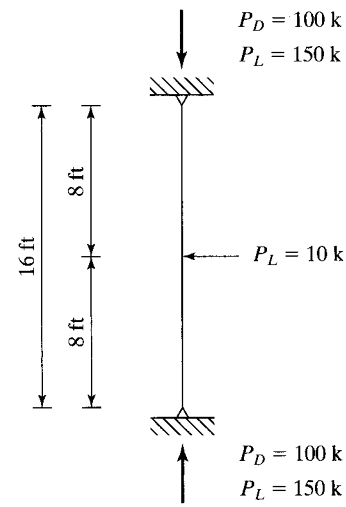


FIGURE P11-12

11-13. A W12 × 96 section consisting of 50 ksi steel is used for a 12 ft long beam-column in a braced frame. The member must support service loads $P_D = 85$ k and $P_L = 125$ k and service moments $M_{Dx} = 70$ ft-k, $M_{Lx} = 120$ ft-k, $M_{Dy} = 20$ ft-k and $M_{Ly} = 35$ ft-k. The moments are applied at both ends of the column so as to put it in single curvature about both axes. The column has $K_x = K_y = 1.0$ and $C_b = 1.0$. Is the member satisfactory? (*Ans.* N.G. 1.048 LRFD, N.G. 1.092 ASD)

11-14. *Analysis problem in an unbraced frame – using loads and moments obtained using the requirements of the Direct Analysis Method.*

A W12 × 72 section is used for a 16 ft long beam-column in an unbraced frame about the *x-x* axis but is braced about the *y-y* axis. Based on a first-order analysis, the member supports the following factored loads: P_{nt} load of 194 k and P_{lt} load of 152 k. The member also supports factored moments: M_{ntx} moment of 110 ft-k and M_{ltx} moment of 76 ft-k. The moments given are at the top of the member. The lower end has moments that are one-half of these values and bend the member in double curvature. There are no transverse loads between the ends. $P_{story} = 3000$ k, and $P_{e\ storyx} = 75{,}000$ k. Using the LRFD procedure, is the member satisfactory if $F_y = 50$ ksi and $C_b = 1.0$?

11-15. *Analysis problem in an unbraced frame – using loads and moments obtained using the requirements of the Effective Length Method.*

A W8 × 48 section consisting of 50 ksi steel is used for a 14 ft long beam-column in a one story building frame. It is unbraced in the plane of the frame (x-x axis) but is braced out of the plane of the frame (y-y axis) so that $K_y = 1.0$. K_x has been determined to equal 1.67. A first-order analysis has been completed and the results yielded a P_{nt} load of 52 k and a P_{lt} load of 16 k. The factored M_{ntx} moment was found to be 96 ft-k and a factored M_{ltx} moment of 50 ft-k. The member is pinned at the base, and is laterally braced only at the top and the bottom. There are no transverse loads between the ends and $C_b = 1.0$. The $P_{story} = 104$ k and the ratio of $P_{mf}/P_{story} = 1.0$. H, the story shear, is equal to 4.4 k, and the drift index (Δ_H/L) is 0.0025. Using the LRFD procedure, is the member adequate? (*Ans.* OK 0.985 LRFD)

11-16 and 11-17. *Design problems in braced frames – using loads and moments obtained using the Direct Analysis Method.*

11-16. Select the lightest W14, 15 ft long, beam-column that is not subjected to sidesway. The service loads are $P_D = 105$ k and $P_L = 120$ k. The service moments are $M_{Dx} = 85$ ft-k and $M_{Lx} = 95$ ft-k. The member consists of $F_y = 50$ ksi steel. Assume $C_b = 1.0$ and $C_{mx} = 0.85$.

11-17. Sidesway is prevented for the beam-column shown in the Fig. P11-17. If the first-order moments shown are about the x axis, select the lightest W8 if it consists of $F_y = 50$ steel. Assume $C_b = 1.0$. (*Ans.* W8 × 35 LRFD and ASD)

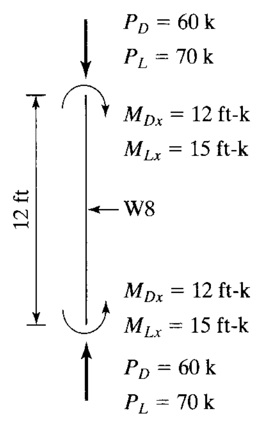


FIGURE P11-17

11-18 and 11-19. *Design problems in braced frames – using loads and moments obtained using the Effective Length Method.*

11-18. Select the lightest W10 beam-column member in a braced frame that supports service loads of $P_D = 50$ k and $P_L = 75$ k. The member also supports the service moments

$M_{Dx} = 27.5$ ft-k, $M_{Lx} = 40$ ft-k, $M_{Dy} = 10$ ft-k and $M_{Ly} = 15$ ft-k. The member is 15 ft long and moments occur at one end while the other end is pinned. There are no transverse loads on the member and assume $C_b = 1.0$. Use 50 ksi steel.

11-19. Select the lightest W12, 15 ft long, $F_y = 50$ ksi steel beam-column for the first-order loads and moments shown in the Fig. P11-19. The column is part of a braced frame system. Assume $C_b = 1.0$. (*Ans.* W12 × 96 LRFD, W12 × 106 ASD)

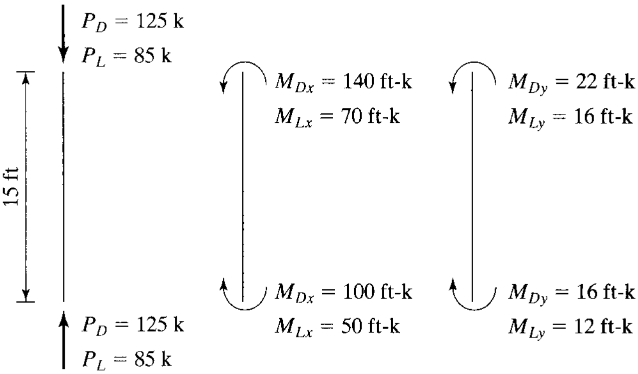


FIGURE P11-19

11-20. *Design problem in an unbraced frame – using loads and moments obtained using the requirements of the Direct Analysis Method.*

11-20. Using the LRFD procedure, select the lightest W12, 10 ft long beam-column in an unbraced frame. Based on a first-order analysis, the member supports the following factored loads: P_{nt} load of 140 k and P_{lt} load of 105 k. The member also supports factored moments: M_{ntx} moment of 60 ft-k, M_{ltx} moment of 90 ft-k, M_{nty} moment of 40 ft-k, and M_{lty} moment of 75 ft-k. $P_{story} = 5000$ k, $P_{e\,story\,x} = 40{,}000$ k, $P_{e\,storyy} = 30{,}000$ k $C_{mx} = C_{my} = 1.0$, $C_b = 1.0$ and $F_y = 50$ ksi steel.

11-21 and 11-22. *Design problems in unbraced frames – using loads and moments obtained using the requirements of the Effective Length Method.*

11-21. Using the LRFD procedure, select the lightest W10, 16 ft long beam-column in an unbraced frame. Based on a first-order analysis, the member supports the following factored loads: P_{nt} load of 148 k and P_{lt} load of 106 k. The member also supports factored moments: M_{ntx} moment of 92 ft-k and M_{ltx} moment of 64 ft-k. The moments are equal at each end and the member is bent in single curvature. There are no transverse loads between the ends. $K_x = 1.5$, $K_y = 1.0$, $P_{story} = 2800$ k, $P_{e\,storyx} = 72{,}800$ k, $C_b = 1.0$ and $F_y = 50$ ksi. (*Ans.* W10 × 68 LRFD)

11-22. Using the LRFD procedure, select the lightest W12 section ($F_y = 50$ ksi steel) for a 14 ft long beam-column that is part of a portal unbraced frame. The given unfactored axial force and moment are due to wind. P_{lt} load is a 175 k wind load and M_{ltx} moment is 85 ft-k wind moment. $C_{mx} = 1.0$, $K_x = 1.875$ and $K_y = 1.0$. The $P_{story} = 1212.5$ k and the ratio of $P_{mf}/P_{story} = 0.20$. H, the story shear, is equal to 15.0 k, and the drift index (Δ_H/L) is 0.0020.

CHAPTER 12

Bolted Connections

12.1 INTRODUCTION

For many years, riveting was the accepted method used for connecting the members of steel structures. For the last few decades, however, bolting and welding have been the methods used for making structural steel connections, and riveting is almost never used. This chapter and the next are almost entirely devoted to bolted connections, although some brief remarks are presented at the end of Chapter 13 concerning rivets.

Bolting of steel structures is a very rapid field erection process that requires less skilled labor than does riveting or welding. This gives bolting a distinct economic advantage over the other connection methods in the United States, where labor costs are so very high. Even though the purchase price of a high-strength bolt is several times that of a rivet, the overall cost of bolted construction is cheaper than that for riveted construction because of reduced labor and equipment costs and the smaller number of bolts required to resist the same loads.

Part 16.2 of the AISC Manual provides a copy of the "Specification for Structural Joints Using ASTM A325 or A490 Bolts," dated June 30, 2004, and published by the Research Council on Structural Connections (RCSC). There, the reader can find almost anything he or she would like to know about steel bolts. Included are types, sizes, steels, preparations needed for bolting, use of washers, tightening procedures, inspection, and so on.

12.2 TYPES OF BOLTS

There are several types of bolts that can be used for connecting steel members. They are described in the paragraphs that follow.

Unfinished bolts are also called *ordinary* or *common bolts*. They are classified by the ASTM as A307 bolts and are made from carbon steels with stress–strain characteristics very similar to those of A36 steel. They are available in diameters from 1/2 to 1 1/2 in in 1/8-in increments.

A307 bolts generally have square heads and nuts to reduce costs, but hexagonal heads are sometimes used because they have a slightly more attractive appearance, are easier to turn and to hold with the wrenches, and require less turning space. As they have relatively large tolerances in shank and thread dimensions, their design strengths are appreciably smaller than those for high-strength bolts. They are primarily used in light structures subjected to static loads and for secondary members (such as purlins, girts, bracing, platforms, small trusses, and so forth).

Designers often are guilty of specifying high-strength bolts for connections when common bolts would be satisfactory. *The strength and advantages of common bolts have usually been greatly underrated in the past.* The analysis and design of A307 bolted connections are handled exactly as are riveted connections in every way, except that the design stresses are slightly different.

High-strength bolts are made from medium carbon heat-treated steel and from alloy steel and have tensile strengths two or more times those of ordinary bolts. There are two basic types, the A325 bolts (made from a heat-treated medium carbon steel) and the higher strength A490 bolts (also heat-treated, but made from an alloy steel). High-strength bolts are used for all types of structures, from small buildings to skyscrapers and monumental bridges. These bolts were developed to overcome the weaknesses of rivets—primarily, insufficient tension in their shanks after cooling. The resulting rivet tensions may not be large enough to hold them in place during the application of severe impactive and vibrating loads. The result is that they may become loose and vibrate and may eventually have to be replaced. High-strength bolts may be tightened until they have very high tensile stresses so that the connected parts are clamped tightly together between the bolt and nut heads, permitting loads to be transferred primarily by friction.

Sometimes high-strength bolts are needed with diameters and lengths larger than those available with A325 and A490 bolts. Should they be required with diameters exceeding 1 1/2 inches or lengths longer than 8 in, A449 bolts may be used, as well as A354 threaded rods. For anchor rods, ASTM F1554 threaded rods are preferred.

12.3 HISTORY OF HIGH-STRENGTH BOLTS

The joints obtained using high-strength bolts are superior to riveted joints in performance and economy, and they are the leading field method of fastening structural steel members. C. Batho and E. H. Bateman first claimed in 1934 that high-strength bolts could satisfactorily be used for the assembly of steel structures,[1] but it was not until 1947 that the Research Council on Riveted and Bolted Structural Joints of the Engineering Foundation was established. This group issued their first specifications in 1951, and high-strength bolts were adopted with amazing speed by building and bridge engineers for both static and dynamic loadings. They not only quickly became the leading method of making field connections, they also were found to have many applications for shop connections. The construction of the Mackinac Bridge in Michigan involved the use of more than one million high-strength bolts.

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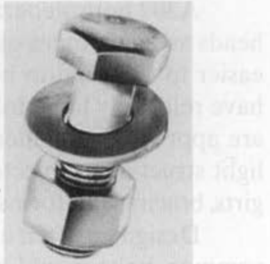
[1]C. Batho and E. H. Bateman, "Investigations on Bolts and Bolted Joints," H. M. Stationery Office (London, 1934). (In the United Kingdom, H. M. Stationery Office is rather like the U.S. Printing Office.)



High-strength bolt. (Courtesy of Bethlehem Steel Corporation.)

Connections that were formerly made with ordinary bolts and nuts were not too satisfactory when they were subjected to vibratory loads, because the nuts frequently became loose. For many years, this problem was dealt with by using some type of lock-nut, but the modern high-strength bolts furnish a far superior solution.

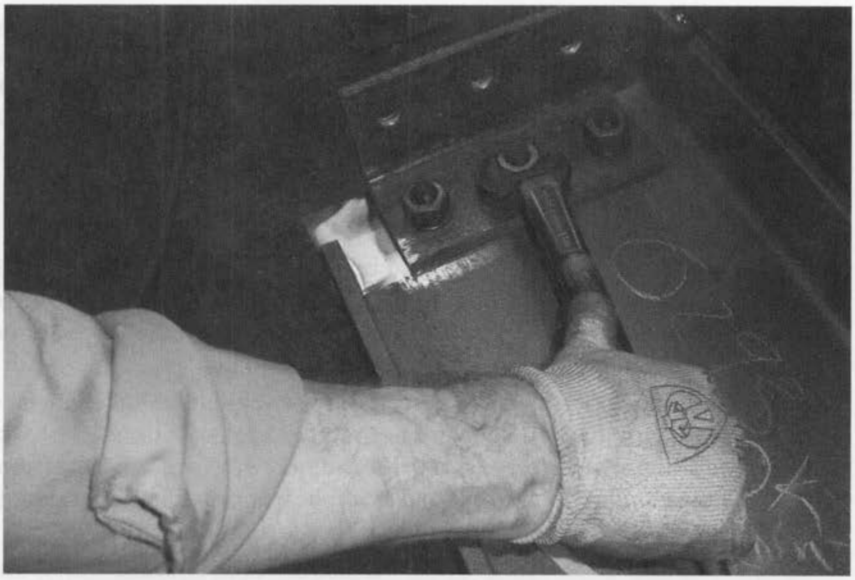
12.4 ADVANTAGES OF HIGH-STRENGTH BOLTS

Among the many advantages of high-strength bolts, which partly explain their great success, are the following:

1. Smaller crews are involved, compared with riveting. Two two-person bolting crews can easily turn out over twice as many bolts in a day as the number of rivets driven by the standard four-person riveting crew. The result is quicker steel erection.
2. Compared with rivets, fewer bolts are needed to provide the same strength.
3. Good bolted joints can be made by people with a great deal less training and experience than is necessary to produce welded and riveted connections of equal quality. The proper installation of high-strength bolts can be learned in a matter of hours.
4. No erection bolts are required that may have to be later removed (depending on specifications), as in welded joints.
5. Though quite noisy, bolting is not nearly as loud as riveting.
6. Cheaper equipment is used to make bolted connections.
7. No fire hazard is present, nor danger from the tossing of hot rivets.
8. Tests on riveted joints and fully tensioned bolted joints under identical conditions show that bolted joints have a higher fatigue strength. Their fatigue strength is also equal to or greater than that obtained with equivalent welded joints.
9. Where structures are to be later altered or disassembled, changes in connections are quite simple because of the ease of bolt removal.

12.5 SNUG-TIGHT, PRETENSIONED, AND SLIP-CRITICAL BOLTS

High-strength bolted joints are said to be *snug-tight*, *pretensioned*, or *slip-critical*. These terms are defined in the paragraphs to follow. The type of joint used is dependent on the type of load that the fasteners will have to carry.



A spud wrench used by ironworkers for erecting structural steel and tightening bolts. One end of the wrench is sized for the hexagonal ends of bolts and nuts, while the other end is tapered to a rounded point and is used to align bolt holes between different connection pieces. (Courtesy of CMC South Carolina Steel.)

a. **Snug-tight bolts**

For most connections, bolts are tightened only to what is called a *snug-tight* condition. Snug-tight is the situation existing when all the plies of a connection are in firm contact with each other. It usually means the tightness produced by the full effort of a person using a spud wrench, or the tightness achieved after a few impacts of the pneumatic wrench. Obviously there is some variation in the degree of tightness achieved under these conditions. Snug-tight bolts must be clearly identified on both design and erection drawings.

Snug-tight bolts are permitted for all situations in which pretensioned or slip-critical bolts are not required. In this type of connection, the plies of steel being connected must be brought together so that they are solidly seated against each other, but they do not have to be in continuous contact. The installed bolts do not have to be inspected to determine their actual pretensioned stresses.

b. **Pretensioned joints**

The bolts in a pretensioned joint are brought to very high tensile stresses equal to approximately 70 percent of their minimum tensile stresses. To properly tighten them, it is necessary to first bring them to a snug-tight condition. Then they are further tightened by one of the four methods described in Section 12.6.

Pretensioned joints are required for connections subjected to appreciable load reversals where nearly full or full design loads are applied to them in one direction, after which the nearly full or full design loads are applied in the other direction. Such a condition is typical of seismic loadings, but not of wind loads. Pretensioned bolts are also required for joints subject to fatigue loads where there is no reversal of the load direction. In addition, they are used where the bolts are subjected to tensile fatigue stresses. A490 bolts should be pretensioned if they are subjected to tension or if they are subjected to combined shear and tension, whether or not there is fatigue. Pretensioned bolts are permitted when slip resistance is of no concern.

c. **Slip-critical joints**

The installation of slip-critical bolts is identical with that for pretensioned joints. The only difference between the two is in the treatment of the contact or faying surfaces. Their inspection is the same, except that the inspector needs to check the faying or contact surface for slip-critical joints.

Slip-critical joints are required only for situations involving shear or combined shear and tension. They are not required for situations involving only tension. In addition, they are to be used for joints with oversized holes and for joints with slotted holes where the load is applied approximately normal (within 80 to 100 degrees) to the long direction of the slot.

When loads are applied to snug-tight bolts, there may be a little slippage, as the holes are a little larger in diameter than the shanks of the bolts. As a result, the parts of the connection may bear against the bolts. You can see that if we have a fatigue situation with constantly changing loads, this is not a desirable situation.

For fatigue situations, and for connections subject to direct tension, it is desirable to use connections that will not slip. These are referred to as *slip-critical connections*. To achieve this situation, the bolts must be tightened until they reach a fully tensioned condition in which they are subject to extremely large tensile forces.

Fully tensioning bolts is an expensive process, as is the inspection necessary to see that they are fully tensioned. Thus, they should be used only where absolutely necessary, as where the working loads cause large numbers of stress changes resulting in fatigue problems. Section J of the AISC Commentary gives a detailed list of connections that must be made with fully tensioned bolts. Included in this list are connections for supports of running machinery or for live loads producing impact and stress reversal; column splices in all tier structures 200 ft or more in height; connections of all beams and girders to columns and other beams or girders on which the bracing of the columns is dependent for structures over 125 ft in height; and so on.

Snug-tight bolts have several advantages over fully tensioned ones. One worker can properly tighten bolts to a snug-tight condition with an ordinary spud wrench or with only a few impacts of an impact wrench. The installation is quick, and only a visual inspection of the work is needed. (Such is not the case for fully tensioned bolts.) Furthermore, snug-tight bolts may be installed with electric wrenches, thus eliminating the need for air compression on the site. As a result, the use of snug-tight bolts saves time and money and is safer than the procedure needed for fully tensioned bolts. *Therefore, for most situations, snug-tight bolts should be used.*

Tables 12.1 and 12.1M provide the minimum fastener tensions required for slip-resistant connections and for connections subject to direct tension. These are, respectively, reproductions of Tables J3.1 and J3.1M of the AISC Specification.

TABLE 12.1 Minimum Bolt Pretension, kips*

Bolt Size, in	Group A-A325 Bolts	Group B-A490 Bolts
$\frac{1}{2}$	12	15
$\frac{5}{8}$	19	24
$\frac{3}{4}$	28	35
$\frac{7}{8}$	39	49
1	51	64
$1\frac{1}{8}$	56	80
$1\frac{1}{4}$	71	102
$1\frac{3}{8}$	85	121
$1\frac{1}{2}$	103	148

* Equal to 0.70 of minimum tensile strength of bolts, rounded off to nearest kip, as specified in ASTM Specifications for A325 and A490M bolts with UNC threads.

TABLE 12.1M Minimum Bolt Pretension, kN*

Bolt Size, mm	Group A-A325M Bolts	Group B-A490M Bolts
M16	91	114
M20	142	179
M22	176	221
M24	205	257
M27	267	334
M30	326	408
M36	475	595

* Equal to 0.70 of minimum tensile strength of bolts, rounded off to nearest kN, as specified in ASTM Specifications for A325M and A490M bolts with UNC threads.
Source: American Institute of Steel Construction, *Manual of Steel Construction* (Chicago: AISC, 2011), Table J3.1 and J3.1M, pp. 16.1–119. "Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved."

The quality-control provisions specified in the manufacture of the A325 and A490 bolts are more stringent than those for the A449 bolts. As a result, despite the method of tightening, the A449 bolts may not be used in slip-resistant connections.

Although many engineers felt that there would be some slippage compared with rivets (because of the fact that the hot driven rivets more nearly filled the holes), it was found that there is less slippage in fully tensioned high-strength bolted joints than in riveted joints under similar conditions.

It is interesting that the nuts used for fully tensioned high-strength bolts almost never need special provisions for locking. Once these bolts are installed and sufficiently tightened to produce the tension required, there is almost no tendency for the nuts to come loose. There are, however, a few situations where they will work loose under heavy vibrating loads. What do we do then? Some steel erectors have replaced the offending bolts with longer ones with two fully tightened nuts. Others have welded the nuts onto the bolts. Apparently, the results have been somewhat successful.

12.6 METHODS FOR FULLY PRETENSIONING HIGH-STRENGTH BOLTS

We have already commented on the tightening required for snug-tight bolts. For fully tensioned bolts, several methods of tightening are available. These methods, including the turn-of-the-nut method, the calibrated wrench method, and the use of alternative design bolts and direct tension indicators, are permitted without preference by the Specification. For A325 and A490 bolts, the minimum pretension equals 70 percent of their specified minimum tensile strength.

12.6.1 Turn-of-the-Nut Method

The bolts are brought to a snug-tight condition and then, with an impact wrench, they are given from one-third to one full turn, depending on their length and the slope of the surfaces under their heads and nuts. Table 8-2, page 16.2-48 in the Manual, presents the amounts of turn to be applied. (The amount of turn given to a particular bolt can easily be controlled by marking the snug-tight position with paint or crayon.)

12.6.2 Calibrated Wrench Method

With this method, the bolts are tightened with an impact wrench that is adjusted to stall at that certain torque which is theoretically necessary to tension a bolt of that diameter and ASTM classification to the desired tension. Also, it is necessary that wrenches be calibrated daily and that hardened washers be used. Particular care needs to be given to protecting the bolts from dirt and moisture at the job site. The reader should refer to the "Specification for Structural Joints Using ASTM A325 or A490 Bolts" in Part 16.2 of the Manual for additional tightening requirements.

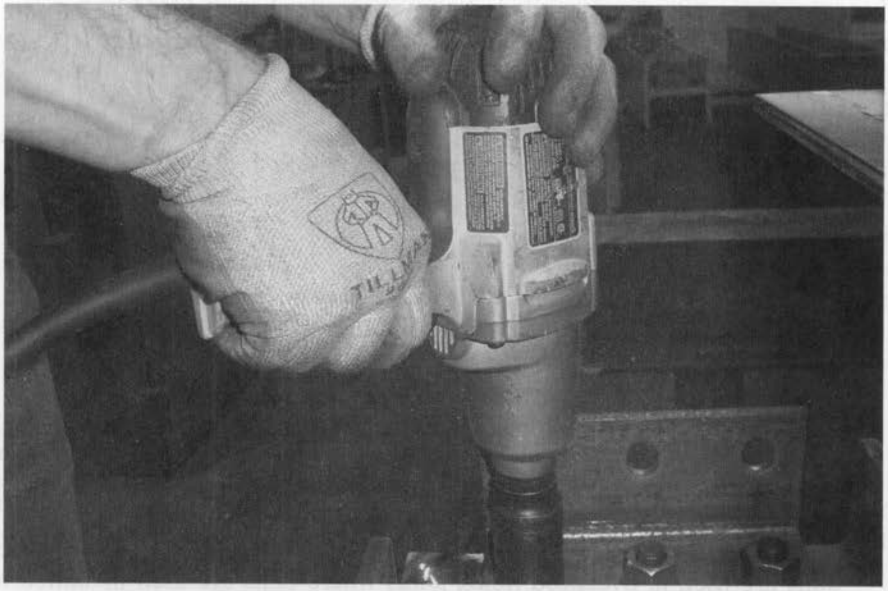
12.6.3 Direct Tension Indicator

The direct tension indicator (which was originally a British device) consists of a hardened washer that has protrusions on one face in the form of small arches. The arches will be flattened as a bolt is tightened. The amount of gap at any one time is a measure of the bolt tension.

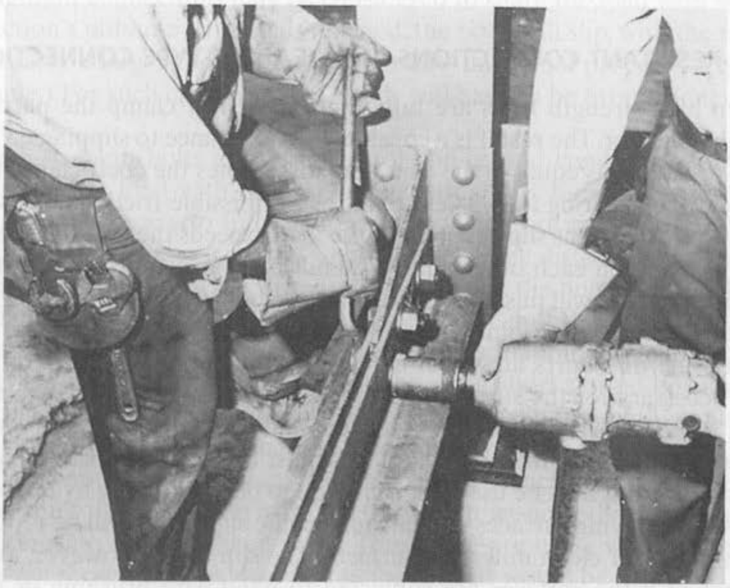
12.6.4 Alternative Design Fasteners

In addition to the preceding methods, there are some alternative design fasteners that can be tensioned quite satisfactorily. Bolts with splined ends that extend beyond the threaded portion of the bolts, called **twist-off bolts**, are one example. Special wrench chucks are used to tighten the nuts until the splined ends shear off. This method of tightening bolts is quite satisfactory and will result in lower labor costs.

A maximum bolt tension is not specified in any of the preceding tightening methods. This means that the bolt can be tightened to the highest load that will not break it, and the bolt still will do the job. Should the bolt break, another one is put in, with no damage done. It might be noted that the nut is stronger than the bolt, and the bolt will break before the nut strips. (The bolt specification mentioned previously requires that a tension measuring device be available at the job site to ensure that specified tensions are achieved.)

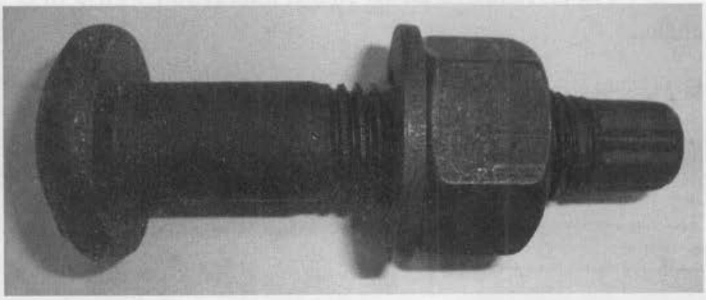


An impact wrench used to tighten bolts to either a snug-tight or a fully tensioned condition. It may be electric, such as this one, or it may be pneumatic. (Courtesy of CMC South Carolina Steel.)



Torquing the nut for a high-strength bolt with an air-driven impact wrench. (Courtesy of Bethlehem Steel Corporation.)

For fatigue situations where members are subjected to constantly fluctuating loads, the slip-resistant connection is very desirable. If, however, the force to be carried is less than the frictional resistance, and thus no forces are applied to the bolts, how



A "twist-off bolt," also called a "load-indicator bolt," or a "tension control bolt." Notice the spline at the end of the bolt shank. (Courtesy of CMC South Carolina Steel.)

could we ever have a fatigue failure of the bolts? Slip-resistant connections can be designed to prevent slipping either at the service load limit state or at the strength load limit state.

Other situations where slip-resistant connections are desirable include joints where bolts are used in oversized holes, joints where bolts are used in slotted holes and the loads are applied parallel or nearly so to the slots, joints that are subjected to significant force reversals, and joints where bolts and welds resist shear together on a common *faying surface*. (The faying surface is the contact, or shear area between the members.)

12.7 SLIP-RESISTANT CONNECTIONS AND BEARING-TYPE CONNECTIONS

When high-strength bolts are fully tensioned, they clamp the parts being connected tightly together. The result is a considerable resistance to slipping on the faying surface. This resistance is equal to the clamping force times the coefficient of friction.

If the shearing load is less than the permissible frictional resistance, the connection is referred to as **slip-resistant**. If the load exceeds the frictional resistance, the members will slip on each other and will tend to shear off the bolts; at the same time, the connected parts will push or bear against the bolts, as shown in Fig. 12.1 on page 401.

The surfaces of joints, including the area adjacent to washers, need to be free of loose scale, dirt, burrs, and other defects that might prevent the parts from solid seating. It is necessary for the surface of the parts to be connected to have slopes of not more than 1 to 20 with respect to the bolt heads and nuts, unless beveled washers are used. For slip-resistant joints, the faying surfaces must also be free from oil, paint, and lacquer. (Actually, paint may be used if it is proved to be satisfactory by test.)

If the faying surfaces are galvanized, the slip factor will be reduced to almost half of its value for clean mill scale surfaces. The slip factor, however, may be significantly improved if the surfaces are subjected to hand wire brushing or to "brush off" grit blasting. However, such treatments do not seem to provide increased slip resistance for sustained loadings where there seems to be a creeplike behavior.[2]

[2] J. W. Fisher and J. H. A. Struik, *Guide to Design Criteria for Bolted and Riveted Joints* (New York: John Wiley & Sons, 1974) pp. 205–206.

The AASHTO Specifications permit hot-dip galvanization if the coated surfaces are scored with wire brushes or sandblasted after galvanization and before steel erection.

The ASTM Specification permits the galvanization of the A325 bolts themselves, but not the A490 bolts. There is a danger of embrittlement of this higher-strength steel during galvanization due to the possibility that hydrogen may be introduced into the steel in the pickling operation of the galvanization process.

If special faying surface conditions (such as blast-cleaned surfaces or blast-cleaned surfaces with special slip-resistant coatings applied) are used to increase the slip resistance, the designer may increase the values used here to the ones given by the Research Council on Structural Joints in Part 16.2 of the AISC Manual.

12.8 MIXED JOINTS

Bolts may on occasion be used in combination with welds and on other occasions with rivets (as where they are added to old riveted connections to enable them to carry increased loads). The AISC Specification contains some specific rules for these situations.

12.8.1 Bolts in Combination with Welds

For new work, neither A307 common bolts nor high-strength bolts designed for bearing or snug-tight connections may be considered to share the load with welds. (Before the connection's ultimate strength is reached, the bolts will slip, with the result that the welds will carry a larger proportion of the load—the actual proportion being difficult to determine.) For such circumstances, welds will have to be proportioned to resist the entire loads.

If high-strength bolts are designed for slip-critical conditions, they may be allowed to share the load with welds. For such situations, the AISC Commentary J1.8 states that it is necessary to fully tighten the bolts before the welds are made. If the weld is made first, the heat from the weld may very well distort the connection so that we will not get the slip-critical resistance desired from the bolts. If the bolts are placed and fully tightened before the welds are made, the heat of the welding will not change the mechanical properties of the bolts. For such a situation, the loads may be shared if the bolts are installed in standard-size holes or in short slotted holes with the slots perpendicular to the load direction. However, the contribution of the bolts is limited to 50 percent of their available strength in a bearing-type connection.[3]

If we are making alterations for an existing structure that is connected with bearing or snug-tight bolts or with rivets, we can assume that any slipping that is going to occur has already taken place. Thus, if we are using welds in the alteration, we will design those welds neglecting the forces that would be produced by the existing dead load.

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[3]Kulak and G. Y. Grondin, "Strength of Joints That Combine Bolts and Welds," *Engineering Journal* (Chicago: AISC, vol. 38 no. 2, 2nd Quarter, 2001) pp. 89–98.

12.8.2 High-Strength Bolts in Combination with Rivets

High-strength bolts may be considered to share loads with rivets for new work or for alterations of existing connections that were designed as slip-critical. (The ductility of the rivets allows the capacity of both sets of fasteners to act together.)

12.9 SIZES OF BOLT HOLES

In addition to the standard size bolt holes (STD), which are 1/16 in larger in diameter than the bolts, there are three types of enlarged holes: oversized, short-slotted, and long-slotted. Oversized holes will on occasion be very useful in speeding up steel erection. In addition, they give some latitude for adjustments in plumbing frames during erection. The use of nonstandard holes requires the approval of the designer and is subject to the requirements of Section J3 of the AISC Specification. Table 12.2 provides the nominal dimensions in inches for the various kinds of enlarged holes permitted by the AISC, while Table 12.2M provides the same information in millimeters. (These tables are, respectively, Tables J3.3 and J3.3M of the AISC Specification.)

The situations in which we may use the various types of enlarged holes are now described.

Oversized holes (OVS) may be used in all plies of connections as long as the applied load does not exceed the permissible slip resistance. They may not be used in bearing-type connections. It is necessary for hardened washers to be used over oversized holes that are located in outer plies. The use of oversized holes permits the use of larger construction tolerances.

Short-slotted holes (SSL) may be used regardless of the direction of the applied load for slip-critical connections. For bearing type connections, however, the slots must be perpendicular to the direction of loading. Should the load be applied in a direction approximately normal (between 80 and 100 degrees) to the slot, these holes may be used in any or all plies of connections for bearing-type connections. It is necessary to

TABLE 12.2 Nominal Hole Dimensions, Inches

	Hole Dimensions			
Bolt Diameter	Standard (Dia.)	Oversize (Dia.)	Short-slot (Width × Length)	Long-slot (Width × Length)
$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{9}{16} \times \frac{11}{16}$	$\frac{9}{16} \times 1\frac{1}{4}$
$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{11}{16} \times \frac{7}{8}$	$\frac{11}{16} \times 1\frac{9}{16}$
$\frac{3}{4}$	$\frac{13}{16}$	$\frac{15}{16}$	$\frac{13}{16} \times 1$	$\frac{13}{16} \times 1\frac{7}{8}$
$\frac{7}{8}$	$\frac{15}{16}$	$1\frac{1}{16}$	$\frac{15}{16} \times 1\frac{1}{8}$	$\frac{15}{16} \times 2\frac{3}{16}$
1	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{1}{16} \times 1\frac{5}{16}$	$1\frac{1}{16} \times 2\frac{1}{2}$
$\geq 1\frac{1}{8}$	$d + \frac{1}{16}$	$d + \frac{5}{16}$	$(d + \frac{1}{16}) \times (d + \frac{3}{8})$	$(d + \frac{1}{16}) \times (2.5 \times d)$

TABLE 12.2M Nominal Hole Dimensions, mm

Bolt Diameter	Hole Dimensions: Standard (Dia.)	Oversize (Dia.)	Short-slot (Width × Length)	Long-slot (Width × Length)
M16	18	20	18 × 22	18 × 40
M20	22	24	22 × 26	22 × 50
M22	24	28	24 × 30	24 × 55
M24	27 [a]	30	27 × 32	27 × 60
M27	30	35	30 × 37	30 × 67
M30	33	38	33 × 40	33 × 75
≥M36	$d + 3$	$d + 8$	$(d + 3) \times (d + 10)$	$(d + 3) \times 2.5d$

[a] Clearance provided allows the use of a 1 in bolt if desirable.

Source: American Institute of Steel Construction, *Manual of Steel Construction* (Chicago: AISC, 2011), Table J3.3 and J3.3M, pp. 16.1–121.

use washers (hardened if high-strength bolts are being used) over short-slotted holes in an outer ply. The use of short-slotted holes provides for some mill and fabrication tolerances, but does not result in the necessity for slip-critical procedures.

Long-slotted holes (LSL) may be used in *only one* of the connected parts of slip-critical or bearing-type connections at any one faying surface. For slip-critical joints these holes may be used in any direction, but for bearing-type connections the loads must be normal (between 80 and 100 degrees) to the axes of the slotted holes. If long-slotted holes are used in an outer ply they will need to be covered with plate washers or a continuous bar with standard holes. For high-strength bolted connections, the washers or bar do not have to be hardened, but they must be made of structural-grade material and may not be less than 5/16 in thick. Long-slotted holes are usually used when connections are being made to existing structures where the exact positions of the members being connected are not known.

Generally, washers are used to prevent scoring or galling of members when bolts are tightened. Most persons think that they also serve the purpose of spreading out the clamping forces more uniformly to the connected members. Tests have shown, however, that standard size washers don't affect the pressure very much, except when oversized or short-slotted holes are used. Sections 2.5 and 2.6 of Part 16.2 of the Manual (page 16.2-13) provide detailed information concerning washers.

12.10 LOAD TRANSFER AND TYPES OF JOINTS

The following paragraphs present a few of the elementary types of bolted joints subjected to axial forces. (That is, the loads are assumed to pass through the centers of gravities of the groups of connectors.) For each of these joint types, some comments are made about the methods of load transfer. Eccentrically loaded connections are discussed in Chapter 13.

For this initial discussion, the reader is referred to part (a) of Fig. 12.1. It is assumed that the plates shown are connected with a group of snug-tight bolts. In other words, the bolts are not tightened sufficiently so as to significantly squeeze the plates together. If there is assumed to be little friction between the plates, they will slip a little due to the applied loads. As a result, the loads in the plates will tend to shear the connectors off on the plane between the plates and press or bear against the sides of the bolts, as shown in part (b) of the figure. These connectors are said to be in *single shear and bearing* (also called *unenclosed bearing*). They must have sufficient strength to satisfactorily resist these forces, and the members forming the joint must be sufficiently strong to prevent the connectors from tearing through.

When rivets were used instead of the snug-tight bolts, the situation was somewhat different because hot-driven rivets would cool and shrink and then squeeze or clamp the connected pieces together with sizable forces that greatly increased the friction

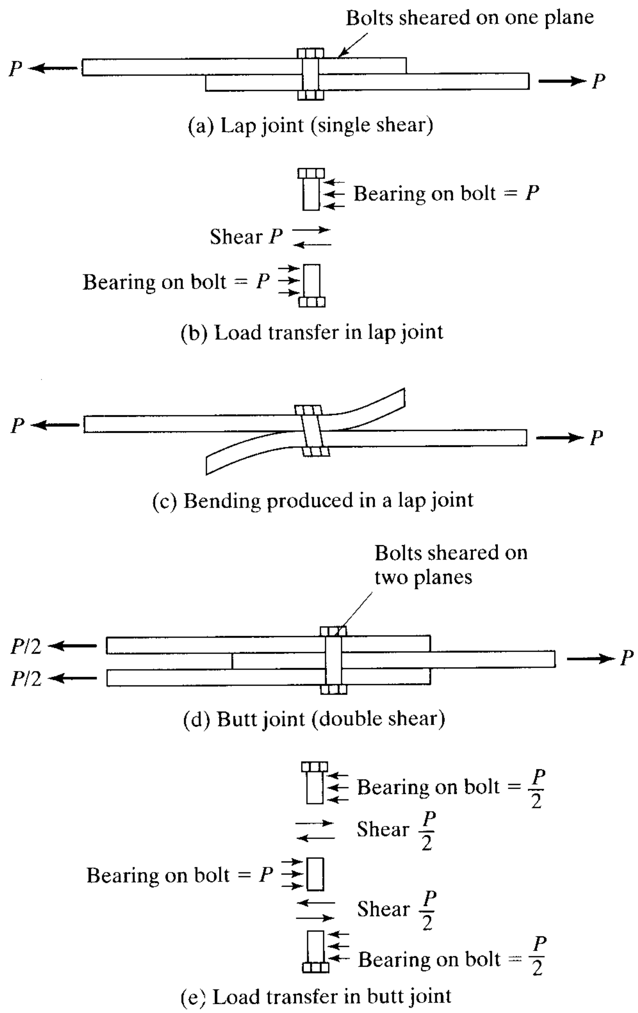


FIGURE 12.1

between the pieces. As a result, a large portion of the loads being transferred between the members was transferred by friction. The clamping forces produced in riveted joints, however, were generally not considered to be dependable, and for this reason specifications normally consider such connections to be snug-tight with no frictional resistance. The same assumption is made for A307 common bolts, as they are not tightened to large dependable tensions.

Fully tensioned high-strength bolts are in a different class altogether. By the tightening methods previously described, a very dependable tension is obtained in the bolts, resulting in large clamping forces and large dependable amounts of frictional resistance to slipping. Unless the loads to be transferred are larger than the frictional resistance, the entire forces are resisted by friction and the bolts are not really placed in shear or bearing. If the load exceeds the frictional resistance, there will be slippage, with the result that the bolts will be placed in shear and bearing.

12.10.1 The Lap Joint

The joint shown in part (a) of Fig. 12.1 is referred to as a *lap joint*. This type of joint has a disadvantage in that the center of gravity of the force in one member is not in line with the center of gravity of the force in the other member. A couple is present that causes an undesirable bending in the connection, as shown in part (c) of the figure. For this reason, the lap joint, which is desirably used only for minor connections, should be designed with at least two fasteners in each line parallel to the length of the member to minimize the possibility of a bending failure.

12.10.2 The Butt Joint

A *butt joint* is formed when three members are connected as shown in Fig. 12.1(d). If the slip resistance between the members is negligible, the members will slip a little and tend to shear off the bolts simultaneously on the two planes of contact between the members. Again, the members are bearing against the bolts, and the bolts are said to be in *double shear and bearing* (also called *enclosed bearing*). The butt joint is more desirable than the lap joint for two main reasons:

1. The members are arranged so that the total shearing force, P, is split into two parts, causing the force on each plane to be only about one-half of what it would be on a single plane if a lap joint were used. From a shear standpoint, therefore, the load-carrying ability of a group of bolts in double shear is theoretically twice as great as the same number of bolts in single shear.
2. A more symmetrical loading condition is provided. (In fact, the butt joint does provide a symmetrical situation if the outside members are the same thickness and resist the same forces. The result is a reduction or elimination of the bending described for a lap joint.)

12.10.3 Double-Plane Connections

The double-plane connection is one in which the bolts are subjected to single shear and bearing, but in which bending moment is prevented. This type of connection, which is shown for a hanger in Fig. 12.2(a), subjects the bolts to single shear.

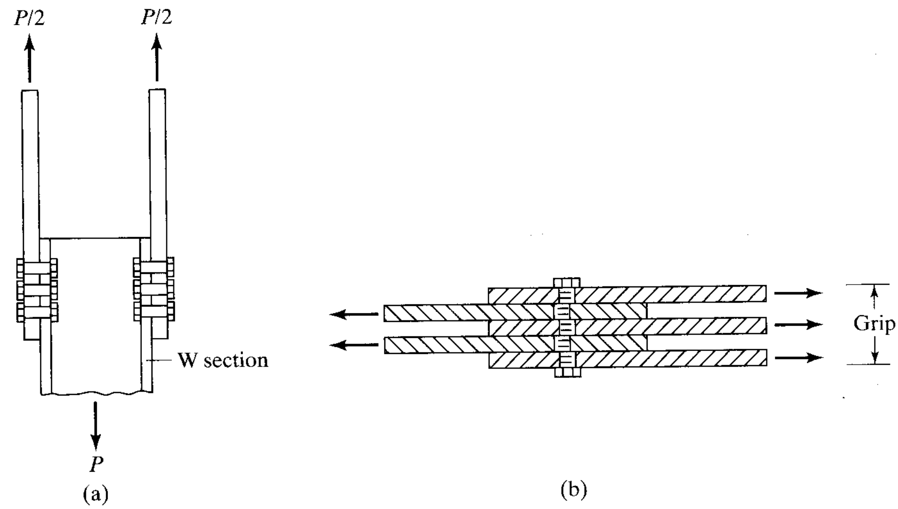


FIGURE 12.2 (a) Hanger connection. (b) Bolts in multiple shear.

12.10.4 Miscellaneous

Bolted connections generally consist of lap or butt joints or some combination of them, but there are other cases. For instance, there are occasionally joints in which more than three members are being connected and the bolts are in multiple shear, as shown in Fig. 12.2(b). In this figure, you can see how the loads are tending to shear this bolt on four separate planes (quadruple shear). Although the bolts in this connection are being sheared on more than two planes, the usual practice is to consider no more than double shear for strength calculations. It seems rather unlikely that shear failures can occur simultaneously on three or more planes. Several other types of bolted connections are discussed in this chapter and the next. These include bolts in tension, bolts in shear and tension, etc.

12.11 FAILURE OF BOLTED JOINTS

Figure 12.3 shows several ways in which failure of bolted joints can occur. To design bolted joints satisfactorily, it is necessary to understand these possibilities. These are described as follows:

1. The possibility of failure in a lap joint by shearing of the bolt on the plane between the members (single shear) is shown in part (a).
2. The possibility of a tension failure of one of the plates through a bolt hole is shown in part (b).
3. A possible failure of the bolts and/or plates by bearing between the two is given in part (c).
4. The possibility of failure due to the shearing out of part of the member is shown in part (d).
5. The possibility of a shear failure of the bolts along two planes (double shear) is shown in part (e).

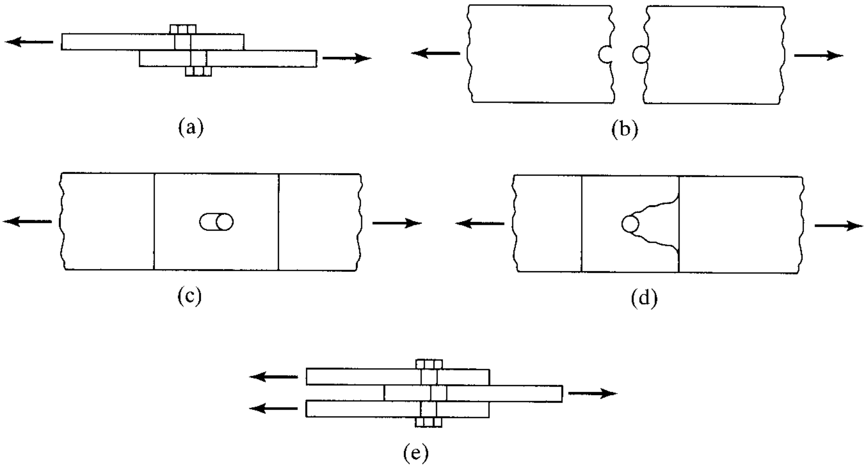


FIGURE 12.3

(a) Failure by single shearing of bolt. (b) Tension failure of plate. (c) Crushing failure of plate. (d) Shear failure of plate behind bolt. (e) Double shear failure of a butt joint.

12.12 SPACING AND EDGE DISTANCES OF BOLTS

Before minimum spacings and edge distances can be discussed, it is necessary for a few terms to be explained. The following definitions are given for a group of bolts in a connection and are shown in Fig. 12.4:

Pitch is the center-to-center distance of bolts in a direction parallel to the axis of the member.

Gage is the center-to-center distance of bolt lines perpendicular to the axis of the member.

The *edge distance* is the distance from the center of a bolt to the adjacent edge of a member.

The *distance between bolts* is the shortest distance between fasteners on the same or different gage lines.

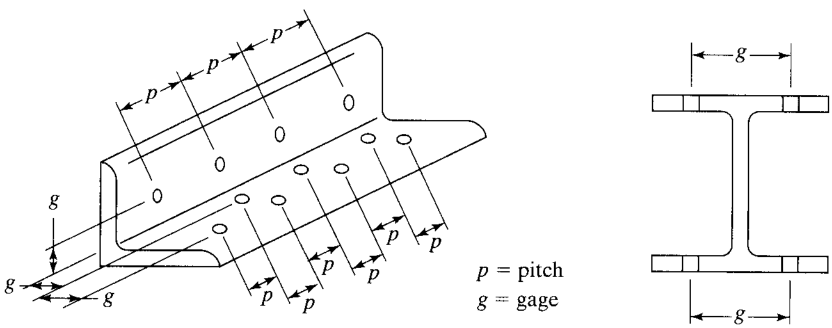


FIGURE 12.4

12.12.1 Minimum Spacings

Bolts should be placed a sufficient distance apart to permit efficient installation and to prevent bearing failures of the members between fasteners. The AISC Specification (J3.3) provides a minimum center-to-center distance for standard, oversized, or slotted fastener holes. For standard, oversized, or slotted holes, the minimum center-to-center distance should not be less than 2 2/3 diameters (with three diameters being preferred). Test results have clearly shown that bearing strengths are directly proportional to the center-to-center spacing up to a maximum of 3d. No additional bearing strength is obtained when spacings greater than 3d are used.

12.12.2 Minimum Edge Distances

Bolts should not be placed too near the edges of a member for two major reasons. First, the punching of holes too close to the edges may cause the steel opposite the hole to bulge out or even crack. The second reason applies to the ends of members where there is danger of the fastener tearing through the metal. The usual practice is to place the fastener a minimum distance from the edge of the plates equal to about 1.5 to 2.0 times the fastener diameter so that the metal there will have a shearing strength at least equal to that of the fasteners. For more exact information, it is necessary to refer to the specification. The AISC Specification (J3.4) states that the distance from the center of a standard hole to the edge of a connected part may not be less than the applicable value given in Table 12.3 or 12.3M. (Tables J3.4 and J3.4M in the Manual.)

The minimum edge distance from the center of an oversized hole or a slotted hole to the edge of a connected part must equal the minimum distance required for a standard hole plus an increment C_2, values of which are provided in Table 12.4 or Table 12.4M. (These tables are, respectively, Tables J3.5 and J3.5M from the AISC

TABLE 12.3 Minimum Edge Distance[a] from Center of Standard Hole[b] to Edge of Connected Part, inches

Bolt Diameter (in)	Minimum Edge Distance (in)
$\frac{1}{2}$	$\frac{3}{4}$
$\frac{5}{8}$	$\frac{7}{8}$
$\frac{3}{4}$	1
$\frac{7}{8}$	$1\frac{1}{8}$
1	$1\frac{1}{4}$
$1\frac{1}{8}$	$1\frac{1}{2}$
$1\frac{1}{4}$	$1\frac{5}{8}$
Over $1\frac{1}{4}$	$1\frac{1}{4} \times$ Diameter

[a] If necessary, lesser edge distances are permitted provided the appropriate provisions from Sections J3.10 and J4 are satisfied, but edge distances less than one bolt diameter are not permitted without approval from the engineer of record.
[b] For oversized or slotted holes, see Table J3.5.

TABLE 12.3M Minimum Edge Distance[a] from Center of Standard Hole[b] to Edge of Connected Part, mm

Bolt Diameter (mm)	Mininmum Edge Distance (mm)
16	22
20	26
22	28
24	30
27	34
30	38
36	46
Over 36	$1.25d$

[a] If necessary, lesser edge distances are permitted provided the appropriate provisions from Sections J3.10 and J4 are satisfied, but edge distances less than one bolt diameter are not permitted without approval from the engineer of record.
[b] For oversized or slotted holes, see Table J3.5M.

Source: American Institute of Steel Construction, *Manual of Steel Construction Load & Resistance Factor Design*, 14th ed. (Chicago: AISC, 2011), Table J3.4 and J3.4M, p. 16.1–123.

TABLE 12.4 Values of Edge Distance Increment C_2, Inches

Normal Diameter of Fastener (in)	Oversized Holes	Slotted Holes: Long Axis Perpendicular to Edge: Short Slots	Slotted Holes: Long Axis Perpendicular to Edge: Long Slots [a]	Long Axis Parallel to Edge
$\leq 7/8$	$1/16$	$1/8$	$3/4d$	0
1	$1/8$	$1/8$		
$\geq 1\,1/8$	$1/8$	$3/16$		

[a] When length of slot is less than maximum allowable (see Table 12.2 herein), C_2 is permitted to be reduced by one-half the difference between the maximum and actual slot lengths.

TABLE 12.4M Values of Edge Distance Increment C_2, mm

Nominal Diameter of Fastener (mm)	Oversized Holes	Slotted Holes: Long Axis Perpendicular to Edge: Short Slots	Slotted Holes: Long Axis Perpendicular to Edge: Long Slots [a]	Long Axis Parallel to Edge
≤ 22	2	3	$0.75d$	0
24	3	3		
≥ 27	3	5		

[a] When length of slot is less than maximum allowable (see Table 12.2M in text), C_2 is permitted to be reduced by one-half the difference between the maximum and actual slot lengths.

Source: American Institute of Steel Construction, *Manual of Steel Construction*, 14th ed. (Chicago: AISC, 2011), Table J3.5 and J3.5M, p. 16.1–124.

Specification.) As will be seen in the pages to follow, the computed bearing strengths of connections will have to be reduced if these requirements are not met.

12.12.3 Maximum Spacing and Edge Distances

Structural steel specifications provide maximum edge distances for bolted connections. The purpose of such requirements is to reduce the chances of moisture getting between the parts. When fasteners are too far from the edges of parts being connected, the edges may sometimes separate, thus permitting the entrance of moisture. When this happens and there is a failure of the paint, corrosion will develop and accumulate, causing increased separations between the parts. The AISC maximum permissible edge distance (J3.5) is 12 times the thickness of the connected part, but not more than 6 in (150 mm).

The maximum edge distances and spacings of bolts used for weathering steel are smaller than they are for regular painted steel subject to corrosion, or for regular unpainted steel not subject to corrosion. One of the requirements for using weathering steel is that it must not be allowed to be constantly in contact with water. For this reason, the AISC Specification tries to insure that the parts of a built-up weathering steel member are connected tightly together at frequent intervals to prevent forming of pockets that might catch and hold water. The AISC Specification (J3.5) states that the maximum spacing of bolts center-to-center for painted members, or for unpainted members not subject to corrosion, is 24 times the thickness of the thinner plate, not to exceed 12 in (305 mm). For unpainted members consisting of weathering steel subject to atmospheric corrosion, the maximum is 14 times the thickness of the thinner plate, not to exceed 7 in (180 mm).

Holes cannot be punched very close to the web-flange junction of a beam or the junction of the legs of an angle. They can be drilled, but this rather expensive practice should not be followed unless there is an unusual situation. Even if the holes are drilled in these locations, there may be considerable difficulty in placing and tightening the bolts in the limited available space.

12.13 BEARING-TYPE CONNECTIONS—LOADS PASSING THROUGH CENTER OF GRAVITY OF CONNECTIONS

12.13.1 Shearing Strength

In bearing-type connections, it is assumed that the loads to be transferred are larger than the frictional resistance caused by tightening the bolts, with the result that the members slip a little on each other, putting the bolts in shear and bearing. The design or LRFD strength of a bolt in single shear equals ϕ times the nominal shearing strength of the bolt in ksi times its cross-sectional area. The allowable ASD strength equals its nominal shearing strength divided by Ω times its cross-sectional area. The LRFD ϕ value is 0.75 for high-strength bolts, while for ASD Ω it is 2.00.

The nominal shear strengths of bolts and rivets are given in Table 12.5 (Table J3.2 in the AISC Specification). For A325 bolts, the values are 54 ksi if threads are not excluded from shear planes and 68 ksi if threads are excluded. (The values are 68 ksi and 84 ksi,

TABLE 12.5 Nominal Strength of Fasteners and Threaded Parts, ksi (MPa)		
Description of Fasteners	Nominal Tensile Strength, F_{nt}, ksi (MPa)[a]	Nominal Shear Strength in Bearing-Type Connections, F_{nv}, ksi (MPa)[b]
A307 bolts	45 (310)	27 (188)[c][d]
Group A (A325 type) bolts, when threads are not excluded from shear planes	90 (620)	54 (372)
Group A (A325 type) bolts, when threads are excluded from shear planes	90 (620)	68 (457)
Group B (A490 type) bolts, when threads are not excluded from shear planes	113 (780)	68 (457)
Group B (A490 type) bolts, when threads are excluded from shear planes	113 (780)	84 (579)
Threaded parts meeting the requirements of Section A3.4 of the Manual, when threads are not excluded from shear planes	$0.75F_u$	$0.450F_u$
Threaded parts meeting the requirements of Section A3.4 of the Manual, when threads are excluded from shear planes	$0.75F_u$	$0.563F_u$

[a] For high-strength bolts subjected to tensile fatigue loading, see Appendix 3.
[b] For end loaded connections with a fastener pattern length greater than 38 in (965 mm), F_{nv} shall be reduced to 83.3 percent of the tabulated values. Fastener pattern length is the maximum distance parallel to the line of force between the centerline of the bolts connecting two parts with one faying surface.
[c] For A307 bolts, the tabulated values shall be reduced by 1 percent for each $\frac{1}{16}$ in (2 mm) over 5 diameters of length in the grip.
[d] Threads permitted in shear planes.

Source: American Institute of Steel Construction, *Manual of Steel Construction*, 14th ed. (Chicago: AISC, 2011), Table J3.2, p. 16.1–120. "Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved."

respectively, for A490 bolts.) Should a bolt be in double shear, its shearing strength is considered to be twice its single shear value.

The student may very well wonder what is done in design practice concerning threads excluded or not excluded from the shear planes. If normal bolt and member sizes are used, the threads will almost always be excluded from the shear plane. It is true, however, that some extremely conservative individuals always assume that the threads are not excluded from the shear plane.

Sometimes the designer needs to use high-strength bolts with diameters larger than those of available A325 and A490 bolts. One example is the use of very large bolts for fastening machine bases. For such situations, AISC Specification A3.3 permits the use of the quenched and tempered A449 bolts. (**Quenching** is the heating of steel to approximately 1650°F, followed by its quick cooling in water, oil, brine, or molten lead. This process produces very strong and hard steels, but, at the same time, steels that are more susceptible to residual stresses. For this reason, after quenching is done, the steel is tempered. **Tempering** is the reheating of steel to a temperature of perhaps 1100° or 1150°F, after which the steel is allowed to air cool. The internal stresses are reduced and the steel is made tougher and more ductile.)

12.13.2 Bearing Strength

The bearing strength of a bolted connection is not, as you might expect, determined from the strength of the bolts themselves; rather, it is based upon the strength of the parts being connected and the arrangement of the bolts. In detail, its computed strength is dependent upon the spacing of the bolts and their edge distances, the specified tensile strength F_u of the connected parts, and the thickness of the connected parts.

Expressions for the nominal bearing strengths (R_n values) at bolt holes are provided in Section J3.10 of the AISC Specification. To determine ϕR_n and $\frac{R_n}{\Omega}$, ϕ is 0.75 and Ω is 2.00. The various expressions listed there include nominal bolt diameters (d), the thicknesses of members bearing against the bolts (t), and the clear distances (l_c) between the edges of holes and the edges of the adjacent holes or edges of the material in the direction of the force. Finally, F_u is the specified minimum tensile strength of the connected material.

To be consistent throughout this text, the author has conservatively assumed that the diameter of a bolt hole equals the bolt diameter, plus 1/8 in. This dimension is used in computing the value of L_c for substituting into the expressions for R_n.

The expressions to follow are used to compute the nominal bearing strengths of bolts used in connections that have standard, oversized, or short-slotted holes, regardless of the direction of loading. They also are applicable to connections with long-slotted holes if the slots are parallel to the direction of the bearing forces.

a. If deformation around bolt holes is a design consideration (that is, if we want deformations to be ≤ 0.25 in), then

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(AISC Equation J3-6a)}$$

For the problems considered in this text, we will normally assume that deformations around the bolt holes are important. Thus, unless specifically stated otherwise, Equation J3-6a will be used for bearing calculations.

If deformation around bolt holes is not a design consideration (that is, if deformations > 0.25 in are acceptable), then

$$R_n = 1.5 l_c t F_u \leq 3.0 d t F_u \qquad \text{(AISC Equation J3-6b)}$$

b. For bolts used in connections with long-slotted holes, the slots being perpendicular to the forces,

$$R_n = 1.0\,l_c t F_u \leq 2.0 d t F_u \qquad \text{(AISC Equation J3-6c)}$$

As described in Section 12.9 of this chapter, oversized holes cannot be used in bearing connections, but short-slotted holes can be used in bearing-type connections—if the loads are perpendicular to the long directions of the slots.

Tests of bolted joints have shown that neither the bolts nor the metal in contact with the bolts actually fail in bearing. However, these tests also have shown that the efficiency of the connected parts in tension and compression is affected by the magnitude of the bearing stress. Therefore, the nominal bearing strengths given by the AISC Specification are values above which they feel the strength of the connected parts is impaired. In other words, these apparently very high design bearing stresses are not really bearing stresses at all, but, rather, indexes of the efficiencies of the connected parts. If bearing stresses larger than the values given are permitted, the holes seem to elongate more than about 1/4 in and impair the strength of the connections.

From the preceding, we can see that the bearing strengths given are not specified to protect fasteners from bearing failures, because they do not need such protection. Thus, the same bearing values will be used for a particular joint, regardless of the grades of bolts used and regardless of the presence or absence of bolt threads in the bearing area.

12.13.3 Minimum Connection Strength

Example 12-1 illustrates the calculations involved in determining the strength of the bearing-type connection shown in Fig. 12.5. By a similar procedure, the number of bolts required for a certain loading condition is calculated in Example 12-2. In each case, the bearing thickness to be used equals the smaller total thickness on one side or the other, since the steel grade for all plates is the same and since the edge distances are identical for all plates. For instance, in Fig. 12.6, the bearing thickness equals the smaller of $2 \times 1/2$ in on the left or 3/4 in on the right.

In the connection tables of the AISC Manual and in various bolt literature, we constantly see abbreviations used when referring to various types of bolts. For instance, we may see A325-SC, A325-N, A325-X, A490-SC, and so on. These are used to represent the following:

A325-SC—slip-critical or fully tensioned A325 bolts

A325-N—snug-tight or bearing A325 bolts with threads *included* in the shear planes

A325-X—snug-tight or bearing A325 bolts with threads *excluded* from the shear planes

---

Example 12-1

Determine the design strength $\phi_c P_n$ and the allowable strength $\dfrac{P_n}{\Omega}$ for the bearing-type connection shown in Fig. 12.5. The steel is A36($F_y = 36$ ksi and $F_u = 58$ ksi), the bolts

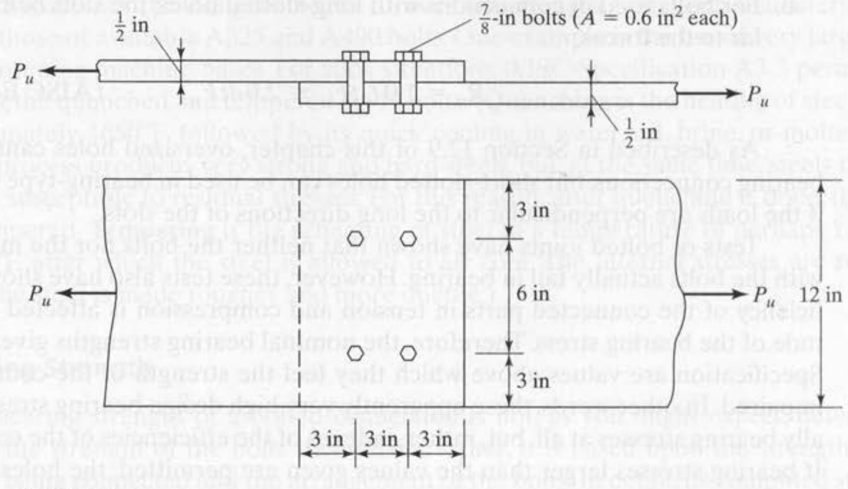


FIGURE 12.5

are 7/8-in A325, the holes are standard sizes, and the threads are excluded from the shear plane. Assume that deformations at bolt holes are a design consideration.

Solution

(a) Gross section yielding of plates

$$P_n = F_y A_g = (36 \text{ ksi})\left(\frac{1}{2} \text{ in} \times 12 \text{ in}\right) = 216 \text{ k}$$

LRFD $\phi_t = 0.9$	ASD $\Omega_t = 1.67$
$\phi_{tn} = (0.9)(216) = 194.4$ k	$\dfrac{P_n}{\Omega} = \dfrac{216}{1.67} = 129.3$ k

(b) Tensile rupture strength of plates

$$A_n = 6.00 \text{ in}^2 - (2)\left(\frac{7}{8} \text{ in} + \frac{1}{8} \text{ in}\right)\left(\frac{1}{2} \text{ in}\right) = 5.00 \text{ in}^2$$

$U = 1.0$ as all parts connected

$$A_e = UA_n = (1.00)(5.00) = 5.00 \text{ in}^2 < 0.85A_g$$
$$= (0.85)(6.00) = 5.10 \text{ in}^2 \text{ as per AISC Spec. J4.1}$$
$$P_n = F_u A_e = (58 \text{ ksi})(5.00 \text{ in}^2) = 290 \text{ k}$$

LRFD $\phi_t = 0.75$	ASD $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(290) = 217.5$ k	$\dfrac{P_n}{\Omega_t} = \dfrac{290}{2.00} = 145$ k

(c) Bearing strength of bolts

$$l_c = \text{lesser of } 3 - \frac{1}{2} \text{ or } 3 - 1 = 2.00 \text{ in}$$

$$R_n = 1.2 l_c t F_u (\text{No. of bolts}) \leq 2.4 dt F_u (\text{No. of bolts})$$

$$= (1.2)(2.00 \text{ in})\left(\frac{1}{2} \text{ in}\right)(58 \text{ ksi})(4)$$

$$= 278.4 \text{ k} > (2.4)\left(\frac{7}{8} \text{ in}\right)\left(\frac{1}{2} \text{ in}\right)(58 \text{ ksi})(4) = 243.6 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(243.6) = 182.7$ k	$\dfrac{R_n}{\Omega} = \dfrac{243.6}{2.00} = 121.8$ k

(d) Shearing strength of bolts

$$R_n = F_{nv} A_b (\text{No. of bolts}) = (68 \text{ ksi})(0.6 \text{ in}^2)(4) = 163.2 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(163.2) = 122.4$ k	$\dfrac{R_n}{\Omega} = \dfrac{163.2 \text{ k}}{2.00} = 81.6$ k
LRFD = 122.4 k (controls)	**ASD = 81.6 k (controls)**

Example 12-2

How many 3/4-in A325 bolts in standard-size holes with threads excluded from the shear plane are required for the bearing-type connection shown in Fig. 12.6? Use $F_u = 58$ ksi and assume edge distances to be 2 in and the distance center-to-center of holes to be 3 in. Assume that deformation at bolt holes is a design consideration. $P_u = 345$ k (LRFD). $P_a = 230$ k (ASD).

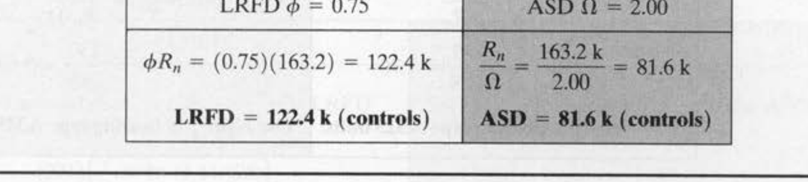


FIGURE 12.6

Solution. Bolts in double shear and bearing on 3/4 in

Bearing strength of 1 bolt

$$L_c = \text{lesser of } 2 - \frac{\frac{3}{4} + \frac{1}{8}}{2} = 1.56 \text{ in} \quad \text{or} \quad 3 - (2)\left(\frac{\frac{3}{4} + \frac{1}{8}}{2}\right) = 2.125 \text{ in}$$

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u$$

$$= (1.2)(1.56 \text{ in})\left(\frac{3}{4} \text{ in}\right)(58 \text{ ksi}) = 81.4 \text{ k} > (2.4)\left(\frac{3}{4} \text{ in}\right)\left(\frac{3}{4} \text{ in}\right)(58 \text{ ksi}) = 78.3 \text{ k}$$

Shearing strength of 1 bolt

$$R_n = (2 \times 0.44 \text{ in}^2)(68 \text{ ksi}) = 59.8 \text{ k} \leftarrow \text{controls}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(59.8) = 44.8$ k	$\frac{R_n}{\Omega} = \frac{59.8}{2.00} = 29.9$ k
No. of bolts reqd. $= \frac{P_u}{\phi R_n}$	No. of bolts reqd. $= \frac{P_a}{R_n/\Omega}$
$= \frac{345}{44.8} = 7.70$	$= \frac{230}{29.9} = 7.69$
Use eight $\frac{3}{4}$-in bearing type A325 bolts.	**Use eight $\frac{3}{4}$-in bearing type A325 bolts.**

Where cover plates are bolted to the flanges of W sections, the bolts must carry the longitudinal shear on the plane between the plates and the flanges. With reference to the cover-plated beam of Fig. 12.7, the unit longitudinal shearing stress to be resisted between a cover plate and the W flange can be determined with the expression $f_v = VQ/Ib$. The total shear force across the flange for a 1-in length of the beam equals $(b)(1.0)(VQ/Ib) = VQ/I$.

The AISC Specification (E6.2) provides a maximum permissible spacing for bolts used in the outside plates of built-up members. It equals the thinner outside plate thickness times $0.75\sqrt{E/F_y}$ and may not be larger than 12 in.

The spacing of pairs of bolts at a particular section in Fig. 12.7 can be determined by dividing the LRFD design shear strength of two bolts by the factored shear per in or by dividing the ASD allowable shear design strength of 2 bolts by the service shear per inch. Theoretically, the spacings will vary as the external shear varies along the span. Example 12-3 illustrates the calculations involved in determining bolt spacing for a cover-plated beam.

The reader should note that AISC Specification F13.3 states that the total cross-sectional area of the cover plates of a bolted girder may not be greater than 70 percent of the total flange area.

Example 12-3

At a certain section in the cover-plated beam of Fig. 12.7, the external factored shears are $V_u = 275$ k and $V_a = 190$ k. Determine the spacing required for 7/8-in A325 bolts used in a bearing-type connection. Assume that the bolt threads are excluded from the shear plane, the edge distance is 3.5 in, $F_y = 50$ ksi, and $F_u = 65$ ksi. Deformation at bolt holes is a design consideration.

Solution

Checking AISC Specification F13.3

$$\text{A of 1 cover plate} = \left(\frac{3}{4}\right)(16) = 12.00 \text{ in}^2$$

$$\text{A of 1 flange} = 12.00 + (12.5)(1.15) = 26.38 \text{ in}^2$$

$$\text{Plate area} \div \text{flange area} = \frac{12.00}{26.38} < 0.70 \qquad \text{(OK)}$$

Computing shearing force to be taken

$$I_g = 3630 + (2)\left(\frac{3}{4} \times 16\right)\left(\frac{22.1}{2} + \frac{0.75}{2}\right)^2 = 6760 \text{ in}^4$$

LRFD	ASD
Factored shear per in $= \dfrac{V_u Q}{I}$ for LRFD	Service load shear per in $= \dfrac{V_a Q}{I}$ for ASD
$= \dfrac{(275)\left(\frac{3}{4} \times 16 \times 11.425\right)}{6760} = 5.578$ k/in	$= \dfrac{(190)\left(\frac{3}{4} \times 16 \times 11.425\right)}{6760} = 3.853$ k/in

Bolts in single shear and bearing on 0.75 in

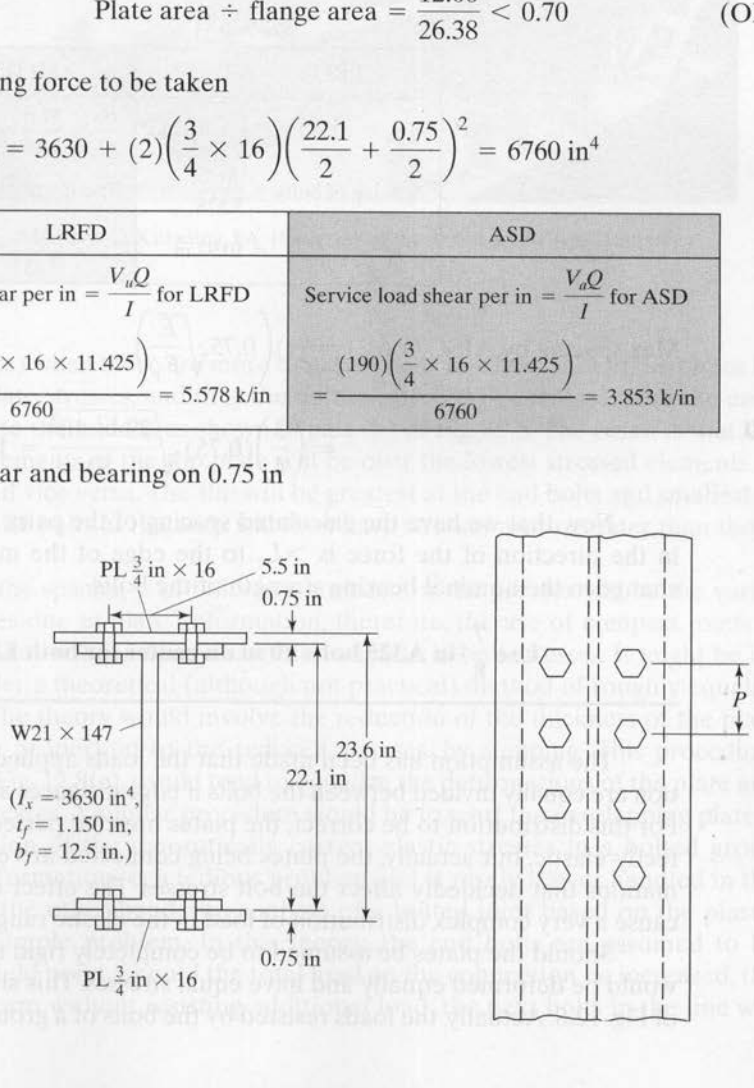


FIGURE 12.7

Bearing strength of 2 bolts

$$L_c = 3.5 - \frac{\frac{7}{8} + \frac{1}{8}}{2} = 3.0 \text{ in}$$

$$R_n = 1.2 L_c t F_u \le 2.4 d t F_u$$

$$= (2)(1.2)(3.0 \text{ in})\left(\frac{3}{4} \text{ in}\right)(65 \text{ ksi})$$

$$= 351 \text{ k} > (2)(2.4)\left(\frac{7}{8} \text{ in}\right)\left(\frac{3}{4} \text{ in}\right)(65 \text{ ksi}) = 204.8 \text{ k}$$

Shearing strength of 2 bolts

$A = 0.60 \text{ in}^2$ each bolt

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(81.6) = 61.2 \text{ k}$	$\frac{R_n}{\Omega} = \frac{81.6}{2.00} = 40.8 \text{ k}$
Spacing of bolts $= \frac{61.2}{5.578}$	Spacing reqd. for bolts
$= 10.97$ in	$= \frac{40.8}{3.853} = 10.59$ in

$$\text{Max spacing by AISC (E6.2)} = (t)\left(0.75\sqrt{\frac{E}{F_y}}\right)$$

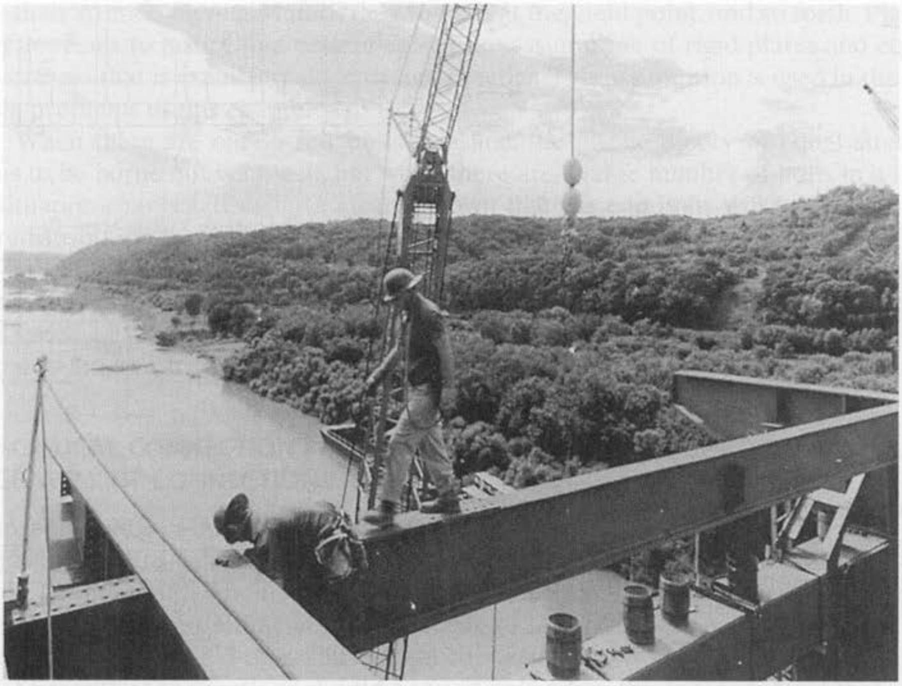
$$= \left(\frac{3}{4}\right)(0.75)\sqrt{\frac{29 \times 10^3}{50}} = 13.55 \text{ in} \le 12 \text{ in}$$

Now that we have the calculated spacing of the pairs of bolts, we can see that L_c in the direction of the force is $>L_c$ to the edge of the member. ∴ there will be no change in the nominal bearing strength of the bolts.

Use $\frac{7}{8}$-in A325 bolts 10 in on center for both LRFD and ASD

The assumption has been made that the loads applied to a bearing-type connection are equally divided between the bolts if edge distances and spacings are satisfactory. For this distribution to be correct, the plates must be perfectly rigid and the bolts perfectly elastic, but actually, the plates being connected are elastic, too, and have deformations that decidedly affect the bolt stresses. The effect of these deformations is to cause a very complex distribution of load in the elastic range.

Should the plates be assumed to be completely rigid and nondeforming, all bolts would be deformed equally and have equal stresses. This situation is shown in part (a) of Fig. 12.8. Actually, the loads resisted by the bolts of a group are probably never equal



Bridge over Allegheny River at Kittaning, PA. (Courtesy of the American Bridge Company.)

(in the elastic range) when there are more than two bolts in a line. Should the plates be deformable, the plate stresses, and thus the deformations, will decrease from the ends of the connection to the middle, as shown in part (b) of Fig. 12.8. The result is that the highest stressed elements of the top plate will be over the lowest stressed elements of the lower plate, and vice versa. The slip will be greatest at the end bolts and smallest at the middle bolts. The bolts at the ends will then have stresses much greater than those in the inside bolts.

The greater the spacing is of bolts in a connection, the greater will be the variation in bolt stresses due to plate deformation; therefore, the use of compact joints is very desirable, as they will tend to reduce the variation in bolt stresses. It might be interesting to consider a theoretical (although not practical) method of roughly equalizing bolt stresses. The theory would involve the reduction of the thickness of the plate toward its end, in proportion to the reduced stresses, by stepping. This procedure, which is shown in Fig. 12.8(c), would tend to equalize the deformations of the plate and thus of the bolt stresses. A similar procedure would be to scarf the overlapping plates.

The calculation of the theoretically correct elastic stresses in a bolted group based on plate deformations is a tedious problem and is rarely if ever handled in the design office. On the other hand, the analysis of a bolted joint based on the plastic theory is a very simple problem. In this theory, the end bolts are assumed to be stressed to their yield point. Should the total load on the connection be increased, the end bolts will deform without resisting additional load, the next bolts in the line will

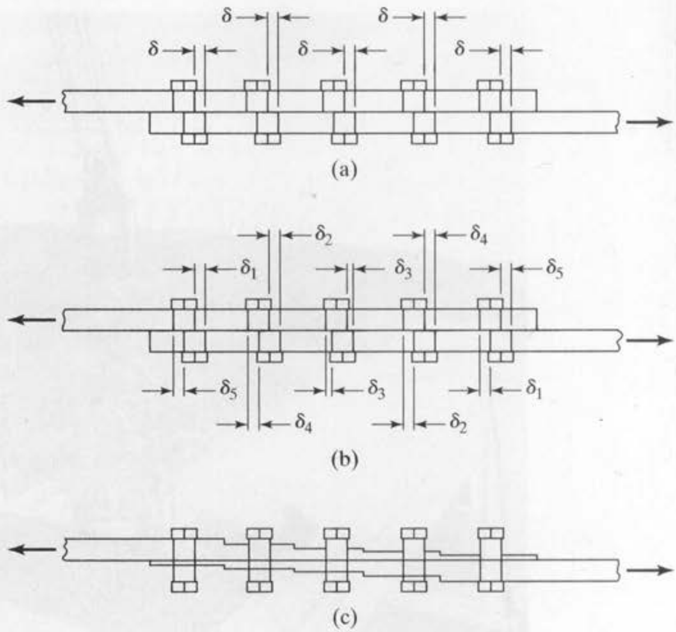


FIGURE 12.8

(a) Assuming nondeforming plates.
(b) Assuming deformable plates.
(c) Stepped joint (impractical).

Three high-strength bolted structures in Constitution Plaza Complex, Hartford, CT, using approximately 195,000 bolts. (Courtesy of Bethlehem Steel Corporation.)

have their stresses increased until they too are at the yield point, and so forth. Plastic analysis seems to justify to a certain extent the assumption of rigid plates and equal bolt stresses that is usually made in design practice. This assumption is used in the example problems of this chapter.

When there are only a few bolts in a line, the plastic theory of equal stresses seems to be borne out very well, but when there are a large number of bolts in a line, the situation changes. Tests have clearly shown that the end bolts will fail before the full redistribution takes place.[4]

For load-carrying bolted joints, it is common for specifications to require a minimum of two or three fasteners. The feeling is that a single connector may fail to live up to its specified strength because of improper installation, material weakness, etc., but if several fasteners are used, the effects of one bad fastener in the group will be overcome.

12.14 SLIP-CRITICAL CONNECTIONS—LOADS PASSING THROUGH CENTER OF GRAVITY OF CONNECTIONS

Almost all bolted connections with standard size holes are designed as bearing-type connections. On some occasions, however, particularly in bridges, it is felt that slipping should be prevented. High-strength bolted connections may be designed such that slipping is prevented either at the service load limit state or at the strength limit state. These are referred to as slip-critical connections.

Slip-critical connections should be used only when the engineer feels that slipping will adversely affect the serviceability of a structure. For such a structure, slipping may cause excessive distortion of the structure or a reduction in strength or stability, even if the strength of the connection is adequate. For one example, it is felt necessary to use slip-critical connections when oversized holes are used or when slots parallel to the load direction of force are planned. Other situations where slip-critical bolts are desirable are described in Section J3.8 of the Commentary to the AISC Specification.

If bolts are tightened to their required tensions for slip-critical connections (see Tables 12.1 and 12.1M), there is very little chance of their bearing against the plates that they are connecting. In fact, tests show that there is very little chance of slip occurring unless there is a calculated shear of at least 50 percent of the total bolt tension. As we have said all along, this means that slip-critical bolts are not stressed in shear; however, AISC Specification J3.8 provides *design shear strengths* (they are really design friction values on the faying surfaces) so that the designer can handle the connections in just about the same manner he or she handles bearing-type connections.

Although there is little or no bearing on the bolts used in slip-critical connections, the AISC in its Section J3.10 states that bearing strength is to be checked for both bearing-type and slip-critical connections because a possibility still exists that slippage could occur; therefore, the connection must have sufficient strength as a bearing-type connection.

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[4]*Trans.* ASCE 105 (1940), p. 1193.

AISC Specification J3.8 states that the nominal slip resistance of a connection (R_n) shall be determined with the expression

$$R_n = \mu D_u h_f T_b n_s \qquad \text{(AISC Equation J3-4)}$$

in which

μ = mean slip coefficient = 0.30 for Class A faying surfaces and 0.5 for Class B faying surfaces. Section 3 of Part 16.2 of the AISC Manual provides detailed information concerning these two surfaces. Briefly, Class A denotes unpainted clean, mill scale surfaces or surfaces with Class A coatings on blast-cleaned steel surfaces. Class B surfaces are unpainted blast-cleaned steel surfaces or surfaces with Class B coatings

D_u = 1.13. This is a multiplier that gives the ratio of the mean installed pretension to the specified minimum pretension given in Table 12.1 of this text (Table J3.1 in the AISC Specification)

h_f = factor for fillers, determined as follows:

(1) Where bolts have been added to distribute loads in the filler, $h_f = 1.0$

(2) Where bolts have not been added to distribute the load in the filler,

(i) For one filler between connected parts, $h_f = 1.0$

(ii) For two or more fillers between connected parts, $h_f = 0.85$

T_b = minimum fastener tension, as given in Table 12.1 of this text

n_s = number of slip planes

For standard size and short-slotted holes perpendicular to the direction of the load

$$\phi = 1.00 \text{ (LRFD)} \qquad \Omega = 1.50 \text{ (ASD)}$$

For oversized and short-slotted holes parallel to the direction of the load

$$\phi = 0.85 \text{ (LRFD)} \qquad \Omega = 1.76 \text{ (ASD)}$$

For long-slotted holes

$$\phi = 0.70 \text{ (LRFD)} \qquad \Omega = 2.14 \text{ (ASD)}$$

It is permissible to introduce finger shims up to 1/4-in thick into slip-critical connections with standard holes without the necessity of reducing the bolt design strength values to those specified for slotted holes (AISC Specification Section J3.2).

The preceding discussion concerning slip-critical joints does not present the whole story, because during erection the joints may be assembled with bolts, and as the members are erected their weights will often push the bolts against the side of the holes before they are tightened and put them in some bearing and shear.

The majority of bolted connections made with standard-size holes can be designed as bearing-type connections without the need to worry about serviceability. Furthermore, if connections are made with three or more bolts in standard-size holes or are used with slots perpendicular to the force direction, slip probably cannot occur, because at least one and perhaps more of the bolts will be in bearing before the external loads are applied.

Sometimes, bolts are used in situations where deformations can cause increasing loads that may be larger than the strength limit states. These situations may occur in connections that make use of oversize holes or slotted holes which are parallel to the load.

When a slip-critical connection is being used with standard or short-slotted holes perpendicular to the direction of the load, the slip will not result in an increased load and a $\phi = 1.0$ or $\Omega = 1.5$ is used. For oversized and short-slotted holes parallel to the direction of load which could result in an increased load situation, a value of $\phi = 0.85$ or $\Omega = 1.76$ is used. Similarly, for long-slotted holes a value of $\phi = 0.70$ or $\Omega = 2.14$ is applicable.

Example 12-4 that follows illustrates the design of slip-critical bolts for a lap joint. The example presents the determination of the number of bolts required for the limit state of slip.

Example 12-4

For the lap joint shown in Fig. 12.9, the axial service loads are $P_D = 27.5$ k and $P_L = 40$ k. Determine the number of 1-in A325 slip-critical bolts in standard-size holes needed for the limit state of slip if the faying surface is Class A. The edge distance is 1.75 in, and the c. to c. spacing of the bolts is 3 in. $F_y = 50$ ksi. $F_u = 65$ ksi.

Solution

Loads to be resisted

LRFD	ASD
$P_u = (1.2)(27.5) + (1.6)(40) = 97$ k	$P_a = 27.5 + 40 = 67.5$ k

Nominal strength of 1 bolt

$R_n = \mu D_u h_f T_b n_s$ (AISC Equation J3-4)

$\mu = 0.30$ for Class A surface

$D_u = 1.13$ multiplier

$h_f = 1.00$ factor for filler

$T_b = 51$ k minimum bolt pretension (AISC Table J3.1)

$n_s = 1.0 =$ number of slip planes

$R_n = (0.30)(1.13)(1.00)(51\text{ k})(1.00) = 17.29$ k/bolt

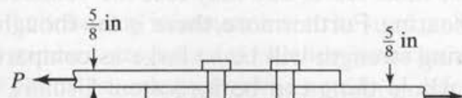

FIGURE 12.9

(a) Slip-critical design to prevent slip

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$\phi R_n = (1.00)(17.29) = 17.29$ k	$\frac{R_n}{\Omega} = \frac{17.29}{1.50} = 11.53$ k
No. of bolts reqd.	No. of bolts reqd.
$= \frac{97}{17.29} = 5.61$	$= \frac{67.5}{11.53} = 5.86$
Use 6 bolts.	**Use 6 bolts.**

Bearing strength of 6 bolts

$$L_c = \text{lesser of } 3 - \left(1 + \frac{1}{8}\right) = 1.875 \text{ in or } 1.75 - \frac{1 + \frac{1}{8}}{2} = 1.187 \text{ in}$$

$$\begin{aligned}\text{Total } R_n &= (6)(1.5l_c tF_u) \leq (6)(2.4dtF_u)\\ &= (6)\left(1.5 \times 1.187 \text{ in} \times \frac{5}{8} \text{ in} \times 65 \text{ ksi}\right)\\ &= 434 \text{ k} < (6)\left(2.4 \times 1.00 \text{ in} \times \frac{5}{8} \text{ in} \times 65 \text{ ksi}\right) = 585 \text{ k}\end{aligned}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(434) = 326 \text{ k} > 97$ k OK	$\frac{R_n}{\Omega} = \frac{434}{2.00} = 217 \text{ k} > 67.5$ k OK

Shearing strength of 6 bolts (single shear)

$$\text{Total } R_n = 6F_{nv}A_b = (6)(68 \text{ ksi})(0.785 \text{ in}^2) = 320.3 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(320.3) = 240.2 \text{ k} > 97$ k OK	$\frac{R_n}{\Omega} = \frac{320.3}{2.00} = 160.2 \text{ k} > 67.5$ k OK
Use 6 bolts.	**Use 6 bolts.**

The reader may think that bearing strength checks for slip-critical connections are a waste of time. He or she may feel the connections are not going to slip and put the bolts in bearing. Furthermore, there is the thought that if slip does occur, the calculated bolt bearing strength will be so large as compared to the calculated shearing strength that the whole thing can be forgotten. Usually, these thoughts are correct, but if for some reason a connection is made with very thin parts, bearing may very well control.

Example 12-5

Repeat Example 12-4 if the plates have long-slotted holes in the direction of the load. Assume that deformations of the connections will cause an increase in the critical load. Therefore, design the connection to prevent slip at the limit state of slip.

Solution

$P_u = 97$ k and $P_a = 67.5$ k from Example 12-4 solution.

Nominal strength of 1 bolt

$$R_n = \mu D_u h_f T_b n_s$$

$$\mu = 0.30 \text{ for Class A surface}$$

$$D_u = 1.13 \text{ multiplier}$$

$$h_f = 1.00 \text{ factor for filler}$$

$$T_b = 51 \text{ k minimum bolt pretension}$$

$$n_s = 1.0 = \text{number of slip planes}$$

$$R_n = (0.30)(1.13)(1.0)(51)(1.0) = 17.29 \text{ k/bolt}$$

Number of bolts required for long-slotted holes

LRFD $\phi = 0.70$	ASD $\Omega = 2.14$
$\phi R_n = (0.70)(17.29) = 12.10$ k	$\dfrac{R_n}{\Omega} = \dfrac{17.29}{2.14} = 8.08$ k
No. reqd. $= \dfrac{97}{12.10} = 8.02$ bolts	No. reqd. $= \dfrac{67.5}{8.08} = 8.35$ bolts

Note: Shear and bearing were checked in Example 12-4 and are obviously ok here as they are higher than they were before.

Ans. **Use 9 bolts.** **Use 9 bolts.**

12.15 PROBLEMS FOR SOLUTION

For each of the problems listed, the following information is to be used, unless otherwise indicated (a) AISC Specification; (b) standard-size holes; (c) members have clean mill-scale surfaces (Class A); (d) $F_y = 36$ ksi and $F_u = 58$ ksi unless otherwise noted, (e) deformation at service loads is a design consideration. Do not consider block shear, unless specifically requested.

12-1 to 12-5. *Determine the LRFD design tensile strength and the ASD allowable tensile strength for the member shown, assuming a bearing-type connection.*

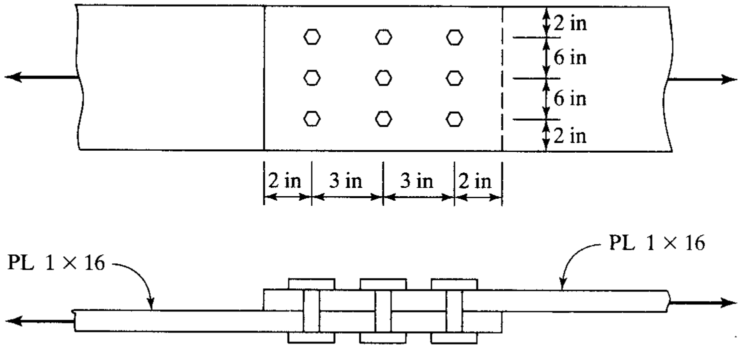

FIGURE P12-1 to 12-5

12-1. A325 $\frac{3}{4}$-in bolts, threads excluded from shear plane. (*Ans.* 202.0 k, 134.7 k)

12-2. A325 1-in bolts, threads excluded from shear plane.

12-3. A490 1-in bolts, threads not excluded from shear plane. (*Ans.* 281.6 k, 190.7 k)

12-4. $\frac{7}{8}$-in A325 bolts, threads excluded from shear plane.

12-5. $\frac{3}{4}$-in A490 bolts, threads not excluded from shear plane. (*Ans.* 202.0 k, 134.7 k)

12-6 to 12-10. *Determine the LRFD design tensile strength and the ASD allowable tensile strength for the member and the bearing-type connections.*

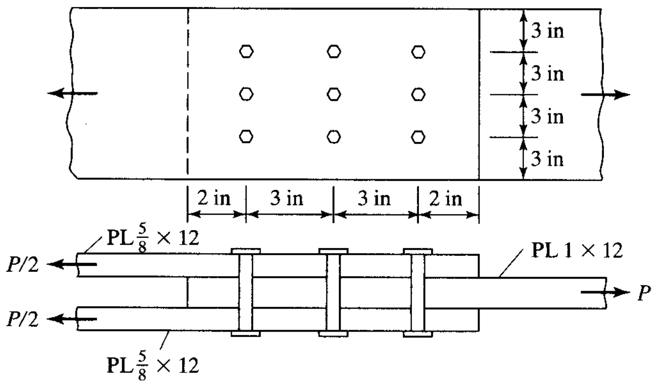

FIGURE P12-6 to 12-10

12-6. A325 $\frac{3}{4}$-in bolts, threads excluded from shear planes.

12-7. A490 $\frac{7}{8}$-in bolts, threads not excluded from shear planes. (*Ans.* 388.8 k, 258.7 k)

12-8. A490 $\frac{3}{4}$-in bolts, threads excluded from shear plane.

12-9. A steel with F_y = 50 ksi, F_u = 70 ksi, $\frac{7}{8}$-in A490 bolts, threads excluded from shear plane. (*Ans.* 472.5 k, 315 k)

12-10. A steel with F_y = 50 ksi, F_u = 70 ksi, 1-in A490 bolts, threads excluded from shear planes.

12-11 to 12-13. *How many bolts are required for LRFD and ASD for the bearing-type connection shown, if $P_D = 50$ k and $P_L = 100$ k?*

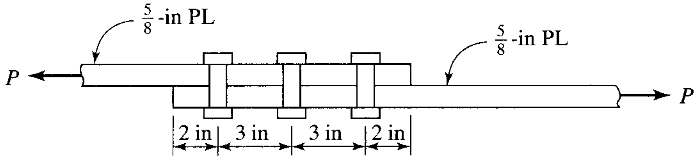

FIGURE P12-11 to 12-13

12-11. A325 $\frac{3}{4}$-in bolts, threads excluded from shear plane. (*Ans.* 10 both LRFD and ASD)
12-12. $F_y = 50$ ksi, $F_u = 70$ ksi, $\frac{3}{4}$-in A325 bolts, threads excluded from shear plane.
12-13. A490 1-in bolts, threads not excluded from shear plane. (*Ans.* 6 both LRFD and ASD)

12-14 to 12-16. *How many bolts are required (LRFD and ASD) for the bearing-type connection shown if $P_D = 120$ k and $P_L = 150$ k?*

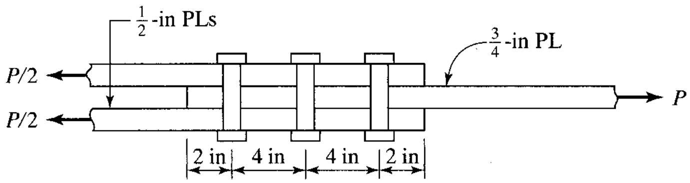

FIGURE P12-14 to 12-16

12-14. A325 $\frac{7}{8}$-in bolts, threads excluded from shear planes.
12-15. A490 $\frac{3}{4}$-in bolts, threads not excluded from shear planes. (*Ans.* 9 or 10 both LRFD and ASD)
12-16. A325 1-in bolts, threads not excluded from shear planes.
12-17. The truss member shown in the accompanying illustration consists of two C12 × 25s (A36 steel) connected to a 1-in gusset plate. How many $\frac{7}{8}$-in A325 bolts (threads excluded from shear plane) are required to develop the full design tensile capacity of the member if it is used as a bearing-type connection? Assume $U = 0.85$. Use both LRFD and ASD methods. (*Ans.* 8 both LRFD and ASD)

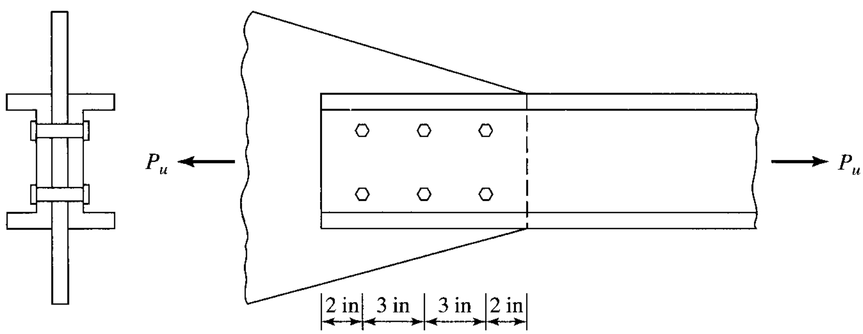

FIGURE P12-17

12-18. Repeat Prob. 12-17, using $\frac{3}{4}$-in A490 bolts (threads excluded).

12-19. Rework Prob. 12-17, if $\frac{7}{8}$-in A490 bolts are used (threads not excluded). $F_y = 50$ ksi and $F_u = 65$ ksi. (*Ans.* 9 both LRFD and ASD)

12-20. For the connection shown in the accompanying illustration, $P_u = 360$ k and $P_a = 260$ k. Determine by LRFD and ASD the number of 1-in A325 bolts required for a bearing-type connection, using A36 steel. Threads are excluded from shear plane.

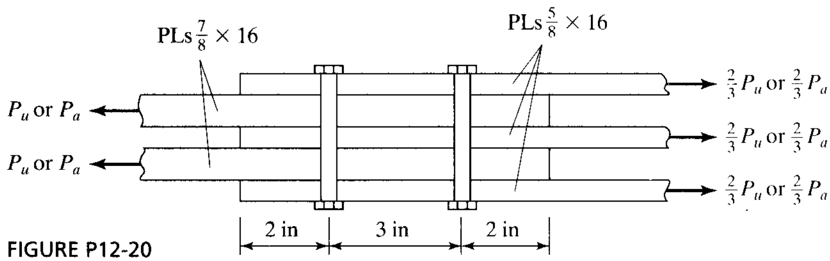

FIGURE P12-20

12-21. Rework Prob. 12-20, using $\frac{7}{8}$-in A490 bolts (threads not excluded from shear plane). (*Ans.* 12 LRFD and 13 or 14 ASD)

12-22. How many $\frac{3}{4}$-in A490 bolts (threads excluded from shear planes) in a bearing-type connection are required to develop the design tensile strength of the member shown? Assume that A36 steel is used and that there are two lines of bolts in each flange (at least three in a line 4 in o.c.). LRFD and ASD. Do not consider block shear.

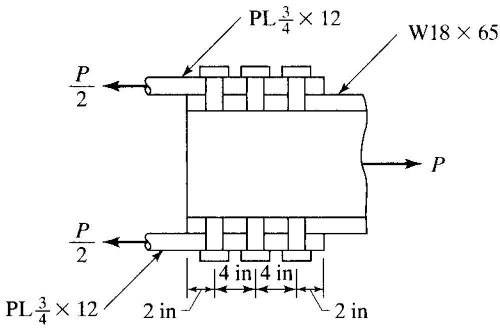

FIGURE P12-22

12-23. For the beam shown in the accompanying illustration, what is the required spacing of $\frac{3}{4}$-in A490 bolts (threads not excluded from shear plane) in a bearing-type

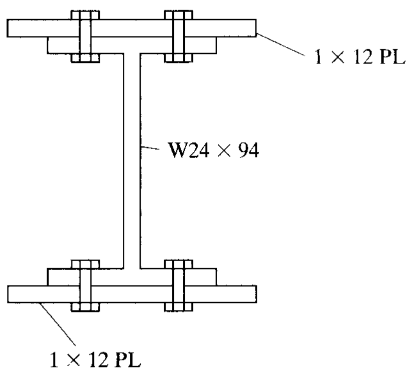

FIGURE P12-23

connection at a section where the external shear $V_D = 80$ k and $V_L = 160$ k? LRFD and ASD. Assume $L_c = 1.50$ in. (*Ans.* 8 in both LRFD and ASD)

12-24. The cover-plated section shown in the accompanying illustration is used to support a uniform load $w_D = 10$ k/ft (includes beam weight effect and $w_L = 12.5$ k/ft for an 24-ft simple span). If $\frac{7}{8}$-in A325 bolts (threads excluded) are used in a bearing-type connection, work out a spacing diagram for the entire span, for LRFD only.

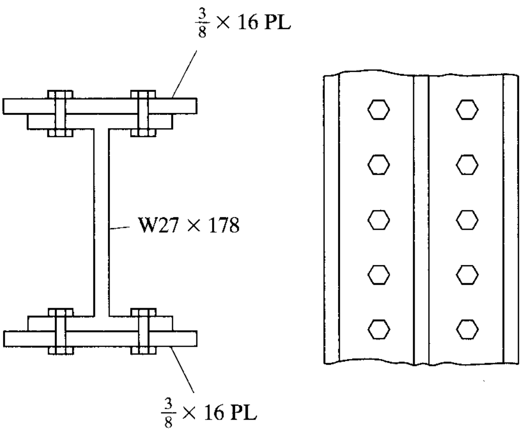

FIGURE P12-24

12-25. For the section shown in the accompanying illustration, determine, for ASD only, the required spacing of $\frac{7}{8}$-in A490 bolts (threads excluded) for a bearing-type connection if the member consists of A572 grade 60 steel ($F_u = 75$ ksi). $V_D = 100$ k and $V_L = 140$ k. Assume Class A surfaces and $L_c = 1.0$ in (*Ans.* 6 in both)

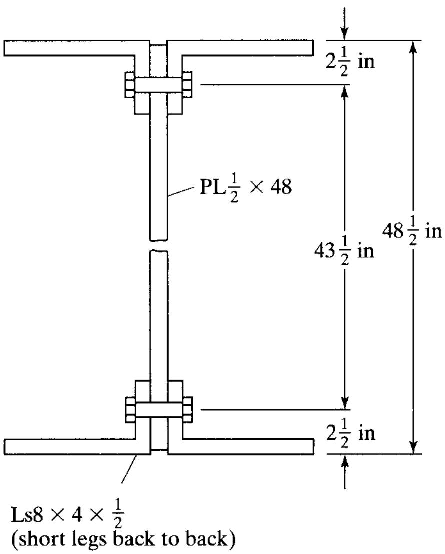

FIGURE P12-25

12-26. For an external shear V_u of 600 k, determine by LRFD the spacing required for 1-in A325 web bolts (threads excluded) in a bearing-type connection for the built-up section shown in the accompanying illustration. Assume that $l_c = 1.5$ in and A36 steel.

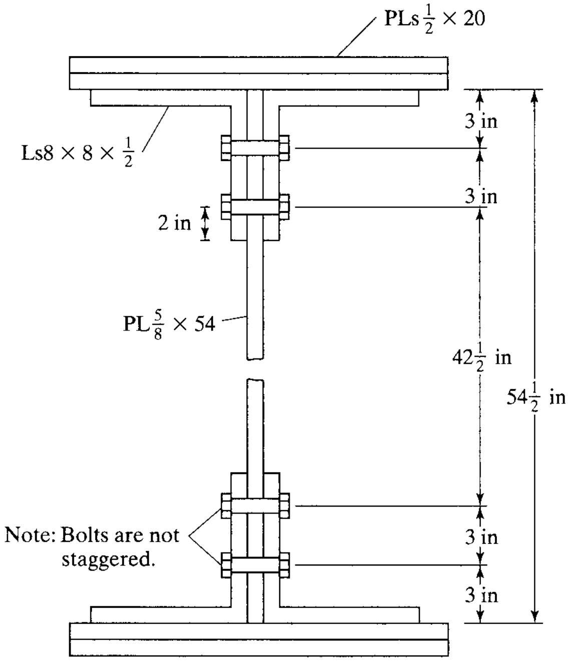

FIGURE P12-26

12-27. Determine the design strength P_u and the allowable strength P_a for the connection shown if $\frac{7}{8}$-in A325 bolts (threads excluded) are used in a slip-critical connection with a factor for fillers, $h_f = 1.0$. Assume A36 steel and Class B faying surface and standard size holes. (*Ans.* 132.2 k, 88.1 k)

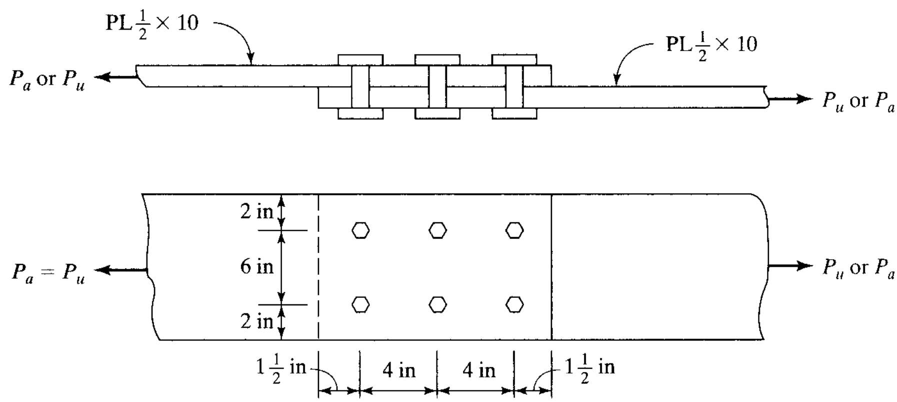

FIGURE P12-27

12-28 to 12-33. *Repeat these problems, using the loads given and determine the number of bolts required for a slip-critical connection. Assume Class A surfaces, standard-size holes, h_f = 1.00, and l_c values of 1.50 in*, UNO.

12-28. Prob. 12-6. $P_D = 100$ k, $P_L = 150$ k

12-29. Prob. 12-11. $P_D = 50$ k, $P_L = 100$ k (*Ans.* 24 both LRFD, ASD)

12-30. Prob. 12-13. $P_D = 75$ k, $P_L = 160$ k

12-31. Prob. 12-14. $P_D = 120$ k, $P_L = 150$ k (*Ans.* 16 both LRFD, ASD)

12-32. Prob. 12-16. $P_D = 40$ k, $P_L = 100$ k

12-33. Prob. 12-20. (*Ans.* 11 LRFD, 12 ASD)

12-34 and 12-35. *Using the bearing-type connection from each problem given, determine the number of 1-in A490 bolts required, by LRFD and ASD, for a slip-critical connection. Assume long-slotted holes in the direction of the load, Class A faying surfaces, h_f = 1.00, and l_c = 1.25 in.*

12-34. Prob. 12-12.

12-35. Prob. 12-15. (*Ans.* 11 or 12 LRFD, 12 ASD)

12-36. Determine the design tensile strength P_u and the allowable tensile strength P_a of the connection shown if eight $\frac{7}{8}$-in A325 bearing-type bolts (threads excluded from shear plane) are used in each flange. Include block shear in your calculations. A36 steel is used.

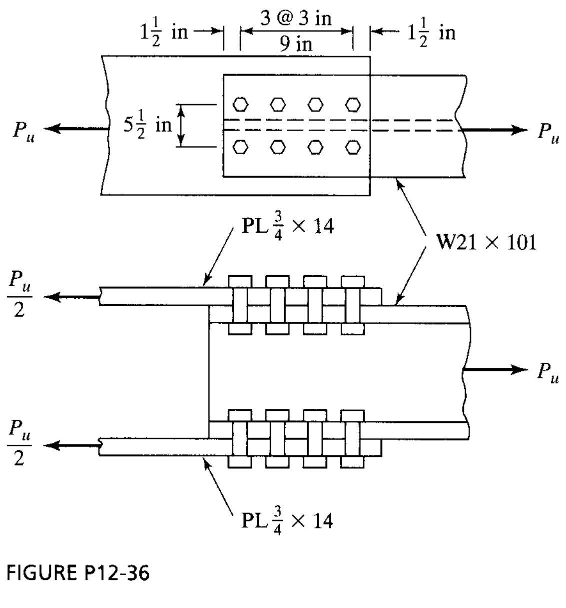

FIGURE P12-36

12-37. Repeat Prob. 12-36, using $\frac{7}{8}$-in A490 bearing-type bolts. $F_y = 50$ ksi and $F_u = 65$ ksi (*Ans.* 604.8 k, 403.2 k)

CHAPTER 13

Eccentrically Loaded Bolted Connections and Historical Notes on Rivets

13.1 BOLTS SUBJECTED TO ECCENTRIC SHEAR

Eccentrically loaded bolt groups are subjected to shears and bending moments. You might think that such situations are rare, but they are much more common than most people suspect. For instance, in a truss it is desirable to have the center of gravity of a member lined up exactly with the center of gravity of the bolts at its end connections. This feat is not quite as easy to accomplish as it may seem, and connections are often subjected to moments.

Eccentricity is quite obvious in Fig. 13.1(a), where a beam is connected to a column with a plate. In part (b) of the figure, another beam is connected to a column with a pair of web angles. It is obvious that this connection must resist some moment, because the center of gravity of the load from the beam does not coincide with the reaction from the column.

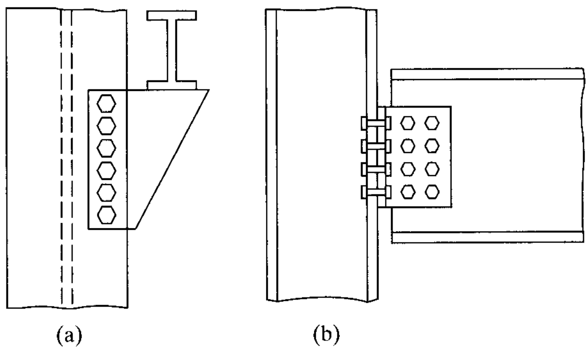

FIGURE 13.1

Eccentrically loaded bolt groups.

Bridge over New River Gorge near Charleston in Fayette County, WV. (Courtesy of the American Bridge Company.)

In general, specifications for bolts and welds clearly state that the center of gravity of the connection should coincide with the center of gravity of the member, unless the eccentricity is accounted for in the calculations. However, Section J1.7 of the AISC Specification provides some exceptions to this rule. It states that the rule is not applicable to the end connections of statically loaded single angles, double angles, and similar members. In other words, the eccentricities between the centers of gravity of these members and the centers of gravity of the connections may be ignored unless fatigue loadings are involved. *Furthermore, the eccentricity between the gravity axes and the gage lines of bolted members may be neglected for statically loaded members.*

The AISC Specification presents values for computing the design strengths of individual bolts, but does not specify a method for computing the forces on these fasteners when they are eccentrically loaded. As a result, the method of analysis to be used is left up to the designer.

Three general approaches for the analysis of eccentrically loaded connections have been developed through the years. The first of the methods is the very conservative *elastic method* in which friction or slip resistance between the connected parts is neglected. In addition, these connected parts are assumed to be perfectly rigid. This type of analysis has been commonly used since at least 1870.[1,2]

Tests have shown that the elastic method usually provides very conservative results. As a consequence, various *reduced* or *effective eccentricity methods* have been proposed.[3] The analysis is handled just as it is in the elastic method, except that smaller eccentricities, and thus smaller moments, are used in the calculations.

The third method, called the *instantaneous center of rotation method*, provides the most realistic values compared with test results, but is extremely tedious to apply, at least with handheld calculators. Tables 7-7 to 7-14 in Part 7 of the Manual for eccentrically loaded bolted connections are based on the ultimate strength method and enable us to solve most of these types of problems quite easily, as long as the bolt patterns are symmetrical. The remainder of this section is devoted to these three analysis methods.

13.1.1 Elastic Analysis

For this discussion, the bolts of Fig. 13.2(a) are assumed to be subjected to a load P that has an eccentricity of e from the c.g. (center of gravity) of the bolt group. To consider the force situation in the bolts, an upward and downward force—each equal to P—is assumed to act at the c.g. of the bolt group. This situation, shown in part (b) of the figure, in no way changes the bolt forces. The force in a particular bolt will, therefore, equal P divided by the number of bolts in the group, as seen in part (c), plus the force due to the moment caused by the couple, shown in part (d) of the figure.

The magnitude of the forces in the bolts due to the moment Pe will now be considered. The distances of each bolt from the c.g. of the group are represented by the values d_1, d_2, etc., in Fig. 13.3. The moment produced by the couple is assumed to cause the plate to rotate about the c.g. of the bolt connection, with the amount of rotation or

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[1] W. McGuire, *Steel Structures* (Englewood Cliffs, NJ: Prentice-Hall, 1968), p. 813.

[2] C. Reilly, "Studies of Iron Girder Bridges," *Proc. Inst. Civil Engrs.* 29 (London, 1870).

[3] T.R. Higgins, "New Formulas for Fasteners Loaded Off Center," *Engr. News Record* (May 21, 1964).

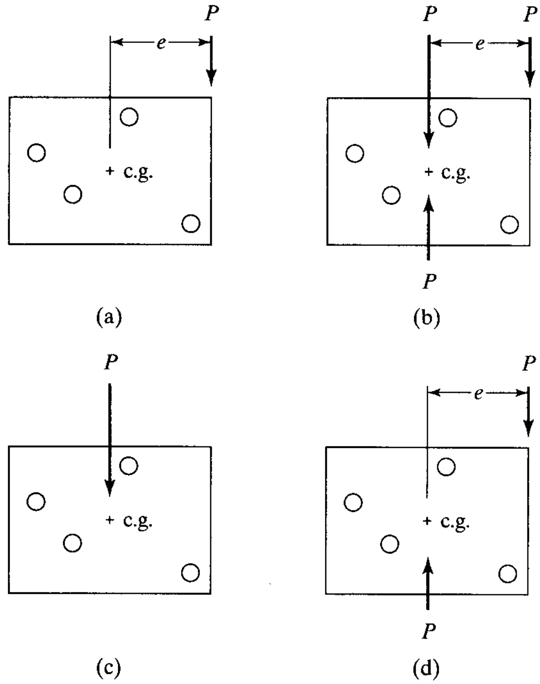


FIGURE 13.2

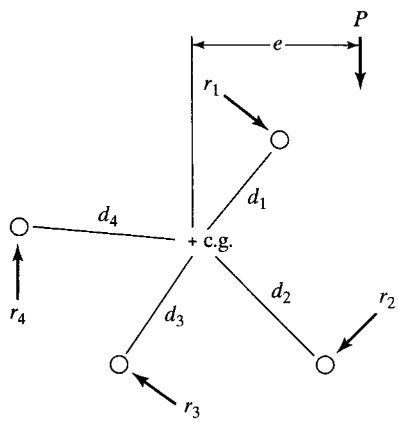


FIGURE 13.3

strain at a particular bolt being proportional to its distance from the c.g. (For this derivation, the gusset plates are again assumed to be perfectly rigid and the bolts are assumed to be perfectly elastic.) Stress is greatest at the bolt that is the greatest distance from the c.g., because stress is proportional to strain in the elastic range.

The rotation is assumed to produce forces of r_1, r_2, r_3, and r_4, respectively, from the bolts in the figure. The moment transferred to the bolts must be balanced by resisting moments of the bolts as shown in Equation (1)

$$M_{\text{c.g.}} = Pe = r_1d_1 + r_2d_2 + r_3d_3 + r_4d_4 \tag{1}$$

As the force caused on each bolt is assumed to be directly proportional to the distance from the c.g., we can write

$$\frac{r_1}{d_1} = \frac{r_2}{d_2} = \frac{r_3}{d_3} = \frac{r_4}{d_4}$$

Writing each r value in terms of r_1 and d_1, we get

$$r_1 = \frac{r_1 d_1}{d_1} \quad r_2 = \frac{r_1 d_2}{d_1} \quad r_3 = \frac{r_1 d_3}{d_1} \quad r_4 = \frac{r_1 d_4}{d_1}$$

Substituting these values into the equation (original) and simplifying yields

$$M = \frac{r_1 d_1^2}{d_1} + \frac{r_1 d_2^2}{d_1} + \frac{r_1 d_3^2}{d_1} + \frac{r_1 d_4^2}{d_1} = \frac{r_1}{d_1}(d_1^2 + d_2^2 + d_3^2 + d_4^2)$$

Therefore,

$$M = \frac{r_1 \Sigma d^2}{d_1}$$

The force on each bolt can now be written as

$$r_1 = \frac{M d_1}{\Sigma d^2} \quad r_2 = \frac{d_2}{d_1} r_1 = \frac{M d_2}{\Sigma d^2} \quad r_3 = \frac{M d_3}{\Sigma d^2} \quad r_4 = \frac{M d_4}{\Sigma d^2}$$

Each value of r is perpendicular to the line drawn from the c.g. to the particular bolt. It is usually more convenient to break these reactions down into vertical and horizontal components. See Fig. 13.4.

In this figure, the horizontal and vertical components of the distance d_1 are represented by h and v, respectively, and the horizontal and vertical components of force

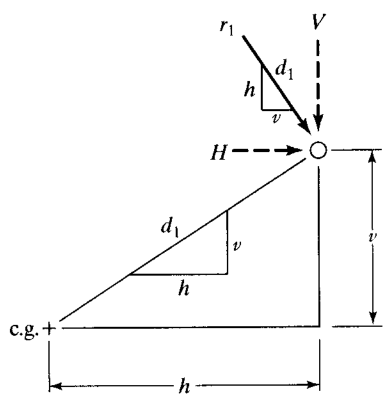


FIGURE 13.4

r_1 are represented by H and V, respectively. It is now possible to write the following ratio from which H can be obtained:

$$\frac{r_1}{d_1} = \frac{H}{v}$$

$$H = \frac{r_1 v}{d_1} = \left(\frac{Md_1}{\Sigma d^2}\right)\left(\frac{v}{d_1}\right)$$

Therefore,

$$H = \frac{Mv}{\Sigma d^2}$$

By a similar procedure,

$$V = \frac{Mh}{\Sigma d^2}$$

Example 13-1

Determine the force in the most stressed bolt of the group shown in Fig. 13.5, using the elastic analysis method.

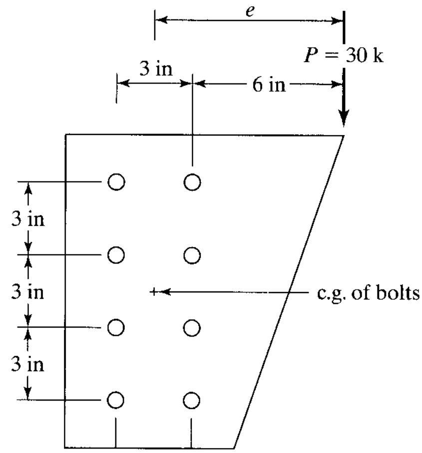


FIGURE 13.5

Solution. A sketch of each bolt and the forces applied to it by the direct load and the clockwise moment are shown in Fig. 13.6. From this sketch, the student can see that the upper right-hand bolt and the lower right-hand bolt are the most stressed and that their respective stresses are equal:

$$e = 6 + 1.5 = 7.5 \text{ in}$$

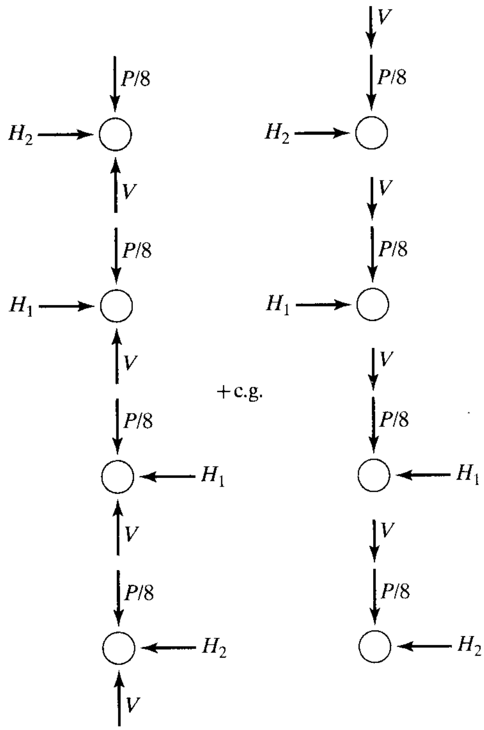


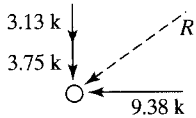
FIGURE 13.6

$$M = Pe = (30\text{ k})(7.5\text{ in}) = 225\text{ in-k}$$
$$\Sigma d^2 = \Sigma h^2 + \Sigma v^2$$
$$\Sigma d^2 = (8)(1.5)^2 + (4)(1.5^2 + 4.5^2) = 108\text{ in}^2$$

For lower right-hand bolt

$$H = \frac{Mv}{\Sigma d^2} = \frac{(225\text{ in-k})(4.5\text{ in})}{108\text{ in}^2} = 9.38\text{ k} \leftarrow$$
$$V = \frac{Mh}{\Sigma d^2} = \frac{(225\text{ in-k})(1.5\text{ in})}{108\text{ in}^2} = 3.13\text{ k} \downarrow$$
$$\frac{P}{8} = \frac{30\text{ k}}{8} = 3.75\text{ k} \downarrow$$

These components for the lower right-hand bolt are sketched as follows:



The resultant force applied to this bolt is

$$R = \sqrt{(3.13 + 3.75)^2 + (9.38)^2} = 11.63 \text{ k}$$

If the eccentric load is inclined, it can be broken down into vertical and horizontal components, and the moment of each about the c.g. of the bolt group can be determined. Various design formulas can be developed that will enable the engineer to directly design eccentric connections, but the process of assuming a certain number and arrangement of bolts, checking stresses, and redesigning probably is just as satisfactory.

The trouble with this inaccurate, but very conservative, method of analysis is that, in effect, we are assuming that there is a linear relation between loads and deformations in the fasteners; further, we assume that their yield stress is not exceeded when the ultimate load on the connection is reached. Various experiments have shown that these assumptions are incorrect.

Summing up this discussion, we can say that the elastic method is easier to apply than the instantaneous center of rotation method to be described in Section 13.1.3. However, it is probably too conservative, as it neglects the ductility of the bolts and the advantage of load redistribution.

13.1.2 Reduced Eccentricity Method

The elastic analysis method just described appreciably overestimates the moment forces applied to the connectors. As a result, quite a few proposals have been made through the years that make use of an effective eccentricity, in effect taking into account the slip resistance on the faying or contact surfaces. One set of reduced eccentricity values that were fairly common at one time follow:

1. With one gage line of fasteners and where n is the number of fasteners in the line,

$$e_{\text{effective}} = e_{\text{actual}} - \frac{1 + 2n}{4}$$

2. With two or more gage lines of fasteners symmetrically placed and where n is the number of fasteners in each line,

$$e_{\text{effective}} = e_{\text{actual}} - \frac{1 + n}{2}$$

The reduced eccentricity values for two fastener arrangements are shown in Fig. 13.7.

To analyze a particular connection with the reduced eccentricity method, the value of $e_{\text{effective}}$ is computed as described before and is used to compute the eccentric moment. Then the elastic procedure is used for the remainder of the calculations.

13.1.3 Instantaneous Center of Rotation Method

Both the elastic and reduced eccentricity methods for analyzing eccentrically loaded fastener groups are based on the assumption that the behavior of the fasteners is elastic. A much more realistic method of analysis is the instantaneous center of

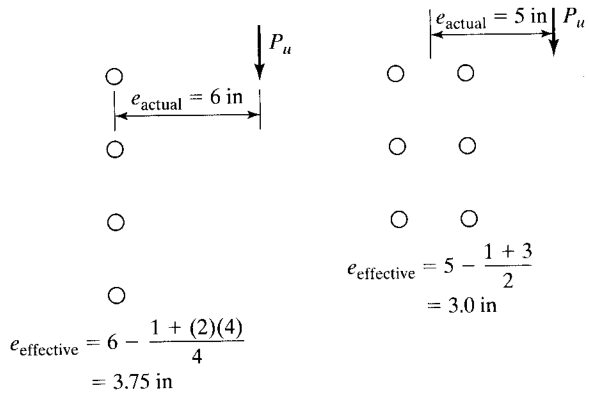


FIGURE 13.7

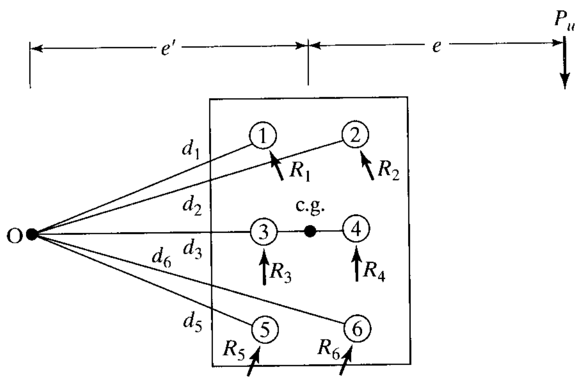


FIGURE 13.8

rotation method, which is described in the next few paragraphs. The values given in the AISC Manual for eccentrically loaded fastener groups were computed by this method.

If one of the outermost bolts in an eccentrically loaded connection begins to slip or yield, the connection will not fail. Instead, the magnitude of the eccentric load may be increased, the inner bolts will resist more load, and failure will not occur until all of the bolts slip or yield.

The eccentric load tends to cause both a relative rotation and translation of the connected material. In effect, this is equivalent to pure rotation of the connection about a single point called the *instantaneous center of rotation*. An eccentrically loaded bolted connection is shown in Fig. 13.8, and the instantaneous center is represented by point 0. It is located a distance e' from the center of gravity of the bolt group.

The deformations of these bolts are assumed to vary in proportion to their distances from the instantaneous center. The ultimate shear force that one of them can resist is not equal to the pure shear force that a bolt can resist. Rather, it is dependent upon the load-deformation relationship in the bolt. Studies by Crawford and Kulak[4] have shown that this force may be closely estimated with the following expression:

$$R = R_{ult}(1 - e^{-10\Delta})^{0.55}$$

In this formula, R_{ult} is the ultimate shear load for a single fastener equaling 74 k for a 3/4-in diameter A325 bolt, e is the base of the natural logarithm (2.718), and Δ is the total deformation of a bolt. Its maximum value experimentally determined is 0.34 in. The Δ values for the other bolts are assumed to be in proportion to R as their d distances are to d for the bolt with the largest d. The coefficients 10.0 and 0.55 also were experimentally obtained. Figure 13.9 illustrates this load-deformation relationship.

This expression clearly shows that the ultimate shear load taken by a particular bolt in an eccentrically loaded connection is affected by its deformation. Thus, the load applied to a particular bolt is dependent upon its position in the connection with respect to the instantaneous center of rotation.

The resisting forces of the bolts of the connection of Fig. 13.8 are represented with the letters R_1, R_2, R_3, and so on. Each of these forces is assumed to act in a direction perpendicular to a line drawn from point 0 to the center of the bolt in question. For this symmetrical connection, the instantaneous center of rotation will fall somewhere on a horizontal line through the center of gravity of the bolt group. This is the case because the sum of the horizontal components of the R forces must be zero, as also must be the sum of the moments of the horizontal components about point 0. The position of point 0 on the horizontal line may be found by a tedious trial-and-error procedure to be described here.

With reference to Fig. 13.8, the moment of the eccentric load about point 0 must be equal to the summation of the moments of the R resisting forces about the same point. If we knew the location of the instantaneous center, we could compute R values for the bolts with the Crawford-Kulak formula and determine P_u from the expression to follow, in which e and e' are distances shown in Figs. 13.8 and 13.11.

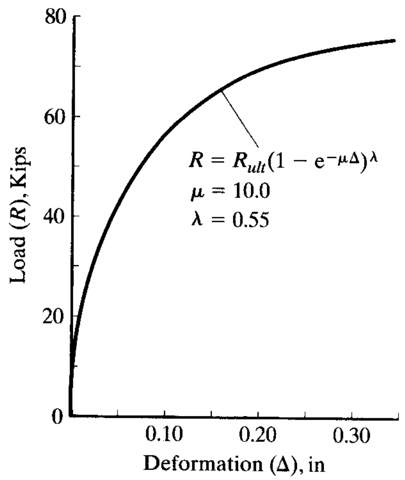


FIGURE 13.9
Ultimate shear force R in a single bolt at any given deformation.

[4]S. F. Crawford and G. L. Kulak, "Eccentrically Loaded Bolt Connections," *Journal of Structural Division*, ASCE 97, ST3 (March 1971), pp. 765–783.

$$P_u(e' + e) = \Sigma Rd$$

$$P_u = \frac{\Sigma Rd}{e' + e}$$

To determine the design strength of such a connection according to the AISC Specification, we can replace R_{ult} in the Crawford-Kulak formula with the design shearing strength of one bolt in a connection where the load is not eccentric. For instance, if we have 7/8-in A325 bolts (threads excluded from shear plane) in single shear bearing on a sufficient thickness so that bearing does not control, R_{ult} will equal, for the LRFD method,

$$R_{ult} = \phi F_n A_b = (0.75)(68\text{ ksi})(0.60\text{ in}^2) = 30.6\text{ k}$$

The location of the instantaneous center is not known, however. Its position is estimated, the R values determined, and P_u calculated as described. It will be noted that P_u must be equal to the summation of the vertical components of the R resisting forces (ΣR_v). If the value is computed and equals the P_u computed by the preceding formula, we have the correct location for the instantaneous center. If not, we try another location, and so on.

In Example 13-2, the author demonstrates the very tedious trial-and-error calculations necessary to locate the instantaneous center of rotation for a symmetrical connection consisting of four bolts. In addition, the LRFD design strength of the connection ϕR_n and the allowable strength R_n/Ω are determined.

To solve such a problem, it is very convenient to set the calculations up in a table similar to the one used in the solution to follow. In the table shown, the h and v values given are the horizontal and vertical components of the d distances from point 0 to the centers of gravity of the individual bolts. The bolt that is located at the greatest distance from point 0 is assumed to have a Δ value of 0.34 in. The Δ values for the other bolts are assumed to be proportional to their distances from point 0. The Δ values so determined are used in the R formula.

A set of tables entitled "Coefficients C for Eccentrically Loaded Bolt Groups" is presented in Tables 7-7 to 7-14 of the AISC Manual. The values in these tables were determined by the procedure described here. A large percentage of the practical cases that the designer will encounter are included in the tables. Should some other situation not covered be faced, the designer may very well decide to use the more conservative elastic procedure previously described.

Example 13-2

The bearing-type 7/8-in A325 bolts of the connection of Fig. 13.10 have a nominal shear strength $r_n = (0.60\text{ in}^2)(68\text{ ksi}) = 40.8\text{ k}$. Locate the instantaneous center of rotation of the connection, using the trial-and-error procedure, and determine the value of P_u.

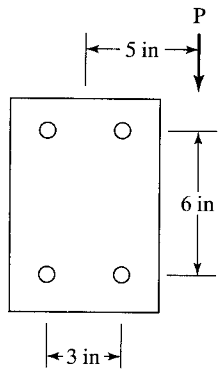


FIGURE 13.10

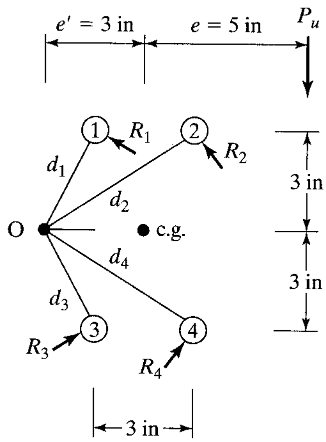


FIGURE 13.11

Solution. *By trial and error:* Try a value of $e' = 3$ in, reference being made to Fig. 13.11. In the accompanying table, Δ for bolt 1 equals (3.3541/5.4083) (0.34) $=$ 0.211 in and R for the same bolt equals $30.6(1 - e^{-(10)(0.211)})^{0.55}$.

Bolt No.	h (in)	v (in)	d (in)	Δ (in)	R(kips)	R_v(kips)	Rd (k-in)
1	1.5	3	3.3541	0.211	28.50	12.74	95.58
2	4.5	3	5.4083	0.34	30.03	24.99	162.43
3	1.5	3	3.3541	0.211	28.50	12.74	95.58
4	4.5	3	5.4083	0.34	30.03	24.99	162.43
						$\Sigma = 75.46$	$\Sigma = 516.03$

$$P_u = \frac{\Sigma Rd}{e' + e} = \frac{516.03}{3 + 5} = 64.50 \text{ k not} = 75.46 \text{ k} \qquad \text{N.G.}$$

After several trials, assume that $e' = 2.40$ in.

Bolt No.	h (in)	v (in)	d (in)	Δ (in)	R (kips)	R_v (kips)	Rd (k-in)
1	0.90	3	3.1321	0.216	28.61	8.22	89.62
2	3.90	3	4.9204	0.34	30.03	23.81	147.78
3	0.90	3	3.1321	0.216	28.61	8.22	89.62
4	3.90	3	4.9204	0.34	30.03	23.81	147.78
						$\Sigma = 64.06$	$\Sigma = 474.80$

Then, we have

$$P_u = \frac{\Sigma Rd}{e' + e} = \frac{474.80}{2.4 + 5} = 64.16 \text{ k almost} = 64.06 \text{ k} \qquad \text{OK}$$

$$P_u = 64.1 \text{ k}$$

Although the development of this method of analysis was actually based on bearing-type connections where slip may occur, both theory and load tests have shown that the method may conservatively be applied to slip-critical connections.[5]

The instantaneous center of rotation may be expanded to include inclined loads and unsymmetrical bolt arrangements, but the trial-and-error calculations with a hand calculator are extraordinarily long for such situations.

Examples 13-3 and 13-4 provide illustrations of the use of the ultimate strength tables in Part 7 of the AISC Manual, for both analysis and design.

Example 13-3

Repeat Example 13-2, using the tables in Part 7 of the Manual. These tables are entitled "Coefficients C for Eccentrically Loaded Bolt Groups." Determine both LRFD design strength and ASD allowable strength of connection.

Solution. Enter Manual Table 7-8 with angle $= 0°$, $s = 6$ in, $e_x = 5$ in, and $n = 2$ vertical rows.

$$C = 2.24$$

$$r_n = F_{nv} A_g = (68 \text{ ksi})(0.6 \text{ in}^2) = 40.8 \text{ k}$$

(From statement in Example 13-2, shear controls are not checked, and thus bearing is not checked.)

$$R_n = Cr_n = (2.24)(40.8) = 91.4 \text{ k}$$

[5]G. L. Kulak, "Eccentrically Loaded Slip-Resistant Connections," *Engineering Journal*, AISC, vol. 12, no. 2 (2nd Quarter, 1975), pp. 52–55.

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(91.4) = \mathbf{68.6\ k}$ Generally agrees with trial-and-error solution in preceding example.	$\dfrac{R_n}{\Omega} = \dfrac{91.4}{2.00} = \mathbf{45.7\ k}$

Example 13-4

Using both LRFD and ASD, determine the number of 7/8-in A325 bolts in standard-size holes required for the connection shown in Fig. 13.12. Use A36 steel and assume that the connection is to be a bearing type with threads excluded from the shear plane. Further assume that the bolts are in single shear and bearing on 1/2 in. Use the instantaneous center of rotation method as presented in the tables of Part 7 of the AISC Manual. Assume that $L_c = 1.0$ in and deformation at bolt holes at service loads is not a design consideration.

Solution

$$e_x = e = 5\tfrac{1}{2}\text{ in} = 5.5\text{ in}$$

Bolts in single shear and bearing on 1/2 in:

$$\begin{aligned} r_n &= \text{nominal shear strength per fastener} \\ &= F_{nv}A_b = (68\text{ ksi})(0.6\text{ in}^2) = 40.8\text{ k} \leftarrow \\ r_n &= \text{nominal bearing strength per fastener} \\ &= 1.5 l_c t F_u = (1.5)(1.0\text{ in})\left(\frac{1}{2}\text{ in}\right)(58\text{ ksi}) = 43.5\text{ k} \\ &< 3.0\,dtF_u = (3.0)\left(\frac{7}{8}\right)\left(\frac{1}{2}\right)(58) = 76.1\text{ k} \end{aligned}$$

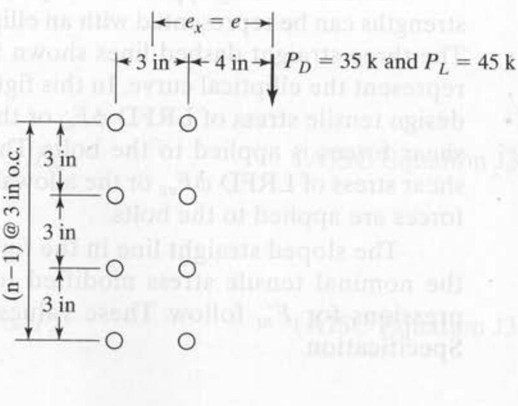


FIGURE 13.12

With reference to Table 7-8 in the Manual, the value of C_{min} required to provide a sufficient number of bolts can be determined as follows:

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$P_u = (1.2)(35) + (1.6)(45) = 114$ k	$P_a = 35 + 45 = 80$ k
$C_{min} = \frac{P_u}{\phi r_n} = \frac{114}{(0.75)(40.8)} = 3.73$	$C_{min} = \frac{\Omega P_a}{r_n} = \frac{(2.00)80}{40.8} = 3.92$
* Use four $\frac{7}{8}$ A325 bolts in each row, as described next.	

* With $e_x = 5\ 1/2$ in and a vertical spacing s of 3 in, we move horizontally in the table until we find the number of bolts in each vertical row so as to provide a C of 3.73 or more. With $e_x = 5$ in and $n = 4$, we find $C = 4.51$. Then, with $e_x = 6$ in and $n = 4$, we find $C = 4.03$. Interpolating for $e_x = 5\ 1/2$ in, we find $C = 4.27 > 3.73$, OK for LRFD.

Since $C = 4.27 > 3.92$, OK for ASD

Use four 7/8 in A325 × bolts in each row (for both LRFD and ASD).

Note: If the situation faced by the designer does not fit the eccentrically loaded bolt group tables given in Part 7 of the AISC Manual, it is recommended that the conservative elastic procedure be used to handle the problem, whether analysis or design.

13.2 BOLTS SUBJECTED TO SHEAR AND TENSION (BEARING-TYPE CONNECTIONS)

The bolts used for a large number of structural steel connections are subjected to a combination of shear and tension. One quite obvious case is shown in Fig. 13.13, where a diagonal brace is attached to a column. The vertical component of force in the figure, V, is trying to shear the bolts off at the face of the column, while the horizontal component of force, H, is trying to fracture them in tension.

Tests on bearing-type bolts subject to combined shear and tension show that their strengths can be represented with an elliptical interaction curve, as shown in Fig. 13.14. The three straight dashed lines shown in the figure can be used quite accurately to represent the elliptical curve. In this figure, the horizontal dashed line represents the design tensile stress of LRFD ϕF_{nt} or the allowable tensile stress of ASD F_{nt}/Ω if no shear forces is applied to the bolts. The vertical dashed line represents the design shear stress of LRFD ϕF_{nv} or the allowable shear stress of the ASD F_{nv}/Ω if no tensile forces are applied to the bolts.

The sloped straight line in the figure is represented by the expression for F'_{nt}, the nominal tensile stress modified to include the effects of shearing force. Expressions for F'_{nt} follow. These values are provided in Section J3.7 of the AISC Specification.

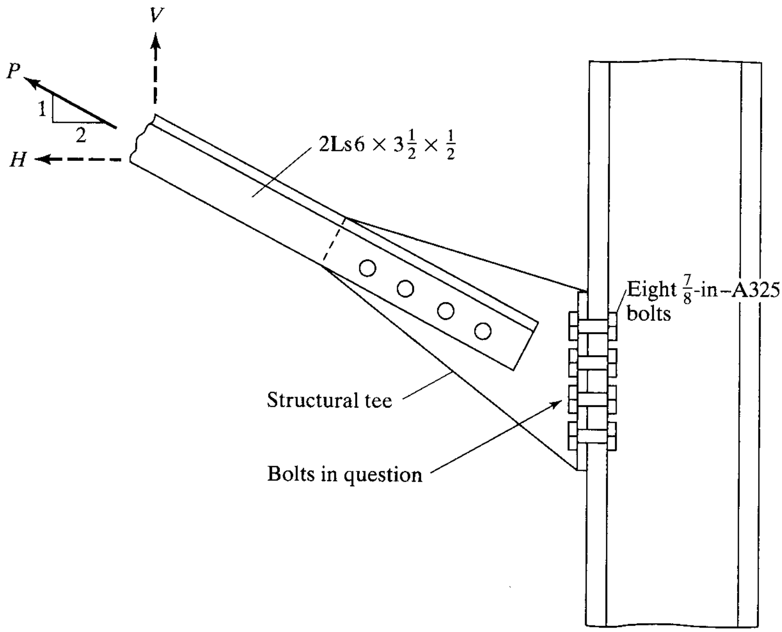


FIGURE 13.13

Combined shear and tension connection.

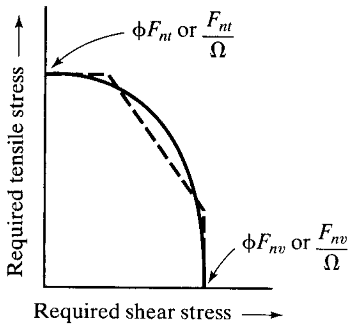


FIGURE 13.14

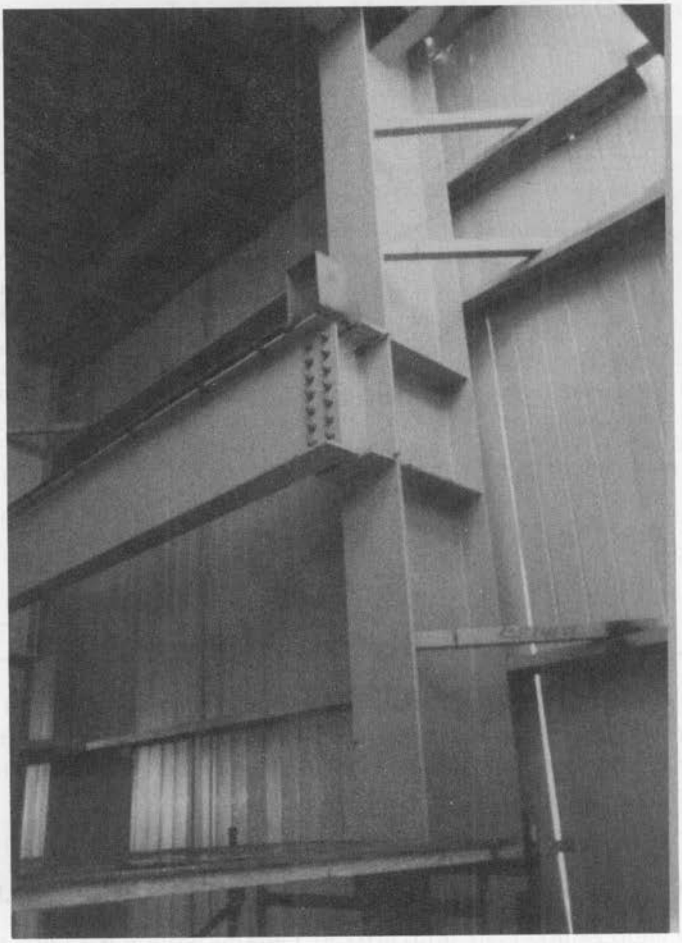
Bolts in a bearing-type connection subject to combined shear and tension.

For LRFD ($\phi = 0.75$)

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt} \qquad \text{(AISC Equation J3-3a)}$$

For ASD ($\Omega = 2.00$)

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt} \qquad \text{(AISC Equation J3-3b)}$$



APD Building, Dublin, GA. (Courtesy Britt, Peters and Associates.)

in which

F_{nt} is the nominal tensile stress from Table 12-5 (AISC Table J3.2), ksi.

F_{nv} is the nominal shear stress from Table 12-5 (AISC Table J3.2), ksi.

f_{rv} is the required shear stress using LRFD or ASD load combinations, ksi.

The available shear stress of the fastener shall equal or exceed the required shear stress, f_{rv}.

The AISC Specification (J3.7) says that if the required stress, f, in either shear or tension, is equal to or less than 30 percent of the corresponding available stress, it is not necessary to investigate the effect of combined stress.

Example 13-5

The tension member previously shown in Fig. 13.13 has eight 7/8-in A325 high-strength bolts in a bearing-type connection. Is this a sufficient number of bolts to resist the applied loads $P_D = 80$ k and $P_L = 100$ k, using the LRFD and ASD specifications, if the bolt threads are excluded from the shear planes?

Solution

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$P_u = (1.2)(80) + (1.6)(100) = 256$ k	$P_a = 80 + 100 = 180$ k
$V = \frac{1}{\sqrt{5}}(256) = 114.5$ k	$V = \frac{1}{\sqrt{5}}(180) = 80.5$ k
$H = \frac{2}{\sqrt{5}}(256) = 229$ k	$H = \frac{2}{\sqrt{5}}(180) = 161$ k
$F_{nt} = 90$ ksi	$F_{nt} = 90$ ksi
$F_{nv} = 68$ ksi	$F_{nv} = 68$ ksi
$f_{rv} = \frac{114.5\text{ k}}{(8)(0.6\text{ in}^2)} = 23.85$ ksi	$f_v = \frac{80.5\text{ k}}{(8)(0.6\text{ in}^2)} = 16.77$ ksi
$f_{rt} = \frac{229\text{ k}}{(8)(0.6\text{ in}^2)} = 47.7$ ksi	$f_t = \frac{161\text{ k}}{(8)(0.6\text{ in}^2)} = 33.54$ ksi
$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$	$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$
$= (1.3)(90) - \frac{90}{(0.75)(68)}(23.85)$	$= (1.3)(90) - \frac{(2.00)(90)}{68}(16.77)$
$= 74.9$ ksi < 90 ksi	$= 72.6$ ksi < 90 ksi
$\phi F'_{nt} = (0.75)(74.9) = 56.2$ ksi > 47.7 ksi	$\frac{F'_{nt}}{\Omega} = \frac{72.6}{2.00} = 36.3$ ksi > 33.54 ksi
Connection is OK.	**Connection is OK.**

13.3 BOLTS SUBJECTED TO SHEAR AND TENSION (SLIP-CRITICAL CONNECTIONS)

When an axial tension force is applied to a slip-critical connection, the clamping force will be reduced, and the design shear strength must be decreased in some proportion to the loss in clamping or prestress. This is accomplished in the AISC Specification (Section J3.9) by multiplying the available slip resistance of the bolts (as determined in AISC Section J.8) by a factor k_{sc}.

For LRFD

$$k_{sc} = 1 - \frac{T_u}{D_u T_b n_b} \qquad \text{(AISC Equation J3-5a)}$$

For ASD

$$k_{sc} = 1 - \frac{1.5T_a}{D_u T_b n_b} \qquad \text{(AISC Equation J3-5b)}$$

Here, the factors are defined as follows:

T_u = the tension force due to the LRFD load combination (that is, $\frac{P_u}{n_b}$)

D_u = a multiplier = 1.13, previously defined in Section 12.14 (AISC Section J3.8)

T_b = minimum fastener tension, as given in Table 12.1 (Table J3.1, AISC)

n_b = the number of bolts carrying the applied tension

T_a = the tension force due to the ASD load combination (that is, $\frac{P_a}{n_b}$)

Example 13-6

A group of twelve 7/8-in A325 high-strength bolts with standard holes is used in a lap joint for a slip-critical joint designed to prevent slip. The connection is to resist the service shear loads $V_D = 40$ k and $V_L = 50$ k, as well as the tensile service loads $T_D = 50$ k and $T_L = 50$ k. Is the connection satisfactory if the faying surface is Class B and the factor for fillers, h_f, is 1.00?

Solution

R_n for 1 bolt in an ordinary slip-critical connection

$$R_n = \mu D_u h_f T_b n_s = (0.50)(1.13)(1.00)(39)(1) = 22.03 \text{ k/bolt}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$V_u = (1.2)(40) + (1.6)(50) = 128$ k	$V_a = 40 + 50 = 90$ k
$T_u = (1.2)(50) + (1.6)(50) = 140$ k	$T_a = 50 + 50 = 100$ k
$\phi R_n = (1.0)(22.03) = 22.03$ k/bolt	$\frac{R_n}{\Omega} = \frac{22.03}{1.50} = 14.69$ k/bolt
Reduction due to tensile load	Reduction due to tensile load
$k_{sc} = 1 - \frac{T_u}{D_u T_b n_b}$	$k_{sc} = 1 - \frac{1.5T_a}{D_u T_b n_b}$
$= 1 - \frac{140}{(1.13)(39)(12)} = 0.735$	$= 1 - \frac{(1.5)(100)}{(1.13)(39)(12)} = 0.716$
Reduced ϕR_n/bolt	Reduced $\frac{R_n}{\Omega}$/bolt
$= (0.735)(22.03 \text{ k}) = 16.20$ k/bolt	$= (0.716)(14.69) = 10.52$ k/bolt
Design slip resistance for 12 bolts = (12)(16.20) = 194.4 k	Allowable slip resistance for 12 bolts = (12)(10.52) = 126.2 k
>128 k **OK**	>90 k **OK**
Connection is satisfactory.	**Connection is satisfactory.**

13.4 TENSION LOADS ON BOLTED JOINTS

Bolted and riveted connections subjected to pure tensile loads have been avoided as much as possible in the past by designers. The use of tensile connections was probably used more often for wind-bracing systems in tall buildings than for any other situation.

Other locations exist, however, where they have been used, such as hanger connections for bridges, flange connection for piping systems, etc. Figure 13.15 shows a hanger-type connection with an applied tensile load.

Hot-driven rivets and fully tensioned high-strength bolts are not free to shorten, with the result that large tensile forces are produced in them during their installation. These initial tensions are actually close to their yield points. There has always been considerable reluctance among designers to apply tensile loads to connectors of this type for fear that the external loads might easily increase their already present tensile stresses and cause them to fail. The truth of the matter, however, is that when external tensile loads are applied to connections of this type, the connectors probably will experience little, if any, change in stress.

Fully tensioned high-strength bolts actually prestress the joints in which they are used against tensile loads. (Think of a prestressed concrete beam that has external compressive loads applied at each end.) The tensile stresses in the connectors squeeze together the members being connected. If a tensile load is applied to this connection at the contact surface, it cannot exert any additional load on the bolts until the members are pulled apart and additional strains put on the bolts. The members cannot be pulled apart until a load is applied that is larger than the total tension in the connectors of the connection. This statement means that the joint is prestressed against tensile forces by the amount of stress initially put in the shanks of the connectors.

Another way of saying this is that if a tensile load P is applied at the contact surface, it tends to reduce the thickness of the plates somewhat, but, at the same time, the contact pressure between the plates will be correspondingly reduced, and the plates will tend to expand by the same amount. The theoretical result, then, is no change in plate thickness and no change in connector tension. This situation continues until P equals the connector tension. At this time an increase in P will result in separation of the plates, and thereafter the tension in the connector will equal P.

Should the load be applied to the outer surfaces, there will be some immediate strain increase in the connector. This increase will be accompanied by an expansion of the plates, even though the load does not exceed the prestress, but the increase will be very slight because the load will go to the plate and connectors roughly in proportion

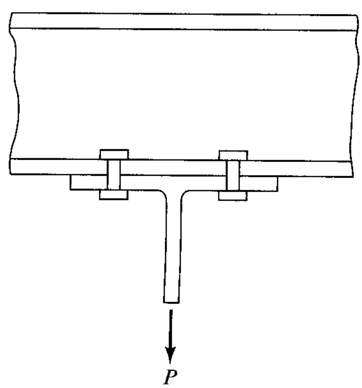


FIGURE 13.15
Hanger connection.

to their stiffness. As the plate is stiffer, it will receive most of the load. An expression can be developed for the elongation of the bolt, on the basis of the bolt area and the assumed contact area between the plates. Depending on the contact area assumed, it will be found that, unless P is greater than the bolt tension, its stress increase will be in the range of 10 percent. Should the load exceed the prestress, the bolt stress will rise appreciably.

The preceding rather lengthy discussion is approximate, but should explain why an ordinary tensile load applied to a bolted joint will not change the stress situation very much.

The AISC nominal tensile strength of bolted or threaded parts is given by the expression to follow, which is independent of any initial tightening force:

$$R_n = F_n A_b, \text{ with } F_n = F_{nt} \text{ for tension or } F_{nv} \text{ for shear.} \quad \text{(AISC Equation J3-1)}$$

When fasteners are loaded in tension, there is usually some bending due to the deformation of the connected parts. As a result, the value of ϕ for LRFD is a rather small 0.75, and Ω for ASD is a rather large 2.00. Table 12.5 of this text (AISC Table J3.2) gives values of F_{nt}, the nominal tensile strength (ksi) for the different kinds of connectors, with the values of threaded parts being quite conservative.

In this expression, A_b is the nominal body area of the unthreaded portion of a bolt, or its threaded part not including upset rods. An upset rod has its ends made larger than the regular rod, and the threads are placed in the enlarged section so that the area at the root of the thread is larger than that of the regular rod. An upset rod was shown in Fig. 4.3. The use of upset rods is not usually economical and should be avoided unless a large order is being made.

If an upset rod is used, the nominal tensile strength of the threaded portion is set equal to $0.75F_u$ times the cross-sectional area at its major thread diameter. This value must be larger than F_y times the nominal body area of the rod before upsetting.

Example 13-7 illustrates the determination of the strength of a tension connection.

Example 13-7

Determine the design tensile strength (LRFD) and the allowable tensile strength (ASD) of the bolts for the hanger connection of Fig. 13.15 if eight 7/8-in A490 high-strength bolts with threads excluded from the shear plane are used. Neglect prying action.

Solution

$$R_n \text{ for 8 bolts} = 8F_{nt}A_b = (8)(113\text{ ksi})(0.6\text{ in}^2) = 542.4\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(542.4) = \mathbf{406.8\ k}$	$\dfrac{R_n}{\Omega} = \dfrac{542.4}{2.00} = \mathbf{271.2\ k}$

13.5 PRYING ACTION

A further consideration that should be given to tensile connections is the possibility of prying action. A tensile connection is shown in Fig. 13.16(a) that is subjected to prying action as illustrated in part (b) of the same figure. Should the flanges of the connection be quite thick and stiff or have stiffener plates like those in Fig. 13.16(c), the prying action will probably be negligible, but this would not be the case if the flanges are thin and flexible and have no stiffeners.

It is usually desirable to limit the number of rows of bolts in a tensile connection, because a large percentage of the load is carried by the inner rows of multi-row connections, even at ultimate load. The tensile connection shown in Fig. 13.17 illustrates this point, as the prying action will throw a large part of the load to the inner connectors, particularly if the plates are thin and flexible. For connections subjected to pure tensile loads, estimates should be made of possible prying action and its magnitude.

The additional force in the bolts resulting from prying action should be added to the tensile force resulting directly from the applied forces. The actual determination

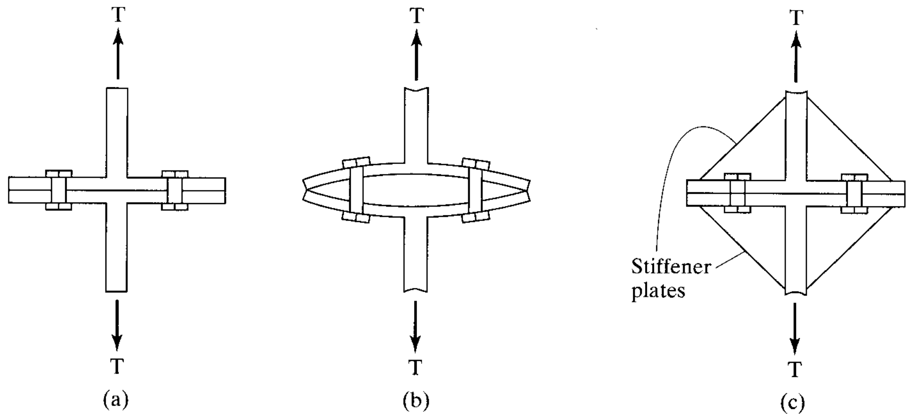


FIGURE 13.16

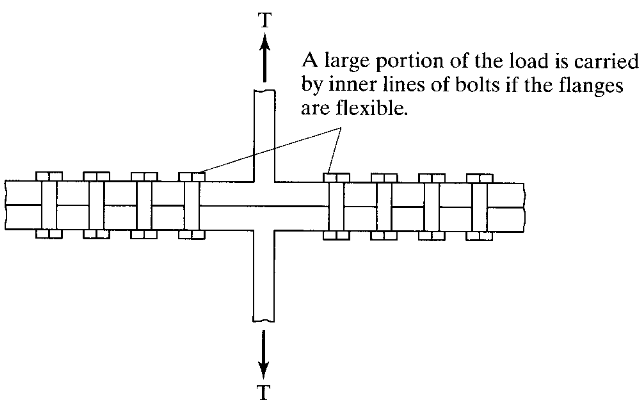


FIGURE 13.17

of prying forces is quite complex, and research on the subject is still being conducted. Several empirical formulas have been developed that approximate test results. Among these are the AISC expressions included in this section.

Hanger and other tension connections should be so designed as to prevent significant deformations. The most important item in such designs is the use of rigid flanges. Rigidity is more important than bending resistance. To achieve this goal, the distance b shown in Fig. 13.18 should be made as small as possible, with a minimum value equal to the space required to use a wrench for tightening the bolts. Information concerning wrench clearance dimensions is presented in a table entitled "Entering and Tightening Clearance" in Tables 7-16 and 7-17 of Part 7 of the AISC Manual.

Prying action, which is present only in bolted connections, is caused by the deformation of the connecting elements when tensile forces are applied. The results are increased forces in some of the bolts above the forces caused directly by the tensile forces. Should the thicknesses of the connected parts be as large as or larger than the values given by the AISC formulas to follow, which are given on pages 9–10 in the Manual, prying action is considered to be negligible. Reference is here made to Fig. 13.18 for the terms involved in the formulas.

For LRFD	**For ASD**
$t_{min} = \sqrt{\dfrac{4.44Tb'}{pF_u}}$	$t_{min} = \sqrt{\dfrac{6.66Tb'}{pF_u}}$

The following terms are defined for these formulas:

$$T = \text{required strength of each bolt} = r_{ut}$$

$$\text{or} \quad r_{at} = \frac{T_u \text{ or } T_a}{\text{no. of bolts}}, \text{ kips}$$

$$b' = \left(b - \frac{d_b}{2}\right), \text{ in}$$

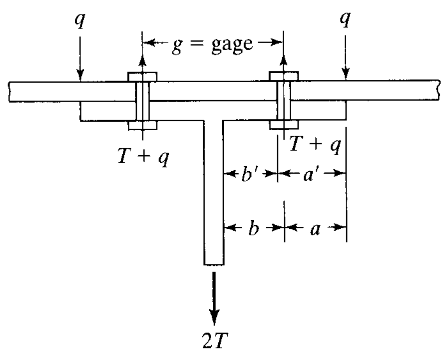


FIGURE 13.18

b = distance from ℄ of bolt to face of tee (for an angle b is measured to ℄ of angle leg) in

d_b = bolt diameter

p = tributary length per pair of bolts (⊥ to plane of paper) preferably not $>g$, in

F_u = specified minimum tensile strength of the connecting element, ksi

Example 13-8, which follows, presents the calculation of the minimum thickness needed for a structural tee flange so that prying action does not have to be considered for the bolts.

Example 13-8

A 10-in long WT8 × 22.5 (t_f = 0.565 in, t_w = 0.345 in, and b_f = 7.04 in) is connected to a W36 × 150 as shown in Fig. 13.19, with six 7/8-in A325 high-strength bolts spaced 3 in o.c. If A36 steel is used, F_u = 58 ksi, is the flange sufficiently thick if prying action is considered? P_D = 30 k and P_L = 40 k.

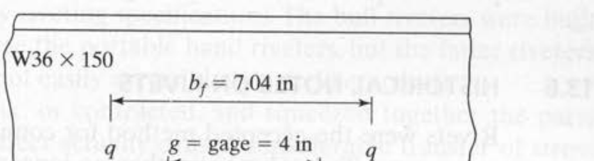


FIGURE 13.19

Solution

LRFD	ASD
$T_u = (1.2)(30) + (1.6)(40) = 100$ k	$T_a = 30 + 40 = 70$ k
$T = r_{ut} = \frac{100}{6} = 16.67$ k each bolt	$T = r_{at} = \frac{70}{6} = 11.67$ k each bolt

$$b' = \left(2 - \frac{0.345}{2}\right) - \frac{0.875}{2} = 1.39 \text{ in}$$

$$d_b = 0.875 \text{ in}$$

$$p = 3 \text{ in}$$

LRFD	ASD
$t_{min} = \sqrt{\frac{(4.44)(16.67)(1.39)}{(3)(58)}}$	$t_{min} = \sqrt{\frac{(6.66)(11.67)(1.39)}{(3)(58)}}$
$= 0.769 \text{ in} > t_f = 0.565 \text{ in}$	$= 0.788 \text{ in} > t_f = 0.565 \text{ in}$
∴ Prying action must be considered.	∴ Prying action must be considered.

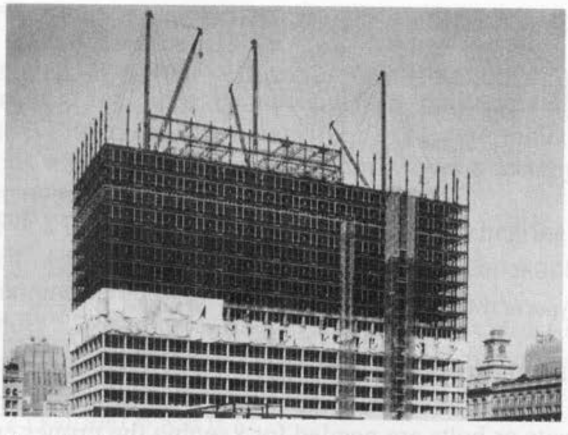
Note: Though not presented here in, pages 9-10 through 9-13 in the AISC Manual provide equations for computing the extra tensile force (q) caused by prying action.

13.6 HISTORICAL NOTES ON RIVETS

Rivets were the accepted method for connecting the members of steel structures for many years. Today, however, they no longer provide the most economical connections and are obsolete. It is doubtful that you could find a steel fabricator who can do riveting. It is, however, desirable for the designer to be familiar with rivets, even though he or she will probably never design riveted structures. He or she may have to analyze an existing riveted structure for new loads or for an expansion of the structure. The purpose of these sections is to present only a very brief introduction to the analysis and design of rivets. One advantage of studying these obsolete connectors is that, while doing so, you automatically learn how to analyze A307 common bolts. These bolts are handled exactly as are rivets, except that the design stresses are slightly different. Example 13-11 illustrates the design of a connection with A307 bolts.

The rivets used in construction work were usually made of a soft grade of steel that would not become brittle when heated and hammered with a riveting gun to form the head. The typical rivet consisted of a cylindrical shank of steel with a rounded head on one end. It was heated in the field to a cherry-red color (approximately 1800°F), inserted in the hole, and a head formed on the far end, probably with a portable rivet gun powered by compressed air. The rivet gun, which had a depression in its head to give the rivet head the desired shape, applied a rapid succession of blows to the rivet.

For riveting done in the shop, the rivets were probably heated to a light cherry-red color and driven with a pressure-type riveter. This type of riveter, usually called a "bull" riveter, squeezed the rivet with a pressure of perhaps as high as 50 to 80 tons (445 to 712 kN) and drove the rivet with one stroke. Because of this great pressure, the rivet in its soft state was forced to fill the hole very satisfactorily. This type of riveting



The U.S. Customs Court, Federal Office Building under construction in New York City. (Courtesy of Bethlehem Steel Corporation.)

was much to be preferred over that done with the pneumatic hammer, but no greater nominal strengths were allowed by riveting specifications. The bull riveters were built for much faster operation than were the portable hand riveters, but the latter riveters were needed for places that were not easily accessible (i.e., field erection).

As the rivet cooled, it shrank, or contracted, and squeezed together the parts being connected. The squeezing effect actually caused considerable transfer of stress between the parts being connected to take place by friction. The amount of friction was not dependable, however, and the specifications did not permit its inclusion in the strength of a connection. Rivets shrink diametrically as well as lengthwise and actually become somewhat smaller than the holes that they are assumed to fill. (Permissible strengths for rivets were given in terms of the nominal cross-sectional areas of the rivets before driving.)

Some shop rivets were driven cold with tremendous pressures. Obviously, the cold-driving process worked better for the smaller-size rivets (probably 3/4 in in diameter or less), although larger ones were successfully used. Cold-driven rivets fill the holes better, eliminate the cost of heating, and are stronger because the steel is cold worked. There is, however, a reduction of clamping force, since the rivets do not shrink after having been driven.

13.7 TYPES OF RIVETS

The sizes of rivets used in ordinary construction work were 3/4 in and 7/8 in in diameter, but they could be obtained in standard sizes from 1/2 in to 1 1/2 in in 1/8-in increments. (The smaller sizes were used for small roof trusses, signs, small towers, etc., while the larger sizes were used for very large bridges or towers and very tall buildings.) The use of more than one or two sizes of rivets or bolts on a single job is usually undesirable, because it is expensive and inconvenient to punch different-size holes in a member in

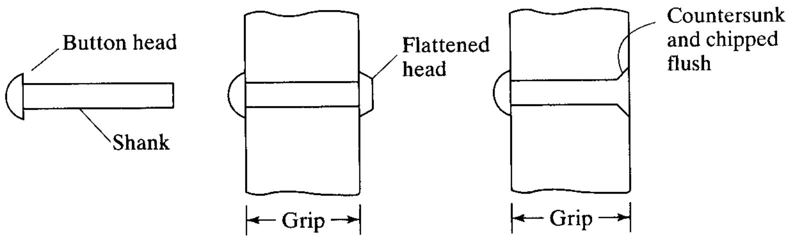


FIGURE 13.20

Types of rivets.

the shop, and the installation of different-size rivets or bolts in the field may be confusing. Some cases arise where it is absolutely necessary to have different sizes, as where smaller rivets or bolts are needed for keeping the proper edge distance in certain sections, but these situations should be avoided if possible.

Rivet heads, usually round in shape, were called *button heads*; but if clearance requirements dictated, the head was flattened or even countersunk and chipped flush. These situations are shown in Fig. 13.20.

The countersunk and chipped-flush rivets did not have sufficient bearing areas to develop full strength, and the designer usually discounted their computed strengths by 50 percent. A rivet with a flattened head was preferred over a countersunk rivet, but if a smooth surface was required, the countersunk and chipped-flush rivet was necessary. This latter type of rivet was appreciably more expensive than the button head type, in addition to being weaker; and it was not used unless absolutely necessary.

There were three ASTM classifications for rivets for structural steel applications, as described in the paragraphs that follow.

13.7.1 ASTM Specification A502, Grade 1

These rivets were used for most structural work. They had a low carbon content of about 0.80 percent, were weaker than the ordinary structural carbon steel, and had a higher ductility. The fact that these rivets were easier to drive than the higher-strength rivets was the main reason that, when rivets were used, they probably were A502, Grade 1, regardless of the strength of the steel used in the structural members.

13.7.2 ASTM Specification A502, Grade 2

These carbon-manganese rivets had higher strengths than the Grade 1 rivets and were developed for the higher-strength steels. Their higher strength permitted the designer to use fewer rivets in a connection and thus smaller gusset plates.

13.7.3 ASTM Specification A502, Grade 3

These rivets had the same nominal strengths as the Grade 2 rivets, but they had much higher resistance to atmospheric corrosion, equal to approximately four times that of carbon steel without copper.

13.8 STRENGTH OF RIVETED CONNECTIONS—RIVETS IN SHEAR AND BEARING

The factors determining the strength of a rivet are its grade, its diameter, and the thickness and arrangement of the pieces being connected. The actual distribution of stress around a rivet hole is difficult to determine, if it can be determined at all; and to simplify the calculations, it is assumed to vary uniformly over a rectangular area equal to the diameter of the rivet times the thickness of the plate.

The strength of a rivet in single shear is the nominal shearing strength times the cross-sectional area of the shank of the rivet. Should a rivet be in double shear, its shearing strength is considered to be twice its single-shear value.

AISC Appendix 5.2.6 indicates that, in checking older structures with rivets, the designer is to assume that the rivets are ASTM A502, grade 1, unless a higher grade is determined by documentation or testing. The nominal shearing strength of A502, grade 1 rivets was 25 ksi, and ϕ was 0.75.

Examples 13-9 and 13-10 illustrate the calculations necessary either to determine the LRFD design and the ASD allowable strengths of existing connections or to design riveted connections. Little comment is made here concerning A307 bolts. The reason is that all the calculations for these fasteners are made exactly as they are for rivets, except that the shearing strengths given by the AISC Specification are different. Only one brief example with common bolts (Example 13-11) is included.

The AISC Specification does not today include rivets, and thus ϕ and Ω values are not included therein. For the example problems to follow, the author uses the rivet ϕ values which were given in the third edition of the LRFD Specification. The Ω values were then determined by the author with the expression $\Omega = 1.50/\phi$, as they were throughout the present specification.

Example 13-9

Determine the LRFD design strength ϕP_n and the ASD allowable strength P_n/Ω of the bearing-type connection shown in Fig. 13.21. A36 steel and A502, Grade 1 rivets

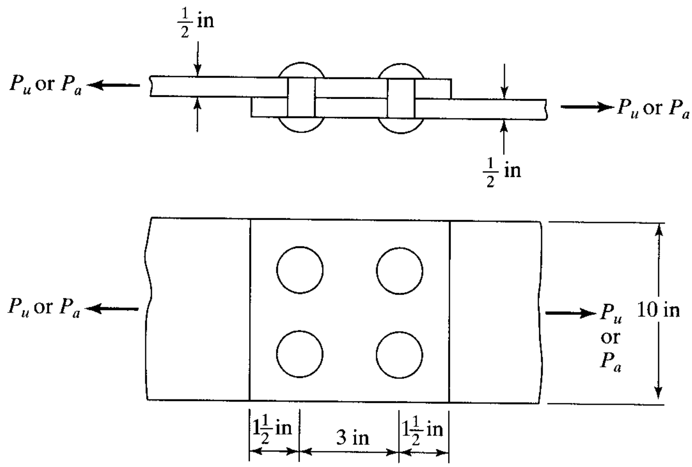


FIGURE 13.21

are used in the connection, and it is assumed that standard-size holes are used and that edge distances and center-to-center distances are = 1.5 in and 3 in, respectively. Neglect block shear. The rivets are 3/4 in in diameter and their F_{nv} is 25 ksi.

Solution. Design tensile force applied to plates

$$A_g = \left(\frac{1}{2}\text{ in}\right)(10\text{ in}) = 5.00\text{ in}^2$$

$$A_n = \left[\left(\frac{1}{2}\text{ in}\right)(10\text{ in}) - (2)\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right)\right] = 4.125\text{ in}^2$$

For tensile yielding

$$P_n = F_y A_g = (36\text{ ksi})(5.00\text{ in}^2) = 180\text{ k}$$

LRFD $\phi_t = 0.90$	ASD $\Omega_t = 1.67$
$\phi_t P_n = (0.90)(180) = 162\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{180}{1.67} = 107.8\text{ k}$

For tensile rupture

$$A_e = UA_n = 1.0 \times 4.125\text{ in}^2 = 4.125\text{ in}^2$$

$$P_n = F_u A_e = (58\text{ ksi})(4.125\text{ in}^2) = 239.25\text{ k}$$

LRFD $\phi_t = 0.75$	ASD $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(239.25) = 179.4\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{239.25}{2.00} = 119.6\text{ k}$

Rivets in single shear and bearing on 1/2 in

$$R_n = (A_{rivet})(F_{nv})(\text{no. of rivets}) = (0.44\text{ in}^2)(25\text{ ksi})(4) = 44.0\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$	
$\phi R_n = (0.75)(44.0) = 33.0\text{ k}$	$\frac{R_n}{\Omega} = \frac{44.0}{2.00} = 22.0\text{ k}$	← Controls

$$\text{For bearing with } l_c = 1.50 - \frac{\frac{3}{4} + \frac{1}{8}}{2} = 1.06\text{ in}$$

$$R_n = 1.2 l_c t F_u\,(\text{no. of rivets}) = (1.2)(1.06)\left(\frac{1}{2}\right)(58)(4) = 147.55\text{ k}$$

$$< 2.4\, dtF_u\,(\text{no. of rivets}) = (2.4)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)(58)(4) = 208.8\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(147.55) = 110.7\text{ k}$	$\frac{R_n}{\Omega} = \frac{147.55}{2.0} = 73.8\text{ k}$

Ans. $P_u = 33.0$ **k** ***Ans.*** $P_a = 22.0$ **k**

Example 13-10

How many 7/8-in A502, Grade 1 rivets are required for the connection shown in Fig. 13.22 if the plates are A36, standard-size holes are used, and the edge distances and center-to-center distances are 1.5 in and 3 in, respectively? Solve by both the LRFD and ASD methods. For ASD, use the Ω values assumed for Example 13-9. $P_u = 170$ k and $P_a = 120$ k.

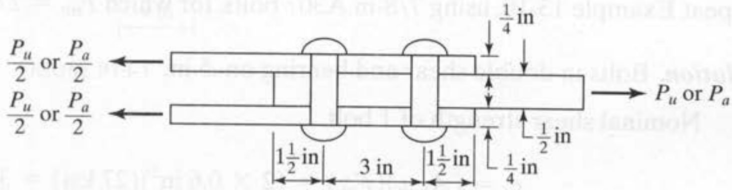


FIGURE 13.22

Solution

Rivets in double shear and bearing on 1/2 in

Nominal double shear strength of 1 rivet

$$r_n = (2A_{rivet})(F_{nv}) = (2 \times 0.6\text{ in}^2)(25.0\text{ ksi}) = 30\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$	
$\phi r_n = (0.75)(30.0) = 22.5\text{ k}$	$\frac{r_n}{\Omega} = \frac{30.00}{2.00} = 15.0\text{ k}$	← Controls

Nominal bearing strength of 1 rivet

$$l_c = 1.50 - \frac{\frac{7}{8} + \frac{1}{8}}{2} = 1.00\text{ in}$$

$$r_n = 1.2 l_c t F_u = (1.2)(1.0)\left(\frac{1}{2}\right)(58) = 34.8\text{ k}$$

$$< 2.4\, dtF_u = (2.4)\left(\frac{7}{8}\right)\left(\frac{1}{2}\right)(58) = 60.9\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi r_n = (0.75)(34.8) = 26.1$ k/rivet	$\dfrac{r_n}{\Omega} = \dfrac{34.8}{2.00} = 17.4$ k/rivet

LRFD	ASD
No. of rivets reqd $= \dfrac{170}{22.5} = 7.56$ **Use eight $\frac{7}{8}$-in A502, grade 1 rivets.**	No of rivets reqd $= \dfrac{120}{15.0} = 8$ **Use eight $\frac{7}{8}$-in A502, grade 1 rivets.**

Example 13-11

Repeat Example 13-10, using 7/8-in A307 bolts, for which $F_{nv} = 27$ ksi.

Solution. Bolts in double shear and bearing on .5 in:

Nominal shear strength of 1 bolt

$$r_n = (A_{bolt})(F_{nv}) = (2 \times 0.6 \text{ in}^2)(27 \text{ ksi}) = 32.4 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi r_n = (0.75)(32.4) = 24.3$ k	$\dfrac{r_n}{\Omega} = \dfrac{32.4}{2.00} = 16.2$ k ← Controls

Nominal bearing strength of 1 bolt

$$r_n = 1.2\, l_c\, t F_u = (1.2)(1.0)\left(\frac{1}{2}\right)(58) = 34.8 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi r_n = (0.75)(34.8) = 26.1$ k	$\dfrac{r_n}{\Omega} = \dfrac{34.8}{2.00} = 17.4$ k

LRFD	ASD
No. of bolts reqd $= \dfrac{170}{24.3} = 7.00$ **Use seven $\frac{7}{8}$-in A307 bolts.**	No. of bolts reqd $= \dfrac{120}{16.2} = 7.41$ **Use eight $\frac{7}{8}$-in A307 bolts.**

13.9 PROBLEMS FOR SOLUTION

For each of the problems listed, the following information is to be used, unless otherwise indicated: (a) A36 steel; (b) standard-size holes; (c) threads of bolts excluded from shear plane.

13-1 to 13-7. *Determine the resultant load on the most stressed bolt in the eccentrically loaded connections shown, using the elastic method.*

13-1. (*Ans.* 23.26 k)

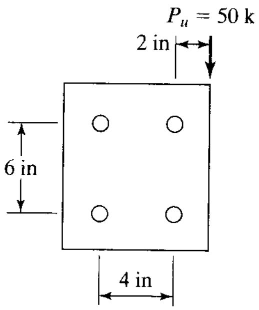


FIGURE P13-1

13-2.

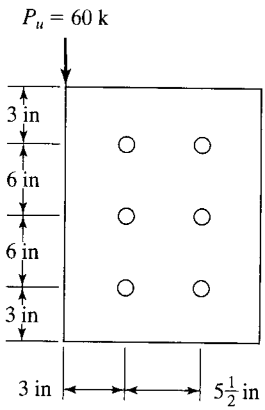


FIGURE P13-2

13-3. (*Ans.* 16.49 k)

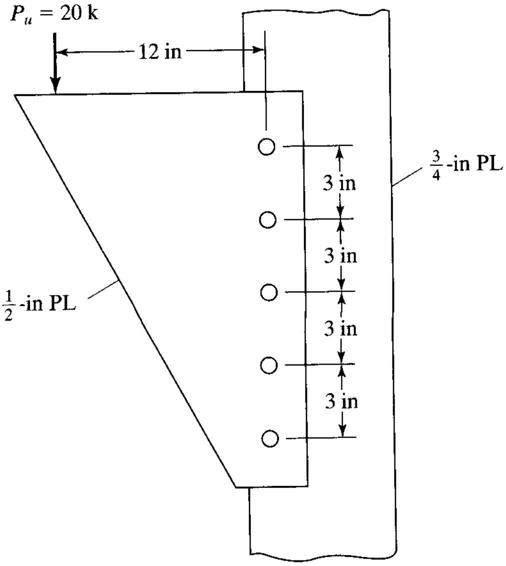


FIGURE P13-3

13-4.

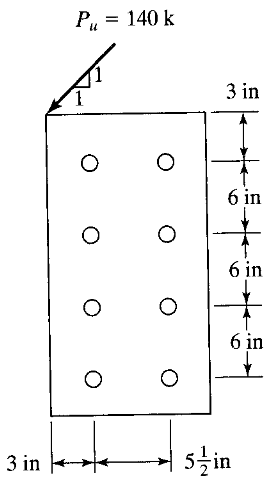


FIGURE P13-4

13-5. (*Ans.* 21.87 k)

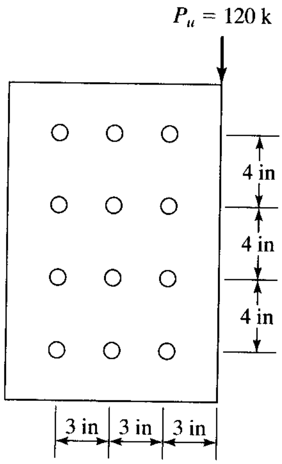


FIGURE P13-5

13-6.

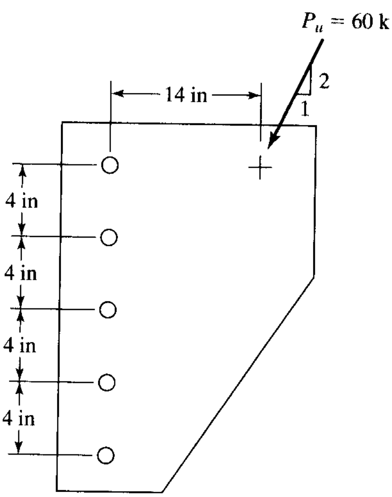


FIGURE P13-6

13-7. (*Ans.* 33.75 k)

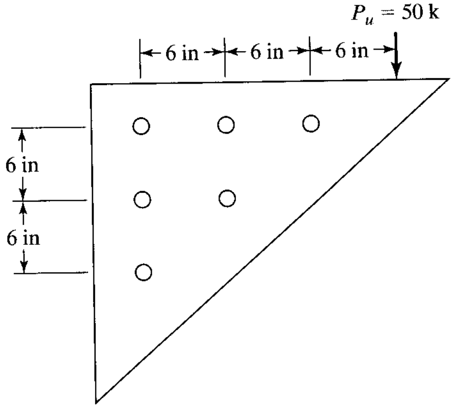


FIGURE P13-7

13-8. Repeat Prob. 13-2, using the reduced eccentricity method given in Section 13.1.2

13-9. Using the elastic method, determine the LRFD design strength and the ASD allowable strength of the bearing-type connection shown. The bolts are 3/4 in A325 and are in single shear and bearing on 5/8 in. The holes are standard sizes and the bolt threads are excluded from the shear plane.(*Ans.* 58.0 k LRFD, 38.7 k ASD)

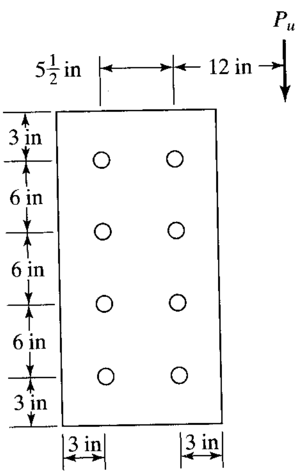


FIGURE P13-9

13-10. Using the elastic method, determine the ASD allowable strength P_n/Ω and the LRFD design strength, ϕP_n for the slip-critical connection shown. The 7/8-in A325 bolts are in "double shear." All plates are 1/2-in thick. Surfaces are Class A. Holes are standard sizes and $h_f = 1.0$.

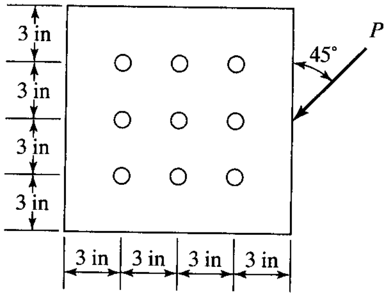


FIGURE P13-10

13-11. Repeat Prob. 13-9, using the ultimate strength tables entitled "Coefficients C for Eccentrically Loaded Bolt Groups" in Part 7 of the AISC Manual. (*Ans.* 73.6 k)

13-12. Repeat Prob. 13-10, using the ultimate strength tables entitled "Coefficients C for Eccentrically Loaded Bolt Groups" in Part 7 of the AISC Manual.

13-13. Is the bearing-type connection shown in the accompanying illustration sufficient to resist the 200 k load that passes through the center of gravity of the bolt group, according to the LRFD and ASD specifications? (*Ans.* $\phi F'_{nt} = 54.7$ ksi, $F'_{nt}/\Omega = 37.8$ ksi. Therefore connection is satisfactory.)

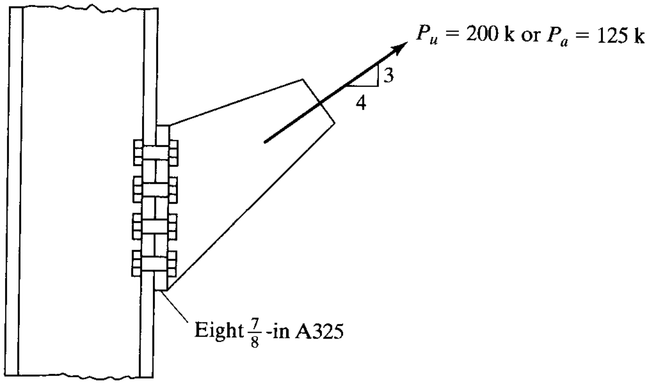


FIGURE P13-13

13-14. Repeat Prob. 13-13 if slip-critical bolts for the required strength level are used and if surfaces are Class A, $h_f = 1.00$ and standard size holes are used.

13-15. If the load shown in the accompanying bearing-type illustration passes through the center of gravity of the bolt group, how large can it be, according to both the LRFD and ASD specifications? Bolt threads are excluded from the shear planes. (*Ans.* 148.7 k LRFD, 99.2 k ASD)

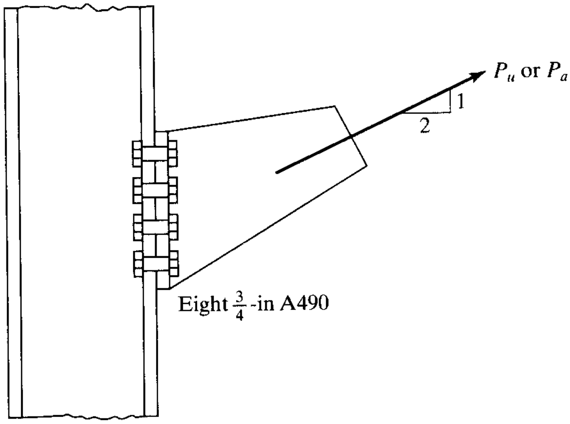


FIGURE P13-15

13-16. Repeat Prob. 13-15 if bolts are A325.

13-17 to 13-24. *Solve these problems by both the LRFD and ASD methods.*

13-17. Determine the number of 3/4-in A325 bolts required in the angles and in the flange of the W shape shown in the accompanying illustration if a bearing-type (snug-tight) connection is used. Use 50 ksi steel, $F_u = 65$ ksi, $L_c = 1.0$ in. Deformation around the bolt holes is a design consideration. (*Ans.* 4 in angle, both LRFD & ASD; 8 in W section, both LRFD & ASD)

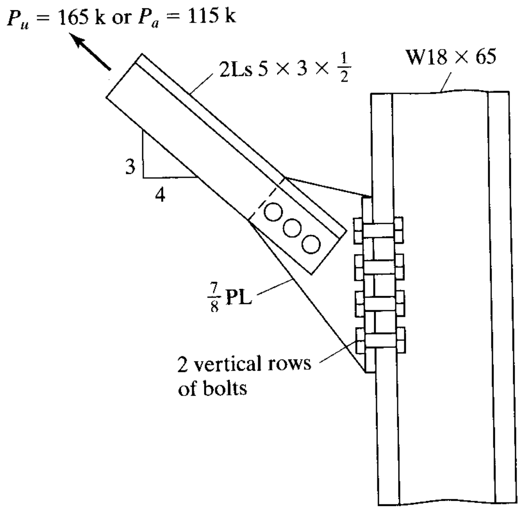


FIGURE P13-17

13-18. Determine the design strength ϕP_n and the allowable strength P_n/Ω of the connection shown if 3/4-in A502, Grade 1 rivets and A36 steel are used. Assume $F_v = 25$ ksi and threads excluded from shear planes.

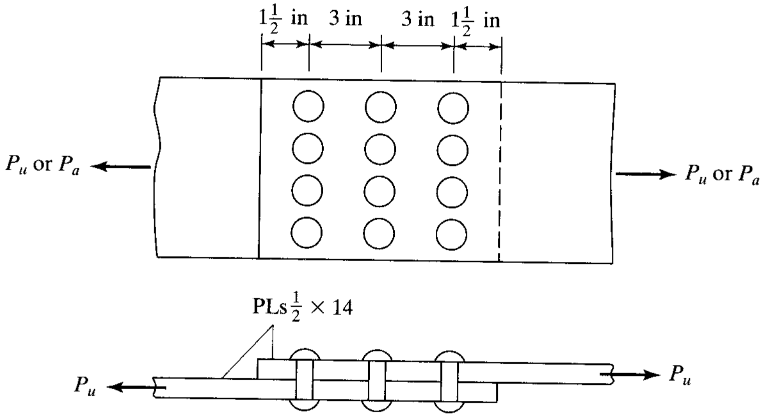


FIGURE P13-18

13-19. The truss tension member shown consists of a single-angle 5 × 3 × 5/16 and is connected to a 1/2-in gusset plate with five 7/8-in A502, Grade 1 rivets. Determine ϕP_n and P_n/Ω if U is assumed to equal 0.9. Neglect block shear. Steel is A36. $F_v = 25$ ksi. (*Ans.* 56.2 k, 37.5 k)

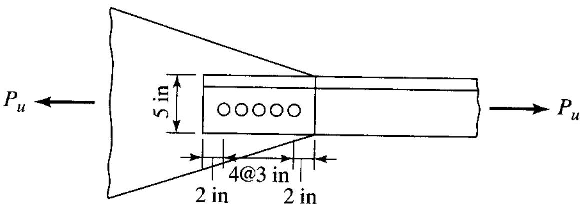


FIGURE P13-19

13-20. How many 7/8-in A502, Grade 1 rivets are needed to carry the load shown in the accompanying illustration if $F_v = 25$ ksi?

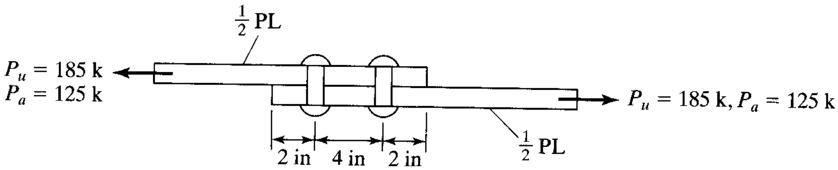


FIGURE P13-20

13-21. Repeat Prob. 13-20 if A307 bolts are used. (*Ans.* 22 LRFD; 22 ASD)

13-22. How many A502, Grade 1 rivets with 1-in diameters need to be used for the butt joint shown? $P_D = 60$ k, $P_L = 80$ k.

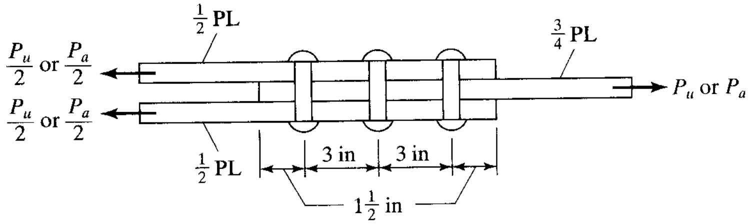


FIGURE P13-22

13-23. For the connection shown in the accompanying illustration, $P_u = 475$ k and $P_a = 320$ k, determine the number of 7/8-in A502, Grade 2 rivets required. $F_v = 25$ ksi (*Ans.* 22 LRFD; 22 ASD)

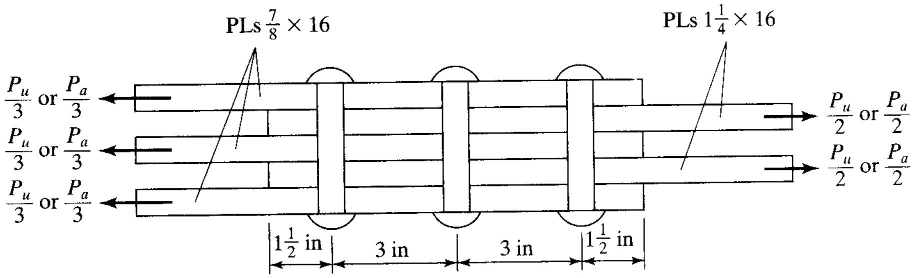


FIGURE P13-23

13-24. For the A36 beam shown in the accompanying illustration, what is the required spacing of 7/8-in A307 bolts if $V_u = 140$ k and $V_a = 100$ k? Assume that $F_v = 25$ ksi, $L_c = 1.50$ in.

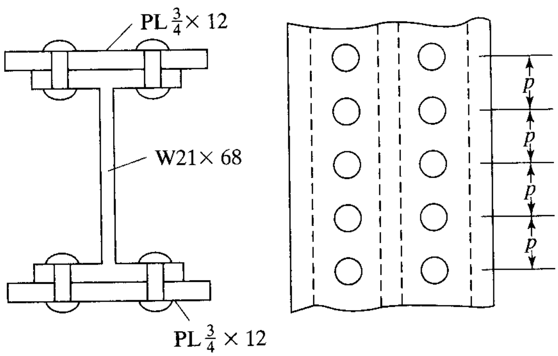


FIGURE P13-24

CHAPTER 14

Welded Connections

14.1 GENERAL

Welding is a process by which metallic parts are connected by heating their surfaces to a plastic or fluid state and allowing the parts to flow together and join (with or without the addition of other molten metal). It is impossible to determine when welding originated, but it was at least several thousand years ago. Metal-working, including welding, was quite an art in ancient Greece three thousand years ago, but welding had undoubtedly been performed for many centuries before that. Ancient welding probably was a forging process in which the metals were heated to a certain temperature (not to the melting stage) and hammered together.

Although modern welding has been available for many years, it has come into its own only in the last few decades for the building and bridge phases of structural engineering. The adoption of structural welding was quite slow for several decades, because many engineers thought that welding had two major disadvantages: (1) Welds had reduced fatigue strength, compared with riveted and bolted connections, and (2) it was impossible to ensure a high quality of welding without unreasonably extensive and costly inspection.

These attitudes persisted for many years, although tests began to indicate that neither reason was valid. Regardless of their validity, these views were widely held and undoubtedly slowed down the use of welding—particularly for highway bridges and, to an even greater extent, railroad bridges. Today, most engineers agree that welded joints have considerable fatigue strength. They will also admit that the rules governing the qualification of welders, the better techniques applied, and the excellent workmanship requirments of the AWS (American Welding Society) specifications make the inspection of welding a much less difficult problem. Furthermore, the chemistry of steels manufactured today is especially formulated to improve their weldability. Consequently, welding is now permitted for almost all structural work.

On the subject of welding, it is interesting to consider welded ships. Ships are subjected to severe impactive loadings that are difficult to predict, yet naval architects use

Roof framing for Cherokee Central Schools, Cherokee, NC. (Courtesy of CMC South Carolina Steel.)

all-welded ships with great success. A similar discussion can be made for airplanes and aeronautical engineers. The slowest adoption of structural welding was for railroad bridges. These bridges are undoubtedly subjected to heavier live loads, larger vibrations, and more stress reversals than highway bridges; but are their stress situations as serious and as difficult to predict as those for ships and planes?

14.2 ADVANTAGES OF WELDING

Today, it is possible to make use of the many advantages that welding offers, since the fatigue and inspection fears have been largely eliminated. Following are several of the many advantages that welding offers:

1. To most designers, the first advantage is economic, because the use of welding permits large savings in pounds of steel used. Welded structures allow the elimination of a large percentage of the gusset and splice plates necessary for bolted structures, as well as the elimination of bolt heads. In some bridge trusses, it may be possible to save up to 15 percent or more of the steel weight by welding.
2. Welding has a much wider range of application than bolting. Consider a steel pipe column and the difficulties of connecting it to other steel members by bolting. A bolted connection may be virtually impossible, but a welded connection presents few difficulties. Many similar situations can be imagined in which welding has a decided advantage.
3. Welded structures are more rigid, because the members often are welded directly to each other. Frequently, the connections for bolted structures are made through

intermediate connection angles or plates that deform due to load transfer, making the entire structure more flexible. On the other hand, greater rigidity can be a disadvantage where simple end connections with little moment resistance are desired. In such cases, designers must be careful as to the type of joints they specify.

4. The process of fusing pieces together creates the most truly continuous structures. Fusing results in one-piece construction, and because welded joints are as strong as or stronger than the base metal, no restrictions have to be placed on the joints. This continuity advantage has permitted the erection of countless slender and graceful statically indeterminate steel frames throughout the world. Some of the more outspoken proponents of welding have referred to bolted structures, with their heavy plates and abundance of bolts, as looking like tanks or armored cars compared with the clean, smooth lines of welded structures. For a graphic illustration of this advantage, compare the moment-resisting connections of Fig. 15.5.
5. It is easier to make changes in design and to correct errors during erection (and less expensive) if welding is used. A closely related advantage has certainly been illustrated in military engagements during the past few wars by the quick welding repairs made to military equipment under battle conditions.
6. Another item that is often important is the relative silence of welding. Imagine the importance of this fact when working near hospitals or schools or when making additions to existing buildings. Anyone with close-to-normal hearing who has attempted to work in an office within several hundred feet of a bolted job can attest to this advantage.
7. Fewer pieces are used, and as a result, time is saved in detailing, fabrication, and field erection.

14.3 AMERICAN WELDING SOCIETY

The American Welding Society's *Structural Welding Code*[1] is the generally recognized standard for welding in the United States. The AISC Specification clearly states that the provisions of the AWS Code apply under the AISC Specification, with only a few minor exceptions, and these are listed in AISC Specification J2. Both the AWS and the AASHTO Specifications cover dynamically loaded structures: Generally, the AWS specification is used for designing the welds for buildings subject to dynamic loads.

14.4 TYPES OF WELDING

Although both gas and arc welding are available, almost all structural welding is arc welding. Sir Humphry Davy discovered in 1801 how to create an electric arc by bringing close together two terminals of an electric circuit of relatively high voltage. Although he is generally given credit for the development of modern welding, a good many years elapsed after his discovery before welding was actually performed with the electric arc. (His work was of the greatest importance to the modern structural world, but it is

[1]American Welding Society, *Structural Welding Code-Steel*, AWS D.1.1-00 (Miami: AWS, 2006).

interesting to note that many people say his greatest discovery was not the electric arc, but rather a laboratory assistant whose name was Michael Faraday.) Several Europeans formed welds of one type or another in the 1880s with the electric arc, while in the United States the first patent for arc welding was given to Charles Coffin of Detroit in 1889.[2]

The figures shown in this chapter illustrate the necessity of supplying additional metal to the joints being welded to give satisfactory connections. In electric-arc welding, the metallic rod, which is used as the electrode, melts off into the joint as it is being made. When gas welding is used, it is necessary to introduce a metal rod known as a *filler* or *welding rod.*

In gas welding, a mixture of oxygen and some suitable type of gas is burned at the tip of a torch or blowpipe held in the welder's hand or by machine. The gas used in structural welding usually is acetylene, and the process is called *oxyacetylene welding.* The flame produced can be used for flame cutting of metals, as well as for welding. Gas welding is fairly easy to learn, and the equipment used is rather inexpensive. It is a slow process, however, compared with other means of welding, and normally it is used for repair and maintenance work and not for the fabrication and erection of large steel structures.

In arc welding, an electric arc is formed between the pieces being welded and an electrode held in the operator's hand with some type of holder, or by an automatic machine. The arc is a continuous spark that, upon contact, brings the electrode and the pieces being welded to the melting point. The resistance of the air or gas between the electrode and the pieces being welded changes the electrical energy into heat. A temperature of 6000 to 10,000°F is produced in the arc. As the end of the electrode melts, small droplets, or globules, of the molten metal are formed and actually are forced by the arc across to the pieces being connected, which penetrate the molten metal to become a part of the weld. The amount of penetration can be controlled by the amount of current consumed. Since the molten droplets of the electrodes actually are propelled into the weld, arc welding can be successfully used for overhead work.

A pool of molten steel can hold a fairly large amount of gases in solution and, if not protected from the surrounding air, will chemically combine with oxygen and nitrogen. After cooling, the welds will be somewhat porous due to the little pockets formed by the gases. Such welds are relatively brittle and have much less resistance to corrosion. A welded joint can be shielded by using an electrode coated with certain mineral compounds. The electric arc causes the coating to melt and creates an inert gas or vapor around the area being welded. The vapor acts as a shield around the molten metal and keeps it from coming freely in contact with the surrounding air. It also deposits a slag in the molten metal, which has less density than the base metal and comes to the surface to protect the weld from the air while the weld cools. After cooling, the slag can easily be removed by peening and wire brushing (such removal being absolutely necessary before painting or application of another weld layer). The elements of the shielded arc welding process are shown in Fig. 14.1. This figure is taken from the *Procedure Handbook of Arc Welding Design and Practice* published by the Lincoln Electric Company. *Shielded metal arc welding* is abbreviated here with the letters SMAW.

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[2]Lincoln Electric Company, *Procedure Handbook of Arc Welding Design and Practice*, 11th ed. Part I (Cleveland, OH, 1957).

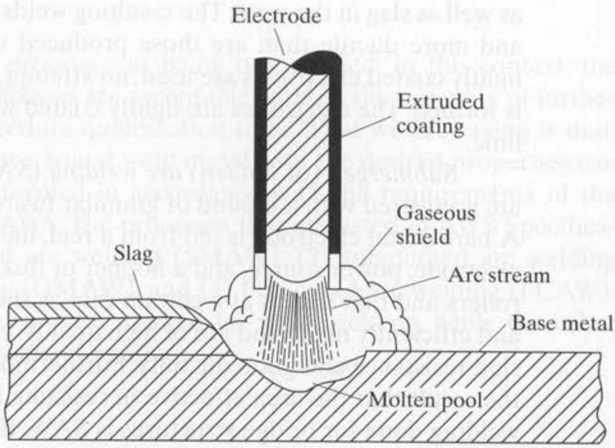


FIGURE 14.1
Elements of the shielded metal arc welding process (SMAW).

The type of welding electrode used is very important because it decidedly affects the weld properties such as strength, ductility, and corrosion resistance. Quite a number of different kinds of electrodes are manufactured, the type to be used for a certain job being dependent upon the metal to be welded, the amount of material that needs to be added, the position of the work, etc. The electrodes fall into two general classes—the *lightly coated electrodes* and the *heavily coated electrodes*.

The heavily coated electrodes are normally used in structural welding because the melting of their coatings produces very satisfactory vapor shields around the work,

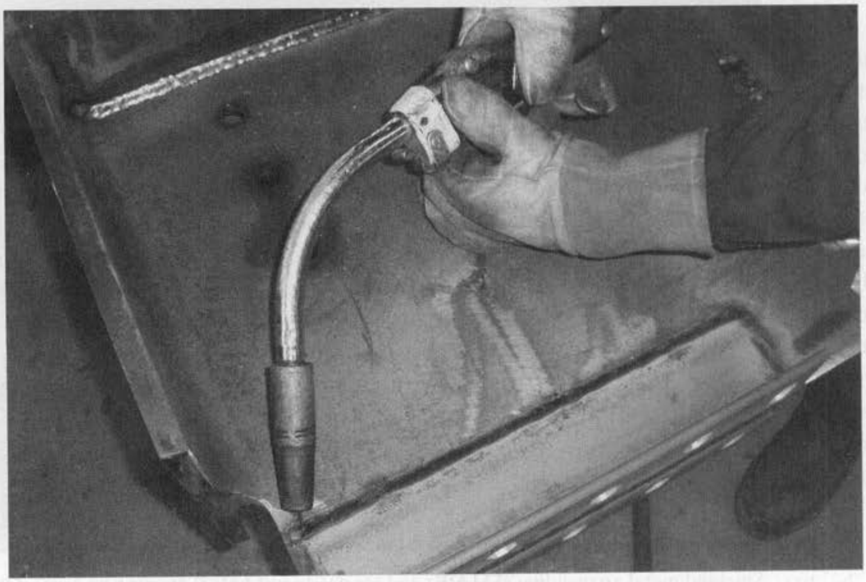
Shielded metal arc welding (SMAW) and electrode just before starting an arc to fillet weld the clip angle to the beam web. (Courtesy of CMC South Carolina Steel.)

as well as slag in the weld. The resulting welds are stronger, more resistant to corrosion, and more ductile than are those produced with lightly coated electrodes. When the lightly coated electrodes are used, no attempt is made to prevent oxidation, and no slag is formed. The electrodes are lightly coated with some arc-stabilizing chemical such as lime.

Submerged (or *hidden*) *arc welding* (SAW) is an automatic process in which the arc is covered with a mound of granular fusible material and is thus hidden from view. A bare metal electrode is fed from a reel, melted, and deposited as filler material. The electrode, power source, and a hopper of flux are attached to a frame that is placed on rollers and that moves at a certain rate as the weld is formed. SAW welds are quickly and efficiently made and are of high quality, exhibiting high impact strength and corrosion resistance and good ductility. Furthermore, they provide deeper penetration, with the result that the area effective in resisting loads is larger. A large percentage of the welding done for bridge structures is SAW. If a single electrode is used, the size of the weld obtained with a single pass is limited. Multiple electrodes may be used, however, permitting much larger welds.

Welds made by the SAW process (automatic or semiautomatic) are consistently of high quality and are very suitable for long welds. One disadvantage is that the work must be positioned for near-flat or horizontal welding.

Another type of welding is *flux-cored arc welding* (FCAW). In this process, a flux-filled steel tube electrode is continuously fed from a reel. Gas shielding and slag are formed from the flux. The AWS Specification (4.14) provides limiting sizes for welding electrode diameters and weld sizes, as well as other requirements pertaining to welding procedures.



Flux-cored arc welding (FCAW). (Courtesy of CMC South Carolina Steel.)

14.5 PREQUALIFIED WELDING

The AWS accepts four welding processes as being prequalified. In this context, the word *prequalified* means that processes are acceptable without the necessity of further proof of their suitability by procedure qualification tests. What we are saying is that, based on many years of experience, sound weld metal with the desired properties can be deposited if the work is performed in accordance with the requirements of the Structural Welding Code of the AWS. The processes that are listed in AWS Specification 1.3.1 are (1) shielded metal arc welding (SMAW), (2) submerged arc welding (SAW), (3) gas metal arc welding (GMAW), and (4) flux-cored arc welding (FCAW). The SMAW process is the usual process applied for hand welding, while the other three are typically automatic or semiautomatic.

14.6 WELDING INSPECTION

Three steps must be taken to ensure good welding for a particular job: (1) establishment of good welding procedures, (2) use of prequalified welders, and (3) employment of competent inspectors in both the shop and the field.

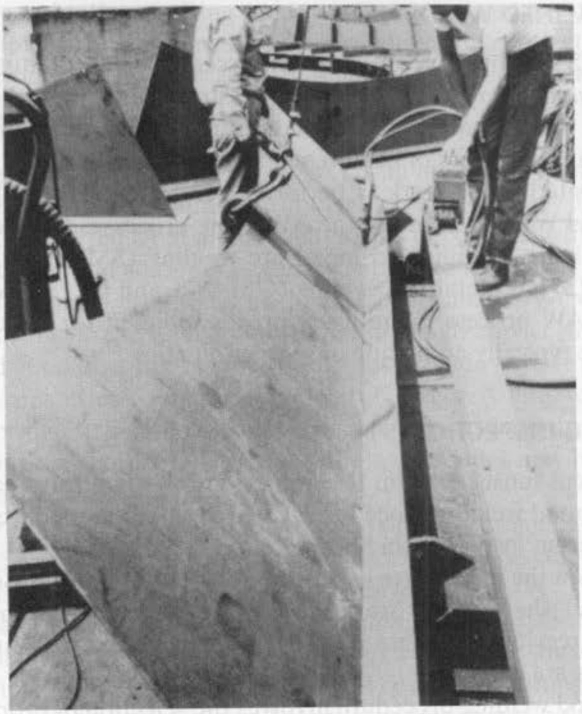
When the procedures established by the AWS and AISC for good welding are followed, and when welders are used who have previously been required to prove their ability, good results usually are obtained. To make absolutely sure, however, well-qualified inspectors are needed.

Good welding procedure involves the selection of proper electrodes, current, and voltage; the properties of base metal and filler; and the position of welding—to name only a few factors. The usual practice for large jobs is to employ welders who have certificates showing their qualifications. In addition, it is not a bad practice to have each person make an identifying mark on each weld so that those frequently doing poor work can be identified. This practice tends to improve the general quality of the work performed.

14.6.1 Visual Inspection

Another factor that will cause welders to perform better work is simply the presence of an inspector who they feel knows good welding when he or she sees it. A good inspector should have done welding and spent much time observing the work of good welders. From this experience, he or she should be able to know if a welder is obtaining satisfactory fusion and penetration. He or she also should be able to recognize good welds in regard to shape, size, and general appearance. For instance, the metal in a good weld should approximate its original color after it has cooled. If it has been overheated, it may have a rusty and reddish-looking color. An inspector can use various scales and gages to check the sizes and shapes of welds.

Visual inspection by a competent person usually gives a good indication of the quality of welds, but is not a perfect source of information, especially regarding the subsurface condition of the weld. It surely is the most economical inspection method and is particularly useful for single-pass welds. This method, however, is good only for picking up surface imperfections. There are several methods for determining the internal



Lincoln ML-3 Squirtwelder mounted on a self-propelled trackless trailer deposits this 1/4-in web-to-flange weld at 28 in/min. (Courtesy of the Lincoln Electric Company.)

soundness of a weld, including the use of penetrating dyes and magnetic particles, ultrasonic testing, and radiographic procedures. These methods can be used to detect internal defects such as porosity, weld penetration, and the presence of slag.

14.6.2 Liquid Penetrants

Various types of dyes can be spread over weld surfaces. These dyes will penetrate into the surface cracks of the weld. After the dye has penetrated into the crack, the excess surface material is wiped off and a powdery developer is used to draw the dye out of the cracks. The outlines of the cracks can then be seen with the eye. Several variations of this method are used to improve the visibility of the defects, including the use of fluorescent dyes. After the dye is drawn from the cracks, they stand out brightly under a black light.[3] Like visual inspection, this method enables us to detect cracks that are open to the surface.

[3]James Hughes, "It's Superinspector." *Steelways*, 25. no. 4 (New York: American Iron and Steel Institute. September/October, 1969), pp. 19–21.

14.6.3 Magnetic Particles

In this method, the weld being inspected is magnetized electrically. Cracks that are at or near the surface of the weld cause north and south poles to form on each side of the cracks. Dry iron powdered filings or a liquid suspension of particles is placed on the weld. These particles form patterns when many of them cling to the cracks, showing the locations of cracks and indicating their size and shape. Only cracks, seams, inclusions, etc., within about 1/10 in of the surface can be located by this method. A disadvantage is that if multilayer welds are used, the method has to be applied to each layer.

14.6.4 Ultrasonic Testing

In recent years, the steel industry has applied ultrasonics to the manufacture of steel. Although the equipment is expensive, the method is quite useful in welding inspections as well. Sound waves are sent through the material being tested and are reflected from the opposite side of the material. These reflections are shown on a cathode ray tube. Defects in the weld will affect the time of the sound transmission. The operator can read the picture on the tube and then locate flaws and learn how severe they are. Ultrasonic testing can sucessfully be used to locate discontinuities in carbon and low-alloy steels, but it doesn't work too well for some stainless steels or for extremely coarse-grained steels.

14.6.5 Radiographic Procedures

The more expensive radiographic methods can be used to check occasional welds in important structures. From these tests, it is possible to make good estimates of the percentage of bad welds in a structure. Portable x-ray machines (where access is not a problem) and radium or radioactive cobalt for making pictures are excellent, but expensive methods of testing welds. These methods are satisfactory for butt welds (such as for the welding of important stainless steel piping at chemical and nuclear projects), but they are not satisfactory for fillet welds, because the pictures are difficult to interpret. A further disadvantage of such methods is the radioactive danger. Careful procedures have to be used to protect the technicians as well as nearby workers. On a construction job, this danger generally requires night inspection of welds, when only a few workers are near the inspection area. (Normally, a very large job would be required before the use of the extremely expensive radioactive materials could be justified.)

A properly welded connection can always be made much stronger—perhaps as much as two times stronger—than the plates being connected. As a result, the actual strength is much greater than is required by the specifications. The reasons for this extra strength are as follows: The electrode wire is made from premium steel, the metal is melted electrically (as is done in the manufacture of high-quality steels), and the cooling rate is quite rapid. As a result, it is rare for a welder to make a weld of less strength than required by the design.

14.7 CLASSIFICATION OF WELDS

Three separate classifications of welds are described in this section. These classifications are based on the types of welds made, the positions of the welds, and the types of joints used.

14.7.1 Type of Weld

The two main types of welds are the *fillet welds* and the *groove welds*. In addition, there are plug and slot welds, which are not as common in structural work. These four types of welds are shown in Fig. 14.2.

Fillet welds are those made where parts lap over each other, as shown in Fig. 14.2(a). They may also be used in tee joints, (as illustrated in Fig. 14.4). Fillet welds are the most economical welds to use, as little preparation of the parts to be connected is necessary. In addition, these welds can be made very well by welders of somewhat lesser skills than those required for good work with other types of welds.

The fillet welds will be shown to be weaker than groove welds, although most structural connections (about 80 percent) are made with fillet welds. Any person who has experience in steel structures will understand why fillet welds are more common than groove welds. Groove welds, as illustrated in Fig. 14.2(b) and (c) (which are welds made in grooves between the members to be joined) are used when the members to be connected are lined up in the same plane. To use them, the members have to fit almost

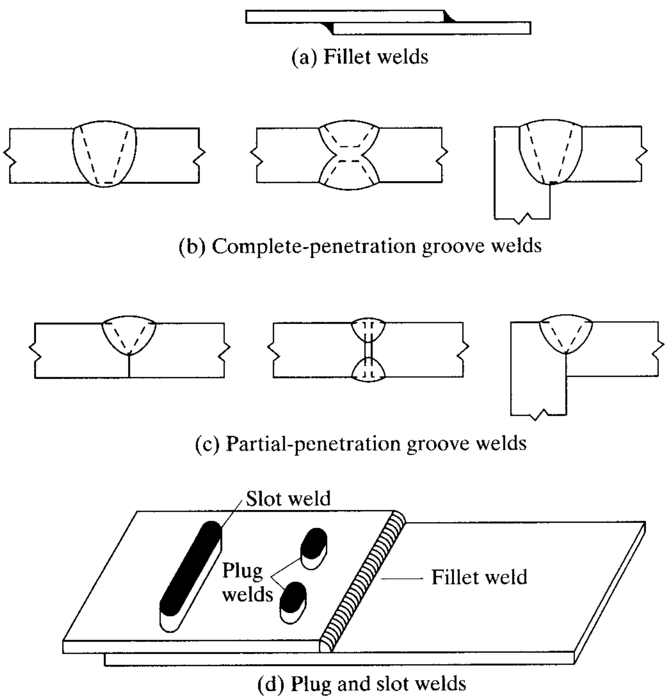


FIGURE 14.2

Four types of structural welds.

perfectly, and unfortunately, the average steel structure does not fit together that way. Have you ever seen steel workers pulling and ramming steel members to get them into position? When members are allowed to lap over each other, larger tolerances are allowable in erection, and fillet welds are used. Nevertheless, groove welds are quite common for many connections, such as column splices, butting of beam flanges to columns, etc., and they make up about 15 percent of structural welding. Groove welds can be either *complete-penetration* welds, which extend for the full thickness of the part being connected, or *partial penetration* welds, which extend for only part of the member thickness.

Groove welds are generally more expensive than fillet welds because of the costs of preparation. In fact, groove welds can cost up to 50 to 100 percent more than fillet welds.

A plug weld is a circular weld that passes through one member into another, thus joining the two together. A slot weld is a weld formed in a slot, or elongated hole, that joins one member to the other member through the slot. The slot may be partly or fully filled with weld material. These welds are shown in Fig. 14.2(d). These two expensive types of welds may occasionally be used when members lap over each other and the desired length of fillet welds cannot be obtained. They may also be used to stitch together parts of a member, such as the fastening of cover plates to a built-up member.

A plug or slot weld is not generally considered suitable for transferring tensile forces perpendicular to the faying surface, because there is not usually much penetration of the weld into the member behind the plug or slot and the fact is that resistance to tension is provided primarily by penetration.

Structural designers accept plug and slot welds as being satisfactory for stitching the different parts of a member together, but many designers are not happy using these welds for the transmission of shear forces. The penetration of the welds from the slots or plugs into the other members is questionable; in addition, there can be critical voids in the welds that cannot be detected with the usual inspection procedures.

14.7.2 Position

Welds are referred to as *flat, horizontal, vertical,* or *overhead*—listed in order of their economy, with the flat welds being the most economical and the overhead welds being the most expensive. A moderately skilled welder can do a very satisfactory job with a flat weld, but it takes the very best to do a good job with an overhead weld. Although the flat welds often are done with an automatic machine, most structural welding is done by hand. We indicated previously that the assistance of gravity is not necessary for the forming of good welds, but it does speed up the process. The globules of the molten electrodes can be forced into the overhead welds against gravity, and good welds will result; however, they are slow and expensive to make, so it is desirable to avoid them whenever possible. These types of welds are shown in Fig. 14.3.

14.7.3 Type of Joint

Welds can be further classified according to the type of joint used: *butt, lap, tee, edge, corner*, etc. See Fig. 14.4.

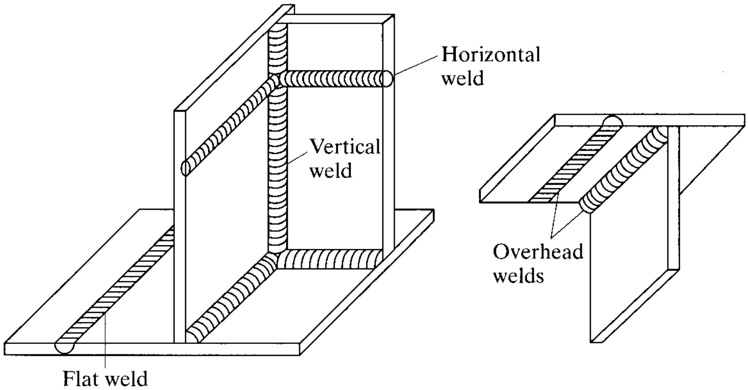


FIGURE 14.3

Weld positions.

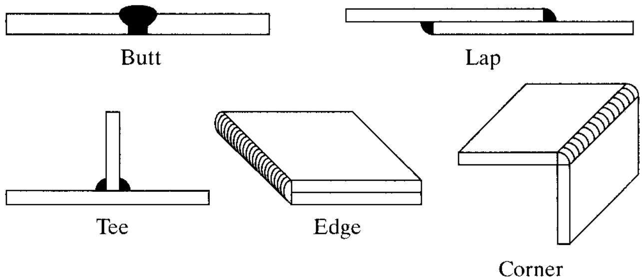


FIGURE 14.4

Types of weld joints.

14.8 WELDING SYMBOLS

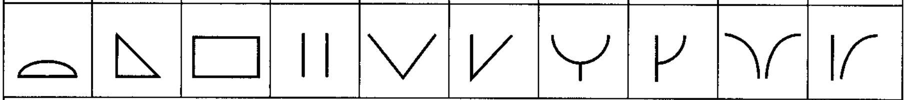
Figure 14.5 presents the various welding symbols developed by the American Welding Society. With this excellent shorthand system, a great deal of information can be presented in a small space on engineering plans and drawings. These symbols require only a few lines and numbers and remove the necessity of drawing in the welds and making long descriptive notes. It is certainly desirable for steel designers and draftsmen to use this standardized system. If most of the welds on a drawing are the same size, a note to that effect can be given and the symbols omitted, except for the off-size welds.

The purpose of this section is to give a general idea of the appearance of welding symbols and the information they can convey. (For more detailed information, refer to the AISC Handbook and to other materials published by the AWS.) The information presented in Fig. 14.5 may be quite confusing; for this reason, a few very common symbols for fillet welds are presented in Fig. 14.6, together with an explanation of each.

Prequalified Welded Joints

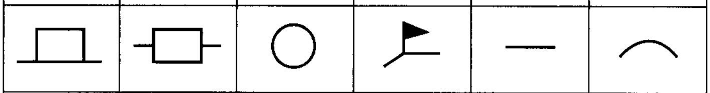
Basic Weld Symbols

Back	Fillet	Plug or Slot	Groove or Butt: Square	V	Bevel	U	J	Flare V	Flare Bevel

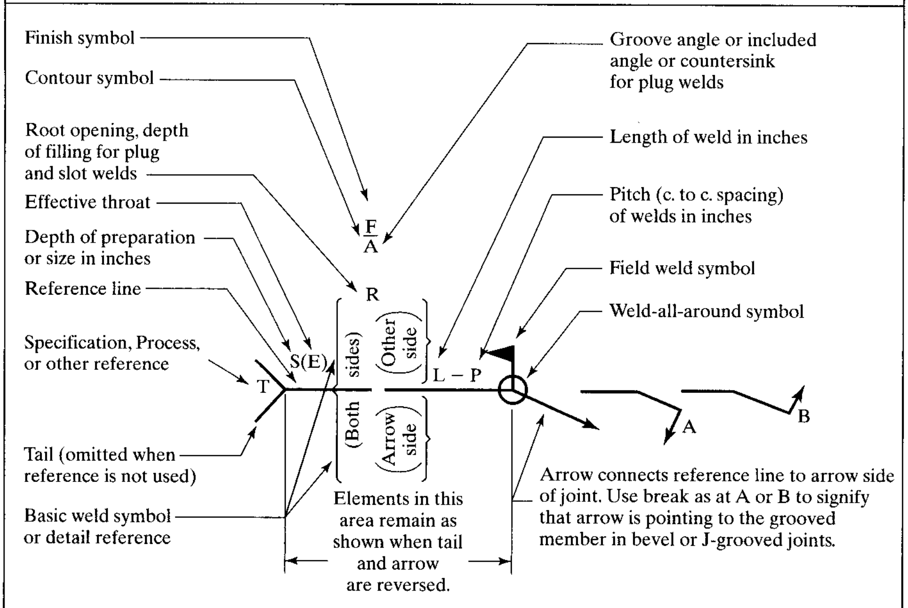


Supplementary Weld symbols

Backing	Spacer	Weld All Around	Field Weld	Contour: Flush	Contour: Convex	For other basic and supplementary weld symbols, see AWS A2.4



Standard Location of Elements of a Welding Symbol



Note:
Size, weld symbol, length of weld, and spacing must read in that order, from left to right, along the reference line. Neither orientation of reference nor location of the arrow alters this rule.
The perpendicular leg of ◺, V, P, I(, weld symbols must be at left.
Dimensions of fillet welds must be shown on both the arrow side and the other side.
Symbols apply between abrupt changes in direction of welding unless governed by the "all around" symbol or otherwise dimensioned.
These symbols do not explicitly provide for the case that frequently occurs in structural work, where duplicate material (such as stiffeners) occurs on the far side of a web or gusset plate. The fabricating industry has adopted this convention: that when the billing of the detail material discloses the existence of a member on the far side as well as on the near side, the welding shown for the near side shall be duplicated on the far side.

FIGURE 14.5

Source: AISC Manual, Table 8–2, p. 8–35, 14th ed, 2011.

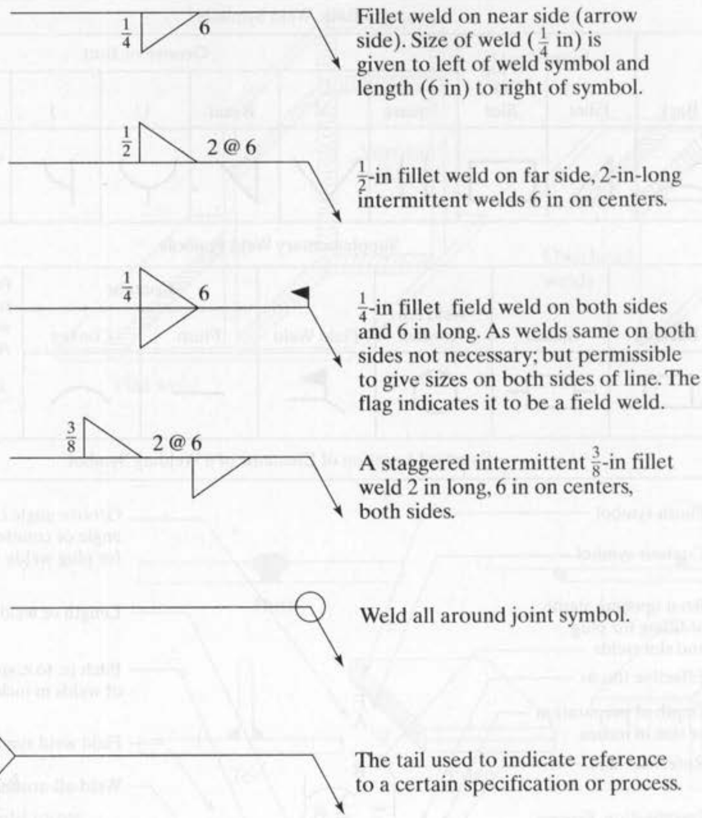


FIGURE 14.6
Sample weld symbols.

14.9 GROOVE WELDS

When complete penetration groove welds are subjected to axial tension or axial compression, the weld stress is assumed to equal the load divided by the net area of the weld. Three types of groove welds are shown in Fig. 14.7. The square groove joint,

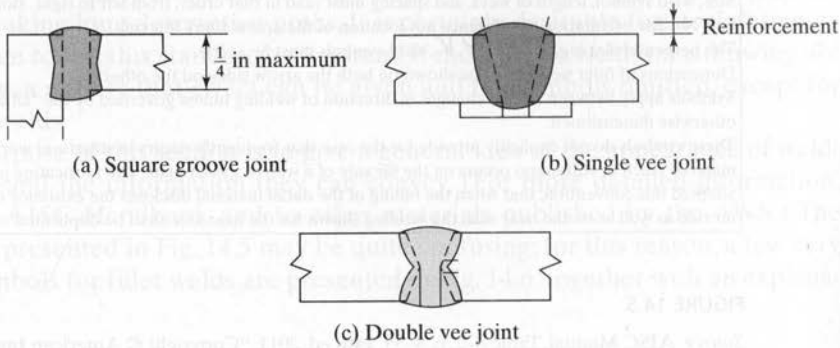


FIGURE 14.7
Groove welds.

shown in part (a) of the figure, is used to connect relatively thin material up to a maximum of 1/4 in thickness. As the material becomes thicker, it is necessary to use the single-vee groove welds and the double-vee groove welds illustrated in parts (b) and (c), respectively, of Fig. 14.7. For these two welds, the members are bevelled before welding to permit full penetration of the weld.

The groove welds shown in Fig. 14.7 are said to have *reinforcement.* Reinforcement is added weld metal that causes the throat dimension to be greater than the thickness of the welded material. Because of reinforcement, groove welds may be referred to as 125 percent, 150 percent, etc., according to the amount of extra thickness at the weld. There are two major reasons for having reinforcement: (1) reinforcement gives a little extra strength, because the extra metal takes care of pits and other irregularities; and (2) the welder can easily make the weld a little thicker than the welded material. It would be a difficult, if not impossible, task to make a perfectly smooth weld with no places that were thinner or thicker than the material welded.

Reinforcement undoubtedly makes groove welds stronger and better when they are subjected to static loads. When the connection is to be subjected to vibrating loads repeatedly, however, reinforcement is not as satisfactory, because stress concentrations develop in the reinforcement and contribute to earlier failure. For these kinds of cases, a common practice is to provide reinforcement and grind it off flush with the material being connected (AASHTO Section 10.34.2.1).

Figure 14.8 shows some of the edge preparations that may be necessary for groove welds. In part (a), a bevel with a feathered edge is shown. When feathered edges are used, there is a problem with burn-through. This may be lessened if a *land* is used, such as the one shown in part (b) of the figure, or a backup strip or backing bar, as shown in part (c). The backup strip is often a 1/4-in copper plate. Weld metal does not stick to copper, and copper has a very high conductivity that is useful in carrying away excess heat and reducing distortion. Sometimes, steel backup strips are used, but they will become a part of the weld and are thus left in place. A land should not be used together with a backup strip, because there is a high possibility that a gas pocket might be formed, preventing full penetration. When double bevels are used, as shown in part (d) of the figure, spacers are sometimes provided to prevent burn-through. The spacers are removed after one side is welded.

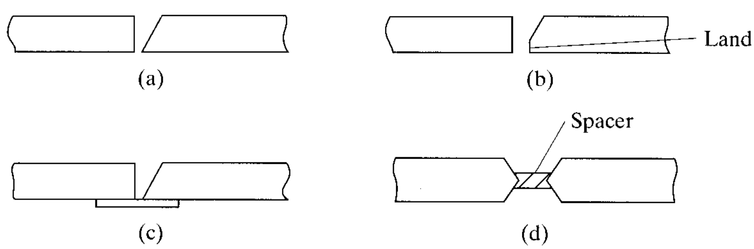


FIGURE 14.8

Edge preparation for groove welds. (a) Bevel with feathered edge. (b) Bevel with a land. (c) Bevel with a backup plate. (d) Double bevel with a spacer.

From the standpoints of strength, resistance to impact stress repetition, and amount of filler metal required, groove welds are preferable to fillet welds. From other standpoints, however, they are not so attractive, and the vast majority of structural welding is fillet welding. Groove welds have higher residual stresses, and the preparations (such as scarfing and veeing) of the edges of members for groove welds are expensive, but the major disadvantages more likely lie in the problems involved with getting the pieces to fit together in the field. (The advantages of fillet welds in this respect were described in Section 14.7.) For these reasons, field groove joints are not used often, except on small jobs and where members may be fabricated a little long and cut in the field to the lengths necessary for precise fitting.

14.10 FILLET WELDS

Tests have shown that fillet welds are stronger in tension and compression than they are in shear, so the controlling fillet weld stresses given by the various specifications are shearing stresses. When practical, it is desirable to try to arrange welded connections so that they will be subjected to shearing stresses only, and not to a combination of shear and tension or shear and compression.

When fillet welds are tested to failure with loads parallel to the weld axes, they seem to fail by shear at angles of about 45° through the throat. Their strength is therefore assumed to equal the design shearing stress or allowable shear stress times the theoretical throat area of the weld. The theoretical throats of several fillet welds are shown in Fig. 14.9. The throat area equals the theoretical throat distance times the length of the weld. In this figure, the root of the weld is the point at which the faces of the original metal pieces intersect, and the theoretical throat of the weld is the shortest distance from the root of the weld to its diagrammatic face.

For the 45° or equal leg fillet, the throat dimension is 0.707 times the leg of the weld, but it has a different value for fillet welds with unequal legs. The desirable fillet weld has a flat or slightly convex surface, although the convexity of the weld does not

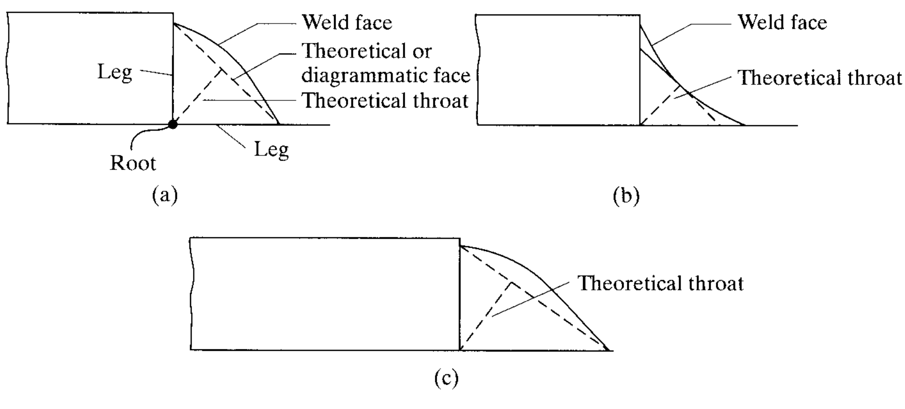
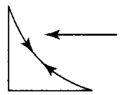


FIGURE 14.9

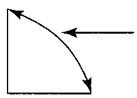
(a) Convex surface. (b) Concave surface. (c) Unequal leg fillet weld.

add to its calculated strength. At first glance, the concave surface would appear to give the ideal fillet weld shape, because stresses could apparently flow smoothly and evenly around the corner, with little stress concentration. Years of experience, however, have shown that single-pass fillet welds of a concave shape have a greater tendency to crack upon cooling, and this factor has proved to be of greater importance than the smoother stress distribution of convex types.

When a concave weld shrinks, the surface is placed in tension, which tends to cause cracks.



When the surface of a convex weld shrinks, it does not place the outer surface in tension; rather, as the face shortens, it is placed in compression.



Another item of importance pertaining to the shape of fillet welds is the angle of the weld with respect to the pieces being welded. The desirable value of this angle is in the vicinity of 45°. For 45° fillet welds, the leg sizes are equal, and such welds are referred to by the leg sizes (e.g., a 1/4-in fillet weld). Should the leg sizes be different (not a 45° weld), both leg sizes are given in describing the weld (e.g., a 3/8-by-1/2-in fillet weld).

The effective throat thickness may be increased in the calculations if it can be shown that consistent penetration beyond the root of the diagrammatic weld is obtained by the weld process being used (AISC specification J2.2a). Submerged arc welding is one area in which consistent extra penetration has, in the past, been assured to occur.

14.11 STRENGTH OF WELDS

In the discussion that follows, reference is made to Fig. 14.10. The stress in a fillet weld is usually said to equal the load divided by the effective throat area of the weld, with no consideration given to the direction of the load. Tests have shown, however, that transversely loaded fillet welds are appreciably stronger than ones loaded parallel to the weld's axis.

Transverse fillet welds are stronger for two reasons: First, they are more uniformly stressed over their entire lengths, while longitudinal fillet welds are stressed unevenly, due to varying deformations along their lengths; second, tests show that failure occurs at angles other than 45°, giving them larger effective throat areas.

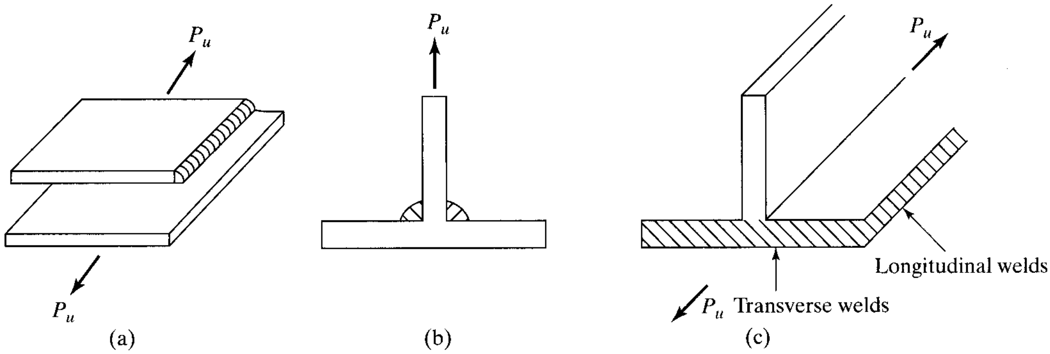


FIGURE 14.10
(a) Longitudinal fillet weld. (b) Transverse fillet weld. (c) Transverse and longitudinal welds.

The method of determining the strength of fillet welds along their longitudinal axes, regardless of the load directions, is usually used to simplify computations. It is rather common for designers to determine the strength of all fillet welds by assuming that the loads are applied in the longitudinal direction.

14.12 AISC REQUIREMENTS

When welds are made, the electrode material should have properties of the base metal. If the properties are comparable, the weld metal is referred to as the *matching base* metal. (That is, their nominal strengths are similar.)

Table 14.1 (which is Table J2.5 of the AISC Specification) provides nominal strengths for various types of welds, including fillet welds, plug and slot welds, and complete-penetration and partial-penetration groove welds.

The design strength of a particular weld (ϕR_n) and the allowable strength R_n/Ω of welded joints shall be the lower value of the base material strength determined according to the limit states of tensile rupture and shear rupture, and the weld metal strength determined according to the limit state of rupture by the expressions to follow:

For the base metal, the nominal strength is

$$R_n = F_{nBM} A_{BM} \qquad \text{(AISC Equation J2-2)}$$

For the weld metal, the nominal strength is

$$R_n = F_{nw} A_{we} \qquad \text{(AISC Equation J2-3)}$$

TABLE 14.1 Available Strength of Welded Joints, ksi (MPa)					
Load Type and Direction Relative to Weld Axis	**Pertinent Metal**	**ϕ and Ω**	**Nominal Strength (F_{nBM} or F_{nw}) ksi (MPa)**	**Effective Area (A_{BM} or A_{we}) in^2 (mm^2)**	**Required Filler Metal Strength Level[a][b]**
COMPLETE-JOINT-PENETRATION GROOVE WELDS					
Tension Normal to weld axis	Strength of the joint is controlled by the base metal.				Matching filler metal shall be used. For T and corner joints with backing left in place, notch tough filler metal is required. See Section J2.6.
Compression Normal to weld axis	Strength of the joint is controlled by the base metal.				Filler metal with a strength level equal to or one strength level less than matching filler metal is permitted.
Tension or Compression Parallel to weld axis	Tension or compression in parts joined parallel to a weld need not be considered in design of welds joining the parts.				Filler metal with a strength level equal to or less than matching filler metal is permitted.
Shear	Strength of the joint is controlled by the base metal.				Matching filler metal shall be used.[c]
PARTIAL-JOINT-PENETRATION GROOVE WELDS INCLUDING FLARE VEE GROOVE AND FLARE BEVEL GROOVE WELDS					
Tension Normal to weld axis	Base	$\phi = 0.75$ $\Omega = 2.00$	F_u	Effective Area	Filler metal with a strength level equal to or less than matching filler metal is permitted.
	Weld	$\phi = 0.80$ $\Omega = 1.88$	$0.60F_{EXX}$	See J2.1a	
Compression Column to Base Plate and column splices designed per J1.4(a)	Compressive stress need not be considered in design of welds joining the parts.				
Compression Connections of members designed to bear other than columns as described in J1.4(b)	Base	$\phi = 0.90$ $\Omega = 1.67$	F_y	See J4	
	Weld	$\phi = 0.80$ $\Omega = 1.88$	$0.60F_{EXX}$	See J2.1a	
Compression Connections not finished-to-bear	Base	$\phi = 0.90$ $\Omega = 1.67$	F_y	See J4	
	Weld	$\phi = 0.80$ $\Omega = 1.88$	$0.90F_{EXX}$	See J2.1a	
Tension or Compression Parallel to weld axis	Tension or compression in parts joined parallel to a weld need not be considered in design of welds joining the parts.				
Shear	Base	Governed by J4			
	Weld	$\phi = 0.75$ $\Omega = 2.00$	$0.60F_{EXX}$	See J2.1a	

(Continued)

TABLE 14.1 Continued

Load Type and Direction Relative to Weld Axis	**Pertinent Metal**	**ϕ and Ω**	**Nominal Strength (F_{nBM} or F_{nw}) ksi (MPa)**	**Effective Area (A_{BM} or A_{we}) in^2 (mm^2)**	**Required Filler Metal Strength Level**[a][b]
FILLET WELDS INCLUDING FILLETS IN HOLES AND SLOTS AND SKEWED T-JOINTS					
Shear	Base	Governed by J4			Filler metal with a strength level equal to or less than matching filler metal is permitted.
	Weld	$\phi = 0.75$ $\Omega = 2.00$	$0.60F_{EXX}$[d]	See J2.2a	
Tension or Compression Parallel to weld axis	Tension or compression in parts joined parallel to a weld need not be considered in design of welds joining the parts.				
PLUG AND SLOT WELDS					
Shear Parallel to faying surface on the effective area	Base	Governed by J4			Filler metal with a strength level equal to or less than matching filler metal is permitted.
	Weld	$\phi = 0.75$ $\Omega = 2.00$	$0.60F_{EXX}$	J2.3a	

[a] For matching weld metal see AWS D1.1, Section 3.3.
[b] Filler metal with a strength level one strength level greater than matching is permitted.
[c] Filler metals with a strength level less than matching may be used for groove welds between the webs and flanges of built-up sections transferring shear loads, or in applications where high restraint is a concern. In these applications, the weld joint shall be detailed and the weld shall be designed using the thickness of the material as the effective throat, $\phi = 0.80$, $\Omega = 1.88$ and $0.60F_{EXX}$ as the nominal strength.
[d] Alternatively, the provisions of J2.4(a) are permitted, provided the deformation compatibility of the various weld elements is considered. Alternatively, Sections J2.4(b) and (c) are special applications of J2.4(a) that provide for deformation compatibility.

Source: AISC Specification, Table J2.5, p. 16.1–114 and 16.1–115, June 22, 2010. "Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved."

In the preceding equations,

$$F_{nBM} = \text{the nominal stress of the base metal, ksi}$$
$$F_{nw} = \text{the nominal stress of the weld metal, ksi}$$
$$A_{BM} = \text{effective area of the base metal, in}^2$$
$$A_{we} = \text{effective area of the weld, in}^2$$

Table 14.1 (Table J2.5 in AISC Specification) provides the weld values needed to use these equations: ϕ, Ω, F_{BM}, and F_w. Limitations on these values are also given in this table.

The filler metal electrodes for shielded arc welding are listed as E60XX, E70XX, etc. In this classification, the letter E represents an electrode, while the first set of digits (60, 70, 80, 90, 100, or 110) indicates the minimum tensile strength of the weld, in ksi.

The remaining digits may be used to specify the type of coating. Because strength is the most important factor to the structural designer, we usually specify electrodes as E70XX, E80XX, or simply E70, E80, and so on. For the usual situation, E70 electrodes are used for steels with F_y values from 36 to 60 ksi, while E80 is used when F_y is 65 ksi.

In addition to the nominal stresses given in Table 14.1, there are several other provisions applying to welding given in Section J2.2b of the LRFD Specification. Among the more important are the following:

1. The minimum length of a fillet weld may not be less than four times the nominal leg size of the weld. Should its length actually be less than this value, the weld size considered effective must be reduced to one-quarter of the weld length.
2. The maximum size of a fillet weld along edges of material less than 1/4 in thick equals the material thickness. For thicker material, it may not be larger than the material thickness less 1/16 in, unless the weld is specially built out to give a full-throat thickness. For a plate with a thickness of 1/4 in or more, it is desirable to keep the weld back at least 1/16 in from the edge so that the inspector can clearly see the edge of the plate and thus accurately determine the dimensions of the weld throat.

As a general statement, the weldability of a material improves as the thickness to be welded decreases. The problem with thicker material is that thick plates take heat from welds more rapidly than thin plates, even if the same weld sizes are used. (The problem can be alleviated somewhat by preheating the metal to be welded to a few hundred degrees Fahrenheit and holding it there during the welding operation.)

3. The minimum permissible size fillet welds of the AISC Specification are given in Table 14.2 (Table J2.4 of the AISC Specification). They vary from 1/8 in for 1/4 in or thinner material up to 5/16 in for material over 3/4 in in thickness. The smallest practical weld size is about 1/8 in, and the most economical size is probably about 1/4 or 5/16 in. The 5/16-in weld is about the largest size that can be made in one

TABLE 14.2 Minimum Size of Fillet Welds

Material Thickness of Thinner Part Joined, in (mm)	Minimum Size of Fillet Weld,[a] in (mm)
To $\frac{1}{4}$ (6) inclusive	$\frac{1}{8}$ (3)
Over $\frac{1}{4}$ (6) to $\frac{1}{2}$ (13)	$\frac{3}{16}$ (5)
Over $\frac{1}{2}$ (13) to $\frac{3}{4}$ (19)	$\frac{1}{4}$ (6)
Over $\frac{3}{4}$ (19)	$\frac{5}{16}$ (8)

[a] Leg dimension of fillet welds. Single pass welds must be used. See Section J2.2b of the LRFD Specification for maximum size of fillet welds.

Source: AISC Specification, Table J2.4, p. 16.1–111, June 22, 2010.

pass with the shielded metal arc welded process (SMAW); with the submerged arc process (SAW), 1/2 in is the largest size.

These minimum sizes were not developed on the basis of strength considerations, but rather because thick materials have a quenching or rapid cooling effect on small welds. If this happens, the result is often a loss in weld ductility. In addition, the thicker material tends to restrain the weld material from shrinking as it cools, with the result that weld cracking can result and present problems.

Note that the minimum sizes given in Table 14.2 are dependent on the thinner of the two parts being joined. It may be larger, however, if so required by the calculated strength.

4. Sometimes, end returns or boxing is used at the end of fillet welds, as shown in Fig. 14.11. In the past, such practices were recommended to provide better fatigue resistance and to make sure that weld thicknesses were maintained over their full lengths. Recent research has shown that such returns are not necessary for developing the capacity of such connections. End returns are also used to increase the plastic deformation capability of such connections (AISC Commentary J2.2b).
5. When longitudinal fillet welds are used for the connection of plates or bars, their length may not be less than the perpendicular distance between them, because of shear lag (discussed in Chapter 3).
6. For lap joints, the minimum amount of lap permitted is equal to five times the thickness of the thinner part joined, but may not be less than 1 in (AISC J2.2b). The purpose of this minimum lap is to keep the joint from rotating excessively.
7. Should the actual length (l) of an end-loaded fillet weld be greater than 100 times its leg size (w), the AISC Specification (J2.2b) states that, due to stress variations along the weld, it is necessary to determine a smaller or effective length for strength determination. This is done by multiplying l by the term β, as given in the following equation in which w is the weld leg size:

$$\beta = 1.2 - 0.002\,(l/w) \leq 1.0 \qquad \text{(AISC Equation J2-1)}$$

If the actual weld length is greater than 300 w, the effective length shall be taken as 180 w.

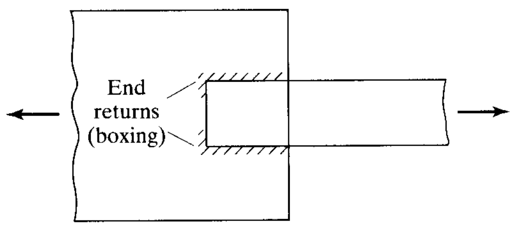
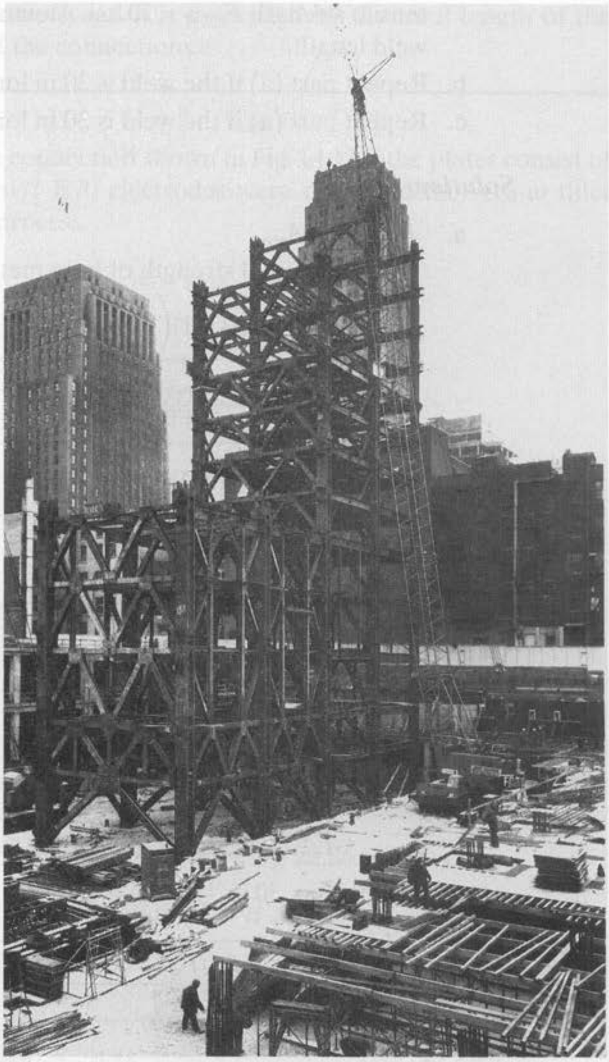


FIGURE 14.11
End returns (or boxing).



The all-welded 56-story Toronto Dominion Bank Tower. (Courtesy of the Lincoln Electric Company.)

14.13 DESIGN OF SIMPLE FILLET WELDS

Examples 14-1 and 14-2 demonstrate the calculations used to determine the strength of various fillet welded connections; Example 14-3 presents the design of such a connection. In these and other problems, weld lengths are selected no closer than the nearest 1/4 in, because closer work cannot be expected in shop or field.

Example 14-1

a. Determine the design strength of a 1-in length of a 1/4-in fillet weld formed by the shielded metal arc process (SMAW) and E70 electrodes with a minimum tensile strength $F_{EXX} = 70$ ksi. Assume that load is to be applied parallel to the weld length.

b. Repeat part (a) if the weld is 20 in long.

c. Repeat part (a) if the weld is 30 in long.

Solution

a. $R_n = F_{nw}A_{we}$

$= (\text{nominal strength of base metal } 0.60\,F_{EXX})(\text{throat } t)(\text{weld length})$

$= (0.60 \times 70 \text{ ksi})\left(\frac{1}{4}\text{in} \times 0.707 \times 1.0\right) = 7.42 \text{ k/in}$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(7.42) = \mathbf{5.56\ k/in}$	$\frac{R_n}{\Omega} = \frac{7.42}{2.00} = \mathbf{3.71\ k/in}$

b. Length, $l = 20$ in

LRFD	ASD
$\frac{l}{w} = \frac{20}{\frac{1}{4}} = 80 < 100$	$\frac{L}{w} = \frac{20}{\frac{1}{4}} = 80 < 100$
$\therefore \beta = 1.0$	$\therefore \beta = 1.0$
$\phi R_n L = (5.56)(20) = \mathbf{111.2\ k}$	$\frac{R_n}{\Omega}L = (3.71)(20) = \mathbf{74.2\ k}$

c. Length, $l = 30$ in

LRFD	ASD
$\frac{l}{w} = \frac{30}{\frac{1}{4}} = 120 > 100$	$\frac{L}{w} = \frac{30}{\frac{1}{4}} = 120 > 100$
$\therefore \beta = 1.2 - (0.002)(120) = 0.96$	$\therefore \beta = 1.2 - (0.002)(120) = 0.96$
$\phi R_n \beta L = (5.56)(0.96)(30) = \mathbf{160.1\ k}$	$\frac{R_n}{\Omega}\beta L = (3.71)(0.96)(30) = \mathbf{106.8\ k}$

Fillet welds may not be designed with a stress that is greater than the design stress on the adjacent members being connected. If the external force applied to the member (tensile or compressive) is parallel to the axis of the weld metal, the design strength may not exceed the axial design strength of the member.

Example 14-2 illustrates the calculations necessary to determine the design strength of plates connected with longitudinal fillet welds. In this example, the shearing strength of the weld per inch controls, and it is multiplied by the total length of the welds to give the total capacity of the connections.

Example 14-2

What is the design strength of the connection shown in Fig. 14.12 if the plates consist of A572 Grade 50 steel (F_u = 65 ksi)? E70 electrodes were used, and the 7/16-in fillet welds were made by the SMAW process.

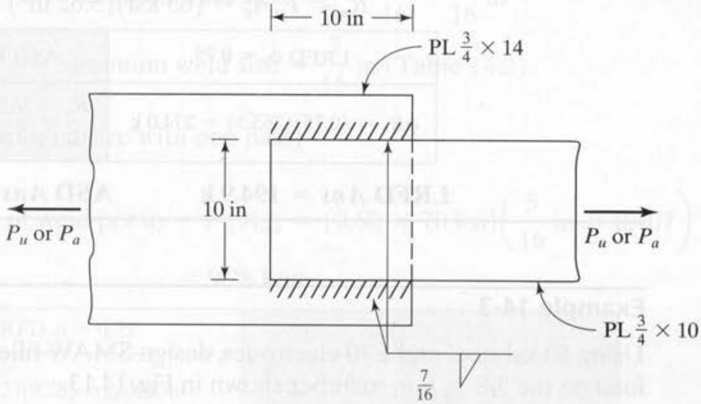


FIGURE 14.12

Solution

$$\text{Weld strength} = F_{we}A_{we} = (0.60 \times 70\text{ ksi})\left(\frac{7}{16}\text{ in} \times 0.707 \times 20\text{ in}\right) = 259.8\text{ k}$$

$$\text{Checking the length to weld size ratio } \frac{L}{w} = \frac{10\text{ in}}{{}^{7}/_{16}\text{ in}} = 22.86 < 100$$

$\therefore$ No reduction in weld strength is required as $\beta = 1.0$.

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$	
$\phi R_n = (0.75)(259.8) = 194.9\text{ k}$	$\frac{R_n}{\Omega} = \frac{259.8}{2.00} = 129.9\text{ k}$	← controls

Check tensile yielding for $\frac{3}{4} \times 10\ PL$

$$R_n = F_yA_g = (50\text{ ksi})\left(\frac{3}{4}\text{ in} \times 10\text{ in}\right) = 375\text{ k}$$

LRFD $\phi_t = 0.90$	ASD $\Omega_t = 1.67$
$\phi_t R_n = (0.90)(375) = 337.5\text{ k}$	$\frac{R_n}{\Omega_t} = \frac{375}{1.67} = 224.6\text{ k}$

Check tensile rupture strength for $\frac{3}{4} \times 10$ PL

$$A_e = A_g U$$

since the weld length, $l = 10$ in, is equal to the distance between the welds, $U = 0.75$ (see Case 4, AISC Table D3.1)

$$A_e = \frac{3}{4} \text{ in} \times 10 \text{ in} \times 0.75 = 5.62 \text{ in}^2$$

$$R_n = F_u A_e = (65 \text{ ksi})(5.62 \text{ in}^2) = 365.3 \text{ k}$$

LRFD $\phi_t = 0.75$	ASD $\Omega_t = 2.00$
$\phi R_n = (0.75)(365.3) = 274.0$ k	$\frac{R_n}{\Omega_t} = \frac{365.3}{2.00} = 182.7$ k

LRFD *Ans* = 194.9 k **ASD *Ans* = 129.9 k**

Example 14-3

Using 50 ksi steel and E70 electrodes, design SMAW fillet welds to resist a full-capacity load on the 3/8 × 6-in member shown in Fig. 14.13.

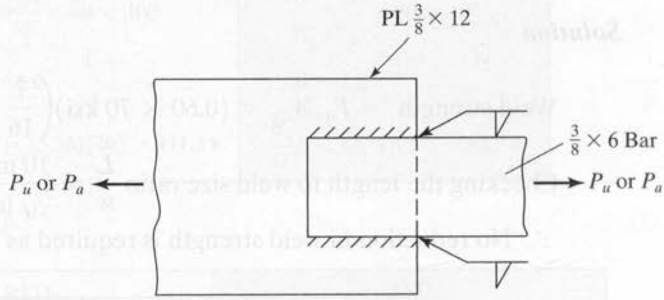


FIGURE 14.13

Solution

Tensile yield strength of gross section of $\frac{3}{8} \times 6$ bar

$$R_n = F_y A_g = (50 \text{ ksi})\left(\frac{3}{8} \text{ in} \times 6 \text{ in}\right) = 112.5 \text{ k}$$

LRFD $\phi_t = 0.90$	ASD $\Omega_t = 1.67$	
$\phi_t R_n = (0.90)(112.5) = 101.2$ k	$\frac{R_n}{\Omega_t} = \frac{112.5}{1.67} = 67.4$ k	← controls

Tensile rupture strength of $\frac{3}{8} \times 6$ bar, assume $U = 1.0$ (conservative)

$$A_e = \frac{3}{8} \text{ in} \times 6 \text{ in} \times 1.0 = 2.25 \text{ in}^2$$

$$R_n = F_u A_e = (65 \text{ ksi})(2.25 \text{ in}^2) = 146.2 \text{ k}$$

LRFD $\phi_t = 0.75$	$\Omega_t = 2.00$
$\phi_t R_n = (0.75)(146.2) = 109.6 \text{ k}$	$\frac{R_n}{\Omega_t} = \frac{146.2}{2.00} = 73.1 \text{ k}$

$\therefore$ Tensile capacity of bar is controlled by yielding.

Design of weld

$$\text{Maximum weld size} = \frac{3}{8} - \frac{1}{16} = \frac{5}{16} \text{ in}$$

$$\text{Minimum weld size} = \frac{3}{16} \text{ in (Table 14.2)}$$

Use $\frac{5}{16}$ weld (maximum size with one pass)

$$R_n \text{ of weld per in} = F_w A_{we} = (0.60 \times 70 \text{ ksi})\left(\frac{5}{16} \text{ in} \times 0.707\right)$$

$$= 9.28 \text{ k/in}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(9.28) = 6.96 \text{ k/in}$	$\frac{R_n}{\Omega} = \frac{9.28}{2.00} = 4.64 \text{ k/in}$
Weld length reqd $= \frac{101.2}{6.96}$	Weld length reqd $= \frac{67.4}{4.64}$
$= 14.54$ in or $7\frac{1}{2}$ in each side	$= 14.53$ in or $7\frac{1}{2}$ in each side
$\frac{L}{w} = \frac{7.5}{\frac{5}{16}} = 24 < 100$ OK $\beta = 1.00$	$\frac{L}{w} = \frac{7.5}{\frac{5}{16}} = 24 < 100$ OK $\beta = 1.00$

Use $7\frac{1}{2}$-in welds each side. **Use $7\frac{1}{2}$-in welds each side.**

Author's Note: With a weld length of only 7 1/2 in and a distance between the welds of 6 in, the U factor will be 0.75 and the tensile capacity will be reduced (controlled by rupture). One might consider using a smaller weld size, possibly the minimum value of 3/16 in, to increase the weld length.

AISC Section J2.4 states that the strength of fillet welds loaded transversely in a plane through their centers of gravity may be determined with the following equation in which $\phi = 0.75$, $\Omega = 2.00$, and θ is the angle between the line of action of the load and the longitudinal axis of the weld:

$$F_{nw} = (0.6F_{EXX})(1.0 + 0.50 \sin^{1.5}\theta) \quad \text{(AISC Equation J2-5)}$$

As the angle θ is increased, the strength of the weld increases. Should the load be perpendicular to the longitudinal axis of the weld, the result will be a 50 percent increase in the computed weld strength. Example 14-4 illustrates the application of this Appendix expression.

Example 14-4

a. Determine the LRFD design and the ASD allowable strengths of the $\frac{1}{4}$-in SMAW fillet welds formed with E70 electrodes, which are shown in part (a) of Fig. 14.14. The load is applied parallel to the longitudinal axis of the welds.

b. Repeat part (a) if the load is applied at a 45° angle, with the longitudinal axis of the welds as shown in part (b) of the figure.

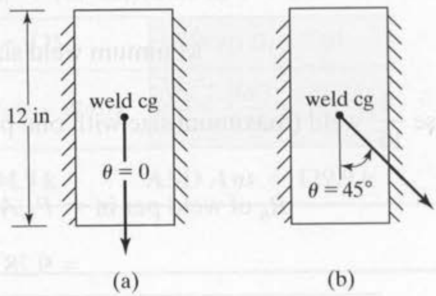


FIGURE 14.14

Solution

a. Load paralled to longitudinal axis of welds

$$\text{Effective throat } t = (0.707)\left(\frac{1}{4}\right) = 0.177 \text{ in}$$

$$R_n = F_{nw}A_{we} = (0.60 \times 70 \text{ ksi})(0.177 \text{ in} \times 24 \text{ in}) = 178.4 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(178.4) = \mathbf{133.8\ k}$	$\frac{R_n}{\Omega} = \frac{178.4}{2.00} = \mathbf{89.2\ k}$

b. Load applied at a 45° angle, with longitudinal axes of welds

$$R_n = 0.60F_{EXX}(1.0 + 0.50 \sin^{1.5}\theta)(0.177 \text{ in} \times 24 \text{ in})$$

$$= (0.60 \times 70 \text{ ksi})(1 + 0.50 \times \sin^{1.5}45°)(0.177 \text{ in} \times 24 \text{ in}) = 231.4 \text{ k}$$

LFRD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(231.4) = \mathbf{173.6\ k}$	$\frac{R_n}{\Omega} = \frac{231.4}{2.00} = \mathbf{115.7\ k}$

The values for ϕR_n and R_n/Ω are increased by almost 30 percent above the values in part (a), where the weld was longitudinally loaded.

14.14 DESIGN OF CONNECTIONS FOR MEMBERS WITH BOTH LONGITUDINAL AND TRANSVERSE FILLET WELDS

For this discussion, the fillet welds of Fig. 14.15 are considered. To determine the total nominal strength of the welds, it seems logical that we should add the nominal strength of the side welds, calculated with $R_n = F_w A_w$ with $F_w = 0.60F_{EXX}$, to the nominal strength of the end or transverse welds, calculated with $R_n = 0.60F_{EXX}(1.0 + 0.50 \sin^{1.5}\theta)A_w$. This procedure, however, is not correct, because the less ductile transverse welds will reach their ultimate deformation capacities before the side or longitudinal welds reach their maximum strengths. As a result of this fact, the AISC in its Section J2.4c states that the total nominal strength of a connection with side and transverse welds is to equal the larger of the values obtained with the following two equations:

$$R_n = R_{nwl} + R_{nwt} \qquad \text{(AISC Equation J2-10a)}$$

$$R_n = 0.85R_{nwl} + 1.5R_{nwt} \qquad \text{(AISC Equation J2-10b)}$$

In these expressions, R_{nwl} is the total nominal strength of the longitudinal or side fillet welds, calculated with $R_{wl} = F_{nw}A_{we}$. The total nominal strength of the transversely loaded fillet welds, R_{wt}, is also calculated with $F_{nw}A_{we}$ and not with $0.60F_w(1 + 0.50 \sin^{1.5}\theta)A_w$. Example 14.5 illustrates the calculation of the LRFD design strength ϕR_n and the ASD allowable strength R_n/Ω for the connection of Fig. 14.15, with its side and transverse welds.

Example 14-5

Determine the total LRFD design strength and the total ASD allowable strength of the 5/16-in E70 fillet welds shown in Fig. 14.15.

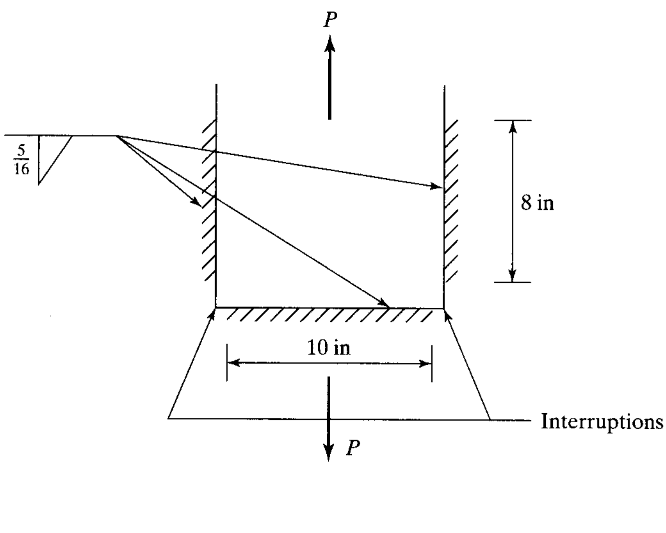


FIGURE 14.15

Solution

$$\text{Effective throat } t = (0.707)\left(\frac{5}{16}\text{ in}\right) = 0.221 \text{ in}$$

$$R_{wl} = R_n \text{ for side welds} = F_{nw}A_{we} = (0.60 \times 70 \text{ ksi})(2 \times 8 \text{ in} \times 0.221 \text{ in})$$
$$= 148.5 \text{ k}$$
$$R_{wt} = R_n \text{ for transverse end weld} = F_{nw}A_{we}$$
$$= (0.60 \times 70 \text{ ksi})(10 \text{ in} \times 0.221 \text{ in}) = 92.8 \text{ k}$$

Applying AISC Equations J2-10a and J2-10b

$$R_n = R_{nwl} + R_{nwt} = 148.5 \text{ k} + 92.8 \text{ k} = 241.3 \text{ k}$$
$$R_n = 0.85R_{nwl} + 1.5R_{nwt} = (0.85)(148.5 \text{ k}) + (1.5)(92.8 \text{ k}) = 265.4 \text{ k} \leftarrow \text{controls}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(265.4) = \mathbf{199\ k}$	$\dfrac{R_n}{\Omega} = \dfrac{265.4}{2.00} = \mathbf{132.7\ k}$

14.15 SOME MISCELLANEOUS COMMENTS

1. Weld Terminations

Generally speaking, the termination points of fillet welds do not have much effect on the strength and serviceability of connections. For certain situations, though, this is not altogether correct. For instance, notches may not only adversely affect the static strength of a connection, but also may decidedly reduce the resistance of the weld to the development of cracks when cyclic loads of sufficient size and frequency are applied. For such situations, it is wise to terminate the welds before the end of a joint is reached.

Should welds be terminated one or two weld sizes (or leg sizes) from the end of joints, their strengths will be only negligibly reduced. In fact, such length reductions are usually not even considered in strength calculations. A detailed discussion of this topic is presented in the Manual in Section J2.2b of the AISC Commentary.

2. Welding around Corners

The reader should be aware of an important fact concerning the use of longitudinal and transverse end welds. It is quite difficult for the welder to deposit continuous welds evenly around the corners between longitudinal and transverse welds without causing a gouge in the welds at the corner. Therefore, it is generally a good practice to interrupt fillet welds at such corners, as shown in Fig. 14.15.

Generally speaking, the location of the ends of fillet welds do not affect their strength or serviceability. However, should cyclic loads of sufficient size and frequency be applied in situations where the welds are uneven or have notches at the corners, there can be a considerable loss of strength. For such situations, the welds are often specified to be terminated before the corners are reached.

3. Strength of 1/16-in Welds for Calculation Purposes

It is rather convenient for design purposes to know the strength of a 1/16-in fillet weld 1 in long. Though this size is below the minimum permissible size given in Table 14.2, the calculated strength of such a weld is useful for determining weld sizes for calculated forces. For a 1-in-long SMAW weld with the load parallel to weld axis, we have the following:

For LRFD

$$\phi F_w A_w = (0.75)(0.60F_{EXX})\left(0.707 \times \frac{1}{16}\right)(1.0) = 0.0199F_{EXX}$$

For ASD

$$\frac{F_w A_w}{\Omega} = \frac{(0.60F_{EXX})\left(0.707 \times \frac{1}{16}\right)(1.00)}{2.00} = 0.0133F_{EXX}$$

For LRFD with E70 electrodes, $R_n = (0.0199)(70) = 1.39$ k/in. If we are designing a fillet weld to resist a factored force of 6.5 k/in, the required LRFD weld size is $6.5/1.39 = 4.68$ sixteenths of an inch, say, 5/16 in.

For ASD with E70 electrodes, $R_n/\Omega = (0.0133)(70) = 0.931$ k/in. If we are designing a fillet weld to resist a service load force of 4.4 k/in, the required ASD weld size is $4.4/0.931 = 4.73$ sixteenths of an inch, say, 5/16 in.

14.16 DESIGN OF FILLET WELDS FOR TRUSS MEMBERS

Should the members of a welded truss consist of single angles, double angles, or similar shapes and be subjected to static axial loads only, the AISC Specification (J1.7) permits the connections to be designed by the procedures described in the preceding section. The designers can select the weld size, calculate the total length of the weld required, and place the welds around the member ends as they see fit. (It would not make sense, of course, to place the weld all on one side of a member, such as for the angle of Fig. 14.16, because of the rotation possibility.)

It should be noted that the centroid of the welds and the centroid of the statically loaded angle do not coincide in the connection shown in this figure. If a welded connection is subjected to varying stresses (such as those occurring in a bridge member), it is essential to place the welds so that their centroid will coincide with the centroid of

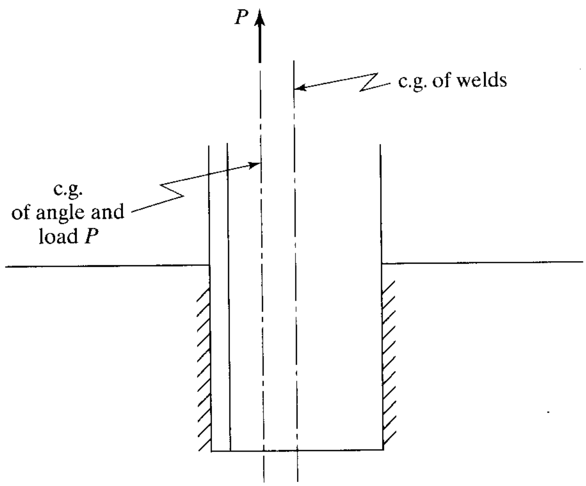


FIGURE 14.16
Eccentrically loaded welds.

the member (or the resulting torsion must be accounted for in design). If the member being connected is symmetrical, the welds will be placed symmetrically; if the member is not symmetrical, the welds will not be symmetrical.

The force in an angle, such as the one shown in Fig. 14.17, is assumed to act along its center of gravity. If the center of gravity of weld resistance is to coincide with the angle force, the welds must be asymmetrically placed, or in this figure L_1 must be longer than L_2. (When angles are connected by bolts, there is usually an appreciable amount of eccentricity, but in a welded joint, eccentricity can be fairly well eliminated.) The information necessary to handle this type of weld design can be easily expressed in equation form, but only the theory behind the equations is presented here.

For the angle shown in Fig. 14.17, the force acting along line L_2 (designated here as P_2) can be determined by taking moments about point A. The member force and the

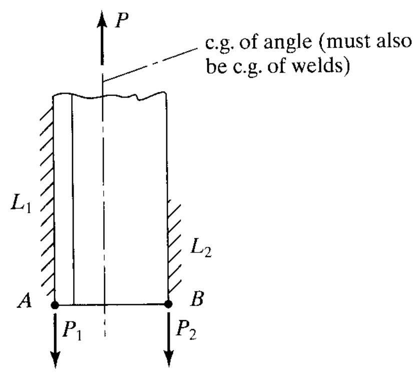


FIGURE 14.17

weld resistance should coincide, and the moments of the two about any point must be zero. If moments are taken about point A, the force P_1 (which acts along line L_1) will be eliminated from the equation, and P_2 can be determined. In a similar manner, P_1 can be determined by taking moments about point B or by $\Sigma V = 0$. Example 14-6 illustrates the design of fillet welds of this type.

There are other possible solutions for the design of the welds for the angle considered in Fig. 14.17. Although the 7/16-in weld is the largest one permitted at the edges of the 1/2-in angle, a larger weld could be used on the other side next to the outstanding angle leg. From a practical point of view, however, the welds should be the same size, because different-size welds slow the welder down due to the need to change electrodes to make different sizes.

Should cyclic or fatigue-type loads be involved, and should the centers of gravity of the loads and welds not coincide, the life of the connection may be severely reduced. Appendix 3 of the AISC Specification addresses the topic of fatigue design.

Example 14-6

Use $F_y = 50$ ksi and $F_u = 65$ ksi, E70 electrodes, and the SMAW process to design side fillet welds for the full capacity of the $5 \times 3 \times 1/2$-in angle tension member shown in Fig. 14.18. Assume that the member is subjected to repeated stress variations, making any connection eccentricity undesirable. Check block shear strength of the member. Assume that the WT chord member has adequate strength to develop the weld strengths and that the thickness of its web is 1/2 in. Assume that $U = 0.87$.

Solution

Tensile yielding on gross section

$$P_n = F_y A_g = (50 \text{ ksi})(3.75 \text{ in}^2) = 187.5 \text{ k}$$

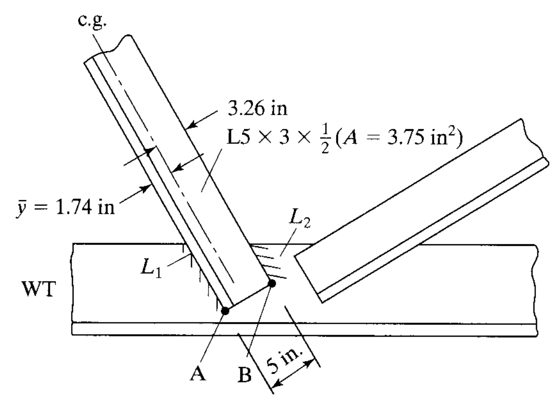


FIGURE 14.18

Tensile rupture on net section

$$A_e = UA_g = (0.87)(3.75 \text{ in}^2) = 3.26 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(3.26 \text{ in}^2) = 211.9 \text{ k}$$

LRFD	ASD
For tensile yielding ($\phi_t = 0.90$)	For tensile yielding ($\Omega_t = 1.67$)
$\phi_t P_n = (0.9)(187.5) = 168.7$ k	$\dfrac{P_n}{\Omega_t} = \dfrac{187.5}{1.67} = 112.3$ k
For tensile rupture ($\phi_t = 0.75$)	For tensile rupture ($\Omega_t = 2.00$)
$\phi_t P_n = (0.75)(211.9) = 158.9 \text{ k} \leftarrow$	$\dfrac{P_n}{\Omega_t} = \dfrac{211.9}{2.00} = 105.9 \text{ k} \leftarrow$

$$\text{Maximum weld size} = \frac{1}{2} - \frac{1}{16} = \frac{7}{16} \text{ in}$$

Use $\frac{5}{16}$-in weld (largest that can be made in single pass)

$$\text{Effective throat } t \text{ of weld} = (0.707)\left(\frac{5}{16} \text{ in}\right) = 0.221 \text{ in}$$

LRFD	ASD
Design strength/in of $\frac{5}{16}$-in welds ($\phi = 0.75$)	Allowable strength/in of $\frac{5}{16}$-in welds ($\Omega = 2.00$)
$= (0.75)(0.60 \times 70)(0.221)(1)$	$= \dfrac{(0.60 \times 70)(0.221)(1)}{2.00}$
$= 6.96$ k/in	$= 4.64$ k/in
Weld length reqd $= \dfrac{158.9}{6.96}$	Weld length reqd $= \dfrac{105.9}{4.64}$
$= 22.83$ in	$= 22.82$ in
Taking moments about point A in Fig. 14.18	Taking moments about point A in Fig. 14.18
$(158.9)(1.74) - 5.00P_2 = 0$	$(105.9)(1.74) - 5.00P_2 = 0$
$P_2 = 55.3$ k	$P_2 = 36.85$ k
$L_2 = \dfrac{55.3 \text{ k}}{6.96 \text{ k/in}} = 7.95$ in (**say, 8 in**)	$L_2 = \dfrac{36.85 \text{ k}}{4.64 \text{ k/in}} = 7.94$ in (**say, 8 in**)
$L_1 = 22.83 - 7.95 = 14.88$ in (**say, 15 in**)	$L_1 = 22.82 - 7.94 = 14.88$ in (**say, 15 in**)

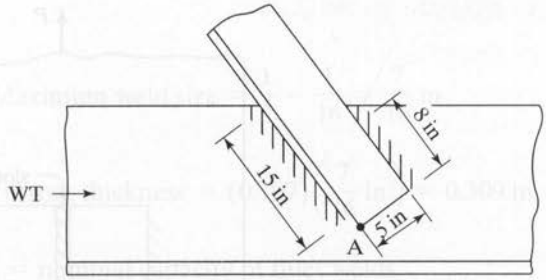


FIGURE 14.19

Welds arranged as shown in Fig. 14.19.

Checking block shearing strength, assuming dimensions previously described

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.6F_yA_{gv} + U_{bs}F_uA_{nt}$$
$$= (0.6)(65)(15 + 8)\left(\frac{1}{2}\right) + (1.00)(65)\left(5 \times \frac{1}{2}\right)$$
$$\leq (0.6)(50)(15 + 8)\left(\frac{1}{2}\right) + (1.00)(65)\left(5 \times \frac{1}{2}\right)$$
$$= 611 \text{ k} > 507.5 \text{ k}$$
$$\therefore R_n = 507.5 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(507.5) = 380.6 \text{ k} > 158.9 \text{ k}$ **OK**	$\frac{R_n}{\Omega} = \frac{507.5}{2.00} = 253.8 \text{ k} > 105.9\ k$ **OK**

14.17 PLUG AND SLOT WELDS

On some occasions, the spaces available for fillet welds may not be sufficient to support the applied loads. The plate shown in Fig. 14.20 falls into this class. Even using the maximum fillet weld size (7/16 in here) will not support the load. It is assumed that space is not available to use transverse welds at the plate ends in this case.

One possibility for solving this problem involves the use of a slot weld, as shown in the figure. There are several AISC requirements pertaining to slot welds that need to be mentioned here. AISC Specification J2.3 states that the width of a slot may not be less than the member thickness plus 5/16 in (rounded off to the next greater odd 1/16 in, since structural punches are made in these diameters), nor may it be greater than 2.25 times the weld thickness. For members up to 5/8-in thickness, the weld thickness

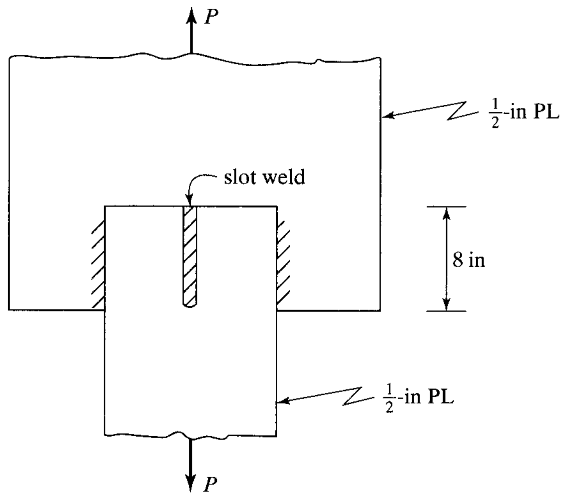


FIGURE 14.20

must equal the plate thickness; for members greater than 5/8-in thickness, the weld thickness may not be less than one-half the member thickness (or 5/8 in). The maximum length permitted for slot welds is ten times the weld thickness. The limitations given in specifications for the maximum sizes of plug or slot welds are caused by the detrimental shrinkage that occurs around these types of welds when they exceed certain sizes. Should holes or slots larger than those specified be used, it is desirable to use fillet welds around the borders of the holes or slots rather than using a slot or plug weld. Slot and plug welds are normally used in conjunction with fillet welds in lap joints. Sometimes, plug welds are used to fill in the holes temporarily used for erection bolts for beam and column connections. They may or may not be included in the calculated strength of these joints.

The LRFD design strength of a plug or slot weld is equal to its design stress ϕF_w times its area in the shearing plane. The ASD allowable strength equals F_w/Ω times the same shearing area. The shearing area is the area of contact at the base of the plug or slot. The length required for a slot weld can be determined from the expression to follow:

$$L = \frac{\text{load}}{(\text{width})(\text{design stress})}$$

Example 14-7

Design SMAW fillet welds and a slot weld to connect the plates shown in Fig. 14.20 if $P_D = 110$ k, $P_L = 120$ k, $F_y = 50$ ksi, $F_u = 65$ ksi, and E70 electrodes are used. As shown in the figure, the plates may lap over each other by only 8 in due to space limitations.

Solution

$$\text{Maximum weld size} = \frac{1}{2} - \frac{1}{16} = \frac{7}{16}\text{ in}$$

$$\text{Effective throat thickness} = (0.707)\left(\frac{7}{16}\text{ in}\right) = 0.309\text{ in}$$

$$R_n = \text{nominal capacity of fillet welds}$$

$$= (0.60 \times 70\text{ ksi})(2 \times 8\text{ in} \times 0.309\text{ in}) = 207.6\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$P_u = (1.2)(110) + (1.6)(120) = 324$ k	$P_a = 110 + 120 = 230$ k
$\phi R_n = (0.75)(207.6) = 155.7$ k	$\frac{R_n}{\Omega} = \frac{207.6}{2.00} = 103.8$ k
< 324 k $\therefore$ **Try slot weld.**	< 230 k $\therefore$ **Try slot weld.**

$$\text{Minimum width of slot} = t \text{ of PL} + \frac{5}{16} = \frac{1}{2} + \frac{5}{16} = \frac{13}{16}\text{ in}$$

$$\text{Maximum width of slot} = \left(2\frac{1}{4}\right)\left(\frac{1}{2}\text{ in}\right) = 1\frac{1}{8}\text{ in}$$

$$\text{Rounding slot width to next larger odd } \frac{1}{16}\text{ in} = \frac{15}{16}\text{ in}$$

$$\text{Try } \frac{15}{16}\text{-in wide slot.}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
ϕR_n for all welds must $= 324$ k	$\frac{R_n}{\Omega}$ for all welds must $= 230$ k
$155.7 + \phi(R_n \text{ of slot weld}) = 324$	$103.8 + \frac{R_n}{\Omega}$ for slot weld $= 230$
$155.7 + (0.75)\left(\frac{15}{16} \times L \times 0.6 \times 70\right) = 324$	$103.8 + \frac{{}^{15}\!/_{16} \times L \times 0.6 \times 70}{2.00} = 230$
L reqd $= 5.70$ in	L reqd $= 6.41$ in
Use 6-in slot weld.	**Use $6\frac{1}{2}$-in slot weld.**

14.18 SHEAR AND TORSION

Fillet welds are frequently loaded with eccentrically applied loads, with the result that the welds are subjected to either shear and torsion or to shear and bending. Figure 14.21 is presented to show the difference between the two situations. Shear and torsion, shown in part (a) of the figure, are the subject of this section, while shear and bending, shown in part (b) of the figure, are discussed in Section 14.19.

As is the case for eccentrically loaded bolt groups (Section 13.1), the AISC Specification provides the design strength of welds, but does not specify a method of analysis for eccentrically loaded welds. It's left to the designer to decide which method to use.

14.18.1 Elastic Method

Initially, the very conservative elastic method is presented. In this method, friction or slip resistance between the connected parts is neglected, the connected parts are assumed to be perfectly rigid, and the welds are assumed to be perfectly elastic.

For this discussion, the welded bracket of part (a) of Fig. 14.21 is considered. The pieces being connected are assumed to be completely rigid, as they were in bolted connections. The effect of this assumption is that all deformation occurs in the weld. The

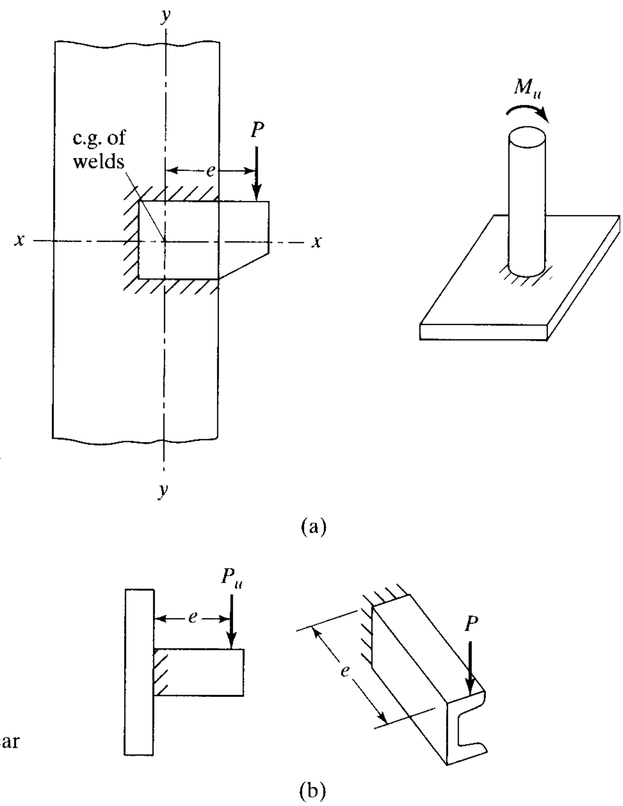


FIGURE 14.21

(a) Welds subjected to shear and torsion. (b) Welds subjected to shear and bending.

weld is subjected to a combination of shear and torsion, as was the eccentrically loaded bolt group considered in Section 13.1. The force caused by torsion can be computed from the following familiar expression:

$$f = \frac{Td}{J}$$

In this expression, T is the torsion, d is the distance from the center of gravity of the weld to the point being considered, and J is the polar moment of inertia of the weld. It is usually more convenient to break the force down into its vertical and horizontal components. In the following expressions, f_h and f_v are the horizontal and vertical components, respectively, of the force f:

$$f_h = \frac{Tv}{J} \quad f_v = \frac{Th}{J}$$

Notice that the formulas are almost identical to those used for determining stresses in bolt groups subject to torsion. These components are combined with the usual direct shearing stress, which is assumed to equal the reaction divided by the total length of the welds. For design of a weld subject to shear and torsion, it is convenient to assume a 1-in weld and to compute the stresses on a weld of that size. Should the assumed weld be overstressed, a larger weld is required; if it is understressed, a smaller one is desirable.

Although the calculations probably will show the weld to be overstressed or understressed, the calculation does not have to be repeated, because a ratio can be set up to give the weld size for which the load would produce a computed stress exactly equal to the design stress. Note that the use of a 1-in weld simplifies the units, because 1 in of length of weld is 1 in^2 of weld, and the computed stresses are said to be either kips per square inch or kips per inch of length. Should the calculations be based on some size other than a 1-in weld, the designer must be very careful to keep the units straight, particularly in obtaining the final weld size. To further simplify the calculations, the welds are assumed to be located at the edges where the fillet welds are placed, rather than at the centers of their effective throats. As the throat dimensions are rather small, this assumption changes the results very little. Example 14-8 illustrates the calculations involved in determining the weld size required for a connection subjected to a combination of shear and torsion.

Example 14-8

For the A36 bracket shown in Fig. 14.22(a), determine the fillet weld size required if E70 electrodes, the AISC Specification, and the SMAW process are used.

Solution. Assuming a 1-in weld as shown in part (b) of Fig. 14.22

$$A = 2(4\text{ in}^2) + 10\text{ in}^2 = 18\text{ in}^2$$

$$\bar{x} = \frac{(4\text{ in}^2)(2\text{ in})(2)}{18\text{ in}^2} = 0.89\text{ in}$$

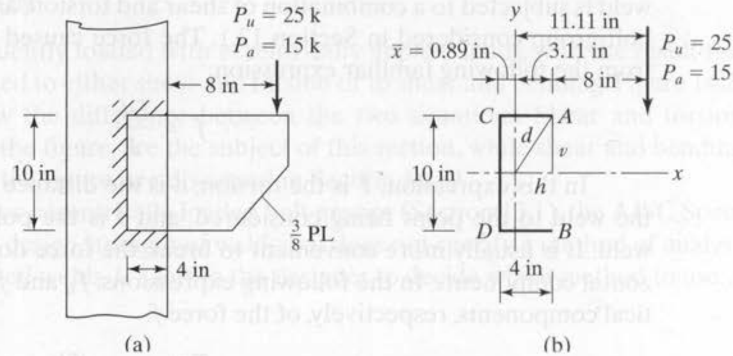


FIGURE 14.22

$$I_x = \left(\frac{1}{12}\right)(1)(10)^3 + (2)(4)(5)^2 = 283.3 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(1)(4)^3 + 2(4)(2 - 0.89)^2 + (10)(0.89)^2 = 28.4 \text{ in}^4$$

$$J = 283.3 + 28.4 = 311.7 \text{ in}^4$$

According to our previous work, the welds perpendicular to the direction of the loads are appreciably stronger than the welds parallel to the loads. However, to simplify the calculations, the author conservatively assumes that all the welds have design strengths or allowable strengths per inch equal to the values for the welds parallel to the loads.

$$R_n \text{ for a 1 in weld} = 0.707 \times 1 \times 0.6 \times 70 = 29.69 \text{ ksi}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(29.69) = 22.27$ ksi	$\dfrac{R_n}{\Omega} = \dfrac{29.69}{2.00} = 14.84$ ksi
Forces @ points *C* & *D*	**Forces @ points *C* & *D***
$f_h = \dfrac{(25 \times 11.11)(5)}{311.7} = 4.46$ k/in	$f_h = \dfrac{(15)(11.11)(5)}{311.7} = 2.67$ k/in
$f_v = \dfrac{(25 \times 11.11)(0.89)}{311.7} = 0.79$ k/in	$f_v = \dfrac{(15)(11.11)(0.89)}{311.7} = 0.48$ k/in
$f_s = \dfrac{25}{18} = 1.39$ k/in	$f_s = \dfrac{15}{18} = 0.83$ k/in
$f_r = \sqrt{(0.79 + 1.39)^2 + (4.46)^2} = 4.96$ k/in	$f_r = \sqrt{(0.48 + 0.83)^2 + (2.67)^2} = 2.97$ k/in
Size $= \dfrac{4.96 \text{ k/in}}{22.27 \text{ k/in}^2} = 0.223$ in, say $\dfrac{1}{4}$ in	Size $= \dfrac{2.97 \text{ k/in}}{14.84 \text{ k/in}^2} = 0.200$ in, say $\dfrac{1}{4}$ in
Forces @ points *A* & *B*	**Forces @ points *A* & *B***
$f_h = \dfrac{(25 \times 11.11)(5)}{311.7} = 4.46$ k/in	$f_h = \dfrac{(15 \times 11.11)(5)}{311.7} = 2.67$ k/in

(Continued)

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$f_v = \dfrac{(25 \times 11.11)(3.11)}{311.7} = 2.77$ k/in	$f_v = \dfrac{(15 \times 11.11)(3.11)}{311.7} = 1.66$ k/in
$f_s = \dfrac{25}{18} = 1.39$ k/in	$f_s = \dfrac{15}{18} = 0.83$ k/in
$f_r = \sqrt{(2.77 + 1.39)^2 + (4.46)^2}$ $= 6.10$ k/in	$f_r = \sqrt{(1.66 + 0.83)^2 + (2.67)^2}$ $= 3.65$ k/in
Size $= \dfrac{6.10}{22.27} = 0.274$ in, say $\dfrac{5}{16}$ in	Size $= \dfrac{3.65}{14.84} = 0.246$ in, say $\dfrac{1}{4}$ in
Use $\frac{5}{16}$-in fillet welds, E70, SMAW.	**Use $\frac{1}{4}$-in fillet weld, E70, SMAW.**

14.18.2 Ultimate Strength Method

An ultimate strength analysis of eccentrically loaded welded connections is more realistic than the more conservative elastic procedure just described. For the discussion that follows, the eccentrically loaded fillet weld of Fig. 14.23 is considered. As for eccentrically loaded bolted connections, the load tends to cause a relative rotation and translation between the parts connected by the weld.

Even if the eccentric load is of such a magnitude that it causes the most stressed part of the weld to yield, the entire connection will not yield. The load may be increased, the less stressed fibers will begin to resist more of the load, and failure will not occur until all the weld fibers yield. The weld will tend to rotate about its instantaneous center of rotation. The location of this point (which is indicated by the letter O in the figure) is dependent upon the location of the eccentric load, the geometry of the weld, and the deformations of the different elements of the weld.

If the eccentric load P_u or P_a is vertical and if the weld is symmetrical about a horizontal axis through its center of gravity, the instantaneous center will fall somewhere on the horizontal x axis. Each differential element of the weld will provide a resisting force R. As shown in Fig. 14.23, each of these resisting forces is assumed to act perpendicular to a ray

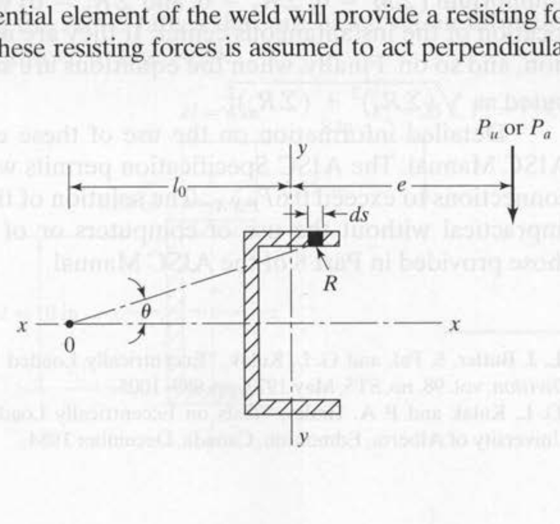


FIGURE 14.23

drawn from the instantaneous center to the center of gravity of the weld element in question.

Studies have been made to determine the maximum shear forces that eccentrically loaded weld elements can withstand.[4, 5] The results, which depend on the load deformation relationship of the weld elements, may be represented either with curves or in formula fashion. The ductility of the entire weld is governed by the maximum deformation of the weld element that first reaches its limit. (The element that is located at the greatest distance from the weld's instantaneous center probably reaches its limit first.)

Just as in eccentrically loaded bolted connections, the location of the instantaneous center of rotation is determined by trial and error. Unlike eccentrically loaded bolt groups, however, the strength and deformation of welds are dependent on the angle θ that the force in each element makes with the axis of that element. The deformation of each element is proportional to its distance from the instantaneous center. At maximum stress, the weld deformation is Δ_{max} which is determined with the following expression, in which w is the weld leg size:

$$\Delta_{max} = 1.087w(\theta + 6)^{-0.65} \leq 0.17w$$

The deformation in a particular element is assumed to vary directly in proportion to its distance from the instantaneous center:

$$\Delta_r = \frac{l_r}{l_m}\Delta_m$$

The nominal shear strength of a weld segment at a deformation Δ is

$$R_n = 0.6\,F_{EXX}A_{we}(1.0 + 0.50\sin^{1.5}\theta)[p(1.9 - 0.9p)]^{0.3}$$

where θ is the angle of loading measured from the weld longitudinal axis, and p is the ratio of the deformation of an element to its deformation at ultimate stress.

We can assume a location of the instantaneous center, determine R_n values for the different elements of the weld, and compute ΣR_x and ΣR_y. The three equations of equilibrium ($\Sigma M = 0$, $\Sigma R_x = 0$, and $\Sigma R_y = 0$) will be satisfied if we have the correct location of the instantaneous center. If they are not satisfied, we will try another location, and so on. Finally, when the equations are satisfied, the value of P_u can be computed as $\sqrt{(\Sigma R_x)^2 + (\Sigma R_y)^2}$.

Detailed information on the use of these expressions is given in Part 8 of the AISC Manual. The AISC Specification permits weld strengths in eccentrically loaded connections to exceed $0.6F_{EXX}$. The solution of these kinds of problems is completely impractical without the use of computers or of computer-generated tables such as those provided in Part 8 of the AISC Manual.

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[4] L. J. Butler, S. Pal, and G. L. Kulak, "Eccentrically Loaded Weld Connections," *Journal of the Structural Division*, vol. 98, no. ST5, May 1972, pp. 989–1005.

[5] G. L. Kulak and P. A. Timler, "Tests on Eccentrically Loaded Fillet Welds," Dept of Civil Engineering, University of Alberta, Edmonton, Canada, December 1984.

TABLE 14.3 Electrode Strength Coefficient, C_1

Electrode	F_{EXX} (ksi)	C_1
E60	60	0.857
E70	70	1.00
E80	80	1.03
E90	90	1.16
E100	100	1.21
E110	110	1.34

Source: AISC Manual, Table 8-3, p. 8-65, 14th ed., 2011.

The values given in the tables of Part 8 of the AISC Manual were developed by the ultimate strength method. Using the tables, the nominal strength R_n of a particular connection can be determined from the following expression in which C is a tabular coefficient, C_1 is a coefficient depending on the electrode number and given in Table 14.3 (Table 8-3 in AISC manual), D is the weld size in sixteenths of an inch, and l is the length of the vertical weld:

$$R_n = CC_1Dl$$

$$\phi = 0.75 \text{ and } \Omega = 2.00$$

The Manual includes tables for both vertical and inclined loads (at angles from the vertical of 0° to 75°). The user is warned not to interpolate for angles in between these values because the results may not be conservative. Therefore, the user is advised to use the value given for the next-lower angle. If the connection arrangement being considered is not covered by the tables, the conservative elastic procedure previously described may be used. Examples 14-9 and 14-10 illustrate the use of these ultimate strength tables.

Example 14-9

Repeat Example 14-8, using the AISC tables that are based on an ultimate strength analysis. The connection is redrawn in Fig. 14.24.

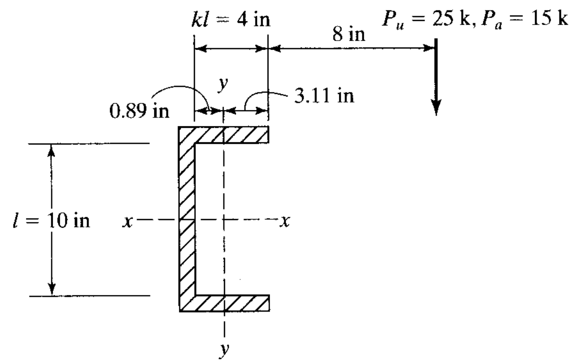

FIGURE 14.24

Solution

$$e_x = 11.11 \text{ in}$$
$$l = 10 \text{ in}$$
$$a = \frac{e_x}{l} = \frac{11.11 \text{ in}}{10 \text{ in}} = 1.11$$
$$k = \frac{kl}{l} = \frac{4 \text{ in}}{10 \text{ in}} = 0.40$$
$$C = 1.31 \text{ from Table 8-8, in the AISC Manual for } \theta = 0° \text{ by straight-line interpolation}$$
$$C_1 = 1.0 \text{ from Table 8-3 in AISC Manual (E70 electrodes)}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$D_{min} = \text{weld size reqd} = \dfrac{P_u}{\phi C C_1 l}$	$D_{min} = \text{weld size reqd} = \dfrac{\Omega P_a}{C C_1 l}$
$= \dfrac{25}{(0.75)(1.31)(1.0)(10)}$	$= \dfrac{(2.00)(15)}{(1.31)(1.0)(10)}$
$= 2.54$ sixteenths $= 0.159$ in	$= 2.29$ sixteenths $= 0.143$ in
(compared with 0.273 in obtained with elastic method)	(compared with 0.246 in obtained with the elastic method)
Use $\frac{3}{16}$-in fillet weld, E70, SMAW.	**Use $\frac{3}{16}$-in fillet weld, E70, SMAW.**

Example 14-10

Determine the weld size required for the situation shown in Fig. 14.25, using the AISC tables that are based on an ultimate strength analysis (A36 steel, E70 electrodes).

Solution

$$e_x = al = 9 \text{ in}$$
$$l = 16 \text{ in}$$
$$kl = 6 \text{ in}$$
$$a = \frac{al}{l} = \frac{9 \text{ in}}{16 \text{ in}} = 0.562$$
$$k = \frac{kl}{l} = \frac{6 \text{ in}}{16 \text{ in}} = 0.375$$
$$C = 3.32 \text{ from Table 8-6 in the AISC Manual for } \theta = 0° \text{ by double interpolation}$$
$$C_1 = 1.0 \text{ from Table 8-3 in AISC Manual (E70 electrodes)}$$

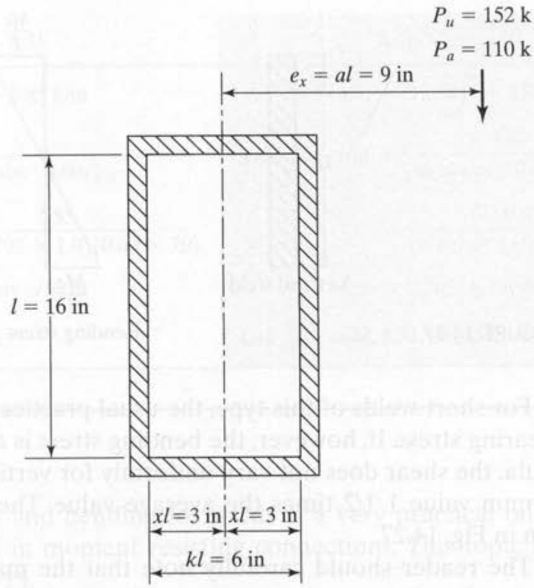

FIGURE 14.25

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$D_{min} = \text{weld size reqd} = \dfrac{P_u}{\phi C C_1 l}$	$D_{min} = \text{weld size reqd} = \dfrac{\Omega P_a}{C C_1 l}$
$= \dfrac{152}{(0.75)(3.32)(1.0)(16)} = 3.82 \text{ sixteenths}$	$= \dfrac{(2.00)(110)}{(3.32)(1.0)(16)} = 4.14 \text{ sixteenths}$
$= 0.239$ in	$= 0.259$ in
Use $\frac{1}{4}$-in fillet weld, E70, SMAW.	**Use $\frac{5}{16}$-in fillet weld, E70, SMAW.**

14.19 SHEAR AND BENDING

The welds shown in Fig. 14.21(b) and in Fig. 14.26 are subjected to a combination of shear and bending.

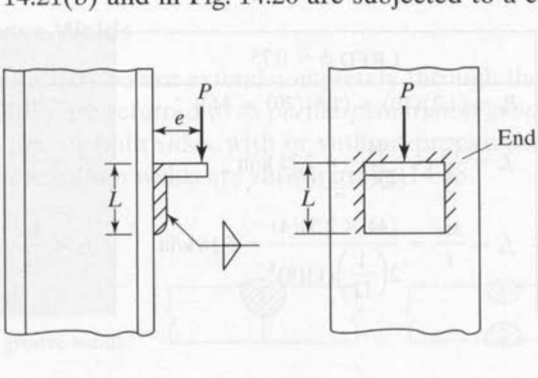

FIGURE 14.26

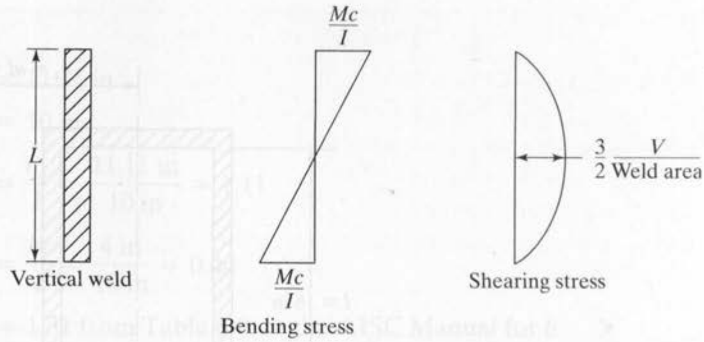

FIGURE 14.27

For short welds of this type, the usual practice is to consider a uniform variation of shearing stress. If, however, the bending stress is assumed to be given by the flexure formula, the shear does not vary uniformly for vertical welds, but as a parabola with a maximum value 1 1/2 times the average value. These stress and shear variations are shown in Fig. 14.27.

The reader should carefully note that the maximum shearing stresses and the maximum bending stresses occur at different locations. Therefore, it probably is not necessary to combine the two stresses at any one point. If the weld is capable of withstanding the worst shear and the worst moment individually, it is probably satisfactory. In Example 14-11, however, a welded connection subjected to shear and bending is designed by the usual practice of assuming a uniform shear distribution in the weld and combining the value vectorially with the maximum bending stress.

Example 14-11

Using E70 electrodes, the SMAW process, and the LRFD Specification, determine the weld size required for the connection of Fig. 14.26 if $P_D = 10$ k, $P_L = 20$ k, $e = 2\ 1/2$ in, and $L = 8$ in. Assume that the member thicknesses do not control weld size.

Solution

Initially, assume fillet welds with 1-in leg sizes.

LRFD $\phi = 0.75$	ASD $\Omega_t = 2.00$
$P_u = (1.2)(10) + (1.6)(20) = 44$ k $f_v = \dfrac{P_u}{A} = \dfrac{44}{(2)(8)} = 2.75$ k/in $f_b = \dfrac{Mc}{I} = \dfrac{(44 \times 2.5)(4)}{2\left(\frac{1}{12}\right)(1)(8)^3} = 5.16$ k/in	$P_a = 10 + 20 = 30$ k $f_v = \dfrac{P_a}{A} = \dfrac{30}{(2)(8)} = 1.88$ k/in $f_b = \dfrac{Mc}{I} = \dfrac{(30 \times 2.5)(4)}{2\left(\frac{1}{12}\right)(1)(8)^3} = 3.52$ k/in

(*Continued*)

LRFD $\phi = 0.75$	ASD $\Omega_r = 2.00$
$f_r = \sqrt{(2.75)^2 + (5.16)^2} = 5.85$ k/in	$f_r = \sqrt{(1.88)^2 + (3.52)^2} = 3.99$ k/in
$\text{weld size reqd} = \dfrac{f_r}{(\phi)(\text{weld size})\, 0.60F_{EXX}}$	$\text{weld size reqd} = \dfrac{\Omega f_r}{(\text{weld size})(0.60\ F_{EXX})}$
$= \dfrac{5.85}{(0.75)(0.707 \times 1.0)(0.60 \times 70)}$	$= \dfrac{(2.00)(3.99)}{(0.707 \times 1.0)(0.60 \times 70)}$
$= 0.263$ in, say 5/16 in	$= 0.269$ in, say 5/16 in
Use $\frac{5}{16}$-in weld, E70, SMAW.	**Use $\frac{5}{16}$-in weld, E70, SMAW.**

The subject of shear and bending for welds is a very practical one, as it is the situation commonly faced in moment resisting connections. This topic is considered at some length in Chapter 15.

14.20 FULL-PENETRATION AND PARTIAL-PENETRATION GROOVE WELDS

14.20.1 Full-Penetration Groove Welds

When plates with different thicknesses are joined, the strength of a full-penetration groove weld is based on the strength of the thinner plate. Similarly, if plates of different strengths are joined, the strength of a full-penetration weld is based on the strength of the weaker plate. Notice that no allowances are made for the presence of reinforcement—that is, for any extra weld thickness.

Full-penetration groove welds are the best type of weld for resisting fatigue failures. In fact, in some specifications they are the only groove welds permitted if fatigue is possible. Furthermore, a study of some specifications shows that allowable stresses for fatigue situations are increased if the crowns or reinforcement of the groove welds have been ground flush.

14.20.2 Partial-Penetration Groove Welds

When we have groove welds that do not extend completely through the full thickness of the parts being joined, they are referred to as *partial-penetration groove welds*. Such welds can be made from one or both sides, with or without preparation of the edges (such as bevels). Partial-penetration welds are shown in Fig. 14.28.

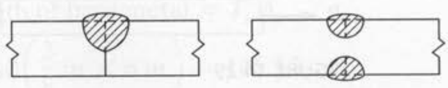

FIGURE 14.28
Partial-penetration groove welds.

Partial-penetration groove welds often are economical in cases in which the welds are not required to develop large forces in the connected materials, such as for column splices and for the connecting together of the various parts of built-up members.

In Table 14.1, we can see that the design stresses are the same as for full-penetration welds when we have compression or tension parallel to the axis of the welds. When we have tension transverse to the weld axis, there is a substantial strength reduction because of the possiblity of high stress concentrations.

Examples 14-12 and 14-13 illustrate the calculations needed to determine the strength of full-penetration and partial-penetration groove welds. The design strengths of both full-penetration and partial-penetration groove welds are given in Table 14.1 of this text (AISC Table J2.5).

In part (b) of Example 14-13, it is assumed that two W sections are spliced together with a partial-penetration groove weld, and the design shear strength of the member is determined. To do this, it is necessary to compute the following: (1) the shear rupture strength of the base material as per AISC Section J2.4, (2) the shear yielding strength of the connecting elements as per AISC Section J4.3, and (3) the shear strength of the weld as per AISC Section J2.2 and AISC Table J2.5. The design shear strength of the member is the least of these three values, which are described as follows:

1. Shear fracture of base material $= F_n A_{ns}$ with $\phi = 0.75$, $\Omega = 2.00$, $F_n = 0.6F_u$, and A_{ns} = net area subject to shear.
2. Shear yielding of connecting elements = ϕR_n = with $(0.60\text{A}_{vg})\ F_y$, with $\phi = 0.75$, $\Omega = 2.00$, and A_{vg} = gross area subjected to shear.
3. Shear yielding of the weld $= F_w = (0.60F_{EXX})A_w$, with $\phi = 0.75$, $\Omega = 2.00$, and $A_w = A_{eff}$ = area of weld.

Example 14-12

a. Determine the LRFD design strength and the ASD allowable strength of a SMAW full-penetration groove weld for the plates shown in Fig. 14.29. Use F_y = 50 ksi and E70 electrodes.

b. Repeat part (a) if a partial-joint-penetration groove weld (45° bevel) is used with a depth of $\frac{1}{2}$ in.

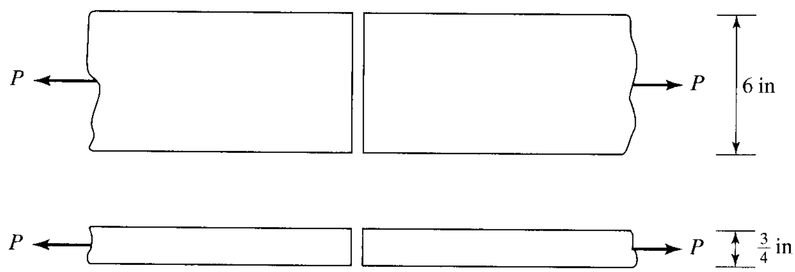

FIGURE 14.29

Solution

a. Full-penetration groove weld

Tension yielding strength

$$R_n = F_y A_g$$

$$= (50 \text{ ksi})\left(\frac{3}{4} \text{ in} \times 6 \text{ in}\right) = 225 \text{ k}$$

LRFD $\phi = 0.90$	ASD $\Omega = 1.67$	
$\phi R_n = (0.9)(225) = \mathbf{202.5\ k}$	$\frac{R_n}{\Omega} = \frac{225}{1.67} = \mathbf{134.7\ k}$	← controls

Tension rupture strength

$$R_n = F_u A_e \quad \text{where } A_e = A_g \text{U}$$

$$\text{and U} = 1.0$$

$$= (65 \text{ ksi})\left(\frac{3}{4} \text{ in}\right)(6 \text{ in})(1.0) = 292.5 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(292.5) = \mathbf{219.4\ k}$	$\frac{R_n}{\Omega} = \frac{292.5}{2.00} = \mathbf{146.2\ k}$

b. Partial-joint-penetration groove weld

Weld values

Effective throat of weld

$$= \frac{1}{2} - \frac{1}{8} = \frac{3}{8} \text{ in as reqd in AISC Table J2.1}$$

$$R_n = (0.60 \times 70 \text{ ksi})\left(\frac{3}{8} \text{ in} \times 6 \text{ in}\right) = 94.5 \text{ k}$$

LRFD $\phi = 0.80$	ASD $\Omega = 1.88$	
$\phi R_n = (0.80)(94.5) = \mathbf{75.6\ k}$	$\frac{R_n}{\Omega} = \frac{94.5}{1.88} = \mathbf{50.3\ k}$	← controls

Base metal values

$$R_n = \text{strength of base metal} = F_u A_e$$

$$= (65 \text{ ksi})\left(\frac{3}{8} \text{ in} \times 6 \text{ in}\right) = 146.3 \text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(146.3) = \mathbf{109.7\ k}$	$\dfrac{R_n}{\Omega} = \dfrac{146.3}{2.00} = \mathbf{73.1\ k}$

Example 14-13

a. A full-penetration groove weld made with E70 electrodes is use to splice together the two halves of a 50 ksi (F_u = 65 ksi) W21 × 166. Determine the shear strength of the splice.

b. Repeat part (a) if two vertical partial-penetration welds (60° V-groove) (E70 electrodes) with throat thicknesses of 1/4 in are used.

Solution

Using a W21 × 166 (d = 22.5 in, t_w = 0.750 in)

a. Strength of the joint is controlled by the base metal (J4.2)

Shear yielding strength

$$R_n = 0.60\ F_y A_{gv}$$

$$= 0.60\ (50\ \text{ksi})\ (0.75\ \text{in})\ (22.5\ \text{in}) = 506.2\ \text{k}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$\phi R_n = (1.0)(506.2) = \mathbf{506.2\ k}$	$\dfrac{R_n}{\Omega} = \dfrac{506.2}{1.50} = \mathbf{337.5\ k}$

Shear rupture strength

$$R_n = 0.60\ F_u A_{nv}$$

$$= 0.60\ (65\ \text{ksi})\ (0.75\ \text{in})\ (22.5\ \text{in}) = 658.1\ \text{k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$	
$\phi R_n = (0.75)(658.1) = \mathbf{493.6\ k}$	$\dfrac{R_n}{\Omega} = \dfrac{658.1}{2.00} = \mathbf{329.0\ k}$	← controls

b. Base metal values (J4.2)

Shear yielding strength

$$R_n = 0.60\ F_y A_{gv}$$
$$= 0.60\ (50\ \text{ksi})\ (0.75\ \text{in})\ (22.5\ \text{in}) = 506.2\ \text{k}$$

LRFD $\phi = 1.00$	ASD $\Omega = 1.50$
$\phi R_n = (1.00)(506.2) = \mathbf{506.2\ k}$	$\frac{R_n}{\Omega} = \frac{506.2}{1.50} = \mathbf{337.5\ k}$

Shear rupture strength

$$R_n = 0.60\ F_u A_{nv}$$
$$= 0.60(65\,\text{ksi})\left(2 \times \frac{1}{4}\,\text{in}\right)(22.5\,\text{in}) = 438.7\ \text{k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$	
$\phi R_n = (0.75)(438.7) = \mathbf{329.0\ k}$	$\frac{R_n}{\Omega} = \frac{438.7}{2.00} = \mathbf{219.4\ k}$	← controls

Weld values (60° V-groove weld)

$$R_n = 0.6\ F_{EXX} A_{we}$$
$$= 0.6(70\,\text{ksi})\left(2 \times \frac{1}{4}\,\text{in}\right)(22.5\,\text{in}) = 472.5\ \text{k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(472.5) = \mathbf{354.4\ k}$	$\frac{R_n}{\Omega} = \frac{472.5}{2.00} = \mathbf{236.3\ k}$

14.21 PROBLEMS FOR SOLUTION

Unless otherwise noted, A36 steel is to be used for all problems.

14-1. A 1/4-in fillet weld, SMAW process, is used to connect the members shown in the accompanying illustration. Determine the LRFD design load and the ASD allowable load that can be applied to this connection, including the plates, using the AISC Specification and E70 electrodes. (*Ans.* 97.2 k, 64.7 k)

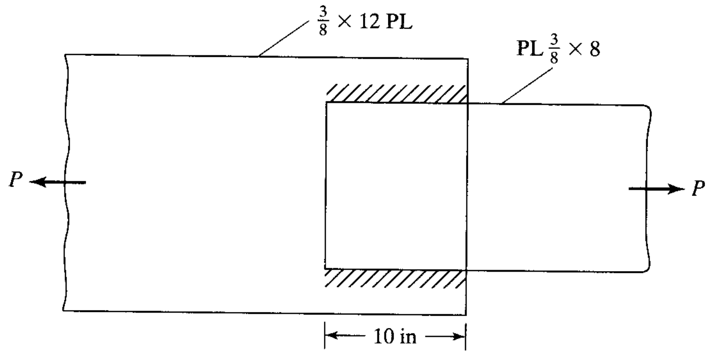

FIGURE P14-1

14-2. Repeat Prob. 14-1 if the weld lengths are 24 in.

14-3. Rework Prob. 14-1 if A572 grade 65 steel and E80 electrodes are used. (*Ans.* 127.3 k, 84.8 k)

14-4. Determine the LRFD design strength and the ASD allowable strength of the 5/16-in fillet welds shown, if E70 electrodes are used.

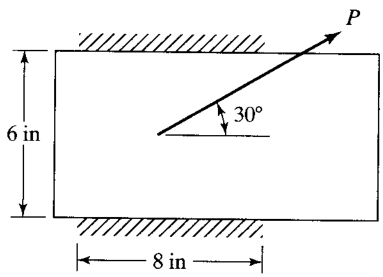

FIGURE P14-4

14-5. (a) Repeat Prob. 14-4 if 1/4-in welds are used and if $\theta = 45°$. (*Ans.* 115.6 k, LRFD; 92.3 k, ASD)

(b) Repeat part (a) if $\theta = 15°$. (*Ans.* 95.0 k, LRFD; 63.3 k, ASD)

14-6. Using both the LRFD and ASD methods, design maximum-size SMAW fillet welds for the plates shown, if $P_D = 40$ k, $P_L = 60$ k and E70 electrodes are used.

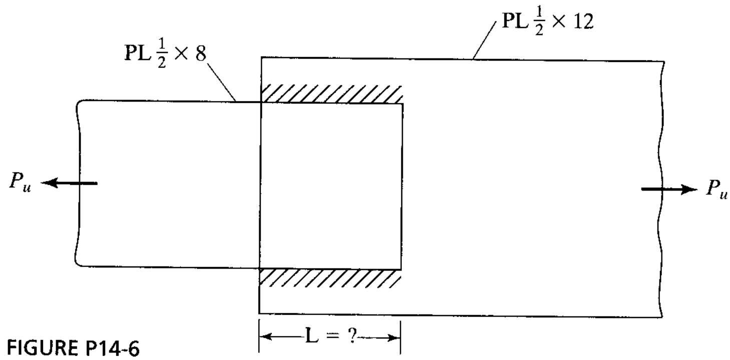

FIGURE P14-6

14-7. Repeat Prob. 14-6 if $\frac{5}{16}$-in welds are used. (*Ans.* 10.5 in, 11 in)

14-8. Calculate ϕR_n for Prob. 14-6, using $\frac{5}{16}$ in side welds 8 in long and a vertical end weld at the end of the $\frac{1}{2} \times 8$ plate. Also use A572 grade 65 steel and E80 electrodes.

14-9. Rework Prob. 14-8, using $\frac{1}{4}$ in side welds 10 in long and welds at the end of the $\frac{1}{2} \times 8$ PL and E70 electrodes. (*Ans.* 161.5 k, 107.6 k)

14-10. The 5/8 × 8-in PL shown in the accompanying illustration is to be connected to a gusset plate with 1/4-in SMAW fillet welds. Determine ϕR_n and $\frac{R_n}{\Omega}$ of the bar if E70 electrodes are used.

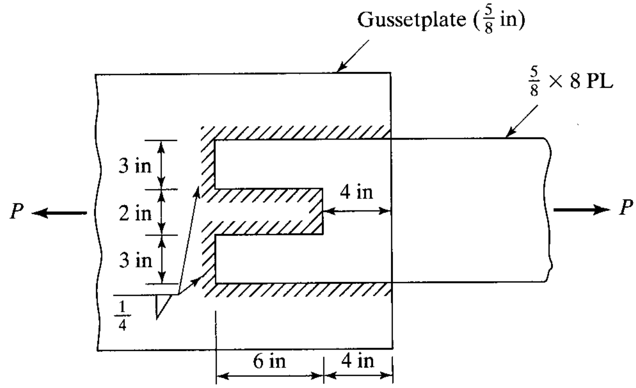

FIGURE P14-10

14-11. Design by LRFD and ASD maximum size side SMAW fillet welds required to develop the loads $P_D = 70$ k and $P_L = 60$ k for an L6 × 4 × 1/2, using E70 electrodes and 50 ksi steel. The member is connected on the sides of the 6-in leg and is subject to alternating loads. (*Ans.* $L_1 = 12.5$ in, $L_2 = 6.5$ in (LRFD); $L_1 = 13.5$ in, $L_2 = 7.0$ in (ASD))

14-12. Rework Prob. 14-11, using side welds and a weld at the end of the angle.

14-13. Rework Prob. 14-11, using E80 electrodes. (*Ans.* $L_1 = 11$ in, $L_2 = 5.5$ in (LRFD); $L_1 = 12$ in, $L_2 = 6$ in (ASD))

14-14. One leg of an 8 × 8 × 3/4 angle is to be connected with side welds and a weld at the end of the angle to a plate behind, to develop the loads $P_D = 170$ k and $P_L = 200$ k. Balance the fillet welds around the center of gravity of the angle. Using LRFD and ASD methods, determine weld lengths if E70 electrodes and maximum weld size is used.

14-15. It is desired to design 5/16-in SMAW fillet welds necessary to connect a C10 × 30 made from A36 steel to a 3/8-in gusset plate. End, side, and slot welds may be used to develop the loads $P_D = 80$ k and $P_L = 120$ k. Use both ASD and LRFD procedures. No welding is permitted on the back of the channel. Use E70 electrodes. It is assumed that, due to space limitations, the channel can lap over the gusset plate by a maximum of 8 in. (*Ans.* $\frac{15}{16} \times 4$ in slot LRFD, $\frac{15}{16} \times 4\frac{1}{4}$ in slot ASD).

14-16. Rework Prob. 14-15, using A572 grade 60 steel, E80 electrodes, and 5/16-in fillet welds.

14-17. Using the elastic method, determine the maximum force per inch to be resisted by the fillet weld shown in the accompanying illustration. (*Ans.* 11.77 k/in)

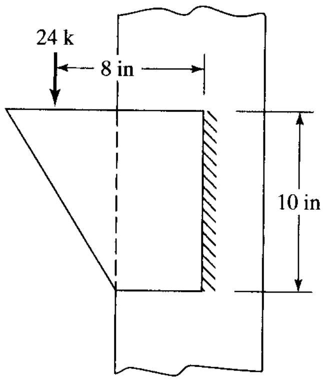

FIGURE P14-17

14-18. Using the elastic method, determine the maximum force to be resisted per inch by the fillet weld shown in the accompanying illustration.

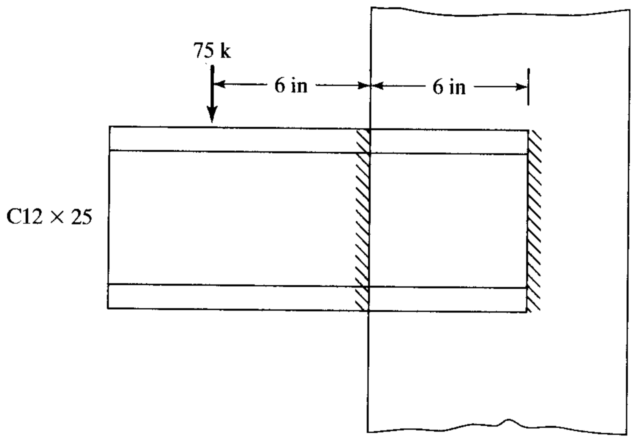

FIGURE P14-18

14-19. Using the elastic method, rework Prob. 14-18 if welds are used on the top and bottom of the channel in addition to those shown in the figure. (*Ans.* 5.88 k/in)

14-20. Using the elastic method, determine the maximum force per inch to be resisted by the fillet welds shown in the accompanying illustration.

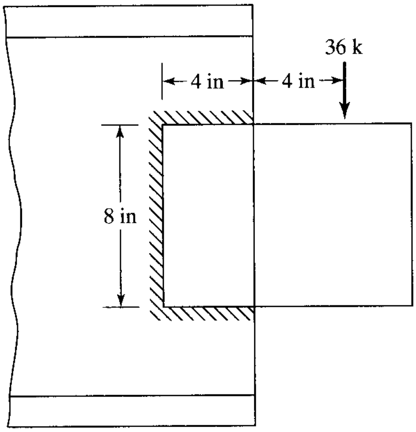

FIGURE P14-20

14-21. Determine the maximum eccentric loads ϕP_n that can be applied to the connection shown in the accompanying illustration if 1/4-in SMAW fillet welds are used. Assume plate thickness is 1/2 in and use E70 electrodes. (a) Use elastic method. (b) Use AISC tables and the ultimate strength method. (*Ans.* (a) 27.0 k, (b) 62.4 k)

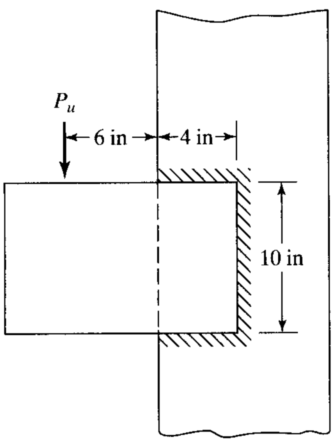

FIGURE P14-21

14-22. Rework Prob. 14-21 if 5/16-in fillet welds are used and the vertical weld is 8 in high.

14-23. Using the LRFD method and E70 electrodes, determine the fillet weld size required for the connection of Prob. 14-17 if $P_D = 10$ k, $P_L = 10$ k, and the height of the weld is 12 in. (a) Use elastic method. (b) Use AISC tables and the ultimate strength method.
(*Ans.* (a) 7/16 in, (b) 1/4 in)

14-24. Repeat part (a) of Prob. 14-23, using the ASD method.

14-25. Using E70 electrodes and the SMAW process, determine the LRFD fillet weld size required for the bracket shown in the accompanying illustration. (a) Use elastic method. (b) Use AISC tables and the ultimate strength method.
(*Ans.* (a) 3/8 in, (b) 1/4 in)

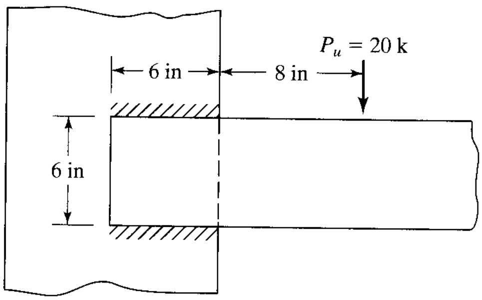

FIGURE P14-25

14-26. Rework Prob. 14-25 if the load is increased from 20 to 25 k and the horizontal weld lengths are increased from 6 to 8 in.

14-27. Rework Prob. 14-25, using ASD with $P_a = 11$ k. (*Ans.* (a) 5/16 in, (b) 3/16 in)

14-28. Using LRFD only, determine the fillet weld size required for the connection shown in the accompanying illustration. E70. (a) Use elastic method. (b) Use AISC tables and the ultimate strength method.

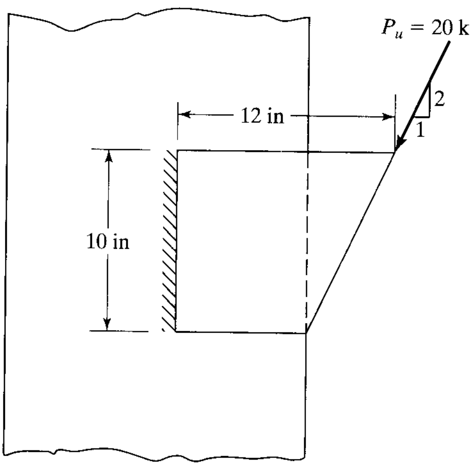

FIGURE P14-28

14-29. Using the ASD method only, determine the SMAW fillet weld size required for the connection shown in the accompanying illustration. E70. What angle thickness should be used? (a) Use elastic method. (b) Use AISC tables and the ultimate strength method. (*Ans.* (a) 3/16 in, (b) 1/8 in)

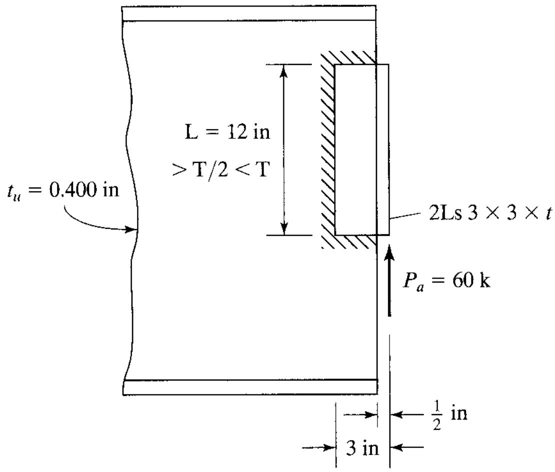

FIGURE P14-29

14-30. Assuming that the LRFD method is to be used, determine the fillet weld size required for the connection shown in the accompanying illustration. Use E70 electrodes and the elastic method.

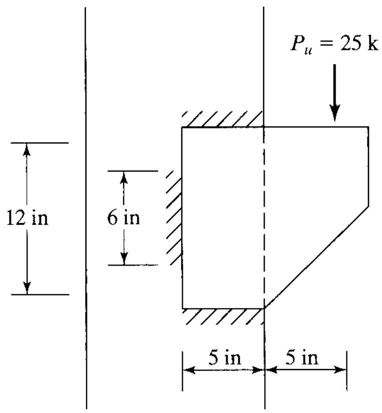

FIGURE P14-30

14-31. Determine the fillet weld size required by the ASD method for the connection shown in the accompanying illustration. E70. The SMAW process is to be used. (a) Use the elastic method. (b) Use ASD tables and the ultimate strength method. (*Ans.* (a) 3/8 in, (b) 3/16 in)

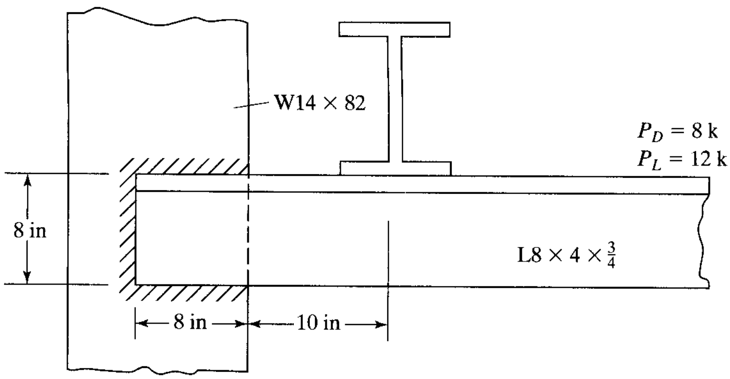

FIGURE P14-31

14-32. Determine the value of the loads ϕP_n and P_n/Ω that can be applied to the connection shown in the accompanying illustration if 3/8-in fillet welds are used. E70. SAW. Use the elastic method.

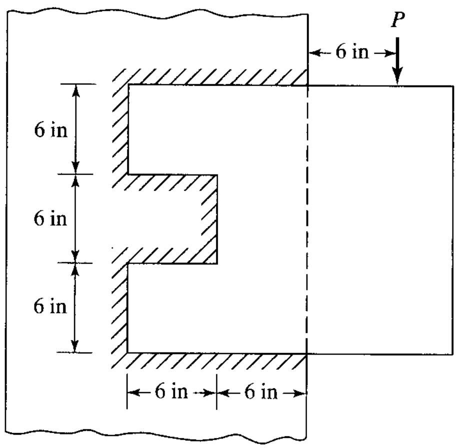

FIGURE P14-32

14-33. Using LRFD and ASD, determine the length of 1/4-in SMAW E70 fillet welds 12 in on center required to connect the cover plates for the section shown in the accompanying illustration at a point where the external shear V_u is 80 k and $V_a = 55$ k. E70. (*Ans.* 2.5 in both LRFD & ASD)

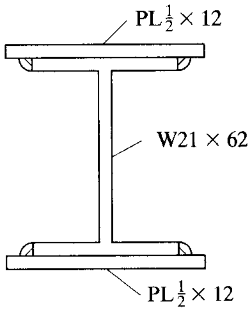

FIGURE P14-33

14-34. The welded girder shown in the accompanying illustration has an external shear $V_D = 300$ k and $V_L = 350$ k at a particular section. Determine the fillet weld size required to fasten the plates to the web if the SMAW process is used. E70. Use LRFD and ASD.

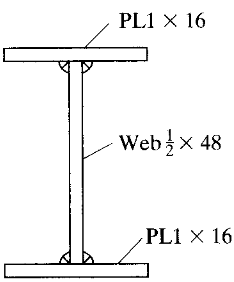

FIGURE P14-34

14-35. (a) Using both LRFD and ASD procedures and assuming the A36 plates in Fig. 14.29 are 12 in wide and 1/2 in thick, determine their design tensile strength and their ASD allowable strength if a full-penetration groove weld is used. Use E70 electodes.

(b) Repeat if a 5/16-in partial-penetration groove weld is used on one side.
(*Ans.* (a) 194.4 k (LRFD); 129.3 k (ASD), (b) 75.6 k (LRFD); 50.3 k (ASD))

14-36. (a) If full-penetration groove welds formed with E70 electrodes are used to splice together the two halves of a W24 × 117, determine the shear strength capacity of the splice using both the LRFD and ASD procedures. Use SMAW weld process, $F_y = 50$ ksi, $F_u = 65$ ksi.

(b) Repeat part (a) if two vertical partial-penetration groove welds (45° Bevel) with 1/4 in throat thickness are used.