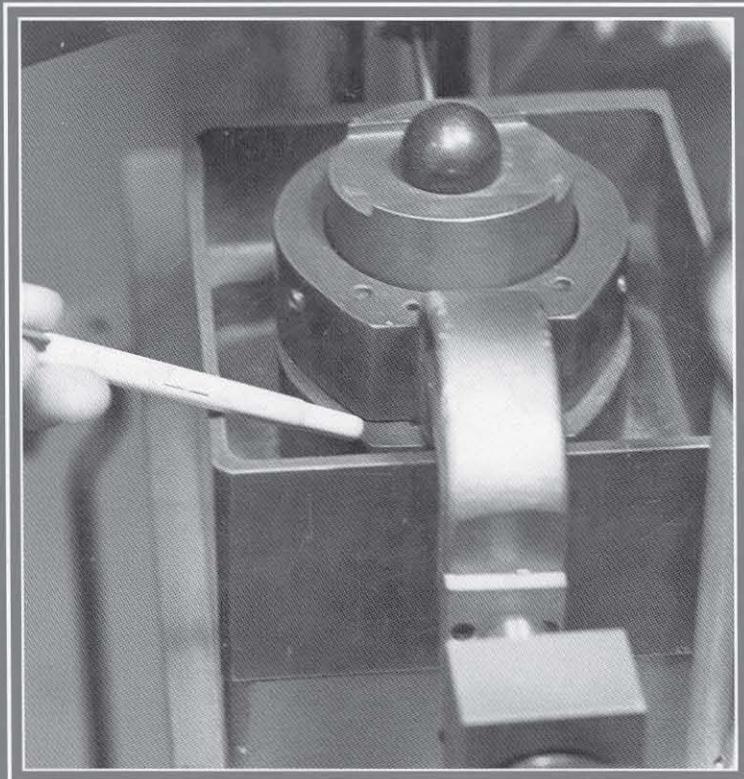


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# EXPERIMENTAL SOIL MECHANICS



JEAN-PIERRE BARDET

# **EXPERIMENTAL SOIL MECHANICS**

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# Preface

In 1983, when I started to teach experimental soil mechanics to the undergraduate students in civil engineering at the University of Southern California, I thought that laboratory soil testing was well covered by textbooks. However, I stumbled on three unexpected but major problems when I attempted to select a textbook for my course: the reliance on standardized testing procedures, outdated data processing, and the absence of typical test results.

In reference to the first problem, most recent textbooks describe soil testing as a set of standardized procedures with little reference to the theories prompting these tests. They are useful for training laboratory technicians in testing companies but are less informative to student engineers. Any standardized testing procedure is not engraved in stone—it only provides guidelines which, when followed carefully, guarantee the quality and repeatability of test results. As an educator I feel strongly that standardized tests should not eclipse the principles that motivated the tests.

In reference to the problem of outdated data processing, the textbooks on soil testing that I examined were filled with complicated and numerous data sheets. They required students to write down their measurements and calculate results with hand calculators and archaic nomographs, such as for the hydrometer test. In this age of personal computers, such tedious data reduction is as outdated as slide rules. To update the data processing I wrote two generations of BASIC programs for personal computers. The first generation produced poor graphics and barely improved the hand calculator results. The second generation had advanced graphic and input/output features but had to be sealed hermetically because it was too complicated. Both versions created many difficulties in entering data and unnecessarily confused soil testing. I found myself asking more fundamental questions. Does the computer improve or hinder students' understanding? Do students benefit from a program they do not understand?

These concerns motivated the use of spreadsheet programs that have an open calculation structure (e.g., Lotus 123, Quattro Pro, and Microsoft Excel). Initially, spreadsheets were intended for business purposes, but now they have mathematical functions and programmable custom functions that make them suit-

able for engineering. They can reduce experimental data, plot graphs, and print reports. The spreadsheet approach eased my apprehensions about using computers in teaching experimental techniques. In fact, spreadsheets enhanced the ways my students understood physical phenomena and processed their measurements. For instance, by using Stokes' and buoyancy laws directly, instead of applying esoteric nomographs, they better understood the principles and limitations of hydrometer analysis. With spreadsheets, I was therefore capable of addressing two of the problems found in textbooks on soil testing: the advanced data processing techniques actually helped to emphasize the principles of soil mechanics. My students also benefited in other ways, enhancing their communication and presentation skills, and producing well-organized laboratory reports.

The third problem with existing textbooks is that there has been no way for students to know if they have performed the test correctly. In the professional world, we look to published experimental results constantly. If our results deviate significantly from normal results, we must either correct our own errors or explain the discrepancy. To establish normal test results for comparison, I have compiled typical results on soil properties from various sources, and included a few useful empirical correlations between soil properties. Empirical correlations are often discredited from the scientific point of view due to their lack of physical and rational explanations; however, some correlations are useful in the laboratory to check approximate agreement between different soil properties. With typical results and empirical correlations, my students have a basic knowledge of values for soil properties and some points of reference for discussing the validity of their experimental results.

Since 1983, my students have been using the spreadsheets programs, comparing their test results, and gaining a thorough understanding of the theories behind the tests. In effect, they have learned much more than basic standardized testing procedures. I hope that *Experimental Soil Mechanics* will benefit other teachers and students in geotechnical engineering.

### **Organization of materials**

The book has nine chapters. The first seven chapters introduce the laboratory experiments (soil classification, density and compaction, permeability and seepage, consolidation, and shear strength) in the order in which most instructors present the material in geotechnical engineering. A systematic and consistent approach to each laboratory experiment covers theory, equipment, experimental procedure, and data processing. This presentation stresses that experiment and theory are meaningful only when considered together. All theories are based on experiments, and all experiments need a theory to explain and apply their results. Each test procedure is illustrated with photographs and line illustrations. Last are review questions and exercises to check understanding.

The eighth chapter covers the basic elements of experimental techniques, such as dimensions and units, data modeling, error analysis, and report writing. The ninth chapter reviews the use of spreadsheet programs relevant to data processing and the tabular and graphical presentation of laboratory results. It is versatile enough to apply not only to soil testing but also to various engineering fields. The spreadsheet experts may skip this section, provided that their expertise takes them through the worked examples.

### **Suggestions**

For students, the theoretical sections should be studied before the experimental procedure and data processing sections. I suggest developing a basic knowledge

of spreadsheet calculation and programming before moving on to the data processing of soil experiments. The book covers a sufficiently large number of experiments to fully occupy a one-semester course. Several unprocessed data sets included in the exercises can be used to substitute for unsuccessful experiments in the laboratory.

My publisher and I have made all possible efforts to correct typographical mistakes. However, there may still be some errors in this first edition. Report them by using the world wide web site for Prentice Hall (<http://www.prenhall.com>) so that I can correct them in the next editions. I also welcome suggestions and comments which may help me to improve the quality and usefulness of this book.

### **Acknowledgments**

I would like to express my thanks to all my students, especially G. Bowyer, P. Chraghchian, G. Gunawan, Q. Huang, M. M. Kapuskar, S. Karimi, N. Mace, A. Mahdi, J. Ramadan, S. Richards, A. Shiv, and J. Young. I extend my thanks to E. Amirkhanian, who assisted in taking photographs. I am also indebted to my wife, Marilyn, for her support and encouragement, and for proofreading the manuscripts.

*Jean-Pierre Bardet*

*To Benoit, Fabienne, Jacques,  
Marcel, Marie-Thérèse, and  
Marilyn.*

# Introduction

In geotechnical engineering, the design and analysis of soil structures, such as earthdams, retaining walls, excavations, slopes, and foundations, are based on the theory of soil mechanics and the experimental determination of soil properties. *Soil mechanics* applies the *principles* of mechanics including kinematics, dynamics, fluid mechanics, and mechanics of materials to soils. Soil mechanics identifies and relates the main engineering properties of soils such as their density, compressibility, and resistance to shear. However, all these theoretical developments largely depend on experiments on soil properties in the laboratory and in the field.

## LABORATORY SOIL TESTING

This book is about *experimental soil mechanics*. It reviews the basic *theory* of soil mechanics and describes the experiments which determine soil properties in the laboratory. It covers the tests listed in Table 1. These tests fall into five major categories: Soil classification, density and compaction, seepage, consolidation, and shear strength. Following is a breakdown of the categories and objectives of the experiments in Table 1.

*Soil classification* categorizes soils according to their probable engineering behavior. By knowing the soil classification, the engineer already has a fairly good idea of the way the soil will behave during construction. However, a soil classification does not eliminate the need for detailed testing for engineering properties.

*Density*, water content, void ratio, specific gravity, etc. are elementary soil properties which characterize the state of soils in the laboratory and in the field. Some of these soil properties such as soil density can be altered by *compaction* to control and improve other types of engineering properties such as seepage, compressibility and shear strength.

*Seepage* refers to the flow of water through soils, which takes place in water-retaining structures such as earthdams and levees. Permeability is the primary soil property required to analyze seepage problems, which can be solved using methods such as electrical analogy and finite differences.

Consolidation is the deferred compressibility of soils following the application of construction loads. This phenomenon is primarily responsible for the long-term settlement of buildings with time, which may take years to be completed.

Shear strength includes the soil properties that characterize the ability of soils to withstand construction loads. Shear strength properties are used in the stability analysis of structures including slopes, retaining walls, and foundations.

### **Field sampling and field testing**

Laboratory testing relies on *field sampling* for taking representative samples of soils from the field. In general, the removal of soil from its natural environment disturbs its natural state. Some sampling techniques minimize this disturbance and produce so-called *undisturbed* samples, which are hermetically sealed in sampling tubes to preserve their moisture content. Other sampling techniques yield *disturbed* samples which are broken into smaller fragments and have density and structures different from the soils in the field. The sampling technique to be used depends on the type of information required and the characteristics of the soil to be sampled. For classification purposes, disturbed samples are sufficient, but for the determination of engineering properties of soil in the laboratory, undisturbed samples are usually required. A description of the types of samplers available and their utilization can be found in Lowe and Zacheo (1991).

*Field testing*, also referred to as *in-situ testing*, is an efficient means to determine soil properties in the field. It produces abundant and detailed information on soil profiles at a location, such as stratigraphy of soil layers and spatial variation of their properties. The determination of soil properties from field measurements is generally based on empirical correlations between quantities measured in the field and soil properties measured in the laboratory (e.g., Kulhawy and Mayne, 1990). Recent developments in field testing are surveyed by Jamiolkowski et al. (1985).

### **Report of Laboratory Tests**

Suggestions and guidelines for the preparation of soil reports can be found in Chapter 8-5. It is useful to compare experimental results recently obtained with those from other sources to check the validity of the results, and to understand possible similarities and discrepancies. This book provides some data on soil properties taken from various sources including Biarez and Hicher, 1994; Bowles, 1992; Head, 1984, 1986, and 1988; Holtz and Kovacz, 1981; Lambe, 1951; Lambe and Whitman, 1979; and Mitchell, 1993.

In the practice of geotechnical engineering, soil testing addresses real problems. In the academic context, it is instructive to recreate such a goal-oriented approach by posing practical yet simple problems which require the measurement of several soil properties. A few examples of this goal-oriented approach are given in Projects.

**TABLE 1**

Categories and objectives of the laboratory experiments in this book

Category and main objectives	Laboratory tests	Specific objectives
<b>Engineering soil classification</b> Categorize soils according to their probable engineering behavior	<ul style="list-style-type: none"> <li>• <u>Grain size analysis</u> <ul style="list-style-type: none"> <li>• Sieve analysis</li> <li>• Sedimentation analysis           <ul style="list-style-type: none"> <li>• Hydrometer</li> <li>• Pipette</li> <li>• Buoyancy</li> </ul> </li> <li>• Combined analysis</li> </ul> </li>   <li>• <u>Atterberg limit tests</u> <ul style="list-style-type: none"> <li>• Plastic limit test</li> <li>• Liquid limit test</li> </ul> </li> <li>• Shrinkage limit analysis           <ul style="list-style-type: none"> <li>• Mercury method</li> <li>• Wax method</li> </ul> </li> <li>• Engineering soil classification</li> </ul>	<ul style="list-style-type: none"> <li>• Determine grain size distribution curve</li> <li>• Test for coarse-grained soils</li> <li>• Test for fine-grained soils           <ul style="list-style-type: none"> <li>• ASTM</li> <li>• British Standards</li> </ul> </li>   <li>• Combine sieve and sedimentation analysis</li> <li>• Determine plasticity of fine-grained soils           <ul style="list-style-type: none"> <li>• Measure plastic limit PL</li> <li>• Measure liquid limit LL</li> </ul> </li> <li>• Define the shrinkage and swelling potential of fine-grained soils</li>   <li>• Identify soil group in USCS and AASHTO engineering soil classification based on results of grain size analysis and Atterberg limit tests</li> </ul>
<b>Density and compaction</b> Determine basic states of soils in the laboratory and in the field	<ul style="list-style-type: none"> <li>• Determination of unit weight</li>   <li>• Determination of specific gravity</li>   <li>• Standard and improved laboratory compaction tests</li> <li>• Sand cone test</li> </ul>	<ul style="list-style-type: none"> <li>• Determine unit weight, void ratio, degree of saturation, and water content of fine-grained soils</li> <li>• Determine the unit weight of soil minerals</li> <li>• Define the optimum water content and maximum density for soils</li> <li>• Control the soil density in the field after field compaction</li> </ul>
<b>Seepage</b> Calculate total head, water pressure, total flow, and hydraulic gradients in seepage problems	<ul style="list-style-type: none"> <li>• Permeability tests           <ul style="list-style-type: none"> <li>• Constant head test</li> <li>• Falling head test</li> </ul> </li> <li>• Electrical analogy of seepage problems</li>   <li>• Finite difference solution of seepage problems</li> </ul>	<ul style="list-style-type: none"> <li>• Measure the permeability coefficient of soils</li> <li>• Test for coarse-grained soils</li> <li>• Test for fine-grained soils</li> <li>• Solve seepage problems (e.g., flow of water under a sheetpile wall) with a physical means</li> <li>• Solve seepage problems with numerical methods and spreadsheets</li> </ul>
<b>Consolidation</b> Calculate long term settlement of structures	<ul style="list-style-type: none"> <li>• Consolidation test</li> </ul>	<ul style="list-style-type: none"> <li>• Determine the properties of fine-grained soils for calculating the amplitude and rate of settlement of structures           <ul style="list-style-type: none"> <li>• Compressibility</li> <li>• Overconsolidation ratio and pressure</li> <li>• Consolidation coefficients (primary and secondary)</li> </ul> </li> </ul>
<b>Shear strength</b> Determine the soil properties (undrained shear strength $S_u$ , friction angle $\phi'$ and cohesion $c'$ ) for analyzing the stability of foundations, excavations, slopes, retaining walls, etc.	<ul style="list-style-type: none"> <li>• Unconfined compression test (UC)</li> <li>• Direct shear test (DS)</li>   <li>• Triaxial tests           <ul style="list-style-type: none"> <li>• CD and CU triaxial tests on coarse-grained soils</li> <li>• CD, CU, and UU triaxial tests on fine-grained soils</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• Measure rapidly but approximately <math>S_u</math></li> <li>• Measure shear strength (<math>S_u</math>, <math>\phi'</math> and <math>c'</math>) on a predetermined surface of rupture (slopes, foundation, etc.)</li> <li>• Measure shear strength (<math>S_u</math>, <math>\phi'</math> and <math>c'</math>) under various stress conditions, including drained and undrained loadings. Better control of initial stresses and loading stress paths than UC and DS tests (except for UU tests)</li> </ul>

## STANDARD PROCEDURES FOR SOIL TESTING

In the United States of America, Great Britain, and many other countries, most of the experimental procedures in laboratory soil testing are described by standards. Standards provide guidelines and minimum requirements to obtain reliable and repeatable test results. Tables 2 and 3 give the correspondence between the laboratory experiments covered in each test, and the relevant test methods and guides of the American Society for Testing and Materials (ASTM) and the British Standard Institutions (BS). Further information can be found in these guidelines and in Head (1984, 1986, and 1988).

TABLE 2

Experiments covered in this book and related ASTM test methods (ASTM, 1995a and 1995b)

Experiment	Chapter	ASTM number	ASTM description
<b>Sieve analysis</b>	1-2	D 422-63	Test Method of Particle-Size Analysis of Soils
		D 1556-90	Test Method for Amount of Material in Soils Finer than the No. 200 (75 mm) Sieve
		D 421-85	Practice for Dry Preparation of Soil Samples for Particle-Size Analysis and Determination of Soil Constants
		D 2217-85	Practice for Wet Preparation of Soil Samples for Particle-Size Analysis and Determination of Soil Constants
		E 11-87	Specifications for Wire-Cloth Sieves for Testing Purposes
<b>Hydrometer analysis</b>	1-4	D 422-63	Test Method of Particle-Size Analysis of Soils
		D 421-85	Practice for Dry Preparation of Soil Samples for Particle-Size Analysis and Determination of Soil Constants
		E 100-94	Specifications for ASTM Hydrometers
<b>Determination of water content</b>	2-2	D 2216-90	Method of Laboratory Determination of Water (Moisture) Content of Soil, Rock, and Soil-Aggregate Mixtures
		D 4944-89	Test Method for Field Determination of Water (Moisture) Content of Soil by Calcium Carbide Gas Pressure Tester Method
		D 4643-93	Test Method for Determination of Water (Moisture) Content of Soil by the Microwave Oven Method
		D 4959-89	Test Method for Determination of Water (Moisture) Content of Soil by Direct Heating Method
<b>Atterberg limit test</b>	2-3 to 2-5	D 4318-93	Test Method for Liquid Limit, Plastic Limit, and Plasticity Index of Soils
<b>Shrinkage limit</b>	2-7 and 2-8	D 427-93	Test Method for Shrinkage Factor of Soils by the Mercury Method
		D 4943-89	Test Method for Shrinkage Factor of Soils by the Wax Method
<b>Engineering classification of soils</b>	2-9	D 2487-93	Test Method for Classification of Soils for Engineering Purposes (Unified Soil Classification System)
		D 3282-93	Test Method for Classification of Soils and Soil-Aggregates Mixtures for Highway Construction Purposes
		D 2488-93	Practice for Description and Identification of Soils (Visual/Manual Procedure)
		D 448-86	Classification for Sizes of Aggregate for Road and Bridge Construction
<b>Determination of density</b>	3-2	D 4253-93	Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table
		D 4254-91	Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density
		D 4718-87	Practice for Correction of Unit Weight and Water Content for Soils Containing Oversize Particles

**TABLE 2 (cont.)**

Experiments covered in this book and related ASTM test methods (ASTM, 1995a and 1995b)

Experiment	Chapter	ASTM number	ASTM description
Determination of specific gravity	3-3	D 854-92 C 127-88 D 5550-94	Standard Test Method for Specific Gravity of Soils Test Method for Specific Gravity and Absorption of Coarse Aggregate Test Method for Soil Solids by Gas Pycnometer
Compaction tests	3-5	D 1140-54 D 1557-91 D 2168-90 D 5080-93	Test Method for Laboratory Compaction Characteristics of Soil Using Standard Effort (12,400 ft-lbf/ft <sup>3</sup> (600 kN-m/m <sup>3</sup> )) Test Method for Laboratory Compaction Characteristics of Soil Using Modified Effort (56,000 ft-lbf/ft <sup>3</sup> (2700 kN-m/m <sup>3</sup> )) Test Method for Calibration of Laboratory Mechanical Hammer Soil Compactors Test Method for Rapid Determination of Percent Compaction
In-situ density determination	3-6	D 1556-90 D 2167-94 D 2922-91 D 5195-91 D 2937-94 D 4564-93 D 4914-89 D 5030-89 D 3017-88 D 5220-92	Test Method for Density of Soil In Place by the Sand-Cone Method Test Method for Density and Unit Weight of Soil In Place by the Rubber Balloon Method Test Method for Density of Soil and Soil-Aggregate In-Place by Nuclear Methods (Shallow Depth) Test Method for Density of Soil and Rock In-Place at Depths Below the Surface by Nuclear Methods Test Method for Density of Soil In Place by the Drive-Cylinder Method Test Method for Density of Soil In Place by the Sleeve Method Test Method for Density of Soil and Rock In Place by the Sand Replacement Method in a Test Pit Test Method for Density of Soil and Rock In Place by the Water Replacement Method in a Test Pit Test Method for Density of Soil and Rock In-Place by Nuclear Methods (Shallow Depth) Test Method for Density of Soil and Rock In-Place by the Neutron Depth Probe Method
Permeability test	4-2	D 2434-68 D 5084-90	Test Method for Permeability of Granular Soils (Constant Head) Test Method for Measurement of Hydraulic Conductivity of Saturated Porous Materials Using a Flexible Wall Permeameter
Consolidation test	6-2	D 2435-90 D 4186-89 D 5333-92 D 4546-90	Test Method for One-Dimensional Consolidation Properties of Soils Test Method for One-Dimensional Consolidation Properties of Soils Using Controlled-Strain Loading Test Method for Measurement of Collapse Potential of Soils Test Method for One-Dimensional Swell or Settlement Potential of Cohesive Soils
Unconfined compression test	7-3	D 2166-91	Test Method for Unconfined Compressive Strength of Cohesive Soils
Direct shear test	7-5	D 3080-90	Test Method for Direct Shear Test of Soils Under Consolidated Drained Conditions
Triaxial tests	7-8	D 4767-88 D 2850-87	Test Method for Consolidated-Undrained Triaxial Compression Test on Cohesive Soils Test Method for Unconsolidated, Undrained Compressive Strength of Cohesive Soils in Triaxial Compression
Units	8-4	E 380	Excerpts from Standard Practice for Use of the International System of Units (SI) (the Modernized Metric System)

**TABLE 3**

Experiments covered in this book and related BS 1377 (1975) test methods

Experiment	Chapter	Test number	BS description
Sieve analysis	1-2	7B 7A 7D	Test Sieves (BS 410, 1969) Dry Sieving Wet Sieving
Hydrometer analysis	1-4	7C	Hydrometer Analysis
Pipette analysis	1-5	7C	Pipette Analysis
Determination of water content	2-2	1A 1B	Oven Drying Sand Bath Method
Atterberg limit test	2-3 to 2-5	2B 2C 2A 3	Casagrande Method Liquid Limit-Casagrande One-point Method Cone Penetrometer Method Plastic Limit
Determination of density	3-2	15F 15E 6B	Water Displacement Weighing in Water Density Bottle
Determination of specific gravity	3-3	6A 6B	Gas Jar Method Pycnometer
Compaction tests	3-5	12 13 14	Ordinary Compaction Test Heavy Compaction Compaction by Vibration; Vibrating Hammer Method
Consolidation test	6-2	17	Oedometer Consolidation
Unconfined compression test	7-3	20	Autographic Unconfined Compression
Triaxial tests	7-8	21	Triaxial Compression

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## PROJECTS

The following projects include several types of experiments on the same soil, and require comprehensive reports with recommendations and conclusions. They may require several weeks to be completed.

1. Classify a soil based on the results of the sieve and sedimentation analysis, then determine if the soil is appropriate for the following applications:
  - 1.1 Core of earthdam
  - 1.2 Shell of earthdam
  - 1.3 Fill resistant to frost heave
  - 1.4 Workable construction materials.
2. Compare the results of hydrometer, pipette and buoyancy analyses on the same soil. Conclude on the pros and cons of each method. Write a recommendation for your company which is about to select one of these methods.
3. In a site which has previously been compacted, compare the optimum water and density from laboratory tests with the corresponding values in the field. Conclude on the degree of compaction in the field.
4. Solve one of the seepage problems of Chapters 4-4 and 4-5 for the permeability you measured in the laboratory.
5. Solve one of the seepage problems of Chapters 4-4 and 4-5 by using both electrical analogy and finite difference. Superimpose and compare the flow nets obtained from each method.
6. Compare the values of permeability coefficients measured in the fixed wall permeameter and triaxial cell.
7. Compare the values of permeability coefficients measured in the falling head permeameter and consolidation cell.
8. For a given embankment or tank, calculate the settlement for the soil properties you measured in the laboratory.
9. Compare the values of undrained shear strength on the same fine-grained material obtained from the unconfined compression test, direct shear test, and CU triaxial test. Compare your results and conclusions with those given in Chapter 7-2.
10. For a given slope stability problem, calculate the factor of safety of the slope based on the laboratory test which you believe is the most appropriate.

# 1

# Grain Size Distribution

- 1-1** Principles of grain size analysis
- 1-2** Sieve analysis
- 1-3** Principles of sedimentation analysis
- 1-4** Hydrometer analysis
- 1-5** Pipette analysis
- 1-6** Buoyancy analysis
- 1-7** Combined grain size analysis

# 1-1 Principles of Grain Size Analysis

## SCOPE

As shown in Figure 1, a soil consists of particles of various shapes, sizes, and quantity. The grain size analysis divides these particles into size groups and determines their relative proportions by weight. Grain size analysis is a basic laboratory test required to identify soils in engineering soil classification systems.



**Figure 1** The particles of this soil were divided into six size groups in the laboratory. The largest particle shown is 15 mm in diameter. The largest sizes in each group from left to right are 0.07, 0.3, 1, 2, 5, and 15 mm, respectively.

## GROUPS OF PARTICLE SIZES

As shown in Table 1, the engineering soil classification systems such as ASTM D 422, BS 1377, AASHTO, and USCS, divide soil particles on the basis of size into

categories—boulders, cobbles, gravel, sand, silt, and clay—with an optional subdivision indicating coarse, medium and fine. Soil particles have sizes ranging from greater than 200 mm down to less than 0.002 mm. The ratio between these extremes is  $10^5 : 1$ . As the particle size  $D$  decreases, the number of particles contained in a unit mass of soil increases proportionally to  $1/D^3$  and their individual mass decreases in the same ratio. For illustration, the mass of several spheres with diameters ranging from 100 to 0.001 mm is calculated in Table 2 for a typical mineral unit mass  $\rho_s = 2.65 \text{ g/cm}^3$  (unit mass is the mass per unit volume). The ratio of smallest and largest masses is enormous  $-10^{15} : 1$ .

**Specific surface**, which is the total surface area of particles per unit mass, is an important characteristic of small soil particles. It largely influences the interaction between small soil particles which depends on electrostatic forces. The dimension of specific surface is squared length divided by mass, and its unit is  $\text{mm}^2/\text{g}$  or  $\text{m}^2/\text{g}$ . As shown in Table 2, the specific surface of spheres increases proportionally to  $1/D$ . However, natural soil particles have much more complicated shapes than those of spheres, and their specific surface is even higher than those shown in Table 2. For instance, the specific surface of fine sand particles is about  $0.03 \text{ m}^2/\text{g}$ , while those of flat and platelike clay particles such as kaolinite and montmorillonite are  $10$  and  $1000 \text{ m}^2/\text{g}$ .

**TABLE 1**  
Classification of particle size in the BS 1377, USCS, AASHTO and ASTM engineering soil classification systems.

**TABLE 2**  
Particle Size, Mass, and Surface Area of Spheres.

Equivalent soil category	Particle size D (mm)	Particle mass m (g)	Number of particles per gram	Specific surface area (m <sup>2</sup> /g)
Cobble	100.0	$1.4 \times 10^3$	$7.2 \times 10^{-4}$	$2.3 \times 10^{-5}$
Gravel	10.0	1.4	$7.2 \times 10^{-1}$	$2.3 \times 10^{-4}$
Coarse sand	1.0	$1.4 \times 10^{-3}$	$7.2 \times 10^2$	$2.3 \times 10^{-3}$
Fine sand	0.1	$1.4 \times 10^{-6}$	$7.2 \times 10^5$	$2.3 \times 10^{-2}$
Medium silt	0.01	$1.4 \times 10^{-9}$	$7.2 \times 10^8$	$2.3 \times 10^{-1}$
Clay	0.001	$1.4 \times 10^{-12}$	$7.2 \times 10^{11}$	2.3

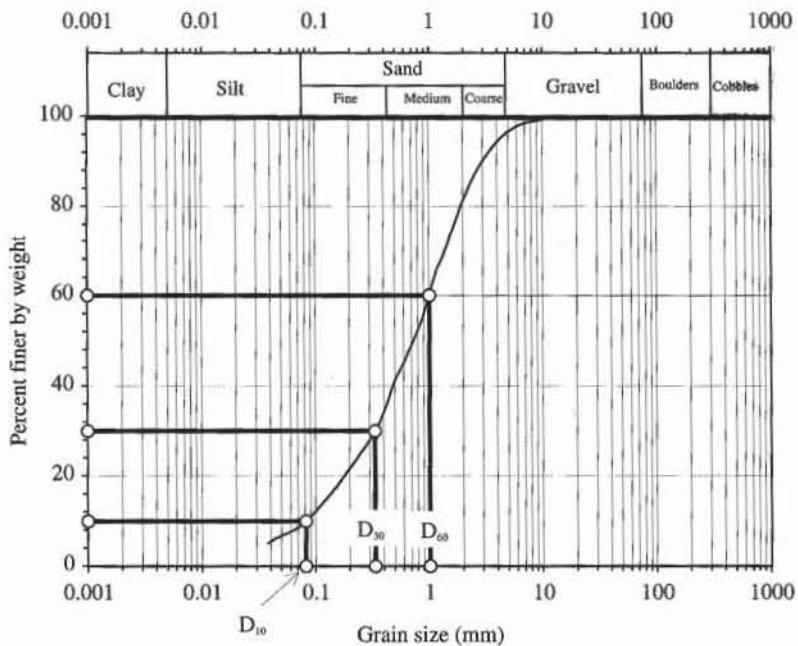
$m = \rho_s \frac{\pi}{6} D^3$  = mass of sphere of diameter  $D$  and unit mass  $\rho_s$  ( $\rho_s = 2.65 \text{ g/cm}^3$ )

$\frac{6}{\rho_s \pi D^3}$  = number of particles per gram

$\frac{6\pi D^2}{\pi \rho_s D^3} = \frac{6}{\rho_s D}$  = specific surface of

### PARTICLE SIZE DISTRIBUTION CURVE

In Table 1, six categories of particle sizes were defined: boulder, cobble, gravel, sand, silt, and clay. However, natural soils are often made of a mixture of particles that do not fall entirely within only one of these size ranges covering two or more categories. It would be too tedious to count individually all the different sizes of particles because of the enormous number (see Table 2). Therefore, the grain distribution within the various size categories is generally represented by using the cumulative distribution of statistics. The grain size distribution curve is plotted as the percentage finer than a given size versus the particle sizes on a logarithmic scale. Figure 2 shows a typical particle size distribution curve. Grain-size distribution curves are always increasing because they are cumulative distribution. Plotting particle size data on a standard chart enables engineers to recognize instantly the grading characteristics of a soil. The position of a curve on the chart indicates the soil fineness or coarseness. The steepness, flatness, and general shape indicate the distribution of grain sizes. Other types of charts are also found in practice. For example, the horizontal axis may have an opposite orientation, with cobbles and gravel to the left. The vertical axis may also represent percent coarser by weight, instead of percent finer by weight.



**Figure 2** Grain size distribution chart.

In addition to the cumulative frequency distributions, there are other graphical representations for particle size distribution, such as frequency distributions. In this case, the percentage by mass between certain sizes is plotted versus the logarithm of the grain size. These representations are used in powder technology (e.g., Allen, 1974). The semilogarithmic cumulative grain size distribution curve remains the most commonly used representation for soils.

### APPLICATION OF GRAIN SIZE DISTRIBUTION CURVES

Grain size distribution curves provide a means by which soils can be classified and their engineering properties assessed. This classification, which applies primarily to sands and gravels, will be completed in Chapter 2-9.

### Classification of Sands and Gravels

Grain size distribution curves enable sands and gravels to be classified into three main types: uniform, well graded, and poorly graded.

**Uniform soils.** In uniform soils, the majority of grains are nearly the same size. The grading curve is very steep, as shown by curve A in Fig. 3, which represents a uniform sand. The uniformity in soils is characterized by the uniformity coefficient  $C_u$ :

$$C_u = \frac{D_{60}}{D_{10}} \quad (1)$$

where  $D_{10}$  is the grain size corresponding to 10% finer and  $D_{60}$  is the grain size corresponding to 60% finer.  $D_{10}$  and  $D_{60}$  are obtained by interpolation between the experimental points of the grain size distribution curve (see Fig. 2).  $C_u$  represents the average slope of the grain size distribution between 10 and 60%. For instance, in Fig. 2,  $D_{10} = 0.082$  mm,  $D_{60} = 1.008$  mm, and  $C_u = 12.35$ . The smallest possible value for  $C_u$  is equal to 1 and corresponds to a perfectly uniform assemblage of grains of identical size.

**Well-graded soils.** Well-graded soils contain a wide and even distribution of particle sizes. A well-graded silty sand and gravel is shown by curve B in Fig. 3. The smooth concave upward-grading curve is typical of well-graded material. Curve B' of Fig. 3 represents an idealized material in which the particles fit together in the densest possible state of packing (Fuller grading), as illustrated in Fig. 4. The interstices of the largest particles of size  $D_{\max}$  are filled in a regular pattern with smaller and smaller particles that occupy the void without holding the neighboring particles apart. The Fuller grading has the smooth shape shown in Fig. 3 and is determined using the equation

$$P = 100 \sqrt{\frac{D}{D_{\max}}} \quad (2)$$

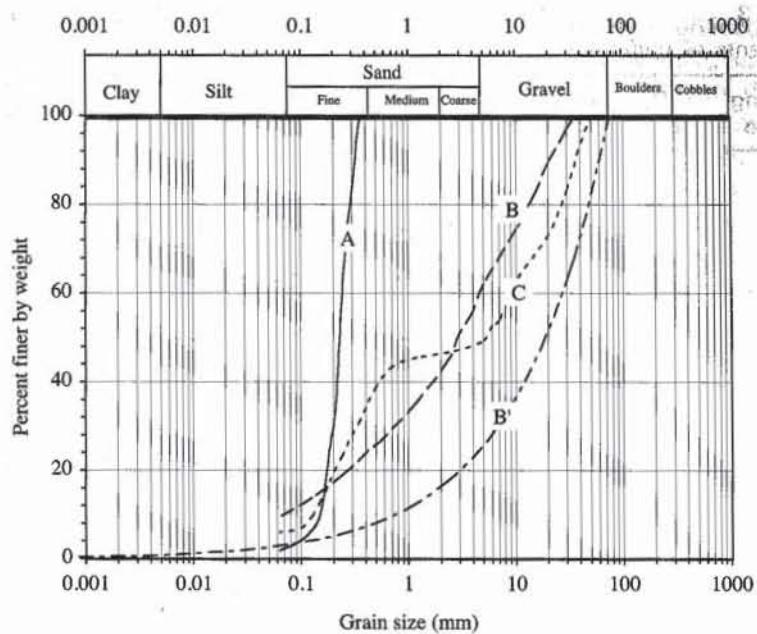
where  $P$  is the percentage by weight of particles finer than diameter  $D$  and  $D_{\max}$  is the maximum particle size ( $D_{\max} = 75$  mm in the example of Fig. 3).

Particle size distribution can be characterized by its curvature, and more specifically by the coefficient of curvature  $C_c$ :

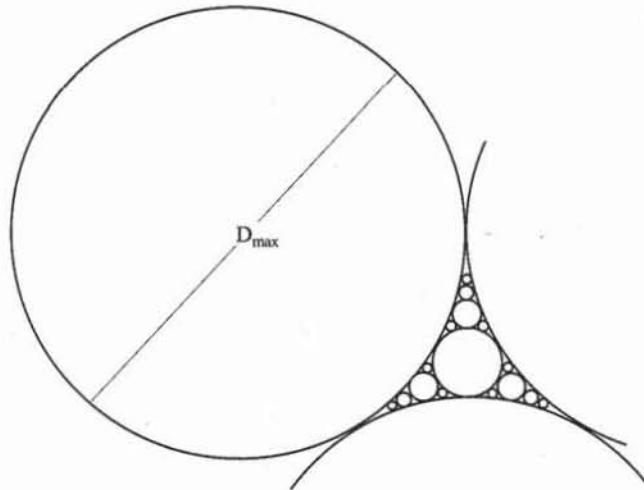
$$C_c = \frac{D_{30}^2}{D_{10} D_{60}} \quad (3)$$

where  $D_{30}$  is the grain size corresponding to 30% finer (see Fig. 2). By definition, gravels are considered to be well graded when  $C_u > 4$  and  $1 < C_c < 3$ . Sands are considered to be well graded when  $C_u > 6$  and  $1 < C_c < 3$ . For example, in Fig. 2,  $D_{10} = 0.082$  mm,  $D_{30} = 0.334$  mm,  $D_{60} = 1.008$  mm, and  $C_c = 1.35$ . As found previously,  $C_u = 12.35$ ; therefore, the sand of Fig. 2 is well graded.

**Poorly graded soils.** The term *poorly graded* applies to any soil, including uniform soil, which does not comply with the description of well graded. Poorly graded soils are deficient in certain sizes. Gap-graded materials are examples of poorly graded materials with missing ranges of particle sizes. For example, curve C in Fig. 3 has a flat part indicating that there are only a few particles in the range 1 to 10 mm. In practice, gap-graded materials are generally found in the coarse



**Figure 3** Examples of grain size distribution curves for sands and gravels (data after Head, 1984).



**Figure 4** Idealized Fuller packing (two-dimensional representation).

sand-fine gravel range. The values of  $D_{10}$ ,  $D_{30}$ ,  $D_{60}$ ,  $C_u$ , and  $C_c$ , for materials A, B, and C, and Fuller packing are summarized in Table 3.

#### Classification of Clays and Silts

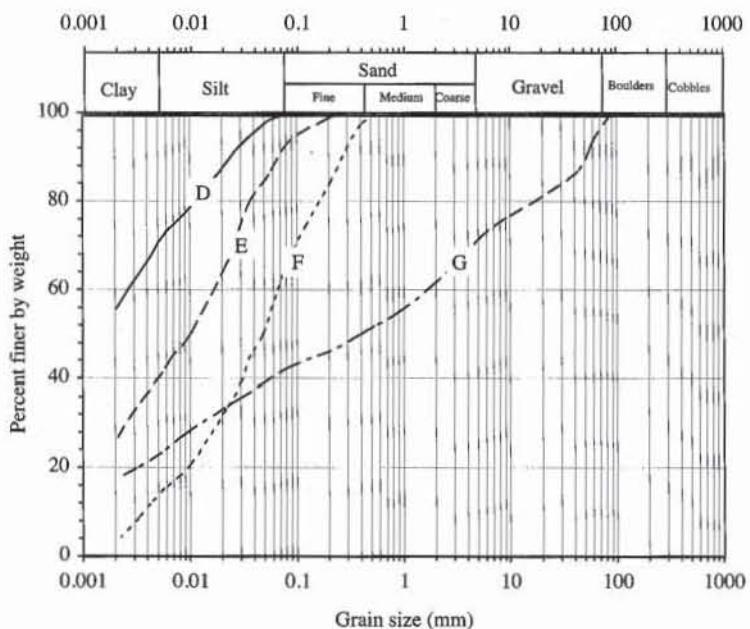
Soils consisting entirely of clay- or silt-size particles are rarely found in nature. Most clays contain silt-size particles, and most material described as silt includes some clay or some sandy material, or both. Some typical grading curves of clays and silts are shown in Fig. 5.

Curve D is described as clay, although it consists of 56% clay-and 44% silt-size particles. Curve E shows a well-graded soil consisting primarily of silt, with a

**TABLE 3**

Coefficients of uniformity and curvature for soils in Figs. 3 and 5.

Grading curve	$D_{10}$ (mm)	$D_{30}$ (mm)	$D_{50}$ (mm)	$D_{60}$ (mm)	$C_u$	$C_c$	Description
A	0.148	0.196	0.229	0.242	1.64	1.07	Uniform fine sand
B	0.069	0.745	2.880	4.642	67.10	1.73	Well-graded silty sand and gravel
B'	0.730	6.551	18.750	26.203	35.89	2.24	Idealized Fuller grading
C	0.125	0.332	5.068	8.971	72.03	0.10	Gap graded silty sand and gravel
D	-	-	0.001	0.003	-	-	Clay
E	0.001	0.003	0.010	0.017	21.68	0.51	Sandy and silty clay
F	0.004	0.018	0.048	0.066	17.97	1.32	Sandy silt
G	0.001	0.013	0.401	1.642	2693.84	0.10	Gravelly sandy silty clay

**Figure 5** Examples of grain size distribution curves (data after Head, 1984).

clay fraction of less than 2% which is described as sandy silt with a trace of clay. Curve *F* has a mixture of clay, silt, and sand. The soil is described as silty clay with sand. Curve *G* represents a well-graded soil containing particles of all sizes from cobbles down to clay. It is described as gravelly sandy silty clay. This type of soil, which is found as glacial till, is often called boulder clay.

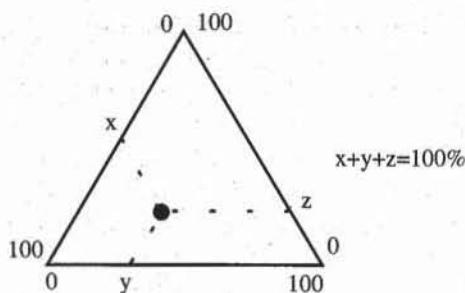
The values of  $D_{10}$ ,  $D_{30}$ ,  $D_{60}$ ,  $C_u$ , and  $C_c$  for materials *D*, *E*, *F*, and *G* are given in Table 3. The particle size corresponding to 10% finer for material *D* is outside the range of the grain size distribution chart;  $D_{10}$ ,  $C_u$ , and  $C_c$  cannot be determined for this material.

The clay fraction is defined as the percent by weight finer than 2  $\mu\text{m}$ . Its values for soils *A* to *G* are listed in Table 4.

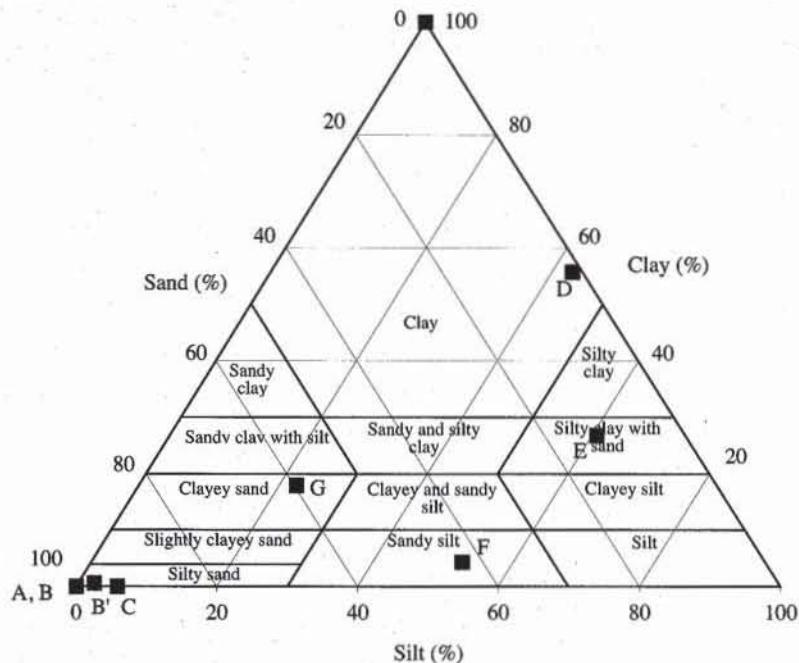
### TRIANGULAR CLASSIFICATION CHART

A triangular classification chart is an alternative representation of the grain size distribution of soils. Less commonly used than the semilogarithmic representation, it is convenient for comparing clay-silt-sand mixtures on the proportions of

each constituent. As shown in Fig. 6, each side of the triangle is divided into 100 parts, representing the percentage of three soil constituents: clay, silt, and sand. A point within the triangle indicates the percentage of these constituents, the sum of which adds up to 100%. The triangular chart can also be used to show (clay + silt)-sand-gravel mixtures, or any other three main constituents of soil. The triangular coordinates of samples *A* through *G* in Figs. 3 and 5, are listed in Table 4 and are plotted as points on the triangular chart of Fig. 7. This triangular chart was introduced by the U.S. Bureau of Reclamation (1974).



**Figure 6** Definition of coordinates on a triangular classification chart.



**Figure 7** Representation of samples *A* to *G* of Table 3 on a triangular classification chart.

## FREQUENCY DISTRIBUTIONS

As noted earlier, grain size distributions are usually represented as cumulative distributions in soil mechanics. Figures 8 and 9 show an alternative representation: the frequency distributions corresponding to the samples of Figs. 3 and 5. The frequency  $f_i$  associated with a particular size  $d_i$  is calculated from the cumulative values  $p_i$  (e.g., percent by weight finer) by using the relation

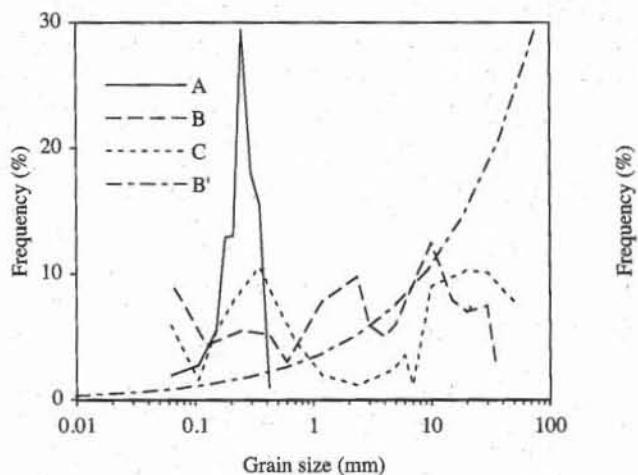
$$f_i = p_{i+1} - p_i \quad (4)$$

**TABLE 4**

Coordinates of samples A to G of Table 3 for triangular classification chart (after Head, 1984)

Material	Sand fraction (%)	Silt fraction (%)	Clay fraction (%)
A	100.0	0.0	0.0
B	100.0	0.0	0.0
B'	97.1	2.3	0.6
C	94.1	5.9	0.0
D	1.4	42.9	55.7
E	12.5	60.7	26.8
F	43.0	52.7	4.3
G	59.7	22.4	17.9

The sum of  $f_i$  should equal 100%. As shown in Fig. 8, sample A (uniform sand) displays a unique peak at 0.2 mm, while sample B' (Fuller material) has a continuous and decreasing distribution. As shown in Figs. 8 and 9, all other samples have a multimodal, rather than a unimodal distribution with several peaks. In all cases, their frequency distribution appears more complicated than their cumulative distribution.



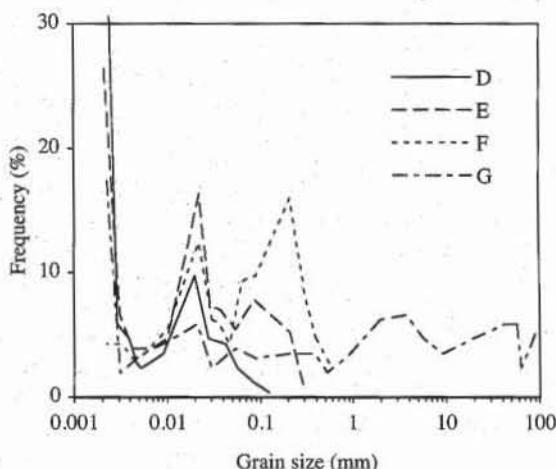
**Figure 8** Frequency distributions corresponding to the cumulative distributions of Fig. 3.

## APPLICATIONS OF GRAIN SIZE ANALYSIS IN ENGINEERING PRACTICE

In geotechnical engineering, particle size analyses are useful for various practical applications, ranging from the selection of fill and aggregate materials, to road construction, drainage, filters, and grouting.

### Selection of Fill Materials

Soils used for the construction of embankments and earth dams are required to be within specified limits as defined by particle size distribution curves. The various zones of an earth dam, for instance, have different gradation characteristics.



**Figure 9** Frequency distributions corresponding to the cumulative distributions of Fig. 5.

### **Selection of Aggregate Materials**

Sands and gravels for use as concrete aggregates are divided by particle size distribution curves into various types. In the exploration for sand and gravel resources, particle size analysis is the main criterion for selection of sites for potential development.

### **Road Subbase Materials**

Each layer of a road or airfield runway subbase must comply to a particular grading specification to provide a mechanical stable foundation (see the description of the AASHTO classification system in Chapter 2-9).

### **Drainage Filters**

The grading specification for a filter layer must be related in certain ways to the grading of the adjacent ground or of the next filter layer. This specification, referred to as the *filter requirement*, prevents small particles from being dragged by seepage forces and avoids the clogging of pores between large particles.

### **Groundwater Drainage**

The drainage characteristics of the ground depend largely on the proportion of fines (silts and clay-size particles) present in the soil.

### **Grouting and Chemical Injection**

Grouting and chemical injections consist of injecting liquids with predefined mechanical or chemical characteristics inside the soil interstices to decrease the ground permeability and/or to improve its mechanical properties. The most suitable grouting process and the extent to which the ground can be impregnated depend mainly on the grading characteristics of soils.

## LIMITATIONS OF PARTICLE SIZE ANALYSIS

Particle size analysis helps to classify soils, especially coarse soils. It is possible to tell from grain size distribution analysis whether the soil consists of predominantly gravel, sand, silt, or clay, and to a limited extent, which of these size ranges is likely to control the soil engineering properties. Particle size analysis is of greater value if supplemented by descriptive details such as color and particle shape. But the engineering behavior of soils also depends on factors other than particle sizes, such as mineral, structural, and geological history. The physical behavior of clays, such as plastic consistency, controls more of its mechanical behavior than its particle size distribution, and for this the Atterberg limits test (see Chapters 2-3 to 2-5) provides more significant information than is provided by grain size analysis.

## TYPES OF GRAIN SIZE ANALYSES

There are two separate procedures for obtaining the grain size distribution of soils: sieve analysis and sedimentation analysis. Sieve analysis is used for gravel- and sand-size particles (coarse-grained soils with grain size larger than 75  $\mu\text{m}$ ) but cannot be used for silt- and clay-size particles (fine-grained soils with grain size smaller than 75  $\mu\text{m}$ ). A sedimentation procedure (e.g., hydrometer, pipette, or buoyancy analysis) is used instead. Sieve and sedimentation analyses are combined to define the grain size distribution of soils having fine and coarse grains. The grain size distribution of soils by sieving, sedimentation, and combined analyses are described in subsequent chapters.

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## REVIEW QUESTIONS

1. Name the six categories of particle sizes that are identified in soil mechanics.
2. What is a specific surface? In what unit is it expressed? Can you give a typical value for the specific surface of montmorillonite?
3. What is a Fuller material?
4. What axes do we generally use in soil mechanics to represent a grain size

- distribution curve? Are there other types of representation?
5. Define *coefficient of uniformity*  $C_u$  and the *coefficient of curvature*  $C_c$ . What are their physical meanings?
  6. Is it always possible to determine the coefficient of uniformity  $C_u$  and the coefficient of curvature  $C_c$  for all types of soils?
  7. What are the two main experimental techniques used in soil mechanics to determine a grain size distribution curve? Define their range of application.
  8. Is it possible for a grain size distribution curve to have a bump (i.e., to increase, then to decrease)?
  9. Define *poorly graded soil* and *well-graded soil*. What is a gap-graded material?
  10. Material A has  $C_u = 1$  and material B has  $C_u = 4$ . Which material is a better filter, and which is a better drain?
  11. Explain why the percent by weight finer value always increases with the particle diameter.
  12. Calculate analytically  $C_u$  and  $C_c$  for a Fuller material ( $P = 100\sqrt{D/D_{\max}}$ ).
  13. What is the coefficient  $C_u$  for a set of marbles having identical size?
  14. Draw the grain size distribution curve of an assembly of marbles knowing that the marbles have only two sizes. There is 1 kg of 8-mm-diameter marbles and 1 kg of 10-mm-diameter marbles.

## EXERCISES

1. The result of a grain size analysis is given below. Plot the grain size distribution curve, and calculate  $D_{10}$ ,  $D_{30}$ ,  $D_{60}$ ,  $C_u$ , and  $C_c$ . Compare your results to those in Fig. 1.

Particle size (mm)	Percent finer (%)
15.00	100.00
10.00	99.22
6.00	97.67
4.00	94.19
2.80	88.76
2.00	81.00
1.40	70.00
1.18	65.89
1.00	59.69
0.85	54.00
0.60	45.00
0.50	41.09
0.36	31.01
0.18	20.00
0.13	15.00
0.08	9.00
0.05	6.20
0.04	5.00

2. The tabulated data of four grain distribution curves of sands and gravels are given below. Plot these grain size distribution curves by using semilogarithmic axes as shown in Fig. 3. Calculate  $D_{10}$ ,  $D_{30}$ ,  $D_{60}$ ,  $C_u$ , and  $C_c$ .

Sample A		Sample B		Sample C		Fuller	
Particle size (mm)	Percent finer (%)						
0.425	100.0	35.000	100.0	50.000	100.0	75.000	100.0
0.355	99.0	30.000	96.9	38.000	92.1	37.500	70.7
0.300	83.5	20.000	89.4	30.000	83.1	18.750	50.0
0.250	65.5	15.000	82.4	20.000	73.0	9.375	35.3
0.212	36.1	10.000	74.5	10.000	62.7	4.688	25.0
0.180	23.1	5.000	62.0	7.000	53.7	2.344	17.6
0.150	10.2	4.000	56.0	6.000	52.5	1.172	12.5
0.106	4.7	3.000	51.0	4.750	49.0	0.586	8.8
0.063	1.9	2.360	45.1	2.360	46.6	0.293	6.2
		1.180	35.3	1.180	45.4	0.146	4.4
		0.600	27.5	0.710	43.5	0.073	3.1
		0.425	24.5	0.500	38.8	0.037	2.2
		0.250	19.3	0.355	31.3	0.018	1.5
		0.125	13.8	0.212	20.7	0.009	1.1
		0.063	9.4	0.150	12.9	0.005	0.7
				0.106	7.4	0.002	0.5
				0.063	5.8	0.001	0.4

3. The tabulated data of four grain distribution curves of silts and clays are given below. Plot these grain size distribution curves by using semilogarithmic axes as shown in Fig. 5. Calculate  $D_{10}$ ,  $D_{30}$ ,  $D_{60}$ ,  $C_u$ , and  $C_c$ , and compare your results to those of Table 3.

Sample D		Sample E		Sample F		Sample G	
Particle size (mm)	Percent finer (%)						
0.124	100	0.289	100.0	0.552	100.0	90.252	100.0
0.085	100	0.209	99.0	0.399	97.2	63.719	94.9
0.057	98	0.088	93.7	0.304	92.6	57.686	92.6
0.043	96	0.055	85.9	0.209	84.4	42.804	86.7
0.028	92	0.038	80.5	0.090	68.4	19.316	80.9
0.020	87	0.029	73.5	0.063	58.7	9.161	76.2
0.009	77	0.022	66.1	0.047	49.4	5.712	72.7
0.006	74	0.010	49.8	0.037	45.1	3.743	68.0
0.005	71	0.007	45.1	0.030	39.3	1.961	61.4
0.005	69	0.006	41.2	0.022	33.0	0.930	55.2
0.004	66	0.004	37.3	0.010	20.6	0.525	51.7
0.003	61	0.003	33.4	0.006	15.1	0.389	49.8
0.002	56	0.002	26.8	0.004	11.6	0.209	46.3
				0.003	8.5	0.090	42.8
				0.002	4.2	0.055	39.6
						0.030	35.7
						0.021	33.4
						0.009	27.6
						0.005	23.3
						0.003	19.8
						0.002	17.8

4. By using the tabulated data of Exercises 2 and 3, compute the sand, silt, and clay fractions of samples A through G. Plot these fractions, each of which should add up to 100%, on a triangular chart. Compare your results with those of Table 4 and Fig. 7.

5. Write a spreadsheet function that calculates the  $x$ - $y$  Cartesian (horizontal and vertical) coordinates of a point to be plotted in a triangular chart.
6. Plot the frequency distributions corresponding to the cumulative distributions of samples  $A, B, C, B', D, E, F$ , and  $G$  that are given in the tables of Exercises 2 and 3. Compare your results to those of Figs. 8 and 9.
7. Derive the equation that relates the percentage by weight finer  $P$  to the grain size  $D$  for a Fuller material having for maximum grain size  $D_{\max}$ .
8. In the case of a Log normal distribution, calculate the analytical expression for the coefficients of uniformity and curvature in terms of the mean value  $\mu$  and standard deviation  $\sigma$ .

# 1-2 Sieve Analysis

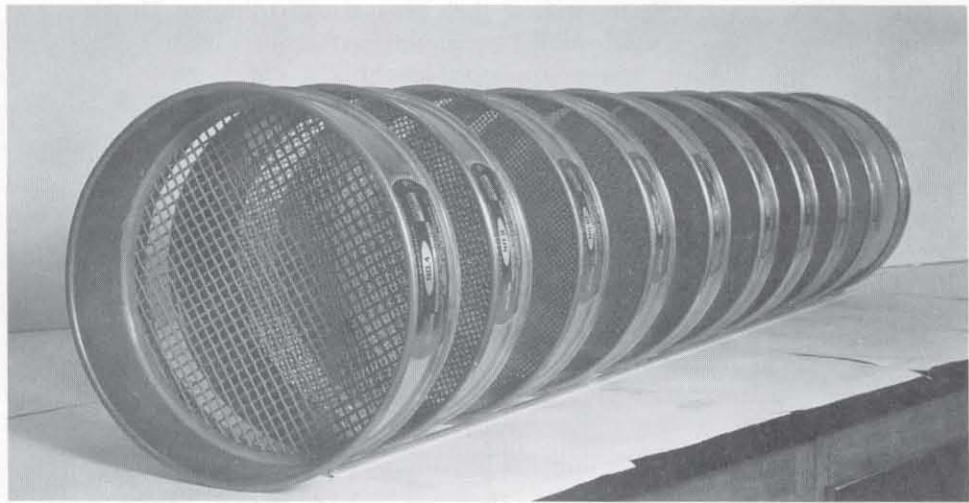
## OBJECTIVE

The sieve analysis determines the grain size distribution curve of soil samples by passing them through a stack of sieves of decreasing mesh opening sizes and by measuring the weight retained on each sieve. The sieve analysis is generally applied to the soil fraction larger than 75  $\mu\text{m}$ . Grains smaller than 75  $\mu\text{m}$  are sorted by using sedimentation (e.g., hydrometer or pipette analysis). Sieving can be performed in either wet or dry conditions. Dry sieving is used only for soils with a negligible amount of plastic fines, such as gravels and clean sands, whereas wet sieving is applied to soils with plastic fines.

## EQUIPMENT

The equipment used in sieve analysis includes:

- Series of standard sieves with openings ranging from 7.5 cm to 75  $\mu\text{m}$ , including a cover plate and bottom pan. Figure 1 shows an example of a stack of sieves. Sieves are generally constructed of wire screens with square openings of standard size. Table 1 lists the number and mesh opening sizes of the U.S. standard sieves. Only a few sieves in Table 1 are selected for the sieve analysis. The total number and mesh sizes of the sieves are selected to cover the range of grain sizes in an even distribution on a logarithmic scale.
- Sieve shake (Fig. 2).
- Balances sensitive to 0.1 g.
- Soft wire brush.
- Sample splitter or riffle for dividing large soil samples into smaller samples with identical grain size distribution.
- Mortar and rubber-covered pestle, for breaking up aggregations of soil particles.



**Figure 1** Typical stack of sieves for grain size analysis. The sieves are stacked by decreasing mesh opening size, with the largest mesh opening at the top of the stack.



**Figure 2** The stack of sieves is mounted on a mechanical shaker.

## PREPARATION OF SOIL SAMPLE

The material to be tested is first air dried. Aggregations or lumps are thoroughly broken up with the fingers or with the mortar and pestle. The specimen to be tested should be large enough to be representative of the soil in the field. It should also be small enough not to overload sieves. Large soil samples are divided by using a

riffle to preserve their grain-size distribution. The size of a representative specimen depends on the maximum particle size. Table 2 gives some guidelines for selecting the minimum sample weight.

**TABLE 1**  
Numbers and openings of U.S. standard sieves

Sieve number	Sieve opening (mm)	Sieve number	Sieve opening (mm)
4	4.75	45	0.355
5	4	50	0.3
6	3.35	60	0.25
7	2.8	70	0.212
8	2.36	80	0.18
10	2	100	0.15
12	1.7	120	0.125
14	1.4	140	0.106
16	1.18	170	0.09
18	1	200	0.075
20	0.85	230	0.063
25	0.71	270	0.053
30	0.6	325	0.045
35	0.5	400	0.038
40	0.425		

**TABLE 2**  
Approximate sample weight for sieve analysis

Maximum particle size	Minimum weight of sample (g)
7.5 cm	6,000
5 cm	4,000
2.5 cm	2,000
1 cm	1,000
Finer than No. 4 sieve	200
Finer than No. 10 sieve	100

## PROCEDURE

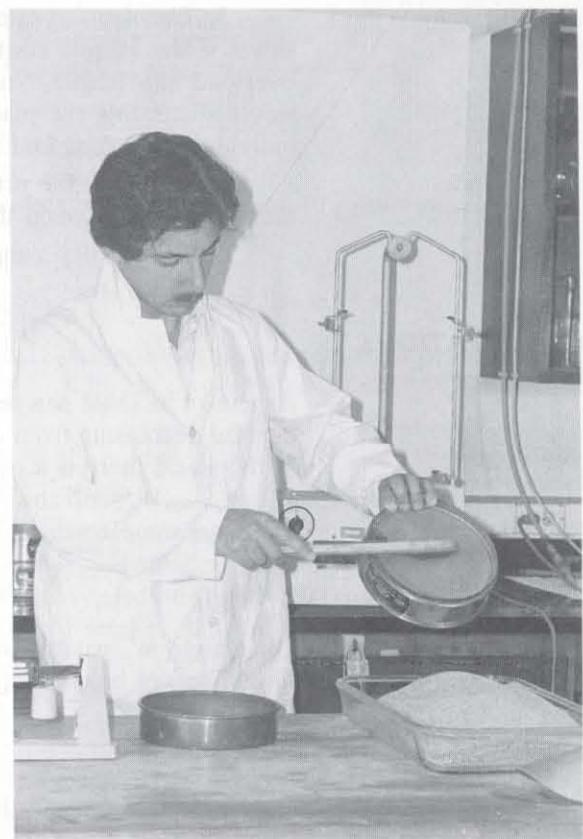
There are two different procedures for dry and wet screenings. Wet sieving is used when the small particles aggregate and form hard lumps, or coat the coarser particles.

### Dry Sieving

1. Oven dry the sample, allow it to cool, and measure its weight.
2. Select a stack of sieves suitable to the soil being tested. The choice of sieves usually depends on experience, judgment, and the intended applications of grain size analysis. A stack of six or seven sieves is generally sufficient for most soils and applications. The top sieve should have an opening slightly larger than the largest particles. Arrange the stack of sieves so that the largest mesh opening is at the top and the smallest is at the bottom (see Fig. 1).
3. Attach a pan at the bottom of the sieve stack. Pour the sample on the top sieve as shown in Fig. 3. Add a cover plate to avoid dust and loss of particles while shaking.



**Figure 3** The soil sample is poured on the stack of sieves with a pan at the bottom.



**Figure 4** The soil particles stuck in the mesh opening of the sieves are removed with a brush.

4. Place the stack of sieves in the mechanical shaker as shown in Fig. 2, and shake for about 10 min or until additional shaking does not produce appreciable changes in the amounts of material retained in each sieve.

5. Remove the stack of sieves from the shaker. Beginning with the top sieve, transfer its contents to a piece of paper or a large recipient. Carefully empty the sieve without losing any material, and use a brush to remove grains stuck in its mesh opening, as shown in Fig. 4. Measure the weight of soil retained in each sieve and note the corresponding sieve mesh opening and number.

6. Repeat step 5 for each sieve. As a preliminary check, the weights retained on all the sieves and the bottom pan are added, and their sum is compared to the initial sample weight. Both weights should be within about 1%. If the difference is greater than 1%, too much material was lost, and weighing and/or sieving should be repeated.

### **Wet Sieving**

1. Weigh the dry specimen as for dry sieving.

2. If the sample contains plastic fines, which tend to form hard lumps or to coat the coarser particles, place the oven-dried sample in a pan filled with enough water to cover all the material and allow it to soak until all the soil lumps or coatings have disintegrated. Soaking may take 2 to 24 hours, depending on the soil.

3. Transfer the sample to a No. 200 sieve, or to a set of No. 4 to No. 200 sieves if the sample contains an appreciable amount of coarse particles. Do not overload the fragile No. 200 sieve. Wash the sample thoroughly through the sieves, discarding the material passing the No. 200 sieve. Larger particles may be individually washed and removed from the sieves.
4. Oven dry the retained material, and weigh it after it has cooled. Record the difference between the dry weights before and after washing.
5. Use the dry sample for dry sieving starting at step 2.

## COMPUTATIONS

As shown in Table 3, a generic stack of sieves is made of  $n$  sieves having an opening size decreasing from top to bottom, and numbered from 1 to  $n$ . At the bottom of the stack, there is a pan numbered  $n + 1$ . The weight retained on each sieve is  $W_1, W_2, \dots, W_n$ , and the weight retained on the pan is  $W_{n+1}$ , as shown in Table 3. The total sample weight  $W_{\text{tot}}$  is the sum of all retained weights:

$$W_{\text{tot}} = W_1 + W_2 + \dots + W_{n-1} + W_n + W_{n+1} \quad (1)$$

The weight  $W'_j$  passing the  $j$ th sieve is the sum of weights retained in the pan and sieves located below the  $j$ th sieve. The weight passing the top sieve ( $j = 1$ ) is

$$W'_1 = W_{\text{tot}} - W_1 \quad (2)$$

and the weight passing the  $j$ th sieve ( $n \geq j \geq 1$ ) is

$$W'_j = W'_{j-1} - W_j \quad (3)$$

$W'_{n+1} = 0$  because no material passes through the pan. The percentage in weight of material finer than the mesh opening of the  $j$ th sieve is  $p_j$ :

$$p_j = 100 \frac{W'_j}{W_{\text{tot}}} \quad (4)$$

As shown in Table 3,  $p_j$  always decreases with  $j$  because  $W'_j$  decreases continuously with  $j$ .

TABLE 3  
Summary of calculations for sieve analysis

Sieve number	Weight retained	Weight passing	Percent finer
1 (top)	$W_1$	$W'_1 = W_2 + W_3 + \dots + W_{n+1} = W_{\text{tot}} - W_1$	$100 W'_1 / W_{\text{tot}}$
2	$W_2$	$W'_2 = W_3 + W_4 + \dots + W_{n+1} = W'_1 - W_2$	$100 W'_2 / W_{\text{tot}}$
$j$	$W_j$	$W'_j = W_{j+1} + \dots + W_{n+1} = W'_{j-1} - W_j$	$100 W'_j / W_{\text{tot}}$
$n$	$W_n$	$W'_n = W_{n+1}$	$100 W'_n / W_{\text{tot}}$
$n + 1$ (pan)	$W_{n+1}$	$W'_{n+1} = 0$	0

The coefficients of uniformity  $C_u$  and curvature  $C_c$  are defined from  $D_{10}$ ,  $D_{30}$ , and  $D_{60}$ , the grain size corresponding to 10, 30, and 60% by weight finer.  $D_{10}$ ,  $D_{30}$ , and  $D_{60}$  are obtained by using a semilogarithmic interpolation between



$$\log y = \log y_1 + (\log w - \log w_1) \left[ \frac{\frac{y_2 - y_1}{\log w_2 - \log w_1}}{\log y_2 - \log y_1} \right]$$

$$\log y = \log y_1 + (w - w_1) \frac{y_2 - y_1}{w_2 - w_1} \quad 27$$

the data points of the grain size distribution curve. For instance, when  $p_i \leq 10 \leq p_{i+1}$ ,

$$\log(D_{10}) = \log(d_i) + \frac{\log(d_{i+1}) - \log(d_i)}{p_{i+1} - p_i} (10 - p_i) \quad (5)$$

where  $d_i$  and  $d_{i+1}$  are the sieve openings corresponding to  $p_i$  and  $p_{i+1}$ . Therefore,  $D_{10}$  is

$$D_{10} = d_i \left( \frac{d_{i+1}}{d_i} \right)^{(10 - p_i)/(p_{i+1} - p_i)} \quad (6)$$

$D_{30}$  and  $D_{60}$  are calculated similarly to  $D_{10}$ .

### Error Analysis

The results of sieve analyses are checked by comparing the accumulated weight  $W_{\text{tot}}$  and initial sample weight  $W_0$ . If  $W_{\text{tot}}$  is smaller than  $W_0$ , material was lost or data were recorded incorrectly.  $W_{\text{tot}}$  may also be slightly larger than  $W_0$ , owing to added measurement errors. Equation 1 implies that the error  $\Delta W_{\text{tot}}$  on  $W_{\text{tot}}$  is

$$\Delta W_{\text{tot}} \leq (n + 1)\Delta W \quad (7)$$

where  $\Delta W$  is the scale accuracy and  $n$  is the number of sieves. Because all weights are measured on the same scale, the error  $\Delta W_0$  on  $W_0$  is also equal to  $\Delta W$ . Therefore, the error  $\Delta(W_{\text{tot}} - W_0)$  on  $W_{\text{tot}} - W_0$  is

$$\Delta(W_{\text{tot}} - W_0) \leq (n + 2)\Delta W \quad (8)$$

If the sieve analysis was correctly performed,  $|W_{\text{tot}} - W_0|$  should be smaller than  $(n + 2)\Delta W$ . The analysis should be repeated if  $|W_{\text{tot}} - W_0|$  is larger than  $(n + 2)\Delta W$ .

### EXAMPLE

Figures 5 and 6 show the results of a sieve analysis for a fine-grained sand. As shown in Fig. 5, the results are presented in the form of a grain size distribution curve, which is obtained by plotting grain size (i.e., sieve opening)

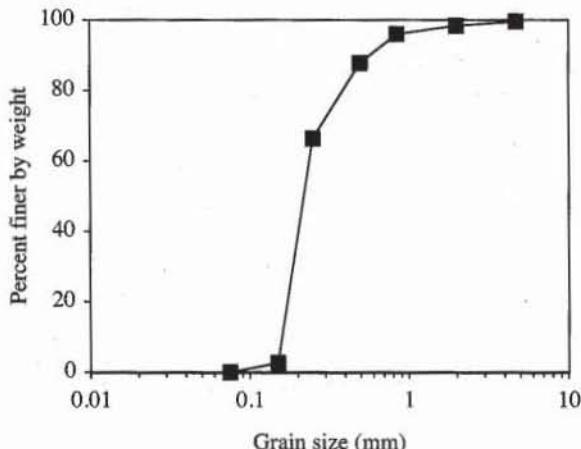


Figure 5 Example of grain size distribution curve obtained by sieve analysis.

on the abscissa and the percent finer by weight on the ordinate. A logarithmic scale is used for the grain size, and a linear scale is used for the percent finer values.

Figure 7 shows the formulas used in Fig. 6. The coefficients  $D_{10}$ ,  $D_{30}$ , and  $D_{60}$  are calculated by using the semilogarithmic interpolation of Eq. 6, which is performed by the user-defined function INTER (See Chapter 8-1).

	A	B	C	D	E
1					<b>Sieve analysis</b>
2					
3					
4					
5					
6					
7					
8	US sieve number	Sieve opening (mm) d	Mass retained (g) M	Mass passing (g) $M_n$	Percent finer by weight p
9					
10	4	4.750	0.00	190.20	99.53
11	10	2.000	2.10	188.10	98.43
12	20	0.850	4.60	183.50	96.02
13	35	0.500	15.80	167.70	87.76
14	60	0.250	40.90	126.80	66.35
15	100	0.150	122.00	4.80	2.51
16	200	0.075	4.70	0.10	0.05
17	pan		0.10	0.00	0.00
18		Total mass $M_{tot} = 190.20$ g			
19		$D_{10} = 0.159$ mm		$C_u = 1.492$	
20		$D_{30} = 0.187$ mm		$C_c = 0.923$	
21		$D_{60} = 0.238$ mm			

Figure 6 Example of data set for grain size analysis.

	A	B	C	D	E
8	US sieve number	Sieve opening (mm) d	Mass retained (g) M	Mass passing (g) $M_n$	Percent finer by weight p
9					
10	4	4.75	0	= $M_{tot}-M$	$=(M_p/M_0)*100$
11	10	2	2.1	= $D_{10}-M$	$=(M_p/M_0)*100$
12	20	0.85	4.6	= $D_{11}-M$	$=(M_p/M_0)*100$
13	35	0.5	15.8	= $D_{12}-M$	$=(M_p/M_0)*100$
14	60	0.25	40.9	= $D_{13}-M$	$=(M_p/M_0)*100$
15	100	0.15	122	= $D_{14}-M$	$=(M_p/M_0)*100$
16	200	0.075	4.7	= $D_{15}-M$	$=(M_p/M_0)*100$
17	pan		0.1	= $D_{16}-M$	$=(M_p/M_0)*100$
18		Total mass $M_{tot} = \text{SUM}(M)$			
19		$D_{10} = \text{inter}(10,p,d)$		$C_u = D_{60}/D_{10}$	
20		$D_{30} = \text{inter}(30,p,d)$		$C_c = D_{30}^2/D_{10}/D_{60}$	
21		$D_{60} = \text{inter}(60,p,d)$			

Figure 7 Formulas used in Fig. 6.

**REFERENCES**

- See Introduction for references to ASTM procedures (pages 4 to 6).
- AASHTO T87, Sample preparation. American Association of State Highway Transportation Officials, Washington, DC.
- AASHTO T88, Standard test method for particle size analysis of soil.
- BS 410, 1969, *Test Sieves*, British Standards Institution, London.

**REVIEW QUESTIONS**

1. What is the purpose of grain size analysis?
2. Under what conditions should you use wet sieving instead of dry sieving?
3. On which basis do you select the number and opening of sieves for the sieve analysis of a given soil?
4. How can you quickly verify the results of dry sieving?
5. What are the smallest and largest mesh openings used in practice for determining grain size distribution?
6. A mass of volcanic ashes with highly crushable grains is brought to the laboratory. What precaution do you take to determine its grain size distribution?
7. On what range of particle size does the sieve analysis apply?
8. Is it possible to carry out a sieve analysis on a sample of clay?

**EXERCISES**

1. The following masses of soil are retained on sieves.

US Sieve No.	4	10	20	40	100	200	pan
Mass retained (g)	100	150	200	250	200	100	5

- Draw the grain size distribution curve and determine  $C_c$  and  $C_u$ .
2. Determine the formulas that define the error on the percent by weight finer in terms of the retained weights and the experimental errors. Include these formulas in the spreadsheet calculation.
  3. Modify the spreadsheet to obtain a grain size distribution curve that has percent coarser by weight, as the vertical axis rather than percent finer by weight.
  4. In your laboratory report, calculate coefficients  $D_{10}$ ,  $D_{30}$ , and  $D_{60}$  by using a linear interpolation instead of the nonlinear interpolation of Eq. 6. Compare the values obtained by both interpolations.
  5. Plot the following grain size distribution as a frequency distribution and a cumulative distribution. Calculate the range, average, and standard deviation of this distribution. Attempt to fit the experimental distribution with a lognormal distribution.

Sieve number	Mass retained (g)
8	76
10	38
20	374
40	538
50	226
60	74
80	142
100	44
200	104
pan	60

Sample mass, 1684 g.

6. Plot the following grain size distributions as cumulative distributions. Calculate the coefficients of uniformity and curvature.

Sieve number	Mass retained (g)	Sieve number	Mass retained (g)
4	0	4	0
8	98	9	118
10	46	16	160
20	394	20	184
40	594	30	214
50	240	40	270
60	88	60	264
80	144	80	118
100	50	100	56
200	104	200	102
pan	56	pan	62

Sample mass, 1814 g.

Sample mass, 1560 g.

7. Write a report on the results of the following sieve analyses. Include a discussion of the experimental errors assuming that the scale accuracy is 0.5 g. Plot the grain size distribution curves, and classify the soils by using their coefficients of uniformity and curvature.

Sieve number	Mass retained (g)	Sieve number	Mass retained (g)
4	0	4	0
8	76	10	124
10	38	16	168
20	374	20	200
40	538	30	240
50	226	40	274
60	74	60	328
80	142	80	136
100	44	100	52
200	104	200	114
pan	60	pan	58

Sample mass, 1684 g.

Sample mass, 1698 g.

## 1-3

# Principles of Sedimentation Analysis

### OBJECTIVE

Sedimentation analysis (hydrometer, pipette, and buoyancy analysis) defines the grain size distribution curve of soils that are too fine to be tested with sieves. Sedimentation analysis sorts soil particles by size using the physical process of sedimentation, a process that is described by Stokes' law (Stokes, 1891). The grain size is calculated from the distance of sedimentation of soil particles. The percent by weight finer is determined by measuring the unit weight of the soil-fluid suspension.

### STOKES' LAW

The rigid sphere of Fig. 1 with diameter  $D$  is immersed in a viscous fluid of viscosity  $\eta$  and having velocity  $U$  far away from the sphere. According to Stokes' law, slowly moving viscous fluids exert drag force  $F$  on the sphere:

$$F = 3\pi\eta UD \quad (1)$$

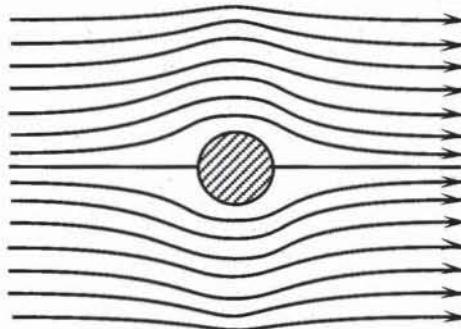


Figure 1 Flow past a fixed sphere for low Reynolds numbers.

The fluid viscosity  $\eta$  has the dimension of mass divided by length and time, and its unit is generally g/cm·s.

Stokes' law is applicable to slowly moving fluids that display the laminar flow patterns of Fig. 1 when the Reynolds number is smaller than 1. By definition, the Reynolds number  $R_e$  is the dimensionless ratio of inertial and viscous forces. In the particular case of the flow around a sphere,  $R_e$  is

$$R_e = \frac{\rho UD}{\eta} \quad (2)$$

where  $\rho$  is the fluid unit mass (i.e., the fluid mass in a unit volume). Stokes' law is no longer applicable when  $R_e \gg 1$  because of the changes taking place in the flow pattern.

These changes in flow patterns around a sphere are illustrated in Fig. 2 by considering the flow around a cylinder. After the laminar flow of Fig. 2a, a circulation appears in Fig. 2b behind the cylinder with two vortices rotating in opposite directions. When  $R_e \approx 40$ , there is a sudden change in the character of the motion. One of the vortices behind the object gets so long that it breaks off and travels downstream with the fluid. Then the fluid curls around behind the sphere and makes another vortex. The vortices peel off alternately on each side, so an instantaneous view of the flow looks roughly like that sketched in Fig. 2c. As the velocity gets higher and higher, there is less and less time for the vorticity to diffuse in a larger region of fluid. When  $R_e \approx 10^4$ , the flow is chaotic and irregular. As  $R_e$  is increased further, the turbulent region moves toward the cylinder and forms a turbulent boundary layer.

The drag force  $F$  defined for spheres in Eq. 1 may be generalized to other shapes by expressing it in terms of a drag coefficient  $C_d$  times the stagnation pressure  $\frac{1}{2}\rho U^2$  and the projected area  $A_p$  of the body normal to the flow:

$$F = \frac{1}{2} C_d A_p \rho U^2 \quad (3)$$

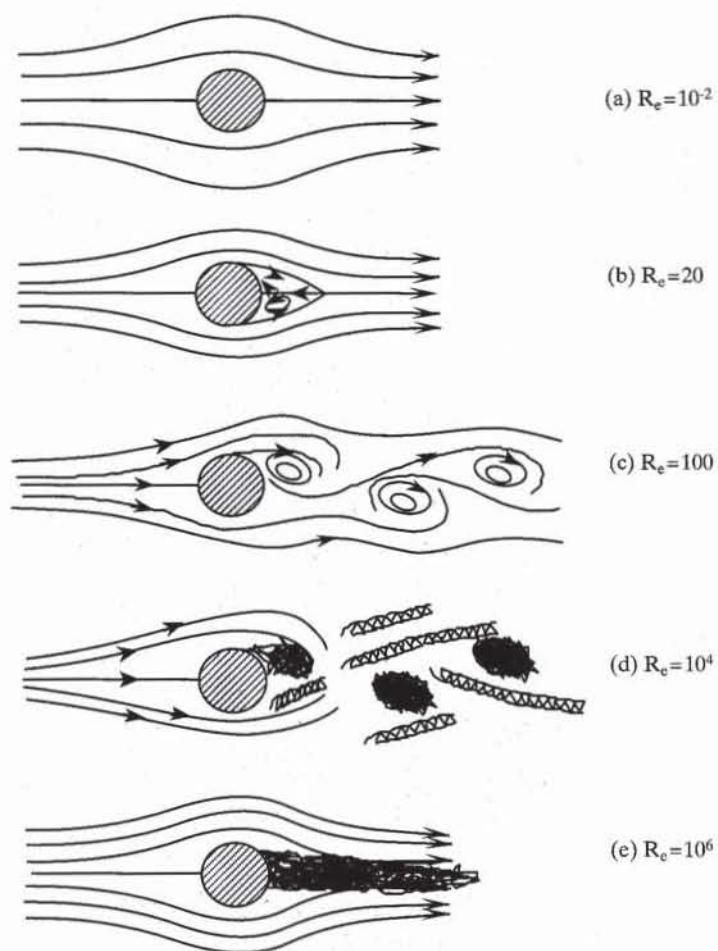
In the case of a sphere,  $A_p$  and  $C_d$  are

$$A_p = \frac{\pi}{4} D^2 \quad \text{and} \quad C_d = 24 \frac{\eta}{\rho UD} = \frac{24}{R_e} \quad (4)$$

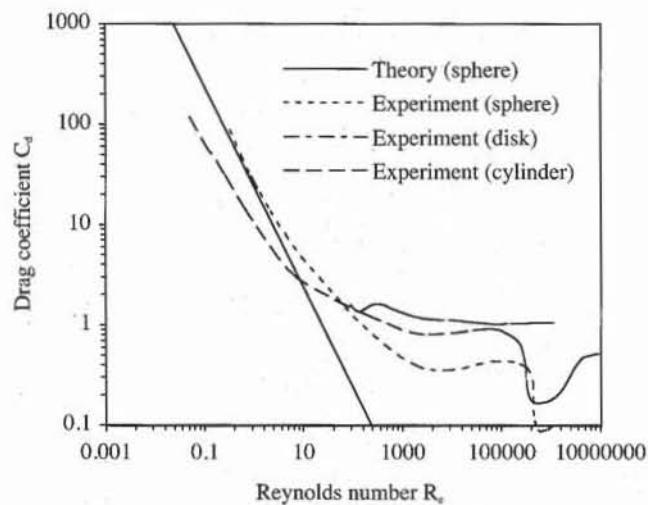
Figure 3 shows the theoretical and measured drag coefficients  $C_d$  for spheres, disks, and long circular cylinders. Equation 4 predicts that  $C_d$  is inversely proportional to  $R_e$ , which translates into a straight line of slope equal to  $-1$  in the log-log scale of Fig. 3. As shown in Fig. 3, Eq. 4 reproduces reasonably well the measured values of  $C_d$  for spheres provided that  $R_e < 1$ . But it breaks down when  $R_e > 1$  for spheres and other shapes, as indicated by departure of experimental points from the theoretical straight line in Fig. 3.

### APPLICATION OF STOKES' LAW TO FALLING SPHERES

When a sphere of radius  $a$  falls at a constant velocity  $U$  in a fluid of viscosity  $\eta$ , it is subjected to three forces: its weight,  $+\frac{4}{3}\pi a^3 \gamma_s$ ; the drag force,  $-6\pi\eta Ua$ ; and the buoyancy force,  $-\frac{4}{3}\pi a^3 \gamma_w$  where  $\gamma_s$  is the sphere unit weight and  $\gamma_w$  is the fluid unit weight. By definition, the unit weight is the weight of a unit volume. The buoyancy and drag forces are acting in the direction opposite to the weight that is arbitrarily taken positive. When the particle velocity reaches a constant velocity, the forces are in equilibrium:



**Figure 2** Flow past a fixed cylinder for various Reynolds numbers.



**Figure 3** Measured and theoretical drag coefficient for spheres, disks, and cylinders (data after Roberson and Crowe, 1993).

$$\frac{4}{3}\pi a^3 \gamma_s - \frac{4}{3}\pi a^3 \gamma_w - 6\pi \eta U a = 0 \quad (5)$$

The sphere velocity  $U$  is proportional to  $a^2$ :

$$U = \frac{2}{9} \frac{\gamma_s - \gamma_w}{\eta} a^2 \quad (6)$$

while  $R_e$  is proportional to  $a^3$ :

$$R_e = \frac{4}{9} \frac{\gamma_s - \gamma_w}{\eta^2} \rho_w a^3 \quad (7)$$

where  $\rho_w$  is the fluid unit mass.

The terminal velocity  $U$  and  $R_e$  values of spheres made of a typical soil mineral and falling in water are given in Table 1. Stokes' law does not apply to spheres with a radius larger than 0.1 mm, because  $R_e > 1$ . The diameter of 0.1 mm gives an approximate upper limit of particle size for which Eq. 6 applies. Equation 6 does not hold also for colloids that are particles smaller than 1  $\mu\text{m}$ . Colloids are influenced predominantly by the electrostatic forces acting on their surfaces. In this case the gravitational forces of Eq. 5, which are proportional to particle volume, become negligible with respect to electrostatic forces. The motion of colloids is random and is referred to as *Brownian*.

As a consequence of Eq. 6, the sphere of radius  $a$  initially at the liquid surface at time  $t = 0$  will reach the depth  $H$  at time  $t$ :

$$H = \frac{2}{9} \frac{\gamma_s - \gamma_w}{\eta} a^2 t \quad (8)$$

**TABLE 1**

Velocity and Reynolds number  
of a sphere falling in water ( $\rho_s = 2.7 \text{ g/cm}^3$ ,  
 $\rho_w = 1.0 \text{ g/cm}^3$ ,  $\eta = 0.01 \text{ g/cm}\cdot\text{s}$ )

$a$ (mm)	$U$ (cm/s)	$R_e$
1.0	$3.7 \times 10^2$	$7.4 \times 10^3$
0.1	3.7	7.4
0.01	$3.7 \times 10^{-2}$	$7.4 \times 10^{-3}$
0.001	$3.7 \times 10^{-4}$	$7.4 \times 10^{-6}$

## THE SEDIMENTATION PROCESS

The model of Fig. 4 illustrates the suspension process with four particle sizes. The particles are settling in a 50-cm-high container filled with water. The particle diameters and terminal velocities calculated by using Eq. 6 are given in Table 2. Although it oversimplifies soils that contain many more particle sizes, this model is still useful to understand what happens in suspensions at various time intervals.

Initially the particles are distributed uniformly by shaking the suspension. The sedimentation process starts at time  $t = 0$  immediately after the suspension stops being shaken. All the particles are assumed to reach their terminal velocity within a very short time. At  $t = 10 \text{ s}$ , the coarse silt particles A have traveled about 1 cm, whereas the clay particles have only fallen 40  $\mu\text{m}$ . At  $t = 15 \text{ min}$ , all

the coarse silt particles have reached the bottom. At  $t = 3$  h, all medium silt particles have settled at the bottom. At  $t = 8$  h, only the clay particles remain in suspension. The sedimentation process sorts small particles by size in the same way that sieves separates coarse particles.

Instead of four sizes, soil may have  $n$  different particle sizes  $a_1, a_2, \dots, a_n$ , where  $a_1 > a_2 > \dots > a_n$ . These particles when mixed with water make a soil–water mixture which is hereafter referred to as a *suspension*. The concentrations  $W_1, W_2, \dots, W_n$  denote the total weights of particles of size  $a_1, a_2, \dots, a_n$  in 1 L.  $W_{\text{tot}} = W_1 + W_2 + \dots + W_n$  is the total weight of particles. The percent  $p_j$  by weight finer than size  $a_j$  is

$$p_j = \frac{100}{W_{\text{tot}}} \sum_{i=j+1}^n W_i \quad (\%) \quad (9)$$

In a random suspension, particles are evenly scattered. The weight of particles and percent finer for a given particle size is identical at all depths. The sedimentation process sorts the particles as illustrated in Fig. 4. At time  $t > 0$ , all the particles above depth  $H$  have a radius smaller than  $a$ :

$$a = \sqrt{\frac{9}{2} \frac{\eta}{\gamma_s - \gamma_w} \frac{H}{t}} \quad (10)$$

At depth  $H$ , the concentration of particles of radius  $a$  is constant from  $t = 0$  until  $t = H/U$ , where  $U$  is the velocity of the particles of radius  $a$ . At time  $t > H/U$ ,

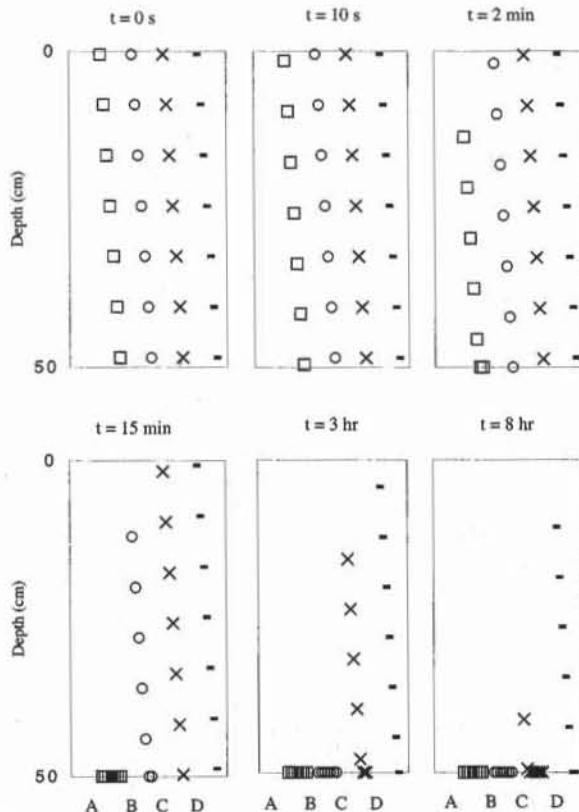


Figure 4 Schematic representation of the sedimentation process of particle sizes A, B, C, and D in Table 2 (after Head, 1984).

there is no longer a particle of radius  $a$  at depth  $H$ . The percent by weight finer than  $a$  can therefore be determined from the concentration of particles of size  $a$  at depth  $H$  at time  $t$ .

TABLE 2

Terminal velocities of particles in suspension in Fig. 4.

Type of particle	Diameter ( $\mu\text{m}$ )	Terminal velocity (cm/s)
A: Coarse silt	35	0.1090
B: Medium silt	12	0.0128
C: Fine silt	4	0.0014
D: Clay	2	0.0004

### UNIT WEIGHT OF A MIXTURE OF FLUID AND PARTICLES

The weight of a mixture of water and soil particles is

$$V\gamma_{av} = x + \left( V - \frac{x}{\gamma_s} \right) \gamma_w \quad (11)$$

where  $\gamma_{av}$  is the average unit weight,  $V$  is the total volume of the suspension,  $x$  the weight of particles in the volume  $V$ ,  $\gamma_s$  the unit weight of the solid particles, and  $\gamma_w$  the unit weight of the water. Therefore, the weight  $x$  of particles in volume  $V$  is

$$x = \frac{G_s}{G_s - 1} (\gamma_{av} - \gamma_w) V \quad (12)$$

where  $G_s$  is the specific gravity of soil grains:

$$G_s = \frac{\gamma_s}{\gamma_w} \quad (13)$$

The average unit weight  $\gamma_{av}$  of the suspension is

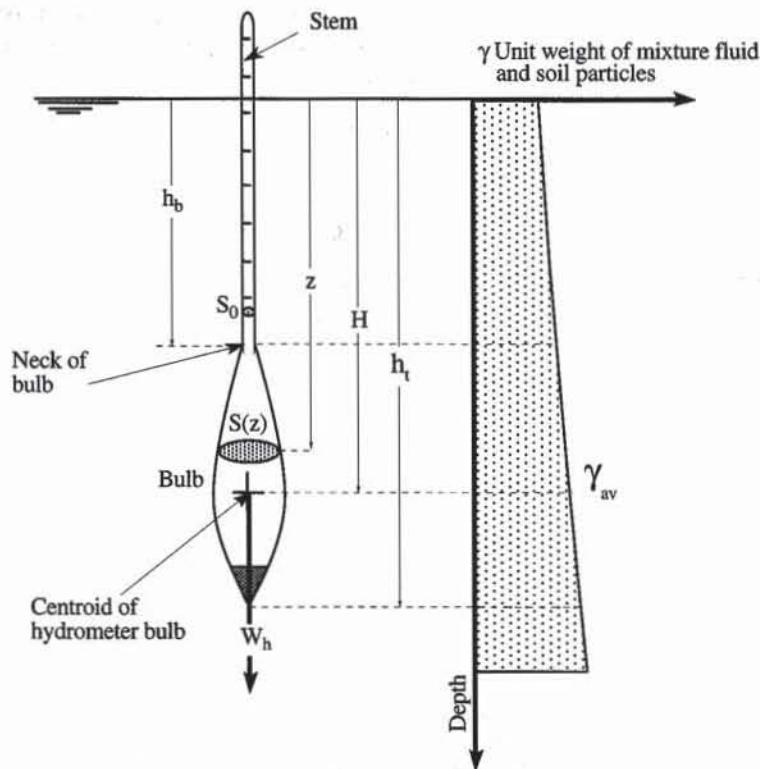
$$\gamma_{av} = \frac{x}{V} \left( 1 - \frac{\gamma_w}{\gamma_s} \right) + \gamma_w \quad (14)$$

Equation 14 implies that  $\gamma_{av} > \gamma_w$  when  $\gamma_s > \gamma_w$ . When particles settle down,  $x$  gradually decreases toward zero and  $\gamma_{av}$  slowly decreases toward  $\gamma_w$ . The unit weight  $\gamma_{av}$  of a sedimenting mixture, which varies with depth and time, can be measured with a hydrometer.

### HYDROMETER

As shown in Fig. 5, the hydrometer has a graduated stem and a weighted bulb. When it floats, its weight  $W_h$  compensates the buoyancy force:

$$W_h = \int_0^{h_t} S(z)\gamma(z) dz \quad (15)$$



**Figure 5** Schematic view of the hydrometer in a sedimenting mixture.

where  $z$  is the depth measured from the free surface,  $S(z)$  the cross-sectional area of the hydrometer at depth  $z$ ,  $\gamma(z)$  the fluid unit weight at depth  $z$ , and  $h_t$  the wetted length of the hydrometer at time  $t$ . The buoyancy force is the sum of the buoyancy forces applied to bulb and stem:

$$W_h = \int_{h_b}^{h_t} S(z)\gamma(z)dz + S_0 \int_0^{h_b} \gamma(z)dz \quad (16)$$

where  $h_b$  is the depth of the bulb neck and  $S_0$  is the constant cross-sectional area of the stem. Because the stem volume is much smaller than the bulb volume  $V_b$ , the second integral of Eq. 16 is small compared to the first one. Therefore, the hydrometer measures the average unit weight  $\gamma_{av}$  of the fluid between depths  $h_b$  and  $h_t$ :

$$\gamma_{av} = \frac{1}{\int_{h_b}^{h_t} S(z)dz} \int_{h_b}^{h_t} S(z)\gamma(z)dz = \frac{1}{V_b} \int_{h_b}^{h_t} S(z)\gamma(z)dz \quad (17)$$

As shown in Fig. 5, when the suspension density gradually varies with depth,  $\gamma_{av}$  is approximately equal to the suspension unit weight at depth  $H$  where the bulb is centered. Since the hydrometer weight  $W_h$  is constant, Eq. 16 implies that a decrease in mixture density between depths  $h_b$  and  $h_t$  is balanced by an increase in the wetted length  $h_t$ . Therefore, the hydrometer stem can be graduated to measure the suspension unit weight.

The 152H hydrometers (ASTM E 100-94) are graduated to measure the weight  $x$  of soil in 1 L assuming that  $G_s = 2.65$ . When the soil has another specific density  $G_s$ , the corrected weight  $x'$  of soil per liter of suspension is

$$x' = \frac{2.65 - 1}{2.65} \frac{G_s}{G_s - 1} x = 0.6226 \frac{G_s}{G_s - 1} x \quad (18)$$

### DETERMINATION OF PERCENT BY WEIGHT FINER

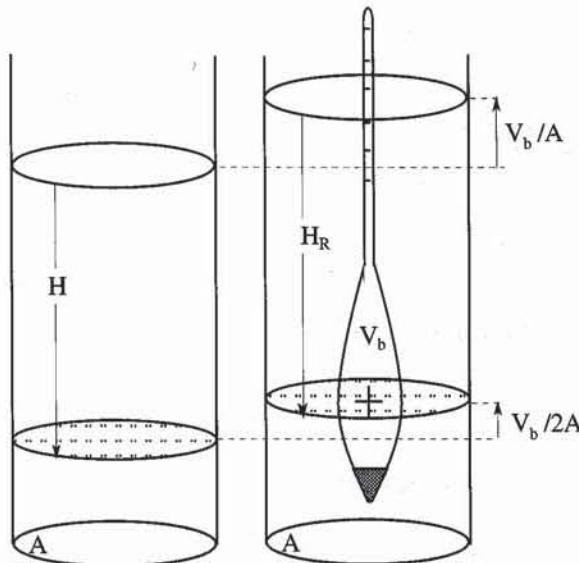
The hydrometer gives the weight  $x$  of particles located around the bulb centroid but not of those located above or below. If  $W_{\text{tot}}$  is the initial concentration of particles, the percent by weight finer than size  $a$  is

$$p = \frac{x}{W_{\text{tot}}} \times 100 \quad (\%) \quad (19)$$

As shown in Fig. 6, when the hydrometer is immersed in a burette of finite size, the water level rises, which changes the distance particles fall relative to the free surface. The water surface rises the distance  $V_b/A$ , where  $V_b$  is the volume of the hydrometer bulb and  $A$  is the cross-sectional area of the sedimentation burette, whereas the water at the elevation of the bulb centroid only rises the amount  $V_b/2A$ . Therefore, the particles at depth  $H$  where the bulb centroid is located have indeed fallen the distance  $H_R$ :

$$H_R = H - \frac{V_b}{2A} \quad (20)$$

The substitution of  $H_R$  into Eq. 10 gives the grain size  $a$  corresponding to  $p$  in Eq. 19.



**Figure 6** Effect of finite size of burette on distance of sedimentation.

## UNIT MASS AND VISCOSITY OF WATER

As indicated in Table 3, the unit mass  $\rho_w$  and the viscosity  $\eta$  of the water are functions of temperature  $T$ . As shown in Figs. 7 and 8, the values of  $\rho_w$  and  $\eta$  that are listed in Table 3 may be fitted by using a cubic polynomial:

$$\eta = 0.0178 - 5.684 \times 10^{-4}T + 1.115 \times 10^{-5}T^2 - 1.017 \times 10^{-7}T^3 \quad (21)$$

$$\rho_w = 0.99991 + 5.202 \times 10^{-5}T - 7.512 \times 10^{-6}T^2 + 3.605 \times 10^{-8}T^3 \quad (22)$$

The values of  $\rho_w$  and  $\eta$  at temperature  $T$  may also be calculated by linear interpolation of values in Table 3. For instance, if  $T$  is between temperatures  $T_i$  and  $T_{i+1}$ , then  $\eta$  is

$$\eta = \eta_i + (\eta_{i+1} - \eta_i) \frac{T - T_i}{T_{i+1} - T} \quad (23)$$

where  $\eta_{i+1}$  is the viscosity at temperature  $T_{i+1}$  and  $\eta_i$  is the viscosity at temperature  $T_i$ . In a similar way,  $\rho_w$  at temperature  $T$  is

$$\rho_w = \rho_i + (\rho_{i+1} - \rho_i) \frac{T - T_i}{T_{i+1} - T} \quad (24)$$

where  $\rho_{i+1}$  is the water unit mass at temperature  $T_{i+1}$ , and  $\rho_i$  is the water unit mass at temperature  $T_i$ .

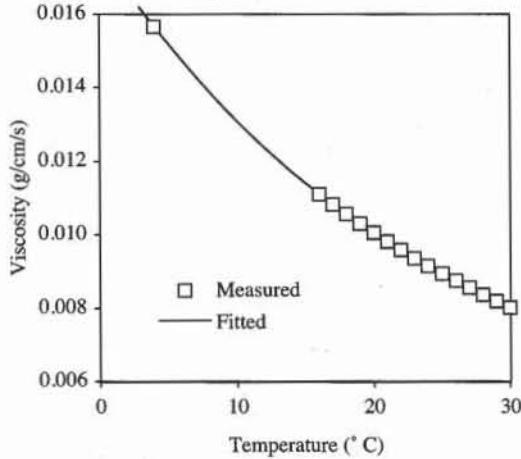


Figure 7 Variation of viscosity of water with temperature.

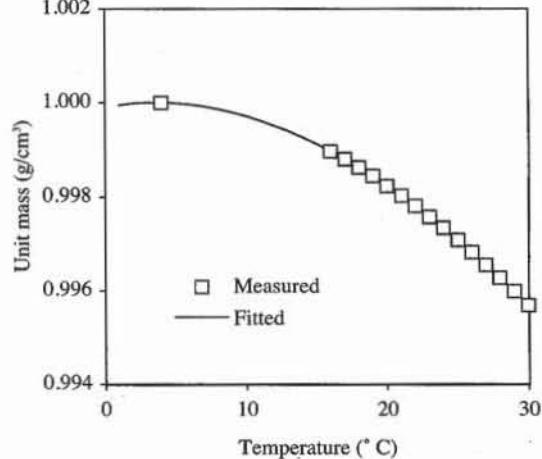


Figure 8 Variation of unit mass of water with temperature.

## LIMITATIONS OF SEDIMENTATION ANALYSIS

The sedimentation analysis assumes that soil particles (1) are spherical, (2) have similar specific gravity  $G_s$ , (3) are separated from one another, and (4) do not interact during sedimentation. These assumptions are not always verified, which limits the application of sedimentation principles to grain size analysis.

**TABLE 3**

Viscosity and density of distilled water  
at various temperatures (International Critical Tables, 1928)

Temperature (°C)	Dynamic viscosity (g/s·cm)	Unit mass (g/cm <sup>3</sup> )
4.0	0.01567	1.00000
16.0	0.01111	0.99897
17.0	0.01083	0.99880
18.0	0.01056	0.99862
19.0	0.01030	0.99844
20.0	0.01005	0.99823
21.0	0.00981	0.99802
22.0	0.00958	0.99780
23.0	0.00936	0.99757
24.0	0.00914	0.99733
25.0	0.00894	0.99708
26.0	0.00874	0.99682
27.0	0.00855	0.99655
28.0	0.00836	0.99627
29.0	0.00818	0.99598
30.0	0.00801	0.99568

1. As described later in Chapter 2-1, small particles such as clay and silt particles are usually not spherical but similar to "corn flakes." Therefore, the drag coefficients  $C_d$  of these non-spherical objects can be different from that of Stokes' law.
2. Natural soils may consist of particles having several minerals, different values of  $G_s$ , and various tendencies to break down into small fragments. In this case, the particle velocity and size may not be described by using an average value of  $G_s$ .
3. Particles in suspension are usually separated by using a chemical agent which creates repulsive forces between particles. However it is difficult to prevent some particles from clustering together and falling faster than individual particles.
4. Finally, the flow patterns around falling particles, which are assumed to be laminar, may become complicated when they interact with one another. It is assumed that this effect is limited when the concentration of soil particles is smaller than 50 g/L.

In spite of these limitations, the principle of sedimentation is still used to determine the grain size distribution of fine-grained soils in geotechnical engineering. It may lead to results which do not fully agree with those of more advanced techniques used in the powder industry (e.g., Allen, 1974). However, the sedimentation analysis is not expensive to perform, and produces consistent results which are sufficient to classify soils for engineering purposes.

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 INTERNATIONAL CRITICAL TABLES, 1928, Vols. III and V, McGraw-Hill Book Co., New York.

- ROBERSON, J. A., and C. T. CROWE, 1993, *Engineering Fluid Mechanics*, Fifth Ed., Houghton Mifflin Company, Boston, pp. 502–508.
- STOKES, G., 1891, *Mathematical and Physical Paper III*, Cambridge University Press, Cambridge, U.K.

**REVIEW QUESTIONS**

1. What is Stokes' law? Which physical quantities does it relate?
2. What is the physical dimension of the viscosity used in Stokes' law? In which unit is it usually expressed? What is the approximate value of the water viscosity at 20°C?
3. State the Reynolds number of a sphere of radius  $R$  in a fluid of viscosity  $\eta$  and unit mass  $\rho$  flowing with velocity  $U$ .
4. What is the approximate range of Reynolds numbers to which Stokes' law applies?
5. What is the drag coefficient of a sphere calculated by using Stokes' law?
6. What is the terminal velocity of a sphere of radius 0.2 mm and specific gravity  $G_s = 2.65$  that falls into pure water at 20°C?
7. Does Stokes' law apply to the particles of medium sand falling in the sedimentation burette?
8. Do the water viscosity and water density increase or decrease with temperature?
9. What physical law prevents the particles smaller than 1  $\mu\text{m}$  from falling when they are in suspension?
10. What shape is assumed for the soil particles when interpreting the results of a sedimentation analysis? Does this assumption apply to clay particles?
11. Does the addition of salt to distilled water increase or decrease the unit mass of water?

# 1-4 Hydrometer Analysis

## OBJECTIVE

Hydrometer analysis is used to determine the grain size distribution of fine-grained soils having particle sizes smaller than 75  $\mu\text{m}$ . If soil samples have particle sizes ranging from sand to silt or clay, sieving and sedimentation analysis are combined as explained in Chapter 1-7. The principle of hydrometer analysis is based on Stokes' law. It assumes that dispersed soil particles of various shapes and sizes fall in water under their own weight as non-interacting spheres.

## EQUIPMENT

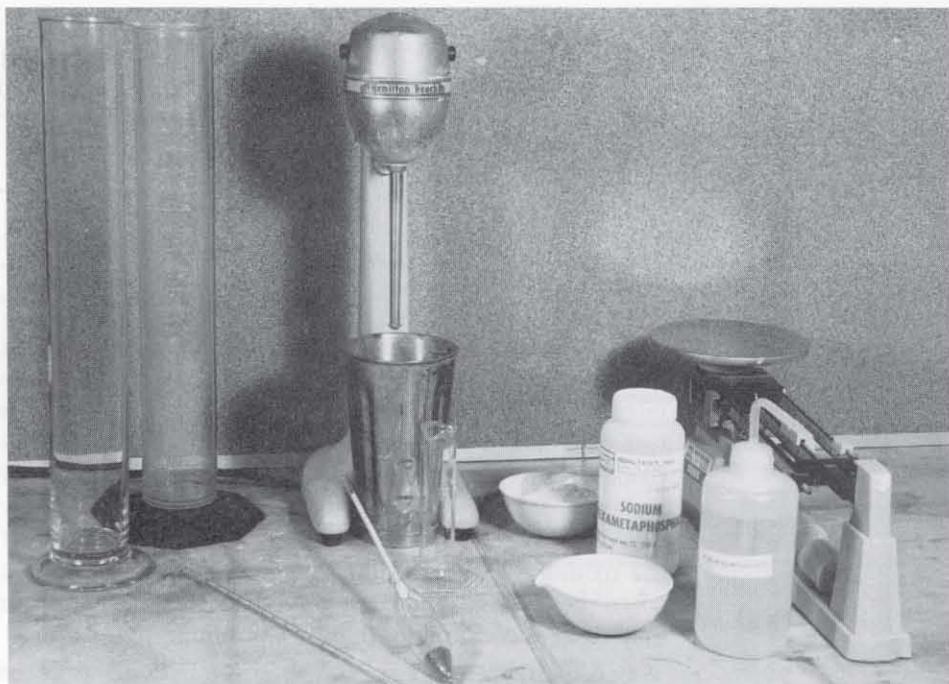
The equipment used in hydrometer analysis includes:

- Soil hydrometer. There are two different types of hydrometers. One is graduated in the specific gravity of fluids and is calibrated to read 1.000 g/cm<sup>3</sup> in pure water at 20°C. The other is graduated in grams of soil and is calibrated at 0 g/L in pure water at 20°C. The capacity and accuracy of both types of hydrometer are indicated in Table 1. Hereafter, we will use the 152H hydrometer graduated in grams of soil per liter (ASTM D422).

**TABLE 1**  
Capacity and Accuracy of Hydrometers

Hydrometer type	Capacity	Accuracy	Unit
Fluid specific gravity	0.995–1.04	0.001	g/cm <sup>3</sup>
Grams of soil per liter	0.0–50.0	1.0	g/L

- Dispersion apparatus, a high-speed mechanical stirrer equipped with an electric motor, which rotates a stirring paddle at high speed (Fig. 1). The dispersion apparatus disperses the mixture of soil, chemicals, and water. If no conventional stirrer is available, a malt mixer or blender can be used.



**Figure 1** Equipment for hydrometer analysis. From left to right, two graduated cylinders of glass, essentially 45 cm high and 6.6 cm in diameter, marked for a volume of 1000 mL; a thermometer, ranging from 0 to 50°C, accurate to 0.5°C; a stopwatch; a balance accurate to 0.01 g; a measuring cylinder, 100 mL; two porcelain evaporating dishes about 100 mm in diameter; a stock solution of sodium hexametaphosphate; a dispersion apparatus; a digital scale; and two sedimentation cylinders.

- Two sedimentation cylinders of glass, essentially 45 cm high and 6.6 cm in diameter, marked for a volume of 1000 mL.
- Thermometer, ranging from 0 to 50°C, accurate to 0.5°C.
- Stopwatch.
- Balance accurate to 0.01 g.
- Measuring cylinder, 100 mL.
- Two porcelain evaporating dishes about 100 mm in diameter.
- Drying oven, 105°-110°C.
- Glass rod about 12 mm in diameter and about 400 mm long.
- 500 mL of hydrogen peroxide.
- 500 mL of stock solution of sodium hexametaphosphate with a concentration of 40 g per liter of distilled or demineralized water. This solution should be prepared frequently, and should be less than a month old.
- 3 L of distilled or demineralized water. The water should be at the temperature that is expected to prevail during the hydrometer test.

## PREPARATION AND PRETREATMENT OF SAMPLE

1. The test specimen is obtained from the fraction of soil sample that is smaller than 75 µm. The approximate weight of the dry specimen may be selected as shown in Table 3.

**TABLE 2**

Stock Solution for Dispersing Agent

Chemical	Quantity	Unit
Sodium hexametaphosphate Buffered with Na <sub>2</sub> CO <sub>3</sub> (commercial name "Calgon")	40.0	g
Water	1.0	L

**TABLE 3**Approximate Quantity of Material  
for Sedimentation Analysis.

Soil type	Dry weight (g)
Fat clays	30.0
Lean clays and silty soils	50.0

2. Organic soils must be treated with chemicals to remove organic matter. The organic matter is removed from soils by oxidation and is accomplished by mixing the soil sample with a solution of 30% hydrogen peroxide. The air-dried test specimen is placed in a 1000-mL conical beaker, 150 mL of the hydrogen peroxide solution is added, and the mixture is stirred gently for a few minutes with a glass rod. The oxidation process may be accelerated by heating the mixture gently. Very organic soils may require several additions of hydrogen peroxide, and the oxidation process may take 2 to 3 days. The oxidation process is completed when there are no more gas bubbles. After the pretreatment, the volume of liquid may be reduced to about 50 mL by boiling.

3. Very fine soil grains will normally tend to flocculate in a suspension (i.e., will adhere to each other and settle together). A dispersing agent is added to all samples to prevent grains from flocculating. A 125-cm<sup>3</sup> quantity of stock solution of sodium hexametaphosphate (40 g/L) is usually sufficient to disperse most soils. After placing the dry sample in a dish, distilled or demineralized water is added until the sample is submerged; then the 125 cm<sup>3</sup> of dispersing agent is added. The sample should be allowed to soak overnight or until all soil lumps have disintegrated.

## HYDROMETER AND CYLINDER CALIBRATION

Prior to the hydrometer test, the hydrometer and sedimentation cylinder are calibrated as follows.

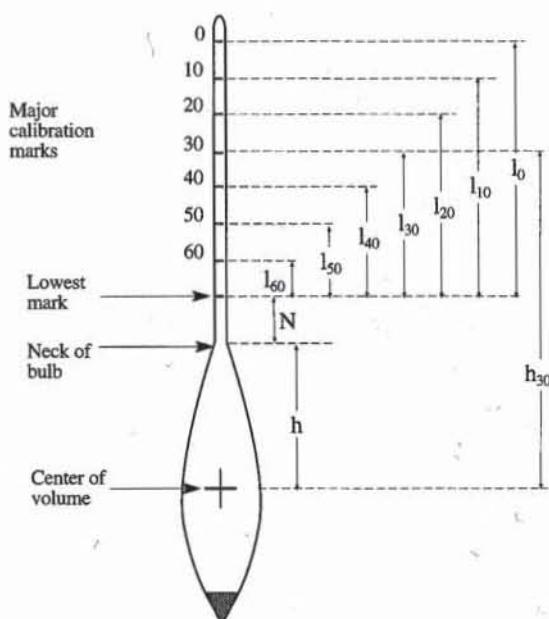
1. The cross-sectional area *A* of the sedimentation cylinder is determined by measuring its internal diameter.

2. The volume of the hydrometer bulb *V<sub>b</sub>* is obtained by immersing it in a graduated cylinder and measuring the rise of water level.

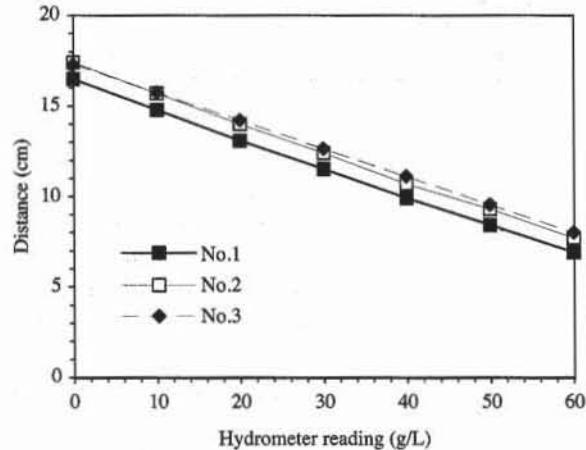
3. The distances between the bulb center and graduation marks 0, 10, 20, 30, 40, 50, and 60 g/L are noted *h<sub>0</sub>*, *h<sub>10</sub>*, *h<sub>20</sub>*, *h<sub>30</sub>*, *h<sub>40</sub>*, *h<sub>50</sub>*, and *h<sub>60</sub>*, respectively. As shown in Fig. 2, they are determined by adding *h* and *N* to *l<sub>0</sub>*, *l<sub>10</sub>*, *l<sub>20</sub>*, *l<sub>30</sub>*, *l<sub>40</sub>*, *l<sub>50</sub>*, and *l<sub>60</sub>*, where *N* is the distance between bulb neck and lowest stem mark and *h* is the distance between the bulb centroid and bulb neck. For instance, the distance between the bulb center and the 30-g/L graduation is *h<sub>30</sub>*:

$$h_{30} = l_{30} + N + h \quad (1)$$

For a symmetrical bulb, *h* is half the bulb height. Typical results of a hydrometer and cylinder calibration are reported in Table 3. As shown in Fig. 3, the distance between the graduation marks and bulb centroid are linearly related and can be fitted by linear regression.



**Figure 2** Measurements for calibration of hydrometer.



**Figure 3** Relation between hydrometer readings and distance to bulb center for the hydrometers of Table 3.

**TABLE 4**

Example of Calibration Results for Three Soil Hydrometers.

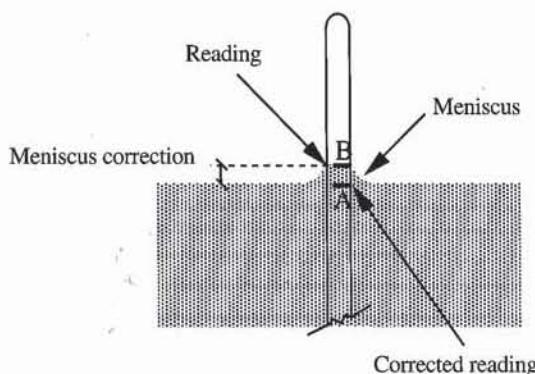
Graduation mark on stem (g/L)	Distance to bulb center (cm)		
	Hydrometer 1	Hydrometer 2	Hydrometer 3
0	16.5	17.4	17.3
10	14.8	15.7	15.75
20	13.1	14	14.2
30	11.5	12.4	12.65
40	9.9	10.7	11.1
50	8.4	9.3	9.55
60	6.9	7.7	8
Bulb volume (mL)	60	60	60
Slope (cm L/g)	-0.16	-0.16	-0.16
Intercept (cm)	16.39	17.30	17.30

## CORRECTION OF HYDROMETER READING

Three corrections are applied to the hydrometer reading  $R$ :  $C_m$ , the meniscus correction;  $m$ , the temperature correction; and  $C_d$ , the dispersing agent correction.  $C_m$  applies to the calculations of grain size and percent by weight finer, whereas  $m$  and  $C_d$  apply only to the calculation of percent by weight finer.

### Meniscus Correction

Hydrometers are calibrated to read correctly at the surface of a transparent liquid, which is represented by level  $A$  in Fig. 4. But soil suspensions are not transparent, making such a reading impossible. Therefore, the hydrometer is read



**Figure 4** Reading a hydrometer, and determination of meniscus correction.

systematically at the upper rim of the meniscus, which corresponds to point *B* in Fig. 4.  $C_m$  is determined by immersing the hydrometer in clear water and measuring the difference between the hydrometer reading made at points *A* and *B*.  $C_m$  is always positive and is a constant for a given hydrometer.  $C_m$  is equal to approximately 0.5 g/L for most 152H hydrometers. The corrected hydrometer reading  $R'$  is

$$R' = R + C_m \quad (2)$$

where  $R$  is the reading above the meniscus.

### Temperature Correction

Temperature influences the solution density and hydrometer volume (caused by thermal contraction or expansion) and consequently the density reading. Since the hydrometer is calibrated at 20°C, a temperature correction factor  $m$  must be added algebraically to each hydrometer reading. The following temperature correction  $m$  (g/L) was determined experimentally for a particular type of hydrometer

$$m = 1000 [0.99823 - \rho_w - 0.000025 (T - 20)] \quad (3)$$

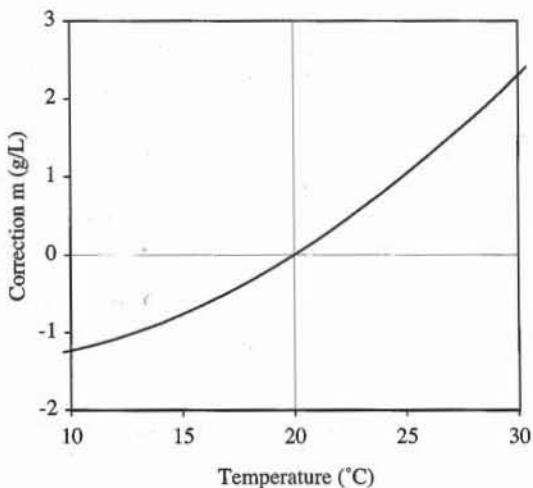
where  $T$  is the temperature of water (°C) and  $\rho_w$  is the water unit mass (g/cm<sup>3</sup>) at temperature  $T$ . As shown in Fig. 5, the correction  $m$  is equal to zero for  $T = 20^\circ\text{C}$ , and is positive or negative, depending on  $T$ .

### Dispersing Agent Correction

The addition of a dispersing agent to water increases the liquid density.  $C_d$  is the hydrometer reading in water and dispersing agent.  $C_d$  is always positive and should be subtracted from the corrected hydrometer reading  $R'$  when calculating percent by weight finer. The maximum value of  $C_d$  is estimated from the concentration of dispersing agent as follows:

$$C_d = 0.001 X_d V_d \quad \text{g/L} \quad (4)$$

where  $X_d$  is the concentration of dispersing agent in stock solution (g/L), and  $V_d$  is the volume of stock solution in 1 L. When  $X_d = 40$  g/L and  $V_d = 125$  mL, Eq. 4 gives  $C_d = 5$  g/L, a value which is slightly different from the measured value of  $C_d$  because the dispersing agent has a smaller specific density than soils.

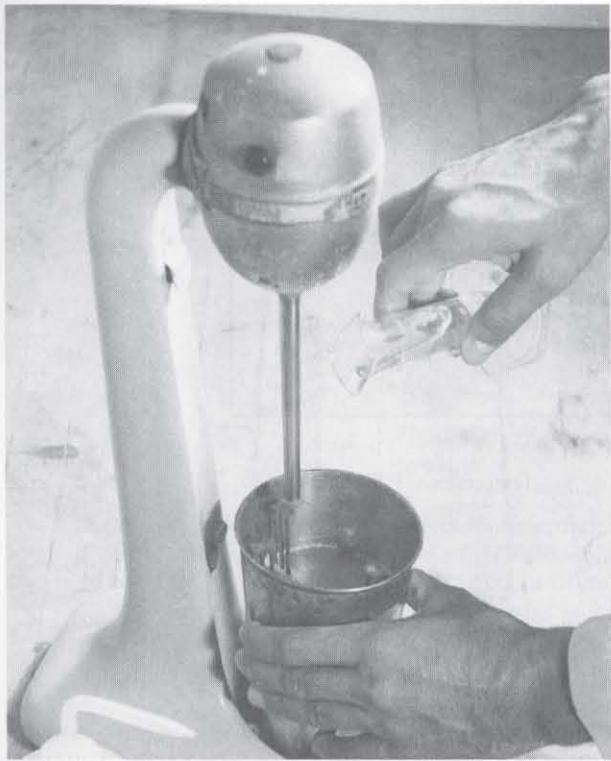


**Figure 5** Variation of temperature correction factor  $m$  for a particular type of hydrometer.

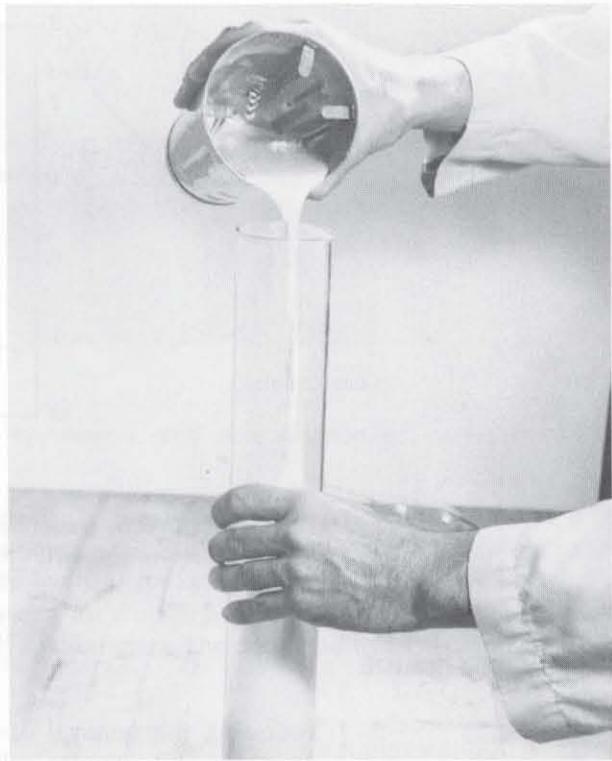
## TEST PROCEDURE

Following the removal of organic matter, addition of dispersing agent, and hydrometer and cylinder calibration, the test procedure consists of the following steps:

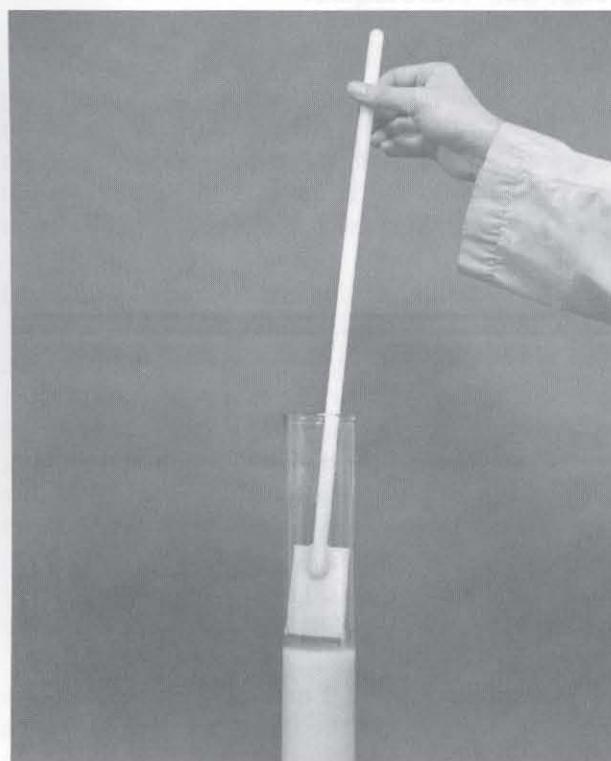
1. Determine the dispersing agent correction  $C_d$  and the meniscus correction  $C_m$ .
2. Measure the specific gravity of solids  $G_s$ .
3. Transfer the soil-water slurry from the dish to a dispersion cup, washing any residue from the dish with distilled or demineralized water. As shown in Fig. 6, add distilled water to the dispersion cup until the water surface is 5 to 8 cm below the top of the cup. If the cup contains too much water, it will splash out while mixing. Place the cup in the dispersing machine and disperse the suspension for 1 to 10 min.
4. Transfer the suspension into a 1000-mL sedimentation cylinder and add distilled or demineralized water to fill the 1000 mL cylinder (Fig. 7).
5. About 1 min before starting the test, take the graduate in one hand and, using the palm of the other hand or a suitable rubber cap as a stopper, shake the suspension vigorously for a few seconds to mix the sediment at the bottom of the graduate into a uniform suspension. Repeat this agitation several times by turning the cylinder upside down. Sometimes it is necessary to loosen the sediment at the bottom of the cylinder by means of a hand agitator (Fig. 8). Sustain a uniform suspension until the test begins.
6. Slowly immerse the hydrometer in the liquid 20 to 25 s before each reading. Immerse and remove it very slowly, as shown in Fig. 9, to prevent disturbance of the suspension.
7. Record the hydrometer reading after 1 and 2 min has elapsed from the time agitation has stopped (Fig. 10). As soon as the 2-min reading has been taken, carefully remove the hydrometer from the suspension and place it in clean water as shown in Fig. 11. If a hydrometer is left in a soil suspension too long, material will settle on or adhere to the hydrometer bulb, and this will cause a sig-



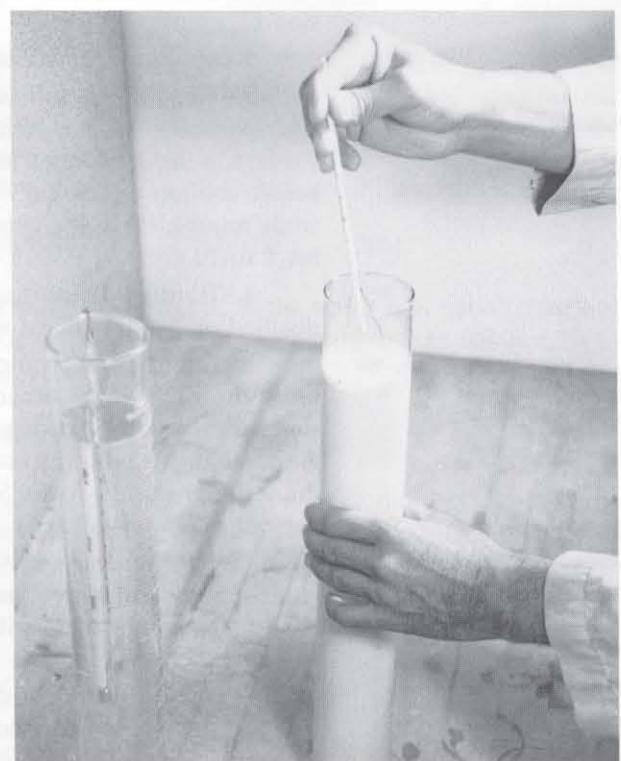
**Figure 6** The soil sample is placed in a dispersion cup and the dispersing agent is added.



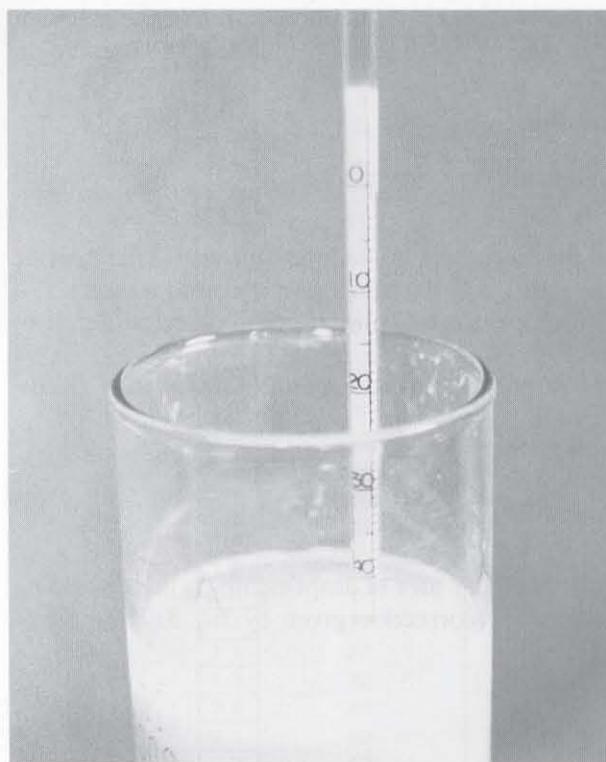
**Figure 7** After dispersion, the soil suspension is poured into the sedimentation cylinder.



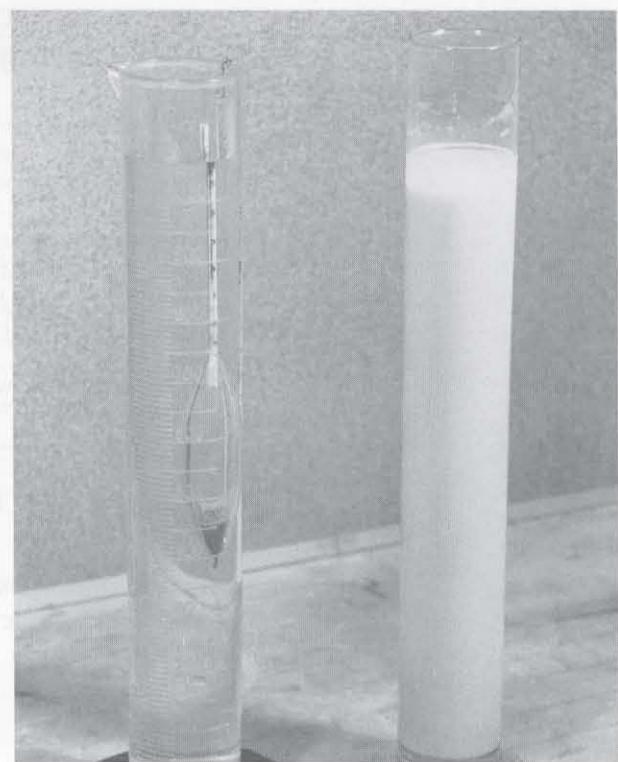
**Figure 8** A uniform suspension may be obtained by using a hand agitator.



**Figure 9** At selected time intervals, the hydrometer is slowly and carefully immersed.



**Figure 10** When the hydrometer stops moving, the reading is made at the top of the meniscus.



**Figure 11** Once the measurement is completed, the hydrometer is removed slowly and carefully, and immersed in a second cylinder filled with clean water.

nificant error in the reading. Insert the hydrometer in the suspension again and record readings after 4, 15, 30, 60, 120, 240, and 1240 min.

8. At the end of 2 min and after each hydrometer reading, record the water temperature. Temperature changes of the soil suspension during the test affect test results. Variations in temperature can be minimized by keeping the suspension away from heat sources, such as radiators, sunlight, or open windows.

## COMPUTATION

The corrected hydrometer reading  $R' = R_t + C_m$ , where  $R_t$  is the hydrometer reading at time  $t$  and  $C_m$  is the meniscus correction. The depth of fall  $H$  is calculated by linear interpolation of  $R'$ . For instance, when  $20 \leq R' \leq 30$  g/L,  $H$  corresponding to  $R'$  is

$$H = h_{20} + (h_{30} - h_{20}) \frac{R' - 20}{30 - 20} \quad (5)$$

where  $h_{20}$  and  $h_{30}$  correspond to the marks 20 and 30 g/L, respectively, as defined in the hydrometer calibration. The corrected depth of fall  $H_R$  is defined to account for the rise of water level when the hydrometer is immersed:

$$H_R = H - \frac{V_b}{2A} \quad (6)$$

where  $V_b$  is the bulb volume,  $A = (\pi/4) d_c^2$  the cross-sectional area of the cylinder, and  $d_c$  the diameter of the cylinder. According to Stokes' equation, the particle diameter  $D$  (mm) is

$$D = \sqrt{\frac{30\eta H_R}{981(G_s - 1)\rho_w t}} \quad (7)$$

where  $t$  is the time (min) after the beginning of sedimentation,  $G_s$  the specific gravity of the soil particles,  $\rho_w$  the unit mass of water ( $\text{g}/\text{cm}^3$ ) at temperature  $T$ ,  $\eta$  the viscosity of water ( $\text{g}/\text{cm}\cdot\text{s}$ ) at temperature  $T$ , and  $H_R$  the corrected depth of fall (cm).

The percentage  $p$  by weight of particles with diameter smaller than  $D$  corresponding to  $R'$  is

$$p = \frac{0.6226}{W_0} (R' - C_d + m) \frac{G_s}{G_s - 1} \times 100 \quad (\%) \quad (8)$$

where  $W_0$  is the oven-dried weight of soil per liter of suspension,  $C_d$  the dispersing agent correction, and  $m$  the temperature correction given by Eq. 3. The unit of  $W_0$ ,  $R'$ ,  $C_d$ , and  $m$  is g/L.

### EXAMPLE

Figures 12 and 13 show the results of a hydrometer analysis of a silt. As shown in Fig. 12, the results are presented on the same type of grain size distribution curve as the sieve analysis. Figure 13 shows the input/output data corresponding to Fig. 12, and Fig. 14 shows the formulas used in Fig. 13. Figure 15 lists the user-defined functions Densi, Visco, and M. Densi and Visco return the water unit mass in ( $\text{g}/\text{cm}^3$ ) and water viscosity in ( $\text{g}/\text{cm}\cdot\text{s}$ ) at temperature  $T$  ( $^\circ\text{C}$ ). M calculates changes in the hydrometer reading as a function of temperature  $T$  ( $^\circ\text{C}$ ) (see Eq. 3). The user-defined function INTERL is also used to calculate the distance of fall by linear interpolation. The clay fraction, which the percent by weight finer than 2  $\mu\text{m}$ , is calculated by using the user-defined INTER (see Chapter 8-1).

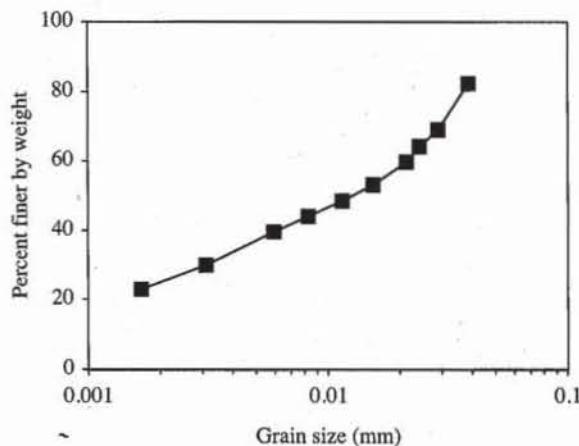


Figure 12 Example of grain size distribution obtained by hydrometer analysis.

	A	B	C	D	E	F
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14	Graduation mark on hydrometer stem (g/L)	Distance to bulb center (cm)				
15	R <sub>s</sub>	H <sub>s</sub>				
16	0	16.5				
17	10	14.8				
18	20	13.1				
19	30	11.5				
20	40	9.9				
21	50	8.4				
22	60	6.9				
23						
24	Time (min)	Hydrometer reading (g/L)	Temperature (°C)	Corrected distance of fall (cm)	Grain size (mm)	Percent finer by weight
25	t	R <sub>t</sub>	T <sub>e</sub>	HR	D	p
26	1	40.0	22.5	8.54	0.0388	82.2
27	2	34.0	22.5	9.56	0.0290	68.8
28	3	32.0	22.0	9.90	0.0243	64.2
29	4	30.0	22.0	10.24	0.0214	59.7
30	8	27.0	22.0	10.75	0.0155	53.1
31	15	25.0	21.5	11.09	0.0116	48.4
32	30	23.0	21.5	11.43	0.0083	43.9
33	60	21.0	21.5	11.77	0.0060	39.5
34	240	17.0	20.0	12.45	0.0031	30.0
35	900	14.0	19.0	12.96	0.0017	22.9
36	Clay fraction (%) = 24.4					

Figure 13 Example of data set.

**REFERENCE**

See Introduction for references on ASTM procedures (pages 4 to 6).

**REVIEW QUESTIONS**

- What is the purpose of hydrometer analysis? On which physical principles is hydrometer analysis founded?
- Does hydrometer analysis determine the size of soil particles exactly?

	D	E
2 4	Corrected distance of fall (cm)	Grain size (mm)
2 5	HR	D
2 6	=INTERL(Rt+ABS(Cm),Rs,Hs)-Vb/(2*PI()*dc^2/4)	=SQRT(30*VISCO(Te)*Hr/(Gs-1)/981/DENSI(Te)/t)
F		
2 4	Percent finer by weight	
2 5	P	
2 6	=100/W0*(Rt-ABS(Cd)+ABS(Cm)+M(Te))*0.6226*Gs/(Gs-1)	3 6 Clay fraction (%) = =INTER(0.002,D,p)
B	C	

Figure 14 Formulas used in Fig. 13.

	A	B
1	VISCO	viscosity of water
2	=RESULT(1)	in g/cm/s as a function of
3	=ARGUMENT("T",1)	temperature T in degree Celsius
4	=RETURN(0.0178-5.684/10^4*T+1.115/10^5*T^2-1.017/10^7*T^3)	
5		
6	DENSI	density of water
7	=RESULT(1)	in gr/cm3 as a function pf
8	=ARGUMENT("T",1)	temperature T in degree Celsius
9	=RETURN(0.99991+5.202/10^5*T-7.512/10^6*T^2+3.605/10^8*T^3)	
10		
11	M	temperature correction
12	=RESULT(1)	factor for hydrometer
13	=ARGUMENT("T",1)	as a function of temperature T
14	=RETURN(1000*(0.99823-DENSI(T)-0.000025*(T-20)))	in degree Celsius

Figure 15 User-defined functions VISCO, DENSI, and M used in Fig. 14.

3. Why do you correct the distance of fall of particles during hydrometer analysis?
4. What physical quantity is read on the stem of a 152H hydrometer? In what unit is this quantity expressed?
5. What corrections are made on the hydrometer reading?
6. What modifications would be required if one wanted to carry out a hydrometer analysis in a 2000-mL cylinder instead of a 1000-mL cylinder?
7. Does the hydrometer go up or down during the sedimentation of soil particles?
8. What is the purpose of the dispersing agent? Does its use require a correction?
9. Why should you remove the hydrometer from the sedimentation burette after each reading?
10. Is there a correction for the specific gravity of soil particles when one uses a 152H hydrometer?
11. What physical quantity does the hydrometer measure? At what location does it measure it?
12. Why must you slowly insert and remove the hydrometer in the sedimentation burette?

13. Why do you measure the water temperature during the hydrometer analysis?
14. What is the purpose of the hydrometer calibration? What quantities does it relate?
15. Why does the meniscus correction always have the same sign? What is this sign?
16. For what reason do you agitate the suspension at the beginning of the hydrometer test?
17. What is the usual duration of a hydrometer analysis? Why does the sedimentation part take so much time?
18. Suppose that one wants to calibrate the hydrometer analysis with an assembly of spherical particles having only one radius: say  $a = 0.05$  mm. Describe qualitatively what will happen to the hydrometer as a function of time. Will it sink gradually from the beginning of the sedimentation? Sketch the variation of hydrometer penetration versus time.

## EXERCISES

1. Equation 3 was assumed to account for the effect  $m$  of temperature on the hydrometer reading. Verify experimentally this relation.
2. The calibration of the hydrometer requires measurement of the distances  $h$  for the readings  $R = 0, 10, 20, 30, 40, 50$ , and  $60$  g/L. By using a linear regression, give an approximate expression that relates  $R$  and  $h$  for the following hydrometer:

$R$ (g/L)	0	10	20	30	40	50	60
$h$ (cm)	17.4	15.7	14.0	12.4	10.7	9.3	7.7

3. Modify the example of Fig. 13 in order to use the fitting between the hydrometer reading  $R$  and the distance  $h$  between readings and the bulb center when processing the experimental results of the hydrometer test.
4. Draw the grain size distribution curve for the following hydrometer analysis results.

Soil data	Hydrometer calibration			Hydrometer test		
	Graduation mark on stem (g/L)	Distance to bulb center (cm)	Time (min)	Hydrometer reading (g/L)	Temperature (°C)	
Mass retained on No. 200 sieve	0 g	0	16.5	1	20.0	24
Mass in suspension	30.02 g	10	14.8	2	19.5	24
Specific unit weight	2.65	20	13.1	4	18.7	24
Dispersing agent correction	3 g/L	30	11.5	15	16.0	24
Meniscus correction	0.5 g/L	40	9.9	30	14.5	24
Burette diameter	5.44 cm	50	8.4	60	13.0	24
Bulb volume	65 cm <sup>3</sup>	60	6.9	120	12.0	23
				240	10.0	29
				1240	9.5	23.5

5. Draw the grain size distribution curve for the following hydrometer analysis results.

Soil data	Hydrometer calibration			Hydrometer test	
	Graduation mark on stem (g/L)	Distance to bulb center (cm)	Time (min)	Hydrometer reading (g/L)	Temperature (°C)
Mass retained on No. 200 sieve	0 g	0	18.15	1	40.0
Mass in suspension	45 g	10	16.7	2	38.5
Specific unit weight	2.65	20	15.25	4	38.0
Dispersing agent correction	3.5 g/L	30	13.8	15	37.5
Meniscus correction	0.5 g/L	40	12.35	30	36.5
Burette diameter	5.44 cm	50	10.9	60	35.5
Bulb volume	65 cm <sup>3</sup>	60		120	33.0
				240	31.2
				1240	26.0
					23

6. Draw the grain size distribution curve for the following hydrometer analysis results.

Soil data	Hydrometer calibration			Hydrometer test	
	Graduation mark on stem (g/L)	Distance to bulb center (cm)	Time (min)	Hydrometer reading (g/L)	Temperature (°C)
Mass retained on No. 200 sieve	0 g	0	24.5	1	20.0
Mass in suspension	30.02 g	10	22.9	2	19.0
Specific unit weight	2.65	20	21.3	4	18.0
Dispersing agent correction	3 g/L	30	19.7	15	16.0
Meniscus correction	0.5 g/L	40	18.1	30	14.5
Burette diameter	5.44 cm	50	16.5	60	13.5
Bulb volume	65 cm <sup>3</sup>	60	9.45	120	12.5
				240	12.0
				1240	9.5
					23

7. Draw the grain size distribution curve for the following hydrometer analysis results.

Soil data	Hydrometer calibration			Hydrometer test	
	Graduation mark on stem (g/L)	Distance to bulb center (cm)	Time (min)	Hydrometer reading (g/L)	Temperature (°C)
Mass retained on No. 200 sieve	0 g	0	17.8	1	30.0
Mass in suspension	30.04 g	10	16.1	2	28.0
Specific unit weight	2.65	20	14.4	4	27.0
Dispersing agent correction	2.25 g/L	30	12.7	15	25.0
Meniscus correction	0.5 g/L	40	11	30	23.0
Burette diameter	5.95 cm	50	9.3	60	20.0
Bulb volume	65 cm <sup>3</sup>	60	7.6	120	17.0
				244	15.0
				1059	11.0
					23

8. Derive Eq. 4 which determines the approximate  $C_d$  dispersing agent correction when a volume  $V_d$  of stock solution with a concentration  $X_d$  is used in a liter of soil suspension.

# 1-5 Pipette Analysis

## OBJECTIVE

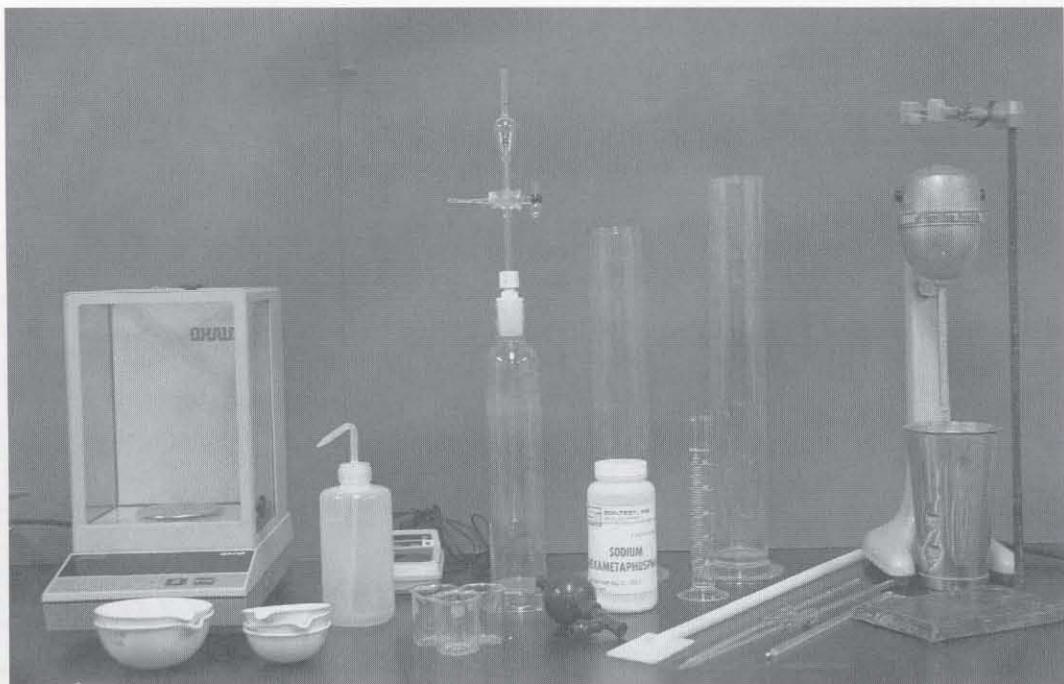
Like hydrometer analysis, pipette analysis is used to determine the grain size distribution of fine-grained soils having particle size smaller than  $75\text{ }\mu\text{m}$ . If soil samples have fine and coarse particles, including sand, silt, and clay particles, sieving and pipette analysis are combined as explained in Chapter 1-7. Pipette analysis is based on Stokes' law and assumes that dispersed soil particles of various shapes and sizes fall in a liquid under the action of gravitational forces as noninteracting spheres.

In the United States, pipette analysis is less commonly used than hydrometer analysis and is not described by ASTM. However, in the United Kingdom, the British Standard (BS 1377, 1975) refers to pipette analysis as the primary method for determining the grain size distribution of fine-grained soils. Pipette analysis gives faster results than hydrometer analysis but requires accurate measurements of small weights.

## EQUIPMENT

The equipment used in pipette analysis includes:

- Sampling pipette, either an Andreasen pipette or a regular pipette, capable of measuring  $10 \pm 0.2\text{ mL}$  of liquid, with a lowering and raising support (Fig. 1). The Andreasen pipette shown in Figs. 1, 2 and 3 has its own sedimentation cylinder and its own support for adjustment of sampling depth. As shown in Figs. 4 and 5, a regular pipette can also be used.
- Dispersion apparatus (same as that used for hydrometer analysis).
- Two sedimentation cylinders (same as those used for hydrometer analysis).
- Thermometer, ranging from 0 to  $50^\circ\text{C}$ , accurate to  $0.5^\circ\text{C}$ .
- Stopwatch.
- Balance accurate to 0.001 g.



**Figure 1** Equipment for pipette analysis. Two sedimentation cylinders, a thermometer, an Andreasen pipette, a 10-mL pipette, a dispersion apparatus, porcelain evaporating dishes, and a stock solution of sodium hexametaphosphate, and a 1-mg sensitive scale.

- Measuring cylinder, 100 mL.
- Porcelain evaporating dishes about 10 cm and 5 cm in diameter.
- Drying oven, 105°–110°C.
- Hand agitator about 400 mm long.
- 500 mL of hydrogen peroxide.
- 500 mL of stock solution of sodium hexametaphosphate prepared as in the hydrometer test.
- 2 L of distilled or demineralized water.

## SAMPLE PREPARATION AND PRETREATMENT

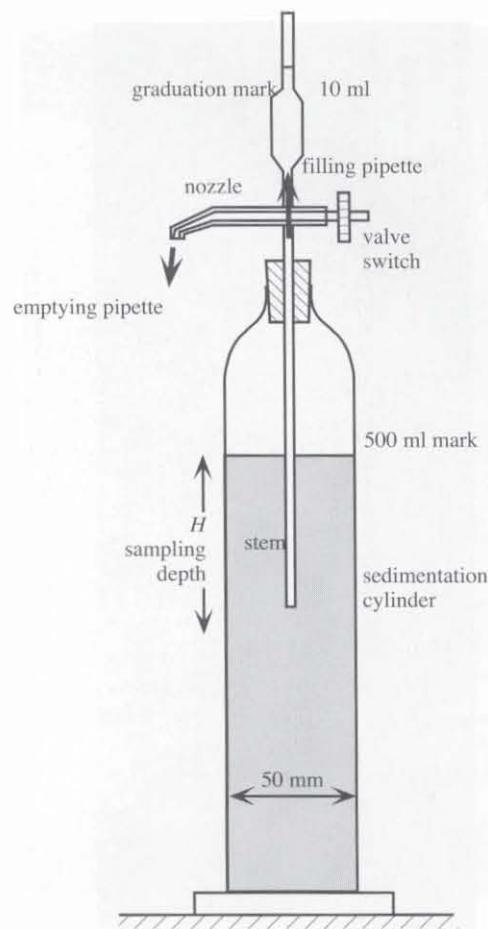
The test specimen is selected, pretreated for removal of organic matter, and mixed with a dispersing agent as described for hydrometer analysis. For the pipette analysis, it is recommended to use 500 mL of soil-water suspension and 65 mL of stock solution of sodium hexametaphosphate (i.e., half the quantities used for hydrometer analysis).

## PIPETTE CALIBRATION

The pipette volume  $V$  must be calibrated. This calibration does not need to be repeated before each experiment. The pipette, which has first been cleaned and dried, is filled with distilled water until the bottom of the water meniscus reaches the graduation mark on the pipette stem. Then the pipette contents are emptied



**Figure 2** The Andreasen pipette, its sedimentation cylinder, and a rubber pumping and sucking device.



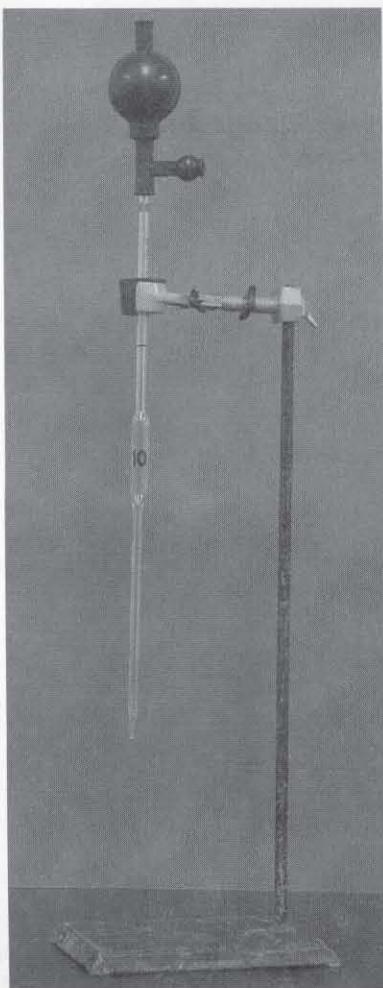
**Figure 3** In the Andreasen pipette, the sample can be taken without removing the pipette. The sampling depth may be adjusted by varying the pipette position.

into a dish and the mass of water is measured. The volume  $V$  in milliliters is equal to the mass of water in grams. Make three determinations of  $V$ , and take the average for volume  $V$ .

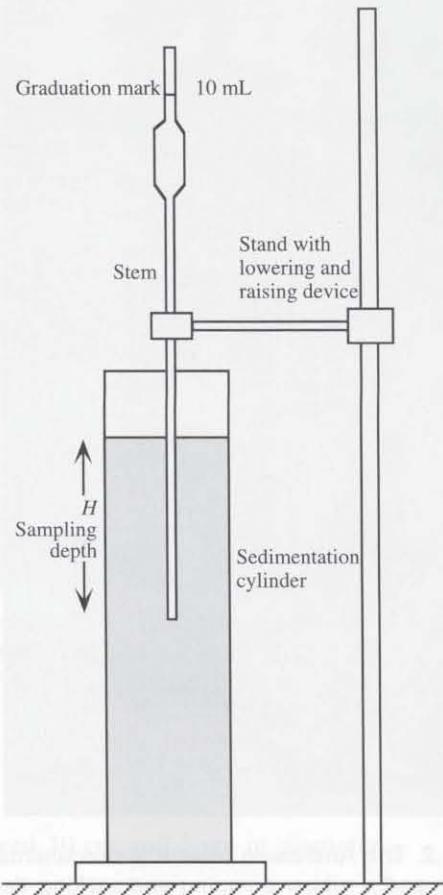
## TEST PROCEDURE

There are two pipette methods: wet and dry. In the *wet method*, samples are weighed without being dried, whereas in the *dry method*, samples are dried prior to being weighed. After the removal of organic matter, addition of dispersing agent, and pipette calibration, the test procedures consist of the following steps:

1. Follow steps 2 to 5 of the hydrometer analysis, except for the total volume of the suspension, which is now 500 mL, and the stock solution volume, which is now 65 mL.
2. Pipette samples are to be taken at several specified time intervals corresponding to the particle size equal to 20, 10, 6, and 2  $\mu\text{m}$ , as shown in Table 1. No sampling is made for 75 and 60  $\mu\text{m}$  because there is simply not enough time. The last sampling operation takes place about 7 h after the be-



**Figure 4** Regular pipette capable of measuring 10 mL of liquid. It is mounted on a stand with a lowering and raising device.



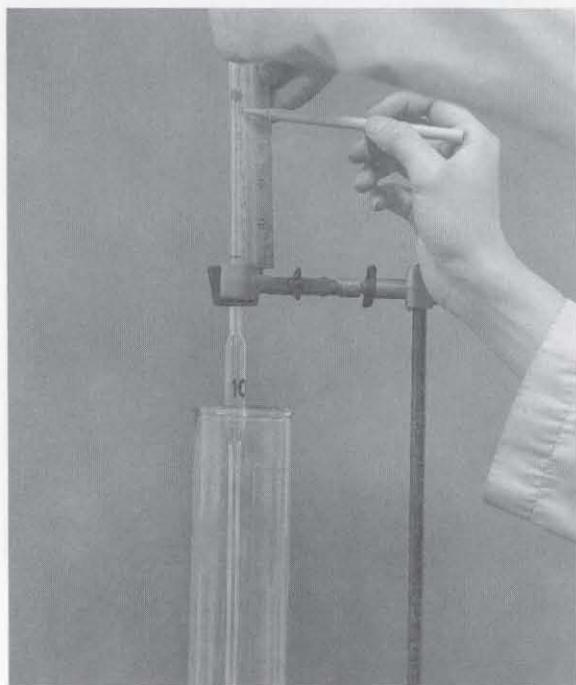
**Figure 5** Regular pipette mounted on a stand and lowered into suspension just before sampling.

ginning of the test. Therefore, the pipette analysis can be completed three times faster than hydrometer analysis. The sampling times  $t$  of Table 1 are calculated by using the following equation for a temperature of 25°C:

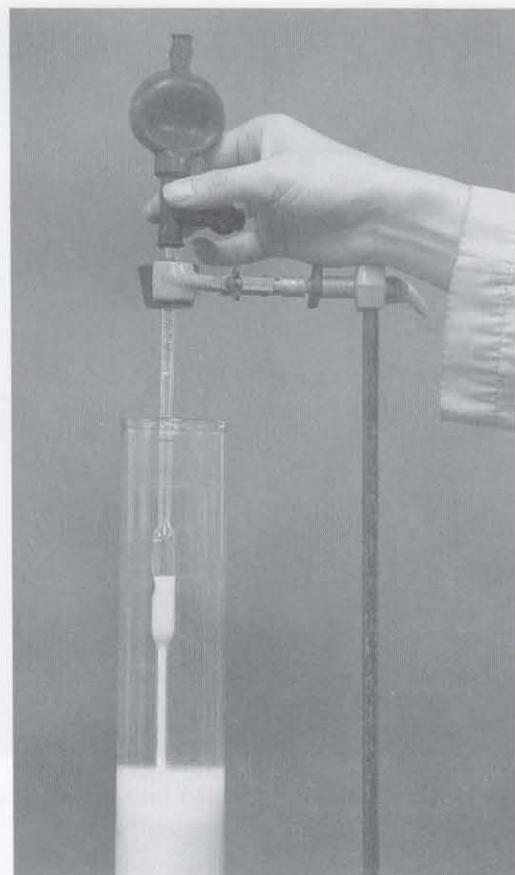
$$t = \frac{18 \times 10^8 \eta H}{(G_s - 1)981 \rho_w D^2} \quad (\text{s}) \quad (1)$$

where  $G_s$  is the specific gravity of the soil particles,  $\rho_w$  the unit mass of the water ( $\text{g}/\text{cm}^3$ ),  $\eta$  the viscosity of the water ( $\text{g}/\text{cm}\cdot\text{s}$ ),  $H$  the sampling depth (cm), and  $D$  the particle size ( $\mu\text{m}$ ).

3. *Regular pipette.* Move the pipette over the sedimentation cylinder, and lower it until its tip touches the water surface (Fig. 6). About 15 s before a sample is due, steadily lower the pipette 100 mm into the suspension, slowly enough not to disturb the suspension. Draw a sample into the pipette until the bottom of the liquid meniscus reaches the graduation mark. A convenient way of drawing the sample is to use a rubber pump attachment (see Fig. 7). The drawing operation should take about 10 s. Then gently withdraw the pipette from the suspension.



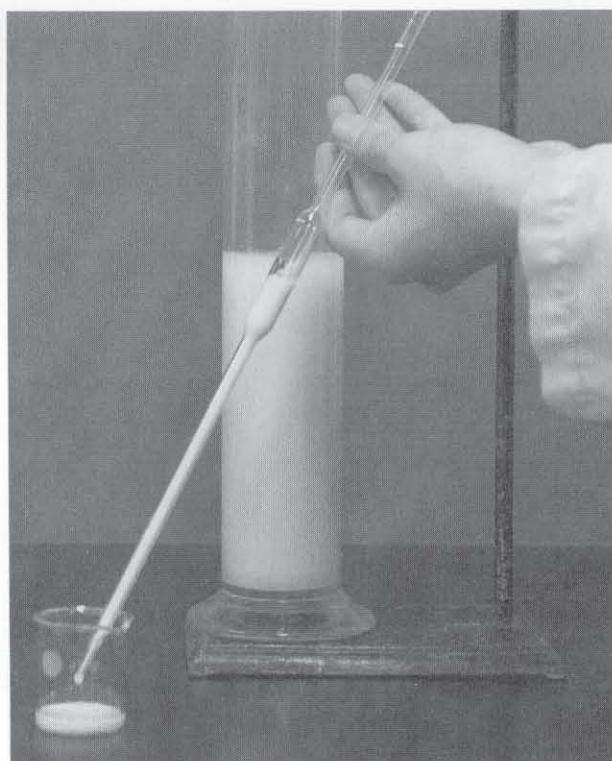
**Figure 6** The pipette is first positioned to touch the liquid surface, then steadily and slowly lowered into the suspension to the sampling depth.



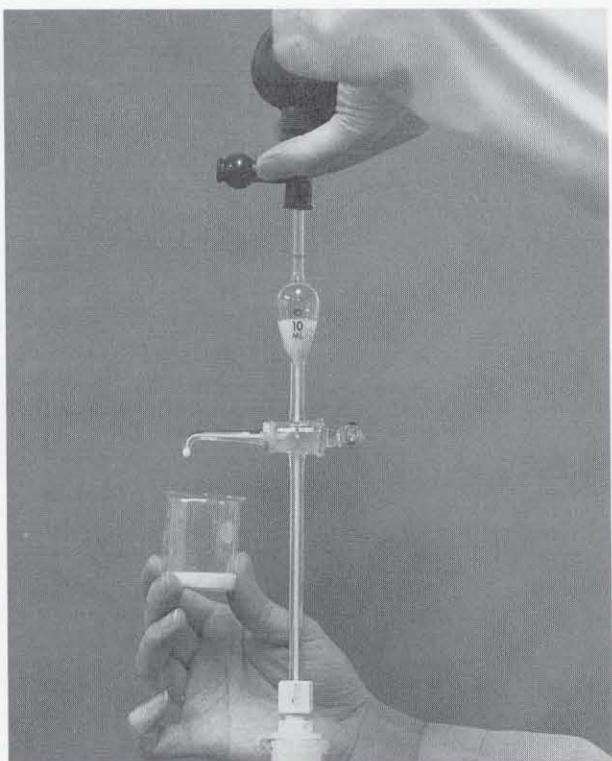
**Figure 7** Open the pipette tap and draw a sample into the pipette until the bottom of the liquid meniscus reaches the graduation mark.

*Andreasen pipette.* Slowly draw a sample into the pipette until the bottom of the liquid meniscus reaches the graduation mark. Switch the two-way valve to empty the pipette.

4. Carefully empty the pipette contents, without losing a drop, into a weighing dish previously weighed (Figs. 8 and 9). Squeeze the rubber pump attachment several times to remove all sample traces from the pipette.
5. *Wet method.* Measure the sample weight with an accuracy smaller than or equal to 1 mg.  
*Dry method.* Place the weighing dish and sample in an oven at 105°C, and weigh it when the sample is dry. Accurate weighing is important because the sample weights are very small.
6. After each sampling, record the water temperature. The temperature should not vary excessively during the experiment.
7. At a convenient time between samplings, the mass of dispersing agent in solution is measured. Take a sample of water and dispersing agent from another sedimentation cylinder, which contains the same amount of dispersing agent and water as the test cylinder but no soil. The sampling time is not critical because the dispersing agent does not settle. Empty the pipette sample into a weighing dish.



**Figure 8** The pipette is emptied into a weighing dish.



**Figure 9** In contrast to a regular pipette, the Andreasen pipette is emptied without being removed from the sedimentation cylinder.

**8. Wet method.** Weigh the sample of dispersing agent.

**Dry method.** Dry the sample of dispersing agent in an oven at 105°C, and weigh it when it is dry.

**TABLE 1**  
Approximate Sampling Times

$G_s$	Diameter of particle ( $\mu\text{m}$ )					
	75	60	20	10	6	2
2.5	20 s	31 s	04 min 35 s	18 min 20 s	50 min 57 s	07 h 38 min
2.55	19 s	30 s	04 min 26 s	17 min 45 s	49 min 18 s	07 h 23 min
2.6	18 s	29 s	04 min 18 s	17 min 12 s	47 min 46 s	07 h 09 min
2.65	18 s	28 s	04 min 10 s	16 min 40 s	46 min 19 s	06 h 56 min
2.7	17 s	27 s	04 min 03 s	16 min 11 s	44 min 57 s	06 h 44 min
2.75	17 s	26 s	03 min 56 s	15 min 43 s	43 min 40 s	06 h 33 min
2.8	16 s	25 s	03 min 49 s	15 min 17 s	42 min 27 s	06 h 22 min
2.85	16 s	25 s	03 min 43 s	14 min 52 s	41 min 18 s	06 h 11 min
2.9	15 s	24 s	03 min 37 s	14 min 29 s	40 min 13 s	06 h 01 min
2.95	15 s	24 s	03 min 32 s	14 min 06 s	39 min 11 s	05 h 52 min
3	15 s	23 s	03 min 26 s	13 min 45 s	38 min 13 s	05 h 43 min
3.05	14 s	22 s	03 min 21 s	13 min 25 s	37 min 17 s	05 h 35 min
3.1	14 s	22 s	03 min 17 s	13 min 06 s	36 min 23 s	05 h 27 min
3.15	14 s	21 s	03 min 12 s	12 min 48 s	35 min 33 s	05 h 19 min
3.2	13 s	21 s	03 min 08 s	12 min 30 s	34 min 44 s	05 h 12 min

Depth of sampling, 10 cm; temperature, 25°C;  $G_s$ , specific gravity.

**COMPUTATION**

For both dry and wet pipette methods, the grain size  $D$  is

$$D = \sqrt{\frac{30 \eta H}{(G_s - 1)981\rho_w t}} \quad (\text{mm}) \quad (2)$$

where  $t$  is the time (min) after the beginning of sedimentation,  $G_s$  the specific gravity of the soil particles,  $\rho_w$  the unit mass of the water ( $\text{g}/\text{cm}^3$ ) at temperature  $T$ ,  $\eta$  the viscosity of the water in ( $\text{g}/\text{cm}\cdot\text{s}$ ) at temperature  $T$ , and  $H$  the sampling depth (cm).

In the dry pipette method, the percentage  $p$  by weight of particles with diameter smaller than  $D$  is

$$p = \frac{V_t}{V} \frac{M_s - M_b - M_d}{M_0} \times 100 \quad (\%) \quad (3)$$

where  $M_0$  is the total mass of the oven-dried soil in suspension (g),  $M_b$  the mass of the empty weighing dish (g),  $M_s$  the mass of the weighing dish and sample of oven-dried soil (g),  $V_t$  the total volume of the suspension (mL),  $V$  the volume of the pipette (mL), and  $M_d$  the mass of the dispersing agent in volume  $V$  (g). The mass  $M_d$  of the dispersing agent in volume  $V$  is measured directly. It can also be estimated as follows:

$$M_d = 0.001 \frac{V}{V_t} C_d V_d \quad (\text{g}) \quad (4)$$

where  $C_d$  is the concentration of the dispersing agent in stock solution ( $\text{g}/\text{L}$ ),  $V_d$  the volume of the stock solution in total volume (mL), and  $V_t$  the volume of the suspension (mL). In general, Eq. 4 slightly overestimates the measured value of  $M_d$  when the dispersing agent is not completely dissolved.

In the wet pipette method, the percentage  $p$  by weight of particles with diameter smaller than  $D$  is

$$p = \frac{\rho_s V_t}{\rho_s V - M_{dw}} \frac{M_{sw} - M_b - M_{dw}}{M_0} \times 100 \quad (\%) \quad (5)$$

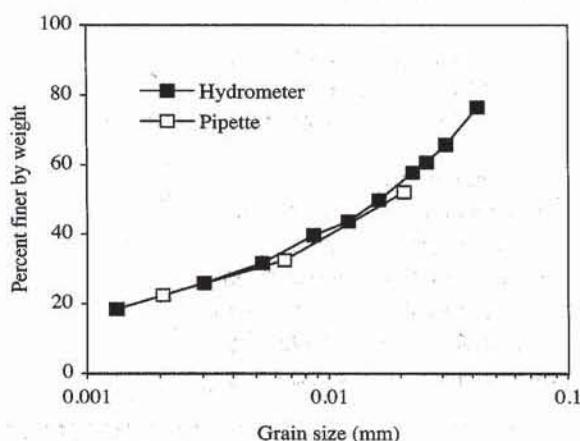
where  $M_0$  is the total mass of the oven-dry soil in suspension (g),  $M_b$  the mass of the empty weighing dish (g),  $M_{sw}$  the mass of the weighing dish and sample of soil suspension (g),  $V_t$  the total volume of suspension (mL),  $V$  the volume of the pipette (mL),  $\rho_s$  the unit mass of the solid (i.e.,  $\rho_s = G_s \rho_w$ ) ( $\text{g}/\text{mL}$ ), and  $M_{dw}$  the mass of water and dispersing agent in volume  $V$  (g).  $M_{dw}$  is measured directly. It can also be estimated as follows:

$$M_{dw} = V \rho_w + 0.001 \frac{V}{V_t} C_d V_d \left(1 - \frac{1}{G_c}\right) \quad (\text{g}) \quad (6)$$

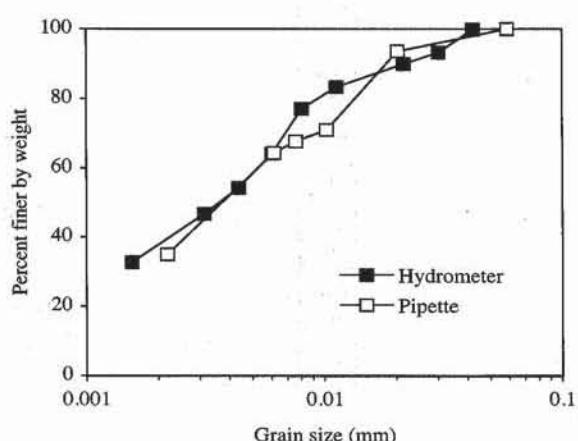
where  $G_c$  is the specific gravity of the dispersing agent. Equation 6 slightly overestimates the measured value of  $M_{dw}$  for the same reason as Eq. 4.

### EXAMPLE

Figures 10 and 12 show the results of a dry pipette analysis obtained for a silt from Lucerne Valley, California. As shown in Fig. 10, the results are presented in a grain size distribution curve similar to that of sieve and hydrometer analyses. Figure 13 shows the formulas used in Fig. 12. Figures 11, 14, and 15 give the results, data set, and formulas of a wet pipette analysis on a silt from Barnard, Vermont. Both dry and wet methods use the user-defined functions DENSI and VISCO, which are defined in Chapter 1-4. DENSI and VISCO return the water unit mass ( $\text{g}/\text{cm}^3$ ) and water viscosity ( $\text{g}/\text{cm}\cdot\text{s}$ ), respectively, at temperature  $T$  ( $^\circ\text{C}$ ).



**Figure 10** Comparison of results for hydrometer and dry pipette analysis on a silt from Lucerne Valley, California.



**Figure 11** Comparison of results for hydrometer and wet pipette analysis on a silt from Barnard, Vermont.

	A	B	C	D	E	F	G
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15	Time (min)	Depth of sampling (cm)	Temperature ( $^\circ\text{C}$ )	Mass of bottle (g)	Mass of dry sample and bottle (g)	Grain size (mm)	Percent finer by weight
16	t	H	$T_e$	$M_b$	$M_s$	D	p
17	4	10.0	23.0	44.5987	44.8772	0.0209	52.06
18	40	10.0	23.0	47.7586	47.9397	0.0066	32.60
19	410	10.0	23.8	43.2441	43.3748	0.0020	22.52

**Figure 12** Example of data set for dry pipette analysis.

	F	G
15	Grain size (mm)	Percent finer by weight
16	D	p
17	=20*SQRT(4.5*VISCO(Te)*H/((Gs-1)*DENSI(Te)*981*60*t))	=100*(Ms-Mb-Md+Mbd)/M0*Vt/V

Figure 13 Formulas used in Fig. 12.

	A	B	C	D	E	F	G
<b>Pipette analysis</b>							
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14	Time (min)	Sampling depth (cm)	Temperature (°C)	Mass of bottle (g)	Mass of sample and bottle (g)	Grain size (mm)	Percent finer by weight
15	t	H	T <sub>a</sub>	M <sub>b</sub>	M <sub>s</sub>	D	p
16	0.5	10.0	23	16.960	27.180	0.0591	100.07
17	4.2	10.0	23	16.000	26.200	0.0205	93.58
18	8.3	5.0	23	86.790	96.920	0.0102	70.83
19	15.0	5.0	23	45.160	55.280	0.0076	67.58
20	23.1	5.0	23	45.950	56.060	0.0061	64.33
21	180.5	5.0	23	47.080	57.100	0.0022	35.09

Figure 14 Example of data set for wet pipette analysis.

	F	G
14	Grain size (mm)	Percent finer by weight
15	D	p
16	=20*SQRT(4.5*VISCO(Te)*H/((Gs-1)*DENSI(Te)*981*60*t))	=100*(Ms-Mb-Md+Mbd)/M0*Vt/(V-(Mdw-Mbd)/Gs/DENSI(Te))

Figure 15 Formulas used in Fig. 14.

**COMPARISON OF HYDROMETER AND PIPETTE ANALYSES**

Pipette analysis has several advantages over hydrometer analysis. It takes less time because the sampling depth is adjustable, whereas it is fixed in hydrometer analysis. The calculations are also simpler and there is no need to account for the correction of meniscus or hydrometer dilation. However, compared to hydrometer analysis, pipette analysis is less adapted to the conditions encountered in a field laboratory. It requires accurate weight measurement. As shown in Figs. 10

and 11, hydrometer and pipette analyses give similar grain size distribution curves. This similarity is not surprising because both analyses are based on sedimentation (Stokes' law) and sample preparation is identical.

### REVIEW QUESTIONS

1. What is the purpose of pipette analysis? On which physical principle is pipette analysis founded?
2. Does pipette analysis determine the size of soil particles exactly?
3. Compare pipette analysis and hydrometer analysis based on their principles and experimental procedures.
4. What is the purpose of a dispersing agent? Does its use require a correction in pipette analysis?
5. Why is a constant temperature required during the sedimentation process?
6. What is the purpose of pipette calibration? Do you have to repeat it before each sampling?
7. For what reason do you agitate the suspension at the beginning of a pipette test?
8. When does the analysis time start in pipette analysis?
9. If one wanted to sample at a depth of 5 cm (instead of 10 cm) during pipette analysis, what would be the effect of this change?
10. What is the average duration of a pipette analysis? Why does pipette analysis require less time than hydrometer analysis?
11. Why is it difficult to measure the particles with a diameter larger than  $75 \mu\text{m}$  by using pipette analysis?

### EXERCISES

1. Using a spreadsheet program, construct a table of sampling times similar to Table 1, but for  $H = 15 \text{ cm}$  and  $T = 20^\circ\text{C}$  instead of  $H = 10 \text{ cm}$  and  $T = 25^\circ\text{C}$ .
2. Derive Eqs. 4 and 6, which estimate the mass of dispersing agent in the pipette for the dry and wet methods.

# 1-6 Buoyancy Analysis

## OBJECTIVE

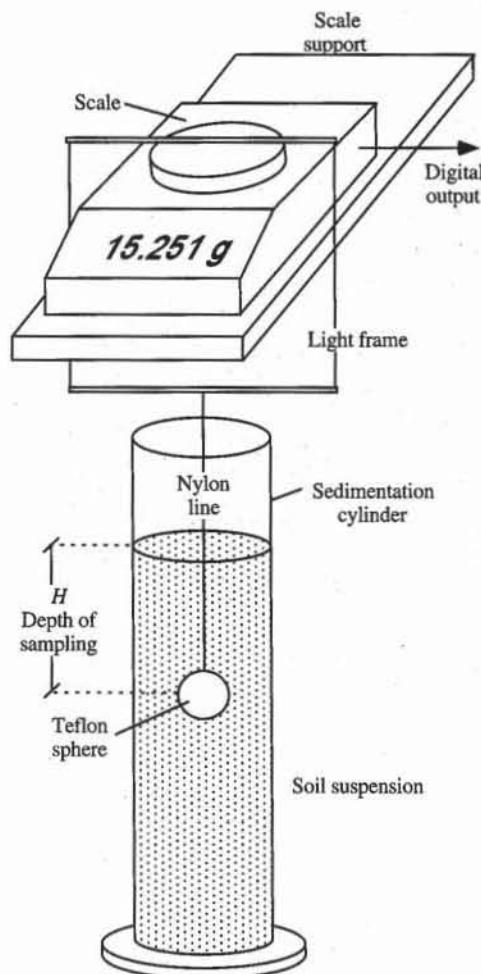
The buoyancy analysis is based on the same physical principles as hydrometer and pipette analyses. The buoyancy analysis can be used to determine the grain size distribution of fine-grained soils having particle size smaller than  $75\mu\text{m}$ . If soil samples have fine and coarse particles including sand, silt and clay particles, sieving and buoyancy analyses are combined as explained in Chapter 1-7.

To our knowledge, the buoyancy analysis is a new type of experiment for grain size analysis. Compared to the hydrometer analysis, it gives faster results but requires accurate weight measurement. Compared to the pipette analysis, it has fewer and less complicated experimental steps.

## EQUIPMENT

The equipment of the buoyancy analysis includes the following:

- Teflon sphere about 2.5 cm in diameter attached to a 0.1 mm thick nylon line by using a small point of rapid glue. As shown in Fig. 1 the line is attached to a light frame which can sit on the platen of a sensitive scale.
- Dispersion apparatus (same as for hydrometer analysis).
- Two 1000 mL sedimentation cylinders (same as for hydrometer analysis).
- Thermometer, ranging from 0 to  $50^\circ\text{C}$ , accurate to  $0.5^\circ\text{C}$ .
- Stop-watch.
- Balance accurate to 1 mg mounted on a cantilever support as shown in Fig. 1.
- Measuring cylinder, 100 mL.
- Drying oven.
- Hand agitator about 400 mm long.
- 500 mL of Hydrogen peroxide.
- 1000 mL of stock solution of sodium hexametaphosphate prepared as in hydrometer test.



**Figure 1** Equipment for buoyancy analysis. A sphere about 2.5 cm in diameter is attached to a nylon line and a light frame.

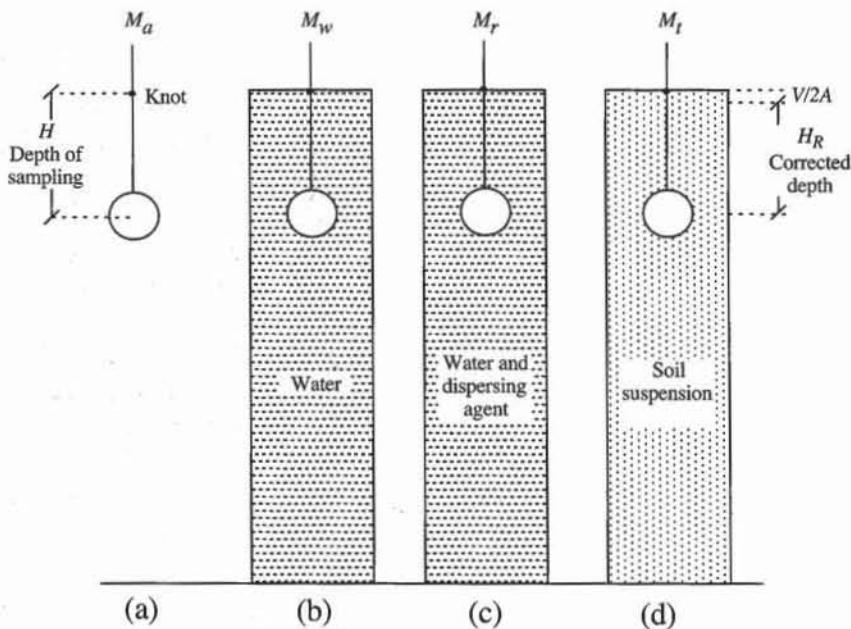
## SAMPLE PREPARATION AND PRETREATMENT

The test specimen is selected, pretreated for removal of organic matter, and mixed with a dispersing agent as described in Chapter 1-4.

## INSTRUMENT CALIBRATION

Before each experiment, the following calibration steps are performed:

1. Clean and dry the sphere and measure its mass  $M_a$  suspended in the air (Fig. 2a).
2. Immerse the sphere in water and measure its mass  $M_w$  and the water temperature (Fig. 2b).
3. Pour 125 mL of stock solution of sodium hexametaphosphate into a 1000 mL graduate, and add distilled or demineralized water to reach the 1000 mL



**Figure 2** Measurements of mass  $M_a$ ,  $M_w$ ,  $M_r$ , and  $M_t$ , and definition of corrected sampling depth during the buoyancy analysis.

mark. Immerse the sphere in the graduate, and measure its mass  $M_r$  and the temperature of the dispersing agent solution (Fig. 2c). The temperature should be similar to the water temperature in step 2.

## TEST PROCEDURE

After the removal of organic matter, addition of dispersing agent (see Chapter 1-4), and calibration, the test procedures consist of the following steps:

1. Follow steps 2 to 5 of the hydrometer analysis.
2. At the following times  $t$  after the beginning of sedimentation:  $t = 0.5, 1, 2, 4, 15, 30, 60, 120, 240$ , and  $480$  minutes, slowly immerse the sphere in the soil suspension to a depth  $H = 10$  cm, and measure mass  $M_t$  (Fig. 2d). The sampling depth and time series defined above should cover grain sizes ranging from 1 to  $75 \mu\text{m}$ . The time  $t$  and depth  $H$  of sampling are related to grain size  $D$  as follows:

$$t = \frac{30\eta H}{(G_s - 1)981\rho_w D^2} \text{ (min)} \quad (1)$$

where  $G_s$  is the specific gravity of soil particles,  $\rho_w$  the unit mass of water ( $\text{g}/\text{cm}^3$ ),  $\eta$  the viscosity of water ( $\text{g}/\text{cm}\cdot\text{s}$ ),  $H$  the sampling depth (cm), and  $D$  the grain size (mm). There should be no air circulation around the scale to avoid fluctuation in readout.

3. After each sampling, measure the water temperature  $T_t$ , which should not vary excessively during the experiment. If  $T_t$  is largely different from the temperature at which  $M_a$ ,  $M_w$  and  $M_r$  were measured then these quantities must be measured again at temperature  $T_t$ .

4. After removing the sphere from the soil suspension, use a wash bottle to clean it of soil particles which may be attached to it. Then immerse the sphere in water at the same temperature as the soil suspension.

## COMPUTATION

The grain size  $D$  (mm) is:

$$D = \sqrt{\frac{30\eta H_R}{(G_s - 1)981\rho_w t}} \quad (2)$$

where  $t$  is the time (min) after the beginning of sedimentation,  $G_s$  the specific gravity of soil particles,  $\rho_w$  the unit mass of water ( $\text{g}/\text{cm}^3$ ) at temperature  $T_t$ , and  $\eta$  the viscosity of water ( $\text{g}/\text{cm}\cdot\text{s}$ ) at temperature  $T_t$ . The corrected sampling depth  $H_R$  (cm), which accounts for the rise in water level when the sphere is immersed in the suspension, is

$$H_R = H - \frac{V}{2A}, \quad V = \frac{M_a - M_w}{\rho_w}, \quad \text{and} \quad A = \frac{\pi}{4} d_c^2 \quad (3)$$

where  $H$  is the sampling depth (cm),  $d_c$  the internal diameter of the sedimentation cylinder (cm),  $M_w$  the mass of the sphere in water (g), and  $M_a$  the mass of the sphere in air (g). The volume of the sphere can also be calculated from its diameter.

The percentage  $p$  by weight of particles with diameter smaller than  $D$  is:

$$p = \frac{V_{tot}}{M_{tot}} \frac{\rho_w G_s (M_r - M_t)}{(G_s - 1)M_a - G_s M_w + M_r} \times 100 \quad (\%) \quad (4)$$

where  $M_{tot}$  is the total mass of oven-dry soil in suspension (g),  $V_{tot}$  the total volume of suspension (mL),  $\rho_w$  the unit mass of water ( $\text{g}/\text{cm}^3$ ) at temperature  $T_t$ ,  $G_s$  the specific gravity of soil particles,  $M_t$  the mass of sphere in suspension at time  $t$  (g), and  $M_w$  the mass of sphere in water and dispersing agent (g).

## EXAMPLE

Figures 3 and 4 show the results of buoyancy analysis obtained for a silt from Los Angeles, California. The results are presented in a grain-size distribution

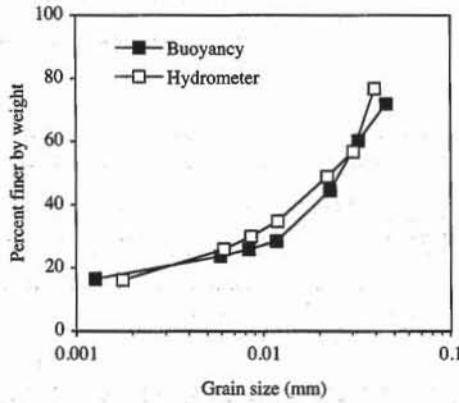


Figure 3 Comparison of results for buoyancy and hydrometer analyses on a silt from Los Angeles, California.

curve similar to that of hydrometer analyses. Figure 5 shows the formulas used in Fig. 3. Figures 3 and 6 give the results of a hydrometer analysis performed on the same material as the buoyancy analysis. Both analyses use the user-defined functions DENSI and VISCO (see Chapter 1-4). DENSI and VISCO return the water unit mass ( $\text{g}/\text{cm}^3$ ) and water viscosity ( $\text{g}/\text{cm}/\text{s}$ ), respectively, at temperature  $T$  ( $^{\circ}\text{C}$ ).

	A	B	C	D	E	F
1						
<b>Buoyancy analysis</b>						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17	Time (min)	Mass of sphere in suspension (g) $M_s$	Temperature ( $^{\circ}\text{C}$ ) $T_a$	Grain size (mm)	Percent finer by weight	
18	t					
19	1	16.827	24.5	0.046	71.94	
20	2	16.858	24.5	0.032	60.33	
21	4	16.9	24.5	0.023	44.59	
22	15	16.943	24.5	0.012	28.48	
23	30	16.95	24.5	0.008	25.86	
24	60	16.956	24.5	0.006	23.61	
25	1300	16.975	24.5	0.001	16.49	

Figure 4 Example of data set for buoyancy analysis.

	D	E	F
13	Unit mass of water and dispersing agent $r_f = (M_a - M_r)/V$		$\text{g}/\text{cm}^3$
14	Volume of sphere $V = (M_a - M_w)/\text{DENSI}(T_a)$		$\text{cm}^3$

	D	E
17	Grain size (mm)	Percent finer by weight
18		
19	=SQRT(30*VISCO(Ta)*(H-V/(2*PI()*D^2/4))/(Gs-1)/981/T)	=DENSI(Ta)*Gs*(Mr-Mt)/(Ma*(Gs-1)-Mw*Gs+Mr)*Vtot/Mtot*100

Figure 5 Formulas used in Fig. 4.

## COMPARISON OF BUOYANCY, HYDROMETER AND PIPETTE ANALYSES

The buoyancy analysis offers advantages over the hydrometer and pipette analyses. It takes less time to complete than the hydrometer analysis, and requires fewer and less complicated steps than the pipette analysis. However, like the pipette analysis and in contrast to the hydrometer analysis, the buoyancy analysis requires accurate weight measurement. As shown in Fig. 3, the buoyancy and hydrometer analyses give similar grain size distribution curves. This similarity is not surprising because both analyses are based on the same physical principle (i.e., Stokes' law), and sample preparation.

## REFERENCES

See Introduction for references to ASTM procedures (pages 4 to 6).

BS 1370, 1975, *Methods of tests for soil for civil engineering purposes*, British Standards Institution, London, UK.

## REVIEW QUESTIONS

1. What is the purpose of the buoyancy analysis? On which physical principle is this analysis based?
2. Does the buoyancy analysis determine exactly the size of soil particles?
3. Compare the principle and experimental procedure of the buoyancy, pipette and hydrometer analyses.
4. What is the purpose of the dispersing agent? Does its use require a correction in the buoyancy analysis?
5. Why is a constant temperature required during the sedimentation process?
6. For what reason do you agitate the suspension at the beginning of the buoyancy test?

## 1-7

# Combined Grain Size Analysis

### OBJECTIVE

A combined grain size analysis is required when neither the fraction of soil particles smaller than 75  $\mu\text{m}$  nor that with particles larger than 75  $\mu\text{m}$  can be neglected. A sieve analysis is performed on the fraction with particles larger than 75  $\mu\text{m}$ , and a sedimentation (hydrometer, pipette, or buoyancy) analysis is performed on the fraction with particles smaller than 75  $\mu\text{m}$ . The combined analysis gives a grain size distribution curve over a wide range of grain size.

### EQUIPMENT

The equipment for the combined analysis is identical to that used for sedimentation and sieve analyses.

### PREPARATION OF SAMPLE

The total amount of the sample should be sufficient to yield the required amounts of material for both sieve and sedimentation analyses. Samples of soils having fines with little or no plasticity are oven dried, weighed, and then separated on a No. 200 sieve. Samples of soils having plastic fines are soaked in water as explained for wet sieving (see Chapter 1-2), then washed over a No. 200 sieve.

### PROCEDURE

1. Perform a sieve analysis on a representative portion of the sample, and measure the weight passing through a No. 200 sieve.
2. Perform a sedimentation (hydrometer, pipette, or buoyancy) analysis on a sample passing through a No. 200 sieve.

## COMPUTATION

Combined analysis and sieve analysis have identical grain size distribution curves for particles retained in No. 200 sieves. However, for particles tested in the sedimentation analysis, the total percent by weight finer becomes:

$$p = \frac{W_{200}}{W_{\text{tot}}} p_h \quad (1)$$

where  $W_{200}$  is the weight of dry sample passing a No. 200 sieve,  $W_{\text{tot}}$  the weight of total dry sample in sieve analysis, and  $p_h$  the percent by weight finer calculated in sedimentation analysis alone. The clay fraction, which is the percent by weight finer than 2  $\mu\text{m}$ , should be calculated by using the combined percent by weight finer calculation of Eq. 1.

## EXAMPLE

The results of the combined analysis are presented on a grain size distribution curve similar to the one used for sieve and sedimentation analyses. As shown in Fig. 1, the curves obtained from sieve and sedimentation analyses may not connect smoothly. This offset is caused partly by the breakdown of Stokes' law for large particles in hydrometer analysis and the difficulty of wet-sieving fine particles in the presence of surficial tension in the sieve analysis. As shown in Fig. 1, both curves are joined by constructing a smooth curve between them. The data of Fig. 1 are listed in Fig. 2, and Fig. 3 shows the formulas used in Fig. 2.

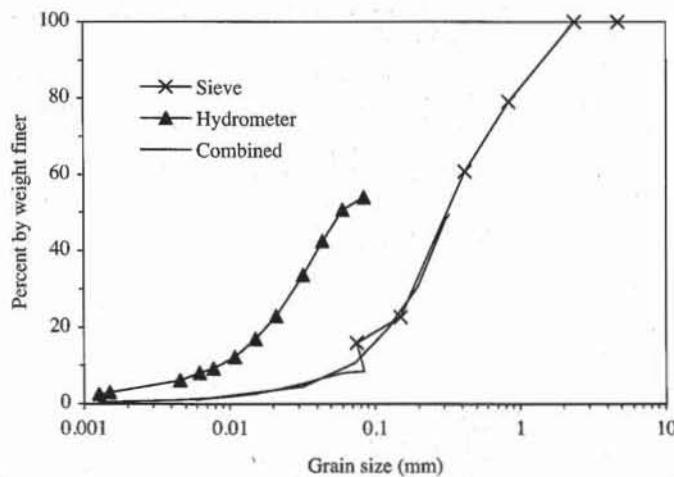


Figure 1 Grain size distribution curves from sieve and hydrometer analyses (after Lambe, 1951).

## REFERENCE

See Introduction for references to ASTM procedures (pages 4 to 6).

LAMBE, T. W., 1951, *Soil Testing for Engineers*, John Wiley & Sons, New York.

	A	B	C	D	E
1	<b>Combined analysis</b>				
2					
3					
4					
5					
6					
7					
8					
9	<b>Sieve analysis</b>				
10	Mass passing No. 200 sieve $W_{N200}$ (g) = 67.6				
11	Total sample mass $W_{tot}$ (g) = 424.7				
12					
13					
14					
15	<b>Hydrometer analysis</b>				
16					
17					
18					
19					
20					
21					
22					
23					
24					
25					
26					
27	Clay fraction (%) = 0.5				

**Figure 2** Example of data set for combined analysis (after Lambe, 1951).

	<b>E</b>	
7	Combined percent finer by weight	
8	$p_c$	
9	$=p$	
10	$=p$	
11	$=p$	
12	$=p$	
13	$=p$	
14	$=p$	
15	$=p \cdot W_{N200} / W_{tot}$	
16	$=p \cdot W_{N200} / W_{tot}$	
17	$=p \cdot W_{N200} / W_{tot}$	
	<b>D</b>	<b>E</b>
27	Clay fraction (%) = =INTER(0.002,d,pc)	

**Figure 3** Formulas used in Fig. 2.

## REVIEW QUESTIONS

- What is the purpose of a combined grain size analysis? On what type of soils do you need to carry out a combined grain size analysis?
- Is it possible for the grain size distribution curves of sieve and sedimentation analyses to overlap?

# 2

# Plasticity, Shrinkage, and Soil Classification

- 2-1** Principles of liquid and plastic limits tests
- 2-2** Determination of water content
- 2-3** Liquid limit test
- 2-4** One-point liquid limit test
- 2-5** Plastic limit test
- 2-6** Principles of shrinkage limit analysis
- 2-7** Shrinkage limit analysis with mercury
- 2-8** Shrinkage limit analysis with wax
- 2-9** Engineering classification of soils

# Principles of Liquid and Plastic Limits Tests

The engineering behavior of fine-grained soils depends on factors other than particle size distribution. It is influenced primarily by their mineral and structural composition and the amount of water they contain, which is referred to as *water content* (or *moisture content*). The liquid and plastic limits tests characterize the effects of water content on fine-grained soils and help to classify fine-grained soils and to assess their mineral composition and engineering properties.

## WATER CONTENT

Soils are made of solid particles with voids between. These voids are generally filled with air and water. The water content  $w$  of a soil is

$$w = \frac{W_w}{W_s} \times 100 \quad (\%) \quad (1)$$

where  $W_w$  is the weight of water removed from the soil by oven drying at 105° to 110°C and  $W_s$  is the weight of the dried soil. A soil is considered to be dry when its mass does not change by oven drying, which may usually require about 12 to 24 h.

Oven drying removes the water completely from soils without clay particles, but partially from soils with clay particles. Clay particles are made of clay minerals and have plate-like shapes smaller than 2  $\mu\text{m}$ . Table 1 lists four common clay minerals and typical values of their specific surfaces. Kaolinite, the largest clay mineral, has a thickness or edge dimension of about 1  $\mu\text{m}$ , while montmorillonite, one of the smallest clay minerals, has a thickness of only a few nanometers.

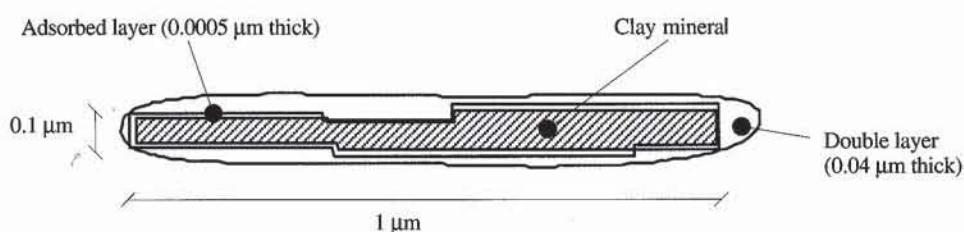
The clay minerals relate to water in several complex ways, which give three main categories of water around a clay particle as shown in Fig. 1:

1. Adsorbed water, held on the particle surface by powerful electrical forces and virtually in a solid state. This layer is about two water molecules thick

**TABLE 1**

Average values of relative sizes, thicknesses, and specific surfaces of four common clay minerals (after Yong and Warkentin, 1975).

Clay mineral	Typical thickness ( $\mu\text{m}$ )	Typical diameter ( $\mu\text{m}$ )	Specific surface ( $\text{m}^2/\text{g}$ )
Montmorillonite	0.003	0.1–1	800
Illite and Chlorite	0.03	10	80
Kaolinite	0.05–2	0.3–4	15



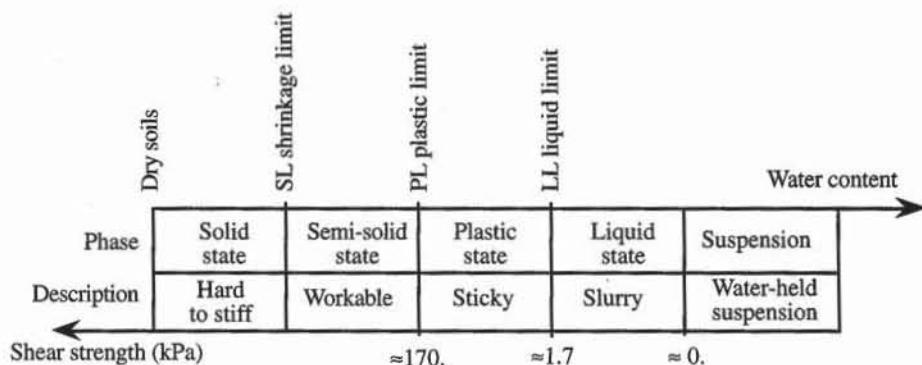
**Figure 1** Schematic side-view representation of a typical particle of kaolinite, with its adsorbed layer and double layer.

(i.e., 0.0005  $\mu\text{m}$ ). The adsorbed water cannot be removed by oven drying at 110°C and is considered to be part of the soil particles.

2. Chemically combined water, in the form of water of hydration within the crystal structure. This layer is referred to as the *double layer*. Its thickness varies with clay minerals, type, and concentration of ions in the water, and other factors (Yeung, 1992). As shown in Fig. 1, the double layer is about 0.04  $\mu\text{m}$  thick for a kaolinite clay particle. Except for gypsum and some tropical clays, this water is not generally removable by oven drying.
3. Interstitial water, not so tightly held as chemically combined and adsorbed waters. It can be removed by drainage, air drying, or oven drying.

## LIQUID AND PLASTIC LIMITS

The mechanical properties of a clay are altered by changing the water content. A clay softens when water is added, and with sufficient water, forms a slurry that behaves as a viscous liquid; this is known as the *liquid state*. If the water content is gradually reduced by drying it slowly, the clay eventually begins to hold together and to offer some resistance to deformation; this is the *plastic state*. With further loss of water, the clay shrinks and its stiffness increases until it becomes brittle; this is the *semisolid state*. As drying continues, the clay continues to shrink until it reaches a constant minimum volume. Beyond that point, further drying causes no further decrease in volume; this is the *solid state*. These four states are shown in Fig. 2. The change from one state to the next is not abrupt, but gradual.



**Figure 2** Variation of consistency of fine-grained soils with water content.

These smooth transitions are empirically defined by introducing the liquid limit  $LL$ , plastic limit  $PL$ , and shrinkage limit  $SL$ . The moisture content between  $PL$  and  $LL$  is the plasticity index  $PI$ :

$$PI = LL - PL \quad (2)$$

$PI$  is a measure of the plasticity of a clay.

The liquid and plastic limit tests provide a means of measuring and describing the plasticity range of clay soils. Liquid and plastic limits are also referred to as *Atterberg limits*, after the Swedish scientist A. Atterberg, who first defined them for the classification of agricultural soils in 1911. Originally, the limits were determined by simple tests using an evaporating dish (Bauer, 1959). The procedures were defined more precisely for engineering purposes by Casagrande (1932). The mechanical device he designed for determining the liquid limit is still known as the Casagrande apparatus (Casagrande, 1958). A cone penetrometer apparatus can also be used instead of the *Casagrande apparatus*, but we will not describe this technique.

## PLASTICITY CHART

Sedimentation tests (e.g., hydrometer tests) give the clay fraction but unfortunately, no information about the type of clay. Clay particles are too small to be examined visually except by using an electron microscope. The identification of clay minerals in soils with x-ray diffraction would also be too lengthy and expensive for engineering practices. The liquid and plastic limit tests are two basic engineering experiments that enable the classification of clay soils and assessment of their probable types of clay minerals.

Fine-grained soils are usually classified by using the plasticity chart. The plasticity chart is a graphical plot of the liquid limit  $LL$  against the plasticity index  $PI$ . The standard plasticity chart is shown in Fig. 3. When the values of  $LL$  and  $PI$  for inorganic clays are plotted on this chart, most of the points lie just above the line marked *A-line* and in a narrow band parallel to it. The *A-line* is defined by the relationship

$$PI = 0.73(LL - 20) \quad (3)$$

where  $PI$  and  $LL$  are in percent. The *A-line* is a reference line derived from experimental observations. It does not represent a well-defined boundary between

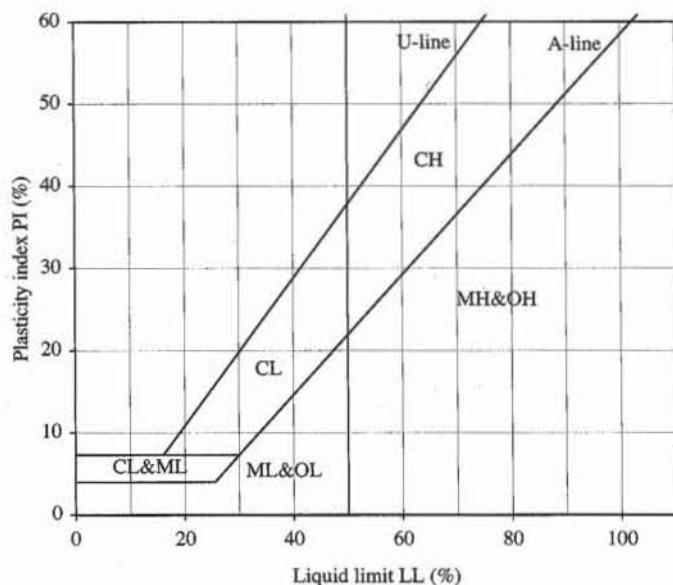


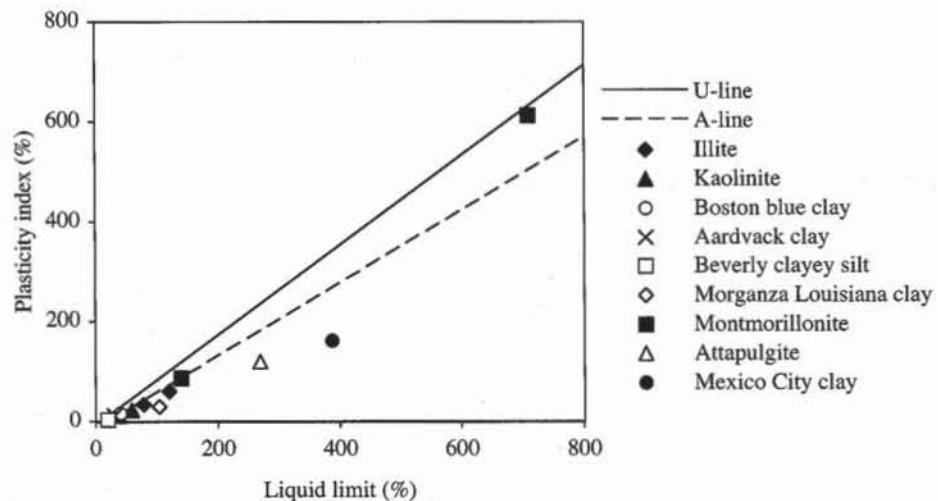
Figure 3 Plasticity chart.

soil types. The *U-line* of Fig. 3 is a tentative upper limit for all soils, which was also drawn from experimental data. The *U-line* has the equation

$$PI = 0.9(LL - 8) \quad (4)$$

Table 2 provides us with a few examples of liquid limits, plastic limits and plastic indexes for various clay minerals and soils. The data points of Table 1 are plotted by using two different scales in Figs. 4 and 5. Most points are lined up along the *A-line*.

Based on a compilation of experimental results on the Atterberg limits of various clay minerals, Holtz and Kovacs (1981) observed that the clay minerals can be determined by using the Atterberg limits and the plasticity chart (see Fig. 6). As shown in Figs. 4 and 5, montmorillonites are very high on the chart, close to the *U-line*, whereas illites and kaolinites are close to the *A-line*.

Figure 4 Representation of various clay minerals and natural soils on the plasticity chart ( $0 < LL < 800\%$ ).

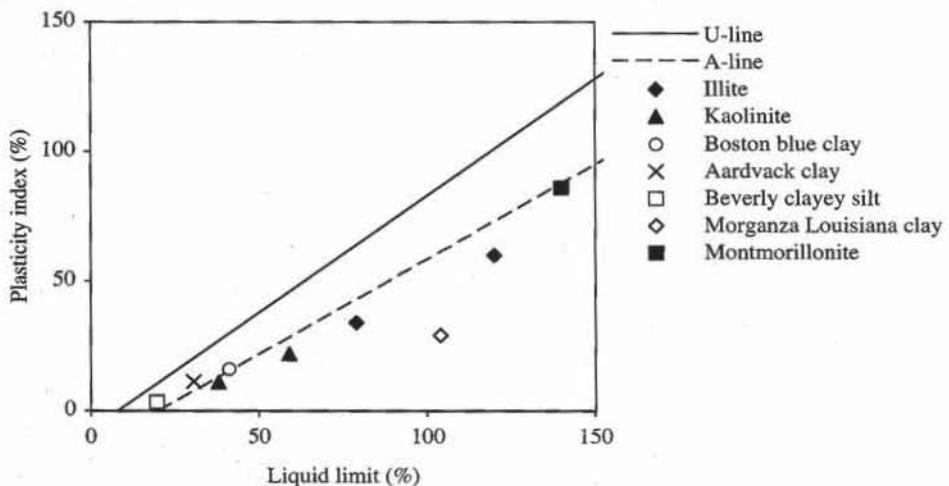


Figure 5 Representation of various clay minerals and natural soils on the plasticity chart ( $0 < LL < 200\%$ ).

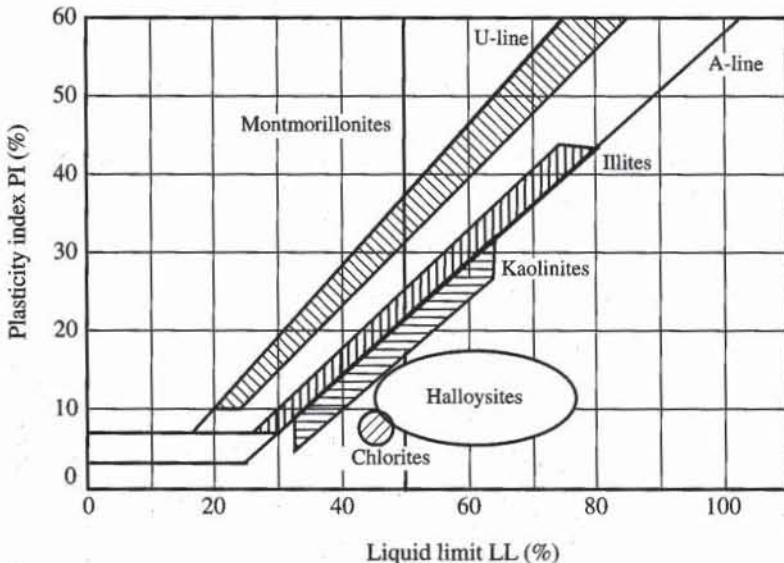


Figure 6 Location of common clay minerals on the plasticity chart (after Holtz and Kovacs, 1981).

## CONSISTENCY OF CLAYS

The state of a clay cannot be defined solely by its water content. Two different clays with identical water content may exhibit quite different characteristics. It is therefore preferable to characterize the state of a clay by the liquidity index  $LI$ , which relates its water content to its liquid and plastic limits as follows:

$$LI = \frac{w - PL}{LL - PL} = \frac{w - PL}{PI} \quad (5)$$

$LI$  provides us with a normalized representation of water content in relation to the plasticity range. Below the plastic range (i.e.,  $w \leq PL$ ),  $LI$  is negative. At the liquid limit (i.e.,  $LI = 1$ ), a slowly drying slurry first begins to show a small but

**TABLE 2**

Typical ranges of index properties of some common clay minerals and natural soils (after Lambe and Whitman, 1979; and Lambe, 1951).

Mineral	Exchangeable ion	Liquid limit (%)	Plastic limit (%)	Plastic index (%)
Montmorillonite	Na	710	54	656
	K	660	98	562
	Ca	510	81	429
	Mg	410	60	350
	Fe	290	75	215
Illite	Na	120	53	67
	K	120	60	60
	Ca	100	45	55
	Mg	95	46	49
	Fe	110	49	61
Kaolinite	Na	53	32	21
	K	49	29	20
	Ca	38	27	11
	Mg	54	31	23
	Fe	59	37	22
Attapulgite	H	270	150	120
Mexico City clay		388	226	162
Boston blue clay (illite)		41	25	16
Aardvack clay		30.6	19.6	11
Morganza Louisiana clay		104	74.8	29.2
Beverly clayey silt (illite)		19.5	16.3	3.2

definite shear strength. As the moisture content decreases and  $LI$  approaches zero, the shear strength increases considerably, and at the plastic limit ( $LI = 0$ ) the shear strength may be 100 times greater than at the liquid limit  $LL$ .

### ACTIVITY OF CLAYS

The Atterberg limits are related to the combined effects of particle size and mineral composition. In Fig. 7, Skempton (1953) showed that the plasticity index depends on the clay fraction—percent by weight of particles finer than  $2 \mu\text{m}$ —and that the plasticity index/clay fraction ratio was constant for a given clay mineral.

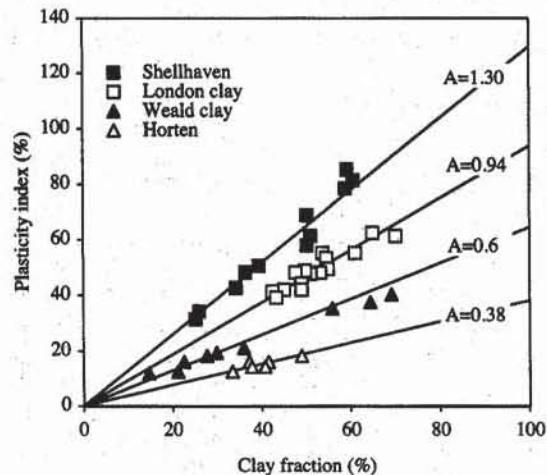


Figure 7 Relation between plasticity index and clay fraction (after Skempton, 1953).

The slope of the linear relationship between *PI* and the clay fraction of a particular clay is called the activity *A*:

$$A = \frac{PI(\%)}{\text{clay fraction (\%)}} \quad (6)$$

On the basis of their *A* values, clays can be classified into the four groups of Table 3. Approximate values of *A* for some clay minerals are listed in Table 4. Montmorillonites are highly active because they have very small particles and large plasticity indices.

**TABLE 3**  
Activity of clays

Description	Activity
Inactive	< 0.75
Normal	0.75–1.25
Active	1.25–2.
Highly active (e.g., bentonite)	> 2 6 or more

**TABLE 4**  
Activity of various minerals  
(after Skempton, 1953; and Mitchell, 1993)

Mineral	Activity
Na-montmorillonite	4–7
Ca-montmorillonite	1.5
Illite	0.5–1.3
Kaolinite	0.3–0.5
Halloysite (dehydrated)	0.5
Halloysite (hydrated)	0.1
Attapulgite	0.5–1.2
Allophane	0.5–1.2
Mica (muscovite)	0.2
Calcite	0.2
Quartz	0.

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## REVIEW QUESTIONS

1. Why do we use Atterberg limits to characterize fine-grained soils? Why is the result of the hydrometer analysis insufficient for this purpose?
2. Define water content? Give an example of a maximum value for the water content in soils.
3. How many categories of water can we distinguish around clay particles? Can you name these categories?
4. What are the definitions of liquid and plastic limits? Are these definitions based on theoretical or empirical concepts?
5. Does the shear strength increase or decrease with water content in fine-grained soils?
6. Is it possible for a soil to have a liquid limit and a plasticity index both equal to 30%? Why?
7. Define *activity of clays*. What is it used for?
8. Is it meaningful to define the activity of a sand?
9. Define *clay fraction*.
10. A sample of wet clay and its container weigh 102 g. After oven drying, the sample and the container weigh 60 g. What is the water content?
11. It is possible for a sample of clay to have a water content equal to 700%? Can you give an example?
12. Which clay mineral has the largest activity? Why is it so active?

## EXERCISES

1. Calculate the activity of the clay from the following test results.

Soils	Clay fraction (%)	Plasticity index (%)
Shellhaven	59.3	85.2
	60.6	81.3
	58.9	78.4
	50.2	68.8
	51.1	61.4
	50.2	58.0
	39.4	50.6
	36.4	48.3
	34.2	42.6
	26.0	34.1
	25.1	31.3

Soils	Clay fraction (%)	Plasticity index (%)
London clay	70.1	61.4
	64.9	62.5
	61.0	55.1
	53.7	55.1
	54.5	53.4
	55.0	49.4
	53.2	48.3
	51.1	47.7
	49.8	48.9
	47.6	48.3
	48.9	44.3
	48.9	42.0
	45.0	42.0
	42.4	41.5
	43.3	39.2
	41.1	37.5

2. Calculate the activity of the clay from the following test results.

Soils	Clay fraction (%)	Plasticity index (%)
Weald clay	69.3	40.3
	64.5	37.5
	55.8	35.2
	35.9	21.0
	29.9	19.3
	27.7	18.2
	22.5	15.9
	21.2	12.5
	14.7	11.9

Soils	Clay fraction (%)	Plasticity index (%)
Horten	48.9	18.2
	41.6	15.9
	40.7	14.2
	37.2	15.9
	37.7	14.2
	33.3	12.5

# Determination of Water Content

## DEFINITION

By definition, water content,  $w$ , is the ratio of the weight of water in a given soil mass to the weight of solid particles. The standard and recommended method for determining the water content of soils is the oven-drying method with a drying temperature of 105° to 110°C. Alternative methods include the sand bath method, the carbide method, and the alcohol method. Detailed instructions on these alternative procedures are given by Head (1984).

## EQUIPMENT

The equipment for determining water content includes:

- Thermostatically controlled drying oven, capable of maintaining a temperature of 105° to 110° C. A microwave oven may be used for fast and approximate determination of water content (see ASTM D6643).
- Balance accurate to 0.2% of the sample weight.
- Small metal containers with lids. Containers and lids should be as light as practicable in relation to the amount of material. They should be washed clean and dried thoroughly before use.

## PROCEDURE

1. Clean, dry, and weigh the container and its lid. Make sure that both have the same labels.
2. Select the test sample to be representative of the soil from which it is taken. It is recommended to determine two or three separate moisture contents and to average them. However, if only a very small quantity of soil is available, it is better to use it all for one measurement. It is recommended to select the approximate mass of the specimen depending on soil types as follows:

Soil type	Mass
Homogeneous clay and silts	30 g
Medium-grained soils	300 g
Coarse-grained soils	3 kg

3. Place the specimen in the container and immediately determine the weight of the container, lid, and wet soil. Each sample should be weighed as soon as possible. If weighing is delayed, the lid must be fitted tightly to avoid loss of water by evaporation.

4. Before placing the specimen in the oven, remove the lid and place it under or next to the container in the oven. Leave the specimen in the oven until it has dried to reach a constant weight. The time required for drying will vary depending on the type of soil, size of specimen, and type of oven. It takes a few minutes in the microwave oven but can take several hours in a standard oven.

5. When the specimen is estimated to be dry, remove its container from the oven and close it with its lid. Allow the container to cool until it can be handled comfortably with bare hands, then determine its dry weight. If the specimen is not weighed immediately after cooling, it must be placed in the oven again to remove the moisture that it has absorbed from the atmosphere.

## COMPUTATION

The water content  $w$  is calculated as follows:

$$w = \frac{W_w - W_d}{W_d - W_c} \times 100 \quad (\%) \quad (1)$$

where  $W_c$  is the weight of the container,  $W_w$  the weight of the container and wet soil, and  $W_d$  the weight of the container and dry soil. If two or three separate measurements have been made on the same soil specimen, the average value of  $w$  is then calculated.

## REFERENCES

See Introduction for references to ASTM procedures (pages 4 to 6).

HEAD, K. H., *Manual of Soil Laboratory Testing*, Pentech Press, London, 1984.

## REVIEW QUESTIONS

- Why is it not recommended to leave an oven-dried sample in the open air for a long time before measuring its dry weight?
- Excluding oven drying, are there other methods to determine the water content of soils?
- Is it possible to measure the water content of sands?
- Why do we use a fixed temperature range to dry soils? What is the effect on soils of microwave drying?
- What is the function of the container lid when determining the water content of a soil?

## 2-3 Liquid Limit Test

### OBJECTIVE

The liquid limit test determines the liquid limit of a soil. By convention, the liquid limit is defined as the water content at which the groove cut into the soil pat in the standard liquid limit device requires 25 blows to close along a distance of 13 mm.

### EQUIPMENT

The equipment for the liquid limit test includes:

- Mechanical device shown in Fig. 1. The cup must fall freely from a height equal to  $10 \pm 0.2$  mm above the base. The material and construction must conform to ASTM D4318-93.
- Grooving tool as illustrated in Fig. 2. The V-groove profile must not differ more than 0.25mm from those specified in Fig. 2. The gage for checking the height of drop of the cup is to be  $10 \pm 0.2$  mm.

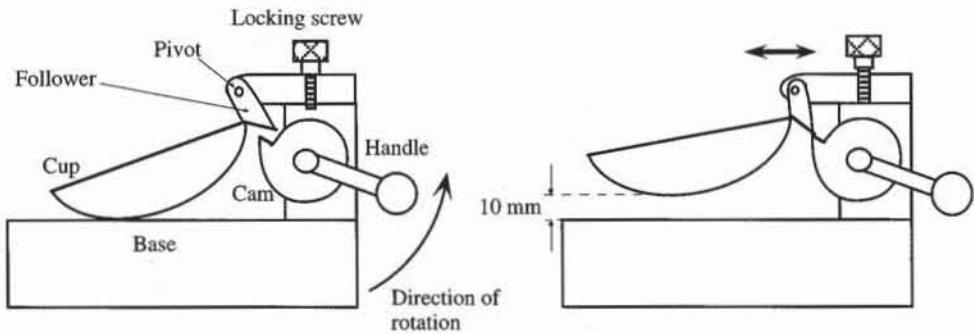
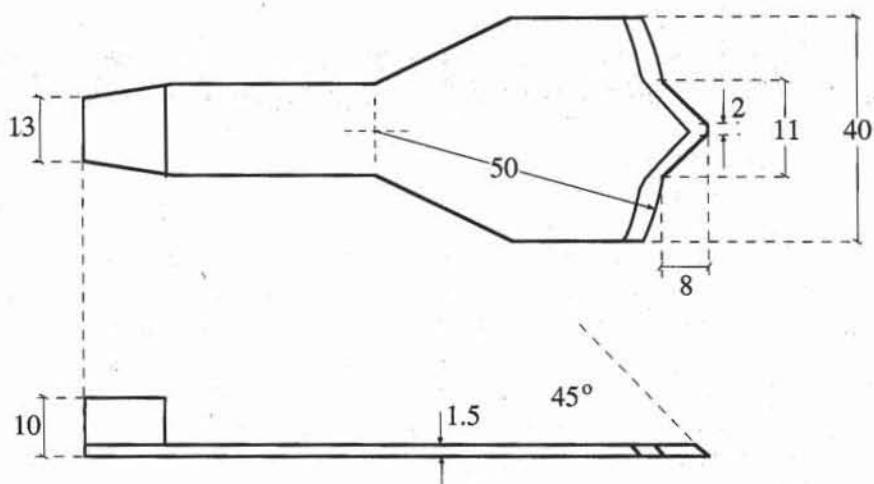


Figure 1 Side-view of Casagrande apparatus.

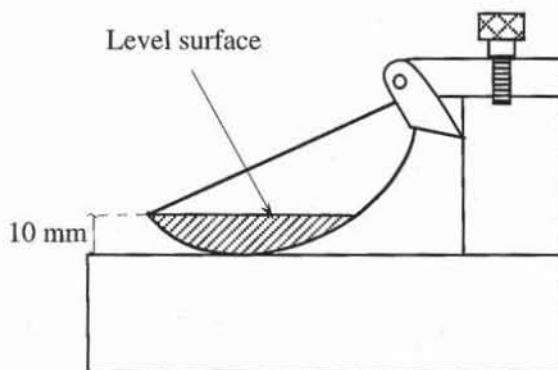


**Figure 2** Grooving tool for Casagrande apparatus (dimensions in mm).

- Spatula, with a blade about 10 cm long and about 2 cm wide.
- Mixing dish or bowl.
- Specimen containers. Metal containers with lids are recommended. The containers should be resistant to corrosion. Containers approximately 2 cm high by 5 cm in diameter are adequate.
- Balance, sensitive to 0.01 g.
- Drying oven.
- Sieve, U.S. Standard No. 40 (0.42 mm).

## PROCEDURE

1. The liquid limit device must first be calibrated. Its bowl must be clean, dry, and oil-free. The height of drop is checked by using the spacer gage on the grooving tool handle (see Fig. 2). This 10-mm-thick steel block should just pass between cup and base when the cup is at its maximum height (see Fig. 1). The locknut is tightened after adjustment and the maximum height rechecked with the gage.
2. Whenever possible, the soil used for Atterberg limits tests should not be dried prior to testing. Oven-drying alters the index properties of soils, especially of organic clays. The test material should be free from coarse particles (e.g., larger than 425  $\mu\text{m}$ ). If sieving is required, the soil may be air dried before testing. About 250 g of dry soil is needed for both the liquid and plastic limit tests. An essential step in the sample preparation is thorough mixing of the soil with water. Natural or distilled water is preferable to tap water, to avoid ion exchange between soil and water impurities, which may affect the soil plasticity. Water and soil are mixed on a glass plate by using a spatula until the mixture is uniform and behaves as a soft paste that can be shaped with a spatula.
3. Place 50 to 80 g of the specimen in the bowl and level it off to a depth of approximately 1 cm. The surface of the soil paste should be smoothed off level and parallel to the base, giving a depth at the greatest thickness of 10 mm (Fig. 3).



**Figure 3** Soil placed in a Casagrande bowl.



**Figure 4** A groove is cut through the sample from back to front.

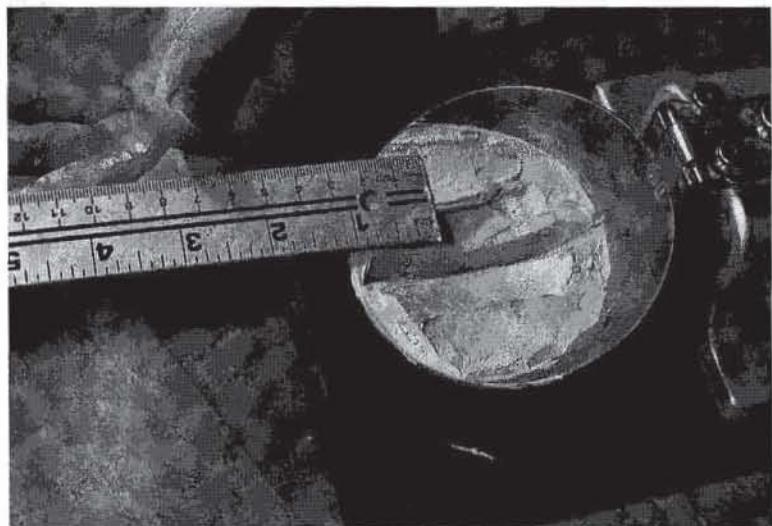
4. As shown in Fig. 4, a groove is cut through the sample from back to front, dividing it into two equal halves. Starting near the hinge, draw the grooving tool toward the front in a continuous movement with a circular motion, keeping the tool normal to the cup surface and its chamfered edge in the direction of movement. The tip of the tool should scrape the bowl lightly. The completed groove must be clean and sharp.
5. Turn the crank handle at a steady rate of two revolutions per second, so that the bowl is lifted and dropped. Continue turning until the two halves of the soil pat come in contact at the bottom of the groove along a distance of 13 mm, as shown in Fig. 5. During the test, the soil should slump and flow plastically in the bowl. It should not slide on the bowl surface. Record the number of blows required to close the groove.
6. Remove 5 to 10 g of soil from the sample and use it to determine the water content of the complete specimen.
7. Repeat the run. Transfer the soil remaining in the cup to the mixing dish, and repeat steps 3 to 6 for three additional specimens with various water contents. The water content can be lowered by drying the specimen through continued mixing with a spatula and a hair dryer. It is recommended that the water content be adjusted to obtain two specimens between 15 and 25 blows, and two others between 25 and 35 blows. Material left over in the mixing dish should be preserved for the plastic limit test.

## COMPUTATION

The water content  $w_i$  corresponding to the blow counts  $N_i$  is calculated as in Chapter 2-2. The line passing through  $n$  data points  $(\log N_i, w_i)$  is determined by linear regression,

$$w = A \log N + B \quad (1)$$

where the slope  $A$  and intercept  $B$  are



**Figure 5** For the liquid limit, the groove is assumed to be closed when the two soil parts come in contact along a distance of 13 mm.

$$A = \frac{n \sum_{i=1}^n w_i \log N_i - \sum_{i=1}^n w_i \sum_{i=1}^n \log N_i}{n \sum_{i=1}^n (\log N_i)^2 - \left( \sum_{i=1}^n \log N_i \right)^2},$$

and

$$B = \frac{\sum_{i=1}^n w_i \sum_{i=1}^n (\log N_i)^2 - \sum_{i=1}^n \log N_i \sum_{i=1}^n w_i \log N_i}{n \sum_{i=1}^n (\log N_i)^2 - \left( \sum_{i=1}^n \log N_i \right)^2} \quad (2)$$

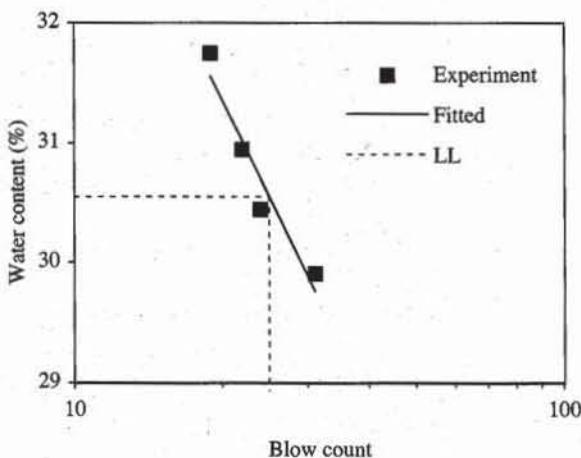
The liquid limit corresponds to  $N = 25$  on the line  $w = A \log N + B$

$$LL = A \log(25) + B \quad (3)$$

## EXAMPLE

As shown in Figs. 6 to 8, four different specimens of a particular clay were used to find the liquid limit. In Fig. 6 the logarithmic horizontal axis represents the number of blows  $N_i$ , and the vertical axis represents the corresponding moisture content  $w_i$ . In Fig. 6, the straight line drawn through the experimental points was defined by using a linear regression on four data points  $(\log N_i, w_i)$ . All the formulas used in Fig. 7 are listed in Fig. 8. The liquid limit is calculated by using Eq. 3 calibrated with the coefficients  $A$  and  $B$  found by linear regression.

Figure 9 shows additional examples of results for the liquid limit tests on several clays (data after Casagrande, 1932). The points  $(N, w)$  are lined up along straight lines, even for a wide range of  $N$  values.



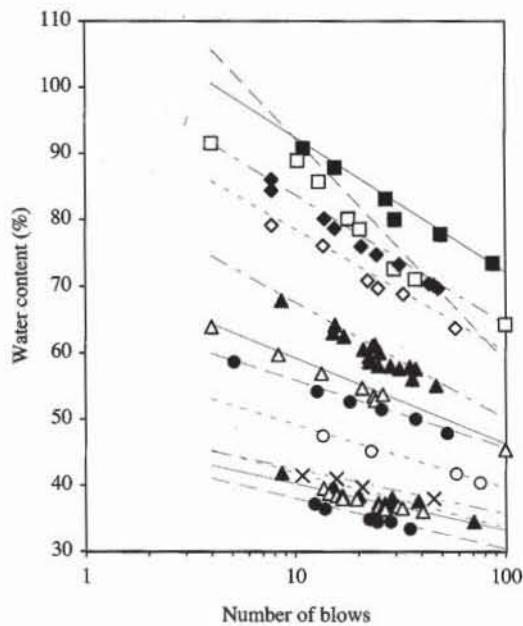
**Figure 6** Example of linear regression for finding the flow line.

	A	B	C	D	E	F	G
<b>1</b>	<b>Liquid limit</b>						
<b>2</b>	Analyst name: <i>Henry T. Guapo</i>						
<b>3</b>	Test date: 20-Feb-94						
<b>4</b>	Sample description: Aardvark modeling clay						
<b>5</b>							
<b>6</b>	Set number	Tare mass (g)	Tare with wet soil (g)	Tare with dry soil (g)	Blow count	Water content (%)	Water content fitted (%)
<b>7</b>	$W_c$	$W_w$	$W_d$	N	w		
<b>8</b>	1	47.72	59.89	57.05	24	30.44	30.70
<b>9</b>	2	43.21	59.76	55.95	31	29.91	29.76
<b>10</b>	3	45.17	61.25	57.45	22	30.94	31.02
<b>11</b>	4	45.81	58.26	55.26	19	31.75	31.56
<b>12</b>	Liquid limit (%) = 30.55						
<b>13</b>	Slope of flow line = 0.119						

**Figure 7** Example of data set for liquid limit test.

	F	G
<b>6</b>	Water content (%)	Water content fitted (%)
<b>7</b>	w	
<b>8</b>	$=100*(Ww-Wd)/(Wd-Wc)$	$=TREND(w,LOG10(N),LOG10(E8))$
<b>9</b>	$=100*(Ww-Wd)/(Wd-Wc)$	$=TREND(w,LOG10(N),LOG10(E9))$
<b>12</b>	B	C
<b>13</b>	Liquid limit (%) = $=TREND(w,LOG10(N),LOG10(25))$	
	Slope of flow line = $=SLOPE(LOG(w),LOG(N))$	

**Figure 8** Formulas used in Fig. 7.



**Figure 9** Additional examples of results for liquid limit tests on several clays (data after Casagrande, 1932).

## REFERENCES

See Introduction for references to ASTM procedures (pages 4 to 6).

CASAGRANDE, A., 1932, Research on the Atterberg limits of soils," *Public Roads*, Vol. 8, pp. 121-136.

## REVIEW QUESTIONS

1. What is the purpose of the liquid limit test?
2. How do you define *liquid limit*?
3. Why do you use a special cup and cranking device to determine the liquid limit? Why not use another shape for the cup?
4. Should you add or remove water to obtain a lower blow count?
5. What is the minimum number of data points required to determine the liquid limit?
6. What is the purpose of calibration of the Atterberg device?
7. Under what conditions would you use seawater to moisten a clay sample?
8. Is there a typical value for the liquid limit for clean fine sand? Justify your answer.

## EXERCISE

1. Determine the liquid limit from the following experimental results.

Set number	Tare mass (g)	Tare with wet soil (g)	Tare with dry soil (g)	Blow count
1	47.11	73.87	67.86	34
2	47.11	82.44	73.86	29
3	47.07	75.70	69.09	20
4	47.07	76.99	69.41	14

## 2-4 One-point Liquid Limit Test

### OBJECTIVE

The one-point liquid limit test is a quick means of determining the liquid limit of a soil. It requires one moisture content measurement instead of four measurements as in the standard liquid limit test in Chapter 2-3. However, the one-point liquid limit test is likely to be less reliable than the standard liquid limit test.

### DEFINITION

In the test results of Fig. 1 obtained for various types of soil, the points ( $\log N$ ,  $\log w$ ) are lined up along parallel straight lines having a constant inclination. These straight lines are called flow lines. Therefore,  $\log(w)$  and  $\log(N)$  are related through

$$\log(w) = A' \log(N) + B'' \quad (1)$$

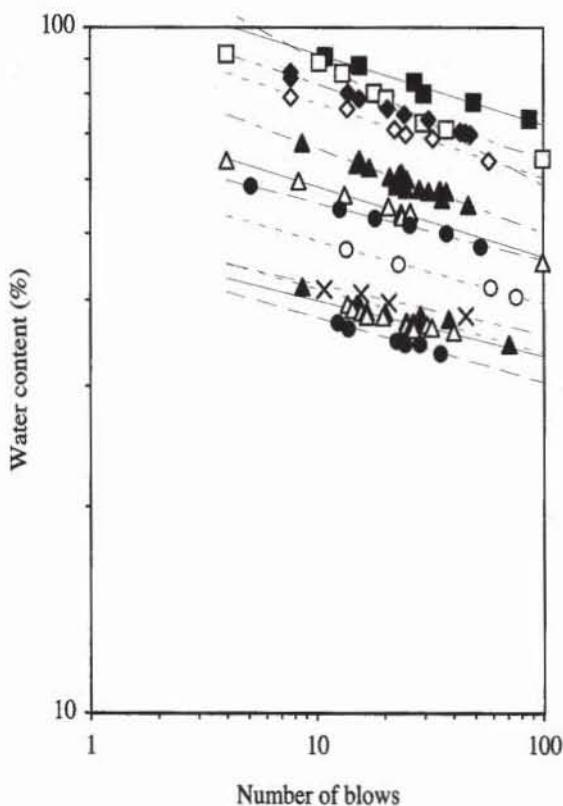
where  $A'$  is the constant slope of flow lines. Because Eq. 1 applies to point  $(25, LL)$ ;

$$\log(w) - A' \log(N) = \log(LL) - A' \log(25) \quad (2)$$

one obtains the following relation between liquid limit  $LL$ , water content  $w$ , and number of blows  $N$ :

$$LL = w \left( \frac{N}{25} \right)^{A'} \quad (3)$$

For most soils,  $A'$  was found approximately equal to 0.104. Therefore,  $LL$  can be determined by using Eq. 3 for only one point  $(N, w)$ . This is the shortcut used by the one-point limit test. As shown in Table 1, the liquid limits calculated by using Eq. 3 and  $A' = 0.104$  are generally close to those determined with the standard liquid limit test.



**Figure 1** Examples of  $\log(N)$  —  $\log(w)$  curves (data after Casagrande, 1932).

**TABLE 1**

Measured liquid limits, calculated one-point liquid limits, and slope of flow lines for several clays  
(data after Casagrande, 1932)

Set	Liquid limit (%)	One point liquid limit (%)	Slope of flow line	Error (%)
1	83.4	83.6	0.103	0.2
2	75.9	76.9	0.182	1.4
3	75.0	76.2	0.110	1.6
4	70.3	70.1	0.110	0.4
5	59.5	60.7	0.123	2.0
6	53.3	53.2	0.102	0.1
7	51.4	49.7	0.084	3.3
8	45.0	44.5	0.091	0.9
9	39.4	38.0	0.073	3.7
10	38.2	38.9	0.093	1.8
11	37.1	37.1	0.080	0.0
12	34.6	34.5	0.094	0.3

#### EQUIPMENT AND PREPARATION OF SAMPLE

The equipment and sample preparation are the same as for the standard liquid limit test, except for the soil sample, which is prepared slightly more plastic.

**PROCEDURE**

Same as for the standard liquid limit test in Chapter 2-3. The number of blows should be between 20 and 30.

**COMPUTATION**

The liquid limit is calculated using Eq. 3 and  $A' = 0.104$ . A liquid limit value should be computed for two trials of water content determination, and the average of the two is taken as the final liquid limit. The test is considered valid when the difference between the two liquid limit values is less than 2% of their average.

**REFERENCE**

CASAGRANDE, A., 1932, Research on the Atterberg limits of soils, *Public Roads*, Vol. 8, pp. 121-136.

**REVIEW QUESTION**

- What is the purpose of the one-point liquid limit test? What is the main difference between the one-point liquid limit test and the usual liquid limit test?

**EXERCISES**

- Verify Eq. 1 from the results of your standard liquid limit tests and calculate the slope of the flow line.
- Verify Eq. 3 for the following results of standard liquid limit tests. Calculate the slope of the flow line. Compare the liquid limits calculated by the four-point and one-point methods.

Set number	Number of blows	Water content (%)	Set number	Number of blows	Water content (%)
1	11	90.8	6	8	59.7
	16	87.9		13	56.8
	27	83.2		21	54.7
	30	80.0		24	53.4
	49	77.8		24	52.8
	87	73.4		26	53.6
2	10	88.9	7	5	58.6
	13	85.7		13	54.2
	18	80.1		18	52.6
	20	78.6		26	51.4
	30	72.6		37	49.9
	37	71.0		53	47.8
3	8	86.1	8	14	47.5
	8	84.4		23	45.1
	14	80.1		59	41.7
	16	78.7		76	40.4
	21	76.0			
	25	74.7			
	31	73.2			
	43	70.4			
	46	70.2			
	48	69.6			

3. For the following test results, compare the liquid limits calculated by the four-point and one point methods.

Set number	Number of blows	Water content (%)	Set number	Number of blows	Water content (%)
4	8	79.2	9	11	41.4
	14	76.1		16	40.9
	22	70.9		21	39.7
	25	69.6		46	37.9
	33	68.8	10	10	43.0
	58	63.7		9	41.8
5	9	67.8		15	39.5
	16	64.1		16	39.2
	15	63.0		20	38.4
	17	62.4		29	38.1
	24	61.1		29	37.3
	23	60.9		38	37.5
	21	60.4		71	34.5
	25	60.0			
	23	59.4			
	23	58.6	Set number	Number of blows	Water content (%)
	25	58.1			
	29	57.9			
	31	57.5	11	14	39.5
	35	57.8		14	38.8
	37	57.5		16	38.5
	36	56.0		17	38.5
	47	55.0		17	37.9
				20	37.8
				25	37.2
				25	36.8
				27	37.1
				26	36.8
				27	36.4
				27	36.3
				31	36.8
				32	36.5
				40	36.0
			12	12	37.1
				14	36.4
				23	34.9
				24	34.9
				25	34.5
				29	34.5
				35	33.4

## 2-5 Plastic Limit Test

### OBJECTIVE

The plastic limit test is used to determine the lowest moisture content at which the soil behaves plastically. It is carried out only on the soil fraction passing No. 40 sieve ( $425\text{ }\mu\text{m}$ ) and is usually performed in conjunction with the liquid limit test. By convention, the plastic limit of a soil is defined as the water content at which the soil begins to crumble when rolled into a thread 3 mm in diameter.

### EQUIPMENT

The equipment for the plastic limit test includes:

- Surface for rolling the thread, such as a glass or plastic plate or smooth linoleum tabletop.
- Short metal rod of 3 mm diameter.
- Spatula with a blade about 10 cm long and about 2 cm wide.
- Specimen containers for determination of water content (see Chapter 2-3).
- Balance sensitive to 0.01 g.
- Hair dryer.
- Drying oven.

### PROCEDURE

1. About 20 g of soil is prepared as for the liquid limit test. The sample may be obtained by air drying and sieving through a No. 40 sieve or by taking the natural soil and removing coarse particles by hand. If it is initially too wet, the sample should be allowed to dry partially in air on the glass plate until the right consistency is achieved. Drying may be accelerated by mixing with a spatula and by using a hair dryer.



**Figure 1** The soil specimen is rolled under the fingers into a thread 3 mm in diameter.



**Figure 2** The 3-mm-diameter soil thread crumbles when the water content is equal to the plastic limit.

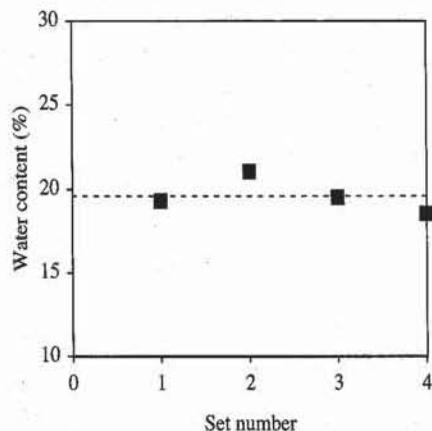
2. When the soil is plastic enough, it is kneaded and shaped into a 1- to 2-cm-diameter ball. The material should be plastic enough not to stick to the fingers when squeezed.
3. The ball is formed into a thread by rolling it under the fingers against the test surface (Fig. 1). Use just enough pressure to roll the soil into a thread 3 mm in diameter as shown in Fig. 2. Gage this diameter by using metal rod 3 mm in diameter as reference. The pressure required for rolling the thread varies greatly depending on the soil toughness. Some tough clays may require firm pressure as they become harder near the plastic limit. Very silty and organic clays that have a soft and spongy consistency at the plastic limit must be rolled gently. If the thread diameter gets smaller than 3 mm without crumbling, fold and knead the thread into a ball again and repeat the rolling process. Knead and roll the soil thread until it has dried to the point of crumbling and breaking into numerous pieces about 3 to 9 mm in length when the thread diameter reaches approximately 3 mm. As shown in Fig. 2, the crumbling of the soil thread, which corresponds to longitudinal and transverse cracking, should be the result of the decrease in water content, not the result of excessive hand pressure.
4. As soon as the soil thread crumbles, collect part of it and determine its water content. Repeat steps 2 and 3 with another portion of the prepared material, and check that two successive runs give approximately the same plastic limit. If the two test values vary more than 5% from the average, additional tests should be performed.

## **COMPUTATION**

The plastic limit  $PL$  is reported as the average of two similar values. If it is not possible to obtain a plastic limit in the plastic limit test, the soil is reported as nonplastic. This also applies if  $PL \geq LL$ . Errors in computing the liquid or plastic limits can be detected by plotting the point  $(LL, PI)$  on the plasticity chart. This point should fall under the U-line.

## **EXAMPLE**

An example of plastic limit determination is given in Figs. 3 to 5. Figure 3 summarizes the measured plastic limits, and Fig. 5 lists the formulas used in Fig. 4. In this example, four measurements were made. The plastic limit is the average of these four measurements. As shown in Fig. 3, all these measurements vary by less than 5% from their mean value, and two tests would have been sufficient to determine the plastic limit.



**Figure 3** Variation of the plastic limits in four different tests.

	A	B	C	D	E
1	Analyst name: <i>Mike Kapuskar</i> Test date: <i>11/13/90</i> Sample description: <i>Aardvark modeling clay</i>				
2					
3					
4					
5					
6	Set number	Mass of container (g)	Mass of container with wet soil (g)	Mass of container with dry soil (g)	Water content (%)
7		M <sub>o</sub>	M <sub>w</sub>	M <sub>d</sub>	w
8	1	20.10	30.00	28.40	19.28
9	2	22.40	29.30	28.10	21.05
10	3	21.20	31.00	29.40	19.51
11	4	23	35.80	33.80	18.52
12	Plastic limit (%) =	19.59			

**Figure 4** Data set for plastic limit test.

E	
Water content (%)	
6	
7	w
8	$=(M_w - M_d) / (M_d - M_c) * 100$

	A	B
12	Plastic limit (%) =	=AVERAGE(w)

**Figure 5** Formulas used in Fig. 4.**REFERENCE**

See Introduction for references to ASTM procedures (pages 4 to 6).

**EXERCISES**

1. Determine the plastic limit from the experimental results of Table E1.

**TABLE E1**

Set number	Tare mass (g)	Tare with wet soil (g)	Tare with dry soil (g)
1	47.12	77.12	71.44
2	45.67	75.67	69.94
3	45.66	75.66	70.05
4	45.67	75.67	69.68

2. Same as Exercise 1 but for the experimental results of Table E2.

**TABLE E2**

Set number	Tare mass (g)	Tare with wet soil (g)	Tare with dry soil (g)
1	47.11	48.63	48.47
2	43.21	44.96	44.75
3	47.12	48.71	48.47
4	104.19	106.49	106.19

3. Same as Exercise 1 but for the experimental results of Table E3.

**TABLE E3**

Set number	Tare mass (g)	Tare with wet soil (g)	Tare with dry soil (g)
1	47.12	54.70	53.59
2	46.69	47.84	47.63
3	45.61	47.03	46.66
4	45.81	47.77	47.49

4. Same as Exercise 1 but for the experimental results of Table E4.

TABLE E4

Set number	Tare mass (g)	Tare with wet soil (g)	Tare with dry soil (g)
1	46.70	49.14	48.85
2	47.72	51.50	51.02
3	47.08	49.33	49.05
4	47.72	50.17	49.82

## 2-6 Principles of Shrinkage Limit Analysis

### DEFINITIONS

Fine-grained soils shrink continuously when the water content decreases, until the water content reaches the shrinkage limit. At that point the soil particles are in close contact, and the soil volume can no longer decrease, even if the water content is reduced further. Clays are more susceptible to shrinkage than are silts and sands. In most cohesive soils, the shrinkage limit is appreciably smaller than the plastic limit, except for silts, which have similar shrinkage and plastic limits.

#### Shrinkage Limit

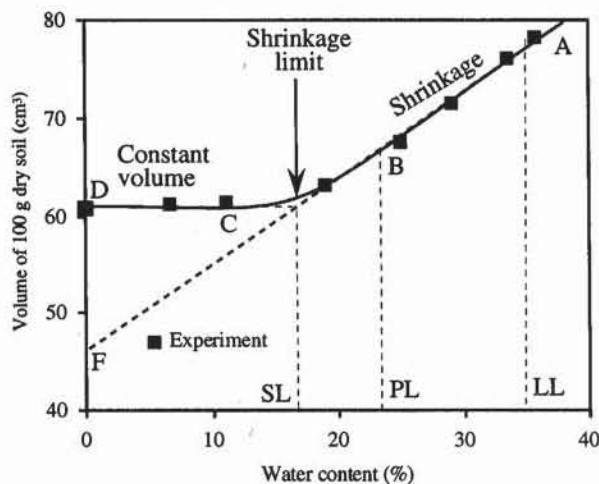
Figure 1 shows the typical variation of volume measured for clays when the water content decreases. The volume change decreases proportionally to the loss of water content  $w$  between points  $A$  and  $B$ , where  $w > PL$ . However, between points  $C$  and  $D$ , where  $w < PL$ , there is no further decrease in volume as the soil dries. The shrinkage limit  $SL$  is the water content at the intersection of lines  $AB$  and  $CD$ . The intercept of line  $AB$  (i.e., point  $F$ ) corresponds to the total volume of dry soil particles.

#### Shrinkage Ratio

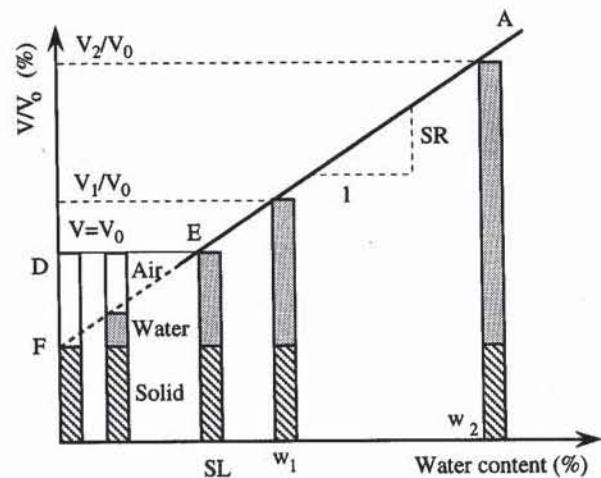
Related to Fig. 1, Fig. 2 shows the corresponding variation of volume ratio  $V/V_0$ , where  $V$  is the total soil volume and  $V_0$  is the dry soil volume. The shrinkage ratio  $SR$  is the slope of line  $AF$  in Fig. 2:

$$SR = \frac{(V_1 - V_2)/V_0}{w_1 - w_2} \quad (1)$$

where  $V_1$  is the volume corresponding to water content  $w_1$  and  $V_2$  is the volume corresponding to water content  $w_2$ . The columns of Fig. 2 illustrate the variation of the volumes of soil and its air, water, and solid constituents at various drying



**Figure 1** Typical shrinkage curve for clay soil (data after Head, 1984).



**Figure 2** Definition of shrinkage ratio SR.

stages. For large water content, the soil is fully saturated, and its volume varies linearly with  $w$ . The volume of the solid fraction is always constant and equal to the volume at point  $F$ . At point  $E$  the soil particles cannot get closer together even though water is being removed, and air starts to fill voids as drying continues. When  $V_1$  and  $V_2$  are both larger than  $V_0$ , the water content decreases linearly with the volume change:

$$w_1 - w_2 = \frac{\rho_w}{M_0} (V_1 - V_2) \quad (2)$$

where  $\rho_w$  is the water unit mass and  $M_0$  is the mass of dry soil. Therefore,  $SR$  is

$$SR = \frac{M_0}{\rho_w V_0} \quad (3)$$

### Linear Shrinkage

The linear shrinkage ratio  $LS$  characterizes the change in length induced by drying a cylindrical sample of soil initially about its liquid limit.  $LS$  is

$$LS = \frac{L - L_0}{L} \times 100 \quad (\%) \quad (4)$$

where  $L$  is the original length of the sample at about the liquid limit and  $L_0$  is the length of the dry sample.  $LS$  gives an indication on the amount of axial strain that drying may cause to soil samples.  $LS$  can also be determined from volume changes provided that the soil shrinks uniformly in all directions:

$$LS = \left( 1 - \sqrt[3]{\frac{V_0}{V}} \right) \times 100 \quad (\%) \quad (5)$$

where  $V$  is the initial soil volume close to the liquid limit and  $V_0$  is the dry soil volume. In the case of most British soils (Head, 1984),  $LS$  was found to be approxi-

mately related to the plasticity index  $PI$  through

$$PI = 2.13LS \quad (\%) \quad (6)$$

### TYPICAL VALUES FOR SHRINKAGE LIMIT

Table 1 lists some typical values of shrinkage limits for clay minerals and soils.

**TABLE 1**

Liquid, plastic, and shrinkage limits for several clay minerals and natural soils  
(after Lambe and Whitman, 1979; and Lambe, 1951)

Mineral	Exchangeable ion	Liquid limit (%)	Plastic limit (%)	Plasticity index (%)	Shrinkage limit (%)	Shrinkage limit calculated by method 1 (%)	Shrinkage limit calculated by method 2 (%)
Montmorillonite	Na	710	54	656	9.9	-132.3	6.2
	K	660	98	562	9.3	-74.8	10.1
	Ca	510	81	429	10.5	-51.3	10.5
	Mg	410	60	350	14.7	-45.3	9.5
	Fe	290	75	215	10.3	2.1	15.7
Illite	Na	120	53	67	15.4	26.0	23.4
	K	120	60	60	17.5	33.0	27.8
	Ca	100	45	55	16.8	23.4	22.1
	Mg	95	46	49	14.7	25.8	23.8
	Fe	110	49	61	15.3	24.7	22.8
Kaolinite	Na	53	32	21	26.8	23.1	22.9
	K	49	29	20	21.2	21.1	
	Ca	38	27	11	24.5	22.1	22.4
	Mg	54	31	23	28.7	21.8	21.7
	Fe	59	37	22	29.2	26.5	26.0
Attapulgite	H	270	150	120	7.6	82.5	43.9
Mexico City clay		388	226	162	43	126.6	52.5
Boston blue clay (illite)		41	25	16	18.7	19.3	19.3
Aardvack clay		30.6	19.6	11	12.17	16.7	16.4
Morganza Louisiana clay		104	74.8	29.2	13.7	52.1	47.0
Beverly clayey silt (illite)		19.5	16.3	3.2	13.3	16.4	15.4

### APPROXIMATE DETERMINATION OF SHRINKAGE LIMIT

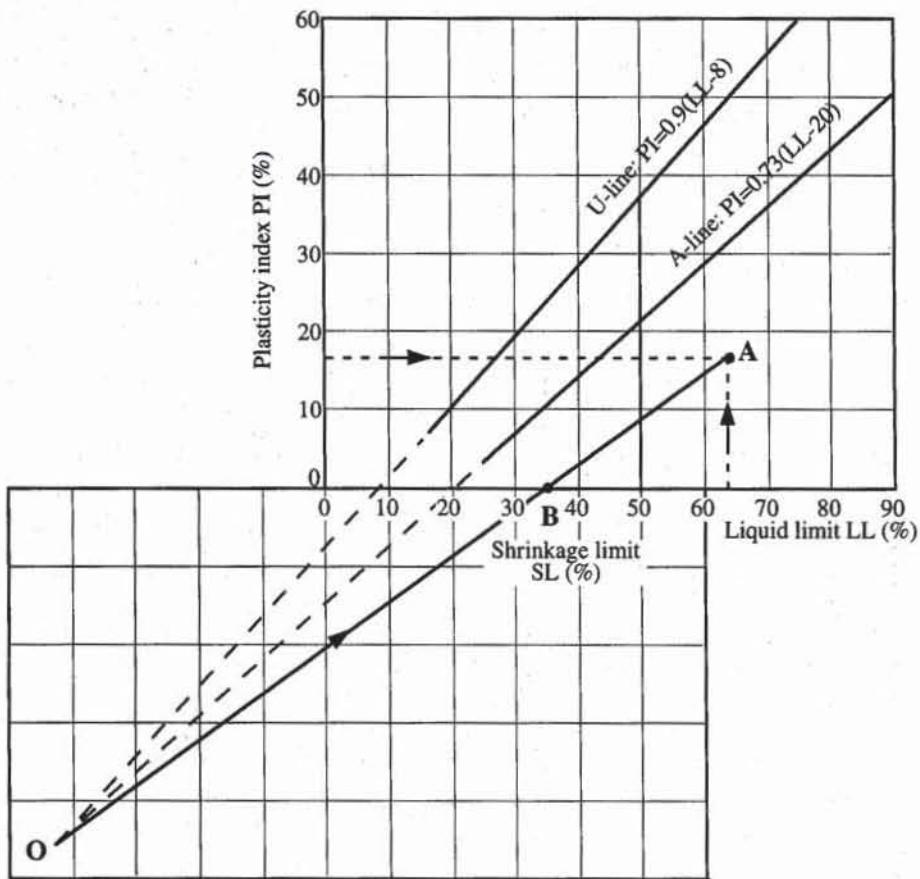
There are two empirical methods to estimate  $SL$  from the liquid and plastic limits.

#### Method 1

The shrinkage limit  $SL$  is estimated as follows (Holtz and Kovacs, 1981):

$$SL = 5.4 + 0.73LL - PI \quad (\%) \quad (7)$$

where  $LL$  is the liquid limit and  $PI$  is the plasticity index. Table 1 compares the measured shrinkage limits with those estimated using Eq. 7. Eq. 7 is a very crude approximation of the measured shrinkage limits. Equation 7 even predicts an unrealistic negative value of the shrinkage limit for montmorillonite.



**Figure 3** Method 2 for estimating the shrinkage limit from liquid and plastic limits.

### Method 2

As shown in Fig. 3, the U- and A-lines of the plasticity chart intersect at point *O* with coordinates  $-43.53$  and  $-46.38$ . The shrinkage limit *SL* of a soil with liquid limit *LL* and plasticity index *PI* can be estimated as the liquid limit of the intersection point *B* between line *OA* and the liquid limit axis, where *A* has for coordinates *LL* and *PI* (Holtz and Kovacs, 1981). This graphical construction corresponds to

$$SL = 46.38 \frac{LL + 43.53}{PI + 46.38} - 43.53 \quad (\%) \quad (8)$$

where *LL* is the liquid limit (%) and *PI* is the plastic index (%). As shown in Table 1, Eq. 8 provides a more accurate approximation of measured shrinkage limits than does Eq. 7.

### ENGINEERING SIGNIFICANCE OF SHRINKAGE

The effects of shrinkage of fine-grained soils are of considerable significance from a geotechnical engineering point of view. Shrinkage cracks are caused by the evaporation from the surface in dry climates and lowering the groundwater table.

When the climate changes and the soils again have access to water, they tend to increase in volume and swell. The volume changes resulting from both shrinkage and swelling of fine-grained soils are often large enough to cause serious damage to small buildings and highway pavements. In 1973, Jones and Holtz estimated that shrinking and swelling soils caused about \$2.3 billion in damage annually in the United States alone, which, to put things in perspective, was more than twice the annual cost of damage from floods, hurricanes, tornadoes, and earthquakes combined.

## REFERENCES

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- LAMBE, T. W., 1951, *Soil Testing for Engineers*, John Wiley & Sons, NY, 165 pp.
- LAMBE, T. W., and R. V. WHITMAN, 1979, *Soil Mechanics, SI Version*, John Wiley & Sons, New York, 553 pp.

## REVIEW QUESTIONS

1. Define *shrinkage limit*. What is the position of the shrinkage limit with respect to the liquid and plastic limits?
2. Define *shrinkage ratio*.
3. Is it meaningful to define a shrinkage limit for sands?
4. Is the shrinkage limit larger or smaller than the plastic limit?
5. Which methods are used to estimate the shrinkage limits from liquid and plastic limits?

# Shrinkage Limit Analysis with Mercury

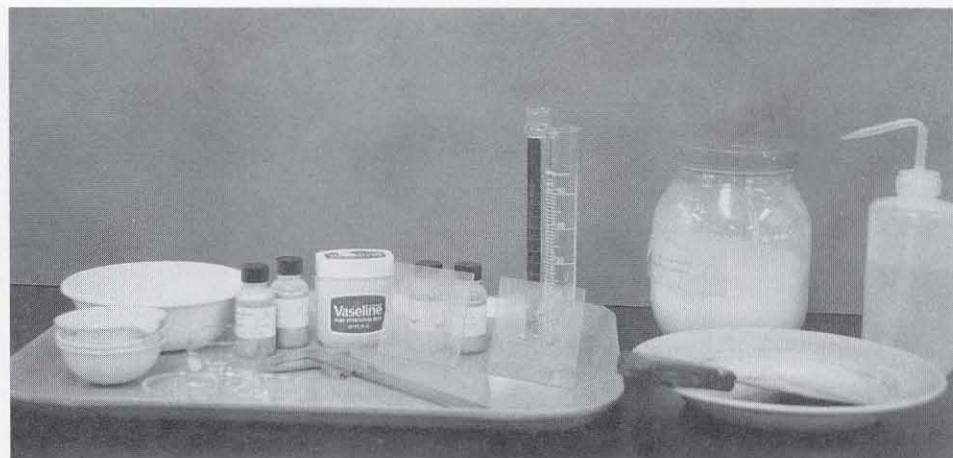
## OBJECTIVE

Shrinkage limit analysis determines the shrinkage limit and shrinkage ratio of fine-grained soils. The *shrinkage limit* is the water content below which a soil undergoes no further volume change. The shrinkage ratio provides an estimate of the amount of volume change during drying and wetting. There are two methods to determine the shrinkage limit: the mercury method and the wax method. The mercury method is described in this chapter, the wax method in Chapter 2-8.

## EQUIPMENT

The equipment for shrinkage limit analysis with mercury includes:

- Shrinkage dish, porcelain or metal, about 42 mm in diameter and 12 mm deep (Fig. 1).
- Immersion glass cup about 57 mm in diameter and 38 mm deep, with the rim ground flat.
- Prong plate, transparent, fitted with three prongs and large enough to cover the immersion glass cup.
- Mercury, a little more than 1 kg, to fill the glass cup completely. Mercury is a harmful substance—avoid direct skin contact with mercury. After the experiment, pour the mercury back into its container without leaving any mercury trace on the laboratory equipment, bench, or floor. Handling of mercury can be hazardous unless the appropriate precautionary measures are taken. Mercury vapour is poisonous when its concentration exceeds 100  $\mu\text{g}/\text{m}^3$ . It is important to control the surface area of mercury exposed to the air (absolutely no spillage of mercury on the floor or laboratory bench) and have a normal ventilation in the room.
- Large evaporating dish and tray, to prevent mercury from spilling.



**Figure 1** Equipment for shrinkage limit analysis. Shrinkage dish, prong plate, container of mercury, petroleum jelly, spatula, large evaporating dish, and large tray.

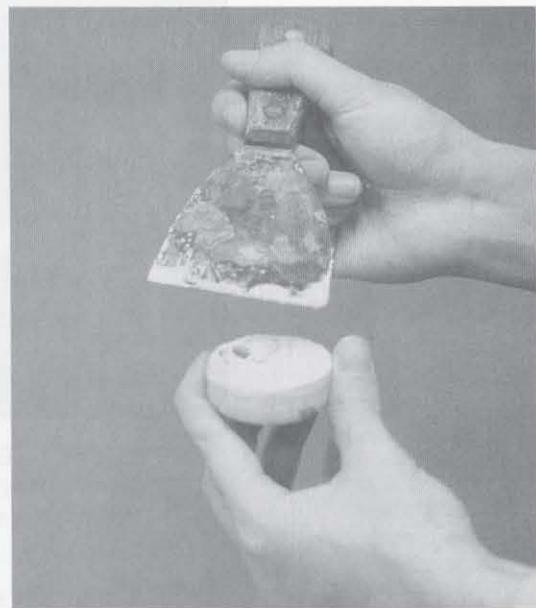
- Petroleum jelly, silicon grease, or Teflon powder spray to lubricate shrinkage dish.
- Balance accurate to 0.01 g.
- Measuring cylinder, 100 mL.
- Drying oven.
- Straight-edge spatula.

## TEST PROCEDURE

1. Take about 40 g of the material used for the liquid and plastic limit tests. This material should pass a No. 40 sieve. Place the soil in an evaporating dish and, use a spatula to mix thoroughly with distilled water to obtain a soil paste with a water content higher than the liquid limit.
2. Measure the weights of the shrinkage limit dish, first dry, then filled with water, to determine its volume.
3. Lightly coat the inside of the dish with petroleum jelly or silicon grease (Fig. 2). This coating will prevent the soil from sticking to the dish or forming cracks upon drying. Measure the weight of the coated dish.
4. Fill about one-third of the dish with the soil paste (Fig. 3). Tap the dish with the spatula handle to cause the soil paste to flow to the edges of the dish and to release air bubbles. Add a second layer of soil, about the same size as the first, and again tap the sample to release entrapped air. Add more soil and overtop the dish slightly. Strike off the excess soil with a straight-edge and clean off adhering soil from the outside of the shrinkage dish. Measure the weight of the wet soil and dish.
5. Leave the soil in the shrinkage dish to dry in the laboratory until the soil surface changes to a light color (about 5 to 6 h). Place it in the oven at 105 to 110°C and dry it to constant weight (about 12 to 18 h). Measure the weight of the dry soil and container.
6. Remove the dried soil-pat carefully from the shrinkage dish. It should be intact if it was adequately air dried before oven drying. Place the immersion cup in a clean evaporating dish, itself located inside a large tray (Fig. 4). Fill



**Figure 2** Lightly coat the inside of the shrinkage limit dish with petroleum jelly or silicon grease.



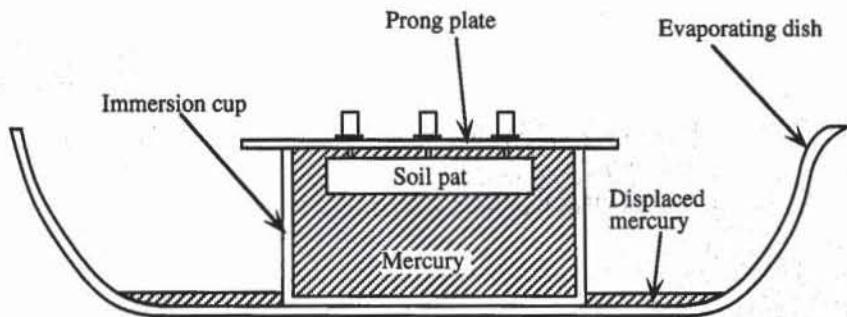
**Figure 3** The shrinkage dish is filled with soil paste. The dish is tapped with a spatula handle to cause the soil to flow to the edges of the dish. When the dish is completely filled with an excess standing out, strike off the excess with a straightedge.



**Figure 4** Place the immersion cup in a clean evaporating dish, itself located inside a large tray. Place the dry soil pat on the mercury surface.



**Figure 5** Press the three prongs of the prong plate carefully onto the sample and force it under the mercury.



**Figure 6** The three prongs push the dry soil pat into the mercury, and displace mercury from the immersion cup to the evaporating dish.

the cup to overflowing with mercury, and remove the mercury excess by pressing the glass prong plate firmly on the top of the cup. Press slowly to avoid trapping air under the glass plate. Carefully remove the prong plate. Transfer the excess mercury collected in the evaporating dish into its original container without spilling.

7. Place the soil pat on the mercury surface (Fig. 4). The soil pat will float because the mercury is much denser than any soil. Press the three prongs of the prong plate carefully on the sample and force it under the mercury (Fig. 5). Avoid trapping any air. Press the plate firmly onto the dish. Displaced mercury will be held in the evaporating dish. Brush off any droplets of mercury adhering to the cup into the evaporating dish (Fig. 6). Transfer all the displaced mercury to the measuring cylinder and measure the weight of displaced mercury. The volume of displaced mercury is equal to the volume of the dry soil pat.
8. Transfer the mercury back into its original container, by working above a large tray, without spilling.

## COMPUTATION

The initial water content  $w$  of the wet soil pat is

$$w = \frac{M - M_0}{M_0} \quad (1)$$

where  $M$  is the mass of the wet soil and  $M_0$  is the mass of the dry soil. The shrinkage limit  $SL$  is calculated from  $w$  by removing the change in water content from  $V$  to  $V_0$ :

$$SL = w - \rho_w \frac{V - V_0}{M_0} \quad (2)$$

where  $V$  is the volume of the wet soil pat,  $V_0$  the volume of the dry soil pat, and  $\rho_w$  the unit mass of water.  $V$  is equal to the volume of the mold where the pat is formed.

$V_0$  is equal to the volume of mercury that is displaced by immersion of the dry pat into the mercury bowl:

$$V_0 = \frac{M_{Hg}}{\rho_{Hg}} \quad (3)$$

where  $M_{Hg}$  is the mass of the displaced mercury and  $\rho_{Hg}$  is the unit mass of the mercury = 13.6 g/cm<sup>3</sup>. The shrinkage ratio is

$$SR = \frac{M_0}{\rho_w V_0} \quad (4)$$

The linear shrinkage ratio  $LS$  is

$$LS = 1 - \sqrt[3]{\frac{V_0}{V}} \quad (5)$$

### EXAMPLE

Figure 7 shows an example of input/output data for the shrinkage limit test, and Fig. 8 shows the formulas used in Fig. 7.

	A	B	C	D
<b>Shrinkage Test</b>				
1				
2				
3				
4				
5				
6				
7	<b>MERCURY METHOD</b>	Sample 1	Sample 2	
8	Mass of coated dish $M_c$ (g)	17.51	15.64	
9	Mass of coated dish and wet soil $M_w$ (g)	78.07	76.47	
10	Mass of coated dish and dry soil $M_d$ (g)	64.41	62.56	
11	Volume of wet soil $V$ (cm <sup>3</sup> )	32.42	32.85	
12	Mass of dish $M_t$ (g)	130.00	130.00	
13	Mass of dish and displaced mercury $M_{Hg}$ (g)	462.00	466.00	
14	Unit mass of Mercury $\rho_{Hg}$ (g/cm <sup>3</sup> )	13.60		
15	Volume of dry soil $V_0$ (cm <sup>3</sup> )	24.41	24.71	Average
16	Initial water content $w$	29.13%	29.64%	29.38%
17	Shrinkage limit $SL$	12.06%	12.28%	12.17%
18	Shrinkage ratio $SR$	1.92	1.90	1.91
19	Linear Shrinkage $LS$	9.02%	9.06%	9.04%

Figure 7 Example of data set.

	A	B	C	D
15	Volume of dry soil $V_0$ (cm <sup>3</sup> )	$=(M_{Hg}-M_t)/\rho_{Hg}$	$=(M_{Hg}-M_t)/\rho_{Hg}$	Average
16	Initial water content $w$	$=(M_w-M_d)/(M_d-M_c)$	$=(M_w-M_d)/(M_d-M_c)$	$=AVERAGE(w)$
17	Shrinkage limit $SL$	$=w-(V-V_0)/(M_d-M_c)$	$=w-(V-V_0)/(M_d-M_c)$	$=AVERAGE(SL)$
18	Shrinkage ratio $SR$	$=(M_d-M_c)/V_0$	$=(M_d-M_c)/V_0$	$=AVERAGE(SR)$
19	Linear Shrinkage $LS$	$=(1-(V_0/V))^{(1/3)}$	$=(1-(V_0/V))^{(1/3)}$	$=AVERAGE(LS)$

Figure 8 List of formulas used in Fig. 7.

**REVIEW QUESTIONS**

1. What is the purpose of the shrinkage limit?
2. Why do we use mercury in the shrinkage limit test? Is there another technique to define the shrinkage limit? Which one?
3. Why do we coat the shrinkage dish with petroleum jelly or silicon grease?

**EXERCISES**

1. Calculate the ratio between the plasticity index  $PI$  and linear shrinkage ratio  $LS$ . Compare your value to that found for British soils ( $PI/LS = 2.13$ ).
2. Calculate the shrinkage limit, shrinkage ratio, and linear shrinkage ratio from the following test results.

	Sample 1	Sample 2
Mass of coated dish (g)	17.04	13.61
Mass of coated dish + wet soil (g)	74.24	68.88
Mass of coated dish + dry soil (g)	56.61	51.62
Volume of wet soil ( $\text{cm}^3$ )	32.41	34.96
Mass of dish (g)	134.00	134.00
Mass of dish + displaced mercury (g)	400.00	438.00
Unit mass of mercury	13.60	13.60

	Sample 1	Sample 2
Mass of coated dish (g)	17.51	13.97
Mass of coated dish + wet soil (g)	78.07	72.97
Mass of coated dish + dry soil (g)	64.41	59.80
Volume of wet soil ( $\text{cm}^3$ )	32.42	34.95
Mass of dish (g)	0.00	0.00
Mass of dish + displaced mercury (g)	316.00	355.00
Unit mass of mercury	13.60	13.60

## 2-8

# Shrinkage Limit Analysis with Wax

### OBJECTIVE

The shrinkage limit analysis with wax has the same objective as the shrinkage limit analysis with mercury (Chapter 2-7). It determines the shrinkage limit and shrinkage ratio of fine-grained soils by using wax instead of mercury. Because wax is less toxic than mercury, this method is recommended in an academic environment. The wax method assumes that wax, similar to mercury, does not wet or expand dry soils.

### EQUIPMENT

The equipment for shrinkage limit analysis with wax includes:

- Shrinkage dish, porcelain or metal, about 42 mm in diameter and 12 mm deep.
- Petroleum jelly, silicon grease, or Teflon powder spray to lubricate shrinkage dish.
- Straight-edge spatula.
- Balance accurate to 0.01 g.
- Support frame for suspending the sample below the balance (see Chapter 3-2).
- Paraffin wax and wax bath.
- Container filled with water.
- Drying oven.
- Evaporating dish.

### TEST PROCEDURE

The procedure combines several steps of the shrinkage limit analysis with mercury and the determination of the unit weight of cohesive soils (refer to Chapters 2-7 and 3-2 for details).

1. Construct the soil pat and dry it as specified in steps 1 to and 4 of the test procedure in Chapter 2-7. Measure the weight of the coated dish, the weight of the coated dish and wet soil, the weight of the coated dish and dry soil, and the volume of wet soil.
2. Measure the volume of the dry soil pat as specified in steps 2 to and 6 of the determination of the unit weight of cohesive soils in Chapter 3-2. If the pat has a tendency to crumble, apply thin layers of hot wax on it with a brush before immersing it completely in the heated wax bath. Measure the total and buoyant weights of the dry pat and wax.
3. The wax unit mass should be determined as described in Chapter 3-2.

## COMPUTATION

The moisture content  $w$  of the initial wet soil pat, the shrinkage limit  $SL$ , the shrinkage ratio  $SR$ , and the linear shrinkage ratio  $LS$  are given in Eqs. 1, 2, 4, and 5 of Chapter 2-7. However, in the wax method,  $V_0$  is calculated from the weight of displaced water corrected for the additional volume of wax:

$$V_0 = \frac{M_{sp} - M_{bsp}}{\rho_w} - \frac{M_p}{\rho_p} \quad (1)$$

where  $M_{sp}$  is the mass of the dry soil covered with wax,  $M_{bsp}$  the buoyant mass of the dry soil covered with wax,  $M_p$  the mass of wax covering the dry pat,  $\rho_w$  the unit mass of water, and  $\rho_p$  the unit mass of wax.

## EXAMPLE

Figure 1 shows an example of input/output data for the shrinkage limit analysis with wax, and Fig. 2 shows the formulas used in Fig. 1.

	A	B	C	D
1	<b>Shrinkage Test</b>			
2	Analyst name: Henry Guapo			
3	Test date: 2/22/93			
4	Sample description: Aardvark modeling clay			
5				
6	<b>WAX METHOD</b>		Sample 1	Sample 2
7	Mass of coated dish $M_c$ (g)	15.64	13.90	
8	Mass of coated dish and wet soil $M_w$ (g)	76.47	76.08	
9	Mass of coated dish and dry soil $M_d$ (g)	62.56	61.43	
10	Volume of wet soil $V$ ( $\text{cm}^3$ )	32.85	33.75	
11	Mass of soil and wax $M_{sp}$ (g)	51.85	48.40	
12	Buoyant mass of soil and wax $M_{bsp}$ (g)	22.05	22.70	
13	Unit mass of wax $\rho_{wax}$ ( $\text{g}/\text{cm}^3$ )	0.95	0.95	
14	Volume of dry soil $V_0$ ( $\text{cm}^3$ )	24.61	24.78	Average
15	Initial water content $w$	29.64%	30.83%	30.23%
16	Shrinkage limit $SL$	12.08%	11.96%	12.02%
17	Shrinkage ratio $SR$	1.91	1.92	1.91
18	Linear Shrinkage $LS$	9.17%	9.78%	9.48%

Figure 1 Example of data set.

	A	B	C	D
14	Volume of dry soil $V_0$ ( $\text{cm}^3$ )	$=(\text{Msp}-\text{Mbsp}) - (\text{Msp}-(\text{Md}-\text{Mc}))/\text{rwax}$	$=(\text{Msp}-\text{Mbsp}) - (\text{Msp}-(\text{Md}-\text{Mc}))/\text{rwax}$	Average
15	Initial water content w	$=(\text{Mw}-\text{Md})/(\text{Md}-\text{Mc})$	$=(\text{Mw}-\text{Md})/(\text{Md}-\text{Mc})$	$=\text{AVERAGE}(w)$
16	Shrinkage limit SL	$=w-(V-V_0)/(\text{Md}-\text{Mc})$	$=w-(V-V_0)/(\text{Md}-\text{Mc})$	$=\text{AVERAGE}(SL)$
17	Shrinkage ratio SR	$=(\text{Md}-\text{Mc})/V_0$	$=(\text{Md}-\text{Mc})/V_0$	$=\text{AVERAGE}(SR)$
18	Linear Shrinkage LS	$=(1-(V_0/V))^{(1/3)}$	$=(1-(V_0/V))^{(1/3)}$	$=\text{AVERAGE}(LS)$

**Figure 2** Formulas used in Fig. 1.**COMPARISON OF WAX AND MERCURY METHODS**

The shrinkage limit analyses with wax and mercury were performed successively for some particular soils and were found to yield similar results. The volume of a dry soil pat can be measured by using first mercury, then wax. As shown in Fig. 3,  $SL = 12.06$  and  $12.28\%$  for the mercury method, while  $SL = 12.08$  and  $11.96\%$  for the wax method.

**Shrinkage Test**Analyst name: *Henry Guapo*.

Test date: 2/22/93

Sample description: *Aardvark modeling clay*

	Sample 1	Sample 2	Sample 3
Mass of coated dish (g)	17.51	15.64	13.90
Mass of coated dish and wet soil (g)	78.07	76.47	76.08
Mass of coated dish and dry soil (g)	64.41	62.56	61.43
Volume of wet soil ( $\text{cm}^3$ )	32.42	32.85	33.75

**MERCURY METHOD**

Mass of dish (g)	130.00	130.00	
Mass of dish and displaced mercury (g)	462.00	466.00	
Unit mass of Mercury ( $\text{g}/\text{cm}^3$ )	13.60	13.60	
Initial water content	29.13%	29.64%	30.83%
Volume of dry soil ( $\text{cm}^3$ )	24.41	24.71	
Shrinkage limit (%)	12.06%	12.28%	
Shrinkage ratio	1.92	1.90	
Linear Shrinkage (%)	9.02%	9.06%	

**WAX METHOD**

Mass of soil and wax (g)		51.85	48.40
Bouyant mass of soil and wax (g)		22.05	22.70
Unit mass of wax ( $\text{g}/\text{cm}^3$ )		0.95	0.95
Volume of dry soil ( $\text{cm}^3$ )		24.61	24.78
Shrinkage limit (%)		12.08%	11.96%
Shrinkage ratio		1.91	1.92
Linear Shrinkage (%)		9.17%	9.78%

**Figure 3** Comparison of shrinkage limits obtained by using the wax and mercury methods.**REVIEW QUESTIONS**

- What are the differences between the wax and mercury methods of determining the shrinkage limit?
- How do you determine the unit weight of wax?
- What is the main function of the wax in the wax method?

**EXERCISE**

1. Calculate the shrinkage limit, shrinkage ratio, and linear shrinkage ratio from the following test results obtained from the wax and mercury methods. Compare the results obtained by the wax method and mercury method.

	Sample 1	Sample 2	Sample 3	Sample 4
Mass of coated dish (g)	17.51	13.97	0.00	0.00
Mass of coated dish + wet soil (g)	78.07	72.97	60.56	59.00
Mass of coated dish + dry soil (g)	64.41	59.80	47.20	45.80
Volume of wet soil ( $\text{cm}^3$ )	32.42	34.95	33.75	33.75
<b>Mercury method</b>				
Mass of dish (g)	0.00	0.00		
Mass of dish + displaced mercury (g)	316.00	355.00		
Unit mass of Mercury ( $\text{g}/\text{cm}^3$ )	13.60	13.60		
<b>Wax method</b>				
Mass of soil and wax (g)			54.55	53.75
Buoyant weight of soil and wax (g)			22.10	20.40
Unit mass of wax ( $\text{g}/\text{cm}^3$ )			0.95	0.95

## **2-9      Engineering Classification of Soils**

### **INTRODUCTION**

Soil classification systems attribute to soils a label or designation that represents their most significant properties for specific engineering applications. They are based on measurable parameters, such as coefficient of uniformity, clay fraction, activity, and liquid limit. Several soil classification systems were developed to meet specific engineering needs. Some examples in the United States are the AASHTO classification system (ASTM D 3282) and the Unified Soil Classification System (ASTM D 2487). These engineering soil classifications are based on the results of Atterberg limits tests and grain size analyses.

### **UNIFIED SOIL CLASSIFICATION SYSTEM**

The Unified Soil Classification System (USCS), originally developed by Casagrande, is one of the most commonly used systems. It classifies soils into groups defined by a primary and a secondary letter. The letters and their meanings are given in Table 1. Normally, two letters are used. For example, SW indicates well-graded sand. However, soils having the characteristics of two groups are classified using dual symbols (e.g., CL-CH). The flowchart of Fig. 1 specifies the steps for classifying soils. Coarse-grained soils (e.g., sands and gravels) are classified using their grain size distribution curve, whereas fine-grained soils (e.g., silts and clays) are classified using their liquid and plastic limits. Under the USCS, many soils fall within one of two general categories. Coarse-grained soils, which include sands and gravels, have either G or S for the first letter and W, P, M, or C for the second. Fine-grained soils have M, C, or O for the first letter and L or H for the second. A third group of soils—highly organic soils, or Pt—consists of peat, muck, and so on. They are typically spongy, crumbly, and compressible and are undesirable for use in supporting structures.

**TABLE 1**

Primary and secondary letters used in the Unified Soil Classification System.

Primary letter		Secondary letter	
G	Gravel	W	Well graded
S	Sand	P	Poorly graded
M	Silt	M	With nonplastic fines
C	Clay	C	With plastic fines
O	Organic	L	Of low plasticity ( $LL < 50\%$ )
Pt	Peat	H	Of high plasticity ( $LL > 50\%$ )

### EXAMPLE

The results of grain size analysis and the Atterberg limit for a soil are as follows:

U.S. sieve size	Percent passing
No. 4	100
No. 10	85.6
No. 40	72.3
No. 200	58.8
Liquid limit	$LL = 46.2\%$
Plastic limit	$PL = 21.9\%$

The plasticity index is  $PI = LL - PL = 46.2 - 21.9 = 24.3\%$ .

The soil is fine-grained (right branch of Fig. 1) because more than 50% passes a No. 200 sieve. Its liquid limit  $LL = 46.2\%$  is less than 50%. The point  $(LL, PI) = (46.2, 24.3)$  falls above the A-line of the plasticity chart. The soil is classified as CL.

### AASHTO CLASSIFICATION SYSTEM

"AASHTO" stands for the "American Association of State Highway and Transportation Officials." This classification system is widely used in highway work. The system intends to indicate a soil's acceptability as a highway and road subgrade and base course, partially by the use of a numerical measure of the soil quality, termed the *group index GI*.

AASHTO classifies soils by using the results of grain size analysis and liquid and plastic limit tests. The necessary parameters are listed in the first column of Table 2. With the values of these parameters known, one enters the column of Table 2 labeled A-1a and determines whether or not the parameters meet the limiting values of that column. If they do, the soil classification is A-1a. If they do not, one enters the next column to the right and determines whether or not the parameters meet the limiting values of that column. The procedure is repeated until the parameters meet all the limiting values of a column. The soil classification is given at the top of that particular column. In addition to its group name, a soil is further classified by its group index *GI*:

$$GI = (F-35)[0.2 + 0.005(LL-40)] + 0.01(F-15)(PI-10) \quad (1)$$

where  $F$  is the percentage of soil passing a No. 200 sieve,  $LL$  the liquid limit, and  $PI$  the plasticity index.  $GI$  is rounded off to the nearest whole number and, if

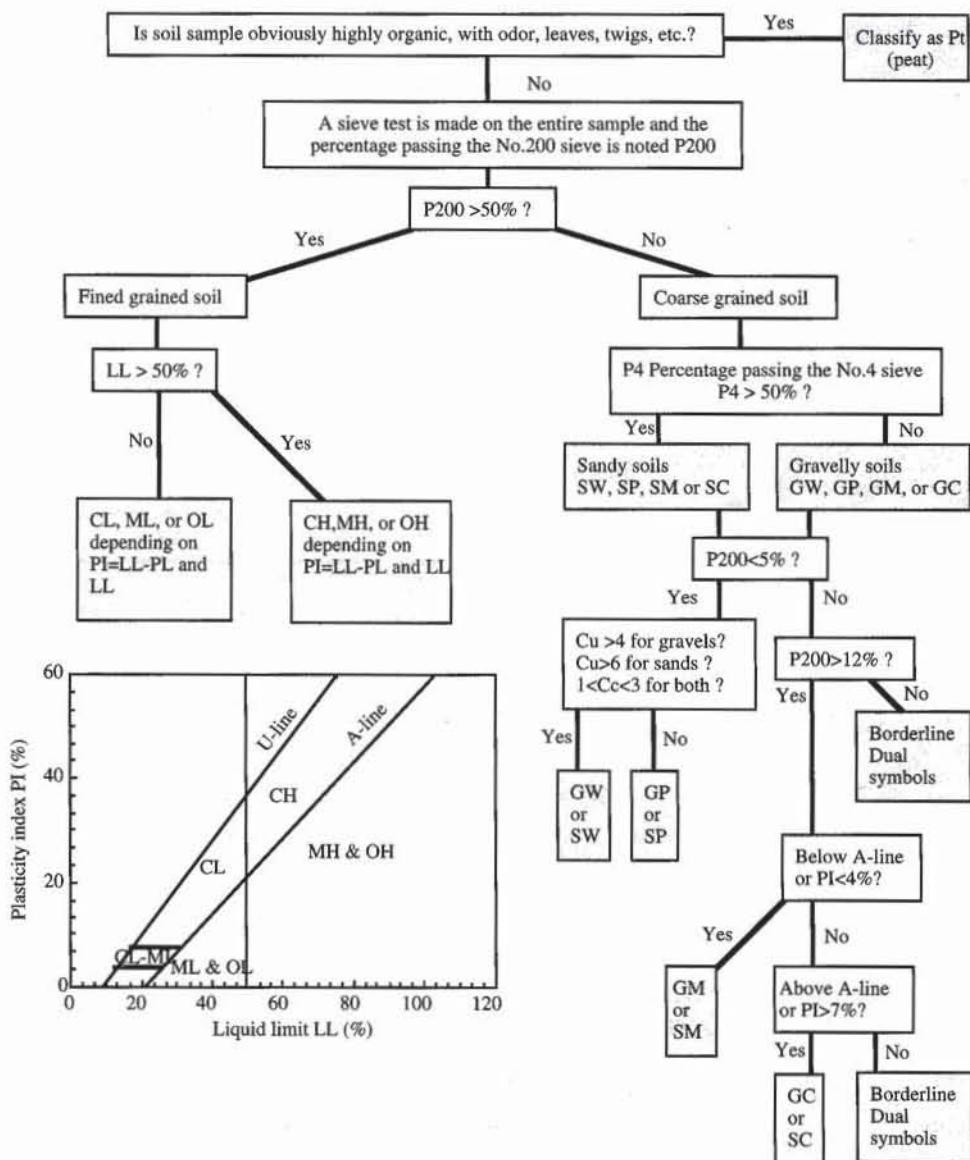


Figure 1 Flowchart of Unified Soil Classification System.

negative, set equal to zero. The result of the AASHTO classification is reported by appending *GI* in parentheses to the group designation.

### EXAMPLE

With 85.6% passing a No. 10 sieve, 72.3% passing a No. 40 sieve, 58.8% passing a No. 200 sieve, a liquid limit of 42.6%, and a plasticity index of 24.3%, one proceeds across Table 2 from left to right until the first column is reached in which these parameters meet the limiting values in that column. The result is A-7. The group index is determined using Eq. 1.

$$GI = (58.8 - 35)[0.2 + 0.005(46.2 - 40)] + 0.01(58.8 - 15)(24.3 - 10) = 11.8 \approx 12$$

Therefore, this soil is classified as A-7 (12).

TABLE 2

AASHTO classification of soils and soil-aggregate mixtures

General classification	Granular materials (35% or less passing .075 mm)							Silt-clay materials (more than 35% passing .075 mm)			
	A-1		A-3	A-2				A-4	A-5	A-6	A-7
Group classification	A-1a	A-1b		A-2-4	A-2-5	A-2-6	A-2-7				A-7-5 A-7-6
Percent passing 2.00 mm (No. 10) 0.425 mm (No. 40) 0.075 mm (No. 200)	50 max. 30 max. 15 max.	50 max. 25 max.	51 max. 10 max.	35 max.	35 max.	35 max.	35 max.	36 min.	36 min.	36 min.	36 min.
Fraction passing 0.425 mm (No. 40) Liquid limit Plasticity index			N.P.	40 max. 10 max.	41 min. 10 max.	40 max. 11 min.	41 min. 11 min.	40 max. 10 max.	41 min. 10 max.	40 max. 11 min.	41 min. 11 min.
Usual types of significant constituent materials	Stone fragments Gravel and sand		Fine sand	Silty or clayey gravel sand				Silty soils		Clayey soils	
General rating as subgrade	Excellent to good							Fair to poor			

Classification procedure: Given the required test data, proceed from left to right in chart; the correct group will be found by the process of elimination. The first group from the left consistent with the test data is the correct classification. The A-7 group is subdivided into A-7-5 or A-7-6, depending on the plastic limit. For  $PL < 30$ , the classification is A-7-6; for  $PL > 30$ , A-7-5. N.P. denotes "nonplastic."

### COMPARISON OF USCS AND AASHTO CLASSIFICATION SYSTEMS

There are significant differences between the USCS sand AASHTO soil classification systems, which result from their different histories and purposes. Tables 3 and 4 compare the two systems in terms of the probable corresponding soil groups.

**TABLE 3**

Comparable soil groups in the AASHTO and USCS systems (after Liu, 1970)

Soil group in USCS	Comparable soil groups in AASHTO system		
	Most probable	Possible	Possible but improbable
GW	A-1a	—	A-2-4, A-2-5, A-2-6, A-2-7
GP	A-1a	A-1b	A-3, A-2-4, A-2-5, A-2-6, A-2-7
GM	A-1b, A-2-4, A-2-5, A-2-7	A-2-6	A-4, A-5, A-6, A-7-5, A-7-6, A-1a
GC	A-2-6, A-2-7	A-2-4, A-6	A-4, A-7-6, A-7-5
SW	A-1b	A-1a	A-3, A-2-4, A-2-5, A-2-6, A-2-7
SP	A-3, A-1b	A-1a	A-2-4, A-2-5, A-2-6, A-2-7
SM	A-1b, A-24, A-2-5, A-2-7	A-2-6, A-4,	A-6, A-7-5, A-5, A-7-6, A-1a
SC	A-2-6, A-2-7	A-2-4, A-6, A-4, A-7-6	A-7-5
ML	A-4, A-5	A-6, A-7-5	—
CL	A-6, A-7-6	A-4	—
OL	A-4, A-5	A-6, A-7-5, A-7-6	—
MH	A-7-5, A-5	—	A-7-6
CH	A-7-6	A-7-5	—
OH	A-7-5, A-5	—	A-7-6
Pt	—	—	—

**TABLE 4**

Comparable soil groups in the AASHTO and USCS systems (after Liu, 1970)

Soil group in AASHTO system	Comparable soil groups in USCS system		
	Most probable	Possible	Possible but improbable
A-1a	GW, GP	SW, SP	GM, SM
A-1b	SW, SP, GM, SM	GP	—
A-3	SP	—	SW, GP
A-2-4	GM, SM	GC, SC	GW, GP, SW, SP
A-2-5	GM, SM	—	GW, GP
A-2-6	GC, SC	GM, SM	GW, GP
A-2-7	GM, GC, SM, SC	—	GW, GP, SW, SP
A-4	ML, OL	CL, SM, SC	GM, GC
A-5	OH, MH, ML, OL	—	SM, GM
A-6	CL	ML, OL, SC	GC, GM, SM
A-7-5	OH, MH	ML, OL, CH	GM, SM, GC, SC
A-7-6	CH, CL	ML, OL, SC	OH, MH, GC, GM, SM

### EXAMPLE OF UNIFIED SOIL CLASSIFICATION

An example of classification with the USCS is given in Figs. 2 to 5. The formulas used in Fig. 2 are listed in Fig. 3. Figure 4 shows the user-defined function for USCS. The input data in Fig. 2 are given in the arguments of the USC function.

In Fig. 2 the soil is classified as CL (i.e., a clay of low plasticity). The results of the USC function can be verified by plotting the point ( $LL, PI$ ) on the plasticity chart of Fig. 5. In this case the point ( $LL, PI$ ) is not a borderline case because it is far from the boundaries between two classifications.

	A	B
1	<b>Example of Unified Soil Classification</b>	
2	Clean Sand (Sieve analysis) SP	
3	Clay sample CL	
4	Liquid limit (%) = 34	
5	Plastic limit (%) = 20	

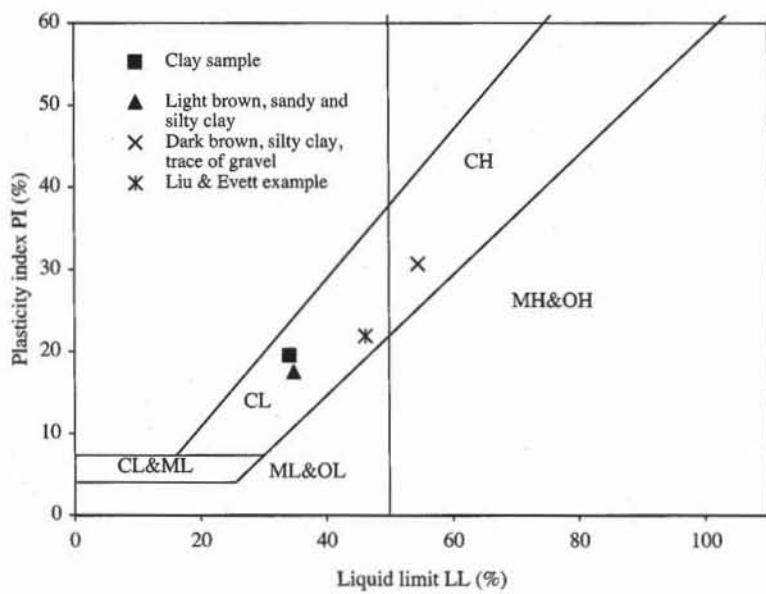
Figure 2 Example of Unified Soil Classification.

	B
2	=USC(0,99,0.05,1.48,0.96,0,0)
3	=USC(0,100,100, 0, 0,B4,B5)

Figure 3 Formulas used in Fig. 2.

	A	B
1	<b>USC</b>	<b>Unified Soil Classification (USC)</b>
2	=RESULT(2)	Returns a classification code
3	=ARGUMENT("Organic",1)	Amount of organic matter in%
4	=ARGUMENT("pf4",1)	Percent passing #4 sieve
5	=ARGUMENT("pf200",1)	Percent passing #200 sieve
6	=ARGUMENT("Cu",1)	Coefficient of uniformity
7	=ARGUMENT("Cc",1)	Coefficient of curvature
8	=ARGUMENT("LL",1)	Liquid limit in %
9	=ARGUMENT("PL",1)	Plastic limit in %
10	=LL-PL	Plasticity index
11	=IF(pf4<pf200,RETURN("ERR: % passing No.4 must be larger than % passing No.200"))	Error messages
12	=IF(LL<PL,RETURN("ERR: liquid limit must be larger or equal to plastic limit"))	Purely organic clay or silt
13	=IF(Organic>99.9,RETURN("Pt"))	Classify according to plasticity chart
14	=IF(pf200>50)	Above U line
15	= IF(AND(A10>0.9*(LL-8),A10>7.3),RETURN("ERR: above U line in plasticity chart"))	Under A line
16	= IF(OR(A10<=0.73*(LL-20),A10<=4))	
17	= IF(LL>50,IF(Organic<=1,RETURN("MH"),RETURN("OH")))	
18	= IF(Organic<=1,RETURN("ML"),RETURN("OL"))	
19	= ELSE.IF(A10>=7.3)	
20	= IF(LL>50,RETURN("CH"),RETURN("CL"))	
21	= ELSE.IF(AND(A10>4,A10<=7.3))	
22	= RETURN("CL-ML")	
23	= END.IF()	
24	=ELSE.IF(AND(pf200>=5,pf200<=12))	
25	= IF(pf4>50,RETURN("SC-SM"),RETURN("GC-GM"))	
26	=ELSE.IF(pf200>12)	
27	= IF(AND(A10>0.9*(LL-8),A10>7.3),RETURN("ERR: above U line in plasticity chart"))	
28	= IF(AND(A10>0.73*(LL-20),A10>7),IF(pf4<50,RETURN("GC"),RETURN("SC"))))	
29	= IF(OR(A10<=0.73*(LL-20),A10<4),IF(pf4<50,RETURN("GM"),RETURN("SM"))))	
30	= IF(pf4<50,RETURN("GM-GC"),RETURN("SM-SC"))	
31	=ELSE()	
32	= IF(Cu<1,RETURN("Cu must be greater than or equal to 1"))	
33	= IF(pf4>50)	
34	= IF(AND(Cu>6,Cc>1,Cc<3),RETURN("SW"),RETURN("SP"))	
35	= ELSE()	
36	= IF(AND(Cu>4,Cc>1,Cc<3),RETURN("GW"),RETURN("GP"))	
37	= END.IF()	
38	=END.IF()	
39	=RETURN("ERR")	

Figure 4 User-defined function for Unified Soil Classification.



**Figure 5** Position of USCS and AASHTO classification examples of Figs. 2 and 7 on plasticity chart.

### EXAMPLE OF AASHTO SOIL CLASSIFICATION

Figure 6 lists the user-defined function for AASHTO classification, Fig. 7 is an example of AASHTO soil classification, and Fig. 8 lists the formulas used in Fig. 7. This function exactly follows the instructions of Table 2.

	A	B
40	<b>AASHTO</b>	<b>AASHTO Classification System</b>
41	=RESULT(2)	Returns a Classification Code
42	=ARGUMENT("PP10",1)	Percent Passing No.10 Sieve
43	=ARGUMENT("PP40",1)	Percent Passing No.40 Sieve
44	=ARGUMENT("PP200",1)	Percent Passing No.200 Sieve
45	=ARGUMENT("LL",1)	Liquid Limit in %
46	=ARGUMENT("PL",1)	Plastic Limit in %
47	=LL-PL	Plasticity Index
48	=("&TEXT(MAX(INT((PP200-35)*(0.2+0.005*(LL-40))+0.01*(PP200-15)*(A47-10)),0),"0")&"	Group Index
49	=IF(PP10<PP40,RETURN("ERROR: % passing No.10 must be larger than % passing No.40"))	
50	=IF(PP40<PP200,RETURN("ERROR: % passing No.40 must be larger than % passing No.200"))	
51	=IF(LL<PL,RETURN("ERROR: liquid limit must be larger or equal plastic limit"))	
52	=IF(AND(PP10<50,PP40<30,PP200<15,A47<6),RETURN("A-1a"))	
53	=IF(AND(PP40<50,PP200<25,A47<6),RETURN("A-1b"))	
54	=IF(AND(PP40<51,PP200<10,PL<0),RETURN("A-3"))	
55	=IF(AND(PP200<35,LL<40,A47<10),RETURN("A-2-4"&A48))	
56	=IF(AND(PP200<35,LL>41,A47<10),RETURN("A-2-5"&A48))	
57	=IF(AND(PP200<35,LL<40,A47>11),RETURN("A-2-6"&A48))	
58	=IF(AND(PP200<35,LL>41,A47>11),RETURN("A-2-7"&A48))	
59	=IF(AND(PP200>=36,LL<40,A47<10),RETURN("A-4"&A48))	
60	=IF(AND(PP200>=36,LL>41,A47<10),RETURN("A-5"&A48))	
61	=IF(AND(PP200>=36,LL<40,A47>11),RETURN("A-6"&A48))	
62	=IF(AND(PP200>=36,LL>41,A47>11,PL>30),RETURN("A-7-5"&A48))	
63	=IF(AND(PP200>=36,LL>41,A47>11,PL<30),RETURN("A-7-6"&A48))	
64	=RETURN("N-C")	

**Figure 6** User-defined function for AASHTO classification.

	A	B
6	<b>Example of AASHTO Classification</b>	
7	Light brown, sandy and silty clay A-2-6(0)	
8	Liquid limit (%) = 34.8	
9	Plastic limit (%) = 17.5	
10	Dark brown, silty clay, trace of gravel A-7-5(10)	
11	Liquid limit (%) = 54.5	
12	Plastic limit (%) = 30.7	
13	Medium brown, very gravelly, coarse sand A-1b	
14	Liu & Evett example A-7-6(11)	
15	Liquid limit (%) = 46.2	
16	Plastic limit (%) = 21.9	

**Figure 7** Result of AASHTO classification.

	B
7	=AASHTO(68.5,36.1,18.1,B8,B9)
8	34.8
9	17.5
10	=AASHTO(79.5,69,54.3,B11,B12)
11	54.5
12	30.7
13	=AASHTO(59.1,38.5,5.1,0,0)
14	=AASHTO(85.6,72.3,58.8,B15,B16)

**Figure 8** Formulas used in Fig. 7.

## USE OF SOIL CLASSIFICATION

Table 5 gives a general indication of engineering properties such as permeability (Chapter 4-1), compressibility (Chapter 6-1), and strength (Chapter 7-1) of the various soil groups in USCS. Table 5 also gives an indication of the relative desirability of each group for use in earth dams, canal sections, foundations and runways. Table 6 lists some typical values of engineering properties for compacted soils (Chapter 3-4) which may be used for preliminary analysis. However, for final analysis, the engineering soil properties must be determined from laboratory or field tests.

Soil classification is a valuable tool which provides the soil engineer with some empirical guidance through the results of field experience. However, as pointed out by Lambe and Whitman (1979), solving flow, compression, and stability problems merely on the basis of soil classification can lead to disastrous results. Empirical correlations between index properties and fundamental soil behavior have many large deviations.

TABLE 5

Engineering use chart based on results of USCS classification (after Lambe and Whitman, 1979; and Wagner, 1957).

**TABLE 6**

Typical properties of compacted soils based on results of USCS classification (after NAVFAC, 1982).

Typical names and soil groups	Group symbol	Range of maximum dry unit weight (kN/m <sup>3</sup> )	Range of optimum moisture content (%)	Typical value of compressibility (1/MPa)		Typical strength characteristics			Typical coefficient of permeability (cm/s)
				m <sub>v</sub> at 140 kPa	m <sub>v</sub> at 350 kPa	Cohesion c' as compacted (kPa)	Cohesion c' as saturated (kPa)	Effective stress envelope ϕ' (deg)	
Well graded gravels, gravel-sand mixtures, little or no fines	GW	19.6-21.2	8-11	0.02	0.02	0	0	> 38	0.03
Poorly graded gravels, gravel-sand mixtures, little or no fines	GP	18.1-19.6	11-14	0.03	0.03	0	0	> 37	0.05
Silty gravels, poorly graded gravel-sand-silt mixtures	GM	18.9-21.2	8-12	0.04	0.03	-	-	> 34	> 5 × 10 <sup>-7</sup>
Clayey gravels, poorly graded gravel-sand-silt mixtures	GC	18.1-20.4	9-14	0.05	0.05	-	-	> 31	> 5 × 10 <sup>-8</sup>
Well graded sands, gravelly sands, little or no fines	SW	17.3-20.4	9-16	0.04	0.03	0	0	38	> 5 × 10 <sup>-4</sup>
Poorly graded sands, gravelly sands, little or no fines	SP	15.7-18.9	12-21	0.06	0.04	0	0	37	> 5 × 10 <sup>-4</sup>
Silty sands, poorly graded sand-silt mixtures	SM	17.3-19.6	11-16	0.06	0.05	50	20	34	> 3 × 10 <sup>-5</sup>
Sand-silt-clay mixture with slightly plastic fines	SM-SC	17.3-20.4	11-15	0.06	0.04	50	14	33	> 1 × 10 <sup>-6</sup>
Clayey sands, poorly graded sand-clay mixtures	SC	16.5-19.6	11-19	0.08	0.06	74	11	31	> 3 × 10 <sup>-7</sup>
Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity	ML	14.9-18.9	12-24	0.06	0.05	67	11	32	> 5 × 10 <sup>-6</sup>
Mixture of inorganic silt and clay	ML-CL	15.7-18.9	12-22	0.07	0.06	65	9	32	> 3 × 10 <sup>-7</sup>
Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays	CL	14.9-18.9	12-24	0.09	0.07	65	22	28	> 5 × 10 <sup>-8</sup>
Organic silts and organic silt-clays of low plasticity	OL	12.6-15.7	21-33	-	-	86	13	-	-
Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	MH	11.0-14.9	24-40	0.14	0.11	72	20	25	> 3 × 10 <sup>-7</sup>
Inorganic clays of high plasticity, fat clays	CH	11.8-16.5	19-36	0.19	0.11	103	11	19	> 5 × 10 <sup>-8</sup>
Organic clays of medium to high plasticity, fat clays	OH	10.2-15.7	21-45	-	-	-	-	-	-

**REFERENCES**

- See Introduction for references to ASTM procedures (pages 4 to 6).
- LAMBE, T.W., and R.V. WHITMAN, 1979, *Soil Mechanics, SI Version*, John Wiley & Sons, Inc., New York, 553 p.
- LIU, T.K., 1970, "A review of Engineering Soil Classification Systems," *Special Procedures for Testing Soil and Rock for Engineering Purposes*, 5th Ed., American Society for Testing and Materials, ASTM Special Technical Publication No. 479, pp. 361-382.
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**REVIEW QUESTIONS**

1. What is the purpose of a soil classification? How many soil classification systems can you name?
2. In the USCS, what material properties are used to classify coarse-grained soils?
3. In the USCS, what material properties are used to classify fine-grained soils?

**EXERCISES**

1. The results of particle size analyses, and where appropriate, limit tests on samples of four soils are given below. Classify each soil according to the USCS.

Particle size	Percentage smaller			
	Soil A	Soil B	Soil C	Soil D
63 mm	100	—	100	—
20 mm	64	—	76	—
6.3 mm	39	100	65	—
2 mm	24	98	59	—
600 $\mu\text{m}$	12	90	54	—
212 $\mu\text{m}$	5	9	47	100
63 $\mu\text{m}$	0	3	34	95
20 $\mu\text{m}$	—	—	23	69
6 $\mu\text{m}$	—	—	14	46
2 $\mu\text{m}$	—	—	7	31
Liquid limit	—	—	26	42
Plastic index	—	—	9	18

2. Same as Exercise 2 but for the AASHTO classification system.
3. Determine the extreme values of soil parameters that give GW in the USCS. You may find the necessary parameters and their extreme values by working the flowchart of Fig. 1 backward or by using the custom function of the USCS. Having determined the necessary parameters and their extreme values, use those to determine the possible AASHTO classifications. Compare your results with those in Table 3.

4. Repeat Exercise 4 for one of the following USCS classifications: GP, GM, GC, SW, SP, SM, SC, ML, CL, O, MH, CH, and OH. Compare your results with those in Table 3.
5. Determine the extreme values of soil parameters that give A-1a in the AASHTO classification system. You may find the necessary parameters and their extreme values in Table 2. Having determined the necessary parameters and their extreme values, use those to determine the possible USCS classifications. Compare your results with those in Table 4.
6. Repeat Exercise 5 for one of the following AASHTO classifications: A-1b, A-3, A-2-4, A-2-5, A-2-6, A-2-7, A-4, A-5, A-6, A-7-5, and A-7-6. Compare your results with those in Table 4.

# 3

# Density and Compaction

- 3-1** Weight-volume relationships
- 3-2** Unit weight of cohesive soils
- 3-3** Determination of specific gravity
- 3-4** Principles of compaction
- 3-5** Compaction tests
- 3-6** Sand cone method

## 3-1

# Weight-Volume Relationships

### DEFINITIONS

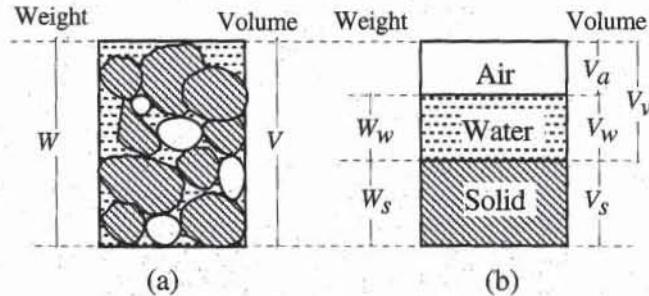
As illustrated in Fig. 1a, soils are made of solid particles with voids in between. For most soils the particles are made of soil minerals, and the voids are filled with air and water. Soils are three-phase materials with solid, liquid, and air constituents. Figure 1a shows a soil sample of total volume  $V$  and weight  $W$ . In Fig. 1b, the volumes and weight of its solid, water, and air constituents are related through

$$W = W_w + W_s \quad V = V_v + V_s = V_a + V_w + V_s \quad V_v = V_a + V_w \quad (1)$$

where  $W_s$  is the weight of solid grains,  $W_w$  the weight of water,  $V_s$  the volume of soil grains,  $V_w$  the volume of water,  $V_a$  the volume of air, and  $V_v$  the volume of voids occupied by water and air. It is assumed that the weight of air  $W_a$  is negligible (i.e.,  $W_a = 0$ ).

The unit weights of the water constituent and solid constituent are denoted  $\gamma_w$  and  $\gamma_s$ , respectively, where

$$\gamma_s = \frac{W_s}{V_s} \quad \text{and} \quad \gamma_w = \frac{W_w}{V_w} \quad (2)$$



**Figure 1** Definition of (a) the weight and volume of a soil sample, and (b) the weights and volumes of its solid, water, and air constituents.

The specific gravity  $G_s$  is the ratio of the solid and water unit weights:

$$G_s = \frac{\gamma_s}{\gamma_w} \quad (3)$$

The total or bulk unit weight  $\gamma$  of the sample of Fig. 1a is

$$\gamma = \frac{W}{V} \quad (4)$$

When the sample of Fig. 1b has no water but the same total volume  $V$ , its dry unit weight  $\gamma_d$  is

$$\gamma_d = \frac{W_s}{V} \quad (5)$$

When the sample has its voids filled completely with water, it is said to be fully saturated. When its total volume  $V$  is still equal to  $V$  after saturation, its saturated unit weight  $\gamma_{\text{sat}}$  is

$$\gamma_{\text{sat}} = \frac{W_s + V_v \gamma_w}{V} \quad (6)$$

In soil mechanics, it is also common to define the buoyant unit weight  $\gamma_b$ , which is the unit weight of the saturated soils immersed in water (i.e., uplifted by the buoyancy force  $= \gamma_w V$ ):

$$\gamma_b = \frac{W_s + V_v \gamma_w - V \gamma_w}{V} = \gamma_{\text{sat}} - \gamma_w \quad (7)$$

The total unit mass  $\rho$ , dry unit mass  $\rho_d$ , saturated unit mass  $\rho_{\text{sat}}$ , buoyant unit mass  $\rho_b$ , water unit mass  $\rho_w$ , and solid unit mass  $\rho_s$  are related to their unit weight counterparts through

$$\gamma = \rho g, \quad \gamma_s = \rho_s g, \quad \gamma_w = \rho_w g, \quad \gamma_{\text{sat}} = \rho_{\text{sat}} g, \quad \gamma_d = \rho_d g, \quad \text{and} \quad \gamma_b = \rho_b g \quad (8)$$

where  $g$  is the earth gravity acceleration (i.e.,  $g = 9.81 \text{ m/s}^2$ ). The dimensions of unit weight is force divided by volume, whereas the dimension of unit mass is mass divided by volume:

$$\begin{aligned} [\gamma] &= [\gamma_s] = [\gamma_w] = [\gamma_{\text{sat}}] = [\gamma_d] = [\gamma_b] = FL^{-3} = MLT^{-2}L^{-3} = MT^{-2}L^{-2} \\ [\rho] &= [\rho_s] = [\rho_w] = [\rho_{\text{sat}}] = [\rho_d] = [\rho_b] = ML^{-3} \end{aligned} \quad (9)$$

where  $F$ ,  $M$ ,  $L$ , and  $T$  represent the dimension of force, mass, length, and time, respectively. The metric unit generally used for all unit weights is  $\text{kN/m}^3$ , whereas the units for unit mass can be  $\text{g/cm}^3$ ,  $\text{kg/m}^3$ , or  $\text{tons/m}^3$ .

In soil mechanics, the proportions of the solid, liquid, and air constituents are characterized by five additional dimensionless quantities:  $e$ ,  $n$ ,  $w$ ,  $S_r$ , and  $D_r$ . The void ratio  $e$  is the ratio of the volume of voids to the solid volume:

$$e = \frac{V_v}{V_s} \quad (10)$$

The porosity  $n$  is the ratio of the volume of voids to the total volume:

$$n = \frac{V_v}{V} \quad (11)$$

The water content  $w$  is the ratio of the weight of water to the dry weight:

$$w = \frac{W_w}{W_s} \times 100 \quad (\%) \quad (12)$$

The degree of saturation  $S_r$  is the ratio of the volume of water to the volume of voids:

$$S_r = \frac{V_w}{V_v} \times 100 \quad (\%) \quad (13)$$

In the case of coarse-grained soils, the relative density  $D_r$  is defined to characterize the degree of compaction:

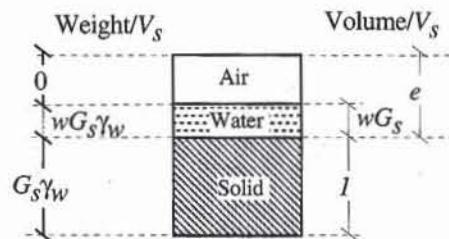
$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100 \quad (\%) \quad (14)$$

where  $e_{\max}$  is the maximum void ratio,  $e_{\min}$  the minimum void ratio, and  $e$  the present void ratio.  $D_r$  varies from 0 when  $e = e_{\max}$  to 100% when  $e = e_{\min}$ . In reality,  $e_{\min}$  and  $e_{\max}$  are not the absolute maximum and minimum void ratios of a soil, respectively. They are just index void ratios obtained by standard procedures. In general, the values of  $w$ ,  $S_r$ , and  $D_r$  are expressed in percent, whereas those of  $e$  and  $n$  are expressed as decimal numbers.

## RELATIONS

Various relations can be obtained directly between  $\gamma$ ,  $\gamma_d$ ,  $\gamma_{sat}$ ,  $\gamma_b$ ,  $G_s$ ,  $e$ ,  $n$ ,  $w$ , and  $S_r$  by using Fig. 2, where all the volumes and weights of Fig. 1b are divided by the solid volume  $V_s$  (i.e., by selecting  $V_s = 1$ ). The following relations were also used in obtaining Fig. 2:

$$\frac{W_w}{V_s} = \frac{W_w}{W_s} \frac{W_s}{V_s} = w\gamma_s = wG_s\gamma_w \quad \text{and} \quad \frac{V_w}{V_s} = \frac{V_w}{W_w} \frac{W_w}{V_s} = \frac{wG_s\gamma_w}{\gamma_w} = wG_s \quad (15)$$



**Figure 2** Schematic definition of the weight and volume for a soil sample having a unit solid volume ( $V_s = 1$ ).

A few useful relations are

$$n = \frac{e}{1+e} \quad \text{and} \quad e = \frac{n}{1-n} \quad (16)$$

$$S_r = \frac{G_s w}{e} \quad (17)$$

$$\gamma = \gamma_w \frac{G_s(1+w)}{1+e} = \gamma_w \frac{G_s + S_r e}{1+e} = \gamma_w \frac{G_s(1+w)}{1+G_s(w/S_r)} \quad (18)$$

$$\gamma_d = \gamma_w \frac{G_s}{1+e} = \frac{\gamma}{1+w} = \gamma_w \frac{G_s}{1+G_s(w/S_r)} \quad (19)$$

$$\gamma_{\text{sat}} = \gamma_w \frac{G_s + e}{1+e} = \gamma_w \frac{G_s(1+w)}{1+G_s w} \quad (20)$$

Table 1 lists some additional useful relations that were obtained after some algebraic manipulations. Note that the water content  $w$  in Table 1 is the saturated water content, which is required to get a full saturation (i.e.,  $S_r = 100\%$ ). When using Eqs. 16 to 20 and those of Table 1, make sure to use decimal values (e.g., 0.1) and not percents (e.g., 10%) for  $w$ ,  $S_r$ , and  $D_r$ .

## TYPICAL VALUES

By definition,  $S_r$  and  $D_r$  must be between 0 and 100%. They cannot be negative or larger than 100%.  $S_r = 0\%$  corresponds to a dry soil, whereas  $S_r = 100\%$  corresponds to a fully saturated soil.  $D_r = 0\%$  corresponds to the loosest state of a coarse-grained soil, whereas  $D_r = 100\%$  corresponds to its densest state. Tables 2 and 3 list some values of specific gravity of solids for minerals and typical soils. Table 4 and 5 list some values of dry unit weight, bulk (or natural) unit weight, saturated unit weight, porosity, and void ratio for typical soils.

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## REVIEW QUESTIONS

1. Define *specific gravity of soil*.
2. Define *total weight*, *dry weight*, and *saturated unit weight*.
3. Define *void ratio*, *porosity*, and *relative density*.
4. What are typical values of specific gravity for soils?

*Continues on page 134*

TABLE 1

Relation between  $G_s$ ,  $\gamma_d$ ,  $\gamma_{\text{sat}}$ , saturated  $w$ ,  $n$ , and  $e$  (After Jumikis, 1962)

	$G_s$	$\gamma_d$	$\gamma_{\text{sat}}$	Saturated $w$	$n$	$e$
$G_s, \gamma_d$			$\left(1 - \frac{1}{G_s}\right) \gamma_d + \gamma_w$	$\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}$	$1 - \frac{\gamma_d}{G_s \gamma_w}$	$\frac{G_s \gamma_w}{\gamma_d} - 1$
$G_s, \gamma_{\text{sat}}$		$\frac{\gamma_{\text{sat}} - \gamma_w}{G_s - 1} G_s$		$\frac{\gamma_w - \gamma_{\text{sat}}/G_s}{\gamma_{\text{sat}} - \gamma_w}$	$\frac{G_s - \gamma_{\text{sat}}/\gamma_w}{G_s - 1}$	$\frac{G_s \gamma_w - \gamma_{\text{sat}}}{\gamma_{\text{sat}} - \gamma_w}$
$G_s, w$		$\frac{G_s}{1 + wG_s} \gamma_w$	$\frac{1+w}{1+wG_s} G_s \gamma_w$		$\frac{wG_s}{1+wG_s}$	$wG_s$
$G_s, n$		$G_s(1-n)\gamma_w$	$(G_s - n(G_s - 1))\gamma_w$	$\frac{n}{(1-n)G_s}$		$\frac{n}{1-n}$
$G_s, e$		$\frac{G_s}{1+e} \gamma_w$	$\frac{G_s + e}{1+e} \gamma_w$	$\frac{e}{G_s}$	$\frac{e}{1+e}$	
$\gamma_d, \gamma_{\text{sat}}$	$\frac{\gamma_d}{\gamma_d + \gamma_w - \gamma_{\text{sat}}}$			$\frac{\gamma_{\text{sat}} - \gamma_d}{\gamma_d} - 1$	$\frac{\gamma_{\text{sat}} - \gamma_d}{\gamma_w}$	$\frac{\gamma_{\text{sat}} - \gamma_d}{\gamma_d + \gamma_w - \gamma_{\text{sat}}}$
$\gamma_d, w$	$\frac{\gamma_d}{\gamma_w - w\gamma_d}$		$(1+w)\gamma_d$		$w \frac{\gamma_d}{\gamma_w}$	$\frac{w\gamma_d}{\gamma_w - w\gamma_d}$
$\gamma_d, n$	$\frac{\gamma_d}{(1-n)\gamma_w}$		$\gamma_d + n\gamma_w$	$\frac{n\gamma_w}{\gamma_d}$		$\frac{n}{1-n}$
$\gamma_d, e$	$(1+e) \frac{\gamma_d}{\gamma_w}$		$\frac{e\gamma_w}{1+e} + \gamma_d$	$\frac{e}{1+e} \frac{\gamma_w}{\gamma_d}$	$\frac{e}{1+e}$	
$\gamma_{\text{sat}}, w$	$\frac{\gamma_{\text{sat}}}{\gamma_w + w(\gamma_{\text{sat}} - \gamma_w)}$	$\frac{\gamma_{\text{sat}}}{1+w}$			$\frac{w\gamma_{\text{sat}}}{(1+w)\gamma_w}$	$\frac{w\gamma_{\text{sat}}}{\gamma_w - w(\gamma_{\text{sat}} - \gamma_w)}$
$\gamma_{\text{sat}}, n$	$\frac{\gamma_{\text{sat}} - n\gamma_w}{(1-n)\gamma_w}$	$\gamma_{\text{sat}} - n\gamma_w$		$\frac{n\gamma_w}{\gamma_{\text{sat}} - n\gamma_w}$		$\frac{n}{1-n}$
$\gamma_{\text{sat}}, e$	$(1+e) \frac{\gamma_{\text{sat}}}{\gamma_w} - e$	$\gamma_{\text{sat}} - \frac{e}{1+e} \gamma_w$		$\frac{e\gamma_w}{\gamma_{\text{sat}} + e(\gamma_{\text{sat}} - \gamma_w)}$	$\frac{e}{1+e}$	
$w, n$	$\frac{n}{(1-n)w}$	$\frac{n}{w} \gamma_w$	$n \frac{1+w}{w} \gamma_w$			$\frac{n}{1-n}$
$w, e$	$\frac{e}{w}$	$\frac{e}{(1-e)w} \gamma_w$	$\frac{(1+w)e}{(1+e)w} \gamma_w$		$\frac{e}{1+e}$	

TABLE 2

Specific gravity of minerals  
(after Lambe and Whitman, 1979)

Mineral	Specific gravity
Quartz	2.65
K-feldspars	2.54–2.57
Na-Ca-feldspars	2.62–2.76
Calcite	2.72
Dolomite	2.85
Muscovite	2.7–3.2
Biotite	2.8–3.2
Chlorite	2.6–2.9
Pyrophyllite	2.84
Serpentine	2.2–2.7
Kaolinite	2.62–2.66
Halloysite	2.55
Illite	2.60–2.86
Montmorillonite	2.75–2.78
Attapulgite	2.30

**TABLE 3**

Typical values of specific gravity of various soils (after Djoenaidi, 1985)

		Type of soil	Specific gravity
Inorganic	Gravel	2.65	
	Coarse and medium sand	2.65	
	Fine sand (silty)	2.65	
	Loess, rock flour, sandy silt	2.67	
Inorganic	Slightly clayey sand	2.65	
	Sandy silt	2.66	
	Silt	2.67–2.70	
	Clayey sand	2.67	
Organic	Clayey sandy silt	2.67	
	Clayey silt	2.68	
	Sand-clay	2.68	
	Sand-silt-clay	2.69	
	Silt-clay	2.71	
	Sandy clay	2.70	
	Silty clay	2.75	
	Lean clay	2.75	
	Clay	2.72–2.80	
	Silts with traces of organic matter	2.30	
Organic	Organic alluvial muds	2.13–2.60	
	Peat	1.50–2.15	

**TABLE 4**

Maximum and minimum values of void ratio and dry unit weight for granular soils (after Lambe and Whitman, 1979)

Description	Void ratio		Dry unit weight ( $\text{kN/m}^3$ )	
	$e_{\max}$	$e_{\min}$	$\gamma_{d\min}$	$\gamma_{d\max}$
Uniform spheres	0.92	0.35	—	—
Standard Ottawa sand	0.80	0.50	14.5	17.3
Clean Uniform sand	1.0	0.4	13.0	18.5
Uniform inorganic silt	1.1	0.4	12.6	18.5
Silty sand	0.9	0.3	13.7	20.0
Fine to coarse sand	0.95	0.2	13.4	21.7
Micaceous sand	1.2	0.4	11.9	18.9
Silty sand and gravel	0.85	0.14	14.0	22.9

**REVIEW QUESTIONS cont.**

5. What are typical values of total, dry, and saturated unit weights?
6. What are typical values of void ratio for sands and clays?
7. Define *degree of saturation*. Is it possible for a soil to have a degree of saturation of 120%?

**EXERCISES**

1. For a moist soil, given  $V = 1.2 \text{ m}^3$ ;  $W = 23.04 \text{ kN}$ ;  $w = 8.6\%$ ; and  $G_s = 2.71$ , determine the bulk unit weight, dry unit weight, void ratio, porosity, degree of saturation, and volume of water  $V_w$ .
2. The following laboratory measurement were made on a sample of soft clay taken under the water table: total volume  $V = 31.3 \text{ cm}^3$ ; total weight  $W = 0.47 \text{ N}$ ; weight after oven drying  $W_{\text{dry}} = 0.258 \text{ N}$ ; and specific gravity

**TABLE 5**

Range of values of void ratio, porosity and unit weight of typical soils (after Kedzi, 1974)

Soil type	State of soil	Porosity (%)	Void ratio	Unit weight ( $\text{kN/m}^3$ )			
				Dry	Natural	Saturated	Buoyant
Sandy gravel	Loose	38–42	0.61–0.72	14–17	18–20	18–21	8–11
	Dense	18–25	0.22–0.33	19–21	20–23	21–23	12–14
Coarse sand, medium sand	Loose	40–45	0.67–0.82	13–15	16–19	17–19	8–10
	Dense	25–32	0.33–0.47	17–18	18–21	20–21	10–11
Uniform fine sand	Loose	45–48	0.82–0.92	14–15	15–19	19–20	9–10
	Dense	33–36	0.49–0.56	17–18	18–21	21–22	11–12
Coarse silt	Loose	45–55	0.82–1.22	13–15	15–19	18–20	8–10
	Dense	35–40	0.54–0.67	16–17	17–21	20–21	10–11
Silt	Soft	45–50	0.82–1.00	13–15	16–20	18–20	8–10
	Slightly plastic	35–40	0.54–0.67	16–17	17–21	20–21	10–11
	Hard	30–35	0.43–0.54	18–19	18–19	21–22	12–13
Lean clay	Soft	50–55	1.00–1.22	13–14	15–18	18–19	8–9
	Slightly plastic	35–45	0.54–0.82	15–18	17–21	19–22	10–13
	Hard	30–35	0.43–0.54	18–19	18–22	21–22	12–13
Fat clay	Soft	60–70	1.50–2.33	9–15	12–18	16–22	6–12
	Slightly plastic	40–55	0.67–1.22	15–18	15–20	20–23	10–13
	Hard	30–40	0.43–0.67	18–20	17–22	22–24	12–14

$G_s = 2.7$ . Determine the the bulk unit weight, water content, void ratio, porosity, and degree of saturation.

- For a given soil, the in situ void ratio  $e$  is 0.8 and the soil specific gravity  $G_s$  is 2.7. Calculate the porosity, dry unit weight, and saturated unit weight. What would the bulk unit weight be when the soil is 60% saturated?
- A base course for a highway is compacted to a unit weight of  $18.25 \text{ kN/m}^3$  at a water content of 14.6%. The specific gravity  $G_s$  of the soil is 2.81. Specifications require compaction to achieve a void ratio of no greater than 0.80. Has this specification been met?
- A soil sample has a mass of 129.1 g and a volume of  $56.4 \text{ cm}^3$ . The mass of the soil grains is 121.5 g. The soil grains' specific gravity  $G_s$  is 2.7. Find the water content  $w$ , the void ratio  $e$ , and the degree of saturation  $S_r$ .
- A cylinder contains  $500 \text{ cm}^3$  of loose dry sand, which weighs 750 g. The volume is reduced by vibration to 10% of the original volume. Assume that the specific gravity of the sand grains is  $G_s = 2.65$ . For loose sand, compute the void ratio, porosity, and dry unit weight. Compute the same quantities for vibrated sand.
- The total volume of a soil specimen is  $85 \text{ cm}^3$ . Its weight is 155 g. The dry weight of the specimen is 122 g. The density of the solid  $G_s$  is 2.75. Calculate the water content, void ratio, porosity, degree of saturation, bulk unit weight, and dry unit weight.
- A soil has a unit weight of  $19.93 \text{ kN/m}^3$ , a specific density  $G_s$  of 2.67, and a water content  $w$  of 12.6%. Determine its dry unit weight, void ratio, porosity, and degree of saturation. What is the weight of water needed to fully saturate  $1 \text{ m}^3$  of this soil?
- Soil has been compacted in an embankment at a bulk unit weight of  $21.09 \text{ kN/m}^3$  and a water content of 12%. The value of soil specific gravity  $G_s$  is 2.65. Calculate the dry unit weight, void ratio, and degree of saturation. Would it be possible to compact this soil at a water content of 13.5% to a dry unit weight of  $19.62 \text{ kN/m}^3$ ? Justify your answer.

## 3-2

# Unit Weight of Cohesive Soils

### OBJECTIVE

The unit weight of cohesive soils is determined by either measuring the weight of sample trimmed to measurable volumes or by weighing in water. The latter method applies to specimens of irregular shapes that cannot be trimmed easily.

### EQUIPMENT

The equipment for measuring the unit weight of cohesive soils (Fig. 1) by trimming is listed below.

- Balance accurate to 0.01 g.
- Ring with cutting edge (about 5 cm in diameter and 2 cm high).
- Calipers.
- Wire saw.
- Drying oven.
- Evaporating dish.

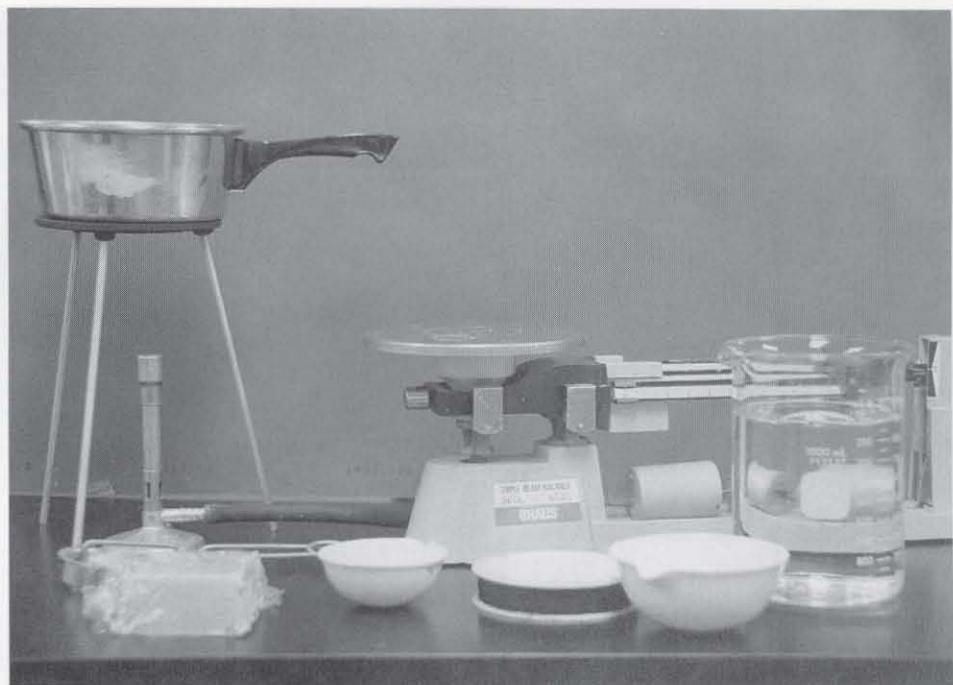
The following equipment is also required for weighing in water:

- Support frame for suspending the sample to a line attached to the balance.
- Wax and pan to melt wax. Blocks of solidified wax are readily available from most hardware stores.
- Container filled with about 1 L of water.

### TEST PROCEDURE

#### Trimming

1. Measure the weight of the clean cutting ring, and determine its internal diameter and height by using a caliper.

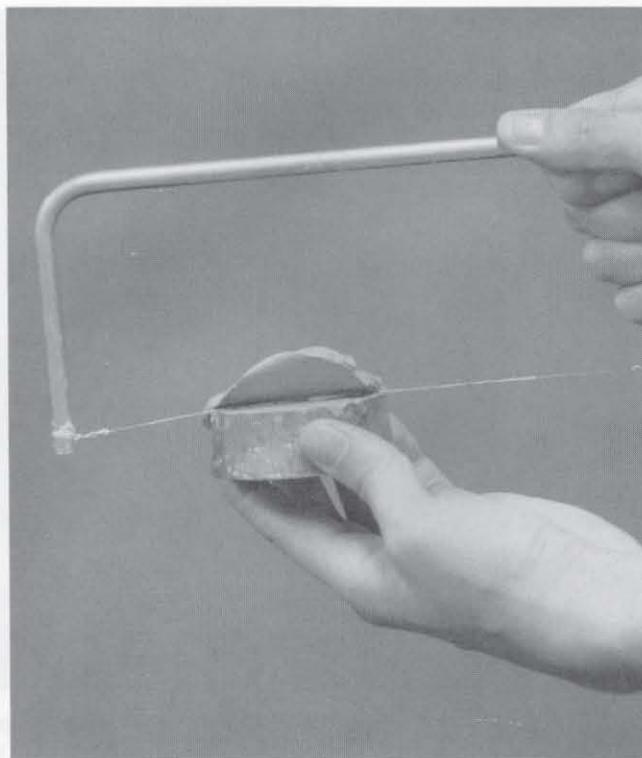


**Figure 1** Equipment for measurement of unit weight of cohesive soils. Balance accurate to 0.01 g, solidified wax block, and wax bath, container filled with water, and evaporating dish

2. Gently push the cutting ring completely into the cohesive soil. With the help of a knife, cut a larger block containing the cutting ring and the sample soil.
3. Cut the excess soil on both ends of the ring by using a wire saw as shown in Fig. 2. Also clean the sides of the cutting ring from any excess soil.
4. Weigh the ring with the soil inside.
5. Oven dry it and measure its dry weight.

#### Weighing in Water

1. Trim the specimen to a convenient bulky shape about 5 cm in size. Avoid having sharp corners or holes, as those create air pockets when coating the specimen with wax.
2. Attach the specimen to 50 cm of fishing line and weigh the specimen as shown in Fig. 3a.
3. Immerse the specimen in a bath of hot wax as shown in Fig. 4. The wax must be heated to just above its melting point. Repeat the immersion process several times to obtain a specimen continuously coated by wax. If the sample is fragile and has a tendency to disintegrate, use a brush to coat it with wax before immersing completely it in the wax bath.
4. After the wax has solidified on the sample, attach the waxed sample to the scale and weigh it again as shown in Figs. 3a and 5.
5. Lift the waxed specimen, put a bucket of water right under the scale, and immerse the specimen in the water as shown in Figs. 3b and 6. Record its immersed weight.



**Figure 2** The unit weight of a cohesive soil can be determined by measuring the weight of a sample of measurable volume. The sample is cut using a ring with a cutting edge, then its ends are trimmed using a wire saw.

6. Remove the sample from the weighing bath, remove the excess water with a rag, and cut a smaller sample that is free of wax for water determination. Measure its weight, oven dry it, and measure its dry weight.

### Determination of Wax Unit Weight

When weighing in water, the wax unit weight  $\gamma_p$  needs to be determined, but only once for a series of tests. Because wax is lighter than water, it does not sink into water and needs to be ballasted. As shown in Fig. 7, (a) measure the submerged weight  $W_1$  of a metal block heavy enough to sink the wax piece in water, (b) measure the weight  $W_2$  of the wax piece alone, and (c) measure the total buoyant unit weight  $W_3$  of both metal and wax pieces. The wax unit weight is

$$\gamma_p = \gamma_w \frac{W_2}{W_1 + W_2 - W_3} \quad (1)$$

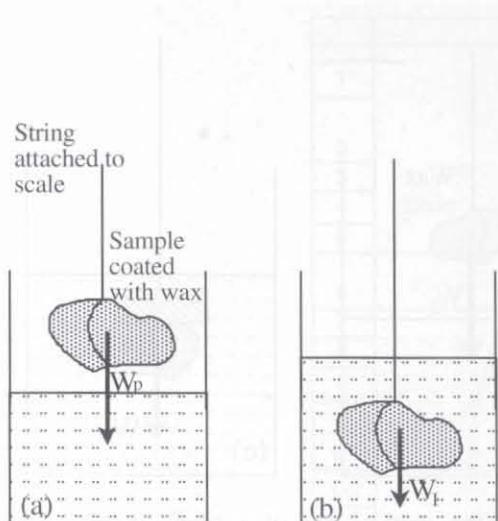
where  $\gamma_w$  is the water unit mass. A typical value for  $\gamma_p$  is 9.32 kN/m<sup>3</sup>.

### COMPUTATION

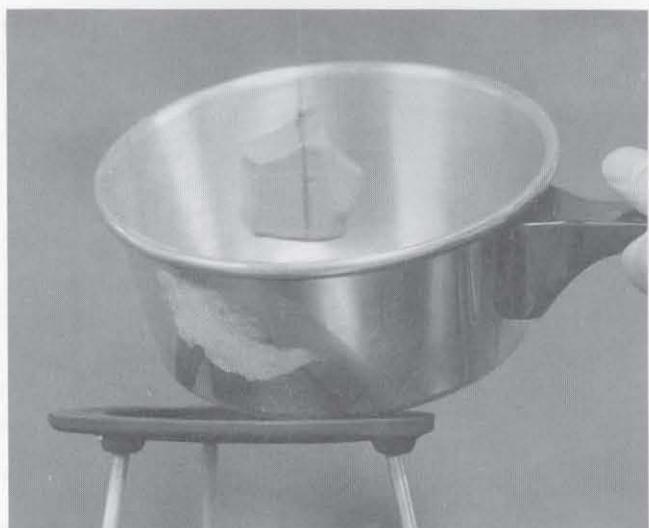
#### Trimming

The total unit weight is calculated as follows:

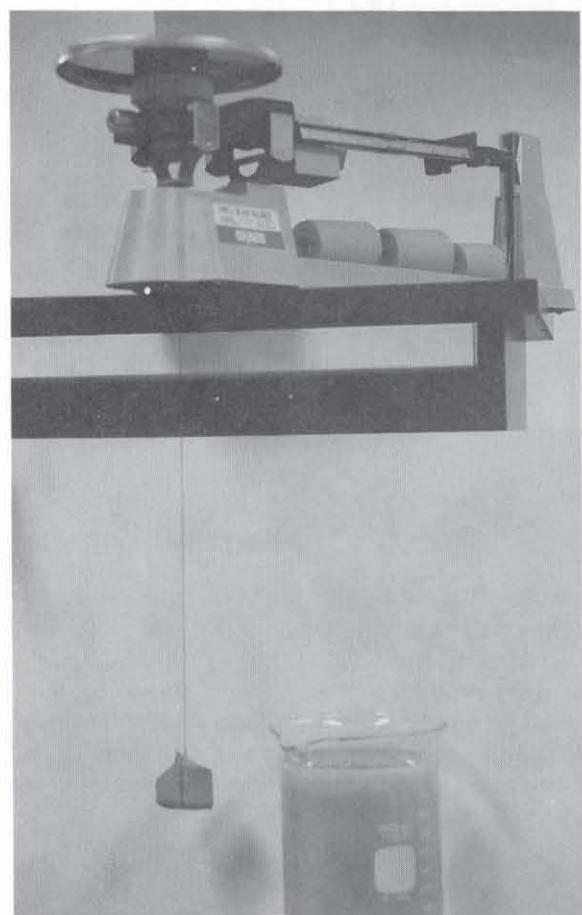
$$\gamma = \frac{W - W_{\text{ring}}}{\pi / 4 D^2 H} \quad (2)$$



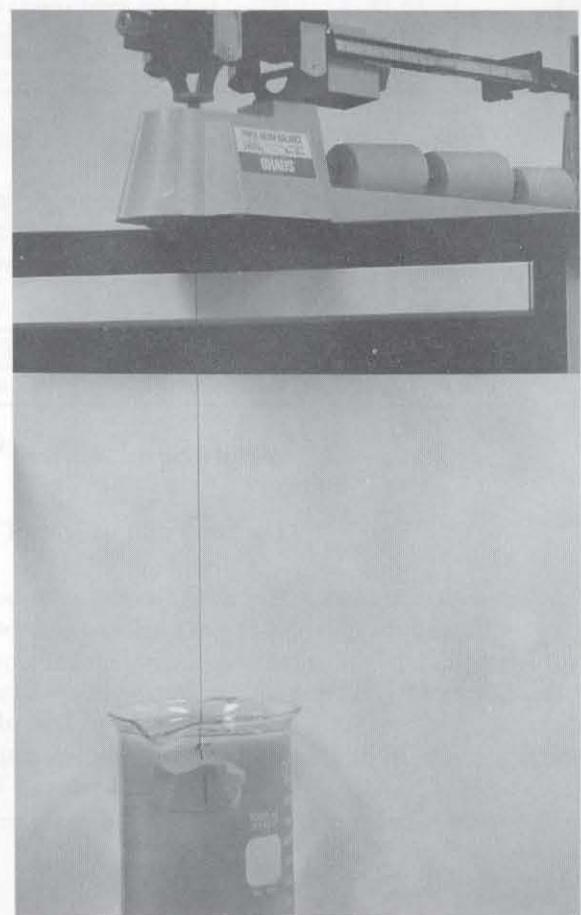
**Figure 3** Use of buoyancy to measure volume.



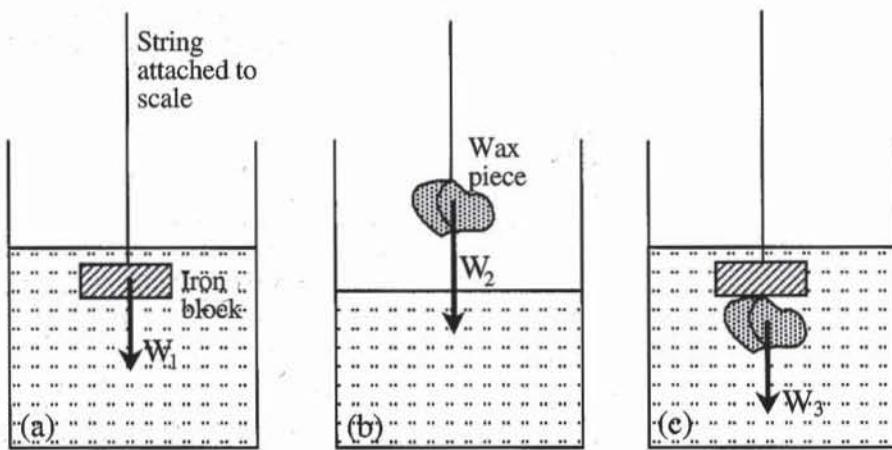
**Figure 4** The specimen is immersed in a bath of melted wax.



**Figure 5** The sample is weighed by suspending it to a fishing line attached to the balance.



**Figure 6** The waxed soil sample is immersed in water and its buoyant weight is measured.



**Figure 7** Steps to determine the unit weight of a body lighter than water.

where  $W$  is the weight of the sample and ring,  $W_{\text{ring}}$  the weight of the sampling ring,  $D$  the diameter of the ring, and  $H$  the height of the ring. The water content is

$$w = \frac{W_w - W_d}{W_d - W_{\text{dish}}} \times 100 \quad (\%) \quad (3)$$

where  $W_w$  is the weight of the soil sample and weighing dish,  $W_d$  the weight of the oven-dried soil sample and weighing dish, and  $W_{\text{dish}}$  the weight of the weighing dish. The soil dry unit weight is

$$\gamma_d = \frac{\gamma}{1 + w} \quad (4)$$

### Weighing in Water

The total unit weight of the soil is calculated as follows:

$$\gamma = \gamma_w \frac{W}{W_p - W_I - (\gamma_w/\gamma_p)(W_p - W)} \quad (5)$$

where  $W$  is the weight of the soil sample,  $W_p$  the weight of the waxed soil sample,  $W_I$  the immersed weight of the waxed soil sample,  $\gamma_w$  the water unit weight, and  $\gamma_p$  the wax unit weight. The water content and dry unit weight are calculated using Eqs. 3 and 4.

### EXAMPLE

Figure 8 shows an example for the measurement of soil unit weight by weighing in water. The formulas used in Fig. 8 are shown in Fig. 9.

	A	B	C	D
1	<b>Unit weight of soils</b>			
2	Analyst name: J.P. Bardet			
3	Test date: 11/13/90			
4	Sample description: Brown clay			
5				
6	<b>SOIL UNIT WEIGHT</b>			
7	Mass of soil sample (g) M	511.40		
8	Mass of waxed soil (g) Mp	519.50		
9	Mass of immersed soil (g) Mi	215.50		
10	Mass of trimmed sample (g) Mt	41.2		
11	Mass of dry sample (g) Md	37.1		
12	Bulk unit weight $\gamma_{sat}$ =	16.53 kN/m <sup>3</sup>		
13	Water content w =	11.05%		
14	Dry unit weight $\gamma_d$ =	14.89 kN/m <sup>3</sup>		
15				
16	<b>WAX UNIT WEIGHT</b>			
17	Mass of immersed iron block (g) Mwi	51.40	51.40	
18	Mass of wax block (g) Mwb	72.10	81.65	
19	Mass of immersed iron and wax blocks (g) Mwbi	48.50	47.56	
20	Unit weight of wax $\gamma_p$ =	9.42 kN/m <sup>3</sup>		

Figure 8 Example of data set for measurement of soil unit weight.

	A	B	C
12	Bulk unit weight $\gamma_{sat}$ = =AVERAGE(M/(Mp-Mi-(Mp-M)/B20)*9.8)	kN/m <sup>3</sup>	
13	Water content w = =AVERAGE((Mt-Md)/Md)		
14	Dry unit weight $\gamma_d$ = =gsat/(1+w)	kN/m <sup>3</sup>	
20	Unit weight of wax $\gamma_p$ = =AVERAGE(9.8*Mwb/(Mwi+Mwb-Mwbi))	kN/m <sup>3</sup>	

Figure 9 Formulas used in Fig. 8.

**REVIEW QUESTIONS**

- What is the simplest method for measuring the unit weight of cohesive soils? Under what conditions do we use weighing in water to determine unit weight?
- What precaution should you take if you must measure the unit weight of soils with many air pockets? Should you or not fill and coat these holes with wax?
- Can you apply the technique described in this chapter to measure the unit weight of sands? Why?
- Derive Eq. 1, which determines the unit weight of a sample lighter than water.

**EXERCISE**

- Calculate the unit weight of a cohesive soil sample by trimming and weighing in water, and compare the results.

## 3-3 Determination of Specific Gravity

### OBJECTIVE

The specific gravity  $G_s$  of a soil is the ratio between the unit masses of soil particles and water.  $G_s$  is useful for determining weight-volume relationships.

$$G_s = \frac{\gamma_s}{\gamma_w}$$

### EQUIPMENT

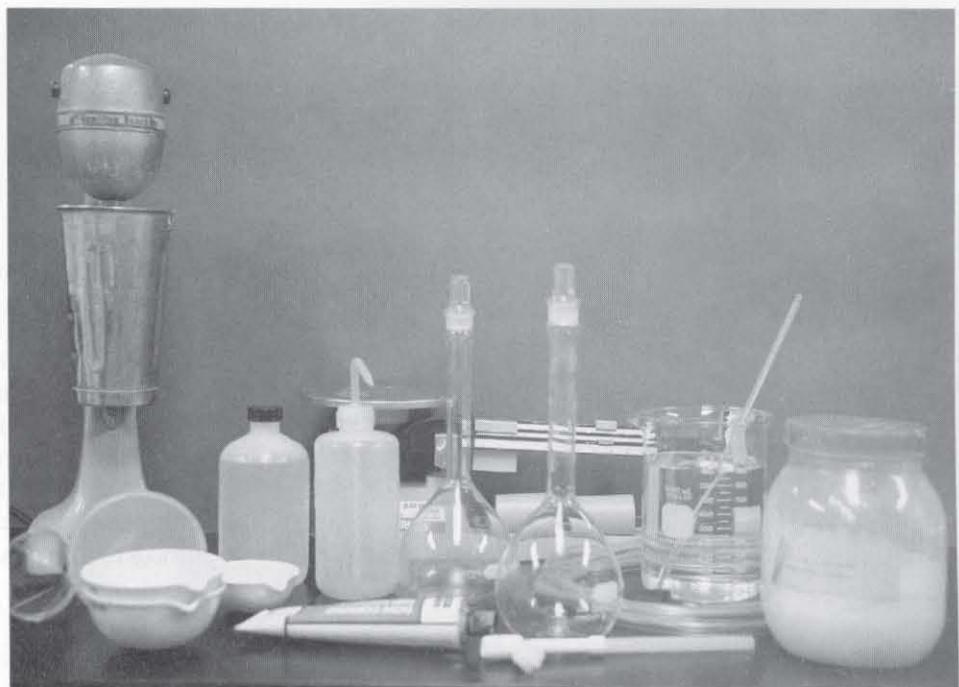
The equipment for determination of specific gravity includes:

- Volumetric flasks (250 or 500 mL) with stoppers, numbered and calibrated (Fig. 1).
- Vacuum pump.
- Balance accurate to 0.01 g.
- Distilled deaired water.
- Thermometer, ranging from 0 to 50°C, accurate to 0.5°C.
- Drying oven.
- Evaporating dish.

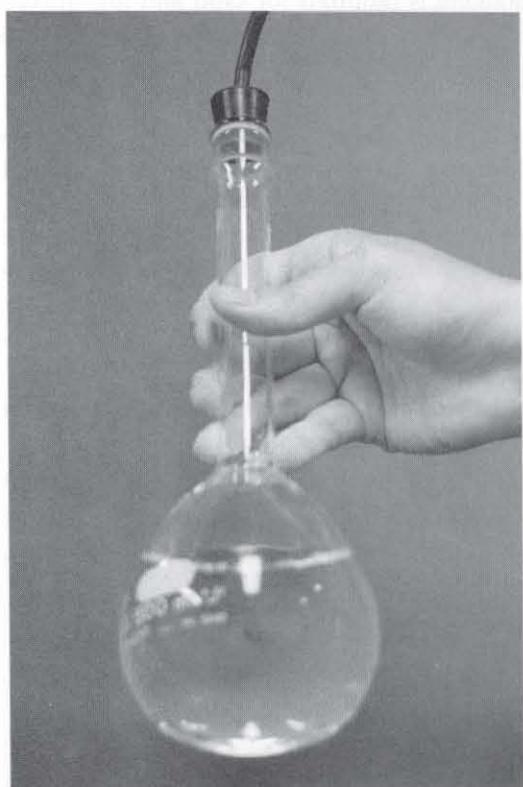
### DEAIRING WATER AND FLASK CALIBRATION

The test water must be deaired and the volumetric flask calibrated. The flask calibration does not need to be repeated before each experiment.

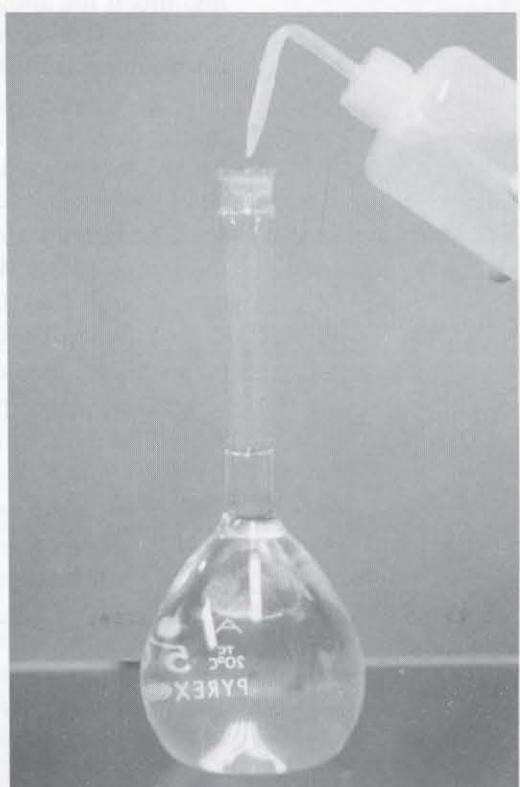
1. As shown in Fig. 2, water is deaired in the flask which is three-fourths filled by applying vacuum through the stopper. While water is deaired, bubbles appear because the reduced air pressure causes the water to boil. Deaired water should be stored in airtight bottles.



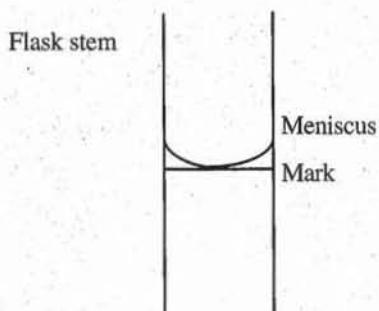
**Figure 1** Equipment for measurement of specific density. Balance sensitive to 0.01g, 500-mL volumetric flask, evaporating dish, supply of deaired water, and thermometer.



**Figure 2** Water is deaired by filling the flask about three-fourths and applying vacuum through the stopper.



**Figure 3** During the flask calibration, the flask is carefully filled with deaired water up to its mark.



**Figure 4** The volumetric flask is filled when the bottom of the water meniscus is aligned with the mark on its stem.

**2.** The volumetric flask is calibrated by measuring its weight when it is filled with deaired water. Wash the volumetric flask. As shown in Fig. 3, carefully fill it to the volume mark with deaired water, without splashing to introduce air bubbles. The flask is filled correctly when the mark on its stem is at the bottom of the water meniscus, as shown in Fig. 4. Put the stopper on the volumetric flask, and measure its weight  $W_{fw}$ . Measure the water temperature  $T$ . The bottle should be calibrated at the same temperature as during the test. Any change in water temperature modifies the flask volume.

### TEST PROCEDURE

**1.** Take a sample of 100 to 120 g of air-dried soil. For fine-grained soils, mix the sample with water in an evaporating dish to make about 200 mL of soil-water mixture. For clays, transfer the soil-water mixture into a malt mixer container and mix it for about 5 to 10 min. No soaking is required for sand and fine gravel. However, the aggregates should be broken into pieces small enough to go into the volumetric flask.

**2.** Transfer the soil-water mixture from the evaporating dish into the volumetric flask. Wash any remaining soil into the flask using a wash bottle. Add sufficient water to fill the flask two-thirds to three-fourths full. Do not fill it completely, because its contents must be agitated under vacuum.

**3.** Attach the flask to a vacuum line and for at least 10 min gently agitate the mixture while keeping it away from the flask stopper. The reduced air pressure should cause the water to boil.

**4.** When the deairing process is complete, add deaired water to fill the calibrated flask volume (see Fig. 3).

**5.** Measure the weight of the flask. Measure the water temperature, which should be close to that of the temperature of flask calibration.

**6.** Empty the flask and its contents into a deep evaporating dish and oven dry. Measure the weight of dry soil.

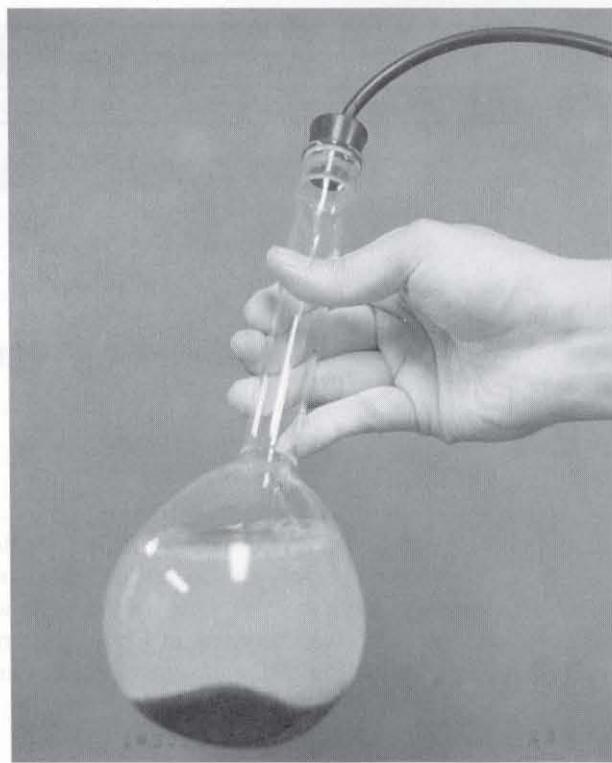
**7.** Repeat the test to calculate additional values of  $G_s$  until the values of  $G_s$  are within 2% of each other.

### COMPUTATION

The specific gravity  $G_s$  of a soil is calculated as follows:

$$G_s = \frac{W_s}{W_s + W_{fw} - W_{fs}} \quad (1)$$

where  $W_s$  is the weight of the dry soil,  $W_{fs}$  the weight of the flask filled with soil and water, and  $W_{fw}$  the weight of the flask filled with deaired water only.



**Figure 5** The flask is attached to a vacuum line, and its mixture is gently agitated by turning the flask for at least 10 min.

### EXAMPLE

Figure 6 shows an example of a data set for the specific gravity of soils. The formulas used in Fig. 6 are shown in Fig. 7.

	A	B	C
1	<b>Specific Gravity</b>		
2	Analyst name: <i>J.E. Bowles</i>		
3	Test date: 11/13/90		
4	Sample description:		
5			
6		Sample 1	Sample 2
7	Mass of flask and water (g) $M_{fw}$	693.27	693.27
8	Mass of flask, soil and water (g) $M_{fs}$	753.66	754.69
9	Mass of evaporating dish (g) $M_c$	254.52	270.52
10	Mass of evaporating dish and dry soil (g) $M_d$	350.11	368.49
11	Specific Gravity $G_s$	2.72	2.68
12	Average Specific Gravity $G_s =$	2.70	

**Figure 6** Example of data set for measurement of specific gravity.

	A	B	C
11	Specific Gravity $G_s = (Md-Mc)/(Md-Mc+Mfw-Mfs)$	$= (Md-Mc)/(Md-Mc+Mfw-Mfs)$	
12	Average Specific Gravity $G_s = \text{AVERAGE}(G_s)$		

**Figure 7** Formulas used in Fig. 6.**REFERENCE**

See Introduction for references to ASTM procedures (pages 4 to 6).

**REVIEW QUESTIONS**

1. Define *specific gravity of soil*.
2. What are typical values of the specific gravity for soils?
3. Can you apply the technique described in this section to measure the specific gravity of material lighter than water?
4. Why do we use vacuum while determining the specific gravity of soils?
5. What is the effect of water temperature on the determination of the specific gravity of soils?

# **3-4 The Principles of Compaction**

## **INTRODUCTION**

For many civil engineering projects, soils have to be compacted to a denser state to improve their engineering properties. They are compacted by mechanical means with rolling, ramming, or vibrating equipment. The soil density to be obtained by field compaction is defined by means of two basic laboratory compaction tests: the standard and modified compaction tests. In 1933, Proctor introduced a laboratory test to control soil compaction, which later became known as the standard Proctor compaction test. The other compaction test, the modified AASHTO test, was later introduced to simulate the compaction of heavy equipment, which produces higher compaction energy.

## **DEFINITIONS**

Soil compaction consists of closely packing the soil particles together by mechanical means, thus increasing the soil dry unit weight. As illustrated in Fig. 1a, soils are made of solid grains with voids filled with air and water. The proportion of solid, liquid, and air is represented in Fig. 1b. As shown in Fig. 1c, compaction only reduces the air fraction. It barely changes the water content and has no effect on the solid volume. In theory, the most efficient compaction process should remove the air fraction completely. However, in practice, compaction cannot completely eliminate the air fraction, but only reduces it to a minimum, provided that appropriate techniques are used.

Compaction should not be confused with consolidation, which corresponds to the drainage of water from soils subjected to static loads. In most soils, compaction is too rapid to allow time for drainage.

As illustrated in the typical compaction curve of Fig. 2, water has an important effect on soil compaction. Even at low water content, the soil grains are surrounded by a thin film of water. A small increase in water content tends to increase the repulsion of particles and to facilitate their orderly arrangement. Un-

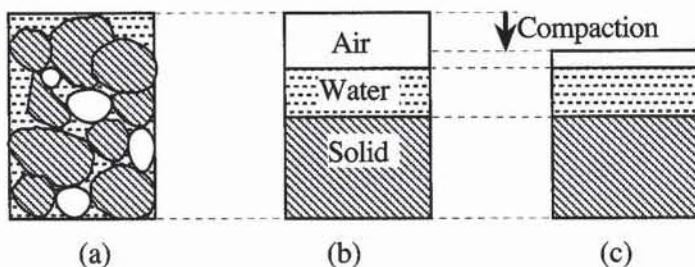


Figure 1 Principle of soil compaction.

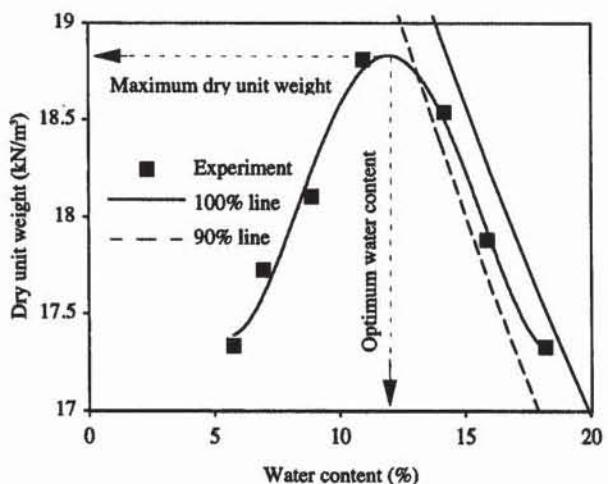


Figure 2 Typical compaction curve (after Lambe, 1951).

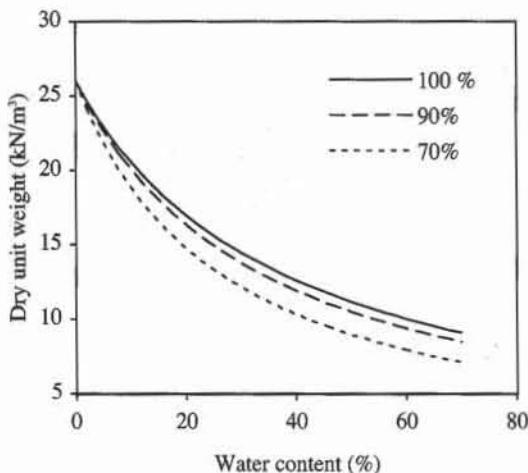
til the optimum water content is reached, the addition of water expells more air from soils, and enables to reach larger dry unit weight. The densest soil is obtained at the optimum water content. When the water content exceeds this optimum value, the water pushes the grains apart. Since water is much more incompressible than the grain assembly and has no time to drain, the dry unit weight starts to decrease.

### Saturation Lines

The dry unit weight  $\gamma_d$ , total unit weight  $\gamma$ , water content  $w$ , degree of saturation  $S_r$ , and specific gravity  $G_s$  are related through

$$\gamma_d = \frac{\gamma}{1+w} = \gamma_w \frac{G_s}{1+G_s w/S_r} \quad (1)$$

When  $S_r$ ,  $\gamma_w$ , and  $G_s$  are given, Eq. 1 defines the saturation lines. Figure 3 shows the saturation lines corresponding to  $S_r = 100\%$ ,  $90\%$  and  $70\%$  for  $G_s = 2.65$  and  $\gamma_w = 9.8 \text{ kN/m}^3$ .  $\gamma_d$  decreases with  $w$  but increases with  $S_r$ . Because  $S_r \leq 100\%$ , all measured points  $(w, \gamma_d)$  must be below the  $100\%$  saturation line. The  $100\%$  saturation line defines the upper limit of compaction curves. No data point can be beyond this line. As shown in Fig. 2, the descending branch of the measured compaction curve falls between the  $100\%$  and  $90\%$  saturation lines. The dry unit weight decreases with water content because water filled the soil voids before drying. The  $100\%$  saturation line is commonly plotted next to compaction curves while reporting the result of compaction tests.



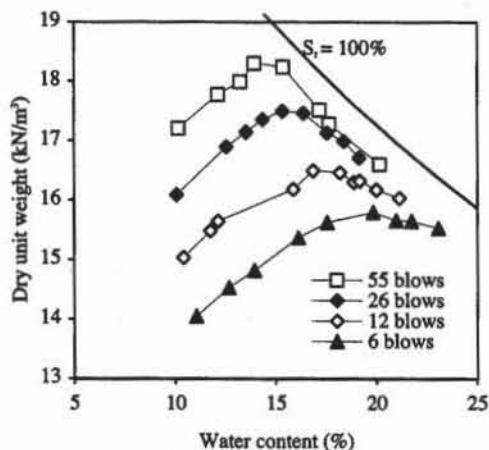
**Figure 3** Variation of dry unit weight versus water content for constant degree of saturation  $S_r = 100\%$ , 90%, and 70%.

### Compacting Efforts

The methods used for various standard types of compaction tests are summarized in Table 1. The compaction methods vary depending on the rammer weight, rammer drop, size and height of mold, and the number of layers and blows per layer. The work done  $E$  by the rammer per unit volume of soil is

$$E = W_r \frac{H}{V} N_B N_L \quad (2)$$

where  $W_r$  is the rammer weight,  $H$  the rammer drop,  $V$  the volume of compacted soil,  $N_B$  the number of blows per layer, and  $N_L$  the number of layers. As shown in Table 1, the work done  $E$  in the modified compaction test is about 4.5 times as much as that in the standard test. According to ASTM the larger mold is used for coarser soils having particles larger than 9.5 mm but smaller than 19 mm. As shown in Fig. 4, the relation between density and water content is influenced by the compactive



**Figure 4** Compaction curves for a silty clay subjected to different numbers of blows per layer in modified compaction test (after Turnbull and Foster, 1956).

**TABLE 1**

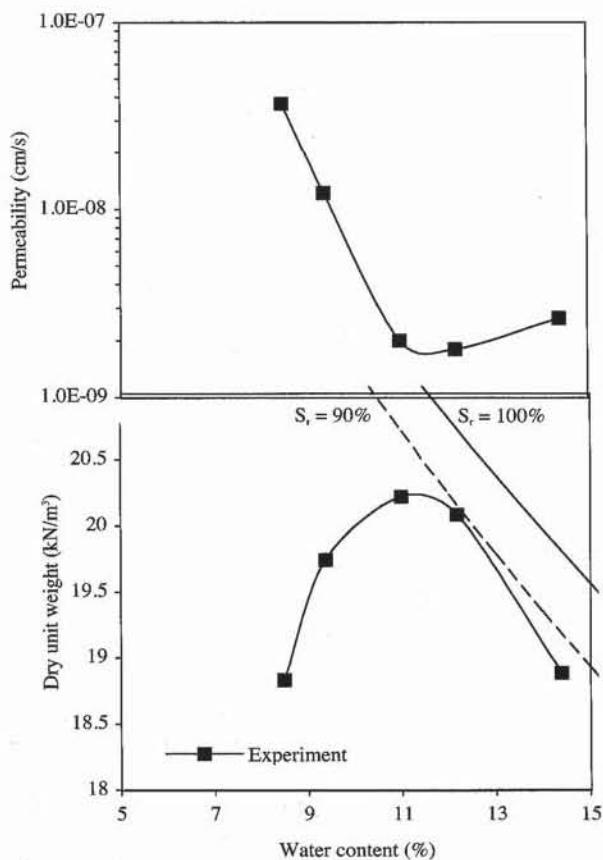
Summary of compaction procedures (ASTM D698 and D1557)

Type of test	$W_r$ Rammer weight (N)	H Hammer drop (mm)	$N_L$ Number of layers	$N_B$ Number of blows per Layer	D Diameter of mold (cm)	L Height of mold (cm)	E Work done per unit volume of soil (kJ/m <sup>3</sup> )
Standard compaction	24.4	305	3	25	10.2	11.6	592
ASTM D 698	24.4	305	3	56	15.2	11.6	589
Modified compaction	44.5	457	5	25	10.2	11.6	2695
ASTM D 1557	44.5	457	5	56	15.2	11.6	2683

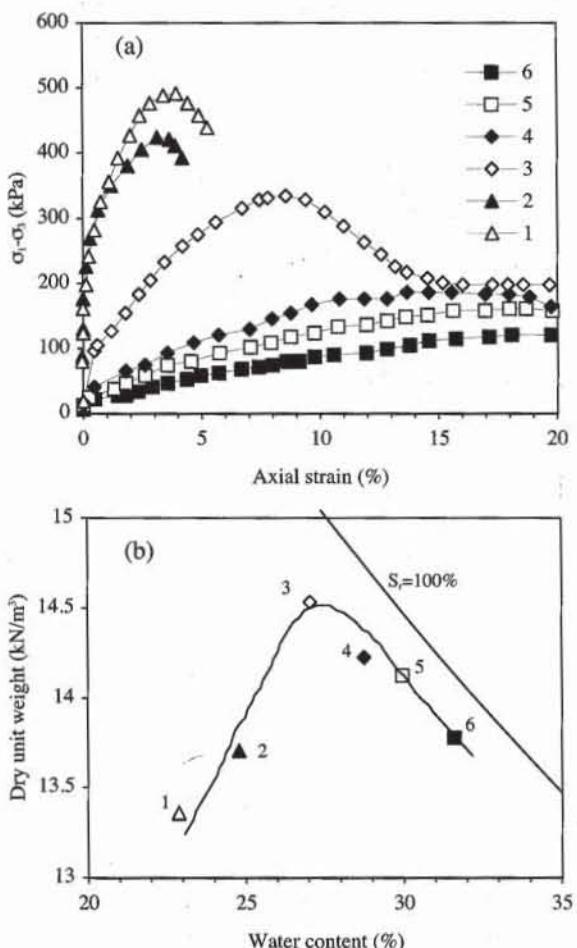
effort, higher compactive efforts giving denser soils. Modified compaction tests yield denser soils than standard compaction tests.

### Influence of soil compaction on soil properties

The nature and magnitude of compaction in fine-grained soils significantly influences their mechanical behavior. A few effects are illustrated in Figs. 5 and 6. For additional information refer to Mitchell et al. (1965), and Hilf (1991).



**Figure 5** Compaction-permeability tests on Siburia clay (Lambe, 1962).



**Figure 6** Influence of water content on the stress-strain relationship for compacted samples of kaolinite (a) stress versus strain relationships for compacted samples, (b) dry unit weight versus water content (from Seed and Chan, 1959).

As shown in Fig. 5, an increase in water content  $w$  during compaction causes a decrease in permeability  $k$  when  $w$  is smaller than the optimum water content  $w_{opt}$ , and a slight increase in  $k$  when  $w > w_{opt}$ . Compaction modifies the permeability by decreasing the voids available for flow, and reorienting soil particles.

Fig. 6 shows the influence of compaction water content  $w$  on the stress-strain response of compacted samples of kaolinite subjected to triaxial tests (see Chapter 7-6). Samples compacted with  $w < w_{opt}$  tend to be more rigid and stronger than samples compacted with  $w > w_{opt}$ .

As pointed by Lambe and Whitman (1969), the engineer must consider the behavior of the soil not only as compacted. Many changes can occur in the compacted soil, such as changes in saturation due to permeating water, which ultimately determine its shear strength and compressibility.

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**REVIEW QUESTIONS**

1. What is the compaction of soils? How is it different from consolidation? Illustrate your answer with a diagram showing the variation of air, water and soil fractions of soil samples during compaction and during consolidation.
2. Who introduced the standard compaction test?
3. What is the main difference between standard and modified compaction tests?
4. What is a compaction curve?
5. What is the 100% saturation line? How is it related to compaction curves?
6. What is the influence of compactive effort on compaction curve?
7. Why do we compact soils in civil engineering?

**EXERCISES**

1. Find the maximum dry unit weight and optimum water content for the compaction test results of Table E1 (data from Lambe, 1962). Plot the data points and the 100% and 90% saturation lines ( $G_s = 2.65$  and  $\gamma_w = 9.8 \text{ kN/m}^3$ ).

**TABLE E1**

Water content (%)	Dry unit weight ( $\text{kN/m}^3$ )
5.75	17.34
6.95	17.72
8.90	18.10
10.97	18.81
14.19	18.54
15.90	17.88
18.21	17.33

2. Find the maximum dry unit weight and optimum water content for the compaction test results of Table E2 on Siburia clay (data from Lambe, 1962). Plot the data points and the 100% and 90% saturation lines ( $G_s = 2.85$  and  $\gamma_w = 9.8 \text{ kN/m}^3$ ). Plot the variation of permeability versus water content, and comment on the effect of compaction on permeability.

**TABLE E2**

Water content (%)	Dry unit weight (kN/m <sup>3</sup> )	Water content (%)	Permeability (cm/s)
8.47	18.83	8.27	$3.7 \times 10^{-8}$
9.37	19.74	9.17	$1.2 \times 10^{-8}$
10.99	20.22	10.98	$2.0 \times 10^{-9}$
11.46	20.23	12.39	$1.8 \times 10^{-9}$
11.85	20.08	14.34	$2.6 \times 10^{-9}$
12.17	20.08		
13.32	19.74		
13.93	19.03		
14.22	19.03		
14.40	18.88		

3. Same as Exercise 1 for the results of Table E3.

**TABLE E3**

Water content (%)	Dry unit weight (kN/m <sup>3</sup> )
22.87	13.36
24.78	13.70
27.04	14.53
28.75	14.22
29.96	14.12
31.62	13.78

4. Find the maximum dry unit weight and optimum water content for the results of four different compaction tests in Table E4. Plot the data points on the same graph, and draw the 100% saturation line ( $G_s = 2.65$  and  $\gamma_w = 9.8 \text{ kN/m}^3$ ).

**TABLE E4**

	Water content (%)	Dry unit weight (kN/m <sup>3</sup> )
Modified 55 blows per layer	10.19	17.20
	12.14	17.77
	13.25	17.98
	13.95	18.30
	15.39	18.24
	17.20	17.52
	17.66	17.28
	20.17	16.60
Modified 26 blows per layer	10.09	16.09
	12.55	16.89
	13.53	17.14
	14.37	17.36
	15.34	17.50
	16.41	17.46
	17.57	17.13
	18.41	16.99

**TABLE E4 (CONT.)**

	Water content (%)	Dry unit weight (kN/m <sup>3</sup> )
	19.15	16.71
Modified	10.42	15.04
6 blows per layer	11.76	15.48
	12.14	15.64
	15.90	16.19
	16.87	16.50
	18.22	16.46
	18.92	16.30
	19.20	16.32
	20.03	16.17
	21.15	16.03
Modified	11.07	14.04
12 blows per layer	12.69	14.53
	13.95	14.82
	16.13	15.37
	17.57	15.62
	19.85	15.80
	21.01	15.66
	21.75	15.64
	23.10	15.52

# 3-5 Compaction Tests

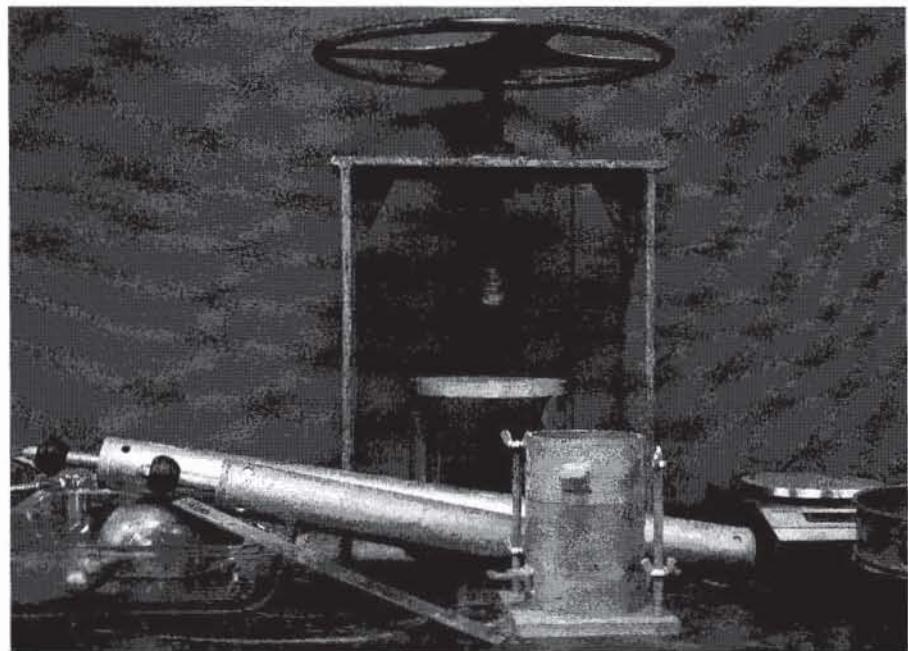
## OBJECTIVE

Laboratory compaction tests are used to determine the relation between water content and dry unit weight and to find the maximum dry unit weight and optimum water content.

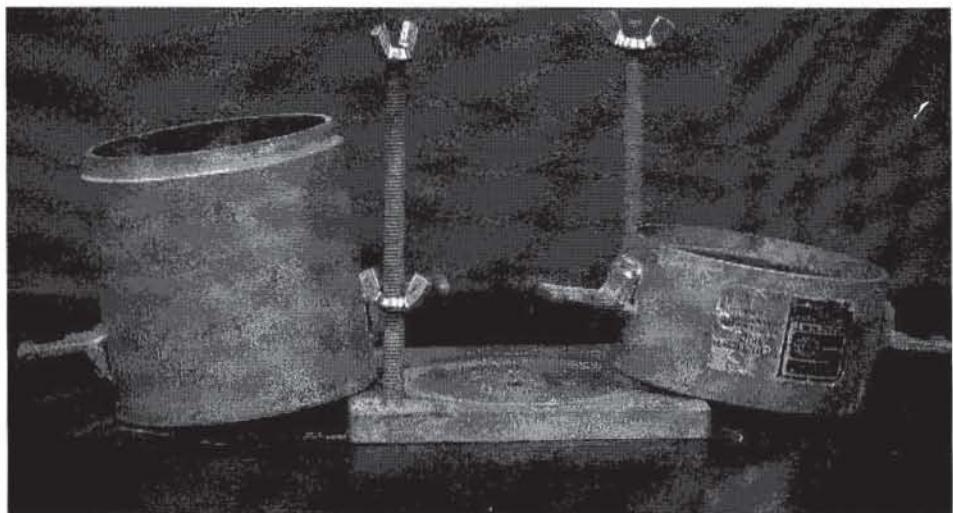
## EQUIPMENT

The equipment used in compaction tests includes:

- Cylindrical metal mold, internal dimension 105 mm in diameter and 115 mm high (volume 1000 cm<sup>3</sup>). The mold is fitted with a detachable base plate and a removable extension collar (Figs. 1 to 3). A split mold (Fig. 4) may be used when an extractor is not available.
- For the standard compaction test, metal rammer with 50-mm-diameter face, weighing 24.4 kN, sliding freely in a tube that controls the height of drop to 300 mm (Fig. 5). For the modified compaction test, the rammer weight is 44.5 kN and the height of drop is 460 mm.
- Extractor apparatus for removing compacted material from the mold (see Figs. 11 and 12).
- Scoop or trowel.
- Steel straightedge, 30 cm long.
- No. 4 sieve.
- Balance, 10 kg capacity, accurate to 1.0 g.
- Drying oven, and evaporating dishes for moisture content determination.
- Ruler and vernier caliper.



**Figure 1** Equipment for compaction test. Cylindrical mold, rammer, scoop, steel straightedge, 10 kg capacity scale, and extractor.



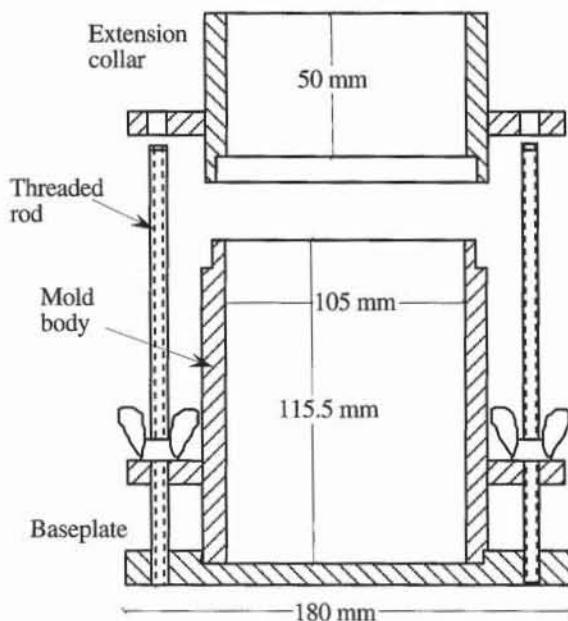
**Figure 2** Base, mold body, and extension collar of cylindrical mold for compaction test.

## PREPARATION OF EQUIPMENT

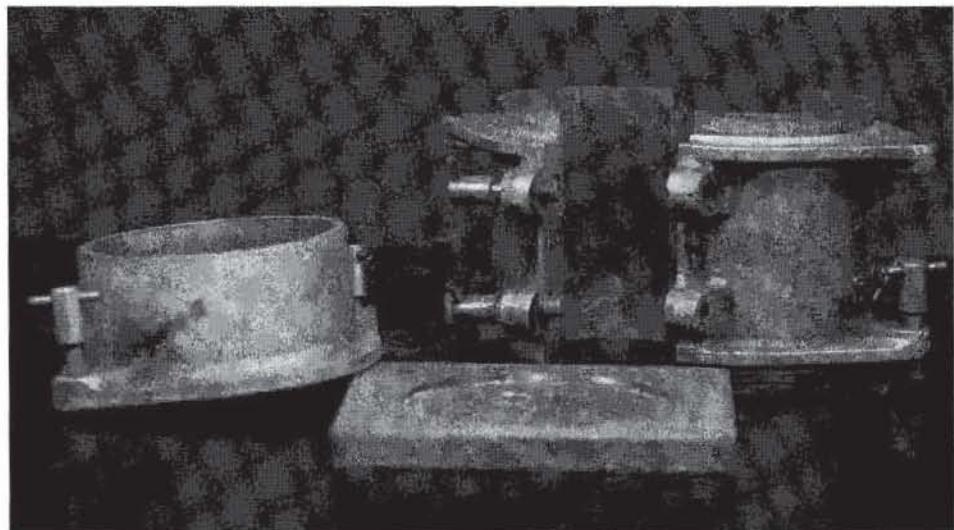
The exact volume of the standard compaction mold is measured before the compaction test. Clean and dry the mold, extension collar, and base plate. Weigh the mold body without the base plate and extension collar. Measure the internal diameter  $D$  and height  $H$  of the mold body. The mold volume  $V$  is

$$V = \frac{\pi}{4} D^2 H \quad (1)$$

Check the rammer to ensure that it falls freely through the correct height of drop (Table 1 of Chapter 3-4).



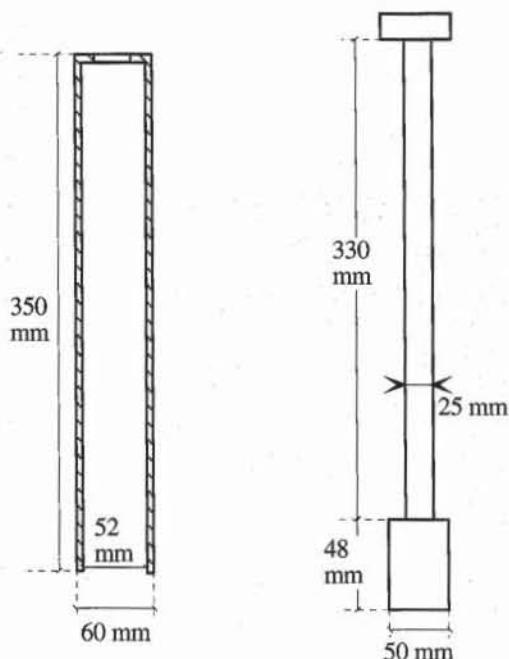
**Figure 3** Dimensions and parts of a standard compaction mold.



**Figure 4** Split mold for compaction test. In contrast to the standard mold, the split mold can be split open to remove the compacted soil sample, and does not require the use of an extruding device.

## PREPARATION OF SOIL SAMPLE

The original bulk sample is air dried and weighed. The large particles are removed by passing the sample through a No. 4 sieve. The mass of material required for the test is about 3 kg when the same soil is used in all the test points. ASTM recommends using a fresh soil sample for each test point, which requires about 15 kg of soils for five test points.



**Figure 5** Rammer for standard compaction test.

## TEST PROCEDURE

### Standard compaction test

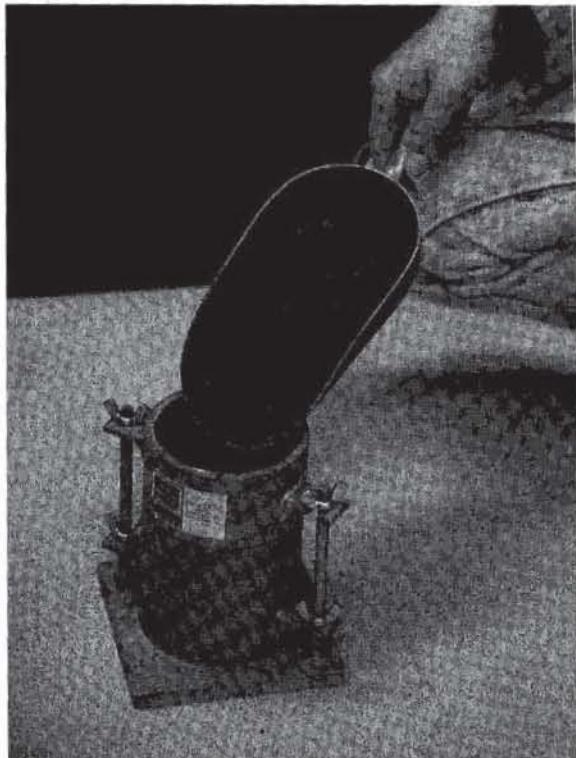
1. To obtain five well-placed points on the compaction curve, the water content is selected low for the first test point and is gradually increased for the other points. It should be about 4 to 5% below the optimum water content for the first point, and 4 to 5% above the optimum water content for the fifth and last point.
2. Add a suitable amount of water and mix thoroughly. Thorough mixing of soil and water is essential. The weight of water  $W_w$  to be added to achieve the water content  $w$  in percent may be estimated as follows:

$$W_w = \frac{W_s(w - w_0)}{100} \quad (2)$$

where  $w_0$  is the previous water content (%) and  $W_s$  is the weight of dry soil.

3. Place the mold assembly on a solid base such as a concrete floor. Add loose soil to the mold so that it is about half full (Fig. 6). Compact the soil by applying 25 blows of the rammer dropped from the controlled height of 300 mm (Fig. 7). The rammer should be positioned properly before releasing. The guide tube must be held vertically. Place the tube gently on the soil surface; the rammer does the compaction, not the tube. To avoid injury, the hand that holds the tube must be kept clear from the falling hammer. As shown in Fig. 8, the rammer should be positioned to evenly distribute the compaction energy into the soil.

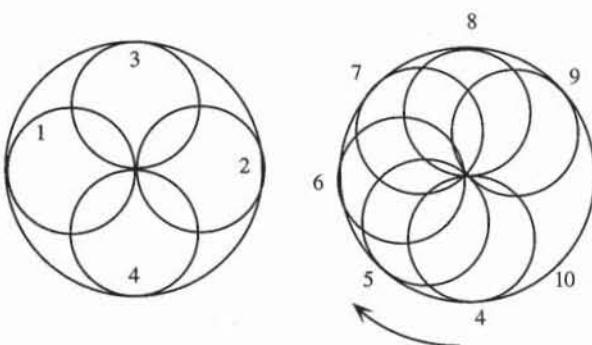
4. Place a second, approximately equal layer of soil in the mold, and compact it with 25 blows as before. Repeat with a third layer, which should bring the



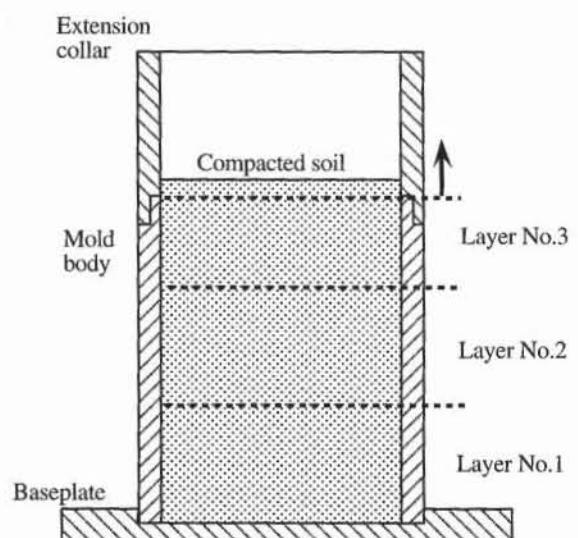
**Figure 6** The compaction mold is half-filled with loose soil.



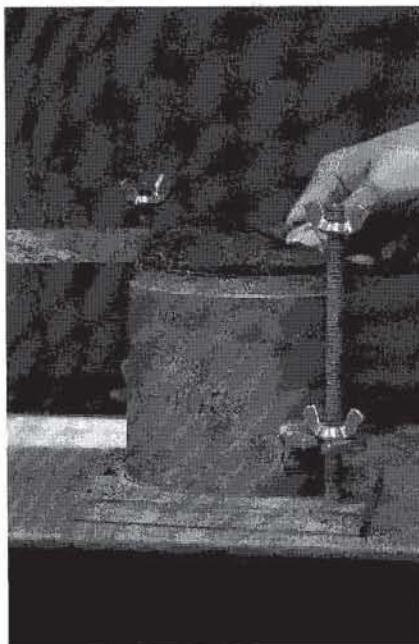
**Figure 7** The soil is compacted by applying 25 blows with the rammer.



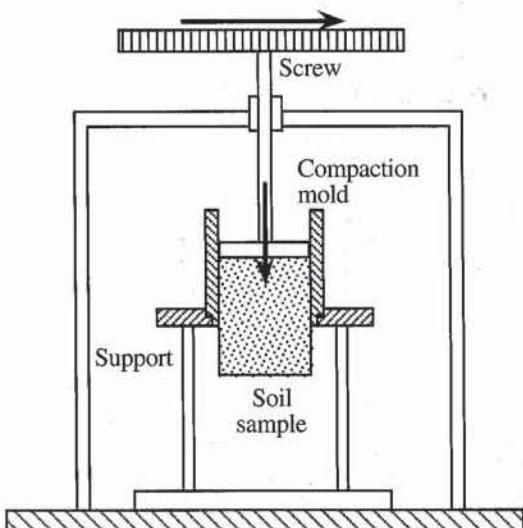
**Figure 8** Sequence of blows using hand rammer.



**Figure 9** At the end of the compaction phases, the compacted soil level should be just above the mold body.



**Figure 10** The excess soil is cut away by leveling off the top of the mold.



**Figure 11** Extractor for removing compacted soil samples.

compacted soil level in the extension collar to about 6 mm above the level of the mold body, as shown in Fig. 9. If the compacted soil level in the extension collar is much higher, the test becomes inaccurate because the compacting energy per unit volume of soil is no longer constant.

5. Remove the extension collar carefully. Cut away the excess soil and level off to the top of the mold (Fig. 10). Any small cavity resulting from the removal of stones should be filled with fine materials.
6. Remove the base plate carefully, and weigh soil and mold.
7. Fit the mold on the extractor and extract the soil from the mold (Figs. 11 and 12).
8. Immediately take up to three representative samples to determine the sample water content (Fig. 13).
9. Break up the material on the tray and add an increment of water to achieve a desirable water content (refer to Eq. 2).
10. Go to step 2 and repeat to obtain five compaction points.

#### Modified compaction test

Follow the same procedure as the standard compaction test, but use the heavier rammer (44.5 kN instead of 24.4 kN) with a larger height of drop (457 mm instead of 305 mm). Also compact the soil in 5 layers (instead of 3) by applying 56 blows per layer (instead of 25).

#### COMPUTATION

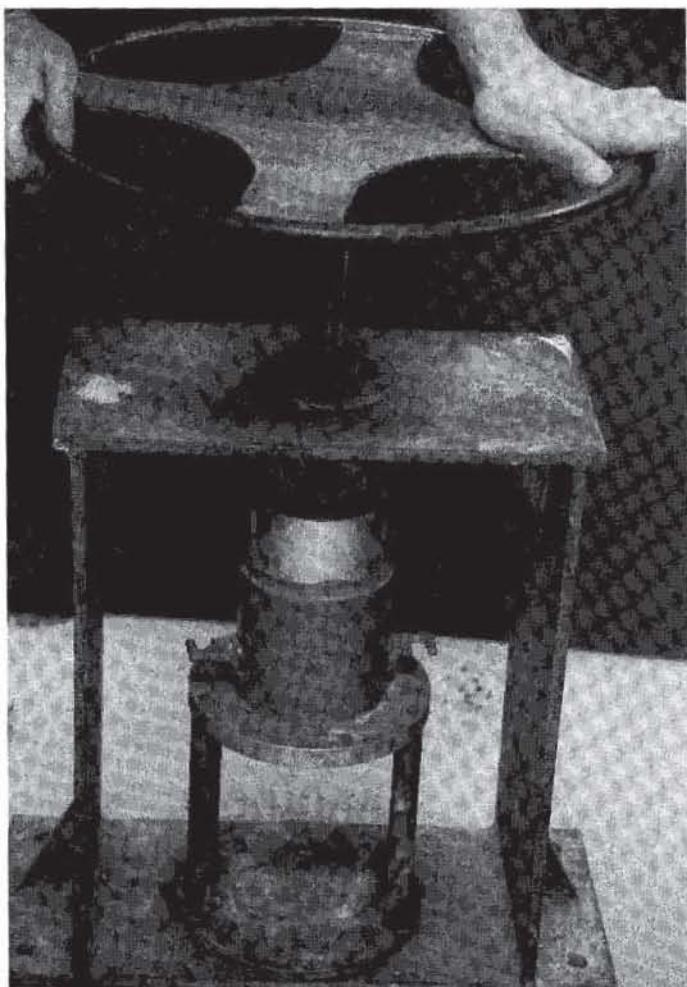
The bulk unit weight  $\gamma$  is calculated as follows:

$$\gamma = \frac{W - W_m}{V} \quad (3)$$

where  $W$  is the weight of the soil and mold,  $W_m$  the weight of the empty mold, and  $V$  the volume of the mold. The dry unit weight  $\gamma_d$  is

$$\gamma_d = \frac{\gamma}{1 + w/100} \quad (4)$$

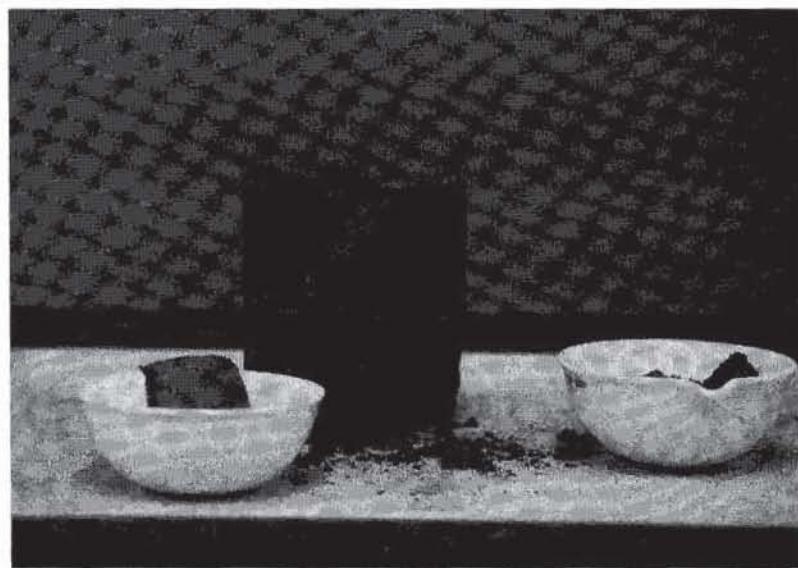
where  $w$  is the water content (%).



**Figure 12** The compacted soil may become very hard; it is removed from the compaction mold by using an extractor.

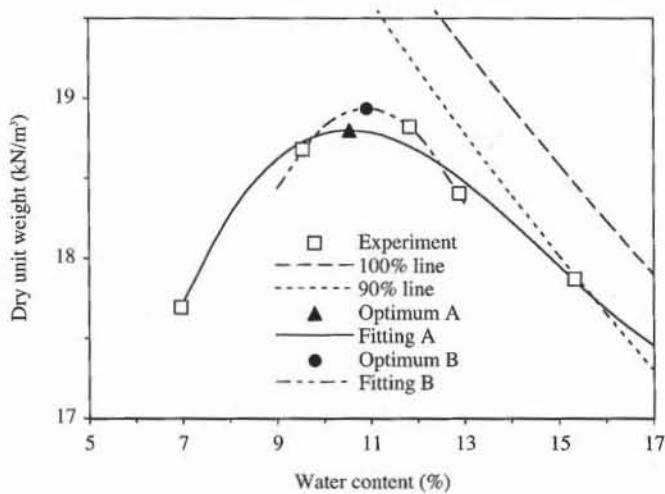
### EXAMPLE

Figures 14 and 15 show an example of a compaction test. The results are presented in the form of a compaction curve: dry unit weight  $\gamma_d$  versus water content  $w$  (%). This curve is obtained by plotting the data points for each compacted sample and connecting these points by a smooth curve. The 100% and 90% saturation lines are also plotted next to the compaction curve. They are obtained by using Eq. 1 of Chapter 3-4 for  $G_s = 2.65$ . All the experimental data points fall under the 100% saturation line, and some fall on the 90% saturation line. Figure 16 shows the formulas used in Figs. 14 and 15, and Fig. 17 shows the user-defined functions used in Fig. 15. These user-defined functions use functions FIT2 and FIT3, which are defined in Chapter 8-1. FIT2



**Figure 13** Take up to three representative samples to determine the moisture content of the sample.

and FIT3 perform a quadratic and a polynomial regression, respectively. The maximum unit weight and optimum water content are determined by using two different methods, referred to as A and B. Method A uses the user-defined function OPTIMUM3, which returns the maximum dry unit weight and optimum water content by fitting all data points with the cubic polynomial regression of function FIT3. The optimum of method A is plotted as a solid triangle in Fig. 14. Method B uses the user-defined function OPTIMUM2, which returns the maximum dry unit weight and optimum water content by fitting only three data points with the quadratic polynomial regression of function FIT2. The three data points are selected close to the maximum. The optimum of method B is plotted as a solid circle. Method B is equivalent to the one specified by ASTM D 5080. In the example of Fig. 14, method B probably gives a better optimum than method A.



**Figure 14** Example of compaction curve with 100% and 90% saturation lines.

	A	B	C	D	E
1	Compaction test				
2	Analyst name: K. Tiel				
3	Test date: 3/1/1993				
4	Sample description: Kaprielian soil				
5					
6	Diameter of mold d =	10.14	cm		
7	Height of mold h =	11.67	cm		
8	Mass of mold M <sub>m</sub> =	4250.00	g		
9	Specific gravity G <sub>s</sub> =	2.65			
10					
11	Mass of soil and mold (g)	Mass of can and wet soil (g) M <sub>w</sub>	Mass of can and dry soil (g) M <sub>d</sub>	Mass of can (g) M <sub>c</sub>	Water content (%)
12					
13	6070.00	81.42	79.18	46.58	6.87
14		95.56	92.19	45.50	7.22
15		90.20	87.49	47.44	6.77
16	6274.00	185.76	177.10	104.90	11.99
17		171.66	164.54	104.20	11.80
18		165.30	156.63	82.46	11.69
19	6218.00	94.21	90.07	46.58	9.52
20		106.93	101.50	45.50	9.70
21		85.04	81.80	47.44	9.43
22	6248.00	169.50	162.56	104.90	12.04
23		214.50	201.79	104.20	13.02
24		163.45	153.75	82.46	13.61
25	6232.00	82.80	78.27	46.58	14.29
26		90.33	84.22	45.50	15.78
27		85.45	80.23	47.44	15.92
28					
29	Mass of soil and mold (g)	Average water content (%)	Dry unit weight (kN/m <sup>3</sup> )		
30	M	w	g <sub>d</sub>		
31	6070.00	6.95	17.70		
32	6274.00	11.83	18.82		
33	6218.00	9.55	18.68		
34	6248.00	12.89	18.40		
35	6232.00	15.33	17.87		
36				Method A	Method B
37	Optimum moisture (%) =			10.55	10.91
38	Maximum dry unit weight (kN/m <sup>3</sup> ) =			18.80	18.93

Figure 15 Example of data set.

**REVIEW QUESTIONS**

- How many data points are necessary to construct a standard compaction curve?
- Why is it important for the final level of compacted soil to be just above the mold body?
- How do you select the water content for the five samples in the compaction test?

		B	C
		Average water content (%)	Dry unit weight ( $\text{kN/m}^3$ )
		W	$g_d$
11	Water content (%)		
12			
13	$=(\text{Mw}-\text{Md})/(\text{Md}-\text{Mc}) * 100$		
14	$=(\text{Mw}-\text{Md})/(\text{Md}-\text{Mc}) * 100$		
29			
30			
31	$=\text{AVERAGE}(\text{E}13:\text{E}15)$	$=(\text{M}-\text{Mm})/(\text{PI})*\text{d}^2/4*\text{h})/(1+\text{w}/100)*9.8$	
32	$=\text{AVERAGE}(\text{E}16:\text{E}18)$	$=(\text{M}-\text{Mm})/(\text{PI})*\text{d}^2/4*\text{h})/(1+\text{w}/100)*9.8$	
33	$=\text{AVERAGE}(\text{E}19:\text{E}21)$	$=(\text{M}-\text{Mm})/(\text{PI})*\text{d}^2/4*\text{h})/(1+\text{w}/100)*9.8$	
34	$=\text{AVERAGE}(\text{E}22:\text{E}24)$	$=(\text{M}-\text{Mm})/(\text{PI})*\text{d}^2/4*\text{h})/(1+\text{w}/100)*9.8$	
35	$=\text{AVERAGE}(\text{E}25:\text{E}27)$	$=(\text{M}-\text{Mm})/(\text{PI})*\text{d}^2/4*\text{h})/(1+\text{w}/100)*9.8$	
36	C	D	E
37	Optimum moisture (%) =	$=\text{OPTIMUM}3(w,gd)$	$=\text{OPTIMUM}2(\text{B}32:\text{B}34,\text{C}32:\text{C}34)$
38	Maximum dry unit weight ( $\text{kN/m}^3$ ) =	$=\text{OPTIMUM}3(w,gd)$	$=\text{OPTIMUM}2(\text{B}32:\text{B}34,\text{C}32:\text{C}34)$

Figure 16 Formulas used in Fig. 15.

		A	B
1	<b>OPTIMUM3</b>		Optimum water content and Maximum unit weight with a cubic fitting
2	$=\text{RESULT}(64)$		A0
3	$=\text{ARGUMENT}("W",64)$		5.15634642049554
4	$=\text{ARGUMENT}("GD",64)$		
5	$=\text{SET.VALUE}(\text{B}5:\text{E}5,\text{TRANSPOSE}(\text{FIT}3(W, \text{GD})))$		
6	$=(-D5+\text{SQRT}(D5^2-3*E5*C5))/3/E5$		
7	$=(-D5-\text{SQRT}(D5^2-3*E5*C5))/3/E5$		
8	$=\text{IF}(\text{AND}(A6>=\text{MIN}(W), W<=\text{MAX}(W)), \text{SET.VALUE}(B9,A6))$		
9	$=\text{IF}(\text{AND}(A7>=\text{MIN}(W), W<=\text{MAX}(W)), \text{SET.VALUE}(B9,A7))$		10.5521567377323
10	$=\text{SET.VALUE}(B10,E5*B9^3+D5*B9^2+C5*B9+B5)$		18.7974176689729
11	$=\text{RETURN}(B9:B10)$		
12			
13	<b>GDFIT3</b>		Unit weight corresponding to water content with a cubic fitting
14	$=\text{RESULT}(1)$		A0
15	$=\text{ARGUMENT}("Wnew",1)$		5.15634642049554
16	$=\text{ARGUMENT}("W",64)$		
17	$=\text{ARGUMENT}("GD",64)$		
18	$=\text{SET.VALUE}(\text{B}18:\text{E}18,\text{TRANSPOSE}(\text{FIT}3(W, \text{GD})))$		
19	$=\text{RETURN}(E18*Wnew^3+D18*Wnew^2+C18*Wnew+B18)$		
20			
21	<b>OPTIMUM2</b>		Optimum water content and Maximum unit weight with a quadratic fitting
22	$=\text{RESULT}(64)$		A0
23	$=\text{ARGUMENT}("W",64)$		2.74895896163071
24	$=\text{ARGUMENT}("GD",64)$		
25	$=\text{SET.VALUE}(\text{B}25:\text{D}25,\text{TRANSPOSE}(\text{FIT}2(W, \text{GD})))$		
26	$=-C25/2/D25$		
27	$=D25*A26^2+C25*A26+B25$		
28	$=\text{RETURN}(A26:A27)$		
29			
30	<b>GDFIT2</b>		Unit weight corresponding to water content with a quadratic fitting
31	$=\text{RESULT}(1)$		A0
32	$=\text{ARGUMENT}("Wnew",1)$		2.74895896163071
33	$=\text{ARGUMENT}("W",64)$		
34	$=\text{ARGUMENT}("GD",64)$		
35	$=\text{SET.VALUE}(\text{B}35:\text{D}35,\text{TRANSPOSE}(\text{FIT}2(W, \text{GD})))$		
36	$=\text{RETURN}(D35*Wnew^2+C35*Wnew+B35)$		

Figure 17 User-defined functions OPTIMUM3 and GDFIT3 for cubic fitting, and OPTIMUM2 and GDFIT2 for quadratic fitting.

4. Will you obtain the same optimum water content and maximum density for the standard and modified compaction tests? How would you expect the values to be different?

**EXERCISE**

1. Calculate the optimum water content and the maximum dry unit weight from the data

Diameter of mold (cm)	10.07		
Height of mold (cm)	11.54		
Mass of mold (g)	4258.00		
Mass of soil and mold (g)	Mass of can and wet soil (g)	Mass of can and dry soil (g)	Mass of can (g)
6070.00	98.03	94.65	46.67
	95.13	91.56	45.67
	105.80	101.67	45.61
6274.00	88.29	84.76	46.69
	72.27	70.07	45.67
	79.00	76.08	45.61
6218.00	73.22	70.29	46.69
	77.32	73.73	45.67
	84.36	77.84	45.61
6248.00	62.62	60.46	46.69
	75.95	71.64	45.67
	62.68	60.33	45.61
6232.00	90.35	83.68	46.69
	82.73	77.09	45.67
	86.33	80.34	45.61
Diameter of mold (cm)	10.16		
Height of mold (cm)	11.65		
Mass of mold (g)	4256.00		
Mass of soil and mold (g)	Mass of can and wet soil (g)	Mass of can and dry soil (g)	Mass of can (g)
6178.00	171.79	166.2	104.87
	129.50	126.14	89.05
	145.91	142.32	104.17
6318.00	145.00	140.93	104.87
	148.26	142.21	89.05
	141.11	137.42	104.17
6318.00	161.02	154.46	104.87
	133.16	128.06	89.05
	135.69	132.06	104.17
6274.00	162.90	155.53	104.87
	131.99	126.53	89.05
	164.55	156.93	104.17
6212.00	169.49	160.41	104.87
	147.39	139.33	89.05
	171.19	162.02	104.17

**REFERENCES**

See Introduction for references to ASTM procedures (pages 4 to 6).

## 3-6 The Sand Cone Method

### PRINCIPLE

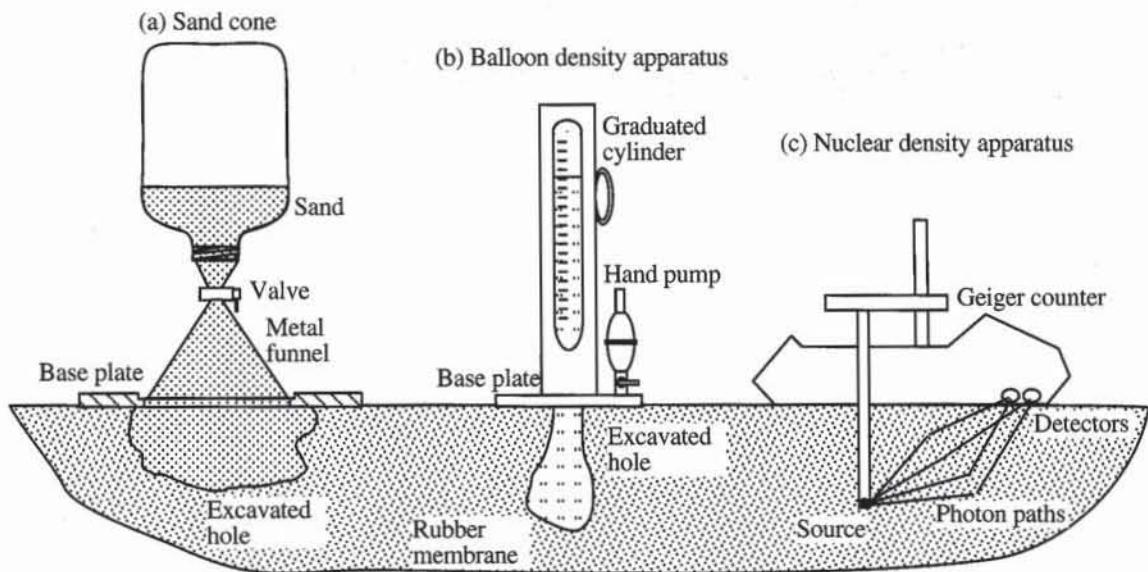
The sand cone method is used to determine soil density in the field and to control the results of field compaction in earth embankments, road fill, and structural backfill. Figure 1 illustrates the principle of the sand cone method and two other techniques: balloon density and nuclear methods. In the sand cone method shown in Fig. 1a, a soil sample is excavated manually and its weight  $W$  measured (Fig. 2). The volume  $V$  of excavated soil is determined from the volume of fine sand required to fill the hole. The bulk unit weight  $\gamma$  and dry unit weight  $\gamma_d$  of the in-place soil is

$$\gamma = \frac{W}{V}, \quad \text{and} \quad \gamma_d = \frac{\gamma}{1 + w/100} \quad (1)$$

where  $w$  is the water content (%), which is usually determined in the laboratory. The volume of fine sand is determined by measuring its weight, which assumes that it has a well-known density.

As shown in Fig. 1b, the rubber balloon method is based on the same principle as the sand cone method but uses a balloon inflated with water to fill the excavated hole. The volume of the hole, which is equal to the volume of injected water, is measured directly on the graduated cylinder of the rubber balloon device.

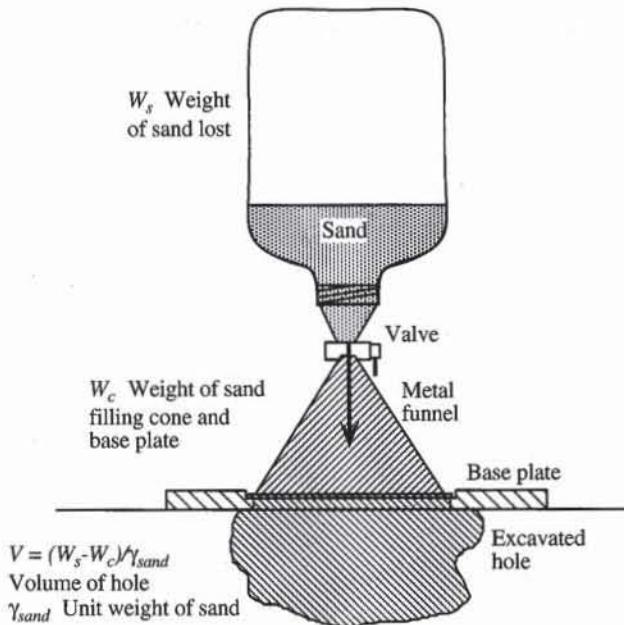
As shown in Fig. 1c, the nuclear density method measures both soil density and water content by using two types of radioactive sources. The radium or cesium isotope source generates gamma radiation, which is scattered by soil particles, whereas the americium–beryllium isotopes source emits neutrons that are scattered by the hydrogen atoms of the soil water. In the field, the radioactive sources, which are stored in a protective container during transport, are pushed at the end of a rod into the soil. The amounts of scatter between sources and detectors are measured by a Geiger counter and are related to soil density and water content after calibration. Nuclear methods have increased in popularity during



**Figure 1** Three methods of determining the unit weight of soils in the field.

the past twenty years, owing to their advantages over traditional techniques. They are conducted rapidly and yield results within minutes. However, their disadvantages include high initial cost and potential danger of radioactive exposure. Strict radiation safety standards, such as carrying safety badges with radiation-sensitive films, must be enforced for these methods.

Only the sand cone method is described hereafter. Although it is not the most efficient and rapid test method, this basic test illustrates well the principle of the determination of density of soil in the field.

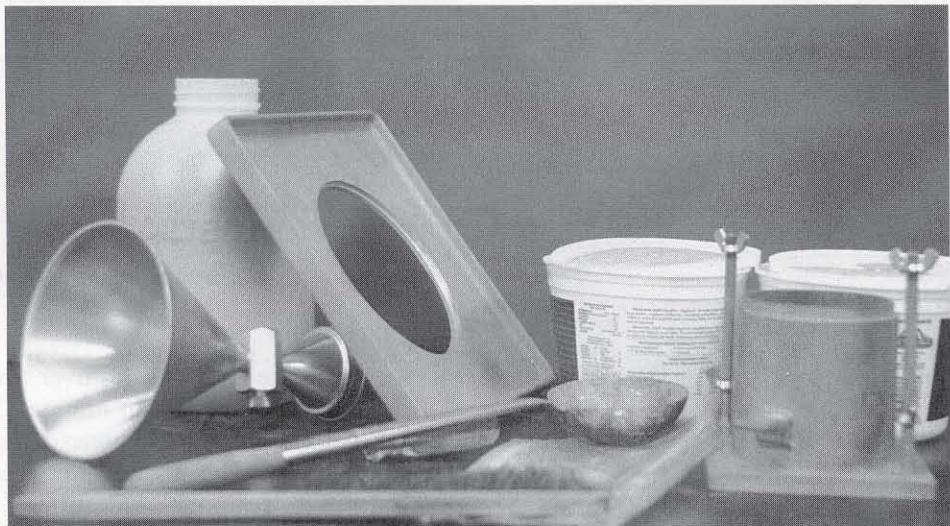


**Figure 2** Principle of sand cone method.

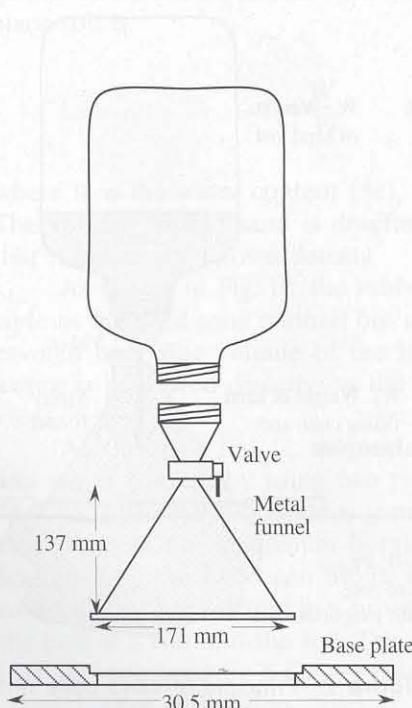
## EQUIPMENT

The equipment for the sand cone method includes:

- Sand cone with fitted valve (Fig. 3). The metal funnel is screwed on a 3.83-L plastic jar. (Fig. 4).
- Base plate about 30 cm wide.
- Uniform fine sand. About 1 kg of sand is required for each field measurement.



**Figure 3** Equipment for sand cone method. Sand cone, 3.83-L plastic jar, digging tools, airtight container, base plate, brush, and compaction mold.



**Figure 4** Sand cone (ASTM dimension)

- Digging tools (large spoons and screwdriver) to dig a hole in the soil.
- Airtight plastic bag or container to collect soil samples.
- Balance, 10 to 25 kg capacity, accurate to 1.0 g. A rugged field balance with leveling capabilities is recommended.
- Drying oven and evaporating dishes for moisture content determination.

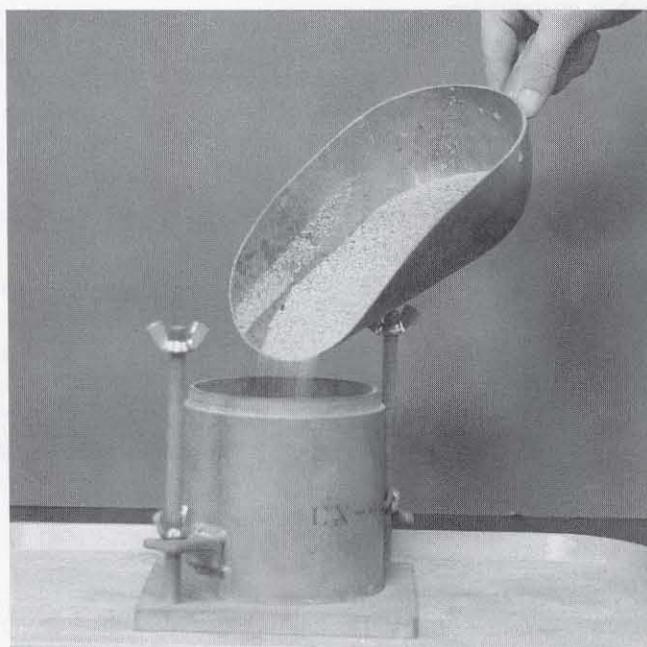
## CALIBRATION OF EQUIPMENT

The sand cone method uses a fine uniform sand that passes through a No.20 sieve but is retained on a No. 30 sieve. The sand grain size ranges from 0.85 to 0.6 mm. Its uniformity helps to keep a constant density, which is a requirement when volumes are to be determined from weight measurements. ASTM requires a coefficient of uniformity  $C_u$  smaller than 2, all particle sizes smaller than 2.0 mm, and no more than 3% smaller than 0.25 mm. The sand cone equipment is calibrated as follows.

### Determination of Sand Unit Weight

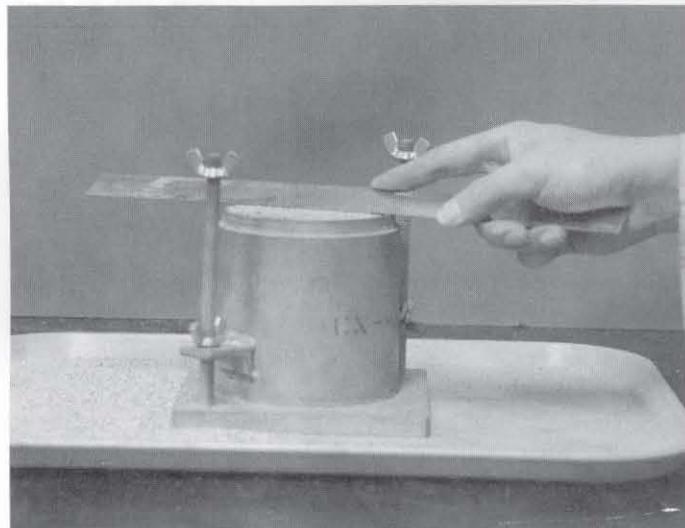
1. Measure the weight  $W_m$  of a standard compaction mold, which includes the mold body and base plate but not its extension collar. Calculate its internal volume  $V$  after having measured its internal height and diameter.
2. As shown in Fig. 5, pour sand into the mold by using a scoop. Ideally, the pouring of sand in the laboratory should be similar to that in the field. Do not shake or vibrate the mold, which may increase the sand density. Fill the mold completely and strike off the excess sand with a straightedge, as shown in Fig. 6. Measure the weight  $W$  of the mold and sand.
3. Repeat step 2 until two weight readings are in good agreement, within 10 g. The sand unit weight  $\gamma_{\text{sand}}$  is

$$\gamma_{\text{sand}} = \frac{W - W_m}{V} \quad (2)$$

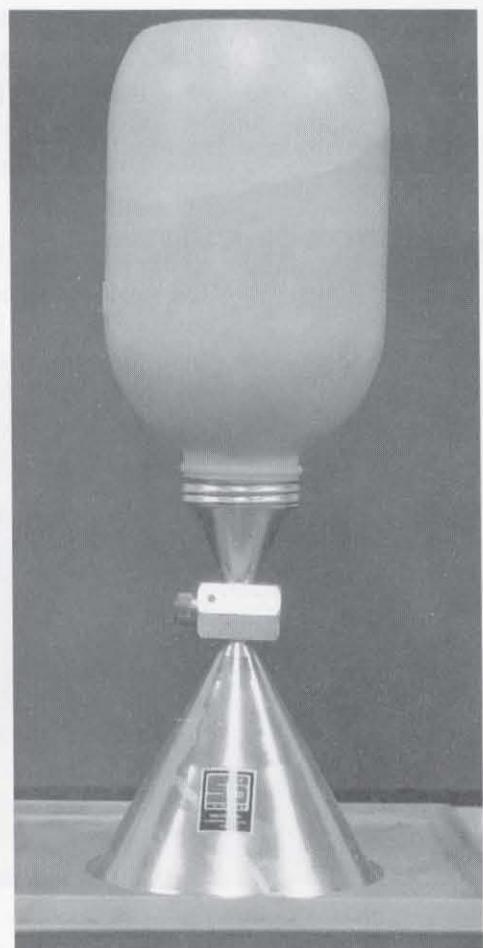


**Figure 5** Filling the mold with sand using a scoop to determine the sand unit weight.

the sand cone method is a simple way of determining soil density. It consists of filling a base plate with sand, leveling off the excess sand with a straightedge, and then filling a jar with sand until it is overflowing. The weight of sand required to fill the jar is measured. This weight is then converted to volume by dividing the weight by the unit weight of sand. The volume of sand is then converted to volume of soil by subtracting the volume of air contained in the sand.



**Figure 6** After filling the mold completely, strike off the excess sand with a straightedge.



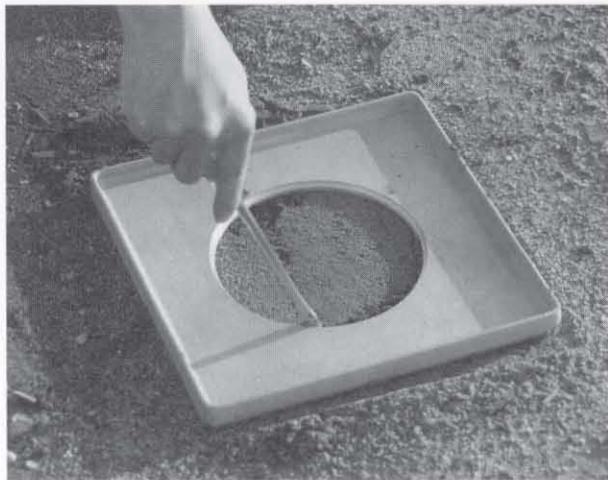
**Figure 7** The sand weight required to fill the volumes of cone and base plate is measured in the laboratory.

#### Determination of Weight of Sand to Fill Jar, Cone, and Base

1. Fill the jar completely with sand, and measure its weight  $W_f$ .
2. Place the base plate on a flat tray. The groove along the circular hole of the base plate should be facing up, as shown in Fig. 7. Turn the sand cone upside down with the valve closed, and position the metal funnel on the base plate. Open the valve to let the sand fill the funnel. Close it when the sand stops flowing. Measure the weight  $W_b$  of the partially empty bottle. The difference  $W_f - W_b$  is the sand weight  $W_c$  required to fill the cone and base plate.
3. Repeat steps 1 and 2 until two weight readings are in good agreement.

#### TEST PROCEDURE

1. At the location where the density is to be determined, level off the ground surface and position the baseplate horizontally. By using a screwdriver, mark the base plate opening on the ground as shown in Fig. 8. Remove the base plate, and dig a hole with an opening size slightly larger than the base plate open-



**Figure 8** In the field, level off the ground surface, position the base plate, and mark the base plate opening using a screwdriver.



**Figure 9** A hole is excavated with an opening size slightly larger than the base plate opening.

ing as shown in Fig. 9. The volume of the excavated hole should be smaller than  $3830 \text{ cm}^3$ , the full capacity of the sand cone jar. As a guide, ASTM suggests the following volumes for the holes:

Maximum grain size (mm)	Volume of test hole ( $\text{cm}^3$ )
4.75	700
12.7	1400
25.0	2100
50.0	2800

2. Carefully place all the soil removed from the hole into the airtight plastic bag or container. It is important not to lose any material. A loss of material would introduce substantial error in the determination of the unit weight for such a relatively small sample.
3. Measure the weight  $W_f$  of the full jar on the field scale.
4. Center the base plate above the hole (see Fig. 10). The base plate opening should be facing up. If necessary, brush soil off the base plate. Turn the sand cone upside down with the valve closed, and position the metal funnel on the groove of the base plate (see Fig. 11). Open the valve to let the sand fill the hole. Close it when the sand stops flowing. Measure the weight  $W_e$  of the partially empty jar and the weight  $W$  of the soil sample.
5. Salvage as much sand from the hole as possible.
6. After returning from the field, determine the water content of the soil samples.

## COMPUTATION

As shown in Fig. 2, the volume  $V$  of the sampling hole is calculated as follows:

$$V = \frac{W_f - W_e - W_c}{\gamma_{\text{sand}}} \quad (3)$$



**Figure 10** Face up, the base plate is centered above the hole and cleaned up by using a brush.



**Figure 11** During the actual field test, the valve of the sand cone is opened and the sand flows down to fill the excavated hole.

jar partially empty,  $W_c$  the sand weight required to fill the cone and base plate, and  $\gamma_{\text{sand}}$  the sand unit weight. The bulk unit weight  $\gamma$  and dry unit weight  $\gamma_d$  of the field sample are calculated as follows:

$$\gamma = \frac{W}{V} \quad \text{and} \quad \gamma_d = \frac{\gamma}{1 + w/100} \quad (4)$$

where  $W$  is the weight of the sample collected and  $w$  is the water content of the sample (%).

### EXAMPLE

The reported test results of the sand cone method should clearly indicate the bulk unit weight  $\gamma$ , dry unit weight  $\gamma_d$ , and water content  $w$  (%). The point  $(w, \gamma_d)$  should be plotted on the compaction curve obtained in a laboratory compaction test to compare the field values to the optimum water content and maximum dry unit weight. Then one can easily verify whether or not the compacted soil in the field meets the compaction requirement. Figure 12 shows an example of the sand cone method test, and Fig. 13 shows the formulas used in Fig. 12.

	A	B	C
1			Sand cone method
2			
3			Analyst name: Kary P. Tiel
4			Test date: 3/1/1993
5			Sample description: Sample from Kaprielian Hall
6	Measurement in the field		
7	Mass of jar and sand before use M <sub>s</sub> =	6804.00	g
8	Mass of jar and sand after use M <sub>e</sub> =	4384.00	g
9	Mass of collected soil M <sub>c</sub> =	1181.69	g
10			
11	Water content in the laboratory	Sample 1	Sample 2
12	Mass of can and wet soil (g) M <sub>w</sub> =	169.81	189.70
13	Mass of can and dry soil (g) M <sub>d</sub> =	165.52	184.05
14	Mass of can (g) M <sub>t</sub> =	104.79	104.19
15	Water content w =	7.06%	7.07%
16	Average water content w =	7.07%	
17	Bulk unit weight $\gamma$ =	20.12	kN/m <sup>3</sup>
18	Dry unit weight $\gamma_d$ =	18.79	kN/m <sup>3</sup>
19			
20	Calibration in the laboratory		
21	Diameter of mold D =	10.14	cm
22	Height of mold H =	11.67	cm
23			
24		Sample 1	Sample 2
25	Mass of mold and sand (g) M <sub>ms</sub> =	5570.0	5580.0
26	Mass of empty mold (g) M <sub>m</sub> =	4242.0	4256.0
27	Mass of jar and sand before filling cone	4946.0	
28	Mass of jar and sand after filling cone (g)	3336.0	
29	Unit weight of sand (kN/m <sup>3</sup> ) $\gamma_{\text{sand}}$ =	13.81	13.77
30	Average unit weight of sand $\gamma_{\text{sand}}$ =	13.79	kN/m <sup>3</sup>
31	Sand mass required to fill cone M <sub>c</sub> =	1610.0	g

Figure 12 Example of data set.

	A	B	C
15	Water content $w = (M_w - M_d) / (M_d - M_t)$		$= (M_w - M_d) / (M_d - M_t)$
16	Average water content $w = \text{AVERAGE}(B15:C15)$		
17	Bulk unit weight $\gamma = M / (M_f - M_e - M_c) * g_{sand}$	$\text{kN/m}^3$	
18	Dry unit weight $\gamma_d = g / (1 + w)$		$\text{kN/m}^3$

	A	B	C
29	Unit weight of sand ( $\text{kN/m}^3$ ) $\gamma_{sand} = 9.8 * (M_{ms} - M_m) / (\rho_l * D^2 * H / 4)$		$= 9.8 * (M_{ms} - M_m) / (\rho_l * D^2 * H / 4)$
30	Average unit weight of sand $\gamma_{sand} = \text{AVERAGE}(B29:C29)$		$\text{kN/m}^3$
31	Sand mass required to fill cone $M_c = \text{AVERAGE}(M_{jb} - M_{ja})$		g

Figure 13 Formulas used in Fig. 12.

**REVIEW QUESTIONS**

1. State the objective and principle of the sand cone method.
2. Why is it important not to lose any soil from the excavated hole during the sand cone method?
3. Why do we use a particular sand for the sand cone method? Why not use any sand?
4. Can you name two other test methods that are used to define soil density in the field?

**EXERCISES**

1. Measure the internal dimension of the cone and base plate and calculate the volumes of the cone and base plate. Compare with the weight found during the equipment calibration.
2. Measure exactly the volume of the 1-gallon jar by measuring its weight empty and filled with water. After drying the jar, fill it with sand and weigh it. Verify that the sand unit weight is approximately equal to the sand density determined during the equipment calibration.
3. Calculate the in situ density from the following results obtained from a sand cone test.

Determination of sand unit mass in the laboratory	Diameter of mold (cm) Height of mold (cm) Mass of mold and sand (g) Mass of mold (g)	10.13 11.65 5602 4252
Determination of sand mass to fill cone	Mass of jar and sand before filling cone (g) Mass of jar and sand after filling cone (g)	3516 1934
In place measurement	Mass of jar and sand before use (g) Mass of jar and sand after use (g) Mass of collected soil (g)	6542 4334 865.52
		Trial 1   Trial 2   Trial 3
Determination of water content in the laboratory	Mass of can and wet soil (g) Mass of can and dry soil (g) Mass of can (g)	146.54   142.52   147.32 144.63   140.27   144.83 113.65   104.89   104.18

4. Calculate the in situ density from the following results obtained from a sand cone test.

Determination of sand unit mass in the laboratory	Diameter of mold (cm)	10.16	Trial 1	Trial 2	Trial 3
	Height of mold (cm)	11.65			
	Mass of mold and sand (g)	5596			
	Mass of mold (g)	4246			
Determination of sand mass to fill cone	Mass of jar and sand before filling cone (g)	6122			
	Mass of jar and sand after filling cone (g)	4460			
In place measurement	Mass of jar and sand before use (g)	6314			
	Mass of jar and sand after use (g)	3594			
	Mass of collected soil (g)	1552			
Determination of water content in the laboratory	Mass of can and wet soil (g)	106	130	122	
	Mass of can and dry soil (g)	104	128	120	
	Mass of can (g)	82	104	104	

# 4

# Permeability and Seepage

- 4-1** Principles of permeability tests
- 4-2** Constant head permeability test
- 4-3** Falling head permeability test
- 4-4** Electrical analogy of seepage problems
- 4-5** Finite difference solutions of seepage problems

# 4-1 Principles of Permeability Tests

## INTRODUCTION

Soils are permeable to water because the voids between soil particles are interconnected. The degree of permeability is characterized by the permeability coefficient  $k$ , also referred to as hydraulic conductivity. In the laboratory,  $k$  is measured by using either the constant head test for soils of high permeability (e.g., sands), or the falling head test for soils of intermediate and low permeability (e.g., silts and clays). Before describing the test procedures, the basic concepts of seepage are reviewed.

## DEFINITIONS

### Fluid Velocity

Figure 1 illustrates a flow of water through an inclined tube filled with soil. The water molecules moving from cross sections  $A$  to  $B$  follow a tortuous path around the soil particles and through the voids. As shown in the cross section of Fig. 2, the velocity of water, denoted by the vector  $\mathbf{v}_f$ , is only defined in the voids through which water travels, and it varies from void to void.

### Seepage Quantity

In Fig. 2, the seepage quantity  $q$  is the volume of water passing through a tube cross section during a unit time interval.  $q$  is the flux of water:

$$q = \int_S v_f dS \quad (1)$$

where  $v_f$  is the component of  $\mathbf{v}_f$  parallel to the tube axis and  $S$  is the total cross-sectional area of the tube.  $v_f$  is assumed equal to zero at the particle locations.

### Discharge Velocity

The discharge velocity  $v$  parallel to the tube axis is defined as

$$v = \frac{q}{S} \quad (2)$$

It is smaller than the average fluid velocity  $\bar{v}_f$  on cross-sectional area  $S_f$  only occupied by water.  $v$  and  $\bar{v}_f$  are related through

$$v = n\bar{v}_f, \quad \bar{v}_f = \frac{1}{S_f} \int_{S_f} v_f dS_f, \quad \text{and} \quad n = \frac{V_v}{V} \quad (3)$$

where  $n$  is the porosity and  $V_v$  is the volume of voids in volume  $V$  between cross sections  $A$  and  $B$  of Fig. 1. As shown in Figs. 1 and 2,  $v$  averages the real water flow. The smoothed trajectory of water particles, which is tangent to the discharge velocity, is called a *flow line*.

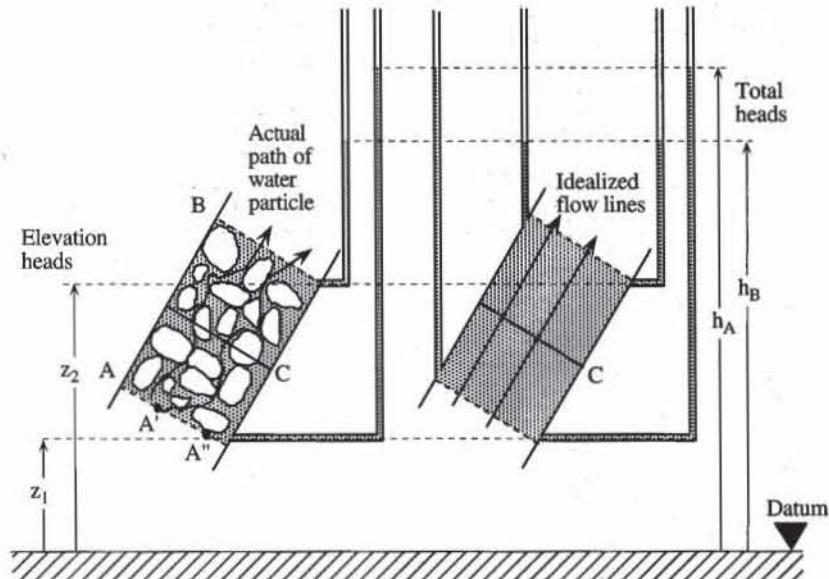


Figure 1 Flow of water through soil.

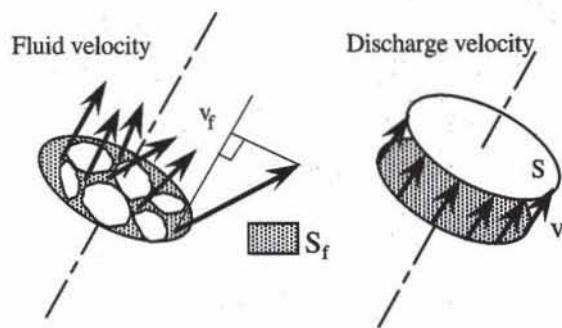


Figure 2 Fluid velocity and discharge velocity.

### Total Head

In hydrostatics (i.e., when the water does not move), the total head  $h$  is the sum of the pressure head  $u/\gamma_w$ , and the elevation head  $z$ :

$$h = \frac{u}{\gamma_w} + z \quad (4)$$

where  $u$  is the pore water pressure,  $\gamma_w$  is the fluid unit weight, and  $z$  the elevation above a given datum. As shown in Fig. 1, pressure, elevation, and total heads must always have the same datum. When water moves at velocity  $v_f$ , the total head is defined using the Bernoulli equation:

$$h = \frac{v_f^2}{2g} + \frac{u}{\gamma_w} + z \quad (5)$$

where  $g$  is the earth gravity. For most soil flow problems,  $v_f^2/2g$  is negligible compared to the pressure and elevation head, because  $v_f$  is much smaller than 1 m/s. Therefore, the total head in soils is given by Eq. 4.

### Piezometric Head

In hydrostatics, the total head  $h$  is equal to the elevation of the free surface of water above the datum. This elevation is called the *piezometric head*. In Fig. 1, the total (or piezometric) head is the elevation above the datum of the water level in the pipes. The total head is constant at all points of the same cross section in Fig. 1 (e.g., points  $A$ ,  $A'$ , and  $A''$  of Fig. 1).

### Hydraulic Gradient

The hydraulic gradient  $i$  is the gradient of total head. In Fig. 1, the hydraulic gradient between sections  $A$  and  $B$  is equal to the head drop  $h_B - h_A$  divided by the distance  $L = AB$  where the head drop takes place:

$$i = \frac{h_B - h_A}{L} \quad (6)$$

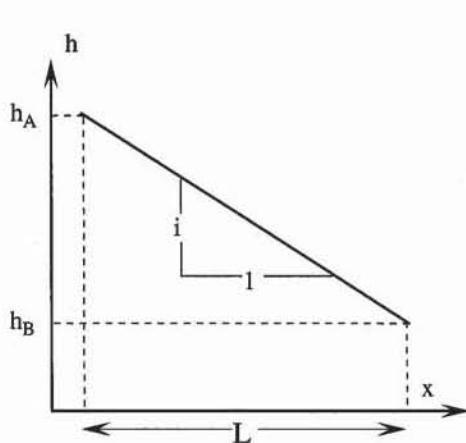
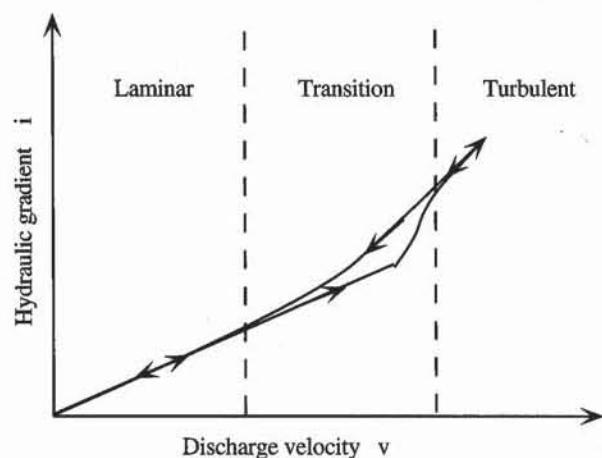
where  $h_A$  is the total head in section  $A$  and  $h_B$  is the total head in section  $B$ . The hydraulic gradient  $i$  is a dimensionless number because  $h$  and  $L$  have the same dimension. In Fig. 3,  $i$  is the slope of the variation of  $h$  versus distance  $x$ .

### Darcy's Law

As described in Chapter 1-3, fluid flows can be laminar, turbulent, or transitional, depending on the Reynolds number. In laminar flows, the fluid flows in parallel layers without mixing. In turbulent flows, random velocity fluctuations result in mixing and internal energy dissipation. In transitional flows, the flows are between the laminar and turbulent regimes. These different flow regimes are also found in soils and influence the relation between discharge velocity and hydraulic gradient. As shown in Fig. 4,  $i$  varies linearly with  $v$  in the laminar regime but varies nonlinearly and irreversibly with  $v$  in the transitional and turbulent zones.

For most flows in soils,  $v$  is so small that  $v$  is proportional to  $i$ ; that is, Darcy's law (Darcy, 1856) applies:

$$v = k i \quad (7)$$

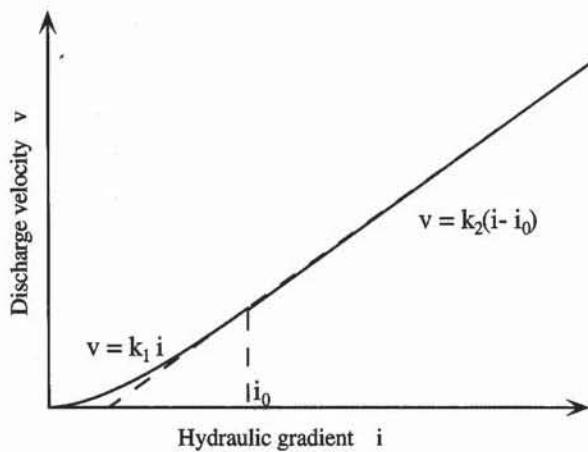
**Figure 3** Variation of total head  $h$  versus distance  $x$ .**Figure 4** Zones of laminar and turbulent flows (after Taylor, 1948).

where  $k$  is the coefficient of permeability. The coefficient of permeability  $k$  is defined as the rate of discharge of water under conditions of laminar flow through a unit cross-sectional area of a soil subjected to a unit hydraulic gradient. The dimension of  $k$  is velocity (i.e.,  $[k] = LT^{-1}$ ), and its unit is usually cm/s.

Experiments have shown that Darcy's law (Eq. 7) is valid for a wide range of soil types and hydraulic gradients. However, Darcy's law no longer applies for large hydraulic gradients in clean gravels and rock fills where flows may be turbulent. It also breaks down for very small hydraulic gradients in clays. As shown in Fig. 5, in Swedish clays, Hansbo (1960) found a nonlinear relation between  $v$  and  $i$  for very small hydraulic gradients (i.e.,  $i < i_0$ ), and a linear relation with an offset for larger hydraulic gradients (i.e.,  $i \geq i_0$ ).

### Critical Hydraulic Gradient

When the water flows upward, frictional drag tends to lift the particles and force them apart. The hydraulic gradient that breaks contact between particles is the

**Figure 5** Deviation from Darcy's law observed in Swedish clays (after Hansbo, 1960).

critical hydraulic gradient  $i_c$ :

$$i_c = \frac{\gamma_b}{\gamma_w} \quad (8)$$

where  $\gamma_b = \gamma_{sat} - \gamma_w$  is the submerged (or buoyant) unit weight of the soil,  $\gamma_w$  the unit weight of the water, and  $\gamma_{sat}$  the saturated unit weight of the soil. For most soils,  $\gamma_b \approx \gamma_w$ ; therefore,  $i_c \approx 1$ . The loss of contact between soil grains results in the *quick condition*, in which soils behave as liquids. A quick condition can occur in any cohesionless soil when the upward hydraulic gradient exceeds  $i_c$ . A hydraulic gradient in excess of  $i_c$  is also responsible for the *boiling* of sand at the bottom of excavations and subsurface erosion known as *piping*.

## EMPIRICAL RELATIONSHIPS

The permeability of a soil depends primarily on the size and shape of grains, shape and arrangement of voids, void ratio, degree of saturation, and temperature. Several equations were proposed to calculate the permeability of soils, especially sands, from their physical characteristics. Two correlations are given below.

### Hazen's Formula

Based on experimental work with fine uniform sand, Hazen (1892) related permeability  $k$  and effective particle size  $D_{10}$  (cm) as follows:

$$k = C_1 D_{10}^2 \quad (\text{cm/s}) \quad (9)$$

where  $C_1 \approx 100$  for fine uniform sand. As shown in Table 1,  $C_1$  is not a constant but varies with soil types. Its average value in Table 1 is 16. Figure 6 suggests that  $k$  is related to not only the particle size  $D_{10}$  but also the void ratio  $e$ .

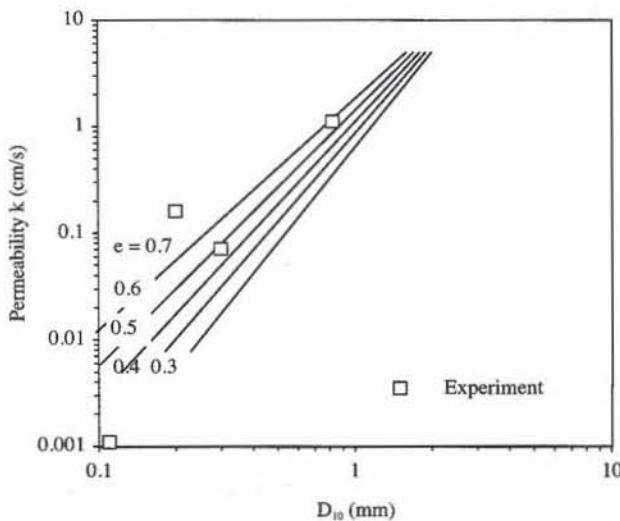


Figure 6 Variation of coefficient of permeability with particle size  $D_{10}$  (after NAVFAC, 1982).

**TABLE 1**

Permeability test data (after Lane and Washburn, 1946; and Lambe and Whitman, 1979)

Soil type	Particle size $D_{10}$ (cm)	Permeability ( $\mu\text{m/s}$ )	$C_1 = k/D_{10}^2$ ( $\text{cm}^{-1}\text{s}^{-1}$ )
Coarse gravel	0.0820	1100	17
Sandy gravel	0.0200	160	40
Fine gravel	0.0300	71	8
Silty gravel	0.0060	4.6	13
Coarse sand	0.0110	1.1	1
Medium sand	0.0020	0.29	7
Fine sand	0.0030	0.096	1
Silt	0.0006	0.15	42

### Kozeny–Carman Formula

The Kozeny–Carman formula (Kozeny, 1927; and Carman, 1939) is

$$k = \frac{\gamma_w}{5f \eta S^2} \frac{e^3}{1+e} \quad (10)$$

where  $e$  is the void ratio, and  $\eta$  the dynamic viscosity of water. The coefficient  $f$  depends on pore shape:  $f = 1.1$  for rounded grains,  $f = 1.25$  for subrounded grains, and  $f = 1.4$  for angular grains (Loudon, 1952). The specific surface area  $S$  (i.e., the surface area per unit volume of grains) is obtained from the equation

$$S = \frac{6}{\sqrt{d_{\max} d_{\min}}} (\text{mm}^{-1}) \quad (11)$$

where  $d_{\max}$  is the maximum grain diameter (mm) and  $d_{\min}$  is the minimum grain diameter (mm).

### Effect of Temperature on Permeability Coefficient

Equation 10 shows that  $k$  is not a constant for a given soil but varies with  $\eta$ , which in turn varies with temperature  $T$ . The permeability at temperature  $T$ ,  $k_T$ , is reduced to that at  $20^\circ\text{C}$ ,  $k_{20^\circ\text{C}}$ , by using

$$k_{20^\circ\text{C}} = \frac{\eta_T}{\eta_{20^\circ\text{C}}} k_T = R k_T \quad (12)$$

where  $\eta_{20^\circ\text{C}}$  is the viscosity of water at  $20^\circ\text{C}$ ,  $\eta_T$  the viscosity of water at temperature  $T$ , and  $R = \eta_T/\eta_{20^\circ\text{C}}$ . The variation of  $R$  with temperature is shown in Fig. 7.

### TYPES OF TESTS

There are two main types of laboratory permeability tests: constant head test and falling head test.

#### Constant Head Test

As schematized in Fig. 8, in the constant head test, a constant head drop is ap-

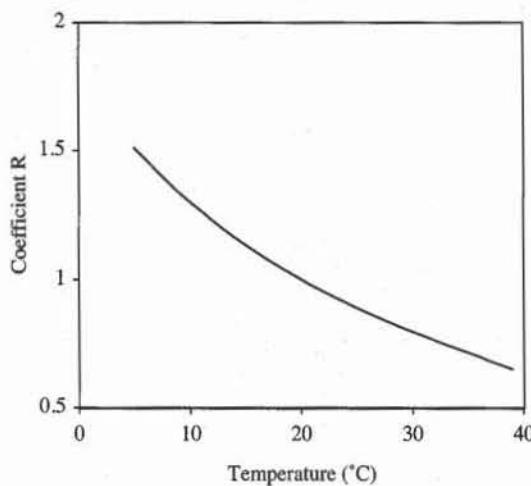


Figure 7 Variation of correction factor  $R$  versus temperature  $T$ .

plied to the soil sample, and the resulting seepage quantity is measured. The constant head test is used primarily for coarse-grained soils (clean sands and gravels) with  $k \geq 10^{-3}$  cm/s. For fine-grained soils, the seepage quantity becomes too small to be measured accurately.

The coefficient of permeability  $k_T$  at temperature  $T$  is

$$k_T = \frac{qL}{(h_1 - h_2)At} \quad (13)$$

where  $q$  is the volume of water collected in a burette during time interval  $t$ ,  $L$  the length of the specimen,  $h_1$  the head at the left end of the specimen,  $h_2$  the head at the right end of the specimen, and  $A$  the cross-sectional area of the specimen.

### Falling Head Test

As illustrated in Fig. 9, the falling head test does not fix the total head. It lets it fall in the standpipe connected to the upper part of the specimen. The falling

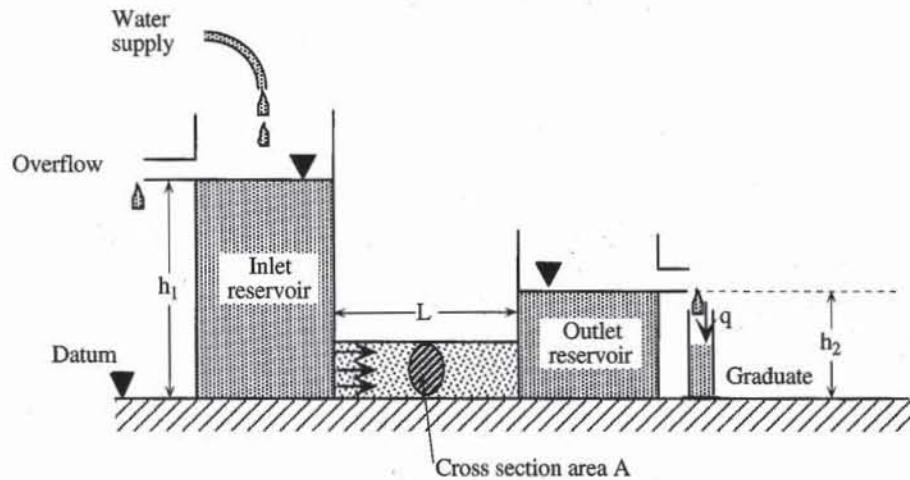


Figure 8 Principle of constant head test.

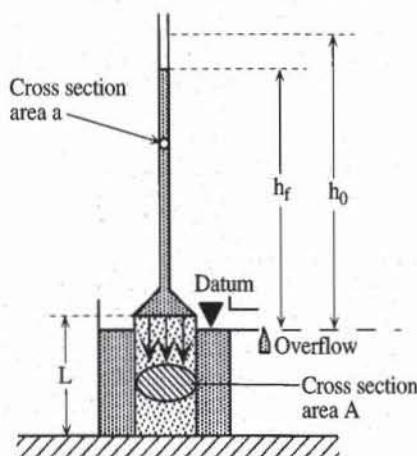


Figure 9 Principle of falling head test.

head test is generally used for less permeable soils (fine sands to fat clays) with  $k \leq 10^{-3}$  cm/s. It is not practical for soils with  $k > 10^{-3}$  cm/s because the head falls too rapidly to be measured. The coefficient of permeability  $k_T$  at temperature  $T$  is calculated as follows:

$$k_T = \frac{aL}{At} \log \frac{h_0}{h_f} \quad (14)$$

where  $a$  is the cross-sectional area of the standpipe,  $A$  the cross-sectional area of the specimen,  $L$  the length of the specimen,  $h_0$  the elevation above the datum of water in the standpipe at the beginning of the experiment ( $t = 0$ ), and  $h_f$  the elevation above the datum of water in the standpipe at time  $t$ .

#### TYPICAL VALUES FOR PERMEABILITY

Table 2 gives some typical values of permeability for various types of soil, and classify them on the basis of permeability. Figures 10 and 11 present permeability data on a variety of soils with different void ratios. The range of permeability covered by Figs. 10 and 11 is 1 cm/s to  $10^{-10}$  cm/s. As shown in Fig. 12, the permeability of clay vary with void ratio, plasticity index  $PI$ , and clay fraction  $CF$ . As shown in Fig. 13, the permeability coefficient is almost isotropic in most clays (i.e., the horizontal permeability  $k_h$  is practically equal to the vertical permeability  $k_v$ ) except for varved clay and stratified deposits where the ratio  $k_h/k_v$  can exceed 10.

**TABLE 2**  
Classification of soils according to their coefficients of permeability  
(after Kulhawy and Mayne, 1990; and Terzaghi and Peck, 1967)

Soil	Coefficient of permeability $k$ (cm/s)	Degree of permeability
Gravel	Over $10^{-1}$	High
Sandy gravel, clean sand, fine sand	$10^{-1}$ to $10^{-3}$	Medium
Sand, dirty sand, silty sand	$10^{-3}$ to $10^{-5}$	Low
Silt, silty clay	$10^{-5}$ to $10^{-7}$	Very low
Clay	Less than $10^{-7}$	Practically impermeable