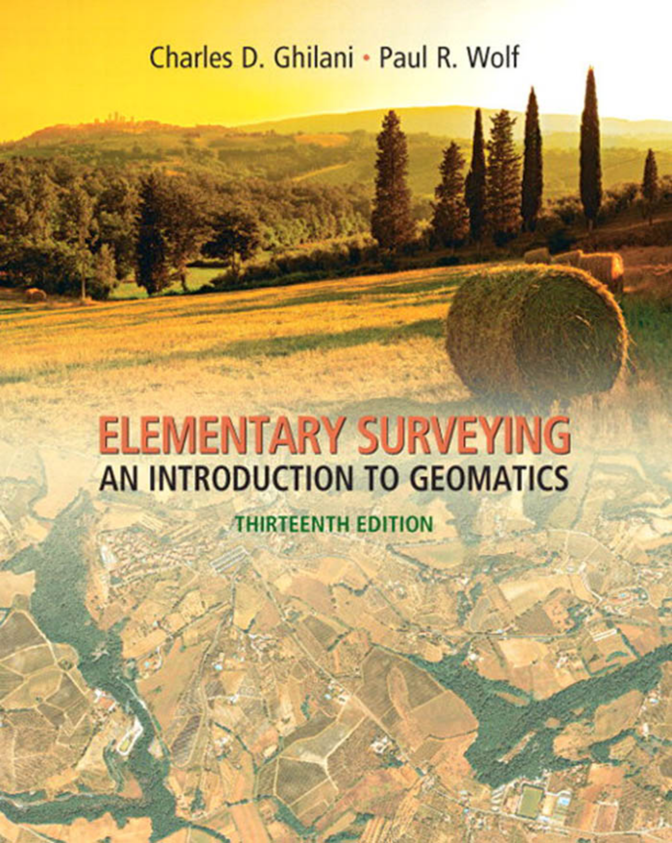
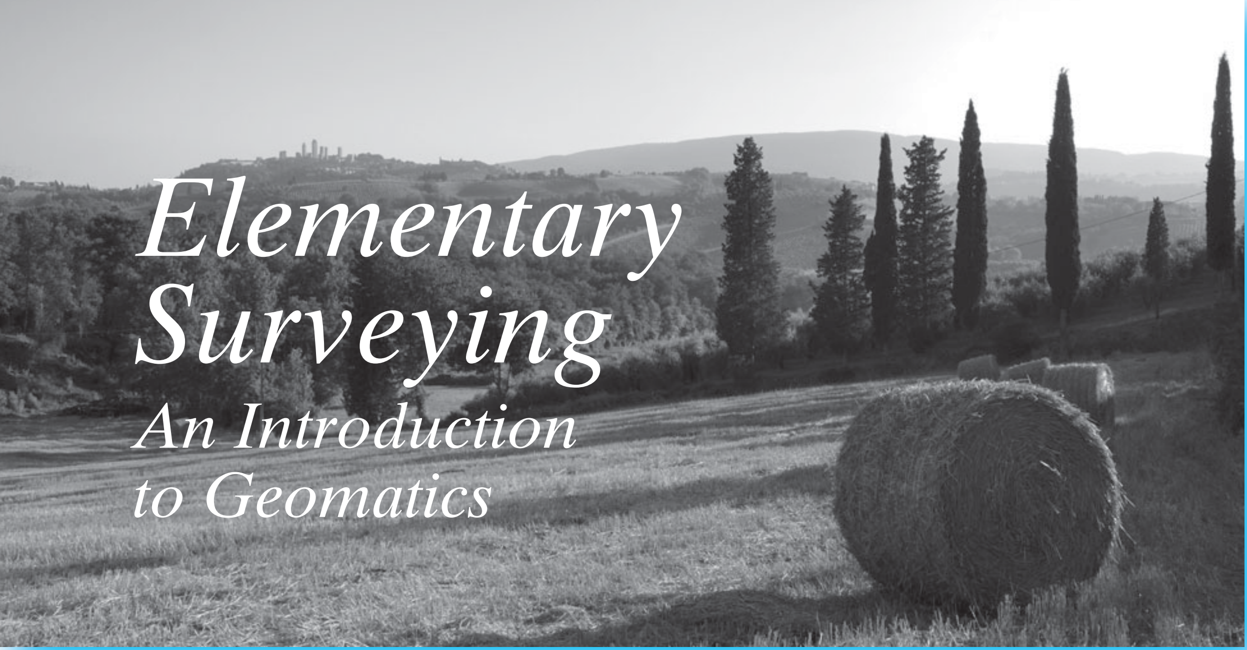

Charles D. Ghilani • Paul R. Wolf
ELEMENTARY SURVEYING
AN INTRODUCTION TO GEOMATICS
THIRTEENTH EDITION



Elementary Surveying

An Introduction to Geomatics

Thirteenth Edition

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Prentice Hall

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Preface

This 13th Edition of *Elementary Surveying: An Introduction to Geomatics* is a readable text that presents basic concepts and practical material in each of the areas fundamental to modern surveying (geomatics) practice. It is written primarily for students beginning their study of surveying (geomatics) at the college level. Although the book is elementary, its depth and breadth also make it ideal for self-study and preparation for licensing examinations. This edition includes more than 400 figures and illustrations to help clarify discussions, and numerous example problems are worked to illustrate computational procedures.

In keeping with the goal of providing an up-to-date presentation of surveying equipment and procedures, total stations are stressed as the instruments for making angle and distance observations. Additionally, mobile mapping has been introduced in this edition. Transits and theodolites, which are not used in practice, are just briefly introduced in the main body of the text. Similarly, automatic levels are now the dominant instruments for elevation determination, and accordingly their use is stressed. Dumpy levels, which are seldom used nowadays, are only briefly mentioned in the main text. For those who still use these instruments, they are covered in more detail in Appendix A of this book. However, this will be the last edition that contains this appendix.

As with past editions, this book continues to emphasize the theory of errors in surveying work. At the end of each chapter, common errors and mistakes related to the topic covered are listed so that students will be reminded to exercise caution in all of their work. Practical suggestions resulting from the authors' many years of experience are interjected throughout the text. Many of the 1000 after-chapter problems have been rewritten so that instructors can create new assignments for their students. An Instructor's Manual is available on the companion website at http://www.pearsonhighered.com/ghilani for this book to instructors who adopt the book by contacting their Prentice Hall sales representative. Also available on this website are short videos presenting the solution of selected problems in this book. These video solutions are indicated by the icon shown here in the margin. There is also a complete Pearson eText available for students.

In addition, updated versions of STATS, WOLFPACK, and MATRIX are available on the companion website for this book at http://www.pearsonhighered.com/ghilani. These programs contain options for statistical computations, traverse computations for polygon, link, and radial traverses; area calculations; astronomical azimuth reduction; two-dimensional coordinate transformations; horizontal and vertical curve computations; and least-squares adjustments. Mathcad® worksheets and Excel® spreadsheets are included on the companion website for this book. These programmed computational sheets demonstrate the solution to many of the example problems discussed herein. For those desiring additional knowledge in map projections, the Mercator, Albers Equal Area, Oblique Stereographic, and Oblique Mercator map projections have been included with these files. Also included are hypertext markup language (html) files of the Mathcad® worksheets for use by those who do not own the software.

WHAT'S NEW IN THIS EDITION?

- Discussion on the impact of the new L2C and L5 signals in GPS
- Discussion on the effects of solar activity in GNSS surveys
- Additional method of computing slope intercepts
- Introduction to mobile mapping systems
- 90% of problems revised
- Video Examples

ACKNOWLEDGMENTS

Past editions of this book, and this current one, have benefited from the suggestions, reviews, and other input from numerous students, educators, and practitioners. For their help, the authors are extremely grateful. In this edition, those professors and graduate students who reviewed material or otherwise assisted include Robert Schultz, Oregon State University; Steven Frank, New Mexico State University; Jeremy Deal, University of Texas-Arlington; Guoqing Zhou, Old Dominion University; Eric Fuller, St. Cloud State University; Loren J. Gibson, Florida Atlantic University; John J. Rose, Phoenix College; Robert Moynihan, University of New Hampshire; Marlee Walton, Iowa State University; Douglas E. Smith, Montana State University; Jean M. Rüeger, The University of New South Wales, Sydney, Australia; Thomas Seybert, The Pennsylvania State University; and Bon Dewitt, University of Florida. The authors would also like to thank the following professionals who have reviewed material and otherwise assisted in the development of this book: Paul Dukas, Professional Surveyor and Mapper, Geomatics Consultant (Chapter 21); Ron Oberlander, Topcon Positioning Systems; Charles Harpster, Pennsylvania Department of Transportation; Preston Hartzell, Eastern States Engineering; Eduardo Fernandez-Falcon, Topcon Positioning Systems; and Joseph Gabor and Brian Naberezny.

In addition, the authors wish to acknowledge the contributions of charts, maps, or other information from the National Geodetic Survey, the U.S. Geological Survey, and the U.S. Bureau of Land Management. Also appreciation is expressed to the many instrument manufacturers who provided pictures and other descriptive information on their equipment for use herein. To all of those named above and to any others who may have been inadvertently omitted, the authors are extremely thankful.

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1 Introduction

1.1 DEFINITION OF SURVEYING

Surveying, which has recently also been interchangeably called *geomatics* (see Section 1.2), has traditionally been defined as the science, art, and technology of determining the relative positions of points above, on, or beneath the Earth's surface, or of establishing such points. In a more general sense, however, surveying (geomatics) can be regarded as that discipline which encompasses all methods for measuring and collecting information about the physical earth and our environment, processing that information, and disseminating a variety of resulting products to a wide range of clients. Surveying has been important since the beginning of civilization. Its earliest applications were in measuring and marking boundaries of property ownership. Throughout the years its importance has steadily increased with the growing demand for a variety of maps and other spatially related types of information and the expanding need for establishing accurate line and grade to guide construction operations.

Today the importance of measuring and monitoring our environment is becoming increasingly critical as our population expands, land values appreciate, our natural resources dwindle, and human activities continue to stress the quality of our land, water, and air. Using modern ground, aerial, and satellite technologies, and computers for data processing, contemporary surveyors are now able to measure and monitor the Earth and its natural resources on literally a global basis. Never before has so much information been available for assessing current conditions, making sound planning decisions, and formulating policy in a host of land-use, resource development, and environmental preservation applications.

Recognizing the increasing breadth and importance of the practice of surveying, the *International Federation of Surveyors* (see Section 1.11) adopted the following definition:

"A surveyor is a professional person with the academic qualifications and technical expertise to conduct one, or more, of the following activities;

- to determine, measure and represent the land, three-dimensional objects, point-fields, and trajectories;
- to assemble and interpret land and geographically related information;
- to use that information for the planning and efficient administration of the land, the sea and any structures thereon; and
- to conduct research into the above practices and to develop them.

Detailed Functions

The surveyor's professional tasks may involve one or more of the following activities, which may occur either on, above, or below the surface of the land or the sea and may be carried out in association with other professionals.

1. The determination of the size and shape of the earth and the measurements of all data needed to define the size, position, shape and contour of any part of the earth and monitoring any change therein.
2. The positioning of objects in space and time as well as the positioning and monitoring of physical features, structures and engineering works on, above or below the surface of the earth.
3. The development, testing and calibration of sensors, instruments and systems for the above-mentioned purposes and for other surveying purposes.
4. The acquisition and use of spatial information from close range, aerial and satellite imagery and the automation of these processes.
5. The determination of the position of the boundaries of public or private land, including national and international boundaries, and the registration of those lands with the appropriate authorities.
6. The design, establishment and administration of geographic information systems (GIS) and the collection, storage, analysis, management, display and dissemination of data.
7. The analysis, interpretation and integration of spatial objects and phenomena in GIS, including the visualization and communication of such data in maps, models and mobile digital devices.
8. The study of the natural and social environment, the measurement of land and marine resources and the use of such data in the planning of development in urban, rural and regional areas.
9. The planning, development and redevelopment of property, whether urban or rural and whether land or buildings.
10. The assessment of value and the management of property, whether urban or rural and whether land or buildings.
11. The planning, measurement and management of construction works, including the estimation of costs.

In application of the foregoing activities surveyors take into account the relevant legal, economic, environmental, and social aspects affecting each project."

The breadth and diversity of the practice of surveying (geomatics), as well as its importance in modern civilization, are readily apparent from this definition.

1.2 GEOMATICS

As noted in Section 1.1, geomatics is a relatively new term that is now commonly being applied to encompass the areas of practice formerly identified as surveying. The name has gained widespread acceptance in the United States, as well as in other English-speaking countries of the world, especially in Canada, the United Kingdom, and Australia. In the United States, the *Surveying Engineering Division* of The American Society of Civil Engineers changed its name to the *Geomatics Division*. Many college and university programs in the United States that were formerly identified as "Surveying" or "Surveying Engineering" are now called "Geomatics" or "Geomatics Engineering."

The principal reason cited for making the name change is that the manner and scope of practice in surveying have changed dramatically in recent years. This has occurred in part because of recent technological developments that have provided surveyors with new tools for measuring and/or collecting information, for computing, and for displaying and disseminating information. It has also been driven by increasing concerns about the environment locally, regionally, and globally, which have greatly exacerbated efforts in monitoring, managing, and regulating the use of our land, water, air, and other natural resources. These circumstances, and others, have brought about a vast increase in demands for new spatially related information.

Historically surveyors made their measurements using ground-based methods and until rather recently the transit and tape[1] were their primary instruments. Computations, analyses, and the reports, plats, and maps they delivered to their clients were prepared (in hard copy form) through tedious manual processes. Today the modern surveyor's arsenal of tools for measuring and collecting environmental information includes electronic instruments for automatically measuring distances and angles, satellite surveying systems for quickly obtaining precise positions of widely spaced points, and modern aerial digital imaging and laser-scanning systems for quickly mapping and collecting other forms of data about the earth upon which we live. In addition, computer systems are available that can process the measured data and automatically produce plats, maps, and other products at speeds unheard of a few years ago. Furthermore, these products can be prepared in electronic formats and be transmitted to remote locations via telecommunication systems.

Concurrent with the development of these new data collection and processing technologies, *geographic information systems* (GISs) have emerged and matured. These computer-based systems enable virtually any type of spatially related information about the environment to be integrated, analyzed,

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[1] These instruments are described in Appendix A and Chapter 6, respectively.

displayed, and disseminated.[2] The key to successfully operating geographic information systems is spatially related data of high quality, and the collection and processing of this data placing great new demands upon the surveying community.

As a result of these new developments noted above, and others, many feel that the name surveying no longer adequately reflects the expanded and changing role of their profession. Hence the new term geomatics has emerged. In this text, the terms surveying and geomatics are both used, although the former is used more frequently. Nevertheless students should understand that the two terms are synonymous as discussed above.

1.3 HISTORY OF SURVEYING

The oldest historical records in existence today that bear directly on the subject of surveying state that this science began in Egypt. Herodotus recorded that Sesostris (about 1400 B.C.) divided the land of Egypt into plots for the purpose of taxation. Annual floods of the Nile River swept away portions of these plots, and surveyors were appointed to replace the boundaries. These early surveyors were called *rope-stretchers*, since their measurements were made with ropes having markers at unit distances.

As a consequence of this work, early Greek thinkers developed the science of geometry. Their advance, however, was chiefly along the lines of pure science. Heron stands out prominently for applying science to surveying in about 120 B.C. He was the author of several important treatises of interest to surveyors, including *The Dioptra*, which related the methods of surveying a field, drawing a plan, and making related calculations. It also described one of the first pieces of surveying equipment recorded, the *diopter* [Figure 1.1(a)]. For many years Heron's work was the most authoritative among Greek and Egyptian surveyors.

Significant development in the art of surveying came from the practical-minded Romans, whose best-known writing on surveying was by Frontinus. Although the original manuscript disappeared, copied portions of his work have been preserved. This noted Roman engineer and surveyor, who lived in the first century, was a pioneer in the field, and his essay remained the standard for many years. The engineering ability of the Romans was demonstrated by their extensive construction work throughout the empire. Surveying necessary for this construction resulted in the organization of a surveyors' guild. Ingenious instruments were developed and used. Among these were the *groma* [Figure 1.1(b)], used for sighting; the *libella*, an A-frame with a plumb bob, for leveling; and the *chorobates*, a horizontal straightedge about 20 ft long with supporting legs and a groove on top for water to serve as a level.

One of the oldest Latin manuscripts in existence is the *Codex Acerianus*, written in about the sixth century. It contains an account of surveying as practiced by the Romans and includes several pages from Frontinus's treatise. The

[2]Geographic information systems are briefly introduced in Section 1.9, and then described in greater detail in Chapter 28.

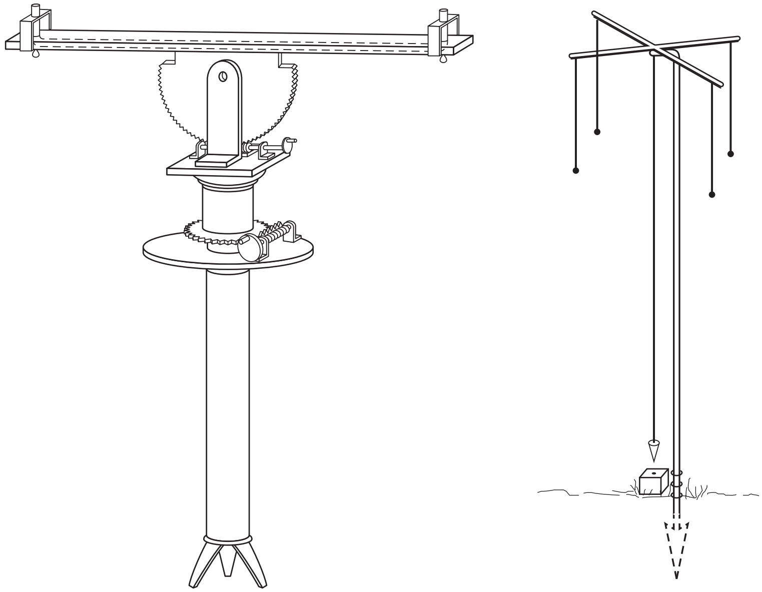

Figure 1.1
Historical surveying instruments: (a) the diopter, (b) the groma.

manuscript was found in the 10th century by Gerbert and served as the basis for his text on geometry, which was largely devoted to surveying.

During the Middle Ages, the Arabs kept Greek and Roman science alive. Little progress was made in the art of surveying, and the only writings pertaining to it were called "practical geometry."

In the 13th century, Von Piso wrote *Practica Geometria*, which contained instructions on surveying. He also authored *Liber Quadratorum*, dealing chiefly with the *quadrans*, a square brass frame having a 90° angle and other graduated scales. A movable pointer was used for sighting. Other instruments of the period were the *astrolabe*, a metal circle with a pointer hinged at its center and held by a ring at the top, and the *cross staff*, a wooden rod about 4 ft long with an adjustable crossarm at right angles to it. The known lengths of the arms of the cross staff permitted distances to be measured by proportion and angles.

Early civilizations assumed the Earth to be a flat surface, but by noting the Earth's circular shadow on the moon during lunar eclipses and watching ships gradually disappear as they sailed toward the horizon, it was slowly deduced that the planet actually curved in all directions.

Determining the true size and shape of the Earth has intrigued humans for centuries. History records that a Greek named Eratosthenes was among the first to compute its dimensions. His procedure, which occurred about 200 B.C., is illustrated in Figure 1.2. Eratosthenes had concluded that the Egyptian cities of Alexandria and Syene were located approximately on the same meridian, and he had also observed that at noon on the summer solstice, the sun was directly overhead at Syene. (This was apparent because at that time of that day, the image of

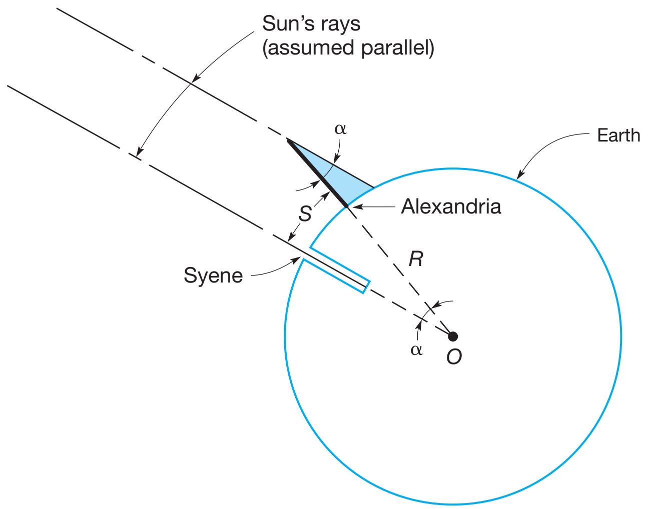

Figure 1.2 Geometry of the procedure used by Eratosthenes to determine the Earth's circumference.

the sun could be seen reflecting from the bottom of a deep vertical well there.) He reasoned that at that moment, the sun, Syene, and Alexandria were in a common meridian plane, and if he could measure the arc length between the two cities, and the angle it subtended at the Earth's center, he could compute the Earth's circumference. He determined the angle by measuring the length of the shadow cast at Alexandria from a vertical staff of known length. The arc length was found from multiplying the number of caravan days between Syene and Alexandria by the average daily distance traveled. From these measurements, Eratosthenes calculated the Earth's circumference to be about 25,000 mi. Subsequent precise geodetic measurements using better instruments, but techniques similar geometrically to Eratosthenes', have shown his value, though slightly too large, to be amazingly close to the currently accepted one. (Actually, as explained in Chapter 19, the Earth approximates an oblate spheroid having an equatorial radius about 13.5 mi longer than the polar radius.)

In the 18th and 19th centuries, the art of surveying advanced more rapidly. The need for maps and locations of national boundaries caused England and France to make extensive surveys requiring accurate triangulation; thus, geodetic surveying began. The U.S. Coast Survey (now the National Geodetic Survey of the U.S. Department of Commerce) was established by an act of Congress in 1807. Initially its charge was to perform hydrographic surveys and prepare nautical charts. Later its activities were expanded to include establishment of reference monuments of precisely known positions throughout the country.

Increased land values and the importance of precise boundaries, along with the demand for public improvements in the canal, railroad, and turnpike eras, brought surveying into a prominent position. More recently, the large volume of general construction, numerous land subdivisions that require precise records, and demands posed by the fields of exploration and ecology have entailed an augmented surveying program. Surveying is still the sign of progress in the development, use, and preservation of the Earth's resources.

In addition to meeting a host of growing civilian needs, surveying has always played an important role in our nation's defense activities. World Wars I and II, the Korean and Vietnam conflicts, and the more recent conflicts in the Middle East and Europe have created staggering demands for precise measurements and accurate maps. These military operations also provided the stimulus for improving instruments and methods to meet these needs. Surveying also contributed to, and benefited from, the space program where new equipment and systems were needed to provide precise control for missile alignment and for mapping and charting portions of the moon and nearby planets.

Developments in surveying and mapping equipment have now evolved to the point where the traditional instruments that were used until about the 1960s or 1970s—the transit, theodolite, dumpy level, and steel tape—have now been almost completely replaced by an array of new "high-tech" instruments. These include electronic *total station instruments*, which can be used to automatically measure and record horizontal and vertical distances, and horizontal and vertical angles; and *global navigation satellite systems* (GNSS) such as the *global positioning system* (GPS) that can provide precise location information for virtually any type of survey. Laser-scanning instruments combine automatic distance and angle measurements to compute dense grids of coordinated points. Also new aerial cameras and remote sensing instruments have been developed, which provide images in digital form, and these images can be processed to obtain spatial information and maps using new *digital photogrammetric restitution instruments* (also called *softcopy plotters*). Figure 1.3, 1.4, 1.5, and 1.6, respectively, show a total station instrument, 3D mobile mapping system, laser-scanning instrument, and modern softcopy plotter. The 3D mobile mapping system in Figure 1.4 is an integrated system consisting of scanners, GNSS receiver, inertial measurement unit, and a high-quality hemispherical digital camera that can map all items within 30 m of the vehicle as the vehicle travels at highway speeds. The system can capture 1.3 million data points per second providing the end user with high-quality, georeferenced coordinates on all items visible in the images.

Figure 1.3 LEICA TPS 1100 total station instrument. (Courtesy Leica Geosystems AG.)

Figure 1.4
The IP-S2 3D mobile mapping system. (Courtesy Topcon Positioning Systems.)

Figure 1.5
LEICA HDS 3000 laser scanner. (Courtesy of Christopher Gibbons, Leica Geosystems AG.)

Figure 1.6 Intergraph Image Station Z softcopy plotter. (From *Elements of Photogrammetry: With Applications in GIS*, by Wolf and Dewitt, 2000, Courtesy Intergraph, Inc., and the McGraw-Hill Companies.)

1.4 GEODETIC AND PLANE SURVEYS

Two general classifications of surveys are *geodetic* and *plane*. They differ principally in the assumptions on which the computations are based, although field measurements for geodetic surveys are usually performed to a higher order of accuracy than those for plane surveys.

In geodetic surveying, the curved surface of the Earth is considered by performing the computations on an *ellipsoid* (curved surface approximating the size and shape of the Earth—see Chapter 19). It is now becoming common to do geodetic computations in a three-dimensional, *Earth-centered, Earth-fixed* (ECEF) Cartesian coordinate system. The calculations involve solving equations derived from solid geometry and calculus. Geodetic methods are employed to determine relative positions of widely spaced monuments and to compute lengths and directions of the long lines between them. These monuments serve as the basis for referencing other subordinate surveys of lesser extents.

In early geodetic surveys, painstaking efforts were employed to accurately observe angles and distances. The angles were measured using precise ground-based theodolites, and the distances were measured using special tapes made from metal having a low coefficient of thermal expansion. From these basic measurements, the relative positions of the monuments were computed. Later, electronic instruments were used for observing the angles and distances. Although these latter types of instruments are still sometimes used on geodetic surveys, satellite positioning has now almost completely replaced other instruments for these types of surveys. Satellite positioning can provide the needed positions with much greater accuracy, speed, and economy. GNSS receivers enable ground stations to be located precisely by observing distances to satellites operating in known positions along their orbits. GNSS surveys are being used in all forms of surveying including geodetic, hydrographic, construction, and boundary surveying. The principles of operation of the global positioning system are given in Chapter 13, field and office procedures used in static GNSS surveys are discussed in Chapter 14, and the methods used in kinematic GNSS surveys are discussed in Chapter 15.

In plane surveying, except for leveling, the reference base for fieldwork and computations is assumed to be a flat horizontal surface. The direction of a plumb line (and thus gravity) is considered parallel throughout the survey region, and all observed angles are presumed to be plane angles. For areas of limited size, the surface of our vast ellipsoid is actually nearly flat. On a line 5 mi long, the ellipsoid arc and chord lengths differ by only about 0.02 ft. A plane surface tangent to the ellipsoid departs only about 0.7 ft at 1 mi from the point of tangency. In a triangle having an area of 75 square miles, the difference between the sum of the three ellipsoidal angles and three plane angles is only about 1 sec. Therefore, it is evident that except in surveys covering extensive areas, the Earth's surface can be approximated as a plane, thus simplifying computations and techniques. In general, algebra, plane and analytical geometry, and plane trigonometry are used in plane-surveying calculations. Even for very large areas, map projections, such as those described in Chapter 20, allow plane-surveying computations to be used. This book concentrates primarily on methods of plane surveying, an approach that satisfies the requirements of most projects.

1.5 IMPORTANCE OF SURVEYING

Surveying is one of the world's oldest and most important arts because, as noted previously, from the earliest times it has been necessary to mark boundaries and divide land. Surveying has now become indispensable to our modern way of life. The results of today's surveys are used to (1) map the Earth above and below sea level; (2) prepare navigational charts for use in the air, on land, and at sea; (3) establish property boundaries of private and public lands; (4) develop data banks of land-use and natural resource information that aid in managing our environment; (5) determine facts on the size, shape, gravity, and magnetic fields of the earth; and (6) prepare charts of our moon and planets.

Surveying continues to play an extremely important role in many branches of engineering. For example, surveys are required to plan, construct, and maintain highways, railroads, rapid-transit systems, buildings, bridges, missile ranges, launching sites, tracking stations, tunnels, canals, irrigation ditches, dams, drainage works, urban land subdivisions, water supply and sewage systems, pipelines, and mine shafts. Surveying methods are commonly employed in laying out industrial assembly lines and jigs.[3] These methods are also used for guiding the fabrication of large equipment, such as airplanes and ships, where separate pieces that have been assembled at different locations must ultimately be connected as a

[3]See footnote 1.

unit. Surveying is important in many related tasks in agronomy, archeology, astronomy, forestry, geography, geology, geophysics, landscape architecture, meteorology, paleontology, and seismology, but particularly in military and civil engineering.

All engineers must know the limits of accuracy possible in construction, plant design and layout, and manufacturing processes, even though someone else may do the actual surveying. In particular, surveyors and civil engineers who are called on to design and plan surveys must have a thorough understanding of the methods and instruments used, including their capabilities and limitations. This knowledge is best obtained by making observations with the kinds of equipment used in practice to get a true concept of the theory of errors and the small but recognizable differences that occur in observed quantities.

In addition to stressing the need for reasonable limits of accuracy, surveying emphasizes the value of significant figures. Surveyors and engineers must know when to work to hundredths of a foot instead of to tenths or thousandths, or perhaps the nearest foot, and what precision in field data is necessary to justify carrying out computations to the desired number of decimal places. With experience, they learn how available equipment and personnel govern procedures and results.

Neat sketches and computations are the mark of an orderly mind, which in turn is an index of sound engineering background and competence. Taking field notes under all sorts of conditions is excellent preparation for the kind of recording and sketching expected of all engineers. Performing later office computations based on the notes underscores their importance. Additional training that has a carryover value is obtained in arranging computations in an organized manner.

Engineers who design buildings, bridges, equipment, and so on are fortunate if their estimates of loads to be carried are correct within 5%. Then a factor of safety of 2 or more is often applied. But except for some topographic work, only exceedingly small errors can be tolerated in surveying, and there is no factor of safety. Traditionally, therefore, both manual and computational precision are stressed in surveying.

■ 1.6 SPECIALIZED TYPES OF SURVEYS

Many types of surveys are so specialized that a person proficient in a particular discipline may have little contact with the other areas. Persons seeking careers in surveying and mapping, however, should be knowledgeable in every phase, since all are closely related in modern practice. Some important classifications are described briefly here.

Control surveys establish a network of horizontal and vertical monuments that serve as a reference framework for initiating other surveys. Many control surveys performed today are done using techniques discussed in Chapter 14 with GNSS instruments.

Topographic surveys determine locations of natural and artificial features and elevations used in map making.

Land, boundary, and *cadastral surveys* establish property lines and property corner markers. The term cadastral is now generally applied to surveys of the public lands systems. There are three major categories: *original surveys* to establish new section corners in unsurveyed areas that still exist in Alaska and several western states; *retracement surveys* to recover previously established boundary lines; and *subdivision surveys* to establish monuments and delineate new parcels of ownership. *Condominium surveys*, which provide a legal record of ownership, are a type of boundary survey.

Hydrographic surveys define shorelines and depths of lakes, streams, oceans, reservoirs, and other bodies of water. *Sea surveying* is associated with port and offshore industries and the marine environment, including measurements and marine investigations made by shipborne personnel.

Alignment surveys are made to plan, design, and construct highways, railroads, pipelines, and other linear projects. They normally begin at one control point and progress to another in the most direct manner permitted by field conditions.

Construction surveys provide line, grade, control elevations, horizontal positions, dimensions, and configurations for construction operations. They also secure essential data for computing construction pay quantities.

As-built surveys document the precise final locations and layouts of engineering works and record any design changes that may have been incorporated into the construction. These are particularly important when underground facilities are constructed, so their locations are accurately known for maintenance purposes, and so that unexpected damage to them can be avoided during later installation of other underground utilities.

Mine surveys are performed above and below ground to guide tunneling and other operations associated with mining. This classification also includes geophysical surveys for mineral and energy resource exploration.

Solar surveys map property boundaries, solar easements, obstructions according to sun angles, and meet other requirements of zoning boards and title insurance companies.

Optical tooling (also referred to as *industrial surveying* or *optical alignment*) is a method of making extremely accurate measurements for manufacturing processes where small tolerances are required.

Except for control surveys, most other types described are usually performed using plane-surveying procedures, but geodetic methods may be employed on the others if a survey covers an extensive area or requires extreme accuracy.

Ground, aerial, and *satellite surveys* are broad classifications sometimes used. Ground surveys utilize measurements made with ground-based equipment such as automatic levels and total station instruments. Aerial surveys are accomplished using either *photogrammetry* or *remote sensing*. Photogrammetry uses cameras that are carried usually in airplanes to obtain images, whereas remote sensing employs cameras and other types of sensors that can be transported in either aircraft or satellites. Procedures for analyzing and reducing the image data are described in Chapter 27. Aerial methods have been used in all the specialized types of surveys listed, except for optical tooling, and in this area *terrestrial* (ground-based) photographs are often used. Satellite surveys include the determination of ground locations from measurements made to satellites using GNSS receivers, or the use of satellite images for mapping and monitoring large regions of the Earth.

1.7 SURVEYING SAFETY

Surveyors (geomatics engineers) generally are involved in both field and office work. The fieldwork consists in making observations with various types of instruments to either (a) determine the relative locations of points or (b) to set out stakes in accordance with planned locations to guide building and construction operations. The office work involves (1) conducting research and analysis in preparing for surveys, (2) computing and processing the data obtained from field measurements, and (3) preparing maps, plats, charts, reports, and other documents according to client specifications. Sometimes the fieldwork must be performed in hostile or dangerous environments, and thus it is very important to be aware of the need to practice safety precautions.

Among the most dangerous of circumstances within which surveyors must sometimes work are job sites that are either on or near highways or railroads, or that cross such facilities. Job sites in construction zones where heavy machinery is operating are also hazardous, and the dangers are often exacerbated by poor hearing conditions from the excessive noise, and poor visibility caused by obstructions and dust, both of which are created by the construction activity. In these situations, whenever possible, the surveys should be removed from the danger areas through careful planning and/or the use of *offset* lines. If the work must be done in these hazardous areas, then certain safety precautions should be followed. Safety vests of fluorescent yellow color should always be worn in these situations, and flagging materials of the same color can be attached to the surveying equipment to make it more visible. Depending on the circumstances, signs can be placed in advance of work areas to warn drivers of the presence of a survey party ahead, cones and/or barricades can be placed to deflect traffic around surveying activities, and flaggers can be assigned to warn drivers, or to slow or even stop them, if necessary. The *Occupational Safety and Health Administration* (OSHA), of the U.S. Department of Labor,[4] has developed safety standards and guidelines that apply to the various conditions and situations that can be encountered.

Besides the hazards described above, depending on the location of the survey and the time of year, other dangers can also be encountered in conducting field surveys. These include problems related to weather such as frostbite and overexposure to the sun's rays, which can cause skin cancers, sunburns, and heat stroke. To help prevent these problems, plenty of fluids should be drunk, large-brimmed hats and sunscreen can be worn, and on extremely hot days surveying should commence at dawn and terminate at midday or early afternoon. Outside work should not be done on extremely cold days, but if it is necessary, warm clothing should be worn and skin areas should not be exposed. Other hazards that can be encountered during field surveys include wild animals, poisonous snakes, bees, spiders, wood ticks, deer ticks (which can carry lyme disease), poison

[4]The mission of OSHA is to save lives, prevent injuries, and protect the health of America's workers. Its staff establishes protective standards, enforces those standards, and reaches out to employers and employees through technical assistance and consultation programs. For more information about OSHA and its safety standards, consult its website http://www.osha.gov.

ivy, and poison oak. Surveyors should be knowledgeable about the types of hazards that can be expected in any local area, and always be alert and on the lookout for them. To help prevent injury from these sources, protective boots and clothing should be worn and insect sprays can be used. Certain tools can also be dangerous, such as chain saws, axes, and machetes that are sometimes necessary for clearing lines of sight. These must always be handled with care. Also, care must be exercised in handling certain surveying instruments, like long-range poles and level rods, especially when working around overhead wires, to prevent accidental electrocutions.

Many other hazards, in addition to those cited above can be encountered when surveying in the field. Thus, it is essential that surveyors always exercise caution in their work, and know and follow accepted safety standards. In addition, a first-aid kit should always accompany a survey party in the field, and it should include all of the necessary antiseptics, ointments, bandage materials, and other equipment needed to render first aid for minor accidents. The survey party should also be equipped with cell phones for more serious situations, and telephone numbers to call in emergencies should be written down and readily accessible.

1.8 LAND AND GEOGRAPHIC INFORMATION SYSTEMS

Land Information Systems (LISs) and *Geographic Information Systems* (GISs) are areas of activity that have rapidly assumed positions of major prominence in surveying. These computer-based systems enable storing, integrating, manipulating, analyzing, and displaying virtually any type of spatially related information about our environment. LISs and GISs are being used at all levels of government, and by businesses, private industry, and public utilities to assist in management and decision making. Specific applications have occurred in many diverse areas and include natural resource management, facilities siting and management, land records modernization, demographic and market analysis, emergency response and fleet operations, infrastructure management, and regional, national, and global environmental monitoring. Data stored within LISs and GISs may be both natural and cultural, and be derived from new surveys, or from existing sources such as maps, charts, aerial and satellite photos, tabulated data and statistics, and other documents. However, in most situations, the needed information either does not exist, or it is unsatisfactory because of age, scale, or other reasons. Thus, new measurements, maps, photos, or other data must be obtained.

Specific types of information (also called *themes* or *layers* of information) needed for land and geographic information systems may include political boundaries, individual property ownership, population distribution, locations of natural resources, transportation networks, utilities, zoning, hydrography, soil types, land use, vegetation types, wetlands, and many, many more. An essential ingredient of all information entered into LIS and GIS databases is that it be spatially related, that is, located in a common geographic reference framework. Only then are the different layers of information physically relatable so they can be analyzed using computers to support decision making. This geographic positional requirement will place a heavy demand upon surveyors (geomatics engineers) in the future, who will play key roles in designing, implementing, and managing these systems. Surveyors from virtually all of the specialized areas described in Section 1.6 will be involved in developing the needed databases. Their work will include establishing the required basic control framework; conducting boundary surveys and preparing legal descriptions of property ownership; performing topographic and hydrographic surveys by ground, aerial, and satellite methods; compiling and digitizing maps; and assembling a variety of other digital data files.

The last chapter of this book, Chapter 28, is devoted to the topic of land and geographic information systems. This subject seems appropriately covered at the end, after each of the other types of surveys needed to support these systems has been discussed.

■ 1.9 FEDERAL SURVEYING AND MAPPING AGENCIES

Several agencies of the U.S. government perform extensive surveying and mapping. Three of the major ones are:

1. The National Geodetic Survey (NGS), formerly the Coast and Geodetic Survey, was originally organized to map the coast. Its activities have included control surveys to establish a network of reference monuments throughout the United States that serve as points for originating local surveys, preparation of nautical and aeronautical charts, photogrammetric surveys, tide and current studies, collection of magnetic data, gravimetric surveys, and worldwide control survey operations. The NGS now plays a major role in coordinating and assisting in activities related to upgrading the national network of reference control monuments, and to the development, storage, and dissemination of data used in modern LISs and GISs.
2. The U.S. Geological Survey (USGS), established in 1879, has as its mission the mapping of our nation and the survey of its resources. It provides a wide variety of maps, from topographic maps showing the geographic relief and natural and cultural features, to thematic maps that display the geology and water resources of the United States, to special maps of the moon and planets. The National Mapping Division of the USGS has the responsibility of producing topographic maps. It currently has nearly 70,000 different topographic maps available, and it distributes approximately 10 million copies annually. In recent years, the USGS has been engaged in a comprehensive program to develop a national digital cartographic database, which consists of map data in computer-readable formats.
3. The Bureau of Land Management (BLM), originally established in 1812 as the General Land Office, is responsible for managing the public lands. These lands, which total approximately 264 million acres and comprise about one eighth of the land in the United States, exist mostly in the western states and Alaska. The BLM is responsible for surveying the land and managing its natural resources, which include minerals, timber, fish and

wildlife, historical sites, and other natural heritage areas. Surveys of most public lands in the conterminous United States have been completed, but much work remains in Alaska.

In addition to these three federal agencies, units of the U.S. Army Corps of Engineers have made extensive surveys for emergency and military purposes. Some of these surveys provide data for engineering projects, such as those connected with flood control. Surveys of wide extent have also been conducted for special purposes by nearly 40 other federal agencies, including the Forest Service, National Park Service, International Boundary Commission, Bureau of Reclamation, Tennessee Valley Authority, Mississippi River Commission, U.S. Lake Survey, and Department of Transportation.

All states have a surveying and mapping section for purposes of generating topographic information upon which highways are planned and designed. Likewise, many counties and cities also have surveying programs, as have various utilities.

1.10 THE SURVEYING PROFESSION

The personal qualifications of surveyors are as important as their technical ability in dealing with the public. They must be patient and tactful with clients and their sometimes-hostile neighbors. Few people are aware of the painstaking research of old records required before fieldwork is started. Diligent, time-consuming effort may be needed to locate corners on nearby tracts for checking purposes as well as to find corners for the property in question.

Land or boundary surveying is classified as a learned profession because the modern practitioner needs a wide background of technical training and experience, and must exercise a considerable amount of independent judgment. Registered (licensed) professional surveyors must have a thorough knowledge of mathematics (particularly geometry, trigonometry, and calculus); competence with computers; a solid understanding of surveying theory, instruments, and methods in the areas of geodesy, photogrammetry, remote sensing, and cartography; some competence in economics (including office management), geography, geology, astronomy, and dendrology; and a familiarity with laws pertaining to land and boundaries. They should be knowledgeable in both field operations and office computations. Above all, they are governed by a professional code of ethics and are expected to charge professional-level fees for their work.

Permission to trespass on private property or to cut obstructing tree branches and shrubbery must be obtained through a proper approach. Such privileges are not conveyed by a surveying license or by employment in a state highway department or other agency (but a court order can be secured if a landowner objects to necessary surveys).

All 50 states, Guam, and Puerto Rico have registration laws for professional surveyors and engineers (as do the provinces of Canada). In general, a surveyor's license is required to make property surveys, but not for construction, topographic, or route work, unless boundary corners are set.

To qualify for registration as either a professional land surveyor (PLS) or a professional engineer (PE), it is necessary to have an appropriate college degree, although some states allow relevant experience in lieu of formal education. In addition, candidates must acquire two or more years of mentored practical experience and must also pass a two-day comprehensive written examination. In most states, common national examinations covering fundamentals and principles and practice of land surveying are now used. However, usually two hours of the principles and practice exam are devoted to local legal customs and aspects. As a result, transfer of registration from one state to another has become easier.

Some states also require continuing education units (CEUs) for registration renewal, and many more are considering legislation that would add this requirement. Typical state laws require that a licensed land surveyor sign all plats, assume responsibility for any liability claims, and take an *active part* in the fieldwork.

■ 1.11 PROFESSIONAL SURVEYING ORGANIZATIONS

There are many professional organizations in the United States and worldwide that serve the interests of surveying and mapping. Generally the objectives of these organizations are the advancement of knowledge in the field, encouragement of communication among surveyors, and upgrading of standards and ethics in surveying practice. The *American Congress on Surveying and Mapping* (ACSM) is the foremost professional surveying organization in the United States. Founded in 1941, ACSM regularly sponsors technical meetings at various locations throughout the country. These meetings bring together large numbers of surveyors for presentation of papers, discussion of new ideas and problems, and exhibition of the latest in surveying equipment. ACSM publishes a quarterly journal, *Surveying and Land Information Science*, and also regularly publishes its newsletter, *The ACSM Bulletin*.

As noted in the preceding section, all states require persons who perform boundary surveys to be licensed. Most states also have professional surveyor societies or organizations with full membership open only to licensed surveyors. These state societies are generally affiliated with ACSM and offer benefits similar to those of ACSM, except that they concentrate on matters of state and local concern.

The *American Society for Photogrammetry and Remote Sensing* (ASPRS) is a sister organization of ACSM. Like ACSM, this organization is also devoted to the advancement of the fields of measurement and mapping, although its major interests are directed toward the use of aerial and satellite imagery for achieving these goals. ASPRS has been cosponsor of many technical meetings with ACSM, and its monthly journal *Photogrammetric Engineering and Remote Sensing* regularly features surveying and mapping articles.

The *Geomatics Division* of the *American Society of Civil Engineers* (ASCE) is also dedicated to professional matters related to surveying and publishes quarterly the *Journal of Surveying Engineering*.

The *Surveying and Geomatics Educators Society* (SAGES) holds pedagogical conferences on the instruction of surveying/geomatics in higher educational institutions.

Another organization in the United States, the *Urban and Regional Information Systems Association* (URISA), also supports the profession of surveying and mapping. This organization uses information technology to solve problems in planning, public works, the environment, emergency services, and utilities. Its *URISA Journal* is published quarterly.

The *Canadian Institute of Geomatics* (CIG) is the foremost professional organization in Canada concerned with surveying. Its objectives parallel those of ACSM. This organization, formerly the *Canadian Institute of Surveying and Mapping* (CISM), disseminates information to its members through its *CIG Journal*.

The *International Federation of Surveyors* (FIG), founded in 1878, fosters the exchange of ideas and information among surveyors worldwide. The acronym *FIG* stems from its French name, *Fédération Internationale des Géométres*. FIG membership consists of professional surveying organizations from many countries throughout the world. ACSM has been a member since 1959. FIG is organized into nine technical commissions, each concerned with a specialized area of surveying. The organization sponsors international conferences, usually at four-year intervals, and its commissions also hold periodic symposia where delegates gather for the presentation of papers on subjects of international interest.

1.12 SURVEYING ON THE INTERNET

The explosion of available information on the Internet has had a significant impact on the field of surveying (geomatics). The Internet enables the instantaneous electronic transfer of documents to any location where the necessary computer equipment is available. It brings resources directly into the office or home, where previously it was necessary to travel to obtain the information or wait for its transfer by mail. Software, educational materials, technical documents, standards, and much more useful information are available on the Internet. As an example of how surveyors can take advantage of the Internet, data from a *Continuously Operating Reference Station* (CORS) can be downloaded from the NGS website for use in a GNSS survey (see Section 14.3.5).

Many agencies and institutions maintain websites that provide data free of charge on the Internet. Additionally, some educational institutions now place credit and noncredit courses on the Internet so that distance education can be more easily achieved. With a web browser, it is possible to research almost any topic from a convenient location, and names, addresses, and phone numbers of goods or services providers in a specific area can be identified. As an example, if it was desired to find companies offering mapping services in a certain region, a web search engine could be used to locate web pages that mention this service. Such a search may result in over a million pages if a very general term such as "mapping services" is used to search, but using more specific terms can narrow the search.

Unfortunately the addresses of particular pages and entire sites, given by their *Universal Resource Locators* (URLs), tend to change with time. However, at the risk of publishing URLs that may no longer be correct, a short list of important websites related to surveying is presented in Table 1.1.

TABLE 1.1 UNIVERSAL RESOURCE LOCATOR ADDRESSES FOR SOME SURVEYING RELATED SITES

Universal Resource Locator	Owner of Site
http://www.ngs.noaa.gov	National Geodetic Survey
http://www.usgs.gov	U.S. Geological Survey
http://www.blm.gov	Bureau of Land Management
http://www.navcen.uscg.mil	U.S. Coast Guard Navigation Center
http://www.usno.navy.mil	U.S. Naval Observatory
http://www.acsm.net	American Congress on Surveying and Mapping
http://www.asprs.org	American Society for Photogrammetry and Remote Sensing
http://www.asce.org	American Society of Civil Engineers
http://www.pearsonhighered.com/ghilani	Companion website for this book

1.13 FUTURE CHALLENGES IN SURVEYING

Surveying is currently in the midst of a revolution in the way data are measured, recorded, processed, stored, retrieved, and shared. This is in large part because of developments in computers and computer-related technologies. Concurrent with technological advancements, society continues to demand more data, with increasingly higher standards of accuracy, than ever before. Consequently, in a few years the demands on surveying engineers (geomatics engineers) will likely be very different from what they are now.

In the future, the National Spatial Reference System, a network of horizontal and vertical control points, must be maintained and supplemented to meet requirements of increasingly higher-order surveys. New topographic maps with larger scales as well as digital map products are necessary for better planning. Existing maps of our rapidly expanding urban areas need revision and updating to reflect changes, and more and better map products are needed of the older parts of our cities to support urban renewal programs and infrastructure maintenance and modernization. Large quantities of data will be needed to plan and design new rapid-transit systems to connect our major cities, and surveyors will face new challenges in meeting the precise standards required in staking alignments and grades for these systems.

In the future, assessment of environmental impacts of proposed construction projects will call for more and better maps and other data. GISs and LISs that contain a variety of land-related data such as ownership, location, acreage, soil types, land uses, and natural resources must be designed, developed, and maintained. Cadastral surveys of the yet unsurveyed public lands are essential. Monuments set years ago by the original surveyors have to be recovered and remonumented for preservation of property boundaries. Appropriate surveys with very demanding accuracies will be necessary to position drilling rigs as mineral and oil explorations press further offshore. Other future challenges include making precise deformation surveys for monitoring existing structures such as dams, bridges, and skyscrapers to detect imperceptible movements that could be precursors to catastrophes caused by their failure. Timely measurements and maps of the effects of natural disasters such as earthquakes, floods, and hurricanes will be needed so that effective relief and assistance efforts can be planned and implemented. In the space program, the desire for maps of neighboring planets will continue. And we must increase our activities in measuring and monitoring natural and human-caused global changes (glacial growth and retreat, volcanic activity, large-scale deforestation, and so on) that can potentially affect our land, water, atmosphere, energy supply, and even our climate.

These and other opportunities offer professionally rewarding indoor or outdoor (or both) careers for numerous people with suitable training in the various branches of surveying.

PROBLEMS

NOTE: Answers for some of these problems, and some in later chapters, can be obtained by consulting the bibliographies, later chapters, websites, or professional surveyors.

1.1 Develop your personal definition for the practice of surveying.

1.2 Explain the difference between geodetic and plane surveys.

1.3 Describe some surveying applications in:
(a) Archeology **(b)** Mining **(c)** Agriculture

1.4 List 10 uses for surveying other than property and construction surveying.

1.5 Why is it important to make accurate surveys of underground utilities?

1.6 Discuss the uses for topographic surveys.

1.7 What are hydrographic surveys, and why are they important?

1.8 Name and briefly describe three different surveying instruments used by early Roman engineers.

1.9 Briefly explain the procedure used by Eratosthenes in determining the Earth's circumference.

1.10 Describe the steps a land surveyor would need to do when performing a boundary survey.

1.11 Do laws in your state specify the accuracy required for surveys made to lay out a subdivision? If so, what limits are set?

1.12 What organizations in your state will furnish maps and reference data to surveyors and engineers?

1.13 List the legal requirements for registration as a land surveyor in your state.

1.14 Briefly describe the European Galileo system and discuss its similarities and differences with GPS.

1.15 List at least five nonsurveying uses for GPS.

1.16 Explain how aerial photographs and satellite images can be valuable in surveying.

1.17 Search the Internet and define a VLBI station. Discuss why these stations are important to the surveying community.

1.18 Describe how a GIS can be used in flood emergency planning.

1.19 Visit one of the surveying websites listed in Table 1.1, and write a brief summary of its contents. Briefly explain the value of the available information to surveyors.

1.20 Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where GPS was used. Write a brief summary of the article.

1.21 Same as Problem 1.20, except the article should be on safety as related to surveying.

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2 Units, Significant Figures, and Field Notes

PART I • UNITS AND SIGNIFICANT FIGURES

2.1 INTRODUCTION

Five types of observations illustrated in Figure 2.1 form the basis of traditional plane surveying: (1) horizontal angles, (2) horizontal distances, (3) vertical (or zenith) angles, (4) vertical distances, and (5) slope distances. In the figure, *OAB* and *ECD* are horizontal planes, and *OACE* and *ABDC* are vertical planes. Then as illustrated, horizontal angles, such as angle *AOB*, and horizontal distances, *OA* and *OB*, are measured in horizontal planes; vertical angles, such as *AOC*, are measured in vertical planes; zenith angles, such as *EOC*, are also measured in vertical planes; vertical lines, such as *AC* and *BD*, are measured vertically (in the direction of gravity); and slope distances, such as *OC*, are determined along inclined planes. By using combinations of these basic observations, it is possible to compute relative positions between any points. Equipment and procedures for making each of these basic kinds of observations are described in later chapters of this book.

2.2 UNITS OF MEASUREMENT

Magnitudes of measurements (or of values derived from observations) must be given in terms of specific units. In surveying, the most commonly employed units are for *length*, *area*, *volume*, and *angle*. Two different systems are in use for specifying units of observed quantities, the *English* and *metric* systems. Because of its widespread adoption, the metric system is called the *International System of Units*, and abbreviated *SI*.

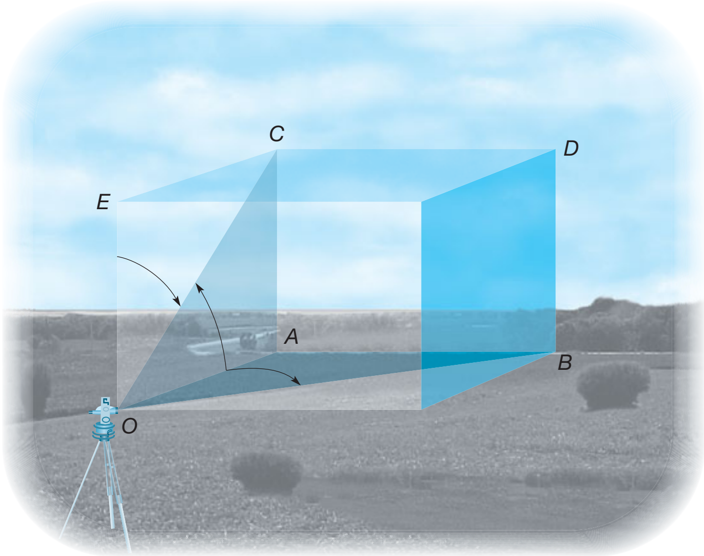


Figure 2.1 Kinds of measurements in surveying.

The basic unit employed for length measurements in the English system is the foot, whereas the meter is used in the metric system. In the past, two different definitions have been used to relate the foot and meter. Although they differ slightly, their distinction must be made clear in surveying. In 1893, the United States officially adopted a standard in which 39.37 in. was exactly equivalent to 1 m. Under this standard, the foot was approximately equal to 0.3048006 m. In 1959, a new standard was officially adopted in which the inch was equal to exactly 2.54 cm. Under this standard, 1 ft equals exactly 0.3048 m. This current unit, known as the international foot, differs from the previous one by about 1 part in 500,000, or approximately 1 foot per 100 miles. This small difference is thus important for very precise surveys conducted over long distances, and for conversions of high elevations or large coordinate values such as those used in State Plane Coordinate Systems as discussed in Chapter 20. Because of the vast number of surveys performed prior to 1959, it would have been extremely difficult and confusing to change all related documents and maps that already existed. Thus the old standard, now called the *U.S. survey foot*, is still used. Individual states have the option of officially adopting either standard. The National Geodetic Survey uses the meter in its distance measurements; thus, it is unnecessary to specify the foot unit. However, those making conversions from metric units must know the adopted standard for their state and use the appropriate conversion factor.

Because the English system has long been the officially adopted standard for measurements in the United States, except for geodetic surveys, the linear units of feet and *decimals* of a foot are most commonly used by surveyors. In construction, feet and inches are often used. Because surveyors perform all types of surveys including geodetic, and they also provide measurements for developing construction plans and guiding building operations, they must understand all the various systems of units and be capable of making conversions between them.

Caution must always be exercised to ensure that observations are recorded in their proper units, and conversions are correctly made.

A summary of the length units used in past and present surveys in the United States includes the following:

1 foot = 12 inches
1 yard = 3 feet
1 inch = 2.54 centimeters (basis of international foot)
1 meter = 39.37 inches (basis of U.S. survey foot)
1 rod = 1 pole = 1 perch = 16.5 feet
1 vara = approximately 33 inches (old Spanish unit often encountered in the southwestern United States)
1 Gunter's chain (ch) = 66 feet = 100 links (lk) = 4 rods
1 mile = 5280 feet = 80 Gunter's chains
1 nautical mile = 6076.10 feet (nominal length of a minute of latitude, or of longitude at the equator)
1 fathom = 6 feet.

In the English system, areas are given in *square feet* or *square yards*. The most common unit for large areas is the *acre*. Ten square chains (Gunter's) equal 1 acre. Thus an acre contains 43,560 ft^2, which is the product of 10 and 66^2. The *arpent* (equal to approximately 0.85 acre, but varying somewhat in different states) was used in land grants of the French crown. When employed as a linear term, it refers to the length of a side of 1 square arpent.

Volumes in the English system can be given in *cubic feet* or *cubic yards*. For very large volumes, for example, the quantity of water in a reservoir, the *acre-foot* unit is used. It is equivalent to the area of an acre having a depth of 1 ft, and thus is 43,560 ft^3.

The unit of angle used in surveying is the *degree*, defined as 1/360 of a circle. One degree (1°) equals 60 min, and 1 min equals 60 sec. Divisions of seconds are given in tenths, hundredths, and thousandths. Other methods are also used to subdivide a circle, for example, 400 *grads* (with 100 *centesimal min*/grad and 100 *centesimal sec*/min. Another term, *gons*, is now used interchangeably with grads. The military services use *mils* to subdivide a circle into 6400 units.

A *radian* is the angle subtended by an arc of a circle having a length equal to the radius of the circle. Therefore, 2π rad = 360°, 1 rad ≈ 57°17′44.8″ ≈ 57.2958°, and 0.01745 rad ≈ 1°.

■ 2.3 INTERNATIONAL SYSTEM OF UNITS (SI)

As noted previously, the meter is the basic unit for length in the metric or SI system. Subdivisions of the meter (m) are the *millimeter* (mm), *centimeter* (cm), and *decimeter* (dm), equal to 0.001, 0.01, and 0.1 m, respectively. A kilometer (km) equals 1000 m, which is approximately five eighths of a mile.

Areas in the metric system are specified using the *square meter* (m^2). Large areas, for example, tracts of land, are given in *hectares* (ha), where one hectare is equivalent to a square having sides of 100 m. Thus, there are 10,000 m^2, or about

2.471 acres per hectare. The *cubic meter* (m^3) is used for volumes in the SI system. Degrees, minutes, and seconds, or the radian, are accepted SI units for angles.

The metric system was originally developed in the 1790s in France. Although other definitions were suggested at that time, the French Academy of Sciences chose to define the meter as 1/10,000,000 of the length of the Earth's meridian through Paris from the equator to the pole. The actual length that was adopted for the meter was based on observations that had been made up to that time to determine the Earth's size and shape. Although later measurements revealed that the initially adopted value was approximately 0.2 mm short of its intended definition related to the meridional quadrant, still the originally adopted length became the standard.

Shortly after the metric system was introduced to the world, Thomas Jefferson who was the then secretary of state, recommended that the United States adopt it, but the proposal lost by one vote in the Congress! When the metric system was finally legalized for use (but not officially adopted) in the United States in 1866, a meter was defined as the interval under certain physical conditions between lines on an international prototype bar made of 90% platinum and 10 percent iridium, and accepted as equal to exactly 39.37 inches. A copy of this bar was held in Washington, D.C. and compared periodically with the international standard held in Paris. In 1960, at the General Conference on Weights and Measures (CGPM), the United States and 35 other nations agreed to redefine the meter as the length of 1,650,763.73 waves of the orange-red light produced by burning the element krypton (Kr-86). That definition permitted industries to make more accurate measurements and to check their own instruments without recourse to the standard meter-bar in Washington. The wavelength of this light is a true constant, whereas there is a risk of instability in the metal meter-bar. The CGPM met again in 1983 and established the current definition of the meter as the length of the path traveled by light in a vacuum during a time interval of 1/299,792,458 sec. Obviously, with this definition, the speed of light in a vacuum becomes exactly 299,792,458 m/sec. The advantage of this latest standard is that the meter is more accurately defined, since it is in terms of time, the most accurate of our basic measurements.

During the 1960s and 1970s, significant efforts were made toward promoting adoption of SI as the legal system for weights and measures in the United States. However, costs and frustrations associated with making the change generated substantial resistance, and the efforts were temporarily stalled. Recognizing the importance to the United States of using the metric system in order to compete in the rapidly developing global economy, in 1988 the Congress enacted the *Omnibus Trade and Competitiveness Act*. It designated the metric system as the *preferred* system of weights and measures for U.S. trade and commerce. The Act, together with a subsequent *Executive Order* issued in 1991, required all federal agencies to develop definite metric conversion plans and to use SI standards in their procurements, grants, and other business-related activities to the extent economically feasible. As an example of one agency's response, the Federal Highway Administration adopted a plan calling for (1) use of metric units in all publications and correspondence after September 30, 1992 and (2) use of metric units on all plans and contracts for federal highways after September 30, 1996. Although the Act and Executive Order did not mandate states, counties, cities, or industries to convert to metric, strong incentives were provided, for example, if SI directives were not complied with, certain federal matching funds could be withheld. In light of these developments, it appeared that the metric system would soon become the official system for use in the United States. However, again much resistance was encountered, not only from individuals but also from agencies of some state, county, and town and city governments, as well as from certain businesses. As a result, the SI still has not been adopted officially in the United States.

Besides the obvious advantage of being better able to compete in the global economy, another significant advantage that would be realized in adopting the SI standard would be the elimination of the confusion that exists in making conversions between the English System and the SI. The 1999 crash of the Mars Orbiter underscores costs and frustrations associated with this confusion. This $125 million satellite was supposed to monitor the Martian atmosphere, but instead it crashed into the planet because its contractor used English units while NASA's Jet Propulsion Laboratory was giving it data in the metric system. For these reasons and others, such as the decimal simplicity of the metric system, surveyors who are presently burdened with unit conversions and awkward computations involving yard, foot, and inch units should welcome official adoption of the SI. However, since this adoption has not yet occurred, this book uses both English and SI units in discussion and example problems.

2.4 SIGNIFICANT FIGURES

In recording observations, an indication of the accuracy attained is the number of digits (significant figures) recorded. By definition, the number of significant figures in any observed value includes the positive (certain) digits plus one (*only one*) digit that is estimated or rounded off, and therefore questionable. For example, a distance measured with a tape whose smallest graduation is 0.01 ft, and recorded as 73.52 ft, is said to have four significant figures; in this case the first three digits are certain, and the last is rounded off and therefore questionable but still significant.

To be consistent with the theory of errors discussed in Chapter 3, it is essential that data be recorded with the correct number of significant figures. If a significant figure is dropped in recording a value, the time spent in acquiring certain precision has been wasted. On the other hand, if data are recorded with more figures than those that are significant, false precision will be implied. The number of significant figures is often confused with the number of decimal places. Decimal places may have to be used to maintain the correct number of significant figures, but in themselves they do not indicate significant figures. Some examples follow:

Two significant figures: 24, 2.4, 0.24, 0.0024, 0.020
Three significant figures: 364, 36.4, 0.000364, 0.0240
Four significant figures: 7621, 76.21, 0.0007621, 24.00.

Zeros at the end of an integer value may cause difficulty because they may or may not be significant. In a value expressed as 2400, for example, it is not known how many figures are significant; there may be two, three, or four, and therefore definite rules must be followed to eliminate the ambiguity. The preferred method of eliminating this uncertainty is to express the value in terms of powers of 10. The significant figures in the measurement are then written in scientific notation as a number between 1 and 10 with the correct number of zeros and power of 10. As an example, 2400 becomes 2.400×10^3 if both zeros are significant, 2.40×10^3 if one is, and 2.4×10^3 if there are only two significant figures. Alternatively, a bar may be placed over the last significant figure, as $240\bar{0}$, $24\bar{0}0$, and $2\bar{4}00$ for 4, 3, and 2 significant figures, respectively.

When observed values are used in the mathematical processes of addition, subtraction, multiplication, and division, it is imperative that the number of significant figures given in answers be consistent with the number of significant figures in the data used. The following three steps will achieve this for addition or subtraction: (1) identify the column containing the rightmost significant digit in each number being added or subtracted, (2) perform the addition or subtraction, and (3) round the answer so that its rightmost significant digit occurs in the leftmost column identified in step (1). Two examples illustrate the procedure.

(a)

$$\begin{array}{r} 46.7418 \\ +\quad 1.03 \\ \underline{+375.0} \\ 422.7718 \end{array}$$

(answer 422.8)

(b)

$$\begin{array}{r} 378. \\ \underline{-2.1} \\ 375.9 \end{array}$$

(answer 376.)

In (**a**), the digits 8, 3, and 0 are the rightmost significant ones in the numbers 46.7418, 1.03, and 375.0, respectively. Of these, the 0 in 375.0 is leftmost with respect to the decimal. Thus, the answer 422.7718 obtained on adding the numbers is rounded to 422.8, with its rightmost significant digit occurring in the same column as the 0 in 375.0. In (**b**), the digits 8 and 1 are rightmost, and of these the 8 is leftmost. Thus, the answer 375.9 is rounded to 376.

In multiplication, the number of significant figures in the answer is equal to the least number of significant figures in any of the factors. For example, $362.56 \times 2.13 = 7721.2528$ when multiplied, but the answer is correctly given as 772. Its three significant figures are governed by the three significant digits in 2.13. Likewise, in division the quotient should be rounded off to contain only as many significant figures as the least number of significant figures in either the divisor or the dividend. These rules for significant figures in computations stem from error propagation theory, which is discussed further in Section 3.17.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The icon in the margin indicates the availability of such videos. The video *significant figures.mp4* discusses the rules applied to significant figures and rounding, which is covered in the following section.

In surveying, four specific types of problems relating to significant figures are encountered and must be understood.

1. Field measurements are given to some specific number of significant figures, thus dictating the number of significant figures in answers derived when the measurements are used in computations. In an intermediate calculation, it is

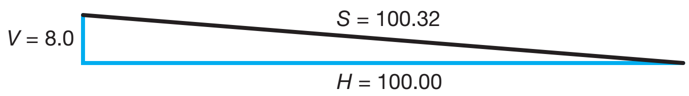


Figure 2.2
Slope correction.

a common practice to carry at least one more digit than required, and then round off the final answer to the correct number of significant figures.

2. There may be an implied number of significant figures. For instance, the length of a football field might be specified as 100 yd. But in laying out the field, such a distance would probably be measured to the nearest hundredth of a foot, not the nearest half-yard.
3. Each factor may not cause an equal variation. For example, if a steel tape 100.00 ft long is to be corrected for a change in temperature of 15°F, one of these numbers has five significant figures while the other has only two. However, a 15° variation in temperature changes the tape length by only 0.01 ft. Therefore, an adjusted tape length to five significant figures is warranted for this type of data. Another example is the computation of a slope distance from horizontal and vertical distances, as in Figure 2.2. The vertical distance V is given to two significant figures, and the horizontal distance H is measured to five significant figures. From these data, the slope distance S can be computed to five significant figures. For small angles of slope, a considerable change in the vertical distance produces a relatively small change in the difference between slope and horizontal distances.
4. Observations are recorded in one system of units but may have to be converted to another. A good rule to follow in making these conversions is to retain in the answer a number of significant figures equal to those in the observed value. As an example, to convert 178 ft 6-3/8 in. to meters, the number of significant figures in the measured value would first be determined by expressing it in its smallest units. In this case, 1/8th in. is the smallest unit and there are $(178 \times 12 \times 8) + (6 \times 8) + 3 = 17{,}139$ of these units in the value. Thus, the measurement contains five significant figures, and the answer is $17{,}139 \div (8 \times 39.37 \text{ in./m}) = 54.416$ m, properly expressed with five significant figures. (Note that 39.37 used in the conversion is an exact constant and does not limit the number of significant figures.)

■ 2.5 ROUNDING OFF NUMBERS

Rounding off a number is the process of dropping one or more digits so the answer contains only those digits that are significant. In rounding off numbers to any required degree of precision in this text, the following procedures will be observed:

1. When the digit to be dropped is lower than 5, the number is written without the digit. Thus, 78.374 becomes 78.37. Also 78.3749 rounded to four figures becomes 78.37.
2. When the digit to be dropped is exactly 5, the nearest even number is used for the preceding digit. Thus, 78.375 becomes 78.38 and 78.385 is also rounded to 78.38.

3. When the digit to be dropped is greater than 5, the number is written with the preceding digit increased by 1. Thus, 78.386 becomes 78.39.

Procedures 1 and 3 are standard practice. When rounding the value 78.375 in procedure 2, however, some people always take the next higher hundredth, whereas others invariably use the next lower hundredth. However, using the nearest even digit establishes a uniform procedure and produces better-balanced results in a series of computations. It is an improper procedure to perform two-stage rounding where, for example, in rounding 78.3749 to four digits it would be first rounded to five figures, yielding 78.375, and then rounded again to 78.38. The correct answer in rounding 78.3749 to four figures is 78.37.

It is important to recognize that rounding should only occur with the final answer. Intermediate computations should be done without rounding to avoid problems that can be caused by rounding too early. Example (**a**) of Section 2.4 is repeated below to illustrate this point. The sum of 46.7418, 1.03, and 375.0 is rounded to 422.8 as shown in the "correct" column. If the individual values are rounded prior to the addition as shown in the "incorrect" column, the incorrect result of 422.7 is obtained.

Correct	Incorrect
46.7418	~~46.7~~
+ 1.03	~~+ 1.0~~
+ 375.0	~~+ 375.0~~
422.7718	~~422.7~~
(answer 422.8)	~~(answer 422.7)~~

PART II • FIELD NOTES

2.6 FIELD NOTES

Field notes are the records of work done in the field. They typically contain measurements, sketches, descriptions, and many other items of miscellaneous information. In the past, field notes were prepared exclusively by hand lettering in field books or special note pads as the work progressed and data were gathered. However, automatic data collectors, also known as electronic field book and survey controllers, have been introduced that can interface with many different modern surveying instruments. As the work progresses, they create computer files containing a record of observed data. Data collectors are rapidly gaining popularity, but when used, manually prepared sketches and descriptions often supplement the numerical data they generate. Regardless of the manner or form in which the notes are taken, they are extremely important.

Whether prepared manually, created by a data collector, or a combination of these forms, surveying field notes are the only permanent records of work done in the field. If the data are incomplete, incorrect, lost, or destroyed, much or all of the time and money invested in making the measurements and records have been wasted. Hence, the job of data recording is frequently the most important and difficult one in a surveying party. Field books and computer files containing information gathered over a period of weeks are worth many thousands of dollars because of the costs of maintaining personnel and equipment in the field.

Recorded field data are used in the office to perform computations, make drawings, or both. The office personnel using the data are usually not the same people who took the notes in the field. Accordingly, it is essential that without verbal explanations notes be intelligible to anyone.

Property surveys are subject to court review under some conditions, so field notes become an important factor in litigation. Also, because they may be used as references in land transactions for generations, it is necessary to index and preserve them properly. The salable "goodwill" of a surveyor's business depends largely on the office library of field books. Cash receipts may be kept in an unlocked desk drawer, but field books are stored in a fireproof safe!

2.7 GENERAL REQUIREMENTS OF HANDWRITTEN FIELD NOTES

The following points are considered in appraising a set of field notes:

Accuracy. This is the most important quality in all surveying operations.

Integrity. A single omitted measurement or detail can nullify use of the notes for computing or plotting. If the project was far from the office, it is time-consuming and expensive to return for a missing measurement. Notes should be checked carefully for completeness before leaving the survey site and never "fudged" to improve closures.

Legibility. Notes can be used only if they are legible. A professional-looking set of notes is likely to be professional in quality.

Arrangement. Note forms appropriate to a particular survey contribute to accuracy, integrity, and legibility.

Clarity. Advance planning and proper field procedures are necessary to ensure clarity of sketches and tabulations, and to minimize the possibility of mistakes and omissions. Avoid crowding notes; paper is relatively cheap. Costly mistakes in computing and drafting are the end results of ambiguous notes.

Appendix B contains examples of handwritten field notes for a variety of surveying operations. Their plate number identifies each. Other example note forms are given at selected locations within the chapters that follow. These notes have been prepared keeping the above points in mind.

In addition to the items stressed in the foregoing, certain other guidelines must be followed to produce acceptable handwritten field notes. The notes should be lettered with a sharp pencil of at least 3H hardness so that an indentation is made in the paper. Books so prepared will withstand damp weather in the field (or even a soaking) and still be legible, whereas graphite from a soft pencil, or ink from a pen or ballpoint, leaves an undecipherable smudge under such circumstances.

Erasures of recorded data are not permitted in field books. If a number has been entered incorrectly, a line is run through it without destroying the number's legibility, and the proper value is noted above it (see Figure 5.5). If a partial or entire page is to be deleted, diagonal lines are drawn through opposite corners, and **VOID** is lettered prominently on the page, giving the reasons.

Field notes are presumed to be "original" unless marked otherwise. Original notes are those taken at the same time the observations are being made. If the original notes are copied, they must be so marked (see Figure 5.11). Copied notes may not be accepted in court because they are open to question concerning possible mistakes, such as interchanging numbers, and omissions. The value of a distance or an angle placed in the field book from memory 10 min after the observation is definitely unreliable. Students are tempted to scribble notes on scrap sheets of paper for later transfer in a neater form to the field book. This practice may result in the loss of some or all of the original data and defeats one purpose of a surveying course—to provide experience in taking notes under actual field conditions. In a real job situation, a surveyor is not likely to spend any time at night transcribing scribbled notes. Certainly, an employer will not pay for this evidence of incompetence.

■ 2.8 TYPES OF FIELD BOOKS

Since field books contain valuable data, suffer hard wear, and must be permanent in nature, only the best should be used for practical work. Various kinds of field books as shown in Figure 2.3 are available, but bound and loose-leaf types are most common. The bound book, a standard for many years, has a sewed binding, a hard cover of leatherette, polyethylene, or covered hardboard, and contains 80 leaves. Its use ensures maximum testimony acceptability for property survey records in courtrooms. Bound duplicating books enable copies of the original notes to be made through carbon paper in the field. The alternate duplicate pages are perforated to enable their easy removal for advance shipment to the office.

Loose-leaf books have come into wide use because of many advantages, which include (1) assurance of a flat working surface, (2) simplicity of filing individual project notes, (3) ready transfer of partial sets of notes between field and office, (4) provision for holding pages of printed tables, diagrams, formulas, and

Figure 2.3
Field books.
(Courtesy Topcon Positioning Systems.)

sample forms, (5) the possibility of using different rulings in the same book, and (6) a saving in sheets and thus cost since none are wasted by filing partially filled books. A disadvantage is the possibility of losing sheets.

Stapled or spiral-bound books are not suitable for practical work. However, they may be satisfactory for abbreviated surveying courses that have only a few field periods, because of limited service required and low cost. Special column and page rulings provide for particular needs in leveling, angle measurement, topographic surveying, cross-sectioning, and so on.

A camera is a helpful note-keeping "instrument." Moderately priced, reliable, lightweight cameras can be used to document monuments set or found and to provide records of other valuable information or admissible field evidence. Recorded images can become part of the final record of survey. Tape recorders can also be used in certain circumstances, particularly where lengthy written explanations would be needed to document conditions or provide detailed descriptions.

■ 2.9 KINDS OF NOTES

Four types of notes are kept in practice: (1) sketches, (2) tabulations, (3) descriptions, and (4) combinations of these. The most common type is a combination form, but an experienced recorder selects the version best fitted to the job at hand. The note forms in Appendix B illustrate some of these types and apply to field problems described in this text. Other examples are included within the text at appropriate locations. Sketches often greatly increase the efficiency with which notes can be taken. They are especially valuable to persons in the office who must interpret the notes without the benefit of the notekeeper's presence. The proverb about one picture being worth a thousand words might well have been intended for notekeepers!

For a simple survey, such as measuring the distances between points on a series of lines, a sketch showing the lengths is sufficient. In measuring the length of a line forward and backward, a sketch together with tabulations properly arranged in columns is adequate, as in Plate B.1 in Appendix B. The location of a reference point may be difficult to identify without a sketch, but often a few lines of description are enough. Photos may be taken to record the location of permanent stations and the surrounding locale. The combination of a sketch with dimensions and photographic images can be invaluable in later station relocation. Benchmarks are usually briefly described, as in Figure 5.5.

In notekeeping this axiom is always pertinent: *When in doubt about the need for any information, include it and make a sketch. It is better to have too much data than not enough.*

■ 2.10 ARRANGEMENTS OF NOTES

Note styles and arrangements depend on departmental standards and individual preference. Highway departments, mapping agencies, and other organizations engaged in surveying furnish their field personnel with sample note forms, similar to those in Appendix B, to aid in preparing uniform and complete records that can be checked quickly.

It is desirable for students to have as guides, predesigned sample sets of note forms covering their first fieldwork to set high standards and save time. The note forms shown in Appendix B are composites of several models. They stress the open style, especially helpful for beginners, in which some lines or spaces are skipped for clarity. Thus, angles observed at a point *A* (see Plate B.4) are placed opposite *A* on the page, but distances observed between *A* and *B* on the ground are recorded on the line between *A* and *B* in the field book.

Left- and right-hand pages are practically always used in pairs and therefore carry the same page number. A complete title should be lettered across the top of the left page and may be extended over the right one. Titles may be abbreviated on succeeding pages for the same survey project. Location and type of work are placed beneath the title. Some surveyors prefer to confine the title on the left page and keep the top of the right one free for date, party, weather, and other items. This design is revised if the entire right page has to be reserved for sketches and benchmark descriptions. Arrangements shown in Appendix B demonstrate the flexibility of note forms. The left page is generally ruled in six columns designed for tabulation only. Column headings are placed between the first two horizontal lines at the page top and follow from left to right in the anticipated order of reading and recording. The upper part of the left or right page must contain the following items:

1. *Project name, location, date, time of day* (A.M. *or* P.M.), *and starting and finishing times*. These entries are necessary to document the notes and furnish a timetable as well as to correlate different surveys. Precision, troubles encountered, and other facts may be gleaned from the time required for a survey.
2. *Weather*. Wind velocity, temperature, and adverse weather conditions such as rain, snow, sunshine, and fog have a decided effect on accuracy in surveying operations. Surveyors are unlikely to do their best possible work at temperatures of 15°F or with rain pouring down their necks. Hence, weather details are important in reviewing field notes, in applying corrections to observations due to temperature variations, and for other purposes.
3. *Party*. The names and initials of party members and their duties are required for documentation and future reference. Jobs can be described by symbols, such as $\overline{\wedge}$ for instrument operator, ϕ for rod person, and N for notekeeper. The party chief is frequently the notekeeper.
4. *Instrument type and number*. The type of instrument used (with its make and serial number) and its degree of adjustment affects the accuracy of a survey. Identification of the specific equipment employed may aid in isolating some errors—for example, a particular tape with an actual length that is later found to disagree with the distance recorded between its end graduations.

To permit ready location of desired data, each field book must have a table of contents that is kept current daily. In practice, surveyors cross-index their notes on days when field work is impossible.

2.11 SUGGESTIONS FOR RECORDING NOTES

Observing the suggestions given in preceding sections, together with those listed here, will eliminate some common mistakes in recording notes.

1. Letter the notebook owner's name and address on the cover and the first inside page using permanent ink. Number all field books for record purposes.
2. Begin a new day's work on a new page. For property surveys having complicated sketches, this rule may be waived.
3. Employ any orderly, standard, familiar note form type, but, if necessary, design a special arrangement to fit the project.
4. Include explanatory statements, details, and additional observations if they might clarify the notes for field and office personnel.
5. Record what is read without performing any mental arithmetic. Write down what you read!
6. Run notes down the page, except in route surveys, where they usually progress upward to conform with sketches made while looking in the forward direction. (See Plate B.5 in Appendix B.)
7. Use sketches instead of tabulations when in doubt. Carry a straightedge for ruling lines and a small protractor to lay off angles.
8. Make drawings to general proportions rather than to exact scale, and recognize that the usual preliminary estimate of space required is too small. Lettering parallel with or perpendicular to the appropriate features, showing clearly to what they apply.
9. Exaggerate details on sketches if clarity is thereby improved, or prepare separate diagrams.
10. Line up descriptions and drawings with corresponding numerical data. For example, a benchmark description should be placed on the right-hand page opposite its elevation, as in Figure 5.5.
11. Avoid crowding. If it is helpful to do so, use several right-hand pages of descriptions and sketches for a single left-hand sheet of tabulation. Similarly, use any number of pages of tabulation for a single drawing. Paper is cheap compared with the value of time that might be wasted by office personnel in misinterpreting compressed field notes, or by requiring a party to return to the field for clarification.
12. Use explanatory notes when they are pertinent, always keeping in mind the purpose of the survey and needs of the office force. Put these notes in open spaces to avoid conflict with other parts of the sketch.
13. Employ conventional symbols and signs for compactness.
14. A meridian arrow is vital for all sketches. Have north at the top and on the left side of sketches if possible.
15. Keep tabulated figures inside of and off column rulings, with decimal points and digits in line vertically.
16. Make a mental estimate of all measurements before receiving and recording them in order to eliminate large mistakes.
17. Repeat aloud values given for recording. For example, before writing down a distance of 124.68, call out "one, two, four, point six, eight" for verification by the person who submitted the measurement.

18. Place a zero before the decimal point for numbers smaller than 1; that is, record 0.37 instead of .37.
19. Show the precision of observations by means of significant figures. For example, record 3.80 instead of 3.8 only if the reading was actually determined to hundredths.
20. Do not superimpose one number over another or on lines of sketches, and do not try to change one figure to another, as a 3 to a 5.
21. Make all possible arithmetic checks on the notes and record them before leaving the field.
22. Compare all misclosures and error ratios while in the field. On large projects where daily assignments are made for several parties, completed work is shown by satisfactory closures.
23. Arrange essential computations made in the field so they can be checked later.
24. Title, index, and cross-reference each new job or continuation of a previous one by client's organization, property owner, and description.
25. Sign surname and initials in the lower right-hand corner of the right page on all original notes. This places responsibility just as signing a check does.

2.12 INTRODUCTION TO DATA COLLECTORS

Advances in computer technology in recent years have led to the development of sophisticated automatic data collection systems for taking field notes. These devices are about the size of a pocket calculator and are produced by a number of different manufacturers. They are available with a variety of features and capabilities. Figure 2.4 illustrates two different data collectors.

Data collectors can be interfaced with modern surveying instruments, and when operated in that mode they can automatically receive and store data in

(a) (b)

Figure 2.4
Various data collectors that are used in the field: (a) Trimble TSC2 data collector (Courtesy of Trimble) and (b) Carlson Explorer data collector.

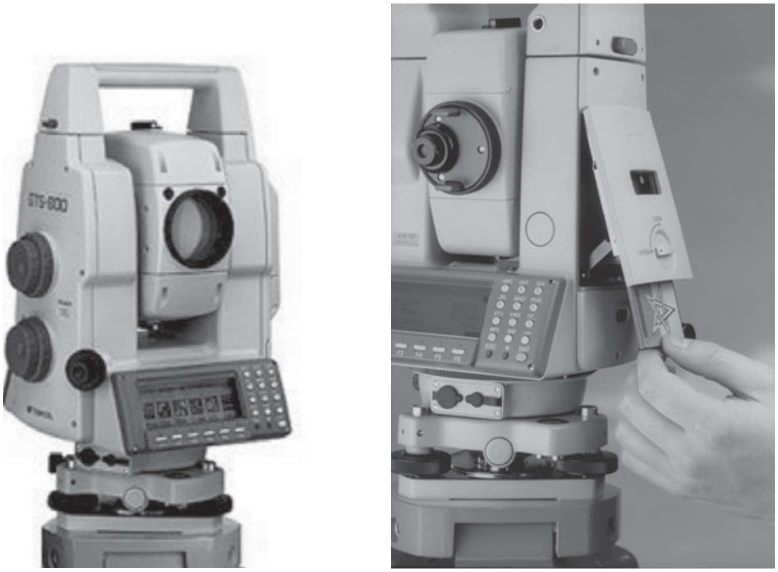
computer compatible files as observations are taken. Control of the measurement and storage operations is maintained through the data collector's keyboard. For clarification of the notes, the operator inputs point identifiers and other descriptive information along with the measurements as they are being recorded automatically. When a job is completed or at day's end, the files can be transferred directly to a computer for further processing.

In using automatic data collectors, the usual preliminary information such as date, party, weather, time, and instrument number is entered manually into the file through the keyboard. For a given type of survey, the data collector's internal microprocessor is programmed to follow a specific sequence of steps. The operator identifies the type of survey to be performed from a menu, or by means of a code, and then follows instructions that appear on the unit's screen. Step-by-step prompts will guide the operator to either (a) input "external" data (which may include station names, descriptions, or other information) or (b) press a key to initiate the automatic recording of observed values. Since data collectors require the users to follow specific steps when performing a survey, they are often referred to as *survey controllers*.

Data collectors store information in either binary or ASCII (American Standard Code for Information Interchange) format. Binary storage is faster and more compact, but usually the data must be translated to ASCII before they can be read or edited. Most data collectors enable an operator to scroll through stored data, displaying them on the screen for review and editing while still at the job site. The organizational structures used by different data collectors in storing information vary considerably from one manufacturer to the next. They all follow specific rules, and once they are understood, the data can be readily interpreted by both field and office personnel. The disadvantage of having varied data structures from different manufacturers is that a new system must be learned with each instrument of different make. Efforts have been made toward standardizing the data structures. The *Survey Data Management System* (SDMS), for example, has been adopted by the *American Association of State Highway and Transportation Officials* (AASHTO) and is recommended for all surveys involving highway work. The example field notes for a radial survey given in Table 17.1 of Section 17.9 are in the SDMS format.

Most manufacturers of modern surveying equipment have developed data collectors specifically to be interfaced with their own instruments, but some are flexible. The Trimble TSC2 survey controller shown in Figure 2.4(a), for example, can be interfaced with Trimble instruments, but it can also be used with others. In addition to serving as a data collector, the TSC2 is able to perform a variety of timesaving calculations directly in the field. It has a Windows CE operating system and thus can run a variety of Windows software programs. Additionally, it has Bluetooth technology so that it can communicate with instruments without using cables and has WiFi capabilities for connecting to the Internet.

Some automatic data collectors can also be operated as electronic field books. In the electronic field book mode, the data collector is not interfaced with a surveying instrument. Instead of handwriting the data in a field book, the notekeeper enters observations into the data collector manually by means of keyboard strokes after readings are taken. This has the advantage of enabling field notes to be recorded directly in a computer format ready for further processing, even though the surveying instruments being used may be older and not



(a) (b)

Figure 2.5
The Topcon GTS 800 total station with internal data collector. (Courtesy Topcon Positioning Systems.)

compatible for direct interfacing with data collectors. However, data collectors provide the utmost in efficiency when they are interfaced with surveying instruments such as total stations that have automatic readout capabilities.

The touch screen of the Carlson Explorer data collector shown in Figure 2.4(b) is a so-called third-party unit; that is, it is made by an independent company to be interfaced with instruments manufactured by others. It also utilizes a Windows CE operating system and has both Bluetooth and WiFi capabilities. It can be either operated in the electronic field book mode or interfaced with a variety of instruments for automatic data collection.

Many instrument manufacturers incorporate data collection systems as internal components directly into their equipment. This incorporates all features of external data collectors, including the display panel, within the instrument. The Topcon GTS 800 shown in Figure 2.5 has an MS-DOS®-based operating system with the ability to run the TDS (Tripod Data System) Survey Pro Software® onboard. It comes standard with 2 MB of program memory and 2 MB of internal data memory. The instrument has a PCMCIA[1] port for use with external data cards to allow for transfer of data from the field to the office without the instrument.

Data collectors currently use the Windows® CE operating system. A pen and pad arrangement enables the user to point on menus and options to run software. The data collectors shown in Figure 2.4 and the Trimble TSC2 data collector shown in Figure 2.6 have this type of interface. A code-based GPS antenna can be inserted into a PCMCIA port of several data collectors to add code-based GPS capabilities to the unit. Most modern data collectors have the capability of

[1]A PCMCIA port conforms to the *Personal Computer Memory Card International Association* standards.

Figure 2.6 Trimble TSC2 with Bluetooth technology. (Courtesy of Trimble.)

running advanced computer software in the field. As one example of their utility, field crews can check their data before sending it to the office (Figure 2.7).

As each new series of data collectors is developed, more sophisticated user interfaces are being designed, and the software that accompanies the systems is being improved. These systems have resulted in increased efficiency and productivity, and have provided field personnel with new features, such as the ability to perform additional field checks. However, the increased complexity of operating surveying instruments with advanced data collectors also requires field personnel with higher levels of education and training.

■ 2.13 TRANSFER OF FILES FROM DATA COLLECTORS

At regular intervals, usually at lunchtime and at the end of a day's work, or when a survey has been completed, the information stored in files within a data collector is transferred to another device. This is a safety precaution to avoid accidentally losing substantial amounts of data. Ultimately, of course, the files are downloaded to a host computer, which will perform computations or generate maps and plots from the data. Depending on the peripheral equipment available, different procedures for data transfer can be used. In one method that is particularly convenient when surveying in remote locations, data can be returned to the home office via telephony technology using devices called *data modems*. (Modems convert computer data into audible tones for transmission via telephone systems.) Thus, office personnel can immediately begin using the data. In areas with cell phone coverage, this operation can be performed in the field. Another method of data transfer consists in downloading data straight into a computer by direct hookup via an RS-232 cable. This can be performed in the office, or it can be done in the field if a laptop computer is available. In areas with

Figure 2.7
Screen of a Trimble TSC2 survey controller. (Courtesy of Trimble.)

wireless Internet, data can be transferred to the office using wireless connections. Data collectors with WiFi capabilities allow field crews to communicate directly with office personnel, thus allowing data to be transferred, checked, and verified before the crews leave the field.

Some surveying instruments, for example, the Topcon GTS 800 Series total station shown in Figure 2.5, are capable of storing data externally on PCMCIA cards. These cards can, in turn, be taken to the office, where the files can be downloaded using a computer with a PCMCIA port. These ports are standard for most laptop computers, and thus allow field crews to download data from the PCMCIA card and external or internal data collector to storage devices on the computer at regular intervals in the field. With the inclusion of a modem, field crews can transfer files to an office computer over phone lines. Office personnel can check field data, or compute additional points to be staked, in the office and return the results to the field crews while they are still on the site.

From the preceding discussion, and as illustrated in Figure 2.8, automatic data collectors are central components of modern computerized surveying systems. In these systems, data flow automatically from the field instrument through the collector to the printer, computer, plotter, and other units in the system. The term

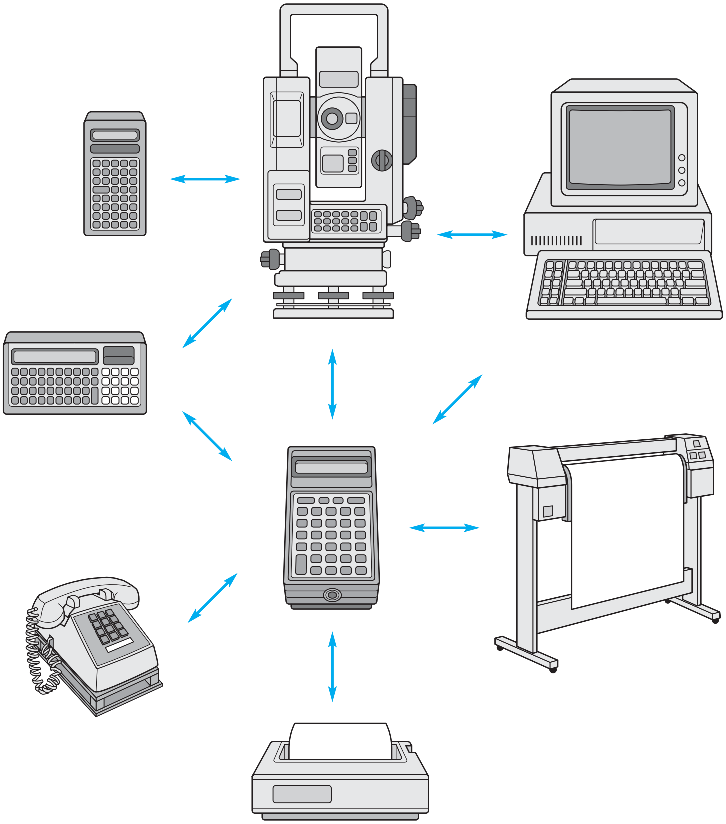


Figure 2.8 Automatic data collector—a central component in modern computerized surveying systems. (Reprinted with permission from Sokkia Corporation.)

"field-to-finish systems" is often applied when this form of instrumentation and software is utilized in surveying.

■ 2.14 DIGITAL DATA FILE MANAGEMENT

Once the observing process is completed in the field, the generated data files must be transferred (downloaded) from the data collector to another secure storage device. Typical information downloaded from a data collector includes a file of computed coordinates and a raw data file. Data collectors generally provide the option of saving these and other types of files. In this case, the coordinate file consists of computed coordinate values generated using the observations and any applied field corrections. Field corrections may include a scale factor, offsets, and Earth curvature and refraction corrections applied to distances. Field crews generally can edit and delete information from the computed file. However, the raw data file consists of the original unreduced observations and cannot be altered in the field. The necessity for each type of data file is dependent on the intended use of the survey. In most surveys, it would be prudent to save both the coordinate and raw files. As an example, for projects that require specific closures, or that are subject to legal review, the raw data file is an essential element of the survey. However, in topographic and GNSS surveys large quantities of data are often generated. In these types of projects, the raw data file can be eliminated to provide more storage space for coordinate files.

With data collectors and digital instruments, personnel in modern surveying offices deal with considerably more data than was customary in the past. This increased volume inevitably raises new concerns about data reliability and safe storage. Many methods can be used to provide backup of digital data. Some storage options include removable media disks and tapes. Since these tend to be magnetic, there is an inevitable danger that data could be lost due to the presence of external magnetic devices, or from the failure of the disk's surface. Because of this problem, it is wise to keep two copies of the files for all jobs. Another inexpensive solution to this problem is the use of compact disk (CD) and digital video disk (DVD) writers. These drives will write an optical image of a project's data on a portable disk media. Since CDs and DVDs are small but have large storage capabilities, entire projects, including drawings, can be recorded in a small space that is easily archived for future reference. However, these disks can fail when scratched. Thus, care must be taken in their handling and storage.

■ 2.15 ADVANTAGES AND DISADVANTAGES OF DATA COLLECTORS

The major advantages of automatic data collection systems are that (1) mistakes in reading and manually recording observations in the field are precluded and (2) the time to process, display, and archive the field notes in the office is reduced significantly. Systems that incorporate computers can execute some programs in the field, which adds a significant advantage. As an example, the data for a survey can be corrected for systematic errors and misclosures computed, so verification that a survey meets closure requirements is made before the crew leaves a site.

Data collectors are most useful when large quantities of information must be recorded, for example, in topographic surveys or cross-sectioning. In Section 17.9, their use in topographic surveying is described, and an example set of notes taken for that purpose is presented and discussed.

Although data collectors have many advantages, they also present some dangers and problems. There is the slight chance, for example, the files could be accidentally erased through carelessness or lost because of malfunction or damage to the unit. Some difficulties are also created by the fact that sketches cannot be entered into the computer. However, this problem can be overcome by supplementing files with sketches made simultaneously with the observations that include field codes. These field codes can instruct the drafting software to draw a map of the data complete with lines, curves, and mapping symbols. The process of collecting field data with field codes that can be interpreted later by software is known as a *field-to-finish* survey. This greatly reduces the time needed to complete a project. Field-to-finish mapping surveys are discussed in more detail in Section 17.12. It is important to realize that not all information can be stored in digital form, and thus it is important to keep a traditional field book to enter sketches, comments, and additional notes when necessary. In any event, these devices should not be used for long-term storage. Rather the data should be downloaded and immediately saved to some permanent storage device such as a CD or DVD once the field collection for a project is complete.

Data collectors are available from numerous manufacturers. They must be capable of transferring data through various hardware in modern surveying systems such as that illustrated in Figure 2.8. Since equipment varies considerably, it is important when considering the purchase of a data collector to be certain it fits the equipment owned or perhaps needed in the future.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

2.1 List the five types of measurements that form the basis of traditional plane surveying.

2.2 Give the basic units that are used in surveying for length, area, volume, and angles in
- **(a)** The English system of units.
- **(b)** The SI system of units.

2.3 Why was the survey foot definition maintained in the United States?

2.4 Convert the following distances given in meters to U.S. survey feet:
*__(a)__ 4129.574 m **(b)** 738.296 m **(c)** 6048.083 m

2.5 Convert the following distances given in feet to meters:
*__(a)__ 537.52 ft **(b)** 9364.87 ft **(c)** 4806.98 ft

2.6 Compute the lengths in feet corresponding to the following distances measured with a Gunter's chain:
*__(a)__ 10 ch 13 lk **(b)** 6 ch 12 lk **(c)** 24 ch 8 lk

2.7 Express 95,748 ft^2 in:
*__(a)__ acres **(b)** hectares **(c)** square Gunter's chains

2.8 Convert 5.6874 ha to:
(a) acres **(b)** square Gunter's chains

2.9 What are the lengths in feet and decimals for the following distances shown on a building blueprint?
(a) 30 ft 9-3/4 in. **(b)** 12 ft 6-1/32 in.

2.10 What is the area in acres of a rectangular parcel of land measured with a Gunter's chain if the recorded sides are as follows:
*__(a)__ 9.17 ch and 10.64 ch **(b)** 12 ch 36 lk and 24 ch 28 lk

2.11 Compute the area in acres of triangular lots shown on a plat having the following recorded right-angle sides:
(a) 208.94 ft and 232.65 ft **(b)** 9 ch 25 lk and 6 ch 16 lk

2.12 A distance is expressed as 125,845.64 U.S. survey feet. What is the length in
*__(a)__ international feet? **(b)** meters?

2.13 What are the radian and degree–minute–second equivalents for the following angles given in grads:
*__(a)__ 136.0000 grads **(b)** 89.5478 grads **(c)** 68.1649 grads

2.14 Give answers to the following problems in the correct number of significant figures:
- *__(a)__ sum of 23.15, 0.984, 124, and 12.5
- **(b)** sum of 36.15, 0.806, 22.4, and 196.458
- **(c)** product of 276.75 and 33.7
- **(d)** quotient of 4930.27 divided by 1.29

2.15 Express the value or answer in powers of 10 to the correct number of significant figures:
- **(a)** 11,432
- **(b)** 4520
- **(c)** square of 11,293
- **(d)** sum of (11.275 + 0.5 + 146.12) divided by 7.2

2.16 Convert the adjusted angles of a triangle to radians and show a computational check:
- ***(a)** 39°41′54″, 91°30′16″, and 48°47′50″
- **(b)** 82°17′43″, 29°05′54″, and 68°36′23″

2.17 Why should a pen not be used in field notekeeping?

2.18 Explain why one number should not be superimposed over another or the lines of sketches.

2.19* Explain why data should always be entered directly into the field book at the time measurements are made, rather than on scrap paper for neat transfer to the field book later.

2.20 Why should a new day's work begin on a new page?

2.21 Explain the reason for item 18 in Section 2.11 when recording field notes.

2.22 Explain the reason for item 24 in Section 2.11 when recording field notes.

2.23 Explain the reason for item 27 in Section 2.11 when recording field notes.

2.24 When should sketches be made instead of just recording data?

2.25 Justify the requirement to list in a field book the makes and serial numbers of all instruments used on a survey.

2.26 Discuss the advantages of survey controllers that can communicate with several different types of instruments.

2.27 Discuss the advantages of survey controllers.

2.28 Search the Internet and find at least two sites related to
- **(a)** Manufacturers of survey controllers.
- **(b)** Manufacturers of total stations.
- **(c)** Manufacturers of global navigation satellite system (GNSS) receivers.

2.29 What advantages are offered to field personnel if the survey controller provides a map of the survey?

2.30 Prepare a brief summary of an article from a professional journal related to the subject matter of this chapter.

2.31 Describe what is meant by the phrase "field-to-finish."

2.32 Why are sketches in field books not usually drawn to scale?

2.33 Create a computational program that solves Problem 2.16.

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3 Theory of Errors in Observations

■ 3.1 INTRODUCTION

Making observations (measurements), and subsequent computations and analyses using them, are fundamental tasks of surveyors. Good observations require a combination of human skill and mechanical equipment applied with the utmost judgment. However, no matter how carefully made, observations are never exact and will always contain errors. Surveyors (geomatics engineers), whose work must be performed to exacting standards, should therefore thoroughly understand the different kinds of errors, their sources and expected magnitudes under varying conditions, and their manner of propagation. Only then can they select instruments and procedures necessary to reduce error sizes to within tolerable limits.

Of equal importance, surveyors must be capable of assessing the magnitudes of errors in their observations so that either their acceptability can be verified or, if necessary, new ones made. The design of measurement systems is now practiced. Computers and sophisticated software are tools now commonly used by surveyors to plan measurement projects and to investigate and distribute errors after results have been obtained.

■ 3.2 DIRECT AND INDIRECT OBSERVATIONS

Observations may be made directly or indirectly. Examples of *direct observations* are applying a tape to a line, fitting a protractor to an angle, or turning an angle with a total station instrument.

An *indirect observation* is secured when it is not possible to apply a measuring instrument directly to the quantity to be observed. The answer is therefore determined by its relationship to some other observed value or values. As an example, we can find the distance across a river by observing the length of a line on one side of the river and the angle at each end of this line to a point on the other side, and then computing the distance by one of the standard trigonometric formulas. Many indirect observations are made in surveying, and since all measurements contain errors, it is inevitable that quantities computed from them will also contain errors. The manner by which errors in measurements combine to produce erroneous computed answers is called error propagation. This topic is discussed further in Section 3.17.

3.3 ERRORS IN MEASUREMENTS

By definition, an error is the difference between an observed value for a quantity and its true value, or

$$E = X - \overline{X} \tag{3.1}$$

where E is the error in an observation, X the observed value, and $\overline{X}$ its true value. It can be unconditionally stated that (1) no observation is exact, (2) every observation contains errors, (3) the true value of an observation is never known, and, therefore, (4) the exact error present is always unknown. These facts are demonstrated by the following. When a distance is observed with a scale divided into tenths of an inch, the distance can be read only to hundredths (by interpolation). However, if a better scale graduated in hundredths of an inch was available and read under magnification, the same distance might be estimated to thousandths of an inch. And with a scale graduated in thousandths of an inch, a reading to ten-thousandths might be possible. Obviously, accuracy of observations depends on the scale's division size, reliability of equipment used, and human limitations in estimating closer than about one tenth of a scale division. As better equipment is developed, observations more closely approach their true values, but they can never be exact. Note that observations, not counts (of cars, pennies, marbles, or other objects), are under consideration here.

3.4 MISTAKES

These are observer blunders and are usually caused by misunderstanding the problem, carelessness, fatigue, missed communication, or poor judgment. Examples include transposition of numbers, such as recording 73.96 instead of the correct value of 79.36; reading an angle counterclockwise, but indicating it as a clockwise angle in the field notes; sighting the wrong target; or recording a measured distance as 682.38 instead of 862.38. Large mistakes such as these are not considered in the succeeding discussion of errors. They must be detected by careful and systematic checking of all work, and eliminated by repeating some or all of the measurements. It is very difficult to detect small mistakes because they merge with errors. When not exposed, these small mistakes will therefore be incorrectly treated as errors.

■ 3.5 SOURCES OF ERRORS IN MAKING OBSERVATIONS

Errors in observations stem from three sources, and are classified accordingly.

Natural errors are caused by variations in wind, temperature, humidity, atmospheric pressure, atmospheric refraction, gravity, and magnetic declination. An example is a steel tape whose length varies with changes in temperature.

Instrumental errors result from any imperfection in the construction or adjustment of instruments and from the movement of individual parts. For example, the graduations on a scale may not be perfectly spaced, or the scale may be warped. The effect of many instrumental errors can be reduced, or even eliminated, by adopting proper surveying procedures or applying computed corrections.

Personal errors arise principally from limitations of the human senses of sight and touch. As an example, a small error occurs in the observed value of a horizontal angle if the vertical crosshair in a total station instrument is not aligned perfectly on the target, or if the target is the top of a rod that is being held slightly out of plumb.

■ 3.6 TYPES OF ERRORS

Errors in observations are of two types: *systematic* and *random*.

Systematic errors, also known as *biases*, result from factors that comprise the "measuring system" and include the environment, instrument, and observer. So long as system conditions remain constant, the systematic errors will likewise remain constant. If conditions change, the magnitudes of systematic errors also change. Because systematic errors tend to accumulate, they are sometimes called *cumulative errors*.

Conditions producing systematic errors conform to physical laws that can be modeled mathematically. Thus, if the conditions are known to exist and can be observed, a correction can be computed and applied to observed values. An example of a constant systematic error is the use of a 100-ft steel tape that has been calibrated and found to be 0.02 ft too long. It introduces a 0.02-ft error each time it is used, but applying a correction readily eliminates the error. An example of variable systematic error is the change in length of a steel tape resulting from temperature differentials that occur during the period of the tape's use. If the temperature changes are observed, length corrections can be computed by a simple formula, as explained in Chapter 6.

Random errors are those that remain in measured values after mistakes and systematic errors have been eliminated. They are caused by factors beyond the control of the observer, obey the laws of probability, and are sometimes called *accidental errors*. They are present in all surveying observations.

The magnitudes and algebraic signs of random errors are matters of chance. There is no absolute way to compute or eliminate them, but they can be estimated using adjustment procedures known as *least squares* (see Section 3.21 and Chapter 16). Random errors are also known as *compensating errors*, since they tend to partially cancel themselves in a series of observations. For example, a person interpolating to hundredths of a foot on a tape graduated only to tenths, or reading a level rod marked in hundredths, will presumably estimate too high on some values and too low on others. However, individual personal characteristics may nullify such partial compensation since some people are inclined to interpolate high, others interpolate low, and many favor certain digits—for example, 7 instead of 6 or 8, 3 instead of 2 or 4, and particularly 0 instead of 9 or 1.

3.7 PRECISION AND ACCURACY

A *discrepancy* is the difference between two observed values of the same quantity. A small discrepancy indicates there are probably no mistakes and random errors are small. However, small discrepancies do not preclude the presence of systematic errors.

Precision refers to the degree of refinement or consistency of a group of observations and is evaluated on the basis of discrepancy size. If multiple observations are made of the same quantity and small discrepancies result, this indicates high precision. The degree of precision attainable is dependent on equipment sensitivity and observer skill.

Accuracy denotes the absolute nearness of observed quantities to their true values. The difference between precision and accuracy is perhaps best illustrated with reference to target shooting. In Figure 3.1(a), for example, all five shots exist in a small group, indicating a precise operation; that is, the shooter was able to repeat the procedure with a high degree of consistency. However, the shots are far from the bull's-eye and therefore not accurate. This probably results from misaligned rifle sights. Figure 3.1(b) shows randomly scattered shots that are neither precise nor accurate. In Figure 3.1(c), the closely spaced grouping, in the bull's-eye, represents both precision and accuracy. The shooter who obtained the results in (a) was perhaps able to produce the shots of (c) after aligning the rifle sights. In surveying, this would be equivalent to the calibration of observing instruments.

As with the shooting example, a survey can be precise without being accurate. To illustrate, if refined methods are employed and readings taken carefully, say to 0.001 ft, but there are instrumental errors in the measuring device and corrections are not made for them, the survey will not be accurate. As a numerical example, two observations of a distance with a tape assumed to be 100.000 ft long, that is actually 100.050 ft, might give results of 453.270 and 453.272 ft. These values are precise, but they are not accurate, since there is a systematic error of approximately $4.53 \times 0.050 = 0.23$ ft in each. The precision obtained would be expressed as $(453.272 - 453.270)/453.271 = 1/220{,}000$, which is excellent, but

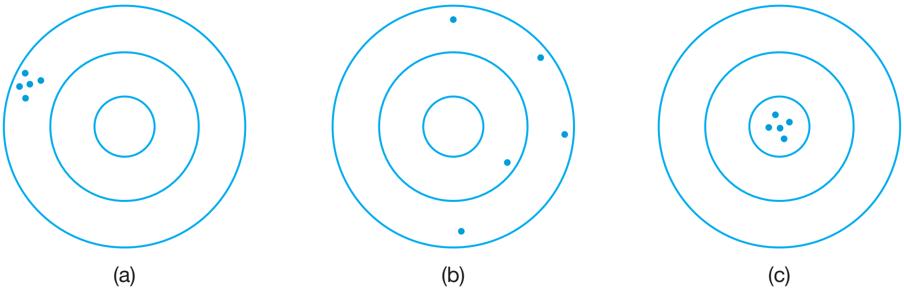

Figure 3.1 Examples of precision and accuracy. (a) Results are precise but not accurate. (b) Results are neither precise nor accurate. (c) Results are both precise and accurate.

accuracy of the distance is only 0.23/453.271 = 1 part in 2000. Also, a survey may appear to be accurate when rough observations have been taken. For example, the angles of a triangle may be read with a compass to only the nearest 1/4 degree and yet produce a sum of exactly 180°, or a zero misclosure error. On good surveys, precision and accuracy are consistent throughout.

■ 3.8 ELIMINATING MISTAKES AND SYSTEMATIC ERRORS

All field operations and office computations are governed by a constant effort to eliminate mistakes and systematic errors. Of course it would be preferable if mistakes never occurred, but because humans are fallible, this is not possible. In the field, experienced observers who alertly perform their observations using standardized repetitive procedures can minimize mistakes. Mistakes that do occur can be corrected only if discovered. Comparing several observations of the same quantity is one of the best ways to identify mistakes. Making a common sense estimate and analysis is another. Assume that five observations of a line are recorded as follows: 567.91, 576.95, 567.88, 567.90, and 567.93. The second value disagrees with the others, apparently because of a transposition of figures in reading or recording. Either casting out the doubtful value, or preferably repeating the observation can eradicate this mistake.

When a mistake is detected, it is usually best to repeat the observation. However, if a sufficient number of other observations of the quantity are available and in agreement, as in the foregoing example, the widely divergent result may be discarded. Serious consideration must be given to the effect on an average before discarding a value. It is seldom safe to change a recorded number, even though there appears to be a simple transposition in figures. Tampering with physical data is always a bad practice and will certainly cause trouble, even if done infrequently.

Systematic errors can be calculated and proper corrections applied to the observations. Procedures for making these corrections to all basic surveying observations are described in the chapters that follow. In some instances, it may be possible to adopt a field procedure that automatically eliminates systematic errors. For example, as explained in Chapter 5, a leveling instrument out of adjustment causes incorrect readings, but if all backsights and foresights are made the same length, the errors cancel in differential leveling.

■ 3.9 PROBABILITY

At one time or another, everyone has had an experience with games of chance, such as coin flipping, card games, or dice, which involve probability. In basic mathematics courses, laws of combinations and permutations are introduced. It is shown that events that happen randomly or by chance are governed by mathematical principles referred to as probability.

Probability may be defined as the ratio of the number of times a result should occur to its total number of possibilities. For example, in the toss of a fair die there is a one-sixth probability that a 2 will come up. This simply means that there are six possibilities, and only one of them is a 2. In general, if a result may occur in m ways and fail to occur in n ways, then the probability of its occurrence is $m/(m + n)$. The probability that any result will occur is a fraction between 0 and 1; 0 indicating impossibility and 1 denoting absolute certainty. Since any result must either occur or fail, the sum of the probabilities of occurrence and failure is 1. Thus if 1/6 is the probability of throwing a 2 with one toss of a die, then $(1-1/6)$, or 5/6 is the probability that a 2 will not come up.

The theory of probability is applicable in many sociological and scientific observations. In Section 3.6, it was pointed out that random errors exist in all surveying work. This can perhaps be better appreciated by considering the measuring process, which generally involves executing several elementary tasks. Besides instrument selection and calibration, these tasks may include setting up, centering, aligning, or pointing the equipment; setting, matching, or comparing index marks; and reading or estimating values from graduated scales, dials, or gauges. Because of equipment and observer imperfections, exact observations cannot be made, so they will always contain random errors. The magnitudes of these errors, and the frequency with which errors of a given size occur, follow the laws of probability.

For convenience, the term error will be used to mean only random error for the remainder of this chapter. It will be assumed that all mistakes and systematic errors have been eliminated before random errors are considered.

3.10 MOST PROBABLE VALUE

It has been stated earlier that in physical observations, the true value of any quantity is never known. However, its *most probable value* can be calculated if redundant observations have been made. *Redundant observations* are measurements in excess of the minimum needed to determine a quantity. For a single unknown, such as a line length that has been directly and independently observed a number of times using the same equipment and procedures,[1] the first observation establishes a value for the quantity and all additional observations are redundant. The most probable value in this case is simply the arithmetic mean, or

$$\overline{M} = \frac{\Sigma M}{n} \tag{3.2}$$

where $\overline{M}$ is the most probable value of the quantity, ΣM the sum of the individual measurements M, and n the total number of observations. Equation (3.2) can be derived using the principle of least squares, which is based on the theory of probability.

As discussed in Chapter 16, in more complicated problems, where the observations are not made with the same instruments and procedures, or if several interrelated quantities are being determined through indirect observations, most probable values are calculated by employing least-squares methods. The

[1]The significance of using the same equipment and procedures is that observations are of equal reliability or weight. The subject of unequal weights is discussed in Section 3.20.

treatment here relates to multiple direct observations of the same quantity using the same equipment and procedures.

3.11 RESIDUALS

Having determined the most probable value of a quantity, it is possible to calculate *residuals*. A residual is simply the difference between the most probable value and any observed value of a quantity, which in equation form is

$$\nu = \overline{M} - M \tag{3.3}$$

where v is the residual in any observation M, and $\overline{M}$ is the most probable value for the quantity. Residuals are theoretically identical to errors, with the exception that residuals can be calculated whereas errors cannot because true values are never known. Thus, residuals rather than errors are the values actually used in the analysis and adjustment of survey data.

3.12 OCCURRENCE OF RANDOM ERRORS

To analyze the manner in which random errors occur, consider the data of Table 3.1, which represents 100 repetitions of an angle observation made with a precise total station instrument (described in Chapter 8). Assume these observations are free from mistakes and systematic errors. For convenience in analyzing the data, except for the first value, only the seconds' portions of the observations are tabulated. The data have been rearranged in column (1) so that entries begin with the smallest observed value and are listed in increasing size. If a certain value was obtained more than once, the number of times it occurred, or its *frequency*, is tabulated in column (2).

From Table 3.1, it can be that the *dispersion* (range in observations from smallest to largest) is $30.8 - 19.5 = 11.3$ sec. However, it is difficult to analyze the distribution pattern of the observations by simply scanning the tabular values; that is, beyond assessing the dispersion and noticing a general trend for observations toward the middle of the range to occur with greater frequency. To assist in studying the data, a *histogram* can be prepared. This is simply a bar graph showing the sizes of the observations (or their residuals) versus their frequency of occurrence. It gives an immediate visual impression of the distribution pattern of the observations (or their residuals).

For the data of Table 3.1, a histogram showing the frequency of occurrence of the residuals has been developed and is plotted in Figure 3.2. To plot a histogram of residuals, it is first necessary to compute the most probable value for the observed angle. This has been done with Equation (3.2). As shown at the bottom of Table 3.1, its value is 27°43′24.9″. Then using Equation (3.3), residuals for all observed values are computed. These are tabulated in column (3) of Table 3.1. The residuals vary from 5.4″ to −5.9″. (The sum of the absolute value of these two extremes is the dispersion, or 11.3″.)

To obtain a histogram with an appropriate number of bars for portraying the distribution of residuals adequately, the interval of residuals represented by

TABLE 3.1 ANGLE OBSERVATIONS FROM PRECISE TOTAL STATION INSTRUMENT

Observed Value (1)	No. (2)	Residual (Sec) (3)	Observed Value (1 Cont.)	No. (2. Cont.)	Residual (Sec) (3 Cont.)
27°43′19.5″	1	5.4	27°43′25.1″	3	−0.2
20.0	1	4.9	25.2	1	−0.3
20.5	1	4.4	25.4	1	−0.5
20.8	1	4.1	25.5	2	−0.6
21.2	1	3.7	25.7	3	−0.8
21.3	1	3.6	25.8	4	−0.9
21.5	1	3.4	25.9	2	−1.0
22.1	2	2.8	26.1	1	−1.2
22.3	1	2.6	26.2	2	−1.3
22.4	1	2.5	26.3	1	−1.4
22.5	2	2.4	26.5	1	−1.6
22.6	1	2.3	26.6	3	−1.7
22.8	2	2.1	26.7	1	−1.8
23.0	1	1.9	26.8	2	−1.9
23.1	2	1.8	26.9	1	−2.0
23.2	2	1.7	27.0	1	−2.1
23.3	3	1.6	27.1	3	−2.2
23.6	2	1.3	27.4	1	−2.5
23.7	2	1.2	27.5	2	−2.6
23.8	2	1.1	27.6	1	−2.7
23.9	3	1.0	27.7	2	−2.8
24.0	5	0.9	28.0	1	−3.1
24.1	3	0.8	28.6	2	−3.7
24.3	1	0.6	28.7	1	−3.8
24.5	2	0.4	29.0	1	−4.1
24.7	3	0.2	29.4	1	−4.5
24.8	3	0.1	29.7	1	−4.8
24.9	2	0.0	30.8	1	−5.9
25.0	2	−0.1	Σ = 2494.0	Σ = 100	

Mean = 2494.0/100 = 24.9″

Most Probable Value = 27°43′24.9″

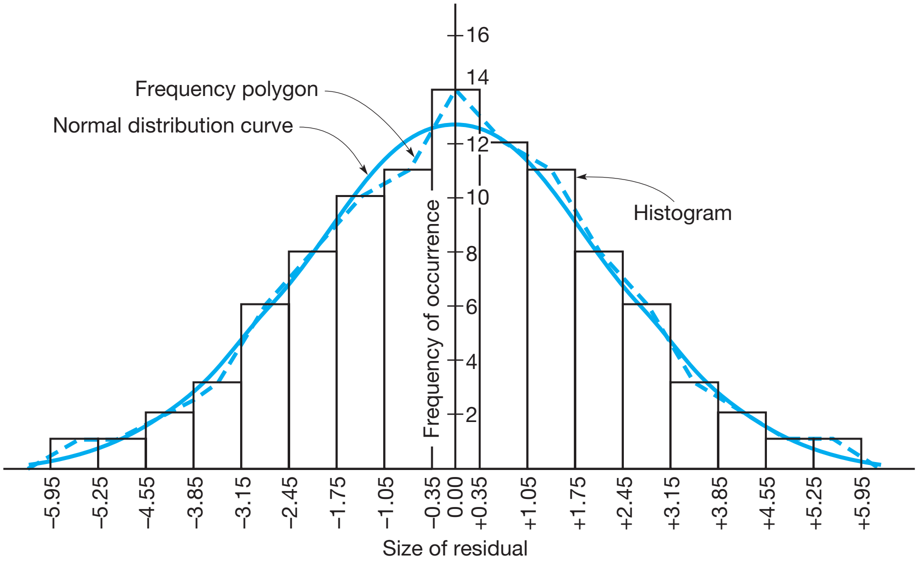

Figure 3.2
Histogram, frequency polygon, and normal distribution curve of residuals from angle measurements made with total station.

each bar, or the *class interval*, was chosen as 0.7″. This produced 17 bars on the graph. The range of residuals covered by each interval, and the number of residuals that occur within each interval, are listed in Table 3.2. By plotting class intervals on the abscissa against the number (frequency of occurrence) of residuals in each interval on the ordinate, the histogram of Figure 3.2 was obtained.

If the adjacent top center points of the histogram bars are connected with straight lines, the so-called *frequency polygon* is obtained. The frequency polygon for the data of Table 3.1 is superimposed as a heavy dashed blue line in Figure 3.2. It graphically displays essentially the same information as the histogram.

If the number of observations being considered in this analysis were increased progressively, and accordingly the histogram's class interval taken smaller and smaller, ultimately the frequency polygon would approach a smooth continuous curve, symmetrical about its center like the one shown with the heavy solid blue line in Figure 3.2. For clarity, this curve is shown separately in Figure 3.3. The curve's "bell shape" is characteristic of a normally distributed group of errors, and thus it is often referred to as the *normal distribution curve*. Statisticians frequently call it the *normal density curve*, since it shows the densities of errors having various sizes. In surveying, normal or very nearly normal error distributions are expected, and henceforth in this book that condition is assumed.

In practice, histograms and frequency polygons are seldom used to represent error distributions. Instead, normal distribution curves that approximate them are preferred. Note how closely the normal distribution curve superimposed on Figure 3.2 agrees with the histogram and the frequency polygon.

As demonstrated with the data of Table 3.1, the histogram for a set of observations shows the probability of occurrence of an error of a given size graphically by bar areas. For example, 14 of the 100 residuals (errors) in Figure 3.2 are between −0.35″ and +0.35″. This represents 14% of the errors, and the center histogram bar, which corresponds to this interval, is 14% of the total area of all

TABLE 3.2 RANGES OF CLASS INTERVALS AND NUMBER OF RESIDUALS IN EACH INTERVAL

Histogram Interval (Sec)	Number of Residuals in Interval
−5.95 to −5.25	1
−5.25 to −4.55	1
−4.55 to −3.85	2
−3.85 to −3.15	3
−3.15 to −2.45	6
−2.45 to −1.75	8
−1.75 to −1.05	10
−1.05 to −0.35	11
−0.35 to +0.35	14
+0.35 to +1.05	12
+1.05 to +1.75	11
+1.75 to +2.45	8
+2.45 to +3.15	6
+3.15 to +3.85	3
+3.85 to +4.55	2
+4.55 to +5.25	1
+5.25 to +5.95	1
	$\Sigma = 100$

bars. Likewise, the area between ordinates constructed at any two abscissas of a normal distribution curve represents the percent probability that an error of that size exists. Since the area sum of all bars of a histogram represents all errors, it therefore represents all probabilities, and thus its sum equals 1. Likewise, the total area beneath a normal distribution curve is also 1.

If the same observations of the preceding example had been taken using better equipment and more caution, smaller errors would be expected and the normal distribution curve would be similar to that in Figure 3.4(a). Compared to Figure 3.3, this curve is taller and narrower, showing that a greater percentage of values have smaller errors, and fewer observations contain big ones. For this comparison, the same ordinate and abscissa scales must be used for both curves. Thus, the observations of Figure 3.4(a) are more precise. For readings taken less precisely, the opposite effect is produced, as illustrated in Figure 3.4(b), which shows a shorter and wider curve. In all three cases, however, the curve maintained its characteristic symmetric bell shape.

From these examples, it is seen that relative precisions of groups of observations become readily apparent by comparing their normal distribution curves. The

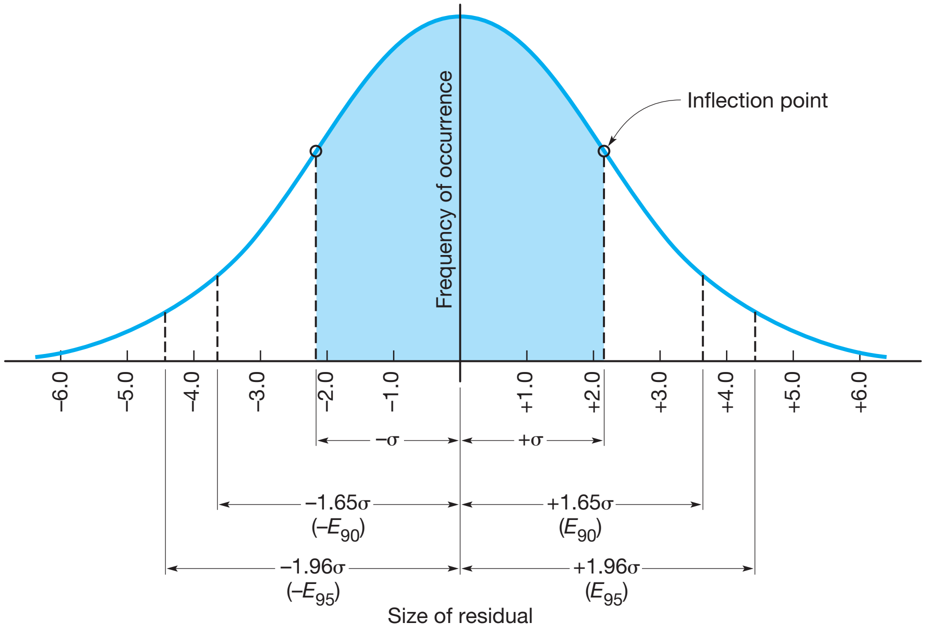

Figure 3.3
Normal distribution curve.

normal distribution curve for a set of observations can be computed using parameters derived from the residuals, but the procedure is beyond the scope of this text.

■ 3.13 GENERAL LAWS OF PROBABILITY

From an analysis of the data in the preceding section and the curves in Figures 3.2 through 3.4, some general laws of probability can be stated:

1. Small residuals (errors) occur more often than large ones; that is, they are more probable.
2. Large errors happen infrequently and are therefore less probable; for normally distributed errors, unusually large ones may be mistakes rather than random errors.
3. Positive and negative errors of the same size happen with equal frequency; that is, they are equally probable. [This enables an intuitive deduction of Equation (3.2) to be made: that is, the most probable value for a group of repeated observations, made with the same equipment and procedures, is the mean.]

■ 3.14 MEASURES OF PRECISION

As shown in Figures 3.3 and 3.4, although the curves have similar shapes, there are significant differences in their dispersions; that is, their abscissa widths differ. The magnitude of dispersion is an indication of the relative precisions of the observations. Other statistical terms more commonly used to express precisions of groups

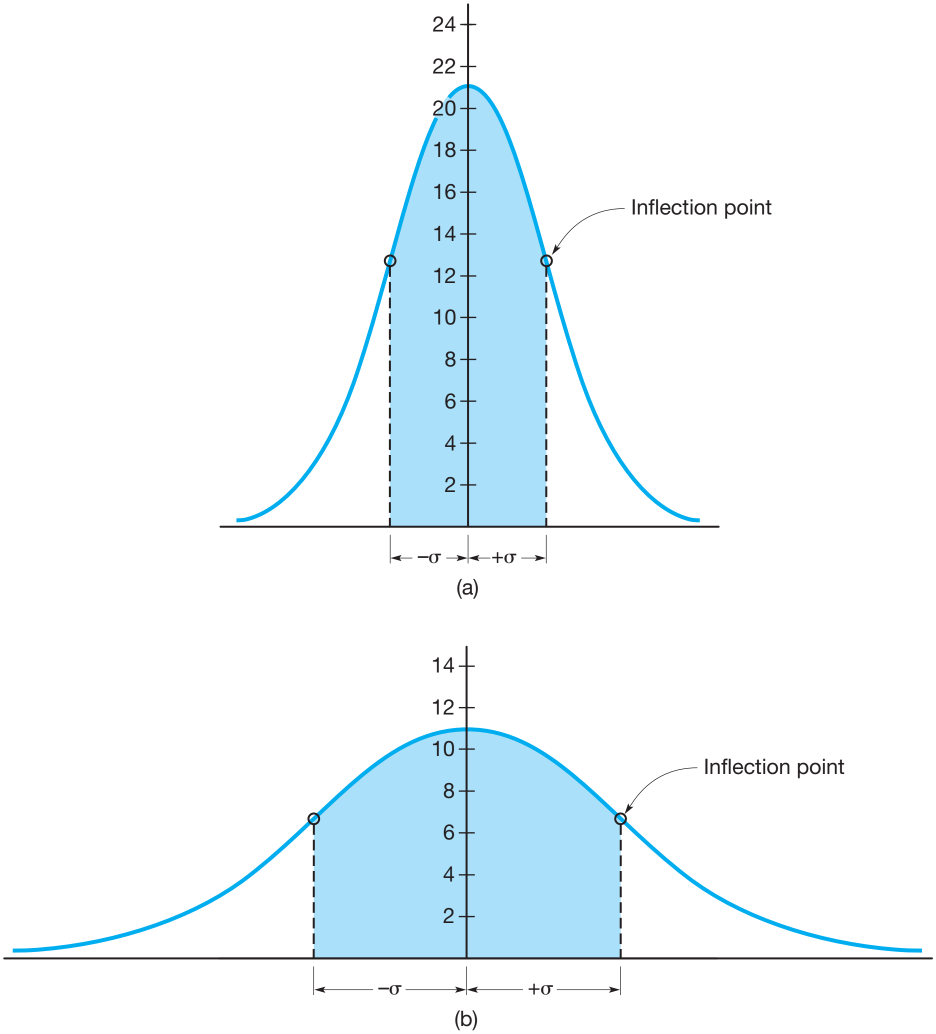

Figure 3.4
Normal distribution curves for:
(a) increased precision,
(b) decreased precision.

of observations are *standard deviation* and *variance*. The equation for the standard deviation is

$$\sigma = \pm\sqrt{\frac{\Sigma\nu^2}{n - 1}} \tag{3.4}$$

where σ is the standard deviation of a group of observations of the same quantity, ν the residual of an individual observation, $\Sigma\nu^2$ the sum of squares of the individual residuals, and n the number of observations. *Variance* is equal to σ^2, the square of the standard deviation.

Note that in Equation (3.4), the standard deviation has both plus and minus values. On the normal distribution curve, the numerical value of the standard deviation is the abscissa at the inflection points (locations where the curvature changes from concave downward to concave upward). In Figures 3.3 and 3.4, these inflection points are shown. Note the closer spacing between them for the more precise observations of Figure 3.4(a) as compared to Figure 3.4(b).

Figure 3.5 is a graph showing the percentage of the total area under a normal distribution curve that exists between ranges of residuals (errors) having equal positive and negative values. The abscissa scale is shown in multiples of the standard deviation. From this curve, the area between residuals of $+\sigma$ and $-\sigma$ equals approximately 68.3% of the total area under the normal distribution curve. Hence, it gives the range of residuals that can be expected to occur 68.3% of the time. This relation is shown more clearly on the curves in Figures 3.3 and 3.4, where the areas between $\pm\sigma$ are shown shaded. The percentages shown in Figure 3.5 apply to all normal distributions; regardless of curve shape or the numerical value of the standard deviation.

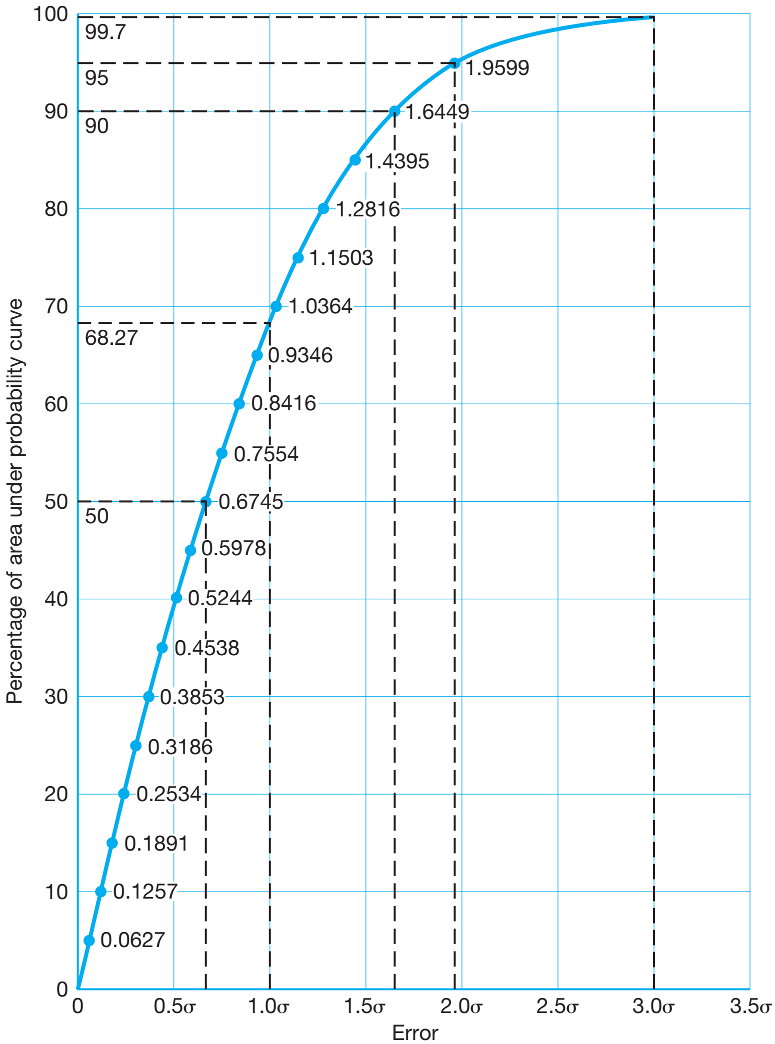

Figure 3.5 Relation between error and percentage of area under normal distribution curve.

3.15 INTERPRETATION OF STANDARD DEVIATION

It has been shown that the standard deviation establishes the limits within which observations are expected to fall 68.3% of the time. In other words, if an observation is repeated ten times, it will be expected that about seven of the results will fall within the limits established by the standard deviation, and conversely about three of them will fall anywhere outside these limits. Another interpretation is that one additional observation will have a 68.3% chance of falling within the limits set by the standard deviation.

When Equation (3.4) is applied to the data of Table 3.1, a standard deviation of ± 2.19 is obtained. In examining the residuals in the table, 70 of the 100 values, or 70%, are actually smaller than 2.19 sec. This illustrates that the theory of probability closely approximates reality.

3.16 THE 50, 90, AND 95 PERCENT ERRORS

From the data given in Figure 3.5, the probability of an error of any percentage likelihood can be determined. The general equation is

$$E_P = C_P \sigma \tag{3.5}$$

where E_P, is a certain percentage error and C_P, the corresponding numerical factor taken from Figure 3.5.

By Equation (3.5), after extracting appropriate multipliers from Figure 3.5, the following are expressions for errors that have a 50%, 90%, and 95% chance of occurring:

$$E_{50} = 0.6745\sigma \tag{3.6}$$

$$E_{90} = 1.6449\sigma \tag{3.7}$$

$$E_{95} = 1.9599\sigma \tag{3.8}$$

The 50 percent error, or E_{50}, is the so-called *probable error*. It establishes limits within which the observations should fall 50% of the time. In other words, an observation has the same chance of coming within these limits as it has of falling outside of them.

The 90 and 95 percent errors are commonly used to specify precisions required on surveying (geomatics) projects. Of these, the 95 percent error, also frequently called the *two-sigma* (2σ) *error*, is most often specified. As an example, a particular project may call for the 95 percent error to be less than or equal to a certain value for the work to be acceptable. For the data of Table 3.1, applying Equations (3.7) and (3.8), the 90 and 95 percent errors are ± 3.60 and ± 4.29 sec, respectively. These errors are shown graphically in Figure 3.3.

The so-called *three-sigma* (3σ) *error* is also often used as a criterion for rejecting individual observations from sets of data. From Figure 3.5, there is a 99.7% probability that an error will be less than this amount. Thus, within a group of observations, any value whose residual exceeds 3σ is considered to be a mistake, and either a new observation must be taken or the computations based on one less value.

The x-axis is an asymptote of the normal distribution curve, so the 100 percent error cannot be evaluated. This means that no matter what size error is found, a larger one is theoretically possible.

Example 3.1

To clarify definitions and use the equations given in Sections 3.10 through 3.16, suppose that a line has been observed 10 times using the same equipment and procedures. The results are shown in column (1) of the following table. It is assumed that no mistakes exist, and that the observations have already been corrected for all systematic errors. Compute the most probable value for the line length, its standard deviation, and errors having 50%, 90%, and 95% probability.

Length (ft)(1)	**Residual ν (ft)(2)**	**ν^2 (3)**
538.57	+0.12	0.0144
538.39	−0.06	0.0036
538.37	−0.08	0.0064
538.39	−0.06	0.0036
538.48	+0.03	0.0009
538.49	+0.04	0.0016
538.33	−0.12	0.0144
538.46	+0.01	0.0001
538.47	+0.02	0.0004
538.55	+0.10	0.0100
$\Sigma = 5384.50$	$\Sigma = 0.00$	$\Sigma\nu^2 = 0.0554$

Solution

By Equation (3.2), $\overline{M} = \dfrac{5384.50}{10} = 538.45$ ft

By Equation (3.3), the residuals are calculated. These are tabulated in column (2) and their squares listed in column (3). Note that in column (2) the algebraic sum of residuals is zero. (For observations of equal reliability, except for round off, this column should always total zero and thus provide a computational check.)

By Equation (3.4), $\sigma = \pm\sqrt{\dfrac{\Sigma\nu^2}{n-1}} = \sqrt{\dfrac{0.0554}{9}} = \pm 0.078 = \pm 0.08$ ft.

By Equation (3.6), $E_{50} = \pm 0.6745\sigma = \pm 0.6745(0.078) = \pm 0.05$ ft.

By Equation (3.7), $E_{95} = \pm 1.6449(0.078) = \pm 0.13$ ft.

By Equation (3.8), $E_{99} = \pm 1.9599(0.078) = \pm 0.15$ ft.

The following conclusions can be drawn concerning this example.

1. The most probable line length is 538.45 ft.
2. The standard deviation of a single observation is ± 0.08 ft. Accordingly, the normal expectation is that 68% of the time a recorded length will lie between $538.45 - 0.08$ and $538.45 + 0.08$ or between 538.37 and 538.53 ft; that is, about seven values should lie within these limits. (Actually seven of them do.)
3. The probable error (E_{50}) is ± 0.05 ft. Therefore, it can be anticipated that half, or five of the observations, will fall in the interval 538.40 to 538.50 ft. (Four values do.)
4. The 90% error is ± 0.13 ft, and thus nine of the observed values can be expected to be within the range of 538.32 and 538.58 ft.
5. The 95% error is ± 0.15 ft, so the length can be expected to lie between 538.30 and 538.60, 95% of the time. (Note that all observations indeed are within the limits of both the 90 and 95 percent errors.)

3.17 ERROR PROPAGATION

It was stated earlier that because all observations contain errors, any quantities computed from them will likewise contain errors. The process of evaluating errors in quantities computed from observed values that contain errors is called *error propagation*. The propagation of random errors in mathematical formulas can be computed using the general law of the propagation of variances. Typically in surveying (geomatics), this formula can be simplified since the observations are usually mathematically independent. For example, let $a, b, c, \ldots, n$ be observed values containing errors $E_a, E_b, E_c, \ldots, E_n$, respectively. Also let Z be a quantity derived by computation using these observed quantities in a function f, such that

$$Z = f(a, b, c, \ldots, n) \tag{3.9}$$

Then assuming that $a, b, c, \ldots, n$ are independent observations, the error in the computed quantity Z is

$$E_Z = \pm\sqrt{\left(\frac{\partial f}{\partial a}E_a\right)^2 + \left(\frac{\partial f}{\partial b}E_b\right)^2 + \left(\frac{\partial f}{\partial c}E_c\right)^2 + \cdots + \left(\frac{\partial f}{\partial n}E_n\right)^2} \tag{3.10}$$

where the terms $\partial f/\partial a, \partial f/\partial b, \partial f/\partial c, \ldots, \partial f/\partial n$ are the partial derivatives of the function f with respect to the variables $a, b, c, \ldots, n$. In the subsections that follow, specific cases of error propagation common in surveying are discussed, and examples are presented.

3.17.1 Error of a Sum

Assume the sum of independently observed observations $a, b, c, \ldots$ is Z. The formula for the computed quantity Z is

$$Z = a + b + c + \cdots$$

The partial derivatives of Z with respect to each observed quantity are $\partial Z/\partial a = \partial Z/\partial b = \partial Z/\partial c = \cdots = 1$. Substituting these partial derivatives into Equation (3.10), the following formula is obtained, which gives the propagated error in the sum of quantities, each of which contains a different random error:

$$E_{Sum} = \pm\sqrt{E_a^2 + E_b^2 + E_c^2 + \cdots} \tag{3.11}$$

where E represents any specified percentage error (such as σ, E_{50}, E_{90}, or E_{95}), and *a, b*, and *c* are the separate, independent observations.

The error of a sum can be used to explain the rules for addition and subtraction using significant figures. Recall the addition of 46.7418, 1.03, and 375.0 from Example (**a**) from Section 2.4. Significant figures indicate that there is uncertainty in the last digit of each number. Thus, assume estimated errors of ±0.0001, ±0.01, and ±0.1 respectively for each number. The error in the sum of these three numbers is $\sqrt{0.0001^2 + 0.01^2 + 0.1^2} = \pm 0.1$. The sum of three numbers is 422.7718, which was rounded, using the rules of significant figures, to 422.8. Its precision matches the estimated accuracy produced by the error in the sum of the three numbers. Note how the least accurate number controls the accuracy in the summation of the three values.

Example 3.2

Assume that a line is observed in three sections, with the individual parts equal to (753.81, ±0.012), (1238.40, ±0.028), and (1062.95, ±0.020) ft, respectively. Determine the line's total length and its anticipated standard deviation.

Solution

Total length $= 753.81 + 1238.40 + 1062.95 = 3055.16$ ft.

By Equation (3.11), $E_{Sum} = \pm\sqrt{0.012^2 + 0.028^2 + 0.020^2} = \pm 0.036$ ft

3.17.2 Error of a Series

Sometimes a series of similar quantities, such as the angles within a closed polygon, are read with each observation being in error by about the same amount. The total error in the sum of all observed quantities of such a series is called the *error of the series*, designated as E_{Series}. If the same error E in each observation is assumed and Equation (3.11) applied, the series error is

$$E_{Series} = \pm\sqrt{E^2 + E^2 + E^2 + \cdots} = \pm\sqrt{nE^2} = \pm E\sqrt{n} \tag{3.12}$$

where E represents the error in each individual observation and n the number of observations.

This equation shows that when the same operation is repeated, random errors tend to balance out and the resulting error of a series is proportional to the square root of the number of observations. This equation has extensive use—for instance, to determine the allowable misclosure error for angles of a traverse, as discussed in Chapter 9.

■ Example 3.3

Assume that any distance of 100 ft can be taped with an error of ±0.02 ft if certain techniques are employed. Determine the error in taping 5000 ft using these skills.

Solution

Since the number of 100 ft lengths in 5000 ft is 50 then by Equation (3.12)

$$E_{Series} = \pm E\sqrt{n} = \pm 0.02\sqrt{50} = \pm 0.14 \text{ ft}$$

■ Example 3.4

A distance of 1000 ft is to be taped with an error of not more than ±0.10 ft. Determine how accurately each 100 ft length must be observed to ensure that the error will not exceed the permissible limit.

Solution

Since by Equation (3.12), $E_{Series} = \pm E\sqrt{n}$ and $n = 10$, the allowable error E in 100 ft is

$$E = \pm\frac{E_{Series}}{\sqrt{n}} \pm \frac{0.10}{\sqrt{10}} = \pm 0.03 \text{ ft}$$

■ Example 3.5

Suppose it is required to tape a length of 2500 ft with an error of not more than ±0.10 ft. How accurately must each tape length be observed?

Solution

Since 100 ft is again considered the unit length, $n = 25$, and by Equation (3.12), the allowable error E in 100 ft is

$$E = \pm\frac{0.10}{\sqrt{25}} = \pm 0.02 \text{ ft}$$

Analyzing Examples 3.4 and 3.5 shows that the larger the number of possibilities, the greater the chance for errors to cancel out.

3.17.3 Error of a Product

The equation for propagated AB, where E_a and E_b are the respective errors in A and B, is

$$E_{prod} = \pm\sqrt{A^2E_b^2 + B^2E_a^2} \tag{3.13}$$

The physical significance of the error propagation formula for a product is illustrated in Figure 3.6, where A and B are shown to be observed sides of a rectangular parcel of land with errors E_a and E_b respectively. The product AB is the parcel area. In Equation (3.13), $\sqrt{A^2E_b^2} = AE_b$ represents either of the longer (horizontal) crosshatched bars and is the error caused by either $-E_b$ or $+E_b$. The term $\sqrt{B^2E_a^2} = BE_a$ is represented by the shorter (vertical) crosshatched bars, which is the error resulting from either $-E_a$ or $+E_a$.

Example 3.6

For the rectangular lot illustrated in Figure 3.6, observations of sides A and B with their 95% errors are (252.46, ±0.053) and (605.08, ±0.072) ft, respectively. Calculate the parcel area and the expected 95% error in the area.

Solution

Area $= 252.46 \times 605.08 = 152{,}760$ ft^2

By Equation (3.13),

$$E_{95} = \pm\sqrt{(252.46)^2(0.072)^2 + (605.08)^2(0.053)^2} = \pm 36.9 \text{ ft}^2$$

Example 3.6 can also be used to demonstrate the validity of one of the rules of significant figures in computation. The computed area is actually 152,758.4968 ft^2. However, the rule for significant figures in multiplication (see Section 2.4) states that there cannot be more significant figures in the answer

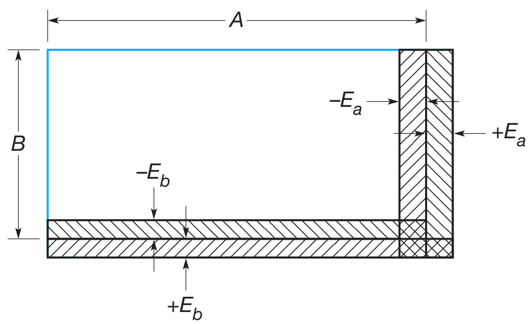

Figure 3.6
Error of area.

than in any of the individual factors used. Accordingly, the area should be rounded off to 152,760 (five significant figures). From Equation (3.13), with an error of ± 36.9 ft^2, the answer could be 152,758.4968 $\pm$ 36.9, or from 152,721.6 to 152,795.4 ft^2. Thus, the fifth digit in the answer is seen to be questionable, and hence the number of significant figures specified by the rule is verified.

3.17.4 Error of the Mean

Equation (3.2) stated that the most probable value of a group of repeated observations of equal weight is the arithmetic mean. Since the mean is computed from individual observed values, each of which contains an error, the mean is also subject to error. By applying Equation (3.12), it is possible to find the error for the sum of a series of observations where each one has the same error. Since the sum divided by the number of observations gives the mean, the error of the mean is found by the relation

$$E_m = \frac{E_{series}}{n}$$

Substituting Equation (3.12) for E_{series}

$$E_m = \frac{E\sqrt{n}}{n} = \frac{E}{\sqrt{n}} \tag{3.14}$$

where E is the specified percentage error of a single observation, E_m the corresponding percentage error of the mean, and n the number of observations.

The error of the mean at any percentage probability can be determined and applied to all criteria that have been developed. For example, the standard deviation of the mean, $(E_{68})_m$ or σ_m is

$$(E_{68})_m = \sigma_m = \frac{\sigma}{\sqrt{n}} = \pm\sqrt{\frac{\Sigma \nu^2}{n(n-1)}} \tag{3.15a}$$

and the 90 and 95 percent errors of the mean are

$$(E_{90})_m = \frac{E_{90}}{\sqrt{n}} = \pm 1.6449\sqrt{\frac{\Sigma \nu^2}{n(n-1)}} \tag{3.15b}$$

$$(E_{95})_m = \frac{E_{95}}{\sqrt{n}} = \pm 1.9599\sqrt{\frac{\Sigma \nu^2}{n(n-1)}}. \tag{3.15c}$$

These equations show that *the error of the mean varies inversely as the square root of the number of repetitions*. Thus, to double the accuracy—that is, to reduce the error by one half—four times as many observations must be made.

■ Example 3.7

Calculate the standard deviation of the mean and the 90% error of the mean for the observations of Example 3.1.

Solution

By Equation (3.15a), $\sigma_m = \dfrac{\sigma}{\sqrt{n}} = \pm\dfrac{0.078}{\sqrt{10}} = \pm 0.025$ ft

Also, by Equation (3.15b), $(E_{90})_m = \pm 1.6449(0.025) = \pm 0.041$ ft

These values show the error limits of 68% and 90% probability for the line's length. It can be said that the true line length has a 68% chance of being within ±0.025 of the mean, and a 90% likelihood of falling not farther than ±0.041 ft from the mean.

■ 3.18 APPLICATIONS

The preceding example problems show that the equations of error probability are applied in two ways:

1. To analyze observations already made, for comparison with other results or with specification requirements.
2. To establish procedures and specifications in order that the required results will be obtained.

The application of the various error probability equations must be tempered with judgment and caution. Recall that they are based on the assumption that the errors conform to a smooth and continuous normal distribution curve, which in turn is based on the assumption of a large number of observations. Frequently in surveying only a few observations—often from two to eight—are taken. If these conform to a normal distribution, then the answer obtained using probability equations will be reliable; if they do not, the conclusions could be misleading. In the absence of knowledge to the contrary, however, an assumption that the errors are normally distributed is still the best available.

■ 3.19 CONDITIONAL ADJUSTMENT OF OBSERVATIONS

In Section 3.3, it was emphasized that the true value of any observed quantity is never known. However, in some types of problems, the sum of several observations must equal a fixed value; for example, the sum of the three angles in a plane triangle has to total 180°. In practice, therefore, the observed angles are adjusted to make them add to the required amount. Correspondingly, distances—either horizontal or vertical—must often be adjusted to meet certain conditional requirements. The methods used will be explained in later chapters, where the operations are taken up in detail.

3.20 WEIGHTS OF OBSERVATIONS

It is evident that some observations are more precise than others because of better equipment, improved techniques, and superior field conditions. In making adjustments, it is consequently desirable to assign *relative weights* to individual observations. It can logically be concluded that if an observation is very precise, it will have a small standard deviation or variance, and thus should be weighted more heavily (held closer to its observed value) in an adjustment than an observation of lower precision. From this reasoning, it is deduced that weights of observations should bear an inverse relationship to precision. In fact, it can be shown that relative weights are inversely proportional to variances, or

$$W_a \propto \frac{1}{\sigma_a^2} \tag{3.16}$$

where W_a is the weight of an observation a, which has a variance of σ_a^2. Thus, the higher the precision (the smaller the variance), the larger should be the relative weight of the observed value being adjusted. In some cases, variances are unknown originally, and weights must be assigned to observed values based on estimates of their relative precision. If a quantity is observed repeatedly and the individual observations have varying weights, the weighted mean can be computed from the expression

$$\overline{M}_W = \frac{\Sigma WM}{\Sigma W} \tag{3.17}$$

where $\overline{M}_W$ is the weighted mean, ΣWM the sum of the individual weights times their corresponding observations, and ΣW the sum of the weights.

Example 3.8

Suppose four observations of a distance are recorded as 482.16, 482.17, 482.20, and 482.18 and given weights of 1, 2, 2, and 4, respectively, by the surveyor. Determine the weighted mean.

Solution

By Equation (3.17)

$$\overline{M}_W = \frac{482.16 + 482.17(2) + 482.20(2) + 482.14(4)}{1 + 2 + 2 + 4} = 482.16 \text{ ft}$$

In computing adjustments involving unequally weighted observations, corrections applied to observed values should be made *inversely proportional to the relative weights*.

Example 3.9

Assume the observed angles of a certain plane triangle, and their relative weights, are $A = 49°51'15''$, $W_a = 1$; $B = 60°32'08''$, $W_b = 2$; and $C = 69°36'33''$, $W_c = 3$. Compute the weighted mean of the angles.

Solution

The sum of the three angles is computed first and found to be 4″ less than the required geometrical condition of exactly 180°. The angles are therefore adjusted in inverse proportion to their relative weights, as illustrated in the accompanying tabulation. Angle C with the greatest weight (3) gets the smallest correction, $2x$; B receives $3x$; and A, $6x$.

	Observed Angle	Wt	Correction	Numerical Corr.	Rounded Corr.	Adjusted Angle
A	49°51′15″	1	$6x$	+2.18″	+2″	49°51′17″
B	60°32′08″	2	$3x$	+1.09″	+1″	60°32′09″
C	69°36′33″	3	$2x$	+0.73″	+1″	69°36′34″
Sum	179°59′56″	$\Sigma = 6$	$11x$	+4.00″	+4″	180°00′00″

$11x = 4''$ and $x = +0.36''$

It must be emphasized again that adjustment computations based on the theory of probability are valid only if systematic errors and employing proper procedures, equipment, and calculations eliminates mistakes.

3.21 LEAST-SQUARES ADJUSTMENT

As explained in Section 3.19, most surveying observations must conform to certain geometrical conditions. The amounts by which they fail to meet these conditions are called misclosures, and they indicate the presence of random errors. In Example 3.9, for example, the misclosure was 4″. Various procedures are used to distribute these misclosure errors to produce mathematically perfect geometrical conditions. Some simply apply corrections of the same size to all observed values, where each correction equals the total misclosure (with its algebraic sign changed), divided by the number of observations. Others introduce corrections in proportion to assigned weights. Still others employ rules of thumb, for example, the "compass rule" described in Chapter 10 for adjusting closed traverses.

Because random errors in surveying conform to the mathematical laws of probability and are "normally distributed," the most appropriate adjustment procedure should be based upon these laws. Least squares is such a method. It is not a new procedure, having been applied by the German mathematician Karl Gauss as early as the latter part of the 18th century. However, until the advent of computers, it was only used sparingly because of the lengthy calculations involved.

Least squares is suitable for adjusting any of the basic types of surveying observations described in Section 2.1, and is applicable to all of the commonly used surveying procedures. The method enforces the condition that *the sum of the weights of the observations times their corresponding squared residuals is minimized*. This fundamental condition, which is developed from the equation for the normal error distribution curve, provides most probable values for the adjusted quantities. In addition, it also (a) enables the computation of precisions of the adjusted values, (b) reveals the presence of mistakes so steps can be taken to eliminate them, and (c) makes possible the optimum design of survey procedures in the office before going to the field to take observations.

The basic assumptions that underlie least-squares theory are as follows: (1) mistakes and systematic errors have been eliminated so only random errors remain; (2) the number of observations being adjusted is large; and (3) the frequency distribution of errors is normal. Although these assumptions are not always met, the least-squares adjustment method still provides the most rigorous error treatment available, and hence it has become very popular and important in modern surveying. A more detailed discussion of the subject is presented in Chapter 16.

3.22 USING SOFTWARE

Computations such as those in Table 3.1 can be long and tedious. Fortunately, spreadsheet software often has the capability of computing the mean and standard deviation of a group of observations. For example, in Microsoft Excel®, the mean of a set of observations can be determined using the average() function and the standard deviation can be determined using the stdev() function. Similarly, histograms of data can also be plotted once the data is organized into classes. The reader can download all of the Excel files for this book by downloading the file *Excel Spreadsheets.zip* from the companion website for this book at http://www.pearsonhighered.com/ghilani. The spreadsheet *c3.xls* demonstrates the use of the functions mentioned previously and also demonstrates the use of a spreadsheet to solve the example problems in this chapter. Also on the companion website for this book is the software STATS. This software can read a text file of data and compute the statistics demonstrated in this chapter. Furthermore, STATS will histogram the data using a user-specified number of class intervals. The help file that accompanies this software describes the file format for the data and the use of the software. For those having the software Mathcad® version 14.0 or higher, an accompanying e-book is available on the companion website. This e-book is in the file *Mathcad files.zip* on the companion website. If this book is decompressed in the Mathcad subdirectory *handbook*, the e-book will be available in the Mathcad help system. This e-book can also be accessed by selecting the file *elemsurv.hbk* in your Windows directory and has a worksheet that demonstrates the examples presented in this chapter. For those who do have Mathcad version 14.0 or higher, a set of hypertext markup language (html) files of the e-book are available on the companion website. These files can be accessed by opening the file *index.html* in your browser.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

3.1 Explain the difference between *direct* and *indirect measurements* in surveying. Give two examples of each.

3.2 Define the term *systematic error*, and give two surveying examples of a systematic error.

3.3 Define the term *random error*, and give two surveying examples of a random error.

3.4 Explain the difference between accuracy and precision.

3.5 Discuss what is meant by the precision of an observation.

A distance AB is observed repeatedly using the same equipment and procedures, and the results, in meters, are listed in Problems 3.6 through 3.10. Calculate **(a)** the line's most probable length, **(b)** the standard deviation, and **(c)** the standard deviation of the mean for each set of results.

3.6* 65.401, 65.400, 65.402, 65.396, 65.406, 65.401, 65.396, 65.401, 65.405, and 65.404

3.7 Same as Problem 3.6, but discard one observation, 65.396.

3.8 Same as Problem 3.6, but discard two observations, 65.396 and 65.406.

3.9 Same as Problem 3.6, but include two additional observations, 65.398 and 65.408.

3.10 Same as Problem 3.6, but include three additional observations, 65.398, 65.408, and 65.406.

In Problems 3.11 through 3.14, determine the range within which observations should fall **(a)** 90% of the time and **(b)** 95% of the time. List the percentage of values that actually fall within these ranges.

3.11* For the data of Problem 3.6.

3.12 For the data of Problem 3.7.

3.13 For the data of Problem 3.8.

3.14 For the data of Problem 3.9.

In Problems 3.15 through 3.17, an angle is observed repeatedly using the same equipment and procedures. Calculate **(a)** the angle's most probable value, **(b)** the standard deviation, and **(c)** the standard deviation of the mean.

3.15* $23°30'00''$, $23°29'40''$, $23°30'15''$, and $23°29'50''$.

3.16 Same as Problem 3.15, but with three additional observations, $23°29'55''$, $23°30'05''$, and $23°30'20''$.

3.17 Same as Problem 3.16, but with two additional observations, $23°30'05''$ and $23°29'55''$.

3.18* A field party is capable of making taping observations with a standard deviation of ± 0.010 ft per 100 ft tape length. What standard deviation would be expected in a distance of 200 ft taped by this party?

3.19 Repeat Problem 3.18, except that the standard deviation per 30-m tape length is ± 0.003 m and a distance of 120 m is taped. What is the expected 95% error in 120 m?

3.20 A distance of 200 ft must be taped in a manner to ensure a standard deviation smaller than ± 0.04 ft. What must be the standard deviation per 100 ft tape length to achieve the desired precision?

3.21 Lines of levels were run requiring n instrument setups. If the rod reading for each backsight and foresight has a standard deviation σ, what is the standard deviation in each of the following level lines?

(a) $n = 26, \sigma = \pm 0.010$ ft

(b) $n = 36, \sigma = \pm 3$ mm

3.22 A line AC was observed in 2 sections AB and BC, with lengths and standard deviations listed below. What is the total length AC, and its standard deviation?

*__(a)__ $AB = 60.00 \pm 0.015$ ft; $BC = 86.13 \pm 0.018$ ft

(b) $AB = 60.000 \pm 0.008$ m; 35.413 ± 0.005 m

3.23 Line AD is observed in three sections, AB, BC, and CD, with lengths and standard deviations as listed below. What is the total length AD and its standard deviation?

(a) $AB = 572.12 \pm 0.02$ ft; $BC = 1074.38 \pm 0.03$ ft; $CD = 1542.78 \pm 0.05$ ft

(b) $AB = 932.965 \pm 0.009$ m; $BC = 945.030$ m ± 0.010 m; $CD = 652.250$ m ± 0.008 m

3.24 A distance AB was observed four times as 236.39, 236.40, 236.36, and 236.38 ft. The observations were given weights of 2, 1, 3, and 2, respectively, by the observer. *__(a)__ Calculate the weighted mean for distance AB. **(b)** What difference results if later judgment revises the weights to 2, 1, 2, and 3, respectively?

3.25 Determine the weighted mean for the following angles:

(a) 89°42′45″, wt 2; 89°42′42″, wt 1; 89°42′44″, wt 3

(b) 36°58′32″ ± 3″; 36°58′28″ ± 2″; 36°58′26″ ± 3″; 36°58′30″ ± 1″

3.26 Specifications for observing angles of an n-sided polygon limit the total angular misclosure to E. How accurately must each angle be observed for the following values of n and E?

(a) $n = 10, E = 8''$

(b) $n = 6, E = 14''$

3.27 What is the area of a rectangular field and its estimated error for the following recorded values:

*__(a)__ 243.89 ± 0.05 ft, by 208.65 ± 0.04 ft

(b) 725.33 ± 0.08 ft by 664.21 ± 0.06 ft

(c) 128.526 ± 0.005 m, by 180.403 ± 0.007 m

3.28 Adjust the angles of triangle ABC for the following angular values and weights:

*__(a)__ $A = 49°24'22''$, wt 2; $B = 39°02'16''$, wt 1; $C = 91°33'00''$, wt 3

(b) $A = 80°14'04''$, wt 2; $B = 38°37'47''$, wt 1; $C = 61°07'58''$, wt 3

3.29 Determine relative weights and perform a weighted adjustment (to the nearest second) for angles A, B, and C of a plane triangle, given the following four observations for each angle:

Angle *A*	Angle *B*	Angle *C*
38°47′58″	71°22′26″	69°50′04″
38°47′44″	71°22′22″	69°50′16″
38°48′12″	71°22′12″	69°50′30″
38°48′02″	71°22′12″	69°50′10″

3.30 A line of levels was run from benchmarks A to B, B to C, and C to D. The elevation differences obtained between benchmarks, with their standard deviations, are listed below. What is the difference in elevation from benchmark A to D and the standard deviation of that elevation difference?

(a) BM A to BM $B = +34.65 \pm 0.10$ ft; BM B to BM $C = -48.23 \pm 0.08$ ft; and BM C to BM $D = -54.90 \pm 0.09$ ft

(b) BM A to BM $B = +27.823 \pm 0.015$ m; BM B to BM $C = +15.620 \pm 0.008$ m; and BM C to BM $D = +33.210 \pm 0.011$ m

(c) BM A to BM $B = -32.688 \pm 0.015$ m; BM B to BM $C = +5.349 \pm 0.022$ m; and BM C to BM $D = -15.608 \pm 0.006$ m

3.31 Create a computational program that solves Problem 3.9.

3.32 Create a computational program that solves Problem 3.17.

3.33 Create a computational program that solves Problem 3.29.

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4 Leveling—Theory, Methods, and Equipment

PART I • LEVELING—THEORY AND METHODS

4.1 INTRODUCTION

Leveling is the general term applied to any of the various processes by which elevations of points or differences in elevation are determined. It is a vital operation in producing necessary data for mapping, engineering design, and construction. Leveling results are used to (1) design highways, railroads, canals, sewers, water supply systems, and other facilities having grade lines that best conform to existing topography; (2) lay out construction projects according to planned elevations; (3) calculate volumes of earthwork and other materials; (4) investigate drainage characteristics of an area; (5) develop maps showing general ground configurations; and (6) study earth subsidence and crustal motion.

4.2 DEFINITIONS

Basic terms in leveling are defined in this section, some of which are illustrated in Figure 4.1.

Vertical line. A line that follows the local direction of gravity as indicated by a plumb line.

Level surface. A curved surface that at every point is perpendicular to the local plumb line (the direction in which gravity acts). Level surfaces are approximately spheroidal in shape. A body of still water is the closest example of a level surface. Within local areas, level surfaces at different

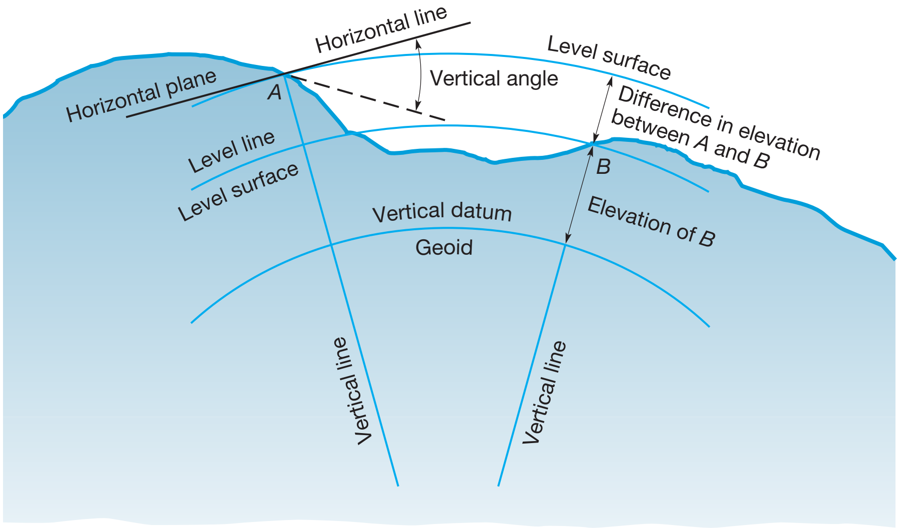


Figure 4.1
Leveling terms.

heights are considered to be concentric.[1] Level surfaces are also known as *equipotential surfaces* since, for a particular surface, the potential of gravity is equal at every point on the surface.

Level line. A line in a level surface—therefore, a curved line.

Horizontal plane. A plane perpendicular to the local direction of gravity. In plane surveying, it is a plane perpendicular to the local vertical line.

Horizontal line. A line in a horizontal plane. In plane surveying, it is a line perpendicular to the local vertical.

Vertical datum. Any level surface to which elevations are referenced. This is the surface that is arbitrarily assigned an elevation of zero (see Section 19.6). This level surface is also known as a reference datum since points using this datum have heights relative to this surface.

Elevation. The distance measured along a vertical line from a vertical datum to a point or object. If the elevation of point *A* is 802.46 ft, *A* is 802.46 ft above the reference datum. The elevation of a point is also called its height above the datum.

Geoid. A particular level surface that serves as a datum for all elevations and astronomical observations.

Mean sea level (*MSL*). The average height for the surface of the seas for all stages of tide over a 19-year period as defined by the National Geodetic Vertical Datum of 1929, further described in Section 4.3. It was derived from readings, usually taken at hourly intervals, at 26 gaging stations along the Atlantic and Pacific oceans and the Gulf of Mexico. The elevation of the sea differs from station to station depending on local influences of the tide; for example, at two points 0.5 mi apart on opposite sides of an island in the Florida Keys, it varies by 0.3 ft. Mean sea level

[1]Due to flattening of the earth in the polar direction, level surfaces at different elevations and different latitudes are not truly concentric. This topic is discussed in more detail in Chapter 19.

was accepted as the vertical datum for North America for many years. However, the current vertical datum uses a single benchmark as a reference (see Section 4.3).

Tidal datum. The vertical datum used in coastal areas for establishing property boundaries of lands bordering waters subject to tides. A tidal datum also provides the basis for locating fishing and oil drilling rights in tidal waters, and the limits of swamp and overflowed lands. Various definitions have been used in different areas for a tidal datum, but the one most commonly employed is the *mean high water* (MHW) line. Others applied include *mean higher high water* (MHHW), *mean low water* (MLW), and *mean lower low water* (MLLW). Interpretations of a tidal datum, and the methods by which they are determined, have been, and continue to be, the subject of numerous court cases.

Benchmark (*BM*). A relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known or assumed. Common examples are metal disks set in concrete (see Figure 20.8), reference marks chiseled on large rocks, nonmovable parts of fire hydrants, curbs, etc.

Leveling. The process of finding elevations of points or their differences in elevation.

Vertical control. A series of benchmarks or other points of known elevation established throughout an area, also termed *basic control* or *level control.* The basic vertical control for the United States was derived from first- and second-order leveling. Less precise third-order leveling has been used to fill gaps between second-order benchmarks, as well as for many other specific projects (see Section 19.10). Elevations of benchmarks, which are part of the National Spatial Reference System, can be obtained online from the National Geodetic Survey at http://www.ngs.noaa.gov. The data sheets for vertical control give the (1) approximate geodetic coordinates for the station, (2) adjusted NAVD88 elevation, (3) observed or modeled gravity reading at the station, and (4) a description of the station and its location among other things. Software plugins for an Internet browser exists which will plot these points in Google Earth to aid in the location of the monuments in the field.

■ 4.3 NORTH AMERICAN VERTICAL DATUM

Precise leveling operations to establish a distributed system of reference benchmarks throughout the United States began in the 1850s. This work was initially concentrated along the eastern seaboard, but in 1887 the U.S. Coast and Geodetic Survey (USC&GS) began its first transcontinental leveling across the country's midsection. That project was completed in the early 1900s. By 1929, thousands of benchmarks had been set. In that year, the USC&GS began a general least-squares adjustment of all leveling completed in the United States and Canada. The adjustment involved over 100,000 km of leveling and incorporated long-term data from the 26 tidal gaging stations; hence, it was related to mean sea level. In fact, that network of benchmarks with their resulting adjusted elevations defined the mean sea level datum. It was called the *National Geodetic Vertical Datum of 1929* (*NGVD29*).

Through the years after 1929, the NGVD29 deteriorated somewhat due to various causes including changes in sea level and shifting of the Earth's crust. Also, more than 625,000 km of additional leveling was completed. To account for these changes and incorporate the additional leveling, the National Geodetic Survey (NGS) performed a new general readjustment. Work on this adjustment, which included more than 1.3 million observed elevation differences, began in 1978. Although not finished until 1991, its planned completion date was 1988, and thus it has been named the *North American Vertical Datum of 1988* (*NAVD88*). Besides the United States and Canada, Mexico was also included in this general readjustment. This adjustment shifted the position of the reference surface from the mean of the 26 tidal gage stations to a single tidal gage benchmark known as *Father Point*, which is in Rimouski, Quebec, Canada, near the mouth of the St. Lawrence Seaway. Thus, elevations in NAVD88 are no longer referenced to mean sea level. Benchmark elevations that were defined by the NGVD29 datum have changed by relatively small, but nevertheless significant amounts in the eastern half of the continental United States (see Figure 20.7). However, the changes are much greater in the western part of the country and reach 1.5 m in the Rocky Mountain region. It is therefore imperative that surveyors positively identify the datum to which their elevations are referred. Listings of the new elevations are available from the NGS.[2]

4.4 CURVATURE AND REFRACTION

From the definitions of a level surface and a horizontal line, it is evident that the horizontal plane departs from a level surface because of curvature of the Earth. In Figure 4.2, the deviation *DB* from a horizontal line through point *A* is expressed approximately by the formulas

$$C_f = 0.667M^2 = 0.0239F^2 \tag{4.1a}$$

or

$$C_m = 0.0785K^2 \tag{4.1b}$$

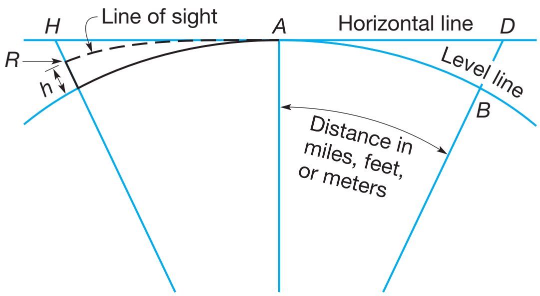


Figure 4.2 Curvature and refraction.

[2]Descriptions and NAVD88 elevations of benchmarks can be obtained from the National Geodetic Information Center at their website address http://www.ngs.noaa.gov/datasheet.html. Information can also be obtained by email at info_center@ngs.noaa.gov, or by writing to the National Geodetic Information Center, NOAA, National Geodetic Survey, 1315 East West Highway, Silver Spring, MD 20910; telephone: (301) 713-3242.

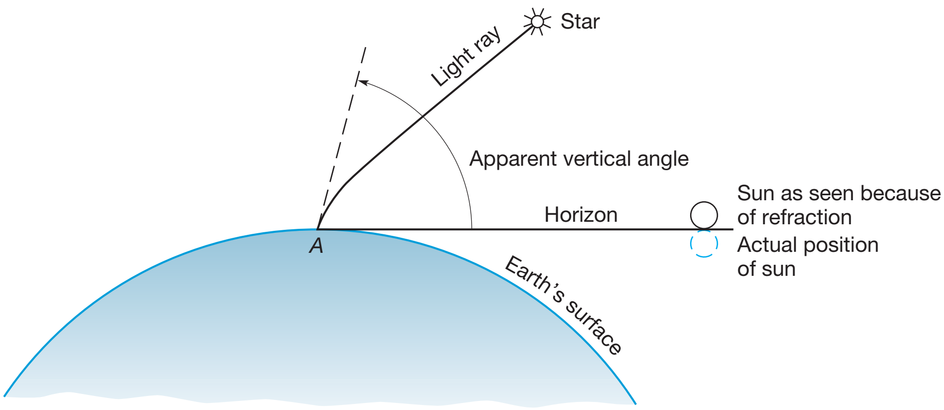


Figure 4.3
Refraction.

where the departure of a level surface from a horizontal line is C_f in feet or C_m in meters, M is the distance AB in miles, F the distance in *thousands of feet*, and K the distance in kilometers.

Since points A and B are on a level line, they have the same elevation. If a graduated rod was held vertically at B and a reading was taken on it by means of a telescope with its line of sight AD horizontal, the Earth's curvature would cause the reading to be read too high by length BD.

Light rays passing through the Earth's atmosphere are bent or refracted toward the Earth's surface, as shown in Figure 4.3. Thus a theoretically horizontal line of sight, like AH in Figure 4.2, is bent to the curved form AR. Hence, the reading on a rod held at R is diminished by length RH.

The effects of refraction in making objects appear higher than they really are (and therefore rod readings too small) can be remembered by noting what happens when the sun is on the horizon, as in Figure 4.3. At the moment when the sun has just passed below the horizon, it is seen just above the horizon. The sun's diameter of approximately 32 min is roughly equal to the average refraction on a horizontal sight. Since the red wavelength of light bends the greatest, it is not uncommon to see a red sun in a clear sky at dusk and dawn.

Displacement resulting from refraction is variable. It depends on atmospheric conditions, length of line, and the angle a sight line makes with the vertical. For a horizontal sight, refraction R_f in feet or R_m in meters is expressed approximately by the formulas

$$R_f = 0.093\ M^2 = 0.0033\ F^2 \tag{4.2a}$$

or

$$R_m = 0.011K^2 \tag{4.2b}$$

This is about one seventh the effect of curvature of the Earth, but in the opposite direction.

The combined effect of curvature and refraction, h in Figure 4.2, is approximately

$$h_f = 0.574\ M^2 = 0.0206\ F^2 \tag{4.3a}$$

or

$$h_m = 0.0675\ K^2 \tag{4.3b}$$

where h_f is in feet and h_m is in meters.

For sights of 100, 200, and 300 ft, $h_f = 0.00021, 0.00082$, and 0.0019 ft, respectively, or 0.00068 m for a 100 m length. It will be explained in Section 5.4 that, although the combined effects of curvature and refraction produce rod readings that are slightly too large, proper field procedures in differential leveling can practically eliminate the error due to these causes. However, this is not true for trigonometric leveling (see Section 4.5.4) where this uncompensated systematic error can result in erroneous elevation determinations. This is one of several reasons why trigonometric leveling has never been used in geodetic surveys.

4.5 METHODS FOR DETERMINING DIFFERENCES IN ELEVATION

Differences in elevation have traditionally been determined by taping, differential leveling, barometric leveling, and indirectly by trigonometric leveling. A newer method involves measuring vertical distances electronically. Brief descriptions of these methods follow. Other new techniques, described in Chapters 13, 14, and 15, utilize satellite systems. Elevation differences can also be determined using photogrammetry, as discussed in Chapter 27.

4.5.1 Measuring Vertical Distances by Taping or Electronic Methods

Application of a tape to a vertical line between two points is sometimes possible. This method is used to measure depths of mine shafts, to determine floor elevations in condominium surveys, and in the layout and construction of multistory buildings, pipelines, etc. When water or sewer lines are being laid, a graduated pole or rod may replace the tape (see Section 23.4). In certain situations, especially on construction projects, reflectorless electronic distance measurement (EDM) devices (see Section 6.22) are replacing the tape for measuring vertical distances on construction sites. This concept is illustrated in Figures 4.4 and 24.4.

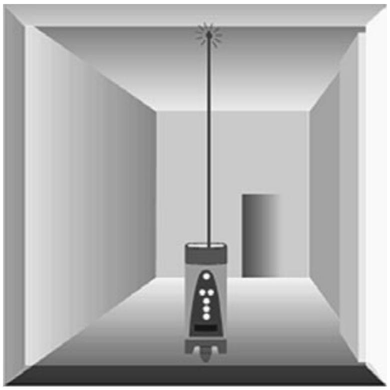


Figure 4.4 Reflectorless EDMs are being used to measure elevation differences in construction applications. (Reprinted with permission from Leica Geosystems, Inc.)

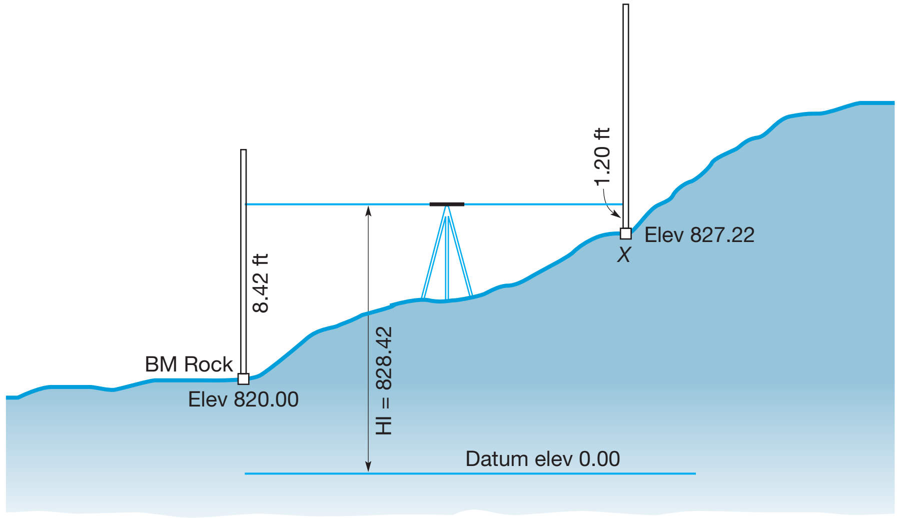


Figure 4.5
Differential leveling.

4.5.2 Differential Leveling

In this most commonly employed method, a telescope with suitable magnification is used to read graduated rods held on fixed points. A horizontal line of sight within the telescope is established by means of a level vial or automatic compensator.

The basic procedure is illustrated in Figure 4.5. An instrument is set up approximately halfway between BM Rock and point *X*.[3] Assume the elevation of BM Rock is known to be 820.00 ft. After leveling the instrument, a plus sight taken on a rod held on the BM gives a reading of 8.42 ft. A *plus sight* (+S), also termed *backsight* (BS), is the reading on a rod held on a point of known or assumed elevation. This reading is used to compute the *height of instrument* (HI), defined as the vertical distance from datum to the instrument line of sight. Direction of the sight—whether forward, backward, or sideways—is not important. The term plus sight is preferable to backsight, but both are used. Adding the plus sight 8.42 ft to the elevation of BM Rock, 820.00, gives an HI of 828.42 ft.

If the telescope is then turned to bring into view a rod held on point *X*, a *minus sight* (−S), also called *foresight* (FS), is obtained. In this example, it is 1.20 ft. A minus sight is defined as the rod reading on a point whose elevation is desired. The term minus sight is preferable to foresight. Subtracting the minus sight, 1.20 ft, from the HI, 828.42, gives the elevation of point *X* as 827.22 ft.

Differential leveling theory and applications can thus be expressed by two equations, which are repeated over and over

$$\text{HI} = \text{elev} + \text{BS} \tag{4.4}$$

and

$$\text{elev} = \text{HI} - \text{FS} \tag{4.5}$$

[3]As noted in Section 4.4, the combination of earth curvature and atmospheric refraction causes rod readings to be too large. However for any setup, if the backsight and foresight lengths are made equal (which is accomplished with the midpoint setup) the error from these sources is eliminated, as described in Section 5.4.

Since differential leveling is by far the most commonly used method to determine differences in elevation, it will be discussed in detail in Chapter 5.

4.5.3 Barometric Leveling

The barometer, an instrument that measures air pressure, can be used to find relative elevations of points on the Earth's surface since a change of approximately 1000 ft in elevation will correspond to a change of about 1 in. of mercury (Hg) in atmospheric pressure. Figure 4.6 shows a surveying altimeter. Calibration of the scale on different models is in multiples of 1 or 2 ft, 0.5 or 1 m. Air pressures are affected by circumstances other than difference in elevation, such as sudden shifts in temperature and changing weather conditions due to storms. Also, during each day a normal variation in barometric pressure amounting to perhaps a 100-ft difference in elevation occurs. This variation is known as the *diurnal range.*

In barometric leveling, various techniques can be used to obtain correct elevation differences in spite of pressure changes that result from weather variations. In one of these, a *control* barometer remains on a benchmark (base) while a *roving* instrument is taken to points whose elevations are desired. Readings are made on the base at stated intervals of time, perhaps every 10 min, and the elevations recorded along with temperature and time. Elevation, temperature, and

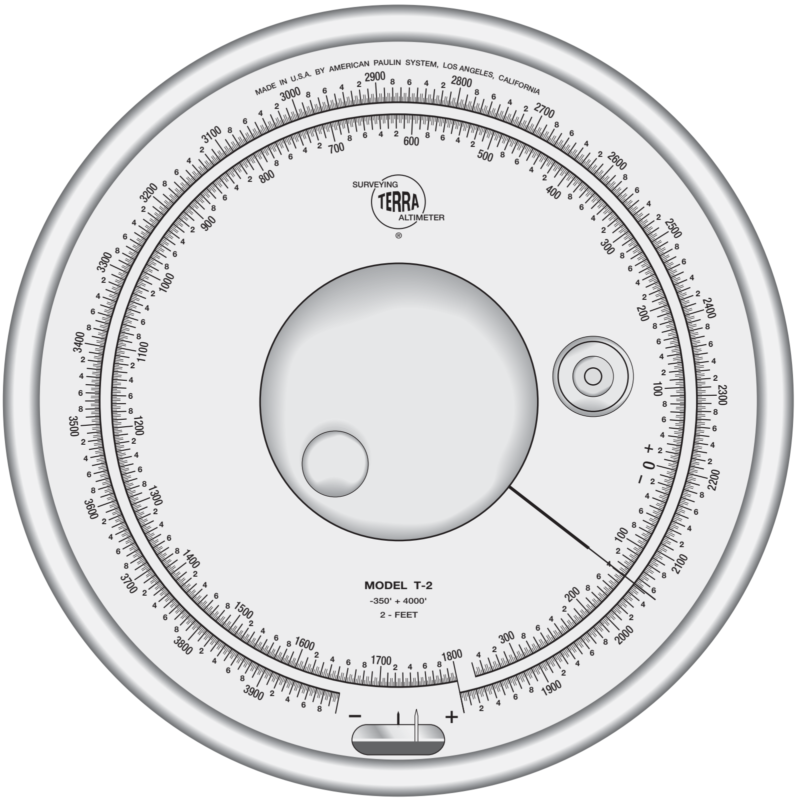


Figure 4.6
Surveying altimeter. (Courtesy American Paulin System.)

time readings with the roving barometer are taken at critical points and adjusted later in accordance with changes observed at the control point. Methods of making field surveys using a barometer have been developed in which one, two, or three bases may be used. Other methods employ leapfrog or semi-leapfrog techniques. In stable weather conditions, and by using several barometers, elevations correct to within ±2 to 3 ft are possible.

Barometers have been used in the past for work in rough country where extensive areas had to be covered but a high order of accuracy was not required. However, they are seldom used today having given way to other more modern and accurate equipment.

4.5.4 Trigonometric Leveling

The difference in elevation between two points can be determined by measuring (1) the inclined or horizontal distance between them and (2) the zenith angle or the altitude angle to one point from the other. (Zenith and altitude angles, described in more detail in Section 8.13, are measured in vertical planes. Zenith angles are observed downward from vertical, and altitude angles are observed up or down from horizontal.) Thus, in Figure 4.7, if slope distance S and zenith angle z or altitude angle α between C and D are observed, then V, the elevation difference between C and D, is

$$V = S \cos z \tag{4.6}$$

or

$$V = S \sin \alpha \tag{4.7}$$

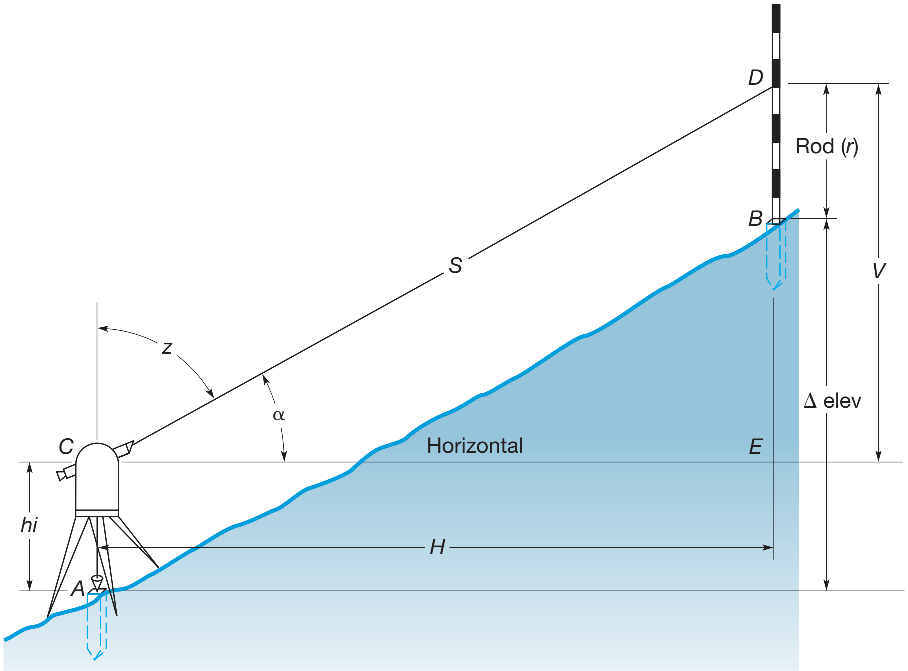


Figure 4.7 Trigonometric leveling—short lines.

Alternatively, if horizontal distance H between C and D is measured, then V is

$$V = H \cot z \tag{4.8}$$

or

$$V = H \tan \alpha \tag{4.9}$$

The difference in elevation (Δelev) between points A and B in Figure 4.7 is given by

$$\Delta\text{elev} = hi + V - r \tag{4.10}$$

where hi is the height of the instrument above point A and r the reading on the rod held at B when zenith angle z or altitude angle α is read. If r is made equal to hi, then these two values cancel in Equation (4.10) and simplify the computations.

Note the distinction in this text between *HI* and *hi*. Although both are called height of instrument, the term *HI is the elevation of the instrument above datum*, as described in Section 4.5.2, while *hi is the height of the instrument above an occupied point*, as discussed here.

For short lines (up to about 1000 ft in length) elevation differences obtained in trigonometric leveling are appropriately depicted by Figure 4.7 and properly computed using Equations (4.6) through (4.10). However, for longer lines Earth curvature and refraction become factors that must be considered. Figure 4.8

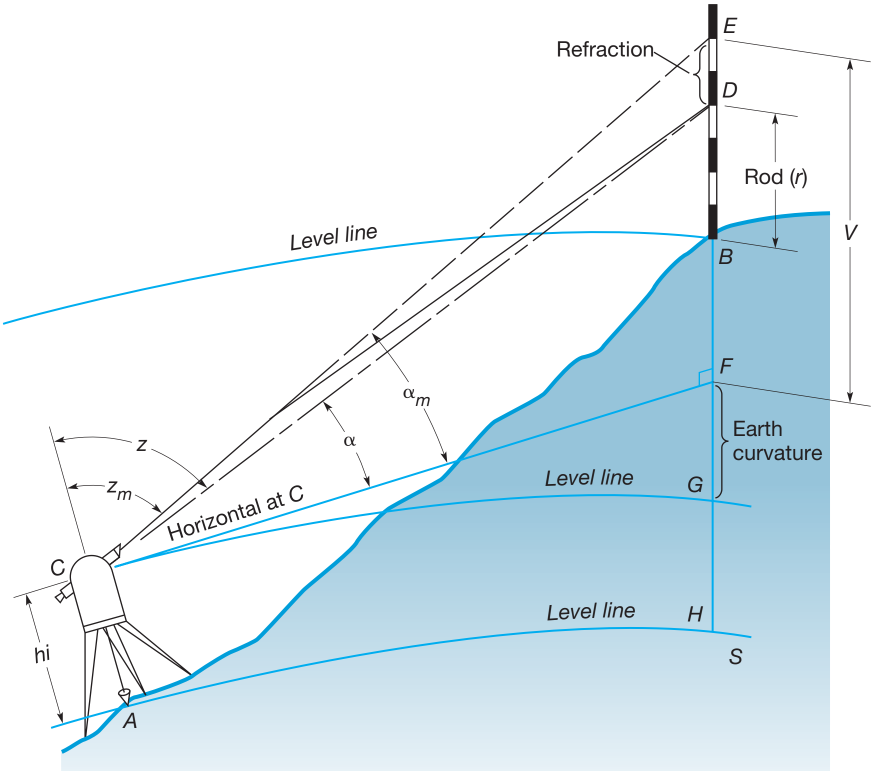


Figure 4.8
Trigonometric leveling—long lines.

illustrates the situation. Here an instrument is set up at C over point A. Sight D is made on a rod held at point B, and zenith angle z_m, or altitude angle α_m, is observed. The true difference in elevation (Δelev) between A and B is vertical distance HB between level lines through A and B, which is equal to $HG + GF + V - ED - r$. Since HG is the instrument height hi, GF is earth curvature C [see Equations (4.1)], and ED is refraction R [see Equations (4.2)], the elevation difference can be written as

$$\Delta\text{elev} = hi + V + h_{CR} - r \tag{4.11}$$

The value of V in Equation (4.11) is obtained using one of Equations (4.6) through (4.9), depending on what quantities are observed. Again if r is made equal to hi, these values cancel. Also, the term h_{CR} is given by Equations (4.3). Thus, except for the addition of the curvature and refraction correction, long and short sights may be treated the same in trigonometric leveling computations. Note that in developing Equation (4.11), angle F in triangle CFE was assumed to be 90°. Of course as lines become extremely long, this assumption does not hold. However, for lengths within a practical range, errors caused by this assumption are negligible.

The hi used in Equation (4.11) can be obtained by simply observing the vertical distance from the occupied point up to the instrument's horizontal axis (axis about which the telescope rotates) using a graduated rule or rod. An alternate method can be used to determine the elevation of a point that produces accurate results and does not require measurement of the hi. In this procedure, which is especially convenient if a total station instrument is used, the instrument is set up at a location where it is approximately equidistant from a point of known elevation (benchmark) and the one whose elevation is to be determined. The slope distance and zenith (or vertical) angle are measured to each point. Because the distances from the two points are approximately equal, curvature and refraction errors cancel. Also, since the same instrument setup applies to both readings, the hi values cancel, and if the same rod reading r is sighted when making both angle readings, they cancel. Thus, the elevation of the unknown point is simply the benchmark elevation, minus V calculated for the benchmark, plus V computed for the unknown point, where the V values are obtained using either Equation (4.6) or (4.7).

Example 4.1

The slope distance and zenith angle between points A and B were observed with a total station instrument as 9585.26 ft and 81°42′20″, respectively. The hi and rod reading r were equal. If the elevation of A is 1238.42 ft, compute the elevation of B.

Solution

By Equation (4.3a), the curvature and refraction correction is

$$h_f = 0.0206\left(\frac{9585.26 \sin 81°42'20''}{1000}\right)^2 = 1.85\,\text{ft}$$

(Theoretically, the horizontal distance should be used in computing curvature and refraction. In practice, multiplying the slope distance by the sine of the zenith angle approximates it.)

By Equations (4.6) and (4.11), the elevation difference is (note that *hi* and *r* cancel)

$$V = 9585.26 \cos 81^\circ 42' 20'' = 1382.77 \text{ ft}$$
$$\Delta\text{elev} = 1382.77 + 1.85 = 1384.62 \text{ ft}$$

Finally, the elevation of *B* is

$$\text{elev}_B = 1238.42 + 1384.62 = 2623.04 \text{ ft}$$

Note that if curvature and refraction had been ignored, an error of 1.85 ft would have resulted in the elevation for *B* in this calculation. Although Equation (4.11) was derived for an uphill sight, it is also applicable to downhill sights. In that case, the algebraic sign of *V* obtained in Equations (4.6) through (4.9) will be negative, however, because α will be negative or z greater than 90°.

For uphill sights curvature and refraction is added to a positive *V* to increase the elevation difference. For downhill sights, it is again added, but to a negative *V*, which decreases the elevation difference. Therefore, if "reciprocal" zenith (or altitude) angles are read (simultaneously observing the angles from both ends of a line), and *V* is computed for each and averaged, the effects of curvature and refraction cancel. Alternatively, the curvature and refraction correction can be completely ignored if one calculation of *V* is made using the average of the reciprocal angles. This assumes atmospheric conditions remain constant, so that refraction is equal for both angles. Hence, they should be observed within as short a time period as possible. This method is preferred to reading the zenith (or altitude) angle from one end of the line and correcting for curvature and refraction, as in Example 4.1. The reason is that Equations (4.3) assume a standard atmosphere, which may not actually exist at the time of observations.

Example 4.2

For Example 4.1, assume that at *B* the slope distance was observed again as 9585.26 ft and the zenith angle was read as 98°19′06″. The instrument height and *r* were equal. Compute (**a**) the elevation difference from this end of the line and (**b**) the elevation difference using the mean of reciprocal angles.

Solution

(a) By Equation (4.3a), $h_f = 1.85$ (the same as for Example 4.1).
By Equations (4.6) and (4.11) (note that *hi* and *r* cancel),

$$\Delta\text{elev} = 9585.26 \cos 98^\circ 19' 06'' + 1.85 = -1384.88 \text{ ft}$$

Note that this disagrees with the value of Example 4.1 by 0.26 ft. (The sight from B to A was downhill, hence the negative sign.) The difference of 0.26 ft is most probably due partly to observational errors and partly to refraction changes that occurred during the time interval between vertical angle observations. The average elevation difference for observations made from the two ends is 1384.75 ft.

(b) The average zenith angle is $\dfrac{81°42'20'' + (180° - 98°19'06'')}{2} = 81°41'37''$

By Equation (4.10), $\Delta\text{elev} = 9585.26 \cos 81°41'37'' = 1384.75$ ft

Note that this checks the average value obtained using the curvature and refraction correction.

With the advent of total station instruments, trigonometric leveling has become an increasingly common method for rapid and convenient observation of elevation differences because slope distances and vertical angles are quickly and easily observed from a single setup. Trigonometric leveling is used for topographic mapping, construction stakeout, control surveys, and other tasks. It is particularly valuable in rugged terrain. In trigonometric leveling, accurate zenith (or altitude) angle observations are critical. For precise work, a 1″ to 3″ total station instrument is recommended and angles should be read direct and reversed from both ends of a line. Also, errors caused by uncertainties in refraction are mitigated if sight lengths are limited to about 1000 ft.

PART II • EQUIPMENT FOR DIFFERENTIAL LEVELING

4.6 CATEGORIES OF LEVELS

Instruments used for differential leveling can be classified into four categories: *dumpy levels*, *tilting levels*, *automatic levels*, and *digital levels*. Although each differs somewhat in design, all have two common components: (1) a *telescope* to create a line of sight and enable a reading to be taken on a graduated rod and (2) a system to orient the line of sight in a horizontal plane. Dumpy and tilting levels use level vials to orient their lines of sight, while automatic levels employ *automatic compensators*. Digital levels also employ automatic compensators, but use bar-coded rods for automated digital readings. Automatic levels are the type most commonly employed today, although tilting levels are still used especially on projects requiring very precise work. Digital levels are rapidly gaining prominence. These three types of levels are described in the sections that follow. Dumpy levels are rarely used today, having been replaced by other newer types. They are discussed in Appendix A. *Hand levels*, although not commonly used for differential leveling, have many special uses where rough elevation differences over short distances are needed. They are also discussed in this chapter. Total station instruments can also be used for differential leveling. These instruments and their uses are described in Section 8.18.

Electronic laser levels that transmit beams of either visible laser or invisible infrared light are another category of leveling instruments. They are not commonly employed in differential leveling, but are used extensively for establishing elevations on construction projects. They are described in Chapter 23.

■ 4.7 TELESCOPES

The telescopes of leveling instruments define the line of sight and magnify the view of a graduated rod against a reference reticle, thereby enabling accurate readings to be obtained. The components of a telescope are mounted in a cylindrical tube. Its four main components are the objective lens, negative lens, reticle, and eyepiece. Two of these parts, the objective lens and eyepiece, are external to the instrument, and are shown on the automatic level illustrated in Figure 4.9.

Objective Lens. This compound lens, securely mounted in the tube's object end, has its optical axis reasonably concentric with the tube axis. Its main function is to gather incoming light rays and direct them toward the negative focusing lens.

Negative Lens. The negative lens is located between the objective lens and reticle, and mounted so its optical axis coincides with that of the objective lens. Its function is to focus rays of light that pass through the objective lens onto the reticle plane. During focusing, the negative lens slides back and forth along the axis of the tube.

Reticle. The reticle consists in a pair of perpendicular reference lines (usually called *crosshairs*) mounted at the principal focus of the objective optical system. The point of intersection of the crosshairs, together with the optical center of the objective system, forms the so-called *line of sight*, also sometimes called the *line of collimation*. The crosshairs are fine lines etched on a thin round glass plate. The glass plate is held in place in the main cylindrical tube by two pairs of opposing screws, which are located at right angles to each other to facilitate adjusting the line of sight. Two additional lines parallel to and equidistant from the primary lines are commonly added to reticles for special purposes such as for *three-wire leveling* (see Section 5.8) and for *stadia* (see Section 5.4). The

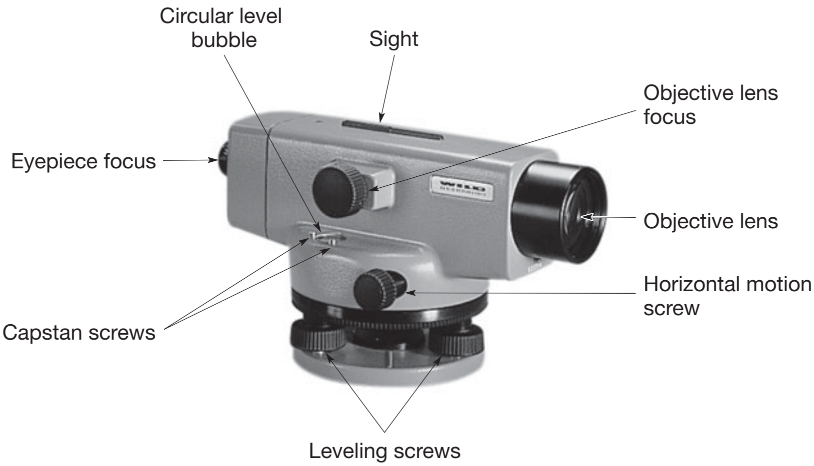


Figure 4.9 Parts of an automatic level. (Courtesy Leica Geosystems AG.)

reticle is mounted within the main telescope tube with the lines placed in a horizontal-vertical orientation.

Eyepiece. The eyepiece is a microscope (usually with magnification from about 25 to 45 power) for viewing the image.

Focusing is an important function to be performed in using a telescope. The process is governed by the fundamental principle of lenses stated in the following formula:

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \tag{4.12}$$

where f_1 is the distance from the lens to the image at the reticle plane, f_2 the distance from the lens to the object, and f the lens *focal length*. The focal length of any lens is a function of the radii of the ground spherical surfaces of the lens, and of the index of refraction of the glass from which it is made. It is a constant for any particular single or compound lens. To focus for each varying f_2 distance, f_1 must be changed to maintain the equality of Equation (4.12).

Focusing the telescope of a level is a two-stage process. First the eyepiece lens must be focused. Since the position of the reticle in the telescope tube remains fixed, the distance between it and the eyepiece lens must be adjusted to suit the eye of an individual observer. This is done by bringing the crosshairs to a clear focus; that is, making them appear as black as possible when sighting at the sky or a distant, light-colored object. Once this has been accomplished, the adjustment need not be changed for the same observer, regardless of sight length, unless the eye fatigues.

The second stage of focusing occurs after the eyepiece has been adjusted. Objects at varying distances from the telescope are brought to sharp focus at the plane of the crosshairs by turning the focusing knob. This moves the negative focusing lens to change f_1 and create the equality in Equation (4.12) for varying f_2 distances.

After focusing, if the crosshairs appear to travel over the object sighted when the eye is shifted slightly in any direction, *parallax* exists. The objective lens, the eyepiece, or both must be refocused to eliminate this effect if accurate work is to be done.

■ 4.8 LEVEL VIALS

Level vials are used to orient many different surveying instruments with respect to the direction of gravity. There are two basic types: the *tube* vial and the *circular* or so-called "bull's-eye" version. Tube vials are used on tilting levels (and also on the older dumpy levels) to precisely orient the line of sight horizontal prior to making rod readings. Bull's-eye vials are also used on tilting levels, and on automatic levels for quick, rough leveling, after which precise final leveling occurs. The principles of both types of vials are identical.

A tube level is a glass tube manufactured so that its upper inside surface precisely conforms to an arc of a given radius (see Figure 4.10). The tube is sealed at both ends, and except for a small air bubble, it is filled with a sensitive liquid.

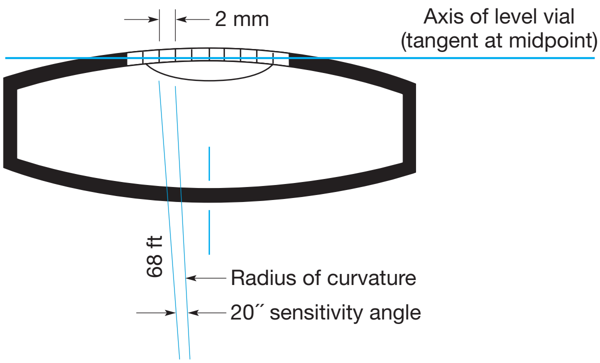


Figure 4.10
Tube-type level vial.

The liquid must be nonfreezing, quick acting, and maintain a bubble of relatively stable length for normal temperature variations. Purified synthetic alcohol is generally used. As the tube is tilted, the bubble moves, always to the highest point in the tube because air is lighter than the liquid. Uniformly spaced graduations etched on the tube's exterior surface, and spaced 2 mm apart, locate the bubble's relative position. The *axis of the level vial* is an imaginary longitudinal line tangent to the upper inside surface at its midpoint. When the bubble is centered in its run, the axis should be a horizontal line, as in Figure 4.10. For a leveling instrument that uses a level vial, if it is in proper adjustment, its line of sight is parallel to its level vial axis. Thus by centering the bubble, the line of sight is made horizontal.

Its radius of curvature, established in manufacture, determines the sensitivity of a level vial; the larger the radius, the more sensitive a bubble. A highly sensitive bubble, necessary for precise work, may be a handicap in rough surveys because more time is required to center it.

A properly designed level has a vial sensitivity correlated with the *resolving power* (resolution) of its telescope. A slight movement of the bubble should be accompanied by a small but discernible change in the observed rod reading at a distance of about 200 ft. Sensitivity of a level vial is expressed in two ways: (1) the angle, in seconds, subtended by one division on the scale and (2) the radius of the tube's curvature. If one division subtends an angle of 20″ at the center, it is called a 20″ bubble. A 20″ bubble on a vial with 2-mm division spacings has a radius of approximately 68 ft.[4] The sensitivity of level vials on most tilting levels (and the older dumpy levels) ranges from approximately 20″ to 40″.

[4]The relationship between sensitivity and radius is readily determined. In radian measure, an angle θ subtended by an arc whose radius and length are R and S, respectively, is given as

$$\theta = \frac{S}{R}$$

Thus for a 20″ bubble with 2-mm vial divisions, by substitution

$$\frac{20''}{206{,}265''/\text{rad}} = \frac{2\ \text{mm}}{R}$$

Solving for R

$$R = \frac{2\ \text{mm}(206{,}265''/\text{rad})}{20''} = 20{,}625\ \text{mm} = 20.6\ \text{m} = 68\ \text{ft (approx.)}$$

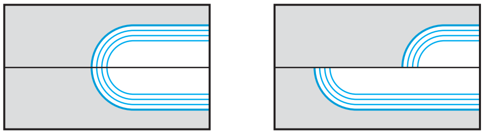


Figure 4.11
Coincidence-type level vial correctly set in left view; twice the deviation of the bubble shown in the right view.

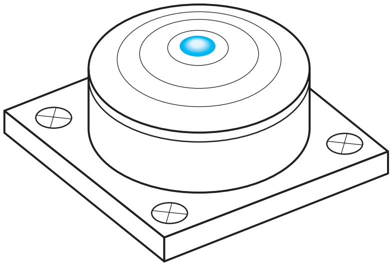


Figure 4.12
Bull's-eye level vial.

Figure 4.11 illustrates the *coincidence-type tube level* vial used on precise equipment. A prism splits the image of the bubble and makes the two ends visible simultaneously. Bringing the two ends together to form a smooth curve centers the bubble. This arrangement enables bubble centering to be done more accurately.

Circular level vials are spherical in shape (see Figure 4.12), the inside surface of the sphere being precisely manufactured to a specific radius. Like the tube version, except for an air bubble, circular vials are filled with liquid. The vial is graduated with concentric circles having 2-mm spacings. Its axis is actually a plane tangent to the radius point of the graduated concentric circles. When the bubble is centered in the smallest circle, the axis should be horizontal. Besides their use for rough leveling of tilting and automatic levels, circular vials are also used on total station instruments, tribrachs, rod levels, prism poles, and many other surveying instruments. Their sensitivity is much lower than that of tube vials—generally in the range from 2′ to 25′ per 2-mm division.

■ 4.9 TILTING LEVELS

Tilting levels were used for the most precise work. With these instruments, an example of which is shown in Figure 4.13, quick approximate leveling is achieved using a circular vial and the leveling screws. On some tilting levels, a ball-and-socket arrangement (with no leveling screws) permits the head to be tilted and quickly locked nearly level. Precise level in preparation for readings is then obtained by carefully centering a telescope bubble. This is done for each sight, after aiming at the rod, by tilting or rotating the telescope slightly in a vertical plane about a fulcrum at the vertical axis of the instrument. A micrometer screw under the eyepiece controls this movement.

The tilting feature saves time and increases accuracy, since only one screw need be manipulated to keep the line of sight horizontal as the telescope is turned about a vertical axis. The telescope bubble is viewed through a system of

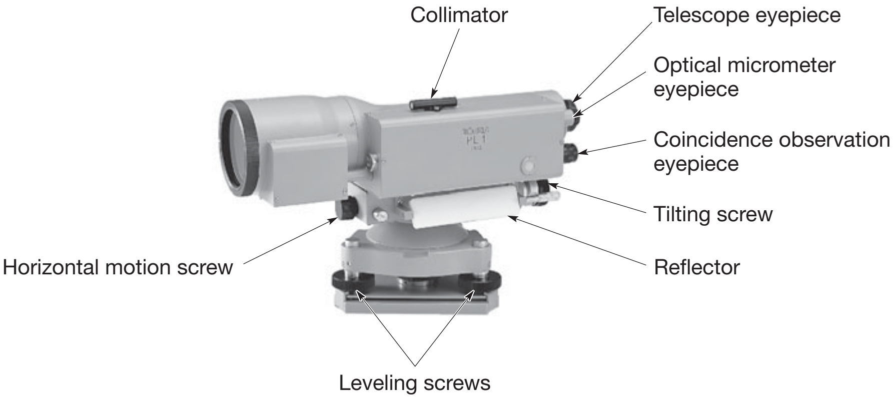


Figure 4.13
Parts of a precise tilting level. (Courtesy Sokkia Corporation.)

prisms from the observer's normal position behind the eyepiece. A prism arrangement splits the bubble image into two parts. Centering the bubble is accomplished by making the images of the two ends coincide, as in Figure 4.11.

The tilting level shown in Figure 4.13 has a three-screw leveling head, 42× magnification, and sensitivity of the level vial equal to 10″/2 mm.

■ 4.10 AUTOMATIC LEVELS

Automatic levels of the type pictured in Figure 4.14 incorporate a self-leveling feature. Most of these instruments have a three-screw leveling head, which is used to quickly center a bull's-eye bubble, although some models have a ball-and-socket arrangement for this purpose. After the bull's-eye bubble is centered manually, an *automatic compensator* takes over, levels the line of sight, and keeps it level.

The operating principle of one type of automatic compensator used in automatic levels is shown schematically in Figure 4.15. The system consists of prisms suspended from wires to create a pendulum. The wire lengths, support locations, and nature of the prisms are such that only horizontal rays reach the intersection of crosshairs. Thus, a horizontal line of sight is achieved even though the telescope itself may be slightly tilted away from horizontal. Damping devices shorten the time for the pendulum to come to rest, so the operator does not have to wait.

Automatic levels have become popular for general use because of the ease and rapidity of their operation. Some are precise enough for second-order and

Figure 4.14
Automatic level with micrometer. (Courtesy Topcon Positioning Systems.)

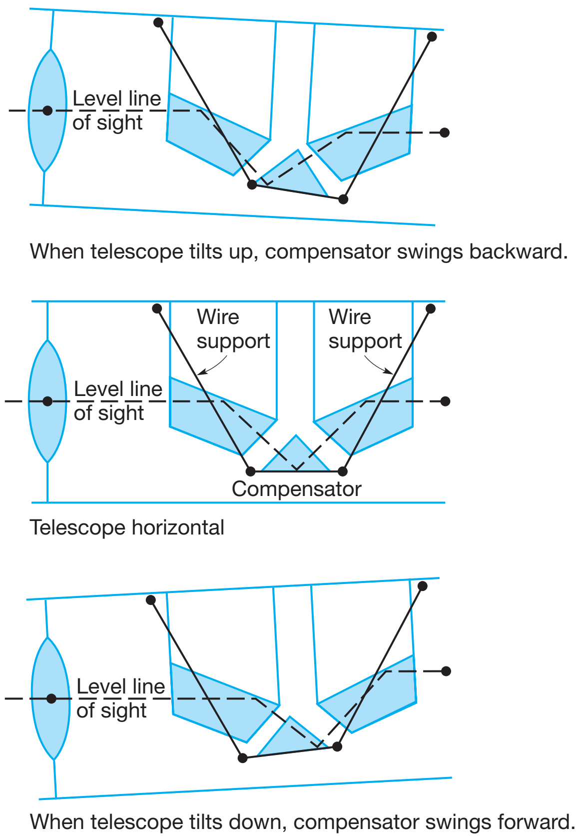


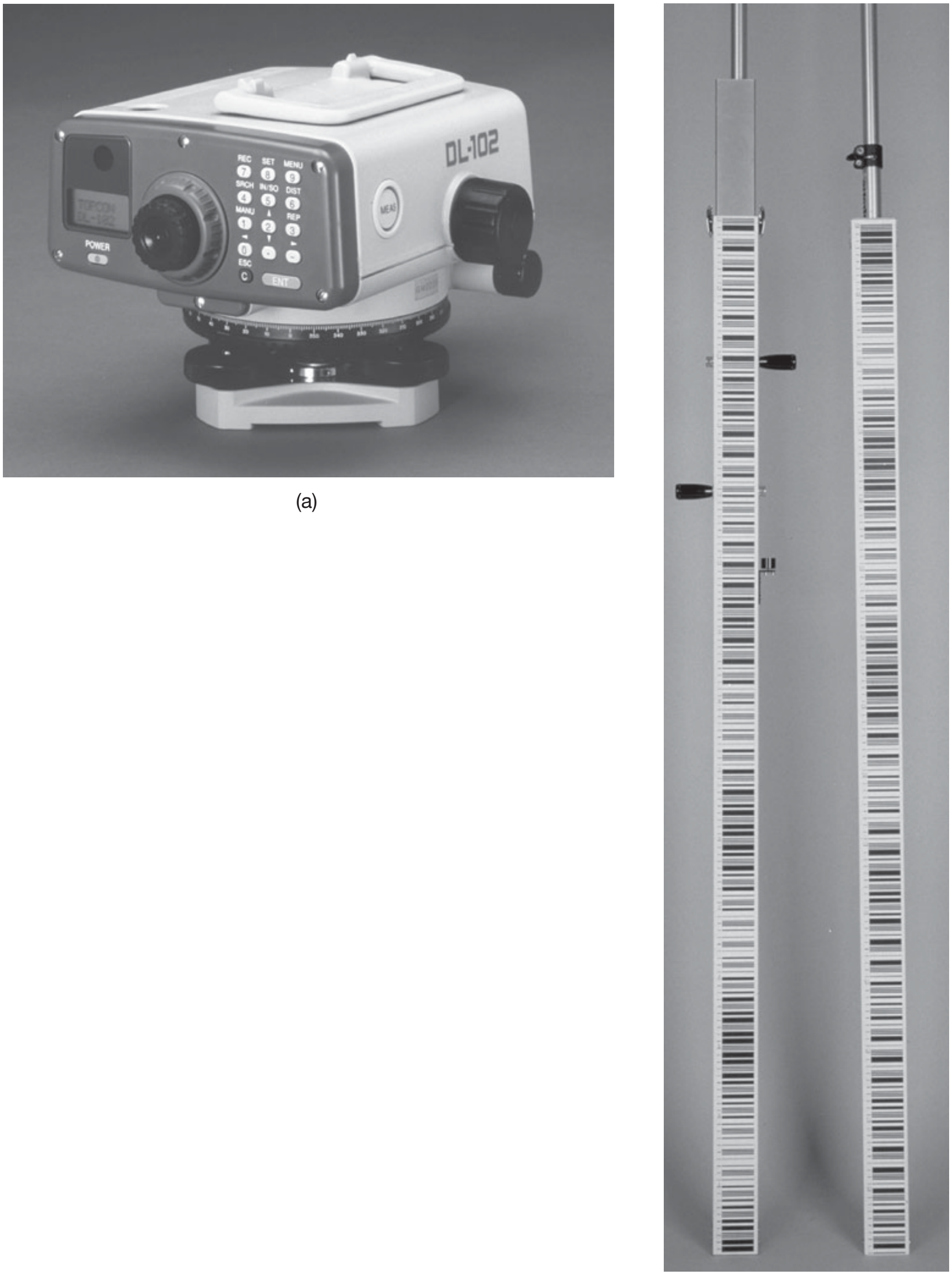
Figure 4.15 Compensator of self-leveling level.

even first-order work if a *parallel-plate micrometer* is attached to the telescope front as an accessory, as with the instrument shown in Figure 4.14. When the micrometer plate is tilted, the line of sight is displaced parallel to itself, and decimal parts of rod graduations can be read by means of a graduated dial.

Under certain conditions, the damping devices of an automatic level compensator can stick. To check, with the instrument leveled and focused, read the rod held on a stable point, lightly tap the instrument, and after it vibrates, determine whether the same reading is obtained. Also, some unique compensator problems, such as residual stresses in the flexible links, can introduce systematic errors if not corrected by an appropriate observational routine on first-order work. Another problem is that some automatic compensators are affected by magnetic fields, which result in systematic errors in rod readings. The sizes of the errors are azimuth-dependent, maximum for lines run north and south, and can exceed 1 mm/km. Thus, it is of concern for high-order control leveling only.

4.11 DIGITAL LEVELS

The newest type of automatic level, the *electronic digital level*, is pictured in Figure 4.16(a). It is classified in the automatic category because it uses a pendulum compensator to level itself, after an operator accomplishes rough leveling



(a)

(b)

Figure 4.16 (a) Electronic digital level and (b) associated level rod. (Courtesy Topcon Positioning Systems.)

with a circular bubble. With its telescope and crosshairs, the instrument could be used to obtain readings manually, just like any of the automatic levels. However, this instrument is designed to operate by employing electronic digital image processing. After leveling the instrument, its telescope is turned toward a special bar-coded rod [Figure 4.16(b)] and focused. At the press of a button, the image of bar codes in the telescope's field of view is captured and processed. This processing consists of an onboard computer comparing the captured image to the rod's entire pattern, which is stored in memory. When a match is found, which takes about 4 sec, the rod reading is displayed digitally. It can be recorded manually or automatically stored in the instrument's data collector.

The length of rod appearing within the telescope's field of view is a function of the distance from the rod. Thus as a part of its image processing, the instrument is also able to automatically compute the sight length, a feature convenient for balancing backsight and foresight lengths (see Section 5.4). The instrument's maximum range is approximately 100 m, and its accuracy in rod readings is ± 0.5 mm. The bar-coded rods can be obtained with English or metric graduations on the side opposite the bar code. The graduated side of the rod can be used by the operator to manually read the rod in situations that prohibit the instrument from reading the bar codes such as when the rod is in heavy brush.

■ 4.12 TRIPODS

Leveling instruments, whether tilting, automatic, or digital, are all mounted on tripods. A sturdy tripod in good condition is essential to obtain accurate results. Several types are available. The legs are made of wood or metal, may be fixed or adjustable in length, and solid or split. All models are shod with metallic conical points and hinged at the top, where they connect to a metal head. An adjustable-leg tripod is advantageous for setups in rough terrain or in a shop, but the type with a fixed-length leg may be slightly more rigid. The split-leg model is lighter than the solid type, but less rugged. (Adjustment of tripods is covered in Section 8.19.2.)

■ 4.13 HAND LEVEL

The hand level (Figure 4.17) is a handheld instrument used on low-precision work, or to obtain quick checks on more precise work. It consists of a brass tube approximately 6 in. long, having a plain glass objective and peep-sight eyepiece. A small level vial mounted above a slot in the tube is viewed through the eyepiece by means of a prism or 45° angle mirror. A horizontal line extends across the tube's center.

The prism or mirror occupies only one half of the tube, and the other part is open to provide a clear sight through the objective lens. Thus the rod being sighted and the reflected image of the bubble are visible beside each other with the horizontal cross line superimposed.

The instrument is held in one hand and leveled by raising or lowering the objective end until the cross line bisects the bubble. Resting the level against a rod or staff provides stability and increases accuracy. This instrument is especially

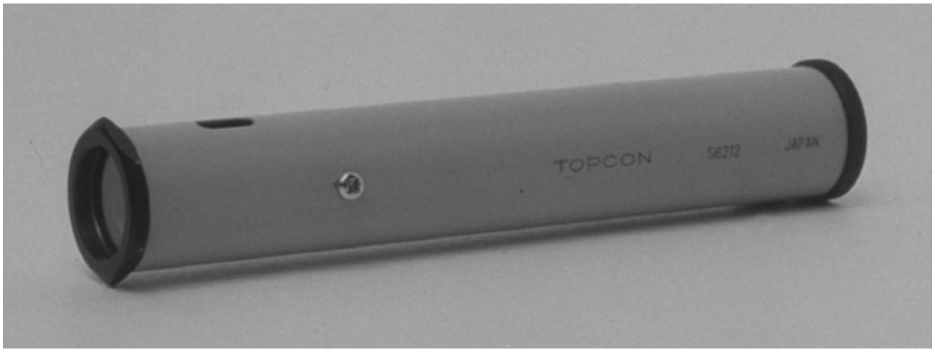


Figure 4.17
Hand level.
(Courtesy Topcon Positioning Systems.)

valuable in quickly checking proposed locations for instrument setups in differential leveling.

■ 4.14 LEVEL RODS

A variety of level rods are available, some of which are shown in Figure 4.18. They are made of wood, fiberglass, or metal and have graduations in feet and decimals, or meters and decimals.

The Philadelphia rod, shown in Figure 4.18(a) and (b), is the type most commonly used in college surveying classes. It consists of two sliding sections graduated in hundredths of a foot and joined by brass sleeves *a* and *b*. The rear section can be locked in position by a clamp screw *c* to provide any length from a *short rod* for readings of 7 ft or less to a *long rod* (*high rod*) for readings up to 13 ft. *When the high rod is needed, it must be extended fully, otherwise a serious mistake will result in its reading.* Graduations on the front faces of the two sections read continuously from zero at the base to 13 ft at the top for the high-rod setting.

Rod graduations are accurately painted, alternate black and white spaces 0.01 ft wide. Spurs extending the black painting emphasize the 0.1- and 0.05-ft marks. Tenths are designated by black figures, and footmarks by red numbers, all straddling the proper graduation. Rodpersons should keep their hands off the painted markings, particularly in the 3- to 5-ft section, where a worn face will make the rod unfit for use. A Philadelphia rod can be read accurately with a level at distances up to about 250 ft.

A wide choice of patterns, colors, and graduations on single-piece, two-piece, three-section, and four-section leveling rods is available. The various types, usually named for cities or states, include the Philadelphia, New York, Boston, Troy, Chicago, San Francisco, and Florida rods.

Philadelphia rods can be equipped with targets [*d* in Figure 4.18(a) and (b)] for use on long sights. When employed, the rodperson sets the target at the instrument's line-of-sight height according to communications or hand signals from the instrument operator. It is fixed using clamp *e*, then read and recorded by the rodperson. The *vernier* at *f*, can be used to obtain readings to the nearest 0.001 ft if desired. (Verniers are described in Section A.4.2.)

The Chicago rod, consisting of independent sections (usually three) that fit together but can be disassembled, is widely used on construction surveys. The

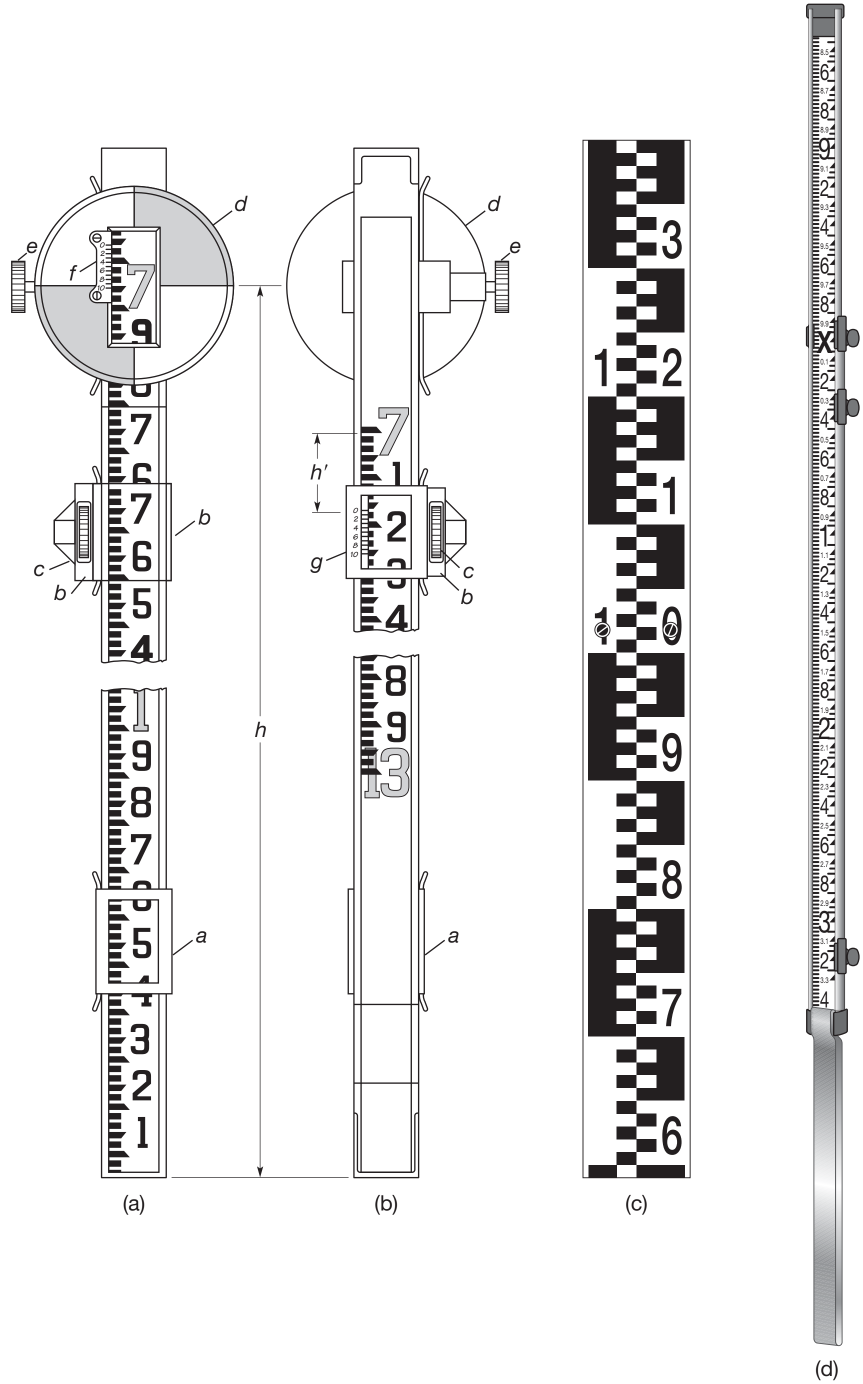


Figure 4.18
(a) Philadelphia rod (front).
(b) Philadelphia rod (rear).
(c) Double-faced leveling rod with metric graduations.
(d) Lenker direct-reading rod.

San Francisco model has separate sections that slide past each other to extend or compress its length, and is generally employed on control, land, and other surveys. Both are conveniently transported in vehicles.

The direct-reading Lenker level rod [Figure 4.18(d)] has numbers in reverse order on an endless graduated steel-band strip that can be revolved on the rod's end rollers. Figures run down the rod and can be brought to a desired reading—for example, the elevation of a benchmark. Rod readings are preset for the backsight, and then, due to the reverse order of numbers, foresight readings give elevations directly without manually adding backsights and subtracting foresights.

A rod consisting of a wooden, or fiberglass, frame and an Invar strip to eliminate the effects of humidity and temperature changes is used on precise work. The Invar strip, attached at its ends only is free to slide in grooves on each side of the wooden frame. Rods for precise work are usually graduated in meters and often have dual scales. Readings of both scales are compared to eliminate mistakes.

As described in Section 1.8, safety in traffic and near heavy equipment is an important consideration. The Quad-pod, an adjustable stand that clamps to any leveling rod, can help to reduce traffic hazards, and in some cases also lower labor costs.

■ 4.15 TESTING AND ADJUSTING LEVELS

Through normal use and wear, all leveling instruments will likely become maladjusted from time to time. The need for some adjustments may be noticed during use, for example, level vials on tilting levels. Others may not be so obvious, and therefore it is important that instruments be checked periodically to determine their state of adjustment. If the tests reveal conditions that should be adjusted, depending on the particular instrument, and the knowledge and experience of its operator, some or all of the adjustments can be made immediately in the field. However, if the parts needing adjustment are not readily accessible, or if the operator is inexperienced in making the adjustments, it is best to send the instruments away for adjustment by qualified technicians.

4.15.1 Requirements for Testing and Adjusting Instruments

Before testing and adjusting instruments, care should be exercised to ensure that any apparent lack of adjustment is actually caused by the instrument's condition and not by test deficiencies. To properly test and adjust leveling instruments in the field, the following rules should be followed:

1. Choose terrain that permits solid setups in a nearly level area enabling sights of at least 200 ft to be made in opposite directions.
2. Perform adjustments when good atmospheric conditions prevail, preferably on cloudy days free of heat waves. No sight line should pass through alternate sun and shadow, or be directed into the sun.
3. Place the instrument in shade, or shield it from direct rays of the sun.

4. Make sure the tripod shoes are tight and the instrument is screwed onto the tripod firmly. Spread the tripod legs well apart and position them so that the tripod plate is nearly level. Press the shoes into the ground firmly.

Standard methods and a prescribed order must be followed in adjusting surveying instruments. Loosening or tightening the proper adjusting nuts and screws with special tools and pins attains correct positioning of parts. Time is wasted if each adjustment is perfected on the first trial, since some adjustments affect others. The complete series of tests may have to be repeated several times if an instrument is badly off. A final check of all adjustments should be made to ensure that all have been completed satisfactorily.

Tools and adjusting pins that fit the capstans and screws should be used, and the capstans and screws handled with care to avoid damaging the soft metal. Adjustment screws are properly set when an instrument is shipped from the factory. Tightening them too much (or not enough) nullifies otherwise correct adjustment procedures and may leave the instrument in worse condition than it was before adjusting.

4.15.2 Adjusting for Parallax

The parallax adjustment is extremely important, and must be kept in mind at all times when using a leveling instrument, but especially during the testing and adjustment process. The adjustment is done by carefully focusing the objective lens and eyepiece so that the crosshairs appear clear and distinct, and so that the crosshairs do not appear to move against a background object when the eye is shifted slightly in position while viewing through the eyepiece.

4.15.3 Testing and Adjusting Level Vials

For leveling instruments that employ a level vial, the axis of the level vial should be perpendicular to the vertical axis of the instrument (axis about which the instrument rotates in azimuth). Then once the bubble is centered, the instrument can be turned about its vertical axis in any azimuth and the bubble will remain centered. Centering the bubble and revolving the telescope 180° about the vertical axis can quickly check this condition. The distance the bubble moves off the central position is twice the error. To correct any maladjustment, turn the capstan nuts at one end of the level vial to move the bubble *halfway back* to the centered position. Level the instrument using the leveling screws. Repeat the test until the bubble remains centered during a complete revolution of the telescope.

4.15.4 Preliminary Adjustment of the Horizontal CrossHair

Although it is good practice to always sight an object at the center of the crosshairs, if this is not done and the horizontal crosshair is not truly horizontal when the instrument is leveled, an error will result. To test for this condition, sight a sharply defined point with one end of the horizontal crosshair. Turn the telescope slowly on its vertical axis so that the crosshair moves across the point. If the crosshair does not remain on the point for its full length, it is out of adjustment.

To correct any maladjustment, loosen the four capstan screws holding the reticle. Rotate the reticle in the telescope tube until the horizontal hair remains on the point as the telescope is turned. The screws should then be carefully tightened in their final position.

4.15.5 Testing and Adjusting the Line of Sight

For tilting levels, described in Section 4.9, when the bubble of the level vial is centered, the line of sight should be horizontal. In other words, for this type of instrument to be in perfect adjustment, the axis of the level vial and the line of sight must be parallel. If they are not, a *collimation error* exists. For the automatic levels, described in Section 4.10, after rough leveling by centering the circular bubble, the automatic compensator must define a horizontal line of sight if it is in proper adjustment. If it does not, the compensator is out of adjustment, and again a collimation error exists. *The collimation error will not cause errors in differential leveling as long as backsight and foresight distances are balanced*. However, it will cause errors when backsights and foresights are not balanced, which sometimes occurs in differential leveling, and cannot be avoided in profile leveling (see Section 5.9), and construction staking (see Chapter 23).

One method of testing a level for collimation error is to stake out four points spaced equally, each about 100 ft apart on approximately level ground as shown in Figure 4.19. The level is then set up at point 1, leveled, and rod readings (r_A) at A, and (R_B) at B are taken. Next the instrument is moved to point 2 and releveled. Readings R_A at A, and r_B at B are then taken. As illustrated in the figure, assume that a collimation error ε exists in the rod readings of the two shorter sights. Then the error caused by this source would be 2ε in the longer sights because their length is double that of the shorter ones. Whether or not there is a collimation error, the difference between the rod readings at 1 should equal the

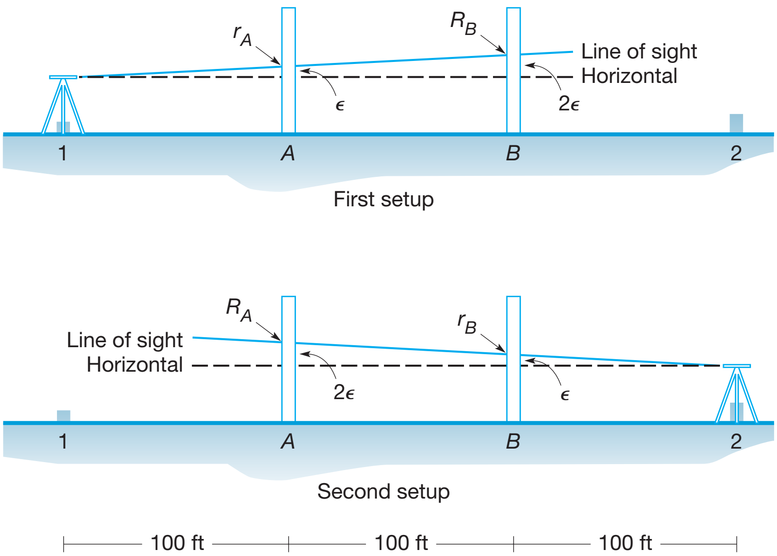


Figure 4.19 Horizontal collimation test.

difference of the two readings at 2. Expressing this equality, with the collimation error included, gives

$$(R_B - 2\varepsilon) - (r_A - \varepsilon) = (r_B - \varepsilon) - (R_A - 2\varepsilon) \tag{4.13}$$

Solving for ε in Equation (4.13) yields

$$\varepsilon = \frac{R_B - r_A - r_B + R_A}{2} \tag{4.14}$$

The corrected reading for the level rod at point A while the instrument is still setup at point 2 should be $R_A - 2\varepsilon$. If an adjustment is necessary, it is done by loosening the top (or bottom) screw holding the reticle, and tightening the bottom (or top) screw to move the horizontal hair up or down until the required reading is obtained on the rod at A. This changes the orientation of the line of sight. Several trials may be necessary to achieve the exact setting. If the reticle is not accessible, or the operator is unqualified, then the instrument should be serviced by a qualified technician.

As discussed in Section 19.13, it is recommended that the level instrument be tested before the observation process when performing precise differential leveling. A correction for the error in the line of sight is then applied to all field observations using the sight distances obtained by reading the stadia wires. The error in the line of sight is expressed in terms of ε per unit sight distance. For example, the collimation error C is unitless and expressed as 0.00005 ft/ft or 0.00005 m/m. Using the sight distances obtained in the leveling process this error can be mathematically eliminated. However, for most common leveling work, this error is removed by simply keeping minus and plus sight distances approximately equal between benchmarks.

Example 4.3

A horizontal collimation test is performed on an automatic level following the procedures just described. With the instrument setup at point 1, the rod reading at A was 5.630 ft, and to B was 5.900 ft. After moving and leveling the instrument at point 2, the rod reading to A was determined to be 5.310 ft and to B 5.560 ft. As shown in Figure 4.19, the distance between the points was 100 ft. What is the collimation error of the instrument, and the corrected reading to A from point 2?

Solution

Substituting the appropriate values into Equation (4.14), the collimation error is

$$\varepsilon = \frac{5.900 - 5.630 - 5.560 + 5.310}{2} = 0.010 \text{ ft}$$

Thus the corrected reading to A from point 2 is

$$R = 5.310 - 2(0.010) = 5.290 \text{ ft}$$

As noted above, if a collimation error exists but the instrument is not adjusted, accurate differential leveling can still be achieved when the plus sight and minus sight distances are balanced. In situations where these distances cannot be balanced, correct rod readings can still be obtained by applying collimation corrections to the rod readings. This procedure is described in Section 5.12.1.

Example 4.4

The instrument in Example 4.3 was used in a survey between two benchmarks before the instrument was adjusted where the sight distance could not be balanced due to the physical conditions. The sum of the plus sights was 900 ft while the sum of the minus sights was 1300 ft between the two benchmarks. The observed elevation difference was 120.64 ft. What is the corrected elevation difference between the two benchmarks?

Solution

In Example 4.3, the error ε was determined to be 0.01 ft/100 ft. Thus the collimation error C is -0.0001 ft/ft, and the corrected elevation difference is

$$\Delta\text{elev} = 120.64 - 0.0001(900 - 1300) = 120.68 \text{ ft}$$

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

4.1 Define the following leveling terms: (a) vertical line, (b) level surface, and (c) leveling.

4.2* How far will a horizontal line depart from the Earth's surface in 1 km? 5 km? 10 km? (Apply both curvature and refraction)

4.3 Visit the website of the National Geodetic Survey, and obtain a data sheet description of a benchmark in your local area.

4.4 Create plot of the curvature and refraction corrections for sight lines going from 0 ft to 10,000 ft in 500-ft increments.

4.5 Create a plot of curvature and refraction corrections for sight lines going from 0 m to 10,000 m in 500-m increments.

4.6 Why is it important for a benchmark to be a stable, relatively permanent object?

4.7* On a large lake without waves, how far from shore is a sailboat when the top of its 30-ft mast disappears from the view of a person lying at the water's edge?

4.8 Similar to Problem 4.7, except for a 8-m mast and a person whose eye height is 1.5 m above the water's edge.

4.9 Readings on a line of differential levels are taken to the nearest 0.01 ft. For what maximum distance can the Earth's curvature and refraction be neglected?

4.10 Similar to Problem 4.9 except readings are to the 3 mm.

4.11 Describe how readings are determined in a digital level when using a bar-coded rod.

Successive plus and minus sights taken on a downhill line of levels are listed in Problems 4.12 and 4.13. The values represent the horizontal distances between the instrument and either the plus or minus sights. What error results from curvature and refraction?

4.12* 20, 225; 50, 195; 40, 135; 30, 250 ft.

4.13 5, 75; 10, 60; 10, 55; 15, 70 m.

4.14 What error results if the curvature and refraction correction is neglected in trigonometric leveling for sights: (**a**) 3000 ft long (**b**) 500 m long (**c**) 5000 ft long?

4.15* The slope distance and zenith angle observed from point *P* to point *Q* were 2013.875 m and 95°13′04″, respectively. The instrument and rod target heights were equal. If the elevation of point *P* is 88.988 m, above datum, what is the elevation of point *Q*?

4.16 The slope distance and zenith angle observed from point *X* to point *Y* were 5401.85 ft and 83°53′16″. The instrument and rod target heights were equal. If the elevation of point *X* is 2045.66 ft above datum, what is the elevation of point *Y*?

4.17 Similar to Problem 4.15, except the slope distance was 854.987 m, the zenith angle was 82°53′48″, and the elevation of point *P* was 354.905 m above datum.

4.18 In trigonometric leveling from point *A* to point *B*, the slope distance and zenith angle measured at *A* were 2504.897 m and 85°08′54″. At *B* these measurements were 2504.891 m and 94°52′10″ respectively. If the instrument and rod target heights were equal, calculate the difference in elevation from *A* to *B*.

4.19 Describe how parallax in the viewing system of a level can be detected and removed.

4.20 What is the sensitivity of a level vial with 2-mm divisions for: (**a**) a radius of 40 m (**b**) a radius of 10 m?

4.21* An observer fails to check the bubble, and it is off two divisions on a 250-ft sight. What error in elevation difference results with a 10-sec bubble?

4.22 An observer fails to check the bubble, and it is off two divisions on a 100-m sight. What error results for a 20-sec bubble?

4.23 Similar to Problem 4.22, except a 10-sec bubble is off three divisions on a 130-m sight.

4.24 With the bubble centered, a 150-m-length sight gives a reading of 1.208 m. After moving the bubble four divisions off center, the reading is 1.243 m. For 2-mm vial divisions, what is: (**a**) the vial radius of curvature in meters (**b**) the angle in seconds subtended by one division?

4.25 Similar to Problem 4.24, except the sight length was 300 ft, the initial reading was 4.889 ft, and the final reading was 5.005 ft.

4.26 Sunshine on the forward end of a 20″/2 mm level vial bubble draws it off two divisions, giving a plus sight reading of 1.632 m on a 100-m shot. Compute the correct reading.

4.27 List in tabular form, for comparison, the advantages and disadvantages of an automatic level versus a digital level.

4.28* If a plus sight of 3.54 ft is taken on BM *A*, elevation 850.48 ft, and a minus sight of 7.84 ft is read on point *X*, calculate the HI and the elevation of point *X*.

4.29 If a plus sight of 2.486 m is taken on BM *A*, elevation 605.348 m, and a minus sight of 0.468 m is read on point *X*, calculate the HI and the elevation of point *X*.

4.30 Similar to Problem 4.28, except a plus sight of 7.44 ft is taken on BM *A* and a minus sight of 1.55 ft read on point *X*.

4.31 Describe the procedure used to test if the level vial is perpendicular to the vertical axis of the instrument.

4.32 A horizontal collimation test is performed on an automatic level following the procedures described in Section 4.15.5. With the instrument setup at point 1, the rod reading at *A* was 4.886 ft, and to *B* it was 4.907 ft. After moving and leveling the instrument at point 2, the rod reading to *A* was 5.094 ft and to *B* was 5.107 ft. What is the collimation error of the instrument and the corrected reading to *B* from point 2?

4.33 The instrument tested in Problem 4.32 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was -30.36 ft. The sum of the plus sight distances between the benchmarks was 1800 ft and the sum of the minus sight distances was 1050 ft. What is the corrected elevation difference between the two benchmarks?

4.34 Similar to Problem 4.32 except that the rod readings are 0.894 and 0.923 m to A and B, respectively, from point 1, and 1.083 and 1.100 m to A and B, respectively, from point 2. The distance between the points in the test was 50 m.

4.35 The instrument tested in Problem 4.34 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was 28.024 m. The sum of the plus sight distances between the benchmarks was 1300 m and the sum of the minus sight distances was 3200 m. What is the corrected elevation difference between the two benchmarks?

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5 Leveling—Field Procedures and Computations

5.1 INTRODUCTION

Chapter 4 covered the basic theory of leveling, briefly described the different procedures used in determining elevations, and showed examples of most types of leveling equipment. This chapter concentrates on differential leveling and discusses handling the equipment, running and adjusting simple leveling loops, and performing some project surveys to obtain data for field and office use. Some special variations of differential leveling, useful or necessary in certain situations, are presented. Profile leveling, to determine the configuration of the ground surface along some established reference line, is described in Section 5.9. Finally, errors in leveling are discussed. Leveling procedures for construction and other surveys, along with those of higher order to establish the nationwide vertical control network, will be covered in later chapters.

5.2 CARRYING AND SETTING UP A LEVEL

The safest way to transport a leveling instrument in a vehicle is to leave it in the container. The case closes properly only when the instrument is set correctly in the padded supports. A level should be removed from its container by lifting from the base, not by grasping the telescope. The head must be screwed snugly on the tripod. If the head is too loose, the instrument is unstable; if too tight, it may "freeze." Once the instrument is removed from the container, the container should be once again closed to prevent dirt and moisture from entering it.

The legs of a tripod must be tightened correctly. If each leg falls slowly of its own weight after being placed in a horizontal position, it is adjusted properly. Clamping them too tightly strains the plate and screws. If the legs are loose, unstable setups result.

Except for a few instruments that employ a ball-and-socket arrangement, all modern levels use a three-screw leveling head for initial rough leveling. Note that each of the levels illustrated in Chapter 4 (see Figures 4.9, 4.13, 4.14, and 4.16) has this type of arrangement. In leveling a three-screw head, the telescope is rotated until it is over two screws as in the direction *AB* of Figure 5.1. Using the thumb and first finger of each hand to adjust simultaneously the opposite screws approximately centers the bubble. This procedure is repeated with the telescope rotated 90° so that it is over *C*, the remaining single screw. Time is wasted by centering the bubble exactly on the first try, since it can be thrown off during the cross leveling. Working with the same screws in succession about three times should complete the job. A simple but useful rule in centering a bubble, illustrated in Figure 5.1, is: *A bubble follows the left thumb when turning the screws*. A circular bubble is centered by alternately turning one screw and then the other two. The telescope need not be rotated during the process.

It is generally unnecessary to set up a level over any particular point. Therefore it is inexcusable to have the base plate badly out of level before using the leveling screws. On sidehill setups, placing one leg on the uphill side and two on the downhill slope eases the problem. On very steep slopes, some instrument operators prefer two legs uphill and one downhill for stability. The most convenient height of setup is one that enables the observer to sight through the telescope without stooping or stretching.

Inexperienced instrument operators running levels up or down steep hillsides are likely to find, after completing the leveling process, that the telescope is too low for sighting the upper turning point or benchmark. To avoid this, a hand level can be used to check for proper height of the setup before leveling the instrument precisely. As another alternative, the instrument can be quickly set up without attempting to level it carefully. Then the rod is sighted making sure the bubble is somewhat back of center. If it is visible for this placement, it obviously will also be seen when the instrument is leveled.

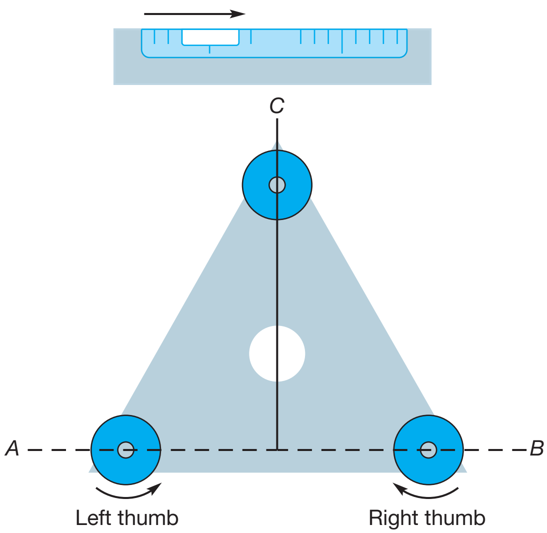

Figure 5.1
Use of leveling screws on a three-screw instrument.

5.3 DUTIES OF A RODPERSON

The duties of a rodperson are relatively simple. However, a careless rodperson can nullify the best efforts of an observer by failing to follow a few basic rules.

A level rod must be held plumb on the correct monument or turning point to give the correct reading. In Figure 5.2, point A is below the line of sight by vertical distance AB. If the rod is tilted to position AD, an erroneous reading AE is obtained. It can be seen that the smallest reading possible, AB, is the correct one and is secured only when the rod is plumb.

A rod level of the type shown in Figure 5.3 ensures *fast* and *correct* rod plumbing. Its L-shape is designed to fit the rear and side faces of a rod, while the bull's-eye bubble is centered to plumb the rod in *both directions*. However if a rod level is not available, one of the following procedures can be used to plumb the rod.

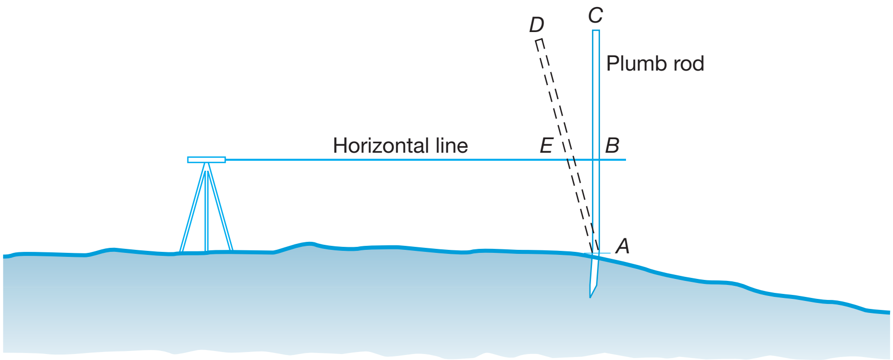

Figure 5.2
Plumbing a level rod.

Figure 5.3
Rod level. (Courtesy Tom Pantages.)

Waving the rod is one procedure that can be used to ensure that the rod is plumb when a reading is taken. The process consists of *slowly* tilting the rod top, first perhaps a foot or two toward the instrument and then just slightly away from it. The observer watches the readings increase and decrease alternately, and then selects the minimum value, the correct one. Beginners tend to swing the rod too fast and through too long an arc. Small errors can be introduced in the process if the bottom of the rod is resting on a flat surface. A rounded-top monument, steel spike, or thin edge makes an excellent benchmark or intermediate point for leveling.

On still days the rod can be plumbed by letting it balance of its own weight while lightly supported by the fingertips. An observer makes certain the rod is plumb in the lateral direction by checking its coincidence with the vertical wire and signals for any adjustment necessary. The rodperson can save time by sighting along the side of the rod to line it up with a telephone pole, tree, or side of a building. Plumbing along the line toward the instrument is more difficult, but holding the rod against the toes, stomach, and nose will bring it close to a plumb position. A plumb bob suspended alongside the rod can also be used, and in this procedure the rod is adjusted in position until its edge is parallel with the string.

Example 5.1

In Figure 5.2, what error results if the rod is held in position AD, and if $AE = 10$ ft and $EB = 6$ in.?

Solution

Using the Pythagorean theorem, the vertical rod is

$$AB = \sqrt{10^2 - 0.5^2} = 9.987 \text{ ft}$$

Thus the error is $10.00 - 9.987 = 0.013$ ft, or 0.01 ft.

Errors of the magnitude of Example 5.1 are serious, whether the results are carried out to hundredths or thousandths. They make careful plumbing necessary, particularly for high-rod readings.

5.4 DIFFERENTIAL LEVELING

Figure 5.4 illustrates the procedure followed in differential leveling. In the figure, the elevation of new BM Oak is to be determined by originating a leveling circuit at established BM Mil. In running this circuit, the first reading, a *plus sight*, is taken on the established benchmark. From it, the *HI* can be computed using Equation (4.4). Then a *minus sight* is taken on the first intermediate point (called a *turning point*, and labeled TP1 in the figure), and by Equation (4.5) its elevation is obtained. The process of taking a plus sight, followed by a minus sight, is repeated over and over until the circuit is completed.

As shown in the example of Figure 5.4, four instrument setups were required to complete half of the circuit (the run from BM Mil to BM Oak). Field

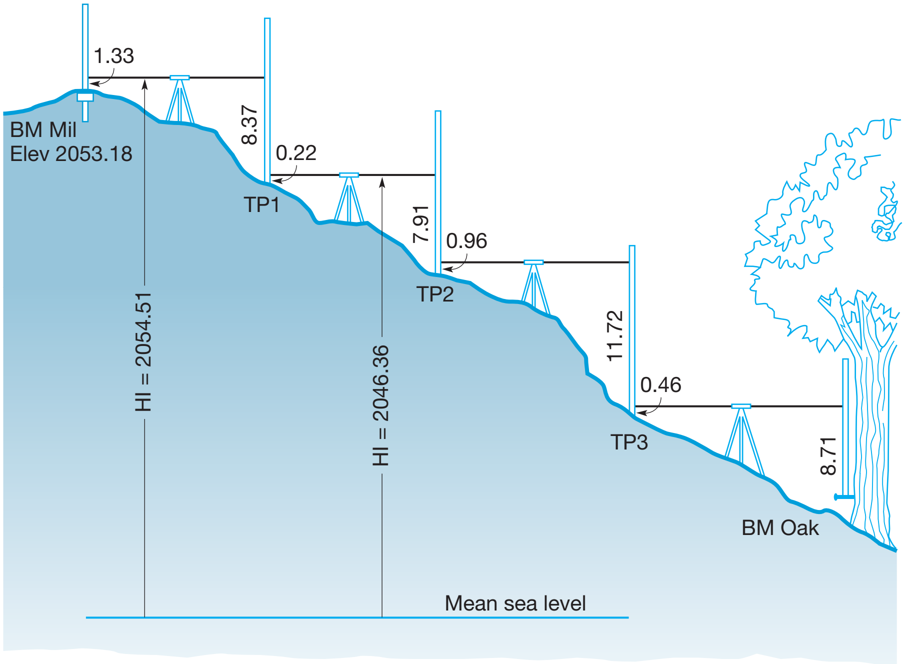

Figure 5.4 Differential leveling.

notes for the example of Figure 5.4 are given in Figure 5.5. As illustrated in this figure, a tabular form of field notes is used for differential leveling, and the addition and subtraction to compute HIs and elevations is done directly in the notes. These notes also show the data for the return run from BM Oak back to BM Mil to complete the circuit. It is important in differential leveling to run closed circuits so that the accuracy of the work can be checked, as will be discussed later.

As noted, the intermediate points upon which the rod is held in running a differential leveling circuit are called *turning points* (*TPs*). Two rod readings are taken on each, a minus sight followed by a plus sight. Turning points should be solid objects with a definite high point. Careful selection of stable turning points is essential to achieve accurate results. Steel turning pins and railroad spikes driven into firm ground make excellent turning points when permanent objects are not conveniently available.

In differential leveling, horizontal lengths for the plus and minus sights should be made about equal. This can be done by pacing, by stadia measurements, by counting rail lengths or pavement joints if working along a track or roadway, or by any other convenient method. Stadia readings are the most precise of these methods and will be discussed in detail.

Stadia was once commonly used for mapping.[1] The stadia method determines the horizontal distance to points through the use of readings on the upper and lower (stadia) wires on the reticle. The method is based on the principle that in

[1]Readers interested in using stadia for mapping purposes should refer to previous editions of this book.

DIFFERENTIAL LEVELS

Sta.	+ B.S.	H.I.	− F.S.	Elev.	Adj. Elev.
BM Mil.	1.33			2053.18	2053.18
		2054.51		(−0.004)	
TP1	0.22		8.37	2046.14	2046.14
		2046.36		(−0.008)	
TP2	0.96		7.91 ~~8.91~~	2038.45	2038.44
		2039.41		(−0.012)	
TP3	0.46		11.72	2027.69	2027.68
		2028.15		(−0.016)	
BM Oak	11.95		8.71	2019.44	2019.42
		2031.39		(−0.022)	
TP4	12.55		2.61	2028.78	2028.76
		2041.33		(−0.026)	
TP5	12.77		0.68	2040.65	2040.62
		2053.42		(−0.030)	
BM Mil.			0.21	2053.21	2053.18
Σ =	+40.24	Σ =	−40.21		

Page Check:
2053.18
+ 40.24
2093.42
− 40.21
2053.21 Check

GRAND LAKES UNIV. CAMPUS

BM Mil. to BM Oak

BM Mil. on GLU Campus	29 Sept. 2000
SW of Engineering Bldg.	Clear, Warm 70° F
9.4 ft. north of sidewalk	T.E. Henderson N
to instrument room and	J.F. King ϕ
1.6 ft. from Bldg. Bronze	D.R. Moore
disk in concrete flush	Lietz Level #6
with ground, stamped "Mil"	

BM Oak is a temporary project bench mark located at corner of Cherry and Pine Sts., 14 ft. West of computer laboratory. Twenty penny spike in 18″ Oak tree, 1 ft. above ground.

Loop Misclosure = 2053.21 − 2053.18 = 0.03

Permissible Misclosure = $0.02\sqrt{n} = 0.02\sqrt{7}$ = 0.05 ft.

Adjustment = $\frac{0.03}{7}$ = 0.004′ per H.I.

J.E. Henderson

Figure 5.5
Differential leveling notes for Figure 5.4.

similar triangles, corresponding sides are proportional. In Figure 5.6, which depicts a telescope with a simple lens, light rays from points A and B pass through the lens center and form a pair of similar triangles AmB and amb. Here $AB = I$ is the rod intercept (stadia interval), and $ab = i$ is the spacing between stadia wires.

Standard symbols used in stadia observations and their definitions are as follows (refer to Figure 5.6):

- f = *focal length* of lens (a constant for any particular compound objective lens)
- i = spacing between stadia wires (ab in Figure 5.6)
- f/i = *stadia interval factor* usually 100 and denoted by K
- I = rod intercept (AB in Figure 5.6), also called *stadia interval*
- c = distance from instrument center (vertical axis) to objective lens center (varies slightly when focusing the objective lens for different sight lengths but is generally considered to be a constant)
- C = stadia constant = $c + f$
- d = distance from the focal point F in front of telescope to face of rod
- D = distance from instrument center to rod face = $C + d$

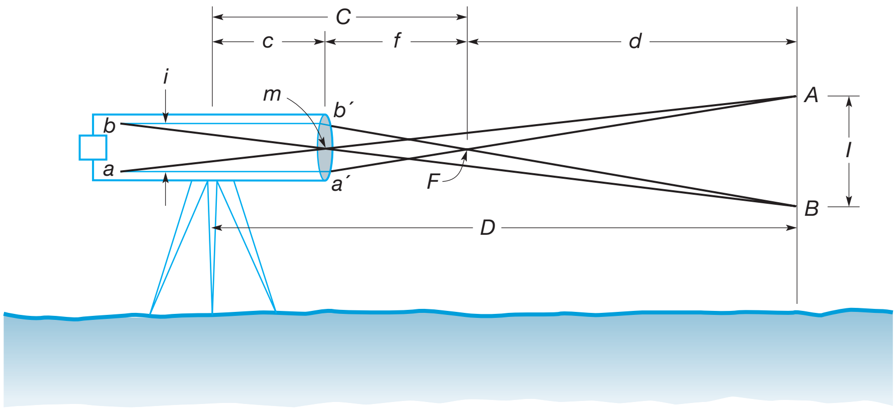

Figure 5.6
Principle of stadia.

From similar triangles of Figure 5.6

$$\frac{d}{f} = \frac{I}{i} \quad \text{or} \quad d = \frac{f}{i}I = KI$$

Thus

$$D = KI + C \tag{5.1}$$

The geometry illustrated in Figure 5.6 pertains to a simplified type of *external focusing telescope*. It has been used because an uncomplicated drawing correctly shows the relationships and aids in deriving the stadia equation. These telescopes are now obsolete in surveying instruments. The objective lens of an *internal focusing telescope* (the type now used in surveying instruments) remains fixed in position, while a movable negative-focusing lens between the objective lens and the plane of the crosshairs changes directions of the light rays. As a result, the stadia constant (C) is so small that it can be assumed equal to zero and drops out of Equation (5.1). Thus the equation for distance on a horizontal stadia sight reduces to

$$D = KI \tag{5.2}$$

Fixed stadia lines in theodolites, transits, levels, and alidades are generally spaced by instrument manufacturers to make the stadia interval factor $f/i = K$ equal to 100. It should be determined the first time an instrument is used, although the manufacturer's specific value posted inside the carrying case will not change unless the crosshairs, reticle, or lenses are replaced or adjusted.

To determine the stadia interval factor K, rod intercept I for a horizontal sight of known distance D is read. Then in an alternate form of Equation (5.2), the stadia interval factor is $K = D/I$. As an example, at a measured distance of 300.0 ft, a rod interval of 3.01 was read. Then $K = 300.0/3.01 = 99.7$. Accuracy in determining K is increased by averaging values from several lines whose observed lengths vary from about 100 to 500 ft by 100-ft increments.

It should be realized by the reader that in differential leveling the actual sight distances to the rod are not important. All one needs to balance is the rod intervals on the plus and minus sights between benchmarks to ensure that the sight distances are balanced.

Balancing plus and minus sights will eliminate errors due to instrument maladjustment (most important) and the combined effects of the Earth's curvature and refraction, as shown in Figure 5.6. Here e_1 and e_2 are the combined curvature and refraction errors for the plus and minus sights, respectively. If D_1 and D_2 are made equal, e_1 and e_2 are also equal. In calculations, e_1 is added and e_2 subtracted; thus they cancel each other. The procedure for reading all three wires of the instrument is known as three-wire leveling, which is discussed in Section 5.8.

Figure 5.7 can also be used to illustrate the importance of balancing sight lengths if a collimation error exists in the instrument's line of sight. This condition exists, if after leveling the instrument, its line of sight is not horizontal. For example, suppose in Figure 5.7 because the line of sight is systematically directed below horizontal, an error e_1 results in the plus sight. But if D_1 and D_2 are made equal, an error e_2 (equal to e_1) will result on the minus sight and the two will cancel, thus eliminating the effect of the instrumental error. On slopes it may be somewhat difficult to balance lengths of plus and minus sights, but following a zigzag path can do it usually. It should be remembered that Earth curvature, refraction, and collimation errors are systematic and will accumulate in long leveling lines if care is not taken to balance the plus and minus sight distances.

A benchmark is described in the field book the first time used, and thereafter by noting the page number on which it was recorded. Descriptions begin with the general location and must include enough details to enable a person unfamiliar with the area to find the mark readily (see the field notes of Figures 5.5 and 5.12). A benchmark is usually named for some prominent object it is on or near, to aid in describing its location; one word is preferable. Examples are BM River, BM Tower, BM Corner, and BM Bridge. On extensive surveys, benchmarks are often numbered consecutively. Although advantageous in identifying relative positions along a line, this method is more subject to mistakes in field marking or recording. Digital images of the benchmark with one showing a close-up of the monument and another showing the horizon of the benchmark with the leveling rod located on the monument can often help in later recovery of the monument.

Turning points are also numbered consecutively but not described in detail, since they are merely a means to an end and usually will not have to be relocated. However, if possible, it is advisable to select turning points that can be relocated, so if reruns on long lines are necessary because of blunders, fieldwork can be reduced. Before a party leaves the field, all possible note checks must be made to detect any mistakes in arithmetic and verify achievement of an acceptable closure. *The algebraic sum of the plus and minus sights applied to the first elevation should*

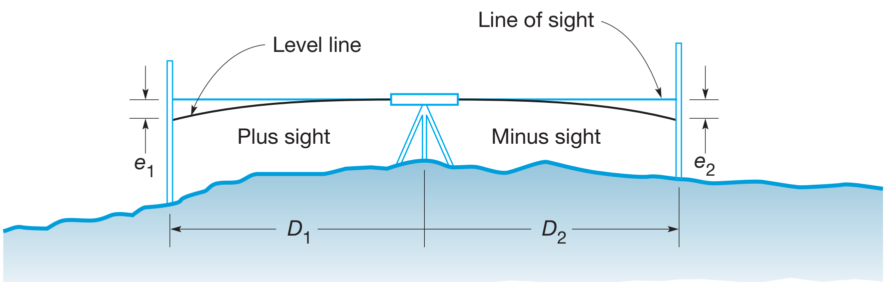

Figure 5.7 Balancing plus and minus sight distances to cancel errors caused by curvature and refraction.

give the last elevation. This computation checks the addition and subtraction for all HIs and turning points unless compensating mistakes have been made. When carried out for each left-hand page of tabulations, it is termed the *page check*. In Figure 5.5, for example, note that the page check is secured by adding the sum of backsights, 40.24, to the starting elevation 2053.18, and then subtracting the sum of foresights, 40.21, to obtain 2053.21, which checks the last elevation.

As previously noted, leveling should always be checked by running closed circuits or loops. This can be done either by returning to the starting benchmark, as demonstrated with the field notes in Figure 5.5, or by ending the circuit at another benchmark of equal or higher reliability. The final elevation should agree with the starting elevation if returning to the initial benchmark. The amount by which they differ is the *loop misclosure*. Note that in Figure 5.5, a loop misclosure of 0.03 ft was obtained.

If closure is made to another benchmark, the *section misclosure* is the difference between the closing benchmark's given elevation and its elevation obtained after leveling through the section. Specifications, or purpose of the survey, fix permissible misclosures (see Section 5.5). If the allowable misclosure is exceeded, one or more additional runs must be made. When acceptable misclosure is achieved, final elevations are obtained by making an adjustment (see Section 5.6).

Note that in running a level circuit between benchmarks, *a new instrument setup has to be made before starting the return run to get a complete check*. In Figure 5.5, for example, a minus sight of 8.71 was read on BM Oak to finish the run out, and a plus sight of 11.95 was recorded to start back, showing that a new setup had been made. Otherwise, an error in reading the final minus sight would be accepted for the first plus sight on the run back. An even better check is secured by tying the run to a different benchmark.

If the elevation above a particular vertical datum (i.e., NAVD88) is available for the starting benchmark, elevations then determined for all intermediate points along the circuit will also be referenced to the same datum. However, if the starting benchmark's elevation above datum is not known, an assumed value may be used and all elevations converted to the datum later by applying a constant.

A lake or pond undisturbed by wind, inflow, or outflow can serve as an extended turning point. Stakes driven flush with the water, or rocks whose high points are at this level should be used. However, this water level as a turning point should be used with caution since bodies of water generally flow to an outlet and thus may have differences in elevations along their surfaces.

Double-rodded lines of levels are sometimes used on important work. In this procedure, plus and minus sights are taken on two turning points, using two rods from each setup, and the readings carried in separate note form columns. A check on each instrument setup is obtained if the HI agrees for both lines. This same result can be accomplished using just one set of turning points, and reading both sides of a single rod that has two faces, that is, one side in feet and the other in meters. These rods are often used in precise leveling.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *differential leveling field notes.mp4* discusses the process of differential leveling, entering readings into your field book, and adjusting a simple differential leveling loop.

5.5 PRECISION

Precision in leveling is increased by repeating observations, making frequent ties to established benchmarks, using high-quality equipment, keeping it in good adjustment, and performing the measurement process carefully. However, no matter how carefully the work is executed, errors will exist and will be evident in the form of misclosures, as discussed in Section 5.4. To determine whether or not work is acceptable, misclosures are compared with permissible values on the basis of either number of setups or distance covered. Various organizations set precision standards based on their project requirements. For example, on a simple construction survey, an allowable misclosure of $C = 0.02\,\text{ft}\sqrt{n}$ might be used, where n is the number of setups. Note that this criterion was applied for the level circuit in the field notes of Figure 5.5.

The Federal Geodetic Control Subcommittee (FGCS) recommends the following formula to compute allowable misclosures:[2]

$$C = m\sqrt{K} \tag{5.3}$$

where C is the allowable loop or section[3] misclosure, in millimeters; m is a constant; and K the total length leveled, in kilometers. For "loops" (circuits that begin and end on the same benchmark), K is the total perimeter distance, and the FGCS specifies constants of 4, 5, 6, 8, and 12 mm for the five classes of leveling, designated, respectively, as (1) first-order class I, (2) first-order class II, (3) second-order class I, (4) second-order class II, and (5) third-order. For "sections" the constants are the same, except that 3 mm applies for first-order class I. The particular order of accuracy recommended for a given type of project is discussed in Section 19.7.

Example 5.2

A differential leveling loop is run from an established BM A to a point 2 mi away and back, with a misclosure of 0.056 ft. What order leveling does this represent?

Solution

$$C = \frac{0.056\ \text{ft}}{0.0028\ \text{ft/mm}} = 17\ \text{mm}$$

$$K = (2\ \text{mi} + 2\ \text{mi}) \times 1.61\ \text{km/mi} = 6.4\ \text{km}$$

By a rearranged form of Equation 5.1, $m = \dfrac{C}{\sqrt{K}} = \dfrac{17}{\sqrt{6.4}} = 6.7$

[2]The FGCS was formerly the FGCC (Federal Geodetic Control Committee). Their complete specifications for leveling are available in a booklet entitled "Standards and Specifications for Geodetic Control Networks" (September 1984). Information on how to obtain this and other related publications can be obtained at the following website: http://www.ngs.noaa.gov. Inquiries can also be made by email at info_center@ngs.noaa.gov, or by writing to the National Geodetic Information Center, NOAA, National Geodetic Survey, 1315 East West Highway, Station 9202, Silver Spring, MD 20910; telephone: (301) 713-3242.

[3]A section consists of a line of levels that begins on one benchmark, and closes on another.

This leveling meets the allowable 8-mm tolerance level for second-order class II work, but does not quite meet the 6-mm level for second-order class I, and if that standard had been specified, the work would have to be repeated. It should be pointed out that even though this survey met the closure tolerance for a second-order class II as specified in the FGCS *Standards and Specifications for Geodetic Control Networks*, other requirements must be met before the survey can be certified to meet any level in the standards. The reader should refer to the standards listed in the bibliography at the end of this chapter.

Since distance leveled is proportional to number of instrument setups, the misclosure criteria can be specified using that variable. As an example, if sights of 200 ft are taken, thereby spacing instrument setups at about 400 ft, approximately 8.2 setups/km will be made. For second-order class II leveling, the allowable misclosure will then be, again by Equation (5.1)

$$C = \frac{8}{\sqrt{8.2}}\sqrt{n} = 2.8\sqrt{n}$$

where C is the allowable misclosure, in millimeters; and n the number of times the instrument is set up.

It is important to point out that meeting FGCS misclosure criterion[4] alone does not guarantee that a certain order of accuracy has been met. Because of compensating errors, it is possible, for example, that crude instruments and low-order techniques can produce small misclosures, yet intermediate elevations along the circuit may contain large errors. To help ensure that a given level of accuracy has indeed been met, besides stating allowable misclosures, the FGCS also specifies equipment and procedures that must be used to achieve a given order of accuracy. These specifications identify calibration requirements for leveling instruments (including rods), and they also outline required field procedures that must be used. Then if the misclosure specified for a given order of accuracy has been met, while employing appropriate instruments and procedures, it can be reasonably expected that all intermediate elevations along the circuit are established to that order.

Field procedures specified by the FGCS include minimum ground clearances for the line of sight, allowable differences between the lengths of pairs of backsight and foresight distances, and maximum sight lengths. As examples, sight lengths of not more than 50 m are permitted for first-order class I, while lengths up to 90 m are allowed for third order. As noted in Sections 5.4 and 5.8, the stadia method is convenient for measuring the lengths of backsights and foresights to verify their acceptance.

5.6 ADJUSTMENTS OF SIMPLE LEVEL CIRCUITS

Since permissible misclosures are based on the lengths of lines leveled, or number of setups, it is logical to adjust elevations in proportion to these values. Observed elevation differences d and lengths of sections L are shown for a circuit in Figure 5.8.

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[4]A complete listing of the specifications for performing geodetic control leveling can be obtained at http://www.ngs.noaa.gov/fgcs/tech_pub/1984-stds-specs-geodetic-control-networks.htm.

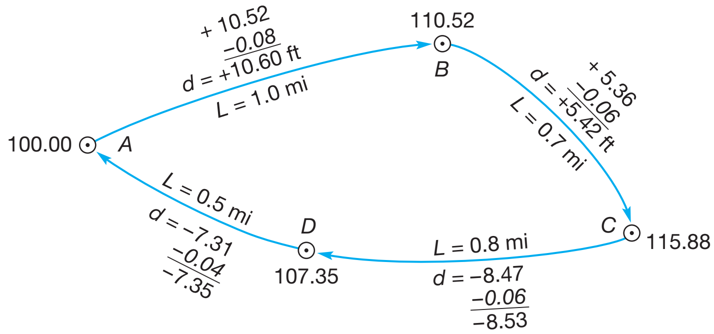

Figure 5.8 Adjustment of level circuit based on lengths of lines.

The misclosure found by algebraic summation of the elevation differences is +0.24 ft. Adding lengths of the sections yields a total circuit length of 3.0 mi. Elevation adjustments are then (0.24 ft/3.0) multiplied by the corresponding lengths, giving corrections of −0.08, −0.06, −0.06, and −0.04 ft (shown in the figure). The adjusted elevation differences (shown in black) are used to get the final elevations of benchmarks (also shown in black in the figure). Any misclosure that fails to meet tolerances may require reruns instead of adjustment. In Figure 5.5, adjustment for misclosure was made based on the number of instrument setups. Thus after verifying that the misclosure of 0.03 ft was within tolerance, the correction per setup was $0.03/7 = 0.004$ ft. Since errors in leveling accumulate, the first point receives a correction of 1×0.004, the second 2×0.004, and so on. The corrections are shown in parenthesis above each unadjusted elevation in Figure 5.5. However, the corrected elevations are rounded off to the nearest hundredth of a foot. Level circuits with different lengths and routes are sometimes run from scattered reference points to obtain the elevation of a given benchmark. The most probable value for a benchmark elevation can then be computed from a weighted mean of the observations, the weights varying inversely with line lengths.

In running level circuits, especially long ones, it is recommended that some turning points or benchmarks used in the first part of the circuit be included again on the return run. This creates a multiloop circuit, and if a blunder or large error exists, its location can be isolated to one of the smaller loops. This saves time because only the smaller loop containing the blunder or error needs to be rerun.

Although the least-squares method (see Section 16.6) is the best method for adjusting circuits that contain two or more loops, an approximate procedure can also be employed. In this method each loop is adjusted separately, beginning with the one farthest from the closing benchmark.

5.7 RECIPROCAL LEVELING

Sometimes in leveling across topographic features such as rivers, lakes, and canyons, it is difficult or impossible to keep plus and minus sights short and equal. Reciprocal leveling may be utilized at such locations.

As shown in Figure 5.9, a level is set up on one side of a river at X, near A, and rod readings are taken on points A and B. Since XB is very long, several readings are taken for averaging. Reading, turning the leveling screws to throw

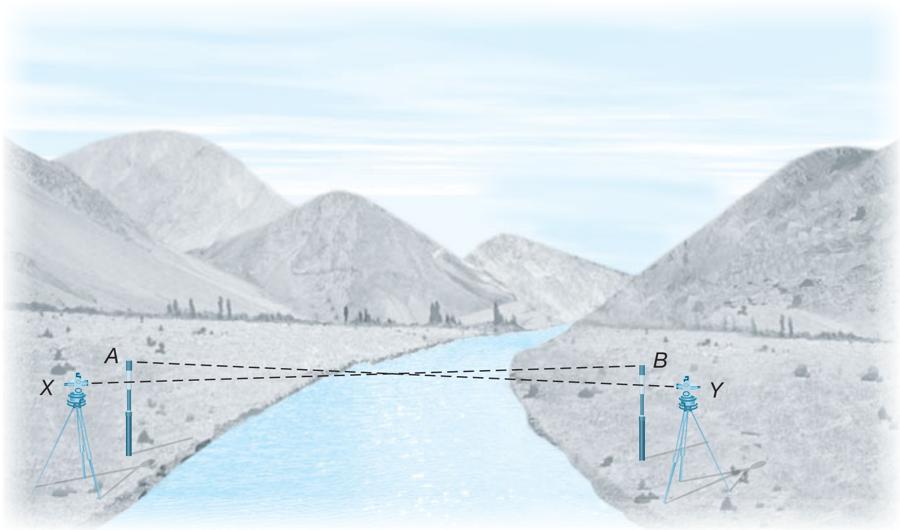

Figure 5.9
Reciprocal leveling.

the instrument out of level, releveling, and reading again, does this. The process is repeated two, three, four, or more times. Then the instrument is moved close to Y and the same procedure followed.

The two differences in elevation between A and B, determined with an instrument first at X and then at Y, will not agree normally because of curvature, refraction, and personal and instrumental errors. However, in the procedure just outlined, the long foresight from X to B is balanced by the long backsight from Y to A. Thus the average of the two elevation differences cancels the effects of curvature, refraction, and instrumental errors, so the result is accepted as the correct value if the precision of the two differences appears satisfactory. Delays at X and Y should be minimized because refraction varies with changing atmospheric conditions.

■ 5.8 THREE-WIRE LEVELING

As implied by its name, three-wire leveling consists in making rod readings on the upper, middle, and lower crosshairs. Formerly it was used mainly for precise work, but it can be used on projects requiring only ordinary precision. The method has the advantages of (1) providing checks against rod reading blunders, (2) producing greater accuracy because averages of three readings are available, and (3) furnishing stadia measurements of sight lengths to assist in balancing backsight and foresight distances. In the three-wire procedure the difference between the upper and middle readings is compared with that between the middle and lower values. They must agree within one or two of the smallest units being recorded (usually 0.1 or 0.2 of the least count of the rod graduations); otherwise the readings are repeated. An average of the three readings is used as a computational check against the middle wire. As noted in Section 5.4, the difference between the upper and lower readings multiplied by the instrument *stadia interval factor* gives the sight distances. In leveling, the distances are often not important. What is important is that the sum of the plus sights is about equal to the sum of the minus sights, which eliminates errors due to curvature, refraction, and collimation errors.

A sample set of field notes for the three-wire method is presented in Figure 5.10. Backsight readings on BM *A* of 0.718, 0.633, and 0.550 m taken on the upper, middle, and lower wires, respectively, give upper and lower differences (multiplied by 100) of 8.5 and 8.3 m, which agree within acceptable tolerance. Stadia measurement of the backsight length (the sum of the upper and lower differences) is 16.8 m. The average of the three backsight readings on BM *A*, 0.6337 m, agrees within 0.0007 m of the middle reading. The stadia foresight length of 15.9 m at this setup is within 0.9 m of the backsight length, and is satisfactory. The HI (104.4769 m) for the first setup is found by adding the backsight reading to the elevation of BM A. Subtracting the foresight reading on TP1 gives its elevation (103.4256 m). This process is repeated for each setup.

THREE-WIRE LEVELING
TAYLOR LAKE ROAD

Sta.	+ Sight	Stadia	− Sight	Stadia	Elev.
BM A					103.8432
	0.718		1.131		
	0.633	8.5	1.051	8.0	+0.6337
	0.550	8.3	0.972	7.9	104.4769
3	)1.901	16.8 3	)3.154	15.9	−1.0513
	+0.6337		−1.0513		
TP1					103.4256
	1.151		1.041		
	1.082	6.9	0.969	7.2	+1.0820
	1.013	6.9	0.897	7.2	104.5076
3	)3.246	13.8 3	)2.907	14.4	−0.9690
	+1.0820		−0.9690		
TP2					103.5386
	1.908		1.264		
	1.841	6.7	1.194	7.0	+1.8410
	1.774	6.7	1.123	7.1	105.3796
3	)5.523	13.4 3	)3.581	14.1	−1.1937
	+1.8410		−1.1937		
BM B					104.1859
Σ	+3.5567	Σ	−3.2140		Check
Page	Check:				
	103.8432	+3.5567	−3.2140	= 104.1	859

Figure 5.10 Sample field notes for three-wire leveling.

■ 5.9 PROFILE LEVELING

Before engineers can properly design linear facilities such as highways, railroads, transmission lines, aqueducts, canals, sewers, and water mains, they need accurate information about the topography along the proposed routes. Profile leveling, which yields elevations at definite points along a reference line, provides the needed data. The subsections that follow discuss topics pertinent to profile leveling and include staking and stationing the reference line, field procedures for profile leveling, and drawing and using the profile.

5.9.1 Staking and Stationing the Reference Line

Depending on the particular project, the reference line may be a single straight segment, as in the case of a short sewer line; a series of connected straight segments which change direction at angle points, as with transmission lines; or straight segments joined by curves, which occur with highways and railroads. The required alignment for any proposed facility will normally have been selected as the result of a preliminary design, which is usually based on a study of existing maps and aerial photos. The reference alignment will most often be the proposed construction centerline, although frequently offset reference lines are used.

To stake the proposed reference line, key points such as the starting and ending points and angle points will be set first. Then intermediate stakes will be placed on line, usually at 100-ft intervals if the English system of units is used, but sometimes at closer spacing. If the metric system is used, stakes are usually placed at 10-, 20-, 30-, or 40-m spacing, depending on conditions. Distances for staking can be taped, or measured using the electronic distance measuring (EDM) component of a total station instrument operating in its tracking mode (see Sections 8.2 and 23.9).

In route surveying, a system called *stationing* is used to specify the relative horizontal position of any point along the reference line. The starting point is usually designated with some arbitrary value, for example in the English system of units, 10 + 00 or 100 + 00, although 0 + 00 can be used. If the beginning point was 10 + 00, a stake 100 ft along the line from it would be designated 11 + 00, the one 200 ft along the line 12 + 00, etc. The term *full station* is applied to each of these points set at 100-ft increments. This is the usual increment staked in rural areas. A point located between two full stations, say 84.90 ft beyond station 17 + 00, would be designated 17 + 84.90. Thus, locations of intermediate points are specified by their nearest preceding full station and their so-called *plus*. For station 17 + 84.90, the plus is 84.90. If the metric system is used, full stations are 1 km (1000 m) apart. The starting point of a reference line might be arbitrarily designated as 1 + 000 or 10 + 000, but again 0 + 000 could be used. In rural areas, intermediate points are normally set at 30- or 40-m increments along the line, and are again designated by their pluses. If the beginning point was 1 + 000, and stakes were being set at 40-m intervals, then 1 + 040, 1 + 080, 1 + 120, etc. would be set.

In rugged terrain and in urban situations, stakes are normally set closer together, for example at *half stations* (50-ft increments) or even *quarter stations*

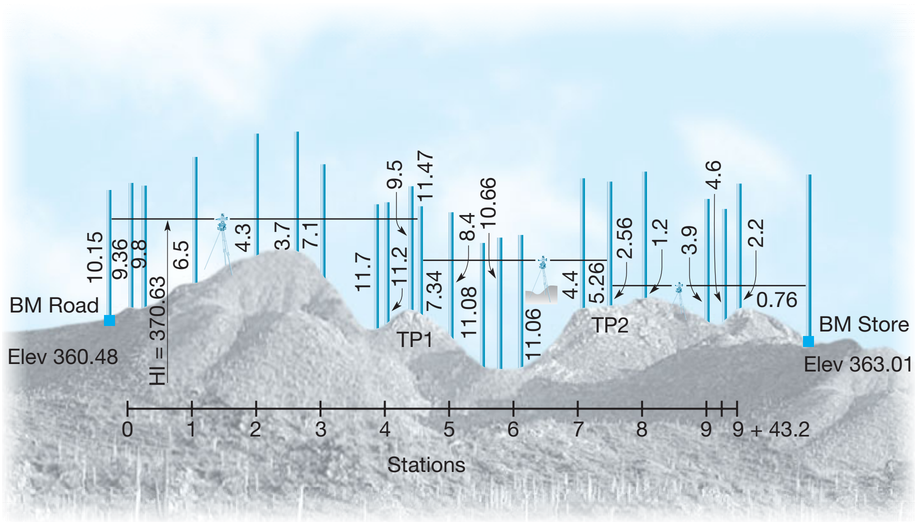

Figure 5.11
Profile leveling.

(25-ft increments) in the English system of units. In the metric system, 20-, 10-, or even 5-m increments may be staked.

Stationing not only provides a convenient unambiguous method for specifying positions of points along the reference line, it also gives the distances between points. For example, in the English system stations 24 + 18.3 and 17 + 84.9 are (2418.3 − 1784.9), or 633.4 ft, apart, and in the metric system stations 1 + 120 and 2 + 040 are 920 m apart.

5.9.2 Field Procedures for Profile Leveling

Profile leveling consists simply of differential leveling with the addition of intermediate minus sights (foresights) taken at required points along the reference line. Figure 5.11 illustrates an example of the field procedure, and the notes in Figure 5.12 relate to this example. Stationing for the example is in feet. As shown in the figure, the leveling instrument is initially set up at a convenient location and a plus sight of 10.15 ft taken on the benchmark. Adding this to the benchmark elevation yields a HI of 370.63 ft. Then intermediate minus sights are taken on points along the profile at stations as 0 + 00, 0 + 20, 1 + 00, etc. (If the reference line's beginning is far removed from the benchmark, differential levels running through several turning points may be necessary to get the instrument into position to begin taking intermediate minus sights on the profile line.) Notice that the note form for profile leveling contains all the same column headings as differential leveling, but is modified to include another column labeled "Intermediate Sight."

When distances to intermediate sights become too long, or if terrain variations or vegetations obstruct rod readings ahead, the leveling instrument must be moved. Establishing a turning point, as TP1 in Figure 5.11, does this. After reading

PROFILE LEVELS						BM ROAD to BM STORE	
Station	+ Sight	HI	− Sight	Int. Sight	Elev.		
BM Road	10.15	(370.62) 370.63			360.48	BM Road 3 miles SW of Mpls. 200 yrds. N of Pine St. over pass 40ft. E of ℄Hwy. 169 Top of RW conc post No.268.	SW Minneapolis on Hwy 169 6 Oct. 2000
0+00				9.36	361.26	℄Hwy. 169, painted "X"	Cool, Sunny, 50° F
0+20				9.8	360.8	West drainage ditch	R.J. Hintz N
1+00				6.5	364.1		N.R. Olson ϕ
2+00				4.3	366.3		R.C. Perry ⊼
2+60				3.7	366.9	Summit	Wild Level #3
3+00				7.1	363.5		
3+90				11.7	358.9	Sag	
4+00				11.2	359.4		COPY
4+35		(366.48)		9.5	361.1	Summit	
TP1	7.34	366.50	11.47		359.16		
5+00				8.4	358.1		Page Check:
5+54				11.08	355.40	E gutter, Maple St.	+20.05
5+74				10.66	355.82	℄Maple St.	−17.49
5+94				11.06	355.42	W gutter, Maple St.	+ 2.56
6+00				10.5	356.0		360.48
7+00		(362.77)		4.4	362.1		363.04
TP2	2.56	368.80	5.26		361.24		
8+00				1.2	362.6	Summit 363.04-363.01=	Misclosure = 0.03
9+00				3.9	359.9		
9+25.2				3.4	360.4	Top of E curb, Elm St.	
9+25.3				4.6	359.2	Bottom of E curb, Elm St.	
9+43.2				2.2	361.6	℄ Elm St.	
BM Store			0.76		363.04	BM Store. NE corner Elm St. & 4th Ave. SE corner	
Σ	20.05		17.49		(363.01)	Store foundation wall. 3″ brass disc set in grout. BM store elev. = 363.01	R.J. Hintz

Figure 5.12
Profile leveling notes for Figure 5.11.

a minus sight on the turning point, the instrument is moved ahead to a good vantage point both for reading the backsight on the turning point, as well as to take additional rod readings along the profile line ahead. The instrument is leveled, the plus sight taken on TP1, the new HI computed, and further intermediate sights taken. This procedure is repeated until the profile is completed.

Whether the stationing is in feet or meters, intermediate sights are usually taken at all full stations. If stationing is in feet and the survey area is in rugged terrain or in an urban area, the specifications may require that readings also be taken at half- or even quarter-stations. If stationing is in meters, depending on conditions, intermediate sights may be taken at 40-, 30-, 20-, or 10-m increments. In any case, sights are also taken at high and low points along the alignment, as well as at changes in slope.

Intermediate sights should always be taken on "critical" points such as railroad tracks, highway centerlines, gutters, and drainage ditches. As presented in

Figure 5.12, rod readings are normally only taken to the nearest 0.1 ft (English system) or nearest cm (metric system) where the rod is held on the ground, but on critical points, and for all plus and minus sights taken on turning points and benchmarks, the readings are recorded to the nearest hundredth of a foot (English) or the nearest mm (metric).

In profile leveling, lengths of intermediate minus sights vary, and in general they will not equal the plus sight length. Thus errors due to an inclined line of sight and to curvature and refraction will occur. Because errors from these sources increase with increasing sight lengths, on important work the instrument's condition of adjustment should be checked (see Section 4.15), and excessively long intermediate foresight distances should be avoided.

Instrument heights (HIs) and elevations of all turning points are computed immediately after each plus sight and minus sight. However, elevations for intermediate minus sights are not computed until after the circuit is closed on either the initial benchmark or another. Then the circuit misclosure is computed, and if acceptable, an adjustment is made and elevations of intermediate points are calculated. The procedure is described in the following subsection.

As in differential leveling, the page check should be made for each left-hand sheet. However in profile leveling, intermediate minus sights play no part in this computation. As illustrated in Figure 5.12, the page check is made by adding the algebraic sum of the column of plus sights and the column of minus sights to the beginning elevation. This should equal the last elevation tabulated on the page for either a turning point or the ending benchmark if that is the case, as it is in the example of Figure 5.12.

5.9.3 Drawing and Using the Profile

Prior to drawing the profile, it is first necessary to compute elevations along the reference line from the field notes. However, this cannot be done until an adjustment has been made to distribute any misclosure in the level circuit. In the adjustment process, HIs are adjusted, because they will affect computed profile elevations. The adjustment is made progressively in proportion to the total number of HIs in the circuit. The procedure is illustrated in Figure 5.12, where the misclosure was 0.03 ft. Since there were three HIs, the correction applied to each is $-0.03/3 = -0.01$ ft per HI. Thus a correction of 0.01 was applied to the first HI, -0.02 ft to the second, and -0.03 ft to the third. Adjusted HIs are shown in Figure 5.12 in parentheses above their unadjusted values. It is unnecessary to correct turning point elevations since they are of no consequence. After adjusting the HIs, profile elevations are computed by subtracting intermediate minus sights from their corresponding *adjusted* HIs. The profile is then drawn by plotting elevations on the ordinate versus their corresponding stations on the abscissa. By connecting adjacent plotted points, the profile is realized.

Until recently, profiles were manually plotted, usually on special paper like the type shown in Figure 5.13. Now with computer-aided drafting and design (CADD) systems (see Section 18.14), it is only necessary to enter the stations and elevations into the computer, and this special software will plot and display the profile on the screen. Hard copies, if desired, may be obtained from plotters interfaced with a

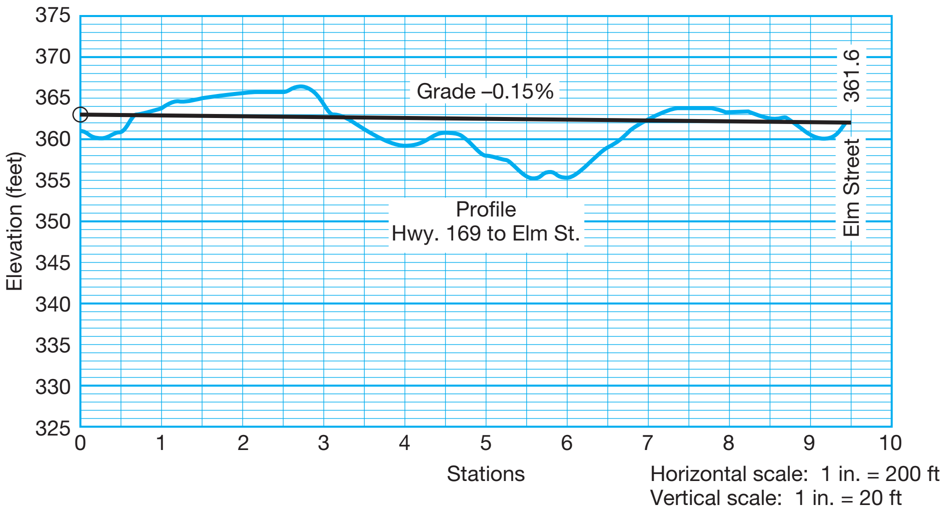

Figure 5.13 Plot of profile.

computer. Often these profiles are generated automatically from the CADD software using only the alignment of the structure and an overlaying topographic map.

In drawing profiles, the vertical scale is generally exaggerated with respect to the horizontal scale to make differences in elevation more pronounced. A ratio of 10:1 is frequently used, but flatness or roughness of the terrain determines the desirable proportions. Thus, for a horizontal scale of 1 in. = 100 ft, the vertical scale might be 1 in. = 10 ft. The scale actually employed should be plainly marked. Plotted profiles are used for many purposes, such as (1) determining depth of cut or fill on proposed highways, railroads, and airports; (2) studying grade-crossing problems; and (3) investigating and selecting the most economical grade, location, and depth for sewers, pipelines, tunnels, irrigation ditches, and other projects.

The *rate of grade* (or *gradient* or *percent grade*) is the rise or fall in feet per 100 ft, or in meters per 100 m. Thus a grade of 2.5% means a 2.5-ft difference in elevation per 100 ft horizontally. Ascending grades are plus; descending grades, minus. A gradeline of -0.15%, chosen to approximately equalize cuts and fills, is shown in Figure 5.13. Along this grade line, elevations drop at the rate of 0.15 ft per 100 ft. The grade begins at station $0 + 00$ where it approximately meets existing ground at elevation 363.0 ft, and ends at station $9 + 43$ and elevation 361.6 ft where again it approximately meets existing ground. The process of staking grades is described in Chapter 23.

The term *grade* is also used to denote the elevation of the finished surface on an engineering project.

■ 5.10 GRID, CROSS-SECTION, OR BORROW-PIT LEVELING

Grid leveling is a method for locating contours (see Section 17.9.3). It is accomplished by staking an area in squares of 10, 20, 50, 100, or more feet (or comparable meter lengths) and determining the corner elevations by differential leveling.

Rectangular blocks, say 50 by 100 ft or 20 by 30 m, that have the longer sides roughly parallel with the direction of most contour lines may be preferable on steep slopes. The grid size chosen depends on the project extent, ground roughness, and accuracy required.

The same process, termed *borrow-pit leveling*, is employed on construction jobs to ascertain quantities of earth, gravel, rock, or other material to be excavated or filled. The procedure is covered in Section 26.10 and Plate B.2.

5.11 USE OF THE HAND LEVEL

A hand level can be used for some types of leveling when a low order of accuracy is sufficient. The instrument operator takes a plus and minus sight while standing in one position, and then moves ahead to repeat the process. A hand level is useful, for example, in cross-sectioning to obtain a few additional rod readings on sloping terrain where a turning point would otherwise be required.

5.12 SOURCES OF ERROR IN LEVELING

All leveling measurements are subject to three sources of error: (1) instrumental, (2) natural, and (3) personal. These are summarized in the subsections that follow.

5.12.1 Instrumental Errors

Line of Sight. As described in Section 4.15, a properly adjusted leveling instrument that employs a level vial should have its line of sight and level vial axis parallel. Then, with the bubble centered, a horizontal plane, rather than a conical surface, is generated as the telescope is revolved. Also, if the compensators of automatic levels are operating properly, they should always produce a truly horizontal line of sight. If these conditions are not met, a *line of sight* (or *collimation*) error exists, and serious errors in rod readings can result. These errors are systematic, but they are canceled in differential leveling if the horizontal lengths of plus and minus sights are kept equal. The error may be serious in going up or down a steep hill where all plus sights are longer or shorter than all minus sights, unless care is taken to run a zigzag line. The size of the collimation error, ε, can be determined in a simple field procedure [see Equation (4.14) and Section 4.15.5]. If backsights and foresights cannot be balanced, a correction for this error can be made.

To apply the collimation correction, the value of ε from Equation (4.14) is divided by the length of the spaces between adjacent stakes in Figure 4.20. This yields the *collimation correction factor* in units of feet per foot, or meters per meter. Then for any backsight or foresight, the correction to be subtracted from the rod reading is obtained by multiplying the length of the sight by this correction factor. As an example, suppose that the distance between stakes in Example 4.3 was 100 ft. Then the collimation correction factor is $0.010/100 = 0.0001$ ft/ft. Suppose that a reading of 5.29 ft was obtained on a backsight of 200 ft length with this instrument. The corrected rod reading would then be $5.29 - (200 \times 0.0001) = 5.27$.

Cross hair Not Exactly Horizontal. Reading the rod near the center of the horizontal crosshair will eliminate or minimize this potential error.

Rod Not Correct Length. Inaccurate divisions on a rod cause errors in observed elevation differences similar to those resulting from incorrect markings on a measuring tape. Uniform wearing of the rod bottom makes HI values too large, but the effect is canceled when included in both plus and minus sights. Rod graduations should be checked by comparing them with those on a standardized tape.

Tripod Legs Loose. Tripod leg bolts that are too loose or too tight allow movement or strain that affects the instrument head. Loose metal tripod shoes cause unstable setups.

5.12.2 Natural Errors

Curvature of the Earth. As noted in Section 4.4, a level surface curves away from a horizontal plane at the rate of 0.667 M^2 or 0.0785 K^2, which is about 0.7 ft/mi or 8 cm/km. The effect of curvature of the earth is to increase the rod reading. Equalizing lengths of plus and minus sights in differential leveling cancels the error due to this cause.

Refraction. Light rays coming from an object to the telescope are bent, making the line of sight a curve concave to the earth's surface, which thereby decreases rod readings. Balancing the lengths of plus and minus sights usually eliminates errors due to refraction. However, large and sudden changes in atmospheric refraction may be important in precise work. Although, errors due to refraction tend to be random over a long period of time, they could be systematic on one day's run.

Temperature Variations. Heat causes leveling rods to expand, but the effect is not important in ordinary leveling. If the level vial of a tilting level is heated, the liquid expands and the bubble shortens. This does not produce an error (although it may be inconvenient), unless one end of the tube is warmed more than the other, and the bubble therefore moves. Other parts of the instrument warp because of uneven heating, and this distortion affects the adjustment. Shading the level by means of a cover when carrying it, and by an umbrella when it is set up, will reduce or eliminate heat effects. These precautions are followed in precise leveling.

Air boiling or heat waves near the ground surface or adjacent to heated objects make the rod appear to wave and prevent accurate sighting. Raising the line of sight by high tripod setups, taking shorter sights, avoiding any that pass close to heat sources (such as buildings and stacks), and using the lower magnification of a variable-power eyepiece reduce the effect.

Wind. Strong wind causes the instrument to vibrate and makes the rod unsteady. Precise leveling should not be attempted on excessively windy days.

Settlement of the Instrument. Settlement of the instrument during the time between a plus sight reading and a minus sight makes the latter too small and therefore the recorded elevation of the next point too high. The error is cumulative in a series of setups on soft material. Therefore setups on spongy ground, blacktop, or ice should be avoided if possible, but if they are necessary, unusual care is required to reduce the resulting errors. This can include taking readings in quick order, using two rods and two observers to preclude walking around the instrument, and alternating the order of taking plus and minus sights. Additionally whenever possible, the instrument tripod's legs can be set on long hubs that are driven to refusal in the soft material.

Settlement of a Turning Point. This condition causes an error similar to that resulting from settlement of the instrument. It can be avoided by selecting firm, solid turning points or, if none are available, using a steel turning pin set firmly in the ground. A railroad spike can also be used in most situations.

5.12.3 Personal Errors

Bubble Not Centered. In working with levels that employ level vials, errors caused by the bubble not being exactly centered at the time of sighting are the most important of any, particularly on long sights. If the bubble runs between the plus and minus sights, *it must be recentered before the minus sight is taken*. Experienced observers develop the habit of checking the bubble before and after each sight, a procedure simplified with some instruments, which have a mirror-prism arrangement permitting a simultaneous view of the level vial and rod.

Parallax. Parallax caused by improper focusing of the objective or eyepiece lens results in incorrect rod readings. Careful focusing eliminates this problem.

Faulty Rod Readings. Incorrect rod readings result from parallax, poor weather conditions, long sights, improper target settings, and other causes, including mistakes such as those due to careless interpolation and transposition of figures. Short sights selected to accommodate weather and instrument conditions reduce the magnitude of reading errors. If a target is used, the rodperson should read the rod, and the observer should check it independently.

Rod Handling. Using a rod level that is in adjustment, or holding the rod parallel to a plumb bob string eliminates serious errors caused by improper plumbing of the rod. Banging the rod on a turning point for the second (plus) sight may change the elevation of a point.

Target Setting. If a target is used, it may not be clamped at the exact place signaled by the observer because of slippage. A check sight should always be taken after the target is clamped.

5.13 MISTAKES

A few common mistakes in leveling are listed here.

Improper Use of a Long Rod. If the vernier reading on the back of a damaged Philadelphia rod with English units is not exactly 6.500 ft or 7.000 ft for the short rod, the target must be set to read the same value before extending the rod.

Holding the Rod in Different Places for the Plus and Minus Sights on a Turning Point. The rodperson can avoid such mistakes by using a well-defined point or by outlining the rod base with lumber crayon, keel, or chalk.

Reading a Foot Too High. This mistake usually occurs because the incorrect footmark is in the telescope's field of view near the cross line; for example, an observer may read 5.98 instead of 4.98. Noting the footmarks both above and below the horizontal cross line will prevent this mistake.

Waving a Flat Bottom Rod while Holding It on a Flat Surface. This action produces an incorrect rod reading because rotation is about the rod edges instead of the center or front face. In precise work, plumbing with a rod level, or other means, is preferable to waving. This procedure also saves time.

Recording Notes. Mistakes in recording, such as transposing figures, entering values in the wrong column, and making arithmetic mistakes, can be minimized by having the notekeeper repeat the value called out by an observer, and by making the standard field-book checks on rod sums and elevations. Digital levels that automatically take rod readings, store the values, and compute the level notes can eliminate these mistakes.

Touching Tripod or Instrument during the Reading Process. Beginners using instruments that employ level vials may center the bubble, put one hand on the tripod or instrument while reading a rod, and then remove the hand while checking the bubble, which has now returned to center but was off during the observation. Of course, the instrument should not be touched when taking readings, but detrimental effects of this bad habit are practically eliminated when using automatic levels.

5.14 REDUCING ERRORS AND ELIMINATING MISTAKES

Errors in running levels are reduced (but never eliminated) by carefully adjusting and manipulating both instrument and rod (see Section 4.15 for procedures) and establishing standard field methods and routines. The following routines prevent most large errors or quickly disclose mistakes: (1) checking the bubble before and after each reading (if an automatic level is not being used), (2) using a rod level, (3) keeping the horizontal lengths of plus and minus sights equal, (4) running lines forward and backward, (5) making the usual field-book arithmetic checks, and (6) breaking long leveling circuits into smaller sections.

5.15 USING SOFTWARE

On the companion website for this book at http://www.pearsonhighered.com/ghilani is the software WOLFPACK. In this software is an option that takes the plus and minus readings from a simple leveling circuit to create a set of field notes and the file appropriate for a least-squares adjustment of the data (see Section 16.6). A sample file of the field notes from Figure 5.5 is depicted in Figure 5.14.

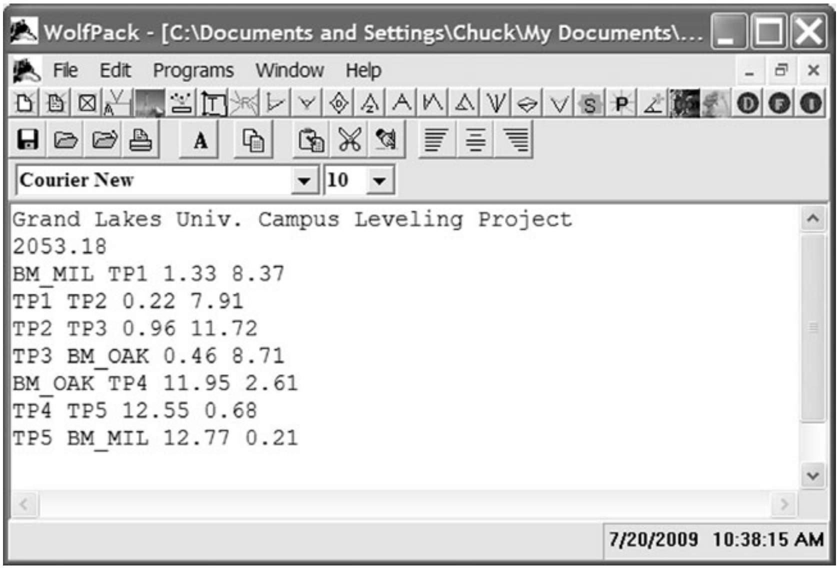

Figure 5.14 Sample data file for field notes in Figure 5.5.

The software limits the length of the station identifiers to 10 characters. These characters must not include a space, comma, or tab since these are used as data delimiters in the file. All benchmark stations must start with the letters *BM*, while all turning points must start with the letters *TP*. This is used by the software to differentiate between a benchmark and a turning point in the data file.

While the format of the file is explained fully in the WOLFPACK help system, it will be presented here as an aid to the reader. The first line of the file shown in Figure 5.14 is a title line, which in this case is "Grand Lakes Univ. Campus Leveling Project." The second line contains starting and ending benchmark elevations. Since this line starts and ends on the same benchmark (BM_MIL), its elevation of 2053.18 need be listed only once. If a level circuit starts at one benchmark, but closes on another, then both the starting and ending elevations of the leveling circuit should be listed on this line. The remainder of the file contains the plus and minus sights between each set of stations. Thus each line contains the readings from one instrument setup. For example, a plus sight of 1.33 was made on BM_MIL and a minus sight of 8.37 was made on TP1, which is the first turning point. Each instrument setup is listed in order following the same procedure. Once the file is created and saved using the WOLFPACK editor, it can be read into the option *Reduction of differential leveling notes* as shown in Figure 5.15. The software then creates notes similar to those shown in Figure 5.5 adjusting the elevations, and demonstrating a page check.

For those who are interested in higher-level programming, the Mathcad® worksheet *C5.xmcd* is available on the companion website for this book at http://www.pearsonhighered.com/ghilani. This worksheet reads a text file of observations that are obtained typically in differential leveling and creates and adjusts the data placing the results in a format typically found in a field book. Additionally, the Excel® spreadsheet *C5.xls* demonstrates how a spreadsheet can be used to reduce the notes in Figure 5.5.

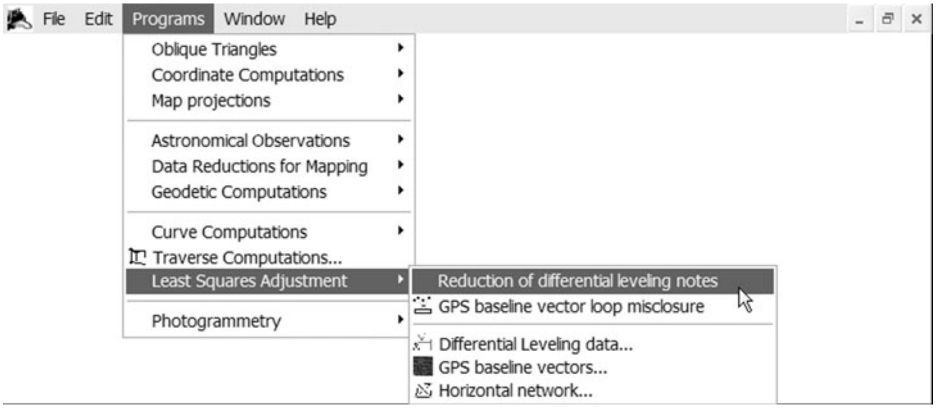

Figure 5.15 Option in WOLFPACK to reduce data file in Figure 5.14.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G .

5.1 What proper field procedures can virtually eliminate Earth curvature and refraction errors in differential leveling?

5.2 Why is it advisable to set up a level with all three tripod legs on, or in, the same material (concrete, asphalt, soil), if possible?

5.3 Explain how a stable setup of the level may be achieved on soft soil such as in a swamp.

5.4 Discuss how errors due to lack of instrument adjustment can be practically eliminated in running a line of differential levels.

5.5 Why is it preferable to use a rod level when plumbing the rod?

5.6 Why are double-rodded lines of levels recommended for precise work?

5.7 List four considerations that govern a rodperson's selection of TPs and BMs.

5.8* What error is created by a rod leaning 10 min from plumb at a 5.513 m reading on the leaning rod?

5.9 Similar to Problem 5.8, except for a 12-ft reading.

5.10 What error results on a 50-m sight with a level if the rod reading is 1.505 m but the top of the 3-m rod is 0.3 m out of plumb?

5.11 What error results on a 200-ft sight with a level if the rod reading is 6.307 ft but the top of the 7-ft rod is 0.2 ft out of plumb?

5.12 Prepare a set of level notes for the data listed. Perform a check and adjust the misclosure. Elevation of BM 7 is 852.045 ft. If the total loop length is 2000 ft, what order of leveling is represented? (Assume all readings are in feet)

Point		−S (FS)
BM 7	9.432	
TP1	6.780	8.363
BM 8	7.263	9.822
TP2	3.915	9.400
TP3	7.223	5.539
BM 7		1.477

5.13 Similar to Problem 5.12, except the elevation of BM 7 is 306.928 m and the loop length 2 km. (Assume all readings are in meters.)

5.14 A differential leveling loop began and closed on BM Tree (elevation 654.07 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 5.06 (+S) on BM Tree, 8.99 (–S) and 7.33 (+S) on TP1, 2.52 (–S) and 4.85 (+S) on BM *X*, 3.61 (–S) and 5.52 (+S) on TP2, and 7.60 (–S) on BM Tree. Prepare, check, and adjust the notes.

5.15 A differential leveling circuit began on BM Hydrant (elevation 1823.65 ft) and closed on BM Rock (elevation 1841.71 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) given in the order taken are 8.04 (+S) on BM Hydrant, 5.63 (–S) and 6.98 (+S) on TP1, 2.11 (–S) and 9.05 (+S) on BM 1, 3.88 (–S) and 5.55 (+S) on BM 2, 5.75 (–S) and 10.44 (+S) on TP2, and 4.68 (–S) on BM Rock. Prepare, check, and adjust the notes.

5.16 A differential leveling loop began and closed on BM Bridge (elevation 103.895 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 1.023 (+S) on BM Bridge, 1.208 (–S) and 0.843 (+S) on TP1, 0.685 (–S) and 0.982 (+S) on BM *X*, 0.944 (–S) and 1.864 (+S) on TP2, and 1.879 (–S) on BM Bridge. Prepare, check, and adjust the notes.

5.17 A differential leveling circuit began on BM Rock (elevation 243.897 m) and closed on BM Manhole (elevation 240.100 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 0.288 (+S) on BM Rock, 0.987 (–S) and 0.305 (+S) on TP1, 1.405 (–S) and 0.596 (+S) on BM 1, 1.605 (–S) and 0.661 (+S) on BM 2, 1.992 (–S) and 1.056 (+S) on TP2, and 0.704 (–S) on BM Manhole. Prepare, check, and adjust the notes.

5.18 A differential leveling loop started and closed on BM Juno, elevation 5007.86 ft. The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 3.00 (+S) on BM Juno, 8.14 (–S) and 5.64 (+S) on TP1, 3.46 (–S) and 6.88 (+S) on TP2, 10.27 (–S) and 8.03 (+S) on BM1, 4.17 (–S) and 7.86 (+S) on TP3, and 5.47 (–S) on BM Juno. Prepare, check, and adjust the notes.

5.19* A level setup midway between *X* and *Y* reads 6.29 ft on *X* and 7.91 ft on *Y*. When moved within a few feet of *X*, readings of 5.18 ft on *X* and 6.76 ft on *Y* are recorded. What is the true elevation difference, and the reading required on *Y* to adjust the instrument?

5.20 To test its line of sight adjustment, a level is set up near *C* (elev 193.436 m) and then near *D*. Rod readings listed in the order taken are $C = 1.256$ m, $D = 1.115$ m, $D = 1.296$ m, and $C = 1.151$ m. Compute the elevation of *D*, and the reading required on *C* to adjust the instrument.

5.21* The line of sight test shows that a level's line of sight is inclined downward 3 mm/50 m. What is the allowable difference between BS and FS distances at each setup (neglecting curvature and refraction) to keep elevations correct within 1 mm?

5.22 Reciprocal leveling gives the following readings in meters from a setup near *A:* on *A*, 2.558; on *B*, 1.883, 1.886, and 1.885. At the setup near *B:* on *B*, 1.555; on *A*, 2.228, 2.226, and 2.229. The elevation of *A* is 158.618 m. Determine the misclosure and elevation of *B*.

5.23* Reciprocal leveling across a canyon provides the data listed (in meters). The elevation of *Y* is 2265.879 ft. The elevation of *X* is required. Instrument at *X*: $+S = 3.182$, $-S = 9.365$, 9.370, and 9.368. Instrument at *Y*: $+S = 10.223$; $-S = 4.037$, 4.041, and 4.038.

5.24 Prepare a set of three-wire leveling notes for the data given and make the page check. The elevation of BM *X* is 106.101 m. Rod readings (in meters) are (*H* denotes upper cross-wire readings, *M* middle wire, and *L* lower wire): +S on BM *X:* $H = 0.965$, $M = 0.736$, $L = 0.507$; –S on TP1: $H = 1.594$, $M = 1.341$, $L = 1.088$;

+S on TP1: $H = 1.876$, $M = 1.676$, $L = 1.476$; –S on BM *Y*: $H = 1.437$, $M = 1.240$, $L = 1.043$.

5.25 Similar to Problem 5.24, except the elevation of BM *X* is 638.437 ft, and rod readings (in feet) are: +S on BM *X*: $H = 4.329$, $M = 3.092$, $L = 1.855$; –S on TP1: $H = 6.083$, $M = 4.918$, $L = 3.753$; +S on TP1: $H = 7.834$, $M = 6.578$, $L = 5.321$; –S on BM *Y*: $H = 4.674$, $M = 3.367$, $L = 2.060$.

5.26 Assuming a stadia constant of 99.987, what is the distance leveled in Problem 5.24?

5.27 Assuming a stadia constant of 101.5, what is the distance leveled in Problem 5.25?

5.28 Prepare a set of profile leveling notes for the data listed and show the page check. All data are given in feet. The elevation of BM *A* is 1364.58, and the elevation of BM *B* is 1349.26. Rod readings are: +S on BM *A*, 2.86 intermediate foresight (IFS) on 11+00, 3.7; –S on TP1, 10.56; +S on TP1, 11.02; intermediate foresight on 12+00, 8.7; on 12+50, 6.5; on 13+00, 5.7; on 14+00, 6.3; −S on TP2, 9.15; +S on TP2, 4.28; intermediate foresight on 14+73, 3.5; on 15+00, 4.2; on 16+00, 6.4; –S on TP3, 8.77; +S on TP3, 4.16; –S on BM *B*, 9.08.

5.29 Same as Problem 5.28, except the elevation of BM *A* is 438.96 ft, the elevation of BM *B* is 427.32 ft, and the +S on BM *A* is 6.56 ft.

5.30 Plot the profile Problem 5.28 and design a grade line between stations 11 + 00 and 16 + 00 that balances cut and fill areas.

5.31* What is the percent grade between stations 11 + 00 and 16 + 00 in Problem 5.28?

5.32 Differential leveling between BMs *A, B, C, D*, and *A* gives elevation differences (in meters) of −6.352, +12.845, +9.241, and −15.717, and distances in km of 0.6, 1.0, 1.3, and 0.5, respectively. If the elevation of *A* is 886.891, compute the adjusted elevations of BMs *B, C*, and *D*, and the order of leveling.

5.33 Leveling from BM *X* to *W*, BM *Y* to *W*, and BM *Z* to *W* gives differences in elevation (in feet) of −30.24, +26.20, and +10.18, respectively. Distances between benchmarks are $XW = 3500$, $YW = 2700$, and $ZW = 4500$. True elevations of the benchmarks are $X = 460.82$, $Y = 404.36$, and $Z = 420.47$. What is the adjusted elevation of *W*? (Note: All data are given in feet.)

5.34 A 3-m level rod was calibrated and its graduated scale was found to be uniformly expanded so that the distance between its 0 and 3.000 marks was actually 3.006 m. How will this affect elevations determined with this rod for (**a**) circuits run on relatively flat ground (**b**) circuits run downhill (**c**) circuits run uphill?

5.35* A line of levels with 42 setups (84 rod readings) was run from BM Rock to BM Pond with readings taken to the nearest 3.0 mm; hence any observed value could have an error of ±1.5 mm. For reading errors only, what total error would be expected in the elevation of BM Pond?

5.36 Same as Problem 5.35, except for 64 setups and readings to the nearest 0.01 ft with possible error of ±0.005 ft each.

5.37 Compute the permissible misclosure for the following lines of levels: (**a**) a 10-km loop of third-order levels (**b**) a 20-km section of second-order class I levels (**c**) a 40-km loop of first-order class I levels.

5.38 Create a computational program that solves Problem 5.12.

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6 Distance Measurement

PART I • METHODS FOR MEASURING DISTANCES

■ 6.1 INTRODUCTION

Distance measurement is generally regarded as the most fundamental of all surveying observations. In traditional ground surveys, even though many angles may be read, the length of at least one line must be measured to supplement the angles in locating points. In plane surveying, the distance between two points means the horizontal distance. If the points are at different elevations, the distance is the horizontal length between vertical lines at the points.

Lengths of lines may be specified in different units. In the United States, the foot, decimally divided, is usually used although the meter is becoming increasingly more common. Geodetic surveys, and many highway surveys employ the meter. In architectural and machine work, and on some construction projects, the unit is a foot divided into inches and fractions of an inch. As discussed in Section 2.2, chains, varas, rods, and other units have been, and still are, utilized in some localities and for special purposes.

■ 6.2 SUMMARY OF METHODS FOR MAKING LINEAR MEASUREMENTS

In surveying, linear measurements have been obtained by many different methods. These include (1) pacing, (2) odometer readings, (3) optical rangefinders, (4) tacheometry (stadia), (5) subtense bars, (6) taping, (7) electronic distance measurement (EDM), (8) satellite systems, and others. Of these, surveyors most commonly use taping, EDM, and satellite systems today. In particular, the satellite-supported *Global Navigation Satellite Systems* (GNSS) are rapidly replacing all other systems due to many advantages, but most notably because of their range, accuracy, and efficiency. Methods (1) through (5) are discussed briefly in the following sections. Taping is discussed in Part II of this chapter, and EDM is described in Part III of this chapter. Satellite systems are described in Chapters 13, 14, and 15.

Triangulation is a method for determining positions of points from which horizontal distances can be computed (see Section 19.12.1). In this procedure, lengths of lines are computed trigonometrically from measured baselines and angles. *Photogrammetry* can also be used to obtain horizontal distances. This topic is covered in Chapter 27. Besides these methods, distances can be estimated, a technique useful in making field note sketches and checking observations for mistakes. With practice, estimating can be done quite accurately.

■ 6.3 PACING

Distances obtained by pacing are sufficiently accurate for many purposes in surveying, engineering, geology, agriculture, forestry, and military field sketching. Pacing is also used to detect blunders that may occur in making distance observations by more accurate methods.

Pacing consists of counting the number of steps, or paces, in a required distance. The length of an individual's pace must be determined first. This is best done by walking with natural steps back and forth over a level course at least 300 ft long, and dividing the known distance by the average number of steps. For short distances, the length of each pace is needed, but the number of steps taken per 100 ft is desirable for checking long lines.

It is possible to adjust one's pace to an even 3 ft, but a person of average height finds such a step tiring if maintained for very long. The length of an individual's pace varies when going uphill or downhill and changes with age. For long distances, a pocket instrument called a *pedometer* can be carried to register the number of paces, or a *passometer* attached to the body or leg counts the steps. Some surveyors prefer to count *strides*, a stride being two paces.

Pacing is one of the most valuable things learned in surveying, since it has practical applications for everybody and requires no equipment. If the terrain is open and reasonably level, experienced pacers can measure distances of 100 ft or longer with an accuracy of 1/50 to 1/100 of the distance.

■ 6.4 ODOMETER READINGS

An odometer converts the number of revolutions of a wheel of known circumference to a distance. Lengths measured by an odometer on a vehicle are suitable for some preliminary surveys in route-location work. They also serve as rough checks on observations made by other methods. Other types of measuring wheels are available and useful for determining short distances, particularly on curved lines. Odometers give surface distances, which should be corrected to horizontal if the ground slopes severely (see Section 6.13). With odometers, an accuracy of approximately 1/200 of the distance is reasonable.

■ 6.5 OPTICAL RANGEFINDERS

These instruments operate on the same principle as rangefinders on single-lens reflex cameras. Basically, when focused, they solve for the object distance f_2 in Equation (4.12), where focal length f and image distance f_1 are known. An operator looks through the lens and adjusts the focus until a distant object viewed is focused in coincidence, whereupon the distance to that object is obtained. These instruments are capable of accuracies of 1 part in 50 at distances up to 150 ft, but accuracy diminishes as the length increases. They are suitable for reconnaissance, sketching, or checking more accurate observations for mistakes.

■ 6.6 TACHEOMETRY

Tacheometry (*stadia* is the more common term in the United States) is a surveying method used to quickly determine the horizontal distance to, and elevation of, a point. As discussed in Section 5.4, stadia observations are obtained by sighting through a telescope equipped with two or more horizontal cross wires at a known spacing. The apparent intercepted length between the top and bottom wires is read on a graduated rod held vertically at the desired point. The distance from telescope to rod is found by proportional relationships in similar triangles. An accuracy of 1/500 of the distance is achieved with reasonable care.

■ 6.7 SUBTENSE BAR

This indirect distance-measuring procedure involves using a theodolite to read the horizontal angle subtended by two targets precisely spaced at a fixed distance apart on a subtense bar. The unknown distance is computed from the known target spacing and the measured horizontal angle. Prior to observing the angle from one end of the line, the bar is centered over the point at the other end of the line, and oriented perpendicular to the line and in a horizontal plane. For sights of 500 ft (150 m) or shorter, and using a 1-in. theodolite, an accuracy of 1 part in 3000 or better can be achieved. Accuracy diminishes with increased line length. Besides only being suitable for relatively short lines, this method of distance measurement is time consuming and is seldom used today, having been replaced by electronic distance measurement.

PART II • DISTANCE MEASUREMENTS BY TAPING

■ 6.8 INTRODUCTION TO TAPING

Observation of horizontal distances by taping consists of applying the known length of a graduated tape directly to a line a number of times. Two types of problems arise: (1) observing an unknown distance between fixed points, such as between two stakes in the ground and (2) laying out a known or required distance with only the starting mark in place.

Taping is performed in six steps: (1) lining in, (2) applying tension, (3) plumbing, (4) marking tape lengths, (5) reading the tape, and (6) recording the distance. The application of these steps in taping on level and sloping ground is detailed in Sections 6.11 and 6.12.

6.9 TAPING EQUIPMENT AND ACCESSORIES

Over the years, various types of tapes and other related equipment have been used for taping in the United States. Tapes in current use are described here, as are other accessories used in taping.

Surveyor's and *engineer's tapes* are made of steel 1/4 to 3/8 in. wide and weigh 2 to 3 lbs/100 ft. Those graduated in feet are most commonly 100 ft long, although they are also available in lengths of 200, 300, and 500 ft. They are marked in feet, tenths and hundredths. Metric tapes have standard lengths of 30, 60, 100, and 150 m. All can either be wound on a reel [see Figure 6.1(a)] or done up in loops.

Invar tapes are made of a special nickel-steel alloy (35% nickel and 65% steel) to reduce length variations caused by differences in temperature. The thermal coefficient of expansion and contraction of this material is only about 1/30 to 1/60 that of an ordinary steel tape. However, the metal is soft and somewhat unstable. This weakness, along with the cost perhaps ten times that of steel tapes, made them suitable only for precise geodetic work and as a standard for comparison with working tapes. Another version, the *Lovar tape*, has properties and a cost between those of steel and Invar tapes.

Cloth (or *metallic*) *tapes* are actually made of high-grade linen, 5/8 in. wide with fine copper wires running lengthwise to give additional strength and prevent excessive elongation. Metallic tapes commonly used are 50, 100, and 200 ft long and come on enclosed reels [see Figure 6.1(b)]. Although not suitable for precise work, metallic tapes are convenient and practical for many purposes.

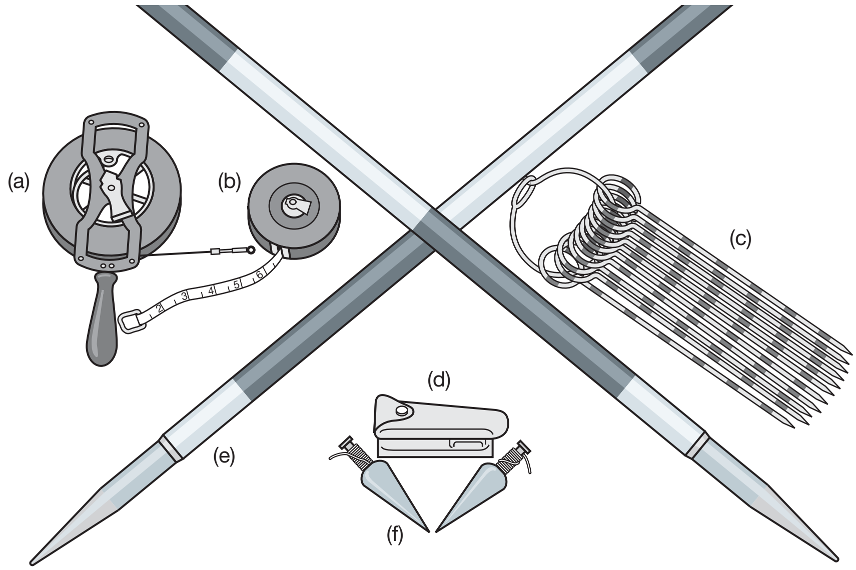

Figure 6.1
Taping equipment for field party.

Fiberglass tapes come in a variety of sizes and lengths and are usually wound on a reel. They can be employed for the same types of work as metallic tapes.

Chaining pins or *taping pins* are used to mark tape lengths. Most taping pins are made of number 12 steel wire, sharply pointed at one end, have a round loop at the other end, and are painted with alternate red and white bands [see Figure 6.1(c)]. Sets of 11 pins carried on a steel ring are standard.

The *hand level*, described in Section 4.13, is a simple instrument used to keep the tape ends at equal elevations when observing over rough terrain [see Figures 4.17 and 6.1(d)].

Tension handles facilitate the application of a desired standard or known tension. A complete unit consists of a wire handle, a clip to fit the ring end of the tape, and a spring balance reading up to 30 lb in 1/2-lb graduations.

Clamp handles are used to apply tension by a positive, quick grip using a scissors-type action on any part of a steel tape. They do not damage the tape and prevent injury to hands and the tape.

A *pocket thermometer* permits reading data for making temperature corrections. It is about 5 in. long, graduated from perhaps $-30°$ to $+120°F$ in $1°$ or $2°$ divisions, and kept in a protective metal case.

Range poles (lining rods) made of wood, steel, or aluminum are about 1 in. thick and 6 to 10 ft long. They are round or hexagonal in cross section and marked with alternate 1-ft long red and white bands that can be used for rough measurements [see Figure 6.1(e)]. The main utility of range poles is to mark the line being measured so that the tape's alignment can be maintained.

Plumb bobs for taping [see Figure 6.1(f)] should weigh a minimum of 8 oz and have a fine point. However, most surveyors use 24-oz plumb bobs for stability reasons. At least 6 ft of good-quality string or cord, free of knots, is necessary for convenient work with a plumb bob. The points of most plumb bobs are removable, which facilitates replacement if they become dull or broken. The string can be wound on a spring-loaded reel that is useful for rough targeting. However, in taping, it is best to not use a reel.

■ 6.10 CARE OF TAPING EQUIPMENT

The following points are pertinent in the care of tapes and range poles:

1. Considering the cross-sectional area of the average surveyor's steel tape and its permissible stress, a pull of 100 lb will do no damage. But if the tape is kinked, a pull of less than 1 lb can break it. Therefore, always check to be certain that any loops and kinks are eliminated before tension is applied.
2. If a tape gets wet, wipe it first with a dry cloth, then with an oily one.
3. Tapes should be either kept on a reel or "*thrown*" into circular loops, but not handled both ways.
4. Each tape should have an individual number or tag to identify it.
5. Broken tapes can be mended by riveting or applying a sleeve device, but a mended tape should not be used on important work.
6. Range poles are made with the metal shoe and point in line with the section above. This alignment may be lost if the pole is used improperly.

6.11 TAPING ON LEVEL GROUND

The subsections that follow describe six steps in taping on level ground using a tape.

6.11.1 Lining In

Using range poles, the line to be measured should be marked at both ends, and at intermediate points where necessary, to ensure unobstructed sight lines. Taping requires a minimum of two people, a *forward tapeperson* and a *rear tapeperson*. The forward tapeperson is lined in by the rear tapeperson. Directions are given by vocal or hand signals.

6.11.2 Applying Tension

The rear tapeperson holding the 100-ft end of a tape over the first (rear) point lines in while the forward tapeperson, holding the zero end of the tape. For accurate results the tape must be straight and the two ends held at the same elevation. A specified tension, generally between 10 and 25 lb, is applied. To maintain a steady pull, tapepersons wrap the leather thong at the tape's end around one hand, keep forearms against their bodies, and face at right angles to the line. In this position, they are off the line of sight. Also, the body need only be tilted to hold, decrease, or increase the pull. Sustaining a constant tension with *outstretched* arms is difficult, if not impossible, for a pull of 15 lb or more. Good communication between forward and rear tapepersons will avoid jerking the tape, save time, and produce better results.

6.11.3 Plumbing

Weeds, brush, obstacles, and surface irregularities may make it undesirable to lay a tape on the ground. In those cases, the tape is held above ground in a horizontal position. Placing the plumb-bob string over the proper tape graduation and securing it with one thumb, mark each end point on the tape. The rear tapeperson continues to hold a plumb bob over the fixed point, while the forward tapeperson marks the length. In measuring a distance shorter than a full tape length, the forward tapeperson moves the plumb-bob string to a point on the tape over the ground mark.

6.11.4 Marking Tape Lengths

When the tape has been lined in properly, tension has been applied, and the rear tapeperson is over the point, "stick" is called out. The forward tapeperson then places a pin exactly opposite the zero mark of the tape and calls "stuck." The marked point is checked by repeating the measurement until certainty of its correct location is assured.

After checking the measurement, the forward tapeperson signals that the point is OK, the rear tapeperson pulls up the rear pin, and they move ahead. The forward tapeperson drags the tape, paces roughly 100 ft, and stops. The rear tapeperson calls "tape" to notify the forward tapeperson that they have gone 100 ft just before the 100-ft end reaches the pin that has been set. The process of measuring 100-ft lengths is repeated until a partial tape length is needed at the end of the line.

6.11.5 Reading the Tape

There are two common styles of graduations on 100-ft surveyor's tapes. *It is necessary to identify the type being used before starting work* to avoid making one-foot mistakes repeatedly.

The more common type of tape has a total graduated length of 101 ft. It is marked from 0 to 100 by full feet in one direction, and has an additional foot preceding the zero mark graduated from 0 to 1 ft in tenths, or in tenths and hundredths in the other direction. In measuring the last partial tape length of a line with this kind of tape, a full-foot graduation is held by the rear tapeperson at the last pin set [like the 87-ft mark in Figure 6.2(a)]. The actual footmark held is the one that causes the graduations on the extra foot between zero and the tape end to straddle the closing point. The forward tapeperson reads the additional length of 0.68 ft beyond the zero mark. In the case illustrated, to ensure correct recording, the rear tapeperson calls "87." The forward tapeperson repeats and adds the partial foot reading, calling "87.68." Since part of a foot has been added, this type of tape is known as an *add tape*.

The other kind of tape found in practice has a total graduated length of 100 ft. It is marked from 0 to 100 with full-foot increments, and the first foot at each end (from 0 to 1 and from 99 to 100) is graduated in tenths, or in tenths and hundredths. With this kind of tape, the last partial tape length is measured by holding a full-foot graduation at the last chaining pin set such that the graduated section of the tape between the zero mark and the 1-ft mark straddles the closing point. This is indicated in Figure 6.2(b), where the 88-ft mark is being held on the last chaining pin and the tack marking the end of the line is opposite 0.32 ft read from the zero end of the tape. The partial tape length is then 88.0 − 0.32 = 87.68 ft. The quantity 0.32 ft is said to be *cut off*; hence this type of tape is called a *cut tape*. To ensure subtraction of a foot from the number at the full-foot graduation used, the following field procedure and calls are recommended: rear tapeperson calls "88"; forward tapeperson says "cut point three-two"; rear tapeperson answers "eighty

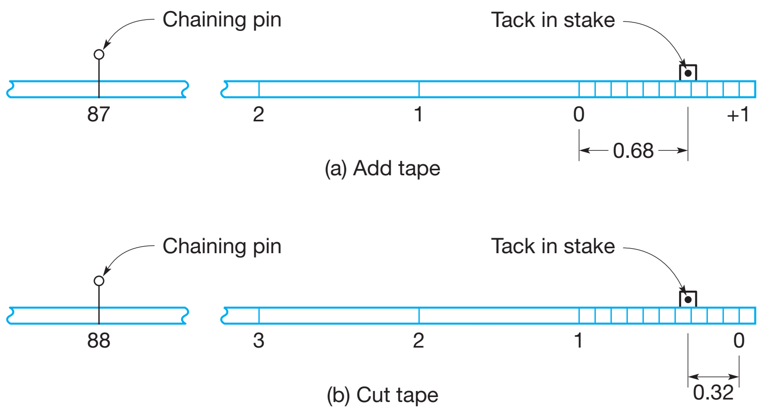

Figure 6.2 Reading partial tape lengths.

seven point six eight"; forward tapeperson confirms the subtraction and replies "check" when satisfied it is correct.

An advantage of the add tape is that it is easier to use because no subtraction is needed when measuring decimal parts of a foot. Its disadvantage is that careless tapepersons will sometimes make measurements of 101.00 ft and record them as 100.00 ft. The cut tape practically eliminates this mistake.

The same routine should be used throughout all taping by a party and the results tested in every possible way. A single mistake in subtracting the partial foot when using a cut tape will destroy the precision of a hundred other good measurements. For this reason, the add tape is more foolproof. The greatest danger for mistakes in taping arises when changing from one style of tape to the other.

6.11.6 Recording the Distance

Accurate fieldwork may be canceled by careless recording. After the partial tape length is obtained at the end of a line, the rear tapeperson determines the number of full 100-ft tape lengths by counting the pins collected from the original set of 11. For distances longer than 1000 ft, a notation is made in the field book when the rear tapeperson has 10 pins and one remains in the ground. This signifies a tally of 10 full tape lengths and has traditionally been called an "out." The forward tapeperson starts out again with 10 pins and the process is repeated. Since long distances are measured electronically today, tapes are typically used for distances less than 100 ft today.

Although taping procedures may appear to be relatively simple, high precision is difficult to achieve, especially for beginners. Taping is a skill that can best be taught and learned by field demonstrations and practice.

■ 6.12 HORIZONTAL MEASUREMENTS ON SLOPING GROUND

In taping on uneven or sloping ground, it is standard practice to hold the tape horizontally and use a plumb bob at one or perhaps both ends. It is difficult to keep the plumb line steady for heights above the chest. Wind exaggerates this problem and may make accurate work impossible.

On steeper slopes, where a 100-ft length cannot be held horizontally without plumbing from above shoulder level, shorter distances are measured and accumulated to total a full tape length. This procedure, called *breaking tape*, is illustrated in Figure 6.3. As an example of this operation, assume that when taping down slope, the 100-ft end of the tape is held at the rear point, and the forward tapeperson can advance only 30 ft without being forced to plumb from above the chest. A pin is therefore set beneath the 70-ft mark, as in Figure 6.4. The rear tapeperson moves ahead to this pin and holds the 70-ft graduation there while another pin is set at, say, the 25-ft mark. Then, with the 25-ft graduation over the second pin, the full 100-ft distance is marked at the zero point. In this way, the partial tape lengths are added mechanically to make a full 100 ft by holding the proper graduations, and no mental arithmetic is required. The rear tapeperson returns the pins set at the intermediate points to the forward tapeperson to keep the tally clear on the number of full tape lengths established. To avoid

Figure 6.3
Breaking tape.

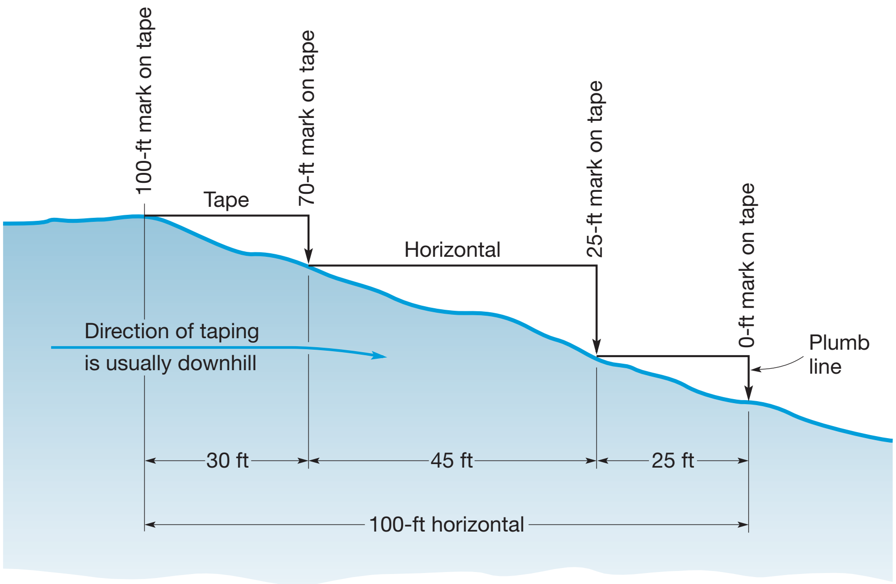

Figure 6.4
Procedure for breaking tape (when tape is not in box or on reel).

kinking the tape, the full 100-ft length is pulled ahead by the forward tapeperson into position for measuring the next tape length. In all cases the tape is leveled by eye or hand level, with the tapepersons remembering the natural tendency to have the downhill end of a tape too low. Practice will improve the knack of holding a tape horizontally by keeping it perpendicular to the vertical plumb-bob string.

Taping downhill is preferable to measuring uphill for two reasons. First, in taping downhill, the rear point is held steady on a fixed object while the other end is plumbed. In taping uphill, the forward point must be set while the other end is wavering somewhat. Second, if breaking tape is necessary, the head tapeperson can more conveniently use the hand level to proceed downhill a distance, which renders the tape horizontal when held comfortably at chest height.

6.13 SLOPE MEASUREMENTS

In measuring the distance between two points on a steep slope, rather than break tape every few feet, it may be desirable to tape along the slope and compute the horizontal component. This requires measurement also of either the altitude angle α or the difference in elevation d (Figure 6.5). Breaking tape is more time consuming and generally less accurate due to the accumulation of random errors from marking tape ends and keeping the tape level and aligned for many short sections.

In Figure 6.5, if altitude angle α is determined, the horizontal distance between points A and B can be computed from the relation

$$H = L \cos \alpha \tag{6.1a}$$

where H is the horizontal distance between points, L the slope length separating them, and α the altitude angle from horizontal, usually obtained with an *Abney hand level* and *clinometer* (hand device for measuring angles of inclination). If the difference in elevation d between the ends of the tape is measured, which is done by leveling (see Chapter 5), the horizontal distance can be computed using the following expression derived from the Pythagorean theorem:

$$H = \sqrt{L^2 - d^2} \tag{6.2a}$$

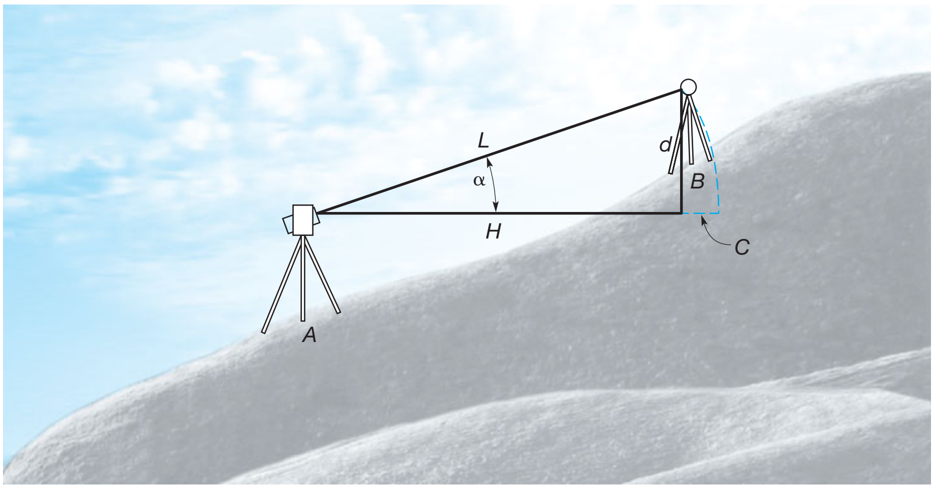

Figure 6.5
Slope measurement.

Another approximate formula, obtained from the first term of a binomial expansion of the Pythagorean theorem, may be used in lower-order surveys to reduce slope distances to horizontal:

$$H = L - \frac{d^2}{2L}(\text{approx.}) \tag{6.2b}$$

In Equation (6.2b) the term $d^2/2L$ equals C in Figure 6.5 and is a correction to be subtracted from the measured slope length to obtain the horizontal distance. The error in using the approximate formula for a 100 ft length grows with increasing slope. Equation (6.2b) is useful for making quick estimates, without a calculator, or error sizes produced for varying slope conditions. It should not be used as an alternate method of Equation (6.2a) when reducing slope distances.

■ 6.14 SOURCES OF ERROR IN TAPING

There are three fundamental sources of error in taping

1. *Instrumental errors*. A tape may differ in actual length from its nominal graduated length because of a defect in manufacture or repair, or as a result of kinks.
2. *Natural errors*. The horizontal distance between end graduations of a tape varies because of the effects of temperature, wind, and weight of the tape itself.
3. *Personal errors*. Tapepersons setting pins, reading the tape, or manipulating the equipment.

The most common types of taping errors are discussed in the subsections that follow. They stem from instrumental, natural, and personal sources. Some types produce systematic errors, others produce random errors.

6.14.1 Incorrect Length of Tape

Incorrect length of a tape can be one of the most important errors. It is systematic. Tape manufacturers do not guarantee steel tapes to be exactly their graduated nominal length—for example, 100.00 ft—nor do they provide a standardization certificate unless requested and paid for as an extra. The true length is obtained by comparing it with a standard tape or distance. The National Institute of Standards and Technology (NIST)[1] of the U.S. Department of Commerce will make such a comparison and certify the exact distance between end graduations under given conditions of temperature, tension, and manner of support. A 100-ft steel tape usually is standardized for each of the two sets of conditions—for example, 68°F, a 12-lb pull, with the tape lying on a flat surface (fully supported throughout); and 68°F, a 20-lb pull, with the tape supported at the ends only. Schools and surveying

[1]Information on tape calibration services of the National Institute of Standards and Technology can be obtained at the following website: http://www.nist.gov. Tapes can be sent for calibration to the National institute of Standards and Technology, Building 220, Room 113, 100 Bureau Dr., Gaithersburg, MD 20899; telephone: (301) 975-2465.

offices often have a precisely measured 100-ft line or at least one standardized tape that is used only to check other tapes subjected to wear.

An error, caused by incorrect length of a tape, occurs each time the tape is used. If the true length, known by standardization, is not exactly equal to its nominal value of 100.00 ft recorded for every full length, the correction can be determined as

$$C_L = \left(\frac{l - l'}{l'}\right)L \tag{6.3}$$

where C_L is the correction to be applied to the measured (recorded) length of a line to obtain the true length, l the actual tape length, l' the nominal tape length, and L the measured (recorded) length of line. Units for the terms in Equation (6.3) can be in either feet or meters.

6.14.2 Temperature Other Than Standard

Steel tapes are standardized for 68°F (20°C) in the United States. A temperature higher or lower than this value causes a change in length that must be considered. The coefficient of thermal expansion and contraction of steel used in ordinary tapes is approximately 0.00000645 per unit length per degree Fahrenheit, and 0.0000116 per unit length per degree Celsius. For any tape, the correction for temperature can be computed as

$$C_T = k(T_1 - T)L \tag{6.4}$$

where C_T is the correction in the length of a line caused by nonstandard temperature, k the coefficient of thermal expansion and contraction of the tape, T_1 the tape temperature at the time of measurement, T the tape temperature when it has standard length, and L the observed (recorded) length of line. The correction C_T will have the same units as L, which can be either feet or meters. Errors caused by temperature change may be practically eliminated by either (**a**) measuring temperature and making corrections according to Equation (6.4) or (**b**) using an Invar tape.

Errors caused by temperature changes are systematic and have the same sign if the temperature is always above 68°F, or always below that standard. When the temperature is above 68°F during part of the time occupied in measuring a long line, and below 68°F for the remainder of the time, the errors tend to partially balance each other, but corrections should still be computed and applied.

Temperature effects are difficult to assess in taping. The air temperature read from a thermometer may be quite different from that of the tape to which it is attached. Sunshine, shade, wind, evaporation from a wet tape, and other conditions make the tape temperature uncertain. Field experiments prove that temperatures on the ground or in the grass may be 10 to 25° higher or lower than those at shoulder height because of a 6-in. "layer of weather" (microclimate) on top of the ground. Since a temperature difference of 15°F produces a change of 0.01 ft per 100 ft tape length, the importance of such large variations is obvious.

Shop measurements made with steel scales and other devices likewise are subject to temperature effects. The precision required in fabricating a large airplane or ship can be lost by this one cause alone.

6.14.3 Inconsistent Pull

When a steel tape is pulled with a tension greater than its *standard pull* (the tension at which it was calibrated), the tape will stretch and become longer than its standard length. Conversely, if less than standard pull is used, the tape will be shorter than its standard length. The *modulus of elasticity* of the tape regulates the amount that it stretches. The correction for pull can be computed and applied using the following formula

$$C_P = (P_1 - P)\frac{L}{AE} \tag{6.5}$$

where C_P is the total elongation in tape length due to pull, in feet; P_1 the pull applied to the tape at the time of the observation, in pounds; P the standard pull for the tape, in pounds; A the cross-sectional area of the tape, in square inches; E the modulus of elasticity of steel, in pounds per square inch; and L the observed (recorded) length of line. An average value of E is 29,000,000 lb/in.^{2} for the kind of steel typically used in tapes. In the metric system, to produce the correction C_P in meters, comparable units of P and P_1 are kilograms, L is meters, A is square centimeters, and E is kilograms per square centimeter. An average value of E for steel in these units is approximately 2,000,000 kg/cm^2. The cross-sectional area of a steel tape can be obtained from the manufacturer, by measuring its width and thickness with calipers, or by dividing the total tape weight by the product of its length (in feet) times the unit weight of steel (490 lb/ft^2), and multiplying by 144 to convert square feet to square inches.

Errors resulting from incorrect tension can be eliminated by (a) using a spring balance to measure and maintain the standard pull or (b) applying a pull other than standard and making corrections for the deviation from standard according to Equation (6.5).

Errors caused by incorrect pull may be either systematic or random. The pull applied by even an experienced tapeperson is sometimes greater or less than the desired value. An inexperienced person, particularly one who has not used a spring balance on a tape, is likely to apply less than the standard tension consistently.

6.14.4 Sag

A steel tape not supported along its entire length sags in the form of a *catenary*, a good example being the cable between two power poles. Because of sag, the horizontal distance (chord length) is less than the graduated distance between tape ends, as illustrated in Figure 6.6. Sag can be reduced by applying greater tension, but not eliminated unless the tape is supported throughout. The following formula is used to compute the sag correction:

$$C_S = -\frac{w^2 L_S^3}{24P_1^2} \tag{6.6}$$

where in the English system C_S is the correction for sag (difference between length of curved tape and straight line from one support to the next), in feet; L_S the

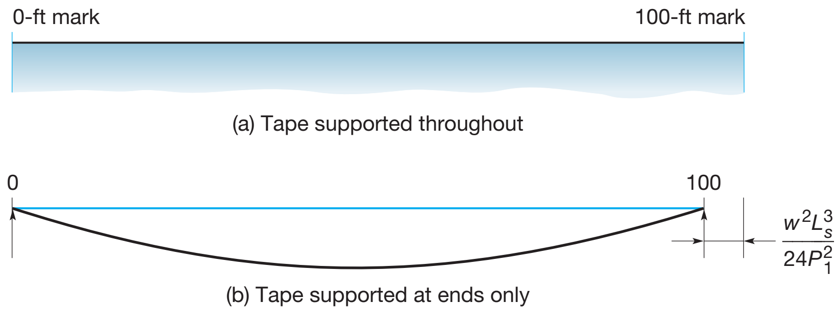

Figure 6.6
Effect of sag.

unsupported length of the tape, in feet; w the weight of the tape per foot of length, in pounds; and P_1 the pull on the tape, in pounds. Metric system units for Equation (6.6) are kg/m for w, kg for P_1, and meters for C_S and L_S.

The effects of errors caused by sag can be eliminated by (a) supporting the tape at short intervals or throughout or (b) by computing a sag correction for each unsupported segment and applying the total to the recorded length according to Equation (6.6). It is important to recognize that Equation (6.6) is nonlinear and thus must be applied to each unsupported section of the tape. It is incorrect to apply it to the overall length of a line unless the line was observed in one section.

As stated previously, when lines of unknown length are being measured, sag corrections are always negative, whereas positive corrections occur if the tension applied exceeds the standard pull. For any given tape, the so-called *normal tension* needed to offset these two factors can be obtained by setting Equations (6.5) and (6.6) equal to each other and solving for P_1. Although applying the normal tension does eliminate the need to make corrections for both pull and sag, it is not commonly used because the required pull is often too great for convenient application.

6.14.5 Tape Not Horizontal and Tape Off-Line

Corrections for errors caused by a tape being inclined in the vertical plane are computed in the same manner as corrections for errors resulting from it being off-line in the horizontal plane. Corrected lengths can be determined by Equation (6.2), where in the vertical plane, d is the difference in elevation between the tape ends, and in the horizontal plane, d is the amount where one end of the tape is off-line. In either case, L is the length of tape involved in the measurement.

Errors caused by the tape not being horizontal are systematic, and always make recorded lengths longer than true lengths. They are reduced by using a hand level to keep elevations of the tape ends equal, or by running differential levels (see Section 5.4) over the taping points, and applying corrections for elevation differences. Errors from the tape being off-line are also systematic, and they too make recorded lengths longer than true lengths. This type of error can be eliminated by careful alignment.

6.14.6 Improper Plumbing

Practice and steady nerves are necessary to hold a plumb bob still long enough to mark a point. The plumb bob will sway, even in calm weather. On very gradual slopes and on smooth surfaces such as pavements, inexperienced tapepersons obtain better results by laying the tape on the ground instead of plumbing. Experienced tapepersons plumb most measurements.

Errors caused by improper plumbing are random, since they may make distances either too long or too short. However, the errors would be systematic when taping directly against or in the direction of a strong wind. Heavier plumb bobs and touching the plumb bob on the ground, or steadying it with one foot, decreases its swing. Practice in plumbing will reduce errors.

6.14.7 Faulty Marking

Chaining pins should be set perpendicular to the taped line but inclined 45° to the ground. This position permits plumbing to the point where the pin enters the ground without interference from the loop.

Brush, stones, and grass or weeds deflect a chaining pin and may increase the effect of incorrect marking. Errors from these sources tend to be random and are kept small by carefully locating a point, then checking it.

When taping on solid surfaces such as pavement or sidewalks, pencil marks or scratches can be used to mark taped segments. Accuracy in taping on the ground can be increased by using tacks in stakes as markers rather than chaining pins.

6.14.8 Incorrect Reading or Interpolation

The process of reading to hundredths on tapes graduated only to tenths, or to thousandths on tapes graduated to hundredths, is called interpolation. Errors from this source are random over the length of a line. They can be reduced by care in reading, employing a magnifying glass, or using a small scale to determine the last figure.

6.14.9 Summary of Effects of Taping Errors

An error of 0.01 ft is significant in many surveying measurements. Table 6.1 lists the nine types of taping errors; classifies them as instrumental (I), natural (N), or personal (P), and systematic (S) or random (R); and gives the departure from normal that produces an error of 0.01 ft in a 100-ft length.

The accepted method of reducing errors on precise work is to make separate measurements of the same line with different tapes, at different times of day, and in opposite directions. An accuracy of 1/10,000 can be obtained by careful attention to details.

6.15 TAPE PROBLEMS

All tape problems develop from the fact that a tape is either longer or shorter than its graduated “nominal” length because of manufacture, temperature changes, tension applied, or some other reason. There are only two basic types of taping tasks: an unknown distance between two fixed points can be *measured*, or a required distance can be *laid off* from one fixed point. Since the tape may be too long or too short for either task, there are four possible versions of taping

TABLE 6.1 SUMMARY OF ERRORS

Error Type	Error Source*	Systematic (S) or Random (R)	Departure from Normal to Produce 0.01-ft Error for 100-ft Tape
Tape length	I	S	0.01 ft
Temperature	N	S or R	15°F
Pull	P	S or R	15 lb
Sag	N, P	S	0.6 ft at center for 100-ft tape standardized by support throughout
Alignment	P	S	1.4 ft at one end of 100-ft tape, or 0.7 ft at midpoint
Tape not level	P	S	1.4-ft elevation difference between ends of 100-ft tape
Plumbing	P	R	0.01 ft
Marking	P	R	0.01 ft
Interpolation	P	R	0.01 ft

*I, instrumental; N, natural; P, personal.

problems, which are: (1) measure with a tape that is too long, (2) measure with a tape that is too short, (3) lay off with a tape that is too long, and (4) lay off with a tape that is too short. The solution of a particular problem is always simplified and verified by drawing a sketch.

Assume that the fixed distance *AB* in Figure 6.7 is measured with a tape that is found to be 100.03 ft as measured between the 0- and 100-ft marks. Then (the conditions in the figure are greatly exaggerated) the first tape length would extend to point 1; the next, to point 2; and the third, to point 3. Since the distance remaining from 3 to *B* is less than the correct distance from the true 300-ft mark to *B*, the *recorded* length *AB* is too small and must be increased by a correction. If the tape had been too short, the *recorded* distance would be too large, and the correction must be subtracted.

In laying out a required distance from one fixed point, the reverse is true. The correction must be subtracted from the desired length for tapes longer than their nominal value and added for tapes that are shorter. A simple sketch like Figure 6.7 makes clear whether the correction should be added or subtracted for any of the four cases.

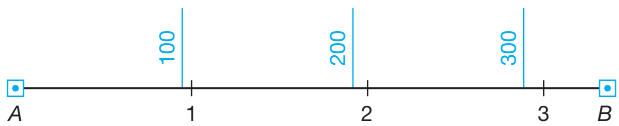

Figure 6.7 Taping between fixed points, tape too long.

6.16 COMBINED CORRECTIONS IN A TAPING PROBLEM

In taping linear distances, several types of systematic errors often occur simultaneously. The following examples illustrate procedures for computing and applying corrections for the two basic types of problems, *measurement* and *layoff*.

Example 6.1

A 30-m steel tape standardized at 20°C and supported throughout under a tension of 5.45 kg was found to be 30.012 m long. The tape had a cross-sectional area of 0.050 cm^2 and a weight of 0.03967 kg/m. This tape was held horizontal, supported at the ends only, with a constant tension of 9.09 kg, to measure a line from A to B in three segments. The data listed in the following table were recorded. Apply corrections for tape length, temperature, pull, and sag to determine the correct length of the line.

(a) The tape length correction by Equation (6.3) is

$$C_L = \left(\frac{30.012 - 30.000}{30.000}\right) 81.151 = +0.0324 \text{ m}$$

(b) Temperature corrections by Equation (6.4) are (Note: separate corrections are required for distances observed at different temperatures.):

Section	Measured (Recorded) Distance (m)	Temperature (°C)
A-1	30.000	14
1-2	30.000	15
2-B	21.151	16
	Σ81.151	

$$C_{T_1} = 0.0000116(14 - 20)30.000 = -0.0021 \text{ m}$$

$$C_{T_2} = 0.0000116(15 - 20)30.000 = -0.0017 \text{ m}$$

$$C_{T_3} = 0.0000116(16 - 20)21.151 = -0.0010 \text{ m}$$

$$\Sigma C_T = -0.0048 \text{ m}$$

(c) The pull correction by Equation (6.5) is

$$C_P\left(\frac{9.09 - 5.45}{0.050 \times 2{,}000{,}000}\right) 81.151 = 0.0030 \text{ m}$$

(d) The sag corrections by Equation (6.6) are (Note: separate corrections are required for the two suspended lengths.):

$$C_{S_1} = -2\left[\frac{(0.03967)^2(30.000)^3}{24(9.09)^2}\right] = -0.0429 \text{ m}$$

$$C_{S_2} = -\frac{(0.03967)^2(21.151)^3}{24(9.09)^2} = -0.0075 \text{ m}$$

$$\Sigma C_S = -0.0504 \text{ m}$$

(e) Finally, corrected distance AB is obtained by adding all corrections to the measured distance, or

$$AB = 81.151 + 0.0324 - 0.0048 + 0.0030 - 0.0504 = 81.131 \text{ m}$$

Example 6.2

A 100-ft steel tape standardized at 68°F and supported throughout under a tension of 20 lb was found to be 100.012 ft long. The tape had a cross-sectional area of 0.0078 in.^{2} and a weight of 0.0266 lb/ft. This tape is used to lay off a horizontal distance CD of exactly 175.00 ft. The ground is on a smooth 3% grade, thus the tape will be used fully supported. Determine the correct slope distance to layoff if a pull of 15 lb is used and the temperature is 87°F.

Solution

(a) The tape length correction, by Equation (6.3), is

$$C_L = \left(\frac{100.012 - 100.000}{100.000}\right)175.00 = +0.021 \text{ ft}$$

(b) The temperature correction, by Equation (6.4), is

$$C_T = 0.00000645(87 - 68)175.00 = +0.021 \text{ ft}$$

(c) The pull correction, by Equation (6.5), is

$$C_P = \frac{(15 - 20)}{0.0078(29{,}000{,}000))}175.00 = -0.0004 \text{ ft}$$

(d) Since this is a layoff problem, all corrections are subtracted. Thus, the required horizontal distance to layoff, rounded to the nearest hundredth of a foot, is

$$CD_h = 175.00 - 0.021 - 0.021 + 0.0004 = 174.96 \text{ ft}$$

(e) Finally, a rearranged form of Equation (6.2) is used to solve for the slope distance (the difference in elevation d for use in this equation, for 174.96 ft on a 3% grade, is $174.96(0.03) = 5.25$ ft):

$$CD_s = \sqrt{(174.96)^2 + (5.25)^2} = 175.04 \text{ ft}$$

PART III • ELECTRONIC DISTANCE MEASUREMENT

6.17 INTRODUCTION

A major advance in surveying instrumentation occurred approximately 60 years ago with the development of electronic distance measuring (EDM) instruments. These devices measure lengths by indirectly determining the number of full and partial waves of transmitted electromagnetic energy required in traveling between the two ends of a line. In practice, the energy is transmitted from one end of the line to the other and returned to the starting point; thus, it travels the double path distance. Multiplying the total number of cycles by its wavelength and dividing by 2, yields the unknown distance.

The Swedish physicist Erik Bergstrand introduced the first EDM instrument in 1948. His device, called the *geodimeter* (an acronym for geodetic distance meter), resulted from attempts to improve methods for measuring the velocity of light. The instrument transmitted visible light and was capable of accurately observing distances up to about 25 mi (40 km) at night. In 1957, a second EDM apparatus, the *tellurometer*, was introduced. Designed in South Africa by Dr T. L. Wadley, this instrument transmitted microwaves, and was capable of observing distances up to 50 mi (80 km) or more, day or night.

The potential value of these early EDM models to the surveying profession was immediately recognized. However, they were expensive and not readily portable for field operations. Furthermore, observing procedures were lengthy, and mathematical reductions to obtain distances from observed values were difficult and time consuming. Continued research and development have overcome all of these deficiencies. Prior to the introduction of EDM instruments, taping made accurate distance measurements. Although seemingly a relatively simple procedure, precise taping is one of the most difficult and painstaking of all surveying tasks. Now EDM instruments have made it possible to obtain accurate distance measurements rapidly and easily. Given a line of sight, long or short lengths can be measured over bodies of water, busy freeways, or terrain that is inaccessible for taping.

In the current generation, EDM instruments are combined with *digital theodolites* and *microprocessors* to produce *total station instruments* (see Figures 1.3 and 2.5). These devices can simultaneously and automatically observe both distances and angles. The microprocessor receives the measured slope length and zenith (or altitude) angle, calculates horizontal and vertical distance components, and displays them in real time. When equipped with *data collectors* (see Section 2.12), they can record field notes electronically for transmission to computers, plotters, and other office equipment for processing. These so-called *field-to-finish* systems are gaining worldwide acceptance and changing the practice of surveying substantially.

6.18 PROPAGATION OF ELECTROMAGNETIC ENERGY

Electronic distance measurement is based on the rate and manner that electromagnetic energy propagates through the atmosphere. The rate of propagation can be expressed with the following equation

$$V = f\lambda \tag{6.7}$$

where V is the velocity of electromagnetic energy, in meters per second; f the modulated frequency of the energy, in hertz;[2] and λ the wavelength, in meters. The velocity of electromagnetic energy in a vacuum is 299,792,458 m/sec. Its speed is slowed somewhat in the atmosphere according to the following equation

$$V = c/n \tag{6.8}$$

[2]The hertz (Hz) is a unit of frequency equal to 1 cycle/sec. The kilohertz (KHz), megahertz (MHz), and gigahertz (GHz) are equal to 10^3, 10^6, and 10^9 Hz, respectively.

where c is the velocity of electromagnetic energy in a vacuum, and n the atmospheric *index of refraction*. The value of n varies from about 1.0001 to 1.0005, depending on pressure and temperature, but is approximately equal to 1.0003. Thus, accurate electronic distance measurement requires that atmospheric pressure and temperature be measured so that the appropriate value of n is known.

Temperature, atmospheric pressure, and relative humidity all have an effect on the index of refraction. Because a light source emits light composed of many wavelengths, and since each wavelength has a different index of refraction, the group of waves has a *group index of refraction*. The value for the group refractivity N_g in *standard air*[3] for electronic distance measurement is

$$N_g = (n_g - 1)10^6 = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4} \tag{6.9}$$

where λ is the wavelength of the light expressed in micrometers (μm) and n_g is the group refractive index. The wavelengths of light sources commonly used in EDMs are 0.6328 μm for red laser and 0.900 to 0.930 μm for infrared.

The actual group refractive index n_a for atmosphere at the time of observation due to variations in temperature, pressure, and humidity can be computed as

$$n_a = 1 + \left(\frac{273.15}{1013.25} \cdot \frac{N_g P}{t + 273.15} - \frac{11.27\ e}{t + 273.15}\right)10^{-6} \tag{6.10}$$

where e is the partial water vapor pressure in hectopascal[4] (hPa) as defined by the temperature and relative humidity at the time of the measurement, P the pressure in hPa, and t the dry bulb temperature in °C. The partial water vapor pressure, e, can be computed with sufficient accuracy for normal operating conditions as

$$e = E \cdot h/100 \tag{6.11}$$

where $E = 10^{[7.5t/(237.3+t)+0.7858]}$ and h is the relative humidity in percent.

Example 6.3

What is the actual wavelength and velocity of a near-infrared beam ($\lambda = 0.915\ \mu$m) of light modulated at a frequency of 320 MHz through an atmosphere with a (dry) temperature t of 34°C, relative humidity h of 56%, and an atmospheric pressure of 1041.25 hPa?

Solution

By Equation (6.9)

$$N_g = 287.6155 + \frac{4.88660}{(0.915)^2} + \frac{0.06800}{(0.915)^4} = 293.5491746$$

[3]A standard air is defined with the following conditions: 0.0375% carbon dioxide, temperature of 0°C, pressure of 760 mm of mercury, and 0% humidity.

[4]1 Atmosphere = 101.325 kPa = 1013.25 hPa = 760 torr = 760 mmHg

By Equation (6.11)

$$a = \frac{7.5 \times 34}{(237.3 + 34)} + 0.7858 = 1.7257$$

$$E = 10^a = 53.18$$

$$e = Eh = 53.18 \times 56/100 = 29.7788$$

By Equation (6.10)

$$n_a = 1 + \left(\frac{273.15}{1013.25} \cdot \frac{293.5492 \times 1041.25}{34 + 273.15} - \frac{11.27 \times 29.7788}{34 + 273.15}\right)10^{-6}$$

$$= 1 + (268.268660 - 1.09265)10^{-6}$$

$$= 1.0002672$$

By Equation (6.8)

$$V = 299{,}792{,}458/1.0002672 = 299{,}712{,}382 \text{ m/sec}$$

Rearranging Equation (6.7) yields an actual wavelength of

$$\lambda = 299{,}712{,}382/320{,}000{,}000 = 0.9366012\ \mu\text{m}$$

Note in the solution of Example 6.3 that the second parenthetical term in Equation (6.10) accounts for the effects of humidity in the atmosphere. In fact, if this term were ignored the actual index of refraction n_a would become 1.0002683 resulting in the same computed wavelength to five decimal places. This demonstrates why, in using EDM instruments that employ near-infrared light, the effects of humidity on the transmission of the wave can be ignored for all but the most precise work. The student should verify this fact.

The manner by which electromagnetic energy propagates through the atmosphere can be represented conceptually by the sinusoidal curve illustrated in Figure 6.8. This figure shows one wavelength, or *cycle*. Portions of wavelengths or

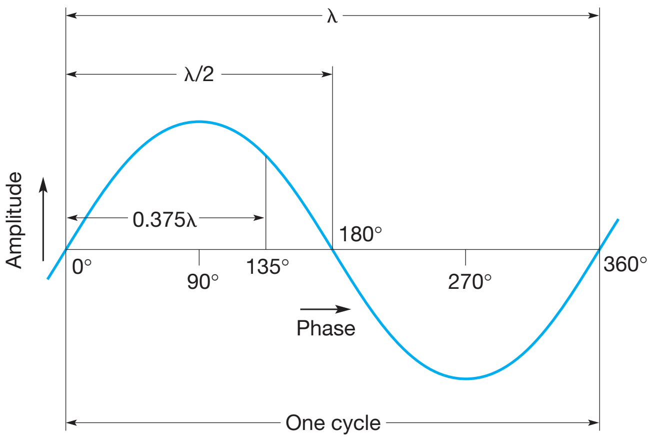

Figure 6.8
A wavelength of electromagnetic energy illustrating phase angles.

the positions of points along the wavelength are given by phase angles. Thus, in Figure 6.8, a 360° *phase angle* represents a full cycle, or a point at the end of a wavelength, while 180° is a half wavelength, or the midpoint. An intermediate position along a wavelength having a phase angle of, say, 135° is 135/360, or 0.375 of a wavelength.

■ 6.19 PRINCIPLES OF ELECTRONIC DISTANCE MEASUREMENT

In Section 6.17, it was stated that distances are observed electronically by determining the number of full and partial waves of transmitted electromagnetic energy that are required in traveling the distance between the two ends of a line. In other words, this process involves determining the number of wavelengths in an unknown distance. Then, knowing the precise length of the wave, the distance can be determined. This is similar to relating an unknown distance to the calibrated length of a steel tape.

The procedure of measuring a distance electronically is depicted in Figure 6.9, where an EDM device has been centered over station A by means of a plumb bob or optical plumbing device. The instrument transmits a *carrier signal* of electromagnetic energy to station B. A *reference frequency* of a precisely regulated wavelength has been superimposed or *modulated* onto the carrier. A reflector at B returns the signal to the receiver, so its travel path is double the slope distance AB. In the figure, the modulated electromagnetic energy is represented by a series of sine waves, each having wavelength λ. The unit at A determines the number of wavelengths in the double path, multiplied by the wavelength in feet or meters, and divided by 2 to obtain distance AB.

Of course, it would be highly unusual if a measured distance was exactly an integral number of wavelengths, as illustrated in Figure 6.9. Rather, some fractional part of a wavelength would in general be expected; for example, the partial

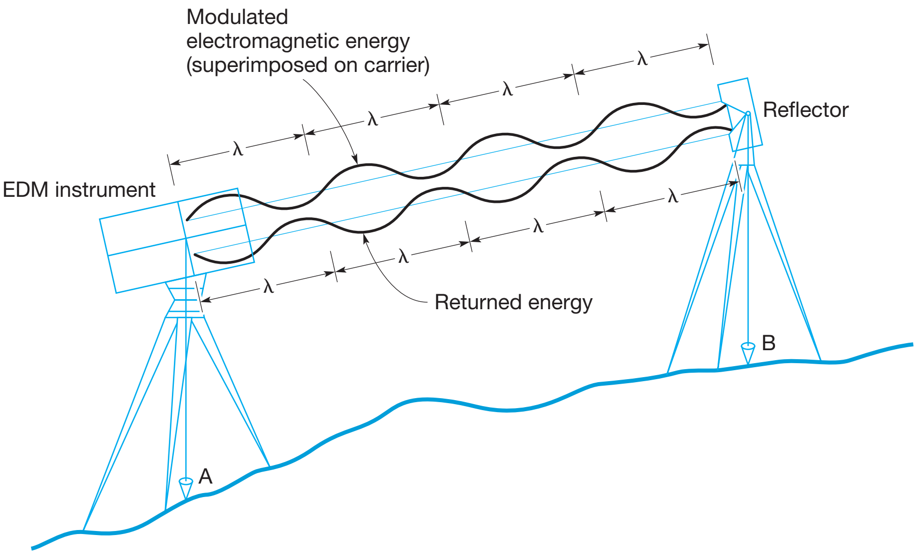

Figure 6.9 Generalized EDM procedure.

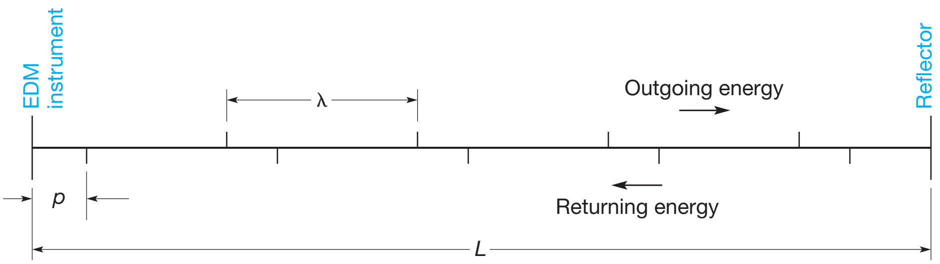

Figure 6.10 Phase difference measurement principle.

value p shown in Figure 6.10. In that figure, distance L between the EDM instrument and reflector would be expressed as

$$L = \frac{n\lambda + p}{2} \tag{6.12}$$

where λ is the wavelength, n the number of full wavelengths, and p the length of the fractional part. The fractional length is determined by the EDM instrument from measurement of the *phase shift* (phase angle) of the returned signal. To illustrate, assume that the wavelength for the example of Figure 6.10 was precisely 20.000 m. Assume also that the phase angle of the returned signal was 115.7°, in which case length p would be $(115.7/360)20.000 = 6.428$ m. Then from the figure, since $n = 9$, by Equation (6.12), length L is

$$L = \frac{9(20.000) + 6.428}{2} = 93.214 \text{ m}$$

Considering the double path distance, the 20-m wavelength used in the example just given has an "effective wavelength" of 10 m. This is one of the fundamental wavelengths used in current EDM instruments. It is generated using a frequency of approximately 15 MHz.

EDM instruments cannot determine the number of full wavelengths in an unknown distance by transmitting only one frequency and wavelength. To resolve the ambiguity n, in Equation (6.12), they must transmit additional signals having longer wavelengths. This procedure is explained in the following section, which describes electro-optical EDM instruments.

6.20 ELECTRO-OPTICAL INSTRUMENTS

The majority of EDM instruments manufactured today are electro-optical and transmit infrared or laser light as a carrier signal. This is primarily because its intensity can be modulated directly, considerably simplifying the equipment. Earlier models used tungsten or mercury lamps. They were bulky, required a large power source, and had relatively short operating ranges, especially during the day because of excessive atmospheric scatter. EDM instruments using coherent light produced by gas lasers followed. These were smaller and more portable and were capable of making observations of long distances in the daytime as well as at night.

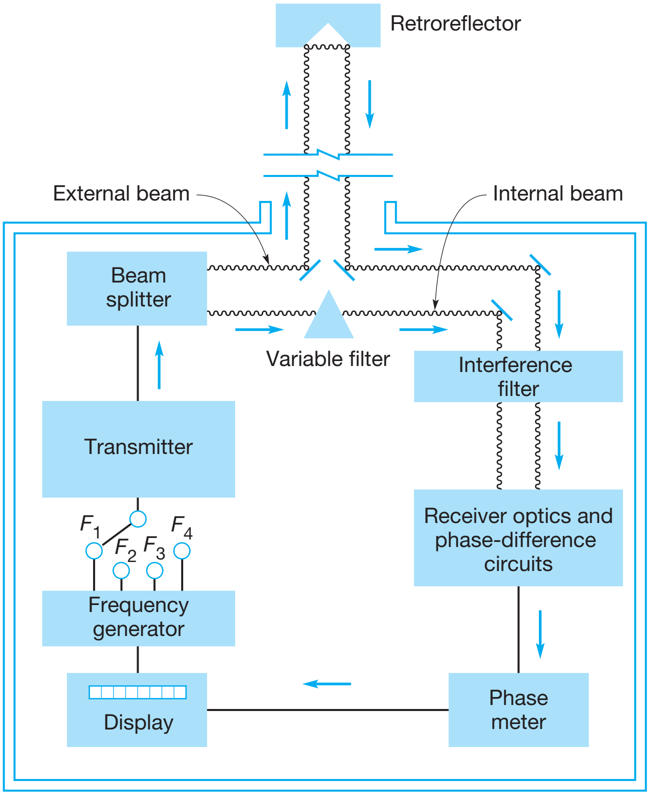

Figure 6.11 Generalized block diagram illustrating operation of electro-optical EDM instrument.

Figure 6.11 is a generalized schematic diagram illustrating the basic method of operation of one particular type of electro-optical instrument. The transmitter uses a GaAs diode that emits *amplitude-modulated* (AM) infrared light. A crystal oscillator precisely controls the frequency of modulation. The modulation process may be thought of as similar to passing light through a stovepipe in which a damper plate is spinning at a precisely controlled rate or frequency. When the damper is closed, no light passes. As it begins to open, light intensity increases to a maximum at a phase angle of 90° with the plate completely open. Intensity reduces to zero again with the damper closed at a phase angle of 180°, and so on. This intensity variation or amplitude modulation is properly represented by sine waves such as those shown in Figures 6.8 and 6.9.

As shown in Figure 6.11, a beam splitter divides the light emitted from the diode into two separate signals: an *external* measurement beam and an *internal* reference beam. By means of a telescope mounted on the EDM instrument, the external beam is carefully aimed at a retroreflector that has been centered over the point at the line's other end. Figure 6.12 shows a triple corner cube retroreflector of the type used to return the external beam, coaxial, to the receiver.

The internal beam passes through a variable-density filter and is reduced in intensity to a level equal to that of the returned external signal, enabling a more accurate observation to be made. Both internal and external signals go through an interference filter, which eliminates undesirable energy such as sunlight. The

Figure 6.12
Triple retroreflector. (Courtesy Topcon Positioning Systems.)

internal and external beams then pass through components to convert them into electric energy while preserving the phase shift relationship resulting from their different travel path lengths. A phase meter converts this phase difference into direct current having a magnitude proportional to the differential phase. This current is connected to a *null meter* that is adjusted to null the current. The fractional wavelength is measured during the nulling process, converted to distance, and displayed.

To resolve the ambiguous number of full cycles a wave has undergone, EDM instruments transmit different modulation frequencies. The unit illustrated in the schematic of Figure 6.11 uses four frequencies: $F1$, $F2$, $F3$, and $F4$, as indicated. If modulation frequencies of 14.984 MHz, 1.4984 MHz, 149.84 KHz, and 14.984 KHz are used, and assuming the index of refraction is 1.0003, then their corresponding "effective" wavelengths are 10.000, 100.00, 1000.0, and 10,000 m, respectively. Assume that a distance of 3867.142 appears on the display as the result of measuring a line. The four rightmost digits, 7.142, are obtained from the phase shift measured while transmitting the 10.000-m wavelength at frequency $F1$. Frequency $F2$, having a 100.00-m wavelength, is then transmitted, yielding a fractional length of 67.14. This provides the digit 6 in the displayed distance. Frequency $F3$ gives a reading of 867.1, which provides the digit 8 in the answer, and finally, frequency $F4$ yields a reading of 3867, which supplies the digit 3, to complete the display. From this example, it should be evident that the high resolution of a measurement (nearest 0.001 m) is secured using the 10.000-m wavelength, and the others simply resolve the ambiguity of the number of these shorter wavelengths in the total distance.

With older instruments, changing of frequencies and nulling were done manually by setting dials and turning knobs. Now modern instruments incorporate microprocessors that control the entire measuring process. Once the instrument is aimed at the reflector and the measurement started, the final distance appears in the display almost instantaneously. Other changes in new instruments include improved electronics to control the amplitude modulation, and replacement of the null meter by an electronic phase detector. These changes have significantly improved the accuracy with which phase shifts can be determined, which in turn has reduced the number of different frequencies that need to be transmitted. Consequently, as few as two frequencies are now used on some instruments: one that produces a short wavelength to provide the high-resolution digits and one with a long wavelength to provide the coarse numbers. To illustrate how this is possible, consider again the example measurement just described which used four frequencies. Recall that a reading of 7.142 was obtained with the 10.000-m wavelength, and that 3867 was read with the 10,000-m wavelength. Note the overlap of the common digit 7 in the two readings. Assuming that both phase shift measurements are reliably made to four significant figures, the leftmost digit of the first reading should indeed be the same as the rightmost one of the second reading. If these digits are the same in the measurement, this provides a check on the operation of the instrument. Modern instruments compare these overlapping digits and will display an error message if they do not agree. If they do check, the displayed distance will take all four digits from the first (short wavelength) reading, and the first three digits from the second reading.

Manufacturers provide a full range of instruments with precisions that vary from $\pm(1\text{ mm} + 1\text{ ppm})$ to $\pm(10\text{ mm} + 5\text{ ppm})$.[5] Earlier versions were manufactured to stand alone on a tripod, and thus from any setup they could only measure distances. Now, as noted earlier, in most instances EDMs are combined with electronic digital theodolites to produce our modern and very versatile total station instruments. These are described in the following section.

■ 6.21 TOTAL STATION INSTRUMENTS

Total station instruments (also sometimes called electronic tacheometers) combine an EDM instrument, an electronic digital theodolite, and a computer in one unit. These devices, described in more detail in Chapter 8, automatically observe horizontal and zenith (or altitude) angles, as well as distances, and transmit the results in real time to a built-in computer. The horizontal and zenith (or altitude) angle and slope distance can be displayed, and then upon keyboard commands, horizontal and vertical distance components can be instantaneously computed from these data and displayed. If the instrument is oriented in direction, and the coordinates of the occupied station are input to the system, the coordinates of

[5]Accuracies in electronic distance measurements are quoted in two parts; the first part is a constant, and the second is proportional to the distance measured. The abbreviation ppm = parts per million. One ppm equals 1 mm/km. In a distance 5000 ft long, a 5-ppm error equals $5000 \times (5 \times 10^{-6}) = 0.025$ ft .

Figure 6.13
The LEICA TC1101 total station. (Courtesy Leica Geosystems AG.)

any point sighted can be immediately obtained. These data can all be stored within the instrument, or in a data collector, thereby eliminating manual recording.

Total station instruments are of tremendous value in all types of surveying, as will be discussed in later portions of this text. Besides automatically computing and displaying horizontal and vertical components of a slope distance, and coordinates of points sighted, total station instruments can be operated in the *tracking mode*. In this mode, sometimes also called *stakeout*, a required distance (horizontal, vertical, or slope) can be entered by means of the control panel, and the instrument's telescope aimed in the proper direction. Then as the reflector is moved forward or back in position, the difference between the desired distance and that to the reflector is rapidly updated and displayed. When the display shows the difference to be zero, the required distance has been established and a stake is set. This feature, extremely useful in construction stakeout, is described further in Section 23.9.

The total station instruments shown in Figures 2.5, 6.13, and 8.2 all have a distance range of approximately 3 km (using a single prism) with an accuracy of $\pm$(2 mm + 2 ppm) and read angles to the nearest 1 in.

■ 6.22 EDM INSTRUMENTS WITHOUT REFLECTORS

Recently some EDM instruments have been introduced that do not require reflectors for distance measurement. These devices use time-pulsed infrared laser signals, and in their *reflectorless* mode of operation, they can observe distances up to 100 m in length. Figure 6.14(a) shows a handheld laser distance-measuring instrument.

(a) (b)

Figure 6.14 (a) The LEICA DISTO handheld laser distance-measuring instrument, (b) using the LECIA DISTO to measure to an inaccessible point. (Courtesy Leica Geosystems AG.)

Some total station instruments, like that shown in Figure 6.13, utilize laser signals and can also observe distances up to 100 m in the reflectorless mode. But as noted earlier, with prisms they can observe lengths up to 3 km.

Using instruments in the reflectorless mode, observations can be made to inaccessible objects such as the features of a building as shown in Figure 6.14(b), faces of dams and retaining walls, structural members being assembled on bridges, and so on. These instruments can increase the speed and efficiency of surveys in any construction or fabrication project, especially when measuring to features that are inaccessible.

■ 6.23 COMPUTING HORIZONTAL LENGTHS FROM SLOPE DISTANCES

All EDM equipment measures the slope distance between two stations. As noted earlier, if the EDM unit is incorporated into a total station instrument, then it can reduce these distances to their horizontal components automatically [if the zenith (or altitude) angle is input]. With some of the earliest EDMs, this could not be done, and reductions were carried out manually. The procedures used, whether performed internally by the microprocessor or done manually, follow those outlined in this section. It is presumed, of course, that slope distances are first corrected for instrumental and atmospheric conditions.

Reduction of slope distances to horizontal can be based on elevation differences, or on zenith (or vertical) angle. Because of Earth curvature, long lines must be treated differently in reduction than short ones and will be discussed in Chapter 19.

6.23.1 Reduction of Short Lines by Elevation Differences

If difference in elevation is used to reduce slope distances to horizontal, during field operations heights h_e of the EDM instrument and h_r of the reflector above their respective stations are measured and recorded (see Figure 6.15). If elevations of

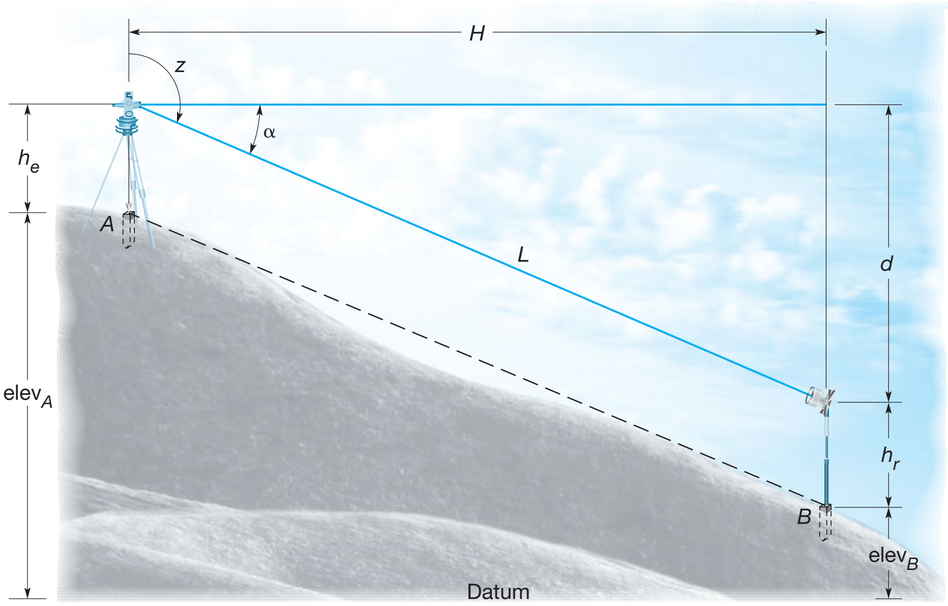

Figure 6.15 Reduction of EDM slope distance to horizontal.

stations A and B in the figure are known, Equation (6.2) will reduce the slope distance to horizontal, with the value of d (difference in elevation between EDM instrument and reflector) computed as follows:

$$d = (\text{elev}_A + h_e) - (\text{elev}_B + h_r) \tag{6.13}$$

Example 6.4

A slope distance of 165.360 m (corrected for meteorological conditions) was measured from A to B, whose elevations were 447.401 and 445.389 m above datum, respectively. Find the horizontal length of line AB if the heights of the EDM instrument and reflector were 1.417 and 1.615 m above their respective stations.

Solution

By Equation (6.13)

$$d = (447.401 + 1.417) - (445.389 + 1.615) = 1.814 \text{ m}$$

By Equation (6.2)

$$H = \sqrt{(165.360)^2 - (1.814)^2} = 165.350 \text{ m}$$

6.23.2 Reduction of Short Lines by Vertical Angles

If zenith angle z (angle measured downward from the upward direction of the plumb line) is observed to the inclined path of the transmitted energy when measuring slope distance L (see Figure 6.15), then the following equation is applicable to reduce the slope length to its horizontal component:

$$H = L\sin(z) \tag{6.14}$$

If altitude angle α (angle between horizontal and the inclined energy path) is observed (see Figure 6.15), then Equation (6.1) is applicable for the reduction. For most precise work, especially on longer lines, the zenith (or altitude) angle should be observed in both the direct and reversed modes, and averaged (see Section 8.13). Also, as discussed in Section 19.14.2, the mean obtained from both ends of the line will compensate for curvature and refraction.

6.24 ERRORS IN ELECTRONIC DISTANCE MEASUREMENT

As noted earlier, accuracies of EDM instruments are quoted in two parts: a constant error and a scalar error proportional to the distance observed. Specified errors vary for different instruments, but the constant portion is usually about ±2 mm, and the proportion is generally about ±2 ppm. The constant error is most significant on short distances; for example, with an instrument having a constant error of ±2 mm, a measurement of 20 m is good to only 2/20,000 = 1/10,000, or 100 ppm. For a long distance, say 2 km, the constant error becomes negligible and the proportional part more important.

The major error components in an observed distance are instrument and target miscentering, and the specified constant and scalar errors of the EDM instrument. Using Equation (3.11), the error in an observed distance is computed as

$$E_d = \sqrt{E_i^2 + E_r^2 + E_c^2 + (ppm \times D)^2} \tag{6.15}$$

where E_i is the estimated miscentering error in the instrument, E_r is the estimated miscentering error in the reflector, E_c the specified constant error for the EDM, ppm the specified scalar error for the EDM, and D the measured slope distance.

Example 6.5

A slope distance of 827.329 m was observed between two stations with an EDM instrument having specified errors of ±(2 mm + 2 ppm). The instrument was centered with an estimated error of ±3 mm. The estimated error in target miscentering was ±5 mm. What is the estimated error in the observed distance?

Solution

By Equation (6.15)

$$E_d = \sqrt{(3)^2 + (5)^2 + (2)^2 + (2 \times 10^{-6} \times 827329)^2} = \pm 6.4 \text{ mm}$$

Note in the solution that the distance of 827.329 m was converted to millimeters to obtain unit consistency. This solution results in a distance precision of 6.4/827,329, or better than 1:129,000.

From the foregoing, it is clear that except for very short distances, the order of accuracy possible with EDM instruments is very high. Errors can seriously degrade the observations, however, and thus care should always be exercised to minimize their effects. Sources of error in EDM work may be personal, instrumental, or natural. The subsections that follow identify and describe errors from each of these sources.

6.24.1 Personal Errors

Personal errors include inaccurate setups of EDM instruments and reflectors over stations, faulty measurements of instrument and reflector heights [needed for computing horizontal lengths (see Section 6.23)], and errors in determining atmospheric pressures and temperatures. These errors are largely random. They can be minimized by exercising utmost care and by using good-quality barometers and thermometers.

Mistakes (not errors) in manually reading and recording displayed distances are common and costly. They can be eliminated with some instruments by obtaining the readings in both feet and meters and comparing them. Of course, data collectors (see Section 2.12) also circumvent this problem. Additionally, as shown in Table 6.2, misalignment of the prism can cause significant errors when the reflector is set in its 0 mm constant position.

An example of a common mistake is failing to set the temperature and pressure in an EDM before obtaining an observation. Assume this occurred with the atmospheric conditions given in Example 6.3. The actual index of refraction was computed as 1.0002672. If the fundamental wavelength for a standard atmosphere was 10.000 m, then the actual wavelength produced by the EDM would be $10.000/1.0002672 = 9.9973$ m. Using Equation (6.3) with an observed distance of 827.329 m, the error, e, in the observed distance would be

$$e = \left(\frac{9.9973 - 10.000}{10.000}\right)827.329 = -0.223 \text{ m}$$

TABLE 6.2 ERROR IN OBSERVED DISTANCE DUE TO MISALIGNMENT OF THE PRISM

Misalignment in Degrees	0 mm Constant Prism Error (mm)	−30 mm Constant Prism Error (mm)
0	0.00	0.00
5	0.1	0.0
10	0.6	0.1
15	1.3	0.2
20	2.3	0.4
25	3.5	0.7
30	5.1	1.1

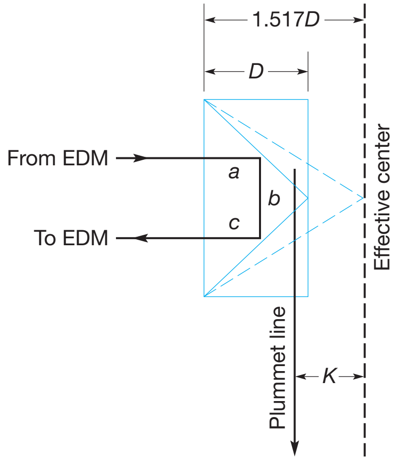

Figure 6.16 Schematic of retroreflector where *D* is the depth of the prism.

The effect of failing to account for the actual atmospheric conditions produces a precision of only $|-0.223|/827.329$, or 1:3700. This is well below the computed precision of 1:129,000 in Example 6.5.

6.24.2 Instrumental Errors

If EDM equipment is carefully adjusted and precisely calibrated, instrumental errors should be extremely small. To assure their accuracy and reliability, EDM instruments should be checked against a first-order baseline at regular time intervals. For this purpose, the National Geodetic Survey has established a number of accurate baselines in each state.[6] These are approximately a mile long and placed in relatively flat areas. Monuments are set at the ends and at intermediate points along the baseline.

Although most EDM instruments are quite stable, occasionally they become maladjusted and generate erroneous frequencies. This results in faulty wavelengths that degrade distance measurements in a manner similar to using a tape of incorrect length. Periodic checking of the equipment against a calibrated baseline will detect the existence of observational errors. It is especially important to make these checks if high-order surveys are being conducted.

The corner cube reflectors used with EDM instruments are another source of instrumental error. Since light travels at a lower velocity in glass than in air, the "effective center" of the reflector is actually behind the prism. Thus, it frequently does not coincide with the plummet, a condition that produces a systematic error in distances known as the *reflector constant*. This situation is shown in Figure 6.16. Notice that because the retroreflector is comprised of mutually perpendicular faces, the light always travels a total distance of $a + b + c = 2D$ in the prism.

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[6]For locations of baselines in your area, contact the NGS National Geodetic Information Center by email at: info_center@ngs.noaa.gov; at their website address: http://www.ngs.noaa.gov/CBLINES/calibration.html; by telephone at (301)713-3242; or by writing to NOAA, National Geodetic Survey, Station 09202, 1315 East West Highway, Silver Spring, MD 20910.

Additionally, given a refractive index for glass, which is greater than air, the velocity of light in the prism is reduced following Equation (6.8) to create an effective distance of nD where n is the index of refraction of the glass (approximately 1.517). The dashed line in Figure 6.16 shows the effective center thus created. The reflector constant, K in the figure, can be as large as 70 mm and will vary with reflectors.

Once known, the *electrical center* of the EDM can be shifted forward to compensate for the reflector constant. However, if an EDM instrument is being used regularly with several unmatched reflectors, this shift is impractical. In this instance, the offset for each reflector should be subtracted from the observed distances to obtain corrected values.

With EDM instruments that are components of total stations and are controlled by microprocessors, this constant can be entered via the keyboard and included in the internally computed corrections. Equipment manufacturers also produce matching reflector sets for which the reflector constant is the same, thus allowing a single constant to be used for a set of reflectors with an instrument.

By comparing precisely known baseline lengths to observed distances, a so-called *system measurement constant* can be determined. This constant can then be applied to all subsequent observations for proper correction. Although calibration using a baseline is preferred, if one is not available, the constant can be obtained with the following procedure. Three stations, A, B, and C, should be established in a straight line on flat ground, with stations A and C at a distance that is multiple units of the fundamental wavelength of the instrument apart. The fundamental wavelength of most instruments today is typically 10 m. Station B should be in between stations A and C also at a multiple of the fundamental wavelength of the EDM. For example, the lengths AB and BC could be set at 40 m and 60 m, respectively, for an instrument with a fundamental wavelength of 10 m. The length of AC and the two components, AB and BC, should be observed several times with the instrument-reflector constant set to zero and the means of each length determined. From these observations, the following equation can be written:

$$AC + K = (AB + K) + (BC + K)$$

from which

$$K = AC - (AB + BC) \tag{6.16}$$

where K is the system measurement constant to be added to correct the observed distances.

The procedure, including centering of the EDM instrument and reflector, should be repeated several times very carefully, and the average value of K adopted. Since different reflectors have varying offsets, the test should be performed with any reflector that will be used with the EDM, and the results marked on each to avoid confusion later. For the most precise calibration, lengths AB and BC should be carefully laid out as even multiples of the instrument's shortest measurement wavelength. Failure to do this can cause an incorrect value of K to be obtained. As shown in Figure 6.16, due to the construction of the reflector and the pole being located near the center of the reflector, the system measurement constant is typically negative.

While the above procedure provides method for determining a specific instrument-reflector constant, it is highly recommended that EDM instruments be calibrated using NGS calibration baselines. These baselines have been established throughout the country for use by surveyors. Their technical manual *Use of Calibration Base Lines*, which is listed in the bibliography at the end of the chapter, provides guidelines on the use of the baselines and reduction of the observations providing both the instrument-reflector offset constant and a scaling factor.

6.24.3 Natural Errors

Natural errors in EDM operations stem primarily from atmospheric variations in temperature, pressure, and humidity, which affect the index of refraction, and modify the wavelength of electromagnetic energy. The values of these variables must be measured and used to correct observed distances. As demonstrated in Example 6.3, humidity can generally be neglected when using electro-optical instruments, but this variable was important when microwave instruments were employed.

The National Weather Service adjusts atmospheric pressure readings to sea level values. Since atmospheric pressure changes by approximately 1 in. of mercury (Hg) per 1000 ft of elevation, under no circumstances should radio broadcast values for atmospheric pressure be used to correct distances. Instead, atmospheric pressure should be measured by an aneroid barometer that is calibrated against a mercurial barometer. Many high school and college physics departments have mercurial barometers.

EDM instruments within total stations have onboard microprocessors that use atmospheric variables, input through the keyboard, to compute corrected distances after making observations but before displaying them. For older instruments, varying the transmission frequency made corrections, or they could be computed manually after the observation. Equipment manufacturers provided tables and charts that assisted in this process. The magnitude of error in electronic distance measurement due to errors in observing atmospheric pressure and temperature is indicated in Figure 6.17. Note that a 10°C temperature error, or a pressure difference of 25 mm (1 in.) of mercury, each produces a distance error of about 10 ppm. Thus, if a radio broadcast atmospheric pressure is entered into an EDM in Denver, CO, the resulting distance error could be as great as 50 ppm and a 200-m distance could in error by as much as 1 cm.

As discussed in Section 6.14.2, a microclimate can exist in the layers of atmosphere immediately above a surface such as the ground. Since this microclimate can substantially change the index of refraction, it is important to maintain a line of sight that is at least 0.5 m above the surface of the ground. On long lines of sight, the observer should be cognizant of intervening ridges or other objects that may exist between the instrument and reflector, which could cause problems in meeting this condition. If this condition cannot be met, the height of the reflector may be increased. Under certain conditions, it may be necessary to set an intermediate point on the encroaching surface to ensure that light from the EDM does not travel through these lower layers.

For the most precise work, on long lines, a sampling of the atmospheric conditions along the line of sight should be observed. In this case, it may be necessary

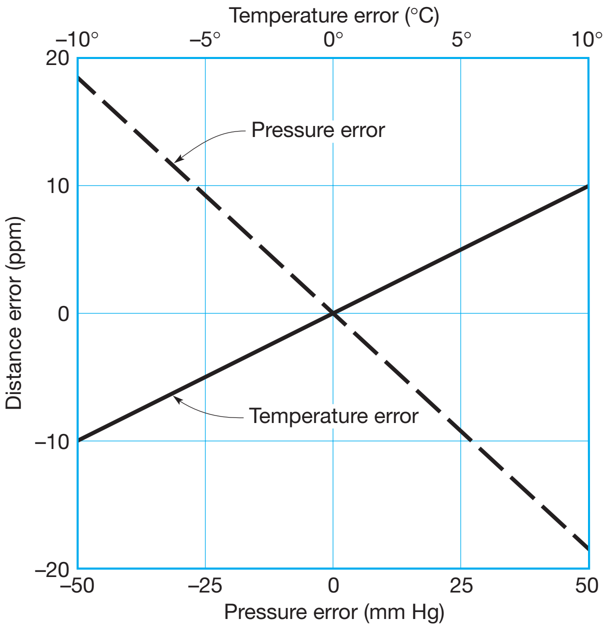

Figure 6.17 Errors in EDM produced by temperature and pressure errors (based on atmospheric temperature and pressure of 15° and 760 mm of mercury).

to elevate the meteorological instruments. This can be difficult where the terrain becomes substantially lower than the sight line. In these cases, the atmospheric measurements for the ends of the line can be measured and averaged.

6.25 USING SOFTWARE

On the companion website at http://www.pearsonhighered.com/ghilani is the Excel® spreadsheet *c6.xls*. This spreadsheet demonstrates the computations in Examples 6.1 and 6.3. For those wishing to see this programmed in a higher-level language, a Mathcad® worksheet *C6.xmcd* is available on the companion website also. This worksheet additionally demonstrates Examples 6.3 and 6.4.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

6.1 What distance in travel corresponds to 1 msec of time for electromagnetic energy?

6.2 A student counted 92, 90, 92, 91, 93, and 91 paces in six trials of walking along a course of 200-ft known length on level ground. Then 85, 86, 86, and 84 paces were counted in walking four repetitions of an unknown distance *AB*. What is **(a)*** the pace length and **(b)** the length of *AB*?

6.3 What difference in temperature from standard, if neglected in use of a steel tape, will cause an error of 1 part in 5000?

6.4 An add tape of 101 ft is incorrectly recorded as 100 ft for a 200-ft distance. What is the correct distance?

6.5* List five types of common errors in taping.

6.6 List the proper procedures for taping a horizontal distance of about 123 ft down a 4% slope.

6.7 For the following data, compute the horizontal distance for a recorded slope distance *AB*,

(**a**) $AB = 385.29$ ft, slope angle $= 6°03'26''$

(**b**) $AB = 186.793$ m, difference in elevation *A* to $B = -8.499$ m.

A 100-ft steel tape of cross-sectional area 0.0025 in.^{2}, weight 2.3 lb, and standardized at 68°F is 99.992 ft between end marks when supported throughout under a 12-lb pull. What is the true horizontal length of a recorded distance *AB* for the conditions given in Problems 6.8 through 6.11? (Assume horizontal taping and all full tape lengths except the last.)

	Recorded Distance *AB* (ft)	Average Temperature (°F)	Means of Support	Tension (lb)
6.8*	86.06	68	Throughout	12
6.9	124.73	85	Throughout	15
6.10	86.35	50	Ends only	22
6.11	94.23	75	Ends only	25

For the tape of Problems 6.8 through 6.11, determine the true horizontal length of the recorded slope distance *BC* for the conditions shown in Problems 6.12 through 6.13. (Assume the tape was fully supported for all measurements.)

	Recorded Slope Distance *BC* (ft)	Average Temperature Per 100 ft (°F)	Tension (lb)	Elevation Difference (ft)
6.12	95.08	48	15	2.45
6.13	65.86	88	20	3.13

A 30-m steel tape measured 29.991 m when standardized fully supported under a 5.500-kg pull at a temperature of 20°C. The tape weighed 1.22 kg and had a cross-sectional area of 0.016 cm^2. What is the corrected horizontal length of a recorded distance *AB* for the conditions given in Problems 6.14 through 6.15?

	Recorded Distance *AB* (m)	Average Temperature (°C)	Tension (kg)	Means of Support
6.14	28.056	18	8.3	Throughout
6.15	16.302	25	7.9	Ends only

For the conditions given in Problems 6.16 through 6.18, determine the horizontal length of *CD* that must be laid out to achieve the required true horizontal distance *CD*. Assume a 100-ft steel tape will be used, with cross-sectional area 0.0025 in.^{2}, weight 2.4 lb, and standardized at 68°F to be 100.008 ft between end marks when supported throughout with a 12-lb pull. (Assume horizontal taping and all full tape lengths except the last.)

	Required Horizontal Distance *CD* (ft)	Average Temperature (°F)	Means of Support	Tension (lb)
6.16	97.54	68	Throughout	12
6.17	68.96	54	Throughout	20
6.18	68.78	91	Throughout	18

6.19* When measuring a distance *AB*, the first taping pin was placed 1.0 ft to the right of line *AB* and the second pin was set 0.5 ft left of line *AB*. The recorded distance was 236.89 ft. Calculate the corrected distance. (Assume three taped segments, the first two 100 ft each.)

6.20 List the possible errors that can occur when measuring a distance with an EDM.

6.21 Briefly describe how a distance can be measured by the method of phase comparison.

6.22 Describe why the sight line for electronic distance measurement should be at least 0.5 m off the edge of a parked vehicle.

6.23* Assume the speed of electromagnetic energy through the atmosphere is 299,784,458 m/sec for measurements with an EDM instrument. What time lag in the equipment will produce an error of 800 m in a measured distance?

6.24 What is the length of the partial wavelength for electromagnetic energy with a frequency of 15 MHz and a phase shift of 263°?

6.25 What "actual" wavelength results from transmitting electromagnetic energy through an atmosphere having an index of refraction of 1.0006, if the frequency is:
(a)* 29.988 MHz **(b)** 2.988 MHz

6.26 Using the speed of electromagnetic energy given in Problem 6.23, what distance corresponds to each nanosecond of time?

6.27 To calibrate an EDM instrument, distances *AC*, *AB*, and *BC* along a straight line were observed as 216.622 m, 130.320 m, and 86.281 m, respectively. What is the system measurement constant for this equipment? Compute the length of each segment corrected for the constant.

6.28 Which causes a greater error in a line measured with an EDM instrument? **(a)** A disregarded 10°C temperature variation from standard or **(b)** a neglected atmospheric pressure difference from standard of 20 mm of mercury?

6.29* In Figure 6.15, h_e, h_r, elev$_A$, elev$_B$, and the measured slope length L were 5.32, 5.18, 1215.37, 1418.68, and 2282.74 ft, respectively. Calculate the horizontal length between *A* and *B*.

6.30 Similar to Problem 6.29, except that the values were 1.535, 1.502, 334.215, 386.289, and 1925.461 m, respectively.

6.31 In Figure 6.15, h_e, h_r, z, and the measured slope length L were 5.25 ft, 5.56 ft, 86°30′46″, and 1598.27 ft, respectively. Calculate the horizontal length between *A* and *B* if a total station measures the distance.

6.32* Similar to Problem 6.31, except that the values were 1.45 m, 1.55 m, 96°05′33″, and 1663.254 m, respectively.

6.33 What is the actual wavelength and velocity of a near-infrared beam ($\lambda = 0.899\ \mu m$) of light modulated at a frequency of 330 MHz through an atmosphere with a dry bulb temperature, T, of 24°C; a relative humidity, h, of 69%; and an atmospheric pressure of 933 hPa?

6.34 If the temperature and pressure at measurement time are 18°C and 760 mm Hg, respectively, what will be the error in electronic measurement of a line 3 km long if the temperature at the time of observing is recorded 10°C too low? Will the observed distance be too long or too short?

6.35 Determine the most probable length of a line AB, the standard deviation, and the 95% error of the measurement for the following series of taped observations made under the same conditions: 632.088, 632.087, 632.089, 632.083, 632.093, 632.088, 632.083, 632.088, 632.092, and 632.091 m.

6.36* The standard deviation of taping a 30-m distance is ±5 mm. What should it be for a 90-m distance?

6.37 If an EDM instrument has a purported accuracy capability of ±(3 mm + 3 ppm), what error can be expected in a measured distance of **(a)** 30 m **(b)** 1586.49 ft **(c)** 975.468 m? (Assume that the instrument and target miscentering errors are equal to zero.)

6.38 The estimated error for both instrument and target miscentering errors is ±3 mm. For the EDM in Problem 6.37, what is the estimated error in the observed distances?

6.39 If a certain EDM instrument has an accuracy capability of ±(1 mm + 2 ppm), what is the precision of measurements, in terms of parts per million, for line lengths of: **(a)** 30.000 m **(b)** 300.000 m **(c)** 3000.000 m? (Assume that the instrument and target miscentering errors are equal to zero.)

6.40 The estimated error for both instrument and target miscentering errors is ±3 mm. For the EDM and distances listed in Problem 6.39, what is the estimated error in each distance? What is the precision of the measurements in terms of parts per million?

6.41 Create a computational program that solves Problem 6.29.

6.42 Create a computational program that solves Problem 6.38.

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7 Angles, Azimuths, and Bearings

7.1 INTRODUCTION

Determining the locations of points and orientations of lines frequently depends on the observation of angles and directions. In surveying, directions are given by *azimuths* and *bearings* (see Sections 7.5 and 7.6).

As described in Section 2.1, and illustrated in Figure 2.1, angles measured in surveying are classified as either *horizontal* or *vertical*, depending on the plane in which they are observed. Horizontal angles are the basic observations needed for determining bearings and azimuths. Vertical angles are used in trigonometric leveling, stadia (see Section 17.9.2), and for reducing slope distances to horizontal (see Section 6.23).

Angles are most often *directly* observed in the field with total station instruments, although in the past transits, theodolites, and compasses have been used. (See Appendix A for descriptions of the transit and theodolite. The surveyor's compass is described in Section 7.10.) Three basic requirements determine an angle. As shown in Figure 7.1, they are (1) *reference* or *starting line*, (2) *direction of turning*, and (3) *angular distance* (value of the angle). Methods of computing bearings and azimuths described in this chapter are based on these three elements.

7.2 UNITS OF ANGLE MEASUREMENT

A purely arbitrary unit defines the value of an angle. The *sexagesimal* system used in the United States, and many other countries, is based on degrees, minutes, and seconds, with the last unit further divided decimally. In Europe the *grad* or *gon* is commonly used (see Section 2.2). Radians may be more suitable in computer computations, but the sexagesimal system continues to be used in most U.S. surveys.

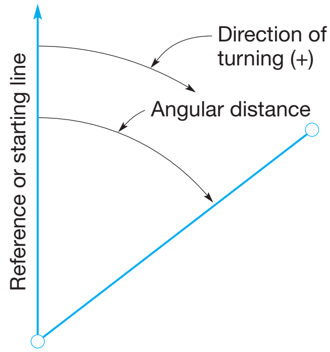

Figure 7.1 Basic requirements in determining an angle.

7.3 KINDS OF HORIZONTAL ANGLES

The kinds of horizontal angles most commonly observed in surveying are (1) *interior angles*, (2) *angles to the right*, and (3) *deflection angles*. Because they differ considerably, the kind used must be clearly indicated in field notes. Interior angles, shown in Figure 7.2, are observed on the inside of a *closed polygon*. Normally the angle at each apex within the polygon is measured. Then, as discussed in Section 9.7, a check can be made on their values because the sum of all interior angles in any polygon must equal $(n - 2)180°$, where n is the number of angles. Polygons are commonly used for boundary surveys and many other types of work. Surveyors (geomatics engineers) normally refer to them as *closed traverses*.

Exterior angles, located outside a closed polygon, are explements of interior angles. The advantage to be gained by observing them is their use as another check, since the sum of the interior and exterior angles at any station must total 360°.

Angles to the right are measured *clockwise from the rear to the forward station*. Note: As a survey progresses, stations are commonly identified by consecutive alphabetical letters (as in Figure 7.2), or by increasing numbers. Thus, the interior angles of Figure 7.2(a) are also angles to the right. Most data collectors require that angles to the right be observed in the field. *Angles to the left*, turned counterclockwise from the rear station, are illustrated in Figure 7.2(b). Note that the polygons of Figure 7.2 are "right" and "left"—that is, similar in shape but

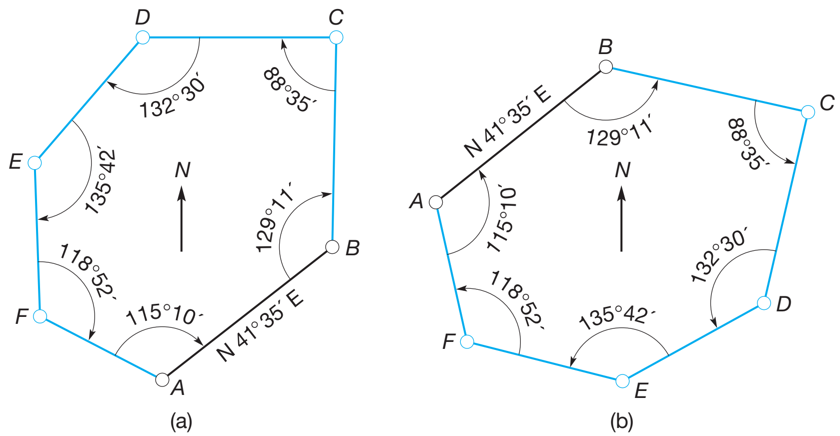

Figure 7.2 Closed polygon. (a) Clockwise interior angles (angles to the right). (b) Counterclockwise interior angles (angles to the left).

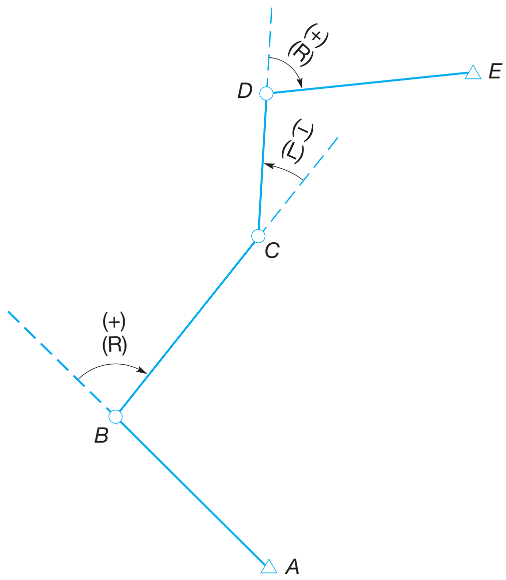

Figure 7.3
Deflection angles.

turned over like the right and left hands. Figure 7.2(b) is shown only to emphasize a serious mistake that occurs if counterclockwise angles are observed and recorded or assumed to be clockwise. *To avoid this confusion, it is recommended that a uniform procedure of always observing angles to the right be adopted* and the direction of turning noted in the field book with a sketch.

Angles to the right can be either interior or exterior angles of a closed-polygon traverse. Whether the angle is an interior or exterior angle depends on the direction the instrument proceeds around the traverse. If the direction around the traverse is counterclockwise, then the angles to the right will be interior angles. However, if the instrument proceeds clockwise around the traverse, then exterior angles will be observed. If this is the case, the sum of the exterior angles for a closed-polygon traverse will be $(n + 2)180°$. Analysis of a simple sketch should make these observations clear.

Deflection angles (Figure 7.3) are observed from an extension of the back line to the forward station. They are used principally on the long linear alignments of route surveys. As illustrated in the figure, deflection angles may be observed to the right (clockwise) or to the left (counterclockwise) depending on the direction of the route. Clockwise angles are considered plus, and counterclockwise ones minus, as shown in the figure. Deflection angles are always smaller than 180° and appending an R or L to the numerical value identifies the direction of turning. Thus the angle at B in Figure 7.3 is (*R*), and that at C is (*L*). Deflection angles are the only exception where counterclockwise observation of angles should be made.

■ 7.4 DIRECTION OF A LINE

The direction of a line is defined by the horizontal angle between the line and an arbitrarily chosen reference line called a *meridian*. Different meridians are used for specifying directions including (a) geodetic (also often called *true*), (b) astronomic, (c) magnetic, (d) grid, (e) record, and (f) assumed.

The *geodetic* meridian is the north-south reference line that passes through a mean position of the Earth's geographic poles. The positions of the poles defined as their mean locations between the period of 1900.0 and 1905.0 (see Section 19.3).

Wobbling of the Earth's rotational axis, also discussed in Section 19.3, causes the position of the Earth's geographic poles to vary with time. At any point, the *astronomic* meridian is the north-south reference line that passes through the instantaneous position of the Earth's geographic poles. Astronomic meridians derive their name from the field operation to obtain them, which consists in making observations on the celestial objects, as described in Appendix C. Geodetic and astronomic meridians are very nearly the same, and the former can be computed from the latter by making small corrections (see Sections 19.3 and 19.5).

A *magnetic* meridian is defined by a freely suspended magnetic needle that is only influenced by the Earth's magnetic field. Magnetic meridians are discussed in Section 7.10.

Surveys based on a state or other plane coordinate system employ a *grid* meridian for reference. Grid north is the direction of geodetic north for a selected *central meridian* and held parallel to it over the entire area covered by a plane coordinate system (see Chapter 20).

In boundary surveys, the term *record* meridian refers to directional references quoted in the recorded documents from a previous survey of a particular parcel of land. Another similar term, *deed* meridian, is used in the description of a parcel of land as recorded in a property deed. Chapters 21 and 22 discuss the use of record meridians and deed meridians in boundary retracement surveys.

An *assumed* meridian can be established by merely assigning any arbitrary direction—for example, taking a certain street line to be north. The directions of all other lines are then found in relation to it.

From the above definitions, it should be obvious that the terms north or due north, if used in a survey, must be defined, since they do not specify a unique line.

■ 7.5 AZIMUTHS

Azimuths are horizontal angles observed clockwise from any reference meridian. In plane surveying, azimuths are generally observed from north, but astronomers and the military have used south as the reference direction. The National Geodetic Survey (NGS) also used south as its reference for azimuths for NAD27, but north has been adopted for NAD83 (see Section 19.6). Examples of azimuths observed from north are shown in Figure 7.4. As illustrated, they can range from 0° to 360° in value. Thus the azimuth of OA is 70°; of OB, 145°; of OC, 235°; and of OD, 330°. Azimuths may be *geodetic, astronomic, magnetic, grid, record*, or *assumed*, depending on the reference meridian used. To avoid any confusion, it is necessary to state in the field notes, at the beginning of work, what reference meridian applies for azimuths, and whether they are observed from north or south.

A line's forward direction can be given by its *forward* azimuth, and its reverse direction by its *back* azimuth. In plane surveying, forward azimuths are converted to back azimuths, and vice versa, by adding or subtracting 180°. For example, if the azimuth of OA is 70°, the azimuth of AO is $70° + 180° = 250°$. If the azimuth of OC is 235°, the azimuth of CO is $235° - 180° = 55°$.

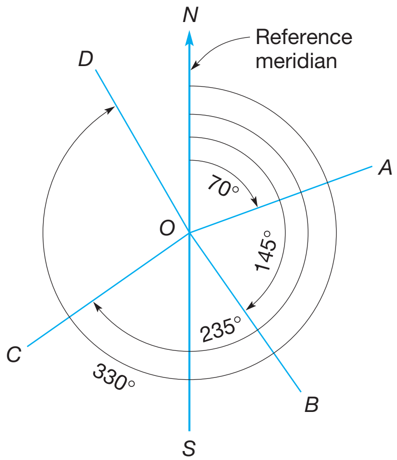

Figure 7.4
Azimuths.

Azimuths can be read directly on the graduated circle of a total station instrument after the instrument has been oriented properly. As explained in Section 9.2.4, this can be done by sighting along a line of known azimuth with that value indexed on the circle, and then turning to the desired course. Azimuths are used advantageously in boundary, topographic, control, and other kinds of surveys, as well as in computations.

7.6 BEARINGS

Bearings are another system for designating directions of lines. *The bearing of a line is defined as the acute horizontal angle between a reference meridian and the line*. The angle is observed from either the north or south toward the east or west, to give a reading smaller than 90°. The letter N or S preceding the angle, and E or W following it shows the proper quadrant. Thus, a properly expressed bearing includes quadrant letters and an angular value. An example is N80°E. In Figure 7.5,

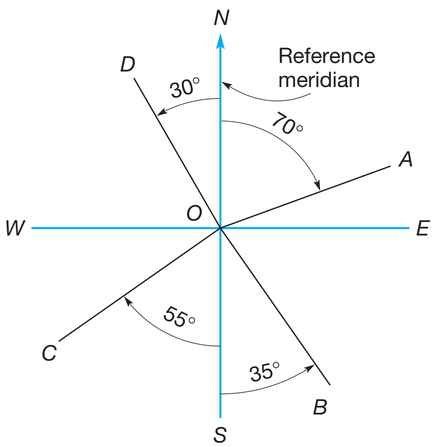

Figure 7.5
Bearing angles.

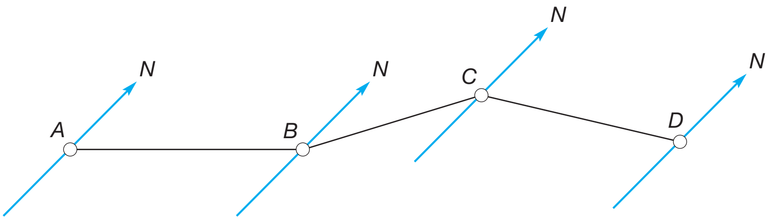

Figure 7.6
Forward and back bearings.

all bearings in quadrant *NOE* are measured clockwise from the meridian. Thus the bearing of line *OA* is N70°E. All bearings in quadrant *SOE* are counterclockwise from the meridian, so *OB* is S35°E. Similarly, the bearing of *OC* is S55°W and that of *OD*, N30°W. When lines are in the cardinal directions, the bearings should be listed as "Due North," "Due East," "Due South," or "Due West."

Geodetic bearings are observed from the geodetic meridian, *astronomic bearings* from the local astronomic meridian, *magnetic bearings* from the local magnetic meridian, *grid bearings* from the appropriate grid meridian, and *assumed bearings* from an arbitrarily adopted meridian. The magnetic meridian can be obtained in the field by observing the needle of a compass, and used along with observed angles to get computed magnetic bearings.

In Figure 7.6 assume that a compass is set up successively at points *A*, *B*, *C*, and *D* and bearings read on lines *AB*, *BA*, *BC*, *CB*, *CD*, and *DC*. As previously noted, bearings *AB*, *BC*, and *CD* are *forward bearings*; those of *BA*, *CB*, and *DC*, *back bearings*. Back bearings should have the same numerical values as forward bearings but opposite letters. Thus if bearing *AB* is N44°E, bearing *BA* is S44°W.

7.7 COMPARISON OF AZIMUTHS AND BEARINGS

Because bearings and azimuths are encountered in so many surveying operations, the comparative summary of their properties given in Table 7.1 should be helpful. Bearings are readily computed from azimuths by noting the quadrant in which the azimuth falls, then converting as shown in the table.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *Angles, Azimuths, and Bearings.mp4* discusses each type of angle typically used in surveying, the different types of azimuths and bearings, and demonstrates how azimuths can be converted to bearings.

Example 7.1

The azimuth of a boundary line is 128°13′46″. Convert this to a bearing.

Solution

The azimuth places the line in the southeast quadrant. Thus, the bearing angle is

$$180° - 128°13'46'' = 51°46'14''$$

and the equivalent bearing is S51°46′14″E.

TABLE 7.1 COMPARISON OF AZIMUTHS AND BEARINGS

Azimuths	Bearings
Vary from 0 to 360°	Vary from 0 to 90°
Require only a numerical value	Require two letters and a numerical value
May be geodetic, astronomic, magnetic, grid, assumed, forward or back	Same as azimuths
Are measured clockwise only	Are measured clockwise and counterclockwise
Are measured either from north only, or from south only on a particular survey	Are measured from north and south

Quadrant	Formulas for computing bearing angles from azimuths
I (NE)	Bearing = Azimuth
II (SE)	Bearing = 180° − Azimuth
III (SW)	Bearing = Azimuth − 180°
IV (NW)	Bearing = 360° − Azimuth

Example directions for lines in the four quadrants (azimuths from north)

Azimuth	Bearing
54°	N54°E
112°	S68°E
231°	S51°W
345°	N15°W

Example 7.2

The first course of a boundary survey is written as N37°13′W. What is its equivalent azimuth?

Solution

Since the bearing is in the northwest quadrant, the azimuth is

$$360° - 37°13' = 322°47'.$$

7.8 COMPUTING AZIMUTHS

Most types of surveys, but especially those that employ traversing, require computation of azimuths (or bearings). A traverse, as described in Chapter 9, is a series of connected lines whose lengths and angles at the junction points have been observed. Figures 7.2 and 7.3 illustrate examples. Traverses have many uses. To

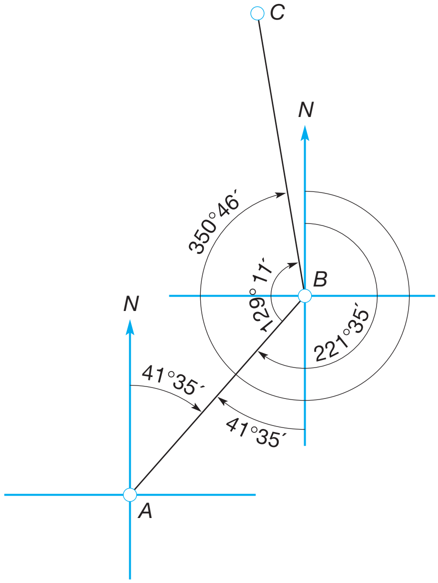

Figure 7.7
Computation of azimuth *BC* of Figure 7.2(a).

survey the boundary lines of a piece of property, for example, a "closed-polygon" type traverse like that of Figure 7.2(a) would normally be used. A highway survey from one city to another would usually involve a traverse like that of Figure 7.3. Regardless of the type used, it is necessary to compute the directions of its lines.

Many surveyors prefer azimuths to bearings for directions of lines because they are easier to work with, especially when calculating traverses with computers. Also sines and cosines of azimuth angles provide correct algebraic signs for departures and latitudes as discussed in Section 10.4.

Azimuth calculations are best made with the aid of a sketch. Figure 7.7 illustrates computations for azimuth BC in Figure 7.2(a). Azimuth BA is found by adding 180° to azimuth AB: $180° + 41°35' = 221°35'$ to yield its back azimuth. Then the angle to the right at B, $129°11'$, is added to azimuth BA to get azimuth BC: $221°35' + 129°11' = 350°46'$. This general process of adding (or subtracting) 180° to obtain the back azimuth and then adding the angle to the right is repeated for each line until the azimuth of the starting line is recomputed. If a computed azimuth exceeds 360°, then 360° is subtracted from it and the computations are continued. These calculations are conveniently handled in tabular form, as illustrated in Table 7.2. This table lists the calculations for all azimuths of Figure 7.2(a). Note that a check was secured by recalculating the beginning azimuth using the last angle. The procedures illustrated in Table 7.2 for computing azimuths are systematic and readily programmed for computer solution. The reader can view a Mathcad® worksheet *Azs.xmcd* on the companion website for this book at http://www.pearsonhighered.com/ghilani to review these computations. Also on this website are instructional videos that can be downloaded. The video *Azimuths from Angles.mp4* discusses the process of computing azimuths around a traverse and demonstrates the tabular method.

Traverse angles must be adjusted to the proper geometric total before azimuths are computed. As noted earlier, in a closed-polygon traverse, the sum of

Table 7.2 Computation of Azimuths (from North) for Lines of Figure 7.2(a)

Angles to the Right [Figure 7.2(a)]

41°35′ = *AB*	211°51′ = *DE*
+180°00′	−180°00′
221°35′ = *BA*	31°51′ = *ED*
+129°11′	+135°42′
350°46′ = *BC*	167°33′ = *EF*
−180°00′	+180°00′
170°46′ = *CB*	347°33′ = *FE*
+88°35′	+118°52′
259°21′ = *CD*	466°25′ − *360° = 106°25′ = *FA*
−180°00′	−180°00′
79°21′ = *DC*	286°25′ = *AF*
+132°30′	+115°10′
211°51′ = *DE*	401°35′ − *360° = 41°35′ = *AB* ✔

*When a computed azimuth exceeds 360°, the correct azimuth is obtained by merely subtracting 360°.

interior angles equals $(n - 2)180°$, where n is the number of angles or sides. If the traverse angles fail to close by say 10″ and are not adjusted prior to computing azimuths, the original and computed check azimuth of *AB* will differ by the same 10″, assuming there are no other calculating errors. *The azimuth of any starting course should always be recomputed as a check using the last angle*. Any discrepancy shows that (a) an arithmetic error was made or (b) the angles were not properly adjusted prior to computing azimuths.

■ 7.9 COMPUTING BEARINGS

Drawing sketches similar to those in Figure 7.8 showing all data simplify computations for bearings of lines. In Figure 7.8(a), the bearing of line *AB* from Figure 7.2(a) is N41°35′E, and the angle at *B* turned clockwise (to the right) from known line *BA* is 129°11′. Then the bearing angle of line *BC* is 180° − (41°35′ + 129°11′) = 9°14′, and from the sketch the bearing of *BC* is N9°14′W.

In Figure 7.8(b), the clockwise angle at *C* from *B* to *D* was observed as 88°35′. The bearing of *CD* is 88°35′ − 9°14′ = S79°21′W. Continuing this technique, the bearings in Table 7.3 have been determined for all lines in Figure 7.2(a).

In Table 7.3, note that the last bearing computed is for *AB*, and it is obtained by employing the 115°10′ angle observed at *A*. It yields a bearing of N41°35′E, which agrees with the starting bearing. Students should compute each bearing of Figure 7.2(a) to verify the values given in Table 7.3.

An alternate method of computing bearings is to determine the azimuths as discussed in Section 7.8, and then convert the computed azimuths to bearings

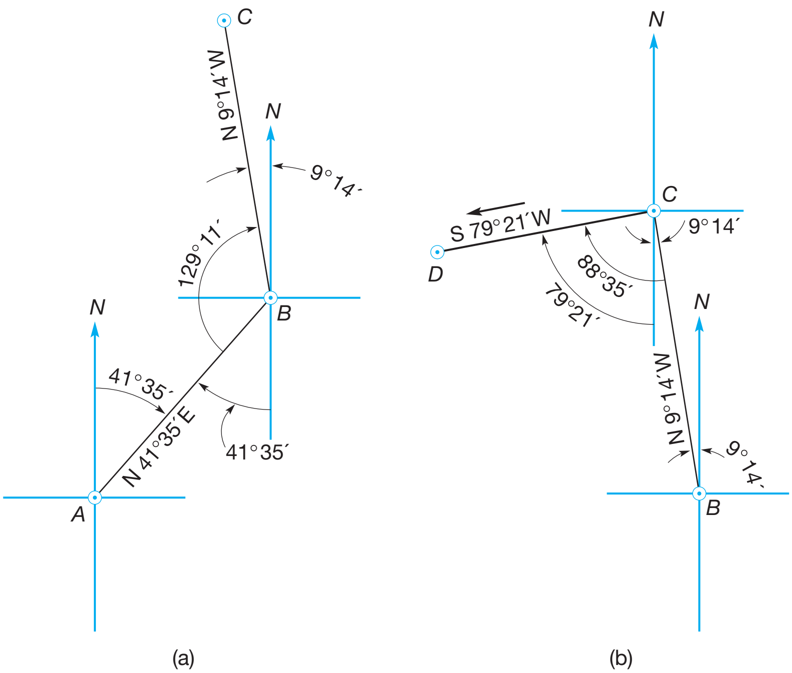

Figure 7.8
(a) Computation of bearing *BC* of Figure 7.2(a).
(b) Computation of bearing *CD* of Figure 7.2(a).

TABLE 7.3 BEARINGS OF LINES IN FIGURE 7.2(a)

Course	Bearing
AB	N41°35′E
BC	N9°14′W
CD	S79°21′W
DE	S31°51′W
EF	S12°27′E
FA	S73°35′E
AB	N41°35′E ✔

using the techniques discussed in Section 7.7. For example in Table 7.2, the azimuth of line *CD* is 259°21′. Using the procedure discussed in Section 7.7, the bearing angle is $259°21' - 180° = 79°21'$, and the bearing is S79°21′W.

Bearings, rather than azimuths, are used predominately in boundary surveying. This practice originated from the period of time when the magnetic bearings of parcel boundaries were determined directly using a surveyor's compass (see Section 7.10). Later, although other instruments (i.e., transits and theodolites) were used to observe the angles, and the astronomic meridian was more commonly used, the practice of using bearings for land surveys continued and is still in common use today. Because boundary retracement surveyors *must follow the footsteps of the* *original surveyor* (see Chapter 21), they need to understand magnetic directions and their nuances. The following sections discuss magnetic directions and explain how to convert directions from magnetic to other reference meridians and vice versa.

■ 7.10 THE COMPASS AND THE EARTH'S MAGNETIC FIELD

Before transits, theodolites, and total station instruments were invented, directions of lines and angles were determined using compasses. Most of the early land-surveying work in the United States was done using these venerable instruments. Figure 7.9(a) shows the *surveyor's compass*. The instrument consists of a metal baseplate (A) with two sight vanes (B) at the ends. The compass box (C) and two small level vials (D) are mounted on the baseplate, the level vials being perpendicular to each other. When the compass was set up and the bubbles in the vials centered, the compass box was horizontal and ready for use.

A single leg called a Jacob staff supported early compasses. A ball-and-socket joint and a clamp were used to rotate the instrument and clamp it in its horizontal position. Later versions, such as that shown in Figure 7.9(a), were mounted on a tripod. This arrangement provided greater stability.

The compass box of the surveyor's compass was covered with glass to protect the magnetized steel needle inside. The needle was mounted on a pivot at the center of a circle that was graduated in degrees. A top view of a surveyor's compass box with its graduations is illustrated in Figure 7.9(b). In the figure, the zero graduations are at the north and south points of the compass and in line with the two sight-vane slits that comprise the line of sight. Graduations are numbered in multiples of 10° clockwise and counterclockwise from 0° at the north and south, to 90° at the east and west.

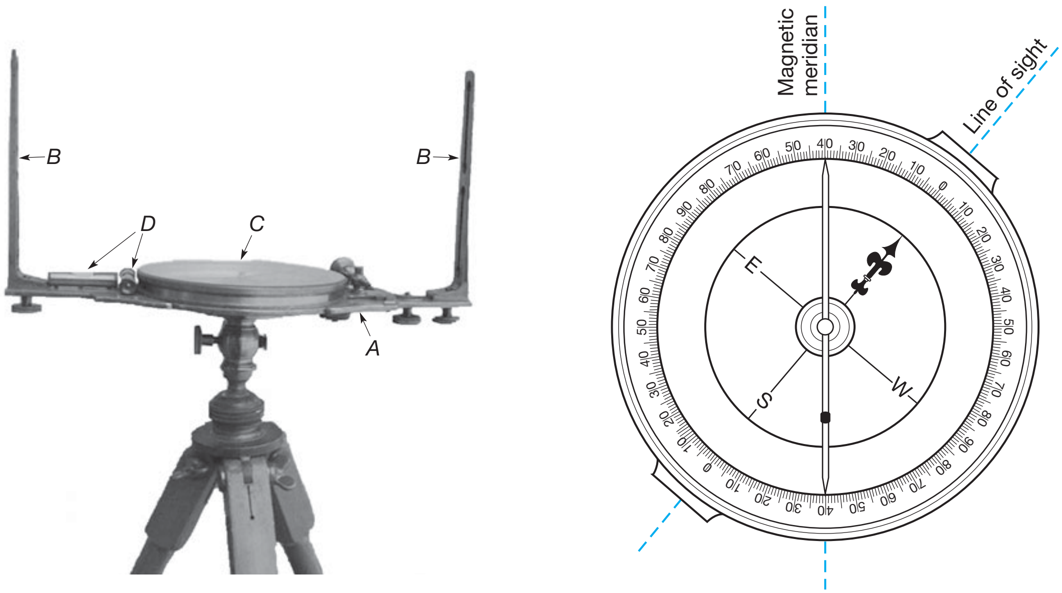

Figure 7.9 (a) Surveyor's compass. (Courtesy W. & L.E. Gurley) (b) Compass box.

In using the compass, the sight vanes and compass box could be revolved to sight along a desired line, and then its magnetic bearing could be read directly. Note in Figure 7.9(b), for example, that the needle is pointing north and that the line of sight is directed in a northeast direction. The magnetic bearing of the line, read directly from the compass, is N 40° E. (Note that the letters *E* and *W* on the face of the compass box are reversed from their normal positions to provide the direct readings of bearings.)

Unless disturbed by *local attraction* (a local anomaly caused from such things as power lines, railroad tracks, metallic belt buckles, and so on that affect the direction a compass needle points at any location), a compass needle is free to spin and align itself with the Earth's magnetic field pointing in the direction of the magnetic meridian (toward the magnetic north pole in the northern hemisphere).[1]

The magnetic forces of the Earth not only align the compass needle, but they also pull or dip one end of it below the horizontal position. The *angle of dip* varies from 0° near the equator to 90° at the magnetic poles. In the northern hemisphere, the south end of the needle is weighted with a very small coil of wire to balance the dip effect and keep it horizontal. The position of the coil can be adjusted to conform to the latitude in which the compass is used. Note the coil (dark spot) on the south end of the needle of the compass of Figure 7.9(b).

The Earth's magnetic field resembles that of a huge dipole magnet located at the Earth's center, with the magnet offset from the Earth's rotational axis by about 11°. This field has been observed at about 200 magnetic observatories around the world, as well at many other temporary stations. At each observation point both the field's intensity and its direction are measured. Based upon many years of data, models of the Earth's magnetic field have been developed. These models are used to compute the *magnetic declination* and *annual change* (see Sections 7.11 and 7.12), which are elements of importance to surveyors (geomatics engineers). The accuracy of the models is affected by several items including the locations of the observations, the types of rocks at the surfaces together with the underlying geological structures in the areas, and local attractions. Today's models give magnetic declinations that are accurate to within about 30 min of arc, however, local anomalies of 3° to 4°, or more, can exist in some areas.

■ 7.11 MAGNETIC DECLINATION

Magnetic declination is the horizontal angle observed from the geodetic meridian to the magnetic meridian. Navigators call this angle *variation* of the compass; the armed forces use the term *deviation.* An east declination exists if the magnetic meridian is east of geodetic north; a west declination occurs if it is west of geodetic north. East declinations are considered positive and west declinations negative. The relationship between geodetic north, magnetic north, and magnetic declination is given by the expression

$$\textit{geodetic azimuth} = \textit{magnetic azimuth} + \textit{magnetic declination} \qquad (7.1)$$

[1]The locations of the north and south geomagnetic poles are continually changing, and in 2005, they were located at approximately 79.74° north latitude and 71.78° west longitude, and 79.74° south latitude and 108.22° east longitude, respectively.

Because the magnetic pole positions are constantly changing, magnetic declinations at all locations also undergo continual changes. Establishing a meridian from astronomical or satellite (GNSS) observations and then reading a compass while sighting along the observed meridian can obtain the current declination at any location obtained baring any local attractions. Another way of determining the magnetic declination at a point is to interpolate it from an *isogonic chart.* An isogonic chart shows magnetic declinations in a certain region for a specific epoch of time. Lines on such maps connecting points that have the same declination are called *isogonic lines.* The isogonic line along which the declination is zero (where the magnetic needle defines geodetic north as well as magnetic north) is termed the *agonic line.* Figure 7.10 is an isogonic chart covering the conterminous (CONUS) 48 states of the United States for the year 1996. On that chart, the agonic line cuts through the central part of the United States. It is gradually moving westward. Points to the west of the agonic line have east declinations and points to the east have west declinations. As a memory aid, the needle can be thought of as pointing toward the agonic line. Note there is about a 40° difference in declination between the northeast portion of Maine and the northwest part of Washington. This is a huge change if a pilot flies by compass between the two states!

The dashed lines in Figure 7.10 show the *annual change* in declination. These lines indicate the amount of *secular change* (see Section 7.12) that is expected in magnetic declination in a period of one year. The annual change at any location can be interpolated between the lines and the value used to estimate the declination a few years before or after the chart date.

■ 7.12 VARIATIONS IN MAGNETIC DECLINATION

It has been stated that magnetic declinations at any point vary over time. These variations can be categorized as *secular, daily, annual*, and *irregular*, and are summarized as follows.

Secular Variation. Because of its magnitude, this is the most important of the variations. Unfortunately, no physical law has been found to enable precise long-term predictions of secular variation, and its past behavior can be described only by means of detailed tables and charts derived from observations. Records, which have been kept at London for four centuries, show a range in magnetic declination from 11°E in 1580, to 24°W in 1820, back to 3°W in 2000. Secular variation changed the magnetic declination at Baltimore, MD, from 5°11′W in 1640 to 0°35′W in 1800, 5°19′W in 1900, 7°25′W in 1950, 8°43′W in 1975, and 11°01′W in 2000.

In retracing old property lines run by compass or based on the magnetic meridian, it is necessary to allow for the difference in magnetic declination at the time of the original survey and at the present date. The difference is attributed mostly to secular variation.

Daily Variation. Daily variation of the magnetic needle's declination causes it to swing through an arc averaging approximately 8′ for the United States. The needle reaches its extreme easterly position at about 8:00 A.M. and its most westerly position at about 1:30 P.M. Mean declination occurs at around 10:30 A.M. and 8:00 P.M. These hours and the daily

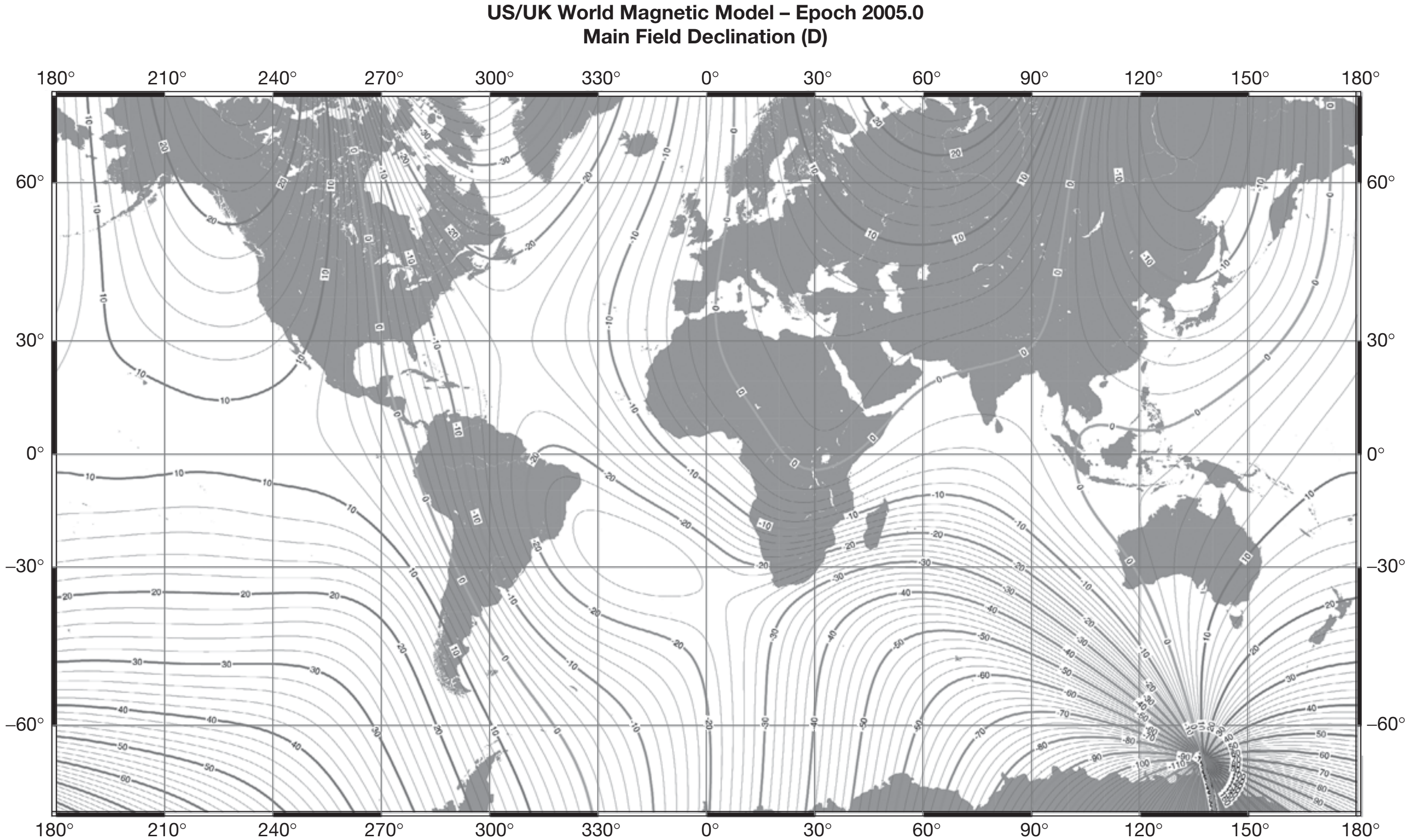

Figure 7.10 Isogonic lines from World Magnetic Model for 2005. This image is from the NOAA National Geophysical Data Center, NGDC on the Internet at http://www.ngdc.noaa.gov/seg/geomag/declination.shtml

variation change with latitude and season of the year. Usually the daily variation is ignored since it is well within the range of error expected in compass readings.

Annual Variation. This periodic swing is less than 1 min of arc and can be neglected. It must not be confused with the annual change (the amount of secular-variation change in one year) shown on some isogonic maps.

Irregular Variations. Unpredictable magnetic disturbances and storms can cause short-term irregular variations of a degree or more.

■ 7.13 SOFTWARE FOR DETERMINING MAGNETIC DECLINATION

As noted earlier, direct observations are only applicable for determining current magnetic declinations. In most situations, however, magnetic declinations that existed years ago, for example on the date of an old property survey, are needed in order to perform retracement surveys. Until recently these old magnetic declinations had to be interpolated from isogonic charts for the approximate time desired, and the lines of annual change used to correct to the specific year required. Now software is available that can quickly provide the needed magnetic declination values. The software uses models that were developed from historical records of magnetic declination and annual change, which have been maintained for the many observation stations throughout the United States and the world.

The program WOLFPACK, which is on the companion website for this book at http://www.pearsonhighered.com/ghilani, contains an option for computing magnetic field elements. This program uses models that span five or more year time frames. Using the World Magnetic Model of 1995 (file: WMM-95.DAT), the declination and annual change for Portland Maine on September 25, 1999 were determined to be about 16°54′W[2] and 0.0′W per year, respectively (see the input data in Figure 7.11). Using this same program, the declinations for various other cities in the United States were determined for January 1, 2000, and are shown in Table 7.4. It is important when using this software to select the appropriate model file for the desired date. Select the appropriate model from a drop-down list for the "Model File." The models are given by their source, and the

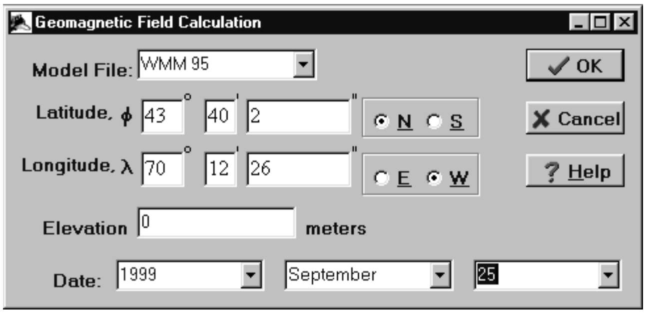

Figure 7.11 Magnetic declination data entry screen in WOLFPACK setup to compute magnetic field values for Portland, Maine.

[2]The software indicates west declination as negative, and east declination as positive.

TABLE 7.4 MAGNETIC DECLINATION AND ANNUAL CHANGE FOR VARIOUS LOCATIONS IN THE UNITED STATES FOR JANUARY 1, 2000

City	Magnetic Declination	Annual Change
Boston, MA	15°48′W	1.0′E
Cleveland, OH	7°45′W	4.1′W
Madison, WI	1°13′W	6.2′W
Denver, CO	10°21′E	4.9′W
San Francisco, CA	15°26′E	3.1′W
Seattle, WA	18°45′E	7.2′W

year. The latitude, longitude, and elevation of the station must be entered in the appropriate data boxes and the time of the desired computation is selected from the drop-down list at the bottom of the box. After computing the magnetic field elements for the particular location and time, the results are displayed for printing. Similar computations to determine magnetic declination and rates of annual change can be made by using the NOAA National Geophysical Data Centers' (NGDC) online computation page at http://www.ngdc.noaa.gov/geomag/geomag.shtml. The location of any U.S. city can be found with the U.S. Gazetteer, which is linked to the software, or can be obtained at http://www.census.gov/cgibin/gazetteer on the Internet page of the U.S. Census Bureau. It should be noted that all of these models are only accurate to the nearest 30 min and should be used with caution.

7.14 LOCAL ATTRACTION

Metallic objects and direct-current electricity, both of which cause a local attraction, affect the main magnetic field. As an example, when set up beside an old-time streetcar with overhead power lines, the compass needle would swing toward the car as it approached, then follow it until it was out of effective range. If the source of an artificial disturbance is fixed, all bearings from a given station will be in error by the same amount. However, angles calculated from bearings taken at the station will be correct.

Local attraction is present if the forward and back bearings of a line differ by more than the normal observation errors. Consider the following compass bearings read on a series of lines:

AB	N24°15′W
BC	N76°40′W
CD	N60°00′E
DE	N88°35′E
BA	S24°10′E
CB	S76°40′E
DC	S61°15′W
ED	S87°25′W

Forward-bearing *AB* and back-bearing *BA* agree reasonably well, indicating that little or no local attraction exists at *A* or *B*. The same is true for point *C*. However, the bearings at *D* differ from corresponding bearings taken at *C* and *E* by roughly 1°15′ to the west of north. Local attraction therefore exists at point *D* and deflects the compass needle by approximately 1°15′ to the west of north.

It is evident that to detect local attraction, successive stations on a compass traverse have to be occupied, and forward and back bearings read, even though the directions of all lines could be determined by setting up an instrument only on alternate stations.

7.15 TYPICAL MAGNETIC DECLINATION PROBLEMS

Typical problems in boundary surveys require the conversion of geodetic bearings to magnetic bearings, magnetic bearings to geodetic bearings, and magnetic bearings to magnetic bearings for the declinations existing at different dates. The following examples illustrate two of these types of problems.

Example 7.3

Assume the magnetic bearing of a property line was recorded as S43°30′E in 1862. At that time the magnetic declination at the survey location was 3°15′W. What geodetic bearing is needed for a subdivision property plan?

Solution

A sketch similar to Figure 7.12 makes the relationship clear and should be used by beginners to avoid mistakes. Geodetic north is designated by a full-headed long arrow and magnetic north by a half-headed shorter arrow. The geodetic bearing is seen to be S43°30′E + 3°15′ = S46°45′E. Using different colored pencils to show the direction of geodetic north, magnetic north, and lines on the ground helps

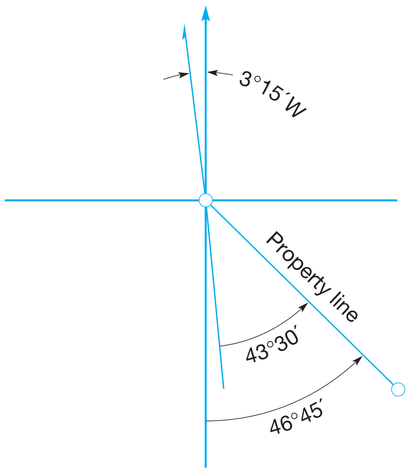

Figure 7.12 Computing geodetic bearings from magnetic bearings and declinations.

clarify the sketch. Although this problem is done using bearings, Equation (7.1) could be applied by converting the bearings to azimuths. That is, the magnetic azimuth of the line is 136°30′. Applying Equation (7.1) using a negative declination angle results in a geodetic azimuth of 136°30′ − 3°15′ = 133°15′, which correctly converts to the geodetic bearing of *S*46°45′E.

Example 7.4

Assume the magnetic bearing of line *AB* read in 1878 was N26°15′E. The declination at the time and place was 7°15′W. In 2000, the declination was 4°30′E. The magnetic bearing in 2000 is needed.

Solution

The declination angles are shown in Figure 7.13. The magnetic bearing of line *AB* is equal to the earlier date bearing minus the sum of the declination angles, or

$$\text{N}26°15'\text{E} - (7°15' + 4°30') = \text{N}14°30'\text{E}$$

Again, the problem can be computed using azimuths as 26°15′ − 7°15′ − 4°30′ = 14°30′, which converts to a bearing of N14°30′E.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *Magnetic Directions.mp4* discusses how to obtain the magnetic declination for any time period, the process of converting magnetic azimuths to their geodetic equivalents, and how to convert magnetic directions between different time periods.

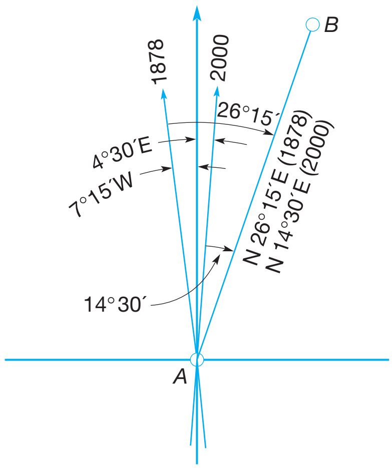

Figure 7.13 Computing magnetic bearing changes due to declination changes.

7.16 MISTAKES

Some mistakes made in using azimuths and bearings are:

1. Confusing magnetic and other reference bearings.
2. Mixing clockwise and counterclockwise angles.
3. Interchanging bearings for azimuths.
4. Listing bearings with angular values greater than 90°.
5. Failing to include both directional letters when listing a bearing.
6. Failing to change bearing letters when using the back bearing of a line.
7. Using an angle at the wrong end of a line in computing bearings—that is, using angle A instead of angle B when starting with line AB as a reference.
8. Not including the last angle to recompute the starting bearing or azimuth as a check—for example, angle A in traverse $ABCDEA$.
9. Subtracting 360°00′ as though it were 359°100′instead of 359°60′, or using 90° instead of 180° in bearing computations.
10. Adopting an assumed reference line that is difficult to reproduce.
11. Reading degrees and decimals from a calculator as though they were degrees, minutes, and seconds.
12. Failing to adjust traverse angles before computing bearings or azimuths if there is a misclosure.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

7.1 List the different reference meridians that can be used for the direction of a line and describe the advantages and disadvantages of each system.

7.2 What are the disadvantages of using an assumed meridian for the starting course in a traverse?

7.3 What is meant by an angle to the right?

7.4 By means of a sketch describe (a) interior angles, (b) angles to the right, and (c) deflection angles.

7.5 Convert: ***(a)** 203°26′48″ to grads **(b)** 339.0648 grads to degrees, minutes, and seconds **(c)** 207°18′45″ to radians.

In Problems 7.6 through 7.7, convert the azimuths from north to bearings, and compute the angles, smaller than 180° between successive azimuths.

7.6 68°06′42″, 133°15′56″, 217°44′05″, and 320°35′18″

7.7 65°12′55″, 146°27′39″, 233°56′12″, and 348°52′11″

Convert the bearings in Problems 7.8 through 7.9 to azimuths from north and compute the angle, smaller than 180°, between successive bearings.

7.8 N27°50′05″E, *S*38°12′44″E, *S*23°16′22″W, and N73°14″30″W

7.9 N12°18′38″E, S14°32′12″E, S82°12′10″W, and N02°15′41″W

Compute the azimuth from north of line CD in Problems 7.10 through 7.12. (Azimuths of AB are also from north.)

7.10* Azimuth AB = 68°26′32″; angles to the right ABC = 45°07′08″, BCD = 36°26′48″.

7.11 Bearing AB = S14°26′12″E; angles to the right ABC = 133°20′46″, BCD = 54°31′28″.

7.12 Azimuth AB = 195°12′07″; angles to the right ABC = 10°36′09″, BCD = 32°16″14″.

7.13* For a bearing DE = N08°53′56″W and angles to the right, compute the bearing of FG if angle DEF = 88°12″29″ and EFG = 40°20′30″.

7.14 Similar to Problem 7.13, except the azimuth of DE is 132°22′48″ and angles to the right DEF and EFG are 101°34′02″ and 51°09′01″, respectively.

Course AB of a five-sided traverse runs due north. From the given balanced interior angles to the right, compute and tabulate the bearings and azimuths from north for each side of the traverses in Problems 7.15 through 7.17.

7.15 A = 77°23′26″, B = 125°58″59″, C = 105°28′32″, D = 116°27′02″, E = 114°42′01″

7.16* A = 90°29′18″, B = 107°54′36″, C = 104°06′37″, D = 129°02′57″, E = 108°26′32″

7.17 A = 98°12′18″, B = 126°08′30″, C = 100°17′44″, D = 110°50′40″, E = 104°30′48″

In Problems 7.18 and 7.19, compute and tabulate the azimuths of the sides of a regular hexagon (polygon with six equal angles), given the starting direction of side AB.

7.18 Bearing of AB = N56°27′13″W (Station C is westerly from B.)

7.19 Azimuth of AB = 87°14′26″ (Station C is westerly from B.)

7.20 Describe the relationship between forward and back azimuths.

Compute azimuths of all lines for a closed traverse $ABCDEFA$ that has the following balanced angles to the right, using the directions listed in Problems 7.21 and 7.22. FAB = 118°26′59″, ABC = 123°20′28″, BCD = 104°10′32″, CDE = 133°52′50″, DEF = 108°21′58″, EFA = 131°47′13″.

7.21 Bearing AB = S28°18′42″W.

7.22 Azimuth DE = 116°10′20″.

7.23 Similar to Problem 7.21, except that bearings are required, and fixed bearing AB = N33°46′25″E.

7.24 Similar to Problem 7.22, except that bearings are required, and fixed azimuth DE = 286°22′40″ (from north).

7.25 Geometrically show how the sum of the interior angles of a pentagon (five sides) can be computed using the formula $(n - 2)180°$?

7.26 Determine the predicted declinations on January 1, 2010 using the WMM-10 model at the following locations.

(a)* latitude = 42°58′28″N, longitude = 77°12′36″W, elevation = 310.0 m;
(b) latitude = 37°56′44″N, longitude = 110°50′40″W, elevation = 1500 m;
(c) latitude = 41°18′15″N, longitude = 76°00′26″W, elevation = 240 m;

7.27 Using Table 7.4, what was the total difference in magnetic declination between Boston, MA and San Francisco, CA on January 1, 2000?

7.28 The magnetic declination at a certain place is 12°06′W. What is the magnetic bearing there: **(a)** of true north **(b)** of true south **(c)** of true west?

7.29 Same as Problem 7.28, except the magnetic declination at the place is 3°30′E.

For Problems 7.30 through 7.32 the observed magnetic bearing of line AB and its true magnetic bearing are given. Compute the amount and direction of local attraction at point A.

	Observed Magnetic Bearing	True Magnetic Bearing
7.30*	N28°15′E	N30°15′E
7.31	S13°25′W	S10°15′W
7.32	N11°56′W	N8°20′E

What magnetic bearing is needed to retrace a line for the conditions stated in Problems 7.33 through 7.36?

	1875 Magnetic Bearing	1875 Declination	Present Declination
7.33*	N32°45′E	8°12′W	2°30′E
7.34	S63°40′W	3°40′E	2°20′W
7.35	N69°20′W	1°20′W	3°30′W
7.36	S24°30′E	12°30′E	22°30′E

In Problems 7.37 through 7.38, calculate the magnetic declination in 1870 based on the following data from an old survey record.

	1870 Magnetic Bearing	Present Magnetic Bearing	Present Magnetic Declination
7.37	S14°20′E	S15°50′E	1°15′W
7.38	S40°30′E	S52°35′E	8°30′E

7.39 An angle *APB* is measured at different times using various instruments and procedures. The results, which are assigned certain weights, are as follows: 46°13′28″, wt 1; 46°13′32″, wt 2; and 43°13′30″, wt 3. What is the most probable value of the angle?
7.40 Similar to Problem 7.39, but with an additional measurement of 43°13′32″, wt 4.
7.41 Explain why the letters E and W on a compass [see Figure 7.9(b)] are reversed from their normal positions.
7.42 Create a computational program that solves Problem 7.21.
7.43 Create a computational program that solves Problem 7.22.

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8 Total Station Instruments; Angle Observations

PART I • TOTAL STATION INSTRUMENTS

■ 8.1 INTRODUCTION

In the past, transits and theodolites were the most commonly used surveying instruments for making angle observations. These two devices were fundamentally equivalent and could accomplish basically the same tasks.[1] Today, the total station instrument has replaced transits and theodolites. Total station instruments can accomplish all of the tasks that could be done with transits and theodolites and do them much more efficiently. In addition, they can also observe distances accurately and quickly. Furthermore, they can make computations with the angle and distance observations, and display the results in real time. These and many other significant advantages have made total stations the predominant instruments used in surveying practice today. They are used for all types of surveys including topographic, hydrographic, cadastral, and construction surveys. The use of total station instruments for specific types of surveys is discussed in later chapters. This chapter describes the general design and characteristics of total station instruments, and also concentrates on procedures for using them in observing angles.

■ 8.2 CHARACTERISTICS OF TOTAL STATION INSTRUMENTS

Total station instruments, as shown in Figure 8.1, combine three basic components—an electronic distance measuring (EDM) instrument, an electronic angle measuring component, and a computer or microprocessor—into one integral unit.

[1] Discussions of the transit and theodolite are given in Appendix A.

Figure 8.1 Parts of a total station instrument, with view of eyepiece end of telescope. (Courtesy Leica Geosystems AG.)

These devices can automatically observe horizontal and vertical angles, as well as slope distances from a single setup (see Chapter 6). From these data they can compute horizontal and vertical distance components instantaneously, elevations and coordinates of points sighted, and display the results on a *liquid crystal display* (LCD). As discussed in Chapter 2, they can also store the data, either on board or in external data collectors connected to their communication ports.

The telescope is an important part of a total station instrument. It is mounted between the instrument's *standards* (see Figure 8.1), and after the instrument has been leveled, it can be revolved (or "plunged") so that its *axis of sight*[2] defines a vertical plane. The axis about which the telescope revolves is called the *horizontal axis*. The telescope can also be rotated in any azimuth about a vertical line called the *vertical axis*. Being able to both revolve and rotate the telescope in this manner makes it possible for an operator to aim the telescope in any azimuth, and along any slope, to sight points. This is essential in making angle observations,

[2]The axis of sight, also often called the "line of sight," is the reference line within the telescope which an observer uses for making pointings with the instrument. As defined in Section 4.7, it is the line connecting the optical center of the objective lens and the intersection of crosshairs in the reticle.

as described in Part II of this chapter. The three reference axes, the axis of sight, the horizontal axis, and the vertical axis, are illustrated in Figure 8.24.

The EDM instruments that are integrated into total station instruments (described in Section 6.21), are relatively small, and as shown in Figure 8.1, are mounted with the telescope between the standards of the instrument. Although the EDM instruments are small, they still have distance ranges adequate for most work. Lengths up to about 4 km can be observed with a single prism, and even farther with a triple prism like the one shown in Figure 6.12.

Total station instruments are manufactured with two graduated circles, mounted in mutually perpendicular planes. Prior to observing angles, the instrument is leveled so that its *horizontal* circle is oriented in a horizontal plane, which automatically puts the vertical circle in a vertical plane. Horizontal and zenith (or altitude) angles can then be observed directly in their respective planes of reference. To increase the precision of the final horizontal angle, repeating instruments had two vertical axes. This resulted in two horizontal motion screws. One set of motion screws allowed the instrument to be turned without changing the value on the horizontal circle. Repeating theodolites are discussed in more detail in Section A.5.2. Today's total station instruments usually have only one vertical axis and thus are considered directional instruments. However, as discussed later, angles can be repeated on a total station by following the procedures described in the instrument's manual. Most early versions of total station instruments employed level vials for orienting the circles in horizontal and vertical planes, but many newer ones now use automatic compensators, or electronic tilt-sensing mechanisms.

The angle resolution of available total stations varies from as low as a half-second for precise instruments suitable for control surveys, up to 20″ for less expensive instruments made specifically for construction work. Formats used for displaying angles also vary with different instruments. For example, the displays of some actually show the degree, minute, and second symbols, but others use only a decimal point to separate the number of degrees from the minutes and seconds. Thus, 315.1743 is actually 315°17′43″. Most instruments allow a choice of units, such as the display of angular measurements in degrees, minutes, and seconds, or in grads (gons). Distances may be shown in either feet or meters. Also, certain instruments enable the choice of displaying either zenith or altitude angles. These choices are entered through the keyboard, and the microprocessor performs the necessary conversions accordingly. The keyboard, used for instrument control and data entry, is located just above the leveling head, as shown in Figure 8.1.

Once the instrument has been set up and a sighting has been made through the telescope, the time required to make and display an angle and distance reading is approximately 2 to 4 sec when a total station instrument is being operated in the *normal mode*, and less than 0.5 sec when operated in the *tracking mode*. The normal mode, which is used in most types of surveys with the exception of construction layout, results in higher precision because multiple observations are made and averages taken. In the tracking mode, used primarily for construction layout, a prism is held on line near the anticipated final location of a stake. An observation is quickly taken to the prism, and the distance that it must be moved forward or back is instantly computed and displayed. The prism is moved ahead or back according to the results of the first observation, and another check of the distance is made. The process is quickly repeated as many times as necessary until the correct distance is obtained, whereupon the stake is set. This procedure is discussed in more detail in Chapter 23.

Robotic total stations, which are further discussed in Section 8.6, have servomotors on both the horizontal and vertical axes that allow the instrument to perform a second pointing on a target or track a roving target without operator interaction. These instruments are often used in construction layout. In fact, robotic total stations are required in machine control on a construction site as discussed in Section 23.11. In machine control, the instrument guides a piece of construction equipment through the site preparation process, informing the construction equipment operator of the equipment's position on the job site and the amount of soil that needs to be removed or added at its location to match the project design.

8.3 FUNCTIONS PERFORMED BY TOTAL STATION INSTRUMENTS

Total station instruments, with their microprocessors, can perform a variety of functions and computations, depending on how they are programmed. Most are capable of assisting an operator, step by step, through several different types of basic surveying operations. After selecting the type of survey from a menu, prompts will automatically appear on the display to guide the operator through each step. An example illustrating a topographic survey conducted using this procedure is given in Section 17.9.1.

In addition to providing guidance to the operator, microprocessors of total stations can perform many different types of computations. The capabilities vary with different instruments, but some standard computations include (1) averaging of multiple angle and distance observations, (2) correcting electronically observed distances for prism constants, atmospheric pressure, and temperature, (3) making curvature and refraction corrections to elevations determined by trigonometric leveling, (4) reducing slope distances to their horizontal and vertical components, (5) calculating point elevations from the vertical distance components (supplemented with keyboard input of instrument and reflector heights), and (6) computing coordinates of surveyed points from horizontal angle and horizontal distance components (supplemented with keyboard input of coordinates for the occupied station, and a reference azimuth. The subject of coordinate computations is covered in Chapters 10 and 11.

Many total stations, but not all, are also capable of making corrections to observed horizontal and vertical angles for various instrumental errors. For example, by going through a simple calibration process, the *indexing error* of the vertical circle can be determined (see Section 8.13), stored in the microprocessor, and then a correction applied automatically each time a vertical angle is observed. A similar calibration and correction procedure applies to errors that exist in horizontal angles due to imperfections in the instrument (see Section 8.8).

Some total stations are also able to correct for personal errors, such as imperfect leveling of the instrument. By means of tilt-sensing mechanisms, they automatically measure the amount and direction of dislevelment, and then make corrections to the observed horizontal and vertical angles for this condition.

8.4 PARTS OF A TOTAL STATION INSTRUMENT

The upper part of the total station instrument, called the *alidade*, includes the telescope, graduated circles, and all other elements necessary for measuring angles and distances. The basic design and appearance of these instruments (see Figures 8.1 and 8.2) are:

1. The *telescopes* are short, have reticles with crosshairs etched on glass, and are equipped with rifle sights or collimators for rough pointing. Most telescopes have two focusing controls. The objective lens control is used to focus on the object being viewed. The eyepiece control is used to focus on the reticle. If the focusing of the two lenses is not coincident, a condition known as *parallax* will

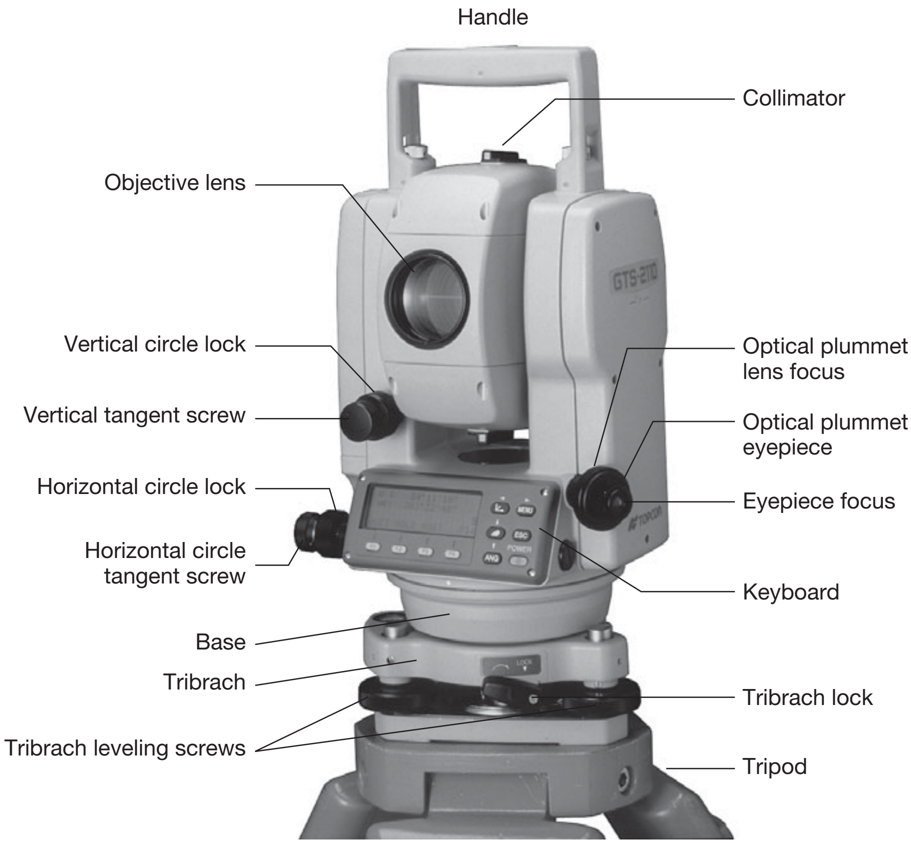


Figure 8.2 Parts of a total station instrument with view of objective end of the telescope. (Courtesy Topcon Positioning Systems.)

exist. Parallax is the apparent motion of an object caused by a movement in the position of the observer's eye. The existence of parallax can be observed by quickly shifting one's eye position slightly and watching for movement of the object in relation to the crosshairs. Careful adjustment of the eyepiece and objective lens will result in a sharp image of both the object and the crosshairs with no visible parallax. Since the eye tends to tire through use, the presence of parallax should be checked throughout the day. A common mistake of beginners is to have a colleague "check" their pointings. This is not recommended for many reasons including the personal focusing differences that exist between different individuals.

With newer instruments, objective lens auto focusing is available. This works in a manner similar to auto focusing for a camera, and increases the rate at which pointings can be made when objects are at variable distances from the instrument.

2. The *angle measurement system* functions by passing a beam of light through finely spaced graduations. The Topcon GTS 210 of Figure 8.2 is representative of the way total stations operate, and is briefly described here. For horizontal angle measurements, two glass circles within the alidade are mounted parallel, one on top of the other, with a slight spacing between them. After the instrument has been leveled, the circles should be in horizontal planes. The *rotor* (lower circle) contains a pattern of equally divided alternate dark lines and light spaces. The *stator* (upper circle) contains a slit-shaped pattern, which has the same pitch as that of the rotor circle. A *light-emitting diode* (LED) directs collimated light through the circles from below toward a photo detector cell above. A modern total station may have as many as 20,000 graduations!

When an angle is turned, the rotor moves with respect to the stator creating alternating variations of light intensity. Photo detectors sense these variations, convert them into electrical pulses, and pass them to a microprocessor for conversion into digital values. The digits are displayed using a *liquid crystal diode* (LCD). Another separate system like that just described is also mounted within the alidade for measuring *zenith* (or *altitude*) *angles*. With the instrument leveled, this vertical circle system is aligned in a vertical plane. After making an observation, horizontal and vertical angles are both displayed, and can be manually read and recorded in field books, or alternatively, the instruments can be equipped with data collectors that eliminate manual reading and recording. (This helps eliminate mistakes!) The Topcon GTS 210 can resolve angles to an accuracy of 5″.

3. The *vertical circle* of most total station instruments is precisely indexed with respect to the direction of gravity by an *automatic compensator*. These devices are similar to those used on automatic levels (see Section 4.10) and automatically align the vertical circle so that 0° is oriented precisely upward toward the *zenith* (opposite the direction of gravity). Thus, the vertical circle readings are actually *zenith angles*, that is, 0° occurs with the telescope pointing vertically upward, and either 90° or 270° is read when it is horizontal. Upon command, the microprocessor can convert zenith angles to *altitude angles* (i.e., values measured up or down from 0° at the horizontal). The *vertical motion*, which contains a *lock* and *tangent screw*, enables the telescope to be released so that it can be revolved

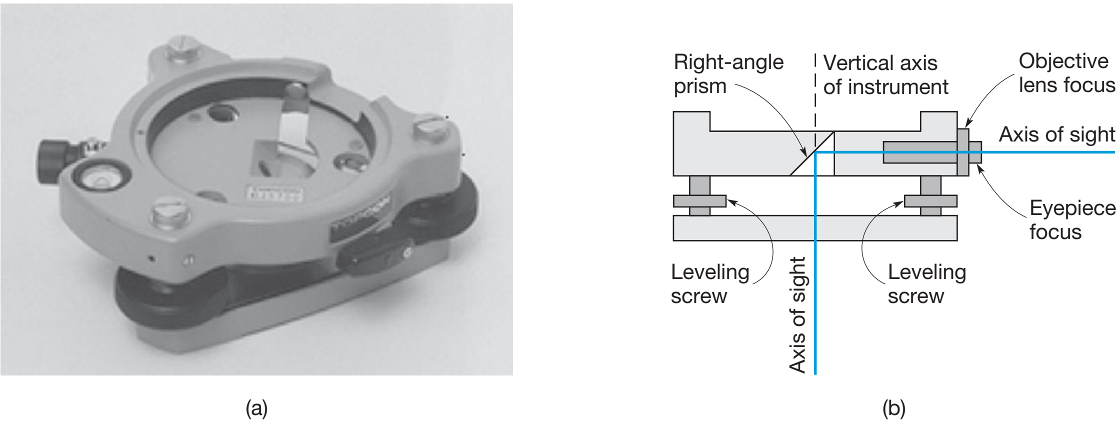
about the horizontal axis, or locked (clamped) to prevent it from revolving. To sight a point, the lock can be opened and the telescope tilted up or down about the horizontal axis as necessary to the approximate position needed to sight a point. The lock is then clamped, and fine pointing completed using the vertical tangent screw.

In servo-driven total stations (see Figure 8.7), the lock and tangent screw are replaced with a jog/shuttle mechanism. This device actuates an internal servo-drive motor that rotates the telescope about its horizontal axis. The speed at which the mechanism rotates determines the speed at which the telescope rotates.

4. Rotation of the telescope about the vertical axis occurs within a steel cylinder or on precision ball bearings, or a combination of both. The *horizontal motion*, which also contains a *lock* and *tangent screw*, controls this rotation. Clamping the lock can prevent rotation. To sight a point, the lock is released and the telescope rotated in azimuth to the approximate direction desired, and the lock clamped again. Then the horizontal tangent screw enables a fine adjustment to be made in the direction of pointing. (Actually when sighting a point, both the vertical and horizontal locks are released so that the telescope can be simultaneously revolved and rotated. Then both are locked and fine pointing made using the two tangent screws.)

Similar to the vertical motion in servo-driven total stations, the horizontal lock and tangent screw is replaced with a jog/shuttle mechanism that actuates an internal servo-drive to rotate the instrument about its vertical axis. Again the speed at which the mechanism is rotated determines the speed at which the instrument rotates.

5. The *tribrach* (see Figures 8.1 and 8.2) consists of three screws or cams for leveling, a circular level, clamping device to secure the base of the total station or accessories (such as prisms and sighting targets), and threads to attach the tribrach to the head of a tripod. As shown in Figure 8.3, some tribrachs also have



(a) (b)

Figure 8.3 (a) Tribrach with optical plummet, (b) schematic of a tribrach optical plummet. [Figure (a), Courtesy Topcon Positioning Systems.]

integral optical plummets (described below) to enable centering accessories over a point without the instrument.

6. The *bases* of total stations are often designed to permit interchange of the instrument with sighting targets and prisms in tribrachs without disturbing previously established centering over survey points. This can save a considerable amount of time. Most manufacturers use a standardized "three-post" arrangement to enable interchangeability between different instruments and accessories.

7. An *optical plummet*, built into either the tribrach or alidade of total station instruments, permits accurate centering over a point. Although either type enables accurate centering, best accuracy is achieved with those that are part of the alidade of the instrument. The optical plummet provides a line of sight that is directed downward, collinear with the vertical axis of the instrument. But the total station instrument or tribrach must be leveled first for the line of sight to be *vertical*. Figures 8.3(a) and (b) show a tribrach with optical plummet, and a schematic of the tribrach optical plummet, respectively. Due to the short length of the telescope in an optical plummet, it is extremely important to remove parallax before centering the instrument with this device.

In newer instruments, laser plummets have replaced the optical plummet. This device produces a beam of collimated light that coincides with the vertical axis of the instrument. Since focusing of the objective and eyepiece lens is not required with a laser plummet, this option will increase both the speed and accuracy of setups. However, the laser mark may be difficult to see in bright sunlight. Shading the mark can help in these situations.

8. When being used, total station instruments stand on *tripods.* The tripods are the wide-frame type, and most have adjustable legs. Their primary composition may be wood, metal, or fiberglass.

9. The *microprocessor* provides several significant advantages to surveyors. As examples, (a) the circles can be zeroed instantaneously by simply pressing a button, or they can be initialized to any value by entry through the keyboard (valuable for setting the reference azimuth for a backsight); (b) angles can be observed with values increasing either left or right; and (c) angles observed by repetition (see Section 8.8), can be added to provide the total, even though 360° may have been passed one or more times. Other advantages include reduction of mistakes in making readings, and an increase in the overall speed of operation.

10. The *keyboard* and *display* (see Figure 8.2) provide the means of communicating with the microprocessor. Most total stations have a keyboard and display on both sides of the instrument, a feature that is especially convenient when operating the instrument in both the *direct* and *reverse* modes (see Section 8.8), as is usually done when observing angles. Some robotic total stations (see Section 8.6) also have a keyboard and display mounted on a remote prism pole for "one-person" operations.

11. The *communication port* (see Figure 8.1) enables external data collectors to be connected to the instrument. Some instruments have internal data collection capabilities, and their communications ports permit them to be interfaced with a computer for direct downloading of data.

■ 8.5 HANDLING AND SETTING UP A TOTAL STATION INSTRUMENT

A total station instrument should be carefully lifted from its carrying case by grasping the standards or handle, and the instrument securely fastened to the tripod by means of the tribrach. For most surveys, prior to observing distances and angles, the instrument must first be carefully set up over a specific point. The setup process using an instrument with an optical plummet, tribrach mount with circular bubble, and adjustable-leg tripod is accomplished most easily using the following steps: (1) extend the legs so that the scope of the instrument will be at an appropriate elevation for view and then adjust the position of the tripod legs by lifting and moving the tripod as a whole until the point is roughly centered beneath the tripod head (beginners can drop a stone from the center of the tripod head, or use a plumb bob to check nearness to the point); (2) firmly place the legs of the tripod in the ground and extend the legs so that the head of the tripod is approximately level; repeat step (1) if the tripod head is not roughly centered over the point; (3) roughly center the tribrach leveling screws on their posts; (4) mount the tribrach approximately in the middle of the tripod head to permit maximum translation in step (9) in any direction; (5) focus the plummet properly on the point, making sure to check for parallax; (6) manipulate the leveling screws to aim the plummet's pointing device at the point below; (7) center the circular bubble by adjusting the lengths of the tripod extension legs; (8) and level the instrument using the plate bubble and leveling screws; and (9) if necessary, loosen the tribrach screw and translate the instrument (do not rotate it) to carefully center the plummet's pointing device on the point; (10) repeat steps (8) and (9) until precise leveling and centering are accomplished. With total stations that have their plummets in the tribrach, the instrument can and should be left in the case until step (8).

To level a total station instrument that has a plate-level vial, the telescope is rotated to place the axis of the level vial parallel to the line through any two leveling screws, as the line through A and B in Figure 8.4(a). The bubble is centered

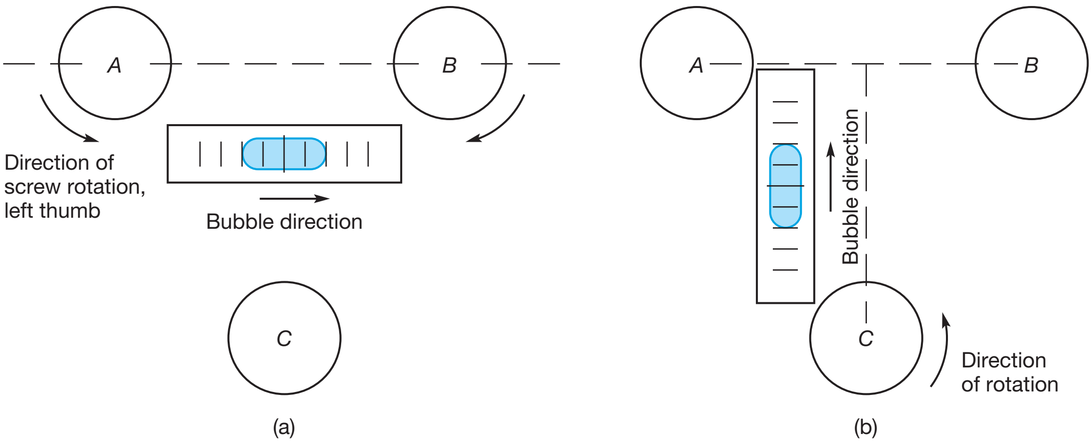


Figure 8.4 Bubble centering with three-screw leveling head.

Figure 8.5
The LEICA TPS 300 electronic leveling system. (Courtesy Leica Geosystems AG.)

by turning these two screws, then rotated 90°, as shown in Figure 8.4(b), and centered again using the third screw (C) only. This process is repeated in the initial two positions and carefully checked to ensure that the bubble remains centered. As illustrated in Figure 8.4, *the bubble moves in the direction of the left thumb when the foot screws are turned*. A solid tripod setup is essential, and the instrument must be shaded if set up in bright sunlight. Otherwise, the bubble will expand and run toward the warmer end as the liquid is heated.

Many instruments, such as the LEICA TPS 300 shown in Figure 8.1, do not have traditional level vials. Rather, they are equipped with an electronic, dual-axis leveling system as shown in Figure 8.5 in which four probes sense a liquid (horizontal) surface. After preliminary leveling is performed by means of the tribrach's circular bubble, signals from the probes are processed to form an image on the LCD display, which guides an operator in performing rough leveling. The three leveling screws are used, but the instrument need not be turned about its vertical axis in the leveling process. After rough leveling, the amount and direction of any residual dislevelment is automatically and continuously received by the microprocessor, which corrects observed horizontal and vertical angles accordingly in real time.

As noted earlier, total stations are controlled with entries made either through their built-in keypads or through the keypads of handheld data collectors. Details for operating each individual total station vary somewhat and therefore are not described here. They are covered in the manuals provided with the purchase of instruments.

When moving between setups in the field, proper care should be taken. Before the total station is removed from the tripod, the foot screws should be returned to the midpoints of the posts. Many instruments have a line on the screw post that indicates the halfway position. The instrument should NEVER

(a) (b)

Figure 8.6 (a) A proper method of transporting a total station in the field. (b) Total station in open case. (Courtesy Leica Geosystems AG.)

be transported on the tripod since this causes stress to tripod head, tribrach, and instrument base. Figure 8.6(a) depicts the proper procedure for carrying equipment in the field. With adjustable-leg tripods, retracting them to their shortest positions and lightly clamping them in position can avoid stress on the legs.

When returning the total station to its case, all locking mechanisms should be released. This procedure protects the threads and reduces wear when the instrument is jostled during transport and also prevents the threads from seizing during long periods of storage. If the instrument is wet, it should be wiped down and left in an open case until it is dry as shown in Figure 8.6(b). When storing tripods, it is important to loosen or lightly clamp all legs. This is especially true with wooden tripods where the wood tends to expand and contract with humidity in the air. Failure to loosen the clamping mechanism on wooden tripods can result in crushed wood fibers, which inhibit the ability of the clamp to hold the leg during future use.

■ 8.6 SERVO-DRIVEN AND REMOTELY OPERATED TOTAL STATION INSTRUMENTS

Manufacturers also produce "robotic" total station instruments equipped with servo-drive mechanisms that enable them to aim automatically at a point to be set. The Geodimeter 4000 Robotic Total Station from Spectra Precision shown on the left in Figure 8.7 is an example. With these instruments, it is only necessary to

Figure 8.7
The Geodimeter robotic total station. (Courtesy of Trimble.)

identify the point's number with a keyboard entry. The computer retrieves the direction to the point from storage or computes it and activates a servomotor to turn the telescope to that direction within a few seconds. This feature is particularly useful for construction stakeout, but it is also convenient in control surveying when multiple observations are made in observing angles. In this instance, final precise pointing is done manually.

The *remote positioning unit* (RPU) shown on the right in Figure 8.7, which is attached to a prism pole, has a built-in telemetry link for communication with the total station. The robotic instrument is equipped with an automatic search and aim function, as well as a link for communication with the RPU. It has servomotors for automatic aiming at the prism both horizontally and vertically. With the RPU, the total station instrument can be controlled from a distance.

To operate the system, the robotic instrument must first be set up and oriented. This consists in entering the coordinates of the point where the total station is located, and taking a backsight along a line of known azimuth. Once oriented, an operator carries the RPU to any convenient location and sights the robotic instrument using the telescope of the RPU. The vertical angle of sight is transmitted to the robotic instrument, whereupon the instrument's vertical servomotor automatically sets its telescope at the required vertical angle. Its horizontal servomotor then activates and swings around until it finds the prism. Once the total station has found the RPU, which only takes a few seconds, and locks onto it, it will automatically follow its further movements. If lock is lost, the search routine is simply repeated. The RPU not only serves as the control unit for the system, but it also operates as a data collector.

With this and similar systems, the total station instrument is completely controlled through the keyboard of the remote unit. These systems enable one person to conduct a complete survey. They are exceptionally well suited for construction surveys and topographic surveys, but can be used advantageously in other types as well. The system not only eliminates one person and speeds the work, but more importantly, it eliminates mistakes in identifying points that can occur when the prism is far from the total station and cannot be seen clearly.

PART II • ANGLE OBSERVATIONS

8.7 RELATIONSHIP OF ANGLES AND DISTANCES

Determining the relative positions of points often involves observing of both angles and distances. The best-quality surveys result when there is compatibility between the accuracies of these two different kinds of measurements. The formula for relating distances to angles is given by the geometric relationship

$$S = R\theta \tag{8.1}$$

In Equation (8.1), S is the arc length subtended at a distance R by an arc of θ in radians. To select instruments and survey procedures necessary for achieving consistency, and to evaluate the effects of errors due to various sources, it is helpful to consider the relationships between angles and distances given here and illustrated in Figure 8.8.

1′ of arc = 0.03 ft at 100 ft, or 3 cm at 100 m (approx.)
1′ of arc = 1 in. at 300 ft (approx.; actually 340 ft)
1″ of arc = 1 ft at 40 mi, or 0.5 m at 100 km, or 1 mm at 200 m (approx.)
1″ of arc = 0.000004848 radians (approx.)
1 radian = 206,264.8″ of arc (approx.)

In accordance with the relationships listed, an error of approximately 1 min results in an observed angle if the line of sight is misdirected by 1 in. over a distance

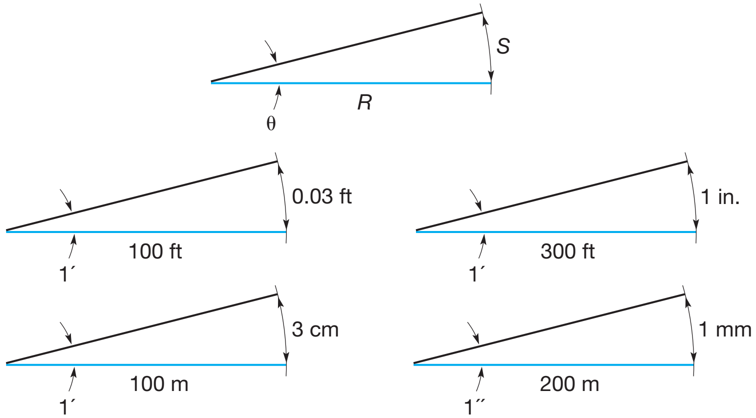


Figure 8.8 Angle and distance relationships.

of 300 ft. This illustrates the importance of setting the instrument and targets over their respective points precisely, especially where short sights are involved. If an angle is expected to be accurate to within $\pm 5''$ for sights of 500 ft, then the distance must be correct to within $500(5'')0.000004848 = \pm 0.01$ ft for compatibility.

To appreciate the precision capabilities of a high-quality total station, an instrument reading to the nearest $0.5''$ is capable of measuring the angle between two points approximately 1 cm apart and 4 km away theoretically! However, as discussed in Sections 8.19 through 8.21, errors from centering the instrument, sighting the point, reading the circle, and other sources, make it difficult, if not impossible, to actually accomplish this accuracy.

8.8 OBSERVING HORIZONTAL ANGLES WITH TOTAL STATION INSTRUMENTS

As stated in Section 2.1, horizontal angles are observed in horizontal planes. After a total station instrument is set up and leveled, its horizontal circle is in a horizontal plane and thus in proper orientation for observing horizontal angles. To observe a horizontal angle, for example, angle *JIK* of Figure 8.9(a), the instrument is first set up and centered over station *I*, and leveled. Then a *backsight* is taken on station *J*. This is accomplished by releasing the horizontal and vertical locks, turning the telescope in the approximate direction of *J*, and clamping both locks. A precise pointing is then made to place the vertical cross hair on the target using the horizontal and vertical tangent screws, and an initial value of $0°00'00''$ is entered in the display. The horizontal motion is then unlocked, and the telescope turned clockwise toward point *K* to make the *foresight*. The vertical circle lock is also usually released to tilt the telescope for sighting point *K*. Again the motions are clamped with the line of sight approximately on station *K*, and precise pointing is made as before using the horizontal tangent screw. When the foresight is completed, the value of the horizontal angle will automatically appear in the display.

To eliminate instrumental errors and increase precision, angle observations should be repeated an equal number of times in each of the *direct* and *reverse*

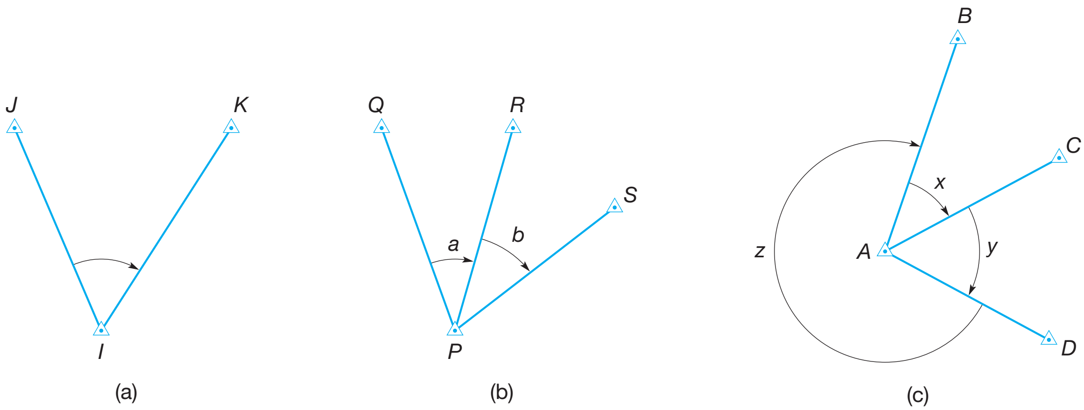


Figure 8.9 Measurement of horizontal angles.

modes, and the average taken. Built-in computers of total station instruments will perform the averaging automatically and display the final results. For instruments that have only a single keyboard and display, the instrument is in its direct mode when the eyepiece and keyboard are on the same side of the instrument. However, instruments do vary by manufacturer, and the operator should refer to the instrument's manual to determine the proper orientation of their instrument when in the direct mode. To get from the direct mode into the reverse mode, the telescope is "plunged" (rotated 180° about the horizontal axis).

Procedures for repeating horizontal angle observations can differ with instruments of different manufacture, and operators must therefore become familiar with the features of their specific instrument by referring to its manual. The following is an example procedure that applies to some instruments. After making the first observation of angle *JIK*, as described above, the angular value in the display is held by pressing a button on the keyboard of the instrument. (Assume the first observation was in the direct mode.) To repeat the angle with the instrument in the same mode, a backsight is again taken on station *J* using the horizontal lock and tangent screw. After the backsight is completed, with the first observed angular value still on the display, the display is released for the next angle observation by again pressing the appropriate button on the keyboard. Using the same procedures described earlier, a foresight is again taken on station *K*, after which the display will read the sum of the two repeated angles. This procedure is repeated until the desired number of angles is observed in the direct mode, whereupon the display will show the sum of these repetitions. Then the telescope is plunged to place it in the reverse mode, and the angle repeated an equal number of times using the same procedure. In the end, the sum of all angles turned, direct and reverse, will be displayed. The final angle is the mean.

The procedure just described for observing horizontal angles is called the *repetition method*. As noted earlier, obtaining an average value from repeated observations increases precision, and by incorporating equal numbers of direct and reverse measurements, certain instrumental errors are eliminated (see Section 8.20).

An example set of field notes for observing the angle of Figure 8.9(a) by the repetition method is shown in Figure 8.10. In the example, four repetitions, two in each of the direct and reverse modes, were taken. In the notes, the identification of the angle being observed is recorded in column (1), the value of the first reading of the angle is placed in column (2) and it is recorded for checking purposes only, the fourth (final) reading is tabulated in column (3), and the mean of the four readings, which produces the final angle, is given in column (4). Note that the

HORIZONTAL ANGLE MEASUREMENT

Angle	First Reading	Fourth Reading	Mean Angle		
(1)	(2)	(3)	(4)		
	° ′ ″	° ′ ″	° ′ ″		
JIK	66 37 40	266 30 48	66 37 42		

Figure 8.10 Field notes for measuring the horizontal angle of Figure 8.9(a) by repetition.

first reading would not have to be recorded, except that it is used as a check against which the mean angle is compared. If these two values agree within tolerable limits, the mean angle is accepted, if not the work is repeated.

Special capabilities are available with many total station instruments to enhance their accuracy and expedite operation. For example, most instruments have a dual-axis automatic compensator that senses any misorientation of the circles. This information is relayed to the built-in computer that corrects for any *indexing error* in the vertical circle (see Section 8.13), and any dislevelment of the horizontal circle, before displaying angular values. This real-time tilt sensing and correction feature makes it necessary to perform rough leveling of the instrument only, thus reducing setup time. In addition, some instruments observe angles by integration of electronic signals over the entire circle simultaneously; thus, errors due to graduations and eccentricities (see Section 8.20.1) are eliminated. Furthermore, the computer also corrects horizontal angles for instrumental errors if the axis of sight is not perpendicular to the horizontal axis, or if the horizontal axis is not perpendicular to the vertical axis. (These conditions are discussed in Sections 8.15 and 8.20.1, respectively.). This feature makes it possible to obtain angle observations free from instrumental errors without averaging equal numbers of direct and reverse readings. With these advantages, and more, it is obvious why these instruments have replaced the older instruments discussed in Appendix A.

■ 8.9 OBSERVING HORIZONTAL ANGLES BY THE DIRECTION METHOD

As an alternative to observing horizontal angles by the repetition method described in the preceding section, total station instruments can be used to determine horizontal angles by the *direction method*. This procedure consists in observing *directions*, which are simply horizontal circle readings taken to successive stations sighted around the horizon. Then by taking the difference in directions between any two stations, the angle between them is determined. The procedure is particularly efficient when multiple angles are being measured at a station. An example of this type of situation is illustrated in Figure 8.9(b), where angles *a* and *b* must both be observed at station *P*. Figure 8.11 shows a set of field notes for observing these angles by direction method. The notes are the results of four repetitions of direction measurements in each of the direct and reverse mode. In these notes the repetition number is listed in column (1), the station sighted in column (2), direction readings taken in the direct and reverse modes in columns (3) and (4), respectively, the mean of direct and reverse readings in column (5), and the computed angles (obtained by subtracting the mean direction for station *Q* from that of station *R*, and subtracting *R* from *S*) in column (6). As a check, the four values for each angle in column (6) should be compared for agreement, and a determination made as to whether they meet acceptance criteria before leaving the occupied station, so that additional readings can be made if necessary. Final values for the two angles are taken as the averages of the four angles in column (6). These are 37°30′28″ and 36°43′14″ for angles *a* and *b*, respectively. Note that in this procedure, as was the case with the repetition method, the multiple readings increase the precisions of the angles, and by taking equal numbers of direct

DIRECTIONS OBSERVED FROM STATION P

Repetition No.	Station Sighted	Reading Direct	Reading Reverse	Mean	Angle
(1)	(2)	(3)	(4)	(5)	(6)
		° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	Q	0 00 00	0 00 00	0 00 00	
	R	37 30 27	37 30 21	37 30 24	37 30 24
	S	74 13 42	74 13 34	74 13 38	36 43 14
2	Q	0 00 00	0 00 00	0 00 00	
	R	37 30 32	37 30 28	37 30 30	37 30 30
	S	74 13 48	74 13 42	74 13 46	36 43 16
3	Q	0 00 00	0 00 00	0 00 00	
	R	37 30 26	37 30 26	37 30 26	37 30 26
	S	74 13 36	74 13 40	74 13 38	36 43 12
4	Q	0 00 00	0 00 00	0 00 00	
	R	37 30 34	37 30 30	37 30 32	37 30 32
	S	74 13 48	74 13 44	74 13 46	36 43 14

Figure 8.11 Field notes for measuring directions for Figure 8.9(b).

and reverse readings, instrumental errors are eliminated. As previously noted, this method of observing directions can significantly reduce the time at a station, especially when several angles with multiple repetitions are needed, for example in triangulation.

The procedures for observing multiple angles with data collectors can vary by manufacturer. The reader should refer to their data collector manual to determine the proper procedures for their situation. One of the advantages of using a data collector to observe multiple angles is that they provide immediate post-observation statistics. The residuals of each observation can be displayed after the observation process before accepting the average observations. The operator can view each residual and decide if any are too large to meet the job specifications, instrument specifications, and field conditions. If a single residual is deemed excessive, that observation can be removed and the observation repeated. If all the residuals are too large, the entire set of observations can be removed and the entire angle observation process repeated.

■ 8.10 CLOSING THE HORIZON

Closing the horizon consists in using the direction method as described in the preceding section, but including *all* angles around a point. Suppose that in Figure 8.9(c) only angles x and y are needed. However, in closing the horizon angle z is also observed thereby providing for additional checks. An example set

CLOSING THE HORIZON
AT STATION A

Position No.	Station Sighted	Reading Direct	Reading Reverse	Mean	Angle
(1)	(2)	(3)	(4)	(5)	(6)
		° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	B	0 00 00	0 00 00	0 00 00	
	C	42 12 12	42 12 14	42 12 13	42 12 13
	D	102 08 26	102 08 28	102 08 27	59 56 14
	B	0 00 02	0 00 02	0 00 02	257 51 35
				Sum	360 00 02
2	B	0 00 00	0 00 00	0 00 00	
	C	42 12 12	42 12 14	42 12 13	42 12 13
	D	102 08 28	102 08 28	102 08 28	59 56 15
	B	0 00 04	0 00 04	0 00 04	257 51 36
				Sum	360 00 04
3	B	0 00 00	0 00 00	0 00 00	
	C	42 12 14	42 12 12	42 12 13	42 12 13
	D	102 08 28	102 08 26	102 08 27	59 56 14
	B	0 00 04	0 00 00	0 00 02	257 51 35
				Sum	360 00 02
4	B	0 00 00	0 00 00	0 00 00	
	C	42 12 14	42 12 12	42 12 13	42 12 13
	D	102 08 32	102 08 28	102 08 30	59 56 17
	B	0 00 04	0 00 04	0 00 04	257 51 34
				Sum	360 00 04

Figure 8.12 Field notes for closing the horizon at station A of Figure 8.9(c).

of field notes for this operation is shown in Figure 8.12. The angles are first turned around the horizon by making a pointing and direction reading at each station with the instrument in the direct mode [see the data entries in column (3) of Figure 8.12]. A final foresight pointing is made on the initial backsight station, and this provides a check because it should give the initial backsight reading (allowing for reasonable random errors). Any difference is the *horizon misclosure*, and if its value exceeds an allowable tolerance, that round of readings should be discarded and the observations repeated. (Note that in the field notes of Figure 8.12, the maximum horizon misclosure was 4″.)

After completing the readings in the direct mode, the telescope is plunged to its reverse position and all directions around the horizon observed again [see the data entries in column (4)]. A set of readings around the horizon in both the direct and reverse modes constitutes a so-called *position*. The notes of Figure 8.12 contain the results of four positions.

The note-reduction process consists of calculating mean values of the direct and reverse directions to each station, [see column (5)], and from them, the individual angles around the horizon are computed as discussed in Section 8.9 [see column (6)]. Finally their sum is calculated, and checked against (360°). Any difference reveals a mistake or mistakes in computing the individual angles. Again, repeat values for each individual angle are obtained, and as another check on the work, these should be compared for their agreement.

As an alternative to closing the horizon by observing directions, each individual angle could be measured independently using the procedures outlined in Section 8.8. After observing all angles around the horizon, their sum could also be computed and compared against 360°. However, this procedure is not as efficient as closing the horizon using directions.

■ 8.11 OBSERVING DEFLECTION ANGLES

A deflection angle is a horizontal angle observed from the prolongation of the preceding line, right or left, to the following line. In Figure 8.13(a) the deflection angle at F is 12°15′10″ to the right (12°15′10″R), and the deflection angle at G is 16°20′27″L.

A straight line between terminal points is theoretically the most economical route to build and maintain for highways, railroads, pipelines, canals, and transmission lines. Practically, obstacles and conditions of terrain and land-use require bends in the route, but deviations from a straight line are kept as small as possible. If an instrument is in perfect adjustment (which is unlikely), the deflection angle at F [see Figure 8.13(a)] is observed by setting the circle to zero and backsighting on point E with the telescope in the direct position. The telescope is then plunged to its reversed position, which places the line of sight on EF extended, as shown dashed in the figure. The horizontal lock is released for the foresight, point G sighted, the horizontal lock clamped, the vertical cross hair set on the mark carefully by means of the horizontal tangent screw, and the angle read.

Deflection angles are subject to serious errors if the instrument is not in adjustment, particularly if the line of sight is not perpendicular to the horizontal axis (see Section 8.15). If this condition exists, deflection angles may be read as larger or smaller than their correct values, depending on whether the line of sight after plunging is to the right or left of the true prolongation [see Figure 8.13(b)]. To eliminate errors from this cause, angles are usually doubled or quadrupled by the following procedure: the first backsight is taken with the circle set at zero and the telescope in the direct position. After plunging the telescope, the angle is observed and kept in the display. Using the procedures specified by the manufacturer for holding an angle in the display, a second backsight is taken, retaining the first angle, and keeping the telescope reverse. The telescope is plunged back to the direct position for the foresight, the display released, and the angle

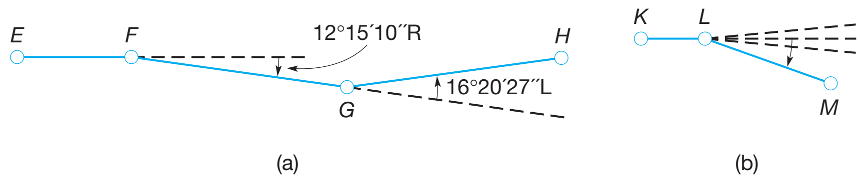


Figure 8.13
Deflection angles.

reobserved. Dividing the total angle by 2 gives an average angle from which instrumental errors have been eliminated by cancellation. In outline fashion, the method is as follows:

1. Backsight with the telescope direct. Plunge to reverse mode and observe the angle. Hold the displayed angle.
2. Backsight with the telescope still reversed. Plunge again to direct mode, release the angle display and observe the angle.
3. Read the total angle and divide by 2 for an average.

Of course, making four, six, or eight repetitions and averaging can increase the precision in direction angle observation.

Figure 8.14 shows the left-hand page of field notes for observing the deflection angles at stations *F* and *G* of Figure 8.13(a). The procedure just outlined was followed. Four repetitions of each angle were taken with the instrument alternated from direct to reverse with each repetition. Readings were recorded only after the first, second, and fourth repetitions. *The final angle is the mean obtained by dividing the last recorded value by the total number of repetitions*—four in this case. The purpose of the first two values is only to provide checks: that is, the second reading should be twice the first, and the final mean angle should be equal to the first, allowing of course for random errors.

If a mistake should occur, as in the first set of angles observed at station *G* of Figure 8.14, lines should be drawn through the incorrect data, the word "VOID" written beside them, and the observations repeated. In this voided data set, half the second recorded value (16°20′28″) agrees reasonably well with the first value (16°20′30″), but the final mean (16°20′00″) does not agree with the first recorded value. Thus, that data set is discarded.

DEFLECTION ANGLES

Sta	BS/FS Sta	No. Reps.	Circle Rdg ° ′ ″	Mean Angle ° ′ ″	Right/ Left
	E	1	12 15 12		
F		2	24 30 20		
	G	4	49 00 40	12 15 10	R
	F	1	16 20 30		
G		2	32 40 56	VOID	
	H	4	65 20 00	16 20 00	L
	F	1	16 20 24		
G		2	32 40 52		
	H	4	65 21 48	16 20 27	L

Figure 8.14
Field notes for measuring deflection angles.

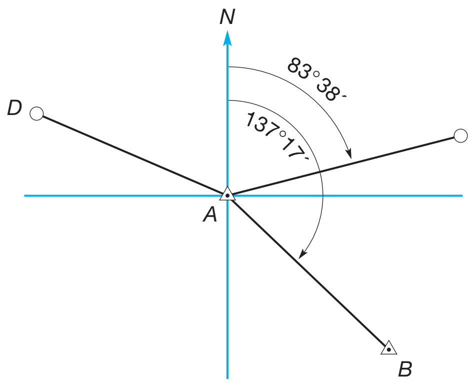


Figure 8.15 Orientation by azimuths.

■ 8.12 OBSERVING AZIMUTHS

Azimuths are observed from a reference direction which itself must be determined from (a) a previous survey, (b) the magnetic needle, (c) a solar or star observation, (d) GPS observations, (e) a north-seeking gyro, or (f) assumption. Suppose that in Figure 8.15 the azimuth of line *AB* is known to be 137°17′00″ from north. The azimuth of any other line that starts at *A*, such as *AC* in the figure, can be found directly using a total station instrument. In this process, with the instrument set up and centered over station *A*, and leveled, a backsight is first taken on point *B*. The azimuth of line *AB* (137°17′00″) is then set on the horizontal circle using the keyboard. The instrument is now "oriented," since the line of sight is in a known direction with the corresponding azimuth on the horizontal circle. If the circle were turned until it read 0°, the telescope would be pointing toward north (along the meridian). The next steps are to loosen the horizontal lock, turn the telescope clockwise to *C* and read the resultant direction, which is the azimuth of *AC*, and in this case is 83°38′00″.

In Figure 8.15, if the instrument is set up at point *B* instead of *A*, the azimuth of *BA* (317°17′00″) or the back azimuth of *AB* is put on the circle and point *A* sighted. The horizontal lock is released, and sights taken on points whose azimuths from *B* are desired. Again, if the instrument is turned until the circle reads zero, the telescope points north (or along the reference meridian). By following this procedure at each successive station of a traverse, for example at *A*, *B*, *C*, *D*, *E*, and *F* of the traverse of Figure 7.2(a), the azimuths of all traverse lines can be determined. With a closed polygon traverse like that of Figure 7.2(a), station *A* should be occupied a second time and the azimuth of *AB* determined again to serve as a check on the work.

■ 8.13 OBSERVING VERTICAL ANGLES

A vertical angle is the difference in direction between two intersecting lines measured in a vertical plane. Vertical angles can be observed as either altitude or zenith angles. An altitude angle is the angle above or below a horizontal plane through the point of observation. Angles above the horizontal plane are called *plus angles*, or *angles of elevation*. Those below it are *minus angles*, or *angles of depression*. Zenith angles are measured with zero on the vertical circle oriented toward the zenith of the instrument and thus go from 0° to 360° in a clockwise circle about the horizontal axis of the instrument.

Most total station instruments are designed so that zenith angles are displayed rather than altitude angles. In equation form, the relationship between altitude angles and zenith angles is

$$\text{Direct mode} \qquad \alpha = 90° - z \tag{8.2a}$$

$$\text{Reverse mode} \qquad \alpha = z - 270° \tag{8.2b}$$

where z and α are the zenith and altitude angles, respectively. With a total station, therefore, a reading of 0° corresponds to the telescope pointing vertically upward. In the direct mode, with the telescope horizontal, the zenith reading is 90°, and if the telescope is elevated 30° above horizontal, the reading is 60°. In the reverse mode, the horizontal reading is 270°, and with the telescope raised 30° above the horizon it is 300°. Altitude angles and zenith angles are observed in trigonometric leveling and in EDM work for reduction of observed slope distances to horizontal.

Observation of zenith angles with a total station instrument follows the same general procedures as those just described for horizontal angles, except that an automatic compensator orients the vertical circle. As with horizontal angles, instrumental errors in vertical angle observations are compensated for by computing the mean from an equal number of direct and reverse measurements. With zenith angles, the mean is computed from

$$\bar{z}_D = \frac{\Sigma z_D}{n} + \frac{n(360°) - (\Sigma z_D + \Sigma z_R)}{2n} \tag{8.3}$$

where $\bar{z}_D$ is the mean value of the zenith angle (expressed according to its direct mode value), Σz_D the sum of direct zenith angles, Σz_R the sum of reverse angles, and n the number of z_D and z_R pairs of zenith angles read. The latter part of Equation (8.3) accounts for the *indexing error* present in the instrument.

An indexing error exists if 0° on the vertical circle is not truly at the zenith with the instrument in the direct mode. This will cause all vertical angles read in this mode to be in error by a constant amount. For any instrument, an error of the same magnitude will also exist in the reverse mode, but it will be of opposite algebraic sign. The presence of an indexing error in an instrument can be detected by observing zenith angles to a well-defined point in both modes of the instrument. If the sum of the two values does not equal 360°, an indexing error exists. To eliminate the effect of the indexing error, equal numbers of direct and reverse angle observations should be made, and averaged. The averaging is normally done by the microprocessor of the total station instrument. Even though an indexing error may not exist, to be safe, *experienced surveyors always adopt field procedures that eliminate errors just in case the instrument is out of adjustment.*

With some total station instruments, indexing errors can be eliminated from zenith angles by computation, after going through a calibration procedure with the instrument. The computations are done by the microprocessor and applied to the angles before they are displayed. Procedures for performing this calibration vary with different manufacturers and are given in the manuals that accompany the equipment.

Example 8.1

A zenith angle was read twice direct giving values of 70°00′10″ and 70°00′12″, and twice reverse yielding readings of 289°59′44″ and 289°59′42″. What is the mean zenith angle?

Solution

Two pairs of zenith angles were read, thus $n = 2$. The sum of direct angles is 140°00′22″ and that of reverse values is 579°59′26″. Then by Equation (8.3)

$$\bar{z}_D = \frac{140°00'22''}{2} + \frac{2(360°) - (140°00'22'' + 579°59'26'')}{2 \times 2}$$
$$= 70°00'11'' + 0°00'03'' = 70°00'14''$$

Note that the value of 03″from the latter part of Equation (8.3) is the index error.

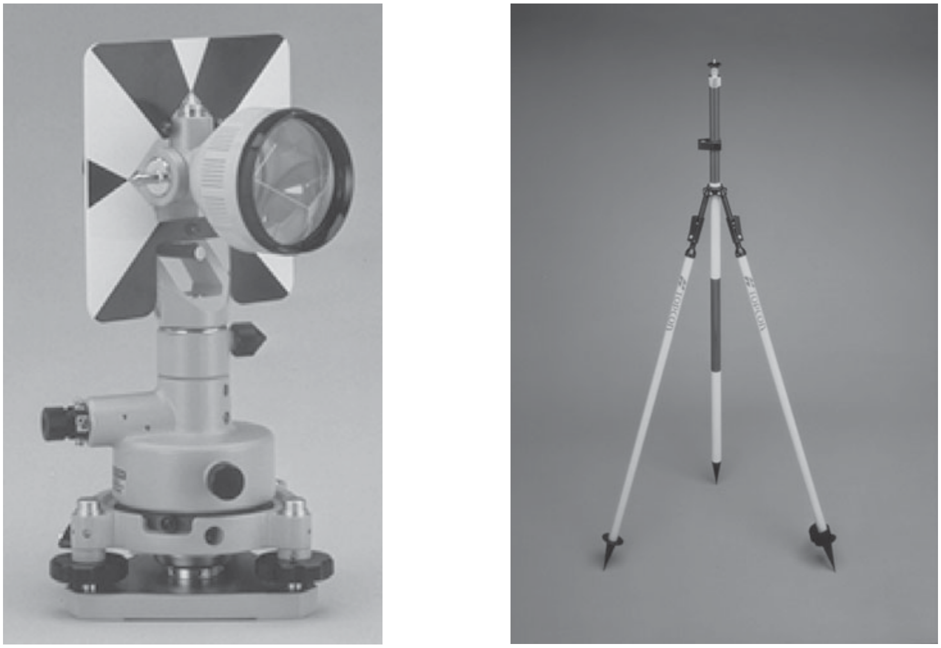
8.14 SIGHTS AND MARKS

Objects commonly used for sights when total station instruments are being used only for angle observations include prism poles, chaining pins, pencils, plumb-bob strings, reflectors, and tripod-mounted targets. For short sights, a string is preferred to a prism pole because the small diameter permits more accurate sighting. Small red and white targets of thin plastic or cardboard placed on the string extend the length of observation possible. Triangular marks placed on prisms as shown in Figure 8.16(a) provide excellent targets at both close and longer sight distances.

An error is introduced if the prism pole sighted is not plumb. The pole is kept vertical by means of a circular bubble. [The bubble should be regularly checked for adjustment, and adjusted if necessary (see Section 8.19.5)]. The person holding the prism has to take special precautions in plumbing the pole, carefully watching the circular bubble on the pole. Bipods like the one shown in Figure 8.16(b) and tripods have been developed to hold the pole during multiple angle observation sessions.

The prism pole shown in Figure 8.16(b) has graduations for easy determination of the prism's height. The tripod mount shown in Figure 8.16(a) is centered over the point using the optical plummet of the tribrach. When sighting a prism pole, the vertical cross hair should bisect the pole just below the prism. Errors can result if the prism itself is sighted, especially on short lines since any misalignment of the face of the prism with the line of sight will cause and offset pointing on the prism.

In construction layout work and in topographic mapping, permanent backsights and foresights may be established. These can be marks on structures such



(a) (b)

Figure 8.16 (a) Prism and sighting target with tribrach and tribrach adapter, and (b) pole and bipod, used when measuring distances and horizontal angles with total station instruments. (Courtesy Topcon Positioning Systems.)

as walls, steeples, water tanks, and bridges, or they can be fixed artificial targets. They provide definite points on which the instrument operator can check orientation without the help of a rodperson.

The error in a horizontal angle due to miscentering of the line of sight on a target, or too large a target, can be determined with Equation (8.1). For instance, assume a prism pole that is 20 mm wide is used as a target on a direction of only 100 m. Assuming that the pointing will be within 1/2 of the width of the pole (10 mm), then according to Equation (8.1) the error in the direction would be (0.01/100) 206,264.8 = 21″! For an angle where both sight distances are 100 m and assuming that the pointings are truly random, the error would propagate according to Equation (3.12), and would result in an estimated error in the angle of $21''\sqrt{2}$, or approximately 30″. From the angle-distance relationships of Section 8.7, it is easy to see why the selection of good targets that are appropriate for the sight distances in angle observations is so important.

■ 8.15 PROLONGING A STRAIGHT LINE

On route surveys, straight lines may be continued from one point through several others. To prolong a straight line from a backsight, the vertical cross wire is aligned on the back point by means of the lower motion, the telescope plunged, and a point, or points, set ahead on line. In plunging the telescope, a serious error can occur if the line of sight is not perpendicular to the horizontal axis. The effects of this error can be eliminated, however, by following proper field procedures. The procedure used is known as the *principle of reversion*. The method applied, actually double reversion, is termed *double centering*. Figure 8.17 shows

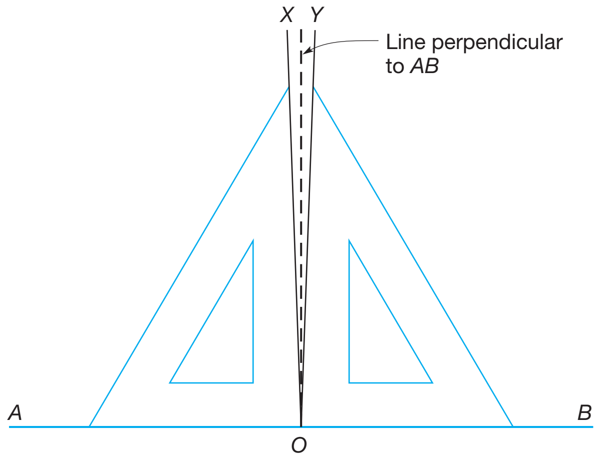


Figure 8.17 Principle of reversion.

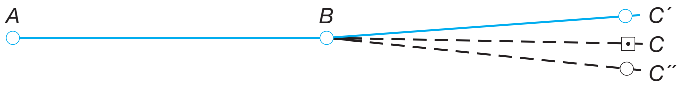


Figure 8.18 Double centering.

a simple use of the principle in drawing a right angle with a defective triangle. Lines *OX* and *OY* are drawn with the triangle in "normal" and "reverse" positions. Angle *XOY* represents twice the error in the triangle at the 90° corner, and its bisector (shown dashed in the figure) establishes a line perpendicular to *AB*.

To prolong line *AB* of Figure 8.18 by double centering with a total station whose line of sight is not perpendicular to its horizontal axis, the instrument is set up at *B*. A backsight is taken on *A* with the telescope in the direct mode, and by plunging the telescope into the reverse position the first point C' is set. The horizontal circle lock is released, and the telescope turned in azimuth to take a second backsight on point *A*, this time with the telescope still plunged. The telescope is plunged again to its direct position and point C'' placed. Distance $C'C''$ is bisected to get point *C*, on line *AB* prolonged. In outline form, the procedure is as follows:

1. Backsight on point *A* with the telescope direct. Plunge to the reverse position and set point C'.
2. Backsight on point *A* with the telescope still reverse. Plunge to a direct position and set point C''.
3. Split the distance $C'C''$ to locate point *C*.

In the above procedure, each time the telescope is plunged, the instrument creates twice the total error in the instrument. Thus, at the end of the procedure, four times the error that exists in the instrument lies between points C' and C''. To adjust the instrument, the reticle must be shifted to bring the vertical cross wire one fourth of the distance back from C'' toward C'. For total station instruments that have exposed capstan screws for adjusting their reticles, an adjustment can be made in the field. Generally, however, it is best to leave this adjustment to qualified experts. If the adjustment is made in the field, it must be

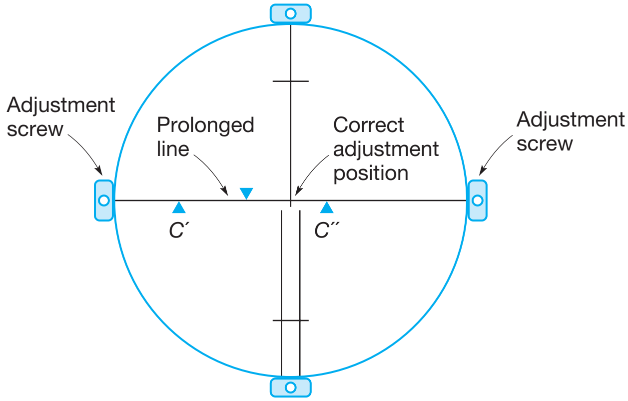


Figure 8.19 The crosshair adjustment procedure.

done very carefully! Figure 8.19 depicts the condition after the adjustment is completed. Since each crosshair has two sets of opposing capstan screws, *it is important to loosen one screw before tightening the opposing one by an equal amount*. After the adjustment is completed, the procedure should be repeated to check the adjustment.

■ 8.16 BALANCING-IN

Occasionally it is necessary to set up an instrument on a line between two points already established but not intervisible—for example, A and B in Figure 8.20. This can be accomplished in a process called *balancing-in* or *wiggling-in*.

Location of a trial point C' on line is estimated and the instrument set over it. A sight is taken on point A from point C' and the telescope plunged. If the line of sight does not pass through B, the instrument is moved laterally a distance CC' estimated from the proportion $CC' = BB' \times AC/AB$, and the process repeated. Several trials may be required to locate point C exactly or close enough for the purpose at hand. The shifting head of the instrument is used to make the final small adjustment. A method for getting a close first approximation of required point C takes two persons, X able to see point A and Y having point B visible, as

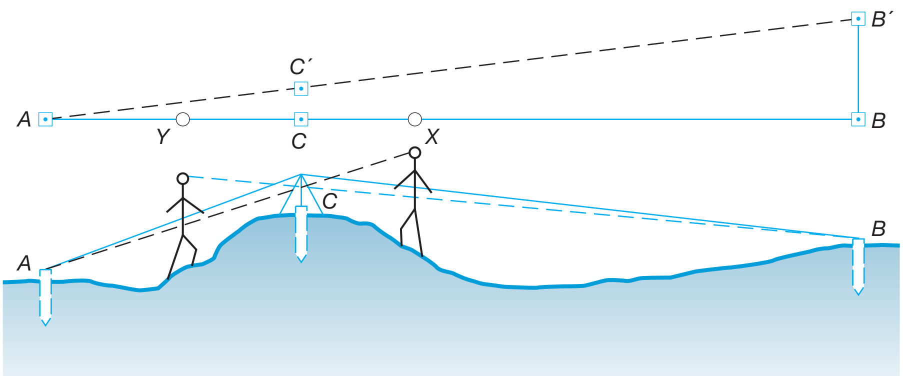


Figure 8.20 Balancing-in.

shown in Figure 8.20. Each aligns the other in with the visible point in a series of adjustments, and two range poles are placed at least 20 ft apart on the course established. An instrument set at point C in line with the poles should be within a few tenths of a foot of the required location. From there the wiggling-in process can proceed more quickly.

■ 8.17 RANDOM TRAVERSE

On many surveys it is necessary to run a line between two established points that are not intervisible because of obstructions. This situation arises repeatedly in property surveys. To solve the problem, a *random traverse* is run from one point in the approximate direction of the other. Using coordinate computation procedures presented in Chapter 10, the coordinates of the stations along the random traverse are computed. Using these same computation procedures, coordinates of the points along the "true" line are computed, and observations necessary to stake out points on the line computed from the coordinates. With data collectors, the computed coordinates can be automatically determined in the field, and then staked out using the functions of the data collector.

As a specific example of a random traverse, consider the case shown in Figure 8.21 where it is necessary to run line *X-Y*. On the basis of a compass bearing, or information from maps or other sources, the general direction to proceed is estimated, and starting line *X-1* is given an assumed azimuth. Random traverse *X-1-2-3-Y* is then run, and coordinates of all points determined. Based upon these computations, coordinates are also computed for points *A* and *B*, which are on line *X-Y*. The distance and direction necessary for setting *A* with an instrument set up at point 1 are then computed using procedures discussed in Chapter 10. Similarly the coordinates of *B* are determined and set from station 2. Using a data collector, these computations can be performed automatically. This procedure, known as *stake out*, is discussed in Chapter 23.

Once the angles and distances have been computed for staking points A and B, the actual stake out procedure is aided by operating the total station instrument in its *tracking mode* (see Section 6.21 and Chapter 23). If a robotic total station instrument is available, one person can perform the layout procedure. *This method of establishing points on a line is only practical when direct sighting along the line is not physically possible*.

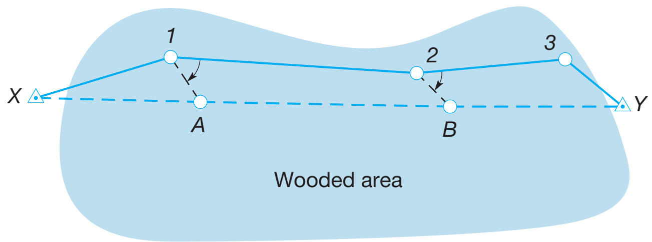


Figure 8.21 Random traverse *X-1-2-3-Y*.

8.18 TOTAL STATIONS FOR DETERMINING ELEVATION DIFFERENCES

With a total station instrument, computed vertical distances between points can be obtained in real time from observed slope distances and zenith angles. In fact, this is the basis for *trigonometric leveling* (see Section 4.5.4). Several studies have compared the accuracies of elevation differences obtained by trigonometric leveling using modern total station instruments to those achieved by differential leveling as discussed in Chapters 4 and 5. Trigonometric leveling accuracies have always been limited by instrumental errors (discussed in Section 8.20) and the effects of refraction (see Section 4.4). Even with these problems, elevations derived from a total station survey are of sufficient accuracy for many applications such as for topographic mapping and other lower-order work.

However, studies have suggested that high-order results can be obtained in trigonometric leveling by following specific procedures. The suggested guidelines are: (1) place the instrument between two prisms so that sight distances are appropriate for the angular accuracy of the instrument, using Figure 8.22 as a guide;[3] (2) use target panels with the prisms; (3) keep rod heights equal so that their measurement is unnecessary; (4) observe the vertical distances between the prisms using two complete sets[4] of observations at a minimum; (5) keep sight distances approximately equal; and (6) apply all necessary atmospheric corrections and reflector constants as discussed in Chapter 6. This type of trigonometric leveling can be done faster than differential leveling, especially in rugged terrain where sight distances are limited due to rapid changes in elevation.

A set of notes from trigonometric leveling is shown in Figure 8.23. Column (a) lists the backsight and foresight station identifiers and the positions of the telescope [direct (D) and reverse (R)] for each observation; (b) tabulates the backsight vertical distances, (BS+); (c) lists the backsight horizontal distances to the nearest decimeter; (d) gives the foresight vertical distances, (FS−); (e) lists the

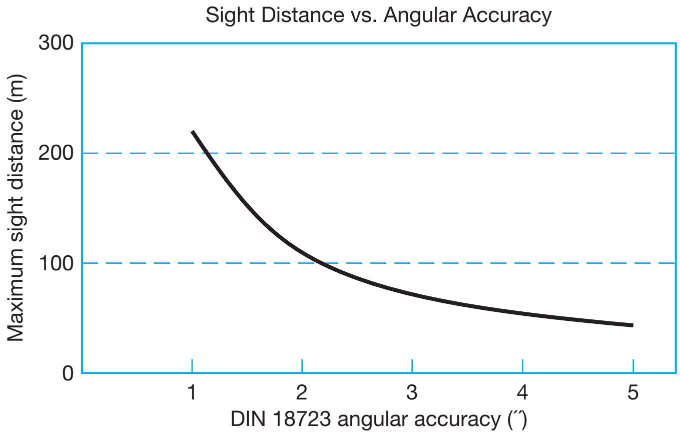


Figure 8.22 Graph of appropriate sight distance versus angular accuracy.

[3]A description of DIN18723 noted in Figure 8.22 is given in Section 8.21.

[4]One set of observations includes an elevation determination in both the direct and reverse positions.

TRIGONOMETRIC
LEVELING NOTES

(a)	(b)	(c)	(d)	(e)	(f)
Sta/Pos	BS(+)	BD	FS(−)	FD	ΔElev
A					
D	1.211	98.12	1.403	86.34	
D	1.210		1.403		
R	1.211		1.404		
R	1.211		1.403		
Mean	1.2108		1.4033		−0.192
B					
D	−5.238	101.543	−9.191	93.171	
D	−5.236		−9.191		
R	−5.238		−9.193		
R	−5.237		−9.192		
Mean	−5.2373		−9.1918		3.954
C					
D	4.087	73.245	−3.849	97.392	
D	4.088		−3.851		
R	4.086		−3.849		
R	4.087		−3.849		
Mean	4.0870		−3.8495		7.936
D					
D	3.214	89.87	6.507	97.392	
D	3.214		6.507		
R	3.214		6.508		
R	3.215		6.507		
Mean	3.2143		6.5072		−3.293
E					
				Sum	8.405

Figure 8.23
Trigonometric leveling field notes.

foresight horizontal distances to the nearest decimeter; and (f) tallies the elevation differences between the stations, computed as the difference of the BS vertical distances, minus the FS vertical distances. The observed elevation difference between stations A and E is 8.405 m.

■ 8.19 ADJUSTMENT OF TOTAL STATION INSTRUMENTS AND THEIR ACCESSORIES

The accuracy achieved with total station instruments is not merely a function of their ability to resolve angles and distances. It is also related to operator procedures and the condition of the total station instrument and other peripheral equipment being used with it. Operator procedure pertains to matters such as careful centering and leveling of the instrument, accurate pointing at targets, and observing proper field procedures such as taking averages of multiple angle observations made in both direct and reverse positions.

In Section 8.2, three reference axes of a total station instrument were defined: (a) the line of sight, (b) the horizontal axis, and (c) the vertical axis. These instruments also have a fourth reference axis, (d) the *axis of the plate-level vial* (see Section 4.8). For a properly adjusted instrument, the following relationships should exist between these axes: (1) the axis of the plate-level vial should be perpendicular to the vertical axis, (2) the horizontal axis should be perpendicular to the vertical axis, and (3) the line of sight should be perpendicular to the horizontal axis. If these conditions do not exist, accurate observations can still be made by following proper procedures. However, it is more convenient if the instrument is in adjustment. Today, most total stations have calibration procedures that can electronically compensate for conditions (1) and (2) using sightings to well-defined targets with menu-defined procedures that can be performed in the field. However, if the operator is in doubt about the calibration procedures, a qualified technician should always be consulted.

The adjustment for making the line of sight perpendicular to the horizontal axis was described in Section 8.15, and the procedure for making the axis of the plate bubble perpendicular to the vertical axis is given in Section 8.19.1. The test to determine if a total station's horizontal axis is perpendicular to its vertical axis is a simple one. With the instrument in the direct mode, it is set up a convenient distance away from a high vertical surface, say the wall of a two- or three-story building. After carefully leveling the instrument, sight a well-defined point, say A, high on the wall, at an altitude angle of at least 30°, and clamp the horizontal lock. Revolve (plunge) the telescope about its horizontal axis to set a point, B, on the wall below A just above ground level. Plunge the telescope to put it in reverse mode, turn the telescope 180° in azimuth, sight point A again, and clamp the horizontal lock. Plunge the telescope to set another point, C, at the same level as B. If B and C coincide, no adjustment is necessary. If the two points do not agree, then the horizontal axis is not perpendicular to the vertical axis. If an adjustment for this condition is necessary, the operator should refer to the manual that came with the instrument, or send the instrument to a qualified technician.

Peripheral equipment that can affect accuracy includes tribrachs, plummets, prisms, and prism poles. Tribrachs must provide a snug fit without slippage. Plummets that are out of adjustment cause instruments to be miscentered over the point. Crooked prism poles or poles with circular bubbles that are out of adjustment also cause errors in placement of the prism over the point being observed. Prisms should be checked periodically to determine their constants (see Section 6.24.2), and their values stored for use in correcting distance observations. Surveyors should always heed the following axiom: *In practice, instruments should always be kept in good adjustment, but used as though they might not be*.

In the following subsections, procedures are described for making some relatively simple adjustments to equipment that can make observing more efficient and convenient, and also improve accuracy in the results.

8.19.1 Adjustment of Plate-Level Vials

As stated earlier, two types of leveling systems are used on total station instruments; (a) plate-level vials, and (b) electronic leveling systems. These systems control the fine level of the instrument. If an instrument is equipped with a plate-level vial, it can easily be tested for its state of adjustment. To make the test, the instrument should first be leveled following the procedures outlined in Section 8.5. Then after carefully centering the bubble, the telescope should be rotated 180° from its first position. If the level vial is in adjustment, the bubble will remain centered. If the bubble deviates from center, the axis of the plate-level vial is not perpendicular to the vertical axis. The amount of bubble run indicates twice the error that exists. Level vials usually have a capstan adjusting screw for raising or lowering one end of the tube. If the level vial is out of adjustment, it can be adjusted by bringing the bubble *halfway back* to the centered position by turning the screw. Repeat the test until the bubble remains centered during a complete revolution of the telescope. If the instrument is equipped with an electronic level, follow the procedures outlined in the operator's manual to adjust the leveling mechanism.

If a plate bubble is out of adjustment, the instrument can be used without adjusting it and accurate results can still be obtained, but specific procedures described in Section 8.20.1 must be followed.

8.19.2 Adjustment of Tripods

The nuts on the tripod legs must be tight to prevent slippage and rotation of the head. They are correctly adjusted if each tripod leg falls slowly of its own weight when placed in a horizontal position. If the nuts are overly tight, or if pressure is applied to the legs crosswise (which can break them) instead of lengthwise to fix them in the ground, the tripod is in a strained position. The result may be an unnoticed movement of the instrument head after the observational process has begun.

Tripod legs should be well spread to furnish stability and set so that the telescope is at a convenient height for the observer. Tripod shoes must be tight. Proper field procedures can eliminate most instrument maladjustments, but there is no method that corrects a poor tripod with dried-out wooden legs, except to discard or repair it.

8.19.3 Adjustment of Tribrachs

The tribrach is an essential component of a secure and accurate setup. It consists of a minimum of three components, which are (1) a clamping mechanism, (2) leveling screws, and (3) a circular level bubble. As shown in Figure 8.3, some tribrachs also contain an optical plummet to center the tribrach over a station. The clamping mechanism consists of three slides that secure three posts that protrude from the base of the instrument or tribrach adapter. As the tribrach wears, the clamping mechanism may not sufficiently secure the instrument during observation procedures. When this happens, the instrument will move in the tribrach after it has been clamped, and the tribrach should be repaired or replaced.

8.19.4 Adjustment of Plummets

The line of sight in a plummet should coincide with the vertical axis of the instrument. Two different situations exist: (1) the plummet is enclosed in the alidade of the instrument and rotates with it when turned in azimuth, or (2) the plummet is part of the tribrach that is fastened to the tripod and does not turn in azimuth.

To adjust a plummet contained in the alidade, set the instrument over a fine point and aim the line of sight exactly at it by turning the leveling screws. Carefully adjust for any existing parallax. Rotate the instrument 180° in azimuth. If the plummet reticle moves off the point, bring it *halfway* back by means of the adjusting screws provided. These screws are similar to those shown in Figure 8.19. As with any adjustment, repeat the test to check the adjustment and correct if necessary.

For the second case where the optical plummet is part of the tribrach, carefully lay the instrument, with the tribrach attached, on its side (horizontally) on a stable base such as a bench or desk, and clamp it securely. Fasten a sheet of paper on a vertical wall at least six feet away, such that it is in the field of view of the optical plummet's telescope. With the horizontal lock clamped, mark the position of the optical plummet's line of sight on the paper. Release the horizontal lock and rotate the tribrach 180°. If the reticle of the optical plummet moves off the point, bring it *halfway* back by means of the adjusting screws. Center the reticle on the point again with the leveling screws, and repeat the test.

8.19.5 Adjustment of Circular Level Bubbles

If a circular-level bubble on a total station does not remain centered when the instrument is rotated in azimuth, the bubble is out of adjustment. It should be corrected, although precise adjustment is unnecessary because it does not control fine leveling of the reference axes. To adjust the bubble, carefully level the instrument using the plate bubble and then center the circular bubble using its adjusting screws.

Circular bubbles used on prism poles and level rods must be in good adjustment for accurate work. To adjust them, carefully orient the rod or pole vertically by aligning it parallel to a long plumb line, and fasten it in that position using shims and C-clamps. Then center the bubble in the vial using the adjusting screws. Special adapters have been made to aid in the adjustment of the circular level bubble on rods or poles by some vendors.

For instruments such as automatic levels that do not have plate bubbles, use the following procedure. To adjust the bubble, carefully center it using the leveling screws and turn the instrument 180° in azimuth. *Half* of the bubble run is corrected by manipulating the vial-adjusting screws. Following the adjustment, the bubble should be centered using the leveling screws, and the test repeated.

■ 8.20 SOURCES OF ERROR IN TOTAL STATION WORK

Errors in using total stations result from *instrumental, natural*, and *personal* sources. These are described in the subsections that follow.

8.20.1 Instrumental Errors

Figure 8.24 shows the fundamental reference axes of a total station. As discussed in Section 8.19, for a properly adjusted instrument, the four axes must bear specific relationships to each other. These are: (1) the vertical axis should be perpendicular to the axis of the plate-level vial, (2) the horizontal axis should be perpendicular to

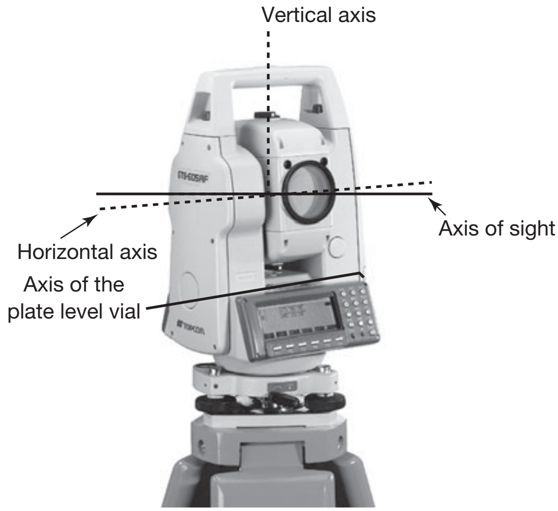


Figure 8.24 Reference axes of a total station instrument. (Courtesy Topcon Positioning Systems.)

the vertical axis, and (3) the axis of sight should be perpendicular to the horizontal axis. If these relationships are not true, errors will result in measured angles unless proper field procedures are observed. A discussion of errors caused by maladjustment of these axes and of other sources of instrumental errors follows.

1. *Plate bubble out of adjustment*. If the axis of the plate bubble is not perpendicular to the vertical axis, the latter will not be truly vertical when the plate bubble is centered. *This condition causes errors in observed horizontal and vertical angles that cannot be eliminated by averaging direct and reverse readings*. The plate bubble is out of adjustment if after centering it runs when the instrument is rotated 180° in azimuth. The situation is illustrated in Figure 8.25. With the telescope

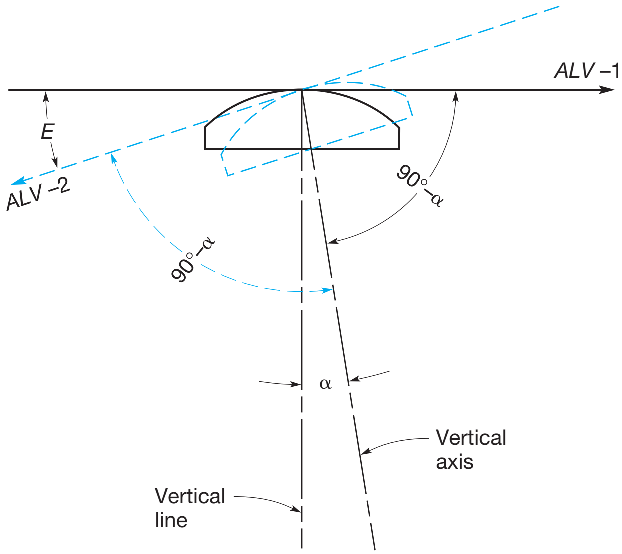


Figure 8.25 Plate bubble out of adjustment.

initially pointing to the right and the bubble centered, the axis of the level vial is horizontal, as indicated by the solid line labeled ALV-1. Because the level vial is out of adjustment, it is not perpendicular to the vertical axis of the instrument, but instead makes an angle of $90° - \alpha$ with it. After turning the telescope 180°, it points left and the axis of the level vial is in the position indicated by the dashed line labeled ALV-2. The angle between the axis of the level vial and vertical axis is still $90° - \alpha$, but as shown in the figure, its indicated dislevelment, or bubble run, is E. From the figure's geometry, $E = 2\alpha$ is double the bubble's maladjustment. The vertical axis can be made truly vertical by bringing the bubble back *half of the bubble run*, using the foot screws. Then, even though it is not centered, the bubble should stay in the same position as the instrument is rotated in azimuth, and accurate angles can be observed.

Although instruments can be used to obtain accurate results with their plate bubbles maladjusted, it is inconvenient and time consuming, so the required adjustment should be made as discussed in Section 8.19.1.

As noted earlier, some total stations are equipped with dual-axis compensators, which are able to sense the amount and direction of vertical axis tilt automatically. They can make corrections computationally in real time to both horizontal and vertical angles for this condition. Instruments equipped with single-axis compensators can only correct vertical angles. Procedures outlined in the manuals that accompany the instruments should be followed to properly remove any error.

As was stated in Section 8.8, total station instruments with dual-axis compensators can apply a mathematical correction to horizontal angles, which accounts for any dislevelment of the horizontal and vertical axes. In Figure 8.26, to sight on point S, the telescope is plunged upward. Because the instrument is

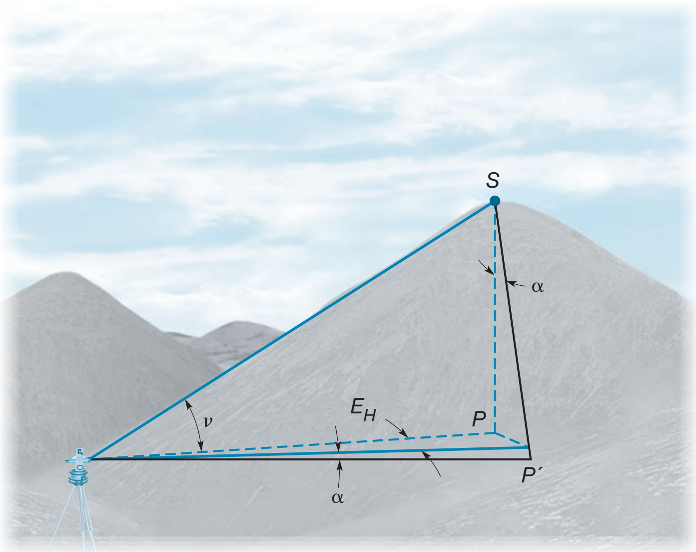


Figure 8.26
Geometry of instrument dislevelment.

misleveled, the line of sight scribes an inclined line SP' instead of the required vertical line SP. The angle between these two lines is α the amount that the instrument is out of level. From this figure, it can be shown that the error in the horizontal direction, E_H, is

$$E_H = \alpha \tan (v) \tag{8.4}$$

In Equation (8.4), v is the altitude angle to point S. For the observation of any horizontal angle if the altitude angles for both the backsight and foresight are nearly the same, the resultant error in the horizontal angle is negligible. In flat terrain, this is approximately the case and the error due to dislevelment can be small. However, in mountainous terrain where the backsight and foresight pointings can vary by large amounts, this error can become substantial. For example, assume that an instrument that is 20″ out of level reads a backsight zenith angle as 93°, and the foresight zenith angle as 80°. The horizontal error in the backsight direction would be $20''\tan(-3°) = -1.0''$ and in the foresight is $20''\tan(10°) = 3.5''$ resulting in a cumulative error in the horizontal angle of $3.5'' - (-1'') = 4.5''$. This is a systematic error that becomes more serious as larger vertical angles are observed. It is critical in astronomical observations for azimuth as discussed in Chapter 19.

Two things should be obvious from this discussion, it is important to check (1) the adjustment of the plate bubble often and (2) check the position of the bubble during the observation process.

2. *Horizontal axis not perpendicular to vertical axis*. This situation causes the axis of sight to define an inclined plane as the telescope is plunged and, therefore, if the backsight and foresight have differing angles of inclination, incorrect horizontal angles will result. Errors from this origin can be canceled by averaging an equal number of direct and reverse readings, or by double centering if prolonging a straight line. With total station instruments having dual-axis compensation, this error can be determined in a calibration process that consists of carefully pointing to the same target in both direct and reverse modes. From this operation the microprocessor can compute and store a correction factor. It is then automatically applied to all horizontal angles subsequently observed.

3. *Axis of sight not perpendicular to horizontal axis*. If this condition exists, as the telescope is plunged, the axis of sight generates a cone whose axis coincides with the horizontal axis of the instrument. The greatest error from this source occurs when plunging the telescope, as in prolonging a straight line or measuring deflection angles. Also, when the angle of inclination of the backsight is not equal to that of the foresight, observed horizontal angles will be incorrect. These errors are eliminated by double centering and by averaging equal numbers of direct and reverse readings.

4. *Vertical-circle indexing error*. As noted in Section 8.13, when the axis of sight is horizontal, an altitude angle of zero, or a zenith angle of either 90° or 270°, should be read; otherwise an indexing error exists. The error can be eliminated by computing the mean from equal numbers of altitude (or zenith) angles read in the direct and reverse modes. With most newer total station instruments, the indexing error can be determined by carefully reading the same zenith angle

both direct and reverse. The value of the indexing error is then computed, stored, and automatically applied to all observed zenith angles. However, the determination of the indexing error should be done carefully during calibration to ensure that an incorrect calibration error is not applied to all subsequent angles observed with the instrument.

5. *Eccentricity of centers*. This condition exists if the geometric center of the graduated horizontal (or vertical) circle does not coincide with its center of rotation. Errors from this source are usually small. Total stations may also be equipped with systems that automatically average readings taken on opposite sides of the circles, thereby compensating for this error.

6. *Circle graduation errors*. If graduations around the circumference of a horizontal or vertical circle are nonuniform, errors in observed angles will result. These errors are generally very small. Some total stations always use readings taken from many locations around the circles for each observed horizontal and vertical angle, thus providing an elegant system for eliminating these errors.

7. *Errors caused by peripheral equipment*. Additional instrumental errors can result from worn tribrachs, plummets that are out of adjustment, unsteady tripods, and sighting poles with maladjusted circular bubbles. This equipment should be regularly checked and kept in good condition or adjustment. Procedures for adjusting these items are outlined in Section 8.19.

8.20.2 Natural Errors

1. *Wind*. Wind vibrates the tripod that the total station instrument rests on. On high setups, light wind can vibrate the instrument to the extent that precise pointings become impossible. Shielding the instrument, or even suspending observations on precise work, may be necessary on windy days. An optical plummet is essential for making setups in this situation.

2. *Temperature effects*. Temperature differentials cause unequal expansion of various parts of total station instruments. This causes bubbles to run, which can produce erroneous observations. Shielding instruments from sources of extreme heat or cold reduces temperature effects.

3. *Refraction*. Unequal refraction bends the line of sight and may cause an apparent shimmering of the observed object. It is desirable to keep lines of sight well above the ground and avoid sights close to buildings, smokestacks, vehicles, and even large individual objects in generally open spaces. In some cases, observations may have to be postponed until atmospheric conditions have improved.

4. *Tripod settlement*. The weight of an instrument may cause the tripod to settle, particularly when set up on soft ground or asphalt highways. When a job involves crossing swampy terrain, stakes should be driven to support the tripod legs and work at a given station completed as quickly as possible. Stepping near a tripod leg or touching one while looking through the telescope will demonstrate the effect of settlement on the position of the bubble and cross wires. Most total station instruments have sensors that tell the operator when dislevelment has become too severe to continue the observation process.

8.20.3 Personal Errors

1. *Instrument not set up exactly over point*. Miscentering of the instrument over a point will result in an incorrect horizontal angle being observed. As shown in Figure 8.27, instrument miscentering will cause errors in both the backsight and foresight directions of an angle. The amount of error is dependent on the position of the instrument in relation to the point. For instance, in Figure 8.27(a), the miscentering that is depicted will have minimal effect on the observed angle since the error on the backsight to P_1 will partially cancel the error on the foresight to P_2. However, in Figures 8.27(b) and (c), the effect of the miscentering has a maximum effect on the observed angular values. Since the position of the instrument is random in relation to the station, it is important to carefully center the instrument over the station when observing angles. The position should be checked at intervals during the time a station is occupied, to be certain it remains centered.

2. *Bubbles not centered perfectly*. The bubbles must be checked frequently, but NEVER releveled between a backsight and a foresight—only *before* starting and *after* finishing an angular position.

3. *Improper use of clamps and tangent screws*. An observer must form good operational habits and be able to identify the various clamps and tangent screws by their touch without looking at them. Final setting of tangent screws is always made with a positive motion to avoid backlash. Clamps should be tightened just once and not checked again to be certain they are secure.

4. *Poor focusing*. Correct focusing of the eyepiece on the crosshairs, and of the objective lens on the target, is necessary to prevent parallax. Objects sighted should be placed as near the center of the field of view as possible. Focusing affects pointing, which is an important source of error. In newer instruments like the Topcon GTS 600-AF shown in Figure 8.24, automatic focusing of the objective lens is provided. These devices are similar to the modern photographic camera and can increase the speed of the survey when sight distances to the targets vary.

5. *Overly careful sights*. Checking and double-checking the position of the cross hair setting on a target wastes time and actually produces poorer results

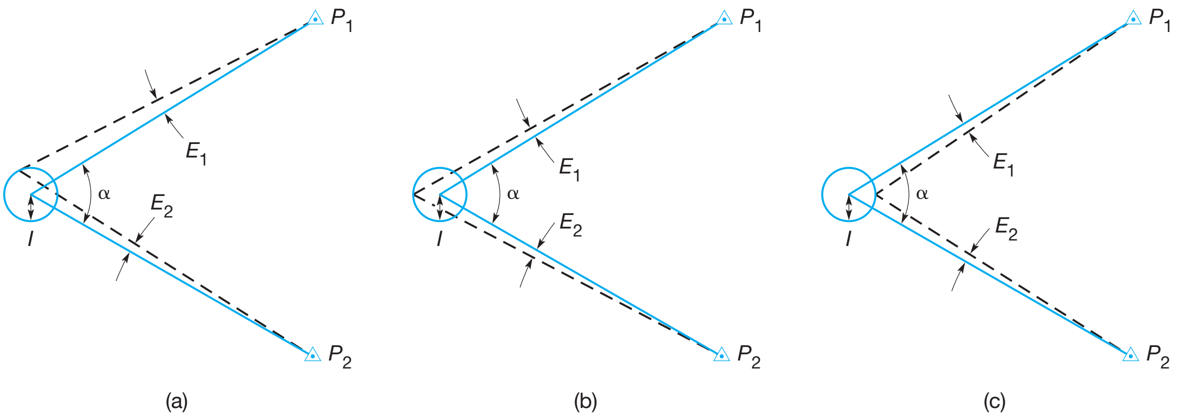


Figure 8.27 Effects of instrument miscentering on an angle.

than one fast observation. The cross hair should be aligned quickly, and the next operation begun promptly.

6. *Careless plumbing and placement of rod*. One of the most common errors results from careless plumbing of a rod when the instrument operator because of brush or other obstacles in the way can only see the top. Another is caused by placing a pole off-line behind a point to be sighted.

8.21 PROPAGATION OF RANDOM ERRORS IN ANGLE OBSERVATIONS

Random errors are present in every horizontal angle observation. Whenever an instrument's circles are read, a small error is introduced into the final measured angle. Similarly, each operator will have some miscentering on the target. These error sources are random. They may be small or large, depending on the instrument, the operator, and the conditions at the time of the angle observation. Increasing the number of angle repetitions can reduce the effects of reading and pointing.

With the introduction of total station instruments, standards were developed for estimating errors in angle observations caused by reading and pointing on a well-defined target. The standards, called DIN 18723, provide values for estimated errors in the mean of two direction observations, one each in the direct and reverse modes. The Leica TPS 300 (Figure 8.1) has a DIN 18723 accuracy of $\pm 2''$, and the Topcon GTS 210A (Figure 8.2) has a DIN 18723 accuracy of $\pm 5''$.

A set of angles observed with a total station will have an estimated error of

$$E = \frac{2E_{DIN}}{\sqrt{n}} \tag{8.5}$$

where E is the estimated error in the angle due to pointing and reading, n is the total number of angles read in both direct and reverse modes, and E_{DIN} is the DIN 18723 error.

Example 8.2

Three sets of angles (3D and 3R) are measured with the Leica TPS 300. What is the estimated error in the angle?

Solution

By Equation (8.5), the estimated error is

$$E = \frac{2(2'')}{\sqrt{6}} = \pm 1.6''$$

8.22 MISTAKES

Some common mistakes in angle observation work are

1. Sighting on or setting up over the wrong point.
2. Calling out or recording an incorrect value.
3. Improper focusing of the eyepiece and objective lenses of the instrument.
4. Leaning on the tripod or placing a hand on the instrument when pointing or taking readings.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

8.1 At what step should the instrument be mounted on the tripod when setting up over a point?

8.2 List the four axes of a total station and their relationship with each other.

8.3 Describe a systematic error that can be present in an angle and describe how it is removed by field procedure.

8.4 Name and briefly describe the three main components of a total station.

8.5 What is the purpose of dual-axis compensation in a total station instrument?

8.6 What is the purpose of the jog/shuttle mechanism on a servo-driven total station?

8.7 Why is it important to remove any parallax from an optical plummet?

8.8 Describe the steps used in setting up a total station with an adjustable leg tripod over a point.

8.9 What is meant by an angular position?

8.10 Why are the bases of total station instruments designed to be interchanged with other accessories?

8.11 Why is it important to keep the circular bubble of a sighting rod in adjustment?

8.12 Determine the angles subtended for the following conditions:
(a)*a 2-cm diameter pipe sighted by total station from 100 m.
(b) a 1/4-in. stake sighted by total station from 400 ft.
(c) a 1/4-in. diameter chaining pin observed by total station from 50 ft.

8.13 What is the error in an observed direction for the situations noted?
(a) setting a total station 3 mm to the side of a tack on a 50-m sight.
(b) lining in the edge (instead of center) of a 1/4-in. diameter chaining pin at 100 ft.
(c) sighting the edge (instead of center) of a 2-cm diameter range pole 100 m.
(d) sighting the top of a 6-ft range pole that is 3′ off-level on a 300-ft sight.

8.14* Intervening terrain obstructs the line of sight so only the top of a 6-ft-long pole can be seen on a 250-ft sight. If the range pole is out of plumb and leaning sideways 0.025 ft per vertical foot, what maximum angular error results?

8.15 Same as Problem 8.14, except that it is a 2-m pole that is out of plumb and leaning sideways 2 cm per meter on a 100 m sight.

8.16 Discuss the advantages of a robotic total station instrument.

8.17 Explain why the level bubble should be shaded when leveling an instrument in bright sun.

8.18 How is a total station with a level bubble off by 2 graduations leveled in the field?

8.19 An interior angle x and its explement y were turned to close the horizon. Each angle was observed once direct and once reverse using the repetition method. Starting with an initial backsight setting of 0°00′00″ for each angle, the readings after the first and second turnings of angle x were 49°36′24″ and 99°13′00″ and the readings after the first and second turnings of angle y were 310°23′28″ and 260°46′56″. Calculate each angle and the horizon misclosure.

8.20* A zenith angle is measured as 284°13′56″ in the reversed position. What is the equivalent zenith angle in the direct position?

8.21 What is the average zenith angle given the following direct and reverse readings
Direct: 94°23′48″, 94°23′42″, 94°23′44″
Reverse: 265°36′24″, 265°36′20″, 265°36′22″

In Figure 8.9(c), direct and reversed directions observed with a total station instrument from A to points B, C, and D are listed in Problems 8.22 and 8.23. Determine the values of the three angles, and the horizon misclosure.

8.22 Direct: 0°00′00″, 191°13′36″, 245°53′44″, 0°00′02″
Reverse: 0°00′00″, 191°13′42″, 245°53′46″, 0°00′00″

8.23 Direct: 0°00′00″, 43°11′12″, 121°36′42″, 0°00′02″
Reverse: 359°59′58″, 43°11′16″, 121°36′48″, 359°59′56″

8.24* The angles at point X were observed with a total station instrument. Based on four readings, the standard deviation of the angle was ±5.6″. If the same procedure is used in observing each angle within a six-sided polygon, what is the estimated standard deviation of closure at a 95% level of probability?

8.25 The line of sight of a total station is out of adjustment by 5″.
- **(a)** In prolonging a line by plunging the telescope between backsight and foresight, but not double centering, what angular error is introduced?
- **(b)** What off-line linear error results on a foresight of 300 m?

8.26 A line PQ is prolonged to point R by double centering. Two foresight points R' and R'' are set. What angular error would be introduced in a single plunging based on the following lengths of QR and $R'R''$, respectively?
- **(a)***650.50 ft and 0.35 ft.
- **(b)** 253.432 m and 23 mm.

8.27 Explain why the "principal of reversion" is important in angle measurement.

8.28 What is indexing error, and how can its value be obtained and eliminated from observed zenith angles?

8.29* A total station with a 20″/div. level bubble is one division out of level on a point with an altitude angle of 38°15′44″. What is the error in the horizontal pointing?

8.30 What is the equivalent altitude angle for a zenith angle of 86°02′06″?

8.31 What error in horizontal angles is consistent with the following linear precisions?
- **(a)** 1/5000, 1/10,000, 1/20,000, and 1/100,000
- **(b)** 1/300, 1/800, 1/1000, 1/3000, and 1/8000

8.32 Why is it important to check if the shoes on a tripod are tight?

8.33 Describe the procedure to adjust an optical plummet on a total station.

8.34 List the procedures for "wiggling-in" a point.

8.35 A zenith angle was read twice direct giving values of 86°34′12″ and 86°34′16″, and twice reverse yielding readings of 273°25′32″ and 273°25′36″. What is the mean zenith angle? What is the indexing error?

8.36 Write a review of an article on total station instruments written in a professional journal.

8.37 Create a computational program that takes the directions in Figure 8.12 and computes the average angles, their standard deviations, and the horizon misclosure.

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9 Traversing

9.1 INTRODUCTION

A traverse is a series of consecutive lines whose ends have been marked in the field and whose lengths and directions have been determined from observations. In traditional surveying by ground methods, *traversing*, the act of marking the lines, that is, establishing traverse stations and making the necessary observations, is one of the most basic and widely practiced means of determining the relative locations of points.

There are two kinds of traverses: *closed* and *open*. Two categories of closed traverses exist: *polygon* and *link*. In the polygon traverse, as shown in Figure 9.1(a), the lines return to the starting point, thus forming a closed figure that is both geometrically and mathematically closed. Link traverses finish upon another station that should have a positional accuracy equal to or greater than that of the starting point. The link type (geometrically open, mathematically closed), as illustrated in Figure 9.1(b), must have a closing reference direction, for example, line $E\text{-}Az\ Mk_2$. Closed traverses provide checks on the observed angles and distances, which is an extremely important consideration. They are used extensively in control, construction, property, and topographic surveys.

If the distance between stations *C* and *E* in Figure 9.1(a) were observed, the resultant set of observations would become what is called a *network*. A network involves the interconnection of stations within the survey to create additional redundant observations. Networks offer more geometric checks than closed traverses. For instance, in Figure 9.1(a), after computing coordinates on stations *C* and *E* using elementary procedures, the observed distance *CE* can be compared against a value obtained by inversing the coordinates (see Chapter 10 for discussion on computation of coordinates and inversing coordinates). Figure 9.7(b) shows another example where a network has been developed. Networks should be adjusted using the method of least squares as presented in Chapter 16.

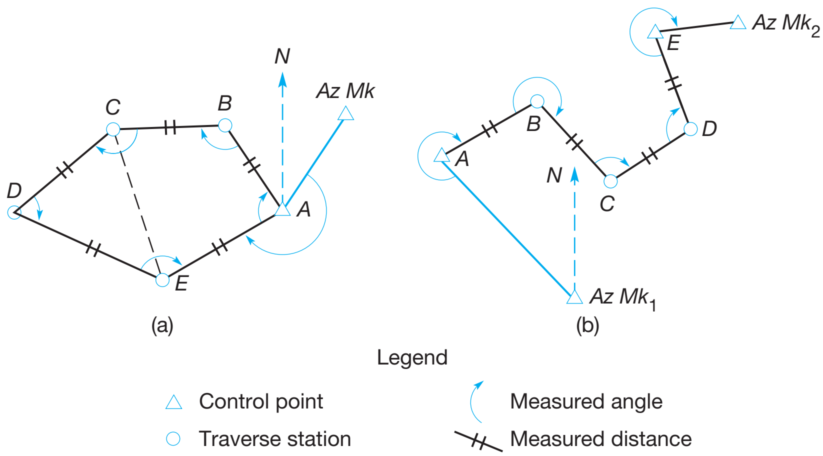


Figure 9.1
Examples of closed traverses.

An open traverse (geometrically and mathematically open) (Figure 9.2) consists of a series of lines that are connected but do not return to the starting point or close upon a point of equal or greater order accuracy. *Open traverses should be avoided because they offer no means of checking for observational errors and mistakes*. If they must be used, observations should be repeated carefully to guard against mistakes. The precise control-traversing techniques presented in Section 19.12.2 should be considered in these situations.

Hubs (wooden stakes with tacks to mark the points), steel stakes, or pipes are typically set at each traverse station *A*, *B*, *C*, etc., in Figures 9.1 and 9.2, where a change in direction occurs. Spikes, "P-K"[1] nails, and scratched crosses are used on blacktop pavement. Chiselled or painted marks are made on concrete. Traverse stations are sometimes interchangeably called *angle points* because an angle is observed at each one.

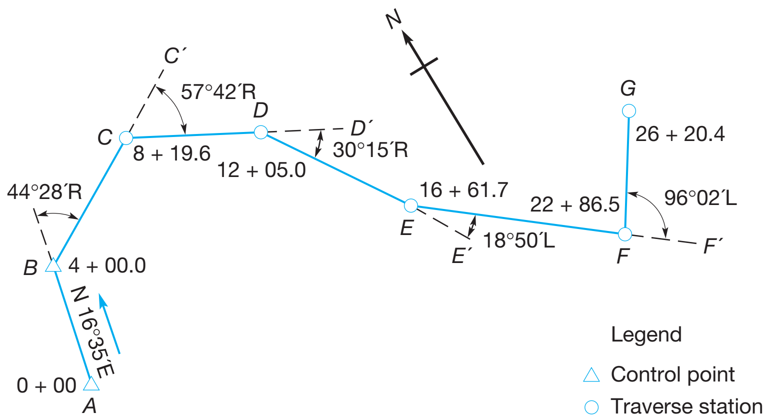


Figure 9.2
Open traverse.

[1]P-K is a trade name for concrete nails. The Parker–Kalon Company originally manufactured these nails. There is a small depression in the center of the nail that serves as a marker for the location of the station. Several companies now manufacture similar or better versions of this nail. Still the original name, P-K, is used to denote this type of nail.

■ 9.2 OBSERVATION OF TRAVERSE ANGLES OR DIRECTIONS

The methods used in observing angles or directions of traverse lines vary and include (1) interior angles, (2) angles to the right, (3) deflection angles, and (4) azimuths. These are described in the following subsections.

9.2.1 Traversing by Interior Angles

Interior-angle traverses are used for many types of work, but they are especially convenient for property surveys. Although interior angles could be observed either clockwise or counterclockwise, *to reduce mistakes in reading, recording, and computing, they should always be turned clockwise* from the backsight station to the foresight station. The procedure is illustrated in Figure 9.1(a). In this text, except for left deflection angles, clockwise turning will always be assumed. Furthermore, when angles are designated by three station letters or numbers in this text, the backsight station will be given first, the occupied station second, and the foresight station third. Thus, angle *EAB* of Figure 9.1(a) was observed at station *A*, with the backsight on station *E* and the foresight at station *B*.

Interior angles may be improved by averaging equal numbers of direct and reversed readings. As a check, exterior angles may also be observed to close the horizon (see Section 8.10). In the traverse of Figure 9.1(a), a reference line *A-Az MK* of known direction exists. Thus, the clockwise angle at *A* from *Az Mk* to *E* must also be observed to enable determining the directions of all other lines. This would not be necessary if the traverse contained a line of known direction, like *AB* of Figure 7.2, for example.

9.2.2 Traversing by Angles to the Right

Angles observed clockwise from a backsight on the "rearward" traverse station to a foresight on the "forward" traverse station [see Figures 9.1(a) and (b)] are called *angles to the right*. According to this definition, to avoid ambiguity in angle-to-the-right designations, the "sense" of the forward traverse direction must be established. This is normally done by consecutive numbering or lettering of traverse stations so that they increase in the forward direction. Depending on the direction of the traversing, angles to the right may be interior or exterior angles in a polygon traverse. If the direction of traversing is counterclockwise around the figure, then clockwise interior angles will be observed. However, if the direction of traversing is clockwise, then exterior angles will be observed. Data collectors generally follow this convention when traversing. Thus, in Figure 9.1(b), for example, the direction from *A* to *B*, *B* to *C*, *C* to *D*, etc., is forward. By averaging equal numbers of direct and reversed readings, observed angles to the right can also be checked and their accuracy improved. From the foregoing definitions of interior angles and angles to the right, it is evident that in a polygon traverse the only difference between the two types of observational procedures may be ordering of the backsight and foresight stations since both procedures observe clockwise angles.

9.2.3 Traversing by Deflection Angles

Route surveys are commonly run by deflection angles observed to the right or left from the lines extended, as indicated in Figure 9.2. A deflection angle is not

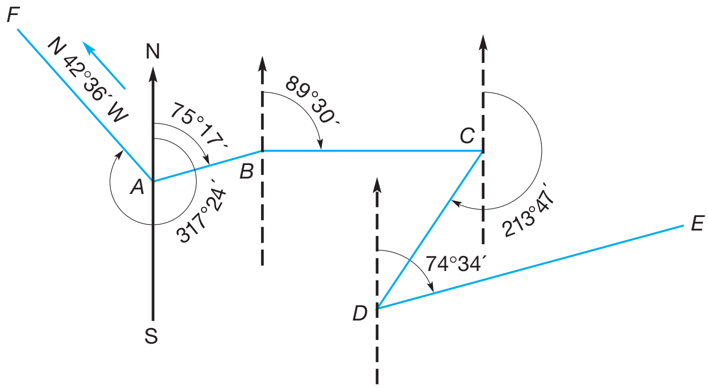


Figure 9.3
Azimuth traverse.

complete without a designation R or L, and, of course, it cannot exceed 180°. Each angle should be doubled or quadrupled, and an average value determined. The angles should be observed an equal number of times in face left and face right to reduce instrumental errors. Deflection angles can be obtained by subtracting 180° from angles to the right. Positive values so obtained denote right deflection angles; negative ones are left.

9.2.4 Traversing by Azimuths

With total station instruments, traverses can be run using azimuths. This process permits reading azimuths of all lines directly and thus eliminates the need to calculate them. In Figure 9.3, azimuths are observed clockwise from the north end of the meridian through the angle points. The instrument is oriented at each setup by sighting on the previous station with either the back azimuth on the circle (if angles to the right are turned) or the azimuth (if deflection angles are turned), as described in Section 8.11. Then the forward station is sighted. The resulting reading on the horizontal circle will be the forward line's azimuth.

9.3 OBSERVATION OF TRAVERSE LENGTHS

The length of each traverse line (also called a course) must be observed, and this is usually done by the simplest and most economical method capable of satisfying the required precision of a given project. Their speed, convenience, and accuracy makes the EDM component of a total station instrument the most often used, although taping or other methods discussed in Chapter 6 could be employed. A distinct advantage of traversing with total station instruments is that both angles and distances can be observed with a single setup at each station. Averages of distances observed both forward and back will provide increased accuracy, and the repeat readings afford a check on the observations.

Sometimes state statutes regulate the precision for a traverse to locate boundaries. On construction work, allowable limits of closure depend on the use and extent of the traverse and project type. Bridge location, for example, demands a high degree of precision.

In closed traverses, each course is observed and recorded as a separate distance. On long link traverses for highways and railroads, distances are carried along continuously from the starting point using stationing (see Section 5.9.1). In Figure 9.2, which uses stationing in feet, for example, beginning with station 0 + 00 at point A, 100-ft stations (1 + 00, 2 + 00, and 3 + 00) are marked until hub B at station 4 + 00 is reached. Then stations 5 + 00, 6 + 00, 7 + 00, 8 + 00, and 8 + 19.60 are set along course BC to C, etc. The length of a line in a stationed link traverse is the difference between stationing at its end points; thus, the length of line BC is $819.60 - 400.00 = 419.60$ ft.

■ 9.4 SELECTION OF TRAVERSE STATIONS

Positions selected for setting traverse stations vary with the type of survey. In general, guidelines to consider in choosing them include accuracy, utility, and efficiency. Of course, intervisibility between adjacent stations, forward and back, must be maintained for angle and distance observations. The stations should also ideally be set in convenient locations that allow for easy access. Ordinarily, stations are placed to create lines that are as long as possible. This not only increases efficiency by reducing the number of instrument setups, but it also increases accuracy in angle observations. However, utility may override using very long lines because intermediate hubs, or stations at strategic locations, may be needed to complete the survey's objectives.

Often the number of stations can be reduced and the length of the sight lines increased by careful reconnaissance. It is always wise to "walk" the area being surveyed and find ideal locations for stations before the traverse stakes are set and the observation process is undertaken.

Each different type of survey will have its unique requirements concerning traverse station placement. On property surveys, for example, traverse stations are placed at each corner if the actual boundary lines are not obstructed and can be occupied. If offset lines are necessary, a stake is located near each corner to simplify the observations and computations. Long lines and rolling terrain may necessitate extra stations.

On route surveys, stations are set at each angle point and at other locations where necessary to obtain topographic data or extend the survey. Usually the centerline is run before construction begins, but it will likely be destroyed and need replacement one or more times during various phases of the project. An offset traverse can be used to avoid this problem.

A traverse run to provide control for topographic mapping serves as a framework to which map details such as roads, buildings, streams, and hills are referenced. Station locations must be selected to permit complete coverage of the area to be mapped. *Spurs* consisting of one or more lines may branch off as open (*stub*) traverses to reach vantage points. However, their use should be discouraged since a check on their positions cannot be made.

■ 9.5 REFERENCING TRAVERSE STATIONS

Traverse stations often must be found and reoccupied months or even years after they are established. Also they may be destroyed through construction or other activity. Therefore, it is important that they be referenced by creating observational *ties* to them so that they can be relocated if obscured or reestablished if destroyed.

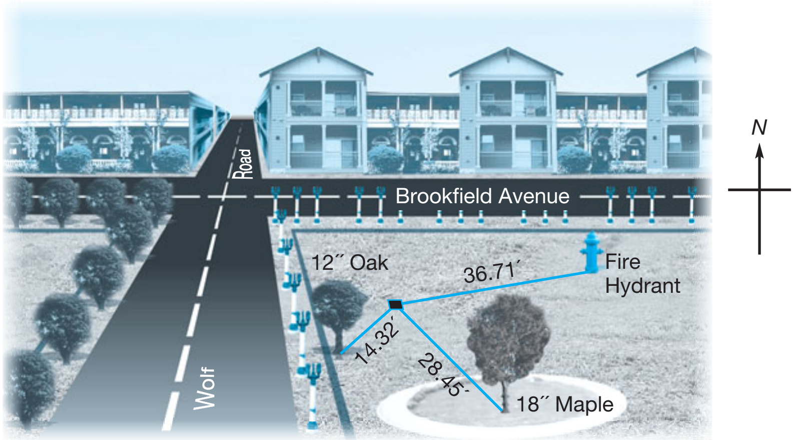


Figure 9.4
Referencing a point.

Figure 9.4 presents a typical traverse tie. As illustrated, these ties consist of distance observations made to nearby fixed objects. Short lengths (less than 100 ft) are convenient if a steel tape is being used, but, of course, the distance to definite and unique points is a controlling factor. Two ties, preferably at about right angles to each other, are sufficient, but three should be used to allow for the possibility that one reference mark may be destroyed. Ties to trees can be observed in hundredths of a foot if nails are driven into them. However, *permission must be obtained from the landowner before driving nails into trees*. It is always important to remember that the surveyor may be held legally responsible for any damages to property that may occur during the survey.

If natural or existing features such as trees, utility poles, or corners of buildings are not available, stakes may be driven and used as ties. Figure 9.5(a) shows an arrangement of *straddle hubs* well suited to tying in a point such as H on a highway centerline or elsewhere. Reference points A and B are carefully set on the line through H, as are C and D. Lines AB and CD should be roughly perpendicular, and the four points should be placed in safe locations, outside of areas likely to be disturbed. It is recommended that a third point be placed on each line to serve as an alternate in the event one point is destroyed. The intersection of the lines of sight of two total stations set up at A and C and simultaneously aimed at B and D, respectively, will recover the point. The traverse hub H can also be found by intersecting strings stretched between diagonally opposite ties if the lengths are not too long. Hubs in the position illustrated by Figure 9.5(a) are sometimes used but are not as desirable as straddle hubs for stringing.

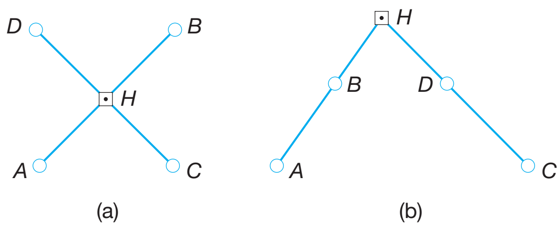


Figure 9.5
Hubs for ties.

9.6 TRAVERSE FIELD NOTES

The importance of notekeeping was discussed in Chapter 2. Since a traverse is itself the end on a property survey and the basis for all other data in mapping, a single mistake or omission in recording is one too many. All possible field and office checks must therefore be made. A partial set of field notes for an interior-angle traverse run using a total station instrument is shown in Figure 9.6. Notice that details such as date, weather, instrument identifications, and party members and their duties are recorded on the right-hand page of the notes. Also a sketch with a north arrow is shown. The observed data is recorded on the left-hand page. First, each station that is occupied is identified, and the heights of the total station instrument and reflector that apply at that station are recorded. Then horizontal circle readings, zenith angles, horizontal distances, and elevation differences observed at each station are recorded. Notice that each horizontal angle is measured twice in the direct mode, and twice in the reversed mode. As noted earlier, this practice eliminates instrumental errors and gives repeat angle values for checking.

TRAVERSING WITH A

Instrument at sta 101
$h_e = 5.3$ $h_r = 5.3$

Sta. Sighted	D/R	Horiz. Circle	Zenith Angle	Horiz. Dist.	Elev. Diff.
104	D	0°00′00″	86°30′01″	324.38	+19.84
102	D	82°18′19″	92°48′17″	216.02	−10.58
104	R	180°00′03″	273°30′00″		
102	R	262°18′18″	267°11′41″		

Instrument at sta 102
$h_e = 5.5$ $h_r = 5.5$

Sta. Sighted	D/R	Horiz. Circle	Zenith Angle	Horiz. Dist.	Elev. Diff.
101	D	0°00′00″	87°11′19″	261.05	+10.61
103	D	95°32′10″	85°19′08″	371.65	+30.43
101	R	180°00′02″	272°48′43″		
103	R	275°32′08″	274°40′50″		

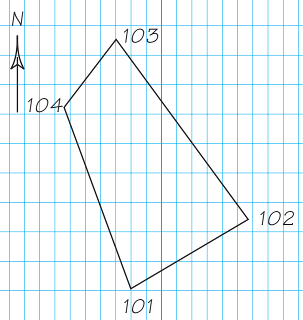
Instrument at sta 103
$h_e = 5.4$ $h_r = 5.4$

Sta. Sighted	D/R	Horiz. Circle	Zenith Angle	Horiz. Dist.	Elev. Diff.
102	D	0°00′00″	94°40′48″	371.63	−30.42
104	D	49°33′46″	90°01′54″	145.03	− 0.08
102	R	180°00′00″	265°19′14″		
104	R	229°33′47″	269°58′00″		

TOTAL STATION INSTRUMENT

Topo Control Survey
19 Oct. 2000
Cool, Sunny, 48° F
Pressure 29.5 in.
Total Station #7
Reflector #7A
M.R. Duckett – ⊼
N. Dahman – ∅
T. Ruhren – N

Sketch



M. R. Duckett

Figure 9.6 Example traverse field notes using a total station instrument.

Zenith angles were also observed twice each direct and reversed. Although not needed for traversing, they are available for checking if larger than tolerable misclosures (see Chapter 10) should exist in the traverse. Details of making traverse observations with a total station instrument are described in Section 9.8.

9.7 ANGLE MISCLOSURE

The angular misclosure for an interior-angle traverse is the difference between the sum of the observed angles and the geometrically correct total for the polygon. The sum, Σ, of the interior angles of a closed polygon should be

$$\Sigma = (n - 2)180^\circ \tag{9.1}$$

where n is the number of sides, or angles, in the polygon. This formula is easily derived from known facts. The sum of the angles in a triangle is 180°; in a rectangle, 360°; and in a pentagon, 540°. Thus, each side added to the three required for a triangle increases the sum of the angles by 180°. As was mentioned in Section 7.3, if the direction about a traverse is clockwise when observing angles to the right, exterior angles will be observed. In this case, the sum of the exterior angles will be

$$\Sigma = (n + 2)\,180^\circ \tag{9.2}$$

Figure 9.1(a) shows a five-sided figure in which, if the sum of the observed interior angles equals 540°00′05″, the angular misclosure is 5″. Misclosures result from the accumulation of random errors in the angle observations. Permissible misclosure can be computed by the formula

$$c = K\sqrt{n} \tag{9.3}$$

where n is the number of angles, and K a constant that depends on the level of accuracy specified for the survey. The Federal Geodetic Control Subcommittee (FGCS) recommends constants for five different orders of traverse accuracy: *first-order, second-order class I, second-order class II, third-order class I, and third-order class II*. Values of K for these orders, from highest to lowest, are 1.7″, 3″, 4.5″, 10″, and 12″, respectively. Thus, if the traverse of Figure 9.1(a) were being executed to second-order class II standards, its allowable misclosure error would be $4.5''\sqrt{5} = \pm 10''$.

The algebraic sum of the deflection angles in a closed-polygon traverse equals 360°, clockwise (right) deflections being considered plus and counterclockwise (left) deflections, minus. This rule applies if lines do not crisscross, or if they cross an even number of times. When lines in a traverse cross an odd number of times, the sum of right deflections equals the sum of left deflections.

A closed-polygon azimuth traverse is checked by setting up on the starting point a second time, after having occupied the successive stations around the traverse, and orienting by back azimuths. The azimuth of the first side is then obtained a second time and compared with its original value. Any difference is the misclosure. If the first point is not reoccupied, the interior angles computed from the azimuths will automatically check the proper geometric total, even though one or more of the azimuths may be incorrect.

Although angular misclosures cannot be directly computed for link traverses, the angles can still be checked. The direction of the first line may be determined from two intervisible stations with a known azimuth between them, or from a sun or Polaris observation, as described in Appendix C. Observed angles are then applied to calculate the azimuths of all traverse lines. The last line's computed azimuth is compared with its known value, or the result obtained from another sun or Polaris observation, On long traverses, intermediate lines can be checked similarly. In using sun or Polaris observations to check angles on traverses of long east-west extent, allowance must be made for *convergence of meridians*. This topic is discussed in Section 19.12.2.

■ 9.8 TRAVERSING WITH TOTAL STATION INSTRUMENTS

Total station instruments, with their combined electronic angle and distance measurement components, speed the process of traversing significantly because both the angles and distances can be observed from a single setup. The observing process is further aided because angles and distances are resolved automatically and displayed. Furthermore, the microprocessors of total stations can perform traverse computations, reduce slope distances to their horizontal and vertical components, and instantaneously calculate and store station coordinates and elevations. The reduction to obtain horizontal and vertical distance components was illustrated with the traverse notes of Figure 9.6.

To illustrate a method of traversing with a total station instrument, refer to the traverse of Figure 9.1(b). With the instrument set up and leveled at station A, a backsight is carefully taken on $Az\ MK_1$. The azimuth of line A-$Az\ MK_1$ is initialized on the horizontal circle by entering it in the unit using the keyboard. The coordinates and elevation of station A are also entered in memory. Next a foresight is made on station B. The azimuth of line AB will now appear on the display, and upon keyboard command, can be stored in the microprocessor's memory. Slope distance AB is then observed and reduced to its horizontal and vertical components by the microprocessor. Then the line's departure and latitude are computed and added to the coordinates of station A to yield the coordinates of station B. (Departures, latitudes, and coordinates are described in Chapter 10.) These procedures should be performed in both the direct and reversed modes and the results averaged to account for instrumental errors.

The procedure outlined for station A is repeated at station B, except that the back azimuth BA and coordinates of station B need not be entered; rather, they are recalled from the instrument's memory. From the setup at B, azimuth BC and coordinates of C are determined and stored. This procedure is continued until a station of known coordinates is reached, as E in Figure 9.1(b). Here the known coordinates of E are entered in the unit's computer and compared to those obtained for E through the traverse observations. Their difference (or misclosure) is computed, displayed, and, if within allowable limits, distributed by the microprocessor to produce final coordinates of intermediate stations. (Procedures for distributing traverse misclosure errors are covered in Chapters 10 and 16.)

Mistakes in orientation can be minimized when a data collector is used in combination with a total station. In this process, the coordinates of each backsight station are checked before proceeding with the angle and distance observations to the next foresight station. For example, in Figure 9.1(a), after the total station is leveled and oriented at station *B*, an observation is taken "back" on *A*. If the newly computed coordinates of *A* do not closely match their previously stored values, the instrument setup, leveling, and orientation should be rechecked, and the problem resolved before proceeding with any further measurements. This procedure often takes a minimal amount of time and typically identifies most field mistakes that occur during the observational process.

If desired, traverse station elevations can also be determined as a part of the procedure (usually the case for topographic surveys). Then entries *hi* (height of instrument) and *hr* (height of reflector) must be input (see Section 6.23). The microprocessor computes the vertical component of the slope distance, which includes a correction for curvature and refraction (see Section 4.5.4). The elevation difference is added to the occupied station's elevation to produce the next point's elevation. At the final station, any misclosure is determined by comparing the computed elevation with its known value, and if within tolerance, the misclosure is distributed to produce adjusted elevations of intermediate traverse stations.

All data from traversing with a total station instrument can be stored in a data collector for printing and transferred to the office for computing and plotting (see Sections 2.12 through 2.15). Alternatively, the traverse notes can be recorded manually as illustrated with Figure 9.6.

9.9 RADIAL TRAVERSING

In certain situations, it may be most convenient to determine the relative positions of points by radial traversing. In this procedure, as illustrated in Figure 9.7(a), some point *O*, whose position is assumed known, is selected from which all points

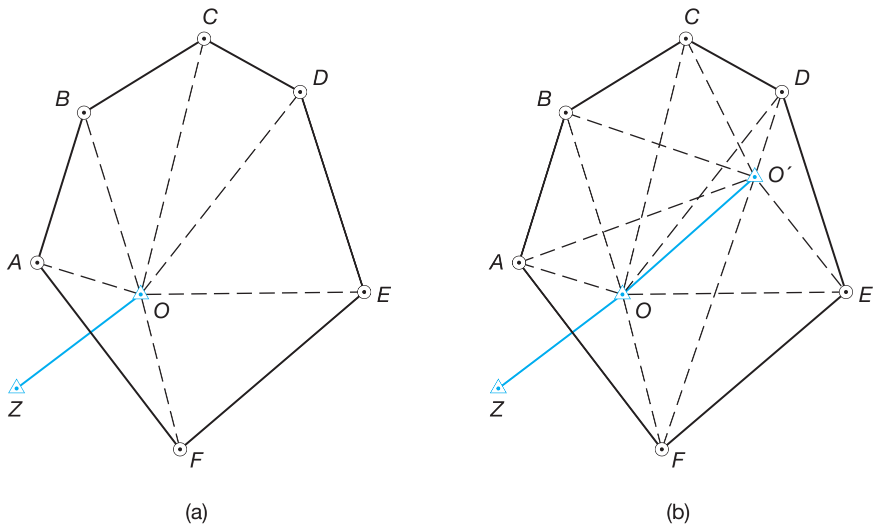


Figure 9.7
Radial traversing. (a) From one occupied station. (b) From two occupied stations.

to be located can be seen. If a point such as O does not exist, it can be established. It is also assumed that a nearby azimuth mark, like Z in Figure 9.7(a), is available, and that reference azimuth OZ is known. With a total station instrument at point O, after backsighting on Z, horizontal angles to all stations A through F are observed. Azimuths of all radial lines from O (as OA, OB, OC, etc.) can then be calculated. The horizontal lengths of all radiating lines are also observed. By using the observed lengths and azimuths, coordinates for each point can be computed. (The subject of coordinate computations is discussed in Chapter 10.)

It should be clear that in the procedure just described, each point A through F has been surveyed independently of all others, and that no checks on their computed positions exist. To provide checks, lengths AB, BC, CD, etc., could be computed from the coordinates of points, and then these same lengths observed. This results in many extra setups and substantially more fieldwork, thus defeating one of the major benefits of radial traversing. To solve the problem of gaining checks with a minimum of extra fieldwork, the method presented in Figure 9.7(b) is recommended. Here a second hub O' is selected from which all points can also be seen. The position of O' is determined by observations of the horizontal angle and distance from station O. This second hub O' is then occupied, and horizontal angles and distances to all stations A through F are observed as before. With the coordinates of both O and O' known, and by using the two independent sets of angles and distances, two sets of coordinates can be computed for each station, thus obtaining the checks. If the two sets for each point agree within a reasonable tolerance, the average can be taken. However, a better adjustment is obtained using the method of least squares (see Section 3.21 and Chapter 16). Although radial traversing can provide coordinates of many points in an area rapidly, the method is not as rigorous as running closed traverses.

Radial traversing is ideal for quickly establishing a large number of points in an area, especially when a total station instrument is employed. They not only enable the angle and distance observations to be made quickly, but they also perform the calculations for azimuth, horizontal distance, and station coordinates in real time. Radial methods are also very convenient for laying out planned construction projects with a total station instrument. In this application, the required coordinates of points to be staked are determined from the design, and the angles and distances that must be observed from a selected station of known position are computed. These are then laid out with a total station to set the stakes. The procedures are discussed in detail in Section 23.9.

■ 9.10 SOURCES OF ERROR IN TRAVERSING

Some sources of error in running a traverse are:

1. Poor selection of stations, resulting in bad sighting conditions caused by (a) alternate sun and shadow, (b) visibility of only the rod's top, (c) line of sight passing too close to the ground, (d) lines that are too short, and (e) sighting into the sun.
2. Errors in observations of angles and distances.
3. Failure to observe angles an equal number of times direct and reversed.

9.11 MISTAKES IN TRAVERSING

Some mistakes in traversing are:

1. Occupying or sighting on the wrong station.
2. Incorrect orientation.
3. Confusing angles to the right and left.
4. Mistakes in note taking.
5. Misidentification of the sighted station.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

9.1 Discuss the differences and similarities between a polygon and link traverse.

9.2 Discuss the differences between an open and closed traverse.

9.3 How can an angular closure be obtained on a link traverse?

9.4 What similarities and differences exist between interior angles and angles to the right in a polygon traverse?

9.5 Draw two five-sided closed polygon traverses with station labels 1 to 5. The first traverse should show angles to the right that are interior angles, and the second should show angles to the right that are exterior angles.

9.6 Discuss the importance of reconnaissance in establishing traverse stations.

9.7 How should traverse stations be referenced?

9.8 Discuss the advantages and dangers of radial traversing.

9.9 What should be the sum of the interior angles for a closed-polygon traverse that has: ***(a)** 6 sides **(b)** 7 sides **(c)** 10 sides.

9.10 What should the sum of the exterior angles for a closed-polygon traverse that are listed in Problem 9.9.

9.11 Four interior angles of a five-sided polygon traverse were observed as $A = 98°33'26''$, $B = 111°04'37''$, $C = 123°43'58''$, and $D = 108°34''25''$. The angle at E was not observed. If all observed angles are assumed to be correct, what is the value of angle E?

9.12 Similar to Problem 9.11, except the traverse had seven sides with observed angles of $A = 138°55'04''$, $B = 125°05'16''$, $C = 104°14'49''$, $D = 129°13'13''$, $E = 138°48'37''$, and $F = 128°08'25''$. Compute the angle at G, which was not observed.

9.13 What is the angular misclosure of a five-sided polygon traverse with observed angles of $83°07'23''$, $105°23'01''$, $124°56'48''$, $111°51'31''$, and $114°41'27''$.

9.14 Show that the sum of the exterior angles for a closed-polygon traverse is $(n + 2)180°$.

9.15* According to FGSC standards, what is the maximum acceptable angular misclosure for a second order, class I traverse having 20 angles?

9.16* What is the angular misclosure for a five-sided polygon traverse with observed exterior angles of $252°26'37''$, $255°55'13''$, $277°15'53''$, $266°35'02''$, and $207°47'05''$?

9.17 What is the angular misclosure for a six-sided polygon traverse with observed interior angles of $121°36'06''$, $125°16'04''$, $123°21'44''$, $121°09'58''$, $120°30'12''$, and $108°06'08''$?

9.18 Discuss how a data collector can be used to check the setup of a total station in traversing.

9.19* If the standard error for each measurement of a traverse angle is $\pm 3.3''$, what is the expected standard error of the misclosure in the sum of the angles for an eight-sided traverse?

9.20 If the angles of a traverse are turned so that the 95% error of any angle is ±2.5″, what is the 95% error in a twelve-sided traverse?

9.21 What criteria should be used when making reference ties to traverse stations?

9.22* The azimuth from station A of a link traverse to an azimuth mark is 212°12′36″. The azimuth from the last station of the traverse to an azimuth mark is 192°12′15″. Angles to the right are observed at each station: $A = 136°15'41''$, $B = 119°15'37''$, $C = 93°48'55''$, $D = 136°04'17''$, $E = 108°30'10''$, $F = 42°48'03''$, and $G = 63°17'17''$. What is the angular misclosure of this link traverse?

9.23 What FGCS order and class does the traverse in Problem 9.22 meet?

9.24* The interior angles in a five-sided closed-polygon traverse were observed as $A = 104°28'36''$, $B = 110°26'54''$, $C = 106°25'58''$, $D = 102°27'02''$, and $E = 112°11'15''$. Compute the angular misclosure. For what FGCS order and class is this survey adequate?

9.25 Similar to Problem 9.24, except for a six-sided traverse with observed exterior angles of $A = 244°28'36''$, $B = 238°26'54''$, $C = 246°25'58''$, $D = 234°27'02''$, $E = 235°08'55''$, and $F = 241°02'45''$.

9.26 In Figure 9.6, what is the average interior angle with the instrument at station 101.

9.27 Same as Problem 9.26 except at instrument station 103.

9.28 Explain why it is advisable to use two instrument stations, as O and O' in Figure 9.7(b), when running radial traverses.

9.29 Create a computational program that computes the misclosure of interior angles in a closed polygon traverse. Use this program to solve Problem 9.24.

9.30 Create a computational program that computes the misclosure of angles in a closed link traverse. Use this program to solve Problem 9.22.

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10 Traverse Computations

■ 10.1 INTRODUCTION

Measured angles or directions of closed traverses are readily investigated before leaving the field. Linear measurements, even though repeated, are more likely a source of error, and must also be checked. Although the calculations are lengthier than angle checks, with today's programmable calculators and portable computers they can also be done in the field to determine, before leaving, whether a traverse meets the required precision. If specifications have been satisfied, the traverse is then adjusted to create perfect "closure" or geometric consistency among angles and lengths; if not, field observations must be repeated until adequate results are obtained.

Investigation of precision and acceptance or rejection of the field data are extremely important in surveying. Adjustment for geometric closure is also crucial. For example, in land surveying the law may require property descriptions to have exact geometric agreement.

Different procedures can be used for computing and adjusting traverses. These vary from elementary methods to more advanced techniques based on the method of least squares (see Chapter 16). This chapter concentrates on elementary procedures. The usual steps followed in making elementary traverse computations are (1) adjusting angles or directions to fixed geometric conditions, (2) determining preliminary azimuths (or bearings) of the traverse lines, (3) calculating departures and latitudes and adjusting them for misclosure, (4) computing rectangular coordinates of the traverse stations, and (5) calculating the lengths and azimuths (or bearings) of the traverse lines after adjustment. These procedures are all discussed in this chapter and are illustrated with several examples. Chapter 16 discusses traverse adjustment using the method of least squares.

10.2 BALANCING ANGLES

In elementary methods of traverse adjustment, the first step is to balance (adjust) the angles to the proper geometric total. For closed traverses, angle balancing is done readily since the total error is known (see Section 9.7), although its exact distribution is not. Angles of a closed traverse can be adjusted to the correct geometric total by applying one of two methods:

1. Applying an average correction to each angle where observing conditions were approximately the same at all stations. The correction for each angle is found by dividing the total angular misclosure by the number of angles.
2. Making larger corrections to angles where poor observing conditions were present.

Of these two methods, the first is almost always applied.

Example 10.1

For the traverse of Figure 10.1, the observed interior angles are given in Table 10.1. Compute the adjusted angles using methods 1 and 2.

Solution

The computations are best arranged as shown in Table 10.1. The first part of the adjustment consists of summing the interior angles and determining the misclosure according to Equation (9.1), which in this instance, as shown beneath column 2, is +11″. The remaining calculations are tabulated, and the rationale for the procedures follows.

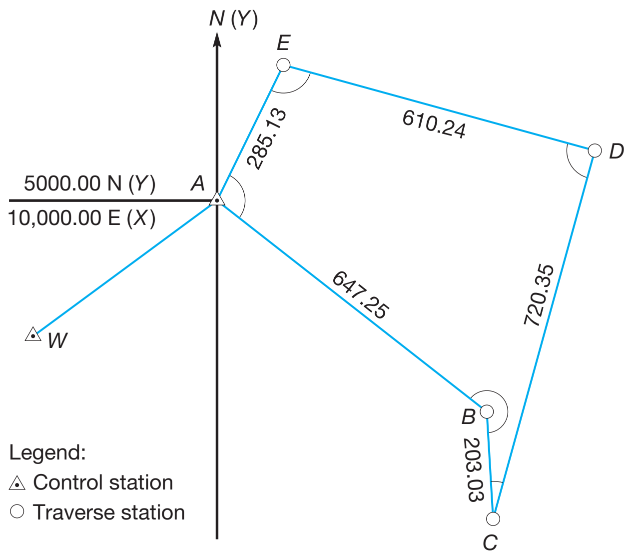

Figure 10.1 Traverse.

TABLE 10.1 ADJUSTMENT OF ANGLES

Point (1)	Measured Interior Angle (2)	Method 1: Multiples of Average Correction (3)	Method 1: Correction Rounded To 1″ (3)	Method 1: Successive Differences (5)	Method 1: Adjusted Angle (6)
A	100°45′37″	2.2″	2″	2″	100°45′35″
B	231°23′43″	4.4″	4″	2″	231°23′41″
C	17°12′59″	6.6″	7″	3″	17°12′56″
D	89°03′28″	8.8″	9″	2″	89°03′26″
E	101°34′24″	11.0″	11″	2″	101°34′22″
	Σ = 540°00′11″			Σ = 11″	Σ = 540°00′00″

Point (1)	Measured Interior Angle (2)	Method 2: Adjustment (7)	Method 2: Adjusted Angle (8)
A	100°45′37″	2″	100°45′35″
B	231°23′43″	3″	231°23′40″
C	17°12′59″	3″	17°12′56″
D	89°03′28″	1″	89°03′27″
E	101°34′24″	2″	101°34′22″
	Σ = 540°00′11″	Σ = 11″	Σ = 540°00′00″

For work of ordinary precision, it is reasonable to adopt corrections that are even multiples of the smallest recorded digit or decimal place for the angle readings. Thus in this example, corrections to the nearest 1″ will be made.

Method 1 consists of subtracting $11''/5 = 2.2''$ from each of the five angles. However, since the angles were read in multiples of 1″, applying corrections to the nearest tenth of a second would give a false impression of their precision. Therefore it is desirable to establish a pattern of corrections to the nearest 1″, as shown in Table 10.1. First multiples of the average correction of 2.2″ are tabulated in column (3). In column (4), each of these multiples has been rounded off to the nearest 1″. Then successive differences (adjustments for each angle) are found by subtracting the preceding value in column (4) from the one being considered. These are tabulated in column (5). Note that as a check, the sum of the corrections in this column must equal the angular misclosure of the traverse, which in this case is 11″. The adjusted interior angles obtained by applying these corrections are listed in column (6). As another check, they must total exactly the true geometric value of $(n - 2)180°$, or 540°00′00″ in this case.

In method 2, judgment is required because corrections are made to the angles expected to contain the largest errors. In this example, 3″ is subtracted from the angles at *B* and *C*, since they have the shortest sights (along line *BC*), and 2″ is subtracted from the angles at *A* and *E*, because they have the next shortest sights (along line *AE*). A 1″ correction was applied to angle *D* because of its long sights. The sum of the corrections must equal the total misclosure. The adjustment made in this manner is shown in columns (7) and (8) of Table 10.1.

It should be noted that, although the adjusted angles by both methods satisfy the geometric condition of a closed figure, they may be no nearer to the true values than before adjustment. Unlike corrections for linear observations (described in Section 10.7), *adjustments applied to angles are independent of the size of the angle*.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *Adjusting Angle Observations.mp4* discusses the use of method 1 to adjust angles in this section.

■ 10.3 COMPUTATION OF PRELIMINARY AZIMUTHS OR BEARINGS

After balancing the angles, the next step in traverse computation is calculation of either preliminary azimuths or preliminary bearings. This requires the direction of at least one course within the traverse to be either known or assumed. For some computational purposes an assumed direction is sufficient, and in that case the usual procedure is to simply assign north as the direction of one of the traverse lines. On certain traverse surveys, the magnetic bearing of one line can be determined and used as a reference for determining the other directions. However, in most instances, as in boundary surveys, true directions are needed. This requirement can be met by (1) incorporating within the traverse a line whose true direction was established through a previous survey; (2) including one end of a line of known direction as a station in the traverse [e.g., station *A* of line *A-Az Mk* of Figure 9.1(a)], and then observing an angle from that reference line to a traverse line; or (3) determining the true direction of one traverse line by astronomical observations (see Appendix C), or by GNSS surveys (see Chapters 13, 14, and 15).

If a line of known direction exists within the traverse, computation of preliminary azimuths (or bearings) proceeds as discussed in Chapter 7. Angles adjusted to the proper geometric total must be used; otherwise the azimuth or bearing of the first line, when recomputed after using all angles and progressing around the traverse, will differ from its fixed value by the angular misclosure. Azimuths or bearings at this stage are called "preliminary" because they will change after the traverse is adjusted, as explained in Section 10.11. It should also be noted that since the azimuth of the courses will change, so will the angles, which were previously adjusted.

■ Example 10.2

Compute preliminary azimuths for the traverse courses of Figure 10.1, based on a fixed azimuth of 234°17′18″ for line *AW*, a measured angle to the right of 151°52′24″ for *WAE*, and the angle adjustment by method 1 of Table 10.1.

TABLE 10.2 COMPUTATION OF PRELIMINARY AZIMUTH USING THE TABULAR METHOD

126°55′17″ = *AB*	+89°03′26″ + *D*
+180°	284°35′20″ = *DE*
306°55′17″ = *BA*	−180°
+231°23′41″ + *B*	104°35′20″ = *ED*
538°18′58″ − 360° = 178°18′58″ − *BC*	+101°34′22″ + *E*
−180°	206°09′42″ = *EA*
358°18′58″ = *CD*	−180°
+17°12′56″ + *C*	26°09′42″ = *AE*
375°31′54″ − 360° = 15°31′54″ = *CD*	+100°45′35″ + *A*
−180°	126°55′17″ = *AB*
195°31′54″ = *DC*	

Solution

Step 1: Compute the azimuth of course AB.

$$Az_{AB} = 234°17'18'' + 151°52'24'' + 100°45'35'' - 360° = 126°55'17''$$

Step 2: Using the tabular method discussed in Section 7.8, compute preliminary azimuths for the remaining lines. The computations for this example are shown in Table 10.2. Figure 10.2 demonstrates the computations for line BC. Note that the azimuth of AB was recalculated as a check at the end of the table.

■ 10.4 DEPARTURES AND LATITUDES

After balancing the angles and calculating preliminary azimuths (or bearings), traverse closure is checked by computing the *departure* and *latitude* of each line. As illustrated in Figure 10.3, the departure of a course is its orthographic projection on the east-west axis of the survey and is equal to the length of the course

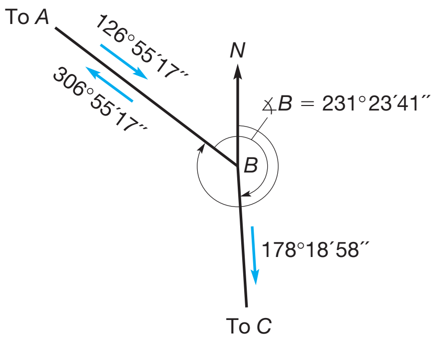

Figure 10.2 Computation of azimuth BC.

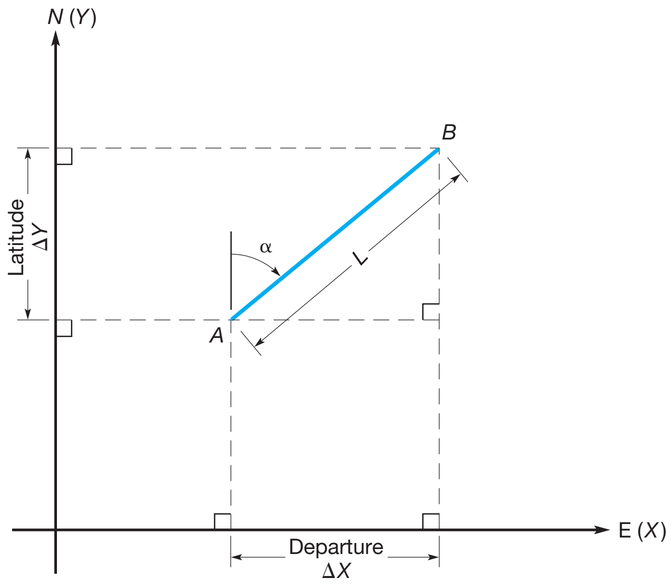

Figure 10.3 Departure and latitude of a line.

multiplied by the sine of its azimuth (or bearing) angle. Departures are sometimes called *eastings* or *westings*.

Also as shown in Figure 10.3, the latitude of a course is its orthographic projection on the north-south axis of the survey, and is equal to the course length multiplied by the cosine of its azimuth (or bearing) angle. Latitude is also called *northing* or *southing*.

In equation form, the departure and latitude of a line are

$$\text{departure} = L \sin \alpha \tag{10.1}$$

$$\text{latitude} = L \cos \alpha \tag{10.2}$$

where L is the horizontal length and α the azimuth of the course. Departures and latitudes are merely changes in the X and Y components of a line in a rectangular grid system, sometimes referred to as ΔX and ΔY. In traverse calculations, east departures and north latitudes are considered plus; west departures and south latitudes, minus. Azimuths (from north) used in computing departures and latitudes range from 0 to 360°, and the algebraic signs of sine and cosine functions automatically produce the proper algebraic signs of the departures and latitudes. Thus, a line with an azimuth of 126°55′17″ has a positive departure and negative latitude (the sine at the azimuth is plus and the cosine minus); a course of 284°35′20″ azimuth has a negative departure and positive latitude. In using bearings for computing departures and latitudes, the angles are always between 0 and 90°; hence their sines and cosines are invariably positive. Proper algebraic signs of departures and latitudes must therefore be assigned on the basis of the bearing angle directions, so a *NE* bearing has a plus departure and latitude, a *SW* bearing gets a minus departure and latitude, and so on. Because computers and hand calculators automatically affix correct algebraic signs to departures and latitudes through the use of azimuth angle sines and cosines, it is more convenient to use azimuths than bearings for traverse computations.

■ 10.5 DEPARTURE AND LATITUDE CLOSURE CONDITIONS

For a closed-polygon traverse like that of Figure 10.1, it can be reasoned that if all angles and distances were measured perfectly, the algebraic sum of the departures of all courses in the traverse should equal zero. Likewise, the algebraic sum of all latitudes should equal zero. And for closed link-type traverses like that of Figure 9.1(b), the algebraic sum of departures should equal the total difference in departure (ΔX) between the starting and ending control points. The same condition applies to latitudes (ΔY) in a link traverse. Because the observations are not perfect and errors exist in the angles and distances, the conditions just stated rarely occur. The amounts by which they fail to be met are termed *departure misclosure* and *latitude misclosure*. Their values are computed by algebraically summing the departures and latitudes, and comparing the totals to the required conditions.

The magnitudes of the departure and latitude misclosures for closed-polygon-type traverses give an "indication" of the precision that exists in the observed angles and distances. Large misclosures certainly indicate that either significant errors or even mistakes exist. Small misclosures usually mean the observed data are precise and free of mistakes, but it is not a guarantee that systematic or compensating errors do not exist.

■ 10.6 TRAVERSE LINEAR MISCLOSURE AND RELATIVE PRECISION

Because of errors in the observed traverse angles and distances, if one were to begin at point A of a closed-polygon traverse like that of Figure 10.1, and progressively follow each course for its observed distance along its preliminary bearing or azimuth, one would finally return not to point A, but to some other nearby point A'. Point A' would be removed from A in an east-west direction by the departure misclosure, and in a north-south direction by the latitude misclosure. The distance between A and A' is termed the linear misclosure of the traverse. It is calculated from the following formula:

$$\text{linear misclosure} = \sqrt{(\text{departure misclosure})^2 + (\text{latitude misclosure})^2} \quad \textbf{(10.3)}$$

The *relative precision* of a traverse is expressed by a fraction that has the linear misclosure as its numerator and the traverse perimeter or total length as its denominator, or

$$\text{relative precision} = \frac{\text{linear misclosure}}{\text{traverse length}} \quad \textbf{(10.4)}$$

The fraction that results from Equation (10.4) is then reduced to reciprocal form, and the denominator rounded to the same number of significant figures as the numerator. This is illustrated in the following example.

■ Example 10.3

Based on the preliminary azimuths from Table 10.2 and lengths shown in Figure 10.1, calculate the departures and latitudes, linear misclosure, and relative precision of the traverse.

TABLE 10.3 COMPUTATION OF DEPARTURES AND LATITUDES

Station	Preliminary Azimuths	Length	Departure	Latitude
A	126°55′17″	647.25	517.451	−388.815
B	178°18′58″	203.03	5.966	−202.942
C	15°31′54″	720.35	192.889	694.045
D	284°35′20″	610.24	−590.565	153.708
E	206°09′42″	285.13	−125.715	−255.919
A		Σ = 2466.00	Σ = 0.026	Σ = 0.077

Solution

In computing departures and latitudes, the data and results are usually listed in a standard tabular form, such as that shown in Table 10.3. The column headings and rulings save time and simplify checking.

In Table 10.3, taking the algebraic sum of east (+) and west (−) departures gives the misclosure, 0.026 ft. Also, summing north (+) and south (−) latitudes gives the misclosure in latitude, 0.077 ft. Linear misclosure is the hypotenuse of a small triangle with sides of 0.026 ft and 0.077 ft, and in this example its value is, by Equation (10.3)

$$\text{linear misclosure} = \sqrt{(0.026)^2 + (0.077)^2} = 0.081 \text{ ft}$$

The relative precision for this traverse, by Equation (10.4), is

$$\text{relative precision} = \frac{0.081}{2466.00} = \frac{1}{30{,}000}$$

10.7 TRAVERSE ADJUSTMENT

For any closed traverse, the linear misclosure must be adjusted (or distributed) throughout the traverse to "close" or "balance" the figure. This is true even though the misclosure is negligible in plotting the traverse at map scale. There are several elementary methods available for traverse adjustment, but the one most commonly used is the *compass rule* (Bowditch method). As noted earlier, adjustment by least squares is a more advanced technique that can also be used. These two methods are discussed in the subsections that follow.

10.7.1 Compass (Bowditch) Rule

The compass, or Bowditch, rule adjusts the departures and latitudes of traverse courses in proportion to their lengths. Although not as rigorous as the least-squares method, it does result in a logical distribution of misclosures. Corrections by this method are made according to the following rules:

$$\text{correction in departure for } AB = -\frac{\text{(total departure misclosure)}}{\text{traverse perimeter}}\text{length of } AB \qquad \textbf{(10.5)}$$

$$\text{correction in latitude for } AB = -\frac{\text{(total latitude misclosure)}}{\text{traverse perimeter}}\text{length of } AB \qquad \textbf{(10.6)}$$

Note that the algebraic signs of the corrections are opposite those of the respective misclosures.

Example 10.4

Using the preliminary azimuths from Table 10.2 and lengths from Figure 10.1, compute departures and latitudes, linear misclosure, and relative precision. Balance the departures and latitudes using the compass rule.

Solution

A tabular solution, which is somewhat different than that used in Example 10.3, is employed for computing departures and latitudes (see Table 10.4). To compute departure and latitude corrections by the compass rule, Equations (10.5) and (10.6) are used as demonstrated. By Equation (10.5) the correction in departure for AB is

$$-\left(\frac{0.026}{2466}\right)647.25 = -0.007 \text{ ft}$$

And by Equation (10.6) the correction for the latitude of AB is

$$-\left(\frac{0.077}{2466}\right)647.25 = -0.020 \text{ ft}$$

The other corrections are likewise found by multiplying a constant—the ratio of misclosure in departure, and latitude, to the perimeter—by the successive course lengths.

In Table 10.4, the departure and latitude corrections are shown in parentheses above their unadjusted values. These corrections are added algebraically to their respective unadjusted values, and the corrected quantities tabulated in the "balanced" departure and latitude columns. A check is made of the computational process by algebraically summing the balanced departure and latitude columns to verify that each is zero. In these columns, if rounding off causes a small excess or deficiency, revising one of the corrections to make the closure perfect eliminates this.

TABLE 10.4 BALANCING DEPARTURES AND LATITUDES BY THE COMPASS (BOWDITCH) RULE

			Unadjusted		Balanced		Coordinates*	
Station	**Preliminary Azimuths**	**Length (ft)**	**Departure**	**Latitude**	**Departure**	**Latitude**	***X* (ft) (easting)**	***Y* (ft) (northing)**
A			(−0.007)	(−0.020)			10,000.00	5000.00
	126°55′17″	647.25	517.451	−388.815	517.444	−388.835		
B			(−0.002)	(−0.006)			10,517.44	4611.16
	178°18′58″	203.03	5.966	−202.942	5.964	−202.948		
C			(−0.008)	(−0.023)			10,523.41	4408.22
	15°31′54″	720.35	192.889	694.045	192.881	694.022		
D			(−0.006)	(−0.019)			10,716.29	5102.24
	284°35′20″	610.24	−590.565	153.708	−590.571	153.689		
E			(−0.003)	(−0.009)			10,125.72	5255.93
	206°09′42″	285.13	−125.715	−255.919	−125.718	−255.928		
A							10,000.00✓	5000.00✓
		Σ = 2466.00	Σ = 0.026	Σ = 0.077	Σ = 0.000	Σ = 0.000		

$$\text{Linear precision} = \sqrt{(0.026)^2 + (-0.077)^2} = 0.081 \text{ ft}$$

$$\text{Relative precision} = \frac{0.081}{2466} = \frac{1}{30{,}000}$$

*Coordinates are rounded to same significance as observed lengths.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *Latitudes and Departures.mp4* demonstrates the computation and adjustment for the traverse shown in Figure 10.1.

10.7.2 Least-Squares Method

As noted in Section 3.21, the method of least squares is based on the theory of probability, which models the occurrence of random errors. This results in adjusted values having the highest probability. Thus the least-squares method provides the best and most rigorous traverse adjustment, but until recently the method has not been widely used because of the lengthy computations required. The availability of computers has now made these calculations routine, and consequently the least-squares method has gained popularity.

In applying the least-squares method to traverses, angle and distance observations are adjusted simultaneously. Thus no preliminary angle adjustment is made, as is done when using the compass rule. The least-squares method is valid for any type of traverse, and has the advantage that observations of varying precisions can be weighted appropriately in the computations. Examples illustrating some elementary least-squares adjustments are presented in Chapter 16.

10.8 RECTANGULAR COORDINATES

Rectangular X and Y coordinates of any point give its position with respect to an arbitrarily selected pair of mutually perpendicular reference axes. The X coordinate is the perpendicular distance, in feet or meters, from the point to the Y axis; the Y coordinate is the perpendicular distance to the X axis. Although the reference axes are discretionary in position, in surveying they are normally oriented so that the Y axis points north-south, with north the positive Y direction. The X axis runs east-west, with positive X being east. Given the rectangular coordinates of a number of points, their relative positions are uniquely defined.

Coordinates are useful in a variety of computations, including (1) determining lengths and directions of lines, and angles (see Section 10.11 and Chapter 11); (2) calculating areas of land parcels (see Section 12.5); (3) making certain curve calculations (see Sections 24.12 and 24.13); and (4) locating inaccessible points (see Section 11.9). Coordinates are also advantageous for plotting maps (see Section 18.8.1).

In practice, *state plane coordinate systems*, as described in Chapter 20, are most frequently used as the basis for rectangular coordinates in plane surveys. However for many calculations, any arbitrary system may be used. As an example, coordinates may be arbitrarily assigned to one traverse station. For example, to avoid negative values of X and Y an origin is assumed south and west of the traverse such that one hub has coordinates $X = 10{,}000.00$, $Y = 5{,}000.00$, or any other suitable values. In a closed traverse, assigning $Y = 0.00$ to the most southerly point and $X = 0.00$ to the most westerly station saves time in hand calculations.

Given the X and Y coordinates of any starting point A, the X coordinate of the next point B is obtained by adding the adjusted departure of course AB to X_A.

Likewise, the Y coordinate of B is the adjusted latitude of AB added to Y_A. In equation form this is

$$\begin{aligned} X_B &= X_A + \text{departure } AB \\ Y_B &= Y_A + \text{latitude } AB \end{aligned} \tag{10.7}$$

For closed polygons, the process is continued around the traverse, successively adding departures and latitudes until the coordinates of starting point A are recalculated. If these recalculated coordinates agree exactly with the starting ones, a check on the coordinates of all intermediate points is obtained (unless compensating mistakes have been made). For link traverses, after progressively computing coordinates for each station, if the calculated coordinates of the closing control point equal that point's control coordinates, a check is obtained.

Example 10.5

Using the balanced departures and latitudes obtained in Example 10.4 (see Table 10.4) and starting coordinates $X_A = 10{,}000.00$ and $Y_A = 5{,}000.00$, calculate coordinates of the other traverse points.

Solution

The process of successively adding balanced departures and latitudes to obtain coordinates is carried out in the two rightmost columns of Table 10.4. Note that the starting coordinates $X_A = 10{,}000.00$ and $Y_A = 5{,}000.00$ are recomputed at the end to provide a check. Note also that X and Y coordinates are frequently referred to as *eastings* and *northings*, respectively, as is indicated in Table 10.4.

10.9 ALTERNATIVE METHODS FOR MAKING TRAVERSE COMPUTATIONS

Procedures for making traverse computations that vary somewhat from those described in preceding sections can be adopted. One alternative is to adjust azimuths or bearings rather than angles. Another is to apply compass rule corrections directly to coordinates. These procedures are described in the subsections that follow.

10.9.1 Balancing Angles by Adjusting Azimuths or Bearings

In this method, "unadjusted" azimuths or bearings are computed based on the observed angles. These azimuths or bearings are then adjusted to secure a geometric closure, and to obtain preliminary values for use in computing departures and latitudes. The method is equally applicable to closed-polygon traverses, like that of Figure 10.1, or to closed-link traverses, as shown in Figure 9.1(b) that begins on one control station and ends on another. The procedure of making the adjustment for angular misclosure in this manner will be explained by an example.

Example 10.6

Table 10.5 lists observed angles to the right for the traverse of Figure 9.1(b). The azimuths of lines A-$Az\,Mk_1$ and E-$Az\,Mk_2$ have known values of 139°05′45″ and 86°20′47″, respectively. Compute unadjusted azimuths and balance them to obtain geometric closure.

Solution

From the observed angles of column (2) in Table 10.5, unadjusted azimuths have been calculated and are listed in column (3). Because of angular errors, the unadjusted azimuth of the final line E-$Az\,Mk_2$ disagrees with its fixed value by 0°00′10″. This represents the angular misclosure, which is divided by 5, the number of observed angles, to yield a correction of −2″ per angle. The corrections to azimuths, which accumulate and increase by −2″ for each angle, are listed in column (4). Thus line AB, which is based on one observed angle, receives a −2″ correction; line BC which uses two observed angles, gets a −4″ correction; and so

TABLE 10.5 BALANCING TRAVERSE AZIMUTHS

Station (1)	Measured Angle* (2)	Unadjusted Azimuth (3)	Azimuth Correction (4)	Preliminary Azimuth (5)
Az MK$_1$				
		319°05′45″		319°05′45″
A	283°50′10″			
		62°55′55″	−2″	62°55′53″
B	256°17′18″			
		139°13′13″	−4″	139°13′09″
C	98°12′36″			
		57°25′49″	−6″	57°25′43″
D	103°30′34″			
		340°56′23″	−8″	340°56′15″
E	285°24′34″			
		86°20′57″	−10″	86°20′47″
Az Mk$_2$				

$$\begin{array}{r} 86°20'57'' \\ -\underline{86°20'47''} \\ \text{misclosure} = 0°00'10'' \\ \text{correction per angle} = -10''/5 = -2'' \end{array}$$

*Observed angles are angles to the right.

on. The final azimuth, $E\text{-}Az\,Mk_2$, receives a $-10''$ correction because all five observed angles have been included in its calculation. The corrected preliminary azimuths are listed in column 5.

10.9.2 Balancing Departures and Latitudes by Adjusting Coordinates

In this procedure, commencing with the known coordinates of a beginning station, unadjusted departures and latitudes for each course are successively added to obtain "preliminary" coordinates for all stations. For closed-polygon traverses, after progressing around the traverse, preliminary coordinates are recomputed for the beginning station. The difference between the computed preliminary X coordinate at this station and its known X coordinate is the departure misclosure. Similarly, the disagreement between the computed preliminary Y coordinate for the beginning station and its known value is the latitude misclosure. Corrections for these misclosures can be calculated using compass-rule Equations (10.5) and (10.6) and applied directly to the preliminary coordinates to obtain adjusted coordinates. The result is exactly the same as if departures and latitudes were first adjusted and coordinates computed from them, as was done in Examples 10.4 and 10.5.

Closed traverses like the one shown in Figure 9.1(b) can be similarly adjusted. For this type of traverse, unadjusted departures and latitudes are also successively added to the beginning station's coordinates to obtain preliminary coordinates for all points, including the final closing station. Differences in preliminary X and Y coordinates, and the corresponding known values for the closing station, represent the departure and latitude misclosures, respectively. These misclosures are distributed directly to preliminary coordinates using the compass rule to obtain final adjusted coordinates. The procedure will be demonstrated by an example.

Example 10.7

Table 10.6 lists the preliminary azimuths (from Table 10.5) and observed lengths (in feet) for the traverse of Figure 9.1(b). The known coordinates of stations A and E are $X_A = 12{,}765.48, Y_A = 43{,}280.21, X_E = 14{,}797.12$, and $Y_E = 44{,}384.51$ ft. Adjust this traverse for departure and latitude misclosures by making corrections to preliminary coordinates.

Solution

From the lengths and azimuths listed in columns (2) and (3) of Table 10.6, departures and latitudes are computed and tabulated in columns (4) and (5). These unadjusted values are progressively added to the known coordinates of station A to obtain preliminary coordinates for all stations, including E, and are listed in columns (6) and (7). Comparing the preliminary X and Y coordinates of station E with its known values yields departure and latitude misclosures of $+0.179$ and -0.024 ft, respectively. From these values, the linear misclosure of 0.181 ft and relative precision of 1/21,000 are computed (see Table 10.6).

TABLE 10.6 TRAVERSE ADJUSTMENT BY COORDINATES

Station (1)	Length (ft) (2)	Preliminary Azimuth (3)	Departure (4)	Latitude (5)	Preliminary Coordinates (ft) X (6)	Preliminary Coordinates (ft) Y (7)	Corrections (ft) X (8)	Corrections (ft) Y (9)	Adjusted Coordinates* X (ft) (10)	Adjusted Coordinates* Y (ft) (11)
A					12,765.48	43,280.21			12,765.48	43,280.21
	1045.50	62°55′53″	930.978	475.762			−0.048	0.006		
B					13,696.458	43,755.972	(−0.048)	(0.006)	13,696.41	43,755.98
	1007.38	139°13′09″	657.988	−762.802			−0.046	0.006		
C					14,354.446	42,993.170	(−0.094)	(0.012)	14,354.35	42,993.18
	897.81	57°25′43″	756.604	483.336			−0.041	0.006		
D					15,111.050	43,476.506	(−0.135)	(0.018)	15,110.92	43,476.52
	960.66	340°56′15″	−313.751	907.980			−0.044	0.006		
E					14,797.299	44,384.486	(−0.179)	(0.024)	14,797.12✓	44,384.51✓
	Σ = 3911.35				−14,797.12	−44,384.51				
				Misclosures	+0.179	−0.024				

$$\text{Linear precision} = \sqrt{(0.179)^2 + (-0.024)^2} = 0.181 \text{ ft}$$

$$\text{Relative precision} = \frac{0.181}{3911} = \frac{1}{21{,}000}$$

*Adjusted coordinates are rounded to same significance as observed lengths.

Compass-rule corrections for each course are computed and listed in columns (8) and (9). Their cumulative values obtained by progressively adding the corrections are given in parentheses in columns (8) and (9). Finally, by applying the cumulative corrections to the preliminary coordinates of columns 6 and 7, final adjusted coordinates (rounded to the nearest hundredth of a foot) listed in columns (10) and (11) are obtained.

10.10 INVERSING

If the departure and latitude of a line AB are known, its length and azimuth or bearing are readily obtained from the following relationships:

$$\tan \text{azimuth (or bearing) } AB = \frac{\text{departure } AB}{\text{latitude } AB} \tag{10.8}$$

$$\begin{aligned} \text{length } AB &= \frac{\text{departure } AB}{\text{sin azimuth (or bearing) } AB} \\ &= \frac{\text{latitude } AB}{\text{cos azimuth (or bearing) } AB} \\ &= \sqrt{(\text{departure } AB)^2 + (\text{latitude } AB)^2} \end{aligned} \tag{10.9}$$

Equations (10.7) can be written to express departures and latitudes in terms of coordinate differences ΔX and ΔY as follows:

$$\begin{aligned} \text{departure}_{AB} &= X_B - X_A = \Delta X \\ \text{latitude}_{AB} &= Y_B - Y_A = \Delta Y \end{aligned} \tag{10.10}$$

Substituting Equations (10.10) into Equations (10.8) and (10.9)

$$\tan \text{azimuth (or bearing) } AB = \frac{X_B - X_A}{Y_B - Y_A} = \frac{\Delta X}{\Delta Y} \tag{10.11}$$

$$\begin{aligned} \text{length } AB &= \frac{X_B - X_A (\text{or } \Delta X)}{\text{sin azimuth (or bearing) } AB} \\ &= \frac{Y_B - Y_A (\text{or } \Delta Y)}{\text{cos azimuth (or bearing) } AB} \\ &= \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \\ &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \end{aligned} \tag{10.12}$$

Equations (10.8) through (10.12) can be applied to any line whose coordinates are known, whether or not it was actually observed in the survey. Note that X_B and Y_B

must be listed first in Equations (10.11) and (10.12), so that ΔX and ΔY will have the correct algebraic signs. Computing lengths and directions of lines from departures and latitudes, or from coordinates, is called *inversing*.

10.11 COMPUTING FINAL ADJUSTED TRAVERSE LENGTHS AND DIRECTIONS

In traverse adjustments, as illustrated in Examples 10.4 and 10.7, corrections are applied to the computed departures and latitudes to obtain adjusted values. These in turn are used to calculate X and Y coordinates of the traverse stations. By changing departures and latitudes of lines in the adjustment process, their lengths and azimuths (or bearings) also change. In many types of surveys, it is necessary to compute the changed, or "final adjusted," lengths and directions. For example, if the purpose of the traverse was to describe the boundaries of a parcel of land, the final adjusted lengths and directions would be used in the recorded deed.

The equations developed in the preceding section permit computation of final values for lengths and directions of traverse lines based either on their adjusted departures and latitudes or on their final coordinates.

Example 10.8

Calculate the final adjusted lengths and azimuths of the traverse of Example 10.4 from the adjusted departures and latitudes listed in Table 10.7.

Solution

Equations (10.8) and (10.9) are applied to calculate the adjusted length and azimuth of line AB. All others were computed in the same manner. The results are listed in Table 10.7.

TABLE 10.7 FINAL ADJUSTED LENGTHS AND DIRECTIONS FOR TRAVERSE OF EXAMPLE 10.4

	Balanced		Balanced	
Line	**Departure**	**Latitude**	**Length (ft)**	**Azimuth**
AB	517.444	−388.835	647.26	126°55′23″
BC	5.964	−202.948	203.04	178°19′00″
CD	192.881	694.022	720.33	15°31′54″
DE	−590.571	153.689	610.24	284°35′13″
EA	−125.718	−255.928	285.14	206°09′41″

By Equation (10.8)

$$\tan \text{azimuth}_{AB} = \frac{517.444}{-388.835} = -1.330755;$$

$$\text{azimuth}_{AB} = -53°04'37'' + 180° = 126°55'23''$$

By Equation (10.9)

$$\text{length}_{AB} = \sqrt{(517.444)^2 + (-388.835)^2} = 647.26 \text{ ft}$$

Comparing the observed lengths of Table 10.4 to the final adjusted values in Table 10.7, it can be seen that, as expected, the values have undergone small changes, some increasing, others decreasing, and length DE remaining the same because of compensating changes.

Example 10.9

Using coordinates, calculate adjusted lengths and azimuths for the traverse of Example 10.7 (see Table 10.6).

Solution

Equations (10.11) and (10.12) are used to demonstrate calculation of the adjusted length and azimuth of line AB. All others were computed in the same way. The results are listed in Table 10.8. Comparing the adjusted lengths and azimuths of this table with their unadjusted values of Table 10.6 reveals that all values have undergone changes of varying amounts.

$$X_B - X_A = 13{,}696.41 - 12{,}765.48 = 930.93 = \Delta X$$

$$Y_B - Y_A = 43{,}755.98 - 43{,}280.21 = 475.77 = \Delta Y$$

By Equation (10.11) $\tan \text{azimuth}_{AB} = 930.93/475.77 = 1.95668075$; $\text{azimuth}_{AB} = 62°55'47''$.

By Equation (10.12), $\text{length}_{AB} = \sqrt{(930.93)^2 + (475.77)^2} = 1045.46$ ft.

TABLE 10.8 FINAL ADJUSTED LENGTHS AND DIRECTIONS FOR TRAVERSE OF EXAMPLE 10.7

	Adjusted		Adjusted	
Line	ΔX	ΔY	Length (ft)	Azimuth
AB	930.93	475.77	1045.46	62°55′47″
BC	657.94	−762.80	1007.35	139°13′16″
CD	756.57	483.34	897.78	57°25′38″
DE	−313.80	907.99	960.68	340°56′06″

TABLE 10.9 FINAL ADJUSTED ANGLES FOR EXAMPLE 10.4

Angle	Foresight Azimuth	Backsight Azimuth	Adjusted Angle	Difference
A (EAB)	$AB = 126°55'23''$	$AE = 26°09'41''$	100°45′42″	7″
B (ABC)	$BC = (178°19'00'' + 360°)$	$BA = 306°55'23''$	231°23′37″	−4″
C (BCD)	$CD = (15°31'54'' + 360°)$	$CB = (178°19'00'' + 180°)$	17°12′54″	−2″
D (CDE)	$DE = 284°35'13''$	$DC = (15°31'54'' + 180°)$	89°03′19″	−7″
E (DEA)	$EA = 206°09'41''$	$ED = (284°35'13'' - 180°)$	101°34′28″	6″
			$\sum = 540°00'00''$	$\sum = 0''$

Because the final adjusted azimuths are different from their preliminary values, the preliminary adjusted angles have also changed. The backsight azimuth must be subtracted from the foresight azimuth to compute the final adjusted angles. A method of listing both the backsight and foresight stations for each angle helps in determining which azimuths should be subtracted. For example, the angle at A in Figure 10.1 is listed as EAB, where E is the backsight station and B is the foresight station for the clockwise interior angle. As a pneumonic, angle A is computed as the difference in azimuths AB and AE, where Az_{AB} is the foresight azimuth of angle A and Az_{AE} is the backsight azimuth. Thus, the angle at A is computed as

$$\begin{aligned}\angle EAB &= Az_{AB} - Az_{AE} \\ &= 126°55'23'' - (206°09'41'' - 180°) \\ &= 100°45'42''\end{aligned}$$

Notice in this example that the back azimuth of EA from Table 10.7 was needed for the backsight, and thus 180° was subtracted from azimuth EA. Also note that the final adjusted value for the angle at A differs from the preliminary adjusted value by 7″. The final adjusted angles for remainder of the traverse are shown in Table 10.9. For each angle the appropriate three-letter designator, which defines the clockwise interior angle, is shown in parentheses. Table 10.8 also shows the appropriate foresight and backsight azimuths and the final adjusted angle at each station. Notice that the sum of the angles again achieves geometric closure with a value of 540°. However, each angle differs from the value given in Table 10.1 by the amount shown in the last column.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *Traverse Computations II.mp4* demonstrates the computations of the adjusted observations for the traverse shown in Figure 10.1.

10.12 COORDINATE COMPUTATIONS IN BOUNDARY SURVEYS

Computation of a bearing from the known coordinates of two points on a line is commonly done in boundary surveys. If the lengths and directions of lines from traverse points to the corners of a field are known, the coordinates of the corners can be determined and the lengths and bearings of all sides calculated.

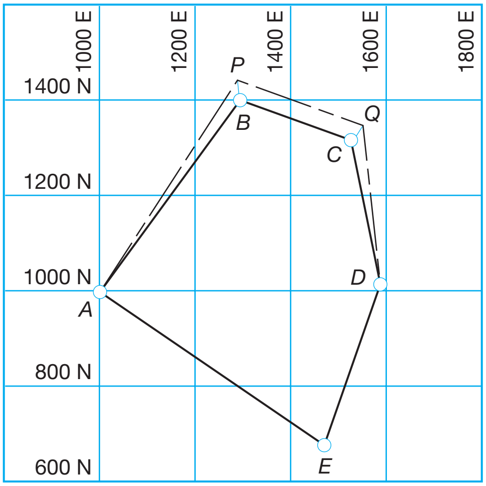

Figure 10.4
Plot of traverse for a boundary survey.

Example 10.10

In Figure 10.4, *APQDEA* is a parcel of land that must be surveyed, but because of obstructions, traverse stations cannot be set at *P* and *Q*. Therefore offset stations *B* and *C* are set nearby, and closed traverse *ABCDE* run. Lengths and azimuths of lines *BP* and *CQ* are observed as 42.50 ft, 354°50′00″ and 34.62 ft, 26°39′54″, respectively. Following procedures demonstrated in earlier examples, traverse *ABCEA* was computed and adjusted, and coordinates were determined for all stations. They are given in the following table.

Point	*X* (ft)	*Y* (ft)
A	1000.00	1000.00
B	1290.65	1407.48
C	1527.36	1322.10
D	1585.70	1017.22
E	1464.01	688.25

Compute the length and bearing of property line *PQ*.

Solution

1. Using Equations (10.1) and (10.2), the departures and latitudes of lines *BP* and *CQ* are:

$$\text{Dep}_{BP} = 42.50 \sin(354°50'00'') = -3.83 \text{ ft}$$
$$\text{Dep}_{CQ} = 34.62 \sin(26°39'54'') = 15.54 \text{ ft}$$

$$\text{Lat}_{BP} = 42.50 \cos(354°50'00'') = 42.33 \text{ ft}$$
$$\text{Lat}_{CQ} = 34.62 \cos(26°39'54'') = 30.94 \text{ ft}$$

2. From the coordinates of stations B and C and the departures and latitudes just calculated, the following tabular solution yields X and Y coordinates for points P and Q:

	X	Y		X	Y
B	1290.65	1407.48	C	1527.36	1322.10
BP	−3.83	+42.33	CQ	+15.54	+30.94
P	1286.82	1449.81	Q	1542.90	1353.04

3. From the coordinates of P and Q, the length and bearing of line PQ are found in the following manner:

	X	Y
Q	1542.90	1353.04
P	−1286.88	−1449.81
PQ	$\Delta X = 256.02$	$\Delta Y = -96.77$

By Equation (10.11), tan bearing$_{PQ}$ = 256.02/−96.77 = −2.64565; bearing$_{PQ}$ = S69°17′40″E

By Equation (10.12), length $PQ = \sqrt{(-96.77)^2 + (256.02)^2} = 273.79\,\text{ft}$

By using Equations (10.11) and (10.12), lengths and bearings of lines AP and QD can also be determined. As stated earlier, extreme caution must be used when employing this procedure, since no checks are obtained on the length and azimuth measurements of lines BP and CQ, nor are there any computational checks on the calculated lengths and bearings.

10.13 USE OF OPEN TRAVERSES

Although open traverses should be used with reluctance, sometimes there are situations where it is very helpful to run one and then compute the length and direction of the "closing line." In Figure 10.5, for example, suppose that improved horizontal alignment is planned for Taylor Lake and Atkins Roads, and a new construction line AE must be laid out. Because of dense forest, visibility between points A and E is not possible. A random line (see Section 8.17) could be run from A toward E and then corrected to the desired line, but that would be very difficult and time consuming due to tree density. One solution to this problem is to run open traverse $ABCDE$, which can be done quite easily along the cleared right-of-way of existing roads.

For this problem an assumed azimuth (e.g., due north) can be taken for line UA, and assumed coordinates (e.g., 10,000.00 and 10,000.00) can be assigned

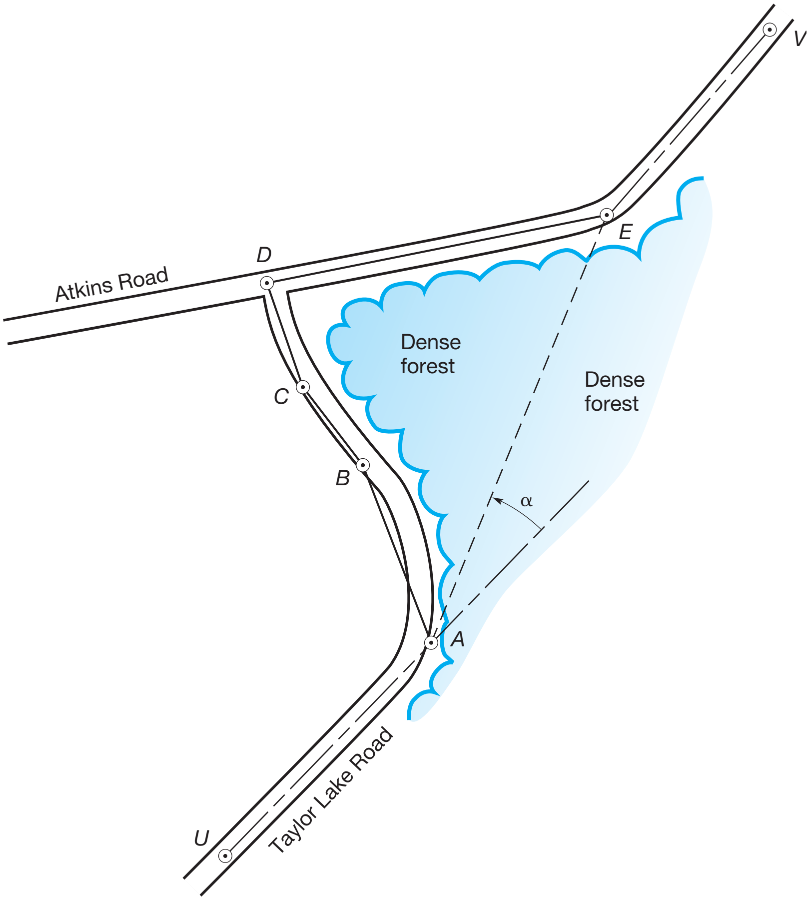

Figure 10.5
Closing line of an open traverse.

to station A. From observed lengths and angles, departures and latitudes of all lines, and coordinates of all points can be computed. From the resulting coordinates of stations A and E, the length and azimuth of closing line AE can be calculated. Finally the deflection angle α needed to reach E from A can be computed and laid off.

In running open traverses, extreme caution must be exercised in all observations, because there is no check, and any errors or mistakes will result in an erroneous length and direction for the closing line. Procedures such as closing the horizon and observing the lengths of the lines from both ends of the lines should be practiced so that independent checks on all observations are obtained. Utmost care must also be exercised in the calculations, although carefully plotting the traverse and scaling the length of the closing line and the deflection angle can secure a rough check on them.

Point	Length (ft)	Angle to the Right
A		115°18′25″
	3305.78	
B		161°24′11″
	1862.40	
C		204°50′09″
	1910.22	
D		273°46′37″
	6001.83	
E		

Example 10.11

Compute the length and azimuth of closing line AE and deflection angle α of Figure 10.5, given the following observed data:

Solution

Table 10.10 presents a tabular solution for computing azimuths, departures and latitudes, and coordinates.

From the coordinates of points A and E, the ΔX and ΔY values of line AE are

$$\Delta X = 7{,}004.05 - 10{,}000.00 = -2{,}995.95 \text{ ft}$$
$$\Delta Y = 17{,}527.05 - 10{,}000.00 = 7{,}527.05 \text{ ft}$$

By Equation (10.12), the length of closing line AE is

$$\text{length}_{AE} = \sqrt{(-2995.95)^2 + (7527.05)^2} = 8101.37 \text{ ft}$$

TABLE 10.10 COMPUTATIONS FOR CLOSING LINE

Point	Azimuth	Departure	Latitude	X (ft)	Y (ft)
U					
	North (assumed)				
A				10,000.00	10,000.00
	295°18′25″	−2988.53	1413.11		
B				7011.47	11,413.11
	276°42′36″	−1849.64	217.61		
C				5161.83	11,630.72
	301°32′45″	−1627.93	999.39		
D				3533.90	12,630.11
	35°19′22″	3470.15	4896.94		
E				7004.05	17,527.05

By Equation (10.11), the azimuth of closing line AE is

$$\tan \text{azimuth}_{AE} = \frac{-2995.95}{7527.05} = -0.39802446;\ \text{azimuth}_{AE} = 338°17'46''$$

(Note that with a negative ΔX and positive ΔY the bearing of AE is northwest, hence the azimuth is 338°17′46″.)

Finally, deflection angle α is the difference between the azimuths of lines AE and UA, or

$$-\alpha = 338°17'46'' - 360° = -21°42'14''(\text{left})$$

With the emergence of GNSS, problems like that illustrated in Example 10.11 will no longer need to be solved using open traverses. Instead, receivers could be set at points U, A, and E of Figure 10.5, and their coordinates determined. From these coordinates the azimuths of lines UA and AE can be calculated, as well as angle α.

■ 10.14 STATE PLANE COORDINATE SYSTEMS

Under ordinary circumstances, rectangular coordinate systems for plane surveys would be limited in size due to earth curvature. However, the National Geodetic Survey (NGS) developed statewide coordinate systems for each state in the United States, which retain an accuracy of 1 part in 10,000 or better while fitting curved geodetic distances to plane grid lengths. However, if reduction of observations is properly performed (see Section 20.8), no accuracy will be lost in the survey.

State plane coordinates are related to the geodetic coordinates of latitude and longitude, so control survey stations set by the NGS, as well as those set by others, can all be tied to the systems. As additional stations are set and their coordinates determined, they too become usable reference points in the state plane systems. These monumented control stations serve as starting points for local surveys, and permit accurate restoration of obliterated or destroyed marks having known coordinates. If state plane coordinates of two intervisible stations are known, like A and *Az Mk* of Figure 9.1(a), the direction of line *A-Az Mk* can be computed and used to orient the total station instrument at A. In this way, azimuths and bearings of traverse lines are obtained without the necessity of making astronomical observations or resorting to other means.

In the past, some cities and counties have used their own local plane coordinate systems for locating street, sewer, property, and other lines. Because of their limited extent and the resultant discontinuity at city or county lines, such local systems are less desirable than a statewide grid. Another plane coordinate system called the *Universal Transverse Mercator* (UTM) (see Section 20.12) is widely used to pinpoint the locations of objects by coordinates. The military and others use this system for a variety of purposes.

■ 10.15 TRAVERSE COMPUTATIONS USING COMPUTERS

Computers of various types and sizes are now widely used in surveying and are particularly convenient for making traverse computations. Small programmable handheld units, data collectors, and laptop computers are commonly taken into the field and used to verify data for acceptable misclosures before returning to the office. In the office, personal computers are widely used. A variety of software is available for use by surveyors. Some manufacturers supply standard programs, which include traverse computations, with the purchase of their equipment. Various softwares are also available for purchase from a number of suppliers. Spreadsheet software can also be conveniently used with personal computers to calculate and adjust traverses. Of course, surveying and engineering firms frequently write programs specifically for their own use. Standard programming languages employed include Fortran, Pascal, BASIC, C, and others.

A traverse computation program is provided in the software WOLFPACK on the companion website for this book at http://www.pearsonhighered.com/ghilani. It computes departures and latitudes, linear misclosure, and relative precision, and performs adjustments by the compass (Bowditch) rule. In addition, the program calculates coordinates of the traverse points and the area within polygon traverses using the coordinate method (discussed in Section 12.5). In Figure 10.6, the input and output files from WOLFPACK are shown for Example 10.4. For the data file of Figure 10.6, the information entered to the right of the numerical data is for explanation only and need not be included in the file. The format of any data file can be found in the accompanying help screen for the desired option.

Also on the companion website for this book, the Excel® file *C10.xls* demonstrates the traverse computations and for the data in Examples 10.4 and 10.6. For those interested in a higher-level programming language, Example 10.4 is computed in the Mathcad® worksheet *TRAV.XMCD*. This example is also demonstrated in the html file *Trav.html*.

Besides performing routine computations such as traverse solutions, personal computers have many other valuable applications in surveying and engineering offices. Two examples are their use with *computer-aided drafting* (CAD) software for plotting maps and drawing contours (see Section 18.14), and with increasing frequency they are also being employed to operate *geographic information system* (GIS) software (see Chapter 28).

■ 10.16 LOCATING BLUNDERS IN TRAVERSE OBSERVATIONS

A numerical or graphic analysis can often be used to determine the location of a mistake, and thereby save considerable field time in making necessary additional observations. For example, if the sum of the interior angles of a five-sided traverse

DATA FILE

```
Figure 10.1, Example 10.4 //title line
5 1  //number of courses; 1 = angles to the right; -1 = clockwise direction
126 55 17 //azimuth of first course in traverse; degrees minutes seconds
647.25 100 45 37 //first distance and angle at control station
203.03 231 23 43 //distance and angle for second course and station, respectively
720.35 17 12 59 //and so on
610.24 89 03 28
285.13 101 34 24
10000.00 5000.00 //coordinates of first control station
```

OUTPUT FILE

```
~~~~~~~~~~~~~~~~~~~~~~~~~~~ Traverse Computation ~~~~~~~~~~~~~~~~~~~~~~~~~~~
Title: Figure 10.1, Example 10.4 //title line Type: Polygon traverse

          Angle Summary
          Station   Unadj. Angle     Adj. Angle
          ---------------------------------------
              1     100°45'37.0"     100°45'34.8"
              2     231°23'43.0"     231°23'40.8"
              3      17°12'59.0"      17°12'56.8"
              4      89° 3'28.0"      89°03'25.8"
              5     101°34'24.0"     101°34'21.8"
          Angular misclosure (sec): 11"

                                           Unbalanced
Course      Length       Azimuth         Dep         Lat
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
 1-2        647.25    126°55'17.0"   517.451   -388.815
 2-3        203.03    178°18'57.8"     5.966   -202.942
 3-4        720.35     15°31'54.6"   192.891    694.044
 4-5        610.24    284°35'20.4"  -590.564    153.709
 5-1        285.13    206°09'42.2"  -125.716   -255.919
         ----------                 ---------  ---------
  Sum =   2,466.00                     0.028      0.077

            Balanced                           Coordinates
        Dep         Lat      Point         X              Y
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
      517.443   -388.835     1       10,000.00       5,000.00
        5.964   -202.949     2       10,517.44       4,611.16
      192.883    694.022     3       10,523.41       4,408.22
     -590.571    153.690     4       10,716.29       5,102.24
     -125.719   -255.928     5       10,125.72       5,255.93

     Linear misclosure  = 0.082
     Relative Precision = 1 in 30,200

     Area: 272,600 sq. ft.
            6.258 acres {if distance units are feet}

Adjusted Observations
~~~~~~~~~~~~~~~~~~~~~
    Course    Distance   Azimuth     Point      Angle
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
       1-2      647.26  126°55'24"      1  100°45'42"
       2-3      203.04  178°19'00"      2  231°23'37"
       3-4      720.33   15°31'54"      3   17°12'54"
       4-5      610.24  284°35'14"      4   89°03'20"
       5-6      285.14  206°09'41"      5  101°34'28"
```

Figure 10.6 Data file and output file of traverse computations using WOLFPACK.

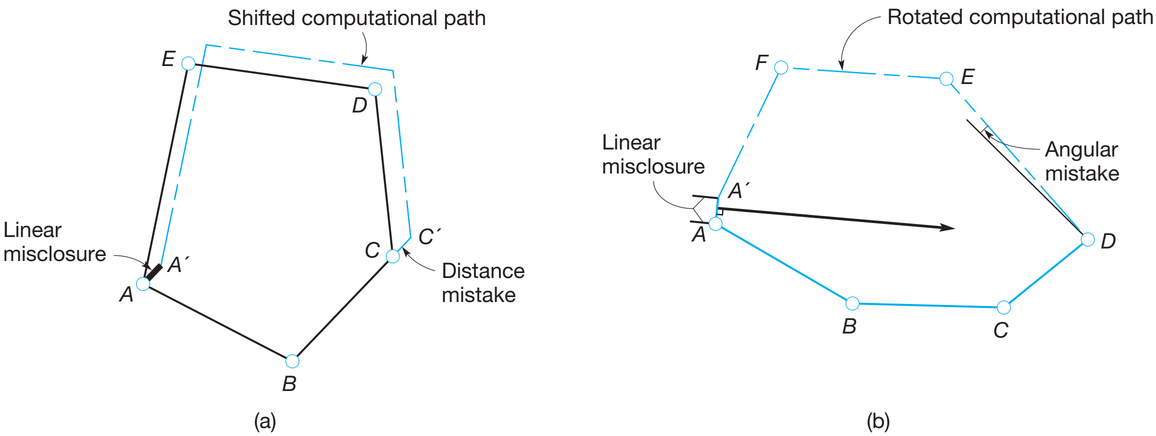

Figure 10.7 Locating a distance (a) or angle (b) blunder.

gives a large misclosure—say 10′11″—it is likely that one mistake of 10′ and several small errors accumulating to 11″ have been made. Methods of graphically locating the station or line where the mistake occurred are illustrated in Figure 10.7. The procedure is shown for a five-sided traverse but can be used for traverses having any number of sides.

In Figure 10.7(a), a blunder in the distance BC has occurred. Notice that the mistake CC' shifts the computed coordinates of the remaining stations in such a manner that the azimuth of the linear misclosure line closely matches the azimuth of the course BC that contains the mistake. If no other errors, random or systematic, occurred in the traverse, there would be a perfect match in the directions of the two lines. However since random errors are inevitable, the direction of the course containing the mistake and that of the linear misclosure line never matches perfectly, but will be close.

As shown in Figure 10.7(b), a mistake in an angle (such as at D) will rotate the computed coordinates of the remaining stations. When this happens, the linear misclosure line AA' is a chord of a circle with radius AD. Thus, the perpendicular bisector of the linear misclosure line will point to the center of the circle, which is the station where the angular mistake occurred. Again if no other errors occurred during the observational process, this perpendicular bisector would point directly to the station. Since other random errors are inevitable, it will most likely point near the station.

Additional observations and careful field practice will help isolate mistakes. For instance, horizon closures often help isolate and eliminate mistakes in the field. A *cutoff line*, such as CE shown dashed in Figure 9.1(a), run between two stations on a traverse, produces smaller closed figures to aid in checking and isolating blunders. Additionally, the extra observations will increase the redundancy in the traverse, and hence the precision of the overall work. These additional observations can be used as checks when performing a compass rule adjustment or can be included in a least-squares adjustment, which is discussed in Chapter 16.

For those wishing to program the computations presented in this chapter, the Mathcad worksheet *TRAV.XMCD*, which is available on the companion website for this book, demonstrates the examples presented in this chapter. Additionally, a traverse with a single angular blunder is used to demonstrate how the perpendicular bisector of the misclosure line seemingly points directly to the angle containing a 1-min blunder.

10.17 MISTAKES IN TRAVERSE COMPUTATIONS

Some of the more common mistakes made in traverse computations are:

1. Failing to adjust the angles before computing azimuths or bearings
2. Applying angle adjustments in the wrong direction and failing to check the angle sum for proper geometric total
3. Interchanging departures and latitudes or their signs
4. Confusing the signs of coordinates

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

10.1 What are the usual steps followed in adjusting a closed traverse?

10.2* The sum of seven interior angles of a closed-polygon traverse each read to the nearest 3″ is 899°59′39″. What is the misclosure, and what correction would be applied to each angle in balancing them by method 1 of Section 10.2?

10.3 Similar to Problem 10.2, except the angles were read to the nearest 3″, and their sum was 720°00′18″ for a six-sided polygon traverse.

10.4 Similar to Problem 10.2, except the angles were read to the nearest 1″, and their sum for a nine-sided polygon traverse was 1259°59′44″.

10.5* Balance the angles in Problem 9.22. Compute the preliminary azimuths for each course.

10.6 Balance the following interior angles (angles-to-the-right) of a five-sided closed polygon traverse using method 1 of Section 10.2. If the azimuth of side AB is fixed at 74°31′17″, calculate the azimuths of the remaining sides. A = 105°13′14″; B = 92°36′06″; C = 67°15′22″; D = 217°24′30″; E = 57°30′38″. (*Note*: Line BC bears NW.)

10.7 Compute departures and latitudes, linear misclosure, and relative precision for the traverse of Problem 10.6 if the lengths of the sides (in feet) are as follows: AB = 2157.34; BC = 1722.58; CD = 1318.15; DE = 1536.06; and EA = 1785.58. (*Note:* Assume units of feet for all distances.)

10.8 Using the compass (Bowditch) rule, adjust the departures and latitudes of the traverse in Problem 10.7. If the coordinates of station A are X = 20,000.00 ft and Y = 15,000.00 ft, calculate (a) coordinates for the other stations, (b) lengths and azimuths of lines AD and EB, and (c) the final adjusted angles at stations A and C.

10.9 Balance the following interior angles-to-the-right for a polygon traverse to the nearest 1″ using method 1 of Section 10.2. Compute the azimuths assuming a fixed azimuth of 277°00′04″ for line AB. A = 119°37′10″; B = 106°12′58″; C = 104°39′22″; D = 130°01′54″; E = 79°28′16″. (*Note*: Line BC bears SW.)

10.10 Determine departures and latitudes, linear misclosure, and relative precision for the traverse of Problem 10.9 if lengths of the sides (in meters) are as follows: $AB = 223.011$; $BC = 168.818$; $CD = 182.358$; $DE = 229.024$; and $EA = 207.930$.

10.11 Using the compass (Bowditch) rule adjust the departures and latitudes of the traverse in Problem 10.10. If the coordinates of station A are $X = 310{,}630.892$ m and $Y = 121{,}311.411$ m, calculate (a) coordinates for the other stations and, from them, (b) the lengths and bearings of lines CA and BD, and (c) the final adjusted angles at B and D.

10.12 Same as Problem 10.9, except assume line AB has a fixed azimuth of 147°36′25″ and line BC bears NE.

10.13 Using the lengths from Problem 10.10 and azimuths from Problem 10.12, calculate departures and latitudes, linear misclosure, and relative precision of the traverse.

10.14 Adjust the departures and latitudes of Problem 10.13 using the compass (Bowditch) rule, and compute coordinates of all stations if the coordinates of station A are $X = 243{,}605.596$ m and $Y = 25{,}393.201$ m. Compute the length and azimuth of line AC.

10.15 Compute and tabulate for the following closed-polygon traverse: (a) preliminary bearings, (b) unadjusted departures and latitudes, (c) linear misclosure, and (d) relative precision. (*Note*: Line BC bears NE.)

Course	Bearing	Length (m)	Interior Angle (Right)
AB	S50°54′23″E	329.722	A = 120°07′10″
BC		210.345	B = 59°39′10″
CD		279.330	C = 248°00′57″
DE		283.426	D = 86°51′04″
EF		433.007	E = 102°09′16″
FA		307.625	F = 103°12′41″

10.16* In Problem 10.15, if one side and/or angle is responsible for most of the error of closure, which is it likely to be?

10.17 Adjust the traverse of Problem 10.15 using the compass rule. If the coordinates in meters of point A are 6521.951 E and 7037.072 N, determine the coordinates of all other points. Find the length and bearing of line AE.

For the closed-polygon traverses given in Problem 10.18 through 10.19 (lengths in feet), compute and tabulate: (a) unbalanced departures and latitudes, (b) linear misclosure, (c) relative precision, and (d) preliminary coordinates if $X_A = 10{,}000.00$ and $Y_A = 5000.00$. Balance the traverses by coordinates using the compass rule.

	Course	AB	BC	CD	DA
10.18	Bearing	N54°07′19″W	S38°52′55″W	S30°38′15″E	N44°47′31″E
	Length	305.55	239.90	283.41	373.00
10.19	Azimuth	124°09′35″	61°57′48″	298°13′52″	238°20′54″
	Length	541.17	612.41	615.35	524.18

10.20 Compute the linear misclosure, relative precision, and adjusted lengths and azimuths for the sides after the departures and latitudes are balanced by the compass rule in the following closed-polygon traverse.

Course	Length (m)	Departure (m)	Latitude (m)
AB	399.233	−367.851	+155.150
BC	572.996	−129.550	−558.152
CA	640.164	+497.402	+403.003

10.21 The following data apply to a closed link traverse [like that of Figure 9.1(b)]. Compute preliminary azimuths, adjust them, and calculate departures and latitudes, misclosures in departure and latitude, and traverse relative precision. Balance the departures and latitudes using the compass rule, and calculate coordinates of points *B*, *C*, and *D*. Compute the final lengths and azimuths of lines *AB*, *BC*, *CD*, and *DE*.

Station	Measured Angle (to the Right)	Adjusted Azimuth	Measured Length (ft)	*X* (ft)	*Y* (ft)
$AzMk_1$					
		342°09′28″			
A	258°12′18″			2,521,005.86	379,490.84
			200.55		
B	215°02′53″				
			253.84		
C	128°19′11″				
			205.89		
D	237°34′05″			2,521,575.16	379,714.76
		101°18′31″			
$AzMk_2$					

10.22 Similar to Problem 10.21, except use the following data:

Station	Measured Angle (to the Right)	Adjusted Azimuth	Measured Length (m)	*X* (m)	*Y* (m)
$AzMk_1$					
		330°40′42″			
A	82°57′54″			185,435.380	24,957.460
			285.993		
B	261°21′42″				
			275.993		
C	149°31′27″				
			318.871		
D	118°33′32″				
			236.504		
E	215°00′51″			184,539.770	24,880.286
		258°05′38″			
$AzMk_2$					

The azimuths (from north of a polygon traverse are $AB = 38°17'02''$, $BC = 121°26'30''$, $CD = 224°56'59''$, and $DA = 308°26'56''$. If one observed distance contains a mistake, which course is most likely responsible for the closure conditions given in Problems 10.23 and 10.24? Is the course too long or too short?

10.23* Algebraic sum of departures = 5.12 ft latitudes = −3.13 ft.

10.24 Algebraic sum of departures = −3.133 m latitudes = +2.487 m.

10.25 Determine the lengths and bearings of the sides of a lot whose corners have the following *X* and *Y* coordinates (in feet): *A* (5000.00, 5000.00); *B* (5289.67, 5436.12); *C* (4884.96, 5354.54); *D* (4756.66, 5068.37).

10.26 Compute the lengths and azimuths of the sides of a closed-polygon traverse whose corners have the following *X* and *Y* coordinates (in meters): *A* (8000.000, 5000.000); *B* (2650.000, 4702.906); *C* (1752.028, 2015.453); *D* (1912.303, 1511.635).

10.27 In searching for a record of the length and true bearing of a certain boundary line which is straight between *A* and *B*, the following notes of an old random traverse were found (survey by compass and Gunter's chain, declination 4°45′ W). Compute the true bearing and length (in feet) of *BA*.

Course	*A*-1	1-2	2-3	3-*B*
Magnetic bearing	Due North	N20°00′E	Due East	S46°30′E
Distance (ch)	11.90	35.80	24.14	12.72

10.28 Describe how a blunder may be located in a traverse.

10.29 Create a computational program that solves Problem 10.18.

10.30 Create a computational program that solves Problem 10.21.

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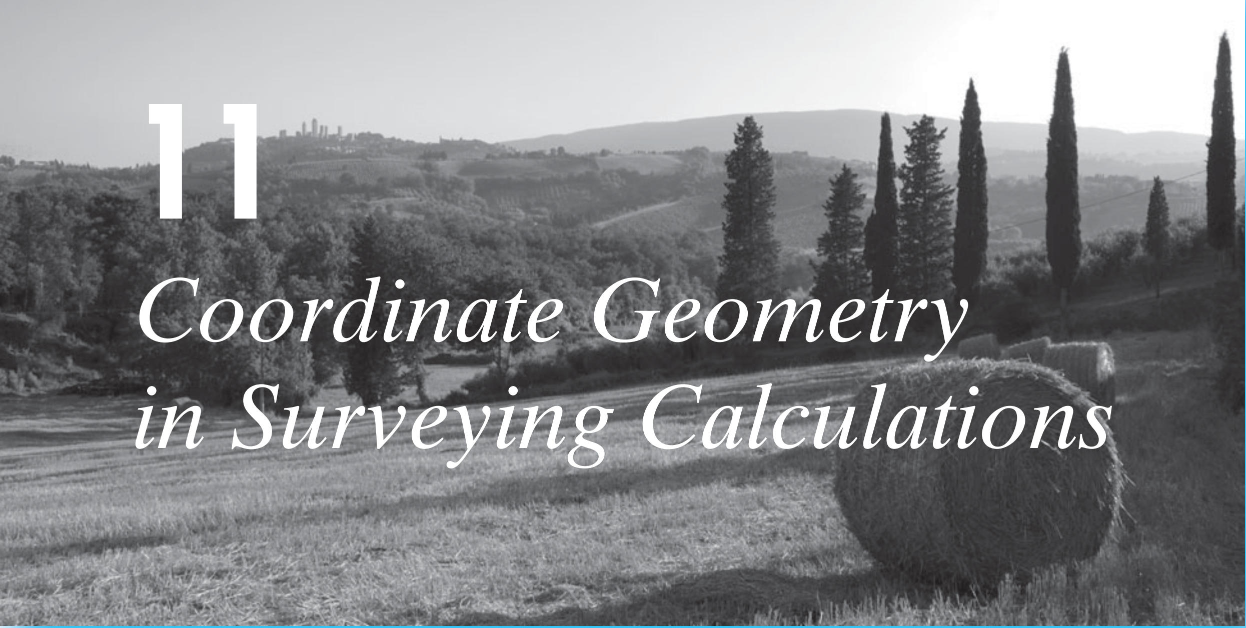


11 Coordinate Geometry in Surveying Calculations

11.1 INTRODUCTION

Except for extensive geodetic control surveys, almost all other surveys are referenced to plane rectangular coordinate systems. State plane coordinates (see Chapter 20) are most frequently employed, although local arbitrary systems can be used. Advantages of referencing points in a rectangular coordinate system are (1) the relative positions of points are uniquely defined, (2) they can be conveniently plotted, (3) if lost in the field, they can readily be recovered from other available points referenced to the same system, and (4) computations are greatly facilitated.

Computations involving coordinates are performed in a variety of surveying problems. Two situations were introduced in Chapter 10, where it was shown that the length and direction (azimuth or bearing) of a line can be calculated from the coordinates of its end points. Area computation using coordinates is discussed in Chapter 12. Additional problems that are conveniently solved using coordinates are determining the point of intersection of (a) two lines, (b) a line and a circle, and (c) two circles. The solutions for these and other coordinate geometry problems are discussed in this chapter. It will be shown that the method employed to determine the intersection point of a line and a circle reduces to finding the intersection of a line of known azimuth and another line of known length. Also, the problem of finding the intersection of two circles consists of determining the intersection point of two lines having known lengths. These types of problems are regularly encountered in the horizontal alignment surveys where it is necessary to compute intersections of tangents and circular curves, and in boundary and subdivision work where parcels of land are often defined by straight lines and circular arcs.

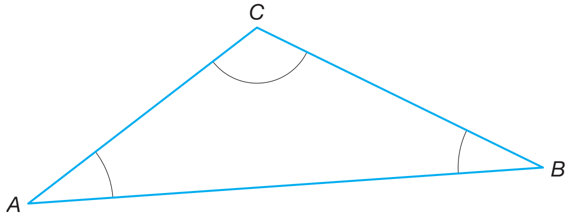


Figure 11.1
An oblique triangle.

The three types of intersection problems noted above are conveniently solved by forming a triangle between two stations of known position from which the observations are made, and then solving for the parts of this triangle. Two important functions used in solving oblique triangles are (1) the law of sines, and (2) the law of cosines. The law of sines relates the lengths of the sides of a triangle to the sines of the opposite angles. For Figure 11.1, this law is

$$\frac{BC}{\sin\ A} = \frac{AC}{\sin\ B} = \frac{AB}{\sin\ C} \tag{11.1}$$

where AB, BC, and AC are the lengths of the three sides of the triangle ABC, and A, B, and C are the angles. The law of cosines relates two sides and the included angle of a triangle to the length of the side opposite the angle. In Figure 11.1, the following three equations can be written that express the law of cosines:

$$\begin{aligned} BC^2 &= AC^2 + AB^2 - 2(AC)(AB)\cos A \\ AC^2 &= BA^2 + BC^2 - 2(BA)(BC)\cos B \\ AB^2 &= CB^2 + CA^2 - 2(CB)(CA)\cos C \end{aligned} \tag{11.2}$$

In some coordinate geometry solutions, the use of the quadratic formula can be used. Examples where this equation simplifies the solution are discussed in Sections 24.16.1 and 25.10. This formula, which gives the solution for x in any quadratic equation of form $ax^2 + bx + c = 0$, is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{11.3}$$

In the remaining sections of this chapter, procedures using triangles and Equations (11.1) through (11.3) are presented for solving each type of standard coordinate geometry problem.

■ 11.2 COORDINATE FORMS OF EQUATIONS FOR LINES AND CIRCLES

In Figure 11.2, straight line AB is referenced in a plane rectangular coordinate system. Coordinates of end points A and B are X_A, Y_A, X_B, and Y_B. Length AB and azimuth Az_{AB} of this line in terms of these coordinates are

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \tag{11.4}$$

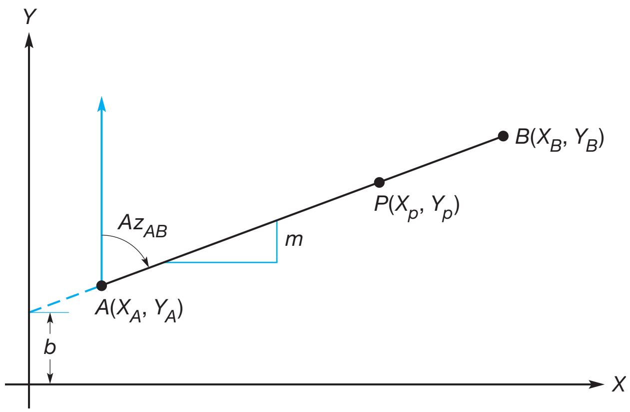


Figure 11.2
Geometry of a straight line in a plane coordinate system.

$$Az_{AB} = \tan^{-1}\left(\frac{\Delta X}{\Delta Y}\right) + C \tag{11.5a}$$

where ΔX is $X_B - X_A$, ΔY is $\Delta X_B - \Delta X_A$, C is 0° if both ΔX and ΔY are greater than zero; C is 180° if ΔY is less than zero, and C is 360° if ΔX is less than zero, and ΔY is greater than zero. Another frequently used equation for determining the azimuth of a course in software is known as the atan2 function which is computed as

$$Az_{AB} = \text{atan2}(\Delta Y, \Delta X) + D = 2\tan^{-1}\left(\frac{\sqrt{\Delta X^2 + \Delta Y^2} - \Delta Y}{\Delta X}\right) + D \tag{11.5b}$$

where D is the 0° if the results of the atan2 function are positive and 360° if the results of the function are negative. The general mathematical expression for a straight line is

$$Y_P = mX_P + b \tag{11.6}$$

where Y_P is the Y coordinate of any point P on the line whose X coordinate is X_P, m the slope of the line, and b the y-intercept of the line. Slope m can be expressed as

$$m = \frac{Y_B - Y_A}{X_B - X_A} = \cot(Az_{AB}) \tag{11.7}$$

From Equations (11.5a) and (11.7), it can be shown that

$$Az_{AB} = \tan^{-1}\left(\frac{1}{m}\right) + C \tag{11.8}$$

The general mathematical expression for a circle in rectangular coordinates can be written as

$$R^2 = (X_P - X_O)^2 + (Y_P - Y_O)^2 \tag{11.9}$$

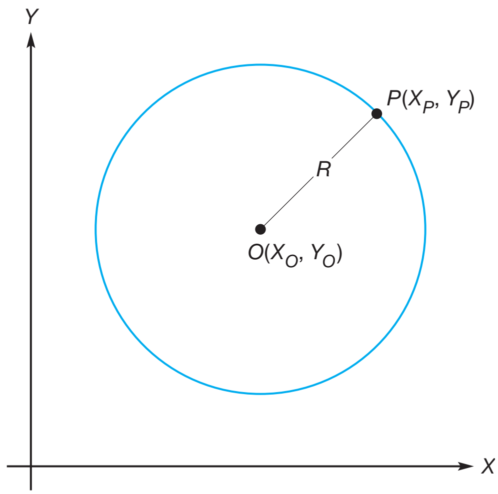


Figure 11.3 Geometry of a circle in a plane coordinate system.

In Equation (11.9), and with reference to Figure 11.3, R is the radius of the circle, X_O and Y_O are the coordinates of the radius point O, and X_P and Y_P the coordinates of any point P on the circle. Another form of the circle equation is

$$X_P^2 + Y_P^2 - 2X_OX_P - 2Y_OY_P + f = 0 \tag{11.10}$$

where the length of the radius of the circle is given as $R = \sqrt{X_O^2 + Y_O^2 - f}$. [Note: Although Equations (11.9) and (11.10) are not used in solving problems in this chapter, they are applied in later chapters.]

11.3 PERPENDICULAR DISTANCE FROM A POINT TO A LINE

A common problem encountered in boundary surveying is determining the perpendicular distance of a point from a line. This procedure can be used to check the alignment of survey markers on a block and is also useful in subdivision design. Assume in Figure 11.4 that points A and B are on the line defined by two block corners whose coordinates are known. Also assume that the coordinates of point P are

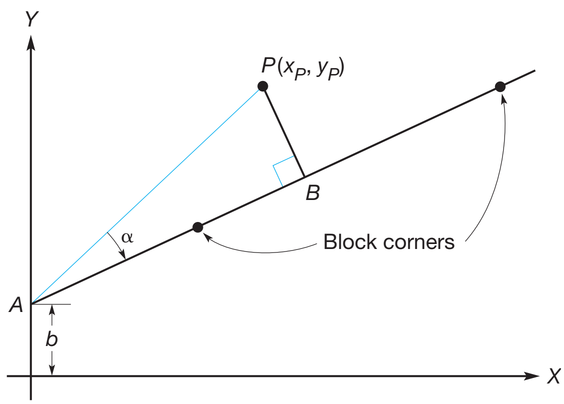


Figure 11.4 Perpendicular distance of a point from a line.

known. The slope, m, and y-intercept, b, of line AB are computed from the coordinates of the block corners. By assigning the X and Y coordinate axes as shown in the figure, the coordinates of point A are $X_A = 0$, and $Y_A = b$. Using Equations (11.4) and (11.5a), the length and azimuth of line AP can be determined from its coordinates. By Equation (11.8), the azimuth of line AB can be determined from the slope of the line AB. Now angle α can be computed as the difference in the azimuth AP and AB, which for the situation depicted in Figure 11.4 is

$$\alpha = Az_{AB} - Az_{AP} \tag{11.11}$$

Recognizing that ABP is a right triangle, length BP is

$$BP = AP \sin \alpha \tag{11.12}$$

where the length of AP is determined from the coordinates of points A and P using Equation (11.4).

Example 11.1

For Figure 11.4, assume that the coordinates (X,Y) of point P are (1123.82, 509.41) and that the coordinates of the block corners are (865.49, 416.73) and (1557.41, 669.09). What is the perpendicular distance of point P from line AB? (All units are in feet.)

Solution

By Equation (11.7), and using the block corner coordinates, the slope of line AB is

$$m = \frac{669.09 - 416.73}{1557.41 - 865.49} = 0.364724245$$

Rearranging Equation (11.6), the y-intercept of line AB is

$$b = 416.73 - 0.364724245(865.49) = 101.065 \text{ ft}$$

By Equations (11.4) and (11.5a), the length and azimuth of line AP is

$$AP = \sqrt{(1123.82 - 0)^2 + (509.41 - 101.065)^2} = 1195.708 \text{ ft}$$

$$Az_{AP} = \tan^{-1}\left(\frac{1123.82 - 0}{509.41 - 101.07}\right) + 0° = 70°01'52.2''$$

By Equation (11.8), the azimuth of line AB is

$$Az_{AB} = \tan^{-1}\left(\frac{1}{0.364724245}\right) + 0° = 69°57'42.7''$$

Using Equation (11.11), angle α is

$$\alpha = 70°01'52.2'' - 69°57'42.7'' = 0°04'09.5''$$

From Equation (11.12), the perpendicular distance from point P to line AB is

$$BP = 1195.708 \sin(0°04'09.5'') = 1.45 \text{ ft}$$

11.4 INTERSECTION OF TWO LINES, BOTH HAVING KNOWN DIRECTIONS

Figure 11.5 illustrates the intersection of two lines AP and BP. Each has known coordinates for one end point, and each has a known direction. Determining the point of intersection for this type of situation is often called the *direction-direction* problem. A simple method of computing the intersection point P is to solve for the parts of oblique triangle ABP. Since the coordinates of A and B are known, the length and azimuth of AB (shown dashed) can be determined using Equations (11.4) and (11.5a), respectively. Then, from the figure it can be seen that angle A is the difference in the azimuths of AB and AP, or

$$A = Az_{AP} - Az_{AB} \tag{11.13}$$

Similarly, angle B is the difference in the azimuths of BA and BP, or

$$B = Az_{BA} - Az_{BP} \tag{11.14}$$

With two angles of the triangle ABP computed, the remaining angle P is

$$P = 180° - A - B \tag{11.15}$$

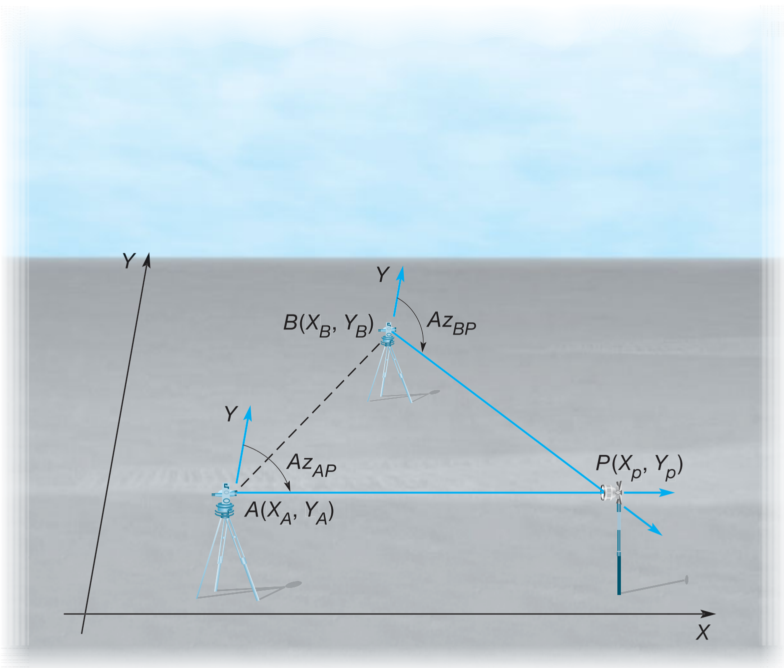


Figure 11.5 Intersection of two lines with known directions.

Substituting into Equation (11.1) and rearranging the length of side AP is

$$AP = AB\frac{\sin(B)}{\sin(P)} \tag{11.16}$$

With both the length and azimuth of AP known, the coordinates of P are

$$\begin{aligned} X_P &= X_A + AP \sin Az_{AP} \\ Y_P &= Y_A + AP \cos Az_{AP} \end{aligned} \tag{11.17}$$

A check on this solution can be obtained by solving for length BP, and using it together with the azimuth of BP to compute the coordinates of P. The two solutions should agree, except for round off.

It should be noted that if the azimuths for lines AP and BP are equal, then the lines are parallel and have no intersection.

Example 11.2

In Figure 11.5, assuming the following information is known for two lines, compute coordinates X_P and Y_P of the intersection point. (Coordinates are in feet.)

$$\begin{aligned} X_A &= 1425.07 \qquad X_B = 7484.80 \qquad Az_{AP} = 76°04'24'' \\ Y_A &= 1971.28 \qquad Y_B = 5209.64 \qquad Az_{BP} = 141°30'16'' \end{aligned}$$

Solution

By Equations (11.4) and (11.5a), the length and azimuth of side AB are

$$AB = \sqrt{(7484.80 - 1425.07)^2 + (5209.64 - 1971.28)^2} = 6870.757 \text{ ft}$$

$$Az_{AB} = \tan^{-1}\left(\frac{7484.80 - 1425.07}{5209.64 - 1971.28}\right) + 0° = 61°52'46.8''$$

By Equations (11.13) through (11.15), the three angles of triangle ABP are

$$A = 76°04'24'' - 61°52'46.8'' = 14°11'37.2''$$

$$B = (180° + 61°52'46.8'') - 141°30'16'' = 100°22'30.8''$$

$$P = 180° - 14°11'37.2'' - 100°22'30.8'' = 65°25'52.0''$$

By Equation (11.16), length AP is

$$AP = 6870.757\frac{\sin 100°22'30.8''}{\sin 65°25'52.0''} = 7431.224 \text{ ft}$$

By Equations (11.17), the coordinates of station P are

$$X_P = 1425.07 + 7431.224 \sin 76°04'24'' = 8637.85 \text{ ft}$$
$$Y_P = 1971.28 + 7431.224 \cos 76°04'24'' = 3759.83 \text{ ft}$$

Check:

$$BP = 6870.757\left[\frac{\sin 14°11'37.2''}{\sin 65°25'52''}\right] = 1852.426 \text{ ft}$$
$$X_P = 7484.80 + (1852.426) \sin 141°30'16'' = 8637.85 \text{ (Check!)}$$
$$Y_P = 5209.64 + (1852.426) \cos 141°30'16'' = 3759.83 \text{ (Check!)}$$

11.5 INTERSECTION OF A LINE WITH A CIRCLE

Figure 11.6 illustrates the intersection of a line (AC) of known azimuth with a circle of known radius ($BP_1 = BP_2$). Finding the intersection for this situation reduces to finding the intersection of a line of known direction with another line of known length and is sometimes referred to as the *direction-distance* problem. As shown in the figure, notice that this problem has two different solutions, but as discussed later, the incorrect one can generally be detected and discarded.

The approach to solving this problem is similar to that employed in Section 11.4; that is, the answer is determined by solving an oblique triangle. This particular solution will demonstrate the use of the quadratic equation to obtain both solutions. In Figure 11.6, the coordinates of B (the radius point of the circle) are known. From the coordinates of points A and B, the length and azimuth of

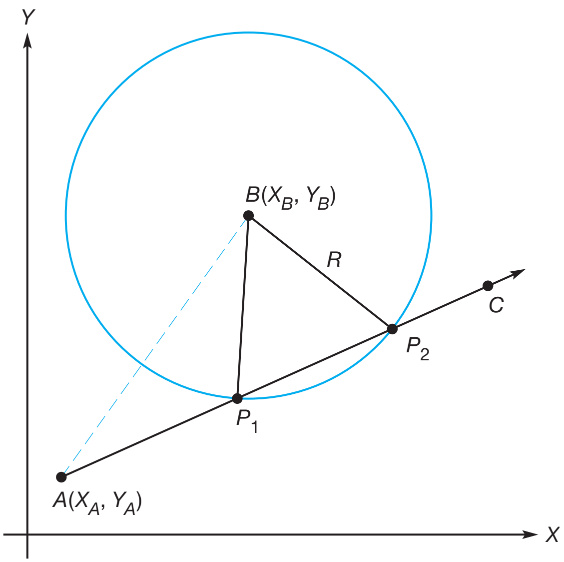


Figure 11.6 Intersection of a line and a circle.

line AB (shown dashed) are determined by employing Equations (11.4) and (11.5a), respectively. Then angle A is computed from the azimuths of AB and AC as follows:

$$A = Az_{AP} - Az_{AB} \tag{11.18}$$

Substituting the known values of A, AB, and BP into the law of cosines [Equation (11.2)] yields

$$BP^2 = AB^2 + AP^2 - 2(AB)(AP)\cos A \tag{11.19}$$

In Equation (11.19), AP is an unknown quantity. Rearranging this equation gives

$$AP^2 - 2(AB)(\cos A)AP + (AB^2 - BP^2) = 0 \tag{11.20}$$

Now Equation (11.20), which is a second-degree expression, can be solved using the quadratic formula [Equation (11.3)] as follows:

$$AP = \frac{2(AB)\cos(A) \pm \sqrt{[2(AB)\cos A]^2 - 4(AB^2 - BP^2)}}{2} \tag{11.21}$$

In comparing Equation (11.21) to Equation (11.3), it can be seen that $a = 1, b = 2(AB)\cos A$, and $c = (AB^2 - BP^2)$. Because of the $\pm$ sign in the formula, there are two solutions for length AP. Once these two lengths are determined, the possible coordinates of station P are

$$\begin{aligned} X_{P1} &= X_A + AP_1 \sin(Az_{AP}) \quad \text{and} \quad Y_{P1} = Y_A + AP_1 \cos(Az_{AP}) \\ X_{P2} &= X_A + AP_2 \sin(Az_{AP}) \quad \text{and} \quad Y_{P2} = Y_A + AP_2 \cos(Az_{AP}). \end{aligned} \tag{11.22}$$

If errors exist in the given data for the problem, or if an impossible design is attempted, the circle will not intersect the line. In this case, the terms under the radical in Equation (11.21) will be negative, that is, $[2(AB)\cos A]^2 - 4(AB^2 - BP^2) < 0$. It is therefore important when solving any of the coordinate geometry problems to be alert for these types of potential problems.

The sine law can also be used to solve this problem. However, care must be exercised when using the sine law since the two solutions will not be readily apparent. The procedure of solving this problem using the sine law is as follows:

1. Compute the length and azimuth of line AB from the coordinates using Equations (11.4) and (11.5a), respectively.
2. Compute the angle at A using Equation (11.18).
3. Using the sine law solve for the angles at P_1 as

$$\sin P = \frac{AB \sin A}{BP} \tag{11.23}$$

4. Note that the sine function has the relationship $\sin(x) = \sin(180° - x)$. Thus, the solution for the angle at B is

$$\begin{aligned} B_1 &= 180° - (A + P) \\ B_2 &= P - A \end{aligned} \tag{11.24}$$

5. Using the two solutions for angle B, determine the azimuth of line BP as

$$\begin{aligned} Az_{BP1} &= Az_{BA} - B_1 \\ Az_{BP2} &= Az_{BA} - B_2 \end{aligned} \tag{11.25}$$

6. Finally using the two azimuths and the observed length of BP determine the two possible solutions for station P as

$$\begin{aligned} X_{P1} &= X_B + BP\sin(Az_{BP1}) \quad \text{and} \quad Y_{P1} = Y_B + BP\sin(Az_{BP1}) \\ X_{P2} &= X_B + BP\sin(Az_{BP2}) \quad \text{and} \quad Y_{P2} = Y_B + BP\sin(Az_{BP2}) \end{aligned} \tag{11.26}$$

Example 11.3

In Figure 11.6, assume the coordinates of point A are $X = 100.00$ and $Y = 130.00$, and that the coordinates of point B are $X = 500.00$ and $Y = 600.00$. If the azimuth of AP is 70°42′36″, and the radius of the circle (length BP) is 350.00, what are the possible coordinates of point P? (Note: Linear units are feet.)

Solution

By Equations (11.4) and (11.5a), the length and azimuth of AB are

$$AB = \sqrt{(500 - 100)^2 + (600 - 130)^2} = 617.171 \text{ ft}$$

$$Az_{AB} = \tan^{-1}\left(\frac{500 - 100}{600 - 130}\right) + 0° = 40°23'59.7''$$

By Equation (11.18), the angle at A is

$$A = 70°42'36'' - 40°23'59.7'' = 30°18'36.3''$$

Substituting appropriate values according to Equation (11.20), the quadratic equation coefficients are

$$\begin{aligned} a &= 1 \\ b &= -2(617.171)\cos 30°18'36.3'' = -1065.616 \\ c &= 617.171^2 - 350.00^2 = 258{,}400.043 \end{aligned}$$

Substituting these values into Equation (11.21) yields

$$AP = \frac{1065.616 \pm \sqrt{1065.616^2 - 4(258{,}400.043)}}{2}$$

$$= \frac{1065.616 \pm 319.276}{2}$$

$$= 373.170 \text{ or } 692.446$$

Using the azimuth and distances for AP, the two possible solutions for the coordinates of P are

$$X_{P1} = 100.00 + 373.170 \sin 70°42'36'' = 452.22 \text{ ft}$$

$$Y_{P1} = 130.00 + 373.170 \cos 70°42'36'' = 253.28 \text{ ft}$$

or

$$X_{P2} = 100.00 + 692.446 \sin 70°42'36'' = 753.57 \text{ ft}$$

$$Y_{P2} = 130.00 + 692.446 \cos 70°42'36'' = 358.75 \text{ ft}$$

In solving a quadratic equation, the decision to add or subtract the value from the radical can be made on the basis of experience, or by using a carefully constructed scaled diagram, which also provides a check on the computations. One answer will be unreasonable and should be discarded. An arithmetic check is possible by solving for the two possible angles at B to P in triangle ABP and determining the coordinates of P from station B, or by solving the problem using the second procedure. Students should verify that the same solution can be obtained using Equations (11.23) through (11.26).

11.6 INTERSECTION OF TWO CIRCLES

In Figure 11.7, the intersection of two circles is illustrated. Note that the circles are obtained by simply radiating two distances (their radius values R_A and R_B) about their radius points A and B. As shown, this geometry again results in two intersection points, P_1 and P_2. As with the two previous cases, these intersection points can again be located by solving for the parts of oblique triangle ABP. In this situation, two sides of the triangle are the known radii, and thus the problem is often called the *distance-distance* problem. The third side of the triangle, AB, can be computed from known coordinates of A and B, or the distance can be observed.

The first step in solving this problem is to compute the length and azimuth of line AB using Equations (11.4) and (11.5a). Then angle A can be determined using the law of cosines (Equation 11.2). As shown in Figure 11.7, the two solutions for P at either P_1 or P_2 are derived by either adding or subtracting angle A from the azimuth of line AB to obtain the direction of AP. By rearranging Equation 11.2, angle A is

$$A = \cos^{-1}\left[\frac{(AB)^2 + (AP)^2 - (BP)^2}{2(AB)(AP)}\right] \quad (11.27)$$

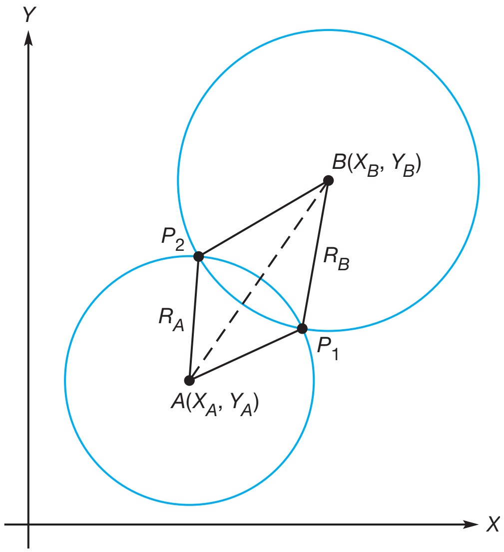


Figure 11.7
Intersection of two circles.

Thus, the azimuth of line AP is either

$$\begin{aligned} Az_{AP1} &= Az_{AB} + A \\ Az_{AP2} &= Az_{AB} - A \end{aligned} \tag{11.28}$$

The possible coordinates of P are

$$\begin{aligned} X_{P1} &= X_A + AP_1 \sin(Az_{AP1}) \quad \text{and} \quad Y_{P1} = Y_A + AP_1 \cos(Az_{AP1}) \\ X_{P2} &= X_A + AP_2 \sin(Az_{AP2}) \quad \text{and} \quad Y_{P2} = Y_A + AP_2 \cos(Az_{AP2}) \end{aligned} \tag{11.29}$$

The decision of whether to add or subtract angle A from the azimuth of line AB can be made on the basis of experience, or through the use of a carefully constructed scaled diagram. One answer will be unreasonable and should be discarded. As can be seen from Figure 11.7, there will be no solution if length of AB is greater than the sum of R_A and R_B.

■ Example 11.4

In Figure 11.7, assume the following data (in meters) are available:

$$\begin{aligned} X_A &= 2851.28 \quad & Y_A &= 299.40 \quad & R_A &= 2000.00 \\ X_B &= 3898.72 \quad & Y_B &= 2870.15 \quad & R_B &= 1500.00 \end{aligned}$$

Compute the X and Y coordinates of point P.

Solution

By Equations (11.4) and (11.5a), the length and azimuth of AB are

$$AB = \sqrt{(3898.72 - 2851.28)^2 + (2870.15 - 299.40)^2} = 2775.948 \text{ m}$$

$$Az_{AB} = \tan^{-1}\left(\frac{3898.72 - 2851.28}{2870.15 - 299.40}\right) + 0° = 22°10'05.6''$$

By Equation (11.27), A is

$$A = \cos^{-1}\left(\frac{2775.948^2 + 2000.00^2 - 1500.00^2}{2(2775.948)2000.00}\right) = 31°36'53.6''$$

By combining Equations (11.28) and (11.29), the possible solutions for P are

$$X_{P_1} = 2851.28 + 2000.00 \sin(22°10'05.6'' + 31°36'53.6'') = 4464.85 \text{ m}$$

$$Y_{P_1} = 299.40 + 2000.00 \cos(22°10'05.6'' + 31°36'53.6'') = 1481.09 \text{ m}$$

or

$$X_{P_2} = 2851.28 + 2000.00 \sin(22°10'05.6'' - 31°36'53.6'') = 2523.02 \text{ m}$$

$$Y_{P_2} = 299.40 + 2000.00 \cos(22°10'05.6'' - 31°36'53.6'') = 2272.28 \text{ m}$$

An arithmetic check on this solution can be obtained by determining the angle and coordinates of P from station B.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *COGO I.mp4* demonstrates the intersection problems presented in the previous sections.

11.7 THREE-POINT RESECTION

This procedure locates a point of unknown position by observing horizontal angles from that point to three visible stations whose positions are known. The situation is illustrated in Figure 11.8, where a total station instrument occupies station P and angles x and y are observed. A summary of the method used to compute the coordinates of station P follows (refer to Figure 11.8):

1. From the known coordinates of A, B, and C calculate lengths a and c, and angle α at station B.
2. Subtract the sum of angles x, y, and α in figure $ABCP$ from 360° to obtain the sum of angles $A + C$

$$A + C = 360° - (\alpha + x + y) \tag{11.30}$$

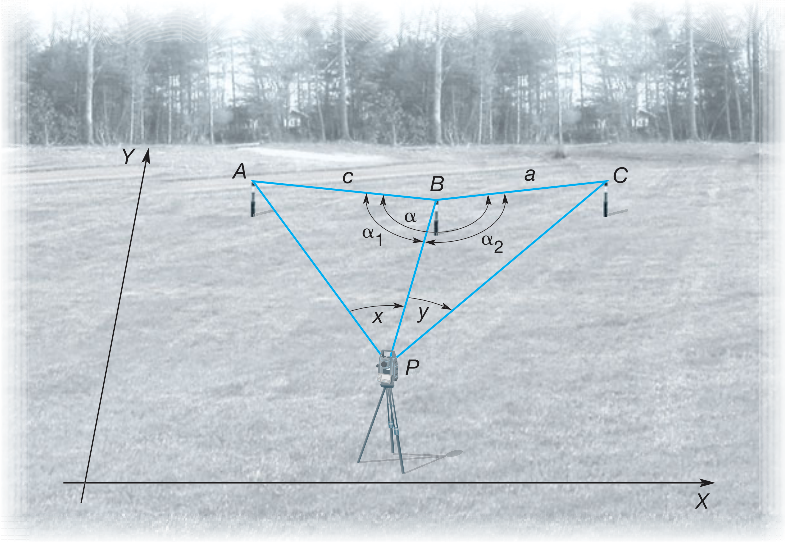


Figure 11.8
The resection problem.

3. Calculate angles A and C using the following:

$$A = \tan^{-1}\left(\frac{a \sin x \sin (A + C)}{c \sin y + a \sin x \cos (A + C)}\right) \tag{11.31}$$

$$C = \tan^{-1}\left(\frac{c \sin y \sin (A + C)}{a \sin x + c \sin y \cos (A + C)}\right) \tag{11.32}$$

4. From angle A and azimuth AB, calculate azimuth AP in triangle ABP. Then solve for length AP using the law of sines, where $\alpha_1 = 180° - A - x$. Calculate the departure and latitude of AP followed by the coordinates of P.
5. In the manner outlined in step 4, use triangle BCP to calculate the coordinates of P to obtain a check.

Example 11.5

In Figure 11.8, angles x and y were measured as 48°53′12″ and 41°20′35″, respectively. Control points A, B, and C have coordinates (in feet) of $X_A = 5721.25$, $Y_A = 21{,}802.48$, $X_B = 13{,}542.99$, $Y_B = 22{,}497.95$, $X_C = 20{,}350.09$, and $Y_C = 24{,}861.22$. Calculate the coordinates of P.

Solution

1. By Equation (11.4)

$$a = \sqrt{(20{,}350.09 - 13{,}542.99)^2 + (24{,}861.22 - 22{,}497.95)^2} = 7205.67 \text{ ft}$$

$$c = \sqrt{(13{,}542.99 - 5721.25)^2 + (22{,}497.95 - 21{,}802.48)^2} = 7852.60 \text{ ft}$$

2. By Equation (11.5a)

$$Az_{AB} = \tan^{-1}\left(\frac{13{,}542.99 - 5721.25}{22{,}497.95 - 21{,}802.48}\right) + 0° = 84°55'08.1''$$

$$Az_{BC} = \tan^{-1}\left(\frac{20{,}350.09 - 13{,}542.99}{24{,}861.22 - 22{,}497.95}\right) + 0° = 70°51'15.0''$$

3. Calculate angle α,

$$\alpha = 180° - (70°51'15.0'' - 84°55'08.1'') = 194°03'53.1''$$

4. By Equation (11.30)

$$A + C = 360° - 194°03'53.1'' - 48°53'12'' - 41°20'35'' = 75°42'19.9''$$

5. By Equation (11.31)

$$A = \tan^{-1}\left(\frac{7250.67 \sin 48°53'12'' \sin 75°42'19.9''}{7852.60 \sin 41°20'35'' + 7205.67 \sin 48°53'12'' \cos 75°42'19.9''}\right)$$
$$= 38°51'58.7''$$

6. By Equation (11.32)

$$C = \tan^{-1}\left(\frac{7852.60 \sin 41°20'35'' \sin 75°42'19.9''}{7205.67 \sin 48°53'12'' + 7852.60 \sin 41°20'35'' \cos 75°42'19.9''}\right)$$
$$= 36°50'21.2''$$

$$(A + C = 38°51'58.7'' + 36°50'21.2'' = 75°42'19.9'' \checkmark)$$

7. Calculate angle α_1

$$\alpha_1 = 180° - 38°51'58.7'' - 48°53'12'' = 92°14'49.3''$$

8. By the law of sines

$$AP = \frac{\sin 92°14'49.3''(7852.60)}{\sin 48°53'12''} = 10{,}414.72 \text{ ft}$$

$$AZ_{AP} = AZ_{AB} + A = 84°55'08.1'' + 38°51'58.7'' = 123°47'06.8''$$

9. By Equations (10.1) and (10.2)

$$\text{Dep}_{AP} = 10{,}414.72 \sin 123°47'06.8'' = 8655.97 \text{ ft}$$

$$\text{Lat}_{AP} = 10{,}414.72 \cos 123°47'06.8'' = -5791.43 \text{ ft}$$

10. By Equation (10.7)

$$X_P = 5721.25 + 8655.97 = 14{,}377.22 \text{ ft}$$

$$Y_P = 21{,}802.48 - 5791.43 = 16{,}011.05 \text{ ft}$$

11. As a check, triangle *BCP* was solved to obtain the same results.

The three-point resection problem just described provides a unique solution for the unknown coordinates of point P, that is, there are no redundant observations, and thus no check can be made on the observations. This is actually a special case of the more general resection problem, which provides redundancy and enables a least-squares solution. In the general resection problem, in addition to observing the angles x and y, distances from P to one or more control stations could also have been observed. Other possible variations in resection that provide redundancy include observing (a) one angle and two distances to two control stations; (b) two angles and one, two, or three distances to three control points; or (c) the use of more than three control stations. Then all observations can be included in a least-squares solution to obtain the most probable coordinates of point P. Resection has become a popular method for quickly orienting total station instruments, as discussed in Section 23.9. The procedure is convenient because these instruments can readily observe both angles and distances, and their on-board microprocessors can instantaneously provide the least-squares solution for the instrument's position.

It should be noted that the resection problem will not have a unique solution if points A, B, C, and P define a circle. Selecting points B and P so that they both lie on the same side of a line connecting points A and C avoids this problem. Additionally, the accuracy of the solution will decrease if the observed angles x and y become small. As a general guideline, the observed angles should be greater than 30° for best results.

11.8 TWO-DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

It is sometimes necessary to convert coordinates of points from one survey coordinate system to another. This happens, for example, if a survey is performed in some local-assumed or arbitrary coordinate system and later it is desired to convert it to state plane coordinates. The process of making these conversions is called *coordinate transformation*. If only planimetric coordinates (i.e., Xs and Ys) are involved, and true shape is retained, it is called *two-dimensional (2D) conformal coordinate transformation*.

The geometry of a 2D conformal coordinate transformation is illustrated in Figure 11.9. In the figure, X-Y represents a local-assumed coordinate system, and E-N a state plane coordinate system. Coordinates of points A through D are known in the X-Y system and those of A and B are also known in the E-N system. Points such as A and B, whose positions are known in both systems, are termed *control points*. At least two control points are required in order to determine E-N coordinates of other points such as C and D.

In general, three steps are involved in coordinate transformation: (1) rotation, (2) scaling, and (3) translation. As shown in Figure 11.9, rotation consists in determining coordinates of points in the rotated X'-Y' axis system (shown dashed). The X'-Y' axes are parallel with E-N but the origin of this system coincides with the origin of X-Y. In the figure, the rotation angle θ, between the X-Y and X'-Y' axis systems, is

$$\theta = \alpha - \beta \tag{11.33}$$

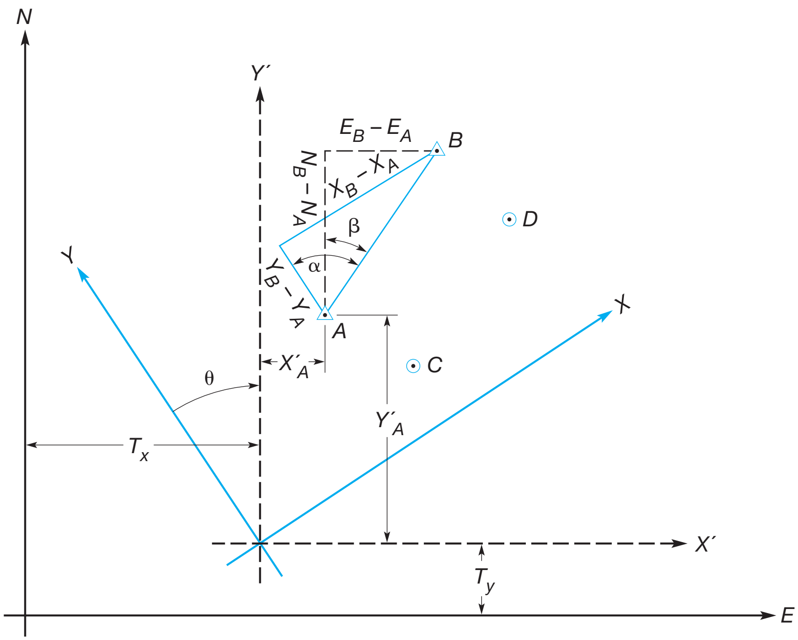


Figure 11.9 Geometry of the 2D coordinate transformation.

In Equation (11.33), azimuths, α and β, are calculated from the two sets of coordinates of control points A and B using Equation (11.5a) as follows:

$$\alpha = \tan^{-1}\left(\frac{X_B - X_A}{Y_B - Y_A}\right) + C$$

$$\beta = \tan^{-1}\left(\frac{E_B - E_A}{N_B - N_A}\right) + C$$

where as explained in Section 11.2, C places the azimuth in the proper quadrant.

In many cases, a scale factor must be incorporated in coordinate transformations. This would occur, for example, in transforming from a local arbitrary coordinate system into a state plane coordinate grid. The scale factor relating any two coordinate systems can be computed according to the ratio of the length of a line between two control points obtained from E-N coordinates to that determined using X-Y coordinates. Thus,

$$s = \frac{\sqrt{(E_B - E_A)^2 + (N_B - N_A)^2}}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}} \tag{11.34}$$

(Note: If the scale factor is unity, the two surveys are of equal scale, and it can be ignored in the coordinate transformation.)

With θ and s known, scaled and rotated X' and Y' coordinates of any point, for example, A, can be calculated from

$$\begin{aligned} X'_A &= sX_A \cos\theta - sY_A \sin\theta \\ Y'_A &= sX_A \sin\theta + sY_A \cos\theta \end{aligned} \tag{11.35}$$

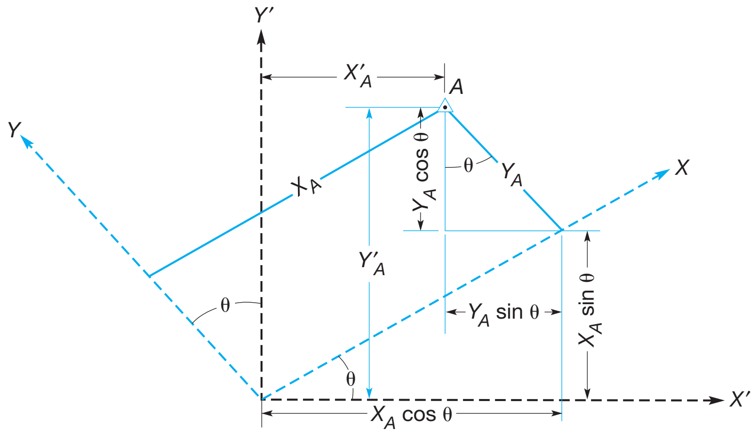


Figure 11.10
Detail of rotation formulas in 2D conformal coordinate transformation.

Individual parts of the rotation formulas [right-hand sides of Equations (11.35)] are developed with reference to Figure 11.10. Translation consists of shifting the origin of the X'-Y'' axes to that in the E-N system. This is achieved by adding translation factors T_X and T_Y (see Figure 11.9) to X' and Y' coordinates to obtain E and N coordinates. Thus, for point A

$$E_A = X'_A + T_X \qquad \textbf{(11.36)}$$
$$N_A = Y'_A + T_Y$$

Rearranging Equations (11.36) and using coordinates of one of the control points (such as A), numerical values for T_X and T_Y can be obtained as

$$T_X = E_A - X'_A \qquad \textbf{(11.37)}$$
$$T_Y = N_A - Y'_A$$

The other control point (i.e., point B) should also be used in Equations (11.37) to calculate T_X and T_Y and thus obtain a computational check.

Substituting Equations (11.35) into Equations (11.36) and dropping subscripts, the following equations are obtained for calculating E and N coordinates of noncontrol points (such as C and D) from their X and Y values:

$$E = sX\cos\theta - sY\sin\theta + T_X \qquad \textbf{(11.38)}$$
$$N = sX\sin\theta + sY\cos\theta + T_Y$$

In summary, the procedure for performing 2D conformal coordinate transformations consists of (1) calculating rotation angle θ using two control points, and Equations (11.5) and (11.33); (2) solving Equations (11.34), (11.35), and (11.37) using control points to obtain scale factor s, and translation factors T_X and T_Y; and (3) applying θ, s, and T_X and T_Y in Equations (11.38) to transform all noncontrol points. If more than two control points are available, an improved solution can be obtained using least squares. Coordinate transformation calculations require a significant amount of time if done by hand, but are easily performed when programmed for computer solution.

Example 11.6

In Figure 11.9, the following *E-N* and *X-Y* coordinates are known for points *A* through *D*. Compute *E* and *N* coordinates for points *C* and *D*.

	State Plane Coordinates (ft)		Arbitrary Coordinates (ft)	
Point	**E**	**N**	**X**	**Y**
A	194,683.50	99,760.22	2848.28	2319.94
B	196,412.80	102,367.61	5720.05	3561.68
C			3541.72	897.03
D			6160.31	1941.26

Solution

1. Determine α, β, and θ from Equations (11.5) and (11.33)

$$\alpha = \tan^{-1}\left(\frac{5720.05 - 2848.28}{3561.68 - 2319.94}\right) + 0° = 66°36'59.7''$$

$$\beta = \tan^{-1}\left(\frac{196{,}412.80 - 194{,}683.50}{102{,}367.61 - 99{,}760.22}\right) + 0° = 33°33'12.7''$$

$$\theta = 66°36'59.7'' - 33°33'12.7'' = 33°03'47''$$

2. Compute the scale factor from Equation (11.34)

$$s = \frac{\sqrt{(196{,}412.80 - 194{,}683.50)^2 + (102{,}367.61 - 99{,}860.22)^2}}{\sqrt{(5720.05 - 2848.28)^2 + (3561.68 - 2319.94)^2}}$$

$$= \frac{3128.73}{3128.73}$$

$$= 1.00000$$

(Since the scale factor is 1, it can be ignored.)

3. Determine T_X and T_Y from Equations (11.35) through (11.37) using point *A*

$$X'_A = 2848.28 \cos 33°03'47'' - 2319.94 \sin 33°03'47'' = 1121.39 \text{ ft}$$

$$Y'_A = 2848.28 \sin 33°03'47'' + 2319.94 \cos 33°03'47'' = 3498.18 \text{ ft}$$

$$T_X = 194{,}683.50 - 1121.39 = 193{,}562.11 \text{ ft}$$

$$T_Y = 99{,}760.22 - 3498.18 = 96{,}262.04 \text{ ft}$$

4. Check T_X and T_Y using point B

$$X'_B = 5720.05 \cos 33°03'47'' - 3561.68 \sin 33°03'47'' = 2850.69 \text{ ft}$$
$$Y'_B = 5720.05 \sin 33°03'47'' + 3561.68 \cos 33°03'47'' = 6105.58 \text{ ft}$$

$$T_X = 196{,}412.80 - 2850.69 = 193{,}562.11 \text{ ft (Check!)}$$
$$T_Y = 102{,}367.61 - 6105.58 = 96{,}262.03 \text{ ft (Check!)}$$

5. Solve Equations (11.38) for E and N coordinates of points C and D

$$\begin{aligned} E_C &= 3541.72 \cos 33°03'47'' - 897.03 \sin 33°03'47'' + 193{,}562.11 \\ &= 196{,}040.93 \text{ ft} \\ N_C &= 3541.72 \sin 33°03'47'' + 897.03 \cos 33°03'47'' + 96{,}262.04 \\ &= 98{,}946.04 \text{ ft} \\ E_D &= 6160.31 \cos 33°03'47'' - 1941.26 \sin 33°03'47'' + 193{,}562.11 \\ &= 197{,}665.81 \text{ ft} \\ N_D &= 6160.31 \sin 33°03'47'' + 1941.26 \cos 33°03'47'' + 96{,}262.04 \\ &= 101{,}249.78 \text{ ft} \end{aligned}$$

With some simple modifications, Equations (11.38) can be rewritten in matrix form as

$$sR\begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \end{bmatrix} = \begin{bmatrix} E \\ N \end{bmatrix} + \begin{bmatrix} v_E \\ v_N \end{bmatrix} \tag{11.39}$$

where the rotation matrix, R, is

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{11.40}$$

Also v_E and v_N are residual errors which must be included if more than two control points are available. Scaling the rotation matrix by s, and substituting a for $(s \cos\theta)$, b for $(s \sin\theta)$, c for T_X, and d for T_Y, Equation (11.39) can be rewritten as

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} E \\ N \end{bmatrix} + \begin{bmatrix} v_E \\ v_N \end{bmatrix} \tag{11.41}$$

With Equation (11.41), a least-squares adjustment (see Chapter 16) can be performed when more than two points are common in both coordinate systems. The program WOLFPACK, which is on the companion website for this book at http://www.pearsonhighered.com/ghilani, has this software option under the *coordinate computations* submenu. It will determine the unknown parameters for the 2D conformal coordinate transformation, and transform any additional

DATA FILE

```
Example 11.6                              {title line}
2                                         {number of control points}
A 194683.50 99760.22 2848.28 2319.94      {Point ID, SPCS E and N, arbitrary X and Y}
B 196412.80 102367.61 5720.05 3561.68
C 3561.68 897.03                          {Point ID, arbitrary system X and Y}
D 6160.31 1941.26
```

RESULTS OF ADJUSTMENT

```
Two Dimensional Conformal Coordinate Transformation of File: Example 11.6
--------------------------------------------------------------------------

ax - by + Tx = X + VX
bx + ay + Ty = Y + VY

Transformed Control Points
   POINT        X          Y        VX        VY
-------------------------------------------------
     A  194,683.50  99,760.22    -0.000    -0.000
     B  196,412.80  102,367.61    0.000     0.000

Transformation Parameters:
 a =  0.83807009
 b =  0.54556070
 Tx =  193562.110
 Ty =  96262.038

Rotation = 33°03'46.9"
  Scale = 1.00000

*******  Unique Solution Obtained !! *******

   POINT      x          y           X            Y
----------------------------------------------------
     C   3,541.72     897.03  196,040.94   98,946.04
     D   6,160.31   1,941.26  197,665.81  101,249.77
```

Figure 11.11 Data file and results of adjustment for Example 11.6 using WOLFPACK.

points. The data file and the results of the adjustment for Example 11.6 are shown in Figure 11.11.

Note that the transformed X and Y coordinates of points C and D obtained using the computer program agree (except for round off) with those computed in Example 11.6. Note also that in this solution with two control points, there are no redundancies and thus the residuals VX and VY are zeros. Also on the companion website are instructional videos that can be downloaded. The video *COGO II.mp4* develops the equations presented in this section and demonstrates the solution to Example 11.6.

■ 11.9 INACCESSIBLE POINT PROBLEM

It is sometimes necessary to determine the elevation of a point that is inaccessible. This task can be accomplished by establishing a baseline such that the inaccessible point is visible from both ends. As an example, assume that the elevation

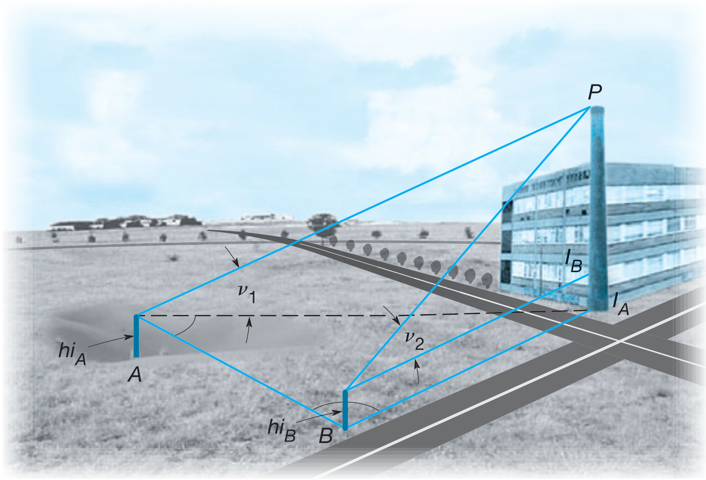


Figure 11.12 Geometry of the inaccessible point problem.

of the chimney shown in Figure 11.12 is desired. Baseline AB is established, its length measured and the elevations of its end points determined. Horizontal angles A and B and altitude angles v_1 and v_2 are observed as shown in the figure. Points I_A and I_B are vertically beneath P. Using the observed values, the law of sines is applied to compute horizontal lengths AI_A and BI_B of triangle ABI as

$$AI_A = \frac{AB\sin(B)}{\sin[180° - (A + B)]} = \frac{AB\sin(B)}{\sin(A + B)} \tag{11.42}$$

$$BI_B = \frac{AB\sin(A)}{\sin(A + B)} \tag{11.43}$$

Length IP can be derived from either triangle AI_AP or triangle BI_BP as

$$I_AP = AI_A \tan(v_1) \tag{11.44}$$

$$I_BP = BI_B \tan(v_2) \tag{11.45}$$

The elevation of point P is computed as the average of the heights from the two triangles, which may differ because of random errors in the observation of v_1 and v_2, as

$$Elev_P = \frac{I_AP + Elev_A + hi_A + I_BP + Elev_B + hi_B}{2} \tag{11.46}$$

In Equation (11.46), hi_A and hi_B are the instrument heights at A and B, respectively.

Example 11.7

Stations A and B have elevations of 298.65 and 301.53 ft, respectively, and the instrument heights at A and B are $hi_A = 5.55$ and $hi_B = 5.48$ ft. The other field observations are

$$AB = 136.45 \text{ ft}$$
$$A = 44°12'34'' \quad B = 39°26'56''$$
$$v_1 = 8°12'47'' \quad v_2 = 5°50'10''$$

What is the elevation of the chimney stack?

Solution

By Equations (11.42) and (11.43), the lengths of AI_A and BI_B are

$$AI_A = \frac{136.45 \sin 39°26'56''}{\sin(44°12'34'' + 39°26'56'')} = 87.233 \text{ ft}$$
$$BI_B = \frac{136.45 \sin 44°12'34''}{\sin(44°12'34'' + 39°26'56'')} = 95.730 \text{ ft}$$

From Equation (11.44), length I_AP is

$$I_AP = 87.233 \tan 8°12'47'' = 12.591 \text{ ft}$$

And from Equation (11.45), length I_BP is

$$I_BP = 95.730 \tan 5°50'10'' = 9.785 \text{ ft}$$

Finally, by Equation (11.46), the elevation of point P is

$$Elev_P = \frac{12.591 + 298.65 + 5.55 + 9.785 + 301.53 + 5.48}{2} = 316.79 \text{ ft}$$

11.10 THREE-DIMENSIONAL TWO-POINT RESECTION

The three-dimensional (3D) coordinates X_P, Y_P, and Z_P of a point such as P of Figure 11.13 can be determined based upon angle and distance observations made from that point to two other stations of known positions. This procedure is convenient for establishing coordinates of occupied stations on elevated structures, or in depressed areas such as in mines. In Figure 11.13, for example, assume that a total station instrument is placed at point P, whose X_P, Y_P, and Z_P coordinates are unknown, and that control points A and B are visible from P. Slope lengths PA

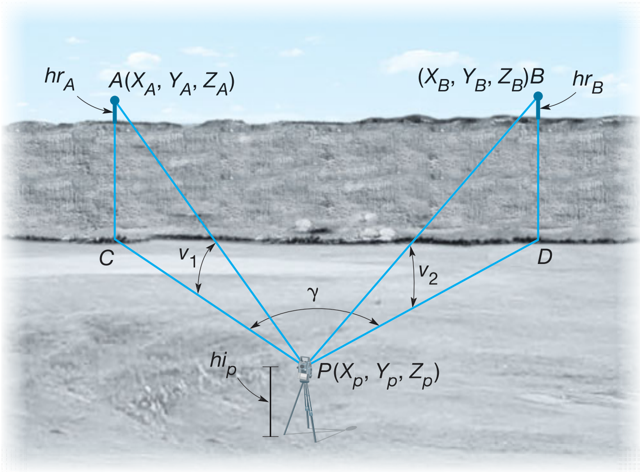


Figure 11.13
Geometry of the 3D two-point resection problem.

and *PB* are observed along with horizontal angle and vertical angles and The computational process for determining X_P, Y_P, and Z_P is as follows.

1. Determine the length and azimuth of *AB* using Equations (11.4) and (11.5).
2. Compute horizontal distances *PC* and *PD* as

$$\begin{aligned} PC &= PA\cos(v_1) \\ PD &= PB\cos(v_2) \end{aligned} \tag{11.47}$$

   where *C* and *D* are vertically beneath *A* and *B*, respectively.
3. Using Equation (11.3), calculate horizontal angle *DCP* as

$$DCP = \cos^{-1}\left(\frac{AB^2 + PC^2 - PD^2}{2(AB)PC}\right) \tag{11.48}$$

4. Determine the azimuth of line *AP* as

$$Az_{AP} = Az_{AB} + DCP \tag{11.49}$$

5. Compute the planimetric (*X*-*Y*) coordinates of point *P* as

$$\begin{aligned} X_P &= X_A + PC\sin Az_{AP} \\ Y_P &= Y_A + PC\cos Az_{AP} \end{aligned} \tag{11.50}$$

6. Determine elevation differences *AC* and *BD* as

$$\begin{aligned} AC &= PA\sin(v_1) \\ BD &= PB\sin(v_2) \end{aligned} \tag{11.51}$$

7. And finally calculate the elevation of *P* as

$$\begin{aligned} Elev_{P1} &= Elev_A + hr_A - AC - hi_P \\ Elev_{P2} &= Elev_B + hr_B - BD - hi_P \\ Elev_P &= \frac{Elev_{P1} + Elev_{P2}}{2} \end{aligned} \tag{11.52}$$

In Equations (11.52), hi_P is the height of instrument above point P, and hr_A and hr_B are the reflector heights above stations A and B, respectively.

Example 11.8

For Figure 11.13, the X, Y, and Z coordinates (in meters) of station A are 7034.982, 5413.896, and 432.173, respectively, and those of B are 7843.745, 5807.242, and 428.795, respectively. Determine the 3D position of a total station instrument at point P based upon the following observations.

$$v_1 = 24°33'42'' \quad PA = 667.413 \text{ m} \quad hr_A = 1.743 \text{ m} \quad \gamma = 77°48'08''$$

$$v_2 = 26°35'08'' \quad PB = 612.354 \text{ m} \quad hr_B = 1.743 \text{ m} \quad hi_P = 1.685 \text{ m}$$

Solution

1. Using Equations (11.4) and (11.5), determine the length and azimuth of line AB.

$$AB = \sqrt{(7843.745 - 7034.982)^2 + (5807.242 - 5413.896)^2} = 899.3435 \text{ m}$$

$$Az_{AB} = \tan^{-1}\left(\frac{7843.745 - 7034.982}{5807.242 - 5413.896}\right) + 0° = 64°03'49.6''$$

2. By Equations (11.47), determine lengths PC and PD.

$$PC = 667.413 \cos(24°33'42'') = 607.0217 \text{ m}$$
$$PD = 612.354 \cos(26°35'08'') = 547.6080 \text{ m}$$

3. From Equation (11.48), compute angle DCP.

$$DCP = \cos^{-1}\left(\frac{899.3435^2 + 607.0217^2 - 547.6080^2}{2(899.3435)607.0217}\right) = 36°31'24.2''$$

Note that this computed angle can be checked by using the law of sines, Equation (11.1), as

$$DCP = \sin^{-1}\left(\frac{547.6080 \sin 77°48'08''}{899.3435}\right) = 36°31'24.2'' \text{ (Check!)}$$

4. Using Equation (11.49), find the azimuth of line AP.

$$Az_{AP} = 64°03'49.6'' + 36°31'24.2'' = 100°35'13.8''$$

5. From Equations (11.50), compute the X-Y coordinates of point P.

$$X_P = 7034.982 + 607.0217 \sin 100°35'13.8'' = 7631.670 \text{ m}$$
$$Y_P = 5413.896 + 607.0217 \cos 100°35'13.8'' = 5302.367 \text{ m}$$

6. By Equations (11.51), compute the vertical distances of AC and BD.

$$AC = 667.413 \sin 24°33'42'' = 277.425 \text{ m}$$
$$BD = 612.354 \sin 26°35'08'' = 274.049 \text{ m}$$

7. And finally, using Equations (11.52), compute and average the elevation of point P.

$$Elev_P = 432.173 + 1.743 - 277.425 - 1.685 = 154.806 \text{ m}$$
$$Elev_P = 428.795 + 1.743 - 274.049 - 1.685 = 154.804 \text{ m}$$
$$\text{Average Elevation} = 154.805 \text{ m}$$

11.11 SOFTWARE

Coordinate geometry provides a convenient approach to solving problems in almost all types of modern surveys. Many problems that otherwise appear difficult can be greatly simplified and readily solved by working with coordinates. Although the calculations are sometimes rather lengthy, this has become inconsequential with the advent of computers and data collectors. Many software packages are available for performing coordinate geometry calculations. However, people involved in surveying (geomatics) must understand the basis for the computations, and they must exercise all possible checks to verify the accuracy of their results.

The Mathcad worksheet *C11.xmcd*, which is available on the companion website for this book at http://www.pearsonhighered.com/ghilani, demonstrates the programming of each example shown in this chapter. This software demonstrates the step-by-step approach in solving these problems. Programming of these problems in a higher-level programming language eliminates many of the mistakes that can occur when solving these problems by conventional methods. Figure 11.14 shows the coordinate geometry submenu from the WOLFPACK program, which is also available on the companion website. Also note in the figure, the menu options for a 2D conformal coordinate transformation, and a quadratic equation solver. The 2D conformal coordinate transformation requires a data file. The format for this file is discussed in the WOLFPACK help system, which is shown in Figure 11.15. This file can be created in a text editor.

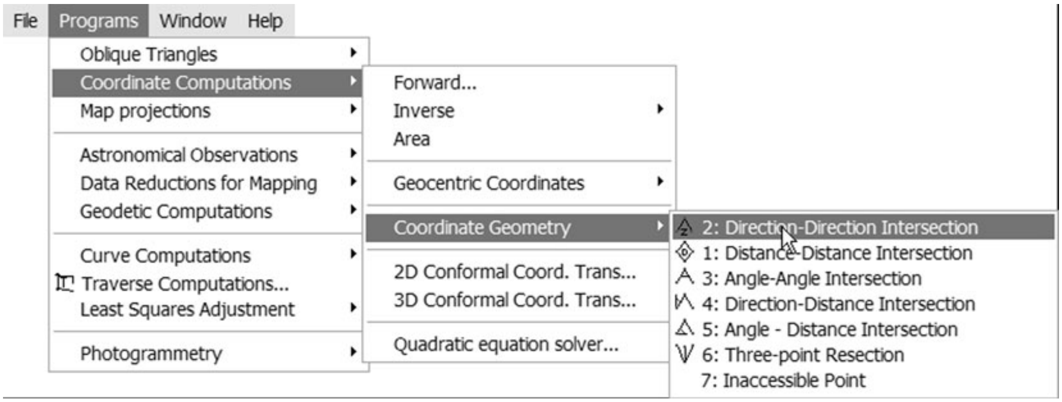


Figure 11.14 Coordinate geometry submenu from WOLFPACK program.

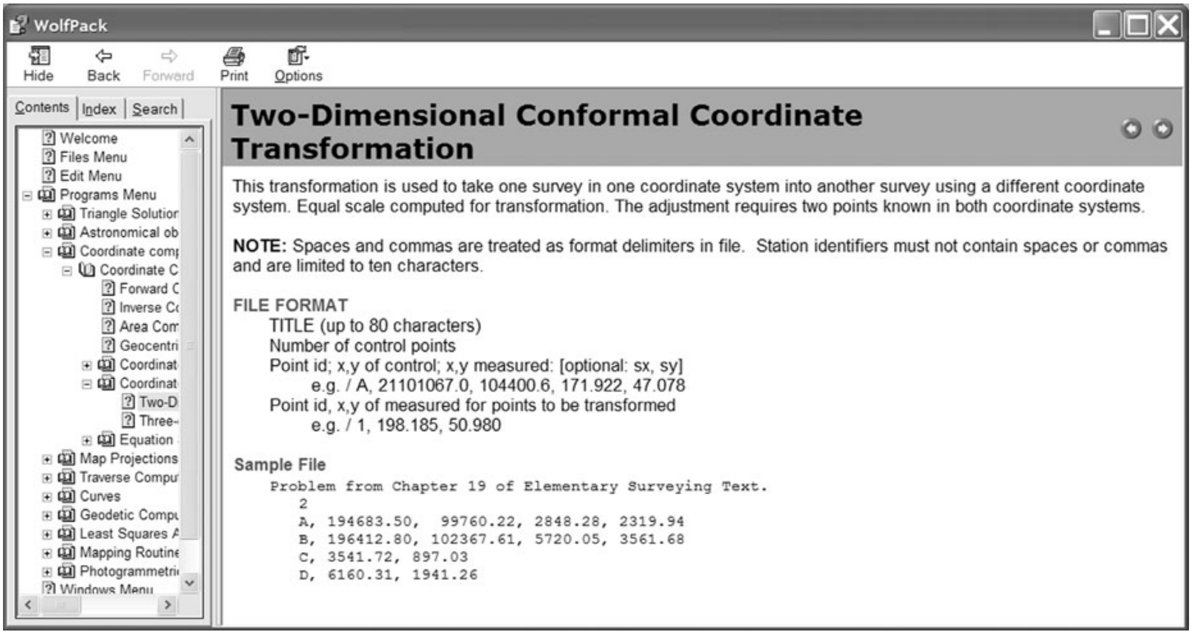


Figure 11.15 Help screen for 2D conformal coordinate transformation from WOLFPACK program.

WOLFPACK contains an editor for this purpose. Its solution is also demonstrated in the Mathcad worksheet *C11-8.XMCD*, which is also available on the companion website for this book, demonstrates the least-squares solution of the example in Section 11.8.

Because of the nature of trigonometric functions, computations in some coordinate geometry problems will become numerically unstable when the angles involved approach 0° or 90°. Thus, if coordinate geometry is intended to be used to determine the locations of points, it is generally prudent to design the survey so that triangles used in the solution have angles between 30° and 60°. Also, it is important to observe good surveying practices in the field, such as taking the averages of equal numbers of direct and reversed angle observations, and exercising other checks and precautions.

As will be seen later, coordinate geometry plays an important role in computing highway alignments, in subdivision designs, and in the operation of geographic information systems.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

11.1 The X and Y coordinates (in meters) of station Shore are 379.241 and 819.457, respectively, and those for station Rock are 437.854 and 973.482, respectively. What are the azimuth, bearing, and length of the line connecting station Shore to station Rock?

11.2 Same as Problem 11.1, except that the X and Y coordinates (in feet) of Shore are 3875.17 and 5678.15, respectively, and those for Rock are 1831.49 and 3849.61, respectively.

11.3* What are the slope, and y-intercept for the line in Problem 11.1?

11.4 What are the slope, and the y-intercept for the line in Problem 11.2?

11.5* If the slope (XY plane) of a line is 0.800946, what is the azimuth of the line to the nearest second of arc? (XY plane)

11.6 If the slope (XY plane) of a line is −0.240864, what is the azimuth of the line to the nearest second of arc? (XY plane)

11.7* What is the perpendicular distance of a point from the line in Problem 11.1, if the X and Y coordinates (in meters) of the point are 422.058 and 932.096, respectively?

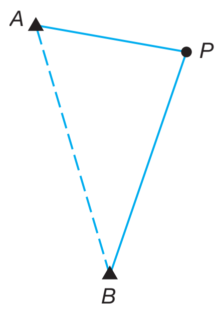
11.8 What is the perpendicular distance of a point from the line in Problem 11.2, if the X and Y coordinates (in feet) of the point are 4651.08 and 2698.98, respectively?

11.9* A line with an azimuth of 105°46′33″ from a station with X and Y coordinates of 5885.31 and 5164.15, respectively, is intersected with a line that has an azimuth of 200°31′24″ from a station with X and Y coordinates of 7337.08 and 5949.99, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

11.10 A line with an azimuth of 74°39′34″ from a station with X and Y coordinates of 1530.66 and 1401.08, respectively, is intersected with a line that has an azimuth of 301°56′04″ from a station with X and Y coordinates of 1895.53 and 1348.16, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

11.11 Same as Problem 11.9 except that the bearing of the first line is S 50°22′44″ E and the bearing of the second line is S 28°42′20″ W.

11.12 In the accompanying figure, the X and Y coordinates (in meters) of station A are 5084.274 and 8579.124, respectively, and those of station B are 6012.870 and 6589.315, respectively. Angle BAP was measured as 315°15′47″ and angle ABP was measured as 41°21′58″. What are the coordinates of station P?



Problems 11.12 through 11.16 Field conditions for intersections.

11.13* In the accompanying figure, the X and Y coordinates (in feet) of station A are 1248.16 and 3133.35, respectively, and those of station B are 1509.15 and 1101.89, respectively. The length of BP is 2657.45 ft, and the azimuth of line AP is 98°25′00″. What are the coordinates of station P?

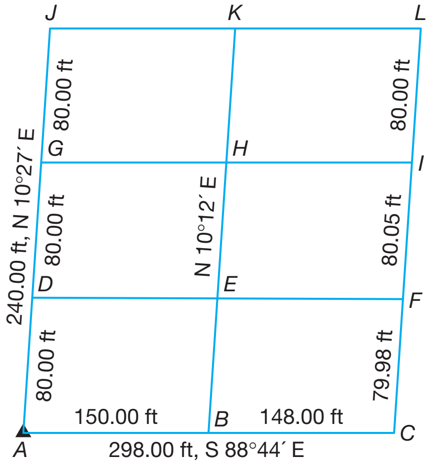
11.14 In the accompanying figure, the X and Y coordinates (in feet) of station A are 7593.15 and 9971.03, respectively, and those of station B are 8401.78 and 7714.63, respectively. The length of AP is 1987.54 ft, and angle ABP is 30°58′26″. What are the possible coordinates for station P?

11.15* A circle of radius 798.25 ft, centered at point A, intersects another circle of radius 1253.64 ft, centered at point B. The X and Y coordinates (in feet) of A are 3548.53

and 2836.49, respectively, and those of B are 4184.62 and 1753.52, respectively. What are the coordinates of station P in the figure?

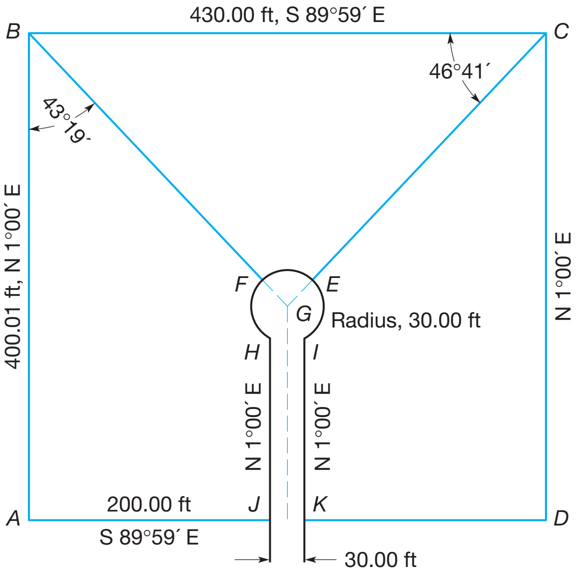
11.16 The same as Problem 11.15, except the radii from A and B are 787.02 ft and 1405.74 ft, respectively, and the X and Y coordinates (in feet) of A are 4058.74 and 6311.32, respectively, and those of station B are 4581.52 and 4345.16, respectively.

11.17 For the subdivision in the accompanying figure, assume that lines AC, DF, GI, and JL are parallel, but that lines BK and CL are parallel to each other, but not parallel to AJ. If the X and Y coordinates (in feet) of station A are (1000.00, 1000.00), what are the coordinates of each lot corner shown?



Problem 11.17 Subdivision.

11.18 If the X and Y coordinates (in feet) of station A are (5000.00, 5000.00), what are the coordinates of the remaining labeled corners in the accompanying figure?



Problem 11.18 Subdivision.

11.19* In Figure 11.8, the X and Y coordinates (in feet) of A are 1234.98 and 5415.48, respectively, those of B are 3883.94 and 5198.47, respectively, and those of C are 6002.77 and 5603.25, respectively. Also angle x is 36°59′21″ and angle y is 44°58′06″. What are the coordinates of station P?

11.20 In Figure 11.8, the X and Y coordinates (in feet) of A are 4371.56 and 8987.63, those of B are 8531.05 and 8312.57, and those of C are 10,240.98 and 8645.07, respectively. Also angle x is 50°12′45″ and angle y is 44°58′06″. What are the coordinates of station P?

11.21 In Figure 11.9, the following EN and XY coordinates for points A through D are given. In a 2D conformal coordinate transformation, to convert the XY coordinates into the EN system, what are the

(a)* Scale factor?
(b) Rotation angle?
(c) Translations in X and Y?
(d) Coordinates of points C in the EN coordinate system?

	State Plane Coordinates (m)		Arbitrary Coordinates (ft)	
Point	***E***	***N***	***X***	***Y***
A	639,940.832	642,213.266	2154.08	5531.88
B	641,264.746	641,848.554	6488.16	4620.34
C			5096.84	5995.7392

11.22 Do Problem 11.21 with the following coordinates.

	State Plane Coordinates (m)		Arbitrary Coordinates (m)	
Point	***E***	***N***	***X***	***Y***
A	588,933.451	418,953.421	5492.081	3218.679
B	588,539.761	420,185.869	6515.987	4009.588
C			4865.191	3649.031

11.23 In Figure 11.12, the elevations of stations A and B are 403.16 and 410.02 ft, respectively. Instrument heights hi_A and hi_B are 5.20 and 5.06 ft, respectively. What is the average elevation of point P if the other field observations are:

$AB = 256.79$ ft
$A = 52°30'08''$ $B = 40°50'51''$
$v_1 = 24°38'15''$ $v_2 = 22°35'42''$

11.24 In Problem 11.23, assume station P is to the left of the line AB, as viewed from station A. If the X and Y coordinates (in feet) of station A are 1245.68 and 543.20, respectively, and the azimuth of line AB is 55°23′44″, what are the X and Y coordinates of the inaccessible point?

11.25 In Figure 11.12, the elevations of stations A and B *are* 1106.78 and 1116.95 ft, respectively. Instrument heights hi_A and hi_B are 5.14 and 5.43 ft, respectively. What is the average elevation of point P if the other field observations are:

$AB = 438.18$ ft
$A = 49°31'00''$ $B = 52°35'26''$
$v_1 = 27°40'57''$ $v_2 = 27°20'51''$

11.26 In Problem 11.25, assume station P is to the left of line AB as viewed from station A. If the X and Y coordinates (in feet) of station A are 8975.18 and 7201.89, respectively, and the azimuth of line AB is 347°22′38″, what are the X and Y coordinates of the inaccessible point?

11.27 In Figure 11.13, the X, Y, and Z coordinates (in feet) of station A are 1816.45, 987.39, and 1806.51, respectively, and those of B are 1633.11, 1806.48, and 1806.48, respectively. Determine the $3D$ position of the occupied station P with the following observations:

$$v_1 = 30°06'22'' \quad PA = 228.50 \text{ ft} \quad hr_A = 5.68 \text{ ft} \quad \gamma = 72°02'28''$$
$$v_2 = 29°33'02'' \quad PB = 232.35 \text{ ft} \quad hr_B = 5.68 \text{ ft} \quad hi_P = 5.34 \text{ ft}$$

11.28 Adapt Equations (11.43) and (11.47) so they are applicable for zenith angles.

11.29 In Figure 11.13, the X, Y, and Z coordinates (in meters) of station A are 135.461, 211.339, and 98.681, respectively, and those of B are 301.204, 219.822, and 100.042, respectively. Determine the $3D$ position of occupied station P with the following observations:

$$z_1 = 119°22'38'' \quad PA = 150.550 \text{ m} \quad hr_A = 1.690 \text{ m} \quad \gamma = 79°05'02''$$
$$z_2 = 120°08'50'' \quad PB = 149.770 \text{ m} \quad hr_B = 1.690 \text{ m} \quad hi_P = 1.685 \text{ m}$$

11.30 Use WOLFPACK to do Problem 11.9.
11.31 Use WOLFPACK to do Problem 11.10.
11.32 Use WOLFPACK to do Problem 11.12.
11.33 Use WOLFPACK to do Problem 11.13.
11.34 Use WOLFPACK to do Problem 11.15.
11.35 Use WOLFPACK to do Problem 11.16.
11.36 Use WOLFPACK to do Problem 11.17.
11.37 Write a computational program that solves Example 11.6 using matrices.
11.38 Write a computational program that solves Example 11.8.

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12 Area

12.1 INTRODUCTION

There are a number of important reasons for determining areas. One is to include the acreage of a parcel of land in the deed describing the property. Other purposes are to determine the acreage of fields, lakes, etc., or the number of square yards to be surfaced, paved, seeded, or sodded. Another important application is determining end areas for earthwork volume calculations (see Chapter 26).

In plane surveying, area is considered to be the orthogonal projection of the surface onto a horizontal plane. As noted in Chapter 2, in the English system the most commonly used units for specifying small areas are the ft^2 and yd^2, and for large tracts the acre is most often used, where $1\,\text{acre} = 43{,}560\,ft^2 = 10\,ch^2$ (Gunter's). An acre lot, if square, would thus be 208.71^+ ft on a side. In the metric system, smaller areas are usually given in m^2, and for larger tracts *hectares* are commonly used, where 1 hectare is equivalent to a square having sides of 100 m, and thus equals 10,000 m^2. In converting areas between the English and metric systems, the conversion factors given in Table 12.1 are useful.

12.2 METHODS OF MEASURING AREA

Both field and map measurements are used to determine area. Field measurement methods are the more accurate and include (1) division of the tract into simple figures (triangles, rectangles, and trapezoids), (2) offsets from a straight line, (3) coordinates, and (4) double-meridian distances. Each of these methods is described in sections that follow.

Methods of determining area from map measurements include (1) counting coordinate squares, (2) dividing the area into triangles, rectangles, or other regular geometric shapes, (3) digitizing coordinates, and (4) running a planimeter over

TABLE 12.1 APPROXIMATE AREA CONVERSION FACTORS

To Convert from	To	Multiply by
ft^2	m^2	$(12/39.37)^2 \approx 0.09291$
m^2	ft^2	$(39.37/12)^2 \approx 10.76364$
yd^2	m^2	$(36/39.37)^2 \approx 0.83615$
m^2	yd^2	$(39.37/36)^2 \approx 1.19596$
acres	hectares	$[39.37/(4.356 \times 12)]^2 \approx 2.47099$
hectares	acres	$(4.356 \times 12/39.37)^2 \approx 0.40470$

the enclosing lines. These processes are described and illustrated in Section 12.9. Because maps themselves are derived from field observations, methods of area determination invariably depend on this basic source of data.

12.3 AREA BY DIVISION INTO SIMPLE FIGURES

A tract can usually be divided into simple geometric figures such as triangles, rectangles, or trapezoids. The sides and angles of these figures can be observed in the field and their individual areas calculated and totaled. An example of a parcel subdivided into triangles is shown in Figure 12.1.

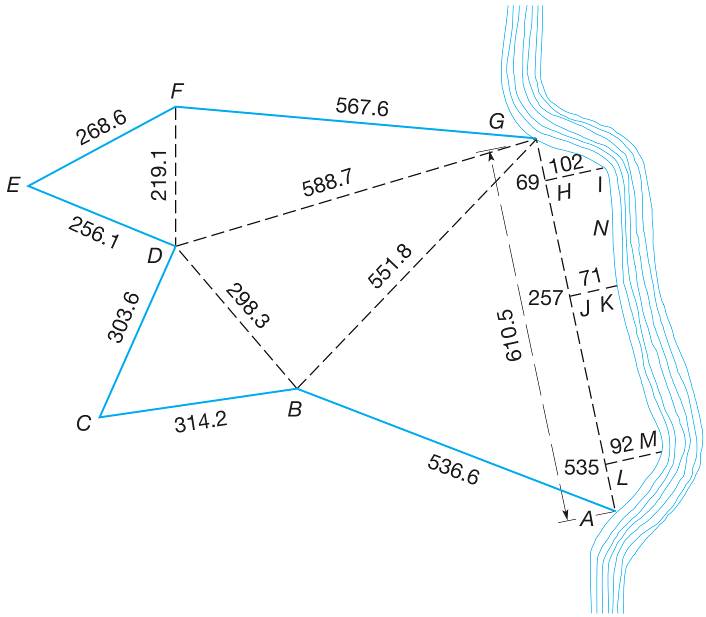

Figure 12.1
Area determination by triangles.

Formulas for computing areas of rectangles and trapezoids are well known. The area of a triangle whose lengths of sides are known can be computed by the formula

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)} \qquad \textbf{(12.1)}$$

where a, b, and c are the lengths of sides of the triangle and $s = 1/2(a + b + c)$. Another formula for the area of a triangle is

$$\text{area} = \frac{1}{2}ab \sin C \qquad \textbf{(12.2)}$$

where C is the angle included between sides a and b.

The choice of whether to use Equation (12.1) or (12.2) will depend on the triangle parts that are most conveniently determined; a decision ordinarily dictated by the nature of the area and the type of equipment available.

■ 12.4 AREA BY OFFSETS FROM STRAIGHT LINES

Irregular tracts can be reduced to a series of trapezoids by observing right-angle offsets from points along a reference line. The reference line is usually marked by stationing (see Section 5.9.1), and positions where offsets are observed are given by their stations and pluses. The spacing between offsets may be either *regular* or *irregular*, depending on the conditions. These two cases are discussed in the subsections that follow.

12.4.1 Regularly Spaced Offsets

Offsets at regularly spaced intervals are shown in Figure 12.2. For this case, the area is found by the formula

$$\text{area} = b\left(\frac{h_0}{2} + h_1 + h_2 + \cdots + \frac{h_n}{2}\right) \qquad \textbf{(12.3)}$$

where b is the length of a common interval between offsets, and $h_0, h_1, \ldots, h_n$ are the offsets. The regular interval for the example of Figure 12.2 is a half-station, or 50 ft.

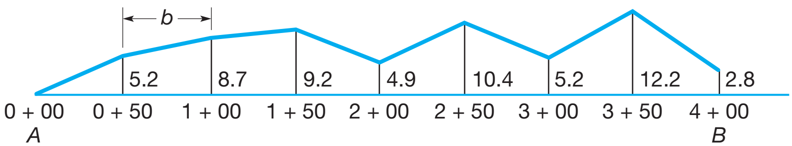

Figure 12.2 Area by offsets.

Example 12.1

Compute the area of the tract shown in Figure 12.2.

Solution

By Equation (12.3)

$$\text{area} = 50\left(0 + 5.2 + 8.7 + 9.2 + 4.9 + 10.4 + 5.2 + 12.2 + \frac{2.8}{2}\right)$$
$$= 2860 \text{ ft}^2$$

In this example, a summation of offsets (terms within parentheses) can be secured by the *paper-strip method*, in which the area is plotted to scale and the midordinate of each trapezoid is successively added by placing tick marks on a long strip of paper. The area is then obtained by making a single measurement between the first and last tick marks, multiplying by the scale to convert it to a field distance, and then multiplying by width b.

12.4.2 Irregularly Spaced Offsets

For irregularly curved boundaries like that in Figure 12.3, the spacing of offsets along the reference line varies. Spacing should be selected so that the curved boundary is accurately defined when adjacent offset points on it are connected by straight lines. A formula for calculating area for this case is

$$\text{area} = \frac{1}{2}[a(h_0 + h_1) + b(h_1 + h_2) + c(h_2 + h_3) + \cdots] \tag{12.4}$$

where $a, b, c, \ldots$ are the varying offset spaces, and $h_0, h_1, h_2, \ldots$ are the observed offsets.

Example 12.2

Compute the area of the tract shown in Figure 12.3.

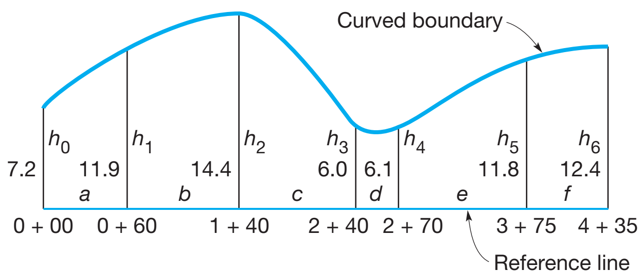

Figure 12.3 Area by offsets for a tract with a curved boundary.

Solution

By Equation (12.4)

$$\begin{aligned}\text{area} &= \frac{1}{2}[60(7.2 + 11.9) + 80(11.9 + 14.4) + 100(14.4 + 6.0) \\ &\quad + 30(6.0 + 6.1) + 105(6.1 + 11.8) + 60(11.8 + 12.4)] \\ &= 4490 \text{ ft}^2\end{aligned}$$

12.5 AREA BY COORDINATES

Computation of area within a closed polygon is most frequently done by the coordinate method. In this procedure, coordinates of each angle point in the figure must be known. They are normally obtained by traversing, although any method that yields the coordinates of these points is appropriate. If traversing is used, coordinates of the stations are computed after adjustment of the departures and latitudes, as discussed and illustrated in Chapter 10. The coordinate method is also applicable and convenient for computing areas of figures whose coordinates have been digitized using an instrument like that shown in Figure 28.9. The coordinate method is easily visualized; it reduces to one simple equation that applies to all geometric configurations of closed polygons and is readily programmed for computer solution.

The procedure for computing areas by coordinates can be developed with reference to Figure 12.4. As shown in that figure, it is convenient (but not necessary) to adopt a reference coordinate system with the X and Y axes passing through the most southerly and the most westerly traverse stations, respectively. Lines BB', CC', DD', and EE' in the figure are constructed perpendicular to the Y axis. These lines create a series of trapezoids and triangles (shown by different color shadings). The area enclosed with traverse $ABCDEA$ can be expressed in terms of the areas of these individual trapezoids and triangles as

$$\begin{aligned}\text{area}_{ABCDEA} &= E'EDD'E' + D'DCC'D' \\ &\quad - AE'EA - CC'B'BC - ABB'A \end{aligned} \qquad \textbf{(12.5)}$$

The area of each trapezoid, for example $E'EDD'E'$ can be expressed in terms of lengths as

$$\text{area}_{E'EDD'E'} = \frac{E'E + DD'}{2} \times E'D'$$

In terms of coordinate values, this same area $E'EDD'E'$ is

$$\text{area}_{E'EDD'E'} = \frac{X_E + X_D}{2}(Y_E - Y_D)$$

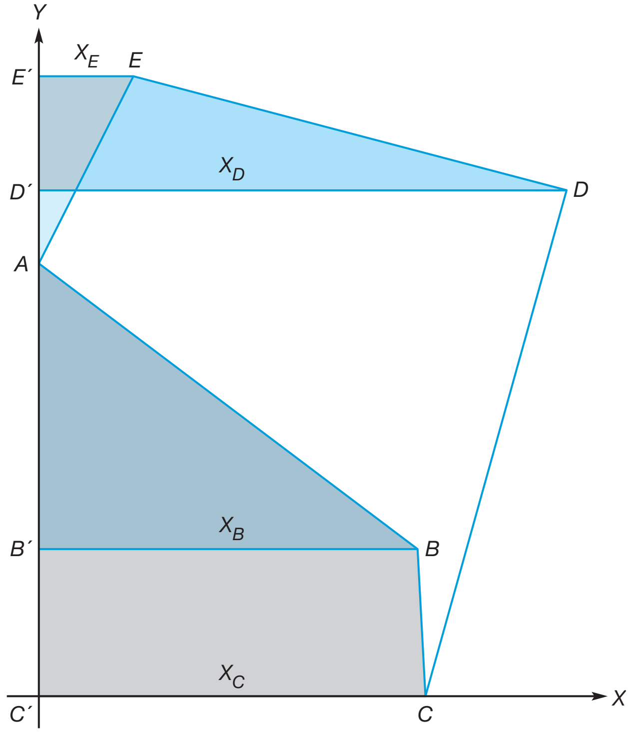

Figure 12.4
Area computation by the coordinate method.

Each of the trapezoids and triangles of Equation (12.5) can be expressed by coordinates in a similar manner. Substituting these coordinate expressions into Equation (12.5), multiplying by 2 to clear fractions, and rearranging

$$
\begin{aligned}
2(\text{area}) = &+X_AY_B + X_BY_C + X_CY_D + X_DY_E + X_EY_A \\
&-X_BY_A - X_CY_B - X_DY_C - X_EY_D - X_AY_E
\end{aligned}
\tag{12.6}
$$

Equation (12.6) can be reduced to an easily remembered form by listing the X and Y coordinates of each point in succession in two columns, as shown in Equation (12.7), *with coordinates of the starting point repeated at the end*. The products noted by diagonal arrows are ascertained with dashed arrows considered plus and solid ones minus. *The algebraic summation of all products is computed and its absolute value divided by 2 to get the area*.

$$
\begin{array}{cc}
X_A & Y_A \\
X_B & Y_B \\
X_C & Y_C \\
X_D & Y_D \\
X_E & Y_E \\
X_A & Y_A
\end{array}
\tag{12.7}
$$

The procedure indicated in Equation (12.7) is applicable to calculating any size traverse. The following formula, easily derived from Equation (12.6), is a variation that can also be used,

$$\text{area} = \frac{1}{2}\big[X_A(Y_E - Y_B) + X_B(Y_A - Y_C) + X_C(Y_B - Y_D) + X_D(Y_C - Y_E) + X_E(Y_D - Y_A)\big] \tag{12.8}$$

It was noted earlier that for convenience, an axis system can be adopted in which $X = 0$ for the most westerly traverse point, and $Y = 0$ for the most southerly station. Magnitudes of coordinates and products are thereby reduced, and the amount of work lessened, since four products will be zero. However, selection of a special origin like that just described is of little consequence if the problem has been programmed for computer solution. Then the coordinates obtained from traverse adjustment can be used directly in the solution. However, a word of caution applies, if coordinate values are extremely large as they would normally be; for example, if state plane values are used (see Chapter 20). In those cases, to ensure sufficient precision and prevent serious round-off errors, double precision should be used. Or, as an alternative, the decimal place in each coordinate can arbitrarily be moved n places to the left, the area calculated, and then multiplied by 10^{2n}.

Either Equation (12.6) or Equation (12.8) can be readily programmed for solution by computer. The program WOLFPACK has this option under its *coordinate computations* menu. The format of the data file for this option is listed in its help screen. As was noted in Chapter 10, the "closed polygon traverse" option of WOLFPACK also computes areas using the coordinates of the adjusted traverse stations. A Mathcad® worksheet C12.xmcd, which is available on the companion website for this book at http://www.pearsonhighered.com/ghilani, demonstrates the computations in Sections 12.3 through 12.5.

■ Example 12.3

Figure 12.5 illustrates the same traverse as Figure 12.4. The computations in Table 10.4 apply to this traverse. Coordinate values shown in Figure 12.5, however, result from shifting the axes so that $X_A = 0.00$ (A is the most westerly station) and $Y_C = 0.00$ (C is the most southerly station). This was accomplished by subtracting 10,000.00 (the value of X_A) from all X coordinates, and subtracting 4408.22 (the value of Y_C) from all Y coordinates. Compute the traverse area by the coordinate method. (Units are feet.)

Solution

These computations are best organized for tabular solution. Table 12.2 shows the procedure. Thus, the area contained within the traverse is

$$\text{area} = \frac{|1{,}044{,}861 - 499{,}684|}{2} = 272{,}588 \text{ ft}^2 \text{ (say } 272{,}600 \text{ ft}^2) = 6.258 \text{ acres}$$

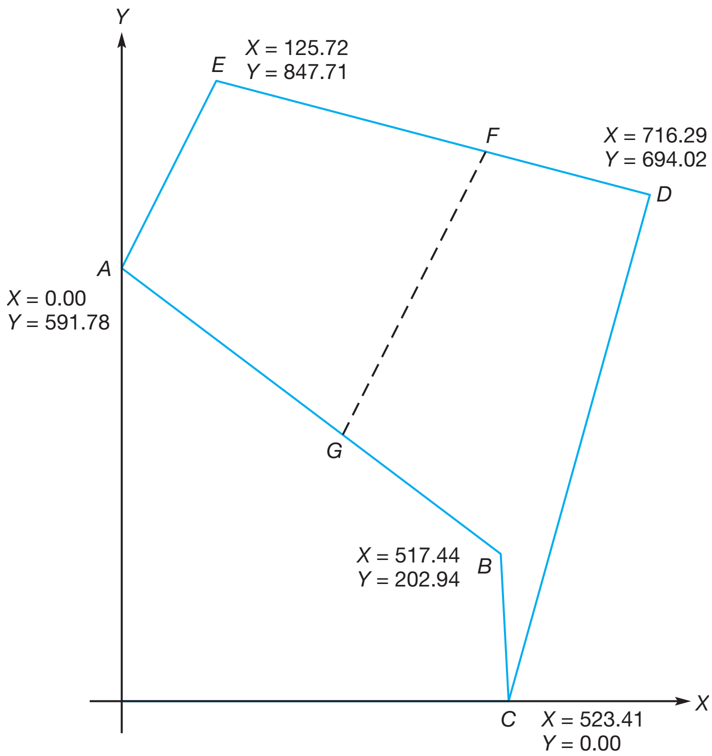

Figure 12.5
Traverse for computation of area by coordinates.

TABLE 12.2 COMPUTATION OF AREA BY COORDINATES

			Double Area (ft^2)	
Point	***X* (ft)**	***Y* (ft)**	**Plus (*XY*)**	**Minus (*YX*)**
A	0.00	591.78		
B	517.44	202.94	0	306,211
C	523.41	0.00	0	106,221
D	716.29	694.02	363,257	0
E	125.72	847.71	607,206	87,252
A	0.00	591.78	74,398	0
			Σ = 1,044,861	Σ = 499,684
			−499,684	
			545,177	

545,177 ÷ 2 = 272,588 ft^2 = 6.258 acres

Notice that the precision of the computations was limited to four digits. This is due to the propagation of errors as discussed in Section 3.17.3. As an example, consider a square that has the same area as the parcel in Table 12.2. The length of its sides would be approximately 522.1 ft. Assuming that these coordinates have uncertainties of about ±0.05 ft, the error in the product as given by Equation (3.13) would be

$$E_{\text{area}} = \sqrt{(522.1 \times 0.05)^2 + (522.1 \times 0.05)^2} = \pm 37\,\text{ft}^2$$

Thus, rounding the computed area to the nearest hundred square feet is justified. As a *rule of thumb*, the accuracy of the area should not be stated any better than

$$E_{\text{area}} = \sigma_S S\sqrt{2} \tag{12.9}$$

where S is the length of the side of a square having an area equivalent to the parcel being considered, and σ_S is the uncertainty in the coordinates of the points that bound the area in question.

Because of the effects of error propagation, it is important to remember that it is better to be conservative when expressing areas, and thus a phrase such as "6.258 acres more or less" is often adopted, especially when writing property descriptions (see Chapter 21).

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The video *Area Computations.mp4* demonstrates the computation of areas in Figures 12.1 and 12.5.

■ 12.6 AREA BY DOUBLE-MERIDIAN DISTANCE METHOD

The area within a closed figure can also be computed by the double-meridian distance (DMD) method. This procedure requires balanced departures and latitudes of the tract's boundary lines, which are normally obtained in traverse computations. The DMD method is not as commonly used as the coordinate method because it is not as convenient, but given the data from an adjusted traverse, it will yield the same answer. The DMD method is useful for checking answers obtained by the coordinate method when performing hand computations.

By definition, *the meridian distance of a traverse course is the perpendicular distance from the midpoint of the course to the reference meridian*. To ease the problem of signs, a reference meridian usually is placed through the most westerly traverse station.

In Figure 12.6, the meridian distances of courses AB, BC, CD, DE, and EA are MM', PP', QQ', RR', and TT', respectively. To express PP' in terms of convenient distances, MF and BG are drawn perpendicular to PP'. Then

$$\begin{aligned} PP' &= P'F + FG + GP \\ &= \text{meridian distance of } AB + \frac{1}{2}\text{departure of } AB + \frac{1}{2}\text{departure of } BC \end{aligned}$$

Thus, the meridian distance for any course of a traverse equals the meridian distance of the preceding course plus one half the departure of the preceding

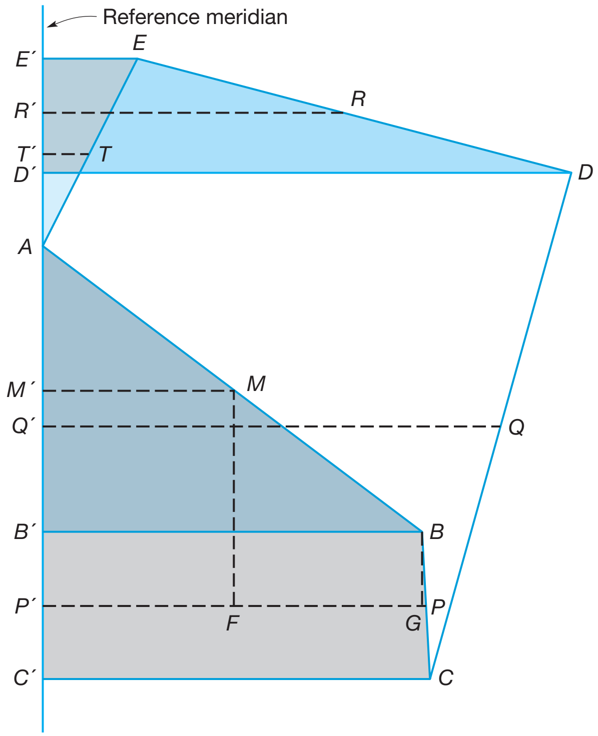

Figure 12.6 Meridian distances and traverse area computation by DMD method.

course plus half the departure of the course itself. It is simpler to employ full departures of courses. Therefore, DMDs equal to twice the meridian distances that are used, and a single division by 2 is made at the end of the computation.

Based on the considerations described, the following general rule can be applied in calculating DMDs: *The DMD for any traverse course is equal to the DMD of the preceding course, plus the departure of the preceding course, plus the departure of the course itself*. Signs of the departures must be considered. When the reference meridian is taken through the most westerly station of a closed traverse and calculations of the DMDs are started with a course through that station, *the DMD of the first course is its departure*. Applying these rules, for the traverse in Figure 12.6

$$\text{DMD of } AB = \text{departure of } AB$$
$$\text{DMD of } BC = \text{DMD of } AB + \text{departure of } AB + \text{departure of } BC$$

A check on all computations is obtained if the DMD of the last course, after computing around the traverse, is also equal to its departure but has the opposite sign. If there is a difference, the departures were not correctly adjusted before starting, or a mistake was made in the computations. With reference to Figure 12.6, the area enclosed by traverse $ABCDEA$ may be expressed in terms of trapezoid areas (shown by different color shadings) as

$$\text{area} = E'EDD'E' + C'CDD'C' - (AB'BA + BB'C'CB + AEE'A) \tag{12.10}$$

The area of each figure equals the meridian distance of a course times its balanced latitude. For example, the area of trapezoid $C'CDD'C' = Q'Q \times C'D'$, where $Q'Q$ and $C'D'$ are the meridian distance and latitude, respectively, of line CD. The DMD of a course multiplied by its latitude equals double the area. Thus, the algebraic summation of all double areas gives *twice the area* inside the entire traverse. Signs of the products of DMDs and latitudes must be considered. If the reference line is passed through the most westerly station, all DMDs are positive. The products of DMDs and north latitudes are therefore plus and those of DMDs and south latitudes are minus.

Example 12.4

Using the balanced departures and latitudes listed in Table 10.4 for the traverse of Figure 12.6, compute the DMDs of all courses.

Solution

The calculations done in tabular form following the general rule, are illustrated in Table 12.3.

Example 12.5

Using the DMDs determined in Example 12.4, calculate the area within the traverse.

TABLE 12.3 COMPUTATION OF DMDs

Departure of *AB* =	+517.444	= DMD of *AB*
Departure of *AB* =	+517.444	
Departure of *BC* =	+5.964	
	+1040.852	= DMD of *BC*
Departure of *BC* =	+5.964	
Departure of *CD* =	+192.881	
	+1239.697	= DMD of *CD*
Departure of *CD* =	+192.881	
Departure of *DE* =	−590.571	
	+842.007	= DMD of *DE*
Departure of *DE* =	−590.571	
Departure of *EA* =	−125.718	
	+125.718	= DMD of *EA* ✓

TABLE 12.4 COMPUTATION OF AREA BY DMDs

Course	Balanced Departure (ft)	Balanced Latitude (ft)	DMD (ft)	Double Areas (ft^2) Plus	Double Areas (ft^2) Minus
AB	517.44	−388.84	517.44		201,201
BC	5.96	−202.95	1040.85		211,240
CD	192.88	694.02	1239.70	860,376	
DE	−590.57	153.69	842.01	129,408	
EA	−125.72	−255.93	125.72	———	32,176
Total	0.00	0.00		989,784	444,617
				−444,617	
				545,167	

545,167/2 = 272,584 ft^2 (say 272,600 ft^2) = 6.258 acres

Solution

Computations for area by DMDs are generally arranged as in Table 12.4, although a combined form may be substituted. Sums of positive and negative double areas are obtained, and the absolute value of the smaller subtracted from that of the larger. The result is divided by 2 to get the area (272,600 ft^2) and by 43,560 to obtain the number of acres (6.258). Note that the answer agrees with the one obtained using the coordinate method.

If the total of minus double areas is larger than the total of plus values, it signifies only that DMDs were computed by going around the traverse in a clockwise direction.

In modern surveying and engineering offices, area calculations are seldom done by hand; rather, they are programmed for computer solution. However, if an area is computed by hand, it should be checked by using different methods or by two persons who employ the same system. As an example, an individual working alone in an office could calculate areas by coordinates and check by DMDs. Those experienced in surveying (geomatics) have learned that a half-hour spent checking computations in the field and office can eliminate lengthy frustrations at a later time. The Mathcad worksheet *C12.XMCD*, which is available on the companion website at http://www.pearsonhighered.com/ghilani, demonstrates the programming of the coordinate method discussed in this book.

■ 12.7 AREA OF PARCELS WITH CIRCULAR BOUNDARIES

The area of a tract that has a circular curve for one boundary, as in Figure 12.7, can be found by dividing the figure into two parts: polygon *ABCDEGFA* and *sector EGF*. The radius $R = EG = FG$ and either central angle $\theta = EGF$ or

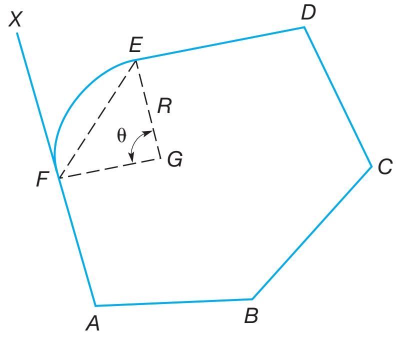

Figure 12.7
Area with circular curve as part of boundary.

length EF must be known or computed to permit calculation of sector area EGF. If R and central angle θ are known, then the area of sector is

$$EGF = \pi R^2(\theta/360°) \tag{12.11}$$

If chord length EF is known, angle $\theta = 2\sin^{-1}(EF/2R)$, and the preceding equation is used to calculate the sector area. To obtain the tract's total area, the sector area is added to area $ABCDEGFA$ found by either the coordinate or DMD method.

Another method that can be used is to compute the area of the traverse $ABCDEFA$, and then add the area of the *segment*, which is the region between the arc and chord EF. The area of a segment is found as

$$\text{Area of segment} = 0.5R^2(\theta - \sin\theta) \tag{12.12}$$

where θ is expressed in radian units.

■ 12.8 PARTITIONING OF LANDS

Calculations for purposes of partitioning land—that is, *cutting off* a portion of a tract for title transfer—can be aided significantly by using coordinates. For example, suppose the owner of the tract of land in Figure 12.5 wishes to subdivide the parcel with a line GF, parallel to AE, and have 3.000 acres in parcel $AEFG$. This problem can be approached by three different methods. The first involves trial and error, and works quite well given today's computing capabilities. The second consists of writing equations for simple geometric figures such as triangles, rectangles, and trapezoids that enable a unique solution to be obtained for the coordinates of points F and G. The third approach involves setting up a series of coordinate geometry equations, together with an area equation, and then solving for the coordinates of F and G. The following subsections describe each of the above procedures.

12.8.1 Trial and Error Method

In this approach, estimated coordinates for the positions of stations F and G are determined, and the area of parcel $AEF'G'$ is computed using Equation (12.6) where F' and G' are the estimated positions of F and G. This procedure is repeated until the area of the parcel equals 3.000 acres, or 130,680 ft^2.

Step 1: Using the final adjusted lengths and directions computed in Example 10.8 and coordinates of A and E from Example 12.3, and estimating the position of the cutoff line to be half the distance along line ED (i.e., 610.24/2 = 305.12 ft), the coordinates of stations F' and G' in parcel $AEF'G'$ are computed as

Station F':

$$X = 125.72 + 305.12 \sin 104°35'13'' = 421.00$$
$$Y = 847.71 + 305.12 \cos 104°35'13'' = 770.87$$

Station G': is determined by direction-direction intersection using procedures discussed in Section 11.4. From WOLFPACK, the coordinates of Station G' are

$$X = 243.24 \text{ and } Y = 408.99$$

Creating a file for area computations, the area contained by these four stations is only 102,874 ft^2. Since 3.000 acres is equivalent to 130,680 ft^2, the estimated distance of 305.12 was short. It can now be increased and the process repeated.

Step 2: To estimate the amount necessary to increase the distance, an assumption that the figure $F'FGG'$ is a rectangle, with one side of length $F'G'$, or 403.18 ft, where that length is obtained by inversing the coordinates of F' and G' from step 1. Thus, the amount to move the line $F'G'$ is determined as

$$(130{,}680 - 102{,}874)/403.18 = 68.97 \text{ ft}$$

Thus, for the second trial, the distance that F' is from E should be $305.12 + 68.97 = 374.09$ ft. Using the same procedure as in step 1, the area of $AEF'G'$ is 131,015 ft^2. The determined area is now too large, and can be reduced using the same assumption, that was used at the beginning of this step. Thus, the distance EF' should be

$$\begin{aligned} EF' &= 374.09 + (130{,}680 - 131{,}015)/(\text{length of } F'G') \\ &= 374.09 - 0.78 = 373.31 \end{aligned}$$

This process is repeated until the final coordinates for F and G are determined. The next iteration yielded coordinates for F' of (487.00, 753.69) and for G' of (297.61, 368.14). Using these coordinates, the area of the parcel was computed to be 130,690 ft^2, or within 10 ft^2. The process is again repeated resulting in a reduction of the distance EF' of 0.02 ft, or $EF' = 373.29$ ft. The resulting area for $AEF'G'$ is 130,679 ft^2. Since this is within 1 ft^2 of the area, the coordinates are accepted as

$$F = (486.98, 753.70)$$
$$G = (297.59, 368.16)$$

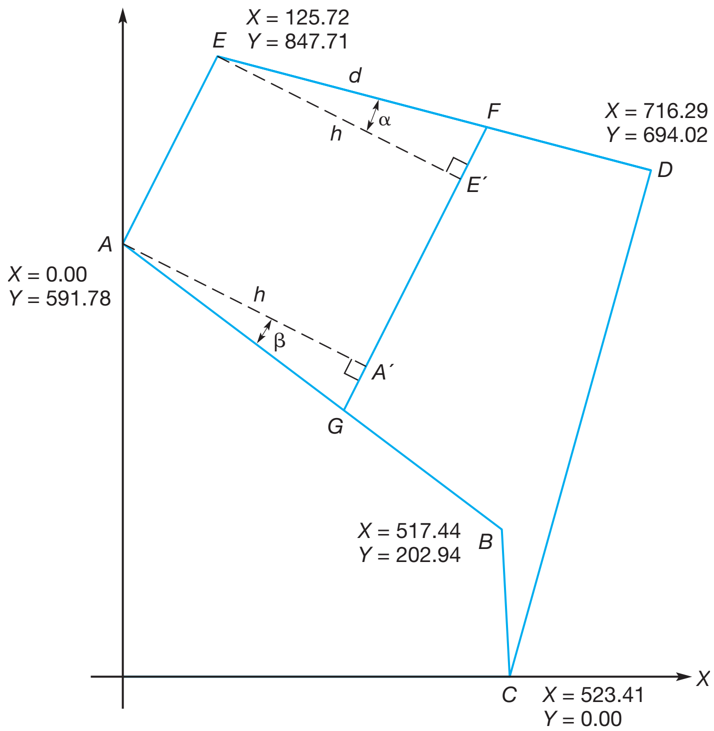

Figure 12.8 Partitioning of lands by simple geometric figures.

The trial and error approach can be applied to solve many different types of land partitioning problems. Although the procedure may appear to involve a significant number of calculations, in many cases it provides the fastest and easiest solution when a computer program such as WOLFPACK is available for doing the coordinate geometry calculations.

12.8.2 Use of Simple Geometric Figures

As can be seen in Figure 12.8, parcel $AEFG$ is a parallelogram. Thus, the formula for the area of a parallelogram $[A = 1/2(b_1 + b_2)h]$ can be employed, where b_1 is AE and b_2 is FG. In this procedure, a trigonometric relationship between the unknown length EF (denoted as d in Figure 12.8) and the missing parts h, FE', and $A'G$ must be determined. From the figure, angles α and β can be determined from azimuth differences, as

$$\alpha = AZ_{EE'} - AZ_{ED}$$

$$\beta = AZ_{AB} - AZ_{AA'}$$

Note in Table 10.7 that AZ_{EA} is 206°09′41″, and thus $AZ_{AA'}$ and $AZ_{EE'}$, which are perpendicular to line EA are 206°09′41″ − 90° = 116°09′41″. Also from Table 10.7, AZ_{ED} and AZ_{AB} are 104°35′13″ and 126°55′23″, respectively. Thus, the numerical values for α and β are:

$$\alpha = 116°09'41'' - 104°35'13'' = 11°34'28''$$
$$\beta = 126°55'23'' - 116°09'41'' = 10°45'42''$$

Now the parts h, FE', and $A'G$ can be expressed in terms of the unknown distance d as

$$\begin{aligned} h &= d\cos\alpha \\ FE' &= d\sin\alpha \\ A'G &= h\tan\beta = d\cos\alpha\tan\beta \end{aligned} \quad \textbf{(12.13)}$$

The formula for the area of parallelogram $AEFG$ is

$$\tfrac{1}{2}(AE + FE' + AE + A'G)h = 130{,}680 \quad \textbf{(12.14)}$$

Substituting Equations (12.13) into Equation (12.14), rearranging yields

$$(\cos^2\alpha\tan\beta + \cos\alpha\sin\alpha)d^2 + [2(AE)\cos\alpha]d - 261{,}360 = 0 \quad \textbf{(12.15)}$$

Expression (12.15) is a quadratic equation and can be solved using Equation (11.3). Substituting the appropriate values into Equation (12.15) and solving yields $d = EF = 373.29$ ft. This is the same answer as was derived in Section 12.8.1.

This approach of using the equations of simple geometric figures is convenient for solving a variety of land partitioning problems.

12.8.3 Coordinate Method

This method involves using Equations (10.11) and (12.8) to obtain four equations with the four unknowns X_F, Y_F, X_G, and Y_G, that can be uniquely solved. By Equation (10.11), the following three coordinate geometry equations can be written:

$$\frac{X_F - X_E}{Y_F - Y_E} = \frac{X_D - X_E}{Y_D - Y_E} \quad \textbf{(12.16)}$$

$$\frac{X_G - X_A}{Y_G - Y_A} = \frac{X_B - X_A}{Y_B - Y_A} \quad \textbf{(12.17)}$$

$$\frac{X_A - X_E}{Y_A - Y_E} = \frac{X_G - X_F}{Y_G - Y_F} \quad \textbf{(12.18)}$$

Also by area Equation (12.8):

$$\begin{aligned} X_A(Y_G - Y_E) + X_E(Y_A - Y_F) + X_F(Y_E - Y_G) \\ + X_G(Y_F - Y_A) = 2 \times \text{area} \end{aligned} \quad \textbf{(12.19)}$$

Substituting the known coordinates X_A, Y_A, X_B, Y_B, X_D, Y_D, X_E, and Y_E into Equations (12.16) through (12.19) yields four equations that can be solved for the four unknown coordinates. The four equations can be solved simultaneously, for example by using matrix methods, to determine the unknown coordinates for points F and G. (The program MATRIX is included on the companion website for this book.)

Alternatively, the four equations can be solved by substitution. In this approach, Equations (12.16) and (12.17) are rewritten in terms of one of the unknowns, say X_F and X_G. These two new equations are then substituted into Equations (12.18) and (12.19). The resultant equations will now contain two unknowns Y_F and Y_G. The equation corresponding to Equation (12.18) can then be solved in terms of unknown, say Y_F, and this can be substituted into the equation corresponding to (12.19). The resultant expression will be a quadratic equation in terms of Y_G which can be solved using Equation (11.3). This solution can then be substituted into the previous equations to derive the remaining three unknowns.

■ 12.9 AREA BY MEASUREMENTS FROM MAPS

To determine the area of a tract of land from map measurements its boundaries must first be identified on an existing map or a plot of the parcel drawn from survey data. Then one of several available methods can be used to determine its area. Accuracy in making area determinations from map measurements is directly related to the accuracy of the maps being used. Accuracy of maps, in turn, depends on the quality of the survey data from which they were produced, map scale, and the precision of the drafting process. Therefore, if existing maps are being used to determine areas, their quality should first be verified.

Even with good-quality maps, areas measured from them will not normally be as accurate as those computed directly from survey data. Map scale and the device used to extract map measurements are major factors affecting the resulting area accuracy. If, for example, a map is plotted to a scale of 1000 ft/1 in., and an engineer's scale is used which produces measurements good to ± 0.02 in., distances or coordinates scaled from this map can be no better than about $(\pm 0.02 \times 1000) = \pm 20$ ft. This uncertainty can produce substantial errors in areas. Differential shrinkage or expansion of the material upon which maps are drafted is another source of error in determining areas from map measurements. Changes in dimensions of 2% to 3% are common for certain types of paper. (The subjects of maps and mapping are discussed in more detail in Chapters 17 and 18.)

Aerial photos can also be used as map substitutes to determine *approximate* areas if the parcel boundaries can be identified. The areas are approximate, as explained in Chapter 26, because except for flat areas the scale of an aerial photo is not uniform throughout. Aerial photos are particularly useful for determining areas of irregularly shaped tracts, such as lakes. Different procedures for determining areas from maps are described in the subsections that follow.

12.9.1 Area by Counting Coordinate Squares

A simple method for determining areas consists in overlaying the mapped parcel with a transparency having a superimposed grid. The number of grid squares included within the tract is then counted, with partial squares estimated and added to the total. Area is the product of the total number of squares times the area represented by each square. As an example, if the grids are 0.20 in. on a side, and a map at a scale of 200 ft/in. is overlaid, each square is equivalent to $(0.20 \times 200)^2 = 1600 \text{ ft}^2$.

12.9.2 Area by Scaled Lengths

If the boundaries of a parcel are identified on a map, the tract can be divided into triangles, rectangles, or other regular figures, the sides measured, and the areas computed using standard formulas and totaled.

12.9.3 Area by Digitizing Coordinates

A mapped parcel can be placed on a digitizing table which is interfaced with a computer, and the coordinates of its corner points quickly and conveniently recorded. From the file of coordinates, the area can be computed using either Equation (12.6) or Equation (12.8). It must be remembered, however, that even though coordinates may be digitized to the nearest 0.001 in., their actual accuracy can be no better than the map from which the data were extracted. Area determination by digitizing existing maps is now being practiced extensively in creating databases of geographic information systems.

12.9.4 Area by Planimeter

A planimeter measures the area contained within any closed figure that is circumscribed by its tracer. There are two types of planimeters: mechanical and electronic. The major parts of the mechanical type are a scale bar, graduated drum and disk, vernier, tracing point and guard, and anchor arm, weight, and point. The scale bar may be fixed or adjustable. For the standard fixed-arm planimeter, one revolution of the disk (dial) represents 100 in.^{2} and one turn of the drum (wheel) represents 10 in.^{2} The adjustable type can be set to read units of area directly for any particular map scale. The instrument touches the map at only three places: anchor point, drum, and tracing-point guard.

Because of its ease of use, the electronic planimeter (Figure 12.9) has replaced its mechanical counterpart. An electronic planimeter operates similarly to the mechanical type, except that the results are given in digital form on a display console. Areas can be measured in units of square inches or square centimeters

Figure 12.9
Electronic planimeter. (Courtesy Topcon Positioning Systems.)

and by setting an appropriate scale factor, they can be obtained directly in acres or hectares. Some instruments feature multipliers that can automatically compute and display volumes.

As an example of using an adjustable type of mechanical planimeter, assume the area within the traverse of Figure 12.5 will be measured. The anchor point beneath the weight is set in a position outside the traverse (if inside, a polar constant must be added), and the tracing point brought over corner *A*. An initial reading of 7231 is taken, the 7 coming from the disk, 23 from the drum, and 1 from the vernier. The tracing point is moved along the traverse lines from *A* to *B*, *C*, *D*, and *E*, and back to *A*. A triangle or a straightedge may guide the point, but normally it is steered freehand. A final reading of 8596 is made. The difference between the initial and final readings, 1365, is multiplied by the planimeter constant to obtain the area. To determine the planimeter constant, a square area is carefully laid out 5 in. on a side, with diagonals of 7.07 in., and its perimeter traced with the planimeter. If the difference between initial and final readings for the 5 in. square is, for example, 1250, then

$$5 \text{ in.} \times 5 \text{ in.} = 25 \text{ in.}^2 = 1250 \text{ units}$$

Thus, the planimeter constant is

$$1 \text{ unit} = \frac{25}{1250} = 0.020 \text{ in.}^2$$

Finally the area within the traverse is

$$\text{area} = 1365 \text{ units} \times 0.020 = 27.3 \text{ in.}^2$$

If the traverse is plotted at a map scale of 1 in. = 100 ft, then 1 in.^{2} = 10,000 ft^2 and the area measured is 273,000 ft^2.

As a check on planimeter operation, the outline may be traced in the opposite direction. The initial and final readings at point *A* should agree within a limit of perhaps two to five units.

The precision obtained in using a planimeter depends on operator skill, accuracy of the plotted map, type of paper, and other factors. Results correct to within 1/2% to 1% can be obtained by careful work.

A planimeter is most useful for irregular areas, such as that in Figure 12.3, and has many applications in surveying and engineering. The planimeter has been widely used in highway offices for determining areas of cross sections, and is also convenient for determining areas of drainage basins and lakes from measurements on aerial photos, checking computed areas in property surveys, etc.

■ 12.10 SOFTWARE

As discussed in this chapter, there are several methods of determining the area of a parcel or figure. The method of area by coordinates is most commonly used in practice. However, other methods are sometimes used in unique situations that require a clever solution. Software typically uses the method of area by coordinates. For example, a computer-aided drafting (CAD) software package can use the coordinates of an irregularly shaped parcel to quickly determine its area by the coordinate method. WOLFPACK uses this method in determining the area enclosed by a figure from a listing of coordinates in sequential order. You may also enter the bounding coordinates of a parcel in a CAD package to determine the area enclosed by a parcel. For those wishing to see a higher-level programming of several of the examples discussed in this chapter, you are encouraged to explore the Mathcad® worksheet *C12.XMCD*, which can be found on the companion website for this book at http://www.pearsonhighered.com/ghilani.

12.11 SOURCES OF ERROR IN DETERMINING AREAS

Some sources of error in area computations are:

1. Errors in the field data from which coordinates or maps are derived.
2. Making a poor selection of intervals and offsets to fit irregular boundaries.
3. Making errors in scaling from maps.
4. Shrinkage and expansion of maps.
5. Using coordinate squares that are too large and therefore make estimation of areas of partial blocks difficult.
6. Making an incorrect setting of the planimeter scale bar.
7. Running off and on the edge of the map sheet with the planimeter drum.
8. Using different types of paper for the map and planimeter calibration sheet.

12.12 MISTAKES IN DETERMINING AREAS

In computing areas, common mistakes include:

1. Forgetting to divide by 2 in the coordinate and DMD methods.
2. Confusing signs of coordinates, departures, latitudes, and DMDs.
3. Forgetting to repeat the coordinates of the first point in the area by coordinates method.
4. Failing to check an area computation by a different method.
5. Not drawing a sketch to scale or general proportion for a visual check.
6. Not verifying the planimeter scale constant by tracing a known area.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

12.1* Compute the area enclosed within polygon *DEFGD* of Figure 12.1 using triangles.

12.2 Similar to Problem 12.1, except for polygon *BCDGB* of Figure 12.1.

12.3 Compute the area enclosed between line *ABGA* and the shoreline of Figure 12.1 using the offset method.

12.4 By rule of thumb, what is the estimated uncertainty in 430,568 ft^2 if the estimated error in the coordinates was ± 0.2 ft?

12.5* Compute the area between a lake and a straight line AG, from which offsets are taken at irregular intervals as follows (all distances in feet):

Offset Point	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Stationing	0.00	0 + 54.80	1 + 32.54	2 + 13.02	2 + 98.74	3 + 45.68	4 + 50.17
Offset	2.3	4.2	6.5	5.4	9.1	8.9	3.9

12.6 Repeat Problem 12.5 with the following offset in meters.

Offset Point	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Stationing	0.00	20.00	78.94	148.96	163.65	203.69	250.45
Offset	1.15	4.51	6.04	9.57	6.87	3.64	0.65

12.7 Use the coordinate method to compute the area enclosed by the traverse of Problem 10.8.

12.8 Calculate by coordinates the area within the traverse of Problem 10.11.

12.9 Compute the area enclosed in the traverse of Problem 10.8 using DMDs.

12.10* Determine the area within the traverse of Problem 10.11 using DMDs.

12.11 By the DMD method, find the area enclosed by the traverse of Problem 10.20.

12.12 Compute the area within the traverse of Problem 10.17 using the coordinate method. Check by DMDs.

12.13 Calculate the area inside the traverse of Problem 10.18 by coordinates and check by DMDs.

12.14 Compute the area enclosed by the traverse of Problem 10.19 using the DMD method. Check by coordinates.

12.15 Find the area of the lot in Problem 10.25.

12.16* Determine the area of the lot in Problem 10.26.

12.17 Calculate the area of Lot 16 in Figure 21.2.

12.18 Plot the lot of Problem 10.25 to a scale of 1 in. = 100 ft. Determine its surrounded area using a planimeter.

12.19 Similar to Problem 12.18, except for the traverse of Problem 10.26.

12.20 Plot the traverse of Problem 10.19 to a scale of 1 in. = 200 ft, and find its enclosed area using a planimeter.

12.21 The (X,Y) coordinates (in feet) for a closed-polygon traverse $ABCDEFA$ follow. A (1000.00, 1000.00), B (1661.73, 1002.89), C (1798.56, 1603.51), D (1289.82, 1623.69), E (1221.89, 1304.24), and F (1048.75, 1301.40). Calculate the area of the traverse by the method of coordinates.

12.22 Compute by DMDs the area in hectares within a closed-polygon traverse $ABCDEFA$ by placing the X and Y axes through the most southerly and most westerly stations, respectively. Departures and latitudes (in meters) follow. AB: E dep. = 50, N lat. = 45; BC: E dep. = 60, N lat. = 55; CD: E dep. = 45, S lat. = 25; DE: W dep. = 70, S lat. = 40; EF: W dep. = 50, S lat. = 30; FA: W dep. = 35, N lat. = 5.

12.23 Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates at angle points: A (1275.11, 1356.11), B (1000.27, 1365.70), C (1000.00, 1000.00), D (1450.00, 1000.00) with a circular arc of radius CD starting at D and ending at A with the curve outside the course AD.

12.24 Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates in feet at angle points: A (526.68, 823.98), B (535.17, 745.61), C (745.17, 745.61), D (745.17, 845.61), E (546.62, 846.14) with a circular arc of radius 25 ft starting at E, tangent to DE, and ending at A.

12.25 Divide the area of the lot in Problem 12.23 into two equal parts by a line through point B. List in order the lengths and azimuths of all sides for each parcel.

12.26 Partition the lot of Problem 12.24 into two equal areas by means of a line parallel to BC. Tabulate in clockwise consecutive order the lengths and azimuths of all sides of each parcel.

12.27 Lot $ABCD$ between two parallel street lines is 350.00 ft deep and has a 220.00 ft frontage (AB) on one street and a 260.00 ft frontage (CD) on the other. Interior angles at A and B are equal, as are those at C and D. What distances AE and BF should be laid off by a surveyor to divide the lot into two equal areas by means of a line EF parallel to AB?

12.28 Partition 1-acre parcel from the northern part of lot $ABCDEFA$ in Problem 12.21 such that its southern line is parallel to the northern line.

12.29 Write a computational program for calculating areas within closed polygon traverses by the coordinate method.

12.30 Write a computational program for calculating areas within closed polygon traverses by the DMD method.

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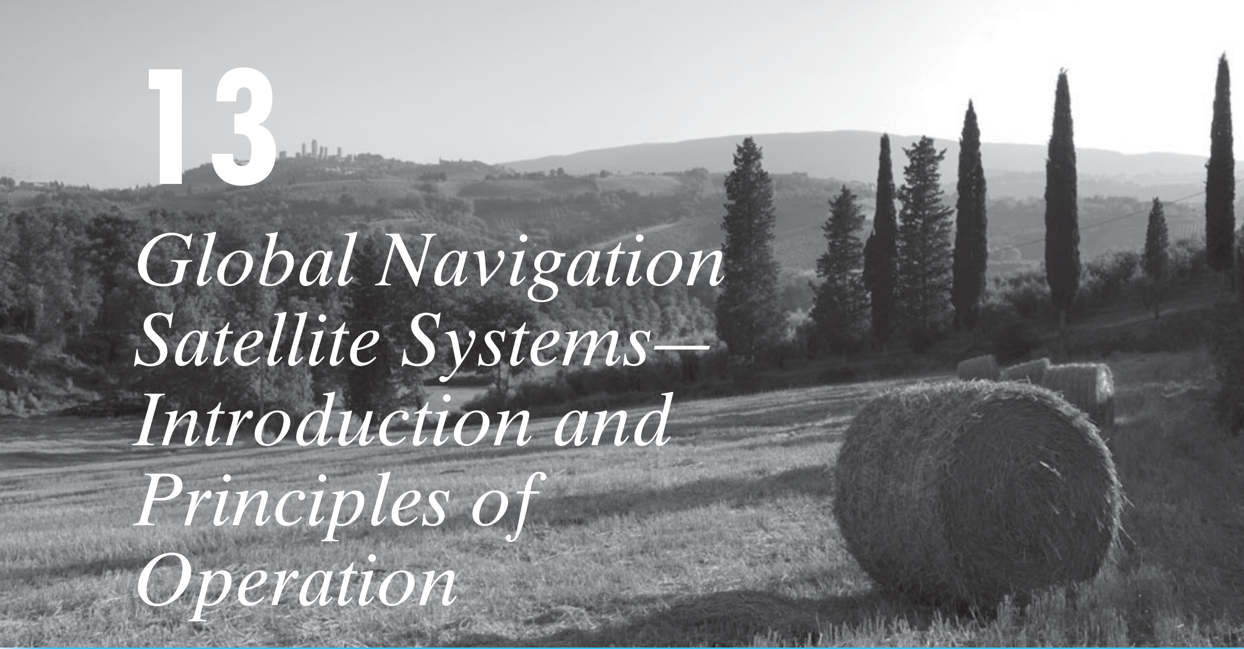


13 Global Navigation Satellite Systems—Introduction and Principles of Operation

■ 13.1 INTRODUCTION

During the 1970s, a new and unique approach to surveying, the *global positioning system* (GPS), emerged. This system, which grew out of the space program, relies upon signals transmitted from satellites for its operation. It has resulted from research and development paid for by the military to produce a system for global navigation and guidance. More recently other countries are developing their own systems. Thus, the entire scope of satellite systems used in positioning is now referred to as *global navigation satellite systems* (GNSS). Receivers that use GPS satellites and another system such as GLONASS (see Section 13.10) are known as GNSS receivers. These systems provide precise timing and positioning information anywhere on the Earth with high reliability and low cost. The systems can be operated day or night, rain or shine, and do not require cleared lines of sight between survey stations. This represents a revolutionary departure from conventional surveying procedures, which rely on observed angles and distances for determining point positions. Since these systems all share similar features, the global positioning system will be discussed in further detail herein.

Development of the first generation of satellite positioning systems began in 1958. This early system, known as the *Navy Navigation Satellite System* (NNSS), more commonly called the *TRANSIT* system, operated on the *Doppler* principle.

In this system, *Doppler shifts* (changes in frequency) of signals transmitted from satellites were observed by receivers located on ground stations. The observed Doppler shifts are a function of the distances to the satellites and their directions of movement with respect to the receivers. The transmitting frequency was known and together with accurate satellite orbital position data and precise timing of observations, the positions of the receiving stations could be determined. The constellation of satellites in the TRANSIT system, which varied between five and seven in number, operated in polar orbits at altitudes of approximately 1100 km. The objective of the TRANSIT system was to aid in the navigation of the U.S. Navy's Polaris submarine fleet. The first authorized civilian use of the system occurred in 1967, and the surveying community quickly adopted the new technology, finding it particularly useful for control surveying. Although these early instruments were bulky and expensive, the observation sessions lengthy, and the accuracy achieved moderate, the Doppler program was nevertheless an important breakthrough in satellite positioning in general, and in surveying in particular.

Because of the success of the Doppler program, the U.S. Department of Defense (DoD) began development of the ***NAV**igation **S**atellite **T**iming **a**nd **R**anging* (NAVSTAR) Global Positioning System (GPS). The first satellite to support the development and testing of the system was placed in orbit in 1978. Since that date many additional satellites have been launched. The global positioning system, developed at a cost of approximately $12 billion, became fully operational in December of 1993. Like the earlier Doppler versions, the global positioning system is based on observations of signals transmitted from satellites whose positions within their orbits are precisely known. Also, the signals are picked up with *receivers* located at ground stations. However, the methods of determining distances from receivers to satellites, and of computing receiver positions, are different. These methods are described in later sections of this chapter. Current generation satellite receivers are illustrated in Figures 1.4 and 13.1. The size and cost of satellite surveying equipment have been substantially reduced from those of the Doppler program, and field and office procedures involved in surveys have been simplified so that now high accuracies can be achieved in real time.

■ 13.2 OVERVIEW OF GPS

As noted in the preceding section, precise distances from the satellites to the receivers are determined from timing and signal information, enabling receiver positions to be computed. In satellite surveying, the satellites become the reference or *control* stations, and the *ranges* (distances) to these satellites are used to compute the positions of the receiver. Conceptually, this is equivalent to resection in traditional ground surveying work, as described in Section 11.7, where distances and/or angles are observed from an unknown ground station to control points of known position.

The global positioning system can be arbitrarily broken into three parts: (a) the *space segment*, (b) the *control segment*, and (c) the *user segment*. The **space segment** consists nominally of 24 satellites operating in six orbital planes spaced at 60° intervals around the equator. Four additional satellites are held in reserve as spares. The orbital planes are inclined to the equator at 55° [see Figure 13.2(b)].

(a) (b)

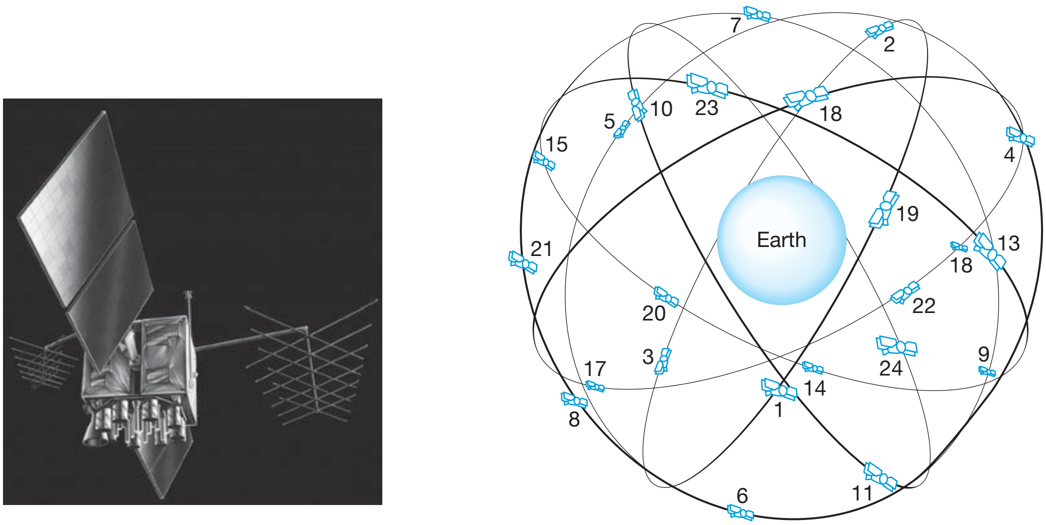
Figure 13.1
(a) The Trimble R8 and (b) the Sokkia GSR2700 receivers. (Courtesy of Trimble Navigation and Sokkia Corp.)

This configuration provides 24-h satellite coverage between the latitudes of 80°N and 80°S. The satellites travel in near-circular orbits that have a mean altitude of 20,200 km above the Earth and an orbital period of 12 sidereal hours.[1] The individual satellites are normally identified by their *PseudoRandom Noise* (PRN) number, (described below), but can also be identified by their satellite vehicle number (SVN) or orbital position.

Precise atomic clocks are used in the satellites to control the timing of the signals they transmit. These are extremely accurate clocks,[2] and extremely expensive as well. If the receivers used these same clocks, they would be cost prohibitive and would also require that users become trained in handling hazardous materials. Thus the clocks in the receivers are controlled by the oscillations of a

[1]A sidereal day is approximately 4 min shorter than a solar day. See Chapter 19 for more information on sidereal years and days.

[2]Atomic clocks are used, which employ either cesium or rubidium. The rubidium clocks may lose 1 sec per 30,000 years, while the cesium type may lose 1 sec only every 300,000 years. Hydrogen maser clocks, which may lose only 1 sec every 30,000,000 years, have been proposed for future satellites. For comparison, quartz crystal clocks used in receivers may lose a second every 30 years.



(a) (b)

Figure 13.2 (a) A GPS satellite and (b) the GPS constellation.

quartz crystal that, although also precise, are less accurate than atomic clocks. However, these relatively low cost timing devices produce a receiver that is also relatively inexpensive.

The **control segment** consists of *monitoring stations* which monitor the signals and track the positions of the satellites over time. The initial GPS monitoring stations are at Colorado Springs, and on the islands of Hawaii, Ascension, Diego Garcia, and Kwajalein. The tracking information is relayed to the *master control station* in the Consolidated Space Operations Center (CSOC) located at Schriever Air Force base in Colorado Springs. The master control station uses this data to make precise, near-future predictions of the satellite orbits, and their clock correction parameters. This information is uploaded to the satellites, and in turn, transmitted by them as part of their *broadcast message* to be used by receivers to predict satellite positions and their clock *biases* (systematic errors).

The **user segment** in GPS consists of two categories of receivers that are classified by their access to two services that the system provides. These services are referred to as the *Standard Position Service* (*SPS*) and the *Precise Positioning Service* (*PPS*). The SPS is provided on the L1 broadcast frequency and more recently the L2 (see Section 13.3) at no cost to the user. This service was initially intended to provide accuracies of 100 m in horizontal positions, and 156 m in vertical positions at the 95% error level. However, improvements in the system and the processing software have substantially reduced these error estimates. The PPS is broadcast on both the L1 and L2 frequencies, and is only available to receivers having valid cryptographic keys, which are reserved almost entirely for DoD use. This message provides a published accuracy of 18 m in the horizontal, and 28 m in the vertical at the 95% error level.

13.3 THE GPS SIGNAL

As the GPS satellites are orbiting, each continually broadcasts a unique signal on the two *carrier frequencies*. The carriers, which are transmitted in the L band of microwave radio frequencies, are identified as the L1 signal with a frequency of 1575.42 MHz and the L2 signal at a frequency of 1227.60 MHz. These frequencies are derived from a fundamental frequency, f_0, of 10.23 MHz. The L1 band has frequency of $154 \times f_0$ and the L2 band has a frequency of $120 \times f_0$.

Much like a radio station broadcasts, several different types of information (messages) are modulated upon these carrier waves using a phase modulation technique. Some of the information included in the broadcast message is the almanac, broadcast ephemeris, satellite clock correction coefficients, ionospheric correction coefficients, and satellite condition (also termed *satellite health*). These terms are defined later in this chapter.

In order for receivers to independently determine the ground positions of the stations they occupy in real time, it was necessary to devise a system for accurate measurement of signal travel time from satellite to receiver. In GPS, this was accomplished by modulating the carriers with *pseudorandom noise* (PRN) codes. The PRN codes consist of unique sequences of binary values (zeros and ones) that appear to be random but, in fact, are generated according to a special mathematical algorithm using devices known as *tapped feedback shift registers*. Each satellite transmits two different PRN codes. The L1 signal is modulated with the *precise code*, or *P code*, and also with the *coarse/acquisition code*, or *C/A code*. The L2 signal was modulated only with the P code. Each satellite broadcasts a unique set of codes known as *GOLD codes* that allow receivers to identify the origins of received signals. This identification is important when tracking several different satellites simultaneously.

The C/A code and P code are older technology. Recent satellites are being equipped with new codes. These satellites include a second civilian code on the L2 signal called the L2C. This code has both a moderate and long version. Additionally, the P code is being replaced by two new military codes, known as *M* codes. In 1999, the Interagency GPS Executive Board (IGEB) decided to add a third civilian signal known as the L5 to provide safety of life applications to GPS. The L5 will be broadcast at a frequency of 1176.45 MHz. Both the L2C and L5 are added to the Block IIF and Block III satellites. The improvements in positioning due to these new codes will be discussed later in this chapter.

The C/A code has a frequency of 1.023 MHz and a wavelength of about 300 m. It is accessible to all users, and is a series of 1023 binary digits (*chips*) that are unique to each satellite. This chip pattern is repeated every millisecond in the C/A code. The P code, with a frequency of 10.23 MHz and a wavelength of about 30 m, is 10 times more accurate for positioning than the C/A code. The P code has a chip pattern that takes 266.4 days to repeat. Each satellite is assigned a unique single-week segment of the pattern that is reinitialized at midnight every Saturday. Table 13.1 lists the GPS frequencies, and gives their factors of the fundamental frequency, f_0, of the P code.

To meet military requirements, the P code is encrypted with a W code to derive the Y code. This Y code can only be read with receivers that have the proper cryptographic keys. This encryption process is known as *anti-spoofing* (A-S). Its

TABLE 13.1 FREQUENCIES TRANSMITTED BY GPS

Code Name	Frequency (MHz)	Factor of f_0
C/A	1.023	Divide by 10
P	10.23	1
L1	1575.42	Multiply by 154
L2	1227.60	Multiply by 120
L5	1176.45	Multiply by 115

purpose is to deny access to the signal by potential enemies who could deliberately modify and retransmit it with the intention of "spoofing" unwary friendly users.

Because of its need for "one-way" communication, the satellite positioning systems depend on precise timing of the transmitted signal. To understand the concepts of the one-way system, consider the following. Imagine that the satellite transmits a series of audible beeps, and that the beeps are broadcast in a known irregular pattern. Now imagine that this same pattern is synchronously duplicated (but not transmitted) at the receiving station. Since the signal of the satellite transmitter must travel to the receiver, its reception there will be delayed in relation to the signal being generated by the receiver. This delay can be measured, and converted to a time difference.

The process described above is similar to that used with GPS. In GPS, the chips of the PRN codes replace the beeps and the precise time of broadcast of the satellite code is placed into the broadcast message with a starting time indicated by the front edge of one of the chips. The receiver simultaneously generates a duplicate PRN code. Matching the incoming satellite signal with the identical receiver-generated signal derives the time it takes for the signal to travel from satellite to receiver. This yields the signal delay that is converted to travel time. From the travel time, and the known signal velocity, the distance to the satellite can be computed.

To aid in matching the codes, the broadcast message from each satellite contains a *Hand-Over Word* (HOW), which consists of some identification bits, flags, and a number. This number, times four, produces the *Time of Week* (TOW), which marks the leading edge of the next section of the message. The HOW and TOW assist the receiver in matching the signal received from the satellite to that generated by the receiver, so the delay can be quickly determined. This matching process is illustrated diagrammatically in Figure 13.3.

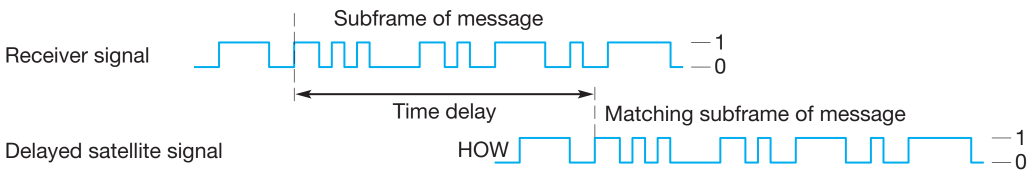


Figure 13.3 Determination of signal travel time by code matching.

■ 13.4 REFERENCE COORDINATE SYSTEMS

In determining the positions of points on Earth from satellite observations, three different reference coordinate systems are important. First of all, satellite positions at the instant they are observed are specified in the "space-related" *satellite reference coordinate systems*. These are three-dimensional rectangular systems defined by the satellite orbits. Satellite positions are then transformed into a three-dimensional rectangular *geocentric coordinate system,* which is physically related to the Earth. As a result of satellite positioning observations, the positions of new points on Earth are determined in this coordinate system. Finally, the geocentric coordinates are transformed into the more commonly used and locally oriented *geodetic coordinate system*. The following subsections describe these three coordinate systems.

13.4.1 The Satellite Reference Coordinate System

Once a satellite is launched into orbit, its movement thereafter within that orbit is governed primarily by the Earth's gravitational force. However, there are a number of other lesser factors involved including the gravitational forces exerted by the sun and moon, as well as forces due to solar radiation. Because of movements of the Earth, sun, and moon with respect to each other, and because of variations in solar radiation, these forces are not uniform and hence satellite movements vary somewhat from their ideal paths. As shown in Figure 13.4, ignoring all forces except the Earth's gravitational pull, a satellite's idealized orbit is elliptical, and has one of its two foci at G, the Earth's mass center. The figure also illustrates a satellite reference coordinate system, X_S, Y_S, Z_S. The *perigee* and *apogee* points are where the satellite is closest to, and farthest away from G, respectively, in its orbit. The *line of apsides* joins these two points, passes through the two foci, and is the reference axis X_S. The origin of the X_S, Y_S, Z_S coordinate system is at G; the Y_S axis is in the mean orbital plane; and Z_S is perpendicular to this plane. Values of Z_S coordinates represent

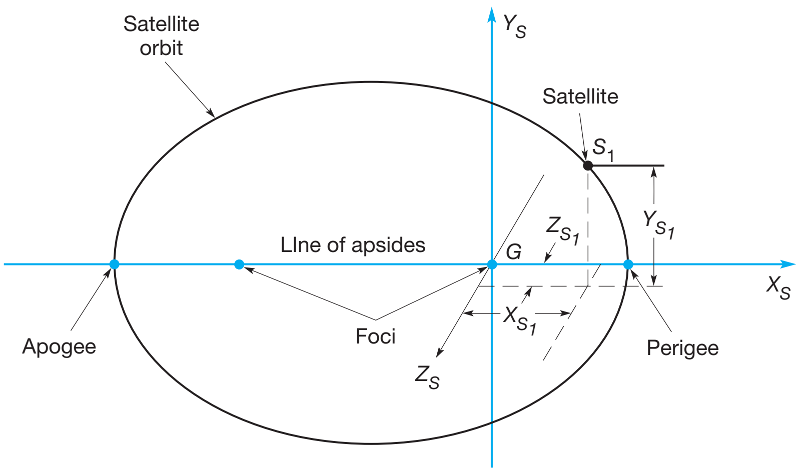


Figure 13.4
Satellite reference coordinate system.

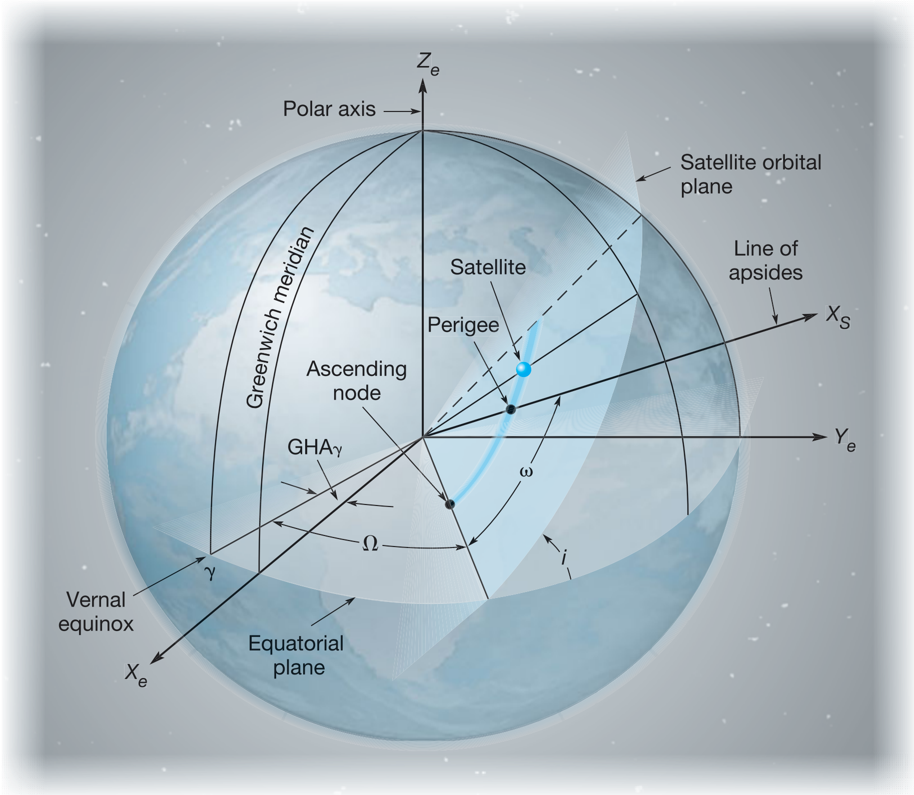


Figure 13.5 Parameters involved in transforming from the satellite reference coordinate system to the geocentric coordinate system.

departures of the satellite from its mean orbital plane, and normally are very small. A satellite at position S_1 would have coordinates X_{S1}, Y_{S1}, and Z_{S1}, as shown in Figure 13.4. For any instant of time, the satellite's position in its orbit can be calculated from its orbital parameters, which are part of the broadcast ephemeris.

13.4.2 The Geocentric Coordinate System

Because the objective of satellite surveys is to locate points on the surface of the Earth, it is necessary to have a so-called *terrestrial* frame of reference, which enables relating points physically to the Earth. The frame of reference used for this is the geocentric coordinate system. Figure 13.5 illustrates a quadrant of a *reference ellipsoid*,[3] with a geocentric coordinate system (X_e, Y_e, Z_e) superimposed. This three-dimensional rectangular coordinate system has its origin at the mass center of the Earth. Its X_e axis passes through the Greenwich meridian in the plane of the equator, and its Z_e axis coincides with the *Conventional Terrestrial Pole* (CTP) (see Section 20.3).

[3]The reference ellipsoid used for most GPS work is the *World Geodetic System of 1984 (WGS84) ellipsoid*. As explained in Section 19.1, any ellipsoid can be defined by two parameters, for example the *semimajor axis (a)*, and the *flattening ratio (f)*. For the WGS84 ellipsoid these values are $a = 6{,}378{,}137$ m (exactly), and $f = 1/298.257223563$.

To make the conversion from the satellite reference coordinate system to the geocentric system, four angular parameters are required which define the relationship between the satellite's orbital coordinate system and key reference planes and lines on the Earth. As shown in Figure 13.5, these parameters are (1) the *inclination* angle, i (angle between the orbital plane and the Earth's equatorial plane), (2) the *argument of perigee*, ω (angle in the orbital plane from the equator to the line of apsides), (3) the *right ascension of the ascending node*, Ω (angle in the plane of the Earth's equator from the vernal equinox to the line of intersection between the orbital and equatorial planes), and (4) the *Greenwich hour angle of the vernal equinox*, GHA_{γ} (angle in the equatorial plane from the Greenwich meridian to the vernal equinox). These parameters are known in real time for each satellite based upon predictive mathematical modeling of the orbits. Where higher accuracy is needed, satellite coordinates in the geocentric system for specific epochs of time are determined from observations at the tracking stations and distributed in precise ephemerides.

The equations for making conversions from satellite reference coordinate systems to the geocentric system are beyond the scope of this text. They are included in the software that accompanies the satellite positioning systems when they are purchased. However, an html file named satellite.html is available on the companion website for this book at http://www.pearsonhighered.com/ghilani, which demonstrates the transformation of satellite coordinates to the terrestrial coordinate system. Although the equations are not presented here, through this discussion students are apprised of the nature of satellite motion, and of the fact that there are definite mathematical relationships between orbiting satellites and the positions of points located on the Earth's surface.

13.4.3 The Geodetic Coordinate System

Although the positions of points in a satellite survey are computed in the geocentric coordinate system described in the preceding subsection, in that form they are inconvenient for use by surveyors (geomatics engineers). This is the case for three reasons: (1) with their origin at the Earth's center, geocentric coordinates are typically extremely large values, (2) with the X-Y plane in the plane of the equator, the axes are unrelated to the conventional directions of north-south or east-west on the surface of the Earth, and (3) geocentric coordinates give no indication about relative elevations between points. For these reasons, the *geocentric* coordinates are converted to *geodetic* coordinates of latitude (ϕ), longitude (λ), and height (h) so that reported point positions become more meaningful and convenient for users.

Figure 13.6 also illustrates a quadrant of the reference ellipsoid, and shows both the geocentric coordinate system (X,Y,Z), and the geodetic coordinate system (ϕ, λ, h). Conversions from geocentric to geodetic coordinates, and vice versa are readily made. From the figure it can be shown that geocentric coordinates of point P can be computed from its geodetic coordinates using the following equations:

$$
\begin{aligned}
X_P &= (R_{N_P} + h_P)\cos\phi_P\cos\lambda_P \\
Y_P &= (R_{N_P} + h_P)\cos\phi_P\sin\lambda_P \\
Z_P &= [R_{N_P}(1 - e^2) + h_P]\sin\phi_P
\end{aligned}
\tag{13.1}
$$