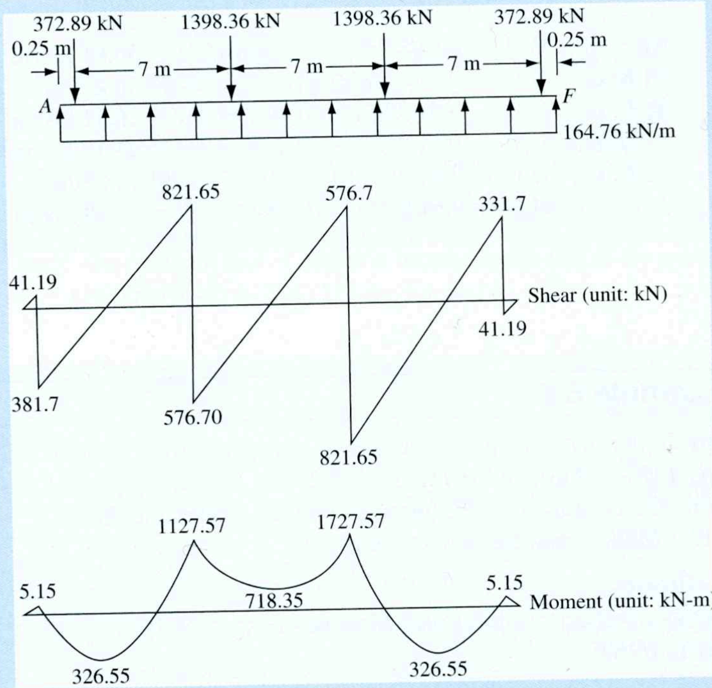

Figure 6.16 Load, shear, and moment diagrams for strip *AGHF*

$$\text{Average load} = \frac{5332 + 4000}{2} = 4666 \text{ kN}$$

$$q_{\text{av(modified)}} = (31.0)\left(\frac{4666}{5332}\right) = 27.12 \text{ kN/m}^2$$

$$F = \frac{4666}{4000} = 1.1665$$

The load, shear, and moment diagrams are shown in Figure 6.17.

Strip *ICDJ*: Figure 6.18 shows the load, shear, and moment diagrams for this strip.

Determination of the Thickness of the Mat

For this problem, the critical section for diagonal tension shear will be at the column carrying 1500 kN of load at the edge of the mat [Figure 6.19]. So

$$b_o = \left(0.5 + \frac{d}{2}\right) + \left(0.5 + \frac{d}{2}\right) + (0.5 + d) = 1.5 + 2d$$

$$U = (b_o d)\left[(\phi)(0.34)\sqrt{f'_c}\right]$$

$$U = (1.7)(1500) = 2550 \text{ kN} = 2.55 \text{ MN}$$

$$2.55 = (1.5 + 2d)(d)\left[(0.85)(0.34)\sqrt{20.7}\right]$$

or

$$(1.5 + 2d)(d) = 1.94; \; d = 0.68 \text{ m}$$

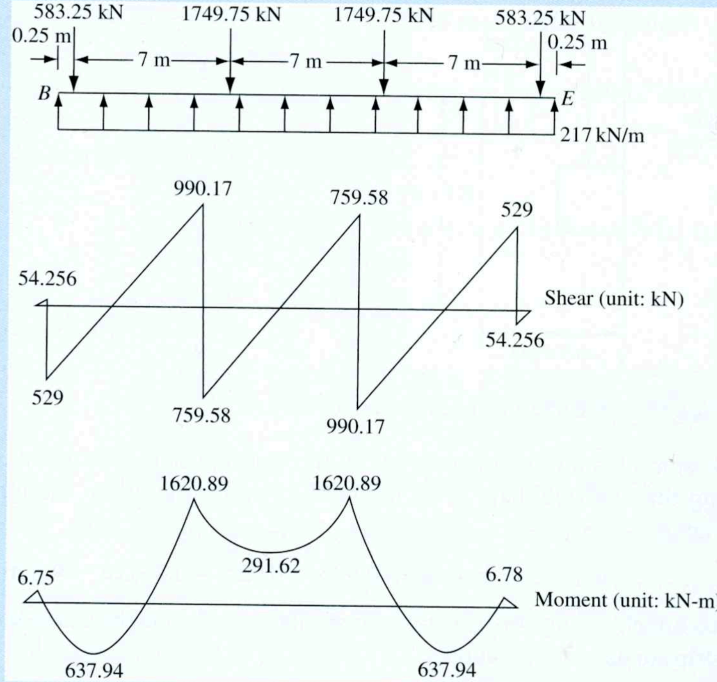

Figure 6.17 Load, shear, and moment diagrams for strip *GIJH*

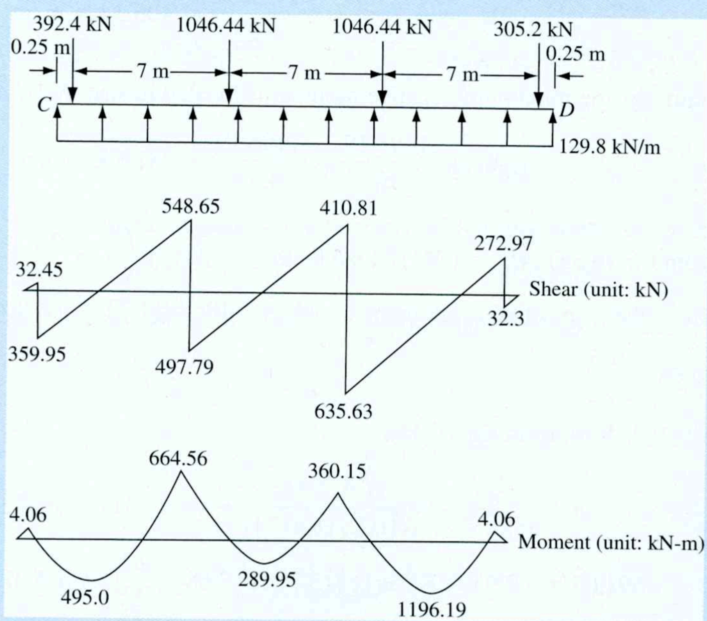

Figure 6.18 Load, shear, and moment diagrams for strip *ICDJ*

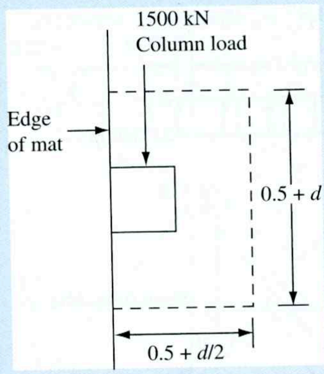

Figure 6.19 Critical perimeter column

Assuming a minimum cover of 76 mm over the steel reinforcement and also assuming that the steel bars to be used are 25 mm in diameter, the total thickness of the slab is

$$h = 0.68 + 0.076 + 0.025 = 0.781 \text{ m} \approx \mathbf{0.8\ m}$$

The thickness of this mat will satisfy the wide beam shear condition across the three strips under consideration.

Determination of Reinforcement

From the moment diagram shown in Figures 6.16, 6.17, and 6.18, it can be seen that the maximum positive moment is located in strip $AGHF$, and its magnitude is

$$M' = \frac{1727.57}{B_1} = \frac{1727.57}{4.25} = 406.5 \text{ kN-m/m}$$

Similarly, the maximum negative moment is located in strip $ICDJ$ and its magnitude is

$$M' = \frac{1196.19}{B_1} = \frac{1196.19}{4.25} = 281.5 \text{ kN-m/m}$$

From Eq. (6.33): $M_u = (M')(\text{load factor}) = \phi A_s f_y\left(d - \frac{a}{2}\right)$.

For the positive moment, $M_u = (406.5)(1.7) = (\phi)(A_s)(413.7 \times 1000)\left(0.68 - \frac{a}{2}\right)$

$\phi = 0.9$. Also, from Eq. (6.34),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.7)(1)} = 23.51A_s; \text{ or } A_s = 0.0425a$$

$$691.05 = (0.9)(0.0425a)(413{,}700)\left(0.68 - \frac{a}{2}\right); \text{ or } a \approx 0.0645$$

So, $A_s = (0.0425)(0.0645) = 0.00274 \text{ m}^2/\text{m} = 2740 \text{ mm}^2/\text{m}$.

Use 25-mm diameter bars at 175 mm center-to-center:

$$\left[A_s \text{ provided} = (491)\left(\frac{1000}{175}\right) = 2805.7 \text{ mm}^2/\text{m}\right]$$

Similarly, for negative reinforcement,

$$M_u = (281.5)(1.7) = (\phi)(A_s)(413.7 \times 1000)\left(0.68 - \frac{a}{2}\right)$$

$$\phi = 0.9.\ A_s = 0.0425a$$

So

$$478.55 = (0.9)(0.0425a)(413.7 \times 1000)\left(0.68 - \frac{a}{2}\right);\ \text{or } a \approx 0.045$$

So, $A_s = (0.045)(0.0425) = 0.001913 \text{ m}^2/\text{m} = 1913 \text{ mm}^2/\text{m}$.

Use 25-mm diameter bars at 255 mm center-to-center:

$$[A_s \text{ provided} = 1925 \text{ mm}^2]$$

Because negative moment occurs at midbay of strip *ICDJ*, reinforcement should be provided. This moment is

$$M' = \frac{289.95}{4.25} = 68.22 \text{ kN-m/m}$$

Hence,

$$M_u = (68.22)(1.7) = (0.9)(0.0425a)(413.7 \times 1000)\left(0.68 - \frac{a}{2}\right);$$

or $a \approx 0.0108$

$$A_s = (0.0108)(0.0425) = 0.000459 \text{ m}^2/\text{m} = 459 \text{ mm}^2/\text{m}$$

Provide 16-mm diameter bars at 400 mm center-to-center:

$$[A_s \text{ provided} = 502 \text{ mm}^2]$$

For general arrangement of the reinforcement see Figure 6.20.

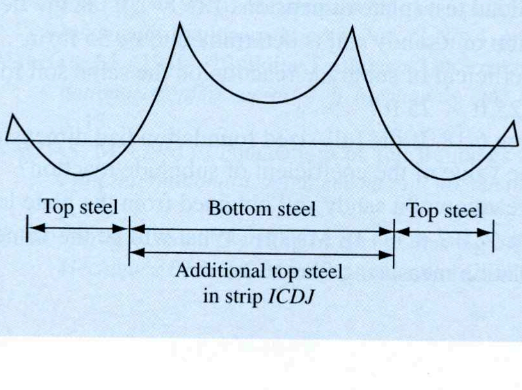

Figure 6.20 General arrangement of reinforcement

Problems

6.1 Determine the net ultimate bearing capacity of mat foundations with the following characteristics:

$$c_u = 120\ \text{kN/m}^2,\ \phi = 0,\ B = 8\ \text{m},\ L = 18\ \text{m},\ D_f = 3\ \text{m}$$

6.2 Following are the results of a standard penetration test in the field (sandy soil):

Depth (m)	Field value of N_{60}
1.5	9
3.0	12
4.5	11
6.0	7
7.5	13
9.0	11
10.5	13

Estimate the net allowable bearing capacity of a mat foundation 6.5 m × 5 m in plan. Here, $D_f = 1.5$ m and allowable settlement = 50 mm. Assume that the unit weight of soil, $\gamma = 16.5\ \text{kN/m}^3$.

6.3 Repeat Problem 6.2 for an allowable settlement of 30 mm.

6.4 A mat foundation on a saturated clay soil has dimensions of 20 m × 20 m. Given: dead and live load = 48 MN, $c_u = 30\ \text{kN/m}^2$, and $\gamma_{\text{clay}} = 18.5\ \text{kN/m}^3$.

a. Find the depth, D_f, of the mat for a fully compensated foundation.

b. What will be the depth of the mat (D_f) for a factor of safety of 2 against bearing capacity failure?

6.5 Repeat Problem 6.4 part b for $c_u = 20\ \text{kN/m}^2$.

6.6 A mat foundation is shown in Figure P6.6. The design considerations are $L = 12$ m, $B = 10$ m, $D_f = 2.2$ m, $Q = 30$ MN, $x_1 = 2$ m, $x_2 = 2$ m, $x_3 = 5.2$ m, and preconsolidation pressure $\sigma'_c \approx 105\ \text{kN/m}^2$. Calculate the consolidation settlement under the center of the mat.

6.7 For the mat foundation in Problem 6.6, estimate the consolidation settlement under the corner of the mat.

6.8 From the plate load test (plate dimensions 1 ft × 1 ft) in the field, the coefficient of subgrade reaction of a sandy soil is determined to be 55 lb/in^3. What will be the value of the coefficient of subgrade reaction on the same soil for a foundation with dimensions of 25 ft × 25 ft?

6.9 Refer to Problem 6.18. If the full-sized foundation had dimensions of 70 ft × 30 ft, what will be the value of the coefficient of subgrade reaction?

6.10 The subgrade reaction of a sandy soil obtained from the plate load test (plate dimensions 1 m × 0.7 m) is 18 MN/m^3. What will be the value of k on the same soil for a foundation measuring 5 m × 3.5 m?

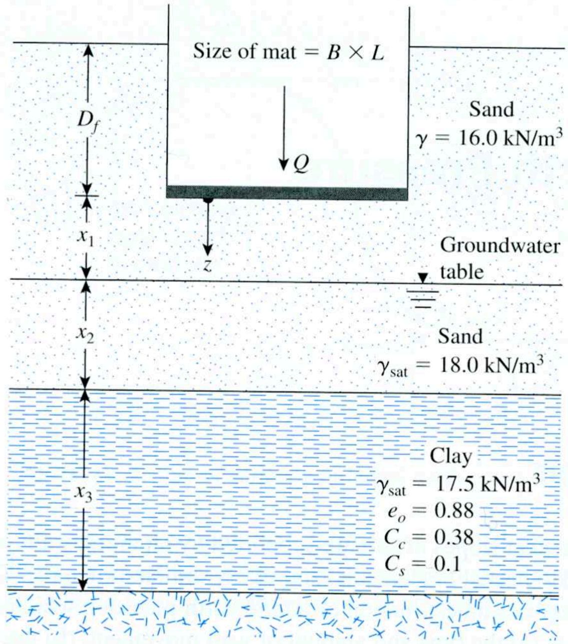

Figure P6.6

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7 Lateral Earth Pressure

7.1 Introduction

Vertical or near-vertical slopes of soil are supported by retaining walls, cantilever sheet-pile walls, sheet-pile bulkheads, braced cuts, and other, similar structures. The proper design of those structures requires an estimation of lateral earth pressure, which is a function of several factors, such as (a) the type and amount of wall movement, (b) the shear strength parameters of the soil, (c) the unit weight of the soil, and (d) the drainage conditions in the backfill. Figure 7.1 shows a retaining wall of height H. For similar types of backfill,

a. The wall may be restrained from moving (Figure 7.1a). The lateral earth pressure on the wall at any depth is called the *at-rest earth pressure.*
b. The wall may tilt away from the soil that is retained (Figure 7.1b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as *active earth pressure.*
c. The wall may be pushed into the soil that is retained (Figure 7.1c). With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to as *passive earth pressure.*

Figure 7.2 shows the nature of variation of the lateral pressure, σ'_h, at a certain depth of the wall with the magnitude of wall movement.

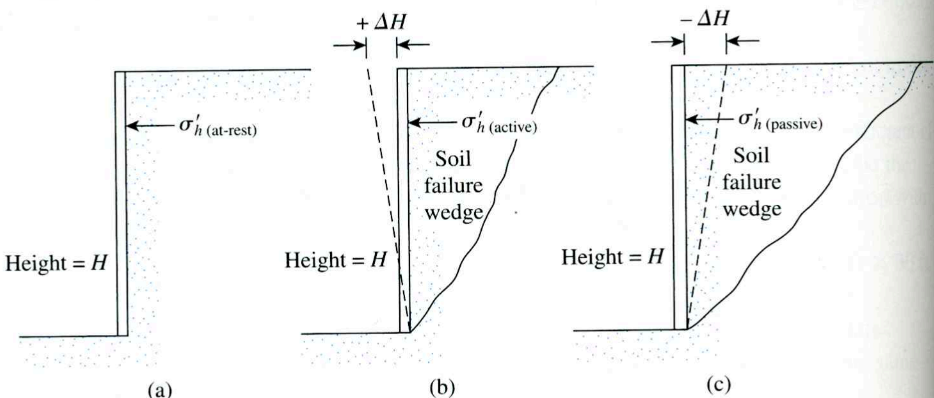


Figure 7.1 Nature of lateral earth pressure on a retaining wall

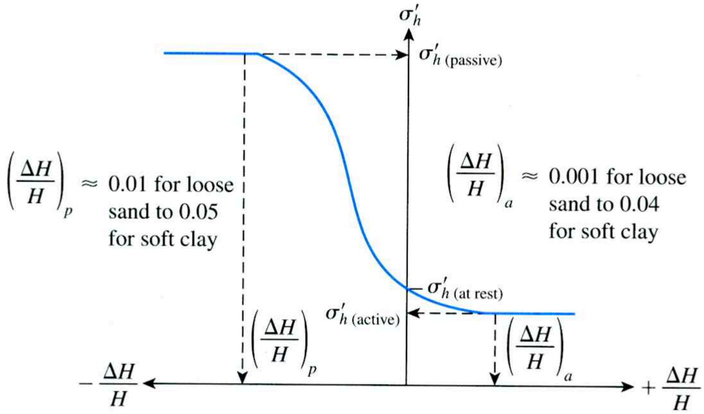


Figure 7.2 Nature of variation of lateral earth pressure at a certain depth

In the sections that follow, we will discuss various relationships to determine the at-rest, active, and passive pressures on a retaining wall. It is assumed that the reader has studied lateral earth pressure in the past, so this chapter will serve as a review.

7.2 Lateral Earth Pressure at Rest

Consider a vertical wall of height H, as shown in Figure 7.3, retaining a soil having a unit weight of γ. A uniformly distributed load, q/unit area, is also applied at the ground surface. The shear strength of the soil is

$$s = c' + \sigma' \tan \phi'$$

where

c' = cohesion
ϕ' = effective angle of friction
σ' = effective normal stress

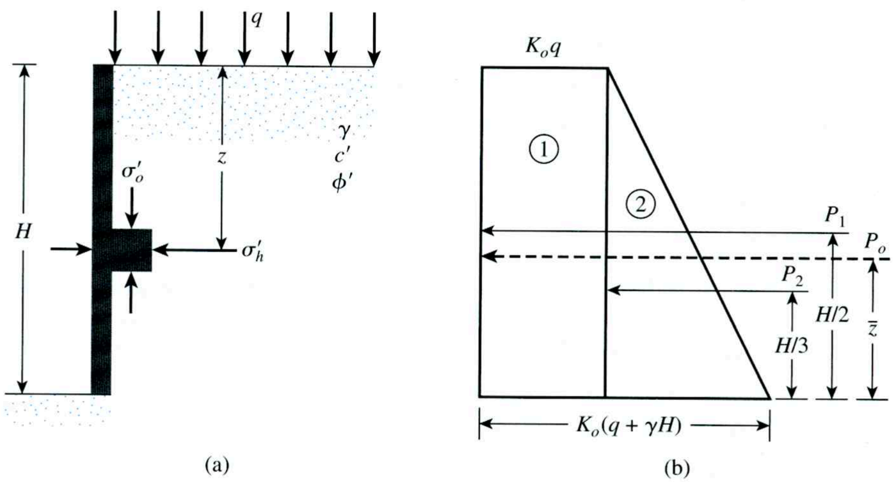


Figure 7.3 At-rest earth pressure

At any depth z below the ground surface, the vertical subsurface stress is

$$\sigma'_o = q + \gamma z \tag{7.1}$$

If the *wall is at rest and is not allowed to move at all,* either away from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth z is

$$\sigma_h = K_o \sigma'_o + u \tag{7.2}$$

where

u = pore water pressure
K_o = coefficient of at-rest earth pressure

For normally consolidated soil, the relation for K_o (Jaky, 1944) is

$$K_o \approx 1 - \sin \phi' \tag{7.3}$$

Equation (7.3) is an empirical approximation.

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin \phi')\text{OCR}^{\sin \phi'} \tag{7.4}$$

where OCR = overconsolidation ratio.

With a properly selected value of the at-rest earth pressure coefficient, Eq. (7.2) can be used to determine the variation of lateral earth pressure with depth z. Figure 7.3b shows the variation of σ'_h with depth for the wall depicted in Figure 7.3a. Note that if the surcharge $q = 0$ and the pore water pressure $u = 0$, the pressure diagram will be a triangle. The total force, P_o, *per unit length* of the wall given in Figure 7.3a can now be obtained from the area of the pressure diagram given in Figure 7.3b and is

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o \tag{7.5}$$

where

P_1 = area of rectangle 1
P_2 = area of triangle 2

The location of the line of action of the resultant force, P_o, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\dfrac{H}{2}\right) + P_2\left(\dfrac{H}{3}\right)}{P_o} \tag{7.6}$$

If the water table is located at a depth $z < H$, the at-rest pressure diagram shown in Figure 7.3b will have to be somewhat modified, as shown in Figure 7.4. If the effective unit weight of soil below the water table equals γ' (i.e., $\gamma_{sat} - \gamma_w$), then

at $z = 0$, $\quad \sigma'_h = K_o\sigma'_o = K_o q$
at $z = H_1$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1)$

and

at $z = H_2$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1 + \gamma' H_2)$

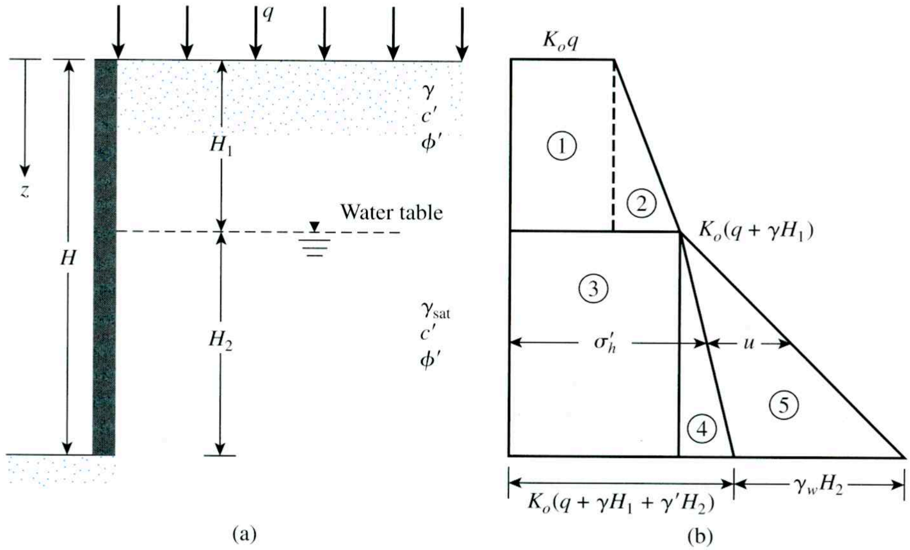


Figure 7.4 At-rest earth pressure with water table located at a depth $z < H$

Note that in the preceding equations, σ'_o and σ'_h are effective vertical and horizontal pressures, respectively. Determining the total pressure distribution on the wall requires adding the hydrostatic pressure u, which is zero from $z = 0$ to $z = H_1$ and is $H_2\gamma_w$ at $z = H_2$. The variation of σ'_h and u with depth is shown in Figure 7.4b. Hence, the total force per unit length of the wall can be determined from the area of the pressure diagram. Specifically,

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where A = area of the pressure diagram.

So,

$$P_o = K_o q H_1 + \tfrac{1}{2}K_o\gamma H_1^2 + K_o(q + \gamma H_1)H_2 + \tfrac{1}{2}K_o\gamma' H_2^2 + \tfrac{1}{2}\gamma_w H_2^2 \tag{7.7}$$

Example 7.1

For the retaining wall shown in Figure 7.5(a), determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume OCR = 1.

Solution

$$K_o = 1 - \sin\phi' = 1 - \sin 30° = 0.5$$

At $z = 0$, $\sigma'_o = 0$; $\sigma'_h = 0$

At $z = 2.5$ m, $\sigma'_o = (16.5)(2.5) = 41.25 \text{ kN/m}^2$;

$$\sigma'_h = K_o\sigma'_o = (0.5)(41.25) = 20.63 \text{ kN/m}^2$$

At $z = 5$ m, $\sigma'_o = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98 \text{ kN/m}^2$;

$$\sigma'_h = K_o\sigma'_o = (0.5)(64.98) = 32.49 \text{ kN/m}^2$$

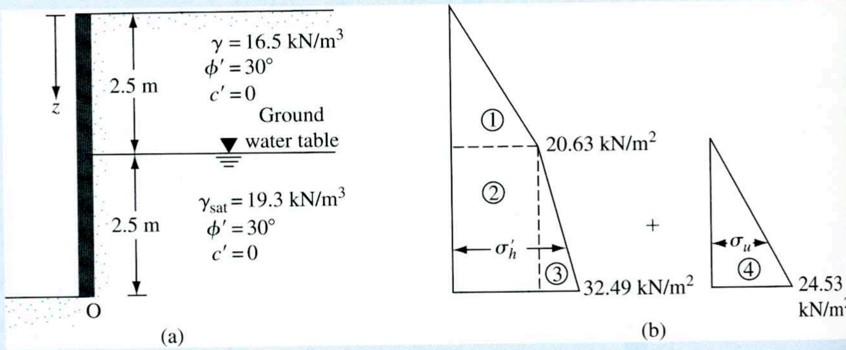


Figure 7.5

The hydrostatic pressure distribution is as follows:

From $z = 0$ to $z = 2.5$ m, $u = 0$. At $z = 5$ m, $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53$ kN/m^2. The pressure distribution for the wall is shown in Figure 7.5b.

The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_o = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$

$$= \tfrac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \tfrac{1}{2}(2.5)(32.49 - 20.63) + \tfrac{1}{2}(2.5)(24.53) = \mathbf{122.85\ kN/m}$$

The location of the center of pressure measured from the bottom of the wall (point O) =

$$\bar{z} = \frac{(\text{Area 1})\left(2.5+\dfrac{2.5}{3}\right)+(\text{Area 2})\left(\dfrac{2.5}{2}\right)+(\text{Area 3}+\text{Area 4})\left(\dfrac{2.5}{3}\right)}{P_o}$$

$$= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85}$$

$$= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53\ m}$$

Active Pressure

7.3 Rankine Active Earth Pressure

The lateral earth pressure described in Section 7.2 involves walls that do not yield at all. However, if a wall tends to move away from the soil a distance Δx, as shown in Figure 7.6a, the soil pressure on the wall at any depth will decrease. For a wall that is *frictionless*, the horizontal stress, σ'_h, at depth z will equal $K_o\sigma'_o(=K_o\gamma z)$ when Δx is zero. However, with $\Delta x > 0, \sigma'_h$ will be less than $K_o\sigma'_o$.

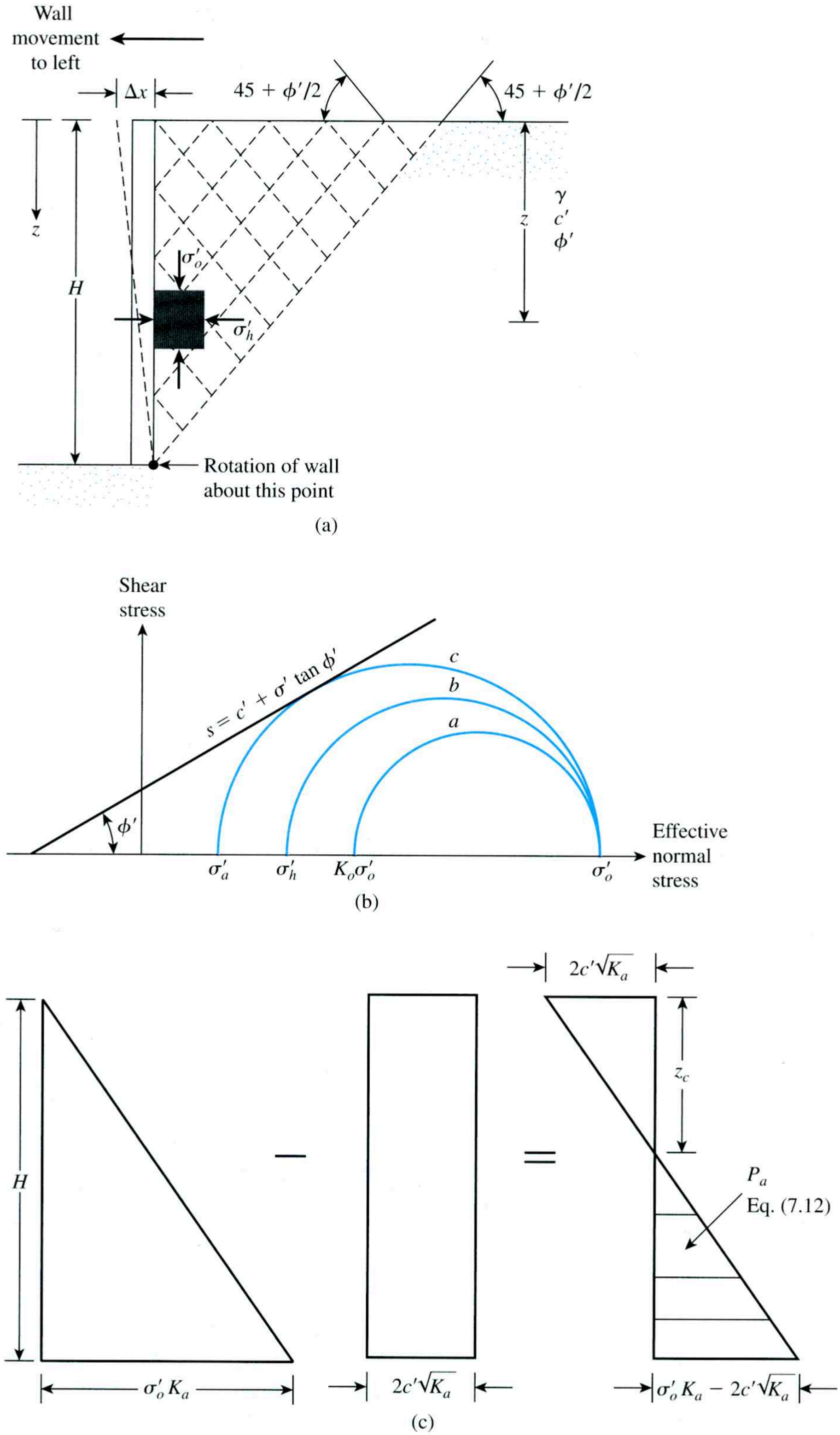


Figure 7.6 Rankine active pressure

The Mohr's circles corresponding to wall displacements of $\Delta x = 0$ and $\Delta x > 0$ are shown as circles *a* and *b*, respectively, in Figure 7.6b. If the displacement of the wall, Δx, continues to increase, the corresponding Mohr's circle eventually will just touch the Mohr–Coulomb failure envelope defined by the equation

$$s = c' + \sigma' \tan \phi'$$

This circle, marked *c* in the figure, represents the failure condition in the soil mass; the horizontal stress then equals σ'_a, referred to as the *Rankine active pressure*. The *slip lines* (failure planes) in the soil mass will then make angles of $\pm(45 + \phi'/2)$ with the horizontal, as shown in Figure 7.6a.

Equation (1.87) relates the principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

For the Mohr's circle *c* in Figure 7.6b,

Major principal stress, $\sigma'_1 = \sigma'_o$

and

Minor principal stress, $\sigma'_3 = \sigma'_a$

Thus,

$$\sigma'_o = \sigma'_a \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma'_a = \frac{\sigma'_o}{\tan^2\left(45 + \frac{\phi'}{2}\right)} - \frac{2c'}{\tan\left(45 + \frac{\phi'}{2}\right)}$$

or

$$\begin{aligned} \sigma'_a &= \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right) \\ &= \sigma'_o K_a - 2c'\sqrt{K_a} \end{aligned} \tag{7.8}$$

where $K_a = \tan^2(45 - \phi'/2)$ = Rankine active-pressure coefficient.

The variation of the active pressure with depth for the wall shown in Figure 7.6a is given in Figure 7.6c. Note that $\sigma'_o = 0$ at $z = 0$ and $\sigma'_o = \gamma H$ at $z = H$. The pressure distribution shows that at $z = 0$ the active pressure equals $-2c'\sqrt{K_a}$, indicating a tensile stress that decreases with depth and becomes zero at a depth $z = z_c$, or

$$\gamma z_c K_a - 2c'\sqrt{K_a} = 0$$

and

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} \tag{7.9}$$

The depth z_c is usually referred to as the *depth of tensile crack,* because the tensile stress in the soil will eventually cause a crack along the soil–wall interface. Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is

$$\begin{aligned} P_a &= \int_0^H \sigma'_a dz = \int_0^H \gamma z K_a dz - \int_0^H 2c'\sqrt{K_a}dz \\ &= \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a} \end{aligned} \tag{7.10}$$

After the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths $z = z_c$ and $z = H$, as shown by the hatched area in Figure 7.6c. This force may be expressed as

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) \tag{7.11}$$

or

$$P_a = \frac{1}{2}\left(H - \frac{2c'}{\gamma\sqrt{K_a}}\right)\left(\gamma H K_a - 2c'\sqrt{K_a}\right) \tag{7.12}$$

However, it is important to realize that the active earth pressure condition will be reached only if the wall is allowed to "yield" sufficiently. The necessary amount of outward displacement of the wall is about $0.001H$ to $0.004H$ for granular soil backfills and about $0.01H$ to $0.04H$ for cohesive soil backfills.

Note further that if the *total stress* shear strength parameters (c, ϕ) were used, an equation similar to Eq. (7.8) could have been derived, namely,

$$\sigma_a = \sigma_o \tan^2\left(45 - \frac{\phi}{2}\right) - 2c\tan\left(45 - \frac{\phi}{2}\right)$$

Example 7.2

A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4\ \text{kN/m}^3$, soil friction angle $\phi' = 26°$, and cohesion $c' = 14.36\ \text{kN/m}^2$. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

Solution

For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

From Figure 7.6c, at $z = 0$,

$$\sigma_a' = -2c'\sqrt{K_a} = -2(14.36)(0.625) = -17.95 \text{ kN/m}^2$$

and at $z = 6$ m,

$$\sigma_a' = (17.4)(6)(0.39) - 2(14.36)(0.625)$$
$$= 40.72 - 17.95 = 22.77 \text{ kN/m}^2$$

Active Force before the Tensile Crack Appeared: Eq. (7.10)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(17.95) = 122.16 - 107.7 = \mathbf{14.46 \text{ kN/m}}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (107.7)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 323.1}{14.46} = \mathbf{-5.45 \text{ m}.}$$

Active Force after the Tensile Crack Appeared: Eq. (7.9)

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(14.36)}{(17.4)(0.625)} = 2.64 \text{ m}$$

Using Eq. (7.11) gives

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \tfrac{1}{2}(6 - 2.64)(22.77) = \mathbf{38.25 \text{ kN/m}}$$

Figure 7.6c indicates that the force $P_a = 38.25$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 2.64}{3} = \mathbf{1.12 \text{ m}}$$

■

Example 7.3

Assume that the retaining wall shown in Figure 7.7a can yield sufficiently to develop an active state. Determine the Rankine active force per unit length of the wall and the location of the resultant line of action.

Solution

If the cohesion, c', is zero, then

$$\sigma_a' = \sigma_o' K_a$$

For the top layer of soil, $\phi_1' = 30°$, so

$$K_{a(1)} = \tan^2\left(45 - \frac{\phi_1'}{2}\right) = \tan^2(45 - 15) = \frac{1}{3}$$

Similarly, for the bottom layer of soil, $\phi_2' = 36°$, and it follows that

$$K_{a(2)} = \tan^2\left(45 - \frac{36}{2}\right) = 0.26$$

The following table shows the calculation of σ_a' and u at various depths below the ground surface.

Depth, z (ft)	σ_o' (lb/ft^2)	K_a	$\sigma_a' = K_a\sigma_o'$ (lb/ft^2)	u (lb/ft^2)
0	0	1/3	0	0
10^-	(102)(10) = 1020	1/3	340	0
10^+	1020	0.26	265.2	0
20	(102)(10) + (121 − 62.4)(10) = 1606	0.26	417.6	(62.4)(10) = 624

The pressure distribution diagram is plotted in Figure 7.7b. The force per unit length is

$$P_a = \text{area 1} + \text{area 2} + \text{area 3} + \text{area 4}$$

$$= \tfrac{1}{2}(10)(340) + (265.2)(10) + \tfrac{1}{2}(417.6 - 265.2)(10) + \tfrac{1}{2}(624)(10)$$

$$= 1700 + 2652 + 762 + 3120 = \mathbf{8234\ lb/ft}$$

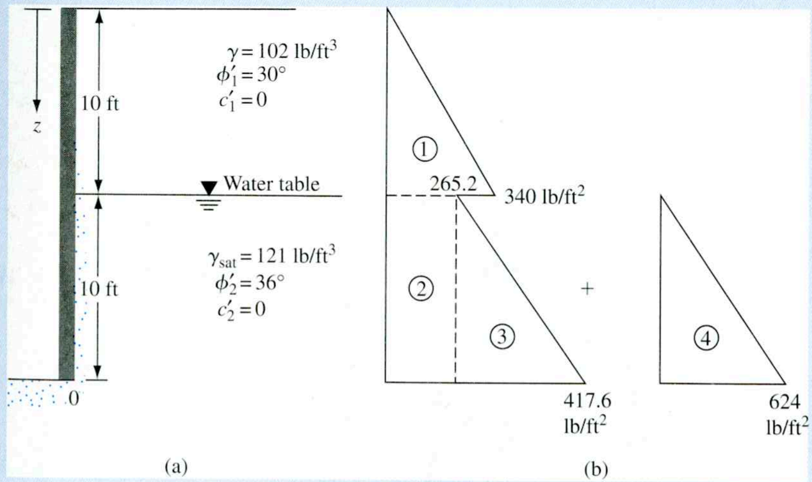


Figure 7.7 Rankine active force behind a retaining wall

The distance of the line of action of the resultant force from the bottom of the wall can be determined by taking the moments about the bottom of the wall (point O in Figure 7.7a) and is

$$\bar{z} = \frac{(1700)\left(10 + \frac{10}{2}\right) + (2652)\left(\frac{10}{2}\right) + (762 + 3120)\left(\frac{10}{3}\right)}{8234} = \mathbf{5.93\ ft}$$

7.4 A Generalized Case for Rankine Active Pressure

In Section 7.3, the relationship was developed for Rankine active pressure for a retaining wall with a vertical back and a horizontal backfill. That can be extended to general cases of frictionless walls with inclined backs and inclined backfills. Some of these cases will be discussed in this section.

Granular Backfill

Figure 7.8 shows a retaining wall whose back is inclined at an angle θ with the vertical. The granular backfill is inclined at an angle α with the horizontal.

For a Rankine active case, the lateral earth pressure (σ'_a) at a depth z can be given as (Chu, 1991),

$$\sigma'_a = \frac{\gamma z \cos\alpha \sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}} \tag{7.13}$$

$$\text{where } \psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta. \tag{7.14}$$

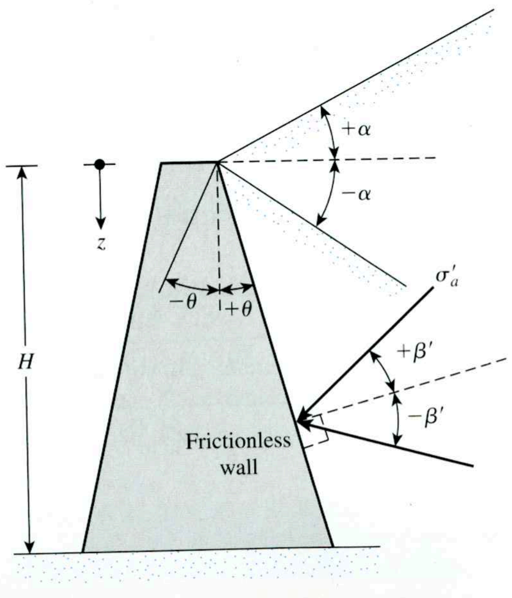


Figure 7.8 General case for a retaining wall with granular backfill

The pressure σ'_a will be inclined at an angle β' with the plane drawn at right angle to the backface of the wall, and

$$\beta' = \tan^{-1}\left(\frac{\sin\phi' \sin\psi_a}{1 - \sin\phi' \cos\psi_a}\right) \tag{7.15}$$

The active force P_a for unit length of the wall then can be calculated as

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{7.16}$$

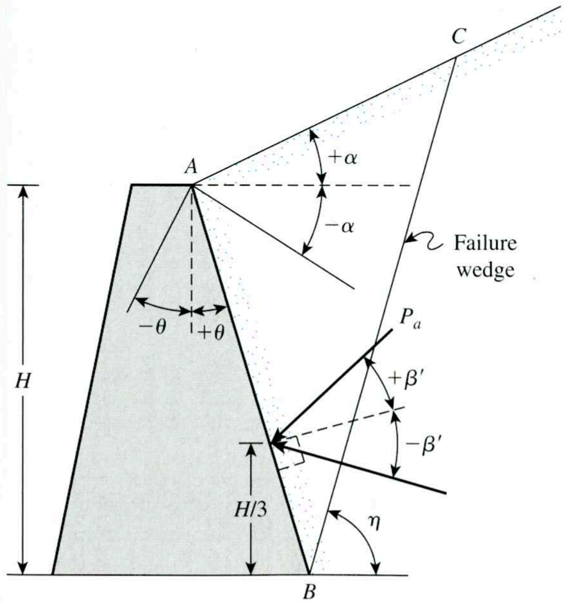
where

$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

$$= \text{Rankine active earth-pressure coefficient for generalized case} \tag{7.17}$$

The location and direction of the resultant force P_a is shown in Figure 7.9. Also shown in this figure is the failure wedge, ABC. Note that BC will be inclined at an angle η. Or

$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) \tag{7.18}$$



$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$

Figure 7.9 Location and direction of Rankine active force

Granular Backfill with Vertical Back Face

As a special case, for a vertical backface of a wall (that is, $\theta = 0$), as shown in Figure 7.10, Eqs. (7.13), (7.16) and (7.17) simplify to the following.

If the backfill of a frictionless retaining wall is a *granular soil* ($c' = 0$) and rises at an angle α with respect to the horizontal (see Figure 7.10), the *active earth-pressure coefficient* may be expressed in the form

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}} \tag{7.19}$$

where ϕ' = angle of friction of soil.

At any depth z, the *Rankine active pressure* may be expressed as

$$\sigma'_a = \gamma z K_a \tag{7.20}$$

Also, the total force per unit length of the wall is

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{7.21}$$

Note that, in this case, the direction of the resultant force P_a is *inclined at an angle α with the horizontal* and intersects the wall at a distance $H/3$ from the base of the wall. Table 7.1 presents the values of K_a (active earth pressure) for various values of α and ϕ'.

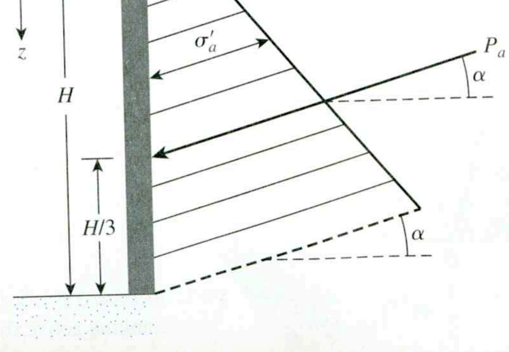


Figure 7.10 Notations for active pressure—Eqs. (7.19), (7.20), (7.21)

Table 7.1 Values of K_a [Eq. (7.19)]

α (deg) ↓	ϕ' (deg) → 28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.2200
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.2209
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.2220
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.2233
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.2247
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.2263
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.2281
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.2301
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.2322
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.2346
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.2373
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.2401
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.2433
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.2467
20	0.4605	0.4365	0.4142	0.3934	0.3739	0.3555	0.3381	0.3216	0.3060	0.2911	0.2769	0.2634	0.2504
21	0.4758	0.4498	0.4259	0.4037	0.3830	0.3637	0.3455	0.3283	0.3120	0.2965	0.2818	0.2678	0.2545
22	0.4936	0.4651	0.4392	0.4154	0.3934	0.3729	0.3537	0.3356	0.3186	0.3025	0.2872	0.2727	0.2590
23	0.5147	0.4829	0.4545	0.4287	0.4050	0.3832	0.3628	0.3438	0.3259	0.3091	0.2932	0.2781	0.2638
24	0.5404	0.5041	0.4724	0.4440	0.4183	0.3948	0.3731	0.3529	0.3341	0.3164	0.2997	0.2840	0.2692
25	0.5727	0.5299	0.4936	0.4619	0.4336	0.4081	0.3847	0.3631	0.3431	0.3245	0.3070	0.2905	0.2750

Vertical Backface with c'–ϕ' Soil Backfill

For a retaining wall with a *vertical back* ($\theta = 0$) and *inclined backfill* of c'–ϕ' soil (Mazindrani and Ganjali, 1997),

$$\sigma'_a = \gamma z K_a = \gamma z K'_a \cos\alpha \tag{7.22}$$

where

$$K'_a = \frac{1}{\cos^2\phi'}\left\{\begin{array}{l} 2\cos^2\alpha + 2\left(\dfrac{c'}{\gamma z}\right)\cos\phi' \sin\phi' \\ -\sqrt{\left[4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\dfrac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\dfrac{c'}{\gamma z}\right)\cos^2\alpha \sin\phi' \cos\phi'\right]}\end{array}\right\} - 1 \tag{7.23}$$

Some values of K'_a are given in Table 7.2. For a problem of this type, the depth of tensile crack is given as

$$z_c = \frac{2c'}{\gamma}\sqrt{\frac{1 + \sin\phi'}{1 - \sin\phi'}} \tag{7.24}$$

For this case, the active pressure is inclined at an angle α with the horizontal.

Table 7.2 Values of K'_a

		$\frac{c'}{\gamma z}$			
ϕ' (deg)	α (deg)	0.025	0.05	0.1	0.5
15	0	0.550	0.512	0.435	−0.179
	5	0.566	0.525	0.445	−0.184
	10	0.621	0.571	0.477	−0.186
	15	0.776	0.683	0.546	−0.196
20	0	0.455	0.420	0.350	−0.210
	5	0.465	0.429	0.357	−0.212
	10	0.497	0.456	0.377	−0.218
	15	0.567	0.514	0.417	−0.229
25	0	0.374	0.342	0.278	−0.231
	5	0.381	0.348	0.283	−0.233
	10	0.402	0.366	0.296	−0.239
	15	0.443	0.401	0.321	−0.250
30	0	0.305	0.276	0.218	−0.244
	5	0.309	0.280	0.221	−0.246
	10	0.323	0.292	0.230	−0.252
	15	0.350	0.315	0.246	−0.263

Example 7.4

Refer to the retaining wall in Figure 7.9. The backfill is granular soil. Given:

$$\text{Wall:} \quad H = 10\,\text{ft}, \quad \theta = +10^\circ$$

$$\text{Backfill:} \quad \alpha = 15^\circ, \quad \phi' = 35^\circ, \quad c' = 0, \quad \gamma = 110\ \text{lb/ft}^3$$

Determine the Rankine active force, P_a, and its location and direction.

Solution

From Eq. (7.14),

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta = \sin^{-1}\left(\frac{\sin 15}{\sin 35}\right) - 15 + (2)(10) = 31.82^\circ$$

From Eq. (7.17),

$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

$$= \frac{\cos(15 - 10)\sqrt{1 + \sin^2 35 - (2)(\sin 35)(\sin 31.82)}}{\cos^2 10\left(\cos 15 + \sqrt{\sin^2 35 - \sin^2 15}\right)} = 0.59$$

$$P_a = \tfrac{1}{2}\gamma H^2 K_a = (\tfrac{1}{2})(110)(10)^2(0.59) = \mathbf{3245\ lb/ft}$$

From Eq. (7.15),

$$\beta' = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_a}{1 - \sin\phi'\cos\psi_a}\right) = \tan^{-1}\left[\frac{(\sin 35)(\sin 31.82)}{1 - (\sin 35)(\cos 31.82)}\right] = \mathbf{30.5^\circ}$$

The force P_a will act at a distance of $10/3 = 3.33$ ft above the bottom of the wall and will be inclined at an angle of $+30.5^\circ$ to the normal drawn to the back face of the wall. ■

Example 7.5

For the retaining wall shown in Figure 7.10, $H = 7.5$ m, $\gamma = 18\ \text{kN/m}^3$, $\phi' = 20^\circ$, $c' = 13.5\ \text{kN/m}^2$, and $\alpha = 10^\circ$. Calculate the Rankine active force, P_a, per unit length of the wall and the location of the resultant force after the occurrence of the tensile crack.

Solution

From Eq. (7.24).

$$z_r = \frac{2c'}{\gamma}\sqrt{\frac{1 + \sin\phi'}{1 - \sin\phi'}} = \frac{(2)(13.5)}{18}\sqrt{\frac{1 + \sin 20}{1 - \sin 20}} = 2.14\ \text{m}$$

At $z = 7.5$ m,

$$\frac{c'}{\gamma z} = \frac{13.5}{(18)(7.5)} = 0.1$$

From Table 7.2, for $\phi' = 20°$, $c'/\gamma z = 0.1$, and $\alpha = 10°$, the value of K_a' is 0.377, so at $z = 7.5$ m,

$$\sigma_a' = \gamma z K_a' \cos\alpha = (18)(7.5)(0.377)(\cos 10) = 50.1 \text{ kN/m}^2$$

After the occurrence of the tensile crack, the pressure distribution on the wall will be as shown in Figure 7.11, so

$$P_a = \left(\frac{1}{2}\right)(50.1)(7.5 - 2.14) = \mathbf{134.3\ kN/m}$$

and

$$\bar{z} = \frac{7.5 - 2.14}{3} = \mathbf{1.79\ m}$$

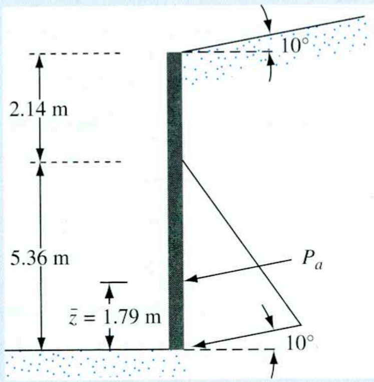


Figure 7.11 Calculation of Rankine active force, $c' - \phi'$ soil

7.5 Coulomb's Active Earth Pressure

The Rankine active earth pressure calculations discussed in the preceding sections were based on the assumption that the wall is frictionless. In 1776, Coulomb proposed a theory for calculating the lateral earth pressure on a retaining wall with granular soil backfill. This theory takes wall friction into consideration.

To apply Coulomb's active earth pressure theory, let us consider a retaining wall with its back face inclined at an angle β with the horizontal, as shown in Figure 7.12a. The backfill is a granular soil that slopes at an angle α with the horizontal.

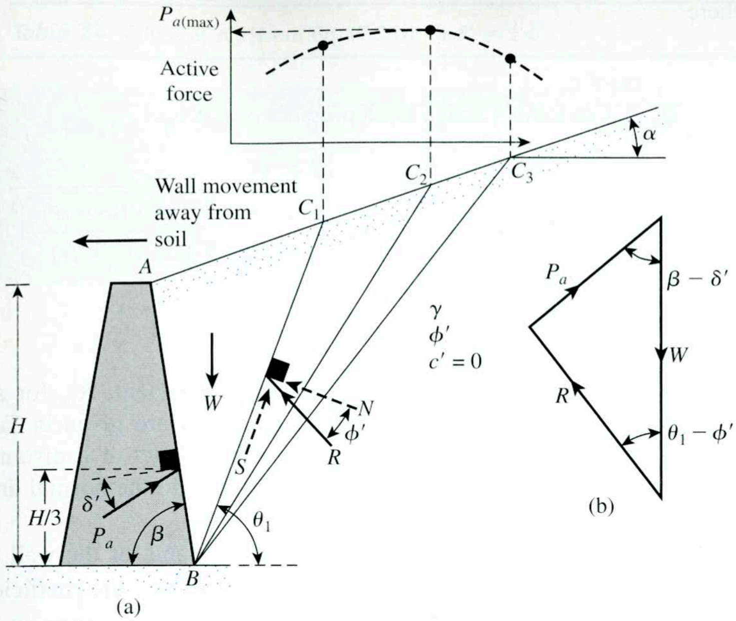


Figure 7.12 Coulomb's active pressure

Also, let δ' be the angle of friction between the soil and the wall (i.e., the angle of wall friction).

Under active pressure, the wall will move away from the soil mass (to the left in the figure). Coulomb assumed that, in such a case, the failure surface in the soil mass would be a plane (e.g., BC_1, BC_2, . . .). So, to find the active force, consider a possible soil failure wedge ABC_1. The forces acting on this wedge (per unit length at right angles to the cross section shown) are as follows:

1. The weight of the wedge, W.
2. The resultant, R, of the normal and resisting shear forces along the surface, BC_1. The force R will be inclined at an angle ϕ' to the normal drawn to BC_1.
3. The active force per unit length of the wall, P_a, which will be inclined at an angle δ' to the normal drawn to the back face of the wall.

For equilibrium purposes, a force triangle can be drawn, as shown in Figure 7.12b. Note that θ_1 is the angle that BC_1 makes with the horizontal. Because the magnitude of W, as well as the directions of all three forces, are known, the value of P_a can now be determined. Similarly, the active forces of other trial wedges, such as ABC_2, ABC_3, . . . , can be determined. The maximum value of P_a thus determined is Coulomb's active force (see top part of Figure 7.12), which may be expressed as

$$P_a = \tfrac{1}{2} K_a \gamma H^2 \qquad (7.25)$$

where

$$K_a = \text{Coulomb's active earth pressure coefficient}$$

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2} \tag{7.26}$$

and H = height of the wall.

The values of the active earth pressure coefficient, K_a, for a vertical retaining wall ($\beta = 90°$) with horizontal backfill ($\alpha = 0°$) are given in Table 7.3. Note that the line of action of the resultant force (P_a) will act at a distance $H/3$ above the base of the wall and will be inclined at an angle δ' to the normal drawn to the back of the wall.

In the actual design of retaining walls, the value of the wall friction angle δ' is assumed to be between $\phi'/2$ and $\frac{2}{3}\phi'$. The active earth pressure coefficients for various values of ϕ', α, and β with $\delta' = \frac{1}{2}\phi'$ and $\frac{2}{3}\phi'$ are respectively given in Tables 7.4 and 7.5. These coefficients are very useful design considerations.

If a uniform surcharge of intensity q is located above the backfill, as shown in Figure 7.13, the active force, P_a, can be calculated as

$$P_a = \tfrac{1}{2}K_a \underset{\substack{\uparrow \\ \text{Eq. (7.25)}}}{\gamma_{eq}} H^2 \tag{7.27}$$

where

$$\gamma_{eq} = \gamma + \left[\frac{\sin\beta}{\sin(\beta + \alpha)}\right]\left(\frac{2q}{H}\right) \tag{7.28}$$

Table 7.3 Values of K_a [Eq. (7.26)] for $\beta = 90°$ and $\alpha = 0°$

	δ' (deg)					
ϕ' (deg)	**0**	**5**	**10**	**15**	**20**	**25**
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2098	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

Table 7.4 Values of K_a [from Eq. (7.26)] for $\delta' = \frac{2}{3}\phi'$

		β (deg)					
α (deg)	ϕ' (deg)	90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
	42	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
5	28	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	29	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	30	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	31	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	32	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	33	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	34	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	35	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	36	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	37	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	38	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	39	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	40	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
	41	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718
	42	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
10	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	29	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	31	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	33	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	35	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	37	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	38	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	39	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
	40	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
	41	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097
	42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
15	28	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

(continued)

Table 7.4 *(Continued)*

α (deg)	ϕ' (deg)	β (deg) 90	85	80	75	70	65
	29	0.3881	0.4397	0.4987	0.5672	0.6483	0.7463
	30	0.3707	0.4219	0.4804	0.5484	0.6291	0.7265
	31	0.3541	0.4049	0.4629	0.5305	0.6106	0.7076
	32	0.3384	0.3887	0.4462	0.5133	0.5930	0.6895
	33	0.3234	0.3732	0.4303	0.4969	0.5761	0.6721
	34	0.3091	0.3583	0.4150	0.4811	0.5598	0.6554
	35	0.2954	0.3442	0.4003	0.4659	0.5442	0.6393
	36	0.2823	0.3306	0.3862	0.4513	0.5291	0.6238
	37	0.2698	0.3175	0.3726	0.4373	0.5146	0.6089
	38	0.2578	0.3050	0.3595	0.4237	0.5006	0.5945
	39	0.2463	0.2929	0.3470	0.4106	0.4871	0.5805
	40	0.2353	0.2813	0.3348	0.3980	0.4740	0.5671
	41	0.2247	0.2702	0.3231	0.3858	0.4613	0.5541
	42	0.2146	0.2594	0.3118	0.3740	0.4491	0.5415
20	28	0.4602	0.5205	0.5900	0.6714	0.7689	0.8880
	29	0.4364	0.4958	0.5642	0.6445	0.7406	0.8581
	30	0.4142	0.4728	0.5403	0.6195	0.7144	0.8303
	31	0.3935	0.4513	0.5179	0.5961	0.6898	0.8043
	32	0.3742	0.4311	0.4968	0.5741	0.6666	0.7799
	33	0.3559	0.4121	0.4769	0.5532	0.6448	0.7569
	34	0.3388	0.3941	0.4581	0.5335	0.6241	0.7351
	35	0.3225	0.3771	0.4402	0.5148	0.6044	0.7144
	36	0.3071	0.3609	0.4233	0.4969	0.5856	0.6947
	37	0.2925	0.3455	0.4071	0.4799	0.5677	0.6759
	38	0.2787	0.3308	0.3916	0.4636	0.5506	0.6579
	39	0.2654	0.3168	0.3768	0.4480	0.5342	0.6407
	40	0.2529	0.3034	0.3626	0.4331	0.5185	0.6242
	41	0.2408	0.2906	0.3490	0.4187	0.5033	0.6083
	42	0.2294	0.2784	0.3360	0.4049	0.4888	0.5930

Table 7.5 Values of K_a [from Eq. (7.26)] for $\delta' = \phi'/2$

α (deg)	ϕ' (deg)	β (deg) 90	85	80	75	70	65
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807
	36	0.2362	0.2718	0.3118	0.3571	0.4092	0.4702

Table 7.5 *(Continued)*

		β (deg)					
α (deg)	ϕ' (deg)	90	85	80	75	70	65
	37	0.2265	0.2620	0.3017	0.3469	0.3990	0.4599
	38	0.2172	0.2524	0.2920	0.3370	0.3890	0.4498
	39	0.2081	0.2431	0.2825	0.3273	0.3792	0.4400
	40	0.1994	0.2341	0.2732	0.3179	0.3696	0.4304
	41	0.1909	0.2253	0.2642	0.3087	0.3602	0.4209
	42	0.1828	0.2168	0.2554	0.2997	0.3511	0.4177
5	28	0.3477	0.3879	0.4327	0.4837	0.5425	0.6115
	29	0.3337	0.3737	0.4185	0.4694	0.5282	0.5972
	30	0.3202	0.3601	0.4048	0.4556	0.5144	0.5833
	31	0.3072	0.3470	0.3915	0.4422	0.5009	0.5698
	32	0.2946	0.3342	0.3787	0.4292	0.4878	0.5566
	33	0.2825	0.3219	0.3662	0.4166	0.4750	0.5437
	34	0.2709	0.3101	0.3541	0.4043	0.4626	0.5312
	35	0.2596	0.2986	0.3424	0.3924	0.4505	0.5190
	36	0.2488	0.2874	0.3310	0.3808	0.4387	0.5070
	37	0.2383	0.2767	0.3199	0.3695	0.4272	0.4954
	38	0.2282	0.2662	0.3092	0.3585	0.4160	0.4840
	39	0.2185	0.2561	0.2988	0.3478	0.4050	0.4729
	40	0.2090	0.2463	0.2887	0.3374	0.3944	0.4620
	41	0.1999	0.2368	0.2788	0.3273	0.3840	0.4514
	42	0.1911	0.2276	0.2693	0.3174	0.3738	0.4410
10	28	0.3743	0.4187	0.4688	0.5261	0.5928	0.6719
	29	0.3584	0.4026	0.4525	0.5096	0.5761	0.6549
	30	0.3432	0.3872	0.4368	0.4936	0.5599	0.6385
	31	0.3286	0.3723	0.4217	0.4782	0.5442	0.6225
	32	0.3145	0.3580	0.4071	0.4633	0.5290	0.6071
	33	0.3011	0.3442	0.3930	0.4489	0.5143	0.5920
	34	0.2881	0.3309	0.3793	0.4350	0.5000	0.5775
	35	0.2757	0.3181	0.3662	0.4215	0.4862	0.5633
	36	0.2637	0.3058	0.3534	0.4084	0.4727	0.5495
	37	0.2522	0.2938	0.3411	0.3957	0.4597	0.5361
	38	0.2412	0.2823	0.3292	0.3833	0.4470	0.5230
	39	0.2305	0.2712	0.3176	0.3714	0.4346	0.5103
	40	0.2202	0.2604	0.3064	0.3597	0.4226	0.4979
	41	0.2103	0.2500	0.2956	0.3484	0.4109	0.4858
	42	0.2007	0.2400	0.2850	0.3375	0.3995	0.4740
15	28	0.4095	0.4594	0.5159	0.5812	0.6579	0.7498
	29	0.3908	0.4402	0.4964	0.5611	0.6373	0.7284
	30	0.3730	0.4220	0.4777	0.5419	0.6175	0.7080
	31	0.3560	0.4046	0.4598	0.5235	0.5985	0.6884
	32	0.3398	0.3880	0.4427	0.5059	0.5803	0.6695
	33	0.3244	0.3721	0.4262	0.4889	0.5627	0.6513
	34	0.3097	0.3568	0.4105	0.4726	0.5458	0.6338
	35	0.2956	0.3422	0.3953	0.4569	0.5295	0.6168

(continued)

Table 7.5 (*Continued*)

α (deg)	ϕ' (deg)	β (deg) 90	85	80	75	70	65
	36	0.2821	0.3282	0.3807	0.4417	0.5138	0.6004
	37	0.2692	0.3147	0.3667	0.4271	0.4985	0.5846
	38	0.2569	0.3017	0.3531	0.4130	0.4838	0.5692
	39	0.2450	0.2893	0.3401	0.3993	0.4695	0.5543
	40	0.2336	0.2773	0.3275	0.3861	0.4557	0.5399
	41	0.2227	0.2657	0.3153	0.3733	0.4423	0.5258
	42	0.2122	0.2546	0.3035	0.3609	0.4293	0.5122
20	28	0.4614	0.5188	0.5844	0.6608	0.7514	0.8613
	29	0.4374	0.4940	0.5586	0.6339	0.7232	0.8313
	30	0.4150	0.4708	0.5345	0.6087	0.6968	0.8034
	31	0.3941	0.4491	0.5119	0.5851	0.6720	0.7772
	32	0.3744	0.4286	0.4906	0.5628	0.6486	0.7524
	33	0.3559	0.4093	0.4704	0.5417	0.6264	0.7289
	34	0.3384	0.3910	0.4513	0.5216	0.6052	0.7066
	35	0.3218	0.3736	0.4331	0.5025	0.5851	0.6853
	36	0.3061	0.3571	0.4157	0.4842	0.5658	0.6649
	37	0.2911	0.3413	0.3991	0.4668	0.5474	0.6453
	38	0.2769	0.3263	0.3833	0.4500	0.5297	0.6266
	39	0.2633	0.3120	0.3681	0.4340	0.5127	0.6085
	40	0.2504	0.2982	0.3535	0.4185	0.4963	0.5912
	41	0.2381	0.2851	0.3395	0.4037	0.4805	0.5744
	42	0.2263	0.2725	0.3261	0.3894	0.4653	0.5582

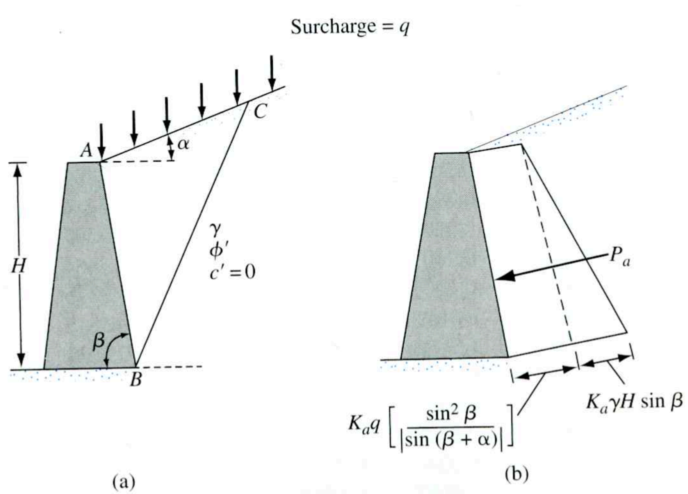


Figure 7.13 Coulomb's active pressure with a surcharge on the backfill

Example 7.6

Consider the retaining wall shown in Figure 7.12a. Given: $H = 4.6$ m; unit weight of soil = 16.5 kN/m^3; angle of friction of soil = 30°; wall friction-angle, $\delta' = \frac{2}{3}\phi'$, soil cohesion, $c' = 0$; $\alpha = 0$, and $\beta = 90°$. Calculate the Coulomb's active force per unit length of the wall.

Solution

From Eq. (7.25)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a$$

From Table 7.4, for $\alpha = 0°$, $\beta = 90°$, $\phi' = 30°$, and $\delta' = \frac{2}{3}\phi' = 20°$, $K_a = 0.297$. Hence,

$$P_a = \tfrac{1}{2}(16.5)(4.6)^2(0.297) = \mathbf{51.85\ kN/m}$$

Example 7.7

Refer to Figure 7.13a. Given: $H = 20$ ft, $\phi' = 30°$, $\delta' = 20°$, $\alpha = 5°$, $\beta = 85°$, $q = 2000$ lb/ft^2, and $\gamma = 115$ lb/ft^3. Determine Coulomb's active force and the location of the line of action of the resultant P_a.

Solution

For $\beta = 85°$, $\alpha = 5°$, $\delta' = 20°$, $\phi' = 30°$, and $K_a = 0.3578$ (Table 7.4). From Eqs. (7.27) and (7.28),

$$p_a = \frac{1}{2}K_a\gamma_{eq}H^2 = \frac{1}{2}K_a\left[\gamma + \frac{2q}{H}\frac{\sin\beta}{\sin(\beta+\alpha)}\right]H^2 = \underbrace{\frac{1}{2}K_a\gamma H^2}_{P_{a(1)}} \underbrace{+ K_a Hq\left[\frac{\sin\beta}{\sin(\beta+\alpha)}\right]}_{P_{a(2)}}$$

$$= (0.5)(0.3578)(115)(20)^2) + (0.3578)(20)(2000)\left[\frac{\sin 85}{\sin(85+5)}\right]$$

$$= 8229.4 + 14{,}257.5 = \mathbf{22{,}486.9\ lb/ft}$$

Location of the line of action of the resultant:

$$P_a\bar{z} = P_{a(1)}\frac{H}{3} + P_{a(2)}\frac{H}{2}$$

or

$$\bar{z} = \frac{(8229.4)\left(\frac{20}{3}\right) + (14{,}257.5)\left(\frac{20}{2}\right)}{22{,}486.9}$$

$$= \mathbf{8.78\ ft} \text{ (measured vertically from the bottom of the wall)}$$

7.6 Lateral Earth Pressure Due to Surcharge

In several instances, the theory of elasticity is used to determine the lateral earth pressure on unyielding retaining structures caused by various types of surcharge loading, such as *line loading* (Figure 7.14a) and *strip loading* (Figure 7.14b).

According to the theory of elasticity, the stress at any depth, z, on a retaining structure caused by a line load of intensity q/unit length (Figure 7.14a) may be given as

$$\sigma = \frac{2q}{\pi H} \frac{a^2 b}{(a^2 + b^2)^2} \tag{7.29}$$

where σ = horizontal stress at depth $z = bH$

(See Figure 7.14a for explanations of the terms a and b.)

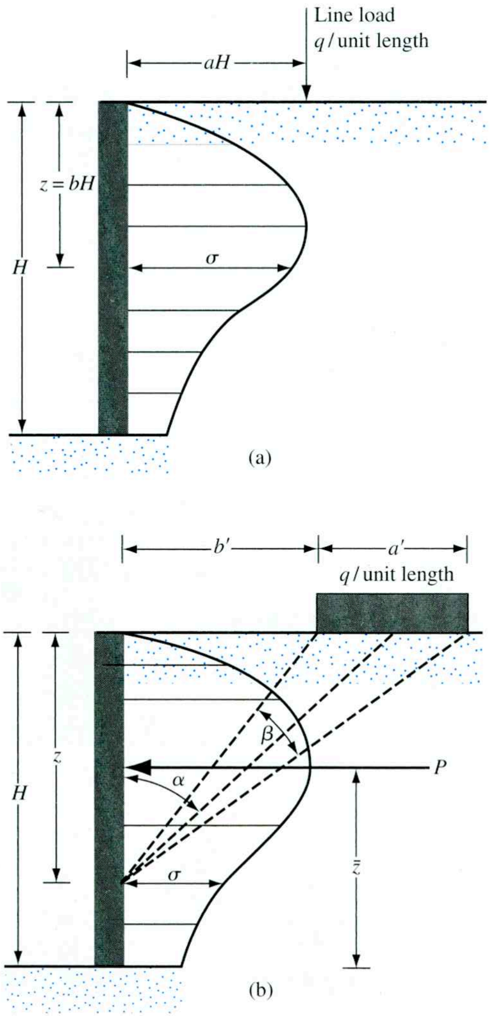


Figure 7.14 Lateral earth pressure caused by (a) line load and (b) strip load

However, because soil is not a perfectly elastic medium, some deviations from Eq. (7.29) may be expected. The modified forms of this equation generally accepted for use with soils are as follows:

$$\sigma = \frac{4a}{\pi H} \frac{a^2 b}{(a^2 + b^2)} \qquad \text{for } a > 0.4 \tag{7.30}$$

and

$$\sigma = \frac{q}{H} \frac{0.203b}{(0.16 + b^2)^2} \qquad \text{for } a \leq 0.4 \tag{7.31}$$

Figure 7.14b shows a strip load with an intensity of q/unit area located at a distance b' from a wall of height H. Based on the theory of elasticity, the horizontal stress, σ, at any depth z on a retaining structure is

$$\sigma = \frac{q}{\pi}(\beta - \sin\beta \cos 2\alpha) \tag{7.32}$$

(The angles α and β are defined in Figure 7.14b.)

However, in the case of soils, the right-hand side of Eq. (7.32) is doubled to account for the yielding soil continuum, or

$$\sigma = \frac{2q}{\pi}(\beta - \sin\beta \cos 2\alpha) \tag{7.33}$$

The total force per unit length (P) due to the *strip loading only* (Jarquio, 1981) may be expressed as

$$P = \frac{q}{90}[H(\theta_2 - \theta_1)] \tag{7.34}$$

where

$$\theta_1 = \tan^{-}\left(\frac{b'}{H}\right) \quad (\text{deg}) \tag{7.35}$$

$$\theta_2 = \tan^{-1}\left(\frac{a' + b'}{H}\right) (\text{deg}) \tag{7.36}$$

The location $\bar{z}$ (see Figure 7.14b) of the resultant force, P, can be given as

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right] \tag{7.37}$$

where

$$R = (a' + b')^2(90 - \theta_2) \tag{7.38}$$

$$Q = b'^2(90 - \theta_1) \tag{7.39}$$

Example 7.8

Refer to Figure 7.14b. Here, $a' = 2$ m, $b' = 1$ m, $q = 40$ kN/m^2, and $H = 6$ m. Determine the total force on the wall (kN/m) caused by the strip loading only.

Solution From Eqs. (7.35) and (7.36),

$$\theta_1 = \tan^{-1}\left(\frac{1}{6}\right) = 9.46°$$

$$\theta_2 = \tan^{-1}\left(\frac{2+1}{6}\right) = 26.57°$$

From Eq. (7.34)

$$P = \frac{q}{90}[H(\theta_2 - \theta_1)] = \frac{40}{90}[6(26.57 - 9.46)] = \mathbf{45.63\ kN/m}$$

Example 7.9

Refer to Example 7.8. Determine the location of the resultant $\bar{z}$.

Solution

From Eqs. (7.38) and (7.39),

$$R = (a' + b')^2(90 - \theta_2) = (2+1)^2(90 - 26.57) = 570.87$$

$$Q = b'^2(90 - \theta_1) = (1)^2(90 - 9.46) = 80.54$$

From Eq. (7.37),

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$

$$= 6 - \left[\frac{(6)^2(26.57 - 9.46) + (570.87 - 80.54) - (57.3)(2)(6)}{(2)(6)(26.57 - 9.46)}\right] = \mathbf{3.96\ m}$$

7.7 Active Earth Pressure for Earthquake Conditions

Coulomb's active earth pressure theory (see Section 7.5) can be extended to take into account the forces caused by an earthquake. Figure 7.15 shows a condition of active pressure with a granular backfill ($c' = 0$). Note that the forces acting on the soil failure wedge in Figure 7.15

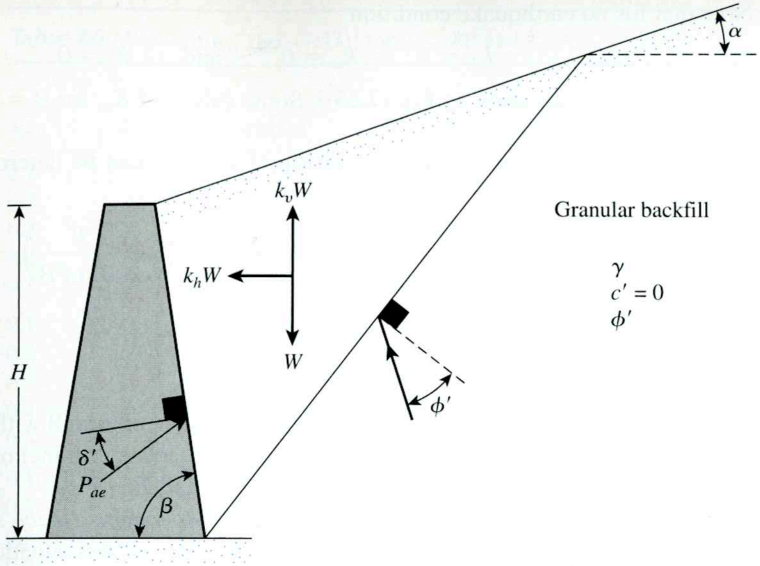


Figure 7.15 Derivation of Eq. (7.42)

are essentially the same as those shown in Figure 7.12a with the addition of k_hW and k_vW in the horizontal and vertical direction respectively; k_h and k_v may be defined as

$$k_h = \frac{\text{horizontal earthquake acceleration component}}{\text{acceleration due to gravity, } g} \tag{7.40}$$

$$k_v = \frac{\text{vertical earthquake acceleration component}}{\text{acceleration due to gravity, } g} \tag{7.41}$$

As in Section 7.5, the relation for the active force per unit length of the wall (P_{ae}) can be determined as

$$P_{ae} = \tfrac{1}{2}\gamma H^2(1 - k_v)K_{ae} \tag{7.42}$$

where

$$K_{ae} = \text{active earth pressure coefficient}$$

$$= \frac{\sin^2(\phi' + \beta - \theta')}{\cos\theta' \sin^2\beta \sin(\beta - \theta' - \delta')\left[1 + \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' - \theta' - \alpha)}{\sin(\beta - \delta' - \theta')\sin(\alpha + \beta)}}\right]^2} \tag{7.43}$$

$$\theta' = \tan^{-1}\left[\frac{k_h}{(1 - k_v)}\right] \tag{7.44}$$

Note that for no earthquake condition

$$k_h = 0, \qquad k_v = 0, \qquad \text{and} \qquad \theta' = 0$$

Hence $K_{ae} = K_a$ [as given by Eq. (7.26)]. Some values of K_{ae} for $\beta = 90°$ and $k_v = 0$ are given in Table 7.6.

The magnitude of P_{ae} as given in Eq. (7.42) also can be determined as (Seed and Whitman, 1970),

$$P_{ae} = \frac{1}{2}\gamma H^2(1 - k_v)[K_a(\beta',\alpha')]\left(\frac{\sin^2\beta'}{\cos\theta' \sin^2\beta}\right) \tag{7.45}$$

where

$$\beta' = \beta - \theta' \tag{7.46}$$

$$\alpha' = \theta' + \alpha \tag{7.47}$$

$K_a(\beta',\alpha')$ = Coulomb's active earth-pressure coefficient on a wall with a back face inclination of β' with the horizontal and with a back fill inclined at an angle α' with the horizontal (such as Tables 7.4 and 7.5)

Equation (7.42) is usually referred to as the *Mononobe–Okabe* solution. Unlike the case shown in Figure 7.12a, the resultant earth pressure in this situation, as calculated by Eq. (7.42) *does not act* at a distance of $H/3$ from the bottom of the wall. The following procedure may be used to obtain the location of the resultant force P_{ae}:

Step 1. Calculate P_{ae} by using Eq. (7.42)
Step 2. Calculate P_a by using Eq. (7.25)
Step 3. Calculate

$$\Delta P_{ae} = P_{ae} - P_a \tag{7.48}$$

Step 4. Assume that P_a acts at a distance of $H/3$ from the bottom of the wall (Figure 7.16)

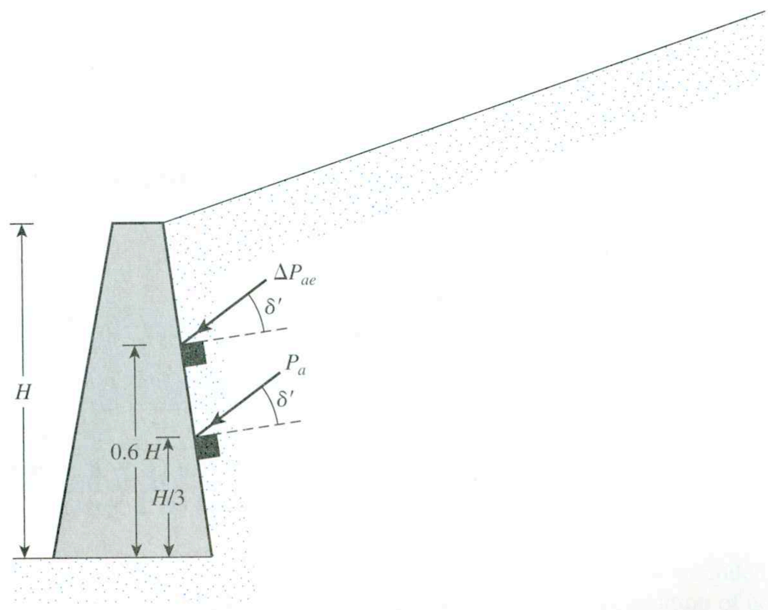


Figure 7.16 Determining the line of action of P_{ae}

Table 7.6 Values of K_{ae} [Eq. (7.43)] for $\beta = 90°$ and $k_v = 0$

k_h	δ' (deg)	α (deg)	ϕ' (deg) 28	30	35	40	45
0.1	0	0	0.427	0.397	0.328	0.268	0.217
0.2			0.508	0.473	0.396	0.382	0.270
0.3			0.611	0.569	0.478	0.400	0.334
0.4			0.753	0.697	0.581	0.488	0.409
0.5			1.005	0.890	0.716	0.596	0.500
0.1	0	5	0.457	0.423	0.347	0.282	0.227
0.2			0.554	0.514	0.424	0.349	0.285
0.3			0.690	0.635	0.522	0.431	0.356
0.4			0.942	0.825	0.653	0.535	0.442
0.5			—	—	0.855	0.673	0.551
0.1	0	10	0.497	0.457	0.371	0.299	0.238
0.2			0.623	0.570	0.461	0.375	0.303
0.3			0.856	0.748	0.585	0.472	0.383
0.4			—	—	0.780	0.604	0.486
0.5			—	—	—	0.809	0.624
0.1	$\phi'/2$	0	0.396	0.368	0.306	0.253	0.207
0.2			0.485	0.452	0.380	0.319	0.267
0.3			0.604	0.563	0.474	0.402	0.340
0.4			0.778	0.718	0.599	0.508	0.433
0.5			1.115	0.972	0.774	0.648	0.522
0.1	$\phi'/2$	5	0.428	0.396	0.326	0.268	0.218
0.2			0.537	0.497	0.412	0.342	0.283
0.3			0.699	0.640	0.526	0.438	0.367
0.4			1.025	0.881	0.690	0.568	0.475
0.5			—	—	0.962	0.752	0.620
0.1	$\phi'/2$	10	0.472	0.433	0.352	0.285	0.230
0.2			0.616	0.562	0.454	0.371	0.303
0.3			0.908	0.780	0.602	0.487	0.400
0.4			—	—	0.857	0.656	0.531
0.5			—	—	—	0.944	0.722
0.1	$\frac{2}{3}\phi'$	0	0.393	0.366	0.306	0.256	0.212
0.2			0.486	0.454	0.384	0.326	0.276
0.3			0.612	0.572	0.486	0.416	0.357
0.4			0.801	0.740	0.622	0.533	0.462
0.5			1.177	1.023	0.819	0.693	0.600
0.1	$\frac{2}{3}\phi'$	5	0.427	0.395	0.327	0.271	0.224
0.2			0.541	0.501	0.418	0.350	0.294
0.3			0.714	0.655	0.541	0.455	0.386
0.4			1.073	0.921	0.722	0.600	0.509
0.5			—	—	1.034	0.812	0.679
0.1	$\frac{2}{3}\phi'$	10	0.472	0.434	0.354	0.290	0.237
0.2			0.625	0.570	0.463	0.381	0.317
0.3			0.942	0.807	0.624	0.509	0.423
0.4			—	—	0.909	0.699	0.573
0.5			—	—	—	1.037	0.800

Step 5. Assume that ΔP_{ae} acts at a distance of $0.6H$ from the bottom of the wall (Figure 7.16)

Step 6. Calculate the location of the resultant as

$$\bar{z} = \frac{(0.6H)(\Delta P_{ae}) + \left(\frac{H}{3}\right)(P_a)}{P_{ae}} \tag{7.49}$$

Example 7.10

Refer to Figure 7.17. For $k_v = 0$ and $k_h = 0.3$, determine:

a. P_{ae} using Eq. (7.45)
b. The location of the resultant, $\bar{z}$, from the bottom of the wall

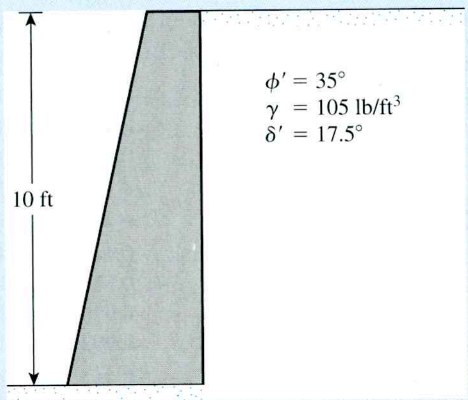


Figure 7.17

Solution

Part a

From Eq. (7.44),

$$\theta' = \tan^{-1}\left(\frac{k_h}{1 - k_v}\right) = \tan^{-1}\left(\frac{0.3}{1 - 0}\right) = 16.7°$$

From Eqs. (7.46) and (7.47),

$$\beta' = \beta - \theta' = 90 - 16.7 = 73.3°$$

$$\alpha' = \theta' + \alpha = 16.7 + 0 = 16.7°$$

$$\frac{\delta'}{\phi'} = \frac{17.5}{35} = 0.5$$

We will refer to Table 7.5. For $\phi' = 35°$, $\delta'/\phi' = 0.5$, $\beta' = 73.3°$, and $\alpha' = 16.7°$, the value of $K_a(\beta',\alpha') \approx 0.495$. Thus, from Eq. (7.45),

$$P_{ae} = \frac{1}{2}\gamma H^2(1 - k_v)[K_a(\beta',\alpha')]\left(\frac{\sin^2\beta'}{\cos\theta'\sin^2\beta}\right)$$
$$= \frac{1}{2}(105)(10^2)(1 - 0)(0.495)\left(\frac{\sin^2 73.3}{\cos 16.7 \sin^2 90}\right) = \mathbf{2489\ lb/ft}$$

Part b

From Eq. (7.25),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a$$

From Eq. (7.26) with $\delta' = 17.5°$, $\beta = 90°$, and $\alpha = 0°$, $K_a \approx 0.246$ (Table 7.5).

$$P_a = \tfrac{1}{2}(105)(10)^2(0.246) = 1292 \text{ lb/ft}$$
$$\Delta P_{ae} = P_{ae} - P_a = 2489 - 1292 = 1197 \text{ lb/ft}$$

From Eq. (7.49),

$$\bar{z} = \frac{(0.6H)(\Delta P_{ae}) + (H/3)(P_a)}{P_{ae}}$$
$$= \frac{[(0.6)(10)](1197) + (10/3)(1292)}{2489} = \mathbf{4.62\ ft}$$

■

7.8 Active Pressure for Wall Rotation about the Top: Braced Cut

In the preceding sections, we have seen that a retaining wall rotates about its bottom. (See Figure 7.18a.) With sufficient yielding of the wall, the lateral earth pressure is approximately equal to that obtained by Rankine's theory or Coulomb's theory. In contrast to retaining walls, braced cuts show a different type of wall yielding. (See Figure 7.18b.) In this case, deformation of the wall gradually increases with the depth of excavation. The variation of the amount of deformation depends on several factors, such as the type of soil, the depth of excavation, and the workmanship involved. However, with very little wall yielding at the top of the cut, the lateral earth pressure will be close to the at-rest pressure. At the bottom of the wall, with a much larger degree of yielding, the lateral earth pressure will be substantially lower than the Rankine active earth pressure. As a result, the distribution of lateral earth pressure will vary substantially in comparison to the linear distribution assumed in the case of retaining walls.

The total lateral force per unit length of the wall, P_a, imposed on a wall may be evaluated theoretically by using Terzaghi's (1943) general wedge theory. (See Figure 7.19.) The failure surface is assumed to be the arc of a logarithmic spiral, defined as

$$r = r_o e^{\theta \tan\phi'} \quad (7.50)$$

where ϕ' = effective angle of friction of soil.

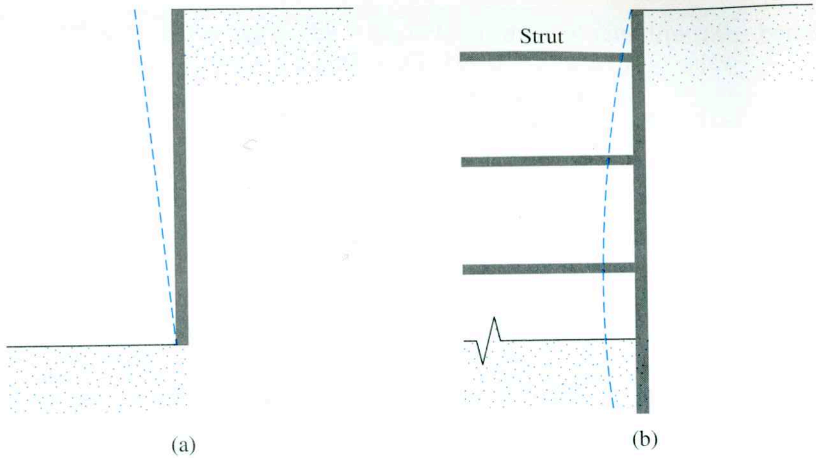


Figure 7.18 Nature of yielding of walls: (a) retaining wall; (b) braced cut

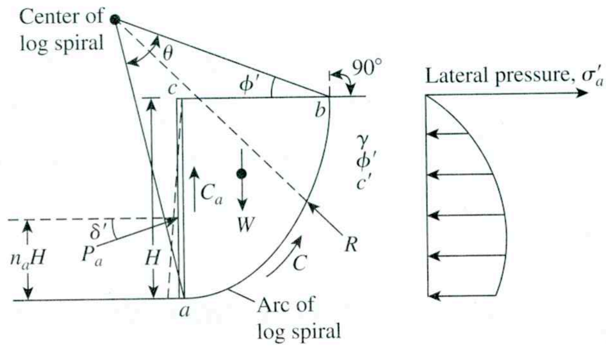


Figure 7.19 Braced cut analysis by general wedge theory: wall rotation about top

In the figure, H is the height of the cut, and the unit weight, angle of friction, and cohesion of the soil are equal to γ, ϕ', and c', respectively. Following are the forces per unit length of the cut acting on the trial failure wedge:

1. Weight of the wedge, W
2. Resultant of the normal and shear forces along ab, R
3. Cohesive force along ab, C
4. Adhesive force along ac, C_a
5. P_a, which is the force acting a distance n_aH from the bottom of the wall and is inclined at an angle δ' to the horizontal

The adhesive force is

$$C_a = c_a'H \tag{7.51}$$

where c_a' = unit adhesion.

A detailed outline for the evaluation of P_a is beyond the scope of this text; those interested should check a soil mechanics text for more information. Kim and Preber (1969) provided tabulated values of $P_a/\frac{1}{2}\gamma H^2$ determined by using the principles of general wedge theory. Table 7.7 gives the variation of $P_a/0.5\gamma H^2$ for granular soil backfill obtained using the general wedge theory.

Table 7.7 Active Pressure for Wall Rotation—General Wedge Theory (Granular Soil Backfill)

Soil friction angle, ϕ' (deg)	δ'/ϕ'	$P_a/0.5\,\gamma H^2$			
		$n_a = 0.3$	$n_a = 0.4$	$n_a = 0.5$	$n_a = 0.6$
25	0	0.371	0.405	0.447	0.499
	½	0.345	0.376	0.413	0.460
	⅔	0.342	0.373	0.410	0.457
	1	0.344	0.375	0.413	0.461
30	0	0.304	0.330	0.361	0.400
	½	0.282	0.306	0.334	0.386
	⅔	0.281	0.305	0.332	0.367
	1	0.289	0.313	0.341	0.377
35	0	0.247	0.267	0.290	0.318
	½	0.231	0.249	0.269	0.295
	⅔	0.232	0.249	0.270	0.296
	1	0.243	0.262	0.289	0.312
40	0	0.198	0.213	0.230	0.252
	½	0.187	0.200	0.216	0.235
	⅔	0.190	0.204	0.220	0.239
	1	0.197	0.211	0.228	0.248
45	0	0.205	0.220	0.237	0.259
	½	0.149	0.159	0.171	0.185
	⅔	0.153	0.164	0.176	0.196
	1	0.173	0.184	0.198	0.215

7.9 Active Earth Pressure for Translation of Retaining Wall—Granular Backfill

Under certain circumstances, retaining walls may undergo lateral translation, as shown in Figure 7.20. A solution to the distribution of active pressure for this case was provided by Dubrova (1963) and was also described by Harr (1966). The solution of Dubrova assumes the validity of Coulomb's solution [Eqs. (7.25) and (7.26)]. In order to understand this procedure, let us consider a vertical wall with a horizontal granular backfill (Figure 7.21). For rotation about the top of the wall, the resultant R of the normal and shear forces along the rupture line AC is inclined at an angle ϕ' to the normal drawn to AC. According to Dubrova there exists infinite number of quasi-rupture lines such as $A'C'$, $A''C''$, . . . for which the resultant force R is inclined at an angle ψ, where

$$\psi = \frac{\phi' z}{H} \tag{7.52}$$

Now, refer to Eqs. (7.25) and (7.26) for Coulomb's active pressure. For $\beta = 90°$ and $\alpha = 0$, the relationship for Coulomb's active force can also be rewritten as

$$P_a = \frac{\gamma}{2\cos\delta'}\left[\frac{H}{\dfrac{1}{\cos\phi'} + (\tan^2\phi' + \tan\phi'\tan\delta')^{0.5}}\right]^2 \tag{7.53}$$

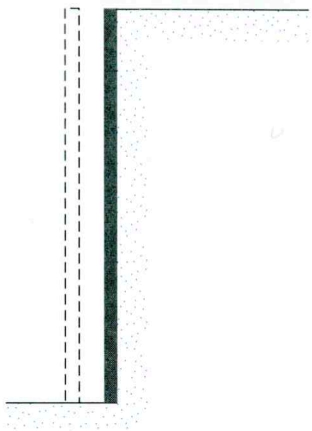


Figure 7.20 Lateral translation of retaining wall

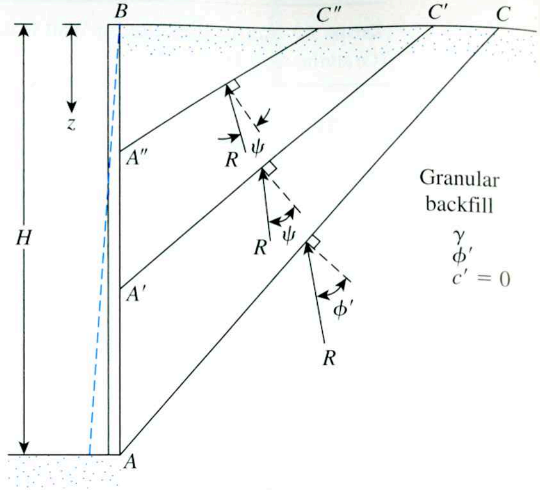


Figure 7.21 Quasi-rupture lines behind a retaining wall

The force against the wall at any z is then given as

$$P_a = \frac{\gamma}{2\cos\delta'}\left[\frac{z}{\dfrac{1}{\cos\psi} + (\tan^2\psi + \tan\psi\tan\delta')^{0.5}}\right]^2 \tag{7.54}$$

The active pressure at any depth z for *wall rotation about the top* is

$$\sigma'_a(z) = \frac{dP_a}{dz} \approx \frac{\gamma}{\cos\delta'}\left[\frac{z\cos^2\psi}{(1 + m\sin\psi)^2} - \frac{z^2\phi'\cos^2\psi}{H(1 + m\sin\psi)}(\sin\psi + m)\right] \tag{7.55}$$

where $m = \left(1 + \dfrac{\tan\delta'}{\tan\psi}\right)^{0.5}$. (7.56)

For frictionless walls, $\delta' = 0$ and Eq. (7.55) simplifies to

$$\sigma'_a(z) = \gamma\tan^2\left(45 - \frac{\psi}{2}\right)\left(z - \frac{\phi' z^2}{H\cos\psi}\right) \tag{7.57}$$

For wall rotation about the bottom, a similar expression can be found in the form

$$\sigma'_a(z) = \frac{\gamma z}{\cos\delta'}\left(\frac{\cos\phi'}{1 + m\sin\phi'}\right)^2 \tag{7.58}$$

For translation of the wall, the active pressure can then be taken as

$$\sigma'_a(z)_{\text{translation}} = \tfrac{1}{2}[\sigma'_a(z)_{\text{rotation about top}} + \sigma'_a(z)_{\text{rotation about bottom}}] \tag{7.59}$$

Example 7.11

Consider a frictionless wall 16 ft high. For the granular backfill, $\gamma = 110\ \text{lb/ft}^3$ and $\phi' = 36°$. Calculate and plot the variation of $\sigma_a(z)$ for a translation mode of the wall movement.

Solution

For a frictionless wall, $\delta' = 0$. Hence, m is equal to one [Eq. (7.56)]. So for rotation about the top, from Eq. (7.57),

$$\sigma'_a(z) = \sigma'_{a(1)} = \gamma \tan^2\left(45 - \frac{\phi' z}{2H}\right)\left[z - \frac{\phi' z^2}{H\cos\left(\frac{\phi' z}{H}\right)}\right]$$

For rotation about the bottom, from Eq. (7.58),

$$\sigma'_a(z) = \sigma'_{a(2)} = \gamma z\left(\frac{\cos\phi'}{1+\sin\phi'}\right)^2$$

$$\sigma'_a(z)_{\text{translation}} = \frac{\sigma'_{a(1)} + \sigma'_{a(2)}}{2}$$

The following table can now be prepared with $\gamma = 110\ \text{lb/ft}^3$, $\phi' = 36°$, and $H = 16$ ft.

z (ft)	$\sigma'_{a(1)}$ (lb/ft^2)	$\sigma'_{a(2)}$ (lb/ft^2)	$\sigma'_{a(z)\text{ translation}}$ (lb/ft^2)
0	0	0	0
4	269.9	114.2	192.05
8	311.2	228.4	269.8
12	233.6	342.6	288.1
16	102.2	456.8	279.5

The plot of $\sigma_a(z)$ versus z is shown in Figure 7.22

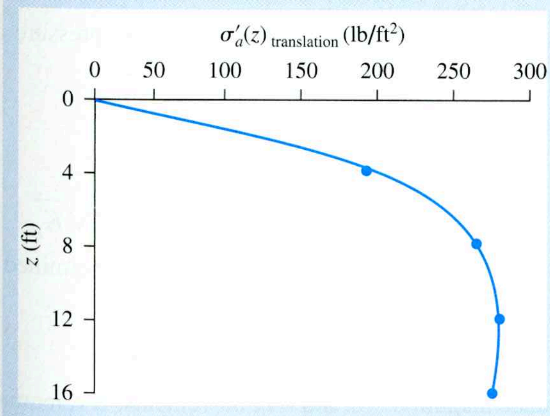


Figure 7.22

Passive Pressure

7.10 Rankine Passive Earth Pressure

Figure 7.23a shows a vertical frictionless retaining wall with a horizontal backfill. At depth z, the effective vertical pressure on a soil element is $\sigma'_o = \gamma z$. Initially, if the wall does not yield at all, the lateral stress at that depth will be $\sigma_h' = K_o \sigma_o'$. This state of stress is illustrated by the Mohr's circle a in Figure 7.23b. Now, if the wall is pushed into the soil mass by an amount Δx, as shown in Figure 7.23a, the vertical stress at depth z will stay the same; however, the horizontal stress will increase. Thus, σ'_h will be greater than $K_o \sigma'_o$. The state of stress can now be represented by the Mohr's circle b in Figure 7.23b. If the wall moves farther inward (i.e., Δx is increased still more), the stresses at depth z will ultimately reach the state represented by Mohr's circle c. Note that this Mohr's circle touches the Mohr–Coulomb failure envelope, which implies that the soil behind the wall will fail by being pushed upward. The horizontal stress, σ'_h, at this point is referred to as the *Rankine passive pressure,* or $\sigma'_h = \sigma'_p$.

For Mohr's circle c in Figure 7.23b, the major principal stress is σ'_p, and the minor principal stress is σ'_o. Substituting these quantities into Eq. (1.87) yields

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right) \tag{7.60}$$

Now, let

$$K_p = \text{Rankine passive earth pressure coefficient} = \tan^2\left(45 + \frac{\phi'}{2}\right) \tag{7.61}$$

Then, from Eq. (7.60), we have

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p} \tag{7.62}$$

Equation (7.62) produces (Figure 7.23c), the passive pressure diagram for the wall shown in Figure 7.23a. Note that at $z = 0$,

$$\sigma'_o = 0 \quad \text{and} \quad \sigma'_p = 2c'\sqrt{K_p}$$

and at $z = H$,

$$\sigma'_o = \gamma H \quad \text{and} \quad \sigma'_p = \gamma H K_p + 2c'\sqrt{K_p}$$

The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \tfrac{1}{2}\gamma H^2 K_p + 2c' H \sqrt{K_p} \tag{7.63}$$

The approximate magnitudes of the wall movements, Δx, required to develop failure under passive conditions are as follows:

Soil type	Wall movement for passive condition, Δx
Dense sand	$0.005H$
Loose sand	$0.01H$
Stiff clay	$0.01H$
Soft clay	$0.05H$

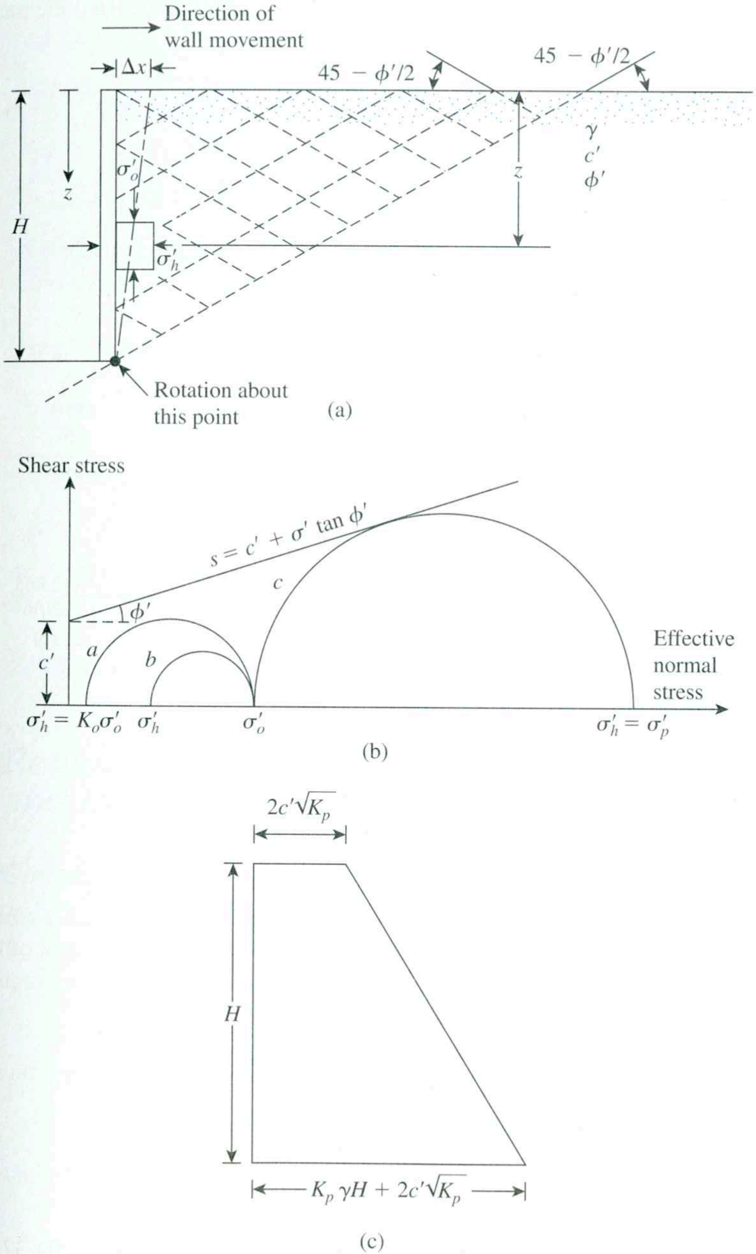


Figure 7.23 Rankine passive pressure

If the backfill behind the wall is a granular soil (i.e., $c' = 0$), then, from Eq. (7.63), the passive force per unit length of the wall will be

$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{7.64}$$

Example 7.12

A 3-m high wall is shown in Figure 7.24a. Determine the Rankine passive force per unit length of the wall.

Solution

For the top layer

$$K_{p(1)} = \tan^2\left(45 + \frac{\phi'_1}{2}\right) = \tan^2(45 + 15) = 3$$

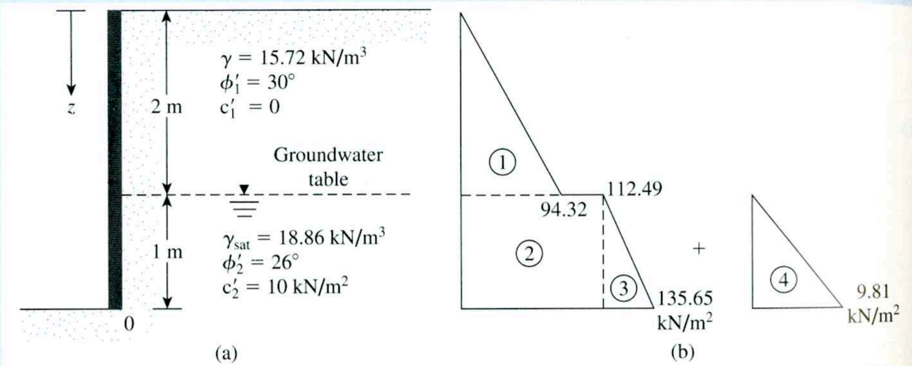


Figure 7.24

From the bottom soil layer

$$K_{p(2)} = \tan^2\left(45 + \frac{\phi'_2}{2}\right) = \tan^2(45 + 13) = 2.56$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

where

σ'_o = effective vertical stress

at $z = 0$, $\sigma'_o = 0$, $c'_1 = 0$, $\sigma'_p = 0$

at $z = 2$ m, $\sigma'_o = (15.72)(2) = 31.44$ kN/m², $c'_1 = 0$

So, for the top soil layer

$$\sigma'_p = 31.44K_{p(1)} + 2(0)\sqrt{K_{p(1)}} = 31.44(3) = 94.32 \text{ kN/m}^2$$

At this depth, that is $z = 2$ m, for the bottom soil layer

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 31.44(2.56) + 2(10)\sqrt{2.56}$$
$$= 80.49 + 32 = 112.49 \text{ kN/m}^2$$

Again, at $z = 3$ m,

$$\sigma'_o = (15.72)(2) + (\gamma_{sat} - \gamma_w)(1)$$
$$= 31.44 + (18.86 - 9.81)(1) = 40.49 \text{ kN/m}^2$$

Hence,

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 40.49(2.56) + (2)(10)(1.6)$$
$$= \mathbf{135.65 \text{ kN/m}^2}$$

Note that, because a water table is present, the hydrostatic stress, u, also has to be taken into consideration. For $z = 0$ to 2 m, $u = 0$; $z = 3$ m, $u = (1)(\gamma_w) = 9.81 \text{ kN/m}^2$.

The passive pressure diagram is plotted in Figure 6.24b. The passive force per unit length of the wall can be determined from the area of the pressure diagram as follows:

Area no.	Area	
1	$(\frac{1}{2})(2)(94.32)$	= 94.32
2	$(112.49)(1)$	= 112.49
3	$(\frac{1}{2})(1)(135.65 - 112.49)$	= 11.58
4	$(\frac{1}{2})(9.81)(1)$	= 4.905
		$P_P \approx 223.3$ **kN/m**

7.11 Rankine Passive Earth Pressure: Vertical Backface and Inclined Backfill

Granular Soil

For a frictionless vertical retaining wall (Figure 7.10) with a *granular backfill* ($c' = 0$), the Rankine passive pressure at any depth can be determined in a manner similar to that done in the case of active pressure in Section 7.4. The pressure is

$$\sigma'_p = \gamma z K_p \tag{7.65}$$

and the passive force is

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \tag{7.66}$$

where

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}} \tag{7.67}$$

Table 7.8 Passive Earth Pressure Coefficient K_p [from Eq. (7.67)]

↓α (deg)	ϕ' (deg)→ 28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

As in the case of the active force, the resultant force, P_p, is inclined at an angle α with the horizontal and intersects the wall at a distance $H/3$ from the bottom of the wall. The values of K_p (the passive earth pressure coefficient) for various values of α and ϕ' are given in Table 7.8.

c'–ϕ' Soil

If the backfill of the frictionless vertical retaining wall is a c'–ϕ' soil (see Figure 7.10), then (Mazindrani and Ganjali, 1997)

$$\sigma'_a = \gamma z K_p = \gamma z K'_p \cos\alpha \tag{7.68}$$

where

$$K'_p = \frac{1}{\cos^2\phi'}\left\{\begin{array}{c} 2\cos^2\alpha + 2\left(\dfrac{c'}{\gamma z}\right)\cos\phi'\sin\phi' \\ + \sqrt{4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\dfrac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\dfrac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'} \end{array}\right\} - 1 \tag{7.69}$$

The variation of K'_p with $\phi', \alpha,$ and $c'/\gamma z$ is given in Table 7.9 (Mazindrani and Ganjali, 1997).

Table 7.9 Values of K'_p

ϕ' (deg)	α (deg)	$c'/\gamma z$ 0.025	0.050	0.100	0.500
15	0	1.764	1.829	1.959	3.002
	5	1.716	1.783	1.917	2.971
	10	1.564	1.641	1.788	2.880
	15	1.251	1.370	1.561	2.732
20	0	2.111	2.182	2.325	3.468
	5	2.067	2.140	2.285	3.435
	10	1.932	2.010	2.162	3.339
	15	1.696	1.786	1.956	3.183
25	0	2.542	2.621	2.778	4.034
	5	2.499	2.578	2.737	3.999
	10	2.368	2.450	2.614	3.895
	15	2.147	2.236	2.409	3.726

Table 7.9 (*Continued*)

		$c'/\gamma z$			
ϕ' (deg)	α (deg)	0.025	0.050	0.100	0.500
30	0	3.087	3.173	3.346	4.732
	5	3.042	3.129	3.303	4.674
	10	2.907	2.996	3.174	4.579
	15	2.684	2.777	2.961	4.394

7.12 Coulomb's Passive Earth Pressure

Coulomb (1776) also presented an analysis for determining the passive earth pressure (i.e., when the wall moves *into* the soil mass) for walls possessing friction (δ' = angle of wall friction) and retaining a granular backfill material similar to that discussed in Section 7.5.

To understand the determination of Coulomb's passive force, P_p, consider the wall shown in Figure 7.25a. As in the case of active pressure, Coulomb assumed that the potential failure surface in soil is a plane. For a trial failure wedge of soil, such as ABC_1, the forces per unit length of the wall acting on the wedge are

1. The weight of the wedge, W
2. The resultant, R, of the normal and shear forces on the plane BC_1, and
3. The passive force, P_p

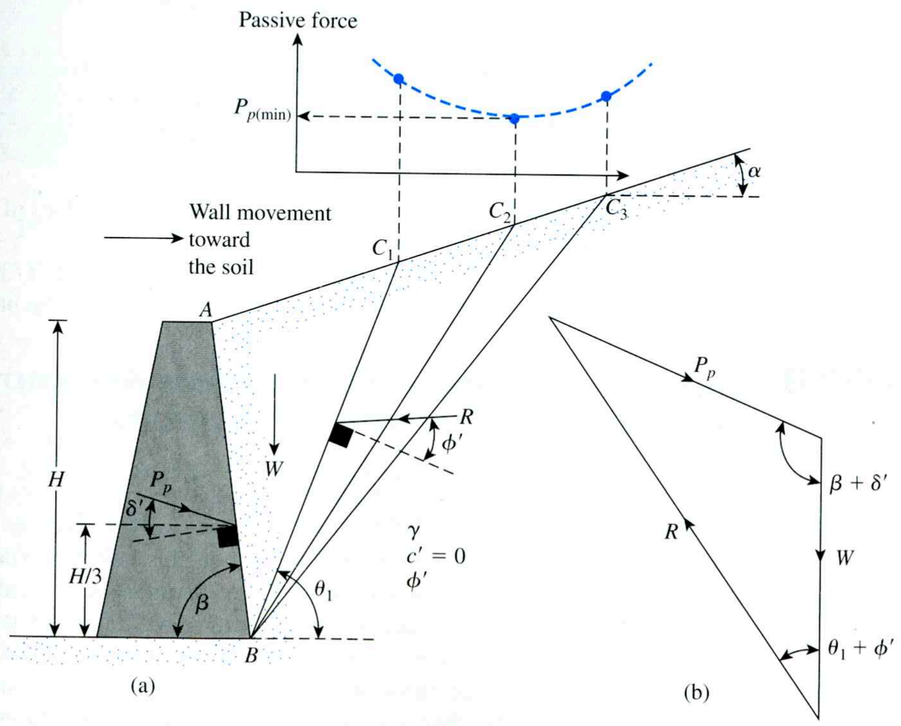


Figure 7.25 Coulomb's passive pressure

Table 7.10 Values of K_p [from Eq. (7.71)] for $\beta = 90°$ and $\alpha = 0°$

	δ' (deg)				
ϕ' (deg)	0	5	10	15	20
15	1.698	1.900	2.130	2.405	2.735
20	2.040	2.313	2.636	3.030	3.525
25	2.464	2.830	3.286	3.855	4.597
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324
40	4.600	5.590	6.946	8.870	11.772

Figure 7.25b shows the force triangle at equilibrium for the trial wedge ABC_1. From this force triangle, the value of P_p can be determined, because the direction of all three forces and the magnitude of one force are known.

Similar force triangles for several trial wedges, such as ABC_1, ABC_2, ABC_3, . . . , can be constructed, and the corresponding values of P_p can be determined. The top part of Figure 7.25a shows the nature of variation of the P_p values for different wedges. The *minimum value of* P_p in this diagram is *Coulomb's passive force,* mathematically expressed as

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \tag{7.70}$$

where

$$K_p = \text{Coulomb's passive pressure coefficient}$$

$$= \frac{\sin^2(\beta - \phi')}{\sin^2\beta \sin(\beta + \delta')\left[1 - \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\sin(\beta + \delta')\sin(\beta + \alpha)}}\right]^2} \tag{7.71}$$

The values of the passive pressure coefficient, K_p, for various values of ϕ' and δ' are given in Table 7.10 ($\beta = 90°, \alpha = 0°$).

Note that the resultant passive force, P_p, will act at a distance $H/3$ from the bottom of the wall and will be inclined at an angle δ' to the normal drawn to the back face of the wall.

7.13 Comments on the Failure Surface Assumption for Coulomb's Pressure Calculations

Coulomb's pressure calculation methods for active and passive pressure have been discussed in Sections 7.5 and 7.12. The fundamental assumption in these analyses is the acceptance of *plane failure surface.* However, for walls with friction, this assumption does not hold in practice. The nature of *actual* failure surface in the soil mass for active and passive pressure is shown in Figure 7.26a and b, respectively (for a vertical wall with a horizontal backfill). Note that the failure surface BC is curved and that the failure surface CD is a plane.

Although the actual failure surface in soil for the case of active pressure is somewhat different from that assumed in the calculation of the Coulomb pressure, the results are not greatly different. However, in the case of passive pressure, as the value of δ' increases, Coulomb's

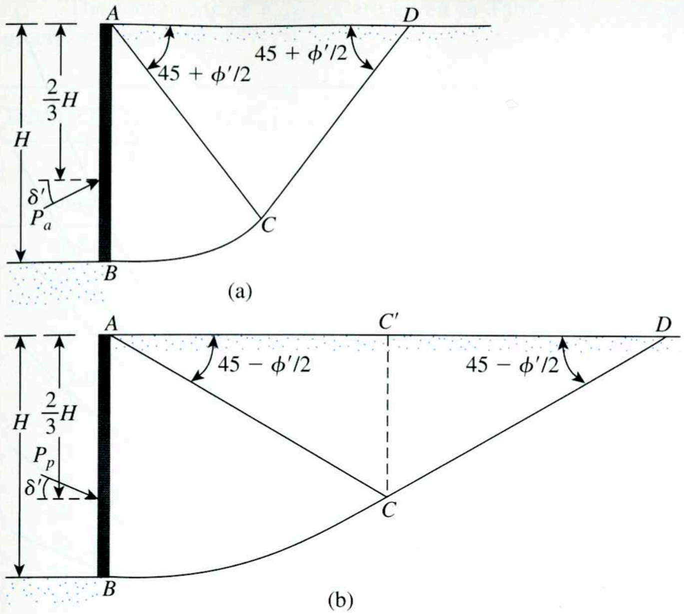


Figure 7.26 Nature of failure surface in soil with wall friction: (a) active pressure; (b) passive pressure

method of calculation gives increasingly erroneous values of P_p. This factor of error could lead to an unsafe condition because the values of P_p would become higher than the soil resistance.

Several studies have been conducted to determine the passive force P_p, assuming that the curved portion BC in Figure 7.26b is an arc of a circle, an ellipse, or a logarithmic spiral.

Shields and Tolunay (1973) analyzed the problem of passive pressure for a *vertical* wall with a *horizontal granular soil* backfill ($c' = 0$). This analysis was done by considering the stability of the wedge $ABCC'$ (see Figure 7.26b), using the *method of slices* and assuming BC as an arc of a logarthmic spiral. From Figure 7.26b, the passive force per unit length of the wall can be expressed as

$$P_p = \frac{1}{2} K_p \gamma H^2 \tag{7.72}$$

The values of the passive earth-pressure coefficient, K_p, obtained by Shields and Tolunay are given in Figure 7.27. These are as good as any for design purposes.

Solution by the Method of Triangular Slices

Zhu and Qian (2000) used the method of triangular slices (such as in the zone of ABC in Fig. 7.28) to obtain the variation of K_p. According to this analysis

$$K_p = K_{p(\delta' = 0)} R \tag{7.73}$$

where

K_p = passive earth-pressure coefficient for a given value of θ, δ', and ϕ'
$K_{p(\delta' = 0)}$ = K_p for a given value of θ, ϕ' with $\delta' = 0$
R = modification factor which is a function of ϕ', θ, δ'/ϕ'

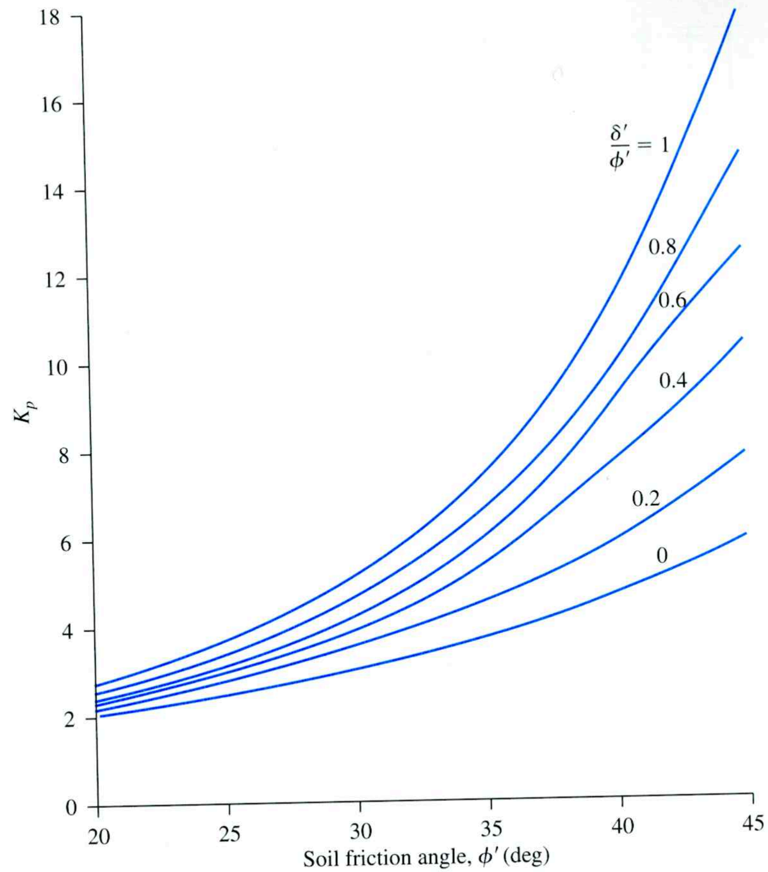


Figure 7.27 K_p based on Shields and Tolunay's analysis

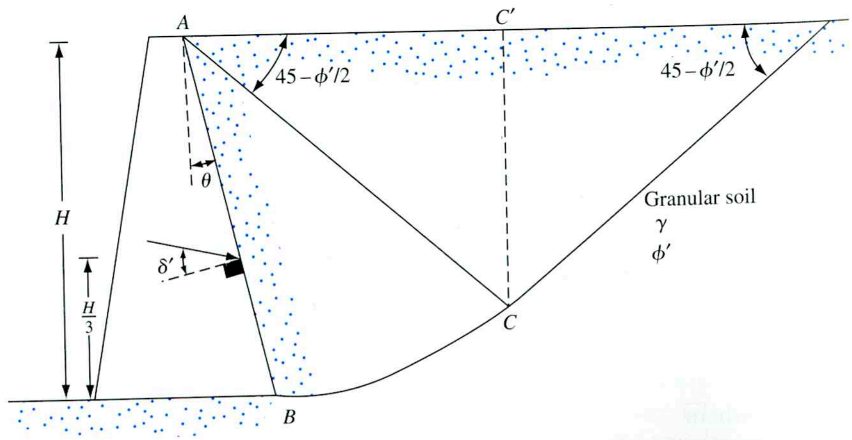


Figure 7.28 Passive pressure solution by the method of triangular slices

(*Note*: BC is arc of a logarthimic spiral)

The variations of $K_{p(\delta'=0)}$ are given in Table 7.11. The interpolated values of R are given in Table 7.12

Table 7.11 Variation of $K_{p(\delta'=0)}$ [see Eq. (7.73) and Figure 7.28]*

	θ (deg)						
ϕ' (deg)	30	25	20	15	10	5	0
20	1.70	1.69	1.72	1.77	1.83	1.92	2.04
21	1.74	1.73	1.76	1.81	1.89	1.99	2.12
22	1.77	1.77	1.80	1.87	1.95	2.06	2.20
23	1.81	1.81	1.85	1.92	2.01	2.13	2.28
24	1.84	1.85	1.90	1.97	2.07	2.21	2.37
25	1.88	1.89	1.95	2.03	2.14	2.28	2.46
26	1.91	1.93	1.99	2.09	2.21	2.36	2.56
27	1.95	1.98	2.05	2.15	2.28	2.45	2.66
28	1.99	2.02	2.10	2.21	2.35	2.54	2.77
29	2.03	2.07	2.15	2.27	2.43	2.63	2.88
30	2.07	2.11	2.21	2.34	2.51	2.73	3.00
31	2.11	2.16	2.27	2.41	2.60	2.83	3.12
32	2.15	2.21	2.33	2.48	2.68	2.93	3.25
33	2.20	2.26	2.39	2.56	2.77	3.04	3.39
34	2.24	2.32	2.45	2.64	2.87	3.16	3.53
35	2.29	2.37	2.52	2.72	2.97	3.28	3.68
36	2.33	2.43	2.59	2.80	3.07	3.41	3.84
37	2.38	2.49	2.66	2.89	3.18	3.55	4.01
38	2.43	2.55	2.73	2.98	3.29	3.69	4.19
39	2.48	2.61	2.81	3.07	3.41	3.84	4.38
40	2.53	2.67	2.89	3.17	3.53	4.00	4.59
41	2.59	2.74	2.97	3.27	3.66	4.16	4.80
42	2.64	2.80	3.05	3.38	3.80	4.34	5.03
43	2.70	2.88	3.14	3.49	3.94	4.52	5.27
44	2.76	2.94	3.23	3.61	4.09	4.72	5.53
45	2.82	3.02	3.32	3.73	4.25	4.92	5.80

*Based on Zhu and Qian, 2000

Table 7.12 Variation of R [Eq. (7.73)]

		R for ϕ' (deg)			
θ (deg)	δ'/ϕ'	30	35	40	45
0	0.2	1.2	1.28	1.35	1.45
	0.4	1.4	1.6	1.8	2.2
	0.6	1.65	1.95	2.4	3.2
	0.8	1.95	2.4	3.15	4.45
	1.0	2.2	2.85	3.95	6.1
5	0.2	1.2	1.25	1.32	1.4
	0.4	1.4	1.6	1.8	2.1
	0.6	1.6	1.9	2.35	3.0
	0.8	1.9	2.35	3.05	4.3
	1.0	2.15	2.8	3.8	5.7

Table 7.12 (*Continued*)

θ (deg)	δ'/ϕ'	R for ϕ' (deg)			
		30	35	40	45
10	0.2	1.15	1.2	1.3	1.4
	0.4	1.35	1.5	1.7	2.0
	0.6	1.6	1.85	2.25	2.9
	0.8	1.8	2.25	2.9	4.0
	1.0	2.05	2.65	3.6	5.3
15	0.2	1.15	1.2	1.3	1.35
	0.4	1.35	1.5	1.65	1.95
	0.6	1.55	1.8	2.2	2.7
	0.8	1.8	2.2	2.8	3.8
	1.0	2.0	2.6	3.4	4.95
20	0.2	1.15	1.2	1.3	1.35
	0.4	1.35	1.45	1.65	1.9
	0.6	1.5	1.8	2.1	2.6
	0.8	1.8	2.1	2.6	3.55
	1.0	1.9	2.4	3.2	4.8

7.14 Passive Pressure under Earthquake Conditions

The relationship for passive earth pressure on a retaining wall with a granular backfill and under earthquake conditions was evaluated by Subba Rao and Choudhury (2005) by the method of limit equilibrium using the pseudo-static approach. Figure 7.29 shows the nature of failure surface in soil considered in this analysis. The passive pressure, P_{pe}, can be expressed as

$$P_{pe} = \left[\tfrac{1}{2}\gamma H^2 K_{p\gamma(e)}\right]\frac{1}{\cos\delta'} \tag{7.74}$$

where $K_{p\gamma(e)}$ = passive earth-pressure coefficient in the normal direction to the wall.

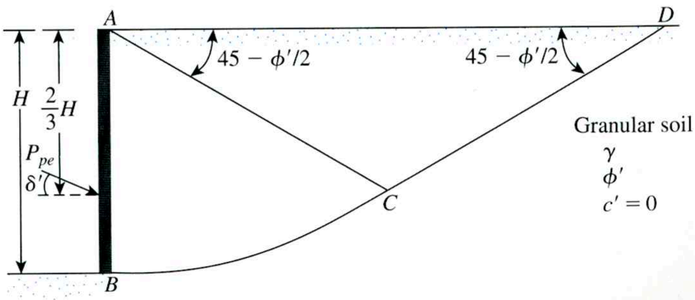


Figure 7.29 Nature of failure surface in soil considered in the analysis to determine P_{pe}

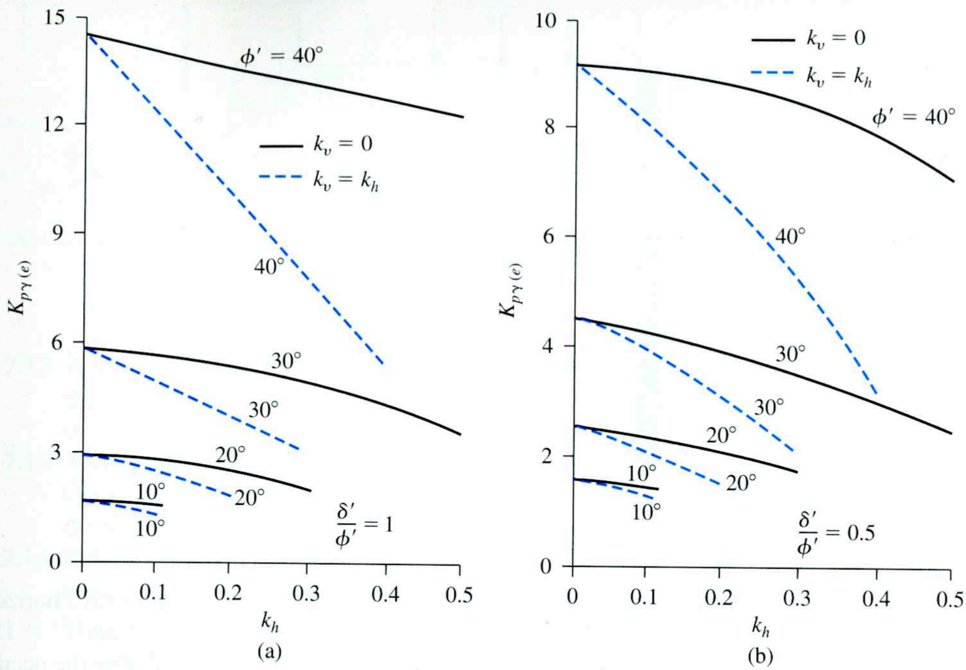


Figure 7.30 Variation of $K_{p\gamma(e)}$: (a) $\frac{\delta'}{\phi'} = 1$, (b) $\frac{\delta'}{\phi'} = 0.5$

$K_{p\gamma(e)}$ is a function of k_h and k_v that are, respectively, coefficient of horizontal and vertical acceleration due to earthquake. The variations of $K_{p\gamma(e)}$ for $\delta'/\phi' = 0.5$ and 1 are shown in Figures 7.30a and b. The passive pressure P_{pe} will be inclined at an angle δ' to the back face of the wall and will act at a distance of $H/3$ above the bottom of the wall.

Problems

7.1 Refer to Figure 7.3a. Given: $H = 3.5$ m, $q = 20$ kN/m^2, $\gamma = 18.2$ kN/m^3 $c' = 0$, and $\phi' = 35°$. Determine the at-rest lateral earth force per meter length of the wall. Also, find the location of the resultant. Use Eq. (7.4) and OCR = 1.5.

7.2 Use Eq. (7.3), Figure P7.2, and the following values to determine the at-rest lateral earth force per unit length of the wall. Also find the location of the resultant.
$H = 5$ m, $H_1 = 2$ m, $H_2 = 3$ m, $\gamma = 15.5$ kN/m^3, $\gamma_{sat} = 18.5$ kN/m^3, $\phi' = 34°$, $c' = 0$, $q = 20$ kN/m^2, and OCR = 1.

7.3 Refer to Figure 7.6a. Given the height of the retaining wall, H is 21 ft; the backfill is a saturated clay with $\phi = 0°$, $c = 630$ lb/ft^2, $\gamma_{sat} = 113$ lb/ft^3,

a. Determine the Rankine active pressure distribution diagram behind the wall.

b. Determine the depth of the tensile crack, z_c.

c. Estimate the Rankine active force per foot length of the wall before and after the occurrence of the tensile crack.

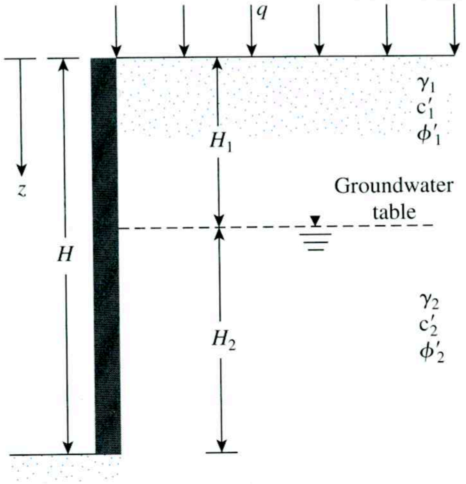


Figure P7.2

7.4 A vertical retaining wall (Figure 7.6a) is 6.3 m high with a horizontal backfill. For the backfill, assume that $\gamma = 17.9\ \text{kN/m}^3$, $\phi' = 26°$, and $c' = 15\ \text{kN/m}^2$. Determine the Rankine active force per unit length of the wall after the occurrence of the tensile crack.

7.5 Refer to Problem 7.2. For the retaining wall, determine the Rankine active force per unit length of the wall and the location of the line of action of the resultant.

7.6 Refer to Figure 7.10. For the retaining wall, $H = 6$ m, $\phi' = 34°$, $\alpha = 10°$, $\gamma = 17\ \text{kN/m}^3$, and $c' = 0$.

a. Determine the intensity of the Rankine active force at $z = 2$ m, 4 m, and 6 m.

b. Determine the Rankine active force per meter length of the wall and also the location and direction of the resultant.

7.7 Refer to Figure 7.10. Given: $H = 22$ ft $\gamma = 115\ \text{lb/ft}^3$, $\phi' = 25°$, $c' = 250\ \text{lb/ft}^2$, and $\alpha = 10°$. Calculate the Rankine active force per unit length of the wall after the occurrence of the tensile crack.

7.8 Refer to Figure 7.12a. Given: $H = 12$ ft, $\gamma = 105\ \text{lb/ft}^3$, $\phi' = 30°$, $c' = 0$, and $\beta = 85°$. Determine the Coulomb's active force per foot length of the wall and the location and direction of the resultant for the following cases:

a. $\alpha = 10°$ and $\delta' = 20°$

b. $\alpha = 20°$ and $\delta' = 15°$

7.9 Refer to Figure 7.13a. Given $H = 3.5$ m, $\alpha = 0$, $\beta = 85°$, $\gamma = 18\ \text{kN/m}^3$, $c' = 0$, $\phi' = 34°$, $\delta'/\phi' = 0.5$, and $q = 30\ \text{kN/m}^2$. Determine the Coulomb's active force per unit length of the wall.

7.10 Refer to Figure 7.14b. Given $H = 3.3$ m, $a' = 1$ m, $b' = 1.5$ m, and $q = 25\ \text{kN/m}^2$. Determine the lateral force per unit length of the unyielding wall caused by the surcharge loading only.

7.11 Refer to Figure 7.15. Here, $H = 6$ m, $\gamma = 17\ \text{kN/m}^3$, $\phi' = 35°$, $\delta' = 17.5°$, $c' = 0$, $\alpha = 10°$, and $\beta = 90°$. Determine the Coulomb's active force for earthquake conditions (P_{ae}) per meter length of the wall and the location and direction of the resultant. Given $k_h = 0.2$ and $k_v = 0$.

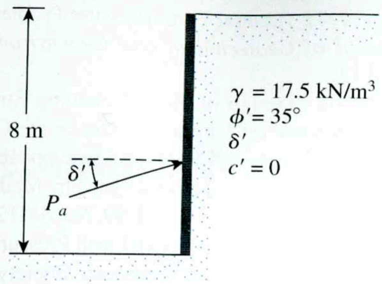


Figure P7.12

7.12 A retaining wall is shown in Figure P7.12. If the wall rotates about its top, determine the magnitude of the active force per unit length of the wall for $n_a = 0.3$, 0.4, and 0.5. Assume $\delta'/\phi' = 0.5$.

7.13 A vertical frictionless retaining wall is 6-m high with a horizontal granular backfill. Given: $\gamma = 16\ \text{kN/m}^3$ and $\phi' = 30°$. For the translation mode of the wall, calculate the active pressure at depths $z = 1.5$ m, 3 m, 4.5 m, and 6 m.

7.14 Refer to Problem 7.3.

a. Draw the Rankine passive pressure distribution diagram behind the wall.

b. Estimate the Rankine passive force per foot length of the wall and also the location of the resultant.

7.15 In Figure 7.28, which shows a vertical retaining wall with a horizontal backfill, let $H = 4$ m, $\theta = 25°$, $\gamma = 16.5\ \text{kN/m}^3$, $\phi' = 35°$, and $\delta' = 10°$. Based on Zhu and Qian's work, what would be the passive force per meter length of the wall?

7.16 Consider a 4-m high retaining wall with a vertical back and horizontal granular backfill, as shown in Figure 7.29. Given: $\gamma = 18\ \text{kN/m}^3$, $\phi' = 40°$, $c' = 0$, $\delta' = 20°$, $k_v = 0$ and $k_h = 0.2$. Determine the passive force P_{pe} per unit length of the wall taking the earthquake effect into consideration.

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8 Retaining Walls

8.1 Introduction

In Chapter 7, you were introduced to various theories of lateral earth pressure. Those theories will be used in this chapter to design various types of retaining walls. In general, retaining walls can be divided into two major categories: (a) conventional retaining walls and (b) mechanically stabilized earth walls.

Conventional retaining walls can generally be classified into four varieties:

1. Gravity retaining walls
2. Semigravity retaining walls
3. Cantilever retaining walls
4. Counterfort retaining walls

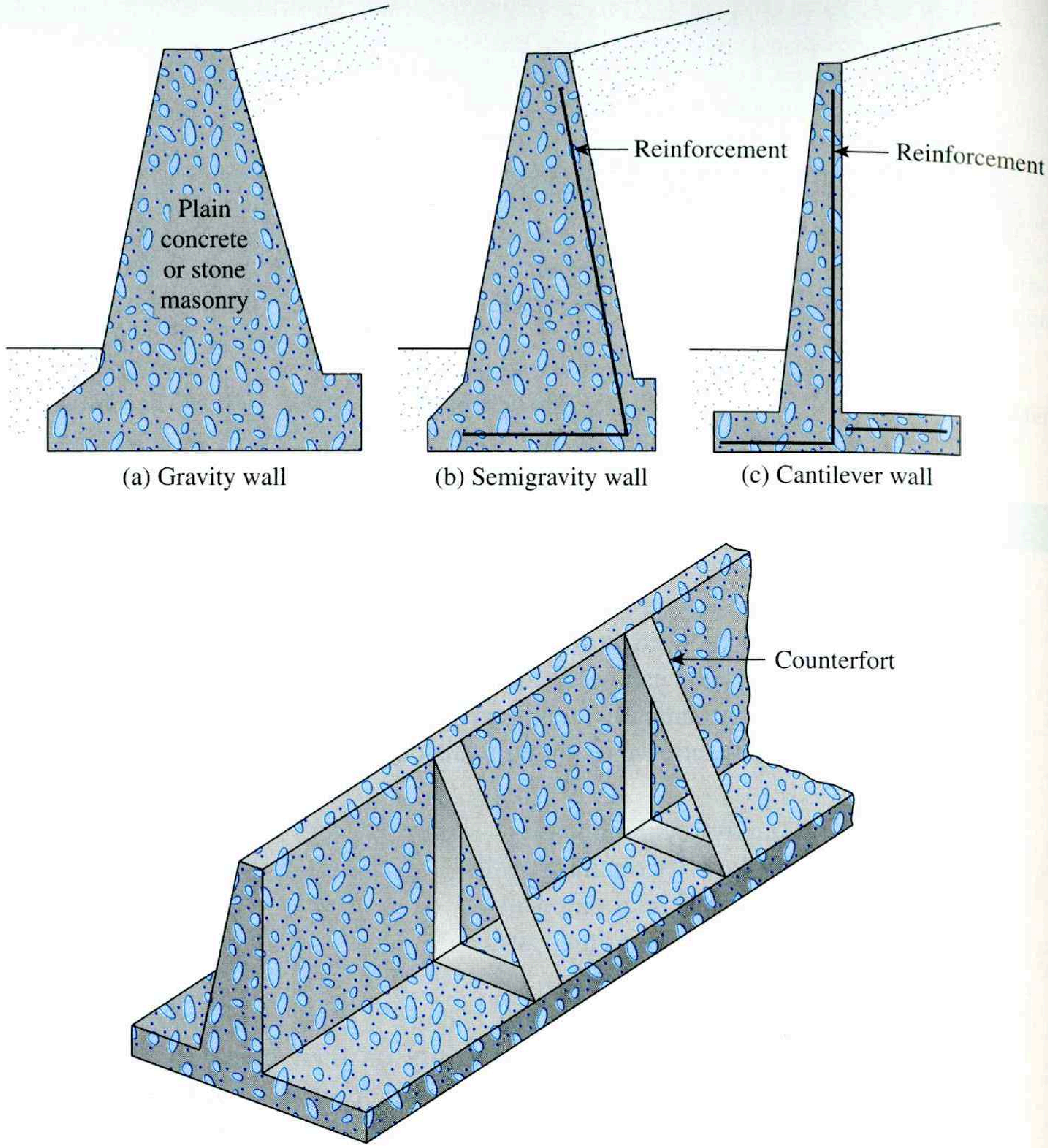
Gravity retaining walls (Figure 8.1a) are constructed with plain concrete or stone masonry. They depend for stability on their own weight and any soil resting on the masonry. This type of construction is not economical for high walls.

In many cases, a small amount of steel may be used for the construction of gravity walls, thereby minimizing the size of wall sections. Such walls are generally referred to as *semigravity walls* (Figure 8.1b).

Cantilever retaining walls (Figure 8.1c) are made of reinforced concrete that consists of a thin stem and a base slab. This type of wall is economical to a height of about 8 m (25 ft). Figure 8.2 shows a cantilever retaining wall under construction.

Counterfort retaining walls (Figure 8.1d) are similar to cantilever walls. At regular intervals, however, they have thin vertical concrete slabs known as *counterforts* that tie the wall and the base slab together. The purpose of the counterforts is to reduce the shear and the bending moments.

To design retaining walls properly, an engineer must know the basic parameters—the *unit weight, angle of friction,* and *cohesion*—of the soil retained behind the wall and the soil below the base slab. Knowing the properties of the soil behind the wall enables the engineer to determine the lateral pressure distribution that has to be designed for.



(a) Gravity wall (b) Semigravity wall (c) Cantilever wall

(d) Counterfort wall

Figure 8.1 Types of retaining wall

There are two phases in the design of a conventional retaining wall. First, with the lateral earth pressure known, the structure as a whole is checked for *stability*. The structure is examined for possible *overturning, sliding,* and *bearing capacity* failures. Second, each component of the structure is checked for *strength,* and the *steel reinforcement* of each component is determined.

This chapter presents the procedures for determining the stability of the retaining wall. Checks for strength can be found in any textbook on reinforced concrete.

Some retaining walls have their backfills stabilized mechanically by including reinforcing elements such as metal strips, bars, welded wire mats, geotextiles, and

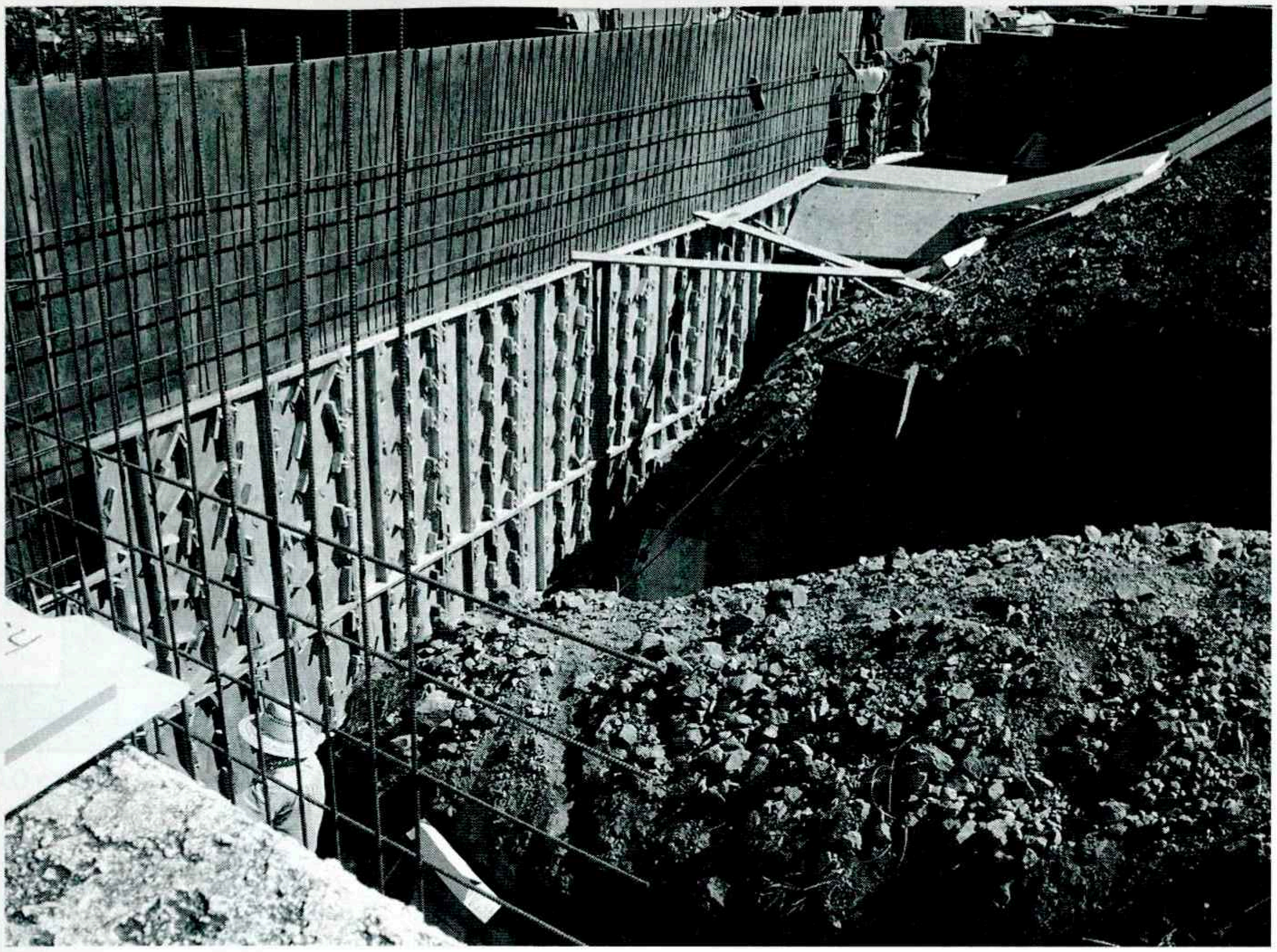


Figure 8.2 A cantilever retaining wall under construction (*Courtesy of Dharma Shakya, Geotechnical Solutions, Inc., Irvine, California*)

geogrids. These walls are relatively flexible and can sustain large horizontal and vertical displacements without much damage.

Gravity and Cantilever Walls

8.2 Proportioning Retaining Walls

In designing retaining walls, an engineer must assume some of their dimensions. Called *proportioning,* such assumptions allow the engineer to check trial sections of the walls for stability. If the stability checks yield undesirable results, the sections can be changed and rechecked. Figure 8.3 shows the general proportions of various retaining-wall components that can be used for initial checks.

Note that the top of the stem of any retaining wall should not be less than about 0.3 m. ($\approx$12 in.) for proper placement of concrete. The depth, *D,* to the bottom of the base slab should be a minimum of 0.6 m($\approx$2 ft). However, the bottom of the base slab should be positioned below the seasonal frost line.

For counterfort retaining walls, the general proportion of the stem and the base slab is the same as for cantilever walls. However, the counterfort slabs may be about 0.3 m ($\approx$12 in.) thick and spaced at center-to-center distances of 0.3*H* to 0.7*H*.

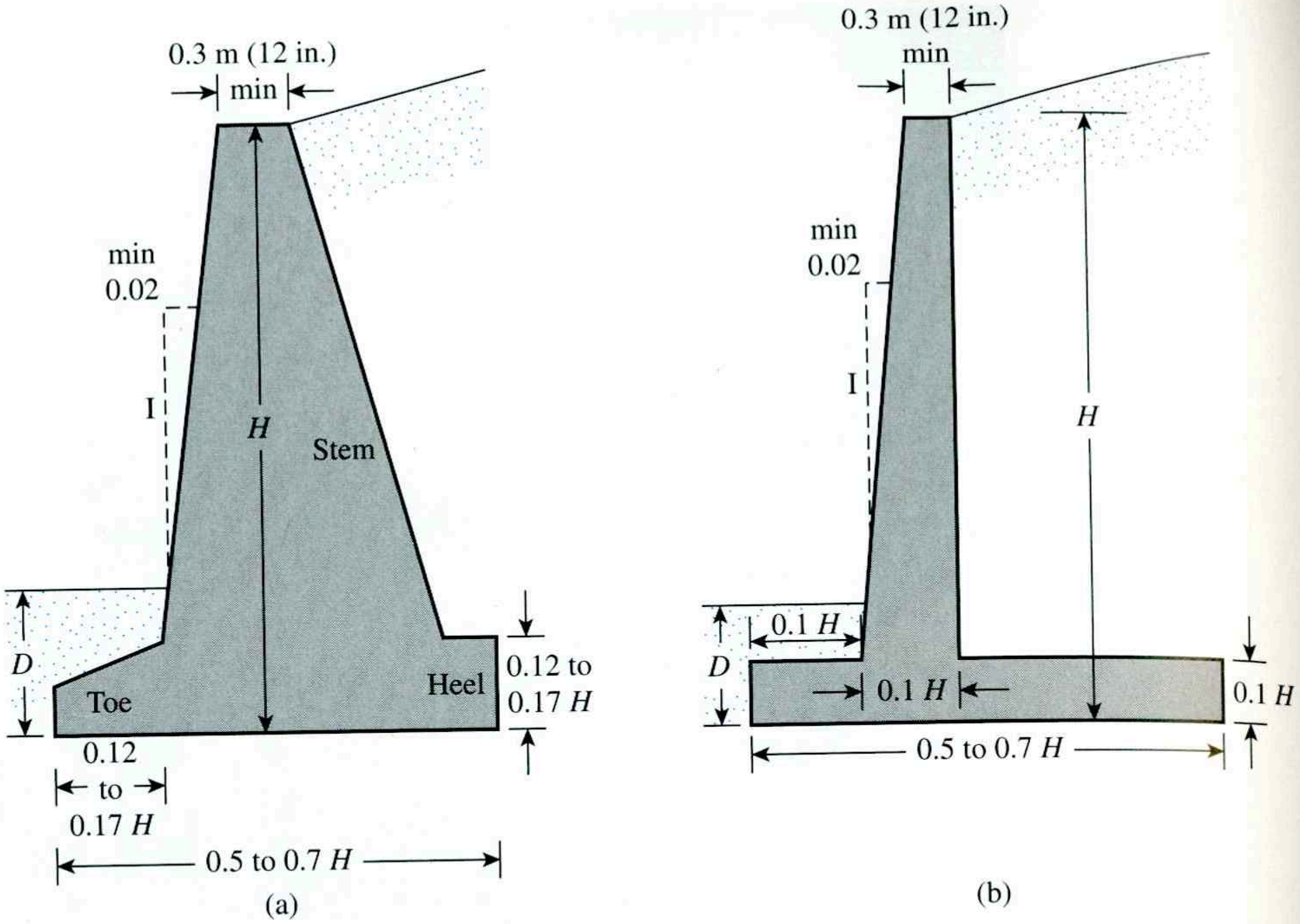


Figure 8.3 Approximate dimensions for various components of retaining wall for initial stability checks: (a) gravity wall; (b) cantilever wall

8.3 Application of Lateral Earth Pressure Theories to Design

The fundamental theories for calculating lateral earth pressure were presented in Chapter 7. To use these theories in design, an engineer must make several simple assumptions. In the case of cantilever walls, the use of the Rankine earth pressure theory for stability checks involves drawing a vertical line *AB* through point *A*, located at the edge of the heel of the base slab in Figure 8.4a. The Rankine active condition is assumed to exist along the vertical plane *AB*. Rankine active earth pressure equations may then be used to calculate the lateral pressure on the face *AB* of the wall. In the analysis of the wall's stability, the force $P_{a(\text{Rankine})}$, the weight of soil above the heel, and the weight W_c of the concrete all should be taken into consideration. The assumption for the development of Rankine active pressure along the soil face *AB* is theoretically correct if the shear zone bounded by the line *AC* is not obstructed by the stem of the wall. The angle, η, that the line *AC* makes with the vertical is

$$\eta = 45 + \frac{\alpha}{2} - \frac{\phi'}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) \tag{8.1}$$

A similar type of analysis may be used for gravity walls, as shown in Figure 8.4b. However, *Coulomb's active earth pressure theory* also may be used, as shown in Figure 8.4c. If it is used, the only forces to be considered are $P_{a(\text{Coulomb})}$ and the weight of the wall, W_c.

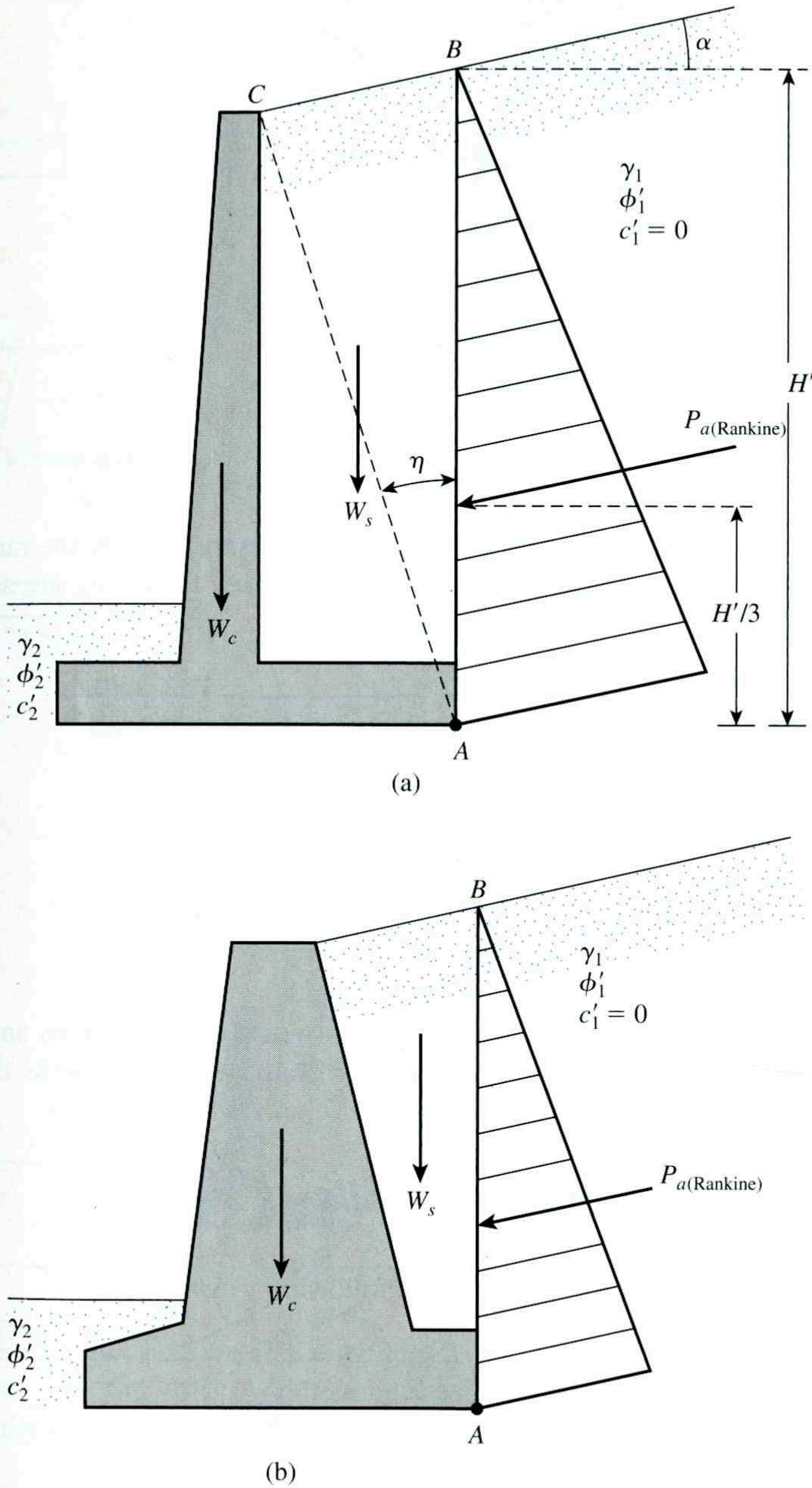
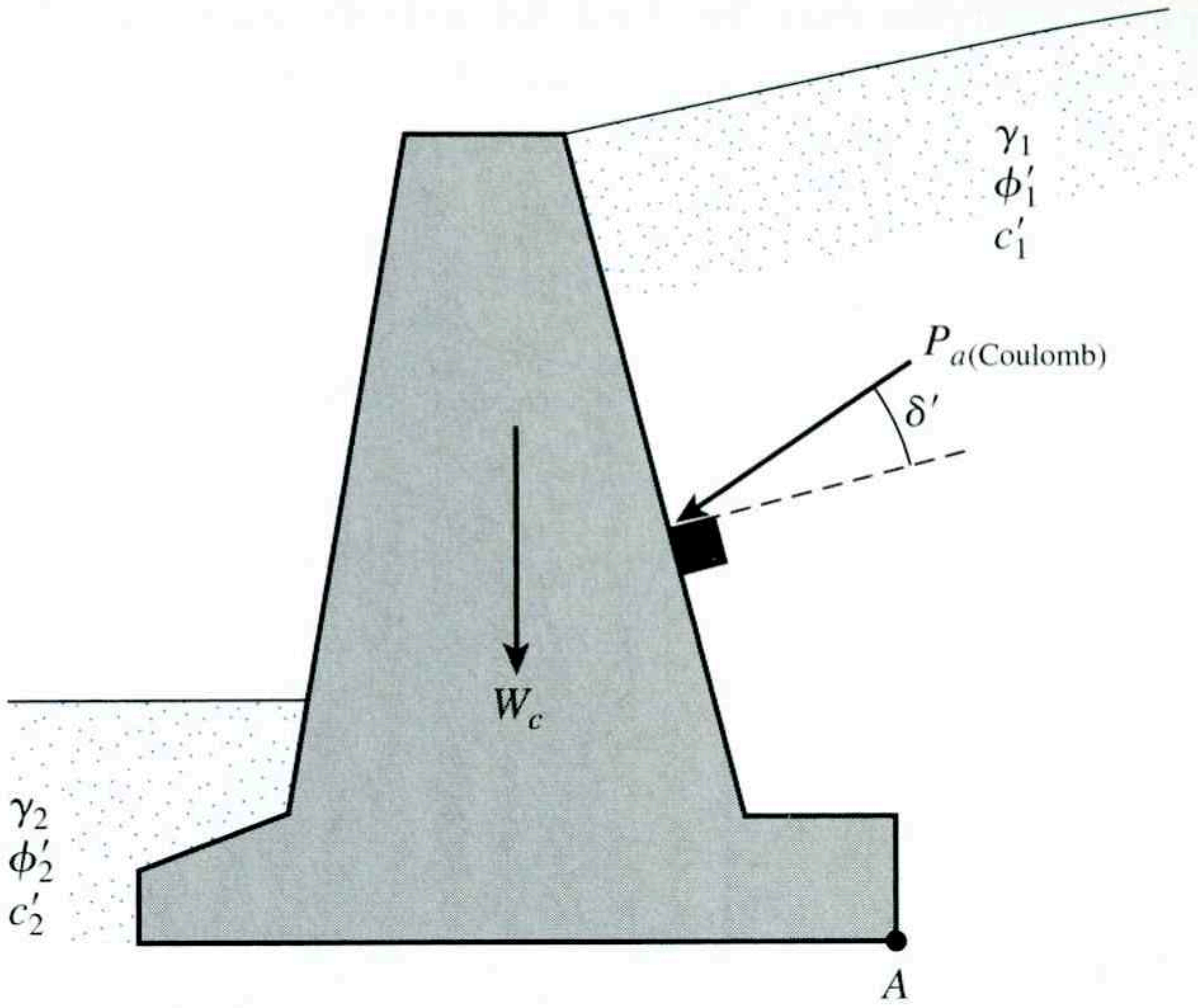


Figure 8.4 Assumption for the determination of lateral earth pressure: (a) cantilever wall; (b) and (c) gravity wall



(c)

Figure 8.4 *(continued)*

If Coulomb's theory is used, it will be necessary to know the range of the wall friction angle δ' with various types of backfill material. Following are some ranges of wall friction angle for masonry or mass concrete walls:

Backfill material	Range of δ' (deg)
Gravel	27–30
Coarse sand	20–28
Fine sand	15–25
Stiff clay	15–20
Silty clay	12–16

In the case of ordinary retaining walls, water table problems and hence hydrostatic pressure are not encountered. Facilities for drainage from the soils that are retained are always provided.

8.4 Stability of Retaining Walls

A retaining wall may fail in any of the following ways:

- It may *overturn* about its toe. (See Figure 8.5a.)
- It may *slide* along its base. (See Figure 8.5b.)
- It may fail due to the loss of *bearing capacity* of the soil supporting the base. (See Figure 8.5c.)
- It may undergo deep-seated shear failure. (See Figure 8.5d.)
- It may go through excessive settlement.

The checks for stability against overturning, sliding, and bearing capacity failure will be described in Sections 8.5, 8.6, and 8.7. The principles used to estimate settlement

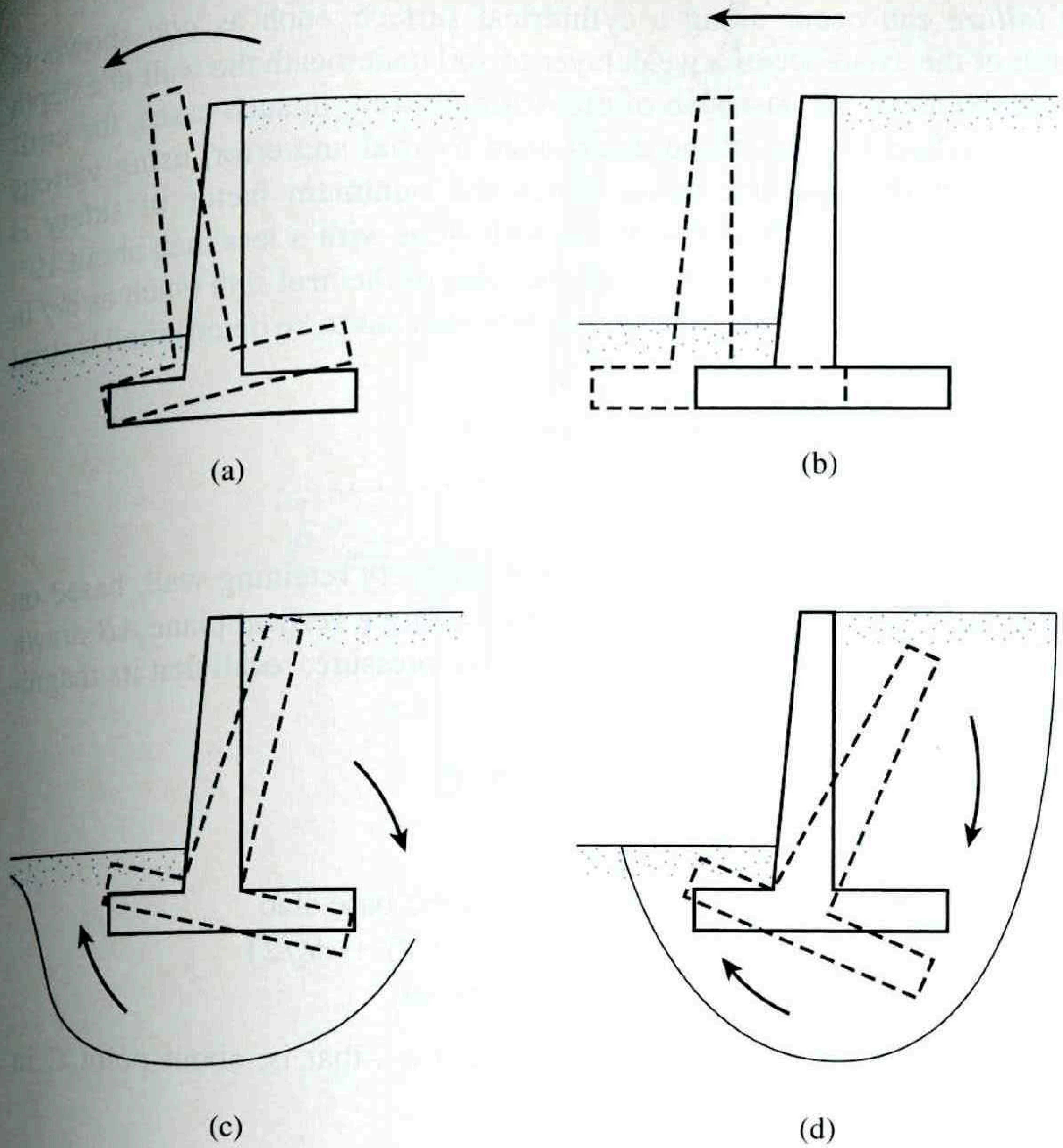


Figure 8.5 Failure of retaining wall: (a) by overturning; (b) by sliding; (c) by bearing capacity failure; (d) by deep-seated shear failure

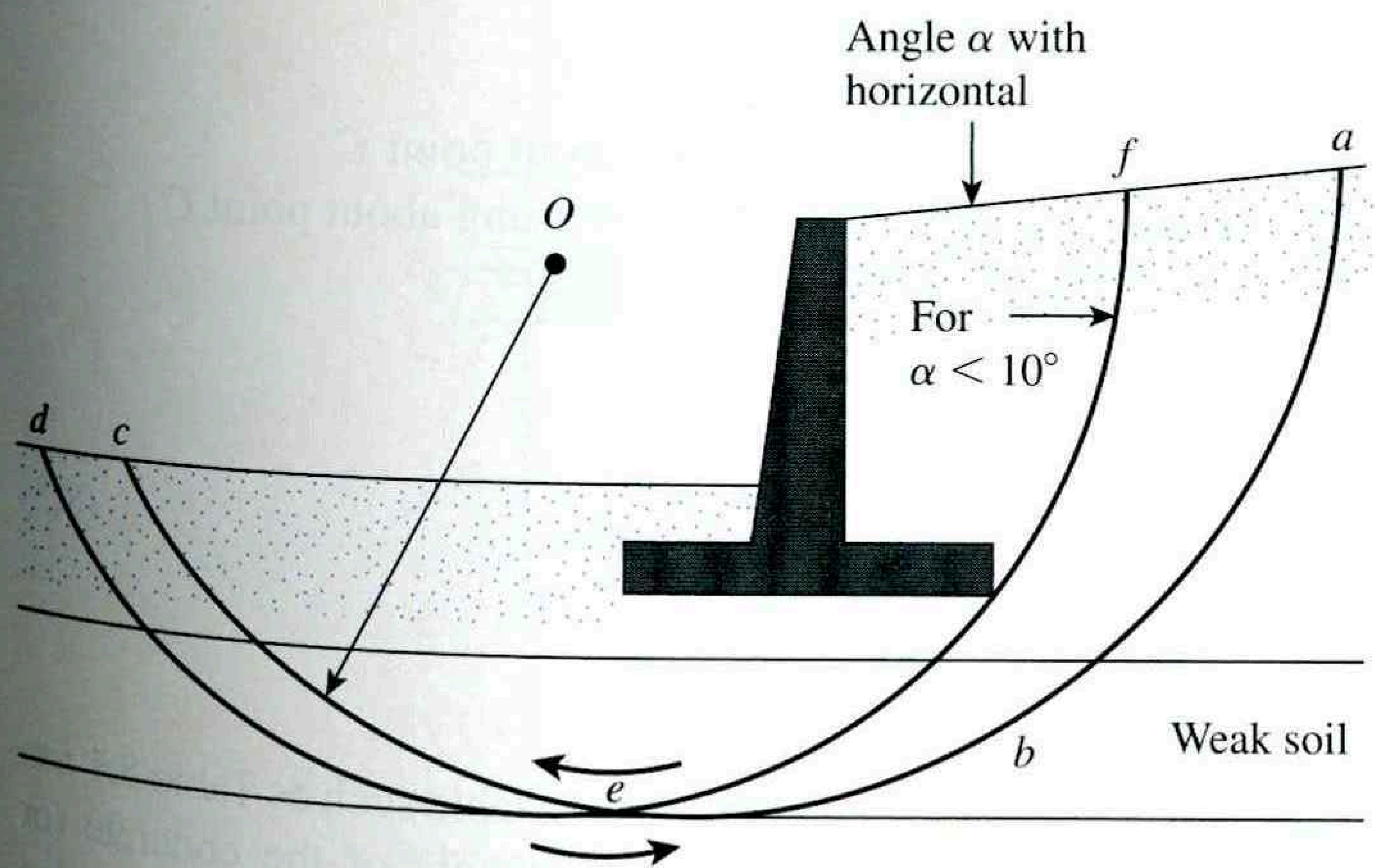


Figure 8.6 Deep-seated shear failure

were covered in Chapter 5 and will not be discussed further. When a weak soil layer is located at a shallow depth—that is, within a depth of 1.5 times the width of the base slab of the retaining wall—the possibility of excessive settlement should be considered. In some cases, the use of lightweight backfill material behind the retaining wall may solve the problem.

Deep shear failure can occur along a cylindrical surface, such as *abc* shown in Figure 8.6, as a result of the existence of a weak layer of soil underneath the wall at a depth of about 1.5 times the width of the base slab of the retaining wall. In such cases, the critical cylindrical failure surface *abc* has to be determined by trial and error, using various centers such as O. The failure surface along which the minimum factor of safety is obtained is the *critical surface of sliding*. For the backfill slope with α less than about 10°, the critical failure circle apparently passes through the edge of the heel slab (such as *def* in the figure). In this situation, the minimum factor of safety also has to be determined by trial and error by changing the center of the trial circle.

8.5 Check for Overturning

Figure 8.7 shows the forces acting on a cantilever and a gravity retaining wall, based on the assumption that the Rankine active pressure is acting along a vertical plane AB drawn through the heel of the structure. P_p is the Rankine passive pressure; recall that its magnitude is [from Eq. (7.63)].

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

where

γ_2 = unit weight of soil in front of the heel and under the base slab
K_p = Rankine passive earth pressure coefficient = $\tan^2(45 + \phi_2'/2)$
c_2', ϕ_2' = cohesion and effective soil friction angle, respectively

The factor of safety against overturning about the toe—that is, about point C in Figure 8.7—may be expressed as

$$\mathrm{FS}_{(\text{overturning})} = \frac{\Sigma M_R}{\Sigma M_o} \tag{8.2}$$

where

ΣM_o = sum of the moments of forces tending to overturn about point C
ΣM_R = sum of the moments of forces tending to resist overturning about point C

The overturning moment is

$$\Sigma M_o = P_h\left(\frac{H'}{3}\right) \tag{8.3}$$

where $P_h = P_a \cos\alpha$.

To calculate the resisting moment, ΣM_R (neglecting P_p), a table such as Table 8.1 can be prepared. The weight of the soil above the heel and the weight of the concrete (or masonry) are both forces that contribute to the resisting moment. Note that the force P_v also contributes to the resisting moment. P_v is the vertical component of the active force P_a, or

$$P_v = P_a \sin\alpha$$

The moment of the force P_v about C is

$$M_v = P_v B = P_a \sin\alpha B \tag{8.4}$$

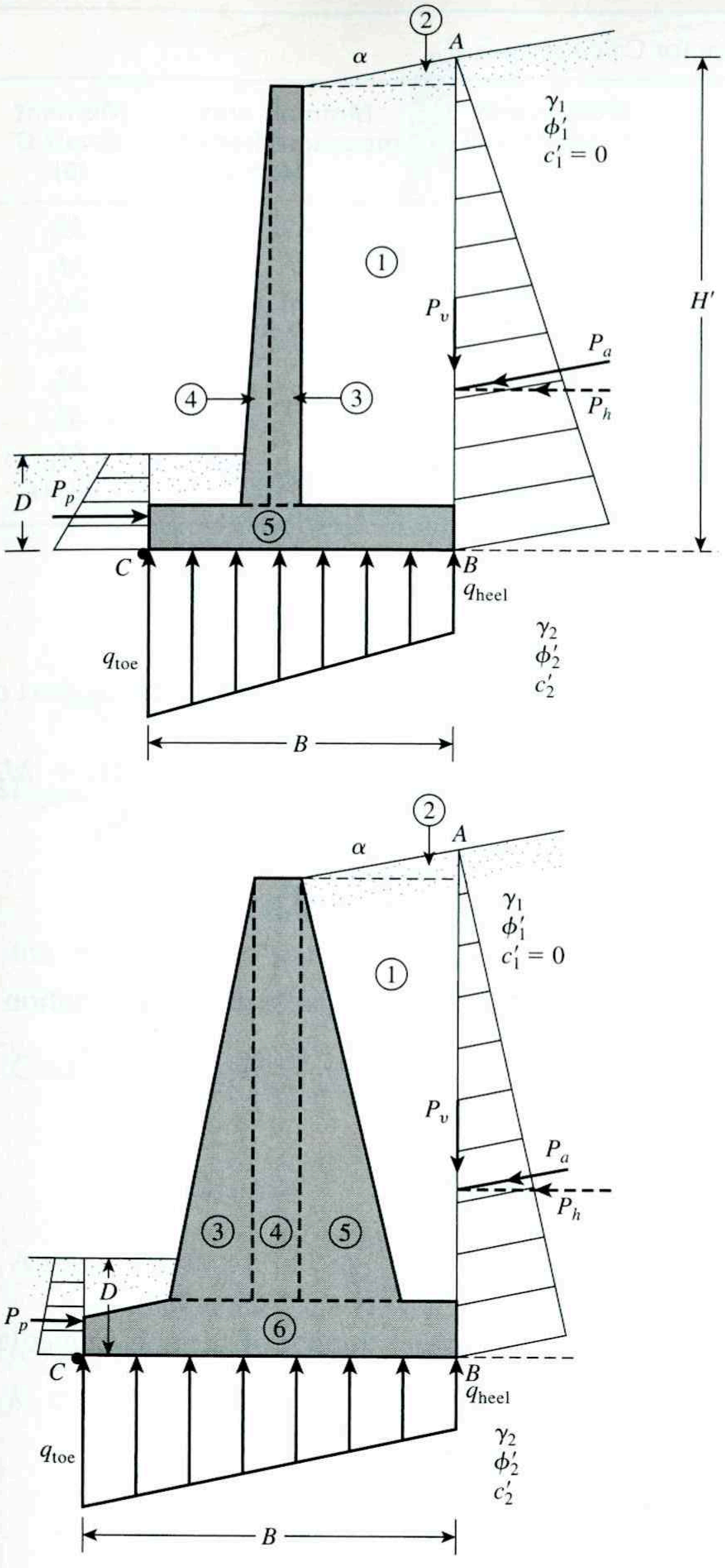


Figure 8.7 Check for overturning, assuming that the Rankine pressure is valid

where B = width of the base slab.

Once ΣM_R is known, the factor of safety can be calculated as

$$\text{FS}_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_v}{P_a \cos\alpha (H'/3)} \tag{8.5}$$

The usual minimum desirable value of the factor of safety with respect to overturning is 2 to 3.

Table 8.1 Procedure for Calculating ΣM_R

Section (1)	Area (2)	Weight/unit length of wall (3)	Moment arm measured from C (4)	Moment about C (5)
1	A_1	$W_1 = \gamma_1 \times A_1$	X_1	M_1
2	A_2	$W_2 = \gamma_1 \times A_2$	X_2	M_2
3	A_3	$W_3 = \gamma_c \times A_3$	X_3	M_3
4	A_4	$W_4 = \gamma_c \times A_4$	X_4	M_4
5	A_5	$W_5 = \gamma_c \times A_5$	X_5	M_5
6	A_6	$W_6 = \gamma_c \times A_6$	X_6	M_6
		P_v	B	M_v
		ΣV		ΣM_R

(*Note:* γ_l = unit weight of backfill
γ_c = unit weight of concrete)

Some designers prefer to determine the factor of safety against overturning with the formula

$$\text{FS}_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6}{P_a \cos\alpha(H'/3) - M_v} \tag{8.6}$$

8.6 Check for Sliding along the Base

The factor of safety against sliding may be expressed by the equation

$$\text{FS}_{(\text{sliding})} = \frac{\Sigma F_{R'}}{\Sigma F_d} \tag{8.7}$$

where

$\Sigma F_{R'}$ = sum of the horizontal resisting forces
ΣF_d = sum of the horizontal driving forces

Figure 8.8 indicates that the shear strength of the soil immediately below the base slab may be represented as

$$s = \sigma' \tan \delta' + c'_a$$

where

δ' = angle of friction between the soil and the base slab
c'_a = adhesion between the soil and the base slab

Thus, the maximum resisting force that can be derived from the soil per unit length of the wall along the bottom of the base slab is

$$R' = s(\text{area of cross section}) = s(B \times 1) = B\sigma' \tan \delta' + Bc'_a$$

However,

$$B\sigma' = \text{sum of the vertical force} = \Sigma V (\text{see Table 8.1})$$

so

$$R' = (\Sigma V) \tan \delta' + Bc'_a$$

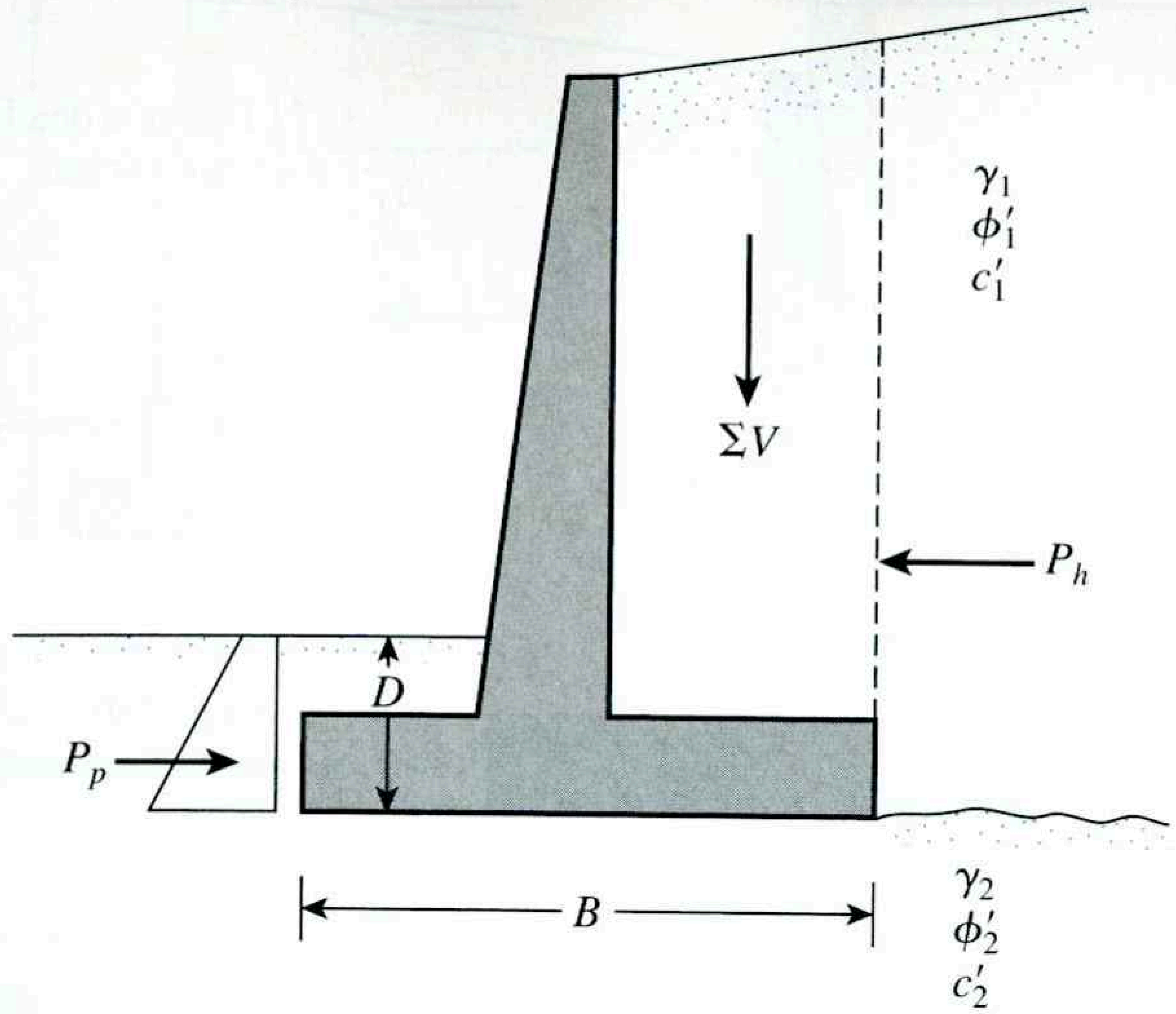


Figure 8.8 Check for sliding along the base

Figure 8.8 shows that the passive force P_p is also a horizontal resisting force. Hence,

$$\Sigma F_{R'} = (\Sigma V)\tan\delta' + Bc'_a + P_p \tag{8.8}$$

The only horizontal force that will tend to cause the wall to slide (a *driving force*) is the horizontal component of the active force P_a, so

$$\Sigma F_d = P_a \cos\alpha \tag{8.9}$$

Combining Eqs. (8.7), (8.8), and (8.9) yields

$$\mathrm{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan\delta' + Bc'_a + P_p}{P_a\cos\alpha} \tag{8.10}$$

A minimum factor of safety of 1.5 against sliding is generally required.

In many cases, the passive force P_p is ignored in calculating the factor of safety with respect to sliding. In general, we can write $\delta' = k_1\phi'_2$ and $c'_a = k_2c'_2$. In most cases, k_1 and k_2 are in the range from $\frac{1}{2}$ to $\frac{2}{3}$. Thus,

$$\mathrm{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan(k_1\phi'_2) + Bk_2c'_2 + P_p}{P_a\cos\alpha} \tag{8.11}$$

If the desired value of $\mathrm{FS}_{(\text{sliding})}$ is not achieved, several alternatives may be investigated (see Figure 8.9):

- Increase the width of the base slab (i.e., the heel of the footing).
- Use a key to the base slab. If a key is included, the passive force per unit length of the wall becomes

$$P_p = \frac{1}{2}\gamma_2 D_1^2 K_p + 2c'_2 D_1\sqrt{K_p}$$

where $K_p = \tan^2\left(45 + \dfrac{\phi'_2}{2}\right)$.

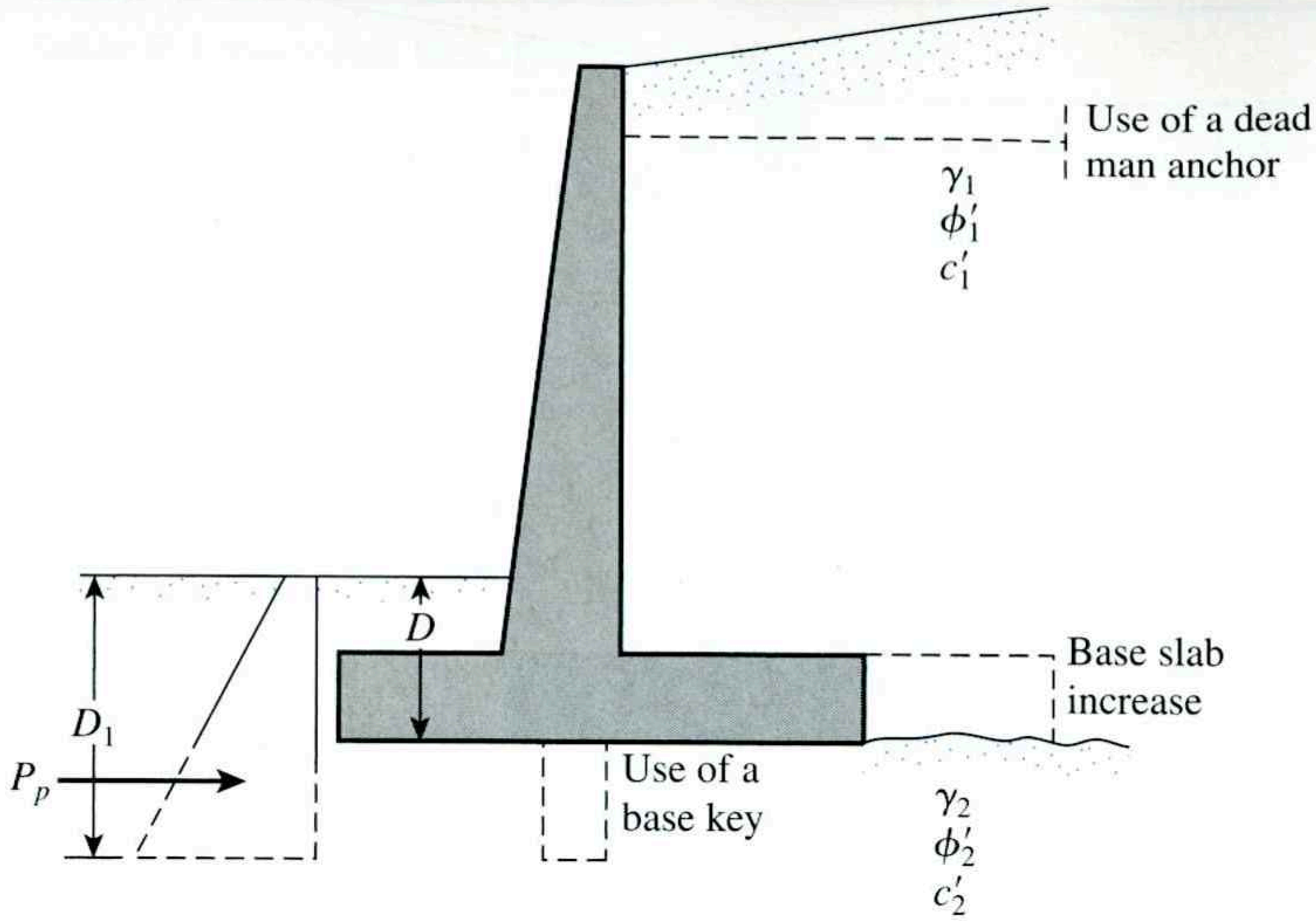


Figure 8.9 Alternatives for increasing the factor of safety with respect to sliding

- Use a *deadman anchor* at the stem of the retaining wall.
- Another possible way to increase the value of $FS_{(sliding)}$ is to consider reducing the value of P_a [see Eq. (8.11)]. One possible way to do so is to use the method developed by Elman and Terry (1988). The discussion here is limited to the case in which the retaining wall has a horizontal granular backfill (Figure 8.10). In Figure 8.10, the active force, P_a, is horizontal ($\alpha = 0$) so that

$$P_a \cos\alpha = P_h = P_a$$

and

$$P_a \sin\alpha = P_v = 0$$

However,

$$P_a = P_{a(1)} + P_{a(2)} \tag{8.12}$$

The magnitude of $P_{a(2)}$ can be reduced if the heel of the retaining wall is sloped as shown in Figure 8.10. For this case,

$$P_a = P_{a(1)} + AP_{a(2)} \tag{8.13}$$

The magnitude of A, as shown in Table 8.2, is valid for $\alpha' = 45°$. However note that in Figure 8.10a

$$P_{a(1)} = \frac{1}{2}\gamma_1 K_a (H' - D')^2$$

and

$$P_a = \frac{1}{2}\gamma_1 K_a H'^2$$

Hence,

$$P_{a(2)} = \frac{1}{2}\gamma_1 K_a[H'^2 - (H' - D')^2]$$

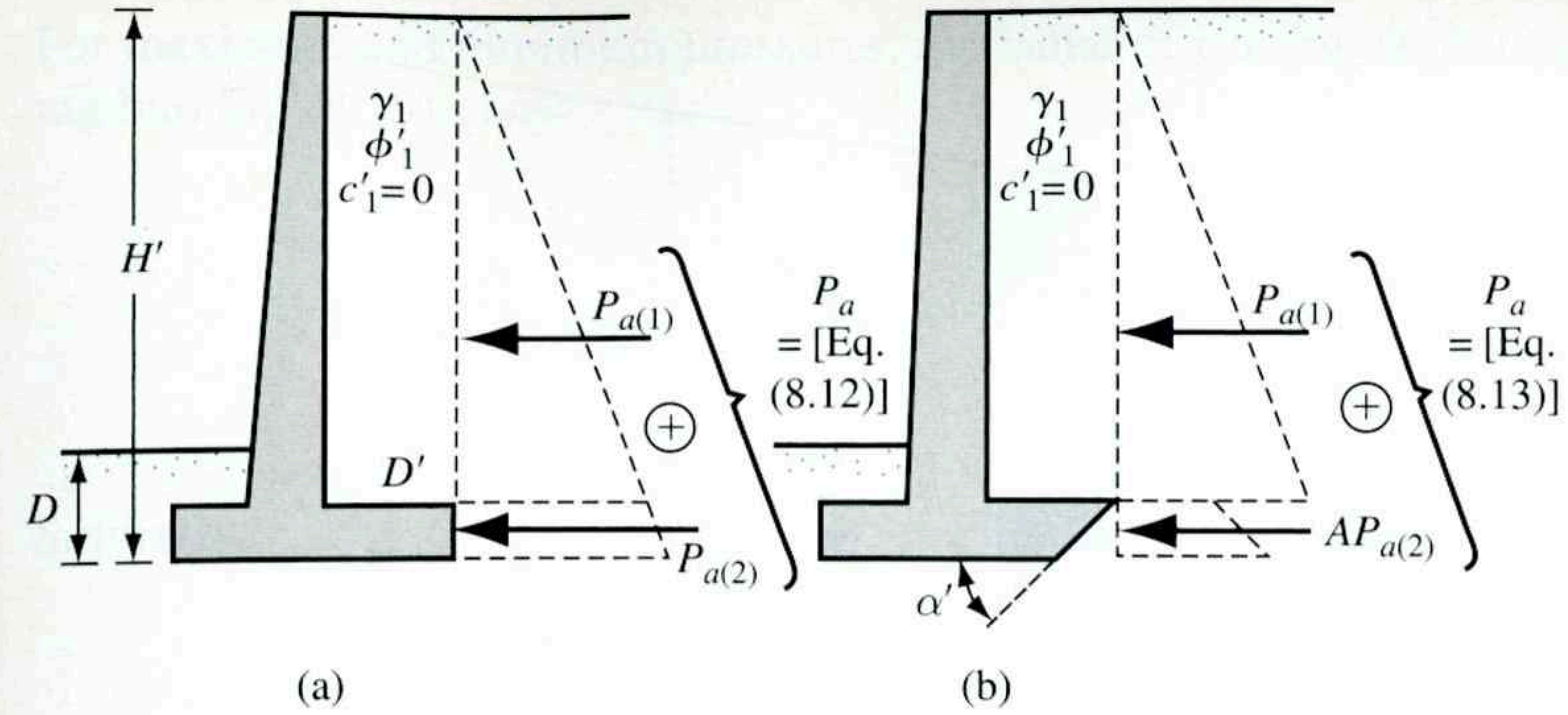


Figure 8.10 Retaining wall with sloped heel

Table 8.2 Variation of A with ϕ'_1 (for $\alpha' = 45°$)

Soil friction angle, ϕ'_1 (deg)	***A***
20	0.28
25	0.14
30	0.06
35	0.03
40	0.018

So, for the active pressure diagram shown in Figure 8.10b,

$$P_a = \frac{1}{2}\gamma_1 K_a (H' - D')^2 + \frac{A}{2}\gamma_1 K_a [H'^2 - (H' - D')^2] \tag{8.14}$$

Sloping the heel of a retaining wall can thus be extremely helpful in some cases.

8.7 Check for Bearing Capacity Failure

The vertical pressure transmitted to the soil by the base slab of the retaining wall should be checked against the ultimate bearing capacity of the soil. The nature of variation of the vertical pressure transmitted by the base slab into the soil is shown in Figure 8.11. Note that q_{toe} and q_{heel} are the *maximum* and the *minimum* pressures occurring at the ends of the toe and heel sections, respectively. The magnitudes of q_{toe} and q_{heel} can be determined in the following manner:

The sum of the vertical forces acting on the base slab is $\Sigma\,\mathbf{V}$ (see column 3 of Table 8.1), and the horizontal force $\mathbf{P}_h$ is $P_a \cos\alpha$. Let

$$\mathbf{R} = \Sigma\,\mathbf{V} + \mathbf{P}_h \tag{8.15}$$

be the resultant force. The net moment of these forces about point C in Figure 8.11 is

$$M_{\text{net}} = \Sigma M_R - \Sigma M_o \tag{8.16}$$

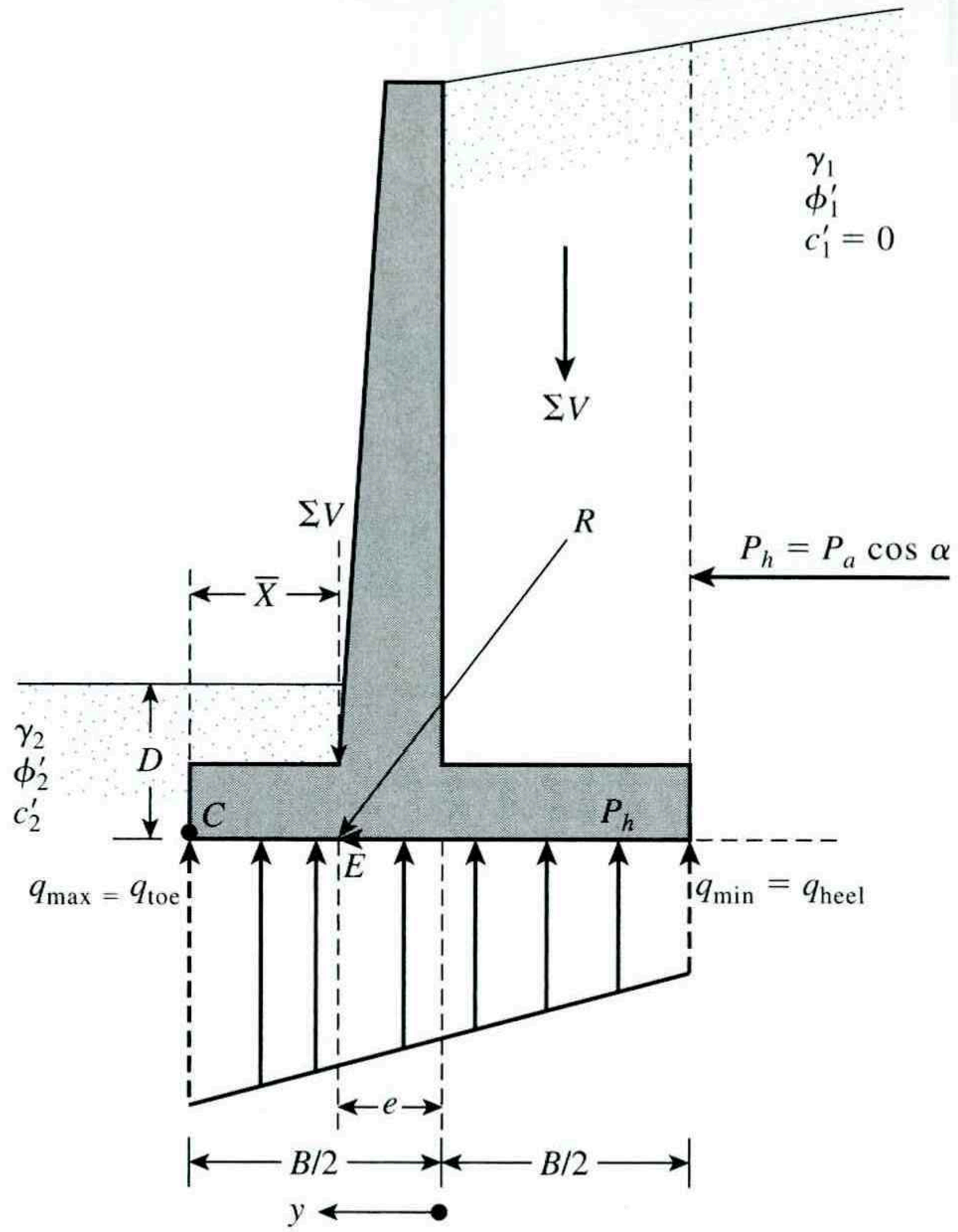


Figure 8.11 Check for bearing capacity failure

Note that the values of ΣM_R and ΣM_o were previously determined. [See Column 5 of Table 8.1 and Eq. (8.3)]. Let the line of action of the resultant R intersect the base slab at E. Then the distance

$$\overline{CE} = \overline{X} = \frac{M_{\text{net}}}{\Sigma V} \tag{8.17}$$

Hence, the eccentricity of the resultant R may be expressed as

$$e = \frac{B}{2} - \overline{CE} \tag{8.18}$$

The pressure distribution under the base slab may be determined by using simple principles from the mechanics of materials. First, we have

$$q = \frac{\Sigma V}{A} \pm \frac{M_{\text{net}} y}{I} \tag{8.19}$$

where

M_{net} = moment = $(\Sigma V)e$
I = moment of inertia per unit length of the base section
$= \frac{1}{12}(1)(B^3)$

For maximum and minimum pressures, the value of y in Eq. (8.19) equals $B/2$. Substituting into Eq. (8.19) gives

$$q_{\max} = q_{\text{toe}} = \frac{\Sigma V}{(B)(1)} + \frac{e(\Sigma V)\frac{B}{2}}{\left(\frac{1}{12}\right)(B^3)} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) \tag{8.20}$$

Similarly,

$$q_{\min} = q_{\text{heel}} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) \tag{8.21}$$

Note that ΣV includes the weight of the soil, as shown in Table 8.1, and that when the value of the eccentricity e becomes greater than $B/6$, $q_{\min}$ [Eq. (8.21)] becomes negative. Thus, there will be some tensile stress at the end of the heel section. This stress is not desirable, because the tensile strength of soil is very small. If the analysis of a design shows that $e > B/6$, the design should be reproportioned and calculations redone.

The relationships pertaining to the ultimate bearing capacity of a shallow foundation were discussed in Chapter 3. Recall that [Eq. (3.40)].

$$q_u = c_2' N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \tfrac{1}{2}\gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i} \tag{8.22}$$

where

$$q = \gamma_2 D$$
$$B' = B - 2e$$
$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi_2'}$$
$$F_{qd} = 1 + 2\tan\phi_2'(1 - \sin\phi_2')^2 \frac{D}{B'}$$
$$F_{\gamma d} = 1$$
$$F_{ci} = F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2$$
$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi_2'^\circ}\right)^2$$
$$\psi^\circ = \tan^{-1}\left(\frac{P_a \cos\alpha}{\Sigma V}\right)$$

Note that the shape factors F_{cs}, F_{qs}, and $F_{\gamma s}$ given in Chapter 3 are all equal to unity, because they can be treated as a continuous foundation. For this reason, the shape factors are not shown in Eq. (8.22).

Once the ultimate bearing capacity of the soil has been calculated by using Eq. (8.22), the factor of safety against bearing capacity failure can be determined:

$$\text{FS}_{(\text{bearing capacity})} = \frac{q_u}{q_{\max}} \tag{8.23}$$

Generally, a factor of safety of 3 is required. In Chapter 3, we noted that the ultimate bearing capacity of shallow foundations occurs at a settlement of about 10% of the foundation width.

In the case of retaining walls, the width B is large. Hence, the ultimate load q_u will occur at a fairly large foundation settlement. A factor of safety of 3 against bearing capacity failure may not ensure that settlement of the structure will be within the tolerable limit in all cases. Thus, this situation needs further investigation.

An alternate relationship to Eq. (8.22) will be Eq. (3.67), or

$$q_u = c'N_{c(ei)}F_{cd} + qN_{q(ei)}F_{qd} + \tfrac{1}{2}\gamma_2 BN_{\gamma(ei)}F_{\gamma d}$$

Since $F_{\gamma d} = 1$,

$$q_u = c'N_{c(ei)}F_{cd} + qN_{q(ei)}F_{qd} + \tfrac{1}{2}\gamma_2 BN_{\gamma(ei)} \tag{8.24}$$

The bearing capacity factors, $N_{c(ei)}$, $N_{q(ei)}$, and $N_{\gamma(ei)}$ were given in Figures 3.26 through 3.28.

Example 8.1

The cross section of a cantilever retaining wall is shown in Figure 8.12. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

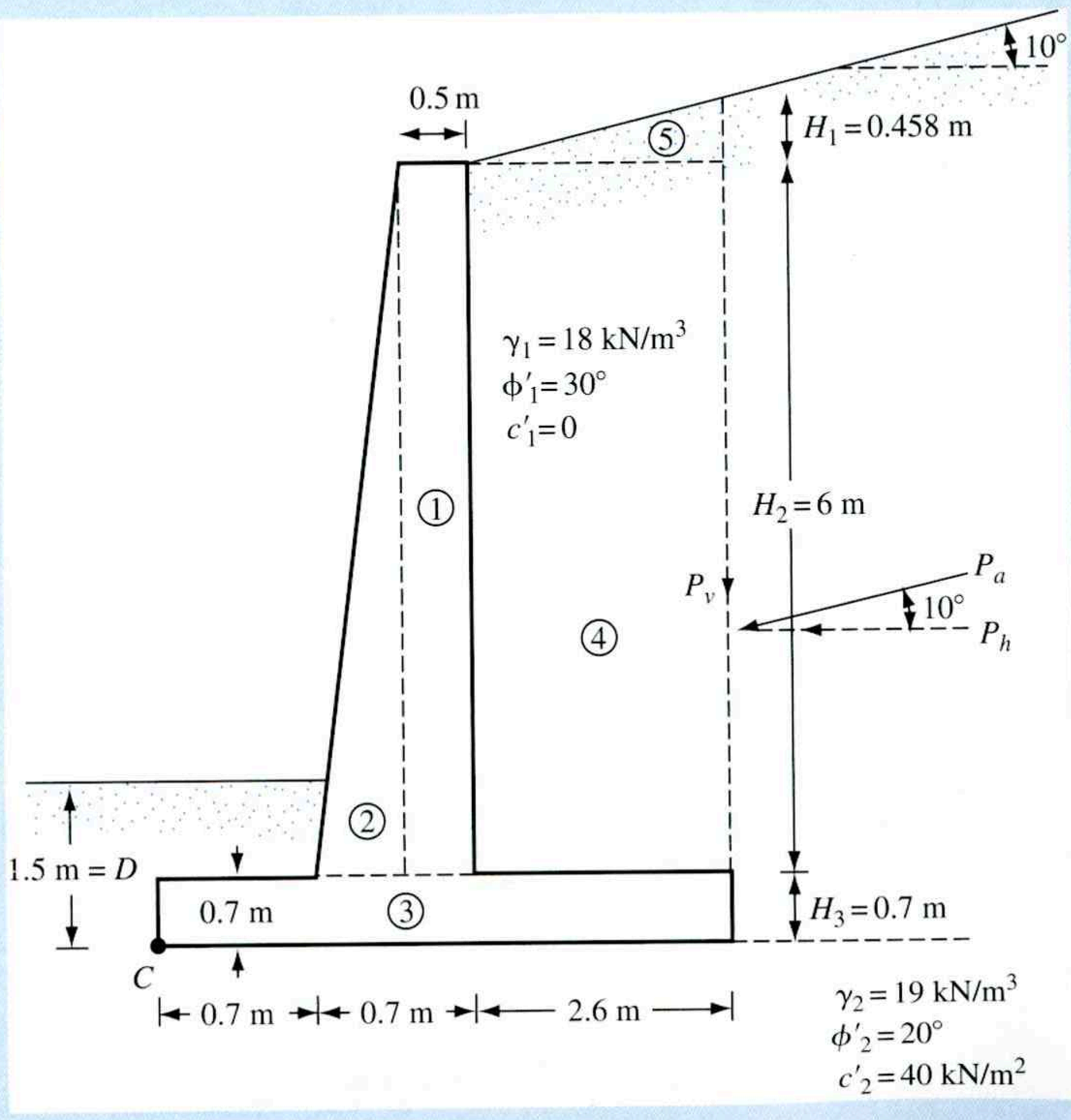


Figure 8.12 Calculation of stability of a retaining wall

Solution

From the figure,

$$H' = H_1 + H_2 + H_3 = 2.6 \tan 10° + 6 + 0.7$$
$$= 0.458 + 6 + 0.7 = 7.158 \text{ m}$$

The Rankine active force per unit length of wall $= P_p = \frac{1}{2}\gamma_1 H'^2 K_a$. For $\phi_1' = 30°$ and $\alpha = 10°$, K_a is equal to 0.3532. (See Table 7.1.) Thus,

$$P_a = \tfrac{1}{2}(18)(7.158)^2(0.3532) = 162.9 \text{ kN/m}$$

$$P_v = P_a \sin 10° = 162.9 (\sin 10°) = 28.29 \text{ kN/m}$$

and

$$P_h = P_a \cos 10° = 162.9 (\cos 10°) = 160.43 \text{ kN/m}$$

Factor of Safety against Overturning

The following table can now be prepared for determining the resisting moment:

Section no.[a]	Area (m²)	Weight/unit length (kN/m)	Moment arm from point *C* (m)	Moment (kN-m/m)
1	$6 \times 0.5 = 3$	70.74	1.15	81.35
2	$\frac{1}{2}(0.2)6 = 0.6$	14.15	0.833	11.79
3	$4 \times 0.7 = 2.8$	66.02	2.0	132.04
4	$6 \times 2.6 = 15.6$	280.80	2.7	758.16
5	$\frac{1}{2}(2.6)(0.458) = 0.595$	10.71	3.13	33.52
		$P_v = 28.29$	4.0	113.16
		$\Sigma V = 470.71$		$1130.02 = \Sigma M_R$

[a]For section numbers, refer to Figure 8.12
$\gamma_{\text{concrete}} = 23.58 \text{ kN/m}^3$

The overturning moment

$$M_o = P_h\left(\frac{H'}{3}\right) = 160.43\left(\frac{7.158}{3}\right) = 382.79 \text{ kN-m/m}$$

and

$$\text{FS}_{\text{(overturning)}} = \frac{\Sigma M_R}{M_o} = \frac{1130.02}{382.79} = \mathbf{2.95 > 2, OK}$$

Factor of Safety against Sliding

From Eq. (8.11),

$$\text{FS}_{\text{(sliding)}} = \frac{(\Sigma V)\tan(k_1\phi_2') + Bk_2c_2' + P_p}{P_a\cos\alpha}$$

Let $k_1 = k_2 = \frac{2}{3}$. Also,

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

$$K_p = \tan^2\left(45 + \frac{\phi_2'}{2}\right) = \tan^2(45 + 10) = 2.04$$

and

$$D = 1.5 \text{ m}$$

So

$$P_p = \tfrac{1}{2}(2.04)(19)(1.5)^2 + 2(40)(\sqrt{2.04})(1.5)$$
$$= 43.61 + 171.39 = 215 \text{ kN/m}$$

Hence,

$$FS_{\text{(sliding)}} = \frac{(470.71)\tan\left(\frac{2 \times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{160.43}$$

$$= \frac{111.56 + 106.67 + 215}{160.43} = \mathbf{2.7 > 1.5, OK}$$

Note: For some designs, the depth D in a passive pressure calculation may be taken to be *equal to the thickness of the base slab.*

Factor of Safety against Bearing Capacity Failure

Combining Eqs. (8.16), (8.17), and (8.18) yields

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{4}{2} - \frac{1130.02 - 382.79}{470.71}$$
$$= 0.411 \text{ m} < \frac{B}{6} = \frac{4}{6} = 0.666 \text{ m}$$

Again, from Eqs. (8.20) and (8.21)

$$q_{\substack{\text{toe}\\ \text{heel}}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right) = \frac{470.71}{4}\left(1 \pm \frac{6 \times 0.411}{4}\right) = 190.2 \text{ kN/m}^2 \text{ (toe)}$$
$$= 45.13 \text{ kN/m}^2 \text{ (heel)}$$

The ultimate bearing capacity of the soil can be determined from Eq. (8.22)

$$q_u = c_2' N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2}\gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

For $\phi_2' = 20°$ (see Table 3.3), $N_c = 14.83$, $N_q = 6.4$, and $N_\gamma = 5.39$. Also,

$$q = \gamma_2 D = (19)(1.5) = 28.5 \text{ kN/m}^2$$

$$B' = B - 2e = 4 - 2(0.411) = 3.178 \text{ m}$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi_2'} = 1.148 - \frac{1 - 1.148}{(14.83)(\tan 20)} = 1.175$$

$$F_{qd} = 1 + 2\tan\phi_2'(1 - \sin\phi_2')^2\left(\frac{D}{B'}\right) = 1 + 0.315\left(\frac{1.5}{3.178}\right) = 1.148$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\psi°}{90°}\right)^2$$

and

$$\psi = \tan^{-1}\left(\frac{P_a \cos\alpha}{\Sigma V}\right) = \tan^{-1}\left(\frac{160.43}{470.71}\right) = 18.82°$$

So

$$F_{ci} = F_{qi} = \left(1 - \frac{18.82}{90}\right)^2 = 0.626$$

and

$$F_{\gamma i} = \left(1 - \frac{\psi}{\phi_2'}\right)^2 = \left(1 - \frac{18.82}{20}\right)^2 \approx 0$$

Hence,

$$\begin{aligned} q_u &= (40)(14.83)(1.175)(0.626) + (28.5)(6.4)(1.148)(0.626) \\ &\quad + \tfrac{1}{2}(19)(5.93)(3.178)(1)(0) \\ &= 436.33 + 131.08 + 0 = 567.41 \text{ kN/m}^2 \end{aligned}$$

and

$$\text{FS}_{\text{(bearing capacity)}} = \frac{q_u}{q_{\text{toe}}} = \frac{567.41}{190.2} = \mathbf{2.98}$$

Note: $\text{FS}_{\text{(bearing capacity)}}$ is less than 3. Some reproportioning will be needed. ■

Example 8.2

A gravity retaining wall is shown in Figure 8.13. Use $\delta' = 2/3\phi_1'$ and Coulomb's active earth pressure theory. Determine

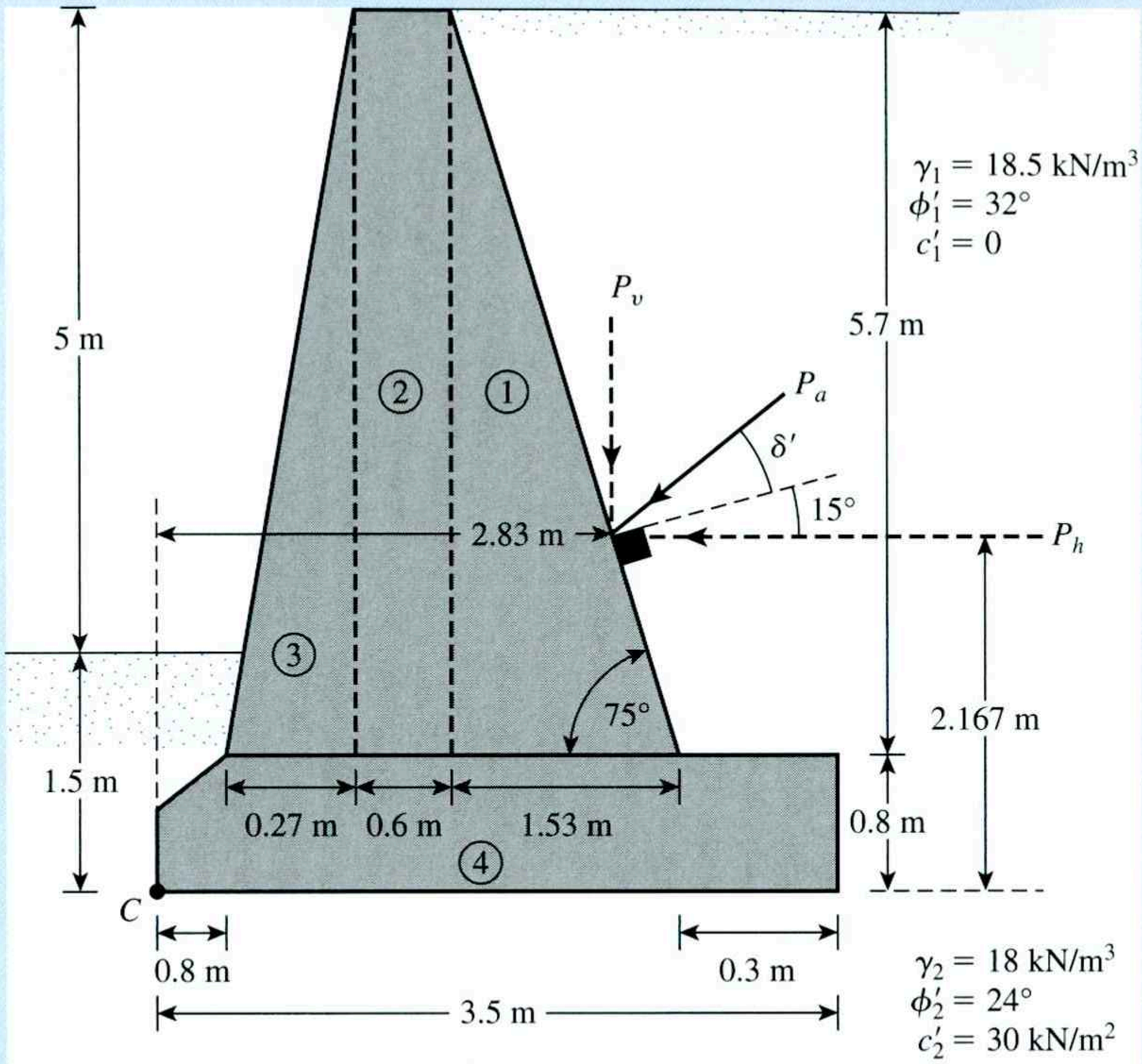


Figure 8.13 Gravity retaining wall (not to scale)

a. The factor of safety against overturning
b. The factor of safety against sliding
c. The pressure on the soil at the toe and heel

Solution

The height

$$H' = 5 + 1.5 = 6.5 \text{ m}$$

Coulomb's active force is

$$P_a = \tfrac{1}{2}\gamma_1 H'^2 K_a$$

With $\alpha = 0°$, $\beta = 75°$, $\delta' = 2/3\phi'_1$, and $\phi'_1 = 32°$, $K_a = 0.4023$. (See Table 7.4.) So,

$$P_a = \tfrac{1}{2}(18.5)(6.5)^2(0.4023) = 157.22 \text{ kN/m}$$

$$P_h = P_a \cos(15 + \tfrac{2}{3}\phi'_1) = 157.22 \cos 36.33 = 126.65 \text{ kN/m}$$

and

$$P_v = P_a \sin(15 + \tfrac{2}{3}\phi'_1) = 157.22 \sin 36.33 = 93.14 \text{ kN/m}$$

Part a: Factor of Safety against Overturning

From Figure 8.13, one can prepare the following table:

Area no.	Area (m^2)	Weight* (kN/m)	Moment arm from *C* (m)	Moment (kN-m/m)
1	$\frac{1}{2}(5.7)(1.53) = 4.36$	102.81	2.18	224.13
2	$(0.6)(5.7) = 3.42$	80.64	1.37	110.48
3	$\frac{1}{2}(0.27)(5.7) = 0.77$	18.16	0.98	17.80
4	$\approx (3.5)(0.8) = 2.8$	66.02	1.75	115.54
		$P_v = 93.14$	2.83	263.59
		$\Sigma V = 360.77$ kN/m		$\Sigma M_R = 731.54$ kN-m/m

*$\gamma_{\text{concrete}} = 23.58\ \text{kN/m}^3$

Note that the weight of the soil above the back face of the wall is not taken into account in the preceding table. We have

$$\text{Overturning moment} = M_o = P_h\left(\frac{H'}{3}\right) = 126.65(2.167) = 274.45\ \text{kN-m/m}$$

Hence,

$$\text{FS}_{(\text{overturning})} = \frac{\Sigma M_R}{\Sigma M_o} = \frac{731.54}{274.45} = \mathbf{2.67 > 2,\ OK}$$

Part b: Factor of Safety against Sliding

We have

$$\text{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan\left(\frac{2}{3}\phi_2'\right) + \frac{2}{3}c_2'B + P_p}{P_h}$$

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

and

$$K_p = \tan^2\left(45 + \frac{24}{2}\right) = 2.37$$

Hence,

$$P_p = \tfrac{1}{2}(2.37)(18)(1.5)^2 + 2(30)(1.54)(1.5) = 186.59\ \text{kN/m}$$

So

$$\text{FS}_{(\text{sliding})} = \frac{360.77\tan\left(\frac{2}{3} \times 24\right) + \frac{2}{3}(30)(3.5) + 186.59}{126.65}$$

$$= \frac{103.45 + 70 + 186.59}{126.65} = \mathbf{2.84}$$

If P_p is ignored, the factor of safety is **1.37**.

Part c: Pressure on Soil at Toe and Heel

From Eqs. (8.16), (8.17), and (8.18),

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{3.5}{2} - \frac{731.54 - 274.45}{360.77} = 0.483 < \frac{B}{6} = 0.583$$

$$q_{\text{toe}} = \frac{\Sigma V}{B}\left[1 + \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 + \frac{(6)(0.483)}{3.5}\right] = \mathbf{188.43\ kN/m^2}$$

and

$$q_{\text{heel}} = \frac{V}{B}\left[1 - \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 - \frac{(6)(0.483)}{3.5}\right] = \mathbf{17.73\ kN/m^2}$$ ■

8.8 Construction Joints and Drainage from Backfill

Construction Joints

A retaining wall may be constructed with one or more of the following joints:

1. *Construction joints* (see Figure 8.14a) are vertical and horizontal joints that are placed between two successive pours of concrete. To increase the shear at the joints, keys may be used. If keys are not used, the surface of the first pour is cleaned and roughened before the next pour of concrete.
2. *Contraction joints* (Figure 8.14b) are vertical joints (grooves) placed in the face of a wall (from the top of the base slab to the top of the wall) that allow the concrete to shrink without noticeable harm. The grooves may be about 6 to 8 mm (≈0.25 to 0.3 in.) wide and 12 to 16 mm (≈0.5 to 0.6 in.) deep.
3. *Expansion joints* (Figure 8.14c) allow for the expansion of concrete caused by temperature changes; vertical expansion joints from the base to the top of the wall may also be used. These joints may be filled with flexible joint fillers. In most cases, horizontal reinforcing steel bars running across the stem are continuous through all joints. The steel is greased to allow the concrete to expand.

Drainage from the Backfill

As the result of rainfall or other wet conditions, the backfill material for a retaining wall may become saturated, thereby increasing the pressure on the wall and perhaps creating an unstable condition. For this reason, adequate drainage must be provided by means of *weep holes* or *perforated drainage pipes.* (See Figure 8.15.)

When provided, weep holes should have a minimum diameter of about 0.1 m (4 in.) and be adequately spaced. Note that there is always a possibility that backfill material may

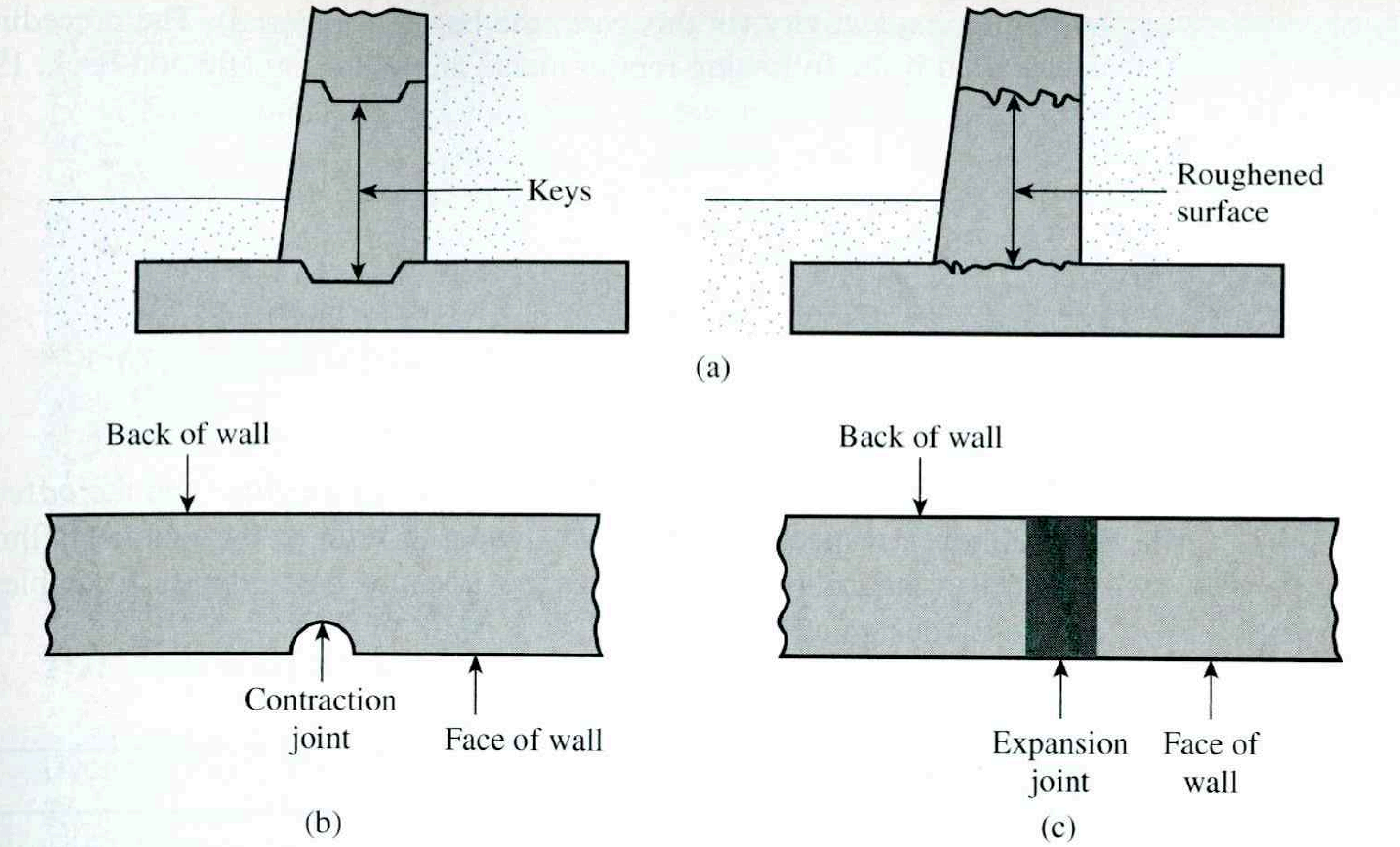


Figure 8.14 (a) Construction joints; (b) contraction joint; (c) expansion joint

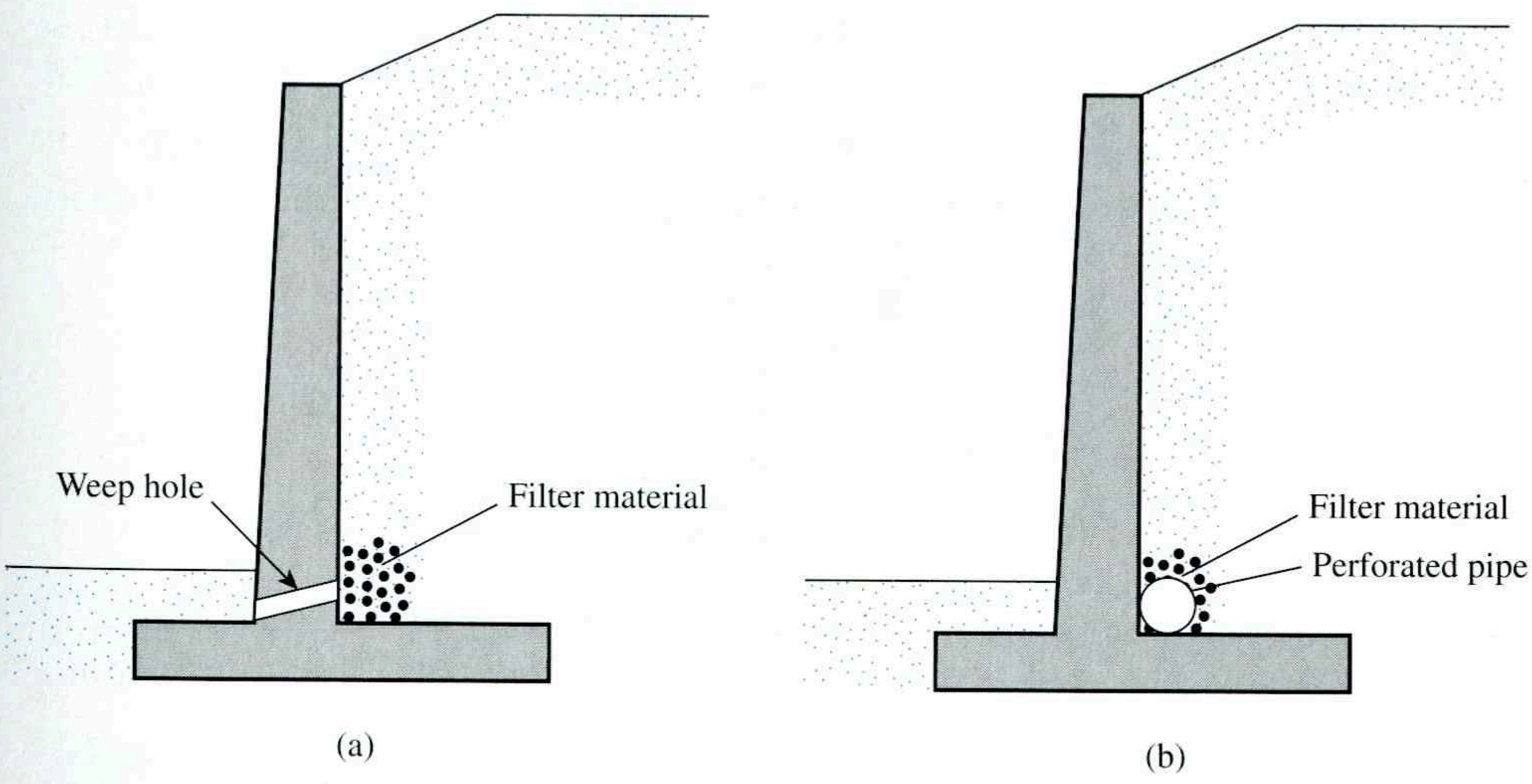


Figure 8.15 Drainage provisions for the backfill of a retaining wall: (a) by weep holes; (b) by a perforated drainage pipe

be washed into weep holes or drainage pipes and ultimately clog them. Thus, a filter material needs to be placed behind the weep holes or around the drainage pipes, as the case may be; geotextiles now serve that purpose.

Two main factors influence the choice of filter material: The grain-size distribution of the materials should be such that (a) the soil to be protected is not washed into the filter and (b) excessive hydrostatic pressure head is not created in the soil with a lower hydraulic conductivity (in this case, the backfill material). The preceding conditions can be satisfied if the following requirements are met (Terzaghi and Peck, 1967):

$$\frac{D_{15(F)}}{D_{85(B)}} < 5 \qquad \text{[to satisfy condition(a)]} \tag{8.25}$$

$$\frac{D_{15(F)}}{D_{15(B)}} > 4 \qquad \text{[to satisfy condition(b)]} \tag{8.26}$$

In these relations, the subscripts F and B refer to the *filter* and the *base* material (i.e., the backfill soil), respectively. Also, D_{15} and D_{85} refer to the diameters through which 15% and 85% of the soil (filter or base, as the case may be) will pass. Example 8.3 gives the procedure for designing a filter.

Example 8.3

Figure 8.16 shows the grain-size distribution of a backfill material. Using the conditions outlined in Section 8.8, determine the range of the grain-size distribution for the filter material.

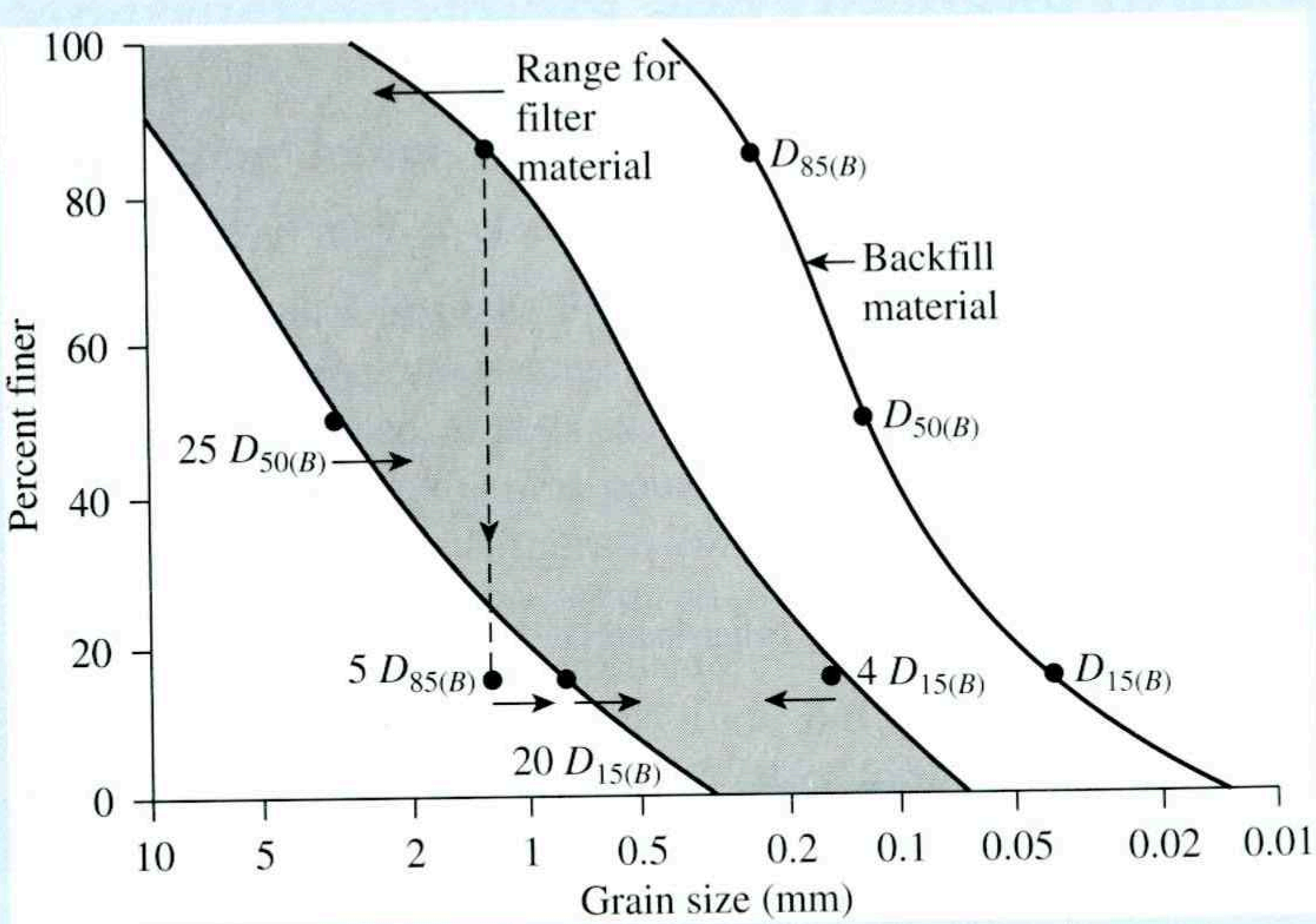


Figure 8.16 Determination of grain-size distribution of filter material

Solution

From the grain-size distribution curve given in the figure, the following values can be determined:

$$D_{15(B)} = 0.04 \text{ mm}$$

$$D_{85(B)} = 0.25 \text{ mm}$$

$$D_{50(B)} = 0.13 \text{ mm}$$

Conditions of Filter

1. $D_{15(F)}$ should be less than $5D_{85(B)}$; that is, $5 \times 0.25 = 1.25$ mm.
2. $D_{15(F)}$ should be greater than $4D_{15(B)}$; that is, $4 \times 0.04 = 0.16$ mm.
3. $D_{50(F)}$ should be less than $25D_{50(B)}$; that is, $25 \times 0.13 = 3.25$ mm.
4. $D_{15(F)}$ should be less than $20D_{15(B)}$; that is, $20 \times 0.04 = 0.8$ mm.

These limiting points are plotted in Figure 8.16. Through them, two curves can be drawn that are similar in nature to the grain-size distribution curve of the backfill material. These curves define the range of the filter material to be used. ■

8.9 Gravity Retaining-Wall Design for Earthquake Conditions

Even in mild earthquakes, most retaining walls undergo limited lateral displacement. Richards and Elms (1979) proposed a procedure for designing gravity retaining walls for earthquake conditions that allows limited lateral displacement. This procedure takes into consideration the wall inertia effect. Figure 8.17 shows a retaining wall with various forces acting on it, which are as follows (per unit length of the wall):

a. W_w = weight of the wall
b. P_{ae} = active force with earthquake condition taken into consideration (Section 7.7)

The backfill of the wall and the soil on which the wall is resting are assumed cohesionless. Considering the equilibrium of the wall, it can be shown that

$$W_w = \left[\tfrac{1}{2}\gamma_1 H^2(1 - k_v)K_{ae}\right]C_{IE} \tag{8.27}$$

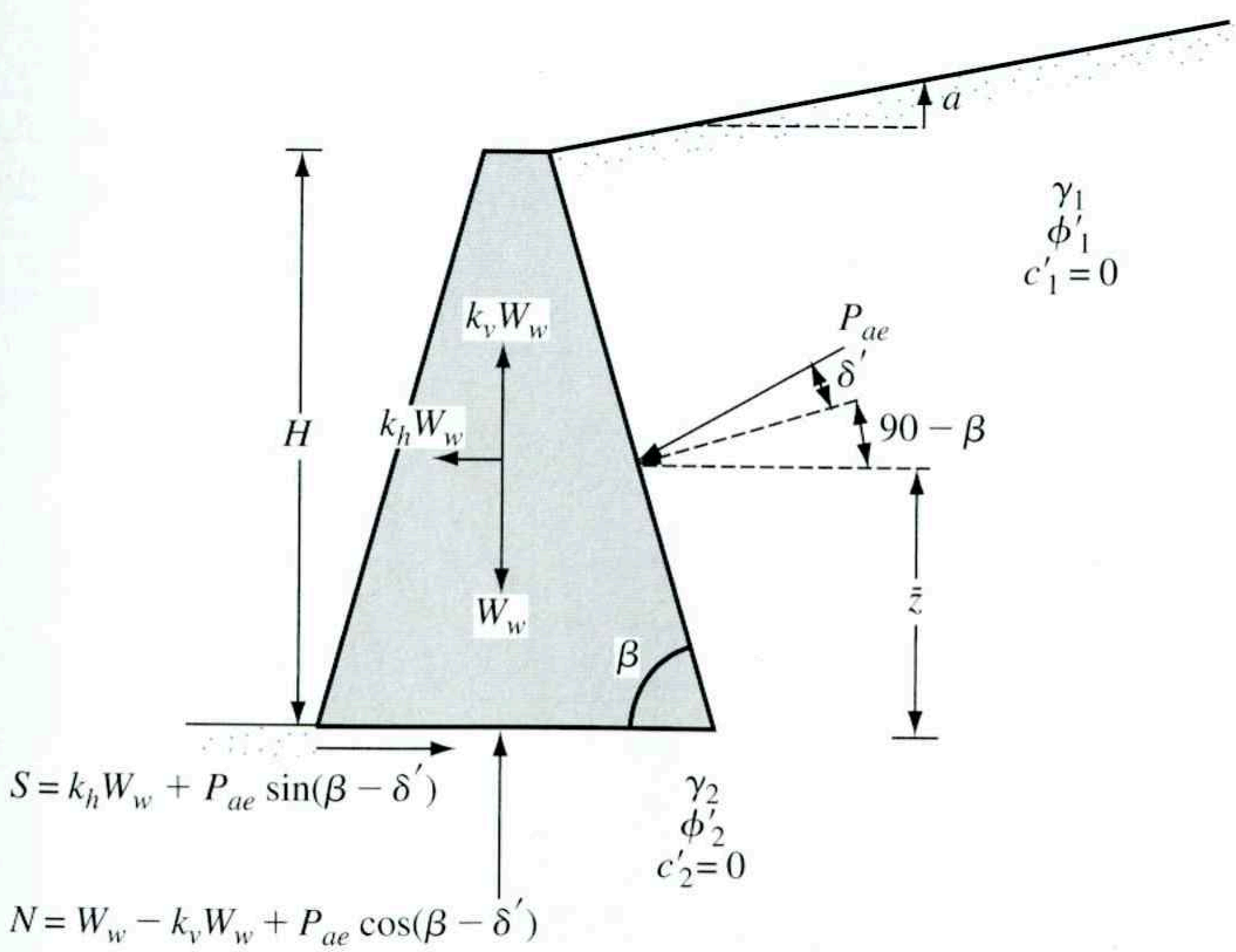


Figure 8.17 Stability of a retaining wall under earthquake forces

where γ_1 = unit weight of the backfill;

$$C_{IE} = \frac{\sin(\beta - \delta') - \cos(\beta - \delta')\tan\phi'_2}{(1 - k_v)(\tan\phi'_2 - \tan\theta')} \quad (8.28)$$

and $\theta' = \tan^{-1}\left(\frac{k_k}{1 - k_v}\right)$

For a detailed derivation of Eq. (8.28), see Das (1983).

Based on Eqs. (8.27) and (8.28), the following procedure may be used to determine the weight of the retaining wall, W_w, for tolerable displacement that may take place during an earthquake.

1. Determine the tolerable displacement of the wall, Δ.
2. Obtain a design value of k_k from

$$k_k = A_a\left(\frac{0.2A_v^2}{A_a\Delta}\right)^{0.25} \quad (8.29)$$

In Eq. (8.29), A and A_a are effective acceleration coefficients and Δ is displacement in inches. The magnitudes of A_a and A_v are given by the Applied Technology Council (1978) for various regions of the United States

3. Assume that $k_v = 0$, and, with the value of k_k obtained, calculate K_{ae} from Eq. (7.43).
4. Use the value of K_{ae} determined in Step 3 to obtain the weight of the wall (W_w)
5. Apply a factor of safety to the value of W_w obtained in Step 4.

Example 8.4

Refer to Figure 8.18. For $k_v = 0$ and $k_k = 0.3$, determine:

a. Weight of the wall for static condition
b. Weight of the wall for zero displacement during an earthquake
c. Weight of the wall for lateral displacement of 38 mm (1.5 in.) during an earthquake

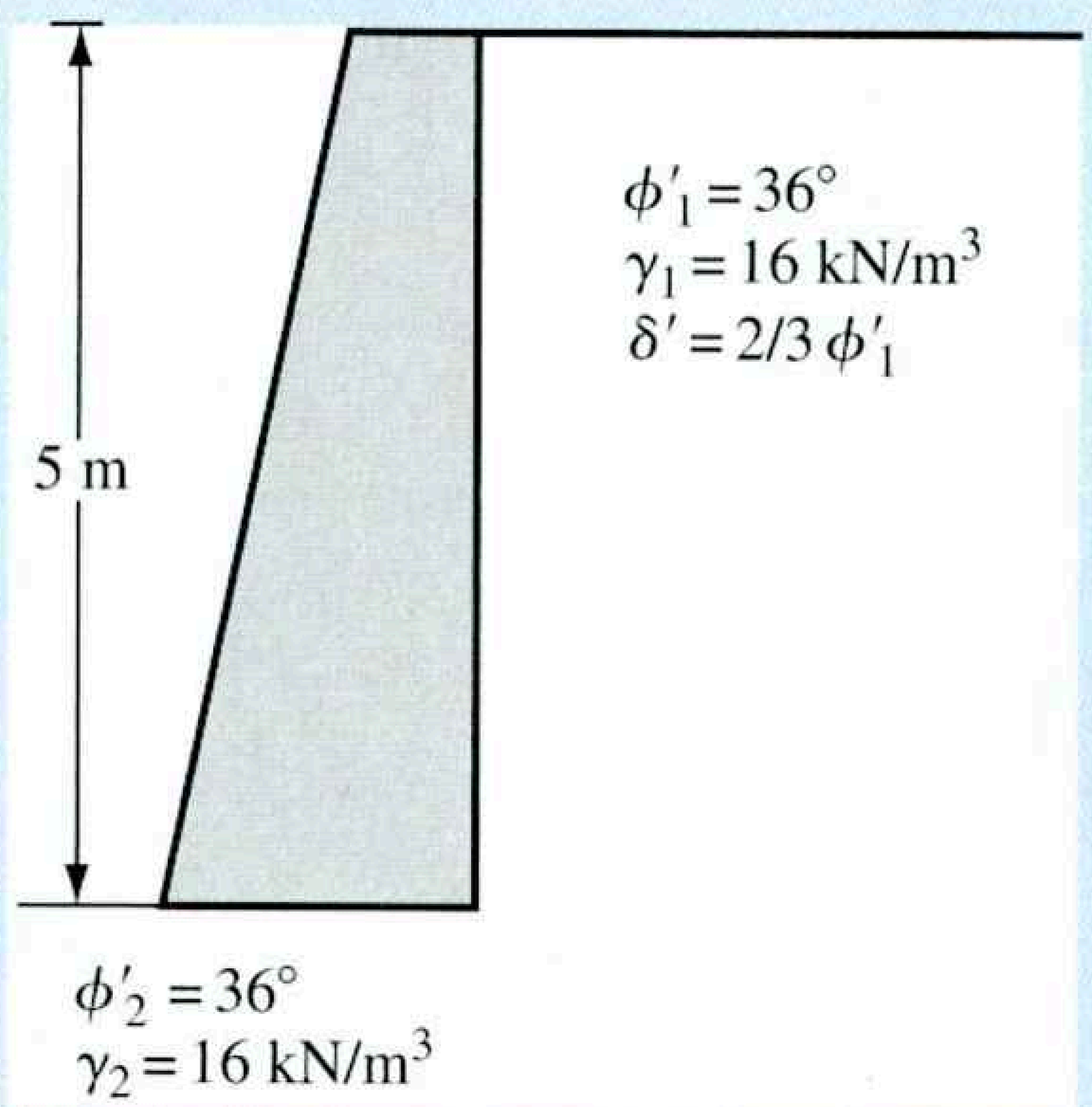


Figure 8.18

For part c, assume that $A_a = 0.2$ and $A_v = 0.2$. For parts a, b, and c, use a factor of safety of 1.5.

Solution

Part a

For static conditions, $\theta' = 0$ and Eq. (8.28) becomes

$$C_{IE} = \frac{\sin(\beta - \delta') - \cos(\beta - \delta')\tan\phi_2'}{\tan\phi_2'}$$

For $\beta = 90°$, $\delta' = 24°$ and $\phi_2' = 36°$,

$$C_{IE} = \frac{\sin(90 - 24) - \cos(90 - 24)\tan 36}{\tan 36} = 0.85$$

For static conditions, $K_{ae} = K_a$, so

$$W_w = \frac{1}{2}\gamma H^2 K_a C_{IE}$$

For $K_a \approx 0.2349$ [Table 7.4],

$$W_w = \frac{1}{2}(16)(5)^2(0.2349)(0.85) = 39.9 \text{ kN/m}$$

With a factor of safety of 1.5,

$$W_w = (39.9)(1.5) = \mathbf{59.9\ kN/m}$$

Part b

For zero displacement, $k_v = 0$,

$$C_{IE} = \frac{\sin(\beta - \delta') - \cos(\beta - \delta')\tan\phi_2'}{\tan\phi_2' - \tan\theta'}$$

$$\tan\theta' = \frac{k_h}{1 - k_v} = \frac{0.3}{1 - 0} = 0.3$$

$$C_{IE} = \frac{\sin(90 - 24) - \cos(90 - 24)\tan 36}{\tan 36 - 0.3} = 1.45$$

For $k_h = 0.3$, $\phi_1' = 36°$ and $\delta' = 2\phi_1'/3$, the value of $K_{ae} \approx 0.48$ (Table 7.6).

$$W_w = \tfrac{1}{2}\gamma_1 H^2(1 - k_v)K_{ae}C_{IE} = \tfrac{1}{2}(16)(5)^2(1 - 0)(0.48)(1.45) = 139.2 \text{ kN/m}$$

With a factor of safety of 1.5, $W_w =$ **208.8 kN/m**

Part c

For a lateral displacement of 38 mm,

$$k_h = A_a\left(\frac{0.2A_v^2}{A_a\Delta}\right)^{0.25} = (0.2)\left[\frac{(0.2)(0.2)^2}{(0.2)(38/25.4)}\right]^{0.25} = 0.081$$

$$\tan \theta' = \frac{k_h}{1 - k_v} = \frac{0.081}{1 - 0} = 0.081$$

$$C_{IE} = \frac{\sin (90 - 24) - \cos (90 - 24) \tan 36}{\tan 36 - 0.081} = 0.957$$

$$W_w = \frac{1}{2}\gamma_1 H^2 \underset{\approx 0.29 \text{ [Table 7.6]}}{\underset{\uparrow}{K_{ae}}} C_{IE}$$

$$W_w = \frac{1}{2}(16)(5)^2(0.29)(0.957) = 55.5 \text{ kN/m}$$

With a factor of safety of 1.5, W_w = **83.3 kN/m** ■

8.10 Comments on Design of Retaining Walls and a Case Study

In Section 8.3, it was suggested that the *active earth pressure coefficient* be used to estimate the lateral force on a retaining wall due to the backfill. It is important to recognize the fact that the active state of the backfill can be established only if the wall yields sufficiently, which does not happen in all cases. The degree to which the wall yields depends on its *height* and the *section modulus*. Furthermore, the lateral force of the backfill depends on several factors identified by Casagrande (1973):

1. Effect of temperature
2. Groundwater fluctuation
3. Readjustment of the soil particles due to creep and prolonged rainfall
4. Tidal changes
5. Heavy wave action
6. Traffic vibration
7. Earthquakes

Insufficient wall yielding combined with other unforeseen factors may generate a larger lateral force on the retaining structure, compared with that obtained from the active earth-pressure theory. This is particularly true in the case of gravity retaining walls, bridge abutments, and other heavy structures that have a large section modulus.

Case Study for the Performance of a Cantilever Retaining Wall

Bentler and Labuz (2006) have reported the performance of a cantilever retaining wall built along Interstate 494 in Bloomington, Minnesota. The retaining wall had 83 panels, each having a length of 9.3 m (30.5 ft). The panel height ranged from 4.0 m to 7.9 m (13 ft to 26 ft). One of the 7.9 m (26 ft) high panels was instrumented with earth pressure cells, tiltmeters,

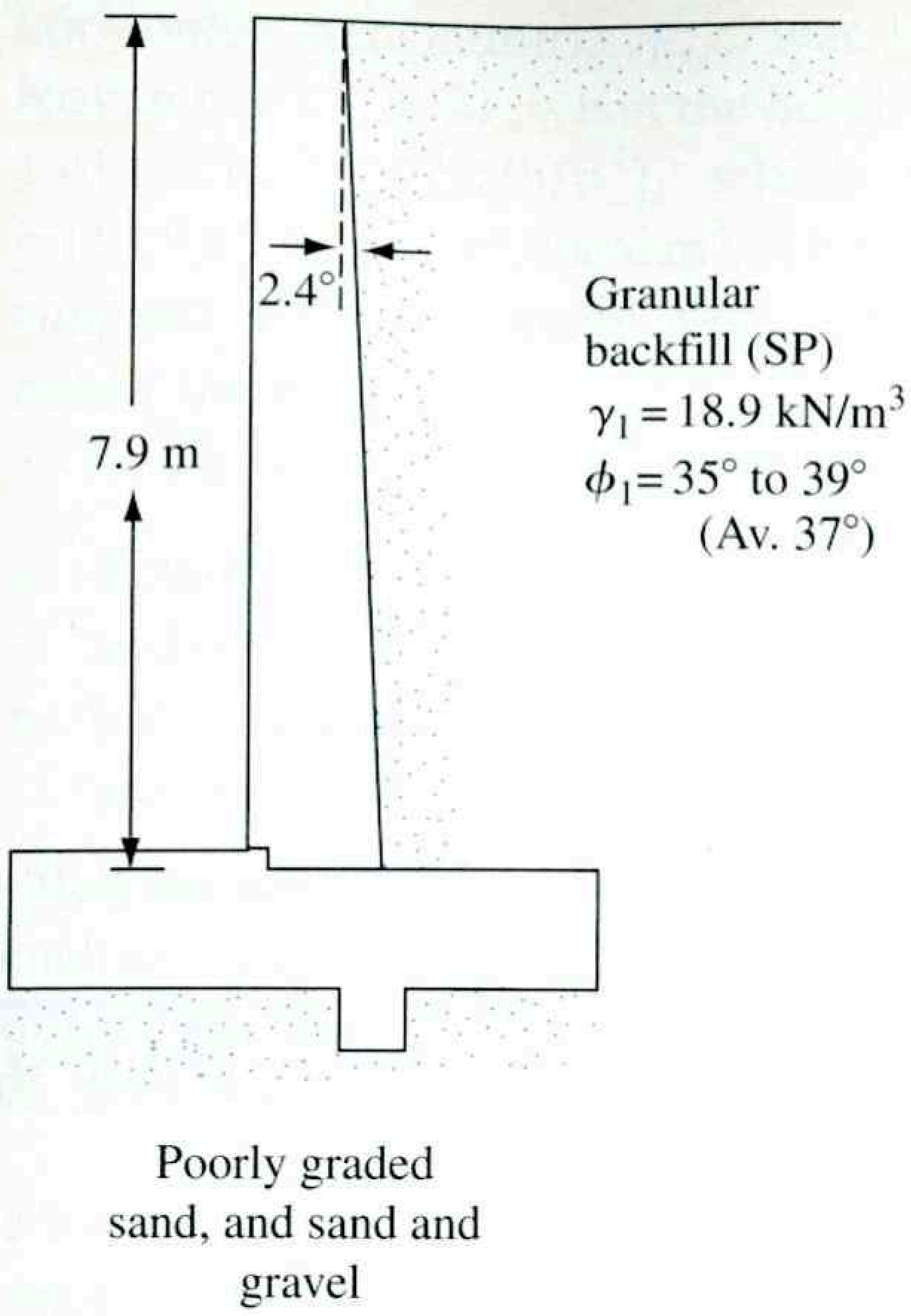


Figure 8.19 Schematic diagram of the retaining wall (drawn to scale)

strain gauges, and inclinometer casings. Figure 8.19 shows a schematic diagram (cross section) of the wall panel. Some details on the backfill and the foundation material are:

- Granular Backfill
  Effective size, $D_{10} = 0.13$ mm
  Uniformity coefficient, $C_u = 3.23$
  Coefficient of gradation, $C_c = 1.4$
  Unified soil classification − SP
  Compacted unit weight, $\gamma_1 = 18.9$ kN/m^3 (120 lb/ft^3)
  Triaxial friction angle, ϕ_1' − 35° to 39° (average 37°)
- Foundation Material
  Poorly graded sand and sand with gravel (medium dense to dense)

The backfill and compaction of the granular material started on October 28, 2001 in stages and reached a height of 7.6 m (25 ft) on November 21, 2001. The final 0.3 m (1 ft) of soil was placed the following spring. During backfilling, the wall was continuously going through translation (see Section 7.9). Table 8.3 is a summary of the backfill height and horizontal translation of the wall.

Table 8.3 Horizontal Translation with Backfill Height

Day	Backfill height (m)	Horizontal translation (mm)
1	0.0	0
2	1.1	0
2	2.8	0
3	5.2	2
4	6.1	4
5	6.4	6
11	6.7	9
24	7.3	12
54	7.6	11

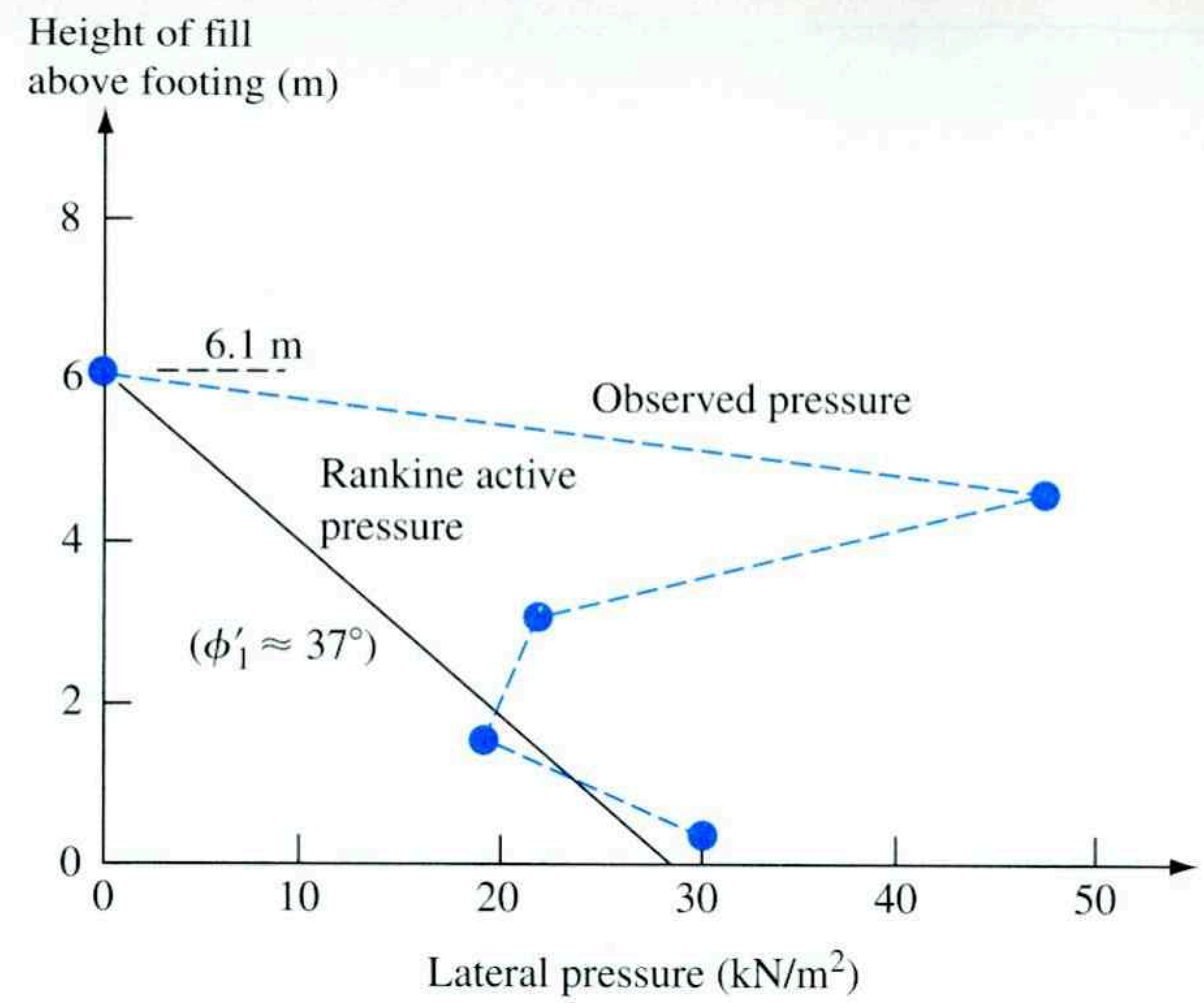


Figure 8.20 Observed lateral pressure distribution after fill height reached 6.1 m (Based on Bentler and Labuz, 2006)

Figure 8.20 shows a typical plot of the variation of lateral earth pressure *after compaction*, σ'_a, when the backfill height was 6.1 m (October 31, 2001) along with the plot of Rankine active earth pressure ($\phi_1' = 37°$). Note that the measured lateral (horizontal) pressure is higher at most heights than that predicted by the Rankine active pressure theory, which may be due to residual lateral stresses caused by compaction. The measured lateral stress gradually reduced with time. This is demonstrated in Figure 8.21 which shows a plot of the variation of σ'_a with depth (November 27, 2001) when the height of the backfill was 7.6 m. The lateral pressure was lower at practically all depths compared to the Rankine active earth pressure.

Another point of interest is the nature of variation of q_{max} and q_{min} (see Figure 8.11). As shown in Figure 8.11, if the wall rotates about C, q_{max} will be at the toe and q_{min} will be at the heel. However, for the case of the retaining wall under consideration (undergoing

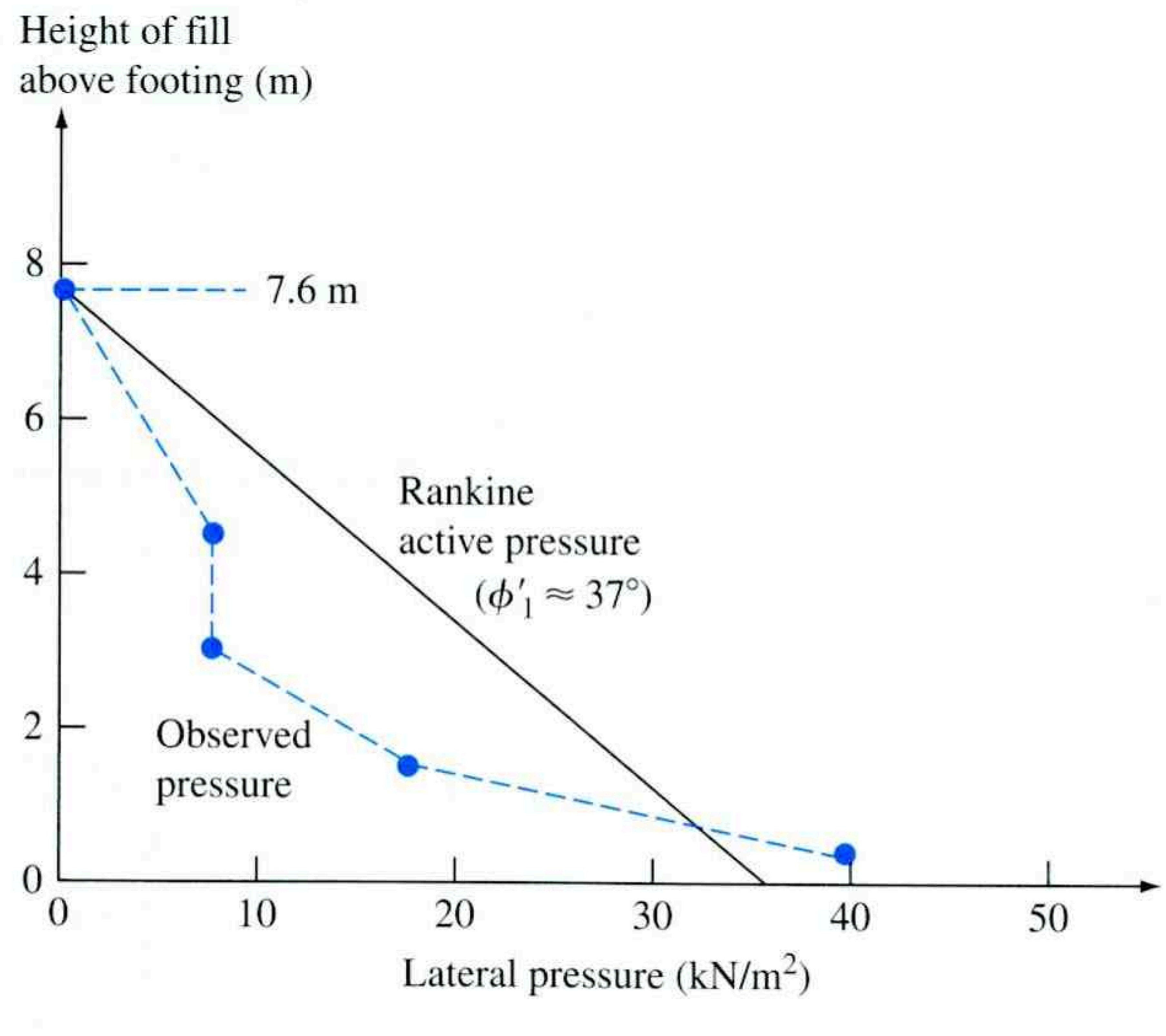


Figure 8.21 Observed pressure distribution on November 27, 2001 (Based on Bentler and Labuz, 2006)