Table 5.2 (*Continued*)

	n										
m	**1.6**	**1.8**	**2.0**	**2.5**	**3.0**	**4.0**	**5.0**	**6.0**	**8.0**	**10.0**	**∞**
0.1	0.03058	0.03090	0.03111	0.03138	0.03150	0.03158	0.03160	0.03161	0.03162	0.03162	0.03162
0.2	0.05994	0.06058	0.06100	0.06155	0.06178	0.06194	0.06199	0.06201	0.06202	0.06202	0.06202
0.3	0.08709	0.08804	0.08867	0.08948	0.08982	0.09007	0.09014	0.09017	0.09018	0.09019	0.09019
0.4	0.11135	0.11260	0.11342	0.11450	0.11495	0.11527	0.11537	0.11541	0.11543	0.11544	0.11544
0.5	0.13241	0.13395	0.13496	0.13628	0.13684	0.13724	0.13737	0.13741	0.13744	0.13745	0.13745
0.6	0.15028	0.15207	0.15326	0.15483	0.15550	0.15598	0.15612	0.15617	0.15621	0.15622	0.15623
0.7	0.16515	0.16720	0.16856	0.17036	0.17113	0.17168	0.17185	0.17191	0.17195	0.17196	0.17197
0.8	0.17739	0.17967	0.18119	0.18321	0.18407	0.18469	0.18488	0.18496	0.18500	0.18502	0.18502
0.9	0.18737	0.18986	0.19152	0.19375	0.19470	0.19540	0.19561	0.19569	0.19574	0.19576	0.19577
1.0	0.19546	0.19814	0.19994	0.20236	0.20341	0.20417	0.20440	0.20449	0.20455	0.20457	0.20458
1.2	0.20731	0.21032	0.21235	0.21512	0.21633	0.21722	0.21749	0.21760	0.21767	0.21769	0.21770
1.4	0.21510	0.21836	0.22058	0.22364	0.22499	0.22600	0.22632	0.22644	0.22652	0.22654	0.22656
1.6	0.22025	0.22372	0.22610	0.22940	0.23088	0.23200	0.23236	0.23249	0.23258	0.23261	0.23263
1.8	0.22372	0.22736	0.22986	0.23334	0.23495	0.23617	0.23656	0.23671	0.23681	0.23684	0.23686
2.0	0.22610	0.22986	0.23247	0.23614	0.23782	0.23912	0.23954	0.23970	0.23981	0.23985	0.23987
2.5	0.22940	0.23334	0.23614	0.24010	0.24196	0.24344	0.24392	0.24412	0.24425	0.24429	0.24432
3.0	0.23088	0.23495	0.23782	0.24196	0.24394	0.24554	0.24608	0.24630	0.24646	0.24650	0.24654
4.0	0.23200	0.23617	0.23912	0.24344	0.24554	0.24729	0.24791	0.24817	0.24836	0.24842	0.24846
5.0	0.23236	0.23656	0.23954	0.24392	0.24608	0.24791	0.24857	0.24885	0.24907	0.24914	0.24919
6.0	0.23249	0.23671	0.23970	0.24412	0.24630	0.24817	0.24885	0.24916	0.24939	0.24946	0.24952
8.0	0.23258	0.23681	0.23981	0.24425	0.24646	0.24836	0.24907	0.24939	0.24964	0.24973	0.24980
10.0	0.23261	0.23684	0.23985	0.24429	0.24650	0.24842	0.24914	0.24946	0.24973	0.24981	0.24989
∞	0.23263	0.23686	0.23987	0.24432	0.24654	0.24846	0.24919	0.24952	0.24980	0.24989	0.25000

[a] After Newmark, 1935.

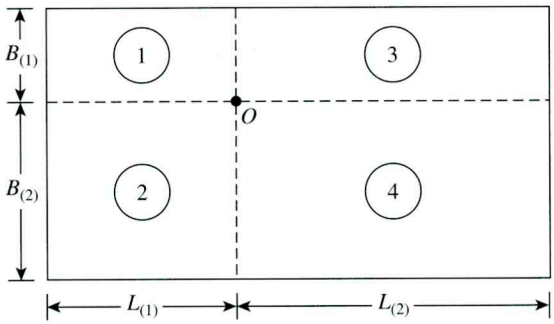


Figure 5.4 Stress below any point of a loaded flexible rectangular area

where

$$I_c = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}}\frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right] \tag{5.11}$$

$$m_1 = \frac{L}{B} \tag{5.12}$$

$$n_1 = \frac{z}{\left(\frac{B}{2}\right)} \tag{5.13}$$

The variation of I_c with m_1 and n_1 is given in Table 5.3.

Table 5.3 Variation of I_c with m_1 and n_1

	m_1									
n_i	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

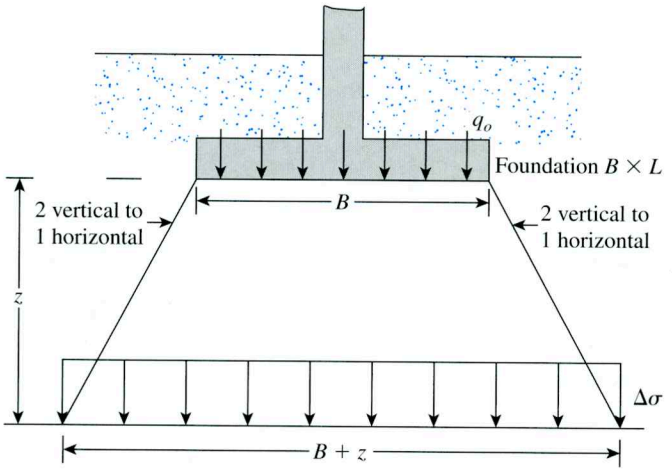


Figure 5.5 2:1 method of finding stress increase under a foundation

Foundation engineers often use an approximate method to determine the increase in stress with depth caused by the construction of a foundation. The method is referred to as the *2:1 method.* (See Figure 5.5.) According to this method, the increase in stress at depth z is

$$\Delta\sigma = \frac{q_o \times B \times L}{(B + z)(L + z)} \tag{5.14}$$

Note that Eq. (5.14) is based on the assumption that the stress from the foundation spreads out along lines with a *vertical-to-horizontal slope of 2:1.*

Example 5.1

A flexible rectangular area measures 2.5 m × 5 m in plan. It supports a load of 150 kN/m^2.

Determine the vertical stress increase due to the load at a depth of 6.25 m below the center of the rectangular area.

Solution

Refer to Figure 5.4. For this case,

$$B_1 = B_2 = \frac{2.5}{2} = 1.25 \text{ m}$$

$$L_1 = L_2 = \frac{5}{2} = 2.5 \text{ m}$$

From Eqs. (5.7) and (5.8),

$$m = \frac{B_1}{z} = \frac{B_2}{z} = \frac{1.25}{6.25} = 0.2$$

$$n = \frac{L_1}{z} = \frac{L_2}{z} = \frac{2.5}{6.25} = 0.4$$

From Table 5.2, for $m = 0.2$ and $n = 0.4$, the value of $I = 0.0328$. Thus,

$$\Delta\sigma = q_o(4I) = (150)(4)(0.0328) = \mathbf{19.68\ kN/m^2}$$

Alternate Solution

From Eq. (5.10),

$$\Delta\sigma = q_o I_c$$

$$m_1 = \frac{L}{B} = \frac{5}{2.5} = 2$$

$$n_1 = \frac{z}{\left(\frac{B}{2}\right)} = \frac{6.25}{\left(\frac{2.5}{2}\right)} = 5$$

From Table 5.3, for $m_1 = 2$ and $n_1 = 5$, the value of $I_c = 0.131$. Thus,

$$\Delta\sigma = (150)(0.131) = \mathbf{19.65\ kN/m^2}$$

■

5.5 Average Vertical Stress Increase Due to a Rectangularly Loaded Area

In Section 5.4, the vertical stress increase below the corner of a uniformly loaded rectangular area was given as

$$\Delta\sigma = q_o I$$

In many cases, one must find the average stress increase, $\Delta\sigma_{av}$, below the corner of a uniformly loaded rectangular area with limits of $z = 0$ to $z = H$, as shown in Figure 5.6. This can be evaluated as

$$\Delta\sigma_{av} = \frac{1}{H}\int_0^H (q_o I)\,dz = q_o I_a \tag{5.15}$$

where

$$I_a = f(m_2, n_2) \tag{5.16}$$

$$m_2 = \frac{B}{H} \tag{5.17}$$

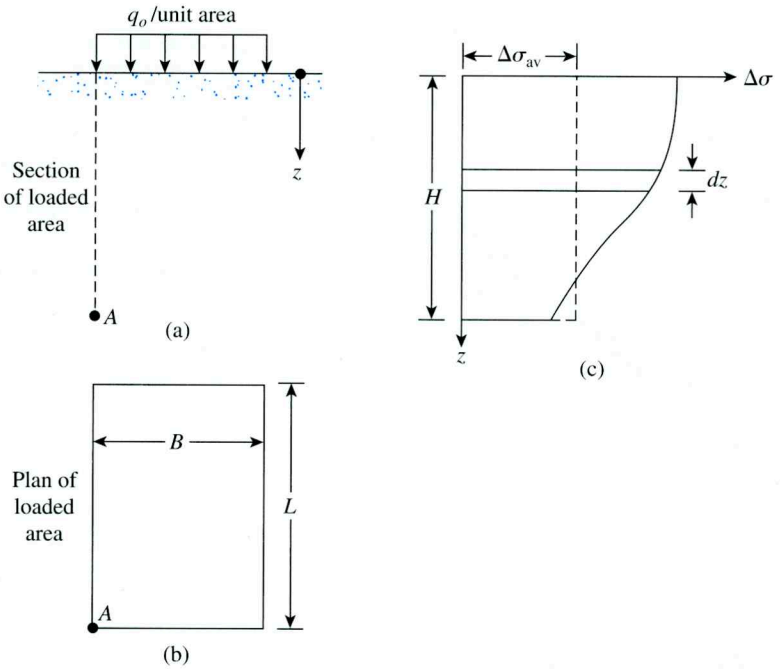


Figure 5.6 Average vertical stress increase due to a rectangularly loaded flexible area

and

$$n_2 = \frac{L}{H} \tag{5.18}$$

The variation of I_a with m_2 and n_2 is shown in Figure 5.7, as proposed by Griffiths (1984).

In estimating the consolidation settlement under a foundation, it may be required to determine the average vertical stress increase in only a given layer—that is, between $z = H_1$ and $z = H_2$, as shown in Figure 5.8. This can be done as (Griffiths, 1984)

$$\Delta\sigma_{av(H_2/H_1)} = q_o\left[\frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1}\right] \tag{5.19}$$

where

$\Delta\sigma_{av(H_2/H_1)}$ = average stress increase immediately below the corner of a uniformly loaded rectangular area between depths $z = H_1$ and $z = H_2$

$$I_{a(H_2)} = I_a \text{ for } z = 0 \text{ to } z = H_2 = f\left(m_2 = \frac{B}{H_2}, n_2 = \frac{L}{H_2}\right).$$

$$I_{a(H_1)} = I_a \text{ for } z = 0 \text{ to } z = H_1 = f\left(m_2 = \frac{B}{H_1}, n_2 = \frac{L}{H_1}\right)$$

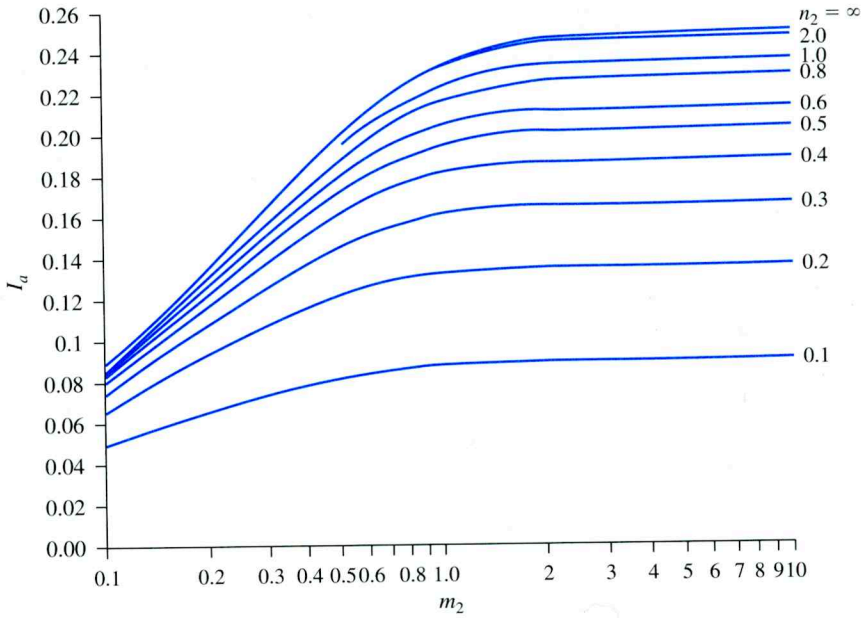


Figure 5.7 Griffiths' influence factor I_a

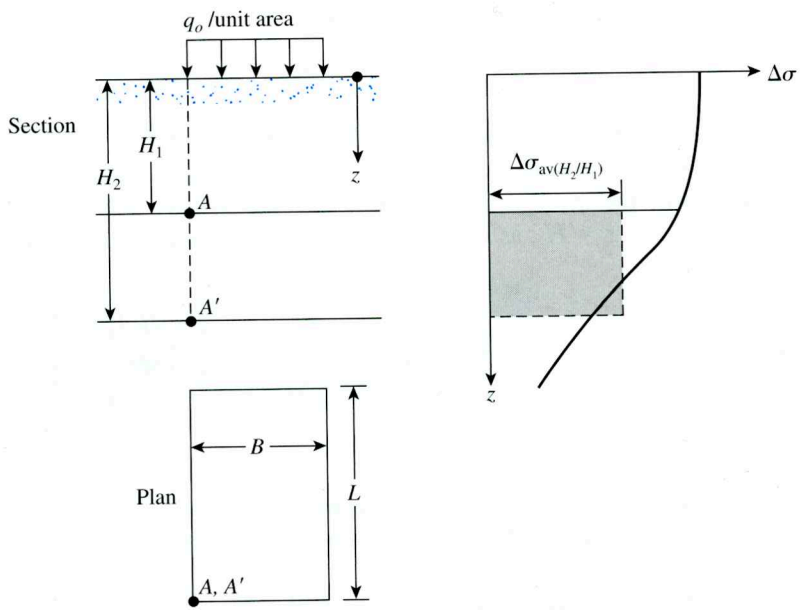


Figure 5.8 Average pressure increase between $z = H_1$ and $z = H_2$ below the corner of a uniformly loaded rectangular area

Example 5.2

Refer to Figure 5.9. Determine the *average* stress increase below the center of the loaded area between $z = 3$ m to $z = 5$ m (that is, between points A and A').

Solution

Refer to Figure 5.9. The loaded area can be divided into four rectangular areas, each measuring 1.5 m × 1.5 m ($L \times B$). Using Eq. (5.19), the average stress increase (between the required depths) below the corner of each rectangular area can be given as

$$\Delta\sigma_{\text{av}\,(H_2/H_1)} = q_o\left[\frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1}\right] = 100\left[\frac{(5)I_{a(H_2)} - (3)I_{a(H_1)}}{5 - 3}\right]$$

For $I_{a(H_2)}$:

$$m_2 = \frac{B}{H_2} = \frac{1.5}{5} = 0.3$$

$$n_2 = \frac{L}{H_2} = \frac{1.5}{5} = 0.3$$

Referring to Figure 5.7, for $m_2 = 0.3$ and $n_2 = 0.3$, $I_{a(H_2)} = 0.126$. For $I_{a(H_1)}$:

$$m_2 = \frac{B}{H_1} = \frac{1.5}{3} = 0.5$$

$$n_2 = \frac{L}{H_1} = \frac{1.5}{3} = 0.5$$

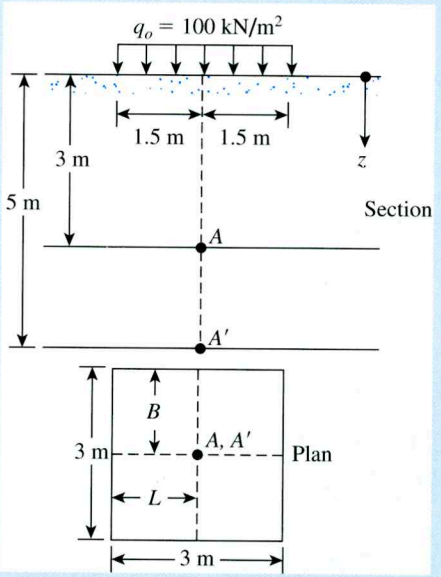


Figure 5.9 Determination of average increase in stress below a rectangular area

Referring to Figure 5.7, $I_{a(H_1)} = 0.175$, so

$$\Delta\sigma_{\text{av}\,(H_2/H_1)} = 100\left[\frac{(5)(0.126) - (3)(0.175)}{5 - 3}\right] = \mathbf{5.25\ kN/m^2}$$

The stress increase between $z = 3$ m to $z = 5$ m below the center of the loaded area is equal to

$$4\Delta\sigma_{\text{av}\,(H_2/H_1)} = (4)(5.25) = \mathbf{21\ kN/m^2}$$

■

5.6 Stress Increase under an Embankment

Figure 5.10 shows the cross section of an embankment of height H. For this two-dimensional loading condition, the vertical stress increase may be expressed as

$$\Delta\sigma = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right] \tag{5.20}$$

where

$q_o = \gamma H$
γ = unit weight of the embankment soil
H = height of the embankment

$$\alpha_1 = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right) \tag{5.21}$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right) \tag{5.22}$$

(Note that α_1 and α_2 are in radians.)

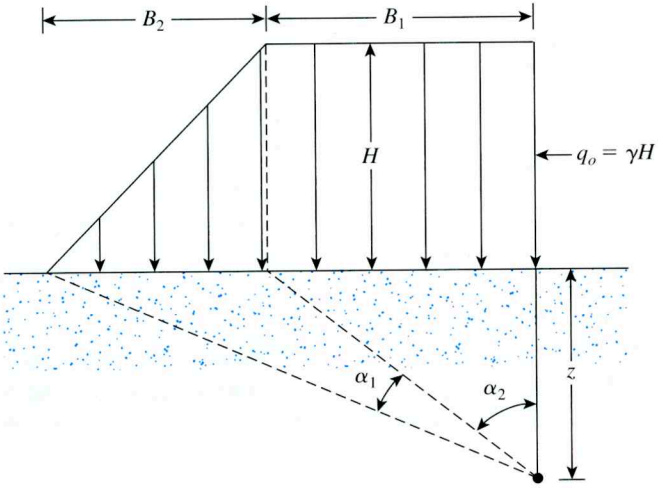


Figure 5.10 Embankment loading

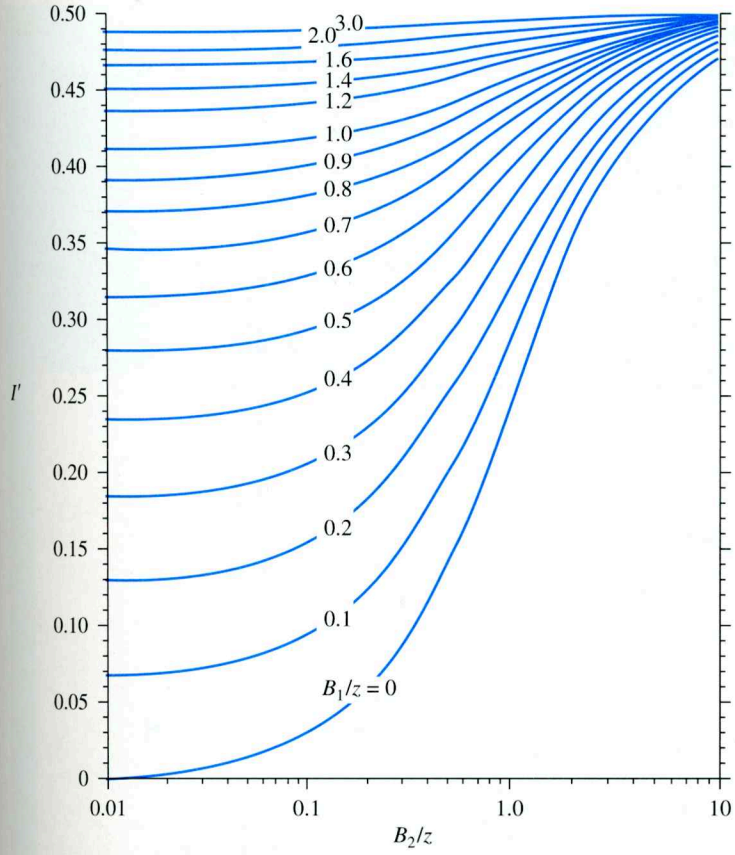


Figure 5.11 Influence value I' for embankment loading (After Osterberg, 1957) (Osterberg, J. O. (1957). "Influence Values for Vertical Stresses in Semi-Infinite Mass Due to Embankment Loading," Proceedings, Fourth International Conference on Soil Mechanics and Foundation Engineering, London, Vol. 1. pp. 393–396. With permission from ASCE.)

For a detailed derivation of Eq. (5.20), see Das (2008). A simplified form of the equation is

$$\Delta\sigma = q_o I' \tag{5.23}$$

where I' = a function of B_1/z and B_2/z.

The variation of I' with B_1/z and B_2/z is shown in Figure 5.11. An application of this diagram is given in Example 5.3.

Example 5.3

An embankment is shown in Figure 5.12a. Determine the stress increase under the embankment at points A_1 and A_2.

Solution

We have

$$\gamma H = (17.5)(7) = 122.5 \text{ kN/m}^2$$

Stress Increase at A_1

The left side of Figure 5.12b indicates that $B_1 = 2.5$ m and $B_2 = 14$ m, so

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5$$

and

$$\frac{B_2}{z} = \frac{14}{5} = 2.8$$

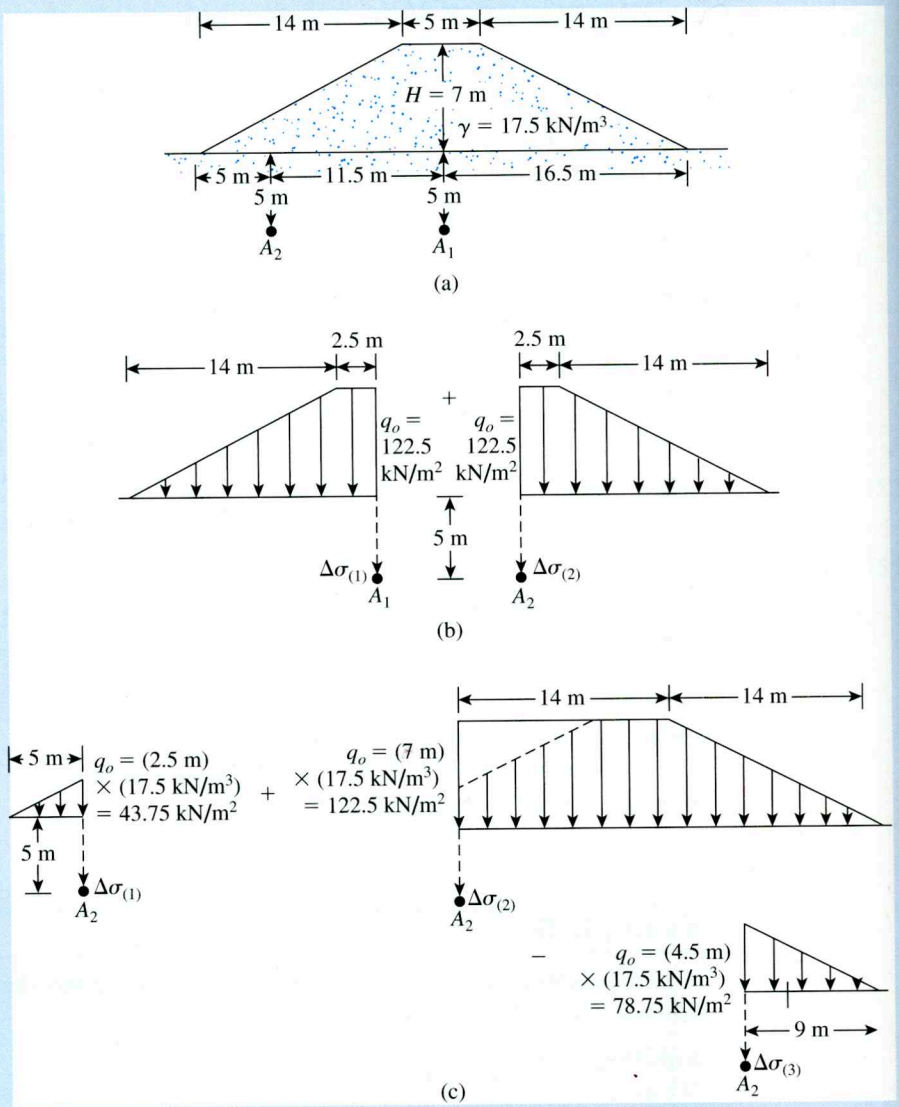


Figure 5.12 Stress increase due to embankment loading

According to Figure 5.11, in this case $I' = 0.445$. Because the two sides in Figure 5.12b are symmetrical, the value of I' for the right side will also be 0.445, so

$$\Delta\sigma = \Delta\sigma_{(1)} + \Delta\sigma_{(2)} = q_o[I'_{(\text{left side})} + I'_{(\text{right side})}]$$
$$= 122.5[0.445 + 0.445] = \mathbf{109.03\ kN/m^2}$$

Stress Increase at A_2

In Figure 5.12c, for the left side, $B_2 = 5$ m and $B_1 = 0$, so

$$\frac{B_2}{z} = \frac{5}{5} = 1$$

and

$$\frac{B_1}{z} = \frac{0}{5} = 0$$

According to Figure 5.11, for these values of B_2/z and B_1/z, $I' = 0.24$; hence,

$$\Delta\sigma_{(1)} = 43.75(0.24) = 10.5\ \text{kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8$$

and

$$\frac{B_1}{z} = \frac{14}{5} = 2.8$$

Thus, $I' = 0.495$, so

$$\Delta\sigma_{(2)} = 0.495(122.5) = 60.64\ \text{kN/m}^2$$

For the right side,

$$\frac{B_2}{z} = \frac{9}{5} = 1.8$$
$$\frac{B_1}{z} = \frac{0}{5} = 0$$

and $I' = 0.335$, so

$$\Delta\sigma_{(3)} = (78.75)(0.335) = 26.38\ \text{kN/m}^2$$

The total stress increase at point A_2 is

$$\Delta\sigma = \Delta\sigma_{(1)} + \Delta\sigma_{(2)} - \Delta\sigma_{(3)} = 10.5 + 60.64 - 26.38 = \mathbf{44.76\ kN/m^2}$$ ■

5.7 Westergaard's Solution for Vertical Stress Due to a Point Load

Boussinesq's solution for stress distribution due to a point load was presented in Section 5.2. The stress distribution due to various types of loading discussed in Sections 5.3 through 5.6 is based on integration of Boussinesq's solution.

Westergaard (1938) has proposed a solution for the determination of the vertical stress due to a point load P in an elastic solid medium in which there exist alternating layers with thin rigid reinforcements (Figure 5.13a). This type of assumption may be an idealization of a clay layer with thin seams of sand. For such an assumption, the vertical stress increase at a point A (Figure 5.13b) can be given as

$$\Delta\sigma = \frac{P\eta}{2\pi z^2}\left[\frac{1}{\eta^2 + (r/z)^2}\right]^{3/2} \tag{5.24}$$

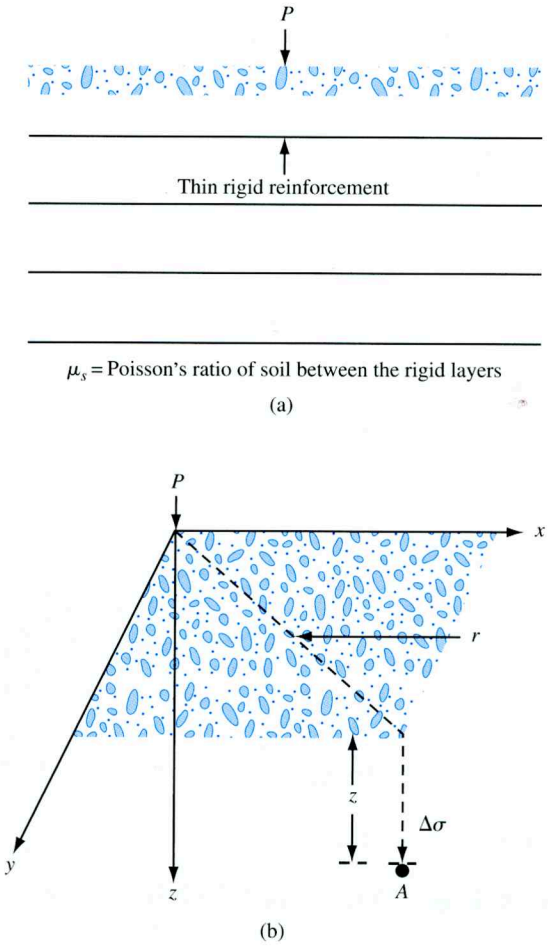


Figure 5.13 Westergaard's solution for vertical stress due to a point load

Table 5.4 Variation of I_1 [Eq. (5.27)].

	I_1		
r/z	$\mu_s = 0$	$\mu_s = 0.2$	$\mu_s = 0.4$
0	0.3183	0.4244	0.9550
0.1	0.3090	0.4080	0.8750
0.2	0.2836	0.3646	0.6916
0.3	0.2483	0.3074	0.4997
0.4	0.2099	0.2491	0.3480
0.5	0.1733	0.1973	0.2416
0.6	0.1411	0.1547	0.1700
0.7	0.1143	0.1212	0.1221
0.8	0.0925	0.0953	0.0897
0.9	0.0751	0.0756	0.0673
1.0	0.0613	0.0605	0.0516
1.5	0.0247	0.0229	0.0173
2.0	0.0118	0.0107	0.0076
2.5	0.0064	0.0057	0.0040
3.0	0.0038	0.0034	0.0023
4.0	0.0017	0.0015	0.0010
5.0	0.0009	0.0008	0.0005

where

$$\eta = \sqrt{\frac{1 - 2\mu_s}{2 - 2\mu_s}} \tag{5.25}$$

μ_s = Poisson's ratio of the solid between the rigid reinforcements

$r = \sqrt{x^2 + y^2}$

Equation (5.24) can be rewritten as

$$\Delta\sigma = \left(\frac{P}{z^2}\right) I_1 \tag{5.26}$$

where

$$I_1 = \frac{1}{2\pi\eta^2}\left[\left(\frac{r}{\eta z}\right)^2 + 1\right]^{-3/2} \tag{5.27}$$

Table 5.4 gives the variation of I_1 with μ_s.

5.8 Stress Distribution for Westergaard Material

Stress Due to a Circularly Loaded Area

Referring to Figure 5.2, if the circular area is located on a Westergaard-type material, the increase in vertical stress, $\Delta\sigma$, at a point located at a depth z immediately below the center of the area can be given as

$$\Delta\sigma = q_o\left\{1 - \frac{\eta}{\left[\eta^2 + \left(\frac{B}{2z}\right)^2\right]^{1/2}}\right\} \tag{5.28}$$

The term η has been defined in Eq. (5.25). The variations of $\Delta\sigma/q_o$ with $B/2z$ and $\mu_s = 0$ are given in Table 5.5.

Stress Due to a Uniformly Loaded Flexible Rectangular Area

Refer to Figure 5.3. If the flexible rectangular area is located on a Westergaard-type material, the stress increase at point A can be given as

$$\Delta\sigma = \frac{q_o}{2\pi}\left[\cot^{-1}\sqrt{\eta^2\left(\frac{1}{m^2} + \frac{1}{n^2}\right) + \eta^4\left(\frac{1}{m^2n^2}\right)}\right] \tag{5.29a}$$

where

$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$

Table 5.5 Variation of $\Delta\sigma/q_o$ with $B/2z$ and $\mu_s = 0$ [Eq. (5.28)]

$B/2z$	$\Delta\sigma/q_o$
0.00	0.0
0.25	0.0572
0.33	0.0938
0.50	0.1835
0.75	0.3140
1.00	0.4227
1.25	0.5076
1.50	0.5736
1.75	0.6254
2.00	0.6667
2.25	0.7002
2.50	0.7278
2.75	0.7510
3.00	0.7706
4.00	0.8259
5.00	0.8600
6.00	0.8830
7.00	0.8995
8.00	0.9120
9.00	0.9217
10.00	0.9295

Table 5.6. Variation of I_w with m and n ($\mu_s = 0$).

	n								
m	**0.1**	**0.2**	**0.4**	**0.5**	**0.6**	**1.0**	**2.0**	**5.0**	**10.0**
0.1	0.0031	0.0061	0.0110	0.0129	0.0144	0.0182	0.0211	0.0211	0.0223
0.2	0.0061	0.0118	0.0214	0.0251	0.0282	0.0357	0.0413	0.0434	0.0438
0.4	0.0110	0.0214	0.0390	0.0459	0.0516	0.0658	0.0768	0.0811	0.0847
0.5	0.0129	0.0251	0.0459	0.0541	0.0610	0.0781	0.0916	0.0969	0.0977
0.6	0.0144	0.0282	0.0516	0.0610	0.0687	0.0886	0.1044	0.1107	0.1117
1.0	0.0183	0.0357	0.0658	0.0781	0.0886	0.1161	0.1398	0.1491	0.1515
2.0	0.0211	0.0413	0.0768	0.0916	0.1044	0.1398	0.1743	0.1916	0.1948
5.0	0.0221	0.0435	0.0811	0.0969	0.1107	0.1499	0.1916	0.2184	0.2250
10.0	0.0223	0.0438	0.0817	0.0977	0.1117	0.1515	0.1948	0.2250	0.2341

or

$$\frac{\Delta\sigma}{q_o} = \frac{1}{2\pi}\left[\cot^{-1}\sqrt{\eta^2\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\eta^4\left(\frac{1}{m^2n^2}\right)}\right] = I_w \tag{5.29b}$$

Table 5.6 gives the variation of I_w with m and n (for $\mu_s = 0$).

Example 5.4

Solve Example 5.1 using Eq. (5.29). Assume $\mu_s = 0$.

Solution

From Example 5.1

$$m = 0.2$$
$$n = 0.4$$
$$\Delta\sigma = q_o(4I_w)$$

From Table 5.6, for $m = 0.2$ and $n = 0.4$, the value of $I_w \approx 0.0214$. So

$$\Delta\sigma = (150)(4 \times 0.0214) = \mathbf{12.84\ kN/m^2}$$

■

Elastic Settlement

5.9 Elastic Settlement of Foundations on Saturated Clay ($\mu_s = 0.5$)

Janbu et al. (1956) proposed an equation for evaluating the average settlement of flexible foundations on saturated clay soils (Poisson's ratio, $\mu_s = 0.5$). For the notation used in Figure 5.14, this equation is

$$S_e = A_1 A_2 \frac{q_o B}{E_s} \tag{5.30}$$

where A_1 is a function of H/B and L/B and A_2 is a function of D_f/B.

Christian and Carrier (1978) modified the values of A_1 and A_2 to some extent as presented in Figure 5.14.

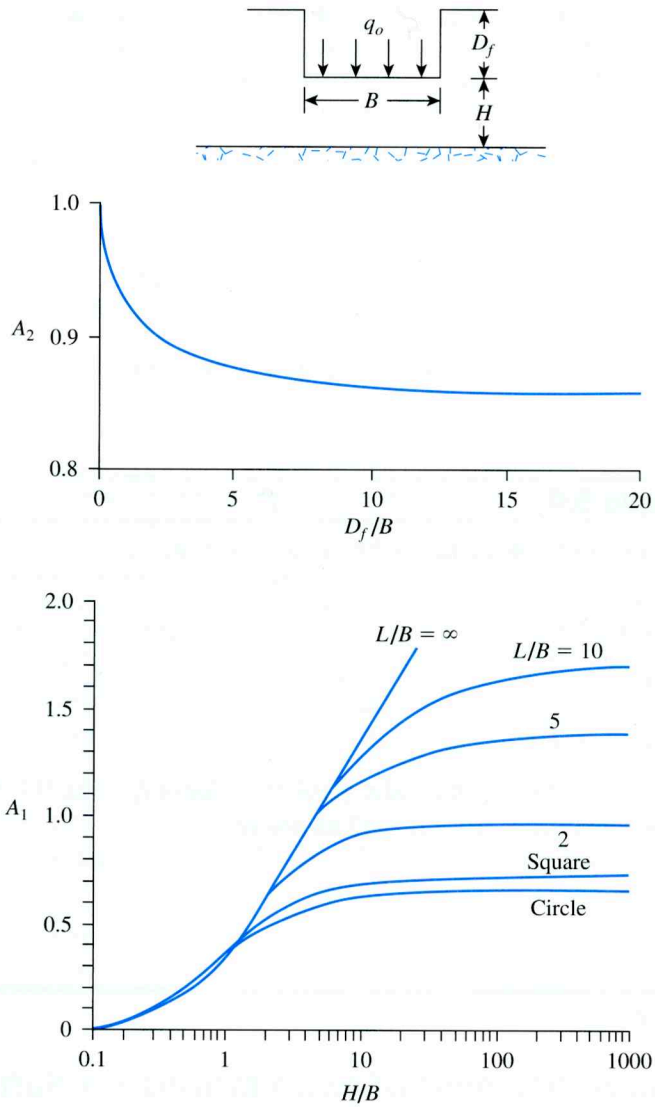


Figure 5.14 Values of A_1 and A_2 for elastic settlement calculation—Eq. (5.30) (After Christian and Carrier, 1978) (Christian, J. T. and and Carrier, W. D. (1978). "Janbu, Bjerrum and Kjaernsli's chart reinterpreted," *Canadian Geotechnical Journal*, Vol. 15, pp. 123–128. © 2008 NRC Canada or its licensors. Reproduced with permission.)

Table 5.7 Range of β for Clay [Eq. (5.31)][a]

Plasticity Index	β				
	OCR = 1	OCR = 2	OCR = 3	OCR = 4	OCR = 5
<30	1500–600	1380–500	1200–580	950–380	730–300
30 to 50	600–300	550–270	580–220	380–180	300–150
>50	300–150	270–120	220–100	180–90	150–75

[a]Interpolated from Duncan and Buchignani, (1976)

The modulus of elasticity (E_s) for clays can, in general, be given as

$$E_s = \beta c_u \tag{5.31}$$

where c_u = undrained shear strength.

The parameter β is primarily a function of the plasticity index and overconsolidation ratio. Table 5.7 provides a general range for β based on that proposed by Duncan and Buchignani (1976). In any case, proper judgment should be used in selecting the magnitude of β.

5.10 Settlement Based on the Theory of Elasticity

The elastic settlement of a shallow foundation can be estimated by using the theory of elasticity. From Hooke's law, as applied to Figure 5.15, we obtain

$$S_e = \int_0^H \varepsilon_z dz = \frac{1}{E_s}\int_0^H (\Delta\sigma_z - \mu_s\Delta\sigma_x - \mu_s\Delta\sigma_y)dz \tag{5.32}$$

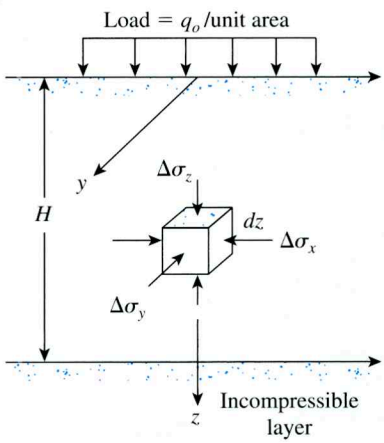


Figure 5.15 Elastic settlement of shallow foundation

where

S_e = elastic settlement
E_s = modulus of elasticity of soil
H = thickness of the soil layer
μ_s = Poisson's ratio of the soil
$\Delta\sigma_x, \Delta\sigma_y, \Delta\sigma_z$ = stress increase due to the net applied foundation load in the x, y, and z directions, respectively

Theoretically, if the foundation is perfectly flexible (see Figure 5.16 and Bowles, 1987), the settlement may be expressed as

$$S_e = q_o(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f \tag{5.33}$$

where

q_o = net applied pressure on the foundation
μ_s = Poisson's ratio of soil
E_s = average modulus of elasticity of the soil under the foundation, measured from $z = 0$ to about $z = 5B$
B' = $B/2$ for center of foundation
= B for corner of foundation
I_s = shape factor (Steinbrenner, 1934)

$$= F_1 + \frac{1-2\mu_s}{1-\mu_s}F_2 \tag{5.34}$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1) \tag{5.35}$$

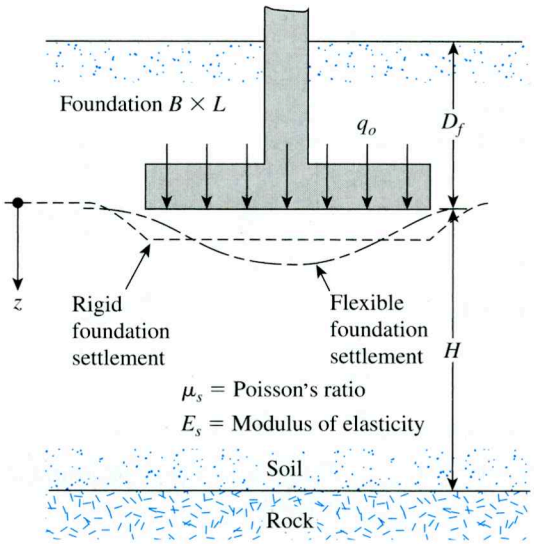


Figure 5.16 Elastic settlement of flexible and rigid foundations

$$F_2 = \frac{n'}{2\pi}\tan^{-1}A_2 \tag{5.36}$$

$$A_0 = m'\ln\frac{\left(1+\sqrt{m'^2+1}\right)\sqrt{m'^2+n'^2}}{m'\left(1+\sqrt{m'^2+n'^2+1}\right)} \tag{5.37}$$

$$A_1 = \ln\frac{\left(m'+\sqrt{m'^2+1}\right)\sqrt{1+n'^2}}{m'+\sqrt{m'^2+n'^2+1}} \tag{5.38}$$

$$A_2 = \frac{m'}{n'\sqrt{m'^2+n'^2+1}} \tag{5.39}$$

$$I_f = \text{depth factor (Fox, 1948)} = f\left(\frac{D_f}{B}, \mu_s, \text{ and } \frac{L}{B}\right) \tag{5.40}$$

α = a factor that depends on the location on the foundation where settlement is being calculated

To calculate settlement at the *center* of the foundation, we use

$$\alpha = 4$$
$$m' = \frac{L}{B}$$

and

$$n' = \frac{H}{\left(\frac{B}{2}\right)}$$

To calculate settlement at a *corner* of the foundation,

$$\alpha = 1$$
$$m' = \frac{L}{B}$$

and

$$n' = \frac{H}{B}$$

The variations of F_1 and F_2 [see Eqs. (5.35) and (5.36)] with m' and n' are given in Tables 5.8 and 5.9. Also, the variation of I_f with D_f/B (for μ_s = 0.3, 0.4, and 0.5) is given in Table 5.10. These values are also given in more detailed form by Bowles (1987).

The elastic settlement of a *rigid foundation* can be estimated as

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})} \tag{5.41}$$

Table 5.8 Variation of F_1 with m' and n'

	m'									
n'	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.014	0.013	0.012	0.011	0.011	0.011	0.010	0.010	0.010	0.010
0.50	0.049	0.046	0.044	0.042	0.041	0.040	0.038	0.038	0.037	0.037
0.75	0.095	0.090	0.087	0.084	0.082	0.080	0.077	0.076	0.074	0.074
1.00	0.142	0.138	0.134	0.130	0.127	0.125	0.121	0.118	0.116	0.115
1.25	0.186	0.183	0.179	0.176	0.173	0.170	0.165	0.161	0.158	0.157
1.50	0.224	0.224	0.222	0.219	0.216	0.213	0.207	0.203	0.199	0.197
1.75	0.257	0.259	0.259	0.258	0.255	0.253	0.247	0.242	0.238	0.235
2.00	0.285	0.290	0.292	0.292	0.291	0.289	0.284	0.279	0.275	0.271
2.25	0.309	0.317	0.321	0.323	0.323	0.322	0.317	0.313	0.308	0.305
2.50	0.330	0.341	0.347	0.350	0.351	0.351	0.348	0.344	0.340	0.336
2.75	0.348	0.361	0.369	0.374	0.377	0.378	0.377	0.373	0.369	0.365
3.00	0.363	0.379	0.389	0.396	0.400	0.402	0.402	0.400	0.396	0.392
3.25	0.376	0.394	0.406	0.415	0.420	0.423	0.426	0.424	0.421	0.418
3.50	0.388	0.408	0.422	0.431	0.438	0.442	0.447	0.447	0.444	0.441
3.75	0.399	0.420	0.436	0.447	0.454	0.460	0.467	0.458	0.466	0.464
4.00	0.408	0.431	0.448	0.460	0.469	0.476	0.484	0.487	0.486	0.484
4.25	0.417	0.440	0.458	0.472	0.481	0.484	0.495	0.514	0.515	0.515
4.50	0.424	0.450	0.469	0.484	0.495	0.503	0.516	0.521	0.522	0.522
4.75	0.431	0.458	0.478	0.494	0.506	0.515	0.530	0.536	0.539	0.539
5.00	0.437	0.465	0.487	0.503	0.516	0.526	0.543	0.551	0.554	0.554
5.25	0.443	0.472	0.494	0.512	0.526	0.537	0.555	0.564	0.568	0.569
5.50	0.448	0.478	0.501	0.520	0.534	0.546	0.566	0.576	0.581	0.584
5.75	0.453	0.483	0.508	0.527	0.542	0.555	0.576	0.588	0.594	0.597
6.00	0.457	0.489	0.514	0.534	0.550	0.563	0.585	0.598	0.606	0.609
6.25	0.461	0.493	0.519	0.540	0.557	0.570	0.594	0.609	0.617	0.621
6.50	0.465	0.498	0.524	0.546	0.563	0.577	0.603	0.618	0.627	0.632
6.75	0.468	0.502	0.529	0.551	0.569	0.584	0.610	0.627	0.637	0.643
7.00	0.471	0.506	0.533	0.556	0.575	0.590	0.618	0.635	0.646	0.653
7.25	0.474	0.509	0.538	0.561	0.580	0.596	0.625	0.643	0.655	0.662
7.50	0.477	0.513	0.541	0.565	0.585	0.601	0.631	0.650	0.663	0.671
7.75	0.480	0.516	0.545	0.569	0.589	0.606	0.637	0.658	0.671	0.680
8.00	0.482	0.519	0.549	0.573	0.594	0.611	0.643	0.664	0.678	0.688
8.25	0.485	0.522	0.552	0.577	0.598	0.615	0.648	0.670	0.685	0.695
8.50	0.487	0.524	0.555	0.580	0.601	0.619	0.653	0.676	0.692	0.703
8.75	0.489	0.527	0.558	0.583	0.605	0.623	0.658	0.682	0.698	0.710
9.00	0.491	0.529	0.560	0.587	0.609	0.627	0.663	0.687	0.705	0.716
9.25	0.493	0.531	0.563	0.589	0.612	0.631	0.667	0.693	0.710	0.723
9.50	0.495	0.533	0.565	0.592	0.615	0.634	0.671	0.697	0.716	0.719
9.75	0.496	0.536	0.568	0.595	0.618	0.638	0.675	0.702	0.721	0.735
10.00	0.498	0.537	0.570	0.597	0.621	0.641	0.679	0.707	0.726	0.740
20.00	0.529	0.575	0.614	0.647	0.677	0.702	0.756	0.797	0.830	0.858
50.00	0.548	0.598	0.640	0.678	0.711	0.740	0.803	0.853	0.895	0.931
100.00	0.555	0.605	0.649	0.688	0.722	0.753	0.819	0.872	0.918	0.956

Table 5.8 (*Continued*)

	m′									
n′	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036
0.75	0.073	0.073	0.072	0.072	0.072	0.072	0.071	0.071	0.071	0.071
1.00	0.114	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
1.25	0.155	0.154	0.153	0.152	0.152	0.151	0.151	0.150	0.150	0.150
1.50	0.195	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
1.75	0.233	0.232	0.229	0.228	0.227	0.226	0.225	0.223	0.223	0.223
2.00	0.269	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
2.25	0.302	0.300	0.296	0.294	0.293	0.291	0.291	0.287	0.287	0.287
2.50	0.333	0.331	0.327	0.324	0.322	0.321	0.320	0.316	0.315	0.315
2.75	0.362	0.359	0.355	0.352	0.350	0.348	0.347	0.343	0.342	0.342
3.00	0.389	0.386	0.382	0.378	0.376	0.374	0.373	0.368	0.367	0.367
3.25	0.415	0.412	0.407	0.403	0.401	0.399	0.397	0.391	0.390	0.390
3.50	0.438	0.435	0.430	0.427	0.424	0.421	0.420	0.413	0.412	0.411
3.75	0.461	0.458	0.453	0.449	0.446	0.443	0.441	0.433	0.432	0.432
4.00	0.482	0.479	0.474	0.470	0.466	0.464	0.462	0.453	0.451	0.451
4.25	0.516	0.496	0.484	0.473	0.471	0.471	0.470	0.468	0.462	0.460
4.50	0.520	0.517	0.513	0.508	0.505	0.502	0.499	0.489	0.487	0.487
4.75	0.537	0.535	0.530	0.526	0.523	0.519	0.517	0.506	0.504	0.503
5.00	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.519	0.519
5.25	0.569	0.568	0.564	0.560	0.556	0.553	0.550	0.537	0.534	0.534
5.50	0.584	0.583	0.579	0.575	0.571	0.568	0.585	0.551	0.549	0.548
5.75	0.597	0.597	0.594	0.590	0.586	0.583	0.580	0.565	0.583	0.562
6.00	0.611	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
6.25	0.623	0.623	0.621	0.618	0.615	0.611	0.608	0.592	0.589	0.588
6.50	0.635	0.635	0.634	0.631	0.628	0.625	0.622	0.605	0.601	0.600
6.75	0.646	0.647	0.646	0.644	0.641	0.637	0.634	0.617	0.613	0.612
7.00	0.656	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
7.25	0.666	0.669	0.669	0.668	0.665	0.662	0.659	0.640	0.635	0.634
7.50	0.676	0.679	0.680	0.679	0.676	0.673	0.670	0.651	0.646	0.645
7.75	0.685	0.688	0.690	0.689	0.687	0.684	0.681	0.661	0.656	0.655
8.00	0.694	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
8.25	0.702	0.706	0.710	0.710	0.708	0.705	0.703	0.682	0.676	0.675
8.50	0.710	0.714	0.719	0.719	0.718	0.715	0.713	0.692	0.686	0.684
8.75	0.717	0.722	0.727	0.728	0.727	0.725	0.723	0.701	0.695	0.693
9.00	0.725	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
9.25	0.731	0.737	0.744	0.746	0.745	0.744	0.742	0.719	0.713	0.711
9.50	0.738	0.744	0.752	0.754	0.754	0.753	0.751	0.728	0.721	0.719
9.75	0.744	0.751	0.759	0.762	0.762	0.761	0.759	0.737	0.729	0.727
10.00	0.750	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
20.00	0.878	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
50.00	0.962	0.989	1.034	1.070	1.100	1.125	1.146	1.265	1.279	1.261
100.00	0.990	1.020	1.072	1.114	1.150	1.182	1.209	1.408	1.489	1.499

Table 5.9 Variation of F_2 with m' and n'

	m'									
n'	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.049	0.050	0.051	0.051	0.051	0.052	0.052	0.052	0.052	0.052
0.50	0.074	0.077	0.080	0.081	0.083	0.084	0.086	0.086	0.0878	0.087
0.75	0.083	0.089	0.093	0.097	0.099	0.101	0.104	0.106	0.107	0.108
1.00	0.083	0.091	0.098	0.102	0.106	0.109	0.114	0.117	0.119	0.120
1.25	0.080	0.089	0.096	0.102	0.107	0.111	0.118	0.122	0.125	0.127
1.50	0.075	0.084	0.093	0.099	0.105	0.110	0.118	0.124	0.128	0.130
1.75	0.069	0.079	0.088	0.095	0.101	0.107	0.117	0.123	0.128	0.131
2.00	0.064	0.074	0.083	0.090	0.097	0.102	0.114	0.121	0.127	0.131
2.25	0.059	0.069	0.077	0.085	0.092	0.098	0.110	0.119	0.125	0.130
2.50	0.055	0.064	0.073	0.080	0.087	0.093	0.106	0.115	0.122	0.127
2.75	0.051	0.060	0.068	0.076	0.082	0.089	0.102	0.111	0.119	0.125
3.00	0.048	0.056	0.064	0.071	0.078	0.084	0.097	0.108	0.116	0.122
3.25	0.045	0.053	0.060	0.067	0.074	0.080	0.093	0.104	0.112	0.119
3.50	0.042	0.050	0.057	0.064	0.070	0.076	0.089	0.100	0.109	0.116
3.75	0.040	0.047	0.054	0.060	0.067	0.073	0.086	0.096	0.105	0.113
4.00	0.037	0.044	0.051	0.057	0.063	0.069	0.082	0.093	0.102	0.110
4.25	0.036	0.042	0.049	0.055	0.061	0.066	0.079	0.090	0.099	0.107
4.50	0.034	0.040	0.046	0.052	0.058	0.063	0.076	0.086	0.096	0.104
4.75	0.032	0.038	0.044	0.050	0.055	0.061	0.073	0.083	0.093	0.101
5.00	0.031	0.036	0.042	0.048	0.053	0.058	0.070	0.080	0.090	0.098
5.25	0.029	0.035	0.040	0.046	0.051	0.056	0.067	0.078	0.087	0.095
5.50	0.028	0.033	0.039	0.044	0.049	0.054	0.065	0.075	0.084	0.092
5.75	0.027	0.032	0.037	0.042	0.047	0.052	0.063	0.073	0.082	0.090
6.00	0.026	0.031	0.036	0.040	0.045	0.050	0.060	0.070	0.079	0.087
6.25	0.025	0.030	0.034	0.039	0.044	0.048	0.058	0.068	0.077	0.085
6.50	0.024	0.029	0.033	0.038	0.042	0.046	0.056	0.066	0.075	0.083
6.75	0.023	0.028	0.032	0.036	0.041	0.045	0.055	0.064	0.073	0.080
7.00	0.022	0.027	0.031	0.035	0.039	0.043	0.053	0.062	0.071	0.078
7.25	0.022	0.026	0.030	0.034	0.038	0.042	0.051	0.060	0.069	0.076
7.50	0.021	0.025	0.029	0.033	0.037	0.041	0.050	0.059	0.067	0.074
7.75	0.020	0.024	0.028	0.032	0.036	0.039	0.048	0.057	0.065	0.072
8.00	0.020	0.023	0.027	0.031	0.035	0.038	0.047	0.055	0.063	0.071
8.25	0.019	0.023	0.026	0.030	0.034	0.037	0.046	0.054	0.062	0.069
8.50	0.018	0.022	0.026	0.029	0.033	0.036	0.045	0.053	0.060	0.067
8.75	0.018	0.021	0.025	0.028	0.032	0.035	0.043	0.051	0.059	0.066
9.00	0.017	0.021	0.024	0.028	0.031	0.034	0.042	0.050	0.057	0.064
9.25	0.017	0.020	0.024	0.027	0.030	0.033	0.041	0.049	0.056	0.063
9.50	0.017	0.020	0.023	0.026	0.029	0.033	0.040	0.048	0.055	0.061
9.75	0.016	0.019	0.023	0.026	0.029	0.032	0.039	0.047	0.054	0.060
10.00	0.016	0.019	0.022	0.025	0.028	0.031	0.038	0.046	0.052	0.059
20.00	0.008	0.010	0.011	0.013	0.014	0.016	0.020	0.024	0.027	0.031
50.00	0.003	0.004	0.004	0.005	0.006	0.006	0.008	0.010	0.011	0.013
100.00	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005	0.006	0.006

Table 5.9 (*Continued*)

	m′									
n′	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
0.50	0.087	0.087	0.088	0.088	0.088	0.088	0.088	0.088	0.088	0.088
0.75	0.109	0.109	0.109	0.110	0.110	0.110	0.110	0.111	0.111	0.111
1.00	0.121	0.122	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.125
1.25	0.128	0.130	0.131	0.132	0.132	0.133	0.133	0.134	0.134	0.134
1.50	0.132	0.134	0.136	0.137	0.138	0.138	0.139	0.140	0.140	0.140
1.75	0.134	0.136	0.138	0.140	0.141	0.142	0.142	0.144	0.144	0.145
2.00	0.134	0.136	0.139	0.141	0.143	0.144	0.145	0.147	0.147	0.148
2.25	0.133	0.136	0.140	0.142	0.144	0.145	0.146	0.149	0.150	0.150
2.50	0.132	0.135	0.139	0.142	0.144	0.146	0.147	0.151	0.151	0.151
2.75	0.130	0.133	0.138	0.142	0.144	0.146	0.147	0.152	0.152	0.153
3.00	0.127	0.131	0.137	0.141	0.144	0.145	0.147	0.152	0.153	0.154
3.25	0.125	0.129	0.135	0.140	0.143	0.145	0.147	0.153	0.154	0.154
3.50	0.122	0.126	0.133	0.138	0.142	0.144	0.146	0.153	0.155	0.155
3.75	0.119	0.124	0.131	0.137	0.141	0.143	0.145	0.154	0.155	0.155
4.00	0.116	0.121	0.129	0.135	0.139	0.142	0.145	0.154	0.155	0.156
4.25	0.113	0.119	0.127	0.133	0.138	0.141	0.144	0.154	0.156	0.156
4.50	0.110	0.116	0.125	0.131	0.136	0.140	0.143	0.154	0.156	0.156
4.75	0.107	0.113	0.123	0.130	0.135	0.139	0.142	0.154	0.156	0.157
5.00	0.105	0.111	0.120	0.128	0.133	0.137	0.140	0.154	0.156	0.157
5.25	0.102	0.108	0.118	0.126	0.131	0.136	0.139	0.154	0.156	0.157
5.50	0.099	0.106	0.116	0.124	0.130	0.134	0.138	0.154	0.156	0.157
5.75	0.097	0.103	0.113	0.122	0.128	0.133	0.136	0.154	0.157	0.157
6.00	0.094	0.101	0.111	0.120	0.126	0.131	0.135	0.153	0.157	0.157
6.25	0.092	0.098	0.109	0.118	0.124	0.129	0.134	0.153	0.157	0.158
6.50	0.090	0.096	0.107	0.116	0.122	0.128	0.132	0.153	0.157	0.158
6.75	0.087	0.094	0.105	0.114	0.121	0.126	0.131	0.153	0.157	0.158
7.00	0.085	0.092	0.103	0.112	0.119	0.125	0.129	0.152	0.157	0.158
7.25	0.083	0.090	0.101	0.110	0.117	0.123	0.128	0.152	0.157	0.158
7.50	0.081	0.088	0.099	0.108	0.115	0.121	0.126	0.152	0.156	0.158
7.75	0.079	0.086	0.097	0.106	0.114	0.120	0.125	0.151	0.156	0.158
8.00	0.077	0.084	0.095	0.104	0.112	0.118	0.124	0.151	0.156	0.158
8.25	0.076	0.082	0.093	0.102	0.110	0.117	0.122	0.150	0.156	0.158
8.50	0.074	0.080	0.091	0.101	0.108	0.115	0.121	0.150	0.156	0.158
8.75	0.072	0.078	0.089	0.099	0.107	0.114	0.119	0.150	0.156	0.158
9.00	0.071	0.077	0.088	0.097	0.105	0.112	0.118	0.149	0.156	0.158
9.25	0.069	0.075	0.086	0.096	0.104	0.110	0.116	0.149	0.156	0.158
9.50	0.068	0.074	0.085	0.094	0.102	0.109	0.115	0.148	0.156	0.158
9.75	0.066	0.072	0.083	0.092	0.100	0.107	0.113	0.148	0.156	0.158
10.00	0.065	0.071	0.082	0.091	0.099	0.106	0.112	0.147	0.156	0.158
20.00	0.035	0.039	0.046	0.053	0.059	0.065	0.071	0.124	0.148	0.156
50.00	0.014	0.016	0.019	0.022	0.025	0.028	0.031	0.071	0.113	0.142
100.00	0.007	0.008	0.010	0.011	0.013	0.014	0.016	0.039	0.071	0.113

Table 5.10 Variation of I_f with D_f/B, B/L, and μ_s

		B/L		
μ_s	D_f/B	0.2	0.5	1.0
0.3	0.2	0.95	0.93	0.90
	0.4	0.90	0.86	0.81
	0.6	0.85	0.80	0.74
	1.0	0.78	0.71	0.65
0.4	0.2	0.97	0.96	0.93
	0.4	0.93	0.89	0.85
	0.6	0.89	0.84	0.78
	1.0	0.82	0.75	0.69
0.5	0.2	0.99	0.98	0.96
	0.4	0.95	0.93	0.89
	0.6	0.92	0.87	0.82
	1.0	0.85	0.79	0.72

Due to the nonhomogeneous nature of soil deposits, the magnitude of E_s may vary with depth. For that reason, Bowles (1987) recommended using a weighted average of E_s in Eq. (5.33), or

$$E_s = \frac{\sum E_{s(i)} \Delta z}{\bar{z}} \tag{5.42}$$

where

$E_{s(i)}$ = soil modulus of elasticity within a depth Δz
$\bar{z}$ = H or $5B$, whichever is smaller

Example 5.5

Refer to Figure 5.16 and consider a rigid square foundation 8 ft × 8 ft in plan (D_f = 4 ft) on a layer of normally consolidated sand. A rock layer is located at z = 36 ft. The following is an approximation of the standard penetration number (N_{60}) with z.

z (ft)	N_{60}
0–8	7
8–21	11
21–36	14

Given: μ_s = 0.3 and q_o = 3500 lb/ft^2. Estimate the elastic settlement of the foundation. Use Eq. (2.29).

Solution

Calculation of Average E_s

From Eq. (2.29),

$$E_s \approx p_a \alpha N_{60}$$

$B = 8$ ft, $p_a \approx 2000$ lb/ft^2, and $\alpha \approx 10$ (since it is a normally consolidated clean sand).

Given: $H = 36 \text{ ft} < 5B$.

The approximate variations of E_s using Eq. (2.29) are given below.

z (ft)	Δz (ft)	N_{60}	E_s (lb/ft^2)
0–8	8	7	140,000
8–21	13	11	220,000
21–36	15	14	280,000

Using Eq. (5.42),

$$E_s = \frac{\Sigma E_{s(i)} \Delta z}{\bar{z}} = \frac{(140{,}000)(8) + (220{,}000)(13) + (280{,}000)(15)}{36}$$

$$= 227{,}200 \text{ lb/ft}^2$$

Calculation of S_e below the Center of the Foundation [Eq. (5.33)]

$$S_e = q_o(\alpha B')\frac{1 - \mu_s^2}{E_s} I_s I_f$$

$$B' = \frac{8}{2} = 4 \text{ ft}$$

$$\alpha = 4$$

$$m' = \frac{L}{B} = 1$$

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{36}{\left(\frac{8}{2}\right)} = 9$$

$$I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

From Table 5.8, $F_1 = 0.491$. From Table 5.9, $F_2 = 0.017$.

$$I_s = 0.491 + \left[\frac{1 - (2)(0.3)}{1 - 0.3}\right](0.017) = 0.5007$$

For $\mu_s = 3$, $D_f/B = 4/8 = 0.5$, and $B/L = 1$, the value of I_f is about 0.78 (Table 5.10).

$$S_e = (3500)(4 \times 4)\left(\frac{1 - 0.3^2}{227{,}200}\right)(0.5007)(0.78) = 0.0876 \text{ ft} \approx 1.05 \text{ in.}$$

Calculation of S_e for Rigid Foundation
From Eq. (5.41),

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})} = (1.05)(0.93) = 0.9765 \text{ in.} \approx \mathbf{0.98 \text{ in.}}$$

5.11 Improved Equation for Elastic Settlement

In 1999, Mayne and Poulos presented an improved formula for calculating the elastic settlement of foundations. The formula takes into account the rigidity of the foundation, the depth of embedment of the foundation, the increase in the modulus of elasticity of the soil with depth, and the location of rigid layers at a limited depth. To use Mayne and Poulos's equation, one needs to determine the equivalent diameter B_e of a rectangular foundation, or

$$B_e = \sqrt{\frac{4BL}{\pi}} \tag{5.43}$$

where

B = width of foundation
L = length of foundation

For circular foundations,

$$B_e = B \tag{5.44}$$

where B = diameter of foundation.

Figure 5.17 shows a foundation with an equivalent diameter B_e located at a depth D_f below the ground surface. Let the thickness of the foundation be t and the modulus of elasticity of the foundation material be E_f. A rigid layer is located at a depth H below the bottom of the foundation. The modulus of elasticity of the compressible soil layer can be given as

$$E_s = E_o + kz \tag{5.45}$$

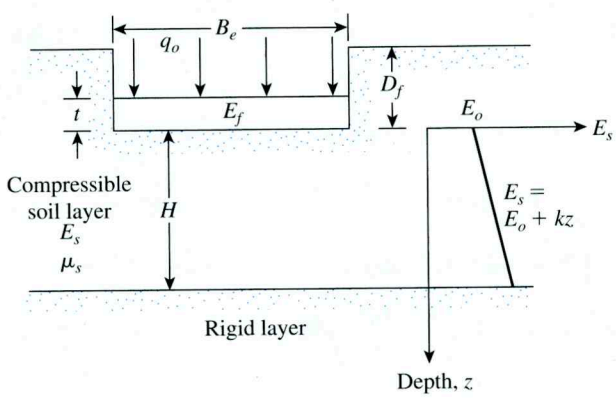


Figure 5.17 Improved equation for calculating elastic settlement: general parameters

With the preceding parameters defined, the elastic settlement below the center of the foundation is

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}\left(1 - \mu_s^2\right) \tag{5.46}$$

where

I_G = influence factor for the variation of E_s with depth

$= f\left(\beta = \frac{E_o}{kB_e}, \frac{H}{B_e}\right)$

I_F = foundation rigidity correction factor

I_E = foundation embedment correction factor

Figure 5.18 shows the variation of I_G with $\beta = E_o/kB_e$ and H/B_e. The foundation rigidity correction factor can be expressed as

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\dfrac{E_f}{E_o + \dfrac{B_e}{2}k}\right)\left(\dfrac{2t}{B_e}\right)^3} \tag{5.47}$$

Similarly, the embedment correction factor is

$$I_E = 1 - \frac{1}{3.5\exp(1.22\mu_s - 0.4)\left(\dfrac{B_e}{D_f} + 1.6\right)} \tag{5.48}$$

Figures 5.19 and 5.20 show the variation of I_F and I_E with terms expressed in Eqs. (5.47) and (5.48).

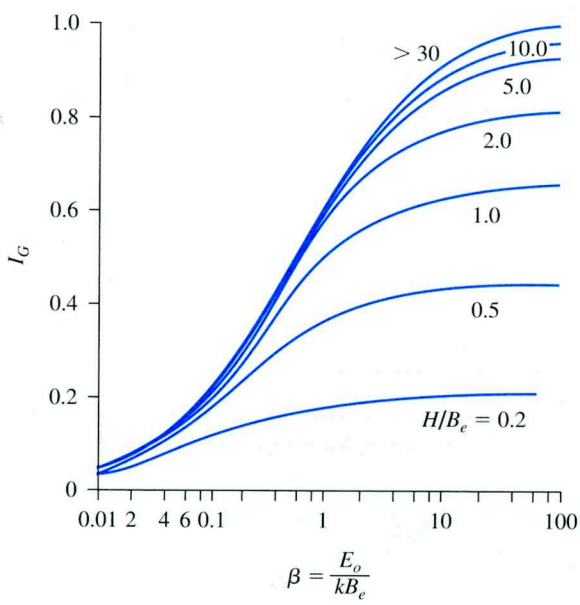


Figure 5.18 Variation of I_G with β

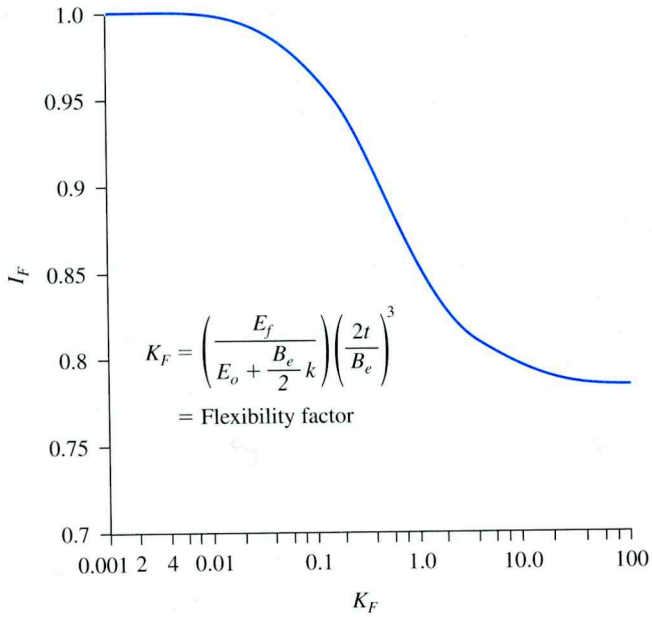


Figure 5.19 Variation of rigidity correction factor I_F with flexibility factor K_F [Eq. (5.47)]

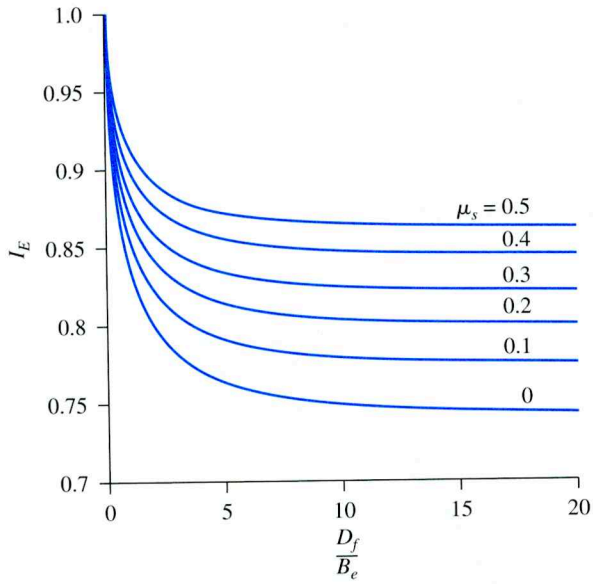


Figure 5.20 Variation of embedment correction factor I_E with D_f/B_e [Eq (5.48)]

Example 5.6

For a shallow foundation supported by a silty clay, as shown in Figure 5.17,

Length = L = 10 ft
Width = B = 5 ft
Depth of foundation = D_f = 5 ft

Thickness of foundation $= t = 1$ ft
Load per unit area $= q_o = 5000$ lb/ft^2
$E_f = 2.3 \times 10^6$ lb/in^2

The silty clay soil has the following properties:

$H = 12$ ft
$\mu_s = 0.3$
$E_o = 1400$ lb/in^2
$k = 25$ lb/in^2/ft

Estimate the elastic settlement of the foundation.

Solution

From Eq. (5.43), the equivalent diameter is

$$B_e = \sqrt{\frac{4BL}{\pi}} = \sqrt{\frac{(4)(5)(10)}{\pi}} = 7.98 \text{ ft}$$

so

$$\beta = \frac{E_o}{kB_e} = \frac{1400}{(25)(7.98)} = 7.02$$

and

$$\frac{H}{B_e} = \frac{12}{7.98} = 1.5$$

From Figure 5.18, for $\beta = 7.02$ and $H/B_e = 1.5$, the value of $I_G \approx 0.69$. From Eq. (5.47),

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\dfrac{E_f}{E_o + \dfrac{B_e}{2}k}\right)\left(\dfrac{2t}{B_e}\right)^3}$$

$$= \frac{\pi}{4} + \frac{1}{4.6 + 10\left[\dfrac{2.3 \times 10^6}{1400 + \left(\dfrac{7.98}{2}\right)(25)}\right]\left[\dfrac{(2)(1.0)}{7.98}\right]^3} = 0.789$$

From Eq. (5.48),

$$I_E = 1 - \frac{1}{3.5 \exp(1.22\mu_s - 0.4)\left(\dfrac{B_e}{D_f} + 1.6\right)}$$

$$= 1 - \frac{1}{3.5\exp[(1.22)(0.3) - 0.4]\left(\frac{7.98}{5} + 1.6\right)} = 0.908$$

From Eq. (5.46),

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}(1 - \mu_s^2)$$

so, with $q_o = 5000\ \text{lb/ft}^2$, it follows that

$$S_e = \frac{(5000)(7.98)(0.69)(0.789)(0.908)}{(1400)(144)}(1 - 0.3^2) = 0.089\ \text{ft} = \mathbf{1.07\ in.}$$ ■

5.12 Settlement of Sandy Soil: Use of Strain Influence Factor

The settlement of granular soils can also be evaluated by the use of a semiempirical *strain influence factor* proposed by Schmertmann et al. (1978). According to this method (Figure 5.21), the settlement is

$$S_e = C_1 C_2 (\bar{q} - q) \sum_{0}^{z_2} \frac{I_z}{E_s} \Delta z \tag{5.49}$$

where

I_z = strain influence factor
C_1 = a correction factor for the depth of foundation embedment $= 1 - 0.5[q/(\bar{q} - q)]$
C_2 = a correction factor to account for creep in soil $= 1 + 0.2 \log(\text{time in years}/0.1)$
$\bar{q}$ = stress at the level of the foundation
$q = \gamma D_f$ = effective stress at the base of the foundation
E_s = modulus of elasticity of soil

The recommended variation of the strain influence factor I_z for square ($L/B = 1$) or circular foundations and for foundations with $L/B \geq 10$ is shown in Figure 5.21. The I_z diagrams for $1 < L/B < 10$ can be interpolated.

Note that the maximum value of I_z [that is, $I_{z(m)}$] occurs at $z = z_1$ and then reduces to zero at $z = z_2$. The maximum value of I_z can be calculated as

$$I_{z(m)} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}} \tag{5.50}$$

where

$q'_{z(1)}$ = effective stress at a depth of z_1 before construction of the foundation

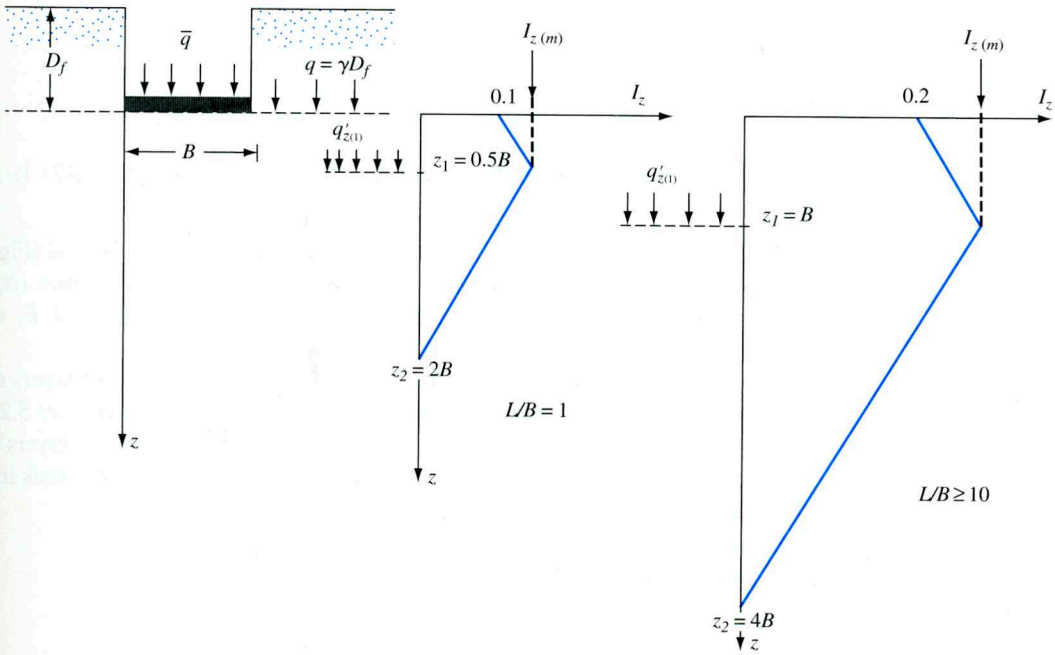


Figure 5.21 Variation of strain influence factor with depth and L/B

The following relations are suggested by Salgado (2008) for interpolation of I_z at $z = 0$, z_1/B, and z_2/B for rectangular foundations.

- I_z at $z = 0$

$$I_z = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) \leq 0.2 \tag{5.51}$$

- Variation of z_1/B for $I_{z(m)}$

$$\frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) \leq 1 \tag{5.52}$$

- Variation of z_2/B

$$\frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) \leq 4 \tag{5.53}$$

Schmertmann et al. (1978) suggested that

$$E_s = 2.5q_c \text{ (for square foundation)} \tag{5.54}$$

and

$$E_s = 3.5q_c \text{ (for } L/B \geq 10) \tag{5.55}$$

where q_c = cone penetration resistance.

It appears reasonable to write (Terzaghi et al., 1996)

$$E_{s(\text{ rectangle})} = \left(1 + 0.4 \log\frac{L}{B}\right) E_{s(\text{ square})} \tag{5.56}$$

The procedure for calculating elastic settlement using Eq. (5.49) is given here (Figure 5.22).

Step 1. Plot the foundation and the variation of I_z with depth to scale (Figure 5.22a).
Step 2. Using the correlation from standard penetration resistance (N_{60}) or cone penetration resistance (q_c), plot the actual variation of E_s with depth (Figure 5.22b).
Step 3. Approximate the actual variation of E_s into a number of layers of soil having a constant E_s, such as $E_{s(1)}, E_{s(2)}, \ldots, E_{s(i)}, \ldots E_{s(n)}$ (Figure 5.22b).
Step 4. Divide the soil layer from $z = 0$ to $z = z_2$ into a number of layers by drawing horizontal lines. The number of layers will depend on the break in continuity in the I_z and E_s diagrams.
Step 5. Prepare a table (such as Table 5.11) to obtain $\Sigma \frac{I_z}{E_s} \Delta z$.
Step 6. Calculate C_1 and C_2.
Step 7. Calculate S_e from Eq. (5.49).

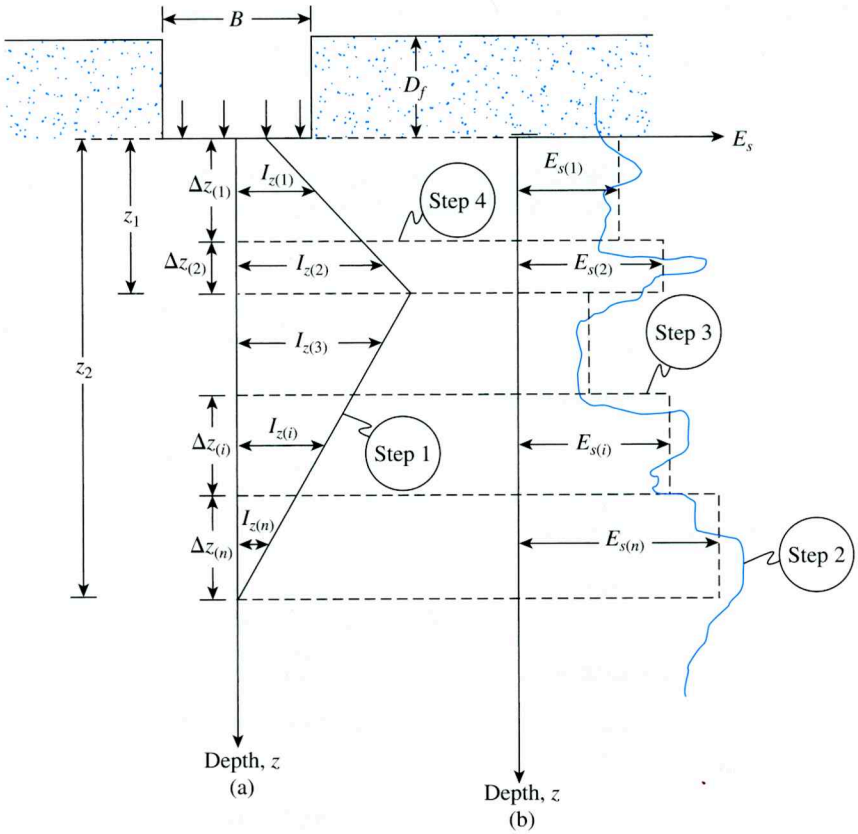


Figure 5.22 Procedure for calculation of S_e using the strain influence factor

Table 5.11 Calculation of $\Sigma \frac{I_z}{E_s} \Delta z$

Layer no.	Δz	E_s	I_z at the middle of the layer	$\frac{I_z}{E_s}\Delta z$
1	$\Delta z_{(1)}$	$E_{s(1)}$	$I_{z(1)}$	$\frac{I_{z(1)}}{E_{s(1)}} \Delta z_1$
2	$\Delta z_{(2)}$	$E_{s(2)}$	$I_{z(2)}$	
⋮	⋮	⋮	⋮	
i	$\Delta z_{(i)}$	$E_{s(i)}$	$I_{z(i)}$	$\frac{I_{z(i)}}{E_{s(i)}} \Delta z_i$
⋮	⋮	⋮	⋮	⋮
n	$\Delta z_{(n)}$	$E_{s(n)}$	$I_{z(n)}$	$\frac{I_{z(n)}}{E_{s(n)}} \Delta z_n$
				$\Sigma \frac{I_z}{E_s} \Delta z$

Example 5.7

Consider a rectangular foundation 2 m × 4 m in plan at a depth of 1.2 m in a sand deposit, as shown in Figure 5.23a. Given: $\gamma = 17.5$ kN/m^3; $\bar{q} = 145$ kN/m^2, and the following approximated variation of q_c with z:

z (m)	q_c (kN/m^2)
0–0.5	2250
0.5–2.5	3430
2.5–5.0	2950

Estimate the elastic settlement of the foundation using the strain influence factor method.

Solution

From Eq. (5.52),

$$\frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) = 0.5 + 0.0555\left(\frac{4}{2} - 1\right) \approx 0.56$$

$$z_1 = (0.56)(2) = 1.12 \text{ m}$$

From Eq. (5.53),

$$\frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) = 2 + 0.222(2 - 1) \doteq 2.22$$

$$z_2 = (2.22)(2) = 4.44 \text{ m}$$

From Eq. (5.51), at $z = 0$,

$$I_z = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) = 0.1 + 0.0111\left(\frac{4}{2} - 1\right) \approx 0.11$$

From Eq. (5.50),

$$I_{z(m)} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}} = 0.5 + 0.1\left[\frac{145 - (1.2 \times 17.5)}{(1.2 + 1.12)(17.5)}\right]^{0.5} = 0.675$$

The plot of I_z versus z is shown in Figure 5.23c. Again, from Eq. (5.56)

$$E_{s(\text{rectangle})} = \left(1 + 0.4\log\frac{L}{B}\right)E_{s(\text{square})} = \left[1 + 0.4\log\left(\frac{4}{2}\right)\right](2.5 \times q_c) = 2.8q_c$$

Hence, the approximated variation of E_s with z is as follows:

z (m)	q_c (kN/m^2)	E_s (kN/m^2)
0–0.5	2250	6300
0.5–2.5	3430	9604
2.5–5.0	2950	8260

The plot of E_s versus z is shown in Figure 5.23b.

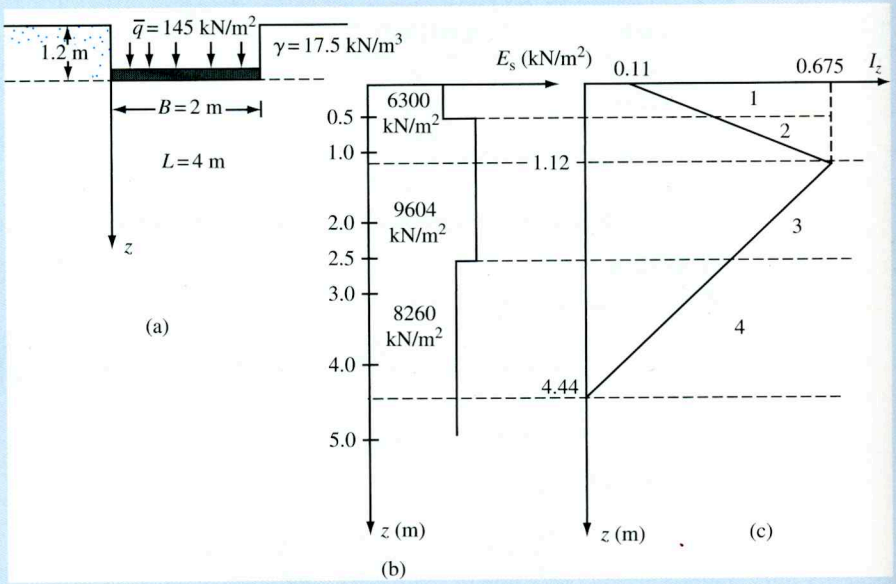


Figure 5.23

The soil layer is divided into four layers as shown in Figures 5.23b and 5.23c. Now the following table can be prepared.

Layer no.	Δz (m)	E_s (kN/m^2)	I_z at middle of layer	$\frac{I_z}{E_s}\Delta z$ (m^3/ kN)
1	0.50	6300	0.236	1.87×10^{-5}
2	0.62	9604	0.519	3.35×10^{-5}
3	1.38	9604	0.535	7.68×10^{-5}
4	1.94	8260	0.197	4.62×10^{-5}
				$\Sigma 17.52 \times 10^{-5}$

$$S_e = C_1 C_2 (\bar{q} - q) \sum \frac{I_z}{E_s} \Delta z$$

$$C_1 = 1 - 0.5\left(\frac{q}{\bar{q} - q}\right) = 1 - 0.5\left(\frac{21}{145 - 21}\right) = 0.915$$

Assume the time for creep is 10 years. So,

$$C_2 = 1 + 0.2\log\left(\frac{10}{0.1}\right) = 1.4$$

Hence,

$$S_e = (0.915)(1.4)(145 - 21)(17.52 \times 10^{-5}) = 2783 \times 10^{-5}\ \text{m} = \mathbf{27.83\ mm}$$ ■

5.13 Settlement of Foundation on Sand Based on Standard Penetration Resistance

Meyerhof's Method

Meyerhof (1956) proposed a correlation for the *net bearing pressure* for foundations with the standard penetration resistance, N_{60}. The net pressure has been defined as

$$q_{\text{net}} = \bar{q} - \gamma D_f$$

where $\bar{q}$ = stress at the level of the foundation.

According to Meyerhof's theory, for 25 mm (1 in.) of estimated maximum settlement,

$$q_{\text{net}}(\text{kip/ft}^2) = \frac{N_{60}}{4} \quad (\text{for } B \leq 4\ \text{ft}) \tag{5.57}$$

and

$$q_{\text{net}}(\text{kip/ft}^2) = \frac{N_{60}}{6}\left(\frac{B + 1}{B}\right)^2 \quad (\text{for } B > 4\ \text{ft}) \tag{5.58}$$

Since the time that Meyerhof proposed his original correlations, researchers have observed that its results are rather conservative. Later, Meyerhof (1965) suggested that the net allowable bearing pressure should be increased by about 50%. Bowles (1977) proposed that the modified form of the bearing equations be expressed as

$$q_{\text{net}}(\text{kip/ft}^2) = \frac{N_{60}}{2.5} F_d S_e \quad (\text{for } B \leq 4 \text{ ft}) \tag{5.59}$$

and

$$q_{\text{net}}(\text{kip/ft}^2) = \frac{N_{60}}{4}\left(\frac{B+1}{B}\right)^2 F_d S_e \quad (\text{for } B > 4 \text{ ft}) \tag{5.60}$$

where

F_d = depth factor = $1 + 0.33(D_f/B)$
B = foundation width, in feet
S_e = settlement, in inches

Hence,

$$S_e(\text{in.}) = \frac{2.5 q_{\text{net}}(\text{kip/ft}^2)}{N_{60} F_d} \quad (\text{for } B \leq 4 \text{ ft}) \tag{5.61}$$

and

$$S_e(\text{in.}) = \frac{4 q_{\text{net}}(\text{kip/ft}^2)}{N_{60} F_d}\left(\frac{B}{B+1}\right)^2 \quad (\text{for } B > 4 \text{ ft}) \tag{5.62}$$

In SI, units, Eqs. (5.59) and (5.60) can be written as

$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22 \text{ m}) \tag{5.63}$$

and

$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m}) \tag{5.64}$$

where B is in meters and S_e is in mm. Hence,

$$S_e(\text{mm}) = \frac{1.25 q_{\text{net}}(\text{kN/m}^2)}{N_{60} F_d} \quad (\text{for } B \leq 1.22 \text{ m}) \tag{5.65}$$

and

$$S_e(\text{mm}) = \frac{2 q_{\text{net}}(\text{kN/m}^2)}{N_{60} F_d}\left(\frac{B}{B+0.3}\right)^2 \quad (\text{for } B > 1.22 \text{ m}) \tag{5.66}$$

The N_{60} referred to in the preceding equations is the standard penetration resistance between the bottom of the foundation and $2B$ below the bottom.

Burland and Burbidge's Method

Burland and Burbidge (1985) proposed a method of calculating the elastic settlement of sandy soil using the *field standard penetration number*, N_{60}. (See Chapter 2.) The method can be summarized as follows:

1. **Variation of Standard Penetration Number with Depth**
   Obtain the field penetration numbers (N_{60}) with depth at the location of the foundation. The following adjustments of N_{60} may be necessary, depending on the field conditions:

   For gravel or sandy gravel,

$$N_{60(a)} \approx 1.25\, N_{60} \tag{5.67}$$

   For fine sand or silty sand below the groundwater table and $N_{60} > 15$,

$$N_{60(a)} \approx 15 + 0.5(N_{60} - 15) \tag{5.68}$$

   where $N_{60(a)}$ = adjusted N_{60} value.
2. **Determination of Depth of Stress Influence (z')**
   In determining the depth of stress influence, the following three cases may arise:

   Case I. If N_{60} [or $N_{60(a)}$] is approximately constant with depth, calculate z' from

$$\frac{z'}{B_R} = 1.4\left(\frac{B}{B_R}\right)^{0.75} \tag{5.69}$$

   where

$$B_R = \text{reference width}\begin{cases} = 1 \text{ ft (if } B \text{ is in ft)} \\ = 0.3 \text{ m (if } B \text{ is in m)} \end{cases}$$

   B = width of the actual foundation

   Case II. If N_{60} [or $N_{60(a)}$] is increasing with depth, use Eq. (5.69) to calculate z'.

   Case III. If N_{60} [or $N_{60(a)}$] is decreasing with depth, $z' = 2B$ or to the bottom of soft soil layer measured from the bottom of the foundation (whichever is smaller).
3. **Calculation of Elastic Settlement S_e**
   The elastic settlement of the foundation, S_e, can be calculated from

$$\frac{S_e}{B_R} = \alpha_1\alpha_2\alpha_3\left[\frac{1.25\left(\frac{L}{B}\right)}{0.25 + \left(\frac{L}{B}\right)}\right]^2\left(\frac{B}{B_R}\right)^{0.7}\left(\frac{q'}{p_a}\right) \tag{5.70}$$

   where

   α_1 = a constant
   α_2 = compressibility index
   α_3 = correction for the depth of influence
   p_a = atmospheric pressure = 100 kN/m^2 ($\approx$2000 lb/ft^2)
   L = length of the foundation

Table 5.12 summarizes the values of q', α_1, α_2, and α_3 to be used in Eq. (5.70) for various types of soils. Note that, in this table, $\bar{N}_{60}$ or $\bar{N}_{60(a)}$ = average value of N_{60} or $N_{60(a)}$ in the depth of stress influence.

Table 5.12 Summary of α_1, α_2, and α_3

Soil type	q'	α_1	α_2	α_3
Normally consolidated sand	q_{net}	0.14	$\dfrac{1.71}{[\bar{N}_{60} \text{ or } \bar{N}_{60(a)}]^{1.4}}$	$\alpha_3 = \dfrac{H}{z'}\left(2 - \dfrac{H}{z'}\right)$ (if $H \leq z'$)
Overconsolidated sand ($q_{net} \leq \sigma_c'$)	q_{net}	0.047	$\dfrac{0.57}{[\bar{N}_{60} \text{ or } \bar{N}_{60(a)}]^{1.4}}$	or $\alpha_3 = 1$ (if $H > z'$)
where σ_c' = preconsolidation pressure				where H = depth of compressible layer
Overconsolidated sand ($q_{net} > \sigma_c'$)	$q_{net} - 0.67\sigma_c'$	0.14	$\dfrac{0.57}{[\bar{N}_{60} \text{ or } \bar{N}_{60(a)}]^{1.4}}$	

Example 5.8

A shallow foundation measuring 1.75 m × 1.75 m is to be constructed over a layer of sand. Given $D_f = 1$ m; N_{60} is generally increasing with depth; $\overline{N}_{60}$ in the depth of stress influence = 10, $q_{net} = 120$ kN/m^2. The sand is normally consolidated. Estimate the elastic settlement of the foundation. Use the Burland and Burbidge method.

Solution

From Eq. (5.69),

$$\frac{z'}{B_R} = 1.4\left(\frac{B}{B_R}\right)^{0.75}$$

Depth of stress influence,

$$z' = 1.4\left(\frac{B}{B_R}\right)^{0.75} B_R = (1.4)(0.3)\left(\frac{1.75}{0.3}\right)^{0.75} \approx 1.58 \text{ m}$$

From Eq. (5.70),

$$\frac{S_e}{B_R} = \alpha_1\alpha_2\alpha_3\left[\frac{1.25\left(\dfrac{L}{B}\right)}{0.25 + \left(\dfrac{L}{B}\right)}\right]^2\left(\frac{B}{B_R}\right)^{0.7}\left(\frac{q'}{p_a}\right)$$

For normally consolidated sand (Table 5.12),

$$\alpha_1 = 0.14$$

$$\alpha_2 = \frac{1.71}{(\overline{N}_{60})^{1.4}} = \frac{1.71}{(10)^{1.4}} = 0.068$$

$$\alpha_3 = 1$$

$$q' = q_{\text{net}} = 120 \text{ kN/m}^2$$

So,

$$\frac{S_e}{0.3} = (0.14)(0.068)(1)\left[\frac{(1.25)\left(\frac{1.75}{1.75}\right)}{0.25 + \left(\frac{1.75}{1.75}\right)}\right]^2 \left(\frac{1.75}{0.3}\right)^{0.7}\left(\frac{120}{100}\right)$$

$$S_e \approx 0.0118 \text{ m} = \mathbf{11.8\ mm}$$

■

Example 5.9

Solve Example 5.8 using Meyerhof's method.

Solution

From Eq. (5.66),

$$S_e = \frac{2q_{\text{net}}}{(N_{60})(F_d)}\left(\frac{B}{B + 0.3}\right)^2$$

$$F_d = 1 + 0.33(D_f/B) = 1 + 0.33(1/1.75) = 1.19$$

$$S_e = \frac{(2)(120)}{(10)(1.19)}\left(\frac{1.75}{1.75 + 0.3}\right)^2 = \mathbf{14.7\ mm}$$

■

5.14 Settlement in Granular Soil Based on Pressuremeter Test (PMT)

Briaud (2007) proposed a method based on Pressuremeter tests (Section 2.22) from which the load-settlement diagrams of foundations can be derived. The following is a step-by-step procedure for performing the analysis.

Step 1. Conduct Pressuremeter tests at varying depths at the desired location and obtain plots of p_p (pressure in the measuring cell for cavity expansion; see Figure 2.32) versus $\Delta R/R_o$ (R_o = initial radius of the PMT cavity, and ΔR = increase in the cavity radius), as shown in Figure 5.24a.

Step 2. Extend the straight line part of the PMT curve to zero pressure and shift the vertical axis, as shown in Figure 5.24a. Re-zero the $\Delta R/R_o$ axis.

Step 3. Draw a strain influence factor diagram for the desired foundation (Section 5.12). Using all Pressuremeter test curves within the depth of influence, develop a mean PMT curve. Referring to Figure 5.24b, this can be done as follows:

For each value of $\Delta R/R_o$, let the p_p values be $p_{p(1)}, p_{p(2)}, p_{p(3)}, \ldots$. The mean value of p_p can be obtained as

$$p_{p(m)} = \frac{A_1}{A}p_{p(1)} + \frac{A_2}{A}p_{p(2)} + \frac{A_3}{A}p_{p(3)} + \cdots \tag{5.71}$$

where

A_1, A_2, A_3 = areas tributary to each test under the strain influence factor diagram

$$A = A_1 + A_2 + A_3 + \cdots \tag{5.72}$$

Step 4. Based on the results of Step 3, develop a mean $p_{p(m)}$ versus $\Delta R/R_o$ plot (Figure 5.24c).

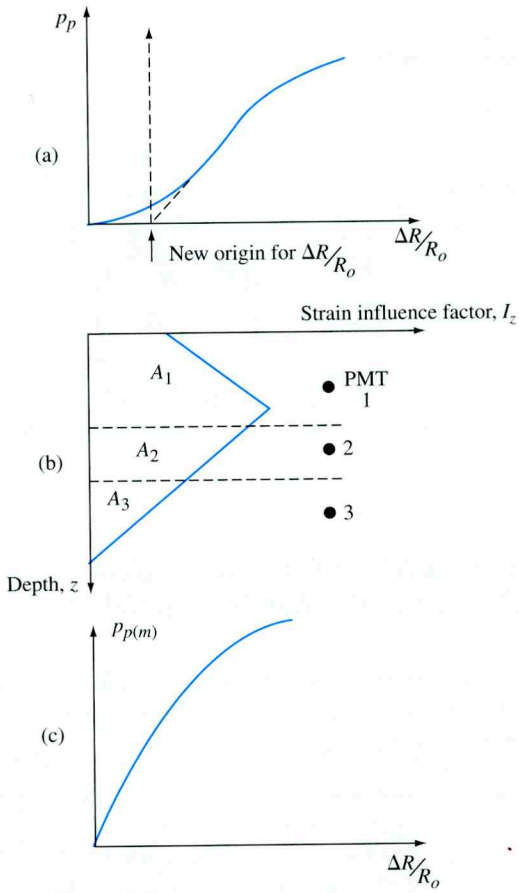


Figure 5.24 (a) Plot of p_p versus $\Delta R/R_o$; (b) averaging the pressuremeter curves within the foundation zone of influence; (c) plot of $p_{p(m)}$ versus $\Delta R/R_o$

Step 5. The mean PMT curve now can be used to develop the load-settlement plot for the foundation via the following equations.

$$\frac{S_e}{B} = 0.24\frac{\Delta R}{R_o} \tag{5.73}$$

and

$$q_o = f_{L/B} f_e f_\delta f_{\beta,d} \Gamma p_{p(m)} \tag{5.74}$$

where

S_e = elastic settlement of the foundation
B = width of foundation
L = length of foundation
q_o = net load per unit area on the foundation
Γ = gamma function linking q_o and $p_{p(m)}$

$$f_{L/B} = \text{shape factor} = 0.8 + 0.2\left(\frac{B}{L}\right) \tag{5.75}$$

$$f_e = \text{eccentricity factor} = 1 - 0.33\left(\frac{e}{B}\right)(\text{center}) \tag{5.76}$$

$$f_e = \text{eccentricity factor} = 1 - \left(\frac{e}{B}\right)^{0.5}(\text{edge}) \tag{5.77}$$

$$f_\delta = \text{load inclination factor} = 1 - \left[\frac{\delta(\text{deg})}{90}\right]^2(\text{center}) \tag{5.78}$$

$$f_\delta = \text{load inclination factor} = 1 - \left[\frac{\delta(\text{deg})}{360}\right]^{0.5}(\text{edge}) \tag{5.79}$$

$$f_{\beta,d} = \text{slope factor} = 0.8\left(1 + \frac{d}{B}\right)^{0.1}(\text{3H:1V slope}) \tag{5.80}$$

$$f_{\beta,d} = \text{slope factor} = 0.7\left(1 + \frac{d}{B}\right)^{0.15}(\text{2H:1V slope}) \tag{5.81}$$

δ = inclination of load with respect to the vertical
β = inclination of a slope with the horizontal if the foundation is located on top of a slope
d = distance of the edge of the foundation from the edge of the slope

The parameters δ, β, d, and e are defined in Figure 5.25. Figure 5.26 shows the design plot for Γ with S_e/B or $0.24\frac{\Delta R}{R_o}$.

Step 6. Based on the values of B/L, e/B, δ, and d/B, calculate the values of $f_{L/B}$, f_e, f_δ, and $f_{\beta,d}$ as needed. Let

$$f = (f_{L/B})(f_e)(f_\delta)(f_{\beta,d}) \tag{5.82}$$

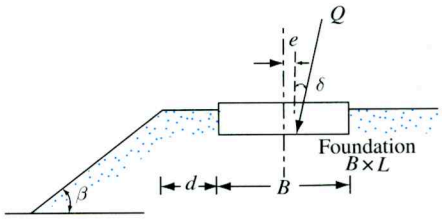


Figure 5.25 Definition of parameters—β, L, d, δ, β, e, and β

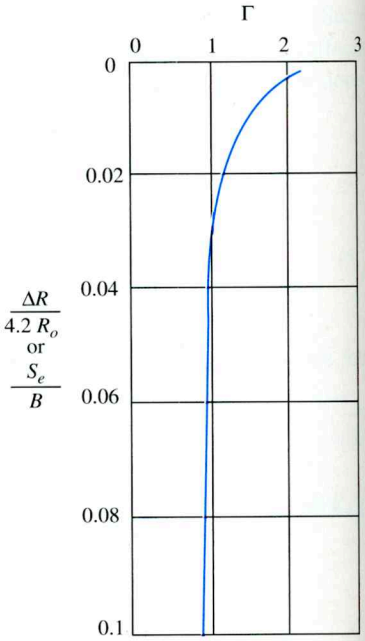


Figure 5.26 Variation of Γ with $S_e/B = 0.24\ \Delta R/R_o$

Thus,

$$q_o = f\Gamma p_{p(m)} \tag{5.83}$$

Step 7. Now prepare a table, as shown in Table 5.13.

Step 8. Complete Table 5.13 as follows:

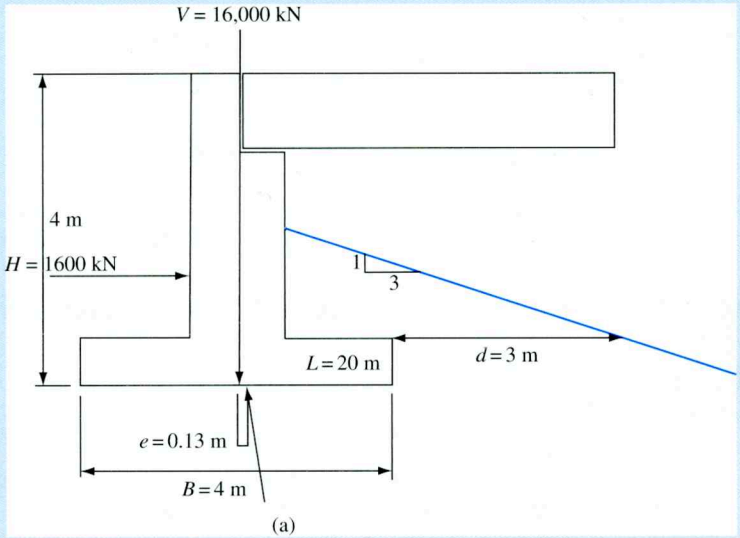
a. Column 1—Assume several values of $\Delta R/R_o$.
b. Column 2—For given values of $\Delta R/R_o$, obtain $p_{p(m)}$ from Figure 5.24c.
c. Column 3—From Eq. (5.73), calculate the values of S_e/B from values of $\Delta R/R_o$ given in Column 1.
d. Column 4—With known values of B, calculate the values of S_e.
e. Column 5—From Figure 5.26, obtain the desired values of Γ.
f. Column 6—Use Eq. (5.83) to obtain q_o.
g. Now plot a graph of S_e (Column 4) versus q_o (Column 6) from which the magnitude of S_e for a given q_o can be determined.

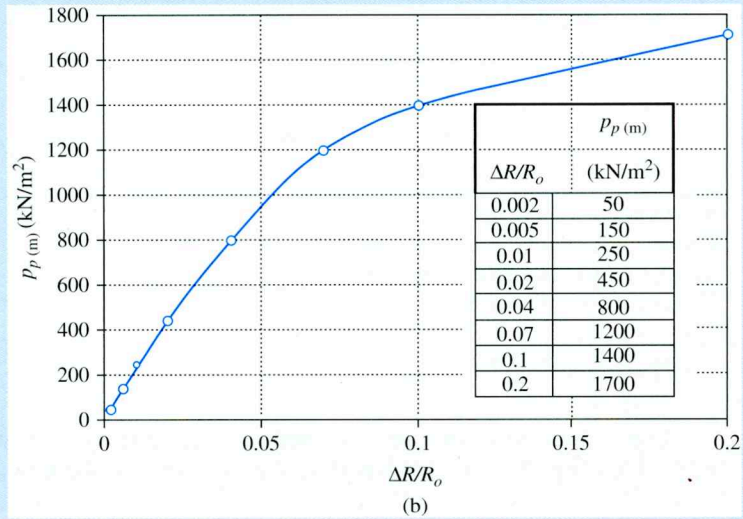
Table 5.13 Calculations to Obtain the Load-Settlement Plot

$\Delta R/R_o$ (1)	$p_{p(m)}$ (2)	S_e/B (3)	S_e (4)	Γ (5)	q_o (6)

Example 5.10

A spread footing, shown in Figure 5.27a with a width of 4 m and a length of 20 m, serves as a bridge abutment foundation. The soil is medium dense sand. A 16,000 kN vertical load acts on the footing. The active pressure on the abutment wall develops a 1,600 kN horizontal load. The resultant reaction force due to the vertical and horizontal load is applied at an eccentricity of 0.13 m. PMT testing at the site produced a mean Pressuremeter curve characterizing the soil and is shown in Figure 5.27b. What is the settlement at the current loading?





$\Delta R/R_o$	p_p (m) (kN/m²)
0.002	50
0.005	150
0.01	250
0.02	450
0.04	800
0.07	1200
0.1	1400
0.2	1700

Figure 5.27

Solution

Given: $B = 4$ m, $L = 20$ m, $d = 3$ m, and slope = 3H:1V.

So

$$f_{L/B} = 0.8 + 0.2\left(\frac{B}{L}\right) = 0.8 + 0.2\left(\frac{4}{20}\right) = 0.84$$

$$f_{e(\text{center})} = 1 - 0.33\left(\frac{e}{B}\right) = 1 - 0.33\left(\frac{0.13}{4}\right) = 0.99$$

$$f_{\delta(\text{center})} = 1 - \left(\frac{\delta}{90}\right)^2$$

$$\delta = \tan^{-1}\left(\frac{H}{V}\right) = \tan^{-1}\left(\frac{1600}{16{,}000}\right) = 5.71°$$

$$f_\delta = 1 - \left(\frac{5.71}{90}\right)^2 = 0.996$$

$$f_{\beta,d} = 0.8\left(1 + \frac{d}{B}\right)^{0.1} = 0.8\left(1 + \frac{3}{4}\right)^{0.1} = 0.846$$

$$f = f_{L/B} f_e f_\delta f_{\beta,d} = (0.84)(0.99)(0.996)(0.845) = 0.7$$

Now the following table can be prepared.

$\Delta R/R_o$ (1)	$p_{p(m)}$ (kN/m²) (2)	S_e/B (3)	S_e (mm) (4)	Γ (5)	q_o (kN/m²) (6)	Q_o (MN) (7)
0.002	50	0.0005	2.0	2.27	79.45	6.36
0.005	150	0.0012	4.8	2.17	227.85	18.23
0.01	250	0.0024	9.6	2.07	362.25	28.98
0.02	450	0.0048	19.2	1.83	576.45	46.12
0.04	800	0.0096	38.4	1.40	784.00	62.72
0.07	1200	0.0168	67.2	1.17	982.8	78.62
0.10	1400	0.024	96.0	1.07	1048.6	83.89
0.20	1700	0.048	192.0	0.90	1071.0	85.68

Note: Columns 1 and 2: From Figure 5.27b
Column 3: (Column 1)(0.24) = S_e/B
Column 4: (Column 3)(B = 4000 mm) = S_e
Column 5: From Figure 5.26
Column 6: $f\Gamma p_{p(m)} = (0.7)(\Gamma)p_{p(m)} = q_o$
Column 7: (Column 6)($B \times L$) = Q_o

Figure 5.28 shows the plot of Q_o versus S_e. From this plot it can be seen that, for a vertical loading of 16,000 kN (16 MN), the value of $S_e \approx$ **4.2 mm**.

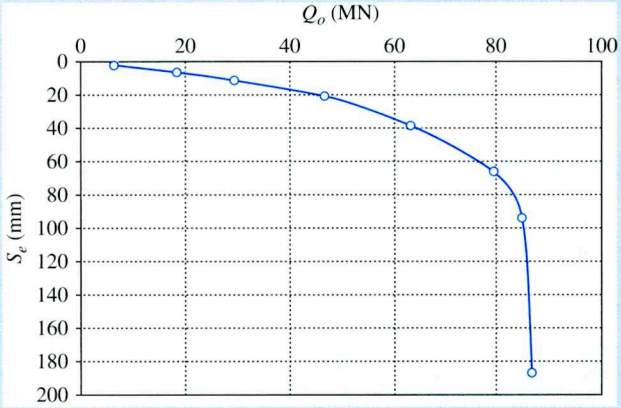


Figure 5.28

Consolidation Settlement

5.15 Primary Consolidation Settlement Relationships

As mentioned before, consolidation settlement occurs over time in saturated clayey soils subjected to an increased load caused by construction of the foundation. (See Figure 5.29.) On the basis of the one-dimensional consolidation settlement equations given in Chapter 1, we write

$$S_{c(p)} = \int \varepsilon_z dz$$

where

$$\varepsilon_z = \text{vertical strain} = \frac{\Delta e}{1 + e_o}$$

$$\Delta e = \text{change of void ratio} = f(\sigma'_o, \sigma'_c, \text{and } \Delta\sigma')$$

So,

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \quad \text{(for normally consolidated clays)} \quad \text{[Eq. (1.61)]}$$

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \quad \text{(for overconsolidated clays with } \sigma'_o + \Delta\sigma'_{av} < \sigma'_c) \quad \text{[Eq. (1.63)]}$$

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c} \quad \text{(for overconsolidated clays with } \sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'_{av}) \quad \text{[Eq. (1.65)]}$$

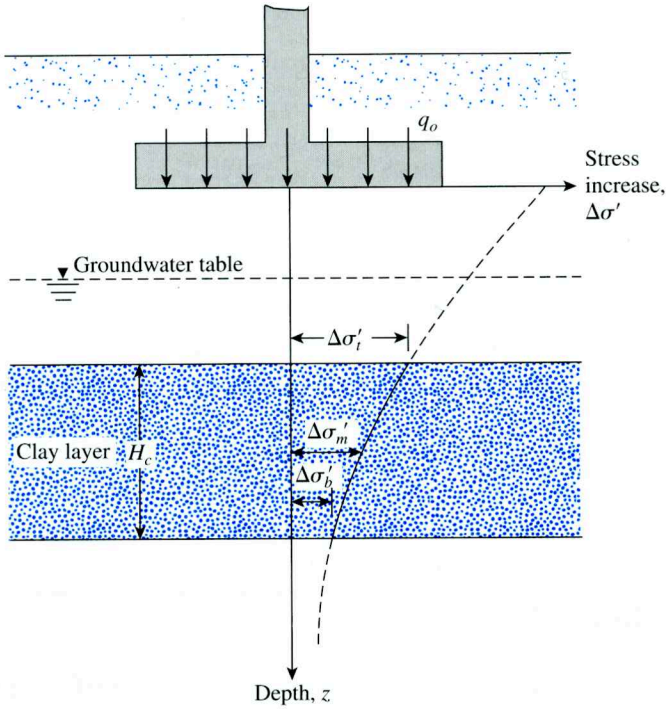


Figure 5.29 Consolidation settlement calculation

where

σ'_o = average effective pressure on the clay layer before the construction of the foundation
$\Delta\sigma'_{av}$ = average increase in effective pressure on the clay layer caused by the construction of the foundation
σ'_c = preconsolidation pressure
e_o = initial void ratio of the clay layer
C_c = compression index
C_s = swelling index
H_c = thickness of the clay layer

The procedures for determining the compression and swelling indexes were discussed in Chapter 1.

Note that the increase in effective pressure, $\Delta\sigma'$, on the clay layer is not constant with depth: The magnitude of $\Delta\sigma'$ will decrease with the increase in depth measured from the bottom of the foundation. However, the average increase in pressure may be approximated by

$$\Delta\sigma'_{av} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b) \tag{5.84}$$

where $\Delta\sigma'_t$, $\Delta\sigma'_m$, and $\Delta\sigma'_b$ are, respectively, the effective pressure increases at the *top*, *middle*, and *bottom* of the clay layer that are caused by the construction of the foundation.

The method of determining the pressure increase caused by various types of foundation load using Boussinesq's solution is discussed in Sections 5.2 through 5.6. $\Delta\sigma'_{av}$ can also be directly obtained from the method presented in Section 5.5.

5.16 Three-Dimensional Effect on Primary Consolidation Settlement

The consolidation settlement calculation presented in the preceding section is based on Eqs. (1.61), (1.63), and (1.65). These equations, as shown in Chapter 1, are in turn based on one-dimensional laboratory consolidation tests. The underlying assumption is that the increase in pore water pressure, Δu, immediately after application of the load equals the increase in stress, $\Delta\sigma$, at any depth. In this case,

$$S_{c(p)-\text{oed}} = \int \frac{\Delta e}{1 + e_o} dz = \int m_v \Delta\sigma'_{(1)} dz$$

where

$S_{c(p)-\text{oed}}$ = consolidation settlement calculated by using Eqs. (1.61), (1.63), and (1.65)
$\Delta\sigma'_{(1)}$ = effective vertical stress increase
m_v = volume coefficient of compressibility (see Chapter 1)

In the field, however, when a load is applied over a limited area on the ground surface, such an assumption will not be correct. Consider the case of a circular foundation on a clay layer, as shown in Figure 5.30. The vertical and the horizontal stress increases at a point in the layer immediately below the center of the foundation are $\Delta\sigma_{(1)}$ and $\Delta\sigma_{(3)}$, respectively. For a saturated clay, the pore water pressure increase at that depth (see Chapter 1) is

$$\Delta u = \Delta\sigma_{(3)} + A[\Delta\sigma_{(1)} - \Delta\sigma_{(3)}] \tag{5.85}$$

where A = pore water pressure parameter. For this case,

$$S_{c(p)} = \int m_v \Delta u \, dz = \int (m_v)\{\Delta\sigma_{(3)} + A[\Delta\sigma_{(1)} - \Delta\sigma_{(3)}]\} \, dz \tag{5.86}$$

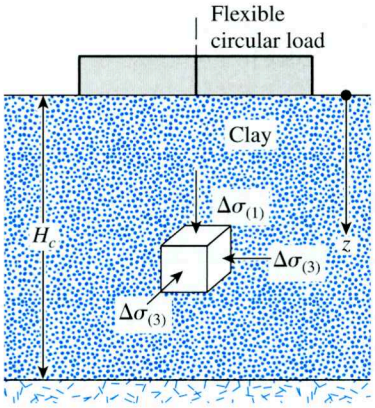


Figure 5.30 Circular foundation on a clay layer

Thus, we can write

$$K_{cir} = \frac{S_{c(p)}}{S_{c(p)-oed}} = \frac{\int_0^{H_c} m_v \Delta u \, dz}{\int_0^{H_c} m_v \Delta\sigma'_{(1)} dz} = A + (1 - A)\left[\frac{\int_0^{H_c} \Delta\sigma'_{(3)} dz}{\int_0^{H_c} \Delta\sigma'_{(1)} dz}\right] \tag{5.87}$$

where K_{cir} = settlement ratio for circular foundations.

The settlement ratio for a continuous foundation, K_{str}, can be determined in a manner similar to that for a circular foundation. The variation of K_{cir} and K_{str} with A and H_c/B is given in Figure 5.31. (*Note:* B = diameter of a circular foundation, and B = width of a continuous foundation.)

The preceding technique is generally referred to as the *Skempton–Bjerrum modification* (1957) for a consolidation settlement calculation.

Leonards (1976) examined the correction factor K_{cr} for a three-dimensional consolidation effect in the field for a circular foundation located over *overconsolidated clay.* Referring to Figure 5.30, we have

$$S_{c(p)} = K_{cr(OC)} S_{c(p)-oed} \tag{5.88}$$

where

$$K_{cr(OC)} = f\left(\text{OCR}, \frac{B}{H_c}\right) \tag{5.89}$$

in which

$$\text{OCR} = \text{overconsolidation ratio} = \frac{\sigma'_c}{\sigma'_o} \tag{5.90}$$

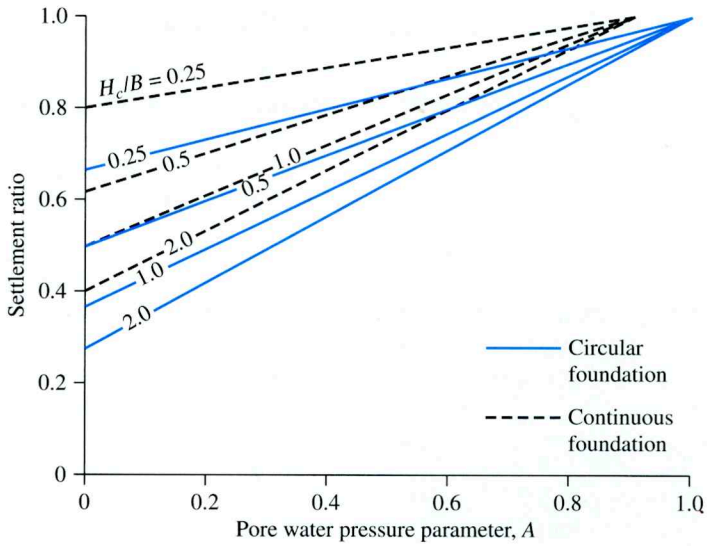


Figure 5.31 Settlement ratios for circular (K_{cir}) and continuous (K_{str}) foundations

Table 5.14 Variation of $K_{cr(OC)}$ with OCR and B/H_c

	$K_{cr(OC)}$		
OCR	$B/H_c = 4.0$	$B/H_c = 1.0$	$B/H_c = 0.2$
1	1	1	1
2	0.986	0.957	0.929
3	0.972	0.914	0.842
4	0.964	0.871	0.771
5	0.950	0.829	0.707
6	0.943	0.800	0.643
7	0.929	0.757	0.586
8	0.914	0.729	0.529
9	0.900	0.700	0.493
10	0.886	0.671	0.457
11	0.871	0.643	0.429
12	0.864	0.629	0.414
13	0.857	0.614	0.400
14	0.850	0.607	0.386
15	0.843	0.600	0.371
16	0.843	0.600	0.357

where

σ'_c = preconsolidation pressure
σ'_o = present average effective pressure

The interpolated values of $K_{cr(OC)}$ from Leonard's 1976 work are given in Table 5.14.

Example 5.11

A plan of a foundation 1 m × 2 m is shown in Figure 5.32. Estimate the consolidation settlement of the foundation, taking into account the three-dimensional effect. Given: $A = 0.6$.

Solution

The clay is normally consolidated. Thus,

$$S_{c(p)-\text{oed}} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{\text{av}}}{\sigma'_o}$$

so

$$\sigma'_o = (2.5)(16.5) + (0.5)(17.5 - 9.81) + (1.25)(16 - 9.81)$$
$$= 41.25 + 3.85 + 7.74 = 52.84 \text{ kN/m}^2$$

From Eq. (5.84),

$$\Delta\sigma'_{\text{av}} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

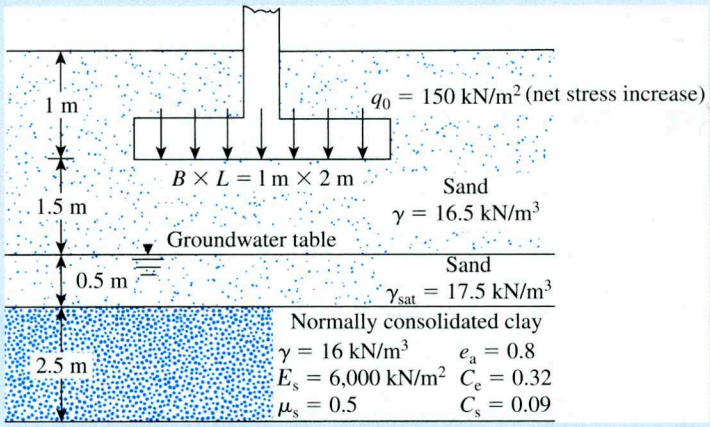


Figure 5.32 Calculation of primary consolidation settlement for a foundation

Now the following table can be prepared (*Note:* $L = 2$ m; $B = 1$ m):

$m_1 = L/B$	z(m)	$z/(B/2) = n_1$	I_c[a]	$\Delta\sigma' = q_o I_c$[b]
2	2	4	0.190	$28.5 = \Delta\sigma'_t$
2	2 + 2.5/2 = 3.25	6.5	≈ 0.085	$12.75 = \Delta\sigma'_m$
2	2 + 2.5 = 4.5	9	0.045	$6.75 = \Delta\sigma'_b$

[a]Table 5.3
[b]Eq. (5.10)

Now,

$$\Delta\sigma'_{av} = \tfrac{1}{6}(28.5 + 4 \times 12.75 + 6.75) = 14.38 \text{ kN/m}^2$$

so

$$S_{c(p)-oed} = \frac{(0.32)(2.5)}{1 + 0.8}\log\left(\frac{52.84 + 14.38}{52.84}\right) = 0.0465 \text{ m}$$

$$= \mathbf{46.5\ mm}$$

Now assuming that the 2:1 method of stress increase (see Figure 5.5) holds good, the area of distribution of stress at the top of the clay layer will have dimensions

$$B' = \text{width} = B + z = 1 + (1.5 + 0.5) = 3 \text{ m}$$

and

$$L' = \text{width} = L + z = 2 + (1.5 + 0.5) = 4 \text{ m}$$

The diameter of an equivalent circular area, B_{eq}, can be given as

$$\frac{\pi}{4}B_{eq}^2 = B'L'$$

so that

$$B_{eq} = \sqrt{\frac{4B'L'}{\pi}} = \sqrt{\frac{(4)(3)(4)}{\pi}} = 3.91 \text{ m}$$

Also,

$$\frac{H_c}{B_{eq}} = \frac{2.5}{3.91} = 0.64$$

From Figure 5.31, for $A = 0.6$ and $H_c/B_{eq} = 0.64$, the magnitude of $K_{cr} \approx 0.78$. Hence,

$$S_{e(p)} = K_{cr}S_{e(p)-\text{oed}} = (0.78)(46.5) \approx \mathbf{36.3\ mm}$$ ■

5.17 Settlement Due to Secondary Consolidation

At the end of primary consolidation (i.e., after the complete dissipation of excess pore water pressure) some settlement is observed that is due to the plastic adjustment of soil fabrics. This stage of consolidation is called *secondary consolidation.* A plot of deformation against the logarithm of time during secondary consolidation is practically linear as shown in Figure 5.33. From the figure, the secondary compression index can be defined as

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log (t_2/t_1)} \tag{5.91}$$

where

C_α = secondary compression index
Δe = change of void ratio
t_1, t_2 = time

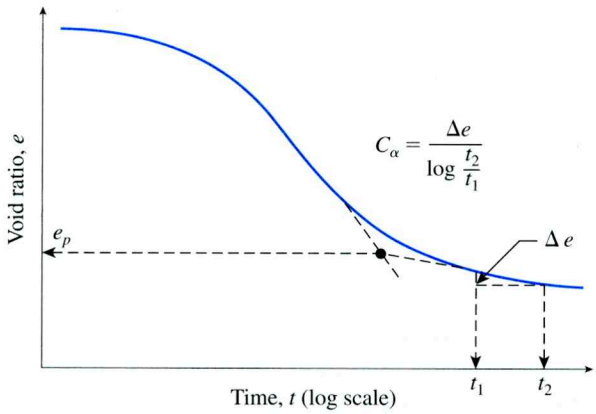


Figure 5.33 Variation of e with log t under a given load increment, and definition of secondary compression index

The magnitude of the secondary consolidation can be calculated as

$$S_{c(s)} = C'_\alpha H_c \log(t_2/t_1) \tag{5.92}$$

where

$$C'_\alpha = C_\alpha/(1 + e_p) \tag{5.93}$$

e_p = void ratio at the end of primary consolidation
H_c = thickness of clay layer

Mesri (1973) correlated C'_α with the natural moisture content (w) of several soils, from which it appears that

$$C'_\alpha \approx 0.0001w \tag{5.94}$$

where w = natural moisture content, in percent. For most overconsolidated soils, C'_a varies between 0.0005 to 0.001.

Mesri and Godlewski (1977) compiled the magnitude of C_a/C_c (C_c = compression index) for a number of soils. Based on their compilation, it can be summarized that

- For inorganic clays and silts:

$$C_\alpha/C_c \approx 0.04 \pm 0.01$$

- For organic clays and silts:

$$C_\alpha/C_c \approx 0.05 \pm 0.01$$

- For peats:

$$C_\alpha/C_c \approx 0.075 \pm 0.01$$

Secondary consolidation settlement is more important in the case of all organic and highly compressible inorganic soils. In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

There are several factors that might affect the magnitude of secondary consolidation, some of which are not yet very clearly understood (Mesri, 1973). The ratio of secondary to primary compression for a given thickness of soil layer is dependent on the ratio of the stress increment, $\Delta\sigma'$, to the initial effective overburden stress, σ'_o. For small $\Delta\sigma'/\sigma'_o$ ratios, the secondary-to-primary compression ratio is larger.

5.18 Field Load Test

The ultimate load-bearing capacity of a foundation, as well as the allowable bearing capacity based on tolerable settlement considerations, can be effectively determined from the field load test, generally referred to as the *plate load test.* The plates that are used for tests in the field are usually made of steel and are 25 mm (1 in.) thick and 150 mm to 762 mm (6 in. to 30 in.) in diameter. Occasionally, square plates that are 305 mm × 305 mm (12 in. × 12 in.) are also used.

To conduct a plate load test, a hole is excavated with a minimum diameter of $4B$ (B is the diameter of the test plate) to a depth of D_f, the depth of the proposed foundation. The plate is placed at the center of the hole, and a load that is about one-fourth to one-fifth of the estimated

ultimate load is applied to the plate in steps by means of a jack. A schematic diagram of the test arrangement is shown in Figure 5.34a. During each step of the application of the load, the settlement of the plate is observed on dial gauges. At least one hour is allowed to elapse between each application. The test should be conducted until failure, or at least until the plate has gone through 25 mm (1 in.) of settlement. Figure 5.34b shows the nature of the load–settlement curve obtained from such tests, from which the ultimate load per unit area can be determined. Figure 5.35 shows a plate load test conducted in the field.

For tests in clay,

$$q_{u(F)} = q_{u(P)} \tag{5.95}$$

where

$q_{u(F)}$ = ultimate bearing capacity of the proposed foundation
$q_{u(P)}$ = ultimate bearing capacity of the test plate

Equation (5.95) implies that the ultimate bearing capacity in clay is virtually independent of the size of the plate.

For tests in sandy soils,

$$q_{u(F)} = q_{u(P)} \frac{B_F}{B_P} \tag{5.96}$$

where

B_F = width of the foundation
B_P = width of the test plate

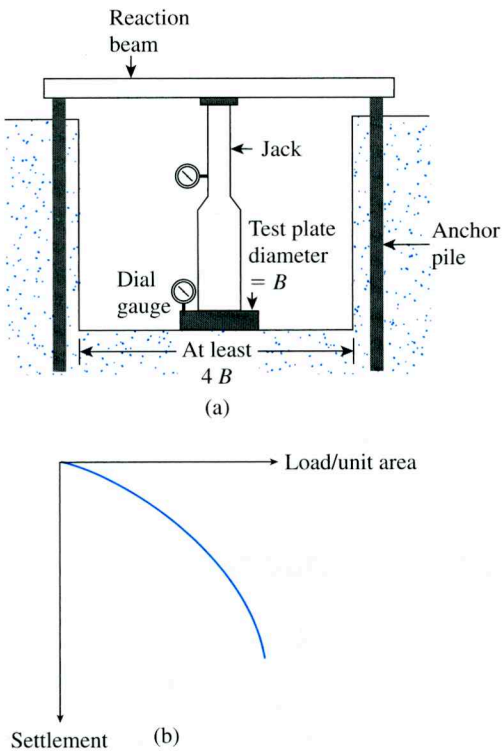


Figure 5.34 Plate load test: (a) test arrangement; (b) nature of load–settlement curve

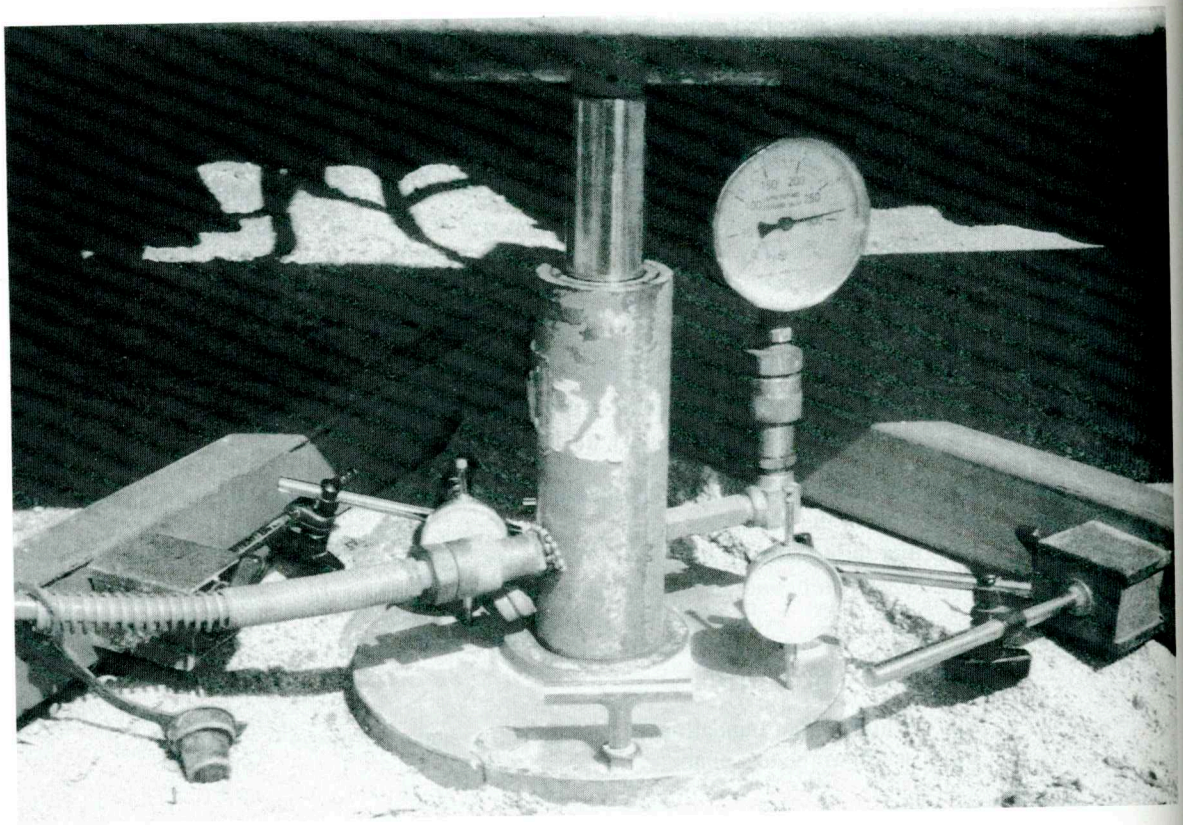


Figure 5.35 Plate load test in the field (*Courtesy of Braja M. Das, Henderson, NV*)

The allowable bearing capacity of a foundation, based on settlement considerations and for a given intensity of load, q_o, is

$$S_F = S_P \frac{B_F}{B_P} \quad \text{(for clayey soil)} \tag{5.97}$$

and

$$S_F = S_P \left(\frac{2B_F}{B_F + B_P} \right)^2 \quad \text{(for sandy soil)} \tag{5.98}$$

5.19 Presumptive Bearing Capacity

Several building codes (e.g., the Uniform Building Code, Chicago Building Code, and New York City Building Code) specify the allowable bearing capacity of foundations on various types of soil. For minor construction, they often provide fairly acceptable guidelines. However, these bearing capacity values are based primarily on the *visual* classification of

near-surface soils and generally do not take into consideration factors such as the stress history of the soil, the location of the water table, the depth of the foundation, and the tolerable settlement. So, for large construction projects, the codes' presumptive values should be used only as guides.

5.20 Tolerable Settlement of Buildings

In most instances of construction, the subsoil is not homogeneous and the load carried by various shallow foundations of a given structure can vary widely. As a result, it is reasonable to expect varying degrees of settlement in different parts of a given building. The *differential settlement* of the parts of a building can lead to damage of the superstructure. Hence, it is important to define certain parameters that quantify differential settlement and to develop limiting values for those parameters in order that the resulting structures be safe. Burland and Worth (1970) summarized the important parameters relating to differential settlement.

Figure 5.36 shows a structure in which various foundations, at A, B, C, D, and E, have gone through some settlement. The settlement at A is AA', at B is BB', etc. Based on this figure, the definitions of the various parameters are as follows:

S_T = total settlement of a given point

ΔS_T = difference in total settlement between any two points

α = gradient between two successive points

β = angular distortion $= \dfrac{\Delta S_{T(ij)}}{l_{ij}}$

($Note$: l_{ij} = distance between points i and j)

ω = tilt

Δ = relative deflection (i.e. movement from a straight line joining two reference points)

$\dfrac{\Delta}{L}$ = deflection ratio

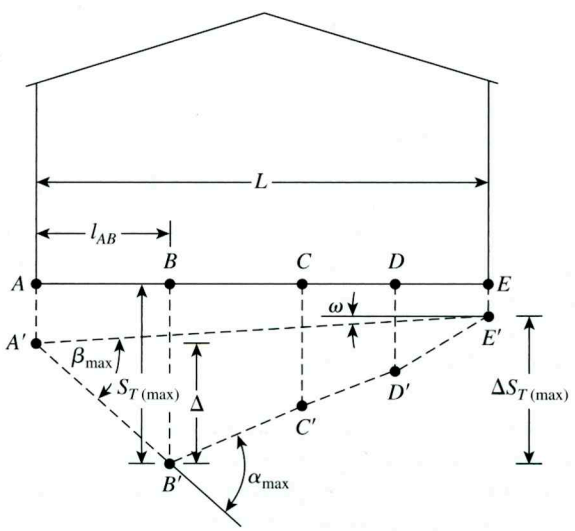


Figure 5.36 Definition of parameters for differential settlement

Since the 1950s, various researchers and building codes have recommended allowable values for the preceding parameters. A summary of several of these recommendations is presented next.

In 1956, Skempton and McDonald proposed the following limiting values for maximum settlement and maximum angular distortion, to be used for building purposes:

Maximum settlement, $S_{T(\max)}$	
In sand	32 mm
In clay	45 mm
Maximum differential settlement, $\Delta S_{T(\max)}$	
Isolated foundations in sand	51 mm
Isolated foundations in clay	76 mm
Raft in sand	51–76 mm
Raft in clay	76–127 mm
Maximum angular distortion, $\beta_{\max}$	1/300

On the basis of experience, Polshin and Tokar (1957) suggested the following allowable deflection ratios for buildings as a function of L/H, the ratio of the length to the height of a building:

$$\Delta/L = 0.0003 \text{ for } L/H \leq 2$$
$$\Delta/L = 0.001 \text{ for } L/H = 8$$

The 1955 Soviet Code of Practice gives the following allowable values:

Type of building	L/H	Δ/L
Multistory buildings and civil dwellings	$\leqslant 3$	0.0003 (for sand) 0.0004 (for clay)
	$\geqslant 5$	0.0005 (for sand) 0.0007 (for clay)
One-story mills		0.001 (for sand and clay)

Bjerrum (1963) recommended the following limiting angular distortion, $\beta_{\max}$ for various structures:

Category of potential damage	$\beta_{\max}$
Safe limit for flexible brick wall ($L/H > 4$)	1/150
Danger of structural damage to most buildings	1/150
Cracking of panel and brick walls	1/150
Visible tilting of high rigid buildings	1/250
First cracking of panel walls	1/300
Safe limit for no cracking of building	1/500
Danger to frames with diagonals	1/600

If the maximum allowable values of $\beta_{\max}$ are known, the magnitude of the allowable $S_{T(\max)}$ can be calculated with the use of the foregoing correlations.

The European Committee for Standardization has also provided limiting values for serviceability and the maximum accepted foundation movements. (See Table 5.15.)

Table 5.15 Recommendations of European Committee for Standardization on Differential Settlement Parameters

Item	Parameter	Magnitude	Comments
Limiting values for serviceability	S_T	25 mm	Isolated shallow foundation
		50 mm	Raft foundation
(European Committee for Standardization, 1994a)	ΔS_T	5 mm	Frames with rigid cladding
		10 mm	Frames with flexible cladding
		20 mm	Open frames
	β	1/500	—
Maximum acceptable foundation movement	S_T	50	Isolated shallow foundation
	ΔS_T	20	Isolated shallow foundation
(European Committee for Standardization, 1994b)	β	≈1/500	—

Problems

5.1 A flexible circular area is subjected to a uniformly distributed load of 150 kN/m^2. The diameter of the loaded area is 2 m. Determine the stress increase in a soil mass at a point located 3 m below the center of the loaded area.
a. by using Eq. (5.3)
b. by using Eq. (5.28). Use $\mu_s = 0$

5.2 Refer to Figure 5.4, which shows a flexible rectangular area. Given: $B_1 = 4$ ft, $B_2 = 6$ ft, $L_1 = 8$ ft, and $L_2 = 10$ ft. If the area is subjected to a uniform load of 3000 lb/ft^2, determine the stress increase at a depth of 10 ft located immediately below point O.

5.3 Repeat Problem 5.2 with the following: $B_1 = 4$ ft, $B_2 = 10$ ft, $L_1 = 8$ ft, $L_2 = 12$ ft, and the uniform load on the flexible area = 2500 lb/ft^2. Determine the stress increase below point O at a depth of 20 ft. Use Eq. (5.29) and $\mu_s = 0$.

5.4 Using Eq. (5.10), determine the stress increase $(\Delta\sigma)$ at $z = 10$ ft below the center of the area described in Problem 5.2.

5.5 Refer to Figure P5.5. Using the procedure outlined in Section 5.5, determine the average stress increase in the clay layer below the center of the foundation due to the net foundation load of 900 kN.

5.6 Solve Problem 5.5 using the 2:1 method [Eqs. (5.14) and (5.84)].

5.7 Figure P5.7 shows an embankment load on a silty clay soil layer. Determine the stress increase at points A, B, and C, which are located at a depth of 5 m below the ground surface.

5.8 A planned flexible load area (see Figure P5.8) is to be 2 m × 3.2 m and carries a uniformly distributed load of 210 kN/m^2. Estimate the elastic settlement below the center of the loaded area. Assume that $D_f = 1.6$ m and $H = \infty$. Use Eq. (5.33).

5.9 Redo Problem 5.8 assuming that $D_f = 1.2$ m and $H = 4$ m.

5.10 Consider a flexible foundation measuring 5 ft × 10 ft in a plan on a soft saturated clay ($\mu_s = 0.5$). The depth of the foundation is 4 ft below the ground surface. A rigid

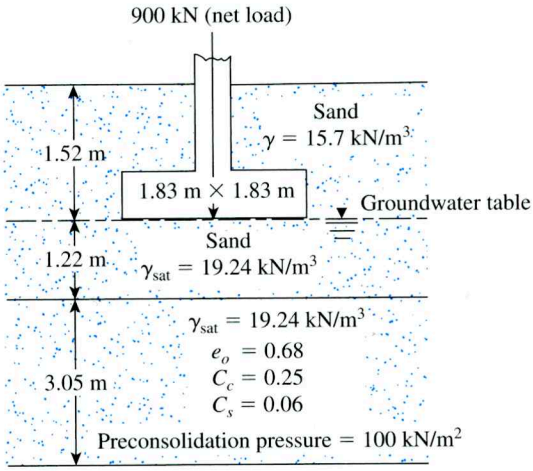


Figure P5.5

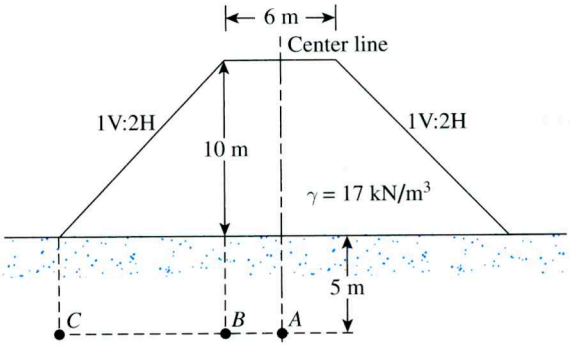


Figure P5.7

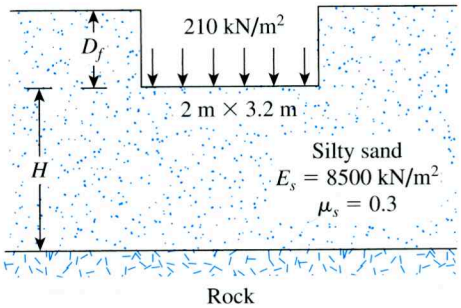


Figure P5.8

rock layer is located at 40 ft below the bottom of the foundation. Given: q_o = 3000 lb/ft² and, for the clay, E_s = 1875 lb/in². Determine the average elastic settlement of the foundation. Use Eq. (5.30).

5.11 Figure 5.16 shows a foundation of 10 ft × 6.25 ft resting on a sand deposit. The net load per unit area at the level of the foundation, q_o, is 3000 lb/ft². For the sand, μ_s = 0.3, E_s = 3200 lb/in.², D_f = 2.5 ft and H = 32 ft. Assume that the foundation is rigid, and determine the elastic settlement the foundation would undergo. Use Eqs. (5.33) and (5.41).

5.12 Repeat Problems 5.11 for a foundation of size = 1.8 m × 1.8 m and with q_o = 190 kN/m², D_f = 1.0 m, H = 15 m; and soil conditions of μ_s = 0.4, E_s = 15,400 kN/m², and γ = 17 kN/m³.

5.13 For a shallow foundation supported by a silty clay, as shown in Figure 5.17, the following are given:

Length, L = 2 m
Width, B = 1 m
Depth of foundation, D_f = 1 m
Thickness of foundation, t = 0.23 m
Load per unit area, q_o = 190 kN/m²
$E_f = 15 \times 10^6$ kN/m²

The silty clay soil has the following properties:

H = 2 m
μ_s = 0.4
E_o = 9000 kN/m²
k = 500 kN/m²/m

Using Eq. (5.46), estimate the elastic settlement of the foundation.

5.14 A plan calls for a square foundation measuring 3 m × 3 m, supported by a layer of sand. (See Figure 5.17.) Led D_f = 1.5 m, t = 0.25 m, E_o = 16,000 kN/m², k = 400 kN/m²/m, μ_s = 0.3, H = 20 m, $E_f = 15 \times 10^6$ kN/m², and q_o = 150 kN/m². Calculate the elastic settlement. Use Eq. (5.46).

5.15 Solve Problem 5.11 with Eq. (5.49). For the correction factor C_{2-}, use a time of 5 years for creep and, for the unit weight of soil, use γ = 115 lb/ft³.

5.16 A continuous foundation on a deposit of sand layer is shown in Figure P5.16 along with the variation of the modulus of elasticity of the soil (E_s). Assuming γ = 18 kN/m³ and C_2 for 10 years, calculate the elastic settlement of the foundation using the strain influence factor method.

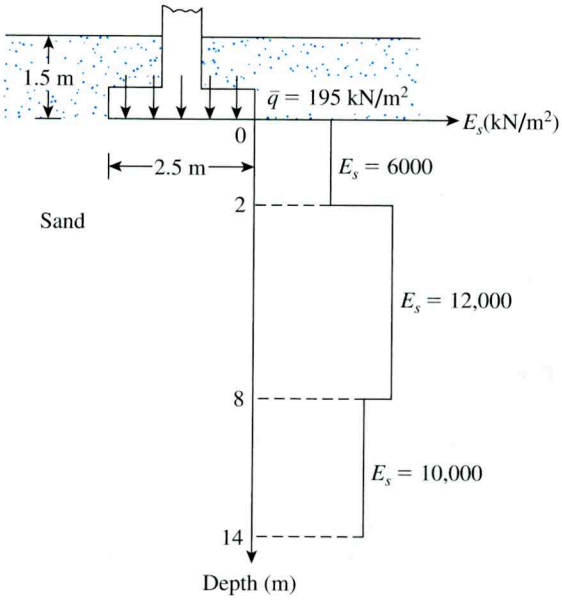


Figure P5.16

5.17 Following are the results of standard penetration tests in a granular soil deposit.

Depth (ft)	Standard penetration number, N_{60}
5	11
10	10
15	12
20	9
25	14

What will be the net allowable bearing capacity of a foundation planned to be 6 ft × 6 ft? Let $D_f = 3$ ft and the allowable settlement = 1 in., and use the relationships presented in Eq. (5.60).

5.18 A shallow foundation measuring 4.2 m × 1.2 m in plan is to be constructed over a normally consolidated sand layer. Given: $D_f = 1$ m, N_{60} increases with depth, $\overline{N}_{60}$ (in the depth of stress influence) = 11, and $q_{net} = 210$ kN/m^2. Estimate the elastic settlement using Burland and Burbidge's method.

5.19 Refer to Figure 5.25. For a foundation on a layer of sand, given: $B = 5$ ft, $L = 10$ ft, $d = 5$ ft, $\beta = 26.6°$, $e = 0.5$ ft, and $\delta = 10^9$. The Pressuremeter testing at the site produced a mean Pressuremeter curve for which the $p_{p(m)}$ versus $\Delta R/R_o$ points are as follow:

$\Delta R/R_o$ (1)	$p_{p(m)}$ (lb/in^2) (2)
0.002	7.2
0.004	24.2
0.008	32.6
0.012	42.4
0.024	68.9
0.05	126.1
0.08	177.65
0.1	210.5
0.2	369.6

What should be the magnitude of Q_o for a settlement (center) of 1 in.?

5.20 Estimate the consolidation settlement of the clay layer shown in Figure P5.5 using the results of Problem 5.5.

5.21 Estimate the consolidation settlement of the clay layer shown in Figure P5.5 using the results of Problem 5.6.

References

AHLVIN, R. G., and ULERY, H. H. (1962). *Tabulated Values of Determining the Composite Pattern of Stresses, Strains, and Deflections beneath a Uniform Load on a Homogeneous Half Space.* Highway Research Board Bulletin 342, pp. 1–13.

BJERRUM, L. (1963). "Allowable Settlement of Structures," *Proceedings, European Conference on Soil Mechanics and Foundation Engineering,* Wiesbaden, Germany, Vol. III, pp. 135–137.

6 Mat Foundations

6.1 Introduction

Under normal conditions, square and rectangular footings such as those described in Chapters 3 and 4 are economical for supporting columns and walls. However, under certain circumstances, it may be desirable to construct a footing that supports a line of two or more columns. These footings are referred to as *combined footings.* When more than one line of columns is supported by a concrete slab, it is called a *mat foundation.* Combined footings can be classified generally under the following categories:

a. Rectangular combined footing
b. Trapezoidal combined footing
c. Strap footing

Mat foundations are generally used with soil that has a low bearing capacity. A brief overview of the principles of combined footings is given in Section 6.2, followed by a more detailed discussion on mat foundations.

6.2 Combined Footings

Rectangular Combined Footing

In several instances, the load to be carried by a column and the soil bearing capacity are such that the standard spread footing design will require extension of the column foundation beyond the property line. In such a case, two or more columns can be supported on a single rectangular foundation, as shown in Figure 6.1. If the net allowable soil pressure is known, the size of the foundation $(B \times L)$ can be determined in the following manner:

a. Determine the area of the foundation

$$A = \frac{Q_1 + Q_2}{q_{\text{net(all)}}} \tag{6.1}$$

where

Q_1, Q_2 = column loads
$q_{\text{net(all)}}$ = net allowable soil bearing capacity

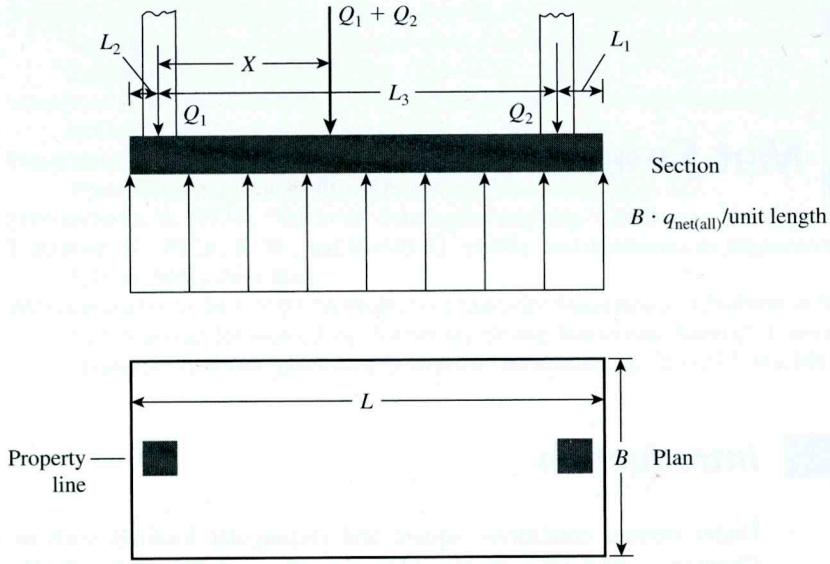


Figure 6.1 Rectangular combined footing

b. Determine the location of the resultant of the column loads. From Figure 6.1,

$$X = \frac{Q_2 L_3}{Q_1 + Q_2} \tag{6.2}$$

c. For a uniform distribution of soil pressure under the foundation, the resultant of the column loads should pass through the centroid of the foundation. Thus,

$$L = 2(L_2 + X) \tag{6.3}$$

where L = length of the foundation.

d. Once the length L is determined, the value of L_1 can be obtained as follows:

$$L_1 = L - L_2 - L_3 \tag{6.4}$$

Note that the magnitude of L_2 will be known and depends on the location of the property line.

e. The width of the foundation is then

$$B = \frac{A}{L} \tag{6.5}$$

Trapezoidal Combined Footing

Trapezoidal combined footing (see Figure 6.2) is sometimes used as an isolated spread foundation of columns carrying large loads where space is tight. The size of the foundation that will uniformly distribute pressure on the soil can be obtained in the following manner:

a. If the net allowable soil pressure is known, determine the area of the foundation:

$$A = \frac{Q_1 + Q_2}{q_{net(all)}}$$

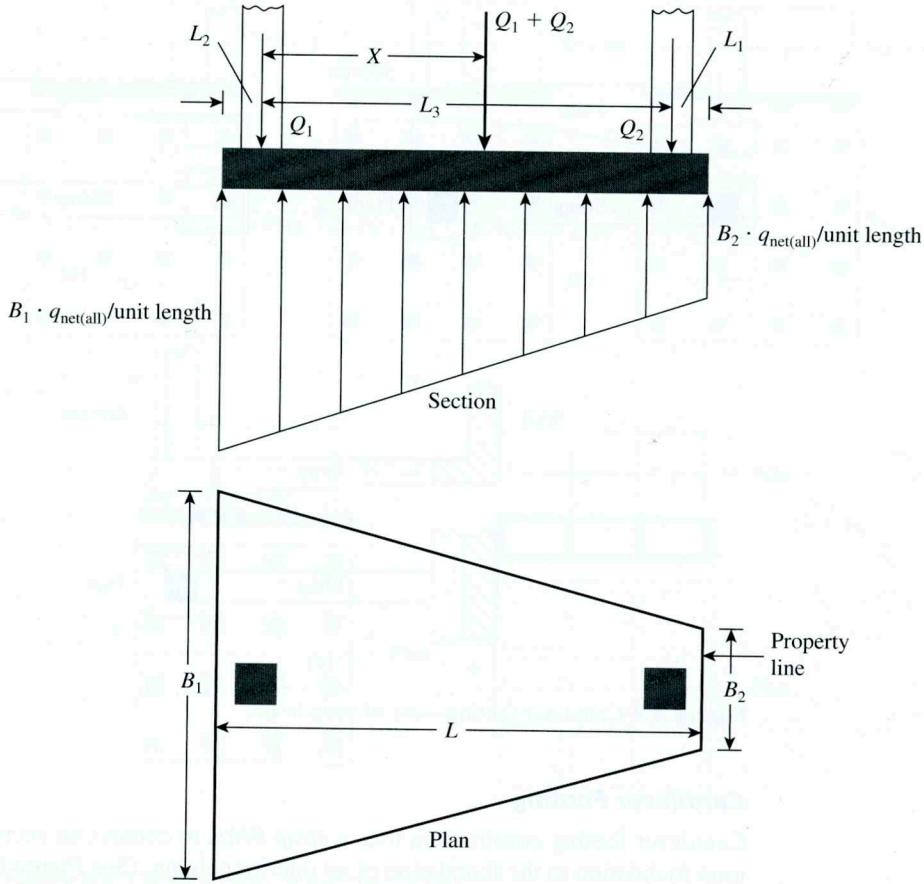


Figure 6.2 Trapezoidal combined footing

From Figure 6.2,

$$A = \frac{B_1 + B_2}{2} L \tag{6.6}$$

b. Determine the location of the resultant for the column loads:

$$X = \frac{Q_2 L_3}{Q_1 + Q_2}$$

c. From the property of a trapezoid,

$$X + L_2 = \left(\frac{B_1 + 2B_2}{B_1 + B_2}\right)\frac{L}{3} \tag{6.7}$$

With known values of A, L, X, and L_2, solve Eqs. (6.6) and (6.7) to obtain B_1 and B_2. Note that, for a trapezoid,

$$\frac{L}{3} < X + L_2 < \frac{L}{2}$$

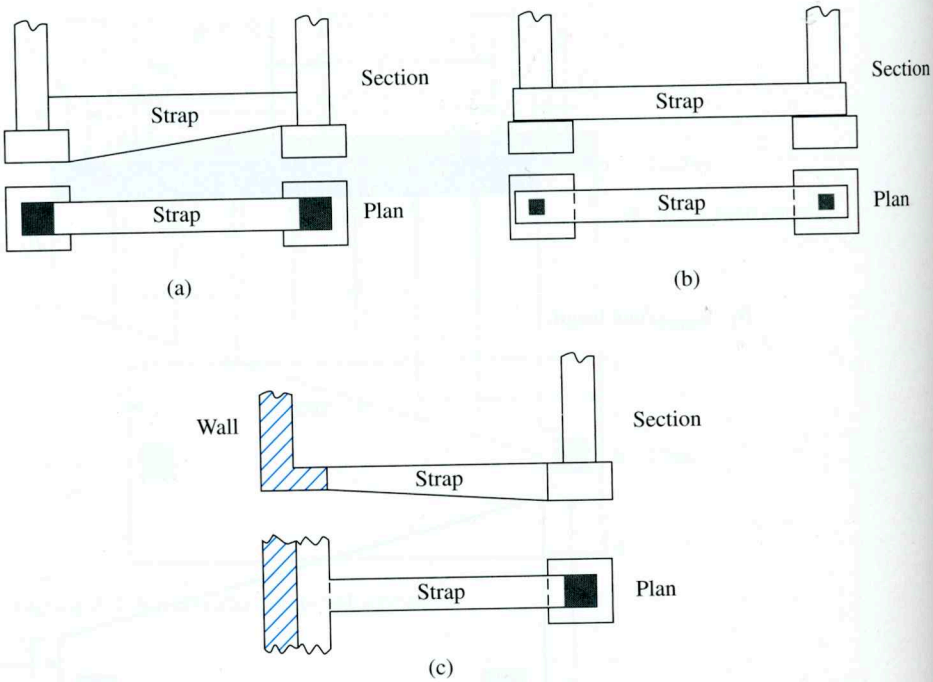


Figure 6.3 Cantilever footing—use of strap beam

Cantilever Footing

Cantilever footing construction uses a *strap beam* to connect an eccentrically loaded column foundation to the foundation of an interior column. (See Figure 6.3). Cantilever footings may be used in place of trapezoidal or rectangular combined footings when the allowable soil bearing capacity is high and the distances between the columns are large.

6.3 Common Types of Mat Foundations

The mat foundation, which is sometimes referred to as a *raft foundation,* is a combined footing that may cover the entire area under a structure supporting several columns and walls. Mat foundations are sometimes preferred for soils that have low load-bearing capacities, but that will have to support high column or wall loads. Under some conditions, spread footings would have to cover more than half the building area, and mat foundations might be more economical. Several types of mat foundations are used currently. Some of the common ones are shown schematically in Figure 6.4 and include the following:

1. Flat plate (Figure 6.4a). The mat is of uniform thickness.
2. Flat plate thickened under columns (Figure 6.4b).
3. Beams and slab (Figure 6.4c). The beams run both ways, and the columns are located at the intersection of the beams.
4. Flat plates with pedestals (Figure 6.4d).
5. Slab with basement walls as a part of the mat (Figure 6.4e). The walls act as stiffeners for the mat.

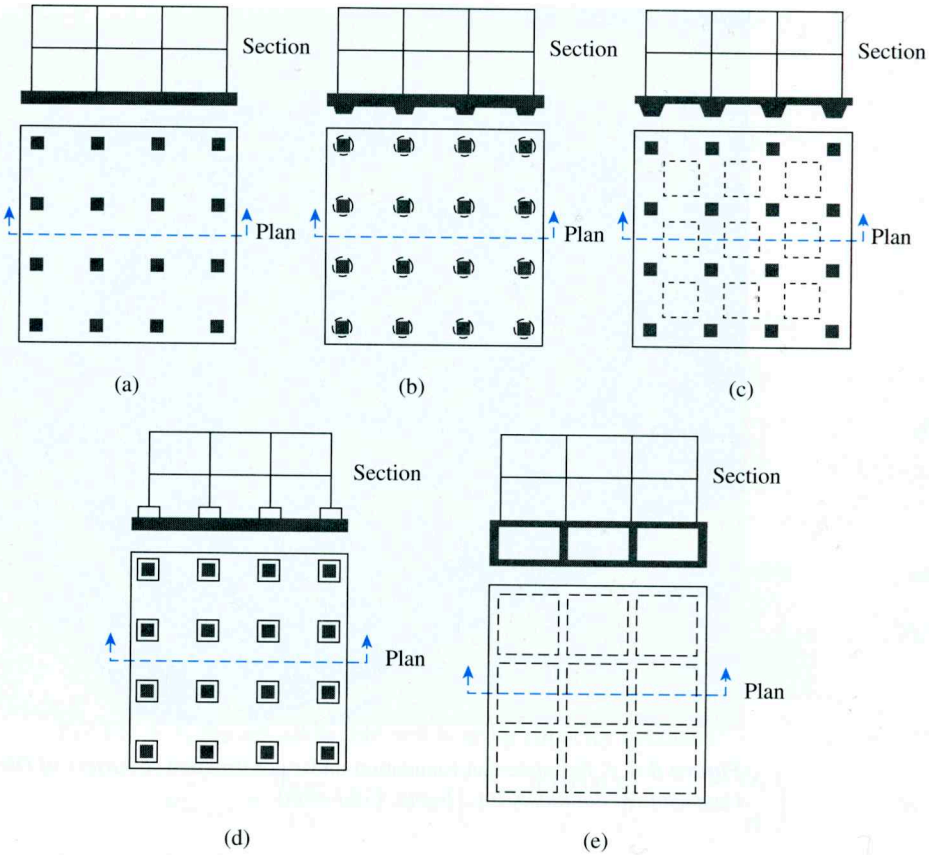


Figure 6.4 Common types of mat foundations

Mats may be supported by piles, which help reduce the settlement of a structure built over highly compressible soil. Where the water table is high, mats are often placed over piles to control buoyancy. Figure 6.5 shows the difference between the depth D_f and the width B of isolated foundations and mat foundations. Figure 6.6 shows a flat-plate mat foundation under construction.

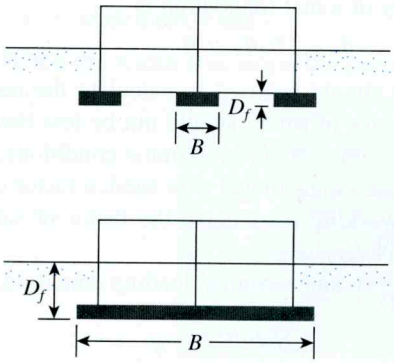


Figure 6.5 Comparison of isolated foundation and mat foundation (B = width, D_f = depth)

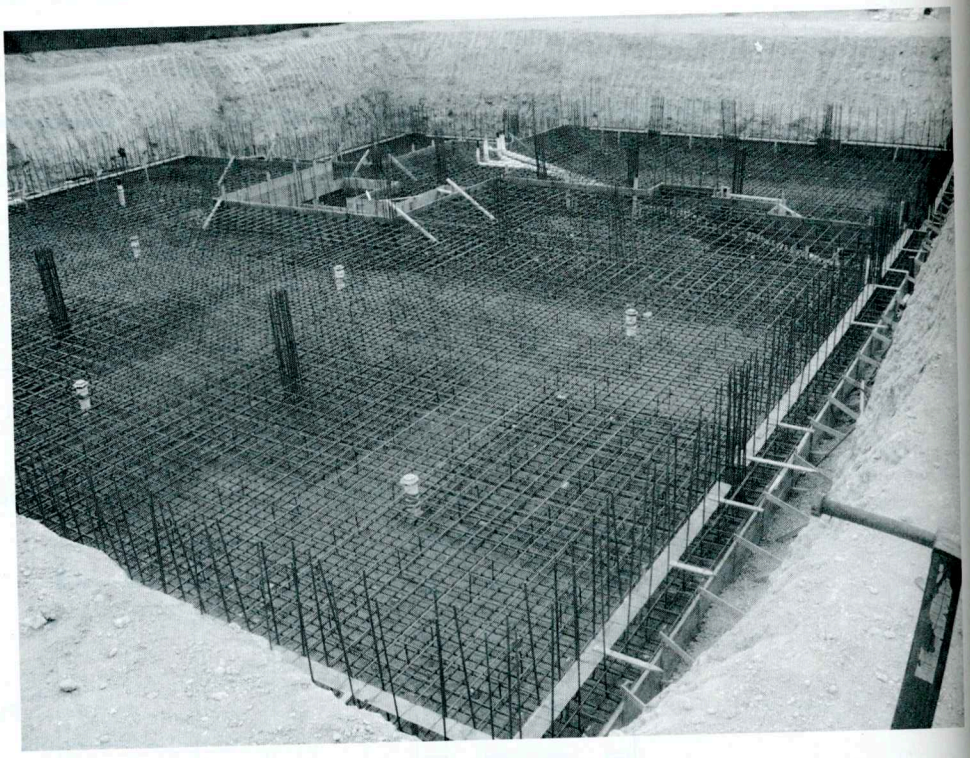


Figure 6.6 A flat plate mat foundation under construction (*Courtesy of Dharma Shakya, Geotechnical Solutions, Inc., Irvine, California*)

6.4 Bearing Capacity of Mat Foundations

The *gross ultimate bearing capacity* of a mat foundation can be determined by the same equation used for shallow foundations (see Section 3.6), or

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i} \qquad \text{[Eq. (3.19)]}$$

(Chapter 3 gives the proper values of the bearing capacity factors, as well as the shape depth, and load inclination factors.) The term B in Eq. (3.19) is the smallest dimension of the mat. The *net ultimate capacity* of a mat foundation is

$$q_{\text{net}(u)} = q_u - q \qquad \text{[Eq. (3.14)]}$$

A suitable factor of safety should be used to calculate the net *allowable* bearing capacity. For mats on clay, the factor of safety should not be less than 3 under dead load or maximum live load. However, under the most extreme conditions, the factor of safety should be at least 1.75 to 2. For mats constructed over sand, a factor of safety of 3 should normally be used. Under most working conditions, the factor of safety against bearing capacity failure of mats on sand is very large.

For saturated clays with $\phi = 0$ and a vertical loading condition, Eq. (3.19) gives

$$q_u = c_uN_cF_{cs}F_{cd} + q \qquad (6.8)$$

where c_u = undrained cohesion. (*Note:* $N_c = 5.14$, $N_q = 1$, and $N_\gamma = 0$.)

From Table 3.4, for $\phi = 0$,

$$F_{cs} = 1 + \frac{B}{L}\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{B}{L}\right)\left(\frac{1}{5.14}\right) = 1 + \frac{0.195B}{L}$$

and

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

Substitution of the preceding shape and depth factors into Eq. (6.8) yields

$$q_u = 5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) + q \tag{6.9}$$

Hence, the net ultimate bearing capacity is

$$q_{\text{net}(u)} = q_u - q = 5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) \tag{6.10}$$

For FS = 3, the net allowable soil bearing capacity becomes

$$q_{\text{net(all)}} = \frac{q_{\text{u(net)}}}{\text{FS}} = 1.713c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) \tag{6.11}$$

The net allowable bearing capacity for mats constructed over granular soil deposits can be adequately determined from the standard penetration resistance numbers. From Eq. (5.64), for shallow foundations,

$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B + 0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \qquad \text{[Eq. (5.64)]}$$

where

N_{60} = standard penetration resistance
B = width (m)
$F_d = 1 + 0.33(D_f/B) \leq 1.33$
S_e = settlement, (mm)

When the width B is large, the preceding equation can be approximated as

$$\begin{aligned} q_{\text{net}}(\text{kN/m}^2) &= \frac{N_{60}}{0.08}F_d\left(\frac{S_e}{25}\right) \\ &= \frac{N_{60}}{0.08}\left[1 + 0.33\left(\frac{D_f}{B}\right)\right]\left[\frac{S_e(\text{mm})}{25}\right] \\ &\leq 16.63N_{60}\left[\frac{S_e(\text{mm})}{25}\right] \end{aligned} \tag{6.12}$$

In English units, Eq. (6.12) may be expressed as

$$q_{\text{net(all)}}(\text{kip/ft}^2) = 0.25N_{60}\left[1 + 0.33\left(\frac{D_f}{B}\right)\right][S_e(\text{in.})] \leq 0.33N_{60}[S_e(\text{in.})] \tag{6.13}$$

Generally, shallow foundations are designed for a maximum settlement of 25 mm (1 in.) and a differential settlement of about 19 mm (0.75 in.).

However, the width of the raft foundations are larger than those of the isolated spread footings. As shown in Table 5.3, the depth of significant stress increase in the soil below a foundation depends on the width of the foundation. Hence, for a raft foundation, the depth of the zone of influence is likely to be much larger than that of a spread footing. Thus, the loose soil pockets under a raft may be more evenly distributed, resulting in a smaller differential settlement. Accordingly, the customary assumption is that, for a maximum raft settlement of 50 mm (2 in.), the differential settlement would be 19 mm (0.75 in.). Using this logic and conservatively assuming that $F_d = 1$, we can respectively approximate Eqs. (6.12) and (6.13) as

$$q_{\text{net(all)}} = q_{\text{net}}(\text{kN/m}^2) \approx 25N_{60} \tag{6.14a}$$

and

$$q_{\text{net(all)}} = q_{\text{net}}(\text{kip/ft}^2) = 0.5N_{60} \tag{6.14b}$$

The net allowable pressure applied on a foundation (see Figure 6.7) may be expressed as

$$q = \frac{Q}{A} - \gamma D_f \tag{6.15}$$

where

Q = dead weight of the structure and the live load
A = area of the raft

In all cases, q should be less than or equal to allowable q_{net}.

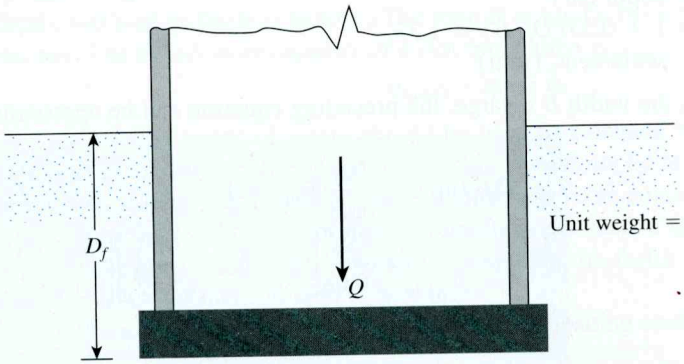


Figure 6.7 Definition of net pressure on soil caused by a mat foundation

Example 6.1

Determine the net ultimate bearing capacity of a mat foundation measuring 15 m × 10 m on a saturated clay with $c_u = 95\ \text{kN/m}^2$, $\phi = 0$, and $D_f = 2$ m.

Solution

From Eq. (6.10),

$$q_{\text{net}(u)} = 5.14c_u\left[1 + \left(\frac{0.195B}{L}\right)\right]\left[1 + 0.4\frac{D_f}{B}\right]$$

$$= (5.14)(95)\left[1 + \left(\frac{0.195 \times 10}{15}\right)\right]\left[1 + \left(\frac{0.4 \times 2}{10}\right)\right]$$

$$= \mathbf{595.9\ kN/m^2}$$

■

Example 6.2

What will be the net allowable bearing capacity of a mat foundation with dimensions of 15 m × 10 m constructed over a sand deposit? Here, $D_f = 2$ m, the allowable settlement is 25 mm, and the average penetration number $N_{60} = 10$.

Solution

From Eq. (6.12),

$$q_{\text{net(all)}} = \frac{N_{60}}{0.08}\left[1 + 0.33\left(\frac{D_f}{B}\right)\right]\left(\frac{S_e}{25}\right) \leq 16.63N_{60}\left(\frac{S_e}{25}\right)$$

or

$$q_{\text{net(all)}} = \frac{10}{0.08}\left[1 + \frac{0.33 \times 2}{10}\right]\left(\frac{25}{25}\right) = \mathbf{133.25\ kN/m^2}$$

■

6.5 Differential Settlement of Mats

In 1988, the American Concrete Institute Committee 336 suggested a method for calculating the differential settlement of mat foundations. According to this method, the rigidity factor K_r is calculated as

$$K_r = \frac{E'I_b}{E_sB^3} \tag{6.16}$$

where

E' = modulus of elasticity of the material used in the structure
E_s = modulus of elasticity of the soil
B = width of foundation
I_b = moment of inertia of the structure per unit length at right angles to B

The term $E'I_b$ can be expressed as

$$E'I_b = E'\left(I_F + \sum I'_b + \sum\frac{ah^3}{12}\right) \tag{6.17}$$

where

$E'I_b$ = flexural rigidity of the superstructure and foundation per unit length at right angles to B

$\Sigma E'I'_b$ = flexural rigidity of the framed members at right angles to B

$\Sigma(E'ah^3/12)$ = flexural rigidity of the shear walls

a = shear wall thickness

h = shear wall height

$E'I_F$ = flexibility of the foundation

Based on the value of K_r, the ratio (δ) of the differential settlement to the total settlement can be estimated in the following manner:

1. If $K_r > 0.5$, it can be treated as a rigid mat, and $\delta = 0$.
2. If $K_r = 0.5$, then $\delta \approx 0.1$.
3. If $K_r = 0$, then $\delta = 0.35$ for square mats $(B/L = 1)$ and $\delta = 0.5$ for long foundations $(B/L = 0)$.

6.6 Field Settlement Observations for Mat Foundations

Several field settlement observations for mat foundations are currently available in the literature. In this section, we compare the observed settlements for some mat foundations constructed over granular soil deposits with those obtained from Eqs. (6.12) and (6.13).

Meyerhof (1965) compiled the observed maximum settlements for mat foundations constructed on sand and gravel, as listed in Table 6.1. In Eq. (6.12), if the depth factor, $1 + 0.33(D_f/B)$, is assumed to be approximately unity, then

$$S_e(\text{mm}) \approx \frac{2q_{\text{net(all)}}}{N_{60}} \tag{6.18}$$

From the values of $q_{\text{net(all)}}$ and N_{60} given in Columns 6 and 5, respectively, of Table 6.1, the magnitudes of S_e were calculated and are given in Column 8.

Column 9 of Table 6.1 gives the ratios of calculated to measured values of S_e. These ratios vary from about 0.79 to 3.39. Thus, calculating the net allowable bearing capacity with the use of Eq. (6.12) or (6.13) will yield safe and conservative values.

6.7 Compensated Foundation

Figure 6.7 and Eq. (6.15) indicate that the net pressure increase in the soil under a mat foundation can be reduced by increasing the depth D_f of the mat. This approach is generally referred to as the *compensated foundation design* and is extremely useful when structures are to be built on very soft clays. In this design, a deeper basement is made below the higher portion of the superstructure, so that the net pressure increase in soil at any depth is relatively uniform. (See Figure 6.8.) From Eq. (6.15) and Figure 6.7, the net average applied pressure on soil is

$$q = \frac{Q}{A} - \gamma D_f$$

Table 6.1 Settlement of Mat Foundations on Sand and Gravel (Based on Meyerhof, 1965) (Based on Meyerhof, G. G., (1965). "Shallow Foundations," *Journal of the Soil Mechanics and Foundation Engineering Divsion*, American Society of Civil Engineers, Vol. 91, No. 2, pp. 21–31, Table 1. With permission from ASCE.)

Case No. (1)	Structure (2)	Reference (3)	B m (ft) (4)	Average N_{60} (5)	$q_{net(all)}$ kN/m^2 (kip/ft^2) (6)	Observed maximum settlement, S_e mm (in.) (7)	Calculated maximum settlement, S_e mm (in.) (8)	calculated S_e / observed S_e (9)
1	T. Edison São Paulo, Brazil	Rios and Silva (1948)	18.29 (60)	15	229.8 (4.8)	15.24 (0.6)	30.64 (1.21)	2.01
2	Banco do Brazil São Paulo, Brazil	Rios and Silva (1948); Vargas (1961)	22.86 (75)	18	239.4 (5.0)	27.94 (1.1)	26.6 (1.05)	0.95
3	Iparanga São Paulo, Brazil	Vargas (1948)	9.14 (30)	9	304.4 (6.4)	35.56 (1.4)	67.64 (2.66)	1.9
4	C.B.I., Esplanda São Paulo, Brazil	Vargas (1961)	14.63 (48)	22	383.0 (8.0)	27.94 (1.1)	34.82 (1.37)	1.25
5	Riscala São Paulo, Brazil	Vargas (1948)	3.96 (13)	20	229.8 (4.8)	12.7 (0.5)	22.98 (0.9)	1.81
6	Thyssen Düsseldorf, Germany	Schultze (1962)	22.55 (74)	25	239.4 (5)	24.13 (0.95)	19.15 (0.75)	0.79
7	Ministry Düsseldorf, Germany	Schultze (1962)	15.85 (52)	20	220.2 (4.6)	20.32 (0.8)	22.02 (0.87)	1.08
8	Chimney Cologne, Germany	Schultze (1962)	20.42 (67)	10	172.4 (3.6)	10.16 (0.4)	34.48 (1.36)	3.39

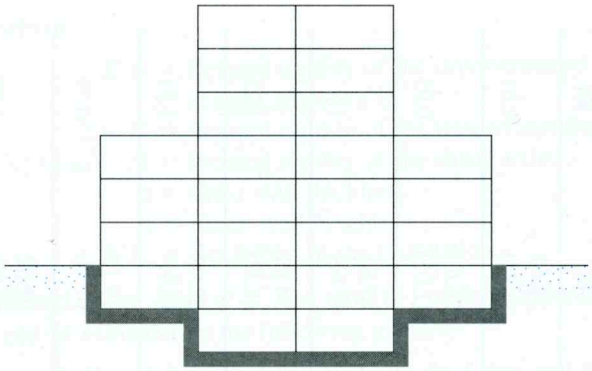


Figure 6.8 Compensated foundation

For no increase in the net pressure on soil below a mat foundation, q should be zero. Thus,

$$D_f = \frac{Q}{A\gamma} \tag{6.19}$$

This relation for D_f is usually referred to as the depth of a *fully compensated foundation.*

The factor of safety against bearing capacity failure for partially compensated foundations (i.e., $D_f < Q/A\gamma$) may be given as

$$\text{FS} = \frac{q_{\text{net}(u)}}{q} = \frac{q_{\text{net}(u)}}{\dfrac{Q}{A} - \gamma D_f} \tag{6.20}$$

where $q_{\text{net}(u)}$ = net ultimate bearing capacity.

For saturated clays, the factor of safety against bearing capacity failure can thus be obtained by substituting Eq. (6.10) into Eq. (6.20):

$$\text{FS} = \frac{5.14c_u\left(1 + \dfrac{0.195B}{L}\right)\left(1 + 0.4\dfrac{D_f}{B}\right)}{\dfrac{Q}{A} - \gamma D_f} \tag{6.21}$$

Example 6.3

The mat shown in Figure 6.7 has dimensions of 60 ft × 100 ft. The total dead and live load on the mat is 25×10^3 kip. The mat is placed over a saturated clay having a unit weight of 120 lb/ft^3 and c_u = 2800 lb/ft^2. Given that D_f = 5 ft, determine the factor of safety against bearing capacity failure.

Solution

From Eq. (6.21), the factor of safety

$$\text{FS} = \frac{5.14c_u\left(1 + \dfrac{0.195B}{L}\right)\left(1 + 0.4\dfrac{D_f}{B}\right)}{\dfrac{Q}{A} - \gamma D_f}$$

We are given that $c_u = 2800\ \text{lb/ft}^2$, $D_f = 5$ ft, $B = 60$ ft, $L = 100$ ft, and $\gamma = 120\ \text{lb/ft}^3$. Hence,

$$\text{FS} = \frac{(5.14)(2800)\left[1 + \dfrac{(0.195)(60)}{100}\right]\left[1 + 0.4\left(\dfrac{5}{60}\right)\right]}{\left(\dfrac{25 \times 10^6\ \text{lb}}{60 \times 100}\right) - (120)(5)} = \mathbf{4.66}$$

■

Example 6.4

Consider a mat foundation 30 m × 40 m in plan, as shown in Figure 6.9. The total dead load and live load on the raft is 200×10^3 kN. Estimate the consolidation settlement at the center of the foundation.

Solution

From Eq. (1.61)

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o}\log\left(\frac{\sigma'_o + \Delta\sigma'_{\text{av}}}{\sigma'_o}\right)$$

$$\sigma'_o = (3.67)(15.72) + (13.33)(19.1 - 9.81) + \frac{6}{2}(18.55 - 9.81) \approx 208\ \text{kN/m}^2$$

$$H_c = 6\ \text{m}$$

$$C_c = 0.28$$

$$e_o = 0.9$$

For $Q = 200 \times 10^3$ kN, the net load per unit area is

$$q = \frac{Q}{A} - \gamma D_f = \frac{200 \times 10^3}{30 \times 40} - (15.72)(2) \approx 135.2\ \text{kN/m}^2$$

In order to calculate $\Delta\sigma'_{\text{av}}$ we refer to Section 5.5. The loaded area can be divided into four areas, each measuring 15 m × 20 m. Now using Eq. (5.19), we can calculate the average stress increase in the clay layer below the corner of each rectangular area, or

$$\Delta\sigma'_{\text{av}(H_2/H_1)} = q_o\left[\frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1}\right]$$

$$= 135.2\left[\frac{(1.67 + 13.33 + 6)I_{a(H_2)} - (1.67 + 13.33)I_{a(H_1)}}{6}\right]$$

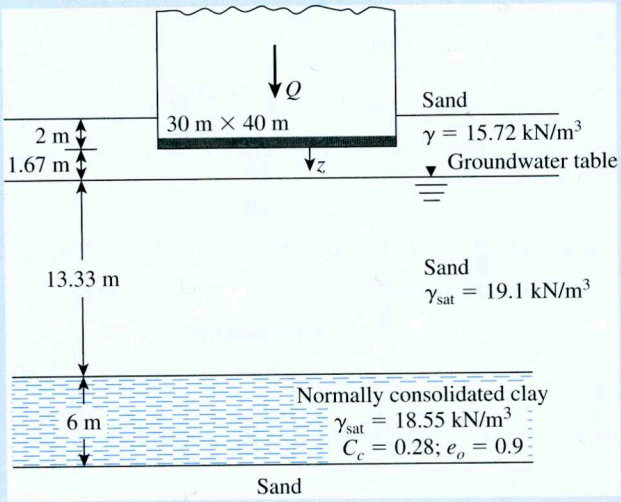


Figure 6.9 Consolidation settlement under a mat foundation

For $I_{a(H_2)}$,

$$m_2 = \frac{B}{H_2} = \frac{15}{1.67 + 13.33 + 6} = 0.71$$

$$n_2 = \frac{L}{H_2} = \frac{20}{21} = 0.95$$

From Fig. 5.7, for $m_2 = 0.71$ and $n_2 = 0.95$, the value of $I_{a(H_2)}$ is 0.21. Again, for $I_{a(H_1)}$,

$$m_2 = \frac{B}{H_1} = \frac{15}{15} = 1$$

$$n_2 = \frac{L}{H_1} = \frac{20}{15} = 1.33$$

From Figure 5.7, $I_{a(H_1)} = 0.225$, so

$$\Delta\sigma'_{av(H_2/H_1)} = 135.2\left[\frac{(21)(0.21) - (15)(0.225)}{6}\right] = 23.32 \text{ kN/m}^2$$

So, the stress increase below the center of the 30 m × 40 m area is $(4)(23.32) = 93.28$ kN/m². Thus

$$S_{c(p)} = \frac{(0.28)(6)}{1 + 0.9}\log\left(\frac{208 + 93.28}{208}\right) = 0.142 \text{ m}$$

$$= \mathbf{142\ mm}$$

■

6.8 Structural Design of Mat Foundations

The structural design of mat foundations can be carried out by two conventional methods: the conventional rigid method and the approximate flexible method. Finite-difference and finite-element methods can also be used, but this section covers only the basic concepts of the first two design methods.

Conventional Rigid Method

The *conventional rigid method* of mat foundation design can be explained step by step with reference to Figure 6.10:

Step 1. Figure 6.10a shows mat dimensions of $L \times B$ and column loads of $Q_1, Q_2, Q_3, \ldots$. Calculate the total column load as

$$Q = Q_1 + Q_2 + Q_3 + \cdots \tag{6.22}$$

Step 2. Determine the pressure on the soil, q, below the mat at points $A, B, C, D, \ldots$, by using the equation

$$q = \frac{Q}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} \tag{6.23}$$

where

$A = BL$
$I_x = (1/12)BL^3$ = moment of inertia about the x-axis
$I_y = (1/12)LB^3$ = moment of inertia about the y-axis
M_x = moment of the column loads about the x-axis $= Qe_y$
M_y = moment of the column loads about the y-axis $= Qe_x$

The load eccentricities, e_x and e_y, in the x and y directions can be determined by using (x', y') coordinates:

$$x' = \frac{Q_1 x_1' + Q_2 x_2' + Q_3 x_3' + \cdots}{Q} \tag{6.24}$$

and

$$e_x = x' - \frac{B}{2} \tag{6.25}$$

Similarly,

$$y' = \frac{Q_1 y_1' + Q_2 y_2' + Q_3 y_3' + \cdots}{Q} \tag{6.26}$$

and

$$e_y = y' - \frac{L}{2} \tag{6.27}$$

Step 3. Compare the values of the soil pressures determined in Step 2 with the net allowable soil pressure to determine whether $q \leqslant q_{\text{all(net)}}$.

Step 4. Divide the mat into several strips in the x and y directions. (See Figure 6.10). Let the width of any strip be B_1.

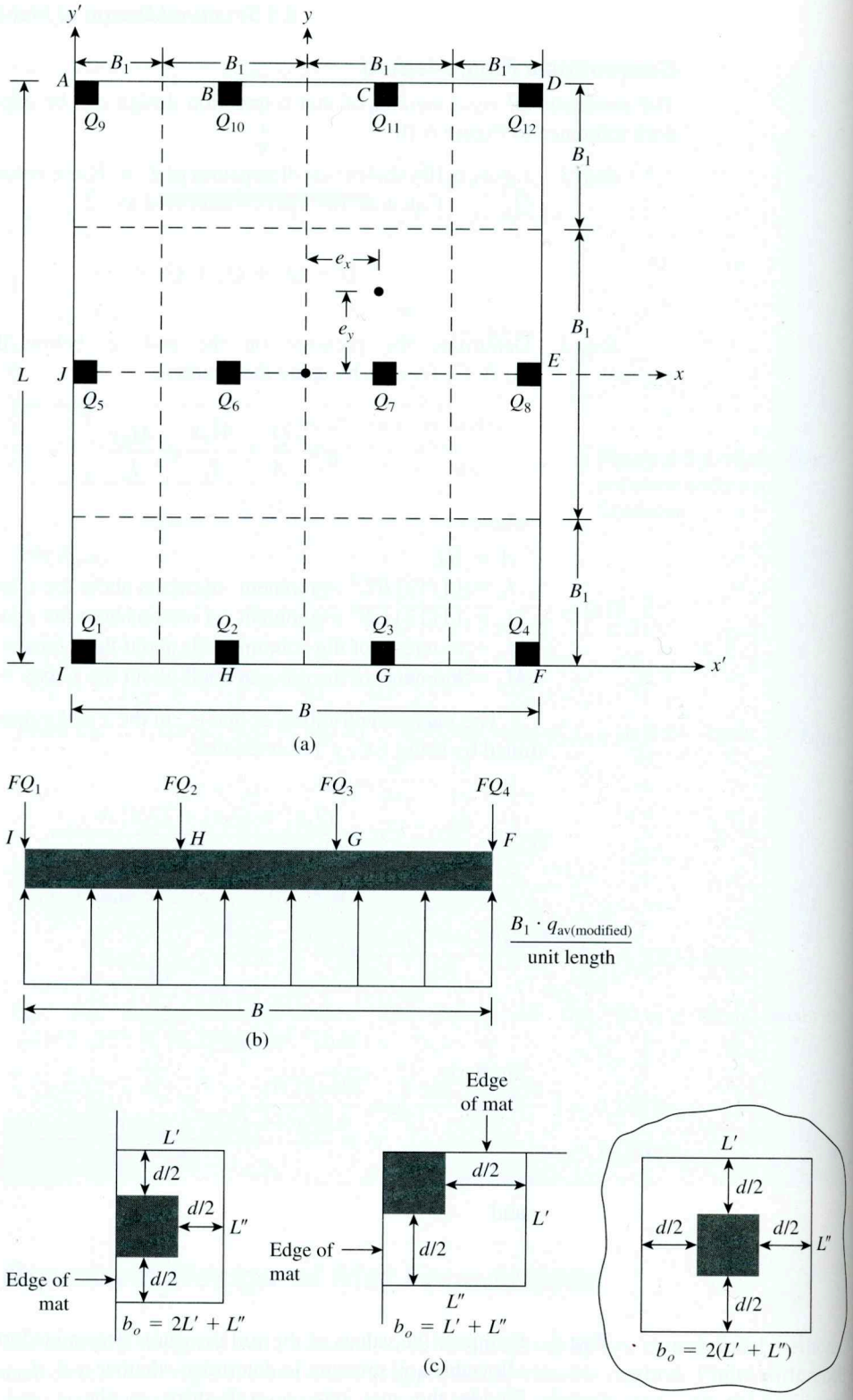


Figure 6.10 Conventional rigid mat foundation design

Step 5. Draw the shear, V, and the moment, M, diagrams for each individual strip (in the x and y directions). For example, the average soil pressure of the bottom strip in the x direction of Figure 6.10a is

$$q_{av} \approx \frac{q_I + q_F}{2} \tag{6.28}$$

where q_I and q_F = soil pressures at points I and F, as determined from Step 2.

The total soil reaction is equal to $q_{av}B_1B$. Now obtain the total column load on the strip as $Q_1 + Q_2 + Q_3 + Q_4$. The sum of the column loads on the strip will not equal $q_{av}B_1B$, because the shear between the adjacent strips has not been taken into account. For this reason, the soil reaction and the column loads need to be adjusted, or

$$\text{Average load} = \frac{q_{av}B_1B + (Q_1 + Q_2 + Q_3 + Q_4)}{2} \tag{6.29}$$

Now, the modified average soil reaction becomes

$$q_{av(\text{modified})} = q_{av}\left(\frac{\text{average load}}{q_{av}B_1B}\right) \tag{6.30}$$

and the column load modification factor is

$$F = \frac{\text{average load}}{Q_1 + Q_2 + Q_3 + Q_4} \tag{6.31}$$

So the modified column loads are FQ_1, FQ_2, FQ_3, and FQ_4. This modified loading on the strip under consideration is shown in Figure 6.10b. The shear and the moment diagram for this strip can now be drawn, and the procedure is repeated in the x and y directions for all strips.

Step 6. Determine the effective depth d of the mat by checking for diagonal tension shear near various columns. According to ACI Code 318-95 (Section 11.12.2.1c, American Concrete Institute, 1995), for the critical section,

$$U = b_o d[\phi(0.34)\sqrt{f'_c}] \tag{6.32a}$$

where

U = factored column load (MN), or (column load) × (load factor)
ϕ = reduction factor = 0.85
f'_c = compressive strength of concrete at 28 days (MN/m^2)

The units of b_o and d in Eq. (6.32a) are in meters. In English units, Eq. (6.32a) may be expressed as

$$U = b_o d(4\phi\sqrt{f'_c}) \tag{6.32b}$$

where U is in lb, b_o and d are in in., and f'_c is in lb/in^2.

The expression for b_o in terms of d, which depends on the location of the column with respect to the plan of the mat, can be obtained from Figure 6.10c.

Step 7. From the moment diagrams of all strips *in one direction* (x or y), obtain the *maximum* positive and negative moments per unit width (i.e., $M' = M/B_1$).

Step 8. Determine the areas of steel per unit width for positive and negative reinforcement in the x and y directions. We have

$$M_u = (M')(\text{load factor}) = \phi A_s f_y \left(d - \frac{a}{2}\right) \tag{6.33}$$

and

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{6.34}$$

where

A_s = area of steel per unit width
f_y = yield stress of reinforcement in tension
M_u = factored moment
ϕ = 0.9 = reduction factor

Examples 6.5 and 6.6 illustrate the use of the conventional rigid method of mat foundation design.

Approximate Flexible Method

In the conventional rigid method of design, the mat is assumed to be infinitely rigid. Also, the soil pressure is distributed in a straight line, and the centroid of the soil pressure is coincident with the line of action of the resultant column loads. (See Figure 6.11a.) In the *approximate flexible method* of design, the soil is assumed to be equivalent to an infinite number of elastic springs, as shown in Figure 6.11b. This assumption is sometimes referred to as the *Winkler foundation.* The elastic constant of these assumed springs is referred to as the *coefficient of subgrade reaction, k.*

To understand the fundamental concepts behind flexible foundation design, consider a beam of width B_1 having infinite length, as shown in Figure 6.11c. The beam is subjected to a single concentrated load Q. From the fundamentals of mechanics of materials,

$$M = E_F I_F \frac{d^2 z}{dx^2} \tag{6.35}$$

where

M = moment at any section
E_F = modulus of elasticity of foundation material
I_F = moment of inertia of the cross section of the beam = $(\frac{1}{12}) B_1 h^3$ (see Figure 6.11c).

However,

$$\frac{dM}{dx} = \text{shear force} = V$$

and

$$\frac{dV}{dx} = q = \text{soil reaction}$$

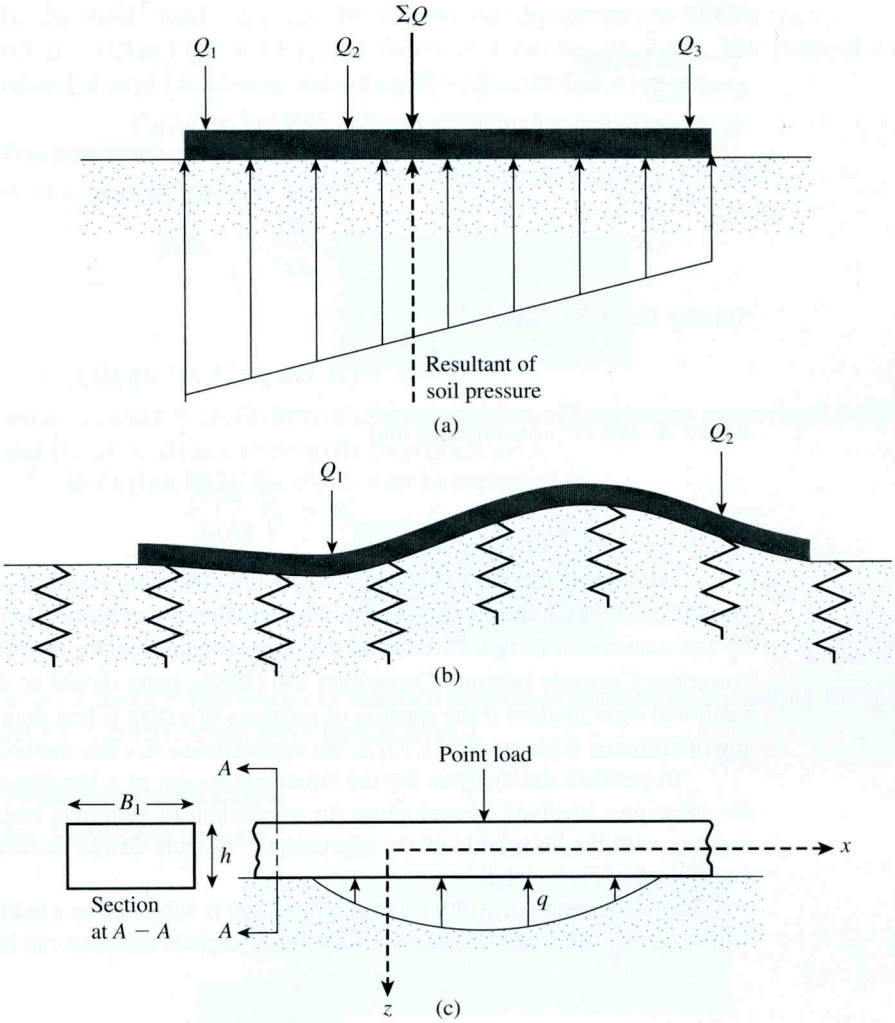


Figure 6.11 (a) Principles of design by conventional rigid method; (b) principles of approximate flexible method; (c) derivation of Eq. (6.39) for beams on elastic foundation

Hence,

$$\frac{d^2M}{dx^2} = q \tag{6.36}$$

Combining Eqs. (6.35) and (6.36) yields

$$E_F I_F \frac{d^4z}{dx^4} = q \tag{6.37}$$

However, the soil reaction is

$$q = -zk'$$

where

z = deflection
$k' = kB_1$
k = coefficient of subgrade reaction (kN/m^3 or lb/in^3)

So,

$$E_F I_F \frac{d^4 z}{dx^4} = -zkB_1 \tag{6.38}$$

Solving Eq. (6.38) yields

$$z = e^{-\alpha x}(A' \cos \beta x + A'' \sin \beta x) \tag{6.39}$$

where A' and A'' are constants and

$$\beta = \sqrt[4]{\frac{B_1 k}{4E_F I_F}} \tag{6.40}$$

The unit of the term β, as defined by the preceding equation, is $(\text{length})^{-1}$. This parameter is very important in determining whether a mat foundation should be designed by the conventional rigid method or the approximate flexible method. According to the American Concrete Institute Committee 336 (1988), mats should be designed by the conventional rigid method if the spacing of columns in a strip is less than $1.75/\beta$. If the spacing of columns is larger than $1.75/\beta$, the approximate flexible method may be used.

To perform the analysis for the structural design of a flexible mat, one must know the principles involved in evaluating the coefficient of subgrade reaction, k. Before proceeding with the discussion of the approximate flexible design method, let us discuss this coefficient in more detail.

If a foundation of width B (see Figure 6.12) is subjected to a load per unit area of q, it will undergo a settlement Δ. The coefficient of subgrade modulus can be defined as

$$k = \frac{q}{\Delta} \tag{6.41}$$

The unit of k is kN/m^3 (or lb/in^3). The value of the coefficient of subgrade reaction is not a constant for a given soil, but rather depends on several factors, such as the length L and width B of the foundation and also the depth of embedment of the foundation. A comprehensive study by Terzaghi (1955) of the parameters affecting the coefficient of subgrade reaction indicated that the value of the coefficient decreases with the width of the foundation.

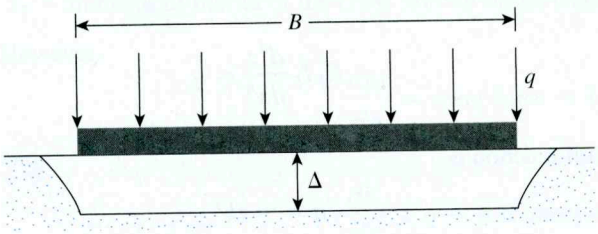


Figure 6.12 Definition of coefficient of subgrade reaction, k

In the field, load tests can be carried out by means of square plates measuring 0.3 m × 0.3 m (1 ft × 1 ft), and values of k can be calculated. The value of k can be related to large foundations measuring $B \times B$ in the following ways:

Foundations on Sandy Soils

For foundations on sandy soils,

$$k = k_{0.3}\left(\frac{B + 0.3}{2B}\right)^2 \tag{6.42}$$

where $k_{0.3}$ and k = coefficients of subgrade reaction of foundations measuring 0.3 m × 0.3 m and B (m) × B (m), respectively (unit is kN/m^3).

In English units, Eq. (6.42) may be expressed as

$$k = k_1\left(\frac{B + 1}{2B}\right)^2 \tag{6.43}$$

where k_1 and k = coefficients of subgrade reaction of foundations measuring 1 ft × 1 ft and B (ft) × B (ft), respectively (unit is lb/in^3).

Foundations on Clays

For foundations on clays,

$$k(\text{kN/m}^3) = k_{0.3}(\text{kN/m}^3)\left[\frac{0.3\ (\text{m})}{B(\text{m})}\right] \tag{6.44a}$$

The definitions of k and $k_{0.3}$ in Eq. (6.44a) are the same as in Eq. (6.42).

In English units,

$$k(\text{lb/in}^3) = k_1\ (\text{lb/in}^3)\left[\frac{1\ (\text{ft})}{B(\text{ft})}\right] \tag{6.44b}$$

The definitions of k and k_1 are the same as in Eq. (6.43).

For rectangular foundations having dimensions of $B \times L$ (for similar soil and q),

$$k = \frac{k_{(B\times B)}\left(1 + 0.5\dfrac{B}{L}\right)}{1.5} \tag{6.45}$$

where

k = coefficient of subgrade modulus of the rectangular foundation $(L \times B)$
$k_{(B \times B)}$ = coefficient of subgrade modulus of a square foundation having dimension of $B \times B$

Equation (6.45) indicates that the value of k for a very long foundation with a width B is approximately $0.67k_{(B \times B)}$.

The modulus of elasticity of granular soils increases with depth. Because the settlement of a foundation depends on the modulus of elasticity, the value of k increases with the depth of the foundation.

Table 6.2 provides typical ranges of values for the coefficient of subgrade reaction, $k_{0.3}(k_1)$, for sandy and clayey soils.

For long beams, Vesic (1961) proposed an equation for estimating subgrade reaction, namely,

$$k' = Bk = 0.65 \sqrt[12]{\frac{E_s B^4}{E_F I_F}} \frac{E_s}{1 - \mu_s^2}$$

or

$$k = 0.65 \sqrt[12]{\frac{E_s B^4}{E_F I_F}} \frac{E_s}{B(1 - \mu_s^2)} \tag{6.46}$$

where

E_s = modulus of elasticity of soil
B = foundation width
E_F = modulus of elasticity of foundation material
I_F = moment of inertia of the cross section of the foundation
μ_s = Poisson's ratio of soil

Table 6.2 Typical Subgrade Reaction Values, $k_{0.3}(k_1)$

	$k_{0.3}(k_1)$	
Soil type	**MN/m³**	**lb/in.³**
Dry or moist sand:		
Loose	8–25	30–90
Medium	25–125	90–450
Dense	125–375	450–1350
Saturated sand:		
Loose	10–15	35–55
Medium	35–40	125–145
Dense	130–150	475–550
Clay:		
Stiff	10–25	40–90
Very stiff	25–50	90–185
Hard	>50	>185

For most practical purposes, Eq. (6.46) can be approximated as

$$k = \frac{E_s}{B(1 - \mu_s^2)} \tag{6.47}$$

Now that we have discussed the coefficient of subgrade reaction, we will proceed with the discussion of the approximate flexible method of designing mat foundations. This method, as proposed by the American Concrete Institute Committee 336 (1988), is described step by step. The use of the design procedure, which is based primarily on the theory of plates, allows the effects (i.e., moment, shear, and deflection) of a concentrated column load in the area surrounding it to be evaluated. If the zones of influence of two or more columns overlap, superposition can be employed to obtain the net moment, shear, and deflection at any point. The method is as follows:

Step 1. Assume a thickness h for the mat, according to Step 6 of the conventional rigid method. (*Note:* h is the *total* thickness of the mat.)

Step 2. Determine the flexural ridigity R of the mat as given by the formula

$$R = \frac{E_F h^3}{12(1 - \mu_F^2)} \tag{6.48}$$

where

E_F = modulus of elasticity of foundation material
μ_F = Poisson's ratio of foundation material

Step 3. Determine the radius of effective stiffness—that is,

$$L' = \sqrt[4]{\frac{R}{k}} \tag{6.49}$$

where k = coefficient of subgrade reaction. The zone of influence of any column load will be on the order of 3 to 4 L'.

Step 4. Determine the moment (in polar coordinates at a point) caused by a column load (see Figure 6.13a). The formulas to use are

$$M_r = \text{radial moment} = -\frac{Q}{4}\left[A_1 - \frac{(1 - \mu_F)A_2}{\frac{r}{L'}}\right] \tag{6.50}$$

and

$$M_t = \text{tangential moment} = -\frac{Q}{4}\left[\mu_F A_1 + \frac{(1 - \mu_F)A_2}{\frac{r}{L'}}\right] \tag{6.51}$$

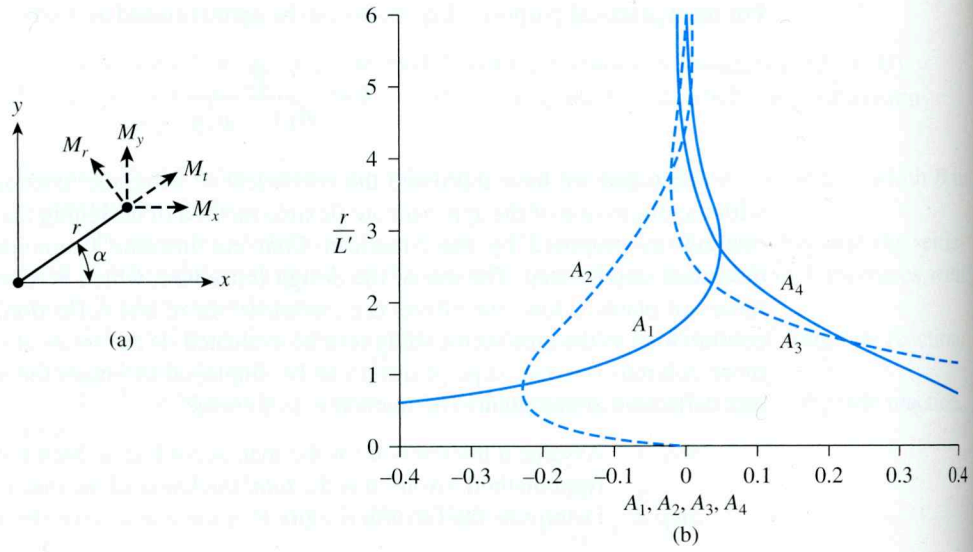


Figure 6.13 Approximate flexible method of mat design

where

$$r = \text{radial distance from the column load}$$
$$Q = \text{column load}$$
$$A_1, A_2 = \text{functions of } r/L'$$

The variations of A_1 and A_2 with r/L' are shown in Figure 6.13b. (For details see Hetenyi, 1946.)

In the Cartesian coordinate system (see Figure 6.13a),

$$M_x = M_t \sin^2\alpha + M_r \cos^2\alpha \tag{6.52}$$

and

$$M_y = M_t \cos^2\alpha + M_r \sin^2\alpha \tag{6.53}$$

Step 5. For the unit width of the mat, determine the shear force V caused by a column load:

$$V = \frac{Q}{4L'} A_3 \tag{6.54}$$

The variation of A_3 with r/L' is shown in Figure 6.13b.

Step 6. If the edge of the mat is located in the zone of influence of a column, determine the moment and shear along the edge. (Assume that the mat is continuous.) Moment and shear opposite in sign to those determined are applied at the edges to satisfy the known conditions.

Step 7. The deflection at any point is given by

$$\delta = \frac{QL'^2}{4R} A_4 \tag{6.55}$$

The variation of A_4 is presented in Figure 6.13b.

Example 6.5

The plan of a mat foundation is shown in Figure 6.14. Calculate the soil pressure at points *A, B, C, D, E,* and *F*. (*Note*: All column sections are planned to be 0.5 m × 0.5 m.)

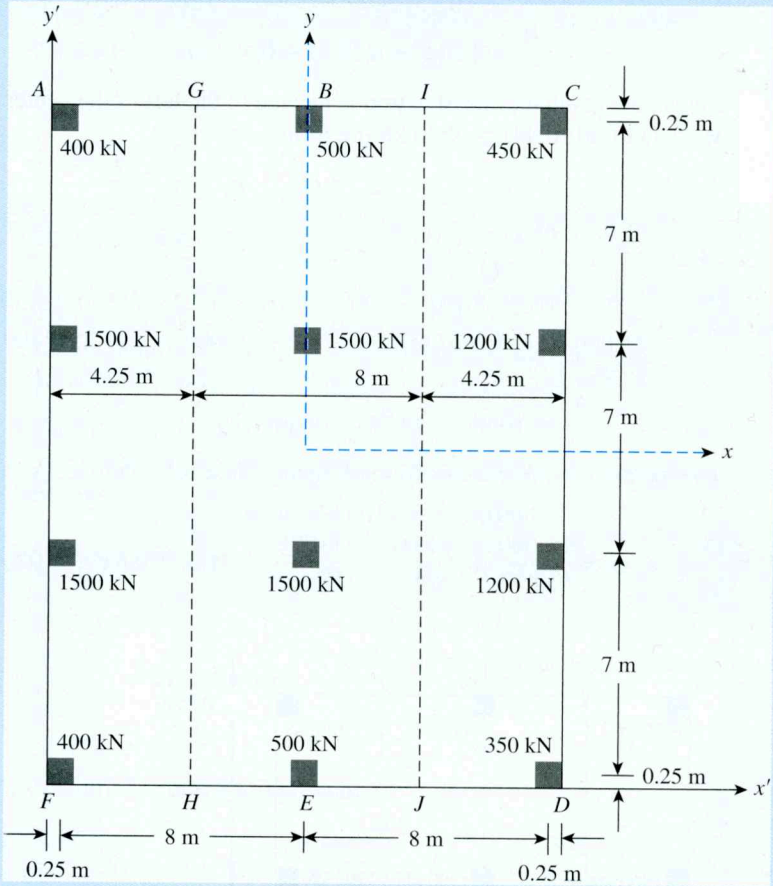


Figure 6.14 Plan of a mat foundation

Solution

Eq. (6.23): $q = \dfrac{Q}{A} \pm \dfrac{M_y\, x}{I_y} \pm \dfrac{M_x\, y}{I_x}$

$$A = (16.5)(21.5) = 354.75 \text{ m}^2$$

$$I_x = \frac{1}{12}BL^3 = \frac{1}{12}(16.5)(21.5)^3 = 13{,}665 \text{ m}^4$$

$$I_y = \frac{1}{12}LB^3 = \frac{1}{12}(21.5)(16.5)^3 = 8{,}050 \text{ m}^4$$

$$Q = 350 + (2)(400) + 450 + (2)(500) + (2)(1200) + (4)(1500) = 11{,}000 \text{ kN}$$

$$M_y = Qe_x; \quad e_x = x' - \frac{B}{2}$$

$$x' = \frac{Q_1x_1' + Q_2x_2' + Q_3x_3' + \cdots}{Q}$$

$$= \frac{1}{11{,}000}\begin{bmatrix} (8.25)(500 + 1500 + 1500 + 500) \\ + (16.25)(350 + 1200 + 1200 + 450) \\ + (0.25)(400 + 1500 + 1500 + 400) \end{bmatrix} = 7.814 \text{ m}$$

$$e_x = x' - \frac{B}{2} = 7.814 - 8.25 = -0.435 \text{ m} \approx -0.44 \text{ m}$$

Hence, the resultant line of action is located to the left of the center of the mat. So $M_y = (11{,}000)(0.44) = 4840$ kN-m. Similarly

$$M_x = Qe_y; \quad e_y = y' - \frac{L}{2}$$

$$y' = \frac{Q_1y_1' + Q_2y_2' + Q_3y_3' + \cdots}{Q}$$

$$= \frac{1}{11{,}000}\begin{bmatrix} (0.25)(400 + 500 + 350) + (7.25)(1500 + 1500 + 1200) \\ +(14.25)(1500 + 1500 + 1200) + (21.25)(400 + 500 + 450) \end{bmatrix}$$

$$= 10.85 \text{ m}$$

$$e_y = y' - \frac{L}{2} = 10.85 - 10.75 = 0.1\text{m}$$

The location of the line of action of the resultant column loads is shown in Figure 6.15.

$M_x = (11{,}000)(0.1) = 1100$ kN-m. So

$$q = \frac{11{,}000}{354.75} \pm \frac{4840x}{8050} \pm \frac{1100y}{13{,}665} = 31.0 \pm 0.6x \pm 0.08y(\text{ kN/m}^2)$$

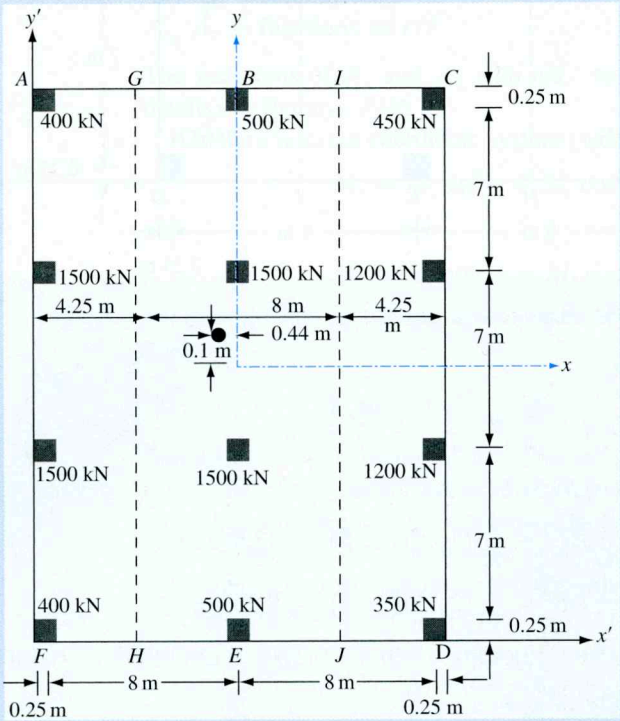


Figure 6.15

Therefore,

At *A*: $q = 31.0 + (0.6)(8.25) + (0.08)(10.75) = \mathbf{36.81\ kN/m^2}$
At *B*: $q = 31.0 + (0.6)(0) + (0.08)(10.75) = \mathbf{31.86\ kN/m^2}$
At *C*: $q = 31.0 - (0.6)(8.25) + (0.08)(10.75) = \mathbf{26.91\ kN/m^2}$
At *D*: $q = 31.0 - (0.6)(8.25) - (0.08)(10.75) = \mathbf{25.19\ kN/m^2}$
At *E*: $q = 31.0 + (0.6)(0) - (0.08)(10.75) = \mathbf{30.14\ kN/m^2}$
At *F*: $q = 31.0 + (0.6)(8.25) - (0.08)(10.75) = \mathbf{35.09\ kN/m^2}$ ■

Example 6.6

Divide the mat shown in Figure 6.14 into three strips, such as *AGHF* ($B_1 = 4.25$ m), *GIJH* ($B_1 = 8$ m), and *ICDJ* ($B_1 = 4.25$ m). Use the result of Example 6.5, and determine the reinforcement requirements in the y direction. Here, $f_c' = 20.7\ \text{MN/m}^2$, $f_y = 413.7\ \text{MN/m}^2$, and the load factor is 1.7.

Solution

Determination of Shear and Moment Diagrams for Strips:

Strip *AGHF*:

$$\text{Average soil pressure} = q_{av} = q_{(\text{at } A)} + q_{(\text{at } F)} = \frac{36.81 + 35.09}{2} = 35.95\ \text{kN/m}^2$$

$$\text{Total soil reaction} = q_{av}B_1L = (35.95)(4.25)(21.50) = 3285\ \text{kN}$$

$$\text{Average load} = \frac{\text{load due to soil reaction} + \text{column loads}}{2} = \frac{3285 + 3800}{2} = 3542.5\ \text{kN}$$

So, modified average soil pressure,

$$q_{av(\text{modified})} = q_{av}\left(\frac{3542.5}{3285}\right) = (35.95)\left(\frac{3542.5}{3285}\right) = 38.768\ \text{kN/m}^2$$

The column loads can be modified in a similar manner by multiplying factor

$$F = \frac{3542.5}{3800} = 0.9322$$

Figure 6.16 shows the loading on the strip and corresponding shear and moment diagrams. Note that the column loads shown in this figure have been multiplied by $F = 0.9322$. Also the load per unit length of the beam is equal to $B_1 q_{av(\text{modified})} = (4.25)(38.768) = 164.76$ kN/m.

Strip *GIJH*: In a similar manner,

$$q_{av} = \frac{q_{(\text{at } B)} + q_{(\text{at } E)}}{2} = \frac{31.86 + 30.14}{2} = 31.0\ \text{kN/m}^2$$

$$\text{Total soil reaction} = (31)(8)(21.5) = 5332\ \text{kN}$$
$$\text{Total column load} = 4000\ \text{kN}$$