STRUCTURAL STEEL DESIGN

FIFTH EDITION

Jack C. McCormac

Stephen F. Csernak

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Preface

This textbook has been prepared with the hope that its readers will, as have so many engineers in the past, become interested in structural steel design and want to maintain and increase their knowledge on the subject throughout their careers in the engineering and construction industries. The material was prepared primarily for an introductory course in the junior or senior year but the last several chapters may be used for a graduate class. The authors have assumed that the student has previously taken introductory courses in mechanics of materials and structural analysis.

The authors' major objective in preparing this new edition was to update the text to conform to both the American Institute of Steel Construction (AISC) 2010 Specification for Structural Steel Buildings and the 14th edition of the AISC Steel Construction Manual published in 2011.

WHAT'S NEW IN THIS EDITION

Several changes to the text were made to the textbook in this edition:

1. End of chapter Problems for Solution have been added for Chapter 1 of the textbook.
2. The load factors and load combinations defined in Chapter 2 of the textbook and used throughout the book in example problems and end of chapter problems for solution have been revised to meet those given in the ASCE 7-10 and Part 2 of the AISC Steel Construction Manual.
3. The classification of compression sections for local buckling defined in Chapter 5 of the textbook has been revised to the new definition given in Section B4.1 of the new AISC Specification. For compression, sections are now classified as *non-slender* element or *slender* element sections.
4. The AISC Specification provides several methods to deal with stability analysis and the design of beam-columns. In Chapter 7 of the textbook, the *Effective Length Method (ELM)* is still used, though a brief introduction to the *Direct Analysis Method (DM)* has been added. A more comprehensive discussion of the DM is reserved for Chapter 11 of the text.
5. In Chapter 11 of the textbook, both the *Direct Analysis Method* and the *Effective Length Method* are presented for the analysis and design of beam-columns. This is to address the fact that the presentation of the *Direct Analysis Method* was moved from an appendix to Chapter C of the new AISC Specification while the *Effective Length Method* moved from Chapter C to Appendix 7.
6. Most of the end of chapter *Problems for Solution* for Chapters 2 through 11 have been revised. For Chapters 12 through 18 about half the problems have been revised.
7. Various photos were updated throughout the textbook.

The authors would like to express appreciation to Dr. Bryant G. Nielson of Clemson University for his assistance in developing the changes to this manuscript and to Sara Elise Roberts, former Clemson University graduate student, for her assistance in the review of the end of chapter problems and their solutions. In addition, the American Institute of Steel Construction was very helpful in providing advance copies of the AISC Specification and Steel Construction Manual revisions. Finally, we would like to thank our families for their encouragement and support in the revising of the manuscript of this textbook.

We also thank the reviewers and users of the previous editions of this book for their suggestions, corrections, and criticisms. We welcome any comments on this edition.

Jack C. McCormac, P.E.
Stephen F. Csernak, P.E.

Contents

CHAPTER 1

Introduction to Structural Steel Design

1.1 ADVANTAGES OF STEEL AS A STRUCTURAL MATERIAL

A person traveling in the United States might quite understandably decide that steel is the perfect structural material. He or she would see an endless number of steel bridges, buildings, towers, and other structures. After seeing these numerous steel structures, this traveler might be surprised to learn that steel was not economically made in the United States until late in the nineteenth century, and the first wide-flange beams were not rolled until 1908.

The assumption of the perfection of this metal, perhaps the most versatile of structural materials, would appear to be even more reasonable when its great strength, light weight, ease of fabrication, and many other desirable properties are considered. These and other advantages of structural steel are discussed in detail in the paragraphs that follow.

1.1.1 High Strength

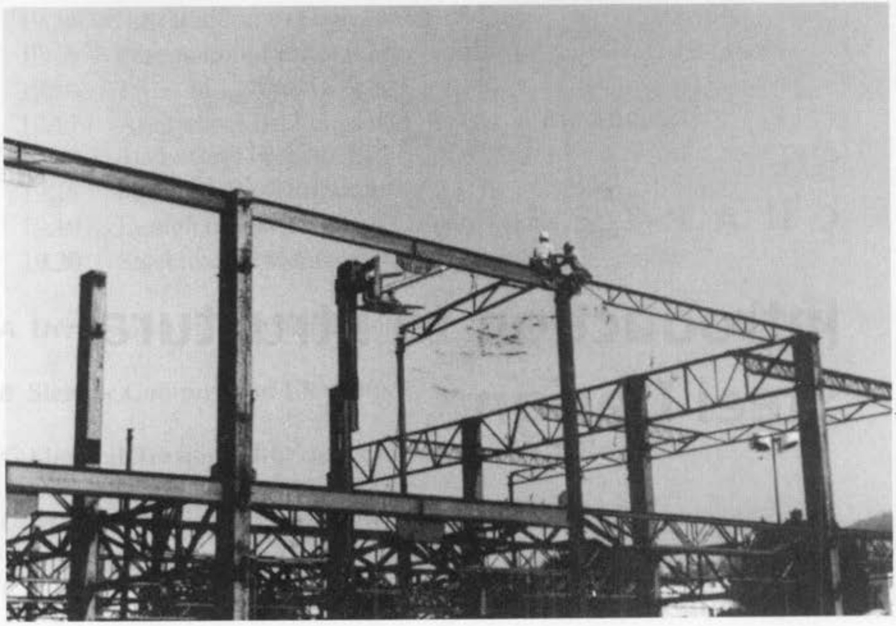
The high strength of steel per unit of weight means that the weight of structures will be small. This fact is of great importance for long-span bridges, tall buildings, and structures situated on poor foundations.

1.1.2 Uniformity

The properties of steel do not change appreciably with time, as do those of a reinforced-concrete structure.

1.1.3 Elasticity

Steel behaves closer to design assumptions than most materials because it follows Hooke's law up to fairly high stresses. The moments of inertia of a steel structure can be accurately calculated, while the values obtained for a reinforced-concrete structure are rather indefinite.



Erection of steel joists. (Courtesy of Vulcraft.)

1.1.4 Permanence

Steel frames that are properly maintained will last indefinitely. Research on some of the newer steels indicates that under certain conditions no painting maintenance whatsoever will be required.

1.1.5 Ductility

The property of a material by which it can withstand extensive deformation without failure under high tensile stresses is its *ductility*. When a *mild* or *low-carbon* structural steel member is being tested in tension, a considerable reduction in cross section and a large amount of elongation will occur at the point of failure before the actual fracture occurs. A material that does not have this property is generally unacceptable and is probably hard and brittle, and it might break if subjected to a sudden shock.

In structural members under normal loads, high stress concentrations develop at various points. The ductile nature of the usual structural steels enables them to yield locally at those points, thus preventing premature failures. A further advantage of ductile structures is that when overloaded, their large deflections give visible evidence of impending failure (sometimes jokingly referred to as "running time").

1.1.6 Toughness

Structural steels are tough—that is, they have both strength and ductility. A steel member loaded until it has large deformations will still be able to withstand large forces. This is a very important characteristic, because it means that steel members can be subjected to large deformations during fabrication and erection without fracture—thus allowing them to be bent, hammered, and sheared, and to have holes punched in them without visible damage. The ability of a material to absorb energy in large amounts is called *toughness*.

1.1.7 Additions to Existing Structures

Steel structures are quite well suited to having additions made to them. New bays or even entire new wings can be added to existing steel frame buildings, and steel bridges may often be widened.

1.1.8 Miscellaneous

Several other important advantages of structural steel are as follows: (a) ability to be fastened together by several simple connection devices, including welds and bolts; (b) adaptation to prefabrication; (c) speed of erection; (d) ability to be rolled into a wide variety of sizes and shapes, as described in Section 1.4 of this chapter; (e) possible reuse after a structure is disassembled; and (f) scrap value, even though not reusable in its existing form. Steel is the ultimate recyclable material.

1.2 DISADVANTAGES OF STEEL AS A STRUCTURAL MATERIAL

In general, steel has the following disadvantages:

1.2.1 Corrosion

Most steels are susceptible to corrosion when freely exposed to air and water, and therefore must be painted periodically. The use of weathering steels, however, in suitable applications tends to eliminate this cost.

Though weathering steels can be quite effective in certain situations for limiting corrosion, there are many cases where their use is not feasible. In some of these situations, corrosion may be a real problem. For instance, corrosion-fatigue failures can occur where steel members are subject to cyclic stresses and corrosive environments. The fatigue strength of steel members can be appreciably reduced when the members are used in aggressive chemical environments and subject to cyclic loads.

The reader should note that steels are available in which copper is used as an anti-corrosion component. The copper is usually absorbed during the steelmaking process.

1.2.2 Fireproofing Costs

Although structural members are incombustible, their strength is tremendously reduced at temperatures commonly reached in fires when the other materials in a building burn. Many disastrous fires have occurred in empty buildings where the only fuel for the fires was the buildings themselves. Furthermore, steel is an excellent heat conductor—nonfireproofed steel members may transmit enough heat from a burning section or compartment of a building to ignite materials with which they are in contact in adjoining sections of the building. As a result, the steel frame of a building may have to be protected by materials with certain insulating characteristics, and the building may have to include a sprinkler system if it is to meet the building code requirements of the locality in question.

1.2.3 Susceptibility to Buckling

As the length and slenderness of a compression member is increased, its danger of buckling increases. For most structures, the use of steel columns is very economical because of their high strength-to-weight ratios. Occasionally, however, some additional steel is needed to stiffen them so they will not buckle. This tends to reduce their economy.

1.2.4 Fatigue

Another undesirable property of steel is that its strength may be reduced if it is subjected to a large number of stress reversals or even to a large number of variations of tensile stress. (Fatigue problems occur only when tension is involved.) The present practice is to reduce the estimations of strength of such members if it is anticipated that they will have more than a prescribed number of cycles of stress variation.

1.2.5 Brittle Fracture

Under certain conditions steel may lose its ductility, and brittle fracture may occur at places of stress concentration. Fatigue-type loadings and very low temperatures aggravate the situation. Triaxial stress conditions can also lead to brittle fracture.

1.3 EARLY USES OF IRON AND STEEL

Although the first metal used by human beings was probably some type of copper alloy such as bronze (made with copper, tin, and perhaps some other additives), the most important metal developments throughout history have occurred in the manufacture and use of iron and its famous alloy called steel. Today, iron and steel make up nearly 95 percent of all the tonnage of metal produced in the world.[1]

Despite diligent efforts for many decades, archaeologists have been unable to discover when iron was first used. They did find an iron dagger and an iron bracelet in the Great Pyramid in Egypt, which they claim had been there undisturbed for at least 5000 years. The use of iron has had a great influence on the course of civilization since the earliest times and may very well continue to do so in the centuries ahead. Since the beginning of the Iron Age in about 1000 BC, the progress of civilization in peace and war has been heavily dependent on what people have been able to make with iron. On many occasions its use has decidedly affected the outcome of military engagements. For instance, in 490 BC in Greece at the Battle of Marathon, the greatly outnumbered Athenians killed 6400 Persians and lost only 192 of their own men. Each of the victors wore 57 pounds of iron armor in the battle. (This was the battle from which the runner Pheidippides ran the approximately 25 miles to Athens and died while shouting news of the victory.) This victory supposedly saved Greek civilization for many years.

[1]American Iron and Steel Institute, *The Making of Steel* (Washington, DC, not dated), p. 6.

The mooring mast of the Empire State Building, New York City. (Courtesy of Getty Images/Hulton Archive Photos.)

According to the classic theory concerning the first production of iron in the world, there was once a great forest fire on Mount Ida in Ancient Troy (now Turkey) near the Aegean Sea. The land surface reportedly had a rich content of iron, and the heat of the fire is said to have produced a rather crude form of iron that could be hammered into various shapes. Many historians believe, however, that human beings first learned to use iron which fell to the earth in the form of meteorites. Frequently, the iron in meteorites is combined with nickel to produce a harder metal. Perhaps, early human beings were able to hammer and chip this material into crude tools and weapons.

Steel is defined as a combination of iron and a small amount of carbon, usually less than 1 percent. It also contains small percentages of some other elements. Although some steel has been made for at least 2000–3000 years, there was really no economical production method available until the middle of the nineteenth century.

The first steel almost certainly was obtained when the other elements necessary for producing it were accidentally present when iron was heated. As the years went by, steel probably was made by heating iron in contact with charcoal. The surface of the iron absorbed some carbon from the charcoal, which was then hammered into the hot iron. Repeating this process several times resulted in a case-hardened exterior of steel. In this way the famous swords of Toledo and Damascus were produced.

The first large volume process for producing steel was named after Sir Henry Bessemer of England. He received an English patent for his process in 1855, but his efforts to obtain a U.S. patent for the process in 1856 were unsuccessful, because it was shown that William Kelly of Eddyville, Kentucky, had made steel by the same process seven years before Bessemer applied for his English patent. Although Kelly was given the patent, the name Bessemer was used for the process.[2]

Kelly and Bessemer learned that a blast of air through molten iron burned out most of the impurities in the metal. Unfortunately, at the same time, the blow eliminated some desirable elements such as carbon and manganese. It was later learned that these needed elements could be restored by adding spiegeleisen, which is an alloy of iron, carbon, and manganese. It was further learned that the addition of limestone in the converter resulted in the removal of the phosphorus and most of the sulfur.

Before the Bessemer process was developed, steel was an expensive alloy used primarily for making knives, forks, spoons, and certain types of cutting tools. The Bessemer process reduced production costs by at least 80 percent and allowed, for the first time, production of large quantities of steel.

The Bessemer converter was commonly used in the United States until the beginning of the twentieth century, but since that time it has been replaced with better methods, such as the open-hearth process and the basic oxygen process.

As a result of the Bessemer process, structural carbon steel could be produced in quantity by 1870, and by 1890, steel had become the principal structural metal used in the United States.

Today, most of the structural steel shapes and plates produced in the United States are made by melting scrap steel. This scrap steel is obtained from junk cars and scrapped structural shapes, as well as from discarded refrigerators, motors, typewriters, bed springs, and other similar items. The molten steel is poured into molds that have approximately the final shapes of the members. The resulting sections, which are run through a series of rollers to squeeze them into their final shapes, have better surfaces and fewer internal or residual stresses than newly made steel.

The shapes may be further processed by cold rolling, by applying various coatings, and perhaps by the process of *annealing*. This is the process by which the steel is heated to an intermediate temperature range (say, 1300–1400°F), held at that temperature for several hours, and then allowed to slowly cool to room temperature. Annealing results in steel with less hardness and brittleness, but greater ductility.

The term **wrought iron** refers to iron with a very low carbon content ($\leq$ 0.15 percent), while iron with a very high carbon content ($\geq$ 2 percent) is referred to as **cast iron**. Steel falls in between cast iron and wrought iron and has carbon contents in the range of 0.15 percent to 1.7 percent (as described in Section 1.8 of this chapter).

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[2]American Iron and Steel Institute, *Steel 76* (Washington, DC, 1976), pp. 5–11.

The first use of metal for a sizable structure occurred in England in Shropshire (about 140 miles northwest of London) in 1779, when cast iron was used for the construction of the 100-ft Coalbrookdale Arch Bridge over the River Severn. It is said that this bridge (which still stands) was a turning point in engineering history because it changed the course of the Industrial Revolution by introducing iron as a structural material. This iron was supposedly four times as strong as stone and thirty times as strong as wood.[3]

A number of other cast-iron bridges were constructed in the following decades, but soon after 1840 the more malleable wrought iron began to replace cast iron. The development of the Bessemer process and subsequent advances, such as the open-hearth process, permitted the manufacture of steel at competitive prices. This encouraged the beginning of the almost unbelievable developments of the last 120 years with structural steel.

1.4 STEEL SECTIONS

The first structural shapes made in the United States were angle irons rolled in 1819. I-shaped steel sections were first rolled in the United States in 1884, and the first skeleton frame structure (the Home Insurance Company Building in Chicago) was erected that same year. Credit for inventing the "skyscraper" is usually given to engineer William LeBaron Jenny, who planned the building, apparently during a bricklayers' strike. Prior to this time, tall buildings in the United States were constructed with load-bearing brick walls that were several feet thick.

For the exterior walls of the 10-story building, Jenny used cast-iron columns encased in brick. The beams for the lower six floors were made from wrought iron, while structural steel beams were used for the upper floors. The first building completely framed with structural steel was the second Rand-McNally building, completed in Chicago in 1890.

An important feature of the 985-ft wrought-iron Eiffel tower constructed in 1889 was the use of mechanically operated passenger elevators. The availability of these machines, along with Jenny's skeleton frame idea, led to the construction of thousands of high-rise buildings throughout the world during the last century.

During these early years, the various mills rolled their own individual shapes and published catalogs providing the dimensions, weight, and other properties of the shapes. In 1896, the Association of American Steel Manufacturers (now the American Iron and Steel Institute, or AISI) made the first efforts to standardize shapes. Today, nearly all structural shapes are standardized, though their exact dimensions may vary just a little from mill to mill.[4]

Structural steel can be economically rolled into a wide variety of shapes and sizes without appreciably changing its physical properties. Usually, the most desirable members are those with large moments of inertia in proportion to their areas. The **I**, **T**, and **C** shapes, so commonly used, fall into this class.

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[3]M. H. Sawyer, "World's First Iron Bridge," *Civil Engineering* (New York: ASCE, December 1979), pp. 46–49.
[4]W. McGuire, *Steel Structures* (Englewood Cliffs, NJ: Prentice-Hall, 1968), pp. 19–21.

Pedestrian bridge for North Carolina Cancer Hospital, Chapel Hill, NC. (Courtesy of CMC South Carolina Steel.)

Steel sections are usually designated by the shapes of their cross sections. As examples, there are angles, tees, zees, and plates. It is necessary, however, to make a definite distinction between American standard beams (called *S beams*) and wide-flange beams (called *W beams*), as they are both I-shaped. The inner surface of the flange of a W section is either parallel to the outer surface or nearly so, with a maximum slope of 1 to 20 on the inner surface, depending on the manufacturer.

The S beams, which were the first beam sections rolled in America, have a slope on their inside flange surfaces of 1 to 6. It might be noted that the constant (or nearly constant) thickness of W-beam flanges compared with the tapered S-beam flanges may facilitate connections. Wide-flange beams comprise nearly 50 percent of the tonnage of structural steel shapes rolled today. The W and S sections are shown in Fig. 1.1, together with several other familiar steel sections. The uses of these various shapes will be discussed in detail in the chapters to follow.

Constant reference is made throughout this book to the 14th edition of the **Steel Construction Manual**, published by the American Institute of Steel Construction (AISC). This manual, which provides detailed information for structural steel shapes, is referred to hereafter as "the AISC Manual," "the Steel Manual," or simply, "the Manual." It is based on the 2010 **Specification for Structural Steel Buildings** (ANSI/AISC 360-10) (hereafter, "the AISC Specification"), published by the AISC on June 22, 2010.

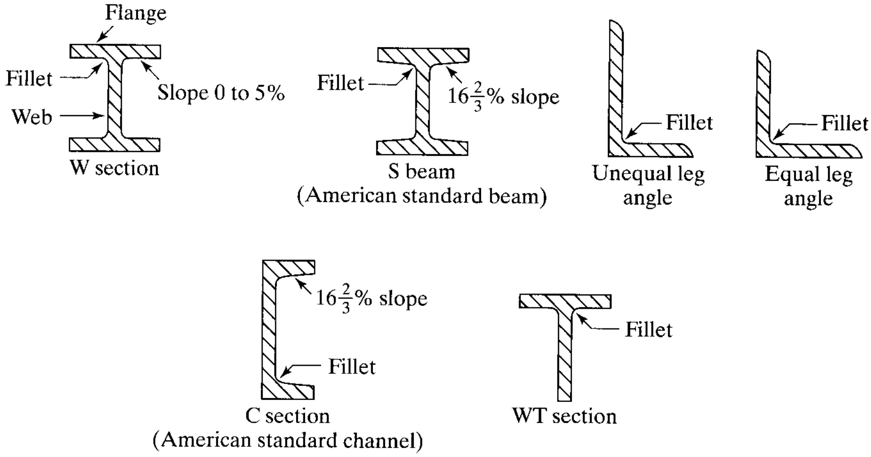
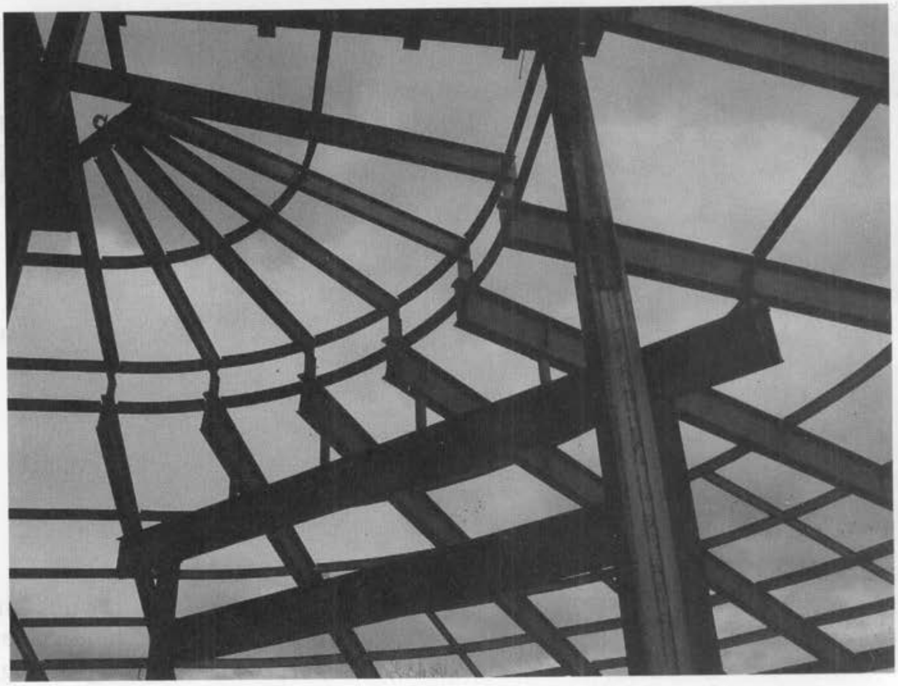


FIGURE 1.1

Rolled-steel shapes.

Structural shapes are identified by a certain system described in the Manual for use in drawings, specifications, and designs. This system is standardized so that all steel mills can use the same identification for purposes of ordering, billing, etc. In addition, so much work is handled today with computers and other automated equipment that it is necessary to have a letter-and-number system which can be printed out with a standard keyboard (as opposed to the old system where certain symbols were used for angles, channels, etc.). Examples of this identification system are as follows:

1. A W27 × 114 is a W section approximately 27 in deep, weighing 114 lb/ft.
2. An S12 × 35 is an S section 12 in deep, weighing 35 lb/ft.
3. An HP12 × 74 is a bearing pile section approximately 12 in deep, weighing 74 lb/ft. Bearing piles are made with the regular W rolls, but with thicker webs to provide better resistance to the impact of pile driving. The width and depth of these sections are approximately equal, and the flanges and webs have equal or almost equal thickness.
4. An M8 × 6.5 is a miscellaneous section 8 in deep, weighing 6.5 lb/ft. It is one of a group of doubly symmetrical H-shaped members that cannot by dimensions be classified as a W, S, or HP section, as the slope of their inner flanges is other than 16 2/3 percent.
5. A C10 × 30 is a channel 10 in deep, weighing 30 lb/ft.
6. An MC18 × 58 is a miscellaneous channel 18 in deep, weighing 58 lb/ft, which cannot be classified as a C shape because of its dimensions.
7. An HSS14 × 10 × 5/8 is a rectangular hollow structural section 14 in deep, 10 in wide, with a 5/8-in wall thickness. It weighs 93.10 lb/ft. Square and round HSS sections are also available.
8. An L6 × 6 × 1/2 is an equal leg angle, each leg being 6 in long and 1/2 in thick.



Roof framing for Glen Oaks School, Bellerose, NY. (Courtesy of CMC South Carolina Steel.)

9. A WT18 × 151 is a tee obtained by splitting a W36 × 302. This type of section is known as a structural tee.
10. Rectangular steel sections are classified as wide *plates* or narrow *bars*.

The only differences between bars and plates are their sizes and production procedures. Historically, flat pieces have been called bars if they are 8 in or less in width. They are plates if wider than 8 in. Tables 1-29, 2-3, and 2-5 in the AISC Manual provide information on bars and plates. Bar and plate thicknesses are usually specified to the nearest 1/16 in for thicknesses less than 3/8 in, to the nearest 1/8 in for thicknesses between 3/8 in and 1 in, and to the nearest 1/4 in for thicknesses greater than 1 in. A plate is usually designated by its thickness, width, and length, in that order; for example, a PL1/2 × 10 × 1 ft 4 in is 1/2 in thick, 10 in wide, and 16 in long. Actually, the term **plate** is almost universally used today, whether a member is fabricated from plate or bar stock. Sheet and strip are usually thinner than bars and plates.

The student should refer to the Steel Manual for information concerning other shapes. Detailed information on these and other sections will be presented herein as needed.

In Part 1 of the Manual, the dimensions and properties of W, S, C, and other shapes are tabulated. The dimensions of the members are given in decimal form (for the use of designers) and in fractions to the nearest sixteenth of an inch (for the use of craftsmen and steel detailers or drafters). Also provided for the use of designers are such items as moments of inertia, section moduli, radii of gyration, and other cross-sectional properties discussed later in this text.

There are variations present in any manufacturing process, and the steel industry is certainly no exception. As a result, the cross-sectional dimensions of steel members may vary somewhat from the values specified in the Manual. Maximum tolerances for the rolling of steel shapes are prescribed by the American Society for Testing and Materials (ASTM) A6 Specification and are presented in Tables 1-22 to 1-28 in the Manual. As a result, calculations can be made on the basis of the properties given in the Manual, regardless of the manufacturer.

Some steel sections listed in the Manual are available in the United States from only one or two steel producers and thus, on occasion, may be difficult to obtain promptly. Accordingly, when specifying sections, the designer would be wise to contact a steel fabricator for a list of sections readily available.

Through the years, there have been changes in the sizes of steel sections. For instance, there may be insufficient demand to continue rolling a certain shape; an existing shape may be dropped because a similar size, but more efficient, shape has been developed, and so forth. Occasionally, designers may need to know the properties of one of the discontinued shapes no longer listed in the latest edition of the Manual or in other tables normally available to them.

For example, it may be desired to add another floor to an existing building that was constructed with shapes no longer rolled. In 1953, the AISC published a book entitled *Iron and Steel Beams 1873 to 1952*, which provides a complete listing of iron and steel beams and their properties rolled in the United States during that period. An up-to-date edition of this book is now available. It is *AISC Design Guide 15* and covers properties of steel shapes produced from 1887 to 2000.[5] There will undoubtedly be many more shape changes in the future. For this reason, the wise structural designer should carefully preserve old editions of the Manual so as to have them available when the older information is needed.

1.5 METRIC UNITS

Almost all of the examples and homework problems presented in this book make use of U.S. customary units. The author, however, feels that today's designer must be able to perform his or her work in either customary or metric units.

The problem of working with metric units when performing structural steel design in the United States has almost been eliminated by the AISC. Almost all of their equations are written in a form applicable to both systems. In addition, the metric equivalents of the standard U.S. shapes are provided in Section 17 of the Manual. For instance, a W36 × 302 section is shown there as a W920 × 449, where the 920 is mm and the 449 is kg/m.

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[5] R. L. Brockenbrough, *AISC Rehabilitation and Retrofit Guide: A Reference for Historic Shapes and Specifications* (Chicago, AISC, 2002).

Mariners, Ballpark, Seattle, WA. (Courtesy of Trade ARBED.)

1.6 COLD-FORMED LIGHT-GAGE STEEL SHAPES

In addition to the hot-rolled steel shapes discussed in the previous section, there are some cold-formed steel shapes. These are made by bending thin sheets of carbon or low-alloy steels into almost any desired cross section, such as the ones shown in Fig. 1.2.[6] These shapes—which may be used for light members in roofs, floors, and walls—vary in thickness from about 0.01 in up to about 0.25 in. The thinner shapes are most often used for some structural panels.

Though cold-working does reduce ductility somewhat, it causes some strength increases. Under certain conditions, design specifications will permit the use of these higher strengths.

Concrete floor slabs are very often cast on formed steel decks that serve as economical forms for the wet concrete and are left in place after the concrete hardens. Several types of decking are available, some of which are shown in Fig. 1.3. The sections

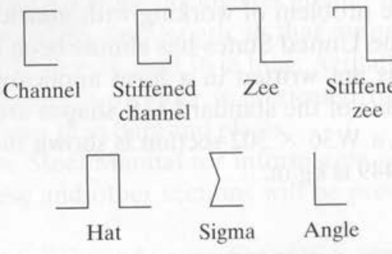

FIGURE 1.2
Cold-formed shapes.

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[6] *Cold-Formed Steel Design Manual* (Washington, DC: American Iron and Steel Institute, 2002).

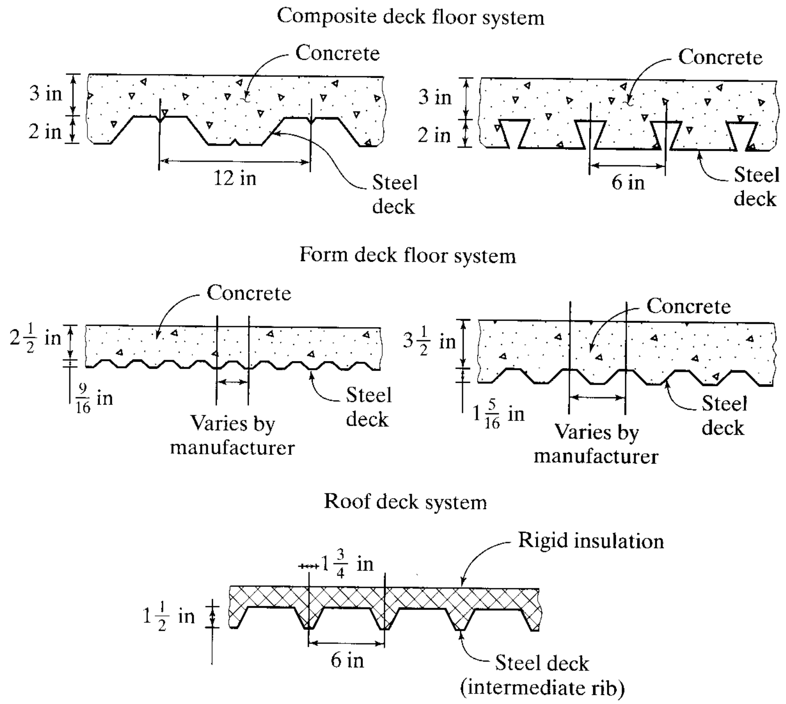


FIGURE 1.3

Some types of steel decks.

with the deeper cells have the useful feature that electrical and mechanical conduits can be placed in them. The use of steel decks for floor slabs is discussed in Chapter 16 of this text. There, composite construction is presented. With such construction steel beams are made composite with concrete slabs by providing for shear transfer between the two so they will act together as a unit.

1.7 STRESS–STRAIN RELATIONSHIPS IN STRUCTURAL STEEL

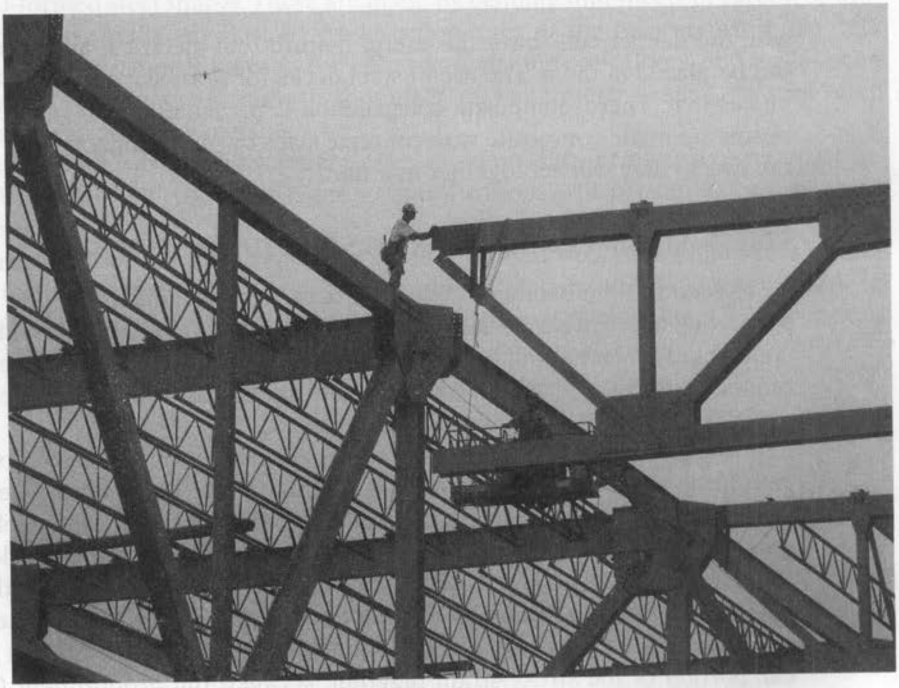
To understand the behavior of steel structures, an engineer must be familiar with the properties of steel. Stress–strain diagrams present valuable information necessary to understand how steel will behave in a given situation. Satisfactory steel design methods cannot be developed unless complete information is available concerning the stress–strain relationships of the material being used.

If a piece of ductile structural steel is subjected to a tensile force, it will begin to elongate. If the tensile force is increased at a constant rate, the amount of elongation will increase linearly within certain limits. In other words, elongation will double when the stress goes from 6000 to 12,000 psi (pounds per square inch). When the tensile stress reaches a value roughly equal to three-fourths of the ultimate strength of the steel, the elongation will begin to increase at a greater rate without a corresponding increase in the stress.

The largest stress for which Hooke's law applies, or the highest point on the linear portion of the stress–strain diagram, is called the *proportional limit.* The largest stress that a material can withstand without being permanently deformed is called the *elastic limit*. This value is seldom actually measured and for most engineering materials, including structural steel, is synonymous with the proportional limit. For this reason, the term *proportional elastic limit* is sometimes used.

The stress at which there is a significant increase in the elongation, or strain, without a corresponding increase in stress is said to be the *yield* stress. It is the first point on the stress–strain diagram where a tangent to the curve is horizontal. The yield stress is probably the most important property of steel to the designer, as so many design procedures are based on this value. Beyond the yield stress there is a range in which a considerable increase in strain occurs without increase in stress. The strain that occurs before the yield stress is referred to as the *elastic strain;* the strain that occurs after the yield stress, with no increase in stress, is referred to as the *plastic strain*. Plastic strains are usually from 10 to 15 times as large as the elastic strains.

Yielding of steel without stress increase may be thought to be a severe disadvantage, when in actuality it is a very useful characteristic. It has often performed the wonderful service of preventing failure due to omissions or mistakes on the designer's part. Should the stress at one point in a ductile steel structure reach the yield point, that part of the structure will yield locally without stress increase, thus preventing premature failure. This ductility allows the stresses in a steel structure to be redistributed. Another way of describing this phenomenon is to say that very high stresses caused by fabrication, erection, or loading will tend to equalize themselves. It might also be said that a



Erection of roof truss, North Charleston, SC. (Courtesy of CMC South Carolina Steel.)

steel structure has a reserve of plastic strain that enables it to resist overloads and sudden shocks. If it did not have this ability, it might suddenly fracture, like glass or other vitreous substances.

Following the plastic strain, there is a range in which additional stress is necessary to produce additional strain. This is called *strain-hardening*. This portion of the diagram is not too important to today's designer, because the strains are so large. A familiar stress–strain diagram for mild or low-carbon structural steel is shown in Fig. 1.4. Only the initial part of the curve is shown here because of the great deformation that occurs before failure. At failure in the mild steels, the total strains are from 150 to 200 times the elastic strains. The curve will actually continue up to its maximum stress value and then "tail off" before failure. A sharp reduction in the cross section of the member (called *necking*) takes place just before the member fractures.

The stress–strain curve of Fig. 1.4 is typical of the usual ductile structural steel and is assumed to be the same for members in tension or compression. (The compression members must be stocky, because slender compression members subjected to compression loads tend to buckle laterally, and their properties are greatly affected by the bending moments so produced.) The shape of the diagram varies with the speed of loading, the type of steel, and the temperature. One such variation is shown in the figure by the dotted line marked *upper yield*.

This shape stress–strain curve is the result when a mild steel has the load applied rapidly, while the *lower yield* is the case for slow loading.

Figure 1.5 shows typical stress–strain curves for several different yield stress steels.

You should note that the stress–strain diagrams of Figs. 1.4 and 1.5 were prepared for a mild steel at room temperature. During welding operations and during fires, structural steel members may be subjected to very high temperatures. Stress–strain diagrams prepared for steels with temperatures above 200°F will be more rounded and nonlinear and will not exhibit well-defined yield points. Steels

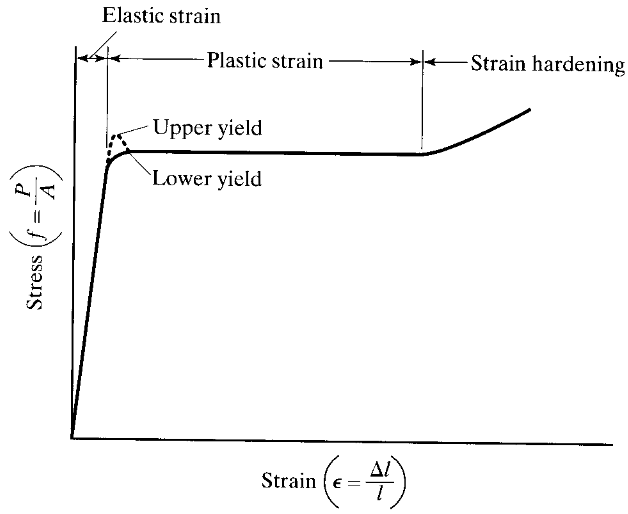
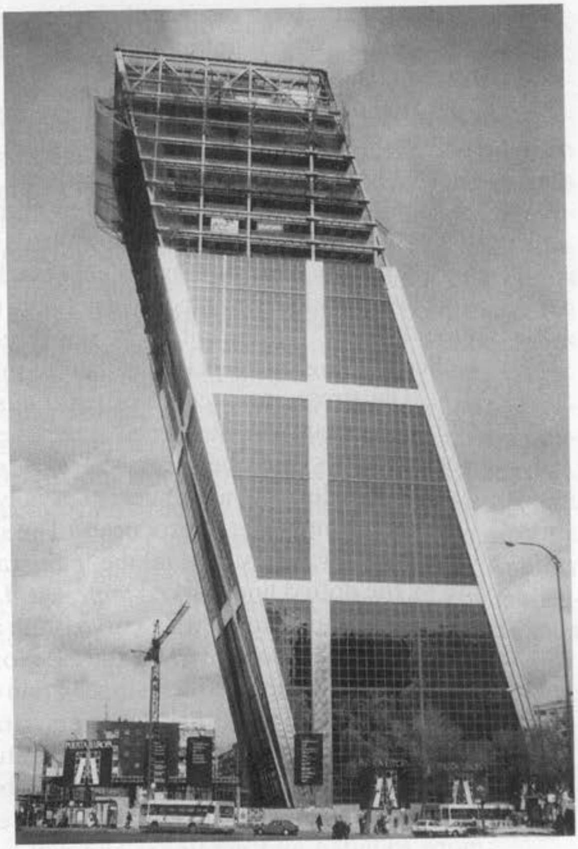


FIGURE 1.4

Typical stress–strain diagram for a mild or low-carbon structural steel at room temperature.



Puerta Europa, Madrid, Spain.
(Courtesy of Trade ARBED.)

(particularly those with high carbon contents) may actually increase a little in tensile strength as they are heated to a temperature of about 700°F. As temperatures are raised into the 800°-to-1000°F range, strengths are drastically reduced, and at 1200°F little strength is left.

Figure 1.6 shows the variation of yield strengths for several grades of steel as their temperatures are raised from room temperature up to 1800° to 1900°F. Temperatures of the magnitudes shown can easily be reached in steel members during fires, in localized areas of members when welding is being performed, in members in foundries over open flame, and so on.

When steel sections are cooled below 32°F, their strengths will increase a little, but they will have substantial reductions in ductility and toughness.

A very important property of a structure that has been stressed, but not beyond its yield point, is that it will return to its original length when the loads are removed. Should it be stressed beyond this point, it will return only part of the way back to its original position. This knowledge leads to the possibility of testing an existing structure by loading and unloading. If, after the loads are removed, the structure does not resume its original dimensions, it has been stressed beyond its yield point.

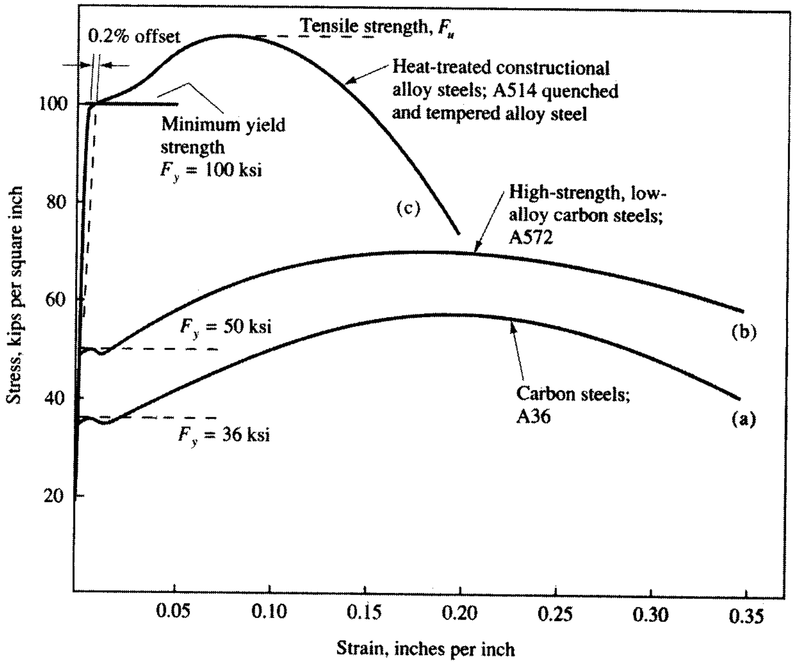


FIGURE 1.5

Typical stress–strain curves. (Based on a figure from Salmon C. G. and J. E. Johnson, *Steel Structures: Design and Behavior, Fourth Edition*. Upper Saddle River, NJ: Prentice Hall, 1996.)

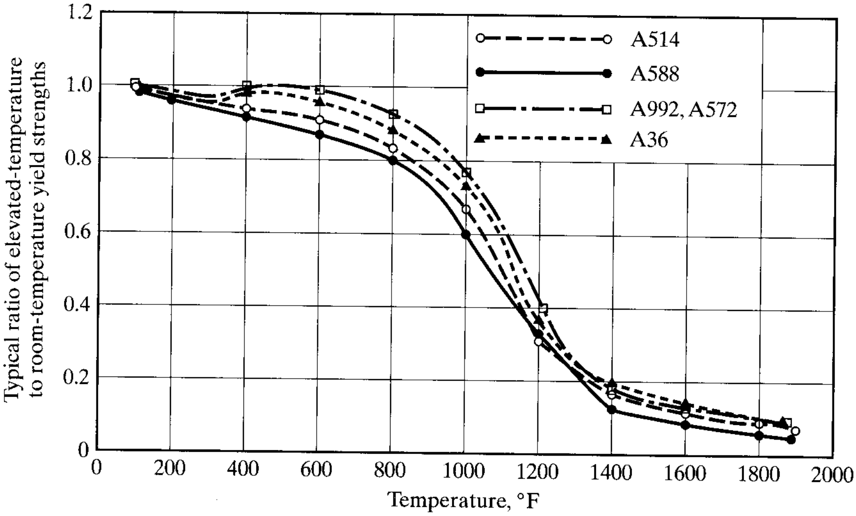


FIGURE 1.6

Effect of temperature on yield strengths.

Steel is an alloy consisting almost entirely of iron (usually over 98 percent). It also contains small quantities of carbon, silicon, manganese, sulfur, phosphorus, and other elements. Carbon is the material that has the greatest effect on the properties of steel. The hardness and strength of steel increase as the carbon content is increased. A 0.01 percent increase in carbon content will cause steel's yield strength to go up about 0.5 kips per square inch (ksi). Unfortunately, however, more carbon will cause steel to be more brittle and will adversely affect its weldability. If the carbon content is reduced, the steel will be softer and more ductile, but also weaker. The addition of such elements as chromium, silicon, and nickel produces steels with considerably higher strengths. Though frequently quite useful, these steels are appreciably more expensive and often are not as easy to fabricate.

A typical stress–strain diagram for a brittle steel is shown in Fig. 1.7. Unfortunately, low ductility, or brittleness, is a property usually associated with high strengths in steels (although not entirely confined to high-strength steels). As it is desirable to have both high strength and ductility, the designer may need to decide between the two extremes or to compromise. A brittle steel may fail suddenly and without warning when over-stressed, and during erection could possibly fail due to the shock of erection procedures.

Brittle steels have a considerable range in which stress is proportional to strain, but do not have clearly defined yield stresses. Yet, to apply many of the formulas given in structural steel design specifications, it is necessary to have definite yield stress values, regardless of whether the steels are ductile or brittle.

If a steel member is strained beyond its elastic limit and then unloaded, it will not return to a condition of zero strain. As it is unloaded, its stress–strain diagram will follow a new path (shown in Fig. 1.7 by the dotted line parallel to the initial straight line). The result is a permanent, or residual, strain.

The line representing the stress–strain ratio for quenched and tempered steels gradually varies from a straight line so that a distinct yield point is not available. For such steels the yield stress is usually defined as the stress at the point of unloading, which corresponds to some arbitrarily defined residual strain (0.002 being the common value). In other words, we increase the strain by a designated amount and from that point draw a line parallel to the straight-line portion of the stress–strain diagram, until

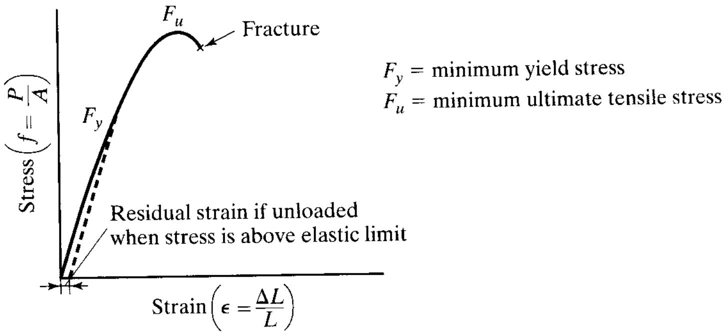


FIGURE 1.7

Typical stress–strain diagram for a brittle steel.

the new line intersects the old. This intersection is the yield stress at that particular strain. If 0.002 is used, the intersection is usually referred to as the yield stress at 0.2 percent offset strain.

1.8 MODERN STRUCTURAL STEELS

The properties of steel can be greatly changed by varying the quantities of carbon present and by adding other elements such as silicon, nickel, manganese, and copper. A steel that has a significant amount of the latter elements is referred to as an *alloy steel*. Although these elements do have a great effect on the properties of steel, the actual quantities of carbon or other alloying elements are quite small. For instance, the carbon content of steel is almost always less than 0.5 percent by weight and is normally from 0.2 to 0.3 percent.

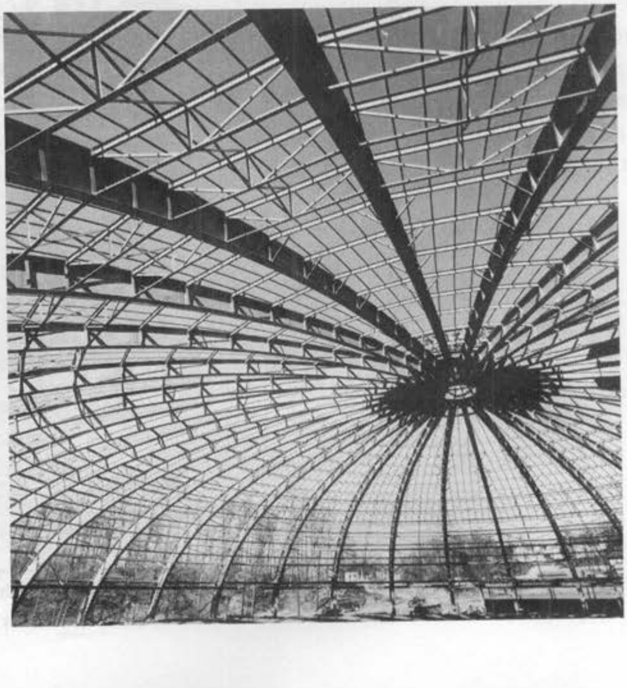
One-half of a 170-ft clear span roof truss for the Athletic and Convention Center, Lehigh University, Bethlehem, PA. (Courtesy of Bethlehem Steel Corporation.)

The chemistry of steel is extremely important in its effect on such properties of the steel as weldability, corrosion resistance, resistance to brittle fracture, and so on. The ASTM specifies the exact maximum percentages of carbon, manganese, silicon, etc., that are permissible for a number of structural steels. Although the physical and mechanical properties of steel sections are primarily determined by their chemical composition, they are also influenced to a certain degree by the rolling process and by their stress history and heat treatment.

In the past few decades, a structural carbon steel designated as A36 and having a minimum yield stress $F_y = 36$ ksi was the commonly used structural steel. More recently, however, most of the structural steel used in the United States is manufactured by melting scrap steel in electric furnaces. With this process, a 50 ksi steel, A992, can be produced and sold at almost the same price as 36 ksi steel.

The 50 ksi steels are the predominant ones in use today. In fact, some of the steel mills charge extra for W sections if they are to consist of A36 steel. On the other hand, 50 ksi angles have on occasion been rather difficult to obtain without special orders to the steel mills. As a result, A36 angles are still frequently used. In addition, 50 ksi plates may cost more than A36 steel.

In recent decades, the engineering and architecture professions have been continually requesting increasingly stronger steels—steels with more corrosion resistance, steels with better welding properties, and various other requirements. Research by the steel industry during this period has supplied several groups of new steels that satisfy many of the demands. Today there are quite a few structural steels designated by the ASTM and included in the AISC Specification.



Steel dome. (Courtesy of Trade ARBED.)

Structural steels are generally grouped into several major ASTM classifications: the carbon steels A36, A53, A500, A501, and A529; the high-strength low-alloy steels A572, A618, A913, and A992; and the corrosion-resistant high-strength low-alloy steels A242, A588, and A847. Considerable information is presented for each of these steels in Part 2 of the Manual. The sections that follow include a few general remarks about these steel classifications.

1.8.1 Carbon Steels

These steels have as their principal strengthening agents carefully controlled quantities of carbon and manganese. Carbon steels have their contents limited to the following maximum percentages: 1.7 percent carbon, 1.65 percent manganese, 0.60 percent silicon, and 0.60 percent copper. These steels are divided into four categories, depending on carbon percentages:

1. Low-carbon steel: < 0.15 percent.
2. Mild steel: 0.15 to 0.29 percent. (The structural carbon steels fall into this category.)
3. Medium-carbon steel: 0.30 to 0.59 percent.
4. High-carbon steel: 0.60 to 1.70 percent.

1.8.2 High-Strength Low-Alloy Steels

There are a large number of high-strength low-alloy steels, and they are included under several ASTM numbers. In addition to containing carbon and manganese, these steels owe their higher strengths and other properties to the addition of one or more alloying agents such as columbium, vanadium, chromium, silicon, copper, and nickel. Included are steels with yield stresses as low as 42 ksi and as high as 70 ksi. These steels generally have much greater atmospheric corrosion resistance than the carbon steels have.

The term *low-alloy* is used arbitrarily to describe steels for which the total of all the alloying elements does not exceed 5 percent of the total composition of the steel.

1.8.3 Atmospheric Corrosion-Resistant High-Strength Low-Alloy Structural Steels

When steels are alloyed with small percentages of copper, they become more corrosion-resistant. When exposed to the atmosphere, the surfaces of these steels oxidize and form a very tightly adherent film (sometimes referred to as a "tightly bound patina" or "a crust of rust"), which prevents further oxidation and thus eliminates the need for painting. After this process takes place (within 18 months to 3 years, depending on the type of exposure—rural, industrial, direct or indirect sunlight, etc.), the steel reaches a deep reddish-brown or black color.

Supposedly, the first steel of this type was developed in 1933 by the U.S. Steel Corporation to provide resistance to the severe corrosive conditions of railroad coal cars.

You can see the many uses that can be made of such a steel, particularly for structures with exposed members which are difficult to paint—bridges, electrical transmission towers, and others. This steel is not considered to be satisfactory for use where it is frequently subject to saltwater sprays or fogs, or continually submerged in water (fresh or salt) or the ground, or where there are severe corrosive industrial fumes. It is also not satisfactory in very dry areas, as in some western parts of the United States. For the patina to form, the steel must be subjected to a wetting and drying cycle. Otherwise, it will continue to look like unpainted steel.

Table 1.1 herein, which is Table 2-4 in the Steel Manual, lists the 12 ASTM steels mentioned earlier in this section, together with their specified minimum yield strengths (F_y) and their specified minimum tensile strengths (F_u). In addition, the right-hand columns of the table provide information regarding the availability of the shapes in the various grades of steels, as well as the preferred grade to use for each of them. The preferred steel in each case is shown with a black box.

You will note by the blackened boxes in the table that A36 is the preferred steel to be used for M, S, HP, C, MC, and L sections, while A992 is the preferred material for the most common shapes, the Ws. The lightly shaded boxes in the table refer to the shapes available in grades of steel other than the preferred grades. Before shapes are specified in these grades, the designer should check on their availability from the steel producers. Finally, the blank, or white, boxes indicate the grades of steel that are not available for certain shapes. Similar information is provided for plates and bars in Table 2-5 of the Steel Manual.

As stated previously, steels may be made stronger by the addition of special alloys. Another factor affecting steel strengths is thickness. The thinner steel is rolled, the stronger it becomes. Thicker members tend to be more brittle, and their slower cooling rates cause the steel to have a coarser microstructure.

Referring back to Table 1.1, you can see that several of the steels listed are available with different yield and tensile stresses with the same ASTM number. For instance, A572 shapes are available with 42, 50, 55, 60, and 65 ksi yield strengths. Next, reading the footnotes in Table 1.1, we note that Grades 60 and 65 steels have the footnote letter "e" by them. This footnote indicates that the only A572 shapes available with these strengths are those thinner ones which have flange thicknesses $\leq$ 2 inches. Similar situations are shown in the table for several other steels, including A992 and A242.

1.9 USES OF HIGH-STRENGTH STEELS

There are indeed other groups of high-strength steels, such as the ultra-high-strength steels that have yield strengths from 160 to 300 ksi. These steels have not been included in the Steel Manual because they have not been assigned ASTM numbers.

It is said that today more than 200 steels exist on the market that provide yield stresses in excess of 36 ksi. The steel industry is now experimenting with steels with yield stresses varying from 200 to 300 ksi, and this may be only the beginning. Many people in the steel industry feel that steels with 500 ksi yield strengths will be made available within a few years. The theoretical binding force between iron atoms has been estimated to be in excess of 4000 ksi.[7]

[7]"L. S. Beedle et al., *Structural Steel Design* (New York: Ronald Press, 1964), p. 44.

TABLE 1.1 Applicable ASTM Specifications for Various Structural Shapes

Steel Typ	ASTM Designatio		F_y Min. Yield Stress (ksi)	F_u Tensile Stress[a] (ksi)	Applicable Shape Series									
												HSS		
					W	M	S	HP	C	MC	L	Rect.	Round	Pipe
Carbon	A36		36	58–80[b]	▒	■	■	▒	■	■	■			
	A53 Gr. B		35	60										■
	A500	Gr. B	42	58									■	
			46	58								■		
		Gr. C	46	62									▒	
			50	62								▒		
	A501	Gr. A	36	58								▒	▒	
		Gr. B	50	70								▒	▒	
	A529[c]	Gr. 50	50	65–100	▒	▒	▒	▒	▒	▒	▒			
		Gr. 55	55	70–100	▒	▒	▒	▒	▒	▒	▒			
High-Strength Low-Alloy	A572	Gr. 42	42	60	▒	▒	▒	▒	▒	▒	▒			
		Gr. 50	50	65[d]	▒	▒	▒	■	▒	▒	▒			
		Gr. 55	55	55	▒	▒	▒	▒	▒	▒	▒			
		Gr. 60[e]	60	60	▒	▒	▒	▒	▒	▒	▒			
		Gr. 65[e]	65	65	▒	▒	▒	▒	▒	▒	▒			
	A618[f]	Gr. I & II	50[g]	70[g]								▒	▒	
		Gr. III	50	50								▒	▒	
	A913	50	50[h]	60[h]	▒	▒	▒	▒	▒	▒	▒			
		60	60	75	▒	▒	▒	▒	▒	▒	▒			
		65	65	80	▒	▒	▒	▒	▒	▒	▒			
		70	70	90	▒	▒	▒	▒	▒	▒	▒			
	A992		50	65[i]	■	▒	▒	▒	▒	▒	▒			
Corrosion Resistant High-Strength Low-Alloy	A242		42[j]	63[j]	▒									
			46[k]	67[k]	▒			▒						
			50[l]	70[l]	▒	▒	▒	▒	▒	▒	▒			
	A588		50	70	▒	▒	▒	▒	▒	▒	▒			
	A847		50	70								▒	▒	

■ = Preferred material specification
▒ = Other applicable material specification, the availability of which should be confirmed prior to specification
□ = Material specification does not apply

[a] Minimum unless a range is shown.
[b] For shapes over 426 lb/ft, only the minimum of 58 ksi applies.
[c] For shapes with a flange thickness less than or equal to 1½ in. only. To improve weldability, a maximum carbon equivalent can be specified (per ASTM Supplementary Requirement S78). If desired, maximum tensile stress of 90 ksi can be specified (per ASTM Supplementary Requirement S79).
[d] If desired, maximum tensile stress of 70 ksi can be specified (per ASTM Supplementary Requirement S91).
[e] For shapes with a flange thickness less than or equal to 2 in. only.
[f] ASTM A618 can also be specified as corrosion-resistant; see ASTM A618.
[g] Minimum applies for walls nominally ¾-in. thick and under. For wall thicknesses over ¾ in., $F_y = 46$ ksi and $F_u = 67$ ksi.
[h] If desired, maximum yield stress of 65 ksi and maximum yield-to-tensile strength ratio of 0.85 can be specified (per ASTM Supplementary Requirement S75).
[i] A maximum yield-to-tensile strength ratio of 0.85 and carbon equivalent formula are included as mandatory in ASTM A992.
[j] For shapes with a flange thickness greater than 2 in. only.
[k] For shapes with a flange thickness greater than 1½ in. and less than or equal to 2 in. only.
[l] For shapes with a flange thickness less than or equal to 1½ in. only.

Source: AISC Manual, Table 2-4, p. 2-48, 14th ed, 2011.

Although the prices of steels increase with increasing yield stresses, the percentage of price increase does not keep up with the percentage of yield-stress increase. The result is that the use of the stronger steels will quite frequently be economical for tension members, beams, and columns. Perhaps the greatest economy can be realized with tension members (particularly those without bolt holes). They may provide a great deal of savings for beams if deflections are not important or if deflections can be controlled (by methods described in later chapters). In addition, considerable economy can frequently be achieved with high-strength steels for short- and medium-length stocky columns. Another application that can provide considerable savings is hybrid construction. In this type of construction two or more steels of different strengths are used, the weaker steels being used where stresses are smaller and the stronger steels where stresses are higher.

Among the other factors that might lead to the use of high-strength steels are the following:

1. Superior corrosion resistance.
2. Possible savings in shipping, erection, and foundation costs caused by weight saving.
3. Use of shallower beams, permitting smaller floor depths.
4. Possible savings in fireproofing because smaller members can be used.

The first thought of most engineers in choosing a type of steel is the direct cost of the members. Such a comparison can be made quite easily, but determining which strength grade is most economical requires consideration of weights, sizes, deflections, maintenance, and fabrication. To make an accurate general comparison of the steels is probably impossible—rather, it is necessary to consider the specific job.

1.10 MEASUREMENT OF TOUGHNESS

The fracture toughness of steel is used as a general measure of its impact resistance, or its ability to absorb sudden increases in stress at a notch. The more ductile steel is, the greater will be its toughness. On the other hand, the lower its temperature, the higher will be its brittleness.

There are several procedures available for estimating notch toughness, but the Charpy V-notch test is the most commonly used. Although this test (which is described in ASTM Specification A6) is somewhat inaccurate, it does help identify brittle steels. With this test, the energy required to fracture a small bar of rectangular cross section with a specified notch (see Fig. 1.8) is measured.

The bar is fractured with a pendulum swung from a certain height. The amount of energy needed to fracture the bar is determined from the height to which the pendulum rises after the blow. The test may be repeated for different temperatures and the fracture energy plotted as a graph, as shown in Fig. 1.9. Such a graph clearly shows the relationship among temperature, ductility, and brittleness. The temperature at the point of steepest slope is referred to as the *transition temperature*.

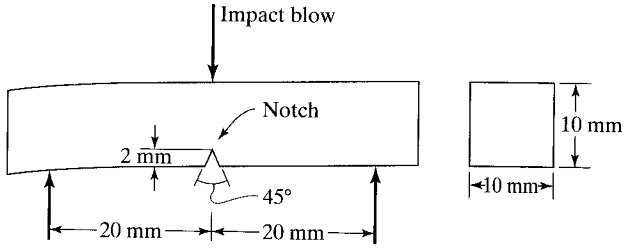


FIGURE 1.8

Specimen for Charpy V-notch test.

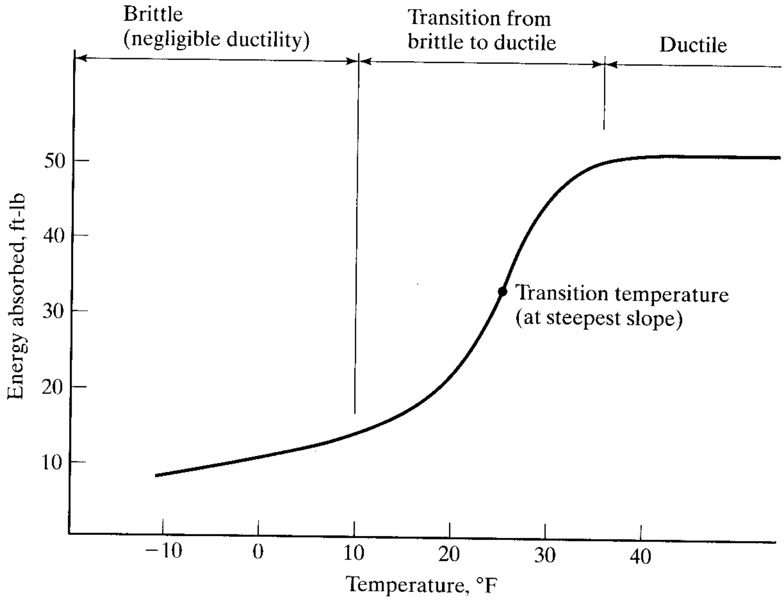


FIGURE 1.9

Results of Charpy V-notch test.

Although the Charpy test is well known, it actually provides a very poor measurement. Other methods for measuring the toughness of steel are considered in articles by Barsom and Rolfe.[8,9]

Different structural steels have different specifications for required absorbed energy levels (say, 20 ft-lb at 20°F), depending on the temperature, stress, and loading conditions under which they are to be used. The topic of brittleness is continued in the next section.

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[8]J. M. Barsom, "Material Considerations in Structural Steel Design," *Engineering Journal*, AISC, 24, 3 (3rd Quarter 1987), pp. 127–139.

[9]S. T. Rolfe, "Fracture and Fatigue Control in Steel Structures," *Engineering Journal*, AISC, 14, 1 (1st Quarter 1977), pp. 2–15.

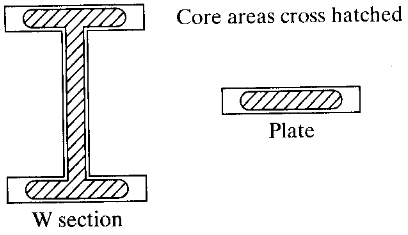


FIGURE 1.10
Core areas where brittle failure may be a problem in thick heavy members.

1.11 JUMBO SECTIONS

Certain heavy W sections with flange thicknesses exceeding 2 inches are often referred to as *jumbo sections*. They are identified with footnotes in the W shape, Table 1.1 of the Steel Manual.

Jumbo sections were originally developed for use as compression members and are quite satisfactory for that purpose. However, designers have frequently used them for tension or flexural members. In these applications, flange and web areas have had some serious cracking problems where welding or thermal cutting has been used. These cracks may result in smaller load-carrying capacities and problems related to fatigue.[10]

Thick pieces of steel tend to be more brittle than thin ones. Some of the reasons for this are that the core areas of thicker shapes (shown in Fig. 1.10) are subject to less rolling, have higher carbon contents (needed to produce required yield stresses), and have higher tensile stresses from cooling. These topics are discussed in later chapters.

Jumbo sections spliced with welds can be satisfactorily used for axial tension or flexural situations if the procedures listed in Specification A3.1c of the AISC Specification are carefully followed. Included among the requirements are the following:

1. The steel used must have certain absorbed energy levels, as determined by the Charpy V-notch test (20 ft-lb at a maximum temperature of 70°F). It is absolutely necessary that the tests be made on specimens taken from the core areas (shown in Fig. 1.10), where brittle fracture has proved to be a problem.
2. Temperature must be controlled during welding, and the work must follow a certain sequence.
3. Special splice details are required.

1.12 LAMELLAR TEARING

The steel specimens used for testing and developing stress–strain curves usually have their longitudinal axes in the direction that the steel was rolled. Should specimens be taken with their longitudinal axes transverse to the rolling direction "through the thickness" of the steel, the results will be lower ductility and toughness. Fortunately, this is a matter of little significance for almost all situations. It can, however, be quite important when thick plates

[10]R. Bjorhovde, "Solutions for the Use of Jumbo Shapes," *Proceedings 1988 National Steel Construction Conference*, AISC, Chicago, June 8–11, pp. 2-1 to 2-20.

and heavy structural shapes are used in highly restrained welded joints. (It can be a problem for thin members, too, but it is much more likely to give trouble in thick members.)

If a joint is highly restrained, the shrinkage of the welds in the through-the-thickness direction cannot be adequately redistributed, and the result can be a tearing of the steel called *lamellar tearing*. (*Lamellar* means "consisting of thin layers.") The situation is aggravated by the application of external tension. Lamellar tearing may show up as fatigue cracking after a number of cycles of load applications.

The lamellar tearing problem can be eliminated or greatly minimized with appropriate weld details and weld procedures. For example, the welds should be detailed so that shrinkage occurs as much as possible in the direction the steel was rolled. Several steel companies produce steels with enhanced through-the-thickness properties that provide much greater resistance to lamellar tearing. Even if such steels are used for heavy restrained joints, the special joint details mentioned before still are necessary.[11]

Figures 8-16 and 8-17 in the Steel Manual show preferred welded joint arrangements that reduce the possibility of lamellar tearing. Further information on the subject is provided in the ASTM A770 specification.

1.13 FURNISHING OF STRUCTURAL STEEL

The furnishing of structural steel consists of the rolling of the steel shapes, the fabrication of the shapes for the particular job (including cutting to the proper dimensions and punching the holes necessary for field connections), and their erection. Very rarely will a single company perform all three of these functions, and the average company performs only one or two of them. For instance, many companies fabricate structural steel and erect it, while others may be only steel fabricators or steel erectors. There are approximately 400 to 500 companies in the United States that make up the fabricating industry for structural steel. Most of them do both fabrication and erection.

Steel fabricators normally carry very little steel in stock because of the high interest and storage charges. When they get a job, they may order the shapes to certain lengths directly from the rolling mill, or they may obtain them from service centers. Service centers, which are an increasingly important factor in the supply of structural steel, buy and stock large quantities of structural steel, which they buy at the best prices they can find anywhere in the world.

Structural steel is usually designed by an engineer in collaboration with an architectural firm. The designer makes design drawings that show member sizes, controlling dimensions, and any unusual connections. The company that is to fabricate the steel makes the detailed drawings subject to the engineer's approval. These drawings provide all the information necessary to fabricate the members correctly. They show the dimensions for each member, the locations of holes, the positions and sizes of connections, and the like. A part of a typical detail drawing for a bolted steel beam is shown in

[11]"Commentary on Highly Restrained Welded Connections," *Engineering Journal*, AISC, vol. 10, no. 3 (3d quarter, 1973), pp. 61–73.

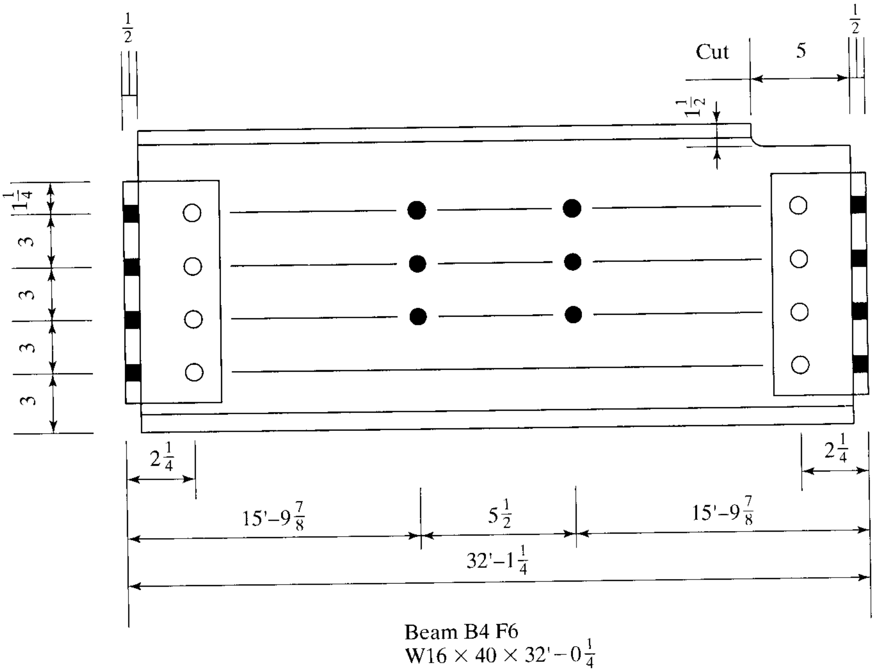


FIGURE 1.11

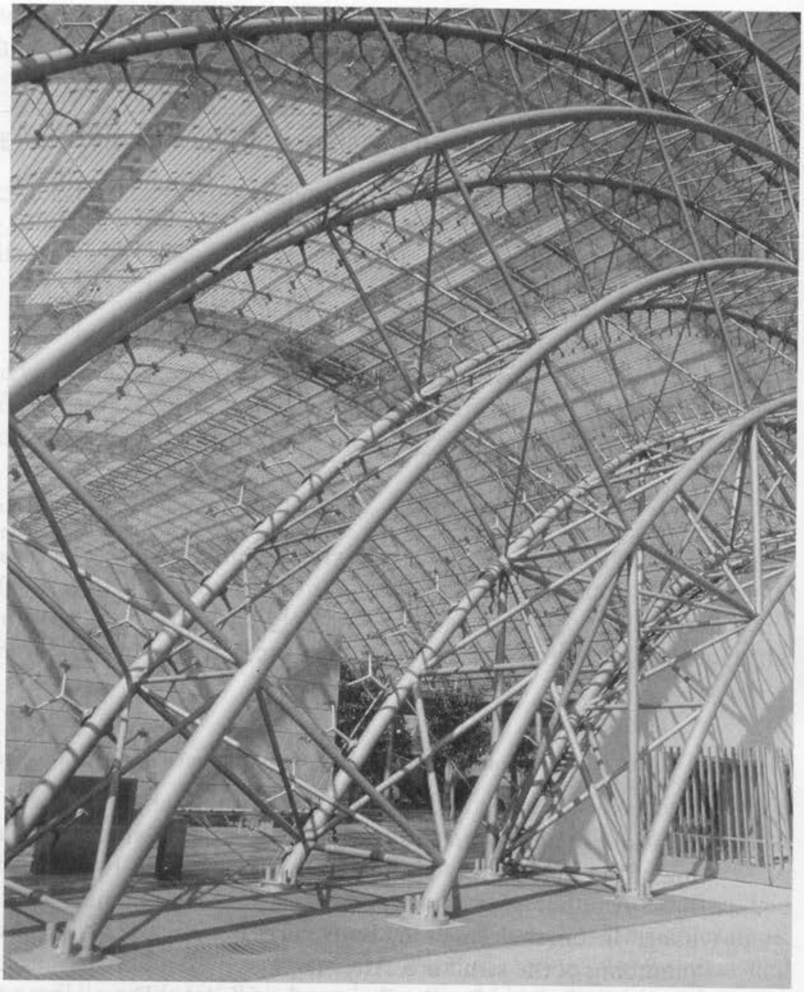
Part of a detail drawing.

Fig. 1.11. There may be a few items included on this drawing that are puzzling to you since you have read only a few pages of this book. However, these items should become clear as you study the chapters to follow.

On actual detail drawings, details probably will be shown for several members. Here, the author has shown only one member, just to indicate the information needed so the shop can correctly fabricate the member. The darkened circles and rectangles indicate that the bolts are to be installed in the field, while the nondarkened ones show the connections which are to be made in the shop.

The erection of steel buildings is more a matter of assembly than nearly any other part of construction work. Each of the members is marked in the shop with letters and numbers to distinguish it from the other members to be used. The erection is performed in accordance with a set of erection plans. These plans are not detailed drawings, but are simple line diagrams showing the position of the various members in the building. The drawings show each separate piece or subassembly of pieces together with assigned shipping or erection marks, so that the steelworkers can quickly identify and locate members in their correct positions in the structure. (Persons performing steel erection often are called *ironworkers*, which is a name held over from the days before structural steel.) Directions (north, south, east, or west) usually are painted on column faces.

Sometimes, the erection drawing gives the sizes of the members, but this is not necessary. They may or may not be shown, depending on the practice of the particular fabricator.



Round Arch Hall at exhibition center, Leipzig, Germany. (© Klaws Hackenberg/Zefa/Corbis. Used by permission.)

Beams, girders, and columns will be indicated on the drawings by the letters B, G, or C, respectively, followed by the number of the particular member as B5, G12, and so on. Often, there will be several members of these same designations where members are repeated in the building.

Multistory steel frames often will have several levels of identical or nearly identical framing systems. Thus, one erection plan may be used to serve several floors. For such situations, the member designations for the columns, beams, and girders will have the level numbers incorporated in them. For instance, column C15(3–5) is column 15, third to fifth floors; while B4F6, or just B4(6), represents beam B4 for the sixth floor. A portion of a building erection drawing is shown in Fig. 1.12.

Next, we describe briefly the erection of the structural steel members for a building. Initially, a group of ironworkers, sometimes called the "raising gang," erects the steel members, installing only a sufficient number of bolts to hold the members in

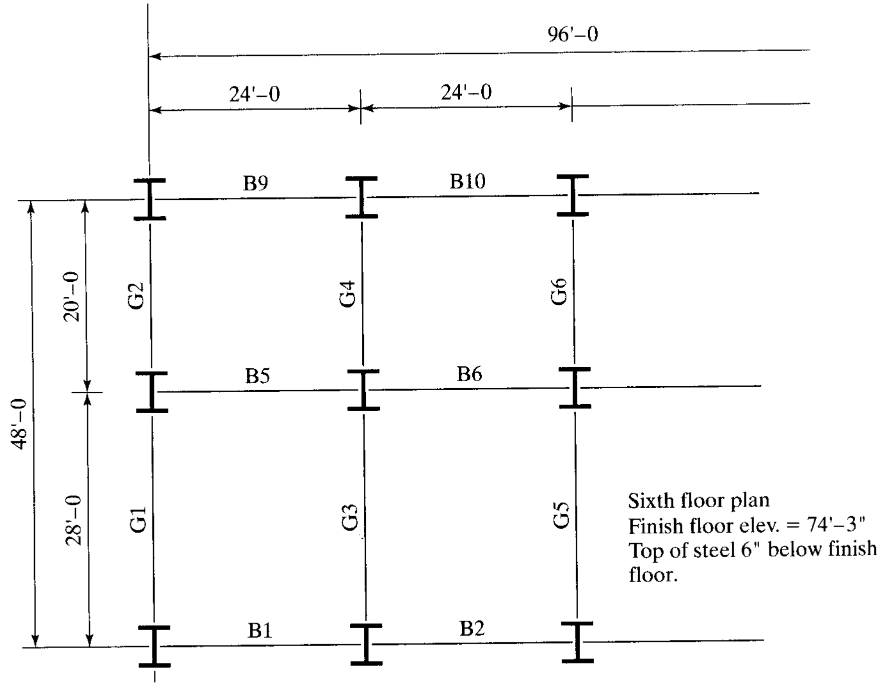


FIGURE 1.12

Part of an erection drawing, showing where each member is to be located.

position. In addition, they place any guy cables where needed for stability and plumbing of the steel frame.

Another group of ironworkers, who are sometimes referred to as the "detail gang," install the remaining bolts, carry out any needed field welding, and complete the plumbing of the structure. After the last two steps are completed, another crew installs the metal decking for the floor and roof slabs. They in turn are followed by the crews who place the necessary concrete reinforcing and the concrete for those slabs.[12]

1.14 THE WORK OF THE STRUCTURAL DESIGNER

The structural designer arranges and proportions structures and their parts so that they will satisfactorily support the loads to which they may feasibly be subjected. It might be said that he or she is involved with the general layout of structures; studies of the possible structural forms that can be used; consideration of loading conditions; analysis of stresses, deflections, and so on; design of parts; and the preparation of design drawings. More precisely, the word *design* pertains to the proportioning of the various parts of a

[12]A. R. Tamboli, editor, *Steel Design Handbook LRFD Method* (New York: McGraw-Hill, 1997), pp. 12–37.

structure after the forces have been calculated, and it is this process which will be emphasized throughout the text, using structural steel as the material.

1.15 RESPONSIBILITIES OF THE STRUCTURAL DESIGNER

The structural designer must learn to arrange and proportion the parts of structures so that they can be practically erected and will have sufficient strength and reasonable economy. These items are discussed briefly next.

1.15.1 Safety

Not only must the frame of a structure safely support the loads to which it is subjected, but it must support them in such a manner that deflections and vibrations are not so great as to frighten the occupants or to cause unsightly cracks.

1.15.2 Cost

The designer needs to keep in mind the factors that can lower cost without sacrifice of strength. These items, which are discussed in more detail throughout the text, include the use of standard-size members, simple connections and details, and members and materials that will not require an unreasonable amount of maintenance through the years.

1.15.3 Constructability

Another objective is the design of structures that can be fabricated and erected without great problems arising. Designers need to understand fabrication methods and should try to fit their work to the fabrication facilities available.

Designers should learn everything possible about the detailing, fabrication, and field erection of steel. The more the designer knows about the problems, tolerances, and clearances in shop and field, the more probable it is that reasonable, practical, and economical designs will be produced. This knowledge should include information concerning the transportation of the materials to the job site (such as the largest pieces that can be transported practically by rail or truck), labor conditions, and the equipment available for erection. Perhaps the designer should ask, "Could I get this thing together if I were sent out to do it?"

Finally, he or she needs to proportion the parts of the structure so that they will not unduly interfere with the mechanical features of the structure (pipes, ducts, etc.) or the architectural effects.

1.16 ECONOMICAL DESIGN OF STEEL MEMBERS

The design of a steel member involves much more than a calculation of the properties required to support the loads and the selection of the lightest section providing these properties. Although at first glance this procedure would seem to give the most economical designs, many other factors need to be considered.

Steel erection for Transamerica Pyramid, San Francisco, CA. (Courtesy of Kaiser Steel Corporation.)

Today, the labor costs involved in the fabrication and erection of structural steel are thought to run close to 60 percent of the total costs of steel structures. On the other hand, material costs represent only about 25 percent of total costs. Thus, we can see that any efforts we make to improve the economy of our work in structural steel should be primarily concentrated in the labor area.

When designers are considering costs, they have a tendency to think only of quantities of materials. As a result, they will sometimes carefully design a structure with the lightest possible members and end up with some very expensive labor situations with only minor material savings. Among the many factors that need to be considered in providing economical steel structures are the following:

1. One of the best ways to achieve economy is to have open communications between designers, fabricators, erectors, and others involved in a particular project. If this is done during the design process, the abilities and experience of each of

the parties may be utilized at a time when it is still possible to implement good economical ideas.

2. The designer needs to select steel sections of sizes that are usually rolled. Steel beams and bars and plates of unusual sizes will be difficult to obtain during boom periods and will be expensive during any period. A little study on the designer's part will enable him or her to avoid these expensive shapes. Steel fabricators are constantly supplied with information from the steel companies and the steel warehousers as to the sizes and lengths of sections available. (Most structural shapes can be procured in lengths from 60 to 75 ft, depending on the producer, while it is possible under certain conditions to obtain some shapes up to 120 ft in length.)
3. A blind assumption that the lightest section is the cheapest one may be in considerable error. A building frame designed by the "lightest-section" procedure will consist of a large number of different shapes and sizes of members. Trying to connect these many-sized members and fit them in the building will be quite complicated, and the pound price of the steel will, in all probability, be rather high. A more reasonable approach would be to smooth out the sizes by selecting many members of the same sizes, although some of them may be slightly overdesigned.
4. The beams usually selected for the floors in buildings will normally be the deeper sections, because these sections for the same weights have the largest moments of inertia and the greatest resisting moments. As building heights increase, however, it may be economical to modify this practice. As an illustration, consider the erection of a 20-story building, for which each floor has a minimum clearance. It is assumed that the depths of the floor beams may be reduced by 6 in without an unreasonable increase in beam weights. The beams will cost more, but the building height will be reduced by 20×6 in $= 120$ in, or 10 ft, with resulting savings in walls, elevator shafts, column heights, plumbing, wiring, and footings.[13]
5. The costs of erection and fabrication for structural steel beams are approximately the same for light and heavy members. Thus, beams should be spaced as far apart as possible to reduce the number of members that have to be fabricated and erected.
6. Structural steel members should be painted only if so required by the applicable specification. You should realize that steel should not be painted if it is to be in contact with concrete. Furthermore, the various fireproofing materials used for protecting steel members adhere better if the surfaces are unpainted.[14]
7. It is very desirable to keep repeating the same section over and over again. Such a practice will reduce the detailing, fabrication, and erection costs.
8. For larger sections, particularly the built-up ones, the designer needs to have information pertaining to transportation problems. The desired information includes the greatest lengths and depths that can be shipped by truck or rail

[13]H. Allison, "Low- and Medium-Rise Steel Buildings" (Chicago: AISC, 1991), pp. 1–5.
[14]Ibid, pp. 1–5.

(see Section 1.18), clearances available under bridges and power lines leading to the project, and allowable loads on bridges. It may be possible to fabricate a steel roof truss in one piece, but is it possible to transport it to the job site and erect it in one piece?

9. Sections should be selected that are reasonably easy to erect and which have no conditions that will make them difficult to maintain. As an example, it is necessary to have access to all exposed surfaces of steel bridge members so that they may be periodically painted (unless one of the special corrosion-resistant steels is used).
10. Buildings are often filled with an amazing conglomeration of pipes, ducts, conduits, and other items. Every effort should be made to select steel members that will fit in with the requirements made by these items.
11. The members of a steel structure are often exposed to the public, particularly in the case of steel bridges and auditoriums. Appearance may often be the major factor in selecting the type of structure, such as where a bridge is desired that will fit in and actually contribute to the appearance of an area. Exposed members may be surprisingly graceful when a simple arrangement, perhaps with curved members, is used; but other arrangements may create a terrible eyesore. The student has certainly seen illustrations of each case. It is very interesting to know that beautiful structures in steel are usually quite reasonable in cost.

The question is often asked, *How do we achieve economy in structural steel design?* The answer is simple: *It lies in what the steel fabricator does not have to do.* (In other words, economy can be realized when fabrication is minimized.)

The April 2000 issue of *Modern Steel Construction* provides several articles which present excellent material on the topic of economy in steel construction.[15] The student can very quickly learn a great deal of valuable information concerning the topic of economy in steel by reading these articles. The author thinks they are a "must read" for anyone practicing steel design.[16–19]

1.17 FAILURE OF STRUCTURES

Many people who are superstitious do not discuss flat tires or make their wills, because they fear they will be tempting fate. These same people would probably not care to discuss the subject of engineering failures. Despite the prevalence of this superstition, an

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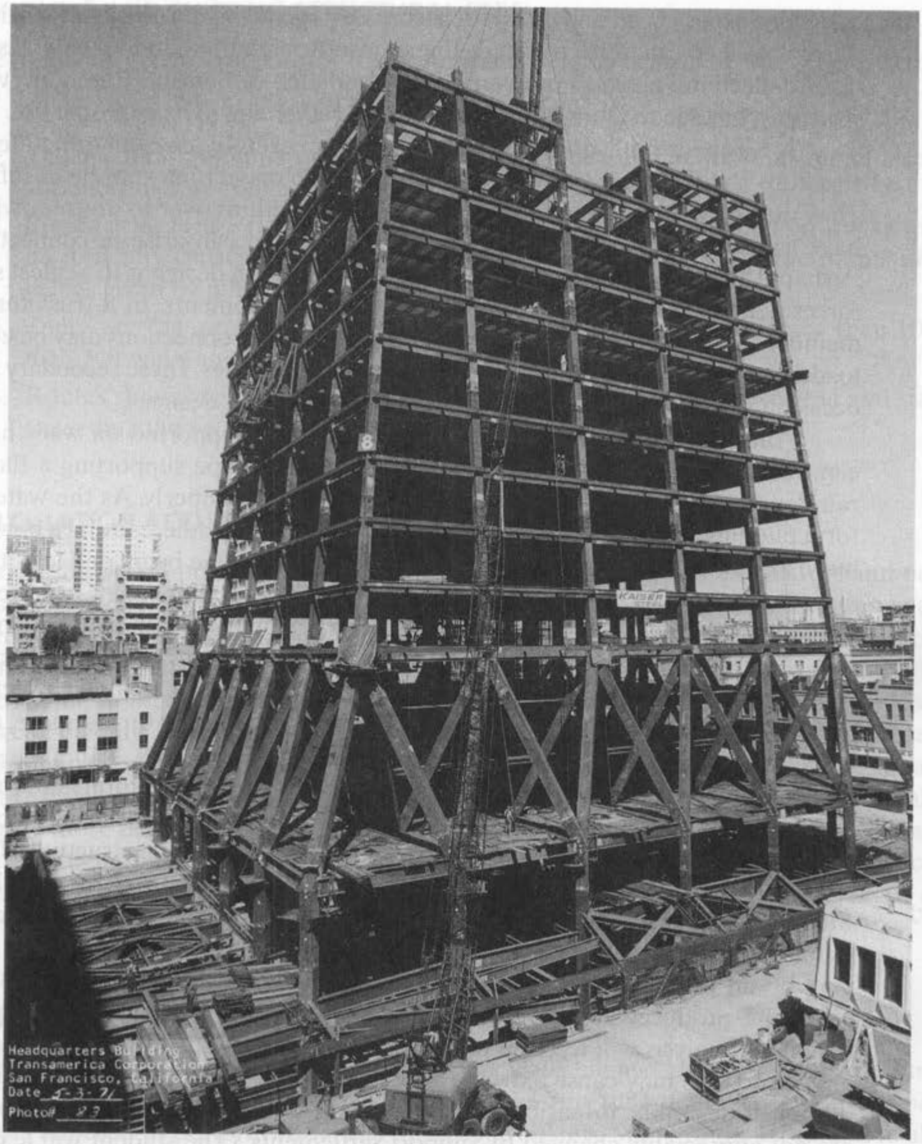
[15] *Modern Steel Construction,* April 2000, vol. 40, no. 4 (Chicago: American Institute of Steel Construction), pp. 6, 25–48, 60.

[16] C. J. Carter, T. M. Murray and W. A. Thornton, "Economy in Steel," in *Modern Steel Construction,* April 2000, vol. 40, no. 4 (Chicago: American Institute of Steel Construction).

[17] D. T. Ricker, "Value Engineering for Steel Construction," in *Modern Steel Construction,* April 2000, vol. 40, no. 4 (Chicago: American Institute of Steel Construction).

[18] J. E. Quinn, "Reducing Fabrication Costs," in *Modern Steel Construction,* April 2000, vol. 40, no. 4 (Chicago: American Institute of Steel Construction).

[19] Steel Joist Institute, "Reducing Joist Cost," in *Modern Steel Construction,* April 2000, vol. 40, no. 4 (Chicago: American Institute of Steel Construction).



Steel erection for Transamerica Pyramid, San Francisco, CA. (Courtesy of Kaiser Steel Corporation.)

awareness of the items that have most frequently caused failures in the past is invaluable to experienced and inexperienced designers alike. Perhaps a study of past failures is more important than a study of past successes. Benjamin Franklin supposedly made the observation that "a wise man learns more from failures than from success."

The designer with little experience particularly needs to know where the most attention should be given and where outside advice is needed. The vast majority of designers, experienced and inexperienced, select members of sufficient size and strength. The collapse of structures is usually due to insufficient attention to the details of connections, deflections, erection problems, and foundation settlement. Rarely, if ever, do steel structures fail due to faults in the material, but rather due to its improper use.

A frequent fault of designers is that after carefully designing the members of a structure, they carelessly select connections which may or may not be of sufficient size. They may even turn the job of selecting the connections over to drafters, who may not have sufficient understanding of the difficulties that can arise in connection design. Perhaps the most common mistake made in connection design is to neglect some of the forces acting on the connections, such as twisting moments. In a truss for which the members have been designed for axial forces only, the connections may be eccentrically loaded, resulting in moments that cause increasing stresses. These secondary stresses are occasionally so large that they need to be considered in design.

Another source of failure occurs when beams supported on walls have insufficient bearing or anchorage. Imagine a beam of this type supporting a flat roof on a rainy night when the roof drains are not functioning properly. As the water begins to form puddles on the roof, the beam tends to sag in the middle, causing a pocket to catch more rain, which creates more beam sag, and so on. As the beam deflects, it pushes out against the walls, possibly causing collapse of walls or slippage of beam ends off the wall. Picture a 60-ft steel beam, supported on a wall with only an inch or two of bearing, that contracts when the temperature drops 50 or 60 degrees overnight. A collapse due to a combination of beam contraction, outward deflection of walls, and vertical deflection caused by precipitation loads is not difficult to visualize; furthermore, actual cases in engineering literature are not difficult to find.

Foundation settlements cause a large number of structural failures, probably more than any other factor. Most foundation settlements do not result in collapse, but they very often cause unsightly cracks and depreciation of the structure. If all parts of the foundation of a structure settle equally, the stresses in the structure theoretically will not change. The designer, usually not able to prevent settlement, has the goal of designing foundations in such a manner that equal settlements occur. Equal settlements may be an impossible goal, and consideration should be given to the stresses that would be produced if settlement variations occurred. The student's background in structural analysis will tell him or her that uneven settlements in statically indeterminate structures may cause extreme stress variations. Where foundation conditions are poor, it is desirable, if feasible, to use statically determinate structures whose stresses are not appreciably changed by support settlements. (The student will learn in subsequent discussions that the ultimate strength of steel structures is usually affected only slightly by uneven support settlements.)

Some structural failures occur because inadequate attention is given to deflections, fatigue of members, bracing against swaying, vibrations, and the possibility of buckling of compression members or the compression flanges of beams. The usual structure, when completed, is sufficiently braced with floors, walls, connections, and special bracing, but there are times during construction when many of these items are not present. As previously indicated, the worst conditions may well occur during erection, and special temporary bracing may be required.

1.18 HANDLING AND SHIPPING STRUCTURAL STEEL

The following general rules apply to the sizes and weights of structural steel pieces that can be fabricated in the shop, shipped to the job, and erected:

1. The maximum weights and lengths that can be handled in the shop and at a construction site are roughly 90 tons and 120 ft, respectively.
2. Pieces as large as 8 ft high, 8 ft wide, and 60 ft long can be shipped on trucks with no difficulty (provided the axle or gross weights do not exceed the permissible values given by public agencies along the designated routes).
3. There are few problems in railroad shipment if pieces are no larger than 10 ft high, 8 ft wide, and 60 ft long, and weigh no more than 20 tons.
4. Routes should be carefully studied, and carriers consulted for weights and sizes exceeding the values given in (2) and (3).

1.19 CALCULATION ACCURACY

A most important point many students with their superb pocket calculators and personal computers have difficulty understanding is that structural design is not an exact science for which answers can confidently be calculated to eight significant figures. Among the reasons for this fact are that the methods of analysis are based on partly true assumptions, the strengths of materials used vary appreciably, and maximum loadings can be only approximated. With respect to this last reason, how many of the users of this book could estimate within 10 percent the maximum load in pounds per square foot that will ever occur on the building floor which they are now occupying? Calculations to more than two or three significant figures are obviously of little value and may actually be harmful in that they mislead the student by giving him or her a fictitious sense of precision. From a practical standpoint, it seems wise to carry all the digits on the calculator for intermediate steps and then round off the final answers.

1.20 COMPUTERS AND STRUCTURAL STEEL DESIGN

The availability of personal computers has drastically changed the way steel structures are analyzed and designed. In nearly every engineering school and design office, computers are used to perform structural analysis problems. Many of the structural analysis programs commercially available also can perform structural design.

Many calculations are involved in structural steel design, and many of these calculations are quite time-consuming. With the use of a computer, the design engineer can greatly reduce the time required to perform these calculations, and likely increase the accuracy of the calculations. In turn, this will then provide the engineer with more time to consider the implications of the design and the resulting performance of the structure, and more time to try changes that may improve economy or behavior.

Although computers do increase design productivity, they also tend to reduce the engineer's "feel" for the structure. This can be a particular problem for young engineers with very little design experience. Unless design engineers have this feel for system behavior, the use of computers can result in large, costly mistakes. Such situations may arise where anomalies and inconsistencies are not immediately apparent to the inexperienced engineer. Theoretically, the computer design of alternative systems for a few projects should substantially improve the engineer's judgment in a short span of time. Without computers, the development of this same judgment would likely require the engineer to work his or her way through numerous projects.

1.21 PROBLEMS FOR SOLUTION

1-1. List the three regions of a stress–strain diagram for mild or low-carbon structural steel.

1-2. List the specifying organization for the following types of steel:

a. Cold-formed steel
b. Hot-rolled steel

1-3. Define the following:

a. Proportional limit
b. Elastic limit
c. Yield stress

1-4. List the preferred steel type (ASTM spec) for the following shapes:

a. Plates
b. W shapes
c. C sections

1-5. List the two methods used to produce steel shapes.

1-6. List four advantages of steel as a structural material.

1-7. What type of steel (ASTM grade) has made the cost of 50 ksi the same as 36 ksi steel because of the use of scrap or recycled steel in the manufacturing process?

1-8. What are the differences between wrought iron, steel, and cast iron?

1-9. What is the range of carbon percentage for *mild* carbon steel?

1-10. List four disadvantages of steel as a structural material.

1-11. List four types of failures for structural steel structures.

CHAPTER 2

Specifications, Loads, and Methods of Design

2.1 SPECIFICATIONS AND BUILDING CODES

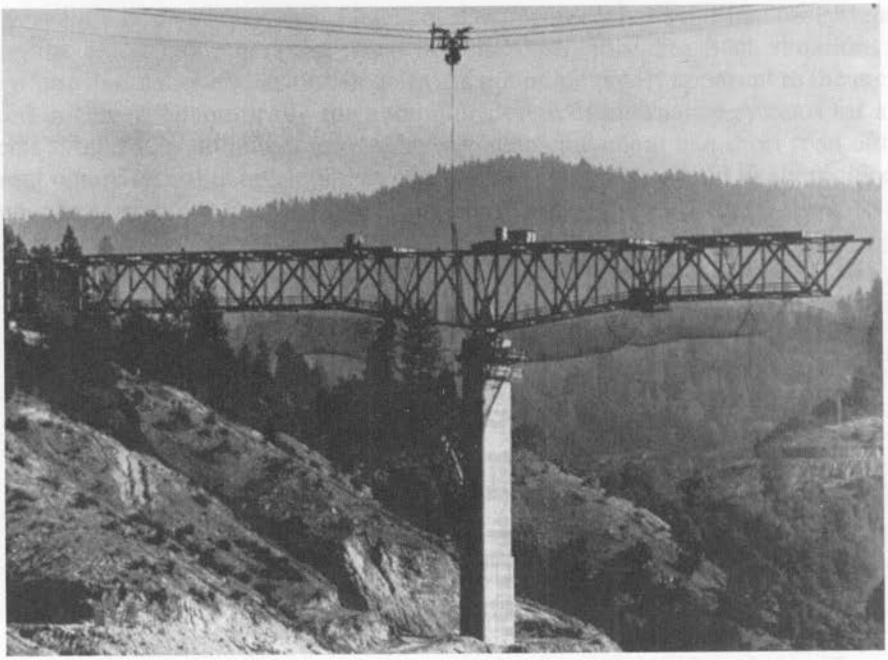
The design of most structures is controlled by building codes and design specifications. Even if they are not so controlled, the designer will probably refer to them as a guide. No matter how many structures a person has designed, it is impossible for him or her to have encountered every situation. By referring to specifications, he or she is making use of the best available material on the subject. Engineering specifications that are developed by various organizations present the best opinion of those organizations as to what represents good practice.

Municipal and state governments concerned with the safety of the public have established building codes with which they control the construction of various structures within their jurisdiction. These codes, which are actually laws or ordinances, specify minimum design loads, design stresses, construction types, material quality, and other factors. They vary considerably from city to city, a fact that causes some confusion among architects and engineers.

Several organizations publish recommended practices for regional or national use. Their specifications are not legally enforceable, unless they are embodied in the local building code or made a part of a particular contract. Among these organizations are the AISC and AASHTO (American Association of State Highway and Transportation Officials). Nearly all municipal and state building codes have adopted the AISC Specification, and nearly all state highway and transportation departments have adopted the AASHTO Specifications.

Readers should note that logical and clearly written codes are quite helpful to design engineers. Furthermore, there are far fewer structural failures in areas that have good building codes that are strictly enforced.

Some people feel that specifications prevent engineers from thinking for themselves—and there may be some basis for the criticism. They say that the ancient engineers who built the great pyramids, the Parthenon, and the great Roman bridges were



South Fork Feather River Bridge in northern California, being erected by use of a 1626-ft-long cableway strung from 210-ft-high masts anchored on each side of the canyon. (Courtesy of Bethlehem Steel Corporation.)

controlled by few specifications, which is certainly true. On the other hand, it should be said that only a few score of these great projects have endured over many centuries, and they were, apparently, built without regard to cost of material, labor, or human life. They were probably built by intuition and by certain rules of thumb that the builders developed by observing the minimum size or strength of members, which would fail only under given conditions. Their likely numerous failures are not recorded in history; only their successes endured.

Today, however, there are hundreds of projects being constructed at any one time in the United States that rival in importance and magnitude the famous structures of the past. It appears that if all engineers in our country were allowed to design projects such as these, without restrictions, there would be many disastrous failures. *The important thing to remember about specifications, therefore, is that they are written, not for the purpose of restricting engineers, but for the purpose of protecting the public.*

No matter how many specifications are written, it is impossible for them to cover every possible design situation. As a result, no matter which building code or specification is or is not being used, the ultimate responsibility for the design of a safe structure lies with the structural designer. Obviously, the intent of these specifications is that the loading used for design be the one that causes the largest stresses.

Another very important code, the *International Building Code*[1] (IBC), was developed because of the need for a modern building code that emphasizes performance.

[1]International Code Council, Inc., *International Building Code* (Washington, DC, 2009).

It is intended to provide a model set of regulations to safeguard the public in all communities.

2.2 LOADS

Perhaps the most important and most difficult task faced by the structural engineer is the accurate estimation of the loads that may be applied to a structure during its life. No loads that may reasonably be expected to occur may be overlooked. After loads are estimated, the next problem is to determine the worst possible combinations of these loads that might occur at one time. For instance, would a highway bridge completely covered with ice and snow be simultaneously subjected to fast-moving lines of heavily loaded trailer trucks in every lane and to a 90-mile lateral wind, or is some lesser combination of these loads more likely?

Section B2 of the AISC Specification states that the nominal loads to be used for structural design shall be the ones stipulated by the applicable code under which the structure is being designed or as dictated by the conditions involved. If there is an absence of a code, the design loads shall be those provided in a publication of the American Society of Civil Engineers entitled *Minimum Design Loads for Buildings and Other Structures*.[2] This publication is commonly referred to as ASCE 7. It was originally published by the American National Standards Institute (ANSI) and referred to as the *ANSI 58.1 Standard*. The ASCE took over its publication in 1988.

In general, loads are classified according to their character and duration of application. As such, they are said to be *dead loads*, *live loads*, and *environmental loads*. Each of these types of loads are discussed in the next few sections.

2.3 DEAD LOADS

Dead loads are loads of constant magnitude that remain in one position. They consist of the structural frame's own weight and other loads that are permanently attached to the frame. For a steel-frame building, the frame, walls, floors, roof, plumbing, and fixtures are dead loads.

To design a structure, it is necessary for the weights, or dead loads, of the various parts to be estimated for use in the analysis. The exact sizes and weights of the parts are not known until the structural analysis is made and the members of the structure selected. The weights, as determined from the actual design, must be compared with the estimated weights. If large discrepancies are present, it will be necessary to repeat the analysis and design with better estimated weights.

Reasonable estimates of structure weights may be obtained by referring to similar types of structures or to various formulas and tables available in several publications. The weights of many materials are given in Part 17 of the Steel Manual. Even more detailed information on dead loads is provided in Tables C3-1 and C3-2 of ASCE 7-10. An experienced engineer can estimate very closely the weights of most materials and will spend little time repeating designs because of poor estimates.

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[2]American Society of Civil Engineers, *Minimum Design Loads for Buildings and Other Structures*. ASCE 7-10. Formerly ANSI A58.1 (Reston, Va.: ASCE, 2010).

TABLE 2.1 Typical Dead Loads for Some Common Building Materials

Reinforced concrete	150 lb/cu ft
Structural steel	490 lb/cu ft
Plain concrete	145 lb/cu ft
Movable steel partitions	4 psf
Plaster on concrete	5 psf
Suspended ceilings	2 psf
5-Ply felt and gravel	6 psf
Hardwood flooring (7/8 in)	4 psf
2 × 12 × 16 in double wood floors	7 psf
Wood studs with 1/2 in gypsum each side	8 psf
Clay brick wythes (4 in)	39 psf

The approximate weights of some common building materials for roofs, walls, floors, and so on are presented in Table 2.1.

2.4 LIVE LOADS

Live loads are loads that may change in position and magnitude. They are caused when a structure is occupied, used, and maintained. Live loads that move under their own power, such as trucks, people, and cranes, are said to be *moving loads*. Those loads that may be moved are *movable loads*, such as furniture and warehouse materials. A great deal of information on the magnitudes of these various loads, along with specified minimum values, are presented in ASCE 7-10.

1. *Floor loads*. The minimum gravity live loads to be used for building floors are clearly specified by the applicable building code. Unfortunately, however, the values given in these various codes vary from city to city, and the designer must be sure that his or her designs meet the requirements of the locality in question. A few of the typical values for floor loadings are listed in Table 2.2. These values were adopted from ASCE 7-10. In the absence of a governing code, this is an excellent one to follow.

Quite a few building codes specify concentrated loads that must be considered in design. Section 4.4 of ASCE 7-10 and Section 1607.4 of IBC-2009 are two such examples. The loads specified are considered as alternatives to the uniform loads previously considered herein.

Some typical concentrated loads taken from Table 4-1 of ASCE 7-10 and Table 1607.1 of IBC-2009 are listed in Table 2.3. These loads are to be placed on floors or roofs at the positions where they will cause the most severe conditions. Unless otherwise specified, each of these concentrated loads is spread over an area 2.5 × 2.5 ft square (6.25 ft^2).

2. *Traffic loads for bridges*. Bridges are subjected to series of concentrated loads of varying magnitude caused by groups of truck or train wheels.
3. *Impact loads*. Impact loads are caused by the vibration of moving or movable loads. It is obvious that a crate dropped on the floor of a warehouse or a truck

TABLE 2.2 Typical Minimum Uniform Live Loads for Design of Buildings

Type of building	LL (psf)
Apartment houses	
Apartments	40
Public rooms	100
Dining rooms and restaurants	100
Garages (passenger cars only)	40
Gymnasiums, main floors, and balconies	100
Office buildings	
Lobbies	100
Offices	50
Schools	
Classrooms	40
Corridors, first floor	100
Corridors above first floor	80
Storage warehouses	
Light	125
Heavy	250
Stores (retail)	
First floor	100
Other floors	75

TABLE 2.3 Typical Concentrated Live Loads for Buildings

Hospitals—operating rooms, private rooms, and wards	1000 lb
Manufacturing building (light)	2000 lb
Manufacturing building (heavy)	3000 lb
Office floors	2000 lb
Retail stores (first floors)	1000 lb
Retail stores (upper floors)	1000 lb
School classrooms	1000 lb
School corridors	1000 lb

bouncing on uneven pavement of a bridge causes greater forces than would occur if the loads were applied gently and gradually. Cranes picking up loads and elevators starting and stopping are other examples of impact loads. Impact loads are equal to the difference between the magnitude of the loads actually caused and the magnitude of the loads had they been dead loads.

Section 4.6 of ASCE 7-10 Specification requires that when structures are supporting live loads that tend to cause impact, it is necessary for those loads to be increased by the percentages given in Table 2.4.

4. *Longitudinal loads.* Longitudinal loads are another type of load that needs to be considered in designing some structures. Stopping a train on a railroad bridge or a truck on a highway bridge causes longitudinal forces to be applied. It is not difficult to imagine the tremendous longitudinal force developed when the driver of a 40-ton-trailer truck traveling 60 mph suddenly has to apply the brakes while

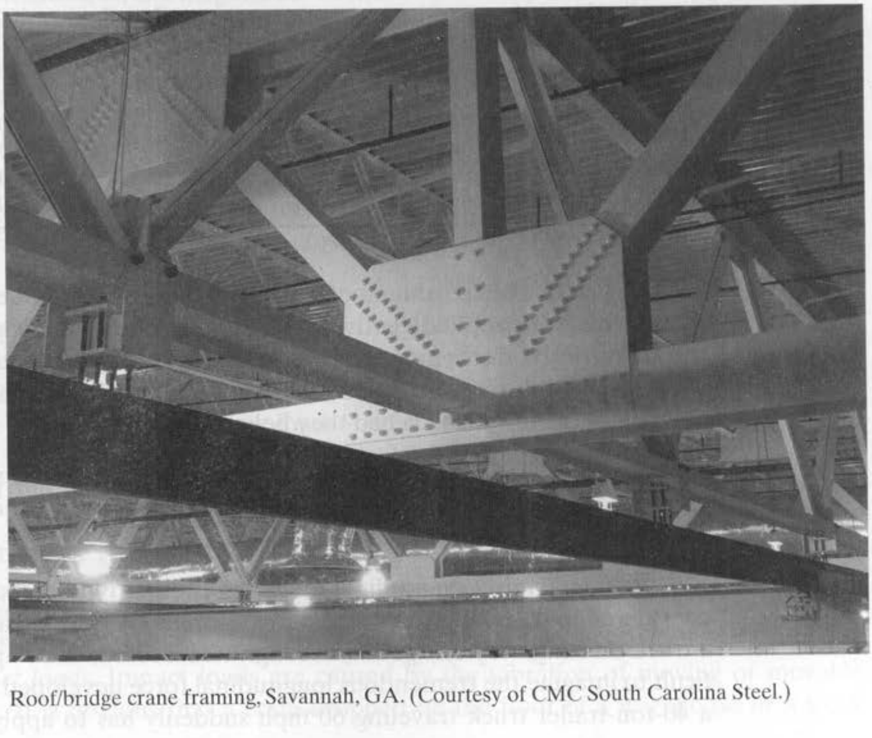
TABLE 2.4 Live Load Impact Factors

Elevator machinery*	100%
Motor-driven machinery	20%
Reciprocating machinery	50%

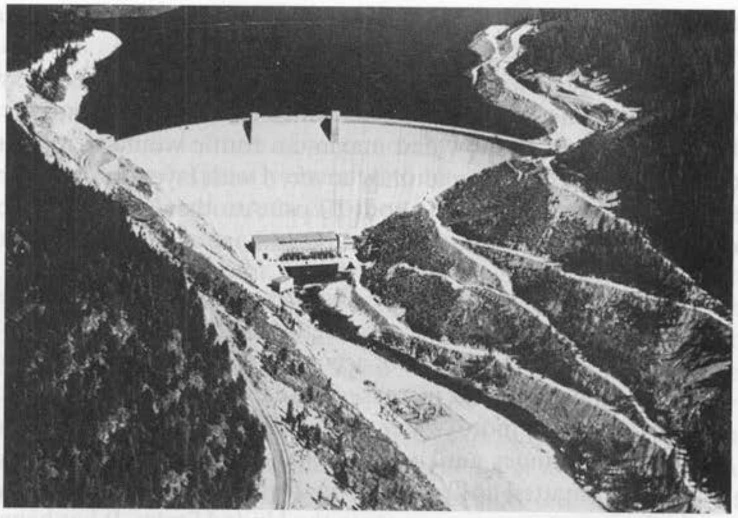
*See Section C4.6, ASCE 7-10 Commentary.

crossing a highway bridge. There are other longitudinal load situations, such as ships bumping a dock during berthing and the movement of traveling cranes that are supported by building frames.

5. *Other live loads*. Among the other types of live loads with which the structural engineer will have to contend are *soil pressures* (such as the exertion of lateral earth pressures on walls or upward pressures on foundations); *hydrostatic pressures* (water pressure on dams, inertia forces of large bodies of water during earthquakes, and uplift pressures on tanks and basement structures); *blast loads* (caused by explosions, sonic booms, and military weapons); *thermal forces* (due to changes in temperature, causing structural deformations and resulting structural forces); and *centrifugal forces* (such as those on curved bridges and caused by trucks and trains, or similar effects on roller coasters, etc.).



Roof/bridge crane framing, Savannah, GA. (Courtesy of CMC South Carolina Steel.)



Hungry Horse Dam and Reservoir, Rocky Mountains, in northwestern Montana. (Courtesy of the Montana Travel Promotion Division.)

2.5 ENVIRONMENTAL LOADS

Environmental loads are caused by the environment in which a particular structure is located. For buildings, environmental loads are caused by rain, snow, wind, temperature change, and earthquakes. Strictly speaking, environmental loads are live loads, but they are the result of the environment in which the structure is located. Even though they do vary with time, they are not all caused by gravity or operating conditions, as is typical with other live loads. A few comments are presented in the paragraphs that follow concerning the different types of environmental loads:

1. *Snow.* In the colder states, snow loads are often quite important. One inch of snow is equivalent to a load of approximately 0.5 psf, but it may be higher at lower elevations where snow is denser. For roof designs, snow loads varying from 10 to 40 psf are commonly used, the magnitude depending primarily on the slope of the roof and, to a lesser degree, on the character of the roof surface. The larger values are used for flat roofs, the smaller ones for sloped roofs. Snow tends to slide off sloped roofs, particularly those with metal or slate surfaces. A load of approximately 10 psf might be used for 45° (degree) slopes and a 40-psf load for flat roofs. Studies of snowfall records in areas with severe winters may indicate the occurrence of snow loads much greater than 40 psf, with values as high as 200 psf in some western states.

Snow is a variable load that may cover an entire roof or only part of it. The snow loads that are applied to a structure are dependent upon many factors, including geographic location, roof pitch, sheltering, and the shape of the roof. Chapter 7 of ASCE 7-10 provides a great deal of information concerning snow loads, including charts and formulas for estimating their magnitudes. There may be drifts against walls or buildup in valleys or between parapets. Snow may slide off one roof onto a lower one. The wind may blow it off one side of a sloping roof, or the snow may crust over and remain in position even during very heavy winds.

Bridges are generally not designed for snow loads, since the weight of the snow is usually not significant in comparison with truck and train loads. In any case, it is doubtful that a full load of snow and maximum traffic would be present at the same time. Bridges and towers are sometimes covered with layers of ice from 1 to 2 in thick. The weight of the ice runs up to about 10 psf. Another factor to be considered is the increased surface area of the ice-coated members, as it pertains to wind loads.

2. *Rain.* Though snow loads are a more severe problem than rain loads for the usual roof, the situation may be reversed for flat roofs, particularly those in warmer climates. If water on a flat roof accumulates faster than it runs off, the result is called *ponding*, because the increased load causes the roof to deflect into a dish shape that can hold more water, which causes greater deflections, and so on. This process continues until equilibrium is reached or until collapse occurs. Ponding is a serious matter, as illustrated by the large number of flat-roof failures that occur during rainstorms every year in the United States. It has been claimed that almost 50 percent of the lawsuits faced by building designers are concerned with roofing systems.[3] Ponding is one of the most common subjects of such litigation.

Ponding will occur on almost any flat roof to a certain degree, even though roof drains are present. Roof drains may very well be used, but they may be inadequate during severe storms, or they may become partially or completely clogged. The best method of preventing ponding is to have an appreciable slope of the roof (1/4 in/ft or more), together with good drainage facilities. In addition to the usual ponding, another problem may occur for very large flat roofs (with perhaps an acre or more of surface area). During heavy rainstorms, strong winds frequently occur. If there is a great deal of water on the roof, a strong wind may very well push a large quantity of it toward one end. The result can be a dangerous water depth as regards the load in psf on that end. For such situations, *scuppers* are sometimes used. These are large holes or tubes in walls or parapets that enable water above a certain depth to quickly drain off the roof.

Chapter 8 of ASCE 7-10 provides information for estimating the magnitude of rain loads that may accumulate on flat roofs.

3. *Wind loads.* A survey of engineering literature for the past 150 years reveals many references to structural failures caused by wind. Perhaps the most infamous of these have been bridge failures, such as those of the Tay Bridge in Scotland in 1879 (which caused the deaths of 75 persons) and the Tacoma Narrows Bridge in Tacoma, Washington, in 1940. But there have also been some disastrous building failures due to wind during the same period, such as that of the Union Carbide Building in Toronto in 1958. It is important to realize that a large percentage of building failures due to wind have occurred during the erection of the building.[4]

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[3]Gary Van Ryzin, 1980, "Roof Design: Avoid Ponding by Sloping to Drain," *Civil Engineering* (New York, ASCE, January), pp. 77–81.

[4]"Wind Forces on Structures, Task Committee on Wind Forces. Committee on Loads and Stresses, Structural Division, ASCE, Final Report," *Transactions ASCE* 126, Part II (1961): 1124–1125.

A great deal of research has been conducted in recent years on the subject of wind loads. Nevertheless, a great deal more work needs to be done, as the estimation of these forces can by no means be classified as an exact science. The magnitudes of wind loads vary with geographical locations, heights above ground, types of terrain surrounding the buildings, the proximity and nature of other nearby structures, and other factors.

Wind pressures are frequently assumed to be uniformly applied to the windward surfaces of buildings and are assumed to be capable of coming from any direction. These assumptions are not very accurate, because wind pressures are not uniform over large areas, the pressures near the corners of buildings being probably greater than elsewhere due to wind rushing around the corners, and so on. From a practical standpoint, therefore, all of the possible variations cannot be considered in design, although today's specifications are becoming more and more detailed in their requirements.

When the designer working with large low-rise buildings makes poor wind estimates, the results are probably not too serious, but this is not the case when tall slender buildings (or long flexible bridges) are being designed. For many years, the average designer ignored wind forces for buildings whose heights were not at least twice their least lateral dimensions. For such cases as these, it was felt that the floors and walls provided sufficient lateral stiffnesses to eliminate the need for definite wind bracing systems. A better practice for designers to follow, however, is to consider all the possible loading conditions that a particular structure may have to resist. If one or more of these conditions (perhaps, wind loading) seem of little significance, they may then be neglected. Should buildings have their walls and floors constructed with modern lightweight materials and/or should they be subjected to unusually high wind loads (as in coastal or mountainous areas), they will probably have to be designed for wind loads even if the height/least lateral dimension ratios are less than two.

Building codes do not usually provide for the estimated forces during tornadoes. The average designer considers the forces created directly in the paths of tornadoes to be so violent that it is not economically feasible to design buildings to resist them. This opinion is undergoing some change, however, as it has been found that the wind resistance of structures (even of small buildings, including houses) can be greatly increased at very reasonable costs by better practices—tying the parts of structures together from the roofs through the walls to the footings—and by making appropriate connections of window frames to walls and, perhaps, other parts of the structure.[5,6]

Wind forces act as pressures on vertical windward surfaces, pressures or suction on sloping windward surfaces (depending on the slope), and suction on flat surfaces and on leeward vertical and sloping surfaces (due to the creation of negative pressures or vacuums). The student may have noticed this definite suction effect where shingles or other roof coverings have been lifted from the leeward roof surfaces of buildings during windstorms. Suction or uplift can easily be demonstrated by holding a piece of paper at two of its corners and blowing above it. For some common structures, uplift loads may be as large as 20 to 30 psf or even more.

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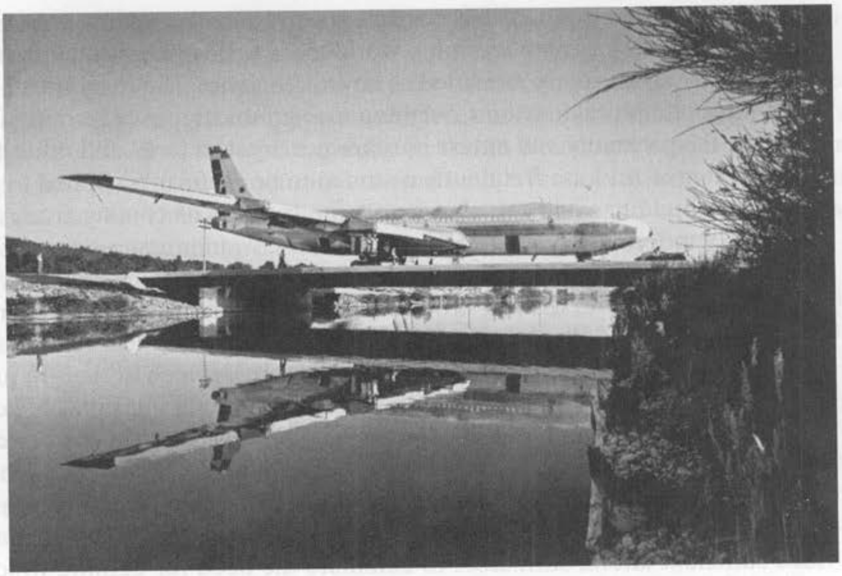
[5]P. R. Sparks, "Wind Induced Instability in Low-Rise Buildings," *Proceedings of the 5th U.S. National Conference on Wind Engineering*, Lubbock, TX, November 6–18, 1985.

[6]P. R. Sparks, "The Risk of Progressive Collapse of Single-Story Buildings in Severe Storms," *Proceedings of the ASCE Structures Congress*, Orlando, FL, August 17–20, 1987.



Access bridge, Renton, WA. (Courtesy of the Bethlehem Steel Corporation.)

During the passing of a tornado or hurricane, a sharp reduction in atmospheric pressure occurs. This decrease in pressure does not penetrate airtight buildings, and the inside pressures, being greater than the external pressures, cause outward forces against the roofs and walls. Nearly everyone has heard stories of the walls of a building "exploding" outward during a storm.

As you can see, the accurate calculation of the most severe wind pressures that need to be considered for the design of buildings and bridges is quite an involved problem. Despite this fact, sufficient information is available today to permit the satisfactory estimation of these pressures in a reasonably efficient manner.

A procedure for estimating the wind pressures applied to buildings is presented in Chapters 26–31 of ASCE 7-10. Several factors are involved when we attempt to account for the effects of wind speed, shape and orientation of the building in question, terrain characteristics around the structure, importance of the building as to human life and welfare, and so on. Though the procedure seems rather complex, it is greatly simplified with the tables presented in the aforementioned specification.

4. *Earthquake loads*. Many areas of the world fall in "earthquake territory," and in those areas it is necessary to consider seismic forces in design for all types of structures. Through the centuries, there have been catastrophic failures of buildings, bridges, and other structures during earthquakes. It has been estimated that as many as 50,000 people lost their lives in the 1988 earthquake in Armenia.[7] The 1989 Loma Prieta and 1994 Northridge earthquakes in California caused many billions of dollars of property damage, as well as considerable loss of life.

[7]V. Fairweather, "The Next Earthquake," *Civil Engineering* (New York: ASCE, March 1990), pp. 54–57.

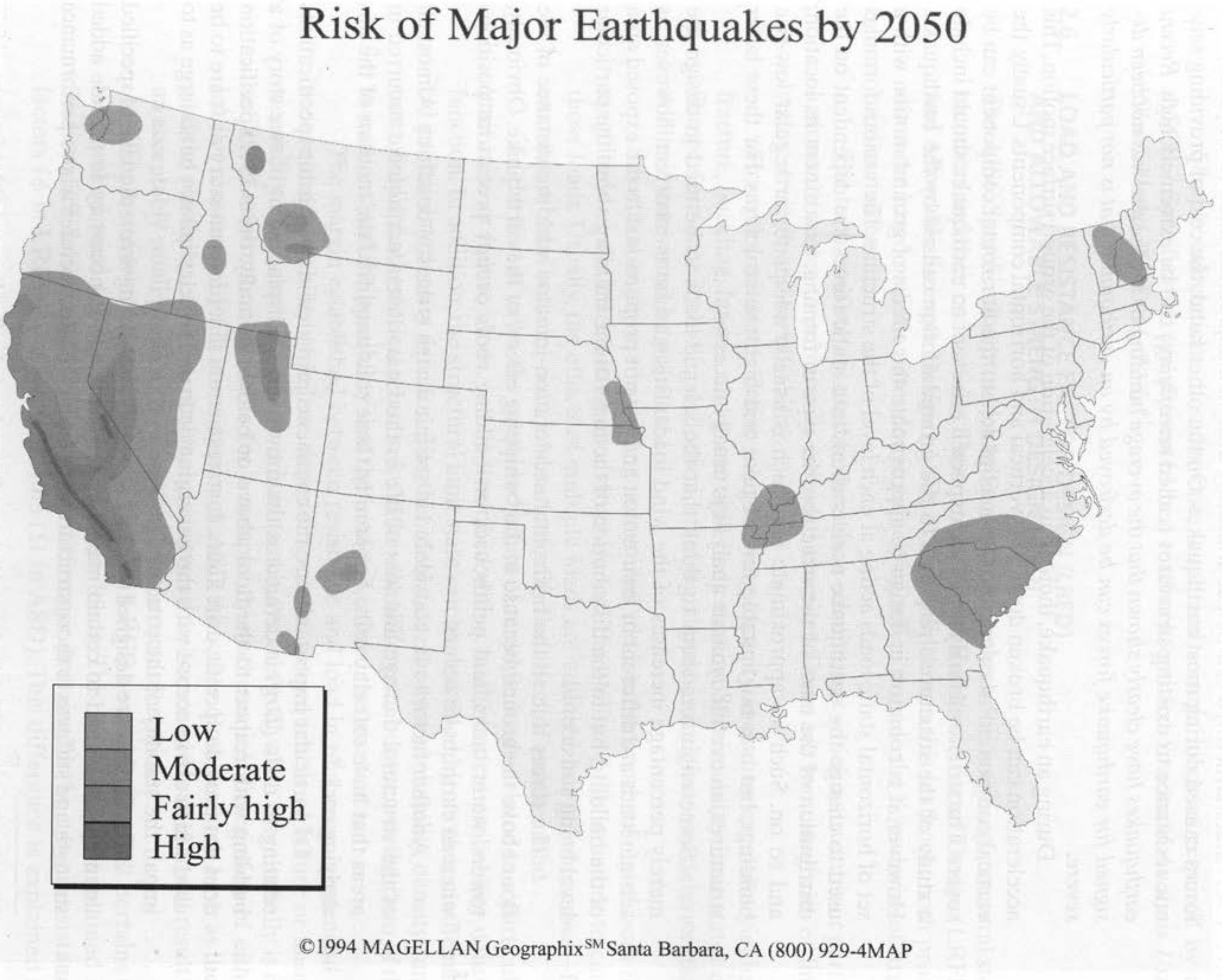

Risk of Major Earthquakes by 2050
Low
Moderate
Fairly high
High
©1994 MAGELLAN Geographix℠ Santa Barbara, CA (800) 929-4MAP

FIGURE 2.1

Figure 2.1 shows regions of the United States that are more susceptible to seismic events. These regions were established on the basis of data from past earthquakes.[8]

Steel structures can be economically designed and constructed to withstand the forces caused during most earthquakes. On the other hand, the cost of providing seismic resistance to existing structures (called *retrofitting*) can be extremely high. *Recent earthquakes have clearly shown that the average building or bridge that has not been designed for earthquake forces can be destroyed by an earthquake that is not particularly severe.*

During an earthquake, there is an acceleration of the ground surface motion. This acceleration can be broken down into vertical and horizontal components. Usually, the vertical component is assumed to be negligible, but the horizontal component can be severe. The structural analysis for the expected effects of an earthquake should include a study of the structure's response to the ground motion caused by the earthquake. However, it is common in design to approximate the effects of ground motion with a set of horizontal static loads acting at each level of the structure. The various formulas used to change the earthquake accelerations into static forces are dependent on the distribution of the mass of the structure, the type of framing, its stiffness, its location, and so on. Such an approximate approach is usually adequate for regular low-rise buildings, but it is not suitable for irregular and high-rise structures. For these latter structures, an overall dynamic analysis is usually necessary.

Some engineers seem to think that the seismic loads to be used in design are merely percentage increases of the wind loads. This surmise is incorrect, however, as seismic loads are different in their action and are not proportional to the exposed area of the building, but to the distribution of the mass of the building above the particular level being considered.

The forces due to the horizontal acceleration increase with the distance of the floor above the ground because of the "whipping effect" of the earthquake. Obviously, towers, water tanks, and penthouses on building roofs occupy precarious positions when an earthquake occurs.

Another factor to be considered in seismic design is the soil condition. Almost all of the structural damage and loss of life in the Loma Prieta earthquake occurred in areas that have soft clay soils. Apparently, these soils amplified the motions of the underlying rock.[9]

Of particular importance are the comments provided in the seismic specifications relating to drift. (*Drift* is defined as the movement or displacement of one story of a building with respect to the floor above or below.) Actually, the AISC Specification does not provide specific drift limits. It states merely that limits on story drift are to be used which are in accord with the governing code and which shall not be so large as to impair the stability of the structure.

If structures are designed so that computed drifts during an earthquake of specified intensity are limited to certain maximum values, it will be necessary to provide added strength and stiffness to those structures. The result will be structures whose performance

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[8]*American Society of Civil Engineers Minimum Design Loads for Buildings and Other Structures*, ASCE 7–88 (New York: ASCE), pp. 33, 34.

[9]Fairweather, op. cit.

is substantially improved during earthquakes. The AISC Manual does not provide detailed specifications for designing structures subject to seismic loads, but such information is presented in the companion *AISC Seismic Design Manual*,[10] as well as in ASCE 7-10.

Sample calculations of snow, rain, wind, and seismic loads as required by the ASCE 7 Specification are presented in a textbook entitled *Structural Analysis Using Classical and Matrix Methods*.[11]

2.6 LOAD AND RESISTANCE FACTOR DESIGN (LRFD) AND ALLOWABLE STRENGTH DESIGN (ASD)

The AISC Specification provides two acceptable methods for designing structural steel members and their connections. These are **Load and Resistance Factor Design** (LRFD) and **Allowable Strength Design** (ASD). As we will learn in this textbook, both procedures are based on limit states design principles, which provide the boundaries of structural usefulness.

The term **limit state** is used to describe a condition at which a structure or part of a structure ceases to perform its intended function. There are two categories of limit states: strength and serviceability.

Strength limit states define load-carrying capacity, including excessive yielding, fracture, buckling, fatigue, and gross rigid body motion. Serviceability limit states define performance, including deflection, cracking, slipping, vibration, and deterioration. All limit states must be prevented.

Structural engineers have long recognized the inherent uncertainty of both the magnitude of the loads acting on a structure and the ability of the structure to carry those loads. Usually, the effects of multiple loads are additive, but in some cases—for example, a beam column—one load can magnify the effect of another load.

In the best of cases, the combined effect of multiple loads, related to a particular limit state or failure mode, can be described with a mathematical probability density function. In addition, the structural limit state can be described by another mathematical probability density function. For this ideal case, the two probability density functions yield a mathematical relation for either the difference between or the ratio of the two means, and the possibility that the load will exceed the resistance.

The margin established between resistance and load in real cases is intended to reduce the probability of failure, depending on the consequences of failure or unserviceability. The question we have is how to achieve this goal when there usually is insufficient information available for a completely mathematical description of either load or resistance. LRFD is one approach; ASD is another. Both methods have as their goal the obtaining of a numerical margin between resistance and load that will result in an acceptably small probability of unacceptable structural response.

There are two major differences between LRFD and ASD. The first pertains to the method used for calculating the design loads. This difference is explained in Sections 2.9, 2.10, and 2.11. The second difference pertains to the use of resistance factors (ϕ in LRFD) and safety factors (Ω in ASD). This difference is explained in

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[10]American Institute of Steel Construction, 2006 (Chicago: AISC).
[11]J. C. McCormac, 2007 (Hoboken, NJ: John Wiley & Sons, Inc.), pp. 24–40.

Sections 2.12 and 2.13. These five sections should clearly fix in the reader's mind an understanding of the differences between LRFD and ASD. It is also important to realize that both LRFD and ASD employ the same methods of structural analysis. Obviously, the behavior of a given structure is independent of the method by which it is designed.

With both the LRFD procedure and the ASD procedure, expected values of the individual loads (dead, live, wind, snow, etc.) are estimated in exactly the same manner as required by the applicable specification. These loads are referred to as **service** or **working loads** throughout the text. Various combinations of these loads, which may feasibly occur at the same time, are grouped together and the largest values so obtained used for analysis and design of structures. The largest load group (in ASD) or the largest linear combination of loads in a group (in LRFD) is then used for analysis and design.

2.7 NOMINAL STRENGTHS

With both LRFD and ASD, the term **nominal strength** is constantly used. The nominal strength of a member is its calculated theoretical strength, with no safety factors (Ω_s) or resistance factors (ϕ_s) applied. In LRFD, a resistance factor, usually less than 1.0, is multiplied by the nominal strength of a member, or in ASD, the nominal strength is divided by a safety factor, usually greater than 1.0, to account for variations in material strength, member dimensions, and workmanship as well as the manner and consequences of failure. The calculation of nominal strengths for tension members is illustrated in Chapter 3, and in subsequent chapters for other types of members.

2.8 SHADING

Though the author does not think the reader will have any trouble whatsoever distinguishing between and making the calculations for the LRFD and ASD methods, in much of the book he has kept them somewhat separate by shading the ASD materials. He selected the ASD method to be shaded because the numbers in the Steel Manual pertaining to that method are shaded (actually, with a green color there).

2.9 COMPUTATION OF LOADS FOR LRFD AND ASD

With both the LRFD and the ASD procedures, expected values of the individual loads (dead, live, wind, snow, etc.) are first estimated in exactly the same manner as required by the applicable specification. These loads are referred to as **service** or **working loads** throughout the text. Various combinations of these loads that feasibly may occur at the same time are grouped together. The largest load group (in ASD) or the largest linear combination of loads in a group (in LRFD) is then used for analysis and design.

In this section and the next two sections, the loading conditions used for LRFD and ASD are presented. In both methods the individual loads (dead, live, and environmental) are estimated in exactly the same manner. After the individual loads are estimated, the next problem is to decide the worst possible combinations of those loads which might occur at the same time and which should be used in analysis and design.

LRFD Load Combinations

With the LRFD method, possible service load groups are formed, and each service load is multiplied by a load factor, normally larger than 1.0. The magnitude of the load factor reflects the uncertainty of that particular load. The resulting linear combination of service loads in a group, each multiplied by its respective load factor, is called a **factored load**. The largest values determined in this manner are used to compute the moments, shears, and other forces in the structure. These controlling values may not be larger than the nominal strengths of the members multiplied by their reduction or ϕ factors. Thus, the factors of safety have been incorporated in the load factors, and we can say

$$(\text{Reduction factor } \phi)(\text{Nominal strength of a member}) \geq \text{computed factored force in member, } R_u$$

$$\phi R_n \geq R_u$$

ASD Load Combinations

With ASD, the service loads are generally not multiplied by load factors or safety factors. Rather, they are summed up, as is, for various feasible combinations, and the largest values so obtained are used to compute the forces in the members. These total forces may not be greater than the nominal strengths of the members, divided by appropriate safety factors. In equation form, the statement may be written as

$$\frac{\text{Nominal strength of member}}{\text{Safety factor } \Omega} \geq \text{largest computed force in member, } R_a.$$

$$\frac{R_n}{\Omega} \geq R_a$$

2.10 COMPUTING COMBINED LOADS WITH LRFD EXPRESSIONS

In Part 2 of the Steel Manual, entitled "General Design Considerations," load factors are calculated to increase the magnitudes of service loads to use with the LRFD procedure. The purpose of these factors is to account for the uncertainties involved in estimating the magnitudes of dead and live loads. To give the reader an idea of what we are talking about, the author poses the following question: "How close, in percent, can you estimate the worst wind or snow load that will ever be applied to the building you are now occupying?" As you think about that for a while, you will probably begin to run up your values considerably.

The required strength of a member for LRFD is determined from the load combinations given in the applicable building code. In the absence of such a code, the values given in ASCE 7 seem to be good ones to use. Part 2 of the AISC Manual provides the following load factors for buildings, which are based on ASCE 7 and are the values used in this text:

1. $U = 1.4D$
2. $U = 1.2D + 1.6L + 0.5\,(L_r \text{ or } S \text{ or } R)$
3. $U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L^* \text{ or } 0.5W)$
4. $U = 1.2D + 1.0W + L^* + 0.5(L_r \text{ or } S \text{ or } R)$

5. $U = 1.2D + 1.0E + L^* + 0.2S$
6. $U = 0.9D + 1.0W$
7. $U = 0.9D + 1.0E$

*The load factor on L in combinations (3.), (4.), and (5.) is to be taken as 1.0 for floors in places of public assembly, for live loads in excess of 100 psf and for parking garage live load. The load factor is permitted to equal 0.5 for other live loads.

In these load combinations, the following abbreviations are used:

U = the design or factored load
D = dead load
L = live load due to occupancy
L_r = roof live load
S = snow load
R = nominal load due to initial rainwater or ice, exclusive of the ponding contribution
W = wind load
E = earthquake load

The load factors for dead loads are smaller than the ones for live loads, because designers can estimate so much more accurately the magnitudes of dead loads than of live loads. In this regard, the student will notice that loads which remain in place for long periods will be less variable in magnitude, while those that are applied for brief periods, such as wind loads, will have larger variations.

It is hoped that the discussion of these load factors will make the designer more conscious of load variations.

The service load values D, L, L_r, S, R, W, and E are all mean values. The different load combinations reflect 50-year recurrence values for different transient loads. In each of these equations, one of the loads is given its maximum estimated value in a 50-year period, and that maximum is combined with several other loads whose magnitudes are estimated at the time of that particular maximum load. You should notice in Equations 4, 5, 6, and 7 that the wind and seismic load factors are given as 1.0. Usually, building codes convert wind and seismic loads to ultimate or factored values. Thus, they have already been multiplied by a load factor. If that is not the case, a load factor larger than 1.0 must be used.

The preceding load factors do not vary in relation to the seriousness of failure. The reader may feel that a higher load factor should be used for a hospital than for a cattle barn, but this is not required. It is assumed, however, that the designer will consider the seriousness of failure when he or she specifies the magnitudes of the service loads. It also should be realized that the ASCE 7 load factors are minimum values, and the designer is perfectly free to use larger ones if it is deemed prudent.

The following are several additional comments regarding the application of the LRFD load combination expressions:

1. It is to be noted that in selecting design loads, adequate allowance must be made for impact conditions before the loads are substituted into the combination expressions.

2. Load combinations 6 and 7 are used to account for the possibilities of uplift. Such a condition is included to cover cases in which tension forces develop, owing to overturning moments. It will govern only for tall buildings where high lateral loads are present. In these combinations, the dead loads are reduced by 10 percent to take into account situations where they may have been overestimated.
3. It is to be clearly noted that the wind and earthquake forces have signs—that is, they may be compressive or they may be tensile (that is, tending to cause uplift). Thus, the signs must be accounted for in substituting into the load combinations. The ± signs aren't so much a matter of tension or compression as they are of saying that wind and earthquake loads can be in any horizontal and sometimes vertical direction. Load combinations 6 and 7 apply specifically to the case in which loads in a member due to wind or earthquake and gravity dead load counteract each other. For a particular column, the maximum tensile W or E force will, in all probability, be different from its maximum compressive force.
4. The magnitudes of the loads (D, L, L_r, etc.) should be obtained from the governing building code or from ASCE 7-10. Wherever applicable, the live loads used for design should be the reduced values specified for large floor areas, multistory buildings, and so on.

Examples 2-1 to 2-3 show the calculation of the factored loads, using the applicable LRFD load combinations. The largest value obtained is referred to as the *critical* or *governing load combination* and is to be used in design.

Example 2-1

The interior floor system shown in Figure 2.2 has W24 × 55 sections spaced 8 ft on center and is supporting a floor dead load of 50 psf and a live floor load of 80 psf. Determine the governing load in lb/ft that each beam must support.

Solution. Note that each foot of the beam must support itself (a dead load) plus $8 \times 1 = 8\text{ ft}^2$ of the building floor.

$$D = 55\text{ lb/ft} + (8\text{ ft})(50\text{ psf}) = 455\text{ lb/ft}$$
$$L = (8\text{ ft})(80\text{ psf}) = 640\text{ lb/ft}$$

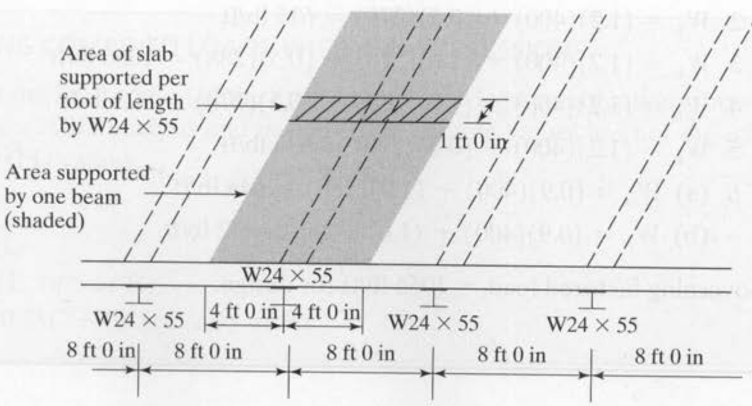


FIGURE 2.2

Computing factored loads, using the LRFD load combinations. In this substitution, the terms having no values are omitted. Note that with a floor live load of 80 psf a load factor of 0.5 has been added to load combinations (3.), (4.), and (5.) per the exception stated in ASCE 7-10 and this text for floor live loads.

1. $W_u = (1.4)(455) = 637$ lb/ft
2. $W_u = (1.2)(455) + (1.6)(640) = 1570$ lb/ft
3. $W_u = (1.2)(455) + (0.5)(640) = 866$ lb/ft
4. $W_u = (1.2)(455) + (0.5)(640) = 866$ lb/ft
5. $W_u = (1.2)(455) + (0.5)(640) = 866$ lb/ft
6. $W_u = (0.9)(455) = 409.5$ lb/ft
7. $W_u = (0.9)(455) = 409.5$ lb/ft

Governing factored load = 1570 lb/ft to be used for design.

Ans. 1570 lb/ft

Example 2-2

A roof system with W16 × 40 sections spaced 9 ft on center is to be used to support a dead load of 40 psf; a roof live, snow, or rain load of 30 psf; and a wind load of ±32 psf. Compute the governing factored load per linear foot.

Solution.

$$D = 40 \text{ lb/ft} + (9 \text{ ft})(40 \text{ psf}) = 400 \text{ lb/ft}$$
$$L = 0$$
$$L_r \text{ or } S \text{ or } R = (9 \text{ ft})(30 \text{ psf}) = 270 \text{ lb/ft}$$
$$W = (9 \text{ ft})(32 \text{ psf}) = 288 \text{ lb/ft}$$

Substituting into the load combination expressions and noting that the wind can be downward, − or uplift, + in Equation 6, we derive the following loads:

1. $W_u = (1.4)(400) = 560$ lb/ft
2. $W_u = (1.2)(400) + (0.5)(270) = 615$ lb/ft
3. $W_u = (1.2)(400) + (1.6)(270) + (0.5)(288) = 1056$ lb/ft
4. $W_u = (1.2)(400) + (1.0)(288) + (0.5)(270) = 903$ lb/ft
5. $W_u = (1.2)(400) + (0.2)(270) = 534$ lb/ft
6. (a) $W_u = (0.9)(400) + (1.0)(288) = 648$ lb/ft

   (b) $W_u = (0.9)(400) + (1.0)(-288) = 72$ lb/ft

Governing factored load = 1056 lb/ft for design.

Ans. 1056 lb/ft

Example 2-3

The various axial loads for a building column have been computed according to the applicable building code, with the following results: dead load = 200 k; load from roof = 50 k (roof live load); live load from floors (reduced as applicable for large floor area and multistory columns) = 250 k; compression wind = 128 k; tensile wind = 104 k; compression earthquake = 60 k; and tensile earthquake = 70 k.

Determine the critical design column load, P_u, using the LRFD load combinations.

Solution.

This problem solution assumes the column floor live load meets the exception for the use of the load factor of 0.5 in load combinations (3.), (4.), and (5.)

1. $P_u = (1.4)(200) = 280$ k
2. $P_u = (1.2)(200) + (1.6)(250) + (0.5)(50) = 665$ k
3. (a) $P_u = (1.2)(200) + (1.6)(50) + (0.5)(250) = 445$ k
   (b) $P_u = (1.2)(200) + (1.6)(50) + (0.5)(128) = 384$ k
4. (a) $P_u = (1.2)(200) + (1.0)(128) + (0.5)(250) + (0.5)(50) = 518$ k
   (b) $P_u = (1.2)(200) - (1.0)(104) + (0.5)(250) + (0.5)(50) = 286$ k
5. (a) $P_u = (1.2)(200) + (1.0)(60) + (0.5)(250) = 425$ k
   (b) $P_u = (1.2)(200) - (1.0)(70) + (0.5)(250) = 295$ k
6. (a) $P_u = (0.9)(200) + (1.0)(128) = 308$ k
   (b) $P_u = (0.9)(200) - (1.0)(104) = 76$ k
7. (a) $P_u = (0.9)(200) + (1.0)(60) = 240$ k
   (b) $P_u = (0.9)(200) - (1.0)(70) = 110$ k

The critical factored load combination, or design strength, required for this column is 665 k, as determined by load combination (2). It will be noted that the results of combination (6a) and (6b) do not indicate an uplift problem.

Ans. 665 k

2.11 COMPUTING COMBINED LOADS WITH ASD EXPRESSIONS

In Part 2 of the 2011 edition of the Steel Manual, the load combinations shown next are presented for ASD analysis and design. The resulting values are not interchangeable with the LRFD values.

1. D
2. D + L
3. D + (L_r or S or R)
4. D + 0.75L + 0.75(L_r or S or R)

5. $D + (0.6W \text{ or } 0.7E)$
6. (a) $D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$

   (b) $D + 0.75L + 0.75(0.7E) + 0.75(S)$
7. $0.6D + 0.6W$
8. $0.6D + 0.7E$

In the seventh and eighth expressions, the reader should note that the full dead load is not used. The variable loads W and E have lateral components and tend to cause the structure to overturn. On the other hand, the dead load is a gravity load, which tends to prevent overturning. Therefore, it can be seen that a more severe condition occurs if for some reason the full dead load is not present.

The student must realize that the AISC Specification provides what the AISC deems to be the maximum loads to be considered for a particular structure. If in the judgment of the designer the loads will be worse than the recommended values, then the values may certainly be increased. As an illustration, if the designer feels that the maximum values of the wind and rain may occur at the same time in his or her area, the 0.75 factor may be neglected. The designer should carefully consider whether the load combinations specified adequately cover all the possible combinations for a particular structure. If it is thought that they do not, he or she is free to consider additional loads and combinations as may seem appropriate. This is true for LRFD and ASD.

Example 2-4, which follows, presents the calculation of the governing ASD load to be used for the roof system of Example 2-2.

Example 2-4

Applying the ASD load combinations recommended by the AISC, determine the load to be used for the roof system of Example 2-2, where $D = 400$ lb/ft, L_r or S or $R = 270$ lb/ft, and $W = 300$ lb/ft. Assume that wind can be plus or minus.

Solution.

1. $W_a = 400$ lb/ft
2. $W_a = 400$ lb/ft
3. $W_a = 400 + 270 = 670$ lb/ft
4. $W_a = 400 + (0.75)(270) = 602.5$ lb/ft
5. $W_a = 400 + (0.6)(300) = 580$ lb/ft
6. (a) $W_a = 400 + 0.75[(0.6)(300)] + 0.75(270) = 737.5$ lb/ft

   (b) $W_a = 400 + 0.75(270) = 602.5$ lb/ft
7. $W_a = (0.6)(400) + (0.6)(-300) = 60$ lb/ft
8. $W_a = (0.6)(400) = 240$ lb/ft

Governing load $= 737.5$ lb/ft.

2.12 TWO METHODS OF OBTAINING AN ACCEPTABLE LEVEL OF SAFETY

The margin established between resistance and load in real cases is intended to reduce the probability of failure or unserviceability to an acceptably small value, depending on the consequences of failure or unserviceability. The question we have is how to achieve this goal when there usually is insufficient information for a complete mathematical description of either load or resistance. LRFD is one approach; ASD is another. Both methods have as their goal the obtaining of a numerical margin between resistance and load that will result in an acceptably small chance of unacceptable structural response.

A safety factor, Ω, is a number usually greater than 1.0 used in the ASD method. The nominal strength for a given limit state is divided by Ω and the result compared with the applicable service load condition.

A resistance factor, ϕ, is a number usually less than 1.0, used in the LRFD method. The nominal strength for a given limit state is multiplied by ϕ and the result compared with the applicable factored load condition.

The relationship between the safety factor Ω and the resistance factor ϕ is one we should remember. In general $\Omega = \frac{1.5}{\phi}$. (For instance if $\phi = 0.9$, Ω will equal $\frac{1.5}{0.9} = 1.67$. If $\phi = 0.75$, Ω will equal $\frac{1.50}{0.75} = 2.00$.)

The load factors in the linear combination of loads in a service load group do not have a standard symbol in the AISC Manual, but the symbol λ will be used here.

Thus if we set

$$Q_i = \text{one of N service loads in a group}$$
$$\lambda_i = \text{load factor associated with loads in LRFD}$$
$$R_n = \text{nominal structural strength}$$

Then for LRFD

$$\phi R_n \geq \sum_{i=1}^{N} \lambda_i Q_i$$

And for ASD

$$\frac{R_n}{\Omega} \geq \sum_{i=1}^{N} Q_i$$

2.13 DISCUSSION OF SIZES OF LOAD FACTORS AND SAFETY FACTORS

Students may sometimes feel that it is foolish to design structures with such large load factors in LRFD design and such large safety factors in ASD design. As the years go by, however, they will learn that these values are subject to so many uncertainties that they may very well spend many sleepless nights wondering if those they have used are sufficient (and they may join other designers in calling them "factors of ignorance"). Some of the uncertainties affecting these factors are as follows:

1. Material strengths may initially vary appreciably from their assumed values, and they will vary more with time due to creep, corrosion, and fatigue.
2. The methods of analysis are often subject to considerable errors.

3. The so-called vagaries of nature, or acts of God (hurricanes, earthquakes, etc.), cause conditions difficult to predict.
4. The stresses produced during fabrication and erection are often severe. Laborers in shop and field seem to treat steel shapes with reckless abandon. They drop them. They ram them. They force the members into position to line up the bolt holes. In fact, the stresses during fabrication and erection may exceed those that occur after the structure is completed. The floors for the rooms of apartment houses and office buildings are probably designed for service live loads varying from 40 to 80 psf (pounds per square foot). During the erection of such buildings, the contractor may have 10 ft of bricks or concrete blocks or other construction materials or equipment piled up on some of the floors (without the knowledge of the structural engineer), causing loads of several hundred pounds per square foot. This discussion is not intended to criticize the practice (not that it is a good one), but rather to make the student aware of the things that happen during construction. (It is probable that the majority of steel structures are overloaded somewhere during construction, but hardly any of them fail. On many of these occasions, the ductility of the steel has surely saved the day.)
5. There are technological changes that affect the magnitude of live loads. The constantly increasing traffic loads applied to bridges through the years is one illustration. The wind also seems to blow harder as the years go by, or at least, building codes keep raising the minimum design wind pressures as more is learned about the subject.
6. Although the dead loads of a structure can usually be estimated quite closely, the estimate of the live loads is more inaccurate. This is particularly true in estimating the worst possible combination of live loads occurring at any one time.
7. Other uncertainties are the presence of residual stresses and stress concentrations, variations in dimensions of member cross sections, and so on.

2.14 AUTHOR'S COMMENT

Should designs be made by both LRFD and ASD, the results will be quite close to each other. On some occasions, the LRFD designs will be slightly more economical. In effect, the smaller load factor used for dead loads in LRFD designs, compared with the load factors used for live loads, gives LRFD a little advantage. With ASD design, the safety factor used for both dead and live loads is constant for a particular problem.

2.15 PROBLEMS FOR SOLUTION

For Probs. 2-1 through 2-4 determine the maximum combined loads using the recommended AISC expressions for LRFD.

2-1 $D = 100$ psf, $L = 70$ psf, $R = 12$ psf, $L_r = 20$ psf and $S = 30$ psf (*Ans.* 247 psf)

2-2 $D = 12{,}000$ lb, $W = \pm 52{,}000$ lb

2-3 $D = 9000$ lb, $L = 5000$ lb, $L_r = 2500$ lb, $E = \pm 6500$ lb (*Ans.* 20,050 lb)

2-4 $D = 24$ psf, $L_r = 16$ psf and $W = \pm 42$ psf

2-5 Structural steel beams are to be placed at 7 ft 6 in on center under a reinforced concrete floor slab. If they are to support a service dead load D = 64 psf of floor area and a service live load L = 100 psf of floor area, determine the factored uniform load per foot which each beam must support. (*Ans.* 1776 plf)

2-6 A structural steel beam supports a roof that weighs 20 psf. An analysis of the loads has the following: S = 12 psf, L_r = 18 psf and W = 38 psf (upwards) or 16 psf (downwards). If the beams are spaced 6 ft 0 in apart, determine the factored uniformly distributed loads per foot (upward and downward, if appropriate) by which each beam should be designed.

For Probs. 2-7 through 2-10 compute the maximum combined loads using the recommended ASD expressions from the AISC.

2-7 Rework Problem 2-1. (*Ans.* 175 psf)

2-8 Rework Problem 2-2.

2-9 Rework Problem 2-3. (*Ans.* 18,037.5 lb)

2-10 Rework Problem 2-4.

2-11 Structural steel beams are to be placed at 7 ft 6 in on center under a reinforced concrete floor slab. If they are to support a service dead load D = 64 psf of floor area and a service live load L = 100 psf of floor area, determine the uniform load per foot which each beam must support using the ASD expressions. (*Ans.* 1230 plf)

2-12 A structural steel beam supports a roof that weighs 20 psf. An analysis of the loads has the following: S = 12 psf, L_r = 18 psf and W = 38 psf (upwards) or 16 psf (downwards). If the beams are spaced 6 ft 0 in apart, determine the uniformly distributed loads per foot (upward and downward, if appropriate) by which each beam should be designed using the ASD expressions.

CHAPTER 3

Analysis of Tension Members

3.1 INTRODUCTION

Tension members are found in bridge and roof trusses, towers, and bracing systems, and in situations where they are used as tie rods. The selection of a section to be used as a tension member is one of the simplest problems encountered in design. As there is no danger of the member buckling, the designer needs to determine only the load to be supported, as previously described in Chapter 2. Then the area required to support that load is calculated as described in Chapter 4, and finally a steel section is selected that provides the required area. Though these introductory calculations for tension members are quite simple, they serve the important tasks of starting students off with design ideas and getting their "feet wet" with the massive Steel Manual.

One of the simplest forms of tension members is the circular rod, but there is some difficulty in connecting it to many structures. The rod has been used frequently in the past, but finds only occasional uses today in bracing systems, light trusses, and timber construction. One important reason rods are not popular with designers is that they have been used improperly so often in the past that they have a bad reputation; however, if designed and installed correctly, they are satisfactory for many situations.

The average-size rod has very little bending stiffness and may quite easily sag under its own weight, injuring the appearance of the structure. The threaded rods formerly used in bridges often worked loose and rattled. Another disadvantage of rods is the difficulty of fabricating them with the exact lengths required and the consequent difficulties of installation.

When rods are used in wind bracing, it is a good practice to produce initial tension in them, as this will tighten up the structure and reduce rattling and swaying. Prestressing the rods limits the amount of compression they will experience during load reversal. (In a similar manner, bicycle wheel spokes are prestressed in tension to prevent the development of compression in them.) To obtain initial tension, the members may be detailed shorter than their required lengths, a method that gives the steel fabricator very little trouble. A common rule of thumb is to detail the rods about 1/16 in short for each 20 ft of length. (Approximate stress

$f = \epsilon E = \dfrac{\left(\dfrac{1}{16}\text{ in}\right)}{\left(12\dfrac{\text{in}}{\text{ft}}\right)(20\text{ ft})}(29 \times 10^6\text{ psi}) = 7550\text{ psi.})$ Another very satisfactory method involves tightening the rods with some sort of sleeve nut or turnbuckle. Table 15-5 of the Steel Manual provides detailed information for such devices.

The preceding discussion on rods should illustrate why rolled shapes such as angles have supplanted rods for most applications. In the early days of steel structures, tension members consisted of rods, bars, and perhaps cables. Today, although the use of cables is increasing for suspended-roof structures, tension members usually consist of single angles, double angles, tees, channels, W sections, or sections built up from plates or rolled shapes. These members look better than the old ones, are stiffer, and are easier to connect. Another type of tension section often used is the welded tension plate, or flat bar, which is very satisfactory for use in transmission towers, signs, foot bridges, and similar structures.

A few of the various types of tension members in general use are illustrated in Fig. 3.1. In this figure, the dotted lines represent the intermittent tie plates or bars used to connect the shapes.

The tension members of steel roof trusses may consist of single angles as small as 2 1/2 × 2 × 1/4 for minor members. A more satisfactory member is made from two angles placed back to back, with sufficient space between them to permit the insertion of plates (called gusset plates) for connection purposes. Where steel sections are used back-to-back in this manner, they should be connected to each other every 4 or 5 ft to prevent rattling, particularly in bridge trusses. Single angles and double angles are probably the most common types of tension members in use. Structural tees make very satisfactory chord members for welded trusses, because the angles just mentioned can conveniently be connected to the webs of the tees.

For bridges and large roof trusses, tension members may consist of channels, W or S shapes, or even sections built up from some combination of angles, channels, and plates. Single channels are frequently used, as they have little eccentricity and are conveniently connected. Although, for the same weight, W sections are stiffer than S sections, they may have a connection disadvantage in their varying depths. For instance,

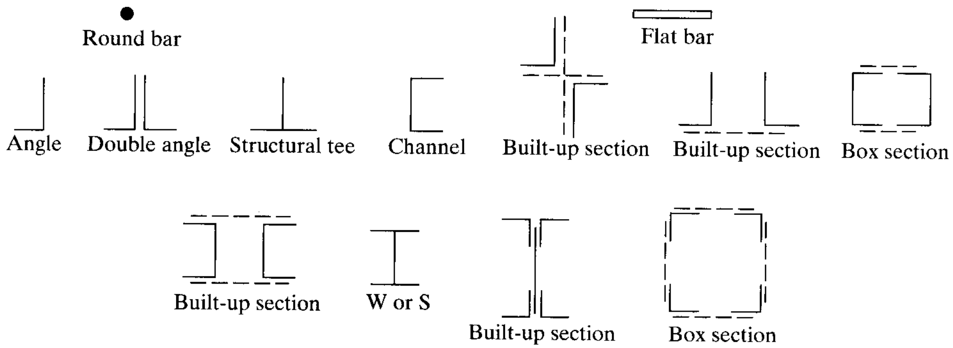


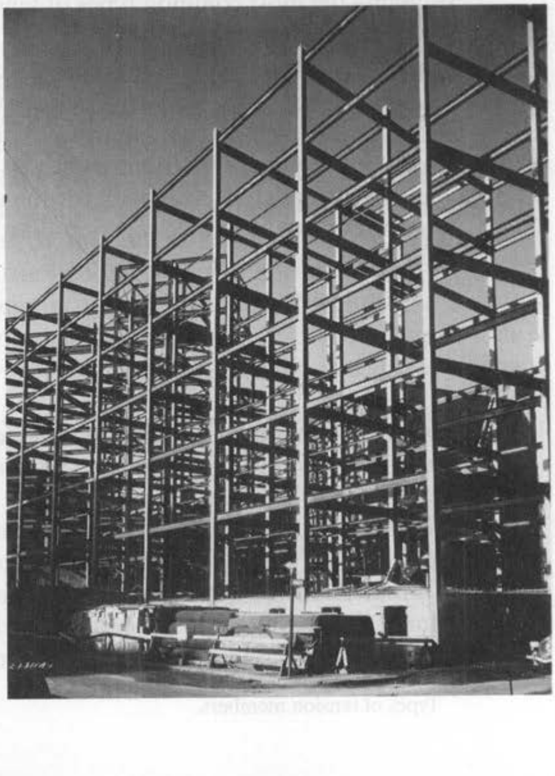
FIGURE 3.1

Types of tension members.

the W12 × 79, W12 × 72, and W12 × 65 all have slightly different depths (12.4 in, 12.3 in, and 12.1 in, respectively), while the S sections of a certain nominal size all have the same depths. For instance, the S12 × 50, the S12 × 40.8, and the S12 × 35 all have 12.00-in depths.

Although single structural shapes are a little more economical than built-up sections, the latter are occasionally used when the designer is unable to obtain sufficient area or rigidity from single shapes. It is important to remember that where built-up sections are used, field connections will have to be made and paint applied; therefore, sufficient space must be available to accomplish these things.

Members consisting of more than one section need to be tied together. Tie plates (also called *tie bars*), located at various intervals, or perforated cover plates serve to hold the various pieces in their correct positions. These plates correct any unequal distribution of loads between the various parts. They also keep the slenderness ratios of the individual parts within limitations, and they may permit easier handling of the built-up members. Long individual members such as angles may be inconvenient to handle due to flexibility, but when four angles are laced together into one member, as shown in Fig. 3.1, the member has considerable stiffness. None of the intermittent tie plates may be considered to increase the effective cross-sectional areas of the sections. As they do not theoretically carry portions of the force in the main sections, their sizes are usually governed by specifications and perhaps by some judgment on the designer's part. Perforated cover plates (see Fig. 6.9) are an exception to this rule, as part of their areas can be considered effective in resisting axial load.



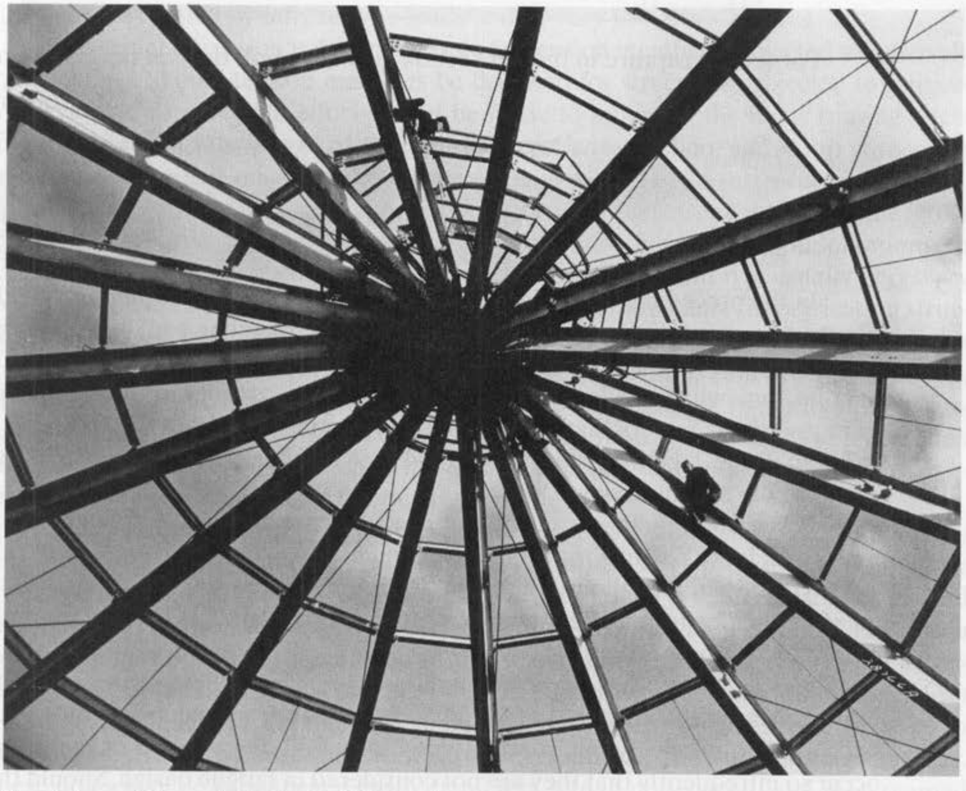
A building framework under construction.
(Courtesy of Bethlehem Steel Corporation.)

Steel cables are made with special steel alloy wire ropes that are cold-drawn to the desired diameter. The resulting wires with strengths of about 200,000 to 250,000 psi can be economically used for suspension bridges, cable supported roofs, ski lifts, and other similar applications.

Normally, to select a cable tension member, the designer uses a manufacturer's catalog. The yield stress of the steel and the cable size required for the design force are determined from the catalog. It is also possible to select clevises or other devices to use for connectors at the cable ends. (See AISC Manual Table 15-3.)

3.2 NOMINAL STRENGTHS OF TENSION MEMBERS

A ductile steel member without holes and subject to a tensile load can resist without fracture a load larger than its gross cross-sectional area times its yield stress because of strain hardening. However, a tension member loaded until strain hardening is reached will lengthen a great deal before fracture—a fact that will, in all probability, end its usefulness and may even cause failure of the structural system of which the member is a part.



The skeleton of the roof of a Ford building under construction. (Courtesy of Bethlehem Steel Corporation.)

If, on the other hand, we have a tension member with bolt holes, it can possibly fail by fracture at the net section through the holes. This failure load may very well be smaller than the load required to yield the gross section, apart from the holes. It is to be realized that the portion of the member where we have a reduced cross-sectional area due to the presence of holes normally is very short compared with the total length of the member. Though the strain-hardening situation is quickly reached at the net section portion of the member, yielding there may not really be a limit state of significance, because the overall change in length of the member due to yielding in this small part of the member length may be negligible.

As a result of the preceding information, the AISC Specification (D2) states that the nominal strength of a tension member, P_n, is to be the smaller of the values obtained by substituting into the following two expressions:

For the limit state of yielding in the gross section (which is intended to prevent excessive elongation of the member),

$$P_n = F_y A_g \qquad \text{(AISC Equation D2-1)}$$

$$\phi_t P_n = \phi_t F_y A_g = \text{design tensile strength by LRFD } (\phi_t = 0.9)$$

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t} = \text{allowable tensile strength for ASD } (\Omega_t = 1.67)$$

For tensile rupture in the net section, as where bolt or rivet holes are present,

$$P_n = F_u A_e \qquad \text{(AISC Equation D2-2)}$$

$$\phi_t P_n = \phi_t F_u A_e = \text{design tensile rupture strength for LRFD } (\phi_t = 0.75)$$

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} \text{ allowable tensile rupture strength for ASD } (\Omega_t = 2.00)$$

In the preceding expressions, F_y and F_u are the specified minimum yield and tensile stresses, respectively, A_g is the gross area of the member, and A_e is the effective net area that can be assumed to resist tension at the section through the holes. This area may be somewhat smaller than the actual net area, A_n, because of stress concentrations and other factors that are discussed in Section 3.5. Values of F_y and F_u are provided in Table 1.1 of this text (Table 2-4 in AISC Manual) for the ASTM structural steels on the market today.

For tension members consisting of rolled steel shapes, there actually is a third limit state, block shear, a topic presented in Section 3.7.

The design and allowable strengths presented here are not applicable to threaded steel rods or to members with pin holes (as in eyebars). These situations are discussed in Sections 4.3 and 4.4.

It is not likely that stress fluctuations will be a problem in the average building frame, because the changes in load in such structures usually occur only occasionally and produce relatively minor stress variations. Full design wind or earthquake loads occur so infrequently that they are not considered in fatigue design. Should there, however, be frequent variations or even reversals in stress, the matter of fatigue must be considered. This subject is presented in Section 4.5.

3.3 NET AREAS

The presence of a hole obviously increases the unit stress in a tension member, even if the hole is occupied by a bolt. (When fully tightened high-strength bolts are used, there may be some disagreement with this statement under certain conditions.) There is still less area of steel to which the load can be distributed, and there will be some concentration of stress along the edges of the hole.

Tension is assumed to be uniformly distributed over the net section of a tension member, although photoelastic studies show there is a decided increase in stress intensity around the edges of holes, sometimes equaling several times what the stresses would be if the holes were not present. For ductile materials, however, a uniform stress distribution assumption is reasonable when the material is loaded beyond its yield stress. Should the fibers around the holes be stressed to their yield stress, they will yield without further stress increase, with the result that there is a redistribution, or balancing, of stresses. At ultimate load, it is reasonable to assume a uniform stress distribution. The influence of ductility on the strength of bolted tension members has been clearly demonstrated in tests. Tension members (with bolt holes) made from ductile steels have proved to be as much as one-fifth to one-sixth stronger than similar members made from brittle steels with the same strengths. We have already shown in Chapter 1 that it is possible for steel to lose its ductility and become subject to brittle fracture. Such a condition can be created by fatigue-type loads and by very low temperatures.

This initial discussion is applicable only for tension members subjected to relatively static loading. Should tension members be designed for structures subjected to fatigue-type loadings, considerable effort should be made to minimize the items causing stress concentrations, such as points of sudden change of cross section, and sharp corners. In addition, as described in Section 4.5, the members may have to be enlarged.

The term "net cross-sectional area," or simply, "net area," refers to the gross cross-sectional area of a member, minus any holes, notches, or other indentations. In considering the area of such items, it is important to realize that it is usually necessary to subtract an area a little larger than the actual hole. For instance, in fabricating structural steel that is to be connected with bolts, the long-used practice was to punch holes with a diameter 1/16 in larger than that of the bolts. When this practice was followed, the punching of a hole was assumed to damage or even destroy 1/16 in more of the surrounding metal. As a result, the diameter of the hole subtracted was 1/8 in larger than the diameter of the bolt. The area of the hole was rectangular and equalled the diameter of the bolt plus 1/8 in times the thickness of the metal.

Today, drills made from very much improved steels enable fabricators to drill very large numbers of holes without resharpening. As a result, a large proportion of bolt holes are now prepared with numerically controlled drills. Though it seems reasonable to add only 1/16 in to the bolt diameters for such holes, to be consistent, the author adds 1/8 in for all standard bolt holes mentioned in this text. (Should the holes be slotted as described in Chapter 12, the usual practice is to add 1/16 in to the actual width of the holes.)

For steel much thicker than bolt diameters, it is difficult to punch out the holes to the full sizes required without excessive deformation of the surrounding material. These holes may be subpunched (with diameters 3/16 in undersized) and then reamed out to full size after the pieces are assembled. Very little material is damaged by this quite expensive process, as the holes are even and smooth, and it is considered unnecessary to subtract the 1/16 in for damage to the sides.

It may be necessary to have an even greater latitude in meeting dimensional tolerances during erection and for high-strength bolts larger than 5/8 in in diameter. For such a situation, holes larger than the standard ones may be used without reducing the performance of the connections. These oversized holes can be short-slotted or long-slotted, as described in Section 12.9.

Example 3-1 illustrates the calculations necessary for determining the net area of a plate type of tension member.

Example 3-1

Determine the net area of the 3/8 × 8-in plate shown in Fig. 3.2. The plate is connected at its end with two lines of 3/4-in bolts.

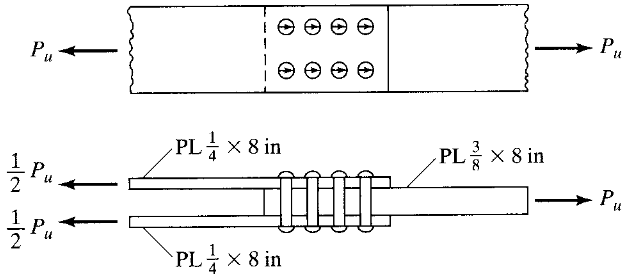


FIGURE 3.2

Solution

$$A_n = \left(\frac{3}{8}\text{ in}\right)(8\text{ in}) - 2\left(\frac{3}{4}\text{ in} + \frac{1}{8}\text{ in}\right)\left(\frac{3}{8}\text{ in}\right) = 2.34\text{ in}^2\ (1510\text{ mm}^2)$$

Ans. 2.34 in^2

The connections of tension members should be arranged so that no eccentricity is present. (An exception to this rule is permitted by the AISC Specification for certain bolted and welded connections, as described in Chapters 13 and 14.) If this arrangement is possible, the stress is assumed to be spread uniformly across the net section of a member. Should the connections have eccentricities, moments will be produced that will cause additional stresses in the vicinity of the connection. Unfortunately, it is often quite difficult to arrange connections without eccentricity. Although specifications cover some situations, the designer may have to give consideration to eccentricities in some cases by making special estimates.

The centroidal axes of truss members meeting at a joint are assumed to coincide. Should they not coincide, eccentricity is present and secondary stresses are the result. The centroidal axes of truss members are assumed to coincide with the lines of action of their respective forces. No problem is present in a symmetrical member, as its centroidal axis is at its center line; but for unsymmetrical members, the problem is a little more difficult. For these members, the center line is not the centroidal axis, but the usual practice is to arrange the members at a joint so that their gage lines coincide. If a

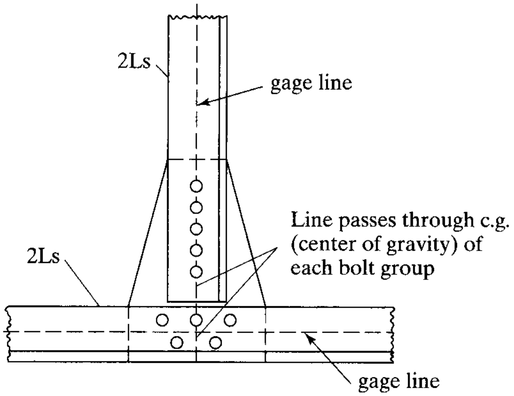


FIGURE 3.3

Lining up centroidal axes of members.

member has more than one gage line, the one closest to the actual centroidal axis of the member is used in detailing. Figure 3.3 shows a truss joint in which the c.g.s coincide.

3.4 EFFECT OF STAGGERED HOLES

Should there be more than one row of bolt holes in a member, it is often desirable to stagger them in order to provide as large a net area as possible at any one section to resist the load. In the preceding paragraphs, tensile members have been assumed to fail transversely, as along line *AB* in either Fig. 3.4(a) or 3.4(b). Figure 3.4(c) shows a member in which a failure other than a transverse one is possible. The holes are staggered, and failure along section *ABCD* is possible unless the holes are a large distance apart.

To determine the critical net area in Fig. 3.4(c), it might seem logical to compute the area of a section transverse to the member (as *ABE*), less the area of one hole, and then the area along section *ABCD*, less two holes. The smallest value obtained along these sections would be the critical value. This method is faulty, however. Along the diagonal line from *B* to *C*, there is a combination of direct stress and shear, and a somewhat smaller area should be used. The strength of the member along section *ABCD* is obviously somewhere between the strength obtained by using a net area computed by subtracting one hole from the transverse cross-sectional area and the value obtained by subtracting two holes from section *ABCD*.

Tests on joints show that little is gained by using complicated theoretical formulas to consider the staggered-hole situation, and the problem is usually handled with an empirical equation. The AISC Specification (B4.3b) and other specifications offer a very simple method for computing the net width of a tension member along a zigzag

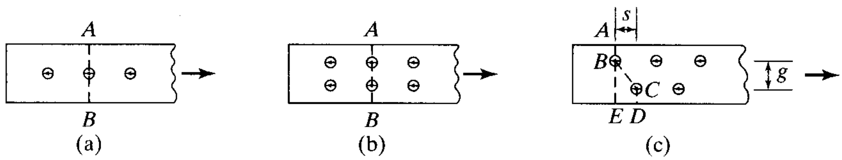
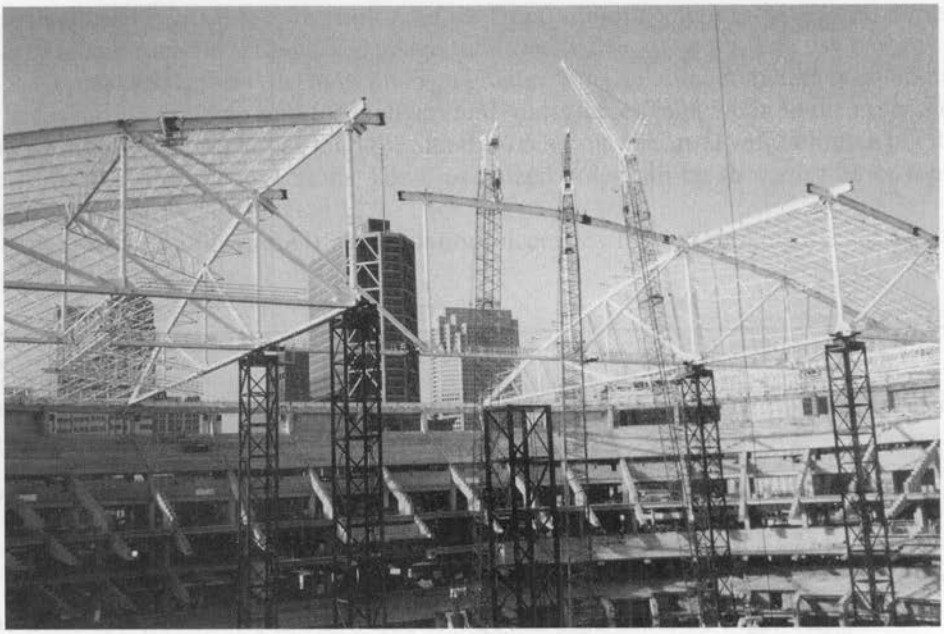


FIGURE 3.4

Possible failure sections in plates.



Trans-World Dome, St. Louis, MO. (Courtesy of Trade ARBED.)

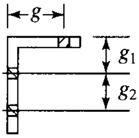
section.[1] The method is to take the gross width of the member, regardless of the line along which failure might occur, subtract the diameter of the holes along the zigzag section being considered, and add for each inclined line the quantity given by the expression $s^2/4g$. (Since this simple expression was introduced in 1922, many investigators have proposed other, often quite complicated rules. However, none of them seems to provide significantly better results.)

In this expression, s is the longitudinal spacing (or pitch) of any two holes and g is the transverse spacing (or gage) of the same holes. The values of s and g are shown in Fig. 3.4(c). There may be several paths, any one of which may be critical at a particular joint. Each possibility should be considered, and the one giving the least value should be used. The smallest net width obtained is multiplied by the plate thickness to give the net area, A_n. Example 3-2 illustrates the method of computing the critical net area of a section that has three lines of bolts. (For angles, the gage for holes in opposite legs is considered to be the sum of the gages from the back of the angle minus the thickness of the angle.)

Holes for bolts and rivets are normally drilled or punched in steel angles at certain standard locations. These locations or gages are dependent on the angle-leg widths and on the number of lines of holes. Table 3.1, which is taken from Table 1-7A, p. 1–48 of the Steel Manual, shows these gages. It is unwise for the designer to require different gages from those given in the table unless unusual situations are present, because of the appreciably higher fabrication costs that will result.

[1]V. H. Cochrane, "Rules for Riveted Hole Deductions in Tension Members," *Engineering News-Record* (New York, November 16, 1922), pp. 847–848.

TABLE 3.1 Workable Gages for Angles, in Inches



Leg	8	7	6	5	4	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{4}$	1
g	$4\frac{1}{2}$	4	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	$1\frac{3}{4}$	$1\frac{3}{8}$	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$
g_1	3	$2\frac{1}{2}$	$2\frac{1}{4}$	2										
g_2	3	3	$2\frac{1}{2}$	$1\frac{3}{4}$										

Example 3-2

Determine the critical net area of the 1/2-in-thick plate shown in Fig. 3.5, using the AISC Specification (Section B4.3b). The holes are punched for 3/4-in bolts.

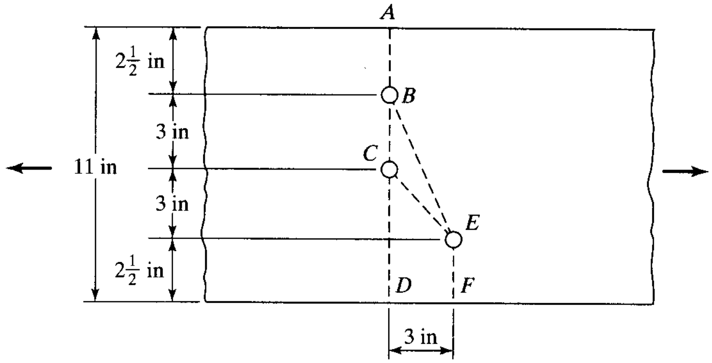


FIGURE 3.5

Solution. The critical section could possibly be $ABCD$, $ABCEF$, or $ABEF$. Hole diameters to be subtracted are 3/4 + 1/8 = 7/8 in. The net areas for each case are as follows:

$$ABCD = (11\text{ in})\left(\frac{1}{2}\text{ in}\right) - 2\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) = 4.63\text{ in}^2$$

$$ABCEF = (11\text{ in})\left(\frac{1}{2}\text{ in}\right) - 3\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) + \frac{(3\text{ in})^2}{4(3\text{ in})}\left(\frac{1}{2}\text{ in}\right) = 4.56\text{ in}^2 \longleftarrow$$

$$ABEF = (11\text{ in})\left(\frac{1}{2}\text{ in}\right) - 2\left(\frac{7}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) + \frac{(3\text{ in})^2}{4(6\text{ in})}\left(\frac{1}{2}\text{ in}\right) = 4.81\text{ in}^2$$

The reader should note that it is a waste of time to check path $ABEF$ for this plate. Two holes need to be subtracted for routes $ABCD$ and $ABEF$. As $ABCD$ is a shorter route, it obviously controls over $ABEF$.

Ans. 4.56 in^2

The problem of determining the minimum pitch of staggered bolts such that no more than a certain number of holes need be subtracted to determine the net section is handled in Example 3-3.

Example 3-3

For the two lines of bolt holes shown in Fig. 3.6, determine the pitch that will give a net area $DEFG$ equal to the one along ABC. The problem may also be stated as follows: Determine the pitch that will give a net area equal to the gross area less one bolt hole. The holes are punched for 3/4-in bolts.

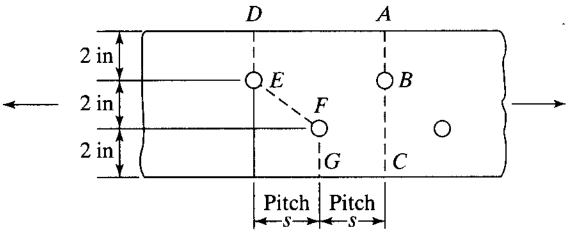


FIGURE 3.6

Solution. The hole diameters to be subtracted are 3/4 in + 1/8 in = 7/8 in.

$$ABC = 6\text{ in} - (1)\left(\frac{7}{8}\text{ in}\right) = 5.13\text{ in}$$

$$DEFG = 6\text{ in} - 2\left(\frac{7}{8}\text{ in}\right) + \frac{s^2}{4(2\text{ in})} = 4.25\text{ in} + \frac{s^2}{8\text{ in}}$$

$$ABC = DEFG$$

$$5.13 = 4.25 + \frac{s^2}{8}$$

$$s = 2.65\text{ in}$$

The $s^2/4g$ rule is merely an approximation or simplification of the complex stress variations that occur in members with staggered arrangements of bolts. Steel specifications can provide only minimum standards, and designers will have to logically apply such information to complicated situations, which the specifications could not cover, in their attempts at brevity and simplicity. The next few paragraphs present a discussion and numerical examples of the $s^2/4g$ rule applied to situations not specifically addressed in the AISC Specification.

The AISC Specification does not include a method for determining the net widths of sections other than plates and angles. For channels, W sections, S sections, and others, the web and flange thicknesses are not the same. As a result, it is necessary to work with net areas rather than net widths. If the holes are placed in straight lines across such a member, the net area can be obtained by simply subtracting the cross-sectional areas of the holes from the gross area of the member. If the holes are staggered, the $\frac{s^2}{4g}$ values must be multiplied by the applicable thickness to change it to an area. Such a procedure is illustrated for a W section in Example 3-4, where bolts pass through the web only.

Example 3-4

Determine the net area of the W12 $\times$ 16 (A_g = 4.71 in^2) shown in Fig. 3.7, assuming that the holes are for 1-in bolts.

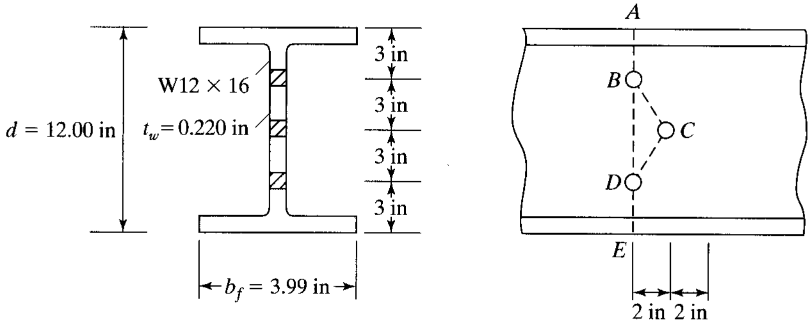


FIGURE 3.7

Solution. Net areas: hole ϕ is 1 in + $\frac{1}{8}$ in = $1\frac{1}{8}$ in

$$ABDE = 4.71 \text{ in}^2 - 2\left(1\frac{1}{8} \text{ in}\right)(0.220 \text{ in}) = 4.21 \text{ in}^2$$

$$ABCDE = 4.72 \text{ in}^2 - 3\left(1\frac{1}{8} \text{ in}\right)(0.220 \text{ in}) + (2)\frac{(2 \text{ in})^2}{4(3 \text{ in})}(0.220 \text{ in}) = 4.11 \text{ in}^2 \leftarrow$$

If the zigzag line goes from a web hole to a flange hole, the thickness changes at the junction of the flange and web. In Example 3-5, the author has computed the net area of a channel that has bolt holes staggered in its flanges and web. The channel is assumed to be flattened out into a single plate, as shown in parts (b) and (c) of Fig. 3.8. The net area along route *ABCDEF* is determined by taking the area of the channel minus the area of the holes along the route in the flanges and web plus the $s^2/4g$ values for each zigzag line times the appropriate thickness. For line *CD*, $s^2/4g$ has been multiplied by the thickness of the web. *For lines BC and DE (which run from holes in the web to holes in the flange), an approximate procedure has been used in which the $s^2/4g$ values have been multiplied by the average of the web and flange thicknesses.*

Example 3-5

Determine the net area along route $ABCDEF$ for the C15 × 33.9 (A_g = 10.00 in^2) shown in Fig. 3.8. Holes are for $\frac{3}{4}$-in bolts.

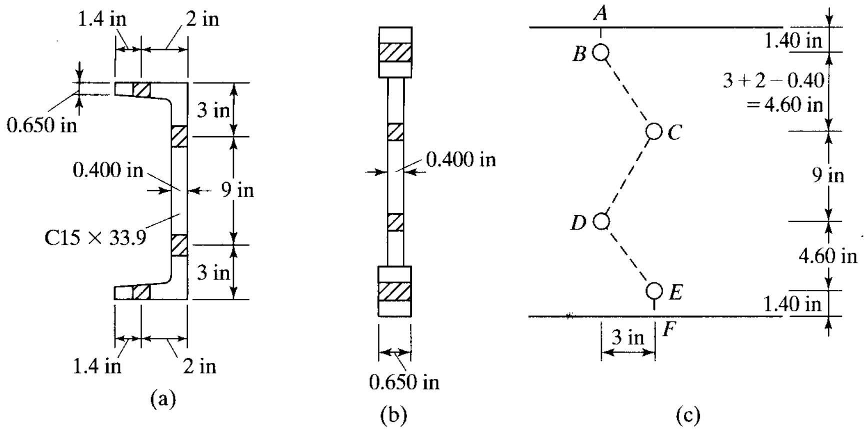


FIGURE 3.8

Solution

Approximate net A along

$$
\begin{aligned}
ABCDEF &= 10.00\text{ in}^2 - 2\left(\frac{7}{8}\text{ in}\right)(0.650\text{ in}) \\
&\quad - 2\left(\frac{7}{8}\text{ in}\right)(0.400\text{ in}) \\
&\quad + \frac{(3\text{ in})^2}{4(9\text{ in})}(0.400\text{ in}) \\
&\quad + (2)\frac{(3\text{ in})^2}{(4)(4.60\text{ in})}\left(\frac{0.650\text{ in} + 0.400\text{ in}}{2}\right) \\
&= 8.78\text{ in}^2
\end{aligned}
$$

Ans. 8.78 in^2

3.5 EFFECTIVE NET AREAS

When a member other than a flat plate or bar is loaded in axial tension until failure occurs across its net section, its actual tensile failure stress will probably be less than the coupon tensile strength of the steel, *unless all of the various elements which make up the section are connected so that stress is transferred uniformly across the section.*

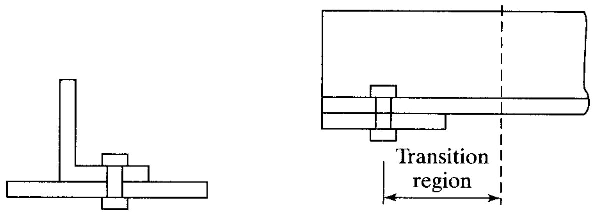


FIGURE 3.9

Shear lag.

(a) Angle connected by one leg only. (b) Stress in transition range > F_y.

If the forces are not transferred uniformly across a member cross section, there will be a transition region of uneven stress running from the connection out along the member for some distance. This is the situation shown in Fig. 3.9(a), where a single angle tension member is connected by one leg only. At the connection more of the load is carried by the connected leg, and it takes the transition distance shown in part (b) of the figure for the stress to spread uniformly across the whole angle.

In the transition region the stress in the connected part of the member may very well exceed F_y and go into the strain-hardening range. Unless the load is reduced, the member may fracture prematurely. The farther we move out from the connection, the more uniform the stress becomes. In the transition region, the shear transfer has "lagged" and the phenomenon is referred to as *shear lag.*

In such a situation, the flow of tensile stress between the full member cross section and the smaller connected cross section is not 100 percent effective. As a result, the AISC Specification (D.3) states that the effective net area, A_e, of such a member is to be determined by multiplying an area A (which is the net area or the gross area or the directly connected area, as described in the next few pages) by a reduction factor U. The use of a factor such as U accounts for the nonuniform stress distribution, in a simple manner.

$$A_e = A_n U \qquad \text{(AISC Equation D3-1)}$$

The value of the reduction coefficient, U, is affected by the cross section of the member and by the length of its connection. An explanation of the way in which U factors are determined follows.

The angle shown in Fig. 3.10(a) is connected at its ends to only one leg. You can easily see that its area effective in resisting tension can be appreciably increased by shortening the width of the unconnected leg and lengthening the width of the connected leg, as shown in Fig. 3.10(b).

Investigators have found that one measure of the effectiveness of a member such as an angle connected by one leg is the distance $\bar{x}$ measured from the plane of the connection to the centroid of the area of the whole section.[2,3] The smaller the value of $\bar{x}$, the larger is the effective area of the member, and thus the larger is the member's design strength.

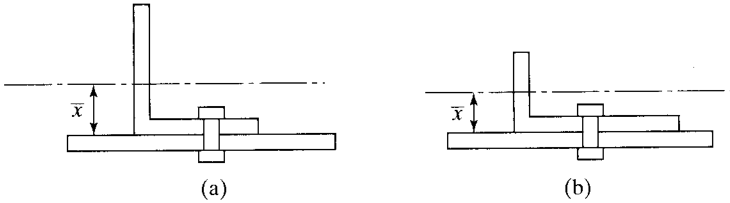
---

[2]E. H. Gaylord, Jr., and C. N. Gaylord, *Design of Steel Structures*, 2d ed. (New York: McGraw-Hill Book Company, 1972), pp. 119–123.

[3]W. H. Munse and E. Chesson, Jr., "Riveted and Bolted Joints: Net Section Design," *Journal of the Structural Division,* ASCE, *89,* STI (February 1963).

FIGURE 3.10

Reducing shear lag by reducing length of unconnected leg and and thus $\bar{x}$.



Another measure of the effectiveness of a member is the length of its connection, L. The greater this length, the smoother will be the transfer of stress to the member's unconnected parts. In other words, if 3 bolts at 3 inches on center are used, the effective area of the member will be less than if 3 bolts at 4 inches on center are used.

The effect of these two parameters, $\bar{x}$ and L, is expressed empirically with the reduction factor

$$U = 1 - \frac{\bar{x}}{L}$$

From this expression, we can see that the smaller the value of $\bar{x}$ and the larger the value of L, the larger will be the value of U, and thus the larger will be the effective area of the member. Section D3 of the AISC Commentary for Section D of the specification has additional explanation of the shear lag effect. Figures C-D3.1 through C-D3.4 show how $\bar{x}$ and L are determined for various bolted and welded tension members.

3.5.1 Bolted Members

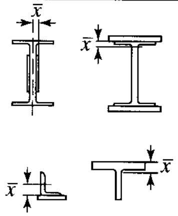
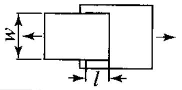
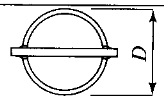
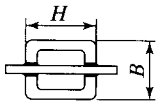
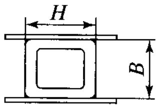
Should a tension load be transmitted by bolts, the gross area is reduced to the net area A_n of the member, and U is computed as follows:

$$U = 1 - \frac{\bar{x}}{L}$$

The length L used in this expression is equal to the distance between the first and last bolts in the line. When there are two or more lines of bolts, L is the length of the line with the maximum number of bolts. Should the bolts be staggered, L is the out-to-out dimension between the extreme bolts in a line. Note that the longer the connection (L) becomes, the larger U will become, as will the effective area of the member. (On the other hand, we will learn in the connection chapters of this text that the effectiveness of connectors is somewhat reduced if very long connections are used.) Insufficient data are available for the case in which only one bolt is used in each line. It is thought that a reasonable approach for this case is to let $A_e = A_n$ of the connected element. Table 3.2 provides a detailed list of shear lag or U factors for different situations. This table is a copy of Table D3.1 of the AISC Specification.

For some problems herein, the authors calculate U with the $1 - \frac{\bar{x}}{L}$ expression, Case 2 from Table 3.2, and then compares it with the value from Case 7 for W, M, S, HP, or tees cut from these shapes and from Case 8 for single angles. He then uses the larger of the two values in his calculations, as permitted by the AISC Specification.

TABLE 3.2 Shear Lag Factors for Connections to Tension Members

Case	Description of Element		Shear Lag Factor, U	Example
1	All tension members where the tension load is transmitted directly to each of the cross-sectional elements by fasteners or welds (except as in Cases 4, 5 and 6).		$U = 1.0$	—
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds or by longitudinal welds in combination with transverse welds. (Alternatively, for W, M, S and HP, Case 7 may be used. For angles, Case 8 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted only by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B + H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B + H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used.)	with flange connected with 3 or more fasteners per line in the direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners per line in the direction of loading	$U = 0.70$	—
8	Single and double angles (If U is calculated per Case 2, the larger value is permitted to be used.)	with 4 or more fasteners per line in the direction of loading	$U = 0.80$	—
		with 3 fasteners per line in the direction of loading (With fewer than 3 fasteners per line in the direction of loading, use Case 2.)	$U = 0.60$	—

l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = eccentricity of connection, in. (mm); B = overall width of rectangular HSS member, measured 90° to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)

Source: AISC Specification, Table D3.1, p. 16.1-28, June 22, 2010.

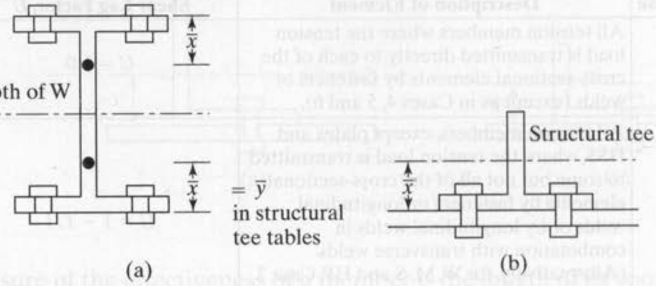


FIGURE 3.11
Values of $\bar{x}$ for different shapes.

In order to calculate U for a W section connected by its flanges only, we will assume that the section is split into two structural tees. Then the value of $\bar{x}$ used will be the distance from the outside edge of the flange to the c.g. of the structural tee, as shown in parts (a) and (b) of Fig. 3.11.

The AISC Specification permits the designer to use larger values of U than obtained from the equation if such values can be justified by tests or other rational criteria.

Section D3 of the AISC Commentary provides suggested $\bar{x}$ values for use in the equation for U for several situations not addressed in the Specification. Included are values for W and C sections bolted only through their webs. Also considered are single angles with two lines of staggered bolts in one of their legs. The basic idea for computing $\bar{x}$ for these cases is presented in the next paragraph.[4]

The channel of Fig. 3.12(a) is connected with two lines of bolts through its web. The "angle" part of this channel above the center of the top bolt is shown darkened in part (b) of the figure. This part of the channel is unconnected. For shear lag purpose, we can determine the horizontal distance from the outside face of the web to the channel centroid. This distance, which is given in the Manual shape tables, will be the $\bar{x}$ used in the equation. It is felt that with this idea in mind, the reader will be able to understand the values shown in the Commentary for other sections.

Example 3-6 illustrates the calculations necessary for determining the effective net area of a W section bolted through its flanges at each end.

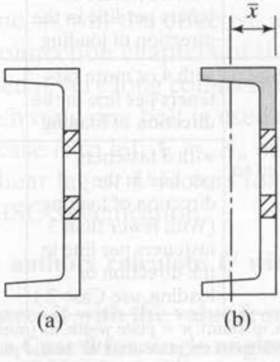


FIGURE 3.12
$\bar{x}$ for a channel bolted through its web.

[4]W. S. Easterling and L. G. Giroux, "Shear Lag Effects in Steel Tension Members," *Engineering Journal*, AISC, no. 3 (3rd Quarter, 1993), pp. 77–89.

Example 3-6

Determine the LRFD design tensile strength and the ASD allowable design tensile strength for a W10 × 45 with two lines of $\frac{3}{4}$-in diameter bolts in each flange using A572 Grade 50 steel, with $F_y = 50$ ksi and $F_u = 65$ ksi, and the AISC Specification. There are assumed to be at least three bolts in each line 4 in on center, and the bolts are not staggered with respect to each other.

Solution. Using a W10 × 45 ($A_g = 13.3 \text{ in}^2$, $d = 10.10$ in, $b_f = 8.02$ in, $t_f = 0.620$ in)

Nominal or available tensile strength of section $P_n = F_y A_g = (50 \text{ ksi})(13.3 \text{ in}^2) = 665 \text{ k}$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(665 \text{ k}) = 598.5 \text{ k}$	$\dfrac{P_n}{\Omega t} = \dfrac{665 \text{ k}}{1.67} = 398.2 \text{ k}$

(b) Tensile rupture strength

$$A_n = 13.3 \text{ in}^2 - (4)\left(\frac{3}{4}\text{ in} + \frac{1}{8}\text{ in}\right)(0.620 \text{ in}) = 11.13 \text{ in}^2$$

Referring to tables in Manual for one-half of a W10 × 45 (or, that is, a WT5 × 22.5), we find that

$$\bar{x} = 0.907 \text{ in } (\bar{y} \text{ from AISC Manual Table 1-8})$$

Length of connection, L = 2 (4 in) = 8 in

$$\text{From Table 3.2 (Case 2), } U = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.907 \text{ in}}{8 \text{ in}} = 0.89$$

$$\text{But } b_f = 8.02 \text{ in} > \frac{2}{3}d = \left(\frac{2}{3}\right)(10.1) = 6.73 \text{ in}$$

$\therefore$ U from Table 3.2 (Case 7) is 0.90 ←

$$A_e = UA_n = (0.90)(11.13 \text{ in}^2) = 10.02 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(10.02 \text{ in}^2) = 651.3 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(651.3 \text{ k}) = 488.5 \text{ k}$ ←	$\dfrac{P_n}{\Omega_t} = \dfrac{651.3 \text{ k}}{2.00} = 325.6 \text{ k}$ ←

Ans. LRFD = 488.5 k (Rupture controls) ASD = 325.6 k (Rupture controls)

Notes:

1. In Table 3.2 (Case 7), it is stated that U may be used equal to 0.90 for W sections if $b_f \geq 2/3d$.
2. Answers to tensile strength problems like this one may be derived from Table 5-1 of the Manual. However, the values in this table are based on the assumptions that $U = 0.9$ and $A_e = 0.75A_g$. As a result, values will vary a little from those determined with calculated values of U and A_e. For this problem, the LRFD values from AISC Table 5-9 are 599 k for tensile yielding and 487 k for tensile rupture. For ASD, the allowable values are 398 k and 324 k, respectively.

Example 3-7

Determine the LRFD design tensile strength and the ASD allowable tensile strength for an A36 ($F_y = 36$ ksi and $F_u = 58$ ksi) L6 × 6 × 3/8 in that is connected at its ends with one line of four 7/8-in-diameter bolts in standard holes 3 in on center in one leg of the angle.

Solution. Using an L6 × 6 × $\frac{3}{8}$ ($A_g = 4.38\text{ in}^2$, $\bar{y} = \bar{x} = 1.62$ in) nominal or available tensile strength of the angle

$$P_n = F_y A_g = (36\text{ ksi})(4.38\text{ in}^2) = 157.7\text{ k}$$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(157.7\ k) = 141.9\text{ k} \leftarrow$	$\dfrac{P_n}{\Omega_t} = \dfrac{157.7\text{ k}}{1.67} = 94.4\text{ k} \leftarrow$

(b) Tensile rupture strength

$$A_n = 4.38\text{ in}^2 - (1)\left(\frac{7}{8}\text{in} + \frac{1}{8}\text{in}\right)\left(\frac{3}{8}\text{in}\right) = 4.00\text{ in}^2$$

$$\text{Length of connection, } L = (3)(3\text{ in}) = 9\text{ in}$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.62\text{ in}}{9\text{ in}} = 0.82$$

From Table 3.2, Case 8, for 4 or more fasteners in the direction of loading, $U = 0.80$. Use calculated $U = 0.82$.

$$A_e = A_n U = (4.00\text{ in}^2)(0.82) = 3.28\text{ in}^2$$

$$P_n = F_u A_e = (58\text{ ksi})(3.28\text{ in}^2) = 190.2\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(190.2 \text{ k}) = 142.6 \text{ k}$	$\frac{P_n}{\Omega_t} = \frac{190.2 \text{ k}}{2.00} = 95.1 \text{ k}$

Ans. LRFD = 141.9 k (Yielding controls) ASD = 94.4 k (Yielding controls)

3.5.2 Welded Members

When tension loads are transferred by welds, the rules from AISC Table D-3.1, Table 3.2 in this text, that are to be used to determine values for A and U (A_e as for bolted connections $= AU$) are as follows:

1. Should the load be transmitted only by longitudinal welds to other than a plate member, or by longitudinal welds in combination with transverse welds, A is to equal the gross area of the member A_g (Table 3.2, Case 2).
2. Should a tension load be transmitted only by transverse welds, A is to equal the area of the directly connected elements and U is to equal 1.0 (Table 3.2, Case 3).
3. Tests have shown that when flat plates or bars connected by longitudinal fillet welds (a term described in Chapter 14) are used as tension members, they may fail prematurely by shear lag at the corners if the welds are too far apart. Therefore, the AISC Specification states that when such situations are encountered, the length of the welds may not be less than the width of the plates or bars. The letter A represents the area of the plate, and UA is the effective net area. For such situations, the values of U to be used (Table 3.2, Case 4) are as follows:

When $l \geq 2w$	$U = 1.0$
When $2w > l \geq 1.5w$	$U = 0.87$
When $1.5w > l \geq w$	$U = 0.75$

   Here, l = weld length, in
   w = plate width (distance between welds), in

For combinations of longitudinal and transverse welds, l is to be used equal to the length of the longitudinal weld, because the transverse weld has little or no effect on the shear lag. (That is, it does little to get the load into the unattached parts of the member.)

Examples 3-8 and 3-9 illustrate the calculations of the effective areas, the LRFD tensile design strengths, and the ASD allowable design strengths of two welded members.

Example 3-8

The 1×6 in plate shown in Fig. 3.13 is connected to a 1×10 in plate with longitudinal fillet welds to transfer a tensile load. Determine the LRFD design tensile strength and the ASD allowable tensile strength of the member if $F_y = 50$ ksi and $F_u = 65$ ksi.

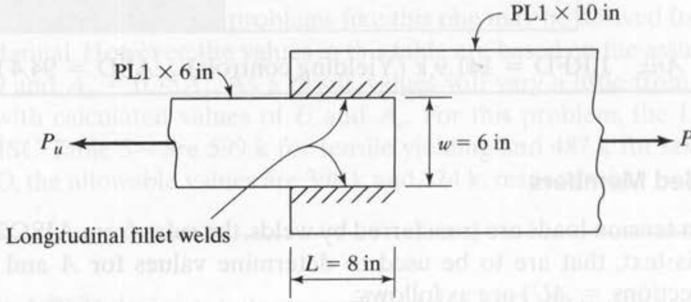


FIGURE 3.13

Solution. Considering the nominal or available tensile strength of the smaller PL 1 in × 6 in

$$P_n = F_y A_g = (50\text{ ksi})(1\text{ in} \times 6\text{ in}) = 300\text{ k}$$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(300\text{ k}) = 270\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{300\text{ k}}{1.67} = 179.6\text{ k}$

(b) Tensile rupture strength

$$1.5w = 1.5 \times 6\text{ in} = 9\text{ in} > L = 8\text{ in} > w = 6\text{ in}$$

$$\therefore U = 0.75 \text{ from Table 3.2, Case 4}$$

$$A_e = A_n U = (6.0\text{ in}^2)(0.75) = 4.50\text{ in}^2$$

$$P_n = F_u A_e = (65\text{ ksi})(4.50\text{ in}^2) = 292.5\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(292.5\text{ k}) = 219.4\text{ k} \leftarrow$	$\frac{P_n}{\Omega_t} = \frac{292.5\text{ k}}{2.00} = 146.2\text{ k} \leftarrow$

Ans. LRFD = 219.4 k (Rupture controls) ASD = 146.2 k (Rupture controls)

Sometimes an angle has one of its legs connected with both longitudinal and transverse welds, but no connections are made to the other leg. To determine U from Table 3.2 for such a case is rather puzzling. The author feels that Case 2 of Table 3.2 (that is, $U = 1 - \frac{\bar{x}}{L}$) should be used for this situation. This is done in Example 3-9.

Example 3-9

Compute the LRFD design tensile strength and the ASD allowable tensile strength of the angle shown in Fig. 3.14. It is welded on the end (transverse) and sides (longitudinal) of the 8-in leg only. $F_y = 50$ ksi and $F_u = 70$ ksi.

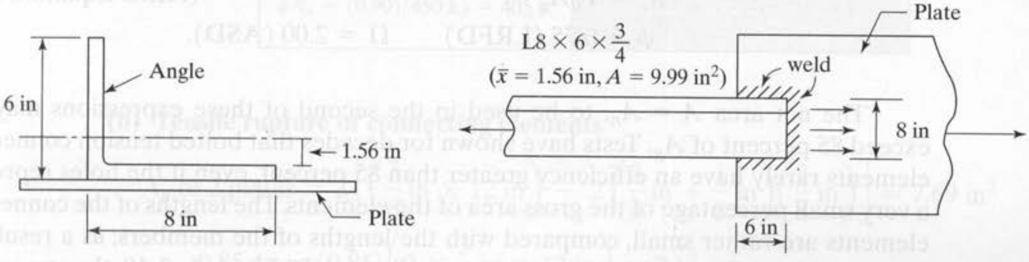


FIGURE 3.14

Solution. Nominal or available tensile strength of the angle

$$= P_n = F_y A_g = (50\text{ ksi})(9.99\text{ in}^2) = 499.5\text{ k}$$

(a) Gross section yielding

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(499.5\text{ k}) = 449.5\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{499.5\text{ k}}{1.67} = 299.1\text{ k}$

(b) Tensile rupture strength (As only one leg of L is connected, a reduced effective area needs to be computed.) Use Table 3.2 (Case 2)

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.56\text{ in}}{6\text{ in}} = 0.74$$

$$A_e = A_g U = (9.99\text{ in}^2)(0.74) = 7.39\text{ in}^2$$

$$P_n = F_u A_e = (70\text{ ksi})(7.39\text{ in}^2) = 517.3\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(517.3\text{ k}) = 388.0\text{ k} \leftarrow$	$P_n = \frac{517.3\text{ k}}{2.00} = 258.6\text{ k} \leftarrow$

Ans. LRFD = 388.0 k (Rupture controls) ASD = 258.6 k (Rupture controls)

3.6 CONNECTING ELEMENTS FOR TENSION MEMBERS

When splice or gusset plates are used as statically loaded tensile connecting elements, their strength shall be determined as follows:

(a) For tensile yielding of connecting elements

$$R_n = F_y A_g \qquad \text{(AISC Equation J4-1)}$$

$$\phi = 0.90 \text{ (LRFD)} \qquad \Omega = 1.67 \text{ (ASD)}$$

(b) For tensile rupture of connecting elements

$$R_n = F_u A_e \qquad \text{(AISC Equation J4-2)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}.$$

The net area $A = A_n$, to be used in the second of these expressions may not exceed 85 percent of A_g. Tests have shown for decades that bolted tension connection elements rarely have an efficiency greater than 85 percent, even if the holes represent a very small percentage of the gross area of the elements. The lengths of the connecting elements are rather small, compared with the lengths of the members; as a result, inelastic deformations of the gross sections are limited. In Example 3-10, the strength of a pair of tensile connecting plates is determined.

Example 3-10

The tension member (F_y = 50 ksi and F_u = 65 ksi) of Example 3-6 is assumed to be connected at its ends with two 3/8 × 12-in plates, as shown in Fig. 3.15. If two lines of 3/4-in bolts are used in each plate, determine the LRFD design tensile force and the ASD allowable tensile force that the two plates can transfer.

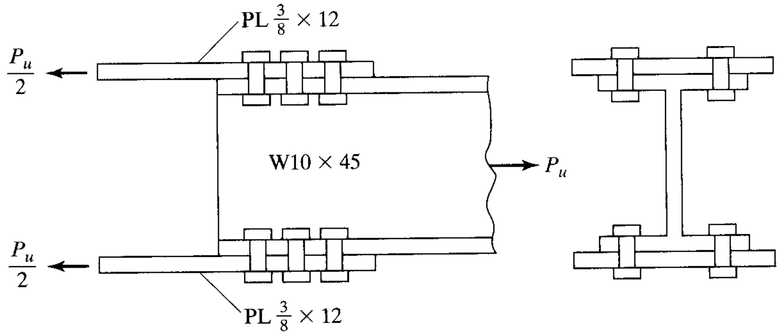


FIGURE 3.15

Solution. Nominal strength of plates

$$R_n = F_y A_g = (50\text{ ksi})\left(2 \times \frac{3}{8}\text{ in} \times 12\text{ in}\right) = 450\text{ k}$$

(a) Tensile yielding of connecting elements

LRFD with $\phi = 0.90$	ASD with $\Omega = 1.67$
$\phi R_n = (0.90)(450\text{ k}) = 405\text{ k}$	$\frac{R_n}{\Omega} = \frac{450\text{ k}}{1.67} = 269.5\text{ k}$

(b) Tensile rupture of connecting elements

$$A_n\text{ of 2 plates} = 2\left[\left(\frac{3}{8}\text{ in} \times 12\text{ in}\right) - 2\left(\frac{3}{4}\text{ in} + \frac{1}{8}\text{ in}\right)\left(\frac{3}{8}\text{ in}\right)\right] = 7.69\text{ in}^2$$

$$0.85A_g = (0.85)\left(2 \times \frac{3}{8}\text{ in} \times 12\text{ in}\right) = 7.65\text{ in}^2 \leftarrow$$

$$R_n = F_u A_e = (65\text{ ksi})(7.65\text{ in}^2) = 497.2\text{ k}$$

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(497.2\text{ k}) = 372.9\text{ k} \leftarrow$	$\frac{R_n}{\Omega} = \frac{497.2\text{ k}}{2.00} = 248.6\text{ k} \leftarrow$

Ans. LRFD = 372.9 k (Rupture controls) ASD = 248.6 k (Rupture controls)

3.7 BLOCK SHEAR

The LRFD design strength and the ASD allowable strengths of tension members are not always controlled by tension yielding, tension rupture, or by the strength of the bolts or welds with which they are connected. They may instead be controlled by *block shear* strength, as described in this section.

The failure of a member may occur along a path involving tension on one plane and shear on a perpendicular plane, as shown in Fig. 3.16, where several possible block shear failures are illustrated. For these situations, it is possible for a "block" of steel to tear out.

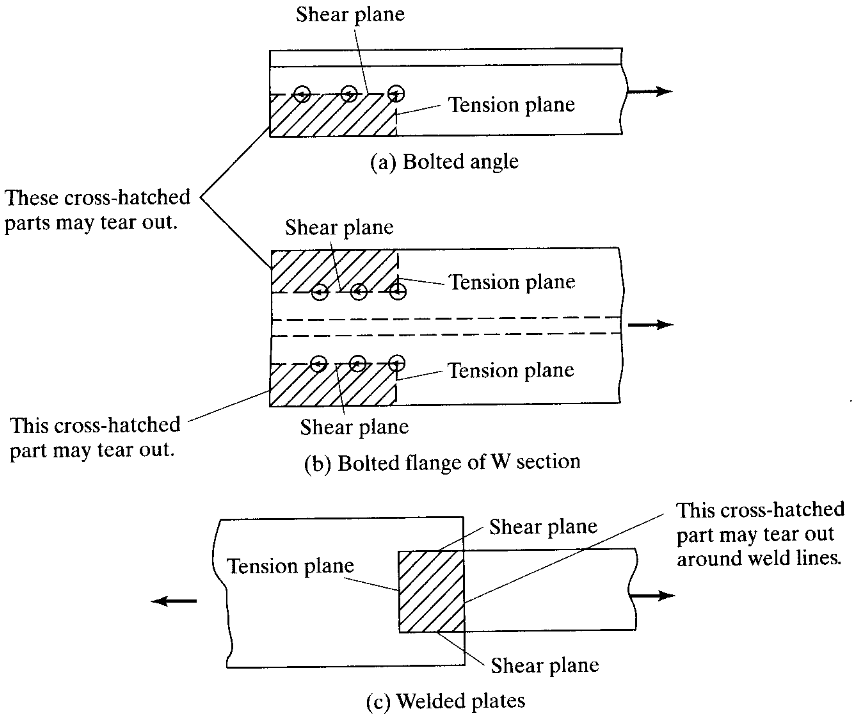


FIGURE 3.16
Block shear.

When a tensile load applied to a particular connection is increased, the fracture strength of the weaker plane will be approached. That plane will not fail then, because it is restrained by the stronger plane. The load can be increased until the fracture strength of the stronger plane is reached. During this time, the weaker plane is yielding. The total strength of the connection equals the fracture strength of the stronger plane plus the yield strength of the weaker plane.[5] Thus, it is not realistic to add the fracture strength of one plane to the fracture strength of the other plane to determine the block shear resistance of a particular member. *You can see that block shear is a tearing, or rupture, situation and not a yielding situation.*

The member shown in Fig. 3.17(a) has a large shear area and a small tensile area; thus, the primary resistance to a block shear failure is shearing and not tensile. The AISC Specification states that it is logical to assume that when shear fracture occurs on this large shear-resisting area, the small tensile area has yielded.

Part (b) of Fig. 3.17 shows, considerably enlarged, a free body of the block that tends to tear out of the angle of part (a). You can see in this sketch that the block shear is caused by the bolts bearing on the back of the bolt holes.

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[5]L. B. Burgett, "Fast Check for Block Shear," *Engineering Journal*, AISC, vol. 29, no. 4 (4th Quarter, 1992), pp. 125–127.

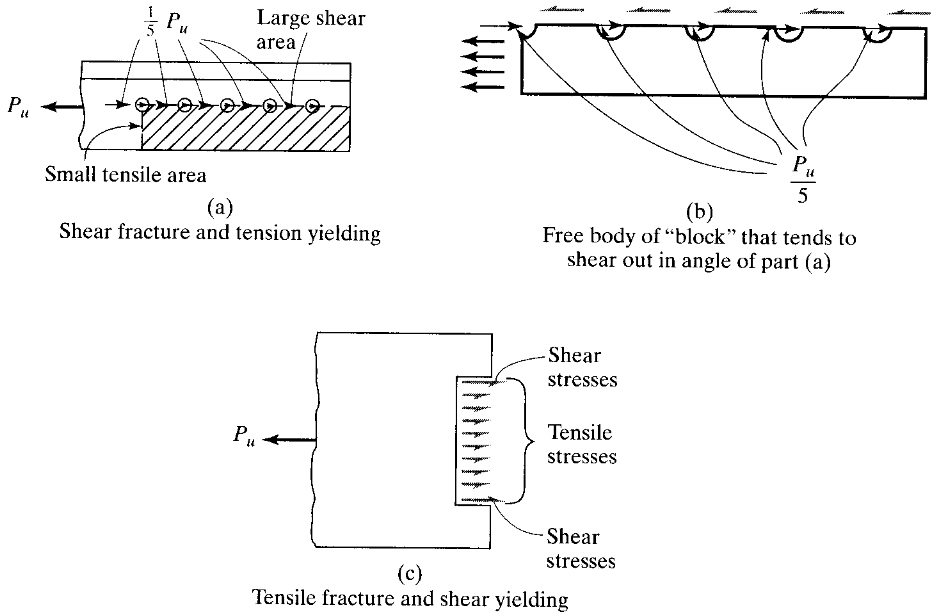


FIGURE 3.17

Block shear.

In part (c) of Fig. 3.17, a member is represented that, so far as block shear goes, has a large tensile area and a small shear area. The AISC feels that for this case the primary resisting force against a block shear failure will be tensile and not shearing. Thus, a block shear failure cannot occur until the tensile area fractures. At that time, it seems logical to assume that the shear area has yielded.

Based on the preceding discussion, the AISC Specification (J4.3) states that the block shear design strength of a particular member is to be determined by (1) computing the tensile fracture strength on the net section in one direction and adding to that value the shear yield strength on the gross area on the perpendicular segment, and (2) computing the shear fracture strength on the gross area subject to tension and adding it to the tensile yield strength on the net area subject to shear on the perpendicular segment. The expression to apply is the one with the larger rupture term.

Test results show that this procedure gives good results. Furthermore, it is consistent with the calculations previously used for tension members, where gross areas are used for one limit state of yielding (F_yA_g), and net area for the fracture limit state (F_uA_e).

The AISC Specification (J4.3) states that the available strength R_n for the block shear rupture design strength is as follows:

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.6F_yA_{gv} + U_{bs}F_uA_{nt}$$

(AISC Equation J4-5)

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

in which

$$A_{gv} = \text{gross area subjected to shear, in}^2\ (\text{mm}^2)$$
$$A_{nv} = \text{net area subjected to shear, in}^2\ (\text{mm}^2)$$
$$A_{nt} = \text{net area subjected to tension, in}^2\ (\text{mm}^2).$$

Another value included in AISC Equation J4-5 is a reduction factor U_{bs}. Its purpose is to account for the fact that stress distribution may not be uniform on the tensile plane for some connections. Should the tensile stress distribution be uniform, U_{bs} will be taken equal to 1.0, according to the AISC Specification (J4.3). The tensile stress is generally considered to be uniform for angles, gusset (or connection) plates, and for coped beams with one line of bolts. The connections of part (a) of Fig. 3.18 fall into this class. Should the tensile stress be nonuniform, U_{bs} is to be set equal to 0.5. Such a situation occurs in coped beams with two lines of bolts as illustrated in part (b) of the figure. The stress there is nonuniform because the row of bolts nearer the end of the beam picks up the largest proportion of the shear load. Should the bolts for coped beams be placed at nonstandard distances from beam ends, the same situation of nonuniform tensile stress can occur, and a U_{bs} value of 0.5 should be used.

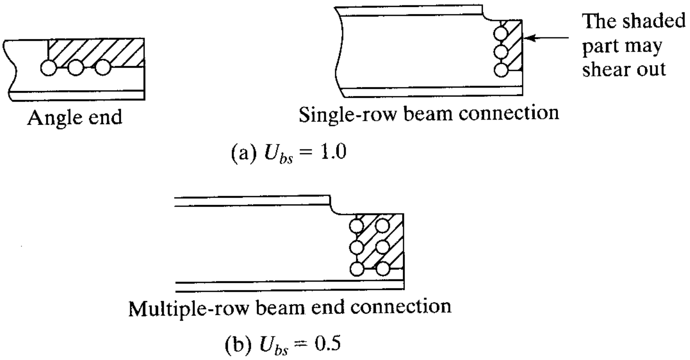


FIGURE 3.18

Block shear.

Examples 3-11 to 3-13 illustrate the determination of the block shear strengths for three members. The topic of block shear is continued in the connection chapters of this text, where we will find that it is absolutely necessary to check beam connections where the top flange of the beams are coped, or cut back, as illustrated in Figs. 10.2(c), 10.6, and 15.6(b). **Should the block shear strength of a connection be insufficient, it may be increased by increasing the edge distance and/or the bolt spacing.**

Example 3-11

The A572 Grade 50 (F_u = 65 ksi) tension member shown in Fig. 3.19 is connected with three 3/4-in bolts. Determine the LRFD block shear rupture strength and the ASD allowable block-shear rupture strength of the member. Also calculate the LRFD design tensile strength and the ASD allowable tensile strength of the member.

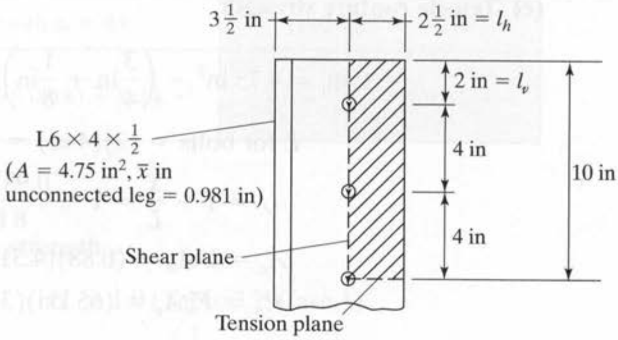


FIGURE 3.19

Solution

$$A_{gv} = (10\text{ in})\left(\frac{1}{2}\text{in}\right) = 5.0\text{ in}^2$$

$$A_{nv} = \left[10\text{ in} - (2.5)\left(\frac{3}{4}\text{in} + \frac{1}{8}\text{in}\right)\right]\left(\frac{1}{2}\text{in}\right) = 3.91\text{ in}^2$$

$$A_{nt} = \left[2.5\text{ in} - \left(\frac{1}{2}\right)\left(\frac{3}{4}\text{in} + \frac{1}{8}\text{in}\right)\right]\left(\frac{1}{2}\text{in}\right) = 1.03\text{ in}^2$$

$$U_{bs} = 1.0$$

$$R_n = (0.6)(65\text{ ksi})(3.91\text{ in}^2) + (1.0)(65\text{ ksi})(1.03\text{ in}^2) = 219.44\text{ k}$$
$$\leq (0.6)(50\text{ ksi})(5.0\text{ in}^2) + (1.0)(65\text{ ksi})(1.03\text{ in}^2) = 216.95\text{ k}$$
$$219.44\text{ k} > 216.95\text{ k}$$
$$\therefore R_n = 216.95\text{ k}$$

(a) Block shear strength

LRFD with ϕ = 0.75	ASD with Ω = 2.00
$\phi R_n = (0.75)(216.95\text{ k}) = 162.7\text{ k} \leftarrow$	$\dfrac{R_n}{\Omega} = \dfrac{216.95\text{ k}}{2.00} = 108.5\text{ k} \leftarrow$

(b) Nominal or available tensile strength of angle

$$P_n = F_y A_g = (50\text{ ksi})(4.75\text{ in}^2) = 237.5\text{ k}$$

Gross section yielding

LRFD with ϕ_t = 0.9	ASD with Ω_t = 1.67
$\phi_t P_n = (0.9)(237.5\text{ k}) = 213.7\text{ k}$	$\dfrac{P_n}{\Omega_t} = \dfrac{237.5\text{ k}}{1.67} = 142.2\text{ k}$

(c) Tensile rupture strength

$$A_n = 4.75\text{ in}^2 - \left(\frac{3}{4}\text{ in} + \frac{1}{8}\text{ in}\right)\left(\frac{1}{2}\text{ in}\right) = 4.31\text{ in}^2$$

$$L \text{ for bolts} = (2)(4\text{ in}) = 8\text{ in}$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.981\text{ in}}{8\text{ in}} = 0.88$$

$$A_e = UA_n = (0.88)(4.31\text{ in}^2) = 3.79\text{ in}^2$$

$$P_n = F_uA_e = (65\text{ ksi})(3.79\text{ in}^2) = 246.4\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(246.4\text{ k}) = 184.8\text{ k}$	$\frac{P_n}{\Omega_t} = \frac{246.4\text{ k}}{2.00} = 123.2\text{ k}$

Ans. LRFD = 162.7 k (Block shear controls) ASD = 108.5 k (Block shear controls)

Example 3-12

Determine the LRFD design strength and the ASD allowable strength of the A36 (F_y = 36 ksi, F_u = 58 ksi) plates shown in Fig. 3.20. Include block shear strength in the calculations.

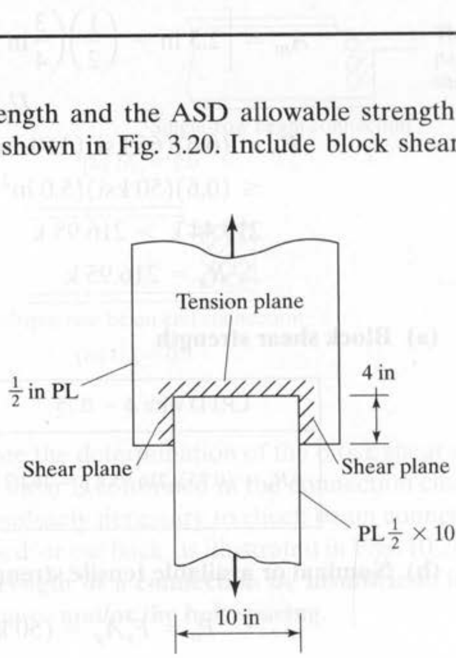


FIGURE 3.20

Solution

(a) Gross section yielding

$$P_n = F_yA_g = (36\text{ ksi})\left(\frac{1}{2}\text{ in} \times 10\text{ in}\right) = 180\text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(180\text{ k}) = 162\text{ k} \leftarrow$	$\dfrac{P_n}{\Omega_t} = \dfrac{180\text{ k}}{1.67} = 107.8\text{ k} \leftarrow$

(b) Tensile rupture strength

$$U = 1.0 \text{ (Table 3.2, Case 1)}$$

$$A_e = (1.0)\left(\frac{1}{2}\text{ in} \times 10\text{ in}\right) = 5.0\text{ in}^2$$

$$P_n = F_u A_e = (58\text{ ksi})(5.0\text{ in}^2) = 290\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(290\text{ k}) = 217.5\text{ k}$	$\dfrac{P_n}{\Omega_t} = \dfrac{290\text{ k}}{2.00} = 145\text{ k}$

(c) Block shear strength

$$A_{gv} = \left(\frac{1}{2}\text{ in}\right)(2 \times 4\text{ in}) = 4.00\text{ in}^2$$

$$A_{nv} = 4.00\text{ in}^2$$

$$A_{nt} = \left(\frac{1}{2}\text{ in}\right)(10\text{ in}) = 5.0\text{ in}^2$$

$$U_{bs} = 1.0$$

$$\begin{aligned} R_n &= (0.6)(58\text{ ksi})(4.0\text{ in}^2) + (1.00)(58\text{ ksi})(5.0\text{ in}^2) = 429.2\text{ k} \\ &\le (0.6)(36\text{ ksi})(4.0\text{ in}^2) + (1.00)(58\text{ ksi})(5.0\text{ in}^2) = 376.4\text{ k} \end{aligned}$$

$$429.2\text{ k} > 376.4\text{ k}$$

$$\therefore R_n = 376.4\text{ k}$$

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(376.4\text{ k}) = 282.3\text{ k}$	$\dfrac{R_n}{\Omega} = \dfrac{376.4\text{ k}}{2.00} = 188.2\text{ k}$

Ans. LRFD = 162 k (Yielding controls) ASD = 107.8 k (Yielding controls)

Example 3-13

Determine the LRFD tensile design strength and the ASD tensile strength of the W12 × 30 (F_y = 50 ksi, F_u = 65 ksi) shown in Fig. 3.21 if $\frac{7}{8}$-in bolts are used in the connection. Include block shear calculations for the flanges.

Solution

(a) Gross section yielding

$$P_n = F_y A_g = (50)(8.79) = 439.5 \text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(439.5) = 395.5 \text{ k}$	$\frac{P_n}{\Omega_t} = \frac{439.5}{1.67} = 263.2 \text{ k}$

(b) Tensile rupture strength

$$A_n = 8.79 \text{ in}^2 - (4)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.440 \text{ in}) = 7.03 \text{ in}^2$$

$$\bar{x} = \bar{y} \text{ in table} = 1.27 \text{ in for WT6} \times 15$$

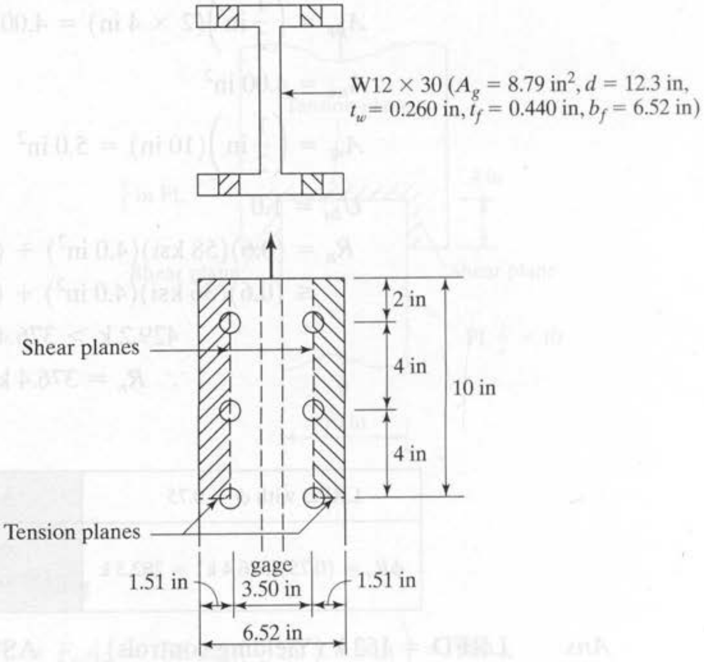


FIGURE 3.21

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.27 \text{ in}}{2 \times 4 \text{ in}} = 0.84$$

$$b_f = 6.52 \text{ in} < \frac{2}{3} \times 12.3 = 8.20 \text{ in}$$

$\therefore$ Use $U = 0.85$ for Case 7 in Table 3.2

$$A_e = UA_n = (0.85)(7.03 \text{ in}^2) = 5.98 \text{ in}^2$$

$$P_n = F_u A_e = (65 \text{ ksi})(5.98 \text{ in}^2) = 388.7 \text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(388.7 \text{ k}) = 291.5 \text{ k} \leftarrow$	$\frac{P_n}{\Omega_t} = \frac{388.7 \text{ k}}{2.00} = 194.3 \text{ k} \leftarrow$

(c) Block shear strength considering both flanges

$$A_{gv} = (4)(10 \text{ in})(0.440 \text{ in}) = 17.60 \text{ in}^2$$

$$A_{nv} = (4)\left[10 \text{ in} - (2.5)\left(\frac{7}{8} \text{ in} + \frac{1}{8} \text{ in}\right)\right]0.440 \text{ in} = 13.20 \text{ in}^2$$

$$A_{nt} = (4)\left[1.51 \text{ in} - \left(\frac{1}{2}\right)\left(\frac{7}{8} \text{ in} + \frac{1}{8} \text{ in}\right)\right]0.440 \text{ in} = 1.78 \text{ in}^2$$

$$R_n = (0.6)(65 \text{ ksi})(13.20 \text{ in}^2) + (1.00)(65 \text{ ksi})(1.78 \text{ in}^2) = 630.5 \text{ k}$$

$$\leq (0.6)(50 \text{ ksi})(17.60 \text{ in}^2) + (1.00)(65 \text{ ksi})(1.78 \text{ in}^2) = 643.7 \text{ k}$$

$$630.5 \text{ k} < 643.7 \text{ k}$$

$$\therefore R_n = 630.5 \text{ k}$$

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(630.5 \text{ k}) = 472.9 \text{ k}$	$\frac{R_n}{\Omega} = \frac{630.5 \text{ k}}{2.00} = 315.2 \text{ k}$

Ans. LRFD = 291.5 k (Rupture controls) ASD = 194.3 k (Rupture controls)

3.8 PROBLEMS FOR SOLUTION (USE STANDARD-SIZE BOLT HOLES FOR ALL PROBLEMS.)

3-1 to 3-12. *Compute the net area of each of the given members.*

3-1. (*Ans.* 5.34 in^2)

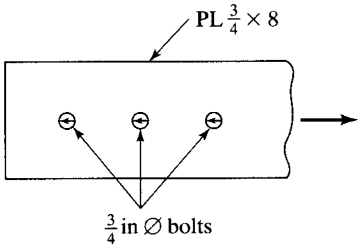


FIGURE P3-1

3-2.

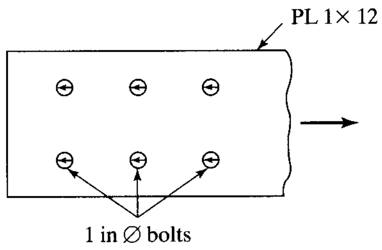


FIGURE P3-2

3-3. (*Ans.* 9.38 in^2)

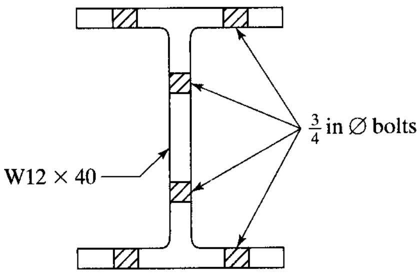


FIGURE P3-3

3-4.

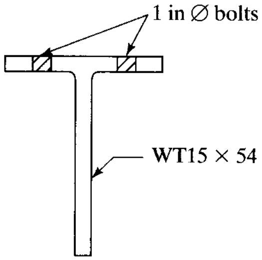


FIGURE P3-4

3-5. An L8 × 4 × 3/4 with two lines of $\frac{3}{4}$-in Ø bolts in the long leg and one line of $\frac{3}{4}$-in Ø bolts in the short leg. (*Ans.* 6.52 in^2)

3-6. A pair of Ls 4 × 4 × $\frac{1}{4}$, with one line of $\frac{7}{8}$-in Ø bolts in each leg.

3-7. A W18 × 35 with two holes in each flange and one in the web, all for $\frac{7}{8}$-in Ø bolts. (*Ans.* 8.30 in^2)

3-8. The built-up section shown in Fig. P3-8 for which $\frac{3}{4}$-in Ø bolts are used.

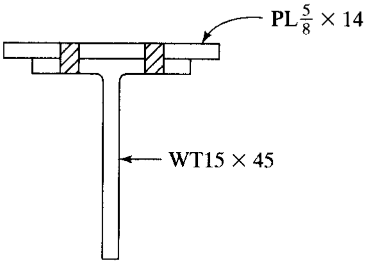


FIGURE P3-8

3-9. The 1 × 8 plate shown in Fig. P3-9. The holes are for $\frac{3}{4}$-in Ø bolts. (*Ans.* 6.44 in^2)

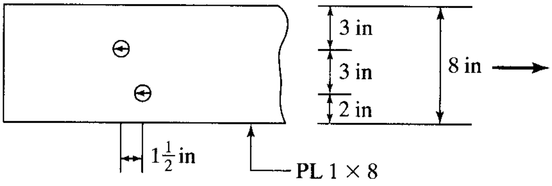


FIGURE P3-9

3-10. The 3/4 × 10 plate shown in Fig. P3-10. The holes are for 7/8-in Ø bolts.

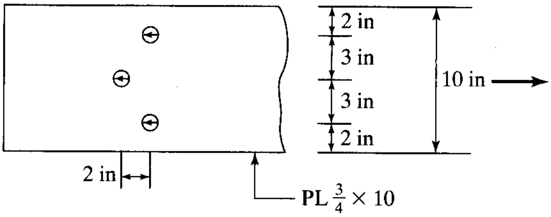


FIGURE P3-10

3-11. The 7/8 × 14 plate shown in Fig. P3-11. The holes are for 7/8-in Ø bolts. (*Ans.* 10.54 in^2)

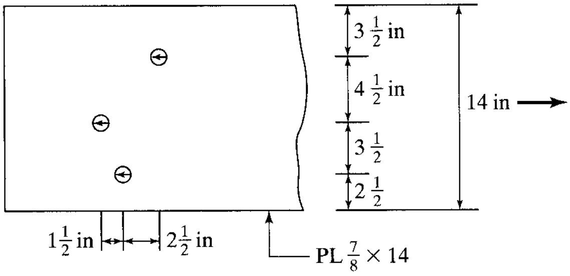


FIGURE P3-11

3-12. The 6 × 4 × 1/2 angle shown has one line of 3/4-in Ø bolts in each leg. The bolts are 4-in on center in each line and are staggered 2 in with respect to each other.

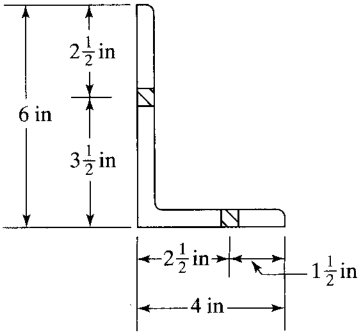


FIGURE P3-12

3-13. The tension member shown in Fig. P3-13 contains holes for 3/4-in Ø bolts. At what spacing, *s*, will the net area for the section through one hole be the same as a rupture line passing though two holes? (*Ans.* 3.24 in)

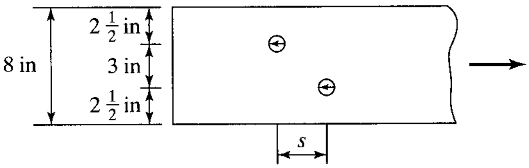


FIGURE P3-13

3-14. The tension member shown in Figure P3-14 contains holes for 7/8-in Ø bolts. At what spacing, *s*, will the net area for the section through two holes be the same as a rupture line passing through all three holes?

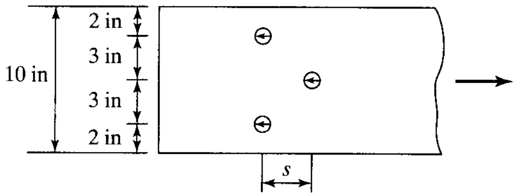


FIGURE P3-14

3-15. An L6 × 6 × 1/2 is used as a tension member, with one gage line of 3/4-in Ø bolts in each leg at the usual gage location (see Table 3.1). What is the minimum amount of stagger, s, necessary so that only one bolt need be subtracted from the gross area of the angle? Compute the net area of this member if the lines of holes are staggered at 3 in. (*Ans.* $s = 4.77$ in, $A_n = 5.05$ in^2)

3-16. An L8 × 4 × 3/4 is used as a tension member, with 7/8-in Ø bolts in each leg at the usual gage location (see Table 3.1). Two rows of bolts are used in the long leg, and one in the short leg. Determine the minimum stagger, s, necessary so that only two holes need be subtracted in determining the net area. What is the net area?

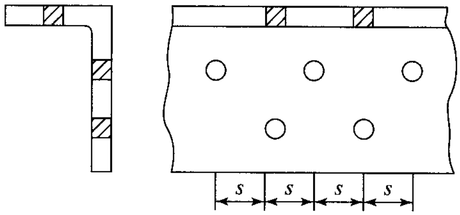


FIGURE P3-16

3-17. Determine the smallest net area of the tension member shown in Fig. P3-17. The holes are for 3/4-in Ø bolts at the usual gage locations. The stagger is 1 1/2 in. (*Ans.* 2.98 in^2)

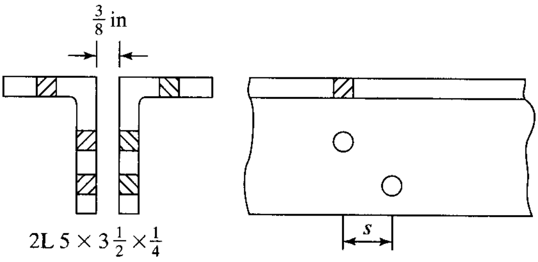


FIGURE P3-17

3-18. Determine the effective net cross-sectional area of the C12 × 25 shown in Fig. P3-18. Holes are for 3/4 in Ø bolts.

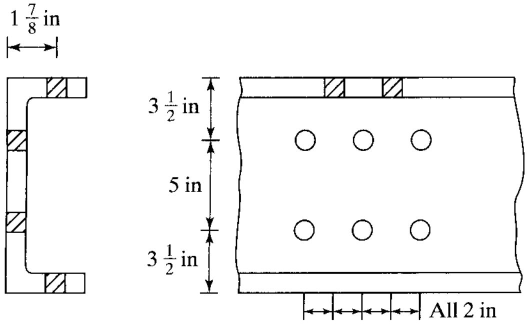


FIGURE P3-18

3-19. Compute the effective net area of the built-up section shown in Fig. P3-19 if the holes are punched for 3/4-in Ø bolts. Assume U = 0.90. (*Ans.* 20.18 in^2)

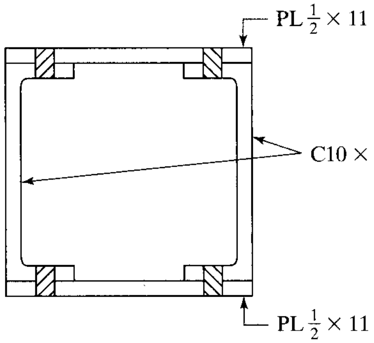


FIGURE P3-19

3-20 to 3-22. *Determine the effective net areas of the sections shown by using the U values given in Table 3.2 of this chapter.*

3-20.

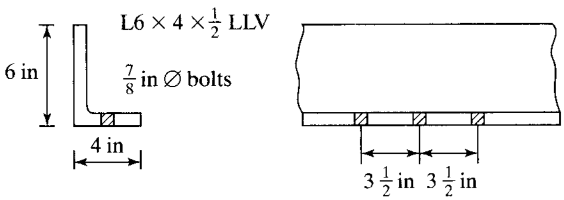


FIGURE P3-20

3-21. Determine the effective net area of the L7 × 4 × $\frac{1}{2}$ shown in Fig. P3-21. Assume the holes are for 1-in Ø bolts. (*Ans.* 3.97 in^2)

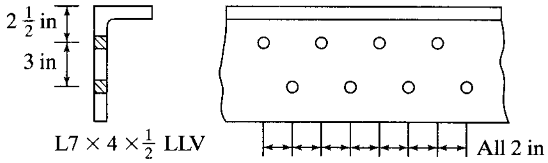


FIGURE P3-21

3-22. An MC12 × 45 is connected through its web with 3 gage lines of 7/8-in Ø bolts. The gage lines are 3 in on center and the bolts are spaced 3 in on center along the gage line. If the center row of bolts is staggered with respect to the outer row, determine the effective net cross-sectional area of the channel. Assume there are four bolts in each line.

3-23. Determine the effective net area of the W16 × 40 shown in Fig. P3-23. Assume the holes are for 3/4-in Ø bolts. (*Ans.* 8.53 in^2)

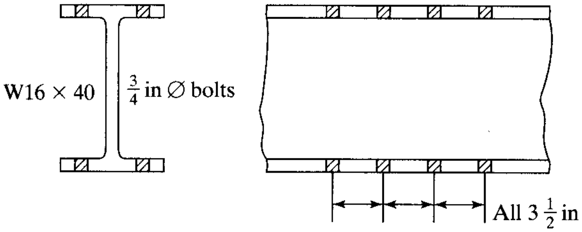


FIGURE P3-23

3-24 to 3-34. *Determine the LRFD design strength and the ASD allowable strength of sections given. Neglect block shear.*

3-24. A36 steel and 7/8-in Ø bolts

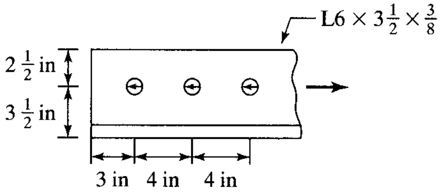


FIGURE P3-24

3-25. A36 steel and 3/4-in Ø bolts *(Ans.* LRFD 170.42 k, ASD 113.39 k)

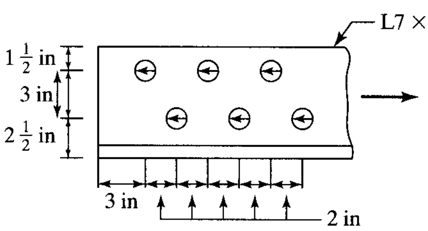


FIGURE P3-25

3-26. A36 steel and 7/8-in Ø bolts

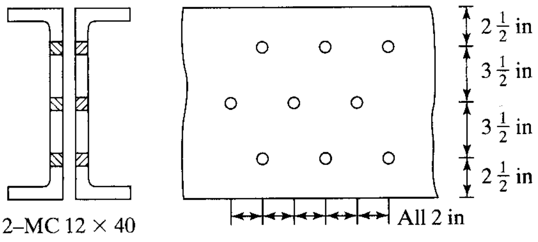


FIGURE P3-26

3-27. A W18 × 40 consisting of A992 steel and having two lines of 1-in Ø bolts in each flange. There are 4 bolts in each line, 3 in on center. (*Ans.* LRFD 391.1 k, ASD 260.7 k)

3-28. A WT8 × 50 of A992 steel having two lines of 7/8-in Ø bolts as shown in Fig. P3-28. There are 4 bolts in each line, 3 in on center.

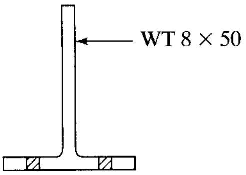


FIGURE P3-28

3-29. A W8 × 40 of A992 steel having two lines of 3/4-in Ø bolts in each flange. There are 3 bolts in each line, 4 in on center. (*Ans.* LRFD 431.2 k, ASD 287.4 k)

3-30. A double angle, ⅃L 7 × 4 × 3/4 in with two gage lines in its long leg and one in its short leg, for 7/8-in Ø bolts as shown in Fig. P3-30. Standard gages are to be used as determined from Table 3.1 in this chapter. A36 steel is used.

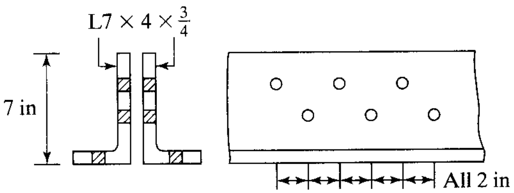


FIGURE P3-30

3-31. A C9 × 20 (F_y = 36 ksi, F_u = 58 ksi) with 2 lines of 7/8-in Ø bolts in the web as shown in Fig. P3-31. (*Ans.* LRFD 190.2 k, ASD 126.5 k)

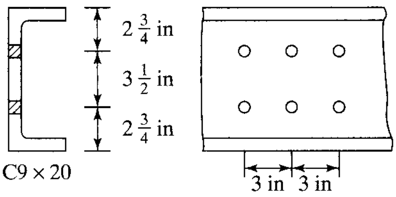


FIGURE P3-31

3-32. A WT5 × 15 consisting of A992 steel with a transverse weld to its flange only as shown in Fig. P3-32.

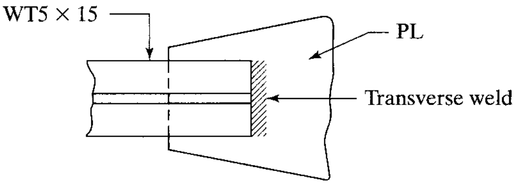


FIGURE P3-32

3-33. A C6 × 10.5 consisting of A36 steel with two longitudinal welds shown in Fig. P3-33. (*Ans.* LRFD 99.5 k, ASD 66.2 k)

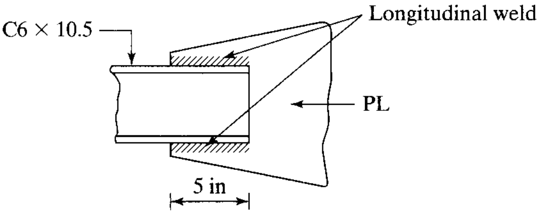


FIGURE P3-33

3-34. A $\frac{3}{8} \times 5$ plate consisting of A36 steel with two longitudinal welds as shown in Fig. P3-34.

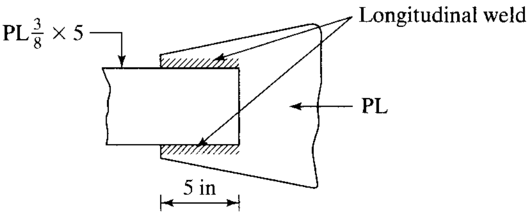


FIGURE P3-34

3-35 to 3-37. *Determine the LRFD design strength and the ASD allowable strength of the sections given, including block shear.*

3-35. A WT6 × 26.5, A992 steel, attached through the flange with six – 1-in Ø bolts as shown in Fig. P3-35. (*Ans.* LRFD 269.2 k, ASD 179.5 k)

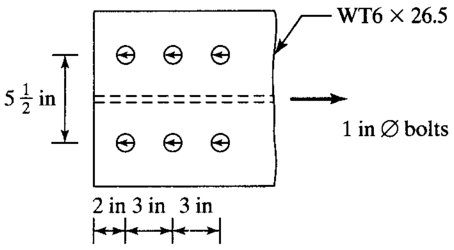


FIGURE P3-35

3-36. A C9 × 15 (A36 steel) with 2 lines of 3/4-in Ø bolts in the web as shown in Fig. P3-36.

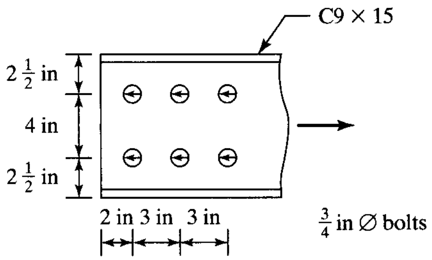


FIGURE P3-36

3-37. An 6 × 6 × 3/8 angle welded to a 3/8 in gusset plate as shown in Fig. P3-37. All steel is $F_y = 36$ ksi and $F_u = 58$ ksi. (*Ans.* LRFD 139.1 k, ASD 92.7 k)

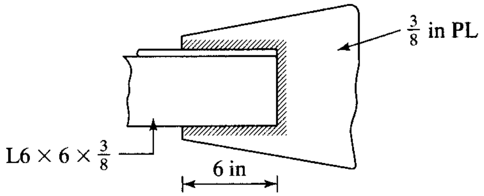


FIGURE P3-37

CHAPTER 4

Design of Tension Members

4.1 SELECTION OF SECTIONS

The determination of the design strengths of various tension members was presented in Chapter 3. In this chapter, the selection of members to support given tension loads is described. Although the designer has considerable freedom in the selection, the resulting members should have the following properties: (a) compactness, (b) dimensions that fit into the structure with reasonable relation to the dimensions of the other members of the structure, and (c) connections to as many parts of the sections as possible to minimize shear lag.

The choice of member type is often affected by the type of connections used for the structure. Some steel sections are not very convenient to bolt together with the required gusset or connection plates, while the same sections may be welded together with little difficulty. Tension members consisting of angles, channels, and W or S sections will probably be used when the connections are made with bolts, while plates, channels, and structural tees might be used for welded structures.

Various types of sections are selected for tension members in the examples to follow, and in each case where bolts are used, some allowance is made for holes. Should the connections be made *entirely* by welding, no holes have to be added to the net areas to give the required gross area. *The student should realize, however, that, very often, welded members may have holes punched in them for temporary bolting during field erection before the permanent field welds are made. These holes need to be considered in design.* It must also be remembered that in AISC Equation D2-2 ($P_n = F_u A_e$) the value of A_e may be less than A_g, even though there are no holes, depending on the arrangement of welds and on whether all of the parts of the members are connected.

The slenderness ratio of a member is the ratio of its unsupported length to its least radius of gyration. Steel specifications give preferable maximum values of slenderness ratios for both tension and compression members. The purpose of such limitations for tension members is to ensure the use of sections with stiffness sufficient to prevent undesirable lateral deflections or vibrations. Although tension members are not subject to buckling under normal loads, stress reversal may occur during shipping and erection and perhaps due to wind or earthquake loads. Specifications usually recommend that slenderness ratios be kept below certain maximum values in order that some minimum compressive strengths be provided in the members. For tension members other than rods, the AISC Specification does not provide a maximum slenderness ratio for tension members, but Section D.1 of the specification suggests that a maximum value of 300 be used.

It should be noted that out-of-straightness does not affect the strength of tension members very much, because the tension loads tend to straighten the members. (The same statement cannot be made for compression members.) For this reason, the AISC Specification is a little more liberal in its consideration of tension members, including those subject to some compressive forces due to transient loads such as wind or earthquake.

The *recommended* maximum slenderness ratio of 300 is not applicable to tension rods. Maximum L/r values for rods are left to the designer's judgment. If a maximum value of 300 were specified for them, they would seldom be used, because of their extremely small radii of gyration, and thus very high slenderness ratios.

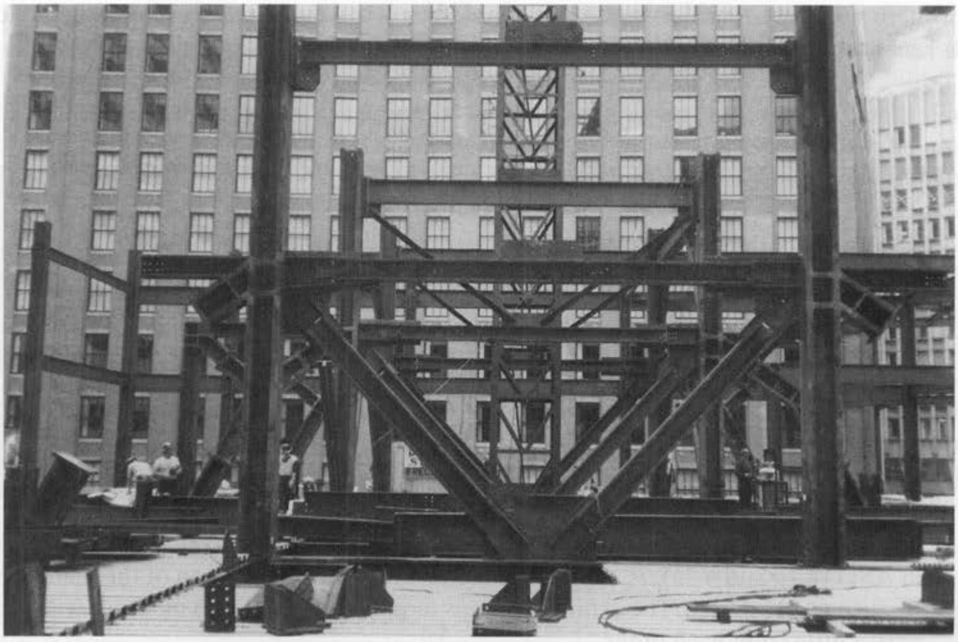
The AASHTO Specifications provide mandatory maximum slenderness ratios of 200 for main tension members and 240 for secondary members. (A *main member* is defined by the AASHTO as one in which stresses result from dead and/or live loads, while *secondary members* are those used to brace structures or to reduce the unbraced length of other members—main or secondary.) *No such distinction is made in the AISC Specification between main and secondary members.* The AASHTO also requires that the maximum slenderness ratio permitted for members subjected to stress reversal be 140.

The design of steel members is, in effect, a trial-and-error process, although tables such as those given in the Steel Manual often enable us to directly select a desirable section. For a tension member, we can estimate the area required, select a section from the Manual providing the corresponding area, and check the section's strength, as described in the previous chapter. After this is done, it may be necessary to try a slightly larger or perhaps smaller section and repeat the checking process. The goal of the design process is to size members such that they are safe by satisfying the failure conditions listed in the AISC Specification. The student must realize that this process is iterative and that there will be some rounding up or down in the process of selecting the final section. The area needed for a particular tension member can be estimated with the LRFD equations or the ASD equations, as described next.

If the LRFD equations are used, the design strength of a tension member is the least of $\phi_t F_y A_g$, $\phi_t F_u A_e$, or its block shear strength. In addition, the slenderness ratio should, preferably, not exceed 300.

a. To satisfy the first of these expressions, the minimum gross area must be at least equal to

$$\min A_g = \frac{P_u}{\phi_t F_y}. \tag{4.1}$$



Transfer truss, 150 Federal Street, Boston, MA. (Courtesy Owen Steel Company, Inc.)

b. To satisfy the second expression, the minimum value of A_e must be at least

$$\min A_e = \frac{P_u}{\phi_t F_u}.$$

And since $A_e = UA_n$ for a bolted member, the minimum value of A_n is

$$\min A_n = \frac{\min A_e}{U} = \frac{P_u}{\phi_t F_u U}.$$

Then the minimum A_g is

$$= \min A_n + \text{estimated area of holes}$$

$$= \frac{P_u}{\phi_t F_u U} + \text{estimated area of holes} \tag{4.2}$$

c. The third expression can be evaluated, once a trial shape has been selected and the other parameters related to the block shear strength are known.

The designer can substitute into Equations 4.1 and 4.2, taking the larger value of A_g so obtained for an initial size estimate. It is, however, well to notice that the maximum preferable slenderness ratio L/r is 300. From this value, it is easy to compute the smallest preferable value of r with respect to each principal axis of the cross section for a particular design—that is, the value of r for which the slenderness ratio will be exactly 300. It is undesirable to consider a section whose least r is less than this value, because its slenderness ratio would exceed the preferable maximum value of 300:

$$\min r = \frac{L}{300} \tag{4.3}$$

If the ASD equations are used for tension member design, the allowable strength is the lesser of $\frac{F_y A_g}{\Omega_t}$ and $\frac{F_u U A_n}{\Omega_t}$. From these expressions, the minimum gross areas required are as follows:

$$\min A_g = \frac{\Omega_t P_a}{F_y} \tag{4.1a}$$

$$\min A_g = \frac{\Omega_t P_a}{F_u U} + \text{estimated area of holes} \tag{4.2a}$$

In the expressions for LRFD (4.1 and 4.2), P_u represents the factored load forces; in ASD (4.1a and 4.2a), P_a represents the result of our application of the load combinations for ASD design. The estimated areas required by these two methods will normally vary a little from each other.

Example 4-1 illustrates the design of a bolted tension member with a W section, while Example 4-2 illustrates the selection of a bolted single angle tension member. In both problems, the areas are estimated with the LRFD expressions. After sections are selected from the Manual, they are checked for their LRFD design strengths and for their allowable ASD strengths. Whichever of the two methods is being used, it may be necessary to try a larger or smaller section and go through the calculations again.

For many of the example design problems presented in this text, the author has used only the LRFD expressions for estimating preliminary member sizes. He could just as well have used only the ASD design expressions. The results by the two methods will be very close to each other. Whatever the estimated sizes, they are carefully checked with both the appropriate LRFD and ASD equations. If the equations are not satisfied, new member sizes will be estimated and checked. We will have the same final results whether we dreamed up an estimated first size out of the blue or used some equation for estimating.

You will find on some occasions that a slightly smaller section will satisfy the LRFD equations than will satisfy the ASD equations. One reason for this is the fact that the load factors required for dead loads are much smaller than those required for live loads. Such is not the case with ASD and its safety factors.

Usually, for the examples in this text only, D and L loads are specified so that we will not have to go through all of the load combination expressions. For such problems, then, we will need only to use the following load combinations:

For LRFD	For ASD
$P_u = 1.4D$	$P_a = D + L$
$P_u = 1.2D + 1.6L$	

As the first of the LRFD expressions will not control unless the dead load is more than eight times as large as the live load, the first expression is omitted for the remaining problems in this text (unless $D > 8L$).

In Example 4-1, a W section is selected for a given set of tensile loads. For this first application of the tension design formulas, the authors have narrowed the problem down to one series of W shapes so that the reader can concentrate on the application of the formulas and not become lost in considering W8s, W10s, W14s, and so on. Exactly the same procedure can be used for trying these other series, as is used here for the W12.

Example 4-1

Select a 30-ft-long W12 section of A992 steel to support a tensile service dead load $P_D = 130$ k and a tensile service live load $P_L = 110$ k. As shown in Fig. 4.1, the member is to have two lines of bolts in each flange for 7/8-in bolts (at least three in a line 4 in on center).

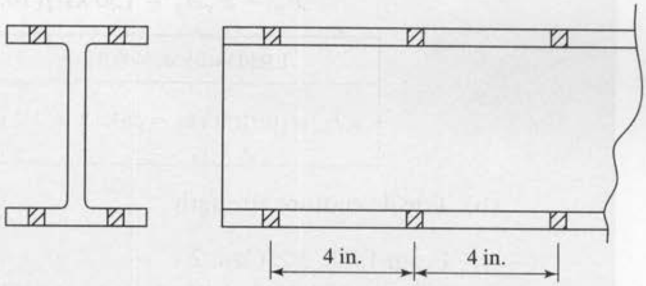


FIGURE 4.1
Cross section of member for Example 4-1.

Solution

(a) Considering the necessary load combinations

LRFD	ASD
$P_u = 1.4D = (1.4)(130\text{ k}) = 182\text{ k}$ $P_u = 1.2D + 1.6L = (1.2)(130\text{ k}) + (1.6)(110\text{ k}) = 332\text{ k}$	$P_a = D + L = 130\text{ k} + 110\text{ k}$ $= 240\text{ k}$

(b) Computing the minimum A_g required, using LRFD Equations 4.1 and 4.2

1. $\text{min } A_g = \dfrac{P_u}{\phi_t F_y} = \dfrac{332\text{ k}}{(0.90)(50\text{ ksi})} = 7.38\text{ in}^2$

2. $\text{min } A_g = \dfrac{P_u}{\phi_t F_u U} + \text{estimated hole areas}$

Assume that $U = 0.85$ from Table 3.2, Case 7, and assume that flange thickness is about 0.380 in after looking at W12 sections in the LRFD Manual which have areas of 7.38 in^2 or more. $U = 0.85$ was assumed since b_f appears to be less than 2/3 d.

$$\min A_g = \frac{332\text{ k}}{(0.75)(65\text{ ksi})(0.85)} + (4)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.380\text{ in}) = 9.53\text{ in}^2 \leftarrow$$

(c) Preferable minimum r

$$\min r = \frac{L}{300} = \frac{(12\text{ in/ft})(30\text{ ft})}{300} = 1.2\text{ in}$$

Try W12 × 35 ($A_g = 10.3\text{ in}^2$, $d = 12.50\text{ in}$, $b_f = 6.56\text{ in}$, $t_f = 0.520\text{ in}$, $r_{\min} = r_y = 1.54\text{ in}$)

Checking

(a) Gross section yielding

$$P_n = F_y A_g = (50\text{ ksi})(10.3\text{ in}^2) = 515\text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(515\text{ k}) = 463.5\text{ k} > 332\text{ k}$ **OK**	$\frac{P_n}{\Omega_t} = \frac{515\text{ k}}{1.67} = 308.4 > 240\text{ k}$ **OK**

(b) Tensile rupture strength

From Table 3.2, Case 2

$\bar{x}$ for half of W12 × 35 or, that is, a WT6 × 17.5 = 1.30 in

$$L = (2)(4\text{ in}) = 8\text{ in}$$

$$U = \left(1 - \frac{\bar{x}}{L}\right) = \left(1 - \frac{1.30\text{ in}}{8\text{ in}}\right) = 0.84$$

From Table 3.2, Case 7

$$U = 0.85,\text{ since } b_f = 6.56\text{ in} < \frac{2}{3}d = \left(\frac{2}{3}\right)(12.50\text{ in}) = 8.33\text{ in},$$

$$A_n = 10.3\text{ in}^2 - (4)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.520\text{ in}) = 8.22\text{ in}^2$$

$$A_e = (0.85)(8.22\text{ in}^2) = 6.99\text{ in}^2$$

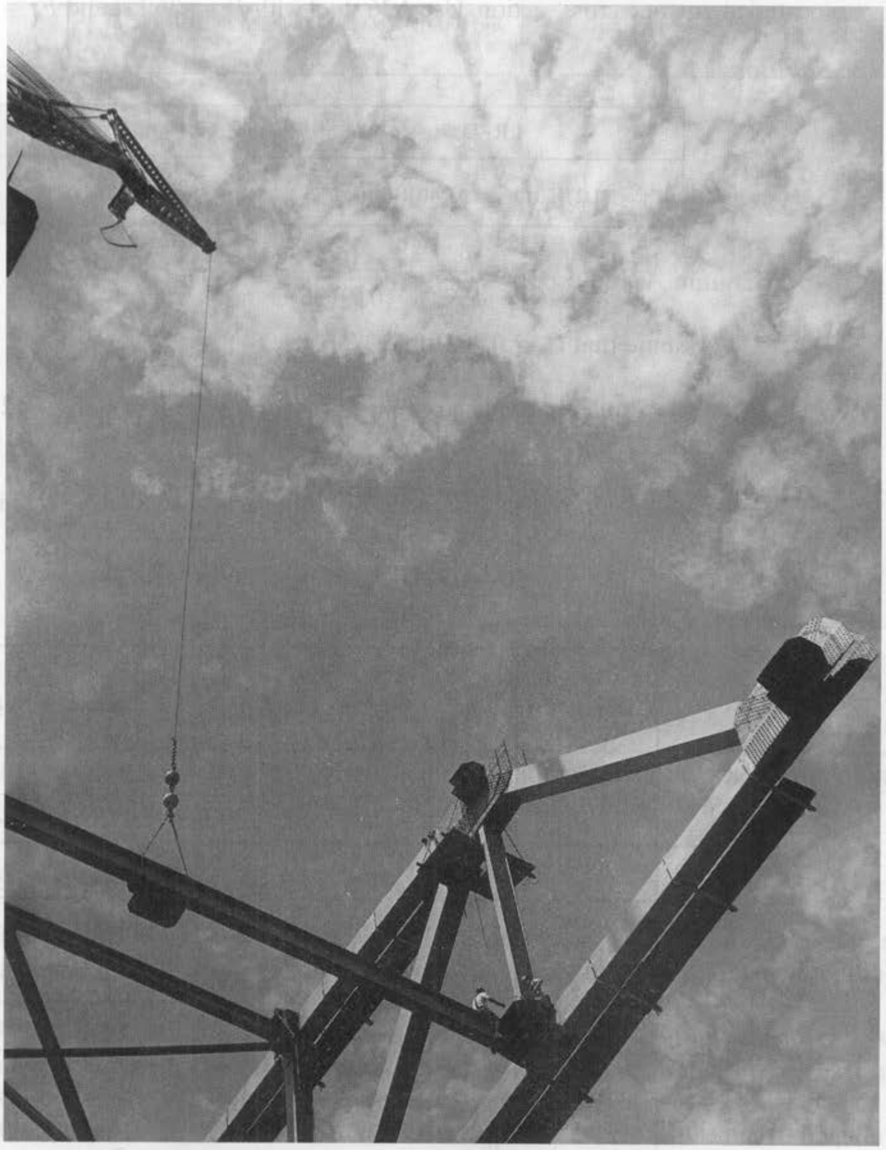
$$P_n = F_u A_e = (65\text{ ksi})(6.99\text{ in}^2) = 454.2\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(454.2\text{ k}) = 340.7\text{ k} > 332\text{ k}$ **OK**	$\frac{P_n}{\Omega_t} = \frac{454.2\text{ k}}{2.00} = 227.1\text{ k} < 240\text{ k}$ **N.G.**

(c) Slenderness ratio

$$\frac{L_y}{r_y} = \frac{12 \text{ in/ft} \times 30 \text{ ft}}{1.54 \text{ in}} = 234 < 300, \textbf{OK} \qquad \text{OK}$$

Ans. By LRFD, use W12 × 35. By ASD, use next larger section W12 × 40.



Bridge over Allegheny River at Kittaning, PA. (Courtesy of the American Bridge Company.)

In Example 4-2, a broader situation is presented, in that the lightest satisfactory angle in the Steel Manual is selected for a given set of tensile loads.

Example 4-2

Design a 9-ft single-angle tension member to support a dead tensile working load of 30 k and a live tensile working load of 40 k. The member is to be connected to one leg only with 7/8-in bolts (at least four in a line 3 in on center). Assume that only one bolt is to be located at any one cross section. Use A36 steel with $F_y = 36$ ksi and $F_u = 58$ ksi.

Solution

LRFD	ASD
$P_u = (1.2)(30) + (1.6)(40) = 100$ k	$P_a = 30 + 40 = 70$ k

1. $$\min A_g \text{ required} = \frac{P_u}{\phi_t F_y} = \frac{100}{(0.9)(36)} = 3.09 \text{ in}^2$$

2. Assume that $U = 0.80$, Table 3.2 (Case 8)

$$\min A_n \text{ required} = \frac{P_u}{\phi_t F_u U} = \frac{100 \text{ k}}{(0.75)(58 \text{ ksi})(0.80)} = 2.87 \text{ in}^2$$

$$\min A_g \text{ required} = 2.87 \text{ in}^2 + \text{bolt hole area} = 2.87 \text{ in}^2 + \left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(t)$$

3. $$\text{Min } r \text{ required} = \frac{(12 \text{ in/ft})(9 \text{ ft})}{300} = 0.36 \text{ in}$$

Angle $t_{(\text{in})}$	Area of one 1-in bolt hole (in^2)	Gross area required = larger of $P_u/\phi_t F_y$ or $P_u/\phi_t F_u U$ + est. hole area (in^2)	Lightest angles available, their areas (in^2) and least radii of gyration (in)
5/16	0.312	3.18	$6 \times 6 \times \frac{5}{16}$ ($A = 3.67$, $r_z = 1.19$)
3/8	0.375	3.25	$6 \times 3\frac{1}{2} \times \frac{3}{8}$ ($A = 3.44$, $r_z = 0.763$)
7/16	0.438	3.30	$4 \times 4 \times \frac{7}{16}$ ($A = 3.30$, $r_z = 0.777$) ←
			$5 \times 3 \times \frac{7}{16}$ ($A = 3.31$, $r_z = 0.644$)
1/2	0.500	3.37	$4 \times 3\frac{1}{2} \times \frac{1}{2}$ ($A = 3.50$, $r_z = 0.716$)
5/8	0.625	3.50	$4 \times 3 \times \frac{5}{8}$ ($A = 3.99$, $r_z = 0.631$)

Try L4 × 4 × $\frac{7}{16}$ ($\bar{x} = 1.15$ in)

Checking

(a) Gross section yielding

$$P_n = F_y A_g = (36\text{ ksi})(3.30\text{ in}^2) = 118.8\text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(118.8\text{ k}) = 106.9\text{ k} > 100\text{ k OK}$	$\dfrac{P_n}{\Omega_t} = \dfrac{118.8\text{ k}}{1.67} = 71.1\text{ k} > 70\text{ k OK}$

(b) Tensile rupture strength

$$A_n = 3.30\text{ in}^2 - (1)\left(\frac{7}{16}\text{ in}\right) = 2.86\text{ in}^2$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{1.15\text{ in}}{(3)(3\text{ in})} = 0.87 \leftarrow$$

U from Table 3.2 (Case 8) $= 0.80$

$A_e = UA_n = (0.87)(2.86\text{ in}^2) = 2.49\text{ in}^2$

$P_n = F_u A_e = (58\text{ ksi})(2.49\text{ in}^2) = 144.4\text{ k}$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(144.4\text{ k}) = 108.3\text{ k} > 100\text{ k OK}$	$\dfrac{P_n}{\Omega_t} = \dfrac{144.4\text{ k}}{2.00} = 72.2\text{ k} > 70\text{ k OK}$

Ans. By LRFD, use L4 $\times$ 4 $\times \frac{7}{16}$. By ASD, select L4 $\times$ 4 $\times \frac{7}{16}$.

In the CD enclosed with the Manual are tension member designs for other steel sections. Included are WT, rectangular and round HSS, and double angle sections.

4.2 BUILT-UP TENSION MEMBERS

Sections D4 and J3.5 of the AISC Specification provide a set of definite rules describing how the different parts of built-up tension members are to be connected together.

1. When a tension member is built up from elements in continuous contact with each other, such as a plate and a shape, or two plates, the longitudinal spacing of connectors between those elements must not exceed 24 times the thickness of the thinner plate—or 12 in if the member is to be painted, or if it is not to be painted and not to be subjected to corrosive conditions.

2. Should the member consist of unpainted weathering steel elements in continuous contact and be subject to atmospheric corrosion, the maximum permissible connector spacings are 14 times the thickness of the thinner plate, or 7 in.
3. Should a tension member be built up from two or more shapes separated by intermittent fillers, the shapes preferably should be connected to each other at intervals such that the slenderness ratio of the individual shapes between the fasteners does not exceed 300.
4. The distance from the center of any bolts to the nearest edge of the connected part under consideration may not be larger than 12 times the thickness of the connected part, or 6 in.
5. For elements in continuous contact with each other, the spacing of connectors are given in Sections J3.3 through J3.5 of the AISC Specification.

Example 4-3 illustrates the review of a tension member that is built up from two channels which are separated from each other. Included in the problem is the design of tie plates or tie bars to hold the channels together, as shown in Fig. 4.2(b). These plates, which are used to connect the parts of built-up members on their open sides, result in more uniform stress distribution among the various parts. Section D4 of the AISC Specification provides empirical rules for their design. (Perforated cover plates may also be used.) The rules are based on many decades of experience with built-up tension members.

In the "Dimensions and Properties" section of Part 1 of the Manual, the usual positions for placing bolts in the flanges of Ws, Cs, WTs, etc., are listed under the heading "Workable Gage." For the channels used in this example, the gage g is given as $1\frac{3}{4}$ in and is shown in Fig. 4.2.

In Fig. 4.2, the distance between the lines of bolts connecting the tie plates in the channels can be seen to equal 8.50 in. The AISC Specification (D4) states that the length of tie plates (lengths are always measured parallel to the long direction of the members, in this text) may not be less than two-thirds the distance between the lines of connectors. Furthermore, their thickness may not be less than one-fiftieth of this distance.

The minimum permissible width of tie plates (not mentioned in the specification) is the width between the lines of connectors plus the necessary edge distance on each side, to keep the bolts from splitting the plate. For this example, this minimum edge distance is taken as $1\frac{1}{2}$ in, from Table J3.4 of the AISC Specification. (Detailed information concerning edge distances for bolts is provided in Chapter 12 .) The plate dimensions are rounded off to agree with the plate sizes available from the steel mills, as given in the Bars and Plates section of Part 1 of the Steel Manual. It is much cheaper to select standard thicknesses and widths rather than to pick odd ones that will require cutting or other operations.

The AISC Specification (D4) provides a maximum spacing between tie plates by stating that the L/r of each individual component of a built-up member running along by itself between tie plates preferably should not exceed 300. If the designer substitutes into this expression ($L/r = 300$), the least r of an individual component of the built-up member, the value of L may be computed. This will be the maximum spacing of the tie plates preferred by the AISC Specification for this member.

Example 4-3

The two C12 × 30s shown in Fig. 4.2 have been selected to support a dead tensile working load of 120 k and a 240-k live tensile working load. The member is 30 ft long, consists of A36 steel, and has one line of three 7/8-in bolts in each channel flange 3 in on center. Using the AISC Specification, determine whether the member is satisfactory and design the necessary tie plates. Assume centers of bolt holes are 1.75 in from the backs of the channels.

Solution. Using C12 × 30s ($A_g = 8.81\text{ in}^2$ each, $t_f = 0.501$ in, $I_x = 162\text{ in}^4$ each, $I_y = 5.12\text{ in}^4$ each, y axis 0.674 in from back of C, $r_y = 0.762$ in)

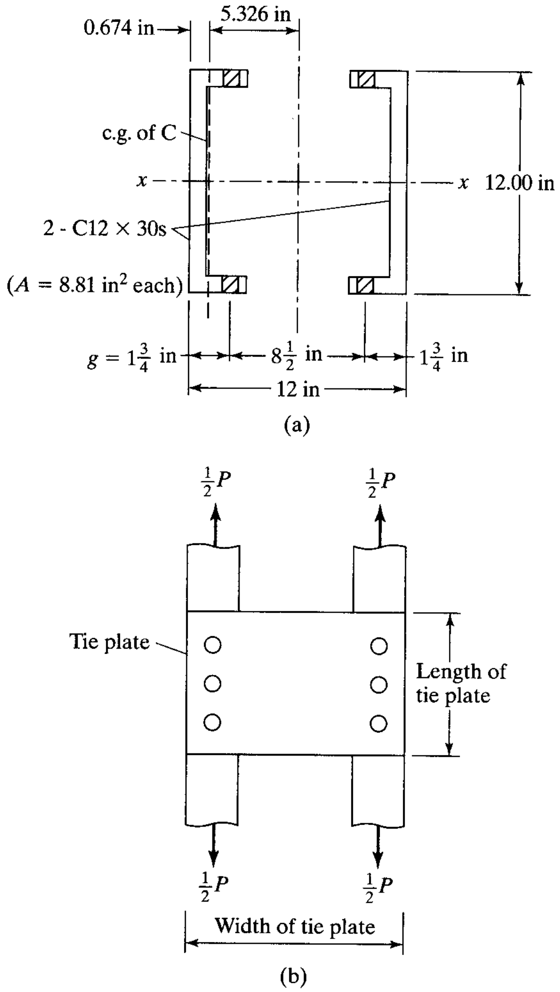


FIGURE 4.2

Built-up section for Example 4-3.

Solution

Loads to be resisted

LRFD	ASD
$P_u = (1.2)(120\text{ k}) + (1.6)(240\text{ k}) = 528\text{ k}$	$P_a = 120\text{ k} + 240\text{ k} = 360\text{ k}$

(a) Gross section yielding

$$P_n = F_y A_g = (36\text{ ksi})(2 \times 8.81\text{ in}^2) = 634.3\text{ k}$$

LRFD with $\phi_t = 0.9$	ASD with $\Omega_t = 1.67$
$\phi_t P_n = (0.9)(634.3\text{ k}) = 570.9\text{ k} > 528\text{ k}$ **OK**	$\dfrac{P_n}{\Omega_t} = \dfrac{634.3\text{ k}}{1.67} = 379.8\text{ k} > 360\text{ k}$ **OK**

(b) Tensile rupture strength

$$A_n = 2\left[8.81\text{ in}^2 - (2)\left(\frac{7}{8}\text{ in} + \frac{1}{8}\text{ in}\right)(0.501\text{ in})\right] = 15.62\text{ in}^2$$

$$U = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.674\text{ in}}{(2)(3\text{ in})} = 0.89 \text{ from Table 3.2 (Case 2)}$$

$$P_n = F_u U A_n = (58\text{ ksi})(15.62\text{ in}^2)(0.89) = 806.3\text{ k}$$

LRFD with $\phi_t = 0.75$	ASD with $\Omega_t = 2.00$
$\phi_t P_n = (0.75)(806.3\text{ k}) = 604.7\text{ k} > 528\text{ k}$ **OK**	$\dfrac{P_n}{\Omega_t} = \dfrac{806.3\text{ k}}{2.00} = 403.1\text{ k} > 360\text{ k}$ **OK**

Slenderness ratio

$$I_x = (2)(162\text{ in}^4) = 324\text{ in}^4$$

$$I_y = (2)(5.12\text{ in}^4) + (2)(8.81\text{ in}^2)(5.326\text{ in})^2 = 510\text{ in}^4$$

$$r_x = \sqrt{\frac{324\text{ in}^4}{17.62\text{ in}^2}} = 4.29\text{ in} < r_y = \sqrt{\frac{510}{17.62}} = 5.38\text{ in}$$

$$\therefore\ r_{\min} = r_x = 4.29\text{ in}$$

$$\frac{L_x}{r_x} = \frac{(12\text{ in/ft} \times 30\text{ ft})}{4.29\text{ in}} = 83.9 < 300$$

Design of tie plates (AISC Specification D4)

Distance between lines of bolts $= 12.00\text{ in} - (2)\left(1\tfrac{3}{4}\text{ in}\right) = 8.50\text{ in}$

Minimum length of tie plates $= \left(\frac{2}{3}\right)(8.50 \text{ in}) = 5.67 \text{ in } (\textit{say 6 in})$

Minimum thickness of tie plates $= \left(\frac{1}{50}\right)(8.50 \text{ in}) = 0.17 \text{ in } \left(\textit{say } \frac{3}{16} \textit{ in}\right)$

Minimum width of tie plates $= 8.50 \text{ in} + (2)\left(1\frac{1}{2} \text{ in}\right) = 11.5 \text{ in } (\textit{say 12 in})$

Maximum preferable spacing of tie plates

Least r of one $C = 0.762 \text{ in} = r_y$

Maximum preferable $\frac{L}{r} = 300$

$$\frac{(12 \text{ in/ft})(L)}{0.762 \text{ in}} = 300$$

$$L = 19.05 \text{ ft } (\textit{say 15 ft})$$

Use $\frac{3}{16} \times 6 \times 1$ ft 0 in tie plate 15 ft 0 in on center.

4.3 RODS AND BARS

When rods and bars are used as tension members, they may be simply welded at their ends, or they may be threaded and held in place with nuts. The AISC nominal tensile design stress for threaded rods, F_{nt}, is given in AISC Table J3.2 and equals $0.75F_u$. This is to be applied to the gross area of the rod A_D, computed with the major thread diameter—that is, the diameter to the outer extremity of the thread. The area required for a particular tensile load can then be calculated as follows:

$$R_n = F_{nt}A_D = 0.75\,F_uA_D$$

$\phi = 0.75$ LRFD	$\Omega = 2.00$ ASD
$A_D \geq \frac{P_u}{\phi\, 0.75F_u}$	$A_D \geq \frac{\Omega P_a}{0.75F_u}$

In Table 7-18 of the Manual, entitled "Threading Dimensions for High-Strength and Non-High-Strength Bolts," properties of standard threaded rods are presented. Example 4-4 illustrates the selection of a rod by the use of this table.

Example 4-4

Using the AISC Specification, select a standard threaded rod of A36 steel to support a tensile working dead load of 10 k and a tensile working live load of 20 k.

Solution

LRFD	ASD
$P_u = (1.2)(10 \text{ k}) + (1.6)(20 \text{ k}) = 44 \text{ k}$	$P_a = 10 \text{ k} + 20 \text{ k} = 30 \text{ k}$

$$A_D \geq \frac{P_u}{\phi\, 0.75F_u} = \frac{44\text{ k}}{(0.75)(0.75)(58\text{ ksi})} = 1.35\text{ in}^2$$

Try $1\frac{3}{8}$ in diameter rod from AISC Table 7-17 using the gross area of the rod 1.49 in^2.

$$R_n = 0.75F_uA_D = (0.75)(58\text{ ksi})(1.49\text{ in}^2) = 64.8\text{ k}$$

LRFD $\phi = 0.75$	ASD $\Omega = 2.00$
$\phi R_n = (0.75)(64.8\text{ k}) = 48.6\text{ k} > 44\text{ k}$ **OK**	$\frac{R_n}{\Omega} = \frac{64.8\text{ k}}{2.00} = 32.4 > 30\text{ k}$ **OK**

Use $1\frac{3}{8}$ in-diameter rod with 6 threads per in.

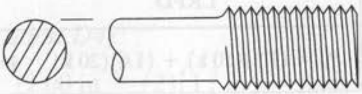
As shown in Fig. 4.3, upset rods sometimes are used, where the rod ends are made larger than the regular rod and the threads are placed in the upset ends. Threads, obviously, reduce the cross-sectional area of a rod. If a rod is upset and the threads are placed in that part of the rod, the result will be a larger cross-sectional area at the root of the thread than we would have if the threads were placed in the regular part of the rod.

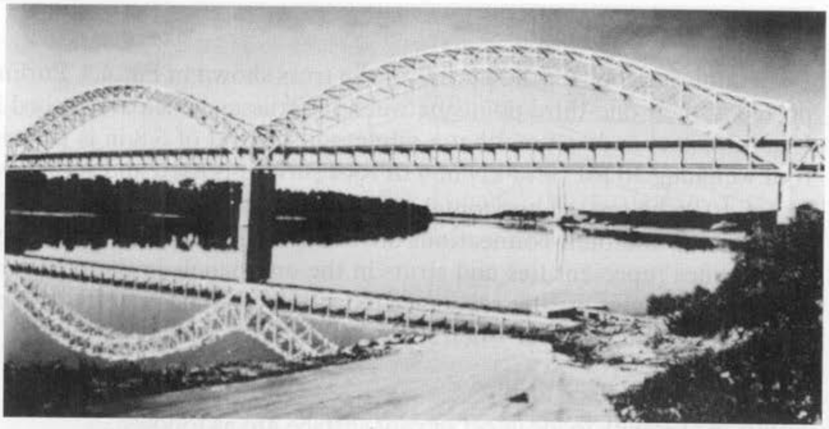
Table J3.2 footnote (d) in the AISC Specification states that the nominal tensile strength of the threaded portion of the upset ends is equal to $0.75F_uA_D$, where A_D is the cross-sectional area of the rod at its major thread diameter. This value must be larger than the nominal body area of the rod (before upsetting) times F_y, so that the net section fracture strength exceeds the gross section yield strength.

Upsetting permits the designer to use the entire area of the regular part of the bar for strength calculations. Nevertheless, the use of upset rods probably is not economical and should be avoided unless a large order is being made.

One situation in which tension rods are sometimes used is in steel-frame industrial buildings with purlins running between their roof trusses to support the roof surface. These types of buildings will also frequently have girts running between the columns along the vertical walls. (Girts are horizontal beams used on the sides of buildings, usually industrial, to resist lateral bending due to wind. They also are often used to support corrugated or other types of siding.) Sag rods may be required to provide support for the purlins parallel to the roof surface and vertical support for the girts along the walls. For roofs with steeper slopes than one vertically to four horizontally, sag rods are often considered necessary to provide lateral support for the purlins, particularly where the purlins consist of steel channels. Steel channels are commonly used as purlins, but they have very little resistance to lateral bending. Although the resisting moment needed parallel to the roof surface is small, an extremely large channel is required to provide

FIGURE 4.3
A round upset rod.





New Albany Bridge crossing the Ohio River between Louisville, KY, and New Albany, IN. (Courtesy of the Lincoln Electric Company.)

such a moment. The use of sag rods for providing lateral support to purlins made from channels usually is economical because of the bending weakness of channels about their y axes. For light roofs (e.g., where trusses support corrugated steel roofs), sag rods will probably be needed at the one-third points if the trusses are more than 20 ft on centers. Sag rods at the midpoints are usually sufficient if the trusses are less than 20 ft on centers. For heavier roofs, such as those made of slate, cement tile, or clay tile, sag rods will probably be needed at closer intervals. The one-third points will likely be necessary if the trusses are spaced at greater intervals than 14 ft, and the midpoints will be satisfactory if truss spacings are less than 14 ft. Some designers assume that the load components parallel to the roof surface can be taken by the roof, particularly if it consists of corrugated steel sheets, and that tie rods are unnecessary. This assumption, however, is open to some doubt and definitely should not be followed if the roof is very steep.

Designers have to use their own judgment in limiting the slenderness values for rods, as they will usually be several times the limiting values mentioned for other types of tension members. A common practice of many designers is to select rod diameters no less than 1/500th of their lengths, to obtain some rigidity, even though design calculations may permit smaller sizes.

It is typically desirable to limit the minimum size of sag rods to 5/8 in, because smaller rods than these are often damaged during construction. The threads on smaller rods are quite easily damaged by overtightening, which seems to be a frequent habit of construction workers. Sag rods are designed for the purlins of a roof truss in Example 4-5. The rods are assumed to support the simple beam reactions for the components of the gravity loads (roofing, purlins, snow, and ice) parallel to the roof surface. Wind forces are assumed to act perpendicular to the roof surfaces and, theoretically, will not affect the sag rod forces. The maximum force in a sag rod will occur in the top sag rod, because it must support the sum of the forces in the lower sag rods. It is theoretically possible to use smaller rods for the lower sag rods, but this reduction in size will probably be impractical.

Example 4-5

Design the sag rods for the purlins of the truss shown in Fig. 4.4. Purlins are to be supported at their one-third points between the trusses, which are spaced 21 ft on centers. Use A36 steel and assume that a minimum-size rod of 5/8 in is permitted. A clay tile roof weighing 16 psf (0.77 kN/m^2) of roof surface is used and supports a snow load of 20 psf (0.96 kN/m^2) of horizontal projection of roof surface. Details of the purlins and the sag rods and their connections are shown in Figs. 4.4 and 4.5. In these figures, the dotted lines represent ties and struts in the end panels in the plane of the roof, commonly used to give greater resistance to loads located on one side of the roof (a loading situation that might occur when snow is blown off one side of the roof during a severe windstorm).

Solution. Gravity loads in psf of roof surface are as follows:

Average weight in psf of the seven purlins on each side of the roof

$$= \frac{(7)(11.5 \text{ lb/ft})}{37.9 \text{ ft}} = 2.1 \text{ psf}$$

$$\text{Snow} = 20 \text{ psf}\left(\frac{3}{\sqrt{10}}\right) = 19.0 \text{ psf of roof surface}$$

Tile roofing = 16.0 psf

$$w_u = (1.2)(2.1 + 16.0) + (1.6)(19.0) = 52.1 \text{ psf}$$

You can see in Figs. 4.4 and 4.5 that half of the load component parallel to the roof surface between the top two purlins on each side of the truss is carried directly to the horizontal sag rod between the purlins. In this example, there are seven purlins (with six spaces between them) on each side of the truss. Thus, 1/12th of the total inclined load goes directly to the horizontal sag rod.

LRFD	ASD
LRFD load on top inclined sag rod, using controlling load factor equation $w_u = (1.2)(2.1 \text{ psf} + 16.0 \text{ psf}) + (1.6)(19.0 \text{ psf})$ $= 52.1$ psf Component of loads parallel to roof surface $= \left(\frac{1}{\sqrt{10}}\right)(52.1 \text{ psf}) = 16.5 \text{ psf}$ Load on top inclined sag rod $= \left(\frac{11}{12}\right)(37.9 \text{ ft})(7 \text{ ft})(16.5 \text{ psf})$ $= 4013 \text{ lbs} = 4.01 \text{ k} = P_u$	ASD load on top inclined sag rod, using controlling ASD load equation $w = 2.1 \text{ psf} + 16.0 \text{ psf} + 19.0 \text{ psf}$ $= 37.1$ psf Component of loads parallel to roof surface $= \left(\frac{1}{\sqrt{10}}\right)(37.1) = 11.7 \text{ psf}$ Load on top inclined sag rod $= \left(\frac{11}{12}\right)(37.9 \text{ ft})(7 \text{ ft})(11.7 \text{ psf})$ $= 2845 \text{ lbs} = 2.85 \text{ k} = P$

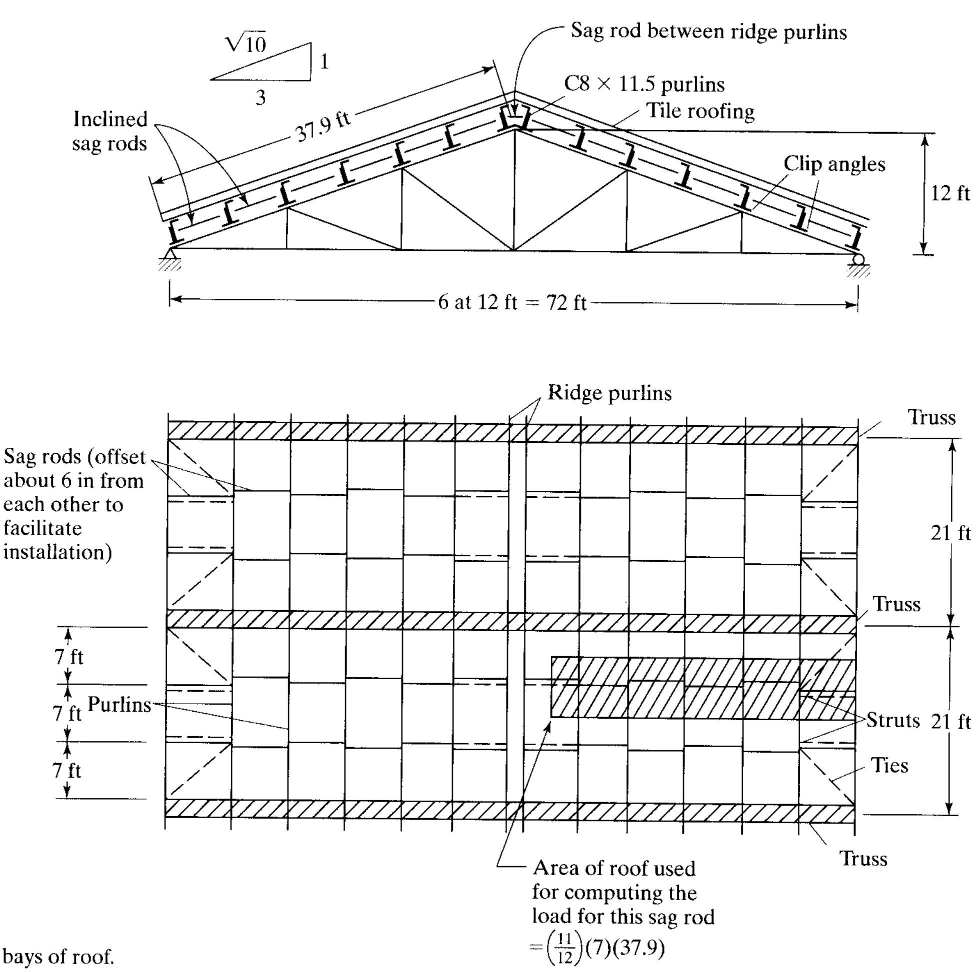
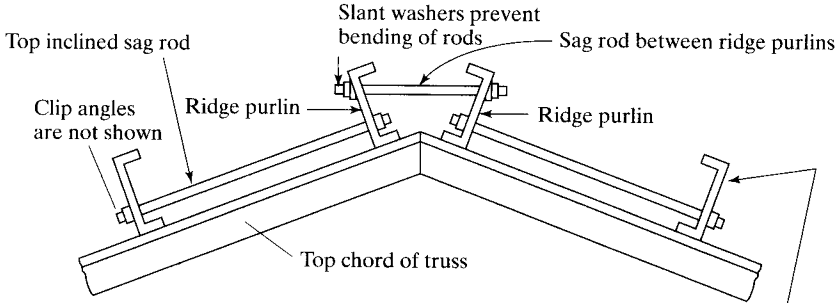


FIGURE 4.4

Plan of two bays of roof.



Note: Some designers prefer that channel purlins be placed with their flanges pointing down roof slopes to avoid the accumulation of debris or condensation on their bottom flanges.

FIGURE 4.5

Details of sag rod connections.

Selecting section with LRFD expression

$$A_D = \frac{P_u}{\phi\, 0.75 F_u} = \frac{4.01\text{ k}}{(0.75)(0.75)(58\text{ ksi})} = 0.12\text{ in}^2$$

Try $\frac{5}{8}$-in rod as minimum practical size, 11 threads per in, from AISC Table 7-18.

$$A_D = 0.307\text{ in}^2$$

$$R_n = 0.75 F_u A_D = (0.75)(58\text{ ksi})(0.307\text{ in}^2) = 13.36\text{ k}$$

LRFD with $\phi = 0.75$	ASD with $\Omega = 2.00$
$\phi R_n = (0.75)(13.36\text{ k}) = 10.02\text{ k} > 4.01\text{ k}$ **OK**	$\frac{R_n}{\Omega} = \frac{13.36\text{ k}}{2.00} = 6.68\text{ k} > 2.85\text{ k}$ **OK**

Use $\frac{5}{8}$-in rod for both LRFD and ASD.

Checking force in tie rods between ridge purlins

LRFD	ASD
$P_u = (37.9\text{ ft})(7\text{ ft})(16.5\text{ psf})\left(\frac{\sqrt{10}}{3}\right)$ $= 4614\text{ lbs} = 4.61\text{ k} < 10.02\text{ k}$ **OK**	$P_a = (37.9\text{ ft})(7\text{ ft})(11.7\text{ psf})\left(\frac{\sqrt{10}}{3}\right)$ $= 3280\text{ lbs} = 3.28\text{ k} < 6.68\text{ k}$ **OK**

Use $\frac{5}{8}$-in rod for both LRFD and ASD.

4.4 PIN-CONNECTED MEMBERS

Until the early years of the twentieth century nearly all bridges in the United States were pin-connected, but today pin-connected bridges are used infrequently because of the advantages of bolted and welded connections. One trouble with the old pin-connected trusses was the wearing of the pins in the holes, which caused looseness of the joints.

An eyebar is a special type of pin-connected member whose ends where the pin holes are located are enlarged, as shown in Fig. 4.6. Though just about obsolete today, eyebars at one time were very commonly used for the tension members of bridge trusses.

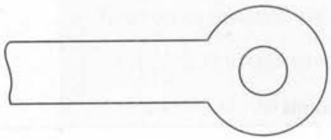


FIGURE 4.6
End of an eyebar.

Pin-connected eyebars are still used occasionally as tension members for long-span bridges and as hangers for some types of bridges and other structures, where they are normally subjected to very large dead loads. As a result, the eyebars are usually prevented from rattling and wearing, as they would under live loads.

Eyebars are not generally made by forging but are thermally cut from plates. As stated in the AISC Commentary (D6), extensive testing has shown that thermally cut members result in more balanced designs. The heads of eyebars are specially shaped so as to provide optimum stress flow around the holes. These proportions are set on the basis of long experience and testing with forged eyebars, and the resulting standards are rather conservative for today's thermally cut members.

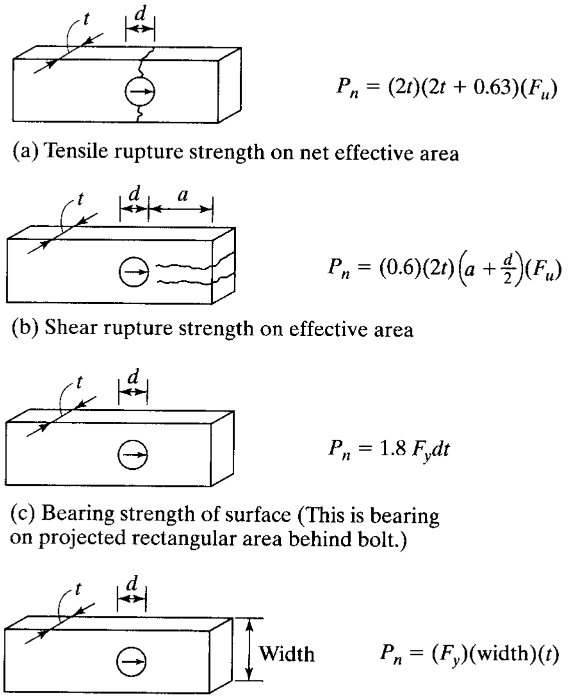
The AISC Specification (D5) provides detailed requirements for pin-connected members as to strength and proportions of the pins and plates. The design strength of such a member is the lowest value obtained from the following equations, where reference is made to Fig. 4.7:

1. Tension rupture on the net effective area. See Fig. 4.7(a).

$$P_n = 2tb_e F_u \qquad \text{(AISC Equation D5–1)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ ASD}$$

in which t = plate thickness and $b_e = 2t + 0.63$, but may not exceed the distance from the hole edge to the edge of the part measured perpendicular to the line of force.



(a) Tensile rupture strength on net effective area

(b) Shear rupture strength on effective area

(c) Bearing strength of surface (This is bearing on projected rectangular area behind bolt.)

(d) Tensile yielding strength in gross section

FIGURE 4.7

Strength of pin-connected tension members.

2. **For shear rupture on the effective area. See Fig. 4.7(b).**

$$P_n = 0.6F_u A_{sf} \qquad \text{(AISC Equation D5–2)}$$
$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ ASD}$$

in which $A_{sf} = 2t(a + d/2)$, where a is the shortest distance from the edge of the pin hole to the member edge, measured parallel to the force.

3. **Strength of surfaces in bearing. See Fig. 4.7(c).**

$$R_n = 1.8F_y A_{pb} \qquad \text{(AISC Equation J7–1)}$$
$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ ASD}$$

in which A_{pb} = projected bearing area = dt. Notice that LRFD Equation J7-1 applies to milled surfaces; pins in reamed, drilled, or bored holes; and ends of fitted bearing stiffeners. (AISC Specification J7(b) provides other equations for determining the bearing strength for expansion rollers and rockers.)

4. **Tensile yielding in the gross section. See Fig. 4.7(d).**

$$P_n = F_y A_g \qquad \text{(AISC Equation D2–1)}$$
$$\phi_t = 0.90 \text{ (LRFD)} \qquad \Omega_t = 1.67 \text{ (ASD)}$$

The AISC Specification D6.2 says that thicknesses < 1/2 in for both eyebars and pin-connected plates are permissible only when external nuts are provided to tighten the pin plates and filler plates into snug contact. The bearing design strength of such plates is provided in AISC Specification J7.

In addition to the other requirements mentioned, AISC Specification D5 specifies certain proportions between the pins and the eyebars. These values are based on long experience in the steel industry, and on experimental work by B. G. Johnston.[1] It has been found that when eyebars and pin-connected members are made from steels with yield stresses greater than 70 kips per square inch (ksi), there may be a possibility of **dishing** (a complicated inelastic stability failure where the head of the eyebar tends to curl laterally into a dish-like shape). For this reason, the AISC Specification requires stockier member proportions for such situations—hole diameter not to exceed five times the plate thickness, and the width of the eyebar to be reduced accordingly.

4.5 DESIGN FOR FATIGUE LOADS

It is not likely that fatigue stresses will be a problem in the average building frame, because the changes in load in such structures usually occur only occasionally and produce relatively minor stress variations. Should there, however, be frequent variations in, or even reversals of, stress, the matter of fatigue must be considered. Fatigue can be a problem in buildings when crane runway girders or heavy vibrating or moving machinery or equipment are supported.

If steel members are subjected to loads that are applied, and then removed, or changed significantly many thousands of times, cracks may occur, and they may spread

[1] B. G. Johnston, "Pin-Connected Plate Links," *Transactions ASCE*, 104 (1939).

so much as to cause failure. The steel must be subjected either to stress reversals or to variations in tension stress, because fatigue problems occur only when tension is present. (Sometimes, however, fatigue cracks occur in members that are subjected only to calculated compression stresses if parts of those members have high residual tension stresses.) The results of fatigue loading is that steel members may fail at stresses well below the stresses at which they would fail if they were subject to static loads. The fatigue strength of a particular member is dependent on the number of cycles of stress change, the range of stress change, and the size of flaws.

In Appendix 3 of the AISC Specification, a simple design method is presented for considering fatigue stresses. For this discussion, the term *stress range* is defined as the magnitude of the change in stress in a member due to the application or removal of service live loads. Should there be stress reversal, the stress range equals the numerical sum of the maximum repeated tensile and compressive stresses.

The fatigue life of members increases as the stress range is decreased. Furthermore, at very low stress ranges, the fatigue life is very large. In fact, there is a range at which the member life appears to be infinite. This range is called the *threshold fatigue stress range.*

If it is anticipated that there will be fewer than 20,000 cycles of loading, no consideration needs to be given to fatigue. (Note that three cycles per day for 25 years equals 27,375 cycles.) If the number of cycles is greater than 20,000, an allowable *stress range*, is calculated as specified in Appendix 3.3 of the AISC Specification. Should a member be selected and found to have a design stress range below the actual stress range, it will be necessary to select a larger member.

The following two additional notes pertain to the AISC fatigue design procedure:

1. The design stress range determined in accordance with the AISC requirements is applicable only to the following situations:
   a. Structures for which the steel has adequate corrosion protection for the conditions expected in that locality.
   b. Structures for which temperatures do not exceed 300°F.
2. The provisions of the AISC Specification apply to stresses that are calculated with service loads, and the maximum permitted stress due to these loads is $0.66F_y$.

   Formulas are given, in Appendix 3 of the AISC Specification, for computing the allowable stress range. For the stress categories A, B, B′, C, D, E, and E′ listed in AISC Appendix Table A3.1,

$$F_{SR} = \left(\frac{C_f}{n_{SR}}\right)^{0.333} \geq F_{TH} \qquad \text{(AISC Equation A-3-1)}$$

in which

F_{SR} = allowable stress range, ksi
C_f = constant from Table A-3.1 in AISC Appendix A
n_{SR} = number of stress range fluctuations in design life
= number of stress range fluctuations per day $\times$ 365 $\times$ years of design life
F_{TH} = threshold allowable stress range, maximum stress range for indefinite design life from AISC Appendix Table A-3.1, ksi

Example 4-6 presents the design of a tension member subjected to fluctuating loads, using Appendix 3 of the AISC Specification. Stress fluctuations and reversals are an everyday problem in the design of bridge structures. The AASHTO Specifications provide allowable stress ranges, determined in a manner similar to that of the AISC Specification.

Example 4-6

A tension member is to consist of a W12 section ($F_y = 50$ ksi) with fillet-welded end connections. The service dead load is 40 k, while it is estimated that the service live load will vary from a compression of 20 k to a tension of 90 k fifty times per day for an estimated design life of 25 years. Select the section, using the AISC procedure.

Solution

$$P_u = (1.2)(40\text{ k}) + (1.6)(90\text{ k}) = 192\text{ k}$$

Estimated section size for gross section tension yield

$$A_g \geq \frac{P_u}{\phi_t F_y} = \frac{192\text{ k}}{(0.9)(50\text{ ksi})} = 4.27\text{ in}^2$$

Try W12 × 16($A_g = 4.71\text{ in}^2$)

$$n_{SR} = (50)(365)(25) = 456{,}250$$

According to Table A-3.1 in Appendix 3 of the AISC Specification, the member falls into Section 1 of the table and into stress category A.

$$C_f = 250 \times 10^8 \text{ from table}$$

$$F_{\text{TH}} = 24 \text{ ksi from table}$$

$$F_{\text{SR}} = \left(\frac{C_f}{n_{SR}}\right)^{0.333} = \left(\frac{250 \times 10^8}{456{,}250}\right)^{0.333} = 37.84\text{ ksi}$$

$$\text{Max service load tension} = \frac{40\text{ k} + 90\text{ k}}{4.71\text{ in}^2} = 27.60\text{ ksi}$$

$$\text{Min service load tension} = \frac{40\text{ k} - 20\text{ k}}{4.71\text{ in}^2} = 4.25\text{ ksi}$$

$$\text{Actual stress range} = 27.60 - 4.25 = 23.35\text{ ksi}$$

$$< F_{SR} = 37.84\text{ ksi} \qquad \text{(OK)}$$

Use W12 × 16.

4.6 PROBLEMS FOR SOLUTION

For all these problems, select sizes with LRFD expressions and check the selected sections with both the LRFD and the ASD expressions.

4-1 to 4-8. *Select sections for the conditions described, using F_y = 50 ksi and F_u = 65 ksi, unless otherwise noted, and neglecting block shear.*

4-1. Select the lightest W12 section available to support working tensile loads of P_D = 120 k and P_W = 288 k. The member is to be 20 ft long and is assumed to have two lines of holes for 3/4-in Ø bolts in each flange. There will be at least three holes in each line 3 in on center. (*Ans.* W12 × 45 LRFD and ASD)

4-2. Repeat Prob. 4-1 selecting a W10 section.

4-3. Select the lightest WT7 available to support a factored tensile load P_u = 250 k, P_a = 160 k. Assume there are two lines of 7/8-in Ø bolts in the flange (at least three bolts in each line 4 in on center). The member is to be 30 ft long. (*Ans.* WT7 × 26.5 LRFD, WT7 × 24 ASD)

4-4. Select the lightest S section that will safely support the service tensile loads P_D = 75 k and P_L = 40 k. The member is to be 20 ft long and is assumed to have one line of holes for 3/4-in Ø bolts in each flange. Assume that there are at least three holes in each line 4 in on center. Use A36 steel.

4-5. Select the lightest C section that will safely support the service tensile loads P_D = 65 k and P_L = 50 k. The member is to be 14 ft long and is assumed to have two lines of holes for 3/4-in Ø bolts in the web. Assume that there are at least three holes in each line 3 in on center. Use A36 steel. (*Ans.* C8 × 18.75 LRFD and ASD)

4-6. Select the lightest W10 section that will resist a service tensile load, P_D = 175 k and P_L = 210 k. The member is to be 25 ft long and is assumed to have two lines of holes in each flange and two lines of holes in the web. Assume there are four bolts in each line 3 in on center. All holes are for 7/8-in Ø bolts. Use A992 – Grade 50 steel.

4-7. Select the lightest C section that will safely support the service tensile loads P_D = 20 k and P_L = 34 k. The member is to be 12 ft long and is assumed to have only a transverse weld at the end of the channel. A36 steel is used. (*Ans.* C6 × 10.5 LRFD and ASD)

4-8. Select the lightest MC12 section that will resist a total factored load of 372 kips and a total service load of 248 kips. The member is to be 20 ft long and is assumed to be welded on the end as well as to each flange for a distance of 6 in along the length of the channel. A36 steel is used.

4-9 to 4-16. *Select the lightest section for each of the situations described in Table 4.1. Assume that bolts are 3 in on center (unless noted otherwise). Do not consider block shear. Determine U from Table 3.2 of this text (except if given).*

TABLE 4.1

Prob. no.	Section	P_D (kips)	P_L (kips)	Length (ft)	Steel	End Connection	Answer
4-9	W8	75	100	24	A992	Two lines of 5/8-in Ø bolts (3 in a line 2 1/2 in on center) each flange	W8 × 28 LRFD and ASD

(Continued)

TABLE 4.1 (*Continued*)

Prob. No.	Section	P_D (kips)	P_L (kips)	Length (ft)	Steel	End Connection	Answer
4-10	W10	120	220	30	A992	Two lines of 3/4-in Ø bolts (3 in a line) each flange	
4-11	W12	150	175	26	A36	Two lines of 7/8-in Ø bolts (2 in a line 4 in on center) each flange	W12 × 58 LRFD W12 × 65 ASD
4-12	W10	135	100	28	A36	Longitudinal weld to flanges only, 6 in long	
4-13	W8	100	80	30	A992	Transverse welds to flanges only	W8 × 24 LRFD W8 × 28 ASD
4-14	S	60	100	22	A36	One line of 3/4-in Ø bolts (3 in a line 4 in on center) each flange	
4-15	WT6	80	120	20	A992	Longitudinal weld to flange only, 6 in long	WT6 × 26.5 LRFD and ASD
4-16	WT4	30	50	18	A36	Transverse weld to flange only	

4-17. Using A36 steel select the lightest equal leg single angle member to resist a tensile load of $P_D = 45$ k, $P_L = 25$ k, and $P_W = 88$ k. The member will be connected through one leg with two lines of three 3/4-in Ø bolts 3 1/2 in on center. The member length is 24 ft. Neglect block shear. (*Ans.* L6 × 6 × 1/2 for LRFD and ASD)

4-18. Select a pair of C10 channels for a tension member subjected to a dead lead of 120 kips and a live load of 275 kips. The channels are placed back to back and connected to a 3/4-in gusset plate by 7/8-in Ø bolts. Assume A588 Grade 50 steel for the channels and assume the gusset plate is sufficient. The member is 25 ft long. The bolts are arranged in two lines parallel to the length of the member. There are two bolts in each line 4 in on center.

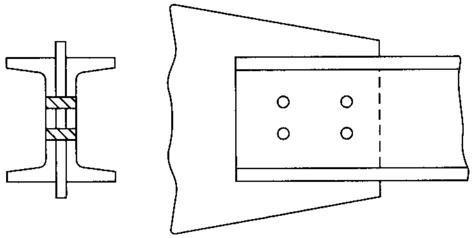


FIGURE P4-18

4-19. Select the lightest C6 channel shape to be used as a 12 ft long tension member to resist the following service loads, $P_D = 20$ k and $P_L = 32$ k. The member will be connected by a transverse weld at the end of the channel only. Use A36 Grade 36 steel with $F_u = 58$ ksi. (*Ans.* C6 × 10.5 LRFD and ASD)

4-20. Design member L_2L_3 of the truss shown in Fig. P4-20. It is to consist of a pair of angles with a 3/8-in gusset plate between the angles at each joint. Use A36 steel and assume two lines of three 3/4-in Ø bolts in each vertical angle leg, 4 in on center. Consider only the angles shown in the double-angle tables of the AISC Manual. For each load, $P_D = 60$ k and $P_L = 48$ k. Do not consider block shear.

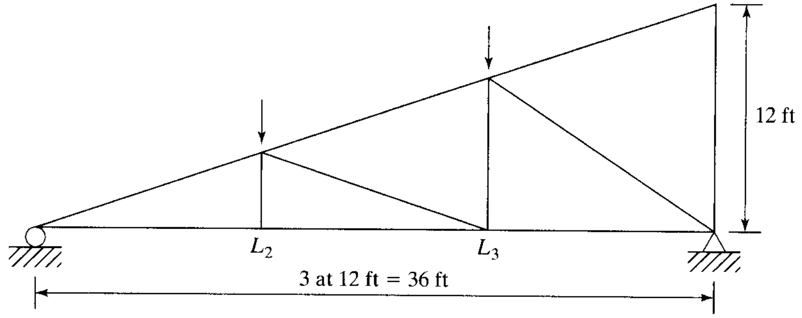


FIGURE P4-20

4-21. Select an ST shape to be used as a 20 ft long tension member that will safely support the service tensile loads: $P_D = 35$ k, $P_L = 115$ k, and $P_S = 65$ k (snow). The connection is through the flange with two lines of three 3/4-in Ø bolts 4 in on center. Use A572 Grade 50 steel. Neglect block shear. (*Ans.* ST10 × 33 LRFD and ASD)

4-22. Select the lightest WT4 shape to be used as a 20 ft long tension member to resist the following service loads: dead load, $D = 20$ k, live load, $L = 35$ k, snow load, $S =$ 25 k, and earthquake, $E = 50$ k. The connection is two lines of bolts through the flange with three 3/4-in Ø bolts in each line spaced at 3 in on center. Use A992 Grade 50 steel. Neglect block shear.

4-23. A tension member is to consist of two C10 channels and two PL 1/2 × 11, arranged as shown in Fig. P4-23 to support the service loads, $P_D = 200$ k and $P_L = 320$ k. The member is assumed to be 30 ft long and is to have four lines of 3/4-in Ø bolts. Assume $U = 0.85$. All steel will be A36. Neglect block shear. (*Ans.* 2 – C10 × 25 LRFD and ASD)

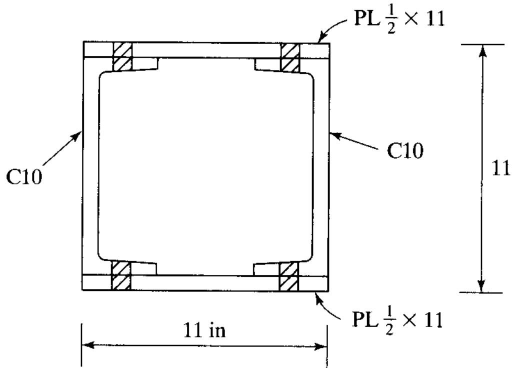


FIGURE P4-23

4-24. A pipe is supported at 25 ft intervals by a pipe strap hung from a threaded rod as shown. A 10-in Ø standard weight steel pipe full of water is used. What size round rod is required? Use A36 steel. Neglect weight of the pipe strap.

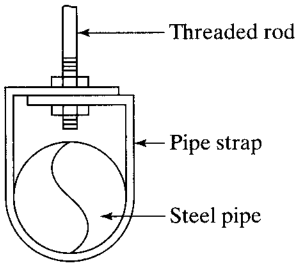


FIGURE P4-24

4-25. Select a standard threaded round rod to support a factored tensile load of 72 kips (service tensile load = 50 kips) using A36 steel. (*Ans.* $1\frac{3}{4}$-in Ø rod LRFD and ASD)

4-26. What size threaded round rod is required for member AC shown in Fig. P4-26? The given load is a service live load. Use A36 steel.

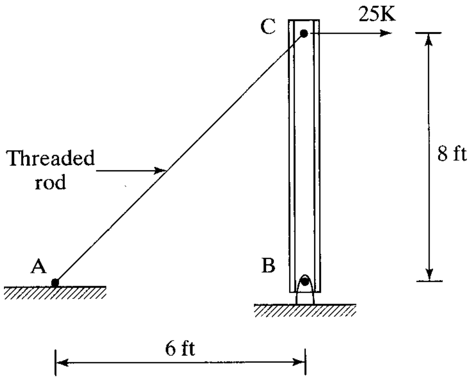


FIGURE P4-26

CHAPTER 5

Introduction to Axially Loaded Compression Members

5.1 GENERAL

There are several types of compression members, the column being the best known. Among the other types are the top chords of trusses and various bracing members. In addition, many other members have compression in some of their parts. These include the compression flanges of rolled beams and built-up beam sections, and members that are subjected simultaneously to bending and compressive loads. Columns are usually thought of as being straight vertical members whose lengths are considerably greater than their thicknesses. Short vertical members subjected to compressive loads are often called struts, or simply, compression members; however, the terms *column* and *compression member* will be used interchangeably in the pages that follow.

There are three general modes by which axially loaded columns can fail. These are flexural buckling, local buckling, and torsional buckling. These modes of buckling are briefly defined as follows:

1. *Flexural buckling* (also called Euler buckling) is the primary type of buckling discussed in this chapter. Members are subject to flexure, or bending, when they become unstable.
2. *Local buckling* occurs when some part or parts of the cross section of a column are so thin that they buckle locally in compression before the other modes of

buckling can occur. The susceptibility of a column to local buckling is measured by the width–thickness ratios of the parts of its cross section. This topic is addressed in Section 5.7.

3. *Flexural torsional buckling* may occur in columns that have certain cross-sectional configurations. These columns fail by twisting (torsion) or by a combination of torsional and flexural buckling. This topic is initially addressed in Section 6.10.

The longer a column becomes for the same cross section, the greater becomes its tendency to buckle and the smaller becomes the load it will support. The tendency of a member to buckle is usually measured by its *slenderness ratio*, which has previously been defined as the ratio of the length of the member to its least radius of gyration. This tendency to buckle is also affected by such factors as the types of end connections, eccentricity of load application, imperfection of column material, initial crookedness of columns, and residual stresses from manufacture.

The loads supported by a building column are applied by the column section above and by the connections of other members directly to the column. The ideal situation is for the loads to be applied uniformly across the column, with the center of gravity of the loads coinciding with the center of gravity of the column. Furthermore, it is desirable for the column to have no flaws, to consist of a homogeneous material, and to be perfectly straight, but these situations are obviously impossible to achieve.

Loads that are exactly centered over a column are referred to as *axial*, or *concentric loads*. The dead loads may or may not be concentrically placed over an interior building column, and the live loads may never be centered. For an outside column, the load situation is probably even more eccentric, as the center of gravity of the loads will often fall well on the inner side of the column. In other words, it is doubtful that a perfect axially loaded column will ever be encountered in practice.

The other desirable situations are also impossible to achieve because of the following conditions: imperfections of cross-sectional dimensions, residual stresses, holes punched for bolts, erection stresses, and transverse loads. It is difficult to take into account all of these imperfections in a formula.

Slight imperfections in tension members and beams can be safely disregarded, as they are of little consequence. On the other hand, slight defects in columns may be of major significance. A column that is slightly bent at the time it is put in place may have significant bending moments equal to the column load times the initial lateral deflection. Mill straightness tolerances, as taken from ASTM A6, are presented in Tables 1-22 through 1-28 of the AISC Manual.

Obviously, a column is a more critical member in a structure than is a beam or tension member, because minor imperfections in materials and dimensions mean a great deal. This fact can be illustrated by a bridge truss that has some of its members damaged by a truck. The bending of tension members probably will not be serious, as the tensile loads will tend to straighten those members; but the bending of any

Two International Place, Boston, MA. (Courtesy of Owen Steel Company, Inc.)

compression members is a serious matter, as compressive loads will tend to magnify the bending in those members.

The preceding discussion should clearly show that column imperfections cause them to bend, and the designer must consider stresses due to those moments as well as those due to axial loads. Chapters 5 to 7 are limited to a discussion of axially loaded columns, while members subjected to a combination of axial loads and bending loads are discussed in Chapter 11.

The spacing of columns in plan establishes what is called a *bay*. For instance, if the columns are 20 ft on center in one direction and 25 ft in the other direction, the bay size is 20 ft × 25 ft. Larger bay sizes increase the user's flexibility in space planning. As to economy, a detailed study by John Ruddy[1] indicates that when shallow spread footings are used, bays with length-to-width ratios of about 1.25 to 1.75, and areas of about 1000 sq ft, are the most cost efficient. When deep foundations are used, his study shows that larger bay areas are more economical.

5.2 RESIDUAL STRESSES

Research at Lehigh University has shown that residual stresses and their distribution are very important factors affecting the strength of axially loaded steel columns. These stresses are of particular importance for columns with slenderness ratios varying from approximately 40 to 120, a range that includes a very large percentage of practical columns. A major cause of residual stress is the uneven cooling of shapes after hot-rolling. For instance, in a W shape the outer tips of the flanges and the middle of the web cool quickly, while the areas at the intersection of the flange and web cool more slowly.

The quicker cooling parts of the sections, when solidified, resist further shortening, while those parts that are still hot tend to shorten further as they cool. The net result is that the areas that cooled more quickly have residual compressive stresses, and the slower cooling areas have residual tensile stresses. The magnitude of these stresses varies from about 10 to 15 ksi (69 to 103 MPa), although some values greater than 20 ksi (138 MPa), have been found.

When rolled-steel column sections with their residual stresses are tested, their proportional limits are reached at P/A values of only a little more than half of their yield stresses, and the stress–strain relationship is nonlinear from there up to the yield stress. Because of the early localized yielding occurring at some points of the column cross sections, buckling strengths are appreciably reduced. Reductions are greatest for columns with slenderness ratios varying from approximately 70 to 90 and may possibly be as high as 25 percent.[2]

As a column load is increased, some parts of the column will quickly reach the yield stress and go into the plastic range because of residual compression stresses. The stiffness of the column will be reduced and become a function of the part of the cross section that is still elastic. A column with residual stresses will behave as though it has a reduced cross section. This reduced section or elastic portion of the column will change as the applied stresses change. The buckling calculations for a particular column with residual stresses can be handled by using an effective moment of inertia I_e of the elastic portion of the cross section or by using the tangent modulus. For the usual sections used as columns, the two methods give fairly close results.

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[1] J. L. Ruddy, "Economics of Low-Rise Steel-Framed Structures," *Engineering Journal*, AISC, vol. 20, no. 3 (3d quarter, 1983), pp. 107–118.

[2] L. S. Beedle and L. Tall, "Basic Column Strength," *Proc. ASCE 86* (July 1960), pp. 139–173.

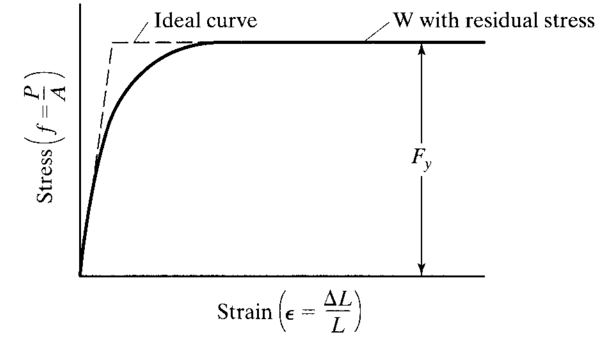
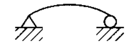
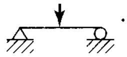


FIGURE 5.1

Effect of residual stresses on column stress–strain diagram.

In columns, welding can produce severe residual stresses that actually may approach the yield point in the vicinity of the weld. Another important fact is that columns may be appreciably bent by the welding process, which can decidedly affect their load-carrying ability. Figure 5.1 shows the effect of residual stresses (due to cooling and fabrication) on the stress–strain diagram for a hot-rolled W shape.

The welding together of built-up shapes frequently causes even higher residual stresses than those caused by the uneven cooling of hot-rolled I-shaped sections.

Residual stresses may also be caused during fabrication when *cambering* is performed by cold bending, and due to cooling after welding. Cambering is the bending of a member in a direction opposite to the direction of bending that will be caused by the service loads. For instance, we may initially bend a beam upward  so that it will be approximately straight when its normal gravity loads are applied .

5.3 SECTIONS USED FOR COLUMNS

Theoretically, an endless number of shapes can be selected to safely resist a compressive load in a given structure. From a practical viewpoint, however, the number of possible solutions is severely limited by such considerations as sections available, connection problems, and type of structure in which the section is to be used. The paragraphs that follow are intended to give a brief résumé of the sections which have proved to be satisfactory for certain conditions. These sections are shown in Fig. 5.2, and the letters in parentheses in the paragraphs to follow refer to the parts of this figure.

The sections used for compression members usually are similar to those used for tension members, with certain exceptions. The exceptions are caused by the fact that the strengths of compression members vary in some inverse relation to the slenderness ratios, and stiff members are required. Individual rods, bars, and plates usually are too slender to make satisfactory compression members, unless they are very short and lightly loaded.

Single-angle members (a) are satisfactory for use as bracing and compression members in light trusses. Equal-leg angles may be more economical than unequal-leg angles, because their least r values are greater for the same area of steel. The top chord

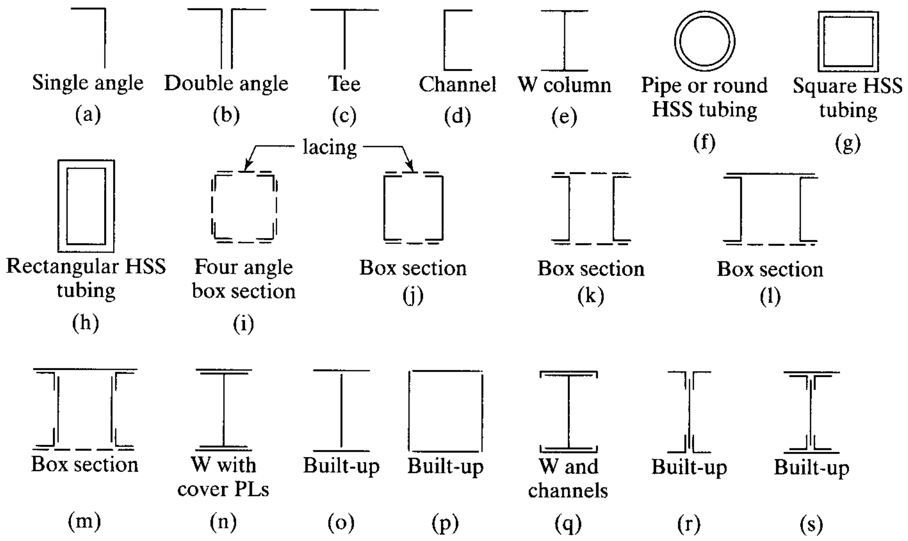


FIGURE 5.2

Types of compression members.

members of bolted roof trusses might consist of a pair of angles back to back (b). There will often be a space between them for the insertion of a gusset or connection plate at the joints necessary for connections to other members. An examination of this section will show that it is probably desirable to use unequal-leg angles with the long legs back to back to give a better balance between the r values about the x and y axes.

If roof trusses are welded, gusset plates may be unnecessary, and structural tees (c) might be used for the top chord compression members because the web members can be welded directly to the stems of the tees. Single channels (d) are not satisfactory for the average compression member because of their almost negligible r values about their web axes. They can be used if some method of providing extra lateral support in the weak direction is available. The W shapes (e) are the most common shapes used for building columns and for the compression members of highway bridges. Their r values, although far from being equal about the two axes, are much more nearly balanced than are the same values for channels.

Several famous bridges constructed during the nineteenth century (such as the Firth of Forth Bridge in Scotland and Ead's Bridge in St. Louis) made extensive use of tube-shaped members. Their use, however, declined due to connection problems and manufacturing costs, but with the development of economical welded tubing, the use of tube-shaped members is again increasing (although the tube shapes of today are very small compared with the giant ones used in those early steel bridges).

Hollow structural sections (square, rectangular, or round) and steel pipe are very valuable sections for buildings, bridges, and other structures. These clean, neat-looking sections are easily fabricated and erected. For small and medium loads, the round sections (f) are quite satisfactory. They are often used as columns in long series of windows, as short columns in warehouses, as columns for the roofs of covered walkways, in the basements and garages of residences, and in other applications. Round columns have the advantage of being equally rigid in all directions and are usually very economical, unless moments are too large for the sizes available. The AISC Manual furnishes the sizes of these sections and classifies them as being either round HSS sections or standard, extra strong, or double extra strong steel pipe.

Square and rectangular tubing (g) and (h) are being used more each year. For many years, only a few steel mills manufactured steel tubing for structural purposes. Perhaps the major reason tubing was not used to a great extent is the difficulty of making connections with rivets or bolts. This problem has been fairly well eliminated, however, by the advent of modern welding. The use of tubing for structural purposes by architects and engineers in the years to come will probably be greatly increased for several reasons:

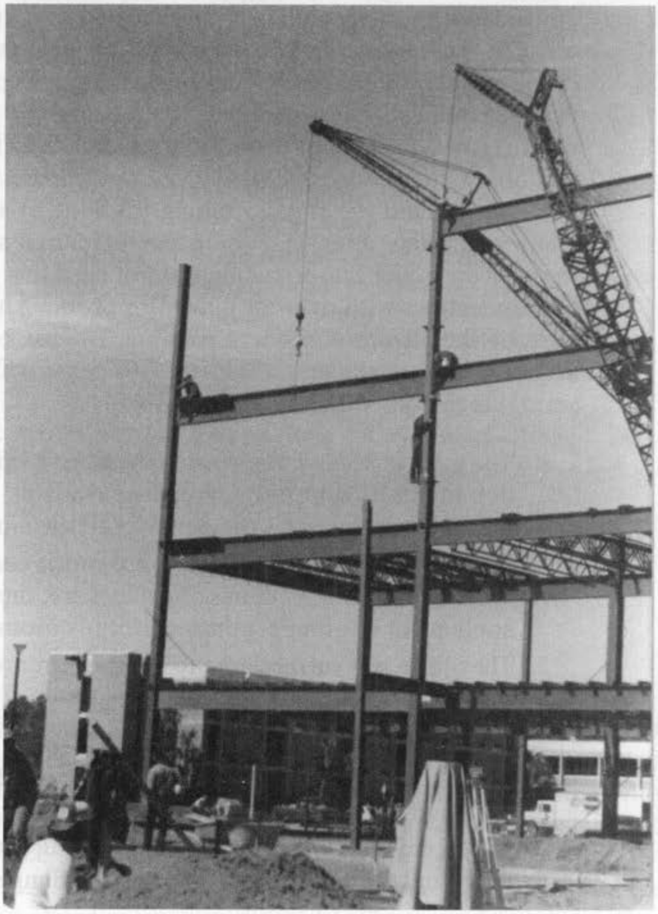
1. The most efficient compression member is one that has a constant radius of gyration about its centroid, a property available in round HSS tubing and pipe sections. Square tubing is the next-most-efficient compression member.
2. Four-sided and round sections are much easier to paint than are the six-sided open W, S, and M sections. Furthermore, the rounded corners make it easier to apply paint or other coatings uniformly around the sections.
3. They have less surface area to paint or fireproof.
4. They have excellent torsional resistance.
5. The surfaces of tubing are quite attractive.
6. When exposed, the round sections have wind resistance of only about two-thirds of that of flat surfaces of the same width.
7. If cleanliness is important, hollow structural tubing is ideal, as it doesn't have the problem of dirt collecting between the flanges of open structural shapes.

A slight disadvantage that comes into play in certain cases is that the ends of tube and pipe sections that are subject to corrosive atmospheres may have to be sealed to protect their inaccessible inside surfaces from corrosion. Although making very attractive exposed members for beams, these sections are at a definite weight disadvantage compared with W sections, which have so much larger resisting moments for the same weights.

For many column situations, the weight of square or rectangular tube sections—usually referred to as *hollow structural sections* (HSS)—can be less than one-half the weights required for open-profile sections (W, M, S, channel, and angle sections). It is true that tube and pipe sections may cost perhaps 25 percent more per pound than open sections, but this still allows the possibility of up to 20 percent savings in many cases.[3]

Hollow structural sections are available with yield strengths up to 50 ksi and with enhanced atmospheric corrosion resistance. Today, they make up a small percentage of

[3]"High Design, Low Cost," *Modern Steel Construction* (Chicago: AISC, March–April 1990), pp. 32–34.



Administration Building Pensacola Christian College, FL. (Courtesy Britt, Peters and Associates.)

the structural steel fabricated for buildings and bridges in the United States. In Japan and Europe, the values are, respectively, 15 percent and 25 percent, and still growing. Thus, it is probable that their use will continue to rise in the United States in the years to come.[4] Detailed information, including various tables on hollow structural sections, can be obtained from the Steel Tube Institute (STI), 2000 Ponce de Leon, Suite 600, Coral Gables, Florida 33134.

Where compression members are designed for very large structures, it may be necessary to use built-up sections. Built-up sections are needed where the members are long and support very heavy loads and/or when there are connection advantages. Generally speaking, a single shape, such as a W section, is more economical than a built-up section having the same cross-sectional area. For heavy column loads, high-strength steels can

[4]Ibid., p. 34.

frequently be used with very economical results if their increased strength permits the use of W sections rather than built-up members.

When built-up sections are used, they must be connected on their open sides with some type of lacing (also called *lattice bars*) to hold the parts together in their proper positions and to assist them in acting together as a unit. The ends of these members are connected with *tie* plates (also called *batten* plates or *stay* plates). Several types of lacing for built-up compression members are shown in Fig. 6.9.

The dashed lines in Fig. 5.2 represent lacing or discontinuous parts, and the solid lines represent parts that are continuous for the full length of the members. Four angles are sometimes arranged as shown in (i) to produce large r values. This type of member may often be seen in towers and in crane booms. A pair of channels (j) is sometimes used as a building column or as a web member in a large truss. It will be noted that there is a certain spacing for each pair of channels at which their r values about the x and y axes are equal. Sometimes the channels may be turned out, as shown in (k).

A section well suited for the top chords of bridge trusses is a pair of channels with a cover plate on top (l) and with lacing on the bottom. The gusset or connection plates at joints are conveniently connected to the insides of the channels and may also be used as splices. When the largest channels available will not produce a top chord member of sufficient strength, a built-up section of the type shown in (m) may be used.

When the rolled shapes do not have sufficient strength to resist the column loads in a building or the loads in a very large bridge truss, their areas may be increased by plates added to the flanges (n). In recent years, it has been found that, for welded construction, a built-up column of the type shown in part (o) is a more satisfactory shape than a W with welded cover plates (n). It seems that in bending (as where a beam frames into the flange of a column) it is difficult to efficiently transfer tensile force through the cover plate to the column without pulling the plate away from the column. For very heavy column loads, a welded box section of the type shown in (p) has proved to be quite satisfactory. Some other built-up sections are shown in parts (q) through (s). The built-up sections shown in parts (n) through (q) have an advantage over those shown in parts (i) through (m) in that they do not require the expense of the lattice work necessary for some of the other built-up sections. Lateral shearing forces are negligible for the single column shapes and for the nonlatticed built-up sections, *but they are definitely not negligible for the built-up latticed columns.*

Today, *composite columns* are being increasingly used. These columns usually consist of steel pipe and structural tubing filled with concrete, or of W shapes encased in concrete, usually square or rectangular in cross section. (These columns are discussed in Chapter 17.)

5.4 DEVELOPMENT OF COLUMN FORMULAS

The use of columns dates to before the dawn of history, but it was not until 1729 that a paper was published on the subject, by Pieter van Musschenbroek, a Dutch mathematician.[5] He presented an empirical column formula for estimating the strength of

[5]L. S. Beedle et al., *Structural Steel Design* (New York: Ronald Press, 1964), p. 269.

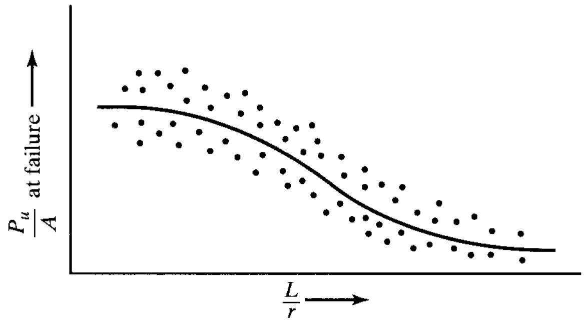


FIGURE 5.3

Test result curve.

rectangular columns. A few years later, in 1757, Leonhard Euler, a Swiss mathematician, wrote a paper of great value concerning the buckling of columns. He was probably the first person to realize the significance of buckling. The Euler formula, the most famous of all column expressions, is derived in Appendix A of this text. This formula, which is discussed in the next section, marked the real beginning of theoretical and experimental investigation of columns.

Engineering literature is filled with formulas developed for ideal column conditions, but these conditions are not encountered in actual practice. Consequently, practical column design is based primarily on formulas that have been developed to fit, with reasonable accuracy, test-result curves. The reasoning behind this procedure is simply that the independent derivation of column expressions does not produce formulas that give results which compare closely with test-result curves for all slenderness ratios.

The testing of columns with various slenderness ratios results in a scattered range of values, such as those shown by the broad band of dots in Fig. 5.3. The dots will not fall on a smooth curve, even if all of the testing is done in the same laboratory, because of the difficulty of exactly centering the loads, lack of perfect uniformity of the materials, varying dimensions of the sections, residual stresses, end restraint variations, and other such issues. The usual practice is to attempt to develop formulas that give results representative of an approximate average of the test results. The student should also realize that laboratory conditions are not field conditions, and column tests probably give the limiting values of column strengths.

The magnitudes of the yield stresses of the sections tested are quite important for short columns, as their failure stresses are close to those yield stresses. For columns with intermediate slenderness ratios, the yield stresses are of lesser importance in their effect on failure stresses, and they are of no significance for long slender columns. For intermediate range columns, residual stresses have more effect on the results, while the failure stresses for long slender columns are very sensitive to end support conditions. In addition to residual stresses and nonlinearity of material, another dominant factor in its effect on column strength is member out-of-straightness.

5.5 THE EULER FORMULA

Obviously, the stress at which a column buckles decreases as the column becomes longer. After it reaches a certain length, that stress will have fallen to the proportional limit of the steel. For that length and greater lengths, the buckling stress will be elastic.

For a column to buckle elastically, it will have to be long and slender. Its buckling load P can be computed with the Euler formula that follows:

$$P = \frac{\pi^2 EI}{L^2}.$$

This formula usually is written in a slightly different form that involves the column's slenderness ratio. Since $r = \sqrt{I/A}$, we can say that $I = Ar^2$. Substituting this value into the Euler formula and dividing both sides by the cross-sectional area, the Euler buckling stress is obtained:

$$\frac{P}{A} = \frac{\pi^2 E}{(L/r)^2} = F_e.$$

Example 5-1 illustrates the application of the Euler formula to a steel column. If the value obtained for a particular column exceeds the steel's proportional limit, the elastic Euler formula is not applicable.

Example 5-1

(a) A W10 × 22 is used as a 15-ft long pin-connected column. Using the Euler expression, determine the column's critical or buckling load. Assume that the steel has a proportional limit of 36 ksi.

(b) Repeat part (a) if the length is changed to 8 ft.

Solution

(a) Using a 15-ft long W10 × 22 ($A = 6.49\text{ in}^2$, $r_x = 4.27\text{ in}$, $r_y = 1.33\text{ in}$)

Minimum $r = r_y = 1.33\text{ in}$

$$\frac{L}{r} = \frac{(12\text{ in/ft})(15\text{ ft})}{1.33\text{ in}} = 135.34$$

Elastic or buckling stress $F_e = \dfrac{(\pi^2)(29 \times 10^3\text{ ksi})}{(135.34)^2}$

$= 15.63\text{ ksi} <$ the proportional limit of 36 ksi

OK column is in elastic range

Elastic or buckling load $= (15.63\text{ ksi})(6.49\text{ in}^2) = 101.4\text{ k}$

(b) Using an 8-ft long W10 × 22,

$$\frac{L}{r} = \frac{(12\text{ in/ft})(8\text{ ft})}{1.33\text{ in}} = 72.18$$

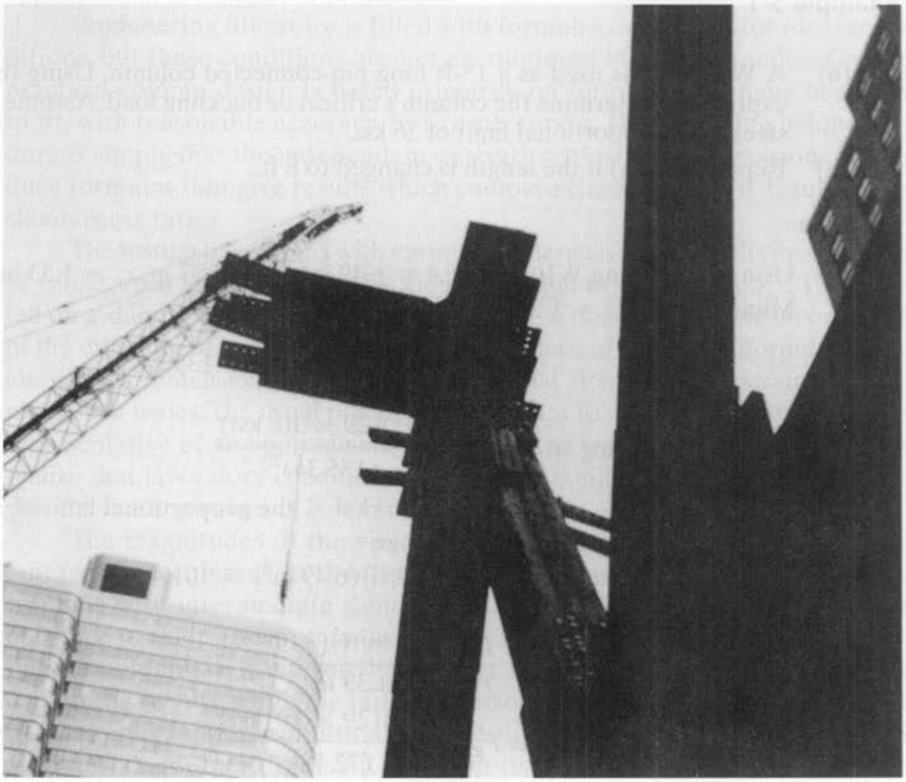
Elastic or buckling stress $F_e = \dfrac{(\pi^2)(29 \times 10^3\text{ ksi})}{(72.18)^2} = 54.94\text{ ksi} > 36\text{ ksi}$

$\therefore$ column is in inelastic range and Euler equation is not applicable.

The student should carefully note that the buckling load determined from the Euler equation is independent of the strength of the steel used.

The Euler equation is useful only if the end support conditions are carefully considered. The results obtained by application of the formula to specific examples compare very well with test results for concentrically loaded, long, slender columns with pinned ends. Designers, however, do not encounter perfect columns of this type. The columns with which they work do not have pinned ends and are not free to rotate, because their ends are bolted or welded to other members. These practical columns have different amounts of restraint against rotation, varying from slight restraint to almost fixed conditions. For the actual cases encountered in practice where the ends are not free to rotate, different length values can be used in the formula, and more realistic buckling stresses will be obtained.

To successfully use the Euler equation for practical columns, the value of L should be the distance between points of inflection in the buckled shape. This distance is referred to as the *effective length* of the column. For a pinned-end column (whose ends can rotate, but cannot translate), the points of inflection, or zero moment, are located at the ends a distance L apart. For columns with different end conditions, the effective lengths may be entirely different. Effective lengths are discussed extensively in the next section.



450 Lexington Ave., New York City. (Courtesy of Owen Steel Company, Inc.)

5.6 END RESTRAINT AND EFFECTIVE LENGTHS OF COLUMNS

End restraint and its effect on the load-carrying capacity of columns is a very important subject indeed. Columns with appreciable rotational and translational end restraint can support considerably more load than can those with little rotational end restraint, as at hinged ends.

The effective length of a column is defined in the previous section as the distance between points of zero moment in the column, that is, the distance between its inflection points. In steel specifications, the effective length of a column is referred to as KL, where K is the *effective length factor.* K is the number that must be multiplied by the length of the column to find its effective length. Its magnitude depends on the rotational restraint supplied at the ends of the column and upon the resistance to lateral movement provided.

The concept of effective lengths is simply a mathematical method of taking a column, whatever its end and bracing conditions, and replacing it with an equivalent pinned-end braced column. A complex buckling analysis could be made for a frame to determine the critical stress in a particular column. The K factor is determined by finding the pinned-end column with an equivalent length that provides the same critical stress. The K factor procedure is a method of making simple solutions for complicated frame-buckling problems.

Columns with different end conditions have entirely different effective lengths. For this initial discussion, it is assumed that no sidesway or joint translation is possible between the member ends. Sidesway or joint translation means that one or both ends of a column can move laterally with respect to each other. Should a column be connected with frictionless hinges, as shown in Fig. 5.4(a), its effective length would be equal to the actual length of the column and K would equal 1.0. If there were such a thing as a perfectly fixed-ended column, its points of inflection (or points of zero moment) would occur at its one-fourth points and its effective length would equal $L/2$, as shown in Fig. 5.4(b). As a result, its K value would equal 0.50.

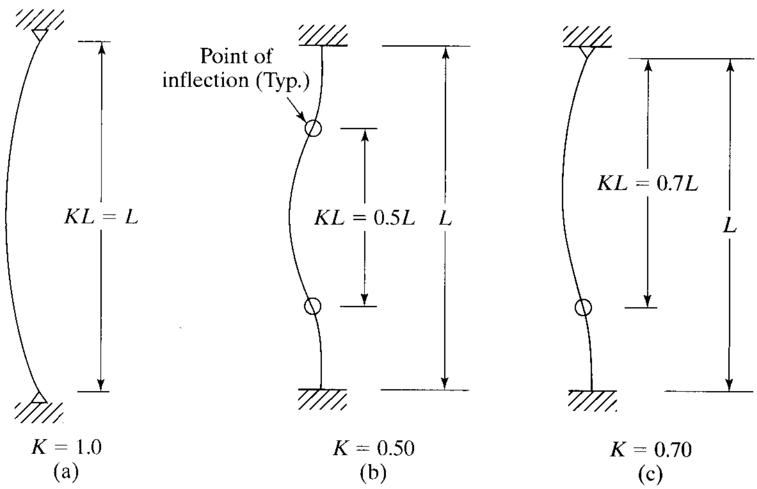


FIGURE 5.4

Effective length (KL) for columns in braced frames (sidesway prevented).

Obviously, the smaller the effective length of a particular column, the smaller its danger of lateral buckling and the greater its load-carrying capacity will be. In Fig. 5.4(c), a column is shown with one end fixed and one end pinned. The K value for this column is theoretically equal to 0.70.

This discussion would seem to indicate that column effective lengths always vary from an absolute minimum of $L/2$ to an absolute maximum of L, but there are many exceptions to this rule. An example is given in Fig. 5.5(a), where a simple bent is shown. The base of each of the columns is pinned, and the other end is free to rotate and move laterally (called *sidesway*). Examination of this figure will show that the effective length will exceed the actual length of the column as the elastic curve will theoretically take the shape of the curve of a pinned-end column of twice its length and K will theoretically equal 2.0. Notice, in part (b) of the figure, how much smaller the lateral deflection of column AB would be if it were pinned at both top and bottom so as to prevent sidesway.

Structural steel columns serve as parts of frames, and these frames are sometimes *braced* and sometimes *unbraced*. A braced frame is one for which sidesway or joint translation is prevented by means of bracing, shear walls, or lateral support from adjoining structures. An unbraced frame does not have any of these types of bracing supplied and must depend on the stiffness of its own members and the rotational rigidity of the joints between the frame members to prevent lateral buckling. For braced frames, K values can never be greater than 1.0, but for unbraced frames, the K values will always be greater than 1.0 because of sidesway.

Table C-C2.2 of the AISC Commentary on the Specification provides recommended effective length factors when ideal conditions are approximated. This table is reproduced here as Table 5.1 with the permission of the AISC. Two sets of K values are provided in the table, the theoretical values and the recommended design values, based on the fact that perfectly pinned and fixed conditions are not possible. If the ends of

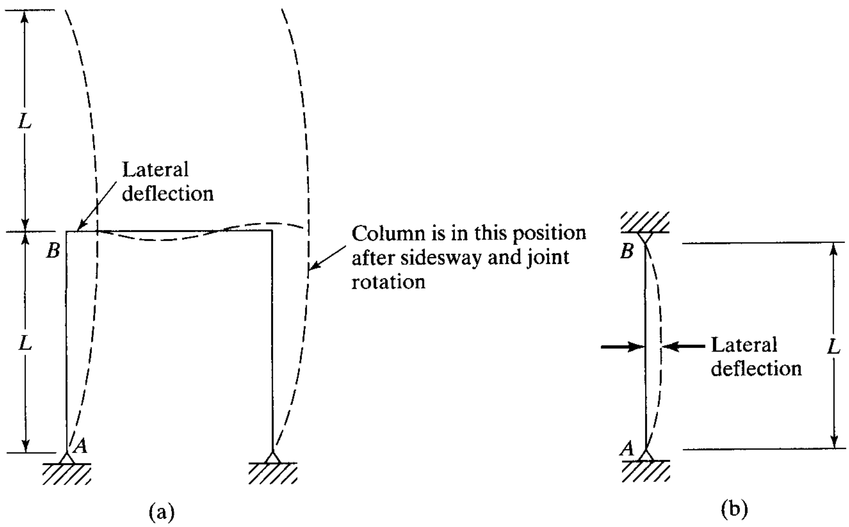
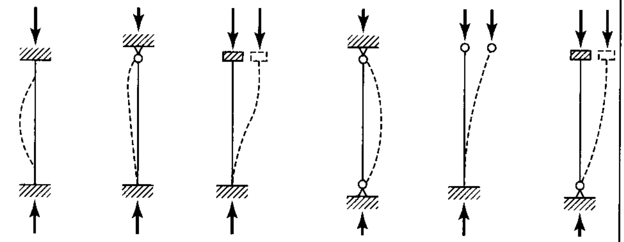


FIGURE 5.5

TABLE 5.1 Approximate Values of Effective Length Factor, K

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	 *Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					

Source: Commentary on the Specification, Appendix 7 – Table C-A-7.1, p. 16.1-511, June 22, 2010.

the column of Fig. 5.4(b) were not quite fixed, the column would be a little freer to bend laterally, and its points of inflection would be farther apart. The recommended design K given in Table 5.1 is 0.65, while the theoretical value is 0.5. As no column ends are perfectly fixed or perfectly hinged, the designer may wish to interpolate between the values given in the table, basing the interpolation on his or her judgment of the actual restraint conditions.

The values in Table 5.1 are very useful for preliminary designs. When using this table, we will almost always apply the design values, and not the theoretical values. In fact, the theoretical values should be used only for those very rare situations where fixed ends are really almost perfectly fixed and/or when simple supports are almost perfectly frictionless. (*This means almost never.*)

You will note in the table that, for cases (a), (b), (c), and (e), the design values are greater than the theoretical values, but this is not the situation for cases (d) and (f), where the values are the same. The reason, in each of these two latter cases, is that if the pinned conditions are not perfectly frictionless, the K values will become smaller, not larger. Thus, by making the design values the same as the theoretical ones, we are staying on the safe side.

The K values in Table 5.1 are probably very satisfactory to use for designing isolated columns, but for the columns in continuous frames, they are probably satisfactory only for making preliminary or approximate designs. Such columns are restrained at their ends by their connections to various beams, and the beams themselves are connected to other columns and beams at their other ends and thus are also restrained. These connections can appreciably affect the K values. As a result, for most situations, the values in Table 5.1 are not sufficient for final designs.

For continuous frames, it is necessary to use a more accurate method for computing K values. Usually, this is done by using the alignment charts that are presented in the first section of Chapter 7. There, we will find charts for determining K values for columns in frames braced against sidesway and for frames not braced against sidesway. These charts should almost always be used for final column designs.

5.7 STIFFENED AND UNSTIFFENED ELEMENTS

Up to this point in the text, the author has considered only the overall stability of members, and yet it is entirely possible for the thin flanges or webs of a column or beam to buckle locally in compression well before the calculated buckling strength of the whole member is reached. When thin plates are used to carry compressive stresses, they are particularly susceptible to buckling about their weak axes due to the small moments of inertia in those directions.

The AISC Specification (Section B4) provides limiting values for the width–thickness ratios of the individual parts of compression members and for the parts of beams in their compression regions. The student should be well aware of the lack of stiffness of thin pieces of cardboard or metal or plastic with free edges. If, however, one of these elements is folded or restrained, its stiffness is appreciably increased. For this reason, two categories are listed in the AISC Manual: *stiffened elements* and *unstiffened elements*.

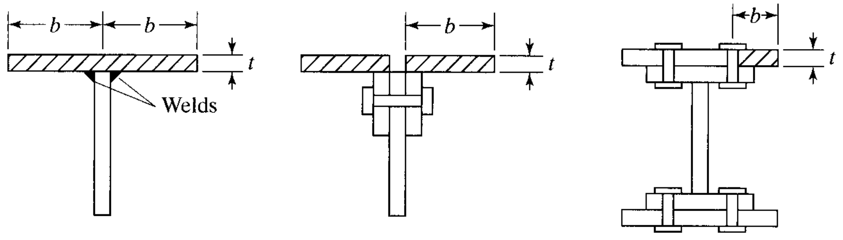
An unstiffened element is a projecting piece with one free edge parallel to the direction of the compression force, while a stiffened element is supported along the two edges in that direction. These two types of elements are illustrated in Fig. 5.6. In each case, the width, b, and the thickness, t, of the elements in question are shown.

Depending on the ranges of different width–thickness ratios for compression elements, and depending on whether the elements are stiffened or unstiffened, the elements will buckle at different stress situations.

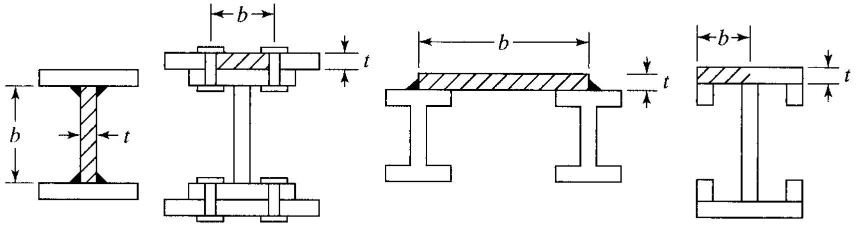
For establishing width–thickness ratio limits for the elements of compression members, the AISC Specification divides members into three classifications, as follows: compact sections, noncompact sections, and slender compression elements. These classifications, which decidedly affect the design compression stresses to be used for columns, are discussed in the paragraphs to follow.

5.7.1 Classification of Compression Sections for Local Buckling

Compression sections are classified as either a nonslender element or a slender element. A nonslender element is one where the width-to-thickness of its compression elements does not exceed λ_r, from Table B4.1a of the AISC Specification. When the width-to-thickness ratio does exceed λ_r, the section is defined as a slender-element



(a) Unstiffened elements



(b) Stiffened elements

FIGURE 5.6

section. The limiting values for λ_r are given in Table 5.2 of this text, which is Table B4.1a of the AISC Specification.

Almost all of the W and HP shapes listed in the Compression Member Section of the AISC Manual are nonslender for 50 ksi yield stress steels. A few of them are slender (and are so indicated in the column tables of the Manual). The values in the tables reflect the reduced design stresses available for slender sections.

If the member is defined as a nonslender element compression member, we should refer to Section E3 of the AISC Specification. The nominal compressive strength is then determined based only on the limit state of flexural buckling.

When the member is defined as a slender element compression member, the nominal compressive strength shall be taken as the lowest value based on the limit states of flexural buckling, torsional buckling, and flexural–torsional buckling. We should refer to Section E7 of the AISC Specification for this condition. Section 6.9 of this text presents an illustration of the determination of the design and allowable strengths of a column which contains slender elements.

5.8 LONG, SHORT, AND INTERMEDIATE COLUMNS

A column subject to an axial compression load will shorten in the direction of the load. If the load is increased until the column buckles, the shortening will stop and the column will suddenly bend or deform laterally and may at the same time twist in a direction perpendicular to its longitudinal axis.

TABLE 5.2 Width-to-Thickness Ratios: Compression Elements in Members Subject to Axial Compression

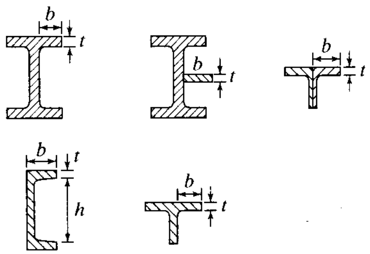
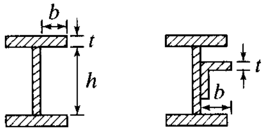
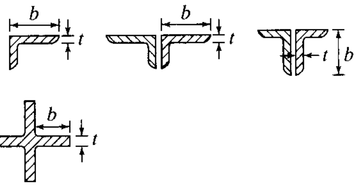
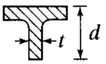
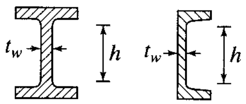
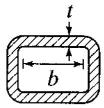
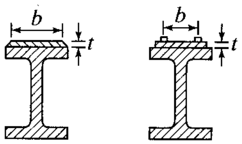
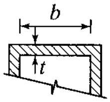
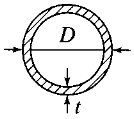
	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender / slender)	Examples
Unstiffened Elements	1	Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections, outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees	b/t	$0.56\sqrt{\dfrac{E}{F_y}}$	
	2	Flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t	$0.64\sqrt{\dfrac{k_c E}{F_y}}$[a]	
	3	Legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	$0.45\sqrt{\dfrac{E}{F_y}}$	
	4	Stems of tees	d/t	$0.75\sqrt{\dfrac{E}{F_y}}$	

TABLE 5.2 Continued

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender / slender)	Examples
Stiffened Elements	5	Webs of doubly symmetric I-shaped sections and channels	h/t_w	$1.49\sqrt{\dfrac{E}{F_y}}$	
	6	Walls of rectangular HSS and boxes of uniform thickness	b/t	$1.40\sqrt{\dfrac{E}{F_y}}$	
	7	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.40\sqrt{\dfrac{E}{F_y}}$	
	8	All other stiffened elements	b/t	$1.49\sqrt{\dfrac{E}{F_y}}$	
	9	Round HSS	D/t	$0.11\dfrac{E}{F_y}$	

The strength of a column and the manner in which it fails are greatly dependent on its effective length. A very short, stocky steel column may be loaded until the steel yields and perhaps on into the strain-hardening range. As a result, it can support about the same load in compression that it can in tension.

As the effective length of a column increases, its buckling stress will decrease. If the effective length exceeds a certain value, the buckling stress will be less than the proportional limit of the steel. Columns in this range are said to fail *elastically*.

As previously shown in Section 5.5, very long steel columns will fail at loads that are proportional to the bending rigidity of the column (EI) and independent of the strength of the steel. For instance, a long column constructed with a 36-ksi yield stress steel will fail at just about the same load as one constructed with a 100-ksi yield stress steel.

Columns are sometimes classed as being long, short, or intermediate. A brief discussion of each of these classifications is presented in the paragraphs to follow.

5.8.1 Long Columns

The Euler formula predicts very well the strength of long columns where the axial buckling stress remains below the proportional limit. Such columns will buckle *elastically*.

5.8.2 Short Columns

For very short columns, the failure stress will equal the yield stress and no buckling will occur. (For a column to fall into this class, it would have to be so short as to have no practical application. Thus, no further reference is made to them here.)

5.8.3 Intermediate Columns

For intermediate columns, some of the fibers will reach the yield stress and some will not. The members will fail by both yielding and buckling, and their behavior is said to be *inelastic*. Most columns fall into this range. (For the Euler formula to be applicable to such columns, it would have to be modified according to the reduced modulus concept or the tangent modulus concept to account for the presence of residual stresses.)

In Section 5.9, formulas are presented with which the AISC estimates the strength of columns in these different ranges.

5.9 COLUMN FORMULAS

The AISC Specification provides one equation (the Euler equation) for long columns with elastic buckling and an empirical parabolic equation for short and intermediate columns. With these equations, a flexural buckling stress, F_{cr}, is determined for a compression member. Once this stress is computed for a particular member, it is multiplied by the cross-sectional area of the member to obtain its nominal strength P_n. The LRFD design strength and ASD allowable strength of a column may be determined as follows:

$$P_n = F_{cr}A_g \qquad \text{(AISC Equation E3-1)}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = \text{LRFD compression strength } (\phi_c = 0.90)$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr}A_g}{\Omega_c} = \text{ASD allowable compression strength } (\Omega_c = 1.67)$$

The following expressions show how F_{cr}, the flexural buckling stress of a column, may be determined for members without slender elements:

(a) If $\dfrac{KL}{r} \leq 4.71\sqrt{\dfrac{E}{F_y}} \left(\text{or } \dfrac{F_y}{F_e} \leq 2.25\right)$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad \text{(AISC Equation E3-2)}$$

(b) If $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}} \left(\text{or } \dfrac{F_y}{F_e} \leq 2.25\right)$

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E3-3)}$$

In these expressions, F_e is the elastic critical buckling stress—that is, the Euler stress—calculated with the effective length of the column KL.

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad \text{(AISC Equation E3-4)}$$

These equations are represented graphically in Fig. 5.7.

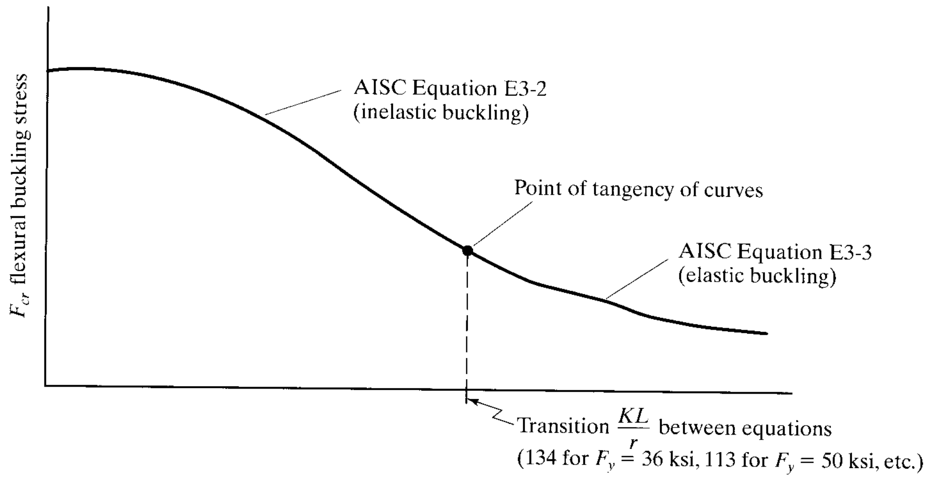


FIGURE 5.7

AISC column curve.

After looking at these column equations, the reader might think that their use would be tedious and time consuming with a pocket calculator. Such calculations, however, rarely have to be made, because the AISC Manual provides computed values of critical stresses $\phi_c F_{cr}$ and $\frac{F_{cr}}{\Omega_c}$ in their Table 4-22. The values are given for practical KL/r values (0 to 200) and for steels with $F_y = 35, 36, 42, 46$, and 50 ksi.

5.10 MAXIMUM SLENDERNESS RATIOS

The AISC Specification no longer provides a specific maximum slenderness ratio, as it formerly did and as is the practice of many other specifications. The AISC Commentary (E2) does indicate, however, that if KL/r is >200, the critical stress F_{cr} will be less than 6.3 ksi. In the past, the AISC maximum permitted KL/r was 200. That value was based on engineering judgment, practical economics, and the fact that special care had to be taken to keep from injuring such a slender member during fabrication, shipping, and erection. As a result of these important practical considerations, the engineer applying the 2010 AISC Specification will probably select compression members with slenderness values below 200, except in certain special situations. For those special cases, both the fabricators and the erectors will be on notice to use extra-special care in handling the members.

5.11 EXAMPLE PROBLEMS

In this section, four simple numerical column problems are presented. In each case, the design strength of a column is calculated. In Example 5-2(a), we determine the strength of a W section. The value of K is calculated as described in Section 5.6, the effective slenderness ratio is computed, and the available critical stresses $\phi_c F_{cr}$ and $\frac{F_{cr}}{\Omega}$ are obtained from Table 4-22 in the Manual.

It will be noted that the Manual, in Part 4, Tables 4-1 to 4-11, has further simplified the needed calculations by computing the LRFD column design strengths ($\phi_c P_n$) and the ASD allowable column strengths $\left(\frac{P_n}{\Omega_c}\right)$ for each of the shapes normally employed as columns for the commonly used effective lengths or KL values. These strengths were determined with respect to the least radius of gyration for each section, and the F_y values used are the preferred ones given in Table 1.1 of this text (Table 2-3 in the Manual). You should also note that the ASD values are shaded in green in the Manual.

Example 5-2

(a) Using the column critical stress values in Table 4-22 of the Manual, determine the LRFD design strength $\phi_c P_n$ and the ASD allowable strength $\frac{P_n}{\Omega_c}$ for the column shown in Fig. 5.8, if a 50-ksi steel is used.

(b) Repeat the problem, using Table 4-1 of the Manual.

(c) Calculate $\phi_c P_n$ and $\frac{P_n}{\Omega_c}$, using the equations of AISC Section E3.

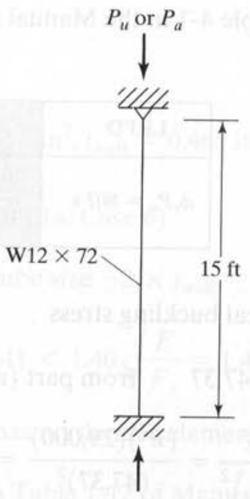


FIGURE 5.8

Solution

(a) Using a W12 × 72 ($A = 21.1 \text{ in}^2$, $r_x = 5.31$ in, $r_y = 3.04$ in, $d = 12.3$ in, $b_f = 12.00$ in, $t_f = 0.670$ in, $k = 1.27$ in, $t_w = 0.430$ in)

$$\frac{b}{t} = \frac{12.00/2}{0.670} = 8.96 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.49$$

$\therefore$ Nonslender unstiffened flange element

$$\frac{h}{t_w} = \frac{d - 2\text{ k}}{t_w} = \frac{12.3 - 2(1.27)}{0.430} = 22.70 < 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.88$$

$\therefore$ Nonslender stiffened web element

$K = 0.80$ from Table 5.1.

Obviously, $(KL/r)_y > (KL/r)_x$ and thus controls

$$\left(\frac{KL}{r}\right)_y = \frac{(0.80)(12 \times 15)\text{ in}}{3.04\text{ in}} = 47.37$$

By straight-line interpolation, $\phi_c F_{cr} = 38.19$ ksi, and $\dfrac{F_{cr}}{\Omega_c} = 25.43$ ksi from Table 4-22 in the Manual using $F_y = 50$ ksi steel.

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} A_g = (38.19)(21.1) = 805.8\text{ k}$	$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr} A_g}{\Omega_c} = (25.43)(21.1) = 536.6\text{ k}$

(b) Entering Table 4-1 in the Manual with KL $(0.8)(15) = 12$ ft

LRFD	ASD
$\phi_t P_n = 807$ k	$\dfrac{P_n}{\Omega_c} = 537$ k

(c) Elastic critical buckling stress

$$\left(\frac{KL}{r}\right)_y = 47.37 \quad \text{from part (a)}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(47.37)^2} = 127.55 \text{ ksi} \qquad \text{(AISC Equation E3-4)}$$

Flexural buckling stress F_{cr}

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 113.43 > \left(\frac{KL}{r}\right)_y = 47.37$$

$$\therefore F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left[0.658^{\frac{50}{127.55}}\right]50 = 42.43 \; ksi \qquad \text{(AISC Equation E3-2)}$$

LRFD $\phi_c = 0.90$	ASD $\Omega_c = 1.67$
$\phi_c F_{cr} = (0.90)(42.43) = 38.19$ ksi	$\dfrac{F_{cr}}{\Omega_c} = \dfrac{42.43}{1.67} = 25.41$ ksi
$\phi_c P_n = \phi_c F_{cr} A = (38.19)(21.1)$	$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}}{\Omega_c} A = (25.41)(21.1)$
$= 805.8$ k	$= 536.2$ k

Example 5-3

An HSS $16 \times 16 \times \frac{1}{2}$ with $F_y = 46$ ksi is used for an 18-ft-long column with simple end supports.

(a) Determine $\phi_c P_n$ and $\dfrac{P_n}{\Omega_c}$ with the appropriate AISC equations.

(b) Repeat part (a), using Table 4-4 in the AISC Manual.

Solution

(a) Using an HSS

$$16 \times 16 \times \frac{1}{2}(A = 28.3 \text{ in}^2, t_{\text{wall}} = 0.465 \text{ in}, r_x = r_y = 6.31 \text{ in})$$

Calculate $\frac{b}{t}$ (AISC Table B4.1a, Case 6)

b is approximated as the tube size $-2 \times t_{\text{wall}}$

$$\frac{b}{t} = \frac{16 - 2(0.465)}{0.465} = 32.41 < 1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000}{46}}$$

$$= 35.15 \quad \therefore \text{ Section has no slender elements}$$

$\frac{b}{t}$ ratio also available from Table 1-12 of Manual

Calculate $\frac{KL}{r}$ and F_{cr}

$K = 1.0$

$$\left(\frac{KL}{r}\right)_x = \left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 18) \text{ in}}{6.31 \text{ in}} = 34.23$$

$$< 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.26$$

$\therefore$ Use AISC Equation E3-2 for F_{cr}

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(34.23)^2} = 244.28 \text{ ksi}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left[0.658^{\frac{46}{244.28}}\right]46$$

$$= 42.51 \text{ ksi}$$

LRFD $\phi_c = 0.90$	ASD $\Omega_c = 1.67$
$\phi_c F_{cr} = (0.90)(42.51) = 38.26$ ksi	$\frac{F_{cr}}{\Omega_c} = \frac{42.51}{1.67} = 25.46$ ksi
$\phi_c P_n = \phi_c F_{cr} A = (38.26)(28.3)$	$\frac{P_n}{\Omega_c} = \frac{F_{cr}}{\Omega_c}A = (25.46)(28.3)$
$= 1082$ k	$= 720$ k

(b) From the Manual, Table 4-4

LRFD	ASD
$\phi_c P_n = 1080$ k	$\dfrac{P_n}{\Omega_c} = 720$ k

Though width–thickness ratios should be checked for all of the column and beam sections used in design, most of these necessary calculations are left out in this text to conserve space. To make this check in most situations, it is necessary only to refer to the Manual column tables where slender sections are clearly indicated for common shapes.

In Example 5-4, the author illustrates the computations necessary to determine the design strength of a built-up column section. Several special requirements for built-up column sections are described in Chapter 6.

Example 5-4

Determine the LRFD design strength $\phi_c P_n$ and the ASD allowable strength $\dfrac{P_n}{\Omega_c}$ for the axially loaded column shown in Fig. 5.9 if $KL = 19$ ft and 50-ksi steel is used.

Solution

$$A_g = (20)\left(\tfrac{1}{2}\right) + (2)(12.6) = 35.2 \text{ in}^2$$

$$\overline{y} \text{ from top} = \frac{(10)(0.25) + (2)(12.6)(9.50)}{35.2} = 6.87 \text{ in}$$

$$I_x = (2)(554) + (2)(12.6)(9.50 - 6.87)^2 + \left(\frac{1}{12}\right)(20)\left(\frac{1}{2}\right)^3 + (10)(6.87 - 0.25)^2$$
$$= 1721 \text{ in}^4$$

$$I_y = (2)(14.3) + (2)(12.6)(6.877)^2 + \left(\tfrac{1}{12}\right)\left(\tfrac{1}{2}\right)(20)^3 = 1554 \text{ in}^4$$

$$r_x = \sqrt{\frac{1721}{35.2}} = 6.99 \text{ in}$$

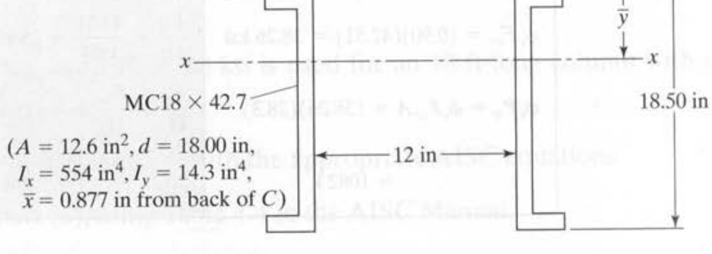


FIGURE 5.9

$$r_y = \sqrt{\frac{1554}{35.2}} = 6.64 \text{ in}$$

$$\left(\frac{KL}{r}\right)_x = \frac{(12)(19)}{6.99} = 32.62$$

$$\left(\frac{KL}{r}\right)_y = \frac{(12)(19)}{6.64} = 34.34 \leftarrow$$

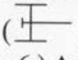
From the Manual, Table 4-22, we read for $\frac{KL}{r} = 34.34$ that $\phi_c F_{cr} = 41.33$ ksi and $\frac{F_{cr}}{\Omega_c} = 27.47$ ksi, for 50 ksi steel.

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} A_g = (41.33)(35.2) = 1455$ k	$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c} = (27.47)(35.2) = 967$ k

To determine the design compression stress needed for a particular column, it is theoretically necessary to compute both $(KL/r)_x$ and $(KL/r)_y$. The reader will notice, however, that for most of the steel sections used for columns, r_y will be much less than r_x. As a result, only $(KL/r)_y$ is calculated for most columns and used in the applicable column formulas.

For some columns, particularly the long ones, bracing is supplied perpendicular to the weak axis, thus reducing the slenderness or the length free to buckle in that direction. Bracing may be accomplished by framing braces or beams into the sides of a column. For instance, horizontal members or *girts* running parallel to the exterior walls of a building frame may be framed into the sides of columns. The result is stronger columns and ones for which the designer needs to calculate both $(KL/r)_x$ and $(KL/r)_y$. The larger ratio obtained for a particular column indicates the weaker direction and will be used for calculating the design stress $\phi_c F_{cr}$ and the allowable stress $\frac{F_{cr}}{\Omega_c}$ for that member.

Bracing members must be capable of providing the necessary lateral forces, without buckling themselves. The forces to be taken are quite small and are often conservatively estimated to equal 0.02 times the column design loads. These members can be selected as are other compression members. A bracing member must be connected to other members that can transfer the horizontal force by shear to the next restrained level. If this is not done, little lateral support will be provided for the original column in question.

If the lateral bracing were to consist of a single bar or rod (), it would not prevent twisting and torsional buckling of the column. (See Chapter 6.) As torsional buckling is a difficult problem to handle, we should provide lateral bracing that prevents lateral movement and twist.[6]

[6]J. A. Yura, "Elements for Teaching Load and Resistance Factor Design" (New York: AISC, August 1987), p. 20.

Steel columns may also be built into heavy masonry walls in such a manner that they are substantially supported in the weaker direction. The designer, however, should be quite careful in assuming that there is complete lateral support parallel to the wall, because the condition of the wall may be unknown and a poorly built wall will not provide 100 percent lateral support.

Example 5-5 illustrates the calculations necessary to determine the LRFD design strength and the ASD allowable strength of a column with two unbraced lengths.

Example 5-5

(a) Determine the LRFD design strength $\phi_c P_n$ and the ASD allowable design strength $\frac{P_n}{\Omega_c}$ for the 50 ksi axially loaded W14 × 90 shown in Fig. 5.10.

Because of its considerable length, this column is braced perpendicular to its weak, or y, axis at the points shown in the figure. These connections are assumed to permit rotation of the member in a plane parallel to the plane of the flanges. At the same time, however, they are assumed to prevent translation or sidesway and twisting of the cross section about a longitudinal axis passing through the shear center of the cross section. (The shear center is the point in the cross section of a member through which the resultant of the transverse loads must pass so that no torsion will occur. See Chapter 10.)

(b) Repeat part (a), using the column tables of Part 4 of the Manual.

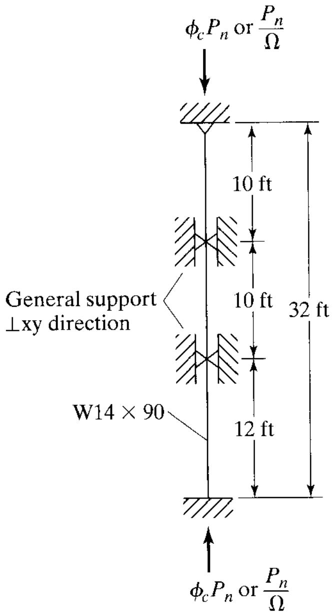


FIGURE 5.10

Solution

(a) Using W14 × 90 ($A = 26.5\text{ in}^2, r_x = 6.14\text{ in}, r_y = 3.70\text{ in}$)
Determining effective lengths

$$K_xL_x = (0.80)(32) = 25.6\text{ ft}$$
$$K_yL_y = (1.0)(10) = 10\text{ ft} \leftarrow \text{governs for } K_yL_y$$
$$K_yL_y = (0.80)(12) = 9.6\text{ ft}$$

Computing slenderness ratios

$$\left(\frac{KL}{r}\right)_x = \frac{(12)(25.6)}{6.14} = 50.03 \leftarrow$$
$$\left(\frac{KL}{r}\right)_y = \frac{(12)(10)}{3.70} = 32.43$$
$$\left.\begin{aligned} \phi_c F_{cr} &= 37.49\text{ ksi} \\ \frac{F_{cr}}{\Omega_c} &= 24.90\text{ ksi} \end{aligned}\right\} \begin{aligned} &\text{from Manual,} \\ &\text{Table 4-22, } F_y = 50\text{ ksi} \end{aligned}$$

LRFD	ASD
$\phi_c P_n = \phi_c F_{cr} A_g = (37.49)(26.5)$ $= 993\text{ k}$	$\frac{P_n}{\Omega_c} = \frac{F_{cr}A_g}{\Omega_c} = (24.90)(26.5) = 660\text{ k}$

(b) Noting from part (a) solution that there are two different KL values

$$K_xL_x = 25.6\text{ ft}$$
$$K_yL_y = 10\text{ ft}$$

We would like to know which of these two values is going to control. This can easily be learned by determining a value of K_xL_x that is equivalent to K_yL_y. The slenderness ratio in the x direction is equated to an equivalent value in the y direction as follows:

$$\frac{K_xL_x}{r_x} = \text{Equivalent } \frac{K_yL_y}{r_y}$$

$$\text{Equivalent } K_yL_y = r_y\frac{K_xL_x}{r_x} = \frac{K_xL_x}{\frac{r_x}{r_y}}$$

Thus, the controlling K_yL_y for use in the tables is the larger of the real $K_yL_y = 10$ ft, or the equivalent K_yL_y.

$$\frac{r_x}{r_y} \text{ for W14} \times 90 \text{ (from the bottom of Table 4-1 of the Manual)} = 1.66$$

$$\text{Equivalent } K_yL_y = \frac{25.6}{1.66} = 15.42 \text{ ft} > K_yL_y \text{ of 10 ft}$$

From column tables with $K_yL_y = 15.42$ ft, we find by interpolation that $\phi_c P_n = 991$ k and $\frac{P_n}{\Omega_c} = 660$ k.

5.12 PROBLEMS FOR SOLUTION

5-1 to 5-4. *Determine the critical buckling load for each of the columns, using the Euler equation. E = 29,000 ksi. Proportional limit = 36,000 psi. Assume simple ends and maximum permissible L/r = 200.*

5-1. A solid round bar of 1¼ in diameter:

a. $L = 4$ ft 0 in (*Ans.* 14.89 kips)
b. $L = 2$ ft 3 in (*Ans.* Euler equation not applicable, F_e exceeds proportional limit)
c. $L = 6$ ft 6 in (*Ans.* Euler equation not applicable, L/r exceeds 200)

5-2. The pipe section shown:

a. $L = 21$ ft 0 in
b. $L = 16$ ft 0 in
c. $L = 10$ ft 0 in

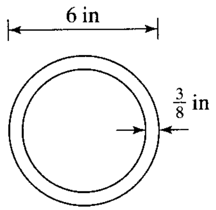


FIGURE P5-2

5-3. A W12 × 50, $L = 20$ ft 0 in. (*Ans.* 278.7 k)

5-4. The four – L4 × 4 × 1/4 shown for $L = 40$ ft 0 in.

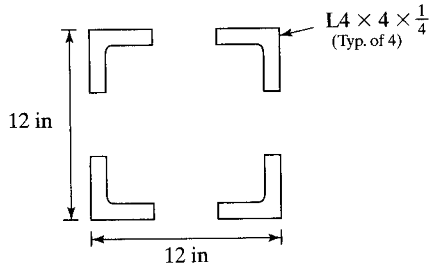


FIGURE P5-4

5-5 to 5-8. *Determine the LRFD design strength, $\Phi_c P_n$, and the ASD allowable strength, P_n/Ω_c, for each of the compression members shown. Use the AISC Specification and a steel with $F_y = 50$ ksi, except for Problem 5-8, $F_y = 46$ ksi.*

5-5. (*Ans.* 212 k LRFD; 141 k ASD)

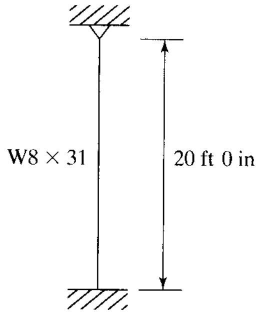


FIGURE P5-5

5-6.

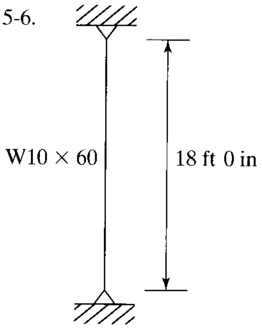


FIGURE P5-6

5-7. (*Ans.* 678.4 k LRFD; 451.5 k ASD)

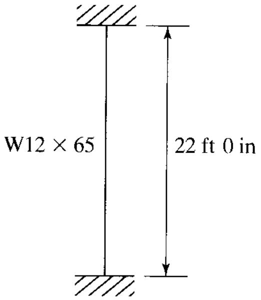


FIGURE P5-7

5-8.

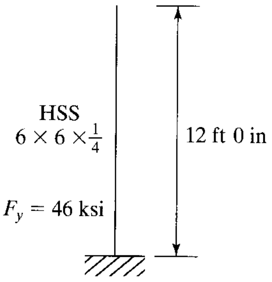


FIGURE P5-8

5-9 to 5-17. *Determine $\Phi_c P_n$, and P_n/Ω_c for each of the columns, using the AISC Specification and $F_y = 50$ ksi, unless noted otherwise.*

5-9. a. W12 × 120 with $KL = 18$ ft. (*Ans.* 1120 k LRFD; 744 k ASD)

b. HP10 × 42 with $KL = 15$ ft. (*Ans.* 371 k LRFD; 247 k ASD)

c. WT8 × 50 with $KL = 20$ ft. (*Ans.* 294 k LRFD; 196 k ASD)

5-10. Note that F_y is different for parts (c) to (e).

a. A W8 × 24 with pinned ends, $L = 12$ ft.

b. A W14 × 109 with fixed ends, $L = 20$ ft.

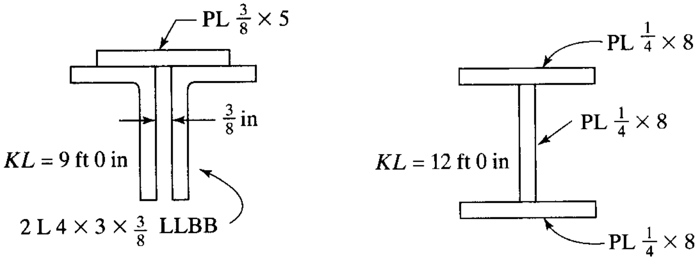
c. An HSS 8 × 6 × 3/8, $F_y = 46$ ksi with pinned ends, $L = 15$ ft.

d. A W12 × 152 with one end fixed and the other end pinned, $L = 25$ ft 0 in, $F_y = 36$ ksi.

e. A Pipe 10 STD with pinned ends, $L = 18$ ft 6 in, $F_y = 35$ ksi.

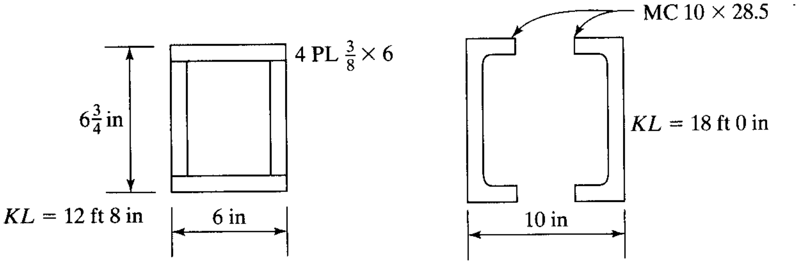
5-11. A W10 × 39 with a 1/2 × 10 in cover plate welded to each flange is to be used for a column with KL = 14 ft. (*Ans.* 685 k LRFD; 455 k ASD)

5-12.



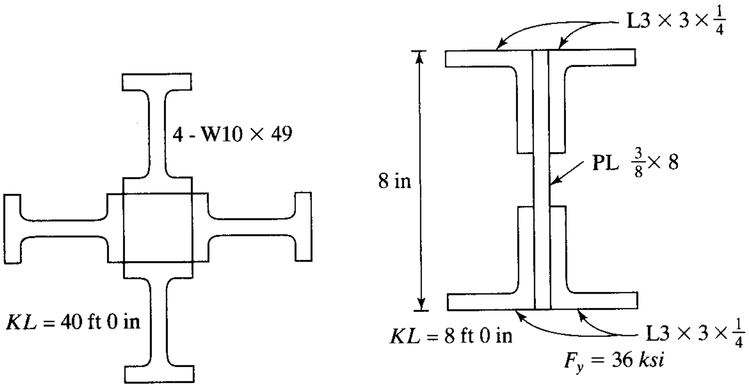
(a) (b)

5-13.



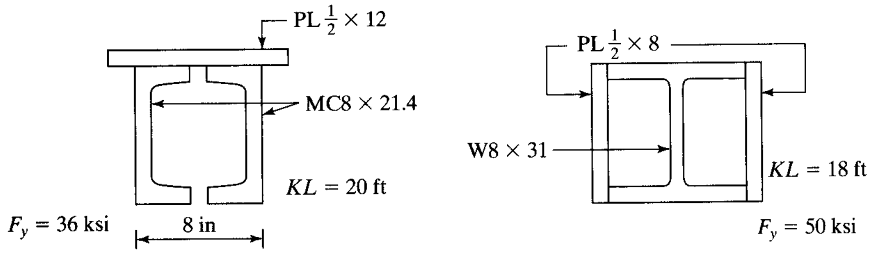
(a) (*Ans.* 297 k LRFD; 198 k ASD) (b) (*Ans.* 601 k LRFD; 400 k ASD)

5-14.



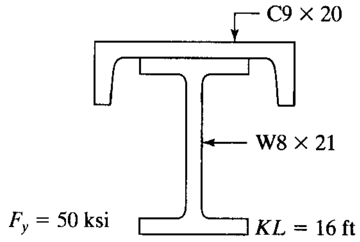
(a) (b)

5-15.

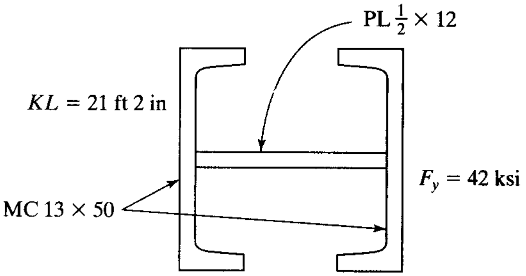


(a) (*Ans.* 451.9 k LRFD; 301.0 k ASD) (b) (*Ans.* 525.9 k LRFD; 350.0 k ASD)

5-16.



(a)



(b)

5-17. A 24 ft axially loaded W12 x 96 column that has the bracing and end support conditions shown in the figure. (*Ans.* 1023.3 k LRFD; 680.4 k ASD)

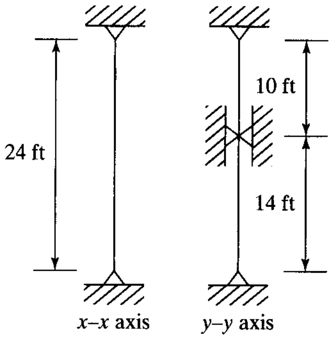


FIGURE P5-17

5-18. Determine the maximum service live load that the column shown can support if the live load is twice the dead load. $K_xL_x = 18$ ft, $K_yL_y = 12$ ft and $F_y = 36$ ksi. Solve by LRFD and ASD methods.

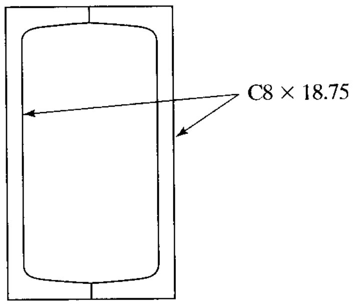


FIGURE P5-18

5-19. Compute the maximum total service live load that can be applied to the A36 section shown in the figure, if $K_xL_x = 12$ ft., $K_yL_y = 10$ ft. Assume the load is 1/2 dead load and 1/2 live load. Solve by both LRFD and ASD methods. (*Ans.* 29.0 k LRFD; 27.0 k ASD)

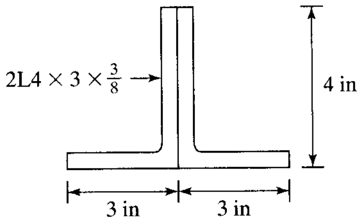


FIGURE P5-19

CHAPTER 6

Design of Axially Loaded Compression Members

6.1 INTRODUCTION

In this chapter, the designs of several axially loaded columns are presented. Included are the selections of single shapes, W sections with cover plates, and built-up sections constructed with channels. Also included are the designs of sections whose unbraced lengths in the x and y directions are different, as well as the sizing of lacing and tie plates for built-up sections with open sides. Another topic that is introduced is flexural torsional buckling of sections.

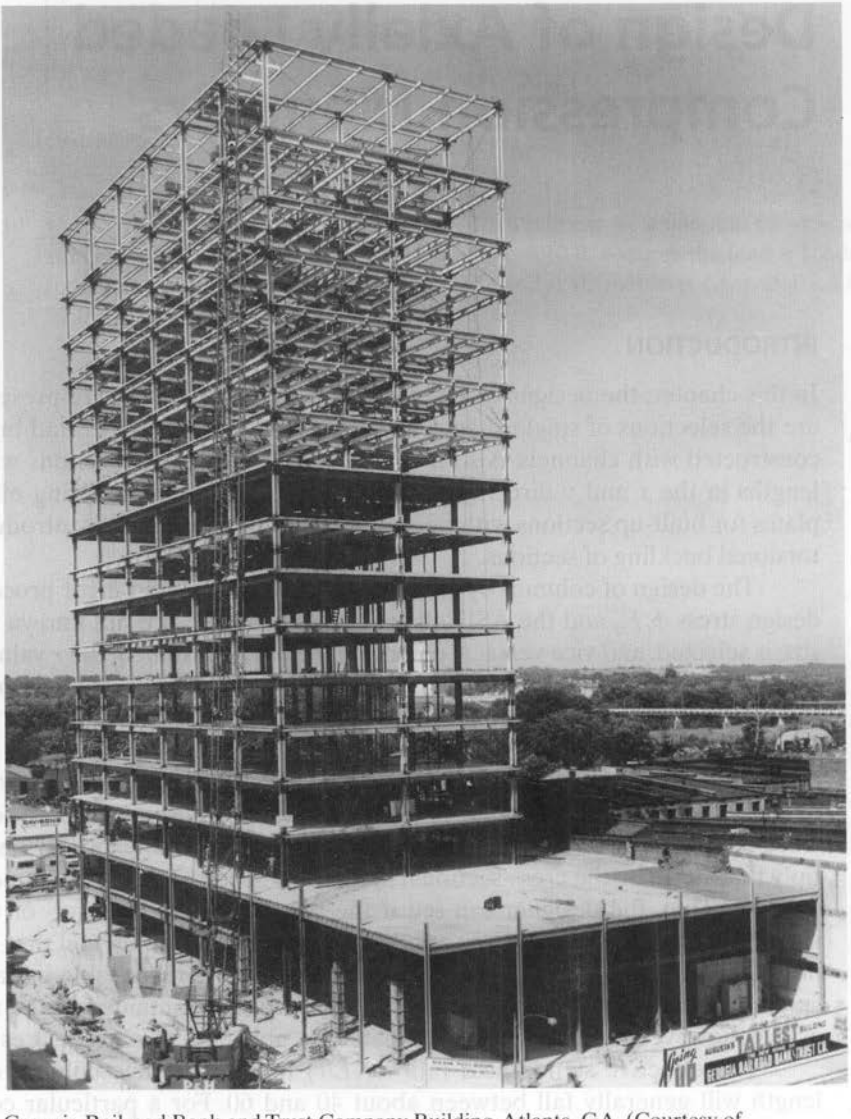
The design of columns by formulas involves a trial-and-error process. The LRFD design stress $\phi_c F_{cr}$ and the ASD allowable stress F_{cr}/Ω_c are not known until a column size is selected, and vice versa. A column size may be assumed, the r values for that section obtained from the Manual or calculated, and the design stress found by substituting into the appropriate column formula. It may then be necessary to try a larger or smaller section. Examples 6-1, 6-3, and 6-4 illustrate this procedure.

The designer may assume an LRFD design stress or an ASD allowable stress and divide that stress into the appropriate column load to give an estimated column area, select a column section with approximately that area, determine its design stress, and multiply that stress by the cross-sectional area of the section to obtain the member's design strength. Thus, the designer can see if the section selected is over- or underdesigned and—if it is appreciably so—try another size. The student may feel that he or she does not have sufficient background or knowledge to make reasonable initial design stress assumptions. If this student will read the information contained in the next few paragraphs, however, he or she will immediately be able to make excellent estimates.

The effective slenderness ratio (KL/r) for the average column of 10- to 15-ft length will generally fall between about 40 and 60. For a particular column, a KL/r

somewhere in this approximate range is assumed and substituted into the appropriate column equation to obtain the design stress. (To do this, you will first note that the AISC for KL/r values from 0 to 200 has substituted into the equations, with the results shown, in AISC Table 4-22. This greatly expedites our calculations.)

To estimate the effective slenderness ratio for a particular column, the designer may estimate a value a little higher than 40 to 60 if the column is appreciably longer than the 10- to 15-ft range, and vice versa. A very heavy factored column load—say, in the 750- to 1000-k range or higher—will require a rather large column for which the



Georgia Railroad Bank and Trust Company Building, Atlanta, GA. (Courtesy of Bethlehem Steel Corporation.)

radii of gyration will be larger, and the designer may estimate a little smaller value of KL/r. For lightly loaded bracing members, he or she may estimate high slenderness ratios of 100 or more.

In Example 6-1, a column is selected by the LRFD method. An effective slenderness ratio of 50 is assumed, and the corresponding design stress $\phi_c F_{cr}$ is picked from AISC Table 4-22. By dividing this value into the factored column load, an estimated required column area is derived and a trial section selected. After a trial section is selected with approximately that area, its actual slenderness ratio and design strength are determined. The first estimated size in Example 6-1, though quite close, is a little too small, but the next larger section in that series of shapes is satisfactory.

The author follows a similar procedure with the ASD formula as follows: Assume $KL/r = 50$, determine F_{cr}/Ω_c from AISC Table 4-22, divide this value into the ASD column load to obtain the estimated area required, and pick a trial section and determine its allowable load.

Example 6-1

Using $F_y = 50$ ksi, select the lightest W14 available for the service column loads $P_D = 130$ k and $P_L = 210$ k. $KL = 10$ ft.

Solution

LRFD	ASD
$P_u = (1.2)(130\text{ k}) + (1.6)(210\text{ k}) = 492\text{ k}$	$P_a = 130\text{ k} + 210\text{ k} = 340\text{ k}$
Assume $\frac{KL}{r} = 50$	Assume $\frac{KL}{r} = 50$
Using $F_y = 50$ ksi steel	Using $F_y = 50$ ksi steel
$\phi_c F_{cr}$ from AISC Table 4-22 = 37.5 ksi	$\frac{F_{cr}}{\Omega_c} = 24.9$ ksi (AISC Table 4-22)
A Reqd $= \frac{P_u}{\phi_c F_{cr}} = \frac{492\text{ k}}{37.5\text{ ksi}} = 13.12\text{ in}^2$	A Reqd $= \frac{P_a}{F_{cr}/\Omega} = \frac{340\text{ k}}{24.9\text{ ksi}} = 13.65\text{ in}^2$
Try W14 × 48 ($A = 14.1\text{ in}^2$, $r_x = 5.85$ in, $r_y = 1.91$ in)	Try W14 × 48 ($A = 14.1\text{ in}^2$, $r_x = 5.85$ in, $r_y = 1.91$ in)
$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(10\text{ ft})}{1.91\text{ in}} = 62.83$	$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(10\text{ ft})}{1.91\text{ in}} = 62.83$
$\phi_c F_{cr} = 33.75$ ksi from AISC Table 4-22	
$\phi_c P_n = (33.75\text{ ksi})(14.1\text{ in}^2) = 476\text{ k} < 492\text{ k}$ N.G.	$\frac{F_{cr}}{\Omega_c} = 22.43$ ksi from AISC Table 4-22

Try next larger section W14 × 53 (A = 15.6 in^2, r_y = 1.92 in)	$\dfrac{P_n}{\Omega_c} = (22.43 \text{ ksi})(14.1 \text{ in}^2) = 316 \text{ k} < 340 \text{ k}$ N.G.
$\left(\dfrac{KL}{r}\right)_y = \dfrac{(12 \text{ in/ft})(10 \text{ ft})}{1.92 \text{ in}} = 62.5$	Try next larger section W14 × 53 (A = 15.6 in^2, r_y = 1.92 in).
$\phi_c F_{cr} = 33.85$ ksi	$\left(\dfrac{KL}{r}\right)_y = \dfrac{(12 \text{ in/ft})(10 \text{ ft})}{1.92 \text{ in}} = 62.5$
$\phi_c P_n = (33.85 \text{ ksi})(15.6 \text{ in}^2)$	$\dfrac{F_{cr}}{\Omega_c} = 22.5$ ksi
$= 528 \text{ k} > 492 \text{ k}$ **OK**	$\dfrac{P_n}{\Omega_c} = (22.5 \text{ ksi})(15.6 \text{ in}^2) = 351 \text{ k} > 340 \text{ k}$ **OK**
Use W14 × 53.	**Use W14 × 53.**

Note: Table 4-1 does not indicate that a W14 × 53 is a slender member for compression.

6.2 AISC DESIGN TABLES

For Example 6-2, Table 4-1 of the Manual is used to select various column sections from tables, without the necessity of using a trial-and-error process. These tables provide axial design strengths ($\phi_c P_n$) and allowable design loads (P_n/Ω_c) for various practical effective lengths of the steel sections commonly used as columns. The values are given with respect to the least radii of gyration, for Ws and WTs with 50 ksi steel. Other grade steels are commonly used for other types of sections, as shown in the Manual and listed here. These include 35 ksi for steel pipe, 36 ksi for Ls, 42 ksi for round HSS sections, and 46 ksi for square and rectangular HSS sections.

For most columns consisting of single steel shapes, the effective slenderness ratio with respect to the y axis $(KL/r)_y$ is larger than the effective slenderness ratio with respect to the x axis $(KL/r)_x$. As a result, the controlling, or smaller, design stress is for the y axis. Because of this, the AISC tables provide design strengths of columns with respect to their y axes. We will learn in the pages to follow how to handle situations in which $(KL/r)_x$ is larger than $(KL/r)_y$.

The resulting tables are very simple to use. The designer takes the KL value for the minor principal axis in feet, enters the table in question from the left-hand side, and moves horizontally across the table. Under each section is listed the design strength $\phi_c P_n$ and the allowable design strength P_n/Ω_c for that KL and steel yield stress. As an illustraion, assume that we have a factored design strength $P_u = 1200$ k, $K_y L_y = 12$ ft, and we want to select the lightest available W14 section, using 50 ksi steel and the LRFD method. We enter with KL = 12 ft into the left-hand column of the first page of AISC Table 4-1 and read from left to right under the $\phi_c P_n$ columns. The values are successively 9030 k, 8220 k, 7440 k, and so on until a few pages later, where the consecutive values 1290 k and 1170 k are found. The 1170 k value is not sufficient, and we go back to the 1290 k value, which falls under the W14 × 109 section.

A similar procedure can be followed in the tables subsequent to AISC Table 4-1 for the selection of rectangular, square, and round HSS sections; pipe; WT shapes; angles; and so on.

Example 6-2 illustrates the selection of various possible sections to be used for a particular column. Among the sections chosen are the round HSS sections in AISC Table 4-5 and the steel pipe sections shown in Table 4-6 of the Manual. It is possible to support a given load with a standard pipe (labeled "std" in the table); with an extra strong pipe (XS), which has a smaller diameter, but thicker walls and thus is heavier and more expensive; or with a double extra-strong pipe (XXS), which has an even smaller diameter, even thicker walls, and greater weight. The XXS sizes are available only in certain sizes (pipes 4, 5, 6, and 8).

Example 6-2

Use the AISC column tables (both LRFD and ASD) for the designs to follow.

(a) Select the lightest W section available for the loads, steel, and KL of Example 6-1. $F_y = 50$ ksi.

(b) Select the lightest satisfactory rectangular or square HSS sections for the situation in part (a). $F_y = 46$ ksi.

(c) Select the lightest satisfactory round HSS section, $F_y = 42$ ksi for the situation in part (a).

(d) Select the lightest satisfactory pipe section, $F_y = 35$ ksi, for the situation in part (a).

Solution

Entering Tables with $K_yL_y = 10$ ft, $P_u = 492$ k for LRFD and $P_a = 340$ k for ASD from Example 6-1 solution.

LRFD	ASD
(a) W8 × 48 ($\phi_c P_n = 497\text{ k} > 492\text{ k}$) from Table 4-1	(a) W10 × 49 $\left(\dfrac{P_n}{\Omega_c} = 366\text{ k} > 340\text{ k}\right)$ from Table 4-1
(b) Rectangular HSS HSS 12 × 8 × $\frac{3}{8}$ @ 47.8 #/ft ($\phi_c P_n = 499\text{ k} > 492\text{ k}$) from Table 4-3	(b) Rectangular HSS HSS 12 × 10 × $\frac{3}{8}$ @ 52.9 #/ft $\left(\dfrac{P_n}{\Omega_c} = 379\text{ k} > 340\text{ k}\right)$ from Table 4-3
Square HSS HSS 10 × 10 × $\frac{3}{8}$ @ 47.8 #/ft ($\phi_c P_n = 513\text{ k} > 492\text{ k}$) from Table 4-4	Square HSS ** HSS 12 × 12 × $\frac{5}{16}$ @ 48.8 #/ft $\left(\dfrac{P_n}{\Omega_c} = 340\text{ k} = 340\text{ k}\right)$ from Table 4-4

(c) Round HSS 16.000 × 0.312 @ 52.3 #/ft ($\phi_c P_n = 529\text{ k} > 492\text{ k}$) from Table 4-5	(c) Round HSS 16.000 × 0.312 @ 52.3 #/ft $\left(\dfrac{P_n}{\Omega_c} = 352\text{ k} > 340\text{ k}\right)$ from Table 4-5
(d) XS Pipe 12 @ 65.5 #/ft ($\phi_c P_n = 530\text{ k} > 492\text{ k}$) from Table 4-6	(d) XS Pipe 12 @ 65.5 #/ft $\left(\dfrac{P_n}{\Omega_c} = 353\text{ k} > 340\text{ k}\right)$ from Table 4-6

**Note: The AISC Column Tables used in this problem indicate that only the HSS 12 × 12 × 5/16 from part (b)—ASD design method is a slender member for compression. The value of $P_n\Omega_c = 340$ k reflects the reduced design strength available for slender sections (per E7 AISC Specification).

An axially loaded column is laterally restrained in its weak direction, as shown in Fig. 6.1. Example 6-3 illustrates the design of such a column with its different unsupported lengths in the x and y directions. The student can easily solve this problem by trial and error. A trial section can be selected, as described in Section 6.1, the slenderness values $\left(\dfrac{KL}{r}\right)_x$ and $\left(\dfrac{KL}{r}\right)_y$ computed, and $\phi_c F_{cr}$ and $\dfrac{F_{cr}}{\Omega_c}$ determined for the larger value and multiplied, respectively, by A_g to determine $\phi_c P_n$ and $\dfrac{P_n}{\Omega_c}$. Then, if necessary, another size can be tried, and so on.

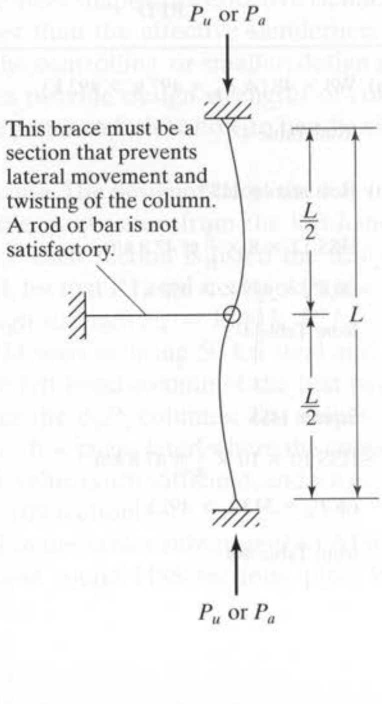


FIGURE 6.1
A column laterally restrained at mid-depth in its weaker direction.

For this discussion, it is assumed that K is the same in both directions. Then, if we are to have equal strengths about the x and y axes, the following relation must hold:

$$\frac{L_x}{r_x} = \frac{L_y}{r_y}$$

For L_y to be equivalent to L_x we would have

$$L_x = L_y \frac{r_x}{r_y}$$

If $L_y(r_x/r_y)$ is less than L_x, then L_x controls; if greater than L_x, then L_y controls.

Consistent with the preceding information, the AISC Manual provides a method by which a section can be selected from its tables, with little trial and error, when the unbraced lengths are different. The designer enters the appropriate table with K_yL_y, selects a shape, takes the r_x/r_y value given in the table for that shape, and multiplies it by L_y. If the result is larger than K_xL_x, then K_yL_y controls and the shape initially selected is the correct one. If the result of the multiplication is less than K_xL_x, then K_xL_x controls and the designer will reenter the tables with a larger K_yL_y equal to $K_xL_x/(r_x/r_y)$ and select the final section.

Example 6-3 illustrates the two procedures described here for selecting a W section that has different effective lengths in the x and y directions.

Example 6-3

Select the lightest available W12 section, using both the LRFD and ASD methods for the following conditions: $F_y = 50$ ksi, $P_D = 250$ k, $P_L = 400$ k, $K_xL_x = 26$ ft and $K_yL_y = 13$ ft.

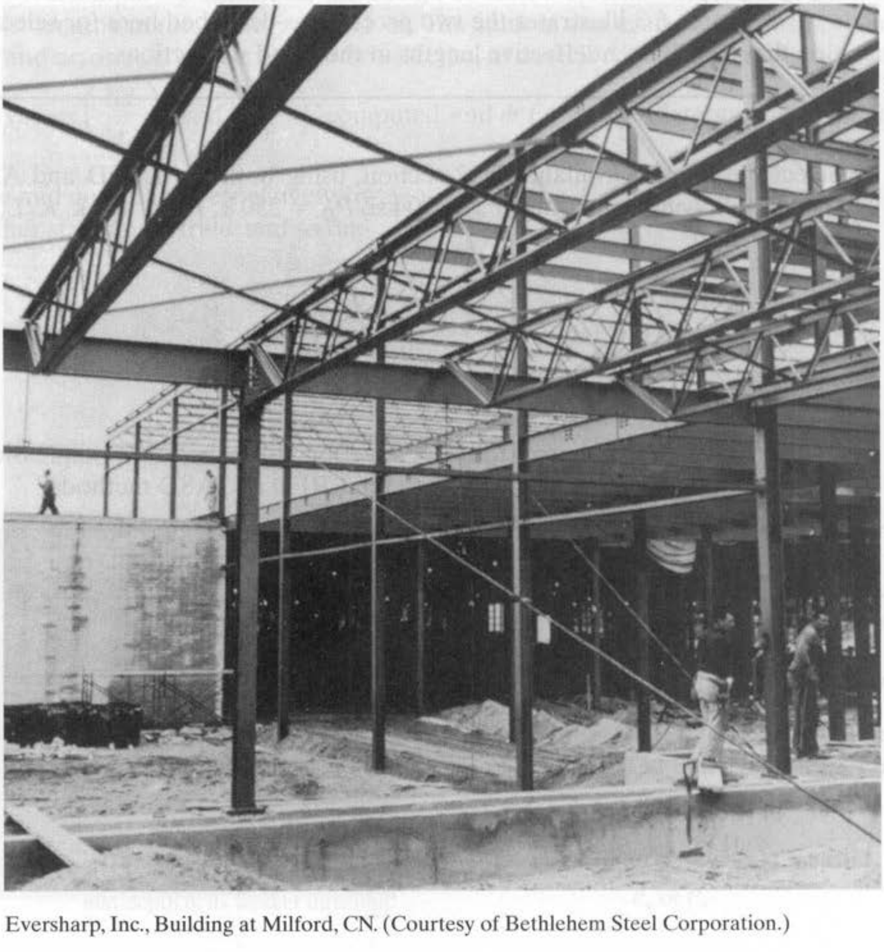
(a) By trial and error
(b) Using AISC tables

Solution

(a) Using trial and error to select a section, using the LRFD expressions, and then checking the section with both the LRFD and ASD methods

LRFD	ASD
$P_u = (1.2)(250\text{ k}) + (1.6)(400\text{ k}) = 940\text{ k}$	$P = 250\text{ k} + 400\text{ k} = 650\text{ k}$
Assume $\frac{KL}{r} = 50$	Assume $\frac{KL}{r} = 50$
Using $F_y = 50$ ksi steel $\phi_c F_{cr} = 37.5$ ksi (AISC Table 4-22)	Using $F_y = 50$ ksi steel $\frac{F_{cr}}{\Omega_c} = 24.9$ ksi (AISC Table 4-22)
$A\text{ Reqd} = \frac{940\text{ k}}{37.5\text{ ksi}} = 25.07\text{ in}^2$	$A\text{ Reqd} = \frac{650\text{ k}}{24.9\text{ ksi}} = 26.10\text{ in}^2$

Try W12 × 87 ($A = 25.6\text{ in}^2$, $r_x = 5.38\text{ in}$, $r_y = 3.07\text{ in}$)	Try W12 × 87 ($A = 25.6\text{ in}^2$, $r_x = 5.38\text{ in}$, $r_y = 3.07\text{ in}$)
$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(26\text{ ft})}{5.38\text{ in}} = 57.99 \leftarrow \therefore \left(\frac{KL}{r}\right)_x$ controls	$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(26\text{ ft})}{5.38\text{ in}} = 57.99 \leftarrow \therefore \left(\frac{KL}{r}\right)_x$ controls
$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(13\text{ ft})}{3.07\text{ in}} = 50.81$	$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(13\text{ ft})}{3.07\text{ in}} = 50.81$
$\phi_c F_{cr} = 35.2\text{ ksi}$ (Table 4-22)	$\frac{F_{cr}}{\Omega_c} = 23.4\text{ ksi}$ (Table 4-22)
$\phi_c P_n = (35.2\text{ ksi})(25.6\text{ in}^2)$	$\frac{P_n}{\Omega_c} = (23.4\text{ ksi})(25.6\text{ in}^2)$
$= 901\text{ k} < 940\text{ k}$ **N.G.**	$= 599\text{ k} < 650\text{ k}$ **N.G.**



Eversharp, Inc., Building at Milford, CN. (Courtesy of Bethlehem Steel Corporation.)

A subsequent check of the next-larger W12 section, a W12 × 96, shows that it will work for both the LRFD and ASD procedures.

(b) Using AISC Tables. Assuming $K_y L_y$ controls

Enter Table 4-1 with $K_y L_y = 13$ ft, $F_y = 50$ ksi and $P_u = 940$ k

LRFD

$$\text{Try W12} \times 87 \left(\frac{r_x}{r_y} = 1.75\right); \phi P_n = 954 \text{ k}$$

$$\text{Equivalent } K_y L_y = \frac{K_x L_x}{\dfrac{r_x}{r_y}}$$

$$= \frac{26}{1.75} = 14.86 > K_y L_y \text{ of } 13 \text{ ft.} \;\therefore\; K_x L_x \text{ controls}$$

Use $K_y L_y = 14.86$ ft and reenter tables

LRFD	ASD
Use W12 × 96	**Use W12 × 96**
$\phi_c P_n = 994 \text{ k} > 940 \text{ k}$ **OK**	$\dfrac{P_n}{\Omega_c} = 662 \text{ k} > 650 \text{ k}$ **OK**

Note: Table 4-1 does not indicate that the W12 × 96 is a slender member for compression.

6.3 COLUMN SPLICES

For multistory buildings, column splices are desirably placed 4 feet above finished floors to permit the attachment of safety cables to the columns, as may be required at floor edges or openings. This offset also enables us to keep the splices from interfering with the beam and column connections.

Typical column splices are shown in Fig. 6.2. Many more, examples are shown in Table 14-3 of the AISC Manual. The column ends are usually milled so they can be placed firmly in contact with each other for purposes of load transfer. When the contact surfaces are milled, a large part of the axial compression (if not all) can be transferred through the contacting areas. It is obvious, however, that splice plates are necessary, even though full contact is made between the columns and only axial loads are involved. For instance, the two column sections need to be held together during erection and afterward. What is necessary to hold them together is decided primarily on the basis of the experience and judgment of the designer. Splice plates are even more necessary when consideration is given to the shears and moments existing in practical columns subjected to off-center loads, lateral forces, moments, and so on.

There is, obviously, a great deal of difference between tension splices and compression splices. In tension splices, all load has to be transferred through the splice, whereas in splices for compression members, a large part of the load can be transferred directly in bearing between the columns. The splice material is then needed to transfer only the remaining part of the load.

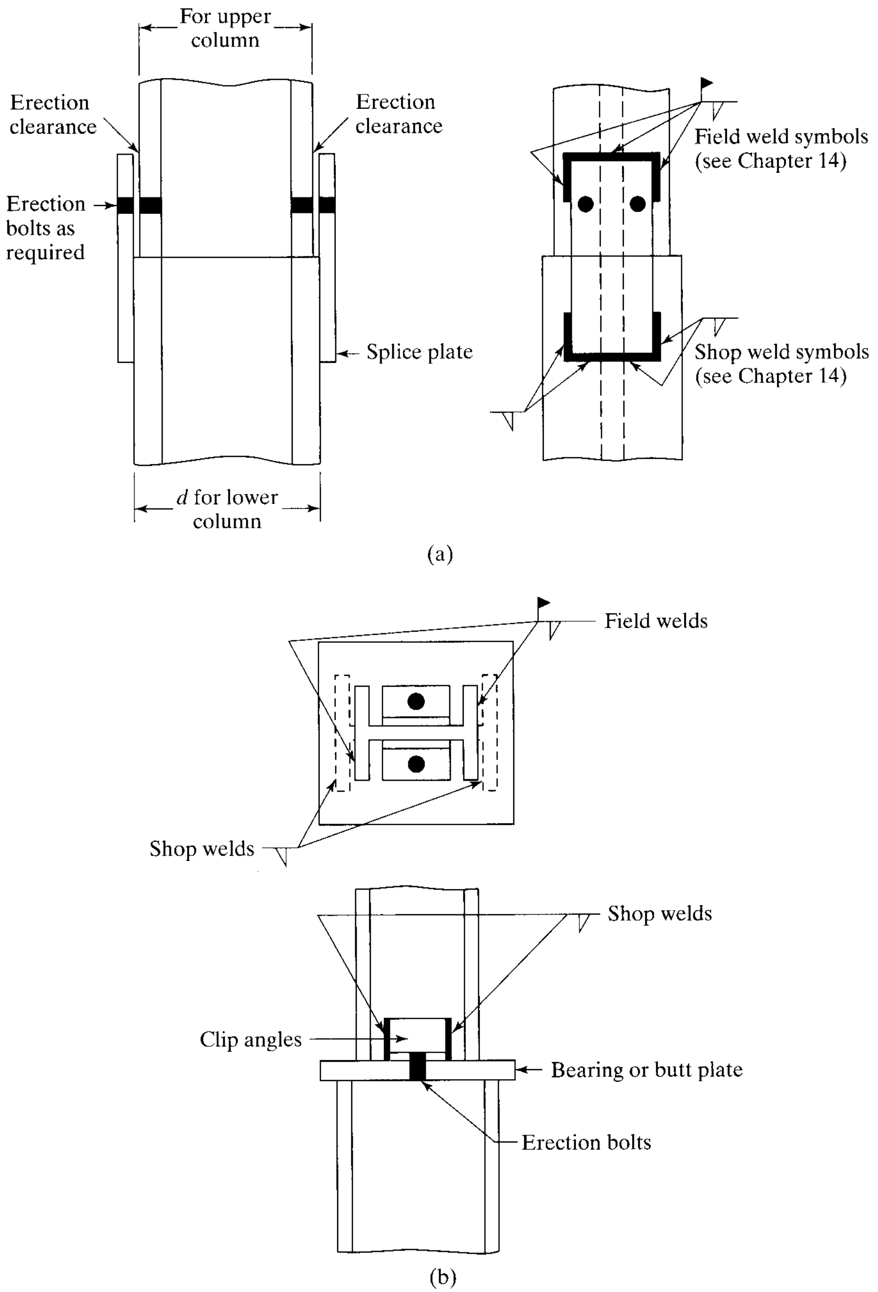


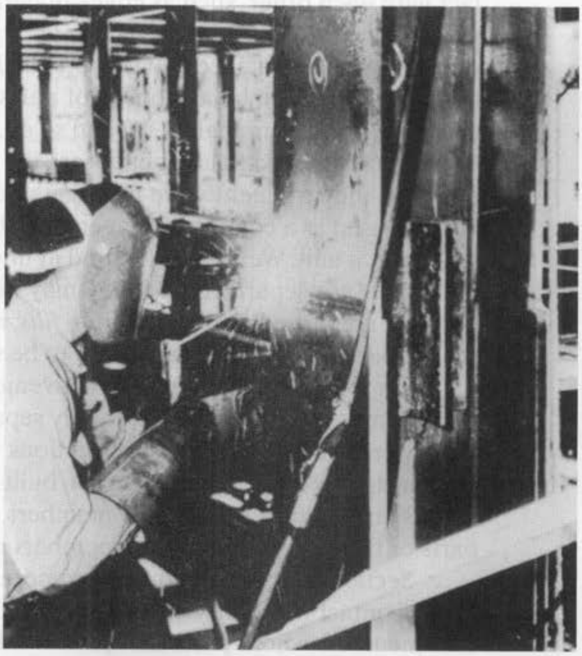
FIGURE 6.2
Column splices. (a) Columns from same W series with total depths close to each other ($d_{\text{lower}} < 2$ in greater than d_{upper}). (b) Columns from different W series.

The amount of load to be carried by the splice plates is difficult to estimate. Should the column ends not be milled, the plates should be designed to carry 100 percent of the load. When the surfaces are milled and axial loads only are involved, the amount of load to be carried by the plates might be estimated to be from 25 to 50 percent of the total load. If bending is involved, perhaps 50 to 75 percent of the total load may have to be carried by the splice material.

The bridge specifications spell out very carefully splice requirements for compression members, but the AISC Specification does not. A few general requirements are given in Section J1.4 of the AISC Specification.

Figure 6.2(a) shows a splice that may be used for columns with substantially the same nominal depths. The student will notice in the AISC Manual that the W shapes of a given nominal size are divided into groups rolled with the same set of rolls. Because of the fixed dimensions of each set of rolls, the clear distances between the flanges are constant for each shape in that group, although their total depths may vary greatly. For instance, the inside distance for each of the 28 shapes (running from the W14 × 61 to the W14 × 730) is approximately 12.60 in, although their total depths vary from 13.9 in to 22.4 in. (Notice that the T values, which are the distances between the web toes of the fillets, are all 10 in for the W14 × 90 through the W14 × 730.)

It is most economical to employ the simple splices of Fig. 6.2(a). This can easily be accomplished by using one series of shapes for as many stories of a building as possible. For instance, we may select a particular W14 column section for the top story or top two stories of a building and then keep selecting heavier and heavier W14s for that column as we come down in the building. We may also switch to higher-strength steel columns as we move down in the building, thus enabling us to stay with the same W series for even more



Welded column splice for Colorado State Building, Denver, CO. (Courtesy of Lincoln Electric Company.)

floors. It will be necessary to use filler plates between the splice plates and the upper column if the upper column has a total depth significantly less than that of the lower column.

Figure 6.2(b) shows a type of splice that can be used for columns of equal or different nominal depths. For this type of splice, the butt plate is shop-welded to the lower column, and the clip angles used for field erection are shop-welded to the upper column. In the field, the erection bolts shown are installed, and the upper column is field-welded to the butt plate. The horizontal welds on this plate resist shears and moments in the columns.

Sometimes, splices are applied to all four sides of columns. The web splices are bolted in place in the field and field-welded to the column webs. The flange splices are shop-welded to the lower column and field-welded to the top column. The web plates may be referred to as *shear plates* and the flange plates as *moment plates*.

For multistory buildings, the columns may be fabricated for one or more stories. Theoretically, column sizes can be changed at each floor level so that the lightest total column weight is used. The splices needed at each floor will be quite expensive, however, and as a result it is usually more economical to use the same column sizes for at least two stories, even though the total steel weight will be higher. Seldom are the same sizes used for as many as three stories, because three-story columns are so difficult to erect. The two-story heights work out very well most of the time.

6.4 BUILT-UP COLUMNS

As previously described in Section 5.3, compression members may be constructed with two or more shapes built up into a single member. They may consist of parts in contact with each other, such as cover-plated sections; or they may consist of parts in near contact with each other, such as pairs of angles that may be separated from each other by a small distance equal to the thickness of the end connections or gusset plates between them. They may consist of parts that are spread well apart, such as pairs of channels or four angles, and so on.

Two-angle sections probably are the most common type of built-up member. (For example, they frequently are used as the members of light trusses.) When a pair of angles are used as a compression member, they need to be fastened together so that they will act as a unit. Welds may be used at intervals (with a spacer bar between the parts if the angles are separated), or they may be connected with *stitch bolts*. When the connections are bolted, washers, or *ring fills*, are placed between the parts to keep them at the proper spacing if the angles are to be separated.

For long columns, it may be convenient to use built-up sections where the parts of the columns are spread out or widely separated from each other. Before heavy W sections were made available, such sections were very commonly used in both buildings and bridges. Today, these types of built-up columns are commonly used for crane booms and for the compression members of various kinds of towers. The widely spaced parts of these types of built-up members must be carefully laced or tied together.

Sections 6.5 and 6.6 concern compression members that are built up from parts in direct contact (or nearly so) with each other. Section 6.7 addresses built-up compression members whose parts are spread widely apart.

6.5 BUILT-UP COLUMNS WITH COMPONENTS IN CONTACT WITH EACH OTHER

Should a column consist of two equal-size plates, as shown in Fig. 6.3, and should those plates not be connected together, each plate will act as a separate column, and each will resist approximately half of the total column load. In other words, the total moment of inertia of the column will equal two times the moment of inertia of one plate. The two "columns" will act the same and have equal deformations, as shown in part (b) of the figure.

Should the two plates be connected together sufficiently to prevent slippage on each other, as shown in Fig. 6.4, they will act as a unit. Their moment of inertia may be computed for the whole built-up section as shown in the figure, and will be four times as large as it was for the column of Fig. 6.3, where slipping between the plates was possible. The reader should also notice that the plates of the column in Fig. 6.4 will deform different amounts as the column bends laterally.

Should the plates be connected in a few places, it would appear that the strength of the resulting column would be somewhere in between the two cases just described.

Reference to Fig. 6.3(b) shows that the greatest displacement between the two plates tends to occur at the ends and the least displacement tends to occur at mid-depth. As a result, connections placed at column ends that will prevent slipping between the parts have the greatest strengthening effect, while those placed at mid-depth have the least effect.

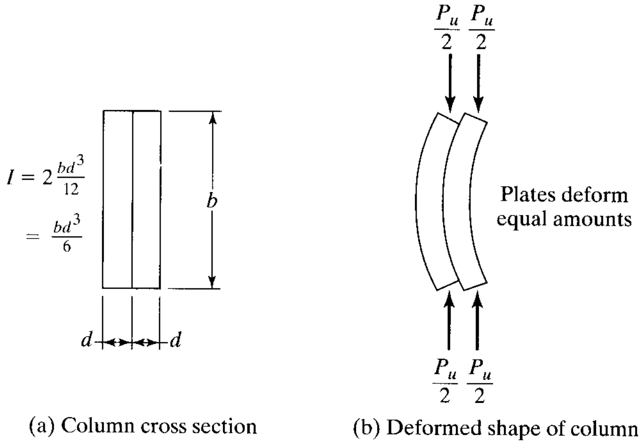


FIGURE 6.3
Column consisting of two plates not connected to each other.

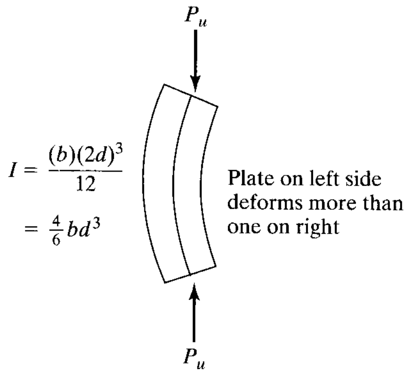


FIGURE 6.4
Column consisting of two plates fully connected to each other.

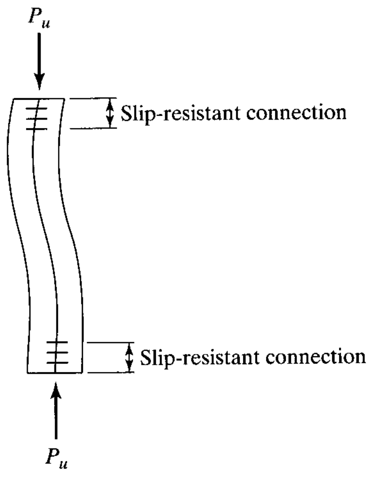


FIGURE 6.5
Column consisting of two plates connected at its ends only.

Should the plates be fastened together at their ends with slip-resistant connectors, those ends will deform together and the column will take the shape shown in Fig. 6.5. As the plates are held together at the ends, the column will bend in an S shape, as shown in the figure.

If the column were to bend in the S shape shown, its K factor would theoretically equal 0.5 and its KL/r value would be the same as the one for the continuously connected column of Fig. 6.4.[1]

$$\frac{KL}{r} \text{ for the column of Fig. 6.4} = \frac{(1)(L)}{\sqrt{\frac{4}{6}bd^3/2bd}} = 1.732L$$

$$\frac{KL}{r} \text{ for the end-fastened column of Fig. 6.5} = \frac{(0.5)(L)}{\sqrt{\frac{1}{6}bd^3/2bd}} = 1.732L$$

Thus, the design stresses are equal for the two cases, and the columns would carry the same loads. This is true for the particular case described here, but is not applicable for the common case where the parts of Fig. 6.5 begin to separate.

6.6 CONNECTION REQUIREMENTS FOR BUILT-UP COLUMNS WHOSE COMPONENTS ARE IN CONTACT WITH EACH OTHER

Several requirements concerning built-up columns are presented in AISC Specification E6. When such columns consist of different components that are in contact with each other and that are bearing on base plates or milled surfaces, they must be connected at their ends with bolts or welds. If welds are used, the weld lengths must at least equal the maximum width of the member. If bolts are used, they may not be spaced longitudinally more than four diameters on center, and the connection must extend for a distance at least equal to $1\frac{1}{2}$ times the maximum width of the member.

[1] J. A. Yura, *Elements for Teaching Load and Resistance Factor Design* (Chicago: AISC, July 1987), pp. 17–19.

The AISC Specification also requires the use of welded or bolted connections between the components of the column end as described in the last paragraph. These connections must be sufficient to provide for the transfer of calculated stresses. If it is desired to have a close fit over the entire faying surfaces between the components, it may be necessary to place the connectors even closer than is required for shear transfer.

When the component of a built-up column consists of an outside plate, the AISC Specification provides specific maximum spacings for fastening. If intermittent welds are used along the edges of the components, or if bolts are provided along all gage lines at each section, their maximum spacing may not be greater than the thickness of the thinner outside plate times $0.75\sqrt{E/F_y}$, nor be greater than 12 in. Should the fasteners be staggered, the maximum spacing along each gage line shall not be greater than the thickness of the thinner outside plate times $1.12\sqrt{E/F_y}$, nor be greater than 18 in (AISC Specification Section E6.2).

In Chapter 12, high-strength bolts are referred to as being *snug-tight* or *slip-critical*. Snug-tight bolts are those that are tightened until all the plies of a connection are in firm contact with each other. This usually means the tightness obtained by the full manual effort of a worker with a spud wrench, or the tightness obtained after a few impacts with a pneumatic wrench.

Slip-critical bolts are tightened much more firmly than are snug-tight bolts. They are tightened until their bodies, or shanks, have very high tensile stresses (approaching the lower bound of their yield stress). Such bolts clamp the fastened parts of a connection so tightly together between the bolt and nut heads that loads are resisted by friction, and slippage is nil. (We will see in Chapter 12 that where slippage is potentially a problem, slip-critical bolts should be used. For example, they should be used if the working or service loads cause a large number of stress changes resulting in a possible fatigue situation in the bolts.)

In the discussion that follows, the letter a represents the distance between connectors, and r_i is the least radius of gyration of an individual component of the column. If compression members consisting of two or more shapes are used, they must be connected together at intervals such that the effective slenderness ratio Ka/r_i of each of the component shapes between the connectors is not larger than 3/4 times the governing or controlling slenderness ratio of the whole built-up member (AISC Commentary E6.1). The end connections must be made with welds or slip-critical bolts with clean mill scale, or blasted, cleaned faying surfaces, with Class A or B faying surfaces. (These surfaces are described in Section J3.8 of the AISC Specification.)

The design strength of compression members built up from two or more shapes in contact with each other is determined with the usual applicable AISC Sections E3, E4 or E7, with one exception. Should the column tend to buckle in such a manner that relative deformations in the different parts cause shear forces in the connectors between the parts, it is necessary to modify the KL/r value for that axis of buckling. This modification is required by Section E6 of the AISC Specification.

Reference is made here to the cover-plated column of Fig. 6.6. If this section tends to buckle about its y axis, the connectors between the W shape and the plates are not subjected to any calculated load. If, on the other hand, it tends to buckle about its x axis, the connectors are subjected to shearing forces. The flanges of the W section and the cover plates will have different stresses and thus different deformations. (In this

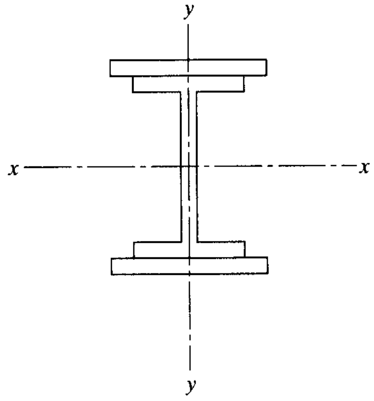


FIGURE 6.6

case, the cover plates and the W flanges to which they are attached bend in the same manner, and thus theoretically no shear or slippage occurs between them.) The result will be shear in the connection between these parts, and $(KL/r)_x$ will have to be modified by AISC Equations E6-1, E6-2a or E6-2b, as described next. (Equation E6-1 is based upon test results that supposedly account for shear deformations in the connectors. Equations E6-2a and E6-2b are based upon theory and was checked by means of tests.)

a. For intermediate connectors that are snug-tight bolted,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \qquad \text{(AISC Equation E6-1)}$$

It is important to remember that the design strength of a built-up column will be reduced if the spacing of connectors is such that one of the components of the column can buckle before the whole column buckles.

b. For intermediate connectors that are welded or have pretensioned bolts, as required for slip-critical joints,

when $\dfrac{a}{r_i} \leq 40$

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_o \qquad \text{(AISC Equation E6-2a)}$$

when $\dfrac{a}{r_i} > 40$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \qquad \text{(AISC Equation E6-2b)}$$

In these two equations,

$\left(\frac{KL}{r}\right)_o$ = column slenderness ratio of the whole built-up member acting as a unit in the buckling direction

$\left(\frac{KL}{r}\right)_m$ = modified slenderness ratio of built-up member because of shear

a = distance between connectors, in

r_i = minimum radius of gyration of individual component, in

K_i = 0.50 for angles back-to-back

= 0.75 for channels back-to-back

= 0.86 for all other cases

For the case in which the column tends to buckle about an axis such as to cause shear in the connection between the column parts, it will be necessary to compute a modified slenderness ratio $(KL/r)_m$ for that axis and to check to see whether that value will cause a change in the design strength of the member. If it does, it may be necessary to revise sizes and repeat the steps just described.

AISC Equation E6-1 is used to compute the modified slenderness ratio $(KL/r)_m$ about the major axis to find out whether it is greater than the slenderness ratio about the minor axis. If it is, that value should be used for determining the design strength of the member.

Section E6 of the AISC Commentary states that, on the basis of judgment and experience, the longitudinal spacing of the connectors for built-up compression members must be such that the slenderness ratios of the individual parts of the members do not exceed three-fourths of the slenderness ratio of the entire member.

Example 6-4 illustrates the design of a column consisting of a W section with cover plates bolted to its flanges, as shown in Fig. 6.7. Even though snug-tight bolts are used for this column, you should realize that AISC Specification E6 states that the end bolts must be pretensioned with Class A or B faying surfaces or the ends must be welded. This is required so that the parts of the built-up section will not slip with respect to each other and thus will act as a unit in resisting loads. (As a practical note, the typical steel company required to tighten the end bolts to a slip-critical condition will probably just go ahead and tighten them all to that condition.)

As this type of built-up section is not shown in the column tables of the AISC Manual, it is necessary to use a trial-and-error design procedure. An effective slenderness ratio is assumed. Then, $\phi_c F_{cr}$ or F_{cr}/Ω_c for that slenderness ratio is determined and divided into the column design load to estimate the column area required. The area of the W section is subtracted from the estimated total area to obtain the estimated cover plate area. Cover plate sizes are then selected to provide the required estimated area.

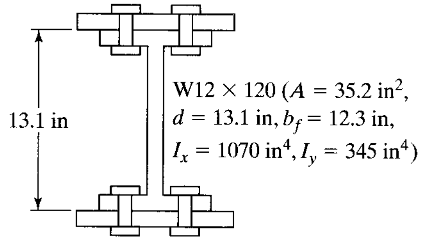


FIGURE 6.7
W section used as column with cover plates.

Example 6-4

You are to design a column for $P_D = 750$ k and $P_L = 1000$ k, using $F_y = 50$ ksi and $KL = 14$ ft. A W12 × 120 (for which $\phi_c P_n = 1290$ k and $P_n/\Omega_c = 856$ k from AISC Manual, Table 4-1) is on hand. Design cover plates to be snug-tight bolted at 6-in spacings to the W section, as shown in Fig. 6.7, to enable the column to support the required load.

Solution

LRFD	ASD
$P_a = (1.2)(750) + (1.6)(1000) = 2500$ k	$P_a = 750 + 1000 = 1750$ k

$$\text{Assume } \frac{KL}{r} = 50$$

$$\phi_c F_{cr} = 37.50 \text{ ksi from AISC Table 4-22}$$

$$A \text{ reqd} = \frac{2500 \text{ k}}{37.50 \text{ ksi}} = 66.67 \text{ in}^2$$

$$-A \text{ of W12} \times 120 = -35.30$$

$$\text{Estimated } A \text{ of 2 plates} = 31.37 \text{ in}^2 \text{ or } 15.69 \text{ in}^2 \text{ each}$$

Try one PL1 × 16 each flange

$$A = 35.20 + (2)(1)(16) = 67.20 \text{ in}^2$$

$$I_x = 1070 + (2)(16)\left(\frac{13.1 + 1.00}{2}\right)^2 = 2660 \text{ in}^4$$

$$r_x = \sqrt{\frac{2660}{67.20}} = 6.29 \text{ in}$$

$$\left(\frac{KL}{r}\right)_x = \frac{(12 \text{ in/ft})(14 \text{ ft})}{6.29 \text{ in}} = 26.71$$

$$I_y = 345 + (2)\left(\frac{1}{12}\right)(1)(16)^3 = 1027.7 \text{ in}^4$$

$$r_y = \sqrt{\frac{1027.7}{67.20}} = 3.91 \text{ in}$$

$$\left(\frac{KL}{r}\right)_y = \frac{(12 \text{ in/ft})(14 \text{ ft})}{3.91 \text{ in}} = 42.97 \leftarrow$$

Computing the modified slenderness ratio yields

$$r_i = \sqrt{\frac{I}{A}} = \sqrt{\frac{\left(\frac{1}{12}\right)(16)(1)^3}{(1)(16)}} = 0.289 \text{ in}$$

$$\frac{a}{r_i} = \frac{6 \text{ in}}{0.289 \text{ in}} = 20.76$$

$$\left(\frac{KL}{r}\right)_x = \sqrt{\left(\frac{KL}{r}\right)_0^2 + \left(\frac{a}{r_i}\right)^2} = \sqrt{(26.71)^2 + (20.76)^2} \qquad \text{(AISC Equation E6-1)}$$

$$= 33.83 < 42.97 \therefore \text{ does not control}$$

Checking the slenderness ratio of the plates, we have

$$\frac{k_a}{r_i} = 20.76 < \frac{3}{4}\left(\frac{KL}{r}\right)_y = \left(\frac{3}{4}\right)(42.97) = 32.23$$

For $\left(\frac{KL}{r}\right)_y = 42.97.$

LRFD	ASD
$\phi_c F_{cr} = 39.31$ ksi from Table 4-22, $F_y = 50$ ksi	$\frac{F_{cr}}{\Omega_c} = 26.2$ ksi from Table 4-22, $F_y = 50$ ksi
$\phi_c P_n = (39.31)(67.30) = 2646 \text{ k} > 2500 \text{ k}$	$\frac{P_n}{\Omega_c} = (26.2)(67.30) = 1763 \text{ k} > 1750 \text{ k}$

Use W12 × 120 with one cover plate 1 × 16 each flange, $F_y = 50$ ksi. (Note: Many other plate sizes could have been selected.)

6.7 BUILT-UP COLUMNS WITH COMPONENTS NOT IN CONTACT WITH EACH OTHER

Example 6-5 presents the design of a member built up from two channels that are not in contact with each other. The parts of such members need to be connected or laced together across their open sides. The design of lacing is discussed immediately after this example and is illustrated in Example 6-6.

Example 6-5

Select a pair of 12-in standard channels for the column shown in Fig. 6.8, using $F_y = 50$ ksi. For connection purposes, the back-to-back distance of the channels is to be 12 in. $P_D = 100$ k and $P_L = 300$ k. Consider both LRFD and ASD procedures.

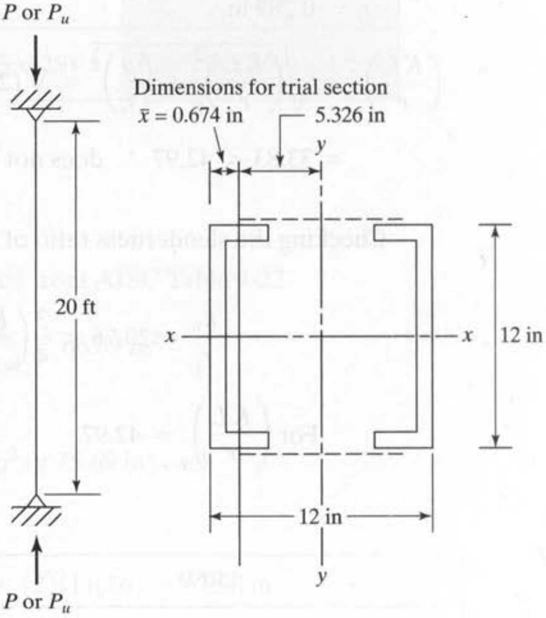


FIGURE 6.8
Column built up from two channels.

Solution

LRFD	ASD
$P_u = (1.2)(100) + (1.6)(300) = 600$ k	$P_a = 100 + 300 = 400$ k

$$\text{Assume } \frac{KL}{r} = 50$$

$$\phi_c F_{cr} = 37.50 \text{ ksi, from Table 4-22 } (F_y = 50 \text{ ksi steel})$$

$$A \text{ reqd} = \frac{600 \text{ k}}{37.50 \text{ ksi}} = 16.00 \text{ in}^2$$

Try 2C12 × 30s. (For each channel, $A = 8.81 \text{ in}^2$, $I_x = 162 \text{ in}^4$, $I_y = 5.12 \text{ in}^4$, $\bar{x} = 0.674 \text{ in.}$)

$$I_x = (2)(162) = 324 \text{ in}^4$$

$$I_y = (2)(5.12) + (2)(8.81)(5.326)^2 = 510 \text{ in}^4$$

$$r_x = \sqrt{\frac{324}{(2)(8.81)}} = 4.29 \text{ in controls}$$

$$KL = (1.0)(20 \text{ ft}) = 20 \text{ ft}$$

$$\frac{KL}{r} = \frac{(12 \text{ in/ft})(20 \text{ ft})}{4.29 \text{ in}} = 55.94$$

LRFD	ASD
$\phi_c F_{cr} = 35.82$ ksi (AISC Table 4-22) $F_y = 50$ ksi	$\frac{F_{cr}}{\Omega_c} = 23.81$ ksi (AISC Table 4-22) $F_y = 50$ ksi
$\phi_c P_n = (35.82)(2 \times 8.81) = 631 \text{ k} > 600 \text{ k}$ **OK**	$\frac{P_n}{\Omega_c} = (23.81)(2 \times 8.81) = 419.5 \text{ k} > 400 \text{ k}$ **OK**

Checking width thickness ratios of channels ($d = 12.00$ in, $b_f = 3.17$ in, $t_f = 0.501$ in, $t_w = 0.510$ in, $k = 1\frac{1}{8}$ in)

Flanges

$$\frac{b}{t} = \frac{3.17}{0.501} = 6.33 < 0.56\sqrt{\frac{29{,}000}{50}} = 13.49 \quad \text{OK (Case 1, AISC Table B4-1a)}$$

Webs

$$\frac{h}{t_w} = \frac{12.00 - (2)(1.125)}{0.510} = 19.12 < 1.49\sqrt{\frac{29{,}000}{50}}$$

$$= 35.88 \text{ (Case 5, AISC Table B4-1a)}$$

∴ Nonslender member

Use 2C12 × 30s.

The open sides of compression members that are built up from plates or shapes may be connected together with continuous cover plates with perforated holes for access purposes, or they may be connected together with lacing and tie plates. (The consideration of lacing is important because of retrofit work where it is used particularly for channels.)

The purposes of the perforated cover plates and the lacing, or latticework, are to hold the various parts parallel and the correct distance apart, and to equalize the stress distribution between the various parts. The student will understand the necessity for lacing if he or she considers a built-up member consisting of several sections (such as the four-angle member of Fig. 5.2(i)) that supports a heavy compressive load. Each of the parts will tend to individually buckle laterally, unless they are tied together to act as a unit in supporting the load. In addition to lacing, it is necessary to have *tie plates* (also called *stay plates* or *batten plates*) as near the ends of the member as possible, and at intermediate points if the lacing is interrupted. Parts (a) and (b) of Fig. 6.9 show arrangements of tie plates and lacing. Other possibilities are shown in parts (c) and (d) of the same figure.

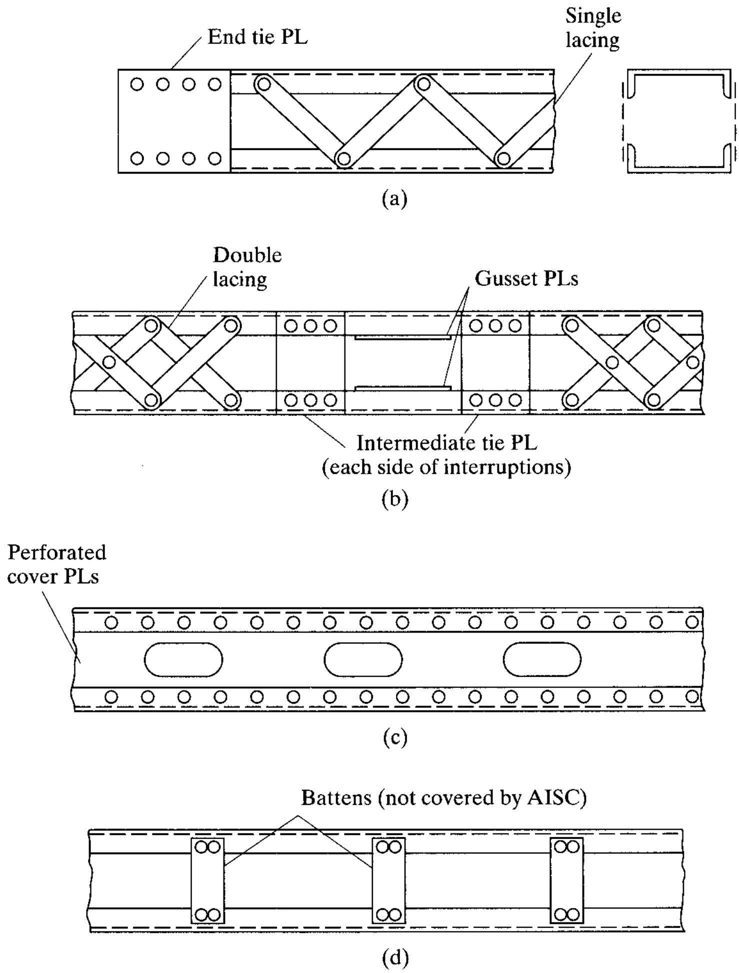


FIGURE 6.9
Lacing and perforated cover plates.

The failure of several structures in the past has been attributed to inadequate lacing of built-up compression members. Perhaps the best-known example was the failure of the Quebec Bridge in 1907. Following its collapse, the general opinion was that the latticework of the compression chords was too weak, and this resulted in failure.

If continuous cover plates perforated with access holes are used to tie the members together, the AISC Specification E6.2 states that (a) they must comply with the limiting width–thickness ratios specified for compression elements in Section B4.1 of the AISC Specification; (b) the ratio of the access hole length (in the direction of stress) to the hole width may not exceed 2; (c) the clear distance between the holes in the direction of stress may not be less than the transverse distance between the nearest lines of connecting fasteners or welds; and (d) the periphery of the holes at all points must have a radius no less than $1\frac{1}{2}$ in. Stress concentrations and secondary bending stresses are usually neglected, but lateral shearing forces must be checked as they are for other types of latticework. (The unsupported width of such plates at access holes is assumed to contribute to the design strength $\phi_c P_n$ of the member if the conditions as to sizes, width–thickness ratios, etc., described in AISC Specification E6 are met.) Perforated cover plates are attractive to many designers because of several advantages they possess:

1. They are easily fabricated with modern gas cutting methods.
2. Some specifications permit the inclusion of their net areas in the effective section of the main members, provided that the holes are made in accordance with empirical requirements, which have been developed on the basis of extensive research.
3. Painting of the members is probably simplified, compared with painting of ordinary lacing bars.

Dimensions of tie plates and lacing are usually controlled by specifications. Section E6 of the AISC Specification states that tie plates shall have a thickness at least equal to one-fiftieth of the distance between the connection lines of welds or other fasteners.

Lacing may consist of flat bars, angles, channels, or other rolled sections. These pieces must be so spaced that the individual parts being connected will not have *L/r* values between connections which exceed three-fourths of the governing value for the entire built-up member. (The governing value is *KL/r* for the whole built-up section.) Lacing is assumed to be subjected to a shearing force normal to the member, equal to not less than 2 percent of the compression design strength $\phi_c P_n$ of the member. The AISC column formulas are used to design the lacing in the usual manner. Slenderness ratios are limited to 140 for single lacing and 200 for double lacing. Double lacing or single lacing made with angles should preferably be used if the distance between connection lines is greater than 15 in.

Example 6-6 illustrates the design of lacing and end tie plates for the built-up column of Example 6-5. Bridge specifications are somewhat different in their lacing requirements from the AISC, but the design procedures are much the same.

Example 6-6

Using the AISC Specification and 36 ksi steel, design bolted single lacing for the column of Example 6-5. Reference is made to Fig. 6.10. Assume that 3/4-in bolts are used.

Solution. Distance between lines of bolts is 8.5 in $<$ 15 in; therefore, single lacing is OK.

Assume that lacing bars are inclined at 60° with axis of member. Length of channels between lacing connections is 8.5/cos 30° $=$ 9.8 in, and KL/r of 1 channel between connections is 9.8/0.762 $=$ 12.9 $<$ 3/4 $\times$ 55.94, which is KL/r of main member previously determined in Example 6-5. Only the LRFD solution is shown.

Force on lacing bar:

V_u = 0.02 times available design compressive strength of member (from Example 6-5), $\phi P_n = 631$ k

$$V_u = (0.02)(631\text{ k}) = 12.62\text{ k}$$

$$\frac{1}{2}V_u = 6.31\text{ k} = \text{shearing force on each plane of lacing}$$

Force in bar (with reference to bar dimensions in Fig. 6.10):

$$\left(\frac{9.8}{8.5}\right)(6.31) = 7.28\text{ k}$$

Properties of flat bar:

$$I = \tfrac{1}{12}bt^3$$

$$A = bt$$

$$r = \sqrt{\frac{\frac{1}{12}bt^3}{bt}} = 0.289t$$

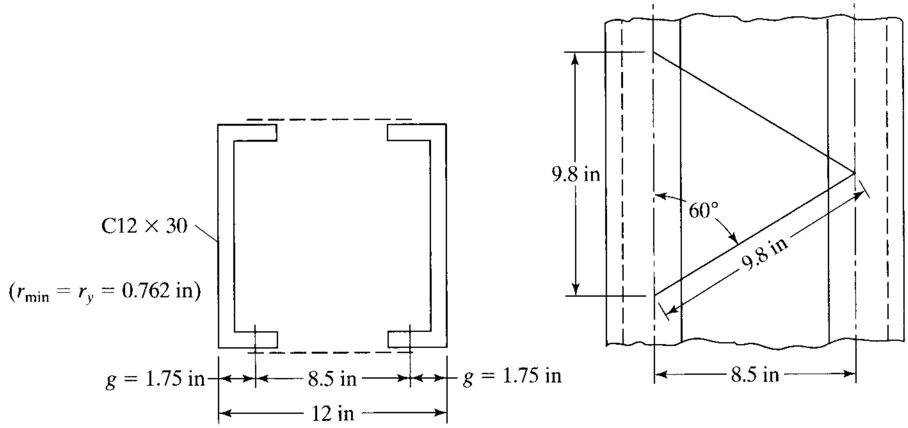


FIGURE 6.10
Two-channel column section with lacing.

Design of bar:

$$\text{Assume } \frac{KL}{r} = \text{maximum value of 140 and } K = 1.0$$

$$\frac{(1.0)\,9.8 \text{ in}}{0.289t \text{ in}} = 140$$

$$t = 0.242 \text{ in (try } \tfrac{1}{4}\text{-in flat bar)}$$

$$\frac{KL}{r} = \frac{(1.0)\,9.8 \text{ in}}{(0.289)(0.250 \text{ in})} = 136$$

$$\phi_c F_{cr} = 12.2 \text{ ksi, from AISC Table 4-22, } F_y = 36 \text{ ksi}$$

$$\text{Area reqd} = \frac{7.28 \text{ k}}{12.2 \text{ ksi}} = 0.597 \text{ in}^2 \left(2.39 \times \tfrac{1}{4} \text{ needed}\right) \text{ Use } \tfrac{1}{4} \times 2\tfrac{1}{2} \text{ bar}$$

Minimum edge distance if $\frac{3}{4}$-in bolt used $= 1\frac{1}{4}$ in AISC Table J3.4

$\therefore$ Minimum length of bar $= 9.8 + (2)\left(1\frac{1}{4}\right) = 12.3$ in, say, 14 in

Use $\frac{1}{4} \times 2\frac{1}{2} \times$ 1-ft 2-in bars, $F_y = 36$ ksi.

Design of end tie plates:

Minimum length $= 8.5$ in

Minimum $t = \left(\frac{1}{50}\right)(8.5) = 0.17$ in

Minimum width $= 8.5 + (2)\left(1\frac{1}{4}\right) = 11$ in

Use $\frac{3}{16} \times 8\frac{1}{2} \times$ 0-ft 12-in end tie plates.

6.8 SINGLE-ANGLE COMPRESSION MEMBERS

You will note that we have not discussed the design of single-angle compression members up to this point. The AISC has long been concerned about the problems involved in loading such members fairly concentrically. It can be done rather well if the ends of the angles are milled and if the loads are applied through bearing plates. In practice, however, single-angle columns are often used in such a manner that rather large eccentricities of load applications are present. The sad result is that it is somewhat easy to greatly underdesign such members.

In Section E5 of the AISC Specification, a special specification is provided for the design of single-angle compression members. Though this specification includes information for tensile, shear, compressive, flexural, and combined loadings, the present discussion is concerned only with the compression case.

In Table 4-11 of the Manual, the calculated strengths of concentrically loaded single angles are provided. The values shown are based on KL/r_z values. So often, though, single angles are connected at their ends by only one leg—an eccentric loading situation. Section E5 of the AISC Specification presents a method for handling such situations where eccentric compression loads are introduced to angles through one connected leg.

The writers of the specification assumed that connections to one leg of an angle provided considerable resistance to bending about the y axis of that angle or that is perpendicular to the connected leg. As a result, the angle was assumed to bend and buckle about the x axis of the member; thus, attention is given to the L/r_x ratio. To account for the eccentricity of loading larger L/r_x ratios for various situations are provided by AISC Equations E5-1 to E5-4 and are to be used for obtaining design stresses.

The first two of the equations apply to equal leg angles and to unequal leg angles connected through their longer legs. Further, the angles are to be used as members in two-dimensional, or planar, trusses, where the other members joining the ones in question are connected at their ends on the same side of gusset plates or on the same side of truss chord members. For these conditions, the following increased slenderness ratios are to be used for strength calculations:

When $L/r_x \leq 80$:

$$\frac{KL}{r} = 72 + 0.75\frac{L}{r_x} \qquad \text{(AISC Equation E5-1)}$$

When $L/r_x > 80$:

$$\frac{KL}{r} = 32 + 1.25\frac{L}{r_x} \leq 200 \qquad \text{(AISC Equation E5-2)}$$

Some variations are given in the specification for unequal leg angles if the leg length ratios are < 1.7 and if the shorter leg is connected. In addition, AISC Equations E5-3 and E5-4 are provided for cases where the single angles are members of box or space trusses. Example 6-7, which follows, illustrates the use of the first of these equations.

Example 6-7

Determine the $\phi_c P_n$ and P_n/Ω_c values for a 10-ft-long A36 angle $8 \times 8 \times 3/4$ with simple end connections, used in a planar truss. The other web members meeting at the ends of this member are connected on the same side of the gusset plates.

Solution

Using an L8 $\times$ 8 $\times$ $\frac{3}{4}$$(A = 11.5 \text{ in}^2, r_x = 2.46 \text{ in})$

$$\frac{L}{r_x} = \frac{(12 \text{ in/ft})(10 \text{ ft})}{2.46 \text{ in}} = 48.78 < 80$$

$$\therefore \quad \frac{KL}{r} = 72 + 0.75\frac{L}{r_x} \qquad \text{(AISC Equation E5-1)}$$

$$= 72 + (0.75)(48.78) = 108.6$$

LRFD	ASD
$\phi_c F_{cr}$ from AISC Table 4-22, $F_y = 36$ ksi	$\frac{F_{cr}}{\Omega_c}$ from AISC Table 4-22, $F_y = 36$ ksi
$= 17.38$ ksi	$= 11.58$ ksi
$\phi_c P_n = \phi_c F_{cr} A_g$	$\frac{P_n}{\Omega_c} = \frac{F_{cr}}{\Omega_c} A_g$
$= (17.38 \text{ ksi})(11.5 \text{ in}^2) = \mathbf{199.9 \text{ k}}$	$= (11.58 \text{ ksi})(11.5 \text{ in}^2) = \mathbf{133.2 \text{ k}}$

Table 4-12 in the Manual provides design values for angles eccentrically loaded in Example 6-7, because the AISC used some different conditions in solving the problem. The values in the table are the lower-bound axial compression strengths of single angles, with no consideration of end restraint. When the conditions described in AISC Specification E5 are not met, this table can be used. The values given were computed, considering biaxial bending about the principal axis of the angle, with the load applied at a given eccentricity, as described on page 4-8 in the Manual.

6.9 SECTIONS CONTAINING SLENDER ELEMENTS

A good many of the square and rectangular HSS have slender walls. The reader will be happy to learn that the effects of slender elements on column strengths have been included in the tables of Part 4 of the Manual. As a result, the designer rarely has to go through the calculations to take into account those items. Several equations are presented in AISC Section E7.1 and E7.2 for the consideration of members containing slender elements. Included are sections with stiffened elements and sections with unstiffened elements. Example 6-8 makes use of the appropriate equations for computing the strengths of such members.

The values obtained in the example problem to follow (Example 6-8) are smaller than the values given in Table 4-3 in the Manual for rectangular HSS sections, because f was assumed to equal F_y, whereas in the proper equations it is actually equal to $\frac{P_n}{A_e}$. This conservative assumption will cause our hand calculations for design strengths, when slender elements are present, to be on the low, or safe, side. To use the correct value of f, it is necessary to use an iterative solution—a procedure for which the computer is ideally suited. In any case, the values calculated by hand, shown as follows, will be several percent on the conservative or low side.

Example 6-8

Determine the axial compressive design strength $\phi_c P_n$ and the allowable design strength $\frac{P_n}{\Omega_c}$ of a 24-ft HSS $14 \times 10 \times \frac{1}{4}$ column section. The base of the column is considered to be fixed, while the upper end is assumed to be pinned. $F_y = 46$ ksi.

Solution

Using an HSS $14 \times 10 \times \frac{1}{4}$ ($A = 10.8$ in^2, $r_x = 5.35$ in, $r_y = 4.14$ in, $t_w = 0.233$ in,

$\frac{b}{t} = 39.9$ and $\frac{h}{t} = 57.1$) All values from Table 1-11

Limiting width–thickness ratio (AISC Table B4.1a, Case 6)

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000}{46}} = 35.15 < \frac{b}{t} \text{ and } \frac{h}{t}$$

$\therefore$ **Both the 10-in walls and the 14-in walls are slender elements.**

Computing b and h and noting that in the absence of the exact fillet dimensions, the AISC recommends that the widths and depths between the web toes of the fillets equal the outside dimensions $-3t_w$.

$$b = 10.00 - (3)(0.233) = 9.30 \text{ in}$$

$$h = 14.00 - (3)(0.233) = 13.30 \text{ in}$$

Computing the effective widths and heights of the walls by using AISC Equation E7-18 yields

$$b_e = 1.92\,t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b$$

b_e for the 10-in wall

$$= (1.92)(0.233)\sqrt{\frac{29{,}000}{46}}\left[1 - \frac{0.38}{39.9}\sqrt{\frac{29{,}000}{46}}\right]$$

$$= 8.55 \text{ in} < 9.30 \text{ in}$$

Length that cannot be used $= 9.30 - 8.55 = 0.75$ in

b_e for the 14-in wall

$$= (1.92)(0.233)\sqrt{\frac{29{,}000}{46}}\left[1 - \frac{0.38}{57.1}\sqrt{\frac{29{,}000}{46}}\right]$$

$$= 9.36 \text{ in} < 13.30 \text{ in}$$

Length that cannot be used = 13.30 − 9.36 = 3.94 in

$$A_e = 10.8 - (2)(0.233)(0.75) - (2)(0.233)(3.94) = 8.61\text{ in}^2$$

$$Q = Q_a = \frac{A_e}{A_g} = \frac{8.61}{10.8} = 0.7972$$

Determine equation to use for F_{cr}

$$\left(\frac{KL}{r}\right)_y = \frac{(0.8)(12\text{ in/ft} \times 24\text{ ft})}{4.14\text{ in}} = 55.65$$

$$< 4.71\sqrt{\frac{29{,}000}{(0.7972)(46)}} = 132.45$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(55.65)^2} = 92.42\text{ ksi} \qquad \text{(AISC Equation E3-4)}$$

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y \qquad \text{(AISC Equation E7-2)}$$

$$= 0.7972\left[0.658^{\frac{0.7972 \times 46}{92.42}}\right]46$$

$$= 31.06\text{ ksi}$$

$$P_n = (10.8)(31.06) = 335.4\text{ k} \qquad \text{(AISC Equation E7-1)}$$

LRFD $\phi_c = 0.90$	ASD $\Omega_c = 1.67$
$\phi_c P_n = (0.90)(335.4\text{ k}) = \mathbf{301.9\text{ k}}$	$\frac{P_n}{\Omega_c} = \frac{335.4\text{ k}}{1.67} = \mathbf{200.8\text{ k}}$

6.10 FLEXURAL-TORSIONAL BUCKLING OF COMPRESSION MEMBERS

Usually symmetrical members such as W sections are used as columns. Torsion will not occur in such sections if the lines of action of the lateral loads pass through their shear centers. The *shear center* is that point in the cross section of a member through which the resultant of the transverse loads must pass so that no torsion will occur. The calculations necessary to locate shear centers are presented in Chapter 10. The shear centers of the commonly used doubly symmetrical sections occur at their centroids. This is not necessarily the case for other sections such as channels and angles. Shear center locations for several types of sections are shown in Fig. 6.11. Also shown in the figure are the coordinates x_0 and y_0 for the shear center of each section with respect to its centroid. These values are needed to solve the flexural-torsional formulas, presented later in this section.

Even though loads pass through shear centers, torsional buckling still may occur. If you load any section through its shear center, no torsion will occur, but one still

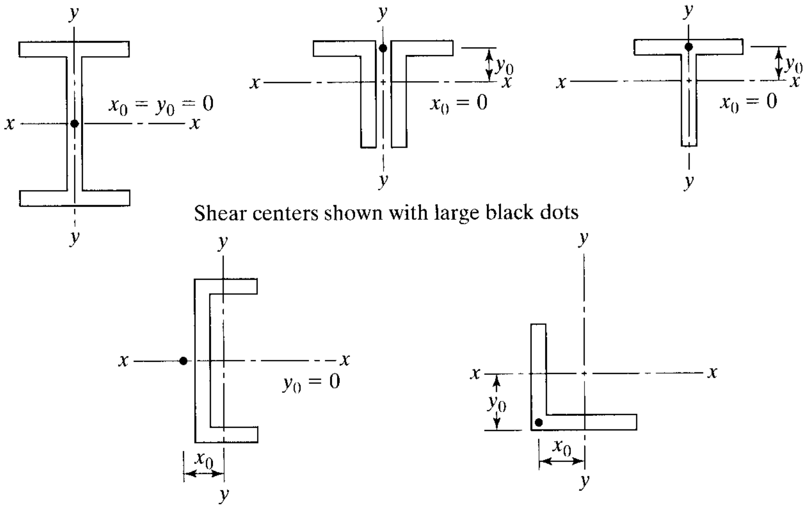


FIGURE 6.11
Shear center locations for some common column sections.

computes torsional buckling strength for these members—that is, buckling load does not depend on the nature of the axial or transverse loading; rather, it depends on the cross-section properties, column length, and support conditions.

Axially loaded compression members can theoretically fail in four different fashions: by local buckling of elements that form the cross section, by flexural buckling, by torsional buckling, or by flexural-torsional buckling.

Flexural buckling (also called *Euler buckling* when elastic behavior occurs) is the situation considered up to this point in our column discussions where we have computed slenderness ratios for the principal column axes and determined $\phi_c F_{cr}$ for the highest ratios so obtained. Doubly symmetrical column members (such as W sections) are subject only to local buckling, flexural buckling, and torsional buckling.

Because torsional buckling can be very complex, it is very desirable to prevent its occurrence. This may be done by careful arrangements of the members and by providing bracing to prevent lateral movement and twisting. If sufficient end supports and intermediate lateral bracing are provided, flexural buckling will always control over torsional buckling. The column design strengths given in the AISC column tables for W, M, S, tube, and pipe sections are based on flexural buckling.

Open sections such as Ws, Ms, and channels have little torsional strength, but box beams have a great deal. Thus, if a torsional situation is encountered, it may be well to use box sections or to make box sections out of W sections by adding welded side plates ( ). Another way in which torsional problems can be reduced is to shorten the lengths of members that are subject to torsion.

For a singly symmetrical section such as a tee or double angle, Euler buckling may occur about the x or y axis. For equal-leg single angles, Euler buckling may occur about the z axis. For all these sections, flexural-torsional buckling is definitely a possibility and may control. (It will always control for unequal-leg single-angle columns.) The values given in the AISC column load tables for double-angle and structural tee sections were computed for buckling about the weaker of the x or y axis and for flexural-torsional buckling.

The average designer does not consider the torsional buckling of symmetrical shapes or the flexural-torsional buckling of unsymmetrical shapes. The feeling is that these conditions don't control the critical column loads, or at least don't affect them very much. This assumption can be far from the truth. Should we have unsymmetrical columns or even symmetrical columns made up of thin plates, however, we will find that torsional buckling or flexural-torsional buckling may significantly reduce column capacities.

Section E4 of the AISC Specification is concerned with the torsional or flexural-torsional buckling of steel columns. Part (b) of the section presents a general method for handling the problem which is applicable to all shapes. Part (a) of the same section is a modification of the procedure presented in part (b) and is applicable specifically to double angles and tee sections used as columns.

The general approach of part (b) is presented here. The procedure involves using AISC Equation E4-9 for the determination of the elastic torsional buckling stress F_{ez} (which is analogous to the Euler buckling stress). After this value is determined, it is used in the appropriate one of AISC Equations E4-4, E4-5, and E4-6 to obtain F_e, the torsional or flexural-torsional elastic buckling stress. The critical stress, F_{cr}, is then determined according to Equation E3-2 or E3-3.

The procedure for part (a), which is for double-angle and tee-shaped compression members, is presented next. The critical stress, F_{cr}, is determined using AISC Equation E4-2. In this equation, F_{cry} is taken as F_{cr} from Equation E3-2 or E3-3, and F_{crz} and H are obtained from Equations E4-3 and E4-10 respectively.

For either procedure, the nominal compressive strength P_n, for the limit states of torsional and flexural-torsional buckling, is determined using AISC Equation E4-1. In the equation, the previously calculated F_{cr} is multiplied by A_g.

Usually it is unnecessary to consider torsional buckling for doubly symmetrical shapes. Furthermore we rarely have to consider the topic for shapes without an axis of symmetry because we probably will not use such members as columns. On some occasions, however, we probably will select sections with one axis of symmetry as columns and for them lateral torsional buckling must be considered.

A numerical example (Example 6-9) for flexural torsional buckling is presented in this section for a WT section used as a column. For such a shape the x axis will be subject to flexural buckling while there may be flexural buckling about the y axis (the axis of symmetry) as well as lateral torsional buckling.

There are four steps involved in solving this type of problem with the AISC Specification. These follow:

1. Determine the flexural buckling strength of the member for its x axis using AISC Equations E3-4, E3-2 or E3-3, as applicable, and E3-1.
2. Determine the flexural buckling strength of the member for its y axis using AISC Equations E3-4, E3-2 or E3-3, as applicable, and E3-1.
3. Determine the flexural torsional buckling strength of the member for its y axis using AISC Equations E4-11, E4-9, E4-10, E4-5, E3-2 or E3-3, as applicable, and E4-1.
4. Select the smallest P_n value determined in the preceding three steps.

Example 6-9

Determine the nominal compressive strength, P_n, of a WT10.5 × 66 with $KL_x = 25$ ft and $KL_y = KL_z = 20$ ft. Use the general approach given in part (b) of AISC Specification E4(b) and A992 steel.

Solution

Using a WT10.5 × 66 ($A = 19.4$ in^2, $t_f = 1.04$ in, $I_x = 181$ in^4, $r_x = 3.06$ in, $I_y = 166$ in^4, $r_y = 2.93$ in, $\bar{y} = 2.33$ in, $J = 5.62$ in^4, $C_w = 23.4$ in^6, and $G = 11{,}200$ ksi)

(1) Determine the flexural buckling strength for the x axis

$$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(25\text{ ft})}{3.06\text{ in}} = 98.04$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)_x^2} = \frac{(\pi^2)(29{,}000)}{(98.04)^2} = 29.78\text{ ksi} \qquad \text{(AISC Equation E3-4)}$$

$$\left(\frac{KL}{r}\right)_x = 98.04 < 4.71\sqrt{\frac{29{,}000}{50}} = 113.43$$

$$\therefore F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad \text{(AISC Equation E3-2)}$$

$$= \left[0.658^{\frac{50}{29.78}}\right]50 = 24.76\text{ ksi}$$

The nominal strength P_n for flexural buckling about x-axis is

$$P_n = F_{cr}A_g = (24.76)(19.4) = \mathbf{480.3\ k} \qquad \text{(AISC Equation E3-1)}$$

(2) Determine the flexural buckling strength for the y axis

$$\left(\frac{KL}{r}\right)_y = \frac{12\text{ in/ft} \times 20\text{ ft}}{2.93\text{ in}} = 81.91$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)_y^2} = \frac{(\pi^2)(29{,}000)}{(81.91)^2} = 42.66\text{ ksi} \qquad \text{(AISC Equation E3-4)}$$

$$\left(\frac{KL}{r}\right)_y = 81.91 < 4.71\sqrt{\frac{29{,}000}{50}} = 113.43$$

$$\therefore F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad \text{(AISC Equation E3-2)}$$

$$= \left[0.658^{\frac{50}{42.66}}\right]50 = 30.61 \text{ ksi}$$

The nominal strength P_n for flexural buckling about y axis is

$$P_n = F_{cr}A_g = (30.61)(19.4) = \mathbf{593.8\ k} \qquad \text{(AISC Equation E3-1)}$$

(3) Determine the flexural–torsional buckling strength of the member about the y axis.

Note that x_0 and y_0 are the coordinates of the shear center with respect to the centroid of the section. Here x_0 equals 0 because the shear center of the WT is located on the y–y axis, while y_0 is $\bar{y} - \frac{t_f}{2}$ since the shear center is located at the intersection of the web and flange center lines as shown in Fig. 6.11.

$$x_0 = 0$$

$$y_0 = \bar{y} - \frac{t_f}{2} = 2.33 - \frac{1.04}{2} = 1.81$$

$\bar{r}_0$ = the polar radius of gyration about the shear center

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} \qquad \text{(AISC Equation E4-11)}$$

$$= 0^2 + 1.81^2 + \frac{181 + 166}{19.4} = 21.16 \text{ in}^2$$

$$F_{ez} = \left(\frac{\pi^2 EC_w}{(K_zL)^2} + GJ\right)\frac{1}{A_g\bar{r}_0^2} \qquad \text{(AISC Equation E4-9)}$$

$$= \left[\frac{\pi^2(29{,}000)(23.4)}{(12 \times 20)^2} + 11{,}200(5.62)\right]\frac{1}{19.4(21.16)} = 153.62 \text{ ksi}$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} \qquad \text{(AISC Equation E4-10)}$$

$$= 1 - \frac{0^2 + 1.81^2}{21.16} = 0.84517$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad \text{(AISC Equation E4-5)}$$

$$= \left(\frac{42.66 + 153.62}{(2)(0.84517)}\right)\left[1 - \sqrt{1 - \frac{(4)(42.66)(153.62)(0.84517)}{(42.66 + 153.62)^2}}\right]$$

$$= 40.42 \text{ ksi}$$

Now we need to go back to either AISC Equation E3-2 or E3-3 to determine the compressive strength of the member.

$$40.42 \text{ ksi} > \frac{F_y}{2.25} = 22.22 \text{ ksi}$$

$$\therefore \text{ Must use AISC Equation E3-2.}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left(0.658^{\frac{50}{40.42}}\right)50 = 29.79 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = (29.79)(19.4) = \mathbf{577.9\ k} \qquad \text{(AISC Equation E4-1)}$$

(4) The smallest one of the P_n values determined in (a), (b), and (c) is our nominal load.

$$P_n = \mathbf{480.3\ k}$$

LRFD $\phi_c = 0.90$	ASD $\Omega_c = 1.67$
$\phi_c P_n = (0.90)(480.3 \text{ k}) = 432.3$	$\frac{P_n}{\Omega_c} = \frac{480.3 \text{ k}}{1.67} = 287.6 \text{ k}$

6.11 PROBLEMS FOR SOLUTION

All of the columns for these problems are assumed to be in frames that are braced against sidesway. Each problem is solved by both the LRFD and the ASD procedures.

6-1 to 6-3. *Use a trial-and-error procedure in which a KL/r value is estimated, the stresses $\phi_c F_{cr}$ and F_{cr}/Ω_c determined from AISC Table 4-22, required areas calculated and trial sections selected–and checked and revised if necessary.*

6-1. Select the lightest available W10 section to support the axial compression loads $P_D = 100$ k and $P_L = 160$ k if $KL = 15$ ft and A992, Grade 50 steel is used. (*Ans.* W10 × 49, LRFD and ASD)

6-2. Select the lightest available W8 section to support the axial loads $P_D = 75$ k and $P_L = 125$ k if $KL = 13$ ft and $F_y = 50$ ksi.

6-3. Repeat Prob. 6-2 if $F_y = 36$ ksi. (*Ans.* W8 × 48, LRFD and ASD)

6-4 to 6-17. *Take advantage of all available column tables in the AISC Manual, particularly those in Part 4.*

6-4. Repeat Prob. 6-1.

6-5. Repeat Prob. 6-2. (*Ans.* W8 × 35, LRFD and ASD)

6-6. Repeat Prob. 6-1 if $P_D = 150$ k and $P_L = 200$ k.

6-7. Several building columns are to be designed, using A992 steel and the AISC Specification. Select the lightest available W sections and state the LRFD design strength, $\phi_c P_n$, and the ASD allowable strength, P_n/Ω_c, for these columns that are described as follows:

a. $P_D = 170$ k, $P_L = 80$ k, $L = 16$ ft, pinned end supports, W8.
(*Ans.* W8 × 48, LRFD $\phi_c P_n = 340\text{ k} > P_u = 332$ k; W8 × 58, ASD $P_n/\Omega_c = 278\text{ k} > P_a = 250$ k)

b. $P_D = 100$ k, $P_L = 220$ k, $L = 25$ ft, fixed at bottom, pinned at top, W14.
(*Ans.* W14 × 74, LRFD $\phi_c P_n = 495\text{ k} > P_u = 472$ k; W14 × 74, ASD $P_n/\Omega_c = 329\text{ k} > P_a = 320$ k)

c. $P_D = 120$ k, $P_L = 100$ k, $L = 25$ ft, fixed end supports, W12.
(*Ans.* W12 × 50, LRFD $\phi_c P_n = 319\text{ k} > P_u = 304$ k; W12 × 53, ASD $P_n/\Omega_c = 297\text{ k} > P_a = 220$ k)

d. $P_D = 250$ k, $P_L = 125$ k, $L = 18.5$ ft, pinned end supports, W14.
(*Ans.* W14 × 74, LRFD $\phi_c P_n = 546\text{ k} > P_u = 500$ k; W14 × 82, ASD $P_n/\Omega_c = 400\text{ k} > P_a = 375$ k)

6-8. Design a column with an effective length of 22 ft to support a dead load of 65 k, a live load of 110 k, and a wind load of 144 k. Select the lightest W12 of A992 steel.

6-9. A W10 section is to be selected to support the loads $P_D = 85$ k and $P_L = 140$ k. The member, which is to be 20 ft long, is fixed at the bottom and fixed against rotation but free to translate at the top. Use A992 steel. (*Ans.* W10 × 68, LRFD and ASD, $\phi_c P_n = 363$ k and $P_n/\Omega_c = 241$ k)

6-10. A W14 section is to be selected to support the loads $P_D = 500$ k and $P_L = 700$ k. The member is 24 ft long with pinned end supports and is laterally supported in the weak direction at the one-third points of the total column length. Use 50 ksi steel.

6-11. Repeat Prob. 6-10 if column length is 18 ft long and $P_D = 250$ k and $P_L = 350$ k. (*Ans.* W14 × 74, LRFD, $\phi_c P_n = 893$ k; W14 × 82, ASD, $P_n/\Omega_c = 655$ k)

6-12. A 28 ft long column is pinned at the top and fixed at the bottom, and has additional pinned support in the weak axis direction at a point 12 ft from the top. Assume the column is part of a braced frame. Axial gravity loads are $P_D = 220$ k and $P_L = 270$ k. Choose the lightest W12 column.

6-13. A 24 ft column in a braced frame building is to be built into a wall in such a manner that it will be continuously braced in its weak axis direction but not about its strong axis direction. If the member is to consist of 50 ksi and is assumed to have fixed ends, select the lightest satisfactory W10 section available using the AISC Specification. Loads are $P_D = 220$ k and $P_L = 370$ k. (*Ans.* W10 × 77 LRFD and ASD)

6-14. Repeat Prob. 6-13 if $P_D = 175$ k and $P_L = 130$ k. Select the lightest satisfactory W8 section available.

6-15. A W12 section of 50 ksi steel is to be selected to support the axial compressive loads of $P_D = 375$ k and $P_L = 535$ k. The member is 36 ft long, is to be pinned top and bottom and is to have lateral support at its one-quarter points, perpendicular to the y-axis (pinned). (*Ans.* W12 × 152 LRFD; W12 × 170 ASD)

6-16. Using the steels for which the column tables are provided in Part 4 of the Manual, select the lightest available rolled sections (W, HP, HSS Square, and HSS Round) that are adequate for the following conditions:

a. $P_D = 150$ k, $P_L = 225$ k, $L = 25$ ft, one end pinned and the other fixed

b. $P_D = 75$ k, $P_L = 225$ k, $L = 16$ ft, fixed ends

c. $P_D = 50$ k, $P_L = 150$ k, $L = 30$ ft, pinned ends

6-17. Assuming axial loads only, select W10 sections for the interior column of the laterally braced frame shown in the accompanying illustration. Use $F_y = 50$ ksi and the LRFD method only. A column splice will be provided just above point B;

therefore, select a column section for column AB and a second different column section for column BC and CD. Miscellaneous data: Concrete weighs 150 lb/ft^3. LL on roof = 30 psf. Roofing DL = 10 psf. LL on floors = 75 psf. Superimposed DL on floors = 12 psf. Partition load on floors = 15 psf. All joints are assumed to be pinned. Frames are 35 feet on center. (*Ans.* Column AB – W10 × 68, Column BC & CD – W10 × 39)

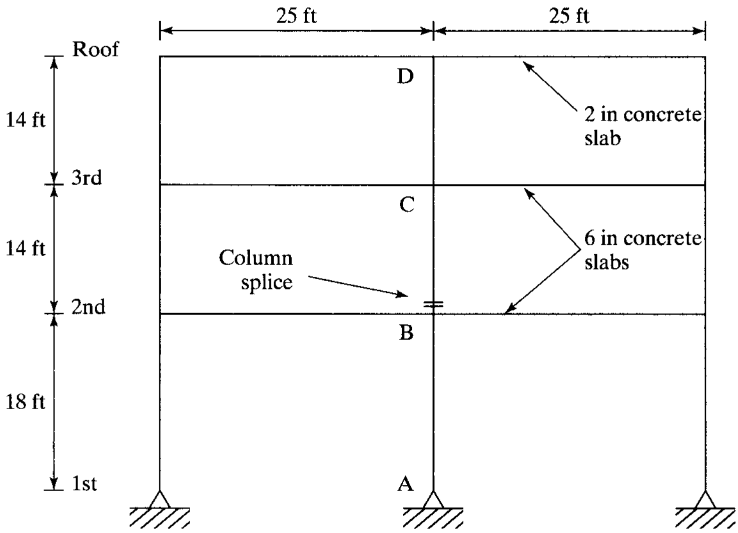


FIGURE P6-17

6-18. You are requested to design a column for $P_D = 225$ k and $P_L = 400$ k, using A992 steel with $KL = 16$ ft. A W14 × 68 is available but may not provide sufficient capacity. If it does not, cover plates may be added to the section to increase the W14's capacity. Design the cover plates to be 12 inch wide and welded to the flanges of the section to enable the column to support the required load (see Fig. P6-18). Determine the minimum plate thickness required, assuming the plates are available in 1/16th-in increments.

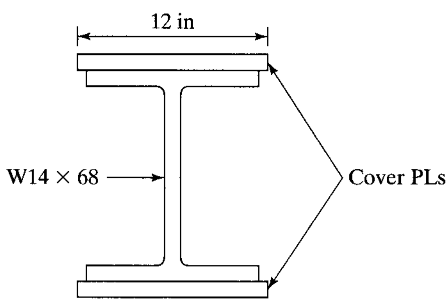


FIGURE P6-18

6-19. Determine the LRFD design strength and the ASD allowable strength of the section shown if snug-tight bolts 3 ft on center are used to connect the A36 angles. The two angles, $5 \times 3\frac{1}{2} \times \frac{1}{2}$, are oriented with the long legs back-to-back (2L $5 \times 3\frac{1}{2} \times \frac{1}{2}$

LLBB) and separated by 3/8 inch. The effective length, $(KL)_x = (KL)_y = 15$ ft. (*Ans.* 101.9 k LRFD; 67.8 k ASD)

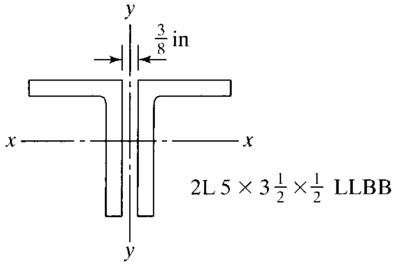


FIGURE P6-19

6-20. Repeat Prob. 6-19 if the angles are welded together with their long legs back-to-back at intervals of 5 ft.

6-21. Four $3 \times 3 \times \frac{1}{4}$ angles are used to form the member shown in the accompanying illustration. The member is 24 ft long, has pinned ends, and consists of A36 steel. Determine the LRFD design strength and the ASD allowable strength of the member. Design single lacing and end tie plates, assuming connections are made to the angles with $\frac{3}{4}$-in diameter bolts. (*Ans.* 159.1 k LRFD; 106.0 k ASD)

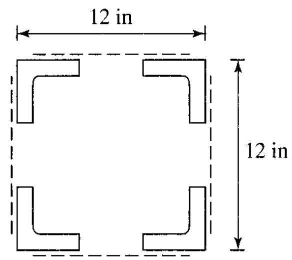


FIGURE P6-21

6-22. Select the lightest pair of C9 channels to support the loads $P_D = 50$ k and $P_L = 90$ k. The member is to be 20 ft long with both ends pinned and is to be arranged as shown in the accompanying illustration. Use A36 steel and design single lacing and end tie plates, assuming that $\frac{3}{4}$-in diameter bolts are to be used for connections. Assume that the bolts are located $1\frac{1}{4}$ in from the back of channels. Solve by LRFD and ASD procedures.

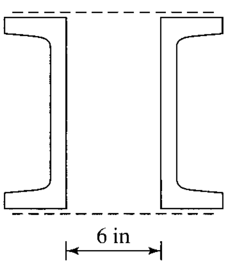


FIGURE P6-22

CHAPTER 7

Design of Axially Loaded Compression Members (Continued) and Column Base Plates

7.1 INTRODUCTION

In this chapter, the available axial strengths of columns used in unbraced steel frames are considered. These frames are also referred to as moment frames or frames with sidesway uninhibited. As the column ends may move laterally the columns must be able to resist both axial loads and bending moments. As a consequence, they are generally referred to as beam–columns. Such members are discussed in detail in Chapter 11 of the text.

The AISC Specification provides several methods to deal with the stability analysis and design of beam–columns. One is the *Direct Analysis Method* (DM) that is specified in Chapter C of the Specification. This approach uses factors required to more accurately determine forces and moments during the analysis phase and eliminates the requirement for calculating the effective length factor, K. This is due to the fact that the effective length of compression members, KL, is taken as the actual length, L, that is, K is taken equal to 1.0. A second method, the *Effective Length Method* (ELM), is given in Appendix 7 of the Specification. In this method, K is calculated using one of the procedures discussed in this chapter.

These two design methods will be discussed further in Chapter 11 of the text. In this chapter, the available strength of compression members, ΦP_n, will be determined in building frames calculating KL using the *Effective Length Method.*

7.2 FURTHER DISCUSSION OF EFFECTIVE LENGTHS

The subject of effective lengths was introduced in Chapter 5, and some suggested K factors were presented in Table 5.1. These factors were developed for columns with certain idealized conditions of end restraint, which may be very different from practical design conditions. The table values are usually quite satisfactory for preliminary designs and for situations in which sidesway is prevented by bracing. Should the columns be part of a continuous frame subject to sidesway, however, it would often be advantageous to make a more detailed analysis, as described in this section. To a lesser extent, this is also desirable for columns in frames braced against sidesway.

Perhaps a few explanatory remarks should be made at this point, defining sidesway as it pertains to effective lengths. For this discussion, sidesway refers to a type of buckling. In statically indeterminate structures, sidesway occurs where the frames deflect laterally due to the presence of lateral loads or unsymmetrical vertical loads, or where the frames themselves are unsymmetrical. Sidesway also occurs in columns whose ends can move transversely when they are loaded to the point that buckling occurs.

Should frames with diagonal bracing or rigid shear walls be used, the columns will be prevented from sidesway and provided with some rotational restraint at their ends. For these situations, pictured in Fig. 7.1, the K factors will fall somewhere between cases (a) and (d) of Table 5.1.

The AISC Specification Appendix 7 (7.2.3(a)) states that $K = 1.0$ should be used for columns in frames with sidesway inhibited, unless an analysis shows that a smaller value can be used. A specification like $K = 1.0$ is often quite conservative, and an analysis made as described herein may result in some savings.

The true effective length of a column is a property of the whole structure, of which the column is a part. In many existing buildings, it is probable that the masonry walls provide sufficient lateral support to prevent sidesway. When light curtain walls are used, however, as they often are in modern buildings, there is probably little resistance to sidesway. Sidesway is also present in tall buildings in appreciable amounts, unless a definite diagonal bracing system or shear walls are used. For these cases, it seems logical to assume that resistance to sidesway is primarily provided by the lateral stiffness of the frame alone.

Theoretical mathematical analyses may be used to determine effective lengths, but such procedures are typically too lengthy and perhaps too difficult for the average designer. The usual procedure is to consult either Table 5.1, interpolating between the idealized values as the designer feels is appropriate, or the alignment charts that are described in this section.

The most common method for obtaining effective lengths is to employ the charts shown in Fig. 7.2. They were developed by O. G. Julian and L. S. Lawrence, and frequently are referred to as the Jackson and Moreland charts, after the firm where Julian and Lawrence worked.[1,2] The charts were developed from a slope-deflection analysis

[1]O. G. Julian and L. S. Lawrence, "Notes on J and L Monograms for Determination of Effective Lengths" (1959). Unpublished.

[2]Structural Stability Research Council, *Guide to Stability Design Criteria for Metal Structures*, 4th ed. T. V. Galambos, ed. (New York: Wiley, 1988).

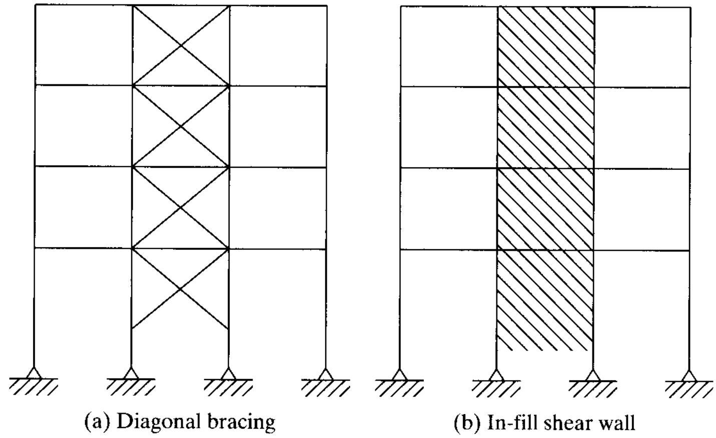


FIGURE 7.1
Sidesway inhibited.

of the frames that included the effect of column loads. One chart was developed for columns braced against sidesway and one for columns subject to sidesway. Their use enables the designer to obtain good K values without struggling through lengthy trial-and-error procedures with the buckling equations.

To use the alignment charts, it is necessary to have preliminary sizes for the girders and columns framing into the column in question before the K factor can be determined for that column. In other words, before the chart can be used, we have to either estimate some member sizes or carry out a preliminary design.

When we say sidesway is *inhibited*, we mean there is something present other than just columns and girders to prevent sidesway or the horizontal translation of the joints. That means we have a definite system of lateral bracing, or we have shear walls. If we say that sidesway is *uninhibited*, we are saying that resistance to horizontal translation is supplied only by the bending strength and stiffness of the girders and beams of the frame in question, with its continuous joints.

The resistance to rotation furnished by the beams and girders meeting at one end of a column is dependent on the rotational stiffnesses of those members. The moment needed to produce a unit rotation at one end of a member if the other end of the member is fixed is referred to as its *rotational stiffness*. From our structural analysis studies, this works out to be equal to $4EI/L$ for a homogeneous member of constant cross section. On the basis of the preceding, we can say that the rotational restraint at the end of a particular column is proportional to the ratio of the sum of the column stiffnesses to the girder stiffnesses meeting at that joint, or

$$G = \frac{\sum \frac{4EI}{L} \text{ for columns}}{\sum \frac{4EI}{L} \text{ for girders}} = \frac{\sum \frac{E_c I_c}{L_c}}{\sum \frac{E_g I_g}{L_g}}.$$

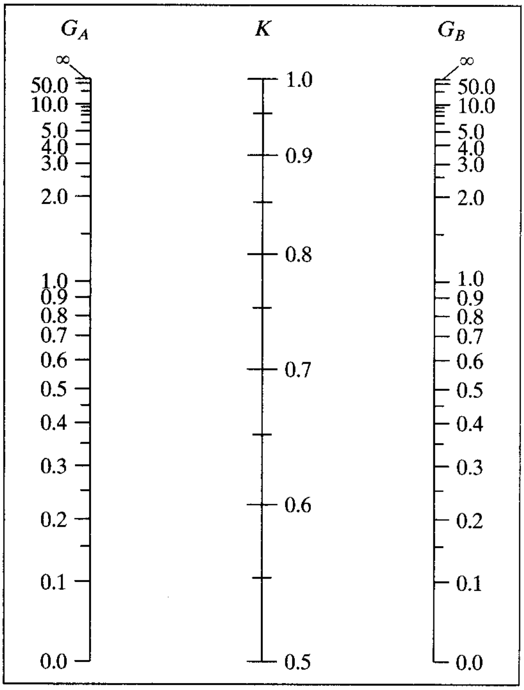
The subscripts A and B refer to the joints at the ends of the columns being considered. G is defined as:

$$G = \frac{\sum\left(\frac{E_c I_c}{L_c}\right)}{\sum\left(\frac{E_g I_g}{L_g}\right)} \quad \text{AISC Equation (C-A-7-2)}$$

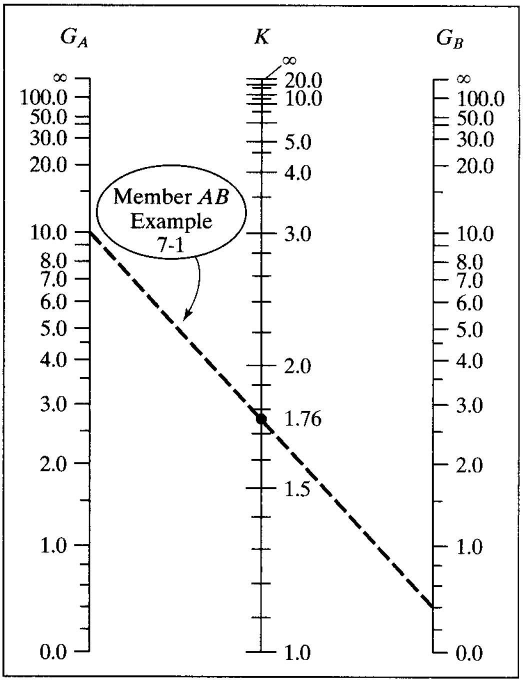
The symbol Σ indicates a summation of all members rigidly connected to that joint and located in the plane in which buckling of the column is being considered. E_c is the elastic modulus of the column, I_c is the moment of inertia of the column, and L_c is the unsupported length of the column. E_g is the elastic modulus of the girder, I_g is the moment of inertia of the girder, and L_g is the unsupported length of the girder or other restraining member. I_c and I_g are taken about axes perpendicular to the plane of buckling being considered. The alignment charts are valid for different materials if an appropriate effective rigidity, EI, is used in the calculation of G.

Adjustments for Columns with Differing End Conditions. For column ends supported by, but not rigidly connected to, a footing or foundation, G is theoretically infinity but unless designed as a true friction-free pin, may be taken as 10 for practical designs. If the column is rigidly attached to a properly designed footing, G may be taken as 1.0. Smaller values may be used if justified by analysis.

From American Institute of Steel Construction Specification, ANSI/AISC 360-10, Commentary to Appendix 7. Fig. C-A-7.1 and C-A-7.2, p. 16.1–512 and 16.1–513 (Chicago: AISC, 2010)



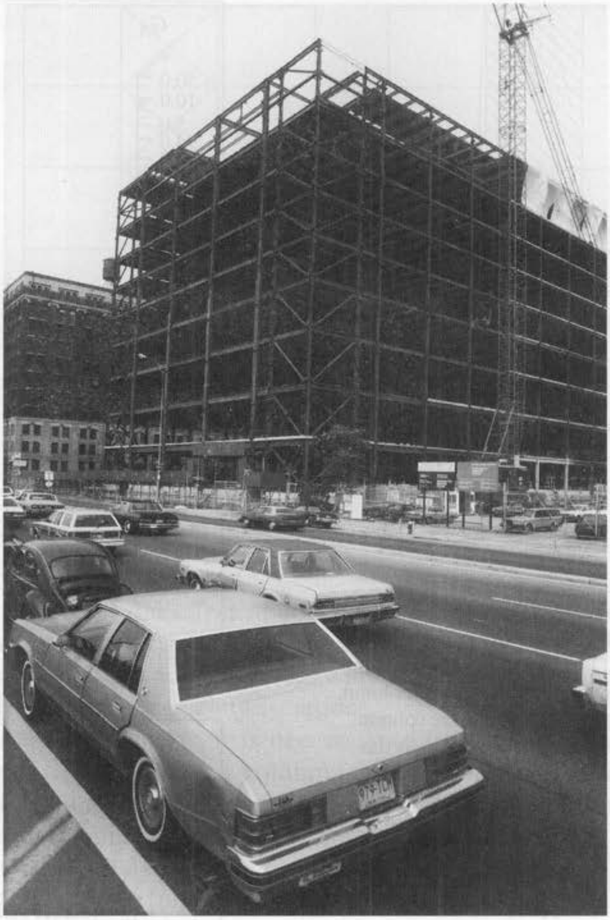
(a) Sidesway inhibited (Braced Frame)



(b) Sidesway uninhibited (Moment Frame)

FIGURE 7.2

Jackson and Moreland alignment charts for effective lengths of columns in continous frames.



Shearson Lehman/American Express Information Services
Center, New York City. (Courtesy of Owen Steel Company, Inc.)

In applying the charts, the G factors at the column bases are quite variable. It is recommended that the following two rules be applied to obtain their values:

1. For pinned columns, G is theoretically infinite, such as when a column is connected to a footing with a frictionless hinge. Since such a connection is not frictionless, it is recommended that G be made equal to 10 where such nonrigid supports are used.
2. For rigid connections of columns to footings, G theoretically approaches zero, but from a practical standpoint, a value of 1.0 is recommended, because no connections are perfectly rigid.

The determination of K factors for the columns of a steel frame by the alignment charts is illustrated in Examples 7-1 and 7-2. The following steps are taken:

1. Select the appropriate chart (sidesway inhibited or sidesway uninhibited).

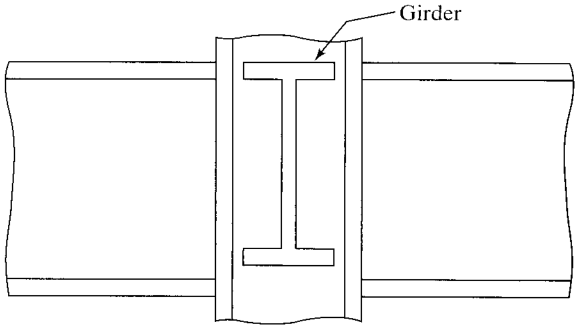


FIGURE 7.3

2. Compute G at each end of the column and label the values G_A and G_B, as desired.
3. Draw a straight line on the chart between the G_A and G_B values, and read K where the line hits the center K scale.

When G factors are being computed for a rigid frame structure (rigid in both directions), the torsional resistance of the perpendicular girders is generally neglected in the calculations. With reference to Fig. 7.3, it is assumed that we are calculating G for the joint shown for buckling in the plane of the paper. For such a case, the torsional resistance of the girder shown, which is perpendicular to the plane being considered, is probably neglected.

If the girders at a joint are very stiff (that is, they have very large EI/L values) the value of $G = \Sigma(E_c I_c/L_c)/\Sigma(E_g I_g/L_g)$ will approach zero and the K factors will be small. If G is very small, the column moments cannot rotate the joint very much; thus, the joint is close to a fixed-end situation. Usually, however, G is appreciably larger than zero, resulting in significantly larger values of K.

The effective lengths of each of the columns of a frame are estimated with the alignment charts in Example 7-1. (When sidesway is possible, it will be found that the effective lengths are always greater than the actual lengths, as is illustrated in this example. When frames are braced in such a manner that sidesway is not possible, K will be less than 1.0.) An initial design has provided preliminary sizes for each of the members in the frame. After the effective lengths are determined, each column can be redesigned. Should the sizes change appreciably, new effective lengths can be determined, the column designs repeated, and so on. Several tables are used in the solution of this example. These should be self-explanatory after the clear directions given on the alignment chart are examined.

7.3 FRAMES MEETING ALIGNMENT CHART ASSUMPTIONS

The Jackson and Moreland charts were developed on the basis of a certain set of assumptions, a complete list of which is given in Section 7.2 of the Commentary of Appendix 7 of the AISC Specification. Among these assumptions are the following:

1. The members are elastic, have constant cross sections, and are connected with rigid joints.

2. All columns buckle simultaneously.
3. For braced frames, the rotations at opposite ends of each beam are equal in magnitude, and each beam bends in single curvature.
4. For unbraced frames, the rotations at opposite ends of each beam are equal in magnitude, but each beam bends in double curvature.
5. Axial compression forces in the girders are neglibible.

The frame of Fig. 7.4 is assumed to meet all of the assumptions on which the alignment charts were developed. From the charts, the column effective length factors are determined, as shown in Example 7-1.

Example 7-1

Determine the effective length factor for each of the columns of the frame shown in Fig. 7.4 if the frame is not braced against sidesway. Use the alignment charts of Fig. 7.2(b).

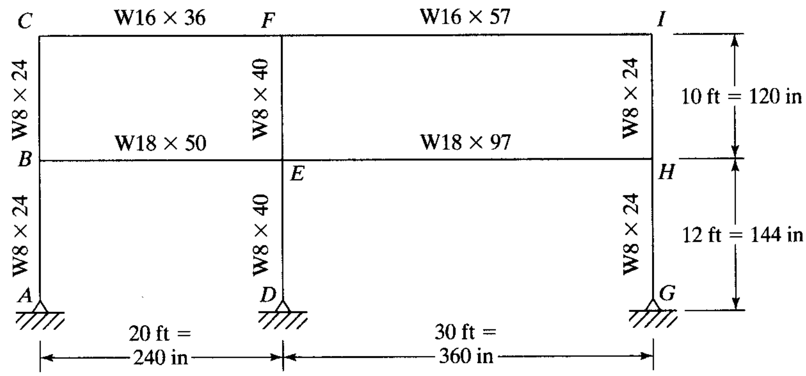


FIGURE 7.4

Solution. Stiffness factors: E is assumed to be 29,000 ksi for all members and is therefore neglected in the equation to calculate G.

	Member	Shape	I	L	I/L
Columns	AB	W8 × 24	82.7	144	0.574
	BC	W8 × 24	82.7	120	0.689
	DE	W8 × 40	146	144	1.014
	EF	W8 × 40	146	120	1.217
	GH	W8 × 24	82.7	144	0.574
	HI	W8 × 24	82.7	120	0.689
Girders	BE	W18 × 50	800	240	3.333
	CF	W16 × 36	448	240	1.867
	EH	W18 × 97	1750	360	4.861
	FI	W16 × 57	758	360	2.106

G factors for each joint:

Joint	$\Sigma(I_c/L_c)/\Sigma(I_g/L_g)$	G
A	Pinned Column, $G = 10$	10.0
B	$\dfrac{0.574 + 0.689}{3.333}$	0.379
C	$\dfrac{0.689}{1.867}$	0.369
D	Pinned Column, $G = 10$	10.0
E	$\dfrac{1.014 + 1.217}{(3.333 + 4.861)}$	0.272
F	$\dfrac{1.217}{(1.867 + 2.106)}$	0.306
G	Pinned Column, $G = 10$	10.0
H	$\dfrac{0.574 + 0.689}{4.861}$	0.260
I	$\dfrac{0.689}{2.106}$	0.327

Column K factors from chart [Fig. 7.2(b)]:

Column	G_A	G_B	K*
AB	10.0	0.379	1.76
BC	0.379	0.369	1.12
DE	10.0	0.272	1.74
EF	0.272	0.306	1.10
GH	10.0	0.260	1.73
HI	0.260	0.327	1.10

*It is a little difficult to read the charts to the three decimal places shown by the author. He has used a larger copy of Fig. 7.2 for his work. For all practical design purposes, the K values can be read to two places, which can easily be accomplished with this figure.

For most buildings, the values of K_x and K_y should be examined separately. The reason for such individual study lies in the different possible framing conditions in the two directions. Many multistory frames consist of rigid frames in one direction and conventionally connected frames with sway bracing in the other. In addition, the points of lateral support may often be entirely different in the two planes.

There is available a set of rather simple equations for computing effective length factors. On some occasions, the designer may find these expressions very convenient to use, compared with the alignment charts just described. Perhaps the most useful situation is for computer programs. You can see that it would be rather inconvenient to stop occasionally in the middle of a computer design to read K factors from the charts and input them to the computer. The equations, however, can easily be included in the programs, eliminating the necessity of using alignment charts.[3]

The alignment chart of Fig. 7.2(b) for frames with sidesway uninhibited always indicates that $K \geq 1.0$. In fact, calculated K factors of 2.0 to 3.0 are common, and even larger values are occasionally obtained. To many designers, such large factors seem completely unreasonable. If the designer derives seemingly high K factors, he or she should carefully review the numbers used to enter the chart (that is, the G values), as well as the basic assumptions made in preparing the charts. These assumptions are discussed in detail in Sections 7.4 and 7.5.

7.4 FRAMES NOT MEETING ALIGNMENT CHART ASSUMPTIONS AS TO JOINT ROTATIONS

In this section, a few comments are presented regarding frames whose joint rotations (and thus their beam stiffnesses) are not in agreement with the assumptions made for developing the charts.

It can be shown by structural analysis that the rotation at point B in the frame of Fig. 7.5 is twice as large as the rotation at B assumed in the construction of the nomographs. Therefore, beam BC in the figure is only one-half as stiff as the value assumed for the development of the alignment charts.

The Jackson and Moreland charts can be accurately used for situations in which the rotations are different from those assumed by making adjustments to the computed beam stiffnesses before the chart values are read. Relative stiffnesses for situations other than the one shown in Fig. 7.5 also can be determined by structural analysis. Table 7.1 presents correction factors to be multiplied by calculated beam stiffnesses, for situations where the beam end conditions are different from those assumed for the development of the charts.

Example 7-2 shows how the correction factors can be applied to a building frame where the rotations at the ends of some of the beams vary from the assumed conditions of the charts.

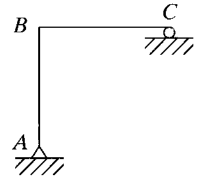


FIGURE 7.5

[3]P. Dumonteil, "Simple Equations for Effective Length Factors," *Engineering Journal*, AISC, vol. 29, no. 3 (3rd Quarter, 1992), pp. 111–115.

TABLE 7.1 Multipliers for Rigidly Attached Members

Condition at Far End of Girder	Sidesway Prevented, Multiply by:	Sidesway Uninhibited, Multiply by:
Pinned	1.5	0.5
Fixed against rotation	2.0	0.67

Example 7-2

Determine K factors for each of the columns of the frame shown in Fig. 7.6. Here, W sections have been tentatively selected for each of the members of the frame and their I/L values determined and shown in the figure.

Solution. First, the G factors are computed for each joint in the frame. In this calculation, the I/L values for members FI and GJ are multiplied by the appropriate factors from Table 7.1.

1. For member FI, the I/L value is multiplied by 2.0, because its far end is fixed and there is no sidesway on that level.
2. For member, GJ, I/L is multiplied by 1.5, because its far end is pinned and there is no sidesway on that level.

$$G_A = 10 \text{ as described in Section 7.2, Pinned Column}$$

$$G_B = \frac{23.2 + 23.2}{70} = 0.663$$

$$G_C = \frac{23.2 + 20.47}{70} = 0.624$$

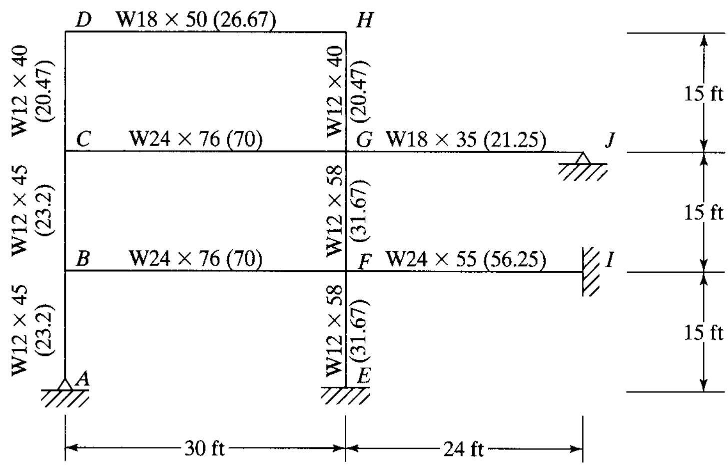


FIGURE 7.6
Steel shapes, including their I/L values.

$$G_D = \frac{20.47}{26.67} = 0.768$$

$$G_E = 1.0 \text{ as described in Section 7.2, Fixed Column}$$

$$G_F = \frac{31.67 + 31.67}{70 + (2.0)(56.25)} = 0.347$$

$$G_G = \frac{31.67 + 20.47}{70 + (1.5)(21.25)} = 0.512$$

$$G_H = \frac{20.47}{26.67} = 0.768$$

Finally, the *K* factors are selected from the appropriate alignment chart of Fig. 7.2.

Column	*G* Factors	Chart used	*K* Factors
AB	10 and 0.663	7.2 (a) no sidesway	0.83
BC	0.663 and 0.624	7.2 (a) no sidesway	0.72
CD	0.624 and 0.768	7.2 (b) has sidesway	1.23
EF	1.0 and 0.347	7.2 (a) no sidesway	0.71
FG	0.347 and 0.512	7.2 (a) no sidesway	0.67
GH	0.512 and 0.768	7.2 (b) has sidesway	1.21

Robins Air Force Base, GA. (Courtesy Britt, Peters and Associates.)

7.5 STIFFNESS-REDUCTION FACTORS

As previously mentioned, the alignment charts were developed according to a set of idealized conditions that are seldom, if ever, completely met in a real structure. Included among those conditions are the following: The column behavior is purely elastic, all columns buckle simultaneously, all members have constant cross sections, all joints are rigid, and so on.

If the actual conditions are different from these assumptions, unrealistically high K factors may be obtained from the charts, and overconservative designs may result. A large percentage of columns will appear in the inelastic range, but the alignment charts were prepared with the assumption of elastic failure. This situation, previously discussed in Chapter 5, is illustrated in Fig. 7.7. For such cases, the chart K values are too conservative and should be corrected as described in this section.

In the elastic range, the stiffness of a column is proportional to EI, where $E = 29{,}000$ ksi; in the inelastic range, its stiffness is more accurately proportional to $E_T I$, where E_T is a reduced or tangent modulus.

The buckling strength of columns in framed structures is shown in the alignment charts to be related to

$$G = \frac{\text{column stiffness}}{\text{girder stiffness}} = \frac{\Sigma(EI/L)\text{ columns}}{\Sigma(EI/L)\text{ girders}}$$

If the columns behave elastically, the modulus of elasticity will be canceled from the preceding expression for G. If the column behavior is inelastic, however, the column stiffness factor will be smaller and will equal $E_T I/L$. As a result, the G factor used to enter the alignment chart will be smaller, and the K factor selected from the chart will be smaller.

Though the alignment charts were developed for elastic column action, they may be used for an inelastic column situation if the G value is multiplied by a correction factor, τ_b. This reduction factor is specified in Section C2-3 of the AISC Specification.

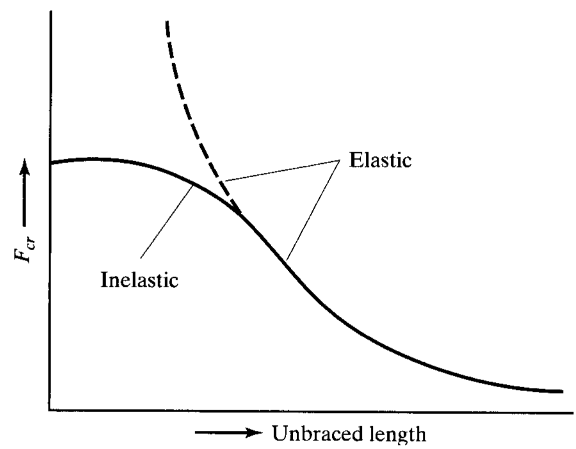


FIGURE 7.7

TABLE 7.2 Stiffness Reduction Factor, τ_b

ASD $\frac{P_a}{A_g}$ / LRFD $\frac{P_u}{A_g}$	F_y, ksi 35 ASD	35 LRFD	36 ASD	36 LRFD	42 ASD	42 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD
45	–	–	–	–	–	–	–	0.0851	–	0.360
44	–	–	–	–	–	–	–	0.166	–	0.422
43	–	–	–	–	–	–	–	0.244	–	0.482
42	–	–	–	–	–	–	–	0.318	–	0.538
41	–	–	–	–	–	0.0930	–	0.388	–	0.590
40	–	–	–	–	–	0.181	–	0.454	–	0.640
39	–	–	–	–	–	0.265	–	0.516	–	0.686
38	–	–	–	–	–	0.345	–	0.575	–	0.730
37	–	–	–	–	–	0.420	–	0.629	–	0.770
36	–	–	–	–	–	0.490	–	0.681	–	0.806
35	–	–	–	0.108	–	0.556	–	0.728	–	0.840
34	–	0.111	–	0.210	–	0.617	–	0.771	–	0.870
33	–	0.216	–	0.306	–	0.673	–	0.811	–	0.898
32	–	0.313	–	0.395	–	0.726	–	0.847	–	0.922
31	–	0.405	–	0.478	–	0.773	–	0.879	0.0317	0.942
30	–	0.490	–	0.556	–	0.816	–	0.907	0.154	0.960
29	–	0.568	–	0.627	–	0.855	–	0.932	0.267	0.974
28	–	0.640	–	0.691	–	0.889	0.102	0.953	0.373	0.986
27	–	0.705	–	0.750	–	0.918	0.229	0.970	0.470	0.994
26	–	0.764	–	0.802	0.0377	0.943	0.346	0.983	0.559	0.998
25	–	0.816	–	0.849	0.181	0.964	0.454	0.992	0.640	1.00
24	–	0.862	–	0.889	0.313	0.980	0.552	0.998	0.713	
23	–	0.901	–	0.923	0.434	0.991	0.640	1.00	0.777	
22	–	0.934	0.0869	0.951	0.543	0.998	0.719		0.834	
21	0.154	0.960	0.249	0.972	0.640	1.00	0.788		0.882	
20	0.313	0.980	0.395	0.988	0.726		0.847		0.922	
19	0.457	0.993	0.525	0.997	0.800		0.896		0.953	
18	0.583	0.999	0.640	1.00	0.862		0.936		0.977	
17	0.693	1.00	0.739		0.913		0.967		0.992	
16	0.786		0.822		0.952		0.987		0.999	
15	0.862		0.889		0.980		0.998		1.00	
14	0.922		0.940		0.996		1.00			
13	0.964		0.976		1.00					
12	0.991		0.996							
11	1.00		1.00							
10										
9										
8										
7										
6										
5										

– Indicates the stiffness reduction parameter is not applicable because the required strength exceeds the available strength for $KL/r = 0$.

Source: AISC Manual, Table 4-21, p. 4-321, 14th ed., 2011.

When $\alpha P_r/P_y$ is less than or equal to 0.5, then τ_b equals 1.0 per AISC Equation C2-2a. When $\alpha P_r/P_y$ is greater than 0.5, then $\tau_b = 4(\alpha P_r/P_y)[1 - (\alpha P_r/P_y)]$ per AISC Equation C2-2b. The factor, α, is taken as 1.0 for the LRFD method and 1.6 for the ASD design basis. P_r is the required axial compressive strength using LRFD or ASD load combinations, P_u or P_a respectively. P_y is the axial yield strength, F_y times the column gross area, A_g. Values of τ_b are shown for various P_u/A_g and P_a/A_g values in Table 7.2, which is Table 4-21 in the AISC Manual.

The τ_b factor is then used to reduce the column stiffness in the equation to calculate G, where $G_{(inelastic)} = \dfrac{\tau_b \sum(I_c/L_c)}{\sum(I_g/L_g)} = \tau_b G_{(elastic)}$. If the end of a column is pinned ($G = 10.0$) or fixed ($G = 1.0$), the value of G at that end should not be multiplied by a stiffness reduction factor.

Example 7-3 illustrates the steps used for the determination of the inelastic effective length factor for a column in a frame subject to sidesway. *In this example, note that the author has considered only in-plane behavior and only bending about the x axis.* As a result of inelastic behavior, the effective length factor is appreciably reduced.

Structures designed by inelastic analysis must meet the provisions of Appendix 1 of the AISC Specification.

Example 7-3

(a) Determine the effective length factor for column AB of the unbraced frame shown in Fig. 7.8, assuming that we have elastic behavior and that all of the other assumptions on which the alignment charts were developed are met. $P_D = 450$ k, $P_L = 700$ k, $F_y = 50$ ksi. Assume that column AB is a W12 × 170 and the columns above and below are as indicated on the figure.

(b) Repeat part (a) if inelastic column behavior is considered.

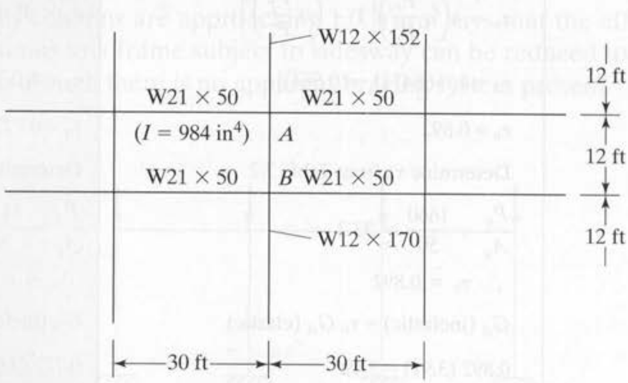


FIGURE 7.8

Solution

LRFD	ASD
$P_u = (1.2)(450) + (1.6)(700) = 1660$ k	$P_a = 450 + 700 = 1150$ k

(a) Assuming that the column is in the elastic range.
Using W12 × 170 ($A = 50$ in², $I_x = 1650$ in⁴) for column AB and the column below.
Using W12 × 152 ($A = 44.7$ in², $I_x = 1430$ in⁴) for column above.

$$G_A = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)} = \frac{\dfrac{1430}{12} + \dfrac{1650}{12}}{2\left(\dfrac{984}{30}\right)} = 3.91$$

$$G_B = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)} = \frac{2\left(\dfrac{1650}{12}\right)}{2\left(\dfrac{984}{30}\right)} = 4.19$$

From Fig. 7.2(b) alignment chart

$$K = 2.05$$

(b) Inelastic solution

LRFD	ASD
$\alpha = 1.0$	$\alpha = 1.6$
$P_r = P_u = 1660$ k	$P_r = P_a = 1150$ k
$P_y = F_y A_g = 50$ ksi (50 in²) = 2500 k	$P_y = F_y A_g = 50$ ksi (50 in²) = 2500 k
$\alpha \dfrac{P_r}{P_y} = \dfrac{1.0(1660)}{2500} = 0.664 > 0.5$	$\alpha \dfrac{P_r}{P_y} = \dfrac{1.6(1150)}{2500} = 0.736 > 0.5$
Use AISC Equation C2-2b	Use AISC Equation C2-2b
$\tau_b = 4\left(\alpha \dfrac{P_r}{P_y}\right)\left[1 - \left(\alpha \dfrac{P_r}{P_y}\right)\right]$	$\tau_b = 4\left(\alpha \dfrac{P_r}{P_y}\right)\left[1 - \left(\alpha \dfrac{P_r}{P_y}\right)\right]$
$\tau_b = 4(0.664)\,[1 - (0.664)]$	$\tau_b = 4(0.736)\,[1 - (0.736)]$
$\tau_b = 0.892$	$\tau_b = 0.777$
Determine τ_b from Table 7.2	Determine τ_b from Table 7.2
$\dfrac{P_u}{A_g} = \dfrac{1660}{50} = 33.2$	$\dfrac{P_a}{A_g} = \dfrac{1150}{50} = 23$
$\therefore\ \tau_b = 0.892$	$\therefore\ \tau_b = 0.777$
G_A (inelastic) = $\tau_b\, G_A$ (elastic)	G_A (inelastic) = $\tau_b\, G_A$ (elastic)
0.892 (3.91) = 3.49	0.777 (3.91) = 3.04
G_B (inelastic) = $\tau_b\, G_B$ (elastic)	G_B (inelastic) = $\tau_b\, G_A$ (elastic)
0.892 (4.19) = 3.74	0.777 (4.19) = 3.26
From Fig. 7.2(b) alignment chart	From Fig. 7.2 (b) alignment chart
$K = 1.96$	$K = 1.86$

7.6 COLUMNS LEANING ON EACH OTHER FOR IN-PLANE DESIGN

When we have an unbraced frame with beams rigidly attached to columns, it is safe to design each column individually, using the sidesway uninhibited alignment chart to obtain the K factors (which will probably be appreciably larger than 1.0).

A column cannot buckle by sidesway unless all of the columns on that story buckle by sidesway. One of the assumptions on which the alignment chart of Fig. 7.2(b) was prepared is that all of the columns in the story in question would buckle at the same time. If this assumption is correct, the columns cannot support or brace each other, because if one gets ready to buckle, they all supposedly are ready to buckle.

In some situations, however, certain columns in a frame have some excess buckling strength. If, for instance, the buckling loads of the exterior columns of the unbraced frame of Fig. 7.9 have not been reached when the buckling loads of the interior columns are reached, the frame will not buckle. The interior columns, in effect, will lean against the exterior columns; that is, the exterior columns will brace the interior ones. For this situation, shear resistance is provided in the exterior columns that resist the sidesway tendency.[4]

A pin-ended column that does not help provide lateral stability to a structure is referred to as a *leaning* column. Such a column depends on the other parts of the structure to provide lateral stability. AISC Commentary Appendix 7 – Section 7.2 states that the effects of gravity-loaded leaning columns shall be included in the design of moment frame columns.

There are many practical situations in which some columns have excessive buckling strength. This might happen when the designs of different columns on a particular story are controlled by different loading conditions. For such cases, failure of the frame will occur only when the gravity loads are increased sufficiently to offset the extra strength of the lightly loaded columns. As a result, the critical loads for the interior columns of Fig. 7.9 are increased, and, in effect, their effective lengths are decreased. In other words, if the exterior columns are bracing the interior ones against sidesway, the K factors for those interior columns are approaching 1.0. Yura[5] says that the effective length of some of the columns in a frame subject to sidesway can be reduced to 1.0 in this type of situation, even though there is no apparent bracing system present.

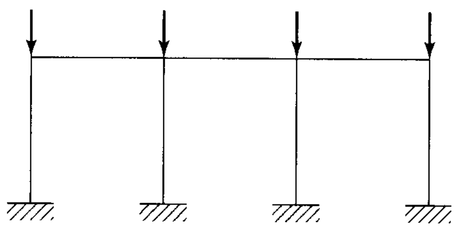


FIGURE 7.9

[4] J. A. Yura, "The Effective Length of Columns in Unbraced Frames," *Engineering Journal*, AISC, vol. 8, no. 2 (second quarter, 1971), pp. 37–4

[5] Ibid., pp. 39–40.

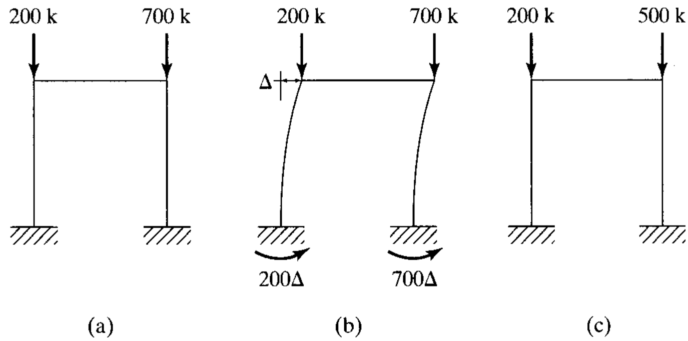


FIGURE 7.10

The net effect of the information presented here is that the total gravity load that an unbraced frame can support equals the sum of the strength of the individual columns. In other words, the total gravity load that will cause sidesway buckling in a frame can be split up among the columns in any proportion, so long as the maximum load applied to any one column does not exceed the maximum load that column could support if it were braced against sidesway, with $K = 1.0$.

For this discussion, the unbraced frame of Fig. 7.10(a) is considered. It is assumed that each column has $K = 2.0$ and will buckle under the loads shown.

When sidesway occurs, the frame will lean to one side, as shown in part (b) of the figure, and $P\Delta$ moments equal to 200Δ and 700Δ will be developed.

Suppose that we load the frame with 200 k on the left-hand column and 500 k on the right-hand column (or 200 k less than we had before). We know that for this situation, which is shown in part (c) of the figure, the frame will not buckle by sidesway until we reach a moment of 700Δ at the right-hand column base. This means that our right-hand column can take an additional moment of 200Δ. Thus, as Yura says, the right-hand column has a reserve of strength that can be used to brace the left-hand column and prevent its sidesway buckling.

Obviously, the left-hand column is now braced against sidesway, and sidesway buckling will not occur until the moment that its base reaches 200Δ. Therefore, it can be designed with a K factor less than 2.0 and can support an additional load of 200 k, giving it a total load of 400 k—*but this load must not be greater than its capacity would be if it were braced against sidesway with* $K = 1.0$. It should be mentioned that the total load the frame can carry is still 900 k, as in part (a) of the figure.

The advantage of the frame behavior described here is illustrated in Fig. 7.11. In this situation, the interior columns of a frame are braced against sidesway by the exterior columns. As a result, the interior columns are assumed to each have K factors equal to 1.0. They are designed for the factored loads shown (660 k each). Then the K factors for the exterior columns are determined with the sidesway uninhibited chart of Fig. 7.2, and they are each designed for column loads equal to $440 + 660 = 1100$ k. To fully understand the benefit of the *leaner column theory*, we must first realize that the frame is assumed to be braced against sidesway in the y or out of plane direction such that $K_y = 1.0$. Each of the end columns of Figure 7.11 will need to support

440 k + 660 k, but these loads are tending to buckle the end columns about their x axes. As a result the $\frac{KL}{r}$ value used to determine $\phi_c F_{cr}$ $\left(\text{or } \frac{F_{cr}}{\Omega_c}\right)$ is $\left(\frac{KL}{r}\right)_x$ and not the much larger $\left(\frac{KL}{r}\right)_y$.

Example 7-4

For the frame of Fig. 7.11, which consists of 50 ksi steel, beams are rigidly connected to the exterior columns, while all other connections are simple. The columns are braced top and bottom against sidesway, out of the plane of the frame, so that $K_y = 1.0$ in that direction. Sidesway is possible in the plane of the frame. Using the LRFD method, design the interior columns assuming that $K_x = K_y = 1.0$, and design the exterior columns with K_x as determined from the alignment chart and $P_u = 1100$ k. (With this approach to column buckling, the interior columns could carry no load at all, since they appear to be unstable under sidesway conditions.) The end columns are assumed to have no bending moment at the top of the member.

Solution. *Design of interior columns:*

Assume $K_x = K_y = 1.0, KL = (1.0)(15) = 15 \text{ ft}, P_u = 660 \text{ k}$.

Use W14 × 74; $\phi P_n = 667 \text{ k} > P_u = 660 \text{ k}$

Design of exterior columns:

In plane $P_u = 440 + 660 = 1100$ k, K_x to be determined from alignment chart. Estimating a column size a little larger than would be required for $P_u = 1100$ k. Try W14 × 120 ($A = 35.3 \text{ in}^2$, $I_x = 1380 \text{ in}^4$, $r_x = 6.24$ in, $r_y = 3.74$ in).

$$G_{\text{top}} = \frac{1380/15}{2100/30 \times 0.5} = 2.63$$

(noting that girder stiffness is multiplied by 0.5, since sidesway is permitted and far end of girder is hinged).

$$G_{\text{bottom}} = 10$$
$$K_x = 2.22 \text{ from Fig. 7.2(b)}$$

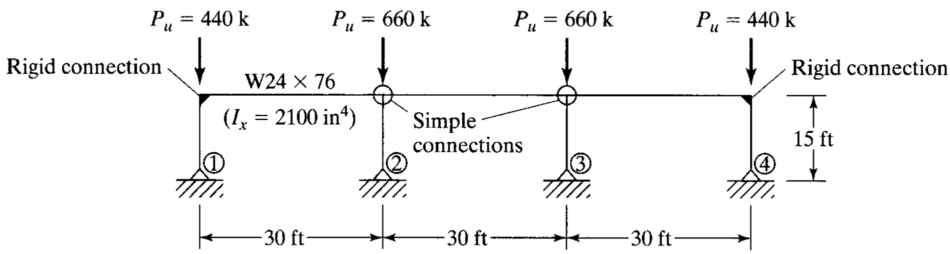


FIGURE 7.11

$$\frac{K_x L_x}{r_x} = \frac{(2.22)(12 \times 15)}{6.24} = 64.04$$

$$\phi_c F_{cr} = 33.38 \text{ ksi}$$

$$\phi_c P_n = (33.38)(35.3) = 1178 \text{ k} > P_u = 1100 \text{ k}$$

Out of plane: $K_y = 1.0, P_u = 440$ k

$$\frac{K_y L_y}{r_y} = \frac{1.0\,(12 \times 15)}{3.74} = 48.13$$

$$\phi F_{cr} = 37.96 \text{ ksi}$$

$$\phi_c P_n = (37.96)\,(35.3) = 1340 \text{ k} > P_u = 440 \text{ k}$$

Use W14 × 120.

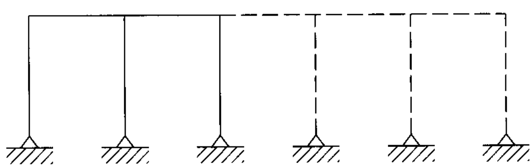
It is rather frightening to think of additions to existing buildings and the leaner column theory. If we have a building (represented by the solid lines in Fig. 7.12) and we decide to add onto it (indicated by the dashed lines in the same figure), we may think that we can use the old frame to brace the new one and that we can keep expanding laterally with no effect on the existing building. Sadly, we may be in for quite a surprise. The leaning of the new columns may cause one of the old ones to fail.

7.7 BASE PLATES FOR CONCENTRICALLY LOADED COLUMNS

The design compressive stress in a concrete or other type of masonry footing is much smaller than it is in a steel column. When a steel column is supported by a footing, it is necessary for the column load to be spread over a sufficient area to keep the footing from being overstressed. Loads from steel columns are transferred through a steel base plate to a fairly large area of the footing below. (Note that a footing performs a related function, in that it spreads the load over an even larger area so that the underlying soil will not be overstressed.)

The base plates for steel columns can be welded directly to the columns, or they can be fastened by means of some type of bolted or welded lug angles. These connection methods are illustrated in Fig. 7.13. A base plate welded directly to the column is shown in part (a) of the figure. For small columns, these plates are probably shop-welded to the columns, but for larger columns it may be necessary to ship the plates separately and set them to the correct elevations. For this second case, the columns are connected to the footing with anchor bolts that pass through the lug angles which have been shop-welded to the columns. This type of arrangement is shown in part (b) of the figure. Some designers like to use lug angles on both flanges

FIGURE 7.12



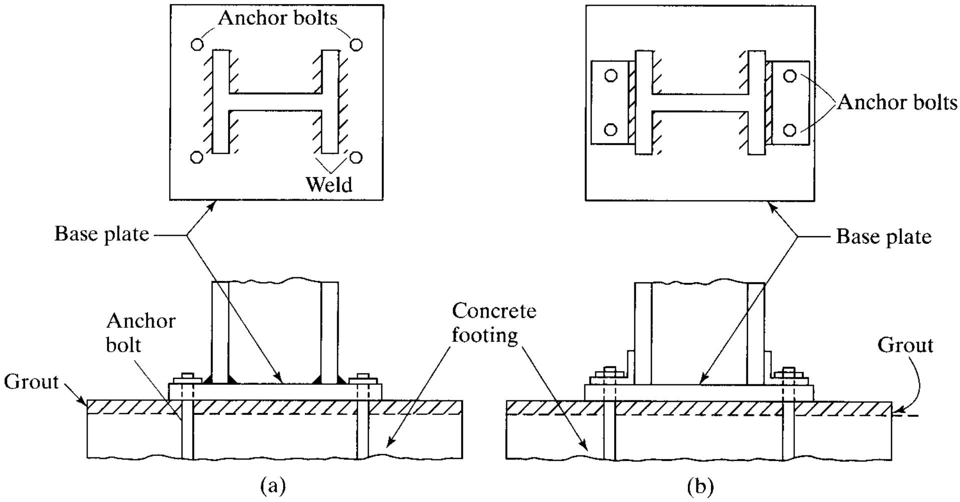


FIGURE 7.13

Column base plates.

and web. (The reader should be aware of OSHA regulations for the safe erection of structural steel, which require the use of no less than four anchor bolts for each column. These bolts will preferably be placed at base plate corners.)

A critical phase in the erection of a steel building is the proper positioning of column base plates. If they are not located at their correct elevations, serious stress changes may occur in the beams and columns of the steel frame. One of the following three methods is used for preparing the site for the erection of a column to its proper elevation: leveling plates, leveling nuts, or preset base plates. An article by Ricker[6] describes these procedures in considerable detail.

For small-to-medium base plates (up to 20 to 22 in), approximately 0.25-in-thick leveling plates with the same dimensions as the base plates (or a little larger) are shipped to the job and carefully grouted in place to the proper elevations. Then the columns with their attached base plates are set on the leveling plates.

As these leveling plates are very light and can be handled manually, they are set by the foundation contractor. This is also true for the lighter base plates. On the other hand, large base plates that have to be lifted with a derrick or crane are usually set by the steel erector.

For larger base plates, up to about 36 in, some types of leveling nuts are used to adjust the base plates up or down. To ensure stability during erection, these nuts must be used on at least four anchor bolts.

If the base plates are larger than about 36 in, the columns with the attached base plates are so heavy and cumbersome that it is difficult to ship them together. For such cases, the base plates are shipped to the job and placed in advance of the steel erection. They can be leveled with shims or wedges.

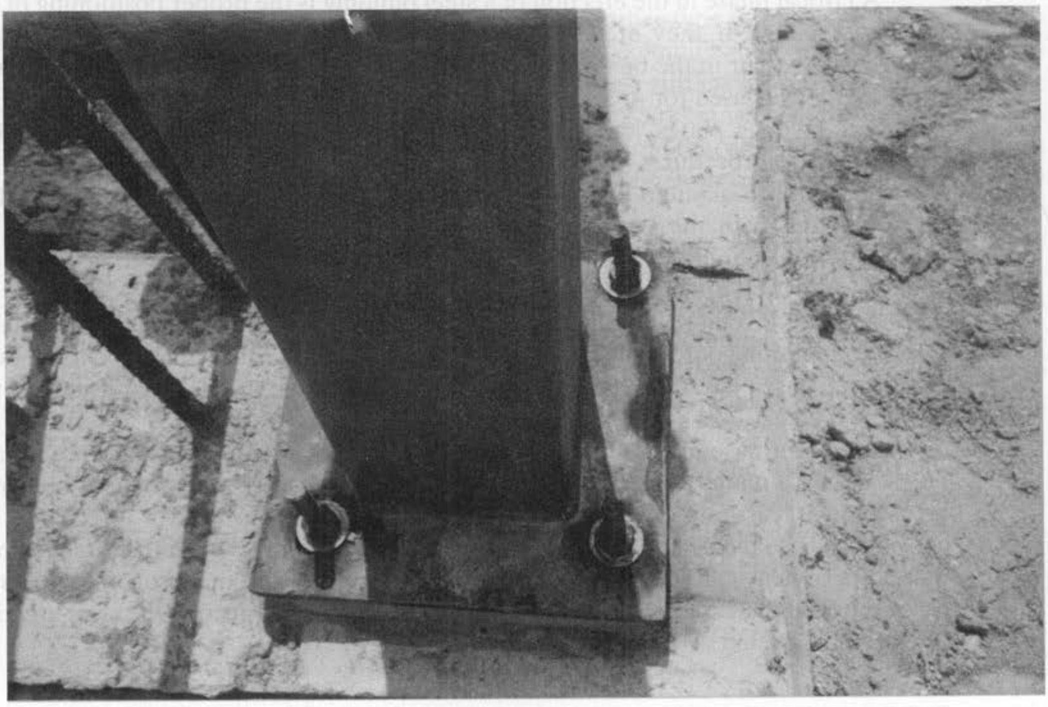
[6] D. T. Ricker, "Some Practical Aspects of Column Bases," *Engineering Journal*, AISC vol. 26, no. 3 (3d quarter, 1989), pp. 81–89.

For tremendously large base plates weighing several tons or more, angle frames may be built to support the plates. They are carefully leveled and filled with concrete, which is screeded off to the correct elevations, and the base plates are set directly on the concrete.

A column transfers its load to the supporting pier or footing through the base plate. Should the supporting concrete area A_2 be larger than the plate area A_1, the concrete strength will be higher. In that case, the concrete surrounding the contact area supplies appreciable lateral support to the directly loaded part, with the result that the loaded concrete can support more load. This fact is reflected in the design stresses.

The lengths and widths of column base plates are usually selected in multiples of even inches, and their thicknesses in multiples of $^1/_8$ in up to 1.25 in, and in multiples of $^1/_4$ in thereafter. To make sure that column loads are spread uniformly over their base plates, it is essential to have good contact between the two. The surface preparation of these plates is governed by Section M2.8 of the AISC Specification. In that section, it is stipulated that bearing plates of 2 in or less in thickness may be used without milling if satisfactory contact is obtained. (Milled surfaces have been accurately sawed or finished to a true plane.) Plates over 2 in thick, up through 4 in, may be pressed to be straightened, or they may be milled at the option of the steel fabricator. Plates thicker than 4 in must be milled if they are beyond the flatness tolerances specified in Table 1-29 of Part 1 of the AISC Manual, entitled "Rectangular Plates."

At least one hole should be provided near the center of large area base plates for placing grout. These holes will permit more even placement of grout under the plates,



Robins Air Force Base, GA. (Courtesy Britt Peters and Associates.)

which will tend to prevent air pockets. Grout holes are not needed if the grout is dry packed. Both anchor bolt holes and grout holes are usually flame cut, because their diameters are often too large for normal punching and drilling. Part 14 of the AISC Manual presents considerably more information concerning the installation of base plates.

If the bottom surfaces of the plates are to be in contact with cement grout, to ensure full bearing contact on the foundation, the plates do not have to be milled. Furthermore, the top surfaces of plates thicker than 4 in do not have to be milled if full-penetration welds (described in Chapter 14) are used. Notice that when finishing is required, as described here, the plates will have to be ordered a little thicker than is necessary for their final dimensions, to allow for the cuts.

Initially, columns will be considered that support average-size loads. Should the loads be very small, so that base plates are very small, the design procedure will have to be revised, as described later in this section.

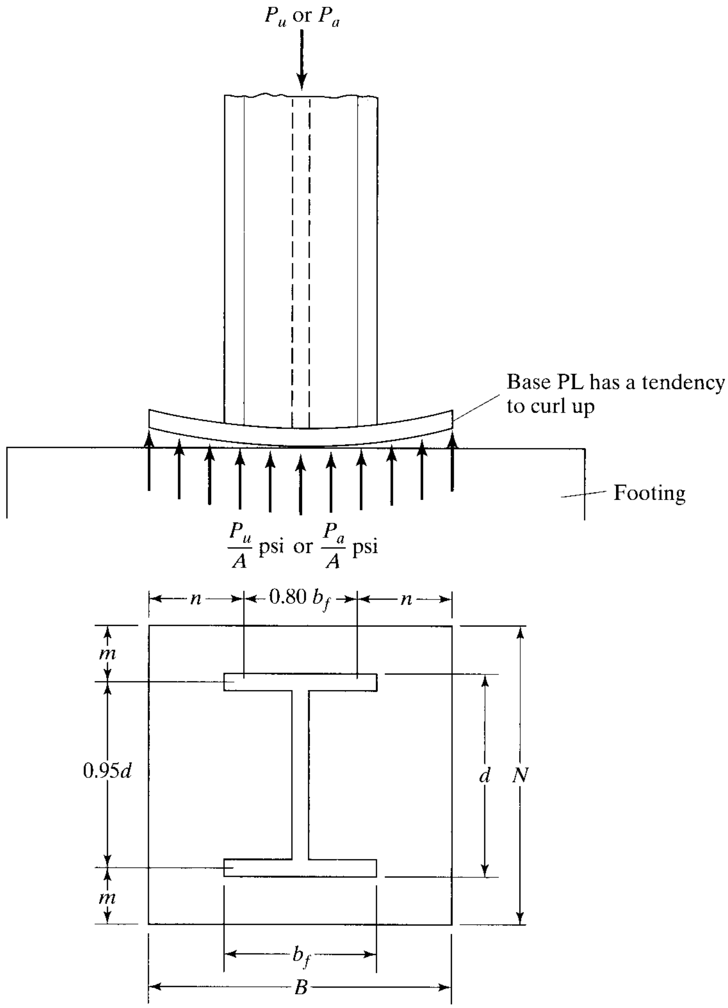


FIGURE 7.14

The AISC Specification does not stipulate a particular method for designing column base plates. The method presented here is based upon example problems presented in the CD accompanying the Manual.

To analyze the base plate shown in Fig. 7.14, note that the column is assumed to apply a total load to the base plate equal to P_u (for LRFD) or P_a (for ASD). Then the load is assumed to be transmitted uniformly through the plate to the footing below, with a pressure equal to P_u/A or P_a/A, where A is the area of the base plate. The footing will push back with an equal pressure and will tend to curl up the cantilevered parts of the base plate outside of the column, as shown in the figure. This pressure will also tend to push up the base plate between the flanges of the column.

With reference to Fig. 7.14, the AISC Manual suggests that maximum moments in a base plate occur at distances $0.80b_f$ and $0.95\,d$ apart. The bending moment can be calculated at each of these sections, and the larger value used to determine the plate thickness needed. This method of analysis is only a rough approximation of the true conditions, because the actual plate stresses are caused by a combination of bending in two directions.

7.7.1 Plate Area

The design strength of the concrete in bearing beneath the base plate must at least equal the load to be carried. When the base plate covers the entire area of the concrete, the nominal bearing strength of the concrete (P_p) is

$$P_p = 0.85 f'_c A_1. \qquad \text{(AISC Equation J8-1)}$$

In this expression, f'_c is the 28-day compression strength of the concrete and A_1 is the area of the base plate. For LRFD design ϕ_c is 0.65, while for ASD design Ω_c is 2.31.

Should the full area of the concrete support not be covered by the plate, the concrete underneath the plate, surrounded by concrete outside, will be somewhat stronger. For this situation, the AISC Specification permits the nominal strength $0.85 f'_c A_1$ to be increased by multiplying it by $\sqrt{A_2/A_1}$. In the resulting expression, A_2 is the maximum area of the portion of the supporting concrete, which is geometrically similar to and concentric with the loaded area. The quantity $\sqrt{A_2/A_1}$ is limited to a maximum value of 2, as shown in the expression that follows. You should note that A_1 may not be less than the depth of the column times its flange width. (Min $A_1 = b_f d$.)

$$P_p = (0.85 f'_c A_1)\sqrt{\frac{A_2}{A_1}} \le 1.7 f'_c A_1 \qquad \text{(LRFD Equation J8-2)}$$

LRFD with $\phi_c = 0.65$	ASD with $\Omega_c = 2.31$
$P_u = \phi_c P_p = \phi_c(0.85 f'_c A_1)\sqrt{\frac{A_2}{A_1}}$	$P_a = \frac{P_p}{\Omega_c} = \frac{0.85 f'_c A_1\sqrt{\frac{A_2}{A_1}}}{\Omega_c}$
$A_1 = \frac{P_u}{\phi_c(0.85 f'_c)\sqrt{\frac{A_2}{A_1}}}$	$A_1 = \frac{P_a \Omega_c}{(0.85 f'_c)\sqrt{\frac{A_2}{A_1}}}.$

After the controlling value of A_1 is determined as just described, the plate dimensions B and N (shown in Fig. 7.14), are selected to the nearest 1 or 2 in so that the values of m and n shown in the figure are roughly equal. Such a procedure will make the cantilever moments in the two directions approximately the same. This will enable us to keep the plate thicknesses to a minimum. The condition $m = n$ can be approached if the following equation is satisfied:

$$N \approx \sqrt{A_1} + \Delta$$

Here,

$$A_1 = \text{area of plate} = BN$$

$$\Delta = 0.5\,(0.95\,d - 0.80\,b_f)$$

$$N = \sqrt{A_1} + \Delta$$

$$B \approx \frac{A_1}{N}$$

From a practical standpoint, designers will often use square base plates with anchor bolts arranged in a square pattern. Such a practice simplifies both field and shop work.

7.7.2 Plate Thickness

To determine the required plate thickness, t, moments are taken in the two directions as though the plate were cantilevered out by the dimensions m and n. Reference is again made here to Figure 7.14. In the expressions to follow, the load P is P_u for LRFD design and P_a for ASD design. The moments in the two directions are $\left(\frac{P_u}{BN}\right)(m)\left(\frac{m}{2}\right) = \frac{P_u m^2}{2BN}$ or $\left(\frac{P}{BN}\right)(n)\left(\frac{n}{2}\right) = \frac{Pm^2}{2BN}$, both computed for a 1-in width of plate.

If lightly loaded base plates, such as those for the columns of low-rise buildings and preengineered metal buildings, are designed by the procedure just described, they will have quite small areas. They will, as a result, extend very little outside the edges of the columns and the computed moments, and the resulting plate thicknesses will be very small, perhaps so small as to be impractical.

Several procedures for handling this problem have been proposed. In 1990, W. A. Thornton[7] combined three of the methods into a single procedure applicable to either heavily loaded or lightly loaded base plates. This modified method is used for the example base plate problems presented on the CD accompanying the AISC manual, as well as for the example problems in this chapter.

Thornton proposed that the thickness of the plates be determined by the largest of m, n, or $\lambda n'$. He called this largest value ℓ.

$$\ell = \max\,(m, n, \text{or } \lambda n')$$

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[7]W. A. Thornton, "Design of Base Plates for Wide Flange Columns—A Concatenation of Methods," *Engineering Journal*, AISC, vol. 27, no. 4 (4th quarter, 1990) pp. 173, 174.

To determine $\lambda n'$, it is necessary to substitute into the following expressions, which are developed in his paper:

$\phi_c P_p = \phi_c 0.85 f'_c A_1$ for plates covering the full area of the concrete support

$\phi_c P_p = \phi_c 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}}$, where $\sqrt{\frac{A_2}{A_1}}$ must be ≤ 2 for plates not covering the entire area of concrete support

LRFD	ASD
$X = \left[\frac{4db_f}{(d+b_f)^2}\right]\frac{P_u}{\phi_c P_p}$	$X = \frac{4db_f}{(d+b_f)^2}\frac{\Omega_c P_a}{P_p}$
$\lambda = \frac{2\sqrt{X}}{1+\sqrt{1-X}} \leq 1$	$\lambda = \frac{2\sqrt{X}}{1+\sqrt{1-X}} \leq 1$
$\lambda n' = \frac{\lambda\sqrt{db_f}}{4}$	$\lambda m' = \frac{\lambda\sqrt{db_f}}{4}$

According to Thornton it is permissible to conservatively assume λ equals 1.0 for all cases; this practice is followed in the examples to follow. As a result, it is unnecessary to substitute into the equations listed for $\overline{X}$, λ and $\lambda n'$. Thus the authors drop the λ from $\lambda n'$ and just uses n'.

Letting the largest value of m, n, or $\lambda n'$ be referred to as ℓ, we find that the largest moment in the plate will equal $\left(\frac{P_u}{BN}\right)(\ell)\left(\frac{\ell}{2}\right) = \frac{P_u \ell^2}{2BN}$ for LRFD and $\frac{P_a \ell^2}{2BN}$ for ASD.

In the next few chapters of this text, the reader will learn how to calculate the resisting moments of plates (as well as the resisting moments for other steel sections). For plates, these values are $\frac{\phi_b F_y bt^2}{4}$ for LRFD, with $\phi_b = 0.9$, and $\frac{F_y bt^2}{4\Omega_b}$ for ASD, with $\Omega_b = 1.67$.

If these resisting moments are equated to the maximum bending moments, the resulting expressions may be solved for the required depth or thickness t with the following results, noting that $b = 1$ in:

LRFD with $\phi_b = 0.9$	ASD with $\Omega_b = 1.67$
$\frac{\phi_b F_y bt^2}{4} = \frac{P_u l^2}{2BN}$	$\frac{F_y bt^2}{4\Omega_b} = \frac{P_a l^2}{2BN}$
$t_{\text{reqd}} = \ell\sqrt{\frac{2P_u}{0.9F_y BN}}$	$t_{\text{reqd}} = \ell\sqrt{\frac{3.33P_a}{F_y BN}}$

Four example base plate designs are presented in the next few pages. Example 7-5 illustrates the design of a base plate supported by a large reinforced concrete footing, with A_2 many times as large as A_1. In Example 7-6, a base plate is designed that is supported by a concrete pedestal, where the plate covers the entire concrete area. In Example 7-7, a base plate is selected for a column that is to be supported on a pedestal 4 in wider on each side than the plate. This means that A_2 cannot be determined until the plate area is computed. Finally, Example 7-8 presents the design of a base plate for an HSS column.

Example 7-5

Design a base plate of A36 steel (F_y = 36 ksi) for a W12 × 65 column (F_y = 50 ksi) that supports the loads P_D = 200 k and P_L = 300 k. The concrete has a compressive strength f'_c = 3 ksi, and the footing has the dimensions 9 ft × 9 ft.

Solution. Using a W12 × 65 column (d = 12.1 in, b_f = 12.0 in)

LRFD	ASD
$P_u = (1.2)(200) + (1.6)(300) = 720 \text{ k}$	$P = 200 + 300 = 500 \text{ k}$
$A_2 = \text{footing area} = \left(12\dfrac{\text{in}}{\text{ft}} \times 9 \text{ ft}\right)\left(12\dfrac{\text{in}}{\text{ft}} \times 9 \text{ ft}\right) = 11{,}664 \text{ in}^2$	$A_2 = 11{,}664 \text{ in}^2$

Determine required base plate area $A_1 = BN$. Note that the area of the supporting concrete is for greater than the base plate area, such that $\sqrt{\dfrac{A_2}{A_1}} = 2.0$.

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$A_1 = \dfrac{P_u}{\phi_c(0.85f'_c)\sqrt{\dfrac{A_2}{A_1}}}$	$A_1 = \dfrac{P_a\Omega_c}{0.85f'_c\sqrt{\dfrac{A_2}{A_1}}} = \dfrac{(500)(2.31)}{(0.85)(3)(2)}$
$= \dfrac{720}{(0.65)(0.85)(3)(2)} = 217.2 \text{ in}^2$	$= 226.5 \text{ in}^2$

The base plate must be at least as large as the column $b_f d = (12.0)(12.1) = 145.2 \text{ in}^2 < 217.2 \text{ in}^2$ and 226.5 in^2 optimize base plate dimensions to make m and n approximately equal. Refer to Fig. 7.15.

LRFD	ASD
$\Delta = \dfrac{0.95d - 0.8b_f}{2}$	$\Delta = 0.947 \text{ in}$
$= \dfrac{(0.95)(12.1) - (0.8)(12.0)}{2} = 0.947 \text{ in}$	
$N = \sqrt{A_1} + \Delta = \sqrt{217.2} + 0.947 = 15.7 \text{ in}$	$N = \sqrt{226.5} + 0.947 = 16.0 \text{ in}$
Say 16 in	Say 16 in
$B = \dfrac{A_1}{N} = \dfrac{217.2}{16} = 13.6 \text{ in}$	$B = \dfrac{226.5}{16} = 14.2 \text{ in}$

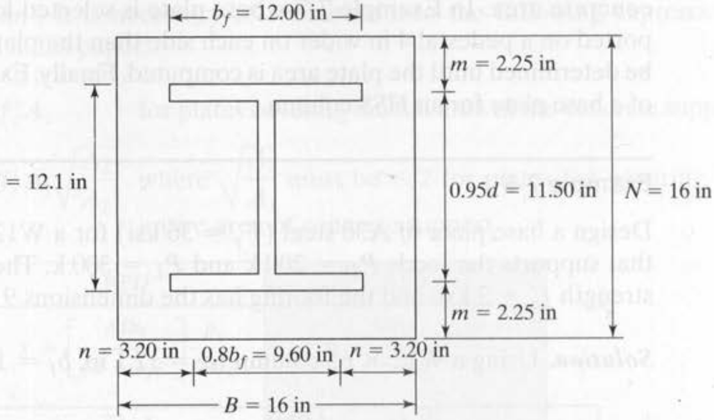


FIGURE 7.15

As previously mentioned, we might very well simplify the plates by making them square – say, 16 in × 16 in.

Check the bearing strength of the concrete

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$\phi_c P_p = \phi_c 0.85 f'_c A_1 \sqrt{\dfrac{A_2}{A_1}}$	$\dfrac{P_p}{\Omega_c} = \dfrac{0.85 f'_c A_1}{\Omega_c}\sqrt{\dfrac{A_2}{A_1}}$
$= (0.65)(0.85)(3)(16 \times 16)(2)$	$= \dfrac{(0.85)(3)(16 \times 16)(2)}{2.31} = 565.2 \text{ k} > 500 \text{ k}$ **OK**
$= 848.6 \text{ k} > 720 \text{ k}$ **OK**	

Computing required base plate thickness

$$m = \frac{N - 0.95d}{2} = \frac{16 - (0.95)(12.1)}{2} = 2.25 \text{ in}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{16 - (0.8)(12.0)}{2} = 3.20 \text{ in}$$

$$n' = \frac{\sqrt{db_f}}{4} = \frac{\sqrt{(12.1)(12.0)}}{4} = 3.01 \text{ in}$$

$$\ell = \text{largest of } m, n, \text{or } n' = 3.20 \text{ in}$$

LRFD	ASD
$t_{\text{reqd}} = \ell\sqrt{\dfrac{2P_u}{0.9F_yBN}}$	$t_{\text{reqd}} = \ell\sqrt{\dfrac{3.33P_a}{F_yBN}}$
$= 3.20\sqrt{\dfrac{(2)(720)}{(0.9)(36)(16 \times 16)}} = 1.33 \text{ in}$	$= 3.20\sqrt{\dfrac{(3.33)(500)}{(36)(16)(16)}} = 1.36 \text{ in}$
Use PL $1\frac{1}{2} \times 16 \times 1$ ft 4 in A36.	Use PL $1\frac{1}{2} \times 16 \times 1$ ft 4 in A36.

Example 7-6

A base plate is to be designed for a W12 × 152 column ($F_y = 50$ ksi) that supports the loads $P_D = 200$ k and $P_L = 450$ k. Select an A36 plate ($F_y = 36$ ksi) to cover the entire area of the 3 ksi concrete pedestal underneath.

Solution. Using a W12 × 152 column ($d = 13.7$ in, $b_f = 12.5$ in)

LRFD	ASD
$P_u = (1.2)(200) + (1.6)(450) = 960$ k	$P_a = 200 + 450 = 650$ k

Determine the required base plate area, noting that the term $\sqrt{\frac{A_2}{A_1}}$ is equal to 1.0, since $A_1 = A_2$.

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$A_1 = \dfrac{P_u}{\phi_c(0.85f'_c)\sqrt{\frac{A_2}{A_1}}}$	$A_1 = \dfrac{P_a\Omega_c}{0.85f'_c\sqrt{\frac{A_2}{A_1}}}$
$= \dfrac{960}{(0.65)(0.85 \times 3)(1)}$	$= \dfrac{(650)(2.31)}{(0.85)(3)(1)}$
$= 579.2 \text{ in}^2 \leftarrow$	$= 588.8 \text{ in}^2 \leftarrow$
$A_1 \min = db_f = (13.7)(12.5)$	$A_1 \min = db_f = (13.7)(12.5)$
$= 171.2 \text{ in}^2$	$= 171.2 \text{ in}^2$

Optimizing base plate dimensions $n \sim m$

LRFD	ASD
$\Delta = \dfrac{0.95d - 0.8b_f}{2}$	
$= \dfrac{(0.95)(13.7) - (0.8)(12.5)}{2} = 1.51$ in	$\Delta = 1.51$ in
$N = \sqrt{A_1} + \Delta = \sqrt{579.2} + 1.51$	$N = \sqrt{588.8} + 1.51$
$= 25.6$ in **Say 26 in**	$= 25.8$ in **Say 26 in**
$B = \dfrac{A_1}{N} = \dfrac{579.2}{26} = 22.3$ in	$B = \dfrac{588.8}{26} = 22.6$ in
Say 23 in	**Say 23 in**

Check the bearing strength of the concrete

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$\phi_c P_p = \phi_c 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}}$	$\frac{P_p}{\Omega_c} = \frac{0.85 f'_c A_1}{\Omega_c}\sqrt{\frac{A_2}{A_1}}$
$= (0.65)(0.85)(3)(23 \times 26)(1.0)$	$= \frac{(0.85)(3)(23 \times 26)}{2.31}(1.0)$
$= 991.2\text{ k} > 960\text{ k}$ **OK**	$= 660.1\text{ k} > 650\text{ k}$ **OK**

Computing required base plate thickness

$$m = \frac{N - 0.95d}{2} = \frac{26 - (0.95)(13.7)}{2} = 6.49\text{ in}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{23 - (0.8)(12.5)}{2} = 6.50\text{ in}$$

$$n' = \frac{\sqrt{db_f}}{4} = \frac{\sqrt{(13.7)(12.5)}}{4} = 3.27\text{ in}$$

$$\ell = \text{maximum of } m, n \text{ or } n' = 6.50\text{ in}$$

LRFD	ASD
$t_{\text{reqd}} = \ell\sqrt{\frac{2P_u}{0.9F_yBN}}$	$t_{\text{reqd}} = \ell\sqrt{\frac{3.33P_a}{F_yBN}}$
$= 6.50\sqrt{\frac{(2)(960)}{(0.9)(36)(26 \times 23)}}$	$= 6.50\sqrt{\frac{(3.33)(650)}{(36)(23 \times 26)}}$
$= 2.05\text{ in}$	$= 2.06\text{ in}$

Use $2\frac{1}{8} \times 23 \times 2$ ft 2 in A36 base plate with 23×26 concrete pedestal ($f'_c = 3$ ksi).

Example 7-7

Repeat Example 7-6 if the column is to be supported by a concrete pedestal 2 in wider on each side than the base plate.

Solution. Using a W12 × 152 ($d = 13.7$ in, $b_f = 12.5$ in)

LRFD	ASD
$P_u = (1.2)(200) + (1.6)(450) = 960\text{ k}$	$P_a = 200 + 450 = 650\text{ k}$
A_1 required from Example 7-6 solution was 579.2 in^2	A_1 required from Example 7-6 solution was 588.8 in^2

If we try a plate $24 \times 25\,(A_1 = 600\text{ in}^2)$, the pedestal area will equal $(24 + 4)(25 + 4) = 812\text{ in}^2$, and $\sqrt{\frac{A_2}{A_1}}$ will equal $\sqrt{\frac{812}{600}} = 1.16$. Recalculating the A_1 values gives

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$A_1 = \dfrac{P_u}{\phi_c(0.85f'_c)\sqrt{\dfrac{A_2}{A_1}}}$	$A_1 = \dfrac{P_a\Omega_c}{0.85f'_c\sqrt{\dfrac{A_2}{A_1}}}$
$= \dfrac{960}{(0.65)(0.85)(3)(1.16)} = 499.3\text{ in}^2$	$= \dfrac{(650)(2.31)}{(0.85)(3)(1.16)} = 507.6\text{ in}^2$

Trying a 22×23 plate (506 in^2), the pedestal area will be $(22 + 4)(23 + 4) = 702\text{ in}^2$, and $\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{702}{506}} = 1.18$. Thus, A_1 (LRFD) will be 490.8 in^2 and A_1 (ASD) will be 499.0 in^2.

Optimizing base plate dimensions $n \sim m$

LRFD	ASD
$\Delta = \dfrac{0.95d - 0.8b_f}{2}$	
$= \dfrac{(0.95)(13.7) - (0.8)(12.5)}{2} = 1.51\text{ in}$	$\Delta = 1.51\text{ in}$
$N = \sqrt{A_1} + \Delta = \sqrt{490.8} + 1.51$	$N = \sqrt{A_1} + \Delta = \sqrt{499.0} + 1.51$
$= 23.66\text{ in}$ **Say, 24 in**	$= 23.85\text{ in.}$ **Say, 24 in**
$B = \dfrac{A_1}{N} = \dfrac{490.8}{24} = 20.45\text{ in}$	$B = \dfrac{A_1}{N} = \dfrac{499}{24} = 20.79\text{ in}$
Say, 21 in	**Say, 21 in**
Use pedestal 25 × 28	
$\sqrt{\dfrac{A_2}{A_1}} = \sqrt{\dfrac{(25)(28)}{(21)(24)}} = 1.18$	Same.

Check the bearing strength of the concrete

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$\phi_c P_p = \phi_c 0.85 f'_c A_1 \sqrt{\dfrac{A_2}{A_1}}$	$\dfrac{P_p}{\Omega_c} = \dfrac{0.85f'_cA_1}{\Omega_c}\sqrt{\dfrac{A_2}{A_1}}$
$= (0.65)(0.85)(3)(21 \times 24)(1.18)$	$= \dfrac{(0.85)(3)(21 \times 24)}{2.31}(1.18)$
$= 985.7\text{ k} > 960\text{ k}$ **OK**	$= 656.5\text{ k} > 650\text{ k}$ **OK**

Computing required base plate thickness

$$m = \frac{N - 0.95d}{2} = \frac{24 - (0.95)(13.7)}{2} = 5.49 \text{ in}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{21 - (0.8)(12.5)}{2} = 5.50 \text{ in}$$

$$n' = \frac{\sqrt{db_f}}{4} = \frac{\sqrt{(13.7)(12.5)}}{4} = 3.27 \text{ in}$$

$$\ell = \text{maximum of } m, n \text{ or } n' = 5.50 \text{ in}$$

LRFD	ASD
$t_{\text{reqd}} = \ell\sqrt{\frac{2P_u}{0.9F_yBN}}$	$t_{\text{reqd}} = \ell\sqrt{\frac{3.33P_a}{F_yBN}}$
$= 5.50\sqrt{\frac{(2)(960)}{(0.9)(36)(21)(24)}}$	$= 5.50\sqrt{\frac{(3.33)(650)}{(36)(21)(24)}}$
$= 1.89$ in	$= 1.90$ in

Use 2 × 21 × 2 ft 0 in A36 base plate with 25 × 28 concrete pedestal ($f'_c = 3$ ksi).

Example 7-8

A HSS 10 × 10 × $\frac{5}{16}$ with $F_y = 46$ ksi is used to support the service loads $P_D = 100$ k and $P_L = 150$ k. A spread footing underneath is 9 ft-0 in × 9 ft-0 in and consists of reinforced concrete with $f'_c = 4000$ psi. Design a base plate for this column with A36 steel ($F_y = 36$ ksi and $F_u = 58$ ksi).

Solution. Required strength

LRFD	ASD
$P_u = (1.2)(100) + (1.6)(150) = 360$ k	$P_a = 100 + 150 = 250$ k

Try a base plate extending 4 in from the face of the column in each direction—that is, an 18 in × 18 in plate.

Determine the available strength of the concrete footing.

$$A_1 = (18)(18) = 324 \text{ in}^2$$

$$A_2 = (12 \times 9)(12 \times 9) = 11{,}664 \text{ in}^2$$

$$P_p = 0.85f'_cA_1\sqrt{\frac{A_2}{A_1}} = (0.85)(4)(324)\sqrt{\frac{11{,}664}{324}} = 6609.6 \text{ K}$$

$$\text{since } \sqrt{\frac{11{,}664}{324}} = 6.0 > 2.0 \therefore P_p = 1.7f'_c A_1$$

$$P_p = 1.7f'_c A_1 = 1.7(4)(324) = 2203.2 \text{ k}$$

LRFD $\phi_c = 0.65$	ASD $\Omega_c = 2.31$
$\phi_c P_p = (0.65)(2203.2)$	$\frac{P_p}{\Omega_c} = \frac{2203.2}{2.31}$
$= 1432.1 \text{ k} > 360 \text{ k}$ **OK**	$= 953.8 \text{ k} > 250 \text{ k}$ **OK**

Determine plate thickness.

$$m = n = \frac{N - (0.95)(\text{outside dimension of HSS})}{2}$$

$$= \frac{18 - (0.95)(10)}{2} = 4.25 \text{ in}$$

Notice that these values for m and n are both less than the distance from the center of the base plate to the center of the HSS walls. However, the moment in the plate outside the walls is greater than the moment in the plate between the walls. You can verify this statement by drawing the moment diagrams for the situation shown in Fig. 7.16.

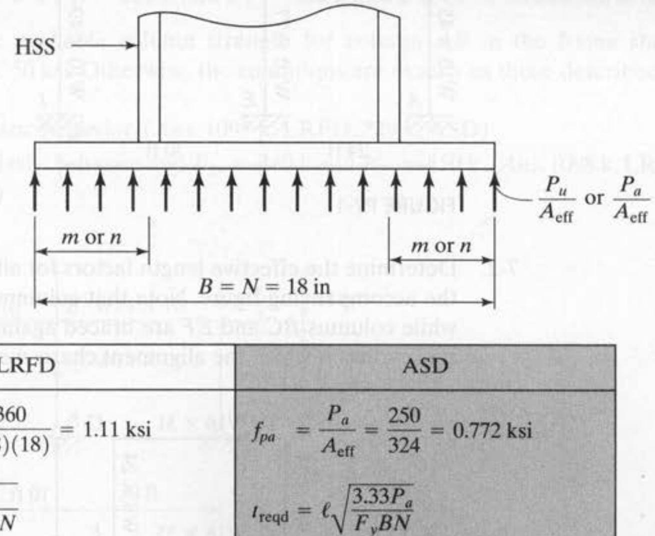


FIGURE 7.16

LRFD	ASD
$f_{pu} = \frac{P_u}{A_{\text{eff}}} = \frac{360}{(18)(18)} = 1.11 \text{ ksi}$	$f_{pa} = \frac{P_a}{A_{\text{eff}}} = \frac{250}{324} = 0.772 \text{ ksi}$
$t_{\text{reqd}} = \ell\sqrt{\frac{2P_u}{0.9F_yBN}}$	$t_{\text{reqd}} = \ell\sqrt{\frac{3.33P_a}{F_yBN}}$
$= 4.25\sqrt{\frac{(2)(360)}{(0.9)(36)(18)(18)}} = 1.11 \text{ in}$	$= 4.25\sqrt{\frac{(3.33)(250)}{(36)(18)(18)}} = 1.14 \text{ in}$

Use $1\frac{1}{4} \times 18 \times 1$ ft 6 in A36 base plate for both LRFD and ASD.

7.7.3 Moment Resisting Column Bases

The designer will often be faced with the need for moment resisting column bases. Before such a topic is introduced, however, the student needs to be familiar with the design of welds (Chapter 14) and moment resisting connections between members (Chapters 14 and 15). For this reason, the subject of moment resisting base plates has been placed in Appendix D.

7.8 PROBLEMS FOR SOLUTION

7-1. Using the alignment chart from the AISC Specification, determine the effective length factors for columns *IJ*, *FG*, and *GH* of the frame shown in the accompanying figure, assuming that the frame is subject to sidesway and that all of the assumptions on which the alignment charts were developed are met. (*Ans.* 1.27, 1.20, and 1.17)

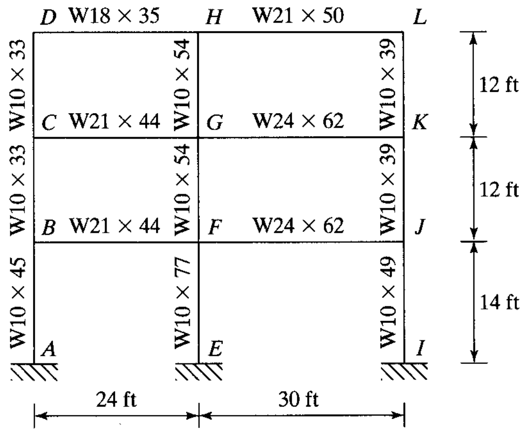


FIGURE P7-1

7-2. Determine the effective length factors for all of the columns of the frame shown in the accompanying figure. Note that columns *CD* and *FG* are subject to sidesway, while columns *BC* and *EF* are braced against sidesway. Assume that all of the assumptions on which the alignment charts were developed are met.

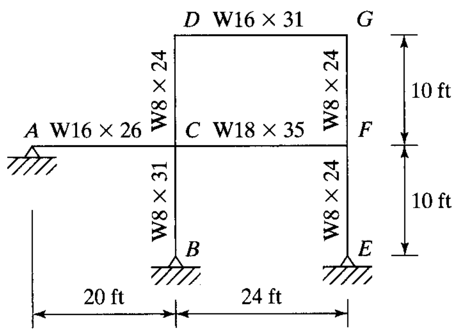


FIGURE P7-2

7-3. to 7-6. *Use both LRFD and ASD methods.*

7-3. a. Determine the available column strength for column AB in the frame shown if $F_y = 50$ ksi, and only in-plane behavior is considered. Furthermore, assume that the column immediately above or below AB are the same size as AB, and also that all the other assumptions on which the alignment charts were developed are met. (*Ans.* 825 k, LRFD; 549 k, ASD)

b. Repeat part (a) if inelastic behavior is considered and $P_D = 200$ k and $P_L = 340$ k. (*Ans.* 838 k, LRFD; 563 k, ASD)

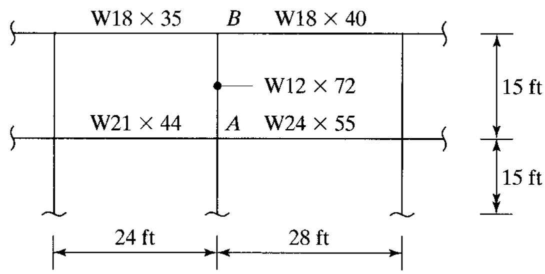


FIGURE P7-3

7-4. Repeat Prob. 7-3 if $P_D = 250$ k and $P_L = 400$ k and a W14 × 90 section is used.

7-5. Determine the available column strength for column AB in the frame shown for which $F_y = 50$ ksi. Otherwise, the conditions are exactly as those described for Prob. 7-3.

a. Assume elastic behavior. (*Ans.* 1095 k, LRFD; 729 k, ASD)

b. Assume inelastic behavior and $P_D = 240$ k and $P_L = 450$ k. (*Ans.* 1098 k, LRFD; 735 k, ASD)

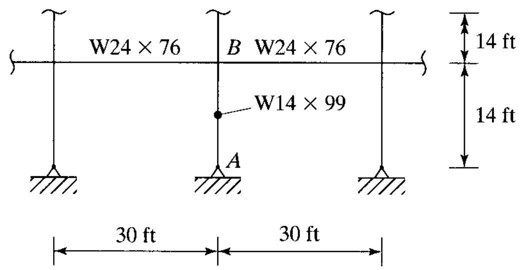


FIGURE P7-5

7-6. Repeat Prob. 7-5 if $P_D = 225$ k and $P_L = 375$ k and a W12 × 87 section is used.

7-7. to 7-13. *Use the Effective Length Method, assume elastic behavior, and use both the LRFD and ASD methods. The columns are assumed to have no bending moments.*

7-7. Design W12 columns for the bent shown in the accompanying figure, with 50 ksi steel. The columns are braced top and bottom against sidesway out of the plane of the frame so that $K_y = 1.0$ in that direction. Sidesway is possible in the plane of the frame, the x-x axis. Design the right-hand column as a leaning column, $K_x = K_y = 1.0$ and the left-hand column as a moment frame column, K_x determined from the alignment chart. $P_D = 350$ k and $P_L = 240$ k for each column. The beam has a moment connection to the left column, and has a simple or pinned connection to the right column. (*Ans.* (Right) W12 × 79, LRFD; W12 × 87, ASD – (Left) W12 × 170, LRFD; W12 × 190, ASD)

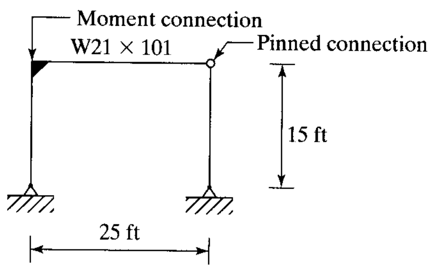


FIGURE P7-7

7-8. Repeat Prob. 7-7 if the loads on each column are $P_D = 120$ k and $P_L = 220$ k, and the girder is a W21 × 68.

7-9. Design W14 columns for the bent shown in the accompanying figure, with 50 ksi steel. The columns are braced top and bottom against sidesway out of the plane of the frame so that $K_y = 1.0$ in that direction. Sidesway is possible in the plane of the frame, the x-x axis. Design the interior column as a leaning column, $K_x = K_y = 1.0$ and the exterior columns as a moment frame columns, K_x determined from the alignment chart. (*Ans.* (Interior) W14 × 176, LRFD; W14 × 193, ASD – (Exterior) W14 × 211, LRFD and ASD)

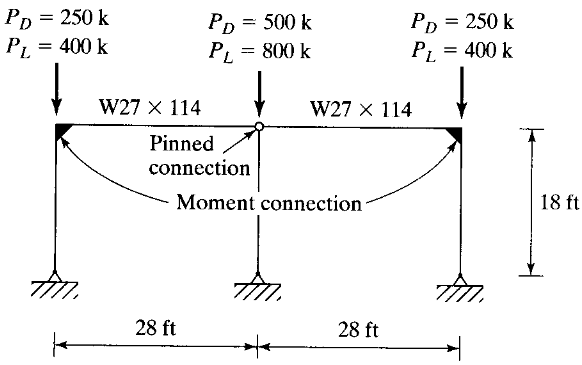


FIGURE P7-9

7-10. Repeat Prob. 7-9, assuming that the outside columns are fixed at the bottom.

7-11. The frame shown in the accompanying figure is unbraced against sidesway about the x-x axis. Determine K_x for column AB. Support conditions in the direction perpendicular to the frame are such that $K_y = 1.0$. Determine if the W14 $\times$ 109 column for member AB is capable of resisting a dead load of 250 k and a live load of 500 k. A992 steel is used. (*Ans.* LRFD W14 $\times$ 109, OK, $\Phi P_n = 1205 \text{ k} > P_u = 1100 \text{ k}$; ASD W14 $\times$ 109, OK, $P_n/\Omega = 803 \text{ k} > P_a = 750 \text{ k}$)

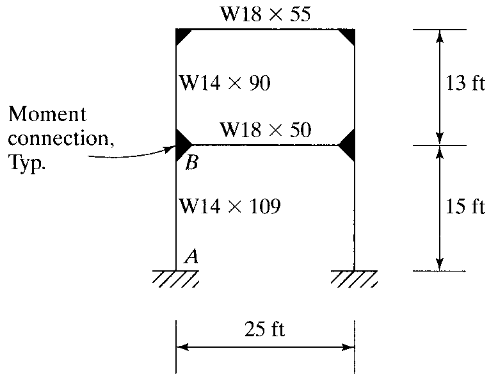


FIGURE P7-11

7-12. The frame shown in the accompanying figure is unbraced against sidesway about the x-x axis. The columns are W8 and the beams are W12 $\times$ 16. ASTM A572 steel is used for the columns and beams. The beams and columns are oriented so that bending is about the x-x axis. Assume that $K_y = 1.0$, and for column AB the service load is 175 k, in which 25 percent is dead load and 75 percent is live load. Select the lightest W8 shape for column AB.

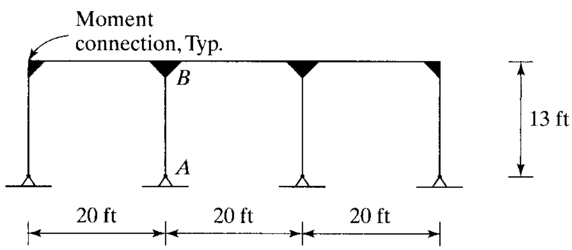


FIGURE P7-12

7-13. Select the lightest W12 shape for column AB of the pinned-base unbraced-moment frame shown in the figure. All steel is ASTM A992. The horizontal girder is a W18 $\times$ 76. The girder and columns are oriented so that bending is about the x-x axis. In the plane perpendicular to the frame, $K_y = 1.0$ and bracing is provided to the y-y axis of the column at the top and mid-height using pinned end connections. The loads on each are P_D = 150 k and P_L = 200 k. (*Ans.* W12 × 53, LRFD; W12 × 58, ASD)

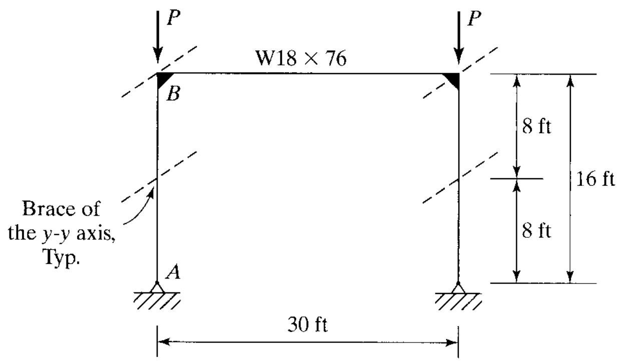


FIGURE P7-13

7-14. Design a square base plate with A36 steel for a W10 × 60 column with a service dead load of 175 k and a service live load of 275 k. The concrete 28-day strength, f'_c, is 3000 psi. The base plate rests on a 12 ft 0 in × 12 ft 0 in concrete footing. Use the LRFD and ASD design methods.

7-15. Repeat Prob. 7-14 if the column is supported by a 24 in × 24 in concrete pedestal. (*Ans.* B PL – 1¾ × 18 × 1 ft 6 in A36 LRFD and ASD)

7-16. Design a rectangular base plate for a W8×28 column with P_D = 80 k and P_L = 150 k if A36 steel is used and f'_c = 3 ksi for the concrete. Assume that the column is supported by a 7ft 0 in × 7 ft 0 in concrete footing. Use the LRFD and ASD design methods.

CHAPTER 8

Introduction to Beams

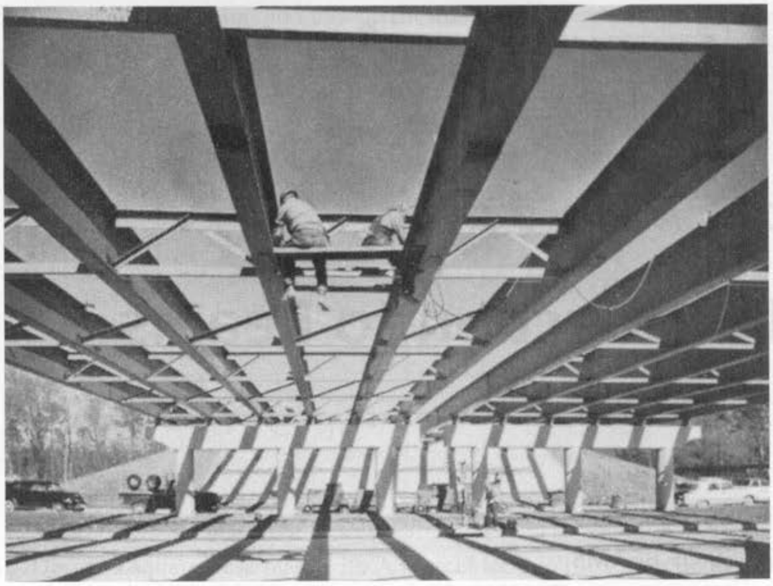
8.1 TYPES OF BEAMS

Beams are usually said to be members that support transverse loads. They are probably thought of as being used in horizontal positions and subjected to gravity or vertical loads, but there are frequent exceptions—roof rafters, for example.

Among the many types of beams are joists, lintels, spandrels, stringers, and floor beams. *Joists* are the closely spaced beams supporting the floors and roofs of buildings, while *lintels* are the beams over openings in masonry walls, such as windows and doors. *Spandrel beams* support the exterior walls of buildings and perhaps part of the floor and hallway loads. The discovery that steel beams as a part of a structural frame could support masonry walls (together with the development of passenger elevators) is said to have permitted the construction of today's high-rise buildings, *Stringers* are the beams in bridge floors running parallel to the roadway, whereas *floor beams* are the larger beams in many bridge floors, which are perpendicular to the roadway of the bridge and are used to transfer the floor loads from the stringers to the supporting girders or trusses. The term *girder* is rather loosely used, but usually indicates a large beam and perhaps one into which smaller beams are framed. These and other types of beams are discussed in the sections to follow.

8.2 SECTIONS USED AS BEAMS

The W shapes will normally prove to be the most economical beam section, and they have largely replaced channels and S sections for beam usage. Channels are sometimes used for beams subjected to light loads, such as purlins, and in places where clearances available require narrow flanges. They have very little resistance to lateral forces and need to be braced, as illustrated by the sag rod problem in Chapter 4. The W shapes have more steel concentrated in their flanges than do S beams and thus have larger moments of inertia and resisting moments for the same weights. They are relatively wide and have appreciable lateral stiffness. (The small amount of space devoted to



Harrison Avenue Bridge, Beaumont, TX. (Courtesy of Bethlehem Steel Corporation.)

S beams in the AISC Manual clearly shows how much their use has decreased from former years. Today, they are used primarily for special situations, such as when narrow flange widths are desirable, where shearing forces are very high, or when the greater flange thickness next to the web may be desirable where lateral bending occurs, as perhaps with crane rails or monorails.)

Another common type of beam section is the open-web steel joist, or bar joist, which is discussed at length in Chapter 19. This type of section, which is commonly used to support floor and roof slabs, is actually a light shop-fabricated parallel chord truss. It is particularly economical for long spans and light loads.

8.3 BENDING STRESSES

For an introduction to bending stresses, the rectangular beam and stress diagrams of Fig. 8.1 are considered. (For this initial discussion, the beam's compression flange is assumed to be fully braced against lateral buckling. Lateral buckling is discussed at length in Chapter 9). If the beam is subjected to some bending moment, the stress at any point may be computed with the usual flexure formula $f_b = Mc/I$. *It is to be remembered, however, that this expression is applicable only when the maximum computed stress in the beam is below the elastic limit.* The formula is based on the usual elastic assumptions: Stress is proportional to strain, a plane section before bending remains a plane section after bending, etc. The value of I/c is a constant for a particular section and is known as the *section modulus* (S). The flexure formula may then be written as follows:

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

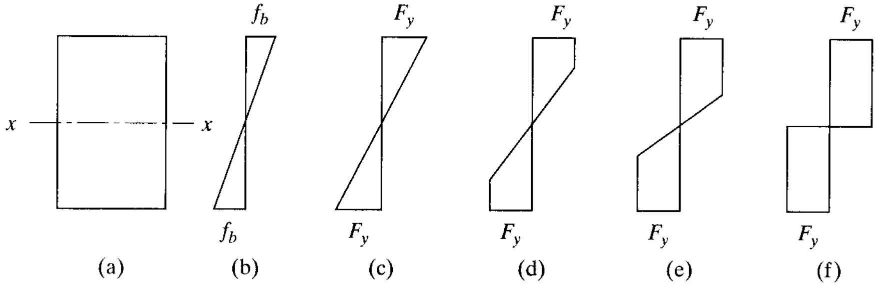


FIGURE 8.1

Variations in bending stresses due to increasing moment about x axis.

Initially, when the moment is applied to the beam, the stress will vary linearly from the neutral axis to the extreme fibers. This situation is shown in part (b) of Fig. 8.1. If the moment is increased, there will continue to be a linear variation of stress until the yield stress is reached in the outermost fibers, as shown in part (c) of the figure. The *yield moment* of a cross section is defined as the moment that will just produce the yield stress in the outermost fiber of the section.

If the moment in a ductile steel beam is increased beyond the yield moment, the outermost fibers that had previously been stressed to their yield stress will continue to have the same stress, but will yield, and the duty of providing the necessary additional resisting moment will fall on the fibers nearer to the neutral axis. This process will continue, with more and more parts of the beam cross section stressed to the yield stress (as shown by the stress diagrams of parts (d) and (e) of the figure), until finally a full plastic distribution is approached, as shown in part (f). Note that the variation of strain from the neutral axis to the outer fibers remains linear for all of these cases. When the stress distribution has reached this stage, a *plastic hinge* is said to have formed, because no additional moment can be resisted at the section. Any additional moment applied at the section will cause the beam to rotate, with little increase in stress.

The *plastic moment* is the moment that will produce full plasticity in a member cross section and create a plastic hinge. The ratio of the plastic moment M_p to the yield moment M_y is called the *shape factor*. The shape factor equals 1.50 for rectangular sections and varies from about 1.10 to 1.20 for standard rolled-beam sections.

8.4 PLASTIC HINGES

This section is devoted to a description of the development of a plastic hinge as in the simple beam shown in Fig. 8.2. The load shown is applied to the beam and increased in magnitude until the yield moment is reached and the outermost fiber is stressed to the yield stress. The magnitude of the load is further increased, with the result that the outer fibers begin to yield. The yielding spreads out to the other fibers, away from the section of maximum moment, as indicated in the figure. The distance in which this yielding occurs away from the section in question is dependent on the loading conditions and the member cross section. For a concentrated load applied at the center line of a simple beam with a rectangular cross section, yielding in the extreme fibers at the time the plastic hinge is formed will extend for one-third of the span. For a W shape in similar

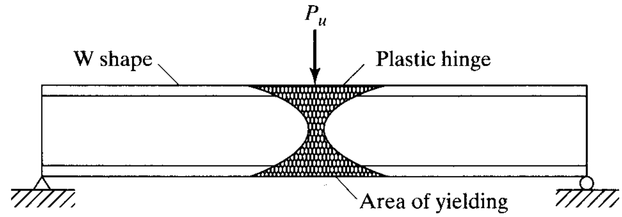


FIGURE 8.2
A plastic hinge.

circumstances, yielding will extend for approximately one-eighth of the span. During this same period, the interior fibers at the section of maximum moment yield gradually, until nearly all of them have yielded and a plastic hinge is formed, as shown in Fig. 8.2.

Although the effect of a plastic hinge may extend for some distance along the beam, for analysis purposes it is assumed to be concentrated at one section. For the calculation of deflections and for the design of bracing, the length over which yielding extends is quite important.

For plastic hinges to form, the sections must be compact. This term was previously introduced in Section 5.7.1. There compact sections were defined as being those which have sufficiently stocky profiles such that they are capable of developing fully plastic stress distributions before they buckle locally. This topic is continued in some detail in Section 9.9.

The student must realize that for plastic hinges to develop the members must not only be compact but also must be braced in such a fashion that lateral buckling is prevented. Such bracing is discussed in Section 9.4.

Finally the effects of shear, torsion, and axial loads must be considered. They may be sufficiently large as to cause the members to fail before plastic hinges can form. In the study of plastic behavior, strain hardening is not considered.

When steel frames are loaded to failure, the points where rotation is concentrated (plastic hinges) become quite visible to the observer before collapse occurs.

8.5 ELASTIC DESIGN

Until recent years, almost all steel beams were designed on the basis of the elastic theory. The maximum load that a structure could support was assumed to equal the load that first caused a stress somewhere in the structure to equal the yield stress of the material. The members were designed so that computed bending stresses for service loads did not exceed the yield stress divided by a safety factor (e.g., 1.5 to 2.0). Engineering structures have been designed for many decades by this method, with satisfactory results. The design profession, however, has long been aware that ductile members do not fail until a great deal of yielding occurs after the yield stress is first reached. This means that such members have greater margins of safety against collapse than the elastic theory would seem to indicate.

8.6 THE PLASTIC MODULUS

The yield moment M_y equals the yield stress times the elastic modulus. The elastic modulus equals I/c or $bd^2/6$ for a rectangular section, and the yield moment equals $F_y bd^2/6$. This same value can be obtained by considering the resisting internal couple shown in Fig. 8.3.

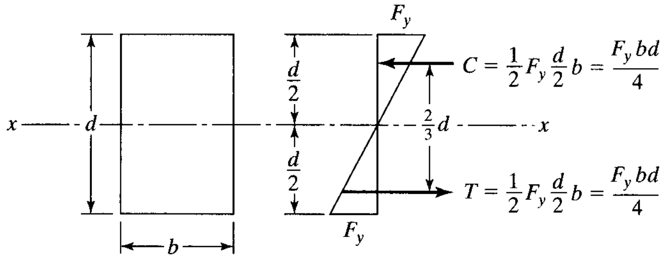


FIGURE 8.3

The resisting moment equals T or C times the lever arm between them, as follows:

$$M_y = \left(\frac{F_y bd}{4}\right)\left(\frac{2}{3}d\right) = \frac{F_y bd^2}{6}$$

The elastic section modulus can again be seen to equal $bd^2/6$ for a rectangular beam.

The resisting moment at full plasticity can be determined in a similar manner. The result is the so-called plastic moment, M_p. *It is also the nominal moment of the section, M_n.* This plastic, or nominal, moment equals T or C times the lever arm between them. For the rectangular beam of Fig. 8.4, we have

$$M_p = M_n = T\frac{d}{2} = C\frac{d}{2} = \left(F_y\frac{bd}{2}\right)\left(\frac{d}{2}\right) = F_y\frac{bd^2}{4}.$$

The plastic moment is said to equal the yield stress times the plastic section modulus. From the foregoing expression for a rectangular section, the plastic section modulus Z can be seen to equal $bd^2/4$. The shape factor, which equals $M_p/M_y = F_yZ/F_yS$, or Z/S, is $(bd^2/4)/(bd^2/6) = 1.50$ for a rectangular section. *A study of the plastic section modulus determined here shows that it equals the statical moment of the tension and compression areas about the plastic neutral axis.* Unless the section is symmetrical, the neutral axis for the plastic condition will not be in the same location as for the elastic condition. The total internal compression must equal the total internal tension. As all fibers are considered to have the same stress (F_y) in the plastic condition, the areas above and below the plastic neutral axis must be equal. This situation does not hold for unsymmetrical sections in the elastic condition. Example 8-1 illustrates the calculations

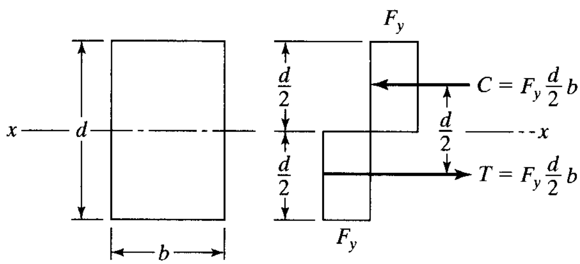


FIGURE 8.4

necessary to determine the shape factor for a tee beam and the nominal uniform load w_n that the beam can theoretically support.

Example 8-1

Determine M_y, M_n, and Z for the steel tee beam shown in Fig. 8.5. Also, calculate the shape factor and the nominal load (w_n) that can be placed on the beam for a 12-ft simple span. $F_y = 50$ ksi.

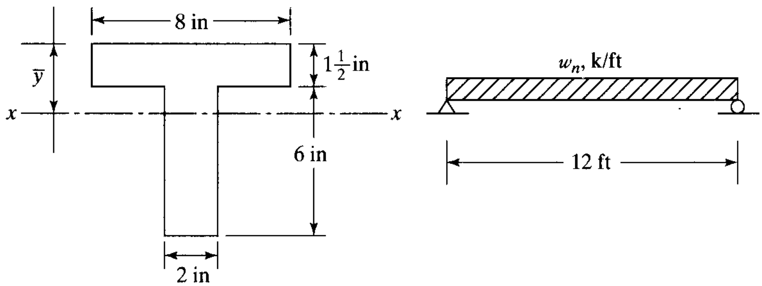


FIGURE 8.5

Solution. Elastic calculations:

$$A = (8\text{ in})\left(1\frac{1}{2}\text{ in}\right) + (6\text{ in})(2\text{ in}) = 24\text{ in}^2$$

$$\bar{y} = \frac{(12\text{ in})(0.75\text{ in}) + (12\text{ in})(4.5\text{ in})}{24\text{ in}^2} = 2.625\text{ in from top of flange}$$

$$\begin{aligned} I &= \frac{1}{12}(8\text{ in})(1.5\text{ in})^3 + (8\text{ in})(1.5\text{ in})(1.875\text{ in})^2 + \frac{1}{12}(2\text{ in})(6\text{ in})^3 \\ &\quad + (2\text{ in})(6\text{ in})(1.875\text{ in})^2 \\ &= 122.6\text{ in}^4 \end{aligned}$$

$$S = \frac{I}{c} = \frac{122.6\text{ in}^4}{4.875\text{ in}} = 25.1\text{ in}^3$$

$$M_y = F_y S = \frac{(50\text{ ksi})(25.1\text{ in}^3)}{12\text{ in/ft}} = 104.6\text{ ft-k}$$

Plastic calculations (plastic neutral axis is at base of flange):

$$Z = (12\text{ in}^2)(0.75\text{ in}) + (12\text{ in}^2)(3\text{ in}) = 45\text{ in}^3$$

$$M_n = M_p = F_y Z = \frac{(50\text{ ksi})(45\text{ in}^3)}{12\text{ in/ft}} = 187.5\text{ ft-k}$$

$$\text{Shape factor} = \frac{M_p}{M_y} \quad\text{or}\quad \frac{Z}{S} = \frac{45\text{ in}^3}{25.1\text{ in}^3} = 1.79$$

$$M_n = \frac{w_n L^2}{8}$$

$$\therefore \quad w_n = \frac{(8)(187.5 \text{ ft-k})}{(12 \text{ ft})^2} = 10.4 \text{ k/ft}$$

The values of the plastic section moduli for the standard steel beam sections are tabulated in Table 3-2 of the AISC Manual, entitled "W Shapes Selection by Z_x", and are listed for each shape in the "Dimensions and Properties" section of the Handbook (Part 1). These Z values will be used continuously throughout the text.

8.7 THEORY OF PLASTIC ANALYSIS

The basic plastic theory has been shown to be a major change in the distribution of stresses after the stresses at certain points in a structure reach the yield stress. The theory is that those parts of the structure that have been stressed to the yield stress cannot resist additional stresses. They instead will yield the amount required to permit the extra load or stresses to be transferred to other parts of the structure where the stresses are below the yield stress, and thus in the elastic range and able to resist increased stress. Plasticity can be said to serve the purpose of equalizing stresses in cases of overload.

As early as 1914, Dr. Gabor Kazinczy, a Hungarian, recognized that the ductility of steel permitted a redistribution of stresses in an overloaded, statically indeterminate structure.[1] In the United States, Prof. J. A. Van den Broek introduced his plastic theory, which he called "limit design." This theory was published in a paper entitled "Theory of Limit Design" in February 1939, in the *Proceedings of the ASCE*.

For this discussion, the stress–strain diagram is assumed to have the idealized shape shown in Fig. 8.6. The yield stress and the proportional limit are assumed to occur at the same point for this steel, and the stress–strain diagram is assumed to be a

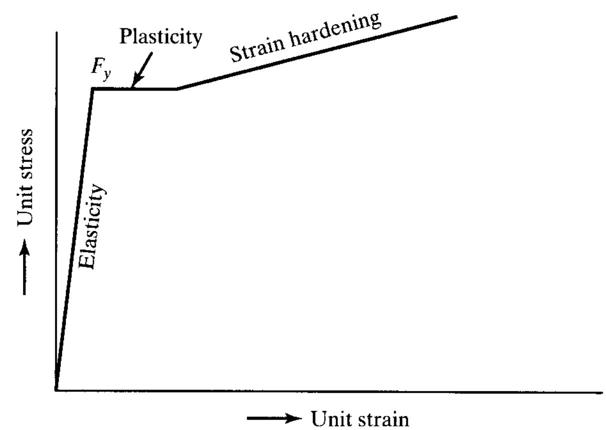


FIGURE 8.6

[1]Lynn S. Beedle, *Plastic Design of Steel Frames* (New York: Wiley, 1958), p. 3.

perfectly straight line in the plastic range. Beyond the plastic range there is a range of strain hardening. This latter range could theoretically permit steel members to withstand additional stress, but from a practical standpoint the strains which arise are so large that they cannot be considered. Furthermore, inelastic buckling will limit the ability of a section to develop a moment greater than M_p, even if strain hardening is significant.

8.8 THE COLLAPSE MECHANISM

A statically determinate beam will fail if one plastic hinge develops. To illustrate this fact, the simple beam of constant cross section loaded with a concentrated load at midspan, shown in Fig. 8.7(a), is considered. Should the load be increased until a plastic hinge is developed at the point of maximum moment (underneath the load in this case), an unstable structure will have been created, as shown in part (b) of the figure. Any further increase in load will cause collapse. P_n represents the nominal, or theoretical, maximum load that the beam can support.

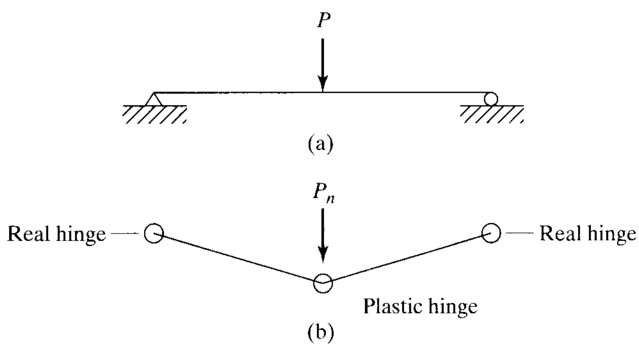


FIGURE 8.7

For a statically indeterminate structure to fail, it is necessary for more than one plastic hinge to form. The number of plastic hinges required for failure of statically indeterminate structures will be shown to vary from structure to structure, but may never be less than two. The fixed-end beam of Fig. 8.8, part (a), cannot fail unless the three plastic hinges shown in part (b) of the figure are developed.

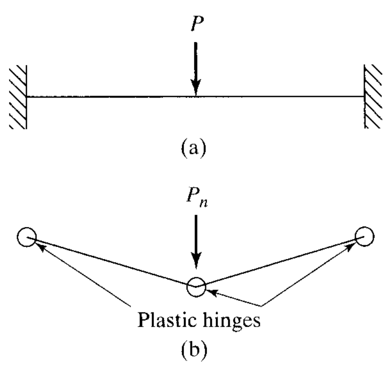


FIGURE 8.8

Although a plastic hinge may have formed in a statically indeterminate structure, the load can still be increased without causing failure if the geometry of the structure permits. The plastic hinge will act like a real hinge insofar as increased loading is concerned. As the load is increased, there is a redistribution of moment, because the plastic hinge can resist no more moment. As more plastic hinges are formed in the structure, there will eventually be a sufficient number of them to cause collapse. Actually, some additional load can be carried after this time, before collapse occurs, as the stresses go into the strain hardening range, but the deflections that would occur are too large to be permissible.

The propped beam of Fig. 8.9, part (a), is an example of a structure that will fail after two plastic hinges develop. Three hinges are required for collapse, but there is a real hinge on the right end. In this beam, the largest elastic moment caused by the design concentrated load is at the fixed end. As the magnitude of the load is increased, a plastic hinge will form at that point.

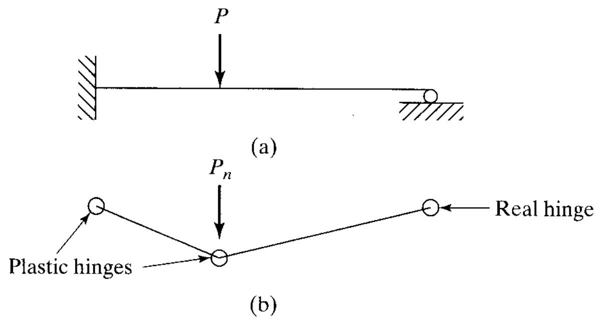


FIGURE 8.9

The load may be further increased until the moment at some other point (here it will be at the concentrated load) reaches the plastic moment. Additional load will cause the beam to collapse. The arrangement of plastic hinges and perhaps real hinges that permit collapse in a structure is called the *mechanism*. Parts (b) of Figs. 8.7, 8.8, and 8.9 show mechanisms for various beams.

After observing the large number of fixed-end and propped beams used for illustration in this text, the student may form the mistaken idea that he or she will frequently encounter such beams in engineering practice. These types of beams are difficult to find in actual structures, but are very convenient to use in illustrative examples. They are particularly convenient for introducing plastic analysis before continuous beams and frames are considered.

8.9 THE VIRTUAL-WORK METHOD

One very satisfactory method used for the plastic analysis of structures is the *virtual-work method*. The structure in question is assumed to be loaded to its nominal capacity, M_n, and is then assumed to deflect through a small additional displacement after the ultimate load is reached. The work performed by the external loads during this displacement is equated to the internal work absorbed by the hinges. For this discussion,

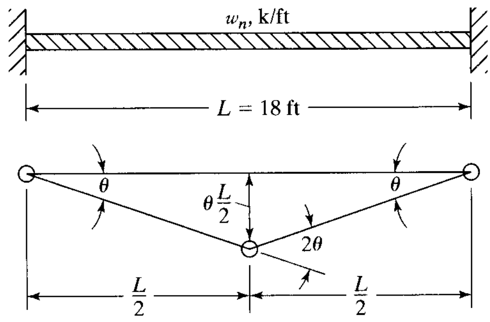


FIGURE 8.10

the *small-angle theory* is used. By this theory, the sine of a small angle equals the tangent of that angle and also equals the same angle expressed in radians. In the pages to follow, the author uses these values interchangeably because the small displacements considered here produce extremely small rotations or angles.

As a first illustration, the uniformly loaded fixed-ended beam of Fig. 8.10 is considered. This beam and its collapse mechanism are shown. Owing to symmetry, the rotations at the end plastic hinges are equal, and they are represented by θ in the figure; thus, the rotation at the middle plastic hinge will be 2θ.

The work performed by the total external load (w_nL) is equal to w_nL times the average deflection of the mechanism. The average deflection equals one-half the deflection at the center plastic hinge $(1/2 \times \theta \times L/2)$. The external work is equated to the internal work absorbed by the hinges, or to the sum of M_n at each plastic hinge times the angle through which it works. The resulting expression can be solved for M_n and w_n as follows:

$$M_n(\theta + 2\theta + \theta) = w_nL\left(\frac{1}{2} \times \theta \times \frac{L}{2}\right)$$

$$M_n = \frac{w_nL^2}{16}$$

$$w_n = \frac{16M_n}{L^2}.$$

For the 18-ft span used in Fig. 8.10, these values become

$$M_n = \frac{(w_n)(18)^2}{16} = 20.25\, w_n$$

$$w_n = \frac{M_n}{20.25}.$$

Plastic analysis can be handled in a similar manner for the propped beam of Fig. 8.11. There, the collapse mechanism is shown, and the end rotations (which are equal to each other) are assumed to equal θ.

The work performed by the external load P_n as it moves through the distance $\theta \times L/2$ is equated to the internal work performed by the plastic moments at the hinges; note that there is no moment at the real hinge on the right end of the beam.

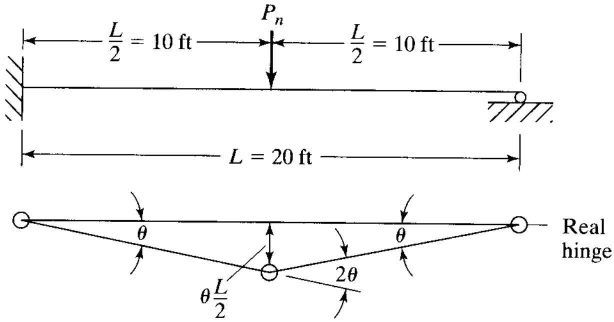


FIGURE 8.11

$$M_n(\theta + 2\theta) = P_n\left(\theta\frac{L}{2}\right)$$

$$M_n = \frac{P_nL}{6} \quad \text{(or } 3.33P_n \text{ for the 20-ft beam shown)}$$

$$P_n = \frac{6M_n}{L} \quad \text{(or } 0.3M_n \text{ for the 20-ft beam shown)}$$

The fixed-end beam of Fig. 8.12, together with its collapse mechanism and assumed angle rotations, is considered next. From this figure, the values of M_n and P_n can be determined by virtual work as follows:

$$M_n(2\theta + 3\theta + \theta) = P_n\left(2\theta \times \frac{L}{3}\right)$$

$$M_n = \frac{P_nL}{9} \quad \text{(or } 3.33P_n \text{ for this beam)}$$

$$P_n = \frac{9M_n}{L} \quad \text{(or } 0.3M_n \text{ for this beam).}$$

A person beginning the study of plastic analysis needs to learn to think of all the possible ways in which a particular structure might collapse. Such a habit is of the greatest importance when one begins to analyze more complex structures. In this light,

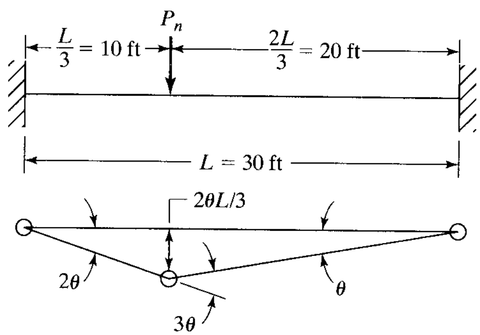


FIGURE 8.12

the plastic analysis of the propped beam of Fig. 8.13 is done by the virtual-work method. The beam with its two concentrated loads is shown, together with four possible collapse mechanisms and the necessary calculations. It is true that the mechanisms of parts (b), (d), and (e) of the figure do not control, but such a fact is not obvious to the average student until he or she makes the virtual-work calculations for each case. Actually, the mechanism of part (e) is based on the assumption that the plastic moment is reached at both of the concentrated loads simultaneously (a situation that might very well occur).

The value for which the collapse load P_n is the smallest in terms of M_n is the correct value (or the value where M_n is the greatest in terms of P_n). For this beam, the second plastic hinge forms at the P_n concentrated load, and P_n equals 0.154 M_n.

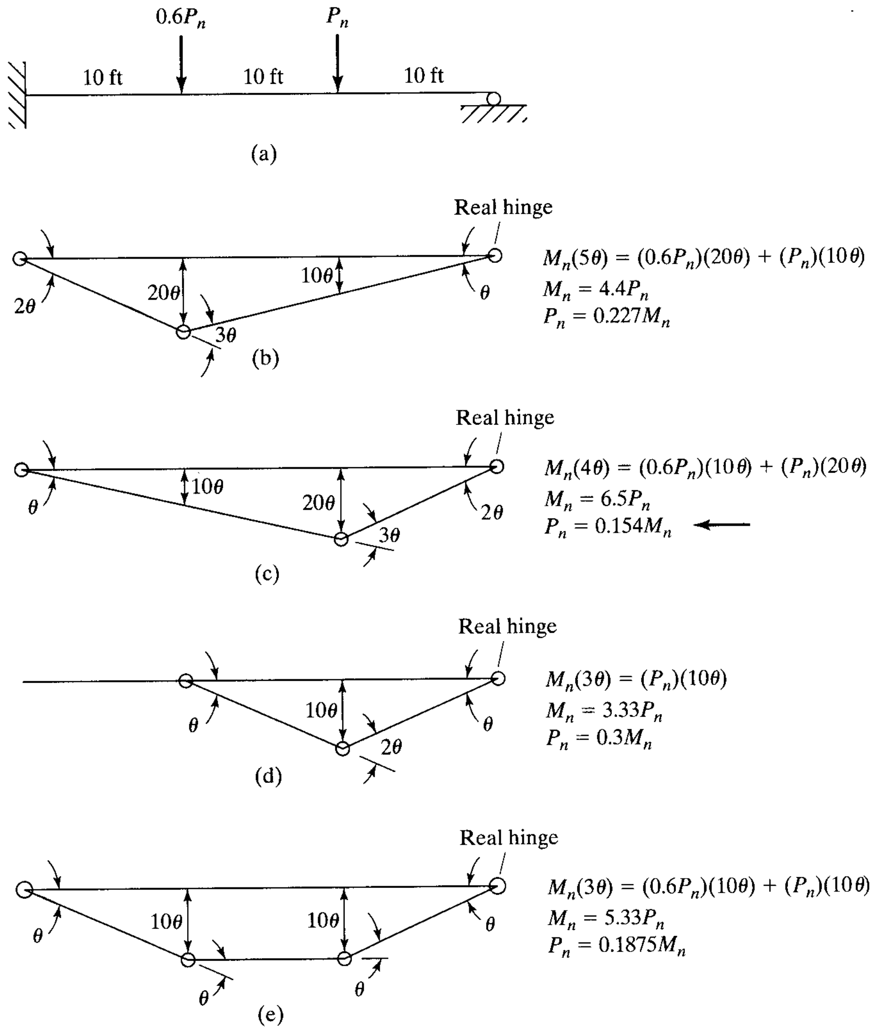


FIGURE 8.13

8.10 LOCATION OF PLASTIC HINGE FOR UNIFORM LOADINGS

There was no difficulty in locating the plastic hinge for the uniformly loaded fixed-end beam, but for other beams with uniform loads, such as propped or continuous beams, the problem may be rather difficult. For this discussion, the uniformly loaded propped beam of Fig. 8.14(a) is considered.

The elastic moment diagram for this beam is shown as the solid line in part (b) of the figure. As the uniform load is increased in magnitude, a plastic hinge will first form at the fixed end. At this time, the beam will, in effect, be a "simple" beam (so far as increased loads are concerned) with a plastic hinge on one end and a real hinge on the other. Subsequent increases in the load will cause the moment to change, as represented by the dashed line in part (b) of the figure. This process will continue until the moment at some other point (a distance x from the right support in the figure) reaches M_n and creates another plastic hinge.

The virtual-work expression for the collapse mechanism of the beam shown in part (c) of Fig. 8.14 is written as follows:

$$M_n\left(\theta + \theta + \frac{L - x}{x}\theta\right) = (w_nL)(\theta)(L - x)\left(\frac{1}{2}\right).$$

Solving this equation for M_n, taking $dM_n/dx = 0$, the value of x can be calculated to equal $0.414L$. This value is also applicable to uniformly loaded end spans of continuous beams with simple end supports, as will be illustrated in the next section.

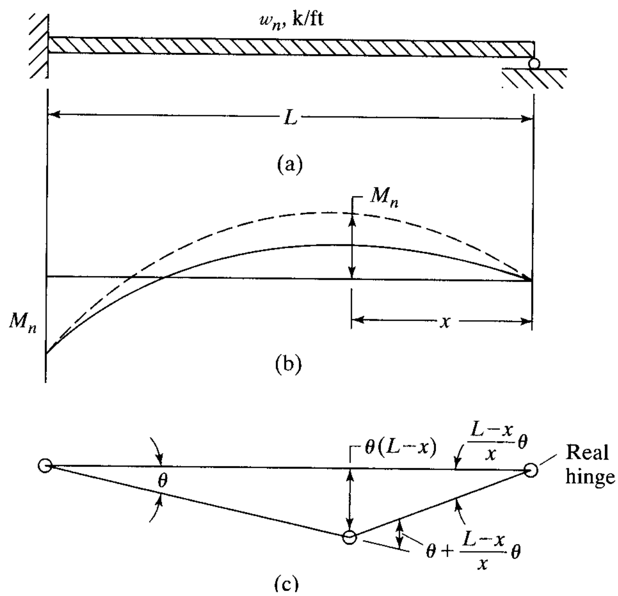


FIGURE 8.14

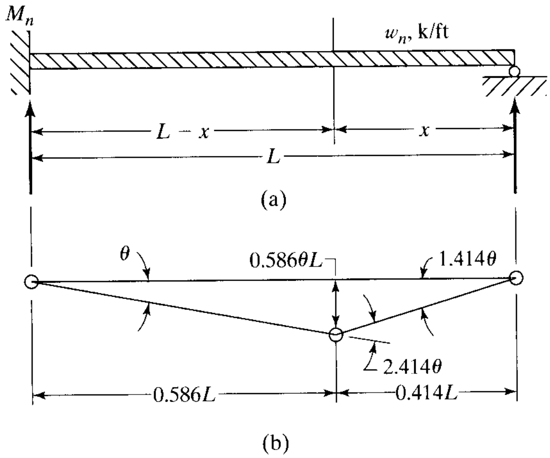


FIGURE 8.15

The beam and its collapse mechanism are redrawn in Fig. 8.15, and the following expression for the plastic moment and uniform load are written by the virtual-work procedure:

$$M_n(\theta + 2.414\theta) = (w_n L)(0.586\theta L)\left(\frac{1}{2}\right)$$

$$M_n = 0.0858 w_n L^2$$

$$w_n = 11.65\frac{M_n}{L^2}$$

8.11 CONTINUOUS BEAMS

Continuous beams are very common in engineering structures. Their continuity causes analysis to be rather complicated in the elastic theory, and even though one of the complex "exact" methods is used for analysis, the resulting stress distribution is not nearly so accurate as is usually assumed.

Plastic analysis is applicable to continuous structures, as it is to one-span structures. The resulting values definitely give a more realistic picture of the limiting strength of a structure than can be obtained by elastic analysis. Continuous, statically indeterminate beams can be handled by the virtual-work procedure as they were for the single-span statically indeterminate beams. As an introduction to continuous beams, Examples 8-2 and 8-3 are presented to illustrate two of the more elementary cases.

Here, it is assumed that if any or all of a structure collapses, failure has occurred. Thus, in the continuous beams to follow, virtual-work expressions are written separately for each span. From the resulting expressions, it is possible to determine the limiting or maximum loads that the beams can support.

Example 8-2

A W18 × 55 ($Z_x = 112 \text{ in}^3$) has been selected for the beam shown in Fig. 8.16. Using 50 ksi steel and assuming full lateral support, determine the value of w_n.

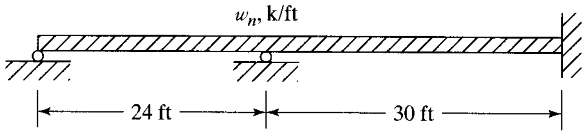
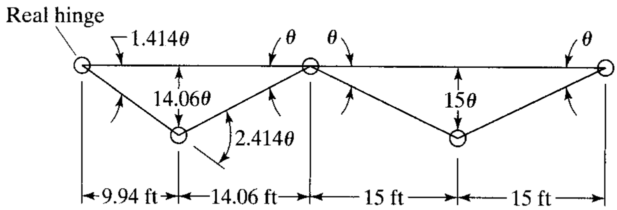


FIGURE 8.16

Solution

$$M_n = F_y Z = \frac{(50 \text{ ksi})(112 \text{ in}^3)}{12 \text{ in/ft}} = 466.7 \text{ ft-k}$$

Drawing the (collapse) mechanisms for the two spans:



Left-hand span:

$$(M_n)(3.414\theta) = (24w_n)\left(\frac{1}{2}\right)(14.06\theta)$$

$$w_n = 0.0202\ M_n = (0.0202)(466.7) = 9.43 \text{ k/ft}$$

Right-hand span:

$$(M_n)(4\theta) = (30w_n)\left(\frac{1}{2}\right)(15\theta)$$

$$w_n = 0.0178\ M_n = (0.0178)(466.7) = 8.31 \text{ k/ft} \leftarrow$$

Additional spans have little effect on the amount of work involved in the plastic analysis procedure. The same cannot be said for elastic analysis. Example 8-3 illustrates the analysis of a three-span beam that is loaded with a concentrated load on each span. The student, from his or her knowledge of elastic analysis, can see that plastic hinges will initially form at the first interior supports and then at the center lines of the end spans, at which time each end span will have a collapse mechanism.

Example 8-3

Using a W21 × 44 ($Z_x = 95.4 \text{ in}^3$) consisting of A992 steel, determine the value of P_n for the beam of Fig. 8.17.

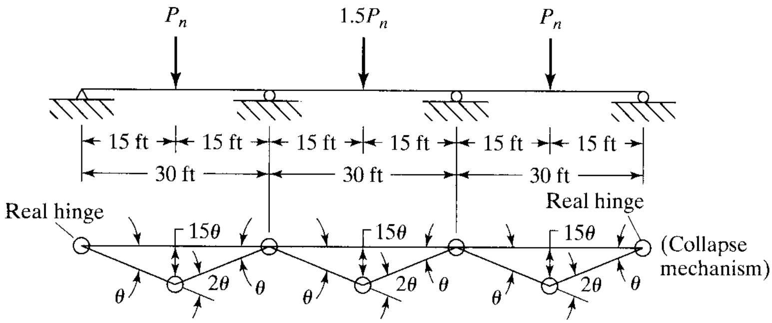


FIGURE 8.17

Solution

$$M_n = F_y Z = \frac{(50 \text{ ksi})(95.4 \text{ in}^3)}{12 \text{ in/ft}} = 397.5 \text{ ft-k}$$

For first and third spans:

$$M_m(3\theta) = (\mathrm{P}_n)(15\theta)$$
$$P_n = 0.2\ M_n = (0.2)(397.5) = 79.5 \text{ k}$$

For center span:

$$(M_n)(4\theta) = (1.5\ \mathrm{P}_n)(15\theta)$$
$$P_n = 0.178\ M_n = (0.178)(397.5) = 70.8 \text{ k} \leftarrow$$

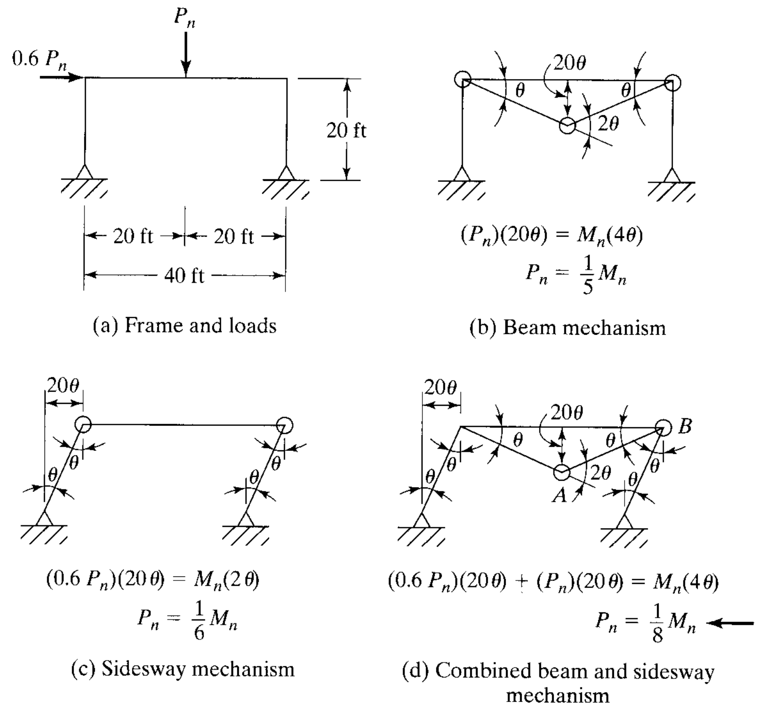
8.12 BUILDING FRAMES

In this section, plastic analysis is applied to a small building frame. It is not the author's purpose to consider frames at length in this chapter. Rather, he wants to show the reader that the virtual-work method is applicable to frames as well as to beams, and that there are other types of mechanisms besides the beam types.

For the frame considered, it is assumed that the same W section is used for both the beam and the columns. If these members differed in size, it would be necessary to take that into account in the analysis.

The pin-supported frame of Fig. 8.18 is statically indeterminate to the first degree. The development of one plastic hinge will cause it to become statically determinate, while the forming of a second hinge can create a mechanism. There are, however, several types of mechanisms that might feasibly occur in this frame. A possible beam mechanism is shown in part (b), a sidesway mechanism is shown in part (c), and a combined beam and sidesway mechanism is shown in part (d). The critical condition is the one that will result in the smallest value of P_n.

Example 8-4 presents the plastic analysis of the frame of Fig. 8.18. The distances through which the loads tend to move in the various mechanisms should be carefully



$(P_n)(20\theta) = M_n(4\theta)$

$P_n = \frac{1}{5}M_n$

(a) Frame and loads

(b) Beam mechanism

$(0.6\,P_n)(20\theta) = M_n(2\theta)$

$P_n = \frac{1}{6}M_n$

$(0.6\,P_n)(20\theta) + (P_n)(20\theta) = M_n(4\theta)$

$P_n = \frac{1}{8}M_n$ ←

(c) Sidesway mechanism

(d) Combined beam and sidesway mechanism

FIGURE 8.18

Possible mechanisms for a frame.

studied. The solution of this problem brings out a point of major significance: *Superposition does not apply to plastic analysis.* You can easily see this by studying the virtual-work expressions for parts (b), (c), and (d) of the figure. The values of P_n obtained for the separate beam and sidesway mechanisms do not add up to the value obtained for the combined beam and sidesway mechanism.

For each mechanism, we want to consider the situation in which we have the fewest possible number of plastic hinges that will cause collapse. If you look at one of the virtual-work expressions, you will note that P_n becomes smaller as the number of plastic hinges decreases. With this in mind, look at part (d) of Fig. 8.18. The frame can feasibly sway to the right without the formation of a plastic hinge at the top of the left column. The two plastic hinges labeled A and B are sufficient for collapse to occur.

Example 8-4

A W12 × 72 (Z_x = 108 in) is used for the beam and columns of the frame shown in Fig. 8.18. If F_y = 50 ksi, determine the value of P_n.

Solution

The virtual-work expressions are written for parts (b), (c), and (d) of Fig. 8.18 and shown with the respective parts of the figure. The combined beam and sidesway case is found to be the critical case, and from it, the value of P_n is determined as follows:

$$P_n = \frac{1}{8}M_n = \left(\frac{1}{8}\right)(F_y Z) = \left(\frac{1}{8}\right)\left(\frac{50 \times 108}{12}\right) = 56.25 \text{ k}$$

8.13 PROBLEMS FOR SOLUTION

8-1 to 8-10. *Find the values of S and Z and the shape factor about the horizontal x axes, for the sections shown in the accompanying figures.*

8-1. (*Ans.* 446.3, 560, 1.25)

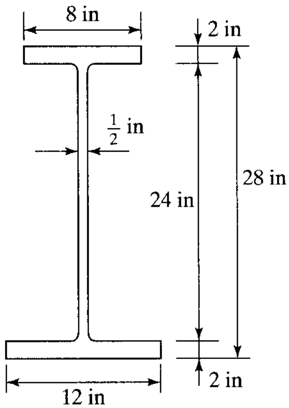


FIGURE P8-1

8-2.

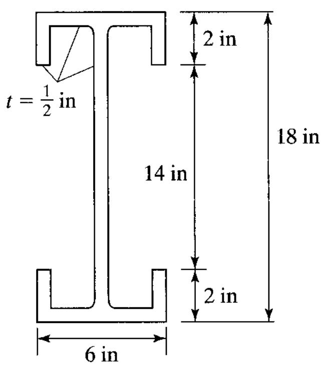


FIGURE P8-2

8-3. (*Ans.* 4.21, 7.15, 1.70)

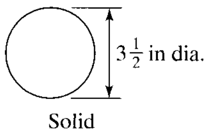


FIGURE P8-3

8-4.

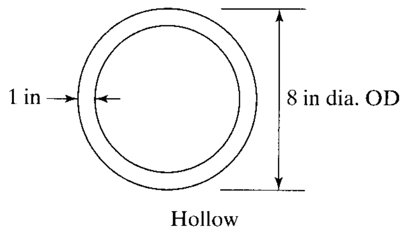


FIGURE P8-4

8-5. (*Ans.* 4.33, 7.78, 1.80)

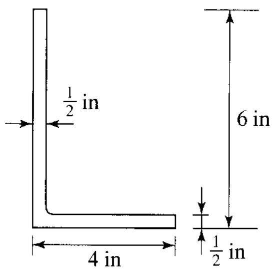


FIGURE P8-5

8-6.

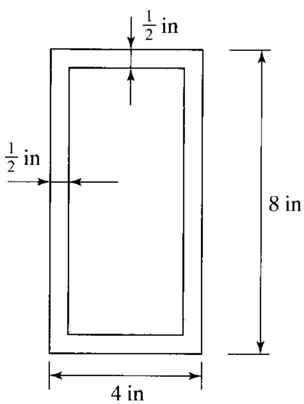


FIGURE P8-6

8-7. (*Ans.* 40.0, 45.8, 1.15)

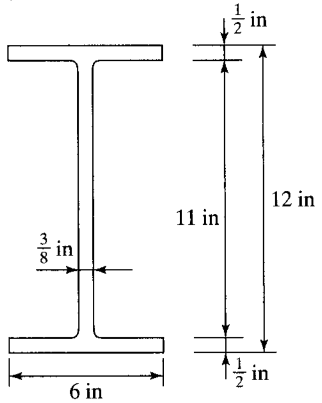


FIGURE P8-7

8-8.

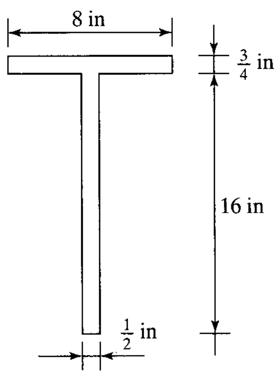


FIGURE P8-8

8-9. (*Ans.* 33.18, 43.0, 1.30)

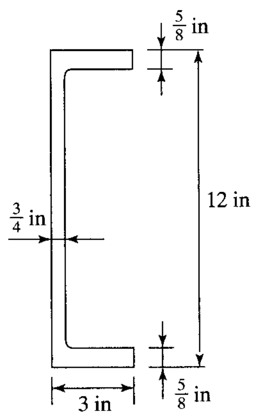


FIGURE P8-9

8-10.

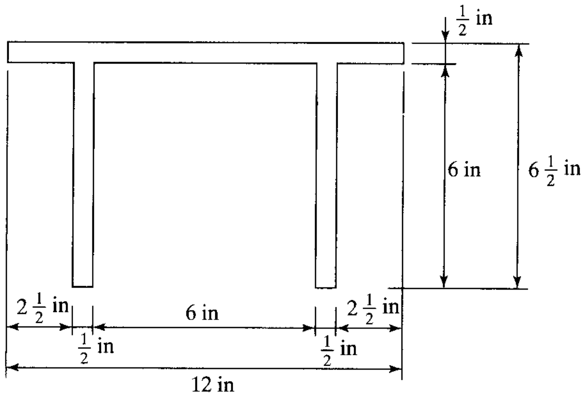


FIGURE P8-10

8-11 to 8-20. *Determine the values of S and Z and the shape factor about the horizontal x axes, unless otherwise directed. Use the web and flange dimensions given in the AISC Manual for making these calculations.*

8-11. A W21 × 122 (*Ans.* 271.8, 305.6, 1.12)

8-12. A W14 × 34 with a cover plate on each flange. The plate is 3/8 × 8 in.

8-13. Two 5 × 3 × 3/8 in Ls long legs vertical (LLV) and back to back. (*Ans.* 4.47, 7.95, 1.78)

8-14. Two C8 × 11.5 channels back to back.

8-15. Four 3 × 3 × 3/8 in Ls arranged as shown in Fig. P8-15. (*Ans.* 4.56, 7.49, 1.64)

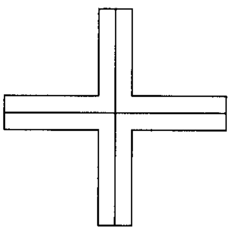


FIGURE P8-15

8-16. A W16 × 31.

8-17. The section of Prob. 8-7, considering the y axis. (*Ans.* 6.02, 9.39, 1.56)

8-18. Rework Prob. 8-9, considering the y axis.

8-19. Rework Prob. 8-12 considering the y axis. (*Ans.* 13.8, 22.6, 1.64)

8-20. Rework Prob. 8-14 considering the y axis.

8-21 to 8-39. *Using the given sections, all of A992 steel, and the plastic theory, determine the values of P_n and w_n as indicated.*

8-21. (*Ans.* 94.3 k)

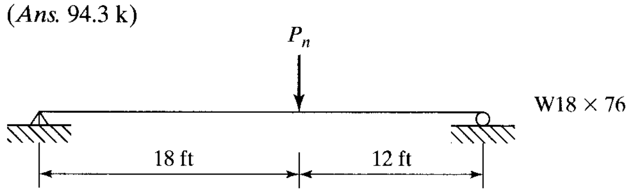


FIGURE P8-21

8-22.

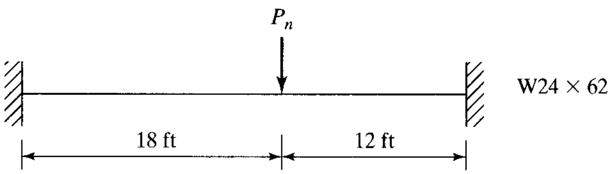


FIGURE P8-22

8-23. (*Ans.* 10.65 k/ft)

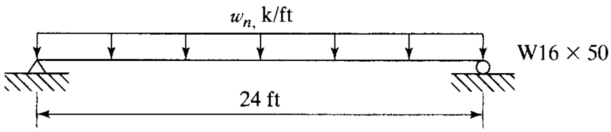


FIGURE P8-23

8-24.

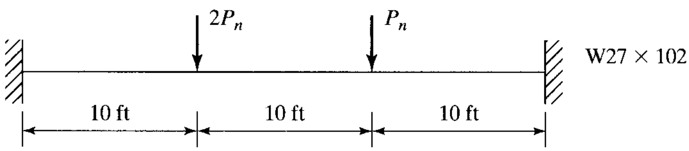


FIGURE P8-24

8-25. (*Ans.* 189.5 k)

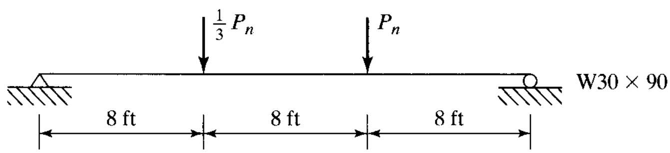


FIGURE P8-25

8-26.

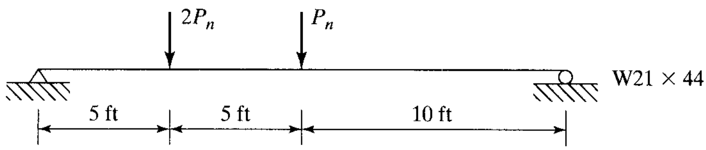


FIGURE P8-26

8-27. (*Ans.* 47.9 k)

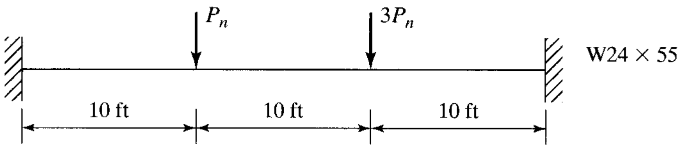


FIGURE P8-27

8-28.

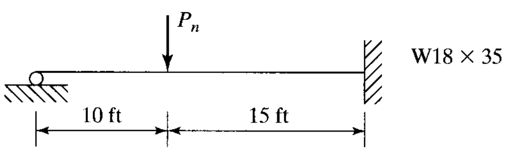


FIGURE P8-28

8-29. (*Ans.* 49.3 k)

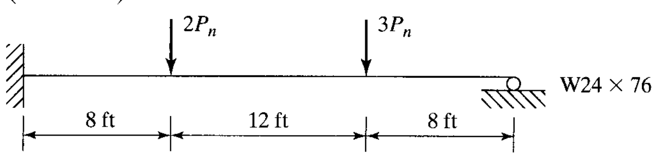


FIGURE P8-29

8-30.

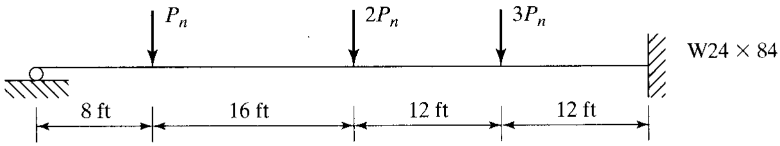


FIGURE P8-30

8-31. (*Ans.* 10.95 k/ft)

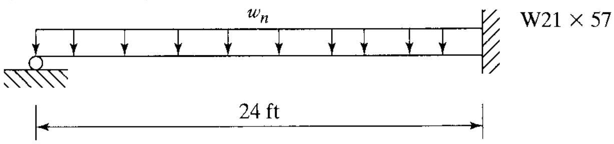


FIGURE P8-31

8-32.

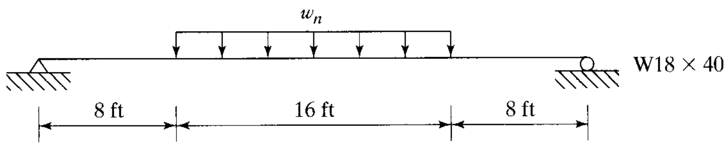


FIGURE P8-32

8-33. (*Ans.* 4.20 k/ft)

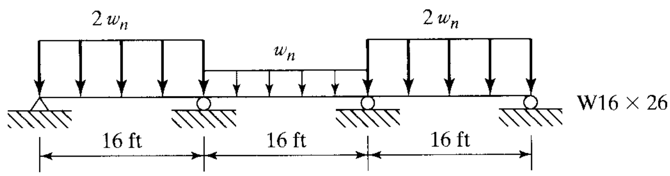


FIGURE P8-33

8-34.

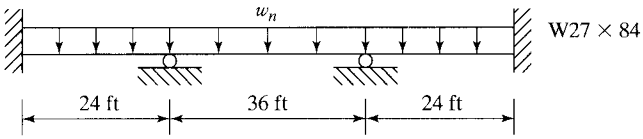


FIGURE P8-34

8-35. (*Ans.* 9.56 k/ft)

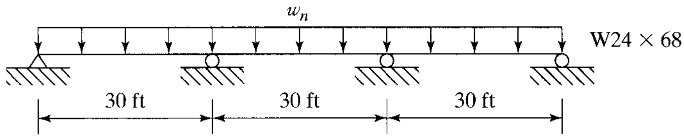


FIGURE P8-35

8-36.

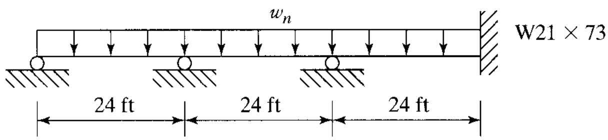


FIGURE P8-36

8-37. (*Ans.* 88.2 k)

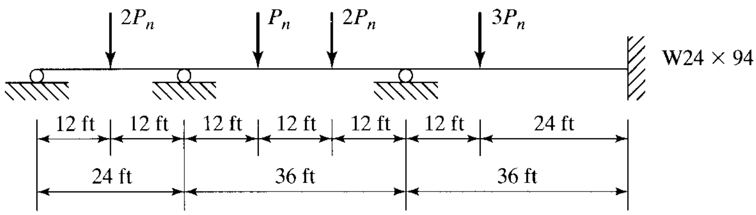


FIGURE P8-37

8-38.

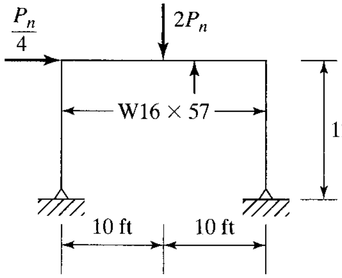


FIGURE P8-38

8-39. Repeat Prob. 8-38 if the column bases are fixed. (*Ans.* 87.5 k)