

**ACI 318-11**

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# **Building Code Requirements for Structural Concrete (ACI 318-11)**

An ACI Standard

## **and Commentary**

Reported by ACI Committee 318



**American Concrete Institute®**



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## Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary

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# **BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-11) AND COMMENTARY**

**REPORTED BY ACI COMMITTEE 318**

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# BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-11) AND COMMENTARY

REPORTED BY ACI COMMITTEE 318

## PREFACE

The “Building Code Requirements for Structural Concrete” (“Code”) covers the materials, design, and construction of structural concrete used in buildings and where applicable in nonbuilding structures. The Code also covers the strength evaluation of existing concrete structures.

Among the subjects covered are: contract documents; inspection; materials; durability requirements; concrete quality, mixing, and placing; formwork; embedded pipes; construction joints; reinforcement details; analysis and design; strength and serviceability; flexural and axial loads; shear and torsion; development and splices of reinforcement; slab systems; walls; footings; precast concrete; composite flexural members; prestressed concrete; shells and folded plate members; strength evaluation of existing structures; provisions for seismic design; structural plain concrete; strut-and-tie modeling in [Appendix A](#); alternative design provisions in [Appendix B](#); alternative load and strength reduction factors in [Appendix C](#); and anchoring to concrete in [Appendix D](#).

The quality and testing of materials used in construction are covered by reference to the appropriate ASTM standard specifications. Welding of reinforcement is covered by reference to the appropriate American Welding Society (AWS) standard.

Uses of the Code include adoption by reference in general building codes, and earlier editions have been widely used in this manner. The Code is written in a format that allows such reference without change to its language. Therefore, background details or suggestions for carrying out the requirements or intent of the Code portion cannot be included. The Commentary is provided for this purpose. Some of the considerations of the committee in developing the Code portion are discussed within the Commentary, with emphasis given to the explanation of new or revised provisions. Much of the research data referenced in preparing the Code is cited for the user desiring to study individual questions in greater detail. Other documents that provide suggestions for carrying out the requirements of the Code are also cited.

**Keywords:** admixtures; aggregates; anchorage (structural); beam-column frame; beams (supports); **building codes**; cements; cold weather construction; columns (supports); combined stress; composite construction (concrete and steel); composite construction (concrete to concrete); compressive strength; **concrete construction**; concrete slabs; **concretes**; construction joints; continuity (structural); contract documents; contraction joints; cover; curing; deep beams; deflections; earthquake-resistant structures; embedded service ducts; flexural strength; floors; folded plates; footings; formwork (construction); frames; hot weather construction; inspection; isolation joints; joints (junctions); joists; lightweight concretes; load tests (structural); loads (forces); materials; mixing; mixture proportioning; modulus of elasticity; moments; pipe columns; pipes (tubing); placing; plain concrete; precast concrete; prestressed concrete; prestressing steels; quality control; **reinforced concrete**; reinforcing steels; roofs; serviceability; shear strength; shear walls; shells (structural forms); spans; splicing; strength; strength analysis; stresses; **structural analysis**; **structural concrete**; **structural design**; structural integrity; T-beams; torsion; walls; water; welded wire reinforcement.

ACI 318-11 was adopted as a standard of the American Concrete Institute May 24, 2011, to supersede ACI 318-08 in accordance with the Institute’s standardization procedure and was published August 2011.

A complete metric companion to ACI 318 has been developed, 318M; metric equivalents are provided only in [Appendix F](#) of this document.

ACI Committee Reports, Manuals, Guides, Standard Practices, and Commentaries are intended for guidance in planning, designing, executing, and inspecting construction. This Commentary is intended for the use of individuals who are competent to evaluate the significance and limitations of its content and recommendations, and who will accept responsibility for the application of the material it contains. The American Concrete Institute disclaims any and all responsibility for the stated principles. The Institute

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The ACI Building Code Requirements for Structural Concrete ("Code") and Commentary are presented in a side-by-side column format, with Code text placed in the left column and the corresponding Commentary text aligned in the right column. To further distinguish the Code from the Commentary, the Code has been printed in Helvetica, the same type face in which this paragraph is set.

This paragraph is set in Times Roman, and all portions of the text exclusive to the Commentary are printed in this type face. Commentary section numbers are preceded by an "R" to further distinguish them from Code section numbers.

Substantive changes from 318-08 are indicated with vertical lines in the margin (editorial changes not indicated).

## INTRODUCTION

This Commentary discusses some of the considerations of Committee 318 in developing the provisions contained in "Building Code Requirements for Structural Concrete (ACI 318-11)," hereinafter called the Code or the 2011 Code. Emphasis is given to the explanation of new or revised provisions that may be unfamiliar to Code users. In addition, comments are included for some items contained in previous editions of the Code to make the present commentary independent of the previous editions. Comments on specific provisions are made under the corresponding chapter and section numbers of the Code.

The Commentary is not intended to provide a complete historical background concerning the development of the Code, nor is it intended to provide a detailed résumé of the studies and research data reviewed by the committee in formulating the provisions of the Code. However, references to some of the research data are provided for those who wish to study the background material in depth.

As the name implies, "Building Code Requirements for Structural Concrete" is meant to be used as part of a legally adopted building code and as such must differ in form and substance from documents that provide detailed specifications, recommended practice, complete design procedures, or design aids.

The Code is intended to cover all buildings of the usual types, both large and small. Requirements more stringent than the Code provisions may be desirable for unusual construction. The Code and Commentary cannot replace sound engineering knowledge, experience, and judgment.

A building code states only the minimum requirements necessary to provide for public health and safety. The Code is based on this principle. For any structure, the owner or the licensed design professional may require the quality of materials and construction to be higher than the minimum requirements necessary to protect the public as stated in the Code. However, lower standards are not permitted.

\*For a history of the ACI Building Code, see Kerekes, F., and Reid, H. B., Jr., "Fifty Years of Development in Building Code Requirements for Reinforced Concrete," ACI JOURNAL, Proceedings V. 50, No. 6, Feb. 1954, p. 441. For a discussion of code philosophy, see Siess, C. P., "Research, Building Codes, and Engineering Practice," ACI JOURNAL, Proceedings V. 56, No. 5, May 1960, p. 1105.

The Commentary directs attention to other documents that provide suggestions for carrying out the requirements and intent of the Code. However, those documents and the Commentary are not a part of the Code.

The Code has no legal status unless it is adopted by the government bodies having the police power to regulate building design and construction. Where the Code has not been adopted, it may serve as a reference to good practice even though it has no legal status.

The Code provides a means of establishing minimum standards for acceptance of designs and construction by legally appointed building officials or their designated representatives. The Code and Commentary are not intended for use in settling disputes between the owner, engineer, architect, contractor, or their agents, subcontractors, material suppliers, or testing agencies. Therefore, the Code cannot define the contract responsibility of each of the parties in usual construction. General references requiring compliance with the Code in the project specifications should be avoided since the contractor is rarely in a position to accept responsibility for design details or construction requirements that depend on a detailed knowledge of the design. Design-build construction contractors, however, typically combine the design and construction responsibility. Generally, the contract documents should contain all of the necessary requirements to ensure compliance with the Code. In part, this can be accomplished by reference to specific Code sections in the project specifications. Other ACI publications, such as "Specifications for Structural Concrete (ACI 301)" are written specifically for use as contract documents for construction.

It is recommended to have testing and certification programs for the individual parties involved with the execution of work performed in accordance with this Code. Available for this purpose are the plant certification programs of the Precast/Prestressed Concrete Institute, the Post-Tensioning Institute, and the National Ready Mixed Concrete Association; the personnel certification programs of the American Concrete Institute and the Post-Tensioning Institute; and the Concrete Reinforcing Steel Institute's Voluntary Certification Program for Fusion-Bonded Epoxy Coating Applicator Plants. In addition, "Standard Specification for Agencies Engaged in Construction Inspecting and/or Testing" (ASTM

E329-09) specifies performance requirements for inspection and testing agencies.

Design reference materials illustrating applications of the Code requirements may be found in the following documents. The design aids listed may be obtained from the sponsoring organization.

#### **Design aids:**

**“ACI Design Handbook,”** Publication SP-17(09), American Concrete Institute, Farmington Hills, MI, 2009, 252 pp. (This provides tables and charts for design of eccentrically loaded columns by the Strength Design Method of the 2005 Code. Provides design aids for use in the engineering design and analysis of reinforced concrete slab systems carrying loads by two-way action. Design aids are also provided for the selection of slab thickness and for reinforcement required to control deformation and assure adequate shear and flexural strengths.)

**“ACI Detailing Manual—2004,”** ACI Committee 315, Publication SP-66(04), American Concrete Institute, Farmington Hills, MI, 2004, 212 pp. (Includes the standard, ACI 315-99, and report, ACI 315R-04. Provides recommended methods and standards for preparing engineering drawings, typical details, and drawings placing reinforcing steel in reinforced concrete structures. Separate sections define responsibilities of both engineer and reinforcing bar detailer.)

**“Guide to Durable Concrete (ACI 201.2R-08),”** ACI Committee 201, American Concrete Institute, Farmington Hills, MI, 2008, 49 pp. (This describes specific types of concrete deterioration. It contains a discussion of the mechanisms involved in deterioration and the recommended requirements for individual components of the concrete, quality considerations for concrete mixtures, construction procedures, and influences of the exposure environment.)

**“Guide for the Design of Durable Parking Structures (362.1R-97 (Reapproved 2002)),”** ACI Committee 362, American Concrete Institute, Farmington Hills, MI, 1997, 33 pp. (This summarizes practical information regarding design of parking structures for durability. It also includes information about design issues related to parking structure construction and maintenance.)

**“CRSI Handbook,”** Concrete Reinforcing Steel Institute, Schaumburg, IL, tenth edition, 2008, 777 pp. (This provides tabulated designs for structural elements and slab systems. Design examples are provided to show the basis and use of the load tables. Tabulated designs are given for beams; square, round, and rectangular columns; one-way slabs; and

one-way joist construction. The design tables for two-way slab systems include flat plates, flat slabs, and waffle slabs. The chapters on foundations provide design tables for square footings, pile caps, drilled piers (caissons), and cantilevered retaining walls. Other design aids are presented for crack control and development of reinforcement and lap splices.)

**“Reinforcement Anchorages and Splices,”** Concrete Reinforcing Steel Institute, Schaumburg, IL, fifth edition, 2008, 100 pp. (This provides accepted practices in splicing reinforcement. The use of lap splices, mechanical splices, and welded splices are described. Design data are presented for development and lap splicing of reinforcement.)

**“Structural Welded Wire Reinforcement Manual of Standard Practice,”** Wire Reinforcement Institute, Hartford, CT, eighth edition, Apr. 2006, 38 pp. (This describes welded wire reinforcement material, gives nomenclature and wire size and weight tables. Lists specifications and properties and manufacturing limitations. Book has latest code requirements as code affects welded wire. Also gives development length and splice length tables. Manual contains customary units and soft metric units.)

**“Structural Welded Wire Reinforcement Detailing Manual,”** Wire Reinforcement Institute, Hartford, CT, 1994, 252 pp. (The manual, in addition to including ACI 318 provisions and design aids, also includes: detailing guidance on welded wire reinforcement in one-way and two-way slabs; precast/prestressed concrete components; columns and beams; cast-in-place walls; and slabs-on-ground. In addition, there are tables to compare areas and spacings of high-strength welded wire with conventional reinforcing.)

**“PCI Design Handbook—Precast and Prestressed Concrete,”** Precast/Prestressed Concrete Institute, Chicago, IL, seventh edition, 2010, 804 pp. (This provides load tables for common industry products, and procedures for design and analysis of precast and prestressed elements and structures composed of these elements. Provides design aids and examples.)

**“Design and Typical Details of Connections for Precast and Prestressed Concrete,”** Precast/Prestressed Concrete Institute, Chicago, IL, second edition, 1988, 270 pp. (This updates available information on design of connections for both structural and architectural products, and presents a full spectrum of typical details. This provides design aids and examples.)

**“Post-Tensioning Manual,”** Post-Tensioning Institute, Farmington Hills, MI, sixth edition, 2006, 354 pp. (This provides comprehensive coverage of post-tensioning systems, specifications, design aids, and construction concepts.)

# CHAPTER 1 — GENERAL REQUIREMENTS

## CODE

### 1.1 — Scope

**1.1.1** — This Code provides minimum requirements for design and construction of structural concrete members of any structure erected under requirements of the legally adopted general building code of which this Code forms a part. In areas without a legally adopted building code, this Code defines minimum acceptable standards for materials, design, and construction practice. This Code also covers the strength evaluation of existing concrete structures.

For structural concrete,  $f'_c$  shall not be less than 2500 psi. No maximum value of  $f'_c$  shall apply unless restricted by a specific Code provision.

## COMMENTARY

### R1.1 — Scope

The American Concrete Institute “**Building Code Requirements for Structural Concrete (ACI 318-11)**,” referred to as the Code or 2011 Code, provides minimum requirements for structural concrete design or construction.

| The 2011 Code revised the previous standard “**Building Code Requirements for Structural Concrete (ACI 318-08)**.” This standard includes in one document the rules for all concrete used for structural purposes including both plain and reinforced concrete. The term “structural concrete” is used to refer to all plain or reinforced concrete used for structural purposes. This covers the spectrum of structural applications of concrete from nonreinforced concrete to concrete containing nonprestressed reinforcement, prestressing steel, or composite steel shapes, pipe, or tubing. Requirements for structural plain concrete are in **Chapter 22**.

Prestressed concrete is included under the definition of reinforced concrete. Provisions of the Code apply to prestressed concrete except for those that are stated to apply specifically to nonprestressed concrete.

| **Chapter 21** of the Code contains provisions for design and detailing of earthquake-resistant structures. See **1.1.9**.

Appendix A of Codes prior to 2002 contained provisions for an alternate method of design for nonprestressed reinforced concrete members using service loads (without load factors) and permissible service load stresses. The Alternate Design Method was intended to give results that were slightly more conservative than designs by the Strength Design Method of the Code. The Alternate Design Method of the 1999 Code may be used in place of applicable sections of this Code.

**Appendix A** of the Code contains provisions for the design of regions near geometrical discontinuities, or abrupt changes in loadings.

**Appendix B** of this Code contains provisions for reinforcement limits based on  $0.75\rho_b$ , determination of the strength reduction factor  $\phi$ , and moment redistribution that have been in the Code for many years, including the 1999 Code. The provisions are applicable to reinforced and prestressed concrete members. Designs made using the provisions of **Appendix B** are equally acceptable as those based on the body of the Code, provided the provisions of **Appendix B** are used in their entirety.

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**1.1.2** — This Code supplements the general building code and shall govern in all matters pertaining to design and construction of structural concrete, except wherever this Code is in conflict with requirements in the legally adopted general building code.

**1.1.3** — This Code shall govern in all matters pertaining to design, construction, and material properties wherever this Code is in conflict with requirements contained in other standards referenced in this Code.

**1.1.4** — For cast-in-place footings, foundation walls, and slabs-on-ground for one- and two-family dwellings and multiple single-family dwellings (townhouses) and their accessory structures, design and construction in accordance with ACI 332-10 shall be permitted.

**1.1.5** — For unusual structures, such as arches, bins and silos, blast-resistant structures, and chimneys, provisions of this Code shall govern where applicable. See also **22.1.3**.

**Appendix C** of the Code allows the use of the factored load combinations given in Chapter 9 of the 1999 Code.

**Appendix D** contains provisions for anchoring to concrete.

**R1.1.2** — The American Concrete Institute recommends that the Code be adopted in its entirety; however, it is recognized that when the Code is made a part of a legally adopted general building code, the general building code may modify provisions of this Code.

**R1.1.4** — “**Residential Code Requirements for Structural Concrete**” reported by ACI Committee 332.<sup>1.1</sup> (This addresses only the design and construction of cast-in-place footings, foundation walls supported on continuous footings, and slabs-on-ground for one- and two-family dwellings and multiple single-family dwellings [townhouses], and their accessory structures.)

**R1.1.5** — Some structures involve unique design and construction problems that are not covered by the Code. However, many Code provisions, such as the concrete quality and design principles, are applicable for these structures. Detailed recommendations for design and construction of some special structures are given in the following ACI publications:

“**Code Requirements for Reinforced Concrete Chimneys and Commentary**” reported by ACI Committee 307.<sup>1.2</sup> (This gives material, construction, and design requirements for circular cast-in-place reinforced chimneys. It sets forth minimum loadings for the design of reinforced concrete chimneys and contains methods for determining the stresses in the concrete and reinforcement required as a result of these loadings.)

“**Standard Practice for Design and Construction of Concrete Silos and Stacking Tubes for Storing Granular Materials and Commentary**” reported by ACI Committee 313.<sup>1.3</sup> (This gives material, design, and construction requirements for reinforced concrete bins, silos, and bunkers and stave silos for storing granular materials. It includes recommended design and construction criteria based on experimental and analytical studies plus worldwide experience in silo design and construction.)

“**Code Requirements for Nuclear Safety-Related Concrete Structures and Commentary**” reported by ACI Committee 349.<sup>1.4</sup> (This provides minimum requirements for design and construction of concrete structures that form part of a nuclear power plant and have nuclear safety-related functions. The

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code does not cover concrete reactor vessels and concrete containment structures, which are covered by ACI 359.)

**“Code for Concrete Containments”** reported by Joint ACI-ASME Committee 359.<sup>1,5</sup> (This provides requirements for the design, construction, and use of concrete reactor vessels and concrete containment structures for nuclear power plants.)

**1.1.6** — This Code does not govern design and installation of portions of concrete piles, drilled piers, and caissons embedded in ground except for structures assigned to Seismic Design Categories D, E, and F. See **21.12.4** for requirements for concrete piles, drilled piers, and caissons in structures assigned to Seismic Design Categories D, E, and F.

**R1.1.6** — The design and installation of piling fully embedded in the ground is regulated by the general building code. For portions of piling in air or water, or in soil not capable of providing adequate lateral restraint throughout the piling length to prevent buckling, the design provisions of this code govern where applicable.

Recommendations for concrete piles are given in detail in **“Design, Manufacture, and Installation of Concrete Piles”** reported by ACI Committee 543.<sup>1,6</sup> (This provides recommendations for the design and use of most types of concrete piles for many kinds of construction.)

Recommendations for drilled piers are given in detail in **“Design and Construction of Drilled Piers”** reported by ACI Committee 336.<sup>1,7</sup> (This provides recommendations for design and construction of foundation piers 2-1/2 ft in diameter or larger made by excavating a hole in the soil and then filling it with concrete.)

Detailed recommendations for precast prestressed concrete piles are given in **“Recommended Practice for Design, Manufacture, and Installation of Prestressed Concrete Piling”** prepared by the PCI Committee on Prestressed Concrete Piling.<sup>1,8</sup>

**1.1.7** — This Code does not govern design and construction of slabs-on-ground, unless the slab transmits vertical loads or lateral forces from other portions of the structure to the soil.

**R1.1.7** — Detailed recommendations for design and construction of slabs-on-ground and floors that do not transmit vertical loads or lateral forces from other portions of the structure to the soil, and residential post-tensioned slabs-on-ground, are given in the following publications:

| **“Guide to Design of Slabs-on-Ground”** reported by ACI Committee 360.<sup>1,9</sup> (This presents information on the design of slabs-on-ground, primarily industrial floors and the slabs adjacent to them. The report addresses the planning, design, and detailing of the slabs. Background information on the design theories is followed by discussion of the soil support system, loadings, and types of slabs. Design methods are given for structural plain concrete, reinforced concrete, shrinkage-compensating concrete, and post-tensioned concrete slabs.)

**“Design of Post-Tensioned Slabs-on-Ground,”** PTI.<sup>1,10</sup> (This provides recommendations for post-tensioned slab-on-ground foundations. Presents guidelines for soil investigation, and design and construction of post-tensioned residential and light commercial slabs on expansive or compressible soils.)

**CODE****COMMENTARY****1.1.8 — Concrete on steel deck**

**1.1.8.1** — Design and construction of structural concrete slabs cast on stay-in-place, noncomposite steel deck are governed by this Code.

**1.1.8.2** — This Code does not govern the composite design of structural concrete slabs cast on stay-in-place, composite steel deck. Concrete used in the construction of such slabs shall be governed by Chapters 1 through 6 of this Code, where applicable. Portions of such slabs designed as reinforced concrete are governed by this Code.

**1.1.9 — Provisions for earthquake resistance**

**1.1.9.1** — The seismic design category of a structure shall be determined in accordance with the legally adopted general building code of which this Code forms a part, or determined by other authority having jurisdiction in areas without a legally adopted building code.

**R1.1.8 — Concrete on steel deck**

In steel framed structures, it is common practice to cast concrete floor slabs on stay-in-place steel deck. In all cases, the deck serves as the form and may, in some cases, serve an additional structural function.

**R1.1.8.1** — In its most basic application, the noncomposite steel deck serves as a form, and the concrete slab is designed to carry all loads, while in other applications the concrete slab may be designed to carry only the superimposed loads. The design of the steel deck for this application is described in “**Standard for Non-Composite Steel Floor Deck**” (ANSI/SDI NC-2010).<sup>1,11</sup> This Standard refers to ACI 318 for the design and construction of the structural concrete slab.

**R1.1.8.2** — Another type of steel deck commonly used develops composite action between the concrete and steel deck. In this type of construction, the steel deck serves as the positive moment reinforcement. The design and construction of composite slabs on steel deck is described in “**Standard for Composite Steel Floor Deck**” (ANSI/SDI C1.0-2006).<sup>1,12</sup> The standard refers to the appropriate portions of ACI 318 for the design and construction of the concrete portion of the composite assembly. Reference 1.13 also provides guidance for design of composite slabs on steel deck. The design of negative moment reinforcement to create continuity at supports is a common example where a portion of the slab is designed in conformance with this Code.

**R1.1.9 — Provisions for earthquake resistance**

**R1.1.9.1** — Design requirements for an earthquake-resistant structure in this Code are determined by the Seismic Design Category (SDC) to which the structure is assigned. In general, the SDC relates to seismic hazard level, soil type, occupancy, and use of the building. Assignment of a building to a SDC is under the jurisdiction of a general building code rather than ACI 318.

Seismic Design Categories in this Code are adopted directly from the 2010 ASCE/SEI 7 standard.<sup>1,14</sup> Similar designations are used by the 2009 edition of the “**International Building Code**” (IBC),<sup>1,15</sup> and the 2009 NFPA 5000 “**Building Construction and Safety Code**.<sup>1,16</sup> The “**BOCA National Building Code**” (NBC)<sup>1,17</sup> and “**Standard Building Code**” (SBC)<sup>1,18</sup> use Seismic Performance Categories. The 1997 “**Uniform Building Code**” (UBC)<sup>1,19</sup> relates seismic design requirements to seismic zones, whereas previous editions of ACI 318 related seismic design requirements to seismic risk levels. Table R1.1.9.1 correlates Seismic Design Categories to the low, moderate/intermediate, and high seismic risk terminology used in ACI 318 for several editions before the 2008 edition, and to the various methods of assigning design requirements in use in the U.S. under the various

**CODE****COMMENTARY****TABLE R1.1.9.1 — CORRELATION BETWEEN SEISMIC-RELATED TERMINOLOGY IN MODEL CODES**

Code, standard, or resource document and edition	Level of seismic risk or assigned seismic performance or design categories as defined in the Code		
ACI 318-08; IBC 2000, 2003, 2006, 2009; NFPA 5000, 2003, 2006, 2009; ASCE 7-98, 7-02, 7-05, 7-10; NEHRP 1997, 2000, 2003, 2009	SDC <sup>*</sup> A, B	SDC C	SDC D, E, F
ACI 318-05 and previous editions	Low seismic risk	Moderate/intermediate seismic risk	High seismic risk
BOCA National Building Code 1993, 1996, 1999; Standard Building Code 1994, 1997, 1999; ASCE 7-93; 7-95; NEHRP 1991, 1994	SPC <sup>†</sup> A, B	SPC C	SPC D, E
Uniform Building Code 1991, 1994, 1997	Seismic Zone 0, 1	Seismic Zone 2	Seismic Zone 3, 4

<sup>\*</sup>SDC = Seismic design category as defined in code, standard, or resource document.<sup>†</sup>SPC = Seismic performance category as defined in code, standard, or resource document.

model building codes, the ASCE/SEI 7 standard, and the NEHRP Recommended Provisions.<sup>1,20</sup>

In the absence of a general building code that prescribes earthquake loads and seismic zoning, it is the intent of Committee 318 that application of provisions for seismic design be consistent with national standards or model building codes such as **References 1.14, 1.15, and 1.16**. The model building codes also specify overstrength factors,  $\Omega_o$ , that are related to the seismic-force-resisting system used for the structure and used for the design of certain elements.

**1.1.9.2** — All structures shall satisfy the applicable provisions of **Chapter 21** except those assigned to Seismic Design Category A and those otherwise exempted by the legally adopted general building code. See **21.1.1**.

**1.1.10** — This Code does not govern design and construction of tanks and reservoirs.

**R1.1.9.2** — Structures assigned to Seismic Design Category (SDC) A have the lowest seismic hazard and performance requirements. Provisions of **Chapters 1 through 19** and **Chapter 22** are considered sufficient for these structures. For structures assigned to other SDCs, the design requirements of **Chapter 21** apply, as delineated in **21.1**.

**R1.1.10** — Detailed recommendations for design and construction of tanks and reservoirs are given in “**Code Requirements for Environmental Engineering Concrete Structures**” reported by ACI Committee 350.<sup>1,21</sup> (This gives material, design and construction recommendations for concrete tanks, reservoirs, and other structures commonly used in water and waste treatment works where dense, impermeable concrete with high resistance to chemical attack is required. Special emphasis is placed on a structural design that minimizes the possibility of cracking and accommodates vibrating equipment and other special loads. Proportioning of concrete, placement, curing, and protection against chemicals are also described. Design and spacing of joints receive special attention.)

Guidance for the design and construction of cooling towers and circular prestressed concrete tanks may be found in the reports of ACI Committees 334,<sup>1,22</sup> 350,<sup>1,21</sup> 372,<sup>1,23</sup> and 373.<sup>1,24</sup>

**CODE****COMMENTARY****1.2 — Contract documents**

**1.2.1** — Contract documents for all structural concrete construction shall bear the seal of a licensed design professional. These contract documents shall show:

- (a) Name and date of issue of code and supplement to which design conforms;
- (b) Live load and other loads used in design;
- (c) Specified compressive strength of concrete at stated ages or stages of construction for which each part of structure is designed;
- (d) Specified strength or grade of reinforcement;
- (e) Size and location of all structural elements and reinforcement;
- (f) Requirements for type, size, location, and installation of anchors; and qualifications for post-installed anchor installers as required by **D.9**;
- (g) Provision for dimensional changes resulting from creep, shrinkage, and temperature;
- (h) Magnitude and location of prestressing forces;
- (i) Anchorage length of reinforcement and location and length of lap splices;
- (j) Type and location of mechanical and welded splices of reinforcement;
- (k) Details and location of all contraction or isolation joints specified for structural plain concrete in **Chapter 22**;
- (l) Minimum concrete compressive strength at time of post-tensioning;
- (m) Stressing sequence for post-tensioning tendons;
- (n) Statement if slab-on-ground is designed as a structural diaphragm, see **21.12.3.4**.

**1.2.2** — Calculations pertinent to design shall be filed with the contract documents when required by the building official. Analyses and designs using computer programs shall be permitted provided design assumptions, user input, and computer-generated output are submitted. Model analysis shall be permitted to supplement calculations.

**R1.2 — Contract documents**

**R1.2.1** — The provisions for preparation of contract documents are, in general, consistent with those of most general building codes and are intended as supplements.

The Code lists some of the more important items of information that should be included in the contract documents. The Code does not imply an all-inclusive list, and additional items may be required by the building official.

**R1.2.2** — Documented computer output is acceptable instead of manual calculations. The extent of input and output information required will vary according to the specific requirements of individual building officials. However, when a computer program has been used, only skeleton data should normally be required. This should consist of sufficient input and output data and other information to allow the building official to perform a detailed review and make comparisons using another program or manual calculations. Input data should be identified as to member designation, applied loads, and span lengths. The related output data should include member designation and

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the shears, moments, and reactions at key points in the span. For column design, it is desirable to include moment magnification factors in the output where applicable.

The Code permits model analysis to be used to supplement structural analysis and design calculations. Documentation of the model analysis should be provided with the related calculations. Model analysis should be performed by an individual having experience in this technique.

**1.3 — Inspection**

**1.3.1** — Concrete construction shall be inspected as required by the legally adopted general building code. In the absence of such inspection requirements, concrete construction shall be inspected throughout the various Work stages by or under the supervision of a licensed design professional or by a qualified inspector.

**R1.3 — Inspection**

The quality of concrete structures depends largely on workmanship in construction. The best of materials and design practices will not be effective unless the construction is performed well. Inspection is necessary to confirm that the construction is in accordance with the contract documents. Proper performance of the structure depends on construction that accurately represents the design and meets code requirements within the tolerances allowed. Qualification of the inspectors can be obtained from a certification program, such as the ACI Certification Program for Concrete Construction Special Inspector.

**R1.3.1** — Inspection of construction by or under the supervision of the licensed design professional responsible for the design should be considered because the person in charge of the design is usually the best qualified to determine if construction is in conformance with construction documents. When such an arrangement is not feasible, inspection of construction through other licensed design professionals or through separate inspection organizations with demonstrated capability for performing the inspection may be used.

Qualified inspectors should establish their qualification by becoming certified to inspect and record the results of concrete construction, including preplacement, placement, and post-placement operations through the ACI Inspector Certification Program: Concrete Construction Special Inspector.

When inspection is done independently of the licensed design professional responsible for the design, it is recommended that the licensed design professional responsible for the design be employed at least to oversee inspection and observe the Work to see that the design requirements are properly executed.

In some jurisdictions, legislation has established registration or licensing procedures for persons performing certain inspection functions. A check should be made in the general building code or with the building official to ascertain if any such requirements exist within a specific jurisdiction.

Inspection reports should be promptly distributed to the owner, licensed design professional responsible for the

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design, contractor, appropriate subcontractors, appropriate suppliers, and the building official to allow timely identification of compliance or the need for corrective action.

Inspection responsibility and the degree of inspection required should be set forth in the contracts between the owner, architect, engineer, contractor, and inspector. Adequate fees should be provided consistent with the work and equipment necessary to properly perform the inspection.

**1.3.2** — The inspector shall require compliance with contract documents. Unless specified otherwise in the legally adopted general building code, inspection records shall include:

- (a) Delivery, placement, and testing reports documenting the quantity, location of placement, fresh concrete tests, strength, and other test of all classes of concrete mixtures;
- (b) Construction and removal of forms and reshoring;
- (c) Placing of reinforcement and anchors;
- (d) Mixing, placing, and curing of concrete;
- (e) Sequence of erection and connection of precast members;
- (f) Tensioning of tendons;
- (g) Any significant construction loadings on completed floors, members, or walls;
- (h) General progress of Work.

**R1.3.2** — By inspection, the Code does not mean that the inspector should supervise the construction. Rather, it means that the one employed for inspection should visit the project with the frequency necessary to observe the various stages of Work and ascertain that it is being done in compliance with contract documents and Code requirements. The frequency should be at least enough to provide general knowledge of each operation, whether this is several times a day or once in several days.

Inspection in no way relieves the contractor from the obligation to follow the contract documents and to provide the designated quality and quantity of materials and workmanship for all job stages. Some of the information regarding designated concrete mixtures on a project is often provided in a preconstruction submittal to the licensed design professional. For instance, concrete mixture ingredients and composition are often described in detail in the submittal and are subsequently identified by a mixture designation (reflected on a delivery ticket) that can also identify the placement location in the structure. The inspector should be present as frequently as necessary to judge whether the quality, as measured by quality assurance tests, quantity, and placement of the concrete comply with the contract documents; to counsel on possible ways of obtaining the desired results; to see that the general system proposed for formwork appears proper (though it remains the contractor's responsibility to design and build adequate forms and to leave them in place until it is safe to remove them); to see that reinforcement is properly installed; to see that concrete is delivered as required and is of the correct quality, properly placed, and cured; and to see that tests for quality assurance are being made as specified.

The Code prescribes minimum requirements for inspection of all structures within its scope. It is not a construction specification and any user of the Code may require higher standards of inspection than cited in the legal code if additional requirements are necessary.

Recommended procedures for organization and conduct of concrete inspection are given in detail in “**Guide for Concrete Inspection**” reported by ACI Committee 311.<sup>1.25</sup> (This sets forth procedures relating to concrete construction to serve as a guide to owners, architects, and engineers in planning an inspection program.)

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Detailed methods of inspecting concrete construction are given in “**ACI Manual of Concrete Inspection**” (SP-2) reported by ACI Committee 311.<sup>1,26</sup> (This describes methods of inspecting concrete construction that are generally accepted as good practice. Intended as a supplement to specifications and as a guide in matters not covered by specifications.)

**1.3.3** — When the ambient temperature falls below 40°F or rises above 95°F, a record shall be kept of concrete temperatures and of protection given to concrete during placement and curing.

**R1.3.3** — The term “ambient temperature” means the temperature of the environment to which the concrete is directly exposed. Concrete temperature as used in this section may be taken as the surface temperature of the concrete. Surface temperatures may be determined by placing temperature sensors in contact with concrete surfaces or between concrete surfaces and covers used for curing, such as insulation blankets or plastic sheeting.

**1.3.4** — Records of inspection required in **1.3.2** and 1.3.3 shall be preserved by the inspecting engineer or architect for at least 2 years after completion of the project.

**R1.3.4** — A record of inspection in the form of a job diary is required in case questions subsequently arise concerning the performance or safety of the structure or members. Photographs documenting job progress may also be desirable.

**1.3.5** — For special moment frames designed in accordance with **Chapter 21**, continuous inspection of the placement of the reinforcement and concrete shall be made by a qualified inspector. The inspector shall be under the supervision of the licensed design professional responsible for the structural design or under the supervision of a licensed design professional with demonstrated capability for supervising inspection of construction of special moment frames.

Records of inspection should be preserved for at least 2 years after the completion of the project. The completion of the project is the date at which the owner accepts the project, or when a certificate of occupancy is issued, whichever date is later. The general building code or other legal requirements may require a longer preservation of such records.

**R1.3.5** — The purpose of this section is to ensure that the detailing required in special moment frames is properly executed through inspection by personnel who are qualified to do this Work. Qualifications of inspectors should be acceptable to the jurisdiction enforcing the general building code.

## **1.4 — Approval of special systems of design or construction**

Sponsors of any system of design or construction within the scope of this Code, the adequacy of which has been shown by successful use or by analysis or test, but which does not conform to or is not covered by this Code, shall have the right to present the data on which their design is based to the building official or to a board of examiners appointed by the building official. This board shall be composed of competent engineers and shall have authority to investigate the data so submitted, to require tests, and to formulate rules governing design and construction of such systems to meet the intent of this Code. These rules, when approved by the building official and promulgated, shall be of the same force and effect as the provisions of this Code.

## **R1.4 — Approval of special systems of design or construction**

New methods of design, new materials, and new uses of materials should undergo a period of development before being specifically covered in a code. Hence, good systems or components might be excluded from use by implication if means were not available to obtain acceptance.

For special systems considered under this section, specific tests, load factors, deflection limits, and other pertinent requirements should be set by the board of examiners, and should be consistent with the intent of the Code.

The provisions of this section do not apply to model tests used to supplement calculations under **1.2.2** or to strength evaluation of existing structures under **Chapter 20**.

**Notes**

# CHAPTER 2 — NOTATION AND DEFINITIONS

## 2.1 — Code notation

The terms in this list are used in the Code and as needed in the Commentary.

$a$	= depth of equivalent rectangular stress block as defined in 10.2.7.1, in., <a href="#">Chapter 10</a>	$A_h$	= total area of shear reinforcement parallel to primary tension reinforcement in a corbel or bracket, in. <sup>2</sup> , see <a href="#">11.8, Chapter 11</a>
$a_v$	= shear span, equal to distance from center of concentrated load to either: (a) face of support for continuous or cantilevered members, or (b) center of support for simply supported members, in., <a href="#">Chapter 11, Appendix A</a>	$A_j$	= effective cross-sectional area within a joint in a plane parallel to plane of reinforcement generating shear in the joint, in. <sup>2</sup> , see <a href="#">21.7.4.1, Chapter 21</a>
$A_b$	= area of an individual bar or wire, in. <sup>2</sup> , <a href="#">Chapters 10, 12</a>	$A_\ell$	= total area of longitudinal reinforcement to resist torsion, in. <sup>2</sup> , <a href="#">Chapter 11</a>
$A_{brg}$	= net bearing area of the head of stud, anchor bolt, or headed deformed bar, in. <sup>2</sup> , <a href="#">Chapter 12, Appendix D</a>	$A_{\ell,min}$	= minimum area of longitudinal reinforcement to resist torsion, in. <sup>2</sup> , see <a href="#">11.5.5.3, Chapter 11</a>
$A_c$	= area of concrete section resisting shear transfer, in. <sup>2</sup> , <a href="#">Chapters 11, 21</a>	$A_{Na}$	= projected influence area of a single adhesive anchor or group of adhesive anchors, for calculation of bond strength in tension, in. <sup>2</sup> , see <a href="#">D.5.5.1, Appendix D</a>
$A_{cf}$	= larger gross cross-sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab, in. <sup>2</sup> , <a href="#">Chapter 18</a>	$A_{Nao}$	= projected influence area of a single adhesive anchor, for calculation of bond strength in tension if not limited by edge distance or spacing, in. <sup>2</sup> , see <a href="#">D.5.5.1, Appendix D</a>
$A_{ch}$	= cross-sectional area of a structural member measured to the outside edges of transverse reinforcement, in. <sup>2</sup> , <a href="#">Chapters 10, 21</a>	$A_{Nc}$	= projected concrete failure area of a single anchor or group of anchors, for calculation of strength in tension, in. <sup>2</sup> , see <a href="#">D.5.2.1, Appendix D</a>
$A_{cp}$	= area enclosed by outside perimeter of concrete cross section, in. <sup>2</sup> , see <a href="#">11.5.1, Chapter 11</a>	$A_{Nco}$	= projected concrete failure area of a single anchor, for calculation of strength in tension if not limited by edge distance or spacing, in. <sup>2</sup> , see <a href="#">D.5.2.1, Appendix D</a>
$A_{cs}$	= cross-sectional area at one end of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut, in. <sup>2</sup> , <a href="#">Appendix A</a>	$A_n$	= area of reinforcement in bracket or corbel resisting tensile force $N_{uc}$ , in. <sup>2</sup> , see <a href="#">11.8, Chapter 11</a>
$A_{ct}$	= area of that part of cross section between the flexural tension face and center of gravity of gross section, in. <sup>2</sup> , <a href="#">Chapter 18</a>	$A_{nz}$	= area of a face of a nodal zone or a section through a nodal zone, in. <sup>2</sup> , <a href="#">Appendix A</a>
$A_{cv}$	= gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered, in. <sup>2</sup> , <a href="#">Chapter 21</a>	$A_o$	= gross area enclosed by shear flow path, in. <sup>2</sup> , <a href="#">Chapter 11</a>
$A_{cw}$	= area of concrete section of an individual pier, horizontal wall segment, or coupling beam resisting shear, in. <sup>2</sup> , <a href="#">Chapter 21</a>	$A_{oh}$	= area enclosed by centerline of the outermost closed transverse torsional reinforcement, in. <sup>2</sup> , <a href="#">Chapter 11</a>
$A_f$	= area of reinforcement in bracket or corbel resisting factored moment, in. <sup>2</sup> , see <a href="#">11.8, Chapter 11</a>	$A_{ps}$	= area of prestressing steel in flexural tension zone, in. <sup>2</sup> , <a href="#">Chapter 18, Appendix B</a>
$A_g$	= gross area of concrete section, in. <sup>2</sup> . For a hollow section, $A_g$ is the area of the concrete only and does not include the area of the void(s), see <a href="#">11.5.1, Chapters 9-11, 14-16, 21, 22, Appendixes B, C</a>	$A_s$	= area of nonprestressed longitudinal tension reinforcement, in. <sup>2</sup> , <a href="#">Chapters 10-12, 14, 15, 18, Appendix B</a>
		$A'_s$	= area of compression reinforcement, in. <sup>2</sup> , <a href="#">Appendix A</a>
		$A_{sc}$	= area of primary tension reinforcement in a corbel or bracket, in. <sup>2</sup> , see <a href="#">11.8.3.5, Chapter 11</a>
		$A_{se,N}$	= effective cross-sectional area of anchor in tension, in. <sup>2</sup> , <a href="#">Appendix D</a>
		$A_{se,V}$	= effective cross-sectional area of anchor in shear, in. <sup>2</sup> , <a href="#">Appendix D</a>

$A_{sh}$	= total cross-sectional area of transverse reinforcement (including crossties) within spacing $s$ and perpendicular to dimension $b_c$ , in. <sup>2</sup> , Chapter 21	$b_c$ = cross-sectional dimension of member core measured to the outside edges of the transverse reinforcement composing area $A_{sh}$ , in., Chapter 21
$A_{si}$	= total area of surface reinforcement at spacing $s_i$ in the $i$ -th layer crossing a strut, with reinforcement at an angle $\alpha_i$ to the axis of the strut, in. <sup>2</sup> , Appendix A	$b_o$ = perimeter of critical section for shear in slabs and footings, in., see 11.11.1.2, Chapters 11, 22
$A_{s,min}$	= minimum area of flexural reinforcement, in. <sup>2</sup> , see 10.5, Chapter 10	$b_s$ = width of strut, in., Appendix A
$A_{st}$	= total area of nonprestressed longitudinal reinforcement (bars or steel shapes), in. <sup>2</sup> , Chapters 10, 21	$b_t$ = width of that part of cross section containing the closed stirrups resisting torsion, in., Chapter 11
$A_{sx}$	= area of structural steel shape, pipe, or tubing in a composite section, in. <sup>2</sup> , Chapter 10	$b_v$ = width of cross section at contact surface being investigated for horizontal shear, in., Chapter 17
$A_t$	= area of one leg of a closed stirrup resisting torsion within spacing $s$ , in. <sup>2</sup> , Chapter 11	$b_w$ = web width, wall thickness, or diameter of circular section, in., Chapters 10-12, 21, 22, Appendix B
$A_{tp}$	= area of prestressing steel in a tie, in. <sup>2</sup> , Appendix A	$b_1$ = dimension of the critical section $b_o$ measured in the direction of the span for which moments are determined, in., Chapter 13
$A_{tr}$	= total cross-sectional area of all transverse reinforcement within spacing $s$ that crosses the potential plane of splitting through the reinforcement being developed, in. <sup>2</sup> , Chapter 12	$b_2$ = dimension of the critical section $b_o$ measured in the direction perpendicular to $b_1$ , in., Chapter 13
$A_{ts}$	= area of nonprestressed reinforcement in a tie, in. <sup>2</sup> , Appendix A	$B_n$ = nominal bearing strength, lb, Chapter 22
$A_v$	= area of shear reinforcement within spacing $s$ , in. <sup>2</sup> , Chapters 11, 17	$B_u$ = factored bearing load, lb, Chapter 22
$A_{vd}$	= total area of reinforcement in each group of diagonal bars in a diagonally reinforced coupling beam, in. <sup>2</sup> , Chapter 21	$c$ = distance from extreme compression fiber to neutral axis, in., Chapters 9, 10, 14, 21
$A_{vf}$	= area of shear-friction reinforcement, in. <sup>2</sup> , Chapters 11, 21	$c_{ac}$ = critical edge distance required to develop the basic strength as controlled by concrete breakout or bond of a post-installed anchor in tension in uncracked concrete without supplementary reinforcement to control splitting, in., see D.8.6, Appendix D
$A_{vh}$	= area of shear reinforcement parallel to flexural tension reinforcement within spacing $s_2$ , in. <sup>2</sup> , Chapter 11	$c_{a,max}$ = maximum distance from center of an anchor shaft to the edge of concrete, in., Appendix D
$A_{v,min}$	= minimum area of shear reinforcement within spacing $s$ , in. <sup>2</sup> , see 11.4.6.3 and 11.4.6.4, Chapter 11	$c_{a,min}$ = minimum distance from center of an anchor shaft to the edge of concrete, in., Appendix D
$A_{vc}$	= projected concrete failure area of a single anchor or group of anchors, for calculation of strength in shear, in. <sup>2</sup> , see D.6.2.1, Appendix D	$c_{a1}$ = distance from the center of an anchor shaft to the edge of concrete in one direction, in. If shear is applied to anchor, $c_{a1}$ is taken in the direction of the applied shear. If tension is applied to the anchor, $c_{a1}$ is the minimum edge distance, Appendix D. Where anchors subject to shear are located in narrow sections of limited thickness, see D.6.2.4
$A_{vco}$	= projected concrete failure area of a single anchor, for calculation of strength in shear, if not limited by corner influences, spacing, or member thickness, in. <sup>2</sup> , see D.6.2.1, Appendix D	$c_{a2}$ = distance from center of an anchor shaft to the edge of concrete in the direction perpendicular to $c_{a1}$ , in., Appendix D
$A_1$	= loaded area, in. <sup>2</sup> , Chapters 10, 22	$c_b$ = smaller of: (a) the distance from center of a bar or wire to nearest concrete surface, and (b) one-half the center-to-center spacing of bars or wires being developed, in., Chapter 12
$A_2$	= area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in. <sup>2</sup> , Chapters 10, 22	$c_c$ = clear cover of reinforcement, in., see 10.6.4, Chapter 10
$b$	= width of compression face of member, in., Chapter 10, Appendix B	$c_{Na}$ = projected distance from center of an anchor shaft on one side of the anchor required to

$c_t$	= develop the full bond strength of a single adhesive anchor, in., see <a href="#">D.5.5.1, Appendix D</a>	$E$	= effects of earthquake, or related internal moments and forces, <a href="#">Chapters 9, 21, Appendix C</a>
$c_1$	= distance from the interior face of the column to the slab edge measured parallel to $c_1$ , but not exceeding $c_1$ , in., <a href="#">Chapter 21</a>	$E_c$	= modulus of elasticity of concrete, psi, see <a href="#">8.5.1, Chapters 8-10, 14, 19</a>
$c_2$	= dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in., <a href="#">Chapters 11, 13, 21</a>	$E_{cb}$	= modulus of elasticity of beam concrete, psi, <a href="#">Chapter 13</a>
$C$	= dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction perpendicular to $c_1$ , in., <a href="#">Chapter 13</a>	$E_{cs}$	= modulus of elasticity of slab concrete, psi, <a href="#">Chapter 13</a>
$C_m$	= cross-sectional constant to define torsional properties of slab and beam, see <a href="#">13.6.4.2, Chapter 13</a>	$E_I$	= flexural stiffness of compression member, in. <sup>2</sup> -lb, see <a href="#">10.10.6, Chapter 10</a>
$d$	= factor relating actual moment diagram to an equivalent uniform moment diagram, <a href="#">Chapter 10</a>	$E_p$	= modulus of elasticity of prestressing steel, psi, see <a href="#">8.5.3, Chapter 8</a>
$d'$	= distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in., <a href="#">Chapters 7, 9-12, 14, 17, 18, 21, Appendixes B, C</a>	$E_s$	= modulus of elasticity of reinforcement and structural steel, psi, see <a href="#">8.5.2, Chapters 8, 10, 14</a>
$d_a$	= distance from extreme compression fiber to centroid of longitudinal compression reinforcement, in., <a href="#">Chapters 9, 18, Appendix C</a>	$f'_c$	= specified compressive strength of concrete, psi, <a href="#">Chapters 4, 5, 8-12, 14, 18, 19, 21, 22, Appendixes A-D</a>
$d'_a$	= outside diameter of anchor or shaft diameter of headed stud, headed bolt, or hooked bolt, in., see <a href="#">D.8.4, Appendix D</a>	$\sqrt{f'_c}$	= square root of specified compressive strength of concrete, psi, <a href="#">Chapters 8, 9, 11, 12, 18, 19, 21, 22, Appendix D</a>
$d_b$	= value substituted for $d_a$ when an oversized anchor is used, in., see <a href="#">D.8.4, Appendix D</a>	$f_{ce}$	= effective compressive strength of the concrete in a strut or a nodal zone, psi, <a href="#">Chapter 15, Appendix A</a>
$d_p$	= nominal diameter of bar, wire, or prestressing strand, in., <a href="#">Chapters 7, 12, 21</a>	$f'_{ci}$	= specified compressive strength of concrete at time of initial prestress, psi, <a href="#">Chapters 7, 18</a>
$d_{pile}$	= distance from extreme compression fiber to centroid of prestressing steel, in., <a href="#">Chapters 11, 18, Appendix B</a>	$\sqrt{f'_{ci}}$	= square root of specified compressive strength of concrete at time of initial prestress, psi, <a href="#">Chapter 18</a>
$d_t$	= diameter of pile at footing base, in., <a href="#">Chapter 15</a>	$f'_{cr}$	= required average compressive strength of concrete used as the basis for selection of concrete proportions, psi, <a href="#">Chapter 5</a>
$D$	= distance from extreme compression fiber to centroid of extreme layer of longitudinal tension steel, in., <a href="#">Chapters 9, 10, Appendix C</a>	$f_{ct}$	= average splitting tensile strength of lightweight concrete, psi, <a href="#">Chapters 5, 9, 11, 12, 22</a>
$e$	= dead loads, or related internal moments and forces, <a href="#">Chapters 8, 9, 20, 21, Appendix C</a>	$f_d$	= stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads, psi, <a href="#">Chapter 11</a>
$e_h$	= base of Napierian logarithms, <a href="#">Chapter 18</a>	$f_{dc}$	= decompression stress; stress in the prestressing steel when stress is zero in the concrete at the same level as the centroid of the prestressing steel, psi, <a href="#">Chapter 18</a>
$e'_N$	= distance from the inner surface of the shaft of a J- or L-bolt to the outer tip of the J- or L-bolt, in., <a href="#">Appendix D</a>	$f_{pc}$	= compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange, psi. (In a composite member, $f_{pc}$ is the resultant compressive stress at centroid of composite section, or at junction of web and flange when the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone), <a href="#">Chapter 11</a>
$e'_V$	= distance between resultant tension load on a group of anchors loaded in tension and the centroid of the group of anchors loaded in tension, in.; $e'_N$ is always positive, <a href="#">Appendix D</a>	$f_{pe}$	= compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of

	section where tensile stress is caused by externally applied loads, psi, <b>Chapter 11</b>	$h_w$	= height of entire wall from base to top, or clear height of wall segment or wall pier considered, in., <b>Chapters 11, 21</b>
$f_{ps}$	= stress in prestressing steel at nominal flexural strength, psi, <b>Chapters 12, 18</b>	$h_x$	= maximum center-to-center horizontal spacing of crossties or hoop legs on all faces of the column, in., <b>Chapter 21</b>
$f_{pu}$	= specified tensile strength of prestressing steel, psi, <b>Chapters 11, 18</b>	$H$	= loads due to lateral pressure of soil, water in soil, or other materials, or related internal moments and forces, lb, <b>Chapter 9, Appendix C</b>
$f_{py}$	= specified yield strength of prestressing steel, psi, <b>Chapter 18</b>	$I$	= moment of inertia of section about centroidal axis, in. <sup>4</sup> , <b>Chapters 10, 11</b>
$f_r$	= modulus of rupture of concrete, psi, see <b>9.5.2.3, Chapters 9, 14, 18, Appendix B</b>	$I_b$	= moment of inertia of gross section of beam about centroidal axis, in. <sup>4</sup> , see <b>13.6.1.6, Chapter 13</b>
$f_s$	= calculated tensile stress in reinforcement at service loads, psi, <b>Chapters 10, 18</b>	$I_{cr}$	= moment of inertia of cracked section transformed to concrete, in. <sup>4</sup> , <b>Chapter 9</b>
$f'_s$	= stress in compression reinforcement under factored loads, psi, <b>Appendix A</b>	$I_e$	= effective moment of inertia for computation of deflection, in. <sup>4</sup> , see <b>9.5.2.3, Chapter 9</b>
$f_{se}$	= effective stress in prestressing steel (after allowance for all prestress losses), psi, <b>Chapters 12, 18, Appendix A</b>	$I_g$	= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in. <sup>4</sup> , <b>Chapters 9, 10, 14</b>
$f_t$	= extreme fiber stress in tension in the precompressed tensile zone calculated at service loads using gross section properties, psi, see <b>18.3.3, Chapter 18</b>	$I_s$	= moment of inertia of gross section of slab about centroidal axis defined for calculating $\alpha_f$ and $\beta_t$ , in. <sup>4</sup> , <b>Chapter 13</b>
$f_{uta}$	= specified tensile strength of anchor steel, psi, <b>Appendix D</b>	$I_{se}$	= moment of inertia of reinforcement about centroidal axis of member cross section, in. <sup>4</sup> , <b>Chapter 10</b>
$f_y$	= specified yield strength of reinforcement, psi, <b>Chapters 3, 7, 9-12, 14, 17-19, 21, Appendices A-C</b>	$I_{sx}$	= moment of inertia of structural steel shape, pipe, or tubing about centroidal axis of composite member cross section, in. <sup>4</sup> , <b>Chapter 10</b>
$f_{ya}$	= specified yield strength of anchor steel, psi, <b>Appendix D</b>	$k$	= effective length factor for compression members, <b>Chapters 10, 14</b>
$f_{yt}$	= specified yield strength $f_y$ of transverse reinforcement, psi, <b>Chapters 10-12, 21</b>	$k_c$	= coefficient for basic concrete breakout strength in tension, <b>Appendix D</b>
$F$	= loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces, <b>Chapter 9, Appendix C</b>	$k_{cp}$	= coefficient for prout strength, <b>Appendix D</b>
$F_n$	= nominal strength of a strut, tie, or nodal zone, lb, <b>Appendix A</b>	$K_{tr}$	= transverse reinforcement index, see <b>12.2.3, Chapter 12</b>
$F_{nn}$	= nominal strength at face of a nodal zone, lb, <b>Appendix A</b>	$l$	= span length of beam or one-way slab; clear projection of cantilever, in., see <b>8.9 and 9.5, Chapters 8 and 9</b>
$F_{ns}$	= nominal strength of a strut, lb, <b>Appendix A</b>	$l_a$	= additional embedment length beyond centerline of support or point of inflection, in., <b>Chapter 12</b>
$F_{nt}$	= nominal strength of a tie, lb, <b>Appendix A</b>	$l_c$	= length of compression member in a frame, measured center-to-center of the joints in the frame, in., <b>Chapters 10, 14, 22</b>
$F_u$	= factored force acting in a strut, tie, bearing area, or nodal zone in a strut-and-tie model, lb, <b>Appendix A</b>	$l_d$	= development length in tension of deformed bar, deformed wire, plain and deformed welded wire reinforcement, or pretensioned strand, in., <b>Chapters 7, 12, 19, 21</b>
$h$	= overall thickness or height of member, in., <b>Chapters 9-12, 14, 17, 18, 20-22, Appendices A, C</b>	$l_{dc}$	= development length in compression of deformed bars and deformed wire, in., <b>Chapter 12</b>
$h_a$	= thickness of member in which an anchor is located, measured parallel to anchor axis, in., <b>Appendix D</b>	$l_{dh}$	= development length in tension of deformed bar or deformed wire with a standard hook, measured from critical section to outside end
$h_{ef}$	= effective embedment depth of anchor, in., see <b>D.1, D.8.5, Appendix D</b> . Where anchors subject to tension are close to three or more edges, see <b>D.5.2.3</b>		
$h_v$	= depth of shearhead cross section, in., <b>Chapter 11</b>		

	of hook (straight embedment length between critical section and start of hook [point of tangency] plus inside radius of bend and one bar diameter), in., see 12.5 and 21.7.5, Chapters 12, 21	<b><math>M_{max}</math></b> = maximum factored moment at section due to externally applied loads, in.-lb, Chapter 11
$l_{dt}$	= development length in tension of headed deformed bar, measured from the critical section to the bearing face of the head, in., see 12.6, Chapter 12	<b><math>M_n</math></b> = nominal flexural strength at section, in.-lb, Chapters 11, 12, 14, 18, 21, 22
$l_e$	= load bearing length of anchor for shear, in., see D.6.2.2, Appendix D	<b><math>M_{nb}</math></b> = nominal flexural strength of beam including slab where in tension, framing into joint, in.-lb, see 21.6.2.2, Chapter 21
$l_n$	= length of clear span measured face-to-face of supports, in., Chapters 8-11, 13, 16, 18, 21	<b><math>M_{nc}</math></b> = nominal flexural strength of column framing into joint, calculated for factored axial force, consistent with the direction of lateral forces considered, resulting in lowest flexural strength, in.-lb, see 21.6.2.2, Chapter 21
$l_o$	= length, measured from joint face along axis of structural member, over which special transverse reinforcement must be provided, in., Chapter 21	<b><math>M_o</math></b> = total factored static moment, in.-lb, Chapter 13
$l_t$	= span of member under load test, taken as the shorter span for two-way slab systems, in. Span is the smaller of: (a) distance between centers of supports, and (b) clear distance between supports plus thickness $h$ of member. Span for a cantilever shall be taken as twice the distance from face of support to cantilever end, Chapter 20	<b><math>M_p</math></b> = required plastic moment strength of shear-head cross section, in.-lb, Chapter 11
$l_u$	= unsupported length of compression member, in., see 10.10.1.1, Chapter 10	<b><math>M_{pr}</math></b> = probable flexural strength of members, with or without axial load, determined using the properties of the member at the joint faces assuming a tensile stress in the longitudinal bars of at least $1.25f_y$ and a strength reduction factor, $\phi$ , of 1.0, in.-lb, Chapter 21
$l_v$	= length of shearhead arm from centroid of concentrated load or reaction, in., Chapter 11	<b><math>M_s</math></b> = factored moment due to loads causing appreciable sway, in.-lb, Chapter 10
$l_w$	= length of entire wall, or length of wall segment or wall pier considered in direction of shear force, in., Chapters 11, 14, 21	<b><math>M_{slab}</math></b> = portion of slab factored moment balanced by support moment, in.-lb, Chapter 21
$l_1$	= length of span in direction that moments are being determined, measured center-to-center of supports, in., Chapter 13	<b><math>M_u</math></b> = factored moment at section, in.-lb, Chapters 10, 11, 13, 14, 21, 22
$l_2$	= length of span in direction perpendicular to $l_1$ , measured center-to-center of supports, in., see 13.6.2.3 and 13.6.2.4, Chapter 13	<b><math>M_{ua}</math></b> = moment at midheight of wall due to factored lateral and eccentric vertical loads, not including $P\Delta$ effects, in.-lb, Chapter 14
$L$	= live loads, or related internal moments and forces, Chapters 8, 9, 20, 21, Appendix C	<b><math>M_v</math></b> = moment resistance contributed by shear-head reinforcement, in.-lb, Chapter 11
$L_r$	= roof live load, or related internal moments and forces, Chapter 9	<b><math>M_1</math></b> = smaller factored end moment on a compression member, to be taken as positive if member is bent in single curvature, and negative if bent in double curvature, in.-lb, Chapter 10
$M_a$	= maximum moment in member due to service loads at stage deflection is computed, in.-lb, Chapters 9, 14	<b><math>M_{1ns}</math></b> = factored end moment on a compression member at the end at which $M_1$ acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb, Chapter 10
$M_c$	= factored moment amplified for the effects of member curvature used for design of compression member, in.-lb, see 10.10.6, Chapter 10	<b><math>M_{1s}</math></b> = factored end moment on compression member at the end at which $M_1$ acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb, Chapter 10
$M_{cr}$	= cracking moment, in.-lb, see 9.5.2.3, Chapters 9, 14	<b><math>M_2</math></b> = larger factored end moment on compression member. If transverse loading occurs between supports, $M_2$ is taken as the largest moment occurring in member. Value of $M_2$ is always positive, in.-lb, Chapter 10
$M_{cre}$	= moment causing flexural cracking at section due to externally applied loads, in.-lb, Chapter 11	<b><math>M_{2,min}</math></b> = minimum value of $M_2$ , in.-lb, Chapter 10
$M_m$	= factored moment modified to account for effect of axial compression, in.-lb, see 11.2.2.2, Chapter 11	<b><math>M_{2ns}</math></b> = factored end moment on compression member at the end at which $M_2$ acts, due to loads that cause no appreciable sidesway,

	calculated using a first-order elastic frame analysis, in.-lb, <b>Chapter 10</b>
$M_{2s}$	factored end moment on compression member at the end at which $M_2$ acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis, in.-lb, <b>Chapter 10</b>
$n$	number of items, such as strength tests, bars, wires, monostrand anchorage devices, anchors, or shearhead arms, <b>Chapters 5, 11, 12, 18, Appendix D</b>
$N_a$	nominal bond strength in tension of a single adhesive anchor, lb, see <b>D.5.5.1, Appendix D</b>
$N_{ag}$	nominal bond strength in tension of a group of adhesive anchors, lb, see <b>D.5.5.1, Appendix D</b>
$N_b$	basic concrete breakout strength in tension of a single anchor in cracked concrete, lb, see <b>D.5.2.2, Appendix D</b>
$N_{ba}$	basic bond strength in tension of a single adhesive anchor, lb, see <b>D.5.5.2, Appendix D</b>
$N_c$	the resultant tensile force acting on the portion of the concrete cross section that is subjected to tensile stresses due to the combined effects of service loads and effective prestress, lb, <b>Chapter 18</b>
$N_{cb}$	nominal concrete breakout strength in tension of a single anchor, lb, see <b>D.5.2.1, Appendix D</b>
$N_{cbg}$	nominal concrete breakout strength in tension of a group of anchors, lb, see <b>D.5.2.1, Appendix D</b>
$N_n$	nominal strength in tension, lb, <b>Appendix D</b>
$N_p$	pullout strength in tension of a single anchor in cracked concrete, lb, see <b>D.5.3.4 and D.5.3.5, Appendix D</b>
$N_{pn}$	nominal pullout strength in tension of a single anchor, lb, see <b>D.5.3.1, Appendix D</b>
$N_{sa}$	nominal strength of a single anchor or individual anchor in a group of anchors in tension as governed by the steel strength, lb, see <b>D.5.1.1 and D.5.1.2, Appendix D</b>
$N_{sb}$	side-face blowout strength of a single anchor, lb, <b>Appendix D</b>
$N_{sbg}$	side-face blowout strength of a group of anchors, lb, <b>Appendix D</b>
$N_u$	factored axial force normal to cross section occurring simultaneously with $V_u$ or $T_u$ ; to be taken as positive for compression and negative for tension, lb, <b>Chapter 11</b>
$N_{ua}$	factored tensile force applied to anchor or individual anchor in a group of anchors, lb, <b>Appendix D</b>
$N_{ua,g}$	total factored tensile force applied to anchor group, lb, <b>Appendix D</b>

$N_{ua,i}$	= factored tensile force applied to most highly stressed anchor in a group of anchors, lb, <b>Appendix D</b>
$N_{ua,s}$	= factored sustained tension load, lb, see <b>D.3.5, Appendix D</b>
$N_{uc}$	= factored horizontal tensile force applied at top of bracket or corbel acting simultaneously with $V_u$ , to be taken as positive for tension, lb, <b>Chapter 11</b>
$p_{cp}$	= outside perimeter of concrete cross section, in., see <b>11.5.1, Chapter 11</b>
$p_h$	= perimeter of centerline of outermost closed transverse torsional reinforcement, in., <b>Chapter 11</b>
$P_b$	= nominal axial strength at balanced strain conditions, lb, see <b>Appendices B, C</b>
$P_c$	= critical buckling load, lb, see <b>10.10.6, Chapter 10</b>
$P_n$	= nominal axial strength of cross section, lb, <b>Chapters 9, 10, 14, 22, Appendixes B, C</b>
$P_{n,max}$	= maximum allowable value of $P_n$ , lb, see <b>10.3.6, Chapter 10</b>
$P_o$	= nominal axial strength at zero eccentricity, lb, <b>Chapter 10</b>
$P_{pu}$	= factored prestressing force at anchorage device, lb, <b>Chapter 18</b>
$P_s$	= unfactored axial load at the design (midheight) section including effects of self-weight, lb, <b>Chapter 14</b>
$P_u$	= factored axial force; to be taken as positive for compression and negative for tension, lb, <b>Chapters 10, 14, 21, 22</b>
$q_{Du}$	= factored dead load per unit area, <b>Chapter 13</b>
$q_{Lu}$	= factored live load per unit area, <b>Chapter 13</b>
$q_u$	= factored load per unit area, <b>Chapter 13</b>
$Q$	= stability index for a story, see <b>10.10.5.2, Chapter 10</b>
$r$	= radius of gyration of cross section of a compression member, in., <b>Chapter 10</b>
$R$	= rain load, or related internal moments and forces, <b>Chapter 9</b>
$s$	= center-to-center spacing of items, such as longitudinal reinforcement, transverse reinforcement, prestressing tendons, wires, or anchors, in., <b>Chapters 10-12, 17-21, Appendix D</b>
$s_i$	= center-to-center spacing of reinforcement in the $i$ -th layer adjacent to the surface of the member, in., <b>Appendix A</b>
$s_o$	= center-to-center spacing of transverse reinforcement within the length $\ell_o$ , in., <b>Chapter 21</b>
$s_s$	= sample standard deviation, psi, <b>Chapter 5, Appendix D</b>
$s_2$	= center-to-center spacing of longitudinal shear or torsion reinforcement, in., <b>Chapter 11</b>
$S$	= snow load, or related internal moments and forces, <b>Chapters 9, 21</b>

$S_e$	= moment, shear, or axial force at connection corresponding to development of probable strength at intended yield locations, based on the governing mechanism of inelastic lateral deformation, considering both gravity and earthquake effects, <b>Chapter 21</b>	$V_p$	= vertical component of effective prestress force at section, lb, <b>Chapter 11</b>
$S_m$	= elastic section modulus, in. <sup>3</sup> , <b>Chapter 22</b>	$V_s$	= nominal shear strength provided by shear reinforcement, lb, <b>Chapter 11</b>
$S_n$	= nominal flexural, shear, or axial strength of connection, <b>Chapter 21</b>	$V_{sa}$	= nominal shear strength of a single anchor or individual anchor in a group of anchors as governed by the steel strength, lb, see <b>D.6.1.1 and D.6.1.2, Appendix D</b>
$S_y$	= yield strength of connection, based on $f_y$ , for moment, shear, or axial force, <b>Chapter 21</b>	$V_u$	= factored shear force at section, lb, <b>Chapters 11-13, 17, 21, 22</b>
$t$	= wall thickness of hollow section, in., <b>Chapter 11</b>	$V_{ua}$	= factored shear force applied to a single anchor or group of anchors, lb, <b>Appendix D</b>
$T$	= cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete, <b>Chapter 9, Appendix C</b>	$V_{ua,g}$	= total factored shear force applied to anchor group, lb, <b>Appendix D</b>
$T_n$	= nominal torsional moment strength, in.-lb, <b>Chapter 11</b>	$V_{ua,i}$	= factored shear force applied to most highly stressed anchor in a group of anchors, lb, <b>Appendix D</b>
$T_u$	= factored torsional moment at section, in.-lb, <b>Chapter 11</b>	$V_{ug}$	= factored shear force on the slab critical section for two-way action due to gravity loads, lb, see <b>21.13.6</b>
$U$	= required strength to resist factored loads or related internal moments and forces, <b>Chapter 9, Appendix C</b>	$V_{us}$	= factored horizontal shear in a story, lb, <b>Chapter 10</b>
$V_n$	= nominal shear stress, psi, see <b>11.11.7.2, Chapters 11, 21</b>	$w_c$	= density (unit weight) of normalweight concrete or equilibrium density of lightweight concrete, lb/ft <sup>3</sup> , <b>Chapters 8, 9</b>
$V_b$	= basic concrete breakout strength in shear of a single anchor in cracked concrete, lb, see <b>D.6.2.2 and D.6.2.3, Appendix D</b>	$w_u$	= factored load per unit length of beam or one-way slab, <b>Chapter 8</b>
$V_c$	= nominal shear strength provided by concrete, lb, <b>Chapters 8, 11, 13, 21</b>	$W$	= wind load, or related internal moments and forces, <b>Chapter 9, Appendix C</b>
$V_{cb}$	= nominal concrete breakout strength in shear of a single anchor, lb, see <b>D.6.2.1, Appendix D</b>	$x$	= shorter overall dimension of rectangular part of cross section, in., <b>Chapter 13</b>
$V_{cbg}$	= nominal concrete breakout strength in shear of a group of anchors, lb, see <b>D.6.2.1, Appendix D</b>	$y$	= longer overall dimension of rectangular part of cross section, in., <b>Chapter 13</b>
$V_{ci}$	= nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment, lb, <b>Chapter 11</b>	$y_t$	= distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in., <b>Chapters 9, 11</b>
$V_{cp}$	= nominal concrete pryout strength of a single anchor, lb, see <b>D.6.3.1, Appendix D</b>	$\alpha$	= angle defining the orientation of reinforcement, <b>Chapters 11, 21, Appendix A</b>
$V_{cpq}$	= nominal concrete pryout strength of a group of anchors, lb, see <b>D.6.3.1, Appendix D</b>	$\alpha_c$	= coefficient defining the relative contribution of concrete strength to nominal wall shear strength, see <b>21.9.4.1, Chapter 21</b>
$V_{cw}$	= nominal shear strength provided by concrete when diagonal cracking results from high principal tensile stress in web, lb, <b>Chapter 11</b>	$\alpha_f$	= ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam, see <b>13.6.1.6, Chapters 9, 13</b>
$V_d$	= shear force at section due to unfactored dead load, lb, <b>Chapter 11</b>	$\alpha_{fm}$	= average value of $\alpha_f$ for all beams on edges of a panel, <b>Chapter 9</b>
$V_e$	= design shear force for load combinations including earthquake effects, lb, see <b>21.5.4.1 and 21.6.5.1, Chapter 21</b>	$\alpha_{f1}$	= $\alpha_f$ in direction of $\ell_1$ , <b>Chapter 13</b>
$V_i$	= factored shear force at section due to externally applied loads occurring simultaneously with $M_{max}$ , lb, <b>Chapter 11</b>	$\alpha_{f2}$	= $\alpha_f$ in direction of $\ell_2$ , <b>Chapter 13</b>
$V_n$	= nominal shear strength, lb, <b>Chapters 8, 10, 11, 21, 22, Appendix D</b>	$\alpha_i$	= angle between the axis of a strut and the bars in the $i$ -th layer of reinforcement crossing that strut, <b>Appendix A</b>
$V_{nh}$	= nominal horizontal shear strength, lb, <b>Chapter 17</b>	$\alpha_s$	= constant used to compute $V_c$ in slabs and footings, <b>Chapter 11</b>

$\alpha_v$	= ratio of flexural stiffness of shearhead arm to that of the surrounding composite slab section, see 11.11.4.5, Chapter 11	$\Delta_{cr}$	= computed, out-of-plane deflection at midheight of wall corresponding to cracking moment, $M_{cr}$ , in., Chapter 14
$\beta$	= ratio of long to short dimensions: clear spans for two-way slabs, see 9.5.3.3 and 22.5.4; sides of column, concentrated load or reaction area, see 11.11.2.1; or sides of a footing, see 15.4.4.2, Chapters 9, 11, 15, 22	$\Delta f_p$	= increase in stress in prestressing steel due to factored loads, psi, Appendix A
$\beta_b$	= ratio of area of reinforcement cut off to total area of tension reinforcement at section, Chapter 12	$\Delta f_{ps}$	= stress in prestressing steel at service loads less decompression stress, psi, Chapter 18
$\beta_{dns}$	= ratio used to account for reduction of stiffness of columns due to sustained axial loads, see 10.10.6.2, Chapter 10	$\Delta_n$	= computed, out-of-plane deflection at midheight of wall corresponding to nominal flexural strength, $M_n$ , in., Chapter 14
$\beta_{ds}$	= ratio used to account for reduction of stiffness of columns due to sustained lateral loads, see 10.10.4.2, Chapter 10	$\Delta_o$	= relative lateral deflection between the top and bottom of a story due to lateral forces computed using a first-order elastic frame analysis and stiffness values satisfying 10.10.5.2, in., Chapter 10
$\beta_n$	= factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone, Appendix A	$\Delta_r$	= difference between initial and final (after load removal) deflections for load test or repeat load test, in., Chapter 20
$\beta_p$	= factor used to compute $V_c$ in prestressed slabs, Chapter 11	$\Delta_s$	= computed, out-of-plane deflection at midheight of wall due to service loads, in., Chapter 14
$\beta_s$	= factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut, Appendix A	$\Delta_u$	= computed deflection at midheight of wall due to factored loads, in., Chapter 14
$\beta_t$	= ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports, see 13.6.4.2, Chapter 13	$\Delta_1$	= measured maximum deflection during first load test, in., see 20.5.2, Chapter 20
$\beta_1$	= factor relating depth of equivalent rectangular compressive stress block to neutral axis depth, see 10.2.7.3, Chapters 10, 18, Appendix B	$\Delta_2$	= maximum deflection measured during second load test relative to the position of the structure at the beginning of second load test, in., see 20.5.2, Chapter 20
$\gamma_f$	= factor used to determine the unbalanced moment transferred by flexure at slab-column connections, see 13.5.3.2, Chapters 11, 13, 21	$\varepsilon_t$	= net tensile strain in extreme layer of longitudinal tension steel at nominal strength, excluding strains due to effective prestress, creep, shrinkage, and temperature, Chapters 8-10, Appendix C
$\gamma_p$	= factor for type of prestressing steel, see 18.7.2, Chapter 18	$\theta$	= angle between axis of strut, compression diagonal, or compression field and the tension chord of the member, Chapter 11, Appendix A
$\gamma_s$	= factor used to determine the portion of reinforcement located in center band of footing, see 15.4.4.2, Chapter 15	$\lambda$	= modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative to normalweight concrete of the same compressive strength, see 8.6.1, 11.6.4.3, 12.2.4(d), 12.5.2, D.3.6, Chapters 9, 11, 12, 19, 21, 22, and Appendixes A, D
$\gamma_v$	= factor used to determine the unbalanced moment transferred by eccentricity of shear at slab-column connections, see 11.11.7.1, Chapter 11	$\lambda_a$	= modification factor reflecting the reduced mechanical properties of lightweight concrete in certain concrete anchorage applications, see D.3.6, Appendix D
$\delta$	= moment magnification factor to reflect effects of member curvature between ends of compression member, Chapter 10	$\lambda_d$	= multiplier for additional deflection due to long-term effects, see 9.5.2.5, Chapter 9
$\delta_s$	= moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads, Chapter 10	$\mu$	= coefficient of friction, see 11.6.4.3, Chapters 11, 21
$\delta_u$	= design displacement, in., Chapter 21	$\xi$	= time-dependent factor for sustained load, see 9.5.2.5, Chapter 9
		$\rho$	= ratio of $A_s$ to $bd$ , Chapters 10, 11, 13, 21, Appendix B
		$\rho'$	= ratio of $A'_s$ to $bd$ , Chapter 9, Appendix B

$\rho_b$	= ratio of $A_s$ to $bd$ producing balanced strain conditions, <a href="#">Appendix B</a>	anchors based on eccentricity of applied loads, see <a href="#">D.6.2.5, Appendix D</a>
$\rho_e$	= ratio of area of distributed longitudinal reinforcement to gross concrete area perpendicular to that reinforcement, <a href="#">Chapters 11, 14, 21</a>	$\psi_{ed,N}$ = factor used to modify tensile strength of anchors based on proximity to edges of concrete member, see <a href="#">D.5.2.5, Appendix D</a>
$\rho_p$	= ratio of $A_{ps}$ to $bd_p$ , <a href="#">Chapter 18</a>	$\psi_{ed,Na}$ = factor used to modify tensile strength of adhesive anchors based on proximity to edges of concrete member, see <a href="#">D5.5.4, Appendix D</a>
$\rho_s$	= ratio of volume of spiral reinforcement to total volume of core confined by the spiral (measured out-to-out of spirals), <a href="#">Chapters 10, 21</a>	$\psi_{ed,V}$ = factor used to modify shear strength of anchors based on proximity to edges of concrete member, see <a href="#">D.6.2.6, Appendix D</a>
$\rho_t$	= ratio of area of distributed transverse reinforcement to gross concrete area perpendicular to that reinforcement, <a href="#">Chapters 11, 14, 21</a>	$\psi_{h,V}$ = factor used to modify shear strength of anchors located in concrete members with $h_a < 1.5c_{a1}$ , see <a href="#">D.6.2.8, Appendix D</a>
$\rho_v$	= ratio of tie reinforcement area to area of contact surface, see <a href="#">17.5.3.3, Chapter 17</a>	$\psi_s$ = factor used to modify development length based on reinforcement size, see <a href="#">12.2.4, Chapter 12</a>
$\rho_w$	= ratio of $A_s$ to $b_w d$ , <a href="#">Chapter 11</a>	$\psi_t$ = factor used to modify development length based on reinforcement location, see <a href="#">12.2.4, Chapter 12</a>
$\phi$	= strength reduction factor, see <a href="#">9.3, Chapters 8-11, 13, 14, 17-22, Appendixes A-D</a>	$\psi_w$ = factor used to modify development length for welded deformed wire reinforcement in tension, see <a href="#">12.7, Chapter 12</a>
$\tau_{cr}$	= characteristic bond stress of adhesive anchor in cracked concrete, psi, see <a href="#">D.5.5.2, Appendix D</a>	$\omega$ = tension reinforcement index, see <a href="#">18.7.2, Chapter 18, Appendix B</a>
$\tau_{uncr}$	= characteristic bond stress of adhesive anchor in uncracked concrete, psi, see <a href="#">D.5.5.2, Appendix D</a>	$\omega'$ = compression reinforcement index, see <a href="#">18.7.2, Chapter 18, Appendix B</a>
$\psi_{c,N}$	= factor used to modify tensile strength of anchors based on presence or absence of cracks in concrete, see <a href="#">D.5.2.6, Appendix D</a>	$\omega_p$ = prestressing steel index, see <a href="#">B.18.8.1, Appendix B</a>
$\psi_{c,P}$	= factor used to modify pullout strength of anchors based on presence or absence of cracks in concrete, see <a href="#">D.5.3.6, Appendix D</a>	$\omega_{pw}$ = prestressing steel index for flanged sections, see <a href="#">B.18.8.1, Appendix B</a>
$\psi_{c,V}$	= factor used to modify shear strength of anchors based on presence or absence of cracks in concrete and presence or absence of supplementary reinforcement, see <a href="#">D.6.2.7</a> for anchors in shear, <a href="#">Appendix D</a>	$\omega_w$ = tension reinforcement index for flanged sections, see <a href="#">B.18.8.1, Appendix B</a>
$\psi_{cp,N}$	= factor used to modify tensile strength of post-installed anchors intended for use in uncracked concrete without supplementary reinforcement to account for the splitting tensile stresses due to installation, see <a href="#">D.5.2.7, Appendix D</a>	$\omega'_w$ = compression reinforcement index for flanged sections, see <a href="#">B.18.8.1, Appendix B</a>
$\psi_{cp,Na}$	= factor used to modify tensile strength of adhesive anchors intended for use in uncracked concrete without supplementary reinforcement to account for the splitting tensile stresses due to installation, see <a href="#">D.5.5.5, Appendix D</a>	$\Omega_o$ = amplification factor to account for overstrength of the seismic-force-resisting system determined in accordance with the legally adopted general building code, <a href="#">Chapter 21, Appendix D</a>
$\psi_e$	= factor used to modify development length based on reinforcement coating, see <a href="#">12.2.4, Chapter 12</a>	
$\psi_{ec,N}$	= factor used to modify tensile strength of anchors based on eccentricity of applied loads, see <a href="#">D.5.2.4, Appendix D</a>	
$\psi_{ec,Na}$	= factor used to modify tensile strength of adhesive anchors based on eccentricity of applied loads, see <a href="#">D5.5.3, Appendix D</a>	
$\psi_{ec,V}$	= factor used to modify shear strength of	

## R2.1 — Commentary notation

The terms used in this list are used in the Commentary, but not in the Code.

Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as feet or kips.

$c'_{a1}$	= limiting value of $c_{a1}$ when anchors are located less than $1.5c_{a1}$ from three or more edges (see <a href="#">Fig. RD.6.2.4), Appendix D</a>
$C$	= compression force acting on a nodal zone, lb, <a href="#">Appendix A</a>
$f_{si}$	= stress in the $i$ -th layer of surface reinforcement, psi, <a href="#">Appendix A</a>

$h_{anc}$	= dimension of anchorage device or single group of closely spaced devices in the direction of bursting being considered, in., <a href="#">Chapter 18</a>	$V$	= shear force acting on anchor or anchor group, <a href="#">Appendix D</a>
$h'_{ef}$	= limiting value of $h_{ef}$ when anchors are located less than $1.5h_{ef}$ from three or more edges (see <a href="#">Fig. RD.5.2.3</a> ), <a href="#">Appendix D</a>	$V_{\parallel}$	= applied shear parallel to the edge, lb, <a href="#">Appendix D</a>
$K_t$	= torsional stiffness of torsional member; moment per unit rotation, see <a href="#">R13.7.5, Chapter 13</a>	$V_{\perp}$	= applied shear perpendicular to the edge, lb, <a href="#">Appendix D</a>
$K_{05}$	= coefficient associated with the 5 percent fractile, <a href="#">Appendix D</a>	$w_s$	= width of a strut perpendicular to the axis of the strut, in., <a href="#">Appendix A</a>
$\ell_{anc}$	= length along which anchorage of a tie must occur, in., <a href="#">Appendix A</a>	$w_t$	= effective height of concrete concentric with a tie, used to dimension nodal zone, in., <a href="#">Appendix A</a>
$\ell_b$	= width of bearing, in., <a href="#">Appendix A</a>	$w_{t,max}$	= maximum effective height of concrete concentric with a tie, in., <a href="#">Appendix A</a>
$M$	= moment acting on anchor or anchor group, <a href="#">Appendix D</a>	$W_a$	= service-level wind load, see <a href="#">R14.8.4</a>
$N$	= tension force acting on anchor or anchor group, <a href="#">Appendix D</a>	$\Delta f_{pt}$	= $f_{ps}$ at the section of maximum moment minus the stress in the prestressing steel due to prestressing and factored bending moments at the section under consideration, psi, see <a href="#">R11.5.3.10, Chapter 11</a>
$R$	= reaction, lb, <a href="#">Appendix A</a>	$\varepsilon_{cu}$	= maximum usable strain at extreme concrete compression fiber, <a href="#">Fig. R10.3.3</a>
$T$	= tension force acting on a nodal zone, lb, <a href="#">Appendix A</a>	$\phi_K$	= stiffness reduction factor, see <a href="#">R10.10, Chapter 10</a>

**CODE****COMMENTARY****2.2 — Definitions**

The following terms are defined for general use in this Code. Specialized definitions appear in individual chapters.

**Admixture** — Material other than water, aggregate, or hydraulic cement, used as an ingredient of concrete and added to concrete before or during its mixing to modify its properties.

**Aggregate** — Granular material, such as sand, gravel, crushed stone, and iron blast-furnace slag, used with a cementing medium to form a hydraulic cement concrete or mortar.

**Aggregate, lightweight** — Aggregate meeting the requirements of ASTM C330 and having a loose bulk density of 70 lb/ft<sup>3</sup> or less, determined in accordance with ASTM C29.

**Anchorage device** — In post-tensioning, the hardware used for transferring a post-tensioning force from the prestressing steel to the concrete.

**Anchorage zone** — In post-tensioned members, the portion of the member through which the concentrated prestressing force is transferred to the concrete and distributed more uniformly across the section. Its extent is equal to the largest dimension of the cross section. For anchorage devices located away from the end of a member, the anchorage zone includes the disturbed regions ahead of and behind the anchorage devices.

**Base of structure** — Level at which the horizontal earthquake ground motions are assumed to be imparted to a building. This level does not necessarily coincide with the ground level. See [Chapter 21](#).

**Basic monostrand anchorage device** — Anchorage device used with any single strand or a single 5/8 in. or smaller diameter bar that satisfies [18.21.1](#) and the anchorage device requirements of ACI 423.7.

**Basic multistrand anchorage device** — Anchorage device used with multiple strands, bars, or wires, or with single bars larger than 5/8 in. diameter, that satisfies

**R2.2 — Definitions**

For consistent application of the Code, it is necessary that terms be defined where they have particular meanings in the Code. The definitions given are for use in application of this Code only and do not always correspond to ordinary usage. A glossary of most-used terms relating to cement manufacturing, concrete design and construction, and research in concrete is contained in “**Concrete Terminology**” available on the ACI Web site.

**Anchorage device** — Most anchorage devices for post-tensioning are standard manufactured devices available from commercial sources. In some cases, “special” details or assemblages are developed that combine various wedges and wedge plates for anchoring prestressing steel. These informal designations as standard anchorage devices or special anchorage devices have no direct relation to the Code and AASHTO “Standard Specifications for Highway Bridges” classification of anchorage devices as Basic Anchorage Devices or Special Anchorage Devices.

**Anchorage zone** — The terminology “ahead of” and “behind” the anchorage device is illustrated in [Fig. R18.13.1\(b\)](#).

**Basic anchorage devices** — Devices that are so proportioned that they can be checked analytically for compliance with bearing stress and stiffness requirements without having to undergo the acceptance-testing program required of special anchorage devices.

**CODE****COMMENTARY**

**18.21.1** and the bearing stress and minimum plate stiffness requirements of AASHTO Bridge Specifications, Division I, Articles 9.21.7.2.2 through 9.21.7.2.4.

**Bonded tendon** — Tendon in which prestressing steel is bonded to concrete either directly or through grouting.

**Boundary element** — Portion along structural wall and structural diaphragm edge strengthened by longitudinal and transverse reinforcement. Boundary elements do not necessarily require an increase in the thickness of the wall or diaphragm. Edges of openings within walls and diaphragms shall be provided with boundary elements as required by **21.9.6** or **21.11.7.5**. See Chapter 21.

**Building official** — The officer or other designated authority charged with the administration and enforcement of this Code, or a duly authorized representative.

**Cementitious materials** — Materials as specified in Chapter 3, which have cementing value when used in concrete either by themselves, such as portland cement, blended hydraulic cements, and expansive cement, or such materials in combination with fly ash, other raw or calcined natural pozzolans, silica fume, and/or ground-granulated blast-furnace slag.

**Collector element** — Element that acts in axial tension or compression to transmit earthquake-induced forces between a structural diaphragm and a vertical element of the seismic-force-resisting system. See Chapter 21.

**Column** — Member with a ratio of height-to-least lateral dimension exceeding 3 used primarily to support axial compressive load. For a tapered member, the least lateral dimension is the average of the top and bottom dimensions of the smaller side.

**Building official** — The term used by many general building codes to identify the person charged with administration and enforcement of provisions of the building code. Such terms as building commissioner or building inspector are variations of the title and the term “building official” as used in this Code, is intended to include those variations, as well as others that are used in the same sense.

**Column** — The term “compression member” is used in the Code to define any member in which the primary stress is longitudinal compression. Such a member need not be vertical but may have any orientation in space. Bearing walls, columns, pedestals, and wall piers qualify as compression members under this definition.

The differentiation between columns and walls in the Code is based on the principal use rather than on arbitrary relationships of height and cross-sectional dimensions. The Code, however, permits walls to be designed using the principles stated for column design (see **14.4**), as well as by the empirical method (see **14.5**).

**CODE****COMMENTARY**

While a wall always encloses or separates spaces, it may also be used to resist horizontal or vertical forces or bending. For example, a retaining wall or a basement wall also supports various combinations of loads.

A column is normally used as a main vertical member carrying axial loads combined with bending and shear. It may, however, form a small part of an enclosure or separation.

In the 2008 Code, the definitions for column and pedestal were revised to provide consistency between the definitions.

**Composite concrete flexural members** — Concrete flexural members of precast or cast-in-place concrete elements, or both, constructed in separate placements but so interconnected that all elements respond to loads as a unit.

**Compression-controlled section** — A cross section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal to the compression-controlled strain limit.

**Compression-controlled strain limit** — The net tensile strain at balanced strain conditions. See 10.3.3.

**Concrete** — Mixture of portland cement or any other hydraulic cement, fine aggregate, coarse aggregate, and water, with or without admixtures.

**Concrete, all-lightweight** — Lightweight concrete containing only lightweight coarse and fine aggregates that conform to ASTM C330.

**Concrete, lightweight** — Concrete containing lightweight aggregate and an equilibrium density, as determined by ASTM C567, between 90 and 115 lb/ft<sup>3</sup>.

**Concrete, lightweight** — In 2000, ASTM C567 adopted “equilibrium density” as the measure for determining compliance with specified in-service density requirements. According to ASTM C567, equilibrium density may be determined by measurement or approximated by calculation using either the measured oven-dry density or the oven-dry density calculated from the mixture proportions. Unless specified otherwise, ASTM C567 requires that equilibrium density be approximated by calculation.

By Code definition, sand-lightweight concrete is structural lightweight concrete with all of the fine aggregate replaced by sand. This definition may not be in agreement with usage by some material suppliers or contractors where the majority, but not all, of the lightweight fines are replaced by sand. For proper application of the Code provisions, the replacement limits should be stated, with interpolation when partial sand replacement is used.

**CODE****COMMENTARY**

**Concrete, normalweight** — Concrete containing only aggregate that conforms to ASTM C33.

**Concrete, sand-lightweight** — Lightweight concrete containing only normalweight fine aggregate that conforms to ASTM C33 and only lightweight aggregate that conforms to ASTM C330.

**Concrete, specified compressive strength of, ( $f'_c$ )** — Compressive strength of concrete used in design and evaluated in accordance with provisions of [Chapter 5](#), expressed in pounds per square inch (psi). Whenever the quantity  $f'_c$  is under a radical sign, square root of numerical value only is intended, and result has units of pounds per square inch (psi).

**Connection** — A region that joins two or more members. In [Chapter 21](#), a connection also refers to a region that joins members of which one or more is precast, for which the following more specific definitions apply:

**Ductile connection** — Connection that experiences yielding as a result of the earthquake design displacements.

**Strong connection** — Connection that remains elastic while adjoining members experience yielding as a result of the earthquake design displacements.

**Contract documents** — Documents, including the project drawings and project specifications, covering the required Work.

**Contraction joint** — Formed, sawed, or tooled groove in a concrete structure to create a weakened plane and regulate the location of cracking resulting from the dimensional change of different parts of the structure.

**Cover, specified concrete** — The distance between the outermost surface of embedded reinforcement and the closest outer surface of the concrete indicated in contract documents.

**Crosstie** — A continuous reinforcing bar having a seismic hook at one end and a hook not less than 90 degrees with at least a six-diameter extension at the other end. The hooks shall engage peripheral longitudinal bars. The 90-degree hooks of two successive crossties engaging the same longitudinal bars shall be alternated end for end. See [Chapters 7, 21](#).

**Curvature friction** — Friction resulting from bends or curves in the specified prestressing tendon profile.

**Concrete, normalweight** — Normalweight concrete typically has a density (unit weight) between 135 and 160 lb/ft<sup>3</sup>, and is normally taken as 145 to 150 lb/ft<sup>3</sup>.

**Cover, specified concrete** — Tolerances on specified concrete cover are provided in [7.5.2.1](#).

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**Deformed reinforcement** — Deformed reinforcing bars, bar mats, deformed wire, and welded wire reinforcement conforming to 3.5.3.

**Deformed reinforcement** — Deformed reinforcement is defined as that meeting the deformed reinforcement specifications of 3.5.3.1, or the specifications of 3.5.3.3, 3.5.3.4, 3.5.3.5, 3.5.3.6, 3.5.3.7, or 3.5.3.11. No other reinforcement qualifies. This definition permits accurate statement of anchorage lengths. Bars or wire not meeting the deformation requirements or welded wire reinforcement not meeting the spacing requirements are “plain reinforcement,” for code purposes, and may be used only for spirals.

**Design displacement** — Total lateral displacement expected for the design-basis earthquake, as required by the governing code for earthquake-resistant design. See Chapter 21.

**Design displacement** — The design displacement is an index of the maximum lateral displacement expected in design for the design-basis earthquake. In documents such as ASCE/SEI 7 and the International Building Code, the design displacement is calculated using static or dynamic linear elastic analysis under code-specified actions considering effects of cracked sections, effects of torsion, effects of vertical forces acting through lateral displacements, and modification factors to account for expected inelastic response. The design displacement generally is larger than the displacement calculated from design-level forces applied to a linear-elastic model of the building.

**Design load combination** — Combination of factored loads and forces in 9.2.

**Design story drift ratio** — Relative difference of design displacement between the top and bottom of a story, divided by the story height. See Chapter 21.

**Development length** — Length of embedded reinforcement, including pretensioned strand, required to develop the design strength of reinforcement at a critical section. See 9.3.3.

**Drop panel** — A projection below the slab used to reduce the amount of negative reinforcement over a column or the minimum required slab thickness, and to increase the slab shear strength. See 13.2.5 and 13.3.7.

**Duct** — A conduit (plain or corrugated) to accommodate prestressing steel for post-tensioned installation. Requirements for post-tensioning ducts are given in 18.17.

**Effective depth of section (*d*)** — Distance measured from extreme compression fiber to centroid of longitudinal tension reinforcement.

**Effective prestress** — Stress remaining in prestressing steel after all losses have occurred.

**Embedment length** — Length of embedded reinforcement provided beyond a critical section.

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**Equilibrium density** — Density of lightweight concrete after exposure to a relative humidity of 50  $\pm$  5 percent and a temperature of 73.5  $\pm$  3.5°F for a period of time sufficient to reach constant density (see ASTM C567).

**Extreme tension steel** — The reinforcement (prestressed or nonprestressed) that is the farthest from the extreme compression fiber.

**Headed deformed bars** — Deformed reinforcing bars with heads that satisfy 3.5.9 attached at one or both ends.

**Headed shear stud reinforcement** — Reinforcement consisting of individual headed studs, or groups of studs, with anchorage provided by a head at each end or by a common base rail consisting of a steel plate or shape.

**Hoop** — A closed tie or continuously wound tie. A closed tie can be made up of several reinforcement elements each having seismic hooks at both ends. A continuously wound tie shall have a seismic hook at both ends. See Chapter 21.

**Isolation joint** — A separation between adjoining parts of a concrete structure, usually a vertical plane, at a designed location such as to interfere least with performance of the structure, yet such as to allow relative movement in three directions and avoid formation of cracks elsewhere in the concrete and through which all or part of the bonded reinforcement is interrupted.

**Jacking force** — In prestressed concrete, temporary force exerted by device that introduces tension into prestressing steel.

**Joint** — Portion of structure common to intersecting members. The effective cross-sectional area of a joint of a special moment frame,  $A_j$ , for shear strength computations is defined in 21.7.4.1. See Chapter 21.

**Licensed design professional** — An individual who is licensed to practice structural design as defined by the statutory requirements of the professional

**Headed deformed bars** — The bearing area of a headed deformed bar is, for the most part, perpendicular to the bar axis. In contrast, the bearing area of the head of headed stud reinforcement is a nonplanar spatial surface of revolution, as shown in Fig. R3.5.5. The two types of reinforcement differ in other ways. The shanks of headed studs are smooth, not deformed as with headed deformed bars. The minimum net bearing area of the head of a headed deformed bar is permitted to be as small as four times the bar area. In contrast, the minimum stud head area is not specified in terms of the bearing area, but by the total head area which must be at least ten times the area of the shank.

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licensing laws of the state or jurisdiction in which the project is to be constructed and who is in responsible charge of the structural design; in other documents, also referred to as *registered design professional*.

**Load, dead** — Dead weight supported by a member, as defined by general building code of which this Code forms a part (without load factors).

**Load, factored** — Load, multiplied by appropriate load factors, used to proportion members by the strength design method of this Code. See 8.1.1 and 9.2.

**Load, live** — Live load specified by general building code of which this Code forms a part (without load factors).

**Load, service** — Load specified by general building code of which this Code forms a part (without load factors).

**Modulus of elasticity** — Ratio of normal stress to corresponding strain for tensile or compressive stresses below proportional limit of material. See 8.5.

**Moment frame** — Frame in which members and joints resist forces through flexure, shear, and axial force. Moment frames designated as part of the seismic-force-resisting system shall be categorized as follows:

**Ordinary moment frame** — A cast-in-place or precast concrete frame complying with the requirements of Chapters 1 through 18, and, in the case of ordinary moment frames assigned to Seismic Design Category B, also complying with 21.2.

**Intermediate moment frame** — A cast-in-place frame complying with the requirements of 21.3 in addition to the requirements for ordinary moment frames.

**Special moment frame** — A cast-in-place frame complying with the requirements of 21.1.3 through 21.1.7, 21.5 through 21.7, or a precast frame complying with the requirements of 21.1.3 through 21.1.7 and 21.5 through 21.8. In addition, the requirements for ordinary moment frames shall be satisfied.

**Net tensile strain** — The tensile strain at nominal strength exclusive of strains due to effective prestress, creep, shrinkage, and temperature.

**Loads** — A number of definitions for loads are given as the Code contains requirements that are to be met at various load levels. The terms “dead load” and “live load” refer to the unfactored loads (service loads) specified or defined by the general building code. Service loads (loads without load factors) are to be used where specified in the Code to proportion or investigate members for adequate serviceability, as in 9.5, Control of Deflections. Loads used to proportion a member for adequate strength are defined as factored loads. Factored loads are service loads multiplied by the appropriate load factors specified in 9.2 for required strength. The term “design loads,” as used in the 1971 Code edition to refer to loads multiplied by the appropriate load factors, was discontinued in the 1977 Code to avoid confusion with the design load terminology used in general building codes to denote service loads, or posted loads in buildings. The factored load terminology, first adopted in the 1977 Code, clarifies when the load factors are applied to a particular load, moment, or shear value as used in the Code provisions.

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**Pedestal** — Member with a ratio of height-to-least lateral dimension less than or equal to 3 used primarily to support axial compressive load. For a tapered member, the least lateral dimension is the average of the top and bottom dimensions of the smaller side.

**Plain concrete** — Structural concrete with no reinforcement or with less reinforcement than the minimum amount specified for reinforced concrete.

**Plain reinforcement** — Reinforcement that does not conform to definition of deformed reinforcement. See 3.5.4.

**Plastic hinge region** — Length of frame element over which flexural yielding is intended to occur due to earthquake design displacements, extending not less than a distance  $h$  from the critical section where flexural yielding initiates. See Chapter 21.

**Post-tensioning** — Method of prestressing in which prestressing steel is tensioned after concrete has hardened.

**Precast concrete** — Structural concrete element cast elsewhere than its final position in the structure.

**Precompressed tensile zone** — Portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present.

**Prestressed concrete** — Structural concrete in which internal stresses have been introduced to reduce potential tensile stresses in concrete resulting from loads.

**Prestressing steel** — High-strength steel element such as wire, bar, or strand, or a bundle of such elements, used to impart prestress forces to concrete.

**Pretensioning** — Method of prestressing in which prestressing steel is tensioned before concrete is placed.

**Reinforced concrete** — Structural concrete reinforced with no less than the minimum amounts of prestressing steel or nonprestressed reinforcement specified in Chapters 1 through 21 and Appendixes A through C.

**Pedestal** — In the 2008 Code, the definitions for column and pedestal were revised to provide consistency between the definitions.

**Plain concrete** — The presence of reinforcement (nonprestressed or prestressed) does not prohibit the member from being classified as plain concrete, provided all requirements of Chapter 22 are satisfied.

**Prestressed concrete** — Reinforced concrete is defined to include prestressed concrete. Although the behavior of a prestressed member with unbonded tendons may vary from that of members with continuously bonded tendons, bonded and unbonded prestressed concrete are combined with conventionally reinforced concrete under the generic term “reinforced concrete.” Provisions common to both prestressed and conventionally reinforced concrete are integrated to avoid overlapping and conflicting provisions.

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**Reinforcement** — Material that conforms to 3.5, excluding prestressing steel unless specifically included.

**Reshores** — Shores placed snugly under a concrete slab or other structural member after the original forms and shores have been removed from a larger area, thus requiring the new slab or structural member to deflect and support its own weight and existing construction loads applied prior to the installation of the reshores.

**Seismic design category** — A classification assigned to a structure based on its occupancy category and the severity of the design earthquake ground motion at the site, as defined by the legally adopted general building code.

**Seismic-force-resisting system** — Portion of the structure designed to resist earthquake design forces required by the legally adopted general building code using the applicable provisions and load combinations.

**Seismic hook** — A hook on a stirrup, or crosstie having a bend not less than 135 degrees, except that circular hoops shall have a bend not less than 90 degrees. Hooks shall have a  $6d_b$  (but not less than 3 in.) extension that engages the longitudinal reinforcement and projects into the interior of the stirrup or hoop. See 7.1.4 and Chapter 21.

**Shear cap** — A projection below the slab used to increase the slab shear strength. See 13.2.6.

**Sheathing** — A material encasing prestressing steel to prevent bonding of the prestressing steel with the surrounding concrete, to provide corrosion protection, and to contain the corrosion inhibiting coating.

**Shores** — Vertical or inclined support members designed to carry the weight of the formwork, concrete, and construction loads above.

**Span length** — See 8.9.

**Special anchorage device** — Anchorage device that satisfies 18.15.1 and the standardized acceptance tests of AASHTO "Standard Specifications for Highway Bridges," Division II, Article 10.3.2.3.

**Sheathing** — Typically, sheathing is a continuous, seam-less, high-density polyethylene material extruded directly on the coated prestressing steel.

**Special anchorage devices** — Special anchorage devices are any devices (monostrand or multistrand) that do not meet the relevant PTI or AASHTO bearing stress and, where applicable, stiffness requirements. Most commercially marketed multibearing surface anchorage devices are special anchorage devices. As provided in 18.15.1, such devices can be used only when they have been shown experimentally to be in compliance with the AASHTO requirements. This demonstration of compliance will ordinarily be furnished by the device manufacturer.

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**Special boundary element** — Boundary element required by 21.9.6.2 or 21.9.6.3.

**Special moment frame** — The provisions of 21.8 are intended to result in a special moment frame constructed using precast concrete having minimum strength and toughness equivalent to that for a special moment frame of cast-in-place concrete.

**Spiral reinforcement** — Continuously wound reinforcement in the form of a cylindrical helix.

**Splitting tensile strength ( $f_{ct}$ )** — Tensile strength of concrete determined in accordance with ASTM C496 as described in ASTM C330. See 5.1.4.

**Steel fiber-reinforced concrete** — Concrete containing dispersed randomly oriented steel fibers.

**Steel fiber-reinforced concrete** — In the Code, discontinuous steel fiber reinforcement conforming to 3.5.8 is permitted only in normalweight concrete proportioned, mixed, sampled, and evaluated in accordance with Chapter 5.

**Stirrup** — Reinforcement used to resist shear and torsion stresses in a structural member; typically bars, wires, or welded wire reinforcement either single leg or bent into L, U, or rectangular shapes and located perpendicular to or at an angle to longitudinal reinforcement.  
|(The term "stirrups" is usually applied to transverse reinforcement in flexural members and the term "ties" to transverse reinforcement in compression members.)  
See also **Tie**.

**Strength, design** — Nominal strength multiplied by a strength reduction factor  $\phi$ . See 9.3.

**Strength, nominal** — Strength of a member or cross section calculated in accordance with provisions and assumptions of the strength design method of this Code before application of any strength reduction factors. See 9.3.1.

**Strength, nominal** — Strength of a member or cross section calculated using standard assumptions and strength equations, and nominal (specified) values of material strengths and dimensions is referred to as "nominal strength." The subscript  $n$  is used to denote the nominal strengths; nominal axial load strength  $P_n$ , nominal moment strength  $M_n$ , and nominal shear strength  $V_n$ . "Design strength" or usable strength of a member or cross section is the nominal strength reduced by the strength reduction factor  $\phi$ .

The required axial load, moment, and shear strengths used to proportion members are referred to either as factored axial loads, factored moments, and factored shears, or required axial loads, moments, and shears. The factored load effects are calculated from the applied factored loads and forces in such load combinations as are stipulated in the code (see 9.2).

The subscript  $u$  is used only to denote the required strengths; required axial load strength  $P_u$ , required moment strength  $M_u$ , and required shear strength  $V_u$ , calculated from the applied factored loads and forces.

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The basic requirement for strength design may be expressed as follows:

$$\text{Design strength} \geq \text{Required strength}$$

$$\phi P_n \geq P_u$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

For additional discussion on the concepts and nomenclature for strength design, see [Commentary Chapter 9](#).

**Strength, required** — Strength of a member or cross section required to resist factored loads or related internal moments and forces in such combinations as are stipulated in this Code. See [9.1.1](#).

**Stress** — Intensity of force per unit area.

**Structural concrete** — All concrete used for structural purposes including plain and reinforced concrete.

**Structural diaphragm** — Structural member, such as a floor or roof slab, that transmits forces acting in the plane of the member to the vertical elements of the seismic-force-resisting system. See [Chapter 21](#) for requirements in the earthquake-resisting structures.

**Structural truss** — Assemblage of reinforced concrete members subjected primarily to axial forces.

**Structural wall** — Wall proportioned to resist combinations of shears, moments, and axial forces. A shear wall is a structural wall. A structural wall designated as part of the seismic-force-resisting system shall be categorized as follows:

**Ordinary structural plain concrete wall** — A wall complying with the requirements of [Chapter 22](#).

**Ordinary reinforced concrete structural wall** — A wall complying with the requirements of [Chapters 1 through 18](#).

**Intermediate precast structural wall** — A wall complying with all applicable requirements of [Chapters 1 through 18](#) in addition to [21.4](#).

**Intermediate precast structural wall** — The provisions of [21.4](#) are intended to result in an intermediate precast structural wall having minimum strength and toughness equivalent to that for an ordinary reinforced concrete structural wall of cast-in-place concrete. A precast concrete wall satisfying only the requirements of [Chapters 1 through 18](#) and not additional requirements of [21.4](#) or [21.10](#) is considered to have ductility and structural integrity less than that for an intermediate precast structural wall.

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**Special structural wall** — A cast-in-place or precast wall complying with the requirements of 21.1.3 through 21.1.7, 21.9, and 21.10, as applicable, in addition to the requirements for ordinary reinforced concrete structural walls.

**Tendon** — In pretensioned applications, the tendon is the prestressing steel. In post-tensioned applications, the tendon is a complete assembly consisting of anchorages, prestressing steel, and sheathing with coating for unbonded applications or ducts with grout for bonded applications.

**Tension-controlled section** — A cross section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005.

**Tie** — Loop of reinforcing bar or wire enclosing longitudinal reinforcement. A continuously wound bar or wire in the form of a circle, rectangle, or other polygon shape without re-entrant corners is acceptable. See also *Stirrup*.

**Transfer** — Act of transferring stress in prestressing steel from jacks or pretensioning bed to concrete member.

**Transfer length** — Length of embedded pretensioned strand required to transfer the effective prestress to the concrete.

**Unbonded tendon** — Tendon in which the prestressing steel is prevented from bonding to the concrete and is free to move relative to the concrete. The prestressing force is permanently transferred to the concrete at the tendon ends by the anchorages only.

**Vertical wall segment** — A segment of a structural wall, bounded horizontally by two openings or by an opening and an edge. Wall piers are vertical wall segments.

**Wall** — Member, usually vertical, used to enclose or separate spaces.

**Wall pier** — A vertical wall segment within a structural wall, bounded horizontally by two openings or by an opening and an edge, with ratio of horizontal length to wall thickness ( $\ell_w/b_w$ ) less than or equal to 6.0, and ratio of clear height to horizontal length ( $h_w/\ell_w$ ) greater than or equal to 2.0.

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**Special precast structural wall** — The provisions of 21.10 are intended to result in a special precast structural wall having minimum strength and toughness equivalent to that for a special reinforced concrete structural wall of cast-in-place concrete.

**Wall** — Openings in walls create vertical and horizontal wall segments. A horizontal wall segment is shown in Fig. R21.9.4.5.

**Wall pier** — Wall piers are vertical wall segments with dimensions and reinforcement intended to result in shear demand being limited by flexural yielding of the vertical reinforcement in the pier.

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**Welded wire reinforcement** — Reinforcing elements consisting of carbon-steel plain or deformed wires, fabricated into sheets or rolls in accordance with ASTM A1064; or reinforcing elements consisting of stainless-steel plain or deformed wires fabricated into sheets or rolls conforming to ASTM A1022.

**Wobble friction** — In prestressed concrete, friction caused by unintended deviation of prestressing sheath or duct from its specified profile.

**Work** — The entire construction or separately identifiable parts thereof that are required to be furnished under the contract documents.

**Yield strength** — Specified minimum yield strength or yield point of reinforcement. Yield strength or yield point shall be determined in tension according to applicable ASTM standards as modified by 3.5 of this Code.

**Notes**

# CHAPTER 3 — MATERIALS

## CODE

### 3.1 — Tests of materials

**3.1.1** — The building official shall have the right to order testing of any materials used in concrete construction to determine if materials are of quality specified.

**3.1.2** — Tests of materials and of concrete shall be made in accordance with standards listed in **3.8**.

**3.1.3** — A complete record of tests of materials and of concrete shall be retained by the inspector for at least 2 years after completion of the project, and made available for inspection during the progress of the Work.

### 3.2 — Cementitious materials

**3.2.1** — Cementitious materials shall conform to the relevant specifications as follows:

- (a) Portland cement: ASTM C150;
- (b) Blended hydraulic cements: ASTM C595 excluding Type IS ( $\geq 70$ ), which is not intended as principal cementing constituents of structural concrete;
- (c) Expansive hydraulic cement: ASTM C845;
- (d) Hydraulic cement: ASTM C1157;
- (e) Fly ash and natural pozzolan: ASTM C618;
- (f) Slag cement: ASTM C989;
- (g) Silica fume: ASTM C1240.

**3.2.2** — Cementitious materials used in the Work shall correspond to those used as the basis for selecting concrete mixture proportions. See **5.2**.

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### R3.1 — Tests of materials

**R3.1.3** — The record of tests of materials and of concrete should be retained for at least 2 years after completion of the project. Completion of the project is the date at which the owner accepts the project or when the certificate of occupancy is issued, whichever date is later. Local legal requirements may require longer retention of such records.

### R3.2 — Cementitious materials

**R3.2.1** — Type IS ( $\geq 70$ ) is a blended cement under ASTM C595 that contains ground-granulated blast-furnace slag as an interground component in a quantity equal to or exceeding 70 percent by weight. This was called Type S or Type SA cement in versions of ASTM C595 before 2006.

**R3.2.2** — Depending on the circumstances, the provision of 3.2.2 may require the same type of cementitious materials or may require cementitious materials from the same respective sources. The latter would be the case if the sample standard deviation<sup>3.1</sup> of strength tests used in establishing the required strength margin was based on cementitious materials from a particular source. If the sample standard deviation was based on tests involving cementitious materials obtained from several sources, the former interpretation would apply.

**CODE****COMMENTARY****3.3 — Aggregates**

**3.3.1** — Concrete aggregates shall conform to one of the following specifications:

- (a) Normalweight: ASTM C33;
- (b) Lightweight: ASTM C330.

Exception: Aggregates that have been shown by test or actual service to produce concrete of adequate strength and durability and approved by the building official.

**3.3.2** — Nominal maximum size of coarse aggregate shall be not larger than:

- (a) 1/5 the narrowest dimension between sides of forms, nor
- (b) 1/3 the depth of slabs, nor
- (c) 3/4 the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, individual tendons, bundled tendons, or ducts.

These limitations shall not apply if, in the judgment of the licensed design professional, workability and methods of consolidation are such that concrete can be placed without honeycombs or voids.

**3.4 — Water**

**3.4.1** — Water used in mixing concrete shall conform to ASTM C1602.

**R3.3 — Aggregates**

**R3.3.1** — Aggregates conforming to ASTM specifications are not always economically available and, in some instances, noncomplying materials have a long history of satisfactory performance. Such nonconforming materials are permitted when acceptable evidence of satisfactory performance is provided. Satisfactory performance in the past, however, does not guarantee good performance under other conditions and in other localities. Whenever possible, aggregates conforming to the designated specifications should be used.

**R3.3.2** — The size limitations on aggregates are provided to ensure proper encasement of reinforcement and to minimize honeycombing. Note that the limitations on maximum size of the aggregate may be waived if, in the judgment of the licensed design professional, the workability and methods of consolidation of the concrete are such that the concrete can be placed without honeycombs or voids.

**R3.4 — Water**

**R3.4.1** — Almost any natural water that is drinkable (potable) and has no pronounced taste or odor is satisfactory as mixing water for making concrete. Excessive impurities in mixing water may affect not only setting time, concrete strength, and volume stability (length change), but may also cause efflorescence or corrosion of reinforcement. Where possible, water with high concentrations of dissolved solids should be avoided.

Salts or other deleterious substances contributed from the aggregate or admixtures are additive to those that might be contained in the mixing water. These additional amounts are to be considered in evaluating the acceptability of the total impurities that may be deleterious to concrete or steel.

ASTM C1602 allows the use of potable water without testing and includes methods for qualifying nonpotable sources of water with consideration of effects on setting time and strength. Testing frequencies are established to ensure continued monitoring of water quality.

ASTM C1602 includes optional limits for chlorides, sulfates, alkalis, and solids in mixing water that can be invoked when appropriate.

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**3.4.2** — Mixing water for prestressed concrete or for concrete that will contain aluminum embedments, including that portion of mixing water contributed in the form of free moisture on aggregates, shall not contain deleterious amounts of chloride ion. See **4.3.1**.

**3.5 — Steel reinforcement**

**3.5.1** — Reinforcement shall be deformed reinforcement, except that plain reinforcement shall be permitted for spirals or prestressing steel; and reinforcement consisting of headed shear studs, structural steel, steel pipe, or steel tubing shall be permitted as specified in this Code. Discontinuous deformed steel fibers shall be permitted only for resisting shear under conditions specified in **11.4.6.1(f)**.

**3.5.2** — Welding of reinforcing bars shall conform to AWS D1.4. Type and location of welded splices and other required welding of reinforcing bars shall be indicated on the contract documents. ASTM specifications for bar reinforcement, except for ASTM A706, shall be supplemented to require a report of material properties necessary to conform to the requirements in AWS D1.4.

**R3.5 — Steel reinforcement**

**R3.5.1** — Discontinuous deformed steel fibers are permitted only for resisting shear in flexural members (see **11.4.6.1(f)**). Fiber-reinforced polymer (FRP) reinforcement is not addressed in this Code. ACI Committee 440 has developed guidelines for the use of FRP reinforcement.<sup>3.2, 3.3</sup>

Materials permitted for use as reinforcement are specified. Other metal elements, such as inserts, anchor bolts, or plain bars for dowels at isolation or contraction joints, are not normally considered to be reinforcement under the provisions of this Code.

**R3.5.2** — When welding of reinforcing bars is required, the weldability of the steel and compatible welding procedures need to be considered. The provisions in AWS D1.4 Welding Code cover aspects of welding reinforcing bars, including criteria to qualify welding procedures.

Weldability of the steel is based on its chemical composition or carbon equivalent (CE). The Welding Code establishes preheat and interpass temperatures for a range of carbon equivalents and reinforcing bar sizes. Carbon equivalent is calculated from the chemical composition of the reinforcing bars. The Welding Code has two expressions for calculating carbon equivalent. A relatively short expression, considering only the elements carbon and manganese, is to be used for bars other than ASTM A706 material. A more comprehensive expression is given for ASTM A706 bars. The CE formula in the Welding Code for ASTM A706 bars is identical to the CE formula in ASTM A706.

The chemical analysis, for bars other than ASTM A706, required to calculate the carbon equivalent is not routinely provided by the producer of the reinforcing bars. For welding reinforcing bars other than ASTM A706 bars, the contract documents should specifically require results of the chemical analysis to be furnished.

ASTM A706 covers low-alloy steel reinforcing bars intended for applications requiring controlled tensile properties or welding. Weldability is accomplished in ASTM A706 by limits or controls on chemical composition and on carbon equivalent.<sup>3.4</sup> The producer is required by ASTM A706 to report the chemical composition and carbon equivalent.

The AWS D1.4 Welding Code requires the contractor to prepare written welding procedure specifications conforming

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to the requirements of the Welding Code. Appendix A of the Welding Code contains a suggested form that shows the information required for such a specification for each joint welding procedure.

Often it is necessary to weld to existing reinforcing bars in a structure when no mill test report of the existing reinforcement is available. This condition is particularly common in alterations or building expansions. AWS D1.4 states for such bars that a chemical analysis may be performed on representative bars. If the chemical composition is not known or obtained, the Welding Code requires a minimum preheat. For bars other than ASTM A706 material, the minimum preheat required is 300°F for bars No. 6 or smaller, and 400°F for No. 7 bars or larger. The required preheat for all sizes of ASTM A706 is to be the temperature given in the Welding Code's table for minimum preheat corresponding to the range of CE "over 45 percent to 55 percent." Welding of the particular bars should be performed in accordance with AWS D1.4. It should also be determined if additional precautions are in order, based on other considerations such as stress level in the bars, consequences of failure, and heat damage to existing concrete due to welding operations.

Welding of wire to wire, and of wire or welded wire reinforcement to reinforcing bars or structural steel elements is not covered by AWS D1.4. If welding of this type is required on a project, the contract documents should specify requirements or performance criteria for this welding. If cold drawn wires are to be welded, the welding procedures should address the potential loss of yield strength and ductility achieved by the cold-working process (during manufacture) when such wires are heated by welding. These potential concerns are not an issue for machine and resistance welding as used in the manufacture of welded plain and deformed wire reinforcement covered by ASTM A1064.

**3.5.3 — Deformed reinforcement**

**3.5.3.1** — Deformed reinforcing bars shall conform to the requirements for deformed bars in one of the following specifications, except as permitted by **3.5.3.3**:

- (a) Carbon steel: ASTM A615;
- (b) Low-alloy steel: ASTM A706;
- (c) Stainless steel: ASTM A955;
- (d) Rail steel and axle steel: ASTM A996. Bars from rail steel shall be Type R.

**R3.5.3 — Deformed reinforcement**

**R3.5.3.1** — Low-alloy steel deformed bars conforming to ASTM A706 are intended for applications where controlled tensile properties, restrictions on chemical composition to enhance weldability, or both, are required.

| Stainless steel deformed bars are used in applications where high corrosion resistance or controlled magnetic permeability are required. The physical and mechanical property requirements for stainless steel bars under ASTM A955 are the same as those for carbon-steel bars under ASTM A615.

| Rail-steel deformed bars used with this Code are required to conform to ASTM A996 including the provisions for Type R bars. Type R bars are required to meet more restrictive provisions for bend tests.

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**3.5.3.2** — Deformed reinforcing bars shall conform to one of the ASTM specifications listed in **3.5.3.1**, except that for bars with  $f_y$  less than 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.5 percent, and for bars with  $f_y$  at least 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.35 percent. See **9.4**.

**R3.5.3.2** — The ASTM specifications require that yield strength be determined by the offset method (0.2 percent offset) and also include, for bars with  $f_y$  at least 60,000 psi, the additional requirement that the stress corresponding to a tensile strain of 0.35 percent be at least  $f_y$ . The 0.35 percent strain limit is necessary to ensure that the assumption of an elasto-plastic stress-strain curve in 10.2.4 will not lead to unconservative values of the member strength. Therefore, the Code defines yield strength in terms of the stress corresponding to a strain of 0.5 percent for  $f_y$  less than 60,000 psi and the stress corresponding to a strain of 0.35 percent for  $f_y$  at least 60,000 psi.

**3.5.3.3** — Deformed reinforcing bars conforming to ASTM A1035 shall be permitted to be used as transverse reinforcement in **21.6.4** or spiral reinforcement in **10.9.3**.

**3.5.3.4** — Bar mats for concrete reinforcement shall conform to ASTM A184. Reinforcing bars used in bar mats shall conform to ASTM A615 or ASTM A706.

**3.5.3.5** — Deformed wire for concrete reinforcement shall conform to ASTM A1064, except that wire shall not be smaller than size D-4 or larger than size D-31 unless as permitted in **3.5.3.7**. For wire with  $f_y$  exceeding 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.35 percent.

**3.5.3.6** — Welded plain wire reinforcement shall conform to ASTM A1064, except that for wire with  $f_y$  exceeding 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.35 percent. Spacing of welded intersections shall not exceed 12 in. in direction of calculated stress, except for welded wire reinforcement used as stirrups in accordance with **12.13.2**.

**3.5.3.7** — Welded deformed wire reinforcement shall conform to ASTM A1064, except that for wire with  $f_y$  exceeding 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.35 percent. Spacing of welded intersections shall not exceed 16 in. in direction of calculated stress, except for welded deformed wire reinforcement used as stirrups in accordance with **12.13.2**. Deformed wire larger than D-31 is permitted when used in welded wire reinforcement conforming to ASTM A1064, but shall be treated as plain wire for development and splice design.

**R3.5.3.5** — An upper limit is placed on the size of deformed wire because tests show that D-45 wire will achieve only approximately 60 percent of the bond strength in tension given by Eq. (12-1).<sup>3.5</sup>

**R3.5.3.6** — Welded plain wire reinforcement is made of wire conforming to ASTM A1064, which specifies a minimum yield strength of 70,000 psi. The Code has assigned a yield strength value of 60,000 psi, but makes provision for the use of higher yield strengths provided the stress corresponds to a strain of 0.35 percent.

**R3.5.3.7** — Welded deformed wire reinforcement should be made of wire conforming to ASTM A1064, which specifies a minimum yield strength of 70,000 psi. The Code has assigned a yield strength value of 60,000 psi, but makes provision for the use of higher yield strengths provided the stress corresponds to a strain of 0.35 percent.

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**3.5.3.8** — Zinc-coated (galvanized) reinforcing bars shall conform to ASTM A767. Epoxy-coated reinforcing bars shall conform to ASTM A775 or to ASTM A934. Zinc and epoxy dual-coated reinforcing bars shall conform to ASTM A1055. Bars to be zinc-coated (galvanized), epoxy-coated, or zinc and epoxy dual-coated shall conform to one of the specifications listed in **3.5.3.1**.

**R3.5.3.8** — Zinc-coated (galvanized) reinforcing bars (ASTM A767), epoxy-coated reinforcing bars (ASTM A775 and A934), and zinc and epoxy dual-coated reinforcing bars (ASTM A1055) are used in applications where corrosion resistance of reinforcement is of particular concern. They have typically been used in parking structures, bridge structures, and other highly corrosive environments. Zinc-coated (galvanized) reinforcing bars conforming to ASTM A767 are coated per the hot-dipped process.

**3.5.3.9** — Epoxy-coated wires and welded wire reinforcement shall conform to ASTM A884. Wires to be epoxy-coated shall conform to **3.5.3.5** and welded wire reinforcement to be epoxy-coated shall conform to **3.5.3.6** or **3.5.3.7**.

**R3.5.3.10** — Stainless steel wire and welded wire are used in applications where high corrosion resistance or controlled magnetic permeability are required. The physical and mechanical property requirements for deformed stainless steel wire and deformed and plain welded wire under ASTM A1022 are the same as those for deformed wire, deformed welded wire, and plain welded wire under ASTM A1064.

**3.5.3.10** — Zinc-coated (galvanized) welded wire reinforcement shall conform to ASTM A1060. Deformed wires to be zinc-coated and fabricated into welded wire reinforcement shall conform to **3.5.3.5**. Plain wires to be zinc-coated and fabricated into welded wire reinforcement shall conform to ASTM A1064, except that for wire with  $f_y$  exceeding 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.35 percent. Welded wire reinforcement to be zinc-coated (galvanized) shall conform to **3.5.3.6** or **3.5.3.7**. Zinc-coated (galvanized) welded deformed wire reinforcement shall be treated as welded plain wire reinforcement for development and splice design.

**3.5.3.11** — Deformed stainless-steel wire and deformed and plain stainless-steel welded wire for concrete reinforcement shall conform to ASTM A1022, except deformed wire shall not be smaller than size D-4 or larger than size D-31, and the yield strength for wire with  $f_y$  exceeding 60,000 psi shall be taken as the stress corresponding to a strain of 0.35 percent. Deformed wire larger than D-31 is permitted where used in welded wire reinforcement conforming to ASTM A1022, but shall be treated as plain wire for development and splice design. Spacing of welded intersections shall not exceed 12 in. for plain welded wire and 16 in. for deformed welded wire in direction of calculated stress, except for welded wire reinforcement used as stirrups in accordance with **12.13.2**.

#### **3.5.4 — Plain reinforcement**

#### **R3.5.4 — Plain reinforcement**

**3.5.4.1** — Plain bars for spiral reinforcement shall conform to ASTM A615, A706, A955, or A1035.

Plain bars and plain wire are permitted only for spiral reinforcement (either as transverse reinforcement for compression members, for torsion members, or for confining reinforcement for splices).

**3.5.4.2** — Plain wire for spiral reinforcement shall conform to ASTM A1064, except that for wire with  $f_y$  exceeding 60,000 psi, the yield strength shall be taken as the stress corresponding to a strain of 0.35 percent.

**CODE****COMMENTARY****3.5.5 — Headed shear stud reinforcement**

**3.5.5.1** — Headed studs and headed stud assemblies shall conform to ASTM A1044.

**R3.5.5 — Headed shear stud reinforcement**

The configuration of the studs for headed shear stud reinforcement differs from the configuration of the headed-type shear studs prescribed in Section 7 of AWS D1.1 and referenced for use in **Appendix D** of this Code (Fig. R3.5.5). Ratios of the head to shank cross-sectional areas of the AWS D1.1 studs range from about 2.5 to 4. In contrast, ASTM A1044 requires the area of the head of headed shear reinforcement studs to be at least 10 times the area of the shank. Thus, according to 3.5.5.1, the AWS D1.1 headed studs are not suitable for use as headed shear stud reinforcement. The base rail, where provided, anchors one end of the studs; ASTM A1044 specifies material width and thickness of the base rail that are sufficient to provide the required anchorage without yielding for stud shank diameters of 0.375, 0.500, 0.625, and 0.750 in. In ASTM A1044, the minimum specified yield strength of headed shear studs is 51,000 psi.

**3.5.6 — Prestressing steel**

**3.5.6.1** — Steel for prestressing shall conform to one of the following specifications:

- (a) Wire: ASTM A421;
- (b) Low-relaxation wire: ASTM A421, including Supplementary Requirement S1 "Low-Relaxation Wire and Relaxation Testing";
- (c) Strand: ASTM A416;
- (d) High-strength bar: ASTM A722.

**3.5.6.2** — Wire, strands, and bars not specifically listed in ASTM A421, A416, or A722 are allowed provided they conform to minimum requirements of these specifications and do not have properties that make them less satisfactory than those listed in ASTM A421, A416, or A722.

**3.5.7 — Structural steel, steel pipe, or tubing**

**3.5.7.1** — Structural steel used with reinforcing bars in composite compression members meeting requirements of **10.13.7** or **10.13.8** shall conform to one of the following specifications:

- (a) Carbon steel: ASTM A36;
- (b) High-strength low-alloy steel: ASTM A242;
- (c) High-strength, low-alloy, Columbium-Vanadium steel: ASTM A572;
- (d) High-strength, low-alloy, 50 ksi steel: ASTM A588;
- (e) Structural shapes: ASTM A992.

**R3.5.6 — Prestressing steel**

**R3.5.6.1** — Because low-relaxation prestressing steel is addressed in a supplementary requirement to ASTM A421, which applies only when low-relaxation material is specified, the appropriate ASTM reference is listed as a separate entity.

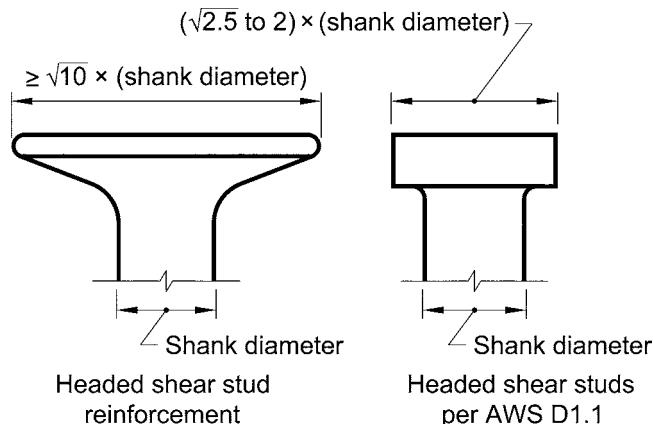


Fig. R3.5.5—Configurations of stud heads.

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**3.5.7.2** — Steel pipe or tubing for composite compression members composed of a steel encased concrete core meeting requirements of **10.13.6** shall conform to one of the following specifications:

- (a) Black steel, hot-dipped, zinc-coated: Grade B of ASTM A53;
- (b) Cold-formed, welded, seamless: ASTM A500;
- (c) Hot-formed, welded, seamless: ASTM A501.

**3.5.8** — Steel discontinuous fiber reinforcement for concrete shall be deformed and conform to ASTM A820. Steel fibers have a length-to-diameter ratio not smaller than 50 and not greater than 100.

**3.5.9** — Headed deformed bars shall conform to ASTM A970 including Annex A1 Requirements for Class HA Head Dimensions.

**3.6 — Admixtures**

**3.6.1** — Admixtures for water reduction and setting time modification shall conform to ASTM C494. Admixtures for use in producing flowing concrete shall conform to ASTM C1017.

**3.6.2** — Air-entraining admixtures shall conform to ASTM C260.

**3.6.3** — Admixtures to be used in concrete that do not conform to 3.6.1 and 3.6.2 shall be subject to prior approval by the licensed design professional.

**3.6.4** — Calcium chloride or admixtures containing chloride from sources other than impurities in admixture ingredients shall not be used in prestressed concrete, in concrete containing embedded aluminum, or in concrete cast against stay-in-place galvanized steel forms. See **4.3.1** and **6.3.2**.

**R3.5.8** — Deformations in steel fibers enhance mechanical anchorage with the concrete. The lower and upper limits for the fiber length-to-diameter ratio are based on available test data.<sup>3,6</sup> Because data are not available on the potential for corrosion problems due to galvanic action, the use of deformed steel fibers in members reinforced with stainless-steel bars or galvanized steel bars is not recommended.

**R3.5.9** — The limitation to Class HA head dimensions from Annex A1 of ASTM A970 is due to a lack of test data for headed deformed bars that do not meet Class HA dimensional requirements. Heads not conforming to Class HA limits on bar deformation obstructions and bearing face features could cause unintended splitting forces in the concrete that may not be characteristic of the heads used in the tests that were the basis for **12.6.1** and **12.6.2**. For heads conforming to Class HA dimensional requirements, the net bearing area of the head can be assumed to be equal to the gross area of the head minus the area of the bar. This assumption may not be valid for heads not conforming to Class HA dimensional requirements.

**R3.6 — Admixtures**

**R3.6.4** — Admixtures containing any chloride, other than impurities from admixture ingredients, should not be used in prestressed concrete or in concrete with aluminum embedments. Concentrations of chloride ion may produce corrosion of embedded aluminum (e.g., conduit), especially if the aluminum is in contact with embedded steel and the concrete is in a humid environment. Corrosion of galvanized steel sheet and galvanized steel stay-in-place forms

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**3.6.5** — Admixtures used in concrete containing expansive cements conforming to ASTM C845 shall be compatible with the cement and produce no deleterious effects.

occurs, especially in humid environments or where drying is inhibited by the thickness of the concrete or coatings or impermeable coverings. See 4.3.1 for specific limits on chloride ion concentration in concrete. See 6.3.2 for requirements of embedded aluminum.

**3.7 — Storage of materials**

**3.7.1** — Cementitious materials and aggregates shall be stored in such manner as to prevent deterioration or intrusion of foreign matter.

**R3.6.5** — The use of admixtures in concrete containing ASTM C845 expansive cements has resulted in reduced levels of expansion or increased shrinkage values. See ACI 223.<sup>3.7</sup>

**3.7.2** — Any material that has deteriorated or has been contaminated shall not be used for concrete.

**3.8 — Referenced standards**

**3.8.1** — Standards of ASTM International referred to in this Code are listed below with their serial designations, including year of adoption or revision, and are declared to be part of this Code as if fully set forth herein:

A36/A36M-08	Standard Specification for Carbon Structural Steel
A53/A53M-10	Standard Specification for Pipe, Steel, Black and Hot-Dipped, Zinc-Coated, Welded and Seamless
A184/A184M-06	Standard Specification for Welded Deformed Steel Bar Mats for Concrete Reinforcement
A242/A242M-04 (2009)	Standard Specification for High-Strength Low-Alloy Structural Steel
A307-10	Standard Specification for Carbon Steel Bolts and Studs, 60,000 psi Tensile Strength
A416/A416M-10	Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete
A421/A421M-10	Standard Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete

**R3.8 — Referenced standards**

ASTM standards are available from ASTM International.

The ASTM standards listed are the latest editions at the time these code provisions were adopted. Because these standards are revised frequently, generally in minor details only, the user of the Code should check directly with ASTM International ([www.astm.org](http://www.astm.org)) if it is desired to reference the latest edition. However, such a procedure obligates the user of the standard to evaluate if any changes in the later edition are significant in the use of the standard.

Many of the ASTM standards are combined standards as denoted by the dual designation, such as ASTM A36/A36M. For simplicity, these combined standards are referenced without the metric (M) designation within the text of the Code and Commentary. In 3.8, however, the complete designation is given because that is the official designation for the standard.

Standard specifications or other material to be legally adopted by reference into a building code should refer to a specific document. This can be done by simply using the complete serial designation since the first part indicates the subject and the second part the year of adoption. All standard documents referenced in this Code are listed in 3.8, with the title and complete serial designation. In other sections of the code, the designations do not include the date so that all may be kept up-to-date by simply revising 3.8.

Type R rail-steel bars are considered a mandatory requirement whenever ASTM A996 is referenced in the Code.

<b>CODE</b>	<b>COMMENTARY</b>
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A500/A500M-10a	Standard Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes
A501-07	Standard Specification for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing
A572/A572M-07	Standard Specification for High-Strength Low-Alloy Columbium-Vanadium Structural Steel
A588/A588M-10	Standard Specification for High-Strength Low-Alloy Structural Steel, up to 50 ksi [345 MPa] Minimum Yield Point, with Atmospheric Corrosion Resistance
A615/A615M-09b	Standard Specification for Deformed and Plain Carbon Steel Bars for Concrete Reinforcement
A706/A706M-09b	Standard Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement
A722/A722M-07	Standard Specification for Uncoated High-Strength Steel Bars for Prestressing Concrete
A767/A767M-09	Standard Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement
A775/A775M-07b	Standard Specification for Epoxy-Coated Steel Reinforcing Bars
A820/A820M-06	Standard Specification for Steel Fibers for Fiber-Reinforced Concrete
A884/A884M-06	Standard Specification for Epoxy-Coated Steel Wire and Welded Wire Reinforcement
A934/A934M-07	Standard Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars
A955/A955M-10a	Standard Specification for Deformed and Plain Stainless-Steel Bars for Concrete Reinforcement

<b>CODE</b>	<b>COMMENTARY</b>
A970/A970M-09	Standard Specification for Headed Steel Bars for Concrete Reinforcement including Annex A1 Requirements for Class HA Head Dimensions
A992/A992M-06a	Standard Specification for Structural Steel Shapes
A996/A996M-09b	Standard Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement
A1022/A1022M-07	Standard Specification for Deformed and Plain Stainless Steel Wire and Welded Wire for Concrete Reinforcement
A1035/A1035M-09	Standard Specification for Deformed and Plain, Low-Carbon, Chromium, Steel Bars for Concrete Reinforcement
A1044/A1044M-05 (2010)	Standard Specification for Steel Stud Assemblies for Shear Reinforcement of Concrete
A1055/A1055M-10	Standard Specification for Zinc and Epoxy Dual-Coated Steel Reinforcing Bars
A1060/A1060M-10	Standard Specification for Zinc-Coated (Galvanized) Steel Welded Wire Reinforcement, Plain and Deformed, for Concrete
A1064/A1064M-10	Standard Specification for Steel Wire and Welded Wire Reinforcement, Plain and Deformed, for Concrete
C29/C29M-09	Standard Test Method for Bulk Density ("Unit Weight") and Voids in Aggregate
C31/C31M-09	Standard Practice for Making and Curing Concrete Test Specimens in the Field
C33/C33M-08	Standard Specification for Concrete Aggregates
C39/C39M-09a	Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens

<b>CODE</b>	<b>COMMENTARY</b>
C42/C42M-04	Standard Test Method for Obtaining and Testing Drilled Cores and Sawed Beams of Concrete
C94/C94M-09a	Standard Specification for Ready-Mixed Concrete
C109/C109M-08	Standard Test Method for Compressive Strength of Hydraulic Cement Mortars (Using 2-in. or [50-mm] Cube Specimens)
C144-04	Standard Specification for Aggregate for Masonry Mortar
C150/C150M-09	Standard Specification for Portland Cement
C172-08	Standard Practice for Sampling Freshly Mixed Concrete
C192/C192M-07	Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory
C231/C231M-09b	Standard Test Method for Air Content of Freshly Mixed Concrete by the Pressure Method
C260-06	Standard Specification for Air-Entraining Admixtures for Concrete
C330/C330M-09	Standard Specification for Lightweight Aggregates for Structural Concrete
C494/C494M-10	Standard Specification for Chemical Admixtures for Concrete
C496/C496M-04 <sup>1</sup>	Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens
C567-05a	Standard Test Method for Determining Density of Structural Lightweight Concrete
C595/C595M-09	Standard Specification for Blended Hydraulic Cements
C618-08a	Standard Specification for Coal Fly Ash and Raw or Calcined Natural Pozzolan for Use in Concrete

<b>CODE</b>	<b>COMMENTARY</b>
C685/C685M-10	Standard Specification for Concrete Made by Volumetric Batching and Continuous Mixing
C845-04	Standard Specification for Expansive Hydraulic Cement
C989-09a	Standard Specification for Slag Cement for Use in Concrete and Mortars
C1012/C1012M-09	Standard Test Method for Length Change of Hydraulic-Cement Mortars Exposed to a Sulfate Solution
C1017/C1017M-07	Standard Specification for Chemical Admixtures for Use in Producing Flowing Concrete
C1077-10	Standard Practice for Laboratories Testing Concrete and Concrete Aggregates for Use in Construction and Criteria for Laboratory Evaluation
C1116/C1116M-09	Standard Specification for Fiber-Reinforced Concrete
C1157/C1157M-09	Standard Performance Specification for Hydraulic Cement
C1218/C1218M-99 (2008)	Standard Test Method for Water-Soluble Chloride in Mortar and Concrete
C1240-05	Standard Specification for Silica Fume Used in Cementitious Mixtures
C1580-09	Standard Test for Water-Soluble Sulfate in Soil
C1602/C1602M-06	Standard Specification for Mixing Water Used in the Production of Hydraulic Cement Concrete
C1609/C1609M-06	Standard Test Method for Flexural Performance of Fiber-Reinforced Concrete (Using Beam With Third-Point Loading)
D516-07	Standard Test Method for Sulfate Ion in Water
D4130-08	Standard Test Method for Sulfate Ion in Brackish Water, Seawater, and Brine

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**3.8.2** — “Structural Welding Code—Reinforcing Steel (AWS D1.4/D1.4M:2011)” of the American Welding Society is declared to be part of this Code as if fully set forth herein.

**R3.8.2** — “Structural Welding Code—Reinforcing Steel (AWS D1.4/D1.4M:2011)” is available from the American Welding Society.

**3.8.3** — Section 2.3.3, Load Combinations Including Flood Loads, and 2.3.4, Load Combinations Including Atmospheric Ice Loads, of “Minimum Design Loads for Buildings and Other Structures” (ASCE/SEI 7-10) is declared to be part of this Code as if fully set forth herein, for the purpose cited in **9.2.6**.

**R3.8.3** — ASCE/SEI 7-10 is available from ASCE.

**3.8.4** — “Specification for Unbonded Single-Strand Tendon Materials (ACI 423.7-07)” is declared to be part of this Code as if fully set forth herein.

**R3.8.5** — The 2002 17th Edition of the AASHTO “Standard Specification for Highway Bridges” is available from AASHTO.

**3.8.5** — Articles 9.21.7.2 and 9.21.7.3 of Division I and Article 10.3.2.3 of Division II of AASHTO “Standard Specification for Highway Bridges” (AASHTO 17th Edition, 2002) are declared to be a part of this Code as if fully set forth herein, for the purpose cited in **18.15.1**.

**R3.8.6** — ACI 355.2 contains requirements for the testing and evaluation of post-installed expansion and undercut anchors for use in both cracked and uncracked concrete.

**3.8.6** — “Qualification of Post-Installed Mechanical Anchors in Concrete (ACI 355.2-07)” is declared to be part of this Code as if fully set forth herein, for the purpose cited in **Appendix D**.

**R3.8.7** — ACI 355.4 contains requirements for the testing and evaluation of adhesive anchors for use in both cracked and uncracked concrete.

**3.8.7** — “Qualification of Post-Installed Adhesive Anchors in Concrete (ACI 355.4-11)” is declared to be a part of this Code as if fully set forth herein, for the purposes cited in **Appendix D**.

**R3.8.8** — “Structural Welding Code—Steel (AWS D1.1/D1.1M:2010)” is available from the American Welding Society.

**3.8.8** — “Structural Welding Code—Steel (AWS D1.1/D1.1M:2010)” of the American Welding Society is declared to be part of this Code as if fully set forth herein.

**3.8.9** — “Acceptance Criteria for Moment Frames Based on Structural Testing (ACI 374.1-05)” is declared to be part of this Code as if fully set forth herein.

**3.8.10** — “Acceptance Criteria for Special Unbonded Post-Tensioned Precast Structural Walls Based on Validation Testing (ACI ITG-5.1-07)” is declared to be part of this Code as if fully set forth herein.

# CHAPTER 4 — DURABILITY REQUIREMENTS

## CODE

### 4.1 — General

**4.1.1** —The value of  $f'_c$  shall be the greatest of the values required by (a) **1.1.1**, (b) for durability in Chapter 4, and (c) for structural strength requirements and shall apply for mixture proportioning in **5.3** and for evaluation and acceptance of concrete in **5.6**. Concrete mixtures shall be proportioned to comply with the maximum water-cementitious material ratio ( $w/cm$ ) and other requirements based on the exposure class assigned to the concrete structural member. All cementitious materials specified in **3.2.1** and the combinations of these materials shall be included in calculating the  $w/cm$  of the concrete mixture.

**4.1.2** — The maximum  $w/cm$  limits in Chapter 4 do not apply to lightweight concrete.

### 4.2 — Exposure categories and classes

## COMMENTARY

### R4.1 — General

Chapters 4 and 5 of earlier editions of the Code were reformatted in 1989 to emphasize the importance of considering durability requirements before selecting  $f'_c$  and concrete cover over the reinforcing steel. In 2008, the format of Chapter 4 was revised extensively by introducing exposure categories and classes with applicable durability requirements for concrete in a unified format.

**R4.1.1** — Maximum water-cementitious material ratios ( $w/cm$ ) of 0.40 to 0.50 that may be required for concretes exposed to freezing and thawing, sulfate soils or waters, or for corrosion protection of reinforcement will typically be equivalent to requiring an  $f'_c$  of 5000 to 4000 psi, respectively. Generally, the required average compressive strengths,  $f'_{cr}$ , will be 500 to 700 psi higher than the specified compressive strength,  $f'_c$ . Because it is difficult to accurately determine the  $w/cm$  of concrete, the  $f'_c$  specified should be reasonably consistent with the  $w/cm$  required for durability. Selection of an  $f'_c$  that is consistent with the maximum permitted  $w/cm$  for durability will help ensure that the maximum  $w/cm$  is not exceeded in the field. For example, a maximum  $w/cm$  of 0.45 and  $f'_c$  of 3000 psi should not be specified for the same concrete mixture. Because the usual emphasis during inspection is on concrete compressive strength, test results substantially higher than the specified compressive strength may lead to a lack of concern for quality and could result in production and delivery of concrete that exceeds the maximum  $w/cm$ .

**R4.1.2** — Maximum  $w/cm$  is not specified for lightweight concrete because of the uncertainty in determining the amount of mixing water that is absorbed by lightweight aggregates before concrete sets. This makes the calculation of the  $w/cm$  uncertain. The use of a minimum specified compressive strength,  $f'_c$ , will ensure the use of a high-quality cement paste. For normalweight concrete, the use of both minimum strength and maximum  $w/cm$  provide additional assurance that the paste is of high quality.

### R4.2 — Exposure categories and classes

Exposure categories defined in **Table 4.2.1** are sub-divided into exposure classes depending on the severity of the exposure. Associated requirements for concrete relative to the exposure classes are provided in **4.3**.

The Code does not include provisions for especially severe exposures, such as acids or high temperatures, and is not concerned with aesthetic considerations such as surface

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## COMMENTARY

**4.2.1** — The licensed design professional shall assign exposure classes based on the severity of the anticipated exposure of structural concrete members for each exposure category according to Table 4.2.1.

**TABLE 4.2.1 — EXPOSURE CATEGORIES AND CLASSES**

Category	Severity	Class	Condition	
<b>F</b> Freezing and thawing	Not applicable	F0	Concrete not exposed to freezing-and-thawing cycles	
	Moderate	F1	Concrete exposed to freezing-and-thawing cycles and occasional exposure to moisture	
	Severe	F2	Concrete exposed to freezing-and-thawing cycles and in continuous contact with moisture	
	Very severe	F3	Concrete exposed to freezing-and-thawing and in continuous contact with moisture and exposed to deicing chemicals	
<b>S</b> Sulfate			<b>Water-soluble sulfate (<math>\text{SO}_4</math>) in soil, percent by mass*</b>	<b>Dissolved sulfate (<math>\text{SO}_4</math>) in water, ppm†</b>
	Not applicable	S0	$\text{SO}_4 < 0.10$	$\text{SO}_4 < 150$
	Moderate	S1	$0.10 \leq \text{SO}_4 < 0.20$	$150 \leq \text{SO}_4 < 1500$ Seawater
	Severe	S2	$0.20 \leq \text{SO}_4 \leq 2.00$	$1500 \leq \text{SO}_4 \leq 10,000$
	Very severe	S3	$\text{SO}_4 > 2.00$	$\text{SO}_4 > 10,000$
<b>P</b> Requiring low permeability	Not applicable	P0	In contact with water where low permeability is not required	
	Required	P1	In contact with water where low permeability is required.	
<b>C</b> Corrosion protection of reinforcement	Not applicable	C0	Concrete dry or protected from moisture	
	Moderate	C1	Concrete exposed to moisture but not to external sources of chlorides	
	Severe	C2	Concrete exposed to moisture and an external source of chlorides from deicing chemicals, salt, brackish water, seawater, or spray from these sources	

\*Percent sulfate by mass in soil shall be determined by ASTM C1580.

†Concentration of dissolved sulfates in water in ppm shall be determined by ASTM D516 or ASTM D4130.

finishes. These items are beyond the scope of the Code and should be covered specifically in the project specifications. Concrete ingredients and proportions are to be selected to meet the minimum requirements stated in the Code and the additional requirements of contract documents.

**R4.2.1** — The Code addresses four exposure categories that affect the requirements for concrete to ensure adequate durability:

**Exposure Category F** applies to exterior concrete that is exposed to moisture and cycles of freezing and thawing, with or without deicing chemicals.

**Exposure Category S** applies to concrete in contact with soil or water containing deleterious amounts of water-soluble sulfate ions as defined in Table 4.2.1.

**Exposure Category P** applies to concrete in contact with water requiring low permeability.

**Exposure Category C** applies to reinforced and prestressed concrete exposed to conditions that require additional protection against corrosion of reinforcement.

Severity of exposure within each category is defined by classes with increasing numerical values representing increasingly severe exposure conditions. A classification of "0" is assigned when the exposure severity has negligible effect or does not apply to the structural member.

**Exposure Category F** is subdivided into four exposure classes: **Exposure Class F0** is assigned to concrete that will not be exposed to cycles of freezing and thawing. **Exposure Class F1** is assigned to concrete exposed to cycles of freezing and thawing and that will be occasionally exposed to moisture before freezing. Examples of Class F1 are exterior walls, beams, girders, and slabs not in direct contact with soil. **Exposure Class F2** is assigned to concrete exposed to cycles of freezing and thawing that is in continuous contact with moisture before freezing. An example is an exterior water tank or vertical members in contact with soil. Exposure Classes F1 and F2 are conditions where exposure to deicing salt is not anticipated. **Exposure Class F3** is assigned to concrete exposed to cycles of freezing and thawing, in continuous contact with moisture, and where exposure to deicing chemicals is anticipated. Examples are horizontal members in parking structures.

**Exposure Category S** is subdivided into four exposure classes: **Exposure Class S0** is assigned for conditions where the water-soluble sulfate concentration in contact with concrete is low and injurious sulfate attack is not a concern. **Exposure Classes S1, S2, and S3** are assigned for structural concrete members in direct contact with soluble sulfates in soil or water. The severity of exposure increases

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from Exposure Class S1 to S3 based on the more critical value of measured water-soluble sulfate concentration in soil or the concentration of dissolved sulfate in water. Sea water exposure is classified as Exposure Class S1.

**Exposure Category P** is subdivided into two exposure classes: Structural members should be assigned to **Exposure Class P0** when there are no specific permeability requirements. **Exposure Class P1** is assigned on the basis of the need for concrete to have a low permeability to water when the permeation of water into concrete might reduce durability or affect the intended function of the structural member. Exposure Class P1 should typically be assigned when other exposure classes do not apply. An example is an interior water tank.

**Exposure Category C** is subdivided into three exposure classes: **Exposure Class C0** is assigned when exposure conditions do not require additional protection against the initiation of corrosion of reinforcement. **Exposure Classes C1 and C2** are assigned to reinforced and prestressed concrete members depending on the degree of exposure to external sources of moisture and chlorides in service. Examples of external sources of chlorides include concrete in direct contact with deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources.

**4.3 — Requirements for concrete mixtures**

**4.3.1** — Based on the exposure classes assigned from **Table 4.2.1**, concrete mixtures shall comply with the most restrictive requirements according to **Table 4.3.1**.

**R4.3 — Requirements for concrete mixtures**

**R4.3.1** — **Table 4.3.1** gives the requirements for concrete on the basis of the assigned exposure classes. When a structural concrete member is assigned more than one exposure class, the most restrictive requirements are applicable. For example, a prestressed concrete member assigned to Exposure Class C2 and Exposure Class F3 would require concrete to comply with a maximum *w/cm* of 0.40 and minimum  $f'_c$  of 5000 psi, respectively. In this case, the requirement for corrosion protection is more restrictive than the requirement for resistance to freezing and thawing.

**Exposure Classes F1, F2, and F3:** In addition to complying with a maximum *w/cm* limit and a minimum strength requirement, concrete for structural members subject to freezing-and-thawing exposures should be air entrained in accordance with **4.4.1**. Structural members assigned to Exposure Class F3 are additionally required to comply with the limitations on the quantity of pozzolans and slag in the composition of the cementitious materials as given in **4.4.2**.

**Exposure Classes S1, S2, and S3:** Concrete exposed to injurious concentrations of sulfates from soil and water should be made with sulfate-resisting cement. **Table 4.3.1** lists the appropriate types of cement and the maximum *w/cm* and minimum specified compressive strengths for various exposure conditions. In selecting cement for sulfate resistance, the principal consideration is its tricalcium aluminate ( $C_3A$ )

**CODE****COMMENTARY****TABLE 4.3.1 — REQUIREMENTS FOR CONCRETE BY EXPOSURE CLASS**

Exposure Class	Max. w/cm*	Min. $f'_c$ , psi	Additional minimum requirements		
			Air content		
F0	N/A	2500	N/A		N/A
F1	0.45	4500	Table 4.4.1		N/A
F2	0.45	4500	Table 4.4.1		N/A
F3	0.45	4500	Table 4.4.1		Table 4.4.2
			Cementitious materials <sup>†</sup> —types		Calcium chloride admixture
			ASTM C150	ASTM C595	ASTM C1157
S0	N/A	2500	No Type restriction	No Type restriction	No Type restriction
S1	0.50	4000	II <sup>‡</sup>	IP(MS), IS (<70) (MS)	MS
S2	0.45	4500	V <sup>§</sup>	IP (HS) IS (<70) (HS)	HS
S3	0.45	4500	V + pozzolan or slag <sup>  </sup>	IP (HS) + pozzolan or slag <sup>  </sup> or IS (<70) (HS) + pozzolan or slag <sup>  </sup>	HS + pozzolan or slag <sup>  </sup>
P0	N/A	2500	None		
P1	0.50	4000	None		
			Maximum water-soluble chloride ion (Cl <sup>-</sup> ) content in concrete, percent by weight of cement <sup>#</sup>		Related provisions
			Reinforced concrete	Prestressed concrete	
C0	N/A	2500	1.00	0.06	None
C1	N/A	2500	0.30	0.06	
C2	0.40	5000	0.15	0.06	

\*For lightweight concrete, see 4.1.2.  
†Alternative combinations of cementitious materials of those listed in Table 4.3.1 shall be permitted when tested for sulfate resistance and meeting the criteria in 4.5.1.  
‡For seawater exposure, other types of portland cements with tricalcium aluminate (C<sub>3</sub>A) contents up to 10 percent are permitted if the w/cm does not exceed 0.40.  
§Other available types of cement such as Type III or Type I are permitted in Exposure Classes S1 or S2 if the C<sub>3</sub>A contents are less than 8 or 5 percent, respectively.  
||The amount of the specific source of the pozzolan or slag to be used shall not be less than the amount that has been determined by service record to improve sulfate resistance when used in concrete containing Type V cement. Alternatively, the amount of the specific source of the pozzolan or slag to be used shall not be less than the amount tested in accordance with ASTM C1012 and meeting the criteria in 4.5.1.  
#Water-soluble chloride ion content that is contributed from the ingredients including water, aggregates, cementitious materials, and admixtures shall be determined on the concrete mixture by ASTM C1218 at age between 28 and 42 days.

Requirements of 7.7.6 shall be satisfied. See 18.16 for unbonded tendons.

content. For Exposure Class S1 (moderate exposure), Type II cement is limited to a maximum C<sub>3</sub>A content of 8.0 percent under ASTM C150. The blended cements under ASTM C595 with the MS designation are appropriate for use in Exposure Class S1. The appropriate types under ASTM C595 are IP(MS) and IS(<70)(MS) and under C1157 is Type MS. For Exposure Class S2 (severe exposure), Type V cement with a maximum C<sub>3</sub>A content of 5 percent is specified. Blended cements Types IP (HS) and IS (<70) (HS) under ASTM C595 and Type HS under ASTM C1157 can also be used. In certain areas, the C<sub>3</sub>A content of other available types such as Type III or Type I may be less than 8 or 5 percent and are usable in moderate or severe sulfate exposures. Note that sulfate-resisting cement will not increase resistance to some chemically aggressive solutions, for example, sulfuric acid. The project specifications should cover all special cases.

The use of fly ash (ASTM C618, Class F), natural pozzolans (ASTM C618, Class N), silica fume (ASTM C1240), or ground-granulated blast-furnace slag (ASTM C989) also has been shown to improve the sulfate resistance of concrete.<sup>4.1-4.3</sup> ASTM C1012 can be used to evaluate the sulfate resistance of mixtures using combinations of cementitious materials as determined in 4.5.1. For Exposure Class S3, the alternative in ACI 318-05 allowing use of Type V plus pozzolan, based on records of successful service, instead of meeting the testing requirements of 4.5.1, still exists and has been expanded to consider the use of slag and the blended cements.

Table 4.2.1 lists seawater under Exposure Class S1 (moderate exposure), even though it generally contains more than 1500 ppm SO<sub>4</sub>. Portland cement with higher C<sub>3</sub>A content improves binding of chlorides present in seawater and the Code permits other types of portland cement with C<sub>3</sub>A up to 10 percent if the maximum w/cm is reduced to 0.40.

In addition to the proper selection of cementitious materials, other requirements for durable concrete exposed to water-soluble sulfate are essential, such as low w/cm, strength, adequate air entrainment, adequate consolidation, uniformity, adequate cover of reinforcement, and sufficient moist curing to develop the potential properties of the concrete.

**Exposure Class P1:** The Code includes an Exposure Class P1 for concrete that needs to have a low permeability when in direct contact with water and where the other exposure conditions defined in Table 4.2.1 do not apply. The primary means to obtain low permeability is to use a low w/cm. Low permeability can be also achieved by optimizing the cementitious materials used in the concrete mixture. One standard method that provides a performance-based indicator of low permeability of concrete is ASTM C1202, which is more reliable in laboratory evaluations than for field-based acceptance.

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**Exposure Class C2:** For reinforced and prestressed concrete in Exposure Class C2, the maximum  $w/cm$ , minimum specified compressive strength, and minimum cover are the basic requirements to be considered. Conditions in structures where chlorides may be applied should be evaluated, such as in parking structures where chlorides may be tracked in by vehicles, or in structures near seawater. Epoxy- or zinc-coated bars or cover greater than the minimum required in 7.7 may be desirable. Use of slag meeting ASTM C989 or fly ash meeting ASTM C618 and increased levels of specified compressive strength provide increased protection. Use of silica fume meeting ASTM C1240 with an appropriate high-range water reducer, ASTM C494, Types F and G, or ASTM C1017 can also provide additional protection.<sup>4,4</sup> The use of ASTM C1202<sup>4,5</sup> to test concrete mixtures proposed for use will provide additional information on the performance of the mixtures.

**Exposure Classes C0, C1, and C2:** For Exposure Classes C0, C1, and C2, the chloride ion limits apply. For reinforced concrete, the permitted maximum amount of water-soluble chloride ions incorporated into the concrete, measured by ASTM C1218 at ages between 28 and 42 days, depend on the degree of exposure to an anticipated external source of moisture and chlorides. For prestressed concrete, the same limit of 0.06 percent chloride ion by weight of cement applies regardless of exposure.

Additional information on the effects of chlorides on the corrosion of reinforcing steel is given in ACI 201.2R,<sup>4,6</sup> which provides guidance on concrete durability, and ACI 222R,<sup>4,7</sup> which provides guidance on factors that impact corrosion of metals in concrete. An initial evaluation of the chloride ion content of the proposed concrete mixture may be obtained by testing individual concrete ingredients for total chloride ion content. If total chloride ion content, calculated on the basis of concrete proportions, exceeds those permitted in Table 4.3.1, it may be necessary to test samples of the hardened concrete for water-soluble chloride ion content. Some of the chloride ions present in the ingredients will either be insoluble in water or will react with the cement during hydration and become insoluble under the test procedures described in ASTM C1218.

When concretes are tested for water-soluble chloride ion content, the tests should be made at an age of 28 to 42 days. The limits in Table 4.3.1 are to be applied to chlorides contributed from the concrete ingredients, not those from the environment surrounding the concrete. For reinforced concrete that will be dry in service (Exposure Class C0), a limit of 1 percent has been included to control the water-soluble chlorides introduced by concrete-making materials. Table 4.3.1 includes limits of 0.30 and 0.15 percent for reinforced concrete subject to Exposure Classes C1 and C2, respectively.

**CODE****COMMENTARY****Table R4.3.1 — Chloride limits for new construction  
(adapted from Table 3.1 of ACI 222R<sup>4,7</sup>)**

Construction type and condition	Chloride limit, percent by mass		
	Test method		
	Acid soluble	Water soluble	ASTM C1152
Prestressed concrete	0.08	0.06	0.06
Reinforced concrete wet in service	0.10	0.08	0.08
Reinforced concrete dry in service	0.20	0.15	0.15

\*The Soxhlet test method is described in ACI 222.1.<sup>4,8</sup>

In **Table 4.2.1**, Exposure Classes C1 and C0 are similar to the categories for reinforced concrete under wet and dry conditions in service as described in ACI 222R. The recommended limit for prestressed concrete in this Code is same as in ACI 222R.

When epoxy- or zinc-coated bars are used, the limits in **Table 4.3.1** may be more restrictive than necessary.

**4.4 — Additional requirements for freezing-and-thawing exposure**

**4.4.1** — Normalweight and lightweight concrete subject to Exposure Classes F1, F2, or F3 shall be air-entrained with air content indicated in Table 4.4.1. Tolerance on air content as delivered shall be  $\pm 1.5$  percent. For  $f'_c$  greater than 5000 psi, reduction of air content indicated in Table 4.4.1 by 1.0 percent shall be permitted.

**TABLE 4.4.1 — TOTAL AIR CONTENT FOR CONCRETE EXPOSED TO CYCLES OF FREEZING AND THAWING**

Nominal maximum aggregate size, in.*	Air content, percent	
	Exposure Class F1	Exposure Classes F2 and F3
3/8	6	7.5
1/2	5.5	7
3/4	5	6
1	4.5	6
1-1/2	4.5	5.5
2†	4	5
3†	3.5	4.5

\*See ASTM C33 for tolerance on oversize for various nominal maximum size designations.

†Air contents apply to total mixture. When testing concretes, however, aggregate particles larger than 1-1/2 in. are removed by sieving and air content is measured on the sieved fraction (tolerance on air content as delivered applies to this value). Air content of total mixture is computed from value measured on the sieved fraction passing the 1-1/2 in. sieve in accordance with ASTM C231.

**R4.4 — Additional requirements for freezing-and-thawing exposure**

**R4.4.1** — A table of required air contents for concrete to resist damage from cycles of freezing and thawing is included in the Code, based on guidance provided for proportioning concrete mixtures in ACI 211.1.<sup>4,9</sup> Target values are provided for Exposure Class F1 (moderate) and both Exposure Classes F2 and F3 (severe) exposures depending on the exposure to moisture or deicing salts. Entrained air will not protect concrete containing coarse aggregates that undergo disruptive volume changes when frozen in a saturated condition.

Section 4.4.1 permits 1 percent lower air content for concrete with  $f'_c$  greater than 5000 psi. Such high-strength concretes will have a lower  $w/cm$  and porosity and, therefore, improved resistance to cycles of freezing and thawing.

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**4.4.2** —The quantity of pozzolans, including fly ash and silica fume, and slag in concrete subject to Exposure Class F3, shall not exceed the limits in Table 4.4.2.

**TABLE 4.4.2 — REQUIREMENTS FOR CONCRETE SUBJECT TO EXPOSURE CLASS F3**

Cementitious materials	Maximum percent of total cementitious materials by weight*
Fly ash or other pozzolans conforming to ASTM C618	25
Slag conforming to ASTM C989	50
Silica fume conforming to ASTM C1240	10
Total of fly ash or other pozzolans, slag, and silica fume	50†
Total of fly ash or other pozzolans and silica fume	35†

\*The total cementitious material also includes ASTM C150, C595, C845, and C1157 cement.  
The maximum percentages above shall include:  
(a) Fly ash or other pozzolans in Type IP, blended cement, ASTM C595, or ASTM C1157;  
(b) Slag used in the manufacture of an IS blended cement, ASTM C595, or ASTM C1157;  
(c) Silica fume, ASTM C1240, present in a blended cement.  
†Fly ash or other pozzolans and silica fume shall constitute no more than 25 and 10 percent, respectively, of the total weight of the cementitious materials.

## 4.5 — Alternative cementitious materials for sulfate exposure

**4.5.1** — Alternative combinations of cementitious materials to those listed in **Table 4.3.1** shall be permitted when tested for sulfate resistance and meeting the criteria in Table 4.5.1.

**TABLE 4.5.1 — REQUIREMENTS FOR ESTABLISHING SUITABILITY OF CEMENTITIOUS MATERIALS COMBINATIONS EXPOSED TO WATER-SOLUBLE SULFATE**

Exposure Class	Maximum expansion when tested using ASTM C1012		
	At 6 months	At 12 months	At 18 months
S1	0.10 percent		
S2	0.05 percent	0.10 percent*	
S3			0.10 percent

\*The 12-month expansion limit applies only when the measured expansion exceeds the 6-month maximum expansion limit.

**R4.4.2** — Table 4.4.2 establishes limitations on the amount of fly ash, other pozzolans, silica fume, and slag that can be included in concrete exposed to deicing chemicals (Exposure Class F3) based on research studies.<sup>4.10,4.11</sup>

## 4.5 — Alternative cementitious materials for sulfate exposure

**R4.5.1** — In the 2008 version of the Code, ASTM C1012 is permitted to be used to evaluate the sulfate resistance of concrete mixtures using alternative combinations of cementitious materials to those listed in **Table 4.3.1** for all classes of sulfate exposure. More detailed guidance on qualification of such mixtures using ASTM C1012 is given in ACI 201.2R.<sup>4.6</sup> The expansion criteria in Table 4.5.1, for testing according to ASTM C1012, are the same as those in ASTM C595 for moderate sulfate resistance (Optional Designation MS) in Exposure Class S1 and for high sulfate resistance (Optional Designation HS) in Exposure Class S2, and the same as in ASTM C1157 for Type MS in Exposure Class S1 and Type HS in Exposure Class S2.

ACI 222R<sup>4.7</sup> has adopted chloride limits, test methods, and construction types and conditions that are slightly different from those in ACI 318, as shown in **Table R4.3.1**. ACI 201.2R<sup>4.6</sup> has adopted these same limits by referring to ACI 222R.

**Notes**

# CHAPTER 5 — CONCRETE QUALITY, MIXING, AND PLACING

## CODE

### 5.1 — General

**5.1.1** — Concrete shall be proportioned to provide an average compressive strength,  $f'_{cr}$ , as prescribed in **5.3.2** and shall satisfy the durability criteria of **Chapter 4**. Concrete shall be produced to minimize the frequency of strength tests below  $f'_c$ , as prescribed in **5.6.3.3**. For concrete designed and constructed in accordance with the Code,  $f'_c$  shall not be less than 2500 psi.

**5.1.2** — Requirements for  $f'_c$  shall be based on tests of cylinders made and tested as prescribed in **5.6.3**.

**5.1.3** — Unless otherwise specified,  $f'_c$  shall be based on 28-day tests. If other than 28 days, test age for  $f'_c$  shall be as indicated in contract documents.

**5.1.4** — Where design criteria in **8.6.1** and **12.2.4(d)**, and **22.2.4** provide for use of a splitting tensile strength value of concrete,  $f'_{ct}$ , laboratory tests shall be made in accordance with ASTM C330 to establish a value of  $f'_{ct}$  corresponding to  $f'_c$ .

**5.1.5** — Splitting tensile strength tests shall not be used as a basis for field acceptance of concrete.

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### R5.1 — General

The requirements for proportioning concrete mixtures are based on the philosophy that concrete should provide both adequate durability (**Chapter 4**) and strength. The criteria for acceptance of concrete are based on the philosophy that the Code is intended primarily to protect the safety of the public. Chapter 5 describes procedures by which concrete of adequate strength can be obtained, and provides procedures for checking the quality of the concrete during and after its placement in the Work.

Chapter 5 also prescribes minimum criteria for mixing and placing concrete.

The provisions of **5.2**, **5.3**, and **5.4**, together with **Chapter 4**, establish required mixture proportions. The basis for determining the adequacy of concrete strength is in **5.6**.

**R5.1.1** — The basic premises governing the designation and evaluation of concrete strength are presented. It is emphasized that the average compressive strength of concrete produced should always exceed the specified value of  $f'_c$  used in the structural design calculations. This is based on probabilistic concepts, and is intended to ensure that adequate concrete strength will be developed in the structure. The durability requirements prescribed in **Chapter 4** are to be satisfied in addition to attaining the average concrete strength in accordance with **5.3.2**.

**R5.1.4** — Equations throughout the code that contain the  $\sqrt{f'_c}$  term have been modified, as necessary, for use with lightweight concrete. Two alternative modification procedures are provided. One alternative is based on laboratory tests to determine the relationship between average splitting tensile strength  $f'_{ct}$  and specified compressive strength  $f'_c$  for the lightweight concrete. For a lightweight aggregate from a given source, it is intended that appropriate values of  $f'_{ct}$  be obtained in advance of design.

**R5.1.5** — Tests for splitting tensile strength of concrete (as required by **5.1.4**) are not intended for control of, or acceptance of, the strength of concrete in the field. Indirect control will be maintained through the normal compressive strength test requirements provided by **5.6**.

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**5.1.6** — Steel fiber-reinforced concrete shall conform to ASTM C1116. The minimum  $f'_c$  for steel fiber-reinforced concrete shall conform to **5.1.1**.

**5.2 — Selection of concrete proportions**

**5.2.1** — Proportions of materials for concrete shall be established to:

- (a) Provide workability and consistency to permit concrete to be worked readily into forms and around reinforcement under conditions of placement to be employed, without segregation or excessive bleeding;
- (b) Meet requirements for applicable exposure categories of **Chapter 4**;
- (c) Conform to strength test requirements of **5.6**.

**5.2.2** — Where different materials are to be used for different portions of proposed Work, each combination shall be evaluated.

**5.2.3** — Concrete proportions shall be established in accordance with 5.3 or, alternatively, **5.4**, and shall meet applicable requirements of **Chapter 4**.

**5.3 — Proportioning on the basis of field experience or trial mixtures, or both****R5.2 — Selection of concrete proportions**

Recommendations for selecting proportions for concrete are given in detail in ACI 211.1.<sup>5.1</sup> (This provides two methods for selecting and adjusting proportions for normalweight concrete: the estimated weight and absolute volume methods. Example calculations are shown for both methods. Proportioning of heavyweight concrete by the absolute volume method is presented in an appendix.)

Recommendations for lightweight concrete are given in ACI 211.2.<sup>5.2</sup> (This provides a method of proportioning and adjusting structural grade concrete containing lightweight aggregates.)

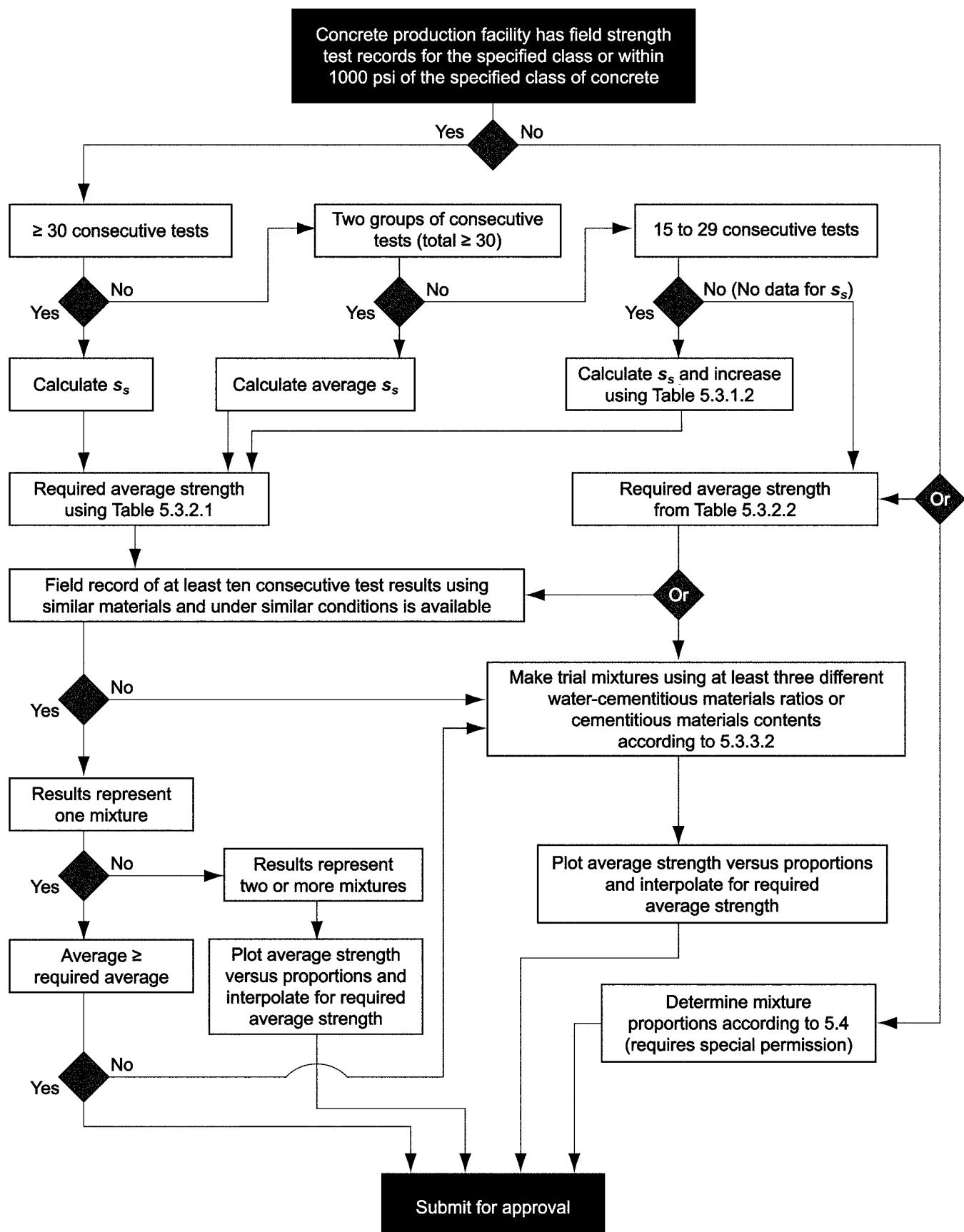
**R5.2.1** — The selected water-cementitious material ratio should be low enough, or in the case of lightweight concrete the compressive strength, high enough to satisfy both the strength criteria (see 5.3 or **5.4**) and the requirements for applicable exposure categories of **Chapter 4**. The code does not include provisions for especially severe exposure conditions, such as acids or high temperatures, and is not concerned with aesthetic considerations such as surface finishes. These items are beyond the scope of the Code and should be covered specifically in the project specifications. Concrete ingredients and proportions are to be selected to meet the minimum requirements stated in the Code and the additional requirements of the contract documents.

**R5.2.3** — The Code emphasizes the use of field experience or laboratory trial mixtures (see 5.3) as the preferred method for selecting concrete mixture proportions.

**R5.3 — Proportioning on the basis of field experience or trial mixtures, or both**

In selecting a suitable concrete mixture there are three basic steps. The first is the determination of the sample standard deviation. The second is the determination of the required average compressive strength. The third is the selection of mixture proportions required to produce that average strength, either by conventional trial mixture procedures or by a suitable experience record. **Figure R5.3** is a flow chart outlining the mixture selection and documentation procedure. The mixture selected should yield an average strength appreciably higher than the specified strength  $f'_c$ . The degree of mixture over design depends on the variability of the test results.

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*Fig. R5.3—Flow chart for selection and documentation of concrete proportions.*

**CODE****COMMENTARY****5.3.1 — Sample standard deviation**

**5.3.1.1** — Where a concrete production facility has strength test records not more than 24 months old, a sample standard deviation,  $s_s$ , shall be established. Test records from which  $s_s$  is calculated:

- (a) Shall represent materials, quality control procedures, and conditions similar to those expected and changes in materials and proportions within the test records shall not have been more restricted than those for proposed Work;
- (b) Shall represent concrete produced to meet a specified compressive strength or strengths within 1000 psi of  $f'_c$ ;
- (c) Shall consist of at least 30 consecutive tests or two groups of consecutive tests totaling at least 30 tests as defined in 5.6.2.4, except as provided in 5.3.1.2.

**5.3.1.2** — Where a concrete production facility does not have strength test records meeting requirements of 5.3.1.1(c), but does have test records not more than 24 months old based on 15 to 29 consecutive tests, a sample standard deviation  $s_s$  shall be established as the product of the calculated sample standard deviation and modification factor of Table 5.3.1.2. To be acceptable, test records shall meet requirements (a) and (b) of 5.3.1.1, and represent only a single record of consecutive tests that span a period of not less than 45 calendar days.

**TABLE 5.3.1.2 —MODIFICATION FACTOR FOR SAMPLE STANDARD DEVIATION WHEN LESS THAN 30 TESTS ARE AVAILABLE**

No. of tests*	Modification factor for sample standard deviation†
Less than 15	Use Table 5.3.2.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

\*Interpolate for intermediate numbers of tests.

†Modified sample standard deviation,  $s_s$ , to be used to determine required average strength,  $f'_{cr}$ , from 5.3.2.1

**R5.3.1 — Sample standard deviation**

The standard deviation established from test records is a measure of the concrete supplier's ability to manage variability of materials, production, and testing of concrete. A test record obtained less than 24 months before a submittal is acceptable.

When a concrete production facility has a suitable record of 30 consecutive tests of similar materials and conditions expected, the sample standard deviation,  $s_s$ , is calculated from those results in accordance with the following formula:

$$s_s = \left[ \frac{\sum(x_i - \bar{x})^2}{(n - 1)} \right]^{1/2}$$

where

- $s_s$  = sample standard deviation, psi
- $x_i$  = individual strength tests as defined in 5.6.2.4
- $\bar{x}$  = average of  $n$  strength test results
- $n$  = number of consecutive strength tests

The sample standard deviation is used to determine the average strength required in 5.3.2.1.

If two test records are used to obtain at least 30 tests, the sample standard deviation used shall be the statistical average of the values calculated from each test record in accordance with the following formula:

$$\bar{s}_s = \left[ \frac{(n_1 - 1)(s_{s1})^2 + (n_2 - 1)(s_{s2})^2}{(n_1 + n_2 - 2)} \right]^{1/2}$$

where

- $\bar{s}_s$  = statistical average standard deviation where two test records are used to estimate the sample standard deviation
- $s_{s1}, s_{s2}$  = sample standard deviations calculated from two test records, 1 and 2, respectively
- $n_1, n_2$  = number of tests in each test record, respectively

If less than 30 tests, but at least 15 tests are available, the calculated sample standard deviation is increased by the factor given in Table 5.3.1.2. This procedure results in a more conservative (increased) required average strength. The factors in Table 5.3.1.2 are based on the sampling distribution of the sample standard deviation and provide protection (equivalent to that from a record of 30 tests) against the possibility that the smaller sample underestimates the true or universe population standard deviation.

The sample standard deviation used in the calculation of required average strength should be developed under

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conditions “similar to those expected” [see 5.3.1.1(a)]. This requirement is important to ensure acceptable concrete.

Concrete for background tests to determine sample standard deviation is considered to be “similar” to that required if made with the same general types of ingredients under no more restrictive conditions of control over material quality and production methods than on the proposed Work, and if its specified strength does not deviate more than 1000 psi from the  $f'_c$  required [see 5.3.1.1(b)]. A change in the type of concrete or a major increase in the strength level may increase the sample standard deviation. Such a situation might occur with a change in type of aggregate (i.e., from natural aggregate to lightweight aggregate or vice versa) or a change from non-air-entrained concrete to air-entrained concrete. Also, there may be an increase in sample standard deviation when the average strength level is raised by a significant amount, although the increment of increase in sample standard deviation should be somewhat less than directly proportional to the strength increase. When there is reasonable doubt, any estimated sample standard deviation used to calculate the required average strength should always be on the conservative (high) side.

Note that the Code uses the sample standard deviation in pounds per square inch instead of the coefficient of variation in percent. The latter is equal to the former expressed as a percent of the average strength.

Even when the average strength and sample standard deviation are of the levels assumed, there will be occasional tests that fail to meet the acceptance criteria prescribed in 5.6.3.3 (perhaps one test in 100).

**5.3.2 — Required average strength**

**5.3.2.1** — Required average compressive strength  $f'_{cr}$  used as the basis for selection of concrete proportions shall be determined from Table 5.3.2.1 using the sample standard deviation,  $s_s$ , calculated in accordance with 5.3.1.1 or 5.3.1.2.

**TABLE 5.3.2.1 — REQUIRED AVERAGE COMPRESSIVE STRENGTH WHEN DATA ARE AVAILABLE TO ESTABLISH A SAMPLE STANDARD DEVIATION**

Specified compressive strength, psi	Required average compressive strength, psi
$f'_c \leq 5000$	Use the larger value computed from Eq. (5-1) and (5-2) $f'_{cr} = f'_c + 1.34s_s \quad (5-1)$ $f'_{cr} = f'_c + 2.33s_s - 500 \quad (5-2)$
$f'_c > 5000$	Use the larger value computed from Eq. (5-1) and (5-3) $f'_{cr} = f'_c + 1.34s_s \quad (5-1)$ $f'_{cr} = 0.90f'_c + 2.33s_s \quad (5-3)$

**R5.3.2 — Required average strength**

**R5.3.2.1** — Once the sample standard deviation has been determined, the required average compressive strength,  $f'_{cr}$ , is obtained from the larger value computed from Eq. (5-1) and (5-2) for  $f'_c$  of 5000 psi or less, or the larger value computed from Eq. (5-1) and (5-3) for  $f'_c$  over 5000 psi. Equation (5-1) is based on a probability of 1-in-100 that the average of three consecutive tests may be below the specified compressive strength  $f'_c$ . Equation (5-2) is based on a similar probability that an individual test may be more than 500 psi below the specified compressive strength  $f'_c$ . Equation (5-3) is based on the same 1-in-100 probability that an individual test may be less than  $0.90f'_c$ . These equations assume that the sample standard deviation used is equal to the population value appropriate for an infinite or very large number of tests. For this reason, use of sample standard deviations estimated from records of 100 or more tests is desirable. When 30 tests are available, the probability of failure will likely be somewhat greater than 1-in-100. The additional refinements required to achieve the 1-in-100 probability are not considered necessary because of the

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**5.3.2.2** — When a concrete production facility does not have field strength test records for calculation of  $s_s$  meeting requirements of **5.3.1.1** or **5.3.1.2**,  $f'_{cr}$  shall be determined from Table 5.3.2.2 and documentation of average strength shall be in accordance with requirements of 5.3.3.

**TABLE 5.3.2.2 — REQUIRED AVERAGE COMPRESSIVE STRENGTH WHEN DATA ARE NOT AVAILABLE TO ESTABLISH A SAMPLE STANDARD DEVIATION**

Specified compressive strength, psi	Required average compressive strength, psi
$f'_c < 3000$	$f'_{cr} = f'_c + 1000$
$3000 \leq f'_c \leq 5000$	$f'_{cr} = f'_c + 1200$
$f'_c > 5000$	$f'_{cr} = 1.10f'_c + 700$

**5.3.3 — Documentation of average compressive strength**

Documentation that proposed concrete proportions will produce an average compressive strength equal to or greater than required average compressive strength  $f'_{cr}$  (see **5.3.2**) shall consist of one or more field strength test record(s) or trial mixtures. The field strength test records or trial mixtures shall not be more than 24 months old and shall conform to 5.3.3.1 and 5.3.3.2, respectively.

**5.3.3.1** — When test records in accordance with **5.3.1.1** or **5.3.1.2** are used to demonstrate that proposed concrete proportions will produce  $f'_{cr}$  (see **5.3.2**), such records shall represent materials and conditions similar to those expected. Changes in materials, conditions, and proportions within the test records shall not have been more restricted than those for proposed Work. For the purpose of documenting average strength potential, test records consisting of less than 30 but not less than 10 consecutive tests are acceptable provided test records encompass a period of time not less than 45 days. Required concrete proportions shall be permitted to be established by interpolation between the strengths and proportions of two or more test records, each of which meets other requirements of this section.

**5.3.3.2** — When an acceptable record of field test results to document the required average strength is not available, concrete proportions established from trial mixtures meeting the following requirements shall be permitted:

uncertainty inherent in assuming that conditions operating when the test record was accumulated will be similar to conditions when the concrete will be produced.

**R5.3.3 — Documentation of average compressive strength**

Once the required average compressive strength  $f'_{cr}$  is known, the next step is to select mixture proportions that will produce an average strength at least as great as the required average strength, and also meet requirements for applicable exposure categories of **Chapter 4**. The documentation may consist of one or more field strength test record(s) in accordance with 5.3.3.1, or suitable laboratory or field trial mixtures in accordance with 5.3.3.2. Generally, if a test record is used, it will be the same one that was used for computation of the standard deviation. However, if this test record shows either lower or higher average compressive strength than the required average compressive strength, different proportions may be necessary or desirable. In such instances, the average from a record of as few as 10 tests may be used, or the proportions may be established by interpolation between the strengths and proportions of two such records of consecutive tests. All field test records for establishing proportions necessary to produce the average compressive strength are to meet the requirements of 5.3.3.1 for “similar materials and conditions.”

For strengths over 5000 psi where the average compressive strength documentation is based on laboratory trial mixtures, it may be appropriate to increase  $f'_{cr}$  calculated in Table 5.3.2.2 to allow for a reduction in strength from laboratory trials to actual concrete production.

**R5.3.3.2** — This section of the Code was modified in ACI 318-08 to clarify the requirements for making trial batches.

(b) For concrete made with more than one type of cementitious material, the concrete supplier must estab-

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- (a) Materials shall be those for proposed Work;
- (b) Trial mixtures with a range of proportions that will produce a range of compressive strengths encompassing  $f'_{cr}$  and meet the durability requirements of Chapter 4;
- (c) Trial mixtures shall have slumps within the range specified for the proposed Work; for air-entrained concrete, air content shall be within the tolerance specified for the proposed Work;
- (d) For each trial mixture, at least two 6 by 12 in. or three 4 by 8 in. cylinders shall be made and cured in accordance with ASTM C192. Cylinders shall be tested at 28 days or at test age designated for  $f'_c$ ;
- (e) The compressive strength results, at designated test age, from the trial mixtures shall be used to establish the composition of the concrete mixture proposed for the Work. The proposed concrete mixture shall achieve an average compressive strength as required in 5.3.2 and satisfy the applicable durability criteria of Chapter 4.

### 5.4 — Proportioning without field experience or trial mixtures

**5.4.1** — If data required by 5.3 are not available, concrete proportions shall be based upon other experience or information, if approved by the licensed design professional. The required average compressive strength  $f'_{cr}$  of concrete produced with materials similar to those proposed for use shall be at least 1200 psi greater than  $f'_c$ . This alternative shall not be used if  $f'_c$  is greater than 5000 psi.

**5.4.2** — Concrete proportioned by this section shall conform to the durability requirements of Chapter 4 and to compressive strength test criteria of 5.6.

### 5.5 — Average compressive strength reduction

As data become available during construction, it shall be permitted to reduce the amount by which the required average concrete strength,  $f'_{cr}$ , must exceed  $f'_c$ , provided:

- (a) Thirty or more test results are available and average of test results exceeds that required by 5.3.2.1, using a sample standard deviation calculated in accordance with 5.3.1.1; or

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lish not only the water-cementitious material ratio but also the relative proportions of cementitious materials and admixtures, if any, that will produce the required average compressive strength and satisfy the durability requirements of Chapter 4. This will require multiple trial batches with different mixture proportions. The exact number of batches will depend on the number of cementitious materials and the range of their relative proportions.

(d) Starting with the 2008 Code, 5.6.2.4 permits two cylinder sizes for preparing test specimens for field acceptance testing.

(e) The compressive strength test results may be analyzed graphically or using regression models to determine the water-cementitious material ratio and the relative proportions of cementitious materials, if other materials in addition to portland cement are used, that will produce concrete satisfying the required average compressive strength  $f'_{cr}$ .

### R5.4 — Proportioning without field experience or trial mixtures

**R5.4.1** — When no prior experience (5.3.3.1) or trial mixture data (5.3.3.2) meeting the requirements of these sections is available, other experience may be used only when permission is given. Because combinations of different ingredients may vary considerably in strength level, this procedure is not permitted for  $f'_c$  greater than 5000 psi and the required average compressive strength should exceed  $f'_c$  by 1200 psi. The purpose of this provision is to allow work to continue when there is an unexpected interruption in concrete supply and there is not sufficient time for tests and evaluation or in small structures where the cost of trial mixture data is not justified.

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- (b) Fifteen to 29 test results are available and average of test results exceeds that required by 5.3.2.1 using a sample standard deviation calculated in accordance with 5.3.1.2; and
- (c) Requirements for exposure categories of Chapter 4 are met.

**5.6 — Evaluation and acceptance of concrete**

**5.6.1** — Concrete shall be tested in accordance with the requirements of 5.6.2 through 5.6.5. The testing agency performing acceptance testing shall comply with ASTM C1077. Qualified field testing technicians shall perform tests on fresh concrete at the job site, prepare specimens required for curing under field conditions, prepare specimens required for testing in the laboratory, and record the temperature of the fresh concrete when preparing specimens for strength tests. Qualified laboratory technicians shall perform all required laboratory tests. All reports of acceptance tests shall be provided to the licensed design professional, contractor, concrete producer, and, when requested, to the owner and the building official.

**5.6.2 — Frequency of testing**

**5.6.2.1** — Samples for strength tests of each class of concrete placed each day shall be taken not less than once a day, nor less than once for each 150  $\text{yd}^3$  of concrete, nor less than once for each 5000  $\text{ft}^2$  of surface area for slabs or walls.

**R5.6 — Evaluation and acceptance of concrete**

Once the mixture proportions have been selected and the job started, the criteria for evaluation and acceptance of the concrete can be obtained from 5.6.

An effort has been made in the Code to provide a clear-cut basis for judging the acceptability of the concrete, as well as to indicate a course of action to be followed when the results of strength tests are not satisfactory.

**R5.6.1** — ASTM C1077<sup>5,3</sup> identifies and defines the duties and minimum technical requirements and qualifications of testing laboratory personnel and requirements for testing concrete and concrete aggregates used in construction. Inspection and accreditation of testing laboratories is a process that ensures that they conform to ASTM C1077. Laboratory and field technicians can establish qualifications by becoming certified through certification programs. Field technicians in charge of sampling concrete; testing for slump, density, yield, air content, and temperature; and making and curing test specimens should be certified in accordance with the requirements of ACI Concrete Field Testing Technician—Grade 1 Certification Program, or an equivalent program. Concrete testing laboratory personnel should be certified in accordance with the requirements of ACI Concrete Laboratory Testing Technician, or Concrete Strength Testing Technician.

The Code requires testing reports to be distributed to the parties responsible for the design, construction, and approval of the work. Such distribution of test reports should be indicated in contracts for inspection and testing services. Prompt distribution of testing reports allows for timely identification of either compliance or the need for corrective action. A complete record of testing allows the concrete producer to reliably establish the required average strength  $f'_{cr}$  for future work.

**R5.6.2 — Frequency of testing**

**R5.6.2.1** — The following three criteria establish the required minimum sampling frequency for each class of concrete:

- (a) Once each day a given class is placed, nor less than
- (b) Once for each 150  $\text{yd}^3$  of each class placed each day, nor less than

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(c) Once for each 5000 ft<sup>2</sup> of slab or wall surface area placed each day.

In calculating surface area, only one side of the slab or wall should be considered. Criteria (c) will require more frequent sampling than once for each 150 yd<sup>3</sup> placed if the average wall or slab thickness is less than 9-3/4 in.

**5.6.2.2** — On a given project, if total volume of concrete is such that frequency of testing required by **5.6.2.1** would provide less than five strength tests for a given class of concrete, tests shall be made from at least five randomly selected batches or from each batch if fewer than five batches are used.

**R5.6.2.2** — Samples for strength tests are to be taken on a strictly random basis if they are to measure properly the acceptability of the concrete. To be representative, the choice of times of sampling, or the batches of concrete to be sampled, are to be made on the basis of chance alone, within the period of placement. Batches should not be sampled on the basis of appearance, convenience, or other possibly biased criteria, because the statistical analyses will lose their validity. Not more than one test (as defined in 5.6.2.4) should be taken from a single batch, and water may not be added to the concrete after the sample is taken.

ASTM D3665<sup>5.4</sup> describes procedures for random selection of the batches to be tested.

**5.6.2.3** — When total quantity of a given class of concrete is less than 50 yd<sup>3</sup>, strength tests are not required when evidence of satisfactory strength is submitted to and approved by the building official.

**5.6.2.4** — A strength test shall be the average of the strengths of at least two 6 by 12 in. cylinders or at least three 4 by 8 in. cylinders made from the same sample of concrete and tested at 28 days or at test age designated for determination of  $f'_c$ .

**R5.6.2.4** — More than the minimum number of specimens may be desirable to allow for discarding an outlying individual cylinder strength in accordance with ACI 214R.<sup>5.5</sup> When individual cylinder strengths are discarded in accordance with ACI 214R, a strength test is valid provided at least two individual 6 by 12 in. cylinder strengths or at least three 4 by 8 in. cylinders are averaged. All individual cylinder strengths that are not discarded in accordance with ACI 214R are to be used to calculate the average strength. The size and number of specimens representing a strength test should remain constant for each class of concrete.

Testing three 4 by 8 in. cylinders preserves the confidence level of the average strength because 4 by 8 in. cylinders tend to have approximately 20 percent higher within-test variability than 6 by 12 in. cylinders.<sup>5.6</sup>

**5.6.3 — Standard-cured specimens****R5.6.3 — Standard-cured specimens**

**5.6.3.1** — Samples for strength tests shall be taken in accordance with ASTM C172.

**5.6.3.2** — Cylinders for strength tests shall be molded and standard-cured in accordance with ASTM C31 and tested in accordance with ASTM C39. Cylinders shall be 4 by 8 in. or 6 by 12 in.

**R5.6.3.2** — The cylinder size should be agreed upon by the owner, licensed design professional, and testing agency before construction.

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**5.6.3.3** — Strength level of an individual class of concrete shall be considered satisfactory if both of the following requirements are met:

- (a) Every arithmetic average of any three consecutive strength tests (see **5.6.2.4**) equals or exceeds  $f'_c$ ;
- (b) No strength test (see **5.6.2.4**) falls below  $f'_c$  by more than 500 psi when  $f'_c$  is 5000 psi or less; or by more than **0.10 $f'_c$**  when  $f'_c$  is more than 5000 psi.

**5.6.3.4** — If either of the requirements of 5.6.3.3 is not met, steps shall be taken to increase the average of subsequent strength test results. Requirements of **5.6.5** shall be observed if requirement of 5.6.3.3(b) is not met.

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**R5.6.3.3** — A single set of criteria is given for acceptability of strength and is applicable to all concrete used in structures designed in accordance with the Code, regardless of design method used. The concrete strength is considered to be satisfactory as long as averages of any three consecutive strength tests remain above the specified  $f'_c$  and no individual strength test falls below the specified  $f'_c$  by more than 500 psi if  $f'_c$  is 5000 psi or less, or falls below  $f'_c$  by more than 10 percent if  $f'_c$  is over 5000 psi. Evaluation and acceptance of the concrete can be judged immediately as test results are received during the course of the Work. Strength tests failing to meet these criteria will occur occasionally (probably about once in 100 tests) even though concrete strength and uniformity are satisfactory. Allowance should be made for such statistically expected variations in deciding whether the strength level being produced is adequate.

**R5.6.3.4** — When concrete fails to meet either of the strength requirements of 5.6.3.3, steps should be taken to increase the average of the concrete test results. If sufficient concrete has been produced to accumulate at least 15 tests, these should be used to establish a new target average strength as described in **5.3**.

If fewer than 15 tests have been made on the class of concrete in question, the new target strength level should be at least as great as the average level used in the initial selection of proportions. If the average of the available tests made on the project equals or exceeds the level used in the initial selection of proportions, a further increase in average level is required.

The steps taken to increase the average level of test results will depend on the particular circumstances, but could include one or more of the following:

- (a) An increase in cementitious materials content;
- (b) Changes in mixture proportions;
- (c) Reductions in or better control of levels of slump supplied;
- (d) A reduction in delivery time;
- (e) Closer control of air content;
- (f) An improvement in the quality of the testing, including strict compliance with standard test procedures.

Such changes in operating and testing procedures, or changes in cementitious materials content, or slump should not require a formal resubmission under the procedures of **5.3**; however, important changes in sources of cement, aggregates, or admixtures should be accompanied by evidence that the average strength level will be improved.

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Laboratories testing cylinders or cores to determine compliance with these requirements should be accredited or inspected for conformance to the requirement of ASTM C1077<sup>5,3</sup> by a recognized agency such as the American Association for Laboratory Accreditation (A2LA), AASHTO Materials Reference Laboratory (AMRL), National Voluntary Laboratory Accreditation Program (NVLAP), Cement and Concrete Reference Laboratory (CCRL), or their equivalent.

**5.6.4 — Field-cured specimens**

**5.6.4.1** — If required by the building official, results of strength tests of cylinders cured under field conditions shall be provided.

**5.6.4.2** — Field-cured cylinders shall be cured under field conditions in accordance with ASTM C31.

**5.6.4.3** — Field-cured test cylinders shall be molded at the same time and from the same samples as laboratory-cured test cylinders.

**5.6.4.4** — Procedures for protecting and curing concrete shall be improved when strength of field-cured cylinders at test age designated for determination of  $f'_c$  is less than 85 percent of that of companion laboratory-cured cylinders. The 85 percent limitation shall not apply if field-cured strength exceeds  $f'_c$  by more than 500 psi.

**R5.6.4 — Field-cured specimens**

**R5.6.4.1** — Strength tests of cylinders cured under field conditions may be required to check the adequacy of curing and protection of concrete in the structure.

**R5.6.4.4** — Positive guidance is provided in the Code concerning the interpretation of tests of field-cured cylinders. Research has shown that cylinders protected and cured to simulate good field practice should test not less than about 85 percent of standard laboratory moist-cured cylinders. This percentage has been set as a rational basis for judging the adequacy of field curing. The comparison is made between the actual measured strengths of companion field-cured and laboratory-cured cylinders, not between field-cured cylinders and the specified value of  $f'_c$ . However, results for the field-cured cylinders are considered satisfactory if the field-cured cylinders exceed the specified  $f'_c$  by more than 500 psi, even though they fail to reach 85 percent of the strength of companion laboratory-cured cylinders.

**5.6.5 — Investigation of low-strength test results**

**5.6.5.1** — If any strength test (see 5.6.2.4) of laboratory-cured cylinders falls below  $f'_c$  by more than the values given in 5.6.3.3(b) or if tests of field-cured cylinders indicate deficiencies in protection and curing (see 5.6.4.4), steps shall be taken to ensure that load-carrying capacity of the structure is not jeopardized.

**5.6.5.2** — If the likelihood of low-strength concrete is confirmed and calculations indicate that load-carrying capacity is significantly reduced, tests of cores drilled from the area in question in accordance with ASTM C42 shall be permitted. In such cases, three cores shall be taken for each strength test that falls below the values given in 5.6.3.3(b).

**R5.6.5 — Investigation of low-strength test results**

Instructions are provided concerning the procedure to be followed when strength tests have failed to meet the specified acceptance criteria. These instructions are applicable only for evaluation of in-place strength at time of construction. Strength evaluation of existing structures is covered by Chapter 20. The building official should apply judgment as to the significance of low test results and whether they indicate need for concern. If further investigation is deemed necessary, such investigation may include nondestructive tests or, in extreme cases, strength tests of cores taken from the structure.

Nondestructive tests of the concrete in place, such as by probe penetration, impact hammer, ultrasonic pulse velocity or pullout may be useful in determining whether or not a portion of the structure actually contains low-strength

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**5.6.5.3** — Cores shall be obtained, moisture conditioned by storage in watertight bags or containers, transported to the laboratory, and tested in accordance with ASTM C42. Cores shall be tested no earlier than 48 hours and not later than 7 days after coring unless approved by the licensed design professional. The specifier of tests referenced in ASTM C42 shall be the licensed design professional.

**5.6.5.4** — Concrete in an area represented by core tests shall be considered structurally adequate if the average of three cores is equal to at least 85 percent of  $f'_c$  and if no single core is less than 75 percent of  $f'_c$ . Additional testing of cores extracted from locations represented by erratic core strength results shall be permitted.

**5.6.5.5** — If criteria of 5.6.5.4 are not met and if the structural adequacy remains in doubt, the responsible authority shall be permitted to order a strength evaluation in accordance with **Chapter 20** for the questionable portion of the structure, or take other appropriate action.

concrete. Such tests are of value primarily for comparisons within the same job rather than as quantitative measures of strength. For cores, if required, conservatively safe acceptance criteria are provided that should ensure structural adequacy for virtually any type of construction.<sup>5.7-5.10</sup> Lower strength may, of course, be tolerated under many circumstances, but this again becomes a matter of judgment on the part of the building official and licensed design professional. When the core tests performed in accordance with 5.6.5.4 fail to provide assurance of structural adequacy, it may be practical, particularly in the case of floor or roof systems, for the building official to require a load test (**Chapter 20**). Short of load tests, if time and conditions permit, an effort may be made to improve the strength of the concrete in place by supplemental wet curing. Effectiveness of such a treatment should be verified by further strength evaluation using procedures previously discussed.

The use of a water-cooled bit results in a core with a moisture gradient between the exterior surface and the interior. This gradient lowers the apparent compressive strength of the core.<sup>5.11</sup> The restriction on the commencement of core testing provides a minimum time for the moisture gradient to dissipate. The maximum time between coring and testing is intended to ensure timely testing of cores when strength of concrete is in question. Research<sup>5.11</sup> has also shown that procedures for soaking or drying cores that were required before ACI 318-02 affect measured compressive strength and result in conditions that are not representative of structures that are dry or wet in service. Thus, to provide reproducible moisture conditions that are representative of in-place conditions, a common moisture conditioning procedure that permits dissipation of moisture gradients is prescribed for cores. ASTM C42 permits the specifier of tests to modify the default duration of moisture conditioning before testing.

Core tests having an average of 85 percent of the specified strength are realistic. To expect core tests to be equal to  $f'_c$  is not realistic, since differences in the size of specimens, conditions of obtaining samples, and procedures for curing, do not permit equal values to be obtained.

The Code, as stated, concerns itself with assuring structural safety, and the instructions in **5.6** are aimed at that objective. It is not the function of the Code to assign responsibility for strength deficiencies, whether or not they are such as to require corrective measures.

Under the requirements of this section, cores taken to confirm structural adequacy will usually be taken at ages later than those specified for determination of  $f'_c$ .

**CODE****COMMENTARY****5.6.6 — Steel fiber-reinforced concrete**

**5.6.6.1** — Acceptance of steel fiber-reinforced concrete used in beams in accordance with **11.4.6.1(f)** shall be determined by testing in accordance with ASTM C1609. In addition, strength testing shall be in accordance with **5.6.1**.

**5.6.6.2** — Steel fiber-reinforced concrete shall be considered acceptable for shear resistance if conditions (a), (b), and (c) are satisfied:

- (a) The weight of deformed steel fibers per cubic yard of concrete is greater than or equal to 100 lb.
- (b) The residual strength obtained from flexural testing in accordance with ASTM C1609 at a midspan deflection of 1/300 of the span length is greater than or equal to 90 percent of the measured first-peak strength obtained from a flexural test or 90 percent of the strength corresponding to  $f_r$  from **Eq. (9-10)**, whichever is larger; and
- (c) The residual strength obtained from flexural testing in accordance with ASTM C1609 at a midspan deflection of 1/150 of the span length is greater than or equal to 75 percent of the measured first-peak strength obtained from a flexural test or 75 percent of the strength corresponding to  $f_r$  from **Eq. (9-10)**, whichever is larger.

**5.7 — Preparation of equipment and place of deposit**

**5.7.1** — Preparation before concrete placement shall include the following:

- (a) All equipment for mixing and transporting concrete shall be clean;
- (b) All debris and ice shall be removed from spaces to be occupied by concrete;
- (c) Forms shall be properly coated;
- (d) Masonry filler units that will be in contact with concrete shall be well drenched;
- (e) Reinforcement shall be thoroughly clean of ice or other deleterious coatings;
- (f) Water shall be removed from place of deposit before concrete is placed unless a tremie is to be used or unless otherwise permitted by the building official;
- (g) All laitance and other unsound material shall be removed before additional concrete is placed against hardened concrete.

**R5.6.6 — Steel fiber-reinforced concrete**

**R5.6.6.1** — The performance criteria are based on results from flexural tests<sup>5,12</sup> conducted on steel fiber-reinforced concretes with fiber types and contents similar to those used in the tests of beams that served as the basis for **11.4.6.1(f)**.

**R5.6.6.2(b),(c)** — The term “residual strength” is defined in ASTM C1609.

**R5.7 — Preparation of equipment and place of deposit**

Recommendations for mixing, handling and transporting, and placing concrete are given in detail in ACI 304R.<sup>5,13</sup> (This presents methods and procedures for control, handling and storage of materials, measurement, batching tolerances, mixing, methods of placing, transporting, and forms.)

Attention is directed to the need for using clean equipment and for cleaning forms and reinforcement thoroughly before beginning to deposit concrete. In particular, sawdust, nails, wood pieces, and other debris that may collect inside the forms should be removed. Reinforcement should be thoroughly cleaned of ice, dirt, loose rust, mill scale, or other coatings. Water should be removed from the forms.

**CODE****COMMENTARY****5.8 — Mixing**

**5.8.1** — All concrete shall be mixed until there is a uniform distribution of materials and shall be discharged completely before mixer is recharged.

**5.8.2** — Ready-mixed concrete shall be mixed and delivered in accordance with requirements of ASTM C94 or C685.

**5.8.3** — Job-mixed concrete shall be mixed in accordance with (a) through (e):

- (a) Mixing shall be done in a batch mixer of approved type;
- (b) Mixer shall be rotated at a speed recommended by the manufacturer;
- (c) Mixing shall be continued for at least 1-1/2 minutes after all materials are in the drum, unless a shorter time is shown to be satisfactory by the mixing uniformity tests of ASTM C94;
- (d) Materials handling, batching, and mixing shall conform to applicable provisions of ASTM C94;
- (e) A detailed record shall be kept to identify:
  - (1) number of batches produced;
  - (2) proportions of materials used;
  - (3) approximate location of final deposit in structure;
  - (4) time and date of mixing and placing.

**5.9 — Conveying**

**5.9.1** — Concrete shall be conveyed from mixer to place of final deposit by methods that will prevent separation or loss of materials.

**5.9.2** — Conveying equipment shall be capable of providing a supply of concrete at site of placement without separation of ingredients and without interruptions sufficient to permit loss of plasticity between successive increments.

**R5.8 — Mixing**

Concrete of uniform and satisfactory quality requires the materials to be thoroughly mixed until uniform in appearance and all ingredients are distributed. Samples taken from different portions of a batch should have essentially the same density, air content, slump, and coarse aggregate content. Test methods for uniformity of mixing are given in ASTM C94. The necessary time of mixing will depend on many factors including batch size, stiffness of the batch, size and grading of the aggregate, and the efficiency of the mixer. Excessively long mixing times should be avoided to guard against grinding of the aggregates.

**R5.9 — Conveying**

Each step in the handling and transporting of concrete needs to be controlled to maintain uniformity within a batch and from batch to batch. It is essential to avoid segregation of the coarse aggregate from the mortar or of water from the other ingredients.

The Code requires the equipment for handling and transporting concrete to be capable of supplying concrete to the place of deposit continuously and reliably under all conditions and for all methods of placement. The provisions of 5.9 apply to all placement methods, including pumps, belt conveyors, pneumatic systems, wheelbarrows, buggies, crane buckets, and tremies.

Serious loss in strength can result when concrete is pumped through pipe made of aluminum or aluminum alloy.<sup>5.14</sup> Hydrogen gas generated by the reaction between the cement alkalies and the aluminum eroded from the interior of the pipe surface has been shown to cause strength reduction as

**CODE****COMMENTARY****5.10 — Depositing**

**5.10.1** — Concrete shall be deposited as nearly as practical in its final position to avoid segregation due to rehandling or flowing.

**5.10.2** — Concreting shall be carried on at such a rate that concrete is at all times plastic and flows readily into spaces between reinforcement.

**5.10.3** — Concrete that has partially hardened or been contaminated by foreign materials shall not be deposited in the structure.

**5.10.4** — Retempered concrete or concrete that has been remixed after initial set shall not be used unless approved by the licensed design professional.

**5.10.5** — After concreting is started, it shall be carried on as a continuous operation until placing of a panel or section, as defined by its boundaries or predetermined joints, is completed except as permitted or prohibited by 6.4.

**5.10.6** — Top surfaces of vertically formed lifts shall be generally level.

**5.10.7** — When construction joints are required, joints shall be made in accordance with 6.4.

**5.10.8** — All concrete shall be thoroughly consolidated by suitable means during placement and shall be thoroughly worked around reinforcement and embedded fixtures and into corners of forms.

**5.11 — Curing**

**5.11.1** — Concrete (other than high-early-strength) shall be maintained above 50°F and in a moist condition for at least the first 7 days after placement, except when cured in accordance with 5.11.3.

**5.11.2** — High-early-strength concrete shall be maintained above 50°F and in a moist condition for at least the first 3 days, except when cured in accordance with 5.11.3.

much as 50 percent. Hence, equipment made of aluminum or aluminum alloys should not be used for pump lines, tremies, or chutes other than short chutes such as those used to convey concrete from a truck mixer.

**R5.10 — Depositing**

Rehandling concrete can cause segregation of the materials. Hence, the Code cautions against this practice. Retempering of partially set concrete with the addition of water should not be permitted unless authorized. This does not preclude the practice (recognized in ASTM C94) of adding water to mixed concrete to bring it up to the specified slump range so long as prescribed limits on the maximum mixing time and *w/cm* are not violated.

Section 5.10.4 of the 1971 Code contained a requirement that “where conditions make consolidation difficult or where reinforcement is congested, batches of mortar containing the same proportions of cement, sand, and water as used in the concrete, shall first be deposited in the forms to a depth of at least 1 in.” That requirement was deleted from the 1977 Code since the conditions for which it was applicable could not be defined precisely enough to justify its inclusion as a code requirement. The practice, however, has merit and should be incorporated in job specifications where appropriate, with the specific enforcement the responsibility of the job inspector. The use of mortar batches aids in preventing honeycomb and poor bonding of the concrete with the reinforcement. The mortar should be placed immediately before depositing the concrete and should be plastic (neither stiff nor fluid) when the concrete is placed.

Recommendations for consolidation of concrete are given in detail in ACI 309R.<sup>5.15</sup> (This presents current information on the mechanism of consolidation and gives recommendations on equipment characteristics and procedures for various classes of concrete.)

**R5.11 — Curing**

Recommendations for curing concrete are given in detail in ACI 308R.<sup>5.16</sup> (This presents basic principles of proper curing and describes the various methods, procedures, and materials for curing of concrete.)

**CODE****COMMENTARY****5.11.3 — Accelerated curing**

**5.11.3.1** — Curing by high-pressure steam, steam at atmospheric pressure, heat and moisture, or other accepted processes, shall be permitted to accelerate strength gain and reduce time of curing.

**5.11.3.2** — Accelerated curing shall provide a compressive strength of the concrete at the load stage considered at least equal to required design strength at that load stage.

**5.11.3.3** — Curing process shall be such as to produce concrete with a durability at least equivalent to the curing method of **5.11.1** or **5.11.2**.

**5.11.4** — When required by the licensed design professional, supplementary strength tests in accordance with **5.6.4** shall be performed to assure that curing is satisfactory.

**R5.11.3 — Accelerated curing**

The provisions of this section apply whenever an accelerated curing method is used, whether for precast or cast-in-place elements. The compressive strength of steam-cured concrete is not as high as that of similar concrete continuously cured under moist conditions at moderate temperatures. Also, the modulus of elasticity  $E_c$  of steam-cured specimens may vary from that of specimens moist-cured at normal temperatures. When steam curing is used, it is advisable to base the concrete mixture proportions on steam-cured test cylinders.

Accelerated curing procedures require careful attention to obtain uniform and satisfactory results. Preventing moisture loss during the curing is essential.

**R5.11.4** — In addition to requiring a minimum curing temperature and time for normal- and high-early-strength concrete, the Code provides a specific criterion in **5.6.4** for judging the adequacy of field curing. At the test age for which the compressive strength is specified (usually 28 days), field-cured cylinders should produce strength not less than 85 percent of that of the standard, laboratory-cured cylinders. For a reasonably valid comparison to be made, field-cured cylinders and companion laboratory-cured cylinders should come from the same sample. Field-cured cylinders should be cured under conditions identical to those of the structure. If the structure is protected from the elements, the cylinder should be protected.

Cylinders related to members not directly exposed to weather should be cured adjacent to those members and provided with the same degree of protection and method of curing. The field cylinders should not be treated more favorably than the elements they represent. (See **5.6.4** for additional information.) If the field-cured cylinders do not provide satisfactory strength by this comparison, measures should be taken to improve the curing. If the tests indicate a possible serious deficiency in strength of concrete in the structure, core tests may be required, with or without supplemental wet curing, to check the structural adequacy, as provided in **5.6.5**.

**5.12 — Cold weather requirements**

**5.12.1** — Adequate equipment shall be provided for heating concrete materials and protecting concrete during freezing or near-freezing weather.

**5.12.2** — All concrete materials and all reinforcement, forms, fillers, and ground with which concrete is to come in contact shall be free from frost.

**5.12.3** — Frozen materials or materials containing ice shall not be used.

**R5.12 — Cold weather requirements**

Recommendations for cold weather concreting are given in detail in ACI 306R.<sup>5.17</sup> (This presents requirements and methods for producing satisfactory concrete during cold weather.)

**CODE****COMMENTARY****5.13 — Hot weather requirements**

During hot weather, proper attention shall be given to ingredients, production methods, handling, placing, protection, and curing to prevent excessive concrete temperatures or water evaporation that could impair required strength or serviceability of the member or structure.

**R5.13 — Hot weather requirements**

Recommendations for hot weather concreting are given in detail in ACI 305R.<sup>5.18</sup> (This defines the hot weather factors that affect concrete properties and construction practices and recommends measures to eliminate or minimize the undesirable effects.)

**Notes**

# CHAPTER 6 — FORMWORK, EMBEDMENTS, AND CONSTRUCTION JOINTS

## CODE

### 6.1 — Design of formwork

**6.1.1** — Forms shall result in a final structure that conforms to shapes, lines, and dimensions of the members as required by the contract documents.

**6.1.2** — Forms shall be substantial and sufficiently tight to prevent leakage of mortar.

**6.1.3** — Forms shall be properly braced or tied together to maintain position and shape.

**6.1.4** — Forms and their supports shall be designed so as not to damage previously placed structure.

**6.1.5** — Design of formwork shall include consideration of the following factors:

- (a) Rate and method of placing concrete;
- (b) Construction loads, including vertical, horizontal, and impact loads;
- (c) Special form requirements for construction of shells, folded plates, domes, architectural concrete, or similar types of elements.

**6.1.6** — Forms for prestressed concrete members shall be designed and constructed to permit movement of the member without damage during application of prestressing force.

### 6.2 — Removal of forms, shores, and reshoring

#### 6.2.1 — Removal of forms

Forms shall be removed in such a manner as not to impair safety and serviceability of the structure. Concrete exposed by form removal shall have sufficient strength not to be damaged by removal operation.

#### 6.2.2 — Removal of shores and reshoring

The provisions of 6.2.2.1 through **6.2.2.3** shall apply to slabs and beams except where cast on the ground.

**6.2.2.1** — Before starting construction, the contractor shall develop a procedure and schedule for removal of shores and installation of reshores and for calculating the loads transferred to the structure during the process.

## COMMENTARY

### R6.1 — Design of formwork

Only minimum performance requirements for formwork, necessary to provide for public health and safety, are prescribed in Chapter 6. Formwork for concrete, including proper design, construction, and removal, demands sound judgment and planning to achieve adequate forms that are both economical and safe. Detailed information on formwork for concrete is given in: “**Guide to Formwork for Concrete**,” reported by Committee 347.<sup>6.1</sup> (This provides recommendations for design, construction, and materials for formwork, forms for special structures, and formwork for special methods of construction. Directed primarily to contractors, the suggested criteria will aid in preparing project specifications for the contractors.)

**Formwork for Concrete**,<sup>6.2</sup> reported by ACI Committee 347. (This is a how-to-do-it handbook for contractors, engineers, and architects following the guidelines established in ACI 347. Planning, building, and using formwork are discussed, including tables, diagrams, and formulas for form design loads.)

### R6.2 — Removal of forms, shores, and reshoring

In determining the time for removal of forms, consideration should be given to the construction loads and to the possibilities of deflections.<sup>6.3</sup> The construction loads are frequently at least as great as the specified live loads. At early ages, a structure may be adequate to support the applied loads but may deflect sufficiently to cause permanent damage.

Evaluation of concrete strength during construction may be demonstrated by field-cured test cylinders or other procedures approved by the building official such as:

- (a) Tests of cast-in-place cylinders in accordance with ASTM C873.<sup>6.4</sup> (This method is limited to use in slabs where the depth of concrete is from 5 to 12 in.);
- (b) Penetration resistance in accordance with ASTM C803<sup>6.5</sup>;

**CODE**

- (a) The structural analysis and concrete strength data used in planning and implementing form removal and shoring shall be furnished by the contractor to the building official when so requested;
- (b) No construction loads shall be supported on, nor any shoring removed from, any part of the structure under construction except when that portion of the structure in combination with remaining forming and shoring system has sufficient strength to support safely its weight and loads placed thereon;
- (c) Sufficient strength shall be demonstrated by structural analysis considering proposed loads, strength of forming and shoring system, and concrete strength data. Concrete strength data shall be based on tests of field-cured cylinders or, when approved by the building official, on other procedures to evaluate concrete strength.

**6.2.2.2** — No construction loads exceeding the combination of superimposed dead load plus specified live load shall be supported on any unshored portion of the structure under construction, unless analysis indicates adequate strength to support such additional loads.

**6.2.2.3** — Form supports for prestressed concrete members shall not be removed until sufficient prestressing has been applied to enable prestressed members to carry their dead load and anticipated construction loads.

**COMMENTARY**

- (c) Pullout strength in accordance with ASTM C900<sup>6,6</sup>;
- (d) Maturity index measurements and correlation in accordance with ASTM C1074.<sup>6,7</sup>

Procedures (b), (c), and (d) require sufficient data, using job materials, to demonstrate correlation of measurements on the structure with compressive strength of molded cylinders or drilled cores.

Where the structure is adequately supported on shores, the side forms of beams, girders, columns, walls, and similar vertical forms may generally be removed after 12 hours of cumulative curing time, provided the side forms support no loads other than the lateral pressure of the plastic concrete. Cumulative curing time represents the sum of time intervals, not necessarily consecutive, during which the temperature of the air surrounding the concrete is above 50°F. The 12-hour cumulative curing time is based on regular cements and ordinary conditions; the use of special cements or unusual conditions may require adjustment of the given limits. For example, concrete made with Type II or V (ASTM C150) or ASTM C595 cements, concrete containing retarding admixtures, and concrete to which ice was added during mixing (to lower the temperature of fresh concrete) may not have sufficient strength in 12 hours and should be investigated before removal of formwork.

The removal of formwork for multistory construction should be a part of a planned procedure considering the temporary support of the entire structure as well as that of each individual member. Such a procedure should be worked out prior to construction and should be based on a structural analysis taking into account the following items, as a minimum:

- (a) The structural system that exists at the various stages of construction and the construction loads corresponding to those stages;
- (b) The strength of the concrete at the various ages during construction;
- (c) The influence of deformations of the structure and shoring system on the distribution of dead loads and construction loads during the various stages of construction;
- (d) The strength and spacing of shores or shoring systems used, as well as the method of shoring, bracing, shore removal, and reshoring including the minimum time intervals between the various operations;
- (e) Any other loading or condition that affects the safety or serviceability of the structure during construction.

For multistory construction, the strength of the concrete during the various stages of construction should be substantiated by field-cured test specimens or other approved methods.

**CODE****COMMENTARY****6.3 — Embedments in concrete**

**6.3.1** — Embedments of any material not harmful to concrete and within limitations of 6.3 shall be permitted in concrete with approval of the licensed design professional, provided they are not considered to replace structurally the displaced concrete, except as provided in **6.3.6**.

**6.3.2** — Any aluminum embedments in structural concrete shall be coated or covered to prevent aluminum-concrete reaction or electrolytic action between aluminum and steel.

**6.3.3** — Conduits, pipes, and sleeves passing through a slab, wall, or beam shall not impair significantly the strength of the construction.

**6.3.4** — Conduits and pipes, with their fittings, embedded within a column shall not displace more than 4 percent of the area of cross section on which strength is calculated or which is required for fire protection.

**6.3.5** — Except when drawings for conduits and pipes are approved by the licensed design professional, conduits and pipes embedded within a slab, wall, or beam (other than those merely passing through) shall satisfy 6.3.5.1 through **6.3.5.3**.

**6.3.5.1** — They shall not be larger in outside dimension than 1/3 the overall thickness of slab, wall, or beam in which they are embedded.

**6.3.5.2** — They shall not be spaced closer than three diameters or widths on center.

**R6.3 — Embedments in concrete**

**R6.3.1** — Any embedments (e.g., conduits, pipes, and sleeves) not harmful to concrete can be placed in the concrete, but the Work should be done in such a manner that the structure will not be endangered. Empirical rules are given in 6.3 for safe installations under common conditions; for other than common conditions, special designs should be made. Many general building codes have adopted ANSI/ASME piping codes B 31.1 for power piping<sup>6,8</sup> and B 31.3 for chemical and petroleum piping.<sup>6,9</sup> The licensed design professional should be sure that the appropriate piping codes are used in the design and testing of the system. The contractor should not be permitted to install conduits, pipes, ducts, or sleeves that are not shown in the contract documents or not approved by the licensed design professional.

For the integrity of the structure, it is important that all conduit and pipe fittings within the concrete be carefully assembled as shown on the plans or called for in the job specifications.

**R6.3.2** — The Code prohibits the use of aluminum in structural concrete unless it is effectively coated or covered. Aluminum reacts with concrete and, in the presence of chloride ions, may also react electrolytically with steel, causing cracking and/or spalling of the concrete. Aluminum electrical conduits present a special problem since stray electric current accelerates the adverse reaction. Provision **3.6.4** prohibits calcium chloride or any admixture containing chloride from being used in concrete with aluminum embedments.

**CODE****COMMENTARY**

**6.3.5.3** — They shall not impair significantly the strength of the construction.

**6.3.6** — Conduits, pipes, and sleeves shall be permitted to be considered as replacing structurally in compression the displaced concrete provided in 6.3.6.1 through 6.3.6.3.

**6.3.6.1** — They are not exposed to rusting or other deterioration.

**6.3.6.2** — They are of uncoated or galvanized iron or steel not thinner than standard Schedule 40 steel pipe.

**6.3.6.3** — They have a nominal inside diameter not over 2 in. and are spaced not less than three diameters on centers.

**6.3.7** — Pipes and fittings shall be designed to resist effects of the material, pressure, and temperature to which they will be subjected.

**6.3.8** — No liquid, gas, or vapor, except water not exceeding 90°F nor 50 psi pressure, shall be placed in the pipes until the concrete has attained its design strength.

**6.3.9** — In solid slabs, piping, unless it is for radiant heating or snow melting, shall be placed between top and bottom reinforcement.

**6.3.10** — Specified concrete cover for pipes, conduits, and fittings shall not be less than 1-1/2 in. for concrete exposed to earth or weather, nor less than 3/4 in. for concrete not exposed to weather or in contact with ground.

**6.3.11** — Reinforcement with an area not less than 0.002 times area of concrete section shall be provided normal to piping.

**6.3.12** — Piping and conduit shall be so fabricated and installed that cutting, bending, or displacement of reinforcement from its proper location will not be required.

## **6.4 — Construction joints**

**6.4.1** — Surface of concrete construction joints shall be cleaned and laitance removed.

**R6.3.7** — The 1983 Code limited the maximum pressure in embedded pipe to 200 psi, which was considered too restrictive. Nevertheless, the effects of such pressures and the expansion of embedded pipe should be considered in the design of the concrete member.

## **R6.4 — Construction joints**

For the integrity of the structure, it is important that all construction joints be defined in construction documents and constructed as required. Any deviations should be approved by the licensed design professional.

**CODE****COMMENTARY**

**6.4.2** — Immediately before new concrete is placed, all construction joints shall be wetted and standing water removed.

**R6.4.2** — The requirements of the 1977 Code for the use of neat cement on vertical joints have been removed, since it is rarely practical and can be detrimental where deep forms and steel congestion prevent proper access. Often wet blasting and other procedures are more appropriate. Because the Code sets only minimum standards, the licensed design professional may have to specify additional procedures if conditions warrant. The degree to which mortar batches are needed at the start of concrete placement depend on concrete proportions, congestion of steel, vibrator access, and other factors.

**6.4.3** — Construction joints shall be so made and located as not to impair the strength of the structure. Provision shall be made for transfer of shear and other forces through construction joints. See **11.6.9**.

**R6.4.3** — Construction joints should be located where they will cause the least weakness in the structure. When shear due to gravity load is not significant, as is usually the case in the middle of the span of flexural members, a simple vertical joint may be adequate. Lateral force design may require special design treatment of construction joints. Shear keys, intermittent shear keys, diagonal dowels, or the shear transfer method of **11.6** may be used whenever a force transfer is required.

**6.4.4** — Construction joints in floors shall be located within the middle third of spans of slabs, beams, and girders.

**R6.4.6** — Delay in placing concrete in members supported by columns and walls is necessary to prevent cracking at the interface of the slab and supporting member caused by bleeding and settlement of plastic concrete in the supporting member.

**6.4.5** — Construction joints in girders shall be offset a minimum distance of two times the width of intersecting beams.

**R6.4.7** — Separate placement of slabs and beams, haunches, and similar elements is permitted when shown on the contract documents and where provision has been made to transfer forces as required in **6.4.3**.

**6.4.6** — Beams, girders, or slabs supported by columns or walls shall not be cast or erected until concrete in the vertical support members is no longer plastic.

**6.4.7** — Beams, girders, haunches, drop panels, shear caps, and capitals shall be placed monolithically as part of a slab system, unless otherwise shown in contract documents.

**Notes**

# CHAPTER 7 — DETAILS OF REINFORCEMENT

## CODE

### 7.1 — Standard hooks

The term “standard hook” as used in this Code shall mean one of the following:

**7.1.1** — 180-degree bend plus  $4d_b$  extension, but not less than 2-1/2 in. at free end of bar.

**7.1.2** — 90-degree bend plus  $12d_b$  extension at free end of bar.

### 7.1.3 — For stirrup and tie hooks

- (a) No. 5 bar and smaller, 90-degree bend plus  $6d_b$  extension at free end of bar; or
- (b) No. 6, No. 7, and No. 8 bar, 90-degree bend plus  $12d_b$  extension at free end of bar; or
- (c) No. 8 bar and smaller, 135-degree bend plus  $6d_b$  extension at free end of bar.

### 7.1.4 — Seismic hooks as defined in 2.2.

## 7.2 — Minimum bend diameters

**7.2.1** — Diameter of bend measured on the inside of the bar, other than for stirrups and ties in sizes No. 3 through No. 5, shall not be less than the values in [Table 7.2](#).

**7.2.2** — Inside diameter of bend for stirrups and ties shall not be less than  $4d_b$  for No. 5 bar and smaller. For bars larger than No. 5, diameter of bend shall be in accordance with [Table 7.2](#).

**7.2.3** — Inside diameter of bend in welded wire reinforcement for stirrups and ties shall not be less than  $4d_b$  for deformed wire larger than D6 and  $2d_b$  for all other wires. Bends with inside diameter of less than  $8d_b$  shall not be less than  $4d_b$  from nearest welded intersection.

## COMMENTARY

### R7.1 — Standard hooks

Recommended methods and standards for preparing design drawings, typical details, and drawings for the fabrication and placing of reinforcing steel in reinforced concrete structures are given in the *ACI Detailing Manual*, reported by ACI Committee 315.<sup>7.1</sup>

All provisions in the Code relating to bar, wire, or strand diameter (and area) are based on the nominal dimensions of the reinforcement as given in the appropriate ASTM specification. Nominal dimensions are equivalent to those of a circular area having the same weight per foot as the ASTM designated bar, wire, or strand sizes. Cross-sectional area of reinforcement is based on nominal dimensions.

**R7.1.3** — Standard stirrup and tie hooks are limited to No. 8 bars and smaller, and the 90-degree hook with  $6d_b$  extension is further limited to No. 5 bars and smaller, in both cases as the result of research showing that larger bar sizes with 90-degree hooks and  $6d_b$  extensions tend to pop out under high load.

### R7.2 — Minimum bend diameters

Standard bends in reinforcing bars are described in terms of the inside diameter of bend because this is easier to measure than the radius of bend. The primary factors affecting the minimum bend diameter are feasibility of bending without breakage and avoidance of crushing the concrete inside the bend.

**R7.2.2** — The minimum  $4d_b$  bend for the bar sizes commonly used for stirrups and ties is based on accepted industry practice in the United States. Use of a stirrup bar size not greater than No. 5 for either the 90-degree or 135-degree standard stirrup hook will permit multiple bending on standard stirrup bending equipment.

**R7.2.3** — Welded wire reinforcement can be used for stirrups and ties. The wire at welded intersections does not have the same uniform ductility and bendability as in areas that were not heated. These effects of the welding temperature are usually dissipated in a distance of approximately four wire diameters. Minimum bend diameters permitted are in most cases the same as those required in the ASTM bend tests for wire material (ASTM A1064).

**CODE****COMMENTARY****TABLE 7.2 — MINIMUM DIAMETERS OF BEND**

Bar size	Minimum diameter
No. 3 through No. 8	$6d_b$
No. 9, No. 10, and No. 11	$8d_b$
No. 14 and No. 18	$10d_b$

**7.3 — Bending**

**7.3.1** — All reinforcement shall be bent cold, unless otherwise permitted by the licensed design professional.

**7.3.2** — Reinforcement partially embedded in concrete shall not be field bent, except as shown in the contract documents or permitted by the licensed design professional.

**7.4 — Surface conditions of reinforcement**

**7.4.1** — At the time concrete is placed, reinforcement shall be free from mud, oil, or other nonmetallic coatings that decrease bond. Epoxy-coating of reinforcement in accordance with standards referenced in [3.5.3.8](#) and [3.5.3.9](#) shall be permitted.

**7.4.2** — Except for prestressing steel, steel reinforcement with rust, mill scale, or a combination of both shall be considered satisfactory, provided the minimum dimensions (including height of deformations) and weight of a hand-wire-brushed test specimen comply with applicable ASTM specifications referenced in [3.5](#).

**R7.3 — Bending**

**R7.3.1** — For unusual bends with inside diameters less than ASTM bend test requirements, special fabrication may be required.

**R7.3.2** — Construction conditions may make it necessary to bend bars that have been embedded in concrete. Such field bending should not be done without authorization of the licensed design professional. Contract documents should specify whether the bars will be permitted to be bent cold or if heating should be used. Bends should be gradual and should be straightened as required.

Tests<sup>7.2,7.3</sup> have shown that A615 Grade 40 and Grade 60 reinforcing bars can be cold bent and straightened up to 90 degrees at or near the minimum diameter specified in [7.2](#). If cracking or breakage is encountered, heating to a maximum temperature of 1500°F may avoid this condition for the remainder of the bars. Bars that fracture during bending or straightening can be spliced outside the bend region.

Heating should be performed in a manner that will avoid damage to the concrete. If the bend area is within approximately 6 in. of the concrete, some protective insulation may need to be applied. Heating of the bar should be controlled by temperature-indicating crayons or other suitable means. The heated bars should not be artificially cooled (with water or forced air) until after cooling to at least 600°F.

**R7.4 — Surface conditions of reinforcement**

Specific limits on rust are based on tests,<sup>7.4</sup> plus a review of earlier tests and recommendations. [Reference 7.4](#) provides guidance with regard to the effects of rust and mill scale on bond characteristics of deformed reinforcing bars. Research has shown that a normal amount of rust increases bond. Normal rough handling generally removes rust that is loose enough to injure the bond between the concrete and reinforcement.

**CODE****COMMENTARY**

**7.4.3** — Prestressing steel shall be clean and free of oil, dirt, scale, pitting and excessive rust. A light coating of rust shall be permitted.

## 7.5 — Placing reinforcement

**7.5.1** — Reinforcement, including tendons, and post-tensioning ducts shall be accurately placed and adequately supported before concrete is placed, and shall be secured against displacement within tolerances permitted in 7.5.2.

**7.5.2** — Unless otherwise specified by the licensed design professional, reinforcement, including tendons, and post-tensioning ducts shall be placed within the tolerances in 7.5.2.1 and 7.5.2.2.

**7.5.2.1** — Tolerances for  $d$  and for concrete cover in flexural members, walls, and compression members shall be as follows:

	Tolerance on $d$	Tolerance on specified concrete cover
$d \leq 8$ in.	$\pm 3/8$ in.	$-3/8$ in.
$d > 8$ in.	$\pm 1/2$ in.	$-1/2$ in.

except that tolerance for the clear distance to formed soffits shall be minus 1/4 in. In addition, tolerance for cover shall also not exceed minus 1/3 the concrete cover specified in the contract documents.

**7.5.2.2** — Tolerance for longitudinal location of bends and ends of reinforcement shall be  $\pm 2$  in., except the tolerance shall be  $\pm 1/2$  in. at the discontinuous ends of brackets and corbels, and  $\pm 1$  in. at the

**R7.4.3** — Guidance for evaluating the degree of rusting on strand is given in Reference 7.5.

## R7.5 — Placing reinforcement

**R7.5.1** — Reinforcement, including tendons, and post-tensioning ducts should be adequately supported in the forms to prevent displacement by concrete placement or workers. Beam stirrups should be supported on the bottom form of the beam by positive supports such as continuous longitudinal beam bolsters. If only the longitudinal beam bottom reinforcement is supported, construction traffic can dislodge the stirrups as well as any prestressing tendons tied to the stirrups.

**R7.5.2** — Generally accepted practice, as reflected in ACI 117,<sup>7.6</sup> has established tolerances on total depth (formwork or finish) and fabrication of truss bent reinforcing bars and closed ties, stirrups, and spirals. The licensed design professional should specify more restrictive tolerances than those permitted by the Code when necessary to minimize the accumulation of tolerances resulting in excessive reduction in effective depth or cover.

More restrictive tolerances have been placed on minimum clear distance to formed soffits because of its importance for durability and fire protection, and because bars are usually supported in such a manner that the specified tolerance is practical.

More restrictive tolerances than those required by the Code may be desirable for prestressed concrete to achieve camber control within limits acceptable to the licensed design professional or owner. In such cases, the contract documents should specify the necessary tolerances. Recommendations are given in Reference 7.7.

**R7.5.2.1** — The Code permits a reinforcement placement tolerance on effective depth  $d$ , which is directly related to the flexural and shear strength of the member. Because reinforcing steel is placed with respect to edges of members and formwork surfaces,  $d$  is not always conveniently measured in the field. Placement tolerances for cover are also provided. For guidance on including field tolerances in project specifications, see ACI 117.<sup>7.6</sup>

**CODE****COMMENTARY**

discontinuous ends of other members. The tolerance for concrete cover of 7.5.2.1 shall also apply at discontinuous ends of members.

**7.5.3** — Welded wire reinforcement (with wire size not greater than W5 or D5) used in slabs not exceeding 10 ft in span shall be permitted to be curved from a point near the top of slab over the support to a point near the bottom of slab at midspan, provided such reinforcement is either continuous over, or securely anchored at support.

**7.5.4** — Welding of crossing bars shall not be permitted for assembly of reinforcement unless authorized by the licensed design professional.

## 7.6 — Spacing limits for reinforcement

**7.6.1** — The minimum clear spacing between parallel bars in a layer shall be  $d_b$ , but not less than 1 in. See also 3.3.2.

**7.6.2** — Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than 1 in.

**7.6.3** — In spirally reinforced or tied reinforced compression members, clear distance between longitudinal bars shall be not less than  $1.5d_b$  nor less than 1-1/2 in. See also 3.3.2.

**7.6.4** — Clear distance limitation between bars shall apply also to the clear distance between a contact lap splice and adjacent splices or bars.

**7.6.5** — In walls and slabs other than concrete joist construction, primary flexural reinforcement shall not be spaced farther apart than three times the wall or slab thickness, nor farther apart than 18 in.

### 7.6.6 — Bundled bars

**7.6.6.1** — Groups of parallel reinforcing bars bundled in contact to act as a unit shall be limited to four in any one bundle.

**7.6.6.2** — Bundled bars shall be enclosed within stirrups or ties.

**7.6.6.3** — Bars larger than No. 11 shall not be bundled in beams.

**R7.5.4** — “Tack” welding (welding crossing bars) can seriously weaken a bar at the point welded by creating a metallurgical notch effect. This operation can be performed safely only when the material welded and welding operations are under continuous competent control, as in the manufacture of welded wire reinforcement.

## R7.6 — Spacing limits for reinforcement

Although the minimum bar spacings are unchanged in this Code, the development lengths given in Chapter 12 became a function of the bar spacings since the 1989 Code. As a result, it may be desirable to use larger than minimum bar spacings in some cases. The minimum limits were originally established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that may cause shear or shrinkage cracking. Use of nominal bar diameter to define minimum spacing permits a uniform criterion for all bar sizes.

### R7.6.6 — Bundled bars

Bond research<sup>7,8</sup> showed that bar cutoffs within bundles should be staggered. Bundled bars should be tied, wired, or otherwise fastened together to ensure remaining in position whether vertical or horizontal.

A limitation that bars larger than No. 11 not be bundled in beams or girders is a practical limit for application to building size members. (The “Standard Specifications for Highway Bridges”<sup>7,9</sup> permits two-bar bundles for No. 14 and No. 18 bars in bridge girders.) Conformance to the crack control requirements of 10.6 will effectively preclude bundling of

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**7.6.6.4** — Individual bars within a bundle terminated within the span of flexural members shall terminate at different points with at least  **$40d_b$**  stagger.

**7.6.6.5** — Where spacing limitations or concrete cover requirements are based on bar diameter,  $d_b$ , a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

**7.6.7 — Tendons and ducts**

**7.6.7.1** — Center-to-center spacing of pretensioning tendons at each end of a member shall be not less than  **$4d_b$**  for strands, or  **$5d_b$**  for wire, except that if specified compressive strength of concrete at time of initial prestress,  $f'_{ci}$ , is 4000 psi or more, minimum center-to-center spacing of strands shall be 1-3/4 in. for strands of 1/2 in. nominal diameter or smaller and 2 in. for strands of 0.6 in. nominal diameter. See also **3.3.2**. Closer vertical spacing and bundling of tendons shall be permitted in the middle portion of a span.

**7.6.7.2** — Bundling of post-tensioning ducts shall be permitted if shown that concrete can be satisfactorily placed and if provision is made to prevent the prestressing steel, when tensioned, from breaking through the duct.

**7.7 — Concrete protection for reinforcement****7.7.1 — Cast-in-place concrete (nonprestressed)**

Unless a greater concrete cover is required by **7.7.6** or **7.7.8**, specified cover for reinforcement shall not be less than the following:

	Concrete cover, in.
(a) Concrete cast against and permanently exposed to earth.....	3
(b) Concrete exposed to earth or weather:	
No. 6 through No. 18 bars.....	2
No. 5 bar, W31 or D31 wire, and smaller .....	1-1/2
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists:	
No. 14 and No. 18 bars .....	1-1/2
No. 11 bar and smaller .....	3/4

bars larger than No. 11 as tensile reinforcement. The Code phrasing “bundled in contact to act as a unit,” is intended to preclude bundling more than two bars in the same plane. Typical bundle shapes are triangular, square, or L-shaped patterns for three- or four-bar bundles. As a practical caution, bundles more than one bar deep in the plane of bending should not be hooked or bent as a unit. Where end hooks are required, it is preferable to stagger the individual bar hooks within a bundle.

**R7.6.7 — Tendons and ducts**

**R7.6.7.1** — The allowed decreased spacing in this section for transfer strengths of 4000 psi or greater is based on References **7.10** and **7.11**.

**R7.6.7.2** — When ducts for prestressing steel in a beam are arranged closely together vertically, provision should be made to prevent the prestressing steel from breaking through the duct when tensioned. Horizontal disposition of ducts should allow proper placement of concrete. A clear spacing of one and one-third times the size of the coarse aggregate, but not less than 1 in., has proven satisfactory. Where concentration of tendons or ducts tends to create a weakened plane in the concrete cover, reinforcement should be provided to control cracking.

**R7.7 — Concrete protection for reinforcement**

Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies. Where concrete cover is prescribed for a class of structural members, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars; to the outermost layer of bars if more than one layer is used without stirrups or ties; or to the metal end fitting or duct on post-tensioned prestressing steel; or to the outermost part of the head on headed bars.

The condition “concrete surfaces exposed to earth or weather” refers to direct exposure to moisture changes and not just to temperature changes. Slab or thin shell soffits are not usually considered directly exposed unless subject to alternate wetting and drying, including that due to condensation conditions or direct leakage from exposed top surface, run off, or similar effects.

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Beams, columns:	
Primary reinforcement, ties, stirrups, spirals.....	1-1/2
Shells, folded plate members:	
No. 6 bar and larger .....	3/4
No. 5 bar, W31 or D31 wire, and smaller .....	1/2

**7.7.2 — Cast-in-place concrete (prestressed)**

Unless a greater concrete cover is required by 7.7.6 or 7.7.8, specified cover for prestressed and nonprestressed reinforcement, ducts, and end fittings shall not be less than the following:

Concrete  
cover, in.

(a) Concrete cast against and permanently exposed to earth .....	3
(b) Concrete exposed to earth or weather:	
Wall panels, slabs, joists.....	1
Other members.....	1-1/2
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists.....	3/4
Beams, columns:	
Primary reinforcement.....	1-1/2
Ties, stirrups, spirals .....	1
Shells, folded plate members:	
No. 5 bar, W31 or D31 wire, and smaller .....	3/8
Other reinforcement .....	$d_b$ but not less than 3/4

**7.7.3 — Precast concrete (manufactured under  
plant control conditions)**

Unless a greater cover is required by 7.7.6 or 7.7.8, specified cover for prestressed and nonprestressed reinforcement, ducts, and end fittings shall not be less than the following:

Concrete  
cover, in.

(a) Concrete exposed to earth or weather:	
Wall panels:	
No. 14 and No. 18 bars, prestressing tendons larger than 1-1/2 in. diameter .....	1-1/2
No. 11 bar and smaller, prestressing tendons 1-1/2 in. diameter and smaller, W31 and D31 wire and smaller.....	3/4
Other members:	
No. 14 and No. 18 bars, prestressing tendons larger than 1-1/2 in. diameter .....	2
No. 6 through No. 11 bars, prestressing tendons larger than 5/8 in. diameter through 1-1/2 in. diameter.....	1-1/2

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Alternative methods of protecting the reinforcement from weather may be provided if they are equivalent to the additional concrete cover required by the Code. When approved by the building official under the provisions of 1.4, reinforcement with alternative protection from the weather may have concrete cover not less than the cover required for reinforcement not exposed to weather.

The development length given in Chapter 12 is now a function of the bar cover. As a result, it may be desirable to use larger than minimum cover in some cases.

**R7.7.3 — Precast concrete (manufactured under plant  
control conditions)**

The lesser cover thicknesses for precast construction reflect the greater convenience of control for proportioning, placing, and curing inherent in precasting. The term “manufactured under plant control conditions” does not specifically imply that precast members should be manufactured in a plant. Structural elements precast at the job site will also qualify under this section if the control of form dimensions, placing of reinforcement, quality control of concrete, and curing procedure are equal to that normally expected in a plant.

Concrete cover to pretensioned strand as described in this section is intended to provide minimum protection against weather and other effects. Such cover may not be sufficient to transfer or develop the stress in the strand, and it may be necessary to increase the cover accordingly.

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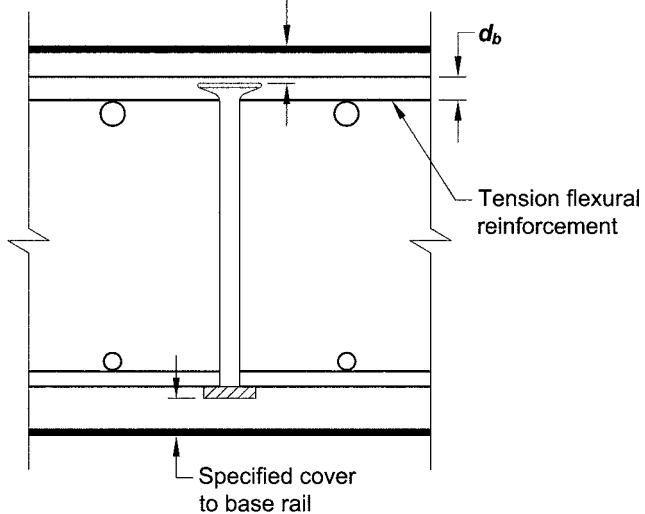
No. 5 bar and smaller, prestressing tendons 5/8 in. diameter and smaller, W31 and D31 wire, and smaller.....	1-1/4
(b) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists:	
No. 14 and No. 18 bars, prestressing tendons larger than 1-1/2 in. diameter .....	1-1/4
Prestressing tendons 1-1/2 in. diameter and smaller.....	3/4
No. 11 bar and smaller, W31 or D31 wire, and smaller.....	5/8
Beams, columns:	
Primary reinforcement .....	$d_b$ but not less than 5/8 and need not exceed 1-1/2
Ties, stirrups, spirals .....	3/8
Shells, folded plate members:	
Prestressing tendons.....	3/4
No. 6 bar and larger .....	5/8
No. 5 bar and smaller, W31 or D31 wire, and smaller.....	3/8

**7.7.4 — Bundled bars**

For bundled bars, minimum specified concrete cover shall not be less than the equivalent diameter of the bundle, but need not be greater than 2 in.; except for concrete cast against and permanently exposed to earth, where specified concrete cover shall not be less than 3 in.

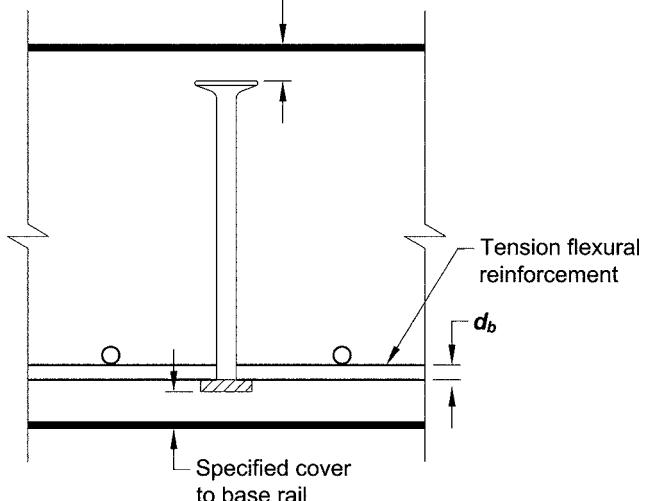
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$$\text{Maximum cover to head (11.11.5)} \\ = (d_b/2) + \text{specified cover}$$



(a) Slab with top and bottom bars

$$\text{Maximum cover to head (11.11.5)} \\ = (d_b/2) + \text{specified cover}$$



(b) Footing with only bottom bars

Fig. R7.7.5—Concrete cover requirements for headed shear stud reinforcement.

**7.7.5 — Headed shear stud reinforcement**

For headed shear stud reinforcement, specified concrete cover for the heads or base rails shall not be less than that required for the reinforcement in the type of member in which the headed shear stud reinforcement is placed.

**R.7.7.5 — Headed shear stud reinforcement**

The shanks, the heads, and the base rails need to be protected by the specified concrete cover. For efficiency in controlling inclined shear cracks, the overall height of the headed stud assembly should be as large as permissible (R11.11.5). The maximum overall height of the headed stud assembly is equal to the thickness of the member less the sum of the specified concrete covers required for the heads and base rails as shown in Fig. R7.7.5.

**CODE****COMMENTARY****7.7.6 — Corrosive environments**

In corrosive environments or other severe exposure conditions, the concrete cover shall be increased as deemed necessary and specified by the licensed design professional. The applicable requirements for concrete based on exposure categories and classes in **Chapter 4** shall be met, or other protection shall be provided.

**7.7.6.1** — For prestressed concrete members exposed to corrosive environments or other severe exposure categories such as those defined in **Chapter 4**, and which are classified as Class T or C in **18.3.3**, specified concrete cover shall not be less than 1.5 times the cover for prestressed reinforcement required by **7.7.2** and **7.7.3**. This requirement shall be permitted to be waived if the precompressed tensile zone is not in tension under sustained loads.

**7.7.7 — Future extensions**

Exposed reinforcement, inserts, and plates intended for bonding with future extensions shall be protected from corrosion.

**7.7.8 — Fire protection**

If the general building code (of which this Code forms a part) requires a thickness of cover for fire protection greater than the concrete cover in **7.7.1** through **7.7.7**, such greater thicknesses shall be specified.

**7.8 — Reinforcement details for columns****7.8.1 — Offset bars**

Offset bent longitudinal bars shall conform to the following:

**7.8.1.1** — Slope of inclined portion of an offset bar with axis of column shall not exceed 1 in 6.

**7.8.1.2** — Portions of bar above and below an offset shall be parallel to axis of column.

**7.8.1.3** — Horizontal support at offset bends shall be provided by transverse ties, spirals, or parts of the floor construction. Horizontal support provided shall be

**R7.7.6 — Corrosive environments**

Where concrete will be exposed to external sources of chlorides in service, such as deicing salts, brackish water, seawater, or spray from these sources, concrete should be proportioned to satisfy the requirements for the applicable exposure class in **Chapter 4**. These include minimum air content, maximum  $w/cm$ , minimum strength for normal-weight and lightweight concrete, maximum chloride ion in concrete, and cement type. Additionally, for corrosion protection, a specified concrete cover for reinforcement not less than 2 in. for walls and slabs and not less than 2-1/2 in. for other members is recommended. For precast concrete members manufactured under plant control conditions, a specified concrete cover not less than 1-1/2 in. for walls and slabs and not less than 2 in. for other members is recommended.

**R7.7.6.1** — Corrosive environments are defined in **Sections 4.2, R4.2.1, and R4.3.1**. Additional information on corrosion in parking structures is given in ACI 362.1R.<sup>7.12</sup>

**R7.8 — Reinforcement details for columns**

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designed to resist 1-1/2 times the horizontal component of the computed force in the inclined portion of an offset bar. Transverse ties or spirals, if used, shall be placed not more than 6 in. from points of bend.

**7.8.1.4** — Offset bars shall be bent before placement in the forms. See **7.3**.

**7.8.1.5** — Where a column face is offset 3 in. or greater, longitudinal bars shall not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, shall be provided. Lap splices shall conform to **12.17**.

### **7.8.2 — Steel cores**

Load transfer in structural steel cores of composite compression members shall be provided by the following:

**7.8.2.1** — Ends of structural steel cores shall be accurately finished to bear at end bearing splices, with positive provision for alignment of one core above the other in concentric contact.

**7.8.2.2** — At end bearing splices, bearing shall be considered effective to transfer not more than 50 percent of the total compressive stress in the steel core.

**7.8.2.3** — Transfer of stress between column base and footing shall be designed in accordance with **15.8**.

**7.8.2.4** — Base of structural steel section shall be designed to transfer the total load from the entire composite member to the footing; or, the base shall be designed to transfer the load from the steel core only, provided ample concrete section is available for transfer of the portion of the total load carried by the reinforced concrete section to the footing by compression in the concrete and by reinforcement.

### **7.9 — Connections**

**7.9.1** — At connections of principal framing elements (such as beams and columns), enclosure shall be provided for splices of continuing reinforcement and for anchorage of reinforcement terminating in such connections.

**7.9.2** — Enclosure at connections shall consist of external concrete or internal closed ties, spirals, or stirrups.

### **R7.8.2 — Steel cores**

The 50 percent limit on transfer of compressive load by end bearing on ends of structural steel cores is intended to provide some tensile strength at such splices (up to 50 percent), since the remainder of the total compressive stress in the steel core are to be transmitted by dowels, splice plates, welds, etc. This provision should ensure that splices in composite compression members meet essentially the same tensile strength as required for conventionally reinforced concrete compression members.

### **R7.9 — Connections**

Confinement is essential at connections to ensure that the flexural strength of the members can be developed without deterioration of the joint under repeated loadings.<sup>7.13,7.14</sup>

**CODE****COMMENTARY****| 7.10 — Transverse reinforcement for compression members**

| **7.10.1** — Transverse reinforcement for compression members shall conform to the provisions of 7.10.4 and **7.10.5** and, where shear or torsion reinforcement is required, shall also conform to provisions of **Chapter 11**.

| **7.10.2** — Transverse reinforcement requirements for composite compression members shall conform to **10.13**. Transverse reinforcement requirements for tendons shall conform to **18.11**.

| **7.10.3** — It shall be permitted to waive the transverse reinforcement requirements of 7.10, **10.13**, and **18.11** where tests and structural analysis show adequate strength and feasibility of construction.

**7.10.4 — Spirals**

Spiral reinforcement for compression members shall conform to **10.9.3** and to the following:

**7.10.4.1** — Spirals shall consist of evenly spaced continuous bar or wire of such size and so assembled to permit handling and placing without distortion from designed dimensions.

**7.10.4.2** — For cast-in-place construction, size of spirals shall not be less than 3/8 in. diameter.

**7.10.4.3** — Clear spacing between spirals shall not exceed 3 in., nor be less than 1 in. See also **3.3.2**.

**7.10.4.4** — Anchorage of spiral reinforcement shall be provided by 1-1/2 extra turns of spiral bar or wire at each end of a spiral unit.

**7.10.4.5** — Spiral reinforcement shall be spliced, if needed, by any one of the following methods:

(a) Lap splices not less than the larger of 12 in. and the length indicated in one of (1) through (5) below:

- (1) deformed uncoated bar or wire,  
or deformed zinc-coated  
(galvanized) bar..... **48d<sub>b</sub>**
- (2) plain uncoated bar or wire,  
or plain zinc-coated (galvanized) bar..... **72d<sub>b</sub>**

**| R7.10 — Transverse reinforcement for compression members**

**R7.10.3** — Precast columns with cover less than 1-1/2 in., prestressed columns without longitudinal bars, columns smaller than minimum dimensions prescribed in earlier Code editions, columns of concrete with small size coarse aggregate, wall-like columns, and other unusual cases may require special designs for transverse reinforcement. Wire, W4, D4, or larger, may be used for ties or spirals. If such unusual columns are considered as spiral columns for load strength in design, the volumetric reinforcement ratio for the spiral,  $\rho_s$ , is to conform to **10.9.3**.

**R7.10.4 — Spirals**

For practical considerations in cast-in-place construction, the minimum diameter of spiral reinforcement is 3/8 in. (3/8 in. round, No. 3 bar, or equivalent deformed or plain wire). This is the smallest size that can be used in a column with 1-1/2 in. or more cover and having concrete compressive strengths of 3000 psi or more if the minimum clear spacing for placing concrete is to be maintained.

Standard spiral sizes are 3/8, 1/2, and 5/8 in. diameter for hot rolled or cold drawn material, plain or deformed.

The Code allows spirals to be terminated at the level of lowest horizontal reinforcement framing into the column. However, if one or more sides of the column are not enclosed by beams or brackets, ties are required from the termination of the spiral to the bottom of the slab, drop panel, or shear cap. If beams or brackets enclose all sides of the column but are of different depths, the ties should extend from the spiral to the level of the horizontal reinforcement of the shallowest beam or bracket framing into the column. These additional ties are to enclose the longitudinal column reinforcement and the portion of bars from beams bent into the column for anchorage. See also **7.9**.

Spirals should be held firmly in place, at proper pitch and alignment, to prevent displacement during concrete placement. The Code has traditionally required spacers to hold the fabricated spiral cage in place but was changed in 1989 to allow alternate methods of installation. When spacers are used, the following may be used for guidance: for spiral bar

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- (3) epoxy-coated deformed bar or wire, or zinc and epoxy dual-coated deformed bar .....  **$72d_b$**
  - (4) plain uncoated bar or wire, or plain zinc-coated (galvanized) bar, which have a standard stirrup or tie hook in accordance with 7.1.3 at ends of lapped spiral reinforcement. The hooks shall be embedded within the core confined by the spiral reinforcement .....  **$48d_b$**
  - (5) epoxy-coated deformed bar or wire, or zinc and epoxy dual-coated deformed bar, which have a standard stirrup or tie hook in accordance with 7.1.3 at ends of lapped spiral reinforcement. The hooks shall be embedded within the core confined by the spiral reinforcement .....  **$48d_b$**
- (b) Full mechanical or welded splices in accordance with 12.14.3.

**7.10.4.6** — Spirals shall extend from top of footing or slab in any story to level of lowest horizontal reinforcement in members supported above.

**7.10.4.7** — Where beams or brackets do not frame into all sides of a column, ties shall extend above termination of spiral to bottom of slab, drop panel, or shear cap.

**7.10.4.8** — In columns with capitals, spirals shall extend to a level at which the diameter or width of capital is two times that of the column.

**7.10.4.9** — Spirals shall be held firmly in place and true to line.

**7.10.5 — Ties**

Tie reinforcement for compression members shall conform to the following:

**7.10.5.1** — All non prestressed bars shall be enclosed by transverse ties, at least No. 3 in size for longitudinal bars No. 10 or smaller, and at least No. 4 in size for No. 11, No. 14, No. 18, and bundled longitudinal bars. Deformed wire or welded wire reinforcement of equivalent area shall be permitted.

**7.10.5.2** — Vertical spacing of ties shall not exceed 16 longitudinal bar diameters, 48 tie bar or wire diameters, or least dimension of the compression member.

**7.10.5.3** — Rectilinear ties shall be arranged such that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie

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or wire smaller than 5/8 in. diameter, a minimum of two spacers should be used for spirals less than 20 in. in diameter, three spacers for spirals 20 to 30 in. in diameter, and four spacers for spirals greater than 30 in. in diameter. For spiral bar or wire 5/8 in. diameter or larger, a minimum of three spacers should be used for spirals 24 in. or less in diameter, and four spacers for spirals greater than 24 in. in diameter. The project specifications or subcontract agreements should be clearly written to cover the supply of spacers or field tying of the spiral reinforcement. In the 1999 Code, splice requirements were modified for epoxy-coated and plain spirals and to allow mechanical splices.

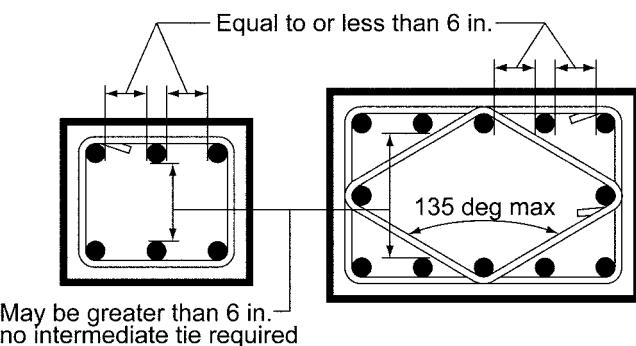
**R7.10.5 — Ties**

All longitudinal bars in compression should be enclosed within transverse ties. Where longitudinal bars are arranged in a circular pattern, only one circular tie per specified spacing is required. This requirement can be satisfied by a continuous circular tie (helix) at larger pitch than required for spirals under 10.9.3, the maximum pitch being equal to the required tie spacing (see also 7.10.4.3).

The 1956 Code required “lateral support equivalent to that provided by a 90-degree corner of a tie,” for every vertical bar. Tie requirements were liberalized in 1963 by increasing the permissible included angle from 90 to 135 degrees and exempting bars that are located within 6 in. clear on each side along the tie from adequately tied bars (see Fig. R7.10.5). Limited tests<sup>7.15</sup> on full-size, axially-loaded, tied columns containing full-length bars (without splices) showed no appreciable difference between ultimate strengths of columns with full tie requirements and no ties at all.

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with an included angle of not more than 135 degrees and no bar shall be farther than 6 in. clear on each side along the tie from such a laterally supported bar.

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*Fig. R7.10.5—Sketch to clarify measurements between laterally supported column bars.*

Since spliced bars and bundled bars were not included in the tests of Reference 7.15, it is prudent to provide a set of ties at each end of lap spliced bars, above and below end-bearing splices, and at minimum spacings immediately below sloping regions of offset bent bars.

Standard tie hooks are intended for use with deformed bars only, and should be staggered where possible. See also 7.9.

Continuously wound bars or wires can be used as ties provided their pitch and area are at least equivalent to the area and spacing of separate ties. Anchorage at the end of a continuously wound bar or wire should be by a standard hook as for separate bars or by one additional turn of the tie pattern. A circular continuously wound bar or wire is considered a spiral if it conforms to 7.10.4, otherwise it is considered a tie.

**7.10.5.4** — Where longitudinal bars are located around the perimeter of a circle, a complete circular tie shall be permitted. The ends of the circular tie shall overlap by not less than 6 in. and terminate with standard hooks that engage a longitudinal column bar. Overlaps at ends of adjacent circular ties shall be staggered around the perimeter enclosing the longitudinal bars.

**7.10.5.5** — Ties shall be located vertically not more than one-half a tie spacing above the top of footing or slab in any story, and shall be spaced as provided herein to not more than one-half a tie spacing below the lowest horizontal reinforcement in slab, drop panel, or shear cap above.

**7.10.5.6** — Where beams or brackets frame from four directions into a column, termination of ties not more than 3 in. below lowest reinforcement in shallowest of such beams or brackets shall be permitted.

**R7.10.5.4** — Vertical splitting and loss of tie restraint are possible where the overlapped ends of adjacent circular ties are anchored at a single longitudinal bar. Adjacent circular ties should not engage the same longitudinal bar with end hook anchorages. While the transverse reinforcement in members with longitudinal bars located around the periphery of a circle can be either spirals or circular ties, spirals are usually more effective.

**R7.10.5.6** — With the 1983 Code, the wording of this section was modified to clarify that ties may be terminated only when elements frame into all four sides of square and rectangular columns; for round or polygonal columns, such elements frame into the column from four directions.

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**7.10.5.7** — Where anchor bolts are placed in the top of columns or pedestals, the bolts shall be enclosed by transverse reinforcement that also surrounds at least four vertical bars of the column or pedestal. The transverse reinforcement shall be distributed within 5 in. of the top of the column or pedestal, and shall consist of at least two No. 4 or three No. 3 bars.

**7.11 — Transverse reinforcement for flexural members**

**7.11.1** — Compression reinforcement in beams shall be enclosed by ties or stirrups satisfying the size and spacing limitations in **7.10.5** or by welded wire reinforcement of equivalent area. Such ties or stirrups shall be provided throughout the distance where compression reinforcement is required.

**7.11.2** — Transverse reinforcement for flexural framing members subject to stress reversals or to torsion at supports shall consist of closed ties, closed stirrups, or spirals extending around the flexural reinforcement.

**7.11.3** — Closed ties or stirrups shall be formed in one piece by overlapping standard stirrup or tie end hooks around a longitudinal bar, or formed in one or two pieces lap spliced with a Class B splice (lap of  $1.3\ell_d$ ) or anchored in accordance with **12.13**.

**7.12 — Shrinkage and temperature reinforcement**

**7.12.1** — Reinforcement for shrinkage and temperature stresses normal to flexural reinforcement shall be provided in structural slabs where the flexural reinforcement extends in one direction only.

**7.12.1.1** — Shrinkage and temperature reinforcement shall be provided in accordance with either **7.12.2** or **7.12.3**.

**7.12.1.2** — Where shrinkage and temperature movements are significantly restrained, the requirements of **8.2.4** and **9.2.3** shall be considered.

**R7.10.5.7** — Provisions for confinement of anchor bolts that are placed in the top of columns or pedestals were added in the 2002 Code. Confinement improves load transfer from the anchor bolts to the column or pier for situations where the concrete cracks in the vicinity of the bolts. Such cracking can occur due to unanticipated forces caused by temperature, restrained shrinkage, and similar effects.

**R7.11 — Transverse reinforcement for flexural members**

**R7.11.1** — Compression reinforcement in beams and girders should be enclosed to prevent buckling; similar requirements for such enclosure have remained essentially unchanged through several editions of the Code, except for minor clarification.

**R7.12 — Shrinkage and temperature reinforcement**

**R7.12.1** — Shrinkage and temperature reinforcement is required at right angles to the principal reinforcement to minimize cracking and to tie the structure together to ensure it is acting as assumed in the design. The provisions of this section are intended for structural slabs only; they are not intended for slabs-on-ground.

**R7.12.1.2** — The area of shrinkage and temperature reinforcement required by **7.12.2.1** has been satisfactory where shrinkage and temperature movements are permitted to occur. Where structural walls or columns provide significant restraint to shrinkage and temperature movements, the restraint of volume changes causes tension in slabs, as well as displacements, shear forces, and flexural moments in columns or walls. In these cases, it may be necessary to increase the amount of slab reinforcement required by **7.12.2.1** due to the shrinkage and thermal effects in both principal directions (see **References 7.7** and **7.16**). Topping slabs also experience tension due to restraint of differential shrinkage between the topping and the precast elements or metal deck (which has zero shrinkage) that should be considered in reinforcing the

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**7.12.2** — Deformed reinforcement conforming to **3.5.3** used for shrinkage and temperature reinforcement shall be provided in accordance with the following:

**7.12.2.1** — Area of shrinkage and temperature reinforcement shall provide at least the following ratios of reinforcement area to gross concrete area, but not less than 0.0014:

(a) Slabs where Grade 40 or 50 deformed bars are used .....	0.0020
(b) Slabs where Grade 60 deformed bars or welded wire reinforcement are used .....	0.0018
(c) Slabs where reinforcement with yield stress exceeding 60,000 psi measured at a yield strain of 0.35 percent is used.....	$\frac{0.0018 \times 60,000}{f_y}$

**7.12.2.2** — Shrinkage and temperature reinforcement shall be spaced not farther apart than five times the slab thickness, nor farther apart than 18 in.

**7.12.2.3** — At all sections where required, reinforcement to resist shrinkage and temperature stresses shall develop  $f_y$  in tension in accordance with **Chapter 12**.

**7.12.3** — Prestressing steel conforming to **3.5.6** used for shrinkage and temperature reinforcement shall be provided in accordance with the following:

**7.12.3.1** — Tendons shall be proportioned to provide a minimum average compressive stress of 100 psi on gross concrete area using effective prestress, after losses, in accordance with **18.6**.

**7.12.3.2** — For monolithic cast-in-place post-tensioned beam-and-slab construction, gross concrete area of a beam and tributary slab shall consist of the total beam area including the slab thickness and the slab within half the clear distance to adjacent beam webs. It shall be permitted to include the effective force in beam tendons in the calculation of total prestress force acting on gross concrete area.

slab. Consideration should be given to strain demands on reinforcement crossing joints of precast elements where most of the restraint is likely to be relieved. Top and bottom reinforcement are both effective in controlling cracks. Control strips during the construction period, which permit initial shrinkage to occur without causing an increase in stresses, are also effective in reducing cracks caused by restraint.

**R7.12.2** — The amounts specified for deformed bars and welded wire reinforcement are empirical but have been used satisfactorily for many years. The area of reinforcement given by 7.12.2.1 may be distributed near the top or bottom of the slab, or may be allocated between the two faces of the slab as deemed appropriate for specific conditions. Splices and end anchorages of shrinkage and temperature reinforcement are to be designed for the full specified yield strength in accordance with **12.1**, **12.15**, **12.18**, and **12.19**.

**R7.12.3** — Prestressed reinforcement requirements have been selected to provide an effective force on the slab approximately equal to the yield strength force for nonprestressed shrinkage and temperature reinforcement. This amount of prestressing, 100 psi on the gross concrete area, has been successfully used on a large number of projects. In monolithic beam-and-slab construction, a minimum of one shrinkage and temperature tendon is required between beams, even if the beam tendons alone provide at least 100 psi average compression stress on the gross concrete area as defined in 7.12.3.2. Any size tendon is permissible as long as all other requirements of 7.12.3 are satisfied. Application of the provisions of 7.12.3.2 to monolithic cast-in-place post-tensioned beam-and-slab construction is illustrated in **Fig. R7.12.3(a)**.

Where the spacing of slab tendons used for shrinkage and temperature reinforcement exceeds 4.5 ft, additional nonprestressed reinforcement is required to extend from the

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**7.12.3.3** — Where slabs are supported on walls or not cast monolithically with beams, gross concrete area is the slab section tributary to the tendon or tendon group.

**7.12.3.4** — In all cases, a minimum of one slab tendon is required between faces of beams or walls. Spacing of slab tendons, and the distance between face of beam or wall to the nearest slab tendon, shall not exceed 6 ft.

**7.12.3.5** — Where spacing of slab tendons exceeds 4.5 ft, additional non prestressed shrinkage and temperature reinforcement conforming to **7.12.2** shall be provided between faces of beams or walls, parallel to the slab shrinkage and temperature tendons. This additional shrinkage and temperature reinforcement shall extend from the slab edges for a distance greater than or equal to the tendon spacing, except **7.12.2.3** shall not apply.

slab edges where the prestressing forces are applied in order to adequately reinforce the area between the slab edge and the point where compressive stresses behind individual anchorages have spread sufficiently such that the slab is uniformly in compression (see Fig. R7.12.3(b)).

Tendons used for shrinkage and temperature reinforcement should be positioned as closely as practicable to mid-depth of the slab. In cases where the shrinkage and temperature tendons are used for supporting the principal tendons, variations from the slab centroid are permissible; however, the resultant of the shrinkage and temperature tendons should not fall outside the kern area of the slab.

The effects of slab shortening should be evaluated to ensure proper action. In most cases, the low level of prestressing recommended should not cause difficulties in a properly detailed structure. Additional attention may be required where thermal effects become significant.

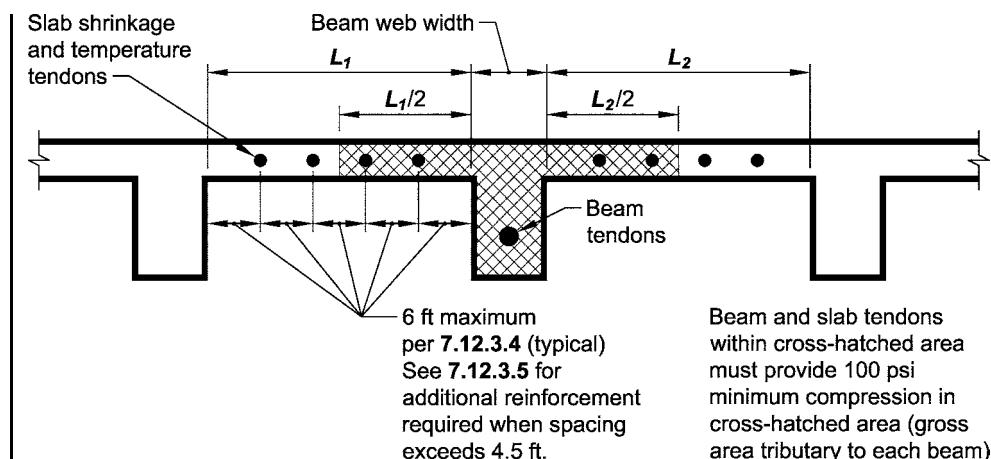


Fig. R7.12.3(a)—Section through beams cast monolithically with slab.

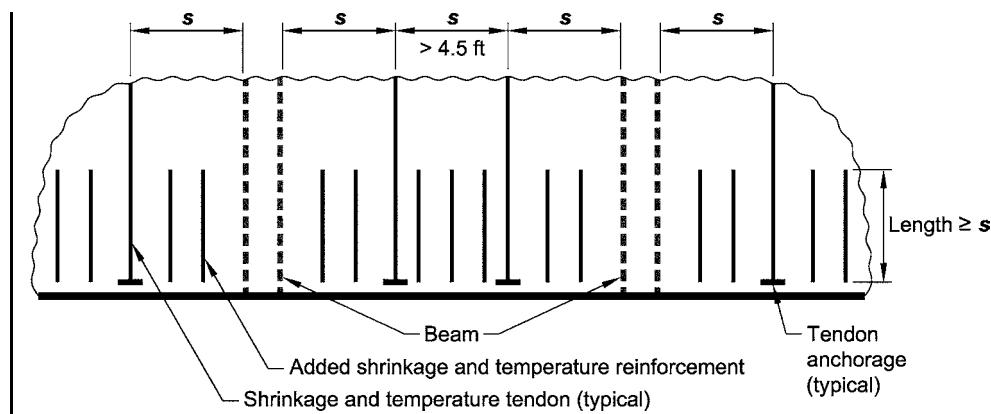


Fig. R7.12.3(b)—Plan at slab edge showing added shrinkage and temperature reinforcement (see 7.12.3.5).

**CODE****COMMENTARY****7.13 — Requirements for structural integrity**

**7.13.1** — In the detailing of reinforcement and connections, members of a structure shall be effectively tied together to improve integrity of the overall structure.

**7.13.2** — For cast-in-place construction, the following shall constitute minimum requirements:

**7.13.2.1** — In joist construction, as defined in [8.13.1](#) through [8.13.3](#), at least one bottom bar shall be continuous or shall be spliced with a Class B tension lap splice or a mechanical or welded splice satisfying [12.14.3](#) and at noncontinuous supports shall be anchored to develop  $f_y$  at the face of the support using a standard hook satisfying [12.5](#) or headed deformed bar satisfying [12.6](#).

**7.13.2.2** — Beams along the perimeter of the structure shall have continuous reinforcement over the span length passing through the region bounded by the longitudinal reinforcement of the column consisting of (a) and (b):

- (a) at least one-sixth of the tension reinforcement required for negative moment at the support, but not less than two bars;
- (b) at least one-quarter of the tension reinforcement required for positive moment at midspan, but not less than two bars.

At noncontinuous supports, the reinforcement shall be anchored to develop  $f_y$  at the face of the support using a standard hook satisfying [12.5](#) or headed deformed bar satisfying [12.6](#).

**7.13.2.3** — The continuous reinforcement required in 7.13.2.2 shall be enclosed by transverse reinforcement of the type specified in [11.5.4.1](#). The transverse reinforcement shall be anchored as specified in [11.5.4.2](#). The transverse reinforcement need not be extended through the column.

**7.13.2.4** — Where splices are required to satisfy 7.13.2.2, the top reinforcement shall be spliced at or near midspan and bottom reinforcement shall be spliced at or near the support. Splices shall be Class B tension lap splices, or mechanical or welded splices satisfying [12.14.3](#).

**R7.13 — Requirements for structural integrity**

Experience has shown that the overall integrity of a structure can be substantially enhanced by minor changes in detailing of reinforcement. It is the intent of this section of the Code to improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance to maintain overall stability.

**R7.13.2** — With damage to a support, top reinforcement that is continuous over the support, but not confined by stirrups, will tend to tear out of the concrete and will not provide the catenary action needed to bridge the damaged support. By making a portion of the bottom reinforcement continuous, catenary action can be provided.

Requiring continuous top and bottom reinforcement in perimeter or spandrel beams provides a continuous tie around the structure. It is not the intent to require a tensile tie of continuous reinforcement of constant size around the entire perimeter of a structure, but simply to require that one half of the top flexural reinforcement required to extend past the point of inflection by [12.12.3](#) be further extended and spliced at or near midspan. Similarly, the bottom reinforcement required to extend into the support by [12.11.1](#) should be made continuous or spliced with bottom reinforcement from the adjacent span. If the depth of a continuous beam changes at a support, the bottom reinforcement in the deeper member should be terminated with a standard hook and bottom reinforcement in the shallower member should be extended into and fully developed in the deeper member.

In the 2002 Code, provisions were added to permit the use of mechanical or welded splices for splicing reinforcement, and the detailing requirements for the longitudinal reinforcement and stirrups in beams were revised. Section 7.13.2 was revised in 2002 to require U-stirrups with not less than 135-degree hooks around the continuous bars, or one-piece closed stirrups to prevent the top continuous bars from tearing out of the top of the beam. Section 7.13.2 was revised in 2008 to require that the transverse reinforcement used to enclose the continuous reinforcement be of the type specified in [11.5.4.1](#) and anchored according to [11.5.4.2](#). [Figure R7.13.2](#) shows an example of a two-piece stirrup that satisfies these requirements. Pairs of U-stirrups lapping one another as defined in [12.13.5](#) are not permitted in perimeter or spandrel beams. In the event of damage to the side concrete cover, the stirrups and top longitudinal reinforcement may tend to tear out of the concrete. Thus, the top longitudinal reinforcement will not provide the catenary action needed to bridge over a damaged region. Further, lapped U-stirrups will not be effective at high torque (see [R11.5.4.1](#)).

**CODE****COMMENTARY**

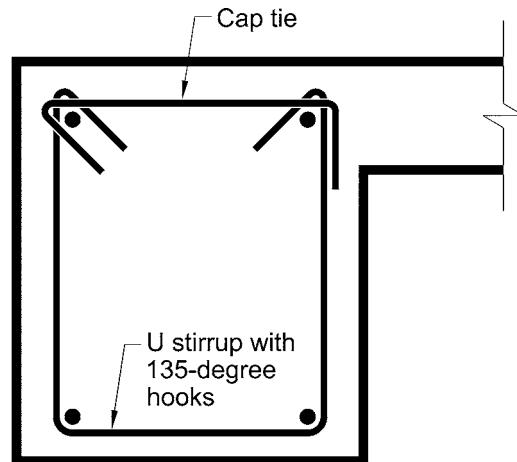
**7.13.2.5** — In other than perimeter beams, where transverse reinforcement as defined in **7.13.2.3** is provided, there are no additional requirements for longitudinal integrity reinforcement. Where such transverse reinforcement is not provided, at least one-quarter of the positive moment reinforcement required at midspan, but not less than two bars, shall pass through the region bounded by the longitudinal reinforcement of the column and shall be continuous or shall be spliced over or near the support with a Class B tension lap splice, or a mechanical or welded splice satisfying **12.14.3**. At noncontinuous supports, the reinforcement shall be anchored to develop  $f_y$  at the face of the support using a standard hook satisfying **12.5** or headed deformed bar satisfying **12.6**.

**7.13.2.6** — For non prestressed two-way slab construction, see **13.3.8.5**.

**7.13.2.7** — For prestressed two-way slab construction, see **18.12.6** and **18.12.7**.

**7.13.3** — For precast concrete construction, tension ties shall be provided in the transverse, longitudinal, and vertical directions and around the perimeter of the structure to effectively tie elements together. The provisions of **16.5** shall apply.

**7.13.4** — For lift-slab construction, see **13.3.8.6** and **18.12.8**.



*Fig. R7.13.2—Example of a two-piece stirrup that complies with the requirements of **7.13.2.3**.*

Lap splices were changed from Class A to Class B in ACI 318-08 to provide similar strength to that provided by mechanical and welded splices satisfying **12.14.3**. Class B lap splices provide a higher level of reliability for abnormal loading events.

**R7.13.3** — The Code requires tension ties for precast concrete buildings of all heights. Details should provide connections to resist applied loads. Connection details that rely solely on friction caused by gravity forces are not permitted by **16.5.1.4**.

Connection details should be arranged so as to minimize the potential for cracking due to restrained creep, shrinkage, and temperature movements. For information on connections and detailing requirements, see **Reference 7.17**.

**Reference 7.18** recommends minimum tie requirements for precast concrete bearing wall buildings.

**Notes**

# CHAPTER 8 — ANALYSIS AND DESIGN — GENERAL CONSIDERATIONS

## CODE

### 8.1 — Design methods

**8.1.1** — In design of structural concrete, members shall be proportioned for adequate strength in accordance with provisions of this Code, using load factors and strength reduction factors  $\phi$  specified in [Chapter 9](#).

**8.1.2** — Design of reinforced concrete using the provisions of [Appendix B](#) shall be permitted.

**8.1.3** — Anchors within the scope of [Appendix D](#) installed in concrete to transfer loads between connected elements shall be designed using [Appendix D](#).

### 8.2 — Loading

**8.2.1** — Design provisions of this Code are based on the assumption that structures shall be designed to resist all applicable loads.

**8.2.2** — Service loads shall be in accordance with the general building code of which this Code forms a part, with such live load reductions as are permitted in the general building code.

**8.2.3** — In design for wind and earthquake loads, integral structural parts shall be designed to resist the total lateral loads.

## COMMENTARY

### R8.1 — Design methods

**R8.1.1** — The strength design method requires service loads or related internal moments and forces to be increased by specified load factors (required strength) and computed nominal strengths to be reduced by specified strength reduction factors  $\phi$  (design strength).

**R8.1.2** — Designs in accordance with [Appendix B](#) are equally acceptable, provided the provisions of [Appendix B](#) are used in their entirety.

An appendix may be judged not to be an official part of a legal document unless specifically adopted. Therefore, specific reference is made to [Appendix B](#) in the main body of the Code to make it a legal part of the Code.

**R8.1.3** — The Code included specific provisions for anchoring to concrete for the first time in the 2002 edition. As has been done in the past with a number of new sections and chapters, new material has been presented as an appendix.

An appendix may be judged not to be an official part of a legal document unless specifically adopted. Therefore, specific reference is made to [Appendix D](#) in the main part of the Code to make it a legal part of the Code.

### R8.2 — Loading

The provisions in the Code are for live, wind, and earthquake loads such as those recommended in “**Minimum Design Loads for Buildings and Other Structures**” (ASCE/SEI 7),<sup>8,1</sup> formerly known as ANSI A58.1. If the service loads specified by the general building code (of which this Code forms a part) differ from those of ASCE/SEI 7, the general building code governs. However, if the nature of the loads contained in a general building code differs considerably from ASCE/SEI 7 loads, some provisions of this Code may need modification to reflect the difference.

Roofs should be designed with sufficient slope or camber to ensure adequate drainage accounting for any long-term deflection of the roof due to the dead loads, or the loads should be increased to account for all likely accumulations of water. If deflection of roof members may result in ponding of water accompanied by increased deflection and additional ponding, the design should ensure that this process is self-limiting.

**R8.2.3** — Any reinforced concrete wall that is monolithic with other structural elements is considered to be an “integral part.” Partition walls may or may not be integral structural

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**8.2.4** — Consideration shall be given to effects of forces due to prestressing, crane loads, vibration, impact, shrinkage, temperature changes, creep, expansion of shrinkage-compensating concrete, and unequal settlement of supports.

### 8.3 — Methods of analysis

**8.3.1** — All members of frames or continuous construction shall be designed for the maximum effects of factored loads as determined by the theory of elastic analysis, except as modified according to **8.4**. It shall be permitted to simplify design by using the assumptions specified in **8.7** through **8.11**.

**8.3.2** — Except for prestressed concrete, approximate methods of frame analysis shall be permitted for buildings of usual types of construction, spans, and story heights.

**8.3.3** — As an alternate to frame analysis, the following approximate moments and shears shall be permitted for design of continuous beams and one-way slabs (slabs reinforced to resist flexural stresses in only one direction), provided (a) through (e) are satisfied:

- (a) There are two or more spans;
- (b) Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent;
- (c) Loads are uniformly distributed;
- (d) Unfactored live load,  $L$ , does not exceed three times unfactored dead load,  $D$ ; and
- (e) Members are prismatic.

For calculating negative moments,  $\ell_n$  is taken as the average of the adjacent clear span lengths.

parts. If partition walls may be removed, the primary lateral load-resisting system should provide all of the required resistance without contribution of the removable partition. However, the effects of all partition walls attached to the structure should be considered in the analysis of the structure because they may lead to increased design forces in some or all elements. Provisions for seismic design are given in **Chapter 21**.

**R8.2.4** — Information is reported on the magnitudes of these various effects, especially the effects of column creep and shrinkage in tall structures,<sup>8.2</sup> and on procedures for including the forces resulting from these effects in design.

As described in **R7.12.1.2**, restraint of shrinkage and temperature movements can cause significant tension in slabs, as well as displacements, shear forces, and flexural moments in columns or walls. In cases of restraint, shrinkage and temperature reinforcement requirements may exceed flexural reinforcement requirements.

### R8.3 — Methods of analysis

**R8.3.1** — Factored loads are service loads multiplied by appropriate load factors. For the strength design method, elastic analysis is used to obtain moments, shears, and reactions.

**R8.3.3** — The approximate moments and shears give reasonably conservative values for the stated conditions if the flexural members are part of a frame or continuous construction. Because the load patterns that produce critical values for moments in columns of frames differ from those for maximum negative moments in beams, column moments should be evaluated separately.

**CODE****COMMENTARY**

Positive moment

End spans

Discontinuous end unrestrained .....	$w_u \ell_n^2 / 11$
Discontinuous end integral with support.....	$w_u \ell_n^2 / 14$
Interior spans.....	$w_u \ell_n^2 / 16$

Negative moments at exterior face of first interior support

Two spans .....	$w_u \ell_n^2 / 9$
More than two spans.....	$w_u \ell_n^2 / 10$

Negative moment at other faces of interior supports.....  $w_u \ell_n^2 / 11$

Negative moment at face of all supports for

Slabs with spans not exceeding 10 ft;
and beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8
at each end of the span .....

$w_u \ell_n^2 / 12$

Negative moment at interior face of exterior support for members built integrally with supports

Where support is spandrel beam .....	$w_u \ell_n^2 / 24$
Where support is a column.....	$w_u \ell_n^2 / 16$

Shear in end members at face of first interior support.....  $1.15 w_u \ell_n / 2$

Shear at face of all other supports.....  $w_u \ell_n / 2$

**8.3.4** — Strut-and-tie models shall be permitted to be used in the design of structural concrete. See [Appendix A](#).

**R8.3.4** — The strut-and-tie model in [Appendix A](#) is based on the assumption that portions of concrete structures can be analyzed and designed using hypothetical pin-jointed trusses consisting of struts and ties connected at nodes. This design method can be used in the design of regions where the basic assumptions of flexure theory are not applicable, such as regions near force discontinuities arising from concentrated forces or reactions, and regions near geometric discontinuities, such as abrupt changes in cross section.

## 8.4 — Redistribution of moments in continuous flexural members

**8.4.1** — Except where approximate values for moments are used, it shall be permitted to decrease factored moments calculated by elastic theory at sections of maximum negative or maximum positive moment in any span of continuous flexural members for any assumed loading arrangement by not more than **1000 $\varepsilon_t$**  percent, with a maximum of 20 percent.

**8.4.2** — Redistribution of moments shall be made only when  $\varepsilon_t$  is equal to or greater than 0.0075 at the section at which moment is reduced.

## R8.4 — Redistribution of moments in continuous flexural members

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at sections of maximum positive or negative moment and cause a shift in the elastic moment diagram. The usual result is a reduction in the values of maximum negative moments in the support regions and an increase in the values of positive moments between supports from those computed by elastic analysis. However, because negative moments are determined for one loading arrangement and positive moments for another (see [13.7.6](#) for an exception), economies in reinforcement can sometimes be realized by reducing maximum elastic positive moments and increasing negative moments, thus narrowing the envelope of maximum negative and positive moments at any section in the span.<sup>8.3</sup> The plastic hinges permit the utilization of the

**CODE****COMMENTARY**

**8.4.3** — The reduced moment shall be used for calculating redistributed moments at all other sections within the spans. Static equilibrium shall be maintained after redistribution of moments for each loading arrangement.

full capacity of more cross sections of a flexural member at ultimate loads.

Before 2008, the Code addressed moment redistribution by permitting an increase or decrease of factored negative moments above or below elastically calculated values, within specified limits. A decrease in negative moment strength implies inelastic behavior in the negative moment region at the support. By increasing the negative moment strength, the positive moments can be reduced but the result is that inelastic behavior will occur in the positive moment region of the member and the percentage change in the positive moment section could be much larger than the 20 percent permitted for negative moment sections.<sup>8.3</sup> The 2008 change places the same percentage limitations on both positive and negative moments.

Using conservative values of limiting concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution up to 20 percent, depending on the reinforcement ratio. The results were found to be conservative (see Fig. R8.4). Studies by Cohn<sup>8.4</sup> and Mattock<sup>8.5</sup> support this conclusion and indicate that cracking and deflection of beams designed for moment redistribution are not significantly greater at service loads than for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the Code is available if the members satisfy the Code requirements.

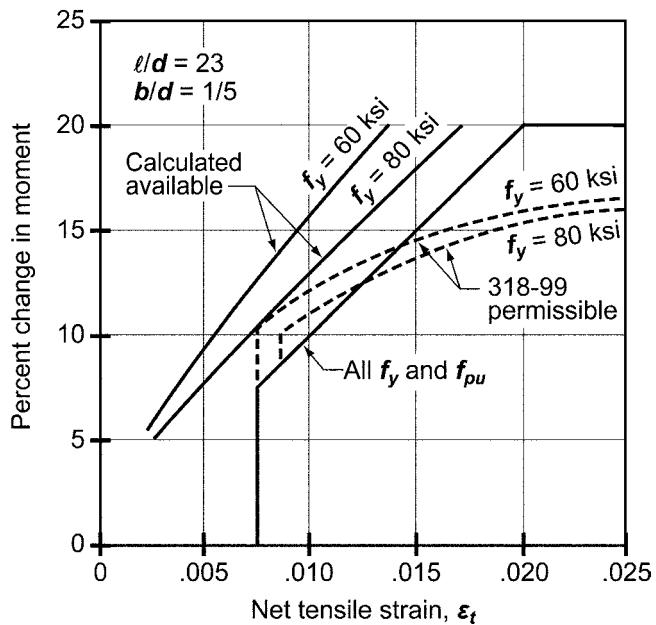


Fig. R8.4—Permissible moment redistribution for minimum rotation capacity.

**CODE****COMMENTARY**

Moment redistribution as permitted by 8.4 is not intended for moments in two-way slab systems that are analyzed using the pattern loadings given in 13.7.6.3 or for use where approximate values of bending moments are used. For the Direct Design Method, 10 percent modification is allowed by 13.6.7.

Before 2002, Section 8.4 specified the permissible redistribution percentage in terms of reinforcement indices. The 2002 Code specified the permissible redistribution percentage in terms of the net tensile strain in extreme tension steel at nominal strength,  $\epsilon_t$ . See Reference 8.6 for a comparison of these moment redistribution provisions.

**8.5 — Modulus of elasticity**

**8.5.1** — Modulus of elasticity,  $E_c$ , for concrete shall be permitted to be taken as  $w_c^{1.5} 33 \sqrt{f'_c}$  (in psi) for values of  $w_c$  between 90 and 160 lb/ft<sup>3</sup>. For normalweight concrete,  $E_c$  shall be permitted to be taken as  $57,000 \sqrt{f'_c}$ .

**8.5.2** — Modulus of elasticity,  $E_s$ , for nonprestressed reinforcement shall be permitted to be taken as 29,000,000 psi.

**8.5.3** — Modulus of elasticity,  $E_p$ , for prestressing steel shall be determined by tests or reported by the manufacturer.

**8.6 — Lightweight concrete**

**8.6.1** — To account for the use of lightweight concrete, unless specifically noted otherwise, a modification factor  $\lambda$  appears as a multiplier of  $\sqrt{f'_c}$  in all applicable equations and sections of this Code, where  $\lambda = 0.85$  for sand-lightweight concrete and 0.75 for all-lightweight concrete. Linear interpolation between 0.75 and 0.85 shall be permitted, on the basis of volumetric fractions, when a portion of the lightweight fine aggregate is replaced with normalweight fine aggregate. Linear interpolation between 0.85 and 1.0 shall be permitted, on the basis of volumetric fractions, for concrete containing normalweight fine aggregate and a blend of lightweight and normalweight coarse aggregates. For normalweight concrete,  $\lambda = 1.0$ . If average splitting tensile strength of lightweight concrete,  $f_{ct}$ , is specified,  $\lambda = f_{ct}/(6.7 \sqrt{f'_c}) \leq 1.0$ .

**R8.5 — Modulus of elasticity**

**R8.5.1** — Studies leading to the expression for modulus of elasticity of concrete in 8.5.1 are summarized in Reference 8.7 where  $E_c$  was defined as the slope of the line drawn from a stress of zero to a compressive stress of  $0.45f'_c$ . The modulus of elasticity for concrete is sensitive to the modulus of elasticity of the aggregate and may differ from the specified value. Measured values range typically from 120 to 80 percent of the specified value. Methods for determining the modulus of elasticity for concrete are described in Reference 8.8.

**R8.6 — Lightweight concrete**

**R8.6.1** — Factor  $\lambda$  reflects the lower tensile strength of lightweight concrete, which can reduce shear strength, friction properties, splitting resistance, bond between concrete and reinforcement, and increase development length, compared with normalweight concrete of the same compressive strength.

Two alternative procedures are provided to determine  $\lambda$ . The first alternative is based on the assumption that the tensile strength of lightweight concrete is a fixed fraction of the tensile strength of normalweight concrete.<sup>8,9</sup> The multipliers are based on data from tests on many types of structural lightweight aggregate.

The second alternative is based on laboratory tests to determine the relationship between average splitting tensile strength  $f_{ct}$  and the specified compressive strength  $f'_c$  for the lightweight concrete being used. For normalweight concrete, the average splitting tensile strength  $f_{ct}$  is approximately equal to  $6.7 \sqrt{f'_c}$ .<sup>8,9,8,10</sup>

**CODE****COMMENTARY****8.7 — Stiffness**

**8.7.1** — Use of any set of reasonable assumptions shall be permitted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions adopted shall be consistent throughout analysis.

**R8.7 — Stiffness**

**R8.7.1** — Ideally, the member stiffnesses  $E_c I$  and  $GJ$  should reflect the degree of cracking and inelastic action that has occurred along each member before yielding. However, the complexities involved in selecting different stiffnesses for all members of a frame would make frame analyses inefficient in design offices. Simpler assumptions are required to define flexural and torsional stiffnesses.

For braced frames, relative values of stiffness are important. Two usual assumptions are to use gross  $E_c I$  values for all members or, to use half the gross  $E_c I$  of the beam stem for beams and the gross  $E_c I$  for the columns.

For frames that are free to sway, a realistic estimate of  $E_c I$  is desirable and should be used if second-order analyses are carried out. Guidance for the choice of  $E_c I$  for this case is given in R10.10.4.

Two conditions determine whether it is necessary to consider torsional stiffness in the analysis of a given structure: (1) the relative magnitude of the torsional and flexural stiffnesses, and (2) whether torsion is required for equilibrium of the structure (equilibrium torsion) or is due to members twisting to maintain deformation compatibility (compatibility torsion). In the case of compatibility torsion, the torsional stiffness may be neglected. For cases involving equilibrium torsion, torsional stiffness should be considered.

**8.7.2** — Effect of haunches shall be considered both in determining moments and in design of members.

**8.8 — Effective stiffness to determine lateral deflections**

**8.8.1** — Lateral deflections of reinforced concrete building systems resulting from service lateral loads shall be computed by either a linear analysis with member stiffness determined using 1.4 times the flexural stiffness defined in 8.8.2 and 8.8.3 or by a more detailed analysis. Member properties shall not be taken greater than the gross section properties.

**8.8.2** — Lateral deflections of reinforced concrete building systems resulting from factored lateral loads shall be computed either by linear analysis with member stiffness defined by (a) or (b), or by a more

**R8.7.2** — Stiffness and fixed-end moment coefficients for haunched members may be obtained from Reference 8.11.

**R8.8 — Effective stiffness to determine lateral deflections**

**R8.8.1** — The selection of appropriate effective stiffness values depends on the intended performance of the structure. For wind loading, it is desirable to maintain elastic behavior in members at service load conditions. When analyzing a structure subjected to earthquake events at short recurrence intervals, some yielding without significant damage to the members may be a tolerable performance objective. As with lateral stability analysis of concrete structures (R10.10.4), a factor of 1.4 times the stiffness used for analysis under factored lateral loads is adequate to model effective section properties for lateral deflection analysis under service loads. Alternatively, a more accurate level of stiffness based on the expected element performance can be determined.

**R8.8.2** — The lateral deflection a structure sustains under factored lateral loads can be substantially different from that calculated using linear analysis in part because of the inelastic response of the members and the decrease in effective

**CODE**

detailed analysis considering the reduced stiffness of all members under the loading conditions:

- (a) By section properties defined in 10.10.4.1(a) through (c); or
- (b) 50 percent of stiffness values based on gross section properties.

**COMMENTARY**

stiffness. The selection of appropriate effective stiffness for reinforced concrete frame members has dual purposes: to provide realistic estimates of lateral deflection and to determine deflection-imposed actions on the gravity system of the structure. A detailed nonlinear analysis of the structure would adequately capture these two effects. A simple way to estimate an equivalent nonlinear lateral deflection ( $\delta_{em}$  at the top story in IBC 2006) using linear analysis is to reduce the modeled stiffness of the concrete members in the structure. The type of lateral load analysis affects the selection of appropriate effective stiffness values. For analyses with wind loading, where it is desirable to prevent nonlinear action in the structure, effective stiffness representative of pre-yield behavior may be appropriate. For earthquake loading, a level of nonlinear behavior is tolerable depending on the intended structural performance and earthquake recurrence interval.

Varying degrees of confidence can be obtained from a simple linear analysis based on the computational rigor used to define the effective stiffness of each member. One option that considers the reduced stiffness of the elements is to calculate the secant stiffness value to the point of yielding of reinforcement for the member, or the secant value to a point before yielding of the reinforcement if analysis demonstrates yielding is not expected for the given loading condition. The alternative options presented in 8.8.2 use values that approximate stiffness for reinforced concrete building systems loaded to near or beyond the yield level and have been shown to produce reasonable correlation with both experimental and detailed analytical results.<sup>8.12,8.13</sup> The effective stiffnesses in Option (a) were developed to represent lower-bound values for stability analysis of concrete building systems subjected to gravity and wind loads. Option (a) is provided so that the model used to calculate slenderness effects may be used to calculate lateral deflections due to factored wind and earthquake loading. In general, for effective section properties,  $E_c$  may be defined as in 8.5.1, A as in 10.10.4.1(c), and the shear modulus may be taken as  $0.4E_c$ .

**8.8.3** — Where two-way slabs without beams are designated as part of the seismic-force-resisting system, lateral deflections resulting from factored lateral loads shall be permitted to be computed by using linear analysis. The stiffness of slab members shall be defined by a model that is in substantial agreement with results of comprehensive tests and analysis and the stiffness of other frame members shall be as defined in 8.8.2.

**R8.8.3** — Analysis of buildings with two-way slab systems without beams requires that the model represent the transfer of lateral loads between vertical members. The model should result in prediction of stiffness in substantial agreement with results of comprehensive tests and analysis. Several acceptable models have been proposed to accomplish this action.<sup>8.14-8.16</sup>

**8.9 — Span length**

**8.9.1** — Span length of members not built integrally with supports shall be considered as the clear span

**R8.9 — Span length**

Beam moments calculated at support centers may be reduced to the moments at support faces for design of

**CODE**

plus the depth of the member, but need not exceed distance between centers of supports.

**8.9.2** — In analysis of frames or continuous construction for determination of moments, span length shall be taken as the distance center-to-center of supports.

**8.9.3** — For beams built integrally with supports, design on the basis of moments at faces of support shall be permitted.

**8.9.4** — It shall be permitted to analyze solid or ribbed slabs built integrally with supports, with clear spans not more than 10 ft, as continuous slabs on knife edge supports with spans equal to the clear spans of the slab and width of beams otherwise neglected.

**8.10 — Columns**

**8.10.1** — Columns shall be designed to resist the axial forces from factored loads on all floors or roof and the maximum moment from factored loads on a single adjacent span of the floor or roof under consideration. Loading condition giving the maximum ratio of moment to axial load shall also be considered.

**8.10.2** — In frames or continuous construction, consideration shall be given to the effect of unbalanced floor or roof loads on both exterior and interior columns and of eccentric loading due to other causes.

**8.10.3** — In computing gravity load moments in columns, it shall be permitted to assume far ends of columns built integrally with the structure to be fixed.

**8.10.4** — Resistance to moments at any floor or roof level shall be provided by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses and conditions of restraint.

**8.11 — Arrangement of live load**

**8.11.1** — It shall be permitted to assume that:

- (a) The live load is applied only to the floor or roof under consideration; and
- (b) The far ends of columns built integrally with the structure are considered to be fixed.

**8.11.2** — It shall be permitted to assume that the arrangement of live load is limited to combinations of:

- (a) Factored dead load on all spans with full factored live load on two adjacent spans; and

**COMMENTARY**

beams. Reference 8.17 provides an acceptable method of reducing moments at support centers to those at support faces.

**R8.10 — Columns**

Section 8.10 has been developed with the intent of making certain that the most demanding combinations of axial load and moments be identified for design.

Section 8.10.4 has been included to make certain that moments in columns are recognized in the design if the girders have been proportioned using 8.3.3. The moment in 8.10.4 refers to the difference between the moments in a given vertical plane, exerted at column centerline by members framing into that column.

**R8.11 — Arrangement of live load**

For determining column, wall, and beam moments and shears caused by gravity loads, the Code permits the use of a model limited to the beams in the level considered and the columns above and below that level. Far ends of columns are to be considered as fixed for the purpose of analysis under gravity loads. This assumption does not apply to lateral load analysis. However, in analysis for lateral loads, simplified methods (such as the portal method) may be used to obtain the moments, shears, and reactions for structures that are symmetrical and satisfy the assumptions used for such simplified methods. For unsymmetrical and high-rise structures, rigorous methods recognizing all structural displacements should be used.

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(b) Factored dead load on all spans with full factored live load on alternate spans.

**COMMENTARY**

The most demanding sets of design forces should be established by investigating the effects of live load placed in various critical patterns.

Most approximate methods of analysis neglect effects of deflections on geometry and axial flexibility. Therefore, beam and column moments may have to be amplified for column slenderness in accordance with 10.10.

**8.12 — T-beam construction**

**8.12.1** — In T-beam construction, the flange and web shall be built integrally or otherwise effectively bonded together.

**8.12.2** — Width of slab effective as a T-beam flange shall not exceed one-quarter of the span length of the beam, and the effective overhanging flange width on each side of the web shall not exceed:

- (a) Eight times the slab thickness; and
- (b) One-half the clear distance to the next web.

**8.12.3** — For beams with a slab on one side only, the effective overhanging flange width shall not exceed:

- (a) One-twelfth the span length of the beam;
- (b) Six times the slab thickness; and
- (c) One-half the clear distance to the next web.

**8.12.4** — Isolated beams, in which the T-shape is used to provide a flange for additional compression area, shall have a flange thickness not less than one-half the width of web and an effective flange width not more than four times the width of web.

**8.12.5** — Where primary flexural reinforcement in a slab that is considered as a T-beam flange (excluding joist construction) is parallel to the beam, reinforcement perpendicular to the beam shall be provided in the top of the slab in accordance with the following:

**8.12.5.1** — Transverse reinforcement shall be designed to carry the factored load on the overhanging slab width assumed to act as a cantilever. For isolated beams, the full width of overhanging flange shall be considered. For other T-beams, only the effective overhanging slab width need be considered.

**8.12.5.2** — Transverse reinforcement shall be spaced not farther apart than five times the slab thickness, nor farther apart than 18 in.

**R8.12 — T-beam construction**

This section contains provisions identical to those of previous Codes for limiting dimensions related to stiffness and flexural calculations. Provisions related to T-beams and other flanged members are stated in 11.5.1 with regard to torsion.

**CODE****COMMENTARY****8.13 — Joist construction**

**8.13.1** — Joist construction consists of a monolithic combination of regularly spaced ribs and a top slab arranged to span in one direction or two orthogonal directions.

**8.13.2** — Ribs shall be not less than 4 in. in width, and shall have a depth of not more than 3-1/2 times the minimum width of rib.

**8.13.3** — Clear spacing between ribs shall not exceed 30 in.

**8.13.4** — Joist construction not meeting the limitations of 8.13.1 through 8.13.3 shall be designed as slabs and beams.

**8.13.5** — When permanent burned clay or concrete tile fillers of material having a unit compressive strength at least equal to  $f'_c$  in the joists are used:

**8.13.5.1** — For shear and negative moment strength computations, it shall be permitted to include the vertical shells of fillers in contact with the ribs. Other portions of fillers shall not be included in strength computations.

**8.13.5.2** — Slab thickness over permanent fillers shall be not less than one-twelfth the clear distance between ribs, nor less than 1-1/2 in.

**8.13.5.3** — In one-way joists, reinforcement normal to the ribs shall be provided in the slab as required by [7.12](#).

**8.13.6** — When removable forms or fillers not complying with 8.13.5 are used:

**8.13.6.1** — Slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 2 in.

**8.13.6.2** — Reinforcement normal to the ribs shall be provided in the slab as required for flexure, considering load concentrations, if any, but not less than required by [7.12](#).

**8.13.7** — Where conduits or pipes as permitted by [6.3](#) are embedded within the slab, slab thickness shall be at least 1 in. greater than the total overall depth of the conduits or pipes at any point. Conduits or pipes shall not impair significantly the strength of the construction.

**R8.13 — Joist construction**

The size and spacing limitations for concrete joist construction meeting the limitations of 8.13.1 through 8.13.3 are based on successful performance in the past.

**R8.13.3** — A limit on the maximum spacing of ribs is required because of the provisions permitting higher shear strengths and less concrete protection for the reinforcement for these relatively small, repetitive members.

**CODE****COMMENTARY**

**8.13.8** — For joist construction,  $V_c$  shall be permitted to be 10 percent more than that specified in [Chapter 11](#).

**R8.13.8** — The increase in shear strength permitted by 8.13.8 is justified on the basis of: (1) satisfactory performance of joist construction with higher shear strengths, designed under previous Codes, which allowed comparable shear stresses, and (2) redistribution of local overloads to adjacent joists.

**8.14 — Separate floor finish**

**8.14.1** — A floor finish shall not be included as part of a structural member unless placed monolithically with the floor slab or designed in accordance with requirements of [Chapter 17](#).

**8.14.2** — It shall be permitted to consider all concrete floor finishes as part of required cover or total thickness for nonstructural considerations.

**R8.14 — Separate floor finish**

The Code does not specify an additional thickness for wearing surfaces subjected to unusual conditions of wear. The need for added thickness for unusual wear is left to the discretion of the licensed design professional.

As in previous editions of the Code, a floor finish may be considered for strength purposes only if it is cast monolithically with the slab. Permission is given to include a separate finish in the structural thickness if composite action is provided for in accordance with [Chapter 17](#).

All floor finishes may be considered for nonstructural purposes such as cover for reinforcement, fire protection, etc. Provisions should be made, however, to ensure that the finish will not spall off, thus causing decreased cover. Furthermore, development of reinforcement considerations requires minimum monolithic concrete cover according to [7.7](#).

**Notes**

# CHAPTER 9 — STRENGTH AND SERVICEABILITY REQUIREMENTS

## CODE

### 9.1 — General

**9.1.1** — Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as are stipulated in this Code.

**9.1.2** — Members also shall meet all other requirements of this Code to ensure adequate performance at service load levels.

**9.1.3** — Design of structures and structural members using the load factor combinations and strength reduction factors of **Appendix C** shall be permitted. Use of load factor combinations from this chapter in conjunction with strength reduction factors of **Appendix C** shall not be permitted.

### 9.2 — Required strength

**9.2.1** — Required strength  **$U$**  shall be at least equal to the effects of factored loads in Eq. (9-1) through (9-7). The effect of one or more loads not acting simultaneously shall be investigated.

$$U = 1.4D \quad (9-1)$$

$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \quad (9-2)$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \quad (9-3)$$

$$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \quad (9-4)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S \quad (9-5)$$

## COMMENTARY

### R9.1 — General

In the 2002 Code, the factored load combinations and strength reduction factors of the 1999 Code were revised and moved to Appendix C. The 1999 combinations were replaced with those of SEI/ASCE 7-02.<sup>9.1</sup> The strength reduction factors were replaced with those of the 1999 Appendix C, except that the factor for flexure was increased. In the 2011 Code, the factored load combinations were revised for consistency with ASCE/SEI 7-10.<sup>9.2</sup>

The changes were made to further unify the design profession on one set of load factors and combinations, and to facilitate the proportioning of concrete building structures that include members of materials other than concrete. When used with the strength reduction factors in 9.3, the designs for gravity loads will be comparable to those obtained using the strength reduction and load factors of the 1999 and earlier Codes. For combinations with lateral loads, some designs will be different, but the results of either set of load factors are considered acceptable.

Chapter 9 defines the basic strength and serviceability conditions for proportioning structural concrete members.

The basic requirement for strength design may be expressed as follows:

$$\text{Design Strength} \geq \text{Required Strength}$$

$$\phi(\text{Nominal Strength}) \geq U$$

In the strength design procedure, the margin of safety is provided by multiplying the service load by a load factor and the nominal strength by a strength reduction factor.

### R9.2 — Required strength

The required strength  **$U$**  is expressed in terms of factored loads, or related internal moments and forces. Factored loads are the loads specified in the general building code multiplied by appropriate load factors.

The factor assigned to each load is influenced by the degree of accuracy to which the load effect usually can be calculated and the variation that might be expected in the load during the lifetime of the structure. Dead loads, because they are more accurately determined and less variable, are assigned a lower load factor than live loads. Load factors also account for variability in the structural analysis used to compute moments and shears.

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$$U = 0.9D + 1.0W \quad (9-6)$$

$$U = 0.9D + 1.0E \quad (9-7)$$

except as follows:

- (a) The load factor on the live load  $L$  in Eq. (9-3) to (9-5) shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where  $L$  is greater than 100 lb/ft<sup>2</sup>.
- (b) Where  $W$  is based on service-level wind loads,  $1.6W$  shall be used in place of  $1.0W$  in Eq. (9-4) and (9-6), and  $0.8W$  shall be used in place of  $0.5W$  in Eq. (9-3).
- (c) Where  $E$  is based on service-level forces,  $1.4E$  shall be used in place of  $1.0E$  in Eq. (9-5) and (9-7).

**COMMENTARY**

The Code gives load factors for specific combinations of loads. In assigning factors to combinations of loading, some consideration is given to the probability of simultaneous occurrence. While most of the usual combinations of loadings are included, it should not be assumed that all cases are covered.

Due regard is to be given to sign in determining  $U$  for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type. The load combinations with  $0.9D$  are specifically included for the case where a higher dead load reduces the effects of other loads. The loading case may also be critical for tension-controlled column sections. In such a case, a reduction in axial load and an increase in moment may result in a critical load combination.

Consideration should be given to various combinations of loading to determine the most critical design condition. This is particularly true when strength is dependent on more than one load effect, such as strength for combined flexure and axial load or shear strength in members with axial load.

If unusual circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the stipulated strength reduction factors  $\phi$  or increase in the stipulated load factors may be appropriate for such members.

In 2011, the Code removed the weight of soil and other fill materials as part of the definition of  $H$ . Consistent with ASCE/SEI 7-10, the weight of these materials is part of dead load,  $D$ . The load factors for  $D$  are appropriate provided the unit weight and thickness of earth or other fill materials are well controlled. If the weight of earth stabilizes the structure, a load factor of zero may be appropriate.

**R9.2.1(a)** — The load modification factor of 9.2.1(a) is different than the live load reductions based on the loaded area that may be allowed in the legally adopted general building code. The live load reduction, based on loaded area, adjusts the nominal live load ( $L_0$  in ASCE/SEI 7) to  $L$ . The live load reduction as specified in the legally adopted general building code can be used in combination with the 0.5 load factor specified in 9.2.1(a).

**R9.2.1(b)** — ASCE/SEI 7-10 has converted wind loads to strength level, and reduced the wind load factor to 1.0. ACI 318 requires use of the previous load factor for wind loads, 1.6, when service-level wind loads are used. For serviceability checks, the commentary to Appendix C of ASCE/SEI 7-10 provides service-level wind loads,  $W_a$ .

**R9.2.1(c)** — In 1993, ASCE 7<sup>9.3</sup> converted earthquake forces to strength level, and reduced the earthquake load factor to 1.0. Model building codes<sup>9.4-9.6</sup> followed. ACI 318 requires use of the previous load factor for earthquake effects, approximately 1.4, when service-level earthquake effects are used.

**CODE****COMMENTARY****9.2.2 — Impact effects**

If resistance to impact effects is taken into account in design, such effects shall be included with  $L$ .

**9.2.3 — Self-straining effects**

Where applicable, the structural effects of  $T$  shall be considered in combination with other loads. The load factor on  $T$  shall be established considering the uncertainty associated with the likely magnitude of  $T$ , the probability that the maximum effect of  $T$  will occur simultaneously with other applied loads, and the potential adverse consequences if the effect of  $T$  is greater than assumed. The load factor on  $T$  shall not have a value less than 1.0.

**R9.2.2 — Impact effects**

If the live load is applied rapidly, as may be the case for parking structures, loading docks, warehouse floors, elevator shafts, etc., impact effects should be considered. In all equations, substitute ( $L$  + impact) for  $L$  when impact should be considered.

**R9.2.3 — Self-straining effects**

Several strategies can be used to accommodate movements due to differential settlement and volume change. Forces due to  $T$  effects are not commonly calculated and combined with other load effects. Rather, designs rely on successful past practices, using compliant structural members and ductile connections to accommodate differential settlement and volume change movement while providing the needed resistance to gravity and lateral loads. Expansion joints and construction closure strips are used to limit volume change movements based on performance of similar structures. Shrinkage and temperature reinforcement is commonly proportioned based on gross concrete area rather than calculated force.

However, where structural movements can lead to damage of nonductile elements, calculation of the predicted force should consider the inherent variability of the expected movement and structure response. A long-term study of the volume change behavior of precast concrete buildings,<sup>9.7</sup> completed in 2009, recommends procedures to account for connection stiffness, thermal exposure, member softening due to creep, and other factors that influence  $T$  forces.

**9.2.4 — Fluid loads**

Where  $F$  is present, it shall be included with the same load factor as  $D$  in Eq. (9-1) through (9-5) and (9-7).

**9.2.5 — Lateral soil pressure**

Where  $H$  is present, it shall be included in the load combinations of 9.2.1 with load factors in accordance with (a), (b), or (c):

- (a) where  $H$  acts alone or adds to the effects of other loads, it shall be included with a load factor of 1.6;
- (b) where the effect of  $H$  is permanent and counteracts the effects of other loads, it shall be included with a load factor of 0.9;
- (c) where the effect of  $H$  is not permanent but, when present, counteracts the effects of other loads,  $H$  shall not be included.

**R9.2.5 — Lateral soil pressure**

The required load factors for lateral pressures from soil, water in soil, and other materials reflect their variability and the possibility that the materials may be removed. The commentary of ASCE/SEI 7-10 includes additional useful discussion pertaining to load factors for  $H$ .

**CODE****COMMENTARY****| 9.2.6 — Flood and ice loads**

If a structure is in a flood zone, or is subjected to forces from atmospheric ice loads, the flood or ice loads and the appropriate load combinations of ASCE/SEI 7 shall be used.

**| 9.2.7 — Prestressing steel jacking force**

For post-tensioned anchorage zone design, a load factor of 1.2 shall be applied to the maximum prestressing steel jacking force.

**9.3 — Design strength**

**9.3.1** — Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall be taken as the nominal strength calculated in accordance with requirements and assumptions of this Code, multiplied by the strength reduction factors  $\phi$  in 9.3.2, **9.3.4**, and **9.3.5**.

**9.3.2** — Strength reduction factor  $\phi$  shall be as given in 9.3.2.1 through **9.3.2.7**:

**9.3.2.1** — Tension-controlled sections as defined in **10.3.4**.....0.90  
(See also **9.3.2.7**)

**| R9.2.6 — Flood and ice loads**

Areas subject to flooding are defined by flood hazard maps, usually maintained by local governmental jurisdictions.

**| R9.2.7 — Prestressing steel jacking force**

The load factor of 1.2 applied to the maximum tendon jacking force results in about 113 percent of the specified prestressing steel yield strength but not more than 96 percent of the nominal ultimate strength of the prestressing steel. This compares well with the maximum attainable jacking force, which is limited by the anchor efficiency factor.

**R9.3 — Design strength**

**R9.3.1** — The design strength of a member refers to the nominal strength calculated in accordance with the requirements stipulated in this Code multiplied by a strength reduction factor  $\phi$ , which is always less than 1.

The purposes of the strength reduction factor  $\phi$  are: (1) to allow for the probability of under-strength members due to variations in material strengths and dimensions, (2) to allow for inaccuracies in the design equations, (3) to reflect the degree of ductility and required reliability of the member under the load effects being considered, and (4) to reflect the importance of the member in the structure.<sup>9.8,9.9</sup>

In the 2002 Code, the strength reduction factors were adjusted to be compatible with the SEI/ASCE 7<sup>9.1</sup> load combinations, which were the basis for the required factored load combinations in model building codes at that time. These factors are essentially the same as those published in Appendix C of the 1995 edition, except the factor for flexure/tension controlled limits is increased from 0.80 to 0.90. This change was based on reliability analyses,<sup>9.8,9.10</sup> statistical study of material properties, as well as the opinion of the committee that the historical performance of concrete structures supports  $\phi = 0.90$ . In 2008,  $\phi$  for spirally reinforced compression-controlled sections was revised based on the reliability analyses reported in **Reference 9.11** and the superior performance of such members when subjected to excessive demand as documented in **Reference 9.12**.

**R9.3.2.1** — In applying 9.3.2.1 and **9.3.2.2**, the axial tensions and compressions to be considered are those caused by external forces. Effects of prestressing forces are not included.

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**9.3.2.2** — Compression-controlled sections, as defined in **10.3.3**:

- (a) Members with spiral reinforcement conforming to **10.9.3**.....0.75
- (b) Other reinforced members .....0.65

For sections in which the net tensile strain in the extreme tension steel at nominal strength,  $\varepsilon_t$ , is between the limits for compression-controlled and tension-controlled sections,  $\phi$  shall be permitted to be linearly increased from that for compression-controlled sections to 0.90 as  $\varepsilon_t$  increases from the compression-controlled strain limit to 0.005.

Alternatively, when **Appendix B** is used, for members in which  $f_y$  does not exceed 60,000 psi, with symmetric reinforcement, and with  $(d - d')/h$  not less than 0.70,  $\phi$  shall be permitted to be increased linearly to 0.90 as  $\phi P_n$  decreases from  $0.10f'_c A_g$  to zero. For other reinforced members,  $\phi$  shall be permitted to be increased linearly to 0.90 as  $\phi P_n$  decreases from  $0.10f'_c A_g$  or  $\phi P_b$ , whichever is smaller, to zero.

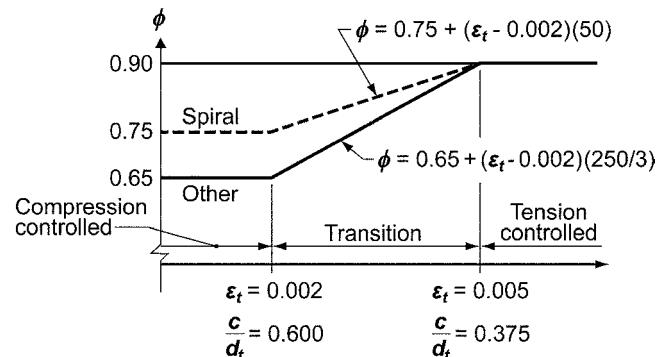
**COMMENTARY**

**R9.3.2.2** — Before the 2002 edition, the Code specified the magnitude of the  $\phi$ -factor for cases of axial load or flexure, or both, in terms of the type of loading. For these cases, the  $\phi$ -factor is now determined by the strain conditions at a cross section, at nominal strength.

A lower  $\phi$ -factor is used for compression-controlled sections than is used for tension-controlled sections because compression-controlled sections have less ductility, are more sensitive to variations in concrete strength, and generally occur in members that support larger loaded areas than members with tension-controlled sections. Members with spiral reinforcement are assigned a higher  $\phi$  than tied columns because they have greater ductility or toughness.

For sections subjected to axial load with flexure, design strengths are determined by multiplying both  $P_n$  and  $M_n$  by the appropriate single value of  $\phi$ . Compression-controlled and tension-controlled sections are defined in **10.3.3** and **10.3.4** as those that have net tensile strain in the extreme tension steel at nominal strength less than or equal to the compression-controlled strain limit, and equal to or greater than 0.005, respectively. For sections with net tensile strain  $\varepsilon_t$  in the extreme tension steel at nominal strength between the above limits, the value of  $\phi$  may be determined by linear interpolation, as shown in Fig. R9.3.2. The concept of net tensile strain  $\varepsilon_t$  is discussed in **R10.3.3**.

Since the compressive strain in the concrete at nominal strength is assumed in **10.2.3** to be 0.003, the net tensile strain limits for compression-controlled members may also be stated in terms of the ratio  $c/d_t$ , where  $c$  is the depth of the neutral axis at nominal strength, and  $d_t$  is the distance from the extreme compression fiber to the extreme tension steel. The  $c/d_t$  limits for compression-controlled and tension-controlled sections are 0.6 and 0.375, respectively. The 0.6 limit applies to sections reinforced with Grade 60



Interpolation on  $c/d_t$ : Spiral  $\phi = 0.75 + 0.15[(1/c/d_t) - (5/3)]$   
Other  $\phi = 0.65 + 0.25[(1/c/d_t) - (5/3)]$

*Fig. R9.3.2—Variation of  $\phi$  with net tensile strain in extreme tension steel,  $\varepsilon_t$ , and  $c/d_t$  for Grade 60 reinforcement and for prestressing steel.*

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<b>9.3.2.3</b> — Shear and torsion .....	0.75
<b>9.3.2.4</b> — Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models) .....	0.65
<b>9.3.2.5</b> — Post-tensioned anchorage zones .....	0.85
<b>9.3.2.6</b> — Strut-and-tie models ( <a href="#">Appendix A</a> ), and struts, ties, nodal zones, and bearing areas in such models .....	0.75
<b>9.3.2.7</b> — Flexural sections in pretensioned members where strand embedment is less than the development length as provided in <a href="#">12.9.1.1</a> :	
(a) From the end of the member to the end of the transfer length .....	0.75
(b) From the end of the transfer length to the end of the development length $\phi$ shall be permitted to be linearly increased from .....	0.75 to 0.9.

Where bonding of a strand does not extend to the end of the member, strand embedment shall be assumed to begin at the end of the debonded length. See also [12.9.3](#).

steel and to prestressed sections. [Figure R9.3.2](#) also gives equations for  $\phi$  as a function of  $c/d_t$ .

The net tensile strain limit for tension-controlled sections may also be stated in terms of the  $\rho/\rho_b$  as defined in the 1999 and earlier editions of the Code. The net tensile strain limit of 0.005 corresponds to a  $\rho/\rho_b$  ratio of 0.63 for rectangular sections with Grade 60 reinforcement. For a comparison of these provisions with the 1999 Code Section 9.3, see [Reference 9.13](#).

<b>9.3.2.3</b> — Shear and torsion .....	0.75
<b>9.3.2.4</b> — Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models) .....	0.65
<b>9.3.2.5</b> — Post-tensioned anchorage zones .....	0.85
<b>9.3.2.6</b> — Strut-and-tie models ( <a href="#">Appendix A</a> ), and struts, ties, nodal zones, and bearing areas in such models .....	0.75

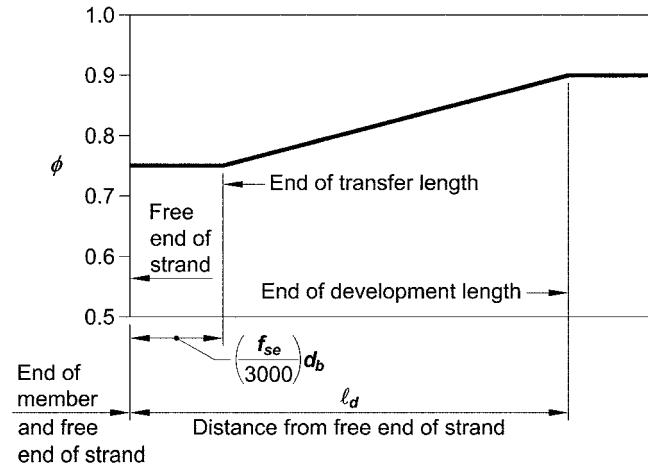
<b>9.3.2.7</b> — Flexural sections in pretensioned members where strand embedment is less than the development length as provided in <a href="#">12.9.1.1</a> :	
(a) From the end of the member to the end of the transfer length .....	0.75
(b) From the end of the transfer length to the end of the development length $\phi$ shall be permitted to be linearly increased from .....	0.75 to 0.9.

**R9.3.2.5** — The  $\phi$ -factor of 0.85 reflects the wide scatter of results of experimental anchorage zone studies. Since [18.13.4.2](#) limits the nominal compressive strength of unconfined concrete in the general zone to  $0.7\lambda f_{ci}'$ , the effective design strength for unconfined concrete is  $0.85 \times 0.7\lambda f_{ci}' \approx 0.6\lambda f_{ci}'$ .

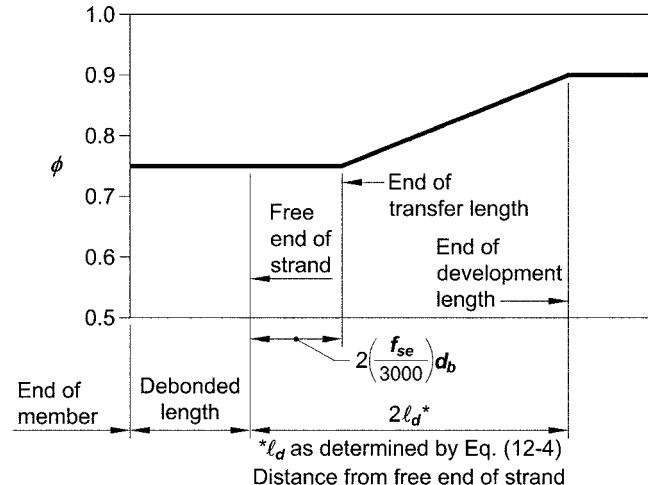
**R9.3.2.6** — The  $\phi$ -factor used in strut-and-tie models is taken equal to the  $\phi$ -factor for shear. The value of  $\phi$  for strut-and-tie models is applied to struts, ties, and bearing areas in such models.

**R9.3.2.7** — If a critical section occurs in a region where strand is not fully developed, failure may be by bond slip. Such a failure resembles a brittle shear failure; hence, the requirements for a reduced  $\phi$ . For sections between the end of the transfer length and the end of the development length, the value of  $\phi$  may be determined by linear interpolation, as shown in [Fig. R9.3.2.7\(a\)](#) and [\(b\)](#).

Where bonding of one or more strands does not extend to the end of the member, instead of a more rigorous analysis,  $\phi$  may be conservatively taken as 0.75 from the end of the member to the end of the transfer length of the strand with the longest debonded length. Beyond this point,  $\phi$  may be varied linearly to 0.9 at the location where all strands are developed, as shown in [Fig. R9.3.2.7\(b\)](#). Alternatively, the contribution of the debonded strands may be ignored until they are fully developed. Embedment of debonded strand is considered to begin at the termination of the debonding sleeves. Beyond this point, the provisions of [12.9.3](#) are applicable.

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*Fig. R9.3.2.7(a)—Variation of  $\phi$  with distance from the free end of strand in pretensioned members with fully bonded strands.*



*Fig. R9.3.2.7(b)—Variation of  $\phi$  with distance from the free end of strand in pretensioned members with debonded strands where 12.9.3 applies.*

**9.3.3** — Development lengths specified in [Chapter 12](#) do not require a  $\phi$ -factor.

**9.3.4** — For structures that rely on intermediate precast structural walls in Seismic Design Category D, E, or F, special moment frames, or special structural walls to resist earthquake effects,  $E$ ,  $\phi$  shall be modified as given in (a) through (c):

- (a) For any structural member that is designed to resist  $E$ ,  $\phi$  for shear shall be 0.60 if the nominal shear strength of the member is less than the shear corresponding to the development of the nominal flexural strength of the member. The nominal flexural strength shall be determined considering the most critical factored axial loads and including  $E$ ;

**R9.3.4** — Section 9.3.4(a) refers to brittle members such as low-rise walls, portions of walls between openings, or diaphragms that are impractical to reinforce to raise their nominal shear strength above nominal flexural strength for the pertinent loading conditions.

Short structural walls were the primary vertical elements of the lateral-force-resisting system in many of the parking structures that sustained damage during the 1994 Northridge earthquake. [Section 9.3.4\(b\)](#) requires the shear strength reduction factor for diaphragms to be 0.60 if the shear strength reduction factor for the walls is 0.60.

**CODE****COMMENTARY**

- (b) For diaphragms,  $\phi$  for shear shall not exceed the minimum  $\phi$  for shear used for the vertical components of the primary seismic-force-resisting system;
- (c) For joints and diagonally reinforced coupling beams,  $\phi$  for shear shall be 0.85.
- 9.3.5** — In Chapter 22,  $\phi$  shall be 0.60 for flexure, compression, shear, and bearing of structural plain concrete.

**9.4 — Design strength for reinforcement**

The values of  $f_y$  and  $f_{yt}$  used in design calculations shall not exceed 80,000 psi, except for prestressing steel and for transverse reinforcement in 10.9.3 and 21.1.5.4.

**9.5 — Control of deflections**

**9.5.1** — Reinforced concrete members subjected to flexure shall be designed to have adequate stiffness to limit deflections or any deformations that adversely affect strength or serviceability of a structure.

**R9.3.5** — The strength reduction factor  $\phi$  for structural plain concrete design is the same for all strength conditions. Since both flexural tension strength and shear strength for plain concrete depend on the tensile strength characteristics of the concrete, with no reserve strength or ductility possible due to the absence of reinforcement, equal strength reduction factors for both bending and shear are considered appropriate. In the 2008 Code, the factor was increased to 0.60 based on reliability analyses and statistical study of concrete properties,<sup>9.11</sup> as well as calibration to past practice.

**R9.4 — Design strength for reinforcement**

In addition to the upper limit of 80,000 psi for yield strength of nonprestressed reinforcement, there are limitations on yield strength in other sections of the Code.

In 11.4.2, 11.5.3.4, 11.6.6, and 18.9.3.2, the maximum value of  $f_y$  or  $f_{yt}$  that may be used in design is 60,000 psi, except that  $f_{yt}$  up to 80,000 psi may be used for shear reinforcement meeting the requirements of ASTM A1064.

In 19.3.2 and 21.1.5.2, the maximum specified yield strength  $f_y$  is 60,000 psi in shells, folded plates, special moment frames, and special structural walls.

The deflection provisions of 9.5 and the limitations on distribution of flexural reinforcement of 10.6 become increasingly critical as  $f_y$  increases.

**R9.5 — Control of deflections**

**R9.5.1** — The provisions of 9.5 are concerned only with deflections or deformations that may occur at service load levels. When long-term deflections are computed, only the dead load and that portion of the live load that is sustained need be considered.

Two methods are given for controlling deflections.<sup>9.14</sup> For nonprestressed beams and one-way slabs, and for composite members, provision of a minimum overall thickness as required by Table 9.5(a) will satisfy the requirements of the Code for members not supporting or attached to partitions or other construction likely to be damaged by large deflections. For nonprestressed two-way construction, minimum thickness as required by 9.5.3.1, 9.5.3.2, and 9.5.3.3 will satisfy the requirements of the Code.

For nonprestressed members that do not meet these minimum thickness requirements, or that support or are attached to partitions or other construction likely to be damaged by large

**CODE****COMMENTARY****9.5.2 — One-way construction (non prestressed)**

**9.5.2.1** — Minimum thickness stipulated in Table 9.5(a) shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

**9.5.2.2** — Where deflections are to be computed, deflections that occur immediately on application of load shall be computed by usual methods or formulas for elastic deflections, considering effects of cracking and reinforcement on member stiffness.

**TABLE 9.5(a) — MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED**

	Minimum thickness, <i>h</i>			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	<i>l/20</i>	<i>l/24</i>	<i>l/28</i>	<i>l/10</i>
Beams or ribbed one-way slabs	<i>l/16</i>	<i>l/18.5</i>	<i>l/21</i>	<i>l/8</i>

Notes:  
Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:  
a) For lightweight concrete having equilibrium density,  $w_c$ , in the range of 90 to 115 lb/ft<sup>3</sup>, the values shall be multiplied by  $(1.65 - 0.005w_c)$  but not less than 1.09.  
b) For  $f_y$  other than 60,000 psi, the values shall be multiplied by  $(0.4 + f_y/100,000)$ .

deflections, and for all prestressed concrete flexural members, deflections should be calculated by the procedures described or referred to in the appropriate sections of the Code, and are limited to the values in Table 9.5(b).

**R9.5.2 — One-way construction (non prestressed)**

**R9.5.2.1** — The minimum thicknesses of Table 9.5(a) apply for non prestressed beams and one-way slabs (see 9.5.2), and for composite members (see 9.5.5). These minimum thicknesses apply only to members not supporting or attached to partitions and other construction likely to be damaged by deflection.

Values of minimum thickness should be modified if other than normalweight concrete and Grade 60 reinforcement are used. The notes beneath the table are essential to its use for reinforced concrete members constructed with structural lightweight concrete or with reinforcement having a specified yield strength,  $f_y$ , other than 60,000 psi. If both of these conditions exist, the corrections in Footnotes (a) and (b) should both be applied.

The modification for lightweight concrete in Footnote (a) is based on studies of the results and discussions in Reference 9.15. No correction is given for concretes with  $w_c$  greater than 115 lb/ft<sup>3</sup> because the correction term would be close to unity in this range.

The modification for  $f_y$  in Footnote (b) is approximate but should yield conservative results for the type of members considered in the table, for typical reinforcement ratios, and for values of  $f_y$  between 40,000 and 80,000 psi.

**R9.5.2.2** — For calculation of immediate deflections of uncracked prismatic members, the usual methods or formulas for elastic deflections may be used with a constant value of  $E_c I_g$  along the length of the member. However, if the member is cracked at one or more sections, or if its depth varies along the span, a more exact calculation becomes necessary.

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**9.5.2.3** — Unless stiffness values are obtained by a more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity for concrete,  $E_c$ , as specified in 8.5.1 (normalweight or lightweight concrete) and with the effective moment of inertia,  $I_e$ , as follows, but not greater than  $I_g$

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \quad (9-8)$$

where

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (9-9)$$

and

$$f_r = 7.5 \lambda \sqrt{f'_c} \quad (9-10)$$

**9.5.2.4** — For continuous members,  $I_e$  shall be permitted to be taken as the average of values obtained from Eq. (9-8) for the critical positive and negative moment sections. For prismatic members,  $I_e$  shall be permitted to be taken as the value obtained from Eq. (9-8) at midspan for simple and continuous spans, and at support for cantilevers.

**9.5.2.5** — Unless values are obtained by a more comprehensive analysis, additional long-term deflection resulting from creep and shrinkage of flexural members (normalweight or lightweight concrete) shall be determined by multiplying the immediate deflection caused by the sustained load considered, by the factor  $\lambda_\Delta$

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} \quad (9-11)$$

where  $\rho'$  shall be the value at midspan for simple and continuous spans, and at support for cantilevers. It shall be permitted to assume  $\xi$ , the time-dependent factor for sustained loads, to be equal to:

5 years or more .....	2.0
12 months.....	1.4
6 months.....	1.2
3 months.....	1.0

**COMMENTARY**

**R9.5.2.3** — The effective moment of inertia procedure described in the Code and developed in Reference 9.16 was selected as being sufficiently accurate for use to control deflections.<sup>9.17-9.19</sup> The effective moment of inertia  $I_e$  was developed to provide a transition between the upper and lower bounds of  $I_g$  and  $I_{cr}$  as a function of the ratio  $M_{cr}/M_a$ . For most cases,  $I_e$  will be less than  $I_g$ .

**R9.5.2.4** — For continuous members, the Code procedure suggests a simple averaging of  $I_e$  values for the positive and negative moment sections. The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity (including the effect of cracking) has the dominant effect on deflections, as shown by ACI Committee 435<sup>9.20,9.21</sup> and SP-43.<sup>9.14</sup>

**R9.5.2.5** — Shrinkage and creep due to sustained loads cause additional long-term deflections over and above those that occur when loads are first placed on the structure. Such deflections are influenced by temperature, humidity, curing conditions, age at time of loading, quantity of compression reinforcement, and magnitude of the sustained load. The expression given in this section is considered satisfactory for use with the Code procedures for the calculation of immediate deflections, and with the limits given in Table 9.5(b). The deflection computed in accordance with this section is the additional long-term deflection due to the dead load and that portion of the live load that will be sustained for a sufficient period to cause significant time-dependent deflections.

Equation (9-11) was developed in Reference 9.22. In Eq. (9-11) the multiplier on  $\xi$  accounts for the effect of compression reinforcement in reducing long-term deflections.  $\xi = 2.0$  represents a nominal time-dependent factor for a 5-year duration of loading. The curve in Fig. R9.5.2.5 may be used to estimate values of  $\xi$  for loading periods less than 5 years.

If it is desired to consider creep and shrinkage separately, approximate equations provided in References 9.16, 9.17, 9.22, and 9.23 may be used.

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**9.5.2.6** — Deflection computed in accordance with **9.5.2.2** through **9.5.2.5** shall not exceed limits stipulated in Table 9.5(b).

**COMMENTARY**

**R9.5.2.6** — It should be noted that the limitations given in this table relate only to supported or attached nonstructural elements. For those structures in which structural members are likely to be affected by deflection or deformation of members to which they are attached in such a manner as to affect adversely the strength of the structure, these deflections and the resulting forces should be considered explicitly in the analysis and design of the structures as required by **9.5.1**. (See Reference **9.19**.)

Where long-term deflections are computed, the portion of the deflection before attachment of the nonstructural elements may be deducted. In making this correction, use may be made of the curve in Fig. R9.5.2.5 for members of usual sizes and shapes.

**9.5.3 — Two-way construction (nonprestressed)**

**9.5.3.1** — Section 9.5.3 shall govern the minimum thickness of slabs or other two-way construction designed in accordance with the provisions of **Chapter 13** and conforming with the requirements of **13.6.1.2**. The thickness of slabs without interior beams spanning between the supports on all sides shall satisfy the requirements of **9.5.3.2** or **9.5.3.4**. The thickness of slabs with beams spanning between the supports on all sides shall satisfy requirements of **9.5.3.3** or **9.5.3.4**.

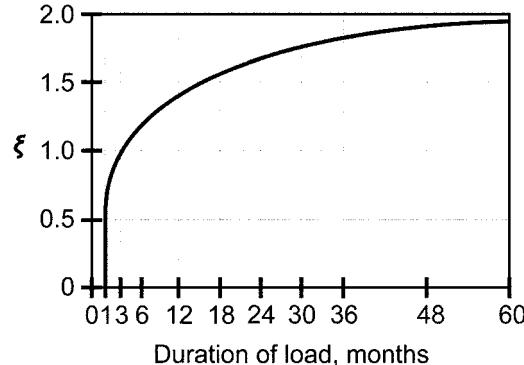
**R9.5.3 — Two-way construction (nonprestressed)**

Fig. R9.5.2.5—Multipliers for long-term deflections.

**TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS**

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load $L$	$\ell/180^*$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load $L$	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) <sup>†</sup>	$\ell/480^{\ddagger}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240^{\$}$

\*Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

<sup>†</sup>Long-term deflection shall be determined in accordance with **9.5.2.5** or **9.5.4.3**, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

<sup>‡</sup>Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

<sup>§</sup>Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

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**TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS\***

$f_y$ , psi <sup>†</sup>	Without drop panels <sup>‡</sup>		With drop panels <sup>‡</sup>			
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>§</sup>		Without edge beams	With edge beams <sup>§</sup>	
40,000	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

\*For two-way construction,  $\ell_n$  is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.  
<sup>†</sup>For  $f_y$  between the values given in the table, minimum thickness shall be determined by linear interpolation.  
<sup>‡</sup>Drop panels as defined in 13.2.5.  
<sup>§</sup>Slabs with beams between columns along exterior edges. The value of  $\alpha_f$  for the edge beam shall not be less than 0.8.

**9.5.3.2** — For slabs without interior beams spanning between the supports and having a ratio of long to short span not greater than 2, the minimum thickness shall be in accordance with the provisions of Table 9.5(c) and shall not be less than the following values:

- (a) Slabs without drop panels as defined in 13.2.5..... 5 in.;
- (b) Slabs with drop panels as defined in 13.2.5 ..... 4 in.

**9.5.3.3** — For slabs with beams spanning between the supports on all sides, the minimum thickness,  $h$ , shall be as follows:

- (a) For  $\alpha_{fm}$  equal to or less than 0.2, the provisions of 9.5.3.2 shall apply;
- (b) For  $\alpha_{fm}$  greater than 0.2 but not greater than 2.0,  $h$  shall not be less than

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{200,000} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad (9-12)$$

and not less than 5 in.;

- (c) For  $\alpha_{fm}$  greater than 2.0,  $h$  shall not be less than

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} \quad (9-13)$$

and not less than 3.5 in.;

- (d) At discontinuous edges, an edge beam shall be provided with a stiffness ratio  $\alpha_f$  not less than 0.80 or

**R9.5.3.2** — The minimum thicknesses in Table 9.5(c) are those that have been developed through the years. Slabs conforming to those limits have not resulted in systematic problems related to stiffness for short- and long-term loads. These limits apply to only the domain of previous experience in loads, environment, materials, boundary conditions, and spans.

**R9.5.3.3** — For panels having a ratio of long to short span greater than 2, the use of Eq. (9-12) and (9-13), which express the minimum thickness as a fraction of the long span, may give unreasonable results. For such panels, the rules applying to one-way construction in 9.5.2 should be used.

The requirement in 9.5.3.3(a) for  $\alpha_{fm}$  equal to 0.2 made it possible to eliminate Eq. (9-13) of the 1989 Code. That equation gave values essentially the same as those in Table 9.5(c), as does Eq. (9-12) at a value of  $\alpha_{fm}$  equal to 0.2.

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the minimum thickness required by [Eq. \(9-12\)](#) or [\(9-13\)](#) shall be increased by at least 10 percent in the panel with a discontinuous edge.

Term  $\ell_n$  in (b) and (c) is length of clear span in long direction measured face-to-face of beams. Term  $\beta$  in (b) and (c) is ratio of clear spans in long to short direction of slab.

**9.5.3.4** — Slab thickness less than the minimum required by [9.5.3.1](#), [9.5.3.2](#), and [9.5.3.3](#) shall be permitted where computed deflections do not exceed the limits of [Table 9.5\(b\)](#). Deflections shall be computed taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. The modulus of elasticity of concrete,  $E_c$ , shall be as specified in [8.5.1](#). The effective moment of inertia,  $I_e$ , shall be that given by [Eq. \(9-8\)](#); other values shall be permitted to be used if they result in computed deflections in reasonable agreement with results of comprehensive tests. Additional long-term deflection shall be computed in accordance with [9.5.2.5](#).

#### 9.5.4 — Prestressed concrete construction

**9.5.4.1** — For flexural members designed in accordance with provisions of [Chapter 18](#), immediate deflection shall be computed by usual methods or formulas for elastic deflections, and the moment of inertia of the gross concrete section,  $I_g$ , shall be permitted to be used for Class U flexural members, as defined in [18.3.3](#).

**9.5.4.2** — For Class C and Class T flexural members, as defined in [18.3.3](#), deflection calculations shall be based on a cracked transformed section analysis. It shall be permitted to base computations on a bilinear moment-deflection relationship, or an effective moment of inertia,  $I_e$ , as defined by [Eq. \(9-8\)](#).

**R9.5.3.4** — The calculation of deflections for slabs is complicated even if linear elastic behavior can be assumed. For immediate deflections, the values of  $E_c$  and  $I_e$  specified in [9.5.2.3](#) may be used.<sup>9.19</sup> However, other procedures and other values of the stiffness  $E_c I_e$  may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

Since available data on long-term deflections of slabs are too limited to justify more elaborate procedures, the additional long-term deflection for two-way construction is required to be computed using the multipliers given in [9.5.2.5](#).

#### R9.5.4 — Prestressed concrete construction

The Code requires deflections for all prestressed concrete flexural members to be computed and compared with the allowable values in [Table 9.5\(b\)](#).

**R9.5.4.1** — Immediate deflections of Class U prestressed concrete members may be calculated by the usual methods or formulas for elastic deflections using the moment of inertia of the gross (uncracked) concrete section and the modulus of elasticity for concrete specified in [8.5.1](#).

**R9.5.4.2** — Class C and Class T prestressed flexural members are defined in [18.3.3](#). [Reference 9.24](#) gives information on deflection calculations using a bilinear moment-deflection relationship and using an effective moment of inertia. [Reference 9.25](#) gives additional information on deflection of cracked prestressed concrete members.

[Reference 9.26](#) shows that the  $I_e$  method can be used to compute deflections of Class T prestressed members loaded above the cracking load. For this case, the cracking moment should take into account the effect of prestress. A method for predicting the effect of nonprestressed tension steel in reducing creep camber is also given in [Reference 9.26](#), with approximate forms given in [References 9.19](#) and [9.27](#).

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**9.5.4.3** — Additional long-term deflection of prestressed concrete members shall be computed taking into account stresses in concrete and steel under sustained load and including effects of creep and shrinkage of concrete and relaxation of steel.

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**R9.5.4.3** — Calculation of long-term deflections of prestressed concrete flexural members is complicated. The calculations should consider not only the increased deflections due to flexural stresses, but also the additional long-term deflections resulting from time-dependent shortening of the flexural member.

Prestressed concrete members shorten more with time than similar nonprestressed members due to the precompression in the slab or beam, which causes axial creep. This creep together with concrete shrinkage results in significant shortening of the flexural members that continues for several years after construction and should be considered in design. The shortening tends to reduce the tension in the prestressing steel, reducing the precompression in the member and thereby causing increased long-term deflections.

Another factor that can influence long-term deflections of prestressed flexural members is adjacent concrete or masonry that is nonprestressed in the direction of the prestressed member. This can be a slab nonprestressed in the beam direction adjacent to a prestressed beam or a nonprestressed slab system. As the prestressed member tends to shrink and creep more than the adjacent nonprestressed concrete, the structure will tend to reach a compatibility of the shortening effects. This results in a reduction of the precompression in the prestressed member as the adjacent concrete absorbs the compression. This reduction in precompression of the prestressed member can occur over a period of years and will result in additional long-term deflections and in increase tensile stresses in the prestressed member.

Any suitable method for calculating long-term deflections of prestressed members may be used, provided all effects are considered. Guidance may be found in [References 9.19, 9.28, 9.29, and 9.30](#).

**9.5.4.4** — Deflection computed in accordance with [9.5.4.1](#) or [9.5.4.2](#), and [9.5.4.3](#) shall not exceed limits stipulated in [Table 9.5\(b\)](#).

**9.5.5 — Composite construction****9.5.5.1 — Shored construction**

If composite flexural members are supported during construction so that, after removal of temporary supports, dead load is resisted by the full composite section, it shall be permitted to consider the composite member equivalent to a monolithically cast member for computation of deflection. For nonprestressed members, the portion of the member in compression shall determine whether values in [Table 9.5\(a\)](#) for normal-weight or lightweight concrete shall apply. If deflection is computed, account shall be taken of curvatures resulting from differential shrinkage of precast and

**R9.5.5 — Composite construction**

Since few tests have been made to study the immediate and long-term deflections of composite members, the rules given in [9.5.5.1](#) and [9.5.5.2](#) are based on the judgment of ACI Committee 318 and on experience.

If any portion of a composite member is prestressed or if the member is prestressed after the components have been cast, the provisions of [9.5.4](#) apply and deflections are to be calculated. For nonprestressed composite members, deflections need to be calculated and compared with the limiting values in [Table 9.5\(b\)](#) only when the thickness of the member or the precast part of the member is less than the minimum thickness given in [Table 9.5\(a\)](#). In unshored construction,

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cast-in-place components, and of axial creep effects in a prestressed concrete member.

**9.5.5.2 — Unshored construction**

If the thickness of a nonprestressed precast flexural member meets the requirements of **Table 9.5(a)**, deflection need not be computed. If the thickness of a nonprestressed composite member meets the requirements of **Table 9.5(a)**, it is not required to compute deflection occurring after the member becomes composite, but the long-term deflection of the precast member shall be investigated for magnitude and duration of load prior to beginning of effective composite action.

**9.5.5.3 — Deflection computed in accordance with 9.5.5.1 or 9.5.5.2 shall not exceed limits stipulated in Table 9.5(b).**

**COMMENTARY**

the thickness of concern depends on whether the deflection before or after the attainment of effective composite action is being considered. (In **Chapter 17**, it is stated that distinction need not be made between shored and unshored members. This refers to strength calculations, not to deflections.)

**Notes**

# CHAPTER 10 — FLEXURE AND AXIAL LOADS

## CODE

## COMMENTARY

### 10.1 — Scope

Provisions of Chapter 10 shall apply for design of members subject to flexure or axial loads or to combined flexure and axial loads.

### 10.2 — Design assumptions

**10.2.1** — Strength design of members for flexure and axial loads shall be based on assumptions given in 10.2.2 through **10.2.7**, and on satisfaction of applicable conditions of equilibrium and compatibility of strains.

**10.2.2** — Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except that, for deep beams as defined in **10.7.1**, an analysis that considers a nonlinear distribution of strain shall be used. Alternatively, it shall be permitted to use a strut-and-tie model. See **10.7**, **11.7**, and **Appendix A**.

**10.2.3** — Maximum usable strain at extreme concrete compression fiber shall be assumed equal to 0.003.

**10.2.4** — Stress in reinforcement below  $f_y$  shall be taken as  $E_s$  times steel strain. For strains greater than that corresponding to  $f_y$ , stress in reinforcement shall be considered independent of strain and equal to  $f_y$ .

### R10.2 — Design assumptions

**R10.2.1** — The strength of a member computed by the strength design method of the Code requires that two basic conditions be satisfied: (1) static equilibrium, and (2) compatibility of strains. Equilibrium between the compressive and tensile forces acting on the cross section at nominal strength should be satisfied. Compatibility between the stress and strain for the concrete and the reinforcement at nominal strength conditions should also be established within the design assumptions allowed by 10.2.

**R10.2.2** — Many tests have confirmed that the distribution of strain is essentially linear across a reinforced concrete cross section, even near ultimate strength.

The strain in both reinforcement and in concrete is assumed to be directly proportional to the distance from the neutral axis. This assumption is of primary importance in design for determining the strain and corresponding stress in the reinforcement.

**R10.2.3** — The maximum concrete compressive strain at crushing of the concrete has been observed in tests of various kinds to vary from 0.003 to higher than 0.008 under special conditions. However, the strain at which ultimate moments are developed is usually about 0.003 to 0.004 for members of normal proportions and materials.

**R10.2.4** — For deformed reinforcement, it is reasonably accurate to assume that the stress in reinforcement is proportional to strain below the specified yield strength  $f_y$ . The increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computations. In strength computations, the force developed in tensile or compressive reinforcement is computed as:

when  $\varepsilon_s < \varepsilon_y$  (yield strain)

$$A_s f_s = A_s E_s \varepsilon_s$$

when  $\varepsilon_s \geq \varepsilon_y$

$$A_s f_s = A_s f_y$$

**CODE****COMMENTARY**

**10.2.5** — Tensile strength of concrete shall be neglected in axial and flexural calculations of reinforced concrete, except when meeting requirements of 18.4.

**10.2.6** — The relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

**10.2.7** — Requirements of 10.2.6 are satisfied by an equivalent rectangular concrete stress distribution defined by the following:

**10.2.7.1** — Concrete stress of  $0.85f'_c$  shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance  $a = \beta_1 c$  from the fiber of maximum compressive strain.

**10.2.7.2** — Distance from the fiber of maximum strain to the neutral axis,  $c$ , shall be measured in a direction perpendicular to the neutral axis.

where  $\varepsilon_s$  is the value from the strain diagram at the location of the reinforcement. For design, the modulus of elasticity of steel reinforcement  $E_s$  may be taken as 29,000,000 psi (see 8.5.2).

**R10.2.5** — The tensile strength of concrete in flexure (modulus of rupture) is a more variable property than the compressive strength and is about 10 to 15 percent of the compressive strength. Tensile strength of concrete in flexure is neglected in strength design. For members with normal percentages of reinforcement, this assumption is in good agreement with tests. For very small percentages of reinforcement, neglect of the tensile strength at ultimate is usually correct.

The strength of concrete in tension, however, is important in cracking and deflection considerations at service loads.

**R10.2.6** — This assumption recognizes the inelastic stress distribution of concrete at high stress. As maximum stress is approached, the stress-strain relationship for concrete is not a straight line but some form of a curve (stress is not proportional to strain). The general shape of a stress-strain curve is primarily a function of concrete strength and consists of a rising curve from zero to a maximum at a compressive strain between 0.0015 and 0.002 followed by a descending curve to an ultimate strain (crushing of the concrete) from 0.003 to higher than 0.008. As discussed under R10.2.3, the Code sets the maximum usable strain at 0.003 for design.

The actual distribution of concrete compressive stress is complex and usually not known explicitly. Research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different assumptions as to the form of stress distribution. The Code permits any particular stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Many stress distributions have been proposed. The three most common are the parabola, trapezoid, and rectangle.

**R10.2.7** — For design, the Code allows the use of an equivalent rectangular compressive stress distribution (stress block) to replace the more exact concrete stress distribution. In the equivalent rectangular stress block, an average stress of  $0.85f'_c$  is used with a rectangle of depth  $a = \beta_1 c$ . The  $\beta_1$  of 0.85 for concrete with  $f'_c \leq 4000$  psi and 0.05 less for each 1000 psi of  $f'_c$  in excess of 4000 psi was determined experimentally.

In the 1976 supplement to the 1971 Code, a lower limit of  $\beta_1$  equal to 0.65 was adopted for concrete strengths greater than 8000 psi. Research data from tests with high-strength concretes<sup>10.1,10.2</sup> supported the equivalent rectangular stress block for concrete strengths exceeding 8000 psi, with a  $\beta_1$

**CODE**

**10.2.7.3** — For  $f'_c$  between 2500 and 4000 psi,  $\beta_1$  shall be taken as 0.85. For  $f'_c$  above 4000 psi,  $\beta_1$  shall be reduced linearly at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but  $\beta_1$  shall not be taken less than 0.65.

**COMMENTARY**

equal to 0.65. Use of the equivalent rectangular stress distribution specified in the 1971 Code, with no lower limit on  $\beta_1$ , resulted in inconsistent designs for high-strength concrete for members subject to combined flexure and axial load.

The equivalent rectangular stress distribution does not represent the actual stress distribution in the compression zone at ultimate, but does provide essentially the same results as those obtained in tests.<sup>10.3</sup>

**10.3 — General principles and requirements**

**10.3.1** — Design of cross sections subject to flexure or axial loads, or to combined flexure and axial loads, shall be based on stress and strain compatibility using assumptions in **10.2**.

**10.3.2** — Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to  $f_y$  just as concrete in compression reaches its assumed ultimate strain of 0.003.

**10.3.3** — Sections are compression-controlled if the net tensile strain in the extreme tension steel,  $\epsilon_t$ , is equal to or less than the compression-controlled strain limit when the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For Grade 60 reinforcement, and for all prestressed reinforcement, it shall be permitted to set the compression-controlled strain limit equal to 0.002.

**R10.3 — General principles and requirements**

**R10.3.1** — Design strength equations for members subject to flexure or combined flexure and axial load are derived in the paper, "Rectangular Concrete Stress Distribution in Ultimate Strength Design."<sup>10.3</sup> Reference 10.3 and previous editions of this Commentary also give the derivations of strength equations for cross sections other than rectangular.

**R10.3.2** — A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber just reaches 0.003 simultaneously with the first yield strain  $f_y/E_s$  in the tension reinforcement. The reinforcement ratio  $\rho_b$ , which produces balanced strain conditions under flexure, depends on the shape of the cross section and the location of the reinforcement.

**R10.3.3** — The nominal flexural strength of a member is reached when the strain in the extreme compression fiber reaches the assumed strain limit 0.003. The net tensile strain  $\epsilon_t$  is the tensile strain in the extreme tension steel at nominal strength, exclusive of strains due to prestress, creep, shrinkage, and temperature. The net tensile strain in the extreme tension steel is determined from a linear strain distribution at nominal strength, shown in Fig. R10.3.3, using similar triangles.

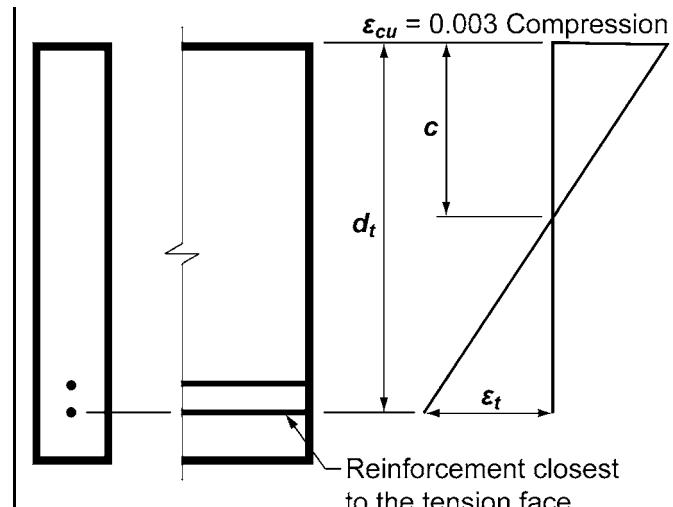


Fig. R10.3.3—Strain distribution and net tensile strain.

**CODE****COMMENTARY**

**10.3.4** — Sections are tension-controlled if the net tensile strain in the extreme tension steel,  $\epsilon_t$ , is equal to or greater than 0.005 when the concrete in compression reaches its assumed strain limit of 0.003. Sections with  $\epsilon_t$  between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

**R10.3.4** — When the net tensile strain in the extreme tension steel is sufficiently large (equal to or greater than 0.005), the section is defined as tension-controlled where ample warning of failure with excessive deflection and cracking may be expected. When the net tensile strain in the extreme tension steel is small (less than or equal to the compression-controlled strain limit), a brittle failure condition may be expected, with little warning of impending failure. Flexural members are usually tension-controlled, whereas compression members are usually compression-controlled. Some sections, such as those with small axial load and large bending moment, will have net tensile strain in the extreme tension steel between the above limits. These sections are in a transition region between compression- and tension-controlled sections. **Section 9.3.2** specifies the appropriate strength reduction factors for tension-controlled and compression-controlled sections, and for intermediate cases in the transition region.

Before the development of these provisions, the limiting tensile strain for flexural members was not stated, but was implicit in the maximum tension reinforcement ratio that was given as a fraction of  $\rho_b$ , which was dependent on the yield strength of the reinforcement. The net tensile strain limit of 0.005 for tension-controlled sections was chosen to be a single value that applies to all types of steel (prestressed and nonprestressed) permitted by this Code.

Unless unusual amounts of ductility are required, the 0.005 limit will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. **Section 8.4** permits redistribution of moments. Since moment redistribution is dependent on adequate ductility in hinge regions, moment redistribution is limited to sections that have a net tensile strain of at least 0.0075.

For beams with compression reinforcement, or T-beams, the effects of compression reinforcement and flanges are automatically accounted for in the computation of net tensile strain  $\epsilon_t$ .

**10.3.5** — For nonprestressed flexural members and nonprestressed members with factored axial compressive load less than  $0.10f'_c A_g$ ,  $\epsilon_t$  at nominal strength shall not be less than 0.004.

**R10.3.5** — The effect of this limitation is to restrict the reinforcement ratio in nonprestressed beams to about the same ratio as in editions of the Code before 2002. The reinforcement limit of  $0.75\rho_b$  results in a net tensile strain in extreme tension steel at nominal strength of 0.00376. The limit of 0.004 is slightly more conservative. This limitation does not apply to prestressed members.

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**10.3.5.1** — Use of compression reinforcement shall be permitted in conjunction with additional tension reinforcement to increase the strength of flexural members.

**10.3.6** — Design axial strength  $\phi P_n$  of compression members shall not be taken greater than  $\phi P_{n,max}$ , computed by Eq. (10-1) or (10-2).

**10.3.6.1** — For non prestressed members with spiral reinforcement conforming to 7.10.4 or composite members conforming to 10.13:

$$\phi P_{n,max} = 0.85\phi[0.85f'_c(A_g - A_{st}) + f_yA_{st}] \quad (10-1)$$

**10.3.6.2** — For non prestressed members with tie reinforcement conforming to 7.10.5:

$$\phi P_{n,max} = 0.80\phi[0.85f'_c(A_g - A_{st}) + f_yA_{st}] \quad (10-2)$$

**10.3.6.3** — For prestressed members, design axial strength,  $\phi P_n$ , shall not be taken greater than 0.85 (for members with spiral reinforcement) or 0.80 (for members with tie reinforcement) of the design axial strength at zero eccentricity,  $\phi P_o$ .

**10.3.7** — Members subject to compressive axial load shall be designed for the maximum moment that can accompany the axial load. The factored axial force  $P_u$  at given eccentricity shall not exceed that given in 10.3.6. The maximum factored moment  $M_u$  shall be magnified for slenderness effects in accordance with 10.10.

**R10.3.6 and R10.3.7** — The minimum design eccentricities included in the 1963 and 1971 Codes were deleted from the 1977 Code except for consideration of slenderness effects in compression members with small or zero computed end moments (see 10.10.6.5). The specified minimum eccentricities were originally intended to serve as a means of reducing the axial load design strength of a section in pure compression to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than  $f'_c$  under sustained high loads. The primary purpose of the minimum eccentricity requirement was to limit the maximum design axial strength of a compression member. This is now accomplished directly in 10.3.6 by limiting the design axial strength of a section in pure compression to 85 or 80 percent of the nominal strength. These percentage values approximate the axial strengths at eccentricity-to-depth ratios of 0.05 and 0.10, specified in the earlier Codes for the spirally reinforced and tied members, respectively. The same axial load limitation applies to both cast-in-place and precast compression members. Design aids and computer programs based on the minimum eccentricity requirement of the 1963 and 1971 Codes are equally applicable.

For prestressed members, the design axial strength in pure compression is computed by the strength design methods of Chapter 10, including the effect of the prestressing force.

Compression member end moments should be considered in the design of adjacent flexural members. In nonsway frames, the effects of magnifying the end moments need not be considered in the design of the adjacent beams. In sway frames, the magnified end moments should be considered in designing the flexural members, as required in 10.10.7.1.

Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load. Satisfactory methods are available in the *ACI Design Handbook*<sup>10.4</sup> and the *CRSI Handbook*.<sup>10.5</sup> The reciprocal load method<sup>10.6</sup> and the load contour method<sup>10.7</sup> are the methods used in those two handbooks. Research<sup>10.8,10.9</sup> indicates that using the equivalent rectangular stress block provisions of 10.2.7 produces satisfactory strength estimates for doubly symmetric sections. A simple and somewhat conservative estimate of nominal strength  $P_{ni}$  can be obtained from the reciprocal load relationship<sup>10.6</sup>

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

**CODE****COMMENTARY**

where:

- $P_{ni}$  = nominal axial load strength at given eccentricity along both axes
- $P_o$  = nominal axial load strength at zero eccentricity
- $P_{nx}$  = nominal axial load strength at given eccentricity along  $x$ -axis
- $P_{ny}$  = nominal axial load strength at given eccentricity along  $y$ -axis

This relationship is most suitable when values  $P_{nx}$  and  $P_{ny}$  are greater than the balanced axial force  $P_b$  for the particular axis.

## 10.4 — Distance between lateral supports of flexural members

**10.4.1** — Spacing of lateral supports for a beam shall not exceed 50 times  $b$ , the least width of compression flange or face.

**10.4.2** — Effects of lateral eccentricity of load shall be taken into account in determining spacing of lateral supports.

## 10.5 — Minimum reinforcement of flexural members

**10.5.1** — At every section of a flexural member where tensile reinforcement is required by analysis, except as provided in 10.5.2, 10.5.3, and 10.5.4,  $A_s$  provided shall not be less than that given by

$$A_{s, min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \quad (10-3)$$

and not less than  $200b_w d/f_y$ .

**10.5.2** — For statically determinate members with a flange in tension,  $A_{s, min}$  shall not be less than the value given by Eq. (10-3), except that  $b_w$  is replaced by either  $2b_w$  or the width of the flange, whichever is smaller.

## R10.4 — Distance between lateral supports of flexural members

Tests<sup>10.10,10.11</sup> have shown that laterally unbraced reinforced concrete beams of any reasonable dimensions, even when very deep and narrow, will not fail prematurely by lateral buckling provided the beams are loaded without lateral eccentricity that causes torsion.

Laterally unbraced beams are frequently loaded off center (lateral eccentricity) or with slight inclination. Stresses and deformations set up by such loading become detrimental for narrow, deep beams, the more so as the unsupported length increases. Lateral supports spaced closer than  $50b$  may be required by loading conditions.

## R10.5 — Minimum reinforcement of flexural members

The provision for a minimum amount of reinforcement applies to flexural members, which for architectural or other reasons, are larger in cross section than required for strength. With a very small amount of tensile reinforcement, the computed moment strength as a reinforced concrete section using cracked section analysis becomes less than that of the corresponding unreinforced concrete section computed from its modulus of rupture. Failure in such a case can be sudden.

To prevent such a failure, a minimum amount of tensile reinforcement is required by 10.5.1 in both positive and negative moment regions. When concrete strength higher than about 5000 psi is used, the  $200/f_y$  value previously prescribed may not be sufficient. Equation (10-3) gives the same amount of reinforcement as  $200b_w d/f_y$  when  $f'_c$  equals 4440 psi. When the flange of a section is in tension, the amount of tensile reinforcement needed to make the strength of the reinforced section equal that of the unreinforced section is about twice that for a rectangular section or that of a flanged section with the flange in compression. A higher amount of minimum tensile reinforcement is particularly necessary in cantilevers and other statically determinate members where there is no possibility for redistribution of moments.

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**10.5.3** — The requirements of **10.5.1** and **10.5.2** need not be applied if, at every section,  $A_s$  provided is at least one-third greater than that required by analysis.

**10.5.4** — For structural slabs and footings of uniform thickness,  $A_{s,min}$  in the direction of the span shall be the same as that required by **7.12.2.1**. Maximum spacing of this reinforcement shall not exceed three times the thickness, nor 18 in.

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**R10.5.3** — The minimum reinforcement required by **Eq. (10-3)** is to be provided wherever reinforcement is needed, except where such reinforcement is at least one-third greater than that required by analysis. This exception provides sufficient additional reinforcement in large members where the amount required by **10.5.1** or **10.5.2** would be excessive.

**R10.5.4** — The minimum reinforcement required for slabs should be equal to the same amount as that required by **7.12.2.1** for shrinkage and temperature reinforcement.

Slabs-on-ground are not considered to be structural slabs in the context of this section, unless they transmit vertical loads or lateral forces from other parts of the structure to the soil. Reinforcement, if any, in slabs-on-ground should be proportioned with due consideration of all design forces. Mat foundations and other slabs that help support the structure vertically should meet the requirements of this section.

In reevaluating the overall treatment of **10.5**, the maximum spacing for reinforcement in structural slabs (including footings) was reduced from the **5h** for temperature and shrinkage reinforcement to the compromise value of **3h**, which is somewhat larger than the **2h** limit of **13.3.2** for two-way slab systems.

**10.6 — Distribution of flexural reinforcement in beams and one-way slabs**

**10.6.1** — This section prescribes rules for distribution of flexural reinforcement to control flexural cracking in beams and in one-way slabs (slabs reinforced to resist flexural stresses in only one direction).

**R10.6 — Distribution of flexural reinforcement in beams and one-way slabs**

**R10.6.1** — Many structures designed by working stress methods and with low steel stress served their intended functions with very limited flexural cracking. When high-strength reinforcing steels are used at high service load stresses, however, visible cracks should be expected, and steps should be taken in detailing of the reinforcement to control cracking. For reasons of durability and appearance, many fine cracks are preferable to a few wide cracks.

Control of cracking is particularly important when reinforcement with a yield strength in excess of 40,000 psi is used. Current good detailing practices will usually lead to adequate crack control even when reinforcement of 60,000 psi yield strength is used.

Extensive laboratory work<sup>10.12-10.14</sup> involving deformed bars has confirmed that crack width at service loads is proportional to steel stress. The significant variables reflecting steel detailing were found to be thickness of concrete cover and the spacing of reinforcement.

Crack width is inherently subject to wide scatter even in careful laboratory work and is influenced by shrinkage and other time-dependent effects. Improved crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension.

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**10.6.2** — Distribution of flexural reinforcement in two-way slabs shall be as required by **13.3**.

**10.6.3** — Flexural tension reinforcement shall be well distributed within maximum flexural tension zones of a member cross section as required by 10.6.4.

**10.6.4** — The spacing of reinforcement closest to the tension face,  $s$ , shall not exceed that given by

$$s = 15\left(\frac{40,000}{f_s}\right) - 2.5c_c \quad (10-4)$$

but not greater than  $12(40,000/f_s)$ , where  $c_c$  is the least distance from surface of reinforcement or prestressing steel to the tension face. If there is only one bar or wire nearest to the extreme tension face,  $s$  used in Eq. (10-4) is the width of the extreme tension face.

Calculated stress  $f_s$  in reinforcement closest to the tension face at service load shall be computed based on the unfactored moment. It shall be permitted to take  $f_s$  as  $2/3f_y$ .

**10.6.5** — Provisions of 10.6.4 are not sufficient for structures subject to very aggressive exposure or designed to be watertight. For such structures, special investigations and precautions are required.

**10.6.6** — Where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in **8.12**, or a width equal to one-tenth the span, whichever is smaller. If the effective flange width exceeds one-tenth the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

**10.6.7** — Where  $h$  of a beam or joist exceeds 36 in., longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member. Skin reinforcement shall extend for a distance  $h/2$  from the tension face. The spacing  $s$  shall be as provided in 10.6.4, where  $c_c$  is the least distance from the surface

**R10.6.3** — Several bars at moderate spacing are much more effective in controlling cracking than one or two larger bars of equivalent area.

**R10.6.4** — This section was updated in the 2005 edition to reflect the higher service stresses that occur in flexural reinforcement with the use of the load combinations introduced in the 2002 Code. The maximum bar spacing is specified directly to control cracking.<sup>10.15-10.17</sup> For the usual case of beams with Grade 60 reinforcement and 2 in. clear cover to the main reinforcement, with  $f_s = 40,000$  psi, the maximum bar spacing is 10 in.

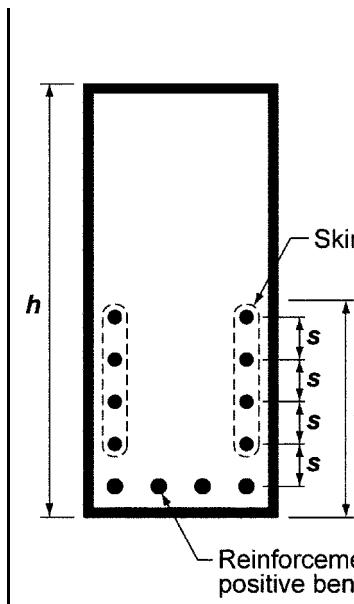
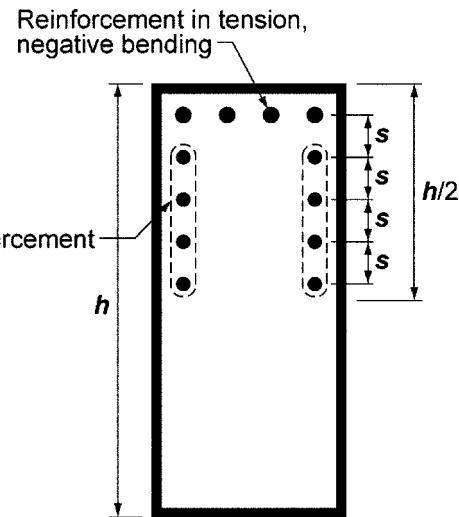
Crack widths in structures are highly variable. In Codes before the 1999 edition, provisions were given for distribution of reinforcement that were based on empirical equations using a calculated maximum crack width of 0.016 in. The current provisions for spacing are intended to limit surface cracks to a width that is generally acceptable in practice but may vary widely in a given structure.

The role of cracks in the corrosion of reinforcement is controversial. Research<sup>10.18,10.19</sup> shows that corrosion is not clearly correlated with surface crack widths in the range normally found with reinforcement stresses at service load levels. For this reason, the former distinction between interior and exterior exposure has been eliminated.

**R10.6.5** — Although a number of studies have been conducted, clear experimental evidence is not available regarding the crack width beyond which a corrosion danger exists. Exposure tests indicate that concrete quality, adequate compaction, and ample concrete cover may be of greater importance for corrosion protection than crack width at the concrete surface.

**R10.6.6** — In major T-beams, distribution of the negative reinforcement for control of cracking should take into account two considerations: (1) wide spacing of the reinforcement across the full effective width of flange may cause some wide cracks to form in the slab near the web, and (2) close spacing near the web leaves the outer regions of the flange unprotected. The one-tenth limitation is to guard against too wide a spacing, with some additional reinforcement required to protect the outer portions of the flange.

**R10.6.7** — For relatively deep flexural members, some reinforcement should be placed near the vertical faces of the tension zone to control cracking in the web.<sup>10.20,10.21</sup> (See Fig. R10.6.7.) Without such auxiliary steel, the width of the cracks in the web may exceed the crack widths at the level of the flexural tension reinforcement. This section was

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*Fig. R10.6.7—Skin reinforcement for beams and joists with  $h > 36$  in.*

of the skin reinforcement or prestressing steel to the side face. It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars or wires.<sup>2</sup>

modified in the 2005 edition to make the skin reinforcement spacing consistent with that of the flexural reinforcement. The size of the skin reinforcement is not specified; research has indicated that the spacing rather than bar size is of primary importance.<sup>10,21</sup> Bar sizes No. 3 to No. 5 (or welded wire reinforcement with a minimum area of 0.1 in.<sup>2</sup> per foot of depth) are typically provided.

Where the provisions for deep beams, walls, or precast panels require more reinforcement, those provisions (along with their spacing requirements) will govern.

## 10.7 — Deep beams

**10.7.1** — Deep beams are members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports, and have either:

- (a) clear spans,  $\ell_n$ , equal to or less than four times the overall member depth  $h$ ; or
- (b) regions with concentrated loads within a distance  $2h$  from the face of the support.

Deep beams shall be designed either taking into account nonlinear distribution of strain, or by [Appendix A](#). (See also [11.7.1](#) and [12.10.6](#).)

**10.7.2** — Deep beams shall satisfy the requirements of [11.7](#).

**10.7.3** — Minimum area of flexural tension reinforcement,  $A_{s,min}$ , shall conform to [10.5](#).

## R10.7 — Deep beams

The span-to-depth ratios used to define deep beams in the 1999 and earlier Codes were based on papers published in 1946 and 1953. The definitions of deep beams given in Chapters 10 and 11 of these earlier Codes were different from each other and different from the current Code definition that is based on D-region behavior (see [Appendix A](#)). Since 2002, the definitions of deep beams in Sections 10.7.1 and [11.7.1](#) are consistent with each other.

This Code does not contain detailed requirements for designing deep beams for flexure except that nonlinearity of strain distribution and lateral buckling is to be considered. Suggestions for the design of deep beams for flexure are given in [References 10.22, 10.23, and 10.24](#).

**CODE****COMMENTARY****10.8 — Design dimensions for compression members****10.8.1 — Isolated compression member with multiple spirals**

Outer limits of the effective cross section of a compression member with two or more interlocking spirals shall be taken at a distance outside the extreme limits of the spirals equal to the minimum concrete cover required by 7.7.

**10.8.2 — Compression member built monolithically with wall**

Outer limits of the effective cross section of a spirally reinforced or tied reinforced compression member built monolithically with a concrete wall or pier shall be taken not greater than 1-1/2 in. outside the spiral or tie reinforcement.

**10.8.3 — Equivalent circular compression member**

As an alternative to using the full gross area for design of a compression member with a square, octagonal, or other shaped cross section, it shall be permitted to use a circular section with a diameter equal to the least lateral dimension of the actual shape. Gross area considered, required percentage of reinforcement, and design strength shall be based on that circular section.

**10.8.4 — Limits of section**

For a compression member with a cross section larger than required by considerations of loading, it shall be permitted to base the minimum reinforcement and strength on a reduced effective area  $A_g$  not less than one-half the total area. This provision shall not apply to special moment frames or special structural walls designed in accordance with Chapter 21.

**10.9 — Limits for reinforcement of compression members****10.9.1 — Area of longitudinal reinforcement,  $A_{st}$ , for noncomposite compression members shall be not less than  $0.01A_g$  or more than  $0.08A_g$ .****R10.8 — Design dimensions for compression members**

With the 1971 Code, minimum sizes for compression members were eliminated to allow wider utilization of reinforced concrete compression members in smaller size and lightly loaded structures, such as low-rise residential and light office buildings. When small sections are used, there is a greater need for careful workmanship and shrinkage stresses have increased significance.

**R10.8.2, R10.8.3, and R10.8.4** — For column design,<sup>10.25</sup> the Code provisions for quantity of reinforcement, both vertical and spiral, are based on the gross column area and core area, and the design strength of the column is based on the gross area of the column section. In some cases, however, the gross area is larger than necessary to carry the factored load. The basis of 10.8.2, 10.8.3, and 10.8.4 is that it is satisfactory to design a column of sufficient size to carry the factored load and then simply add concrete around the designed section without increasing the reinforcement to meet the minimum percentages required by 10.9.1. The additional concrete should not be considered as carrying load; however, the effects of the additional concrete on member stiffness should be included in the structural analysis. The effects of the additional concrete also should be considered in design of the other parts of the structure that interact with the oversize member.

**R10.9 — Limits for reinforcement of compression members**

**R10.9.1** — This section prescribes the limits on the amount of longitudinal reinforcement for noncomposite compression members. If the use of high reinforcement ratios would involve practical difficulties in the placing of concrete, a lower percentage and hence a larger column, or higher strength concrete or reinforcement (see R9.4) should be considered. The percentage of reinforcement in columns should usually not exceed 4 percent if the column bars are required to be lap spliced.

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**Minimum reinforcement** — Since the design methods for columns incorporate separate terms for the load carried by concrete and by reinforcement, it is necessary to specify some minimum amount of reinforcement to ensure that only reinforced concrete columns are designed by these procedures. Reinforcement is necessary to provide resistance to bending, which may exist whether or not computations show that bending exists, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses. Tests have shown that creep and shrinkage tend to transfer load from the concrete to the reinforcement, with a consequent increase in stress in the reinforcement, and that this increase is greater as the ratio of reinforcement decreases. Unless a lower limit is placed on this ratio, the stress in the reinforcement may increase to the yield level under sustained service loads. This phenomenon was emphasized in the report of ACI Committee 105<sup>10.26</sup> and minimum reinforcement ratios of 0.01 and 0.005 were recommended for spiral and tied columns, respectively. However, in all editions of the Code since 1936, the minimum ratio has been 0.01 for both types of transversely reinforced columns.

**Maximum reinforcement** — Extensive tests of the ACI column investigation<sup>10.26</sup> included reinforcement ratios no greater than 0.06. Although other tests with as much as 17 percent reinforcement in the form of bars produced results similar to those obtained previously, it is necessary to note that the loads in these tests were applied through bearing plates on the ends of the columns and the problem of transferring a proportional amount of the load to the bars was thus minimized or avoided. Maximum ratios of 0.08 and 0.03 were recommended by ACI Committee 105<sup>10.26</sup> for spiral and tied columns, respectively. In the 1936 Code, these limits were made 0.08 and 0.04, respectively. In the 1956 Code, the limit for tied columns with bending was raised to 0.08. Since the 1963 Code, it has been required that bending be considered in the design of all columns, and the maximum ratio of 0.08 has been applied to both types of columns. This limit can be considered a practical maximum for reinforcement in terms of economy and requirements for placing.

**10.9.2** — Minimum number of longitudinal bars in compression members shall be 4 for bars within rectangular or circular ties, 3 for bars within triangular ties, and 6 for bars enclosed by spirals conforming to **10.9.3**.

**R10.9.2** — For compression members, a minimum of four longitudinal bars are required when bars are enclosed by rectangular or circular ties. For other shapes, one bar should be provided at each apex or corner and proper transverse reinforcement provided. For example, tied triangular columns require three longitudinal bars, one at each apex of the triangular ties. For bars enclosed by spirals, six bars are required.

When the number of bars in a circular arrangement is less than eight, the orientation of the bars will affect the moment strength of eccentrically loaded columns and should be considered in design.

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**10.9.3** — Volumetric spiral reinforcement ratio,  $\rho_s$ , shall be not less than the value given by

$$\rho_s = 0.45 \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f'_{yt}} \quad (10-5)$$

where the value of  $f'_{yt}$  used in Eq. (10-5) shall not exceed 100,000 psi. For  $f'_{yt}$  greater than 60,000 psi, lap splices according to 7.10.4.5(a) shall not be used.

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**R10.9.3** — The effect of spiral reinforcement in increasing the load-carrying strength of the concrete within the core is not realized until the column has been subjected to a load and deformation sufficient to cause the concrete shell outside the core to spall off. The amount of spiral reinforcement required by Eq. (10-5) is intended to provide additional load-carrying strength for concentrically loaded columns equal to or slightly greater than the strength lost when the shell spalls off. This principle was recommended by ACI Committee 105<sup>10,26</sup> and has been a part of the Code since 1936. The derivation of Eq. (10-5) is given in the ACI Committee 105 report. Tests and experience show that columns containing the amount of spiral reinforcement required by this section exhibit considerable toughness and ductility. Research<sup>10,27-10,29</sup> has indicated that 100,000 psi yield strength reinforcement can be used for confinement. For the 2005 Code, the limit in yield strength for spiral reinforcement was increased from 60,000 to 100,000 psi.

## 10.10 — Slenderness effects in compression members

**10.10.1** — Slenderness effects shall be permitted to be neglected in the following cases:

(a) for compression members not braced against sidesway when:

$$\frac{k\ell_u}{r} \leq 22 \quad (10-6)$$

(b) for compression members braced against sidesway when:

$$\frac{k\ell_u}{r} \leq 34 - 12(M_1/M_2) \leq 40 \quad (10-7)$$

where  $M_1/M_2$  is positive if the column is bent in single curvature, and negative if the member is bent in double curvature.

It shall be permitted to consider compression members braced against sidesway when bracing elements have a total stiffness, resisting lateral movement of that story, of at least 12 times the gross stiffness of the columns within the story.

## R10.10 — Slenderness effects in compression members

The slenderness provisions have been reorganized in the 2008 edition of the Code to reflect the evolution of current practice where second-order effects are considered primarily using computer analysis techniques.

**R10.10.1** — Second-order effects in many structures are negligible. In these cases, it is not necessary to consider slenderness effects and compression members can be designed based on forces determined from first-order analyses. Slenderness effects can be neglected in both braced and unbraced systems depending on the  $k\ell_u/r$  of the member. In evaluation of unbraced systems, moments are based on first-order elastic analysis.

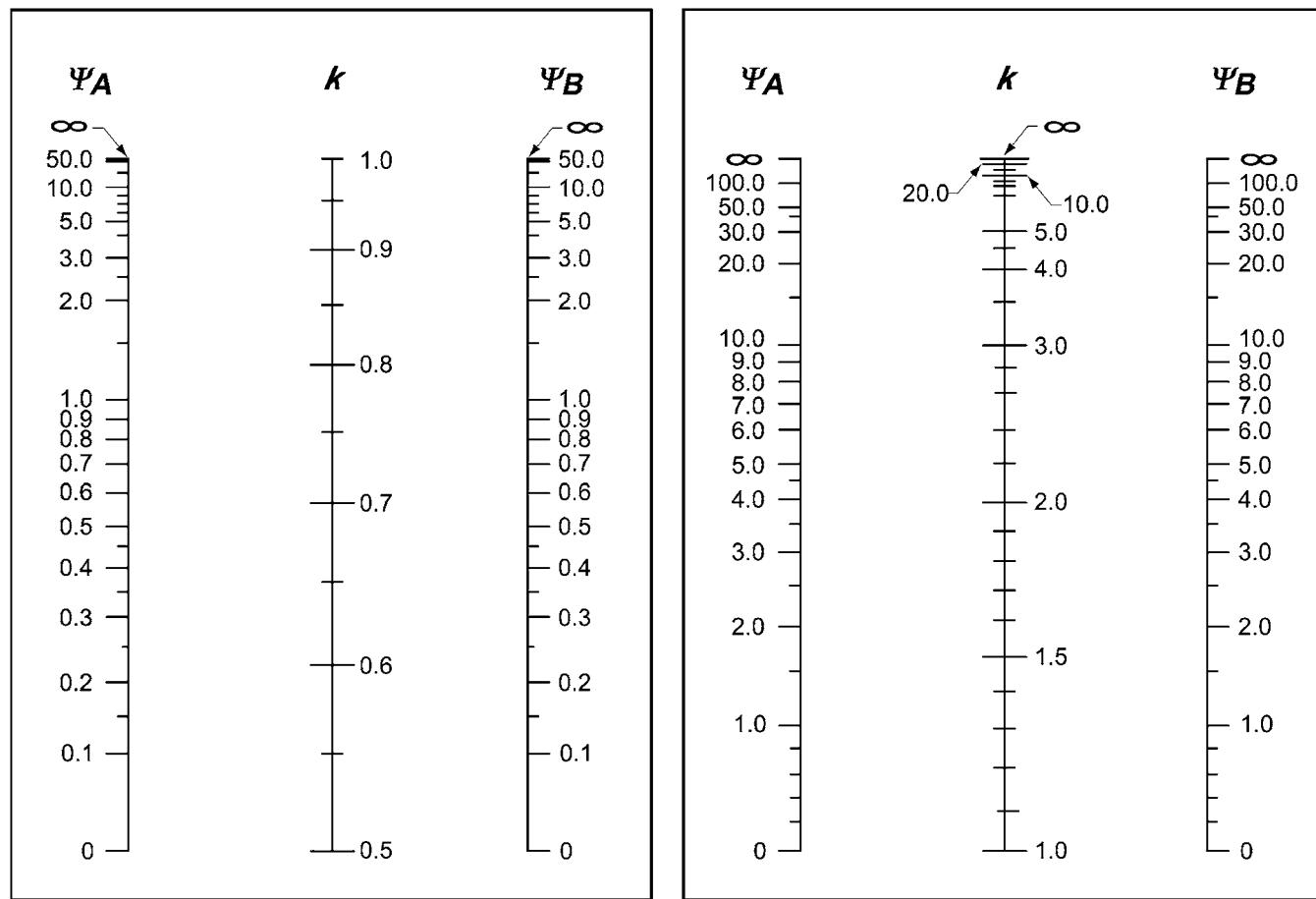
The primary design aid to estimate the effective length factor  $k$  is the Jackson and Moreland Alignment Charts (Fig. R10.10.1.1), which allow a graphical determination of  $k$  for a column of constant cross section in a multibay frame.<sup>10,4,10,30</sup>

Equation (10-7) is based on Eq. (10-11) assuming that a 5 percent increase in moments due to slenderness is acceptable.<sup>10,31</sup> As a first approximation,  $k$  may be taken equal to 1.0 in Eq. (10-7).

The Commentary used to state that a compression member may be assumed braced if located in a story in which the bracing elements have a total stiffness, resisting lateral movement of the story, at least six times the sum of the stiffnesses of all the columns in the story. In ACI 318-95, the language was changed to: "... the bracing elements have such substantial lateral stiffness to resist the lateral deflections of the story that any resulting lateral deflection is not large enough to affect the column strength substantially." The

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(a)  
Nonsway Frames(b)  
Sway Frames

$\Psi$  = ratio of  $\Sigma(EI/\ell_c)$  of compression members to  $\Sigma(EI/\ell)$  of flexural members in a plane at one end of a compression member

$\ell$  = span length of flexural member measured center to center of joints

Fig. R10.10.1.1—Effective length factors  $k$ .

**10.10.1.1** — The unsupported length of a compression member,  $\ell_u$ , shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered. Where column capitals or haunches are present,  $\ell_u$  shall be measured to the lower extremity of the capital or haunch in the plane considered.

**10.10.1.2** — It shall be permitted to take the radius of gyration,  $r$ , equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it shall be permitted to compute  $r$  for the gross concrete section.

change was made because of some concern that the multiplier of six might not be conservative enough. For the 2008 Code, a more conservative multiplier of 12 was chosen. The stiffness of the lateral bracing is considered based on the principal directions of the framing system. Bracing elements in typical building structures consist of shear walls or lateral braces. Torsional eccentricity of the structural system can increase second-order effects and should be considered.

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**10.10.2** — When slenderness effects are not neglected as permitted by **10.10.1**, the design of compression members, restraining beams, and other supporting members shall be based on the factored forces and moments from a second-order analysis satisfying **10.10.3**, **10.10.4**, or **10.10.5**. These members shall also satisfy **10.10.2.1** and **10.10.2.2**. The dimensions of each member cross section used in the analysis shall be within 10 percent of the dimensions of the members shown on the contract documents or the analysis shall be repeated.

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**10.10.2.1** — Total moment including second-order effects in compression members, restraining beams, or other structural members shall not exceed 1.4 times the moment due to first-order effects.

**10.10.2.2** — Second-order effects shall be considered along the length of compression members. It shall be permitted to account for these effects using the moment magnification procedure outlined in **10.10.6**.

**10.10.3 — Nonlinear second-order analysis**

Second-order analysis shall consider material nonlinearity, member curvature and lateral drift, duration of loads, shrinkage and creep, and interaction with the supporting foundation. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with results of comprehensive tests of columns in statically indeterminate reinforced concrete structures.

**R10.10.2** — Design may be based on a nonlinear second-order analysis, an elastic second-order analysis, or the moment magnifier approach.<sup>10.31-10.33</sup> The structure that is analyzed should have members similar to those in the final structure. If the members in the final structure have cross-sectional dimensions more than 10 percent different from those assumed in the analysis, new member properties should be computed and the analysis repeated.

Several methods have been developed to evaluate slenderness effects in compression members that are subject to biaxial bending. A review of some of these methods is presented in **Reference 10.34**.

**R10.10.2.1** — If the weight of a structure is high in proportion to its lateral stiffness, excessive  $P\Delta$  effects (where secondary moments are more than 25 percent of the primary moments) may result, which will eventually introduce singularities into the solution to the equations of equilibrium, indicating physical structural instability.<sup>10.35</sup> Analytical research<sup>10.36</sup> on reinforced concrete frames showed that the probability of stability failure increases rapidly when the stability index  $Q$  exceeds 0.2, which is equivalent to a secondary-to-primary moment ratio of 1.25. According to ASCE/SEI 7-10,<sup>10.37</sup> the maximum value of the stability coefficient  $\theta$ , which is close to the ACI stability coefficient  $Q$ , is 0.25. This value is equivalent to a secondary-to-primary moment ratio of 1.33. The upper limit of 1.4 on the secondary-to-primary moment ratio was chosen considering the above. By providing an upper limit on the second-order moment, it was unnecessary to retain the stability check given in 10.13.6 of the 2005 Code.

**R10.10.2.2** — The maximum moment in a compression member may occur between its ends. While second-order computer analysis programs may be used to evaluate magnification of the end moments, magnification between the ends may not be accounted for unless the member is subdivided along its length. The magnification may be evaluated using the procedure outlined in **10.10.6**.

**R10.10.3 — Nonlinear second-order analysis**

The nonlinear second-order analysis procedure should have been shown to predict ultimate loads within 15 percent of those reported in tests of indeterminate reinforced concrete structures. At the very least, the comparison should include tests of columns in planar nonsway frames, sway frames, and frames with varying column stiffnesses. To allow for variability in the actual member properties and in the analysis, the member properties used in analysis should be multiplied by a stiffness reduction factor  $\phi_K$  less than 1. The concept of a stiffness reduction factor  $\phi_K$  is discussed in **R10.10.4**. For consistency with the second-order analysis in **10.10.4**, the stiffness reduction factor  $\phi_K$  can be taken as 0.80.

**CODE****COMMENTARY****10.10.4 — Elastic second-order analysis**

Elastic second-order analysis shall consider section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and the effects of load duration.

**R10.10.4 — Elastic second-order analysis**

The stiffnesses  $EI$  used in an analysis for strength design should represent the stiffnesses of the members immediately prior to failure. This is particularly true for a second-order analysis that should predict the lateral deflections at loads approaching ultimate. The  $EI$  values should not be based totally on the moment-curvature relationship for the most highly loaded section along the length of each member. Instead, they should correspond to the moment-end rotation relationship for a complete member.

Design computations for slender columns and frames include both a strength reduction factor  $\phi$  for the cross-sectional strength and a stiffness reduction factor  $\phi_K$  for the member stiffnesses. The variability in the cross-sectional strength is accounted for by  $\phi$  in the interaction diagrams while the variability of member stiffness is accounted for by  $\phi_K$  in the structural analysis.

**10.10.4.1** — It shall be permitted to use the following properties for the members in the structure:

- (a) Modulus of elasticity .....  $E_c$  from 8.5.1
- (b) Moments of inertia,  $I$ 
  - Compression members:
 

Columns .....	<b>0.70<math>I_g</math></b>
Walls—Uncracked .....	<b>0.70<math>I_g</math></b>
—Cracked.....	<b>0.35<math>I_g</math></b>
  - Flexural members:
 

Beams .....	<b>0.35<math>I_g</math></b>
Flat plates and flat slabs.....	<b>0.25<math>I_g</math></b>
  - (c) Area..... **1.0 $A_g$**

Alternatively, the moments of inertia of compression and flexural members,  $I$ , shall be permitted to be computed as follows:

Compression members:

$$I = \left(0.80 + 25\frac{A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - 0.5\frac{P_u}{P_o}\right)I_g \leq 0.875I_g \quad (10-8)$$

where  $P_u$  and  $M_u$  shall be determined from the particular load combination under consideration, or the combination of  $P_u$  and  $M_u$  resulting in the smallest value of  $I$ .  $I$  need not be taken less than **0.35 $I_g$** .

Flexural members:

$$I = (0.10 + 25\rho)\left(1.2 - 0.2\frac{b_w}{d}\right)I_g \leq 0.5I_g \quad (10-9)$$

**R10.10.4.1** — The values of  $E_c$ ,  $I$ , and  $A$  have been chosen from the results of frame tests and analyses and include an allowance for the variability of the computed deflections. The modulus of elasticity of the concrete,  $E_c$ , is based on the specified concrete compressive strength while the sway deflections are a function of the average concrete strength, which is higher. The moments of inertia are taken from Reference 10.36, which are multiplied by the stiffness reduction factor  $\phi_K = 0.875$ . For example, the moment of inertia for columns is **0.875(0.80 $I_g$ ) = 0.70 $I_g$** . These two effects result in an overestimation of the second-order deflections on the order of 20 to 25 percent, corresponding to an implicit stiffness reduction of 0.80 to 0.85 on the stability calculation.

The moment of inertia of T-beams should be based on the effective flange width defined in 8.12. It is generally sufficiently accurate to take  $I_g$  of a T-beam as two times the  $I_g$  for the web,  $2(b_w h^3/12)$ .

If the factored moments and shears from an analysis based on the moment of inertia of a wall, taken equal to **0.70 $I_g$** , indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with  $I = 0.35I_g$  in those stories where cracking is predicted using factored loads.

The values of the moments of inertia were derived for nonprestressed members. For prestressed members, the moments of inertia may differ depending on the amount, location, and type of the reinforcement and the degree of cracking prior to ultimate. The stiffness values for prestressed concrete members should include an allowance for the variability of the stiffnesses.

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For continuous flexural members,  $I$  shall be permitted to be taken as the average of values obtained from Eq. (10-9) for the critical positive and negative moment sections.  $I$  need not be taken less than  $0.25I_g$ .

The cross-sectional dimensions and reinforcement ratio used in the above formulas shall be within 10 percent of the dimensions and reinforcement ratio shown on the contract documents or the stiffness evaluation shall be repeated.

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Section 10.10 provides requirements for strength and assumes frame analyses will be carried out using factored loads. Analyses of deflections, vibrations, and building periods are needed at various service (unfactored) load levels<sup>10,38,10,39</sup> to determine the serviceability of the structure and to estimate the wind forces in wind tunnel laboratories. The moments of inertia of the structural members in the service load analyses should be representative of the degree of cracking at the various service load levels investigated. Unless a more accurate estimate of the degree of cracking at service load level is available, it is satisfactory to use  $1.0/0.70 = 1.43$  times the moments of inertia given here for service load analyses.

Equations (10-8) and (10-9) provide more refined values of  $EI$  considering axial load, eccentricity, reinforcement ratio, and concrete compressive strength as presented in References 10.40 and 10.41. The stiffnesses provided in these references are applicable for all levels of loading, including service and ultimate, and consider a stiffness reduction factor  $\phi_K$  comparable to that included in 10.10.4.1(b). For use at load levels other than ultimate,  $P_u$  and  $M_u$  should be replaced with their appropriate values at the desired load level.

**10.10.4.2** — When sustained lateral loads are present,  $I$  for compression members shall be divided by  $(1 + \beta_{ds})$ . The term  $\beta_{ds}$  shall be taken as the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination, but shall not be taken greater than 1.0.

### 10.10.5 — Moment magnification procedure

Columns and stories in structures shall be designated as nonsway or sway columns or stories. The design of columns in nonsway frames or stories shall be based on 10.10.6. The design of columns in sway frames or stories shall be based on 10.10.7.

**10.10.5.1** — It shall be permitted to assume a column in a structure is nonsway if the increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments.

**10.10.5.2** — It also shall be permitted to assume a story within a structure is nonsway if:

$$Q = \frac{\sum P_u \Delta_o}{V_{us} \ell_c} \leq 0.05 \quad (10-10)$$

where  $\sum P_u$  and  $V_{us}$  are the total factored vertical load and the horizontal story shear, respectively, in the story being evaluated, and  $\Delta_o$  is the first-order relative lateral deflection between the top and the bottom of that story due to  $V_{us}$ .

**R10.10.4.2** — The unusual case of sustained lateral loads might exist, for example, if there were permanent lateral loads resulting from unequal earth pressures on two sides of a building.

### R10.10.5 — Moment magnification procedure

This section describes an approximate design procedure that uses the moment magnifier concept to account for slenderness effects. Moments computed using an ordinary first-order frame analysis are multiplied by a moment magnifier that is a function of the factored axial load  $P_u$  and the critical buckling load  $P_c$  for the column. Nonsway and sway frames are treated separately. A first-order frame analysis is an elastic analysis that does not include the internal force effects resulting from deflections.

The moment magnifier design method requires the designer to distinguish between nonsway frames, which are designed according to 10.10.6, and sway frames, which are designed according to 10.10.7. Frequently this can be done by inspection by comparing the total lateral stiffness of the columns in a story to that of the bracing elements. A compression member may be assumed nonsway by inspection if it is located in a story in which the bracing elements (shear walls, shear trusses, or other types of lateral bracing) have such substantial lateral stiffness to resist the lateral deflections of the story that any resulting lateral deflection is not large enough to affect the column strength substantially. If not

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readily apparent by inspection, 10.10.5.1 and 10.10.5.2 give two possible ways of doing this. In 10.10.5.1, a story in a frame is said to be nonsway if the increase in the lateral load moments resulting from  $P\Delta$  effects does not exceed 5 percent of the first-order moments.<sup>10.36</sup> Section 10.10.5.2 gives an alternative method of determining this based on the stability index for a story  $Q$ . In computing  $Q$ ,  $\Sigma P_u$  should correspond to the lateral loading case for which  $\Sigma P_u$  is greatest. A frame may contain both nonsway and sway stories. This test would not be suitable if  $V_{us}$  is zero.

If the lateral load deflections of the frame have been computed using service loads and the service load moments of inertia given in 10.10.4, it is permissible to compute  $Q$  in Eq. (10-10) using 1.2 times the sum of the service gravity loads, the service load story shear, and 1.43 times the first-order service load story deflections.

**10.10.6 — Moment magnification procedure — Nonsway**

Compression members shall be designed for factored axial force  $P_u$  and the factored moment amplified for the effects of member curvature  $M_c$  where

$$M_c = \delta M_2 \quad (10-11)$$

where

$$\delta = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad (10-12)$$

and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \quad (10-13)$$

**10.10.6.1 —  $EI$  shall be taken as**

$$EI = \frac{(0.2E_c I_g + E_s I_{se})}{1 + \beta_{dns}} \quad (10-14)$$

or

$$EI = \frac{0.4E_c I_g}{1 + \beta_{dns}} \quad (10-15)$$

Alternatively,  $EI$  shall be permitted to be computed using the value of  $I$  from Eq. (10-8) divided by  $(1 + \beta_{dns})$ .

The  $\phi$ -factors used in the design of slender columns represent two different sources of variability. First, the stiffness reduction  $\phi_K$ -factor accounts for the variability in the stiffness  $EI$  and the moment magnification analysis. Second, the strength reduction  $\phi$ -factor for tied and spiral columns accounts for the variability of the strength of the cross section. Studies reported in Reference 10.42 indicate that the stiffness reduction factor  $\phi_K$  and the cross-sectional strength reduction  $\phi$ -factors do not have the same values. These studies suggest the stiffness reduction factor  $\phi_K$  for an isolated column should be 0.75 for both tied and spiral columns. The 0.75 factor in Eq. (10-12) is the stiffness reduction factor  $\phi_K$ . The factor is based on the probability of understrength of a single isolated slender column. In the case of a multistory frame, the column and frame deflections depend on the average concrete strength, which is higher than the strength of the concrete in the critical single understrength column. For this reason, the value of  $\phi_K$  in 10.10.4 is 0.875.

**R10.10.6.1** — In defining the critical load, the main problem is the choice of a stiffness  $EI$  that reasonably approximates the variations in stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve. Either Eq. (10-14) or (10-15) may be used to compute  $EI$ . Equation (10-14) was derived for small eccentricity ratios and high levels of axial load where slenderness effects are most pronounced. Equation (10-15) is a simplified approximation to Eq. (10-14) and is less accurate.<sup>10.43</sup> For improved accuracy,  $EI$  can be approximated using the suggested  $E$  and  $I$  values provided by Eq. (10-8) divided by  $(1 + \beta_{dns})$ .

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**10.10.6.2** — The term  $\beta_{dns}$  shall be taken as the ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination, but shall not be taken greater than 1.0.

**10.10.6.3** — The effective length factor,  $k$ , shall be permitted to be taken as 1.0.

**10.10.6.4** — For members without transverse loads between supports,  $C_m$  shall be taken as

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \quad (10-16)$$

where  $M_1/M_2$  is positive if the column is bent in single curvature, and negative if the member is bent in double curvature. For members with transverse loads between supports,  $C_m$  shall be taken as 1.0.

**10.10.6.5** — Factored moment,  $M_2$ , in Eq. (10-11) shall not be taken less than

$$M_{2,min} = P_u(0.6 + 0.03h) \quad (10-17)$$

about each axis separately, where 0.6 and  $h$  are in inches. For members in which  $M_{2,min}$  exceeds  $M_2$ , the value of  $C_m$  in Eq. (10-16) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments,  $M_1/M_2$ .

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**R10.10.6.2** — Creep due to sustained load will increase the lateral deflections of a column and hence the moment magnification. This is approximated for design by reducing the stiffness  $EI$  used to compute  $P_c$  and hence  $\delta$  by dividing  $EI$  by  $(1 + \beta_{dns})$ . Both the concrete and steel terms in Eq. (10-14) are divided by  $(1 + \beta_{dns})$  to reflect the premature yielding of steel in columns subjected to sustained load. For simplification, it can be assumed that  $\beta_{dns} = 0.6$ . In this case, Eq. (10-15) becomes

$$EI = 0.25E_c I_g$$

**R10.10.6.3** — The effective length factor for a compression member considering braced behavior ranges from 0.5 and 1.0. While lower values can be justified, it is recommended that a  $k$  value of 1.0 be used. If lower values are used, the calculation of  $k$  should be based on analysis of the frame using  $E_c$  and  $I$  values given in 10.10.4. The Jackson and Moreland Alignment Charts (Fig. R10.10.1.1) can be used to estimate lower values of  $k$ .<sup>10.4,10.30</sup>

**R10.10.6.4** — The factor  $C_m$  is a correction factor relating the actual moment diagram to an equivalent uniform moment diagram. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design should be based on an equivalent uniform moment  $C_m M_2$  that would lead to the same maximum moment when magnified.<sup>10.31</sup>

In the case of compression members that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of  $M_2$  in Eq. (10-11).  $C_m$  is to be taken as 1.0 for this case.

**R10.10.6.5** — In the Code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are very small or zero, the design of slender columns should be based on the minimum eccentricity given in this section. It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (10-16) in determining the ratio  $M_1/M_2$  for the column when the design should be based on minimum eccentricity. This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than the minimum eccentricity and columns with computed eccentricities equal to or greater than the minimum eccentricity.

**CODE****COMMENTARY****10.10.7 — Moment magnification procedure — Sway**

Moments  $M_1$  and  $M_2$  at the ends of an individual compression member shall be taken as

$$M_1 = M_{1ns} + \delta_s M_{1s} \quad (10-18)$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \quad (10-19)$$

where  $\delta_s$  is computed according to 10.10.7.3 or 10.10.7.4.

**10.10.7.1** — Flexural members shall be designed for the total magnified end moments of the compression members at the joint.

**10.10.7.2** — The effective length factor  $k$  shall be determined using the values of  $E_c$  and  $I$  given in 10.10.4 and shall not be less than 1.0.

**10.10.7.3** — The moment magnifier  $\delta_s$  shall be calculated as

$$\delta_s = \frac{1}{1 - Q} \geq 1 \quad (10-20)$$

If  $\delta_s$  calculated by Eq. (10-20) exceeds 1.5,  $\delta_s$  shall be calculated using second-order elastic analysis or 10.10.7.4.

**R10.10.7 — Moment magnification procedure — Sway**

The analysis described in this section deals only with plane frames subjected to loads causing deflections in that plane. If torsional displacements are significant, a three-dimensional second-order analysis should be used.

**R10.10.7.1** — The strength of a sway frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, the structure approaches a failure mechanism and its axial load capacity is drastically reduced. This section provides that the designer make certain that the restraining flexural members have the strength to resist the magnified column moments.

**R10.10.7.3** — The iterative  $P\Delta$  analysis for second-order moments can be represented by an infinite series. The solution of this series is given by Eq. (10-20).<sup>10.36</sup> Reference 10.44 shows that Eq. (10-20) closely predicts the second-order moments in a sway frame until  $\delta_s$  exceeds 1.5.

The  $P\Delta$  moment diagrams for deflected columns are curved, with  $\Delta$  related to the deflected shape of the columns. Equation (10-20) and most commercially available second-order frame analyses have been derived assuming that the  $P\Delta$  moments result from equal and opposite forces of  $P\Delta/\ell_c$  applied at the bottom and top of the story. These forces give a straight line  $P\Delta$  moment diagram. The curved  $P\Delta$  moment diagrams lead to lateral displacements on the order of 15 percent larger than those from the straight line  $P\Delta$  moment diagrams. This effect can be included in Eq. (10-20) by writing the denominator as  $(1 - 1.15Q)$  rather than  $(1 - Q)$ . The 1.15 factor has been left out of Eq. (10-20) for simplicity.

If deflections have been calculated using service loads,  $Q$  in Eq. (10-20) should be calculated in the manner explained in R10.10.5.

The  $Q$  factor analysis is based on deflections calculated using the values of  $E_c$  and  $I$  from 10.10.4, which include the equivalent of a stiffness reduction factor  $\phi_K$ . These values of  $E_c$  and  $I$  lead to a 20 to 25 percent overestimation of the lateral deflections that corresponds to a stiffness reduction factor  $\phi_K$  between 0.80 and 0.85 on the  $P\Delta$  moments. As a result, no additional  $\phi$ -factor is needed. Once the moments

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**10.10.7.4** — Alternatively, it shall be permitted to calculate  $\delta_s$  as

$$\delta_s = \frac{1}{\frac{\Sigma P_u}{1 - \frac{0.75 \Sigma P_c}} \geq 1} \quad (10-21)$$

where  $\Sigma P_u$  is the summation for all the factored vertical loads in a story and  $\Sigma P_c$  is the summation for all sway-resisting columns in a story.  $P_c$  is calculated using Eq. (10-13) with  $k$  determined from 10.10.7.2 and  $EI$  from 10.10.6.1, where  $\beta_{ds}$  shall be substituted for  $\beta_{dns}$ .

are established using Eq. (10-20), selection of the cross sections of the columns involves the strength reduction factors  $\phi$  from 9.3.2.2.

**R10.10.7.4** — To check the effects of story stability,  $\delta_s$  is computed as an averaged value for the entire story based on use of  $\Sigma P_u / \Sigma P_c$ . This reflects the interaction of all sway-resisting columns in the story in the  $P\Delta$  effects since the lateral deflection of all columns in the story should be equal in the absence of torsional displacements about a vertical axis. In addition, it is possible that a particularly slender individual column in a sway frame could have substantial midheight deflections even if adequately braced against lateral end deflections by other columns in the story. Such a column should be checked using 10.10.6.

If the lateral load deflections involve a significant torsional displacement, the moment magnification in the columns farthest from the center of twist may be underestimated by the moment magnifier procedure. In such cases, a three-dimensional second-order analysis should be considered.

The 0.75 in the denominator of Eq. (10-21) is a stiffness reduction factor  $\phi_K$  as explained in R10.10.6.

In the calculation of  $EI$ ,  $\beta_{ds}$  will normally be zero for a sway frame because the lateral loads are generally of short duration. Sway deflections due to short-term loads such as wind or earthquake are a function of the short-term stiffness of the columns following a period of sustained gravity load. For this case, the definition of  $\beta_{ds}$  in 10.10.4.2 gives  $\beta_{ds} = 0$ . In the unusual case of a sway frame where the lateral loads are sustained,  $\beta_{ds}$  will not be zero. This might occur if a building on a sloping site is subjected to earth pressure on one side but not on the other.

## 10.11 — Axially loaded members supporting slab system

Axially loaded members supporting a slab system included within the scope of 13.1 shall be designed as provided in Chapter 10 and in accordance with the additional requirements of Chapter 13.

## 10.12 — Transmission of column loads through floor system

If  $f'_c$  of a column is greater than 1.4 times that of the floor system, transmission of load through the floor system shall be provided by 10.12.1, 10.12.2, or 10.12.3.

## R10.12 — Transmission of column loads through floor system

The requirements of this section are based on the effect of floor concrete strength on column strength.<sup>10.45</sup> The provisions mean that where the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken. For higher column concrete strengths, methods in 10.12.1 or 10.12.2 should be used for corner or edge columns. Methods in 10.12.1, 10.12.2, or 10.12.3 should be used for interior columns with adequate restraint on all four sides.

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**10.12.1** — Concrete of strength specified for the column shall be placed in the floor at the column location. Top surface of the column concrete shall extend 2 ft into the slab from face of column. Column concrete shall be well integrated with floor concrete, and shall be placed in accordance with 6.4.6 and 6.4.7.

**R10.12.1** — Application of the concrete placement procedure described in 10.12.1 requires the placing of two different concrete mixtures in the floor system. The lower-strength mixture should be placed while the higher-strength concrete is still plastic and should be adequately vibrated to ensure the concretes are well integrated. This requires careful coordination of the concrete deliveries and the possible use of retarders. In some cases, additional inspection services will be required when this procedure is used. It is important that the higher-strength concrete in the floor in the region of the column be placed before the lower-strength concrete in the remainder of the floor to prevent accidental placing of the low-strength concrete in the column area. It is the responsibility of the licensed design professional to indicate on the contract documents where the high- and low-strength concretes are to be placed.

**10.12.2** — Strength of a column through a floor system shall be based on the lower value of concrete strength with vertical dowels and spirals as required.

**10.12.3** — For columns laterally supported on four sides by beams of approximately equal depth or by slabs, it shall be permitted to base strength of the column on an assumed concrete strength in the column joint equal to 75 percent of column concrete strength plus 35 percent of floor concrete strength. In the application of 10.12.3, the ratio of column concrete strength to slab concrete strength shall not be taken greater than 2.5 for design.

### **10.13 — Composite compression members**

**10.13.1** — Composite compression members shall include all such members reinforced longitudinally with structural steel shapes, pipe, or tubing with or without longitudinal bars.

**10.13.2** — Strength of a composite member shall be computed for the same limiting conditions applicable to ordinary reinforced concrete members.

Beginning with the 1983 Code, the amount of column concrete to be placed within the floor is expressed as a simple 2 ft extension from face of the column. Since the concrete placement requirement should be carried out in the field, it is now expressed in a way that is directly evident to workers. The new requirement will also locate the interface between column and floor concrete farther out into the floor, away from regions of very high shear.

**R10.12.3** — Research<sup>10.46</sup> has shown that heavily loaded slabs do not provide as much confinement as lightly loaded slabs when ratios of column concrete strength to slab concrete strength exceed about 2.5. Consequently, a limit is placed on the concrete strength ratio assumed in design.

### **R10.13 — Composite compression members**

**R10.13.1** — Composite columns are defined without reference to classifications of combination, composite, or concrete-filled pipe column. Reference to other metals used for reinforcement has been omitted because they are seldom used in concrete construction.

**R10.13.2** — The same rules used for computing the load-moment interaction strength for reinforced concrete sections can be applied to composite sections. Interaction charts for concrete-filled tubing would have a form identical to those of the *ACI Design Handbook*<sup>10.4</sup> but with  $\gamma$  slightly greater than 1.0.

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**10.13.3** — Any axial load strength assigned to concrete of a composite member shall be transferred to the concrete by members or brackets in direct bearing on the composite member concrete.

**10.13.4** — All axial load strength not assigned to concrete of a composite member shall be developed by direct connection to the structural steel shape, pipe, or tube.

**10.13.5** — For evaluation of slenderness effects, radius of gyration,  $r$ , of a composite section shall be not greater than the value given by

$$r = \sqrt{\frac{(E_c I_g/5) + E_s I_{sx}}{(E_c A_g/5) + E_s A_{sx}}} \quad (10-22)$$

and, as an alternative to a more accurate calculation,  $EI$  in Eq. (10-13) shall be taken either as Eq. (10-14) or

$$EI = \frac{(E_c I_g/5)}{1 + \beta_{dns}} + E_s I_{sx} \quad (10-23)$$

#### 10.13.6 — Structural steel-encased concrete core

**10.13.6.1** — For a composite member with a concrete core encased by structural steel, the thickness of the steel encasement shall be not less than

$$b \sqrt{\frac{f_y}{3E_s}} \text{ for each face of width } b$$

nor

$$h \sqrt{\frac{f_y}{8E_s}} \text{ for circular sections of diameter } h$$

**10.13.6.2** — Longitudinal bars located within the encased concrete core shall be permitted to be used in computing  $A_{sx}$  and  $I_{sx}$ .

#### 10.13.7 — Spiral reinforcement around structural steel core

A composite member with spirally reinforced concrete around a structural steel core shall conform to 10.13.7.1 through 10.13.7.4.

**R10.13.3 and R10.13.4** — Direct bearing or direct connection for transfer of forces between steel and concrete can be developed through lugs, plates, or reinforcing bars welded to the structural shape or tubing before the concrete is cast. Flexural compressive stress need not be considered a part of direct compression load to be developed by bearing. A concrete encasement around a structural steel shape may stiffen the shape, but it would not necessarily increase its strength.

**R10.13.5** — Equation (10-22) is given because the rules of 10.10.1.2 for estimating the radius of gyration are overly conservative for concrete-filled tubing and are not applicable for members with enclosed structural shapes.

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the steel, increasing the steel stresses. In the case of lightly reinforced columns, this load transfer may cause the compression steel to yield prematurely, resulting in a loss in the effective  $EI$ . Accordingly, both the concrete and steel terms in Eq. (10-14) are reduced to account for creep. For heavily reinforced columns or for composite columns in which the pipe or structural shape makes up a large percentage of the cross section, the load transfer due to creep is not significant. Accordingly, Eq. (10-23) was revised in the 1980 Code supplement so that only the  $EI$  of the concrete is reduced for sustained load effects.

#### R10.13.6 — Structural steel-encased concrete core

Steel-encased concrete sections should have a metal wall thickness large enough to attain longitudinal yield stress before buckling outward.

#### R10.13.7 — Spiral reinforcement around structural steel core

| Concrete that is transversely confined by a spiral has increased strength, and the size of the spiral required can be regulated on the basis of the strength of the concrete outside

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**10.13.7.1** — Design yield strength of structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi.

**10.13.7.2** — Spiral reinforcement shall conform to 10.9.3.

**10.13.7.3** — Longitudinal bars located within the spiral shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

**10.13.7.4** — Longitudinal bars located within the spiral shall be permitted to be used in computing  $A_{sx}$  and  $I_{sx}$ .

**10.13.8 — Tie reinforcement around structural steel core**

| A composite member with transversely tied concrete around a structural steel core shall conform to 10.13.8.1 through 10.13.8.7.

**10.13.8.1** — Design yield strength of structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi.

| **10.13.8.2** — Transverse ties shall extend completely around the structural steel core.

| **10.13.8.3** — Transverse ties shall have a diameter not less than 0.02 times the greatest side dimension of composite member, except that ties shall not be smaller than No. 3 and are not required to be larger than No. 5. Welded wire reinforcement of equivalent area shall be permitted.

| **10.13.8.4** — Vertical spacing of transverse ties shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or 0.5 times the least side dimension of the composite member.

**10.13.8.5** — Longitudinal bars located within the ties shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

**10.13.8.6** — A longitudinal bar shall be located at every corner of a rectangular cross section, with other longitudinal bars spaced not farther apart than one-half the least side dimension of the composite member.

**10.13.8.7** — Longitudinal bars located within the ties shall be permitted to be used in computing  $A_{sx}$  and  $I_{sx}$ .

the spiral—the same reasoning that applies for columns reinforced only with longitudinal bars. The radial pressure provided by the spiral ensures interaction between concrete, reinforcing bars, and steel core such that longitudinal bars will both stiffen and strengthen the cross section.

**R10.13.8 — Tie reinforcement around structural steel core**

| The design yield strength of the steel core should be limited to that which would not generate spalling of the concrete. It has been assumed that axially compressed concrete will not spall at strains less than 0.0018. The yield strength of  $0.0018 \times 29,000,000$ , or 52,000 psi, represents an upper limit of the useful maximum steel stress.

| Research<sup>10.47</sup> has shown that the required amount of tie reinforcement around the structural steel core is sufficient for the longitudinal steel bars to be included in the flexural stiffness of the composite column.

**CODE****COMMENTARY****10.14 — Bearing strength**

**10.14.1** — Design bearing strength of concrete shall not exceed  $\phi(0.85f'_c A_1)$ , except when the supporting surface is wider on all sides than the loaded area, then the design bearing strength of the loaded area shall be permitted to be multiplied by  $\sqrt{A_2/A_1}$  but by not more than 2.

**10.14.2** — Section 10.14 does not apply to post-tensioning anchorages.

**R10.14 — Bearing strength**

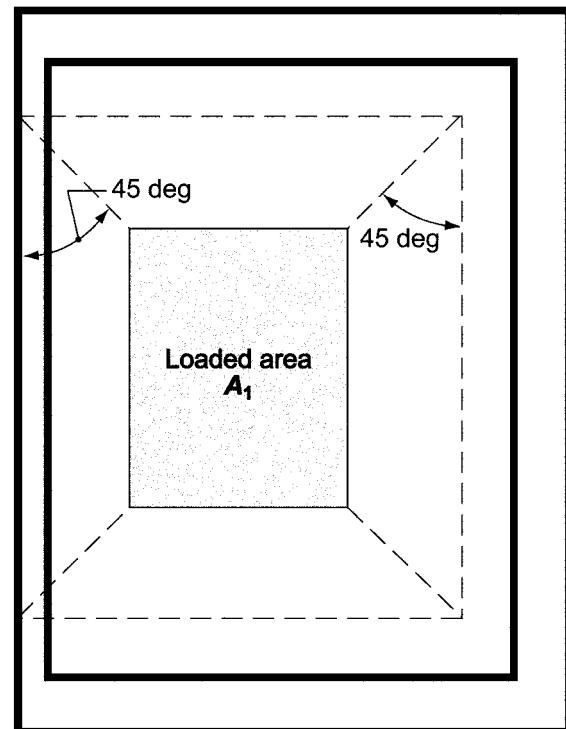
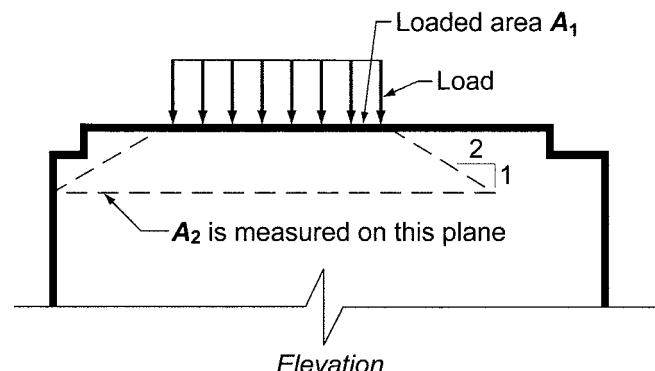
**R10.14.1** — This section deals with bearing strength of concrete supports. The permissible bearing stress of  $0.85f'_c$  is based on tests reported in Reference 10.48. (See also 15.8).

When the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in bearing strength. No minimum depth is given for a supporting member. The minimum depth of support will be controlled by the shear requirements of 11.11.

When the top of the support is sloped or stepped, advantage may still be taken of the condition that the supporting member is larger than the loaded area, provided the supporting member does not slope at too great an angle.

Figure R10.14 illustrates the application of the frustum to find  $A_2$ . The frustum should not be confused with the path by which a load spreads out as it travels downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes to ensure that there is concrete immediately surrounding the zone of high stress at the bearing.  $A_1$  is the loaded area but not greater than the bearing plate or bearing cross-sectional area.

**R10.14.2** — Post-tensioning anchorages are usually laterally reinforced, in accordance with 18.13.

**CODE****COMMENTARY***Plan**Elevation*

*Fig. R10.14—Application of frustum to find  $A_2$  in stepped or sloped supports.*

**Notes**

# CHAPTER 11 — SHEAR AND TORSION

## CODE

### 11.1 — Shear strength

**11.1.1** — Except for members designed in accordance with [Appendix A](#), design of cross sections subject to shear shall be based on

$$\phi V_n \geq V_u \quad (11-1)$$

where  $V_u$  is the factored shear force at the section considered and  $V_n$  is nominal shear strength computed by

$$V_n = V_c + V_s \quad (11-2)$$

where  $V_c$  is nominal shear strength provided by concrete calculated in accordance with [11.2](#), [11.3](#), or [11.11](#), and  $V_s$  is nominal shear strength provided by shear reinforcement calculated in accordance with [11.4](#), [11.9.9](#), or [11.11](#).

**11.1.1.1** — In determining  $V_n$ , the effect of any openings in members shall be considered.

**11.1.1.2** — In determining  $V_c$ , whenever applicable, effects of axial tension due to creep and shrinkage in restrained members shall be considered and effects of inclined flexural compression in variable depth members shall be permitted to be included.

**11.1.2** — The values of  $\sqrt{f'_c}$  used in this chapter shall not exceed 100 psi except as allowed in [11.1.2.1](#).

## COMMENTARY

### R11.1 — Shear strength

This chapter includes shear and torsion provisions for both nonprestressed and prestressed concrete members. The shear-friction concept ([11.6](#)) is particularly applicable to design of reinforcement details in precast structures. Provisions are included for deep flexural members ([11.7](#)), brackets and corbels ([11.8](#)), and shear walls ([11.9](#)). Shear provisions for slabs and footings are given in [11.11](#).

The shear strength is based on an average shear stress on the full effective cross section  $b_w d$ . In a member without shear reinforcement, shear is assumed to be carried by the concrete web. In a member with shear reinforcement, a portion of the shear strength is assumed to be provided by the concrete and the remainder by the shear reinforcement.

The shear strength provided by concrete  $V_c$  is assumed to be the same for beams with and without shear reinforcement and is taken as the shear causing significant inclined cracking. These assumptions are discussed in [References 11.1, 11.2, and 11.3](#).

[Appendix A](#) allows the use of strut-and-tie models in the shear design of disturbed regions. The traditional shear design procedures, which ignore D-regions, are acceptable in shear spans that include B-regions.

**R11.1.1.1** — Openings in the web of a member can reduce its shear strength. The effects of openings are discussed in Section 4.7 of [Reference 11.1](#) and in [References 11.4 and 11.5](#).

**R11.1.1.2** — In a member of variable depth, the internal shear at any section is increased or decreased by the vertical component of the inclined flexural stresses. Computation methods are outlined in various textbooks and in the 1940 Joint Committee Report.<sup>11.6</sup>

**R11.1.2** — Because of a lack of test data and practical experience with concretes having compressive strengths greater than 10,000 psi, the 1989 edition of the Code imposed a maximum value of 100 psi on  $\sqrt{f'_c}$  for use in the calculation of shear strength of concrete beams, joists, and slabs. Exceptions to this limit were permitted in beams and joists when the transverse reinforcement satisfied an increased value for the minimum amount of web reinforcement. There are limited test data on the two-way shear strength of high-strength concrete slabs. Until more experience is obtained for two-way slabs built with concretes that have strengths greater than 10,000 psi, it is prudent to limit  $\sqrt{f'_c}$  to 100 psi for the calculation of shear strength.

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**11.1.2.1** — Values of  $\sqrt{f'_c}$  greater than 100 psi shall be permitted in computing  $V_c$ ,  $V_{ci}$ , and  $V_{cw}$  for reinforced or prestressed concrete beams and concrete joist construction having minimum web reinforcement in accordance with 11.4.6.3, 11.4.6.4, or 11.5.5.2.

**11.1.3** — Computation of maximum  $V_u$  at supports in accordance with 11.1.3.1 or 11.1.3.2 shall be permitted if all conditions (a), (b), and (c) are satisfied:

- (a) Support reaction, in direction of applied shear, introduces compression into the end regions of member;
- (b) Loads are applied at or near the top of the member;
- (c) No concentrated load occurs between face of support and location of critical section defined in 11.1.3.1 or 11.1.3.2.

**11.1.3.1** — For nonprestressed members, sections located less than a distance  $d$  from face of support shall be permitted to be designed for  $V_u$  computed at a distance  $d$ .

**R11.1.2.1** — Based on the test results in References 11.7, 11.8, 11.9, 11.10, and 11.11, an increase in the minimum amount of transverse reinforcement is required for high-strength concrete. These tests indicated a reduction in the reserve shear strength as  $f'_c$  increased in beams reinforced with the specified minimum amount of transverse reinforcement, which is equivalent to an effective shear stress of 50 psi. A provision introduced in the 1989 edition of the Code required an increase in the minimum amount of transverse reinforcement for concrete strengths between 10,000 and 15,000 psi. This provision, which led to a sudden increase in the minimum amount of transverse reinforcement at a compressive strength of 10,000 psi, has been replaced by a gradual increase in the minimum  $A_v$  as  $f'_c$  increases, as given by Eq. (11-13).

**R11.1.3.1** — The closest inclined crack to the support of the beam in Fig. R11.1.3.1(a) will extend upward from the face of the support reaching the compression zone about  $d$  from the face of the support. If loads are applied to the top of this beam, the stirrups across this crack are stressed by loads acting on the lower freebody in Fig. R11.1.3.1(a). The loads applied to the beam between the face of the column and the point  $d$  away from the face are transferred directly to the support by compression in the web above the crack. Accordingly, the Code permits design for a maximum factored shear force  $V_u$  at a distance  $d$  from the support for nonprestressed members, and at a distance  $h/2$  for prestressed members. Two things are emphasized: first, stirrups are required across the potential crack designed for the shear at  $d$  from the support, and second, a tension force exists in the longitudinal reinforcement at the face of the support.

In Fig. R11.1.3.1(b), loads are shown acting near the bottom of a beam. In this case, the critical section is taken at the face of the support. Loads acting near the support should be transferred across the inclined crack extending upward from the support face. The shear force acting on the critical section should include all loads applied below the potential inclined crack.

Typical support conditions where the shear force at a distance  $d$  from the support may be used include: (1)

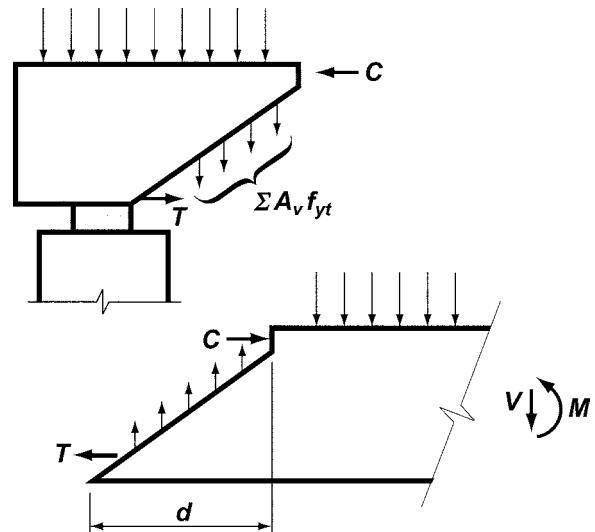
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Fig. R11.1.3.1(a)—Free body diagrams of the end of a beam.

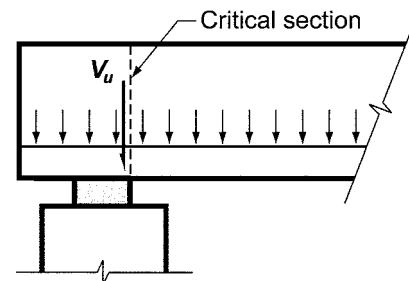
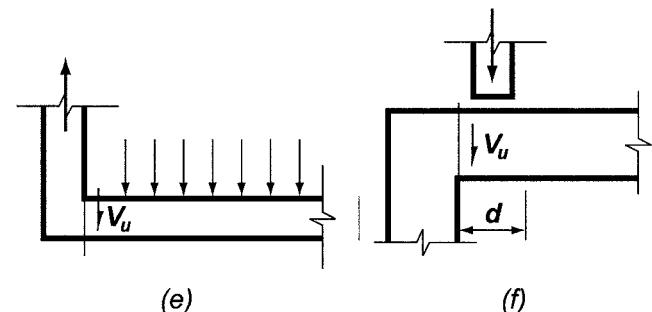
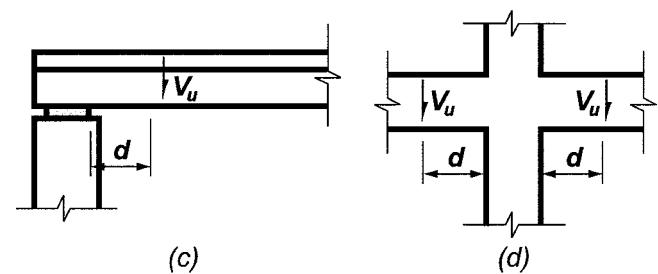


Fig. R11.1.3.1(b)—Location of critical section for shear in a member loaded near bottom.

Fig. R11.1.3.1(c), (d), (e), (f)—Typical support conditions for locating factored shear force  $V_u$ .

**CODE****COMMENTARY**

members supported by bearing at the bottom of the member, such as shown in Fig. R11.1.3.1(c); and (2) members framing monolithically into another member as illustrated in Fig. R11.1.3.1(d).

Support conditions where this provision should not be applied include: (1) Members framing into a supporting member in tension, such as shown in Fig. R11.1.3.1(e). For this case, the critical section for shear should be taken at the face of the support. Shear within the connection should also be investigated and special corner reinforcement should be provided. (2) Members for which loads are not applied at or near the top of the member. This is the condition referred to in Fig. 11.1.3.1(b). For such cases, the critical section is taken at the face of the support. Loads acting near the support should be transferred across the inclined crack extending upward from the support face. The shear force acting on the critical section should include all loads applied below the potential inclined crack. (3) Members loaded such that the shear at sections between the support and a distance  $d$  from the support differs radically from the shear at distance  $d$ . This commonly occurs in brackets and in beams where a concentrated load is located close to the support, as shown in Fig. R11.1.3.1(f) or in footings supported on piles. In this case, the shear at the face of the support should be used.

**11.1.3.2** — For prestressed members, sections located less than a distance  $h/2$  from face of support shall be permitted to be designed for  $V_u$  computed at a distance  $h/2$ .

**11.1.4** — For deep beams, brackets and corbels, walls, and slabs and footings, the special provisions of 11.7 through 11.11 shall apply.

## **11.2 — Shear strength provided by concrete for nonprestressed members**

**11.2.1** —  $V_c$  shall be computed by provisions of 11.2.1.1 through 11.2.1.3, unless a more detailed calculation is made in accordance with 11.2.2. Throughout this chapter, except in 11.6,  $\lambda$  shall be as defined in 8.6.1.

**11.2.1.1** — For members subject to shear and flexure only,

$$V_c = 2\lambda \sqrt{f'_c} b_w d \quad (11-3)$$

**11.2.1.2** — For members subject to axial compression,

$$V_c = 2 \left( 1 + \frac{N_u}{2000 A_g} \right) \lambda \sqrt{f'_c} b_w d \quad (11-4)$$

Quantity  $N_u/A_g$  shall be expressed in psi.

## **R11.2 — Shear strength provided by concrete for nonprestressed members**

**R11.2.1.1** — See R11.2.2.1.

**R11.2.1.2 and R11.2.1.3** — See R11.2.2.2.

**CODE****COMMENTARY**

**11.2.1.3** — For members subject to significant axial tension,  $V_c$  shall be taken as zero unless a more detailed analysis is made using **11.2.2.3**.

**11.2.2** —  $V_c$  shall be permitted to be computed by the more detailed calculation of 11.2.2.1 through **11.2.2.3**.

**11.2.2.1** — For members subject to shear and flexure only,

$$V_c = \left( 1.9 \lambda \sqrt{f'_c} + 2500 \rho_w \frac{V_u d}{M_u} \right) b_w d \quad (11-5)$$

but not greater than  $3.5 \lambda \sqrt{f'_c} b_w d$ . When computing  $V_c$  by Eq. (11-5),  $V_u d/M_u$  shall not be taken greater than 1.0, where  $M_u$  occurs simultaneously with  $V_u$  at section considered.

**11.2.2.2** — For members subject to axial compression, it shall be permitted to compute  $V_c$  using Eq. (11-5) with  $M_m$  substituted for  $M_u$  and  $V_u d/M_u$  not then limited to 1.0, where

$$M_m = M_u - N_u \frac{(4h-d)}{8} \quad (11-6)$$

However,  $V_c$  shall not be taken greater than

$$V_c = 3.5 \lambda \sqrt{f'_c} b_w d \sqrt{1 + \frac{N_u}{500 A_g}} \quad (11-7)$$

$N_u/A_g$  shall be expressed in psi. When  $M_m$  as computed by Eq. (11-6) is negative,  $V_c$  shall be computed by Eq. (11-7).

**R11.2.2.1** — Equation (11-5) is the basic expression for shear strength of members without shear reinforcement.<sup>11.3</sup> The three variables in Eq. (11-5),  $\lambda \sqrt{f'_c}$  (as a measure of concrete tensile strength),  $\rho_w$ , and  $V_u d/M_u$ , are known to affect shear strength, although some research data<sup>11.11.12</sup> indicate that Eq. (11-5) overestimates the influence of  $f'_c$  and underestimates the influence of  $\rho_w$  and  $V_u d/M_u$ . Further information<sup>11.13</sup> has indicated that shear strength decreases as the overall depth of the member increases.

The minimum value of  $M_u$  equal to  $V_u d$  in Eq. (11-5) is to limit  $V_c$  near points of inflection.

For most designs, it is convenient to assume that the second term of Eq. (11-5) equals  $0.1 \sqrt{f'_c}$  and use  $V_c$  equal to  $2\lambda \sqrt{f'_c} b_w d$  as permitted in **11.2.1.1**.

**R11.2.2.2** — Equations (11-6) and (11-7), for members subject to axial compression in addition to shear and flexure, are derived in the Joint ACI-ASCE Committee 326 report.<sup>11.3</sup> As  $N_u$  is increased, the value of  $V_c$  computed from Eq. (11-5) and (11-6) will exceed the upper limit given by Eq. (11-7) before the value of  $M_m$  given by Eq. (11-6) becomes negative. The value of  $V_c$  obtained from Eq. (11-5) has no physical significance if a negative value of  $M_m$  is substituted. For this condition, Eq. (11-7) or **Eq. (11-4)** should be used to calculate  $V_c$ . Values of  $V_c$  for members subject to shear and axial load are illustrated in Fig. R11.2.2.2. The background for these equations is discussed and comparisons are made with test data in **Reference 11.2**.

Because of the complexity of Eq. (11-5) and (11-6), an alternative design provision, **Eq. (11-4)**, is permitted.

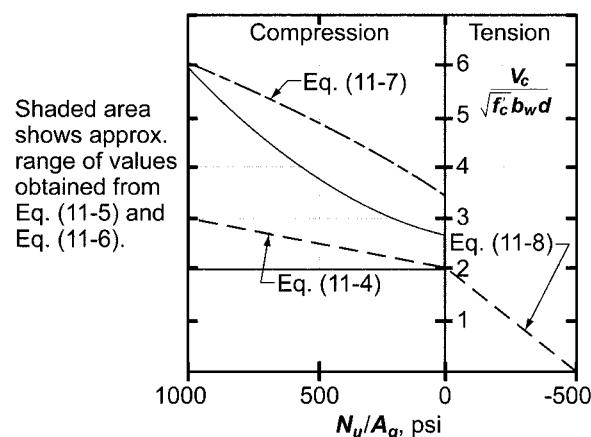


Fig. R11.2.2.2—Comparison of shear strength equations for members subject to axial load.

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**11.2.2.3** — For members subject to significant axial tension,

$$V_c = 2 \left( 1 + \frac{N_u}{500 A_g} \right) \lambda \sqrt{f'_c} b_w d \quad (11-8)$$

but not less than zero, where  $N_u$  is negative for tension.  $N_u/A_g$  shall be expressed in psi.

**11.2.3** — For circular members, the area used to compute  $V_c$  shall be taken as the product of the diameter and effective depth of the concrete section. It shall be permitted to take  $d$  as 0.80 times the diameter of the concrete section.

## 11.3 — Shear strength provided by concrete for prestressed members

**11.3.1** — For the provisions of 11.3,  $d$  shall be taken as the distance from extreme compression fiber to centroid of prestressed and nonprestressed longitudinal tension reinforcement, if any, but need not be taken less than **0.80h**.

**11.3.2** — For members with effective prestress force not less than 40 percent of the tensile strength of flexural reinforcement, unless a more detailed calculation is made in accordance with 11.3.3,

$$V_c = \left( 0.6 \lambda \sqrt{f'_c} + 700 \frac{V_u d_p}{M_u} \right) b_w d \quad (11-9)$$

but  $V_c$  need not be taken less than  $2\lambda \sqrt{f'_c} b_w d$ .  $V_c$  shall not be taken greater than  $5\lambda \sqrt{f'_c} b_w d$  or the value given in **11.3.4** or **11.3.5**.  $V_u d_p/M_u$  shall not be taken greater than 1.0, where  $M_u$  occurs simultaneously with  $V_u$  at the section considered.

**11.3.3** —  $V_c$  shall be permitted to be computed in accordance with **11.3.3.1** and **11.3.3.2**, where  $V_c$  shall be the lesser of  $V_{ci}$  and  $V_{cw}$ .

## COMMENTARY

**R11.2.2.3** — Equation (11-8) may be used to compute  $V_c$  for members subject to significant axial tension. Shear reinforcement may then be designed for  $V_n - V_c$ . The term “significant” is used to recognize judgment is required in deciding whether axial tension needs to be considered. Low levels of axial tension often occur due to volume changes, but are not important in structures with adequate expansion joints and minimum reinforcement. It may be desirable to design shear reinforcement to carry total shear if there is uncertainty about the magnitude of axial tension.

**R11.2.3** — Shear tests of members with circular sections indicate that the effective area can be taken as the gross area of the section or as an equivalent rectangular area.<sup>11.1,11.14,11.15</sup>

## R11.3 — Shear strength provided by concrete for prestressed members

**R11.3.2** — Equation (11-9) offers a simple means of computing  $V_c$  for prestressed concrete beams.<sup>11.2</sup> It may be applied to beams having prestressed reinforcement only, or to members reinforced with a combination of prestressed reinforcement and nonprestressed deformed bars. Equation (11-9) is most applicable to members subject to uniform loading and may give conservative results when applied to composite girders for bridges.

In applying Eq. (11-9) to simply supported members subject to uniform loads,  $V_u d_p/M_u$  can be expressed as

$$\frac{V_u d_p}{M_u} = \frac{d_p(\ell - 2x)}{x(\ell - x)}$$

where  $\ell$  is the span length and  $x$  is the distance from the section being investigated to the support. For concrete with  $f'_c$  equal to 5000 psi,  $V_c$  from 11.3.2 varies as shown in Fig. R11.3.2. Design aids based on this equation are given in Reference 11.16.

**R11.3.3** — Two types of inclined cracking occur in concrete beams: web-shear cracking and flexure-shear cracking. These two types of inclined cracking are illustrated in Fig. R11.3.3.

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**11.3.3.1** —  $V_{ci}$  shall be computed by

$$V_{ci} = 0.6\lambda\sqrt{f'_c} b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \quad (11-10)$$

where  $d_p$  need not be taken less than **0.80h** and

$$M_{cre} = (I/y_t)(6\lambda\sqrt{f'_c} + f_{pe} - f_d) \quad (11-11)$$

and values of  $M_{max}$  and  $V_i$  shall be computed from the load combination causing maximum factored moment to occur at the section.  $V_{ci}$  need not be taken less than  $1.7\lambda\sqrt{f'_c} b_w d$ .

**11.3.3.2** —  $V_{cw}$  shall be computed by

$$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p \quad (11-12)$$

where  $d_p$  need not be taken less than **0.80h**.

Alternatively,  $V_{cw}$  shall be computed as the shear force corresponding to dead load plus live load that results in a principal tensile stress of  $4\lambda\sqrt{f'_c}$  at the centroidal axis of member, or at the intersection of flange and web when the centroidal axis is in the flange. In composite members, the principal tensile stress shall be computed using the cross section that resists live load.

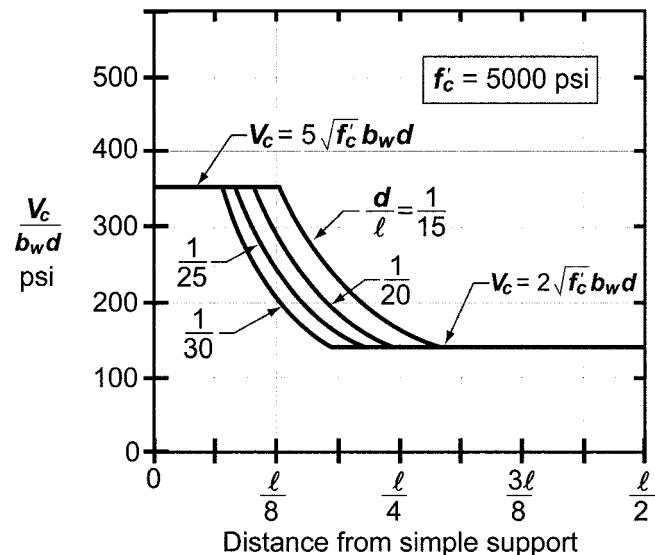
**COMMENTARY**

Fig. R11.3.2—Application of Eq. (11-9) to uniformly loaded prestressed members.

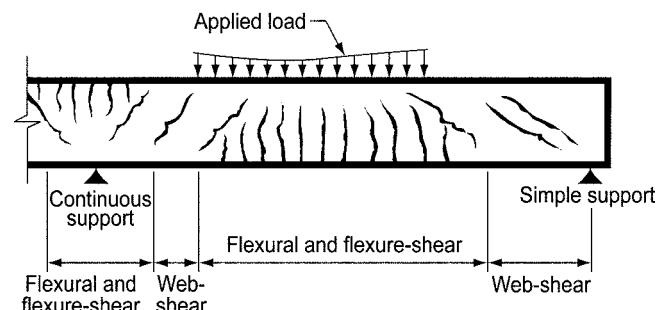


Fig. R11.3.3—Types of cracking in concrete beams.

Web-shear cracking begins from an interior point in a member when the principal tensile stresses exceed the tensile strength of the concrete. Flexure-shear cracking is initiated by flexural cracking. When flexural cracking occurs, the shear stresses in the concrete above the crack are increased. The flexure-shear crack develops when the combined shear and tensile stress exceeds the tensile strength of the concrete.

Equations (11-10) and (11-12) may be used to determine the shear forces causing flexure-shear and web-shear cracking, respectively. The nominal shear strength provided by the concrete  $V_c$  is assumed equal to the lesser of  $V_{ci}$  and  $V_{cw}$ . The derivations of Eq. (11-10) and (11-12) are summarized in Reference 11.17.

In deriving Eq. (11-10) it was assumed that  $V_{ci}$  is the sum of the shear required to cause a flexural crack at the point in question given by

$$V = \frac{V_i M_{cre}}{M_{max}}$$

**CODE****COMMENTARY**

plus an additional increment of shear required to change the flexural crack to a flexure-shear crack. The externally applied factored loads, from which  $V_i$  and  $M_{max}$  are determined, include superimposed dead load, earth pressure, and live load. In computing  $M_{cre}$  for substitution into Eq. (11-10),  $I$  and  $\gamma$  are the properties of the section resisting the externally applied loads.

For a composite member, where part of the dead load is resisted by only a part of the section, appropriate section properties should be used to compute  $f_d$ . The shear due to dead loads,  $V_d$ , and that due to other loads,  $V_i$ , are separated in this case.  $V_d$  is then the total shear force due to unfactored dead load acting on that part of the section carrying the dead loads acting prior to composite action plus the unfactored superimposed dead load acting on the composite member. The terms  $V_i$  and  $M_{max}$  may be taken as

$$V_i = V_u - V_d$$

$$M_{max} = M_u - M_d$$

where  $V_u$  and  $M_u$  are the factored shear and moment due to the total factored loads, and  $M_d$  is the moment due to unfactored dead load (the moment corresponding to  $f_d$ ).

For noncomposite, uniformly loaded beams, the total cross section resists all the shear and the live and dead load shear force diagrams are similar. In this case, Eq. (11-10) reduces to

$$V_{ci} = 0.6 \lambda \sqrt{f'_c} b_w d + \frac{V_u M_{ct}}{M_u}$$

where

$$M_{ct} = (I/y_t)(6\lambda \sqrt{f'_c} + f_{pe})$$

The symbol  $M_{ct}$  in the two preceding equations represents the total moment, including dead load, required to cause cracking at the extreme fiber in tension. This is not the same as  $M_{cre}$  in Code Eq. (11-10) where the cracking moment is that due to all loads except the dead load. In Eq. (11-10), the dead load shear is added as a separate term.

$M_u$  is the factored moment on the beam at the section under consideration, and  $V_u$  is the factored shear force occurring simultaneously with  $M_u$ . Since the same section properties apply to both dead and live load stresses, there is no need to compute dead load stresses and shears separately. The cracking moment  $M_{ct}$  reflects the total stress change from effective prestress to a tension of  $6\lambda \sqrt{f'_c}$ , assumed to cause flexural cracking.

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**11.3.4** — In a pretensioned member in which the section at a distance  $h/2$  from face of support is closer to the end of member than the transfer length of the prestressing steel, the reduced prestress shall be considered when computing  $V_{cw}$ . This value of  $V_{cw}$  shall also be taken as the maximum limit for Eq. (11-9). The prestress force shall be assumed to vary linearly from zero at end of the prestressing steel, to a maximum at a distance from end of the prestressing steel equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

**11.3.5** — In a pretensioned member where bonding of some tendons does not extend to the end of member, a reduced prestress shall be considered when computing  $V_c$  in accordance with 11.3.2 or 11.3.3. The value of  $V_{cw}$  calculated using the reduced prestress shall also be taken as the maximum limit for Eq. (11-9). The prestress force due to tendons for which bonding does not extend to the end of member shall be assumed to vary linearly from zero at the point at which bonding commences to a maximum at a distance from this point equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

## **11.4 — Shear strength provided by shear reinforcement**

### **11.4.1 — Types of shear reinforcement**

**11.4.1.1** — Shear reinforcement consisting of the following shall be permitted:

- (a) Stirrups perpendicular to axis of member;
- (b) Welded wire reinforcement with wires located perpendicular to axis of member;
- (c) Spirals, circular ties, or hoops.

**11.4.1.2** — For nonprestressed members, shear reinforcement shall be permitted to also consist of:

- (a) Stirrups making an angle of 45 degrees or more with longitudinal tension reinforcement;
- (b) Longitudinal reinforcement with bent portion making an angle of 30 degrees or more with the longitudinal tension reinforcement;

Equation (11-12) is based on the assumption that web-shear cracking occurs due to the shear causing a principal tensile stress of approximately  $4\lambda\sqrt{f'_c}$  at the centroidal axis of the cross section.  $V_p$  is calculated from the effective prestress force without load factors.

**R11.3.4 and R11.3.5** — The effect of the reduced prestress near the ends of pretensioned beams on the shear strength should be taken into account. Section 11.3.4 relates to the shear strength at sections within the transfer length of prestressing steel when bonding of prestressing steel extends to the end of the member.

Section 11.3.5 relates to the shear strength at sections within the length over which some of the prestressing steel is not bonded to the concrete, or within the transfer length of the prestressing steel for which bonding does not extend to the end of the beam.

### **R11.4 — Shear strength provided by shear reinforcement**

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(c) Combinations of stirrups and bent longitudinal reinforcement.

**11.4.2** — The values of  $f_y$  and  $f_{yt}$  used in design of shear reinforcement shall not exceed 60,000 psi, except the value shall not exceed 80,000 psi for welded deformed wire reinforcement.

**11.4.3** — Where the provisions of **11.4** are applied to prestressed members,  $d$  shall be taken as the distance from extreme compression fiber to centroid of the prestressed and nonprestressed longitudinal tension reinforcement, if any, but need not be taken less than **0.80h**.

**11.4.4** — Stirrups and other bars or wires used as shear reinforcement shall extend to a distance  $d$  from extreme compression fiber and shall be developed at both ends according to **12.13**.

#### **11.4.5 — Spacing limits for shear reinforcement**

**11.4.5.1** — Spacing of shear reinforcement placed perpendicular to axis of member shall not exceed  **$d/2$**  in nonprestressed members or  **$0.75h$**  in prestressed members, nor 24 in.

**11.4.5.2** — Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45-degree line, extending toward the reaction from mid-depth of member  **$d/2$**  to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement.

**11.4.5.3** — Where  $V_s$  exceeds  $4\sqrt{f'_c} b_w d$ , maximum spacings given in **11.4.5.1** and **11.4.5.2** shall be reduced by one-half.

#### **11.4.6 — Minimum shear reinforcement**

**11.4.6.1** — A minimum area of shear reinforcement,  $A_{v,min}$ , shall be provided in all reinforced concrete flexural members (prestressed and nonprestressed) where  $V_u$  exceeds  **$0.5\phi V_c$** , except in members satisfying one or more of (a) through (f):

(a) Footings and solid slabs;

**R11.4.2** — Limiting the values of  $f_y$  and  $f_{yt}$  used in design of shear reinforcement to 60,000 psi provides a control on diagonal crack width. In the 1995 Code, the limitation of 60,000 psi for shear reinforcement was raised to 80,000 psi for welded deformed wire reinforcement. Research<sup>11.18-11.20</sup> has indicated that the performance of higher-strength steels as shear reinforcement has been satisfactory. In particular, full-scale beam tests described in [Reference 11.19](#) indicated that the widths of inclined shear cracks at service load levels were less for beams reinforced with smaller-diameter welded deformed wire reinforcement cages designed on the basis of a yield strength of 75,000 psi than beams reinforced with deformed Grade 60 stirrups.

**R11.4.3** — Although the value of  $d$  may vary along the span of a prestressed beam, studies<sup>11.2</sup> have shown that, for prestressed concrete members,  $d$  need not be taken less than  **$0.80h$** . The beams considered had some straight tendons or reinforcing bars at the bottom of the section and had stirrups that enclosed the steel.

**R11.4.4** — It is essential that shear (and torsion) reinforcement be adequately anchored at both ends to be fully effective on either side of any potential inclined crack. This generally requires a hook or bend at the end of the reinforcement as provided by **12.13**.

#### **R11.4.6 — Minimum shear reinforcement**

**R11.4.6.1** — Shear reinforcement restrains the growth of inclined cracking. Ductility is increased and a warning of failure is provided. In an unreinforced web, the sudden formation of inclined cracking might lead directly to failure without warning. Such reinforcement is of great value if a member is subjected to an unexpected tensile force or an overload. Accordingly, a minimum area of shear reinforcement

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- (b) Hollow-core units with total untopped depth not greater than 12.5 in. and hollow-core units where  $V_u$  is not greater than  $0.5\phi V_{cw}$ ;
- (c) Concrete joist construction defined by 8.13;
- (d) Beams with  $h$  not greater than 10 in.;
- (e) Beam integral with slabs with  $h$  not greater than 24 in. and not greater than the larger of 2.5 times thickness of flange, and 0.5 times width of web;
- (f) Beams constructed of steel fiber-reinforced, normalweight concrete with  $f'_c$  not exceeding 6000 psi,  $h$  not greater than 24 in., and  $V_u$  not greater than  $\phi 2 \sqrt{f'_c} b_w d$ .

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not less than that given by Eq. (11-13) or (11-14) is required wherever  $V_u$  is greater than  $0.5\phi V_c$ . Solid slabs, footings and joists are excluded from the minimum shear reinforcement requirement because there is a possibility of load sharing between weak and strong areas. However, research<sup>11.21-11.23</sup> has shown that deep, lightly reinforced one-way slabs and beams, particularly if constructed with high-strength concrete, or concrete having a small coarse aggregate size, may fail at shear loads less than  $V_c$ , calculated from Eq. (11-3) especially if subjected to concentrated loads. Because of this, the exclusion for certain beam types in 11.4.6.1(e) is restricted to cases in which  $h$  does not exceed 24 in. For beams where  $f'_c$  is greater than 7000 psi, consideration should be given to providing minimum shear reinforcement when  $h$  is greater than 18 in. and  $V_u$  is greater than  $0.5\phi V_c$ .

Even when  $V_u$  is less than  $0.5\phi V_c$ , the use of some web reinforcement is recommended in all thin-web post-tensioned prestressed concrete members (joists, waffle slabs, beams, and T-beams) to reinforce against tensile forces in webs resulting from local deviations from the design tendon profile, and to provide a means of supporting the tendons in the design profile during construction. If sufficient support is not provided, lateral wobble and local deviations from the smooth parabolic tendon profile assumed in design may result during placement of the concrete. In such cases, the deviations in the tendons tend to straighten out when the tendons are stressed. This process may impose large tensile stresses in webs, and severe cracking may develop if no web reinforcement is provided. Unintended curvature of the tendons, and the resulting tensile stresses in webs, may be minimized by securely tying tendons to stirrups that are rigidly held in place by other elements of the reinforcing cage and held down in the forms. The maximum spacing of stirrups used for this purpose should not exceed the smaller of  $1.5h$  or 4 ft. When applicable, the shear reinforcement provisions of 11.4.5 and 11.4.6 will require closer stirrup spacings.

For repeated loading of flexural members, the possibility of inclined diagonal tension cracks forming at stresses appreciably smaller than under static loading should be taken into account in the design. In these instances, it would be prudent to use at least the minimum shear reinforcement expressed by Eq. (11-13) or (11-14), even though tests or calculations based on static loads show that shear reinforcement is not required.

**R11.4.6.1(b)** — Test results of hollow core units<sup>11.24,11.25</sup> with  $h$  values of 12.5 in. and less have shown shear strengths greater than those calculated by Eq. (11-12) and (11-10). Test results of precast prestressed concrete hollow core units<sup>11.26</sup> with greater depths have shown that web-shear strengths in end regions can be less than strengths computed by Eq. (11-12). By contrast, flexure-shear strengths in those tests equaled or exceeded strengths computed by Eq. (11-10).

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**11.4.6.2** — Minimum shear reinforcement requirements of **11.4.6.1** shall be permitted to be waived if shown by test that required  $M_n$  and  $V_n$  can be developed when shear reinforcement is omitted. Such tests shall simulate effects of differential settlement, creep, shrinkage, and temperature change, based on a realistic assessment of such effects occurring in service.

**11.4.6.3** — Where shear reinforcement is required by **11.4.6.1** or for strength and where **11.5.1** allows torsion to be neglected,  $A_{v,min}$  for prestressed (except as provided in 11.4.6.4) and nonprestressed members shall be computed by

$$A_{v,min} = 0.75 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \quad (11-13)$$

but shall not be less than  $(50b_w s)/f_{yt}$ .

**11.4.6.4** — For prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement,  $A_{v,min}$  shall not be less than the smaller value from Eq. (11-13) and (11-14).

$$A_{v,min} = \frac{A_{ps} f_{pu} s}{80 f_{yt} d} \sqrt{\frac{d}{b_w}} \quad (11-14)$$

**R11.4.6.1(f)** — This exception is intended to provide a design alternative to the use of shear reinforcement, as defined in **11.4.1.1**, for members with longitudinal flexural reinforcement in which  $V_u$  does not exceed  $\phi 2 \sqrt{f'_c} b_w d$ . Fiber-reinforced concrete beams with hooked or crimped steel fibers in dosages as required by **5.6.6.2** have been shown, through laboratory tests, to exhibit shear strengths larger than  $3.5 \sqrt{f'_c} b_w d$ .<sup>11.27</sup> There are no data for the use of steel fibers as shear reinforcement in concrete members exposed to chlorides from deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources. Where steel fibers are used as shear reinforcement in corrosive environments, corrosion protection should be considered.

**R11.4.6.2** — When a member is tested to demonstrate that its shear and flexural strengths are adequate, the actual member dimensions and material strengths are known. The strength used as a basis for comparison should therefore be that corresponding to a strength reduction factor of unity ( $\phi = 1.0$ ), i.e. the required nominal strength  $V_n$  and  $M_n$ . This ensures that if the actual material strengths in the field were less than specified, or the member dimensions were in error such as to result in a reduced member strength, a satisfactory margin of safety will be retained.

**R11.4.6.3** — Previous versions of the Code have required a minimum area of transverse reinforcement that is independent of concrete strength. Tests<sup>11.9</sup> have indicated the need to increase the minimum area of shear reinforcement as concrete strength increases to prevent sudden shear failures when inclined cracking occurs. Equation (11-13) provides for a gradual increase in the minimum area of transverse reinforcement, while maintaining the previous minimum value.

**R11.4.6.4** — Tests<sup>11.28</sup> of prestressed beams with minimum web reinforcement based on Eq. (11-13) and (11-14) indicated that the smaller  $A_v$  from these two equations was sufficient to develop ductile behavior.

Equation (11-14) may be used only for prestressed members meeting the minimum prestress force requirements given in 11.4.6.4. This equation is discussed in **Reference 11.28**.

**CODE****COMMENTARY****11.4.7 — Design of shear reinforcement**

**11.4.7.1** — Where  $V_u$  exceeds  $\phi V_c$ , shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where  $V_s$  shall be computed in accordance with 11.4.7.2 through 11.4.7.9.

**11.4.7.2** — Where shear reinforcement perpendicular to axis of member is used,

$$V_s = \frac{A_v f_{yt} d}{s} \quad (11-15)$$

where  $A_v$  is the area of shear reinforcement within spacing  $s$ .

**11.4.7.3** — Where circular ties, hoops, or spirals are used as shear reinforcement,  $V_s$  shall be computed using Eq. (11-15) where  $d$  is defined in 11.2.3 for circular members,  $A_v$  shall be taken as two times the area of the bar in a circular tie, hoop, or spiral at a spacing  $s$ ,  $s$  is measured in a direction parallel to longitudinal reinforcement, and  $f_{yt}$  is the specified yield strength of circular tie, hoop, or spiral reinforcement.

**11.4.7.4** — Where inclined stirrups are used as shear reinforcement,

$$V_s = \frac{A_v f_{yt} (\sin \alpha + \cos \alpha) d}{s} \quad (11-16)$$

where  $\alpha$  is angle between inclined stirrups and longitudinal axis of the member, and  $s$  is measured in direction parallel to longitudinal reinforcement.

**11.4.7.5** — Where shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support,

$$V_s = A_v f_y \sin \alpha \quad (11-17)$$

but not greater than  $3\sqrt{f'_c b_w d}$ , where  $\alpha$  is angle between bent-up reinforcement and longitudinal axis of the member.

**R11.4.7 — Design of shear reinforcement**

Design of shear reinforcement is based on a modified truss analogy. The truss analogy assumes that the total shear is carried by shear reinforcement. However, considerable research on both nonprestressed and prestressed members has indicated that shear reinforcement needs to be designed to carry only the shear exceeding that which causes inclined cracking, provided the diagonal members in the truss are assumed to be inclined at 45 degrees.

Equations (11-15), (11-16), and (11-17) are presented in terms of nominal shear strength provided by shear reinforcement  $V_s$ . When shear reinforcement perpendicular to axis of member is used, the required area of shear reinforcement  $A_v$  and its spacing  $s$  are computed by

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{yt} d}$$

Research<sup>11.29,11.30</sup> has shown that shear behavior of wide beams with substantial flexural reinforcement is improved if the transverse spacing of stirrup legs across the section is reduced.

**R11.4.7.3** — Although the transverse reinforcement in a circular section may not consist of straight legs, tests indicate that Eq. (11-15) is conservative if  $d$  is taken as defined in 11.2.3.<sup>11.14,11.15</sup>

**CODE****COMMENTARY**

**11.4.7.6** — Where shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support,  $V_s$  shall be computed by Eq. (11-16).

**11.4.7.7** — Only the center three-fourths of the inclined portion of any longitudinal bent bar shall be considered effective for shear reinforcement.

**11.4.7.8** — Where more than one type of shear reinforcement is used to reinforce the same portion of a member,  $V_s$  shall be computed as the sum of the values computed for the various types of shear reinforcement.

**11.4.7.9** —  $V_s$  shall not be taken greater than  $8\sqrt{f'_c} b_w d$ .

**11.5 — Design for torsion**

Design for torsion shall be in accordance with 11.5.1 through 11.5.6, or 11.5.7.

**R11.5 — Design for torsion**

The design for torsion in 11.5.1 through 11.5.6 is based on a thin-walled tube, space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. R11.5(a). Once a reinforced concrete beam has cracked in torsion, its torsional resistance is provided primarily by closed stirrups and longitudinal bars located near the surface

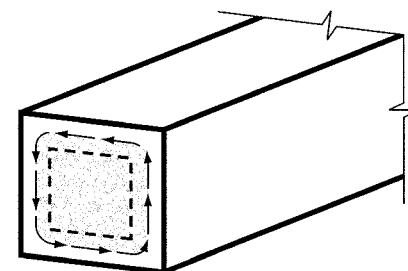
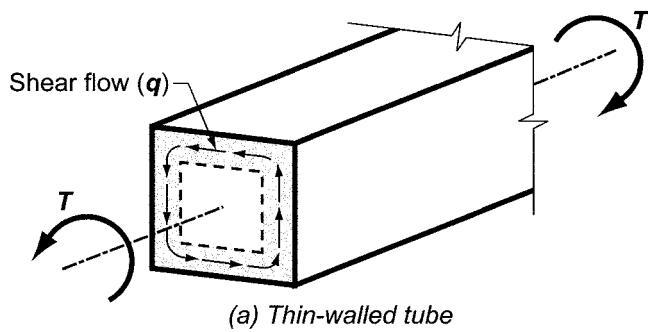


Fig. R11.5—(a) Thin-walled tube; (b) area enclosed by shear flow path.

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of the member. In the thin-walled tube analogy, the resistance is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.

In a closed thin-walled tube, the product of the shear stress  $\tau$  and the wall thickness  $t$  at any point in the perimeter is known as the shear flow,  $q = \tau t$ . The shear flow  $q$  due to torsion acts as shown in Fig. R11.5(a) and is constant at all points around the perimeter of the tube. The path along which it acts extends around the tube at midthickness of the walls of the tube. At any point along the perimeter of the tube the shear stress due to torsion is  $\tau = T/(2A_o t)$  where  $A_o$  is the gross area enclosed by the shear flow path, shown shaded in Fig. R11.5(b), and  $t$  is the thickness of the wall at the point where  $\tau$  is being computed. The shear flow follows the midthickness of the walls of the tube and  $A_o$  is the area enclosed by the path of the shear flow. For a hollow member with continuous walls,  $A_o$  includes the area of the hole.

In the 1995 Code, the elliptical interaction between the nominal shear strength provided by the concrete,  $V_c$ , and the nominal torsion strength provided by the concrete was eliminated.  $V_c$  remains constant at the value it has when there is no torsion, and the torsion carried by the concrete is always taken as zero.

The design procedure is derived and compared with test results in References 11.31 and 11.32.

**11.5.1 — Threshold torsion**

It shall be permitted to neglect torsion effects if the factored torsional moment  $T_u$  is less than:

(a) For nonprestressed members

$$\phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

(b) For prestressed members

$$\phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_c}}}$$

(c) For nonprestressed members subjected to an axial tensile or compressive force

$$\phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4A_g \lambda \sqrt{f'_c}}}$$

Torques that do not exceed approximately one-quarter of the cracking torque  $T_{cr}$  will not cause a structurally significant reduction in either the flexural or shear strength and can be ignored. The cracking torsion under pure torsion  $T_{cr}$  is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness  $t$  prior to cracking of  $0.75A_{cp}/p_{cp}$  and an area enclosed by the wall centerline  $A_o$  equal to  $2A_{cp}/3$ . Cracking is assumed to occur when the principal tensile stress reaches  $4\lambda \sqrt{f'_c}$ . In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equal to the torsional shear stress,  $\tau = T/(2A_o t)$ . Thus, cracking occurs when  $\tau$  reaches  $4\lambda \sqrt{f'_c}$ , giving the cracking torque  $T_{cr}$  as

$$T_{cr} = 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

For solid members, the interaction between the cracking torsion and the inclined cracking shear is approximately circular or elliptical. For such a relationship, a torque of  $0.25T_{cr}$ , as used in 11.5.1, corresponds to a reduction of

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For members cast monolithically with a slab, the overhanging flange width used in computing  $A_{cp}$  and  $p_{cp}$  shall conform to 13.2.4. For a hollow section,  $A_g$  shall be used in place of  $A_{cp}$  in 11.5.1, and the outer boundaries of the section shall conform to 13.2.4.

**11.5.1.1** — For isolated members with flanges and for members cast monolithically with a slab, the overhanging flange width used to compute  $A_{cp}$  and  $p_{cp}$  shall conform to 13.2.4, except that the overhanging flanges shall be neglected in cases where the parameter  $A_{cp}^2/p_{cp}$  calculated for a beam with flanges is less than that computed for the same beam ignoring the flanges.

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3 percent in the inclined cracking shear. This reduction in the inclined cracking shear was considered negligible. The stress at cracking  $4\lambda\sqrt{f'_c}$  has purposely been taken as a lower bound value.

For prestressed members, the torsional cracking load is increased by the prestress. A Mohr's Circle analysis based on average stresses indicates the torque required to cause a principal tensile stress equal to  $4\lambda\sqrt{f'_c}$  is  $\sqrt{1+f_{pc}/(4\lambda\sqrt{f'_c})}$  times the corresponding torque in a nonprestressed beam. A similar modification is made in part (c) of 11.5.1 for members subjected to axial load and torsion.

For torsion, a hollow member is defined as having one or more longitudinal voids, such as a single-cell or multiple-cell box girder. Small longitudinal voids, such as ungrouted post-tensioning ducts that result in  $A_g/A_{cp}$  greater than or equal to 0.95, can be ignored when computing the threshold torque in 11.5.1. The interaction between torsional cracking and shear cracking for hollow sections is assumed to vary from the elliptical relationship for members with small voids, to a straight-line relationship for thin-walled sections with large voids. For a straight-line interaction, a torque of  $0.25T_{cr}$  would cause a reduction in the inclined cracking shear of about 25 percent. This reduction was judged to be excessive.

In the 2002 Code, two changes were made to modify 11.5.1 to apply to hollow sections. First, the minimum torque limits from the 1999 Code were multiplied by  $(A_g/A_{cp})$  because tests of solid and hollow beams<sup>11.33</sup> indicate that the cracking torque of a hollow section is approximately  $(A_g/A_{cp})$  times the cracking torque of a solid section with the same outside dimensions. The second change was to multiply the cracking torque by  $(A_g/A_{cp})$  a second time to reflect the transition from the circular interaction between the inclined cracking loads in shear and torsion for solid members, to the approximately linear interaction for thin-walled hollow sections.

### 11.5.2 — Calculation of factored torsional moment

**11.5.2.1** — If the factored torsional moment,  $T_u$ , in a member is required to maintain equilibrium and exceeds the minimum value given in 11.5.1, the member shall be designed to carry  $T_u$  in accordance with 11.5.3 through 11.5.6.

**11.5.2.2** — In a statically indeterminate structure where reduction of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the maximum  $T_u$  shall be permitted to be reduced to the values given in (a), (b), or (c), as applicable:

- (a) For nonprestressed members, at the sections described in 11.5.2.4

### R11.5.2 — Calculation of factored torsional moment

**R11.5.2.1 and R11.5.2.2** — In designing for torsion in reinforced concrete structures, two conditions may be identified:<sup>11.34,11.35</sup>

- (a) The torsional moment cannot be reduced by redistribution of internal forces (11.5.2.1). This is referred to as equilibrium torsion, since the torsional moment is required for the structure to be in equilibrium.

For this condition, illustrated in Fig. R11.5.2.1, torsion reinforcement designed according to 11.5.3 through 11.5.6 must be provided to resist the total design torsional moments.

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$$\phi 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

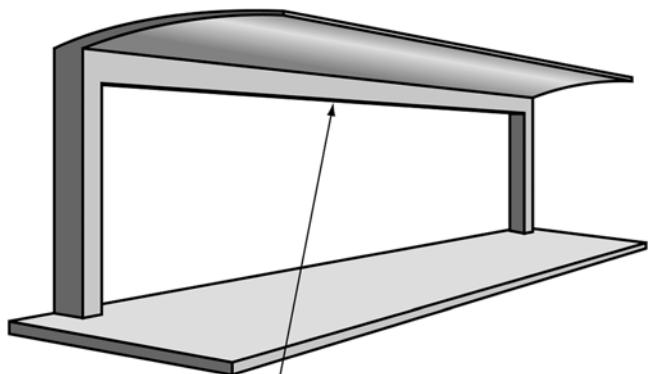
(b) For prestressed members, at the sections described in 11.5.2.5

$$\phi 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_c}}}$$

(c) For nonprestressed members subjected to an axial tensile or compressive force

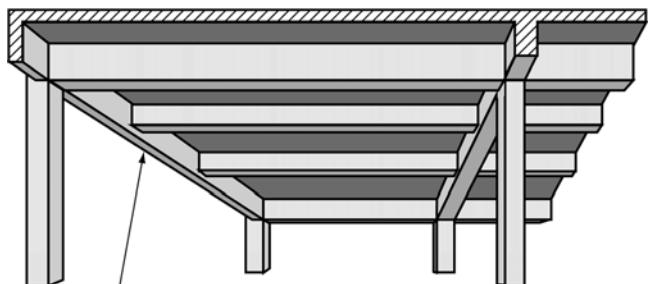
$$\phi 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4A_g \lambda \sqrt{f'_c}}}$$

In (a), (b), or (c), the correspondingly redistributed bending moments and shears in the adjoining members shall be used in the design of these members. For hollow sections,  $A_{cp}$  shall not be replaced with  $A_g$  in 11.5.2.2.

**COMMENTARY**

Design torque may **not** be reduced because moment redistribution is **not** possible

Fig. R11.5.2.1—Design torque may not be reduced (11.5.2.1).



Design torque for this spandrel beam may be reduced because moment redistribution is possible

Fig. R11.5.2.2—Design torque may be reduced (11.5.2.2).

(b) The torsional moment can be reduced by redistribution of internal forces after cracking (11.5.2.2) if the torsion arises from the member twisting to maintain compatibility of deformations. This type of torsion is referred to as compatibility torsion.

For this condition, illustrated in Fig. R11.5.2.2, the torsional stiffness before cracking corresponds to that of the uncracked section according to St. Venant's theory. At torsional cracking, however, a large twist occurs under an essentially constant torque, resulting in a large redistribution of forces in the structure.<sup>11.34,11.35</sup> The cracking torque under combined shear, flexure, and torsion corresponds to a principal tensile stress somewhat less than the  $4\lambda \sqrt{f'_c}$  quoted in R11.5.1.

When the torsional moment exceeds the cracking torque, a maximum factored torsional moment equal to the cracking torque may be assumed to occur at the critical sections near the faces of the supports. This limit has been established to control the width of torsional cracks. The replacement of  $A_{cp}$  with  $A_g$ , as in the calculation of the threshold torque for hollow sections in 11.5.1, is not applied here. Thus, the torque after redistribution is larger and hence more conservative.

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**11.5.2.3** — Unless determined by a more exact analysis, it shall be permitted to take the torsional loading from a slab as uniformly distributed along the member.

**11.5.2.4** — In nonprestressed members, sections located less than a distance  $d$  from the face of a support shall be designed for not less than  $T_u$  computed at a distance  $d$ . If a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support.

**11.5.2.5** — In prestressed members, sections located less than a distance  $h/2$  from the face of a support shall be designed for not less than  $T_u$  computed at a distance  $h/2$ . If a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support.

### 11.5.3 — Torsional moment strength

**11.5.3.1** — The cross-sectional dimensions shall be such that:

(a) For solid sections

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 8 \sqrt{f'_c} \right) \quad (11-18)$$

(b) For hollow sections

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right) \leq \phi \left( \frac{V_c}{b_w d} + 8 \sqrt{f'_c} \right) \quad (11-19)$$

For prestressed members,  $d$  shall be determined in accordance with **11.4.3**.

**Section 11.5.2.2** applies to typical and regular framing conditions. With layouts that impose significant torsional rotations within a limited length of the member, such as a heavy torque loading located close to a stiff column, or a column that rotates in the reverse directions because of other loading, a more exact analysis is advisable.

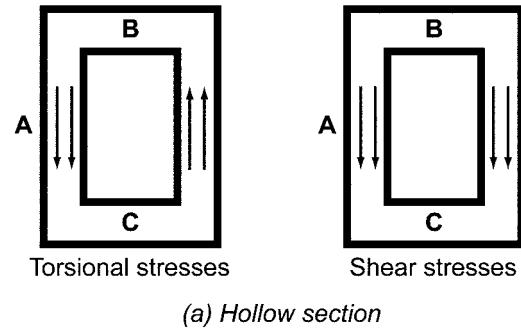
When the factored torsional moment from an elastic analysis based on uncracked section properties is between the values in **11.5.1** and the values given in this section, torsion reinforcement should be designed to resist the computed torsional moments.

**R11.5.2.4 and R11.5.2.5** — It is not uncommon for a beam to frame into one side of a girder near the support of the girder. In such a case, a concentrated shear and torque are applied to the girder.

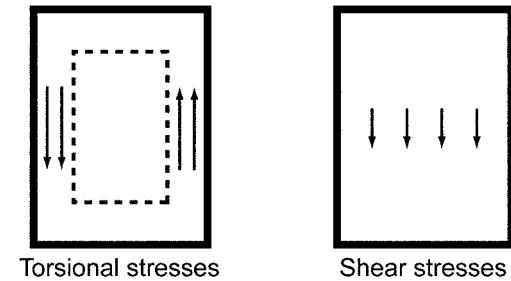
### R11.5.3 — Torsional moment strength

**R11.5.3.1** — The size of a cross section is limited for two reasons: first, to reduce unsightly cracking, and second, to prevent crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. In Eq. (11-18) and (11-19), the two terms on the left-hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus  $8 \sqrt{f'_c}$ , similar to the limiting strength given in **11.4.7.9** for shear without torsion. The limit is expressed in terms of  $V_c$  to allow its use for nonprestressed or prestressed concrete. It was originally derived on the basis of crack control. It is not necessary to check against crushing of the web because this happens at higher shear stresses.

In a hollow section, the shear stresses due to shear and torsion both occur in the walls of the box as shown in **Fig. 11.5.3.1(a)** and hence are directly additive at point A as given in Eq. (11-19). In a solid section, the shear stresses due to torsion act in the “tubular” outside section while the shear stresses due to  $V_u$  are spread across the width of the section as shown in **Fig. R11.5.3.1(b)**. For this reason, stresses are combined in Eq. (11-18) using the square root of the sum of the squares rather than by direct addition.

**CODE****COMMENTARY**

(a) Hollow section



(b) Solid section

Fig. R11.5.3.1—Addition of torsional and shear stresses.

**11.5.3.2** — If the wall thickness varies around the perimeter of a hollow section, [Eq. \(11-19\)](#) shall be evaluated at the location where the left-hand side of [Eq. \(11-19\)](#) is a maximum.

**R11.5.3.2** — Generally, the maximum will be on the wall where the torsional and shearing stresses are additive [Point A in Fig. R11.5.3.1(a)]. If the top or bottom flanges are thinner than the vertical webs, it may be necessary to evaluate [Eq. \(11-19\)](#) at points B and C in Fig. R11.5.3.1(a). At these points, the stresses due to the shear force are usually negligible.

**11.5.3.3** — If the wall thickness is less than  $A_{oh}/p_h$ , the second term in [Eq. \(11-19\)](#) shall be taken as

$$\left( \frac{T_u}{1.7A_{oh}t} \right)$$

where  $t$  is the thickness of the wall of the hollow section at the location where the stresses are being checked.

**11.5.3.4** — The values of  $f_y$  and  $f_{yt}$  used for design of torsional reinforcement shall not exceed 60,000 psi.

**11.5.3.5** — Where  $T_u$  exceeds the threshold torsion, design of the cross section shall be based on

$$\phi T_n \geq T_u \quad (11-20)$$

**R11.5.3.4** — Limiting the values of  $f_y$  and  $f_{yt}$  used in design of torsion reinforcement to 60,000 psi provides a control on diagonal crack width.

**R11.5.3.5** — The factored torsional resistance  $\phi T_n$  must equal or exceed the torsion  $T_u$  due to the factored loads. In the calculation of  $T_n$ , all the torque is assumed to be resisted by stirrups and longitudinal steel with  $T_c = 0$ . At the same

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**11.5.3.6** —  $T_n$  shall be computed by

$$T_n = \frac{2A_o A_t f_y t \cot\theta}{s} \quad (11-21)$$

where  $A_o$  shall be determined by analysis except that it shall be permitted to take  $A_o$  equal to  $0.85A_{oh}$ ;  $\theta$  shall not be taken smaller than 30 degrees nor larger than 60 degrees. It shall be permitted to take  $\theta$  equal to:

- (a) 45 degrees for non prestressed members or members with less prestress than in (b); or
- (b) 37.5 degrees for prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the longitudinal reinforcement.

time, the nominal shear strength provided by concrete,  $V_c$ , is assumed to be unchanged by the presence of torsion. For beams with  $V_u$  greater than about  $0.8\phi V_c$ , the resulting amount of combined shear and torsional reinforcement is essentially the same as required by the 1989 Code. For smaller values of  $V_u$ , more shear and torsion reinforcement will be required.

**R11.5.3.6** — Equation (11-21) is based on the space truss analogy shown in Fig. R11.5.3.6(a) with compression diagonals at an angle  $\theta$ , assuming the concrete carries no tension and the reinforcement yields. After torsional cracking develops, the torsional resistance is provided mainly by closed stirrups, longitudinal bars, and compression diagonals. The concrete outside these stirrups is relatively ineffective. For this reason  $A_o$ , the gross area enclosed by the shear flow path around the perimeter of the tube, is defined after cracking in terms of  $A_{oh}$ , the area enclosed by the centerline of the outermost closed transverse torsional reinforcement. The area  $A_{oh}$  is shown in Fig. R11.5.3.6(b) for various cross sections. In an I-, T-, or L-shaped section,  $A_{oh}$  is taken as that area enclosed by the outermost legs of interlocking stirrups as shown in Fig. R11.5.3.6(b). The expression for  $A_o$  given by Hsu<sup>11.36</sup> may be used if greater accuracy is desired.

The shear flow  $q$  in the walls of the tube, discussed in R11.5, can be resolved into the shear forces  $V_1$  to  $V_4$  acting

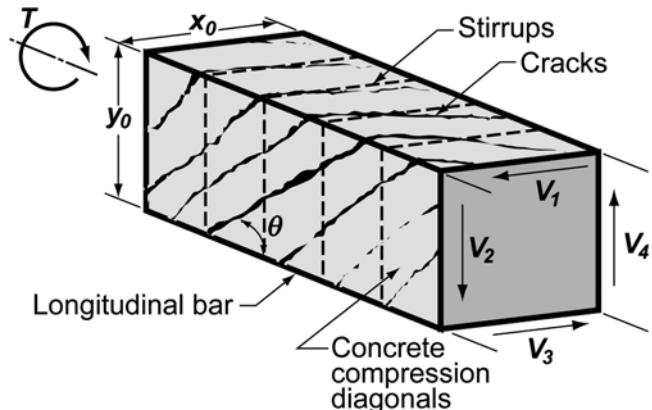
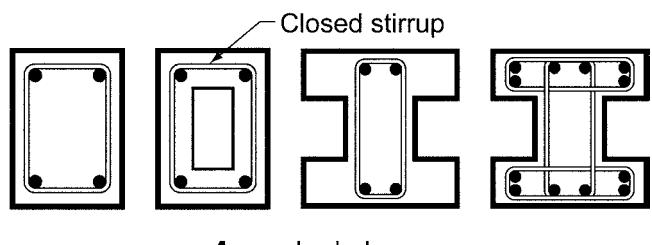


Fig. R11.5.3.6(a)—Space truss analogy.



$$A_{oh} = \text{shaded area}$$

Fig. R11.5.3.6(b)—Definition of  $A_{oh}$ .

**CODE****COMMENTARY**

in the individual sides of the tube or space truss, as shown in Fig. R11.5.3.6(a).

The angle  $\theta$  can be obtained by analysis<sup>11.36</sup> or may be taken to be equal to the values given in 11.5.3.6(a) or (b). The same value of  $\theta$  should be used in both Eq. (11-21) and (11-22). As  $\theta$  gets smaller, the amount of stirrups required by Eq. (11-21) decreases. At the same time, the amount of longitudinal steel required by Eq. (11-22) increases.

**11.5.3.7** — The additional area of longitudinal reinforcement to resist torsion,  $A_\ell$ , shall not be less than

$$A_\ell = \frac{A_t}{s} p_h \left( \frac{f_{yt}}{f_y} \right) \cot^2 \theta \quad (11-22)$$

where  $\theta$  shall be the same value used in Eq. (11-21) and  $A_t/s$  shall be taken as the amount computed from Eq. (11-21) not modified in accordance with 11.5.5.2 or 11.5.5.3;  $f_{yt}$  refers to closed transverse torsional reinforcement, and  $f_y$  refers to longitudinal torsional reinforcement.

**R11.5.3.7** — Figure R11.5.3.6(a) shows the shear forces  $V_1$  to  $V_4$  resulting from the shear flow around the walls of the tube. On a given wall of the tube, the shear flow  $V_i$  is resisted by a diagonal compression component,  $D_i = V_i / \sin \theta$ , in the concrete. An axial tension force,  $N_i = V_i / (\cot \theta)$ , is needed in the longitudinal steel to complete the resolution of  $V_i$ .

Figure R11.5.3.7 shows the diagonal compressive stresses and the axial tension force,  $N_i$ , acting on a short segment along one wall of the tube. Because the shear flow due to torsion is constant at all points around the perimeter of the tube, the resultants of  $D_i$  and  $N_i$  act through the midheight of side  $i$ . As a result, half of  $N_i$  can be assumed to be resisted by each of the top and bottom chords as shown. Longitudinal reinforcement with a strength  $A_\ell f_y$  should be provided to resist the sum of the  $N_i$  forces,  $\Sigma N_i$ , acting in all of the walls of the tube.

In the derivation of Eq. (11-22), axial tension forces are summed along the sides of the area  $A_o$ . These sides form a perimeter length,  $p_o$ , approximately equal to the length of the line joining the centers of the bars in the corners of the tube. For ease in computation, this has been replaced with the perimeter of the closed stirrups,  $p_h$ .

Frequently, the maximum allowable stirrup spacing governs the amount of stirrups provided. Furthermore, when combined shear and torsion act, the total stirrup area is the sum of the amounts provided for shear and torsion. To avoid the need to provide excessive amounts of longitudinal reinforcement, 11.5.3.7 states that the  $A_t/s$  used in calculating  $A_\ell$  at any given section should be taken as the  $A_t/s$  calculated at that section using Eq. (11-21).

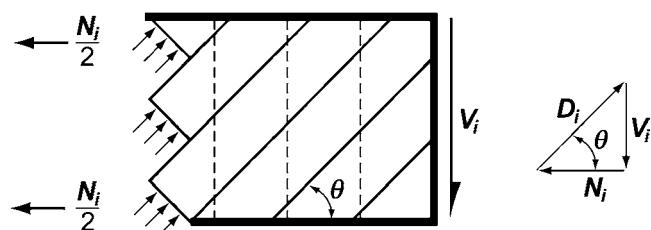


Fig. R11.5.3.7—Resolution of shear force  $V_i$  into diagonal compression force  $D_i$  and axial tension force  $N_i$  in one wall of the tube.

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**11.5.3.8** — Reinforcement required for torsion shall be added to that required for the shear, moment, and axial force that act in combination with the torsion. The most restrictive requirements for reinforcement spacing and placement shall be met.

**COMMENTARY**

**R11.5.3.8** — The stirrup requirements for torsion and shear are added and stirrups are provided to supply at least the total amount required. Since the stirrup area  $A_v$  for shear is defined in terms of all the legs of a given stirrup while the stirrup area  $A_t$  for torsion is defined in terms of one leg only, the addition of stirrups is carried out as follows

$$\text{Total } \left( \frac{A_v + t}{s} \right) = \frac{A_v}{s} + 2 \frac{A_t}{s}$$

If a stirrup group had four legs for shear, only the legs adjacent to the sides of the beam would be included in this summation since the inner legs would be ineffective for torsion.

The longitudinal reinforcement required for torsion is added at each section to the longitudinal reinforcement required for bending moment that acts at the same time as the torsion. The longitudinal reinforcement is then chosen for this sum, but should not be less than the amount required for the maximum bending moment at that section if this exceeds the moment acting at the same time as the torsion. If the maximum bending moment occurs at one section, such as the midspan, while the maximum torsional moment occurs at another, such as the support, the total longitudinal steel required may be less than that obtained by adding the maximum flexural steel plus the maximum torsional steel. In such a case, the required longitudinal steel is evaluated at several locations.

The most restrictive requirements for spacing, cut-off points, and placement for flexural, shear, and torsional steel should be satisfied. The flexural steel should be extended a distance  $d$ , but not less than  $12d_b$ , past where it is no longer needed for flexure as required in **12.10.3**.

**R11.5.3.9** — The longitudinal tension due to torsion is offset in part by the compression in the flexural compression zone, allowing a reduction in the longitudinal torsion steel required in the compression zone.

**11.5.3.9** — It shall be permitted to reduce the area of longitudinal torsion reinforcement in the flexural compression zone by an amount equal to  $M_u/(0.9df_y)$ , where  $M_u$  occurs at the section simultaneously with  $T_u$ , except that the reinforcement provided shall not be less than that required by **11.5.5.3** or **11.5.6.2**.

**11.5.3.10** — In prestressed beams:

- (a) The total longitudinal reinforcement including prestressing steel at each section shall resist  $M_u$  at that section plus an additional concentric longitudinal tensile force equal to  $A_\ell f_y$ , based on  $T_u$  at that section;
- (b) The spacing of the longitudinal reinforcement including tendons shall satisfy the requirements in **11.5.6.2**.

**R11.5.3.10** — As explained in **R11.5.3.7**, torsion causes an axial tension force. In a nonprestressed beam, this force is resisted by longitudinal reinforcement having an axial tensile strength of  $A_\ell f_y$ . This steel is in addition to the flexural reinforcement and is distributed uniformly around the sides of the perimeter so that the resultant of  $A_\ell f_y$  acts along the axis of the member.

In a prestressed beam, the same technique (providing additional reinforcing bars with capacity  $A_\ell f_y$ ) can be followed, or overstrength of the prestressing steel can be used to resist some of the axial force  $A_\ell f_y$  as outlined in the next paragraph.

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In a prestressed beam, the stress in the prestressing steel at nominal strength will be between  $f_{se}$  and  $f_{ps}$ . A portion of the  $A_e f_y$  force can be resisted by a force of  $A_{ps} \Delta f_{pt}$  in the prestressing steel, where  $\Delta f_{pt}$  is the difference between the stress which can be developed in the strand at the section under consideration and the stress required to resist the bending moment at this section,  $M_u$ . The stress required to resist the bending moment can be calculated as  $[M_u / (\phi 0.9 d_p A_{ps})]$ . For pretensioned strands, the stress which can be developed near the free end of the strand can be calculated using the procedure illustrated in Fig. R12.9. Note that near the ends of a pretensioned member, the available stress in the prestressing steel will need to be reduced to account for lack of full development, and should be determined in conjunction with 9.3.2.7.

**11.5.3.11** — In prestressed beams, it shall be permitted to reduce the area of longitudinal torsional reinforcement on the side of the member in compression due to flexure below that required by 11.5.3.10 in accordance with 11.5.3.9.

#### 11.5.4 — Details of torsional reinforcement

**11.5.4.1** — Torsion reinforcement shall consist of longitudinal bars or tendons and one or more of the following:

- (a) Closed stirrups or closed ties, perpendicular to the axis of the member;
- (b) A closed cage of welded wire reinforcement with transverse wires perpendicular to the axis of the member;
- (c) In nonprestressed beams, spiral reinforcement.

**11.5.4.2** — Transverse torsional reinforcement shall be anchored by one of the following:

- (a) A 135-degree standard hook conforming to 7.1.3(c) or seismic hook confirming to 7.1.4 around a longitudinal bar;
- (b) According to 12.13.2.1, 12.13.2.2, or 12.13.2.3 in regions where the concrete surrounding the anchorage is restrained against spalling by a flange or slab or similar member.

#### R11.5.4 — Details of torsional reinforcement

**R11.5.4.1** — Both longitudinal and closed transverse reinforcement are required to resist the diagonal tension stresses due to torsion. The stirrups must be closed, since inclined cracking due to torsion may occur on all faces of a member.

In the case of sections subjected primarily to torsion, the concrete side cover over the stirrups spalls off at high torques.<sup>11.37</sup> This renders lapped-spliced stirrups ineffective, leading to a premature torsional failure.<sup>11.38</sup> In such cases, closed stirrups should not be made up of pairs of U-stirrupslapping one another.

**R11.5.4.2** — When a rectangular beam fails in torsion, the corners of the beam tend to spall off due to the inclined compressive stresses in the concrete diagonals of the space truss changing direction at the corner as shown in Fig. R11.5.4.2(a). In tests,<sup>11.37</sup> closed stirrups anchored by 90-degree hooks failed when this occurred. For this reason, 135-degree standard hooks or seismic hooks are preferable for torsional stirrups in all cases. In regions where this spalling is prevented by an adjacent slab or flange, 11.5.4.2(b) relaxes this and allows 90-degree hooks (see Fig. R11.5.4.2(b)).

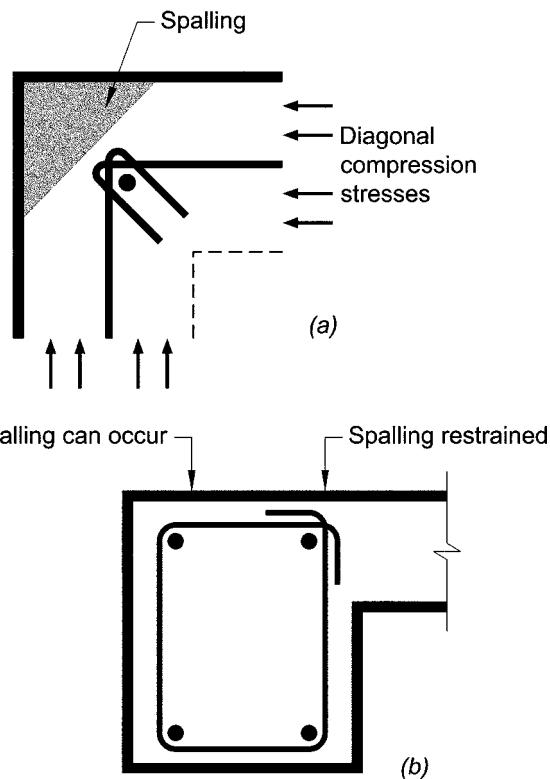
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Fig. R11.5.4.2—Spalling of corners of beams loaded in torsion.

**11.5.4.3** — Longitudinal torsion reinforcement shall be developed at both ends.

**11.5.4.4** — For hollow sections in torsion, the distance from the centerline of the transverse torsional reinforcement to the inside face of the wall of the hollow section shall not be less than  $0.5A_{oh}/p_h$ .

### 11.5.5 — Minimum torsion reinforcement

**11.5.5.1** — A minimum area of torsional reinforcement shall be provided in all regions where  $T_u$  exceeds the threshold torsion given in 11.5.1.

**11.5.5.2** — Where torsional reinforcement is required by 11.5.5.1, the minimum area of transverse closed stirrups shall be computed by

$$(A_v + 2A_t) = 0.75 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \quad (11-23)$$

but shall not be less than  $(50b_w s)/f_{yt}$ .

**R11.5.4.3** — If high torsion acts near the end of a beam, the longitudinal torsion reinforcement should be adequately anchored. Sufficient development length should be provided outside the inner face of the support to develop the needed tension force in the bars or tendons. In the case of bars, this may require hooks or horizontal U-shaped bars lapped with the longitudinal torsion reinforcement.

**R11.5.4.4** — The closed stirrups provided for torsion in a hollow section should be located in the outer half of the wall thickness effective for torsion where the wall thickness can be taken as  $A_{oh}/p_h$ .

### R11.5.5 — Minimum torsion reinforcement

**R11.5.5.1 and R11.5.5.2** — If a member is subject to a factored torsional moment  $T_u$  greater than the values specified in 11.5.1, the minimum amount of transverse web reinforcement for combined shear and torsion is  $50b_w s/f_{yt}$ . The differences in the definition of  $A_v$  and the symbol  $A_t$  should be noted;  $A_v$  is the area of two legs of a closed stirrup whereas  $A_t$  is the area of only one leg of a closed stirrup.

Tests<sup>11.9</sup> of high-strength reinforced concrete beams have indicated the need to increase the minimum area of shear reinforcement to prevent shear failures when inclined cracking occurs. Although there are a limited number of tests of high-strength concrete beams in torsion, the equation

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**11.5.5.3** — Where torsional reinforcement is required by **11.5.5.1**, the minimum total area of longitudinal torsional reinforcement,  $A_{\ell,min}$ , shall be computed by

$$A_{\ell,min} = \frac{5\sqrt{f_c'A_{cp}}}{f_y} - \left(\frac{A_t}{s}\right)p_h \frac{f_{yt}}{f_y} \quad (11-24)$$

where  $A_t/s$  shall not be taken less than  $25b_w/f_{yt}$ ;  $f_{yt}$  refers to closed transverse torsional reinforcement, and  $f_y$  refers to longitudinal reinforcement.

**11.5.6 — Spacing of torsion reinforcement**

**11.5.6.1** — The spacing of transverse torsion reinforcement shall not exceed the smaller of  $p_h/8$  or 12 in.

**11.5.6.2** — The longitudinal reinforcement required for torsion shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 12 in. The longitudinal bars or tendons shall be inside the stirrups. There shall be at least one longitudinal bar or tendon in each corner of the stirrups. Longitudinal bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than 3/8 in.

**11.5.6.3** — Torsional reinforcement shall be provided for a distance of at least  $(b_t + d)$  beyond the point required by analysis.

**11.5.7 — Alternative design for torsion**

For torsion design of solid sections within the scope of this Code with an aspect ratio,  $h/b_t$ , of 3 or greater, it shall be permitted to use another procedure, the adequacy of which has been shown by analysis and substantial agreement with results of comprehensive tests. **Sections 11.5.4** and **11.5.6** shall apply.

for the minimum area of transverse closed stirrups has been changed for consistency with calculations required for minimum shear reinforcement.

**R11.5.5.3** — Reinforced concrete beam specimens with less than 1 percent torsional reinforcement by volume have failed in pure torsion at torsional cracking.<sup>11.31</sup> In the 1989 and prior Codes, a relationship was presented that required about 1 percent torsional reinforcement in beams loaded in pure torsion and less in beams with combined shear and torsion, as a function of the ratio of shear stresses due to torsion and shear. Equation (11-24) was simplified by assuming a single value of this reduction factor and results in a volumetric ratio of about 0.5 percent.

**R11.5.6 — Spacing of torsion reinforcement**

**R11.5.6.1** — The spacing of the stirrups is limited to ensure the development of the ultimate torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking, and to control crack widths. For a square cross section, the  $p_h/8$  limitation requires stirrups at  $d/2$ , which corresponds to **11.4.5.1**.

**R11.5.6.2** — In **R11.5.3.7**, it was shown that longitudinal reinforcement is needed to resist the sum of the longitudinal tensile forces due to torsion in the walls of the thin-walled tube. Since the force acts along the centroidal axis of the section, the centroid of the additional longitudinal reinforcement for torsion should approximately coincide with the centroid of the section. The Code accomplishes this by requiring the longitudinal torsional reinforcement to be distributed around the perimeter of the closed stirrups. Longitudinal bars or tendons are required in each corner of the stirrups to provide anchorage for the legs of the stirrups. Corner bars have also been found to be very effective in developing torsional strength and in controlling cracks.

**R11.5.6.3** — The distance  $(b_t + d)$  beyond the point theoretically required for torsional reinforcement is larger than that used for shear and flexural reinforcement because torsional diagonal tension cracks develop in a helical form.

**R11.5.7 — Alternative design for torsion**

Examples of such procedures are to be found in **References 11.39** to **11.41**, which have been extensively and successfully used for design of precast, prestressed concrete beams with ledges. The procedure described in **References 11.39** and **11.40** is an extension to prestressed concrete sections of the torsion procedures of pre-1995 editions of the Code. The sixth edition of the *PCI Design Handbook*<sup>11.16</sup> describes the procedure of **References 11.40** and **11.41**. This procedure was experimentally verified by the tests described in **Reference 11.42**.

**CODE****COMMENTARY****11.6 — Shear-friction**

**11.6.1** — Provisions of 11.6 are to be applied where it is appropriate to consider shear transfer across a given plane, such as: an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times.

**11.6.2** — Design of cross sections subject to shear transfer as described in 11.6.1 shall be based on **Eq. (11-1)**, where  $V_n$  is calculated in accordance with provisions of 11.6.3 or **11.6.4**.

**11.6.3** — A crack shall be assumed to occur along the shear plane considered. The required area of shear-friction reinforcement  $A_{vf}$  across the shear plane shall be designed using either **11.6.4** or any other shear transfer design methods that result in prediction of strength in substantial agreement with results of comprehensive tests.

**11.6.3.1** — Provisions of **11.6.5** through **11.6.10** shall apply for all calculations of shear transfer strength.

**R11.6 — Shear-friction**

**R11.6.1** — With the exception of 11.6, virtually all provisions regarding shear are intended to prevent diagonal tension failures rather than direct shear transfer failures. The purpose of 11.6 is to provide design methods for conditions where shear transfer should be considered: an interface between concretes cast at different times, an interface between concrete and steel, reinforcement details for precast concrete structures, and other situations where it is considered appropriate to investigate shear transfer across a plane in structural concrete. (See **References 11.43** and **11.44**.)

**R11.6.3** — Although uncracked concrete is relatively strong in direct shear, there is always the possibility that a crack will form in an unfavorable location. The shear-friction concept assumes that such a crack will form, and that reinforcement must be provided across the crack to resist relative displacement along it. When shear acts along a crack, one crack face slips relative to the other. If the crack faces are rough and irregular, this slip is accompanied by separation of the crack faces. At ultimate, the separation is sufficient to stress the reinforcement crossing the crack to its yield point. The reinforcement provides a clamping force  $A_{vffy}$  across the crack faces. The applied shear is then resisted by friction between the crack faces, by resistance to the shearing off of protrusions on the crack faces, and by dowel action of the reinforcement crossing the crack. Successful application of 11.6 depends on proper selection of the location of an assumed crack.<sup>11.16,11.43</sup>

The relationship between shear-transfer strength and the reinforcement crossing the shear plane can be expressed in various ways. **Equations (11-25)** and **(11-26)** of **11.6.4** are based on the shear-friction model. This gives a conservative prediction of shear-transfer strength. Other relationships that give a closer estimate of shear-transfer strength<sup>11.16,11.45,11.46</sup> can be used under the provisions of 11.6.3. For example, when the shear-friction reinforcement is perpendicular to the shear plane, the nominal shear strength  $V_n$  is given by<sup>11.45,11.46</sup>

$$V_n = 0.8A_{vf}f_y + A_cK_1$$

where  $A_c$  is the area of concrete section resisting shear transfer ( $\text{in.}^2$ ) and  $K_1 = 400$  psi for normalweight concrete, 200 psi for all-lightweight concrete, and 250 psi for sand-lightweight concrete. These values of  $K_1$  apply to both monolithically cast concrete and to concrete cast against hardened concrete with a rough surface, as defined in **11.6.9**.

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In this equation, the first term represents the contribution of friction to shear-transfer resistance (0.8 representing the coefficient of friction). The second term represents the sum of the resistance to shearing of protrusions on the crack faces and the dowel action of the reinforcement.

When the shear-friction reinforcement is inclined to the shear plane, such that the shear force produces tension in that reinforcement, the nominal shear strength  $V_n$  is given by

$$V_n = A_{vf} f_y (0.8 \sin \alpha + \cos \alpha) + A_c K_1 \sin^2 \alpha$$

where  $\alpha$  is the angle between the shear-friction reinforcement and the shear plane (that is,  $0 < \alpha < 90$  degrees).

When using the modified shear-friction method, the terms  $(A_{vf} f_y / A_c)$  or  $(A_{vf} f_y \sin \alpha / A_c)$  should not be less than 200 psi for the design equations to be valid.

**11.6.4 — Shear-friction design method**

**11.6.4.1** — Where shear-friction reinforcement is perpendicular to the shear plane,  $V_n$  shall be computed by

$$V_n = A_{vf} f_y \mu \quad (11-25)$$

where  $\mu$  is coefficient of friction in accordance with 11.6.4.3.

**11.6.4.2** — Where shear-friction reinforcement is inclined to the shear plane, such that the shear force produces tension in shear-friction reinforcement,  $V_n$  shall be computed by

$$V_n = A_{vf} f_y (\mu \sin \alpha + \cos \alpha) \quad (11-26)$$

where  $\alpha$  is angle between shear-friction reinforcement and shear plane.

**R11.6.4 — Shear-friction design method**

**R11.6.4.1** — The required area of shear-friction reinforcement  $A_{vf}$  is computed using

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

The specified upper limit on shear strength should also be observed.

**R11.6.4.2** — When the shear-friction reinforcement is inclined to the shear plane, such that the component of the shear force parallel to the reinforcement tends to produce tension in the reinforcement, as shown in Fig. R11.6.4, part of the shear is resisted by the component parallel to the shear plane of the tension force in the reinforcement.<sup>11.46</sup>

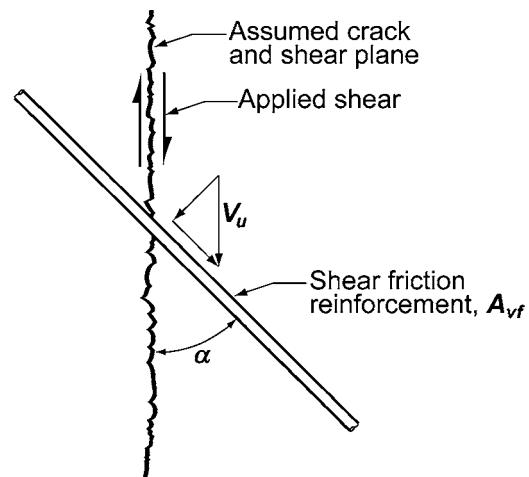


Fig. R11.6.4—Shear-friction reinforcement at an angle to assumed crack.

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**11.6.4.3** — The coefficient of friction  $\mu$  in Eq. (11-25) and Eq. (11-26) shall be taken as:

Concrete placed monolithically .....	<b>1.4<math>\lambda</math></b>
Concrete placed against hardened concrete with surface intentionally roughened as specified in 11.6.9 .....	<b>1.0<math>\lambda</math></b>
Concrete placed against hardened concrete not intentionally roughened .....	<b>0.6<math>\lambda</math></b>
Concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars (see 11.6.10) .....	<b>0.7<math>\lambda</math></b>

where  $\lambda = 1.0$  for normalweight concrete and 0.75 for all lightweight concrete. Otherwise,  $\lambda$  shall be determined based on volumetric proportions of lightweight and normalweight aggregates as specified in 8.6.1, but shall not exceed 0.85.

**11.6.5** — For normalweight concrete either placed monolithically or placed against hardened concrete with surface intentionally roughened as specified in 11.6.9,  $V_n$  shall not exceed the smallest of  $0.2f'_c A_c$ ,  $(480 + 0.08f'_c)A_c$  and  $1600A_c$ , where  $A_c$  is area of concrete section resisting shear transfer. For all other cases,  $V_n$  shall not exceed the smaller of  $0.2f'_c A_c$  or  $800A_c$ . Where concretes of different strengths are cast against each other, the value of  $f'_c$  used to evaluate  $V_n$  shall be that of the lower-strength concrete.

**11.6.6** — The value of  $f_y$  used for design of shear-friction reinforcement shall not exceed 60,000 psi.

**11.6.7** — Net tension across shear plane shall be resisted by additional reinforcement. Permanent net compression across shear plane shall be permitted to be taken as additive to  $A_{vf}f_y$ , the force in the shear-friction reinforcement, when calculating required  $A_{vf}$ .

Equation (11-26) should be used only when the shear force component parallel to the reinforcement produces tension in the reinforcement, as shown in Fig. R11.6.4. When  $\alpha$  is greater than 90 degrees, the relative movement of the surfaces tends to compress the bar and Eq. (11-26) is not valid.

**R11.6.4.3** — In the shear-friction method of calculation, it is assumed that all the shear resistance is due to the friction between the crack faces. It is therefore necessary to use artificially high values of the coefficient of friction in the shear-friction equations so that the calculated shear strength will be in reasonable agreement with test results. For concrete cast against hardened concrete not roughened in accordance with 11.6.9, shear resistance is primarily due to dowel action of the reinforcement and tests<sup>11.47</sup> indicate that reduced value of  $\mu = 0.6\lambda$  specified for this case is appropriate.

The value of  $\mu$  for concrete placed against as-rolled structural steel relates to the design of connections between precast concrete members, or between structural steel members and structural concrete members. The shear-transfer reinforcement may be either reinforcing bars or headed stud shear connectors; also, field welding to steel plates after casting of concrete is common. The design of shear connectors for composite action of concrete slabs and steel beams is not covered by these provisions, but should be in accordance with Reference 11.48.

**R11.6.5** — These upper limits on shear friction strength are necessary as Eq. (11-25) and (11-26) may become unconservative for some cases. Test data<sup>11.49,11.50</sup> on normal-weight concrete either placed monolithically or placed against hardened concrete with surface intentionally roughened as specified in 11.6.9 show that a higher upper limit can be used on shear friction strength for concrete with  $f'_c$  greater than 4000 psi than was allowed before the 2008 revisions. In higher-strength concretes, additional effort may be required to achieve the roughness specified in 11.6.9.

**R11.6.7** — If a resultant tensile force acts across a shear plane, reinforcement to carry that tension should be provided in addition to that provided for shear transfer. Tension may be caused by restraint of deformations due to temperature change, creep, and shrinkage. Such tensile forces have caused failures, particularly in beam bearings.

When moment acts on a shear plane, the flexural tension stresses and flexural compression stresses are in equilibrium. There is no change in the resultant compression  $A_{vf}f_y$  acting across the shear plane and the shear-transfer strength is not changed. It is therefore not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount

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of shear-transfer reinforcement provided in the flexural tension zone. This has been demonstrated experimentally.<sup>11.51</sup>

It has also been demonstrated experimentally<sup>11.44</sup> that if a resultant compressive force acts across a shear plane, the shear-transfer strength is a function of the sum of the resultant compressive force and the force  $A_{vf}f_y$  in the shear-friction reinforcement. In design, advantage should be taken of the existence of a compressive force across the shear plane to reduce the amount of shear-friction reinforcement required, only if it is certain that the compressive force is permanent.

**11.6.8** — Shear-friction reinforcement shall be appropriately placed along the shear plane and shall be anchored to develop  $f_y$  on both sides by embedment, hooks, or welding to special devices.

**R11.6.8** — If no moment acts across the shear plane, reinforcement should be uniformly distributed along the shear plane to minimize crack widths. If a moment acts across the shear plane, it is desirable to distribute the shear-transfer reinforcement primarily in the flexural tension zone.

Since the shear-friction reinforcement acts in tension, it should have full tensile anchorage on both sides of the shear plane. Further, the shear-friction reinforcement anchorage should engage the primary reinforcement, otherwise a potential crack may pass between the shear-friction reinforcement and the body of the concrete. This requirement applies particularly to welded headed studs used with steel inserts for connections in precast and cast-in-place concrete. Anchorage may be developed by bond, by a welded mechanical anchorage, or by threaded dowels and screw inserts. Space limitations often require a welded mechanical anchorage. For anchorage of headed studs in concrete, see Reference 11.16.

**11.6.9** — For the purpose of 11.6, when concrete is placed against previously hardened concrete, the interface for shear transfer shall be clean and free of laitance. If  $\mu$  is assumed equal to **1.0λ**, interface shall be roughened to a full amplitude of approximately 1/4 in.

**11.6.10** — When shear is transferred between as-rolled steel and concrete using headed studs or welded reinforcing bars, steel shall be clean and free of paint.

## 11.7 — Deep beams

**11.7.1** — The provisions of 11.7 shall apply to members with  $\ell_n$  not exceeding **4h** or regions of beams with concentrated loads within a distance **2h** from the support that are loaded on one face and supported on the opposite face so that compression struts can develop between the loads and supports. See also 12.10.6.

## R11.7 — Deep beams

**R11.7.1** — The behavior of deep beams is discussed in References 11.5 and 11.52 through 11.54. For a deep beam supporting gravity loads, 11.7.1 applies if the loads are applied on the top of the beam and the beam is supported on its bottom face. If the loads are applied through the sides or bottom of such a member, strut-and-tie models, as defined in Appendix A, should be used to design reinforcement to suspend the loads within the beam and transfer them to adjacent supports.

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**11.7.2** — Deep beams shall be designed either by taking into account nonlinear distribution of strain or by [Appendix A](#). In all cases, minimum distribution reinforcement shall be provided in accordance with 11.7.4

**11.7.3** — Deep beams shall be proportioned such that  $V_u$  is less than or equal to  $\phi 10 \sqrt{f'_c} b_w d$ .

**11.7.4** — Total distributed reinforcement along the two side faces of deep beams shall not be less than that required in 11.7.4.1 and 11.7.4.2.

**11.7.4.1** — The area of shear reinforcement perpendicular to the longitudinal axis of the beam,  $A_v$ , shall not be less than  $0.0025 b_w s$ , and  $s$  shall not exceed the smaller of  $d/5$  and 12 in.

**11.7.4.2** — The area of shear reinforcement parallel to the longitudinal axis of the beam,  $A_{vh}$ , shall not be less than  $0.0025 b_w s_2$ , and  $s_2$  shall not exceed the smaller of  $d/5$  and 12 in.

## 11.8 — Provisions for brackets and corbels

**11.8.1** — Brackets and corbels with a shear span-to-depth ratio  $a_v/d$  less than 2 shall be permitted to be designed using [Appendix A](#). Design shall be permitted using [11.8.3](#) and [11.8.4](#) for brackets and corbels with:

- (a)  $a_v/d$  not greater than 1, and
- (b) subject to factored horizontal tensile force,  $N_{uc}$ , not larger than  $V_u$ .

The longitudinal reinforcement in deep beams should be extended to the supports and adequately anchored by embedment, hooks, headed deformed bars, or welding to special devices. Bent-up bars are not recommended.

**R11.7.2** — Deep beams can be designed using strut-and-tie models, regardless of how they are loaded and supported. [Section 10.7.1](#) allows the use of analyses that take into account nonlinear strain and stress distributions when proportioning deep beams. Such analyses, including nonlinear finite element analyses, should consider the effects of cracking on the stress distribution.

**R11.7.3** — This limit is imposed to control cracking under service loads and guard against diagonal compression failures in deep beams.

**R11.7.4** — The amount of shear reinforcement required for strength shall be proportioned to be consistent with the analysis method used. The minimum reinforcement requirements in 11.7.4.1 and 11.7.4.2 are to be used irrespective of the analysis method and are intended to control the width and propagation of inclined cracks. Tests<sup>11.52-11.54</sup> have shown that vertical shear reinforcement (perpendicular to the longitudinal axis of the member) is more effective for member strength than horizontal shear reinforcement (parallel to the longitudinal axis of the member) in a deep beam, but the specified minimum reinforcement in both directions is required to control the growth and width of diagonal cracks.

## R11.8 — Provisions for brackets and corbels

Brackets and corbels are cantilevers having shear span-to-depth ratios not greater than unity, which tend to act as simple trusses or deep beams, rather than flexural members designed for shear according to [11.2](#).

The corbel shown in [Fig. R11.8.1](#) may fail by shearing along the interface between the column and the corbel, by yielding of the tension tie, by crushing or splitting of the compression strut, or by localized bearing or shearing failure under the loading plate. These failure modes are illustrated and are discussed more fully in [Reference 11.1](#). The notation used in 11.8 is illustrated in [Fig. R11.8.2](#).

**R11.8.1** — An upper limit of 1.0 for  $a_v/d$  is imposed for design by [11.8.3](#) and [11.8.4](#) for two reasons. First, for  $a_v/d$  shear span-to-depth ratios exceeding unity, the diagonal tension cracks are less steeply inclined and the use of horizontal stirrups alone as specified in [11.8.4](#) is not appropriate. Second, this method of design has only been validated experimentally for  $a_v/d$  of unity or less. An upper limit is provided for  $N_{uc}$  because this method of design has only been validated experimentally for  $N_{uc}$  less than or equal to  $V_u$ , including  $N_{uc}$  equal to zero.

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The requirements of 11.8.2, 11.8.3.2.1, 11.8.3.2.2, 11.8.5, 11.8.6, and 11.8.7 shall apply to design of brackets and corbels. Effective depth  $d$  shall be determined at the face of the support.

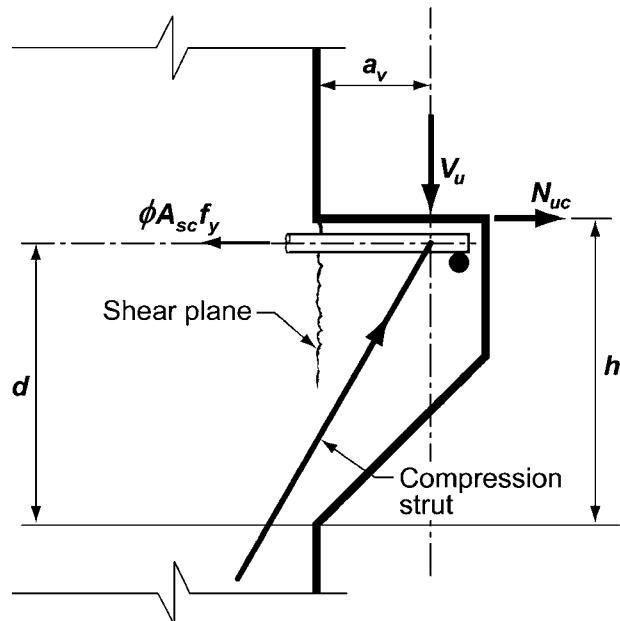


Fig. R11.8.1—Structural action of a corbel.

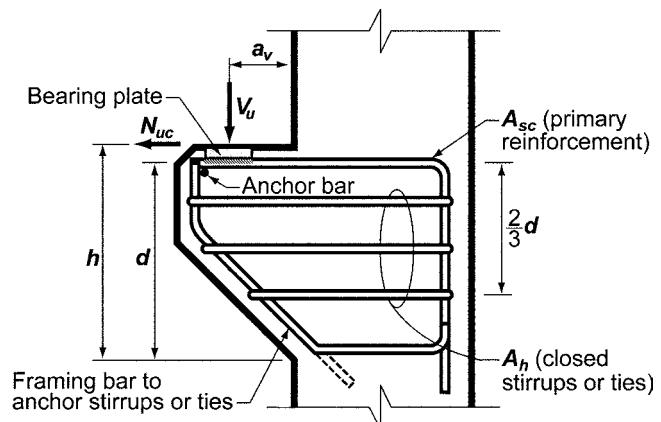


Fig. R11.8.2—Notation used in Section 11.8.

**11.8.2** — Depth at outside edge of bearing area shall not be less than **0.5d**.

**R11.8.2** — A minimum depth is required at the outside edge of the bearing area so that a premature failure will not occur due to a major diagonal tension crack propagating from below the bearing area to the outer sloping face of the corbel or bracket. Failures of this type have been observed<sup>11.55</sup> in corbels having depths at the outside edge of the bearing area less than required in this section of the Code.

**11.8.3** — Section at face of support shall be designed to resist simultaneously  $V_u$ , a factored moment [ $V_u a_v + N_{uc}(h - d)$ ], and a factored horizontal tensile force,  $N_{uc}$ .

**11.8.3.1** — In all design calculations in accordance with 11.8,  $\phi$  shall be taken equal to 0.75.

**R11.8.3.1** — Corbel and bracket behavior is predominantly controlled by shear; therefore, a single value of  $\phi = 0.75$  is required for all design conditions.

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**11.8.3.2** — Design of shear-friction reinforcement,  $A_{vf}$ , to resist  $V_u$  shall be in accordance with 11.6.

**11.8.3.2.1** — For normalweight concrete,  $V_n$  shall not exceed the smallest of  $0.2f'_c b_w d$ ,  $(480 + 0.08f'_c)b_w d$ , and  $1600b_w d$ .

**11.8.3.2.2** — For all-lightweight or sand-lightweight concrete,  $V_n$  shall not be taken greater than the smaller of  $(0.2 - 0.07a_v/d)f'_c b_w d$  and  $(800 - 280a_v/d)b_w d$ .

**11.8.3.3** — Reinforcement  $A_f$  to resist factored moment  $[V_u a_v + N_{uc}(h - d)]$  shall be computed in accordance with 10.2 and 10.3.

**11.8.3.4** — Reinforcement  $A_n$  to resist factored tensile force  $N_{uc}$  shall be determined from  $\phi A_n f_y \geq N_{uc}$ . Factored tensile force,  $N_{uc}$ , shall not be taken less than  $0.2V_u$  unless provisions are made to avoid tensile forces.  $N_{uc}$  shall be regarded as a live load even if tension results from restraint of creep, shrinkage, or temperature change.

**11.8.3.5** — Area of primary tension reinforcement  $A_{sc}$  shall not be less than the larger of  $(A_f + A_n)$  and  $(2A_{vf}/3 + A_n)$ .

**11.8.4** — Total area,  $A_h$ , of closed stirrups or ties parallel to primary tension reinforcement shall not be less than  $0.5(A_{sc} - A_n)$ . Distribute  $A_h$  uniformly within  $(2/3)d$  adjacent to primary tension reinforcement.

**R11.8.3.2.2** — Tests<sup>11.56</sup> have shown that the maximum shear strength of lightweight concrete corbels or brackets is a function of both  $f'_c$  and  $a_v/d$ . No data are available for corbels or brackets made of sand-lightweight concrete. As a result, the same limitations have been placed on both all-lightweight and sand-lightweight brackets and corbels.

**R11.8.3.3** — Reinforcement required to resist moment can be calculated using flexural theory. The factored moment is calculated by summing moments about the flexural reinforcement at the face of support.

**R11.8.3.4** — Because the magnitude of horizontal forces acting on corbels or brackets cannot usually be determined with great accuracy, it is required that  $N_{uc}$  be regarded as a live load.

**R11.8.3.5** — Tests<sup>11.56</sup> suggest that the total amount of reinforcement ( $A_{sc} + A_h$ ) required to cross the face of support should be the greater of:

- (a) The sum of  $A_{vf}$  calculated according to 11.8.3.2 and  $A_n$  calculated according to 11.8.3.4;
- (b) The sum of 1.5 times  $A_f$  calculated according to 11.8.3.3 and  $A_n$  calculated according to 11.8.3.4.

If (a) controls,  $A_{sc} = (2A_{vf}/3 + A_n)$  is required as primary tensile reinforcement, and the remaining  $A_{vf}/3$  should be provided as closed stirrups parallel to  $A_{sc}$  and distributed within  $2d/3$ , adjacent to  $A_{sc}$ . Section 11.8.4 satisfies this by requiring  $A_h = 0.5(2A_{vf}/3)$ .

If (b) controls,  $A_{sc} = (A_f + A_n)$  is required as primary tension reinforcement, and the remaining  $A_f/2$  should be provided as closed stirrups parallel to  $A_{sc}$  and distributed within  $2d/3$ , adjacent to  $A_{sc}$ . Again, 11.8.4 satisfies this requirement.

**R11.8.4** — Closed stirrups parallel to the primary tension reinforcement are necessary to prevent a premature diagonal tension failure of the corbel or bracket. The required area of closed stirrups  $A_h = 0.5(A_{sc} - A_n)$  automatically yields the appropriate amounts, as discussed in R11.8.3.5 above.

**CODE****COMMENTARY**

**11.8.5** —  $A_{sc}/bd$  shall not be less than **0.04( $f'_c/f_y$ )**.

**11.8.6** — At front face of bracket or corbel, primary tension reinforcement shall be anchored by one of the following:

- (a) By a structural weld to a transverse bar of at least equal size; weld to be designed to develop  $f_y$  of primary tension reinforcement;
- (b) By bending primary tension reinforcement back to form a horizontal loop; or
- (c) By some other means of positive anchorage.

**11.8.7** — Bearing area on bracket or corbel shall not project beyond straight portion of primary tension reinforcement, nor project beyond interior face of transverse anchor bar (if one is provided).

**R11.8.5** — A minimum amount of reinforcement is required to prevent the possibility of sudden failure should the bracket or corbel concrete crack under the action of flexural moment and outward tensile force  $N_{uc}$ .

**R11.8.6** — Because the horizontal component of the inclined concrete compression strut (see Fig. R11.8.1) is transferred to the primary tension reinforcement at the location of the vertical load, the primary tension reinforcement is essentially uniformly stressed from the face of the support to the point where the vertical load is applied. It should, therefore, be anchored at its outer end and in the supporting column, so as to be able to develop its specified yield strength from the face of support to the vertical load. Satisfactory anchorage at the outer end can be obtained by bending the primary tension reinforcement bars in a horizontal loop as specified in (b), or by welding a bar of equal diameter or a suitably sized angle across the ends of the primary tension reinforcement bars. The welds should be designed to develop the yield strength of the primary tension reinforcement. The weld detail used successfully in the corbel tests reported in Reference 11.56 is shown in Fig. R11.8.6. The primary tension reinforcement should be anchored within the supporting column in accordance with the requirements of Chapter 12. See additional discussion on end anchorage in R12.10.6.

**R11.8.7** — The restriction on the location of the bearing area is necessary to ensure development of the specified yield strength of the primary tension reinforcement near the load. When corbels are designed to resist horizontal forces, the bearing plate should be welded to the primary tension reinforcement.

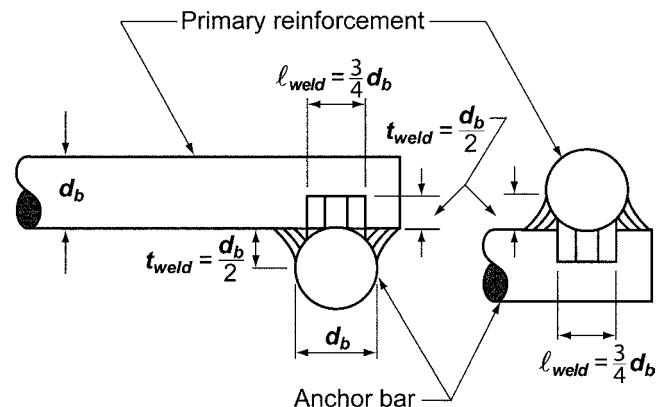


Fig. R11.8.6—Weld details used in tests of Reference 11.56.

**CODE****11.9 — Provisions for walls**

**11.9.1** — Design for shear forces perpendicular to face of wall shall be in accordance with provisions for slabs in **11.11**. Design for horizontal in-plane shear forces in a wall shall be in accordance with 11.9.2 through **11.9.9**. Alternatively, it shall be permitted to design walls with a height not exceeding two times the length of the wall for horizontal shear forces in accordance with **Appendix A** and **11.9.9.2** through **11.9.9.5**.

**11.9.2** — Design of horizontal section for shear in plane of wall shall be based on **Eq. (11-1)** and **(11-2)**, where  $V_c$  shall be in accordance with 11.9.5 or 11.9.6 and  $V_s$  shall be in accordance with **11.9.9**.

**11.9.3** —  $V_n$  at any horizontal section for shear in plane of wall shall not be taken greater than  $10\sqrt{f'_c} hd$ , where  $h$  is thickness of wall, and  $d$  is defined in 11.9.4.

**11.9.4** — For design for horizontal shear forces in plane of wall,  $d$  shall be taken equal to  $0.8\ell_w$ . A larger value of  $d$ , equal to the distance from extreme compression fiber to center of force of all reinforcement in tension, shall be permitted to be used when determined by a strain compatibility analysis.

**11.9.5** — Unless a more detailed calculation is made in accordance with 11.9.6,  $V_c$  shall not be taken greater than  $2\lambda\sqrt{f'_c} hd$  for walls subject to axial compression, or  $V_c$  shall not be taken greater than the value given in **11.2.2.3** for walls subject to axial tension.

**11.9.6** —  $V_c$  shall be permitted to be the lesser of the values computed from Eq. (11-27) and (11-28)

$$V_c = 3.3\lambda\sqrt{f'_c} hd + \frac{N_u d}{4\ell_w} \quad (11-27)$$

or

$$V_c = \left[ 0.6\lambda\sqrt{f'_c} + \frac{\ell_w(1.25\lambda\sqrt{f'_c} + 0.2\frac{N_u}{\ell_w h})}{\frac{M_u}{V_u} - \frac{\ell_w}{2}} \right] hd \quad (11-28)$$

where  $\ell_w$  is the overall length of the wall, and  $N_u$  is positive for compression and negative for tension. If  $(M_u/V_u - \ell_w/2)$  is negative, Eq. (11-28) shall not apply.

**COMMENTARY****R11.9 — Provisions for walls**

**R11.9.1** — Shear in the plane of the wall is primarily of importance for shear walls with a small height-to-length ratio. The design of higher walls, particularly walls with uniformly distributed reinforcement, will probably be controlled by flexural considerations.

**R11.9.3** — Although the width-to-depth ratio of shear walls is less than that for ordinary beams, tests<sup>11.57</sup> on shear walls with a thickness equal to  $\ell_w/25$  have indicated that ultimate shear stresses in excess of  $10\sqrt{f'_c}$  can be obtained.

**R11.9.5 and R11.9.6** — Equations (11-27) and (11-28) may be used to determine the inclined cracking strength at any section through a shear wall. Equation (11-27) corresponds to the occurrence of a principal tensile stress of approximately  $4\lambda\sqrt{f'_c}$  at the centroid of the shear wall cross section. Equation (11-28) corresponds approximately to the occurrence of a flexural tensile stress of  $6\lambda\sqrt{f'_c}$  at a section  $\ell_w/2$  above the section being investigated. As the term

$$\left( \frac{M_u}{V_u} - \frac{\ell_w}{2} \right)$$

decreases, Eq. (11-27) will control before this term becomes negative. When this term becomes negative, Eq. (11-27) should be used.

**CODE****COMMENTARY**

**11.9.7** — Sections located closer to wall base than a distance  $\ell_w/2$  or one-half the wall height, whichever is less, shall be permitted to be designed for the same  $V_c$  as that computed at a distance  $\ell_w/2$  or one-half the height.

**R11.9.7** — The values of  $V_c$  computed from Eq. (11-27) and (11-28) at a section located a lesser distance of  $\ell_w/2$  or  $h_w/2$  above the base apply to that and all sections between this section and the base. However, the maximum factored shear force  $V_u$  at any section, including the base of the wall, is limited to  $\phi V_n$  in accordance with 11.9.3.

**11.9.8** — Where  $V_u$  is less than  $0.5\phi V_c$ , reinforcement shall be provided in accordance with 11.9.9 or in accordance with Chapter 14. Where  $V_u$  exceeds  $0.5\phi V_c$ , wall reinforcement for resisting shear shall be provided in accordance with 11.9.9.

### 11.9.9 — Design of shear reinforcement for walls

**11.9.9.1** — Where  $V_u$  exceeds  $\phi V_c$ , horizontal shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where  $V_s$  shall be computed by

$$V_s = \frac{A_v f_y d}{s} \quad (11-29)$$

where  $A_v$  is area of horizontal shear reinforcement within spacing  $s$ , and  $d$  is determined in accordance with 11.9.4. Vertical shear reinforcement shall be provided in accordance with 11.9.9.4.

**11.9.9.2** — Ratio of horizontal shear reinforcement area to gross concrete area of vertical section,  $\rho_t$ , shall not be less than 0.0025.

**11.9.9.3** — Spacing of horizontal shear reinforcement shall not exceed the smallest of  $\ell_w/5$ ,  $3h$ , and 18 in., where  $\ell_w$  is the overall length of the wall.

**11.9.9.4** — Ratio of vertical shear reinforcement area to gross concrete area of horizontal section,  $\rho_\ell$ , shall not be less than the larger of

$$\rho_\ell = 0.0025 + 0.5 \left( 2.5 - \frac{h_w}{\ell_w} \right) (\rho_t - 0.0025) \quad (11-30)$$

and 0.0025. The value of  $\rho_\ell$  calculated by Eq. (11-30) need not be greater than  $\rho_t$  required by 11.9.9.1. In Eq. (11-30),  $\ell_w$  is the overall length of the wall, and  $h_w$  is the overall height of the wall.

**11.9.9.5** — Spacing of vertical shear reinforcement shall not exceed the smallest of  $\ell_w/3$ ,  $3h$ , and 18 in., where  $\ell_w$  is the overall length of the wall.

### R11.9.9 — Design of shear reinforcement for walls

Both horizontal and vertical shear reinforcement are required for all walls. The notation used to identify the direction of the distributed shear reinforcement in walls was updated in 2005 to eliminate conflicts between the notation used for ordinary structural walls in Chapters 11 and 14 and the notation used for special structural walls in Chapter 21. The distributed reinforcement is now identified as being oriented parallel to either the longitudinal or transverse axis of the wall. Therefore, for vertical wall segments, the notation used to describe the horizontal distributed reinforcement ratio is  $\rho_t$ , and the notation used to describe the vertical distributed reinforcement ratio is  $\rho_\ell$ .

For low walls, test data<sup>11.58</sup> indicate that horizontal shear reinforcement becomes less effective with vertical reinforcement becoming more effective. This change in effectiveness of the horizontal versus vertical reinforcement is recognized in Eq. (11-30); if  $h_w/\ell_w$  is less than 0.5, the amount of vertical reinforcement is equal to the amount of horizontal reinforcement. If  $h_w/\ell_w$  is greater than 2.5, only a minimum amount of vertical reinforcement is required (0.0025sh).

Equation (11-29) is presented in terms of shear strength  $V_s$  provided by the horizontal shear reinforcement for direct application in Eq. (11-1) and (11-2).

Vertical shear reinforcement also should be provided in accordance with 11.9.9.4 within the spacing limitation of 11.9.9.5.

**CODE****COMMENTARY****11.10 — Transfer of moments to columns**

**11.10.1** — When gravity load, wind, earthquake, or other lateral forces cause transfer of moment at connections of framing elements to columns, the shear resulting from moment transfer shall be considered in the design of transverse reinforcement in the columns.

**11.10.2** — Except for connections not part of a primary seismic load-resisting system that are restrained on four sides by beams or slabs of approximately equal depth, connections shall have transverse reinforcement not less than that required by Eq. (11-13) within the column for a depth not less than that of the deepest connection of framing elements to the columns. See also 7.9.

**11.11 — Provisions for slabs and footings**

**11.11.1** — The shear strength of slabs and footings in the vicinity of columns, concentrated loads, or reactions is governed by the more severe of two conditions:

**11.11.1.1** — Beam action where each critical section to be investigated extends in a plane across the entire width. For beam action, the slab or footing shall be designed in accordance with 11.1 through 11.4.

**11.11.1.2** — For two-way action, each of the critical sections to be investigated shall be located so that its perimeter  $b_o$  is a minimum but need not approach closer than  $d/2$  to:

- (a) Edges or corners of columns, concentrated loads, or reaction areas; and
- (b) Changes in slab thickness such as edges of capitals, drop panels, or shear caps.

For two-way action, the slab or footing shall be designed in accordance with 11.11.2 through 11.11.6.

**R11.10 — Transfer of moments to columns**

**R11.10.1** — Tests<sup>11.59</sup> have shown that the joint region of a beam-to-column connection in the interior of a building does not require shear reinforcement if the joint is confined on four sides by beams of approximately equal depth. However, joints without lateral confinement, such as at the exterior of a building, need shear reinforcement to prevent deterioration due to shear cracking.<sup>11.60</sup>

For regions where strong earthquakes may occur, joints may be required to withstand several reversals of loading that develop the flexural strength of the adjoining beams. See Chapter 21 for provisions for seismic design.

**R11.11 — Provisions for slabs and footings**

**R11.11.1** — Differentiation should be made between a long and narrow slab or footing acting as a beam, and a slab or footing subject to two-way action where failure may occur by punching along a truncated cone or pyramid around a concentrated load or reaction area.

**R11.11.1.2** — The critical section for shear in slabs subjected to bending in two directions follows the perimeter at the edge of the loaded area.<sup>11.3</sup> The shear stress acting on this section at factored loads is a function of  $\sqrt{f'_c}$  and the ratio of the side dimension of the column to the effective slab depth. A much simpler design equation results by assuming a pseudocritical section located at a distance  $d/2$  from the periphery of the concentrated load. When this is done, the shear strength is almost independent of the ratio of column size to slab depth. For rectangular columns, this critical section was defined by straight lines drawn parallel to and at a distance  $d/2$  from the edges of the loaded area. Section 11.11.1.3 allows the use of a rectangular critical section.

For slabs of uniform thickness, it is sufficient to check shear on one section. For slabs with changes in thickness, such as the edge of drop panels or shear caps, it is necessary to check shear at several sections.

**CODE****COMMENTARY**

For edge columns at points where the slab cantilevers beyond the column, the critical perimeter will either be three-sided or four-sided.

**11.11.1.3** — For square or rectangular columns, concentrated loads, or reaction areas, the critical sections with four straight sides shall be permitted.

**11.11.2** — The design of a slab or footing for two-way action is based on Eq. (11-1) and (11-2).  $V_c$  shall be computed in accordance with 11.11.2.1, 11.11.2.2, or 11.11.3.1.  $V_s$  shall be computed in accordance with 11.11.3. For slabs with shearheads,  $V_n$  shall be in accordance with 11.11.4. When moment is transferred between a slab and a column, 11.11.7 shall apply.

**11.11.2.1** — For nonprestressed slabs and footings,  $V_c$  shall be the smallest of (a), (b), and (c):

$$(a) \quad V_c = \left(2 + \frac{4}{\beta}\right) \lambda \sqrt{f'_c} b_o d \quad (11-31)$$

where  $\beta$  is the ratio of long side to short side of the column, concentrated load or reaction area;

$$(b) \quad V_c = \left(\frac{\alpha_s d}{b_o} + 2\right) \lambda \sqrt{f'_c} b_o d \quad (11-32)$$

where  $\alpha_s$  is 40 for interior columns, 30 for edge columns, 20 for corner columns; and

$$(c) \quad V_c = 4\lambda \sqrt{f'_c} b_o d \quad (11-33)$$

**11.11.2.2** — At columns of two-way prestressed slabs and footings that meet the requirements of 18.9.3

$$V_c = (\beta_p \lambda \sqrt{f'_c} + 0.3 f_{pc}) b_o d + V_p \quad (11-34)$$

where  $\beta_p$  is the smaller of 3.5 and  $(\alpha_s d/b_o + 1.5)$ ,  $\alpha_s$  is 40 for interior columns, 30 for edge columns, and 20 for corner columns,  $b_o$  is perimeter of critical section defined in 11.11.1.2,  $f_{pc}$  is taken as the average value of  $f_{pc}$  for the two directions, and  $V_p$  is the vertical component of all effective prestress forces crossing the critical section.  $V_c$  shall be permitted to be computed by Eq. (11-34) if the following are satisfied; otherwise, 11.11.2.1 shall apply:

**R11.11.2.1** — For square columns, the shear stress due to ultimate loads in slabs subjected to bending in two directions is limited to  $4\lambda \sqrt{f'_c}$ . However, tests<sup>11.61</sup> have indicated that the value of  $4\lambda \sqrt{f'_c}$  is unconservative when the ratio  $\beta$  of the lengths of the long and short sides of a rectangular column or loaded area is larger than 2.0. In such cases, the actual shear stress on the critical section at punching shear failure varies from a maximum of about  $4\lambda \sqrt{f'_c}$  around the corners of the column or loaded area, down to  $2\lambda \sqrt{f'_c}$  or less along the long sides between the two end sections. Other tests<sup>11.62</sup> indicate that  $v_c$  decreases as the ratio  $b_o/d$  increases. Equations (11-31) and (11-32) were developed to account for these two effects. The words “interior,” “edge,” and “corner columns” in 11.11.2.1(b) refer to critical sections with four, three, and two sides, respectively.

For shapes other than rectangular,  $\beta$  is taken to be the ratio of the longest overall dimension of the effective loaded area to the largest overall perpendicular dimension of the effective loaded area, as illustrated for an L-shaped reaction area in Fig. R11.11.2. The effective loaded area is that area totally enclosing the actual loaded area, for which the perimeter is a minimum.

**R11.11.2.2** — For prestressed slabs and footings, a modified form of Code Eq. (11-31) and (11-34) is specified for two-way action shear strength. Research<sup>11.63,11.64</sup> indicates that the shear strength of two-way prestressed slabs around interior columns is conservatively predicted by Eq. (11-34).  $V_c$  from Eq. (11-34) corresponds to a diagonal tension failure of the concrete initiating at the critical section defined in 11.11.1.2. The mode of failure differs from a punching shear failure of the concrete compression zone around the perimeter of the loaded area predicted by Eq. (11-31). Consequently, the term  $\beta$  does not enter into Eq. (11-34). Values for  $\sqrt{f'_c}$  and  $f_{pc}$  are restricted in design due to limited test data available for higher values. When computing  $f_{pc}$ , loss of prestress due to restraint of the slab by shear walls and other structural elements should be taken into account.

**CODE**

- (a) No portion of the column cross section shall be closer to a discontinuous edge than four times the slab thickness;
- (b) The value of  $\sqrt{f'_c}$  used in Eq. (11-34) shall not be taken greater than 70 psi; and
- (c) In each direction,  $f_{pc}$  shall not be less than 125 psi, nor be taken greater than 500 psi.

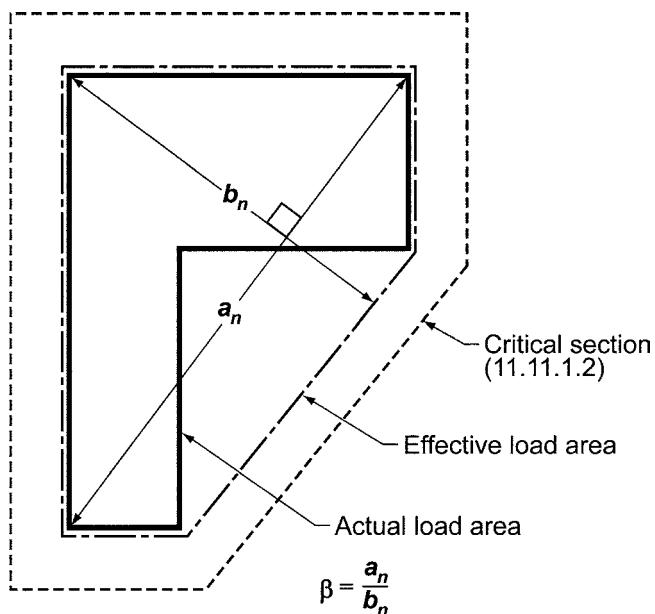
**COMMENTARY**

Fig. R11.11.2—Value of  $\beta$  for a nonrectangular loaded area.

In a prestressed slab with distributed tendons, the  $V_p$  term in Eq. (11-34) contributes only a small amount to the shear strength; therefore, it may be conservatively taken as zero. If  $V_p$  is to be included, the tendon profile assumed in the calculations should be noted.

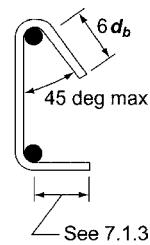
For an exterior column support where the distance from the outside of the column to the edge of the slab is less than four times the slab thickness, the prestress is not fully effective around  $b_o$ , the total perimeter of the critical section. Shear strength in this case is therefore conservatively taken the same as for a nonprestressed slab.

**11.11.3** — Shear reinforcement consisting of bars or wires and single- or multiple-leg stirrups shall be permitted in slabs and footings with  $d$  greater than or equal to 6 in., but not less than 16 times the shear reinforcement bar diameter. Shear reinforcement shall be in accordance with 11.11.3.1 through 11.11.3.4.

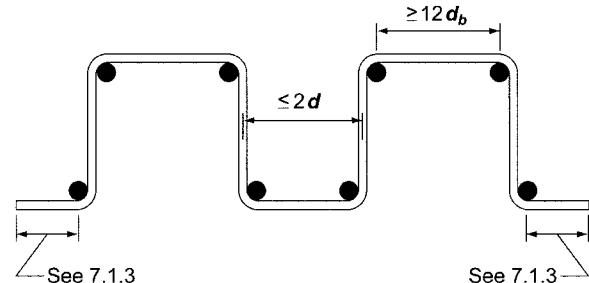
**11.11.3.1** —  $V_n$  shall be computed by Eq. (11-2), where  $V_c$  shall not be taken greater than  $2\lambda\sqrt{f'_c} b_o d$ , and  $V_s$  shall be calculated in accordance with 11.4. In Eq. (11-15),  $A_v$  shall be taken as the cross-sectional area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section.

**11.11.3.2** —  $V_n$  shall not be taken greater than  $6\sqrt{f'_c} b_o d$ .

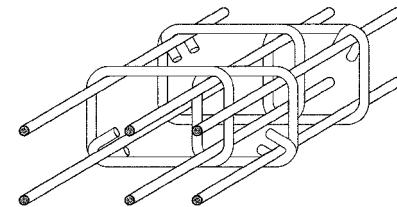
**R11.11.3** — Research<sup>11.65-11.69</sup> has shown that shear reinforcement consisting of properly anchored bars or wires and single- or multiple-leg stirrups, or closed stirrups, can increase the punching shear resistance of slabs. The spacing limits given in 11.11.3.3 correspond to slab shear reinforcement details that have been shown to be effective. Sections 12.13.2 and 12.13.3 give anchorage requirements for stirrup-type shear reinforcement that should also be applied for bars or wires used as slab shear reinforcement. It is essential that this shear reinforcement engage longitudinal reinforcement at both the top and bottom of the slab, as shown for typical details in Fig. R11.11.3(a) to (c). Anchorage of shear reinforcement according to the requirements of 12.13 is difficult in slabs thinner than 10 in. Shear reinforcement consisting of vertical bars mechanically anchored at each end by a plate or head capable of developing the yield strength of the bars has been used successfully.<sup>11.69</sup>

**CODE****COMMENTARY**

(a) single-leg stirrup or bar



(b) multiple-leg stirrup or bar



(c) closed stirrups

Fig. R11.11.3(a)-(c): Single- or multiple-leg stirrup-type slab shear reinforcement.

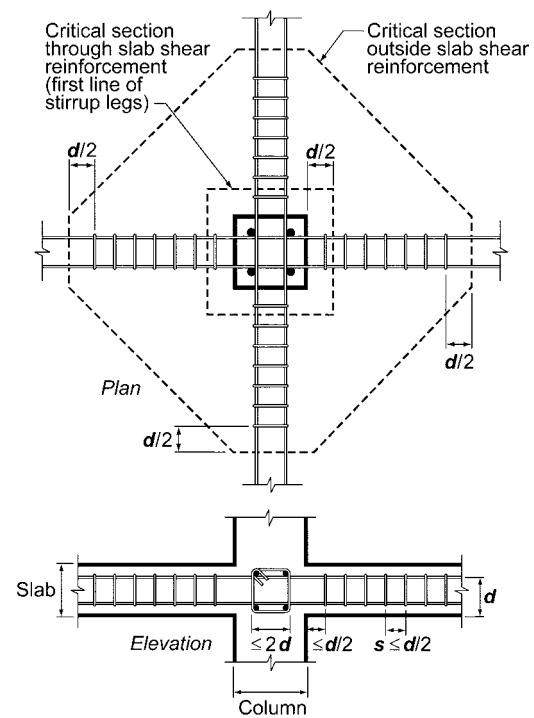


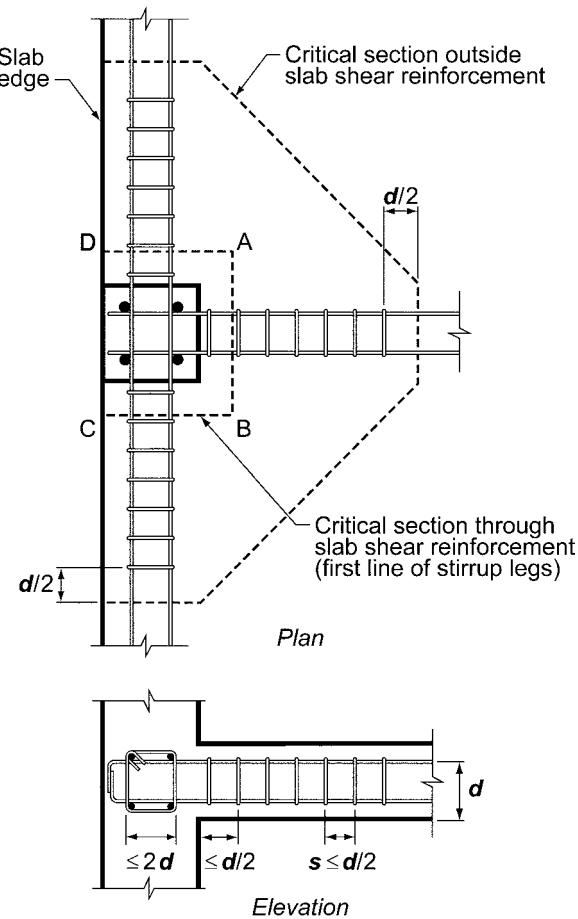
Fig. R11.11.3(d)—Arrangement of stirrup shear reinforcement, interior column.

## CODE

**11.11.3.3** — The distance between the column face and the first line of stirrup legs that surround the column shall not exceed  $d/2$ . The spacing between adjacent stirrup legs in the first line of shear reinforcement shall not exceed  $2d$  measured in a direction parallel to the column face. The spacing between successive lines of shear reinforcement that surround the column shall not exceed  $d/2$  measured in a direction perpendicular to the column face.

**11.11.3.4** — Slab shear reinforcement shall satisfy the anchorage requirements of [12.13](#) and shall engage the longitudinal flexural reinforcement in the direction being considered.

## COMMENTARY



*Fig. R11.11.3(e)—Arrangement of stirrup shear reinforcement, edge column.*

**11.11.4** — Shear reinforcement consisting of structural steel I- or channel-shaped sections (shearheads) shall be permitted in slabs. The provisions of 11.11.4.1 through [11.11.4.9](#) shall apply where shear due to gravity load is transferred at interior column supports. Where moment is transferred to columns, [11.11.7.3](#) shall apply.

**11.11.4.1** — Each shearhead shall consist of steel shapes fabricated by welding with a full penetration weld into identical arms at right angles. Shearhead arms shall not be interrupted within the column section.

In a slab-column connection for which the moment transfer is negligible, the shear reinforcement should be symmetrical about the centroid of the critical section ([Fig. R11.11.3\(d\)](#)). Spacing limits defined in 11.11.3.3 are also shown in [Fig. R11.11.3\(d\)](#) and [\(e\)](#). At edge columns or for interior connections where moment transfer is significant, closed stirrups are recommended in a pattern as symmetrical as possible. Although the average shear stresses on faces *AD* and *BC* of the exterior column in [Fig. R11.11.3\(e\)](#) are lower than on face *AB*, the closed stirrups extending from faces *AD* and *BC* provide some torsional strength along the edge of the slab.

**R11.11.4** — Based on reported test data,<sup>11.70</sup> design procedures are presented for shearhead reinforcement consisting of structural steel shapes. For a column connection transferring moment, the design of shearheads is given in [11.11.7.3](#).

Three basic criteria should be considered in the design of shearhead reinforcement for connections transferring shear due to gravity load. First, a minimum flexural strength should be provided to ensure that the required shear strength of the slab is reached before the flexural strength of the shearhead is exceeded. Second, the shear stress in the slab at

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**11.11.4.2** — A shearhead shall not be deeper than 70 times the web thickness of the steel shape.

**11.11.4.3** — The ends of each shearhead arm shall be permitted to be cut at angles not less than 30 degrees with the horizontal, provided the plastic moment strength of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead.

**11.11.4.4** — All compression flanges of steel shapes shall be located within **0.3d** of compression surface of slab.

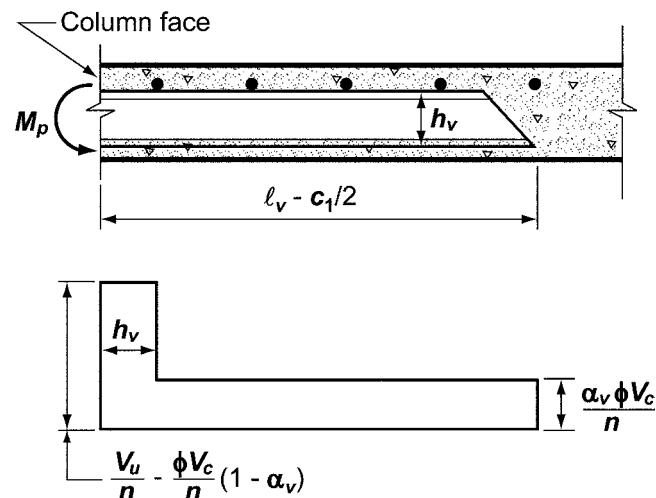


Fig. R11.11.4.5—Idealized shear acting on shearhead.

**11.11.4.5** — The ratio  $\alpha_v$  between the flexural stiffness of each shearhead arm and that of the surrounding composite cracked slab section of width ( $c_2 + d$ ) shall not be less than 0.15.

**11.11.4.6** — Plastic moment strength,  $M_p$ , required for each arm of the shearhead shall be computed by

$$M_p = \frac{V_u}{2\phi n} \left[ h_v + \alpha_v \left( l_v - \frac{c_1}{2} \right) \right] \quad (11-35)$$

where  $\phi$  is for tension-controlled members,  $n$  is number of shearhead arms, and  $l_v$  is minimum length of each shearhead arm required to comply with requirements of 11.11.4.7 and 11.11.4.8.

**11.11.4.7** — The critical slab section for shear shall be perpendicular to the plane of the slab and shall cross each shearhead arm at three-quarters the distance [ $l_v - (c_1/2)$ ] from the column face to the end of the shearhead arm. The critical section shall be located so that its perimeter  $b_o$  is a minimum, but need not be closer than the perimeter defined in 11.11.1.2(a).

the end of the shearhead reinforcement should be limited. Third, after these two requirements are satisfied, the negative moment slab reinforcement can be reduced in proportion to the moment contribution of the shearhead at the design section.

**R11.11.4.5 and R11.11.4.6** — The assumed idealized shear distribution along an arm of a shearhead at an interior column is shown in Fig. R11.11.4.5. The shear along each of the arms is taken as  $\alpha_v \phi V_c / n$ , where  $V_c$  is defined in 11.11.2.1(c). However, the peak shear at the face of the column is taken as the total shear considered per arm  $V_u / n$  minus the shear considered carried to the column by the concrete compression zone of the slab. The latter term is expressed as  $\phi(V_c / n)(1 - \alpha_v)$ , so that it approaches zero for a heavy shearhead and approaches  $V_u / n$  when a light shearhead is used. Equation (11-35) then follows from the assumption that  $\phi V_c$  is about one-half the factored shear force  $V_u$ . In this equation,  $M_p$  is the required plastic moment strength of each shearhead arm necessary to ensure that  $V_u$  is attained as the moment strength of the shearhead is reached. The quantity  $l_v$  is the length from the center of the column to the point at which the shearhead is no longer required, and the distance  $c_1/2$  is one-half the dimension of the column in the direction considered.

**R11.11.4.7** — The test results<sup>11.70</sup> indicated that slabs containing under-reinforcing shearheads failed at a shear stress on a critical section at the end of the shearhead reinforcement less than  $4\sqrt{f'_c}$ . Although the use of over-reinforcing shearheads brought the shear strength back to about the equivalent of  $4\sqrt{f'_c}$ , the limited test data suggest that a conservative design is desirable. Therefore, the shear strength is calculated as  $4\sqrt{f'_c}$  on an assumed critical section located inside the end of the shearhead reinforcement.

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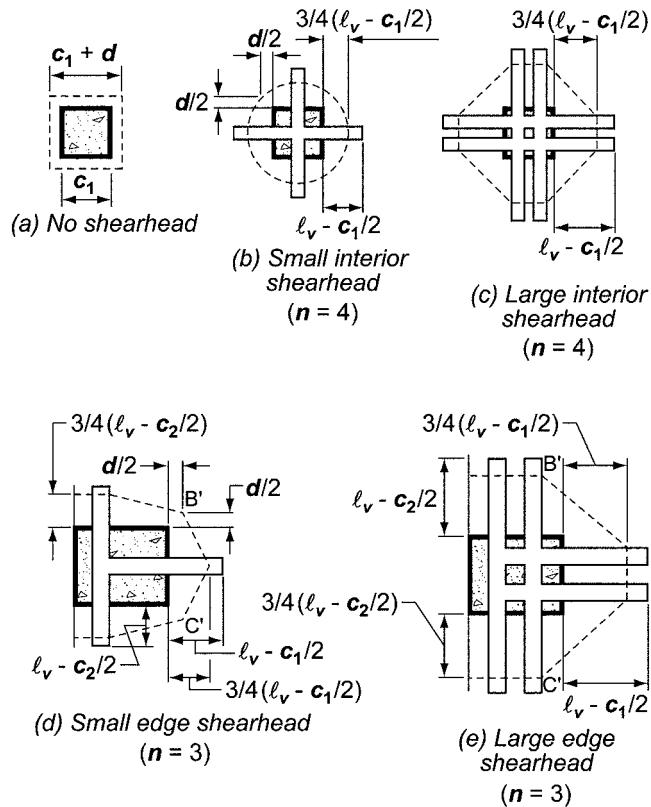


Fig. R11.11.4.7—Location of critical section defined in 11.11.4.7.

**11.11.4.8** —  $V_n$  shall not be taken greater than  $4\sqrt{f'_c} b_o d$  on the critical section defined in 11.11.4.7. When shearhead reinforcement is provided,  $V_n$  shall not be taken greater than  $7\sqrt{f'_c} b_o d$  on the critical section defined in 11.11.1.2(a).

**11.11.4.9** — Moment resistance  $M_v$  contributed to each slab column strip by a shearhead shall not be taken greater than

$$M_v = \frac{\phi \alpha_v V_u}{2n} \left( l_v - \frac{c_1}{2} \right) \quad (11-36)$$

where  $\phi$  is for tension-controlled members,  $n$  is number of shearhead arms, and  $l_v$  is length of each shearhead arm actually provided. However,  $M_v$  shall not be taken larger than the smallest of:

- (a) 30 percent of the total factored moment required for each slab column strip;

The critical section is taken through the shearhead arms three-fourths of the distance [ $l_v - (c_1/2)$ ] from the face of the column to the end of the shearhead. However, this assumed critical section need not be taken closer than  $d/2$  to the column. See Fig. R11.11.4.7.

**R11.11.4.9** — If the peak shear at the face of the column is neglected, and  $\phi V_c$  is again assumed to be about one-half of  $V_u$ , the moment resistance contribution of the shearhead  $M_v$  can be conservatively computed from Eq. (11-36), in which  $\phi$  is the factor for flexure.

**CODE****COMMENTARY**

(b) The change in column strip moment over the length  $\ell_v$ ;

(c)  $M_p$  computed by Eq. (11-35).

**11.11.4.10** — When unbalanced moments are considered, the shearhead must have adequate anchorage to transmit  $M_p$  to the column.

**11.11.5** — Headed shear stud reinforcement, placed perpendicular to the plane of a slab or footing, shall be permitted in slabs and footings in accordance with 11.11.5.1 through 11.11.5.4. The overall height of the shear stud assembly shall not be less than the thickness of the member less the sum of: (1) the concrete cover on the top flexural reinforcement; (2) the concrete cover on the base rail; and (3) one-half the bar diameter of the tension flexural reinforcement. Where flexural tension reinforcement is at the bottom of the section, as in a footing, the overall height of the shear stud assembly shall not be less than the thickness of the member less the sum of: (1) the concrete cover on the bottom flexural reinforcement; (2) the concrete cover on the head of the stud; and (3) one-half the bar diameter of the bottom flexural reinforcement.

**11.11.5.1** — For the critical section defined in 11.11.1.2,  $V_n$  shall be computed using Eq. (11-2), with  $V_c$  and  $V_n$  not exceeding  $3\lambda\sqrt{f'_c} b_o d$  and  $8\sqrt{f'_c} b_o d$ , respectively.  $V_s$  shall be calculated using Eq. (11-15) with  $A_v$  equal to the cross-sectional area of all the shear reinforcement on one peripheral line that is approximately parallel to the perimeter of the column section, where  $s$  is the spacing of the peripheral lines of headed shear stud reinforcement.  $A_v f_{yt} l / (b_o s)$  shall not be less than  $2\sqrt{f'_c}$ .

**11.11.5.2** — The spacing between the column face and the first peripheral line of shear reinforcement shall not exceed  $d/2$ . The spacing between peripheral lines of shear reinforcement, measured in a direction perpendicular to any face of the column, shall be constant. For prestressed slabs or footings satisfying 11.11.2.2, this spacing shall not exceed  $0.75d$ ; for all other slabs and footings, the spacing shall be based on the value of the shear stress due to factored shear force and unbalanced moment at the critical section defined in 11.11.1.2, and shall not exceed:

(a)  $0.75d$  where maximum shear stresses due to factored loads are less than or equal to  $6\phi\sqrt{f'_c}$ ; and

**R11.11.4.10** — See R11.11.7.3.

**R11.11.5** — Headed shear stud reinforcement was introduced in the 2008 Code. Using headed stud assemblies, as shear reinforcement in slabs and footings, requires specifying the stud shank diameter, the spacing of the studs, and the height of the assemblies for the particular applications.

Tests<sup>11.69</sup> show that vertical studs mechanically anchored as close as possible to the top and bottom of slabs are effective in resisting punching shear. The bounds of the overall specified height achieve this objective while providing a reasonable tolerance in specifying that height as shown in Fig. R7.7.5.

Compared with a leg of a stirrup having bends at the ends, a stud head exhibits smaller slip, and thus results in smaller shear crack widths. The improved performance results in larger limits for shear strength and spacing between peripheral lines of headed shear stud reinforcement. Typical arrangements of headed shear stud reinforcement are shown in Fig. R11.11.5. The critical section beyond the shear reinforcement generally has a polygonal shape. Equations for calculating shear stresses on such sections are given in Reference 11.69.

**R11.11.5.1** — When there is unbalanced moment transfer, the design will be based on stresses. The maximum shear stress due to a combination of  $V_u$  and the fraction of unbalanced moment  $\gamma_v M_u$  should not exceed  $\phi v_n$ , where  $v_n$  is taken as the sum of  $3\lambda\sqrt{f'_c}$  and  $A_v f_{yt} l / (b_o s)$ .

**R11.11.5.2** — The specified spacings between peripheral lines of shear reinforcement are justified by experiments.<sup>11.69</sup> The clear spacing between the heads of the studs should be adequate to permit placing of the flexural reinforcement.

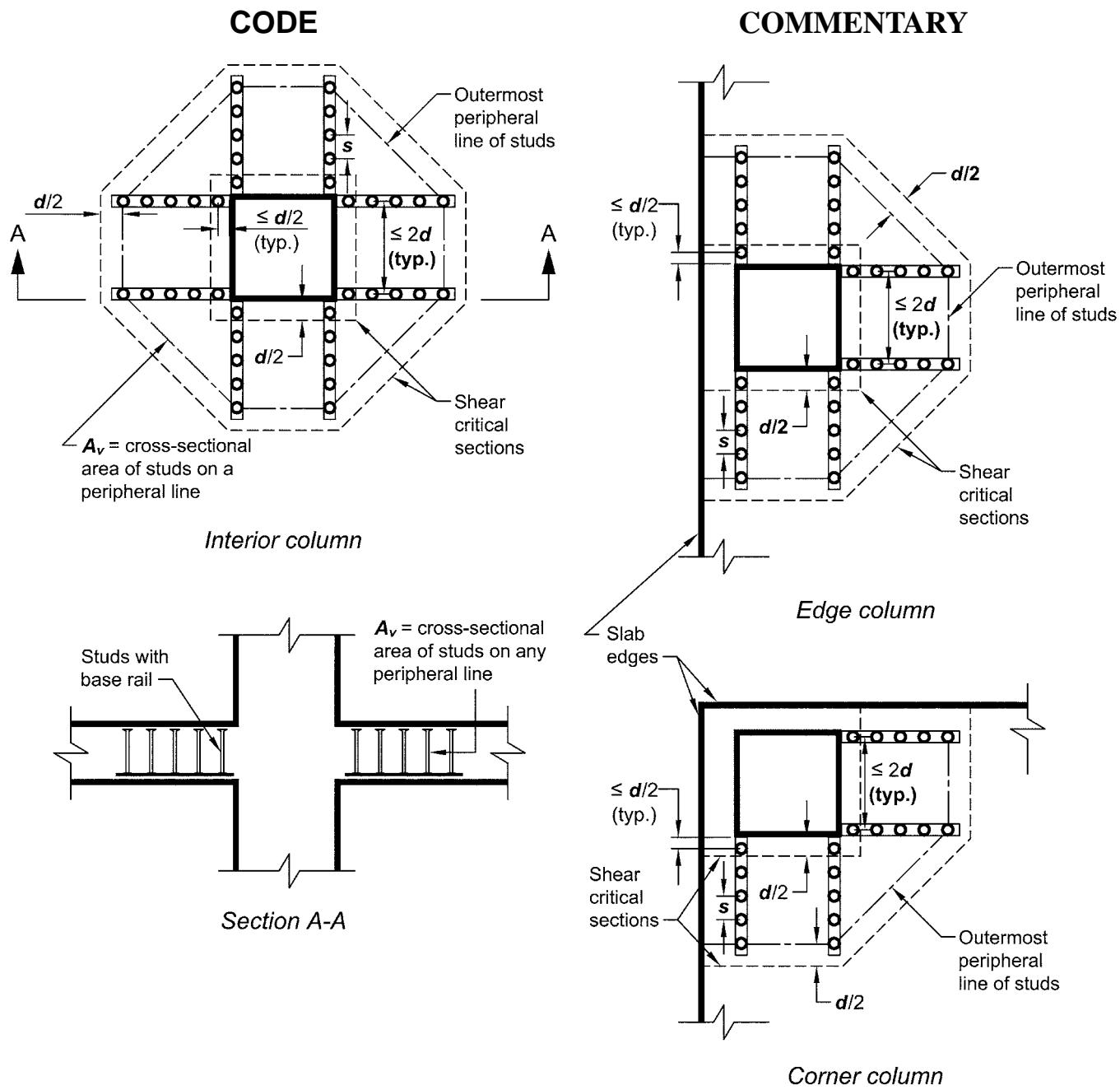
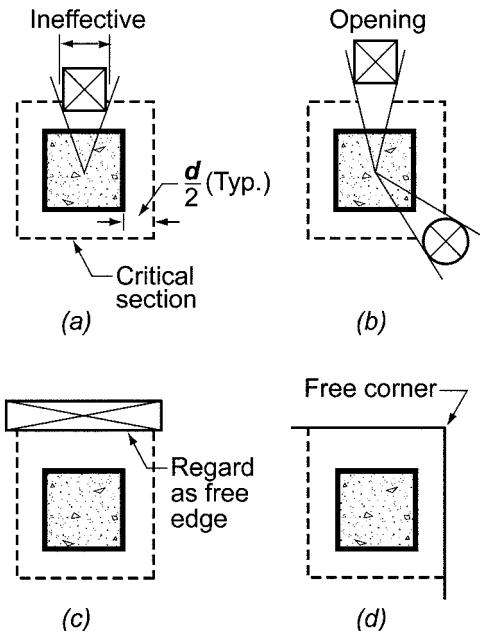


Fig. R11.11.5—Typical arrangements of headed shear stud reinforcement and critical sections.

(b)  $0.5d$  where maximum shear stresses due to factored loads are greater than  $6\phi\sqrt{f'_c}$ .

**11.11.5.3** — The spacing between adjacent shear reinforcement elements, measured on the perimeter of the first peripheral line of shear reinforcement, shall not exceed  $2d$ .

**11.11.5.4** — Shear stress due to factored shear force and moment shall not exceed  $2\phi\lambda\sqrt{f'_c}$  at the critical section located  $d/2$  outside the outermost peripheral line of shear reinforcement.

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*Fig. R11.11.6—Effect of openings and free edges (effective perimeter shown with dashed lines).*

### 11.11.6 — Openings in slabs

When openings in slabs are located at a distance less than 10 times the slab thickness from a concentrated load or reaction area, or when openings in flat slabs are located within column strips as defined in [Chapter 13](#), the critical slab sections for shear defined in [11.11.1.2](#) and [11.11.4.7](#) shall be modified as follows:

**11.11.6.1** — For slabs without shearheads, that part of the perimeter of the critical section that is enclosed by straight lines projecting from the centroid of the column, concentrated load, or reaction area and tangent to the boundaries of the openings shall be considered ineffective.

**11.11.6.2** — For slabs with shearheads, the ineffective portion of the perimeter shall be one-half of that defined in [11.11.6.1](#).

### 11.11.7 — Transfer of moment in slab-column connections

**11.11.7.1** — Where gravity load, wind, earthquake, or other lateral forces cause transfer of unbalanced moment  $M_u$  between a slab and column,  $\gamma_f M_u$  shall be transferred by flexure in accordance with [13.5.3](#). The remainder of the unbalanced moment,  $\gamma_v M_u$ , shall be considered to be transferred by eccentricity of shear about the centroid of the critical section defined in [11.11.1.2](#) where

$$\gamma_v = (1 - \gamma_f) \quad (11-37)$$

### R11.11.6 — Openings in slabs

Provisions for design of openings in slabs (and footings) were developed in [Reference 11.3](#). The locations of the effective portions of the critical section near typical openings and free edges are shown by the dashed lines in Fig. R11.11.6. Additional research<sup>11.61</sup> has confirmed that these provisions are conservative.

### R11.11.7 — Transfer of moment in slab-column connections

**R11.11.7.1** — In [Reference 11.71](#) it was found that where moment is transferred between a column and a slab, 60 percent of the moment should be considered transferred by flexure across the perimeter of the critical section defined in [11.11.1.2](#), and 40 percent by eccentricity of the shear about the centroid of the critical section. For rectangular columns, the portion of the moment transferred by flexure increases as the width of the face of the critical section resisting the moment increases, as given by [Eq. \(13-1\)](#).

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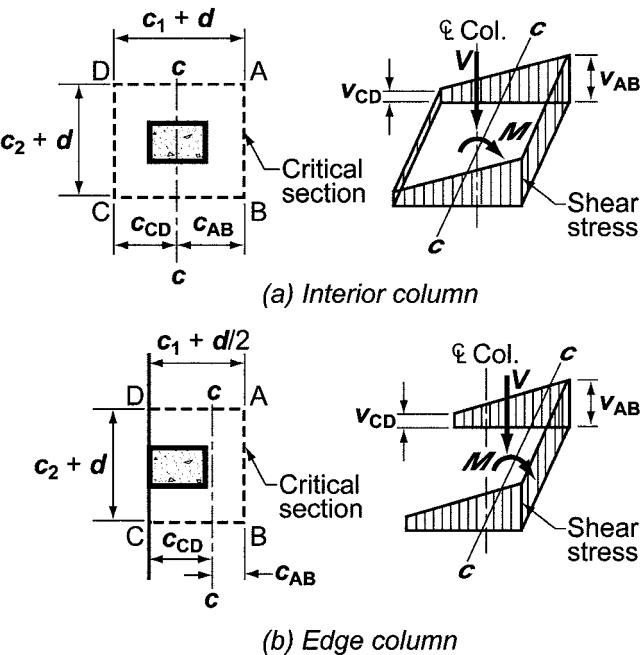


Fig. R11.11.7.2—Assumed distribution of shear stress.

**11.11.7.2** — The shear stress resulting from moment transfer by eccentricity of shear shall be assumed to vary linearly about the centroid of the critical sections defined in **11.11.1.2**. The maximum shear stress due to  $V_u$  and  $M_u$  shall not exceed  $\phi v_n$ :

(a) For members without shear reinforcement,

$$\phi v_n = \phi V_c / (b_o d) \quad (11-38)$$

where  $V_c$  is as defined in **11.11.2.1** or **11.11.2.2**.

(b) For members with shear reinforcement other than shearheads,

$$\phi v_n = \phi (V_c + V_s) / (b_o d) \quad (11-39)$$

where  $V_c$  and  $V_s$  are defined in **11.11.3.1**. The design shall take into account the variation of shear stress around the column. The shear stress due to factored shear force and moment shall not exceed  $\phi(2\lambda\sqrt{f'_c})$  at the critical section located  $d/2$  outside the outermost line of stirrup legs that surround the column.

Most of the data in [Reference 11.71](#) were obtained from tests of square columns, and little information is available for round columns. These can be approximated as square columns. [Figure R13.6.2.5](#) shows square supports having the same area as some nonrectangular members.

**R11.11.7.2** — The stress distribution is assumed as illustrated in Fig. R11.11.7.2 for an interior or exterior column. The perimeter of the critical section,  $ABCD$ , is determined in accordance with **11.11.1.2**. The factored shear force  $V_u$  and unbalanced factored moment  $M_u$  are determined at the centroidal axis  $c-c$  of the critical section. The maximum factored shear stress may be calculated from

$$v_{u(AB)} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

or

$$v_{u(CD)} = \frac{V_u}{A_c} - \frac{\gamma_v M_u c_{CD}}{J_c}$$

where  $\gamma_v$  is given by [Eq. \(11-37\)](#). For an interior column,  $A_c$  and  $J_c$  may be calculated by

$A_c$  = area of concrete of assumed critical section  
=  $2d(c_1 + c_2 + 2d)$

$J_c$  = property of assumed critical section analogous to polar moment of inertia

**CODE****COMMENTARY**

$$= \frac{d(c_1 + d)^3}{6} + \frac{(c_1 + d)d^3}{6} + \frac{d(c_2 + d)(c_1 + d)^2}{2}$$

Similar equations may be developed for  $A_c$  and  $J_c$  for columns located at the edge or corner of a slab.

The fraction of the unbalanced moment between slab and column not transferred by eccentricity of the shear should be transferred by flexure in accordance with 13.5.3. A conservative method assigns the fraction transferred by flexure over an effective slab width defined in 13.5.3.2. Often column strip reinforcement is concentrated near the column to accommodate this unbalanced moment. Available test data<sup>11.71</sup> seem to indicate that this practice does not increase shear strength but may be desirable to increase the stiffness of the slab-column junction.

Test data<sup>11.72</sup> indicate that the moment transfer strength of a prestressed slab-to-column connection can be calculated using the procedures of 11.11.7 and 13.5.3.

Where shear reinforcement has been used, the critical section beyond the shear reinforcement generally has a polygonal shape (Fig. R11.11.3(d) and (e)). Equations for calculating shear stresses on such sections are given in Reference 11.69.

**11.11.7.3** — When shear reinforcement consisting of structural steel I- or channel-shaped sections (shearheads) is provided, the sum of the shear stresses due to vertical load acting on the critical section defined by 11.11.4.7 and the shear stresses resulting from moment transferred by eccentricity of shear about the centroid of the critical section defined in 11.11.1.2(a) and 11.11.1.3 shall not exceed  $\phi 4\lambda \sqrt{f'_c}$ .

**R11.11.7.3** — Tests<sup>11.73</sup> indicate that the critical sections are defined in 11.11.1.2(a) and 11.11.1.3 and are appropriate for calculations of shear stresses caused by transfer of moments even when shearheads are used. Then, even though the critical sections for direct shear and shear due to moment transfer differ, they coincide or are in close proximity at the column corners where the failures initiate. Because a shearhead attracts most of the shear as it funnels toward the column, it is conservative to take the maximum shear stress as the sum of the two components.

Section 11.11.4.10 requires the moment  $M_p$  to be transferred to the column in shearhead connections transferring unbalanced moments. This may be done by bearing within the column or by mechanical anchorage.

**Notes**

# CHAPTER 12 — DEVELOPMENT AND SPLICES OF REINFORCEMENT

## CODE

### 12.1 — Development of reinforcement — General

**12.1.1** — Calculated tension or compression in reinforcement at each section of structural concrete members shall be developed on each side of that section by embedment length, hook, headed deformed bar or mechanical device, or a combination thereof. Hooks and heads shall not be used to develop bars in compression.

**12.1.2** — The values of  $\sqrt{f'_c}$  used in this chapter shall not exceed 100 psi.

**12.1.3** — In addition to requirements in this chapter that affect detailing of reinforcement, structural integrity requirements of 7.13 shall be satisfied.

## COMMENTARY

### R12.1 — Development of reinforcement — General

The development length concept for anchorage of reinforcement was first introduced in the 1971 Code, to replace the dual requirements for flexural bond and anchorage bond contained in earlier editions. It is no longer necessary to consider the flexural bond concept, which placed emphasis on the computation of nominal peak bond stresses. Consideration of an average bond resistance over a full development length of the reinforcement is more meaningful, partially because all bond tests consider an average bond resistance over a length of embedment of the reinforcement, and partially because uncalculated extreme variations in local bond stresses exist near flexural cracks.<sup>12.1</sup>

The development length concept is based on the attainable average bond stress over the length of embedment of the reinforcement. Development lengths are required because of the tendency of highly stressed bars to split relatively thin sections of restraining concrete. A single bar embedded in a mass of concrete should not require as great a development length; although a row of bars, even in mass concrete, can create a weakened plane with longitudinal splitting along the plane of the bars.

In application, the development length concept requires minimum lengths or extensions of reinforcement beyond all points of peak stress in the reinforcement. Such peak stresses generally occur at the points in 12.10.2.

Structural integrity requirements of 7.13 may control detailing of reinforcement at splices and terminations.

The strength reduction factor  $\phi$  is not used in the development length and lap splice equations. An allowance for strength reduction is already included in the expressions for determining development and splice lengths. Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as ft or kip.

From a point of peak stress in reinforcement, some length of reinforcement or anchorage is necessary to develop the stress. This development length or anchorage is necessary on both sides of such peak stress points. Often the reinforcement continues for a considerable distance on one side of a critical stress point so that calculations need involve only the other side, for example, the negative moment reinforcement continuing through a support to the middle of the next span.

**CODE****12.2 — Development of deformed bars and deformed wire in tension**

**12.2.1** — Development length for deformed bars and deformed wire in tension,  $\ell_d$ , shall be determined from either 12.2.2 or 12.2.3 and applicable modification factors of 12.2.4 and 12.2.5, but  $\ell_d$  shall not be less than 12 in.

**12.2.2** — For deformed bars or deformed wire,  $\ell_d$  shall be as follows:

Spacing and cover	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars or wires being developed or spliced not less than $d_b$ , clear cover not less than $d_b$ , and stirrups or ties throughout $\ell_d$ not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than $d_b$	$\left( \frac{f_y \psi_t \psi_e}{25 \lambda \sqrt{f'_c}} \right) d_b$	$\left( \frac{f_y \psi_t \psi_e}{20 \lambda \sqrt{f'_c}} \right) d_b$
Other cases	$\left( \frac{3 f_y \psi_t \psi_e}{50 \lambda \sqrt{f'_c}} \right) d_b$	$\left( \frac{3 f_y \psi_t \psi_e}{40 \lambda \sqrt{f'_c}} \right) d_b$

**12.2.3** — For deformed bars or deformed wire,  $\ell_d$  shall be

$$\ell_d = \left( \frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{(c_b + K_{tr})/d_b} \right) d_b \quad (12-1)$$

in which the confinement term  $(c_b + K_{tr})/d_b$  shall not be taken greater than 2.5, and

$$K_{tr} = \frac{40 A_{tr}}{sn} \quad (12-2)$$

where  $n$  is the number of bars or wires being spliced or developed along the plane of splitting. It shall be permitted to use  $K_{tr} = 0$  as a design simplification even if transverse reinforcement is present.

**COMMENTARY****R12.2 — Development of deformed bars and deformed wire in tension**

The general development length equation (Eq. (12-1)) is given in 12.2.3. The equation is based on the expression for development length previously endorsed by Committee 408.<sup>12.2,12.3</sup> In Eq. (12-1),  $c_b$  is a factor that represents the smallest of the side cover, the cover over the bar or wire (in both cases measured to the center of the bar or wire), or one-half the center-to-center spacing of the bars or wires.  $K_{tr}$  is a factor that represents the contribution of confining reinforcement across potential splitting planes.  $\psi_t$  is the traditional reinforcement location factor to reflect the adverse effects of the top reinforcement casting position.  $\psi_e$  is a coating factor reflecting the effects of epoxy coating. There is a limit on the product  $\psi_t \psi_e$ . The reinforcement size factor  $\psi_s$  reflects the more favorable performance of smaller-diameter reinforcement. In 2008, a revision was made to the  $\lambda$  term which is essentially the inverse of the  $\lambda$  used previously in Chapter 12. A limit of 2.5 is placed on the term  $(c_b + K_{tr})/d_b$ . When  $(c_b + K_{tr})/d_b$  is less than 2.5, splitting failures are likely to occur. For values above 2.5, a pullout failure is expected and an increase in cover or transverse reinforcement is unlikely to increase the anchorage capacity.

Equation (12-1) includes the effects of all variables controlling the development length. Terms in Eq. (12-1) may be disregarded when such omission results in longer and hence, more conservative, development lengths.

The provisions of 12.2.2 and 12.2.3 give a two-tier approach. The user can either calculate  $\ell_d$  based on the actual  $(c_b + K_{tr})/d_b$  (12.2.3) or calculate  $\ell_d$  using 12.2.2, which is based on two preselected values of  $(c_b + K_{tr})/d_b$ .

Section 12.2.2 recognizes that many current practical construction cases utilize spacing and cover values along with confining reinforcement, such as stirrups or ties, that result in a value of  $(c_b + K_{tr})/d_b$  of at least 1.5. Examples include a minimum clear cover of  $d_b$  along with either minimum clear spacing of  $2d_b$ , or a combination of minimum clear spacing of  $d_b$  and minimum ties or stirrups. For these frequently occurring cases, the development length for larger bars can be taken as  $\ell_d = [f_y \psi_t \psi_e / (20 \lambda \sqrt{f'_c})] d_b$ . In the development of ACI 318-95, a comparison with past provisions and a check of a database of experimental results maintained by ACI Committee 408<sup>12.2</sup> indicated that for No. 6 deformed bars and smaller, as well as for deformed wire, the development lengths could be reduced 20 percent using  $\psi_s = 0.8$ . This is the basis for the middle column of the table in 12.2.2. With less cover and in the absence of minimum ties or stirrups, the minimum clear spacing limits of 7.6.1 and the minimum concrete cover requirements of 7.7 result in minimum values of  $c_b$  equal to  $d_b$ . Thus, for “other cases,” the values are based on using  $(c_b + K_{tr})/d_b = 1.0$  in Eq. (12-1).

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The user may easily construct simple, useful expressions. For example, in all structures with normalweight concrete ( $\lambda = 1.0$ ), uncoated reinforcement ( $\psi_e = 1.0$ ), No. 7 or larger bottom bars ( $\psi_t = 1.0$ ) with  $f'_c = 4000$  psi and Grade 60 reinforcement, the equations reduce to

$$\ell_d = \frac{(60,000)(1.0)(1.0)}{20(1.0)\sqrt{4000}} d_b = 47d_b$$

or

$$\ell_d = \frac{3(60,000)(1.0)(1.0)}{40(1.0)\sqrt{4000}} d_b = 71d_b$$

Thus, as long as minimum cover of  $d_b$  is provided along with a minimum clear spacing of  $2d_b$ , or a minimum clear cover of  $d_b$  and a minimum clear spacing of  $d_b$  are provided along with minimum ties or stirrups, then  $\ell_d = 47d_b$ . The penalty for spacing bars closer or providing less cover is the requirement that  $\ell_d = 71d_b$ .

Many practical combinations of side cover, clear cover, and confining reinforcement can be used with 12.2.3 to produce significantly shorter development lengths than allowed by 12.2.2. For example, bars or wires with minimum clear cover not less than  $2d_b$  and minimum clear spacing not less than  $4d_b$  and without any confining reinforcement would have a  $(c_b + K_{tr})/d_b$  value of 2.5 and would require a development length of only  $28d_b$  for the example above.

Before ACI 318-08, Eq. (12-2) for  $K_{tr}$  included the yield strength of transverse reinforcement. The current expression includes only the area and spacing of the transverse reinforcement and the number of wires or bars being developed or lap spliced because tests demonstrate that transverse reinforcement rarely yields during a bond failure.<sup>12.4</sup>

**R12.2.4** — The reinforcement location factor  $\psi_t$  accounts for position of the reinforcement in freshly placed concrete. The factor was reduced to 1.3 in the 1989 Code to reflect research.<sup>12.5,12.6</sup>

The factor  $\lambda$  for lightweight concrete was made the same for all types of lightweight aggregates in the 1989 Code. Research on hooked bar anchorages did not support the variations in previous Codes for all-lightweight and sand-lightweight concrete and a single value, 1.3 (used at that time as a multiplier in the numerator of development length equations), was selected. A unified definition of  $\lambda$  was adopted in the 2008 Code. Because a single definition of  $\lambda$  is now used in the Code, the term  $\lambda$  has been moved from the numerator to the denominator in the development length equations ( $1/0.75 = 1.33$ ). Section 12.2.4 allows a higher factor to be used when the splitting tensile strength of the lightweight concrete is specified. See 5.1.4.

**12.2.4** — The factors used in the expressions for development of deformed bars and deformed wires in tension in 12.2 are as follows:

(a) Where horizontal reinforcement is placed such that more than 12 in. of fresh concrete is cast below the development length or splice,  $\psi_t = 1.3$ . For other situations,  $\psi_t = 1.0$ .

(b) For epoxy-coated bars, zinc and epoxy dual-coated bars, or epoxy-coated wires with cover less than  $3d_b$ , or clear spacing less than  $6d_b$ ,  $\psi_e = 1.5$ . For all other epoxy-coated bars, zinc and epoxy dual-coated bars, or epoxy-coated wires,  $\psi_e = 1.2$ . For uncoated and zinc-coated (galvanized) reinforcement,  $\psi_e = 1.0$ .

However, the product  $\psi_t\psi_e$  need not be greater than 1.7.

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- (c) For No. 6 and smaller bars and deformed wires,  $\psi_s = 0.8$ . For No. 7 and larger bars,  $\psi_s = 1.0$ .
- (d) Where lightweight concrete is used,  $\lambda$  shall not exceed **0.75** unless  $f_{ct}$  is specified (see **8.6.1**). Where normalweight concrete is used,  $\lambda = 1.0$ .

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Studies<sup>12.7-12.9</sup> of the anchorage of epoxy-coated bars show that bond strength is reduced because the coating prevents adhesion and friction between the bar and the concrete. The factors reflect the type of anchorage failure likely to occur. When the cover or spacing is small, a splitting failure can occur and the anchorage or bond strength is substantially reduced. If the cover and spacing between bars is large, a splitting failure is precluded and the effect of the epoxy coating on anchorage strength is not as large. Studies<sup>12.10</sup> have shown that although the cover or spacing may be small, the anchorage strength may be increased by adding transverse steel crossing the plane of splitting, and restraining the splitting crack.

Because the bond of epoxy-coated bars or zinc and epoxy dual-coated bars is already reduced due to the loss of adhesion between the bar and the concrete, an upper limit of 1.7 is established for the product of the factors for top reinforcement and epoxy-coated reinforcement or zinc and epoxy dual-coated reinforcement.

Although there is no requirement for transverse reinforcement along the tension development or splice length, recent research<sup>12.11,12.12</sup> indicates that in concrete with very high compressive strength, brittle anchorage failure occurred in bars with inadequate transverse reinforcement. In splice tests of No. 8 and No. 11 bars in concrete with an  $f'_c$  of approximately 15,000 psi, transverse reinforcement improved ductile anchorage behavior.

### 12.2.5 — Excess reinforcement

Reduction in  $\ell_d$  shall be permitted where reinforcement in a flexural member is in excess of that required by analysis except where anchorage or development for  $f_y$  is specifically required or the reinforcement is designed under provisions of **21.1.1.6** ..... **( $A_s$  required)/( $A_s$  provided)**.

### 12.3 — Development of deformed bars and deformed wire in compression

**12.3.1** — Development length for deformed bars and deformed wire in compression,  $\ell_{dc}$ , shall be determined from 12.3.2 and applicable modification factors of 12.3.3, but  $\ell_{dc}$  shall not be less than 8 in.

**12.3.2** — For deformed bars and deformed wire,  $\ell_{dc}$  shall be taken as the larger of  $(0.02f_y/\lambda\sqrt{f'_c})d_b$  and  $(0.0003f_y)d_b$ , with  $\lambda$  as given in 12.2.4(d) and the constant 0.0003 carries the unit of in.<sup>2</sup>/lb.

**12.3.3** — Length  $\ell_{dc}$  in 12.3.2 shall be permitted to be multiplied by the applicable factors for:

### R12.2.5 — Excess reinforcement

The reduction factor based on area is not to be used in those cases where anchorage development for full  $f_y$  is required. For example, the excess reinforcement factor does not apply for development of positive moment reinforcement at supports according to 12.11.2, for development of shrinkage and temperature reinforcement according to 7.12.2.3, or for development of reinforcement provided according to 7.13 and 13.3.8.5.

### R12.3 — Development of deformed bars and deformed wire in compression

The weakening effect of flexural tension cracks is not present for bars and wire in compression, and usually end bearing of the bars on the concrete is beneficial. Therefore, shorter development lengths are specified for compression than for tension. The development length may be reduced 25 percent when the reinforcement is enclosed within spirals or ties. A reduction in development length is also permitted if excess reinforcement is provided.

In 2008, the term  $\lambda$  was added to the expression for development in 12.3.2 recognizing that there is no known test data on compression development in lightweight concrete but that splitting is more likely in lightweight concrete.

**CODE****COMMENTARY**

- |  |  |
|--|--|
| (a) Reinforcement in excess of that required by analysis .....   | <b>(<math>A_s</math> required)/(<math>A_s</math> provided)</b> |
| (b) Reinforcement enclosed within spiral reinforcement not less than 1/4 in. diameter and not more than 4 in. pitch or within No. 4 ties in conformance with 7.10.5 and spaced at not more than 4 in. on center..... | <b>0.75</b>  |

**12.4 — Development of bundled bars**

**12.4.1** — Development length of individual bars within a bundle, in tension or compression, shall be that for the individual bar, increased 20 percent for three-bar bundle, and 33 percent for four-bar bundle.

**12.4.2** — For determining the appropriate spacing and cover values in 12.2.2, the confinement term in 12.2.3, and the  $\psi_e$  factor in 12.2.4(b), a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area and having a centroid that coincides with that of the bundled bars.

**12.5 — Development of standard hooks in tension**

**12.5.1** — Development length for deformed bars in tension terminating in a standard hook (see 7.1),  $\ell_{dh}$ , shall be determined from 12.5.2 and the applicable modification factors of 12.5.3, but  $\ell_{dh}$  shall not be less than the larger of  $8d_b$  and 6 in.

**12.5.2** — For deformed bars,  $\ell_{dh}$  shall be  $(0.02\psi_e f_y/\lambda \sqrt{f'_c})d_b$  with  $\psi_e$  taken as 1.2 for epoxy-coated reinforcement, and  $\lambda$  taken as 0.75 for lightweight concrete. For other cases,  $\psi_e$  and  $\lambda$  shall be taken as 1.0.

**12.5.3** — Length  $\ell_{dh}$  in 12.5.2 shall be permitted to be multiplied by the following applicable factors:

- (a) For No. 11 bar and smaller hooks with side cover (normal to plane of hook) not less than 2-1/2 in., and

**R12.4 — Development of bundled bars**

**R12.4.1** — An increased development length for individual bars is required when three or four bars are bundled together. The extra extension is needed because the grouping makes it more difficult to mobilize bond resistance from the core between the bars.

It is important to also note 7.6.6.4 relating to the cutoff points of individual bars within a bundle and 12.14.2.2 relating to splices of bundled bars. The increases in development length of 12.4 do apply when computing splice lengths of bundled bars in accordance with 12.14.2.2. The development of bundled bars by a standard hook of the bundle is not covered by the provisions of 12.5.

**R12.4.2** — Although splice and development lengths of bundled bars are a multiple of the diameter of the individual bars being spliced increased by 20 or 33 percent, as appropriate, it is necessary to use an equivalent diameter of the entire bundle derived from the equivalent total area of bars when determining the spacing and cover values in 12.2.2, the confinement term,  $[(c_b + K_{tr})/d_b]$ , in 12.2.3, and the  $\psi_e$  factor in 12.2.4(b). For bundled bars, bar diameter,  $d_b$ , outside the brackets in the expressions of 12.2.2 and of Eq. (12-1) is that of a single bar.

**R12.5 — Development of standard hooks in tension**

The provisions for hooked bar anchorage were extensively revised in the 1983 Code. Study of failures of hooked bars indicate that splitting of the concrete cover in the plane of the hook is the primary cause of failure and that splitting originates at the inside of the hook where the local stress concentrations are very high. Thus, hook development is a direct function of bar diameter  $d_b$ , which governs the magnitude of compressive stresses on the inside of the hook. Only standard hooks (see 7.1) are considered and the influence of larger radius of bend cannot be evaluated by 12.5.

The hooked bar anchorage provisions give the total hooked bar embedment length as shown in Fig. R12.5. The development length  $\ell_{dh}$  is measured from the critical section to the outside end (or edge) of the hook.

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for 90-degree hook with cover on bar extension beyond hook not less than 2 in. .... **0.7**

(b) For 90-degree hooks of No. 11 and smaller bars that are either enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than  $3d_b$  along  $\ell_{dh}$ ; or enclosed within ties or stirrups parallel to the bar being developed, spaced not greater than  $3d_b$  along the length of the tail extension of the hook plus bend..... **0.8**

(c) For 180-degree hooks of No. 11 and smaller bars that are enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than  $3d_b$  along  $\ell_{dh}$ ..... **0.8**

(d) Where anchorage or development for  $f_y$  is not specifically required, reinforcement in excess of that required by analysis.....  **$(A_s \text{ required})/(A_s \text{ provided})$**

In 12.5.3(b) and 12.5.3(c),  $d_b$  is the diameter of the hooked bar, and the first tie or stirrup shall enclose the bent portion of the hook, within  $2d_b$  of the outside of the bend.

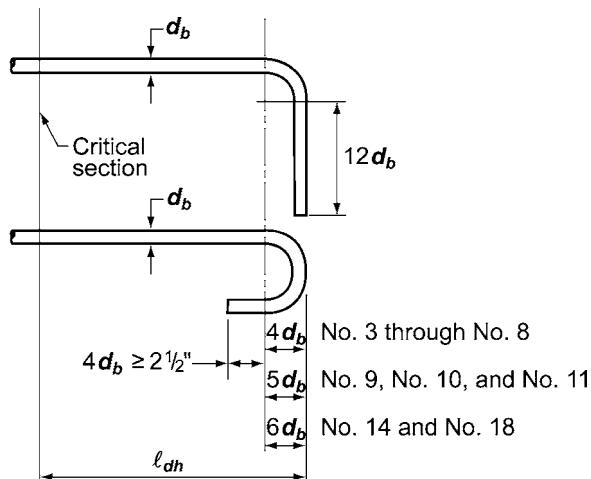
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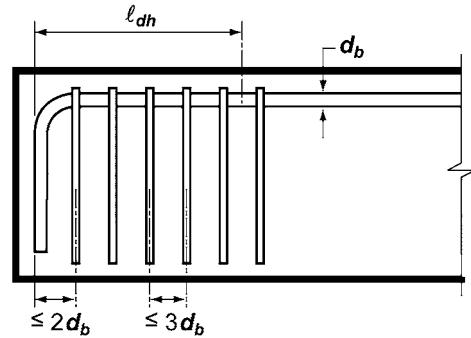
Fig. R12.5—Hooked bar details for development of standard hooks.

The development length for standard hooks  $\ell_{dh}$  of 12.5.2 can be reduced by all applicable modification factors of 12.5.3. As an example, if the conditions of both 12.5.3(a) and (c) are met, both factors may be applied.

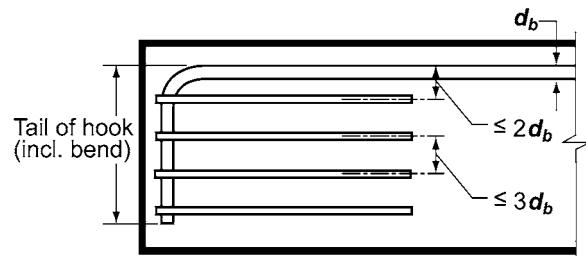
The effects of bar yield strength, excess reinforcement, lightweight concrete, and factors to reflect the resistance to splitting provided from confinement by concrete and transverse ties or stirrups are based on recommendations from References 12.2 and 12.3.

Tests<sup>12,13</sup> indicate that closely spaced ties at or near the bend portion of a hooked bar are most effective in confining the hooked bar. For construction purposes, this is not always practicable. The cases where the modification factor of 12.5.3(b) may be used are illustrated in Fig. R12.5.3(a) and (b). Figure R12.5.3(a) shows placement of ties or stirrups perpendicular to the bar being developed, spaced along the development length,  $\ell_{dh}$ , of the hook. Figure R12.5.3(b) shows placement of ties or stirrups parallel to the bar being developed along the length of the tail extension of the hook plus bend. The latter configuration would be typical in a beam column joint.

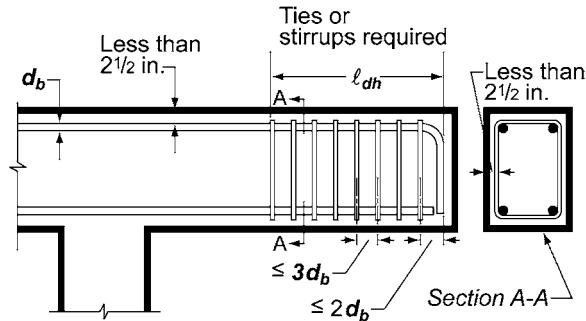
The factor for excess reinforcement in 12.5.3(d) applies only where anchorage or development for full  $f_y$  is not specifically required. The  $\lambda$  factor for lightweight concrete is a simplification over the procedure in 12.2.3.3 of ACI 318-83 in which the increase varies from 18 to 33 percent, depending on the amount of lightweight aggregate used. Unlike straight bar development, no distinction is made between top bars and other bars; such a distinction is difficult for hooked bars in any case. A minimum value of  $\ell_{dh}$  is specified to prevent failure by direct pullout in cases where a hook may be located very near the critical section. Hooks cannot be considered effective in compression.

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*Fig. R12.5.3(a)—Ties or stirrups placed perpendicular to the bar being developed, spaced along the development length  $\ell_{dh}$ .*



*Fig. R12.5.3(b)—Ties or stirrups placed parallel to the bar being developed, spaced along the length of the tail extension of the hook plus bend.*



*Fig. R12.5.4—Concrete cover per 12.5.4.*

Tests<sup>12,14</sup> indicate that the development length for hooked bars should be increased by 20 percent to account for reduced bond when reinforcement is epoxy coated.

**12.5.4** — For bars being developed by a standard hook at discontinuous ends of members with both side cover and top (or bottom) cover over hook less than 2-1/2 in., the hooked bar shall be enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than  $3d_b$  along  $\ell_{dh}$ . The first tie or stirrup shall enclose the bent portion of the hook, within  $2d_b$  of the outside of the bend, where  $d_b$  is the diameter of the hooked bar. For this case, the factors of 12.5.3(b) and (c) shall not apply.

**R12.5.4** — Bar hooks are especially susceptible to a concrete splitting failure if both side cover (normal to plane of hook) and top or bottom cover (in plane of hook) are small. See Fig. R12.5.4. With minimum confinement provided by concrete, additional confinement provided by ties or stirrups is essential, especially if full bar strength should be developed by a hooked bar with such small cover. Cases where hooks may require ties or stirrups for confinement are at ends of simply supported beams, at free end of cantilevers, and at ends of members framing into a joint

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**12.5.5** — Hooks shall not be considered effective in developing bars in compression.

## **12.6 — Development of headed and mechanically anchored deformed bars in tension**

**12.6.1** — Development length for headed deformed bars in tension,  $\ell_{dt}$ , shall be determined from 12.6.2. Use of heads to develop deformed bars in tension shall be limited to conditions satisfying (a) through (f):

- (a) Bar  $f_y$  shall not exceed 60,000 psi;
- (b) Bar size shall not exceed No. 11;
- (c) Concrete shall be normalweight;
- (d) Net bearing area of head  $A_{brg}$  shall not be less than  $4A_b$ ;
- (e) Clear cover for bar shall not be less than  $2d_b$ ; and
- (f) Clear spacing between bars shall not be less than  $4d_b$ .

**12.6.2** — For headed deformed bars satisfying 3.5.9, development length in tension  $\ell_{dt}$  shall be  $(0.016\psi_e f_y / \sqrt{f'_c})d_b$ , where the value of  $f'_c$  used to calculate  $\ell_{dt}$  shall not exceed 6000 psi, and factor  $\psi_e$  shall be taken as 1.2 for epoxy-coated reinforcement and 1.0 for other cases. Length  $\ell_{dt}$  shall not be less than the larger of  $8d_b$  and 6 in.

where members do not extend beyond the joint. In contrast, if calculated bar stress is so low that the hook is not needed for bar anchorage, the ties or stirrups are not necessary. Also, provisions of 12.5.4 do not apply for hooked bars at discontinuous ends of slabs with confinement provided by the slab continuous on both sides normal to the plane of the hook.

**R12.5.5** — In compression, hooks are ineffective and may not be used as anchorage.

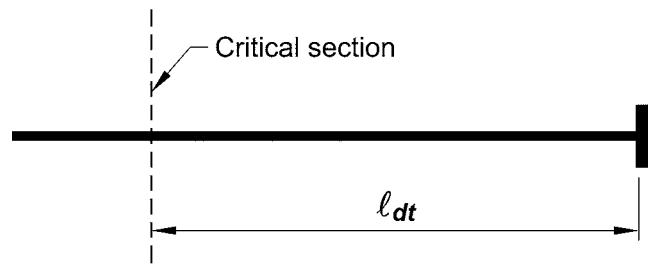
## **R12.6 — Development of headed and mechanically anchored deformed bars in tension**

The development of headed deformed bars and the development and anchorage of deformed bars through the use of mechanical devices within concrete are addressed in 12.6. As used in 12.6, *development* describes cases in which the force in the bar is transferred to the concrete through a combination of a bearing force at the head and bond forces along the bar. Such cases are covered in 12.6.1 and 12.6.2. In contrast, *anchorage* describes cases in which the force in the bar is transferred through bearing to the concrete at the head alone. Design requirements for anchors are given in [Appendix D](#). Headed bars are limited to those types that meet the requirements of HA heads in ASTM A970 because a wide variety of methods are used to attach heads to bars, some of which involve significant obstructions or interruptions of the bar deformations. Headed bars with significant obstructions or interruptions of the bar deformations were not evaluated in the tests used to formulate the provisions in 12.6.2. The headed bars evaluated in the tests were limited to those types that meet the criteria in 3.5.9 for HA heads.

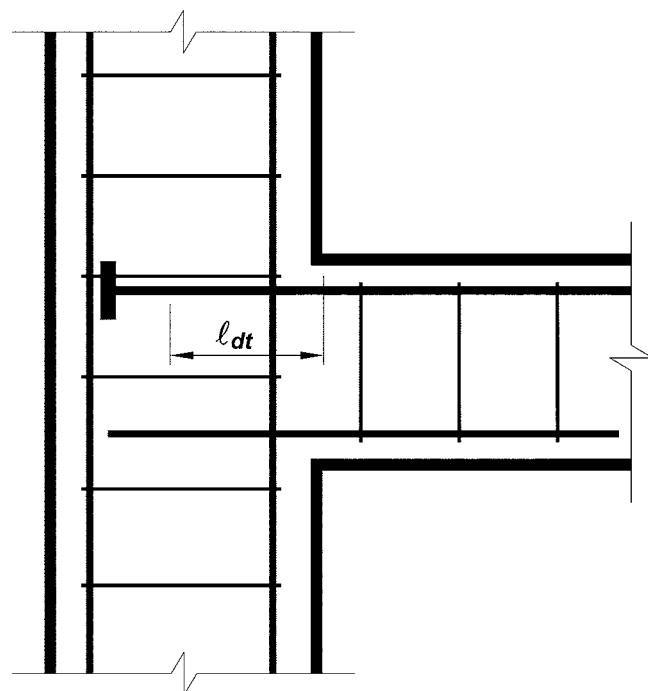
The provisions for headed deformed bars were written with due consideration of the provisions for anchorage in [Appendix D](#) and the bearing strength provisions of 10.14.<sup>12.15,12.16</sup> [Appendix D](#) contains provisions for headed anchors related to the individual failure modes of concrete breakout, side-face blowout, and pullout, all of which were considered in the formulation of 12.6.2. The restrictions on normalweight concrete, maximum bar size of No. 11, and upper limit of 60,000 psi for  $f_y$  are based on the available data from tests.<sup>12.15-12.17</sup>

The provisions for developing headed deformed bars give the length of bar  $\ell_{dt}$  measured from the critical section to the bearing face of the head, as shown in [Fig. R12.6\(a\)](#).

For bars in tension, heads allow the bars to be developed in a shorter length than required for standard hooks.<sup>12.15-12.17</sup> The minimum limits on clear cover, clear spacing, and head size are based on the lower limits of these parameters used in the tests to establish the expression for  $\ell_{dt}$  in 12.6.2. The clear cover and clear spacing requirements in 12.6.1 are

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*Fig. R12.6(a)—Development of headed deformed bars.*



*Fig. R12.6(b)—Headed deformed bar extended to far side of column core with anchorage length that exceeds  $\ell_{dt}$ .*

based on dimensions measured to the bar, not to the head. The head is considered to be part of the bar for the purposes of satisfying the specified cover requirements in 7.7, and aggregate size requirements of 3.3.2(c). To avoid congestion, it may be desirable to stagger the heads. Headed bars with  $A_{bkg} < 4A_b$  have been used in practice, but their performance is not accurately represented by the provisions in 12.6.2, and they should be used only with designs that are supported by test results under 12.6.4. These provisions do not address the design of studs or headed stud assemblies used for shear reinforcement.

A 1.2 factor is conservatively used for epoxy-coated headed deformed reinforcing bars, the same value used for epoxy-coated standard hooks. The upper limit on the value of  $f'_c$  in 12.6.2 for use in calculating  $\ell_{dt}$  is based on the concrete strengths used in the tests.<sup>12.15-12.17</sup> Because transverse reinforcement has been shown to be largely ineffective in improving the anchorage of headed deformed bars,<sup>12.15-12.17</sup>

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additional reductions in development length, such as those allowed for standard hooks with additional confinement provided by transverse reinforcement in 12.5.3, are not used for headed deformed reinforcing bars. Transverse reinforcement, however, helps limit splitting cracks in the vicinity of the head and for that reason is recommended.

In 2011, the excess reinforcement factor for headed bars was removed from the Code. The excess reinforcement factor ( $A_s \text{ required} / A_s \text{ provided}$ ), applicable to deformed bars without heads, is not applicable for headed bars where force is transferred through a combination of bearing at the head and bond along the bar. Concrete breakout due to bearing at the head was considered in developing the provisions of 12.6. Because the concrete breakout capacity of a headed bar is a function of the embedment depth to the 1.5 power (see Appendix D Eq. (D-6)), a reduction in development length with the application of the excess reinforcement factor could result in a potential concrete breakout failure.

Where longitudinal headed deformed bars from a beam or a slab terminate at a supporting member, such as the column shown in Fig. R12.6(b), the bars should extend through the joint to the far face of the confined core of the supporting member, allowing for cover and avoidance of interference with column reinforcement, even though the resulting anchorage length exceeds  $\ell_{dt}$ . Extending the bar to the far side of the column core helps to anchor compressive forces (as identified in a strut-and-tie model) that are likely to form in such a connection and improves the performance of the joint.

**12.6.3** — Heads shall not be considered effective in developing bars in compression.

**12.6.4** — Any mechanical attachment or device capable of developing  $f_y$  of deformed bars is allowed, provided that test results showing the adequacy of such attachment or device are approved by the building official. Development of deformed bars shall be permitted to consist of a combination of mechanical anchorage plus additional embedment length of deformed bars between the critical section and the mechanical attachment or device.

## **12.7 — Development of welded deformed wire reinforcement in tension**

**12.7.1** — Development length for welded deformed wire reinforcement in tension,  $\ell_d$ , measured from the point of critical section to the end of wire shall be computed as the product of  $\ell_d$ , from 12.2.2 or 12.2.3, times welded deformed wire reinforcement factor,  $\psi_w$ , from 12.7.2 or 12.7.3. It shall be permitted to reduce  $\ell_d$  in accordance

**R12.6.3** — No data are available that demonstrate that the use of heads adds significantly to anchorage strength in compression.

**R12.6.4** — Headed deformed bars that do not meet the requirements in 3.5.9, or are not anchored in accordance with 12.6.1 and 12.6.2, may be used if tests demonstrate the ability of the head and bar system to develop or anchor the desired force in the bar, as described in 12.6.4.

## **R12.7 — Development of welded deformed wire reinforcement in tension**

Figure R12.7 shows the development requirements for welded deformed wire reinforcement with one cross wire within the development length. ASTM A1064 for welded deformed wire reinforcement requires the same strength of the weld as required for welded plain wire reinforcement. Some of the development is assigned to welds and some assigned to the

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with 12.2.5 when applicable, but  $\ell_d$  shall not be less than 8 in. except in computation of lap splices by 12.18. When using  $\psi_w$  from 12.7.2, it shall be permitted to use an epoxy-coating factor  $\psi_e$  of 1.0 for epoxy-coated welded deformed wire reinforcement in 12.2.2 and 12.2.3.

**12.7.2** — For welded deformed wire reinforcement with at least one cross wire within  $\ell_d$  and not less than 2 in. from the point of the critical section,  $\psi_w$  shall be the greater of

$$\left( \frac{f_y - 35,000}{f_y} \right)$$

and

$$\left( \frac{5d_b}{s} \right)$$

but not greater than 1.0, where  $s$  is the spacing between the wires to be developed.

**12.7.3** — For welded deformed wire reinforcement with no cross wires within  $\ell_d$  or with a single cross wire less than 2 in. from the point of the critical section,  $\psi_w$  shall be taken as 1.0, and  $\ell_d$  shall be determined as for deformed wire.

**12.7.4** — Where any plain wires, or deformed wires larger than D-31, are present in the welded deformed wire reinforcement in the direction of the development length, the reinforcement shall be developed in accordance with 12.8.

length of deformed wire. The development computations are simplified from earlier Code provisions for wire development by assuming that only one cross wire is contained in the development length. The welded deformed wire reinforcement factor,  $\psi_w$ , in 12.7.2 is applied to the deformed wire development length computed from 12.2. The factor  $\psi_w$  was derived using the general relationships between welded deformed wire reinforcement and deformed wires in the  $\ell_{db}$  values of the 1983 Code.

Tests<sup>12.18</sup> have indicated that epoxy-coated welded wire reinforcement has essentially the same development and splice strengths as uncoated welded wire reinforcement because the cross wires provide the primary anchorage for the wire. Therefore, an epoxy-coating factor of 1.0 is used for development and splice lengths of epoxy-coated welded wire reinforcement with cross wires within the splice or development length.

Deformed wire larger than D-31 is treated as plain wire because tests show that D-45 wire will achieve only approximately 60 percent of the bond strength in tension given by Eq. (12-1).<sup>12.19</sup>

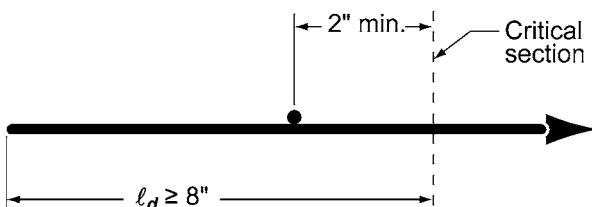


Fig. R12.7—Development of welded deformed wire reinforcement.

**CODE****COMMENTARY****12.8 — Development of welded plain wire reinforcement in tension**

Yield strength of welded plain wire reinforcement shall be considered developed by embedment of two cross wires with the closer cross wire not less than 2 in. from the point of the critical section. However,  $\ell_d$  shall not be less than

$$\ell_d = 0.27 \frac{A_b f_y}{s \lambda \sqrt{f'_c}} \quad (12-3)$$

where  $\ell_d$  is measured from the point of the critical section to the outermost crosswire,  $s$  is the spacing between the wires to be developed, and  $\lambda$  as given in 12.2.4(d). Where reinforcement provided is in excess of that required,  $\ell_d$  may be reduced in accordance with 12.2.5. Length,  $\ell_d$ , shall not be less than 6 in. except in computation of lap splices by 12.19.

**12.9 — Development of prestressing strand**

**12.9.1** — Except as provided in 12.9.1.1, seven-wire strand shall be bonded beyond the critical section, a distance not less than

$$\ell_d = \left( \frac{f_{se}}{3000} \right) d_b + \left( \frac{f_{ps} - f_{se}}{1000} \right) d_b \quad (12-4)$$

The expressions in parentheses are used as constants without units.

**12.9.1.1** — Embedment less than  $\ell_d$  shall be permitted at a section of a member provided the design strand stress at that section does not exceed values obtained from the bilinear relationship defined by Eq. (12-4).

**R12.8 — Development of welded plain wire reinforcement in tension**

Figure R12.8 shows the development requirements for welded plain wire reinforcement with development primarily dependent on the location of cross wires. For welded plain wire reinforcement made with the smaller wires, an embedment of at least two cross wires 2 in. or more beyond the point of critical section is adequate to develop the full yield strength of the anchored wires. However, for welded plain wire reinforcement made with larger closely spaced wires, a longer embedment is required and a minimum development length is provided for this reinforcement.

**R12.9 — Development of prestressing strand**

The development requirements for prestressing strand are intended to provide bond integrity for the strength of the member. The provisions are based on tests performed on normalweight concrete members with a minimum cover of 2 in. These tests may not represent the behavior of strand in low water-cementitious material ratio, no-slump, concrete. Fabrication methods should ensure consolidation of concrete around the strand with complete contact between the steel and concrete. Extra precautions should be exercised when low water-cementitious material ratio, no-slump concrete is used.

The first term in Eq. (12-4) represents the transfer length of the strand, that is, the distance over which the strand should be bonded to the concrete to develop the effective prestress in the prestressing steel  $f_{se}$ . The second term represents the additional length over which the strand should be bonded so that a stress in the prestressing steel at nominal strength of the member,  $f_{ps}$ , may develop.

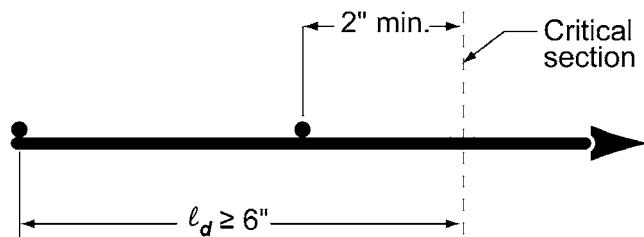
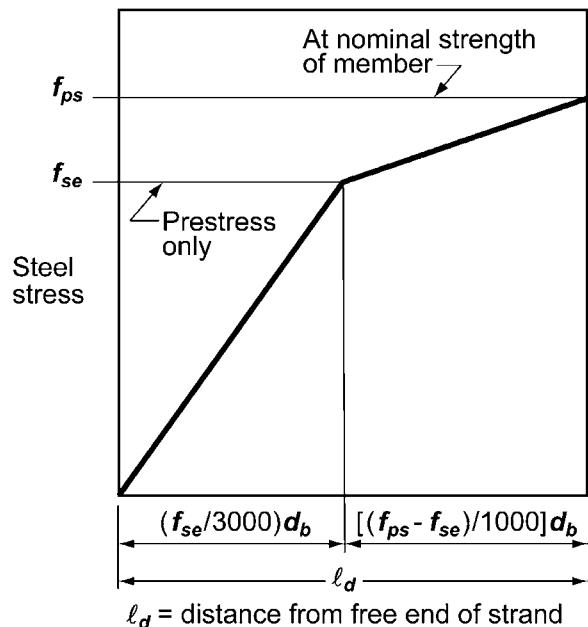


Fig. R12.8—Development of welded plain wire reinforcement.

**CODE****COMMENTARY**

*Fig. R12.9—Idealized bilinear relationship between steel stress and distance from the free end of strand.*

The bond of strand is a function of a number of factors, including the configuration and surface condition of the steel, the stress in the steel, the depth of concrete beneath the strand, and the method used to transfer the force in the strand to the concrete. For bonded applications, quality assurance procedures should be used to confirm that the strand is capable of adequate bond.<sup>12.20,12.21</sup> The precast concrete manufacturer may rely on certification from the strand manufacturer that the strand has bond characteristics that comply with this section. Strand with a slightly rusted surface can have an appreciably shorter transfer length than clean strand. Gentle release of the strand will permit a shorter transfer length than abruptly cutting the strands.

The provisions of 12.9 do not apply to plain wires or to end-anchored tendons. The length for smooth wire could be expected to be considerably greater due to the absence of mechanical interlock. Flexural bond failure would occur with plain wire when first slip occurred.

**R12.9.1.1** — Figure R12.9 shows the relationship between steel stress and the distance over which the strand is bonded to the concrete represented by Eq. (12-4). This idealized variation of strand stress may be used for analyzing sections within the development region.<sup>12.22,12.23</sup> The expressions for transfer length, and for the additional bonded length necessary to develop an increase in stress of  $(f_{ps} - f_{se})$ , are based on tests of members prestressed with clean, 1/4, 3/8, and 1/2 in. diameter strands for which the maximum value of  $f_{ps}$  was 275 kips/in.<sup>2</sup> See References 12.24, 12.25, and 12.26.

**CODE****COMMENTARY**

**12.9.2** — Limiting the investigation to cross sections nearest each end of the member that are required to develop full design strength under specified factored loads shall be permitted except where bonding of one or more strands does not extend to the end of the member, or where concentrated loads are applied within the strand development length.

**12.9.3** — Where bonding of a strand does not extend to end of member, and design includes tension at service load in precompressed tensile zone as permitted by **18.3.3**,  $l_d$  specified in **12.9.1** shall be doubled.

## **12.10 — Development of flexural reinforcement — General**

**12.10.1** — Development of tension reinforcement by bending across the web to be anchored or made continuous with reinforcement on the opposite face of member shall be permitted.

**12.10.2** — Critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent. Provisions of **12.11.3** must be satisfied.

**12.10.3** — Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to  $d$  or  $12d_b$ , whichever is greater, except at supports of simple spans and at free end of cantilevers.

**R12.9.2** — Where bonding of one or more strands does not extend to the end of the member, critical sections may be at locations other than where full design strength is required to be developed, and detailed analysis may be required. **References 12.22** and **12.23** show a method that may be used in the case of strands with different points of full development. Conservatively, only the strands that are fully developed at a section may be considered effective at that section. If critical sections occur in the transfer region, additional considerations may be necessary. Some loading conditions, such as where heavy concentrated loads occur within the strand development length, may cause critical sections to occur away from the section that is required to develop full design strength.

**R12.9.3** — Exploratory tests conducted in 1965<sup>12.24</sup> that study the effect of debonded strand (bond not permitted to extend to the ends of members) on performance of pretensioned girders indicated that the performance of these girders with embedment lengths twice those required by **12.9.1** closely matched the flexural performance of similar pretensioned girders with strand fully bonded to ends of girders. Accordingly, doubled development length is required for strand not bonded through to the end of a member. Subsequent tests<sup>12.27</sup> indicated that in pretensioned members designed for zero tension in the concrete under service load conditions (see **18.3.3**), the development length for debonded strands need not be doubled. For analysis of sections with debonded strands at locations where strand is not fully developed, it is usually assumed that both the transfer length and development length are doubled.

## **R12.10 — Development of flexural reinforcement — General**

**R12.10.2** — Critical sections for a typical continuous beam are indicated with a “c” or an “x” in **Fig. R12.10.2**. For uniform loading, the positive reinforcement extending into the support is more apt to be governed by the requirements of **12.11.3** rather than by development length measured from a point of maximum moment or bar cutoff.

**R12.10.3** — The moment diagrams customarily used in design are approximate; some shifting of the location of maximum moments may occur due to changes in loading, settlement of supports, lateral loads, or other causes. A diagonal tension crack in a flexural member without stirrups may shift the location of the calculated tensile stress approximately a distance  $d$  toward a point of zero moment. When stirrups are provided, this effect is less severe, although still present to some extent.

**CODE****COMMENTARY**

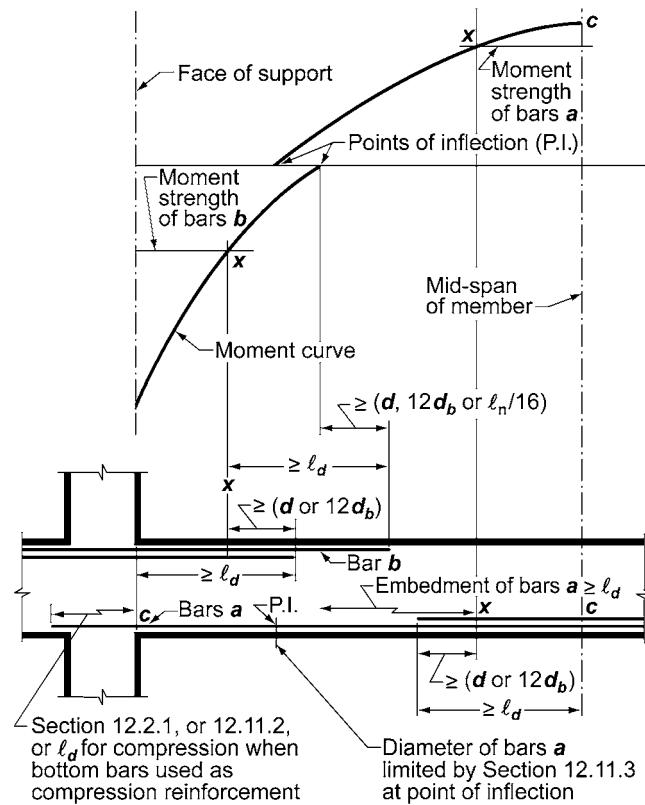
To provide for shifts in the location of maximum moments, the Code requires the extension of reinforcement a distance  $d$  or  $12d_b$  beyond the point at which it is theoretically no longer required to resist flexure, except as noted.

Cutoff points of bars to meet this requirement are illustrated in Fig. R12.10.2.

When bars of different sizes are used, the extension should be in accordance with the diameter of bar being terminated. A bar bent to the far face of a beam and continued there may logically be considered effective, in satisfying this section, to the point where the bar crosses the mid-depth of the member.

**12.10.4** — Continuing reinforcement shall have an embedment length not less than  $\ell_d$  beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

**R12.10.4** — Peak stresses exist in the remaining bars wherever adjacent bars are cut off, or bent, in tension regions. In Fig. R12.10.2, an “x” is used to indicate the peak stress points remaining in continuing bars after part of the bars have been cut off. If bars are cut off as short as the moment diagrams allow, these peak stresses become the full  $f_y$ , which requires a full  $\ell_d$  extension as indicated. This extension may exceed the length required for flexure.



*Fig. R12.10.2—Development of flexural reinforcement in a typical continuous beam.*

**CODE****COMMENTARY**

**12.10.5** — Flexural reinforcement shall not be terminated in a tension zone unless 12.10.5.1, 12.10.5.2, or 12.10.5.3 is satisfied.

**12.10.5.1** —  $V_u$  at the cutoff point does not exceed  $(2/3)\phi V_n$ .

**12.10.5.2** — Stirrup area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance  $(3/4)d$  from the termination point. Excess stirrup area shall be not less than  $60b_w s/f_{yt}$ . Spacing  $s$  shall not exceed  $d/(8\beta_b)$ .

**12.10.5.3** — For No. 11 bars and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and  $V_u$  does not exceed  $(3/4)\phi V_n$ .

**12.10.6** — Adequate anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as: sloped, stepped, or tapered footings; brackets; deep flexural members; or members in which tension reinforcement is not parallel to compression face. See 12.11.4 and 12.12.4 for deep flexural members.

**R12.10.5** — Reduced shear strength and loss of ductility when bars are cut off in a tension zone, as in Fig. R12.10.2, have been reported. The Code does not permit flexural reinforcement to be terminated in a tension zone unless additional conditions are satisfied. Flexure cracks tend to open early wherever any reinforcement is terminated in a tension zone. If the steel stress in the continuing reinforcement and the shear strength are each near their limiting values, diagonal tension cracking tends to develop prematurely from these flexure cracks. Diagonal cracks are less likely to form where shear stress is low (see 12.10.5.1). Diagonal cracks can be restrained by closely spaced stirrups (see 12.10.5.2). A lower steel stress reduces the probability of such diagonal cracking (see 12.10.5.3). These requirements are not intended to apply to tension splices that are covered by 12.2, 12.13.5, and the related 12.15.

**R12.10.6** — Brackets, members of variable depth, and other members where  $f_s$ , calculated stress in reinforcement at service loads, does not decrease linearly in proportion to a decreasing moment, require additional consideration for proper development of the flexural reinforcement. For the bracket shown in Fig. R12.10.6, the stress at ultimate in the reinforcement is almost constant at approximately  $f_y$  from the face of support to the load point. In such a case, development of the flexural reinforcement depends largely on the end anchorage provided at the loaded end. Reference 12.1 suggests a welded cross bar of equal diameter as a means of providing effective end anchorage. An end hook in the vertical plane, with the minimum diameter bend, is not totally effective because an essentially plain concrete corner will exist near loads applied close to the corner. For wide brackets (perpendicular to the plane of the figure) and loads not applied close to the corners, U-shaped bars in a horizontal plane provide effective end hooks.

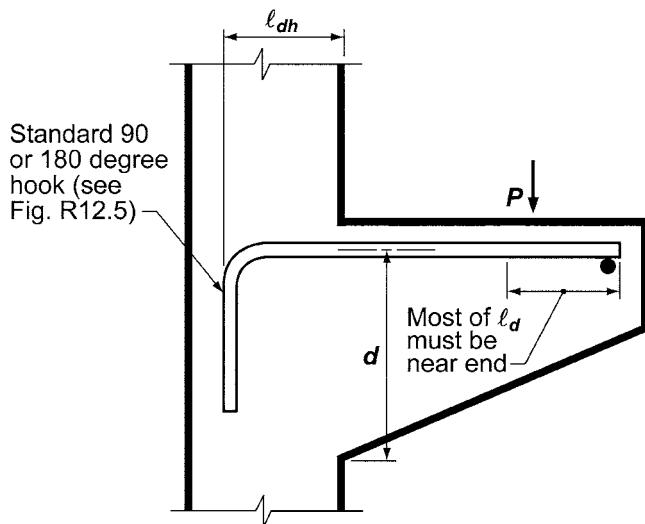


Fig. R12.10.6—Member largely dependent on end anchorage.

**CODE****COMMENTARY****12.11 — Development of positive moment reinforcement**

**12.11.1** — At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of member into the support. In beams, such reinforcement shall extend into the support at least 6 in.

**12.11.2** — When a flexural member is part of a primary seismic-load-resisting system, positive moment reinforcement required to be extended into the support by 12.11.1 shall be anchored to develop  $f_y$  in tension at the face of support.

**12.11.3** — At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that  $\ell_d$  computed for  $f_y$  by 12.2 satisfies Eq. (12-5); except, Eq. (12-5) need not be satisfied for reinforcement terminating beyond center-line of simple supports by a standard hook, or a mechanical anchorage at least equivalent to a standard hook.

$$\ell_d \leq \frac{M_n}{V_u} + \ell_a \quad (12-5)$$

where:

$M_n$  is calculated assuming all reinforcement at the section to be stressed to  $f_y$ ;

$V_u$  is calculated at the section;

$\ell_a$  at a support shall be the embedment length beyond center of support; or

$\ell_a$  at a point of inflection shall be limited to  $d$  or  $12d_b$ , whichever is greater.

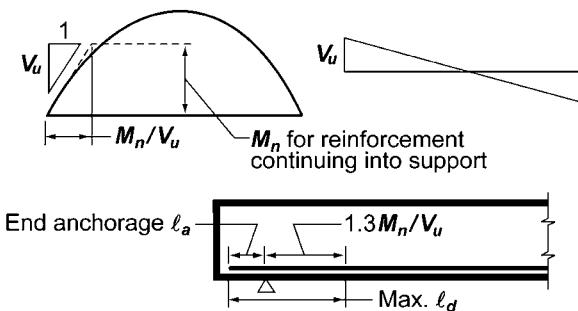
An increase of 30 percent in the value of  $M_n/V_u$  shall be permitted when the ends of reinforcement are confined by a compressive reaction.

**R12.11 — Development of positive moment reinforcement**

**R12.11.1** — Positive moment reinforcement is carried into the support to provide for some shifting of the moments due to changes in loading, settlement of supports, and lateral loads.

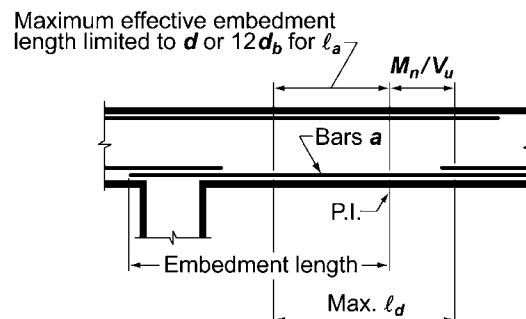
**R12.11.2** — When a flexural member is part of a primary seismic-load-resisting system, loads greater than those anticipated in design may cause reversal of moment at supports; some positive reinforcement should be well anchored into the support. This anchorage is required to ensure ductility of response in the event of serious over-stress, such as from blast or earthquake. It is not sufficient to use more reinforcement at lower stresses.

**R12.11.3** — At simple supports and points of inflection such as "P.I." in Fig. R12.10.2, the diameter of the positive reinforcement should be small enough so that computed development length of the bar  $\ell_d$  does not exceed  $M_n/V_u + \ell_a$ , or under favorable support conditions,  $1.3M_n/V_u + \ell_a$ . Figure R12.11.3(a) illustrates the use of the provision.



Note: The 1.3 factor is usable only if the reaction confines the ends of the reinforcement.

(a) Maximum size of bar at simple support



(b) Maximum size of bar "a" at point of inflection

Fig. R12.11.3—Concept for determining maximum bar size per 12.11.3.

**CODE****COMMENTARY**

At the point of inflection, the value of  $\ell_a$  should not exceed the actual bar extension used beyond the point of zero moment. The  $M_n/V_u$  portion of the available length is a theoretical quantity not generally associated with an obvious maximum stress point.  $M_n$  is the nominal flexural strength of the cross section without the  $\phi$ -factor and is not the applied factored moment.

The length  $M_n/V_u$  corresponds to the development length for the maximum size bar obtained from the previously used flexural bond equation  $\Sigma_o = V/ujd$ , where  $u$  is bond stress, and  $jd$  is the moment arm. In the 1971 Code, this anchorage requirement was relaxed from previous Codes by crediting the available end anchorage length  $\ell_a$  and by including a 30 percent increase for  $M_n/V_u$  when the ends of the reinforcement are confined by a compressive reaction.

For example, a bar size is provided at a simple support such that  $\ell_d$  is computed in accordance with 12.2. The bar size provided is satisfactory only if computed  $\ell_d$  does not exceed  $1.3M_n/V_u + \ell_a$ .

The  $\ell_a$  to be used at points of inflection is limited to the effective depth of the member  $d$  or 12 bar diameters ( $12d_b$ ), whichever is greater. Fig. R12.11.3(b) illustrates this provision at points of inflection. The  $\ell_a$  limitation is added since test data are not available to show that a long end anchorage length will be fully effective in developing a bar that has only a short length between a point of inflection and a point of maximum stress.

**12.11.4** — At simple supports of deep beams, positive moment tension reinforcement shall be anchored to develop  $f_y$  in tension at the face of the support except that if design is carried out using Appendix A, the positive moment tension reinforcement shall be anchored in accordance with A.4.3. At interior supports of deep beams, positive moment tension reinforcement shall be continuous or be spliced with that of the adjacent spans.

## 12.12 — Development of negative moment reinforcement

**12.12.1** — Negative moment reinforcement in a continuous, restrained, or cantilever member, or in any member of a rigid frame, shall be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage.

**12.12.2** — Negative moment reinforcement shall have an embedment length into the span as required by 12.1 and 12.10.3.

**R12.11.4** — The use of the strut-and-tie model for the design of reinforced concrete deep flexural members clarifies that there is significant tension in the reinforcement at the face of the support. This requires the tension reinforcement to be continuous or be developed through and beyond the support.<sup>12.28</sup>

## R12.12 — Development of negative moment reinforcement

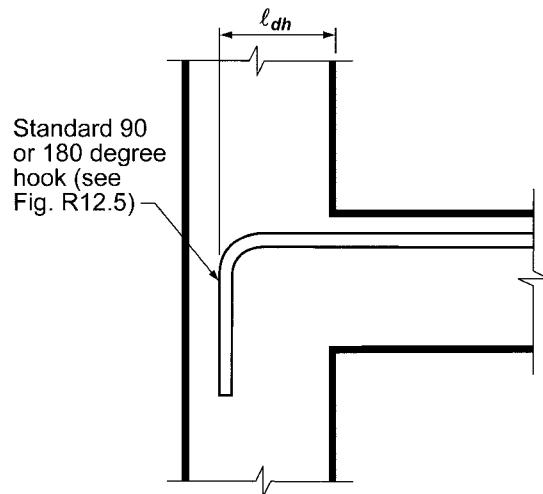
Figure R12.12 illustrates two methods of satisfying requirements for anchorage of tension reinforcement beyond the face of support. For anchorage of reinforcement with hooks, see R12.5.

Section 12.12.3 provides for possible shifting of the moment diagram at a point of inflection, as discussed under R12.10.3. This requirement may exceed that of 12.10.3, and the more restrictive of the two provisions governs.

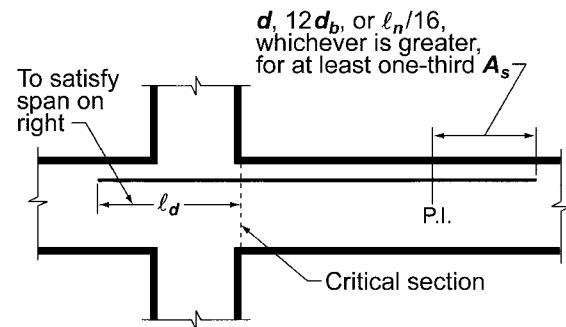
**CODE****COMMENTARY**

**12.12.3** — At least one-third the total tension reinforcement provided for negative moment at a support shall have an embedment length beyond the point of inflection not less than  $d$ ,  $12d_b$ , or  $\ell_n/16$ , whichever is greater.

**12.12.4** — At interior supports of deep flexural members, negative moment tension reinforcement shall be continuous with that of the adjacent spans.



(a) Anchorage into exterior column



**Note:** Usually such anchorage becomes part of the adjacent beam reinforcement.

(b) Anchorage into adjacent beam

Fig. R12.12—Development of negative moment reinforcement.

**12.13 — Development of web reinforcement**

**12.13.1** — Web reinforcement shall be as close to the compression and tension surfaces of the member as cover requirements and proximity of other reinforcement permits.

**12.13.2** — Ends of single leg, simple U-, or multiple U-stirrups shall be anchored as required by 12.13.2.1 through 12.13.2.5.

**R12.13 — Development of web reinforcement**

**R12.13.1** — Stirrups should be carried as close to the compression face of the member as possible because near ultimate load the flexural tension cracks penetrate deeply.

**R12.13.2** — The anchorage or development requirements for stirrups composed of bars or deformed wire were changed in the 1989 Code to simplify the requirements. The straight anchorage was deleted as this stirrup is difficult to hold in place during concrete placement and the lack of a hook may make the stirrup ineffective as it crosses shear cracks near the end of the stirrup.

**CODE****COMMENTARY**

**12.13.2.1** — For No. 5 bar and D31 wire, and smaller, and for No. 6, No. 7, and No. 8 bars with  $f_{yt}$  of 40,000 psi or less, a standard hook around longitudinal reinforcement.

**12.13.2.2** — For No. 6, No. 7, and No. 8 stirrups with  $f_{yt}$  greater than 40,000 psi, a standard stirrup hook around a longitudinal bar plus an embedment between midheight of the member and the outside end of the hook equal to or greater than  $0.014d_b f_{yt}/(\lambda \sqrt{f'_c})$ .

**12.13.2.3** — For each leg of welded plain wire reinforcement forming simple U-stirrups, either:

- (a) Two longitudinal wires spaced at a 2 in. spacing along the member at the top of the U; or
- (b) One longitudinal wire located not more than  $d/4$  from the compression face and a second wire closer to the compression face and spaced not less than 2 in. from the first wire. The second wire shall be permitted to be located on the stirrup leg beyond a bend, or on a bend with an inside diameter of bend not less than  $8d_b$ .

**R12.13.2.1** — For a No. 5 bar or smaller, anchorage is provided by a standard stirrup hook, as defined in 7.1.3, hooked around a longitudinal bar. The 1989 Code eliminated the need for a calculated straight embedment length in addition to the hook for these small bars, but 12.13.1 requires a full-depth stirrup. Likewise, larger stirrups with  $f_{yt}$  equal to or less than 40,000 psi are sufficiently anchored with a standard stirrup hook around the longitudinal reinforcement.

**R12.13.2.2** — Since it is not possible to bend a No. 6, No. 7, or No. 8 stirrup tightly around a longitudinal bar and due to the force in a bar with a design stress greater than 40,000 psi, stirrup anchorage depends on both the value of the hook and whatever development length is provided. A longitudinal bar within a stirrup hook limits the width of any flexural cracks, even in a tensile zone. Since such a stirrup hook cannot fail by splitting parallel to the plane of the hooked bar, the hook strength as utilized in 12.5.2 has been adjusted to reflect cover and confinement around the stirrup hook.

For stirrups with  $f_{yt}$  of only 40,000 psi, a standard stirrup hook provides sufficient anchorage and these bars are covered in 12.13.2.1. For bars with higher strength, the embedment should be checked. A 135-degree or 180-degree hook is preferred, but a 90-degree hook may be used provided the free end of the 90-degree hook is extended the full 12 bar diameters as required in 7.1.3.

**R12.13.2.3** — The requirements for anchorage of welded plain wire reinforcement stirrups are illustrated in Fig. R12.13.2.3.

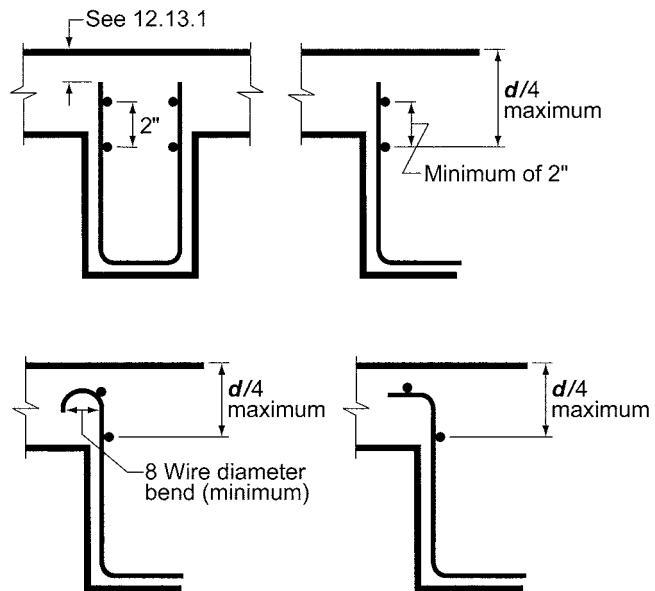
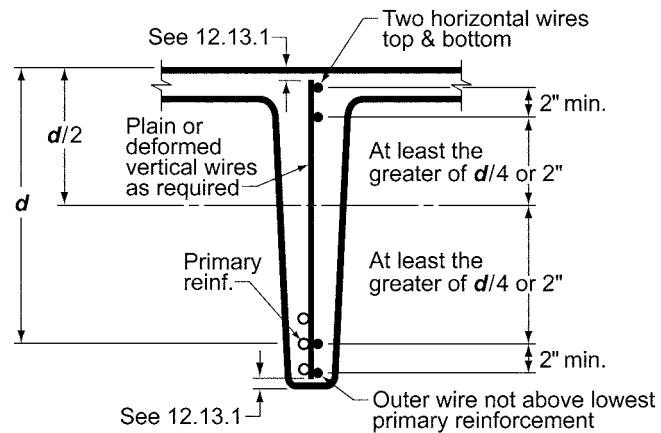


Fig. R12.13.2.3—Anchorage in compression zone of welded plain wire reinforcement U-stirrups.

**CODE****COMMENTARY**

*Fig. R12.13.2.4—Anchorage of single leg welded wire reinforcement shear reinforcement.*

**12.13.2.4** — For each end of a single leg stirrup of welded wire reinforcement, two longitudinal wires at a minimum spacing of 2 in. and with the inner wire at least the greater of  $d/4$  or 2 in. from  $d/2$ . Outer longitudinal wire at tension face shall not be farther from the face than the portion of primary flexural reinforcement closest to the face.

**R12.13.2.4** — Use of welded wire reinforcement for shear reinforcement has become commonplace in the precast, prestressed concrete industry. The rationale for acceptance of straight sheets of welded wire reinforcement as shear reinforcement is presented in a report by a joint PCI/WRI Ad Hoc Committee on Welded Wire Fabric for Shear Reinforcement.<sup>12.29</sup>

The provisions for anchorage of single leg welded wire reinforcement in the tension face emphasize the location of the longitudinal wire at the same depth as the primary flexural reinforcement to avoid a splitting problem at the tension steel level. Figure R12.13.2.4 illustrates the anchorage requirements for single leg, welded wire reinforcement. For anchorage of single leg, welded wire reinforcement, the Code has permitted hooks and embedment length in the compression and tension faces of members (see **12.13.2.1** and **12.13.2.3**), and embedment only in the compression face (see **12.13.2.2**). Section 12.13.2.4 provides for anchorage of straight, single leg, welded wire reinforcement using longitudinal wire anchorage with adequate embedment length in compression and tension faces of members.

| **12.13.2.5** — In joist construction as defined in **8.13**, for No. 4 bar and D20 wire and smaller, a standard hook.

**12.13.3** — Between anchored ends, each bend in the continuous portion of a simple U-stirrup or multiple U-stirrup shall enclose a longitudinal bar.

**12.13.4** — Longitudinal bars bent to act as shear reinforcement, if extended into a region of tension, shall be continuous with longitudinal reinforcement and, if extended into a region of compression, shall be anchored beyond mid-depth  $d/2$  as specified for development length in **12.2** for that part of  $f_y$  required to satisfy **Eq. (11-17)**.

**R12.13.2.5** — In joists, a small bar or wire can be anchored by a standard hook not engaging longitudinal reinforcement, allowing a continuously bent bar to form a series of single-leg stirrups in the joist.

**CODE****COMMENTARY**

**12.13.5** — Pairs of U-stirrups or ties so placed as to form a closed unit shall be considered properly spliced when length of laps are  $1.3\ell_d$ . In members at least 18 in. deep, such splices with  $A_{bfyt}$  not more than 9000 lb per leg shall be considered adequate if stirrup legs extend the full available depth of member.

**12.14 — Splices of reinforcement — General**

**12.14.1** — Splices of reinforcement shall be made only as required or permitted in contract documents, or as authorized by the licensed design professional.

**12.14.2 — Lap splices**

**12.14.2.1** — Lap splices shall not be used for bars larger than No. 11 except as provided in **12.16.2** and **15.8.2.3**.

**12.14.2.2** — Lap splices of bars in a bundle shall be based on the lap splice length required for individual bars within the bundle, increased in accordance with **12.4**. Individual bar splices within a bundle shall not overlap. Entire bundles shall not be lap spliced.

**12.14.2.3** — Bars spliced by noncontact lap splices in flexural members shall not be spaced transversely farther apart than the smaller of one-fifth the required lap splice length, and 6 in.

**12.14.3 — Mechanical and welded splices**

**12.14.3.1** — Mechanical and welded splices shall be permitted.

**12.14.3.2** — A full mechanical splice shall develop in tension or compression, as required, at least  $1.25f_y$  of the bar.

**12.14.3.3** — Except as provided in this Code, all welding shall conform to "Structural Welding Code—Reinforcing Steel" (AWS D1.4).

**R12.13.5** — These requirements for lapping of double U-stirrups to form closed stirrups control over the provisions of **12.15**.

**R12.14 — Splices of reinforcement — General**

Splices should, if possible, be located away from points of maximum tensile stress. The lap splice requirements of **12.15** encourage this practice.

**R12.14.2 — Lap splices**

**R12.14.2.1** — Because of lack of adequate experimental data on lap splices of No. 14 and No. 18 bars in compression and in tension, lap splicing of these bar sizes is prohibited except as permitted in **12.16.2** and **15.8.2.3** for compression lap splices of No. 14 and No. 18 bars with smaller bars.

**R12.14.2.2** — The increased length of lap required for bars in bundles is based on the reduction in the exposed perimeter of the bars. Only individual bars are lap spliced along the bundle.

**R12.14.2.3** — If individual bars in noncontact lap splices are too widely spaced, an unreinforced section is created. Forcing a potential crack to follow a zigzag line (5-to-1 slope) is considered a minimum precaution. The 6 in. maximum spacing is added because most research available on the lap splicing of deformed bars was conducted with reinforcement within this spacing.

**R12.14.3 — Mechanical and welded splices**

**R12.14.3.2** — The maximum reinforcement stress used in design under the Code is the specified yield strength. To ensure sufficient strength in splices so that yielding can be achieved in a member and thus brittle failure avoided, the 25 percent increase above the specified yield strength was selected as both an adequate minimum for safety and a practicable maximum for economy.

**R12.14.3.3** — See **R3.5.2** for discussion on welding.

**CODE**

**12.14.3.4** — A full welded splice shall develop at least  $1.25f_y$  of the bar.

**COMMENTARY**

**R12.14.3.4** — A full welded splice is primarily intended for large bars (No. 6 and larger) in main members. The tensile strength requirement of 125 percent of specified yield strength is intended to provide sound welding that is also adequate for compression. See the discussion on strength in **R12.14.3.2**. The 1995 Code eliminated a requirement that the bars be butted since indirect butt welds are permitted by AWS D1.4, although AWS D1.4 does indicate that wherever practical, direct butt splices are preferable for No. 7 bars and larger.

**12.14.3.5** — Mechanical or welded splices not meeting requirements of **12.14.3.2** or 12.14.3.4 shall be permitted only for No. 5 bars and smaller and in accordance with **12.15.5**.

**R12.14.3.5** — The use of mechanical or welded splices of less strength than 125 percent of specified yield strength is permitted if the minimum design criteria of **12.15.5** are met. Therefore, lap welds of reinforcing bars, either with or without backup material, welds to plate connections, and end-bearing splices are allowed under certain conditions. The 1995 Code limited these lower strength welds and connections to No. 5 bars and smaller due to the potentially brittle nature of failure at these welds.

## **12.15 — Splices of deformed bars and deformed wire in tension**

**12.15.1** — Minimum length of lap for tension lap splices shall be as required for Class A or B splice, but not less than 12 in., where:

Class A splice .....	<b><math>1.0\ell_d</math></b>
Class B splice .....	<b><math>1.3\ell_d</math></b>

where  $\ell_d$  is calculated in accordance with **12.2** to develop  $f_y$ , but without the 12 in. minimum of **12.2.1** and without the modification factor of **12.2.5**.

## **R12.15 — Splices of deformed bars and deformed wire in tension**

**R12.15.1** — Lap splices in tension are classified as Type A or B, with length of lap a multiple of the tensile development length  $\ell_d$  calculated in accordance with **12.2.2** or **12.2.3**. The development length  $\ell_d$  used to obtain lap length should be based on  $f_y$  because the splice classifications already reflect any excess reinforcement at the splice location; therefore, the factor from **12.2.5** for excess  $A_s$  should not be used. When multiple bars located in the same plane are spliced at the same section, the clear spacing is the minimum clear distance between the adjacent splices. For splices in columns with offset bars, **Fig. R12.15.1(a)** illustrates the clear spacing to be used. For staggered splices, the clear spacing is taken as the minimum distance between adjacent splices [**Fig. R12.15.1(b)**].

The 1989 Code contained several changes in development length in tension that eliminated many of the concerns regarding tension splices due to closely spaced bars with minimal cover. Thus, the Class C splice was eliminated although development lengths, on which splice lengths are based, have in some cases increased. Committee 318 considered suggestions from many sources, including ACI Committee 408, but has retained a two-level splice length primarily to encourage splicing bars at points of minimum stress and staggering splices to improve behavior of critical details.

**12.15.2** — Lap splices of deformed bars and deformed wire in tension shall be Class B splices except that Class A splices are allowed when:

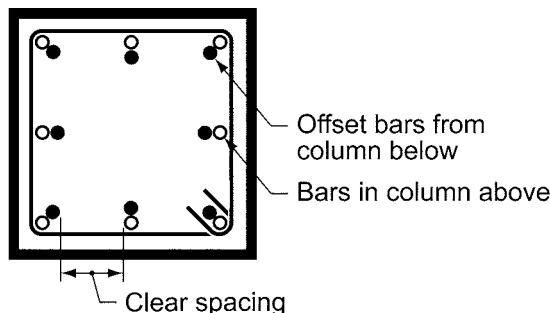
**R12.15.2** — The tension lap splice requirements of 12.15.1 encourage the location of splices away from regions of high tensile stress to locations where the area of steel provided is at least twice that required by analysis. **Table R12.15.2** presents the splice requirements in tabular form as presented in earlier Code editions.

**CODE**

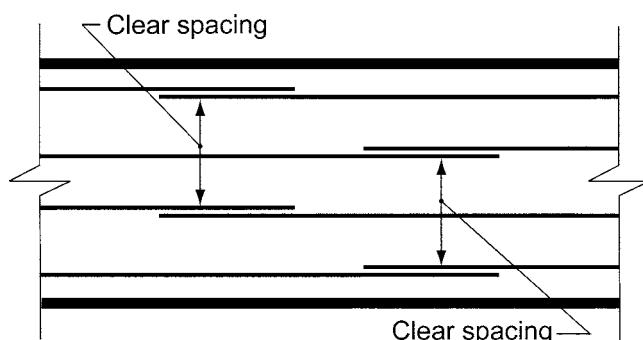
(a) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice; and

(b) one-half or less of the total reinforcement is spliced within the required lap length.

**12.15.3** — When bars of different size are lap spliced in tension, splice length shall be the larger of  $\ell_d$  of larger bar and tension lap splice length of smaller bar.

**COMMENTARY**

(a) Offset column bars



(b) Staggered splices

Fig. R12.15.1—Clear spacing of spliced bars.

**TABLE R12.15.2 — TENSION LAP SPLICES**

$\frac{A_s \text{ provided}^*}{A_s \text{ required}}$	Maximum percent of $A_s$ spliced within required lap length	
	50	100
Equal to or greater than 2	Class A	Class B
Less than 2	Class B	Class B

\*Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice locations.

**12.15.4** — Mechanical or welded splices used where area of reinforcement provided is less than twice that required by analysis shall meet requirements of **12.14.3.2** or **12.14.3.4**.

**12.15.5** — Mechanical or welded splices not meeting the requirements of **12.14.3.2** or **12.14.3.4** shall be permitted for No. 5 bars and smaller if the requirements of 12.15.5.1 through **12.15.5.3** are met:

**12.15.5.1** — Splices shall be staggered at least 24 in.

**12.15.5.2** — In computing the tensile forces that can be developed at each section, the spliced reinforcement stress shall be taken as the specified splice strength, but not greater than  $f_y$ . The stress in the unspliced reinforcement shall be taken as  $f_y$  times the

**R12.15.4** — A mechanical or welded splice should develop at least 125 percent of the specified yield strength when located in regions of high tensile stress in the reinforcement. Such splices need not be staggered, although such staggering is encouraged where the area of reinforcement provided is less than twice that required by the analysis.

**R12.15.5** — See **R12.14.3.5**. Section 12.15.5 concerns the situation where mechanical or welded splices of strength less than 125 percent of the specified yield strength of the reinforcement may be used. It provides a relaxation in the splice requirements where the splices are staggered and excess reinforcement area is available. The criterion of twice the computed tensile force is used to cover sections containing partial tensile splices with various percentages of total continuous steel. The usual partial tensile splice is a flare groove weld between bars or bar and structural steel piece.

**CODE**

ratio of the shortest length embedded beyond the section to  $\ell_d$ , but not greater than  $f_y$ .

**12.15.5.3** — The total tensile force that can be developed at each section must be at least twice that required by analysis, and at least 20,000 psi times the total area of reinforcement provided.

**12.15.6** — Splices in tension tie members shall be made with a full mechanical or full welded splice in accordance with **12.14.3.2** or **12.14.3.4** and splices in adjacent bars shall be staggered at least 30 in.

**COMMENTARY**

To detail such welding, the length of weld should be specified. Such welds are rated at the product of total weld length times effective size of groove weld (established by bar size) times allowable stress permitted by “**Structural Welding Code—Reinforcing Steel**” (AWS D1.4).

A full mechanical or welded splice conforming to **12.14.3.2** or **12.14.3.4** can be used without the stagger requirement instead of the lower strength mechanical or welded splice.

**R12.15.6** — A tension tie member has the following characteristics: member having an axial tensile force sufficient to create tension over the cross section; a level of stress in the reinforcement such that every bar must be fully effective; and limited concrete cover on all sides. Examples of members that may be classified as tension ties are arch ties, hangers carrying load to an overhead supporting structure, and main tension elements in a truss.

In determining if a member should be classified as a tension tie, consideration should be given to the importance, function, proportions, and stress conditions of the member related to the above characteristics. For example, a usual large circular tank, with many bars and with splices well staggered and widely spaced should not be classified as a tension tie member, and Class B splices may be used.

## **12.16 — Splices of deformed bars in compression**

**12.16.1** — Compression lap splice length shall be  $0.0005f_yd_b$ , for  $f_y$  of 60,000 psi or less, or  $(0.0009f_y - 24)d_b$  for  $f_y$  greater than 60,000 psi, but not less than 12 in. For  $f'_c$  less than 3000 psi, length of lap shall be increased by one-third.

## **R12.16 — Splices of deformed bars in compression**

Bond research has been primarily related to bars in tension. Bond behavior of compression bars is not complicated by the problem of transverse tension cracking and thus compression splices do not require provisions as strict as those specified for tension splices. The minimum lengths for column splices contained originally in the 1956 Code have been carried forward in later Codes, and extended to compression bars in beams and to higher strength steels. No changes have been made in the provisions for compression splices since the 1971 Code.

**R12.16.1** — Essentially, lap requirements for compression splices have remained the same since the 1963 Code.

The 1963 Code values were modified in the 1971 Code to recognize various degrees of confinement and to permit design with reinforcement having a specified yield strength up to 80,000 psi. Tests<sup>12.1,12.30</sup> have shown that splice strengths in compression depend considerably on end bearing and do not increase proportionally in strength when the splice length is doubled. Accordingly, for specified yield strengths above 60,000 psi, compression lap lengths are significantly increased, except where spiral enclosures are used (as in spiral columns) the where the increase is about 10 percent for an increase in specified yield strength from 60,000 to 75,000 psi.

**CODE****COMMENTARY**

**12.16.2** — When bars of different size are lap spliced in compression, splice length shall be the larger of  $\ell_{dc}$  of larger bar and compression lap splice length of smaller bar. Lap splices of No. 14 and No. 18 bars to No. 11 and smaller bars shall be permitted.

**12.16.3** — Mechanical or welded splices used in compression shall meet requirements of **12.14.3.2** or **12.14.3.4**.

**12.16.4 — End-bearing splices**

**12.16.4.1** — In bars required for compression only, transmission of compressive stress by bearing of square cut ends held in concentric contact by a suitable device shall be permitted.

**12.16.4.2** — Bar ends shall terminate in flat surfaces within 1.5 degrees of a right angle to the axis of the bars and shall be fitted within 3 degrees of full bearing after assembly.

**12.16.4.3** — End-bearing splices shall be used only in members containing closed ties, closed stirrups, or spirals.

**12.17 — Splice requirements for columns**

**12.17.1** — Lap splices, mechanical splices, butt-welded splices, and end-bearing splices shall be used with the limitations of **12.17.2** through **12.17.4**. A splice shall satisfy requirements for all load combinations for the column.

**R12.16.2** — The lap splice length is to be computed based on the larger of the compression splice length of the smaller bar; or the compression development length of the larger bar. Lap splices are generally prohibited for No. 14 or No. 18 bars; however, for compression only, lap splices are permitted for No. 14 or No. 18 bars to No. 11 or smaller bars.

**R12.16.4 — End-bearing splices**

**R12.16.4.1** — Experience with end-bearing splices has been almost exclusively with vertical bars in columns. If bars are significantly inclined from the vertical, attention is required to ensure that adequate end-bearing contact can be achieved and maintained.

**R12.16.4.2** — These tolerances were added in the 1971 Code, representing practice based on tests of full-size members containing No. 18 bars.

**R12.16.4.3** — This limitation was added in the 1971 Code to ensure a minimum shear resistance in sections containing end-bearing splices.

**R12.17 — Splice requirements for columns**

In columns subject to flexure and axial loads, tension stresses may occur on one face of the column for moderate and large eccentricities as shown in Fig. R12.17. When such tensions occur, 12.17 requires tension splices to be used or an adequate tensile resistance to be provided. Furthermore, a minimum tension strength is required in each face of all columns even where analysis indicates compression only.

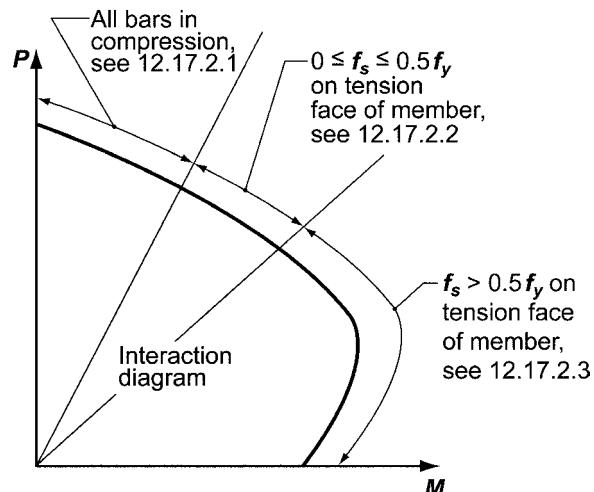


Fig. R12.17—Special splice requirements for columns.

**CODE****COMMENTARY**

The 1989 Code clarifies this section on the basis that a compressive lap splice has a tension strength of at least one-quarter  $f_y$ , which simplifies the calculation requirements in previous Codes.

Note that the column splice should satisfy requirements for all load combinations for the column. Frequently, the basic gravity load combination will govern the design of the column itself, but a load combination including wind or seismic loads may induce greater tension in some column bars, and the column splice should be designed for this tension.

**12.17.2 — Lap splices in columns**

**12.17.2.1** — Where the bar stress due to factored loads is compressive, lap splices shall conform to **12.16.1**, **12.16.2**, and, where applicable, to **12.17.2.4** or **12.17.2.5**.

**12.17.2.2** — Where the bar stress due to factored loads is tensile and does not exceed **0.5f<sub>y</sub>** in tension, lap splices shall be Class B tension lap splices if more than one-half of the bars are spliced at any section, or Class A tension lap splices if half or fewer of the bars are spliced at any section and alternate lap splices are staggered by  $\ell_d$ .

**12.17.2.3** — Where the bar stress due to factored loads is greater than **0.5f<sub>y</sub>** in tension, lap splices shall be Class B tension lap splices.

**12.17.2.4** — In tied reinforced compression members, where ties throughout the lap splice length have an effective area not less than **0.0015hs** in both directions, lap splice length shall be permitted to be multiplied by 0.83, but lap length shall not be less than 12 in. Tie legs perpendicular to dimension **h** shall be used in determining effective area.

**12.17.2.5** — In spirally reinforced compression members, lap splice length of bars within a spiral shall be permitted to be multiplied by 0.75, but lap length shall not be less than 12 in.

**12.17.3 — Mechanical or welded splices in columns**

Mechanical or welded splices in columns shall meet the requirements of **12.14.3.2** or **12.14.3.4**.

**R12.17.2 — Lap splices in columns**

**R12.17.2.1** — The 1989 Code was simplified for column bars always in compression on the basis that a compressive lap splice is adequate for sufficient tension to preclude special requirements.

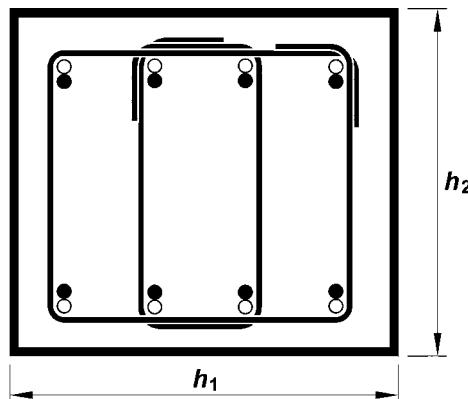
**R12.17.2.4** — Reduced lap lengths are allowed when the splice is enclosed throughout its length by minimum ties.

The tie legs perpendicular to each direction are computed separately and the requirement satisfied in each direction to apply the 0.83 reduction factor. This is illustrated in **Fig. R12.17.2**, where four legs are effective in one direction and two legs in the other direction. This calculation is critical in one direction, which normally can be determined by inspection.

**R12.17.2.5** — Compression lap lengths may be reduced when the lap splice is enclosed throughout its length by spirals because of increased splitting resistance. Spirals should meet requirements of **7.10.4** and **10.9.3**.

**R12.17.3 — Mechanical or welded splices in columns**

Mechanical or welded splices are allowed for splices in columns but should be designed as a full mechanical splice or a full welded splice developing 125 percent  $f_y$  as required by **12.14.3.2** or **12.14.3.4**. Splice strength is traditionally tested in tension and full strength is required to reflect the high compression loads possible in column reinforcement.

**CODE****COMMENTARY**

Direction 1:  $4A_b \geq 0.0015h_1s$

Direction 2:  $2A_b \geq 0.0015h_2s$

where  $A_b$  is the area of the tie

*Fig. R.12.17.2—Example application of 12.17.2.4.*

#### 12.17.4 — End-bearing splices in columns

End-bearing splices complying with 12.16.4 shall be permitted to be used for column bars stressed in compression provided the splices are staggered or additional bars are provided at splice locations. The continuing bars in each face of the column shall have a tensile strength, based on  $f_y$ , not less than 0.25 $f_y$  times the area of the vertical reinforcement in that face.

#### 12.18 — Splices of welded deformed wire reinforcement in tension

**12.18.1** — Minimum lap splice length of welded deformed wire reinforcement measured between the ends of each reinforcement sheet shall be not less than the larger of  $1.3\ell_d$  and 8 in., and the overlap measured between outermost cross wires of each reinforcement sheet shall be not less than 2 in., where  $\ell_d$  is calculated in accordance with 12.7 to develop  $f_y$ .

**12.18.2** — Lap splices of welded deformed wire reinforcement, with no cross wires within the lap splice length, shall be determined as for deformed wire.

**12.18.3** — Where any plain wires, or deformed wires larger than D-31, are present in the welded deformed wire reinforcement in the direction of the lap splice or where welded deformed wire reinforcement is lap spliced to welded plain wire reinforcement, the reinforcement shall be lap spliced in accordance with 12.19.

due to creep effects. If a mechanical splice developing less than a full mechanical splice is used, then the splice is required to conform to all requirements of end-bearing splices of 12.16.4 and 12.17.4.

#### R12.17.4 — End-bearing splices in columns

End-bearing splices used to splice column bars always in compression should have a tension strength of 25 percent of the specified yield strength of the steel area on each face of the column, either by staggering the end-bearing splices or by adding additional steel through the splice location. The end-bearing splice should conform to 12.16.4.

#### R12.18 — Splices of welded deformed wire reinforcement in tension

Splice provisions for welded deformed wire reinforcement are based on available tests.<sup>12.31</sup> The requirements were simplified (1976 Code supplement) from provisions of the 1971 Code by assuming that only one cross wire of each welded wire reinforcement sheet is overlapped and by computing the splice length as  $1.3\ell_d$ . The development length  $\ell_d$  is that computed in accordance with the provisions of 12.7 without regard to the 8 in. minimum. The 8 in. applies to the overall splice length. See Fig. R12.18. If no cross wires are within the lap length, the provisions for deformed wire apply.

Deformed wire larger than D-31 is treated as plain wire because tests show that D-45 wire will achieve only approximately 60 percent of the bond strength in tension given by Eq. (21-1).<sup>12.19</sup>

**CODE****COMMENTARY****12.19 — Splices of welded plain wire reinforcement in tension**

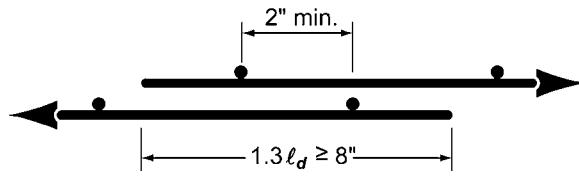
| **12.19.1** — Minimum length of lap for lap splices of welded plain wire reinforcement shall be in accordance with 12.19.1.1 and 12.19.1.2.

| **12.19.1.1** — Where  $A_s$  provided is less than twice that required by analysis at splice location, length of overlap measured between outermost cross wires of each reinforcement sheet shall be not less than the largest of one spacing of cross wires plus 2 in.,  $1.5\ell_d$ , and 6 in., where  $\ell_d$  is calculated in accordance with 12.8 to develop  $f_y$ .

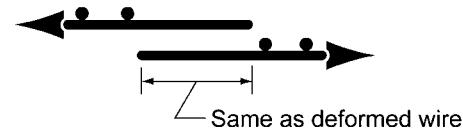
| **12.19.1.2** — Where  $A_s$  provided is at least twice that required by analysis at splice location, length of overlap measured between outermost cross wires of each reinforcement sheet shall not be less than the larger of  $1.5\ell_d$ , and 2 in., where  $\ell_d$  is calculated in accordance with 12.8 to develop  $f_y$ .

**R12.19 — Splices of welded plain wire reinforcement in tension**

The strength of lap splices of welded plain wire reinforcement is dependent primarily on the anchorage obtained from the cross wires rather than on the length of wire in the splice. For this reason, the lap is specified in terms of overlap of cross wires rather than in wire diameters or inches. The 2 in. additional lap required is to assure overlapping of the cross wires and to provide space for satisfactory consolidation of the concrete between cross wires. Research<sup>12.32</sup> has shown an increased splice length is required when welded wire reinforcement of large, closely spaced wires is lapped and, as a consequence, additional splice length requirements are provided for this reinforcement in addition to an absolute minimum of 6 in. The development length  $\ell_d$  is that computed in accordance with the provisions of 12.8 without regard to the 6 in. minimum. Splice requirements are illustrated in Fig. R12.19.

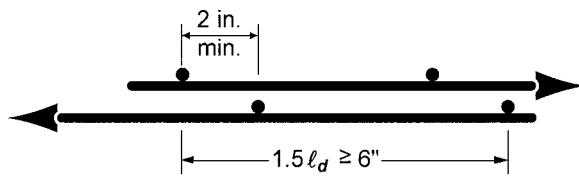


(a) Section 12.18.1

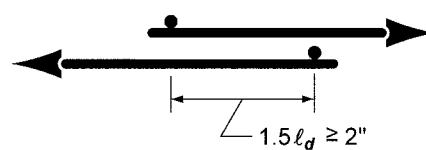


(b) Section 12.18.2

Fig. R12.18—Lap splices of welded deformed wire reinforcement.



(a) Section 12.19.1.1



(b) Section 12.19.1.2

Fig. R12.19—Lap splices of plain welded wire reinforcement.

**Notes**

# CHAPTER 13 — TWO-WAY SLAB SYSTEMS

## CODE

### 13.1 — Scope

**13.1.1** — Provisions of Chapter 13 shall apply for design of slab systems reinforced for flexure in more than one direction, with or without beams between supports.

**13.1.2** — For a slab system supported by columns or walls, dimensions  $c_1$ ,  $c_2$ , and  $\ell_n$  shall be based on an effective support area defined by the intersection of the bottom surface of the slab, or of the drop panel or shear cap if present, with the largest right circular cone, right pyramid, or tapered wedge whose surfaces are located within the column and the capital or bracket and are oriented no greater than 45 degrees to the axis of the column.

**13.1.3** — Solid slabs and slabs with recesses or pockets made by permanent or removable fillers between ribs or joists in two directions are included within the scope of Chapter 13.

**13.1.4** — Minimum thickness of slabs designed in accordance with Chapter 13 shall be as required by 9.5.3.

## COMMENTARY

### R13.1 — Scope

The design methods given in Chapter 13 are based on analysis of the results of an extensive series of tests<sup>13.1-13.7</sup> and the well-established performance record of various slab systems. Much of Chapter 13 is concerned with the selection and distribution of flexural reinforcement. Safety of a slab system requires consideration of the transmission of load from the slab to the columns by flexure, torsion, and shear. Design criteria for shear and torsion in slabs are given in Chapter 11.

The fundamental design principles contained in Chapter 13 are applicable to all planar structural systems subjected to transverse loads. Some of the specific design rules, as well as historical precedents, limit the types of structures to which Chapter 13 applies. General characteristics of slab systems that may be designed according to Chapter 13 are described in this section. These systems include flat slabs, flat plates, two-way slabs, and waffle slabs. Slabs with paneled ceilings are two-way wide-band beam systems.

True one-way slabs, slabs reinforced to resist flexural stresses in only one direction, are excluded. Also excluded are slabs-on-ground that do not transmit vertical loads from other parts of the structure to the soil.

For slabs with beams, the explicit design procedures of Chapter 13 apply only when the beams are located at the edges of the panel and when the beams are supported by columns or other essentially nondeflecting supports at the corners of the panel. Two-way slabs with beams in one direction, with both slab and beams supported by girders in the other direction, may be designed under the general requirements of Chapter 13. Such designs should be based upon analysis compatible with the deflected position of the supporting beams and girders.

For slabs supported on walls, the explicit design procedures in this chapter treat the wall as a beam of infinite stiffness; therefore, each wall should support the entire length of an edge of the panel (see 13.2.3). Wall-like columns less than a full panel length can be treated as columns.

Design aids for use in the engineering analysis and design of two-way slab systems are given in the *ACI Design Handbook*.<sup>13.8</sup> Design aids are provided to simplify application of the direct design and equivalent frame methods of Chapter 13.

**CODE****COMMENTARY****13.2 — General**

**13.2.1** — Column strip is a design strip with a width on each side of a column centerline equal to  $0.25\ell_2$  or  $0.25\ell_1$ , whichever is less. Column strip includes beams, if any.

**13.2.2** — Middle strip is a design strip bounded by two column strips.

**13.2.3** — A panel is bounded by column, beam, or wall centerlines on all sides.

**13.2.4** — For monolithic or fully composite construction, a beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

**13.2.5** — When used to reduce the amount of negative moment reinforcement over a column or minimum required slab thickness, a drop panel shall:

- (a) project below the slab at least one-quarter of the adjacent slab thickness; and
- (b) extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

**13.2.6** — When used to increase the critical condition section for shear at a slab-column joint, a shear cap shall project below the slab and extend a minimum horizontal distance from the face of the column that is equal to the thickness of the projection below the slab soffit.

**R13.2 — General**

**R13.2.3** — A panel includes all flexural elements between column centerlines. Thus, the column strip includes the beam, if any.

**R13.2.4** — For monolithic or fully composite construction, the beams include portions of the slab as flanges. Two examples of the rule are provided in Fig. R13.2.4.

**R13.2.5 and R13.2.6** — Drop panel dimensions specified in 13.2.5 are necessary when reducing the amount of negative moment reinforcement following 13.3.7 or to satisfy some minimum slab thicknesses permitted in 9.5.3. If the dimensions are less than specified in 13.2.5, the projection may be used as a shear cap to increase the shear strength of the slab. For slabs with changes in thickness it is necessary to check the shear strength at several sections. See 11.11.1.2.

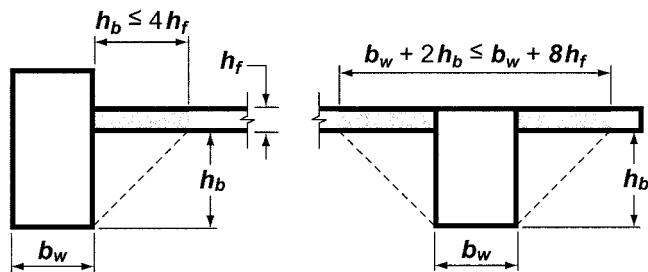


Fig. R13.2.4—Examples of the portion of slab to be included with the beam under 13.2.4.

**CODE****COMMENTARY****13.3 — Slab reinforcement**

**13.3.1** — Area of reinforcement in each direction for two-way slab systems shall be determined from moments at critical sections, but shall not be less than required by **7.12.2.1**.

**13.3.2** — Spacing of reinforcement at critical sections shall not exceed two times the slab thickness, except for portions of slab area of cellular or ribbed construction. In the slab over cellular spaces, reinforcement shall be provided as required by **7.12**.

**13.3.3** — Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of slab and have embedment, straight or hooked, at least 6 in. in spandrel beams, columns, or walls.

**13.3.4** — Negative moment reinforcement perpendicular to a discontinuous edge shall be bent, hooked, or otherwise anchored in spandrel beams, columns, or walls, and shall be developed at face of support according to provisions of **Chapter 12**.

**13.3.5** — Where a slab is not supported by a spandrel beam or wall at a discontinuous edge, or where a slab cantilevers beyond the support, anchorage of reinforcement shall be permitted within the slab.

**13.3.6** — At exterior corners of slabs supported by edge walls or where one or more edge beams have a value of  $\alpha_f$  greater than 1.0, top and bottom slab reinforcement shall be provided at exterior corners in accordance with 13.3.6.1 through 13.3.6.4.

**13.3.6.1** — Corner reinforcement in both top and bottom of slab shall be sufficient to resist a moment per unit of width equal to the maximum positive moment per unit width in the slab panel.

**13.3.6.2** — The moment shall be assumed to be about an axis perpendicular to the diagonal from the corner in the top of the slab and about an axis parallel to the diagonal from the corner in the bottom of the slab.

**13.3.6.3** — Corner reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

**13.3.6.4** — Corner reinforcement shall be placed parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab.

**R13.3 — Slab reinforcement**

**R13.3.2** — The requirement that the center-to-center spacing of the reinforcement be not more than two times the slab thickness applies only to the reinforcement in solid slabs, and not to reinforcement joists or waffle slabs. This limitation is to ensure slab action, cracking, and provide for the possibility of loads concentrated on small areas of the slab. See also **R10.6**.

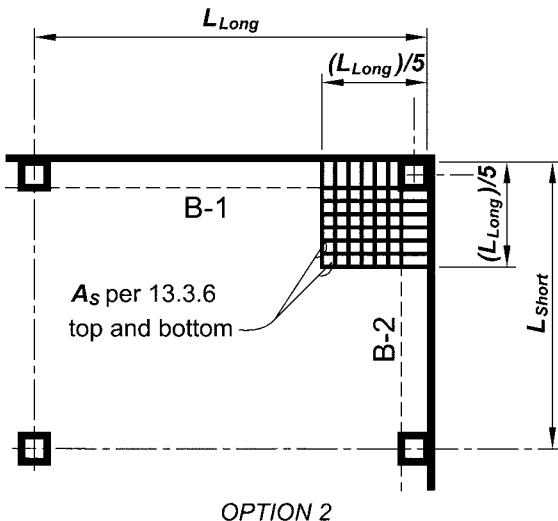
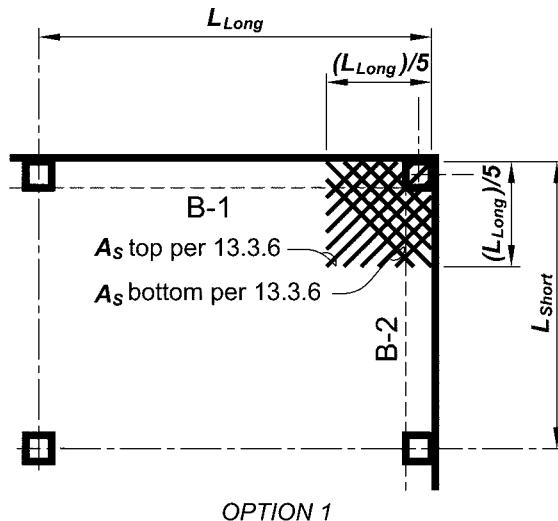
**R13.3.3-R13.3.5** — Bending moments in slabs at spandrel beams can be subject to great variation. If spandrel beams are built solidly into walls, the slab approaches complete fixity. Without an integral wall, the slab could approach simply supported, depending on the torsional rigidity of the spandrel beam or slab edge. These requirements provide for unknown conditions that might normally occur in a structure.

**R13.3.6** — Unrestrained corners of two-way slabs tend to lift when loaded. If this lifting tendency is restrained by edge walls or beams, bending moments result in the slab. This section provides steel to resist these moments and control cracking. Reinforcement provided for flexure in the primary directions may be used to satisfy this requirement. See **Fig. R13.3.6**.

**CODE**

Alternatively, reinforcement shall be placed in two layers parallel to the sides of the slab in both the top and bottom of the slab.

**13.3.7** — When a drop panel is used to reduce the amount of negative moment reinforcement over the column of a flat slab, the dimensions of the drop panel shall be in accordance with **13.2.5**. In computing required slab reinforcement, the thickness of the drop panel below the slab shall not be assumed to be greater than one-quarter the distance from the edge of drop panel to the face of column or column capital.

**COMMENTARY**Notes:

1. Applies where B-1 or B-2 has  $\alpha_f > 1.0$
2. Max. bar spacing  $2h$ , where  $h$  = slab thickness.

*Fig. R13.3.6—Slab corner reinforcement.*

**13.3.8 — Details of reinforcement in slabs without beams**

**13.3.8.1** — In addition to the other requirements of 13.3, reinforcement in slabs without beams shall have minimum extensions as prescribed in **Fig. 13.3.8**.

**13.3.8.2** — Where adjacent spans are unequal, extensions of negative moment reinforcement beyond the face of support as prescribed in **Fig. 13.3.8** shall be based on requirements of the longer span.

**R13.3.8 — Details of reinforcement in slabs without beams**

In the 1989 Code, bent bars were removed from **Fig. 13.3.8**. This was done because bent bars are seldom used and are difficult to place properly. Bent bars are permitted, however, if they comply with **13.3.8.3**. Refer to 13.4.8 of the 1983 Code.

## CODE

## COMMENTARY

STRIP	LOCATION	MINIMUM - $A_s$ AT SECTION	WITHOUT DROP PANELS	WITH DROP PANELS
COLUMN STRIP	TOP	50% REMAINDER		
	BOTTOM	100%		
MIDDLE STRIP	TOP	100%		
	BOTTOM	50% REMAINDER		

Fig. 13.3.8—Minimum extensions for reinforcement in slabs without beams. (See 12.11.1 for reinforcement extension into supports).

**13.3.8.3** — Bent bars shall be permitted only when depth-span ratio permits use of bends of 45 degrees or less.

**13.3.8.4** — In frames where two-way slabs act as primary members resisting lateral loads, lengths of reinforcement shall be determined by analysis but shall not be less than those prescribed in Fig. 13.3.8.

**13.3.8.5** — All bottom bars or wires within the column strip, in each direction, shall be continuous or spliced with Class B tension lap splices or with mechanical or welded splices satisfying 12.14.3. Splices shall be located as shown in Fig. 13.3.8. At least two of the column strip bottom bars or wires in each direction shall pass within the region bounded by the longitudinal reinforcement of the column and shall be anchored at exterior supports.

**R13.3.8.4** — For moments resulting from combined lateral and gravity loadings, the minimum lengths and extensions of bars in Fig. 13.3.8 may not be sufficient.

**R13.3.8.5** — The continuous column strip bottom reinforcement provides the slab some residual ability to span to the adjacent supports should a single support be damaged. The two continuous column strip bottom bars or wires through the column may be termed “integrity steel,” and are provided to give the slab some residual strength following a single punching shear failure at a single support.<sup>13.9</sup> In the 2002 Code, mechanical and welded splices were explicitly recognized as alternative methods of splicing reinforcement.

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**13.3.8.6** — In slabs with shearheads and in lift-slab construction where it is not practical to pass the bottom bars required by **13.3.8.5** through the column, at least two bonded bottom bars or wires in each direction shall pass through the shearhead or lifting collar as close to the column as practicable and be continuous or spliced with a Class B tension lap splice or with mechanical or welded splices satisfying **12.14.3**. At exterior columns, the reinforcement shall be anchored at the shearhead or lifting collar.

**13.4 — Openings in slab systems**

**13.4.1** — Openings of any size shall be permitted in slab systems if shown by analysis that the design strength is at least equal to the required strength set forth in **9.2** and **9.3**, and that all serviceability conditions, including the limits on deflections, are met.

**13.4.2** — As an alternate to analysis as required by 13.4.1, openings shall be permitted in slab systems without beams only, in accordance with 13.4.2.1 through 13.4.2.4.

**13.4.2.1** — Openings of any size shall be permitted in the area common to intersecting middle strips, provided total amount of reinforcement required for the panel without the opening is maintained.

**13.4.2.2** — In the area common to intersecting column strips, not more than one-eighth the width of column strip in either span shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.

**13.4.2.3** — In the area common to one column strip and one middle strip, not more than one-quarter of the reinforcement in either strip shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.

**13.4.2.4** — Shear requirements of **11.11.6** shall be satisfied.

**13.5 — Design procedures**

**13.5.1** — A slab system shall be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if shown that the design strength at every section is at least equal to the required strength set forth in **9.2** and **9.3**, and that all serviceability conditions, including limits on deflections, are met.

**R13.3.8.6** — In the 1992 Code, this provision was added to require the same integrity steel as for other two-way slabs without beams in case of a punching shear failure at a support.

In some instances, there is sufficient clearance so that the bonded bottom bars can pass under shearheads and through the column. Where clearance under the shearhead is inadequate, the bottom bars should pass through holes in the shearhead arms or within the perimeter of the lifting collar. Shearheads should be kept as low as possible in the slab to increase their effectiveness.

**R13.4 — Openings in slab systems**

See **R11.11.6**.

**R13.5 — Design procedures**

**R13.5.1** — This section permits a design to be based directly on fundamental principles of structural mechanics, provided it can be demonstrated explicitly that all strength and serviceability criteria are satisfied. The design of the slab may be achieved through the combined use of classic solutions based on a linearly elastic continuum, numerical

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**13.5.1.1** — Design of a slab system for gravity loads, including the slab and beams (if any) between supports and supporting columns or walls forming orthogonal frames, by either the Direct Design Method of 13.6 or the Equivalent Frame Method of 13.7, shall be permitted.

**13.5.1.2** — For lateral loads, analysis of frames shall take into account effects of cracking and reinforcement on stiffness of frame members.

**13.5.1.3** — Combining the results of the gravity load analysis with the results of the lateral load analysis shall be permitted.

solutions based on discrete elements, or yield-line analyses, including, in all cases, evaluation of the stress conditions around the supports in relation to shear and torsion as well as flexure. The design of a slab system involves more than its analysis, and any deviations in physical dimensions of the slab from common practice should be justified on the basis of knowledge of the expected loads and the reliability of the calculated stresses and deformations of the structure.

**R13.5.1.1** — For gravity load analysis of two-way slab systems, two analysis methods are given in 13.6 and 13.7. The specific provisions of both design methods are limited in application to orthogonal frames subject to gravity loads only. Both methods apply to two-way slabs with beams as well as to flat slabs and flat plates. In both methods, the distribution of moments to the critical sections of the slab reflects the effects of reduced stiffness of elements due to cracking and support geometry.

**R13.5.1.2** — During the life of a structure, construction loads, ordinary occupancy loads, anticipated overloads, and volume changes will cause cracking of slabs. Cracking reduces stiffness of slab members, and increases lateral flexibility when lateral loads act on the structure. Cracking of slabs should be considered in stiffness assumptions so that drift caused by wind or earthquake is not grossly underestimated.

The structure may be modeled for lateral load analysis using any approach that is shown to satisfy equilibrium and geometric compatibility and to be in reasonable agreement with test data.<sup>13.10,13.11</sup> The selected approach should recognize effects of cracking as well as parameters such as  $\ell_2/\ell_1$ ,  $c_1/\ell_1$ , and  $c_2/c_1$ . Some of the available approaches are summarized in Reference 13.12, which includes a discussion on the effects of cracking. Acceptable approaches include plate-bending finite-element models, the effective beam width model, and the equivalent frame model. In all cases, framing member stiffnesses should be reduced to account for cracking.

For nonprestressed slabs, it is normally appropriate to reduce slab bending stiffness to between one-half and one-quarter of the uncracked stiffness. For prestressed construction, stiffnesses greater than those of cracked, nonprestressed slabs may be appropriate. When the analysis is used to determine design drifts or moment magnification, lower-bound slab stiffnesses should be assumed. When the analysis is used to study interactions of the slab with other framing elements, such as structural walls, it may be appropriate to consider a range of slab stiffnesses so that the relative importance of the slab on those interactions can be assessed.

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**13.5.2** — The slab and beams (if any) between supports shall be proportioned for factored moments prevailing at every section.

**13.5.3** — When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of the unbalanced moment shall be transferred by flexure in accordance with 13.5.3.2 through **13.5.3.4**.

**13.5.3.1** — The fraction of unbalanced moment not transferred by flexure shall be transferred by eccentricity of shear in accordance with **11.11.7**.

**13.5.3.2** — A fraction of the unbalanced moment given by  $\gamma_f M_u$  shall be considered to be transferred by flexure within an effective slab width between lines that are one and one-half slab or drop panel thickness (**1.5h**) outside opposite faces of the column or capital, where  $M_u$  is the factored moment to be transferred and

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \quad (13-1)$$

**13.5.3.3** — For non prestressed slabs with unbalanced moments transferred between the slab and columns, it shall be permitted to increase the value of  $\gamma_f$  given by Eq. (13-1) in accordance with the following:

(a) For edge columns with unbalanced moments about an axis parallel to the edge,  $\gamma_f = 1.0$  provided that  $V_u$  at an edge support does not exceed **0.75φVc**, or at a corner support does not exceed **0.5φVc**.

(b) For unbalanced moments at interior supports, and for edge columns with unbalanced moments about an axis perpendicular to the edge, increase  $\gamma_f$  to as much as 1.25 times the value from Eq. (13-1), but not more than  $\gamma_f = 1.0$ , provided that  $V_u$  at the support does not exceed **0.4φVc**. The net tensile strain  $\epsilon_t$  calculated for the effective slab width defined in 13.5.3.2 shall not be less than 0.010.

The value of  $V_c$  in items (a) and (b) shall be calculated in accordance with **11.11.2.1**.

**R13.5.3** — This section is concerned primarily with slab systems without beams. Tests and experience have shown that, unless measures are taken to resist the torsional and shear stresses, all reinforcement resisting that part of the moment to be transferred to the column by flexure should be placed between lines that are one and one-half the slab or drop panel thickness, **1.5h**, on each side of the column. The calculated shear stresses in the slab around the column are required to conform to the requirements of **11.11.2**. See **R11.11.1.2** and **R11.11.2.1** for more details on application of this section.

**R13.5.3.3** — The 1989 Code procedures remain unchanged, except that under certain conditions it is permitted to adjust the level of moment transferred by shear without revising member sizes. Tests indicate that some flexibility in distribution of unbalanced moments transferred by shear and flexure at both exterior and interior supports is possible. Interior, exterior, and corner supports refer to slab-column connections for which the critical perimeter for rectangular columns has four, three, or two sides, respectively. Changes in the 1995 Code recognized, to some extent, design practices before the 1971 Code.<sup>13.13</sup>

At exterior supports, for unbalanced moments about an axis parallel to the edge, the portion of moment transferred by eccentricity of shear  $\gamma_v M_u$  may be reduced provided that the factored shear at the support (excluding the shear produced by moment transfer) does not exceed 75 percent of the shear strength  $\phi V_c$  as defined in **11.11.2.1** for edge columns or 50 percent for corner columns. Tests<sup>13.14,13.15</sup> indicate that there is no significant interaction between shear and unbalanced moment at the exterior support in such cases. Note that as  $\gamma_v M_u$  is decreased,  $\gamma_f M_u$  is increased.

Evaluation of tests of interior supports indicate that some flexibility in distributing unbalanced moments transferred by shear and flexure is possible, but with more severe limitations than for exterior supports. For interior supports, the unbalanced moment transferred by flexure is permitted to be increased up to 25 percent provided that the factored shear (excluding the shear caused by the moment transfer) at the interior supports does not exceed 40 percent of the shear strength  $\phi V_c$  as defined in **11.11.2.1**.

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When the factored shear for a slab-column connection is large, the slab-column joint cannot always develop all of the reinforcement provided in the effective width. The modifications for interior slab-column connections in 13.5.3.3 are permitted only when the reinforcement (within the effective width) required to develop the unbalanced moment  $\gamma_f M_u$  has a net tensile strain  $\epsilon_t$  not less than 0.010. The use of Eq. (13-1) without the modification permitted in 13.5.3.3 will generally indicate overstress conditions on the joint. The provisions of 13.5.3.3 are intended to improve ductile behavior of the slab-column joint. When a reversal of moments occurs at opposite faces of an interior support, both top and bottom reinforcement should be concentrated within the effective width. A ratio of top to bottom reinforcement of approximately 2 has been observed to be appropriate.

For the 2008 Code, two changes were introduced to 13.5.3.3: (1) the limitation for the amount of reinforcement in the effective slab width to 37.5 percent of the balanced steel ratio was updated to refer to a minimum net tensile strain of 0.010 to be consistent with the unified design approach adopted in the 2002 Code, and (2) the requirement for the minimum net tensile strain was eliminated for moment transfer about the slab edge for edge and corner connections based on the original recommendations from Joint ACI-ASCE Committee 352.<sup>13.15</sup>

**13.5.3.4** — Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist moment on the effective slab width defined in 13.5.3.2.

**13.5.4** — Design for transfer of load from slabs to supporting columns or walls through shear and torsion shall be in accordance with Chapter 11.

**13.6 — Direct design method****R13.6 — Direct design method**

The direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

- (1) Determination of the total factored static moment (see 13.6.2);
- (2) Distribution of the total factored static moment to negative and positive sections (see 13.6.3);
- (3) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any (see 13.6.4 through 13.6.6). The distribution of moments to column and middle strips is also used in the equivalent frame method (see 13.7).

**CODE****COMMENTARY****13.6.1 — Limitations**

Design of slab systems within the limitations of 13.6.1.1 through 13.6.1.8 by the direct design method shall be permitted.

**13.6.1.1** — There shall be a minimum of three continuous spans in each direction.

**13.6.1.2** — Panels shall be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.

**13.6.1.3** — Successive span lengths center-to-center of supports in each direction shall not differ by more than one-third the longer span.

**13.6.1.4** — Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns shall be permitted.

**13.6.1.5** — All loads shall be due to gravity only and uniformly distributed over an entire panel. The unfactored live load shall not exceed two times the unfactored dead load.

**13.6.1.6** — For a panel with beams between supports on all sides, Eq. (13-2) shall be satisfied for beams in the two perpendicular directions

$$0.2 \leq \frac{\alpha_{f1} \ell_2^2}{\alpha_{f2} \ell_1^2} \leq 5.0 \quad (13-2)$$

where  $\alpha_{f1}$  and  $\alpha_{f2}$  are calculated in accordance with Eq. (13-3).

**R13.6.1 — Limitations**

The direct design method was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple design and construction procedures, and precedents supplied by performance of slab systems. Consequently, the slab systems to be designed using the direct design method should conform to the limitations in this section.

**R13.6.1.1** — The primary reason for the limitation in this section is the magnitude of the negative moments at the interior support in a structure with only two continuous spans. The rules given for the direct design method assume that the slab system at the first interior negative moment section is neither fixed against rotation nor discontinuous.

**R13.6.1.2** — If the ratio of the two spans (long span/short span) of a panel exceeds 2, the slab resists the moment in the shorter span essentially as a one-way slab.

**R13.6.1.3** — The limitation in this section is related to the possibility of developing negative moments beyond the point where negative moment reinforcement is terminated, as prescribed in Fig. 13.3.8.

**R13.6.1.4** — Columns can be offset within specified limits from a regular rectangular array. A cumulative total offset of 20 percent of the span is established as the upper limit.

**R13.6.1.5** — The direct design method is based on tests<sup>13,16</sup> for uniform gravity loads and resulting column reactions determined by statics. Lateral loads such as wind or seismic require a frame analysis. Inverted foundation mats designed as two-way slabs (see 15.10) involve application of known column loads. Therefore, even where the soil reaction is assumed to be uniform, a frame analysis should be performed.

In the 1995 Code, the limit of applicability of the direct design method for ratios of live load to dead load was reduced from 3 to 2. In most slab systems, the live to dead load ratio will be less than 2 and it will not be necessary to check the effects of pattern loading.

**R13.6.1.6** — The elastic distribution of moments will deviate significantly from those assumed in the direct design method unless the requirements for stiffness are satisfied.

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$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \quad (13-3)$$

**13.6.1.7** — Moment redistribution as permitted by 8.4 shall not be applied for slab systems designed by the direct design method. See 13.6.7.

**13.6.1.8** — Variations from the limitations of 13.6.1 shall be permitted if demonstrated by analysis that requirements of 13.5.1 are satisfied.

**13.6.2 — Total factored static moment for a span**

**13.6.2.1** — Total factored static moment,  $M_o$ , for a span shall be determined in a strip bounded laterally by centerline of panel on each side of centerline of supports.

**13.6.2.2** — Absolute sum of positive and average negative factored moments in each direction shall not be less than

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} \quad (13-4)$$

where  $\ell_n$  is length of clear span in direction that moments are being determined.

**13.6.2.3** — Where the transverse span of panels on either side of the centerline of supports varies,  $\ell_2$  in Eq. (13-4) shall be taken as the average of adjacent transverse spans.

**13.6.2.4** — When the span adjacent and parallel to an edge is being considered, the distance from edge to panel centerline shall be substituted for  $\ell_2$  in Eq. (13-4).

**13.6.2.5** — Clear span  $\ell_n$  shall extend from face to face of columns, capitals, brackets, or walls. Value of  $\ell_n$  used in Eq. (13-4) shall not be less than 0.65 $\ell_1$ . Circular or regular polygon-shaped supports shall be treated as square supports with the same area.

**13.6.3 — Negative and positive factored moments**

**13.6.3.1** — Negative factored moments shall be located at face of rectangular supports. Circular or regular polygon-shaped supports shall be treated as square supports with the same area.

**R13.6.1.7** — Moment redistribution as permitted by 8.4 is not intended for use where approximate values for bending moments are used. For the direct design method, 10 percent modification is allowed by 13.6.7.

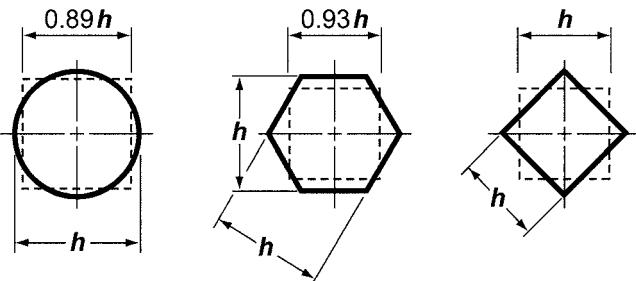
**R13.6.1.8** — It is permitted to use the direct design method even if the structure does not fit the limitations in this section, provided it can be shown by analysis that the particular limitation does not apply to that structure. For a slab system carrying a nonmovable load (such as a water reservoir in which the load on all panels is expected to be the same), live load limitation of 13.6.1.5 need not be satisfied.

**R13.6.2 — Total factored static moment for a span**

**R13.6.2.2** — Equation (13-4) follows directly from Nichol's derivation<sup>f3.17</sup> with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, it will be expedient to calculate static moments for two adjacent half panels that include a column strip with a half middle strip along each side.

**R13.6.3 — Negative and positive factored moments**

**R13.6.2.5** — If a supporting member does not have a rectangular cross section or if the sides of the rectangle are not parallel to the spans, it is to be treated as a square support having the same area, as illustrated in Fig. R13.6.2.5.

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*Fig. R13.6.2.5—Examples of equivalent square section for supporting members.*

**13.6.3.2** — In an interior span, total static moment,  $M_o$ , shall be distributed as follows:

Negative factored moment ..... 0.65

Positive factored moment ..... 0.35

**13.6.3.3** — In an end span, total factored static moment,  $M_o$ , shall be distributed as follows:

	(1)	(2)	(3)	(4)	(5)
Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports			Exterior edge fully restrained
		Without edge beam	With edge beam		
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

**R13.6.3.3** — The moment coefficients for an end span are based on the equivalent column stiffness expressions from References 13.18, 13.19, and 13.20. The coefficients for an unrestrained edge would be used, for example, if the slab were simply supported on a masonry or concrete wall. Those for a fully restrained edge would apply if the slab were constructed integrally with a concrete wall having a flexural stiffness so large compared to that of the slab that little rotation occurs at the slab-to-wall connection.

For other than unrestrained or fully restrained edges, coefficients in the table were selected to be near the upper bound of the range for positive moments and interior negative moments. As a result, exterior negative moments were usually closer to a lower bound. The exterior negative moment strength for most slab systems is governed by minimum reinforcement to control cracking. The final coefficients in the table have been adjusted so that the absolute sum of the positive and average moments equal  $M_o$ .

For two-way slab systems with beams between supports on all sides (two-way slabs), moment coefficients of Column (2) of the table apply. For slab systems without beams between interior supports (flat plates and flat slabs), the moment coefficients of Column (3) or (4) apply, without or with an edge (spandrel) beam, respectively.

In the 1977 Code, distribution factors defined as a function of the stiffness ratio of the equivalent exterior support were used for proportioning the total static moment  $M_o$  in an end span. The approach may be used in place of values in 13.6.3.3.

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**13.6.3.4** — Negative moment sections shall be designed to resist the larger of the two interior negative factored moments determined for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.

**13.6.3.5** — Edge beams or edges of slab shall be proportioned to resist in torsion their share of exterior negative factored moments.

**13.6.3.6** — The gravity load moment to be transferred between slab and edge column in accordance with **13.5.3.1** shall be **0.3M<sub>o</sub>**.

### 13.6.4 — Factored moments in column strips

**13.6.4.1** — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

$\ell_2/\ell_1$	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

**13.6.4.2** — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

$\ell_2/\ell_1$		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Linear interpolations shall be made between values shown, where  $\beta_t$  is calculated in Eq. (13-5) and  $C$  is calculated in Eq. (13-6).

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \quad (13-5)$$

$$C = \sum \left( 1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3} \quad (13-6)$$

The constant  $C$  for T- or L-sections shall be permitted to be evaluated by dividing the section into separate rectangular parts, as defined in **13.2.4**, and summing the values of  $C$  for each part.

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**R13.6.3.4** — The differences in slab moment on either side of a column or other type of support should be accounted for in the design of the support. If an analysis is made to distribute unbalanced moments, flexural stiffness may be obtained on the basis of the gross concrete section of the members involved.

**R13.6.3.5** — Moments perpendicular to, and at the edge of, the slab structure should be transmitted to the supporting columns or walls. Torsional stresses caused by the moment assigned to the slab should be investigated.

### R13.6.4–R13.6.6 — Factored moments in column strips, beams, and middle strips

The rules given for assigning moments to the column strips, beams, and middle strips are based on studies<sup>13.21</sup> of moments in linearly elastic slabs with different beam stiffness tempered by the moment coefficients that have been used successfully.

For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall,  $\ell_n$  in Eq. (13-4) may be assumed equal to  $\ell_n$  of the parallel adjacent column to column span, and the wall may be considered as a beam having a moment of inertia  $I_b$  equal to infinity.

**R13.6.4.2** — The effect of the torsional stiffness parameter  $\beta_t$  is to assign all of the exterior negative factored moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of  $\beta_t$ , the shear modulus has been taken as  $E_{cb}/2$ .

Where walls are used as supports along column lines, they can be regarded as very stiff beams with an  $\alpha_{f1}\ell_2/\ell_1$  value greater than 1. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined,  $\beta_t$  may be taken as zero if the wall is of masonry without torsional resistance, and  $\beta_t$  may be taken as 2.5 for a concrete wall with great torsional resistance that is monolithic with the slab.

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**13.6.4.3** — Where supports consist of columns or walls extending for a distance equal to or greater than  $(3/4)\ell_2$  used to compute  $M_o$ , negative moments shall be considered to be uniformly distributed across  $\ell_2$ .

**13.6.4.4** — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

$\ell_2/\ell_1$	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

**13.6.4.5** — For slabs with beams between supports, the slab portion of column strips shall be proportioned to resist that portion of column strip moments not resisted by beams.

### 13.6.5 — Factored moments in beams

**13.6.5.1** — Beams between supports shall be proportioned to resist 85 percent of column strip moments if  $\alpha_{f1}\ell_2/\ell_1$  is equal to or greater than 1.0.

**13.6.5.2** — For values of  $\alpha_{f1}\ell_2/\ell_1$  between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

**13.6.5.3** — In addition to moments calculated for uniform loads according to 13.6.2.2, 13.6.5.1, and 13.6.5.2, beams shall be proportioned to resist all moments caused by concentrated or linear loads applied directly to beams, including weight of projecting beam stem above or below the slab.

### 13.6.6 — Factored moments in middle strips

**13.6.6.1** — That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

**13.6.6.2** — Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

**13.6.6.3** — A middle strip adjacent to and parallel with a wall-supported edge shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

### R13.6.5 — Factored moments in beams

Loads assigned directly to beams are in addition to the uniform dead load of the slab; uniform superimposed dead loads, such as the ceiling, floor finish, or assumed equivalent partition loads; and uniform live loads. All of these loads are normally included with  $q_u$  in Eq. (13-4). Linear loads applied directly to beams include partition walls over or along beam centerlines and additional dead load of the projecting beam stem. Concentrated loads include posts above or hangers below the beams. For the purpose of assigning directly applied loads, only loads located within the width of the beam stem should be considered as directly applied to the beams. (The effective width of a beam as defined in 13.2.4 is solely for strength and relative stiffness calculations.) Line loads and concentrated loads located on the slab away from the beam stem require consideration to determine their apportionment to slab and beams.

**CODE****COMMENTARY****13.6.7 — Modification of factored moments**

Modification of negative and positive factored moments by 10 percent shall be permitted provided the total static moment for a panel,  $M_o$ , in the direction considered is not less than that required by Eq. (13-4).

**13.6.8 — Factored shear in slab systems with beams**

**13.6.8.1** — Beams with  $\alpha_{f1}\ell_2/\ell_1$  equal to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on tributary areas which are bounded by 45-degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.

**13.6.8.2** — In proportioning beams with  $\alpha_{f1}\ell_2/\ell_1$  less than 1.0 to resist shear, linear interpolation, assuming beams carry no load at  $\alpha_{f1} = 0$ , shall be permitted.

**13.6.8.3** — In addition to shears calculated according to 13.6.8.1 and 13.6.8.2, beams shall be proportioned to resist shears caused by factored loads applied directly on beams.

**13.6.8.4** — Computation of slab shear strength on the assumption that load is distributed to supporting beams in accordance with 13.6.8.1 or 13.6.8.2 shall be permitted. Resistance to total shear occurring on a panel shall be provided.

**13.6.8.5** — Shear strength shall satisfy the requirements of Chapter 11.

**R13.6.7 — Modification of factored moments**

This section permits a reduction of 10 percent in negative or positive factored moments, calculated in accordance with 13.6.3, provided that the total static moment for a panel in the direction considered is not less than  $M_o$  required by Eq. (13-4). This is intended to recognize a limited amount of inelastic behavior and moment redistribution can occur in slabs that were analyzed with the direct design method.

**R13.6.8 — Factored shear in slab systems with beams**

The tributary area for computing shear on an interior beam is shown shaded in Fig. R13.6.8. If the stiffness for the beam  $\alpha_{f1}\ell_2/\ell_1$  is less than 1.0, the shear on the beam may be obtained by linear interpolation. In such cases, the beams framing into the column will not account for all of the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column that should be checked in the same manner as for flat slabs, as required by 13.6.8.4. Sections 13.6.8.1 through 13.6.8.3 do not apply to the calculation of torsional moments on the beams. These moments should be based on the calculated flexural moments acting on the sides of the beam.

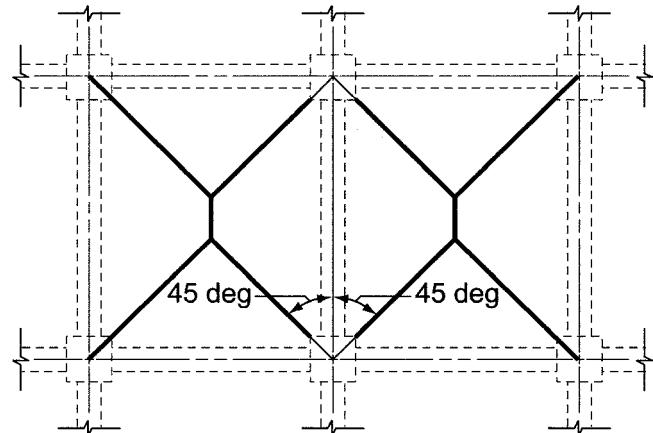


Fig. R13.6.8—Tributary area for shear on an interior beam.

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### 13.6.9 — Factored moments in columns and walls

**13.6.9.1** — Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.

**13.6.9.2** — At an interior support, supporting elements above and below the slab shall resist the factored moment specified by Eq. (13-7) in direct proportion to their stiffnesses unless a general analysis is made.

$$M_u = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2 \ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2] \quad (13-7)$$

where  $q_{Du}'$ ,  $\ell_2'$ , and  $\ell_n'$  refer to shorter span.

## 13.7 — Equivalent frame method

**13.7.1** — Design of slab systems by the equivalent frame method shall be based on assumptions given in 13.7.2 through **13.7.6**, and all sections of slabs and supporting members shall be proportioned for moments and shears thus obtained.

**13.7.1.1** — Where metal column capitals are used, it shall be permitted to take account of their contributions to stiffness and resistance to moment and to shear.

**13.7.1.2** — It shall be permitted to neglect the change in length of columns and slabs due to direct stress, and deflections due to shear.

### 13.7.2 — Equivalent frame

**13.7.2.1** — The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building.

## COMMENTARY

### R13.6.9 — Factored moments in columns and walls

Equation (13-7) refers to two adjoining spans, with one span longer than the other, and with full dead load plus one-half live load applied on the longer span and only dead load applied on the shorter span.

Design and detailing of the reinforcement transferring the moment from the slab to the edge column is critical to both the performance and the safety of flat slabs or flat plates without edge beams or cantilever slabs. It is important that complete design details be shown in the contract documents, such as concentration of reinforcement over the column by closer spacing or additional reinforcement.

### R13.7 — Equivalent frame method

The equivalent frame method involves the representation of the three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with **13.6.4** (column strips), **13.6.5** (beams), and **13.6.6** (middle strips). The equivalent frame method is based on studies reported in References **13.18**, **13.19**, and **13.20**. Many of the details of the equivalent frame method given in the Commentary in the 1989 Code were removed in the 1995 Code.

### R13.7.2 — Equivalent frame

Application of the equivalent frame to a regular structure is illustrated in Fig. R13.7.2. The three-dimensional building is divided into a series of two-dimensional frame bents (equivalent frames) centered on column or support centerlines

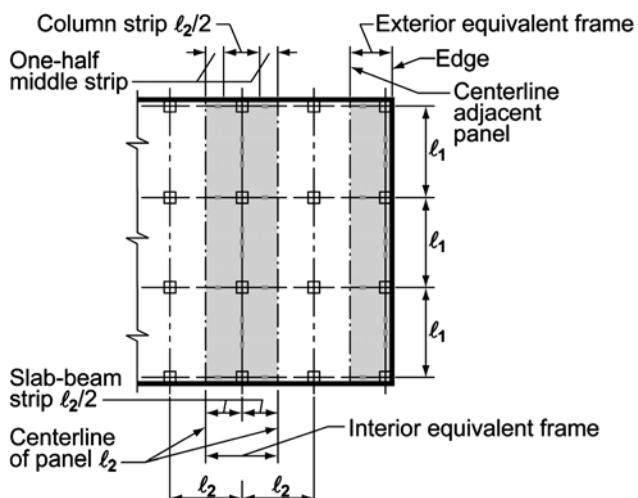


Fig. R13.7.2—Definitions of equivalent frame.

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**13.7.2.2** — Each frame shall consist of a row of columns or supports and slab-beam strips, bounded laterally by the centerline of panel on each side of the centerline of columns or supports.

**13.7.2.3** — Columns or supports shall be assumed to be attached to slab-beam strips by torsional members (see **13.7.5**) transverse to the direction of the span for which moments are being determined and extending to bounding lateral panel centerlines on each side of a column.

**13.7.2.4** — Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of adjacent panel.

**13.7.2.5** — Analysis of each equivalent frame in its entirety shall be permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with far ends of columns considered fixed shall be permitted.

**13.7.2.6** — Where slab-beams are analyzed separately, determination of moment at a given support assuming that the slab-beam is fixed at any support two panels distant therefrom, shall be permitted, provided the slab continues beyond that point.

**13.7.3 — Slab-beams**

**13.7.3.1**—Determination of the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

**13.7.3.2** — Variation in moment of inertia along axis of slab-beams shall be taken into account.

**13.7.3.3** — Moment of inertia of slab-beams from center of column to face of column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at face of column, bracket, or capital divided by the quantity  $(1 - c_2/\ell_2)^2$ , where  $c_2$  and  $\ell_2$  are measured transverse to the direction of the span for which moments are being determined.

**13.7.4 — Columns**

**13.7.4.1** — Determination of the moment of inertia of columns at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

**13.7.4.2** — Variation in moment of inertia along axis of columns shall be taken into account.

**COMMENTARY**

with each frame extending the full height of the building. The width of each equivalent frame is bounded by the centerlines of the adjacent panels. The complete analysis of a slab system for a building consists of analyzing a series of equivalent (interior and exterior) frames spanning longitudinally and transversely through the building.

The equivalent frame comprises three parts: (1) the horizontal slab strip, including any beams spanning in the direction of the frame, (2) the columns or other vertical supporting members, extending above and below the slab, and (3) the elements of the structure that provide moment transfer between the horizontal and vertical members.

**R13.7.3 — Slab-beams**

**R13.7.3.3** — A support is defined as a column, capital, bracket, or wall. A beam is not considered to be a support member for the equivalent frame.

**R13.7.4 — Columns**

Column stiffness is based on the length of the column from mid-depth of slab above to mid-depth of slab below. Column moment of inertia is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any.

When slab-beams are analyzed separately for gravity loads, the concept of an equivalent column, combining the stiffness

**CODE****COMMENTARY**

**13.7.4.3** — Moment of inertia of columns from top to bottom of the slab-beam at a joint shall be assumed to be infinite.

**13.7.5 — Torsional members**

**13.7.5.1** — Torsional members (see **13.7.2.3**) shall be assumed to have a constant cross section throughout their length consisting of the largest of (a), (b), and (c):

- (a) A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined;
- (b) For monolithic or fully composite construction, the portion of slab specified in (a) plus that part of the transverse beam above and below the slab;
- (c) The transverse beam as defined in **13.2.4**.

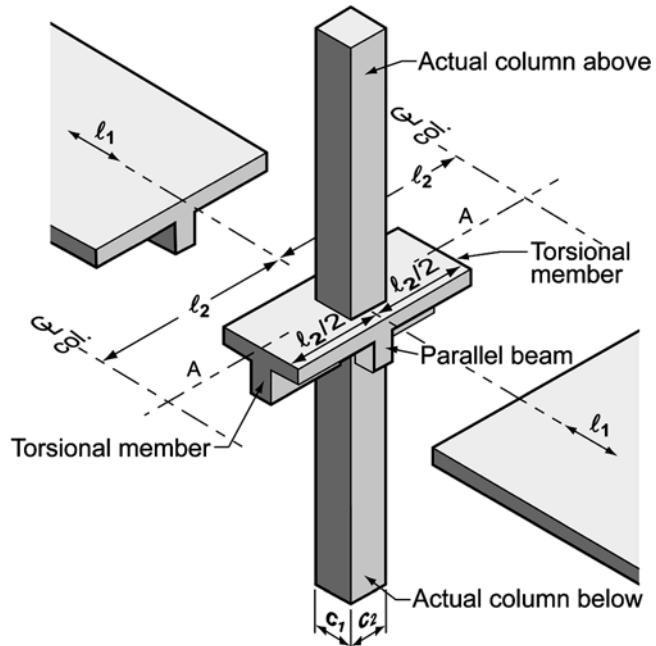
**13.7.5.2** — Where beams frame into columns in the direction of the span for which moments are being determined, the torsional stiffness shall be multiplied by the ratio of the moment of inertia of the slab with such a beam to the moment of inertia of the slab without such a beam.

of the slab-beam and torsional member into a composite element, is used. The column flexibility is modified to account for the torsional flexibility of the slab-to-column connection that reduces its efficiency for transmission of moments. The equivalent column consists of the actual columns above and below the slab-beam, plus attached torsional members on each side of the columns extending to the centerline of the adjacent panels as shown in Fig. R13.7.4.

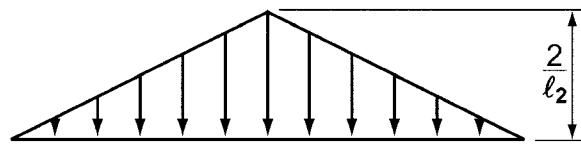
**R13.7.5 — Torsional members**

Computation of the stiffness of the torsional member requires several simplifying assumptions. If no transverse beam frames into the column, a portion of the slab equal to the width of the column or capital is assumed to be the torsional member. If a beam frames into the column, T-beam or L-beam action is assumed, with the flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab but not greater than four times the thickness of the slab. Furthermore, it is assumed that no torsional rotation occurs in the beam over the width of the support.

The member sections to be used for calculating the torsional stiffness are defined in 13.7.5.1. Up to the 1989 Code, Eq. (13-6) specified the stiffness coefficient  $K_t$  of the torsional members. In 1995, the approximate expression for  $K_t$  was moved to the Commentary.



*Fig. R13.7.4—Equivalent column (column plus torsional members).*

**CODE****COMMENTARY**

*Fig. R13.7.5—Distribution of unit twisting moment along column centerline AA shown in Fig. R13.7.4.*

Studies of three-dimensional analyses of various slab configurations suggest that a reasonable value of the torsional stiffness can be obtained by assuming a moment distribution along the torsional member that varies linearly from a maximum at the center of the column to zero at the middle of the panel. The assumed distribution of unit twisting moment along the column centerline is shown in Fig. R13.7.5.

An approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analyses of various slab configurations (References 13.18, 13.19, and 13.20) is given below as

$$K_t = \sum \frac{9E_{cs}C}{l_2 \left(1 - \frac{c_2}{l_2}\right)^3}$$

### 13.7.6 — Arrangement of live load

**13.7.6.1** — When the loading pattern is known, the equivalent frame shall be analyzed for that load.

**13.7.6.2** — When the unfactored live load is variable but does not exceed three-quarters of the unfactored dead load, or the nature of live load is such that all panels will be loaded simultaneously, it shall be permitted to assume that maximum factored moments occur at all sections with full factored live load on entire slab system.

**13.7.6.3** — For loading conditions other than those defined in 13.7.6.2, it shall be permitted to assume that maximum positive factored moment near midspan of a panel occurs with three-quarters of the full factored live load on the panel and on alternate panels; and it shall be permitted to assume that maximum negative factored moment in the slab at a support occurs with three-quarters of the full factored live load on adjacent panels only.

**13.7.6.4** — Factored moments shall be taken not less than those occurring with full factored live load on all panels.

### R13.7.6 — Arrangement of live load

The use of only three-quarters of the full factored live load for maximum moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is thus possible before failure occurs. This procedure, in effect, permits some local overstress under the full factored live load if it is distributed in the prescribed manner, but still ensures that the design strength of the slab system after redistribution of moment is not less than that required to carry the full factored dead and live loads on all panels.

**CODE****COMMENTARY****13.7.7 — Factored moments**

**13.7.7.1** — At interior supports, the critical section for negative factored moment (in both column and middle strips) shall be taken at face of rectilinear supports, but not farther away than  $0.175\ell_1$  from the center of a column.

**13.7.7.2** — At exterior supports with brackets or capitals, the critical section for negative factored moment in the span perpendicular to an edge shall be taken at a distance from face of supporting element not greater than one-half the projection of bracket or capital beyond face of supporting element.

**13.7.7.3** — Circular or regular polygon-shaped supports shall be treated as square supports with the same area for location of critical section for negative design moment.

**13.7.7.4** — Where slab systems within limitations of 13.6.1 are analyzed by the equivalent frame method, it shall be permitted to reduce the resulting computed moments in such proportion that the absolute sum of the positive and average negative moments used in design need not exceed the value obtained from Eq. (13-4).

**13.7.7.5** — Distribution of moments at critical sections across the slab-beam strip of each frame to column strips, beams, and middle strips as provided in 13.6.4, 13.6.5, and 13.6.6 shall be permitted if the requirement of 13.6.1.6 is satisfied.

**R13.7.7 — Factored moments**

**R13.7.7.1-R13.7.7.3** — These Code sections adjust the negative factored moments to the face of the supports. The adjustment is modified at an exterior support to limit reductions in the exterior negative moment. Figure R13.6.2.5 illustrates several equivalent rectangular supports for use in establishing faces of supports for design with nonrectangular supports.

**R13.7.7.4** — Previous Codes have contained this section. It is based on the principle that if two different methods are prescribed to obtain a particular answer, the Code should not require a value greater than the least acceptable value. Due to the long satisfactory experience with designs having total factored static moments not exceeding those given by Eq. (13-4), it is considered that these values are satisfactory for design when applicable limitations are met.

# CHAPTER 14 — WALLS

## CODE

### 14.1 — Scope

**14.1.1** — Provisions of Chapter 14 shall apply for design of walls subjected to axial load, with or without flexure.

**14.1.2** — Cantilever retaining walls are designed according to flexural design provisions of **Chapter 10** with minimum horizontal reinforcement according to **14.3.3**.

### 14.2 — General

**14.2.1** — Walls shall be designed for eccentric loads and any lateral or other loads to which they are subjected.

**14.2.2** — Walls subject to axial loads shall be designed in accordance with **14.2**, **14.3**, and either **14.4**, **14.5**, or **14.8**.

**14.2.3** — Design for shear shall be in accordance with **11.9**.

**14.2.4** — Unless otherwise demonstrated by an analysis, the horizontal length of wall considered as effective for each concentrated load shall not exceed the smaller of the center-to-center distance between loads, and the bearing width plus four times the wall thickness.

**14.2.5** — Compression members built integrally with walls shall conform to **10.8.2**.

**14.2.6** — Walls shall be anchored to intersecting elements, such as floors and roofs; or to columns, pilasters, buttresses, or intersecting walls; and to footings.

**14.2.7** — Quantity of reinforcement and limits of thickness required by **14.3** and **14.5** shall be permitted to be waived where structural analysis shows adequate strength and stability.

**14.2.8** — Transfer of force to footing at base of wall shall be in accordance with **15.8**.

## COMMENTARY

### R14.1 — Scope

Chapter 14 applies generally to walls as vertical load-carrying members. Cantilever retaining walls are designed according to the flexural design provisions of **Chapter 10**. Walls designed to resist shear forces, such as shear walls, should be designed in accordance with Chapter 14 and **11.9** as applicable.

In the 1977 Code, walls could be designed according to Chapter 14 or 10.15. In the 1983 Code, these two were combined in Chapter 14.

### R14.2 — General

Walls should be designed to resist all loads to which they are subjected, including eccentric axial loads and lateral forces. Design is to be carried out in accordance with **14.4** unless the wall meets the requirements of **14.5.1**.

**CODE****COMMENTARY****14.3 — Minimum reinforcement**

**14.3.1** — Minimum vertical and horizontal reinforcement shall be in accordance with 14.3.2 and 14.3.3 unless a greater amount is required for shear by 11.9.8 and 11.9.9.

**14.3.2** — Minimum ratio of vertical reinforcement area to gross concrete area,  $\rho_v$ , shall be:

- (a) 0.0012 for deformed bars not larger than No. 5 with  $f_y$  not less than 60,000 psi; or
- (b) 0.0015 for other deformed bars; or
- (c) 0.0012 for welded wire reinforcement not larger than W31 or D31.

**14.3.3** — Minimum ratio of horizontal reinforcement area to gross concrete area,  $\rho_h$ , shall be:

- (a) 0.0020 for deformed bars not larger than No. 5 with  $f_y$  not less than 60,000 psi; or
- (b) 0.0025 for other deformed bars; or
- (c) 0.0020 for welded wire reinforcement not larger than W31 or D31.

**14.3.4** — Walls more than 10 in. thick, except basement walls, shall have reinforcement for each direction placed in two layers parallel with faces of wall in accordance with the following:

- (a) One layer consisting of not less than one-half and not more than two-thirds of total reinforcement required for each direction shall be placed not less than 2 in. nor more than one-third the thickness of wall from the exterior surface;
- (b) The other layer, consisting of the balance of required reinforcement in that direction, shall be placed not less than 3/4 in. nor more than one-third the thickness of wall from the interior surface.

**14.3.5** — Vertical and horizontal reinforcement shall not be spaced farther apart than three times the wall thickness, nor farther apart than 18 in.

**14.3.6** — Vertical reinforcement need not be enclosed by transverse ties if vertical reinforcement area is not greater than 0.01 times gross concrete area, or where vertical reinforcement is not required as compression reinforcement.

**14.3.7** — In addition to the minimum reinforcement required by 14.3.1, not less than two No. 5 bars in walls having two layers of reinforcement in both directions

**R14.3 — Minimum reinforcement**

The requirements of 14.3 are similar to those in previous codes. These apply to walls designed according to 14.4, 14.5, or 14.8. For walls resisting horizontal shear forces in the plane of the wall, reinforcement designed according to 11.9.9.2 and 11.9.9.4 may exceed the minimum reinforcement in 14.3.

The notation used to identify the direction of the distributed reinforcement in walls was updated in 2005 to eliminate conflicts between the notation used for ordinary structural walls in Chapters 11 and 14 and the notation used for special structural walls in Chapter 21. The distributed reinforcement is now identified as being oriented parallel to either the longitudinal or transverse axis of the wall. Therefore, for vertical wall segments, the notation used to describe the horizontal distributed reinforcement ratio is  $\rho_h$ , and the notation used to describe the vertical distributed reinforcement ratio is  $\rho_v$ .

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and one No. 5 bar in walls having a single layer of reinforcement in both directions shall be provided around window, door, and similar sized openings. Such bars shall be anchored to develop  $f_y$  in tension at the corners of the openings.

#### **14.4 — Walls designed as compression members**

Except as provided in 14.5, walls subject to axial load or combined flexure and axial load shall be designed as compression members in accordance with provisions of 10.2, 10.3, 10.10, 10.11, 10.14, 14.2, and 14.3.

#### **14.5 — Empirical design method**

**14.5.1** — Walls of solid rectangular cross section shall be permitted to be designed by the empirical provisions of 14.5 if the resultant of all factored loads is located within the middle third of the overall thickness of the wall and all limits of 14.2 and 14.3 are satisfied.

**14.5.2** — Design axial strength  $\phi P_n$  of a wall satisfying limitations of 14.5.1 shall be computed by Eq. (14-1) unless designed in accordance with 14.4.

$$\phi P_n = 0.55 \phi f'_c A_g \left[ 1 - \left( \frac{k \ell_c}{32 h} \right)^2 \right] \quad (14-1)$$

where  $\phi$  shall correspond to compression-controlled sections in accordance with 9.3.2.2 and effective length factor  $k$  shall be:

For walls braced top and bottom against lateral translation and

- (a) Restrained against rotation at one or both ends (top, bottom, or both) ..... 0.8
- (b) Unrestrained against rotation at both ends .... 1.0

For walls not braced against lateral translation.....2.0

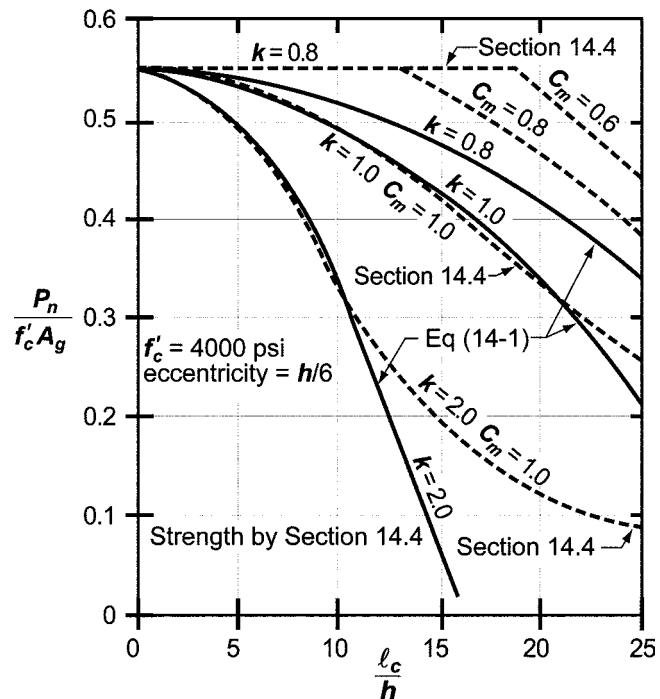
#### **R14.5 — Empirical design method**

The empirical design method applies only to solid rectangular cross sections. All other shapes should be designed according to 14.4.

Eccentric loads and lateral forces are used to determine the total eccentricity of the factored axial force  $P_u$ . When the resultant load for all applicable load combinations falls within the middle third of the wall thickness (eccentricity not greater than  $h/6$ ) at all sections along the length of the undeformed wall, the empirical design method may be used. The design is then carried out considering  $P_u$  as the concentric load. The factored axial force  $P_u$  should be less than or equal to the design axial strength  $\phi P_n$  computed by Eq. (14-1),  $P_u \leq \phi P_n$ .

With the 1980 Code supplement, Eq. (14-1) was revised to reflect the general range of end conditions encountered in wall designs. The wall strength equation in the 1977 Code was based on the assumption of a wall with top and bottom fixed against lateral movement, and with moment restraint at one end corresponding to an effective length factor between 0.8 and 0.9. Axial strength values determined from the original equation were unconservative when compared to test results<sup>14.1</sup> for walls with pinned conditions at both ends, as occurs with some precast and tilt-up applications, or when the top of the wall is not effectively braced against translation, as occurs with free-standing walls or in large structures where significant roof diaphragm deflections occur due to wind and seismic loads. Equation (14-1) gives the same results as the 1977 Code for walls braced against translation and with reasonable base restraint against rotation.<sup>14.2</sup> Values of effective length factors  $k$  are given for commonly occurring wall end conditions. The end condition “restrained against rotation” required for a  $k$  of 0.8 implies attachment to a member having flexural stiffness  $EI/\ell$  at least as large as that of the wall.

The slenderness portion of Eq. (14-1) results in relatively comparable strengths by 14.4 for members loaded at the

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*Fig. R14.5—Empirical design of walls, Eq. (14-1) versus 14.4.*

#### 14.5.3 — Minimum thickness of walls designed by empirical design method

**14.5.3.1** — Thickness of bearing walls shall not be less than 1/25 the supported height or length, whichever is shorter, nor less than 4 in.

**14.5.3.2** — Thickness of exterior basement walls and foundation walls shall not be less than 7-1/2 in.

#### 14.6 — Nonbearing walls

**14.6.1** — Thickness of nonbearing walls shall not be less than 4 in., nor less than 1/30 the least distance between members that provide lateral support.

#### 14.7 — Walls as grade beams

**14.7.1** — Walls designed as grade beams shall have top and bottom reinforcement as required for moment in accordance with provisions of **10.2** through **10.7**. Design for shear shall be in accordance with provisions of **Chapter 11**.

**14.7.2** — Portions of grade beam walls exposed above grade shall also meet requirements of **14.3**.

middle third of the thickness with different braced and restrained end conditions. See Fig. R14.5.

#### R14.5.3 — Minimum thickness of walls designed by empirical design method

The minimum thickness requirements need not be applied to walls designed according to **14.4**.

**CODE****COMMENTARY****14.8 — Alternative design of slender walls**

**14.8.1** — When flexural tension controls the out-of-plane design of a wall, the requirements of 14.8 are considered to satisfy **10.10**.

**14.8.2** — Walls designed by the provisions of 14.8 shall satisfy 14.8.2.1 through 14.8.2.6.

**14.8.2.1** — The wall panel shall be designed as a simply supported, axially loaded member subjected to an out-of-plane uniform lateral load, with maximum moments and deflections occurring at midspan.

**14.8.2.2** — The cross section shall be constant over the height of the panel.

**14.8.2.3** — The wall shall be tension-controlled.

**14.8.2.4** — Reinforcement shall provide a design strength

$$\phi M_n \geq M_{cr} \quad (14-2)$$

where  $M_{cr}$  shall be obtained using the modulus of rupture,  $f_r$ , given by **Eq. (9-10)**.

**14.8.2.5** — Concentrated gravity loads applied to the wall above the design flexural section shall be assumed to be distributed over a width:

- (a) Equal to the bearing width, plus a width on each side that increases at a slope of 2 vertical to 1 horizontal down to the design section; but
- (b) Not greater than the spacing of the concentrated loads; and
- (c) Not extending beyond the edges of the wall panel.

**14.8.2.6** — Vertical stress  $P_u/A_g$  at the midheight section shall not exceed **0.06f'\_c**.

**R14.8 — Alternative design of slender walls**

Section 14.8 was introduced in the 1999 edition and the provisions are based on requirements in the 1997 Uniform Building Code (UBC)<sup>14.3</sup> and experimental research.<sup>14.4</sup> Changes were included in the 2008 edition to reduce differences in the serviceability provisions and ensure that the intent of the UBC provisions is included in future editions of the International Building Code.

The procedure is presented as an alternative to the requirements of **10.10** for the out-of-plane design of slender wall panels, where the panels are restrained against overturning at the top.

Panels that have windows or other large openings are not considered to have constant cross section over the height of the panel. Such walls are to be designed taking into account the effects of openings.

Many aspects of the design of tilt-up walls and buildings are discussed in **References 14.5** and **14.6**.

**R14.8.2.3** — This section was updated in the 2005 Code to reflect the change in design approach that was introduced in 10.3 of the 2002 Code. The previous requirement that the reinforcement ratio should not exceed **0.6ρ<sub>bal</sub>** was replaced by the requirement that the wall be tension-controlled, leading to approximately the same reinforcement ratio.

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**14.8.3** — Design moment strength  $\phi M_n$  for combined flexure and axial loads at midheight shall be

$$\phi M_n \geq M_u \quad (14-3)$$

where

$$M_u = M_{ua} + P_u \Delta_u \quad (14-4)$$

$M_{ua}$  is the maximum factored moment at midheight of wall due to lateral and eccentric vertical loads, not including  $P\Delta$  effects, and  $\Delta_u$  is

$$\Delta_u = \frac{5 M_{ua} l_c^2}{(0.75)48 E_c I_{cr}} \quad (14-5)$$

$M_u$  shall be obtained by iteration of deflections, or by Eq. (14-6).

$$M_u = \frac{M_{ua}}{1 - \frac{5 P_u l_c^2}{(0.75)48 E_c I_{cr}}} \quad (14-6)$$

where

$$I_{cr} = \frac{E_s}{E_c} \left( A_s + \frac{P_u h}{f_y 2d} \right) (d - c)^2 + \frac{l_w c^3}{3} \quad (14-7)$$

and the value of  $E_s/E_c$  shall not be taken less than 6.

**14.8.4** — Maximum out-of-plane deflection,  $\Delta_s$ , due to service loads, including  $P\Delta$  effects, shall not exceed  $\ell_c/150$ .

If  $M_a$ , maximum moment at midheight of wall due to service lateral and eccentric vertical loads, including  $P\Delta$  effects, exceeds  $(2/3)M_{cr}$ ,  $\Delta_s$  shall be calculated by Eq. (14-8)

$$\Delta_s = (2/3)\Delta_{cr} + \frac{(M_a - (2/3)M_{cr})}{(M_n - (2/3)M_{cr})} (\Delta_n - (2/3)\Delta_{cr}) \quad (14-8)$$

If  $M_a$  does not exceed  $(2/3)M_{cr}$ ,  $\Delta_s$  shall be calculated by Eq. (14-9)

$$\Delta_s = \left( \frac{M_a}{M_{cr}} \right) \Delta_{cr} \quad (14-9)$$

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**R14.8.3** — Before the 2008 edition, the effective area of longitudinal reinforcement in a slender wall for obtaining an approximate cracked moment of inertia was calculated using an effective area of tension reinforcement defined as

$$A_{se,w} = A_s + \frac{P_u}{f_y} \quad$$

However, this term overestimated the contribution of axial load in many cases where two layers of reinforcement were used in the slender wall. Therefore, the effective area of longitudinal reinforcement was modified in 2008

$$A_{se,w} = A_s + \frac{P_u}{f_y} \left( \frac{h/2}{d} \right)$$

The neutral axis depth,  $c$ , in Eq. (14-7) corresponds to this effective area of longitudinal reinforcement.

**R14.8.4** — Before the 2008 edition, out-of-plane deflections in wall panels were calculated using the procedures in [Section 9.5.2.3](#). However, re-evaluation of the original test data<sup>14.4</sup> demonstrated that out-of-plane deflections increase rapidly when the service-level moment exceeds  $2/3M_{cr}$ . A linear interpolation between  $\Delta_{cr}$  and  $\Delta_n$  is used to determine  $\Delta_s$  to simplify the design of slender walls if  $M_a > 2/3M_{cr}$ .

Service-level load combinations are not defined in [Chapter 9](#) of ACI 318, but they are discussed in Appendix C of ASCE/SEI 7-10.<sup>14.7</sup> Unlike ACI 318, however, appendixes to ASCE/SEI 7 are not considered to be mandatory parts of the standard. For calculating service-level lateral deflections of structures, Appendix C of ASCE/SEI 7-10 recommends using the following load combination

$$D + 0.5L + W_a$$

in which  $W_a$  is wind load based on serviceability wind speeds provided in the commentary to Appendix C of ASCE/SEI

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where

$$\Delta_{cr} = \frac{5M_{cr}\ell_c^2}{48E_c I_g} \quad (14-10)$$

$$\Delta_n = \frac{5M_n\ell_c^2}{48E_c I_{cr}} \quad (14-11)$$

$I_{cr}$  shall be calculated by Eq. (14-7), and  $M_a$  shall be obtained by iteration of deflections.

7-10. If the slender wall is designed to resist earthquake effects,  $E$ , and  $E$  is based on strength-level earthquake effects, the following load combination is considered to be appropriate for evaluating the service-level lateral deflections

$$D + 0.5L + 0.7E$$

**Notes**

# CHAPTER 15 — FOOTINGS

## CODE

### 15.1 — Scope

**15.1.1** — Provisions of Chapter 15 shall apply for design of isolated footings and, where applicable, to combined footings and mats.

**15.1.2** — Additional requirements for design of combined footings and mats are given in **15.10**.

### 15.2 — Loads and reactions

**15.2.1** — Footings shall be proportioned to resist the factored loads and induced reactions, in accordance with the appropriate design requirements of this Code and as provided in Chapter 15.

**15.2.2** — Base area of footing or number and arrangement of piles shall be determined from unfactored forces and moments transmitted by footing to soil or piles and permissible soil pressure or permissible pile capacity determined through principles of soil mechanics.

**15.2.3** — For footings on piles, computations for moments and shears shall be permitted to be based on the assumption that the reaction from any pile is concentrated at pile center.

## COMMENTARY

### R15.1 — Scope

While the provisions of Chapter 15 apply to isolated footings supporting a single column or wall, most of the provisions are generally applicable to combined footings and mats supporting several columns or walls or a combination thereof.<sup>15.1,15.2</sup>

### R15.2 — Loads and reactions

Footings are required to be proportioned to sustain the applied factored loads and induced reactions, which include axial loads, moments, and shears that have to be resisted at the base of the footing or pile cap.

After the permissible soil pressure or the permissible pile capacity has been determined by principles of soil mechanics and in accordance with the general building code, the size of the base area of a footing on soil or the number and arrangement of the piles should be established on the basis of unfactored (service) loads such as **D**, **L**, **W**, and **E** in whatever combination that governs the design.

Only the computed end moments that exist at the base of a column (or pedestal) need to be transferred to the footing; the minimum moment requirement for slenderness considerations given in **10.10.6.5** need not be considered for transfer of forces and moments to footings.

In cases in which eccentric loads or moments are to be considered, the extreme soil pressure or pile reaction obtained from this loading should be within the permissible values. Similarly, the resultant reactions due to service loads combined with moments, shears, or both, caused by wind or earthquake loads should not exceed the increased values that may be permitted by the general building code.

To proportion a footing or pile cap for strength, the contact soil pressure or pile reaction due to the applied factored loading (see **8.1.1**) should be determined. For a single concentrically loaded spread footing, the soil reaction  $q_s$  due to the factored loading is  $q_s = U/A_f$ , where **U** is the factored concentric load to be resisted by the footing, and  $A_f$  is the base area of the footing as determined by the principles stated in 15.2.2 using the unfactored loads and the permissible soil pressure.

$q_s$  is a calculated reaction to the factored loading used to produce the same required strength conditions regarding flexure, shear, and development of reinforcement in the footing or pile cap, as in any other member.

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**15.3 — Footings supporting circular or regular polygon-shaped columns or pedestals**

For location of critical sections for moment, shear, and development of reinforcement in footings, it shall be permitted to treat circular or regular polygon-shaped concrete columns or pedestals as square members with the same area.

**15.4 — Moment in footings**

**15.4.1** — External moment on any section of a footing shall be determined by passing a vertical plane through the footing, and computing the moment of the forces acting over entire area of footing on one side of that vertical plane.

**15.4.2** — Maximum factored moment,  $M_u$ , for an isolated footing shall be computed as prescribed in 15.4.1 at critical sections located as follows:

- (a) At face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall;
- (b) Halfway between middle and edge of wall, for footings supporting a masonry wall;
- (c) Halfway between face of column and edge of steel base plate, for footings supporting a column with steel base plate.

**15.4.3** — In one-way footings and two-way square footings, reinforcement shall be distributed uniformly across entire width of footing.

**15.4.4** — In two-way rectangular footings, reinforcement shall be distributed in accordance with 15.4.4.1 and 15.4.4.2.

**15.4.4.1** — Reinforcement in long direction shall be distributed uniformly across entire width of footing.

**15.4.4.2** — For reinforcement in short direction, a portion of the total reinforcement,  $\gamma_s A_s$ , shall be distributed uniformly over a band width (centered on centerline of column or pedestal) equal to the length of short side of footing. Remainder of reinforcement required in short direction,  $(1 - \gamma_s) A_s$ , shall be distributed uniformly outside center band width of footing.

In the case of eccentric loading, load factors may cause eccentricities and reactions that are different from those obtained by unfactored loads.

**R15.4 — Moment in footings**

**R15.4.4** — In previous Codes, the reinforcement in the short direction of rectangular footings should be distributed so that an area of steel given by Eq. (15-1) is provided in a band width equal to the length of the short side of the footing. The band width is centered about the column centerline.

The remaining reinforcement required in the short direction is to be distributed equally over the two segments outside the band width, one-half to each segment.

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$$\gamma_s = \frac{2}{(\beta + 1)} \quad (15-1)$$

where  $\beta$  is ratio of long to short sides of footing.

**15.5 — Shear in footings**

**15.5.1** — Shear strength of footings supported on soil or rock shall be in accordance with [11.11](#).

**15.5.2** — Location of critical section for shear in accordance with [Chapter 11](#) shall be measured from face of column, pedestal, or wall, for footings supporting a column, pedestal, or wall. For footings supporting a column or pedestal with steel base plates, the critical section shall be measured from location defined in [15.4.2\(c\)](#).

**15.5.3** — Where the distance between the axis of any pile to the axis of the column is more than two times the distance between the top of the pile cap and the top of the pile, the pile cap shall satisfy [11.11](#) and [15.5.4](#). Other pile caps shall satisfy either [Appendix A](#), or both [11.11](#) and [15.5.4](#). If [Appendix A](#) is used, the effective concrete compression strength of the struts,  $f_{ce}$ , shall be determined using [A.3.2.2\(b\)](#).

**15.5.4** — Computation of shear on any section through a footing supported on piles shall be in accordance with [15.5.4.1](#), [15.5.4.2](#), and [15.5.4.3](#).

**15.5.4.1** — Entire reaction from any pile with its center located  $d_{pile}/2$  or more outside the section shall be considered as producing shear on that section.

**15.5.4.2** — Reaction from any pile with its center located  $d_{pile}/2$  or more inside the section shall be considered as producing no shear on that section.

**15.5.4.3** — For intermediate positions of pile center, the portion of the pile reaction to be considered as producing shear on the section shall be based on straight-line interpolation between full value at  $d_{pile}/2$  outside the section and zero value at  $d_{pile}/2$  inside the section.

**R15.5 — Shear in footings**

**R15.5.1 and R15.5.2** — The shear strength of footings are determined for the more severe condition of [11.11.1.1](#) or [11.11.1.2](#). The critical section for shear is measured from the face of supported member (column, pedestal, or wall), except for supported members on steel base plates.

Computation of shear requires that the soil reaction  $q_s$  be obtained from the factored loads and the design be in accordance with the appropriate equations of [Chapter 11](#).

Where necessary, shear around individual piles may be investigated in accordance with [11.11.1.2](#). If shear perimeters overlap, the modified critical perimeter  $b_o$  should be taken as that portion of the smallest envelope of individual shear perimeter that will actually resist the critical shear for the group under consideration. One such situation is illustrated in Fig. R15.5.

**R15.5.3** — Pile caps supported on piles in more than one plane can be designed using three-dimensional strut-and-tie models satisfying [Appendix A](#).<sup>15.3</sup> The effective concrete compressive strength is from [A.3.2.2\(b\)](#) because it is generally not feasible to provide confining reinforcement satisfying [A.3.3.1](#) and [A.3.3.2](#) in a pile cap.

**R15.5.4** — When piles are located inside the critical sections  $d$  or  $d/2$  from face of column, for one-way or two-way shear, respectively, an upper limit on the shear strength at a section adjacent to the face of the column should be considered. The *CRSI Handbook*<sup>15.4</sup> offers guidance for this situation.

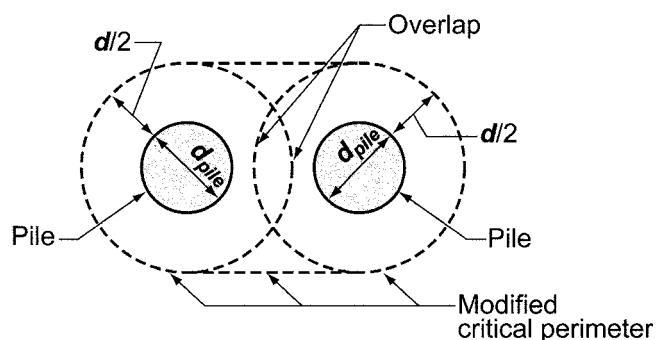


Fig. R15.5—Modified critical perimeter for shear with overlapping critical perimeters.

**CODE****COMMENTARY****15.6 — Development of reinforcement in footings**

**15.6.1** — Development of reinforcement in footings shall be in accordance with **Chapter 12**.

**15.6.2** — Calculated tension or compression in reinforcement at each section shall be developed on each side of that section by embedment length, hook (tension only) or mechanical device, or a combination thereof.

**15.6.3** — Critical sections for development of reinforcement shall be assumed at the same locations as defined in **15.4.2** for maximum factored moment, and at all other vertical planes where changes of section or reinforcement occur. See also **12.10.6**.

**15.7 — Minimum footing depth**

Depth of footing above bottom reinforcement shall not be less than 6 in. for footings on soil, or less than 12 in. for footings on piles.

**15.8 — Transfer of force at base of column, wall, or reinforced pedestal**

**15.8.1** — Forces and moments at base of column, wall, or pedestal shall be transferred to supporting pedestal or footing by bearing on concrete and by reinforcement, dowels, and mechanical connectors.

**15.8.1.1** — Bearing stress on concrete at contact surface between supported and supporting member shall not exceed concrete bearing strength for either surface as given by **10.14**.

**R15.8 — Transfer of force at base of column, wall, or reinforced pedestal**

Section 15.8 provides the specific requirements for force transfer from a column, wall, or pedestal (supported member) to a pedestal or footing (supporting member). Force transfer should be by bearing on concrete (compressive force only) and by reinforcement (tensile or compressive force). Reinforcement may consist of extended longitudinal bars, dowels, anchor bolts, or suitable mechanical connectors.

The requirements of 15.8.1 apply to both cast-in-place construction and precast construction. Additional requirements for cast-in-place construction are given in **15.8.2**. **Section 15.8.3** gives additional requirements for precast construction.

**R15.8.1.1** — Compressive force may be transmitted to a supporting pedestal or footing by bearing on concrete. For strength design, allowable bearing stress on the loaded area is equal to **0.85 $\phi f'_c$** , if the loaded area is equal to the area on which it is supported.

In the common case of a column bearing on a footing larger than the column, bearing strength should be checked at the base of the column and the top of the footing. Strength in the lower part of the column should be checked since the column reinforcement cannot be considered effective near the column base because the force in the reinforcement is not developed for some distance above the base, unless dowels are provided, or the column reinforcement is

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**15.8.1.2** — Reinforcement, dowels, or mechanical connectors between supported and supporting members shall be adequate to transfer:

- (a) All compressive force that exceeds concrete bearing strength of either member;
- (b) Any computed tensile force across interface.

In addition, reinforcement, dowels, or mechanical connectors shall satisfy 15.8.2 or 15.8.3.

**15.8.1.3** — If calculated moments are transferred to supporting pedestal or footing, then reinforcement, dowels, or mechanical connectors shall be adequate to satisfy 12.17.

**15.8.1.4** — Lateral forces shall be transferred to supporting pedestal or footing in accordance with shear-friction provisions of 11.6, or by other appropriate means.

**15.8.2** — In cast-in-place construction, reinforcement required to satisfy 15.8.1 shall be provided either by extending longitudinal bars into supporting pedestal or footing, or by dowels.

**15.8.2.1** — For cast-in-place columns and pedestals, area of reinforcement across interface shall be not less than **0.005A<sub>g</sub>**, where **A<sub>g</sub>** is the gross area of the supported member.

**15.8.2.2** — For cast-in-place walls, area of reinforcement across interface shall be not less than minimum vertical reinforcement given in 14.3.2.

**15.8.2.3** — At footings, it shall be permitted to lap splice No. 14 and No. 18 longitudinal bars, in compression only, with dowels to provide reinforcement required to satisfy 15.8.1. Dowels shall not be larger than No. 11 bar and shall extend into supported

extended into the footing. The unit bearing stress on the column will normally be **0.85φf'<sub>c</sub>**. The permissible bearing strength on the footing may be increased in accordance with 10.14 and will usually be two times **0.85φf'<sub>c</sub>**. The compressive force that exceeds that developed by the permissible bearing strength at the base of the column or at the top of the footing should be carried by dowels or extended longitudinal bars.

**R15.8.1.2** — All tensile forces, whether created by uplift, moment, or other means, should be transferred to supporting pedestal or footing entirely by reinforcement or suitable mechanical connectors. Generally, mechanical connectors would be used only in precast construction.

**R15.8.1.3** — If computed moments are transferred from the column to the footing, the concrete in the compression zone of the column will be stressed to **0.85φf'<sub>c</sub>** under factored load conditions and, as a result, all the reinforcement will generally have to be doweled into the footing.

**R15.8.1.4** — The shear-friction method given in 11.6 may be used to check for transfer of lateral forces to supporting pedestal or footing. Shear keys may be used, provided that the reinforcement crossing the joint satisfies 15.8.2.1, 15.8.3.1, and the shear-friction requirements of 11.6. In precast construction, resistance to lateral forces may be provided by shear-friction, shear keys, or mechanical devices.

**R15.8.2.1 and R15.8.2.2** — A minimum amount of reinforcement is required between all supported and supporting members to ensure ductile behavior. The Code does not require that all bars in a column be extended through and be anchored into a footing. However, reinforcement with an area of 0.005 times the column area or an equal area of properly spliced dowels is required to extend into the footing with proper anchorage. This reinforcement is required to provide a degree of structural integrity during the construction stage and during the life of the structure.

**R15.8.2.3** — Lap splices of No. 14 and No. 18 longitudinal bars in compression only to dowels from a footing are specifically permitted in 15.8.2.3. The dowel bars should be No. 11 or smaller in size. The dowel lap splice length should meet the larger of the two criteria: (a) be able to transfer the

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member a distance not less than the larger of  $\ell_{dc}$ , of No. 14 or No. 18 bars and compression lap splice length of the dowels, whichever is greater, and into the footing a distance not less than  $\ell_{dc}$  of the dowels.

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stress in the No. 14 and No. 18 bars, and (b) fully develop the stress in the dowels as a splice.

**15.8.2.4** — If a pinned or rocker connection is provided in cast-in-place construction, connection shall conform to **15.8.1** and 15.8.3.

**15.8.3** — In precast construction, anchor bolts or suitable mechanical connectors shall be permitted for satisfying **15.8.1**. Anchor bolts shall be designed in accordance with **Appendix D**.

**15.8.3.1** — Connection between precast columns or pedestals and supporting members shall meet the requirements of **16.5.1.3(a)**.

**15.8.3.2** — Connection between precast walls and supporting members shall meet the requirements of **16.5.1.3(b)** and **(c)**.

**15.8.3.3** — Anchor bolts and mechanical connections shall be designed to reach their design strength before anchorage failure or failure of surrounding concrete. Anchor bolts shall be designed in accordance with **Appendix D**.

## **15.9 — Sloped or stepped footings**

**15.9.1** — In sloped or stepped footings, angle of slope or depth and location of steps shall be such that design requirements are satisfied at every section. (See also **12.10.6**.)

**15.9.2** — Sloped or stepped footings designed as a unit shall be constructed to ensure action as a unit.

This provision is an exception to **12.14.2.1**, which prohibits lap splicing of No. 14 and No. 18 bars. This exception results from many years of successful experience with the lap splicing of these large column bars with footing dowels of the smaller size. The reason for the restriction on dowel bar size is recognition of the anchorage length problem of the large bars, and to allow use of the smaller size dowels. A similar exception is allowed for compression splices between different size bars in **12.16.2**.

**R15.8.3.1 and R15.8.3.2** — For cast-in-place columns, **15.8.2.1** requires a minimum area of reinforcement equal to **0.005A<sub>g</sub>** across the column-footing interface to provide some degree of structural integrity. For precast columns, this requirement is expressed in terms of an equivalent tensile force that should be transferred. Thus, across the joint,  $A_s f_y = 200A_g$  [see **16.5.1.3(a)**]. The minimum tensile strength required for precast wall-to-footing connection [see **16.5.1.3(b)**] is somewhat less than that required for columns, since an overload would be distributed laterally and a sudden failure would be less likely. Since the tensile strength values of **16.5.1.3** have been arbitrarily chosen, it is not necessary to include a strength reduction factor  $\phi$  for these calculations.

**CODE****COMMENTARY****15.10 — Combined footings and mats**

**15.10.1** — Footings supporting more than one column, pedestal, or wall (combined footings or mats) shall be proportioned to resist the factored loads and induced reactions, in accordance with appropriate design requirements of the code.

**R15.10 — Combined footings and mats**

**R15.10.1** — Any reasonable assumption with respect to the distribution of soil pressure or pile reactions can be used as long as it is consistent with the type of structure and the properties of the soil, and conforms with established principles of soil mechanics (see [Reference 15.1](#)). Similarly, as prescribed in [15.2.2](#) for isolated footings, the base area or pile arrangement of combined footings and mats should be determined using the unfactored forces, moments, or both, transmitted by the footing to the soil, considering permissible soil pressures and pile reactions.

Design methods using factored loads and strength reduction factors  $\phi$  can be applied to combined footings or mats, regardless of the soil pressure distribution.

Detailed recommendations for design of combined footings and mats are reported by ACI Committee 336.<sup>15.1</sup> See also [Reference 15.2](#).

**15.10.2** — The direct design method of [Chapter 13](#) shall not be used for design of combined footings and mats.

**15.10.3** — Distribution of soil pressure under combined footings and mats shall be consistent with properties of the soil and the structure and with established principles of soil mechanics.

**15.10.4** — Minimum reinforcing steel in non prestressed mat foundations shall meet the requirements of [7.12.2](#) in each principal direction. Maximum spacing shall not exceed 18 in.

**R15.10.4** — Minimum reinforcing steel may be distributed near the top or bottom of the section, or may be allocated between the two faces of the section as deemed appropriate for specific conditions, such that the total area of continuous reinforcing steel satisfies [7.12.2](#).

**Notes**

# CHAPTER 16 — PRECAST CONCRETE

## CODE

### 16.1 — Scope

**16.1.1** — All provisions of this Code, not specifically excluded and not in conflict with the provisions of Chapter 16, shall apply to structures incorporating precast concrete structural members.

## COMMENTARY

### R16.1 — Scope

**R16.1.1** — See 2.2 for definition of precast concrete.

Design and construction requirements for precast concrete structural members differ in some respects from those for cast-in-place concrete structural members and these differences are addressed in this chapter. Where provisions for cast-in-place concrete applied to precast concrete, they have not been repeated. Similarly, items related to composite concrete in [Chapter 17](#) and to prestressed concrete in [Chapter 18](#) that apply to precast concrete are not restated.

More detailed recommendations concerning precast concrete are given in [References 16.1](#) through [16.7](#). Tilt-up concrete construction is a form of precast concrete. It is recommended that [Reference 16.8](#) be reviewed for tilt-up structures.

## 16.2 — General

**16.2.1** — Design of precast members and connections shall include loading and restraint conditions from initial fabrication to end use in the structure, including form removal, storage, transportation, and erection.

### R16.2 — General

**R16.2.1** — Stresses developed in precast members during the period from casting to final connection may be greater than the service load stresses. Handling procedures may cause undesirable deformations. Care should be given to the methods of storing, transporting, and erecting precast members so that performance at service loads and strength under factored loads meet Code requirements.

**16.2.2** — When precast members are incorporated into a structural system, the forces and deformations occurring in and adjacent to connections shall be included in the design.

**R16.2.2** — The structural behavior of precast members may differ substantially from that of similar members that are cast-in-place. Design of connections to minimize or transmit forces due to shrinkage, creep, temperature change, elastic deformation, differential settlement, wind, and earthquake require consideration in precast construction.

**16.2.3** — Tolerances for both precast members and interfacing members shall be specified. Design of precast members and connections shall include the effects of these tolerances.

**R16.2.3** — Design of precast members and connections is particularly sensitive to tolerances on the dimensions of individual members and on their location in the structure. To prevent misunderstanding, the tolerances used in design should be specified in the contract documents. Instead of specifying individual tolerances, the tolerance standard assumed in design may be specified. It is important to specify any deviations from accepted standards.

The tolerances required by [7.5](#) are considered to be a minimum acceptable standard for reinforcement in precast concrete. Refer to publications of the Precast/Prestressed Concrete Institute (PCI) ([References 16.9](#) through [16.11](#)) for guidance on industry-established standard product and erection tolerances. Added guidance is given in [Reference 16.12](#).

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**16.2.4** — In addition to the requirements for contract documents in 1.2, (a) and (b) shall be included in either the contract documents or shop drawings:

- (a) Details of reinforcement, inserts and lifting devices required to resist temporary loads from handling, storage, transportation, and erection;
- (b) Required concrete strength at stated ages or stages of construction.

**16.3 — Distribution of forces among members**

**16.3.1** — Distribution of forces that are perpendicular to the plane of members shall be established by analysis or by test.

**16.3.2** — Where the system behavior requires in-plane forces to be transferred between the members of a precast floor or wall system, 16.3.2.1 and 16.3.2.2 shall apply.

**16.3.2.1** — In-plane force paths shall be continuous through both connections and members.

**16.3.2.2** — Where tension forces occur, a continuous path of steel or steel reinforcement shall be provided.

**16.4 — Member design**

**16.4.1** — In one-way precast floor and roof slabs and in one-way precast, prestressed wall panels, all not wider than 12 ft, and where members are not mechanically connected to cause restraint in the transverse direction, the shrinkage and temperature reinforcement

**R16.2.4** — The additional requirements may be included in either contract documents or shop drawings, depending on the assignment of responsibility for design.

**R16.3 — Distribution of forces among members**

**R16.3.1** — Concentrated point and line loads can be distributed among members provided they have sufficient torsional stiffness and that shear can be transferred across joints. Torsionally stiff members such as hollow-core or solid slabs have more favorable load distribution properties than do torsionally flexible members such as double tees with thin flanges. The actual distribution of the load depends on many factors discussed in detail in References 16.13 through 16.19. Large openings can cause significant changes in distribution of forces.

**R16.3.2** — In-plane forces result primarily from diaphragm action in floors and roofs, causing tension or compression in the chords and shear in the body of the diaphragm. A continuous path of steel, steel reinforcement, or both, using lap splices, mechanical or welded splices, or mechanical connectors, should be provided to carry the tension, whereas the shear and compression may be carried by the net concrete section. A continuous path of steel through a connection includes bolts, weld plates, headed studs, or other steel devices. Tension forces in the connections are to be transferred to the primary reinforcement in the members.

In-plane forces in precast wall systems result primarily from diaphragm reactions and external lateral loads.

Connection details should provide for the forces and deformations due to shrinkage, creep, and thermal effects. Connection details may be selected to accommodate volume changes and rotations caused by temperature gradients and long-term deflections. When these effects are restrained, connections and members should be designed to provide adequate strength and ductility.

**R16.4 — Member design**

**R16.4.1** — For prestressed concrete members not wider than 12 ft, such as hollow-core slabs, solid slabs, or slabs with closely spaced ribs, there is usually no need to provide transverse reinforcement to withstand shrinkage and temperature stresses in the short direction. This is generally

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requirements of 7.12 in the direction normal to the flexural reinforcement shall be permitted to be waived. This waiver shall not apply to members that require reinforcement to resist transverse flexural stresses.

true also for nonprestressed floor and roof slabs. The 12 ft width is less than that in which shrinkage and temperature stresses can build up to a magnitude requiring transverse reinforcement. In addition, much of the shrinkage occurs before the members are tied into the structure. Once in the final structure, the members are usually not as rigidly connected transversely as monolithic concrete, thus the transverse restraint stresses due to both shrinkage and temperature change are significantly reduced.

The waiver does not apply to members such as single and double tees with thin, wide flanges.

**16.4.2** — For precast, nonprestressed walls the reinforcement shall be designed in accordance with the provisions of Chapters 10 or 14, except that the area of horizontal and vertical reinforcement each shall be not less than **0.001A<sub>g</sub>**, where **A<sub>g</sub>** is the gross cross-sectional area of the wall panel. Spacing of reinforcement shall not exceed 5 times the wall thickness nor 30 in. for interior walls nor 18 in. for exterior walls.

**R16.4.2** — This minimum area of wall reinforcement, instead of the minimum values in 14.3, has been used for many years and is recommended by the PCI<sup>16.4</sup> and the Canadian Building Code.<sup>16.20</sup> The provisions for reduced minimum reinforcement and greater spacing recognize that precast wall panels have very little restraint at their edges during early stages of curing and develop less shrinkage stress than comparable cast-in-place walls.

## 16.5 — Structural integrity

## R16.5 — Structural integrity

**16.5.1** — Except where the provisions of 16.5.2 govern, the minimum provisions of 16.5.1.1 through 16.5.1.4 for structural integrity shall apply to all precast concrete structures.

**R16.5.1** — The provisions of 7.13.3 apply to all precast concrete structures. Sections 16.5.1 and 16.5.2 give minimum requirements to satisfy 7.13.3. It is not intended that these minimum requirements override other applicable provisions of the Code for design of precast concrete structures.

**16.5.1.1** — Longitudinal and transverse ties required by 7.13.3 shall connect members to a lateral load-resisting system.

The overall integrity of a structure can be substantially enhanced by minor changes in the amount, location, and detailing of member reinforcement and in the detailing of connection hardware.

**16.5.1.2** — Where precast elements form floor or roof diaphragms, the connections between diaphragm and those members being laterally supported shall have a nominal tensile strength capable of resisting not less than 300 lb per linear ft.

**R16.5.1.1** — Individual members may be connected into a lateral load-resisting system by alternative methods. For example, a load-bearing spandrel could be connected to a diaphragm (part of the lateral load-resisting system). Structural integrity could be achieved by connecting the spandrel into all or a portion of the deck members forming the diaphragm. Alternatively, the spandrel could be connected only to its supporting columns, which in turn is connected to the diaphragm.

**16.5.1.3** — Vertical tension tie requirements of 7.13.3 shall apply to all vertical structural members, except cladding, and shall be achieved by providing

**R16.5.1.2** — Diaphragms are typically provided as part of the lateral load-resisting system. The ties prescribed in 16.5.1.2 are the minimum required to attach members to the floor or roof diaphragms.

**R16.5.1.3** — Base connections and connections at horizontal joints in precast columns and wall panels, including shear walls, are designed to transfer all design

**CODE**

connections at horizontal joints in accordance with (a) through (c):

- (a) Precast columns shall have a nominal strength in tension not less than  $200A_g$ , in lb. For columns with a larger cross section than required by consideration of loading, a reduced effective area  $A_g$ , based on cross section required but not less than one-half the total area, shall be permitted;
- (b) Precast wall panels shall have a minimum of two ties per panel, with a nominal tensile strength not less than 10,000 lb per tie;
- (c) When design forces result in no tension at the base, the ties required by 16.5.1.3(b) shall be permitted to be anchored into an appropriately reinforced concrete floor slab-on-ground.

**16.5.1.4** — Connection details that rely solely on friction caused by gravity loads shall not be permitted.

**COMMENTARY**

forces and moments. The minimum tie requirements of 16.5.1.3 are not additive to these design requirements. Common practice is to place the wall ties symmetrically about the vertical centerline of the wall panel and within the outer quarters of the panel width, wherever possible.

**16.5.2** — For precast concrete bearing wall structures three or more stories in height, the minimum provisions of 16.5.2.1 through 16.5.2.5 shall apply.

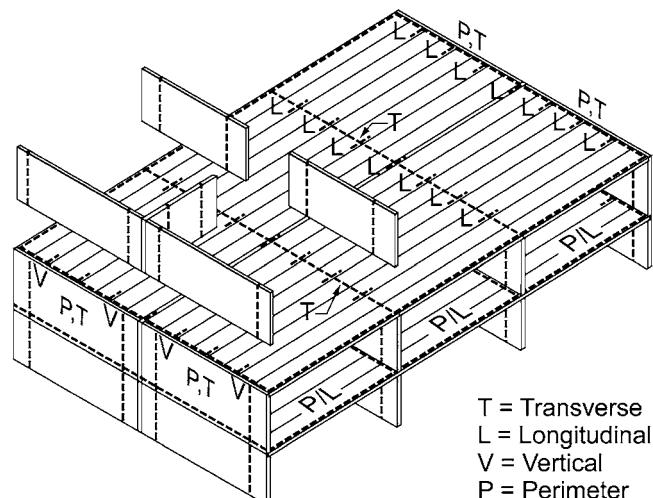
**16.5.2.1** — Longitudinal and transverse ties shall be provided in floor and roof systems to provide a nominal strength of 1500 lb per foot of width or length. Ties shall be provided over interior wall supports and between members and exterior walls. Ties shall be positioned in or within 2 ft of the plane of the floor or roof system.

**R16.5.1.4** — In the event of damage to a beam, it is important that displacement of its supporting members be minimized, so that other members will not lose their load-carrying ability. This situation shows why connection details that rely solely on friction caused by gravity loads are not used. An exception could be heavy modular unit structures (one or more cells in cell-type structures) where resistance to overturning or sliding in any direction has a large factor of safety. Acceptance of such systems should be based on the provisions of 1.4.

**R16.5.2** — The structural integrity minimum tie provisions for bearing wall structures, often called large panel structures, are intended to provide catenary hanger supports in case of loss of a bearing wall support, as shown by test.<sup>16.21</sup> Forces induced by loading, temperature change, creep, and wind or seismic action may require a larger amount of tie force. It is intended that the general precast concrete provisions of 16.5.1 apply to bearing wall structures less than three stories in height.

Minimum ties in structures three or more stories in height, in accordance with 16.5.2.1, 16.5.2.2, 16.5.2.3, 16.5.2.4, and 16.5.2.5, are required for structural integrity (Fig. R16.5.2). These provisions are based on PCI's recommendations for design of precast concrete bearing wall buildings.<sup>16.22</sup> Tie strength is based on yield strength.

**R16.5.2.1** — Longitudinal ties may project from slabs and be lap spliced, welded, or mechanically connected, or they may be embedded in grout joints, with sufficient length and cover to develop the required force. Bond length for unstressed prestressing steel should be sufficient to develop the yield strength.<sup>16.23</sup> It is not uncommon to have ties positioned in the walls reasonably close to the plane of the floor or roof system.

**CODE****COMMENTARY**

*Fig. R16.5.2—Typical arrangement of tensile ties in large panel structures.*

**16.5.2.2** — Longitudinal ties parallel to floor or roof slab spans shall be spaced not more than 10 ft on centers. Provisions shall be made to transfer forces around openings.

**16.5.2.3** — Transverse ties perpendicular to floor or roof slab spans shall be spaced not greater than the bearing wall spacing.

**16.5.2.4** — Ties around the perimeter of each floor and roof, within 4 ft of the edge, shall provide a nominal strength in tension not less than 16,000 lb.

**16.5.2.5** — Vertical tension ties shall be provided in all walls and shall be continuous over the height of the building. They shall provide a nominal tensile strength not less than 3000 lb per horizontal foot of wall. Not less than two ties shall be provided for each precast panel.

## 16.6 — Connection and bearing design

**16.6.1** — Forces shall be permitted to be transferred between members by grouted joints, shear keys, mechanical connectors, reinforcing steel connections, reinforced topping, or a combination of these means.

**16.6.1.1** — The adequacy of connections to transfer forces between members shall be determined by analysis or by test. Where shear is the primary result of imposed loading, it shall be permitted to use the provisions of **11.6** as applicable.

**16.6.1.2** — When designing a connection using materials with different structural properties, their relative stiffnesses, strengths, and ductilities shall be considered.

**R16.5.2.3** — Transverse ties may be uniformly spaced either encased in the panels or in a topping, or they may be concentrated at the transverse bearing walls.

**R16.5.2.4** — The perimeter tie requirements need not be additive with the longitudinal and transverse tie requirements.

## R16.6 — Connection and bearing design

**R16.6.1** — The Code permits a variety of methods for connecting members. These are intended for transfer of forces both in-plane and perpendicular to the plane of the members.

**R16.6.1.2** — Various components in a connection (such as bolts, welds, plates, and inserts) have different properties that can affect the overall behavior of the connection.

**CODE****COMMENTARY**

**16.6.2** — Bearing for precast floor and roof members on simple supports shall satisfy 16.6.2.1 and 16.6.2.2.

**16.6.2.1** — The allowable bearing stress at the contact surface between supported and supporting members and between any intermediate bearing elements shall not exceed the bearing strength for either surface or the bearing element, or both. Concrete bearing strength shall be as given in 10.14.

**16.6.2.2** — Unless shown by test or analysis that performance will not be impaired, (a) and (b) shall be met:

(a) Each member and its supporting system shall have design dimensions selected so that, after consideration of tolerances, the distance from the edge of the support to the end of the precast member in the direction of the span is at least  $\ell_n/180$ , but not less than:

For solid or hollow-core slabs..... 2 in.  
For beams or stemmed members ..... 3 in.

(b) Bearing pads at unarmored edges shall be set back a minimum of 1/2 in. from the face of the support, or at least the chamfer dimension at chamfered edges.

**16.6.2.3** — The requirements of 12.11.1 shall not apply to the positive bending moment reinforcement for statically determinate precast members, but at least one-third of such reinforcement shall extend to the center of the bearing length, taking into account permitted tolerances in 7.5.2.2 and 16.2.3.

**R16.6.2.1** — When tensile forces occur in the plane of the bearing, it may be desirable to reduce the allowable bearing stress, provide confinement reinforcement, or both. Guidelines are provided in Reference 16.4.

**R16.6.2.2** — This section differentiates between bearing length and length of the end of a precast member over the support (Fig. R16.6.2). Bearing pads distribute concentrated loads and reactions over the bearing area, and allow limited horizontal and rotational movements for stress relief. To prevent spalling under heavily loaded bearing areas, bearing pads should not extend to the edge of the support unless the edge is armored. Edges can be armored with anchored steel plates or angles. Section 11.9.7 gives requirements for bearing on brackets or corbels.

**R16.6.2.3** — It is unnecessary to develop positive bending moment reinforcement beyond the ends of the precast element if the system is statically determinate. Tolerances need to be considered to avoid bearing on structural plain concrete where reinforcement has been discontinued.

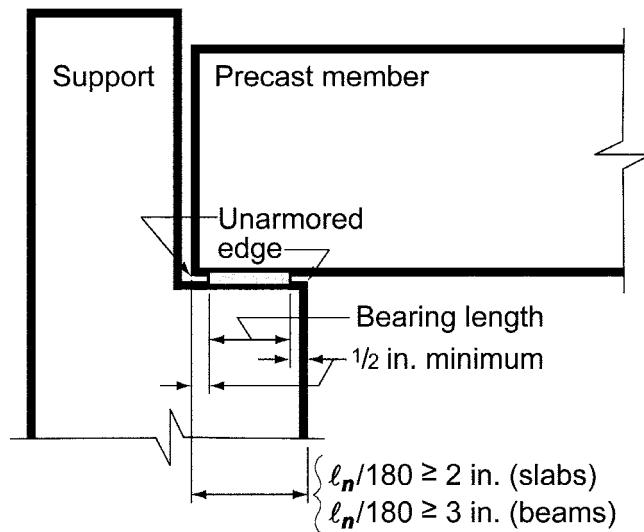


Fig. R16.6.2—Bearing length on support.

**CODE****COMMENTARY****16.7 — Items embedded after concrete placement**

**16.7.1** — When approved by the licensed design professional, embedded items (such as dowels or inserts) that either protrude from the concrete or remain exposed for inspection shall be permitted to be embedded while the concrete is in a plastic state provided that 16.7.1.1, 16.7.1.2, and 16.7.1.3 are met.

**16.7.1.1** — Embedded items are not required to be hooked or tied to reinforcement within the concrete.

**16.7.1.2** — Embedded items are maintained in the correct position while the concrete remains plastic.

**16.7.1.3** — The concrete is properly consolidated around the embedded item.

**16.8 — Marking and identification**

**16.8.1** — Each precast member shall be marked to indicate its location and orientation in the structure and date of manufacture.

**16.8.2** — Identification marks shall correspond to placing drawings.

**16.9 — Handling**

**16.9.1** — Member design shall consider forces and distortions during curing, stripping, storage, transportation, and erection so that precast members are not overstressed or otherwise damaged.

**16.9.2** — During erection, precast members and structures shall be adequately supported and braced to ensure proper alignment and structural integrity until permanent connections are completed.

**16.10 — Strength evaluation of precast construction**

**16.10.1** — A precast element to be made composite with cast-in-place concrete shall be permitted to be tested in flexure as a precast element alone in accordance with **16.10.1.1** and **16.10.1.2**.

**R16.7 — Items embedded after concrete placement**

**R16.7.1** — Section 16.7.1 is an exception to the provisions of **7.5.1**. Many precast products are manufactured in such a way that it is difficult, if not impossible, to position reinforcement that protrudes from the concrete before the concrete is placed. Such items as ties for horizontal shear and inserts can be placed while the concrete is plastic, if proper precautions are taken. This exception is not applicable to reinforcement that is completely embedded, or to embedded items that will be hooked or tied to embedded reinforcement.

**R16.9 — Handling**

**R16.9.1** — The Code requires acceptable performance at service loads and adequate strength under factored loads. However, handling loads should not produce permanent stresses, strains, cracking, or deflections inconsistent with the provisions of the Code. A precast member should not be rejected for minor cracking or spalling where strength and durability are not affected. Guidance on assessing cracks is given in PCI reports on fabrication and shipment cracks.<sup>16.24,16.25</sup>

**R16.9.2** — All temporary erection connections, bracing, shoring as well as the sequencing of removal of these items are shown on contract or erection drawings.

**R16.10 — Strength evaluation of precast construction**

The strength evaluation procedures of **Chapter 20** are applicable to precast members.

<b>CODE</b>	<b>COMMENTARY</b>
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**16.10.1.1** — Test loads shall be applied only when calculations indicate the isolated precast element will not be critical in compression or buckling.

**16.10.1.2** — The test load shall be that load which, when applied to the precast member alone, induces the same total force in the tension reinforcement as would be induced by loading the composite member with the test load required by **20.3.2**.

**16.10.2** — The provisions of **20.5** shall be the basis for acceptance or rejection of the precast element.

# CHAPTER 17 — COMPOSITE CONCRETE FLEXURAL MEMBERS

## CODE

### 17.1 — Scope

**17.1.1** — Provisions of Chapter 17 shall apply for design of composite concrete flexural members defined as precast concrete, cast-in-place concrete elements, or both, constructed in separate placements but so interconnected that all elements respond to loads as a unit.

**17.1.2** — All provisions of the Code shall apply to composite concrete flexural members, except as specifically modified in Chapter 17.

### 17.2 — General

**17.2.1** — The use of an entire composite member or portions thereof for resisting shear and moment shall be permitted.

**17.2.2** — Individual elements shall be investigated for all critical stages of loading.

**17.2.3** — If the specified strength, unit weight, or other properties of the various elements are different, properties of the individual elements or the most critical values shall be used in design.

**17.2.4** — In strength computations of composite members, no distinction shall be made between shored and unshored members.

**17.2.5** — All elements shall be designed to support all loads introduced prior to full development of design strength of composite members.

**17.2.6** — Reinforcement shall be provided as required to minimize cracking and to prevent separation of individual elements of composite members.

**17.2.7** — Composite members shall meet requirements for control of deflections in accordance with **9.5.5**.

## COMMENTARY

### R17.1 — Scope

**R17.1.1** — The scope of Chapter 17 is intended to include all types of composite concrete flexural members. In some cases with fully cast-in-place concrete, it may be necessary to design the interface of consecutive placements of concrete as required for composite members. Composite structural steel-concrete members are not covered in this chapter. Design provisions for such composite members are covered in **Reference 17.1**.

### R17.2 — General

**R17.2.4** — Tests have indicated that the strength of a composite member is the same whether or not the first element cast is shored during casting and curing of the second element.

**R17.2.6** — The extent of cracking is dependent on such factors as environment, aesthetics, and occupancy. In addition, composite action should not be impaired.

**R17.2.7** — The premature loading of precast elements can cause excessive creep and shrinkage deflections. This is especially so at early ages when the moisture content is high and the strength low.

The transfer of shear by direct bond is important if excessive deflection from slippage is to be prevented. A shear key is an added mechanical factor of safety but it does not operate until slippage occurs.

**CODE****COMMENTARY****17.3 — Shoring**

When used, shoring shall not be removed until supported elements have developed design properties required to support all loads and limit deflections and cracking at time of shoring removal.

**17.4 — Vertical shear strength**

**17.4.1** — Where an entire composite member is assumed to resist vertical shear, design shall be in accordance with requirements of **Chapter 11** as for a monolithically cast member of the same cross-sectional shape.

**17.4.2** — Shear reinforcement shall be fully anchored into interconnected elements in accordance with **12.13**.

**17.4.3** — Extended and anchored shear reinforcement shall be permitted to be included as ties for horizontal shear.

**17.5 — Horizontal shear strength**

**17.5.1** — In a composite member, full transfer of horizontal shear forces shall be ensured at contact surfaces of interconnected elements.

**17.5.2** — For the provisions of 17.5, **d** shall be taken as the distance from extreme compression fiber for entire composite section to centroid of prestressed and nonprestressed longitudinal tension reinforcement, if any, but need not be taken less than **0.80h** for prestressed concrete members.

**17.5.3** — Unless calculated in accordance with **17.5.4**, design of cross sections subject to horizontal shear shall be based on

$$V_u \leq \phi V_{nh} \quad (17-1)$$

where **V<sub>nh</sub>** is nominal horizontal shear strength in accordance with 17.5.3.1 through **17.5.3.4**.

**17.5.3.1** — Where contact surfaces are clean, free of laitance, and intentionally roughened, **V<sub>nh</sub>** shall not be taken greater than **80b<sub>v</sub>d**.

**17.5.3.2** — Where minimum ties are provided in accordance with **17.6**, and contact surfaces are clean and free of laitance, but not intentionally roughened, **V<sub>nh</sub>** shall not be taken greater than **80b<sub>v</sub>d**.

**R17.3 — Shoring**

The provisions of **9.5.5** cover the requirements pertaining to deflections of shored and unshored members.

**R17.5 — Horizontal shear strength**

**R17.5.1** — Full transfer of horizontal shear between segments of composite members should be ensured by horizontal shear strength at contact surfaces or properly anchored ties, or both.

**R17.5.2** — Prestressed members used in composite construction may have variations in depth of tension reinforcement along member length due to draped or depressed tendons. Because of this variation, the definition of **d** used in **Chapter 11** for determination of vertical shear strength is also appropriate when determining horizontal shear strength.

**R17.5.3** — The nominal horizontal shear strengths **V<sub>nh</sub>** apply when the design is based on the load factors and  $\phi$ -factors of **Chapter 9**.

**CODE**

**17.5.3.3** — Where ties are provided in accordance with 17.6, and contact surfaces are clean, free of laitance, and intentionally roughened to a full amplitude of approximately 1/4 in.,  $V_{nh}$  shall be taken equal to  $(260 + 0.6\rho_v f_y)\lambda b_v d$ , but not greater than  $500b_v d$ . Values for  $\lambda$  in 11.6.4.3 shall apply and  $\rho_v$  is  $A_v/(b_v s)$ .

**17.5.3.4** — Where  $V_u$  at section considered exceeds  $\phi(500b_v d)$ , design for horizontal shear shall be in accordance with 11.6.4.

**17.5.4** — As an alternative to 17.5.3, horizontal shear shall be permitted to be determined by computing the actual change in compressive or tensile force in any segment, and provisions shall be made to transfer that force as horizontal shear to the supporting element. The factored horizontal shear force  $V_u$  shall not exceed horizontal shear strength  $\phi V_{nh}$  as given in 17.5.3.1 through 17.5.3.4, where area of contact surface shall be substituted for  $b_v d$ .

**17.5.4.1** — Where ties provided to resist horizontal shear are designed to satisfy 17.5.4, the tie area to tie spacing ratio along the member shall approximately reflect the distribution of shear forces in the member.

**17.5.5** — Where tension exists across any contact surface between interconnected elements, shear transfer by contact shall be permitted only when minimum ties are provided in accordance with 17.6.

**17.6 — Ties for horizontal shear**

**17.6.1** — Where ties are provided to transfer horizontal shear, tie area shall not be less than that required by 11.4.6.3, and tie spacing shall not exceed four times the least dimension of supported element, nor exceed 24 in.

**17.6.2** — Ties for horizontal shear shall consist of single bars or wire, multiple leg stirrups, or vertical legs of welded wire reinforcement.

**17.6.3** — All ties shall be fully anchored into interconnected elements in accordance with 12.13.

**COMMENTARY**

**R17.5.3.3** — The permitted horizontal shear strengths and the requirement of 1/4 in. amplitude for intentional roughness are based on tests discussed in References 17.2 through 17.4.

**R17.5.4.1** — The distribution of horizontal shear stresses along the contact surface in a composite member will reflect the distribution of shear along the member. Horizontal shear failure will initiate where the horizontal shear stress is a maximum and will spread to regions of lower stress. Because the slip at peak horizontal shear resistance is small for a concrete-to-concrete contact surface, longitudinal redistribution of horizontal shear resistance is very limited. The spacing of the ties along the contact surface should, therefore, be such as to provide horizontal shear resistance distributed approximately as the shear acting on the member is distributed.

**R17.5.5** — Proper anchorage of ties extending across interfaces is required to maintain contact of the interfaces.

**R17.6 — Ties for horizontal shear**

The minimum areas and maximum spacings are based on test data given in References 17.2 through 17.6.

**Notes**

# CHAPTER 18 — PRESTRESSED CONCRETE

## CODE

### 18.1 — Scope

**18.1.1** — Provisions of Chapter 18 shall apply to members prestressed with wire, strands, or bars conforming to provisions for prestressing steel in 3.5.6.

**18.1.2** — All provisions of this Code not specifically excluded, and not in conflict with provisions of Chapter 18, shall apply to prestressed concrete.

**18.1.3** — The following provisions of this Code shall not apply to prestressed concrete, except as specifically noted: Sections 6.4.4, 7.6.5, 8.12.2, 8.12.3, 8.12.4, 8.13, 10.5, 10.6, 10.9.1, and 10.9.2; Chapter 13; and Sections 14.3, 14.5, and 14.6, except that certain sections of 10.6 apply as noted in 18.4.4.

## COMMENTARY

### R18.1 — Scope

**R18.1.1** — The provisions of Chapter 18 were developed primarily for structural members such as slabs, beams, and columns that are commonly used in buildings. Many of the provisions may be applied to other types of construction, such as pressure vessels, pavements, pipes, and crossties. Application of the provisions is left to the judgment of the licensed design professional in cases not specifically cited in the Code.

**R18.1.3** — Some sections of the Code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides explanation for such exclusions:

**Section 6.4.4** — Tendons of continuous post-tensioned beams and slabs are usually stressed at a point along the span where the tendon profile is at or near the centroid of the concrete cross section. Therefore, interior construction joints are usually located within the end thirds of the span, rather than the middle third of the span as required by 6.4.4. Construction joints located as described in continuous post-tensioned beams and slabs have a long history of satisfactory performance. Thus, 6.4.4 is excluded from application to prestressed concrete.

**Section 7.6.5** — Section 7.6.5 of the Code is excluded from application to prestressed concrete because the requirements for bonded reinforcement and unbonded tendons for cast-in-place members are provided in 18.9 and 18.12, respectively.

**Sections 8.12.2, 8.12.3, and 8.12.4** — The empirical provisions of 8.12.2, 8.12.3, and 8.12.4 for T-beams were developed for nonprestressed reinforced concrete, and if applied to prestressed concrete would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding 8.12.2, 8.12.3, and 8.12.4, no special requirements for prestressed concrete T-beams appear in the Code. Instead, the determination of an effective width of flange is left to the experience and judgment of the licensed design professional. Where possible, the flange widths in 8.12.2, 8.12.3, and 8.12.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 8.12.2.

**CODE****COMMENTARY**

**Sections 8.12.1 and 8.12.5** provide general requirements for T-beams that are also applicable to prestressed concrete members. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

**Section 8.13** — The empirical limits established for nonprestressed reinforced concrete joist floors are based on successful past performance of joist construction using standard joist forming systems. See R8.13. For prestressed joist construction, experience and judgment should be used. The provisions of 8.13 may be used as a guide.

**Sections 10.5, 10.9.1, and 10.9.2** — For prestressed concrete, the limitations on reinforcement given in 10.5, 10.9.1, and 10.9.2 are replaced by those in 18.8.3, 18.9, and 18.11.2.

**Section 10.6** — This section does not apply to prestressed members in its entirety. However, 10.6.4 and 10.6.7 are referenced in 18.4.4 pertaining to Class C prestressed flexural members.

**Chapter 13** — The design of continuous prestressed concrete slabs requires recognition of secondary moments. Also, volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of 18.12.

**Sections 14.5 and 14.6** — The requirements for wall design in 14.5 and 14.6 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

**18.2 — General**

**18.2.1** — Prestressed members shall meet the strength requirements of this Code.

**18.2.2** — Design of prestressed members shall be based on strength and on behavior at service conditions at all stages that will be critical during the life of the structure from the time prestress is first applied.

**R18.2 — General**

**R18.2.1 and R18.2.2** — The design investigation should include all stages that may be significant. The three major stages are: (1) jacking stage, or prestress transfer stage—when the tensile force in the prestressing steel is transferred to the concrete and stress levels may be high relative to concrete strength; (2) service load stage—after long-term volume changes have occurred; and (3) the factored load stage—when the strength of the member is checked. There may be other load stages that require investigation. For example, if the cracking load is significant, this load stage may require study, or the handling and transporting stage may be critical.

From the standpoint of satisfactory behavior, the two stages of most importance are those for service load and factored load.

**CODE****COMMENTARY**

Service load stage refers to the loads defined in the general building code (without load factors), such as live load and dead load, while the factored load stage refers to loads multiplied by the appropriate load factors.

Section 18.3.2 provides assumptions that may be used for investigation at service loads and after transfer of the prestressing force.

**18.2.3** — Stress concentrations due to prestressing shall be considered in design.

**18.2.4** — Provisions shall be made for effects on adjoining construction of elastic and plastic deformations, deflections, changes in length, and rotations due to prestressing. Effects of temperature and shrinkage shall also be included.

**18.2.5** — The possibility of buckling in a member between points where there is intermittent contact between the prestressing steel and an oversize duct, and buckling in thin webs and flanges shall be considered.

**18.2.6** — In computing section properties before bonding of prestressing steel, effect of loss of area due to open ducts shall be considered.

**R18.2.5** — Section 18.2.5 refers to the type of post-tensioning where the prestressing steel makes intermittent contact with an oversize duct. Precautions should be taken to prevent buckling of such members.

If the prestressing steel is in complete contact with the member being prestressed, or is unbonded with the sheathing not excessively larger than the prestressing steel, it is not possible to buckle the member under the prestressing force being introduced.

**R18.2.6** — In considering the area of the open ducts, the critical sections should include those that have coupler sheaths that may be of a larger size than the duct containing the prestressing steel. Also, in some instances, the trumpet or transition piece from the conduit to the anchorage may be of such a size as to create a critical section. If the effect of the open duct area on design is deemed negligible, section properties may be based on total area.

In post-tensioned members after grouting and in pretensioned members, section properties may be based on effective sections using transformed areas of bonded prestressing steel and nonprestressed reinforcement gross sections, or net sections.

### **18.3 — Design assumptions**

**18.3.1** — Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in **10.2**, except that **10.2.4** shall apply only to reinforcement conforming to **3.5.3**.

**18.3.2** — For investigation of stresses at transfer of prestress, at service loads, and at cracking loads, elastic theory shall be used with the assumptions of **18.3.2.1** and **18.3.2.2**.

### **R18.3 — Design assumptions**

**CODE****COMMENTARY**

**18.3.2.1** — Strains vary linearly with depth through the entire load range.

**18.3.2.2** — At cracked sections, concrete resists no tension.

**18.3.3** — Prestressed flexural members shall be classified as Class U, Class T, or Class C based on  $f_t$ , the computed extreme fiber stress in tension in the precompressed tensile zone calculated at service loads, as follows:

- (a) Class U:  $f_t \leq 7.5\sqrt{f'_c}$
- (b) Class T:  $7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$
- (c) Class C:  $f_t > 12\sqrt{f'_c}$

Prestressed two-way slab systems shall be designed as Class U with  $f_t \leq 6\sqrt{f'_c}$ .

**18.3.4** — For Class U and Class T flexural members, stresses at service loads shall be permitted to be calculated using the uncracked section. For Class C flexural members, stresses at service loads shall be calculated using the cracked transformed section.

**18.3.5** — Deflections of prestressed flexural members shall be calculated in accordance with 9.5.4.

## **18.4 — Serviceability requirements — Flexural members**

**18.4.1** — Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses):

- (a) Extreme fiber stress in compression except as permitted in (b) shall not exceed ..... **0.60 $f'_{ci}$**
- (b) Extreme fiber stress in compression at ends of simply supported members shall not exceed .. **0.70 $f'_{ci}$**
- (c) Where computed concrete tensile strength,  $f_t$ , exceeds  **$6\sqrt{f'_c}$**  at ends of simply supported

**R18.3.3** — This section defines three classes of behavior of prestressed flexural members. Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. The behavior of Class T members is assumed to be in transition between uncracked and cracked. The serviceability requirements for each class are summarized in [Table R18.3.3](#). For comparison, [Table R18.3.3](#) also shows corresponding requirements for nonprestressed members.

These classes apply to both bonded and unbonded prestressed flexural members, but prestressed two-way slab systems must be designed as Class U.

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

**R18.3.4** — A method for computing stresses in a cracked section is given in [Reference 18.1](#).

**R18.3.5** — [Reference 18.2](#) provides information on computing deflections of cracked members.

## **R18.4 — Serviceability requirements — Flexural members**

Permissible stresses in concrete address serviceability. Permissible stresses do not ensure adequate structural strength, which should be checked in conformance with other Code requirements.

**R18.4.1** — The concrete stresses at this stage are caused by the force in the prestressing steel at transfer reduced by the losses due to elastic shortening of the concrete, relaxation of the prestressing steel, seating at transfer, and the stresses due to the weight of the member. Generally, shrinkage and creep effects are not included at this stage. These stresses apply to both pretensioned and post-tensioned concrete with proper modifications of the losses at transfer. The compressive transfer stress at ends of simply supported members was raised from **0.60 $f'_{ci}$**  to **0.70 $f'_{ci}$**  in the 2008 Code to reflect research in the precast, prestressed concrete industry practice.<sup>18.3-18.5</sup>

**CODE****COMMENTARY****TABLE R18.3.3 — SERVICEABILITY DESIGN REQUIREMENTS**

	Prestressed			Non prestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\leq 7.5 \sqrt{f'_c}$	$7.5 \sqrt{f'_c} < f_t \leq 12 \sqrt{f'_c}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of $\Delta f_{ps}$ or $f_s$ for crack control	—	—	Cracked section analysis	$M/(A_s \times \text{lever arm})$ , or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

members, or  $3\sqrt{f'_{ci}}$  at other locations, additional bonded reinforcement shall be provided in the tensile zone to resist the total tensile force in concrete computed with the assumption of an uncracked section.

**18.4.2** — For Class U and Class T prestressed flexural members, stresses in concrete at service loads (based on uncracked section properties, and after allowance for all prestress losses) shall not exceed the following:

- (a) Extreme fiber stress in compression due to prestress plus sustained load ..... **0.45f'\_c**
- (b) Extreme fiber stress in compression due to prestress plus total load ..... **0.60f'\_c**

**R18.4.1(c)** — The tension stress limits of  $3\sqrt{f'_{ci}}$  and  $6\sqrt{f'_{ci}}$  refer to tensile stress at locations other than the precompressed tensile zone. Where tensile stresses exceed the permissible values, the total force in the tensile stress zone may be calculated and reinforcement proportioned on the basis of this force at a stress of  $0.6f_y$ , but not more than 30,000 psi. The effects of creep and shrinkage begin to reduce the tensile stress almost immediately; however, some tension remains in these areas after allowance is made for all prestress losses.

**R18.4.2(a) and (b)** — The compression stress limit of  $0.45f'_c$  was conservatively established to decrease the probability of failure of prestressed concrete members due to repeated loads. This limit seemed reasonable to preclude excessive creep deformation. At higher values of stress, creep strains tend to increase more rapidly as applied stress increases.

The change in allowable stress in the 1995 Code recognized that fatigue tests of prestressed concrete beams have shown that concrete failures are not the controlling criterion. Designs with transient live loads that are large compared to sustained live and dead loads have been penalized by the previous single compression stress limit. Therefore, the stress limit of  $0.60f'_c$  permits a one-third increase in allowable compression stress for members subject to transient loads.

Sustained live load is any portion of the service live load that will be sustained for a sufficient period to cause significant time-dependent deflections. Thus, when the sustained live and dead loads are a large percentage of total service load, the  $0.45f'_c$  limit of 18.4.2(a) may control. On the other hand, when a large portion of the total service load consists of a transient or temporary service live load, the increased stress limit of 18.4.2(b) may apply.

The compression limit of  $0.45f'_c$  for prestress plus sustained loads will continue to control the long-term behavior of prestressed members.

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**18.4.3** — Permissible stresses in 18.4.1 and 18.4.2 shall be permitted to be exceeded if shown by test or analysis that performance will not be impaired.

**18.4.4** — For Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by 10.6.4.

For structures subject to fatigue or exposed to corrosive environments, investigations and precautions are required.

**18.4.4.1** — The spacing requirements shall be met by nonprestressed reinforcement and bonded tendons. The spacing of bonded tendons shall not exceed 2/3 of the maximum spacing permitted for nonprestressed reinforcement.

Where both reinforcement and bonded tendons are used to meet the spacing requirement, the spacing between a bar and a tendon shall not exceed 5/6 of that permitted by 10.6.4. See also 18.4.4.3.

**18.4.4.2** — In applying Eq. (10-4) to prestressing tendons,  $\Delta f_{ps}$  shall be substituted for  $f_s$ , where  $\Delta f_{ps}$  shall be taken as the calculated stress in the prestressing steel at service loads based on a cracked section analysis minus the decompression stress  $f_{dc}$ . It shall be permitted to take  $f_{dc}$  equal to the effective stress in the prestressing steel  $f_{se}$ . See also 18.4.4.3.

**18.4.4.3** — In applying Eq. (10-4) to prestressing tendons, the magnitude of  $\Delta f_{ps}$  shall not exceed 36,000 psi. When  $\Delta f_{ps}$  is less than or equal to 20,000 psi, the spacing requirements of 18.4.4.1 and 18.4.4.2 shall not apply.

**18.4.4.4** — Where  $h$  of a beam exceeds 36 in., the area of longitudinal skin reinforcement consisting of reinforcement or bonded tendons shall be provided as required by 10.6.7.

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**R18.4.3** — This section provides a mechanism whereby development of new products, materials, and techniques in prestressed concrete construction need not be inhibited by Code limits on stress. Approvals for the design should be in accordance with 1.4 of the Code.

**R18.4.4** — Spacing requirements for prestressed members with calculated tensile stress exceeding  $12\sqrt{f'_c}$  were introduced in the 2002 edition of the Code.

For conditions of corrosive environments, defined as an environment in which chemical attack (such as seawater, corrosive industrial atmosphere, or sewer gas) is encountered, cover greater than that required by 7.7.2 should be used, and tension stresses in the concrete reduced to eliminate possible cracking at service loads. Judgment should be used to determine the amount of increased cover and whether reduced tension stresses are required.

**R18.4.4.1** — Only tension steel nearest the tension face need be considered in selecting the value of  $c_c$  used in computing spacing requirements. To account for prestressing steel, such as strand, having bond characteristics less effective than deformed reinforcement, a 2/3 effectiveness factor is used.

For post-tensioned members designed as cracked members, it will usually be advantageous to provide crack control by the use of deformed reinforcement, for which the provisions of 10.6 may be used directly. Bonded reinforcement required by other provisions of this Code may also be used as crack control reinforcement.

**R18.4.4.2** — It is conservative to take the decompression stress  $f_{dc}$  equal to  $f_{se}$ , the effective stress in the prestressing steel.

**R18.4.4.3** — The maximum limitation of 36,000 psi for  $\Delta f_{ps}$  and the exemption for members with  $\Delta f_{ps}$  less than 20,000 psi are intended to be similar to the Code requirements before the 2002 edition.

**R18.4.4.4** — The steel area of reinforcement, bonded tendons, or a combination of both may be used to satisfy this requirement.

**CODE****COMMENTARY****18.5 — Permissible stresses in prestressing steel****R18.5 — Permissible stresses in prestressing steel**

The Code does not distinguish between temporary and effective prestressing steel stresses. Only one limit on prestressing steel stress is provided because the initial prestressing steel stress (immediately after transfer) can prevail for a considerable time, even after the structure has been put into service. This stress, therefore, should have an adequate safety factor under service conditions and cannot be considered as a temporary stress. Any subsequent decrease in prestressing steel stress due to losses can only improve conditions and no limit on such stress decrease is provided in the Code.

**18.5.1 —** Tensile stress in prestressing steel shall not exceed the following:

(a) Due to prestressing steel jacking force .....  **$0.94f_{py}$**

but not greater than the lesser of  **$0.80f_{pu}$**  and the maximum value recommended by the manufacturer of prestressing steel or anchorage devices.

| (b) Post-tensioning tendons, at anchorage devices and couplers, immediately after force transfer.....  **$0.70f_{pu}$**

**R18.5.1 —** Because of the high yield strength of low-relaxation wire and strand meeting the requirements of ASTM A421 and A416, it is appropriate to specify permissible stresses in terms of specified minimum ASTM yield strength along with the specified minimum ASTM tensile strength.

Because of the higher allowable initial prestressing steel stresses permitted since the 1983 Code, final stresses can be greater. Structures subject to corrosive conditions or repeated loadings should be of concern when setting a limit on final stress.

**18.6 — Loss of prestress****R18.6 — Loss of prestress**

**18.6.1 —** To determine effective stress in the prestressing steel,  $f_{se}$ , allowance for the following sources of loss of prestress shall be considered:

- (a) Prestressing steel seating at transfer;
- (b) Elastic shortening of concrete;
- (c) Creep of concrete;
- (d) Shrinkage of concrete;
- (e) Relaxation of prestressing steel stress;
- (f) Friction loss due to intended or unintended curvature in post-tensioning tendons.

**R18.6.1 —** For an explanation of how to compute prestress losses, see [References 18.6 through 18.9](#). Lump sum values of prestress losses for both pretensioned and post-tensioned members that were indicated before the 1983 Commentary are considered obsolete. Reasonably accurate estimates of prestress losses can be calculated in accordance with the recommendations in [Reference 18.9](#), which include consideration of initial stress level ( $0.7f_{pu}$  or higher), type of steel (stress-relieved or low-relaxation wire, strand, or bar), exposure conditions, and type of construction (pretensioned, bonded post-tensioned, or unbonded post-tensioned).

Actual losses, greater or smaller than the computed values, have little effect on the design strength of the member, but affect service load behavior (deflections, camber, cracking load) and connections. At service loads, overestimation of prestress losses can be almost as detrimental as underestimation, since the former can result in excessive camber and horizontal movement.

**18.6.2 — Friction loss in post-tensioning tendons****R18.6.2 — Friction loss in post-tensioning tendons**

**18.6.2.1 —** The required effective prestress force shall be indicated in the contract documents.

Estimation of friction losses in post-tensioned tendons is addressed in [Reference 18.10](#). Values of the wobble and

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**18.6.2.2** — Computed friction loss shall be based on experimentally determined wobble and curvature friction coefficients.

**18.6.2.3** — The prestress force and friction losses shall be verified during tendon stressing operations as specified in 18.20.

**18.6.3** — Where loss of prestress in a member occurs due to connection of the member to adjoining construction, such loss of prestress shall be allowed for in design.

## 18.7 — Flexural strength

**18.7.1** — Design moment strength of flexural members shall be computed by the strength design methods of the Code. For prestressing steel,  $f_{ps}$  shall be substituted for  $f_y$  in strength computations.

**18.7.2** — As an alternative to a more accurate determination of  $f_{ps}$  based on strain compatibility, the following approximate values of  $f_{ps}$  shall be permitted to be used if  $f_{se}$  is not less than  $0.5f_{pu}$ .

(a) For members with bonded tendons

$$f_{ps} = f_{pu} \left\{ 1 - \frac{\gamma_p}{\beta_1} \left[ \rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p} (\omega - \omega') \right] \right\} \quad (18-1)$$

where  $\omega$  is  $\rho f_y/f'_c$ ,  $\omega'$  is  $\rho' f_y/f'_c$ , and  $\gamma_p$  is 0.55 for  $f_{py}/f_{pu}$  not less than 0.80; 0.40 for  $f_{py}/f_{pu}$  not less than 0.85; and 0.28 for  $f_{py}/f_{pu}$  not less than 0.90.

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curvature friction coefficients to be used for the particular types of prestressing steel and particular types of ducts should be obtained from the manufacturers of the tendons. An unrealistically low estimate of the friction loss can lead to improper camber, or potential deflection, of the member and inadequate prestress. Overestimation of the friction may result in extra prestressing force. This could lead to excessive camber and excessive shortening of a member. If the friction factors are determined to be less than those assumed in the design, the tendon stressing should be adjusted to give only that prestressing force in the critical portions of the structure required by the design.

When the safety or serviceability of the structure may be involved, the acceptable range of prestressing steel jacking forces or other limiting requirements should either be given or approved by the licensed design professional in conformance with the permissible stresses of 18.4 and 18.5.

## R18.7 — Flexural strength

**R18.7.1** — Design moment strength of prestressed flexural members may be computed using strength equations similar to those for nonprestressed concrete members. The 1983 Code provided strength equations for rectangular and flanged sections, with tension reinforcement only and with tension and compression reinforcement. When part of the prestressing steel is in the compression zone, a method based on applicable conditions of equilibrium and compatibility of strains at a factored load condition should be used.

For other cross sections, the design moment strength  $\phi M_n$  is computed by an analysis based on stress and strain compatibility, using the stress-strain properties of the prestressing steel and the assumptions given in 10.2.

**R18.7.2** — Equation (18-1) may underestimate the strength of beams with high percentages of reinforcement and, for more accurate evaluations of their strength, the strain compatibility and equilibrium method should be used. Use of Eq. (18-1) is appropriate when all of the prestressed reinforcement is in the tension zone. When part of the prestressed reinforcement is in the compression zone, a strain compatibility and equilibrium method should be used.

The  $\gamma_p$  term in Eq. (18-1) reflects the influence of different types of prestressing reinforcement on the value of  $f_{ps}$ . For high-strength prestressing bars conforming to ASTM A722 (Type I),  $f_{py}/f_{pu}$  is equal to or greater than 0.85; for high-strength prestressing bars conforming to ASTM A722 (Type II),  $f_{py}/f_{pu}$  is equal to or greater than 0.80; for stress-

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If any compression reinforcement is taken into account when calculating  $f_{ps}$  by Eq. (18-1), the term

$$\left[ \rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p} (\omega - \omega') \right]$$

shall be taken not less than 0.17 and  $d'$  shall be no greater than  $0.15d_p$ .

(b) For members with unbonded tendons and with a span-to-depth ratio of 35 or less:

$$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{100\rho_p} \quad (18-2)$$

but  $f_{ps}$  in Eq. (18-2) shall not be taken greater than the lesser of  $f_{py}$  and  $(f_{se} + 60,000)$ .

(c) For members with unbonded tendons and with a span-to-depth ratio greater than 35:

$$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{300\rho_p} \quad (18-3)$$

but  $f_{ps}$  in Eq. (18-3) shall not be taken greater than the lesser of  $f_{py}$  and  $(f_{se} + 30,000)$ .

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relieved strand and wire conforming to ASTM A416 and A421,  $f_{py}/f_{pu}$  is equal to or greater than 0.85; and for low-relaxation strand and wire conforming to ASTM A416 and A421,  $f_{py}/f_{pu}$  is equal to or greater than 0.90.

By inclusion of the  $\omega'$  term, Eq. (18-1) reflects the increased value of  $f_{ps}$  obtained when compression reinforcement is provided in a beam with a large reinforcement index. When the term  $[\rho_p (f_{pu}/f'_c) + (d/d_p)(\omega - \omega')]$  in Eq. (18-1) is small, the neutral axis depth is small, the compressive reinforcement does not develop its yield strength, and Eq. (18-1) becomes unconservative. This is the reason why the term  $[\rho_p (f_{pu}/f'_c) + (d/d_p)(\omega - \omega')]$  in Eq. (18-1) may not be taken less than 0.17 if compression reinforcement is taken into account when computing  $f_{ps}$ . If the compression reinforcement is neglected when using Eq. (18-1),  $\omega'$  is taken as zero, then the term  $[\rho_p (f_{pu}/f'_c) + (d/d_p)\omega]$  may be less than 0.17 and an increased and correct value of  $f_{ps}$  is obtained.

When  $d'$  is large, the strain in compression reinforcement can be considerably less than its yield strain. In such a case, the compression reinforcement does not influence  $f_{ps}$  as favorably as implied by Eq. (18-1). For this reason, the applicability of Eq. (18-1) is limited to beams in which  $d'$  is less than or equal to  $0.15d_p$ .

The term  $[\rho_p (f_{pu}/f'_c) + (d/d_p)(\omega - \omega')]$  in Eq. (18-1) may also be written  $[\rho_p (f_{pu}/f'_c) + A_s f_y / (bd_p f'_c) - A_s' f_y / (bd_p f'_c)]$ . This form may be more convenient, such as when there is no unprestressed tension reinforcement.

Equation (18-3) reflects results of tests on members with unbonded tendons and span-to-depth ratios greater than 35 (one-way slabs, flat plates, and flat slabs).<sup>18.11</sup> These tests also indicate that Eq. (18-2), formerly used for all span-depth ratios, overestimates the amount of stress increase in such members. Although these same tests indicate that the moment strength of those shallow members designed using Eq. (18-2) meets the factored load strength requirements, this reflects the effect of the Code requirements for minimum bonded reinforcement, as well as the limitation on concrete tensile stress that often controls the amount of prestressing force provided.

**18.7.3 — Nonprestressed reinforcement conforming to 3.5.3, if used with prestressing steel, shall be permitted to be considered to contribute to the tensile force and to be included in moment strength computations at a stress equal to  $f_y$ . Other nonprestressed reinforcement shall be permitted to be included in strength computations only if a strain compatibility analysis is performed to determine stresses in such reinforcement.**

**CODE****18.8 — Limits for reinforcement of flexural members**

**18.8.1** — Prestressed concrete sections shall be classified as either tension-controlled, transition, or compression-controlled sections, in accordance with 10.3.3 and 10.3.4. The appropriate strength reduction factors,  $\phi$ , from 9.3.2 shall apply.

**18.8.2** — Total amount of prestressed and nonprestressed reinforcement in members with bonded prestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture  $f_r$  specified in 9.5.2.3. This provision shall be permitted to be waived for flexural members with shear and flexural strength at least twice that required by 9.2.

**18.8.3** — Part or all of the bonded reinforcement consisting of bars or tendons shall be provided as close as practicable to the tension face in prestressed flexural members. In members prestressed with unbonded tendons, the minimum bonded reinforcement consisting of bars or tendons shall be as required by 18.9.

**18.9 — Minimum bonded reinforcement**

**18.9.1** — A minimum area of bonded reinforcement shall be provided in all flexural members with unbonded tendons as required by 18.9.2 and 18.9.3.

**COMMENTARY****R18.8 — Limits for reinforcement of flexural members**

**R18.8.1** — The net tensile strain limits for compression- and tension-controlled sections given in 10.3.3 and 10.3.4 apply to prestressed sections. These provisions take the place of maximum reinforcement limits used in the 1999 Code.

The net tensile strain limits for tension-controlled sections given in 10.3.4 may also be stated in terms of  $\omega_p$  as defined in the 1999 and earlier editions of the Code. The net tensile strain limit of 0.005 corresponds to  $\omega_p = 0.32\beta_1$  for prestressed rectangular sections.

**R18.8.2** — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to Code provisions requires considerable additional load beyond cracking to reach its flexural strength. Thus, considerable deflection would warn that the member strength is approaching. If the flexural strength were reached shortly after cracking, the warning deflection would not occur. Transfer of force between the concrete and the prestressing steel, and abrupt flexural failure immediately after cracking, does not occur when the prestressing steel is unbonded<sup>18.12</sup>; therefore, this requirement does not apply to members with unbonded tendons.

**R18.8.3** — Some bonded steel is required to be placed near the tension face of prestressed flexural members. The purpose of this bonded steel is to control cracking under full service loads or overloads.

**R18.9 — Minimum bonded reinforcement**

**R18.9.1** — Some bonded reinforcement is required by the Code in members prestressed with unbonded tendons to ensure flexural performance at ultimate member strength, rather than as a tied arch, and to limit crack width and spacing at service load when concrete tensile stresses exceed the modulus of rupture. Providing the minimum bonded reinforcement as stipulated in 18.9 helps to ensure adequate performance.

Research has shown that unbonded post-tensioned members do not inherently provide large capacity for energy dissipation under severe earthquake loadings because the member response is primarily elastic. For this reason, unbonded post-tensioned structural elements reinforced in accordance with the provisions of this section should be assumed to carry only vertical loads and to act as horizontal diaphragms between energy dissipating elements under earthquake loadings of the magnitude defined in 21.1.1. The minimum bonded reinforcement areas required by Eq. (18-4) and (18-6) are absolute minimum areas independent of grade of steel or design yield strength.

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**18.9.2** — Except as provided in 18.9.3, minimum area of bonded reinforcement shall be computed by

$$A_s = 0.004 A_{ct} \quad (18-4)$$

**18.9.2.1** — Bonded reinforcement required by Eq. (18-4) shall be uniformly distributed over precompressed tensile zone as close as practicable to extreme tension fiber.

**18.9.2.2** — Bonded reinforcement shall be required regardless of service load stress conditions.

**18.9.3** — For two-way flat slab systems, minimum area and distribution of bonded reinforcement shall be as required in 18.9.3.1, 18.9.3.2, and 18.9.3.3.

**18.9.3.1** — Bonded reinforcement shall not be required in positive moment areas where  $f_t$ , the extreme fiber stress in tension in the precompressed tensile zone at service loads, (after allowance for all prestress losses) does not exceed  $2\sqrt{f'_c}$ .

**18.9.3.2** — In positive moment areas where computed tensile stress in concrete at service load exceeds  $2\sqrt{f'_c}$ , minimum area of bonded reinforcement shall be computed by

$$A_s = \frac{N_c}{0.5f_y} \quad (18-5)$$

where the value of  $f_y$  used in Eq. (18-5) shall not exceed 60,000 psi. Bonded reinforcement shall be uniformly distributed over precompressed tensile zone as close as practicable to the extreme tension fiber.

**18.9.3.3** — In negative moment areas at column supports, the minimum area of bonded reinforcement  $A_s$  in the top of the slab in each direction shall be computed by

$$A_s = 0.00075 A_{cf} \quad (18-6)$$

where  $A_{cf}$  is the larger gross cross-sectional area of the slab-beam strips in two orthogonal equivalent frames intersecting at a column in a two-way slab.

Bonded reinforcement required by Eq. (18-6) shall be distributed between lines that are  $1.5h$  outside opposite faces of the column support. At least four bars or wires

**R18.9.2** — The minimum amount of bonded reinforcement for members other than two-way flat slab systems is based on research comparing the behavior of bonded and unbonded post-tensioned beams.<sup>18,13</sup> Based on this research, it is advisable to apply the provisions of 18.9.2 also to one-way slab systems.

**R18.9.3** — The minimum amount of bonded reinforcement in two-way flat slab systems is based on reports by Joint ACI-ASCE Committee 423.<sup>18,6,18,12</sup> Limited research available for two-way flat slabs with drop panels<sup>18,14</sup> indicates that behavior of these particular systems is similar to the behavior of flat plates. Reference 18.12 was revised by Committee 423 in 1983 to clarify that Section 18.9.3 applies to two-way flat slab systems.

**R18.9.3.1** — For usual loads and span lengths, flat plate tests summarized in the Committee 423 report<sup>18,3</sup> and experience since the 1963 Code was adopted indicate satisfactory performance without bonded reinforcement in the areas described in 18.9.3.1.

**R18.9.3.2** — In positive moment areas, where the maximum computed concrete tensile stress exceeds  $2\sqrt{f'_c}$  but is less than  $6\sqrt{f'_c}$ , in accordance with 18.3.3, a minimum bonded reinforcement area proportioned to resist  $N_c$  according to Eq. (18-5) is required. The tensile force  $N_c$  is computed at service load on the basis of an uncracked, homogeneous section.

**R18.9.3.3** — Research on unbonded post-tensioned two-way flat slab systems evaluated by Committee 423<sup>18,1,18,6,18,12,18,14</sup> shows that bonded reinforcement in negative moment regions, proportioned on the basis of 0.075 percent of the cross-sectional area of the slab-beam strip, provides sufficient ductility and reduces crack width and spacing. To account for different adjacent tributary spans, Eq. (18-6) is given on the basis of the equivalent frame as defined in 13.7.2 and pictured in Fig. R13.7.2. For rectangular slab panels, Eq. (18-6) is conservatively based upon the larger of the cross-sectional areas of the two intersecting equivalent frame slab-beam strips at the column. This ensures that the minimum percentage of steel recommended by research is provided in both directions. Concentration of this reinforcement in the top

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shall be provided in each direction. Spacing of bonded reinforcement shall not exceed 12 in.

**18.9.4** — Minimum length of bonded reinforcement required by **18.9.2** and **18.9.3** shall be as required in 18.9.4.1, 18.9.4.2, and 18.9.4.3.

**18.9.4.1** — In positive moment areas, minimum length of bonded reinforcement shall be one-third the clear span length,  $\ell_n$ , and centered in positive moment area.

**18.9.4.2** — In negative moment areas, bonded reinforcement shall extend one-sixth the clear span,  $\ell_n$ , on each side of support.

**18.9.4.3** — Where bonded reinforcement is provided for  $\phi M_n$  in accordance with **18.7.3**, or for tensile stress conditions in accordance with **18.9.3.2**, minimum length also shall conform to provisions of **Chapter 12**.

**18.10 — Statically indeterminate structures**

**18.10.1** — Frames and continuous construction of prestressed concrete shall be designed for satisfactory performance at service load conditions and for adequate strength.

**18.10.2** — Performance at service load conditions shall be determined by elastic analysis, considering reactions, moments, shears, and axial forces induced by prestressing, creep, shrinkage, temperature change, axial deformation, restraint of attached structural elements, and foundation settlement.

**18.10.3** — Moments used to compute required strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads. Adjustment of the sum of these moments shall be permitted as allowed in **18.10.4**.

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of the slab directly over and immediately adjacent to the column is important. Research also shows that where low tensile stresses occur at service loads, satisfactory behavior has been achieved at factored loads without bonded reinforcement. However, the Code requires minimum bonded reinforcement regardless of service load stress levels to help ensure flexural continuity and ductility, and to limit crack widths and spacing due to overload, temperature, or shrinkage. Research on post-tensioned flat plate-to-column connections is reported in **References 18.15 through 18.19**.

**R18.9.4** — Bonded reinforcement should be adequately anchored to develop factored load forces. The requirements of **Chapter 12** will ensure that bonded reinforcement required for flexural strength under factored loads in accordance with **18.7.3**, or for tensile stress conditions at service load in accordance with **18.9.3.2**, will be adequately anchored to develop tension or compression forces. The minimum lengths apply for bonded reinforcement required by **18.9.2** or **18.9.3.3**, but not required for flexural strength in accordance with **18.7.3**. Research<sup>18.14</sup> on continuous spans shows that these minimum lengths provide adequate behavior under service load and factored load conditions.

**R18.10 — Statically indeterminate structures**

**R18.10.3** — For statically indeterminate structures, the moments due to reactions induced by prestressing forces, referred to as secondary moments, are significant in both the elastic and inelastic states (see **References 18.20 through 18.22**). The elastic deformations caused by a nonconcordant tendon change the amount of inelastic rotation required to obtain a given amount of moment redistribution. Conversely, for a beam with a given inelastic rotational capacity, the amount by which the moment at the support may be varied is changed by an amount equal to the secondary moment at the support due to prestressing. Thus, the Code requires that secondary moments be included in determining design moments.

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To determine the moments used in design, the order of calculation should be: (a) determine moments due to dead and live load; (b) modify by algebraic addition of secondary moments; (c) redistribute as permitted. A positive secondary moment at the support caused by a tendon transformed downward from a concordant profile will reduce the negative moments near the supports and increase the positive moments in the midspan regions. A tendon that is transformed upward will have the reverse effect.

**18.10.4 — Redistribution of moments in continuous prestressed flexural members**

**18.10.4.1** — Where bonded reinforcement is provided at supports in accordance with 18.9, it shall be permitted to decrease negative or positive moments calculated by elastic theory for any assumed loading, in accordance with 8.4.

**18.10.4.2** — The reduced moment shall be used for calculating redistributed moments at all other sections within the spans. Static equilibrium shall be maintained after redistribution of moments for each loading arrangement.

**18.11 — Compression members — Combined flexure and axial loads**

**18.11.1** — Prestressed concrete members subject to combined flexure and axial load, with or without nonprestressed reinforcement, shall be proportioned by the strength design methods of this Code. Effects of prestress, creep, shrinkage, and temperature change shall be included.

**18.11.2 — Limits for reinforcement of prestressed compression members**

**18.11.2.1** — Members with average compressive stress in concrete less than 225 psi, due to effective prestress force only, shall have minimum reinforcement in accordance with 7.10, 10.9.1 and 10.9.2 for columns, or 14.3 for walls.

**18.11.2.2** — Except for walls, members with average compressive stress in concrete, due to effective prestress force only, equal to or greater than 225 psi, shall have all tendons enclosed by spirals or transverse ties in accordance with (a) through (d):

- (a) Spirals shall conform to 7.10.4;
- (b) Transverse ties shall be at least No. 3 in size or welded wire reinforcement of equivalent area, and shall be spaced vertically not to exceed 48 tie bar or wire diameters, or the least dimension of the compression member;

**R18.10.4 — Redistribution of moments in continuous prestressed flexural members**

The provisions for redistribution of moments given in 8.4 apply equally to prestressed members. See Reference 18.23 for a comparison of research results and to Section 18.10.4 of the 1999 Code.

For the moment redistribution principles of 18.10.4 to be applicable to beams with unbonded tendons, it is necessary that such beams contain sufficient bonded reinforcement to ensure they will act as beams after cracking and not as a series of tied arches. The minimum bonded reinforcement requirements of 18.9 serves this purpose.

**R18.11 — Compression members — Combined flexure and axial loads****R18.11.2 — Limits for reinforcement of prestressed compression members**

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(c) Ties shall be located vertically not more than half a tie spacing above top of footing or slab in any story, and not more than half a tie spacing below the lowest horizontal reinforcement in members supported above;

(d) Where beams or brackets frame into all sides of a column, ties shall be terminated not more than 3 in. below lowest reinforcement in such beams or brackets.

**18.11.2.3** — For walls with average compressive stress in concrete due to effective prestress force only equal to or greater than 225 psi, minimum reinforcement required by **14.3** shall not apply where structural analysis shows adequate strength and stability.

## 18.12 — Slab systems

**18.12.1** — Factored moments and shears in prestressed slab systems reinforced for flexure in more than one direction shall be determined in accordance with provisions of **13.7** (excluding **13.7.7.4** and **13.7.7.5**), or by more detailed design procedures.

**18.12.2** —  $\phi M_n$  of prestressed slabs required by **9.3** at every section shall be greater than or equal to  $M_u$  considering **9.2**, **18.10.3**, and **18.10.4**.  $\phi V_n$  of prestressed slabs at columns required by **9.3** shall be greater than or equal to  $V_u$  considering **9.2**, **11.1**, **11.11.2**, and **11.11.6.2**.

**18.12.3** — At service load conditions, all serviceability limitations, including limits on deflections, shall be met, with appropriate consideration of the factors listed in **18.10.2**.

**R18.11.2.3** — The minimum amounts of reinforcement in **14.3** need not apply to prestressed concrete walls, provided the average compressive stress in concrete due to effective prestress force only is 225 psi or greater and a structural analysis is performed to show adequate strength and stability with lower amounts of reinforcement.

## R18.12 — Slab systems

**R18.12.1** — Use of the equivalent frame method of analysis (see **13.7**) or more precise analysis procedures is required for determination of both service and factored moments and shears for prestressed slab systems. The equivalent frame method of analysis has been shown by tests of large structural models to satisfactorily predict factored moments and shears in prestressed slab systems. (See **References 18.15** through **18.17**, and **18.24** through **18.26**.) The referenced research also shows that analysis using prismatic sections or other approximations of stiffness may provide erroneous results on the unsafe side. **Section 13.7.7.4** is excluded from application to prestressed slab systems because it relates to reinforced slabs designed by the direct design method, and because moment redistribution for prestressed slabs is covered in **18.10.4**. **Section 13.7.7.5** does not apply to prestressed slab systems because the distribution of moments between column strips and middle strips required by **13.7.7.5** is based on tests for nonprestressed concrete slabs. Simplified methods of analysis using average coefficients do not apply to prestressed concrete slab systems.

**R18.12.2** — Tests indicate that the moment and shear strength of prestressed slabs is controlled by total prestressing steel strength and by the amount and location of nonprestressed reinforcement, rather than by tendon distribution. (See **References 18.15** through **18.17**, and **18.24** through **18.26**.)

**R18.12.3** — For prestressed flat slabs continuous over two or more spans in each direction, the span-thickness ratio generally should not exceed 42 for floors and 48 for roofs; these limits may be increased to 48 and 52, respectively, if calculations verify that both short- and long-term deflection, camber, and vibration frequency and amplitude are not objectionable.

Short- and long-term deflection and camber should be computed and checked against the requirements of serviceability of the structure.

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**18.12.4** — For uniformly distributed loads, spacing of tendons or groups of tendons in at least one direction shall not exceed the smaller of eight times the slab thickness and 5 ft. Spacing of tendons also shall provide a minimum average effective prestress of 125 psi on the slab section tributary to the tendon or tendon group. For slabs with varying cross section along the slab span, either parallel or perpendicular to the tendon or tendon group, the minimum average effective prestress of 125 psi is required at every cross section tributary to the tendon or tendon group along the span. Concentrated loads and opening in slabs shall be considered when determining tendon spacing.

The maximum length of a slab between construction joints is generally limited to 100 to 150 ft to minimize the effects of slab shortening, and to avoid excessive loss of prestress due to friction.

**R18.12.4** — This section provides specific guidance concerning tendon distribution that will permit the use of banded tendon distributions in one direction. This method of tendon distribution has been shown to provide satisfactory performance by structural research. The minimum average effective prestress of 125 psi was used in two-way test panels in the early 70s to address punching shear concerns of lightly reinforced slabs. For this reason, the minimum effective prestress must be provided at every cross section.

If the slab thickness varies along the span of a slab or perpendicular to the span of a slab, resulting in a varying slab cross section, the 125 psi minimum effective prestress and the maximum tendon spacing is required at every cross section tributary to the tendon or group of tendons along the span, considering both the thinner and the thicker slab sections. Note that this may result in higher than the minimum  $f_{pc}$  in thinner cross sections, and tendons spaced at less than the maximum in thicker cross sections along a span with varying thickness, due to the practical aspects of tendon placement in the field.

**18.12.5** — In slabs with unbonded tendons, bonded reinforcement shall be provided in accordance with 18.9.3 and 18.9.4.

**18.12.6** — Except as permitted in 18.12.7, in slabs with unbonded tendons, a minimum of two 1/2 in. diameter or larger, seven-wire post-tensioned strands shall be provided in each direction at columns, either passing through or anchored within the region bounded by the longitudinal reinforcement of the column. Outside column and shear cap faces, these two structural integrity tendons shall pass under any orthogonal tendons in adjacent spans. Where the two structural integrity tendons are anchored within the region bounded by the longitudinal reinforcement of the column, the anchorage shall be located beyond the column centroid and away from the anchored span.

**R18.12.6** — Unbonded prestressing tendons that pass through the slab-column joint at any location over the depth of the slab suspend the slab following a punching shear failure, provided the tendons are continuous through or anchored within the region bounded by the longitudinal reinforcement of the column and are prevented from bursting through the top surface of the slab.<sup>18.27</sup> Between column or shear cap faces, structural integrity tendons should pass below the orthogonal tendons from adjacent spans so that vertical movements of the integrity tendons are restrained by the orthogonal tendons. Where tendons are distributed in one direction and banded in the orthogonal direction, this requirement can be satisfied by first placing the integrity tendons for the distributed tendon direction and then placing the banded tendons. Where tendons are distributed in both directions, weaving of tendons is necessary and use of 18.12.7 may be an easier approach.

**18.12.7** — Prestressed slabs not satisfying 18.12.6 shall be permitted provided they contain bottom reinforcement in each direction passing within the region bounded by the longitudinal reinforcement of the column and anchored at exterior supports as required by 13.3.8.5. The area of bottom reinforcement in each direction shall be not less than 1.5 times

**R18.12.7** — In some prestressed slabs, tendon layout constraints make it difficult to provide the structural integrity tendons required by 18.12.6. In such situations, the structural integrity tendons can be replaced by deformed bar bottom reinforcement.<sup>18.27</sup>

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that required by [Eq. \(10-3\)](#) and not less than  $300b_w d f_y$ , where  $b_w$  is the width of the column face through which the reinforcement passes. Minimum extension of these bars beyond the column or shear cap face shall be equal to or greater than the bar development length required by [12.2.1](#).

**18.12.8** — In lift slabs, bonded bottom reinforcement shall be detailed in accordance with [13.3.8.6](#).

**18.13 — Post-tensioned tendon anchorage zones**

**18.13.1 — Anchorage zone**

The anchorage zone shall be considered as composed of two zones:

- (a) The local zone is the rectangular prism (or equivalent rectangular prism for circular or oval anchorages) of concrete immediately surrounding the anchorage device and any confining reinforcement;
- (b) The general zone is the anchorage zone as defined in [2.2](#) and includes the local zone.

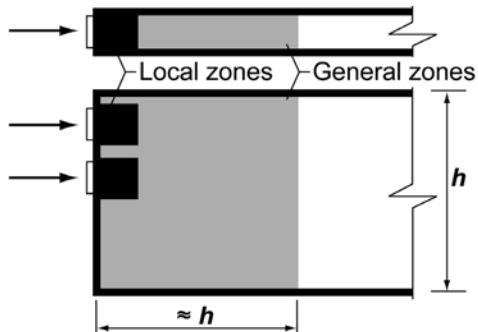
**R18.13 — Post-tensioned tendon anchorage zones**

Section 18.13 was extensively revised in the 1999 Code and was made compatible with the 1996 AASHTO “Standard Specifications for Highway Bridges”<sup>18.28</sup> and the recommendations of NCHRP Report 356.<sup>18.29</sup>

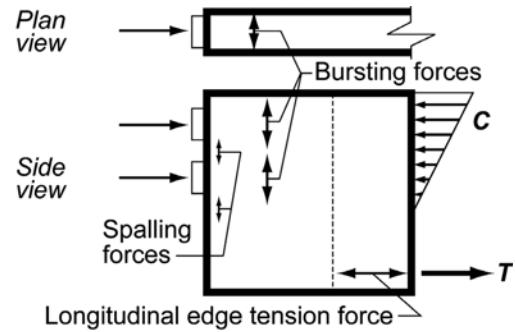
Following the adoption by AASHTO 1994 of comprehensive provisions for post-tensioned anchorage zones, ACI Committee 318 revised the Code to be generally consistent with the AASHTO requirements. Thus, the highly detailed AASHTO provisions for analysis and reinforcement detailing are deemed to satisfy the more general ACI 318 requirements. In the specific areas of anchorage device evaluation and acceptance testing, ACI 318 incorporates the detailed AASHTO provisions by reference.

**R18.13.1 — Anchorage zone**

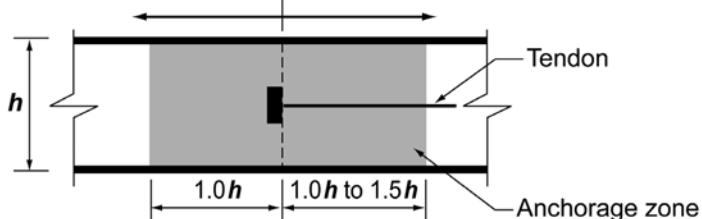
Based on the Principle of Saint-Venant, the extent of the anchorage zone may be estimated as approximately equal to the largest dimension of the cross section. Local zones and general zones are shown in [Fig. R18.13.1\(a\)](#). When anchorage devices located away from the end of the member are tensioned, large tensile stresses exist locally behind and ahead of the device. These tensile stresses are induced by incompatibility of deformations ahead of [as shown in [Fig. R18.13.1\(b\)](#)] and behind the anchorage device. The entire shaded region should be considered, as shown in [Fig. R18.13.1\(b\)](#).

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(a) Local and general zones concept

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(c) Tensile stress zones

**Behind anchorage device      Ahead of anchorage device**

(b) General zone for anchorage device located away from the end of a member

Fig. R18.13.1—Anchorage zones.

**18.13.2 — Local zone**

**18.13.2.1** — Design of local zones shall be based upon the factored prestressing force,  $P_{pu}$ , and the requirements of 9.2.5 and 9.3.2.5.

**18.13.2.2** — Local-zone reinforcement shall be provided where required for proper functioning of the anchorage device.

**18.13.2.3** — Local-zone requirements of 18.13.2.2 are satisfied by 18.14.1 or 18.15.1 and 18.15.2.

**R18.13.2 — Local zone**

The local zone resists the very high local stresses introduced by the anchorage device and transfers them to the remainder of the anchorage zone. The behavior of the local zone is strongly influenced by the specific characteristics of the anchorage device and its confining reinforcement, and less influenced by the geometry and loading of the overall structure. Local-zone design sometimes cannot be completed until specific anchorage devices are determined at the shop drawing stage. When special anchorage devices are used, the anchorage device supplier should furnish the test information to show the device is satisfactory under AASHTO "Standard Specifications for Highway Bridges," Division II, Article 10.3.2.3 and provide information regarding necessary conditions for use of the device. The main considerations in local-zone design are the effects of the high bearing pressure and the adequacy of any confining reinforcement provided to increase the capacity of the concrete resisting bearing stresses.

The factored prestressing force  $P_{pu}$  is the product of the load factor (1.2 from [Section 9.2.7](#)) and the maximum prestressing force allowed. Under 18.5.1, this is usually overstressing due to  $0.94f_{py}$ , but not greater than  $0.8f_{pu}$ , which is permitted for short periods of time.

$$P_{pu} = (1.2)(0.80)f_{pu}A_{ps} = 0.96f_{pu}A_{ps}$$

**CODE****COMMENTARY****18.13.3 — General zone**

**18.13.3.1** — Design of general zones shall be based upon the factored prestressing force,  $P_{pu}$ , and the requirements of **9.2.5** and **9.3.2.5**.

**18.13.3.2** — General-zone reinforcement shall be provided where required to resist bursting, spalling, and longitudinal edge tension forces induced by anchorage devices. Effects of abrupt change in section shall be considered.

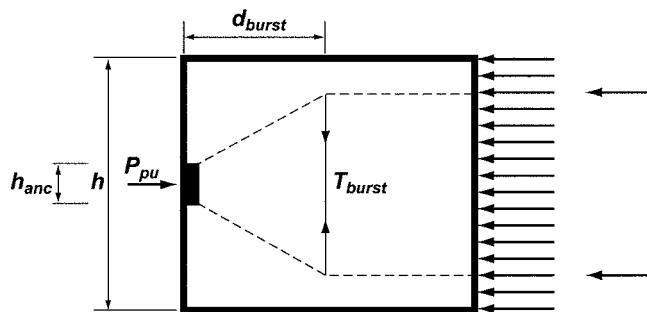
**18.13.3.3** — The general-zone requirements of 18.13.3.2 are satisfied by **18.13.4**, **18.13.5**, **18.13.6** and whichever one of **18.14.2** or **18.14.3** or **18.15.3** is applicable.

**R18.13.3 — General zone**

Within the general zone, the usual assumption of beam theory that plane sections remain plane is not valid.

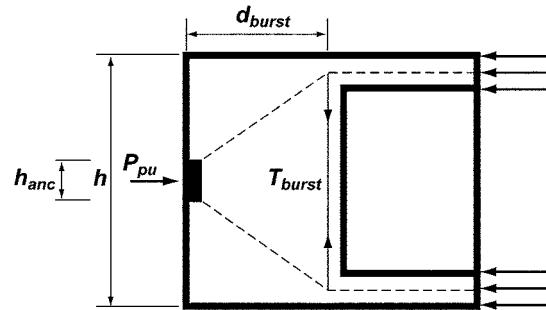
Design should consider all regions of tensile stresses that can be caused by the tendon anchorage device, including bursting, spalling, and edge tension as shown in Fig. R18.13.1(c). Also, the compressive stresses immediately ahead [as shown in Fig. R18.13.1(b)] of the local zone should be checked. Sometimes, reinforcement requirements cannot be determined until specific tendon and anchorage device layouts are determined at the shop-drawing stage. Design and approval responsibilities should be clearly assigned in the contract documents.

Abrupt changes in section can cause substantial deviation in force paths. These deviations can greatly increase tension forces as shown in Fig. R18.13.3.



(a) Rectangular section

$$T_{burst} \sim 0.25 P_{pu}$$



(b) Flanged section with end diaphragm

$$T_{burst} \sim 0.50 P_{pu}$$

Fig. R18.13.3—Effect of cross section change.

**CODE****COMMENTARY****18.13.4 — Nominal material strengths**

**18.13.4.1** — Tensile stress at nominal strength of bonded reinforcement is limited to  $f_y$  for nonprestressed reinforcement and to  $f_{py}$  for prestressed reinforcement. Tensile stress at nominal strength of unbonded prestressed reinforcement for resisting tensile forces in the anchorage zone shall be limited to  $f_{ps} = f_{se} + 10,000$ .

**18.13.4.2** — Except for concrete confined within spirals or hoops providing confinement equivalent to that corresponding to Eq. (10-5), compressive strength in concrete at nominal strength in the general zone shall be limited to  $0.7\lambda f_{ci}$ .

**18.13.4.3** — Compressive strength of concrete at time of post-tensioning shall be specified in the contract documents. Unless oversize anchorage devices are sized to compensate for the lower compressive strength of the prestressing steel is stressed to no more than 50 percent of the final prestressing force, prestressing steel shall not be stressed until compressive strength of concrete as indicated by tests consistent with the curing of the member, is at least 4000 psi for multistrand tendons or at least 2500 psi for single-strand or bar tendons.

**18.13.5 — Design methods**

**18.13.5.1** — The following methods shall be permitted for the design of general zones provided that the specific procedures used result in prediction of strength in substantial agreement with results of comprehensive tests:

- (a) Equilibrium-based plasticity models (strut-and-tie models);
- (b) Linear stress analysis (including finite element analysis or equivalent); or
- (c) Simplified equations where applicable.

**18.13.5.2** — Simplified equations shall not be used where member cross sections are nonrectangular, where discontinuities in or near the general zone cause deviations in the force flow path, where minimum edge distance is less than 1-1/2 times the anchorage device lateral dimension in that direction, or where multiple anchorage devices are used in other than one closely spaced group.

**R18.13.4 — Nominal material strengths**

Some inelastic deformation of concrete is expected because anchorage zone design is based on a strength approach. The low value for the nominal compressive strength for unconfined concrete reflects this possibility. For well-confined concrete, the effective compressive strength could be increased (See Reference 18.29). The value for nominal tensile strength of bonded prestressing steel is limited to the yield strength of the prestressing steel because Eq. (18-1) may not apply to these nonflexural applications. The value for unbonded prestressing steel is based on the values of 18.7.2(b) and (c), but is somewhat limited for these short-length, nonflexural applications. Test results given in Reference 18.29 indicate that the compressive stress introduced by auxiliary prestressing applied perpendicular to the axis of the main tendons is effective in increasing the anchorage zone strength. The inclusion of the  $\lambda$  factor for lightweight concrete reflects its lower tensile strength, which is an indirect factor in limiting compressive stresses, as well as the wide scatter and brittleness exhibited in some lightweight concrete anchorage zone tests.

To limit early shrinkage cracking, monostrand tendons are sometimes stressed at concrete strengths less than 2500 psi. In such cases, either oversized monostrand anchorages are used, or the strands are stressed in stages, often to levels 1/3 to 1/2 the final prestressing force.

**R18.13.5 — Design methods**

The list of design methods in 18.13.5.1 includes those procedures for which fairly specific guidelines have been given in References 18.28 and 18.29. These procedures have been shown to be conservative predictors of strength when compared to test results.<sup>18.29</sup> The use of strut-and-tie models is especially helpful for general zone design.<sup>18.29</sup> In many anchorage applications, where substantial or massive concrete regions surround the anchorages, simplified equations can be used except in the cases noted in 18.13.5.2.

For many cases, simplified equations based on References 18.28 and 18.29 can be used. Values for the magnitude of the bursting force,  $T_{burst}$ , and for its centroidal distance from the major bearing surface of the anchorage,  $d_{burst}$ , may be estimated from Eq. (R18-1) and (R18-2), respectively. The terms of Eq. (R18-1) and (R18-2) are shown in Fig. R18.13.5 for a prestressing force with small eccentricity. In the applications of Eq. (R18-1) and (R18-2), the specified stressing sequence should be considered if more than one tendon is present.

$$T_{burst} = 0.25 \Sigma P_{pu} \left( 1 - \frac{h_{anc}}{h} \right) \quad (\text{R18-1})$$

$$d_{burst} = 0.5(h - 2e_{anc}) \quad (\text{R18-2})$$

**CODE****COMMENTARY**

where

$\Sigma P_{pu}$  = the sum of the  $P_{pu}$  forces from the individual tendons, lb;

$h_{anc}$  = the depth of anchorage device or single group of closely spaced devices in the direction considered, in.;

$e_{anc}$  = the eccentricity (always taken as positive) of the anchorage device or group of closely spaced devices with respect to the centroid of the cross section, in.;

$h$  = the depth of the cross section in the direction considered, in.

Anchorage devices should be treated as closely spaced if their center-to-center spacing does not exceed 1.5 times the width of the anchorage device in the direction considered.

The spalling force for tendons for which the centroid lies within the kern of the section may be estimated as 2 percent of the total factored prestressing force, except for multiple anchorage devices with center-to-center spacing greater than 0.4 times the depth of the section. For large spacings and for cases where the centroid of the tendons is located outside the kern, a detailed analysis is required. In addition, in the post-tensioning of thin sections, or flanged sections, or irregular sections, or when the tendons have appreciable curvature within the general zone, more general procedures such as those of AASHTO Articles 9.21.4 and 9.21.5 will be required. Detailed recommendations for design principles that apply to all design methods are given in Article 9.21.3.4 of Reference 18.28.

**18.13.5.3** — The stressing sequence shall be specified in the contract documents and considered in the design.

**R18.13.5.3** — The sequence of anchorage device stressing can have a significant effect on the general zone stresses. Therefore, it is important to consider not only the final stage of a stressing sequence with all tendons stressed, but also intermediate stages during construction. The most critical bursting forces caused by each of the sequentially post-tensioned tendon combinations, as well as that of the entire group of tendons, should be taken into account.

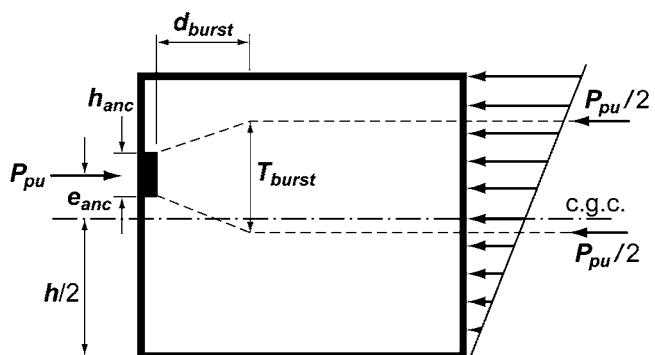


Fig. R18.13.5—Strut-and-tie model example.

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**18.13.5.4** — Three-dimensional effects shall be considered in design and analyzed using three-dimensional procedures or approximated by considering the summation of effects for two orthogonal planes.

**18.13.5.5** — For anchorage devices located away from the end of the member, bonded reinforcement shall be provided to transfer at least  $0.35P_{pu}$  into the concrete section behind the anchor. Such reinforcement shall be placed symmetrically around the anchorage devices and shall be fully developed both behind and ahead of the anchorage devices.

**18.13.5.6** — Where tendons are curved in the general zone, except for monostrand tendons in slabs or where analysis shows reinforcement is not required, bonded reinforcement shall be provided to resist radial and splitting forces.

**18.13.5.7** — Except for monostrand tendons in slabs or where analysis shows reinforcement is not required, minimum reinforcement with a nominal tensile strength equal to 2 percent of each factored prestressing force shall be provided in orthogonal directions parallel to the back face of all anchorage zones to limit spalling.

**18.13.5.8** — Tensile strength of concrete shall be neglected in calculations of reinforcement requirements.

**18.13.6 — Detailing requirements**

Selection of reinforcement sizes, spacings, cover, and other details for anchorage zones shall make allowances for tolerances on the bending, fabrication, and placement of reinforcement, for the size of aggregate, and for adequate placement and consolidation of the concrete.

**18.14 — Design of anchorage zones for monostrand or single 5/8 in. diameter bar tendons****18.14.1 — Local zone design**

Monostrand or single 5/8 in. or smaller diameter bar anchorage devices and local zone reinforcement shall meet the requirements of ACI 423.7 or the special anchorage device requirements of **18.15.2**.

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**R18.13.5.4** — The provision for three-dimensional effects was included so that the effects perpendicular to the main plane of the member, such as bursting forces in the thin direction of webs or slabs are not neglected. In many cases these effects can be determined independently for each direction, but some applications require a fully three-dimensional analysis (for example, diaphragms for the anchorage of external tendons).

**R18.13.5.5** — Where anchorages are located away from the end of a member, local tensile stresses are generated behind these anchorages (see **Fig. R18.13.1(b)**) due to compatibility requirements for deformations ahead of and behind the anchorages. Bonded tie-back reinforcement is required in the immediate vicinity of the anchorage to limit the extent of cracking behind the anchorage. The requirement of  $0.35P_{pu}$  was developed using 25 percent of the unfactored prestressing force being resisted by reinforcement at  $0.6f_y$ .

**R18.14 — Design of anchorage zones for monostrand or single 5/8 in. diameter bar tendons**

**CODE****COMMENTARY****18.14.2 — General-zone design for slab tendons**

**18.14.2.1** — For anchorage devices for 0.5 in. or smaller diameter strands in normalweight concrete slabs, minimum reinforcement meeting the requirements of 18.14.2.2 and 18.14.2.3 shall be provided unless a detailed analysis satisfying 18.13.5 shows such reinforcement is not required.

**18.14.2.2** — Two horizontal bars at least No. 4 in size shall be provided parallel to the slab edge. They shall be permitted to be in contact with the front face of the anchorage device and shall be within a distance of  $1/2h$  ahead of each device. Those bars shall extend at least 6 in. either side of the outer edges of each device.

**18.14.2.3** — If the center-to-center spacing of anchorage devices is 12 in. or less, the anchorage devices shall be considered as a group. For each group of six or more anchorage devices,  $n + 1$  hairpin bars or closed stirrups at least No. 3 in size shall be provided, where  $n$  is the number of anchorage devices. One hairpin bar or stirrup shall be placed between each anchorage device and one on each side of the group. The hairpin bars or stirrups shall be placed with the legs extending into the slab perpendicular to the edge. The center portion of the hairpin bars or stirrups shall be placed perpendicular to the plane of the slab from  $3h/8$  to  $h/2$  ahead of the anchorage devices.

**18.14.2.4** — For anchorage devices not conforming to 18.14.2.1, minimum reinforcement shall be based upon a detailed analysis satisfying 18.13.5.

**18.14.3 — General-zone design for groups of monostrand tendons in beams and girders**

Design of general zones for groups of monostrand tendons in beams and girders shall meet the requirements of 18.13.3 through 18.13.5.

**R18.14.2 — General-zone design for slab tendons**

For monostrand slab tendons, the general-zone minimum reinforcement requirements are based on the recommendations of Joint ACI-ASCE Committee 423,<sup>18.12</sup> which shows typical details. The horizontal bars parallel to the edge required by 18.14.2.2 should be continuous where possible.

The tests on which the recommendations of Reference 18.29 were based were limited to anchorage devices for 1/2 in. diameter, 270,000 psi strand, unbonded tendons in normal-weight concrete. Thus, for larger strand anchorage devices and for all use in lightweight concrete slabs, Committee 423 recommended that the amount and spacing of reinforcement should be conservatively adjusted to provide for the larger anchorage force and smaller splitting tensile strength of lightweight concrete.<sup>18.12</sup>

Both References 18.12 and 18.29 recommend that hairpin bars also be furnished for anchorages located within 12 in. of slab corners to resist edge tension forces. The words “ahead of” in 18.14.2.3 have the meaning shown in Fig. R18.13.1.

In those cases where multistrand anchorage devices are used for slab tendons, 18.15 is applicable.

The bursting reinforcement perpendicular to the plane of the slab required by 18.14.2.3 for groups of relatively closely spaced tendons should also be provided in the case of widely spaced tendons if an anchorage device failure could cause more than local damage.

**R18.14.3 — General-zone design for groups of monostrand tendons in beams and girders**

Groups of monostrand tendons with individual monostrand anchorage devices are often used in beams and girders. Anchorage devices can be treated as closely spaced if their center-to-center spacing does not exceed 1.5 times the width of the anchorage device in the direction considered. If a beam or girder has a single anchorage device or a single group of closely spaced anchorage devices, the use of simplified equations such as those given in R18.13.5 is allowed, unless 18.13.5.2 governs. More complex conditions can be designed using strut-and-tie models. Detailed recommendations for use of such models are given in References 18.27 and 18.30 as well as in R18.13.5.

**CODE****COMMENTARY****18.15 — Design of anchorage zones for multistrand tendons****18.15.1 — Local zone design**

Basic multistrand anchorage devices and local zone reinforcement shall meet the requirements of AASHTO "Standard Specification for Highway Bridges," Division I, Articles 9.21.7.2.2 through 9.21.7.2.4.

Special anchorage devices shall satisfy the tests required in AASHTO "Standard Specification for Highway Bridges," Division I, Article 9.21.7.3 and described in AASHTO "Standard Specification for Highway Bridges," Division II, Article 10.3.2.3.

**18.15.2 — Use of special anchorage devices**

Where special anchorage devices are to be used, supplemental skin reinforcement shall be furnished in the corresponding regions of the anchorage zone, in addition to the confining reinforcement specified for the anchorage device. This supplemental reinforcement shall be similar in configuration and at least equivalent in volumetric ratio to any supplementary skin reinforcement used in the qualifying acceptance tests of the anchorage device.

**18.15.3 — General-zone design**

Design for general zones for multistrand tendons shall meet the requirements of 18.13.3 through 18.13.5.

**18.16 — Corrosion protection for unbonded tendons**

**18.16.1** — Unbonded prestressing steel shall be encased with sheathing. The prestressing steel shall be completely coated and the sheathing around the prestressing steel filled with suitable material to inhibit corrosion.

**18.16.2** — Sheathing shall be watertight and continuous over entire length to be unbonded.

**18.16.3** — For applications in corrosive environments, the sheathing shall be connected to all stressing, intermediate and fixed anchorages in a watertight fashion.

**18.16.4** — Unbonded single-strand tendons shall be protected against corrosion in accordance with ACI 423.7.

**R18.15 — Design of anchorage zones for multistrand tendons****R18.15.1 — Local zone design**

See R18.13.2.

**R18.15.2 — Use of special anchorage devices**

Skin reinforcement is reinforcement placed near the outer faces in the anchorage zone to limit local crack width and spacing. Reinforcement in the general zone for other actions (flexure, shear, shrinkage, temperature, and similar) may be used in satisfying the supplementary skin reinforcement requirement. Determination of the supplementary skin reinforcement depends on the anchorage device hardware used and frequently cannot be determined until the shop-drawing stage.

**R18.16 — Corrosion protection for unbonded tendons**

**R18.16.1** — Suitable material for corrosion protection of unbonded prestressing steel should have the properties identified in Section 5.1 of Reference 18.30.

**R18.16.2** — Typically, sheathing is a continuous, seamless, high-density polyethylene material that is extruded directly onto the coated prestressing steel.

**R18.16.4** — In the 1989 Code, corrosion protection requirements for unbonded single-strand tendons were added in accordance with the Post-Tensioning Institute's "Specification for Unbonded Single Strand Tendons." In the 2002 Code, the reference changed to ACI 423.6. In the 2008 Code, the reference was changed to ACI 423.7.

**CODE****COMMENTARY****18.17 — Post-tensioning ducts**

**18.17.1** — Ducts for grouted tendons shall be mortartight and nonreactive with concrete, prestressing steel, grout, and corrosion inhibitor.

**18.17.2** — Ducts for grouted single-wire, single-strand, or single-bar tendons shall have an inside diameter at least 1/4 in. larger than the prestressing steel diameter.

**18.17.3** — Ducts for grouted multiple wire, multiple strand, or multiple bar tendons shall have an inside cross-sectional area at least two times the cross-sectional area of the prestressing steel.

**18.17.4** — Ducts shall be maintained free of ponded water if members to be grouted are exposed to temperatures below freezing prior to grouting.

**18.18 — Grout for bonded tendons**

**18.18.1** — Grout shall consist of portland cement and water; or portland cement, sand, and water.

**18.18.2** — Materials for grout shall conform to 18.18.2.1 through 18.18.2.4.

**18.18.2.1** — Portland cement shall conform to 3.2.

**18.18.2.2** — Water shall conform to 3.4.

**18.18.2.3** — Sand, if used, shall conform to ASTM C144 except that gradation shall be permitted to be modified as necessary to obtain satisfactory workability.

**18.18.2.4** — Admixtures conforming to 3.6 and known to have no injurious effects on grout, steel, or concrete shall be permitted. Calcium chloride shall not be used.

**R18.17 — Post-tensioning ducts**

**R18.17.4** — Water in ducts may cause distress to the surrounding concrete upon freezing. When strands are present, ponded water in ducts should also be avoided. A corrosion inhibitor should be used to provide temporary corrosion protection if prestressing steel is exposed to prolonged periods of moisture in the ducts before grouting.<sup>18.31</sup>

**R18.18 — Grout for bonded tendons**

Proper grout and grouting procedures are critical to post-tensioned construction.<sup>18.32,18.33</sup> Grout provides bond between the prestressing steel and the duct, and provides corrosion protection to the prestressing steel.

Past success with grout for bonded tendons has been with portland cement. A blanket endorsement of all cementitious materials (defined in 2.1) for use with this grout is deemed inappropriate because of a lack of experience or tests with cementitious materials other than portland cement and a concern that some cementitious materials might introduce chemicals listed as harmful to tendons in R18.18.2. Thus, portland cement in 18.18.1 and water-cement ratio in 18.18.3.3 are retained in the Code.

**R18.18.2** — The limitations on admixtures in 3.6 apply to grout. Substances known to be harmful to tendons, grout, or concrete are chlorides, fluorides, sulfites, and nitrates. Aluminum powder or other expansive admixtures, when approved, should produce an unconfined expansion of 5 to 10 percent. Neat cement grout is used in almost all building construction. Use of finely graded sand in the grout should only be considered with large ducts having large void areas.

**CODE****COMMENTARY****18.18.3 — Selection of grout proportions**

**18.18.3.1** — Proportions of materials for grout shall be based on either (a) or (b):

- (a) Results of tests on fresh and hardened grout prior to beginning grouting operations; or
- (b) Prior documented experience with similar materials and equipment and under comparable field conditions.

**18.18.3.2** — Cement used in the Work shall correspond to that on which selection of grout proportions was based.

**18.18.3.3** — Water content shall be minimum necessary for proper pumping of grout; however, water-cement ratio shall not exceed 0.45 by weight.

**18.18.3.4** — Water shall not be added to increase grout flowability that has been decreased by delayed use of the grout.

**18.18.4 — Mixing and pumping grout**

**18.18.4.1** — Grout shall be mixed in equipment capable of continuous mechanical mixing and agitation that will produce uniform distribution of materials, passed through screens, and pumped in a manner that will completely fill the ducts.

**18.18.4.2** — Temperature of members at time of grouting shall be above 35°F and shall be maintained above 35°F until field-cured 2 in. cubes of grout reach a minimum compressive strength of 800 psi.

**18.18.4.3** — Grout temperatures shall not be above 90°F during mixing and pumping.

**18.19 — Protection for prestressing steel**

Burning or welding operations in the vicinity of prestressing steel shall be performed so that prestressing steel is not subject to excessive temperatures, welding sparks, or ground currents.

**18.20 — Application and measurement of prestressing force**

**18.20.1** — Prestressing force shall be determined by both of (a) and (b):

- (a) Measurement of steel elongation. Required elongation shall be determined from average load-elongation curves for the prestressing steel used;

**R18.18.3 — Selection of grout proportions**

Grout proportioned in accordance with these provisions will generally lead to 7-day compressive strength on standard 2 in. cubes in excess of 2500 psi and 28-day strengths of about 4000 psi. The handling and placing properties of grout are usually given more consideration than strength when designing grout mixtures.

**R18.18.4 — Mixing and pumping grout**

In an ambient temperature of 35°F, grout with an initial minimum temperature of 60°F may require as much as 5 days to reach 800 psi. A minimum grout temperature of 60°F is suggested because it is consistent with the recommended minimum temperature for concrete placed at an ambient temperature of 35°F. Quickset grouts, when approved, may require shorter periods of protection, and the recommendations of the suppliers should be followed. Test cubes should be cured under temperature and moisture conditions as close as possible to those of the grout in the member. Grout temperatures in excess of 90°F will lead to difficulties in pumping.

**R18.20 — Application and measurement of prestressing force**

**R18.20.1** — Elongation measurements for prestressed elements should be in accordance with the procedures outlined in the *Manual for Quality Control for Plants and Production Structural Precast Concrete Products*, published by the Precast/Prestressed Concrete Institute.<sup>18.34</sup>

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(b) Observation of jacking force on a calibrated gage or load cell or by use of a calibrated dynamometer.

Cause of any difference in force determination between (a) and (b) that exceeds 5 percent for pretensioned elements or 7 percent for post-tensioned construction shall be ascertained and corrected.

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Section 18.18.1 of the 1989 Code was revised to permit 7 percent tolerance in prestressing steel force determined by gauge pressure and elongation measurements for post-tensioned construction. Elongation measurements for post-tensioned construction are affected by several factors that are less significant, or that do not exist, for pretensioned elements. The friction along prestressing steel in post-tensioning applications may be affected to varying degrees by placing tolerances and small irregularities in tendon profile due to concrete placement. The friction coefficients between the prestressing steel and the duct are also subject to variation. The 5 percent tolerance that has appeared since the 1963 Code was proposed by Committee 423 in 1958,<sup>18.6</sup> and primarily reflected experience with production of pretensioned concrete elements. Because the tendons for pretensioned elements are usually stressed in the air with minimal friction effects, the 5 percent tolerance for such elements was retained.

**18.20.2** — Where the transfer of force from the bulkheads of pretensioning bed to the concrete is accomplished by flame cutting prestressing steel, cutting points and cutting sequence shall be predetermined to avoid undesired temporary stresses.

**18.20.3** — Long lengths of exposed pretensioned strand shall be cut near the member to minimize shock to concrete.

**18.20.4** — Total loss of prestress due to unreplaced broken prestressing steel shall not exceed 2 percent of total prestress.

## **18.21 — Post-tensioning anchorages and couplers**

**18.21.1** — Anchorages and couplers for bonded and unbonded tendons shall develop at least 95 percent of the  $f_{pu}$  when tested in an unbonded condition, without exceeding anticipated set. For bonded tendons, anchorages and couplers shall be located so that 100 percent of  $f_{pu}$  shall be developed at critical sections after the prestressing steel is bonded in the member.

**R18.20.4** — This provision applies to all prestressed concrete members. For cast-in-place post-tensioned slab systems, a member should be that portion considered as an element in the design, such as the joist and effective slab width in one-way joist systems, or the column strip or middle strip in two-way flat plate systems.

## **R18.21 — Post-tensioning anchorages and couplers**

**R18.21.1** — In the 1986 interim Code, the separate provisions for strength of unbonded and bonded tendon anchorages and couplers presented in 18.19.1 and 18.19.2 of the 1983 Code were combined into a single revised 18.21 covering anchorages and couplers for both unbonded and bonded tendons. Since the 1989 Code, the required strength of the tendon-anchor or tendon-coupler assemblies for both unbonded and bonded tendons, when tested in an unbonded state, is based on 95 percent of the specified breaking strength of the prestressing steel in the test. The prestressing steel material should comply with the minimum provisions of the applicable ASTM specifications as outlined in 3.5.5. The specified strength of anchorages and couplers exceeds the maximum design strength of the prestressing steel by a substantial margin and, at the same time, recognizes the stress-riser effects associated with most available post-

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tensioning anchorages and couplers. Anchorage and coupler strength should be attained with a minimum amount of permanent deformation and successive set, recognizing that some deformation and set will occur when testing to failure. Tendon assemblies should conform to the 2 percent elongation requirements in ACI 301<sup>18.35</sup> and industry recommendations.<sup>18.30</sup> Anchorages and couplers for bonded tendons that develop less than 100 percent of the specified breaking strength of the prestressing steel should be used only where the bond transfer length between the anchorage or coupler and critical sections equals or exceeds that required to develop the prestressing steel strength. This bond length may be calculated by the results of tests of bond characteristics of un tensioned prestressing strand,<sup>18.36</sup> or by bond tests on other prestressing steel materials, as appropriate.

**18.21.2** — Couplers shall be placed in areas approved by the licensed design professional and enclosed in housing long enough to permit necessary movements.

**18.21.3** — In unbonded construction subject to repetitive loads, attention shall be given to the possibility of fatigue in anchorages and couplers.

**18.21.4** — Anchorages, couplers, and end fittings shall be permanently protected against corrosion.

**18.22 — External post-tensioning**

**18.22.1** — Post-tensioning tendons shall be permitted to be external to any concrete section of a member. The strength and serviceability design methods of this Code shall be used in evaluating the effects of external tendon forces on the concrete structure.

**18.22.2** — External tendons shall be considered as unbonded tendons when computing flexural strength unless provisions are made to effectively bond the external tendons to the concrete section along its entire length.

**18.22.3** — External tendons shall be attached to the concrete member in a manner that maintains the desired eccentricity between the tendons and the concrete centroid throughout the full range of anticipated member deflection.

**R18.21.3** — For discussion on fatigue loading, see [Reference 18.37](#).

For detailed recommendations on tests for static and cyclic loading conditions for tendons and anchorage fittings of unbonded tendons, see Section 4.1.3 of [Reference 18.12](#), and Section 15.2.2 of [Reference 18.35](#).

**R18.21.4** — For recommendations regarding protection see Sections 4.2 and 4.3 of [Reference 18.12](#), and Sections 3.4, 3.6, 5, 6, and 8.3 of [Reference 18.30](#).

**R18.22 — External post-tensioning**

External attachment of tendons is a versatile method of providing additional strength, or improving serviceability, or both, in existing structures. It is well suited to repair or upgrade existing structures and permits a wide variety of tendon arrangements.

Additional information on external post-tensioning is given in [Reference 18.38](#).

**R18.22.3** — External tendons are often attached to the concrete member at various locations between anchorages (such as midspan, quarter points, or third points) for desired load balancing effects, for tendon alignment, or to address tendon vibration concerns. Consideration should be given to the effects caused by the tendon profile shifting in relationship to the concrete centroid as the member deforms under effects of post-tensioning and applied load.

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**18.22.4** — External tendons and tendon anchorage regions shall be protected against corrosion, and the details of the protection method shall be indicated in the contract documents.

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**R18.22.4** — Permanent corrosion protection can be achieved by a variety of methods. The corrosion protection provided should be suitable to the environment in which the tendons are located. Some conditions will require that the prestressing steel be protected by concrete cover or by cement grout in polyethylene or metal tubing; other conditions will permit the protection provided by coatings such as paint or grease. Corrosion protection methods should meet the fire protection requirements of the general building code, unless the installation of external post-tensioning is to only improve serviceability.

# CHAPTER 19 — SHELLS AND FOLDED PLATE MEMBERS

## CODE

### 19.1 — Scope and definitions

**19.1.1** — Provisions of Chapter 19 shall apply to thin shell and folded plate concrete structures, including ribs and edge members.

**19.1.2** — All provisions of this Code not specifically excluded, and not in conflict with provisions of Chapter 19, shall apply to thin-shell structures.

**19.1.3** — ***Thin shells*** — Three-dimensional spatial structures made up of one or more curved slabs or folded plates whose thicknesses are small compared to their other dimensions. Thin shells are characterized by their three-dimensional load-carrying behavior, which is determined by the geometry of their forms, by the manner in which they are supported, and by the nature of the applied load.

**19.1.4** — ***Folded plates*** — A class of shell structure formed by joining flat, thin slabs along their edges to create a three-dimensional spatial structure.

**19.1.5** — ***Ribbed shells*** — Spatial structures with material placed primarily along certain preferred rib lines, with the area between the ribs filled with thin slabs or left open.

## COMMENTARY

### R19.1 — Scope and definitions

The Code and Commentary provide information on the design, analysis, and construction of concrete thin shells and folded plates. The process began in 1964 with the publication of a practice and commentary by ACI Committee 334,<sup>19.1</sup> and continued with the inclusion of Chapter 19 in the 1971 Code. The 1982 revision of ACI 334.1R reflected additional experience in design, analysis, and construction and was influenced by the publication of the “**Recommendations for Reinforced Concrete Shells and Folded Plates**” of the International Association for Shell and Spatial Structures (IASS) in 1979.<sup>19.2</sup>

Since Chapter 19 applies to concrete thin shells and folded plates of all shapes, extensive discussion of their design, analysis, and construction in the Commentary is not possible. Additional information can be obtained from the references. Performance of shells and folded plates requires attention to detail.<sup>19.3</sup>

**R19.1.3** — Common types of thin shells are domes (surfaces of revolution),<sup>19.4,19.5</sup> cylindrical shells,<sup>19.5</sup> barrel vaults,<sup>19.6</sup> conoids,<sup>19.6</sup> elliptical paraboloids,<sup>19.6</sup> hyperbolic paraboloids,<sup>19.7</sup> and groined vaults.<sup>19.7</sup>

**R19.1.4** — Folded plates may be prismatic,<sup>19.4,19.5</sup> nonprismatic,<sup>19.5</sup> or faceted. The first two types consist generally of planar thin slabs joined along their longitudinal edges to form a beam-like structure spanning between supports. Faceted folded plates are made up of triangular or polygonal planar thin slabs joined along their edges to form three-dimensional spatial structures.

**R19.1.5** — Ribbed shells<sup>19.6,19.7</sup> generally have been used for larger spans where the increased thickness of the curved slab alone becomes excessive or uneconomical. Ribbed shells are also used because of the construction techniques employed and to enhance the aesthetic impact of the completed structure.

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**19.1.6 — *Auxiliary members*** — Ribs or edge beams that serve to strengthen, stiffen, or support the shell; usually, auxiliary members act jointly with the shell.

**19.1.7 — *Elastic analysis*** — An analysis of deformations and internal forces based on equilibrium, compatibility of strains, and assumed elastic behavior, and representing to a suitable approximation the three-dimensional action of the shell together with its auxiliary members.

**19.1.8 — *Inelastic analysis*** — An analysis of deformations and internal forces based on equilibrium, nonlinear stress-strain relations for concrete and reinforcement, consideration of cracking and time-dependent effects, and compatibility of strains. The analysis shall represent to a suitable approximation three-dimensional action of the shell together with its auxiliary members.

**19.1.9 — *Experimental analysis*** — An analysis procedure based on the measurement of deformations or strains, or both, of the structure or its model; experimental analysis is based on either elastic or inelastic behavior.

**R19.1.6** — Most thin shell structures require ribs or edge beams at their boundaries to carry the shell boundary forces, to assist in transmitting them to the supporting structure, and to accommodate the increased amount of reinforcement in these areas.

**R19.1.7** — Elastic analysis of thin shells and folded plates can be performed using any method of structural analysis based on assumptions that provide suitable approximations to the three-dimensional behavior of the structure. The method should determine the internal forces and displacements needed in the design of the shell proper, the rib or edge members, and the supporting structure. Equilibrium of internal forces and external loads and compatibility of deformations should be satisfied.

Methods of elastic analysis based on classical shell theory, simplified mathematical or analytical models, or numerical solutions using finite element,<sup>19.8</sup> finite differences,<sup>19.6</sup> or numerical integration techniques,<sup>19.6,19.9</sup> are described in the cited references.

The choice of the method of analysis and the degree of accuracy required depends on certain critical factors. These include: the size of the structure, the geometry of the thin shell or folded plate, the manner in which the structure is supported, the nature of the applied load, and the extent of personal or documented experience regarding the reliability of the given method of analysis in predicting the behavior of the specific type of shell<sup>19.6</sup> or folded plate.<sup>19.5</sup>

**R19.1.8** — Inelastic analysis of thin shells and folded plates can be performed using a refined method of analysis based on the specific nonlinear material properties, nonlinear behavior due to the cracking of concrete, and time-dependent effects such as creep, shrinkage, temperature, and load history. These effects are incorporated in order to trace the response and crack propagation of a reinforced concrete shell through the elastic, inelastic, and ultimate ranges. Such analyses usually require incremental loading and iterative procedures to converge on solutions that satisfy both equilibrium and strain compatibility.<sup>19.10,19.11</sup>

**CODE****COMMENTARY****19.2 — Analysis and design**

**19.2.1** — Elastic behavior shall be an accepted basis for determining internal forces and displacements of thin shells. This behavior shall be permitted to be established by computations based on an analysis of the uncracked concrete structure in which the material is assumed linearly elastic, homogeneous, and isotropic. Poisson's ratio of concrete shall be permitted to be taken equal to zero.

**19.2.2** — Inelastic analyses shall be permitted to be used where it can be shown that such methods provide a safe basis for design.

**19.2.3** — Equilibrium checks of internal resistances and external loads shall be made to ensure consistency of results.

**19.2.4** — Experimental or numerical analysis procedures shall be permitted where it can be shown that such procedures provide a safe basis for design.

**19.2.5** — Approximate methods of analysis shall be permitted where it can be shown that such methods provide a safe basis for design.

**19.2.6** — In prestressed shells, the analysis shall also consider behavior under loads induced during prestressing, at cracking load, and at factored load. Where tendons are draped within a shell, design shall take into account force components on the shell resulting from the tendon profile not lying in one plane.

**R19.2.2 — Analysis and design**

**R19.2.1** — For types of shell structures where experience, tests, and analyses have shown that the structure can sustain reasonable overloads without undergoing brittle failure, elastic analysis is an acceptable procedure. In such cases, it may be assumed that reinforced concrete is ideally elastic, homogeneous, and isotropic, having identical properties in all directions. An analysis should be performed for the shell considering service load conditions. The analysis of shells of unusual size, shape, or complexity should consider behavior through the elastic, cracking, and inelastic stages.

**R19.2.2** — Several inelastic analysis procedures contain possible solution methods.<sup>19.10,19.11</sup>

**R19.2.4** — Experimental analysis of elastic models<sup>19.12</sup> has been used as a substitute for an analytical solution of a complex shell structure. Experimental analysis of reinforced microconcrete models through the elastic, cracking, inelastic, and ultimate stages should be considered for important shells of unusual size, shape, or complexity.

For model analysis, only those portions of the structure that significantly affect the items under study need be simulated. Every attempt should be made to ensure that the experiments reveal the quantitative behavior of the prototype structure.

Wind tunnel tests of a scaled-down model do not necessarily provide usable results and should be conducted by a recognized expert in wind tunnel testing of structural models.

**R19.2.5** — Solutions that include both membrane and bending effects and satisfy conditions of compatibility and equilibrium are encouraged. Approximate solutions that satisfy statics but not the compatibility of strains may be used only when extensive experience has proved that safe designs have resulted from their use. Such methods include beam-type analysis for barrel shells and folded plates having large ratios of span to either width or radius of curvature, simple membrane analysis for shells of revolution, and others in which the equations of equilibrium are satisfied, while the strain compatibility equations are not.

**R19.2.6** — If the shell is prestressed, the analysis should include its strength at factored loads as well as its adequacy under service loads, under the load that causes cracking, and under loads induced during prestressing. Axial forces due to draped tendons may not lie in one plane, and due consideration should be given to the resulting force components. The effects of post-tensioning of shell-supporting members should be taken into account.

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**19.2.7** — The thickness of a shell and its reinforcement shall be proportioned for the required strength and serviceability, using either the strength design method of 8.1.1 or the design method of 8.1.2.

**19.2.8** — Shell instability shall be investigated and shown by design to be precluded.

**19.2.9** — Auxiliary members shall be designed according to the applicable provisions of the Code. It shall be permitted to assume that a portion of the shell equal to the flange width, as specified in 8.12, acts with the auxiliary member. In such portions of the shell, the reinforcement perpendicular to the auxiliary member shall be at least equal to that required for the flange of a T-beam by 8.12.5.

**19.2.10** — Strength design of shell slabs for membrane and bending forces shall be based on the distribution of stresses and strains as determined from either an elastic or an inelastic analysis.

**R19.2.7** — The thin shell's thickness and reinforcement are required to be proportioned to satisfy the strength provisions of this Code, and to resist internal forces obtained from an analysis, an experimental model study, or a combination thereof. Reinforcement sufficient to minimize cracking under service load conditions should be provided. The thickness of the shell is often dictated by the required reinforcement and the construction constraints, by 19.2.8, or by the Code minimum thickness requirements.

**R19.2.8** — Thin shells, like other structures that experience in-plane membrane compressive forces, are subject to buckling when the applied load reaches a critical value. Because of the surface-like geometry of shells, the problem of calculating buckling load is complex. If one of the principal membrane forces is tensile, the shell is less likely to buckle than if both principal membrane forces are compressive. The kinds of membrane forces that develop in a shell depend on its initial shape and the manner in which the shell is supported and loaded. In some types of shells, post-buckling behavior should be considered in determining safety against instability.<sup>19.2</sup>

Investigation of thin shells for stability should consider the effect of: (1) anticipated deviation of the geometry of the shell surface as-built from the idealized geometry; (2) large deflections; (3) creep and shrinkage of concrete; (4) inelastic properties of materials; (5) cracking of concrete; (6) location, amount, and orientation of reinforcement; and (7) possible deformation of supporting elements.

Measures successfully used to improve resistance to buckling include the provision of two mats of reinforcement—one near each outer surface of the shell, a local increase of shell curvatures, the use of ribbed shells, and the use of concrete with high tensile strength and low creep.

A procedure for determining critical buckling loads of shells is given in the IASS recommendations.<sup>19.2</sup> Some recommendations for buckling design of domes used in industrial applications are given in References 19.13 and 19.14.

**R19.2.10** — The stresses and strains in the shell slab used for design are those determined by analysis (elastic or inelastic) multiplied by appropriate load factors. Because of detrimental effects of membrane cracking, the computed tensile strain in the reinforcement under factored loads should be limited.

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**19.2.11** — In a region where membrane cracking is predicted, the nominal compressive strength parallel to the cracks shall be taken as  $0.4f'_c$ .

**19.3 — Design strength of materials**

**19.3.1** — Specified compressive strength of concrete  $f'_c$  at 28 days shall not be less than 3000 psi.

**19.3.2** — Specified yield strength of nonprestressed reinforcement  $f_y$  shall not exceed 60,000 psi.

**19.4 — Shell reinforcement**

**19.4.1** — Shell reinforcement shall be provided to resist tensile stresses from internal membrane forces, to resist tension from bending and twisting moments, to limit shrinkage and temperature crack width and spacing, and as reinforcement at shell boundaries, load attachments, and shell openings.

**19.4.2** — Tensile reinforcement shall be provided in two or more directions and shall be proportioned such that its resistance in any direction equals or exceeds the component of internal forces in that direction.

Alternatively, reinforcement for the membrane forces in the slab shall be calculated as the reinforcement required to resist axial tensile forces plus the tensile force due to shear-friction required to transfer shear across any cross section of the membrane. The assumed coefficient of friction,  $\mu$ , shall not exceed that specified in **11.6.4.3**.

**COMMENTARY**

**R19.2.11** — When principal tensile stress produces membrane cracking in the shell, experiments indicate the attainable compressive strength in the direction parallel to the cracks is reduced.<sup>19.15,19.16</sup>

**R19.4 — Shell reinforcement**

**R19.4.1** — At any point in a shell, two different kinds of internal forces may occur simultaneously: those associated with membrane action, and those associated with bending of the shell. The membrane forces are assumed to act in the tangential plane midway between the surfaces of the shell, and are the two axial forces and the membrane shears. Flexural effects include bending moments, twisting moments, and the associated transverse shears. Limiting membrane crack width and spacing due to shrinkage, temperature, and service load conditions is a major design consideration.

**R19.4.2** — The requirement of ensuring strength in all directions is based on safety considerations. Any method that ensures sufficient strength consistent with equilibrium is acceptable. The direction of the principal membrane tensile force at any point may vary depending on the direction, magnitudes, and combinations of the various applied loads.

The magnitude of the internal membrane forces, acting at any point due to a specific load, is generally calculated on the basis of an elastic theory in which the shell is assumed as uncracked. The computation of the required amount of reinforcement to resist the internal membrane forces has been traditionally based on the assumption that concrete does not resist tension. The associated deflections, and the possibility of cracking, should be investigated in the serviceability phase of the design. Achieving this may require a working stress design for steel selection.

Where reinforcement is not placed in the direction of the principal tensile forces and where cracks at the service load level are objectionable, the computation of reinforcement may have to be based on a more refined approach<sup>19.15,19.17,19.18</sup> that considers the existence of cracks. In the cracked state, the concrete is assumed to be unable to resist either tension or shear. Thus, equilibrium is attained by equating tensile-resisting forces in reinforcement and compressive-resisting forces in concrete.

The alternative method to calculate orthogonal reinforcement is the shear-friction method. It is based on the assumption

**CODE****COMMENTARY**

**19.4.3** — The area of shell reinforcement at any section as measured in two orthogonal directions shall not be less than the slab shrinkage or temperature reinforcement required by 7.12.

**19.4.4** — Reinforcement for shear and bending moments about axes in the plane of the shell slab shall be calculated in accordance with Chapters 10, 11, and 13.

**19.4.5** — The area of shell tension reinforcement shall be limited so that the reinforcement will yield before either crushing of concrete in compression or shell buckling can take place.

**19.4.6** — In regions of high tension, membrane reinforcement shall, if practical, be placed in the general directions of the principal tensile membrane forces. Where this is not practical, it shall be permitted to place membrane reinforcement in two or more component directions.

**19.4.7** — If the direction of reinforcement varies more than 10 degrees from the direction of principal tensile membrane force, the amount of reinforcement shall be reviewed in relation to cracking at service loads.

**19.4.8** — Where the magnitude of the principal tensile membrane stress within the shell varies greatly over the area of the shell surface, reinforcement resisting the total tension shall be permitted to be concentrated in the regions of largest tensile stress where it can be shown that this provides a safe basis for design. However, the ratio of shell reinforcement in any portion of the tensile zone shall be not less than 0.0035 based on the overall thickness of the shell.

that shear integrity of a shell should be maintained at factored loads. It is not necessary to calculate principal stresses if the alternative approach is used.

**R19.4.3** — Minimum membrane reinforcement corresponding to slab shrinkage and temperature reinforcement are to be provided in at least two approximately orthogonal directions even if the calculated membrane forces are compressive in one or more directions.

**R19.4.5** — The requirement that the tensile reinforcement yields before the concrete crushes anywhere is consistent with 10.3.3. Such crushing can also occur in regions near supports and, for some shells, where the principal membrane forces are approximately equal and opposite in sign.

**R19.4.6** — Generally, for all shells, and particularly in regions of substantial tension, the orientation of reinforcement should approximate the directions of the principal tensile membrane forces. However, in some structures it is not possible to detail the reinforcement to follow the stress trajectories. For such cases, orthogonal component reinforcement is allowed.

**R19.4.7** — When the directions of reinforcement deviate significantly (more than 10 degrees) from the directions of the principal membrane forces, higher strains in the shell occur to develop the reinforcement. This might lead to the development of unacceptable wide cracks. The crack width should be estimated and limited if necessary.

Permissible crack widths for service loads under different environmental conditions are given in a report of ACI Committee 224.<sup>19,19</sup> Crack width can be limited by an increase in the amount of reinforcement used, by reducing the stress at the service load level, by providing reinforcement in three or more directions in the plane of the shell, or by using closer spacing of smaller-diameter bars.

**R19.4.8** — The practice of concentrating tensile reinforcement in the regions of maximum tensile stress has led to a number of successful and economical designs, primarily for long folded plates, long barrel vault shells, and for domes. The requirement of providing the minimum reinforcement in the remaining tensile zone is intended to limit crack width and spacing.

**CODE****COMMENTARY**

**19.4.9** — Reinforcement required to resist shell bending moments shall be proportioned with due regard to the simultaneous action of membrane axial forces at the same location. Where shell reinforcement is required in only one face to resist bending moments, equal amounts shall be placed near both surfaces of the shell even though a reversal of bending moments is not indicated by the analysis.

**R19.4.9** — The design method should ensure that the concrete sections, including consideration of the reinforcement, are capable of developing the internal forces required by the equations of equilibrium.<sup>19,20</sup> The sign of bending moments may change rapidly from point to point of a shell. For this reason, reinforcement to resist bending, where required, is to be placed near both outer surfaces of the shell. In many cases, the thickness required to provide proper cover and spacing for the multiple layers of reinforcement may govern the design of the shell thickness.

**19.4.10** — Shell reinforcement in any direction shall not be spaced farther apart than 18 in. nor farther apart than five times the shell thickness. Where the principal membrane tensile stress on the gross concrete area due to factored loads exceeds  $4\phi\lambda\sqrt{f'_c}$ , reinforcement shall not be spaced farther apart than three times the shell thickness.

**R19.4.10** — The value of  $\phi$  to be used is that prescribed in [9.3.2.1](#) for axial tension.

**19.4.11** — Shell reinforcement at the junction of the shell and supporting members or edge members shall be anchored in or extended through such members in accordance with the requirements of [Chapter 12](#), except that the minimum development length shall be  $1.2\ell_d$  but not less than 18 in.

**R19.4.11 and R19.4.12** — On curved shell surfaces it is difficult to control the alignment of precast reinforcement. This should be considered to avoid insufficient splice and development lengths. Sections 19.4.11 and 19.4.12 require extra reinforcement length to maintain the minimum lengths on curved surfaces.

**19.4.12** — Splice lengths of shell reinforcement shall be governed by the provisions of [Chapter 12](#), except that the minimum splice length of tension bars shall be 1.2 times the value required by [Chapter 12](#) but not less than 18 in. The number of splices in principal tensile reinforcement shall be kept to a practical minimum. Where splices are necessary they shall be staggered at least  $\ell_d$  with not more than one-third of the reinforcement spliced at any section.

## 19.5 — Construction

## R19.5 — Construction

**19.5.1** — When removal of formwork is based on a specific modulus of elasticity of concrete because of stability or deflection considerations, the value of the modulus of elasticity,  $E_c$ , used shall be determined from flexural tests of field-cured beam specimens. The number of test specimens, the dimensions of test beam specimens, and test procedures shall be specified by the licensed design professional.

**R19.5.1** — When early removal of forms is necessary, the magnitude of the modulus of elasticity at the time of proposed form removal should be investigated to ensure safety of the shell with respect to buckling, and to restrict deflections.<sup>19,3,19,21</sup> The value of the modulus of elasticity  $E_c$  should be obtained from a flexural test of field-cured specimens. It is not sufficient to determine the modulus from the formula in [8.5.1](#), even if the compressive strength of concrete is determined for the field-cured specimen.

**19.5.2** — Contract documents shall specify the tolerances for the shape of the shell. If construction results in deviations from the shape greater than the specified tolerances, an analysis of the effect of the deviations shall be made and any required remedial actions shall be taken to ensure safe behavior.

**R19.5.2** — In some types of shells, small local deviations from the theoretical geometry of the shell can cause relatively large changes in local stresses and in overall safety against instability. These changes can result in local cracking and yielding that may make the structure unsafe or can greatly affect the critical load, producing instability. The effect of such deviations should be evaluated and any necessary remedial actions should be taken. Attention is needed when using air-supported form systems.<sup>19,22</sup>

**Notes**

# CHAPTER 20 — STRENGTH EVALUATION OF EXISTING STRUCTURES

## CODE

## COMMENTARY

### 20.1 — Strength evaluation — General

**20.1.1** — If there is doubt that a part or all of a structure meets the safety requirements of this Code, a strength evaluation shall be carried out as required by the licensed design professional or building official.

### R20.1 — Strength evaluation — General

Chapter 20 does not cover load testing for the approval of new design or construction methods. (See 16.10 for recommendations on strength evaluation of precast concrete members.) Provisions of Chapter 20 may be used to evaluate whether a structure or a portion of a structure satisfies the safety requirements of this Code. A strength evaluation may be required if the materials are considered to be deficient in quality, if there is evidence indicating faulty construction, if a structure has deteriorated, if a building will be used for a new function, or if, for any reason, a structure or a portion of it does not appear to satisfy the requirements of the Code. In such cases, Chapter 20 provides guidance for investigating the safety of the structure.

If the safety concerns are related to an assembly of elements or an entire structure, it is not feasible to load test every element and section to the maximum. In such cases, it is appropriate that an investigation plan be developed to address the specific safety concerns. If a load test is described as part of the strength evaluation process, it is desirable for all parties involved to come to an agreement about the region to be loaded, the magnitude of the load, the load test procedure, and acceptance criteria before any load tests are conducted.

**R20.1.2** — Strength considerations related to axial load, flexure, and combined axial load and flexure are well understood. There are reliable theories relating strength and short-term displacement to load in terms of dimensional and material data for the structure.

To determine the strength of the structure by analysis, calculations should be based on data gathered on the actual dimensions of the structure, properties of the materials in place, and all pertinent details. Requirements for data collection are in 20.2.

**R20.1.3** — If the shear or bond strength of an element is critical in relation to the doubt expressed about safety, a test may be the most efficient solution to eliminate or confirm the doubt. A test may also be appropriate if it is not feasible to determine the material and dimensional properties required for analysis, even if the cause of the concern relates to flexure or axial load.

Wherever possible and appropriate, support the results of the load test by analysis.

**CODE****COMMENTARY**

**20.1.4** — If the doubt about safety of a part or all of a structure involves deterioration, and if the observed response during the load test satisfies the acceptance criteria, the structure or part of the structure shall be permitted to remain in service for a specified time period. If deemed necessary by the licensed design professional, periodic reevaluations shall be conducted.

**R20.1.4** — For a deteriorating structure, the acceptance provided by the load test may not be assumed to be without limits in terms of time. In such cases, a periodic inspection program is useful. A program that involves physical tests and periodic inspection can justify a longer period in service. Another option for maintaining the structure in service, while the periodic inspection program continues, is to limit the live load to a level determined to be appropriate.

## **20.2 — Determination of required dimensions and material properties**

**20.2.1** — Dimensions of the structural elements shall be established at critical sections.

**20.2.2** — Locations and sizes of the reinforcing bars, welded wire reinforcement, or tendons shall be determined by measurement. It shall be permitted to base reinforcement locations on available drawings if spot checks are made confirming the information on the drawings.

**20.2.3** — If required, concrete strength shall be based on results of cylinder tests from the original construction or tests of cores removed from the part of the structure where the strength is in question. For strength evaluation of an existing structure, cylinder or core test data shall be used to estimate an equivalent  $f'_c$ . The method for obtaining and testing cores shall be in accordance with ASTM C42.

The length of the specified time period should be based on consideration of: (a) the nature of the problem; (b) environmental and load effects; (c) service history of the structure; and (d) scope of the periodic inspection program. At the end of a specified time period, further strength evaluation is required if the structure is to remain in service.

With the agreement of all concerned parties, procedures may be devised for periodic testing that do not necessarily conform to the loading and acceptance criteria specified in [Chapter 20](#).

## **R20.2 — Determination of required dimensions and material properties**

This section applies if it is decided to make an analytical evaluation (see [20.1.2](#)).

**R20.2.1** — Critical sections are where each type of stress calculated for the load in question reaches its maximum value.

**R20.2.2** — For individual elements, amount, size, arrangement, and location should be determined at the critical sections for reinforcement or tendons, or both, designed to resist applied load. Nondestructive investigation methods are acceptable. In large structures, determination of these data for approximately 5 percent of the reinforcement or tendons in critical regions may suffice if these measurements confirm the data provided in the construction drawings.

**R20.2.3** — ACI Committee 214 has developed two methods for determining  $f'_c$  from cores taken from an existing structure. These methods are described in ACI 214.4R<sup>20.1</sup> and rely on statistical analysis techniques. The procedures described are only appropriate where the determination of an equivalent  $f'_c$  is necessary for the strength evaluation of an existing structure and should not be used to investigate low cylinder strength test results in new construction, which is considered in [5.6.5](#).

The number of core tests may depend on the size of the structure and the sensitivity of structural safety to concrete strength. In cases where the potential problem involves flexure only, investigation of concrete strength can be minimal for a lightly reinforced section ( $\rho f_y / f'_c \leq 0.15$  for rectangular section).

**CODE**

**20.2.4** — If required, reinforcement or prestressing steel strength shall be based on tensile tests of representative samples of the material in the structure in question.

**20.2.5** — If the required dimensions and material properties are determined through measurements and testing, and if calculations can be made in accordance with **20.1.2**, it shall be permitted to increase  $\phi$  from those specified in **9.3**, but  $\phi$  shall not be more than:

Tension-controlled sections, as defined in <b>10.3.4</b> .....	1.0
Compression-controlled sections, as defined in <b>10.3.3</b> :	
Members with spiral reinforcement conforming to <b>10.9.3</b> .....	0.9
Other reinforced members .....	0.8
Shear and/or torsion .....	0.8
Bearing on concrete .....	0.8

**20.3 — Load test procedure****20.3.1 — Load arrangement**

The number and arrangement of spans or panels loaded shall be selected to maximize the deflection and stresses in the critical regions of the structural elements of which strength is in doubt. More than one test load arrangement shall be used if a single arrangement will not simultaneously result in maximum values of the effects (such as deflection, rotation, or stress) necessary to demonstrate the adequacy of the structure.

**20.3.2 — Load intensity**

The total test load (including dead load already in place) shall not be less than the larger of (a), (b), and (c):

- (a)  **$1.15D + 1.5L + 0.4(L_r \text{ or } S \text{ or } R)$**
- (b)  **$1.15D + 0.9L + 1.5(L_r \text{ or } S \text{ or } R)$**
- (c)  **$1.3D$**

The load factor on the live load  $L$  in (b) shall be permitted to be reduced to 0.45 except for garages, areas occupied as places of public assembly, and all areas where  $L$  is greater than 100 lb/ft<sup>2</sup>. It shall be permitted to reduce  $L$  in accordance with the provisions of the applicable general building code.

**COMMENTARY**

**R20.2.4** — The number of tests required depends on the uniformity of the material and is best determined by the licensed design profession responsible for the evaluation.

**R20.2.5** — Strength reduction factors given in 20.2.5 are larger than those specified in **Chapter 9**. These increased values are justified by the use of accurate field-obtained material properties, actual in-place dimensions, and well-understood methods of analysis.

The strength reduction factors in 20.2.5 were changed for the 2002 edition to be compatible with the load combinations and strength reduction factors of **Chapter 9**, which were revised at that time. For the 2008 edition, the strength reduction factor in 20.2.5 for members with spiral reinforcement was increased to correspond to an increase in this strength reduction factor in **Chapter 9**.

**R20.3 — Load test procedure****R20.3.1—Load arrangement**

It is important to apply the load at locations so that its effects on the suspected defect are a maximum and the probability of unloaded members sharing the applied load is a minimum. In cases where it is shown by analysis that adjoining unloaded elements will help carry some of the load, the load should be placed to develop effects consistent with the intent of the load factor.

**R20.3.2 — Load intensity**

The required load intensity follows previous load test practice. The live load  $L$  may be reduced as permitted by the general building code governing safety considerations for the structure. The test load should be increased to compensate for resistance provided by unloaded portions of the structure in question. The increase in test load is determined from analysis of the loading conditions in relation to the selected pass/fail criterion for the test.

For the 2008 edition, the former test load intensity,  **$0.85(1.4D + 1.7L)$** , was revised to be consistent with the load combinations in **Chapter 9**, which include rain and snow load in some combinations. These test loads are considered appropriate for designs using the load combinations and strength reduction factors of **Chapter 9** or **Appendix C**.

**CODE****COMMENTARY**

**20.3.3** — A load test shall not be made until that portion of the structure to be subjected to load is at least 56 days old. If the owner of the structure, the contractor, and all involved parties agree, it shall be permitted to make the test at an earlier age.

**20.4 — Loading criteria**

**20.4.1** — The initial value for all applicable response measurements (such as deflection, rotation, strain, slip, crack widths) shall be obtained not more than 1 hour before application of the first load increment. Measurements shall be made at locations where maximum response is expected. Additional measurements shall be made if required.

**20.4.2** — Test load shall be applied in not less than four approximately equal increments.

**20.4.3** — Uniform test load shall be applied in a manner to ensure uniform distribution of the load transmitted to the structure or portion of the structure being tested. Arching of the applied load shall be avoided.

**20.4.4** — A set of response measurements shall be made after each load increment is applied and after the total load has been applied on the structure for at least 24 hours.

**20.4.5** — Total test load shall be removed immediately after all response measurements defined in 20.4.4 are made.

**20.4.6** — A set of final response measurements shall be made 24 hours after the test load is removed.

**20.5 — Acceptance criteria**

**20.5.1** — The portion of the structure tested shall show no evidence of failure. Spalling and crushing of compressed concrete shall be considered an indication of failure.

**R20.4 — Loading criteria**

**R20.4.2** — Inspecting the structure after each load increment is advisable.

**R20.4.3** — Arching refers to the tendency for the load to be transmitted nonuniformly to the flexural element being tested. For example, if a slab is loaded by a uniform arrangement of bricks with the bricks in contact, arching would result in reduction of the load on the slab near the midspan of the slab.

**R20.5 — Acceptance criteria**

**R20.5.1** — A general acceptance criterion for the behavior of a structure under the test load is that it does not show evidence of failure. Evidence of failure includes cracking, spalling, or deflection of such magnitude and extent that the observed result is obviously excessive and incompatible with the safety requirements of the structure. No simple rules have been developed for application to all types of structures and conditions. If sufficient damage has occurred so that the structure is considered to have failed that test, retesting is not permitted because it is considered that damaged members should not be put into service even at a lower load rating.

Local spalling or flaking of the compressed concrete in flexural elements related to casting imperfections need not indicate overall structural distress. Crack widths are good indicators

**CODE****COMMENTARY**

**20.5.2** — Measured deflections shall satisfy either Eq. (20-1) or (20-2):

$$\Delta_1 \leq \frac{\ell_t^2}{20,000h} \quad (20-1)$$

$$\Delta_r \leq \frac{\Delta_1}{4} \quad (20-2)$$

If the measured maximum and residual deflections,  $\Delta_1$  and  $\Delta_r$ , do not satisfy Eq. (20-1) or (20-2), it shall be permitted to repeat the load test.

The repeat test shall be conducted not earlier than 72 hours after removal of the first test load. The portion of the structure tested in the repeat test shall be considered acceptable if deflection recovery  $\Delta_r$  satisfies the condition:

$$\Delta_r \leq \frac{\Delta_2}{5} \quad (20-3)$$

where  $\Delta_2$  is the maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test.

**20.5.3** — Structural members tested shall not have cracks indicating the imminence of shear failure.

**20.5.4** — In regions of structural members without transverse reinforcement, appearance of structural cracks inclined to the longitudinal axis and having a horizontal projection longer than the depth of the member at midpoint of the crack shall be evaluated.

of the state of the structure and should be observed to help determine whether the structure is satisfactory. However, exact prediction or measurement of crack widths in reinforced concrete elements is not likely to be achieved under field conditions. Establish criteria before the test, relative to the types of cracks anticipated; where the cracks will be measured; how they will be measured; and approximate limits or criteria to evaluate new cracks or limits for the changes in crack width.

**R20.5.2** — The deflection limits and the retest option follow past practice. If the structure shows no evidence of failure, recovery of deflection after removal of the test load is used to determine whether the strength of the structure is satisfactory. In the case of a very stiff structure, however, the errors in measurements under field conditions may be of the same order as the actual deflections and recovery. To avoid penalizing a satisfactory structure in such a case, recovery measurements are waived if the maximum deflection is less than  $\ell_t^2/(20,000h)$ . The residual deflection  $\Delta_r$  is the difference between the initial and final (after load removal) deflections for the load test or the repeat load test.

**R20.5.3** — Forces are transmitted across a shear crack plane by a combination of aggregate interlock at the interface of the crack that is enhanced by clamping action of transverse stirrup reinforcing and by dowel action of stirrups crossing the crack. As crack lengths increase to approach a horizontal projected length equal to the depth of the member and concurrently widen to the extent that aggregate interlock cannot occur, and as transverse stirrups if present begin to yield or display loss of anchorage so as to threaten their integrity, the member is assumed to be approaching imminent shear failure.

**R20.5.4** — The intent of 20.5.4 is to make the professionals in charge of the test pay attention to the structural implication of observed inclined cracks that may lead to brittle collapse in members without transverse reinforcement.

**CODE****COMMENTARY**

**20.5.5** — In regions of anchorage and lap splices, the appearance along the line of reinforcement of a series of short inclined cracks or horizontal cracks shall be evaluated.

**R20.5.5** — Cracking along the axis of the reinforcement in anchorage zones may be related to high stresses associated with the transfer of forces between the reinforcement and the concrete. These cracks may be indicators of pending brittle failure of the element if they are associated with the main reinforcement. It is important that their causes and consequences be evaluated.

**20.6 — Provision for lower load rating**

If the structure under investigation does not satisfy conditions or criteria of **20.1.2**, **20.5.2**, or **20.5.3**, the structure shall be permitted for use at a lower load rating based on the results of the load test or analysis, if approved by the building official.

**R20.6 — Provision for lower load rating**

Except for load tested members that have failed under a test (see **20.5**), the building official may permit the use of a structure or member at a lower load rating that is judged to be safe and appropriate on the basis of the test results.

**20.7 — Safety**

**20.7.1** — Load tests shall be conducted in such a manner as to provide for safety of life and structure during the test.

**20.7.2** — Safety measures shall not interfere with load test procedures or affect results.

# CHAPTER 21 — EARTHQUAKE-RESISTANT STRUCTURES

## CODE

### 21.1 — General requirements

#### 21.1.1 — Scope

**21.1.1.1** — Chapter 21 contains requirements for design and construction of reinforced concrete members of a structure for which the design forces, related to earthquake motions, have been determined on the basis of energy dissipation in the nonlinear range of response.

**21.1.1.2** — All structures shall be assigned to a seismic design category (SDC) in accordance with 1.1.9.1.

**21.1.1.3** — All members shall satisfy requirements of Chapters 1 to 19 and 22. Structures assigned to SDC B, C, D, E, or F also shall satisfy 21.1.1.4 through 21.1.1.8, as applicable.

**21.1.1.4** — Structures assigned to SDC B shall satisfy 21.1.2.

**21.1.1.5** — Structures assigned to SDC C shall satisfy 21.1.2 and 21.1.8.

**21.1.1.6** — Structures assigned to SDC D, E, or F shall satisfy 21.1.2 through 21.1.8, and 21.11 through 21.13.

**21.1.1.7** — Structural systems designated as part of the seismic-force-resisting system shall be restricted to those designated by the legally adopted general building code of which this Code forms a part, or determined by other authority having jurisdiction in areas without a legally adopted building code. Except for SDC A, for which Chapter 21 does not apply, the following provisions shall be satisfied for each structural system designated as part of the seismic-force-resisting system, regardless of the SDC:

- (a) Ordinary moment frames shall satisfy 21.2.
- (b) Ordinary reinforced concrete structural walls need not satisfy any provisions in Chapter 21.
- (c) Intermediate moment frames shall satisfy 21.3.
- (d) Intermediate precast walls shall satisfy 21.4.
- (e) Special moment frames shall satisfy 21.5 through 21.8.
- (f) Special structural walls shall satisfy 21.9.

## COMMENTARY

### R21.1 — General requirements

#### R21.1.1 — Scope

Chapter 21 contains provisions considered to be the minimum requirements for a cast-in-place or precast concrete structure capable of sustaining a series of oscillations into the inelastic range of response without critical deterioration in strength. The integrity of the structure in the inelastic range of response should be maintained because the design earthquake forces defined in documents such as ASCE/SEI 7,<sup>21.1</sup> the IBC,<sup>21.2</sup> the UBC,<sup>21.3</sup> and the NEHRP<sup>21.4</sup> provisions are considered less than those corresponding to linear response at the anticipated earthquake intensity.<sup>21.4-21.7</sup>

As a properly detailed cast-in-place or precast concrete structure responds to strong ground motion, its effective stiffness decreases and its energy dissipation increases. These changes tend to reduce the response accelerations and lateral inertia forces relative to values that would occur were the structure to remain linearly elastic and lightly damped.<sup>21.7</sup> Thus, the use of design forces representing earthquake effects such as those in ASCE/SEI 7 requires that the seismic-force-resisting system retain a substantial portion of its strength into the inelastic range under displacement reversals.

The provisions of Chapter 21 relate detailing requirements to type of structural framing and seismic design category (SDC). SDCs are adopted directly from ASCE/SEI 7, and relate to considerations of seismic hazard level, soil type, occupancy, and use. Before the 2008 Code, low, intermediate, and high seismic risk designations were used to delineate detailing requirements. For a qualitative comparison of SDCs and seismic risk designations, see Table R1.1.9.1. The assignment of a structure to a SDC is regulated by the legally adopted general building code of which this Code forms a part (see 1.1.9).

The design and detailing requirements should be compatible with the level of energy dissipation (or toughness) assumed in the computation of the design earthquake forces. The terms “ordinary,” “intermediate,” and “special” are specifically used to facilitate this compatibility. The degree of required toughness and, therefore, the level of required detailing, increases for structures progressing from ordinary through intermediate to special categories. It is essential that structures assigned to higher SDCs possess a higher degree of toughness. It is permitted, however, to design for higher toughness in the lower SDCs and take advantage of the lower design force levels.

**CODE**

(g) Special structural walls constructed using precast concrete shall satisfy **21.10**.

All special moment frames and special structural walls shall also satisfy **21.1.3** through **21.1.7**.

**21.1.8** — A reinforced concrete structural system not satisfying the requirements of this chapter shall be permitted if it is demonstrated by experimental evidence and analysis that the proposed system will have strength and toughness equal to or exceeding those provided by a comparable monolithic reinforced concrete structure satisfying this chapter.

**COMMENTARY**

The provisions of **Chapters 1** through **19** and **22** are considered to be adequate for structures assigned to SDC A (corresponding to lowest seismic hazard). For structures assigned to SDC B, additional requirements apply.

Structures assigned to SDC C may be subjected to moderately strong ground shaking. The designated seismic-force-resisting system typically comprises some combination of ordinary cast-in-place structural walls, intermediate precast structural walls, and intermediate moment frames. The legally adopted general building code of which this Code forms a part also may contain provisions for use of other seismic-force-resisting systems in SDC C. **Section 21.1.1.7** defines requirements for whatever system is selected.

Structures assigned to SDC D, E, or F may be subjected to strong ground shaking. It is the intent of Committee 318 that the seismic-force-resisting system of structural concrete buildings assigned to SDC D, E, or F be provided by special moment frames, special structural walls, or a combination of the two. In addition to **21.1.2** through **21.1.8**, these structures also are required to satisfy requirements for continuous inspection (**1.3.5**), diaphragms and trusses (**21.11**), foundations (**21.12**), and gravity-load-resisting elements that are not designated as part of the seismic-force-resisting system (**21.13**). These provisions have been developed to provide the structure with adequate toughness for the high demands expected for these SDCs.

The legally adopted general building code of which this Code forms a part may also permit the use of intermediate moment frames as part of dual systems for some buildings assigned to SDC D, E, or F. It is not the intention of Committee 318 to recommend the use of intermediate moment frames as part of moment-resisting frame or dual systems in SDC D, E, or F. The legally adopted general building code may also permit substantiated alternative or nonprescriptive designs or, with various supplementary provisions, the use of ordinary or intermediate systems for nonbuilding structures in the higher SDCs. These are not the typical applications around which this chapter is written, but wherever the term “ordinary” or “intermediate” moment frame is used in reference to reinforced concrete, **21.2** or **21.3** apply.

**Table R21.1.1** summarizes the applicability of the provisions of **Chapter 21** as they are typically applied where using minimum requirements in the various SDCs. Where special systems are used for structures in SDC B or C, it is not required to satisfy the requirements of **21.13**, although it should be verified that members not designated as part of the seismic-force-resisting system will be stable under design displacements.

The proportioning and detailing requirements in **Chapter 21** are based predominantly on field and laboratory experience with monolithic reinforced concrete building structures and

**CODE****COMMENTARY****TABLE R21.1.1 — SECTIONS OF **CHAPTER 21** TO BE SATISFIED IN TYPICAL APPLICATIONS\***

Component resisting earthquake effect, unless otherwise noted	Seismic Design Category			
	A (None)	B (21.1.1.4)	C (21.1.1.5)	D, E, F (21.1.1.6)
Analysis and design requirements	None	21.1.2	21.1.2	21.1.2, 21.1.3
Materials		None	None	21.1.4 - 21.1.7
Frame members		21.2	21.3	21.5, 21.6, 21.7, 21.8
Structural walls and coupling beams		None	None	21.9
Precast structural walls		None	21.4	21.4, † 21.10
Structural diaphragms and trusses		None	None	21.11
Foundations		None	None	21.12
Frame members not proportioned to resist forces induced by earthquake motions		None	None	21.13
Anchors		None	21.1.8	21.1.8

\*In addition to requirements of Chapters 1 through 19, except as modified by Chapter 21. Section 22.10 also applies in SDC D, E, and F.

†As permitted by the legally adopted general building code of which this Code forms a part.

precast concrete building structures designed and detailed to behave like monolithic building structures. Extrapolation of these requirements to other types of cast-in-place or precast concrete structures should be based on evidence provided by field experience, tests, or analysis. The acceptance criteria for moment frames given in ACI 374.1 can be used in conjunction with Chapter 21 to demonstrate that the strength and toughness of a proposed frame system equals or exceeds that provided by a comparable monolithic concrete system. ACI ITG-5.1 provides similar information for precast wall systems.

The toughness requirements in 21.1.1.8 refer to the concern for the structural integrity of the entire seismic-force-resisting system at lateral displacements anticipated for ground motions corresponding to the design earthquake. Depending on the energy-dissipation characteristics of the structural system used, such displacements may be larger than for a monolithic reinforced concrete structure.

### **21.1.2 — Analysis and proportioning of structural members**

**21.1.2.1** — The interaction of all structural and nonstructural members that affect the linear and nonlinear response of the structure to earthquake motions shall be considered in the analysis.

**21.1.2.2** — Rigid members assumed not to be a part of the seismic-force-resisting system shall be permitted provided their effect on the response of the

### **R21.1.2 — Analysis and proportioning of structural members**

It is assumed that the distribution of required strength to the various components of a seismic-force-resisting system will be guided by the analysis of a linearly elastic model of the system acted upon by the factored forces required by the legally adopted general building code. If nonlinear response history analyses are to be used, base motions should be selected after a detailed study of the site conditions and local seismic history.

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system is considered and accommodated in the structural design. Consequences of failure of structural and nonstructural members that are not a part of the seismic-force-resisting system shall be considered.

**21.1.2.3** — Structural members extending below the base of structure that are required to transmit forces resulting from earthquake effects to the foundation shall comply with the requirements of **Chapter 21** that are consistent with the seismic-force-resisting system above the base of structure.

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Because the design basis earthquake admits nonlinear response, it is necessary to investigate the stability of the seismic-force-resisting system as well as its interaction with other structural and nonstructural members at displacements larger than those indicated by linear analysis. To handle this without having to resort to nonlinear response analysis, one option is to multiply by a factor of at least two the displacements from linear analysis by using the factored lateral forces, unless the legally adopted general building code specifies the factors to be used as in the IBC or the UBC. For lateral displacement calculations, assuming all the horizontal structural members to be fully cracked is likely to lead to better estimates of the possible drift than using uncracked stiffness for all members. The analysis assumptions described in 8.8 also may be used to estimate lateral deflections of reinforced concrete building systems.

The main objective of **Chapter 21** is the safety of the structure. The intent of **21.1.2.1** and **21.1.2.2** is to draw attention to the influence of nonstructural members on structural response and to hazards from falling objects.

Section 21.1.2.3 serves as an alert that the base of structure as defined in analysis may not necessarily correspond to the foundation or ground level. Details of columns and walls extending below the base of structure to the foundation are required to be consistent with those above the base of structure.

In selecting member sizes for earthquake-resistant structures, it is important to consider constructability problems related to congestion of reinforcement. The design should be such that all reinforcement can be assembled and placed in the proper location and that concrete can be cast and consolidated properly. Use of upper limits of reinforcement ratios permitted is likely to lead to insurmountable construction problems, especially at frame joints.

**21.1.3 — Strength reduction factors**

Strength reduction factors shall be as given in **9.3.4**.

**21.1.4 — Concrete in special moment frames and special structural walls**

**21.1.4.1** — Requirements of 21.1.4 apply to special moment frames, special structural walls, and all components of special structural walls including coupling beams and wall piers.

**21.1.4.2** — Specified compressive strength of concrete,  $f'_c$ , shall be not less than 3000 psi.

**21.1.4.3** — Specified compressive strength of lightweight concrete,  $f'_c$ , shall not exceed 5000 psi unless demonstrated by experimental evidence that structural

**R21.1.4 — Concrete in special moment frames and special structural walls**

Requirements of this section refer to concrete quality in frames and walls that resist earthquake-induced forces. The maximum specified compressive strength of lightweight concrete to be used in structural design calculations is limited to 5000 psi, primarily because of paucity of experimental and field data on the behavior of members made with lightweight concrete subjected to displacement reversals in the nonlinear range. If convincing evidence is developed for a specific application, the limit on maximum specified compressive strength of lightweight concrete may be increased to a level justified by the evidence.

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members made with that lightweight concrete provide strength and toughness equal to or exceeding those of comparable members made with normalweight concrete of the same strength. Modification factor  $\lambda$  for lightweight concrete in this Chapter shall be in accordance with 8.6.1 unless specifically noted otherwise.

**21.1.5 — Reinforcement in special moment frames and special structural walls**

**21.1.5.1** — Requirements of 21.1.5 apply to special moment frames, special structural walls, and all components of special structural walls including coupling beams and wall piers.

**21.1.5.2** — Deformed reinforcement resisting earthquake-induced flexure, axial force, or both, shall comply with ASTM A706, Grade 60. ASTM A615 Grades 40 and 60 reinforcement shall be permitted if:

- (a) The actual yield strength based on mill tests does not exceed  $f_y$  by more than 18,000 psi; and
- (b) The ratio of the actual tensile strength to the actual yield strength is not less than 1.25.

**21.1.5.3** — Prestressing steel resisting earthquake-induced flexural and axial loads in frame members and in precast structural walls shall comply with ASTM A416 or A722.

**21.1.5.4** — The value of  $f_{yt}$  used to compute the amount of confinement reinforcement in 21.6.4.4 shall not exceed 100,000 psi.

**21.1.5.5** — The values of  $f_y$  and  $f_{yt}$  used in design of shear reinforcement shall conform to 11.4.2.

**R21.1.5 — Reinforcement in special moment frames and special structural walls**

Use of longitudinal reinforcement with strength substantially higher than that assumed in design will lead to higher shear and bond stresses at the time of development of yield moments. These conditions may lead to brittle failures in shear or bond and should be avoided even if such failures may occur at higher loads than those anticipated in design. Therefore, a ceiling is placed on the actual yield strength of the steel [see 21.1.5.2(a)]. ASTM A706 for low-alloy steel reinforcing bars now includes both Grade 60 (442) and Grade 80 (550); however, only Grade 60 is generally permitted because of insufficient data to confirm applicability of existing code provisions for structures using the higher grade. **Section 21.1.1.8** permits alternative material such as ASTM A706 Grade 80 if results of tests and analytical studies are presented in support of its use.

The requirement for a tensile strength larger than the yield strength of the reinforcement [21.1.5.2(b)] is based on the assumption that the capability of a structural member to develop inelastic rotation capacity is a function of the length of the yield region along the axis of the member. In interpreting experimental results, the length of the yield region has been related to the relative magnitudes of nominal and yield moments.<sup>21.8</sup> According to this interpretation, the larger the ratio of nominal to yield moment, the longer the yield region. **Chapter 21** requires that the ratio of actual tensile strength to actual yield strength is not less than 1.25. Members with reinforcement not satisfying this condition can also develop inelastic rotation, but their behavior is sufficiently different to exclude them from direct consideration on the basis of rules derived from experience with members reinforced with strain-hardening steel.

The restrictions on the values of  $f_y$  and  $f_{yt}$  apply to all types of transverse reinforcement, including spirals, circular hoops, rectilinear hoops, and crossties. The restrictions on the values of  $f_y$  and  $f_{yt}$  in 11.4.2 for computing nominal shear strength are intended to limit the width of shear cracks. Research results<sup>21.9-21.11</sup> indicate that higher yield strengths can be used effectively as confinement reinforcement as specified in 21.6.4.4.

**CODE****21.1.6 — Mechanical splices in special moment frames and special structural walls**

**21.1.6.1** — Mechanical splices shall be classified as either Type 1 or Type 2 mechanical splices, as follows:

- (a) Type 1 mechanical splices shall conform to [12.14.3.2](#);
- (b) Type 2 mechanical splices shall conform to [12.14.3.2](#) and shall develop the specified tensile strength of the spliced bar.

**21.1.6.2** — Type 1 mechanical splices shall not be used within a distance equal to twice the member depth from the column or beam face for special moment frames or from sections where yielding of the reinforcement is likely to occur as a result of inelastic lateral displacements. Type 2 mechanical splices shall be permitted to be used at any location.

**21.1.7 — Welded splices in special moment frames and special structural walls**

**21.1.7.1** — Welded splices in reinforcement resisting earthquake-induced forces shall conform to [12.14.3.4](#) and shall not be used within a distance equal to twice the member depth from the column or beam face for special moment frames or from sections where yielding of the reinforcement is likely to occur as a result of inelastic lateral displacements.

**21.1.7.2** — Welding of stirrups, ties, inserts, or other similar elements to longitudinal reinforcement that is required by design shall not be permitted.

**21.1.8 — Anchoring to concrete**

Anchors resisting earthquake-induced forces in structures assigned to SDC C, D, E, or F shall conform to the requirements of [D.3.3](#).

**COMMENTARY****R21.1.6 — Mechanical splices in special moment frames and special structural walls**

In a structure undergoing inelastic deformations during an earthquake, the tensile stresses in reinforcement may approach the tensile strength of the reinforcement. The requirements for Type 2 mechanical splices are intended to avoid a splice failure when the reinforcement is subjected to expected stress levels in yielding regions. Type 1 splices are not required to satisfy the more stringent requirements for Type 2 splices, and may not be capable of resisting the stress levels expected in yielding regions. The locations of Type 1 splices are restricted because tensile stresses in reinforcement in yielding regions can exceed the strength requirements of [12.14.3.2](#). The restriction on Type 1 splices applies to all reinforcement resisting earthquake effects, including transverse reinforcement.

Recommended detailing practice would preclude the use of splices in regions of potential yield in members resisting earthquake effects. If use of mechanical splices in regions of potential yielding cannot be avoided, there should be documentation on the actual strength characteristics of the bars to be spliced, on the force-deformation characteristics of the spliced bar, and on the ability of the Type 2 splice to be used to meet the specified performance requirements.

**R21.1.7 — Welded splices in special moment frames and special structural walls**

**R21.1.7.1** — Welding of reinforcement should be according to AWS D1.4 as required in [Chapter 3](#). The locations of welded splices are restricted because reinforcement tension stresses in yielding regions can exceed the strength requirements of [12.14.3.4](#). The restriction on welded splices applies to all reinforcement resisting earthquake effects, including transverse reinforcement.

**R21.1.7.2** — Welding of crossing reinforcing bars can lead to local embrittlement of the steel. If welding of crossing bars is used to facilitate fabrication or placement of reinforcement, it should be done only on bars added for such purposes. The prohibition of welding crossing reinforcing bars does not apply to bars that are welded with welding operations under continuous, competent control as in the manufacture of welded wire reinforcement.

**CODE****COMMENTARY****21.2 — Ordinary moment frames****21.2.1 — Scope**

Requirements of 21.2 apply to ordinary moment frames forming part of the seismic-force-resisting system.

**21.2.2** — Beams shall have at least two of the longitudinal bars continuous along both the top and bottom faces. These bars shall be developed at the face of support.

**21.2.3** — Columns having clear height less than or equal to five times the dimension  $c_1$  shall be designed for shear in accordance with 21.3.3.2.

**21.3 — Intermediate moment frames****21.3.1 — Scope**

Requirements of 21.3 apply to intermediate moment frames forming part of the seismic-force-resisting system.

**21.3.2** — Reinforcement details in a frame member shall satisfy **21.3.4** if the factored axial compressive load,  $P_u$ , for the member does not exceed  $A_g f'_c / 10$ . If  $P_u$  is larger, frame reinforcement details shall satisfy **21.3.5**. Where a two-way slab system without beams forms a part of the seismic-force-resisting system, reinforcement details in any span resisting moments caused by  $E$  shall satisfy **21.3.6**.

**21.3.3 — Shear strength**

**21.3.3.1** —  $\phi V_n$  of beams resisting earthquake effect,  $E$ , shall not be less than the smaller of (a) and (b):

(a) The sum of the shear associated with development of  $M_n$  of the beam at each restrained end of the clear span due to reverse curvature bending and the shear calculated for factored gravity loads;

(b) The maximum shear obtained from design load combinations that include  $E$ , with  $E$  assumed to be twice that prescribed by the legally adopted general building code for earthquake-resistant design.

**21.3.3.2** —  $\phi V_n$  of columns resisting earthquake effect,  $E$ , shall not be less than the smaller of (a) and (b):

(a) The shear associated with development of nominal moment strengths of the column at each restrained end of the unsupported length due to reverse curvature bending. Column flexural strength shall be

**R21.2 — Ordinary moment frames**

These provisions were introduced in the 2008 Code and apply only to ordinary moment frames assigned to SDC B. The provisions for beam reinforcement are intended to improve continuity in the framing members as compared with the provisions of **Chapters 1 through 18** and thereby improve lateral force resistance and structural integrity; these provisions do not apply to slab-column moment frames. The provisions for columns are intended to provide additional toughness to resist shear for columns with proportions that would otherwise make them more susceptible to shear failure under earthquake loading.

**R21.3 — Intermediate moment frames**

The objective of the requirements in 21.3.3 is to reduce the risk of failure in shear in beams and columns during an earthquake. Two options are provided to determine the factored shear force.

According to 21.3.3.1(a) and 21.3.3.2(a), the factored shear force is determined from a free-body diagram obtained by cutting through the member ends, with end moments assumed equal to the nominal moment strengths acting in reverse curvature bending. Examples for a beam and a column are illustrated in **Fig. R21.3.3**. In all applications of 21.3.3.1(a) and 21.3.3.2(a), shears are required to be calculated for moments due to reverse curvature bending, acting both clockwise and counterclockwise. **Figure R21.3.3** demonstrates only one of the two options that are to be considered for every member. The factored axial force,  $P_u$ , should be chosen to develop the largest moment strength of the column.

To determine the maximum beam shear, it is assumed that its nominal moment strengths ( $\phi = 1.0$ ) are developed simultaneously at both ends of its clear span. As indicated in **Fig. R21.3.3**, the shear associated with this condition [ $(M_{nl} + M_{nr})/\ell_n$ ] is added algebraically to the shear due to the factored gravity loads to obtain the design shear for the beam. For the example shown, both the dead load  $w_D$  and the live load  $w_L$  have been assumed to be uniformly distributed. Effects of  $E$  acting vertically are to be included if required by the general building code.

Option 21.3.3.1(b) for beams bases  $V_u$  on the load combination including the earthquake effect,  $E$ , which should be doubled. For example, the load combination defined by **Eq. (9-5)** would be

$$U = 1.2D + 2.0E + 1.0L + 0.2S$$

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calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the highest flexural strength;

- (b) The maximum shear obtained from design load combinations that include  $E$ , with  $E$  increased by  $\Omega_o$ .

### 21.3.4 — Beams

**21.3.4.1** — The positive moment strength at the face of the joint shall be not less than one-third the negative moment strength provided at that face of the joint. Neither the negative nor the positive moment strength at any section along the length of the beam shall be less than one-fifth the maximum moment strength provided at the face of either joint.

**21.3.4.2** — At both ends of the beam, hoops shall be provided over lengths not less than  $2h$  measured from the face of the supporting member toward midspan. The first hoop shall be located not more than 2 in. from the face of the supporting member. Spacing of hoops shall not exceed the smallest of (a), (b), (c), and (d):

- (a)  $d/4$ ;
- (b) Eight times the diameter of the smallest longitudinal bar enclosed;
- (c) 24 times the diameter of the hoop bar;
- (d) 12 in.

**21.3.4.3** — Stirrups shall be spaced not more than  $d/2$  throughout the length of the beam.

### 21.3.5 — Columns

**21.3.5.1** — Columns shall be spirally reinforced in accordance with 7.10.4 or shall conform with 21.3.5.2 through 21.3.5.4. Section 21.3.5.5 shall apply to all columns, and 21.3.5.6 shall apply to all columns supporting discontinuous stiff members.

**21.3.5.2** — At both ends of the column, hoops shall be provided at spacing  $s_o$  over a length  $\ell_o$  measured from the joint face. Spacing  $s_o$  shall not exceed the smallest of (a), (b), (c), and (d):

- (a) Eight times the diameter of the smallest longitudinal bar enclosed;
- (b) 24 times the diameter of the hoop bar;
- (c) One-half of the smallest cross-sectional dimension of the column;
- (d) 12 in.

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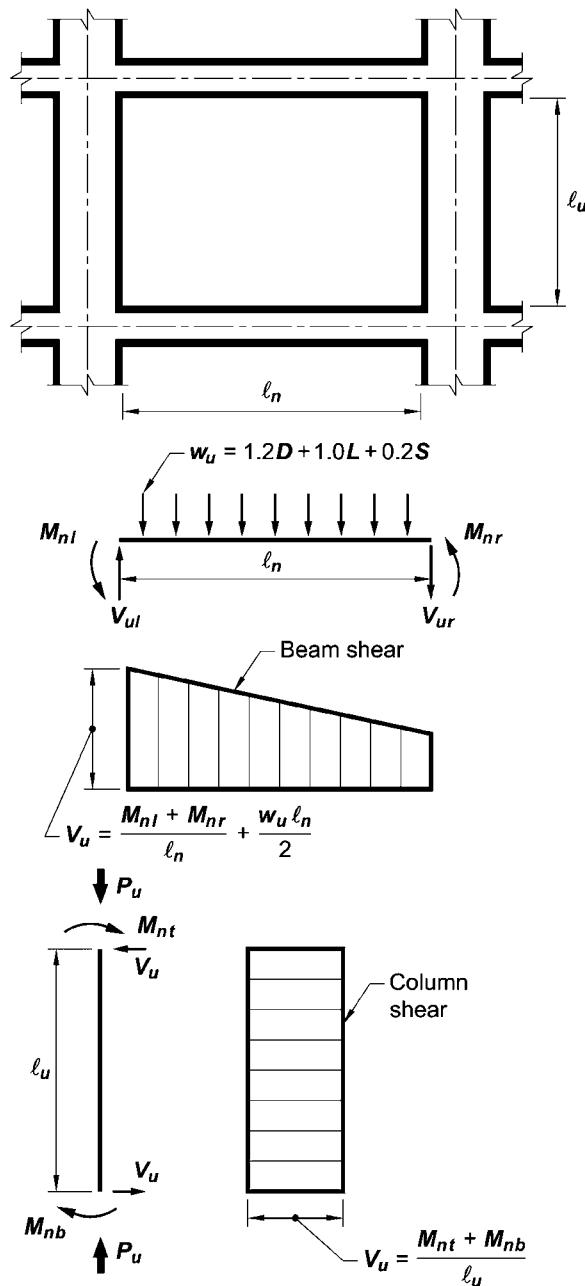


Fig. R21.3.3—Design shears for intermediate moment frames.

where  $E$  is the value specified by the governing code. The factor of 1.0 applied to  $L$  is allowed to be reduced to 0.5 in accordance with 9.2.1(a).

Option 21.3.3.2(b) for columns is similar to that for beams except it bases  $V_u$  on load combinations including the earthquake effect,  $E$ , with  $E$  increased by the overstrength factor  $\Omega_o$ , rather than the factor 2.0. In ASCE 7-05,<sup>21.1</sup>  $\Omega_o = 3.0$  for intermediate moment frames. The higher factor for columns relative to beams is because of greater concerns about shear failure in columns.

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Length  $\ell_o$  shall not be less than the largest of (e), (f), and (g):

- (e) One-sixth of the clear span of the column;
- (f) Maximum cross-sectional dimension of the column;
- (g) 18 in.

**21.3.5.3** — The first hoop shall be located not more than  $s_o/2$  from the joint face.

**21.3.5.4** — Outside the length  $\ell_o$ , spacing of transverse reinforcement shall conform to **7.10** and **11.4.5.1**.

**21.3.5.5** — Joint transverse reinforcement shall conform to **11.10**.

**21.3.5.6** — Columns supporting reactions from discontinuous stiff members, such as walls, shall be provided with transverse reinforcement at the spacing,  $s_o$ , as defined in **21.3.5.2** over the full height beneath the level at which the discontinuity occurs if the portion of factored axial compressive force in these members related to earthquake effects exceeds  $A_g f'_c / 10$ . Where design forces have been magnified to account for the overstrength of the vertical elements of the seismic-force-resisting system, the limit of  $A_g f'_c / 10$  shall be increased to  $A_g f'_c / 4$ . This transverse reinforcement shall extend above and below the columns as required in **21.6.4.6(b)**.

#### **21.3.6 — Two-way slabs without beams**

**21.3.6.1** — Factored slab moment at support including earthquake effects,  $E$ , shall be determined for load combinations given in **Eq. (9-5)** and **(9-7)**. Reinforcement provided to resist  $M_{slab}$  shall be placed within the column strip defined in **13.2.1**.

**21.3.6.2** — Reinforcement placed within the effective width specified in **13.5.3.2** shall be proportioned to resist  $\gamma_f M_{slab}$ . Effective slab width for exterior and corner connections shall not extend beyond the column face a distance greater than  $c_t$  measured perpendicular to the slab span.

**21.3.6.3** — Not less than one-half of the reinforcement in the column strip at support shall be placed within the effective slab width given in **13.5.3.2**.

**21.3.6.4** — Not less than one-quarter of the top reinforcement at the support in the column strip shall be continuous throughout the span.

**Section 21.3.4** contains requirements for providing beams with a threshold level of toughness. Transverse reinforcement at the ends of the beam is required to be hoops. In most cases, stirrups required by **21.3.3** for design shear force will be more than those required by **21.3.4**. Requirements of **21.3.5** serve the same purpose for columns.

Discontinuous structural walls and other stiff members can impose large axial forces on supporting columns during earthquakes. The required transverse reinforcement in **21.3.5.6** is to improve column toughness under anticipated demands. The factored axial compressive force related to earthquake effect should include the factor  $\Omega_o$  if required by the legally adopted general building code of which this Code forms a part.

Section 21.3.6 applies to two-way slabs without beams, such as flat plates.

Using load combinations of **Eq. (9-5)** and **(9-7)** may result in moments requiring top and bottom reinforcement at the supports.

The moment  $M_{slab}$  refers, for a given design load combination with  $E$  acting in one horizontal direction, to that portion of the factored slab moment that is balanced by the supporting members at a joint. It is not necessarily equal to the total design moment at support for a load combination including earthquake effect. In accordance with **13.5.3.2**, only a fraction of the moment  $M_{slab}$  is assigned to the slab effective width. For edge and corner connections, flexural reinforcement perpendicular to the edge is not considered fully effective unless it is placed within the effective slab width.<sup>21.12,21.13</sup> See **Fig. R21.3.6.1**.

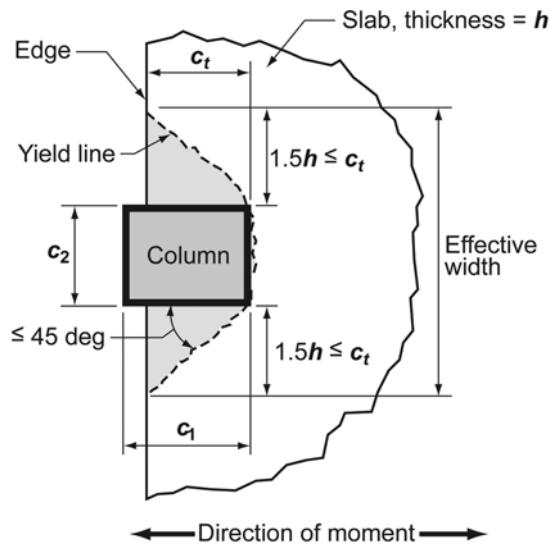
Application of the provisions of 21.3.6 is illustrated in **Fig. R21.3.6.2** and **R21.3.6.3**.

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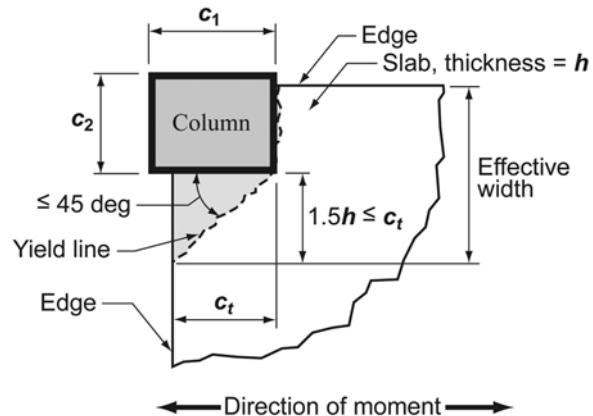
**21.3.6.5** — Continuous bottom reinforcement in the column strip shall be not less than one-third of the top reinforcement at the support in the column strip.

**21.3.6.6** — Not less than one-half of all bottom middle strip reinforcement and all bottom column strip reinforcement at midspan shall be continuous and shall develop  $f_y$  at the face of the support as defined in **13.6.2.5**.

**21.3.6.7** — At discontinuous edges of the slab, all top and bottom reinforcement at support shall be developed at the face of support as defined in **13.6.2.5**.

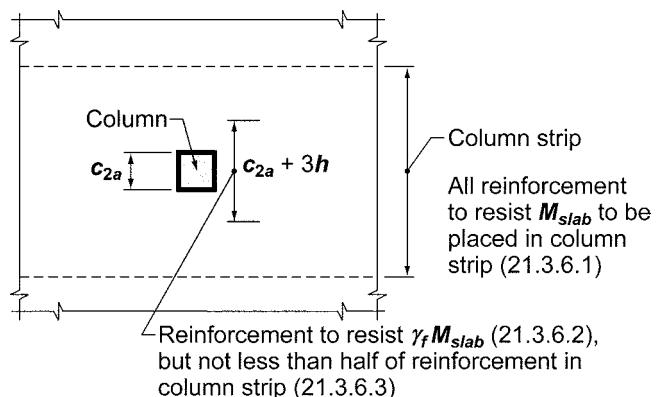


(a) Edge connection



(b) Corner connection

Fig. R21.3.6.1—Effective width for reinforcement placement in edge and corner connections.



**Note:** Applies to both top and bottom reinforcement

Fig. R21.3.6.2—Location of reinforcement in slabs.

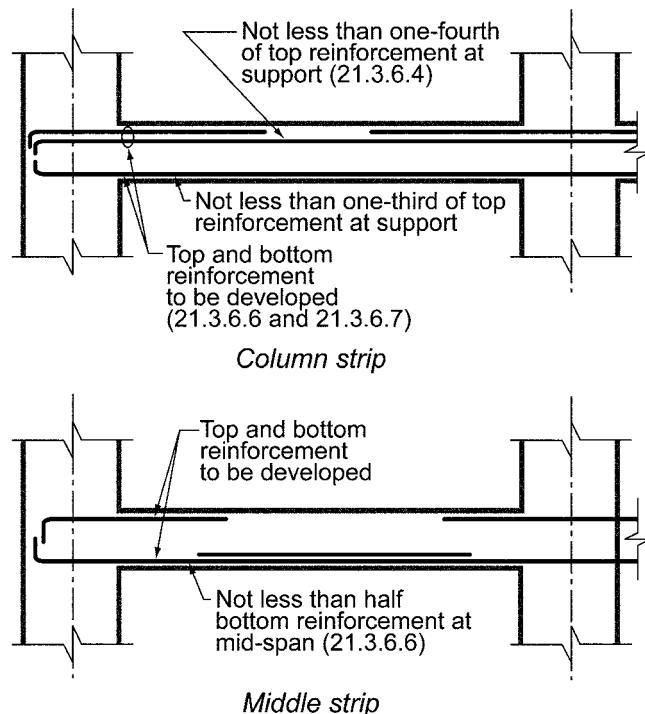
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Fig. R21.3.6.3—Arrangement of reinforcement in slabs.

**21.3.6.8** — At the critical sections for columns defined in 11.11.1.2, two-way shear caused by factored gravity loads shall not exceed  $0.4\phi V_c$ , where  $V_c$  shall be calculated as defined in 11.11.2.1 for nonprestressed slabs and in 11.11.2.2 for prestressed slabs. It shall be permitted to waive this requirement if the slab design satisfies requirements of 21.13.6.

## 21.4 — Intermediate precast structural walls

### 21.4.1 — Scope

Requirements of 21.4 apply to intermediate precast structural walls forming part of the seismic-force-resisting system.

**21.4.2** — In connections between wall panels, or between wall panels and the foundation, yielding shall be restricted to steel elements or reinforcement.

**21.4.3** — Elements of the connection that are not designed to yield shall develop at least  $1.5S_y$ .

**21.4.4** — In structures assigned to SDC D, E, or F, wall piers shall be designed in accordance with 21.9 or 21.13.

**R21.3.6.8** — The requirements apply to two-way slabs that are designated part of the seismic-force-resisting system. Slab-column connections in laboratory tests<sup>21.13</sup> exhibited reduced lateral displacement ductility when the shear at the column connection exceeded the recommended limit. Slab-column connections also must satisfy shear and moment strength requirements of Chapters 11 and 13 under load combinations including earthquake effect.

## R21.4 — Intermediate precast structural walls

Connections between precast wall panels or between wall panels and the foundation are required to resist forces induced by earthquake motions and to provide for yielding in the vicinity of connections. When Type 2 mechanical splices are used to directly connect primary reinforcement, the probable strength of the splice should be at least 1-1/2 times the specified yield strength of the reinforcement.

**CODE****COMMENTARY****21.5 — Flexural members of special moment frames****21.5.1 — Scope**

Requirements of 21.5 apply to special moment frame members that form part of the seismic-force-resisting system and are proportioned primarily to resist flexure. These frame members shall also satisfy the conditions of 21.5.1.1 through 21.5.1.4.

**21.5.1.1** — Factored axial compressive force on the member,  $P_u$ , shall not exceed  $A_g f'_c / 10$ .

**21.5.1.2** — Clear span for member,  $\ell_n$ , shall not be less than four times its effective depth.

**21.5.1.3** — Width of member,  $b_w$ , shall not be less than the smaller of **0.3h** and 10 in.

**21.5.1.4** — Width of member,  $b_w$ , shall not exceed width of supporting member,  $c_2$ , plus a distance on each side of supporting member equal to the smaller of (a) and (b):

(a) Width of supporting member,  $c_2$ , and

(b) 0.75 times the overall dimension of supporting member,  $c_1$ .

**21.5.2 — Longitudinal reinforcement**

**21.5.2.1** — At any section of a flexural member, except as provided in **10.5.3**, for top as well as for bottom reinforcement, the amount of reinforcement shall not be less than that given by **Eq. (10-3)** but not less than **200b<sub>w</sub>d/f<sub>y</sub>**, and the reinforcement ratio,  $\rho$ , shall not exceed 0.025. At least two bars shall be provided continuously at both top and bottom.

**R21.5 — Flexural members of special moment frames****R21.5.1 — Scope**

This section refers to beams of special moment frames resisting lateral loads induced by earthquake motions. Any frame member subjected to a factored axial compressive force exceeding ( $A_g f'_c / 10$ ) under any load combination is to be proportioned and detailed as described in **21.6**.

Experimental evidence<sup>21.14</sup> indicates that, under reversals of displacement into the nonlinear range, behavior of continuous members having length-to-depth ratios of less than 4 is significantly different from the behavior of relatively slender members. Design rules derived from experience with relatively slender members do not apply directly to members with length-to-depth ratios less than 4, especially with respect to shear strength.

Geometric constraints indicated in 21.5.1.3 and 21.5.1.4 were derived from practice and research<sup>21.8</sup> on reinforced concrete frames resisting earthquake-induced forces. The limits in 21.5.1.4 recognize that the maximum effective beam width depends principally on the column dimensions rather than on the depth of the beam, as suggested in the 2005 and earlier versions of the Code. An example of maximum effective beam width is shown in **Fig. R21.5.1**.

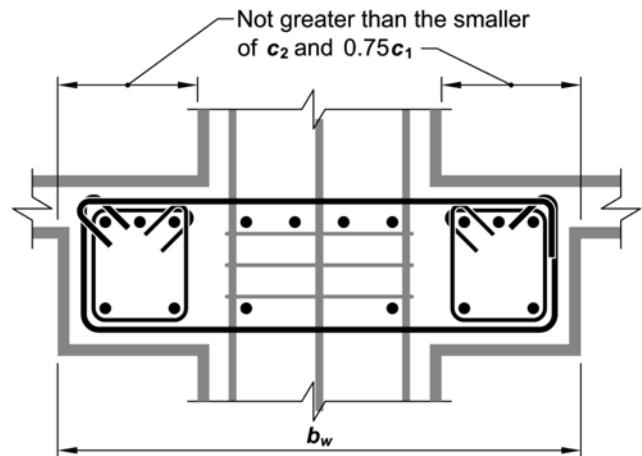
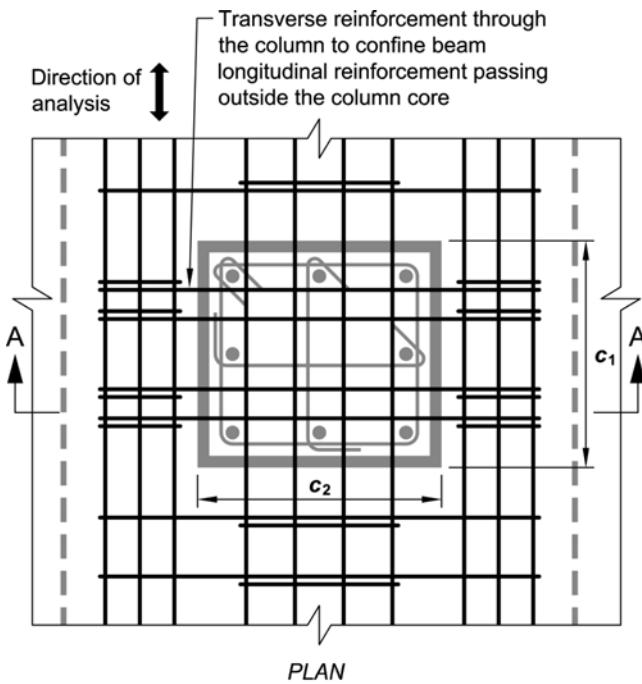
**R21.5.2 — Longitudinal reinforcement**

**Section 10.3.5** limits the net tensile strain,  $\epsilon_t$ , thereby indirectly limiting the tensile reinforcement ratio in a flexural member to a fraction of the amount that would produce balanced conditions. For a section subjected to bending only and loaded monotonically to yielding, this approach is feasible because the likelihood of compressive failure can be estimated reliably with the behavioral model assumed for determining the reinforcement ratio corresponding to balanced failure. The same behavioral model (because of incorrect assumptions such as linear strain distribution, well-defined yield point for the steel, limiting compressive strain in the concrete of 0.003, and compressive stresses in the shell concrete) does not describe the conditions in a flexural member subjected to reversals of displacements well into the inelastic range. Thus, there is little rationale for continuing to refer to balanced conditions in earthquake-resistant design of reinforced concrete structures.

**R21.5.2.1** — The limiting reinforcement ratio of 0.025 is based primarily on considerations of steel congestion and, indirectly, on limiting shear stresses in beams of typical proportions. The requirement of at least two bars, top and bottom, refers again to construction rather than behavioral requirements.

**CODE**

**21.5.2.2** — Positive moment strength at joint face shall be not less than one-half the negative moment strength provided at that face of the joint. Neither the negative nor the positive moment strength at any section along member length shall be less than one-fourth the maximum moment strength provided at face of either joint.

**COMMENTARY**

SECTION A-A

*Fig. R21.5.1—Maximum effective width of wide beam and required transverse reinforcement.*

**21.5.2.3** — Lap splices of flexural reinforcement shall be permitted only if hoop or spiral reinforcement is provided over the lap length. Spacing of the transverse reinforcement enclosing the lap-spliced bars shall not exceed the smaller of  $d/4$  and 4 in. Lap splices shall not be used:

- (a) Within the joints;
- (b) Within a distance of twice the member depth from the face of the joint; and

**R21.5.2.3** — Lap splices of reinforcement are prohibited at regions where flexural yielding is anticipated because such splices are not reliable under conditions of cyclic loading into the inelastic range. Transverse reinforcement for lap splices at any location is mandatory because of the likelihood of loss of shell concrete.

**CODE****COMMENTARY**

(c) Where analysis indicates flexural yielding is caused by inelastic lateral displacements of the frame.

**21.5.2.4** — Mechanical splices shall conform to **21.1.6** and welded splices shall conform to **21.1.7**.

**21.5.2.5** — Prestressing, where used, shall satisfy (a) through (d), unless used in a special moment frame as permitted by **21.8.4**:

(a) The average prestress,  $f_{pc}$ , calculated for an area equal to the smallest cross-sectional dimension of the member multiplied by the perpendicular cross-sectional dimension shall not exceed the smaller of 500 psi and  $f'_c/10$ .

(b) Prestressing steel shall be unbonded in potential plastic hinge regions, and the calculated strains in prestressing steel under the design displacement shall be less than 1 percent.

(c) Prestressing steel shall not contribute to more than one-quarter of the positive or negative flexural strength at the critical section in a plastic hinge region and shall be anchored at or beyond the exterior face of the joint.

(d) Anchorages of the post-tensioning tendons resisting earthquake-induced forces shall be capable of allowing tendons to withstand 50 cycles of loading, bounded by 40 and 85 percent of the specified tensile strength of the prestressing steel.

### **21.5.3 — Transverse reinforcement**

**21.5.3.1** — Hoops shall be provided in the following regions of frame members:

(a) Over a length equal to twice the member depth measured from the face of the supporting member toward midspan, at both ends of the flexural member;

**R21.5.2.5** — These provisions were developed, in part, based on observations of building performance in earthquakes.<sup>21.15</sup> For calculating the average prestress, the smallest cross-sectional dimension in a beam normally is the web dimension, and is not intended to refer to the flange thickness. In a potential plastic hinge region, the limitation on strain and the requirement for unbonded tendons are intended to prevent fracture of tendons under inelastic earthquake deformation. Calculation of the strain in the prestressing steel is required considering the anticipated inelastic mechanism of the structure. For prestressing steel unbonded along the full beam span, strains generally will be well below the specified limit. For prestressing steel with short unbonded length through or adjacent to the joint, the additional strain due to earthquake deformation is calculated as the product of the depth to the neutral axis and the sum of plastic hinge rotations at the joint, divided by the unbonded length.

The restrictions on the flexural strength provided by the tendons are based on the results of analytical and experimental studies.<sup>21.16-21.18</sup> Although satisfactory seismic performance can be obtained with greater amounts of prestressing steel, this restriction is needed to allow the use of the same response modification and deflection amplification factors as those specified in model codes for special moment frames without prestressing steel. Prestressed special moment frames will generally contain continuous prestressing steel that is anchored with adequate cover at or beyond the exterior face of each beam-column connection located at the ends of the moment frame.

Fatigue testing for 50 cycles of loading between 40 and 80 percent of the specified tensile strength of the prestressing steel has been an industry practice of long standing.<sup>21.15,21.19</sup> The 80 percent limit was increased to 85 percent to correspond to the 1 percent limit on the strain in prestressing steel. Testing over this range of stress is intended to conservatively simulate the effect of a severe earthquake. Additional details on testing procedures, but to different stress levels, are provided in Reference **21.19**.

### **R21.5.3 — Transverse reinforcement**

Transverse reinforcement is required primarily to confine the concrete and maintain lateral support for the reinforcing bars in regions where yielding is expected. Examples of hoops suitable for flexural members of frames are shown in Fig. **R21.5.3**.

**CODE**

(b) Over lengths equal to twice the member depth on both sides of a section where flexural yielding is likely to occur in connection with inelastic lateral displacements of the frame.

**21.5.3.2** — The first hoop shall be located not more than 2 in. from the face of a supporting member. Spacing of the hoops shall not exceed the smallest of (a), (b), and (c):

(a)  $d/4$ ;

(b) Six times the diameter of the smallest primary flexural reinforcing bars excluding longitudinal skin reinforcement required by 10.6.7; and

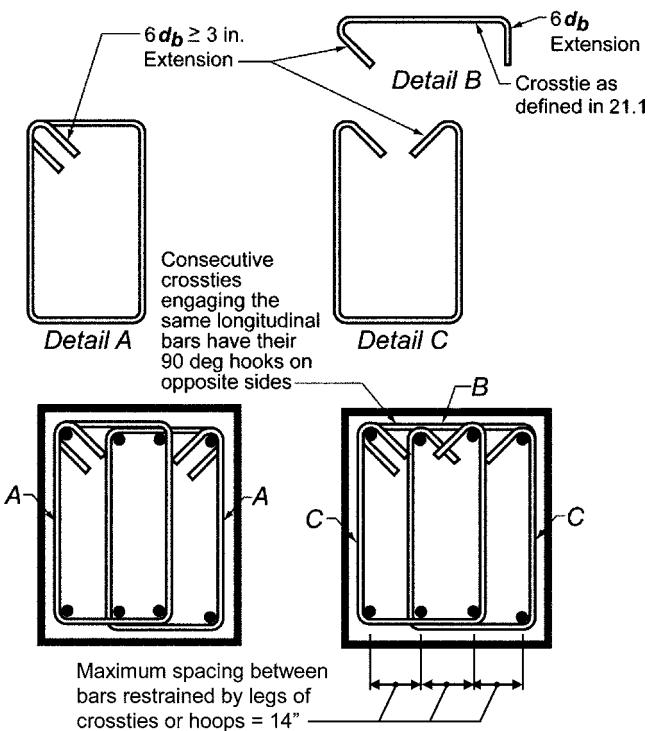
(c) 6 in.

**21.5.3.3** — Where hoops are required, primary flexural reinforcing bars closest to the tension and compression faces shall have lateral support conforming to 7.10.5.3 or 7.10.5.4. The spacing of transversely supported flexural reinforcing bars shall not exceed 14 in. Skin reinforcement required by 10.6.7 need not be laterally supported.

**21.5.3.4** — Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced at a distance not more than  $d/2$  throughout the length of the member.

**21.5.3.5** — Stirrups or ties required to resist shear shall be hoops over lengths of members in 21.5.3.1.

**21.5.3.6** — Hoops in flexural members shall be permitted to be made up of two pieces of reinforcement: a stirrup having seismic hooks at both ends and closed by a crosstie. Consecutive crossties engaging the same longitudinal bar shall have their 90-degree hooks at opposite sides of the flexural member. If the longitudinal reinforcing bars secured by the crossties are confined by a slab on only one side of the flexural frame member, the 90-degree hooks of the crossties shall be placed on that side.

**COMMENTARY**

*Fig. R21.5.3—Examples of overlapping hoops and illustration of limit on maximum horizontal spacing of supported longitudinal bars.*

For many years, the upper limit on hoop spacing was the smallest of  $d/4$ , 8 longitudinal bar diameters, 24 tie bar diameters, and 12 in. The upper limits were changed because of concerns about adequacy of longitudinal bar buckling restraint and confinement of large beams.

In the case of members with varying strength along the span or members for which the permanent load represents a large proportion of the total design load, concentrations of inelastic rotation may occur within the span. If such a condition is anticipated, transverse reinforcement also should be provided in regions where yielding is expected.

Because spalling of the concrete shell is anticipated during strong motion, especially at and near regions of flexural yielding, all web reinforcement should be provided in the form of closed hoops as defined in 21.5.3.5.

**CODE****COMMENTARY****21.5.4 — Shear strength requirements****21.5.4.1 — Design forces**

The design shear force,  $V_e$ , shall be determined from consideration of the statical forces on the portion of the member between faces of the joints. It shall be assumed that moments of opposite sign corresponding to probable flexural moment strength,  $M_{pr}$ , act at the joint faces and that the member is loaded with the factored tributary gravity load along its span.

**R21.5.4 — Shear strength requirements****R21.5.4.1 — Design forces**

In determining the equivalent lateral forces representing earthquake effects for the type of frames considered, it is assumed that frame members will dissipate energy in the nonlinear range of response. Unless a frame member possesses a strength that is on the order of 3 or 4 of the design forces, it should be assumed that it will yield in the event of a major earthquake. The design shear force should be a good approximation of the maximum shear that may develop in a member. Therefore, required shear strength for frame members is related to flexural strengths of the designed member rather than to factored shear forces indicated by lateral load analysis. The conditions described by 21.5.4.1 are illustrated in Fig. R21.5.4.

Because the actual yield strength of the longitudinal reinforcement may exceed the specified yield strength and because strain hardening of the reinforcement is likely to take place at a joint subjected to large rotations, required shear strengths are determined using a stress of at least  $1.25f_y$  in the longitudinal reinforcement.

**21.5.4.2 — Transverse reinforcement**

Transverse reinforcement over the lengths identified in 21.5.3.1 shall be proportioned to resist shear assuming  $V_c = 0$  when both (a) and (b) occur:

- (a) The earthquake-induced shear force calculated in accordance with 21.5.4.1 represents one-half or more of the maximum required shear strength within those lengths;
- (b) The factored axial compressive force,  $P_u$ , including earthquake effects is less than  $A_g f'_c / 20$ .

**R21.5.4.2 — Transverse reinforcement**

Experimental studies<sup>21.20,21.21</sup> of reinforced concrete members subjected to cyclic loading have demonstrated that more shear reinforcement is required to ensure a flexural failure if the member is subjected to alternating nonlinear displacements than if the member is loaded in only one direction: the necessary increase of shear reinforcement being higher in the case of no axial load. This observation is reflected in the Code (see 21.5.4.2) by eliminating the term representing the contribution of concrete to shear strength. The added conservatism on shear is deemed necessary in locations where potential flexural hinging may occur. However, this stratagem, chosen for its relative simplicity, should not be interpreted to mean that no concrete is required to resist shear. On the contrary, it may be argued that the concrete core resists all the shear with the shear (transverse) reinforcement confining and strengthening the concrete. The confined concrete core plays an important role in the behavior of the beam and should not be reduced to a minimum just because the design expression does not explicitly recognize it.

**CODE****COMMENTARY****Notes on Fig. R21.5.4:**

1. Direction of shear force  $V_e$  depends on relative magnitudes of gravity loads and shear generated by end moments.
2. End moments  $M_{pr}$  based on steel tensile stress of  $1.25 f_y$ , where  $f_y$  is specified yield strength. (Both end moments should be considered in both directions, clockwise and counter-clockwise).
3. End moment  $M_{pr}$  for columns need not be greater than moments generated by the  $M_{pr}$  of the beams framing into the beam-column joints.  $V_e$  should not be less than that required by analysis of the structure.

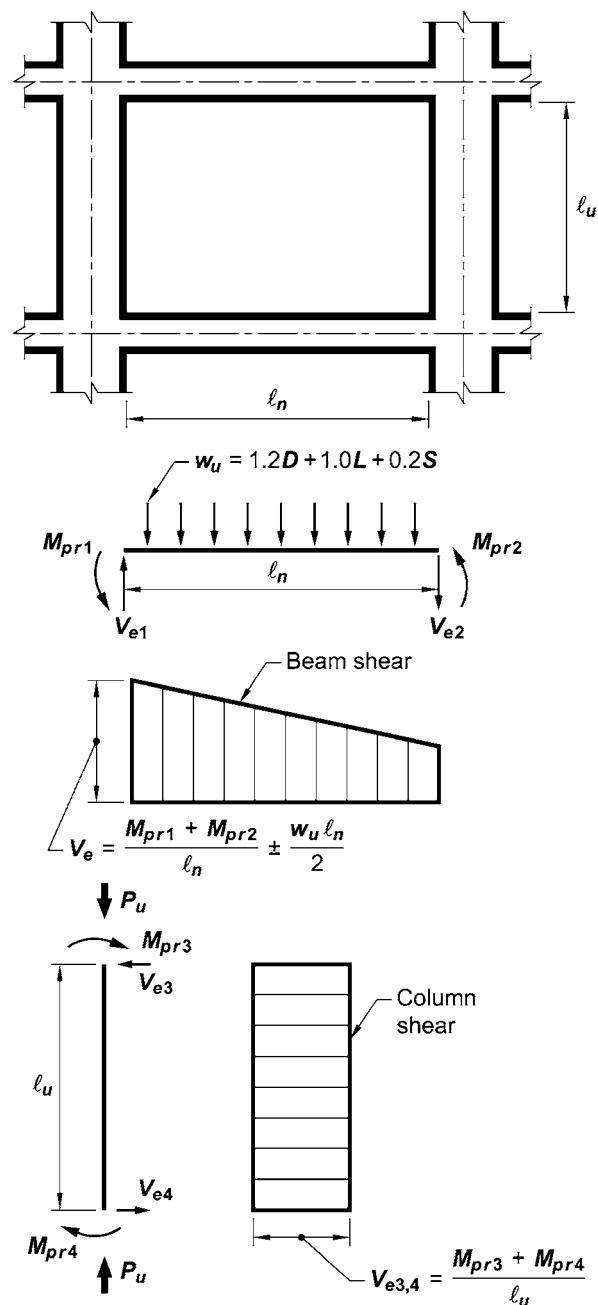


Fig. R21.5.4—Design shears for beams and columns.

## CODE

### 21.6 — Special moment frame members subjected to bending and axial load

#### 21.6.1 — Scope

Requirements of this section apply to special moment frame members that form part of the seismic-force-resisting system and that resist a factored axial compressive force  $P_u$  under any load combination exceeding  $A_g f'_c / 10$ . These frame members shall also satisfy the conditions of 21.6.1.1 and 21.6.1.2.

**21.6.1.1** — The shortest cross-sectional dimension, measured on a straight line passing through the geometric centroid, shall not be less than 12 in.

**21.6.1.2** — The ratio of the shortest cross-sectional dimension to the perpendicular dimension shall not be less than 0.4.

#### 21.6.2 — Minimum flexural strength of columns

**21.6.2.1** — Columns shall satisfy 21.6.2.2 or 21.6.2.3.

**21.6.2.2** — The flexural strengths of the columns shall satisfy Eq. (21-1)

$$\Sigma M_{nc} \geq (6/5) \Sigma M_{nb} \quad (21-1)$$

$\Sigma M_{nc}$  = sum of nominal flexural strengths of columns framing into the joint, evaluated at the faces of the joint. Column flexural strength shall be calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength.

$\Sigma M_{nb}$  = sum of nominal flexural strengths of the beams framing into the joint, evaluated at the faces of the joint. In T-beam construction, where the slab is in tension under moments at the face of the joint, slab reinforcement within an effective slab width defined in 8.12 shall be assumed to contribute to  $M_{nb}$  if the slab reinforcement is developed at the critical section for flexure.

Flexural strengths shall be summed such that the column moments oppose the beam moments. Equation (21-1) shall be satisfied for beam moments acting in both directions in the vertical plane of the frame considered.

**21.6.2.3** — If 21.6.2.2 is not satisfied at a joint, the lateral strength and stiffness of the columns framing into that joint shall be ignored when determining the calculated strength and stiffness of the structure. These columns shall conform to 21.13.

## COMMENTARY

### R21.6 — Special moment frame members subjected to bending and axial load

#### R21.6.1 — Scope

Section 21.6.1 is intended primarily for columns of special moment frames. Frame members, other than columns, that do not satisfy 21.5.1 are to be proportioned and detailed according to this section. These provisions apply to the frame member for all load combinations if the axial load exceeds  $0.1 A_g f'_c$  in any load combination.

The geometric constraints in 21.6.1.1 and 21.6.1.2 follow from previous practice.<sup>21,22</sup>

#### R21.6.2 — Minimum flexural strength of columns

The intent of 21.6.2.2 is to reduce the likelihood of yielding in columns that are considered as part of the seismic-force-resisting system. If columns are not stronger than beams framing into a joint, there is likelihood of inelastic action. In the worst case of weak columns, flexural yielding can occur at both ends of all columns in a given story, resulting in a column failure mechanism that can lead to collapse.

In 21.6.2.2, the nominal strengths of the girders and columns are calculated at the joint faces, and those strengths are compared directly using Eq. (21-1). The 1995 Code required design strengths to be compared at the center of the joint, which typically produced similar results but with added computational effort.

When determining the nominal flexural strength of a girder section in negative bending (top in tension), longitudinal reinforcement contained within an effective flange width of a top slab that acts monolithically with the girder increases the girder strength. Research<sup>21,23</sup> on beam-column subassemblies under lateral loading indicates that using the effective flange widths defined in 8.10 gives reasonable estimates of girder negative bending strengths of interior connections at interstory displacement levels approaching 2 percent of story height. This effective width is conservative where the slab terminates in a weak spandrel.

If 21.6.2.2 cannot be satisfied at a joint, 21.6.2.3 requires that any positive contribution of the column or columns involved to the lateral strength and stiffness of the structure is to be ignored. Negative contributions of the column or columns should not be ignored. For example, ignoring the stiffness of the columns ought not be used as a justification

**CODE****COMMENTARY****21.6.3 — Longitudinal reinforcement**

**21.6.3.1** — Area of longitudinal reinforcement,  $A_{st}$ , shall not be less than  $0.01A_g$  or more than  $0.06A_g$ .

**21.6.3.2** — In columns with circular hoops, the minimum number of longitudinal bars shall be 6.

**21.6.3.3** — Mechanical splices shall conform to 21.1.6 and welded splices shall conform to 21.1.7. Lap splices shall be permitted only within the center half of the member length, shall be designed as tension lap splices, and shall be enclosed within transverse reinforcement conforming to 21.6.4.2 and 21.6.4.3.

**21.6.4 — Transverse reinforcement**

**21.6.4.1** — Transverse reinforcement required in 21.6.4.2 through 21.6.4.4 shall be provided over a length  $\ell_o$  from each joint face and on both sides of any section where flexural yielding is likely to occur as a result of inelastic lateral displacements of the frame. Length  $\ell_o$  shall not be less than the largest of (a), (b), and (c):

- (a) The depth of the member at the joint face or at the section where flexural yielding is likely to occur;
- (b) One-sixth of the clear span of the member; and
- (c) 18 in.

**21.6.4.2** — Transverse reinforcement shall be provided by either single or overlapping spirals satisfying 7.10.4, circular hoops, or rectilinear hoops with or without crossties. Crossties of the same or smaller bar size as the hoops shall be permitted. Each end of the crosstie shall engage a peripheral longitudinal reinforcing bar. Consecutive crossties shall be alternated end for end along the longitudinal reinforcement. Spacing of crossties or legs of rectilinear hoops,  $h_x$ , within a cross section of the member shall not exceed 14 in. on center.

for reducing the design base shear. If inclusion of those columns in the analytical model of the building results in an increase in torsional effects, the increase should be considered as required by the governing code. Furthermore, the column must be provided with transverse reinforcement to increase its toughness to resist shear and axial forces.

**R21.6.3 — Longitudinal reinforcement**

The lower limit of the area of longitudinal reinforcement is to control time-dependent deformations and to have the yield moment exceed the cracking moment. The upper limit of the section reflects concern for steel congestion, load transfer from floor elements to column (especially in low-rise construction) and the development of high shear stresses.

Spalling of the shell concrete, which is likely to occur near the ends of the column in frames of typical configuration, makes lap splices in these locations vulnerable. If lap splices are to be used at all, they should be located near the midheight where stress reversal is likely to be limited to a smaller stress range than at locations near the joints. Transverse reinforcement is required along the lap-splice length because of the uncertainty in moment distributions along the height and the need for confinement of lap splices subjected to stress reversals.<sup>21.24</sup>

**R21.6.4 — Transverse reinforcement**

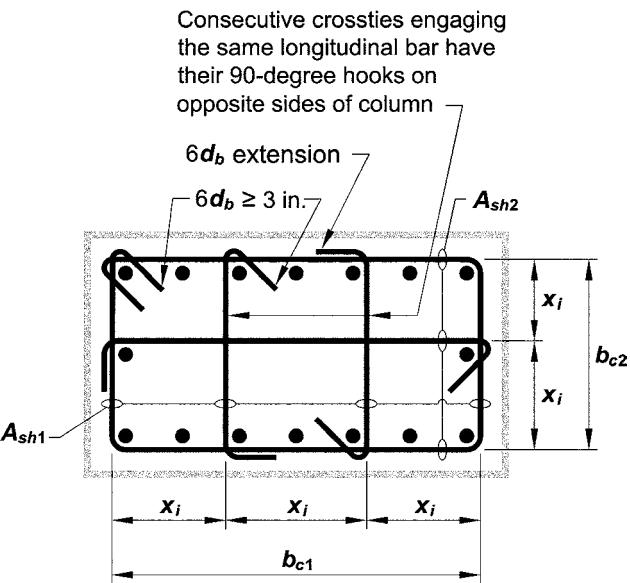
Requirements of this section are concerned with confining the concrete and providing lateral support to the longitudinal reinforcement.

**R21.6.4.1** — Section 21.6.4.1 stipulates a minimum length over which to provide closely-spaced transverse reinforcement at the member ends, where flexural yielding normally occurs. Research results indicate that the length should be increased by 50 percent or more in locations, such as the base of the building, where axial loads and flexural demands may be especially high.<sup>21.25</sup>

**R21.6.4.2** — Sections 21.6.4.2 and 21.6.4.3 provide requirements for configuration of transverse reinforcement for columns and joints of special moment frames. Figure R21.6.4.2 shows an example of transverse reinforcement provided by one hoop and three crossties. Crossties with a 90-degree hook are not as effective as either crossties with 135-degree hooks or hoops in providing confinement. Tests show that if crosstie ends with 90-degree hooks are alternated, confinement will be sufficient.

## CODE

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The dimension  $x_i$  from centerline to centerline of tie legs is not to exceed 14 inches. The term  $h_x$  used in equation 21-2 is taken as the largest value of  $x_i$ .

*Fig. R21.6.4.2—Example of transverse reinforcement in columns.*

**21.6.4.3** — Spacing of transverse reinforcement along the length  $\ell_o$  of the member shall not exceed the smallest of (a), (b), and (c):

- (a) One-quarter of the minimum member dimension;
- (b) Six times the diameter of the smallest longitudinal bar; and
- (c)  $s_o$ , as defined by Eq. (21-2)

$$s_o = 4 + \left( \frac{14 - h_x}{3} \right) \quad (21-2)$$

The value of  $s_o$  shall not exceed 6 in. and need not be taken less than 4 in.

**21.6.4.4** — The amount of transverse reinforcement required in (a) or (b) shall be provided unless a larger amount is required by 21.6.5.

- (a) The volumetric ratio of spiral or circular hoop reinforcement,  $\rho_s$ , shall not be less than required by Eq. (21-3)

$$\rho_s = 0.12 f'_c / f_{yt} \quad (21-3)$$

and shall not be less than required by Eq. (10-5).

**R21.6.4.3** — The requirement that spacing not exceed one-quarter of the minimum member dimension is to obtain adequate concrete confinement. The requirement that spacing not exceed six bar diameters is intended to restrain longitudinal reinforcement buckling after spalling. The 4 in. spacing is for concrete confinement; 21.6.4.3 permits this limit to be relaxed to a maximum of 6 in. if the spacing of crossties or legs of overlapping hoops is less than 8 in.

**R21.6.4.4** — The effect of helical (spiral) reinforcement and adequately configured rectilinear hoop reinforcement on strength and ductility of columns is well established.<sup>21.26</sup> While analytical procedures exist for calculation of strength and ductility capacity of columns under axial and moment reversals,<sup>21.27</sup> the axial load and deformation demands during earthquake loading are not known with sufficient accuracy to justify calculation of required transverse reinforcement as a function of design earthquake demands. Instead, Eq. (10-5) and (21-4) are required, with the intent that spalling of shell concrete will not result in a loss of axial load strength of the column. Equations (21-3) and (21-5) govern for large-diameter columns, and are intended to ensure adequate flexural curvature capacity in yielding regions.

**CODE**

(b) The total cross-sectional area of rectangular hoop reinforcement,  $A_{sh}$ , shall not be less than required by Eq. (21-4) and (21-5)

$$A_{sh} = 0.3 \frac{sb_c f'_c}{f_{yt}} \left[ \left( \frac{A_g}{A_{ch}} \right) - 1 \right] \quad (21-4)$$

$$A_{sh} = 0.09 \frac{sb_c f'_c}{f_{yt}} \quad (21-5)$$

**21.6.4.5** — Beyond the length  $\ell_o$  specified in **21.6.4.1**, the column shall contain spiral or hoop reinforcement satisfying **7.10** with center-to-center spacing,  $s$ , not exceeding the smaller of six times the diameter of the smallest longitudinal column bars and 6 in., unless a larger amount of transverse reinforcement is required by **21.6.3.3** or **21.6.5**.

**21.6.4.6** — Columns supporting reactions from discontinued stiff members, such as walls, shall satisfy (a) and (b):

(a) Transverse reinforcement as required in **21.6.4.2** through **21.6.4.4** shall be provided over their full height at all levels beneath the discontinuity if the factored axial compressive force in these members, related to earthquake effect, exceeds  $A_g f'_c / 10$ . Where design forces have been magnified to account for the overstrength of the vertical elements of the seismic-force-resisting system, the limit of  $A_g f'_c / 10$  shall be increased to  $A_g f'_c / 4$ .

(b) The transverse reinforcement shall extend into the discontinued member at least a distance equal to  $\ell_d$ , where  $\ell_d$  is determined in accordance with **21.7.5** for the largest longitudinal column bar. Where the lower end of the column terminates on a wall, the required transverse reinforcement shall extend into the wall at least  $\ell_d$  of the largest longitudinal column bar at the point of termination. Where the column terminates on a footing, mat, or pile cap, the required transverse reinforcement shall extend at least 12 in. into the footing, mat, or pile cap.

**21.6.4.7** — If the concrete cover outside the confining transverse reinforcement specified in **21.6.4.1**, **21.6.4.5**, and **21.6.4.6** exceeds 4 in., additional transverse reinforcement shall be provided. Concrete cover for additional transverse reinforcement shall not exceed 4 in. and spacing of additional transverse reinforcement shall not exceed 12 in.

**COMMENTARY**

Equations (21-4) and (21-5) are to be satisfied in both cross-sectional directions of the rectangular core. For each direction,  $b_c$  is the core dimension perpendicular to the tie legs that constitute  $A_{sh}$ , as shown in **Fig. R21.6.4.2**.

Research results indicate that yield strengths higher than those specified in **11.4.2** can be used effectively as confinement reinforcement. A  $f_{yt}$  of 100,000 psi is permitted in **Eq. (21-3)**, (21-4), and (21-5) where ASTM A1035 is used as confinement reinforcement.

**R21.6.4.5** — The provisions of 21.6.4.5 are intended to provide reasonable protection and ductility to the midheight of columns outside the length  $\ell_o$ . Observations after earthquakes have shown significant damage to columns in this region, and the minimum ties or spirals required should provide a more uniform toughness of the column along its length.

**R21.6.4.6** — Columns supporting discontinued stiff members, such as walls or trusses, may develop considerable inelastic response. Therefore, it is required that these columns have the specified reinforcement throughout their length. This covers all columns beneath the level at which the stiff member has been discontinued, unless the factored forces corresponding to earthquake effect are low. See **R21.11.7.5** for discussion of the overstrength factor  $\Omega_o$  applied in some codes.

**R21.6.4.7** — The unreinforced shell may spall as the column deforms to resist earthquake effects. Separation of portions of the shell from the core caused by local spalling creates a falling hazard. The additional reinforcement is required to reduce the risk of portions of the shell falling away from the column.

**CODE****COMMENTARY****21.6.5 — Shear strength requirements****21.6.5.1 — Design forces**

The design shear force,  $V_e$ , shall be determined from consideration of the maximum forces that can be generated at the faces of the joints at each end of the member. These joint forces shall be determined using the maximum probable moment strengths,  $M_{pr}$ , at each end of the member associated with the range of factored axial loads,  $P_u$ , acting on the member. The member shears need not exceed those determined from joint strengths based on  $M_{pr}$  of the transverse members framing into the joint. In no case shall  $V_e$  be less than the factored shear determined by analysis of the structure.

**21.6.5.2 — Transverse reinforcement**

Transverse reinforcement over the lengths  $\ell_o$ , identified in 21.6.4.1, shall be proportioned to resist shear assuming  $V_c = 0$  when both (a) and (b) occur:

- (a) The earthquake-induced shear force, calculated in accordance with 21.6.5.1, represents one-half or more of the maximum required shear strength within  $\ell_o$ ;
- (b) The factored axial compressive force,  $P_u$ , including earthquake effects is less than  $A_g f'_c / 20$ .

**21.7 — Joints of special moment frames****21.7.1 — Scope**

Requirements of 21.7 apply to beam-column joints of special moment frames forming part of the seismic-force-resisting system.

**21.7.2 — General requirements**

**21.7.2.1** — Forces in longitudinal beam reinforcement at the joint face shall be determined by assuming that the stress in the flexural tensile reinforcement is  $1.25f_y$ .

**21.7.2.2** — Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and anchored in tension according to 21.7.5 and in compression according to Chapter 12.

**R21.6.5 — Shear strength requirements****R21.6.5.1 — Design forces**

The procedures of 21.5.4.1 also apply to members subjected to axial loads (for example, columns). Above the ground floor, the moment at a joint may be limited by the flexural strength of the beams framing into the joint. Where beams frame into opposite sides of a joint, the combined strength may be the sum of the negative moment strength of the beam on one side of the joint and the positive moment strength of the beam on the other side of the joint. Moment strengths are to be determined using a strength reduction factor of 1.0 and reinforcing steel stress equal to at least  $1.25f_y$ . Distribution of the combined moment strength of the beams to the columns above and below the joint should be based on analysis. The value of  $M_{pr}$  in Fig. R21.5.4 may be computed from the flexural member strengths at the beam-column joints.

**R21.7 — Joints of special moment frames****R21.7.2 — General requirements**

Development of inelastic rotations at the faces of joints of reinforced concrete frames is associated with strains in the flexural reinforcement well in excess of the yield strain. Consequently, joint shear force generated by the flexural reinforcement is calculated for a stress of  $1.25f_y$  in the reinforcement (see 21.7.2.1). A detailed explanation of the reasons for the possible development of stresses in excess of the yield strength in beam tensile reinforcement is provided in Reference 21.8.

## CODE

**21.7.2.3** — Where longitudinal beam reinforcement extends through a beam-column joint, the column dimension parallel to the beam reinforcement shall not be less than 20 times the diameter of the largest longitudinal beam bar for normalweight concrete. For lightweight concrete, the dimension shall be not less than 26 times the bar diameter.

## COMMENTARY

**R21.7.2.3** — Research<sup>21.28-21.32</sup> has shown that straight beam bars may slip within the beam-column joint during a series of large moment reversals. The bond stresses on these straight bars may be very large. To reduce slip substantially during the formation of adjacent beam hinging, it would be necessary to have a ratio of column dimension to bar diameter of approximately 1/32, which would result in very large joints. On reviewing the available tests, the limit of 1/20 of the column depth in the direction of loading for the maximum size of beam bars for normalweight concrete and a limit of 1/26 for lightweight concrete were chosen. Due to the lack of specific data for beam bars through lightweight concrete joints, the limit was based on the amplification factor of 1.3 in Chapter 12 starting with the 1989 Code. The amplification factor was modified slightly in 2008 to 1/0.75 = 1.33, which did not affect this Code section. These limits provide reasonable control on the amount of potential slip of the beam bars in a beam-column joint, considering the number of anticipated inelastic excursions of the building frames during a major earthquake. A thorough treatment of this topic is given in Reference 21.33.

### 21.7.3 — Transverse reinforcement

**21.7.3.1** — Joint transverse reinforcement shall satisfy either 21.6.4.4(a) or 21.6.4.4(b), and shall also satisfy 21.6.4.2, 21.6.4.3, and 21.6.4.7, except as permitted in 21.7.3.2.

**21.7.3.2** — Where members frame into all four sides of the joint and where each member width is at least three-fourths the column width, the amount of reinforcement specified in 21.6.4.4(a) or 21.6.4.4(b) shall be permitted to be reduced by half, and the spacing required in 21.6.4.3 shall be permitted to be increased to 6 in. within the overall depth  $h$  of the shallowest framing member.

**21.7.3.3** — Longitudinal beam reinforcement outside the column core shall be confined by transverse reinforcement passing through the column that satisfies spacing requirements of 21.5.3.2, and requirements of 21.5.3.3 and 21.5.3.6, if such confinement is not provided by a beam framing into the joint.

### 21.7.4 — Shear strength

**21.7.4.1** — For normalweight concrete,  $V_n$  of the joint shall not be taken as greater than the values specified below.

For joints confined by beams  
on all four faces .....  $20\sqrt{f'_c} A_j$

For joints confined by beams  
on three faces or on two opposite faces ....  $15\sqrt{f'_c} A_j$

### R21.7.3 — Transverse reinforcement

The Code requires transverse reinforcement in a joint regardless of the magnitude of the calculated shear force. In 21.7.3.2, the amount of confining reinforcement may be reduced and the spacing may be increased if horizontal members frame into all four sides of the joint.

Section 21.7.3.3 refers to a joint where the width of the beam exceeds the corresponding column dimension. In that case, beam reinforcement not confined by the column reinforcement should be provided lateral support either by a beam framing into the same joint or by transverse reinforcement.

An example of transverse reinforcement through the column provided to confine the beam reinforcement passing outside the column core is shown in Fig. R21.5.1. Additional detailing guidance and design recommendations for both interior and exterior wide-beam connections with beam reinforcement passing outside the column core may be found in Reference 21.8.

### R21.7.4 — Shear strength

The requirements in Chapter 21 for proportioning joints are based on Reference 21.8 in that behavioral phenomena within the joint are interpreted in terms of a nominal shear strength of the joint. Because tests of joints<sup>21.28</sup> and deep beams<sup>21.14</sup> indicated that shear strength was not as sensitive to joint (shear) reinforcement as implied by the expression developed by Joint ACI-ASCE Committee 326<sup>21.34</sup> for beams, Committee 318 set the strength of the joint as a function of only the compressive strength of the concrete

**CODE**

For other cases .....  $12\sqrt{f'_c} A_j$

A beam that frames into a face is considered to provide confinement to the joint if it covers at least three-quarters of the face of the joint. Extensions of beams at least one overall beam depth  $h$  beyond the joint face are permitted to be considered adequate for confining that joint face. Extensions of beams shall satisfy 21.5.1.3, 21.5.2.1, 21.5.3.2, 21.5.3.3, and 21.5.3.6.

$A_j$  is the effective cross-sectional area within a joint computed from joint depth times effective joint width. Joint depth shall be the overall depth of the column,  $h$ . Effective joint width shall be the overall width of the column, except where a beam frames into a wider column, effective joint width shall not exceed the smaller of (a) and (b):

- (a) Beam width plus joint depth
- (b) Twice the smaller perpendicular distance from longitudinal axis of beam to column side.

**21.7.4.2** — For lightweight concrete, the nominal shear strength of the joint shall not exceed three-quarters of the limits given in 21.7.4.1.

**COMMENTARY**

(see 21.7.4) and requires a minimum amount of transverse reinforcement in the joint (see 21.7.3). The effective area of joint  $A_j$  is illustrated in Fig. R21.7.4. In no case is  $A_j$  greater than the column cross-sectional area. A circular column should be considered as having a square section of equivalent area.

The three levels of shear strength required by 21.7.4.1 are based on the recommendation of ACI Committee 352.<sup>21.8</sup> Test data reviewed by the committee<sup>21.35</sup> indicate that the lower value given in 21.7.4.1 of the 1983 Code was conservative when applied to corner joints.

Cyclic loading tests of joints with extensions of beams with lengths at least equal to their depths have indicated similar joint shear strengths to those of joints with continuous beams. These findings suggest that extensions of beams, when properly dimensioned and reinforced with longitudinal and transverse bars, provide effective confinement to the joint faces, thus delaying joint strength deterioration at large deformations.<sup>21.36</sup>

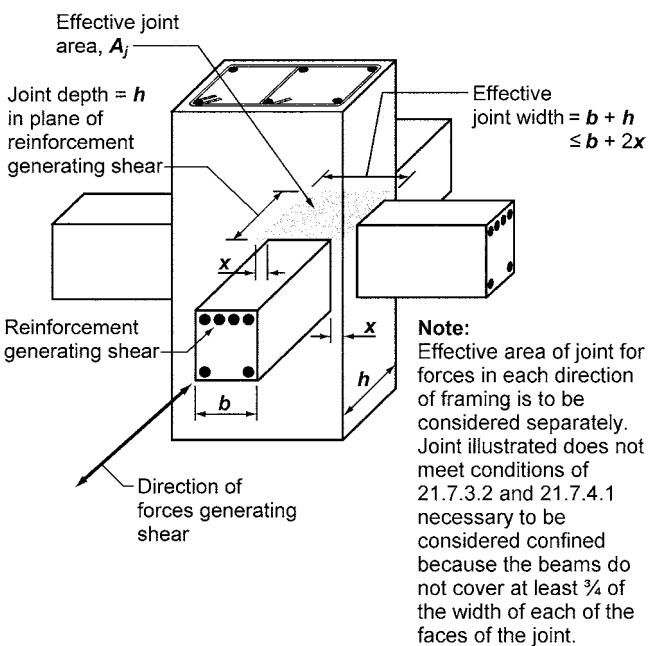


Fig. R21.7.4—Effective joint area.

**CODE****COMMENTARY****21.7.5 — Development length of bars in tension**

**21.7.5.1** — For bar sizes No. 3 through No. 11, the development length,  $\ell_{dh}$ , for a bar with a standard 90-degree hook in normalweight concrete shall not be less than the largest of  $8d_b$ , 6 in., and the length required by Eq. (21-6)

$$\ell_{dh} = f_y d_b / (65 \sqrt{f'_c}) \quad (21-6)$$

For lightweight concrete,  $\ell_{dh}$  for a bar with a standard 90-degree hook shall not be less than the largest of  $10d_b$ , 7-1/2 in., and 1.25 times the length required by Eq. (21-6).

The 90-degree hook shall be located within the confined core of a column or of a boundary element.

**21.7.5.2** — For bar sizes No. 3 through No. 11,  $\ell_d$ , the development length in tension for a straight bar, shall not be less than the larger of (a) and (b):

- (a) 2.5 times the length required by 21.7.5.1 if the depth of the concrete cast in one lift beneath the bar does not exceed 12 in.;
- (b) 3.25 times the length required by 21.7.5.1 if the depth of the concrete cast in one lift beneath the bar exceeds 12 in.

**21.7.5.3** — Straight bars terminated at a joint shall pass through the confined core of a column or of a boundary element. Any portion of  $\ell_d$  not within the confined core shall be increased by a factor of 1.6.

**21.7.5.4** — If epoxy-coated or zinc and epoxy dual-coated reinforcement is used, the development lengths in 21.7.5.1 through 21.7.5.3 shall be multiplied by applicable factors in 12.2.4 or 12.5.2.

**R21.7.5 — Development length of bars in tension**

Minimum development length in tension for deformed bars with standard hooks embedded in normalweight concrete is determined using Eq. (21-6), which is based on the requirements of 12.5. Because Chapter 21 stipulates that the hook is to be embedded in confined concrete, the coefficients 0.7 (for concrete cover) and 0.8 (for ties) have been incorporated in the constant used in Eq. (21-6). The development length that would be derived directly from 12.5 is increased to reflect the effect of load reversals.

The development length in tension of a deformed bar with a standard hook is defined as the distance, parallel to the bar, from the critical section (where the bar is to be developed) to a tangent drawn to the outside edge of the hook. The tangent is to be drawn perpendicular to the axis of the bar (see Fig. R12.5).

Factors such as the actual stress in the reinforcement being more than the yield stress and the effective development length not necessarily starting at the face of the joint were implicitly considered in the development of the expression for basic development length that has been used as the basis for Eq. (21-6).

For lightweight concrete, the length required by Eq. (21-6) is to be increased by 25 percent to compensate for variability of bond characteristics of reinforcing bars in various types of lightweight concrete.

Section 21.7.5.2 specifies the minimum development length in tension for straight bars as a multiple of the length indicated by 21.7.5.1. Section 21.7.5.2(b) refers to top bars.

If the required straight embedment length of a reinforcing bar extends beyond the confined volume of concrete (as defined in 21.5.3, 21.6.4, or 21.7.3), the required development length is increased on the premise that the limiting bond stress outside the confined region is less than that inside.

$$\ell_{dm} = 1.6(\ell_d - \ell_{dc}) + \ell_{dc}$$

or

$$\ell_{dm} = 1.6\ell_d - 0.6\ell_{dc}$$

where

- $\ell_{dm}$  = required development length if bar is not entirely embedded in confined concrete;
- $\ell_d$  = required development length in tension for straight bar embedded in confined concrete;
- $\ell_{dc}$  = length of bar embedded in confined concrete.

Lack of reference to No. 14 and No. 18 bars in 21.7.5 is due to the paucity of information on anchorage of such bars subjected to load reversals simulating earthquake effects.

**CODE****21.8 — Special moment frames  
constructed using precast concrete****21.8.1 — Scope**

Requirements of 21.8 apply to special moment frames constructed using precast concrete forming part of the seismic-force-resisting system.

**21.8.2 — Special moment frames with ductile connections constructed using precast concrete shall satisfy (a) and (b) and all requirements for special moment frames constructed with cast-in-place concrete:**

(a)  $V_n$  for connections computed according to 11.6.4 shall not be less than  $2V_e$ , where  $V_e$  is calculated according to 21.5.4.1 or 21.6.5.1;

(b) Mechanical splices of beam reinforcement shall be located not closer than  $h/2$  from the joint face and shall meet the requirements of 21.1.6.

**21.8.3 — Special moment frames with strong connections constructed using precast concrete shall satisfy all requirements for special moment frames constructed with cast-in-place concrete, as well as (a), (b), (c), and (d).**

(a) Provisions of 21.5.1.2 shall apply to segments between locations where flexural yielding is intended to occur due to design displacements;

(b) Design strength of the strong connection,  $\phi S_n$ , shall be not less than  $S_e$ ;

(c) Primary longitudinal reinforcement shall be made continuous across connections and shall be developed outside both the strong connection and the plastic hinge region; and

(d) For column-to-column connections,  $\phi S_n$  shall not be less than  $1.4S_e$ . At column-to-column connections,  $\phi M_n$  shall be not less than  $0.4M_{pr}$  for the column within the story height, and  $\phi V_n$  of the connection shall be not less than  $V_e$  determined by 21.6.5.1.

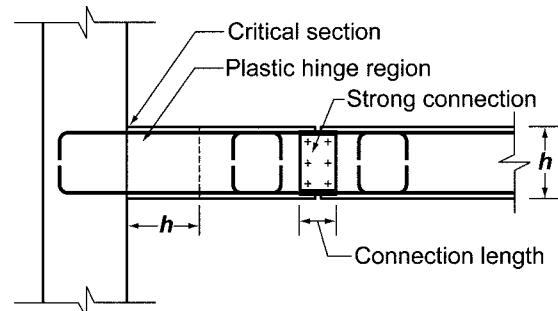
**COMMENTARY****R21.8 — Special moment frames  
constructed using precast concrete**

The detailing provisions in 21.8.2 and 21.8.3 are intended to produce frames that respond to design displacements essentially like monolithic special moment frames.

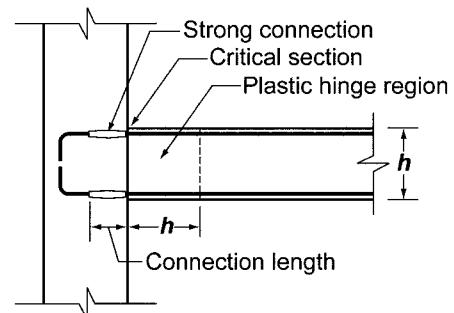
Precast frame systems composed of concrete elements with ductile connections are expected to experience flexural yielding in connection regions. Reinforcement in ductile connections can be made continuous by using Type 2 mechanical splices or any other technique that provides development in tension or compression of at least 125 percent of the specified yield strength  $f_y$  of bars and the specified tensile strength of bars.<sup>21.37-21.40</sup> Requirements for mechanical splices are in addition to those in 21.1.6 and are intended to avoid strain concentrations over a short length of reinforcement adjacent to a splice device. Additional requirements for shear strength are provided in 21.8.2 to prevent sliding on connection faces. Precast frames composed of elements with ductile connections may be designed to promote yielding at locations not adjacent to the joints. Therefore, design shear,  $V_e$ , as computed according to 21.5.4.1 or 21.6.5.1, may be conservative.

Precast concrete frame systems composed of elements joined using strong connections are intended to experience flexural yielding outside the connections. Strong connections include the length of the coupler hardware as shown in Fig. R21.8.3. Capacity-design techniques are used in 21.8.3(b) to ensure the strong connection remains elastic following formation of plastic hinges. Additional column requirements are provided to avoid hinging and strength deterioration of column-to-column connections.

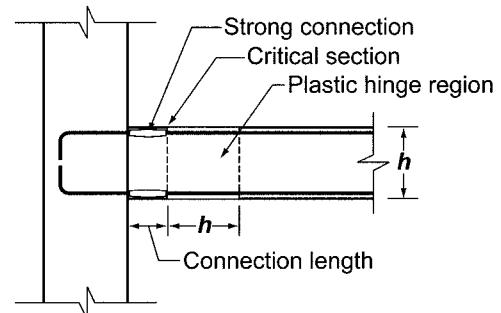
Strain concentrations have been observed to cause brittle fracture of reinforcing bars at the face of mechanical splices in laboratory tests of precast beam-column connections.<sup>21.41</sup> Locations of strong connections should be selected carefully or other measures should be taken, such as debonding of reinforcing bars in highly stressed regions, to avoid strain concentrations that can result in premature fracture of reinforcement.

**CODE****COMMENTARY**

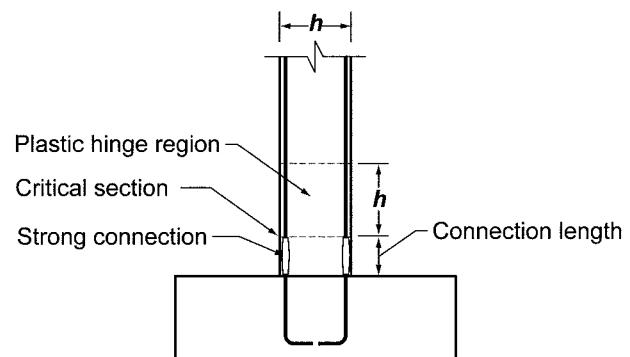
(a) Beam-to-beam connection



(b) Beam-to-column connection



(c) Beam-to-column connection



(d) Column-to-footing connection

Fig. R21.8.3—Strong connection examples.

## CODE

**21.8.4** — Special moment frames constructed using precast concrete and not satisfying the requirements of **21.8.2** or **21.8.3** shall satisfy the requirements of ACI 374.1 and the requirements of (a) and (b):

(a) Details and materials used in the test specimens shall be representative of those used in the structure; and

(b) The design procedure used to proportion the test specimens shall define the mechanism by which the frame resists gravity and earthquake effects, and shall establish acceptance values for sustaining that mechanism. Portions of the mechanism that deviate from Code requirements shall be contained in the test specimens and shall be tested to determine upper bounds for acceptance values.

## COMMENTARY

**R21.8.4** — Precast frame systems not satisfying the prescriptive requirements of **Chapter 21** have been demonstrated in experimental studies to provide satisfactory seismic performance characteristics.<sup>21.42,21.43</sup> ACI 374.1 defines a protocol for establishing a design procedure, validated by analysis and laboratory tests, for such frames. The design procedure should identify the load path or mechanism by which the frame resists gravity and earthquake effects. The tests should be configured to test critical behaviors, and the measured quantities should establish upper-bound acceptance values for components of the load path, which may be in terms of limiting stresses, forces, strains, or other quantities. The design procedure used for the structure should not deviate from that used to design the test specimens, and acceptance values should not exceed values that were demonstrated by the tests to be acceptable. Materials and components used in the structure should be similar to those used in the tests. Deviations may be acceptable if the licensed design professional can demonstrate that those deviations do not adversely affect the behavior of the framing system.

ACI ITG-1.2<sup>21.44</sup> defines design requirements for one type of special precast concrete moment frame for use in accordance with 21.8.4.

## 21.9 — Special structural walls and coupling beams

### 21.9.1 — Scope

Requirements of 21.9 apply to special structural walls and all components of special structural walls including coupling beams and wall piers forming part of the seismic-force-resisting system. Special structural walls constructed using precast concrete shall also comply with **21.10**.

### R21.9 — Special structural walls and coupling beams

#### R21.9.1 — Scope

This section contains requirements for the dimensions and details of special structural walls and all components including coupling beams and wall piers. Wall piers are defined in **2.2**. Design provisions for vertical wall segments depend on the aspect ratio of the wall segment in the plane of the wall ( $h_w/\ell_w$ ), and the aspect ratio of the horizontal cross section ( $\ell_w/b_w$ ), and generally follow the descriptions in Table R21.9.1. The limiting aspect ratios for wall piers are based on engineering judgment. It is intended that flexural yielding of the vertical reinforcement in the pier should limit shear demand on the pier.

**TABLE R21.9.1 — GOVERNING DESIGN PROVISIONS FOR VERTICAL WALL SEGMENTS\***

Clear height of vertical wall segment/length of vertical wall segment ( $h_w/\ell_w$ )	Length of vertical wall segment/wall thickness ( $\ell_w/b_w$ )		
	( $\ell_w/b_w$ ) ≤ 2.5	2.5 < ( $\ell_w/b_w$ ) ≤ 6.0	( $\ell_w/b_w$ ) > 6.0
$h_w/\ell_w < 2.0$	Wall	Wall	Wall
$h_w/\ell_w \geq 2.0$	Wall pier required to satisfy specified column design requirements, see <b>21.9.8.1</b>	Wall pier required to satisfy specified column design requirements or alternative requirements, see <b>21.9.8.1</b>	Wall

\* $h_w$  is the clear height,  $\ell_w$  is the horizontal length, and  $b_w$  is the width of the web of the wall segment.

**CODE****COMMENTARY****21.9.2 — Reinforcement**

**21.9.2.1** — The distributed web reinforcement ratios,  $\rho_e$  and  $\rho_t$ , for structural walls shall not be less than 0.0025, except that if  $V_u$  does not exceed  $A_{cv}\lambda\sqrt{f'_c}$ ,  $\rho_e$  and  $\rho_t$  shall be permitted to be reduced to the values required in 14.3. Reinforcement spacing each way in structural walls shall not exceed 18 in. Reinforcement contributing to  $V_n$  shall be continuous and shall be distributed across the shear plane.

**21.9.2.2** — At least two curtains of reinforcement shall be used in a wall if  $V_u$  exceeds  $2A_{cv}\lambda\sqrt{f'_c}$ .

**21.9.2.3** — Reinforcement in structural walls shall be developed or spliced for  $f_y$  in tension in accordance with Chapter 12, except:

- (a) The effective depth of the member referenced in 12.10.3 shall be permitted to be taken as  $0.8\ell_w$  for walls.
- (b) The requirements of 12.11, 12.12, and 12.13 need not be satisfied.
- (c) At locations where yielding of longitudinal reinforcement is likely to occur as a result of lateral displacements, development lengths of longitudinal reinforcement shall be 1.25 times the values calculated for  $f_y$  in tension.
- (d) Mechanical splices of reinforcement shall conform to 21.1.6 and welded splices of reinforcement shall conform to 21.1.7.

**21.9.3 — Design forces**

$V_u$  shall be obtained from the lateral load analysis in accordance with the factored load combinations.

**21.9.4 — Shear strength**

**21.9.4.1** —  $V_n$  of structural walls shall not exceed

$$V_n = A_{cv}(\alpha_c\lambda\sqrt{f'_c} + \rho_t f_y) \quad (21-7)$$

where the coefficient  $\alpha_c$  is 3.0 for  $h_w/\ell_w \leq 1.5$ , is 2.0 for  $h_w/\ell_w \geq 2.0$ , and varies linearly between 3.0 and 2.0 for  $h_w/\ell_w$  between 1.5 and 2.0.

**R21.9.2 — Reinforcement**

Minimum reinforcement requirements in 21.9.2.1 follow from preceding Codes. The requirement for distributed shear reinforcement is related to the intent to control the width of inclined cracks. The requirement for two layers of reinforcement in walls carrying substantial design shears in 21.9.2.2 is based on the observation that, under ordinary construction conditions, the probability of maintaining a single layer of reinforcement near the middle of the wall section is quite low. Furthermore, presence of reinforcement close to the surface tends to inhibit fragmentation of the concrete in the event of severe cracking during an earthquake.

**R21.9.2.3** — Requirements are based on provisions in Chapter 12. Because actual forces in longitudinal reinforcement of structural walls may exceed calculated forces, reinforcement should be developed or spliced to reach the yield strength of the bar in tension. Requirements of 12.11, 12.12, and 12.13 address issues related to beams and do not apply to walls. At locations where yielding of longitudinal reinforcement is expected, a 1.25 multiplier is applied to account for the likelihood that the actual yield strength exceeds the specified yield strength of the bar, as well as the influence of strain hardening and cyclic load reversals. Where transverse reinforcement is used, development lengths for straight and hooked bars may be reduced as permitted in 12.2 and 12.5, respectively, because closely spaced transverse reinforcement improves the performance of splices and hooks subjected to repeated inelastic demands.<sup>21.45</sup>

**R21.9.3 — Design forces**

Design shears for structural walls are obtained from lateral load analysis with the appropriate load factors. However, the possibility of yielding in components of such structures should be considered, as in the portion of a wall between two window openings, in which case the actual shear may be in excess of the shear indicated by lateral load analysis based on factored design forces.

**R21.9.4 — Shear strength**

Equation (21-7) recognizes the higher shear strength of walls with high shear-to-moment ratios.<sup>21.14, 21.34, 21.46</sup> The nominal shear strength is given in terms of the net area of the section resisting shear. For a rectangular section without openings, the term  $A_{cv}$  refers to the gross area of the cross section rather than to the product of the width and the effective depth. The definition of  $A_{cv}$  in Eq. (21-7) facilitates design calculations for walls with uniformly distributed reinforcement and walls with openings.

**CODE****COMMENTARY**

**21.9.4.2** — In 21.9.4.1, the value of ratio  $h_w/\ell_w$  used for determining  $V_n$  for segments of a wall shall be the larger of the ratios for the entire wall and the segment of wall considered.

**21.9.4.3** — Walls shall have distributed shear reinforcement providing resistance in two orthogonal directions in the plane of the wall. If  $h_w/\ell_w$  does not exceed 2.0, reinforcement ratio  $\rho_t$  shall not be less than reinforcement ratio  $\rho_t$ .

**21.9.4.4** — For all vertical wall segments resisting a common lateral force, combined  $V_n$  shall not be taken larger than  $8A_{cv}\sqrt{f'_c}$ , where  $A_{cv}$  is the gross combined area of all vertical wall segments. For any one of the individual vertical wall segments,  $V_n$  shall not be taken larger than  $10A_{cw}\sqrt{f'_c}$ , where  $A_{cw}$  is the area of concrete section of the individual vertical wall segment considered.

**21.9.4.5** — For horizontal wall segments, including coupling beams,  $V_n$  shall not be taken larger than  $10A_{cw}\sqrt{f'_c}$ , where  $A_{cw}$  is the area of concrete section of a horizontal wall segment or coupling beam.

A vertical wall segment refers to a part of a wall bounded horizontally by openings or by an opening and an edge. When designing an isolated wall or a vertical wall segment,  $\rho_t$  refers to horizontal reinforcement and  $\rho_t$  refers to vertical reinforcement.

The ratio  $h_w/\ell_w$  may refer to overall dimensions of a wall, or of a segment of the wall bounded by two openings, or an opening and an edge. The intent of 21.9.4.2 is to make certain that any segment of a wall is not assigned a unit strength larger than that for the entire wall. However, a wall segment with a ratio of  $h_w/\ell_w$  higher than that of the entire wall should be proportioned for the unit strength associated with the ratio  $h_w/\ell_w$  based on the dimensions for that segment.

To restrain the inclined cracks effectively, reinforcement included in  $\rho_t$  and  $\rho_t$  should be appropriately distributed along the length and height of the wall (see 21.9.4.3). Chord reinforcement provided near wall edges in concentrated amounts for resisting bending moment is not to be included in determining  $\rho_t$  and  $\rho_t$ . Within practical limits, shear reinforcement distribution should be uniform and at a small spacing.

If the factored shear force at a given level in a structure is resisted by several walls or several vertical wall segments of a perforated wall, the average unit shear strength assumed for the total available cross-sectional area is limited to  $8\sqrt{f'_c}$  with the additional requirement that the unit shear strength assigned to any single vertical wall segment does not exceed  $10\sqrt{f'_c}$ . The upper limit of strength to be assigned to any one member is imposed to limit the degree of redistribution of shear force.

“Horizontal wall segments” in 21.9.4.5 refers to wall sections between two vertically aligned openings (see Fig. R21.9.4.5). It is, in effect, a vertical wall segment rotated through 90 degrees. A horizontal wall segment is also referred to as a coupling beam when the openings are aligned vertically over the building height. When designing a horizontal wall segment or coupling beam,  $\rho_t$  refers to vertical reinforcement and  $\rho_t$  refers to horizontal reinforcement.

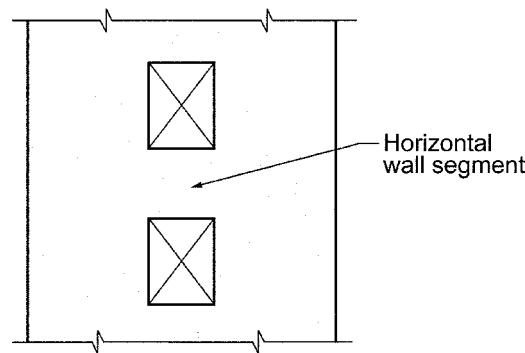


Fig. R21.9.4.5—Wall with openings.

**CODE****COMMENTARY****21.9.5 — Design for flexure and axial loads**

**21.9.5.1** — Structural walls and portions of such walls subject to combined flexural and axial loads shall be designed in accordance with 10.2 and 10.3 except that 10.3.7 and the nonlinear strain requirements of 10.2.2 shall not apply. Concrete and developed longitudinal reinforcement within effective flange widths, boundary elements, and the wall web shall be considered effective. The effects of openings shall be considered.

**21.9.5.2** — Unless a more detailed analysis is performed, effective flange widths of flanged sections shall extend from the face of the web a distance equal to the smaller of one-half the distance to an adjacent wall web and 25 percent of the total wall height.

**21.9.6 — Boundary elements of special structural walls**

**21.9.6.1** — The need for special boundary elements at the edges of structural walls shall be evaluated in accordance with 21.9.6.2 or 21.9.6.3. The requirements of 21.9.6.4 and 21.9.6.5 also shall be satisfied.

**21.9.6.2** — This section applies to walls or wall piers that are effectively continuous from the base of structure to top of wall and designed to have a single critical section for flexure and axial loads. Walls not satisfying these requirements shall be designed by 21.9.6.3.

(a) Compression zones shall be reinforced with special boundary elements where

$$c \geq \frac{\ell_w}{600(\delta_u/h_w)} \quad (21-8)$$

**R21.9.5 — Design for flexure and axial loads**

**R21.9.5.1** — Flexural strength of a wall or wall segment is determined according to procedures commonly used for columns. Strength should be determined considering the applied axial and lateral forces. Reinforcement concentrated in boundary elements and distributed in flanges and webs should be included in the strength computations based on a strain compatibility analysis. The foundation supporting the wall should be designed to develop the wall boundary and web forces. For walls with openings, the influence of the opening or openings on flexural and shear strengths is to be considered and a load path around the opening or openings should be verified. Capacity-design concepts and strut-and-tie models may be useful for this purpose.<sup>21.47</sup>

**R21.9.5.2** — Where wall sections intersect to form L-, T-, C-, or other cross-sectional shapes, the influence of the flange on the behavior of the wall should be considered by selecting appropriate flange widths. Tests<sup>21.48</sup> show that effective flange width increases with increasing drift level and the effectiveness of a flange in compression differs from that for a flange in tension. The value used for the effective compression flange width has little impact on the strength and deformation capacity of the wall; therefore, to simplify design, a single value of effective flange width based on an estimate of the effective tension flange width is used in both tension and compression.

**R21.9.6 — Boundary elements of special structural walls**

**R21.9.6.1** — Two design approaches for evaluating detailing requirements at wall boundaries are included in 21.9.6.1. Section 21.9.6.2 allows the use of displacement-based design of walls, in which the structural details are determined directly on the basis of the expected lateral displacements of the wall. The provisions of 21.9.6.3 are similar to those of the 1995 Code, and have been retained because they are conservative for assessing required transverse reinforcement at wall boundaries for many walls. Requirements of 21.9.6.4 and 21.9.6.5 apply to structural walls designed by either 21.9.6.2 or 21.9.6.3.

**R21.9.6.2** — Section 21.9.6.2 is based on the assumption that inelastic response of the wall is dominated by flexural action at a critical, yielding section. The wall should be proportioned so that the critical section occurs where intended.

Equation (21-8) follows from a displacement-based approach.<sup>21.49,21.50</sup> The approach assumes that special boundary elements are required to confine the concrete where the strain at the extreme compression fiber of the wall exceeds a critical value when the wall is displaced to the design displacement. The horizontal dimension of the special boundary element is intended to extend at least over the length where the compression strain exceeds the critical

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**c** in Eq. (21-8) corresponds to the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with the design displacement  $\delta_u$ . Ratio  $\delta_u/h_w$  in Eq. (21-8) shall not be taken less than 0.007;

(b) Where special boundary elements are required by 21.9.6.2(a), the special boundary element reinforcement shall extend vertically from the critical section a distance not less than the larger of  $\ell_w$  or  $M_u/4V_u$ .

**21.9.6.3** — Structural walls not designed to the provisions of 21.9.6.2 shall have special boundary elements at boundaries and edges around openings of structural walls where the maximum extreme fiber compressive stress, corresponding to load combinations including earthquake effects,  $E$ , exceeds  $0.2f'_c$ . The special boundary element shall be permitted to be discontinued where the calculated compressive stress is less than  $0.15f'_c$ . Stresses shall be calculated for the factored forces using a linearly elastic model and gross section properties. For walls with flanges, an effective flange width as defined in 21.9.5.2 shall be used.

**21.9.6.4** — Where special boundary elements are required by 21.9.6.2 or 21.9.6.3, (a) through (e) shall be satisfied:

(a) The boundary element shall extend horizontally from the extreme compression fiber a distance not less than the larger of  $c - 0.1\ell_w$  and  $c/2$ , where  $c$  is the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with  $\delta_u$ ;

(b) In flanged sections, the boundary element shall include the effective flange width in compression and shall extend at least 12 in. into the web;

(c) The boundary element transverse reinforcement shall satisfy the requirements of 21.6.4.2 through 21.6.4.4, except Eq. (21-4) need not be satisfied and the transverse reinforcement spacing limit of 21.6.4.3(a) shall be one-third of the least dimension of the boundary element;

(d) The boundary element transverse reinforcement at the wall base shall extend into the support at least  $\ell_d$ , according to 21.9.2.3, of the largest longitudinal

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value. The height of the special boundary element is based on upper bound estimates of plastic hinge length and extends beyond the zone over which concrete spalling is likely to occur. The lower limit of 0.007 on the quantity  $\delta_u/h_w$  requires moderate wall deformation capacity for stiff buildings.

The neutral axis depth  $c$  in Eq. (21-8) is the depth calculated according to 10.2, except the nonlinear strain requirements of 10.2.2 need not apply, corresponding to development of nominal flexural strength of the wall when displaced in the same direction as  $\delta_u$ . The axial load is the factored axial load that is consistent with the design load combination that produces the design displacement  $\delta_u$ .

**R21.9.6.3** — By this procedure, the wall is considered to be acted on by gravity loads and the maximum shear and moment induced by earthquake in a given direction. Under this loading, the compressed boundary at the critical section resists the tributary gravity load plus the compressive resultant associated with the bending moment.

Recognizing that this loading condition may be repeated many times during the strong motion, the concrete is to be confined where the calculated compressive stresses exceed a nominal critical value equal to  $0.2f'_c$ . The stress is to be calculated for the factored forces on the section assuming linear response of the gross concrete section. The compressive stress of  $0.2f'_c$  is used as an index value and does not necessarily describe the actual state of stress that may develop at the critical section under the influence of the actual inertia forces for the anticipated earthquake intensity.

**R21.9.6.4** — The value of  $c/2$  in 21.9.6.4(a) is to provide a minimum length of the special boundary element. Where flanges are heavily stressed in compression, the web-to-flange interface is likely to be heavily stressed and may sustain local crushing failure unless special boundary element reinforcement extends into the web. Equation (21-4) does not apply to walls.

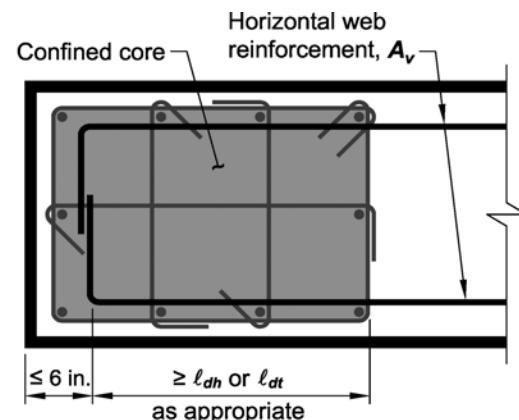
The horizontal reinforcement in a structural wall with low shear to moment ratio resists shear through truss action, with the horizontal bars acting like the stirrups in a beam. Thus, the horizontal bars provided for shear reinforcement must be developed within the confined core of the boundary element and extended as close to the end of the wall as cover requirements and proximity of other reinforcement permit. The requirement that the horizontal web reinforcement be anchored within the confined core of the boundary element and extended to within 6 in. from the end of the wall applies to all horizontal bars whether straight, hooked, or headed, as illustrated in Fig. R21.9.6.4.

Tests<sup>21.51</sup> show that adequate performance can be achieved using spacing larger than permitted by 21.6.4.3(a).

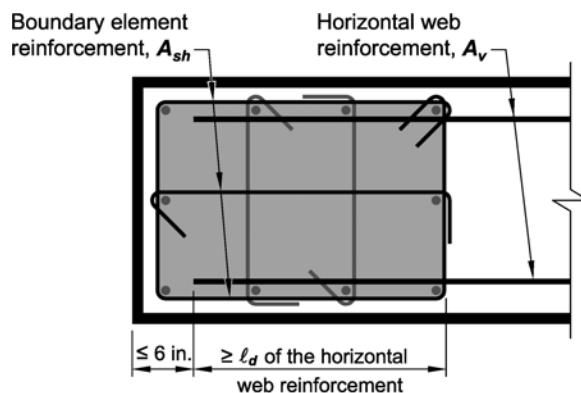
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reinforcement in the special boundary element unless the special boundary element terminates on a footing, mat, or pile cap, where special boundary element transverse reinforcement shall extend at least 12 in. into the footing, mat, or pile cap;

(e) Horizontal reinforcement in the wall web shall extend to within 6 in. of the end of the wall. Reinforcement shall be anchored to develop  $f_y$  in tension within the confined core of the boundary element using standard hooks or heads. Where the confined boundary element has sufficient length to develop the horizontal web reinforcement, and  $A_v f_y / s$  of the web reinforcement is not greater than  $A_{sh} f_{yt} / s$  of the boundary element transverse reinforcement parallel to the web reinforcement, it shall be permitted to terminate the web reinforcement without a standard hook or head.

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**(a)**  
Option with standard hooks or headed reinforcement



**(b)**  
Option with straight developed reinforcement

*Fig. R.21.9.6.4—Development of wall horizontal reinforcement in confined boundary element.*

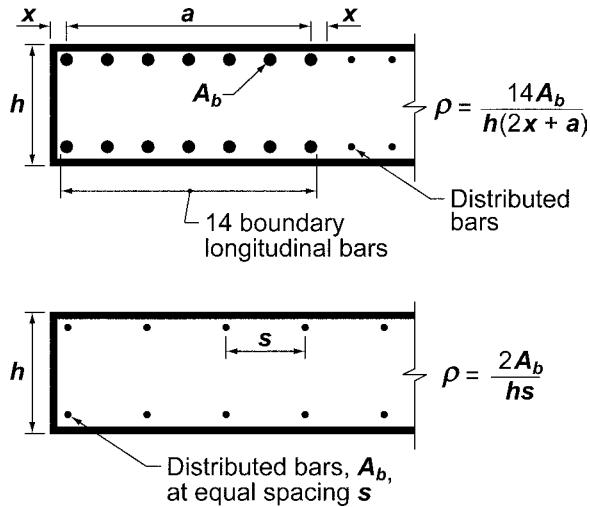
**21.9.6.5** — Where special boundary elements are not required by 21.9.6.2 or 21.9.6.3, (a) and (b) shall be satisfied:

(a) If the longitudinal reinforcement ratio at the wall boundary is greater than  $400/f_y$ , boundary transverse reinforcement shall satisfy 21.6.4.2 and 21.9.6.4(a). The maximum longitudinal spacing of transverse reinforcement in the boundary shall not exceed 8 in.;

(b) Except when  $V_u$  in the plane of the wall is less than  $A_{cv} \lambda \sqrt{f'_c}$ , horizontal reinforcement terminating at the edges of structural walls without boundary elements shall have a standard hook engaging the edge reinforcement or the edge reinforcement shall be enclosed in U-stirrups having the same size and spacing as, and spliced to, the horizontal reinforcement.

**R21.9.6.5** — Cyclic load reversals may lead to buckling of boundary longitudinal reinforcement even in cases where the demands on the boundary of the wall do not require special boundary elements. For walls with moderate amounts of boundary longitudinal reinforcement, ties are required to inhibit buckling. The longitudinal reinforcement ratio is intended to include only the reinforcement at the wall boundary as indicated in Fig. R21.9.5. A larger spacing of ties relative to 21.9.6.4(c) is allowed due to the lower deformation demands on the walls.

The addition of hooks or U-stirrups at the ends of horizontal wall reinforcement provides anchorage so that the reinforcement will be effective in resisting shear forces. It will also tend to inhibit the buckling of the vertical edge reinforcement. In walls with low in-plane shear, the development of horizontal reinforcement is not necessary.

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*Fig. R21.9.6.5—Longitudinal reinforcement ratios for typical wall boundary conditions.*

### 21.9.7 — Coupling beams

**21.9.7.1** — Coupling beams with  $(\ell_n/h) \geq 4$  shall satisfy the requirements of **21.5**. The provisions of **21.5.1.3** and **21.5.1.4** need not be satisfied if it can be shown by analysis that the beam has adequate lateral stability.

**21.9.7.2** — Coupling beams with  $(\ell_n/h) < 2$  and with  $V_u$  exceeding  $4\lambda\sqrt{f'_c} A_{cw}$  shall be reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan, unless it can be shown that loss of stiffness and strength of the coupling beams will not impair the vertical load-carrying ability of the structure, the egress from the structure, or the integrity of nonstructural components and their connections to the structure.

**21.9.7.3** — Coupling beams not governed by **21.9.7.1** or **21.9.7.2** shall be permitted to be reinforced either with two intersecting groups of diagonally placed bars symmetrical about the midspan or according to **21.5.2** through **21.5.4**.

**21.9.7.4** — Coupling beams reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan shall satisfy (a), (b), and either (c) or (d). Requirements of **11.7** shall not apply.

(a)  $V_n$  shall be determined by

$$V_n = 2A_{vd}f_y \sin \alpha \leq 10\sqrt{f'_c} A_{cw} \quad (21-9)$$

where  $\alpha$  is the angle between the diagonal bars and the longitudinal axis of the coupling beam.

### R21.9.7 — Coupling beams

Coupling beams connecting structural walls can provide stiffness and energy dissipation. In many cases, geometric limits result in coupling beams that are deep in relation to their clear span. Deep coupling beams may be controlled by shear and may be susceptible to strength and stiffness deterioration under earthquake loading. Test results<sup>21.52,21.53</sup> have shown that confined diagonal reinforcement provides adequate resistance in deep coupling beams.

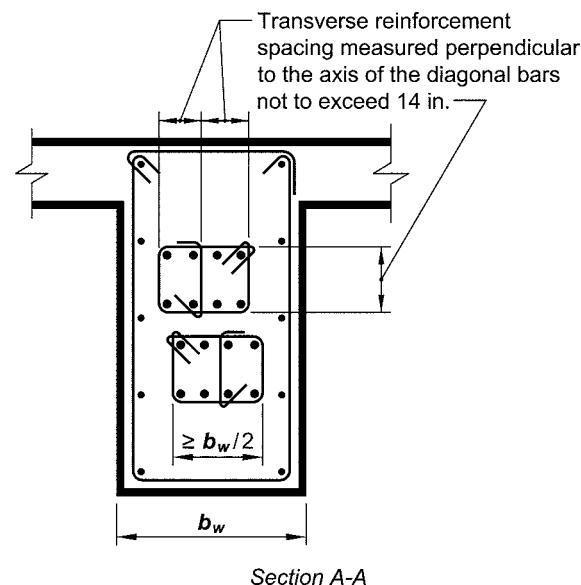
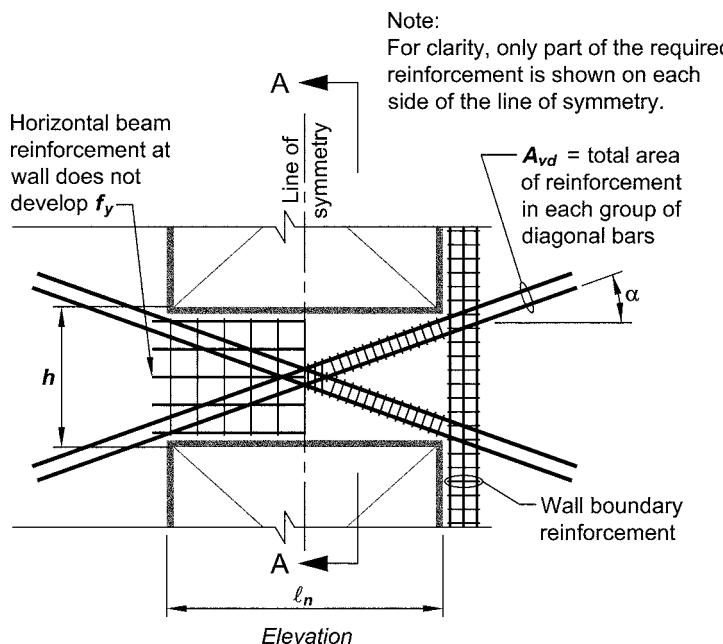
Experiments show that diagonally oriented reinforcement is effective only if the bars are placed with a large inclination. Therefore, diagonally reinforced coupling beams are restricted to beams having aspect ratio  $\ell_n/h < 4$ . The 2008 edition of this Code was changed to clarify that coupling beams of intermediate aspect ratio can be reinforced according to **21.5.2** through **21.5.4**.

Diagonal bars should be placed approximately symmetrically in the beam cross section, in two or more layers. The diagonally placed bars are intended to provide the entire shear and corresponding moment strength of the beam; designs deriving their moment strength from combinations of diagonal and longitudinal bars are not covered by these provisions.

Two confinement options are described. According to **21.9.7.4(c)**, each diagonal element consists of a cage of longitudinal and transverse reinforcement as shown in **Fig. R21.9.7(a)**. Each cage contains at least four diagonal bars and confines a concrete core. The requirement on side dimensions of the cage and its core is to provide adequate toughness and stability to the cross section when the bars are loaded beyond yielding. The minimum dimensions and required reinforcement clearances may control the wall

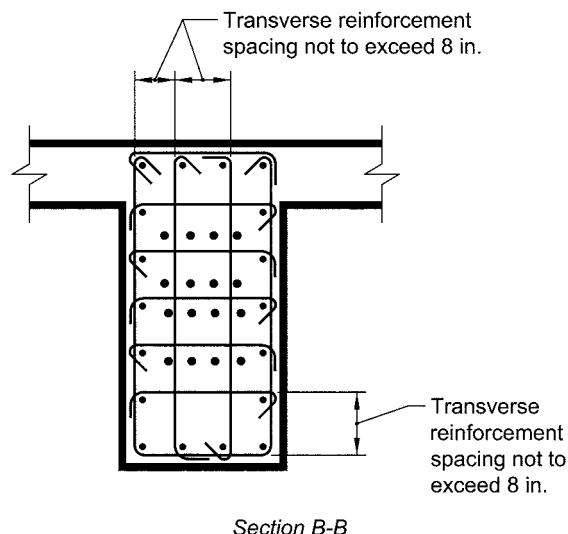
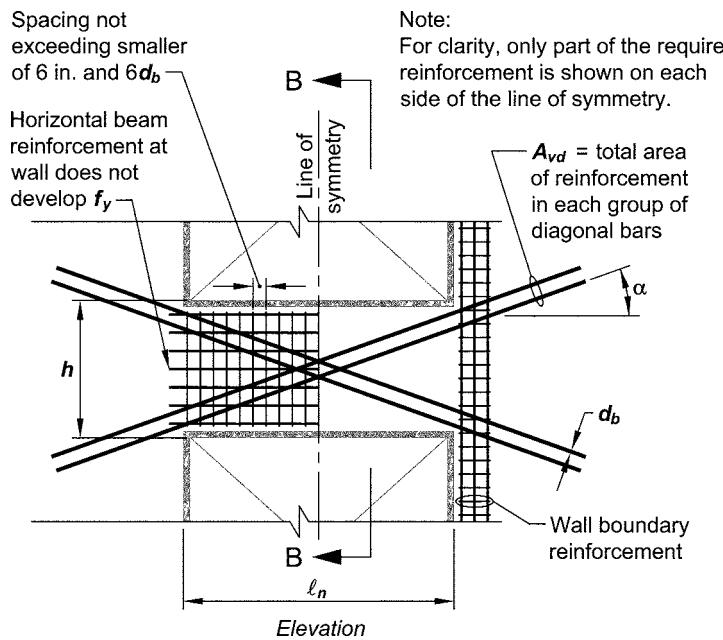
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(a) Confinement of individual diagonals.

**Note:** For clarity in the elevation view, only part of the total required reinforcement is shown on each side of the line of symmetry.



(b) Full confinement of diagonally reinforced concrete beam section.

**Note:** Consecutive crossties engaging the same longitudinal bar have their 90-degree hooks on opposite sides of beam.

Fig. R21.9.7—Coupling beams with diagonally oriented reinforcement. Wall boundary reinforcement shown on one side only for clarity.

(b) Each group of diagonal bars shall consist of a minimum of four bars provided in two or more layers. The diagonal bars shall be embedded into the wall not less than 1.25 times the development length for  $f_y$  in tension.

width. Revisions were made in the 2008 Code to relax spacing of transverse reinforcement confining the diagonal bars, to clarify that confinement is required at the intersection of the diagonals, and to simplify design of the longitudinal and transverse reinforcement around the beam perimeter; beams with these new details are expected to perform acceptably.

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(c) Each group of diagonal bars shall be enclosed by transverse reinforcement having out-to-out dimensions not smaller than  $b_w/2$  in the direction parallel to  $b_w$  and  $b_w/5$  along the other sides, where  $b_w$  is the web width of the coupling beam. The transverse reinforcement shall satisfy 21.6.4.2 and 21.6.4.4, shall have spacing measured parallel to the diagonal bars satisfying 21.6.4.3(c) and not exceeding six times the diameter of the diagonal bars, and shall have spacing of crossties or legs of hoops measured perpendicular to the diagonal bars not exceeding 14 in. For the purpose of computing  $A_g$  for use in Eq. (10-5) and (21-4), the concrete cover as required in 7.7 shall be assumed on all four sides of each group of diagonal bars. The transverse reinforcement, or its alternatively configured transverse reinforcement satisfying the spacing and volume ratio requirements of the transverse reinforcement along the diagonals, shall continue through the intersection of the diagonal bars. Additional longitudinal and transverse reinforcement shall be distributed around the beam perimeter with total area in each direction not less than  $0.002b_w s$  and spacing not exceeding 12 in.

(d) Transverse reinforcement shall be provided for the entire beam cross section satisfying 21.6.4.2, 21.6.4.4, and 21.6.4.7, with longitudinal spacing not exceeding the smaller of 6 in. and six times the diameter of the diagonal bars, and with spacing of crossties or legs of hoops both vertically and horizontally in the plane of the beam cross section not exceeding 8 in. Each crosstie and each hoop leg shall engage a longitudinal bar of equal or larger diameter. It shall be permitted to configure hoops as specified in 21.5.3.6.

### 21.9.8 — Wall piers

**21.9.8.1** — Wall piers shall satisfy the special moment frame requirements for columns of 21.6.3, 21.6.4, and 21.6.5, with joint faces taken as the top and bottom of the clear height of the wall pier. Alternatively, wall piers with  $(\ell_w/b_w) > 2.5$  shall satisfy (a) through (f):

(a) Design shear force shall be determined in accordance with 21.6.5.1 with joint faces taken as the top and bottom of the clear height of the wall pier. Where the legally adopted general building code includes provisions to account for overstrength of the seismic-force-resisting system, the design shear force need not exceed  $\Omega_0$  times the factored shear determined by analysis of the structure for earthquake effects.

(b)  $V_n$  and distributed shear reinforcement shall satisfy 21.9.4.

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Section 21.9.7.4(d) describes a second option for confinement of the diagonals introduced in the 2008 Code (Fig. R21.9.7(b)). This second option is to confine the entire beam cross section instead of confining the individual diagonals. This option can considerably simplify field placement of hoops, which can otherwise be especially challenging where diagonal bars intersect each other or enter the wall boundary.

When coupling beams are not used as part of the lateral-force-resisting system, the requirements for diagonal reinforcement may be waived.

Test results<sup>21.53</sup> demonstrate that beams reinforced as described in Section 21.9.7 have adequate ductility at shear forces exceeding  $10\sqrt{f'_c} b_w d$ . Consequently, the use of a limit of  $10\sqrt{f'_c} A_{cw}$  provides an acceptable upper limit.

### R21.9.8 — Wall piers

Door and window placements in structural walls sometimes lead to narrow vertical wall segments that are considered to be wall piers. The dimensions defining wall piers are given in 2.2. Shear failures of wall piers have been observed in previous earthquakes. The intent of 21.9.8 is to provide sufficient shear strength to wall piers so that a flexural yielding mechanism will develop. The provisions apply to wall piers designated as part of the seismic-force-resisting system. Provisions for wall piers not designated as part of the seismic-force-resisting system are given in 21.13. The effect of all vertical wall segments on the response of the structural system, whether designated as part of the seismic-force-resisting system or not, should be considered as required by 21.1.2.

Wall piers having  $(\ell_w/b_w) \leq 2.5$  behave essentially as columns. Section 21.9.8.1 requires that such members satisfy reinforcement and shear strength requirements of 21.6.3 through 21.6.5. Alternative provisions are provided for wall piers having  $(\ell_w/b_w) > 2.5$ .

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(c) Transverse reinforcement shall be in the form of hoops except it shall be permitted to use single-leg horizontal reinforcement parallel to  $\ell_w$  where only one curtain of distributed shear reinforcement is provided. Single-leg horizontal reinforcement shall have 180-degree bends at each end that engage wall pier boundary longitudinal reinforcement.

(d) Vertical spacing of transverse reinforcement shall not exceed 6 in.

(e) Transverse reinforcement shall extend at least 12 in. above and below the clear height of the wall pier.

(f) Special boundary elements shall be provided if required by 21.9.6.3.

**21.9.8.2** — For wall piers at the edge of a wall, horizontal reinforcement shall be provided in adjacent wall segments above and below the wall pier and be proportioned to transfer the design shear force from the wall pier into the adjacent wall segments.

**21.9.9 — Construction joints**

All construction joints in structural walls shall conform to 6.4 and contact surfaces shall be roughened as in 11.6.9.

**| 21.9.10 — Discontinuous walls**

Columns supporting discontinuous structural walls shall be reinforced in accordance with 21.6.4.6.

**21.10 — Special structural walls  
constructed using precast concrete****21.10.1 — Scope**

Requirements of 21.10 apply to special structural walls constructed using precast concrete forming part of the seismic-force-resisting system.

**21.10.2** — Special structural walls constructed using precast concrete shall satisfy all requirements of 21.9 in addition to 21.4.2 and 21.4.3.

**21.10.3** — Special structural walls constructed using precast concrete and unbonded post-tensioning tendons and not satisfying the requirements of 21.10.2 are permitted provided they satisfy the requirements of ACI ITG-5.1.

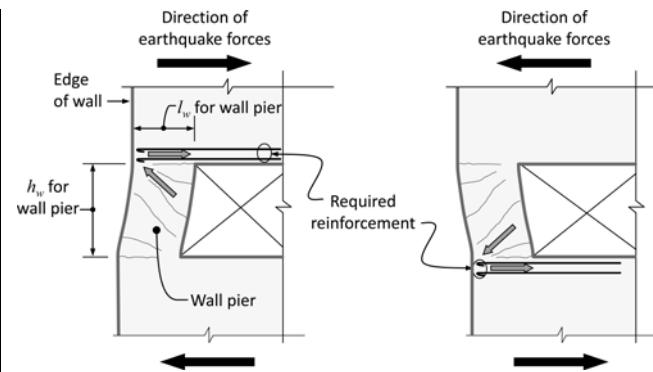
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Fig. R21.9.8—Required horizontal reinforcement in wall segments above and below wall piers at the edge of a wall.

The design shear force determined according to 21.6.5.1 may be unrealistically large in some cases. As an alternative, 21.9.8.1(a) permits the design shear force to be determined using load combinations in which the earthquake effect has been amplified to account for system overstrength. Documents such as the NEHRP provisions,<sup>21.4</sup> ASCE/SEI 7,<sup>21.1</sup> and the International Building Code<sup>21.2</sup> represent the amplified earthquake effect using the factor  $\Omega_o$ .

Section 21.9.8.2 addresses wall piers at the edge of a wall. Under in-plane shear, inclined cracks can propagate into segments of the wall directly above and below the wall pier. Unless there is sufficient reinforcement in the adjacent wall segments, shear failure within the adjacent wall segments can occur. The length of embedment of the provided reinforcement into the adjacent wall segments should be determined considering both development length requirements and shear strength of the wall segments. See Fig. R21.9.8.

**R21.10 — Special structural walls  
constructed using precast concrete**

**R21.10.3** — Experimental and analytical studies<sup>21.54-21.56</sup> have demonstrated that some types of precast structural walls post-tensioned with unbonded tendons, and not satisfying the prescriptive requirements of Chapter 21, provide satisfactory seismic performance characteristics. ACI ITG-5.1 defines a protocol for establishing a design procedure, validated by analysis and laboratory tests, for such walls, with or without coupling beams.

**CODE****COMMENTARY****21.11 — Structural diaphragms and trusses****21.11.1 — Scope**

Floor and roof slabs acting as structural diaphragms to transmit forces induced by earthquake ground motions in structures assigned to SDC D, E, or F shall be designed in accordance with this section. This section also applies to collector elements and trusses forming part of the seismic-force-resisting system.

**21.11.2 — Design forces**

The earthquake design forces for structural diaphragms shall be obtained from the legally adopted general building code using the applicable provisions and load combinations.

ACI ITG-5.2<sup>21.57</sup> defines design requirements for one type of special structural wall constructed using precast concrete and unbonded post-tensioning tendons, and validated for use in accordance with 21.10.3.

**R21.11 — Structural diaphragms and trusses****R21.11.1 — Scope**

Diaphragms as used in building construction are structural elements (such as a floor or roof) that provide some or all of the following functions:

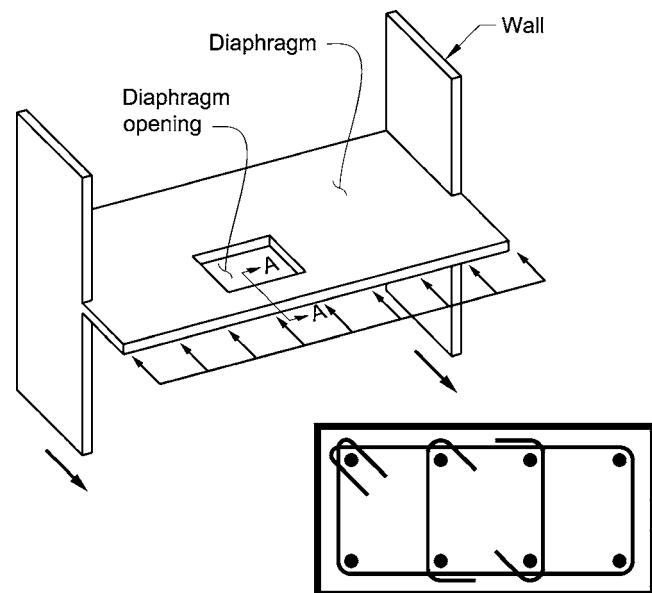
- (a) Support for building elements (such as walls, partitions, and cladding) resisting horizontal forces but not acting as part of the seismic-force-resisting system;
- (b) Transfer of lateral forces from the point of application to the vertical elements of the seismic-force-resisting system;
- (c) Connection of various components of the vertical seismic-force-resisting system with appropriate strength, stiffness, and toughness so the building responds as intended in the design.<sup>21.58</sup>

**R21.11.2 — Design forces**

In the general building codes, earthquake design forces for floor and roof diaphragms typically are not computed directly during the lateral-force analysis that provides story forces and story shears. Instead, diaphragm design forces at each level are computed by a formula that amplifies the story forces recognizing dynamic effects and includes minimum and maximum limits. These forces are used with the governing load combinations to design diaphragms for shear and moment.

For collector elements, general building codes in use in the U.S. specify load combinations that amplify earthquake forces by a factor  $\Omega_v$ . The forces amplified by  $\Omega_v$  are also used for local diaphragm shear force resulting from the transfer of collector forces, and for local diaphragm flexural moments resulting from any eccentricity of collector forces. The specific requirements for earthquake design forces for diaphragms and collectors depend on which general building code is used. The requirements may also vary according to the SDC.

For most concrete buildings subjected to inelastic earthquake demands, it is desirable to limit inelastic behavior of floor and roof diaphragms under the imposed earthquake forces and deformations. It is preferable for inelastic behavior to occur only in the intended locations of the vertical earthquake-force-resisting system that are detailed for ductile response, such as in the beam plastic hinges of special moment frames, or in flexural plastic hinging at the base of structural walls or in coupling beams. For buildings without long diaphragm spans between lateral-force-resisting elements, elastic diaphragm behavior is typically

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*Fig. R21.11.3.2—Example of diaphragm subject to the requirements of 21.11.3.2 and showing an element having confinement as required by 21.11.7.5.*

not difficult to achieve. For buildings where diaphragms could reach their flexural or shear strength before yielding occurs in the vertical seismic systems, designers should consider providing increased diaphragm strength.

**21.11.3 — Seismic load path**

**21.11.3.1** — All diaphragms and their connections shall be proportioned and detailed to provide for a complete transfer of forces to collector elements and to the vertical elements of the seismic-force-resisting system.

**21.11.3.2** — Elements of a structural diaphragm system that are subjected primarily to axial forces and used to transfer diaphragm shear or flexural forces around openings or other discontinuities, shall comply with the requirements for collectors in **21.11.7.5** and **21.11.7.6**.

**21.11.4 — Cast-in-place composite-topping slab diaphragms**

A composite-topping slab cast in place on a precast floor or roof shall be permitted to be used as a structural diaphragm, provided the topping slab is reinforced and the surface of the previously hardened concrete on which the topping slab is placed is clean, free of laitance, and intentionally roughened.

**R21.11.3 — Seismic load path**

**R21.11.3.2** — Section 21.11.3.2 applies to strut-like elements that often are present around openings, diaphragm edges, or other discontinuities in diaphragms. Figure R21.11.3.2 shows an example. Such elements can be subjected to earthquake axial forces in combination with bending and shear from earthquake or gravity loads.

**R21.11.4 — Cast-in-place composite-topping slab diaphragms**

A bonded topping slab is required so that the floor or roof system can provide restraint against slab buckling. Reinforcement is required to ensure the continuity of the shear transfer across precast joints. The connection requirements are introduced to promote a complete system with necessary shear transfers.

**CODE****COMMENTARY****21.11.5 — Cast-in-place topping slab diaphragms**

A cast-in-place noncomposite topping on a precast floor or roof shall be permitted to serve as a structural diaphragm, provided the cast-in-place topping acting alone is proportioned and detailed to resist the design earthquake forces.

**21.11.6 — Minimum thickness of diaphragms**

Concrete slabs and composite topping slabs serving as structural diaphragms used to transmit earthquake forces shall not be less than 2 in. thick. Topping slabs placed over precast floor or roof elements, acting as structural diaphragms and not relying on composite action with the precast elements to resist the design earthquake forces, shall have thickness not less than 2-1/2 in.

**21.11.7 — Reinforcement**

**21.11.7.1** — The minimum reinforcement ratio for structural diaphragms shall be in conformance with **7.12**. Except for post-tensioned slabs, reinforcement spacing each way in floor or roof systems shall not exceed 18 in. Where welded wire reinforcement is used as the distributed reinforcement to resist shear in topping slabs placed over precast floor and roof elements, the wires parallel to the span of the precast elements shall be spaced not less than 10 in. on center. Reinforcement provided for shear strength shall be continuous and shall be distributed uniformly across the shear plane.

**21.11.7.2** — Bonded tendons used as reinforcement to resist collector forces or diaphragm shear or flexural tension shall be proportioned such that the stress due to design earthquake forces does not exceed 60,000 psi. Precompression from unbonded tendons shall be permitted to resist diaphragm design forces if a seismic load path is provided.

**R21.11.5 — Cast-in-place topping slab diaphragms**

Composite action between the topping slab and the precast floor elements is not required, provided that the topping slab is designed to resist the design seismic forces.

**R21.11.6 — Minimum thickness of diaphragms**

The minimum thickness of concrete diaphragms reflects current practice in joist and waffle systems and composite topping slabs on precast floor and roof systems. Thicker slabs are required when the topping slab does not act compositely with the precast system to resist the design seismic forces.

**R21.11.7 — Reinforcement**

**R21.11.7.1** — Minimum reinforcement ratios for diaphragms correspond to the required amount of temperature and shrinkage reinforcement (**7.12**). The maximum spacing for web reinforcement is intended to control the width of inclined cracks. Minimum average prestress requirements (**7.12.3**) are considered to be adequate to limit the crack widths in post-tensioned floor systems; therefore, the maximum spacing requirements do not apply to these systems.

The minimum spacing requirement for welded wire reinforcement in topping slabs on precast floor systems (see **21.11.7.1**) is to avoid fracture of the distributed reinforcement during an earthquake. Cracks in the topping slab open immediately above the boundary between the flanges of adjacent precast members, and the wires crossing those cracks are restrained by the transverse wires.<sup>21.59</sup> Therefore, all the deformation associated with cracking should be accommodated in a distance not greater than the spacing of the transverse wires. A minimum spacing of 10 in. for the transverse wires is required in **21.11.7.1** to reduce the likelihood of fracture of the wires crossing the critical cracks during a design earthquake. The minimum spacing requirements do not apply to diaphragms reinforced with individual bars, because strains are distributed over a longer length.

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**21.11.7.3** — All reinforcement used to resist collector forces, diaphragm shear, or flexural tension shall be developed or spliced for  $f_y$  in tension.

**21.11.7.4** — Type 2 splices are required where mechanical splices are used to transfer forces between the diaphragm and the vertical elements of the seismic-force-resisting system.

**21.11.7.5** — Collector elements with compressive stresses exceeding  $0.2f'_c$  at any section shall have transverse reinforcement satisfying **21.9.6.4(c)** over the length of the element. The specified transverse reinforcement is permitted to be discontinued at a section where the calculated compressive stress is less than  $0.15f'_c$ .

Where design forces have been amplified to account for the overstrength of the vertical elements of the seismic-force-resisting system, the limit of  $0.2f'_c$  shall be increased to  $0.5f'_c$ , and the limit of  $0.15f'_c$  shall be increased to  $0.4f'_c$ .

**21.11.7.6** — Longitudinal reinforcement for collector elements at splices and anchorage zones shall have either:

- (a) A minimum center-to-center spacing of three longitudinal bar diameters, but not less than 1-1/2 in., and a minimum concrete clear cover of two and one-half longitudinal bar diameters, but not less than 2 in.; or
- (b) Transverse reinforcement as required by **11.4.6.3**, except as required in 21.11.7.5.

## 21.11.8 — Flexural strength

Diaphragms and portions of diaphragms shall be designed for flexure in accordance with 10.2 and 10.3 except that the nonlinear distribution of strain requirements of **10.2.2** for deep beams need not apply. The effects of openings shall be considered.

## COMMENTARY

**R21.11.7.3** — Bar development and lap splices are designed according to requirements of **Chapter 12** for reinforcement in tension. Reductions in development or splice length for calculated stresses less than  $f_y$  are not permitted, as indicated in **12.2.5**.

**R21.11.7.5** — In documents such as the NEHRP provisions,<sup>21.4</sup> ASCE/SEI 7,<sup>21.1</sup> the International Building Code,<sup>21.2</sup> and the Uniform Building Code,<sup>21.3</sup> collector elements of diaphragms are designed for forces amplified by a factor,  $\Omega_o$ , to account for the overstrength in the vertical elements of the seismic-force-resisting systems. The amplification factor  $\Omega_o$  ranges between 2 and 3 for most concrete structures, depending on the document selected and on the type of seismic system. In some documents, the factor can be calculated based on the maximum forces that can be developed by the elements of the vertical seismic-force-resisting system.

Compressive stress calculated for the factored forces on a linearly elastic model based on gross section of the structural diaphragm is used as an index value to determine whether confining reinforcement is required. A calculated compressive stress of  $0.2f'_c$  in a member, or  $0.5f'_c$  for forces amplified by  $\Omega_o$ , is assumed to indicate that integrity of the entire structure depends on the ability of that member to resist substantial compressive force under severe cyclic loading. Therefore, transverse reinforcement is required in such members to provide confinement for the concrete and the reinforcement.

**R21.11.7.6** — Section 21.11.7.6 is intended to reduce the possibility of bar buckling and provide adequate bar development conditions in the vicinity of splices and anchorage zones.

## R21.11.8 — Flexural strength

Flexural strength for diaphragms is calculated using the same assumptions as for walls, columns, or beams. The design of diaphragms for flexure and other actions uses the applicable load combinations of **9.2** to consider earthquake forces acting concurrently with gravity or other loads.

**CODE****COMMENTARY**

The influence of slab openings on flexural and shear strength is to be considered, including evaluating the potential critical sections created by the openings. Strut-and-tie models are potentially useful for designing diaphragms with openings.

Earlier design practice assumed design moments for structural diaphragms were resisted entirely by chord forces acting at opposite edges of the diaphragm. This idealization was implicit in earlier versions of the Code, but has been replaced by an approach in which all longitudinal reinforcement, within the limits of 21.11.7, is assumed to contribute to the flexural strength of the diaphragm. This change reduces the required area of longitudinal reinforcement concentrated near the edge of the diaphragm, but should not be interpreted as a requirement to eliminate all boundary reinforcement.

**21.11.9 — Shear strength**

**21.11.9.1** —  $V_n$  of structural diaphragms shall not exceed

$$V_n = A_{cv}(2\lambda\sqrt{f'_c} + \rho_t f_y) \quad (21-10)$$

For cast-in-place topping slab diaphragms on precast floor or roof members,  $A_{cv}$  shall be computed using the thickness of topping slab only for noncomposite topping slab diaphragms and the combined thickness of cast-in-place and precast elements for composite topping slab diaphragms. For composite topping slab diaphragms, the value of  $f'_c$  used to determine  $V_n$  shall not exceed the smaller of  $f'_c$  for the precast members and  $f'_c$  for the topping slab.

**21.11.9.2** —  $V_n$  of structural diaphragms shall not exceed  $8A_{cv}\sqrt{f'_c}$ .

**21.11.9.3** — Above joints between precast elements in noncomposite and composite cast-in-place topping slab diaphragms,  $V_n$  shall not exceed

$$V_n = A_{vf}f_y\mu \quad (21-11)$$

where  $A_{vf}$  is total area of shear friction reinforcement within topping slab, including both distributed and boundary reinforcement, that is oriented perpendicular to joints in the precast system and coefficient of friction,  $\mu$ , is **1.0** $\lambda$ , where  $\lambda$  is given in 11.6.4.3. At least one-half of  $A_{vf}$  shall be uniformly distributed along the length of the potential shear plane. Area of distributed reinforcement in topping slab shall satisfy 7.12.2.1 in each direction.

**R21.11.9 — Shear strength**

The shear strength requirements for diaphragms are similar to those for slender structural walls and are based on the shear provisions for beams. The term  $A_{cv}$  refers to the gross area of the diaphragm, but may not exceed the thickness times the width of the diaphragm. This corresponds to the gross area of the effective deep beam that forms the diaphragm. Distributed slab reinforcement,  $\rho_t$ , used to calculate shear strength of a diaphragm in Eq. (21-10) is positioned perpendicular to the diaphragm flexural reinforcement. Section 21.11.9.2 limits the maximum shear strength of the diaphragm.

In addition to satisfying the provisions in 21.11.9.1 and 21.11.9.2, cast-in-place topping slab diaphragms must also satisfy 21.11.9.3 and 21.11.9.4. Cast-in-place topping slabs on a precast floor or roof system tend to have shrinkage cracks that are aligned with the joints between adjacent precast members. Therefore, the additional shear strength requirements for topping slab diaphragms in 21.11.9.3 are based on a shear friction model,<sup>21.59</sup> and the assumed crack plane corresponds to joints in the precast system along the direction of the applied shear, as shown in Fig. R11.6.4. The coefficient of friction,  $\mu$ , in the shear friction model is taken equal to 1.0 for normalweight concrete due to the presence of these shrinkage cracks.

Both distributed and boundary reinforcement in the topping slab may be considered as shear friction reinforcement,  $A_{vf}$ . Boundary reinforcement within the diaphragm was called chord reinforcement in ACI 318 before 2008. Although the boundary reinforcement also resists flexural forces in the diaphragm, the reduction in the shear friction resistance in the tension zone is offset by the increase in shear friction resistance in the compression zone. Therefore, the area of boundary reinforcement used to resist shear friction need not be added to the area of boundary reinforcement used to resist flexural forces. The distributed topping slab reinforce-

**CODE****COMMENTARY**

**21.11.9.4** — Above joints between precast elements in noncomposite and composite cast-in-place topping slab diaphragms,  $V_n$  shall not exceed the limits in 11.6.5 where  $A_c$  is computed using the thickness of the topping slab only.

ment must contribute at least half of the nominal shear strength. It is assumed that connections between the precast elements do not contribute to the shear strength of the topping slab diaphragm.

Section 21.11.9.4 limits the maximum shear that may be transmitted by shear friction within a topping slab diaphragm.

**21.11.10 — Construction joints**

All construction joints in diaphragms shall conform to 6.4 and contact surfaces shall be roughened as in 11.6.9.

**21.11.11 — Structural trusses**

**21.11.11.1** — Structural truss elements with compressive stresses exceeding  $0.2f'_c$  at any section shall have transverse reinforcement, as given in 21.6.4.2 through 21.6.4.4 and 21.6.4.7, over the length of the element.

**21.11.11.2** — All continuous reinforcement in structural truss elements shall be developed or spliced for  $f_y$  in tension.

**21.12 — Foundations****21.12.1 — Scope**

**21.12.1.1** — Foundations resisting earthquake-induced forces or transferring earthquake-induced forces between structure and ground in structures assigned to SDC D, E, or F shall comply with 21.12 and other applicable Code provisions.

**21.12.1.2** — The provisions in this section for piles, drilled piers, caissons, and slabs-on-ground shall supplement other applicable Code design and construction criteria. See 1.1.6 and 1.1.7.

**21.12.2 — Footings, foundation mats, and pile caps**

**21.12.2.1** — Longitudinal reinforcement of columns and structural walls resisting forces induced by earthquake effects shall extend into the footing, mat, or pile cap, and shall be fully developed for tension at the interface.

**21.12.2.2** — Columns designed assuming fixed-end conditions at the foundation shall comply with 21.12.2.1 and, if hooks are required, longitudinal reinforcement resisting flexure shall have 90-degree hooks near the bottom of the foundation with the free end of the bars oriented toward the center of the column.

**R21.12 — Foundations****R21.12.1 — Scope**

Requirements for foundations supporting buildings assigned to SDC D, E, or F were added to the 1999 Code. They represent a consensus of a minimum level of good practice in designing and detailing concrete foundations including piles, drilled piers, and caissons. It is desirable that inelastic response in strong ground shaking occurs above the foundations, as repairs to foundations can be extremely difficult and expensive.

**R21.12.2 — Footings, foundation mats, and pile caps**

**R21.12.2.2** — Tests<sup>21.60</sup> have demonstrated that flexural members terminating in a footing, slab, or beam (a T-joint) should have their hooks turned inward toward the axis of the member for the joint to be able to resist the flexure in the member forming the stem of the T.

**CODE****COMMENTARY**

**21.12.2.3** — Columns or boundary elements of special structural walls that have an edge within one-half the footing depth from an edge of the footing shall have transverse reinforcement in accordance with 21.6.4.2 through 21.6.4.4 provided below the top of the footing. This reinforcement shall extend into the footing, mat, or pile cap and be developed for  $f_y$  in tension.

**21.12.2.4** — Where earthquake effects create uplift forces in boundary elements of special structural walls or columns, flexural reinforcement shall be provided in the top of the footing, mat, or pile cap to resist actions resulting from the design load combinations, and shall not be less than required by 10.5.

**21.12.2.5** — See 22.10 for use of structural plain concrete in footings and basement walls.

### 21.12.3 — Grade beams and slabs-on-ground

**21.12.3.1** — Grade beams designed to act as horizontal ties between pile caps or footings shall have continuous longitudinal reinforcement that shall be developed within or beyond the supported column or anchored within the pile cap or footing at all discontinuities.

**21.12.3.2** — Grade beams designed to act as horizontal ties between pile caps or footings shall be proportioned such that the smallest cross-sectional dimension shall be equal to or greater than the clear spacing between connected columns divided by 20, but need not be greater than 18 in. Closed ties shall be provided at a spacing not to exceed the lesser of one-half the smallest orthogonal cross-sectional dimension and 12 in.

**21.12.3.3** — Grade beams and beams that are part of a mat foundation subjected to flexure from columns that are part of the seismic-force-resisting system shall conform to 21.5.

**21.12.3.4** — Slabs-on-ground that resist seismic forces from walls or columns that are part of the seismic-force-resisting system shall be designed as structural diaphragms in accordance with 21.11. The contract documents shall clearly state that the slab-on-ground is a structural diaphragm and part of the seismic-force-resisting system.

### 21.12.4 — Piles, piers, and caissons

**21.12.4.1** — Provisions of 21.12.4 shall apply to concrete piles, piers, and caissons supporting structures designed for earthquake resistance.

**R21.12.2.3** — Columns or boundary members supported close to the edge of the foundation, as often occurs near property lines, should be detailed to prevent an edge failure of the footing, pile cap, or mat.

**R21.12.2.4** — The purpose of 21.12.2.4 is to emphasize that top reinforcement should be provided as well as other required reinforcement.

**R21.12.2.5** — Committee 318 recommends that foundation or basement walls be reinforced in buildings assigned to SDC D, E, or F.

### R21.12.3 — Grade beams and slabs-on-ground

For seismic conditions, slabs-on-ground (soil-supported slabs) are often part of the lateral-force-resisting system and should be designed in accordance with this Code as well as other appropriate standards or guidelines. See 1.1.7.

**R21.12.3.2** — Grade beams between pile caps or footings can be separate beams beneath the slab-on-ground or can be a thickened portion of the slab-on-ground. The cross-sectional limitation and minimum tie requirements provide reasonable proportions.

**R21.12.3.3** — Grade beams resisting seismic flexural stresses from column moments should have reinforcement details similar to the beams of the frame above the foundation.

**R21.12.3.4** — Slabs-on-ground often act as a diaphragm to hold the building together at the ground level and minimize the effects of out-of-phase ground motion that may occur over the footprint of the building. In these cases, the slab-on-ground should be adequately reinforced and detailed. The contract documents should clearly state that these slabs-on-ground are structural members so as to prohibit sawcutting of the slab.

### R21.12.4 — Piles, piers, and caissons

Adequate performance of piles and caissons for seismic loadings requires that these provisions be met in addition to other applicable standards or guidelines. See R1.1.6.

**CODE**

**21.12.4.2** — Piles, piers, or caissons resisting tension loads shall have continuous longitudinal reinforcement over the length resisting design tension forces. The longitudinal reinforcement shall be detailed to transfer tension forces within the pile cap to supported structural members.

**21.12.4.3** — Where tension forces induced by earthquake effects are transferred between pile cap or mat foundation and precast pile by reinforcing bars grouted or post-installed in the top of the pile, the grouting system shall have been demonstrated by test to develop at least  $1.25f_y$  of the bar.

**21.12.4.4** — Piles, piers, or caissons shall have transverse reinforcement in accordance with **21.6.4.2** through **21.6.4.4** at locations (a) and (b):

(a) At the top of the member for at least 5 times the member cross-sectional dimension, but not less than 6 ft below the bottom of the pile cap;

(b) For the portion of piles in soil that is not capable of providing lateral support, or in air and water, along the entire unsupported length plus the length required in 21.12.4.4(a).

**21.12.4.5** — For precast concrete driven piles, the length of transverse reinforcement provided shall be sufficient to account for potential variations in the elevation in pile tips.

**21.12.4.6** — Concrete piles, piers, or caissons in foundations supporting one- and two-story stud bearing wall construction are exempt from the transverse reinforcement requirements of 21.12.4.4 and 21.12.4.5.

**21.12.4.7** — Pile caps incorporating batter piles shall be designed to resist the full compressive strength of the batter piles acting as short columns. The slenderness effects of batter piles shall be considered for the portion of the piles in soil that is not capable of providing lateral support, or in air or water.

**COMMENTARY**

**R21.12.4.2** — A load path is necessary at pile caps to transfer tension forces from the reinforcing bars in the column or boundary member through the pile cap to the reinforcement of the pile or caisson.

**R21.12.4.3** — Grouted dowels in a blockout in the top of a precast concrete pile need to be developed, and testing is a practical means of demonstrating strength. Alternatively, reinforcing bars can be cast in the upper portion of the pile, exposed by chipping of concrete and mechanically spliced or welded to an extension.

**R21.12.4.4** — During earthquakes, piles can be subjected to extremely high flexural demands at points of discontinuity, especially just below the pile cap and near the base of a soft or loose soil deposit. The Code requirement for confinement reinforcement at the top of the pile is based on numerous failures observed at this location in earthquakes. Transverse reinforcement is required in this region to provide ductile performance. Possible inelastic action in the pile at abrupt changes in soil deposits should also be considered, such as changes from soft to firm or loose to dense soil layers. Where precast piles are to be used, the potential for the pile tip to be driven to an elevation different than that specified in the contract documents needs to be considered when detailing the pile. If the pile reaches refusal at a shallower depth, a longer length of pile will need to be cut off. If this possibility is not foreseen, the length of transverse reinforcement required by 21.12.4.4 may not be available after the excess pile length is cut off.

**R21.12.4.7** — Extensive structural damage has often been observed at the junction of batter piles and the buildings. The pile cap and surrounding structure should be designed for the potentially large forces that can be developed in batter piles.

**CODE****COMMENTARY****21.13 — Members not designated as part of the seismic-force-resisting system****21.13.1 — Scope**

Requirements of 21.13 apply to members not designated as part of the seismic-force-resisting system in structures assigned to SDC D, E, and F.

**21.13.2 — Members assumed not to contribute to lateral resistance, except two-way slabs without beams and wall piers, shall be detailed according to 21.13.3 or 21.13.4 depending on the magnitude of moments induced in those members when subjected to the design displacement  $\delta_u$ . If effects of  $\delta_u$  are not explicitly checked, it shall be permitted to apply the requirements of 21.13.4. Slab-column connections of two-way slabs without beams shall satisfy the requirements of 21.13.6. Wall piers shall satisfy the requirements of 21.13.7.**

**21.13.3 — Where the induced moments and shears under design displacements,  $\delta_u$ , combined with the factored gravity moments and shears do not exceed the design moment and shear strength of the frame member, the conditions of 21.13.3.1, 21.13.3.2, and 21.13.3.3 shall be satisfied. The gravity load combinations of **(1.2D + 1.0L + 0.2S)** or **0.9D**, whichever is critical, shall be used. The load factor on the live load,  $L$ , shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where  $L$  is greater than 100 lb/ ft<sup>2</sup>.**

**21.13.3.1 — Members with factored gravity axial forces not exceeding  $A_g f'_c / 10$  shall satisfy 21.5.2.1. Stirrups shall be spaced not more than  $d/2$  throughout the length of the member.**

**21.13.3.2 — Members with factored gravity axial forces exceeding  $A_g f'_c / 10$  shall satisfy 21.6.3.1, 21.6.4.2, and 21.6.5. The maximum longitudinal spacing of ties shall be  $s_o$  for the full member length. Spacing  $s_o$  shall not exceed the smaller of six diameters of the smallest longitudinal bar enclosed and 6 in.**

**21.13.3.3 — Members with factored gravity axial forces exceeding  $0.35P_o$  shall satisfy 21.13.3.2 and 21.6.4.7. The amount of transverse reinforcement provided shall be one-half of that required by 21.6.4.4 but shall not be spaced greater than  $s_o$  for the full member length.**

**21.13.4 — If the induced moment or shear under design displacements,  $\delta_u$ , exceeds  $\phi M_n$  or  $\phi V_n$  of the frame member, or if induced moments are not calculated, the conditions of 21.13.4.1, 21.13.4.2, and 21.13.4.3 shall be satisfied.**

**R21.13 — Members not designated as part of the seismic-force-resisting system**

This section applies only to structures assigned to SDC D, E, or F. Model building codes, such as the 2006 IBC, require all structural members not designated as a part of the seismic-force-resisting system to be designed to support gravity loads while subjected to the design displacement. For concrete structures, the provisions of 21.13 satisfy this requirement for columns, beams, slabs, and wall piers of the gravity system. The **design displacement is defined in 2.2**.

The provisions of 21.13 are based on the principle enabling flexural yielding of columns, beams, slabs, and wall piers under the design displacement, by providing sufficient confinement and shear strength in elements that yield. By the provisions of 21.13.2 through 21.13.4 and 21.13.7, columns, beams, and wall piers, respectively, are assumed to yield if the combined effects of factored gravity loads and design displacements exceed the strengths specified in those provisions, or if the effects of design displacements are not calculated. Requirements for transverse reinforcement and shear strength vary with the axial load on the member and whether the member yields under the design displacement.

Models used to determine design displacement of buildings should be chosen to produce results that conservatively bound the values expected during the design earthquake and should include, as appropriate, effects of concrete cracking, foundation flexibility, and deformation of floor and roof diaphragms.

**CODE****COMMENTARY**

**21.13.4.1** — Materials shall satisfy **21.1.4.2**, **21.1.4.3**, **21.1.5.2**, **21.1.5.4**, and **21.1.5.5**. Mechanical splices shall satisfy **21.1.6** and welded splices shall satisfy **21.1.7.1**.

**21.13.4.2** — Members with factored gravity axial forces not exceeding  $A_g f'_c / 10$  shall satisfy **21.5.2.1** and **21.5.4**. Stirrups shall be spaced at not more than  $d/2$  throughout the length of the member.

**21.13.4.3** — Members with factored gravity axial forces exceeding  $A_g f'_c / 10$  shall satisfy **21.6.3**, **21.6.4**, **21.6.5**, and **21.7.3.1**.

**21.13.5** — Precast concrete frame members assumed not to contribute to lateral resistance, including their connections, shall satisfy (a), (b), and (c), in addition to **21.13.2** through **21.13.4**:

- (a) Ties specified in **21.13.3.2** shall be provided over the entire column height, including the depth of the beams;
- (b) Structural integrity reinforcement, as specified in **16.5**, shall be provided; and
- (c) Bearing length at support of a beam shall be at least 2 in. longer than determined from calculations using bearing strength values from **10.14**.

**21.13.6** — For slab-column connections of two-way slabs without beams, slab shear reinforcement satisfying the requirements of **11.11.3** and **11.11.5** and providing  $V_s$  not less than  $3.5 \sqrt{f'_c} b_o d$  shall extend at least four times the slab thickness from the face of the support, unless either (a) or (b) is satisfied:

- (a) The requirements of **11.11.7** using the design shear  $V_{ug}$  and the induced moment transferred between the slab and column under the design displacement;
- (b) The design story drift ratio does not exceed the larger of 0.005 and  $[0.035 - 0.05(V_{ug}/\phi V_c)]$ .

Design story drift ratio shall be taken as the larger of the design story drift ratios of the adjacent stories above and below the slab-column connection.  $V_c$  is defined in **11.11.2**.  $V_{ug}$  is the factored shear force on the slab critical section for two-way action, calculated for the load combination **1.2D + 1.0L + 0.2S**.

The load factor on the live load,  $L$ , shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where  $L$  is greater than 100 lb/ft<sup>2</sup>.

**R21.13.5** — Damage to some buildings with precast concrete gravity systems during the 1994 Northridge earthquake was attributed to several factors addressed in **21.13.5**. Columns should contain ties over their entire height, frame members not proportioned to resist earthquake forces should be tied together, and longer bearing lengths should be used to maintain integrity of the gravity system during shaking. The 2 in. increase in bearing length is based on an assumed 4 percent story drift ratio and 50 in. beam depth, and is considered to be conservative for the ground motions expected for structures assigned to SDC D, E, or F. In addition to the provisions of **21.13.5**, precast frame members assumed not to contribute to lateral resistance should also satisfy **21.13.2** through **21.13.4**, as applicable.

**R21.13.6** — Provisions for shear reinforcement at slab-column connections were added in 2005 to reduce the likelihood of slab punching shear failure. The shear reinforcement is required unless either **21.13.6(a)** or **(b)** is satisfied.

Section 21.13.6(a) requires calculation of shear stress due to the factored shear force and induced moment according to **11.11.7.2**. The induced moment is the moment that is calculated to occur at the slab-column connection when subjected to the design displacement. **Section 13.5.1.2** and the accompanying Commentary provide guidance on selection of the stiffness of the slab-column connection for the purpose of this calculation.

Section 21.13.6(b) does not require the calculation of induced moments, and is based on research<sup>21.61,21.62</sup> that identifies the likelihood of punching shear failure considering the story drift ratio and shear due to gravity loads. **Figure R21.13.6** illustrates the requirement. The requirement can be satisfied by adding slab shear reinforcement, increasing slab thickness, changing the design to reduce the design story drift ratio, or a combination of these.

If column capitals, drop panels, shear caps, or other changes in slab thickness are used, the requirements of **21.13.6** are evaluated at all potential critical sections, as required by **11.11.1.2**.

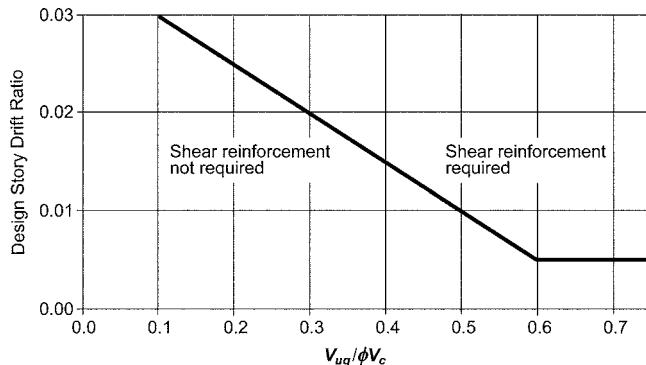
**CODE****COMMENTARY**

Fig. R21.13.6—Illustration of the criterion of 21.13.6(b).

**21.13.7** — Wall piers not designated as part of the seismic-force-resisting system shall satisfy the requirements of 21.9.8. Where the legally adopted general building code includes provisions to account for overstrength of the seismic-force-resisting system, it shall be permitted to determine the design shear force as  $\Omega_o$  times the shear induced under design displacements,  $\delta_u$ .

**R21.13.7** — Section 21.9.8 requires that the design shear force be determined according to 21.6.5.1, which in some cases may result in unrealistically large forces. As an alternative, the design shear force can be determined as the product of an overstrength factor and the shear induced when the wall pier is displaced by  $\delta_u$ . The overstrength factor  $\Omega_o$  included in documents such as the NEHRP provisions,<sup>21.4</sup> ASCE/SEI 7,<sup>21.1</sup> and the International Building Code<sup>21.2</sup> can be used for this purpose.

# CHAPTER 22 — STRUCTURAL PLAIN CONCRETE

## CODE

### 22.1 — Scope

**22.1.1** — This chapter provides minimum requirements for design and construction of structural plain concrete members (cast-in-place or precast).

**22.1.2** — Unless in conflict with the provisions of Chapter 22, the following provisions of this Code shall apply to structural plain concrete members: **Sections 1.1 through 7.5, 7.6.1, 7.6.2, 7.6.4, 7.7, 9.1.3, 9.2, 9.3.5, Chapter 20, 21.12.2.5, C.9.2, C.9.3.5, and Appendix D.**

**22.1.3** — For unusual structures, such as arches, underground utility structures, gravity walls, and shielding walls, provisions of this chapter shall govern where applicable.

## COMMENTARY

### R22.1 — Scope

Before the 1995 Code, requirements for plain concrete were set forth in “**Building Code Requirements for Structural Plain Concrete (ACI 318.1-89) (Revised 1992)**.” Requirements for plain concrete are now in this Code (see **2.2 for definition of plain concrete**). Limitations are provided in **22.2** to clarify the scope and applicability of this chapter. See **1.1.4** for requirements for residential construction within the scope of ACI 332.

**R22.1.2** — Sections of the Code do not apply to the design of structural plain concrete for the following specific reasons:

**Sections 7.6 and 7.8 through 7.13** — These sections contain requirements intended for reinforced concrete members and, with the exception of **7.6.1, 7.6.2, and 7.6.4**, are not applicable to structural plain concrete members, for which strength is determined only by member size and concrete strength, and not by reinforcement (see **22.5**). Concrete cover requirements of **7.7** apply to structural plain concrete members if reinforcement is provided.

**Chapters 8 through 19** — These chapters of the Code contain general and specific design requirements for reinforced and prestressed concrete members, including walls and foundations, which are not applicable to structural plain concrete, except for **9.1.3, 9.2, and 9.3.5** where applicable load factors and strength reduction factors for structural plain concrete are specified.

**Chapter 21** — This chapter contains seismic design requirements for reinforced concrete members and is not applicable to structural plain concrete except in the sections of the chapter where structural plain concrete is specifically addressed.

**Appendices A and B** — **Appendices A and B** are intended for reinforced concrete members and are not applicable to structural plain concrete.

**CODE****COMMENTARY****22.2 — Limitations**

**22.2.1** — Use of structural plain concrete shall be limited to (a), (b), or (c):

- (a) Members that are continuously supported by soil or supported by other structural members capable of providing continuous vertical support;
- (b) Members for which arch action provides compression under all conditions of loading;
- (c) Walls and pedestals. See [22.6](#) and [22.8](#).

The use of structural plain concrete columns shall not be permitted.

**22.2.2** — This chapter shall not govern design and installation of cast-in-place concrete piles and piers embedded in ground.

**22.2.3 — Minimum specified strength**

Specified compressive strength of structural plain concrete shall not be less than the larger of that given in [1.1.1](#) and that required for durability in [Chapter 4](#).

**22.2.4 — Lightweight concrete**

Modification factor  $\lambda$  for lightweight concrete in this Chapter shall be in accordance with [8.6.1](#).

**22.3 — Joints**

**22.3.1** — Contraction or isolation joints shall be provided to divide structural plain concrete members into flexurally discontinuous elements. The size of each element shall be chosen to limit stress caused by restraint to movements from creep, shrinkage, and temperature effects.

**R22.2 — Limitations**

**R22.2.1 and R22.2.2** — Because the strength and structural integrity of structural plain concrete members is based solely on the member size, concrete strength, and other concrete properties, use of structural plain concrete should be limited to members that are primarily in a state of compression, members that can tolerate random cracks without detriment to their structural integrity, and members where ductility is not an essential feature of design. The tensile strength of concrete can be used in design of structural plain concrete members. Tensile stresses due to restraint from creep, shrinkage, or temperature effects are to be considered to avoid uncontrolled cracks or structural failure.

Plain concrete walls are permitted (see [22.6](#)) without a height limitation. However, for multistory construction and other major structures, ACI Committee 318 encourages the use of walls designed in accordance with [Chapter 14](#) (see [R22.6](#)).

Since plain concrete lacks the necessary ductility that columns should possess and because a random crack in an unreinforced column will most likely endanger its structural integrity, the Code does not permit use of plain concrete for columns. It does allow its use for pedestals limited to a ratio of unsupported height to least lateral dimension of 3 or less (see [22.8.2](#)).

Structural elements such as cast-in-place concrete piles and piers in ground or other material sufficiently stiff to provide adequate lateral support to prevent buckling are not covered by this Code. Such elements are covered by the general building code.

**R22.2.3 — Minimum specified strength**

A minimum specified compressive strength requirement for structural plain concrete construction is considered necessary because performance and safety is based solely on strength and quality of concrete treated as a homogeneous material. Lean concrete mixtures may not produce adequately homogeneous material or well-formed surfaces.

**R22.3 — Joints**

Joints in plain concrete construction are an important design consideration. In reinforced concrete, reinforcement is provided to resist the stresses due to restraint of creep, shrinkage, and temperature effects. In plain concrete, joints are the only means of controlling and, thereby, relieving the buildup of such tensile stresses. A plain concrete member

**CODE**

**22.3.2** — In determining the number and location of contraction or isolation joints, consideration shall be given to: influence of climatic conditions; selection and proportioning of materials; mixing, placing, and curing of concrete; degree of restraint to movement; stresses due to loads to which an element is subject; and construction techniques.

**COMMENTARY**

should therefore be small enough, or divided into smaller elements by joints, to control the buildup of internal stresses. The joint may be a contraction joint or an isolation joint. A minimum 25 percent reduction of member thickness is considered sufficient for contraction joints to be effective. The jointing should be such that no axial tension or flexural tension can be developed across a joint after cracking, if applicable—a condition referred to as flexural discontinuity. Where random cracking due to creep, shrinkage, and temperature effects will not affect the structural integrity, and is otherwise acceptable, such as transverse cracks in a continuous wall footing, transverse contraction, or isolation joints are not necessary.

**22.4 — Design method**

**22.4.1** — Factored loads and forces shall be in combinations as in **9.2**.

**22.4.2** — Where required strength exceeds design strength, reinforcement shall be provided and the member designed as a reinforced concrete member in accordance with appropriate design requirements of this Code.

**22.4.3** — Strength design of structural plain concrete members for flexure and axial loads shall be based on a linear stress-strain relationship in both tension and compression.

**22.4.4** — Tensile strength of concrete shall be permitted to be considered in design of plain concrete members when provisions of **22.3** have been followed.

**22.4.5** — No strength shall be assigned to steel reinforcement that may be present.

**22.4.6** — Tension shall not be transmitted through outside edges, construction joints, contraction joints, or isolation joints of an individual plain concrete element. No flexural continuity due to tension shall be assumed between adjacent structural plain concrete elements.

**22.4.7** — When computing strength in flexure, combined flexure and axial load, and shear, the entire cross section of a member shall be considered in design, except for concrete cast against soil where overall thickness ***h*** shall be taken as 2 in. less than actual thickness.

**R22.4 — Design method**

Plain concrete members are proportioned for adequate strength using factored loads and forces. When the design strength is exceeded, the section should be increased or the specified strength of concrete increased, or both, or the member designed as a reinforced concrete member in accordance with the Code. An increase in concrete section may have a detrimental effect; stress due to load will decrease but stresses due to creep, shrinkage, and temperature effects may increase.

**R22.4.3** — Flexural tension may be considered in design of plain concrete members to sustain loads, provided the computed stress does not exceed the permissible stress, and construction, contraction, or isolation joints are provided to relieve the resulting tensile stresses due to restraint of creep, temperature, and shrinkage effects.

**R22.4.7** — The reduced overall thickness ***h*** for concrete cast against earth is to allow for unevenness of excavation and for some contamination of the concrete adjacent to the soil.

**CODE****COMMENTARY****22.5 — Strength design**

**22.5.1** — Design of cross sections subject to flexure shall be based on

$$\phi M_n \geq M_u \quad (22-1)$$

where

$$M_n = 5\lambda \sqrt{f'_c} S_m \quad (22-2)$$

if tension controls, and

$$M_n = 0.85 f'_c S_m \quad (22-3)$$

if compression controls, where  $S_m$  is the corresponding elastic section modulus.

**22.5.2** — Design of cross sections subject to compression shall be based on

$$\phi P_n \geq P_u \quad (22-4)$$

where  $P_n$  is computed by

$$P_n = 0.60 f'_c \left[ 1 - \left( \frac{\ell_c}{32h} \right)^2 \right] A_1 \quad (22-5)$$

and  $A_1$  is the loaded area.

**22.5.3** — Members subject to combined flexure and axial load in compression shall be proportioned such that on the compression face:

$$P_u / \phi P_n + M_u / \phi M_n \leq 1 \quad (22-6)$$

and on the tension face

$$M_u / S_m - P_u / A_g \leq 5\phi \lambda \sqrt{f'_c} \quad (22-7)$$

**22.5.4** — Design of rectangular cross sections subject to shear shall be based on

$$\phi V_n \geq V_u \quad (22-8)$$

where  $V_n$  is computed by

$$V_n = 4/3 \lambda \sqrt{f'_c} b_w h \quad (22-9)$$

**R22.5 — Strength design**

**R22.5.2** — Equation (22-5) is presented to reflect the general range of braced and restrained end conditions encountered in structural plain concrete elements. The effective length factor was omitted as a modifier of  $\ell_c$ , the vertical distance between supports, because this is conservative for walls with assumed pin supports that are required to be braced against lateral translation as in [22.6.6.4](#).

**R22.5.3** — Plain concrete members subject to combined flexure and axial compressive load are proportioned such that on the compression face

$$\frac{P_u}{0.60 \phi f'_c \left[ 1 - \left( \frac{\ell_c}{32h} \right)^2 \right] A_1} + \frac{M_u}{0.85 \phi f'_c S_m} \leq 1$$

and that on the tension face

$$\left( \frac{\text{Calculated bending stress}}{\text{axial stress}} \right) - \left( \frac{\text{Calculated axial stress}}{\text{axial stress}} \right) \leq 5\phi \lambda \sqrt{f'_c}$$

**R22.5.4** — Proportions of plain concrete members usually are controlled by tensile strength rather than shear strength. Shear stress (as a substitute for principal tensile stress) rarely will control. However, because it is difficult to foresee all possible conditions where shear may have to be investigated (such as shear keys), Committee 318 maintains the investigation of this basic stress condition.

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for beam action and by

$$V_n = \left[ \frac{4}{3} + \frac{8}{3\beta} \right] \lambda \sqrt{f'_c} b_o h \quad (22-10)$$

for two-way action, but not greater than  $2.66\lambda\sqrt{f'_c}b_oh$ . In Eq. (22-10),  $\beta$  corresponds to ratio of long side to short side of concentrated load or reaction area.

**COMMENTARY**

The shear requirements for plain concrete assume an uncracked section. Shear failure in plain concrete will be a diagonal tension failure, occurring when the principal tensile stress near the centroidal axis becomes equal to the tensile strength of the concrete. Since the major portion of the principal tensile stress comes from the shear, the Code safeguards against tensile failure by limiting the permissible shear at the centroidal axis as calculated from the equation for a section of homogeneous material:

$$v = VQ/Ib$$

where  $v$  and  $V$  are the shear stress and shear force, respectively, at the section considered;  $Q$  is the statical moment of the area outside the section being considered about centroidal axis of the gross section;  $I$  is the moment of inertia of the gross section; and  $b$  is the width where shear stress is being computed.

**22.5.5** — Design of bearing areas subject to compression shall be based on

$$\phi B_n \geq B_u \quad (22-11)$$

where  $B_u$  is factored bearing load and  $B_n$  is nominal bearing strength of loaded area  $A_1$  calculated by

$$B_n = 0.85f'_c A_1 \quad (22-12)$$

except where the supporting surface is wider on all sides than the loaded area, then  $B_n$  shall be multiplied by  $\sqrt{A_2/A_1}$  but not more than 2.

**22.6 — Walls**

**22.6.1** — Structural plain concrete walls shall be continuously supported by soil, footings, foundation walls, grade beams, or other structural members capable of providing continuous vertical support.

**22.6.2** — Structural plain concrete walls shall be designed for vertical, lateral, and other loads to which they are subjected.

**22.6.3** — Structural plain concrete walls shall be designed for an eccentricity corresponding to the maximum moment that can accompany the axial load but not less than **0.10h**. If the resultant of all factored loads is located within the middle third of the overall wall thickness, the design shall be in accordance with **22.5.3** or **22.6.5**. Otherwise, walls shall be designed in accordance with **22.5.3**.

**22.6.4** — Design for shear shall be in accordance with **22.5.4**.

**R22.6 — Walls**

Plain concrete walls are commonly used for basement wall construction for residential and light commercial buildings in low or nonseismic areas. Although the Code imposes no absolute maximum height limitation on the use of plain concrete walls, experience with use of plain concrete in relatively minor structures should not be extrapolated to using plain concrete walls in multistory construction and other major structures where differential settlement, wind, earthquake, or other unforeseen loading conditions require the walls to possess some ductility and ability to maintain their integrity when cracked. For such conditions, ACI Committee 318 strongly encourages the use of walls designed in accordance with [Chapter 14](#).

The provisions for plain concrete walls are applicable only for walls laterally supported in such a manner as to prohibit relative lateral displacement at top and bottom of individual wall elements (see **22.6.6.4**). The Code does not cover walls without horizontal support to prohibit relative displacement at top and bottom of wall elements. Such laterally unsupported walls are to be designed as reinforced concrete members in accordance with the Code.

**CODE****COMMENTARY****22.6.5 — Empirical design method**

**22.6.5.1** — Structural plain concrete walls of solid rectangular cross section shall be permitted to be designed by Eq. (22-13) if the resultant of all factored loads is located within the middle-third of the overall thickness of wall.

**22.6.5.2** — Design of walls subject to axial loads in compression shall be based on

$$\phi P_n \geq P_u \quad (22-13)$$

where  $P_u$  is factored axial force and  $P_n$  is nominal axial strength calculated by

$$P_n = 0.45 f'_c A_g \left[ 1 - \left( \frac{\ell_c}{32h} \right)^2 \right] \quad (22-14)$$

**22.6.6 — Limitations**

**22.6.6.1** — Unless demonstrated by a detailed analysis, horizontal length of wall to be considered effective for each vertical concentrated load shall not exceed center-to-center distance between loads, nor width of bearing plus four times the wall thickness.

**22.6.6.2** — Except as provided in 22.6.6.3, thickness of bearing walls shall be not less than 1/24 the unsupported height or length, whichever is shorter, nor less than 5-1/2 in.

**22.6.6.3** — Thickness of exterior basement walls and foundation walls shall be not less than 7-1/2 in.

**22.6.6.4** — Walls shall be braced against lateral translation. See [22.3](#) and [22.4.7](#).

**22.6.6.5** — Not less than two No. 5 bars shall be provided around all window and door openings. Such bars shall extend at least 24 in. beyond the corners of openings.

**22.7 — Footings**

**22.7.1** — Structural plain concrete footings shall be designed for factored loads and induced reactions in accordance with appropriate design requirements of this Code and as provided in 22.7.2 through [22.7.8](#).

**22.7.2** — Base area of footing shall be determined from unfactored forces and moments transmitted by footing to soil and permissible soil pressure selected through principles of soil mechanics.

**R22.6.5 — Empirical design method**

When the resultant load falls within the middle-third of the wall thickness (kern of wall section), plain concrete walls may be designed using the simplified Eq. (22-14). Eccentric loads and lateral forces are used to determine the total eccentricity of the factored axial force  $P_u$ . If the eccentricity does not exceed  $h/6$ , Eq. (22-14) may be applied, and design performed considering  $P_u$  as a concentric load. The factored axial load  $P_u$  should not exceed the design axial strength  $\phi P_n$ . Equation (22-14) reflects the range of braced and restrained end conditions encountered in wall design. The limitations of 22.6.6 apply whether the wall is proportioned by [22.5.3](#) or by the empirical method of 22.6.5.

**R22.7 — Footings**

**CODE****COMMENTARY**

**22.7.3** — Plain concrete shall not be used for footings on piles.

**22.7.4** — Thickness of structural plain concrete footings shall not be less than 8 in. See **22.4.7**.

**22.7.5** — Maximum factored moment shall be computed at (a), (b), and (c):

- (a) At the face of the column, pedestal, or wall, for footing supporting a concrete column, pedestal, or wall;
- (b) Halfway between center and face of the wall, for footing supporting a masonry wall;
- (c) Halfway between face of column and edge of steel base plate, for footing supporting a column with steel base plate.

**22.7.6 — Shear in plain concrete footings**

**22.7.6.1** —  $V_u$  shall be computed in accordance with 22.7.6.2, with location of critical section measured from face of column, pedestal, or wall for footing supporting a column, pedestal, or wall. For footing supporting a column with steel base plates, the critical section shall be measured at location defined in 22.7.5(c).

**22.7.6.2** —  $\phi V_n$  of structural plain concrete footings in the vicinity of concentrated loads or reactions shall be governed by the more severe of two conditions:

- (a) Beam action for footing, with a critical section extending in a plane across the entire footing width and located at a distance  $h$  from face of concentrated load or reaction area. For this condition, the footing shall be designed in accordance with **Eq. (22-9)**;
- (b) Two-way action for footing, with a critical section perpendicular to plane of footing and located so that its perimeter  $b_o$  is a minimum, but need not approach closer than  $h/2$  to perimeter of concentrated load or reaction area. For this condition, the footing shall be designed in accordance with **Eq. (22-10)**.

**22.7.7** — Circular or regular polygon-shaped concrete columns or pedestals shall be permitted to be treated as square members with the same area for location of critical sections for moment and shear.

**R22.7.4** — Thickness of plain concrete footings will be controlled by flexural strength (extreme fiber stress in tension not greater than  $5\phi\lambda\sqrt{f'_c}$ ) rather than shear strength for the usual proportions of plain concrete footings. Shear rarely will control (see **R22.5.4**). For footings cast against soil, overall thickness  $h$  used for strength computations is specified in **22.4.7**.

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**22.7.8** — Factored bearing load,  $B_u$ , on concrete at contact surface between supporting and supported member shall not exceed design bearing strength,  $\phi B_n$ , for either surface as given in 22.5.5.

**22.8 — Pedestals**

**22.8.1** — Plain concrete pedestals shall be designed for vertical, lateral, and other loads to which they are subjected.

**22.8.2** — Ratio of unsupported height to average least lateral dimension of plain concrete pedestals shall not exceed 3.

**22.8.3** — Maximum factored axial load,  $P_u$ , applied to plain concrete pedestals shall not exceed design bearing strength,  $\phi B_n$ , given in 22.5.5.

**22.9 — Precast members**

**22.9.1** — Design of precast plain concrete members shall consider all loading conditions from initial fabrication to completion of the structure, including form removal, storage, transportation, and erection.

**22.9.2** — Limitations of 22.2 apply to precast members of plain concrete not only to the final condition but also during fabrication, transportation, and erection.

**22.9.3** — Precast members shall be connected securely to transfer all lateral forces into a structural system capable of resisting such forces.

**22.9.4** — Precast members shall be adequately braced and supported during erection to ensure proper alignment and structural integrity until permanent connections are completed.

**22.10 — Plain concrete in earthquake-resisting structures**

**22.10.1** — Structures assigned to Seismic Design Category D, E, or F shall not have foundation elements of structural plain concrete, except as follows:

- (a) For detached one- and two-family dwellings three stories or less in height and constructed with stud bearing walls, plain concrete footings without longitudinal reinforcement supporting walls and isolated plain concrete footings supporting columns or pedestals are permitted;
- (b) For all other structures, plain concrete footings supporting cast-in-place reinforced concrete or

**R22.8 — Pedestals**

The height-thickness limitation for plain concrete pedestals does not apply for portions of pedestals embedded in soil capable of providing lateral restraint.

**R22.9 — Precast members**

Precast structural plain concrete members are subject to all limitations and provisions for cast-in-place concrete contained in this chapter.

The approach to contraction or isolation joints is expected to be somewhat different than for cast-in-place concrete since the major portion of shrinkage stresses takes place prior to erection. To ensure stability, precast members should be connected to other members. The connection should transfer no tension.

**CODE****COMMENTARY**

reinforced masonry walls are permitted provided the footings are reinforced longitudinally with not less than two continuous reinforcing bars. Bars shall not be smaller than No. 4 and shall have a total area of not less than 0.002 times the gross cross-sectional area of the footing. Continuity of reinforcement shall be provided at corners and intersections;

(c) For detached one- and two-family dwellings three stories or less in height and constructed with stud bearing walls, plain concrete foundations or basement walls are permitted provided the wall is not less than 7-1/2 in. thick and retains no more than 4 ft of unbalanced fill.

**Notes**

# APPENDIX A — STRUT-AND-TIE MODELS

## CODE

### A.1 — Definitions

**B-region** — A portion of a member in which the plane sections assumption of flexure theory from 10.2.2 can be applied.

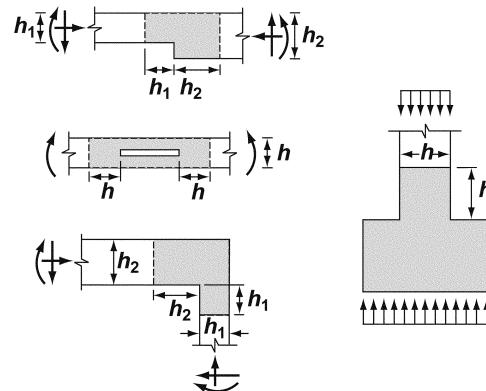
**Discontinuity** — An abrupt change in geometry or loading.

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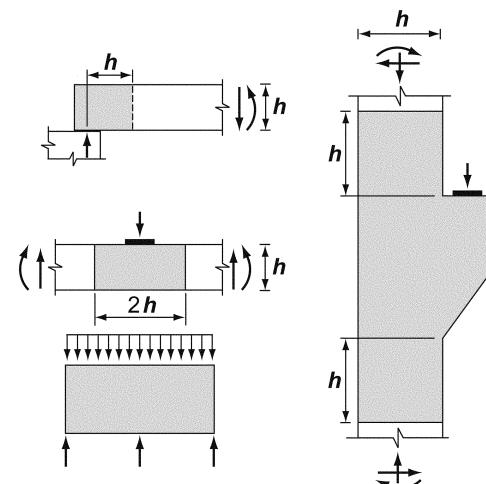
### RA.1 — Definitions

**B-region** — In general, any portion of a member outside of a D-region is a B-region.

**Discontinuity** — A discontinuity in the stress distribution occurs at a change in the geometry of a structural element or at a concentrated load or reaction. St. Venant's principle indicates that the stresses due to axial load and bending approach a linear distribution at a distance approximately equal to the overall height of the member,  $h$ , away from the discontinuity. For this reason, discontinuities are assumed to extend a distance  $h$  from the section where the load or change in geometry occurs. Figure RA.1.1(a) shows typical geometric discontinuities, and Fig. RA.1.1(b) shows combined geometrical and loading discontinuities.



(a) Geometric discontinuities



(b) Loading and geometric discontinuities

Fig. RA.1.1—D-regions and discontinuities.

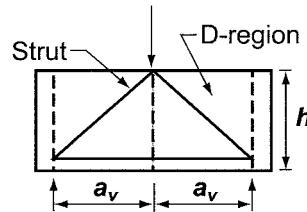
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**D-region** — The portion of a member within a distance,  $h$ , from a force discontinuity or a geometric discontinuity.

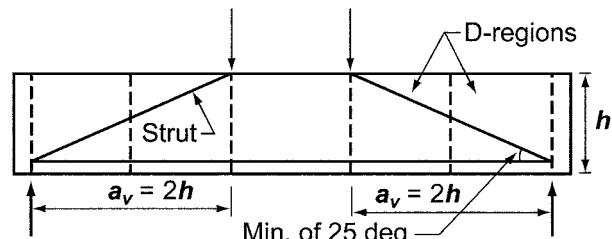
**D-region** — The shaded regions in Fig. RA.1.1(a) and (b) show typical D-regions.<sup>A.1</sup> The plane sections assumption of 10.2.2 is not applicable in such regions.

Each shear span of the beam in Fig. RA.1.2(a) is a D-region. If two D-regions overlap or meet as shown in Fig. RA.1.2(b), they can be considered as a single D-region for design purposes. The maximum length-to-depth ratio of such a D-region would be approximately 2. Thus, the smallest angle between the strut and the tie in a D-region is  $\arctan 1/2 = 26.5$  degrees, rounded to 25 degrees.

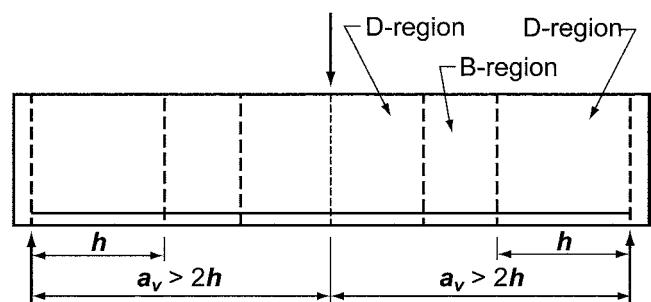
If there is a B-region between the D-regions in a shear span, as shown in Fig. RA.1.2(c), the strength of the shear span is governed by the strength of the B-region if the B- and D-regions have similar geometry and reinforcement.<sup>A.2</sup> This is because the shear strength of a B-region is less than the shear strength of a comparable D-region. Shear spans



(a) Shear span,  $a_v < 2h$ , deep beam



(b) Shear span,  $a_v = 2h$ , limit for a deep beam



(c) Shear span,  $a_v > 2h$ , slender beam

Fig. RA.1.2—Description of deep and slender beams.

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**Deep beam** — See 10.7.1 and 11.7.1.

**Nodal zone** — The volume of concrete around a node that is assumed to transfer strut-and-tie forces through the node.

containing B-regions—the usual case in beam design—are designed for shear using the traditional shear design procedures from 11.1 through 11.4 ignoring D-regions.

**Deep beam** — See Fig. RA.1.2(a), RA.1.2(b), and RA.1.3, and Sections 10.7 and 11.7.

**Nodal zone** — Historically, hydrostatic nodal zones as shown in Fig. RA.1.4 were used. These were largely superseded by what are called extended nodal zones, shown in Fig. RA.1.5.

A **hydrostatic nodal zone** has loaded faces perpendicular to the axes of the struts and ties acting on the node and has equal stresses on the loaded faces. Figure RA.1.4(a) shows a **C-C-C** nodal zone. If the stresses on the face of the nodal zone are the same in all three struts, the ratios of the lengths of the sides of the nodal zone,  $w_{n1}: w_{n2}: w_{n3}$  are in the same proportions as the three forces  $C_1: C_2: C_3$ . The faces of a hydrostatic nodal zone are perpendicular to the axes of the struts and ties acting on the nodal zone.

These nodal zones are called hydrostatic nodal zones because the in-plane stresses are the same in all directions. Strictly speaking, this terminology is incorrect because the in-plane stresses are not equal to the out-of-plane stresses.

A **C-C-T** nodal zone can be represented as a hydrostatic nodal zone if the tie is assumed to extend through the node to be anchored by a plate on the far side of the node, as shown in Fig. RA.1.4(b), provided that the size of the plate results in bearing stresses that are equal to the stresses in the struts. The bearing plate on the left side of Fig. RA.1.4(b) is used to represent an actual tie anchorage. The tie force can be anchored by a plate, or through development of straight or hooked bars, as shown in Fig. RA.1.4(c).

The shaded areas in Fig. RA.1.5(a) and (b) are extended nodal zones. An **extended nodal zone** is that portion of a member bounded by the intersection of the effective strut width,  $w_s$ , and the effective tie width,  $w_t$  (see RA.4.2).

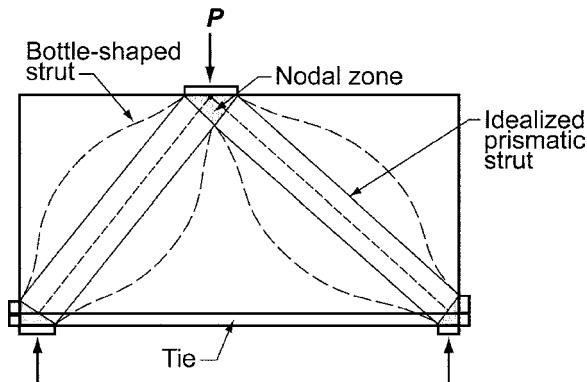


Fig. RA.1.3—Description of strut-and-tie model.

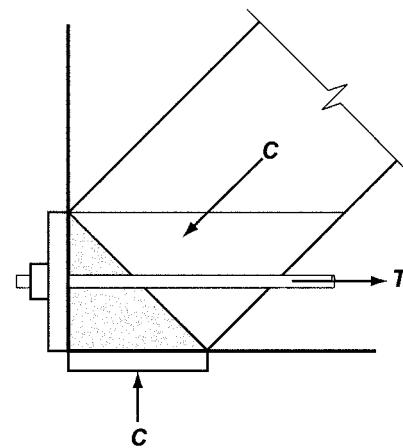
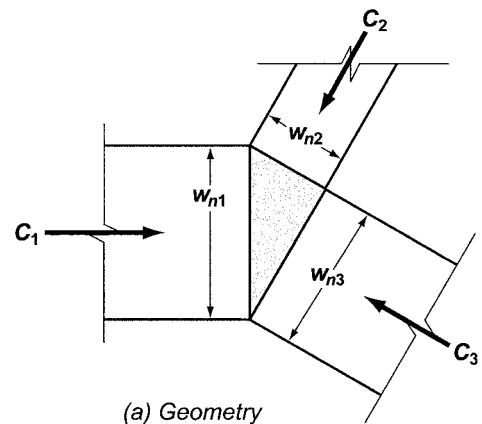
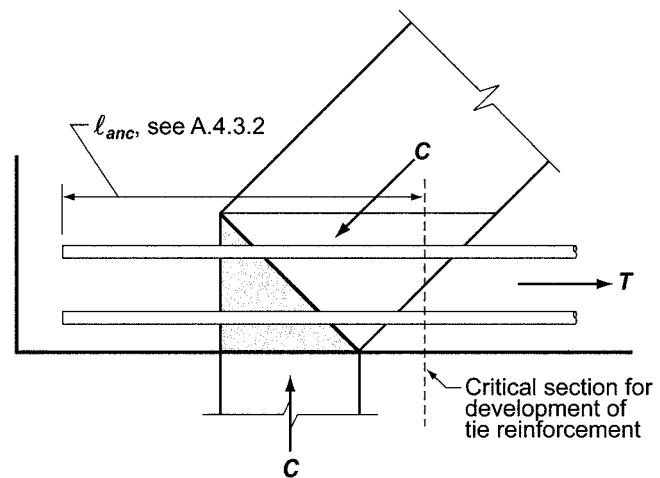
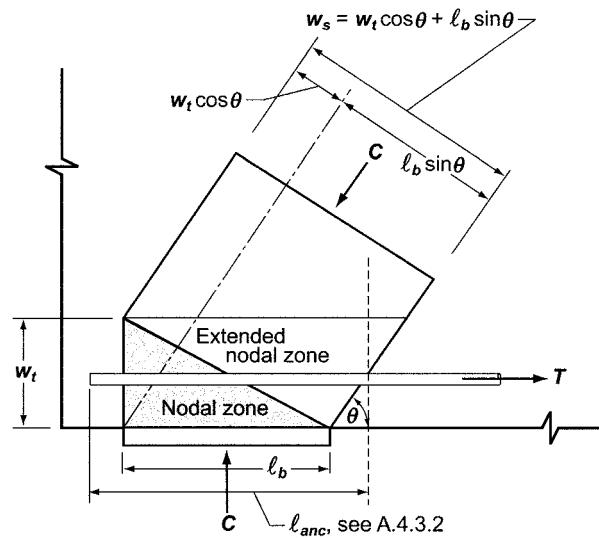
**CODE****COMMENTARY**(b) *Tension force anchored by a plate*(c) *Tension force anchored by bond*

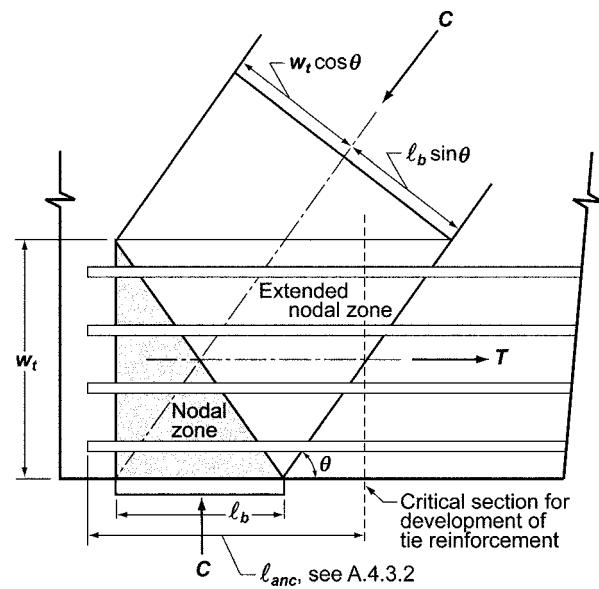
Fig. RA.1.4—Hydrostatic nodes.

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In the nodal zone shown in Fig. RA.1.6(a), the reaction  $R$  equilibrates the vertical components of the forces  $C_1$  and  $C_2$ . Frequently, calculations are easier if the reaction  $R$  is divided into  $R_1$ , which equilibrates the vertical component of  $C_1$  and  $R_2$ , which equilibrates the vertical component of the force  $C_2$ , as shown in Fig. RA1.6(b).



(a) One layer of steel



(b) Distributed steel

*Fig. RA.1.5—Extended nodal zone showing the effect of the distribution of the force.*

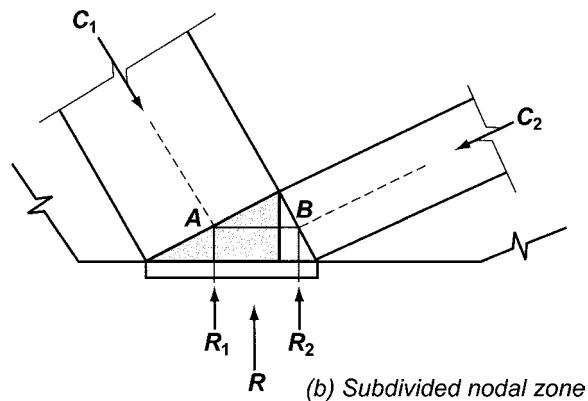
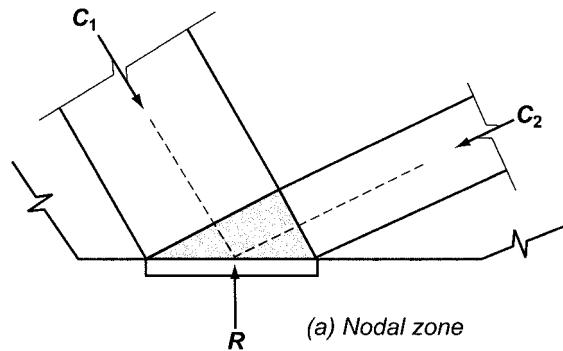
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Fig. RA.1.6—Subdivision of nodal zone.

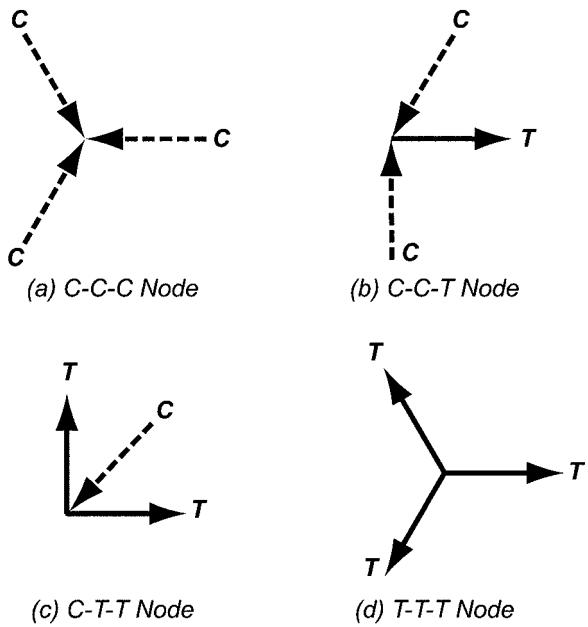


Fig. RA.1.7—Classification of nodes.

**Node** — The point in a joint in a strut-and-tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect.

**Node** — For equilibrium, at least three forces should act on a node in a strut-and-tie model, as shown in Fig. RA.1.7. Nodes are classified according to the signs of these forces. A **C-C-C** node resists three compressive forces, a **C-C-T** node resists two compressive forces and one tensile force, and so on.

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**Strut** — A compression member in a strut-and-tie model. A strut represents the resultant of a parallel or a fan-shaped compression field.

**Strut** — In design, struts are usually idealized as prismatic compression members, as shown by the straight line outlines of the struts in Fig. RA.1.2 and RA.1.3. If the effective compression strength  $f_{ce}$  differs at the two ends of a strut, due either to different nodal zone strengths at the two ends, or to different bearing lengths, the strut is idealized as a uniformly tapered compression member.

**Bottle-shaped strut** — A strut that is wider at mid-length than at its ends.

**Bottle-shaped struts** — A bottle-shaped strut is a strut located in a part of a member where the width of the compressed concrete at midlength of the strut can spread laterally.<sup>A.1,A.3</sup> The curved dashed outlines of the struts in Fig. RA.1.3 and the curved solid outlines in Fig. RA.1.8 approximate the boundaries of bottle-shaped struts. A split cylinder test is an example of a bottle-shaped strut. The internal lateral spread of the applied compression force in such a test leads to a transverse tension that splits the specimen.

To simplify design, bottle-shaped struts are idealized either as prismatic or as uniformly tapered, and crack-control reinforcement from A.3.3 is provided to resist the transverse tension. The amount of confining transverse reinforcement can be computed using the strut-and-tie model shown in Fig. RA.1.8(b) with the struts that represent the spread of the compression force acting at a slope of 1:2 to the axis of the applied compressive force. Alternatively for  $f'_c$  not exceeding 6000 psi, Eq. (A-4) can be used. The cross-sectional area  $A_c$  of a bottle-shaped strut is taken as the smaller of the cross-sectional areas at the two ends of the strut. See Fig. RA.1.8(a).

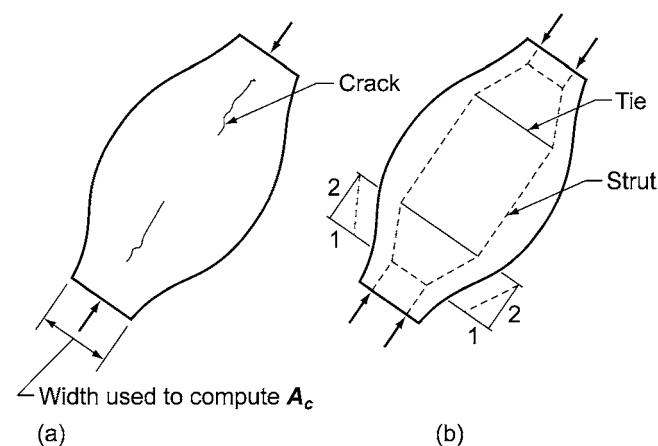


Fig. RA.1.8—Bottle-shaped strut: (a) cracking of a bottle-shaped strut; and (b) strut-and-tie model of a bottle-shaped strut.

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**Strut-and-tie model** — A truss model of a structural member, or of a D-region in such a member, made up of struts and ties connected at nodes, capable of transferring the factored loads to the supports or to adjacent B-regions.

**Tie** — A tension member in a strut-and-tie model.

## A.2 — Strut-and-tie model design procedure

**A.2.1** — It shall be permitted to design structural concrete members, or D-regions in such members, by modeling the member or region as an idealized truss. The truss model shall contain struts, ties, and nodes as defined in A.1. The truss model shall be capable of transferring all factored loads to the supports or adjacent B-regions.

**A.2.2** — The strut-and-tie model shall be in equilibrium with the applied loads and the reactions.

**A.2.3** — In determining the geometry of the truss, the dimensions of the struts, ties, and nodal zones shall be taken into account.

**Strut-and-tie model** — The components of a strut-and-tie model of a single-span deep beam loaded with a concentrated load are identified in Fig. RA.1.3. The cross-sectional dimensions of a strut or tie are designated as thickness and width, both perpendicular to the axis of the strut or tie. Thickness is perpendicular to the plane of the truss model, and width is in the plane of the truss model.

**Tie** — A tie consists of reinforcement or prestressing steel plus a portion of the surrounding concrete that is concentric with the axis of the tie. The surrounding concrete is included to define the zone in which the forces in the struts and ties are to be anchored. The concrete in a tie is not used to resist the axial force in the tie. Although not considered in design, the surrounding concrete will reduce the elongations of the tie, especially at service loads.

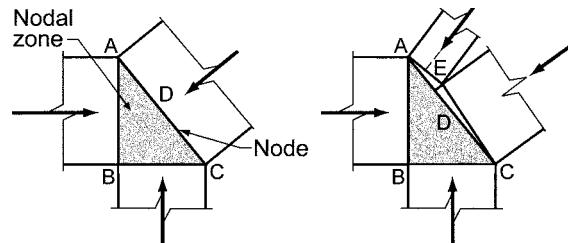
## RA.2 — Strut-and-tie model design procedure

**RA.2.1** — The truss model described in A.2.1 is referred to as a strut-and-tie model. Details of the use of strut-and-tie models are given in References A.1 through A.7. The design of a D-region includes the following four steps:

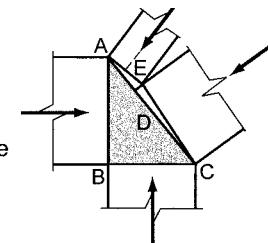
1. Define and isolate each D-region;
2. Compute resultant forces on each D-region boundary;
3. Select a truss model to transfer the resultant forces across the D-region. The axes of the struts and ties, respectively, are chosen to approximately coincide with the axes of the compression and tension fields. The forces in the struts and ties are computed.
4. The effective widths of the struts and nodal zones are determined considering the forces from Step 3 and the effective concrete strengths defined in A.3.2 and A.5.2, and reinforcement is provided for the ties considering the steel strengths defined in A.4.1. The reinforcement should be anchored in the nodal zones.

Strut-and-tie models represent strength limit states and Code requirements for serviceability should be satisfied. Deflections of deep beams or similar members can be estimated using an elastic analysis to analyze the strut-and-tie model. In addition, the crack widths in a tie can be controlled using 10.6.4, assuming the tie is encased in a prism of concrete corresponding to the area of tie from RA.4.2.

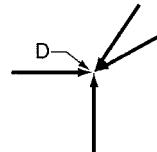
**RA.2.3** — The struts, ties, and nodal zones making up the strut-and-tie model all have finite widths that should be taken into account in selecting the dimensions of the truss. Figure RA.2.3(a) shows a node and the corresponding nodal zone. The vertical and horizontal forces equilibrate the force

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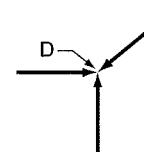
(a) Three struts acting on a nodal zone



(b) Struts A-E and C-E may be replaced by A-C



(c) Four forces acting on node D



(d) Forces on right side of node shown in (c) resolved

Fig. RA.2.3—Resolution of forces on a nodal zone.

in the inclined strut. If the stresses are equal in all three struts, a hydrostatic nodal zone can be used and the widths of the struts will be in proportion to the forces in the struts.

If more than three forces act on a nodal zone in a two-dimensional structure, as shown in Fig. RA.2.3(b), it is generally necessary to resolve some of the forces to end up with three intersecting forces. The strut forces acting on Faces A-E and C-E in Fig. RA.2.3(b) can be replaced with one force acting on Face A-C. This force passes through the node at D.

Alternatively, the strut-and-tie model could be analyzed assuming all the strut forces acted through the node at D, as shown in Fig. RA.2.3(c). In this case, the forces in the two struts on the right side of Node D can be resolved into a single force acting through Point D, as shown in Fig. RA.2.3(d).

If the width of the support in the direction perpendicular to the member is less than the width of the member, transverse reinforcement may be required to restrain vertical splitting in the plane of the node. This can be modeled using a transverse strut-and-tie model.

**A.2.4** — Ties shall be permitted to cross struts. Struts shall cross or overlap only at nodes.

**A.2.5** — The angle,  $\theta$ , between the axes of any strut and any tie entering a single node shall not be taken as less than 25 degrees.

**RA.2.5** — The angle between the axes of struts and ties acting on a node should be large enough to mitigate cracking and to avoid incompatibilities due to shortening of the struts and lengthening of the ties occurring in almost the same directions. This limitation on the angle prevents modeling the shear spans in slender beams using struts inclined at less than 25 degrees from the longitudinal steel. See Reference A.6.

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**A.2.6** — Design of struts, ties, and nodal zones shall be based on

$$\phi F_n \geq F_u \quad (\text{A-1})$$

where  $F_u$  is the factored force acting in a strut, in a tie, or on one face of a nodal zone;  $F_n$  is the nominal strength of the strut, tie, or nodal zone; and  $\phi$  is specified in [9.3.2.6](#).

## A.3 — Strength of struts

**A.3.1** — The nominal compressive strength of a strut without longitudinal reinforcement,  $F_{ns}$ , shall be taken as the smaller value of

$$F_{ns} = f_{ce} A_{cs} \quad (\text{A-2})$$

at the two ends of the strut, where  $A_{cs}$  is the cross-sectional area at one end of the strut, and  $f_{ce}$  is the smaller of (a) and (b):

- (a) the effective compressive strength of the concrete in the strut given in [A.3.2](#);
- (b) the effective compressive strength of the concrete in the nodal zone given in [A.5.2](#).

**A.3.2** — The effective compressive strength of the concrete,  $f_{ce}$ , in a strut shall be taken as

$$f_{ce} = 0.85 \beta_s f'_c \quad (\text{A-3})$$

**A.3.2.1** — For a strut of uniform cross-sectional area over its length .....  $\beta_s = 1.0$

**A.3.2.2** — For struts located such that the width of the midsection of the strut is larger than the width at the nodes (bottle-shaped struts):

- (a) With reinforcement satisfying [A.3.3](#) .....  $\beta_s = 0.75$
- (b) Without reinforcement satisfying [A.3.3](#) .....  $\beta_s = 0.60\lambda$

where the value of  $\lambda$  is defined in [8.6.1](#).

**A.3.2.3** — For struts in tension members, or the tension flanges of members .....  $\beta_s = 0.40$

## COMMENTARY

**RA.2.6** — Factored loads are applied to the strut-and-tie model, and the forces in all the struts, ties, and nodal zones are computed. If several loading cases exist, each should be investigated. The strut-and-tie model, or models, are analyzed for the loading cases and, for a given strut, tie, or nodal zone,  $F_u$  is the largest force in that element for all loading cases.

## RA.3 — Strength of struts

**RA.3.1** — The width of strut  $w_s$  used to compute  $A_{cs}$  is the smaller dimension perpendicular to the axis of the strut at the ends of the strut. This strut width is illustrated in [Fig. RA.1.4\(a\)](#) and [Fig. RA.1.5\(a\)](#) and [\(b\)](#). In two-dimensional structures, such as deep beams, the thickness of the struts may be taken as the width of the member.

**RA.3.2** — The strength coefficient,  $0.85f'_c$ , in Eq. (A-3) represents the effective concrete strength under sustained compression, similar to that used in [Eq. \(10-1\)](#) and [\(10-2\)](#).

**RA.3.2.1** — The value of  $\beta_s$  in A.3.2.1 applies to a strut equivalent to the rectangular stress block in a compression zone in a beam or column.

**RA.3.2.2** — The value of  $\beta_s$  in A.3.2.2 applies to bottle-shaped struts as shown in [Fig. RA.1.3](#). The internal lateral spread of the compression forces can lead to splitting parallel to the axis of the strut near the ends of the strut, as shown in [Fig. RA.1.8](#). Reinforcement placed to resist the splitting force restrains crack width, allows the strut to resist more axial load, and permits some redistribution of force.

The value of  $\beta_s$  in A.3.2.2(b) includes the correction factor,  $\lambda$ , for lightweight concrete because the strength of a strut without transverse reinforcement is assumed to be limited to less than the load at which longitudinal cracking develops.

**RA.3.2.3** — The value of  $\beta_s$  in A.3.2.3 applies, for example, to compression struts in a strut-and-tie model used to design the longitudinal and transverse reinforcement of the tension flanges of beams, box girders, and walls. The low value of  $\beta_s$  reflects that these struts need to transfer compression across cracks in a tension zone.

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**A.3.2.4** — For all other cases .....  $\beta_s = 0.60\lambda$

**A.3.3** — If the value of  $\beta_s$  specified in A.3.2.2(a) is used, the axis of the strut shall be crossed by reinforcement proportioned to resist the transverse tensile force resulting from the compression force spreading in the strut. It shall be permitted to assume the compressive force in the strut spreads at a slope of 2 longitudinal to 1 transverse to the axis of the strut.

**A.3.3.1** — For  $f'_c$  not greater than 6000 psi, the requirement of A.3.3 shall be permitted to be satisfied by the axis of the strut being crossed by layers of reinforcement that satisfy Eq. (A-4)

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i \geq 0.003 \quad (\text{A-4})$$

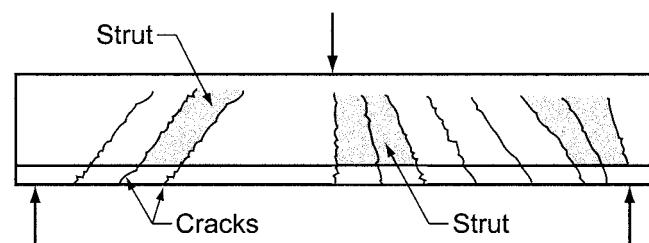
where  $A_{si}$  is the total area of surface reinforcement at spacing  $s_i$  in the  $i$ -th layer of reinforcement crossing a strut at an angle  $\alpha_i$  to the axis of the strut.

**RA.3.2.4** — The value of  $\beta_s$  in A.3.2.4 applies to strut applications not included in A.3.2.1, A.3.2.2, and A.3.2.3. Examples are struts in a beam web compression field in the web of a beam where parallel diagonal cracks are likely to divide the web into inclined struts, and struts are likely to be crossed by cracks at an angle to the struts (see Fig. RA.3.2(a) and (b)). Section A.3.2.4 gives a reasonable lower limit on  $\beta_s$  except for struts described in A.3.2.2(b) and A.3.2.3.

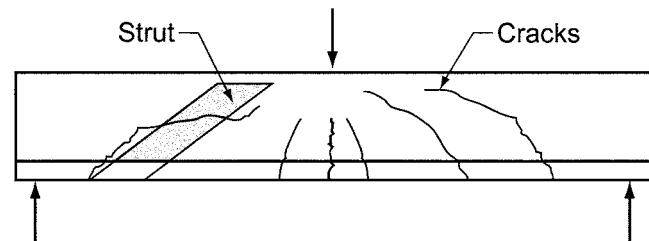
**RA.3.3** — The reinforcement required by A.3.3 is related to the tension force in the concrete due to the spreading of the strut, as shown in the strut-and-tie model in Fig. RA.1.8(b). Section RA.3.3 allows the use of local strut-and-tie models to compute the amount of transverse reinforcement needed in a given strut. The compressive forces in the strut may be assumed to spread at a 2:1 slope, as shown in Fig. RA.1.8(b). For specified concrete compressive strengths not exceeding 6000 psi, the amount of reinforcement required by Eq. (A-4) is deemed to satisfy A.3.3.

Figure RA.3.3 shows two layers of reinforcement crossing a cracked strut. If the crack opens without shear slip along the crack, bars in layer  $i$  in the figure will cause a stress perpendicular to the strut of

$$\frac{A_{si} f_{si}}{b_s s_i} \sin \alpha_i$$



(a) Struts in a beam web with inclined cracks parallel to struts - Section A.3.2.4



(b) Struts crossed by skew cracks - Section A.3.2.4

Fig. RA.3.2—Types of struts.

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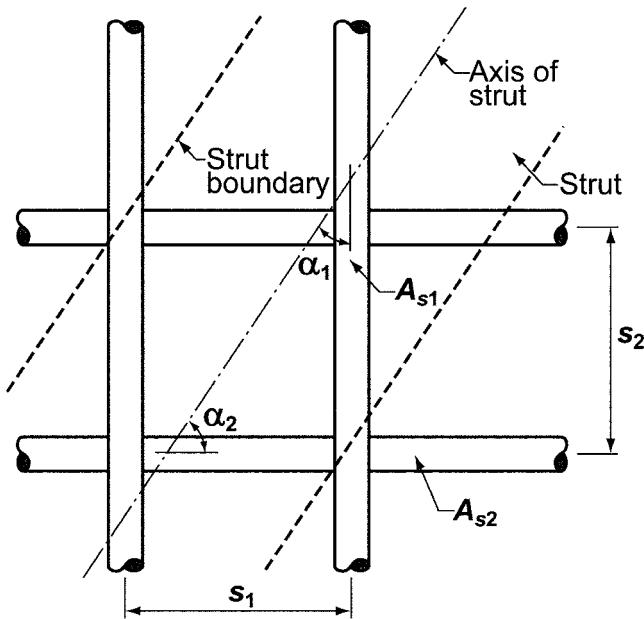


Fig. RA.3.3—Reinforcement crossing a strut.

where the subscript *i* takes on the values of 1 and 2 for the vertical and horizontal bars, respectively, as shown in Fig. RA.3.3. [Equation \(A-4\)](#) is written in terms of a reinforcement ratio rather than a stress to simplify the calculation.

Often, the confinement reinforcement given in [A.3.3](#) is difficult to place in three-dimensional structures such as pile caps. If this reinforcement is not provided, the value of  $f_{ce}$  given in [A.3.2.2\(b\)](#) is used.

**A.3.3.2** — The reinforcement required in [A.3.3](#) shall be placed in either two orthogonal directions at angles  $\alpha_1$  and  $\alpha_2$  to the axis of the strut, or in one direction at an angle  $\alpha$  to the axis of the strut. If the reinforcement is in only one direction,  $\alpha$  shall not be less than 40 degrees.

**A.3.4** — If documented by tests and analyses, it shall be permitted to use an increased effective compressive strength of a strut due to confining reinforcement.

**A.3.5** — The use of compression reinforcement shall be permitted to increase the strength of a strut. Compression reinforcement shall be properly anchored, parallel to the axis of the strut, located within the strut, and enclosed in ties or spirals satisfying [7.10](#). In such cases, the nominal strength of a longitudinally reinforced strut is

$$F_{ns} = f_{ce} A_{cs} + A'_s f'_s \quad (\text{A-5})$$

**RA.3.3.2** — In a corbel with a shear span-to-depth ratio less than 1.0, the confinement reinforcement required to satisfy [A.3.3](#) is usually provided in the form of horizontal stirrups crossing the inclined compression strut, as shown in [Fig. R11.8.2](#).

**RA.3.4** — The design of tendon anchorage zones for prestressed concrete sometimes uses confinement to enhance the compressive strength of the struts in the local zone. Confinement of struts is discussed in [References A.4](#) and [A.8](#).

**RA.3.5** — The strength added by the reinforcement is given by the last term in Eq. (A-5). The stress  $f'_s$  in the reinforcement in a strut at nominal strength can be obtained from the strains in the strut when the strut crushes. For Grade 40 or 60 reinforcement,  $f'_s$  can be taken as  $f_y$ .

**CODE****COMMENTARY****A.4 — Strength of ties**

**A.4.1** — The nominal strength of a tie,  $F_{nt}$ , shall be taken as

$$F_{nt} = A_{ts}f_y + A_{tp}(f_{se} + \Delta f_p) \quad (\text{A-6})$$

where  $(f_{se} + \Delta f_p)$  shall not exceed  $f_{py}$ , and  $A_{tp}$  is zero for nonprestressed members.

In Eq. (A-6), it shall be permitted to take  $\Delta f_p$  equal to 60,000 psi for bonded prestressed reinforcement, or 10,000 psi for unbonded prestressed reinforcement. Other values of  $\Delta f_p$  shall be permitted when justified by analysis.

**A.4.2** — The axis of the reinforcement in a tie shall coincide with the axis of the tie in the strut-and-tie model.

**RA.4 — Strength of ties**

**RA.4.2** — The effective tie width assumed in design  $w_t$  can vary between the following limits, depending on the distribution of the tie reinforcement:

(a) If the bars in the tie are in one layer, the effective tie width can be taken as the diameter of the bars in the tie plus twice the cover to the surface of the bars, as shown in Fig. RA.1.5(a); and

(b) A practical upper limit of the tie width can be taken as the width corresponding to the width in a hydrostatic nodal zone, calculated as

$$w_{t,max} = F_{nt}/(f_{ce}b_s)$$

where  $f_{ce}$  is computed for the nodal zone in accordance with A.5.2. If the tie width exceeds the value from (a), the tie reinforcement should be distributed approximately uniformly over the width and thickness of the tie, as shown in Fig. RA.1.5(b).

**RA.4.3** — Anchorage of ties often requires special attention in nodal zones of corbels or in nodal zones adjacent to exterior supports of deep beams. The reinforcement in a tie should be anchored before it leaves the extended nodal zone at the point defined by the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or the bearing area. This length is  $\ell_{anc}$ . In Fig. RA.1.5(a) and (b), this occurs where the outline of the extended nodal zone is crossed by the centroid of the reinforcement in the tie. Some of the anchorage may be achieved by extending the reinforcement through the nodal zone, as shown in Fig. RA.1.4(c), and developing it beyond the nodal zone. If the tie is anchored using 90-degree hooks, the hooks should be confined within the reinforcement extending into the beam from the supporting member to avoid cracking along the outside of the hooks in the support region.

**A.4.3** — Tie reinforcement shall be anchored by mechanical devices, post-tensioning anchorage devices, standard hooks, or straight bar development as required by A.4.3.1 through A.4.3.4.

**A.4.3.1** — Nodal zones shall develop the difference between the tie force on one side of the node and the tie force on the other side.

**A.4.3.2** — At nodal zones anchoring one tie, the tie force shall be developed at the point where the centroid of the reinforcement in a tie leaves the extended nodal zone and enters the span.

**A.4.3.3** — At nodal zones anchoring two or more ties, the tie force in each direction shall be developed at the point where the centroid of the reinforcement in the tie leaves the extended nodal zone.

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**A.4.3.4** — The transverse reinforcement required by A.3.3 shall be anchored in accordance with 12.13.

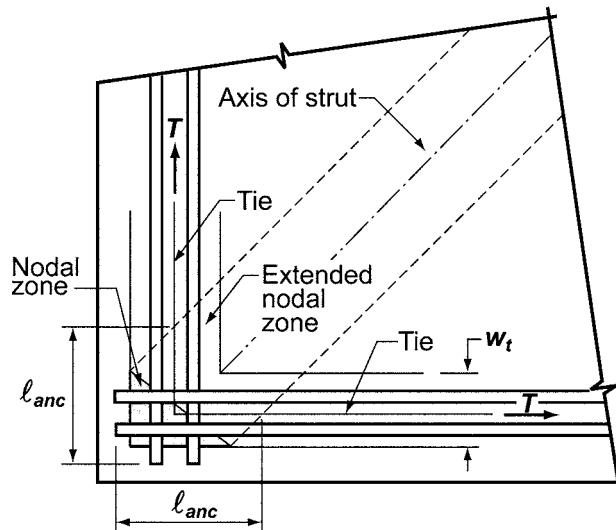


Fig. RA.4.3—Extended nodal zone anchoring two ties.

In deep beams, hairpin bars spliced with the tie reinforcement can be used to anchor the tension tie forces at exterior supports, provided the beam width is large enough to accommodate such bars.

Figure RA.4.3 shows two ties anchored at a nodal zone. Development is required where the centroid of the tie crosses the outline of the extended nodal zone.

The development length of the tie reinforcement can be reduced through hooks, mechanical devices, additional confinement, or by splicing it with several layers of smaller bars.

## A.5 — Strength of nodal zones

**A.5.1** — The nominal compression strength of a nodal zone,  $F_{nn}$ , shall be

$$F_{nn} = f_{ce} A_{nz} \quad (\text{A-7})$$

where  $f_{ce}$  is the effective compressive strength of the concrete in the nodal zone as given in A.5.2, and  $A_{nz}$  is the smaller of (a) and (b):

- (a) The area of the face of the nodal zone on which  $F_u$  acts, taken perpendicular to the line of action of  $F_u$ ;
- (b) The area of a section through the nodal zone, taken perpendicular to the line of action of the resultant force on the section.

## RA.5 — Strength of nodal zones

**RA.5.1** — If the stresses in all the struts meeting at a node are equal, a hydrostatic nodal zone can be used. The faces of such a nodal zone are perpendicular to the axes of the struts, and the widths of the faces of the nodal zone are proportional to the forces in the struts.

Assuming the principal stresses in the struts and ties act parallel to the axes of the struts and ties, the stresses on faces perpendicular to these axes are principal stresses, and A.5.1(a) is used. If, as shown in Fig. RA.1.5(b), the face of a nodal zone is not perpendicular to the axis of the strut, there will be both shear stresses and normal stresses on the face of the nodal zone. Typically, these stresses are replaced by the normal (principal compression) stress acting on the cross-sectional area  $A_c$  of the strut, taken perpendicular to the axis of the strut as given in A.5.1(a).

In some cases, A.5.1(b) requires that the stresses be checked on a section through a subdivided nodal zone. The stresses are checked on the least area section which is perpendicular

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**A.5.2** — Unless confining reinforcement is provided within the nodal zone and its effect is supported by tests and analysis, the calculated effective compressive stress,  $f_{ce}$ , on a face of a nodal zone due to the strut-and-tie forces shall not exceed the value given by

$$f_{ce} = 0.85\beta_n f'_c \quad (\text{A-8})$$

where the value of  $\beta_n$  is given in A.5.2.1 through A.5.2.3.

**A.5.2.1** — In nodal zones bounded by struts or bearing areas, or both.....  $\beta_n = 1.0$ ;

**A.5.2.2** — In nodal zones anchoring one tie .....  $\beta_n = 0.80$ ;

or

**A.5.2.3** — In nodal zones anchoring two or more ties .....  $\beta_n = 0.60$ .

**A.5.3** — In a three-dimensional strut-and-tie model, the area of each face of a nodal zone shall not be less than that given in A.5.1, and the shape of each face of the nodal zones shall be similar to the shape of the projection of the end of the struts onto the corresponding faces of the nodal zones.

to a resultant force in the nodal zone. In Fig. RA.1.6(b), the vertical face which divide the nodal zone into two parts is stressed by the resultant force acting along A-B. The design of the nodal zone is governed by the critical section from A.5.1(a) or A.5.1(b), whichever gives the highest stress.

**RA.5.2** — The nodes in two-dimensional members, such as deep beams, can be classified as **C-C-C** if all the members intersecting at the node are in compression; as **C-C-T** nodes if one of the members acting on the node is in tension; and so on, as shown in Fig. RA.1.7. The effective compressive strength of the nodal zone is given by Eq. (A-8), as modified by A.5.2.1 through A.5.2.3 apply to **C-C-C** nodes, **C-C-T** nodes, and **C-T-T** or **T-T-T** nodes, respectively.

The  $\beta_n$  values reflect the increasing degree of disruption of the nodal zones due to the incompatibility of tension strains in the ties and compression strains in the struts. The stress on any face of the nodal zone or on any section through the nodal zone should not exceed the value given by Eq. (A-8), as modified by A.5.2.1 through A.5.2.3.

**RA.5.3** — This description of the shape and orientation of the faces of the nodal zones is introduced to simplify the calculations of the geometry of a three-dimensional strut-and-tie model.

**Notes**

# APPENDIX B — ALTERNATIVE PROVISIONS FOR REINFORCED AND PRESTRESSED CONCRETE FLEXURAL AND COMPRESSION MEMBERS

## CODE

## COMMENTARY

### B.1 — Scope

Design for flexure and axial load by provisions of Appendix B shall be permitted. When Appendix B is used in design, B.8.4, B.8.4.1, B.8.4.2, and B.8.4.3 shall replace the corresponding numbered sections in Chapter 8; B.10.3.3 shall replace 10.3.3, 10.3.4, and 10.3.5, except 10.3.5.1 shall remain; B.18.1.3, B.18.8.1, B.18.8.2, and B.18.8.3 shall replace the corresponding numbered sections in Chapter 18; B.18.10.4.1, B.18.10.4.2, and B.18.10.4.3 shall replace 18.10.4.1 and 18.10.4.2. If any section in this appendix is used, all sections in this appendix shall be substituted in the body of the Code, and all other sections in the body of the Code shall be applicable.

### B.8.4 — Redistribution of moments in continuous non prestressed flexural members

For criteria on moment redistribution for prestressed concrete members, see B.18.10.4.

**B.8.4.1** — Except where approximate values for moments are used, it shall be permitted to decrease factored moments calculated by elastic theory at sections of maximum negative or maximum positive moment and in any span of continuous flexural members for any assumed loading arrangement by not more than

$$20 \left(1 - \frac{\rho - \rho'}{\rho_b}\right) \text{ percent}$$

**B.8.4.2** — Redistribution of moments shall be made only when the section at which moment is reduced is so designed that  $\rho$  or  $\rho - \rho'$  is not greater than  $0.50\rho_b$ , where

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y} \left( \frac{87,000}{87,000 + f_y} \right) \quad (\text{B-1})$$

**B.8.4.3** — The reduced moment shall be used for calculating redistributed moments at all other sections within the spans. Static equilibrium shall be maintained after redistribution of moments at each loading arrangement.

### RB.1 — Scope

Reinforcement limits, strength reduction factors  $\phi$ , and moment redistribution in Appendix B differ from those in the main body of the Code. Appendix B contains the reinforcement limits, strength reduction factors  $\phi$ , and moment redistribution used in the Code for many years. Designs using the provisions of Appendix B satisfy the Code, and are equally acceptable.

When this appendix is used, the corresponding Commentary sections apply. The load factors and strength reduction factors of either Chapter 9 or Appendix C are applicable.

### RB.8.4 — Redistribution of moments in continuous non prestressed flexural members

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at sections of maximum positive or negative moment and cause a shift in the elastic moment diagram. The usual results are reduction in the values of maximum negative moments in the support regions and an increase in the values of positive moments between supports from those computed by elastic analysis. However, because negative moments are determined for one loading arrangement and positive moments for another (see 13.7.6 for an exception), economies in reinforcement can sometimes be realized by reducing maximum elastic positive moments and increasing negative moments, thus narrowing the envelope of maximum negative and positive moments at any section in the span. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads.

Before 2008, the Code addressed moment redistribution by permitting an increase or decrease of factored negative moments above or below elastically calculated values, within specified limits. A decrease in negative moment strength implies inelastic behavior in the negative moment region at the support. By increasing the negative moment strength, the positive moments can be reduced but the result is that inelastic behavior will occur in the positive moment region of the member and the percentage change in the positive moment section could be much larger than the 20 percent permitted for negative moment sections.<sup>B.1</sup> The 2008 change places the same percentage limitations on both positive and negative moments.

## CODE

## COMMENTARY

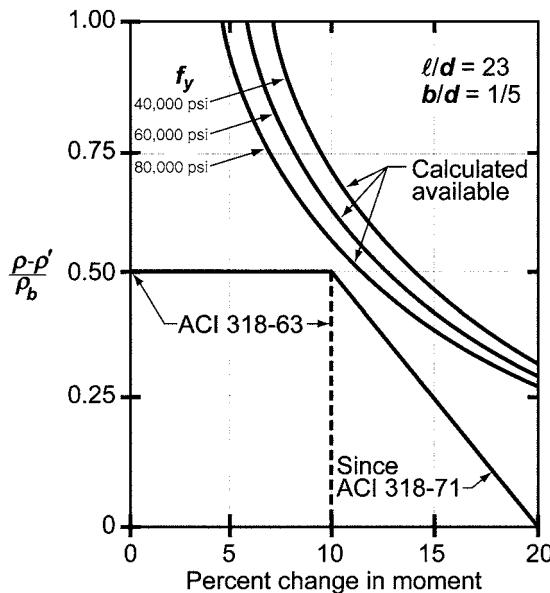


Fig. RB.8.4—Permissible moment redistribution for minimum rotation capacity.

Using conservative values of ultimate concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution up to 20 percent, depending on the reinforcement ratio. The results were found to be conservative (see Fig. RB.8.4). Studies by Cohn<sup>B.2</sup> and Mattock<sup>B.3</sup> support this conclusion and indicate that cracking and deflection of beams designed for moment redistribution are not significantly greater at service loads than for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the Code is available if the members satisfy the Code requirements. This appendix maintains the same limit on redistribution as used in previous Code editions.

Moment redistribution may not be used for slab systems designed by the direct design method (see 13.6.1.7).

### B.10.3 — General principles and requirements

**B.10.3.3** — For flexural members and members subject to combined flexure and compressive axial load where  $\phi P_n$  is less than the smaller of  $0.10f'_c A_g$  and  $\phi P_b$ , the ratio of reinforcement,  $\rho$ , provided shall not exceed 0.75 of the ratio  $\rho_b$  that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, the portion of  $\rho_b$  equalized by compression reinforcement need not be reduced by the 0.75 factor.

### RB.10.3 — General principles and requirements

**RB.10.3.3** — The maximum amount of tension reinforcement in flexural members is limited to ensure a level of ductile behavior.

The nominal flexural strength of a section is reached when the strain in the extreme compression fiber reaches the limiting strain in the concrete. At ultimate strain of the concrete, the strain in the tension reinforcement could just reach the strain at first yield, be less than the yield strain (elastic), or exceed the yield strain (inelastic). The steel strain that exists at limiting concrete strain depends on the relative proportion of steel to concrete and material strengths  $f'_c$  and  $f_y$ .

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If  $\rho(f_y/f'_c)$  is sufficiently low, the strain in the tension steel will greatly exceed the yield strain when the concrete strain reaches its limiting value, with large deflection and ample warning of impending failure (ductile failure condition). With a larger  $\rho(f_y/f'_c)$ , the strain in the tension steel may not reach the yield strain when the concrete strain reaches its limiting value, with consequent small deflection and little warning of impending failure (brittle failure condition). For design it is considered more conservative to restrict the nominal strength condition so that a ductile failure mode can be expected.

Unless unusual amounts of ductility are required, the  $0.75\rho_b$  limitation will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. **Section B.8.4** permits negative moment redistribution. Since moment redistribution is dependent on adequate ductility in hinge regions, the amount of tension reinforcement in hinging regions is limited to  $0.5\rho_b$ .

For ductile behavior of beams with compression reinforcement, only that portion of the total tension steel balanced by compression in the concrete need be limited; that portion of the total tension steel where force is balanced by compression reinforcement need not be limited by the 0.75 factor.

**B.18.1 — Scope**

**B.18.1.3** — The following provisions of this Code shall not apply to prestressed concrete, except as specifically noted: **Sections 6.4.4, 7.6.5, B.8.4, 8.12.2, 8.12.3, 8.12.4, 8.13, B.10.3.3, 10.5, 10.6, 10.9.1, and 10.9.2; Chapter 13; and Sections 14.3, 14.5, and 14.6.**

**RB.18.1 — Scope**

**RB.18.1.3** — Some sections of the Code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides an explanation for such exclusions:

**Section 6.4.4** — Tendons of continuous post-tensioned beams and slabs are usually stressed at a point along the span where the tendon profile is at or near the centroid of the concrete cross section. Therefore, interior construction joints are usually located within the end thirds of the span, rather than the middle third of the span as required by **6.4.4**. Construction joints located as described in continuous post-tensioned beams and slabs have a long history of satisfactory performance. Thus, **6.4.4** is excluded from application to prestressed concrete.

**Section 7.6.5** — **Section 7.6.5** is excluded from application to prestressed concrete since the requirements for bonded reinforcement and unbonded tendons for cast-in-place members are provided in **18.9** and **18.12**, respectively.

**Section B.8.4** — Moment redistribution for prestressed concrete is provided in **B.8.4**.

**Sections 8.12.2, 8.12.3, and 8.12.4** — The empirical provisions of **8.12.2, 8.12.3**, and **8.12.4** for T-beams were developed for conventionally reinforced concrete and, if applied to

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prestressed concrete, would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding 8.12.2, 8.12.3, and 8.12.4, no special requirements for prestressed concrete T-beams appear in the Code. Instead, the determination of an effective width of flange is left to the experience and judgment of the licensed design professional. Where possible, the flange widths in 8.12.2, 8.12.3, and 8.12.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 8.12.2.

Sections 8.12.1 and 8.12.5 provide general requirements for T-beams that are also applicable to prestressed concrete units. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

**Section 8.13** — The empirical limits established for conventionally reinforced concrete joist floors are based on successful past performance of joist construction using “standard” joist forming systems. See R8.13. For prestressed joist construction, experience and judgment should be used. The provisions of 8.13 may be used as a guide.

**Sections B.10.3.3, 10.5, 10.9.1, and 10.9.2** — For prestressed concrete, the limitations on reinforcement given in B.10.3.3, 10.5, 10.9.1, and 10.9.2 are replaced by those in B.18.8, 18.9, and 18.11.2.

**Section 10.6** — When originally prepared, the provisions of 10.6 for distribution of flexural reinforcement were not intended for prestressed concrete members. The behavior of a prestressed member is considerably different from that of a nonprestressed member. Experience and judgment should be used for proper distribution of reinforcement in a prestressed member.

**Chapter 13** — The design of prestressed concrete slabs requires recognition of secondary moments induced by the undulating profile of the prestressing tendons. Also, volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of 18.12.

**Sections 14.5 and 14.6** — The requirements for wall design in 14.5 and 14.6 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

**CODE****COMMENTARY****B.18.8 — Limits for reinforcement of flexural members**

**B.18.8.1** — Ratio of prestressed and nonprestressed reinforcement used for computation of moment strength of a member, except as provided in B.18.8.2, shall be such that  $\omega_p$ ,  $[\omega_p + (d/d_p)(\omega - \omega')]$ , or  $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$  is not greater than **0.36** $\beta_1$ , except as permitted in B.18.8.2.

Ratio  $\omega_p$  is computed as  $\rho_p f_{ps}/f'_c$ . Ratios  $\omega_w$  and  $\omega_{pw}$  are computed as  $\omega$  and  $\omega_p$ , respectively, except that when computing  $\rho$  and  $\rho_p$ ,  $b_w$  shall be used in place of  $b$  and the area of reinforcement or prestressing steel required to develop the compressive strength of the web only shall be used in place of  $A_s$  or  $A_{ps}$ . Ratio  $\omega'_w$  is computed as  $\omega'$ , except that when computing  $\rho'$ ,  $b_w$  shall be used in place of  $b$ .

**B.18.8.2** — When a reinforcement ratio exceeds the limit specified in B.18.8.1 is provided, design moment strength shall not exceed the moment strength based on the compression portion of the moment couple.

**B.18.8.3** — Total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture  $f_r$  in **9.5.2.3**. This provision shall be permitted to be waived for:

- (a) two-way, unbonded post-tensioned slabs; and
- (b) flexural members with shear and flexural strength at least twice that required by **9.2**.

**RB.18.8 — Limits for reinforcement of flexural members**

**RB.18.8.1** — The terms  $\omega_p$ ,  $[\omega_p + (d/d_p)(\omega - \omega')]$  and  $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$  are each equal to **0.85a/d<sub>p</sub>**, where  $a$  is the depth of the equivalent rectangular stress block for the section under consideration, as defined in **10.2.7.1**. Use of this relationship can simplify the calculations necessary to check compliance with RB.18.8.1.

**RB.18.8.2** — Design moment strength of over-reinforced sections may be computed using strength equations similar to those for nonprestressed concrete members. The 1983 Code provided strength equations for rectangular and flanged sections.

**RB.18.8.3** — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to Code provisions requires considerable additional load beyond cracking to reach its flexural strength. This additional load should result in considerable deflection that would warn when the member nominal strength is being approached. If the flexural strength is reached shortly after cracking, the warning deflection would not occur.

Due to the very limited extent of initial cracking in the negative moment region near columns of two-way flat plates, deflection under load does not reflect any abrupt change in stiffness as the modulus of rupture of concrete is reached.

Only at load levels beyond the factored loads is the additional cracking extensive enough to cause an abrupt change in the deflection under load. Tests have shown that it is not possible to rupture (or even yield) unbonded post-tensioning tendons in two-way slabs before a punching shear failure.<sup>B.4-B.9</sup> The use of unbonded tendons in combination with the minimum bonded reinforcement requirements of **18.9.3** and **18.9.4** has been shown to ensure post-cracking ductility and that a brittle failure mode will not develop at first cracking.

**CODE****COMMENTARY****B.18.10 — Statically indeterminate structures**

**B.18.10.1** — Frames and continuous construction of prestressed concrete shall be designed for satisfactory performance at service load conditions and for adequate strength.

**B.18.10.2** — Performance at service load conditions shall be determined by elastic analysis, considering reactions, moments, shears, and axial forces produced by prestressing, creep, shrinkage, temperature change, axial deformation, restraint of attached structural elements, and foundation settlement.

**B.18.10.3** — Moments to be used to compute required strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads. Adjustment of the sum of these moments shall be permitted as allowed in B.18.10.4.

**B.18.10.4 — Redistribution of moments in continuous prestressed flexural members**

**B.18.10.4.1** — Where bonded reinforcement is provided at supports in accordance with 18.9, negative or positive moments calculated by elastic theory for any assumed loading, arrangement shall be permitted to be increased or decreased by not more than

$$20 \left[ 1 - \frac{\omega_p + \frac{d}{d_p}(\omega - \omega')}{0.36\beta_1} \right] \text{ percent}$$

**B.18.10.4.2** — Redistribution of moments shall be made only when the section at which moment is

**RB.18.10 — Statically indeterminate structures**

**RB.18.10.3** — For statically indeterminate structures, the moments due to reactions induced by prestressing forces, referred to as secondary moments, are significant in both the elastic and inelastic states. When hinges and full redistribution of moments occur to create a statically determinate structure, secondary moments disappear. However, the elastic deformations caused by a nonconcordant tendon change the amount of inelastic rotation required to obtain a given amount of moment redistribution. Conversely, for a beam with a given inelastic rotation capacity, the amount by which the moment at the support may be varied is changed by an amount equal to the secondary moment at the support due to prestressing. Thus, the Code requires that secondary moments be included in determining design moments.

To determine the moments used in design, the order of calculation should be: (a) determine moments due to dead load and live load; (b) modify by algebraic addition of secondary moments; and (c) redistribute as permitted. A positive secondary moment at the support caused by a tendon transformed downward from a concordant profile will reduce the negative moments near the supports and increase the positive moments in the midspan regions. A tendon that is transformed upward will have the reverse effect.

**RB.18.10.4 — Redistribution of moments in continuous prestressed flexural members**

As member strength is approached, inelastic behavior at some sections can result in a redistribution of moments in prestressed concrete beams and slabs. Recognition of this behavior can be advantageous in design under certain circumstances. A rigorous design method for moment redistribution is complex. However, recognition of moment redistribution can be accomplished by permitting a reasonable adjustment of the sum of the elastically calculated factored gravity load moments and the unfactored secondary moments due to prestress. The amount of adjustment should be kept within predetermined safety limits.

The amount of redistribution allowed depends on the ability of the critical sections to deform inelastically by a sufficient

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reduced is so designed that  $\omega_p$ ,  $[\omega_p + (d/d_p)(\omega - \omega')]$  or  $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$ , whichever is applicable, is not greater than **0.24 $\beta_1$** .

**B.18.10.4.3** — The reduced moment shall be used for calculating redistributed moments at all other sections within the spans. Static equilibrium shall be maintained after redistribution of moments for each loading arrangement.

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amount. Serviceability is addressed in **18.4**. The choice of **0.24 $\beta_1$**  as the largest tension reinforcement index,  $\omega_p$ ,  $[\omega_p + (d/d_p)(\omega - \omega')]$  or  $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$ , for which redistribution of moments is allowed, is in agreement with the requirements for nonprestressed concrete of **0.5 $\rho_b$**  stated in **B.8.4**.

The terms  $\omega_p$ ,  $[\omega_p + (d/d_p)(\omega - \omega')]$ , and  $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$  appear in **B.18.10.4.1** and **B.18.10.4.3** and are each equal to **0.85a/d<sub>p</sub>**, where *a* is the depth of the equivalent rectangular stress distribution for the section under consideration, as defined in **10.2.7.1**. Use of this relationship can simplify the calculations necessary to determine the amount of moment redistribution permitted by **B.18.10.4.1** and to check compliance with the limitation on flexural reinforcement contained in **B.18.10.4.3**.

For the moment redistribution principles of **B.18.10.4** to be applicable to beams and slabs with unbonded tendons, it is necessary that such beams and slabs contain sufficient bonded reinforcement to ensure that they act as flexural members after cracking and not as a series of tied arches. The minimum bonded reinforcement requirements of **18.9** serve this purpose.

**Notes**

## APPENDIX C — ALTERNATIVE LOAD AND STRENGTH REDUCTION FACTORS

### CODE

#### C.9.1 — Scope

Structural concrete shall be permitted to be designed using the load combinations and strength reduction factors of Appendix C. When Appendix C is used in design, C.9.2.1 through C.9.2.7 shall replace 9.2.1 through 9.2.7 and C.9.3.1 through C.9.3.5 shall replace 9.3.1 through 9.3.5.

### COMMENTARY

#### RC.9.1 — General

**RC.9.1.1** — In the 2002 Code, the load and strength reduction factors formerly in Chapter 9 were revised and moved to this appendix. They have evolved since the early 1960s and are considered to be reliable for concrete construction.

#### C.9.2 — Required strength

**C.9.2.1** — Required strength  $U$  to resist dead load  $D$  and live load  $L$  shall not be less than

$$U = 1.4D + 1.7L \quad (\text{C.9-1})$$

**C.9.2.2** — For structures that also resist  $W$ , wind load, or  $E$ , the effects of earthquake,  $U$  shall not be less than the largest of Eq. (C.9-1), (C.9-2), and (C.9-3)

$$U = 0.75(1.4D + 1.7L) + (1.0W \text{ or } 1.0E) \quad (\text{C.9-2})$$

and

$$U = 0.9D + (1.0W \text{ or } 1.0E) \quad (\text{C.9-3})$$

Where  $W$  is based on service-level wind loads,  $1.6W$  shall be used in place of  $1.0W$  in Eq. (C.9-2) and (C.9-3). Where  $E$  is based on service-level earthquake effects,  $1.4E$  shall be used in place of  $1.0E$  in Eq. (C.9-2) and (C.9-3).

**C.9.2.3** — For structures that resist  $H$ , loads due to lateral pressure of soil, water in soil, or other related materials,  $U$  shall not be less than the larger of Eq. (C.9-1) and (C.9-4):

$$U = 1.4D + 1.7L + 1.7H \quad (\text{C.9-4})$$

In Eq. (C.9-4), where  $D$  or  $L$  reduce the effect of  $H$ ,  $0.9D$  shall be substituted for  $1.4D$ , and zero value of  $L$  shall be used to determine the greatest required strength  $U$ .

#### RC.9.2 — Required strength

ASCE/SEI 7-10 has converted wind loads to strength level, and reduced the wind load factor to 1.0. ACI 318 requires use of the previous load factor for wind loads, 1.6, when service-level wind loads are used.

Model building codes and design load references have converted earthquake forces to strength level, and reduced the earthquake load factor to 1.0 (ASCE 7-93<sup>C.2</sup>; BOCA/NBC 93<sup>C.3</sup>; SBC 94<sup>C.4</sup>; UBC 97<sup>C.5</sup>; and IBC 2000<sup>C.1</sup>). The Code requires use of the previous load factor for earthquake loads, approximately 1.4, when service-level earthquake forces from earlier editions of these references are used.

**RC.9.2.3** — If effects  $H$  caused by earth pressure, ground-water pressure, or pressure caused by granular materials are included in design, the required strength equations become

$$U = 1.4D + 1.7L + 1.7H$$

and where  $D$  or  $L$  reduce the effect of  $H$

$$U = 0.9D + 1.7H$$

but for any combination of  $D$ ,  $L$ , or  $H$

$$U = 1.4D + 1.7L$$

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**C.9.2.4** — For structures that resist  $F$ , load due to weight and pressure of fluids with well-defined densities, the load factor for  $F$  shall be 1.4, and  $F$  shall be added to all loading combinations that include  $L$ .

**RC.9.2.4** — This section addresses the need to consider loading due to weight of liquid or liquid pressure. It specifies a load factor for such loadings with well-defined densities and controllable maximum heights equivalent to that used for dead load. Such reduced factors would not be appropriate where there is considerable uncertainty of pressures, as with groundwater pressures, or uncertainty as to the possible maximum liquid depth, as in ponding of water. See R8.2.

For well-defined fluid pressures, the required strength equations become

$$U = 1.4D + 1.7L + 1.4F$$

and where  $D$  or  $L$  reduce the effect of  $F$

$$U = 0.9D + 1.4F$$

but for any combination of  $D$ ,  $L$ , or  $F$

$$U = 1.4D + 1.7L$$

**C.9.2.5** — If resistance to impact effects is taken into account in design, such effects shall be included with  $L$ .

**RC.9.2.5** — If the live load is applied rapidly, as may be the case for parking structures, loading docks, warehouse floors, elevator shafts, etc., impact effects should be considered. In all equations, substitute ( $L$  + impact) for  $L$  when impact must be considered.

**C.9.2.6** — Where structural effects of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change,  $T$ , are significant,  $U$  shall not be less than the larger of Eq. (C.9-5) and (C.9-6)

$$U = 0.75(1.4D + 1.4T + 1.7L) \quad (\text{C.9-5})$$

$$U = 1.4(D + T) \quad (\text{C.9-6})$$

Estimations of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change shall be based on realistic assessment of such effects occurring in service.

**C.9.2.7** — For post-tensioned anchorage zone design, a load factor of 1.2 shall be applied to the maximum prestressing steel jacking force.

**RC.9.2.6** — The effects of differential settlement, creep, shrinkage, temperature, and shrinkage-compensating concrete should be considered. The term “realistic assessment” is used to indicate that the most probable values, rather than the upper bound values, of the variables should be used.

Equation (C.9-6) is to prevent a design for load

$$U = 0.75(1.4D + 1.4T + 1.7L)$$

to approach

$$U = 1.05(D + T)$$

when live load is negligible.

**RC.9.2.7** — The load factor of 1.2 applied to the maximum prestressing steel jacking force results in a design load of 113 percent of the specified yield strength of prestressing steel but not more than 96 percent of the nominal ultimate strength of the tendon. This compares well with a maximum attainable jacking force, which is limited by the anchor efficiency factor.

**CODE****COMMENTARY****C.9.3 — Design strength**

**C.9.3.1** — Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall be taken as the nominal strength calculated in accordance with requirements and assumptions of this Code, multiplied by the  $\phi$  factors in C.9.3.2, **C.9.3.4**, and **C.9.3.5**.

**C.9.3.2** — Strength reduction factor  $\phi$  shall be as follows:

**C.9.3.2.1** — Tension-controlled sections, as defined in **10.3.4** (See also **C.9.3.2.7**) ..... 0.90

**C.9.3.2.2** — Compression-controlled sections, as defined in **10.3.3**:

- (a) Members with spiral reinforcement conforming to **10.9.3** ..... 0.75
- (b) Other reinforced members ..... 0.70

For sections in which the net tensile strain in the extreme tension steel at nominal strength,  $\varepsilon_t$ , is between the limits for compression-controlled and tension-controlled sections,  $\phi$  shall be permitted to be linearly increased from that for compression-controlled sections to 0.90 as  $\varepsilon_t$  increases from the compression-controlled strain limit to 0.005.

Alternatively, when **Appendix B** is used, for members in which  $f_y$  does not exceed 60,000 psi, with symmetric reinforcement, and with  $(d - d')/h$  not less than 0.70,  $\phi$  shall be permitted to be increased linearly to 0.90 as  $\phi P_n$  decreases from **0.10f'\_c A\_g** to zero. For other reinforced members,  $\phi$  shall be permitted to be increased linearly to 0.90 as  $\phi P_n$  decreases from **0.10f'\_c A\_g** or  $\phi P_b$ , whichever is smaller, to zero.

**RC.9.3 — Design strength**

**RC.9.3.1** — The term “design strength” of a member refers to the nominal strength calculated in accordance with the requirements stipulated in this Code multiplied by a strength reduction factor  $\phi$  that is always less than 1.

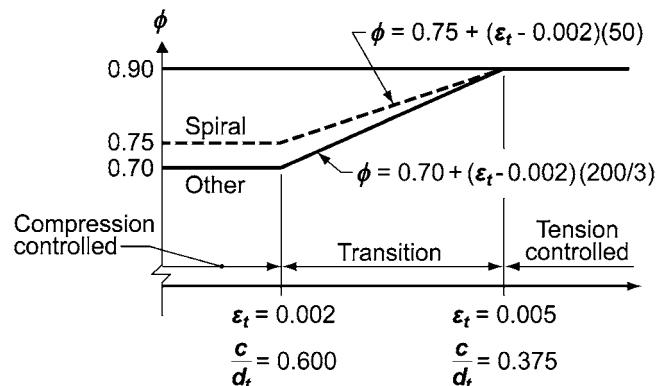
The purposes of the strength reduction factor  $\phi$  are: (1) to allow for the probability of understrength members due to variations in material strengths and dimensions; (2) to allow for inaccuracies in the design equations; (3) to reflect the degree of ductility and required reliability of the member under the load effects being considered; and (4) to reflect the importance of the member in the structure. For example, a lower  $\phi$  is used for columns than for beams because columns generally have less ductility, are more sensitive to variations in concrete strength, and generally support larger loaded areas than beams. Furthermore, spiral columns are assigned a higher  $\phi$  than tied columns since they have greater ductility or toughness.

**RC.9.3.2.1** — In applying C.9.3.2.1 and C.9.3.2.2, the axial tensions and compressions to be considered are those caused by external forces. Effects of prestressing forces are not included.

**RC.9.3.2.2** — Before the 2002 edition, the Code gave the magnitude of the  $\phi$ -factor for cases of axial load or flexure, or both, in terms of the type of loading. For these cases, the  $\phi$ -factor is now determined by the strain conditions at a cross section, at nominal strength.

A lower  $\phi$ -factor is used for compression-controlled sections than is used for tension-controlled sections because compression-controlled sections have less ductility, are more sensitive to variations in concrete strength, and generally occur in members that support larger loaded areas than members with tension-controlled sections. Members with spiral reinforcement are assigned a higher  $\phi$  than tied columns since they have greater ductility or toughness.

For sections subjected to axial load with flexure, design strengths are determined by multiplying both  $P_n$  and  $M_n$  by the appropriate single value of  $\phi$ . Compression-controlled and tension-controlled sections are defined in **10.3.3** and **10.3.4** as those that have net tensile strain in the extreme tension steel at nominal strength less than or equal to the compression-controlled strain limit, and equal to or greater than 0.005, respectively. For sections with net tensile strain  $\varepsilon_t$  in the extreme tension steel at nominal strength between the above limits, the value of  $\phi$  may be determined by linear interpolation, as shown in **Fig. RC.9.3.2**. The concept of net tensile strain  $\varepsilon_t$  is discussed in **R10.3.3**.

**CODE****COMMENTARY**

Interpolation on  $c/d_t$ : Spiral  $\phi = 0.75 + 0.15[(1/c/d_t) - (5/3)]$   
Other  $\phi = 0.70 + 0.20[(1/c/d_t) - (5/3)]$

Fig. RC.9.3.2—Variation of  $\phi$  with net tensile strain  $\epsilon_t$  and  $c/d_t$  for Grade 60 reinforcement and for prestressing steel.

Since the compressive strain in the concrete at nominal strength is assumed in 10.2.3 to be 0.003, the net tensile strain limits for compression-controlled members may also be stated in terms of the ratio  $c/d_t$ , where  $c$  is the distance from the extreme compression fiber to the neutral axis at nominal strength, and  $d_t$  is the distance from the extreme compression fiber to the centroid of the extreme layer of longitudinal tension steel. The  $c/d_t$  limits for compression-controlled and tension-controlled sections are 0.6 and 0.375, respectively. The 0.6 limit applies to sections reinforced with Grade 60 steel and to prestressed sections. Figure RC.9.3.2 also gives equations for  $\phi$  as a function of  $c/d_t$ .

The net tensile strain limit for tension-controlled sections may also be stated in terms of the  $\rho/\rho_b$  as defined in the 1999 and earlier editions of the Code. The net tensile strain limit of 0.005 corresponds to a  $\rho/\rho_b$  ratio of 0.63 for rectangular sections with Grade 60 reinforcement. For a comparison of these provisions with those of the body of the Code, the 1999 ACI Code Section 9.3, see Reference C.6.

**C.9.3.2.3** — Shear and torsion.....0.85

**C.9.3.2.4** — Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models) .....0.70

**C.9.3.2.5** — Post-tensioned anchorage zones.....0.85

**C.9.3.2.6** — Strut-and-tie models (Appendix A), and struts, ties, nodal zones, and bearing areas in such models.....0.85

**RC.9.3.2.5** — The  $\phi$ -factor of 0.85 reflects the wide scatter of results of experimental anchorage zone studies. Since 18.13.4.2 limits the nominal compressive strength of unconfined concrete in the general zone to  $0.7\lambda f'_{ci}$ , the effective design strength for unconfined concrete is  $0.85 \times 0.7\lambda f'_{ci} \approx 0.6\lambda f'_{ci}$ .

**CODE****COMMENTARY**

**C.9.3.2.7** — Flexure sections without axial load in pretensioned members where strand embedment is less than the development length as provided in 12.9.1.1.....0.85

**C.9.3.3** — Development lengths specified in Chapter 12 do not require a  $\phi$ -factor.

**C.9.3.4** — For structures that rely on intermediate precast structural walls in Seismic Design Category D, E, or F, special moment frames, or special structural walls to resist **E**,  $\phi$  shall be modified as given in (a) through (c):

- (a) For any structural member that is designed to resist **E**,  $\phi$  for shear shall be 0.60 if the nominal shear strength of the member is less than the shear corresponding to the development of the nominal flexural strength of the member. The nominal flexural strength shall be determined considering the most critical factored axial loads and including **E**;
- (b) For diaphragms,  $\phi$  for shear shall not exceed the minimum  $\phi$  for shear used for the vertical components of the primary lateral-force-resisting system;
- (c) For joints and diagonally reinforced coupling beams,  $\phi$  for shear shall be 0.85.

**C.9.3.5** — In Chapter 22,  $\phi$  shall be 0.65 for flexure, compression, shear, and bearing of structural plain concrete.

**RC.9.3.2.7** — If a critical section occurs in a region where strand is not fully developed, failure may be by bond slip. Such a failure resembles a brittle shear failure; hence the requirement for a reduced  $\phi$ .

**RC.9.3.4** — Section C.9.3.4(a) refers to brittle members, such as low-rise walls or portions of walls between openings, or diaphragms that are impractical to reinforce to raise their nominal shear strength above nominal flexural strength for the pertinent loading conditions.

Short structural walls were the primary vertical elements of the lateral-force-resisting system in many of the parking structures that sustained damage during the 1994 Northridge earthquake. Section C.9.3.4(b) requires the shear strength reduction factor for diaphragms to be 0.60 if the shear strength reduction factor for the walls is 0.60.

**RC.9.3.5** — The strength reduction factor  $\phi$  for structural plain concrete design is the same for all strength conditions. Since both flexural tension strength and shear strength for plain concrete depend on the tensile strength characteristics of the concrete, with no reserve strength or ductility possible due to the absence of reinforcement, equal strength reduction factors for both bending and shear are considered appropriate.

**Notes**

## APPENDIX D — ANCHORING TO CONCRETE

### CODE

#### D.1 — Definitions

**Adhesive** — Chemical components formulated from organic polymers, or a combination of organic polymers and inorganic materials that cure when blended together.

**Adhesive anchor** — A post-installed anchor, inserted into hardened concrete with an anchor hole diameter not greater than 1.5 times the anchor diameter, that transfers loads to the concrete by bond between the anchor and the adhesive, and bond between the adhesive and the concrete.

**Anchor** — A steel element either cast into concrete or post-installed into a hardened concrete member and used to transmit applied loads to the concrete. Cast-in anchors include headed bolts, hooked bolts (J- or L-bolt), and headed studs. Post-installed anchors included expansion anchors, undercut anchors, and adhesive anchors. Steel elements for adhesive anchors include threaded rods, deformed reinforcing bars, or internally threaded steel sleeves with external deformations.

**Anchor group** — A number of similar anchors having approximately equal effective embedment depths with spacing  $s$  between adjacent anchors such that the protected areas overlap. See D.3.1.1.

**Anchor pullout strength** — The strength corresponding to the anchoring device or a major component of the device sliding out from the concrete without breaking out a substantial portion of the surrounding concrete.

**Anchor reinforcement** — Reinforcement used to transfer the full design load from the anchors into the structural member. See D.5.2.9 or D.6.2.9.

### COMMENTARY

#### RD.1 — Definitions

**Adhesive** — Organic polymers used in adhesives can include, but are not limited to, epoxies, polyurethanes, polyesters, methyl methacrylates, and vinyl esters.

**Adhesive anchor** — The design model included in Appendix D for adhesive anchors is based on the behavior of anchors with hole diameters not exceeding 1.5 times the anchor diameter. Anchors with hole diameters exceeding 1.5 times the anchor diameter behave differently and are therefore excluded from the scope of Appendix D and ACI 355.4. To limit shrinkage and reduce displacement under load, most adhesive anchor systems require the annular gap to be as narrow as practical while still maintaining sufficient clearance for insertion of the anchor element in the adhesive-filled hole and ensuring complete coverage of the bonded area over the embedded length. The annular gap for reinforcing bars is generally larger than that for threaded rods. The required hole size is provided in the Manufacturer's Printed Installation Instructions (MPII).

**Anchor group** — For all potential failure modes (steel, concrete breakout, pullout, side-face blowout, and prayout), only those anchors susceptible to a particular failure mode should be considered when evaluating the strength associated with that failure mode.

**Anchor reinforcement** — Anchor reinforcement is designed and detailed specifically for the purpose of transferring anchor loads from the anchors into the structural member. Hairpins are generally used for this purpose (see RD.5.2.9 and RD.6.2.9); however, other configurations that can be shown to effectively transfer the anchor load are acceptable.

**CODE****COMMENTARY**

**Attachment** — The structural assembly, external to the surface of the concrete, that transmits loads to or receives loads from the anchor.

**Brittle steel element** — An element with a tensile test elongation of less than 14 percent, or reduction in area of less than 30 percent, or both.

**Cast-in anchor** — A headed bolt, headed stud, or hooked bolt installed before placing concrete.

**Concrete breakout strength** — The strength corresponding to a volume of concrete surrounding the anchor or group of anchors separating from the member.

**Concrete pryout strength** — The strength corresponding to formation of a concrete spall behind short, stiff anchors displaced in the direction opposite to the applied shear force.

**Distance sleeve** — A sleeve that encases the center part of an undercut anchor, a torque-controlled expansion anchor, or a displacement-controlled expansion anchor, but does not expand.

**Ductile steel element** — An element with a tensile test elongation of at least 14 percent and reduction in area of at least 30 percent. A steel element meeting the requirements of ASTM A307 shall be considered as a ductile steel element. Except as modified by D.3.3.4.3(a)6 for earthquake effects, deformed reinforcing bars meeting the requirements of ASTM A615, A706, or A955 shall be considered as ductile steel elements.

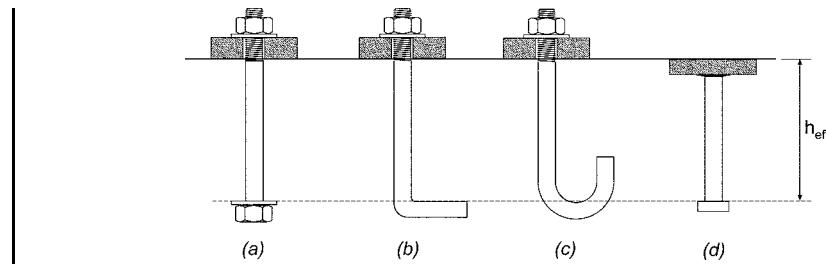
**Edge distance** — The distance from the edge of the concrete surface to the center of the nearest anchor.

**Effective embedment depth** — The overall depth through which the anchor transfers force to or from the surrounding concrete. The effective embedment depth will normally be the depth of the concrete failure surface in tension applications. For cast-in headed anchor bolts and headed studs, the effective embedment depth is measured from the bearing contact surface of the head.

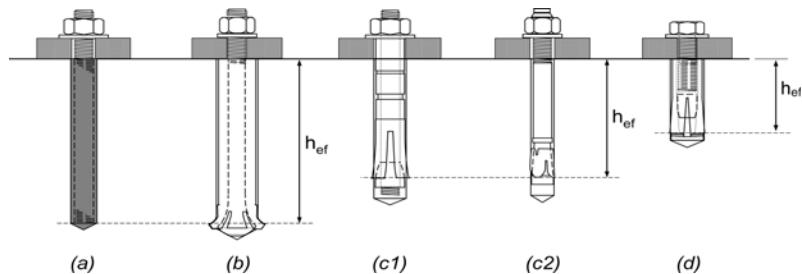
**Brittle steel element** — The 14 percent elongation should be measured over the gauge length specified in the appropriate ASTM standard for the steel.

**Ductile steel element** — The 14 percent elongation should be measured over the gauge length specified in the appropriate ASTM standard for the steel. Due to concerns over fracture in cut threads, it should be verified that threaded deformed reinforcing bars satisfy the strength requirement of 12.14.3.2.

**Effective embedment depth** — Effective embedment depths for a variety of anchor types are shown in Fig. RD.1.1.

**CODE****COMMENTARY**

(A) Cast-in anchors: (a) hex head bolt with washer; (b) L-bolt; (c) J-bolt; and (d) welded headed stud.



(B) Post-installed anchors: (a) adhesive anchor; (b) undercut anchor; (c) torque-controlled expansion anchors ((c1) sleeve-type and (c2) stud-type); and (d) drop-in type displacement-controlled expansion anchor.

*Fig. RD.1.1—Types of anchors.*

**Expansion anchor** — A post-installed anchor, inserted into hardened concrete that transfers loads to or from the concrete by direct bearing or friction or both. Expansion anchors may be torque-controlled, where the expansion is achieved by a torque acting on the screw or bolt; or displacement-controlled, where the expansion is achieved by impact forces acting on a sleeve or plug and the expansion is controlled by the length of travel of the sleeve or plug.

**Expansion sleeve** — The outer part of an expansion anchor that is forced outward by the center part, either by applied torque or impact, to bear against the sides of the predrilled hole.

**Five percent fractile** — A statistical term meaning 90 percent confidence that there is 95 percent probability of the actual strength exceeding the nominal strength.

**Five percent fractile** — The determination of the coefficient  $K_{05}$  associated with the 5 percent fractile,  $\bar{x} - K_{05}s_s$  depends on the number of tests,  $n$ , used to compute the sample mean,  $\bar{x}$ , and sample standard deviation,  $s_s$ . Values of  $K_{05}$  range, for example, from 1.645 for  $n = \infty$ , to 2.010 for  $n = 40$ , and 2.568 for  $n = 10$ . With this definition of the 5 percent fractile, the nominal strength in D.4.2 is the same as the characteristic strength in ACI 355.2.

**CODE****COMMENTARY**

**Headed stud** — A steel anchor conforming to the requirements of AWS D1.1 and affixed to a plate or similar steel attachment by the stud arc welding process before casting.

**Hooked bolt** — A cast-in anchor anchored mainly by bearing of the 90-degree bend (L-bolt) or 180-degree bend (J-bolt) against the concrete, at its embedded end, and having a minimum  $e_h$  of  $3d_a$ .

**Horizontal or upwardly inclined anchor** — An anchor installed in a hole drilled horizontally or in a hole drilled at any orientation above horizontal.

**Manufacturer's Printed Installation Instructions (MPII)** — Published instructions for the correct installation of the anchor under all covered installation conditions as supplied in the product packaging.

**Post-installed anchor** — An anchor installed in hardened concrete. Expansion, undercut, and adhesive anchors are examples of post-installed anchors.

**Projected area** — The area on the free surface of the concrete member that is used to represent the larger base of the assumed rectilinear failure surface. See D.5.2.1 and D.6.2.1.

**Projected influence area** — The rectilinear area on the free surface of the concrete member that is used to calculate the bond strength of adhesive anchors. See D.5.5.1.

**Side-face blowout strength** — The strength of anchors with deeper embedment but thinner side cover corresponding to concrete spalling on the side face around the embedded head while no major breakout occurs at the top concrete surface.

**Specialty insert** — Predesigned and prefabricated cast-in anchors specifically designed for attachment of bolted or slotted connections. Specialty inserts are often used for handling, transportation, and erection, but are also used for anchoring structural elements. Specialty inserts are not within the scope of this appendix.

**Horizontal or upwardly inclined anchor** — Figure RD.1.2 illustrates the potential hole orientations for horizontal or upwardly inclined anchors.

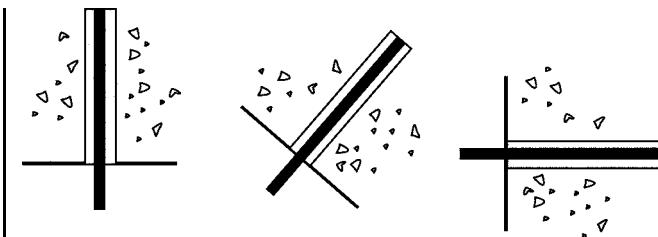


Fig. RD.1.2—Possible orientations of horizontal or upwardly inclined anchors.

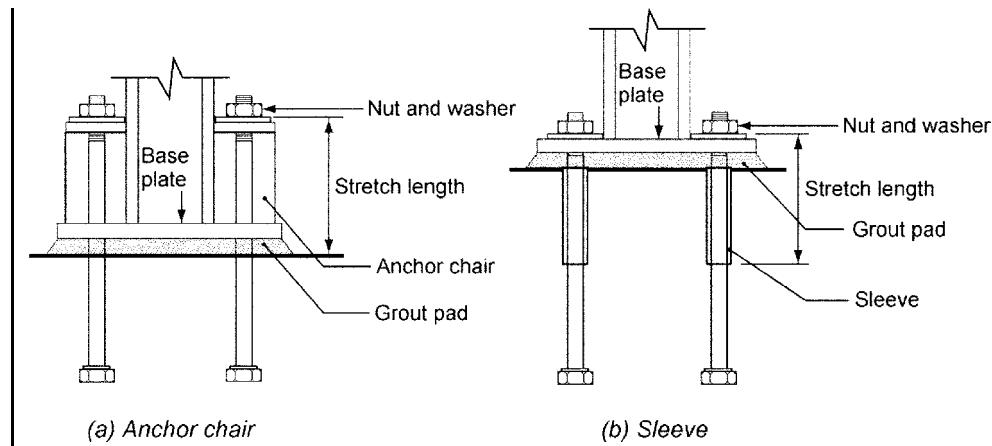
**CODE****COMMENTARY**

Fig. RD.1.3—Illustrations of stretch length (see D.3.3.4.3(a)).

**Stretch length** — Length of anchor, extending beyond concrete in which it is anchored, subject to full tensile load applied to anchor, and for which cross-sectional area is minimum and constant.

**Supplementary reinforcement** — Reinforcement that acts to restrain the potential concrete breakout but is not designed to transfer the full design load from the anchors into the structural member.

**Undercut anchor** — A post-installed anchor that develops its tensile strength from the mechanical interlock provided by undercutting of the concrete at the embedded end of the anchor. The undercutting is achieved with a special drill before installing the anchor or alternatively by the anchor itself during its installation.

## D.2 — Scope

**D.2.1** — This appendix provides design requirements for anchors in concrete used to transmit structural loads by means of tension, shear, or a combination of tension and shear between: (a) connected structural elements; or (b) safety-related attachments and structural elements. Safety levels specified are intended for in-service conditions, rather than for short-term handling and construction conditions.

**Stretch length** — Length of an anchor over which inelastic elongations are designed to occur for earthquake loadings. Examples illustrating stretch length are shown in Fig. RD.1.3.

**Supplementary reinforcement** — Supplementary reinforcement has a configuration and placement similar to anchor reinforcement but is not specifically designed to transfer loads from the anchors into the structural member. Stirrups, as used for shear reinforcement, may fall into this category.

## RD.2 — Scope

**RD.2.1** — Appendix D is restricted in scope to structural anchors that transmit structural loads related to strength, stability, or life safety. Two types of applications are envisioned. The first is connections between structural elements where the failure of an anchor or an anchor group could result in loss of equilibrium or stability of any portion of the structure. The second is where safety-related attachments that are not part of the structure (such as sprinkler systems, heavy suspended pipes, or barrier rails) are attached to structural elements. The levels of safety defined by the combinations of load factors and  $\phi$ -factors are appropriate for structural applications. Other standards may require more stringent safety levels during temporary handling.

**CODE****COMMENTARY**

**D.2.2** — This appendix applies to cast-in anchors and to post-installed expansion (torque-controlled and displacement-controlled), undercut, and adhesive anchors. Adhesive anchors shall be installed in concrete having a minimum age of 21 days at time of anchor installation. Specialty inserts, through-bolts, multiple anchors connected to a single steel plate at the embedded end of the anchors, grouted anchors, and direct anchors such as powder or pneumatic actuated nails or bolts are not included in the provisions of **Appendix D**. Reinforcement used as part of the embedment shall be designed in accordance with other parts of this Code.

**D.2.3** — Design provisions are included for the following types of anchors:

- (a) Headed studs and headed bolts having a geometry that has been demonstrated to result in a pullout strength in uncracked concrete equal to or exceeding  $1.4N_p$ , where  $N_p$  is given in **Eq. (D-14)**;
- (b) Hooked bolts having a geometry that has been demonstrated to result in a pullout strength without the benefit of friction in uncracked concrete equal to or exceeding  $1.4N_p$ , where  $N_p$  is given in **Eq. (D-15)**;
- (c) Post-installed expansion and undercut anchors that meet the assessment criteria of ACI 355.2; and
- (d) Adhesive anchors that meet the assessment criteria of ACI 355.4.

**D.2.4** — Load applications that are predominantly high cycle fatigue or impact loads are not covered by this appendix.

### **D.3 — General requirements**

**D.3.1** — Anchors and anchor groups shall be designed for critical effects of factored loads as determined by elastic analysis. Plastic analysis approaches are permitted where nominal strength is controlled by ductile steel elements, provided that deformational compatibility is taken into account.

**RD.2.2** — Provisions for design of adhesive anchors were added in the 2011 Code. Adhesive anchors are particularly sensitive to a number of factors including installation direction and loading type. Where adhesive anchors are used to resist sustained tension, the provisions include testing requirements for horizontal and upwardly inclined installations in **D.3.4** and design and certification requirements for sustained tension load cases in **D.3.5** and **D.9.2.2** through **D.9.2.4**, respectively. Adhesive anchors qualified in accordance with ACI 355.4 are tested in concrete with compressive strengths within two ranges: 2500 to 4000 psi and 6500 to 8500 psi. Bond strength is in general not highly sensitive to concrete compressive strength. The design performance of adhesive anchors cannot be ensured by establishing a minimum concrete compressive strength at the time of installation in early-age concrete. Therefore, a minimum concrete age of 21 days at the time of adhesive anchor installation was adopted.

The wide variety of shapes and configurations of specialty inserts precludes prescription of generalized tests and design equations. Specialty inserts are not covered by **Appendix D** provisions.

**RD.2.3** — Typical cast-in headed studs and headed bolts with geometries consistent with ANSI/ASME B1.1,<sup>D.1</sup> B18.2.1,<sup>D.2</sup> and B18.2.6<sup>D.3</sup> have been tested and proven to behave predictably, so calculated pullout strengths are acceptable.

Post-installed anchors do not have predictable pullout strengths, and therefore qualification tests to establish the pullout strengths per ACI 355.2 are required. For a post-installed anchor to be used in conjunction with the requirements of this appendix, the results of the ACI 355.2 tests have to indicate that pullout failures exhibit an acceptable load-displacement characteristic or that pullout failures are precluded by another failure mode. For adhesive anchors, the characteristic bond stress and suitability for structural applications are established by testing in accordance with ACI 355.4.

**RD.2.4** — The exclusion from the scope of load applications producing high cycle fatigue or extremely short duration impact (such as blast or shock wave) are not meant to exclude seismic load effects. **D.3.3** presents additional requirements for design when seismic loads are included.

### **RD.3 — General requirements**

**RD.3.1** — When the strength of an anchor group is governed by breakage of the concrete, the behavior is brittle and there is limited redistribution of the forces between the highly stressed and less stressed anchors. In this case, the theory of elasticity is required to be used assuming the attachment that distributes loads to the anchors is sufficiently

**CODE****COMMENTARY**

**D.3.1.1** — Anchor group effects shall be considered wherever two or more anchors have spacing less than the critical spacing as follows:

Failure mode under investigation	Critical spacing
Concrete breakout in tension	$3h_{ef}$
Bond strength in tension	$2c_{Na}$
Concrete breakout in shear	$3c_{a1}$

Only those anchors susceptible to the particular failure mode under investigation shall be included in the group.

**D.3.2** — The design strength of anchors shall equal or exceed the largest required strength calculated from the applicable load combinations in **9.2** or **C.9.2**.

### D.3.3 — Seismic design requirements

**D.3.3.1** — Anchors in structures assigned to Seismic Design Category C, D, E, or F shall satisfy the additional requirements of **D.3.3.2** through **D.3.3.7**.

stiff. The forces in the anchors are considered to be proportional to the external load and its distance from the neutral axis of the anchor group.

If anchor strength is governed by ductile yielding of the anchor steel, significant redistribution of anchor forces can occur. In this case, an analysis based on the theory of elasticity will be conservative. **References D.4** to **D.6** discuss nonlinear analysis, using theory of plasticity, for the determination of the capacities of ductile anchor groups.

### RD.3.3 — Seismic design requirements

Unless **D.3.3.4.1** or **D.3.3.5.1** apply, all anchors in structures assigned to Seismic Design Categories C, D, E, or F are required to satisfy the additional requirements of **D.3.3.1** through **D.3.3.7** regardless of whether earthquake loads are included in the controlling load combination for the anchor design. In addition, all post-installed anchors in structures assigned to Seismic Design Categories C, D, E, or F must meet the requirements of ACI 355.2 or ACI 355.4 for prequalification of anchors to resist earthquake loads. Ideally, for tension loadings, anchor strength should be governed by yielding of the ductile steel element of the anchor. If the anchor cannot meet the specified ductility requirements of **D.3.3.4.3(a)**, then the attachment should be either designed to yield if it is structural or light gauge steel, or designed to crush if it is wood. If ductility requirements of **D.3.3.4.3(a)** are satisfied, then any attachments to the anchor should be designed not to yield. In designing attachments using yield mechanisms to provide adequate ductility, as permitted by **D.3.3.4.3(b)** and **D.3.3.5.3(a)**, the ratio of specified yield strength to expected strength for the material of the attachment should be considered in determining the design force. The value used for the expected strength should consider both material overstrength and strain-hardening effects. For example, the material in a connection element could yield and, due to an increase in its strength with strain hardening, cause a secondary failure of a sub-element or place extra force or deformation demands on the anchors. For a structural steel attachment, if only the specified yield strength of the steel is known, the expected strength should be taken as about 1.5 times the specified yield strength. If the actual yield strength of the steel is known, the expected strength should be taken as about 1.25 times the actual yield strength.

Under seismic conditions, the direction of shear may not be predictable. The full shear force should be assumed in any direction for a safe design.

**CODE****COMMENTARY**

| **D.3.3.2** — The provisions of [Appendix D](#) do not apply to the design of anchors in plastic hinge zones of concrete structures under earthquake forces.

**RD.3.3.2** — The design provisions in [Appendix D](#) do not apply for anchors in plastic hinge zones. The possible higher levels of cracking and spalling in plastic hinge zones are beyond the conditions for which the nominal concrete-governed strength values in [Appendix D](#) are applicable. Plastic hinge zones are considered to extend a distance equal to twice the member depth from any column or beam face, and also include any other sections in walls, frames, and slabs where yielding of reinforcement is likely to occur as a result of lateral displacements.

Where anchors must be located in plastic hinge regions, they should be detailed so that the anchor forces are transferred directly to anchor reinforcement that is specifically designed to carry the anchor forces into the body of the member beyond the anchorage region. Configurations that rely on concrete tensile strength should not be used.

**D.3.3.3** — Post-installed anchors shall be qualified for earthquake loading in accordance with ACI 355.2 or ACI 355.4. The pullout strength  $N_p$  and steel strength in shear  $V_{sa}$  of expansion and undercut anchors shall be based on the results of the ACI 355.2 Simulated Seismic Tests. For adhesive anchors, the steel strength in shear  $V_{sa}$  and the characteristic bond stresses  $\tau_{uncr}$  and  $\tau_{cr}$  shall be based on results of the ACI 355.4 Simulated Seismic Tests.

**RD.3.3.3** — Anchors that are not suitable for use in cracked concrete should not be used to resist earthquake loads. Qualification of post-installed anchors for use in cracked concrete is an integral part of the qualification for resisting earthquake loads in ACI 355.2 and ACI 355.4. The design values obtained from the Simulated Seismic Tests of ACI 355.2 and ACI 355.4 are expected to be less than those for static load applications.

**D.3.3.4 — Requirements for tensile loading**

**D.3.3.4.1** — Where the tensile component of the strength-level earthquake force applied to a single anchor or group of anchors is equal to or less than 20 percent of the total factored anchor tensile force associated with the same load combination, it shall be permitted to design a single anchor or group of anchors to satisfy [D.5](#) and the tensile strength requirements of [D.4.1.1](#).

**D.3.3.4.2** — Where the tensile component of the strength-level earthquake force applied to anchors exceeds 20 percent of the total factored anchor tensile force associated with the same load combination, anchors and their attachments shall be designed in accordance with [D.3.3.4.3](#). The anchor design tensile strength shall be determined in accordance with [D.3.3.4.4](#).

**RD.3.3.4 — Requirements for tensile loading**

**RD.3.3.4.1** — The requirements of [D.3.3.4.3](#) need not apply where the applied earthquake tensile force is a small fraction of the total factored tension force.

**RD.3.3.4.2** — If the ductile steel element is ASTM A36 or ASTM A307 steel, the  $f_{uta}/f_y$  value is typically about 1.5 and the anchor can stretch considerably before rupturing at the threads. For other steels, calculations may need to be made to ensure that a similar behavior can occur. [RD.5.1.2](#) provides additional information on the steel properties of anchors. Provision of upset threaded ends, whereby the threaded end of the rod is enlarged to compensate for the area reduction associated with threading, can ensure that yielding occurs over the stretch length regardless of the ratio of the yield to ultimate strength of the anchor.

**CODE**

**D.3.3.4.3** — Anchors and their attachments shall satisfy one of options (a) through (d):

(a) For single anchors, the concrete-governed strength shall be greater than the steel strength of the anchor. For anchor groups, the ratio of the tensile load on the most highly stressed anchor to the steel strength of that anchor shall be equal to or greater than the ratio of the tensile load on tension-loaded anchors to the concrete-governed strength of those anchors. In each case:

1. The steel strength shall be taken as 1.2 times the nominal steel strength of the anchor.
2. The concrete-governed strength shall be taken as the nominal strength considering pullout, side-face blowout, concrete breakout, and bond strength as applicable. For consideration of pullout in groups, the ratio shall be calculated for the most highly stressed anchor.

In addition, the following shall be satisfied:

3. Anchors shall transmit tensile loads via a ductile steel element with a stretch length of at least eight anchor diameters unless otherwise determined by analysis.
4. Where anchors are subject to load reversals, the anchor shall be protected against buckling.
5. Where connections are threaded and the ductile steel elements are not threaded over their entire length, the ratio of  $f_{uta}/f_{ya}$  shall not be less than 1.3 unless the threaded portions are upset. The upset portions shall not be included in the stretch length.
6. Deformed reinforcing bars used as ductile steel elements to resist earthquake effects shall be limited to ASTM A615 Grades 40 and 60 satisfying the requirements of **21.1.5.2(a)** and **(b)** or ASTM A706 Grade 60.

(b) The anchor or group of anchors shall be designed for the maximum tension that can be transmitted to the anchor or group of anchors based on the development of a ductile yield mechanism in the attachment in flexure, shear, or bearing, or a combination of those conditions, and considering both material overstrength and strain hardening effects for the attachment. The anchor design tensile strength shall be calculated from **D.3.3.4.4**.

(c) The anchor or group of anchors shall be designed for the maximum tension that can be transmitted to the anchors by a non-yielding attachment. The anchor design tensile strength shall be calculated from **D.3.3.4.4**.

**COMMENTARY**

**RD.3.3.4.3** — Four options are provided for determining the required anchor or attachment strength to protect against nonductile tension failure:

In option (a), anchor ductility requirements are imposed and the required anchor strength is determined using strength-level earthquake forces acting on the structure. Research<sup>D.7,D.8</sup> has shown that if the steel of the anchor yields before the concrete anchorage fails, no reduction in the anchor tensile strength is needed for earthquake loadings. Ductile steel anchors should satisfy the **definition for ductile steel elements in D.1**. To facilitate comparison between steel strength, which is based on the most highly-stressed anchor, and concrete strength based on group behavior, the design is performed on the basis of the ratio of applied load to strength for the steel and concrete, respectively.

For some structures, anchors provide the best locations for energy dissipation in the nonlinear range of response. The stretch length of the anchor affects the lateral displacement capacity of the structure and therefore that length needs to be sufficient such that the displacement associated with the design-basis earthquake can be achieved.<sup>D.9</sup> Observations from earthquakes indicate that the provision of a stretch length of eight anchor diameters results in good structural performance. Where the required stretch length is calculated, the relative stiffness of the connected elements needs to be considered. When an anchor is subject to load reversals, and its yielding length outside the concrete exceeds six anchor diameters, buckling of the anchor in compression is likely. Buckling can be restrained by placing the anchor in a tube. However, care must be taken that the tube does not share in resisting the tensile load assumed to act on the anchor. For anchor bolts that are not threaded over their length, it is important to ensure that yielding occurs over the unthreaded portion of the bolt within the stretch length prior to failure in the threads. This is accomplished by maintaining sufficient margin between the specified yield and ultimate strengths of the bolt. It should be noted that the available stretch length may be adversely influenced by construction techniques (for example, the addition of leveling nuts to the examples shown in **Fig. RD.1.3**).

In option (b), the anchor is designed for the tension force associated with the expected strength of the metal or similar material of the attachment. For option (b), as discussed in **RD.3.3**, care must be taken in design to consider the consequences of potential differences between the specified yield strength and the expected strength of the attachment. An example is **21.4.3** for the design of connections of intermediate precast walls where a connection not designed to yield should develop at least  $1.5S_y$ , where  $S_y$  is the nominal strength of the yielding element based on its specified yield strength. Similarly, steel design manuals require structural steel connections that are designated nonyielding and part of the seismic load path to have design strengths that exceed a

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(d) The anchor or group of anchors shall be designed for the maximum tension obtained from design load combinations that include  $E$ , with  $E$  increased by  $\Omega_0$ . The anchor design tensile strength shall satisfy the tensile strength requirements of D.4.1.1.

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multiple of the nominal strength. That multiple depends on a factor relating the likely actual to specified yield strength of the material and an additional factor exceeding unity to account for material strain hardening. For attachments of cold-formed steel or wood, similar principles should be used for determining the expected strength of the attachment in order to determine the required strength of the anchorage.

Additional guidance on the use of options (a) through (d) is provided in the *2009 NEHRP Provisions*<sup>D.9</sup>. The design of anchors in accordance with option (a) should be used only where the anchor yield behavior is well defined and where the interaction of the yielding anchor with other elements in the load path has been adequately addressed. For the design of anchors per option (b), the force associated with yield of a steel attachment, such as an angle, baseplate, or web tab, should be the expected strength, rather than the specified yield strength of the steel. Option (c) may apply to a variety of special cases, such as the design of sill bolts where the crushing of the wood limits the force that can be transferred to the bolt, or where the provisions of AISC 341, Seismic Provisions for Structural Steel Buildings,<sup>D.10</sup> specify loads based on member strengths.

**D.3.3.4.4** — The anchor design tensile strength for resisting earthquake forces shall be determined from consideration of (a) through (e) for the failure modes given in Table D.4.1.1 assuming the concrete is cracked unless it can be demonstrated that the concrete remains uncracked:

- (a)  $\phi N_{sa}$  for a single anchor, or for the most highly stressed individual anchor in a group of anchors;
- (b)  $0.75\phi N_{cb}$  or  $0.75\phi N_{cbg}$ , except that  $N_{cb}$  or  $N_{cbg}$  need not be calculated where anchor reinforcement satisfying D.5.2.9 is provided;
- (c)  $0.75\phi N_{pn}$  for a single anchor, or for the most highly stressed individual anchor in a group of anchors;
- (d)  $0.75\phi N_{sb}$  or  $0.75\phi N_{sbg}$ ; and
- (e)  $0.75\phi N_a$  or  $0.75\phi N_{ag}$

where  $\phi$  is in accordance with D.4.3 or D.4.4.

**D.3.3.4.5** — Where anchor reinforcement is provided in accordance with D.5.2.9, no reduction in design tensile strength beyond that specified in D.5.2.9 shall be required.

**RD.3.3.4.4** — The reduced anchor nominal tensile strengths associated with concrete failure modes is to account for increased cracking and spalling in the concrete resulting from seismic actions. Because seismic design generally assumes that all or portions of the structure are loaded beyond yield, it is likely that the concrete is cracked throughout for the purpose of determining the anchor strength. In locations where it can be demonstrated that the concrete does not crack, uncracked concrete may be assumed for determining the anchor strength as governed by concrete failure modes.

**RD.3.3.4.5** — Where anchor reinforcement as defined in D.5.2.9 and D.6.2.9 is used, with the properties as defined in 21.1.5.2, no separation of the potential breakout prism from the substrate is likely to occur provided the anchor reinforcement is designed for a load greater than the concrete breakout strength.

**CODE****COMMENTARY****D.3.3.5 — Requirements for shear loading****RD.3.3.5 — Requirements for shear loading**

Where the shear component of the earthquake force applied to the anchor exceeds 20 percent of the total anchor shear force, three options are recognized for determining the required shear strength to protect the anchor or group of anchors against premature shear failure. There is no option corresponding to option (a) of D.3.3.4.3 because the cross section of the steel element of the anchor cannot be configured so that steel failure in shear provides any meaningful degree of ductility.

Design of the anchor or group of anchors for the strength associated with force-limiting mechanisms under option (b), such as the bearing strength at holes in a steel attachment or the combined crushing and bearing strength for wood members may be particularly relevant. Tests on typical anchor bolt connections for wood framed shear walls<sup>D.11</sup> showed that wood components attached to concrete with minimum edge distances exhibited ductile behavior. Wood “yield” (crushing) was the first limiting state and resulted in nail slippage in shear. Nail slippage combined with bolt bending provided the required ductility and toughness for the shear walls and limited the loads acting on the bolts. Procedures for defining bearing and shear limit states for connections to cold-formed steel are described in AISI S100-07<sup>D.12</sup> and examples of strength calculations are provided in the AISI “Cold-Formed Steel Design Manual.”<sup>D.13</sup> In such cases, consideration should be given to whether exceedance of the bearing strength may lead to tearing and an unacceptable loss of connectivity. Where anchors are located far from edges it may not be possible to design such that anchor reinforcement controls the anchor strength. In such cases, anchors should be designed for overstrength in accordance with option (c).

**D.3.3.5.1** — Where the shear component of the strength-level earthquake force applied to the anchor or group of anchors is equal to or less than 20 percent of the total factored anchor shear force associated with the same load combination, it shall be permitted to design the anchor or group of anchors to satisfy D.6 and the shear strength requirements of D.4.1.1.

**D.3.3.5.2** — Where the shear component of the strength-level earthquake force applied to anchors exceeds 20 percent of the total factored anchor shear force associated with the same load combination, anchors and their attachments shall be designed in accordance with D.3.3.5.3. The anchor design shear strength for resisting earthquake forces shall be determined in accordance with D.6.

**RD.3.3.5.1** — The requirements of D.3.3.5.3 need not apply where the applied earthquake shear force is a small fraction of the total factored shear force.

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**D.3.3.5.3** — Anchors and their attachments shall be designed using one of options (a) through (c):

- (a) The anchor or group of anchors shall be designed for the maximum shear that can be transmitted to the anchor or group of anchors based on the development of a ductile yield mechanism in the attachment in flexure, shear, or bearing, or a combination of those conditions, and considering both material overstrength and strain hardening effects in the attachment.
- (b) The anchor or group of anchors shall be designed for the maximum shear that can be transmitted to the anchors by a non-yielding attachment.
- (c) The anchor or group of anchors shall be designed for the maximum shear obtained from design load combinations that include  $E$ , with  $E$  increased by  $\Omega_0$ . The anchor design shear strength shall satisfy the shear strength requirements of D.4.1.1.

**D.3.3.5.4** — Where anchor reinforcement is provided in accordance with D.6.2.9, no reduction in design shear strength beyond that specified in D.6.2.9 shall be required.

**D.3.3.6** — Single anchors or groups of anchors that are subjected to both tension and shear forces shall be designed to satisfy the requirements of D.7, with the anchor design tensile strength calculated from D.3.3.4.4.

**D.3.3.7** — Anchor reinforcement used in structures assigned to Seismic Design Category C, D, E, or F shall be deformed reinforcement and shall be limited to ASTM A615 Grades 40 and 60 satisfying the requirements of 21.1.5.2(a) and (b) or ASTM A706 Grade 60.

**D.3.4** — Adhesive anchors installed horizontally or upwardly inclined shall be qualified in accordance with ACI 355.4 requirements for sensitivity to installation direction.

**D.3.5** — For adhesive anchors subjected to sustained tension loading, D.4.1.2 shall be satisfied. For groups of adhesive anchors, Eq. (D-1) shall be satisfied for the anchor that resists the highest sustained tension load. Installer certification and inspection requirements for horizontal and upwardly inclined adhesive anchors subjected to sustained tension loading shall be in accordance with D.9.2.2 through D.9.2.4.

**RD.3.4** — ACI 355.4 includes optional tests to confirm the suitability of adhesive anchors for horizontal and upwardly inclined installations.

**RD.3.5** — For adhesive anchors subjected to sustained tension loading, an additional calculation for the sustained portion of the factored load for a reduced bond resistance is required to account for possible bond strength reductions under sustained load. The resistance of adhesive anchors to sustained tension load is particularly dependent on correct installation, including hole cleaning, adhesive metering and mixing, and prevention of voids in the adhesive bond line (annular gap). In addition, care should be taken in the selection of the correct adhesive and bond strength for the expected conditions on-site such as the concrete condition during

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installation (dry or saturated, cold or hot), the drilling method used (rotary impact drill/rock drill or core drill), and anticipated in-service temperature variations in the concrete. Installer certification and inspection requirements associated with the use of adhesive anchors for horizontal and upwardly inclined installations to resist sustained tension loads are addressed in D.9.2.2 through D.9.2.4.

Adhesive anchors are particularly sensitive to installation direction and loading type. Adhesive anchors installed overhead that resist sustained tension loads are of concern since previous applications of this type have led to failures. Other anchor types may be more appropriate for such cases. Where adhesive anchors are used in overhead applications subjected to sustained tension loading, it is essential to meet test requirements of ACI 355.4 for sensitivity to installation direction, use certified installers, and require special inspection.

**D.3.6** — Modification factor  $\lambda_a$  for lightweight concrete shall be taken as:

Cast-in and undercut anchor concrete failure....	<b>1.0</b> $\lambda$
Expansion and adhesive anchor concrete failure.....	<b>0.8</b> $\lambda$
Adhesive anchor bond failure per Eq. (D-22) ....	<b>0.6</b> $\lambda$

where  $\lambda$  is determined in accordance with 8.6.1. It shall be permitted to use an alternate value of  $\lambda_a$  where tests have been performed and evaluated in accordance with ACI 355.2 or ACI 355.4.

**D.3.7** — The values of  $f'_c$  used for calculation purposes in this appendix shall not exceed 10,000 psi for cast-in anchors, and 8000 psi for post-installed anchors. Testing is required for post-installed anchors when used in concrete with  $f'_c$  greater than 8000 psi.

**RD.3.6** — The number of tests available to establish the strength of anchors in lightweight concrete is limited. Lightweight concrete tests of cast-in headed studs indicate that the present reduction factor  $\lambda$  adequately captures the influence of lightweight concrete.<sup>D.14,D.15</sup> Anchor manufacturer data developed for evaluation reports on both post-installed expansion and adhesive anchors indicate that a reduced  $\lambda$  is needed to provide the necessary safety factor for the respective design strength. ACI 355.2 and ACI 355.4 provide procedures whereby a specific value of  $\lambda_a$  can be used based on testing, assuming the lightweight concrete is similar to the reference test material.

**RD.3.7** — A limited number of tests of cast-in and post-installed anchors in high-strength concrete<sup>D.16</sup> indicate that the design procedures contained in this appendix become unconservative, particularly for cast-in anchors in concrete with compressive strengths in the range of 11,000 to 12,000 psi. Until further tests are available, an upper limit on  $f'_c$  of 10,000 psi has been imposed in the design of cast-in anchors. This limitation is consistent with Chapters 11 and 12. ACI 355.2 and ACI 355.4 do not require testing of post-installed anchors in concrete with  $f'_c$  greater than 8000 psi. Some post-installed expansion anchors may have difficulty expanding in very high-strength concretes and the bond strength of adhesive anchors may be negatively affected by very high-strength concrete. Therefore,  $f'_c$  is limited to 8000 psi in the design of post-installed anchors unless testing is performed.

**CODE****D.4 — General requirements for strength of anchors**

**D.4.1** — Strength design of anchors shall be based either on computation using design models that satisfy the requirements of **D.4.2**, or on test evaluation using the 5 percent fractile of applicable test results for the following:

- (a) Steel strength of anchor in tension (**D.5.1**);
- (b) Concrete breakout strength of anchor in tension (**D.5.2**);
- (c) Pullout strength cast-in, post-installed expansion or undercut anchor in tension (**D.5.3**);
- (d) Concrete side-face blowout strength of headed anchor in tension (**D.5.4**);
- (e) Bond strength of adhesive anchor in tension (**D.5.5**);
- (f) Steel strength of anchor in shear (**D.6.1**);
- (g) Concrete breakout strength of anchor in shear (**D.6.2**); and
- (h) Concrete pryout strength of anchor in shear (**D.6.3**).

In addition, anchors shall satisfy the required edge distances, spacings, and thicknesses to preclude splitting failure, as required in **D.8**.

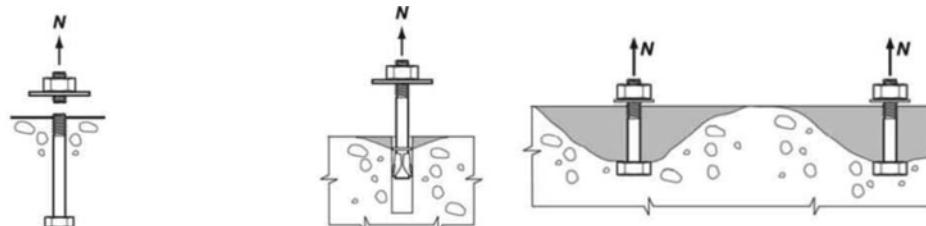
**D.4.1.1** — The design of anchors shall be in accordance with **Table D.4.1.1**. In addition, the design of anchors shall satisfy **D.3.3** for earthquake loading and **D.4.1.2** for adhesive anchors subject to sustained tensile loading.

**COMMENTARY****RD.4 — General requirements for strength of anchors**

**RD.4.1** — This section provides requirements for establishing the strength of anchors in concrete. The various types of steel and concrete failure modes for anchors are shown in **Fig. RD.4.1(a)** and **RD.4.1(b)**. Comprehensive discussions of anchor failure modes are included in **References D.17** to **D.21**. Tension failure modes related to concrete capacity include concrete breakout failure in **D.5.2** (applicable to all anchor types), pullout failure in **D.5.3** (applicable to cast-in anchors and post-installed expansion and undercut anchors), side-face blowout failure in **D.5.4** (applicable to headed anchors), and bond failure in **D.5.5** (applicable to adhesive anchors). Shear failure modes related to concrete capacity include concrete breakout failure and concrete pryout in **D.6.2** and **D.6.3**, respectively (applicable to all anchor types). Any model that complies with the requirements of **D.4.1.3** and **D.4.2** can be used to establish the concrete-related strengths. Additionally, anchor tensile and shear strengths are limited by the minimum spacings and edge distances of **D.8** as required to preclude splitting. The design of post-installed anchors recognizes that the strength of anchors is sensitive to appropriate installation; installation requirements are included in **D.9**. Some post-installed anchors are less sensitive to installation errors and tolerances. This is reflected in varied  $\phi$ -factors, given in **D.4.3** and **D.4.4**, based on the assessment criteria of ACI 355.2 and ACI 355.4.

Test procedures can also be used to determine the single-anchor breakout strength in tension and in shear. The test results, however, are required to be evaluated on a basis statistically equivalent to that used to select the values for the concrete breakout method “considered to satisfy” provisions of **D.4.2**. The basic strength cannot be taken greater than the 5 percent fractile. The number of tests has to be sufficient for statistical validity and should be considered in the determination of the 5 percent fractile.

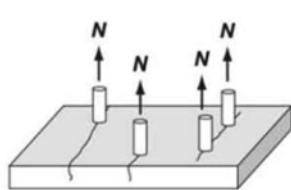
Under combined tension and bending, individual anchors in a group are subjected to different magnitude tensile forces. Similarly, under combined shear and torsion, individual anchors in a group are subjected to different magnitude shear forces. **Table D.4.1.1** includes requirements to design single anchors and individual anchors in a group to safeguard against all potential failure modes. For steel and pullout failure modes, the most highly stressed anchor in the group should be checked to ensure it has sufficient capacity to carry its required load, whereas for concrete breakout, the anchors should be checked as a group. Elastic analysis or plastic analysis of ductile anchors as described in **D.3.1** may be used to determine the loads carried by each anchor.

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(i) Steel failure

(ii) Pullout

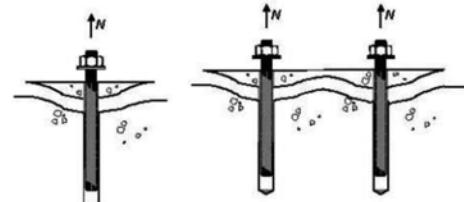
(iii) Concrete breakout



(iv) Concrete splitting



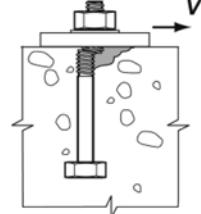
(v) Side-face blowout



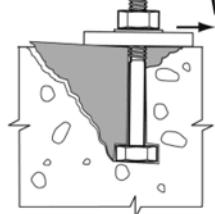
Single

Group

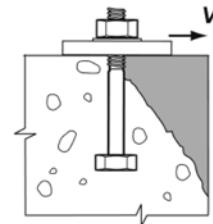
(vi) Bond failure

**(a) tensile loading**

(i) Steel failure preceded by concrete spall



(ii) Concrete pryout for anchors far from a free edge



(iii) Concrete breakout

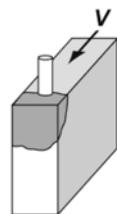
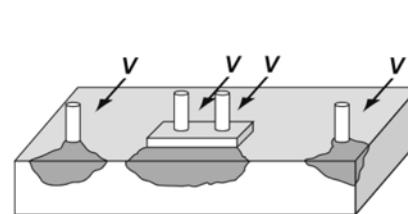
**(b) shear loading**

Fig. RD.4.1—Failure modes for anchors.

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**TABLE D.4.1.1 — REQUIRED STRENGTH OF ANCHORS, EXCEPT AS NOTED IN D.3.3**

Failure mode	Single anchor	Anchor group*	
		Individual anchor in a group	Anchors as a group
Steel strength in tension (D.5.1)	$\phi N_{sa} \geq N_{ua}$	$\phi N_{sa} \geq N_{ua,i}$	
Concrete breakout strength in tension (D.5.2)	$\phi N_{cb} \geq N_{ua}$		$\phi N_{cbg} \geq N_{ua,g}$
Pullout strength in tension (D.5.3)	$\phi N_{pn} \geq N_{ua}$	$\phi N_{pn} \geq N_{ua,i}$	
Concrete side-face blowout strength in tension (D.5.4)	$\phi N_{sb} \geq N_{ua}$		$\phi N_{sbg} \geq N_{ua,g}$
Bond strength of adhesive anchor in tension (D.5.5)	$\phi N_a \geq N_{ua}$		$\phi N_{ag} \geq N_{ua,g}$
Steel strength in shear (D.6.1)	$\phi V_{sa} \geq V_{ua}$	$\phi V_{sa} \geq V_{ua,i}$	
Concrete breakout strength in shear (D.6.2)	$\phi V_{cb} \geq V_{ua}$		$\phi V_{cbg} \geq V_{ua,g}$
Concrete pryout strength in shear (D.6.3)	$\phi V_{cp} \geq V_{ua}$		$\phi V_{cpq} \geq V_{ua,g}$

\* Required strengths for steel and pullout failure modes shall be calculated for the most highly stressed anchor in the group.

**D.4.1.2** — For the design of adhesive anchors to resist sustained tensions loads, in addition to D.4.1.1,

$$0.55\phi N_{ba} \geq N_{ua,s} \quad (\text{D-1})$$

where  $N_{ba}$  is determined in accordance with D.5.5.2.

**D.4.1.3** — When both  $N_{ua}$  and  $V_{ua}$  are present, interaction effects shall be considered using an interaction expression that results in computation of strength in substantial agreement with results of comprehensive tests. This requirement shall be considered satisfied by D.7.

**D.4.2** — The nominal strength for any anchor or group of anchors shall be based on design models that result in predictions of strength in substantial agreement with results of comprehensive tests. The materials used in the tests shall be compatible with the materials used in the structure. The nominal strength shall be based on the 5 percent fractile of the basic individual anchor strength. For nominal strengths related to concrete strength, modifications for size effects, the number of anchors, the effects of close spacing of anchors, proximity to edges, depth of the concrete member, eccentric loadings of anchor groups, and presence or absence of cracking shall be taken into account. Limits

**RD.4.1.2** — The 0.55 factor used for the additional calculation for sustained loads is correlated with ACI 355.4 test requirements and provides satisfactory performance of adhesive anchors under sustained tension loads when used in accordance with ACI 355.4. Product evaluation according to ACI 355.4 is based on sustained tension loading being present for a minimum of 50 years at a standard temperature of 70°F and a minimum of ten years at a temperature of 110°F. For longer life spans (for example, > 50 years) or higher temperatures, lower factors should be considered.

**RD.4.1.3 and RD.4.2** — D.4.1.3 and D.4.2 establish the performance factors for which anchor design models are required to be verified. Many possible design approaches exist and the user is always permitted to “design by test” using D.4.2 as long as sufficient data are available to verify the model.

The method for concrete breakout design included as “considered to satisfy” D.4.2 was developed from the Concrete Capacity Design (CCD) Method,<sup>D.18,D.19</sup> which was an adaptation of the  $\kappa$  Method<sup>D.20,D.23</sup> and is considered to be accurate, relatively easy to apply, and capable of extension to irregular layouts. The CCD Method predicts the strength of an anchor or group of anchors by using a basic equation for tension, or for shear for a single anchor in cracked concrete, and multiplied by factors that account for the number of anchors, edge distance, spacing, eccentricity, and absence of cracking. Experimental and numerical investigations have demonstrated the applicability of the CCD Method to adhesive anchors as well.<sup>D.20</sup>

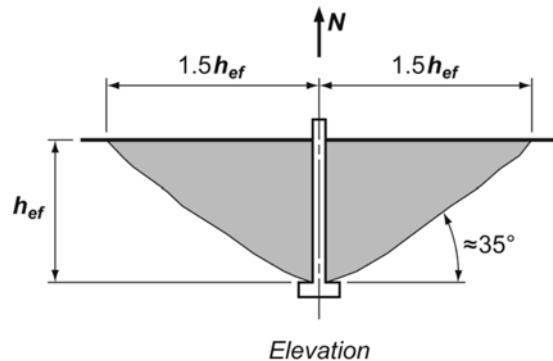
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Fig. RD.4.2(a)—Breakout cone for tension.

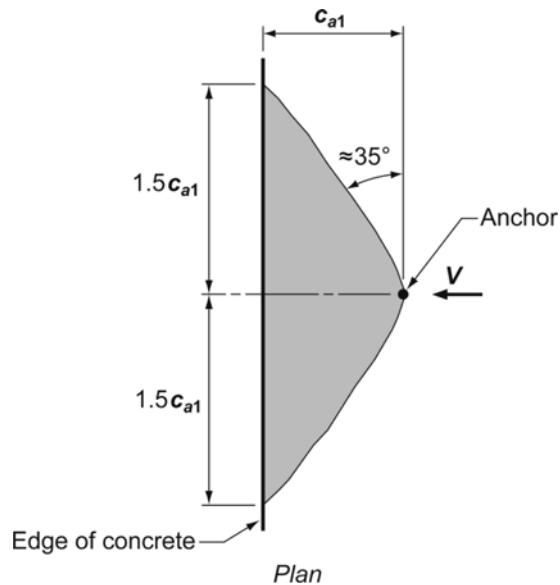


Fig. RD.4.2(b)—Breakout cone for shear.

on edge distances and anchor spacing in the design models shall be consistent with the tests that verified the model.

**D.4.2.1** — The effect of reinforcement provided to restrain the concrete breakout shall be permitted to be included in the design models used to satisfy D.4.2. Where anchor reinforcement is provided in accordance with D.5.2.9 and D.6.2.9, calculation of the concrete breakout strength in accordance with D.5.2 and D.6.2 is not required.

The breakout strength calculations are based on a model suggested in the  $\kappa$ -Method. It is consistent with a breakout prism angle of approximately 35 degrees (Fig. RD.4.2 (a) and (b)).

**RD.4.2.1** — The addition of reinforcement in the direction of the load to restrain concrete breakout can greatly enhance the strength and deformation capacity of the anchor connection. Such enhancement is practical with cast-in anchors such as those used in precast sections.

References D.17, D.24, D.25, D.26, and D.27 provide information regarding the effect of reinforcement on the behavior of anchors. The effect of reinforcement is not included in the ACI 355.2 and ACI 355.4 anchor acceptance tests or in the concrete breakout calculation method of D.5.2 and D.6.2. The beneficial effect of supplementary reinforcement is recognized by the Condition A  $\phi$ -factors in D.4.3 and D.4.4. Anchor reinforcement may be provided instead of calculating breakout strength using the provisions of Chapter 12 in conjunction with D.5.2.9 and D.6.2.9.

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**D.4.2.2** — For anchors with diameters not exceeding 4 in., the concrete breakout strength requirements shall be considered satisfied by the design procedure of D.5.2 and D.6.2.

The breakout strength of an unreinforced connection can be taken as an indication of the load at which significant cracking will occur. Such cracking can represent a serviceability problem if not controlled. (See RD.6.2.1.)

**RD.4.2.2** — The limitation on anchor diameter is based on the current range of test data. In the 2002 through 2008 editions of the Code, there were limitations on the diameter and embedment of anchors to compute the concrete breakout strength. These limitations were necessitated by the lack of test results on anchors with diameter larger than 2 in. and embedment length longer than 24 in. In 2011, limitations on anchor diameter and embedment length were revised to limit the diameter to 4 in. diameter based on the results of tension and shear tests on large-diameter anchors with deep embedments.<sup>D.28,D.29</sup> These tests included 4.25 in. diameter anchors embedded 45 in. in tension tests and 3.5 in. diameter anchors in shear tests. The reason for this 4 in. diameter limit is that the largest diameter anchor in ASTM F1554, “Standard Specification for Anchor Bolts, Steel, 36, 55, and 105-ksi Yield Strength,” is 4 in. whereas other ASTM specifications permit up to 8 in. diameter anchors that have not been tested to ensure applicability of the D.5.2 and D.6.2 concrete breakout provisions.

**RD.4.2.3** — ACI 355.4 limits the embedment depth of adhesive anchors to  $4d_a \leq h_{ef} \leq 20d_a$ , which represents the theoretical limits of the bond model.<sup>D.20</sup>

**D.4.2.3** — For adhesive anchors with embedment depths  $4d_a \leq h_{ef} \leq 20d_a$ , the bond strength requirements shall be considered satisfied by the design procedure of D.5.5.

**D.4.3** — Strength reduction factor  $\phi$  for anchors in concrete shall be as follows when the load combinations of 9.2 are used:

- a) Anchor governed by strength of a ductile steel element
  - i) Tension loads ..... 0.75
  - ii) Shear loads ..... 0.65
- b) Anchor governed by strength of a brittle steel element
  - i) Tension loads ..... 0.65
  - ii) Shear loads ..... 0.60
- c) Anchor governed by concrete breakout, side-face blowout, pullout, or pryout strength

<u>Condition A</u>	<u>Condition B</u>
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i) Shear loads	0.75	0.70
ii) Tension loads		

Cast-in headed studs, headed bolts, or hooked bolts	0.75	0.70
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**RD.4.3** — The  $\phi$ -factors for steel strength are based on using  $f_{uta}$  to determine the nominal strength of the anchor (see D.5.1 and D.6.1) rather than  $f_{ya}$  as used in the design of reinforced concrete members. Although the  $\phi$ -factors for use with  $f_{uta}$  appear low, they result in a level of safety consistent with the use of higher  $\phi$ -factors applied to  $f_{ya}$ . The smaller  $\phi$ -factors for shear than for tension do not reflect basic material differences but rather account for the possibility of a non-uniform distribution of shear in connections with multiple anchors. It is acceptable to have a ductile failure of a steel element in the attachment if the attachment is designed so that it will undergo ductile yielding at a load level corresponding to anchor forces no greater than the minimum design strength of the anchors specified in D.3.3 (see D.3.3.4.3 and D.3.3.5.3).

For anchors governed by the more brittle concrete breakout or blowout failure, two conditions are recognized. If supplementary reinforcement is present (Condition A), greater deformation capacity is provided than in the case where such supplementary reinforcement is not present (Condition B). An explicit design of supplementary reinforcement is not required. However, the arrangement of supplementary reinforcement should generally conform to that of the anchor reinforcement shown in Fig. RD.5.2.9 and RD.6.2.9(b). Full development is not required.

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Post-installed anchors  
with category as determined  
from ACI 355.2 or ACI 355.4

<b>Category 1</b> (Low sensitivity to installation and high reliability)	0.75	0.65
<b>Category 2</b> (Medium sensitivity to installation and medium reliability)	0.65	0.55
<b>Category 3</b> (High sensitivity to installation and lower reliability)	0.55	0.45

Condition A applies where supplementary reinforcement is present except for pullout and pryout strengths.

Condition B applies where supplementary reinforcement is not present, and for pullout or pryout strength.

The strength reduction factors for anchor reinforcement are given in D.5.2.9 and D.6.2.9. Further discussion of strength reduction factors is presented in RD.4.4.

The ACI 355.2 tests for sensitivity to installation procedures determine the reliability category appropriate for a particular expansion or undercut anchoring device. In the ACI 355.2 tests for expansion and undercut anchors, the effects of variability in anchor torque during installation, tolerance on drilled hole size, and energy level used in setting anchors are considered; for expansion and undercut anchors approved for use in cracked concrete, increased crack widths are considered. ACI 355.4 tests for sensitivity of installation procedures determine the category for a particular adhesive anchor system considering the influence of adhesive mixing and the influence of hole cleaning in dry, saturated and water-filled/underwater bore holes. The three categories of acceptable post-installed anchors are:

**Category 1** — low sensitivity to installation and high reliability;

**Category 2** — medium sensitivity to installation and medium reliability; and

**Category 3** — high sensitivity to installation and lower reliability.

The strengths of anchors under shear loads are not as sensitive to installation errors and tolerances. Therefore, for shear calculations of all anchors,  $\phi = 0.75$  for Condition A and  $\phi = 0.70$  for Condition B.

**D.4.4** — Strength reduction factor  $\phi$  for anchors in concrete shall be as follows when the load combinations referenced in Appendix C are used:

- a) Anchor governed by strength of a ductile steel element
  - i) Tension loads.....0.80
  - ii) Shear loads.....0.75
- b) Anchor governed by strength of a brittle steel element
  - i) Tension loads.....0.70
  - ii) Shear loads.....0.65
- c) Anchor governed by concrete breakout, side-face blowout, pullout, or pryout strength

**RD.4.4** — As noted in R9.1, the 2002 Code incorporated the load factors of SEI/ASCE 7-02 and the corresponding strength reduction factors provided in the 1999 Appendix C into 9.2 and 9.3, except that the factor for flexure has been increased. Developmental studies for the  $\phi$ -factors to be used for Appendix D were based on the 1999 9.2 and 9.3 load and strength reduction factors. The resulting  $\phi$ -factors are presented in D.4.4 for use with the load factors of Appendix C, starting with the 2002 Code. The  $\phi$ -factors for use with the load factors of the 1999 Appendix C were determined in a manner consistent with the other  $\phi$ -factors of the 1999 Appendix C. These  $\phi$ -factors are presented in D.4.3 for use with the load factors of 9.2, starting with the 2002 Code. Since developmental studies for  $\phi$ -factors to be used with Appendix D, for brittle concrete failure modes, were performed for the load and strength reduction factors now given in Appendix C, the discussion of the selection of these  $\phi$ -factors appears in this section.

Even though the  $\phi$ -factor for structural plain concrete in Appendix C is 0.65, the basic factor for brittle concrete failures ( $\phi = 0.75$ ) was chosen based on results of probabilistic

	<u>Condition A</u>	<u>Condition B</u>
i) Shear loads	0.85	0.75

**CODE****COMMENTARY**

## ii) Tension loads

Cast-in headed studs, headed bolts, or hooked bolts	0.85	0.75
Post-installed anchors with category as determined from ACI 355.2 or ACI 355.4		
<b>Category 1</b> (Low sensitivity to installation and high reliability)	0.85	0.75
<b>Category 2</b> (Medium sensitivity to installation and medium reliability)	0.75	0.65
<b>Category 3</b> (High sensitivity to installation and lower reliability)	0.65	0.55

Condition A applies where supplementary reinforcement is present except for pullout and pryout strengths.

Condition B applies where supplementary reinforcement is not present, and for pullout and pryout strengths.

## D.5 — Design requirements for tensile loading

### D.5.1 — Steel strength of anchor in tension

**D.5.1.1** — The nominal strength of an anchor in tension as governed by the steel,  $N_{sa}$ , shall be evaluated by calculations based on the properties of the anchor material and the physical dimensions of the anchor.

**D.5.1.2** — The nominal strength of an anchor in tension,  $N_{sa}$ , shall not exceed

$$N_{sa} = A_{se,N} f_{uta} \quad (\text{D-2})$$

where  $A_{se,N}$  is the effective cross-sectional area of an anchor in tension, in.<sup>2</sup>, and  $f_{uta}$  shall not be taken greater than the smaller of  $1.9f_{ya}$  and 125,000 psi.

studies<sup>D.30</sup> that indicated the use of  $\phi = 0.65$  with mean values of concrete-controlled failures produced adequate safety levels. Because the nominal resistance expressions used in this appendix and in the test requirements are based on the 5 percent fractiles, the  $\phi = 0.65$  value would be overly conservative. Comparison with other design procedures and probabilistic studies<sup>D.30</sup> indicated that the choice of  $\phi = 0.75$  was justified. Applications with supplementary reinforcement (Condition A) provide more deformation capacity, permitting the  $\phi$ -factors to be increased. The value of  $\phi = 0.85$  is compatible with the level of safety for shear failures in concrete beams, and has been recommended in the *PCI Design Handbook*<sup>D.31</sup> and by ACI 349.<sup>D.24</sup>

## RD.5 — Design requirements for tensile loading

### RD.5.1 — Steel strength of anchor in tension

**RD.5.1.2** — The nominal strength of anchors in tension is best represented as a function of  $f_{uta}$  rather than  $f_{ya}$  because the large majority of anchor materials do not exhibit a well-defined yield point. The American Institute of Steel Construction (AISC) has based tension strength of anchors on  $A_{se,N} f_{uta}$  since the 1986 edition of their specifications. The use of Eq. (D-2) with 9.2 load factors and the  $\phi$ -factors of D.4.3 give design strengths consistent with the AISC Load and Resistance Factor Design Specifications.<sup>D.32</sup>

The limitation of  $1.9f_{ya}$  on  $f_{uta}$  is to ensure that, under service load conditions, the anchor does not exceed  $f_{ya}$ . The limit on  $f_{uta}$  of  $1.9f_{ya}$  was determined by converting the LRFD provisions to corresponding service level conditions.

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For **Section 9.2**, the average load factor of 1.4 (from **1.2D + 1.6L**) divided by the highest  $\phi$ -factor (0.75 for tension) results in a limit of  $f_{uta}/f_{ya}$  of  $1.4/0.75 = 1.87$ . For **Appendix C**, the average load factor of 1.55 (from **1.4D + 1.7L**), divided by the highest  $\phi$ -factor (0.80 for tension), results in a limit of  $f_{uta}/f_{ya}$  of  $1.55/0.8 = 1.94$ . For consistent results, the serviceability limitation of  $f_{uta}$  was taken as **1.9f<sub>ya</sub>**. If the ratio of  $f_{uta}$  to  $f_{ya}$  exceeds this value, the anchoring may be subjected to service loads above  $f_{ya}$  under service loads. Although not a concern for standard structural steel anchors (maximum value of  $f_{uta}/f_{ya}$  is 1.6 for ASTM A307), the limitation is applicable to some stainless steels.

For post-installed anchors having a reduced cross-sectional area anywhere along the anchor length, such as wedge-type anchors, the effective cross-sectional area of the anchor should be provided by the manufacturer. For threaded rods and headed bolts, ANSI/ASME B1.1<sup>D.1</sup> defines  $A_{se,N}$  as

$$A_{se,N} = \frac{\pi}{4} \left( d_a - \frac{0.9743}{n_t} \right)^2$$

where  $n_t$  is the number of threads per inch.

### **D.5.2 — Concrete breakout strength of anchor in tension**

**D.5.2.1** — The nominal concrete breakout strength in tension,  $N_{cb}$  of a single anchor or  $N_{cbg}$  of a group of anchors, shall not exceed:

(a) For a single anchor

$$N_{cb} = \frac{A_{Nc}}{A_{Nco}} \psi_{ed,N} \psi_{ec,N} \psi_{cp,N} N b \quad (\text{D-3})$$

(b) For a group of anchors

$$N_{cbg} = \frac{A_{Nc}}{A_{Nco}} \psi_{ed,N} \psi_{ec,N} \psi_{cp,N} N b \quad (\text{D-4})$$

Factors  $\psi_{ec,N}$ ,  $\psi_{ed,N}$ ,  $\psi_{cp,N}$ , and  $\psi_{ec,N}$  are defined in **D.5.2.4**, **D.5.2.5**, **D.5.2.6**, and **D.5.2.7**, respectively.  $A_{Nc}$  is the projected concrete failure area of a single anchor or group of anchors that shall be approximated as the base of the rectilinear geometrical figure that results from projecting the failure surface outward **1.5h<sub>ef</sub>** from the centerlines of the anchor, or in the case of a group of anchors, from a line through a row of adjacent anchors.

$A_{Nc}$  shall not exceed  $nA_{Nco}$ , where  $n$  is the number of anchors in the group that resist tension.  $A_{Nco}$  is the projected concrete failure area of a single anchor with an edge distance equal to or greater than **1.5h<sub>ef</sub>**

$$A_{Nco} = 9h_{ef}^2 \quad (\text{D-5})$$

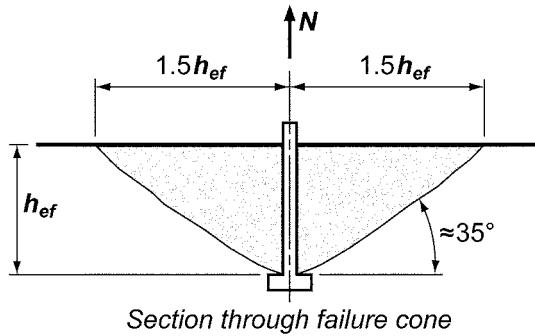
**RD.5.2 — Concrete breakout strength of anchor in tension**

**RD.5.2.1** — The effects of multiple anchors, spacing of anchors, and edge distance on the nominal concrete breakout strength in tension are included by applying the modification factors  $A_{Nc}/A_{Nco}$  and  $\psi_{ed,N}$  in Eq. (D-3) and (D-4).

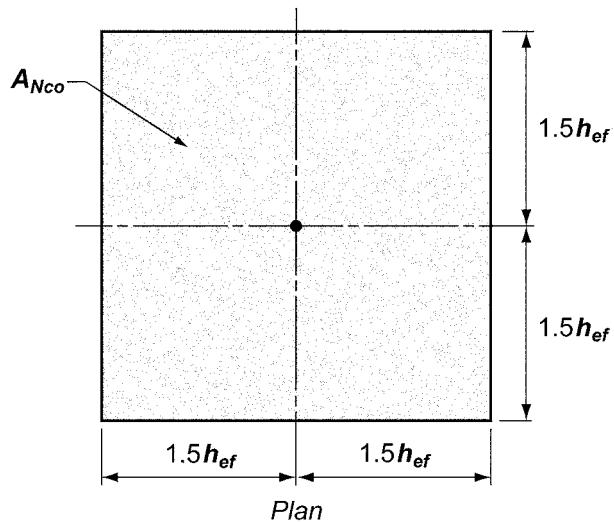
**Figure RD.5.2.1(a)** shows  $A_{Nco}$  and the development of Eq. (D-5).  $A_{Nco}$  is the maximum projected area for a single anchor. **Figure RD.5.2.1(b)** shows examples of the projected areas for various single-anchor and multiple-anchor arrangements. Because  $A_{Nc}$  is the total projected area for a group of anchors, and  $A_{Nco}$  is the area for a single anchor, there is no need to include  $n$ , the number of anchors, in Eq. (D-3) or (D-4). If anchor groups are positioned in such a way that their projected areas overlap, the value of  $A_{Nc}$  is required to be reduced accordingly.

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The critical edge distance for headed studs, headed bolts, expansion anchors, and undercut anchors is  $1.5 h_{ef}$

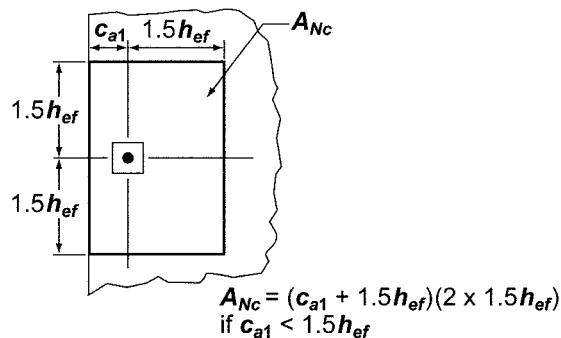


### *Section through failure cone*



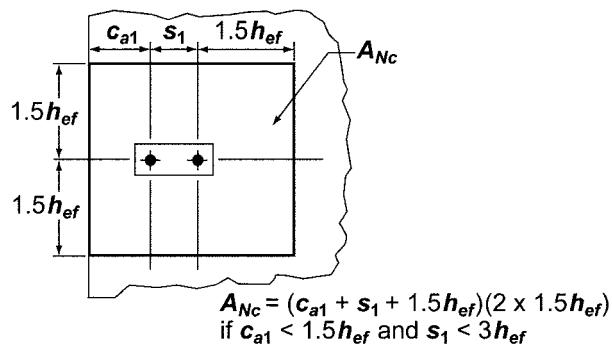
$$A_{Nco} = (2 \times 1.5 h_{ef}) \times (2 \times 1.5 h_{ef}) = 9 h_{ef}^2$$

(a)



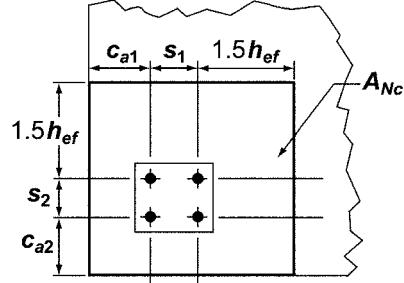
$$A_{Nc} = (c_{a1} + 1.5h_{ef})(2 \times 1.5h_{ef})$$

if  $c_{a1} < 1.5h_{ef}$



$$A_{Nc} = (c_{a1} + s_1 + 1.5h_{ef})(2 \times 1.5h_{ef})$$

if  $c_{a1} < 1.5h_{ef}$  and  $s_1 < 3h_{ef}$



$$A_{Nc} = (c_{a1} + s_1 + 1.5h_{ef})(c_{a2} + s_2 + 1.5h_{ef})$$

if  $c_{a1}$  and  $c_{a2} < 1.5h_{ef}$   
and  $s_1$  and  $s_2 < 3h_{ef}$

(b)

*Fig. RD.5.2.1—(a) Calculation of  $A_{Nco}$ ; and (b) calculation of  $A_{Nc}$  for single anchors and groups of anchors.*

**D.5.2.2** — The basic concrete breakout strength of a single anchor in tension in cracked concrete,  $N_b$ , shall not exceed

$$N_b = k_c \lambda_a \sqrt{f_c} h_{ef}^{1.5} \quad (D-6)$$

where

**$k_c$**  = 24 for cast-in anchors; and  
 **$k_c$**  = 17 for post-installed anchors.

The value of  $k_c$  for post-installed anchors shall be permitted to be increased above 17 based on ACI

**RD.5.2.2** — The equation for the basic concrete breakout strength was derived<sup>D.18,19,D.23,D.24</sup> assuming a concrete failure prism with an angle of about 35 degrees, considering fracture mechanics concepts.

The values of  $k_c$  in Eq. (D-6) were determined from a large database of test results in uncracked concrete<sup>D.18</sup> at the 5 percent fractile. The values were adjusted to corresponding  $k_c$  values for cracked concrete.<sup>D.19,D.33</sup> Tests have shown that the values of  $k_c$  applicable to adhesive anchors are approximately equal to those derived for expansion anchors.<sup>D.20,D.34</sup> Higher  $k_c$  values for post-installed anchors

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| 355.2 or ACI 355.4 product-specific tests, but shall in no case exceed 24.

Alternatively, for cast-in headed studs and headed bolts with 11 in.  $\leq h_{ef} \leq 25$  in.,  $N_b$  shall not exceed

$$N_b = 16\lambda_a \sqrt{f'_c} h_{ef}^{5/3} \quad (\text{D-7})$$

may be permitted, provided they have been determined from testing in accordance with ACI 355.2 or ACI 355.4. For anchors with a deeper embedment ( $h_{ef} > 11$  in.), test evidence indicates the use of  $h_{ef}^{1.5}$  can be overly conservative for some cases. An alternative expression (Eq. (D-7)) is provided using  $h_{ef}^{5/3}$  for evaluation of cast-in headed studs and headed bolts with 11 in.  $\leq h_{ef} \leq 25$  in. This expression can also be appropriate for some undercut post-installed anchors. However, for such anchors, the use of Eq. (D-7) should be justified by test results in accordance with D.4.2. Experimental and numerical investigations indicate that Eq. (D-7) may be unconservative for  $h_{ef} > 25$  in. where bearing pressure on the anchor head is at or near the limit permitted by Eq. (D-14).<sup>D.35</sup>

**D.5.2.3** — Where anchors are located less than  $1.5h_{ef}$  from three or more edges, the value of  $h_{ef}$  used for the calculation of  $A_{Nc}$  in accordance with D.5.2.1, as well as in Eq. (D-3) through (D-10) shall be the larger of  $c_{a,max}/1.5$  and  $s/3$ , where  $s$  is the maximum spacing between anchors within the group.

**RD.5.2.3** — For anchors located less than  $1.5h_{ef}$  from three or more edges, the tensile breakout strength computed by the CCD Method, which is the basis for Eq. (D-3) to (D-10), gives overly conservative results.<sup>D.36</sup> This occurs because the ordinary definitions of  $A_{Nc}/A_{Nco}$  do not correctly reflect the edge effects. This problem is corrected by limiting the value of  $h_{ef}$  used in Eq. (D-3) through (D-10) to  $c_{a,max}/1.5$ , where  $c_{a,max}$  is the largest of the influencing edge distances that are less than or equal to the actual  $1.5h_{ef}$ . In no case should  $c_{a,max}/1.5$  be taken less than one-third of the maximum spacing between anchors within the group. The limit on  $h_{ef}$  of at least one-third of the maximum spacing between anchors within the group prevents the use of a calculated strength based on individual breakout prisms for a group anchor configuration.

This approach is illustrated in Fig. RD.5.2.3. In this example, the proposed limit on the value of  $h_{ef}$  to be used in the computations where  $h_{ef} = c_{a,max}/1.5$ , results in  $h_{ef} = h'_{ef} = 4$  in. For this example, this would be the proper value to be used for  $h_{ef}$  in computing the resistance even if the actual embedment depth is larger.

The requirement of D.5.2.3 may be visualized by moving the actual concrete breakout surface, which originates at the actual  $h_{ef}$ , toward the surface of the concrete parallel to the applied tension load. The value of  $h_{ef}$  used in Eq. (D-3) to (D-10) is determined when either: (a) the outer boundaries of the failure surface first intersect a free edge; or (b) the intersection of the breakout surface between anchors within the group first intersects the surface of the concrete. For the example shown in Fig. RD.5.2.3, Point "A" defines the intersection of the assumed failure surface for limiting  $h_{ef}$  with the concrete surface.

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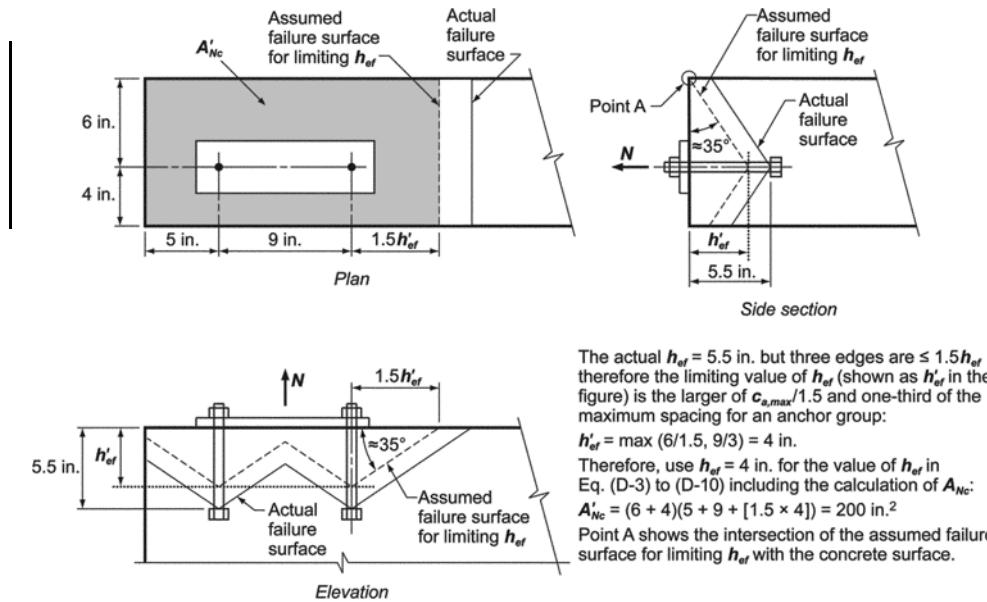


Fig. RD.5.2.3—Example of tension where anchors are located in narrow members.

The actual  $h_{ef} = 5.5$  in. but three edges are  $\leq 1.5 h_{ef}$  therefore the limiting value of  $h_{ef}$  (shown as  $h'_ef$  in the figure) is the larger of  $c_{a,max}/1.5$  and one-third of the maximum spacing for an anchor group:  
 $h'_ef = \max(6/1.5, 9/3) = 4$  in.  
 Therefore, use  $h_{ef} = 4$  in. for the value of  $h_{ef}$  in Eq. (D-3) to (D-10) including the calculation of  $A'_{Nc}$ :  
 $A'_{Nc} = (6 + 4)(5 + 9 + [1.5 \times 4]) = 200$  in.<sup>2</sup>  
 Point A shows the intersection of the assumed failure surface for limiting  $h_{ef}$  with the concrete surface.

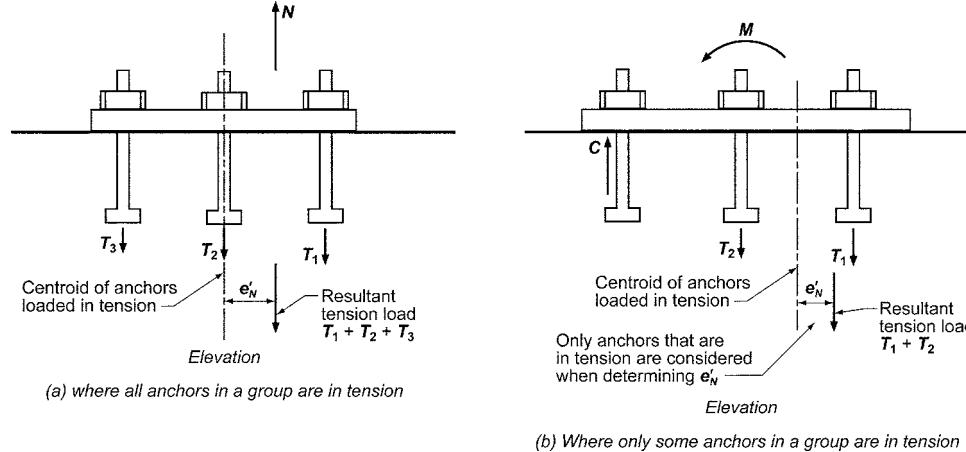


Fig. RD.5.2.4—Definition of  $e'_N$  for a group of anchors.

**D.5.2.4** — The modification factor for anchor groups loaded eccentrically in tension,  $\psi_{ec,N}$ , shall be computed as

$$\psi_{ec,N} = \frac{1}{\left(1 + \frac{2e'_N}{3h_{ef}}\right)} \quad (\text{D-8})$$

but  $\psi_{ec,N}$  shall not be taken greater than 1.0.

If the loading on an anchor group is such that only some anchors are in tension, only those anchors that are in tension shall be considered when determining the eccentricity  $e'_N$  for use in Eq. (D-8) and for the calculation of  $N_{cbg}$  according to Eq. (D-4).

**RD.5.2.4** — Figure RD.5.2.4(a) shows a group of anchors that are all in tension but the resultant force is eccentric with respect to the centroid of the anchor group. Groups of anchors can also be loaded in such a way that only some of the anchors are in tension (Fig. RD.5.2.4(b)). In this case, only the anchors in tension are to be considered in the determination of  $e'_N$ . The anchor loading has to be determined as the resultant anchor tension at an eccentricity with respect to the center of gravity of the anchors in tension.

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In the case where eccentric loading exists about two axes, the modification factor,  $\psi_{ec,N}$ , shall be computed for each axis individually and the product of these factors used as  $\psi_{ec,N}$  in Eq. (D-4).

**D.5.2.5** — The modification factor for edge effects for single anchors or anchor groups loaded in tension,  $\psi_{ed,N}$ , shall be computed as

If  $c_{a,min} \geq 1.5h_{ef}$

$$\text{then } \psi_{ed,N} = 1.0 \quad (\text{D-9})$$

If  $c_{a,min} < 1.5h_{ef}$

$$\text{then } \psi_{ed,N} = 0.7 + 0.3 \frac{c_{a,min}}{1.5h_{ef}} \quad (\text{D-10})$$

**D.5.2.6** — For anchors located in a region of a concrete member where analysis indicates no cracking at service load levels, the following modification factor shall be permitted:

$\psi_{c,N} = 1.25$  for cast-in anchors; and

$\psi_{c,N} = 1.4$  for post-installed anchors, where the value of  $k_c$  used in Eq. (D-6) is 17.

Where the value of  $k_c$  used in Eq. (D-6) is taken from the ACI 355.2 or ACI 355.4 product evaluation report for post-installed anchors qualified for use in both cracked and uncracked concrete, the values of  $k_c$  and  $\psi_{c,N}$  shall be based on the ACI 355.2 or ACI 355.4 product evaluation report.

Where the value of  $k_c$  used in Eq. (D-6) is taken from the ACI 355.2 or ACI 355.4 product evaluation report for post-installed anchors qualified for use in uncracked concrete,  $\psi_{c,N}$  shall be taken as 1.0.

When analysis indicates cracking at service load levels,  $\psi_{c,N}$  shall be taken as 1.0 for both cast-in anchors and post-installed anchors. Post-installed anchors shall be qualified for use in cracked concrete in accordance with ACI 355.2 or ACI 355.4. The cracking in the concrete shall be controlled by flexural reinforcement distributed in accordance with 10.6.4, or equivalent crack control shall be provided by confining reinforcement.

**D.5.2.7** — The modification factor for post-installed anchors designed for uncracked concrete in accordance with D.5.2.6 without supplementary reinforcement to control splitting,  $\psi_{cp,N}$ , shall be computed as follows using the critical distance  $c_{ac}$  as defined in D.8.6

**RD.5.2.5** — If anchors are located close to an edge so that there is not enough space for a complete breakout prism to develop, the strength of the anchor is further reduced beyond that reflected in  $A_{Nc}/A_{Nco}$ . If the smallest side cover distance is greater than or equal to  $1.5h_{ef}$ , a complete prism can form and there is no reduction ( $\psi_{ed,N} = 1$ ). If the side cover is less than  $1.5h_{ef}$ , the factor  $\psi_{ed,N}$  is required to adjust for the edge effect.<sup>D.18</sup>

**RD.5.2.6** — Post-installed anchors that have not met the requirements for use in cracked concrete according to ACI 355.2 or ACI 355.4 should be used only in regions that will remain uncracked. The analysis for the determination of crack formation should include the effects of restrained shrinkage (see 7.12.1.2). The anchor qualification tests of ACI 355.2 or ACI 355.4 require that anchors in cracked concrete zones perform well in a crack that is 0.012 in. wide. If wider cracks are expected, confining reinforcement to control the crack width to about 0.012 in. should be provided.

The concrete breakout strengths given by Eq. (D-6) and (D-7) assume cracked concrete (that is,  $\psi_{c,N} = 1.0$ ) with  $\psi_{c,N}k_c = 24$  for cast-in-place, and 17 for post-installed (cast-in 40 percent higher). When the uncracked concrete  $\psi_{c,N}$  factors are applied (1.25 for cast-in, and 1.4 for post-installed), the results are  $\psi_{c,N}k_c$  factors of 30 for cast-in and 24 for post-installed (25 percent higher for cast-in). This agrees with field observations and tests that show cast-in anchor strength exceeds that of post-installed for both cracked and uncracked concrete.

**RD.5.2.7** — The design provisions in D.5 are based on the assumption that the basic concrete breakout strength can be achieved if the minimum edge distance,  $c_{a,min}$ , equals  $1.5h_{ef}$ . However, test results<sup>D.37</sup> indicate that many torque-controlled and displacement-controlled expansion anchors

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If  $c_{a,min} \geq c_{ac}$

$$\text{then } \psi_{cp,N} = 1.0 \quad (\text{D-11})$$

If  $c_{a,min} < c_{ac}$

$$\text{then } \psi_{cp,N} = \frac{c_{a,min}}{c_{ac}} \quad (\text{D-12})$$

but  $\psi_{cp,N}$  determined from Eq. (D-12) shall not be taken less than  $1.5h_{ef}/c_{ac}$ , where the critical distance  $c_{ac}$  is defined in D.8.6.

For all other cases, including cast-in anchors,  $\psi_{cp,N}$  shall be taken as 1.0.

**D.5.2.8** — Where an additional plate or washer is added at the head of the anchor, it shall be permitted to calculate the projected area of the failure surface by projecting the failure surface outward  $1.5h_{ef}$  from the effective perimeter of the plate or washer. The effective perimeter shall not exceed the value at a section projected outward more than the thickness of the washer or plate from the outer edge of the head of the anchor.

**D.5.2.9** — Where anchor reinforcement is developed in accordance with Chapter 12 on both sides of the breakout surface, the design strength of the anchor reinforcement shall be permitted to be used instead of the concrete breakout strength in determining  $\phi N_n$ . A strength reduction factor of 0.75 shall be used in the design of the anchor reinforcement.

and some undercut anchors require minimum edge distances exceeding  $1.5h_{ef}$  to achieve the basic concrete breakout strength when tested in uncracked concrete without supplementary reinforcement to control splitting. When a tension load is applied, the resulting tensile stresses at the embedded end of the anchor are added to the tensile stresses induced due to anchor installation, and splitting failure may occur before reaching the concrete breakout strength defined in D.5.2.1. To account for this potential splitting mode of failure, the basic concrete breakout strength is reduced by a factor  $\psi_{cp,N}$  if  $c_{a,min}$  is less than the critical edge distance  $c_{ac}$ . If supplementary reinforcement to control splitting is present or if the anchors are located in a region where analysis indicates cracking of the concrete at service loads, then the reduction factor  $\psi_{cp,N}$  is taken as 1.0. The presence of supplementary reinforcement to control splitting does not affect the selection of Condition A or B in D.4.3 or D.4.4.

**RD.5.2.9** — For conditions where the factored tensile force exceeds the concrete breakout strength of the anchor(s) or where the breakout strength is not evaluated, the nominal strength can be that of anchor reinforcement properly anchored as illustrated in Fig. RD.5.2.9. Care needs to be taken in the selection and positioning of the anchor reinforcement. The anchor reinforcement should consist of stirrups, ties, or hairpins placed as close as practicable to the anchor. Only reinforcement spaced less than  $0.5h_{ef}$  from the anchor centerline should be included as anchor reinforcement. The research<sup>D.27</sup> on which these provisions is based was limited to anchor reinforcement with maximum diameter similar to a No. 5 bar. It is beneficial for the anchor reinforcement to enclose the surface reinforcement. In sizing the anchor reinforcement, use of a 0.75 strength reduction factor  $\phi$  is recommended as is used for strut-and-tie models. If the alternate load factors of Appendix C are used, the corresponding strength reduction factor of 0.85 for strut-and-tie models should be used. As a practical matter, use of anchor reinforcement is generally limited to cast-in-place anchors.

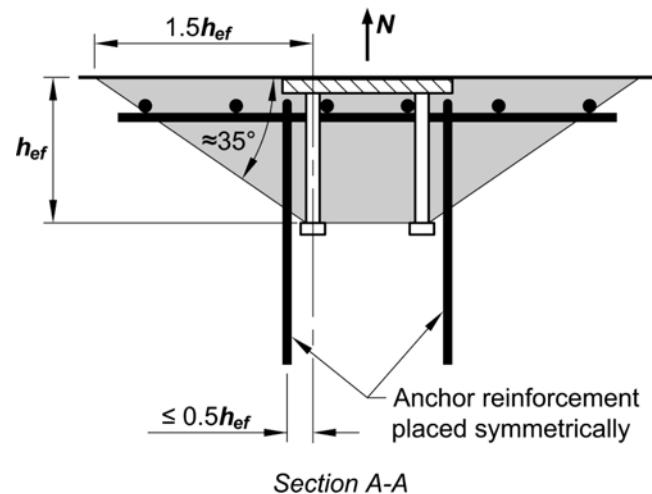
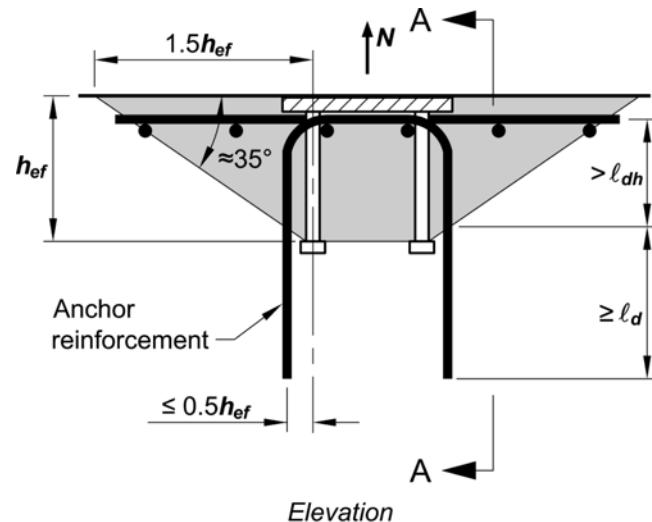
**CODE****COMMENTARY**

Fig. RD.5.2.9—Anchor reinforcement for tension.

**D.5.3 — Pullout strength of cast-in, post-installed expansion and undercut anchors in tension**

**D.5.3.1** — The nominal pullout strength of a single cast-in, post-installed expansion, and post-installed undercut anchor in tension,  $N_{pn}$ , shall not exceed

$$N_{pn} = \gamma_{c,P} N_p \quad (\text{D-13})$$

where  $\gamma_{c,P}$  is defined in D.5.3.6.

**D.5.3.2** — For post-installed expansion and undercut anchors, the values of  $N_p$  shall be based on the 5 percent fractile of results of tests performed and evaluated according to ACI 355.2. It is not permissible to calculate the pullout strength in tension for such anchors.

**RD.5.3 — Pullout strength of cast-in, post-installed expansion and undercut anchors in tension**

**RD.5.3.1** — The design requirements for pullout are applicable to cast-in, post-installed expansion, and post-installed undercut anchors. They are not applicable to adhesive anchors, which are instead evaluated for bond failure in accordance with D.5.5.

**RD.5.3.2** — The pullout strength equations given in D.5.3.4 and D.5.3.5 are only applicable to cast-in headed and hooked anchors;<sup>D.17,D.38</sup> they are not applicable to expansion and undercut anchors that use various mechanisms for end anchorage unless the validity of the pullout strength equations are verified by tests.

## CODE

**D.5.3.3** — For single cast-in headed studs and headed bolts, it shall be permitted to evaluate the pullout strength in tension using D.5.3.4. For single J- or L-bolts, it shall be permitted to evaluate the pullout strength in tension using D.5.3.5. Alternatively, it shall be permitted to use values of  $N_p$  based on the 5 percent fractile of tests performed and evaluated in the same manner as the ACI 355.2 procedures but without the benefit of friction.

**D.5.3.4** — The pullout strength in tension of a single headed stud or headed bolt,  $N_p$ , for use in Eq. (D-13), shall not exceed

$$N_p = 8A_{brg}f'_c \quad (\text{D-14})$$

**D.5.3.5** — The pullout strength in tension of a single hooked bolt,  $N_p$ , for use in Eq. (D-13) shall not exceed

$$N_p = 0.9f'_c e_h d_a \quad (\text{D-15})$$

where  $3d_a \leq e_h \leq 4.5d_a$ .

**D.5.3.6** — For an anchor located in a region of a concrete member where analysis indicates no cracking at service load levels, the following modification factor shall be permitted

$$\psi_{c,P} = 1.4$$

Where analysis indicates cracking at service load levels,  $\psi_{c,P}$  shall be taken as 1.0.

## D.5.4 — Concrete side-face blowout strength of a headed anchor in tension

**D.5.4.1** — For a single headed anchor with deep embedment close to an edge ( $h_{ef} > 2.5c_{a1}$ ), the nominal side-face blowout strength,  $N_{sb}$ , shall not exceed

$$N_{sb} = (160c_{a1}\sqrt{A_{brg}})\lambda_a\sqrt{f'_c} \quad (\text{D-16})$$

If  $c_{a2}$  for the single headed anchor is less than  $3c_{a1}$ , the value of  $N_{sb}$  shall be multiplied by the factor  $(1 + c_{a2}/c_{a1})/4$  where  $1.0 \leq c_{a2}/c_{a1} \leq 3.0$ .

## COMMENTARY

**RD.5.3.3** — The pullout strength in tension of headed studs or headed bolts can be increased by providing confining reinforcement, such as closely spaced spirals, throughout the head region. This increase can be demonstrated by tests.

**RD.5.3.4** — The value computed from Eq. (D-14) corresponds to the load at which crushing of the concrete occurs due to bearing of the anchor head.<sup>D.17,D.26</sup> It is not the load required to pull the anchor completely out of the concrete, so the equation contains no term relating to embedment depth. Local crushing of the concrete greatly reduces the stiffness of the connection, and generally will be the beginning of a pullout failure.

**RD.5.3.5** — Equation (D-15) for hooked bolts was developed by Lutz based on the results of Reference D.37. Reliance is placed on the bearing component only, neglecting any frictional component because crushing inside the hook will greatly reduce the stiffness of the connection, and generally will be the beginning of pullout failure. The limits on  $e_h$  are based on the range of variables used in the three test programs reported in Reference D.38.

## RD.5.4 — Concrete side-face blowout strength of a headed anchor in tension

The design requirements for side-face blowout are based on the recommendations of Reference D.39. These requirements are applicable to headed anchors that usually are cast-in anchors. Splitting during installation rather than side-face blowout generally governs post-installed anchors, and is evaluated by the ACI 355.2 requirements.

**CODE**

**D.5.4.2** — For multiple headed anchors with deep embedment close to an edge ( $h_{ef} > 2.5c_{a1}$ ) and anchor spacing less than  $6c_{a1}$ , the nominal strength of those anchors susceptible to a side-face blowout failure  $N_{sbg}$  shall not exceed

$$N_{sbg} = \left(1 + \frac{s}{6c_{a1}}\right) N_{sb} \quad (\text{D-17})$$

where  $s$  is the distance between the outer anchors along the edge, and  $N_{sb}$  is obtained from Eq. (D-16) without modification for a perpendicular edge distance.

**D.5.5 — Bond strength of adhesive anchor in tension**

**D.5.5.1** — The nominal bond strength in tension,  $N_a$  of a single adhesive anchor or  $N_{ag}$  of a group of adhesive anchors, shall not exceed

(a) For a single adhesive anchor:

$$N_a = \frac{A_{Na}}{A_{Nao}} \psi_{ed, Na} \psi_{cp, Na} N_{ba} \quad (\text{D-18})$$

(b) For a group of adhesive anchors:

$$N_{ag} = \frac{A_{Na}}{A_{Nao}} \psi_{ec, Na} \psi_{ed, Na} \psi_{cp, Na} N_{ba} \quad (\text{D-19})$$

Factors  $\psi_{ec, Na}$ ,  $\psi_{ed, Na}$ , and  $\psi_{cp, Na}$  are defined in D.5.5.3, D.5.5.4, and D.5.5.5, respectively.  $A_{Na}$  is the projected influence area of a single adhesive anchor or group of adhesive anchors that shall be approximated as a rectilinear area that projects outward a distance  $c_{Na}$  from the centerline of the adhesive anchor, or in the case of a group of adhesive anchors, from a line through a row of adjacent adhesive anchors.  $A_{Na}$  shall not exceed  $nA_{Nao}$ , where  $n$  is the number of adhesive anchors in the group that resist tension loads.  $A_{Nao}$  is the projected influence area of a single adhesive anchor with an edge distance equal to or greater than  $c_{Na}$ :

$$A_{Nao} = (2c_{Na})^2 \quad (\text{D-20})$$

where

$$c_{Na} = 10d_a \sqrt{\frac{\tau_{uncr}}{1100}} \quad (\text{D-21})$$

and constant 1100 carries the unit of lb/in.<sup>2</sup>

**COMMENTARY**

**RD.5.4.2** — In determining nominal side-face blowout strength for multiple headed anchors, only those anchors close to an edge ( $h_{ef} > 2.5c_{a1}$ ) that are loaded in tension should be considered. Their strength should be compared to the proportion of the tensile load applied to those anchors.

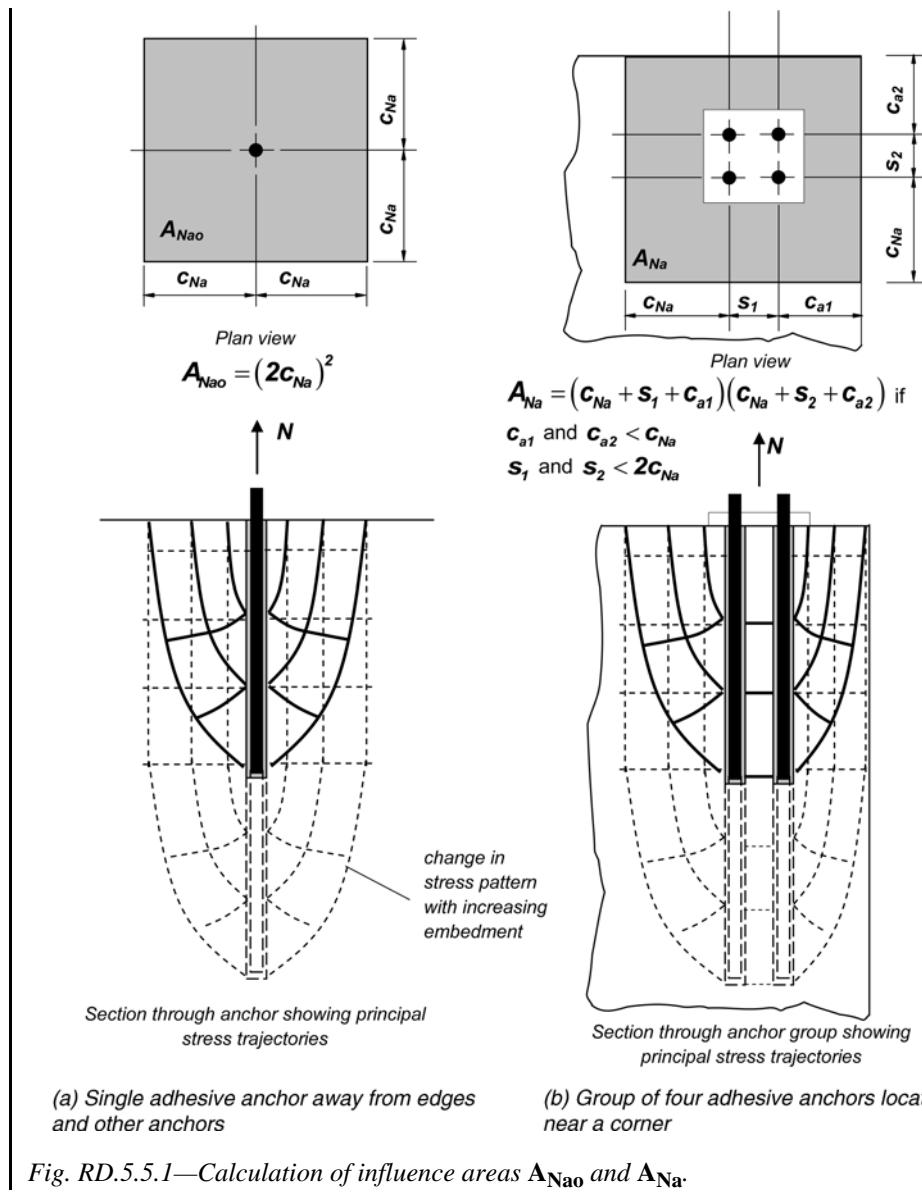
**RD.5.5 — Bond strength of adhesive anchor in tension**

**RD.5.5.1** — Evaluation of bond strength applies only to adhesive anchors. Single anchors with small embedment loaded to failure in tension may exhibit concrete breakout failures, while deeper embedments produce bond failures. Adhesive anchors that exhibit bond failures when loaded individually may exhibit concrete failures when in a group or in a near-edge condition. In all cases, the strength in tension of adhesive anchors is limited by the concrete breakout strength as given by Eq. (D-3) and (D-4).<sup>D.20</sup> The influences of anchor spacing and edge distance on both bond strength and concrete breakout strength must be evaluated for adhesive anchors. The influences of anchor spacing and edge distance on the nominal bond strength of adhesive anchors in tension are included in the modification factors  $A_{Na}/A_{Nao}$  and  $\psi_{ed, Na}$  in Eq. (D-18) and (D-19).

The influence of nearby edges and adjacent loaded anchors on bond strength is dependent on the volume of concrete mobilized by a single adhesive anchor. In contrast to the projected concrete failure area concept used in Eq. (D-3) and (D-4) to compute the breakout strength of an adhesive anchor, the influence area associated with the bond strength of an adhesive anchor used in Eq. (D-18) and (D-19) is not a function of the embedment depth but rather a function of the anchor diameter and the characteristic bond stress. The critical distance  $c_{Na}$  is assumed the same whether the concrete is cracked or uncracked; for simplicity, the relationship for  $c_{Na}$  in Eq. (D-21) uses  $\tau_{uncr}$ . This has been verified by experimental and numerical studies.<sup>D.20</sup> Figure RD.5.5.1(a) shows  $A_{Nao}$  and the development of Eq. (D-20).  $A_{Nao}$  is the projected influence area for the bond strength of a single adhesive anchor. Figure RD.5.5.1(b) shows an example of the projected influence area for an anchor group. Because, in this case,  $A_{Na}$  is the projected influence area for a group of anchors, and  $A_{Nao}$  is the projected influence area for a single anchor, there is no need to include  $n$ , the number of anchors, in Eq. (D-19). If anchors in a group (anchors loaded by a common base plate or attachment) are positioned in such a way that the projected influence areas of the individual anchors overlap, the value of  $A_{Na}$  is less than  $nA_{Nao}$ .

## CODE

## COMMENTARY

Fig. RD.5.5.1—Calculation of influence areas  $A_{Nao}$  and  $A_{Na}$ .

**D.5.5.2** — The basic bond strength of a single adhesive anchor in tension in cracked concrete,  $N_{ba}$ , shall not exceed

$$N_{ba} = \lambda_a \tau_{cr} \pi d_a h_{ef} \quad (\text{D-22})$$

The characteristic bond stress,  $\tau_{cr}$ , shall be taken as the 5 percent fractile of results of tests performed and evaluated according to ACI 355.4.

The tensile strength of closely spaced adhesive anchors with low bond strength may significantly exceed the value given by Eq. (D-19). A correction factor is given in the literature<sup>D.20</sup> to address this issue, but for simplicity, this factor is not included in the Code.

**RD.5.5.2** — The equation for basic bond strength of adhesive anchors as given in Eq. (D-22) represents a uniform bond stress model that has been shown to provide the best prediction of adhesive anchor bond strength through numerical studies and comparisons of different models to an international database of experimental results.<sup>D.21</sup> The basic bond strength is valid for bond failures that occur between the concrete and the adhesive as well as between the anchor and the adhesive.

**CODE**

Where analysis indicates cracking at service load levels, adhesive anchors shall be qualified for use in cracked concrete in accordance with ACI 355.4.

For adhesive anchors located in a region of a concrete member where analysis indicates no cracking at service load levels,  $\tau_{uncr}$  shall be permitted to be used in place of  $\tau_{cr}$  in Eq. (D-22) and shall be taken as the 5 percent fractile of results of tests performed and evaluated according to ACI 355.4.

It shall be permitted to use the minimum characteristic bond stress values in Table D.5.5.2 provided (a) through (e) are satisfied:

- (a) Anchors shall meet the requirements of ACI 355.4;
- (b) Anchors shall be installed in holes drilled with a rotary impact drill or rock drill;
- (c) Concrete at time of anchor installation shall have a minimum compressive strength of 2500 psi;
- (d) Concrete at time of anchor installation shall have a minimum age of 21 days;
- (e) Concrete temperature at time of anchor installation shall be at least 50°F.

**COMMENTARY**

Characteristic bond stresses should be based on tests performed in accordance with ACI 355.4 and should reflect the particular combination of installation and use conditions anticipated during construction and during the anchor service life. For those cases where product-specific information is unavailable at the time of design, Table D.5.5.2 provides lower-bound default values. The characteristic bond stresses in Table D.5.5.2 are the minimum values permitted for adhesive anchor systems qualified in accordance with ACI 355.4 for the tabulated installation and use conditions. Use of these values is restricted to the combinations of specific conditions listed; values for other combinations of installation and use conditions should not be inferred. Where both sustained loading and earthquake loading are present, the applicable factors given in the footnotes of Table D.5.5.2 should be multiplied together. The table assumes that all concrete has a minimum age of 21 days and a minimum concrete compressive strength of 2500 psi. See RD.2.2.

The terms “indoor” and “outdoor” as used in Table D.5.5.2 refer to a specific set of installation and service environments. Indoor conditions represent anchors installed in dry concrete with a rotary impact drill or rock drill and subjected to limited concrete temperature variations over the service life of the anchor. Outdoor conditions are assumed to occur, when at the time of anchor installation, the concrete is exposed to weather and may therefore be wet. Anchors installed in outdoor conditions are also assumed to be subject to greater concrete temperature variations such as might be associated with freezing and thawing or elevated temperatures resulting from direct sun exposure. While the indoor/outdoor characterization is useful for many applications, there may be situations in which a literal interpretation of the terms “indoor” and “outdoor” do not apply. For example, anchors installed before the building envelope is completed may involve drilling in saturated concrete. As such, the minimum characteristic bond stress associated with the outdoor condition in Table D.5.5.2 applies, regardless of whether the service environment is “indoor” and “outdoor.” Rotary impact drills and rock drills produce non-uniform hole geometries that are generally favorable for bond. Installation of adhesive anchors in core drilled holes may result in substantially lower characteristic bond stresses. Because this effect is highly product dependent, design of anchors to be installed in core drilled holes should adhere to the product-specific characteristic bond stresses established through testing in accordance with ACI 355.4.

The characteristic bond stresses associated with specific adhesive anchor systems are dependent on a number of parameters. Consequently, care should be taken to include all parameters relevant to the value of characteristic bond stress used in the design. These parameters include but are not limited to:

**TABLE D.5.5.2 — MINIMUM CHARACTERISTIC BOND STRESSES<sup>\*†</sup>**

Installation and service environment	Moisture content of concrete at time of anchor installation	Peak in-service temperature of concrete, °F	$\tau_{cr}$ , psi	$\tau_{uncr}$ , psi
Outdoor	Dry to fully saturated	175	200	650
Indoor	Dry	110	300	1000

<sup>\*</sup>Where anchor design includes sustained tension loading, multiply values of  $\tau_{cr}$  and  $\tau_{uncr}$  by 0.4.

<sup>†</sup>Where anchor design includes earthquake loads for structures assigned to Seismic Design Category C, D, E, or F, multiply values of  $\tau_{cr}$  by 0.8 and  $\tau_{uncr}$  by 0.4.

**CODE****COMMENTARY**

1. Type and duration of loading — bond strength is reduced for sustained tension loading;
2. Concrete cracking — bond strength is higher in uncracked concrete;
3. Anchor size — bond strength is generally inversely proportional to anchor diameter;
4. Drilling method — bond strength may be lower for anchors installed in core drilled holes;
5. Degree of concrete saturation at time of hole drilling and anchor installation — bond strength may be reduced due to concrete saturation;
6. Concrete temperature at time of installation — installation of anchors in cold conditions may result in retarded adhesive cure and reduced bond strength;
7. Concrete age at time of installation — installation in early-age concrete may result in reduced bond strength (see RD.2.2);
8. Peak concrete temperatures during anchor service life — under specific conditions (for example, anchors in thin concrete members exposed to direct sunlight), elevated concrete temperatures can result in reduced bond strength; and
9. Chemical exposure — anchors used in industrial environments may be exposed to increased levels of contaminants that can reduce bond strength over time.

Anchors tested and assessed under ACI 355.4 may in some cases not be qualified for all of the installation and service environments represented in Table D.5.5.2. Therefore, even where the minimum values given in Table D.5.5.2 are used for design, the relevant installation and service environments should be specified in accordance with D.9.2.1 and only anchors that have been qualified under ACI 355.4 for the installation and service environments corresponding to the characteristic bond stress taken from Table D.5.5.2 should be specified.

Characteristic bond stresses associated with qualified adhesive anchor systems for a specific set of installation and use conditions may substantially exceed the minimum values provided in Table D.5.5.2. For example, 1/2 in. to 3/4 in. diameter anchors installed in impact-drilled holes in dry concrete where use is limited to indoor conditions in uncracked concrete as described above may exhibit characteristic bond stresses,  $\tau_{uncr}$ , in the range of 2000 to 2500 psi.

**CODE****COMMENTARY**

**D.5.5.3** — The modification factor for adhesive anchor groups loaded eccentrically in tension,  $\psi_{ec,Na}$ , shall be computed as:

$$\psi_{ec,Na} = \frac{1}{\left(1 + \frac{e'_N}{c_{Na}}\right)} \quad (\text{D-23})$$

but  $\psi_{ec,Na}$  shall not be taken greater than 1.0.

If the loading on an adhesive anchor group is such that only some adhesive anchors are in tension, only those adhesive anchors that are in tension shall be considered when determining the eccentricity  $e'_N$  for use in Eq. (D-23) and for the calculation of  $N_{ag}$  according to Eq. (D-19).

In the case where eccentric loading exists about two orthogonal axes, the modification factor,  $\psi_{ec,Na}$ , shall be computed for each axis individually and the product of these factors used as  $\psi_{ec,Na}$  in Eq. (D-19).

**D.5.5.4** — The modification factor for edge effects for single adhesive anchors or adhesive anchor groups loaded in tension,  $\psi_{ed,Na}$ , shall be computed as

If  $c_{a,min} \geq c_{Na}$

$$\text{then } \psi_{ed,Na} = 1.0 \quad (\text{D-24})$$

If  $c_{a,min} < c_{Na}$

$$\text{then } \psi_{ed,Na} = 0.7 + 0.3 \frac{c_{a,min}}{c_{Na}} \quad (\text{D-25})$$

**D.5.5.5** — The modification factor for adhesive anchors designed for uncracked concrete in accordance with D.5.5.2 without supplementary reinforcement to control splitting,  $\psi_{cp,Na}$ , shall be computed as:

If  $c_{a,min} \geq c_{ac}$

$$\text{then } \psi_{cp,Na} = 1.0 \quad (\text{D-26})$$

If  $c_{a,min} < c_{ac}$

$$\text{then } \psi_{cp,Na} = \frac{c_{a,min}}{c_{ac}} \quad (\text{D-27})$$

but  $\psi_{cp,Na}$  determined from Eq. (D-27) shall not be taken less than  $c_{Na}/c_{ac}$ , where the critical edge distance,  $c_{ac}$ , is defined in D.8.6. For all other cases,  $\psi_{cp,Na}$  shall be taken as 1.0.

| **RD.5.5.3** — Refer to RD.5.2.4.

**RD.5.5.4** — If anchors are located close to an edge, their strength is further reduced beyond that reflected in  $A_{Na}/A_{Nao}$ . If the smallest side cover distance is greater than or equal to  $c_{Na}$ , there is no reduction ( $\psi_{ed,Na} = 1$ ). If the side cover is less than  $c_{Na}$ , the factor  $\psi_{ed,Na}$  accounts for the edge effect.<sup>D.18,D.20</sup>

**CODE****COMMENTARY****D.6 — Design requirements for shear loading****D.6.1 — Steel strength of anchor in shear**

**D.6.1.1** — The nominal strength of an anchor in shear as governed by steel,  $V_{sa}$ , shall be evaluated by calculations based on the properties of the anchor material and the physical dimensions of the anchor. Where concrete breakout is a potential failure mode, the required steel shear strength shall be consistent with the assumed breakout surface.

**D.6.1.2** — The nominal strength of an anchor in shear,  $V_{sa}$ , shall not exceed (a) through (c):

(a) For cast-in headed stud anchor

$$V_{sa} = A_{se,V} f_{uta} \quad (\text{D-28})$$

where  $A_{se,V}$  is the effective cross-sectional area of an anchor in shear, in.<sup>2</sup>, and  $f_{uta}$  shall not be taken greater than the smaller of  $1.9f_{ya}$  and 125,000 psi.

(b) For cast-in headed bolt and hooked bolt anchors and for post-installed anchors where sleeves do not extend through the shear plane

$$V_{sa} = 0.6A_{se,V} f_{uta} \quad (\text{D-29})$$

where  $A_{se,V}$  is the effective cross-sectional area of an anchor in shear, in.<sup>2</sup>, and  $f_{uta}$  shall not be taken greater than the smaller of  $1.9f_{ya}$  and 125,000 psi.

(c) For post-installed anchors where sleeves extend through the shear plane,  $V_{sa}$  shall be based on the results of tests performed and evaluated according to ACI 355.2. Alternatively, Eq. (D-29) shall be permitted to be used.

**D.6.1.3** — Where anchors are used with built-up grout pads, the nominal strengths of D.6.1.2 shall be multiplied by a 0.80 factor.

**D.6.2 — Concrete breakout strength of anchor in shear**

**D.6.2.1** — The nominal concrete breakout strength in shear,  $V_{cb}$  of a single anchor or  $V_{cbg}$  of a group of anchors, shall not exceed:

(a) For shear force perpendicular to the edge on a single anchor

**RD.6 — Design requirements for shear loading****RD.6.1 — Steel strength of anchor in shear**

**RD.6.1.1** — The shear load applied to each anchor in a group may vary depending on assumptions for the concrete breakout surface and load redistribution (see RD.6.2.1).

**RD.6.1.2** — The nominal shear strength of anchors is best represented as a function of  $f_{uta}$  rather than  $f_{ya}$  because the large majority of anchor materials do not exhibit a well-defined yield point. Welded studs develop a higher steel shear strength than headed anchors due to the fixity provided by the weld between the studs and the base plate. The use of Eq. (D-28) and (D-29) with 9.2 load factors and the  $\phi$ -factors of D.4.3 give design strengths consistent with the AISC Load and Resistance Factor Design Specifications.<sup>D.32</sup>

The limitation of  $1.9f_{ya}$  on  $f_{uta}$  is to ensure that, under service load conditions, the anchor stress does not exceed  $f_{ya}$ . The limit on  $f_{uta}$  of  $1.9f_{ya}$  was determined by converting the LRFD provisions to corresponding service level conditions as discussed in RD.5.1.2.

For post-installed anchors having a reduced cross-sectional area anywhere along the anchor length, the effective cross-sectional area of the anchor should be provided by the manufacturer. For threaded rods and headed bolts, ANSI/ASME B1.1<sup>D.1</sup> defines  $A_{se,V}$  as

$$A_{se,V} = \frac{\pi}{4} \left( d_a - \frac{0.9743}{n_t} \right)^2$$

where  $n_t$  is the number of threads per inch.

**RD.6.2 — Concrete breakout strength of anchor in shear**

**RD.6.2.1** — The shear strength equations were developed from the CCD Method. They assume a breakout cone angle of approximately 35 degrees (see Fig. RD.4.2(b)), and consider fracture mechanics theory. The effects of multiple anchors, spacing of anchors, edge distance, and thickness of the concrete member on nominal concrete breakout strength

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$$V_{cb} = \frac{A_{Vc}}{A_{Vco}} \psi_{ed,V} \psi_{ec,V} \psi_{c,V} \psi_{h,V} V_b \quad (\text{D-30})$$

(b) For shear force perpendicular to the edge on a group of anchors

$$V_{cbg} = \frac{A_{Vc}}{A_{Vco}} \psi_{ec,V} \psi_{ed,V} \psi_{c,V} \psi_{h,V} V_b \quad (\text{D-31})$$

(c) For shear force parallel to an edge,  $V_{cb}$  or  $V_{cbg}$  shall be permitted to be twice the value of the shear force determined from Eq. (D-30) or (D-31), respectively, with the shear force assumed to act perpendicular to the edge and with  $\psi_{ed,V}$  taken equal to 1.0.

(d) For anchors located at a corner, the limiting nominal concrete breakout strength shall be determined for each edge, and the minimum value shall be used.

Factors  $\psi_{ec,V}$ ,  $\psi_{ed,V}$ ,  $\psi_{c,V}$ , and  $\psi_{h,V}$  are defined in D.6.2.5, D.6.2.6, D.6.2.7, and D.6.2.8, respectively.  $V_b$  is the basic concrete breakout strength value for a single anchor.  $A_{Vc}$  is the projected area of the failure surface on the side of the concrete member at its edge for a single anchor or a group of anchors. It shall be permitted to evaluate  $A_{Vc}$  as the base of a truncated half pyramid projected on the side face of the member where the top of the half pyramid is given by the axis of the anchor row selected as critical. The value of  $c_{a1}$  shall be taken as the distance from the edge to this axis.  $A_{Vc}$  shall not exceed  $nA_{Vco}$ , where  $n$  is the number of anchors in the group.

$A_{Vco}$  is the projected area for a single anchor in a deep member with a distance from edges equal or greater than  $1.5c_{a1}$  in the direction perpendicular to the shear force. It shall be permitted to evaluate  $A_{Vco}$  as the base of a half pyramid with a side length parallel to the edge of  $3c_{a1}$  and a depth of  $1.5c_{a1}$ .

$$A_{Vco} = 4.5(c_{a1})^2 \quad (\text{D-32})$$

Where anchors are located at varying distances from the edge and the anchors are welded to the attachment so as to distribute the force to all anchors, it shall be permitted to evaluate the strength based on the distance to the farthest row of anchors from the edge. In this case, it shall be permitted to base the value of  $c_{a1}$  on the distance from the edge to the axis of the farthest anchor row that is selected as critical, and all of the shear shall be assumed to be carried by this critical anchor row alone.

**COMMENTARY**

in shear are included by applying the reduction factor of  $A_{Vc}/A_{Vco}$  in Eq. (D-30) and (D-31), and  $\psi_{ec,V}$  in Eq. (D-31). For anchors far from the edge, D.6.2 usually will not govern. For these cases, D.6.1 and D.6.3 often govern.

Figure RD.6.2.1(a) shows  $A_{Vco}$  and the development of Eq. (D-32).  $A_{Vco}$  is the maximum projected area for a single anchor that approximates the surface area of the full breakout prism or cone for an anchor unaffected by edge distance, spacing, or depth of member. Figure RD.6.2.1(b) shows examples of the projected areas for various single-anchor and multiple-anchor arrangements.  $A_{Vc}$  approximates the full surface area of the breakout cone for the particular arrangement of anchors. Because  $A_{Vc}$  is the total projected area for a group of anchors, and  $A_{Vco}$  is the area for a single anchor, there is no need to include the number of anchors in the equation.

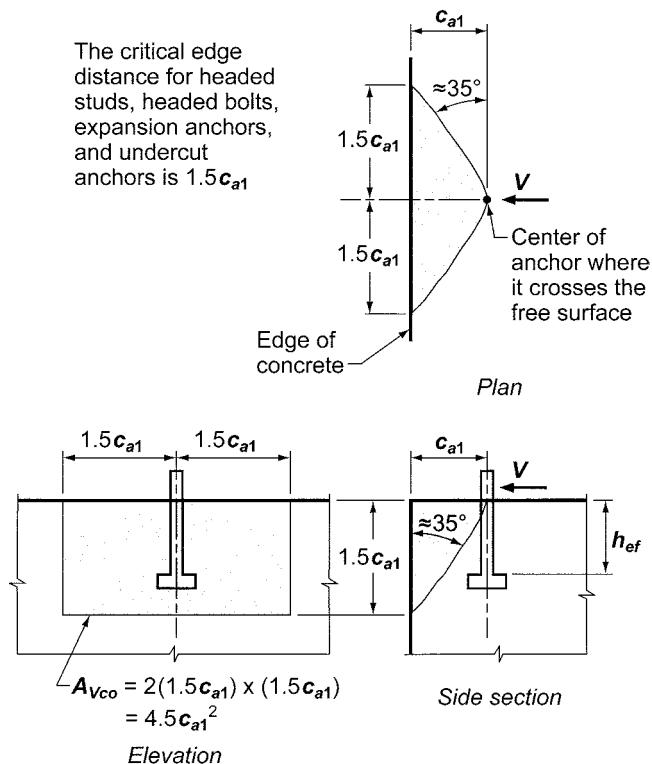
As shown in the examples in Fig. RD.6.2.1(b) of two-anchor groups loaded in shear, when using Eq. (D-31) for cases where the anchor spacing  $s$  is greater than the edge distance to the near edge anchor  $c_{a1,1}$ , both assumptions for load distribution illustrated in Cases 1 and 2 should be considered. This is because the anchors nearest to the free edge could fail first or the whole group could fail as a unit with the failure surface originating from the anchors farthest from the edge. For Case 1, the steel shear strength is provided by both anchors. For Case 2, the steel shear strength is provided entirely by the anchor farthest from the edge. No contribution of the anchor near the edge is then considered. In addition, checking the near-edge anchor for concrete breakout under service loads is advisable to preclude undesirable cracking at service conditions. If the anchor spacing  $s$  is less than the edge distance to the near-edge anchor, then the failure surfaces may merge<sup>D.27</sup> and Case 3 of Fig. RD.6.2.1(b) may be taken as a conservative approach.

If the anchors are welded to a common plate (regardless of anchor spacing  $s$ ), when the anchor nearest the front edge begins to form a failure cone, shear load is transferred to the stiffer and stronger rear anchor. For this reason, only Case 2 need be considered, which is consistent with Section 6.5.5 of the *PCI Design Handbook*.<sup>D.31</sup> For determination of steel shear strength, it is conservative to consider only the anchor farthest from the edge. However, for anchors having a ratio of  $s/c_{a,1,1}$  less than 0.6, both the front and rear anchors may be assumed to resist the shear.<sup>D.40</sup> For ratios of  $s/c_{a,1,1}$  greater than 1, it is advisable to check concrete breakout of the near-edge anchor to preclude undesirable cracking at service conditions.

Further discussion of design for multiple anchors is given in Reference D.16.

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Fig. RD.6.2.1(a)—Calculation of  $A_{Vco}$ .

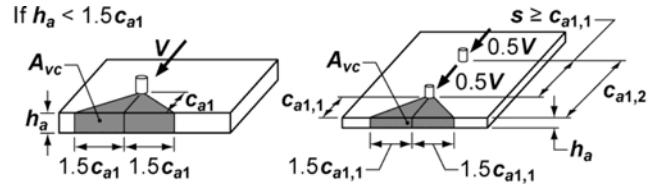
For the case of anchors near a corner subjected to a shear force with components normal to each edge, a satisfactory solution is to check independently the connection for each component of the shear force. Other specialized cases, such as the shear resistance of anchor groups where all anchors do not have the same edge distance, are treated in Reference D.20.

The detailed provisions of D.6.2.1(a) apply to the case of shear force directed toward an edge. When the shear force is directed away from the edge, the strength will usually be governed by D.6.1 or D.6.3.

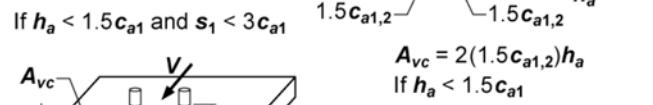
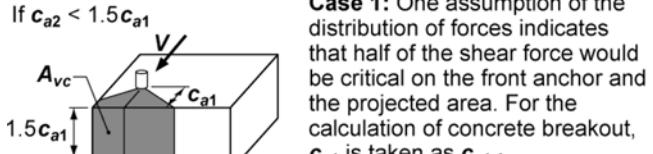
The case of shear force parallel to an edge is shown in Fig. RD.6.2.1(c). The maximum shear force that can be applied parallel to the edge,  $V_{\parallel}$ , as governed by concrete breakout, is twice the maximum shear force that can be applied perpendicular to the edge,  $V_{\perp}$ . A special case can arise with shear force parallel to the edge near a corner. In the example of a single anchor near a corner (see Fig. RD.6.2.1(d)), the provisions for shear force applied perpendicular to the edge should be checked in addition to the provisions for shear force applied parallel to the edge.

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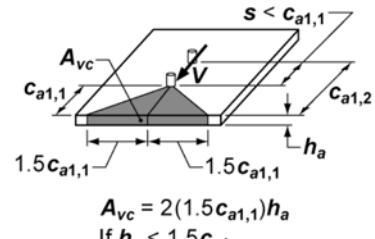
If  $h_a < 1.5c_{a1}$

$$A_{vc} = 2(1.5c_{a1,1})h_a$$


**Case 1:** One assumption of the distribution of forces indicates that half of the shear force would be critical on the front anchor and the projected area. For the calculation of concrete breakout,  $c_{a1}$  is taken as  $c_{a1,1}$ .

**Case 2:** Another assumption of the distribution of forces indicates that the total shear force would be critical on the rear anchor and its projected area. Only this assumption needs to be considered when anchors are welded to a common plate independent of  $s$ . For the calculation of concrete breakout,  $c_{a1}$  is taken as  $c_{a1,2}$ .

**Note:** For  $s \geq c_{a1,1}$ , both Case 1 and Case 2 should be evaluated to determine which controls for design except as noted for anchors welded to a common plate.



**Case 3:** Where  $s < c_{a1,1}$ , apply the entire shear load  $V$  to the front anchor. This case does not apply for anchors welded to a common plate. For the calculation of concrete breakout,  $c_{a1}$  is taken as  $c_{a1,1}$ .

Fig. RD.6.2.1(b)—Calculation of  $A_{vc}$  for single anchors and groups of anchors.

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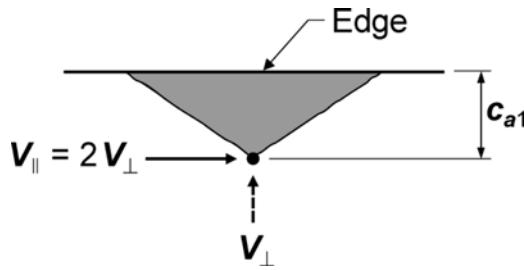


Fig. RD.6.2.1(c)—Shear force parallel to an edge.

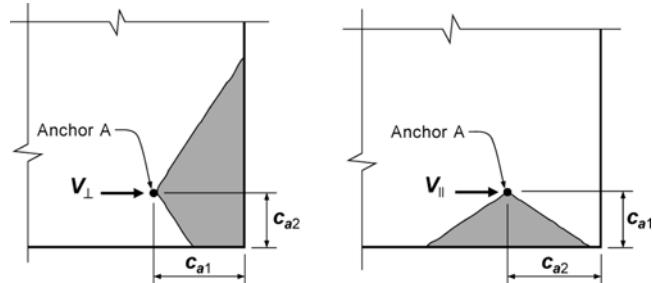


Fig. RD.6.2.1(d)—Shear force near a corner.

**D.6.2.2** — The basic concrete breakout strength in shear of a single anchor in cracked concrete,  $V_b$ , shall be the smaller of (a) and (b):

(a)

$$V_b = \left( 7 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5} \right) \quad (\text{D-33})$$

where  $\ell_e$  is the load-bearing length of the anchor for shear:

$\ell_e = h_{ef}$  for anchors with a constant stiffness over the full length of embedded section, such as headed studs and post-installed anchors with one tubular shell over full length of the embedment depth,

$\ell_e = 2d_a$  for torque-controlled expansion anchors with a distance sleeve separated from expansion sleeve,

and

$\ell_e \leq 8d_a$  in all cases.

(b)

$$V_b = 9 \lambda_a \sqrt{f'_c} (c_{a1})^{1.5} \quad (\text{D-34})$$

**RD.6.2.2** — Like the concrete breakout tensile strength, the concrete breakout shear strength does not increase with the failure surface, which is proportional to  $(c_{a1})^2$ . Instead, the strength increases proportionally to  $(c_{a1})^{1.5}$  due to size effect. The strength is also influenced by the anchor stiffness and the anchor diameter.<sup>D.18,D.19,D.22,D.27</sup> The influence of anchor stiffness and diameter is not apparent in large-diameter anchors,<sup>D.29</sup> resulting in a limitation on the shear breakout strength provided by Eq. (D-34).

The constant, 7, in the shear strength equation was determined from test data reported in Reference D.18 at the 5 percent fractile adjusted for cracking.

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**D.6.2.3** — For cast-in headed studs, headed bolts, or hooked bolts that are continuously welded to steel attachments having a minimum thickness equal to the greater of 3/8 in. and half of the anchor diameter, the basic concrete breakout strength in shear of a single anchor in cracked concrete,  $V_b$ , shall be the smaller of Eq. (D-34) and Eq. (D-35)

$$V_b = \left( 8 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \right) \lambda_a \sqrt{f'_c} (c_{a1})^{1.5} \quad (\text{D-35})$$

where  $\ell_e$  is defined in D.6.2.2

provided that:

- (a) for groups of anchors, the strength is determined based on the strength of the row of anchors farthest from the edge;
- (b) anchor spacing,  $s$ , is not less than 2.5 in.; and
- (c) reinforcement is provided at the corners if  $c_{a2} \leq 1.5h_{ef}$ .

**D.6.2.4** — Where anchors are located in narrow sections of limited thickness such that both edge distances  $c_{a2}$  and thickness  $h_a$  are less than  $1.5c_{a1}$ , the value of  $c_{a1}$  used for the calculation of  $A_{vc}$  in accordance with D.6.2.1 as well as in Eq. (D-32) through (D-39) shall not exceed the largest of:

- (a)  $c_{a2}/1.5$ , where  $c_{a2}$  is the largest edge distance;
- (b)  $h_a/1.5$ ; and
- (c)  $s/3$ , where  $s$  is the maximum spacing perpendicular to direction of shear, between anchors within a group.

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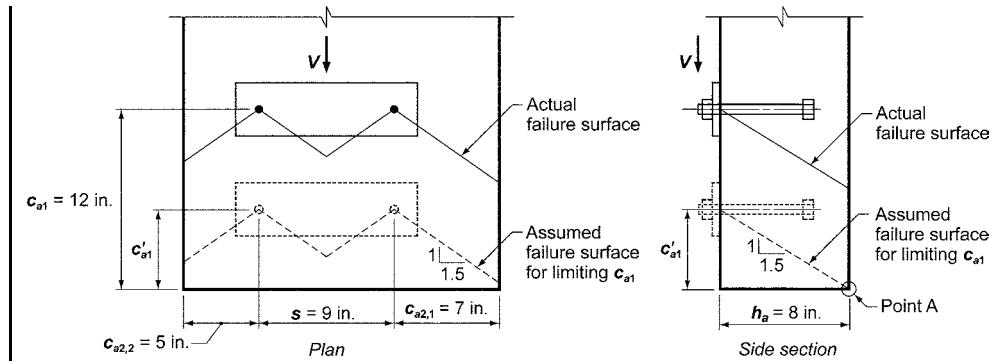
**RD.6.2.3** — For the case of cast-in headed bolts continuously welded to an attachment, test data<sup>D.41</sup> show that somewhat higher shear strength exists, possibly due to the stiff welding connection clamping the bolt more effectively than an attachment with an anchor gap. Because of this, the basic shear value for such anchors is increased but the upper limit of Eq. (D-34) is imposed because tests on large-diameter anchors welded to steel attachments are not available to justify any higher value than Eq. (D-34). The design of supplementary reinforcement is discussed in References D.17, D.22, and D.23.

**RD.6.2.4** — For the case of anchors located in narrow sections of limited thickness where the edge distances perpendicular to the direction of load and the member thickness are less than  $1.5c_{a1}$ , the shear breakout strength computed by the basic CCD Method is overly conservative. These cases were studied for the  $\kappa$  Method<sup>D.23</sup> and the problem was pointed out by Lutz.<sup>D.36</sup> Similar to the approach used for concrete breakout strength in tension in D.5.2.3, the concrete breakout strength in shear for this case is more accurately evaluated if the value of  $c_{a1}$  used in Eq. (D-30) to (D-39) and in the calculation of  $A_{vc}$  is limited to the maximum of two-thirds of the larger of the two edge distances perpendicular to the direction of shear, two-thirds of the member thickness, and one-third of the maximum spacing between anchors within the group, measured perpendicular to the direction of shear. The limit on  $c_{a1}$  of at least one-third of the maximum spacing between anchors within the group prevents the use of a calculated strength based on individual breakout prisms for a group anchor configuration.

This approach is illustrated in Fig. RD.6.2.4. In this example, the limiting value of  $c_{a1}$  is denoted as  $c'_{a1}$  and is used for the calculation of  $A_{vc}$ ,  $A_{vc0}$ ,  $\psi_{ed,V}$ , and  $\psi_{h,V}$  as well as for  $V_b$  (not shown). The requirement of D.6.2.4 may be visualized by moving the actual concrete breakout surface originating at the actual  $c_{a1}$  toward the surface of the concrete in the direction of the applied shear load. The value of  $c_{a1}$  used for the calculation of  $A_{vc}$  and in Eq. (D-30) to (D-39) is determined when either: (a) an outer boundary of the failure surface first intersects the concrete surface; or (b) the intersection of the breakout surface between anchors

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1. The actual  $c_{a1} = 12$  in.
2. The two edge distances  $c_{a2}$  as well as  $h_a$  are all less than  $1.5c_{a1}$ .
3. The limiting value of  $c_{a1}$  (shown as  $c'_{a1}$  in the figure) to be used for the calculation of  $A_{Vc}$  and in Eq. (D-30) to (D-39) is determined as the largest of the following:

$$(c_{a2,\max})/1.5 = (7)/1.5 = 4.67 \text{ in.}$$

$$(h_a)/1.5 = (8)/1.5 = 5.33 \text{ in. (controls)}$$

$$s/3 = 1/3(9) = 3 \text{ in.}$$

4. For this case,  $A_{Vc}$ ,  $A_{Vco}$ ,  $\psi_{ed,V}$ , and  $\psi_{h,V}$  are determined as follows:

$$A_{Vc} = (5 + 9 + 7)(1.5 \times 5.33) = 168 \text{ in.}^2$$

$$A_{Vco} = 4.5(5.33)^2 = 128 \text{ in.}^2$$

$$\psi_{ed,V} = 0.7 + 0.3(5)/5.33 = 0.98$$

$\psi_{h,V} = 1.0$  because  $c_{a1} = (h_a)/1.5$ . Point A shows the intersection of the assumed failure surface with the concrete surface that establishes the limiting value of  $c_{a1}$ .

Fig. RD.6.2.4—Example of shear where anchors are located in narrow members of limited thickness.

within the group first intersects the concrete surface. For the example shown in Fig. RD.6.2.4, Point “A” shows the intersection of the assumed failure surface for limiting  $c_{a1}$  with the concrete surface.

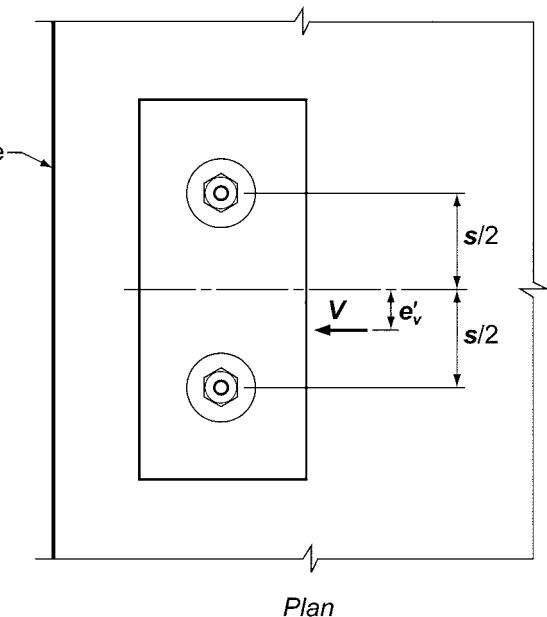
**D.6.2.5** — The modification factor for anchor groups loaded eccentrically in shear,  $\psi_{ec,V}$ , shall be computed as

$$\psi_{ec,V} = \frac{1}{\left(1 + \frac{2e'_V}{3c_{a1}}\right)} \quad (\text{D-36})$$

but  $\psi_{ec,V}$  shall not be taken greater than 1.0.

If the loading on an anchor group is such that only some anchors are loaded in shear in the same direction, only those anchors that are loaded in shear in the same direction shall be considered when determining the eccentricity of  $e'_V$  for use in Eq. (D-36) and for the calculation of  $V_{cbg}$  according to Eq. (D-31).

**RD.6.2.5** — This section provides a modification factor for an eccentric shear force toward an edge on a group of anchors. If the shear force originates above the plane of the concrete surface, the shear should first be resolved as a shear in the plane of the concrete surface, with a moment that may or may not also cause tension in the anchors, depending on the normal force. Figure RD.6.2.5 defines the term  $e'_V$  for calculating the  $\psi_{ec,V}$  modification factor that accounts for the fact that more shear is applied to one anchor than others, tending to split the concrete near an edge.

**CODE****COMMENTARY**Fig. RD.6.2.5—Definition of  $e'_v$  for a group of anchors.

**D.6.2.6** — The modification factor for edge effect for a single anchor or group of anchors loaded in shear,  $\psi_{ed,V}$ , shall be computed as follows using the smaller value of  $c_{a2}$ .

If  $c_{a2} \geq 1.5c_{a1}$

$$\text{then } \psi_{ed,V} = 1.0 \quad (\text{D-37})$$

If  $c_{a2} < 1.5c_{a1}$

$$\text{then } \psi_{ed,V} = 0.7 + 0.3 \frac{c_{a2}}{1.5c_{a1}} \quad (\text{D-38})$$

**D.6.2.7** — For anchors located in a region of a concrete member where analysis indicates no cracking at service loads, the following modification factor shall be permitted

$$\psi_{c,V} = 1.4$$

For anchors located in a region of a concrete member where analysis indicates cracking at service load levels, the following modification factors shall be permitted:

$\psi_{c,V} = 1.0$  for anchors in cracked concrete without supplementary reinforcement or with edge reinforcement smaller than a No. 4 bar;

$\psi_{c,V} = 1.2$  for anchors in cracked concrete with reinforcement of a No. 4 bar or greater between the anchor and the edge; and

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$\psi_{c,v} = 1.4$  for anchors in cracked concrete with reinforcement of a No. 4 bar or greater between the anchor and the edge, and with the reinforcement enclosed within stirrups spaced at not more than 4 in.

**D.6.2.8** — The modification factor for anchors located in a concrete member where  $h_a < 1.5c_{a1}$ ,  $\psi_{h,v}$  shall be computed as

$$\psi_{h,v} = \sqrt{\frac{1.5c_{a1}}{h_a}} \quad (\text{D-39})$$

but  $\psi_{h,v}$  shall not be taken less than 1.0.

**D.6.2.9** — Where anchor reinforcement is either developed in accordance with [Chapter 12](#) on both sides of the breakout surface, or encloses the anchor and is developed beyond the breakout surface, the design strength of the anchor reinforcement shall be permitted to be used instead of the concrete breakout strength in determining  $\phi V_n$ . A strength reduction factor of 0.75 shall be used in the design of the anchor reinforcement.

**RD.6.2.8** — For anchors located in a concrete member where  $h_a < 1.5c_{a1}$ , tests<sup>D.17,D.27</sup> have shown that the concrete breakout strength in shear is not directly proportional to the member thickness  $h_a$ . The factor  $\psi_{h,v}$  accounts for this effect.

**RD.6.2.9** — For conditions where the factored shear force exceeds the concrete breakout strength of the anchor(s) in shear, or where the breakout strength is not evaluated, the nominal strength can be that of anchor reinforcement properly anchored as shown in [Fig. RD.6.2.9\(a\)](#) and [\(b\)](#). To ensure yielding of the anchor reinforcement, the enclosing anchor reinforcement in [Fig. RD.6.2.9\(a\)](#) should be in contact with the anchor and placed as close as practicable to the concrete surface. The research<sup>D.27</sup> on which the provisions for enclosing reinforcement (see [Fig. RD.6.2.9\(a\)](#)) are based was limited to anchor reinforcement with maximum diameter similar to a No. 5 bar. The larger bend radii associated with larger bar diameters may significantly reduce the effectiveness of the anchor reinforcement, and therefore anchor reinforcement with a diameter larger than No. 6 is not recommended.

The reinforcement could also consist of stirrups and ties (as well as hairpins) enclosing the edge reinforcement embedded in the breakout cone and placed as close to the anchors as practicable (see [Fig. RD.6.2.9\(b\)](#)). Only reinforcement spaced less than the lesser of  $0.5c_{a1}$  and  $0.3c_{a2}$  from the anchor centerline should be included as anchor reinforcement. In this case, the anchor reinforcement must be developed on both sides of the breakout surface. For equilibrium reasons, an edge reinforcement must be present. The research on which these provisions are based was limited to anchor reinforcement with maximum diameter similar to a No. 6 bar.

Because the anchor reinforcement is placed below where the shear is applied (see [Fig. RD.6.2.9\(b\)](#)), the force in the anchor reinforcement will be larger than the shear force. In sizing the anchor reinforcement, use of a 0.75 strength reduction factor  $\phi$  is recommended as used for shear and for strut-and-tie models. If the alternate load factors of [Appendix C](#) are used, the corresponding strength reduction factor of 0.85 for shear and strut-and-tie models should be used. As a practical matter, the use of anchor reinforcement is generally limited to cast-in-place anchors.

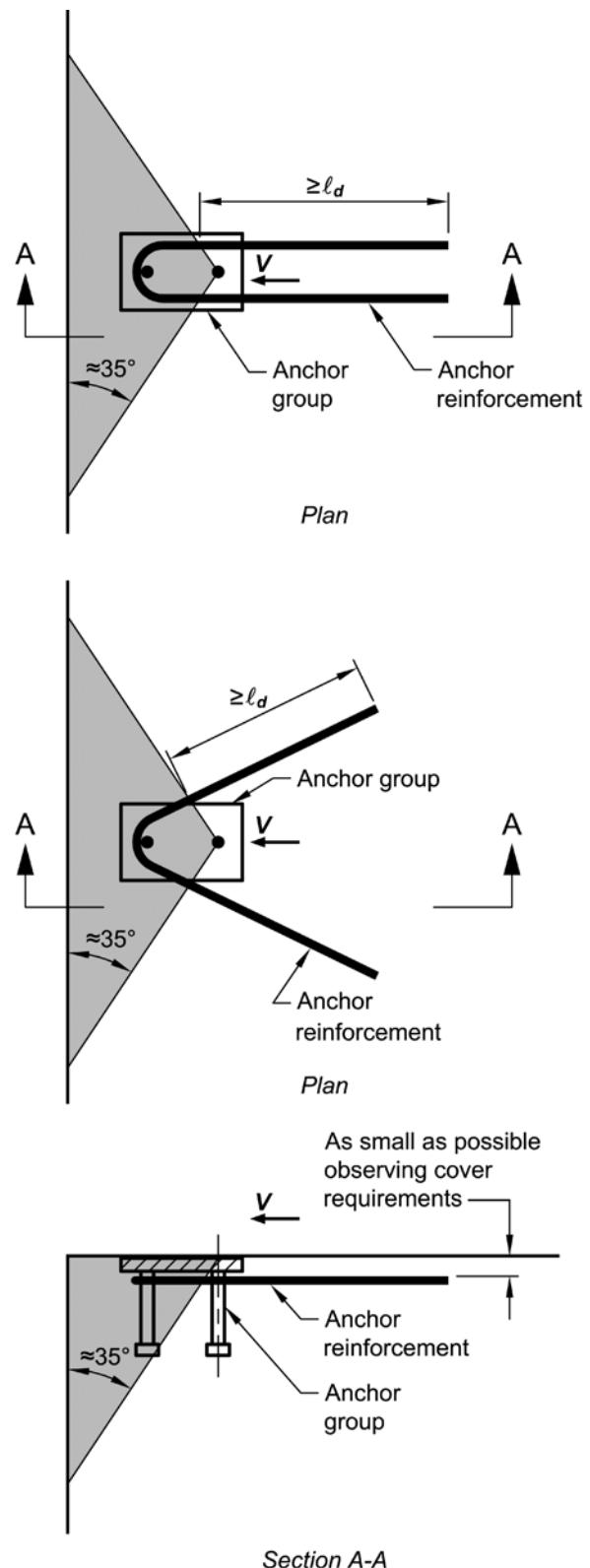
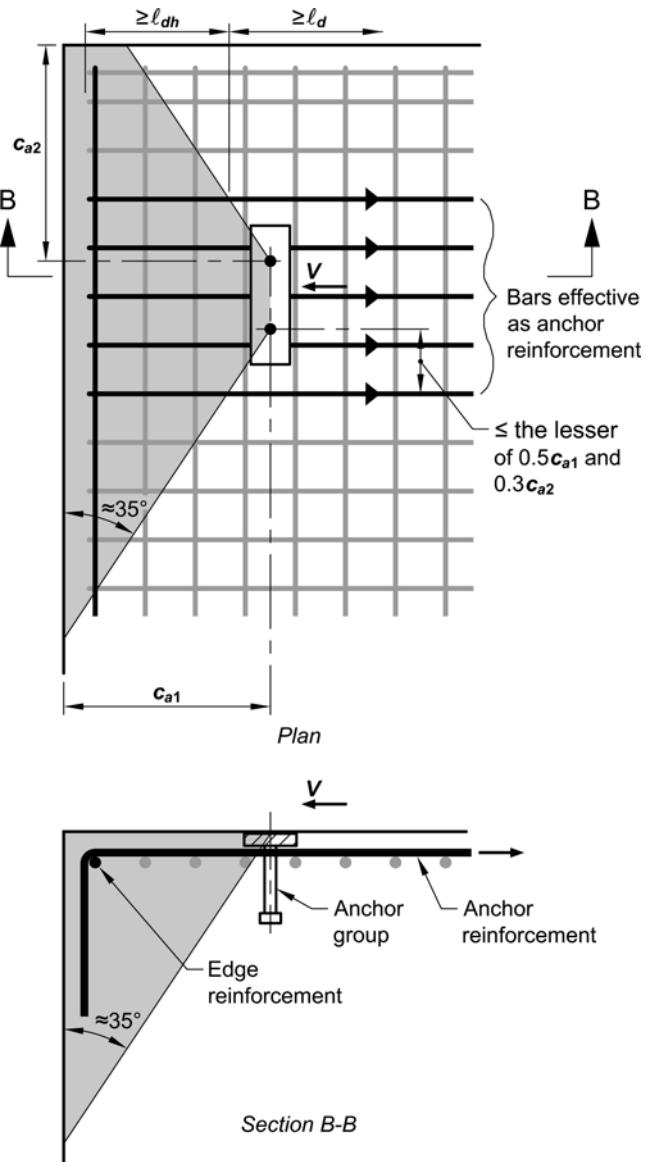
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Fig. RD.6.2.9(a)—Hairpin anchor reinforcement for shear.

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*Fig. RD.6.2.9(b)—Edge reinforcement and anchor reinforcement for shear.*

### D.6.3 — Concrete prout strength of anchor in shear

**D.6.3.1** — The nominal prout strength,  $V_{cp}$  for a single anchor or  $V_{cpq}$  for a group of anchors, shall not exceed:

(a) For a single anchor

$$V_{cp} = k_{cp} N_{cp} \quad (\text{D-40})$$

For cast-in, expansion, and undercut anchors,  $N_{cp}$  shall be taken as  $N_{cb}$  determined from Eq. (D-3),

### RD.6.3 — Concrete prout strength of anchor in shear

**RD.6.3.1** — Reference D.18 indicates that the prout shear resistance can be approximated as one to two times the anchor tensile resistance with the lower value appropriate for  $h_{ef}$  less than 2.5 in. Because it is possible that the bond strength of adhesive anchors could be less than the concrete breakout strength, it is necessary to consider both D.5.2.1 and D.5.5.1 for determination of the prout strength.

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and for adhesive anchors,  $N_{cp}$  shall be the lesser of  $N_a$  determined from Eq. (D-18) and  $N_{cb}$  determined from Eq. (D-3).

(b) For a group of anchors

$$V_{cpg} = k_{cp} N_{cpg} \quad (\text{D-41})$$

For cast-in, expansion, and undercut anchors,  $N_{cpg}$  shall be taken as  $N_{cbg}$  determined from Eq. (D-4), and for adhesive anchors,  $N_{cpg}$  shall be the lesser of  $N_{ag}$  determined from Eq. (D-19) and  $N_{cbg}$  determined from Eq. (D-4).

In Eq. (D-40) and (D-41),  $k_{cp} = 1.0$  for  $h_{ef} < 2.5$  in.; and  $k_{cp} = 2.0$  for  $h_{ef} \geq 2.5$  in.

**D.7 — Interaction of tensile and shear forces**

Unless determined in accordance with D.4.1.3, anchors or groups of anchors that are subjected to both shear and axial loads shall be designed to satisfy the requirements of D.7.1 through D.7.3. The values of  $\phi N_n$  and  $\phi V_n$  shall be the required strengths as determined from D.4.1.1 or from D.3.3.

**D.7.1** — If  $V_{ua}/(\phi V_n) \leq 0.2$  for the governing strength in shear, then full strength in tension shall be permitted:  $\phi N_n \geq N_{ua}$ .

**D.7.2** — If  $N_{ua}/(\phi N_n) \leq 0.2$  for the governing strength in tension, then full strength in shear shall be permitted:  $\phi V_n \geq V_{ua}$ .

**D.7.3** — If  $V_{ua}/(\phi V_n) > 0.2$  for the governing strength in shear and  $N_{ua}/(\phi N_n) > 0.2$  for the governing strength in tension, then

$$\frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2 \quad (\text{D-42})$$

**RD.7 — Interaction of tensile and shear forces**

The shear-tension interaction expression has traditionally been expressed as

$$\left( \frac{N_{ua}}{N_n} \right)^{\zeta} + \left( \frac{V_{ua}}{V_n} \right)^{\zeta} \leq 1.0$$

where  $\zeta$  varies from 1 to 2. The current trilinear recommendation is a simplification of the expression where  $\zeta = 5/3$  (Fig. RD.7). The limits were chosen to eliminate the requirement for computation of interaction effects where very small values of the second force are present. Any other interaction expression that is verified by test data, however, can be used to satisfy D.4.1.3.

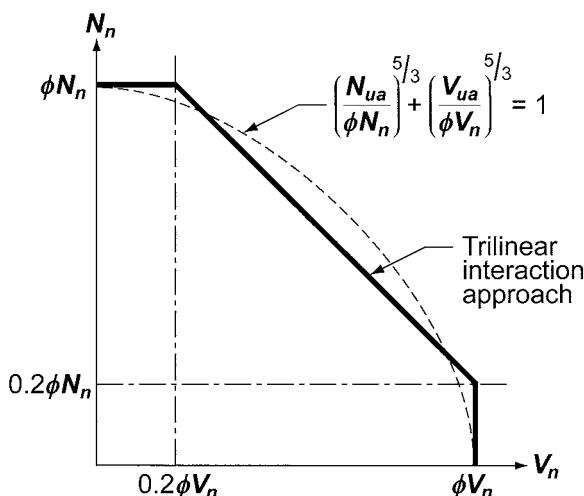


Fig. RD.7—Shear and tensile load interaction equation.

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**D.8 — Required edge distances, spacings, and thicknesses to preclude splitting failure**

Minimum spacings and edge distances for anchors and minimum thicknesses of members shall conform to D.8.1 through **D.8.6**, unless supplementary reinforcement is provided to control splitting. Lesser values from product-specific tests performed in accordance with ACI 355.2 or ACI 355.4 shall be permitted.

**D.8.1** — Unless determined in accordance with D.8.4, minimum center-to-center spacing of anchors shall be  **$4d_a$**  for cast-in anchors that will not be torqued, and  **$6d_a$**  for torqued cast-in anchors and post-installed anchors.

**D.8.2** — Unless determined in accordance with D.8.4, minimum edge distances for cast-in anchors that will not be torqued shall be based on specified cover requirements for reinforcement in **7.7**. For cast-in anchors that will be torqued, the minimum edge distances shall be  **$6d_a$** .

**D.8.3** — Unless determined in accordance with D.8.4, minimum edge distances for post-installed anchors shall be based on the greater of specified cover requirements for reinforcement in **7.7**, or minimum edge distance requirements for the products as determined by tests in accordance with ACI 355.2 or ACI 355.4, and shall not be less than twice the maximum aggregate size. In the absence of product-specific ACI 355.2 or ACI 355.4 test information, the minimum edge distance shall not be less than:

Adhesive anchors.....	<b><math>6d_a</math></b>
Undercut anchors.....	<b><math>6d_a</math></b>
Torque-controlled anchors.....	<b><math>8d_a</math></b>
Displacement-controlled anchors.....	<b><math>10d_a</math></b>

**D.8.4** — For anchors where installation does not produce a splitting force and that will not be torqued, if the edge distance or spacing is less than those specified in D.8.1 to D.8.3, calculations shall be performed by substituting for  $d_a$  a smaller value  $d'_a$  that meets the requirements of D.8.1 to D.8.3. Calculated forces applied to the anchor shall be limited to the values corresponding to an anchor having a diameter of  $d'_a$ .

**D.8.5** — Unless determined from tests in accordance with ACI 355.2, the value of  $h_{ef}$  for an expansion or undercut post-installed anchor shall not exceed the greater of 2/3 of the member thickness,  $h_a$ , and the member thickness minus 4 in.

**RD.8 — Required edge distances, spacings, and thicknesses to preclude splitting failure**

The minimum spacings, edge distances, and thicknesses are very dependent on the anchor characteristics. Installation forces and torques in post-installed anchors can cause splitting of the surrounding concrete. Such splitting also can be produced in subsequent torquing during connection of attachments to anchors including cast-in anchors. The primary source of values for minimum spacings, edge distances, and thicknesses of post-installed anchors should be the product-specific tests of ACI 355.2 and ACI 355.4. In some cases, however, specific products are not known in the design stage. Approximate values are provided for use in design.

**RD.8.2** — Because the edge cover over a deep embedment close to the edge can have a significant effect on the side-face blowout strength of **D.5.4**, in addition to the normal concrete cover requirements, it may be advantageous to use larger cover to increase the side-face blowout strength.

**RD.8.3** — Drilling holes for post-installed anchors can cause microcracking. The requirement for a minimum edge distance twice the maximum aggregate size is to minimize the effects of such microcracking.

**RD.8.4** — In some cases, it may be desirable to use a larger-diameter anchor than the requirements on D.8.1 to D.8.3 permit. In these cases, it is permissible to use a larger-diameter anchor provided the design strength of the anchor is based on a smaller assumed anchor diameter,  $d'_a$ .

**RD.8.5** — Splitting failures are caused by the load transfer between the bolt and the concrete. The limitations on the value of  $h_{ef}$  do not apply to cast-in and adhesive anchors because the splitting forces associated with these anchor types are less than for expansion and undercut anchors.

**CODE****COMMENTARY**

For all post-installed anchors, the maximum embedment depth for a given member thickness should be limited as required to avoid back-face blowout on the opposite side of the concrete member during hole drilling and anchor setting. This is dependent on many variables such as the anchor type, drilling method, drilling technique, type and size of drilling equipment, presence of reinforcement, and strength and condition of the concrete.

**D.8.6** — Unless determined from tension tests in accordance with ACI 355.2 or ACI 355.4, the critical edge distance,  $c_{ac}$ , shall not be taken less than:

Adhesive anchors .....	$2h_{ef}$
Undercut anchors .....	$2.5h_{ef}$
Torque-controlled expansion anchors.....	$4h_{ef}$
Displacement-controlled expansion anchors .....	$4h_{ef}$

**RD.8.6** — The critical edge distance  $c_{ac}$  is determined by the corner test in ACI 355.2 or ACI 355.4, and is only applicable to designs for uncracked concrete. To permit the design of these types of anchors when product-specific information is not available, conservative default values for  $c_{ac}$  are provided. Research has indicated that the corner-test requirements are not met with  $c_{a,min} = 1.5h_{ef}$  for many expansion anchors and some undercut anchors because installation of these types of anchors introduces splitting tensile stresses in the concrete that are increased during load application, potentially resulting in a premature splitting failure. Similarly, adhesive anchors that meet the maximum embedment depth requirement of D.8.5 may not fulfill the corner test requirements with  $c_{a,min} = c_{Na}$  due to the additional flexural stresses induced in the member by the anchor.

**D.8.7** — Contract documents shall specify use of anchors with a minimum edge distance as assumed in design.

**D.9 — Installation and inspection of anchors**

**D.9.1** — Anchors shall be installed by qualified personnel in accordance with the contract documents. The contract documents shall require installation of post-installed anchors in accordance with the Manufacturer's Printed Installation Instructions (MPII). Installation of adhesive anchors shall be performed by personnel trained to install adhesive anchors.

**RD.9 — Installation and inspection of anchors**

**RD.9.1** — Many anchor performance characteristics depend on proper installation of the anchor. Installation of adhesive anchors should be performed by personnel qualified for the adhesive anchor system and installation procedures being used. Construction personnel can establish qualifications by becoming certified through certification programs. For cast-in anchors, care must be taken that the anchors are securely positioned in the formwork and oriented in accordance with the contract documents. Furthermore, it should be ensured that the concrete around the anchors is properly consolidated. Inspection is particularly important for post-installed anchors to make certain that the Manufacturer's Printed Installation Instructions (MPII) are followed. For adhesive anchors, continuous monitoring of installations by qualified inspectors is recommended to ensure required installation procedures are followed. Post-installed anchor strength and deformation capacity are assessed by acceptance testing under ACI 355.2 or ACI 355.4. These tests are carried out assuming installation in accordance with the MPII. Certain types of anchors can be sensitive to variations in hole diameter, cleaning conditions, orientation of the axis, magnitude of the installation torque, crack width, and other variables. Some of this sensitivity is indirectly accounted for in the assigned  $\phi$  values for the different anchor categories, which

**CODE****COMMENTARY**

**D.9.2** — Installation of anchors shall be inspected in accordance with 1.3 and the general building code. Adhesive anchors shall be subject to the following additional requirements:

**D.9.2.1** — For adhesive anchors, the contract documents shall specify proof loading where required in accordance with ACI 355.4. The contract documents shall also specify all parameters associated with the characteristic bond stress used for the design according to D.5.5 including minimum age of concrete; concrete temperature range; moisture condition of concrete at time of installation; type of lightweight concrete, if applicable; and requirements for hole drilling and preparation.

depend in part on the results of the installation safety tests. Gross deviations from the ACI 355.2 or ACI 355.4 acceptance testing results could occur if anchor components are altered, or if anchor installation criteria or procedures vary from those specified in the MPII.

**RD.9.2.1** — Due to the sensitivity of bond strength to installation, on-site quality control is important for adhesive anchors. Where appropriate, a proof loading program should be specified in the contract documents. For adhesive anchors, the contract documents must also provide all parameters relevant to the characteristic bond stress used in the design. These parameters may include, but are not limited to:

1. Acceptable anchor installation environment (dry or saturated concrete; concrete temperature range);
2. Acceptable drilling methods;
3. Required hole cleaning procedures; and
4. Anchor type and size range (threaded rod or reinforcing bar).

Hole cleaning is intended to ensure that drilling debris and dust do not impair bond. Depending on the on-site conditions, hole cleaning may involve operations to remove drilling debris from the hole with vacuum or compressed air, mechanical brushing of the hole wall to remove surface dust, and a final step to evacuate any remaining dust or debris, usually with compressed air. Where wet core drilling is used, holes may be flushed with water and then dried with compressed air. If anchors are installed in locations where the concrete is saturated (for example, outdoor locations exposed to rainfall), the resulting drilling mud must be removed by other means. In all cases, the procedures used should be clearly described by the manufacturer in printed installation instructions accompanying the product. These printed installation instructions, which also describe the limits on concrete temperature and the presence of water during installation as well as the procedures necessary for void-free adhesive injection and adhesive cure requirements, constitute an integral part of the adhesive anchor system and are part of the assessment performed in accordance with ACI 355.4.

**CODE****COMMENTARY**

**D.9.2.2** — Installation of adhesive anchors horizontally or upwardly inclined to support sustained tension loads shall be performed by personnel certified by an applicable certification program. Certification shall include written and performance tests in accordance with the ACI/CRSI Adhesive Anchor Installer Certification program, or equivalent.

**D.9.2.3** — The acceptability of certification other than the ACI/CRSI Adhesive Anchor Installer Certification shall be the responsibility of the licensed design professional.

**D.9.2.4** — Adhesive anchors installed in horizontal or upwardly inclined orientations to resist sustained tension loads shall be continuously inspected during installation by an inspector specially approved for that purpose by the building official. The special inspector shall furnish a report to the licensed design professional and building official that the work covered by the report has been performed and that the materials used and the installation procedures used conform with the approved contract documents and the Manufacturer's Printed Installation Instructions (MPII).

**RD.9.2.2** — The sensitivity of adhesive anchors to installation orientation combined with sustained tension loading warrants installer certification. Certification may also be appropriate for other safety-related applications. Certification is established through an independent assessment such as the ACI/CRSI Adhesive Anchor Installation Certification Program, or similar program with equivalent requirements. In addition, installers should obtain instruction through product-specific training offered by manufacturers of qualified adhesive anchor systems.

**RD.9.2.3** — For the purposes of satisfying D.9.2.3, an equivalent certified installer program should test the adhesive anchor installer's knowledge and skill by an objectively fair and unbiased administration and grading of a written and performance exam. Programs should reflect the knowledge and skill required to install available commercial anchor systems. The effectiveness of a written exam should be verified through statistical analysis of the questions and answers. An equivalent program should provide a responsive and accurate mechanism to verify credentials, which are renewed on a periodic basis.

**RD.9.2.4** — The IBC<sup>D.42</sup> requires special inspection of all post-installed anchors. The installation of adhesive anchors in horizontal or upwardly inclined orientations poses special challenges to the installer and requires particular attention to execution quality as well as an enhanced level of oversight. It is expected that these anchor installations will be inspected by a certified special inspector who is continuously present when and where the installations are being performed.

**Notes**

## APPENDIX E — STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

### ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in. <sup>2</sup>	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

### ASTM STANDARD PRESTRESSING TENDONS

Type*	Nominal diameter, in.	Nominal area, in. <sup>2</sup>	Nominal weight, lb/ft
Seven-wire strand (Grade 250)	1/4 (0.250)	0.036	0.122
	5/16 (0.313)	0.058	0.197
	3/8 (0.375)	0.080	0.272
	7/16 (0.438)	0.108	0.367
	1/2 (0.500)	0.144	0.490
	(0.600)	0.216	0.737
Seven-wire strand (Grade 270)	3/8 (0.375)	0.085	0.290
	7/16 (0.438)	0.115	0.390
	1/2 (0.500)	0.153	0.520
	(0.600)	0.217	0.740
Prestressing wire	0.192	0.029	0.098
	0.196	0.030	0.100
	0.250	0.049	0.170
	0.276	0.060	0.200
Prestressing bars (plain)	3/4	0.44	1.50
	7/8	0.60	2.04
	1	0.78	2.67
	1-1/8	0.99	3.38
	1-1/4	1.23	4.17
	1-3/8	1.48	5.05
Prestressing bars (deformed)	5/8	0.28	0.98
	3/4	0.42	1.49
	1	0.85	3.01
	1-1/4	1.25	4.39
	1-3/8	1.58	5.56

\* Availability of some tendon sizes should be investigated in advance.

**ASTM STANDARD WIRE REINFORCEMENT**

W & D size		Nominal diameter, in.	Nominal area, in. <sup>2</sup>	Nominal weight, lb/ft	Area, in. <sup>2</sup> /ft of width for various spacings						
					Center-to-center spacing, in.						
Plain	Deformed				2	3	4	6	8	10	12
W31	D31	0.628	0.310	1.05	1.86	1.24	0.93	0.62	0.46	0.37	0.31
W30	D30	0.618	0.300	1.02	1.80	1.20	0.90	0.60	0.45	0.36	0.30
	D29	0.608	0.290	0.986	1.74	1.16	0.87	0.58	0.43	0.34	0.29
W28	D28	0.597	0.280	0.952	1.68	1.12	0.84	0.56	0.42	0.33	0.28
	D27	0.586	0.270	0.918	1.62	1.08	0.81	0.54	0.40	0.32	0.27
W26	D26	0.575	0.260	0.884	1.56	1.04	0.78	0.52	0.39	0.31	0.26
	D25	0.564	0.250	0.850	1.50	1.00	0.75	0.50	0.37	0.30	0.25
W24	D24	0.553	0.240	0.816	1.44	0.96	0.72	0.48	0.36	0.28	0.24
	D23	0.541	0.230	0.782	1.38	0.92	0.69	0.46	0.34	0.27	0.23
W22	D22	0.529	0.220	0.748	1.32	0.88	0.66	0.44	0.33	0.26	0.22
	D21	0.517	0.210	0.714	1.26	0.84	0.63	0.42	0.31	0.25	0.21
W20	D20	0.505	0.200	0.680	1.20	0.80	0.60	0.40	0.30	0.24	0.20
	D19	0.492	0.190	0.646	1.14	0.76	0.57	0.38	0.28	0.22	0.19
W18	D18	0.479	0.180	0.612	1.08	0.72	0.54	0.36	0.27	0.21	0.18
	D17	0.465	0.170	0.578	1.02	0.68	0.51	0.34	0.25	0.20	0.17
W16	D16	0.451	0.160	0.544	0.96	0.64	0.48	0.32	0.24	0.19	0.16
	D15	0.437	0.150	0.510	0.90	0.60	0.45	0.30	0.22	0.18	0.15
W14	D14	0.422	0.140	0.476	0.84	0.56	0.42	0.28	0.21	0.16	0.14
	D13	0.407	0.130	0.422	0.78	0.52	0.39	0.26	0.19	0.15	0.13
W12	D12	0.391	0.120	0.408	0.72	0.48	0.36	0.24	0.18	0.14	0.12
W11	D11	0.374	0.110	0.374	0.66	0.44	0.33	0.22	0.16	0.13	0.11
W10	D10	0.357	0.100	0.340	0.60	0.40	0.30	0.20	0.15	0.12	0.10
	D9	0.339	0.090	0.306	0.54	0.36	0.27	0.18	0.13	0.10	0.09
W8	D8	0.319	0.080	0.272	0.48	0.32	0.24	0.16	0.12	0.09	0.08
	D7	0.299	0.070	0.238	0.42	0.28	0.21	0.14	0.10	0.08	0.07
W6	D6	0.276	0.060	0.204	0.36	0.24	0.18	0.12	0.09	0.07	0.06
W5.5		0.265	0.055	0.187	0.33	0.22	0.165	0.11	0.08	0.06	0.055
W5	D5	0.252	0.050	0.170	0.30	0.20	0.15	0.10	0.07	0.06	0.05
W4.5		0.239	0.045	0.153	0.27	0.18	0.135	0.09	0.06	0.05	0.045
W4	D4	0.226	0.040	0.136	0.24	0.16	0.12	0.08	0.06	0.04	0.04

## APPENDIX F — EQUIVALENCE BETWEEN SI-METRIC, MKS-METRIC, AND U.S. CUSTOMARY UNITS OF NONHOMOGENEOUS EQUATIONS IN THE CODE

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
	<b>1 MPa</b>	10 kgf/cm <sup>2</sup>	<b>145 psi</b>
	$f'_c = 21 \text{ MPa}$	$f'_c = 210 \text{ kgf/cm}^2$	$f'_c = 3000 \text{ psi}$
	$f'_c = 28 \text{ MPa}$	$f'_c = 280 \text{ kgf/cm}^2$	$f'_c = 4000 \text{ psi}$
	$f'_c = 35 \text{ MPa}$	$f'_c = 350 \text{ kgf/cm}^2$	$f'_c = 5000 \text{ psi}$
	$f'_c = 40 \text{ MPa}$	$f'_c = 420 \text{ kgf/cm}^2$	$f'_c = 6000 \text{ psi}$
	$f_y = 280 \text{ MPa}$	$f_y = 2800 \text{ kgf/cm}^2$	$f_y = 40,000 \text{ psi}$
	$f_y = 420 \text{ MPa}$	$f_y = 4200 \text{ kgf/cm}^2$	$f_y = 60,000 \text{ psi}$
	$f_{pu} = 1725 \text{ MPa}$	$f_{pu} = 17,600 \text{ kgf/cm}^2$	$f_{pu} = 250,000 \text{ psi}$
	$f_{pu} = 1860 \text{ MPa}$	$f_{pu} = 19,000 \text{ kgf/cm}^2$	$f_{pu} = 270,000 \text{ psi}$
	$\sqrt{f'_c} \text{ in MPa}$	$3.18\sqrt{f'_c} \text{ in kgf/cm}^2$	$12\sqrt{f'_c} \text{ in psi}$
	$0.313\sqrt{f'_c} \text{ in MPa}$	$\sqrt{f'_c} \text{ in kgf/cm}^2$	$3.77\sqrt{f'_c} \text{ in psi}$
	$0.083\sqrt{f'_c} \text{ in MPa}$	$0.27\sqrt{f'_c} \text{ in kgf/cm}^2$	$\sqrt{f'_c} \text{ in psi}$
	$0.17\sqrt{f'_c} \text{ in MPa}$	$0.53\sqrt{f'_c} \text{ in kgf/cm}^2$	$2\sqrt{f'_c} \text{ in psi}$
(5-2)	$f'_{cr} = f'_c + 2.33s_s - 3.5$	$f'_{cr} = f'_c + 2.33s_s - 35$	$f'_{cr} = f'_c + 2.33s_s - 500$
Table 5.3.2.2	$f'_{cr} = f'_c + 7.0$	$f'_{cr} = f'_c + 70$	$f'_{cr} = f'_c + 1000$
	$f'_{cr} = f'_c + 8.3$	$f'_{cr} = f'_c + 84$	$f'_{cr} = f'_c + 1200$
	$f'_{cr} = 1.10f'_c + 5.0$	$f'_{cr} = 1.10f'_c + 50$	$f'_{cr} = 1.10f'_c + 700$
Section 7.12.2.1(c)	$\frac{0.0018 \times 420}{f_y}$	$\frac{0.0018 \times 4200}{f_y}$	$\frac{0.0018 \times 60,000}{f_y}$
Section 8.5.1	$E_c = w_c^{1.5} 0.043\sqrt{f'_c}$	$E_c = w_c^{1.5} 0.14\sqrt{f'_c}$	$E_c = w_c^{1.5} 33\sqrt{f'_c}$
	$E_c = 4700\sqrt{f'_c}$	$E_c = 15,100\sqrt{f'_c}$	$E_c = 57,000\sqrt{f'_c}$
Section 8.6.1	$\lambda = f_{ct}/(0.56\sqrt{f'_c}) \leq 1.0$	$\lambda = f_{ct}/(1.78\sqrt{f'_c}) \leq 1.0$	$\lambda = f_{ct}/(6.7\sqrt{f'_c}) \leq 1.0$
Table 9.5(a)	$(1.65 - 0.0003w_c) \geq 1.09$ $(0.4 + \frac{f_y}{700})$	$(1.65 - 0.0003w_c) \geq 1.09$ $(0.4 + \frac{f_y}{7000})$	$(1.65 - 0.005w_c) \geq 1.09$ $(0.4 + \frac{f_y}{100,000})$
(9-10)	$f_r = 0.62\lambda\sqrt{f'_c}$	$f_r = 2\lambda\sqrt{f'_c}$	$f_r = 7.5\lambda\sqrt{f'_c}$
(9-12)	$h = \frac{\ell_n(0.8 + \frac{f_y}{1400})}{36 + 5\beta(\alpha_{fm} - 0.2)} \geq 125 \text{ mm}$	$h = \frac{\ell_n(0.8 + \frac{f_y}{14,000})}{36 + 5\beta(\alpha_{fm} - 0.2)} \geq 12.5 \text{ cm}$	$h = \frac{\ell_n(0.8 + \frac{f_y}{200,000})}{36 + 5\beta(\alpha_{fm} - 0.2)} \geq 5 \text{ in.}$
(9-13)	$h = \frac{\ell_n(0.8 + \frac{f_y}{1400})}{36 + 9\beta} \geq 90 \text{ mm}$	$h = \frac{\ell_n(0.8 + \frac{f_y}{14,000})}{36 + 9\beta} \geq 9 \text{ cm}$	$h = \frac{\ell_n(0.8 + \frac{f_y}{200,000})}{36 + 9\beta} \geq 3.5 \text{ in.}$
(10-3)	$A_{s,min} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$	$A_{s,min} = \frac{0.8\sqrt{f'_c}}{f_y} b_w d \geq \frac{14}{f_y} b_w d$	$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200}{f_y} b_w d$
(10-4)	$s = 380\left(\frac{280}{f_s}\right) - 2.5c_c \leq 300\left(\frac{280}{f_s}\right)$	$s = 38\left(\frac{2800}{f_s}\right) - 2.5c_c \leq 30\left(\frac{2800}{f_s}\right)$	$s = 15\left(\frac{40,000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40,000}{f_s}\right)$

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
(10-17)	$M_{2,min} = P_u(15 + 0.03h)$	$M_{2,min} = P_u(1.5 + 0.03h)$	$M_{2,min} = P_u(0.6 + 0.03h)$
Section 11.1.2	$\sqrt{f'_c} \leq 8.3 \text{ MPa}$	$\sqrt{f'_c} \leq 27 \text{ kgf/cm}^2$	$\sqrt{f'_c} \leq 100 \text{ psi}$
(11-3)	$V_c = 0.17\lambda\sqrt{f'_c} b_w d$	$V_c = 0.53\lambda\sqrt{f'_c} b_w d$	$V_c = 2\lambda\sqrt{f'_c} b_w d$
(11-4)	$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f'_c} b_w d$	$V_c = 0.53\left(1 + \frac{N_u}{140A_g}\right)\lambda\sqrt{f'_c} b_w d$	$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\lambda\sqrt{f'_c} b_w d$
(11-5)	$V_c = \left(0.16\lambda\sqrt{f'_c} + 17\rho_w \frac{V_u d}{M_u}\right)b_w d$ $\leq 0.29\lambda\sqrt{f'_c} b_w d$	$V_c = \left(0.5\lambda\sqrt{f'_c} + 176\rho_w \frac{V_u d}{M_u}\right)b_w d$ $\leq 0.93\lambda\sqrt{f'_c} b_w d$	$V_c = \left(1.9\lambda\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right)b_w d$ $\leq 3.5\lambda\sqrt{f'_c} b_w d$
(11-7)	$V_c = 0.29\lambda\sqrt{f'_c} b_w d \sqrt{1 + \frac{0.29N_u}{A_g}}$	$V_c = 0.93\lambda\sqrt{f'_c} b_w d \sqrt{1 + \frac{N_u}{35A_g}}$	$V_c = 3.5\lambda\sqrt{f'_c} b_w d \sqrt{1 + \frac{N_u}{500A_g}}$
(11-8)	$V_c = 0.17\left(1 + \frac{0.29N_u}{A_g}\right)\lambda\sqrt{f'_c} b_w d \geq 0$	$V_c = 0.53\left(1 + \frac{N_u}{35A_g}\right)\lambda\sqrt{f'_c} b_w d \geq 0$	$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\lambda\sqrt{f'_c} b_w d \geq 0$
(11-9)	$V_c = \left(0.05\lambda\sqrt{f'_c} + 4.8\frac{V_u d_p}{M_u}\right)b_w d$ $0.17\lambda\sqrt{f'_c} b_w d \leq V_c \leq 0.42\lambda\sqrt{f'_c} b_w d$	$V_c = \left(0.16\lambda\sqrt{f'_c} + 49\frac{V_u d_p}{M_u}\right)b_w d$ $0.53\lambda\sqrt{f'_c} b_w d \leq V_c \leq 1.33\lambda\sqrt{f'_c} b_w d$	$V_c = \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d_p}{M_u}\right)b_w d$ $2\lambda\sqrt{f'_c} b_w d \leq V_c \leq 5\lambda\sqrt{f'_c} b_w d$
(11-10)	$V_{ci} = 0.05\lambda\sqrt{f'_c} b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ $\geq 0.14\lambda\sqrt{f'_c} b_w d$	$V_{ci} = 0.16\lambda\sqrt{f'_c} b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ $\geq 0.45\lambda\sqrt{f'_c} b_w d$	$V_{ci} = 0.6\lambda\sqrt{f'_c} b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ $\geq 1.7\lambda\sqrt{f'_c} b_w d$
(11-11)	$M_{cre} = \left(\frac{I}{Y_t}\right)(0.5\lambda\sqrt{f'_c} + f_{pe} - f_d)$	$M_{cre} = \left(\frac{I}{Y_t}\right)(1.6\lambda\sqrt{f'_c} + f_{pe} - f_d)$	$M_{cre} = \left(\frac{I}{Y_t}\right)(6\lambda\sqrt{f'_c} + f_{pe} - f_d)$
(11-12)	$V_{cw} = (0.29\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$ $0.33\lambda\sqrt{f'_c}$	$V_{cw} = (0.93\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$ $1.1\lambda\sqrt{f'_c}$	$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$ $4\lambda\sqrt{f'_c}$
Section 11.4.5.3	$0.33\sqrt{f'_c} b_w d$	$1.1\sqrt{f'_c} b_w d$	$4\sqrt{f'_c} b_w d$
Section 11.4.6.1(f)	$\phi 0.17\sqrt{f'_c} b_w d$	$\phi 0.53\sqrt{f'_c} b_w d$	$\phi 2\sqrt{f'_c} b_w d$
(11-13)	$A_{v,min} = 0.062\sqrt{f'_c} \frac{b_w s}{f_{yt}}$ $\geq 0.35\frac{b_w s}{f_{yt}}$	$A_{v,min} = 0.2\sqrt{f'_c} \frac{b_w s}{f_{yt}}$ $\geq 3.5\frac{b_w s}{f_{yt}}$	$A_{v,min} = 0.75\sqrt{f'_c} \frac{b_w s}{f_{yt}}$ $\geq \frac{50b_w s}{f_{yt}}$
(11-17)	$V_s = A_v f_y \sin\alpha \leq 0.25\sqrt{f'_c} b_w d$	$V_s = A_v f_y \sin\alpha \leq 0.8\sqrt{f'_c} b_w d$	$V_s = A_v f_y \sin\alpha \leq 3\sqrt{f'_c} b_w d$
Section 11.4.7.9	$0.66\sqrt{f'_c} b_w d$	$2.2\sqrt{f'_c} b_w d$	$8\sqrt{f'_c} b_w d$
Section 11.5.1(a)	$T_u < \phi 0.083\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right)$	$T_u < \phi 0.27\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right)$	$T_u < \phi\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right)$
Section 11.5.1(b)	$T_u < \phi 0.083\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right) \sqrt{1 + \frac{f_{pc}}{0.33\lambda\sqrt{f'_c}}}$	$T_u < \phi 0.27\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right) \sqrt{1 + \frac{f_{pc}}{\lambda\sqrt{f'_c}}}$	$T_u < \phi\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right) \sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
Section 11.5.1(c)	$T_u < \phi 0.083 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}} \quad T_u < \phi 0.27 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{A_g \lambda \sqrt{f'_c}}} \quad T_u < \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4 A_g \lambda \sqrt{f'_c}}} \quad T_u < \phi 0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \quad T_u = \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \quad T_u = \phi 4 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \quad T_u = \phi 0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{0.33 \lambda \sqrt{f'_c}}} \quad T_u = \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{\lambda \sqrt{f'_c}}} \quad T_u = \phi 4 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4 \lambda \sqrt{f'_c}}} \quad T_u = \phi 33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}} \quad T_u = \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{A_g \lambda \sqrt{f'_c}}} \quad T_u = \phi 4 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4 A_g \lambda \sqrt{f'_c}}} \quad \sqrt{\left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \quad \sqrt{\left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 2 \sqrt{f'_c} \right) \quad \sqrt{\left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 8 \sqrt{f'_c} \right) \quad \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right) \leq \phi \left( \frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \quad \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right) \leq \phi \left( \frac{V_c}{b_w d} + 2 \sqrt{f'_c} \right) \quad \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right) \leq \phi \left( \frac{V_c}{b_w d} + 8 \sqrt{f'_c} \right) \quad (A_v + 2A_t) = 0.062 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{0.35 b_w s}{f_{yt}} \quad (A_v + 2A_t) = 0.2 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{3.5 b_w s}{f_{yt}} \quad (A_v + 2A_t) = 0.75 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{50 b_w s}{f_{yt}} \quad A_{\ell,min} = \frac{0.42 \sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \quad A_{\ell,min} = \frac{1.33 \sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \quad A_{\ell,min} = \frac{5 \sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \quad \frac{A_t}{s} \geq \frac{0.175 b_w}{f_{yt}} \quad \frac{A_t}{s} \geq \frac{1.75 b_w}{f_{yt}} \quad \frac{A_t}{s} \geq \frac{25 b_w}{f_{yt}} \quad (3.3 + 0.08 f'_c') A_c \quad 11 A_c \quad 5.5 A_c \quad (34 + 0.08 f'_c') A_c \quad 110 A_c \quad 55 A_c \quad (480 + 0.08 f'_c') A_c \quad 1600 A_c \quad 800 A_c \quad 0.83 \sqrt{f'_c} b_w d \quad (3.3 + 0.08 f'_c') b_w d \quad 11 b_w d \quad 2.65 \sqrt{f'_c} b_w d \quad (34 + 0.08 f'_c') b_w d \quad 110 b_w d \quad (480 + 0.08 f'_c') b_w d \quad 1600 b_w d \quad (5.5 - 1.9 \frac{a_v}{d}) b_w d \quad 0.83 \sqrt{f'_c} h d \quad 0.17 \lambda \sqrt{f'_c} h d \quad (55 - 20 \frac{a_v}{d}) b_w d \quad 2.65 \sqrt{f'_c} h d \quad (800 - 280 \frac{a_v}{d}) b_w d \quad 10 \sqrt{f'_c} h d \quad 2 \lambda \sqrt{f'_c} h d \quad Section 11.6.5$	$\phi 0.083 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}} \quad T_u < \phi 0.27 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{A_g \lambda \sqrt{f'_c}}} \quad T_u < \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4 A_g \lambda \sqrt{f'_c}}} \quad T_u < \phi 0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \quad T_u = \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \quad T_u = \phi 4 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \quad T_u = \phi 0.33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{0.33 \lambda \sqrt{f'_c}}} \quad T_u = \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{\lambda \sqrt{f'_c}}} \quad T_u = \phi 4 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4 \lambda \sqrt{f'_c}}} \quad T_u = \phi 33 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{0.33 A_g \lambda \sqrt{f'_c}}} \quad T_u = \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{A_g \lambda \sqrt{f'_c}}} \quad T_u = \phi 4 \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4 A_g \lambda \sqrt{f'_c}}} \quad \sqrt{\left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \quad \sqrt{\left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 2 \sqrt{f'_c} \right) \quad \sqrt{\left( \frac{V_u}{b_w d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right)^2} \leq \phi \left( \frac{V_c}{b_w d} + 8 \sqrt{f'_c} \right) \quad \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right) \leq \phi \left( \frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \quad \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right) \leq \phi \left( \frac{V_c}{b_w d} + 2 \sqrt{f'_c} \right) \quad \left( \frac{V_u}{b_w d} \right) + \left( \frac{T_u p_h}{1.7 A_{oh}^2} \right) \leq \phi \left( \frac{V_c}{b_w d} + 8 \sqrt{f'_c} \right) \quad (A_v + 2A_t) = 0.062 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{0.35 b_w s}{f_{yt}} \quad (A_v + 2A_t) = 0.2 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{3.5 b_w s}{f_{yt}} \quad (A_v + 2A_t) = 0.75 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq \frac{50 b_w s}{f_{yt}} \quad A_{\ell,min} = \frac{0.42 \sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \quad A_{\ell,min} = \frac{1.33 \sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \quad A_{\ell,min} = \frac{5 \sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \quad \frac{A_t}{s} \geq \frac{0.175 b_w}{f_{yt}} \quad \frac{A_t}{s} \geq \frac{1.75 b_w}{f_{yt}} \quad \frac{A_t}{s} \geq \frac{25 b_w}{f_{yt}} \quad (3.3 + 0.08 f'_c') A_c \quad 11 A_c \quad 5.5 A_c \quad (34 + 0.08 f'_c') A_c \quad 110 A_c \quad 55 A_c \quad (480 + 0.08 f'_c') A_c \quad 1600 A_c \quad 800 A_c \quad 0.83 \sqrt{f'_c} b_w d \quad (3.3 + 0.08 f'_c') b_w d \quad 11 b_w d \quad 2.65 \sqrt{f'_c} b_w d \quad (34 + 0.08 f'_c') b_w d \quad 110 b_w d \quad (480 + 0.08 f'_c') b_w d \quad 1600 b_w d \quad (5.5 - 1.9 \frac{a_v}{d}) b_w d \quad 0.83 \sqrt{f'_c} h d \quad 0.17 \lambda \sqrt{f'_c} h d \quad (55 - 20 \frac{a_v}{d}) b_w d \quad 2.65 \sqrt{f'_c} h d \quad (800 - 280 \frac{a_v}{d}) b_w d \quad 10 \sqrt{f'_c} h d \quad 2 \lambda \sqrt{f'_c} h d \quad Section 11.6.5$	
(11-18)			
(11-19)			
(11-23)			
(11-24)			
Section 11.6.5			
Section 11.7.3			
Section 11.8.3.2.1			
Section 11.8.3.2.2			
Section 11.9.3			
Section 11.9.5			

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
(11-27)	$V_c = 0.27\lambda\sqrt{f'_c} hd + \frac{N_u d}{4\ell_w}$	$V_c = 0.88\lambda\sqrt{f'_c} hd + \frac{N_u d}{4\ell_w}$	$V_c = 3.3\lambda\sqrt{f'_c} hd + \frac{N_u d}{4\ell_w}$
(11-28)	$V_c = \left[ 0.05\lambda\sqrt{f'_c} + \frac{\ell_w \left( 0.1\lambda\sqrt{f'_c} + \frac{0.2N_u}{\ell_w h} \right)}{\frac{M_u - \ell_w}{V_u - \frac{1}{2}}} \right] hd$	$V_c = \left[ 0.16\lambda\sqrt{f'_c} + \frac{\ell_w \left( 0.33\lambda\sqrt{f'_c} + \frac{0.2N_u}{\ell_w h} \right)}{\frac{M_u - \ell_w}{V_u - \frac{1}{2}}} \right] hd$	$V_c = \left[ 0.6\lambda\sqrt{f'_c} + \frac{\ell_w \left( 1.25\lambda\sqrt{f'_c} + \frac{0.2N_u}{\ell_w h} \right)}{\frac{M_u - \ell_w}{V_u - \frac{1}{2}}} \right] hd$
(11-31)	$V_c = 0.17\left(1 + \frac{2}{\beta}\right)\lambda\sqrt{f'_c} b_o d$	$V_c = 0.53\left(1 + \frac{2}{\beta}\right)\lambda\sqrt{f'_c} b_o d$	$V_c = \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f'_c} b_o d$
(11-32)	$V_c = 0.083\left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f'_c} b_o d$	$V_c = 0.27\left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f'_c} b_o d$	$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f'_c} b_o d$
(11-33)	$V_c = 0.33\lambda\sqrt{f'_c} b_o d$	$V_c = \lambda\sqrt{f'_c} b_o d$	$V_c = 4\lambda\sqrt{f'_c} b_o d$
(11-34)	$V_c = (\beta_p\lambda\sqrt{f'_c} + 0.3f_{pc})b_o d + V_p$ Note: $\beta_p$ has units	$V_c = (3.16\beta_p\lambda\sqrt{f'_c} + 0.3f_{pc})b_o d + V_p$ Note: $\beta_p$ has the same units as the SI-metric version	$V_c = (\beta_p\lambda\sqrt{f'_c} + 0.3f_{pc})b_o d + V_p$ Note: $\beta_p$ has units
Section 11.11.2.2	$\sqrt{f'_c} \leq 5.8$ MPa	$\sqrt{f'_c} \leq 18.6$ kgf/cm <sup>2</sup>	$\sqrt{f'_c} \leq 70$ psi
Section 11.11.3.1	$0.17\lambda\sqrt{f'_c} b_o d$	$0.53\lambda\sqrt{f'_c} b_o d$	$2\lambda\sqrt{f'_c} b_o d$
Section 11.11.3.2	$0.5\sqrt{f'_c} b_o d$	$1.6\sqrt{f'_c} b_o d$	$6\sqrt{f'_c} b_o d$
Section 11.11.4.8	$0.33\sqrt{f'_c} b_o d$ $0.58\sqrt{f'_c} b_o d$	$1.1\sqrt{f'_c} b_o d$ $1.9\sqrt{f'_c} b_o d$	$4\sqrt{f'_c} b_o d$ $7\sqrt{f'_c} b_o d$
Section 11.11.5.1	$0.25\lambda\sqrt{f'_c} b_o d$ $0.66\sqrt{f'_c} b_o d$ $0.17\sqrt{f'_c}$	$0.8\lambda\sqrt{f'_c} b_o d$ $2.1\sqrt{f'_c} b_o d$ $0.53\sqrt{f'_c}$	$3\lambda\sqrt{f'_c} b_o d$ $8\sqrt{f'_c} b_o d$ $2\sqrt{f'_c}$
Section 11.11.5.2	$0.5\phi\sqrt{f'_c}$	$1.6\phi\sqrt{f'_c}$	$6\phi\sqrt{f'_c}$
Section 11.11.5.4	$0.17\phi\lambda\sqrt{f'_c}$	$0.53\phi\lambda\sqrt{f'_c}$	$2\phi\lambda\sqrt{f'_c}$
Section 11.11.7.2	$0.17\phi\lambda\sqrt{f'_c}$	$0.53\phi\lambda\sqrt{f'_c}$	$2\phi\lambda\sqrt{f'_c}$
Section 11.11.7.3	$0.33\phi\lambda\sqrt{f'_c}$	$1.1\phi\lambda\sqrt{f'_c}$	$4\phi\lambda\sqrt{f'_c}$
Section 12.1.2	$\sqrt{f'_c} \leq 8.3$ MPa	$\sqrt{f'_c} \leq 26.5$ kgf/cm <sup>2</sup>	$\sqrt{f'_c} \leq 100$ psi
Section 12.2.2	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{2.1\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{6.6\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{25\lambda\sqrt{f'_c}} \right) d_b$
Section 12.2.2	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{1.7\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{5.3\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{20\lambda\sqrt{f'_c}} \right) d_b$
Section 12.2.2	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{1.4\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{4.4\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{3f_y \Psi_t \Psi_e}{50\lambda\sqrt{f'_c}} \right) d_b$
Section 12.2.2	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{1.1\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{f_y \Psi_t \Psi_e}{3.5\lambda\sqrt{f'_c}} \right) d_b$	$\ell_d = \left( \frac{3f_y \Psi_t \Psi_e}{40\lambda\sqrt{f'_c}} \right) d_b$

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
(12-1)	$\ell_d = \frac{f_y}{1.1\lambda\sqrt{f'_c}} \frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} d_b$	$\ell_d = \frac{f_y}{3.5\lambda\sqrt{f'_c}} \frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} d_b$	$\ell_d = \frac{3f_y}{40\lambda\sqrt{f'_c}} \frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} d_b$
Section 12.3.2	$\left(\frac{0.24f_y}{\lambda\sqrt{f'_c}}\right)d_b \geq (0.043f_y)d_b$	$\left(\frac{0.075f_y}{\lambda\sqrt{f'_c}}\right)d_b \geq (0.0044f_y)d_b$	$\left(\frac{0.02f_y}{\lambda\sqrt{f'_c}}\right)d_b \geq (0.0003f_y)d_b$
Section 12.5.2	$\ell_{dh} = \left(\frac{0.24\Psi_e f_y}{\lambda\sqrt{f'_c}}\right)d_b$	$\ell_{dh} = \left(\frac{0.075\Psi_e f_y}{\lambda\sqrt{f'_c}}\right)d_b$	$\ell_{dh} = \left(\frac{0.02\Psi_e f_y}{\lambda\sqrt{f'_c}}\right)d_b$
Section 12.6.2	$\ell_{dh} = \left(\frac{0.19\Psi_e f_y}{\sqrt{f'_c}}\right)d_b$	$\ell_{dh} = \left(\frac{0.06\Psi_e f_y}{\sqrt{f'_c}}\right)d_b$	$\ell_{dh} = \left(\frac{0.016\Psi_e f_y}{\sqrt{f'_c}}\right)d_b$
Section 12.7.2	$\left(\frac{f_y - 240}{f_y}\right)$	$\left(\frac{f_y - 2460}{f_y}\right)$	$\left(\frac{f_y - 35,000}{f_y}\right)$
(12-3)	$\ell_d = 3.3\left(\frac{A_b}{s}\right)\left(\frac{f_y}{\lambda\sqrt{f'_c}}\right)$	$\ell_d = \left(\frac{A_b}{s}\right)\left(\frac{f_y}{\lambda\sqrt{f'_c}}\right)$	$\ell_d = 0.27\left(\frac{A_b}{s}\right)\left(\frac{f_y}{\lambda\sqrt{f'_c}}\right)$
(12-4)	$\ell_d = \left(\frac{f_{se}}{21}\right)d_b + \left(\frac{f_{ps} - f_{se}}{7}\right)d_b$	$\ell_d = \left(\frac{f_{se}}{210}\right)d_b + \left(\frac{f_{ps} - f_{se}}{70}\right)d_b$	$\ell_d = \left(\frac{f_{se}}{3000}\right)d_b + \left(\frac{f_{ps} - f_{se}}{1000}\right)d_b$
Section 12.10.5.2	$0.41\frac{b_w s}{f_{yt}}$	$4.2\frac{b_w s}{f_{yt}}$	$60\frac{b_w s}{f_{yt}}$
Section 12.13.2.2	$0.17\frac{d_b f_{yt}}{\lambda\sqrt{f'_c}}$	$0.053\frac{d_b f_{yt}}{\lambda\sqrt{f'_c}}$	$0.014\frac{d_b f_{yt}}{\lambda\sqrt{f'_c}}$
Section 12.16.1	$0.071f_y d_b$ $(0.13f_y - 24)d_b$	$0.0073f_y d_b$ $(0.013f_y - 24)d_b$	$0.0005f_y d_b$ $(0.0009f_y - 24)d_b$
Section 17.5.3.1 and 17.5.3.2	$0.55b_v d$	$5.6b_v d$	$80b_v d$
Section 17.5.3.3	$(1.8 + 0.6\rho_v f_y)\lambda b_v d \leq 3.5b_v d$	$(18 + 0.6\rho_v f_y)\lambda b_v d \leq 35b_v d$	$(260 + 0.6\rho_v f_y)\lambda b_v d \leq 500b_v d$
Section 18.3.3	(a) Class U : $f_t \leq 0.62\sqrt{f'_c}$	(a) Class U : $f_t \leq 2\sqrt{f'_c}$	(a) Class U : $f_t \leq 7.5\sqrt{f'_c}$
	(b) Class T : $0.62\sqrt{f'_c} < f_t \leq 1.0\sqrt{f'_c}$	(b) Class T : $2\sqrt{f'_c} < f_t \leq 3.2\sqrt{f'_c}$	(b) Class T : $7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$
	(c) Class C : $f_t > 1.0\sqrt{f'_c}$ $f_t \leq 0.50\sqrt{f'_{ci}}$	(c) Class C : $f_t > 3.2\sqrt{f'_c}$ $f_t \leq 1.6\sqrt{f'_{ci}}$	(c) Class C : $f_t > 12\sqrt{f'_c}$ $f_t \leq 6\sqrt{f'_{ci}}$
Section 18.4.1(c)	$0.50\sqrt{f'_{ci}}$ $0.25\sqrt{f'_{ci}}$	$1.6\sqrt{f'_{ci}}$ $0.8\sqrt{f'_{ci}}$	$6\sqrt{f'_{ci}}$ $3\sqrt{f'_{ci}}$
(18-2)	$f_{ps} = f_{se} + 70 + \frac{f'_c}{100\rho_p}$	$f_{ps} = f_{se} + 700 + \frac{f'_c}{100\rho_p}$	$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{100\rho_p}$
(18-3)	$f_{ps} = f_{se} + 70 + \frac{f'_c}{300\rho_p}$	$f_{ps} = f_{se} + 700 + \frac{f'_c}{300\rho_p}$	$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{300\rho_p}$
Section 18.9.3.1 and 18.9.3.2	$0.17\sqrt{f'_c}$	$0.53\sqrt{f'_c}$	$2\sqrt{f'_c}$

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
Section 18.12.7	$\frac{2.1 b_w d}{f_y}$	$\frac{21 b_w d}{f_y}$	$\frac{300 b_w d}{f_y}$
Section 18.13.4.1	$f_{ps} = f_{se} + 70$	$f_{ps} = f_{se} + 700$	$f_{ps} = f_{se} + 10,000$
Section 19.4.10	$0.33\phi\lambda\sqrt{f'_c}$	$1.1\phi\lambda\sqrt{f'_c}$	$4\phi\lambda\sqrt{f'_c}$
Section 21.5.2.1	$\frac{1.4 b_w d}{f_y}$	$\frac{14 b_w d}{f_y}$	$\frac{200 b_w d}{f_y}$
(21-2)	$s_o = 100 + \left(\frac{350 - h_x}{3}\right)$  $100 \text{ mm} \leq s_o \leq 150 \text{ mm}$	$s_o = 10 + \left(\frac{35 - h_x}{3}\right)$  $10 \text{ cm} \leq s_o \leq 15 \text{ cm}$	$s_o = 4 + \left(\frac{14 - h_x}{3}\right)$  $4 \text{ in.} \leq s_o \leq 6 \text{ in.}$
Section 21.7.4.1	$1.7\sqrt{f'_c} A_j$ $1.2\sqrt{f'_c} A_j$ $1.0\sqrt{f'_c} A_j$	$5.3\sqrt{f'_c} A_j$ $4\sqrt{f'_c} A_j$ $3.2\sqrt{f'_c} A_j$	$20\sqrt{f'_c} A_j$ $15\sqrt{f'_c} A_j$ $12\sqrt{f'_c} A_j$
(21-6)	$\ell_{dh} = \frac{f_y d_b}{5.4\sqrt{f'_c}}$	$\ell_{dh} = \frac{f_y d_b}{17.2\sqrt{f'_c}}$	$\ell_{dh} = \frac{f_y d_b}{65\sqrt{f'_c}}$
Section 21.9.2.1	$0.083 A_{cv} \lambda \sqrt{f'_c}$	$0.27 A_{cv} \lambda \sqrt{f'_c}$	$A_{cv} \lambda \sqrt{f'_c}$
Section 21.9.2.2	$0.17 A_{cv} \lambda \sqrt{f'_c}$	$0.53 A_{cv} \lambda \sqrt{f'_c}$	$2 A_{cv} \lambda \sqrt{f'_c}$
(21-7)	$V_n = A_{cv} (\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y)$  $\alpha_c = 0.25 \text{ for } \frac{h_w}{\ell_w} \leq 1.5$  $\alpha_c = 0.17 \text{ for } \frac{h_w}{\ell_w} \geq 2.0$	$V_n = A_{cv} (\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y)$  $\alpha_c = 0.80 \text{ for } \frac{h_w}{\ell_w} \leq 1.5$  $\alpha_c = 0.53 \text{ for } \frac{h_w}{\ell_w} \geq 2.0$	$V_n = A_{cv} (\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y)$  $\alpha_c = 3.0 \text{ for } \frac{h_w}{\ell_w} \leq 1.5$  $\alpha_c = 2.0 \text{ for } \frac{h_w}{\ell_w} \geq 2.0$
Section 21.9.4.4	$0.66 A_{cv} \sqrt{f'_c}$ $0.83 A_{cw} \sqrt{f'_c}$	$2.12 A_{cv} \sqrt{f'_c}$ $2.65 A_{cw} \sqrt{f'_c}$	$8 A_{cv} \sqrt{f'_c}$ $10 A_{cw} \sqrt{f'_c}$
Section 21.9.4.5	$0.83 A_{cw} \sqrt{f'_c}$	$2.65 A_{cw} \sqrt{f'_c}$	$10 A_{cw} \sqrt{f'_c}$
Section 21.9.6.5(a)	$2.8/f_y$	$28/f_y$	$400/f_y$
Section 21.9.6.5(b)	$0.083 A_{cv} \lambda \sqrt{f'_c}$	$0.27 A_{cv} \lambda \sqrt{f'_c}$	$A_{cv} \lambda \sqrt{f'_c}$
Section 21.9.7.2	$0.33 \lambda \sqrt{f'_c} A_{cw}$	$\lambda \sqrt{f'_c} A_{cw}$	$4 \lambda \sqrt{f'_c} A_{cw}$
(21-9)	$V_n = 2A_{vd} f_y \sin\alpha \leq 0.83 \sqrt{f'_c} A_{cw}$	$V_n = 2A_{vd} f_y \sin\alpha \leq 2.65 \sqrt{f'_c} A_{cw}$	$V_n = 2A_{vd} f_y \sin\alpha \leq 10 \sqrt{f'_c} A_{cw}$
(21-10)	$V_n = A_{cv} (0.17 \lambda \sqrt{f'_c} + \rho_t f_y)$	$V_n = A_{cv} (0.53 \lambda \sqrt{f'_c} + \rho_t f_y)$	$V_n = A_{cv} (2 \lambda \sqrt{f'_c} + \rho_t f_y)$
Section 21.11.9.2	$0.66 A_{cv} \sqrt{f'_c}$	$2.12 A_{cv} \sqrt{f'_c}$	$8 A_{cv} \sqrt{f'_c}$
Section 21.13.6	$0.29 \sqrt{f'_c} b_o d$	$0.93 \sqrt{f'_c} b_o d$	$3.5 \sqrt{f'_c} b_o d$
(22-2)	$M_n = 0.42 \lambda \sqrt{f'_c} S_m$	$M_n = 1.33 \lambda \sqrt{f'_c} S_m$	$M_n = 5 \lambda \sqrt{f'_c} S_m$

	SI-metric stress in MPa	mks-metric stress in kgf/cm <sup>2</sup>	U.S. Customary units stress in pounds per square inch (psi)
(22-7)	$\frac{M_u}{S_m} - \frac{P_u}{A_g} \leq \phi 0.42 \lambda_a \sqrt{f'_c}$	$\frac{M_u}{S_m} - \frac{P_u}{A_g} \leq \phi 1.33 \lambda_a \sqrt{f'_c}$	$\frac{M_u}{S_m} - \frac{P_u}{A_g} \leq \phi 5 \lambda_a \sqrt{f'_c}$
(22-9)	$V_n = 0.11 \lambda_a \sqrt{f'_c} b_w h$	$V_n = 0.35 \lambda_a \sqrt{f'_c} b_w h$	$V_n = \frac{4 \lambda_a \sqrt{f'_c}}{3} b_w h$
(22-10)	$V_n = 0.11 \left[ 1 + \frac{2}{\beta} \right] \lambda_a \sqrt{f'_c} b_o h$ $\leq 0.22 \lambda_a \sqrt{f'_c} b_o h$	$V_n = 0.35 \left[ 1 + \frac{2}{\beta} \right] \lambda_a \sqrt{f'_c} b_o h$ $\leq 0.71 \lambda_a \sqrt{f'_c} b_o h$	$V_n = \left[ \frac{4}{3} + \frac{8}{3\beta} \right] \lambda_a \sqrt{f'_c} b_o h$ $\leq 2.66 \lambda_a \sqrt{f'_c} b_o h$
(B-1)	$\rho_b = \frac{0.85 \beta_1 f'_c}{f_y} \left( \frac{600}{600 + f_y} \right)$	$\rho_b = \frac{0.85 \beta_1 f'_c}{f_y} \left( \frac{6120}{6120 + f_y} \right)$	$\rho_b = \frac{0.85 \beta_1 f'_c}{f_y} \left( \frac{87,000}{87,000 + f_y} \right)$
(D-6)	$N_b = k_c \lambda_a \sqrt{f'_c} h_{ef}^{1.5}$ $k_c = 10 \text{ or } 7$	$N_b = k_c \lambda_a \sqrt{f'_c} h_{ef}^{1.5}$ $k_c = 10 \text{ or } 7$	$N_b = k_c \lambda_a \sqrt{f'_c} h_{ef}^{1.5}$ $k_c = 24 \text{ or } 17$
(D-7)	$N_b = 3.9 \lambda_a \sqrt{f'_c} h_{ef}^{5/3}$	$N_b = 5.8 \lambda_a \sqrt{f'_c} h_{ef}^{5/3}$	$N_b = 16 \lambda_a \sqrt{f'_c} h_{ef}^{5/3}$
(D-16)	$N_{sb} = 13 c_{a1} \sqrt{A_{brg}} \lambda_a \sqrt{f'_c}$	$N_{sb} = 42.5 c_{a1} \sqrt{A_{brg}} \lambda_a \sqrt{f'_c}$	$N_{sb} = 160 c_{a1} \sqrt{A_{brg}} \lambda_a \sqrt{f'_c}$
(D-33)	$V_b = 0.6 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$	$V_b = 1.9 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$	$V_b = 7 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$
(D-34)	$V_b = 3.7 \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$	$V_b = 3.8 \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$	$V_b = 9 \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$
(D-35)	$V_b = 0.66 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$	$V_b = 2.1 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$	$V_b = 8 \left( \frac{\ell_e}{d_a} \right)^{0.2} \sqrt{d_a} \lambda_a \sqrt{f'_c} (c_{a1})^{1.5}$

**Notes**

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