Unrestrained expansion of a gas or liquid, including spray paint, is one of the *irreversibilities* listed on p. 243. **Image Source//Getty Images, Inc.**

ENGINEERING CONTEXT The presentation to this point has considered thermodynamic analysis using the conservation of mass and conservation of energy principles together with property relations. In Chaps. 2 through 4 these fundamentals are applied to increasingly complex situations. The conservation principles do not always suffice, however, and the second law of thermodynamics is also often required for thermodynamic analysis. The **objective** of this chapter is to introduce the second law of thermodynamics. A number of deductions that may be called corollaries of the second law are also considered, including performance limits for thermodynamic cycles. The current presentation provides the basis for subsequent developments involving the second law in Chaps. 6 and 7.

5

The Second Law of Thermodynamics

▶ LEARNING OUTCOMES

When you complete your study of this chapter, you will be able to...

- demonstrate understanding of key concepts related to the second law of thermodynamics, including alternative statements of the second law, the internally reversible process, and the Kelvin temperature scale.
- list several important irreversibilities.
- assess the performance of power cycles and refrigeration and heat pump cycles using, as appropriate, the corollaries of Secs. 5.6.2 and 5.7.2, together with Eqs. 5.9–5.11.
- describe the Carnot cycle.
- interpret the Clausius inequality as expressed by Eq. 5.13.

5.1 Introducing the Second Law

The objectives of the present section are to

1. motivate the need for and the usefulness of the second law.
2. introduce statements of the second law that serve as the point of departure for its application.

5.1.1 Motivating the Second Law

It is a matter of everyday experience that there is a definite direction for *spontaneous* processes. This can be brought out by considering the three systems pictured in Fig. 5.1.

▶ System a. An object at an elevated temperature T_i placed in contact with atmospheric air at temperature T_0 eventually cools to the temperature of its much larger surroundings, as illustrated in Fig. 5.1*a*. In conformity with the conservation of energy principle, the decrease in internal energy of the body appears as an increase in the internal energy of the surroundings. The *inverse* process would not take place *spontaneously,* even though energy could be conserved: The internal energy of the surroundings would not decrease spontaneously while the body warmed from T_0 to its initial temperature.

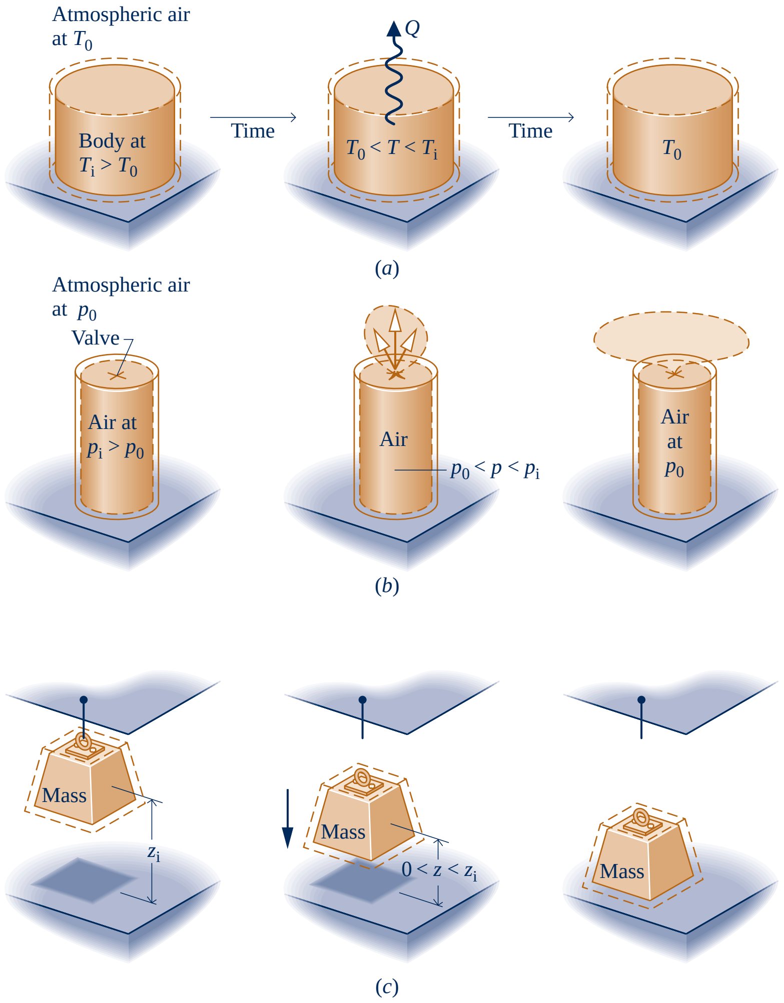


Fig. 5.1 Illustrations of spontaneous processes and the eventual attainment of equilibrium with the surroundings. (*a*) Spontaneous heat transfer. (*b*) Spontaneous expansion. (*c*) Falling mass.

- System b. Air held at a high pressure p_i in a closed tank flows spontaneously to the lower pressure surroundings at p_0 when the interconnecting valve is opened, as illustrated in Fig. 5.1*b*. Eventually fluid motions cease and all of the air is at the same pressure as the surroundings. Drawing on experience, it should be clear that the *inverse* process would not take place *spontaneously,* even though energy could be conserved: Air would not flow spontaneously from the surroundings at p_0 into the tank, returning the pressure to its initial value.
- System c. A mass suspended by a cable at elevation z_i falls when released, as illustrated in Fig. 5.1*c*. When it comes to rest, the potential energy of the mass in its initial condition appears as an increase in the internal energy of the mass and its surroundings, in accordance with the conservation of energy principle. Eventually, the mass also comes to the temperature of its much larger surroundings. The *inverse* process would not take place *spontaneously,* even though energy could be conserved: The mass would not return spontaneously to its initial elevation while its internal energy and/or that of its surroundings decreases.

In each case considered, the initial condition of the system can be restored, but not in a spontaneous process. Some auxiliary devices would be required. By such auxiliary means the object could be reheated to its initial temperature, the air could be returned to the tank and restored to its initial pressure, and the mass could be lifted to its initial height. Also in each case, a fuel or electrical input normally would be required for the auxiliary devices to function, so a permanent change in the condition of the surroundings would result.

Further Conclusions

The foregoing discussion indicates that not every process consistent with the principle of energy conservation can occur. Generally, an energy balance alone neither enables the preferred direction to be predicted nor permits the processes that can occur to be distinguished from those that cannot. In elementary cases, such as the ones considered in Fig. 5.1, experience can be drawn upon to deduce whether particular spontaneous processes occur and to deduce their directions. For more complex cases, where experience is lacking or uncertain, a guiding principle is necessary. This is provided by the *second law*.

The foregoing discussion also indicates that when left alone systems tend to undergo spontaneous changes until a condition of equilibrium is achieved, both internally and with their surroundings. In some cases equilibrium is reached quickly, in others it is achieved slowly. For example, some chemical reactions reach equilibrium in fractions of seconds; an ice cube requires a few minutes to melt; and it may take years for an iron bar to rust away. Whether the process is rapid or slow, it must of course satisfy conservation of energy. However, that alone would be insufficient for determining the final equilibrium state. Another general principle is required. This is provided by the *second law*.

BIOCONNECTIONS Did you ever wonder why a banana placed in a closed bag or in the refrigerator quickly ripens? The answer is in the ethylene, C_2H_4, naturally produced by bananas, tomatoes, and other fruits and vegetables. Ethylene is a plant hormone that affects growth and development. When a banana is placed in a closed container, ethylene accumulates and stimulates the production of more ethylene. This positive feedback results in more and more ethylene, hastening ripening, aging, and eventually spoilage. In thermodynamic terms, if left alone, the banana tends to undergo spontaneous changes until equilibrium is achieved. Growers have learned to use this natural process to their advantage. Tomatoes picked while still green and shipped to distant markets may be red by the time they arrive; if not, they can be induced to ripen by means of an ethylene spray.

5.1.2 Opportunities for Developing Work

By exploiting the spontaneous processes shown in Fig. 5.1, it is possible, in principle, for work to be developed as equilibrium is attained.

▸ FOR EXAMPLE instead of permitting the body of Fig. 5.1*a* to cool spontaneously with no other result, energy could be delivered by heat transfer to a system undergoing a power cycle that would develop a net amount of work (Sec. 2.6). Once the object attained equilibrium with the surroundings, the process would cease. Although there is an *opportunity* for developing work in this case, the opportunity would be wasted if the body were permitted to cool without developing any work. In the case of Fig. 5.1*b*, instead of permitting the air to expand aimlessly into the lower-pressure surroundings, the stream could be passed through a turbine and work could be developed. Accordingly, in this case there is also a possibility for developing work that would not be exploited in an uncontrolled process. In the case of Fig. 5.1*c*, instead of permitting the mass to fall in an uncontrolled way, it could be lowered gradually while turning a wheel, lifting another mass, and so on. ◂ ◂ ◂ ◂ ◂

These considerations can be summarized by noting that when an imbalance exists between two systems, there is an opportunity for developing work that would be irrevocably lost if the systems were allowed to come into equilibrium in an uncontrolled way. Recognizing this possibility for work, we can pose two questions:

1. What is the theoretical maximum value for the work that could be obtained?
2. What are the factors that would preclude the realization of the maximum value?

That there should be a maximum value is fully in accord with experience, for if it were possible to develop unlimited work, few concerns would be voiced over our dwindling fossil fuel supplies. Also in accord with experience is the idea that even the best devices would be subject to factors such as friction that would preclude the attainment of the theoretical maximum work. The second law of thermodynamics provides the means for determining the theoretical maximum and evaluating quantitatively the factors that preclude attaining the maximum.

5.1.3 Aspects of the Second Law

We conclude our introduction to the second law by observing that the second law and deductions from it have many important uses, including means for:

1. predicting the direction of processes.
2. establishing conditions for equilibrium.
3. determining the best *theoretical* performance of cycles, engines, and other devices.
4. evaluating quantitatively the factors that preclude the attainment of the best theoretical performance level.

Other uses of the second law include:

5. defining a temperature scale independent of the properties of any thermometric substance.
6. developing means for evaluating properties such as u and h in terms of properties that are more readily obtained experimentally.

Scientists and engineers have found additional uses of the second law and deductions from it. It also has been used in philosophy, economics, and other disciplines far removed from engineering thermodynamics.

The six points listed can be thought of as aspects of the second law of thermodynamics and not as independent and unrelated ideas. Nonetheless, given the variety of these topic areas, it is easy to understand why there is no single statement of the second law that brings out each one clearly. There are several alternative, yet equivalent, formulations of the second law.

In the next section, three statements of the second law are introduced as *points of departure* for our study of the second law and its consequences. Although the exact relationship of these particular formulations to each of the second law aspects listed above may not be immediately apparent, all aspects listed can be obtained by deduction from these formulations or their corollaries. It is important to add that in every instance where a consequence of the second law has been tested directly or indirectly by experiment, it has been unfailingly verified. Accordingly, the basis of the second law of thermodynamics, like every other physical law, is experimental evidence.

TAKE NOTE...
No single statement of the second law brings out each of its many aspects.

5.2 Statements of the Second Law

Three alternative statements of the second law of thermodynamics are given in this section. They are the (1) Clausius, (2) Kelvin–Planck, and (3) entropy statements. The Clausius and Kelvin–Planck statements are traditional formulations of the second law. You have likely encountered them before in an introductory physics course.

Although the Clausius statement is more in accord with experience and thus easier to accept, the Kelvin–Planck statement provides a more effective means for bringing out second law deductions related to thermodynamic cycles that are the focus of the current chapter. The Kelvin–Planck statement also underlies the entropy statement, which is the most effective form of the second law for an extremely wide range of engineering applications. The entropy statement is the focus of Chap. 6.

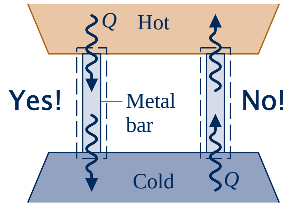
5.2.1 Clausius Statement of the Second Law

The **Clausius statement** of the second law asserts that:

Clausius statement

It is impossible for any system to operate in such a way that the sole result would be an energy transfer by heat from a cooler to a hotter body.

The Clausius statement does not rule out the possibility of transferring energy by heat from a cooler body to a hotter body, for this is exactly what refrigerators and heat pumps accomplish. However, as the words "sole result" in the statement suggest, when a heat transfer from a cooler body to a hotter body occurs, there must be *other effects* within the system accomplishing the heat transfer, its surroundings, or both. If the system operates in a thermodynamic cycle, its initial state is restored after each cycle, so the only place that must be examined for such *other* effects is its surroundings.



FOR EXAMPLE cooling of food is most commonly accomplished by refrigerators driven by electric motors requiring power from their surroundings to operate. The Clausius statement implies it is impossible to construct a refrigeration cycle that operates without a power input.

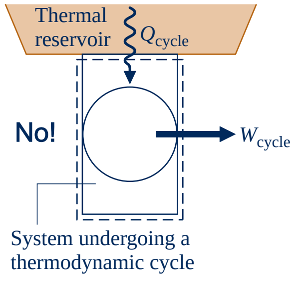
5.2.2 Kelvin–Planck Statement of the Second Law

thermal reservoir

Before giving the Kelvin–Planck statement of the second law, the concept of a **thermal reservoir** is introduced. A thermal reservoir, or simply a reservoir, is a special kind of system that always remains at constant temperature even though energy is added or removed by heat transfer. A reservoir is an idealization of course, but such a system can be approximated in a number of ways—by the earth's atmosphere, large bodies of water (lakes, oceans), a large block of copper, and a system consisting of two phases at a specified pressure (while the ratio of the masses of the two phases changes as the system is heated or cooled at constant pressure, the temperature remains constant as long as both phases coexist). Extensive properties of a thermal reservoir such as internal energy can change in interactions with other systems even though the reservoir temperature remains constant.

Kelvin–Planck statement

Having introduced the thermal reservoir concept, we give the **Kelvin–Planck statement** of the second law:



It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of energy by work to its surroundings while receiving energy by heat transfer from a single thermal reservoir.

The Kelvin–Planck statement does not rule out the possibility of a system developing a net amount of work from a heat transfer drawn from a single reservoir. It only denies this possibility if the system undergoes a thermodynamic cycle.

The Kelvin–Planck statement can be expressed analytically. To develop this, let us study a system undergoing a cycle while exchanging energy by heat transfer with a *single* reservoir, as shown by the adjacent figure. The first and second laws each impose constraints:

- ▶ A constraint is imposed by the first law on the net work and heat transfer between the system and its surroundings. According to the cycle energy balance (see Eq. 2.40 in Sec. 2.6),

$$W_{cycle} = Q_{cycle}$$

  In words, the net work done by (or on) the system undergoing a cycle equals the net heat transfer to (or from) the system. Although the cycle energy balance allows the net work W_{cycle} to be positive *or* negative, the second law imposes a constraint, as considered next.
- ▶ According to the Kelvin–Planck statement, a system undergoing a cycle while communicating thermally with a single reservoir *cannot* deliver a net amount of work to its surroundings: The net work of the cycle *cannot be positive.* However, the Kelvin–Planck statement does not rule out the possibility that there is a net work transfer of energy *to* the system during the cycle *or* that the net work is zero. Thus, the **analytical form of the Kelvin–Planck statement** is

analytical form of the Kelvin–Planck statement

$$W_{cycle} \leq 0 \qquad \text{(single reservoir)} \tag{5.1}$$

where the words *single reservoir* are added to emphasize that the system communicates thermally only with a single reservoir as it executes the cycle. In Sec. 5.4, we associate the "less than" and "equal to" signs of Eq. 5.1 with the presence and absence of *internal irreversibilities,* respectively. The concept of irreversibilities is considered in Sec. 5.3.

The equivalence of the Clausius and Kelvin–Planck statements can be demonstrated by showing that the violation of each statement implies the violation of the other. For details, see the box.

Demonstrating the Equivalence of the Clausius and Kelvin–Planck Statements

The equivalence of the Clausius and Kelvin–Planck statements is demonstrated by showing that the violation of each statement implies the violation of the other. That a violation of the Clausius statement implies a violation of the Kelvin–Planck statement is readily shown using Fig. 5.2, which pictures a hot reservoir, a cold reservoir, and two systems.

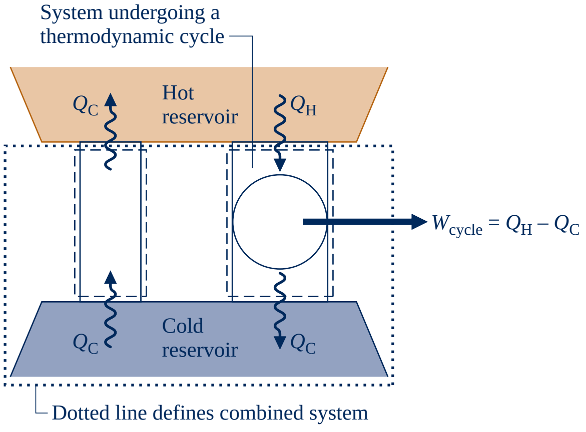


Fig. 5.2 Illustration used to demonstrate the equivalence of the Clausius and Kelvin–Planck statements of the second law.

The system on the left transfers energy Q_C from the cold reservoir to the hot reservoir by heat transfer without other effects occurring and thus *violates the Clausius statement*. The system on the right operates in a cycle while receiving Q_H (greater than Q_C) from the hot reservoir, rejecting Q_C to the cold reservoir, and delivering work W_{cycle} to the surroundings. The energy flows labeled on Fig. 5.2 are in the directions indicated by the arrows.

Consider the *combined* system shown by a dotted line on Fig. 5.2, which consists of the cold reservoir and the two devices. The combined system can be regarded as executing a cycle because one part undergoes a cycle and the other two parts experience no net change in their conditions. Moreover, the combined system receives energy $(Q_H - Q_C)$ by heat transfer from a single reservoir, the hot reservoir, and produces an equivalent amount of work. Accordingly, the combined system violates the Kelvin–Planck statement. Thus, a violation of the Clausius statement implies a violation of the Kelvin–Planck statement. The equivalence of the two second-law statements is demonstrated completely when it is also shown that a violation of the Kelvin–Planck statement implies a violation of the Clausius statement. This is left as an exercise (see end-of-chapter Prob. 5.1).

5.2.3 Entropy Statement of the Second Law

Mass and energy are familiar examples of extensive properties of systems. Entropy is another important extensive property. We show how entropy is evaluated and applied for engineering analysis in Chap. 6. Here we introduce several important aspects.

Just as mass and energy are *accounted for* by mass and energy balances, respectively, entropy is accounted for by an *entropy balance*. In words, the entropy balance states:

$$\begin{bmatrix} \textit{change} \text{ in the amount} \\ \text{of entropy contained} \\ \text{within the system} \\ \text{during some time} \\ \text{interval} \end{bmatrix} = \begin{bmatrix} \text{net amount of} \\ \text{entropy } \textit{transferred} \\ \textit{in} \text{ across the system} \\ \text{boundary during the} \\ \text{time interval} \end{bmatrix} + \begin{bmatrix} \text{amount of entropy} \\ \textit{produced within} \text{ the} \\ \text{system during the} \\ \text{time interval} \end{bmatrix} \tag{5.2}$$

Like mass and energy, *entropy can be transferred* across the system boundary. For closed systems, there is a single means of entropy transfer—namely, entropy transfer accompanying heat transfer. For control volumes entropy also is transferred in and out by streams of matter. These entropy transfers are considered further in Chap. 6.

Unlike mass and energy, which are conserved, *entropy is produced* (or *generated*) within systems whenever *nonidealities* (called *irreversibilities*) such as friction are present. The **entropy statement of the second law** states:

entropy statement of the second law

It is impossible for any system to operate in a way that entropy is destroyed.

It follows that the entropy production term of Eq. 5.2 may be positive or zero but *never* negative. Thus, entropy production is an indicator of whether a process is possible or impossible.

5.2.4 Second Law Summary

In the remainder of this chapter, we apply the Kelvin–Planck statement of the second law to draw conclusions about systems undergoing thermodynamic cycles. The chapter concludes with a discussion of the *Clausius inequality* (Sec. 5.11), which provides the basis for developing the entropy concept in Chap. 6. This is a traditional approach to the second law in engineering thermodynamics. However, the order can be reversed—namely, the entropy statement can be adopted as the starting point for study of the second law aspects of systems.

5.3 Irreversible and Reversible Processes

One of the important uses of the second law of thermodynamics in engineering is to determine the best theoretical performance of systems. By comparing actual performance with the best theoretical performance, insights often can be gained into the potential for improvement. As might be surmised, the best performance is evaluated in terms of idealized processes. In this section such idealized processes are introduced and distinguished from actual processes that invariably involve *irreversibilities*.

5.3.1 Irreversible Processes

irreversible process

reversible processes

A process is called **irreversible** if the system and all parts of its surroundings cannot be exactly restored to their respective initial states after the process has occurred. A process is **reversible** if both the system and surroundings can be returned to their initial states. Irreversible processes are the subject of the present discussion. Reversible processes are considered again in Sec. 5.3.3.

A system that has undergone an irreversible process is not necessarily precluded from being restored to its initial state. However, were the system restored to its initial state, it would not be possible also to return the surroundings to the state they were in initially. As demonstrated in Sec. 5.3.2, the second law can be used to determine whether both the system and surroundings can be returned to their initial states after a process has occurred: the second law can be used to determine whether a given process is reversible or irreversible.

It might be apparent from the discussion of the Clausius statement of the second law that any process involving spontaneous heat transfer from a hotter body to a cooler body is irreversible. Otherwise, it would be possible to return this energy from the cooler body to the hotter body with no other effects within the two bodies or their surroundings. However, this possibility is denied by the Clausius statement.

Processes involving other kinds of spontaneous events, such as an unrestrained expansion of a gas or liquid, are also irreversible. Friction, electrical resistance, hysteresis, and inelastic deformation are examples of additional effects whose presence during a process renders it irreversible.

In summary, irreversible processes normally include one or more of the following **irreversibilities**:

irreversibilities

1. Heat transfer through a finite temperature difference
2. Unrestrained expansion of a gas or liquid to a lower pressure
3. Spontaneous chemical reaction
4. Spontaneous mixing of matter at different compositions or states
5. Friction—sliding friction as well as friction in the flow of fluids
6. Electric current flow through a resistance
7. Magnetization or polarization with hysteresis
8. Inelastic deformation

Although the foregoing list is not exhaustive, it does suggest that *all actual processes are irreversible*. That is, every process involves effects such as those listed, whether it is a naturally occurring process or one involving a device of our construction, from the simplest mechanism to the largest industrial plant. The term *irreversibility* is used to identify any of these effects. The above list comprises a few of the irreversibilities that are commonly encountered.

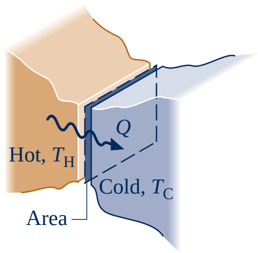
As a system undergoes a process, irreversibilities may be found within the system and its surroundings, although they may be located predominately in one place or the other. For many analyses it is convenient to divide the irreversibilities present into two classes. **Internal irreversibilities** are those that occur within the system. **External irreversibilities** are those that occur within the surroundings, often the immediate surroundings. As this distinction depends solely on the location of the boundary, there is some arbitrariness in the classification, for by extending the boundary to take in a portion of the surroundings, all irreversibilities become "internal." Nonetheless, as shown by subsequent developments, this distinction between irreversibilities is often useful.

internal and external irreversibilities

Engineers should be able to recognize irreversibilities, evaluate their influence, and develop practical means for reducing them. However, certain systems, such as brakes, rely on the effect of friction or other irreversibilities in their operation. The need to achieve profitable rates of production, high heat transfer rates, rapid accelerations, and so on invariably dictates the presence of significant irreversibilities.

Furthermore, irreversibilities are tolerated to some degree in every type of system because the changes in design and operation required to reduce them would be too costly. Accordingly, although improved thermodynamic performance can accompany the reduction of irreversibilities, steps taken in this direction are constrained by a number of practical factors often related to costs.

▶ **FOR EXAMPLE** consider two bodies at different temperatures that are able to communicate thermally. With a *finite* temperature difference between them, a spontaneous heat transfer would take place and, as discussed previously, this would be a source of irreversibility. It might be expected that the importance of this irreversibility diminishes as the temperature difference between the bodies diminishes, and while this *is* the case, there are practical consequences: From the study of heat transfer (Sec. 2.4), we know that the transfer of a finite amount of energy by heat transfer between bodies whose temperatures differ only slightly requires a considerable amount of time, a large (costly) heat transfer surface area, or both. In the limit as the temperature difference between the bodies vanishes, the amount of time and/or surface area required approach infinity. Such options are clearly impractical; still, they must be imagined when thinking of heat transfer approaching reversibility. ◀ ◀ ◀ ◀ ◀



5.3.2 Demonstrating Irreversibility

Whenever an irreversibility is present during a process, that process must necessarily be irreversible. However, the irreversibility of a process can be *demonstrated* rigorously using the Kelvin–Planck statement of the second law and the following procedure: (1) Assume there is a way to return the system and surroundings to their respective initial states. (2) Show that as a consequence of this assumption, it is possible to devise a cycle that violates the Kelvin–Planck statement—namely, a cycle that produces work while interacting thermally with only a single reservoir. Since the existence of such a cycle is denied by the Kelvin–Planck statement, the assumption must be in error and it follows that the process is irreversible.

This procedure can be used to demonstrate that processes involving friction, heat transfer through a finite temperature difference, the unrestrained expansion of a gas or liquid to a lower pressure, and other effects from the list given previously are irreversible. A case involving friction is discussed in the box.

While use of the Kelvin–Planck statement to demonstrate irreversibility is part of a traditional presentation of thermodynamics, such demonstrations can be unwieldy. It is normally easier to use the *entropy production* concept (Sec. 6.7).

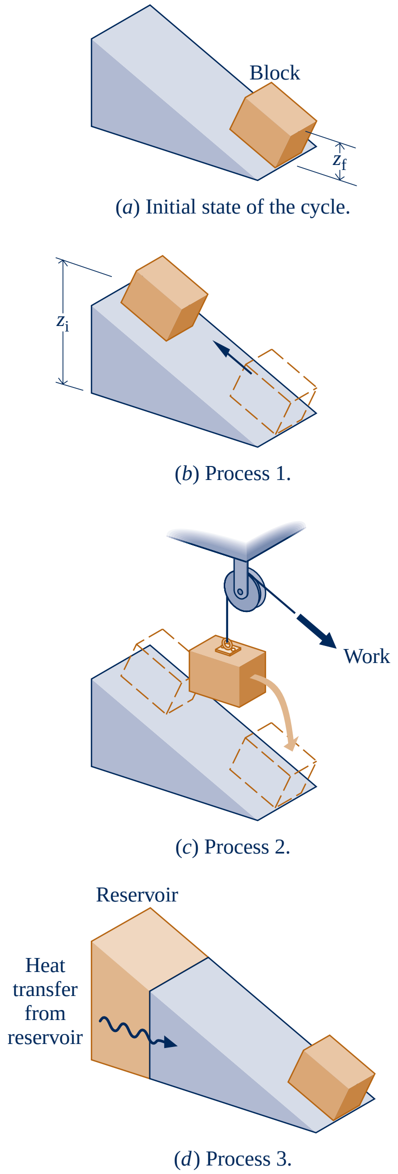


Fig. 5.3 Figure used to demonstrate the irreversibility of a process involving friction.

Demonstrating Irreversibility: Friction

Let us use the Kelvin–Planck statement to demonstrate the irreversibility of a process involving friction. Consider a system consisting of a block of mass m and an inclined plane. To begin, the block is at rest at the top of the incline. The block then slides down the plane, eventually coming to rest at a lower elevation. There is no significant work or heat transfer between the block–plane system and its surroundings during the process.

Applying the closed system energy balance to the system, we get

$$(U_f - U_i) + mg(z_f - z_i) + (\cancel{KE_f - KE_i})^0 = \cancel{Q}^0 - \cancel{W}^0$$

or

$$U_f - U_i = mg(z_i - z_f) \qquad \textbf{(a)}$$

where U denotes the internal energy of the block–plane system and z is the elevation of the block. Thus, friction between the block and plane during the process acts to convert the potential energy decrease of the block to internal energy of the overall system.

Since no work or heat interactions occur between the block–plane system and its surroundings, the condition of the surroundings remains unchanged during the process. This allows attention to be centered on the system only in demonstrating that the process is irreversible, as follows:

When the block is at rest after sliding down the plane, its elevation is z_f and the internal energy of the block–plane system is U_f. To demonstrate that the process is irreversible using the Kelvin–Planck statement, let us take this condition of the system, shown in Fig. 5.3*a*, as the initial state of a cycle consisting of three processes. We imagine that a pulley–cable arrangement and a thermal reservoir are available to assist in the demonstration.

Process 1: Assume the inverse process occurs with no change in the surroundings: As shown in Fig. 5.3*b*, the block returns *spontaneously* to the top of the plane while the internal energy of the system decreases to its initial value, U_i. (This is the process we want to demonstrate is impossible.)

Process 2: As shown in Fig. 5.3*c*, we use the pulley–cable arrangement provided to lower the block from z_i to z_f, while allowing the block–plane system to do work by lifting another mass located in the surroundings. The work done equals the decrease in potential energy of the block. This is the only work for the cycle. Thus, $W_{cycle} = mg(z_i - z_f)$.

Process 3: The internal energy of the system is increased from U_i to U_f by bringing it into communication with the reservoir, as shown in Fig. 5.3*d*. The heat transfer equals $(U_f - U_i)$. This is the only heat transfer for the cycle. Thus, $Q_{cycle} = (U_f - U_i)$, which with Eq. (a) becomes $Q_{cycle} = mg(z_i - z_f)$. At the conclusion of this process the block is again at elevation z_f and the internal energy of the block–plane system is restored to U_f.

The net result of this cycle is to draw energy from a single reservoir by heat transfer, Q_{cycle}, and produce an equivalent amount of work, W_{cycle}. There are no other effects. However, such a cycle is denied by the Kelvin–Planck statement. Since both the heating of the system by the reservoir (Process 3) and the lowering of the mass by the pulley–cable while work is done (Process 2) are possible, we conclude it is Process 1 that is impossible. Since Process 1 is the inverse of the original process where the block slides down the plane, it follows that the original process is irreversible.

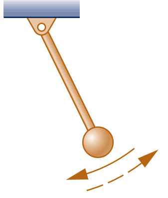
5.3.3 Reversible Processes

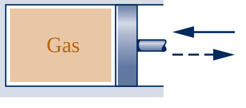
A process of a system is *reversible* if the system and all parts of its surroundings can be exactly restored to their respective initial states after the process has taken place. It should be evident from the discussion of irreversible processes that reversible processes are purely hypothetical. Clearly, no process can be reversible that involves spontaneous heat transfer through a finite temperature difference, an unrestrained expansion of a gas or liquid, friction, or any of the other irreversibilities listed previously. In a strict sense of the word, a reversible process is one that is *perfectly executed*.

All actual processes are irreversible. Reversible processes do not occur. Even so, certain processes that do occur are approximately reversible. The passage of a gas through a properly designed nozzle or diffuser is an example (Sec. 6.12). Many other devices also can be made to approach reversible operation by taking measures to reduce the significance of irreversibilities, such as lubricating surfaces to reduce friction. A reversible process is the *limiting case* as irreversibilities, both internal and external, are reduced further and further.

Although reversible processes cannot actually occur, they can be imagined. In Sec. 5.3.1, we considered how heat transfer would approach reversibility as the temperature difference approaches zero. Let us consider two additional examples:

- A particularly elementary example is a pendulum oscillating in an evacuated space. The pendulum motion approaches reversibility as friction at the pivot point is reduced. In the limit as friction is eliminated, the states of both the pendulum and its surroundings would be completely restored at the end of each period of motion. By definition, such a process is reversible.
- A system consisting of a gas adiabatically compressed and expanded in a frictionless piston–cylinder assembly provides another example. With a very small increase in the external pressure, the piston would compress the gas slightly. At each intermediate volume during the compression, the intensive properties T, p, v, etc. would be uniform throughout: The gas would pass through a series of equilibrium states. With a small decrease in the external pressure, the piston would slowly move out as the gas expands. At each intermediate volume of the expansion, the intensive properties of the gas would be at the same uniform values they had at the corresponding step during the compression. When the gas volume returned to its initial value, all properties would be restored to their initial values as well. The work done *on* the gas during the compression would equal the work done *by* the gas during the expansion. If the work between the system and its surroundings were delivered to, and received from, a frictionless pulley–mass assembly, or the equivalent, there also would be no net change in the surroundings. This process would be reversible.





Horizons

Second Law Takes Big Bite from Hydrogen

Hydrogen is not naturally occurring and thus must be produced. Hydrogen can be produced today from water by *electrolysis* and from natural gas by chemical processing called *reforming*. Hydrogen produced by these means and its subsequent utilization is burdened by the second law.

In electrolysis, an electrical input is employed to dissociate water to hydrogen according to $H_2O \rightarrow H_2 + {}^1/_2O_2$. When the hydrogen is subsequently used by a fuel cell to generate electricity, the cell reaction is $H_2 + {}^1/_2O_2 \rightarrow H_2O$. Although the cell reaction is the inverse of that occurring in electrolysis, the overall loop from electrical input–to hydrogen–to fuel cell-generated electricity is *not* reversible. Irreversibilities in the electrolyzer and the fuel cell conspire to ensure that the fuel cell-generated electricity is much less than the initial electrical input. This is wasteful because the electricity provided for electrolysis could instead be *fully* directed to most applications envisioned for hydrogen, including transportation. Further, when fossil fuel is burned in a power plant to generate electricity for electrolysis, the greenhouse gases produced can be associated with fuel cells by virtue of the hydrogen they consume. Although technical details differ, similar findings apply to the reforming of natural gas to hydrogen.

While hydrogen and fuel cells are expected to play a role in our energy future, second law barriers and other technical and economic issues stand in the way.

5.3.4 Internally Reversible Processes

internally reversible process

A reversible process is one for which no irreversibilities are present within the system *or* its surroundings. An **internally reversible process** is one for which *there are no irreversibilities within the system*. Irreversibilities may be located within the surroundings, however.

FOR EXAMPLE think of water condensing from saturated vapor to saturated liquid at 100°C while flowing through a copper tube whose outer surface is exposed to the ambient at 20°C. The water undergoes an internally reversible process, but there is heat transfer from the water to the ambient through the tube. For a control volume enclosing the water within the tube, such heat transfer is an *external* irreversibility.

At every intermediate state of an internally reversible process of a closed system, all intensive properties are uniform throughout each phase present. That is, the temperature, pressure, specific volume, and other intensive properties do not vary with position. If there were a spatial variation in temperature, say, there would be a tendency for a spontaneous energy transfer by conduction to occur *within* the system in the direction of decreasing temperature. For reversibility, however, no spontaneous processes can be present. From these considerations it can be concluded that the internally reversible process consists of a series of equilibrium states: It is a quasiequilibrium process.

TAKE NOTE...
The terms *internally reversible process* and *quasiequilibrium process* can be used interchangeably. However, to avoid having two terms that refer to the same thing, in subsequent sections we will refer to any such process as an internally reversible process.

The use of the internally reversible process concept in thermodynamics is comparable to idealizations made in mechanics: point masses, frictionless pulleys, rigid beams, and so on. In much the same way as idealizations are used in mechanics to simplify an analysis and arrive at a manageable model, simple thermodynamic models of complex situations can be obtained through the use of internally reversible processes. Calculations based on internally reversible processes often can be adjusted with efficiencies or correction factors to obtain reasonable estimates of actual performance under various operating conditions. Internally reversible processes are also useful for investigating the best thermodynamic performance of systems.

Finally, using the internally reversible process concept, we refine the definition of the thermal reservoir introduced in Sec. 5.2.2 as follows: In subsequent discussions we assume no internal irreversibilities are present within a thermal reservoir. That is, every process of a thermal reservoir is *internally reversible*.

5.4 Interpreting the Kelvin–Planck Statement

In this section, we recast Eq. 5.1, the analytical form of the Kelvin–Planck statement, into a more explicit expression, Eq. 5.3. This expression is applied in subsequent sections to obtain a number of significant deductions. In these applications, the following idealizations are assumed: The thermal reservoir and the portion of the surroundings with which work interactions occur are free of irreversibilities. This allows the "less than" sign to be associated with irreversibilities *within* the system of interest and the "equal to" sign to apply when no internal irreversibilites are present.

Accordingly, the **analytical form of the Kelvin–Planck statement** now takes the form

$$W_{\text{cycle}} \leq 0 \begin{cases} < 0: & \text{Internal irreversibilities present.} \\ = 0: & \text{No internal irreversibilities.} \end{cases} \text{(single reservoir)} \tag{5.3}$$

analytical form: Kelvin–Planck statement

For details, see the *Kelvin–Planck* box below.

Associating Signs with the Kelvin–Planck Statement

Consider a system that undergoes a cycle while exchanging energy by heat transfer with a single reservoir, as shown in Fig. 5.4. Work is delivered to, or received from, the pulley–mass assembly located in the surroundings. A flywheel, spring, or some other device also can perform the same function. The pulley–mass assembly, flywheel, or other device to which work is delivered, or from which it is received, is idealized as free of irreversibilities. The thermal reservoir is also assumed free of irreversibilities.

To demonstrate the correspondence of the "equal to" sign of Eq. 5.3 with the absence of irreversibilities, consider a cycle operating as shown in Fig. 5.4 for which the equality applies. At the conclusion of one cycle,

- ▶ The system would necessarily be returned to its initial state.
- ▶ Since $W_{\text{cycle}} = 0$, there would be no *net* change in the elevation of the mass used to store energy in the surroundings.
- ▶ Since $W_{\text{cycle}} = Q_{\text{cycle}}$, it follows that $Q_{\text{cycle}} = 0$, so there also would be no *net* change in the condition of the reservoir.

Thus, the system and all elements of its surroundings would be exactly restored to their respective initial conditions. By definition, such a cycle is reversible. Accordingly, there can be no irreversibilities present within the system or its surroundings. It is left as an exercise to show the converse: If the cycle occurs reversibly, the equality applies (see end-of-chapter Problem 5.7).

Since a cycle is reversible *or* irreversible and we have linked the equality with reversible cycles, we conclude the inequality corresponds to the presence of internal irreversibilities. Moreover, the inequality can be interpreted as follows: Net work done *on* the system per cycle is converted by action of internal irreversibilities to internal energy that is discharged by heat transfer *to* the thermal reservoir in an amount equal to net work.

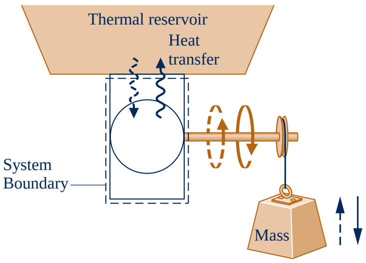


Fig. 5.4 System undergoing a cycle while exchanging energy by heat transfer with a single thermal reservoir.

Concluding Comment

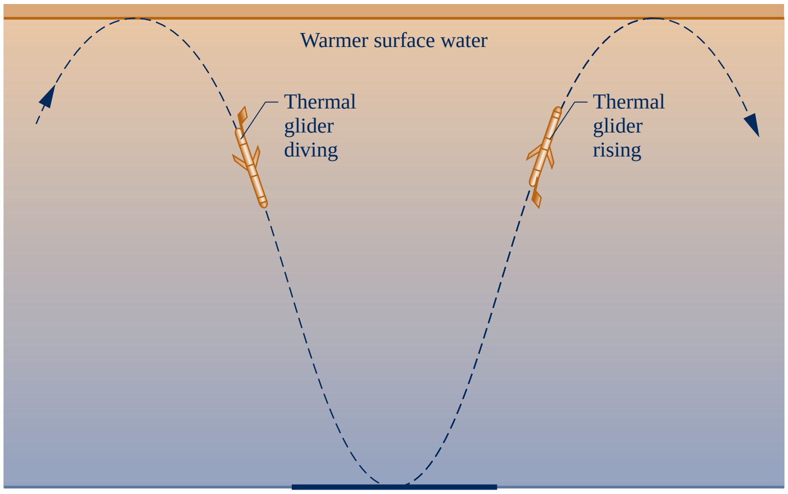
The Kelvin–Planck statement considers systems undergoing *thermodynamic* cycles while exchanging energy by heat transfer with *one* thermal reservoir. These restrictions must be strictly observed—see the *thermal glider* box.

Does the *Thermal Glider* Challenge the Kelvin–Planck Statement?

A 2008 *Woods Hole Oceanographic Institute* news release, "Researchers Give New Hybrid Vehicle Its First Test-Drive in the Ocean," announced the successful testing of an underwater *thermal glider* that "harvests . . . energy from the ocean (thermally) to propel itself." Does this submersible vehicle challenge the Kelvin–Planck statement of the second law?

Study of the thermal glider shows it is capable of sustaining forward motion underwater for weeks while interacting thermally only with the ocean and undergoing a *mechanical* cycle. Still, the glider does not mount a challenge to the Kelvin–Planck statement because it does not exchange energy by heat transfer with a *single* thermal reservoir and does not execute a *thermodynamic* cycle.

The glider propels itself by interacting thermally with warmer surface waters and colder, deep-ocean layers to change its buoyancy to dive, rise toward the surface, and dive again, as shown on the accompanying figure. Accordingly, the glider does not interact thermally with a single reservoir as required by the Kelvin–Planck statement. The glider also does not satisfy all energy needs by interacting with the ocean: Batteries are required to power on-board electronics. Although these power needs are relatively minor, the batteries lose charge with use, and so the glider does not execute a thermodynamic cycle as required by the Kelvin–Planck statement.



5.5 Applying the Second Law to Thermodynamic Cycles

While the Kelvin–Planck statement of the second law (Eq. 5.3) provides the foundation for the rest of this chapter, application of the second law to thermodynamic cycles is by no means limited to the case of heat transfer with a *single* reservoir or even with *any* reservoirs. Systems undergoing cycles while interacting thermally with *two* thermal reservoirs are considered from a second-law viewpoint in Secs. 5.6 and 5.7, providing results having important applications. Moreover, the one- and two-reservoir discussions pave the way for Sec. 5.11, where the *general* case is considered—namely, what the second law says about *any* thermodynamic cycle without regard to the nature of the body or bodies with which energy is exchanged by heat transfer.

In the sections to follow, applications of the second law to power cycles and refrigeration and heat pump cycles are considered. For this content, familiarity with rudimentary thermodynamic cycle principles is required. We recommend you review Sec. 2.6, where cycles are considered from an energy perspective and the thermal efficiency of power cycles and coefficients of performance for refrigeration and heat pump systems are introduced. In particular, Eqs. 2.40–2.48 and the accompanying discussions should be reviewed.

5.6 Second Law Aspects of Power Cycles Interacting with Two Reservoirs

5.6.1 Limit on Thermal Efficiency

A significant limitation on the performance of systems undergoing power cycles can be brought out using the Kelvin–Planck statement of the second law. Consider Fig. 5.5, which shows a system that executes a cycle while communicating thermally with *two* thermal reservoirs, a hot reservoir and a cold reservoir, and developing net work W_{cycle}. The thermal efficiency of the cycle is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{H}}} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \tag{5.4}$$

where Q_{H} is the amount of energy received by the system from the hot reservoir by heat transfer and Q_{C} is the amount of energy discharged from the system to the cold reservoir by heat transfer.

TAKE NOTE...
The energy transfers labeled on Fig 5.5 are positive in the directions indicated by the arrows.

If the value of Q_{C} were zero, the system of Fig. 5.5 would withdraw energy Q_{H} from the hot reservoir and produce an equal amount of work, while undergoing a cycle. The thermal efficiency of such a cycle would be unity (100%). However, this method of operation violates the Kelvin–Planck statement and thus is not allowed.

It follows that for *any* system executing a power cycle while operating between two reservoirs, only a portion of the heat transfer Q_{H} can be obtained as work, and the remainder, Q_{C}, must be discharged by heat transfer to the cold reservoir. That is, the thermal efficiency must be less than 100%.

In arriving at this conclusion it was *not* necessary to

- ▶ identify the nature of the substance contained within the system,
- ▶ specify the exact series of processes making up the cycle,
- ▶ indicate whether the processes are actual processes or somehow idealized.

The conclusion that the thermal efficiency must be less than 100% applies to *all* power cycles whatever their details of operation. This may be regarded as a corollary of the second law. Other corollaries follow.

5.6.2 Corollaries of the Second Law for Power Cycles

Since no power cycle can have a thermal efficiency of 100%, it is of interest to investigate the maximum theoretical efficiency. The maximum theoretical efficiency for systems undergoing power cycles while communicating thermally with two thermal reservoirs at different temperatures is evaluated in Sec. 5.9 with reference to the following two corollaries of the second law, called the **Carnot corollaries**.

Carnot corollaries

1. The thermal efficiency of an irreversible power cycle is always less than the thermal efficiency of a reversible power cycle when each operates between the same two thermal reservoirs.
2. All reversible power cycles operating between the same two thermal reservoirs have the same thermal efficiency.

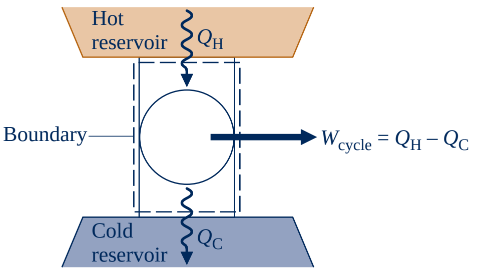


Fig. 5.5 System undergoing a power cycle while exchanging energy by heat transfer with two reservoirs.

A cycle is considered *reversible* when there are no irreversibilities within the system as it undergoes the cycle and heat transfers between the system and reservoirs occur reversibly.

The idea underlying the first Carnot corollary is in agreement with expectations stemming from the discussion of the second law thus far. Namely, the presence of irreversibilities during the execution of a cycle is expected to exact a penalty: If two systems operating between the same reservoirs each receive the same amount of energy Q_H and one executes a reversible cycle while the other executes an irreversible cycle, it is in accord with intuition that the net work developed by the irreversible cycle will be less, and thus the irreversible cycle has the smaller thermal efficiency.

The second Carnot corollary refers only to reversible cycles. All processes of a reversible cycle are perfectly executed. Accordingly, if two reversible cycles operating between the same reservoirs each receive the same amount of energy Q_H but one could produce more work than the other, it could only be as a result of more advantageous selections for the substance making up the system (it is conceivable that, say, air might be better than water vapor) *or* the series of processes making up the cycle (nonflow processes might be preferable to flow processes). This corollary denies both possibilities and indicates that the cycles must have the same efficiency whatever the choices for the working substance or the series of processes.

The two Carnot corollaries can be demonstrated using the Kelvin–Planck statement of the second law. For details, see the box.

Demonstrating the Carnot Corollaries

The first Carnot corollary can be demonstrated using the arrangement of Fig. 5.6. A reversible power cycle R and an irreversible power cycle I operate between the same two reservoirs and each receives the same amount of energy Q_H from the hot reservoir. The reversible cycle produces work W_R while the irreversible cycle produces work W_I. In accord with the conservation of energy principle, each cycle discharges energy to the cold reservoir equal to the difference between Q_H and the work produced. Let R now operate in the opposite direction as a refrigeration (or heat pump) cycle. Since R is reversible, the magnitudes of the energy transfers W_R, Q_H, and Q_C remain the same, but the energy transfers are oppositely directed, as shown by the dashed lines on Fig. 5.6. Moreover, with R operating in the opposite direction, the hot reservoir would experience *no net change* in its condition since it would receive Q_H *from* R while passing Q_H *to* I.

The demonstration of the first Carnot corollary is completed by considering the *combined system* shown by the dotted line on Fig. 5.6, which consists of the two cycles and the hot reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, the combined system exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy Eq. 5.3 expressed as

$$W_{\text{cycle}} < 0 \qquad \text{(single reservoir)}$$

where the inequality is used because the combined system is irreversible in its operation since irreversible cycle I is one of its parts. Evaluating W_{cycle} for the combined system in terms of the work amounts W_I and W_R, the above inequality becomes

$$W_I - W_R < 0$$

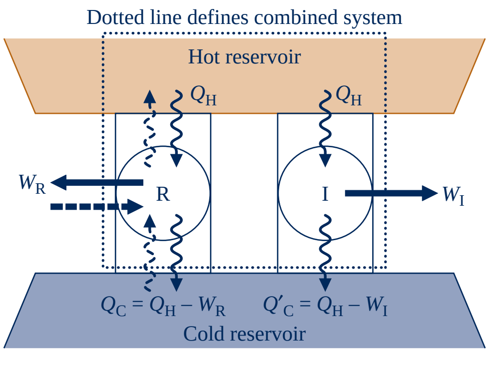


Fig. 5.6 Sketch for demonstrating that a reversible cycle R is more efficient than an irreversible cycle I when they operate between the same two reservoirs.

which shows that W_I must be less than W_R. Since each cycle receives the same energy input, Q_H, it follows that $\eta_I < \eta_R$ and this completes the demonstration.

The second Carnot corollary can be demonstrated in a parallel way by considering any two reversible cycles R_1 and R_2 operating between the same two reservoirs. Then, letting R_1 play the role of R and R_2 the role of I in the previous development, a combined system consisting of the two cycles and the hot reservoir may be formed that must obey Eq. 5.3. However, in applying Eq. 5.3 to this combined system, the equality is used because the system is reversible in operation. Thus, it can be concluded that $W_{R1} = W_{R2}$, and therefore, $\eta_{R1} = \eta_{R2}$. The details are left as an exercise (see end-of-chapter Problem 5.10).

5.7 Second Law Aspects of Refrigeration and Heat Pump Cycles Interacting with Two Reservoirs

5.7.1 Limits on Coefficients of Performance

The second law of thermodynamics places limits on the performance of refrigeration and heat pump cycles as it does for power cycles. Consider Fig. 5.7, which shows a system undergoing a cycle while communicating thermally with two thermal reservoirs, a hot and a cold reservoir. The energy transfers labeled on the figure are in the directions indicated by the arrows. In accord with the conservation of energy principle, the cycle discharges energy Q_H by heat transfer to the hot reservoir equal to the sum of the energy Q_C received by heat transfer from the cold reservoir and the net work input. This cycle might be a refrigeration cycle or a heat pump cycle, depending on whether its function is to remove energy Q_C from the cold reservoir or deliver energy Q_H to the hot reservoir.

For a refrigeration cycle the coefficient of performance is

$$\beta = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} \tag{5.5}$$

The coefficient of performance for a heat pump cycle is

$$\gamma = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} \tag{5.6}$$

As the net work input to the cycle W_{cycle} tends to zero, the coefficients of performance given by Eqs. 5.5 and 5.6 approach a value of infinity. If W_{cycle} were identically zero, the system of Fig. 5.7 would withdraw energy Q_C from the cold reservoir and deliver that energy to the hot reservoir, while undergoing a cycle. However, this method of operation violates the Clausius statement of the second law and thus is not allowed. It follows that the coefficients of performance β and γ must invariably be finite in value. This may be regarded as another corollary of the second law. Further corollaries follow.

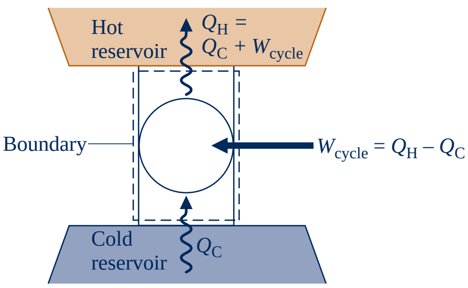


Fig. 5.7 System undergoing a refrigeration or heat pump cycle while exchanging energy by heat transfer with two reservoirs.

5.7.2 Corollaries of the Second Law for Refrigeration and Heat Pump Cycles

The maximum theoretical coefficients of performance for systems undergoing refrigeration and heat pump cycles while communicating thermally with two reservoirs at different temperatures are evaluated in Sec. 5.9 with reference to the following corollaries of the second law:

1. The coefficient of performance of an irreversible refrigeration cycle is always less than the coefficient of performance of a reversible refrigeration cycle when each operates between the same two thermal reservoirs.
2. All reversible refrigeration cycles operating between the same two thermal reservoirs have the same coefficient of performance.

By replacing the term *refrigeration* with *heat pump,* we obtain counterpart corollaries for heat pump cycles.

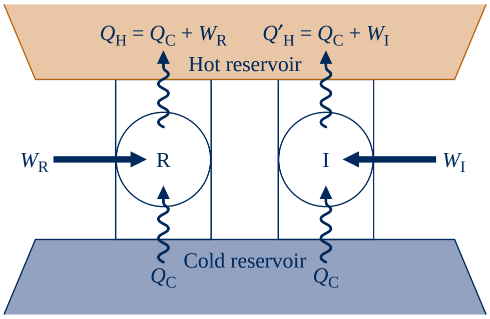


Fig. 5.8 Sketch for demonstrating that a reversible refrigeration cycle R has a greater coefficient of performance than an irreversible cycle I when they operate between the same two reservoirs.

The first of these corollaries agrees with expectations stemming from the discussion of the second law thus far. To explore this, consider Fig. 5.8, which shows a reversible refrigeration cycle R and an irreversible refrigeration cycle I operating between the same two reservoirs. Each cycle removes the same energy Q_C from the cold reservoir. The net work input required to operate R is W_R, while the net work input for I is W_I. Each cycle discharges energy by heat transfer to the hot reservoir equal to the sum of Q_C and the net work input. The directions of the energy transfers are shown by arrows on Fig. 5.8. The presence of irreversibilities during the operation of a refrigeration cycle is expected to exact a penalty: If two refrigerators working between the same reservoirs each receive an identical energy transfer from the cold reservoir, Q_C, and one executes a reversible cycle while the other executes an irreversible cycle, we expect the irreversible cycle to require a greater net work input and thus have the smaller coefficient of performance. By a simple extension it follows that all reversible refrigeration cycles operating between the same two reservoirs have the same coefficient of performance. Similar arguments apply to the counterpart heat pump cycle statements.

These corollaries can be demonstrated formally using the Kelvin–Planck statement of the second law and a procedure similar to that employed for the Carnot corollaries. The details are left as an exercise (see end-of-chapter Problem 5.11).

ENERGY & ENVIRONMENT Warm blankets of pollution-laden air surround major cities. Sunlight-absorbing rooftops and expanses of pavement, together with little greenery, conspire with other features of city living to raise urban temperatures several degrees above adjacent suburban areas. Figure 5.9 shows the variation of surface temperature in the vicinity of a city as measured by infrared measurements made from low-level flights over the area. Health-care professionals worry about the impact of these "heat islands," especially on the elderly. Paradoxically, the hot exhaust from the air conditioners city dwellers use to keep cool also make sweltering neighborhoods even hotter. Irreversibilities within air conditioners contribute to the warming effect. Air conditioners may account for as much as 20% of the urban temperature rise. Vehicles and commercial activity also are contributors. Urban planners are combating heat islands in many ways, including the use of highly-reflective colored roofing products and the installation of roof-top gardens. The shrubs and trees of roof-top gardens absorb solar energy, leading to summer roof temperatures significantly below those of nearby buildings without roof-top gardens, reducing the need for air conditioning.

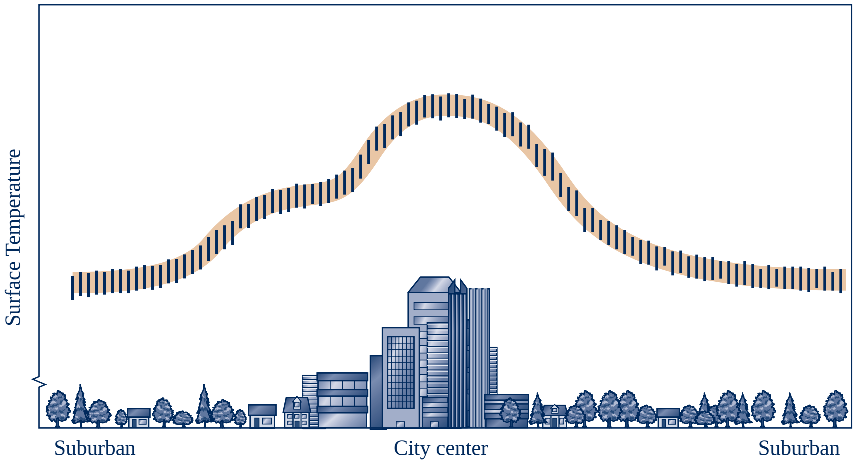


Fig. 5.9 **Surface temperature variation in an urban area.**

5.8 The Kelvin and International Temperature Scales

The results of Secs. 5.6 and 5.7 establish theoretical upper limits on the performance of power, refrigeration, and heat pump cycles communicating thermally with two reservoirs. Expressions for the *maximum* theoretical thermal efficiency of power cycles and the *maximum* theoretical coefficients of performance of refrigeration and heat pump cycles are developed in Sec. 5.9 using the Kelvin temperature scale considered next.

5.8.1 The Kelvin Scale

From the second Carnot corollary we know that all reversible power cycles operating between the same two thermal reservoirs have the same thermal efficiency, regardless of the nature of the substance making up the system executing the cycle or the series of processes. Since the thermal efficiency is independent of these factors, its value can be related only to the nature of the reservoirs themselves. Noting that it is the difference in *temperature* between the two reservoirs that provides the impetus for heat transfer between them, and thereby for the production of work during the cycle, we reason that the thermal efficiency depends *only* on the temperatures of the two reservoirs.

From Eq. 5.4 it also follows that for such reversible power cycles the ratio of the heat transfers Q_C/Q_H depends *only* on the temperatures of the two reservoirs. That is

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\\text{cycle}}} = \psi(\theta_C, \theta_H) \tag{a}$$

where θ_H and θ_C denote the temperatures of the reservoirs and the function ψ is for the present unspecified. Note that the words "rev cycle" are added to this expression to emphasize that it applies only to systems undergoing reversible cycles while operating between two thermal reservoirs.

Equation (a) provides a basis for defining a *thermodynamic* temperature scale: a scale independent of the properties of any substance. There are alternative choices for the function ψ that lead to this end. The **Kelvin scale** is obtained by making a

Kelvin scale

particularly simple choice, namely, $\psi = T_C/T_H$, where T is the symbol used by international agreement to denote temperatures on the Kelvin scale. With this, we get

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \frac{T_C}{T_H} \tag{5.7}$$

Thus, two temperatures on the Kelvin scale are in the same ratio as the values of the heat transfers absorbed and rejected, respectively, by a system undergoing a reversible cycle while communicating thermally with reservoirs at these temperatures.

TAKE NOTE... Some readers may prefer to proceed directly to Sec. 5.9, where Eq. 5.7 is applied.

If a reversible power cycle were operated in the opposite direction as a refrigeration or heat pump cycle, the magnitudes of the energy transfers Q_C and Q_H would remain the same, but the energy transfers would be oppositely directed. Accordingly, Eq. 5.7 applies to each type of cycle considered thus far, provided the system undergoing the cycle operates between two thermal reservoirs and the cycle is reversible.

More on the Kelvin Scale

Equation 5.7 gives only a ratio of temperatures. To complete the definition of the Kelvin scale, it is necessary to proceed as in Sec. 1.7.3 by assigning the value 273.16 K to the temperature at the triple point of water. Then, if a reversible cycle is operated between a reservoir at 273.16 K and another reservoir at temperature T, the two temperatures are related according to

$$T = 273.16\left(\frac{Q}{Q_{tp}}\right)_{\substack{\text{rev}\\ \text{cycle}}} \tag{5.8}$$

where Q_{tp} and Q are the heat transfers between the cycle and reservoirs at 273.16 K and temperature T, respectively. In the present case, the heat transfer Q plays the role of the *thermometric property.* However, since the performance of a reversible cycle is independent of the makeup of the system executing the cycle, the definition of temperature given by Eq. 5.8 depends in no way on the properties of any substance or class of substances.

In Sec. 1.7 we noted that the Kelvin scale has a zero of 0 K, and lower temperatures than this are not defined. Let us take up these points by considering a reversible power cycle operating between reservoirs at 273.16 K and a lower temperature T. Referring to Eq. 5.8, we know that the energy rejected from the cycle by heat transfer Q would not be negative, so T must be nonnegative. Equation 5.8 also shows that the smaller the value of Q, the lower the value of T, and conversely. Accordingly, as Q approaches zero the temperature T approaches zero. It can be concluded that a temperature of zero on the Kelvin scale is the lowest conceivable temperature. This temperature is called the *absolute* zero, and the Kelvin scale is called an *absolute temperature scale.*

When numerical values of the thermodynamic temperature are to be determined, it is not possible to use reversible cycles, for these exist only in our imaginations. However, temperatures evaluated using the constant-volume gas thermometer discussed in Sec. 5.8.2 to follow are identical to those of the Kelvin scale in the range of temperatures where the gas thermometer can be used. Other empirical approaches can be employed for temperatures above and below the range accessible to gas thermometry. The Kelvin scale provides a continuous definition of temperature valid over all ranges and provides an essential connection between the several empirical measures of temperature.

5.8.2 The Gas Thermometer

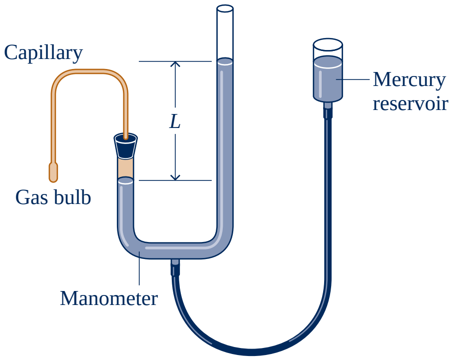


Fig. 5.10 Constant-volume gas thermometer.

The constant-volume gas thermometer shown in Fig. 5.10 is so exceptional in terms of precision and accuracy that it has been adopted internationally as the standard instrument for calibrating other thermometers. The *thermometric substance* is the gas (normally hydrogen or helium), and the *thermometric property* is the pressure exerted by the gas. As shown in the figure, the gas is contained in a bulb, and the pressure exerted by it is measured by an open-tube mercury manometer. As temperature increases, the gas expands, forcing mercury up in the open tube. The gas is kept at constant volume by raising or lowering the reservoir. The gas thermometer is used as a standard worldwide by bureaus of standards and research laboratories. However, because gas thermometers require elaborate apparatus and are large, slowly responding devices that demand painstaking experimental procedures, smaller, more rapidly responding thermometers are used for most temperature measurements and they are calibrated (directly or indirectly) against gas thermometers. For further discussion of gas thermometry, see the box.

Measuring Temperature with the Gas Thermometer—The Gas Scale

It is instructive to consider how numerical values are associated with levels of temperature by the gas thermometer shown in Fig. 5.10. Let p stand for the pressure in the bulb of a constant-volume gas thermometer in thermal equilibrium with a bath. A value can be assigned to the bath temperature by a linear relation

$$T = \alpha p \tag{a}$$

where α is an arbitrary constant.

The value of α is determined by inserting the thermometer into another bath maintained at the triple point of water and measuring the pressure, call it p_{tp}, of the confined gas at the triple point temperature, 273.16 K. Substituting values into Eq. (a) and solving for α

$$\alpha = \frac{273.16}{p_{tp}}$$

Inserting this in Eq. (a), the temperature of the original bath, at which the pressure of the confined gas is p, is then

$$T = 273.16\left(\frac{p}{p_{tp}}\right) \tag{b}$$

However, since the values of both pressures, p and p_{tp}, depend *in part* on the amount of gas in the bulb, the value assigned by Eq. (b) to the bath temperature varies with the amount of gas in the thermometer. This difficulty is overcome in precision thermometry by repeating the measurements (in the original bath and the reference bath) several times with less gas in the bulb in each successive attempt. For each trial the ratio p/p_{tp} is calculated from Eq. (b) and plotted versus the corresponding reference pressure p_{tp} of the gas at the triple point temperature. When several such points have been plotted, the resulting curve is extrapolated to the ordinate where $p_{tp} = 0$. This is illustrated in Fig. 5.11 for constant-volume thermometers with a number of different gases.

Inspection of Fig. 5.11 shows that at each nonzero value of the reference pressure, the p/p_{tp} values differ with the gas employed in the thermometer. However, as pressure decreases, the p/p_{tp} values from thermometers with different gases approach one another, and in the limit as pressure tends to zero, *the same value for p/p_{tp} is obtained*

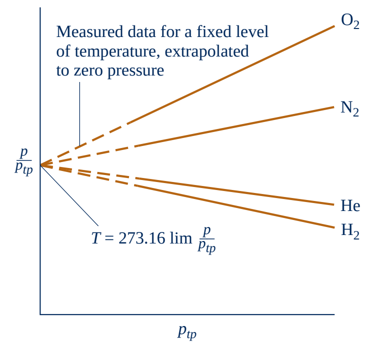


Fig. 5.11 Readings of constant-volume gas thermometers, when several gases are used.

for each gas. Based on these general results, the *gas temperature scale* is defined by the relationship

$$T = 273.16 \lim \frac{p}{p_{\text{tp}}} \tag{c}$$

where "lim" means that both p and p_{tp} tend to zero. It should be evident that the determination of temperatures by this means requires extraordinarily careful and elaborate experimental procedures.

Although the temperature scale of Eq. (c) is independent of the properties of any one gas, it still depends on the properties of gases in general. Accordingly, the measurement of low temperatures requires a gas that does not condense at these temperatures, and this imposes a limit on the range of temperatures that can be measured by a gas thermometer. The lowest temperature that can be measured with such an instrument is about 1 K, obtained with helium. At high temperatures gases dissociate, and therefore these temperatures also cannot be determined by a gas thermometer. Other empirical means, utilizing the properties of other substances, must be employed to measure temperature in ranges where the gas thermometer is inadequate. For further discussion see Sec. 5.8.3.

5.8.3 International Temperature Scale

To provide a standard for temperature measurement taking into account both theoretical and practical considerations, the International Temperature Scale (ITS) was adopted in 1927. This scale has been refined and extended in several revisions, most recently in 1990. *The International Temperature Scale of 1990 (ITS-90)* is defined in such a way that the temperature measured on it conforms with the thermodynamic temperature, the unit of which is the kelvin, to within the limits of accuracy of measurement obtainable in 1990. The ITS-90 is based on the assigned values of temperature of a number of reproducible *fixed points* (Table 5.1). Interpolation between the fixed-point temperatures is accomplished by formulas that give the relation between readings of standard instruments and values of the ITS. In the range from 0.65 to 5.0 K, ITS-90 is defined by equations giving the temperature as functions of the vapor pressures of particular helium isotopes. The range from 3.0 to 24.5561 K is based on measurements using a helium constant-volume gas thermometer. In the range from 13.8033 to 1234.93 K, ITS-90 is defined by means of certain platinum resistance thermometers. Above 1234.93 K the temperature is defined using *Planck's equation for blackbody radiation* and measurements of the intensity of visible-spectrum radiation.

5.9 Maximum Performance Measures for Cycles Operating Between Two Reservoirs

The discussion continues in this section with the development of expressions for the maximum thermal efficiency of power cycles and the maximum coefficients of performance of refrigeration and heat pump cycles in terms of reservoir temperatures evaluated on the Kelvin scale. These expressions can be used as standards of comparison for actual power, refrigeration, and heat pump cycles.

TABLE 5.1

Defining Fixed Points of the International Temperature Scale of 1990

T (K)	Substance[a]	State[b]
3 to 5	He	Vapor pressure point
13.8033	$e\text{-}H_2$	Triple point
≈ 17	$e\text{-}H_2$	Vapor pressure point
≈ 20.3	$e\text{-}H_2$	Vapor pressure point
24.5561	Ne	Triple point
54.3584	O_2	Triple point
83.8058	Ar	Triple point
234.3156	Hg	Triple point
273.16	H_2O	Triple point
302.9146	Ga	Melting point
429.7485	In	Freezing point
505.078	Sn	Freezing point
692.677	Zn	Freezing point
933.473	Al	Freezing point
1234.93	Ag	Freezing point
1337.33	Au	Freezing point
1357.77	Cu	Freezing point

[a]He denotes ^3He or ^4He; $e\text{-}H_2$ is hydrogen at the equilibrium concentration of the ortho- and para-molecular forms.

[b]Triple point: temperature at which the solid, liquid, and vapor phases are in equilibrium. Melting point, freezing point: temperature, at a pressure of 101.325 kPa, at which the solid and liquid phases are in equilibrium.

Source: H. Preston-Thomas, "The International Temperature Scale of 1990 (ITS-90)," *Metrologia* 27, 3–10 (1990). See also www.ITS-90.com.

5.9.1 Power Cycles

The use of Eq. 5.7 in Eq. 5.4 results in an expression for the thermal efficiency of a system undergoing a reversible *power cycle* while operating between thermal reservoirs at temperatures T_H and T_C. That is

$$\eta_{max} = 1 - \frac{T_C}{T_H} \tag{5.9}$$

Carnot efficiency

which is known as the **Carnot efficiency**. As temperatures on the Rankine scale differ from Kelvin temperatures only by the factor 1.8, the T's in Eq. 5.9 may be on either scale of temperature.

Recalling the two Carnot corollaries, it should be evident that the efficiency given by Eq. 5.9 is the thermal efficiency of *all* reversible power cycles operating between two reservoirs at temperatures T_H and T_C, and the *maximum* efficiency *any* power cycle can have while operating between the two reservoirs. By inspection, the value of the Carnot efficiency increases as T_H increases and/or T_C decreases.

Equation 5.9 is presented graphically in Fig. 5.12. The temperature T_C used in constructing the figure is 298 K in recognition that actual power cycles ultimately discharge energy by heat transfer at about the temperature of the local atmosphere or cooling water drawn from a nearby river or lake. Note that the possibility of increasing the thermal efficiency by reducing T_C below that of the environment is

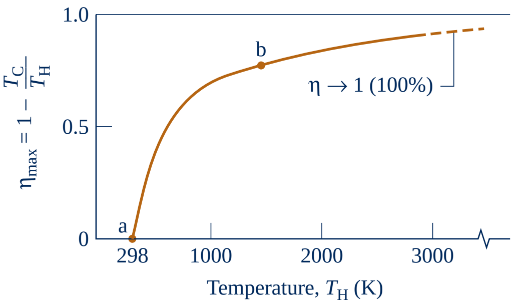


Fig. 5.12 **Carnot efficiency versus T_H, for T_C = 298 K.**

not practical, for maintaining T_C lower than the ambient temperature would require a refrigerator that would have to be supplied work to operate.

Figure 5.12 shows that the thermal efficiency increases with T_H. Referring to segment a–b of the curve, where T_H and η are relatively low, we see that η increases rapidly as T_H increases, showing that in this range even a small increase in T_H can have a large effect on efficiency. Though these conclusions, drawn as they are from Eq. 5.9, apply strictly only to systems undergoing reversible cycles, they are qualitatively correct for actual power cycles. The thermal efficiencies of actual cycles are observed to increase as the *average* temperature at which energy is added by heat transfer increases and/or the *average* temperature at which energy is discharged by heat transfer decreases. However, maximizing the thermal efficiency of a power cycle may not be the only objective. In practice, other considerations such as cost may be overriding.

A Power_Cycle A.9 – Tab c

Conventional power-producing cycles have thermal efficiencies ranging up to about 40%. This value may seem low, but the comparison should be made with an appropriate limiting value and not 100%.

▸ FOR EXAMPLE consider a system executing a power cycle for which the average temperature of heat addition is 745 K and the average temperature at which heat is discharged is 298 K. For a reversible cycle receiving and discharging energy by heat transfer at these temperatures, the thermal efficiency given by Eq. 5.9 is 60%. When compared to this value, an actual thermal efficiency of 40% does not appear to be so low. The cycle would be operating at two-thirds of the theoretical maximum. ◂ ◂ ◂ ◂ ◂

In the next example, we evaluate an inventor's claim about the performance of a power cycle, illustrating the use of the Carnot corollaries (Sec. 5.6.2) and the Carnot efficiency, Eq. 5.9.

EXAMPLE 5.1

Evaluating a Power Cycle Performance Claim

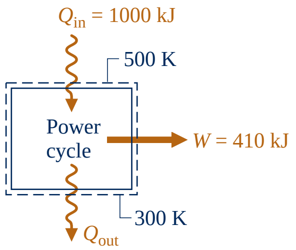
An inventor claims to have developed a power cycle capable of delivering a net work output of 410 kJ for an energy input by heat transfer of 1000 kJ. The system undergoing the cycle receives the heat transfer from hot gases at a temperature of 500 K and discharges energy by heat transfer to the atmosphere at 300 K. Evaluate this claim.

SOLUTION

Known: A system operates in a cycle and produces a net amount of work while receiving and discharging energy by heat transfer at fixed temperatures.

Find: Evaluate the claim that the cycle can develop 410 kJ of work for an energy input by heat of 1000 kJ.

Schematic and Given Data:



Engineering Model:

1. The system shown on the accompanying figure executes a power cycle.
2. The hot gases and the atmosphere play the roles of hot and cold reservoirs, respectively.

Fig. E5.1

Analysis: Inserting the values supplied by the inventor into Eq. 5.4, the cycle thermal efficiency is

$$\eta = \frac{410 \text{ kJ}}{1000 \text{ kJ}} = 0.41\,(41\%)$$

The maximum thermal efficiency *any* power cycle can have while operating between reservoirs at $T_H = 500$ K and $T_C = 300$ K is given by Eq. 5.9.

1 $$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{300 \text{ K}}{500 \text{ K}} = 0.40\ (40\%)$$

The Carnot corollaries provide a basis for evaluating the claim: Since the thermal efficiency of the actual cycle exceeds the maximum theoretical value, the claim cannot be valid.

1 The temperatures T_C and T_H used in evaluating η_{max} *must* be in K or °R.

Skills Developed

Ability to...

- ❑ apply the Carnot corollaries, using Eqs. 5.4 and 5.9 appropriately.

QuickQUIZ If the cycle receives heat transfer from a hot gas at 600 K while all other data remain unchanged, evaluate the inventor's claim. **Ans.** Claim is in accord with the second law.

5.9.2 Refrigeration and Heat Pump Cycles

Equation 5.7 is also applicable to reversible refrigeration and heat pump cycles operating between two thermal reservoirs, but for these Q_C represents the heat added to the cycle from the cold reservoir at temperature T_C on the Kelvin scale and Q_H is the heat discharged to the hot reservoir at temperature T_H. Introducing Eq. 5.7 in Eq. 5.5 results in the following expression for the coefficient of performance of any system undergoing a reversible refrigeration cycle while operating between the two reservoirs

$$\beta_{max} = \frac{T_C}{T_H - T_C} \tag{5.10}$$

Similarly, substituting Eq. 5.7 into Eq. 5.6 gives the following expression for the coefficient of performance of any system undergoing a reversible heat pump cycle while operating between the two reservoirs

$$\gamma_{max} = \frac{T_H}{T_H - T_C} \tag{5.11}$$

Note that the temperatures used to evaluate β_{max} and γ_{max} must be absolute temperatures on the Kelvin or Rankine scale.

From the discussion of Sec. 5.7.2, it follows that Eqs. 5.10 and 5.11 are the maximum coefficients of performance that any refrigeration and heat pump cycles can have while operating between reservoirs at temperatures T_H and T_C. As for the case of the Carnot efficiency, these expressions can be used as standards of comparison for actual refrigerators and heat pumps.

In the next example, we evaluate the coefficient of performance of a refrigerator and compare it with the maximum theoretical value, illustrating the use of the second law corollaries of Sec. 5.7.2 together with Eq. 5.10.

Refrig_Cycle A.10 – Tab c
Heat_Pump_Cycle A.11 – Tab c

EXAMPLE 5.2

Evaluating Refrigerator Performance

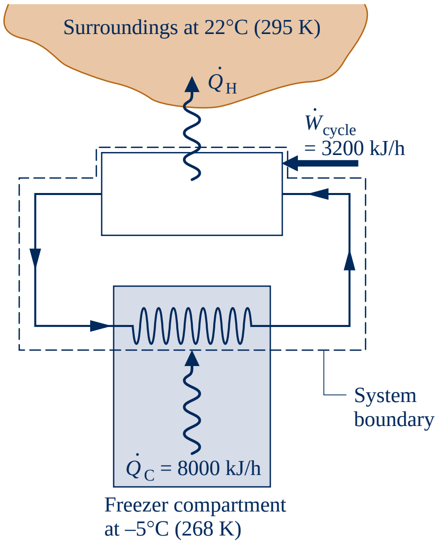
By steadily circulating a refrigerant at low temperature through passages in the walls of the freezer compartment, a refrigerator maintains the freezer compartment at −5°C when the air surrounding the refrigerator is at 22°C. The rate of heat transfer from the freezer compartment to the refrigerant is 8000 kJ/h and the power input required to operate the refrigerator is 3200 kJ/h. Determine the coefficient of performance of the refrigerator and compare with the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the same two temperatures.

SOLUTION

Known: A refrigerator maintains a freezer compartment at a specified temperature. The rate of heat transfer from the refrigerated space, the power input to operate the refrigerator, and the ambient temperature are known.

Find: Determine the coefficient of performance and compare with that of a reversible refrigerator operating between reservoirs at the same two temperatures.

Schematic and Given Data:



Engineering Model:

1. The system shown on the accompanying figure is at steady state.
2. The freezer compartment and the surrounding air play the roles of cold and hot reservoirs, respectively.

Fig. E5.2

Analysis: Inserting the given operating data into Eq. 5.5 expressed on a *time-rate* basis, the coefficient of performance of the refrigerator is

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{8000 \text{ kJ/h}}{3200 \text{ kJ/h}} = 2.5$$

Substituting values into Eq. 5.10 gives the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at $T_C = 268$ K and $T_H = 295$ K

1 $$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{268 \text{ K}}{295 \text{ K} - 268 \text{ K}} = 9.9$$

2 In accord with the corollaries of Sec. 5.7.2, the coefficient of performance of the refrigerator is less than for a reversible refrigeration cycle operating between reservoirs at the same two temperatures. That is, irreversibilities are present within the system.

1 The temperatures T_C and T_H used in evaluating β_{max} *must* be in K or °R.

2 The difference between the actual and maximum coefficients of performance suggests that there may be some potential for improving the thermodynamic performance. This objective should be approached judiciously, however, for improved performance may require increases in size, complexity, and cost.

Skills Developed

Ability to...

- ❑ apply the second law corollaries of Sec. 5.7.2, using Eqs. 5.5 and 5.10 appropriately.

QuickQUIZ An inventor claims the power required to operate the refrigerator can be reduced to 800 kJ/h while all other data remain the same. Evaluate this claim using the second law. **Ans.** $\beta = 10$. Claim invalid.

In Example 5.3, we determine the minimum theoretical work input and cost for one day of operation of an electric heat pump, illustrating the use of the second law corollaries of Sec. 5.7.2 together with Eq. 5.11.

EXAMPLE 5.3

Evaluating Heat Pump Performance

A dwelling requires 6×10^5 Btu per day to maintain its temperature at 70°F when the outside temperature is 32°F. **(a)** If an electric heat pump is used to supply this energy, determine the minimum theoretical work input for one day of operation, in Btu/day. **(b)** Evaluating electricity at 8 cents per kW · h, determine the minimum theoretical cost to operate the heat pump, in $/day.

SOLUTION

Known: A heat pump maintains a dwelling at a specified temperature. The energy supplied to the dwelling, the ambient temperature, and the unit cost of electricity are known.

Find: Determine the *minimum* theoretical work required by the heat pump and the corresponding electricity cost.

Schematic and Given Data:

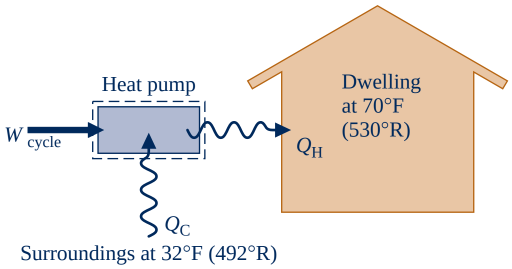


Fig. E5.3

Engineering Model:

1. The system shown on the accompanying figure executes a heat pump cycle.
2. The dwelling and the outside air play the roles of hot and cold reservoirs, respectively.
3. The value of electricity is 8 cents per kW · h.

Analysis:

(a) Using Eq. 5.6, the work for any heat pump cycle can be expressed as $W_{cycle} = Q_H/\gamma$. The coefficient of performance γ of an actual heat pump is less than, or equal to, the coefficient of performance γ_{max} of a reversible heat pump cycle when each operates between the same two thermal reservoirs: $\gamma \leq \gamma_{max}$. Accordingly, for a given value of Q_H, and using Eq. 5.11 to evaluate γ_{max}, we get

$$W_{cycle} \geq \frac{Q_H}{\gamma_{max}}$$

$$\geq \left(1 - \frac{T_C}{T_H}\right) Q_H$$

Inserting values

1 $$W_{cycle} \geq \left(1 - \frac{492°R}{530°R}\right)\left(6 \times 10^5 \frac{Btu}{day}\right) = 4.3 \times 10^4 \frac{Btu}{day}$$

The *minimum* theoretical work input is 4.3×10^4 Btu/day.

(b) Using the result of part (a) together with the given cost data and an appropriate conversion factor

2 $$\begin{bmatrix} \text{minimum} \\ \text{theoretical} \\ \text{cost per day} \end{bmatrix} = \left(4.3 \times 10^4 \frac{Btu}{day} \left| \frac{1 \text{ kW} \cdot \text{h}}{3413 \text{ Btu}} \right| \right)\left(0.08 \frac{\$}{\text{kW} \cdot \text{h}}\right) = 1.01 \frac{\$}{day}$$

1. Note that the temperatures T_C and T_H *must* be in °R or K.
2. Because of irreversibilities, an actual heat pump must be supplied more work than the minimum to provide the same heating effect. The actual daily cost could be substantially greater than the minimum theoretical cost.

Skills Developed

Ability to...

- ❑ apply the second law corollaries of Sec. 5.7.2, using Eqs. 5.6 and 5.11 appropriately.
- ❑ conduct an elementary economic evaluation.

QuickQUIZ If the cost of electricity is 10 cents per kW · h, evaluate the minimum theoretical cost to operate the heat pump, in $/day, keeping all other data the same. **Ans.** $1.26/day.

5.10 Carnot Cycle

Carnot cycle

The Carnot cycles introduced in this section provide specific examples of reversible cycles operating between two thermal reservoirs. Other examples are provided in Chap. 9: the Ericsson and Stirling cycles. In a **Carnot cycle**, the system executing the cycle undergoes a series of four internally reversible processes: two adiabatic processes alternated with two isothermal processes.

5.10.1 Carnot Power Cycle

Figure 5.13 shows the p–v diagram of a Carnot power cycle in which the system is a gas in a piston–cylinder assembly. Figure 5.14 provides details of how the cycle is executed. The piston and cylinder walls are nonconducting. The heat transfers are in the directions of the arrows. Also note that there are two reservoirs at temperatures T_H and T_C, respectively, and an insulating stand. Initially, the piston–cylinder assembly is on the insulating stand and the system is at state 1, where the temperature is T_C. The four processes of the cycle are

Process 1–2: The gas is compressed *adiabatically* to state 2, where the temperature is T_H.

Process 2–3: The assembly is placed in contact with the reservoir at T_H. The gas expands *isothermally* while receiving energy Q_H from the hot reservoir by heat transfer.

Process 3–4: The assembly is again placed on the insulating stand and the gas is allowed to continue to expand *adiabatically* until the temperature drops to T_C.

Process 4–1: The assembly is placed in contact with the reservoir at T_C. The gas is compressed *isothermally* to its initial state while it discharges energy Q_C to the cold reservoir by heat transfer.

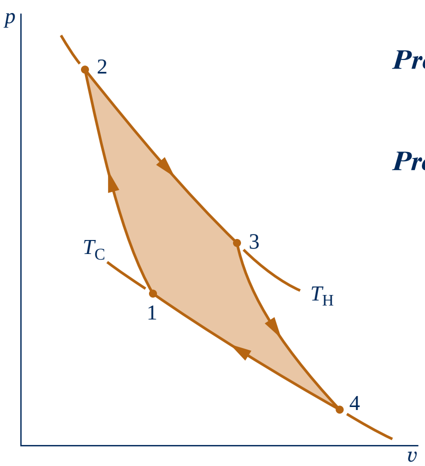


Fig. 5.13 ***p–v*** **diagram for a Carnot gas power cycle.**

For the heat transfer during Process 2–3 to be reversible, the difference between the gas temperature and the temperature of the hot reservoir must be vanishingly small. Since the reservoir temperature remains constant, this implies that the temperature of the gas also remains constant during Process 2–3. The same can be concluded for the gas temperature during Process 4–1.

For each of the four internally reversible processes of the Carnot cycle, the work can be represented as an area on Fig. 5.13. The area under the adiabatic process line 1–2 represents the work done per unit of mass to compress the gas in this process. The areas under process lines 2–3 and 3–4 represent the

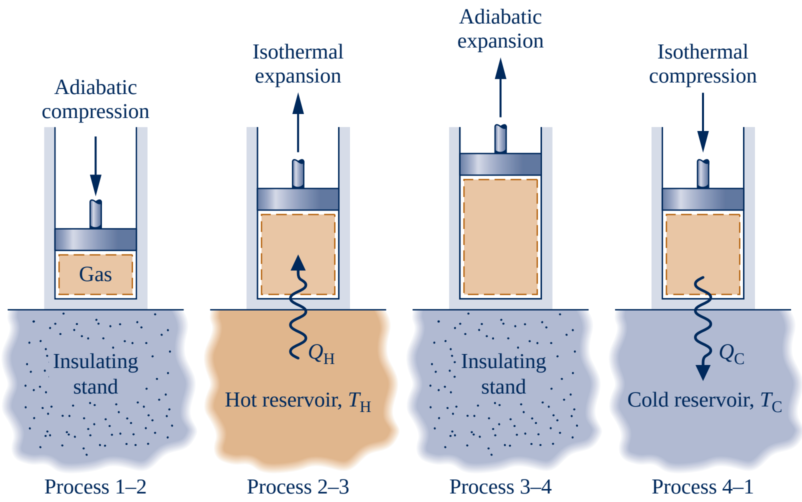


Fig. 5.14 **Carnot power cycle executed by a gas in a piston–cylinder assembly.**

work done per unit of mass by the gas as it expands in these processes. The area under process line 4–1 is the work done per unit of mass to compress the gas in this process. The enclosed area on the *p–v* diagram, shown shaded, is the net work developed by the cycle per unit of mass. The thermal efficiency of this cycle is given by Eq. 5.9.

The Carnot cycle is not limited to processes of a closed system taking place in a piston–cylinder assembly. Figure 5.15 shows the schematic and accompanying *p–v* diagram of a Carnot cycle executed by water steadily circulating through a series of four interconnected components that has features in common with the simple vapor power plant shown in Fig. 4.16. As the water flows through the boiler, a *change of phase* from liquid to vapor at constant temperature T_H occurs as a result of heat transfer from the hot reservoir. Since temperature remains constant, pressure also remains constant during the phase change. The steam exiting the boiler expands adiabatically through the turbine and work is developed. In this process the temperature decreases to the temperature of the cold reservoir, T_C, and there is an accompanying decrease in pressure. As the steam passes through the condenser, a heat transfer to the cold reservoir occurs and some of the vapor condenses at constant temperature T_C. Since temperature remains constant, pressure also remains constant as the water passes through the condenser. The fourth component is a pump (or compressor) that receives a two-phase liquid–vapor mixture from the condenser and returns it adiabatically

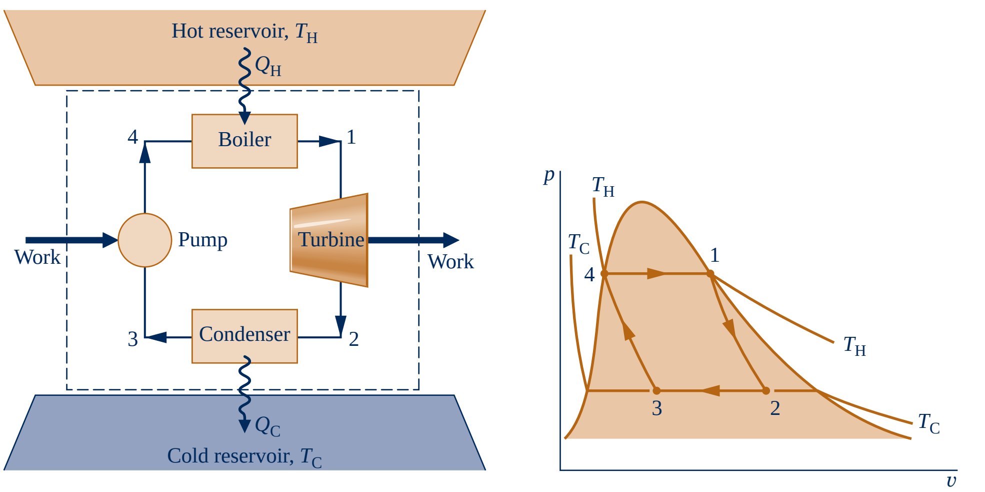


Fig. 5.15 **Carnot vapor power cycle.**

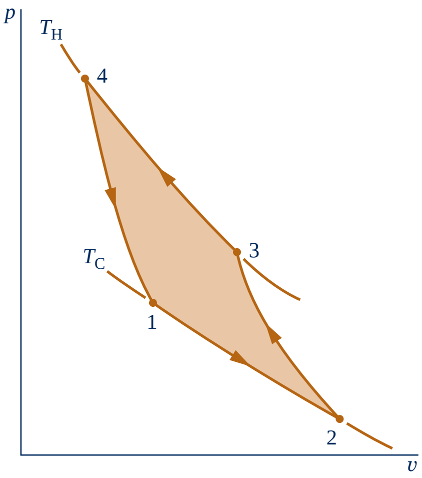


Fig. 5.16 ***p*–*v* diagram for a Carnot gas refrigeration or heat pump cycle.**

to the state at the boiler entrance. During this process, which requires a work input to increase the pressure, the temperature increases from T_C to T_H. The thermal efficiency of this cycle also is given by Eq. 5.9.

5.10.2 Carnot Refrigeration and Heat Pump Cycles

If a Carnot power cycle is operated in the opposite direction, the magnitudes of all energy transfers remain the same but the energy transfers are oppositely directed. Such a cycle may be regarded as a reversible refrigeration or heat pump cycle, for which the coefficients of performance are given by Eqs. 5.10 and 5.11, respectively. A Carnot refrigeration or heat pump cycle executed by a gas in a piston–cylinder assembly is shown in Fig. 5.16. The cycle consists of the following four processes in series:

Process 1–2: The gas expands *isothermally* at T_C while *receiving* energy Q_C from the cold reservoir by heat transfer.

Process 2–3: The gas is compressed *adiabatically* until its temperature is T_H.

Process 3–4: The gas is compressed *isothermally* at T_H while it *discharges* energy Q_H to the hot reservoir by heat transfer.

Process 4–1: The gas expands *adiabatically* until its temperature decreases to T_C.

A refrigeration or heat pump effect can be accomplished in a cycle only if a net work input is supplied to the system executing the cycle. In the case of the cycle shown in Fig. 5.16, the shaded area represents the net work input per unit of mass.

5.10.3 Carnot Cycle Summary

In addition to the configurations discussed previously, Carnot cycles also can be devised that are composed of processes in which a capacitor is charged and discharged, a paramagnetic substance is magnetized and demagnetized, and so on. However, regardless of the type of device or the working substance used,

1. the Carnot cycle *always* has the same four internally reversible processes: two adiabatic processes alternated with two isothermal processes.
2. the thermal efficiency of the Carnot power cycle is *always* given by Eq. 5.9 in terms of the temperatures evaluated on the Kelvin or Rankine scale.
3. the coefficients of performance of the Carnot refrigeration and heat pump cycles are *always* given by Eqs. 5.10 and 5.11, respectively, in terms of temperatures evaluated on the Kelvin or Rankine scale.

5.11 Clausius Inequality

Corollaries of the second law developed thus far in this chapter are for systems undergoing cycles while communicating thermally with *one* or *two* thermal energy reservoirs. In the present section a corollary of the second law known as the *Clausius inequality* is introduced that is applicable to *any* cycle without regard for the body, or bodies, from which the cycle receives energy by heat transfer or to which the cycle rejects energy by heat transfer. The Clausius inequality provides the basis for further development in Chap. 6 of the entropy, entropy production, and entropy balance concepts introduced in Sec. 5.2.3.

The *Clausius inequality* states that for any thermodynamic cycle

$$\oint \left(\frac{\delta Q}{T}\right)_{\mathrm{b}} \leq 0 \tag{5.12}$$

where δQ represents the heat transfer at a part of the system boundary during a portion of the cycle, and T is the absolute temperature at that part of the boundary. The subscript "b" serves as a reminder that the integrand is evaluated at the boundary of the system executing the cycle. The symbol $\oint$ indicates that the integral is to be performed over all parts of the boundary and over the entire cycle. The equality and inequality have the same interpretation as in the Kelvin–Planck statement: the equality applies when there are no internal irreversibilities as the system executes the cycle, and the inequality applies when internal irreversibilities are present. The Clausius inequality can be demonstrated using the Kelvin–Planck statement of the second law. See the box for details.

The **Clausius inequality** can be expressed equivalently as

$$\oint \left(\frac{\delta Q}{T}\right)_{\mathrm{b}} = -\sigma_{\mathrm{cycle}} \tag{5.13}$$

Clausius inequality

where σ_{cycle} can be interpreted as representing the "strength" of the inequality. The value of σ_{cycle} is positive when internal irreversibilities are present, zero when no internal irreversibilities are present, and can never be negative.

In summary, the nature of a cycle executed by a system is indicated by the value for σ_{cycle} as follows:

$$\begin{aligned} \sigma_{\mathrm{cycle}} &= 0 && \text{no irreversibilities present within the system} \\ \sigma_{\mathrm{cycle}} &> 0 && \text{irreversibilities present within the system} \\ \sigma_{\mathrm{cycle}} &< 0 && \text{impossible} \end{aligned} \tag{5.14}$$

▸ FOR EXAMPLE applying Eq. 5.13 to the cycle of Example 5.1, we get

$$\oint \left(\frac{\delta Q}{T}\right)_{\mathrm{b}} = \frac{Q_{\mathrm{in}}}{T_{\mathrm{H}}} - \frac{Q_{\mathrm{out}}}{T_{\mathrm{C}}} = -\sigma_{\mathrm{cycle}}$$

$$= \frac{1000\ \mathrm{kJ}}{500\ \mathrm{K}} - \frac{590\ \mathrm{kJ}}{300\ \mathrm{K}} = 0.033\ \mathrm{kJ/K}$$

giving $\sigma_{\mathrm{cycle}} = -0.033$ kJ/K, where the negative value indicates the proposed cycle is impossible. This is in keeping with the conclusion of Example 5.1. Applying Eq. 5.13 *on a time-rate basis* to the cycle of Example 5.2, we get $\dot{\sigma}_{\mathrm{cycle}} = 8.12\ \mathrm{kJ/h \cdot K}$. The positive value indicates irreversibilities are present within the system undergoing the cycle, which is in keeping with the conclusions of Example 5.2. ◂ ◂ ◂ ◂ ◂

In Sec. 6.7, Eq. 5.13 is used to develop the closed system entropy balance. From that development, the term σ_{cycle} of Eq. 5.13 can be interpreted as the entropy *produced (generated)* by internal irreversibilities during the cycle.

Developing the Clausius Inequality

The Clausius inequality can be demonstrated using the arrangement of Fig. 5.17. A system receives energy δQ at a location on its boundary where the absolute temperature is T while the system develops work δW. In keeping with our sign convention for heat transfer, the phrase *receives energy* δQ includes the possibility of heat transfer *from* the system. The energy δQ is received from a thermal reservoir at T_{res}. To ensure that no irreversibility is introduced as a result of heat transfer between the reservoir and the system, let it be accomplished through an intermediary system that undergoes a cycle

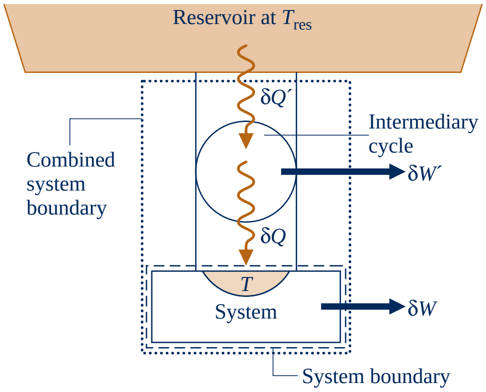


Fig. 5.17 Illustration used to develop the Clausius inequality.

without irreversibilities of any kind. The cycle receives energy $\delta Q'$ from the reservoir and supplies δQ to the system while producing work $\delta W'$. From the definition of the Kelvin scale (Eq. 5.7), we have the following relationship between the heat transfers and temperatures:

$$\frac{\delta Q'}{T_{res}} = \left(\frac{\delta Q}{T}\right)_b \tag{a}$$

As temperature T may vary, a multiplicity of such reversible cycles may be required.

Consider next the combined system shown by the dotted line on Fig. 5.17. An energy balance for the combined system is

$$dE_C = \delta Q' - \delta W_C$$

where δW_C is the total work of the combined system, the sum of δW and $\delta W'$, and dE_C denotes the change in energy of the combined system. Solving the energy balance for δW_C and using Eq. (a) to eliminate $\delta Q'$ from the resulting expression yields

$$\delta W_C = T_{res}\left(\frac{\delta Q}{T}\right)_b - dE_C$$

Now, let the system undergo a single cycle while the intermediary system undergoes one or more cycles. The total work of the combined system is

$$W_C = \oint T_{res}\left(\frac{\delta Q}{T}\right)_b - \cancel{\oint dE_C}^{\,0} = T_{res}\oint\left(\frac{\delta Q}{T}\right)_b \tag{b}$$

Since the reservoir temperature is constant, T_{res} can be brought outside the integral. The term involving the energy of the combined system vanishes because the energy change for any cycle is zero. The combined system operates in a cycle because its parts execute cycles. Since the combined system undergoes a cycle and exchanges energy by heat transfer with a single reservoir, Eq. 5.3 expressing the Kelvin–Planck statement of the second law must be satisfied. Using this, Eq. (b) reduces to give Eq. 5.12, where the equality applies when there are *no irreversibilities within the system* as it executes the cycle and the inequality applies when *internal irreversibilities are present*. This interpretation actually refers to the combination of system plus intermediary cycle. However, the intermediary cycle is free of irreversibilities, so the only possible site of irreversibilities is the system alone.

▶ CHAPTER SUMMARY AND STUDY GUIDE

In this chapter, we motivate the need for and usefulness of the second law of thermodynamics, and provide the basis for subsequent applications involving the second law in Chaps. 6 and 7. Three statements of the second law, the Clausius, Kelvin–Planck, and entropy statements, are introduced together with several corollaries that establish the best theoretical performance for systems undergoing cycles while interacting with thermal reservoirs. The irreversibility concept is introduced and the related notions of irreversible, reversible, and internally reversible processes are discussed. The Kelvin temperature scale is defined and used to obtain expressions for maximum performance measures of power, refrigeration, and heat pump cycles operating between two thermal reservoirs. The Carnot cycle is introduced to provide a specific example of a reversible cycle operating between two thermal reservoirs. Finally, the Clausius inequality providing a bridge from Chap. 5 to Chap. 6 is presented and discussed.

The following checklist provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed you should be able to

- write out the meanings of the terms listed in the margins throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important in subsequent chapters.
- give the Kelvin–Planck statement of the second law, correctly interpreting the "less than" and "equal to" signs in Eq. 5.3.
- list several important irreversibilities.
- apply the corollaries of Secs. 5.6.2 and 5.7.2 together with Eqs. 5.9, 5.10, and 5.11 to assess the performance of power cycles and refrigeration and heat pump cycles.
- describe the Carnot cycle.
- interpret the Clausius inequality.

▶ KEY ENGINEERING CONCEPTS

second law statements, p. 239
thermal reservoir, p. 239
irreversible process, p. 242
reversible process, p. 242
irreversibilities, p. 243
internal and external irreversibilities, p. 243
internally reversible process, p. 246
Carnot corollaries, p. 249
Kelvin scale, p. 253
Carnot efficiency, p. 257
Carnot cycle, p. 262
Clausius inequality, p. 265

▶ KEY EQUATIONS

$W_{\text{cycle}} \leq 0 \begin{cases} < 0: & \text{Internal irreversibilities present.} \\ = 0: & \text{No internal irreversibilities.} \end{cases}$ (single reservoir)	**(5.3) p. 247**	**Analytical form of the Kelvin–Planck statement.**
$\eta_{\max} = 1 - \dfrac{T_C}{T_H}$	**(5.9) p. 257**	**Maximum thermal efficiency: power cycle operating between two reservoirs.**
$\beta_{\max} = \dfrac{T_C}{T_H - T_C}$	**(5.10) p. 259**	**Maximum coefficient of performance: refrigeration cycle operating between two reservoirs.**
$\gamma_{\max} = \dfrac{T_H}{T_H - T_C}$	**(5.11) p. 259**	**Maximum coefficient of performance: heat pump cycle operating between two reservoirs.**
$\oint \left(\dfrac{\delta Q}{T}\right)_b = -\sigma_{\text{cycle}}$	**(5.13) p. 265**	**Clausius inequality.**

▶ EXERCISES: THINGS ENGINEERS THINK ABOUT

1. Extending the discussion of Sec. 5.1.2, how might work be developed when (a) T_i is less than T_0 in Fig. 5.1a, (b) p_i is less than p_0 in Fig. 5.1b?

2. Are health risks associated with consuming tomatoes induced to ripen by an ethylene spray? Explain.

3. What irreversibilities are found in living things?

4. In what ways are irreversibilities associated with the operation of an automobile beneficial?

5. Use the *second law* to explain which species coexisting in a wilderness will be *least* numerous—foxes or rabbits?

6. Is the power generated by fuel cells limited by the Carnot efficiency? Explain.

7. Does the second law impose performance limits on elite athletes seeking world records in events such as track and field and swimming? Explain.

8. Which method of heating is better in terms of operating cost: electric-resistance baseboard heating or a heat pump? Explain.

9. What options exist for effectively using energy discharged by heat transfer from electricity-generating power plants?

10. When would the power input to a basement sump pump be greater—in the presence or absence of internal irreversibilities? Explain.

11. One automobile make recommends 5W20 motor oil while another make specifies 5W30 oil. What do these designations mean and why might they differ for the two makes?

12. What factors influence the *actual* coefficient of performance achieved by refrigerators in family residences?

13. What is the SEER rating labeled on refrigerators seen in appliance showrooms?

14. How does the *thermal glider* (Sec. 5.4) sustain underwater motion for scientific missions lasting weeks?

▶ PROBLEMS: DEVELOPING ENGINEERING SKILLS

Exploring the Second Law

5.1 Complete the demonstration of the equivalence of the Clausius and Kelvin–Planck statements of the second law given in Sec. 5.2.2 by showing that a violation of the Kelvin–Planck statement implies a violation of the Clausius statement.

5.2 An inventor claims to have developed a device that undergoes a thermodynamic cycle while communicating thermally with two reservoirs. The system receives energy Q_C from the cold reservoir and discharges energy Q_H to the hot reservoir while delivering a net amount of work to its surroundings. There are no other energy transfers between the device and its surroundings. Evaluate the inventor's claim using (a) the Clausius statement of the second law, and (b) the Kelvin–Planck statement of the second law.

5.3 Classify the following processes of a closed system as *possible, impossible,* or *indeterminate*.

	Entropy Change	Entropy Transfer	Entropy Production
(a)	>0	0	
(b)	<0		>0
(c)	0	>0	
(d)	>0	>0	
(e)	0	<0	
(f)	>0		<0
(g)	<0	<0	

5.4 As shown in Fig. P5.4, a hot thermal reservoir is separated from a cold thermal reservoir by a cylindrical rod insulated on its lateral surface. Energy transfer by conduction between the two reservoirs takes place through the rod, which remains at steady state. Using the Kelvin–Planck statement of the second law, demonstrate that such a process is irreversible.

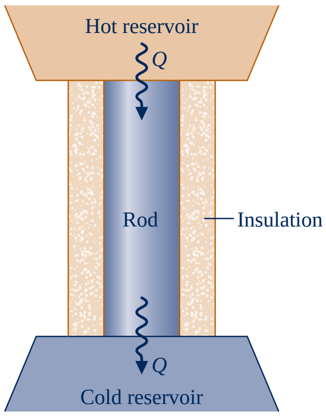


Fig. P5.4

5.5 As shown in Fig. P5.5, a rigid insulated tank is divided into halves by a partition. On one side of the partition is a gas. The other side is initially evacuated. A valve in the partition is opened and the gas expands to fill the entire volume. Using the Kelvin–Planck statement of the second law, demonstrate that this process is irreversible.

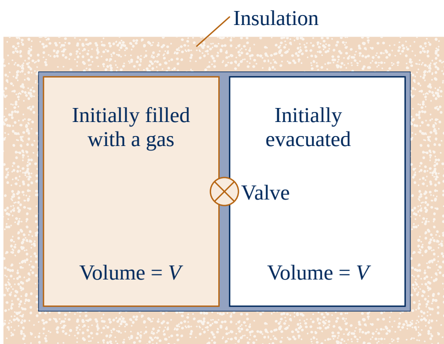


Fig. P5.5

5.6 Answer the following true or false.

(a) A process that violates the second law of thermodynamics violates the first law of thermodynamics.
(b) When a net amount of work is done on a closed system undergoing an *internally reversible* process, a net heat transfer of energy from the system also occurs.
(c) A closed system can experience an increase in entropy only when a net amount of entropy is transferred into the system.
(d) The change in entropy of a closed system is the same for every process between two specified end states.

5.7 Complete the discussion of the Kelvin–Planck statement of the second law in the box of Sec. 5.4 by showing that if a system undergoes a thermodynamic cycle reversibly while communicating thermally with a single reservoir, the equality in Eq. 5.3 applies.

5.8 A reversible power cycle R and an irreversible power cycle I operate between the same two reservoirs.

(a) If each cycle receives the same amount of energy Q_H from the hot reservoir, show that cycle I necessarily discharges more energy Q_C to the cold reservoir than cycle R. Discuss the implications of this for actual power cycles.
(b) If each cycle develops the same net work, show that cycle I necessarily receives more energy Q_H from the hot reservoir than cycle R. Discuss the implications of this for actual power cycles.

5.9 A power cycle I and a reversible power cycle R operate between the same two reservoirs, as shown in Fig. 5.6. Cycle I has a thermal efficiency equal to two-thirds of that for cycle R. Using the Kelvin–Planck statement of the second law, prove that cycle I must be irreversible.

5.10 Provide the details left to the reader in the demonstration of the second Carnot corollary given in the box of Sec. 5.6.2.

5.11 Using the Kelvin–Planck statement of the second law of thermodynamics, demonstrate the following corollaries:

(a) The coefficient of performance of an irreversible refrigeration cycle is always less than the coefficient of performance of a reversible refrigeration cycle when both exchange energy by heat transfer with the same two reservoirs.
(b) All reversible refrigeration cycles operating between the same two reservoirs have the same coefficient of performance.
(c) The coefficient of performance of an irreversible heat pump cycle is always less than the coefficient of performance of a reversible heat pump cycle when both exchange energy by heat transfer with the same two reservoirs.
(d) All reversible heat pump cycles operating between the same two reservoirs have the same coefficient of performance.

5.12 Before introducing the temperature scale now known as the Kelvin scale, Kelvin suggested a *logarithmic* scale in which the function ψ of Sec. 5.8.1 takes the form

$$\psi = \exp \theta_C / \exp \theta_H$$

where θ_H and θ_C denote, respectively, the temperatures of the hot and cold reservoirs on this scale.

(a) Show that the relation between the Kelvin temperature T and the temperature θ on the logarithmic scale is

$$\theta = \ln T + C$$

where C is a constant.
(b) On the Kelvin scale, temperatures vary from 0 to $+\infty$. Determine the range of temperature values on the logarithmic scale.
(c) Obtain an expression for the thermal efficiency of any system undergoing a reversible power cycle while operating between reservoirs at temperatures θ_H and θ_C on the logarithmic scale.

5.13 Demonstrate that the *gas temperature scale* (Sec. 5.8.2) is identical to the *Kelvin temperature scale* (Sec. 5.8.1).

5.14 The platinum resistance thermometer is said to be the most important of the three thermometers specified in ITS-90 because it covers the broad, practically significant interval from 13.8 K to 1234.93 K. What is the operating principle of resistance thermometry and why is platinum specified for use in ITS-90?

5.15 The relation between resistance R and temperature T for a *thermistor* closely follows

$$R = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

where R_0 is the resistance, in ohms (Ω), measured at temperature T_0 (K) and β is a material constant with units of K. For a particular thermistor $R_0 = 2.2\ \Omega$ at $T_0 = 310$ K. From a calibration test, it is found that $R = 0.31\ \Omega$ at $T = 422$ K. Determine the value of β for the thermistor and make a plot of resistance versus temperature.

5.16 Over a limited temperature range, the relation between electrical resistance R and temperature T for a *resistance temperature detector* is

$$R = R_0[1 + \alpha(T - T_0)]$$

where R_0 is the resistance, in ohms (Ω), measured at reference temperature T_0 (in °F) and α is a material constant with units of $(°F)^{-1}$. The following data are obtained for a particular resistance thermometer:

	T (°F)	R (Ω)
Test 1	32	51.39
Test 2	196	51.72

What temperature would correspond to a resistance of 51.47 Ω on this thermometer?

Power Cycle Applications

5.17 The data listed below are claimed for a power cycle operating between hot and cold reservoirs at 1000 K and 300 K, respectively. For each case, determine whether the cycle operates *reversibly*, operates *irreversibly*, or is *impossible*.

(a) $Q_H = 600$ kJ, $W_{cycle} = 300$ kJ, $Q_C = 300$ kJ
(b) $Q_H = 400$ kJ, $W_{cycle} = 280$ kJ, $Q_C = 120$ kJ
(c) $Q_H = 700$ kJ, $W_{cycle} = 300$ kJ, $Q_C = 500$ kJ
(d) $Q_H = 800$ kJ, $W_{cycle} = 600$ kJ, $Q_C = 200$ kJ

5.18 A power cycle receives energy Q_H by heat transfer from a hot reservoir at $T_H = 1500°R$ and rejects energy Q_C by heat transfer to a cold reservoir at $T_C = 500°R$. For each of the following cases, determine whether the cycle operates *reversibly,* operates *irreversibly,* or is *impossible*.

(a) $Q_H = 900$ Btu, $W_{cycle} = 450$ Btu
(b) $Q_H = 900$ Btu, $Q_C = 300$ Btu
(c) $W_{cycle} = 600$ Btu, $Q_C = 400$ Btu
(d) $\eta = 70\%$

5.19 A power cycle operating at steady state receives energy by heat transfer at a rate $\dot{Q}_H$ at $T_H = 1000$ K and rejects energy by heat transfer to a cold reservoir at a rate $\dot{Q}_C$ at $T_C = 300$ K. For each of the following cases, determine whether the cycle operates *reversibly,* operates *irreversibly,* or is *impossible*.

(a) $\dot{Q}_H = 500$ kW, $\dot{Q}_C = 100$ kW
(b) $\dot{Q}_H = 500$ kW, $\dot{W}_{cycle} = 250$ kW, $\dot{Q}_C = 200$ kW
(c) $\dot{W}_{cycle} = 350$ kW, $\dot{Q}_C = 150$ kW
(d) $\dot{Q}_H = 500$ kW, $\dot{Q}_C = 200$ kW

5.20 As shown in Fig. P5.20, a reversible power cycle receives energy Q_H by heat transfer from a hot reservoir at T_H and rejects energy Q_C by heat transfer to a cold reservoir at T_C.

(a) If $T_H = 1200$ K and $T_C = 300$ K, what is the thermal efficiency?
(b) If $T_H = 500°C$, $T_C = 20°C$, and $W_{cycle} = 1000$ kJ, what are Q_H and Q_C, each in kJ?
(c) If $\eta = 60\%$ and $T_C = 40°F$, what is T_H, in °F?
(d) If $\eta = 40\%$ and $T_H = 727°C$, what is T_C, in °C?

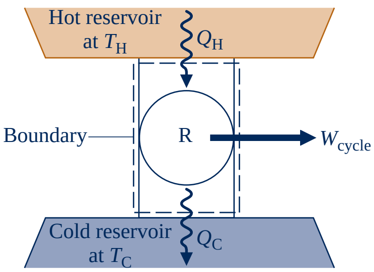


Fig. P5.20

5.21 A reversible power cycle whose thermal efficiency is 40% receives 50 kJ by heat transfer from a hot reservoir at 600 K and rejects energy by heat transfer to a cold reservoir at temperature T_C. Determine the energy rejected, in kJ, and T_C, in K.

5.22 Determine the maximum theoretical thermal efficiency for *any* power cycle operating between hot and cold reservoirs at 602°C and 112°C, respectively.

5.23 A reversible power cycle operating as in Fig. 5.5 receives energy Q_H by heat transfer from a hot reservoir at T_H and rejects energy Q_C by heat transfer to a cold reservoir at 40°F. If $W_{cycle} = 3\ Q_C$, determine (a) the thermal efficiency and (b) T_H, in °F.

5.24 A reversible power cycle has the same thermal efficiency for hot and cold reservoirs at temperature T and 500 K, respectively, as for hot and cold reservoirs at 2000 and 1000 K, respectively. Determine T, in K.

5.25 As shown in Fig. P5.25, two reversible cycles arranged in series each produce the same net work, W_{cycle}. The first cycle receives energy Q_H by heat transfer from a hot reservoir at 1000°R and rejects energy Q by heat transfer to a reservoir at an intermediate temperature, T. The second cycle receives energy Q by heat transfer from the reservoir at temperature T and rejects energy Q_C by heat transfer to a reservoir at 400°R. All energy transfers are positive in the directions of the arrows. Determine

(a) the intermediate temperature T, in °R, and the thermal efficiency for each of the two power cycles.
(b) the thermal efficiency of a *single* reversible power cycle operating between hot and cold reservoirs at 1000°R and 400°R, respectively. Also, determine the net work developed by the single cycle, expressed in terms of the net work developed by each of the two cycles, W_{cycle}.

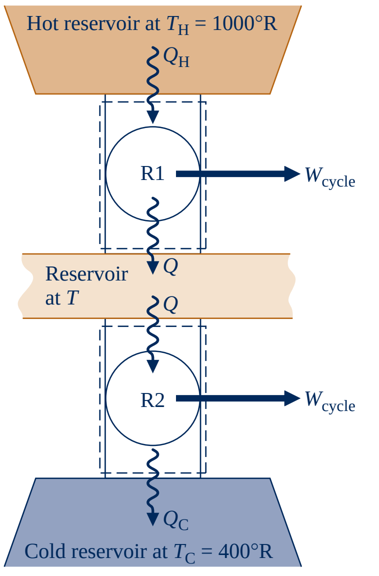


Fig. P5.25

5.26 Two reversible power cycles are arranged in series. The first cycle receives energy by heat transfer from a hot reservoir at 1000°R and rejects energy by heat transfer to a reservoir at temperature T (<1000°R). The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a cold reservoir at 500°R (<T). The thermal efficiency of the first cycle is 50% greater than that of the second cycle. Determine

(a) the intermediate temperature T, in °R, and the thermal efficiency for each of the two power cycles.
(b) the thermal efficiency of a *single* reversible power cycle operating between hot and cold reservoirs at 1000°R and 500°R, respectively.

5.27 A reversible power cycle operating between hot and cold reservoirs at 1000 K and 300 K, respectively, receives 100 kJ by heat transfer from the hot reservoir for each cycle of operation. Determine the net work developed in 10 cycles of operation, in kJ.

5.28 A reversible power cycle operating between hot and cold reservoirs at 1040°F and 40°F, respectively, develops net work in the amount of 600 Btu for each cycle of operation. For three cycles of operation, determine the energy received by heat transfer from the hot reservoir, in Btu.

5.29 A power cycle operates between a lake's surface water at a temperature of 300 K and water at a depth whose temperature is 285 K. At steady state the cycle develops a power output of 10 kW, while rejecting energy by heat transfer to the lower-temperature water at the rate 14,400 kJ/min. Determine (a) the thermal efficiency of the power cycle and (b) the maximum thermal efficiency for any such power cycle.

5.30 An inventor claims to have developed a power cycle having a thermal efficiency of 40%, while operating between hot and cold reservoirs at temperature T_H and $T_C = 300$ K, respectively, where T_H is (a) 600 K, (b) 500 K, (c) 400 K. Evaluate the claim for each case.

5.31 Referring to the cycle of Fig. 5.13, if $p_1 = 2$ bar, $\upsilon_1 = 0.31$ m^3/kg, $T_H = 475$ K, $Q_H = 150$ kJ, and the gas is air obeying the ideal gas model, determine T_C, in K, the net work of the cycle, in kJ, and the thermal efficiency.

5.32 An inventor claims to have developed a power cycle operating between hot and cold reservoirs at 1000 K and 250 K, respectively, that develops net work equal to a multiple of the amount of energy, Q_C, rejected to the cold reservoir—that is $W_{cycle} = NQ_C$, where all quantities are positive. What is the maximum theoretical value of the number N for any such cycle?

5.33 A power cycle operates between hot and cold reservoirs at 500 K and 310 K, respectively. At steady state the cycle develops a power output of 0.1 MW. Determine the minimum theoretical rate at which energy is rejected by heat transfer to the cold reservoir, in MW.

5.34 At steady state, a new power cycle is claimed by its inventor to develop power at a rate of 100 hp for a heat addition rate of 5.1×10^5 Btu/h, while operating between hot and cold reservoirs at 1000 and 500 K, respectively. Evaluate this claim.

5.35 An inventor claims to have developed a power cycle operating between hot and cold reservoirs at 1175 K and 295 K, respectively, that provides a steady-state power output of 32 kW while receiving energy by heat transfer from the hot reservoir at the rate 150,000 kJ/h. Evaluate this claim.

5.36 At steady state, a power cycle develops a power output of 10 kW while receiving energy by heat transfer at the rate of 10 kJ *per cycle of operation* from a source at temperature T. The cycle rejects energy by heat transfer to cooling water at a lower temperature of 300 K. If there are 100 cycles per minute, what is the minimum theoretical value for T, in K?

5.37 A power cycle operates between hot and cold reservoirs at 600 K and 300 K, respectively. At steady state the cycle develops a power output of 0.45 MW while receiving energy by heat transfer from the hot reservoir at the rate of 1 MW.

(a) Determine the thermal efficiency and the rate at which energy is rejected by heat transfer to the cold reservoir, in MW.
(b) Compare the results of part (a) with those of a reversible power cycle operating between these reservoirs and receiving the same rate of heat transfer from the hot reservoir.

5.38 As shown in Fig. P5.38, a system undergoing a power cycle develops a net power output of 1 MW while receiving energy by heat transfer from steam condensing from saturated vapor to saturated liquid at a pressure of 100 kPa. Energy is discharged from the cycle by heat transfer to a nearby lake at 17°C. These are the only significant heat transfers. Kinetic and potential energy effects can be ignored. For operation at steady state, determine the minimum theoretical steam mass flow rate, in kg/s, required by any such cycle.

5.39 A power cycle operating at steady state receives energy by heat transfer from the combustion of fuel at an average temperature of 1000 K. Owing to environmental considerations, the cycle discharges energy by heat transfer to the atmosphere at 300 K at a rate no greater than 60 MW. Based on the cost of fuel, the cost to supply the heat transfer is \$4.50 per GJ. The power developed by the cycle is valued at \$0.08 per kW · h. For 8000 hours of operation annually, determine for any such cycle, in \$ per year, (a) the maximum value of the power generated and (b) the corresponding fuel cost.

5.40 At steady state, a 750-MW power plant receives energy by heat transfer from the combustion of fuel at an average temperature of 317°C. As shown in Fig. P5.40, the plant discharges energy by heat transfer to a river whose mass flow rate is 1.65×10^5 kg/s. Upstream of the power plant the river is at 17°C. Determine the increase in the temperature of the river, ΔT, traceable to such heat transfer, in °C, if the thermal efficiency of the power plant is (a) the Carnot

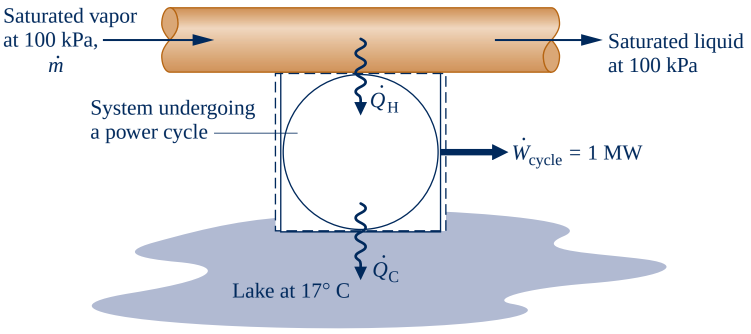


Fig. P5.38

efficiency of a power cycle operating between hot and cold reservoirs at 317°C and 17°C, respectively, (b) two-thirds of the Carnot efficiency found in part (a). Comment.

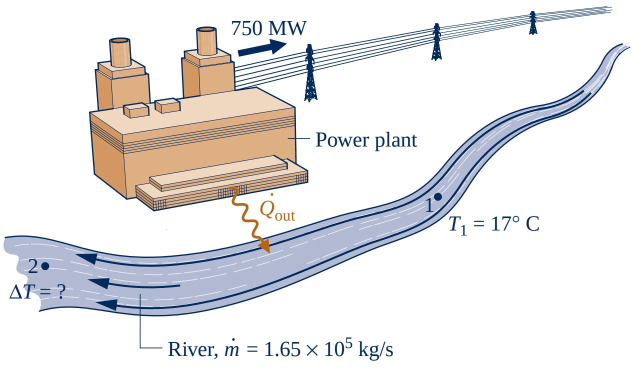


Fig. P5.40

5.41 To increase the thermal efficiency of a reversible power cycle operating between reservoirs at T_H and T_C, would you increase T_H while keeping T_C constant, or decrease T_C while keeping T_H constant? Are there any *natural* limits on the increase in thermal efficiency that might be achieved by such means?

5.42 Two reversible power cycles are arranged in series. The first cycle receives energy by heat transfer from a hot reservoir at temperature T_H and rejects energy by heat transfer to a reservoir at an intermediate temperature $T < T_H$. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a cold reservoir at temperature $T_C < T$.

(a) Obtain an expression for the thermal efficiency of a *single* reversible power cycle operating between hot and cold reservoirs at T_H and T_C, respectively, in terms of the thermal efficiencies of the two cycles.
(b) Obtain an expression for the intermediate temperature T in terms of T_H and T_C for the *special case* where the thermal efficiencies of the two cycles are equal.

Refrigeration and Heat Pump Cycle Applications

5.43 A refrigeration cycle operating between two reservoirs receives energy Q_C from a cold reservoir at $T_C = 275$ K and rejects energy Q_H to a hot reservoir at $T_H = 315$ K. For each of the following cases, determine whether the cycle operates *reversibly,* operates *irreversibly,* or is *impossible*:

(a) $Q_C = 1000$ kJ, $W_{cycle} = 80$ kJ.
(b) $Q_C = 1200$ kJ, $Q_H = 2000$ kJ.
(c) $Q_H = 1575$ kJ, $W_{cycle} = 200$ kJ.
(d) $\beta = 6$.

5.44 A reversible refrigeration cycle operates between cold and hot reservoirs at temperatures T_C and T_H, respectively.

(a) If the coefficient of performance is 3.5 and $T_H = 80$°F, determine T_C, in °F.
(b) If $T_C = -30$°C and $T_H = 30$°C, determine the coefficient of performance.
(c) If $Q_C = 500$ Btu, $Q_H = 800$ Btu, and $T_C = 20$°F, determine T_H, in °F.
(d) If $T_C = 30$°F and $T_H = 100$°F, determine the coefficient of performance.
(e) If the coefficient of performance is 8.9 and $T_C = -5$°C, find T_H, in °C.

5.45 At steady state, a reversible heat pump cycle discharges energy at the rate $\dot{Q}_H$ to a hot reservoir at temperature T_H, while receiving energy at the rate $\dot{Q}_C$ from a cold reservoir at temperature T_C.

(a) If $T_H = 21$°C and $T_C = 7$°C, determine the coefficient performance.
(b) If $\dot{Q}_H = 10.5$ kW, $\dot{Q}_C = 8.75$ kW, and $T_C = 0$°C, determine T_H, in °C.
(c) If the coefficient of performance is 10 and $T_H = 27$°C, determine T_C, in °C.

5.46 Two reversible cycles operate between hot and cold reservoirs at temperature T_H and T_C, respectively.

(a) If one is a power cycle and the other is a heat pump cycle, what is the relation between the coefficient of performance of the heat pump cycle and the thermal efficiency of the power cycle?
(b) If one is a refrigeration cycle and the other is a heat pump cycle, what is the relation between their coefficients of performance?

5.47 A refrigeration cycle rejects $Q_H = 500$ Btu per cycle to a hot reservoir at $T_H = 540$°R, while receiving $Q_C = 375$ Btu per cycle from a cold reservoir at temperature T_C. For 10 cycles of operation, determine (a) the net work input, in Btu, and (b) the minimum theoretical temperature T_C, in °R.

5.48 A reversible heat pump cycle operates as in Fig. 5.7 between hot and cold reservoirs at $T_H = 27$°C and $T_C = -3$°C, respectively. Determine the fraction of the heat transfer Q_H discharged at T_H provided by (a) the net work input, (b) the heat transfer Q_C from the cold reservoir T_C.

5.49 A reversible power cycle and a reversible heat pump cycle operate between hot and cold reservoirs at temperature $T_H = 1000$°R and T_C, respectively. If the thermal efficiency of the power cycle is 60%, determine (a) T_C, in °R, and (b) the coefficient of performance of the heat pump.

5.50 An inventor has developed a refrigerator capable of maintaining its freezer compartment at 20°F while operating in a kitchen at 70°F, and claims the device has a coefficient of performance of (a) 10, (b) 9.6, (c) 4. Evaluate the claim in each of the three cases.

5.51 An inventor claims to have developed a food freezer that at steady state requires a power input of 0.6 kW to extract energy by heat transfer at a rate of 3000 J/s from freezer contents at 270 K. Evaluate this claim for an ambient temperature of 293 K.

5.52 An inventor claims to have developed a refrigerator that at steady state requires a net power input of 0.7 horsepower to remove 12,000 Btu/h of energy by heat transfer from the freezer compartment at 0°F and discharge energy by heat transfer to a kitchen at 70°F. Evaluate this claim.

5.53 An inventor claims to have devised a refrigeration cycle operating between hot and cold reservoirs at 300 K and 250 K, respectively, that removes an amount of energy Q_C by heat transfer from the cold reservoir that is a multiple of the net work input—that is, $Q_C = NW_{cycle}$, where all quantities are positive. Determine the maximum theoretical value of the number N for any such cycle.

5.54 Data are provided for two reversible refrigeration cycles. One cycle operates between hot and cold reservoirs at 27°C and −8°C, respectively. The other cycle operates between the same hot reservoir at 27°C and a cold reservoir at −28°C. If each refrigerator removes the same amount of energy by heat transfer from its cold reservoir, determine the ratio of the net work input values of the two cycles.

5.55 By removing energy by heat transfer from its freezer compartment at a rate of 1.25 kW, a refrigerator maintains the freezer at −26°C on a day when the temperature of the surroundings is 22°C. Determine the minimum theoretical power, in kW, required by the refrigerator at steady state.

5.56 At steady state, a refrigeration cycle maintains a *clean room* at 55°F by removing energy entering the room by heat transfer from adjacent spaces at the rate of 0.12 Btu/s. The cycle rejects energy by heat transfer to the outdoors where the temperature is 80°F.

(a) If the rate at which the cycle rejects energy by heat transfer to the outdoors is 0.16 Btu/s, determine the power required, in Btu/s.
(b) Determine the power required to maintain the clean room's temperature by a reversible refrigeration cycle operating between cold and hot reservoirs at 55°F and 80°F, respectively, and the corresponding rate at which energy is rejected by heat transfer to the outdoors, each in Btu/s.

5.57 For each kW of power input to an ice maker at steady state, determine the maximum rate that ice can be produced, in lb/h, from liquid water at 32°F. Assume that 144 Btu/lb of energy must be removed by heat transfer to freeze water at 32°F, and that the surroundings are at 78°F.

5.58 At steady state, a refrigeration cycle operates between hot and cold reservoirs at 300 K and 270 K, respectively. Determine the minimum theoretical net power input required, in kW per kW of heat transfer from the cold reservoir.

5.59 At steady state, a refrigeration cycle operating between hot and cold reservoirs at 300 K and 275 K, respectively, removes energy by heat transfer from the cold reservoir at a rate of 600 kW.

(a) If the cycle's coefficient of performance is 4, determine the power input required, in kW.
(b) Determine the minimum theoretical power required, in kW, for *any* such cycle.

5.60 An air conditioner operating at steady state maintains a dwelling at 20°C on a day when the outside temperature is 35°C. Energy is removed by heat transfer from the dwelling at a rate of 2800 J/s while the air conditioner's power input is 0.8 kW. Determine (a) the coefficient of performance of the air conditioner and (b) the power input required by a reversible refrigeration cycle providing the same cooling effect while operating between hot and cold reservoirs at 35°C and 20°C, respectively.

5.61 As shown in Fig P5.61, an air conditioner operating at steady state maintains a dwelling at 70°F on a day when the outside temperature is 90°F. If the rate of heat transfer into the dwelling through the walls and roof is 30,000 Btu/h, might a net power input to the air conditioner compressor of 3 hp be sufficient? If yes, determine the coefficient of performance. If no, determine the minimum theoretical power input, in hp.

5.62 A heat pump cycle is used to maintain the interior of a building at 20°C. At steady state, the heat pump receives energy by heat transfer from well water at 10°C and discharges energy by heat transfer to the building at a rate

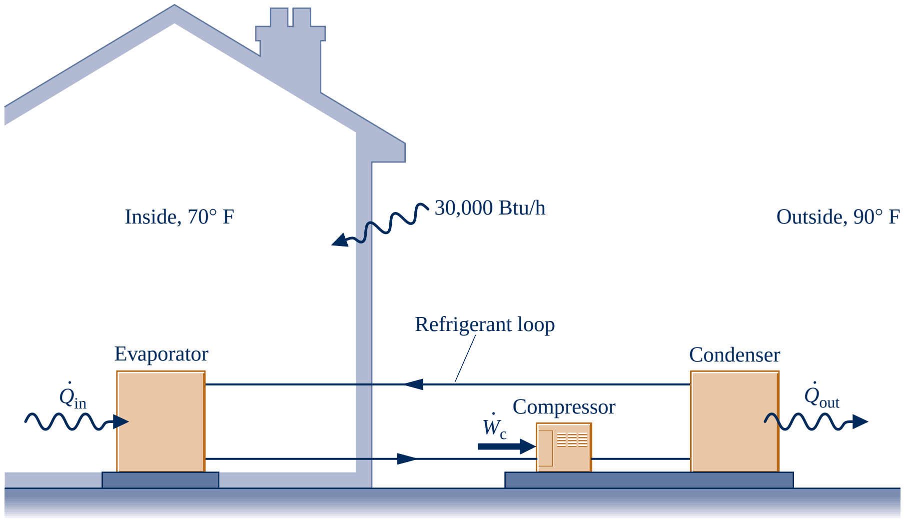


Fig. P5.61

of 120,000 kJ/h. Over a period of 14 days, an electric meter records that 1490 kW · h of electricity is provided to the heat pump. Determine

(a) the amount of energy that the heat pump receives over the 14-day period from the well water by heat transfer, in kJ.
(b) the heat pump's coefficient of performance.
(c) the coefficient of performance of a reversible heat pump cycle operating between hot and cold reservoirs at 20°C and 10°C.

5.63 A refrigeration cycle has a coefficient of performance equal to 75% of the value for a reversible refrigeration cycle operating between cold and hot reservoirs at −5°C and 40°C, respectively. For operation at steady state, determine the net power input, in kW per kW of cooling, required by (a) the actual refrigeration cycle and (b) the reversible refrigeration cycle. Compare values.

5.64 By removing energy by heat transfer from a room, a window air conditioner maintains the room at 22°C on a day when the outside temperature is 32°C.

(a) Determine, in kW per kW of cooling, the *minimum* theoretical power required by the air conditioner.
(b) To achieve required rates of heat transfer with practical-sized units, air conditioners typically receive energy by heat transfer at a temperature *below* that of the room being cooled and discharge energy by heat transfer at a temperature *above* that of the surroundings. Consider the effect of this by determining the *minimum* theoretical power, in kW per kW of cooling, required when T_C = 18°C and T_H = 36°C, and compare with the value found in part (a).

5.65 The refrigerator shown in Fig. P5.65 operates at steady state with a coefficient of performance of 5.0 within a kitchen at 23°C. The refrigerator rejects 4.8 kW by heat transfer to its surroundings from metal coils located on its exterior. Determine

(a) the power input, in kW.
(b) the lowest theoretical temperature *inside* the refrigerator, in K.

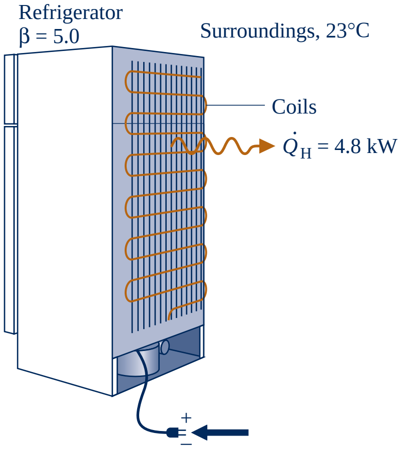


Fig. P5.65

5.66 At steady state, a heat pump provides energy by heat transfer at the rate of 25,000 Btu/h to maintain a dwelling at 70°F on a day when the outside temperature is 30°F. The power input to the heat pump is 4.5 hp. Determine

(a) the coefficient of performance of the heat pump.
(b) the coefficient of performance of a reversible heat pump operating between hot and cold reservoirs at 70°F and 30°F, respectively, and the corresponding rate at which energy would be provided by heat transfer to the dwelling for a power input of 4.5 hp.

5.67 By supplying energy at an average rate of 24,000 kJ/h, a heat pump maintains the temperature of a dwelling at 20°C. If electricity costs 8.5 cents per kW · h, determine the minimum theoretical operating cost for each day of operation if the heat pump receives energy by heat transfer from

(a) the outdoor air at −7°C.
(b) the ground at 5°C.

5.68 A heat pump with a coefficient of performance of 3.5 provides energy at an average rate of 70,000 kJ/h to maintain a building at 20°C on a day when the outside temperature is −5°C. If electricity costs 8.5 cents per kW · h,

(a) determine the actual operating cost and the minimum theoretical operating cost, each in $/day.
(b) compare the results of part (a) with the cost of electrical-resistance heating.

5.69 A heat pump is under consideration for heating a research station located on an Antarctica ice shelf. The interior of the station is to be kept at 15°C. Determine the maximum theoretical rate of heating provided by a heat pump, in kW per kW of power input, in each of two cases: The role of the cold reservoir is played by (a) the atmosphere at −20°C, (b) ocean water at 5°C.

5.70 As shown in Fig. P5.70, a heat pump provides energy by heat transfer to water vaporizing from saturated liquid to saturated vapor at a pressure of 2 bar and a mass flow rate of 0.05 kg/s. The heat pump receives energy by heat transfer from a pond at 16°C. These are the only significant heat transfers. Kinetic and potential energy effects can be ignored. A faded, hard-to-read data sheet indicates the power required by the pump is 35 kW. Can this value be correct? Explain.

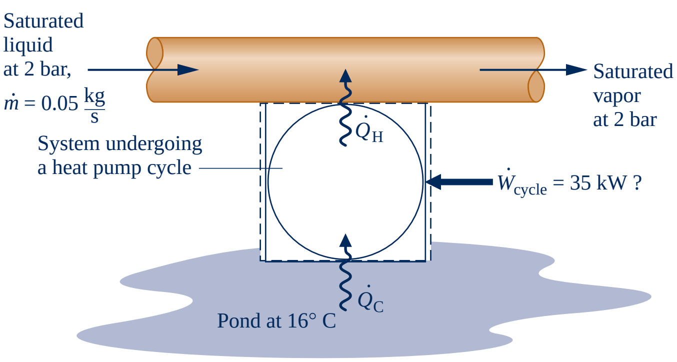


Fig. P5.70

5.71 To maintain a dwelling steadily at 68°F on a day when the outside temperature is 32°F, heating must be provided at an average rate of 700 Btu/min. Compare the electrical power required, in kW, to deliver the heating using (a) electrical-resistance heating, (b) a heat pump whose coefficient of performance is 3.5, (c) a reversible heat pump.

5.72 Referring to the heat pump cycle of Fig. 5.16, if $p_1 = 14.7$ and $p_4 = 18.7$, each in lbf/in.^{2}, $v_1 = 12.6$ and $v_4 = 10.6$, each in ft^3/lb, and the gas is air obeying the ideal gas model, determine T_H and T_C, each in °R, and the coefficient of performance.

5.73 Two reversible refrigeration cycles operate in series. The first cycle receives energy by heat transfer from a cold reservoir at 300 K and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than 300 K. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at 883 K. If the refrigeration cycles have the same coefficient of performance, determine (a) T, in K, and (b) the value of each coefficient of performance.

5.74 Two reversible heat pump cycles operate in series. The first cycle receives energy by heat transfer from a cold reservoir at 250 K and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than 250 K. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at 1440 K. If the heat pump cycles have the same coefficient of performance, determine (a) T, in K, and (b) the value of each coefficient of performance.

5.75 Two reversible refrigeration cycles are arranged in series. The first cycle receives energy by heat transfer from a cold reservoir at temperature T_C and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than T_C. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at T_H. Obtain an expression for the coefficient of performance of a *single* reversible refrigeration cycle operating directly between cold and hot reservoirs at T_C and T_H, respectively, in terms of the coefficients of performance of the two cycles.

5.76 Repeat Problem 5.75 for the case of two reversible heat pump cycles.

Carnot Cycle Applications

5.77 A quantity of water within a piston–cylinder assembly executes a Carnot power cycle. During isothermal expansion, the water is heated from saturated liquid at 50 bar until it is a saturated vapor. The vapor then expands adiabatically to a pressure of 5 bar while doing 364.31 kJ/kg of work.

(a) Sketch the cycle on p–v coordinates.
(b) Evaluate the heat transfer per unit mass and work per unit mass for each process, in kJ/kg.
(c) Evaluate the thermal efficiency.

5.78 One and one-half pounds of water within a piston–cylinder assembly execute a Carnot power cycle. During isothermal expansion, the water is heated at 500°F from saturated liquid to saturated vapor. The vapor then expands adiabatically to a temperature of 100°F and a quality of 70.38%.

(a) Sketch the cycle on p–v coordinates.
(b) Evaluate the heat transfer and work for each process, in Btu.
(c) Evaluate the thermal efficiency.

5.79 Two kilograms of air within a piston–cylinder assembly execute a Carnot power cycle with maximum and minimum temperatures of 750 K and 300 K, respectively. The heat transfer to the air during the isothermal expansion is 60 kJ. At the end of the isothermal expansion, the pressure is 600 kPa and the volume is 0.4 m^3. Assuming the ideal gas model for the air, determine

(a) the thermal efficiency.
(b) the pressure and volume at the beginning of the isothermal expansion, in kPa and m^3, respectively.
(c) the work and heat transfer for each of the four processes, in kJ.
(d) Sketch the cycle on p–V coordinates.

5.80 The pressure–volume diagram of a Carnot power cycle executed by an ideal gas with constant specific heat ratio k is shown in Fig. P5.80. Demonstrate that

(a) $V_4V_2 = V_1V_3$.
(b) $T_2/T_3 = (p_2/p_3)^{(k-1)/k}$.
(c) $T_2/T_3 = (V_3/V_2)^{(k-1)}$.

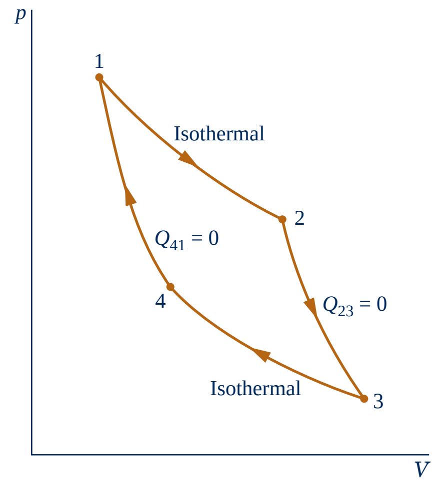


Fig. P5.80

5.81 Carbon dioxide (CO_2) as an ideal gas executes a Carnot power cycle while operating between thermal reservoirs at 450 and 100°F. The pressures at the initial and final states of the isothermal expansion are 400 and 200 lbf/in.^{2}, respectively. The specific heat ratio is $k = 1.24$. Using the results of Problem 5.80 as needed, determine

(a) the work and heat transfer for each of the four processes, in Btu/lb.
(b) the thermal efficiency.
(c) the pressures at the initial and final states of the isothermal compression, in lbf/in.^{2}

5.82 One-tenth kilogram of air as an ideal gas with $k = 1.4$ executes a Carnot refrigeration cycle, as shown in Fig. 5.16. The isothermal expansion occurs at $-23°C$ with a heat transfer to the air of 3.4 kJ. The isothermal compression occurs at 27°C to a final volume of 0.01 m^3. Using the results of Prob. 5.80 adapted to the present case, determine

(a) the pressure, in kPa, at each of the four principal states.
(b) the work, in kJ, for each of the four processes.
(c) the coefficient of performance.

Clausius Inequality Applications

5.83 A system executes a power cycle while receiving 1000 kJ by heat transfer at a temperature of 500 K and discharging energy by heat transfer at a temperature of 300 K. There are no other heat transfers. Applying Eq. 5.13, determine σ_{cycle} if the thermal efficiency is (a) 100%, (b) 40%, (c) 30%. Identify cases (if any) that are internally reversible or impossible.

5.84 A system executes a power cycle while receiving 1050 kJ by heat transfer at a temperature of 525 K and discharging 700 kJ by heat transfer at 350 K. There are no other heat transfers.

(a) Using Eq. 5.13, determine whether the cycle is *internally reversible*, *irreversible*, or *impossible*.
(b) Determine the thermal efficiency using Eq. 5.4 and the given heat transfer data. Compare this value with the *Carnot efficiency* calculated using Eq. 5.9 and comment.

5.85 As shown in Fig. P5.85, a system executes a power cycle while receiving 750 kJ by heat transfer at a temperature of 1500 K and discharging 100 kJ by heat transfer at a temperature of 500 K. Another heat transfer from the system occurs at a temperature of 1000 K. Using Eq. 5.13, determine the thermal efficiency if σ_{cycle} is (a) 0 kJ/K, (b) 0.1 kJ/K, (c) 0.2 kJ/K, (d) 0.35 kJ/K.

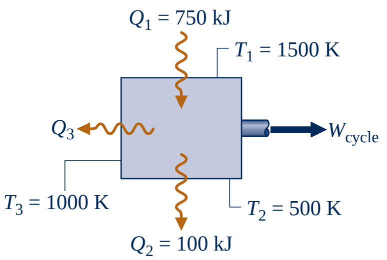


Fig. P5.85

5.86 Figure P5.86 gives the schematic of a vapor power plant in which water steadily circulates through the four components shown. The water flows through the boiler and condenser at constant pressure and through the turbine and pump adiabatically. Kinetic and potential energy effects can be ignored. Process data follow:

Process 4–1: constant-pressure at 1 MPa from saturated liquid to saturated vapor

Process 2–3: constant-pressure at 20 kPa from $x_2 = 88\%$ to $x_3 = 18\%$

(a) Using Eq. 5.13 expressed on a time-rate basis, determine if the cycle is *internally reversible, irreversible,* or *impossible*.
(b) Determine the thermal efficiency using Eq. 5.4 expressed on a time-rate basis and steam table data.
(c) Compare the result of part (b) with the *Carnot efficiency* calculated using Eq. 5.9 with the boiler and condenser temperatures and comment.

5.87 Repeat Problem 5.86 for the following case:

Process 4–1: constant-pressure at 8 MPa from saturated liquid to saturated vapor

Process 2–3: constant-pressure at 8 kPa from $x_2 = 67.5\%$ to $x_3 = 34.2\%$

5.88 Repeat Problem 5.86 for the following case:

Process 4–1: constant-pressure at 0.15 MPa from saturated liquid to saturated vapor

Process 2–3: constant-pressure at 20 kPa from $x_2 = 90\%$ to $x_3 = 10\%$

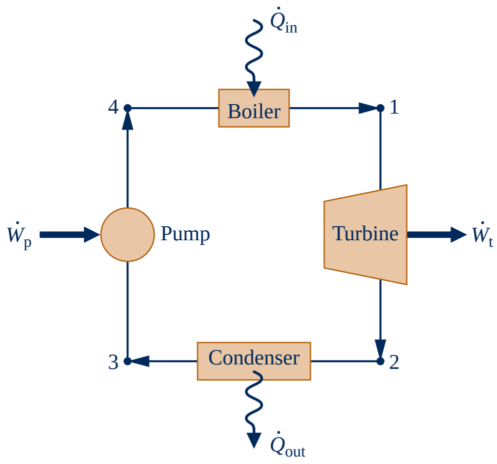


Fig. P5.86–88

5.89 A reversible power cycle R and an irreversible power cycle I operate between the same two reservoirs. Each receives Q_H from the hot reservoir. The reversible cycle develops work W_R, while the irreversible cycle develops work W_I. The reversible cycle discharges Q_C to the cold reservoir, while the irreversible cycle discharges Q'_C.

(a) Using Eq. 5.13, evaluate σ_{cycle} for cycle I in terms of W_I, W_R, and temperature T_C of the cold reservoir only.
(b) Demonstrate that $W_I < W_R$ and $Q'_C > Q_C$.

5.90 A reversible refrigeration cycle R and an irreversible refrigeration cycle I operate between the same two reservoirs and each removes Q_C from the cold reservoir. The net work input required by R is W_R, while the net work input for I is W_I. The reversible cycle discharges Q_H to the hot reservoir, while the irreversible cycle discharges Q'_H. Using Eq. 5.13, show that $W_I > W_R$ and $Q'_H > Q_H$.

5.91 Using Eq. 5.13, complete the following involving reversible and irreversible cycles:

(a) Reversible and irreversible *power cycles* each discharge energy Q_C to a cold reservoir at temperature T_C and receive energy Q_H from hot reservoirs at temperatures T_H and T'_H, respectively. There are no other heat transfers. Show that $T'_H > T_H$.

(b) Reversible and irreversible *refrigeration* cycles each discharge energy Q_H to a hot reservoir at temperature T_H and receive energy Q_C from cold reservoirs at temperatures T_C and T'_C, respectively. There are no other heat transfers. Show that $T'_C > T_C$.

(c) Reversible and irreversible *heat pump* cycles each receive energy Q_C from a cold reservoir at temperature T_C and discharge energy Q_H to hot reservoirs at temperatures T_H and T'_H, respectively. There are no other heat transfers. Show that $T'_H < T_H$.

5.92 Figure P5.92 shows a system consisting of a power cycle driving a heat pump. At steady state, the power cycle receives $\dot{Q}_s$ by heat transfer at T_s from the high-temperature source and delivers $\dot{Q}_1$ to a dwelling at T_d. The heat pump receives $\dot{Q}_0$ from the outdoors at T_0, and delivers $\dot{Q}_2$ to the dwelling. Using Eq. 5.13 on a time rate basis, obtain an expression for the maximum theoretical value of the performance parameter $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ in terms of the temperature ratios T_s/T_d and T_0/T_d.

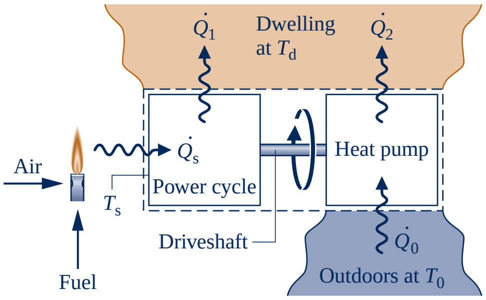


Fig. P5.92

Reviewing Concepts

5.93 Answer the following true or false. Explain.

(a) The maximum thermal efficiency of *any* power cycle operating between hot and cold thermal reservoirs at 1000°C and 500°C, respectively, is 50%.

(b) A process of a closed system that violates the second law of thermodynamics necessarily violates the first law of thermodynamics.

(c) One statement of the second law of thermodynamics recognizes that the extensive property entropy is *produced* within systems whenever friction and other nonidealties are present there.

(d) In principle, the *Clausius inequality* is applicable to *any* thermodynamic cycle.

(e) When a net amount of work is done on a system undergoing an internally reversible process, a net heat transfer from the system necessarily occurs.

5.94 Answer the following true or false. Explain.

(a) The Kelvin scale is the only absolute temperature scale.

(b) In certain instances, domestic refrigerators violate the *Clausius* statement of the second law of thermodynamics.

(c) Friction associated with flow of fluids through channels and around objects is one type of *irreversibility*.

(d) A product website claims that a heat pump capable of maintaining a dwelling at 70°F on a day when the outside temperature is 32°F has a coefficient of performance of 3.5. Still, such a claim is *not* in accord the second law of thermodynamics.

(e) There are no irreversibilities within a system undergoing an *internally reversible process*.

5.95 Answer the following true or false. Explain.

(a) The *second* Carnot corollary states that all power cycles operating between the same two thermal reservoirs have the same thermal efficiency.

(b) When left alone, systems tend to undergo spontaneous changes until equilibrium is attained, both internally and with their surroundings.

(c) Internally reversible processes do not actually occur but serve as hypothetical limiting cases as internal irreversibilities are reduced further and further.

(d) The energy of an *isolated* system remains constant, but its entropy can only decrease.

(e) The maximum coefficient of performance of *any* refrigeration cycle operating between cold and hot reservoirs at 40°F and 80°F, respectively, is closely 12.5.

▶ DESIGN & OPEN-ENDED PROBLEMS: EXPLORING ENGINEERING PRACTICE

5.1D The second law of thermodynamics is sometimes cited in publications of disciplines far removed from engineering and science, including but not limited to philosophy, economics, and sociology. Investigate use of the second law in peer-reviewed nontechnical publications. For three such publications, each in different disciplines, write a three-page critique. For each publication, identify and comment on the key objectives and conclusions. Clearly explain how the second law is used to inform the reader and propel the presentation. Score each publication on a 10-point scale, with 10 denoting a highly effective use of the second law and 1 denoting an ineffective use. Provide a rationale for each score.

5.2D The U.S. Food and Drug Administration (FDA) has long permitted the application of citric acid, ascorbic acid, and other substances to keep fresh meat looking red longer. In 2002, the FDA began allowing meat to be treated with carbon monoxide. Carbon monoxide reacts with *myoglobin* in the meat to produce a substance that resists the natural browning of meat, thereby giving meat a longer shelf life. Investigate the use of carbon monoxide for this purpose. Identify the nature of myoglobin and explain its role in the reactions that cause meat to brown or, when treated with carbon monoxide, allows the meat to appear red longer. Consider the hazards, if any, that may accompany this practice for consumers and for meat industry workers. Report your findings in a memorandum.

5.3D Investigate adverse health conditions that might be exacerbated for persons living in urban *heat islands*. Write a report including at least three references.

5.4D For a refrigerator in your home, dormitory, or workplace, use a plug-in appliance load tester (Fig. P5.4D) to determine the appliance's power requirements, in kW. Estimate *annual* electrical usage for the refrigerator, in kW · h. Compare your estimate of annual electricity use with that for the same or a similar refrigerator posted on the ENERGY STAR® website. Rationalize any significant discrepancy between these values. Prepare a poster presentation detailing your methodologies and findings.

5.5D The objective of this project is to identify a commercially available heat pump system that will meet annual heating and cooling needs of an existing dwelling in a locale of your choice. Consider each of two types of heat pump: air source and ground source. Estimate installation costs, operating costs, and other pertinent costs for each type of heat pump. Assuming a 12-year life, specify the more economical heat pump system. What if electricity were to cost twice its current cost? Prepare a poster presentation of your findings.

5.6D Insulin and several other pharmaceuticals required daily by those suffering from diabetes and other medical conditions have relatively low thermal stability. Those living and traveling in hot climates are especially at risk by heat-induced loss of potency of their pharmaceuticals. Design a wearable, lightweight, and reliable cooler for transporting temperature-sensitive pharmaceuticals. The cooler also must be solely powered by human motion. While the long-term goal is a moderately-priced consumer product, the final project report need only provide the costing of a single prototype.

5.7D Over the years, claimed *perpetual motion* machines have been rejected because they violate physical laws, primarily the first or second laws of thermodynamics, or both. Yet, while skepticism is deeply ingrained about perpetual motion, the *ATMOS* clock is said to enjoy a nearly unlimited operational service life, and advertisements characterize it as a *perpetual motion clock*. Investigate how the ATMOS operates. Provide a complete explanation of its operation, including sketches and references to the first and second laws, as appropriate. Clearly establish whether the ATMOS can justifiably be called a perpetual motion machine, closely approximates one, or only appears to be one. Prepare a memorandum summarizing your findings.

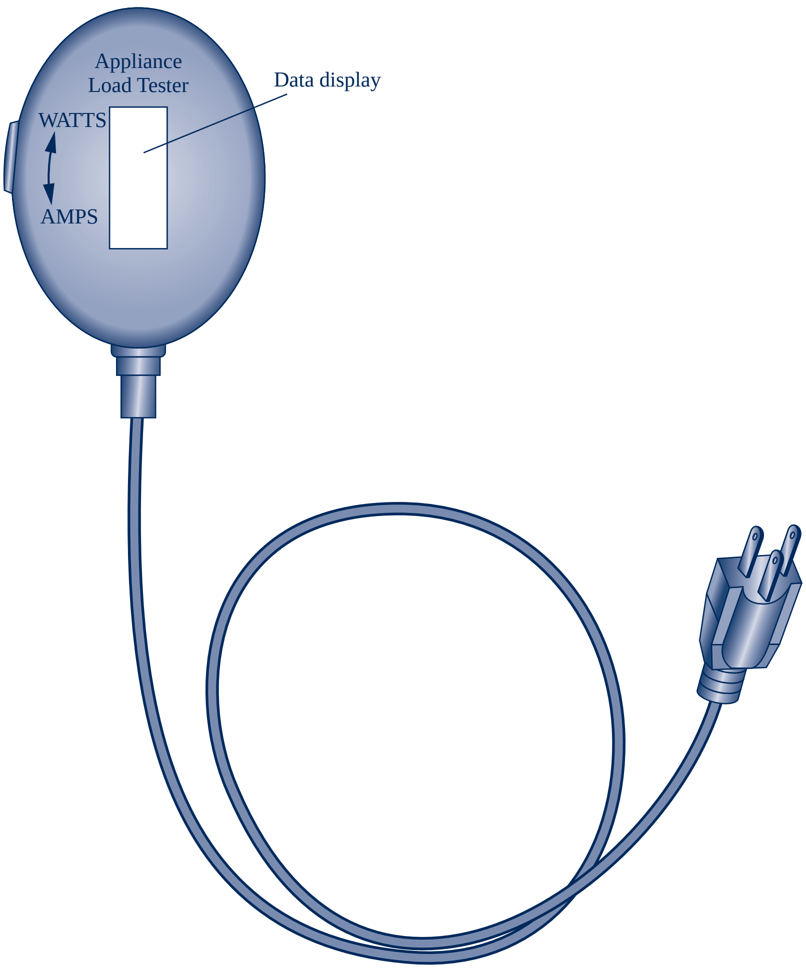


Fig. P5.4D

5.8D For a dwelling in a locale of your choice, determine on a *total cost* basis the feasibility of adapting a commercially available heat pump, working in air-conditioning mode, to heat an outdoor swimming pool on the same property. The aim is to cost-effectively reduce or eliminate heating required from a separate commercially available pool heater, while maintaining the interior of the dwelling at a desired temperature and keeping the pool temperature in a comfortable range. Estimate the cost to adapt the heat pump for such double duty. Also evaluate the cost to operate a pool heater when assisted by the heat pump and the cost to operate a pool heater when not heat pump–assisted. Consider other pertinent costs. Report your findings in an executive summary and a classroom presentation.

5.9D A technical article considers hurricanes as an example of a *natural* Carnot engine: K. A. Emmanuel, "Toward a General Theory of Hurricanes," *American Scientist,* 76, 371–379, 1988. Also see *Physics Today,* 59, No. 8, 74–75, 2006, for a related discussion by this author. U.S. Patent (No. 4,885,913) is said to have been inspired by such an analysis. Does the concept have scientific merit? Engineering merit? Summarize your conclusions in a memorandum.

5.10D Figure P5.10D shows one of those bobbing toy birds that seemingly takes an endless series of sips from a cup filled with water. Prepare a 30-min presentation suitable for a middle school science class explaining the operating principles of this device and whether or not its behavior is at odds with the second law.

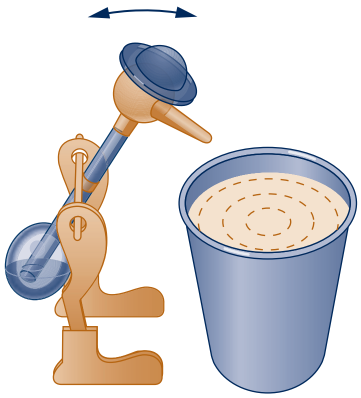


Fig. P5.10D

Directionality **of processes can be determined using entropy, as discussed in Sec. 6.8.** **© Georg Winkens/iStockphoto**

ENGINEERING CONTEXT Up to this point, our study of the second law has been concerned primarily with what it says about systems undergoing thermodynamic cycles. In this chapter means are introduced for analyzing systems from the second law perspective as they undergo processes that are not necessarily cycles. The property *entropy* and the *entropy production* concept introduced in Chap. 5 play prominent roles in these considerations.

The **objective** of this chapter is to develop an understanding of entropy concepts, including the use of entropy balances for closed systems and control volumes in forms effective for the analysis of engineering systems. The Clausius inequality developed in Sec. 5.11, expressed as Eq. 5.13, provides the basis.

6

Using Entropy

▶ LEARNING OUTCOMES

When you complete your study of this chapter, you will be able to...

- demonstrate understanding of key concepts related to entropy and the second law . . . including entropy transfer, entropy production, and the increase in entropy principle.
- evaluate entropy, evaluate entropy change between two states, and analyze isentropic processes, using appropriate property data.
- represent heat transfer in an internally reversible process as an area on a temperature-entropy diagram.
- apply entropy balances to closed systems and control volumes.
- evaluate isentropic efficiencies for turbines, nozzles, compressors, and pumps.

6.1 Entropy–A System Property

The word *energy* is so much a part of the language that you were undoubtedly familiar with the term before encountering it in early science courses. This familiarity probably facilitated the study of energy in these courses and in the current course in engineering thermodynamics. In the present chapter you will see that the analysis of systems from a second law perspective is effectively accomplished in terms of the property *entropy*. Energy and entropy are both abstract concepts. However, unlike energy, the word entropy is seldom heard in everyday conversation, and you may never have dealt with it quantitatively before. Energy and entropy play important roles in the remaining chapters of this book.

6.1.1 Defining Entropy Change

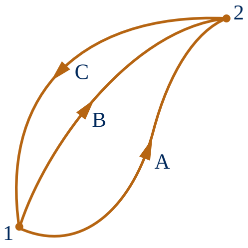


Fig. 6.1 Two internally reversible cycles.

A quantity is a property if, and only if, its change in value between two states is independent of the process (Sec. 1.3.3). This aspect of the property concept is used in the present section together with the Clausius inequality to introduce entropy change as follows:

Two cycles executed by a closed system are represented in Fig. 6.1. One cycle consists of an internally reversible process A from state 1 to state 2, followed by internally reversible process C from state 2 to state 1. The other cycle consists of an internally reversible process B from state 1 to state 2, followed by the same process C from state 2 to state 1 as in the first cycle. For the first cycle, Eq. 5.13 (the Clausius inequality) takes the form

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_{\text{A}} + \left(\int_2^1 \frac{\delta Q}{T}\right)_{\text{C}} = -\cancel{\sigma}^{\,0}_{\text{cycle}} \tag{6.1a}$$

$$\oint \left(\frac{\delta Q}{T}\right)_{\text{b}} = -\sigma_{\text{cycle}}$$
(Eq. 5.13)

For the second cycle, Eq. 5.13 takes the form

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_{\text{B}} + \left(\int_2^1 \frac{\delta Q}{T}\right)_{\text{C}} = -\cancel{\sigma}^{\,0}_{\text{cycle}} \tag{6.1b}$$

In writing Eqs. 6.1, the term σ_{cycle} has been set to zero since the cycles are composed of internally reversible processes.

When Eq. 6.1b is subtracted from Eq. 6.1a, we get

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_{\text{A}} = \left(\int_1^2 \frac{\delta Q}{T}\right)_{\text{B}}$$

This shows that the integral of $\delta Q/T$ is the same for both processes. Since A and B are arbitrary, it follows that the integral of $\delta Q/T$ has the same value for *any* internally reversible process between the two states. In other words, the value of the integral depends on the end states only. It can be concluded, therefore, that the integral represents the change in some property of the system.

Selecting the symbol S to denote this property, which is called *entropy,* the **change in entropy** is given by

definition of entropy change

$$S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} \tag{6.2a}$$

where the subscript "int rev" is added as a reminder that the integration is carried out for any internally reversible process linking the two states. On a differential basis, the defining equation for entropy change takes the form

$$dS = \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} \tag{6.2b}$$

Entropy is an extensive property.

The **SI unit for entropy** is J/K. However, in this book it is convenient to work in terms of kJ/K. A commonly employed **English unit for entropy** is Btu/°R. Units in SI for *specific* entropy are kJ/kg · K for s and kJ/kmol · K for $\bar{s}$. Commonly used English units for *specific* entropy are Btu/lb · °R and Btu/lbmol · °R.

units for entropy

It should be clear that entropy is defined and evaluated in terms of a particular expression (Eq. 6.2a) for which *no accompanying physical picture is given.* We encountered this previously with the property enthalpy. Enthalpy is introduced without physical motivation in Sec. 3.6.1. Then, in Chap. 4, we learned how enthalpy is used for thermodynamic analysis of control volumes. As for the case of enthalpy, to gain an appreciation for entropy you need to understand *how* it is used and *what* it is used for. This is the aim of the rest of this chapter.

6.1.2 Evaluating Entropy

Since entropy is a property, the change in entropy of a system in going from one state to another is the same for *all* processes, both internally reversible and irreversible, between these two states. Thus, Eq. 6.2a allows the determination of the change in entropy, and once it has been evaluated, this is the magnitude of the entropy change for all processes of the system between the two states.

The defining equation for entropy change, Eq. 6.2a, serves as the basis for evaluating entropy relative to a reference value at a reference state. Both the reference value and the reference state can be selected arbitrarily. The value of entropy at any state y relative to the value at the reference state x is obtained in principle from

$$S_y = S_x + \left(\int_x^y \frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} \tag{6.3}$$

where S_x is the reference value for entropy at the specified reference state.

The use of entropy values determined relative to an arbitrary reference state is satisfactory as long as they are used in calculations involving entropy differences, for then the reference value cancels. This approach suffices for applications where composition remains constant. When chemical reactions occur, it is necessary to work in terms of *absolute* values of entropy determined using the *third law of thermodynamics* (Chap. 13).

6.1.3 Entropy and Probability

The presentation of engineering thermodynamics provided in this book takes a *macroscopic* view as it deals mainly with the gross, or overall, behavior of matter. The macroscopic concepts of engineering thermodynamics introduced thus far, including energy and entropy, rest on operational definitions whose validity is shown directly or indirectly through experimentation. Still, insights concerning energy and entropy can result from considering the microstructure of matter. This brings in the use of *probability* and the notion of *disorder*. Further discussion of entropy, probability, and disorder is provided in Sec. 6.8.2.

6.2 Retrieving Entropy Data

In Chap. 3, we introduced means for retrieving property data, including tables, graphs, equations, and the software available with this text. The emphasis there is on evaluating the properties p, v, T, u, and h required for application of the conservation of mass and energy principles. For application of the second law, entropy values are usually required. In this section, means for retrieving entropy data for water and several refrigerants are considered.

Tables of thermodynamic data are introduced in Secs. 3.5 and 3.6 (Tables A-2 through A-18). Specific entropy is tabulated in the same way as considered there for the properties v, u, and h, and entropy values are retrieved similarly. The specific entropy values given in Tables A-2 through A-18 are relative to the following *reference states and values.* For water, the entropy of saturated liquid at 0.01°C (32.02°F) is set to zero. For the refrigerants, the entropy of the saturated liquid at −40°C (−40°F) is assigned a value of zero.

6.2.1 Vapor Data

In the superheat regions of the tables for water and the refrigerants, specific entropy is tabulated along with v, u, and h versus temperature and pressure.

▶ FOR EXAMPLE consider two states of water. At state 1 the pressure is 3 MPa and the temperature is 500°C. At state 2, the pressure is 0.3 MPa and the specific entropy is the same as at state 1, $s_2 = s_1$. The object is to determine the temperature at state 2. Using T_1 and p_1, we find the specific entropy at state 1 from Table A-4 as $s_1 = 7.2338$ kJ/kg · K. State 2 is fixed by the pressure, $p_2 = 0.3$ MPa, and the specific entropy, $s_2 = 7.2338$ kJ/kg · K. Returing to Table A-4 at 0.3 MPa and interpolating with s_2 between 160 and 200°C results in $T_2 = 183$°C. ◀ ◀ ◀ ◀ ◀

6.2.2 Saturation Data

For saturation states, the values of s_f and s_g are tabulated as a function of either saturation pressure or saturation temperature. The specific entropy of a two-phase liquid–vapor mixture is calculated using the quality

$$\begin{aligned} s &= (1 - x)s_f + xs_g \\ &= s_f + x(s_g - s_f) \end{aligned} \tag{6.4}$$

These relations are identical in form to those for v, u, and h (Secs. 3.5 and 3.6).

▶ FOR EXAMPLE let us determine the specific entropy of Refrigerant 134a at a state where the temperature is 0°C and the specific internal energy is 138.43 kJ/kg. Referring to Table A-10, we see that the given value for u falls between u_f and u_g at 0°C, so the system is a two-phase liquid–vapor mixture. The quality of the mixture can be determined from the known specific internal energy

$$x = \frac{u - u_f}{u_g - u_f} = \frac{138.43 - 49.79}{227.06 - 49.79} = 0.5$$

Then with values from Table A-10, Eq. 6.4 gives

$$\begin{aligned} s &= (1 - x)s_f + xs_g \\ &= (0.5)(0.1970) + (0.5)(0.9190) = 0.5580 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$ ◀ ◀ ◀ ◀ ◀

6.2.3 Liquid Data

Compressed liquid data are presented for water in Tables A-5. In these tables s, v, u, and h are tabulated versus temperature and pressure as in the superheat tables, and the tables are used similarly. In the absence of compressed liquid data, the value of the specific entropy can be estimated in the same way as estimates for v and u are obtained for liquid states (Sec. 3.10.1), by using the saturated liquid value at the given temperature

$$s(T, p) \approx s_f(T) \tag{6.5}$$

▶ **FOR EXAMPLE** suppose the value of specific entropy is required for water at 25 bar, 200°C. The specific entropy is obtained directly from Table A-5 as $s = 2.3294$ kJ/kg · K. Using the saturated liquid value for specific entropy at 200°C from Table A-2, the specific entropy is approximated with Eq. 6.5 as $s = 2.3309$ kJ/kg · K, which agrees closely with the previous value. ◀ ◀ ◀ ◀ ◀

6.2.4 Computer Retrieval

The software available with this text, *Interactive Thermodynamics: IT,* provides data for the substances considered in this section. Entropy data are retrieved by simple call statements placed in the workspace of the program.

▶ **FOR EXAMPLE** consider a two-phase liquid–vapor mixture of H_2O at $p = 1$ bar, $v = 0.8475$ m^3/kg. The following illustrates how specific entropy and quality x are obtained using *IT*

```
p = 1 // bar
v = 0.8475 // m³/kg
v = vsat_Px("Water/Steam",p,x)
s = ssat_Px("Water/Steam",p,x)
```

The software returns values of $x = 0.5$ and $s = 4.331$ kJ/kg · K, which can be checked using data from Table A-3. Note that quality x is implicit in the expression for specific volume, and it is not necessary to solve explicitly for x. As another example, consider superheated ammonia vapor at $p = 1.5$ bar, $T = 8$°C. Specific entropy is obtained from *IT* as follows:

```
p = 1.5 // bar
T = 8 // °C
s = s_PT ("Ammonia", p,T)
```

The software returns $s = 5.981$ kJ/kg · K, which agrees closely with the value obtained by interpolation in Table A-15. ◀ ◀ ◀ ◀ ◀

TAKE NOTE...
Note that *IT* does not provide compressed liquid data for *any* substance. *IT* returns liquid entropy data using the approximation of Eq. 6.5. Similarly, Eqs. 3.11, 3.12, and 3.14 are used to return liquid values for v, u, and h, respectively.

6.2.5 Using Graphical Entropy Data

The use of property diagrams as an adjunct to problem solving is emphasized throughout this book. When applying the second law, it is frequently helpful to locate states and plot processes on diagrams having entropy as a coordinate. Two commonly used figures having entropy as one of the coordinates are the temperature–entropy diagram and the enthalpy–entropy diagram.

Temperature–Entropy Diagram

T–s **diagram**

The main features of a **temperature–entropy diagram** are shown in Fig. 6.2. For detailed figures for water in SI and English units, see Figs. A-7. Observe that lines of constant enthalpy are shown on these figures. Also note that in the superheated vapor region constant specific volume lines have a steeper slope than constant-pressure lines. Lines of constant quality are shown in the two-phase liquid–vapor region. On some figures, lines of constant quality are marked as *percent moisture* lines. The percent moisture is defined as the ratio of the mass of liquid to the total mass.

In the superheated vapor region of the *T–s* diagram, constant specific enthalpy lines become nearly horizontal as pressure is reduced. These superheated vapor states are shown as the shaded area on Fig. 6.2. For states in this region of the diagram, the enthalpy is determined primarily by the temperature: $h(T, p) \approx h(T)$. This is the

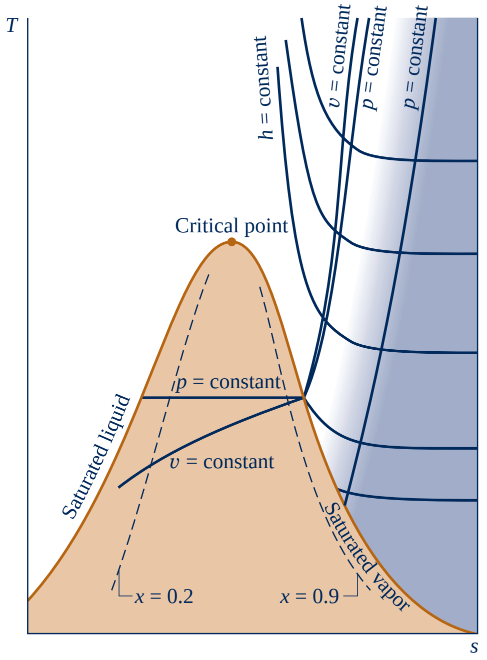


Fig. 6.2 **Temperature–entropy diagram.**

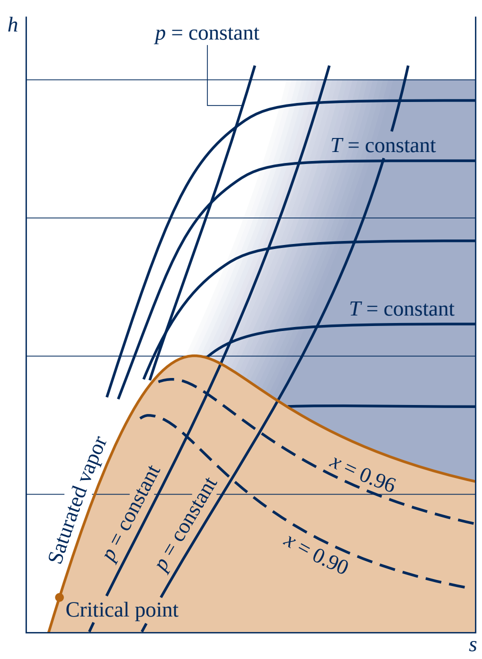


Fig. 6.3 **Enthalpy–entropy diagram.**

region of the diagram where the ideal gas model provides a reasonable approximation. For superheated vapor states outside the shaded area, both temperature and pressure are required to evaluate enthalpy, and the ideal gas model is not suitable.

Enthalpy–Entropy Diagram

Mollier diagram

The essential features of an enthalpy–entropy diagram, commonly known as a **Mollier diagram**, are shown in Fig. 6.3. For detailed figures for water in SI and English units, see Figs. A-8. Note the location of the critical point and the appearance of lines of constant temperature and constant pressure. Lines of constant quality are shown in the two-phase liquid–vapor region (some figures give lines of constant percent moisture). The figure is intended for evaluating properties at superheated vapor states and for two-phase liquid–vapor mixtures. Liquid data are seldom shown. In the superheated vapor region, constant-temperature lines become nearly horizontal as pressure is reduced. These superheated vapor states are shown, approximately, as the shaded area on Fig. 6.3. This area corresponds to the shaded area on the temperature–entropy diagram of Fig. 6.2, where the ideal gas model provides a reasonable approximation.

▸ **FOR EXAMPLE** to illustrate the use of the Mollier diagram in SI units, consider two states of water. At state 1, $T_1 = 240°C$, $p_1 = 0.10$ MPa. The specific enthalpy and quality are required at state 2, where $p_2 = 0.01$ MPa and $s_2 = s_1$. Turning to Fig. A-8, state 1 is located in the superheated vapor region. Dropping a vertical line into the two-phase liquid–vapor region, state 2 is located. The quality and specific enthalpy at state 2 read from the figure agree closely with values obtained using Tables A-3 and A-4: $x_2 = 0.98$ and $h_2 = 2537$ kJ/kg. ◂ ◂ ◂ ◂ ◂

6.3 Introducing the *T dS* Equations

Although the change in entropy between two states can be determined in principle by using Eq. 6.2a, such evaluations are generally conducted using the *T dS* equations developed in this section. The *T dS* equations allow entropy changes to be evaluated from other more readily determined property data. The use of the *T dS* equations to evaluate entropy changes for an incompressible substance is illustrated in Sec. 6.4 and for ideal gases in Sec. 6.5. The importance of the $T\,dS$ equations is greater than their role in assigning entropy values, however. In Chap. 11 they are used as a point of departure for deriving many important property relations for pure, simple compressible systems, including means for constructing the property tables giving u, h, and s.

The $T\,dS$ equations are developed by considering a pure, simple compressible system undergoing an internally reversible process. In the absence of overall system motion and the effects of gravity, an energy balance in differential form is

$$(\delta Q)_{\substack{\text{int}\\ \text{rev}}} = dU + (\delta W)_{\substack{\text{int}\\ \text{rev}}} \tag{6.6}$$

By definition of simple compressible system (Sec. 3.1.2), the work is

$$(\delta W)_{\substack{\text{int}\\ \text{rev}}} = p\,dV \tag{6.7a}$$

On rearrangement of Eq. 6.2b, the heat transfer is

$$(\delta Q)_{\substack{\text{int}\\ \text{rev}}} = T\,dS \tag{6.7b}$$

Substituting Eqs. 6.7 into Eq. 6.6, the **first $T\,dS$ equation** results

$$\boxed{T\,ds = dU + p\,dV} \tag{6.8}$$

first $T\,dS$ equation

The *second* $T\,dS$ equation is obtained from Eq. 6.8 using $H = U + pV$. Forming the differential

$$dH = dU + d(pV) = dU + p\,dV + V\,dp$$

On rearrangement

$$dU + p\,dV = dH - V\,dp$$

Substituting this into Eq. 6.8 gives the **second $T\,dS$ equation**

$$\boxed{T\,dS = dH - V\,dp} \tag{6.9}$$

second $T\,dS$ equation

The $T\,dS$ equations can be written on a unit mass basis as

$$T\,ds = du + p\,dv \tag{6.10a}$$
$$T\,ds = dh - v\,dp \tag{6.10b}$$

or on a per mole basis as

$$T\,d\bar{s} = d\bar{u} + p\,d\bar{v} \tag{6.11a}$$
$$T\,d\bar{s} = d\bar{h} - \bar{v}dp \tag{6.11b}$$

Although the $T\,dS$ equations are derived by considering an internally reversible process, an entropy change obtained by integrating these equations is the change for *any* process, reversible or irreversible, between two equilibrium states of a system. Because entropy is a property, the change in entropy between two states is independent of the details of the process linking the states.

To show the use of the $T\,dS$ equations, consider a change in phase from saturated liquid to saturated vapor at constant temperature and pressure. Since pressure is constant, Eq. 6.10b reduces to give

$$ds = \frac{dh}{T}$$

Then, because temperature is also constant during the phase change

$$s_g - s_f = \frac{h_g - h_f}{T} \tag{6.12}$$

This relationship shows how $s_g - s_f$ is calculated for tabulation in property tables.

▶ **FOR EXAMPLE** consider Refrigerant 134a at 0°C. From Table A-10, $h_g - h_f =$ 197.21 kJ/kg, so with Eq. 6.12

$$s_g - s_f = \frac{197.21 \text{ kJ/kg}}{273.15 \text{ K}} = 0.7220 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

which is the value calculated using s_f and s_g from the table. To give a similar example in English units, consider Refrigerant 134a at 0°F. From Table A-10E, $h_g - h_f =$ 90.12 Btu/lb, so

$$s_g - s_f = \frac{90.12 \text{ Btu/lb}}{459.67°\text{R}} = 0.1961 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}$$

which agrees with the value calculated using s_f and s_g from the table. ◀ ◀ ◀ ◀ ◀

6.4 Entropy Change of an Incompressible Substance

In this section, Eq. 6.10a of Sec. 6.3 is used to evaluate the entropy change between two states of an incompressible substance. The incompressible substance model introduced in Sec. 3.10.2 assumes that the specific volume (density) is constant and the specific internal energy depends solely on temperature. Thus, $du = c(T)dT$, where c denotes the specific heat of the substance, and Eq. 6.10a reduces to give

$$ds = \frac{c(T)dT}{T} + \cancel{\frac{p\,dv}{T}}^{\,0} = \frac{c(T)dT}{T}$$

On integration, the change in specific entropy is

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c(T)}{T} dT$$

When the specific heat is constant, this becomes

$$s_2 - s_1 = c \ln \frac{T_2}{T_1} \quad \text{(incompressible, constant } c\text{)} \tag{6.13}$$

Equation 6.13, along with Eqs. 3.20 giving Δu and Δh, respectively, are applicable to liquids and solids modeled as incompressible. Specific heats of some common liquids and solids are given in Table A-19.

▶ **FOR EXAMPLE** consider a system consisting of liquid water initially at $T_1 = 300$ K, $p_1 = 2$ bar undergoing a process to a final state at $T_2 = 323$ K, $p_2 = 1$ bar. There are two ways to evaluate the change in specific entropy in this case. The first approach is to use Eq. 6.5 together with saturated liquid data from Table A-2. That is, $s_1 \approx s_f(T_1) =$ 0.3954 KJ/kg · K and $s_2 \approx s_f(T_2) = 0.7038$ KJ/kg · K, giving $s_2 - s_1 = 0.308$ KJ/kg · K. The second approach is to use the incompressible model. That is, with Eq. 6.13 and $c = 4.18$ KJ/kg · K from Table A-19, we get

$$s_2 - s_1 = c \ln \frac{T_2}{T_1}$$

$$= \left(4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) \ln\left(\frac{323 \text{ K}}{300 \text{ K}}\right) = 0.309 \text{ KJ/kg} \cdot \text{K}$$

Comparing the values obtained for the change in specific entropy using the two approaches considered here, we see they are in agreement. ◀ ◀ ◀ ◀ ◀

6.5 Entropy Change of an Ideal Gas

In this section, the $T\,dS$ equations of Sec. 6.3, Eqs. 6.10, are used to evaluate the entropy change between two states of an ideal gas. For a quick review of ideal gas model relations, see Table 6.1.

It is convenient to begin with Eqs. 6.10 expressed as

$$ds = \frac{du}{T} + \frac{p}{T} dv \tag{6.14}$$

$$ds = \frac{dh}{T} - \frac{v}{T} dp \tag{6.15}$$

For an ideal gas, $du = c_v(T)\,dT$, $dh = c_p(T)\,dT$, and $pv = RT$. With these relations, Eqs. 6.14 and 6.15 become, respectively

$$ds = c_v(T)\frac{dT}{T} + R\frac{dv}{v} \quad \text{and} \quad ds = c_p(T)\frac{dT}{T} - R\frac{dp}{p} \tag{6.16}$$

On integration, Eqs. 6.16 give, respectively

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R \ln\frac{v_2}{v_1} \tag{6.17}$$

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R \ln\frac{p_2}{p_1} \tag{6.18}$$

Since R is a constant, the last terms of Eqs. 6.16 can be integrated directly. However, because c_v and c_p are functions of temperature for ideal gases, it is necessary to have information about the functional relationships before the integration of the first term in these equations can be performed. Since the two specific heats are related by

$$c_p(T) = c_v(T) + R \tag{3.44}$$

where R is the gas constant, knowledge of either specific function suffices.

6.5.1 Using Ideal Gas Tables

As for internal energy and enthalpy changes of ideal gases, the evaluation of entropy changes for ideal gases can be reduced to a convenient tabular approach. We begin by introducing a new variable $s°(T)$ as

$$s°(T) = \int_{T'}^{T} \frac{c_p(T)}{T} dT \tag{6.19}$$

where T' is an arbitrary reference temperature.

TABLE 6.1

Ideal Gas Model Review

Equations of state:

$$pv = RT \tag{3.32}$$
$$pV = mRT \tag{3.33}$$

Changes in u and h:

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)dT \tag{3.40}$$

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)dT \tag{3.43}$$

Constant Specific Heats	Variable Specific Heats
$u(T_2) - u(T_1) = c_v(T_2 - T_1)$ (3.50) $h(T_2) - h(T_1) = c_p(T_2 - T_1)$ (3.51) See Tables A-20, 21 for c_v and c_p data.	$u(T)$ and $h(T)$ are evaluated from Tables A-22 for air (mass basis) and Tables A-23 for several other gases (molar basis).

The integral of Eq. 6.18 can be expressed in terms of $s°$ as follows

$$\int_{T_1}^{T_2} c_p \frac{dT}{T} = \int_{T'}^{T_2} c_p \frac{dT}{T} - \int_{T'}^{T_1} c_p \frac{dT}{T}$$
$$= s°(T_2) - s°(T_1)$$

Thus, Eq. 6.18 can be written as

$$s(T_2, p_2) - s(T_1, p_1) = s°(T_2) - s°(T_1) - R \ln \frac{p_2}{p_1} \tag{6.20a}$$

or on a per mole basis as

$$\bar{s}(T_2, p_2) - \bar{s}(T_1, p_1) = \bar{s}°(T_2) - \bar{s}°(T_1) - \bar{R} \ln \frac{p_2}{p_1} \tag{6.20b}$$

Because $s°$ depends only on temperature, it can be tabulated versus temperature, like h and u. For air as an ideal gas, $s°$ with units of kJ/kg · K or Btu/lb · °R is given in Table A-22 and A-22E, respectively. Values of $\bar{s}°$ for several other common gases are given in Tables A-23 with units of kJ/kmol · K or Btu/lbmol · °R. In passing, we note the arbitrary reference temperature T' of Eq. 6.19 is specified differently in Tables A-22 than in Tables A-23. As discussed in Sec. 13.5.1, Tables A-23 give *absolute entropy* values.

Using Eqs. 6.20 and the tabulated values for $s°$ or $\bar{s}°$, as appropriate, entropy changes can be determined that account explicitly for the variation of specific heat with temperature.

▶ **FOR EXAMPLE** let us evaluate the change in specific entropy, in kJ/kg · K, of air modeled as an ideal gas from a state where $T_1 = 300$ K and $p_1 = 1$ bar to a state where $T_2 = 1000$ K and $p_2 = 3$ bar. Using Eq. 6.20a and data from Table A-22

$$s_2 - s_1 = s°(T_2) - s°(T_1) - R \ln \frac{p_2}{p_1}$$
$$= (2.96770 - 1.70203)\frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln \frac{3 \text{ bar}}{1 \text{ bar}}$$
$$= 0.9504 \text{ kJ/kg} \cdot \text{K}$$ ◀ ◀ ◀ ◀ ◀

If a table giving $s°$ (or $\bar{s}°$) is not available for a particular gas of interest, the integrals of Eqs. 6.17 and 6.18 can be performed analytically or numerically using specific heat data such as provided in Tables A-20 and A-21.

6.5.2 Assuming Constant Specific Heats

When the specific heats c_v and c_p are taken as constants, Eqs. 6.17 and 6.18 reduce, respectively, to

$$s(T_2, v_2) - s(T_1, v_1) = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \tag{6.21}$$

$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \tag{6.22}$$

These equations, along with Eqs. 3.50 and 3.51 giving Δu and Δh, respectively, are applicable when assuming the ideal gas model with constant specific heats.

▶ **FOR EXAMPLE** let us determine the change in specific entropy, in KJ/kg · K, of air as an ideal gas undergoing a process from $T_1 = 300$ K, $p_1 = 1$ bar to $T_2 = 400$ K, $p_2 = 5$ bar. Because of the relatively small temperature range, we assume a constant value of c_p evaluated at 350 K. Using Eq. 6.22 and $c_p = 1.008$ KJ/kg · K from Table A-20

$$\begin{aligned}\Delta s &= c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\\ &= \left(1.008\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{400\text{ K}}{300\text{ K}}\right) - \left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{5\text{ bar}}{1\text{ bar}}\right)\\ &= -0.1719\text{ kJ/kg}\cdot\text{K}\end{aligned}$$ ◀ ◀ ◀ ◀ ◀

6.5.3 Computer Retrieval

For gases modeled as ideal gases, *IT directly* returns $s(T, p)$ using the following special form of Eq. 6.18:

$$s(T, p) - s(T_{\text{ref}}, p_{\text{ref}}) = \int_{T_{\text{ref}}}^{T} \frac{c_p(T)}{T}dT - R\ln\frac{p}{p_{\text{ref}}}$$

and the following choice of reference state and reference value: $T_{\text{ref}} = 0$ K (0°R), $p_{\text{ref}} = 1$ atm, and $s(T_{\text{ref}}, p_{\text{ref}}) = 0$, giving

$$s(T, p) = \int_0^T \frac{c_p(T)}{T}dT - R\ln\frac{p}{p_{\text{ref}}} \tag{a}$$

Such reference state and reference value choices equip *IT* for use in combustion applications. See the discussion of *absolute entropy* in Sec. 13.5.1.

Changes in specific entropy evaluated using *IT* agree with entropy *changes* evaluated using ideal gas tables.

▶ **FOR EXAMPLE** consider a process of air as an ideal gas from $T_1 = 300$ K, $p_1 = 1$ bar to $T_2 = 1000$ K, $p_2 = 3$ bar. The change in specific entropy, denoted as dels, is determined in SI units using *IT* as follows:

```
p1 = 1//bar
T1 = 300//K
p2 = 3
T2 = 1000
s1 = s_TP("Air",T1,p1)
s2 = s_TP("Air",T2,p2)
dels = s2 − s1
```

The software returns values of $s_1 = 1.706$, $s_2 = 2.656$, and dels $= 0.9501$, all in units of kJ/kg · K. This value for Δs agrees with the value obtained using Table A-22: 0.9504 kJ/kg · K, as shown in the concluding example of Sec. 6.5.1. ◀ ◀ ◀ ◀ ◀

Note again that *IT* returns specific entropy directly using Eq. (a) above. *IT* does not use the special function $s°$.

6.6 Entropy Change in Internally Reversible Processes of Closed Systems

In this section the relationship between entropy change and heat transfer for internally reversible processes is considered. The concepts introduced have important applications in subsequent sections of the book. The present discussion is limited to the case of closed systems. Similar considerations for control volumes are presented in Sec. 6.13.

As a closed system undergoes an internally reversible process, its entropy can increase, decrease, or remain constant. This can be brought out using

$$dS = \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} \tag{6.2b}$$

entropy transfer

isentropic process

which indicates that when a closed system undergoing an internally reversible process receives energy by heat transfer, the system experiences an increase in entropy. Conversely, when energy is removed from the system by heat transfer, the entropy of the system decreases. This can be interpreted to mean that an **entropy transfer** *accompanies* heat transfer. The direction of the entropy transfer is the same as that of the heat transfer. In an *adiabatic* internally reversible process, entropy remains constant. A constant-entropy process is called an **isentropic process**.

6.6.1 Area Representation of Heat Transfer

On rearrangement, Eq. 6.2b gives

$$(\delta Q)_{\substack{\text{int}\\\text{rev}}} = T\,dS$$

Integrating from an initial state 1 to a final state 2

$$Q_{\substack{\text{int}\\\text{rev}}} = \int_1^2 T\,dS \tag{6.23}$$

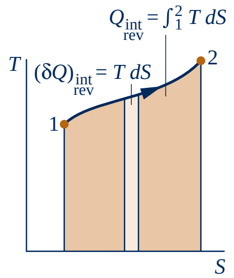


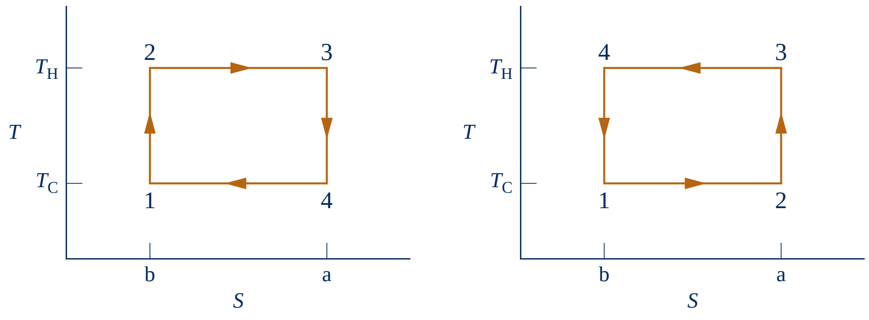
Fig. 6.4 **Area representation of heat transfer for an internally reversible process of a closed system.**

From Eq. 6.23 it can be concluded that an energy transfer by heat to a closed system during an internally reversible process can be represented as an area on a temperature–entropy diagram. Figure 6.4 illustrates the area representation of heat transfer for an arbitrary internally reversible process in which temperature varies. Carefully note that temperature must be in kelvins or degrees Rankine, and the area is the entire area under the curve (shown shaded). Also note that the area representation of heat transfer is not valid for irreversible processes, as will be demonstrated later.

6.6.2 Carnot Cycle Application

Carnot cycle

To provide an example illustrating both the entropy change that accompanies heat transfer and the area representation of heat transfer, consider Fig. 6.5*a*, which shows a **Carnot power cycle** (Sec. 5.10.1). The cycle consists of four internally reversible processes in series: two isothermal processes alternated with two adiabatic processes. In Process 2–3, heat transfer to the system occurs while the temperature of the system remains constant



(*a*) (*b*)

Fig. 6.5 Carnot cycles on the temperature–entropy diagram. (*a*) Power cycle. (*b*) Refrigeration or heat pump cycle.

at T_H. The system entropy increases due to the accompanying entropy transfer. For this process, Eq. 6.23 gives $Q_{23} = T_H(S_3 - S_2)$, so area 2–3–a–b–2 on Fig. 6.5*a* represents the heat transfer during the process. Process 3–4 is an adiabatic and internally reversible process and thus is an isentropic (constant-entropy) process. Process 4–1 is an isothermal process at T_C during which heat is transferred *from* the system. Since entropy transfer accompanies the heat transfer, system entropy decreases. For this process, Eq. 6.23 gives $Q_{41} = T_C(S_1 - S_4)$, which is negative in value. Area 4–1–b–a–4 on Fig. 6.5*a* represents the *magnitude* of the heat transfer Q_{41}. Process 1–2, which completes the cycle, is adiabatic and internally reversible (isentropic).

The net work of any cycle is equal to the net heat transfer, so *enclosed* area 1–2–3–4–1 represents the net work of the cycle. The thermal efficiency of the cycle also can be expressed in terms of areas:

$$\eta = \frac{W_{\text{cycle}}}{Q_{23}} = \frac{\text{area } 1-2-3-4-1}{\text{area } 2-3-a-b-2}$$

The numerator of this expression is $(T_H - T_C)(S_3 - S_2)$ and the denominator is $T_H(S_3 - S_2)$, so the thermal efficiency can be given in terms of temperatures only as $\eta = 1 - T_C/T_H$. This of course agrees with Eq. 5.9.

If the cycle were executed as shown in Fig. 6.5*b*, the result would be a Carnot refrigeration or heat pump cycle. In such a cycle, heat is transferred to the system while its temperature remains at T_C, so entropy increases during Process 1–2. In Process 3–4 heat is transferred from the system while the temperature remains constant at T_H and entropy decreases.

6.6.3 Work and Heat Transfer in an Internally Reversible Process of Water

To further illustrate concepts introduced in this section, Example 6.1 considers water undergoing an internally reversible process while contained in a piston–cylinder assembly.

EXAMPLE 6.1

Evaluating Work and Heat Transfer for an Internally Reversible Process of Water

Water, initially a saturated liquid at 150°C (423.15 K), is contained in a piston–cylinder assembly. The water undergoes a process to the corresponding saturated vapor state, during which the piston moves freely in the cylinder. If the change of state is brought about by heating the water as it undergoes an internally reversible process at constant pressure and temperature, determine the work and heat transfer per unit of mass, each in kJ/kg.

SOLUTION

Known: Water contained in a piston–cylinder assembly undergoes an internally reversible process at 150°C from saturated liquid to saturated vapor.

Find: Determine the work and heat transfer per unit mass.

Schematic and Given Data:

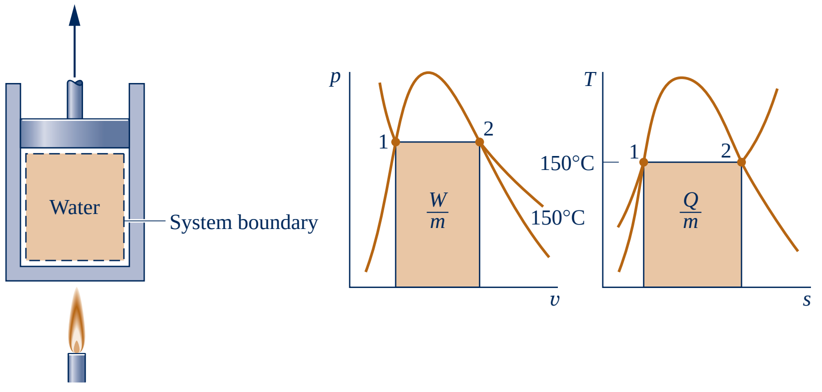


Fig. E6.1

Engineering Model:

1. The water in the piston–cylinder assembly is a closed system.
2. The process is internally reversible.
3. Temperature and pressure are constant during the process.
4. There is no change in kinetic or potential energy between the two end states.

Analysis: At constant pressure the work is

$$\frac{W}{m} = \int_1^2 p\, dv = p(v_2 - v_1)$$

With values from Table A-2 at 150°C

$$\frac{W}{m} = (4.758\ \text{bar})(0.3928 - 1.0905 \times 10^{-3})\left(\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N} \cdot \text{m}}\right|$$

$$= 186.38\ \text{kJ/kg}$$

Since the process is internally reversible and at constant temperature, Eq. 6.23 gives

$$Q = \int_1^2 T\, dS = m \int_1^2 T\, ds$$

or

$$\frac{Q}{m} = T(s_2 - s_1)$$

With values from Table A-2

1 $$\frac{Q}{m} = (423.15\ \text{K})(6.8379 - 1.8418)\ \text{kJ/kg} \cdot \text{K} = 2114.1\ \text{kJ/kg}$$

As shown in the accompanying figure, the work and heat transfer can be represented as areas on p–v and T–s diagrams, respectively.

1 The heat transfer can be evaluated alternatively from an energy balance written on a unit mass basis as

$$u_2 - u_1 = \frac{Q}{m} - \frac{W}{m}$$

Introducing $W/m = p(v_2 - v_1)$ and solving

$$\begin{aligned}\frac{Q}{m} &= (u_2 - u_1) + p(v_2 - v_1) \\ &= (u_2 + pv_2) - (u_1 + pv_1) \\ &= h_2 - h_1\end{aligned}$$

From Table A-2 at 150°C, $h_2 - h_1 = 2114.3$ kJ/kg, which agrees with the value for Q/m obtained in the solution.

Skills Developed

Ability to...

- ❑ evaluate work and heat transfer for an internally reversible process, and represent them as areas on p–v and T–s diagrams, respectively.
- ❑ retrieve entropy data for water.

QuickQUIZ If the initial and final states were saturation states at 100°C (373.15 K), determine the work and heat transfer per unit of mass, each in kJ/kg. Ans. 170 kJ/kg, 2257 kJ/kg.

6.7 Entropy Balance for Closed Systems

In this section, we begin our study of the **entropy balance**. The entropy balance is an expression of the second law that is particularly effective for thermodynamic analysis. The current presentation is limited to closed systems. The entropy balance is extended to control volumes in Sec. 6.9.

entropy balance

Just as mass and energy are accounted for by mass and energy balances, respectively, entropy is accounted for by an entropy balance. In Eq. 5.2, the entropy balance is introduced in words as

$$\left[\begin{array}{c}\textit{change}\text{ in the amount of}\\ \text{entropy contained within}\\ \text{the system during some}\\ \text{time interval}\end{array}\right] = \left[\begin{array}{c}\text{net amount of}\\ \text{entropy }\textit{transferred in}\\ \text{across the system}\\ \text{boundary during the}\\ \text{time interval}\end{array}\right] + \left[\begin{array}{c}\text{amount of }\textit{entropy produced}\\ \textit{within}\text{ the system}\\ \text{during the time interval}\end{array}\right]$$

In symbols, the **closed system entropy balance** takes the form

$$\underbrace{S_2 - S_1}_{\substack{\text{entropy}\\ \text{change}}} = \underbrace{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}_{\substack{\text{entropy}\\ \text{transfer}}} + \underbrace{\sigma}_{\substack{\text{entropy}\\ \text{production}}} \tag{6.24}$$

closed system entropy balance

where subscript b signals that the integrand is evaluated at the system boundary. For the development of Eq. 6.24, see the box.

It is sometimes convenient to use the entropy balance expressed in differential form

$$dS = \left(\frac{\delta Q}{T}\right)_b + \delta\sigma \tag{6.25}$$

Note that the differentials of the nonproperties Q and σ are shown, respectively, as δQ and $\delta\sigma$. When there are no internal irreversibilities, $\delta\sigma$ vanishes and Eq. 6.25 reduces to Eq. 6.2b.

In each of its alternative forms the entropy balance can be regarded as a statement of the second law of thermodynamics. For the analysis of engineering systems, the entropy balance is a more effective means for applying the second law than the Clausius and Kelvin–Planck statements given in Chap. 5.

Developing the Closed System Entropy Balance

The entropy balance for closed systems can be developed using the *Clausius inequality* expressed by Eq. 5.13 (Sec. 5.11) and the defining equation for entropy change, Eq. 6.2a, as follows:

Shown in Fig. 6.6 is a cycle executed by a closed system. The cycle consists of process I, during which internal irreversibilities are present, followed by internally reversible process R. For this cycle, Eq. 5.13 takes the form

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_b + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\ \text{rev}}} = -\sigma \tag{a}$$

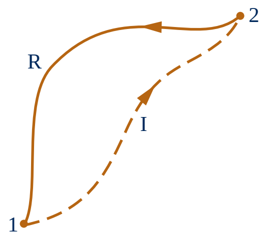


Fig. 6.6 Cycle used to develop the entropy balance.

where the first integral is for process I and the second is for process R. The subscript b in the first integral serves as a reminder that the integrand is evaluated at the system boundary. The subscript is not required in the second integral because temperature is uniform throughout the system at each intermediate state of an internally reversible process. Since no irreversibilities are associated with process R, the term σ_{cycle} of Eq. 5.13, which accounts for the effect of irreversibilities during the cycle, refers only to process I and is shown in Eq. (a) simply as σ.

Applying the definition of entropy change, Eq. 6.2a, we can express the second integral of Eq. (a) as

$$\int_2^1 \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\ \text{rev}}} = S_1 - S_2 \tag{b}$$

With this, Eq. (a) becomes

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_b + (S_1 - S_2) = -\sigma \tag{c}$$

On rearrangement, Eq. (c) gives Eq. 6.24, the closed system entropy balance.

6.7.1 Interpreting the Closed System Entropy Balance

If the end states are fixed, the entropy change on the left side of Eq. 6.24 can be evaluated independently of the details of the process. However, the two terms on the right side depend explicitly on the nature of the process and cannot be determined solely from knowledge of the end states. The first term on the right side of Eq. 6.24 is associated with heat transfer to or from the system during the process. This term can be interpreted as the **entropy transfer accompanying heat transfer**. The direction of entropy transfer is the same as the direction of the heat transfer, and the same sign convention applies as for heat transfer: A positive value means that entropy is transferred into the system, and a negative value means that entropy is transferred out. When there is no heat transfer, there is no entropy transfer.

entropy transfer accompanying heat transfer

The entropy change of a system is not accounted for solely by the entropy transfer, but is due in part to the second term on the right side of Eq. 6.24 denoted by σ. The term σ is positive when internal irreversibilities are present during the process and vanishes when no internal irreversibilities are present. This can be described by saying that **entropy is produced** (or *generated*) within the system by the action of irreversibilities.

entropy production

The second law of thermodynamics can be interpreted as requiring that entropy is produced by irreversibilities and conserved only in the limit as irreversibilities are reduced to zero. Since σ measures the effect of irreversibilities present within the system during a process, its value depends on the nature of the process and not solely on the end states. Entropy production is *not* a property.

When applying the entropy balance to a closed system, it is essential to remember the requirements imposed by the second law on entropy production: The second law requires that entropy *production* be positive, or zero, in value

$$\sigma: \begin{cases} > 0 & \text{irreversibilities present within the system} \\ = 0 & \text{no irreversibilities present within the system} \end{cases} \quad (6.26)$$

The value of the entropy production cannot be negative. In contrast, the *change* in entropy of the system may be positive, negative, or zero:

$$S_2 - S_1: \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \quad (6.27)$$

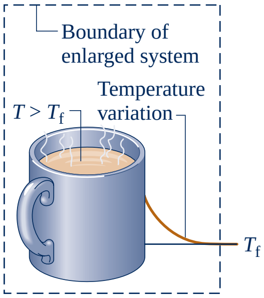
Like other properties, entropy change for a process between two specified states can be determined without knowledge of the details of the process.

Entropy_Bal_Closed _Sys A.24 – All Tabs

6.7.2 Evaluating Entropy Production and Transfer

The objective in many applications of the entropy balance is to evaluate the entropy production term. However, the value of the entropy production for a given process of a system often does not have much significance *by itself*. The significance is normally determined through comparison. For example, the entropy production within a given component might be compared to the entropy production values of the other components included in an overall system formed by these components. By comparing entropy production values, the components where appreciable irreversibilities occur can be identified and rank ordered. This allows attention to be focused on the components that contribute most to inefficient operation of the overall system.

To evaluate the entropy transfer term of the entropy balance requires information regarding both the heat transfer and the temperature on the boundary where the heat transfer occurs. The entropy transfer term is not always subject to direct evaluation, however, because the required information is either unknown or not defined, such as when the system passes through states sufficiently far from equilibrium. In such applications, it may be convenient, therefore, to enlarge the system to include enough of the immediate surroundings that the temperature on the boundary of the *enlarged system* corresponds to the temperature of the surroundings away from the immediate vicinity of the system, T_f. The entropy transfer term is then simply Q/T_f. However, as the irreversibilities present would not be just for the system of interest but for the enlarged system, the entropy production term would account for the effects of internal irreversibilities within the original system and external irreversibilities present within that portion of the surroundings included within the enlarged system.



TAKE NOTE...

On property diagrams, solid lines are used for *internally reversible* processes. A dashed line signals only that a process has occurred between initial and final equilibrium states and does not define a *path* for the process.

6.7.3 Applications of the Closed System Entropy Balance

The following examples illustrate the use of the energy and entropy balances for the analysis of closed systems. Property relations and property diagrams also contribute significantly in developing solutions. Example 6.2 reconsiders the system and end states of Example 6.1 to demonstrate that entropy is produced when internal irreversibilities are present and that the amount of entropy production is not a property. In Example 6.3, the entropy balance is used to determine the minimum theoretical compression work.

EXAMPLE 6.2

Determining Work and Entropy Production for an Irreversible Process of Water

Water initially a saturated liquid at 150°C is contained within a piston–cylinder assembly. The water undergoes a process to the corresponding saturated vapor state, during which the piston moves freely in the cylinder. There is no heat transfer with the surroundings. If the change of state is brought about by the action of a paddle wheel, determine the net work per unit mass, in kJ/kg, and the amount of entropy produced per unit mass, in kJ/kg · K.

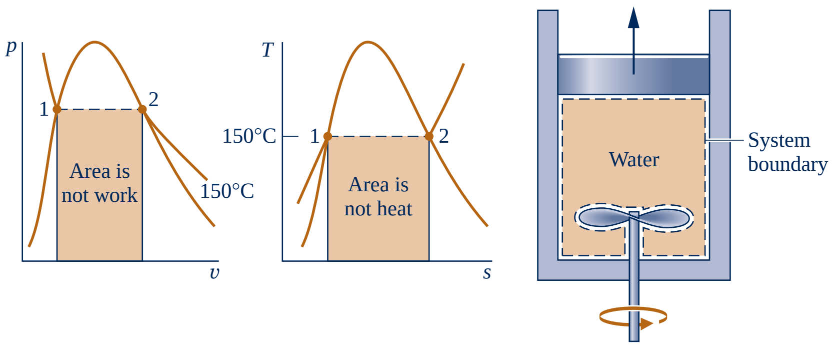
SOLUTION

Known: Water contained in a piston–cylinder assembly undergoes an adiabatic process from saturated liquid to saturated vapor at 150°C. During the process, the piston moves freely, and the water is rapidly stirred by a paddle wheel.

Find: Determine the net work per unit mass and the entropy produced per unit mass.

Schematic and Given Data:

1



Engineering Model:

1. The water in the piston–cylinder assembly is a closed system.
2. There is no heat transfer with the surroundings.
3. The system is at an equilibrium state initially and finally. There is no change in kinetic or potential energy between these two states.

Fig. E6.2

Analysis: As the volume of the system increases during the process, there is an energy transfer by work from the system during the expansion, as well as an energy transfer by work to the system via the paddle wheel. The *net* work can be evaluated from an energy balance, which reduces with assumptions 2 and 3 to

$$\Delta U + \cancel{\Delta \mathrm{KE}}^{0} + \cancel{\Delta \mathrm{PE}}^{0} = \cancel{Q}^{0} - W$$

On a unit mass basis, the energy balance is then

$$\frac{W}{m} = -(u_2 - u_1)$$

With specific internal energy values from Table A-2 at 150°C, $u_1 = 631.68$ kJ/kg, $u_2 = 2559.5$ kJ/kg, we get

$$\frac{W}{m} = -1927.82\,\frac{\text{kJ}}{\text{kg}}$$

The minus sign indicates that the work input by stirring is greater in magnitude than the work done by the water as it expands.

The amount of entropy produced is evaluated by applying the entropy balance Eq. 6.24. Since there is no heat transfer, the term accounting for entropy transfer vanishes

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_{\mathrm{b}}}^{0} + \sigma$$

On a unit mass basis, this becomes on rearrangement

$$\frac{\sigma}{m} = s_2 - s_1$$

With specific entropy values from Table A-2 at 150°C, $s_1 = 1.8418$ kJ/kg · K, $s_2 = 6.8379$ kJ/kg · K, we get

❷
$$\frac{\sigma}{m} = 4.9961 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

---

❶ Although each end state is an equilibrium state at the same pressure and temperature, the pressure and temperature are not necessarily uniform throughout the system at *intervening* states, nor are they necessarily constant in value during the process. Accordingly, there is no well-defined "path" for the process. This is emphasized by the use of dashed lines to represent the process on these p–v and T–s diagrams. The dashed lines indicate only that a process has taken place, and no "area" should be associated with them. In particular, note that the process is adiabatic, so the "area" below the dashed line on the T–s diagram can have no significance as heat transfer. Similarly, the work cannot be associated with an area on the p–v diagram.

❷ The change of state is the same in the present example as in Example 6.1. However, in Example 6.1 the change of state is brought about by heat transfer while the system undergoes an internally reversible process. Accordingly, the value of entropy production for the process of Example 6.1 is zero. Here, fluid friction is present during the process and the entropy production is positive in value. Accordingly, different values of entropy production are obtained for two processes between the *same* end states. This demonstrates that entropy production is not a property.

Skills Developed

Ability to...

- ❑ apply the closed system energy and entropy balances.
- ❑ retrieve property data for water.

QuickQUIZ If the initial and final states were saturation states at 100°C, determine the net work, in kJ/kg, and the amount of entropy produced, in kJ/kg · K. **Ans.** −2087.56 kJ/kg, 6.048 kJ/kg · K.

As an illustration of second law reasoning, minimum theoretical compression work is evaluated in Example 6.3 using the fact that the entropy production term of the entropy balance cannot be negative.

EXAMPLE 6.3

Evaluating Minimum Theoretical Compression Work

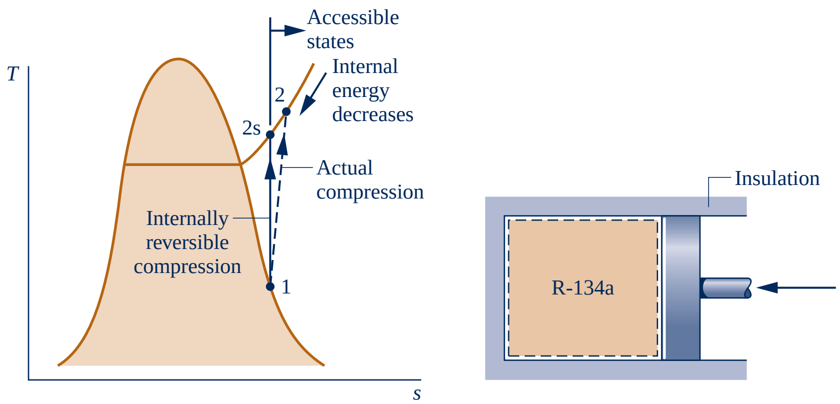
Refrigerant 134a is compressed adiabatically in a piston–cylinder assembly from saturated vapor at 10°F to a final pressure of 120 lbf/in.^{2} Determine the minimum theoretical work input required per unit mass of refrigerant, in Btu/lb.

SOLUTION

Known: Refrigerant 134a is compressed without heat transfer from a specified initial state to a specified final pressure.

Find: Determine the minimum theoretical work input required per unit of mass.

Schematic and Given Data:



Engineering Model:

1. The Refrigerant 134a is a closed system.
2. There is no heat transfer with the surroundings.
3. The initial and final states are equilibrium states. There is no change in kinetic or potential energy between these states.

Fig. E6.3

Analysis: An expression for the work can be obtained from an energy balance. By applying assumptions 2 and 3, we get

$$\Delta U + \cancel{\Delta \mathrm{KE}}^{0} + \cancel{\Delta \mathrm{PE}}^{0} = \cancel{Q}^{0} - W$$

When written on a unit mass basis, the work *input* is then

$$\left(-\frac{W}{m}\right) = u_2 - u_1$$

The specific internal energy u_1 can be obtained from Table A-10E as $u_1 = 94.68$ Btu/lb. Since u_1 is known, the value for the work input depends on the specific internal energy u_2. The minimum work input corresponds to the smallest allowed value for u_2, determined using the second law as follows.

Applying the entropy balance, Eq. 6.24, we get

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$$

where the entropy transfer term is set equal to zero because the process is adiabatic. Thus, the *allowed* final states must satisfy

$$s_2 - s_1 = \frac{\sigma}{m} \geq 0$$

The restriction indicated by the foregoing equation can be interpreted using the accompanying T–s diagram. Since σ cannot be negative, states with $s_2 < s_1$ are not accessible adiabatically. When irreversibilities are present during the compression, entropy is produced, so $s_2 > s_1$. The state labeled 2s on the diagram would be attained in the limit as irreversibilities are reduced to zero. This state corresponds to an *isentropic* compression.

By inspection of Table A-12E, we see that when pressure is fixed, the specific internal energy decreases as specific entropy decreases. Thus, the smallest allowed value for u_2 corresponds to state 2s. Interpolating in Table A-12E at 120 lb/in.^{2}, with $s_{2s} = s_1 = 0.2214$ Btu/lb · °R, we find that $u_{2s} = 107.46$ Btu/lb, which corresponds to a temperature at state 2s of about 101°F. Finally, the *minimum* work input is

1 $$\left(-\frac{W}{m}\right)_{\min} = u_{2s} - u_1 = 107.46 - 94.68 = 12.78 \text{ Btu/lb}$$

1 The effect of irreversibilities exacts a penalty on the work input required: A greater work input is needed for the actual adiabatic compression process than for an internally reversible adiabatic process between the same initial state and the same final pressure. See the Quick Quiz to follow.

Skills Developed

Ability to...

- apply the closed system energy and entropy balances.
- retrieve property data for Refrigerant 134a.

QuickQUIZ If the refrigerant were compressed adiabatically to a final state where $p_2 = 120$ lbf/in.^{2}, $T_2 = 120$°F, determine the work input, in Btu/lb, and the amount of entropy produced, in Btu/lb · °R. **Ans.** 17.16 Btu/lb, 0.0087 Btu/lb · °R.

6.7.4 Closed System Entropy Rate Balance

If the temperature T_b is constant, Eq. 6.24 reads

$$S_2 - S_1 = \frac{Q}{T_b} + \sigma$$

where Q/T_b represents the *amount* of entropy transferred through the portion of the boundary at temperature T_b. Similarly, the quantity $\dot{Q}/T_j$ represents the *time rate* of entropy transfer through the portion of the boundary whose instantaneous temperature is T_j. This quantity appears in the closed system entropy rate balance considered next.

On a time rate basis, the **closed system entropy rate balance** is

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma} \tag{6.28}$$

closed system entropy rate balance

where dS/dt is the time rate of change of entropy of the system. The term $\dot{Q}_j/T_j$ represents the time rate of entropy transfer through the portion of the boundary whose instantaneous temperature is T_j. The term $\dot{\sigma}$ accounts for the time rate of entropy production due to irreversibilities within the system.

To pinpoint the relative significance of the internal and external irreversibilities, Example 6.4 illustrates the application of the entropy rate balance to a system and to an enlarged system consisting of the system and a portion of its immediate surroundings.

EXAMPLE 6.4

Pinpointing Irreversibilities

Referring to Example 2.4, evaluate the rate of entropy production $\dot{\sigma}$, in kW/K, for **(a)** the gearbox as the system and **(b)** an enlarged system consisting of the gearbox and enough of its surroundings that heat transfer occurs at the temperature of the surroundings away from the immediate vicinity of the gearbox, $T_f = 293$ K (20°C).

SOLUTION

Known: A gearbox operates at steady state with known values for the power input through the high-speed shaft, power output through the low-speed shaft, and heat transfer rate. The temperature on the outer surface of the gearbox and the temperature of the surroundings away from the gearbox are also known.

Find: Evaluate the entropy production rate $\dot{\sigma}$ for each of the two specified systems shown in the schematic.

Schematic and Given Data:

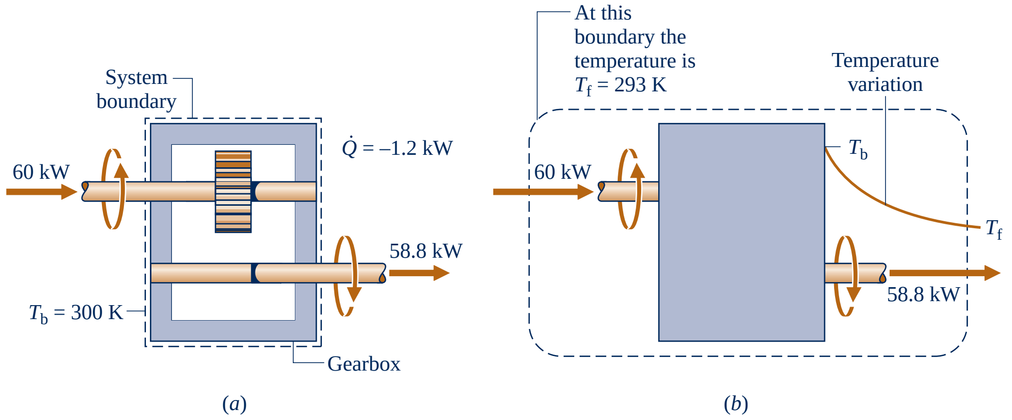


Fig. E6.4

Engineering Model:

1. In part (a), the gearbox is taken as a closed system operating at steady state, as shown on the accompanying sketch labeled with data from Example 2.4.
2. In part (b) the gearbox and a portion of its surroundings are taken as a closed system, as shown on the accompanying sketch labeled with data from Example 2.4.
3. The temperature of the outer surface of the gearbox and the temperature of the surroundings do not vary.

Analysis:

(a) To obtain an expression for the entropy production rate, begin with the entropy balance for a closed system on a time rate basis: Eq. 6.28. Since heat transfer takes place only at temperature T_b, the entropy rate balance reduces at steady state to

$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma}$$

Solving

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b}$$

Introducing the known values for the heat transfer rate $\dot{Q}$ and the surface temperature T_b

$$\dot{\sigma} = -\frac{(-1.2 \text{ kW})}{(300 \text{ K})} = 4 \times 10^{-3} \text{ kW/K}$$

(b) Since heat transfer takes place at temperature T_f for the enlarged system, the entropy rate balance reduces at steady state to

$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_f} + \dot{\sigma}$$

Solving

$$\dot{\sigma} = -\frac{\dot{Q}}{T_f}$$

Introducing the known values for the heat transfer rate $\dot{Q}$ and the temperature T_f

1 $$\dot{\sigma} = -\frac{(-1.2 \text{ kW})}{(293 \text{ K})} = 4.1 \times 10^{-3} \text{ kW/K}$$

1 The value of the entropy production rate calculated in part (a) gauges the significance of irreversibilities associated with friction and heat transfer *within* the gearbox. In part (b), an additional source of irreversibility is included in the enlarged system, namely the irreversibility associated with the heat transfer from the outer surface of the gearbox at T_b to the surroundings at T_f. In this case, the irreversibilities within the gearbox are dominant, accounting for about 98% of the total rate of entropy production.

Skills Developed

Ability to...

- ❑ apply the closed system entropy rate balance.
- ❑ develop an engineering model.

QuickQUIZ If the power delivered were 59.32 kW, evaluate the outer surface temperature, in K, and the rate of entropy production, in kW/K, for the gearbox as the system, keeping input power, h, and A from Example 2.4 the same. Ans. 297 K, 2.3×10^{-3} kW/K.

6.8 Directionality of Processes

Our study of the second law of thermodynamics began in Sec. 5.1 with a discussion of the *directionality* of processes. In this section we consider two related aspects for which there are significant applications: the increase in entropy principle and a statistical interpretation of entropy.

6.8.1 Increase of Entropy Principle

In the present discussion, we use the closed system energy and entropy balances to introduce the increase of entropy principle. Discussion centers on an enlarged system consisting of a system and that portion of the surroundings affected by the system as it undergoes a process. Since all energy and mass transfers taking place are included within the boundary of the enlarged system, the enlarged system is an *isolated* system.

An energy balance for the isolated system reduces to

$$\Delta E\big]_{\text{isol}} = 0 \tag{6.29a}$$

because no energy transfers take place across its boundary. Thus, the energy of the isolated system remains constant. Since energy is an extensive property, its value for the isolated system is the sum of its values for the system and surroundings, respectively, so Eq. 6.29a can be written as

$$\Delta E\big]_{\text{system}} + \Delta E\big]_{\text{surr}} = 0 \tag{6.29b}$$

In either of these forms, the conservation of energy principle places a constraint on the processes that can occur. For a process to take place, it is necessary for the energy of the system plus the surroundings to remain constant. However, not all processes for which this constraint is satisfied can actually occur. Processes also must satisfy the second law, as discussed next.

An entropy balance for the isolated system reduces to

$$\Delta S\big]_{\text{isol}} = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{b}}}^{\,0} + \sigma_{\text{isol}}$$

or

$$\Delta S\big]_{\text{isol}} = \sigma_{\text{isol}} \tag{6.30a}$$

where σ_{isol} is the total amount of entropy produced within the system and its surroundings. Since entropy is produced in all actual processes, the only processes that can occur are those for which the entropy of the isolated system increases. This is known as the **increase of entropy principle**. The increase of entropy principle is sometimes considered an alternative statement of the second law.

increase of entropy principle

Since entropy is an extensive property, its value for the isolated system is the sum of its values for the system and surroundings, respectively, so Eq. 6.30a can be written as

$$\Delta S\big]_{\text{system}} + \Delta S\big]_{\text{surr}} = \sigma_{\text{isol}} \tag{6.30b}$$

Notice that this equation does not require the entropy change to be positive for both the system and surroundings but only that the *sum* of the changes is positive. In either of these forms, the increase of entropy principle dictates the direction in which any process can proceed: Processes occur only in such a direction that the total entropy of the system *plus* surroundings increases.

We observed previously the tendency of systems left to themselves to undergo processes until a condition of equilibrium is attained (Sec. 5.1). The increase of entropy principle suggests that the entropy of an isolated system increases as the state of equilibrium is approached, with the equilibrium state being attained when the entropy reaches a maximum. This interpretation is considered again in Sec. 14.1, which deals with equilibrium criteria.

Example 6.5 illustrates the increase of entropy principle.

EXAMPLE 6.5

Quenching a Hot Metal Bar

A 0.8-lb metal bar initially at 1900°R is removed from an oven and quenched by immersing it in a closed tank containing 20 lb of water initially at 530°R. Each substance can be modeled as incompressible. An appropriate constant specific heat value for the water is c_w = 1.0 Btu/lb · °R, and an appropriate value for the metal is c_m = 0.1 Btu/lb · °R, Heat transfer from the tank contents can be neglected. Determine **(a)** the final equilibrium temperature of the metal bar and the water, in °R, and **(b)** the amount of entropy produced, in Btu/°R.

SOLUTION

Known: A hot metal bar is quenched by immersing it in a closed tank containing water.

Find: Determine the final equilibrium temperature of the metal bar and the water, and the amount of entropy produced.

Schematic and Given Data:

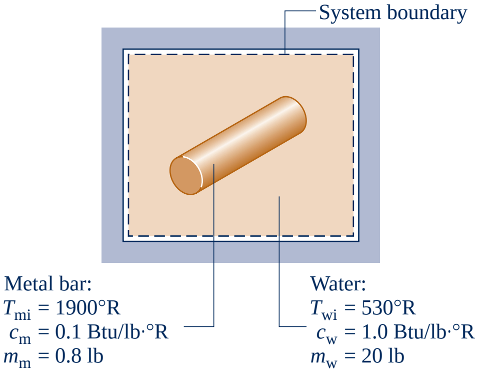


Fig. E6.5

Engineering Model:

1. The metal bar and the water within the tank form a system, as shown on the accompanying sketch.
2. There is no energy transfer by heat or work: The system is isolated.
3. There is no change in kinetic or potential energy.
4. The water and metal bar are each modeled as incompressible with known specific heats.

Analysis:

(a) The final equilibrium temperature can be evaluated from an energy balance for the isolated system

$$\cancel{\Delta \text{KE}}^{0} + \cancel{\Delta \text{PE}}^{0} + \Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$$

where the indicated terms vanish by assumptions 2 and 3. Since internal energy is an extensive property, its value for the isolated system is the sum of the values for the water and metal, respectively. Thus, the energy balance becomes

$$\Delta U]_{\text{water}} + \Delta U]_{\text{metal}} = 0$$

Using Eq. 3.20a to evaluate the internal energy changes of the water and metal in terms of the constant specific heats

$$m_w c_w (T_f - T_{wi}) + m_m c_m (T_f - T_{mi}) = 0$$

where T_f is the final equilibrium temperature, and T_{wi} and T_{mi} are the initial temperatures of the water and metal, respectively. Solving for T_f and inserting values

$$\begin{aligned} T_f &= \frac{m_w (c_w/c_m) T_{wi} + m_m T_{mi}}{m_w (c_w/c_m) + m_m} \\ &= \frac{(20\text{ lb})(10)(530°\text{R}) + (0.8\text{ lb})(1900°\text{R})}{(20\text{ lb})(10) + (0.8\text{ lb})} \\ &= 535°\text{R} \end{aligned}$$

(b) The amount of entropy production can be evaluated from an entropy balance. Since no heat transfer occurs between the isolated system and its surroundings, there is no accompanying entropy transfer, and an entropy balance for the isolated system reduces to

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$$

Entropy is an extensive property, so its value for the isolated system is the sum of its values for the water and the metal, respectively, and the entropy balance becomes

$$\Delta S]_{\text{water}} + \Delta S]_{\text{metal}} = \sigma$$

Evaluating the entropy changes using Eq. 6.13 for incompressible substances, the foregoing equation can be written as

$$\sigma = m_w c_w \ln \frac{T_f}{T_{wi}} + m_m c_m \ln \frac{T_f}{T_{mi}}$$

Inserting values

$$\sigma = (20\text{ lb})\left(1.0\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{535}{530} + (0.8\text{ lb})\left(0.1\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{535}{1900}$$

1 2
$$= \left(0.1878\frac{\text{Btu}}{{}^\circ\text{R}}\right) + \left(-0.1014\frac{\text{Btu}}{{}^\circ\text{R}}\right) = 0.0864\frac{\text{Btu}}{{}^\circ\text{R}}$$

1 The metal bar experiences a *decrease* in entropy. The entropy of the water *increases*. In accord with the increase of entropy principle, the entropy of the isolated system *increases*.

2 The value of σ is sensitive to roundoff in the value of T_f.

Skills Developed

Ability to...

- ❑ apply the closed system energy and entropy balances.
- ❑ apply the incompressible substance model.

QuickQUIZ If the mass of the metal bar were 0.45 lb, determine the final equilibrium temperature, in °R, and the amount of entropy produced, in Btu/°R, keeping all other given data the same. **Ans.** 533°R, 0.0557 Btu/°R.

6.8.2 Statistical Interpretation of Entropy

Building on the increase of entropy principle, in this section we introduce an interpretation of entropy from a microscopic perspective based on *probability*.

In *statistical thermodynamics*, entropy is associated with the notion of microscopic *disorder*. From previous considerations we know that in a spontaneous process of an isolated system, the system moves toward equilibrium and the entropy increases. From the microscopic viewpoint, this is equivalent to saying that as an isolated system moves toward equilibrium our knowledge of the condition of individual particles making up the system decreases, which corresponds to an increase in microscopic disorder and a related increase in entropy.

We use an elementary thought experiment to bring out some basic ideas needed to understand this view of entropy. Actual microscopic analysis of systems is more complicated than the discussion given here, but the essential concepts are the same.

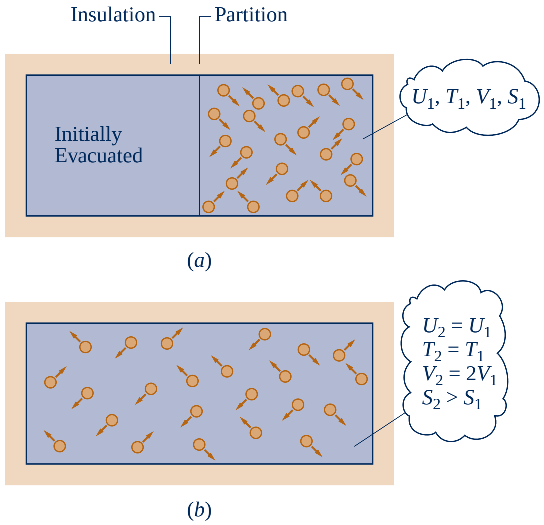


Fig. 6.7 **N molecules a box.**

Consider N molecules initially contained in one half of the box shown in Fig. 6.7*a*. The entire box is considered an isolated system. We assume that the ideal gas model applies. In the initial condition, the gas appears to be at equilibrium in terms of temperature, pressure, and other properties. But, on the microscopic level the molecules are moving about randomly. We do know *for sure*, though, that initially all molecules are on the right side of the vessel.

Suppose we remove the partition and wait until equilibrium is reached again, as in Fig. 6.7*b*. Because the system is isolated, the internal energy U does not change: $U_2 = U_1$. Further, because the internal energy of an ideal gas depends on temperature alone, the temperature is unchanged: $T_2 = T_1$. Still, at the final state a given molecule has twice the volume in which to move: $V_2 = 2V_1$. Just like a coin toss, the probability that the molecule is on one side or the other is now ½, which is the same as the volume ratio V_1/V_2. In the final condition, we have *less knowledge* about where each molecule is than we did originally.

We can evaluate the change in entropy for the process of Fig. 6.7 by applying Eq. 6.17, expressed in terms of volumes and on a molar basis. The entropy change for

Horizons

“Breaking” the Second Law Has Implications for Nanotechnology

Some 135 years ago, renowned nineteenth-century physicist J. C. Maxwell wrote, “. . . the second law is . . . a statistical . . . truth, for it depends on the fact that the bodies we deal with consist of millions of molecules . . . [Still] the second law is continually being violated . . . in any sufficiently small group of molecules belonging to a real body.” Although Maxwell’s view was bolstered by theorists over the years, experimental confirmation proved elusive. Then, in 2002, experimenters reported they had demonstrated violations of the second law: at the micron scale over time intervals of up to 2 seconds, entropy was consumed, not produced [see Phys. Rev. Lett. **89**, 050601 (2002)].

While few were surprised that experimental confirmation had at last been achieved, some *were* surprised by implications of the research for the twenty-first-century field of nanotechnology: The experimental results suggest inherent limitations on nanomachines. These tiny devices—only a few molecules in size—may not behave simply as miniaturized versions of their larger counterparts; and the smaller the device, the more likely its motion and operation could be disrupted unpredictably. Occasionally and uncontrollably, nanomachines may not perform as designed, perhaps even capriciously running backward. Still, designers of these machines will applaud the experimental results if they lead to deeper understanding of behavior at the nanoscale.

the constant-temperature process is

$$(S_2 - S_1)/n = \bar{R}\ln(V_2/V_1) \tag{6.31}$$

where n is the amount of substance on a molar basis (Eq. 1.8). Next, we consider how the entropy change would be evaluated from a microscopic point of view.

microstates
thermodynamic probability

Through more complete molecular modeling and statistical analysis, the total number of positions and velocities − **microstates** − available to a single molecule can be calculated. This total is called the **thermodynamic probability**, w. For a system of N molecules, the thermodynamic probability is w^{N}. In statistical thermodynamics, entropy is considered to be proportional to $\ln(w)^{\mathrm{N}}$. That is, $S \propto \mathrm{N}\ln(w)$. This gives the **Boltzmann relation**

Boltzmann relation

$$S/N = \mathrm{k}\ln w \tag{6.32}$$

where the proportionality factor, k, is called *Boltzmann’s constant.*

Applying Eq. 6.32 to the process of Fig. 6.7, we get

$$\begin{aligned}(S_2 - S_1)/N &= \mathrm{k}\ln(w_2) - \mathrm{k}\ln(w_1)\\ &= \mathrm{k}\ln(w_2/w_1)\end{aligned} \tag{6.33}$$

Comparing Eqs. 6.31 and 6.33, the expressions for entropy change coincide when $\mathrm{k} = n\bar{R}/N$ and $w_2/w_1 = V_2/V_1$. The first of these expressions allows Boltzmann’s constant to be evaluated, giving $\mathrm{k} = 1.3806 \times 10^{-23}$ J/K. Also, since $V_2 > V_1$ and $w_2 > w_1$, Eqs. 6.31 and 6.33 each predict an increase of entropy owing to entropy production during the irreversible adiabatic expansion in this example.

disorder

From Eq. 6.33, we see that any process that increases the number of possible microstates of a system increases its entropy and conversely. Hence, for an *isolated* system, processes occur only in such a direction that the number of microstates available to the system increases, resulting in our having less knowledge about the condition of individual particles. Because of this concept of decreased knowledge, entropy reflects the microscopic **disorder** of the system. We can then say that the only processes an isolated system can undergo are those that increase the disorder of the system. This interpretation is consistent with the idea of directionality of processes discussed previously.

The notion of entropy as a measure of disorder is sometimes used in fields other than thermodynamics. The concept is employed in information theory, statistics, biology, and even in some economic and social modeling. In these applications, the term entropy is used as a measure of disorder without the physical aspects of the thought experiment used here necessarily being implied.

BIOCONNECTIONS Do living things violate the second law of thermodynamics because they seem to create order from disorder? Living things are not isolated systems as considered in the previous discussion of entropy and disorder. Living things interact with their surroundings and are influenced by their surroundings. For instance, plants grow into highly ordered cellular structures synthesized from atoms and molecules originating in the earth and its atmosphere. Through interactions with their surroundings, plants exist in highly organized states and are able to produce within themselves even more organized, lower entropy states. In keeping with the second law, states of lower entropy can be realized within a system as long as the *total* entropy of the system and its surroundings increases. The self-organizing tendency of living things is widely observed and fully in accord with the second law.

6.9 Entropy Rate Balance for Control Volumes

Thus far the discussion of the entropy balance concept has been restricted to the case of closed systems. In the present section the entropy balance is extended to control volumes.

Like mass and energy, entropy is an extensive property, so it too can be transferred into or out of a control volume by streams of matter. Since this is the principal difference between the closed system and control volume forms, the **control volume entropy rate balance** can be obtained by modifying Eq. 6.28 to account for these entropy transfers. The result is

$$\underbrace{\frac{dS_{\text{cv}}}{dt}}_{\substack{\text{rate of}\\\text{entropy}\\\text{change}}} = \underbrace{\sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e}_{\substack{\text{rates of}\\\text{entropy}\\\text{transfer}}} + \underbrace{\dot{\sigma}_{\text{cv}}}_{\substack{\text{rate of}\\\text{entropy}\\\text{production}}} \tag{6.34}$$

control volume entropy rate balance

where dS_{cv}/dt represents the time rate of change of entropy within the control volume. The terms $\dot{m}_i s_i$ and $\dot{m}_e s_e$ account, respectively, for rates of **entropy transfer accompanying mass flow** into and out of the control volume. The term $\dot{Q}_j$ represents the time rate of heat transfer at the location on the boundary where the instantaneous temperature is T_j. The ratio $\dot{Q}_j/T_j$ accounts for the accompanying rate of entropy *transfer*. The term $\dot{\sigma}_{\text{cv}}$ denotes the time rate of entropy *production* due to irreversibilities *within* the control volume.

entropy transfer accompanying mass flow

Entropy_Rate_Bal_CV
A.25 – Tabs a & b

Integral Form of the Entropy Rate Balance

As for the cases of the control volume mass and energy rate balances, the entropy rate balance can be expressed in terms of local properties to obtain forms that are more generally applicable. Thus, the term $S_{\text{cv}}(t)$, representing the total entropy associated with the control volume at time t, can be written as a volume integral

$$S_{\text{cv}}(t) = \int_V \rho s \, dV$$

where ρ and s denote, respectively, the local density and specific entropy. The rate of entropy transfer accompanying heat transfer can be expressed more generally as an integral over the surface of the control volume

$$\begin{bmatrix}\text{time rate of entropy}\\\text{transfer accompanying}\\\text{heat transfer}\end{bmatrix} = \int_A \left(\frac{\dot{q}}{T}\right)_{\text{b}} dA$$

where $\dot{q}$ is the *heat flux,* the time rate of heat transfer per unit of surface area, through the location on the boundary where the instantaneous temperature is T. The subscript "b" is added as a reminder that the integrand is evaluated on the boundary of the control volume. In addition, the terms accounting for entropy transfer accompanying mass flow can be expressed as integrals over the inlet and exit flow areas, resulting in the following form of the entropy rate balance

$$\frac{d}{dt}\int_V \rho s\, dV = \int_A \left(\frac{\dot{q}}{T}\right)_b dA + \sum_i \left(\int_A s\rho V_n\, dA\right)_i - \sum_e \left(\int_A s\rho V_n\, dA\right)_e + \dot{\sigma}_{cv} \qquad \textbf{(6.35)}$$

where V_n denotes the normal component in the direction of flow of the velocity relative to the flow area. In some cases, it is also convenient to express the entropy production rate as a volume integral of the local volumetric rate of entropy production within the control volume. The study of Eq. 6.35 brings out the assumptions underlying Eq. 6.34. Finally, note that for a closed system the sums accounting for entropy transfer at inlets and exits drop out, and Eq. 6.35 reduces to give a more general form of Eq. 6.28.

6.10 Rate Balances for Control Volumes at Steady State

Since a great many engineering analyses involve control volumes at steady state, it is instructive to list steady-state forms of the balances developed for mass, energy, and entropy. At steady state, the conservation of mass principle takes the form

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e \qquad \textbf{(4.6)}$$

The energy rate balance at steady state is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \qquad \textbf{(4.18)}$$

Finally, the **steady-state form of the entropy rate balance** is obtained by reducing Eq. 6.34 to give

steady-state entropy rate balance

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv} \qquad \textbf{(6.36)}$$

These equations often must be solved simultaneously, together with appropriate property relations.

Mass and energy are conserved quantities, but entropy is not conserved. Equation 4.6 indicates that at steady state the total rate of mass flow into the control volume equals the total rate of mass flow out of the control volume. Similarly, Eq. 4.18 indicates that the total rate of energy transfer into the control volume equals the total rate of energy transfer out of the control volume. However, Eq. 6.36 requires that the rate at which entropy is transferred out must *exceed* the rate at which entropy enters, the difference being the rate of entropy production within the control volume owing to irreversibilities.

6.10.1 One-Inlet, One-Exit Control Volumes at Steady State

Since many applications involve one-inlet, one-exit control volumes at steady state, let us also list the form of the entropy rate balance for this important case. Thus, Eq. 6.36 reduces to read

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \tag{6.37}$$

Entropy_Rate_Bal_CV
A.25 – Tab c

Or, on dividing by the mass flow rate $\dot{m}$ and rearranging

$$s_2 - s_1 = \frac{1}{\dot{m}}\left(\sum_j \frac{\dot{Q}_j}{T_j}\right) + \frac{\dot{\sigma}_{cv}}{\dot{m}} \tag{6.38}$$

The two terms on the right side of Eq. 6.38 denote, respectively, the rate of entropy transfer accompanying heat transfer and the rate of entropy production within the control volume, each *per unit of mass flowing through the control volume*. From Eq. 6.38 it can be concluded that the entropy of a unit of mass passing from inlet to exit can increase, decrease, or remain the same. Furthermore, because the value of the second term on the right can never be negative, a decrease in the specific entropy from inlet to exit can be realized only when more entropy is transferred out of the control volume accompanying heat transfer than is produced by irreversibilities within the control volume. When the value of this entropy transfer term is positive, the specific entropy at the exit is greater than the specific entropy at the inlet whether internal irreversibilities are present or not. In the special case where there is no entropy transfer accompanying heat transfer, Eq. 6.38 reduces to

$$s_2 - s_1 = \frac{\dot{\sigma}_{cv}}{\dot{m}} \tag{6.39}$$

Accordingly, when irreversibilities are present within the control volume, the entropy of a unit of mass increases as it passes from inlet to exit. In the limiting case in which no irreversibilities are present, the unit mass passes through the control volume with no change in its entropy—that is, isentropically.

6.10.2 Applications of the Rate Balances to Control Volumes at Steady State

The following examples illustrate the use of the mass, energy, and entropy balances for the analysis of control volumes at steady state. Carefully note that property relations and property diagrams also play important roles in arriving at solutions.

Turbine
A.19 – Tab d

In Example 6.6, we evaluate the rate of entropy production within a turbine operating at steady state when there is heat transfer from the turbine.

EXAMPLE 6.6

Determining Entropy Production in a Steam Turbine

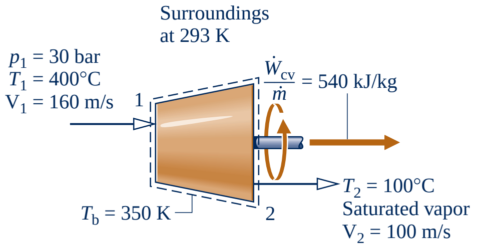
Steam enters a turbine with a pressure of 30 bar, a temperature of 400°C, and a velocity of 160 m/s. Saturated vapor at 100°C exits with a velocity of 100 m/s. At steady state, the turbine develops work equal to 540 kJ per kg of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 350 K. Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in kJ/kg · K. Neglect the change in potential energy between inlet and exit.

SOLUTION

Known: Steam expands through a turbine at steady state for which data are provided.

Find: Determine the rate of entropy production per kg of steam flowing.

Schematic and Given Data:



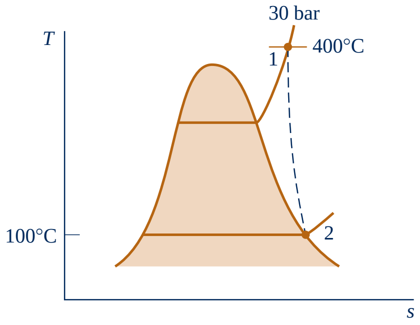


Fig. E6.6

Engineering Model:

1. The control volume shown on the accompanying sketch is at steady state.
2. Heat transfer from the turbine to the surroundings occurs at a specified average outer surface temperature.
3. The change in potential energy between inlet and exit can be neglected.

Analysis: To determine the entropy production per unit mass flowing through the turbine, begin with mass and entropy rate balances for the one-inlet, one-exit control volume at steady state:

$$\dot{m}_1 = \dot{m}_2$$

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{\sigma}_{cv}$$

Since heat transfer occurs only at T_b = 350 K, the first term on the right side of the entropy rate balance reduces to $\dot{Q}_{cv}/T_b$. Combining the mass and entropy rate balances

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

where $\dot{m}$ is the mass flow rate. Solving for $\dot{\sigma}_{cv}/\dot{m}$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_b} + (s_2 - s_1)$$

The heat transfer rate, $\dot{Q}_{cv}/\dot{m}$, required by this expression is evaluated next.

Reduction of the mass and energy rate balances results in

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

where the potential energy change from inlet to exit is dropped by assumption 3. From Table A-4 at 30 bar, 400°C, h_1 = 3230.9 kJ/kg, and from Table A-2, $h_2 = h_g(100°C)$ = 2676.1 kJ/kg. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = 540\frac{\text{kJ}}{\text{kg}} + (2676.1 - 3230.9)\left(\frac{\text{kJ}}{\text{kg}}\right) + \left[\frac{(100)^2 - (160)^2}{2}\right]\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 540 - 554.8 - 7.8 = -22.6\text{ kJ/kg}$$

From Table A-2, s_2 = 7.3549 kJ/kg · K, and from Table A-4, s_1 = 6.9212 kJ/kg · K. Inserting values into the expression for entropy production

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{(-22.6\text{ kJ/kg})}{350\text{ K}} + (7.3549 - 6.9212)\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)$$

$$= 0.0646 + 0.4337 = 0.498\text{ kJ/kg}\cdot\text{K}$$

Skills Developed

Ability to...

- apply the control volume mass, energy and entropy rate balances.
- retrieve property data for water.

QuickQUIZ If the boundary were located to include the turbine and a portion of the immediate surroundings so heat transfer occurs at the temperature of the surroundings, 293 K, determine the rate at which entropy is produced within the enlarged control volume, in kJ/K per kg of steam flowing, keeping all other given data the same. **Ans.** 0.511 kJ/kg · K.

In Example 6.7, the mass, energy, and entropy rate balances are used to evaluate a performance claim for a device producing hot and cold streams of air from a single stream of air at an intermediate temperature.

Heat_Exchanger
A.22 – Tab d

EXAMPLE 6.7

Evaluating a Performance Claim

An inventor claims to have developed a device requiring no energy transfer by work, $\dot{W}_{cv}$, or heat transfer, yet able to produce hot and cold streams of air from a single stream of air at an intermediate temperature. The inventor provides steady-state test data indicating that when air enters at a temperature of 70°F and a pressure of 5.1 atm, separate streams of air exit at temperatures of 0 and 175°F, respectively, and each at a pressure of 1 atm. Sixty percent of the mass entering the device exits at the lower temperature. Evaluate the inventor's claim, employing the ideal gas model for air and ignoring changes in the kinetic and potential energies of the streams from inlet to exit.

SOLUTION

Known: Data are provided for a device that at steady state produces hot and cold streams of air from a single stream of air at an intermediate temperature without energy transfers by work or heat.

Find: Evaluate whether the device can operate as claimed.

Schematic and Given Data:

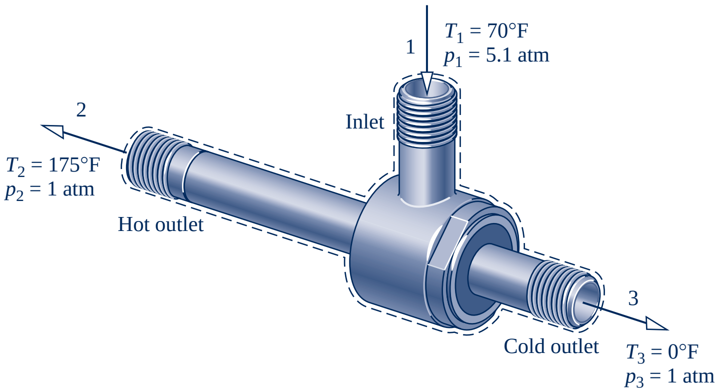


Fig. E6.7

Engineering Model:

1. The control volume shown on the accompanying sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.
3. Changes in the kinetic and potential energies from inlet to exit can be ignored.
4. (1) The air is modeled as an ideal gas with constant $c_p = 0.24$ Btu/lb · °R.

Analysis: For the device to operate as claimed, the conservation of mass and energy principles must be satisfied. The second law of thermodynamics also must be satisfied; and in particular the rate of entropy production cannot be negative. Accordingly, the mass, energy and entropy rate balances are considered in turn.

With assumptions 1–3, the mass and energy rate balances reduce, respectively, to

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$
$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Since $\dot{m}_3 = 0.6\dot{m}_1$, it follows from the mass rate balance that $\dot{m}_3 = 0.4\dot{m}_1$. By combining the mass and energy rate balances and evaluating changes in specific enthalpy using constant c_p, the energy rate balance is also satisfied. That is

$$\begin{aligned} 0 &= (\dot{m}_2 + \dot{m}_3)h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \\ &= \dot{m}_2(h_1 - h_2) + \dot{m}_3(h_1 - h_3) \\ &= 0.4\dot{m}_1[c_p(T_1 - T_2)] + 0.6\dot{m}_1[c_p(T_1 - T_3)] \\ &= 0.4(T_1 - T_2) + 0.6(T_1 - T_3) \\ &= 0.4(-105) + 0.6(70) \\ &= 0 \end{aligned}$$

(2)

Accordingly, with the given data the conservation of mass and energy principles are satisfied.

Since no significant heat transfer occurs, the entropy rate balance at steady state reads

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{\text{cv}}$$

Combining the mass and entropy rate balances

$$\begin{aligned} 0 &= (\dot{m}_2 + \dot{m}_3)s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{\text{cv}} \\ &= \dot{m}_2(s_1 - s_2) + \dot{m}_3(s_1 - s_3) + \dot{\sigma}_{\text{cv}} \\ &= 0.4\dot{m}_1(s_1 - s_2) + 0.6\dot{m}_1(s_1 - s_3) + \dot{\sigma}_{\text{cv}} \end{aligned}$$

Solving for $\dot{\sigma}_{\text{cv}}/\dot{m}_1$ and using Eq. 6.22 to evaluate changes in specific entropy

$$\frac{\dot{\sigma}_{\text{cv}}}{\dot{m}_1} = 0.4\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right] + 0.6\left[c_p \ln\frac{T_3}{T_1} - R\ln\frac{p_3}{p_1}\right]$$

3
$$= 0.4\left[\left(0.24\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{635}{530} - \left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{1}{5.1}\right]$$

$$+ 0.6\left[\left(0.24\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{460}{530} - \left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{1}{5.1}\right]$$

4
$$= 0.1086\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Thus, the second law of thermodynamics is also satisfied.

5 On the basis of this evaluation, the inventor's claim does not violate principles of thermodynamics.

---

1 Since the specific heat c_p of air varies little over the temperature interval from 0 to 175°F, c_p can be taken as constant. From Table A-20E, $c_p = 0.24$ Btu/lb · °R.

2 Since temperature *differences* are involved in this calculation, the temperatures can be either in °R or °F.

3 In this calculation involving temperature *ratios*, the temperatures are in °R. Temperatures in °F should not be used.

4 If the value of the rate of entropy production had been negative or zero, the claim would be rejected. A negative value is impossible by the second law and a zero value would indicate operation without irreversibilities.

5 Such devices *do* exist. They are known as *vortex tubes* and are used in industry for *spot cooling*.

Skills Developed

Ability to...

- ❑ apply the control volume mass, energy and entropy rate balances.
- ❑ apply the ideal gas model with constant c_p.

QuickQUIZ If the inventor would claim that the hot and cold streams exit the device at 5.1 atm, evaluate the revised claim, keeping all other given data the same. Ans. Claim invalid.

A Compressor A.20 – Tab d

Throttling_Dev A.23 – Tab d

In Example 6.8, we evaluate and compare the rates of entropy production for three components of a heat pump system. Heat pumps are considered in detail in Chap. 10.

EXAMPLE 6.8

Determining Entropy Production in Heat Pump Components

Components of a heat pump for supplying heated air to a dwelling are shown in the schematic below. At steady state, Refrigerant 22 enters the compressor at −5°C, 3.5 bar and is compressed adiabatically to 75°C, 14 bar. From the compressor, the refrigerant passes through the condenser, where it condenses to liquid at 28°C, 14 bar. The refrigerant then expands through a throttling valve to 3.5 bar. The states of the refrigerant are shown on the accompanying T–s diagram. Return air from the dwelling enters the condenser at 20°C, 1 bar with a volumetric flow rate of 0.42 m^3/s and exits at 50°C with a negligible change in pressure. Using the ideal gas model for the air and neglecting kinetic and potential energy effects, **(a)** determine the rates of entropy production, in kW/K, for control volumes enclosing the condenser, compressor, and expansion valve, respectively. **(b)** Discuss the sources of irreversibility in the components considered in part (a).

SOLUTION

Known: Refrigerant 22 is compressed adiabatically, condensed by heat transfer to air passing through a heat exchanger, and then expanded through a throttling valve. Steady-state operating data are known.

Find: Determine the entropy production rates for control volumes enclosing the condenser, compressor, and expansion valve, respectively, and discuss the sources of irreversibility in these components.

Schematic and Given Data:

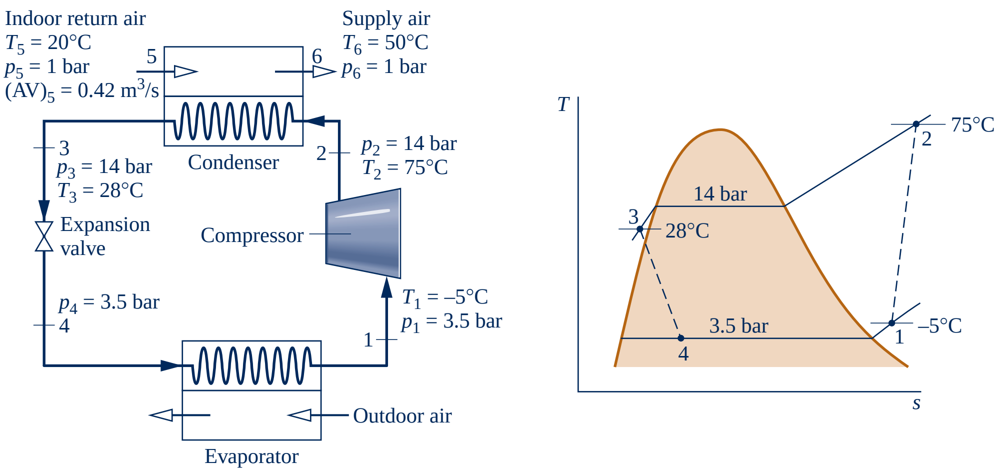


Fig. E6.8

Engineering Model:

1. Each component is analyzed as a control volume at steady state.
2. The compressor operates adiabatically, and the expansion across the valve is a *throttling process*.
3. For the control volume enclosing the condenser, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.
4. Kinetic and potential energy effects can be neglected.
5. (1) The air is modeled as an ideal gas with constant $c_p = 1.005$ kJ/kg · K.

Analysis:

(a) Let us begin by obtaining property data at each of the principal refrigerant states located on the accompanying schematic and T–s diagram. At the inlet to the compressor, the refrigerant is a superheated vapor at −5°C, 3.5 bar, so from Table A-9, $s_1 = 0.9572$ kJ/kg · K. Similarly, at state 2, the refrigerant is a superheated vapor at 75°C, 14 bar, so interpolating in Table A-9 gives $s_2 = 0.98225$ kJ/kg · K and $h_2 = 294.17$ kJ/kg.

State 3 is compressed liquid at 28°C, 14 bar. From Table A-7, $s_3 \approx s_f(28°C) = 0.2936$ kJ/kg · K and $h_3 \approx h_f(28°C) = 79.05$ kJ/kg. The expansion through the valve is a *throttling process,* so $h_3 = h_4$. Using data from Table A-8, the quality at state 4 is

$$x_4 = \frac{(h_4 - h_{f4})}{(h_{fg})_4} = \frac{(79.05 - 33.09)}{(212.91)} = 0.216$$

and the specific entropy is

$$s_4 = s_{f4} + x_4(s_{g4} - s_{f4}) = 0.1328 + 0.216(0.9431 - 0.1328) = 0.3078 \text{ kJ/kg} \cdot \text{K}$$

Condenser

Consider the control volume enclosing the condenser. With assumptions 1 and 3, the entropy rate balance reduces to

$$0 = \dot{m}_{ref}(s_2 - s_3) + \dot{m}_{air}(s_5 - s_6) + \dot{\sigma}_{cond}$$

To evaluate $\dot{\sigma}_{cond}$ requires the two mass flow rates, $\dot{m}_{air}$ and $\dot{m}_{ref}$, and the change in specific entropy for the air. These are obtained next.

Evaluating the mass flow rate of air using the ideal gas model and the known volumetric flow rate

$$\dot{m}_{air} = \frac{(AV)_5}{v_5} = (AV)_5 \frac{p_5}{RT_5}$$
$$= \left(0.42 \frac{m^3}{s}\right) \frac{(1 \text{ bar})}{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(293 \text{ K})} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 0.5 \text{ kg/s}$$

The refrigerant mass flow rate is determined using an energy balance for the control volume enclosing the condenser together with assumptions 1, 3, and 4 to obtain

$$\dot{m}_{ref} = \frac{\dot{m}_{air}(h_6 - h_5)}{(h_2 - h_3)}$$

With assumption 5, $h_6 - h_5 = c_p(T_6 - T_5)$. Inserting values

2
$$\dot{m}_{ref} = \frac{\left(0.5 \frac{kg}{s}\right)\left(1.005 \frac{kJ}{kg \cdot K}\right)(323 - 293)\text{K}}{(294.17 - 79.05) \text{ kJ/kg}} = 0.07 \text{ kg/s}$$

Using Eq. 6.22, the change in specific entropy of the air is

$$s_6 - s_5 = c_p \ln \frac{T_6}{T_5} - R \ln \frac{p_6}{p_5}$$
$$= \left(1.005 \frac{kJ}{kg \cdot K}\right) \ln\left(\frac{323}{293}\right) - R \ln \left(\cancel{\frac{1.0}{1.0}}\right)^0 = 0.098 \text{ kJ/kg} \cdot \text{K}$$

Finally, solving the entropy balance for $\dot{\sigma}_{cond}$ and inserting values

$$\dot{\sigma}_{cond} = \dot{m}_{ref}(s_3 - s_2) + \dot{m}_{air}(s_6 - s_5)$$
$$= \left[\left(0.07 \frac{kg}{s}\right)(0.2936 - 0.98225)\frac{kJ}{kg \cdot K} + (0.5)(0.098)\right] \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$
$$= 7.95 \times 10^{-4} \frac{kW}{K}$$

Compressor

For the control volume enclosing the compressor, the entropy rate balance reduces with assumptions 1 and 3 to

$$0 = \dot{m}_{ref}(s_1 - s_2) + \dot{\sigma}_{comp}$$

or

$$\begin{aligned}\dot{\sigma}_{\text{comp}} &= \dot{m}_{\text{ref}}(s_2 - s_1)\\ &= \left(0.07\,\frac{\text{kg}}{\text{s}}\right)(0.98225 - 0.9572)\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|\\ &= 17.5 \times 10^{-4}\ \text{kW/K}\end{aligned}$$

Valve

Finally, for the control volume enclosing the throttling valve, the entropy rate balance reduces to

$$0 = \dot{m}_{\text{ref}}(s_3 - s_4) + \dot{\sigma}_{\text{valve}}$$

Solving for $\dot{\sigma}_{\text{valve}}$ and inserting values

$$\begin{aligned}\dot{\sigma}_{\text{valve}} = \dot{m}_{\text{ref}}(s_4 - s_3) &= \left(0.07\,\frac{\text{kg}}{\text{s}}\right)(0.3078 - 0.2936)\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|\\ &= 9.94 \times 10^{-4}\ \text{kW/K}\end{aligned}$$

(b) The following table summarizes, in rank order, the calculated entropy production rates:

3

Component	$\dot{\sigma}_{\text{cv}}$ (kW/K)
compressor	17.5×10^{-4}
valve	9.94×10^{-4}
condenser	7.95×10^{-4}

Entropy production in the compressor is due to fluid friction, mechanical friction of the moving parts, and internal heat transfer. For the valve, the irreversibility is primarily due to fluid friction accompanying the expansion across the valve. The principal source of irreversibility in the condenser is the temperature difference between the air and refrigerant streams. In this example, there are no pressure drops for either stream passing through the condenser, but slight pressure drops due to fluid friction would normally contribute to the irreversibility of condensers. The evaporator shown in Fig. E6.8 has not been analyzed.

1 Due to the relatively small temperature change of the air, the specific heat c_p can be taken as constant at the average of the inlet and exit air temperatures.

2 Temperatures in K are used to evaluate $\dot{m}_{\text{ref}}$, but since a temperature *difference* is involved the same result would be obtained if temperatures in °C were used. Temperatures in K, and not °C, are required when a temperature *ratio* is involved, as in Eq. 6.22 used to evaluate $s_6 - s_5$.

3 By focusing attention on reducing irreversibilities at the sites with the highest entropy production rates, *thermodynamic* improvements may be possible. However, costs and other constraints must be considered, and can be overriding.

Skills Developed

Ability to...

- ❑ apply the control volume mass, energy and entropy rate balances.
- ❑ develop an engineering model.
- ❑ retrieve property data for Refrigerant 22.
- ❑ apply the ideal gas model with constant c_p.

QuickQUIZ If the compressor operated adiabatically *and* without internal irreversibilities, determine the temperature of the refrigerant at the compressor exit, in °C, keeping the compressor inlet state and exit pressure the same. **Ans.** 65°C.

6.11 Isentropic Processes

The term *isentropic* means constant entropy. Isentropic processes are encountered in many subsequent discussions. The object of the present section is to show how properties are related at any two states of a process in which there is no change in specific entropy.

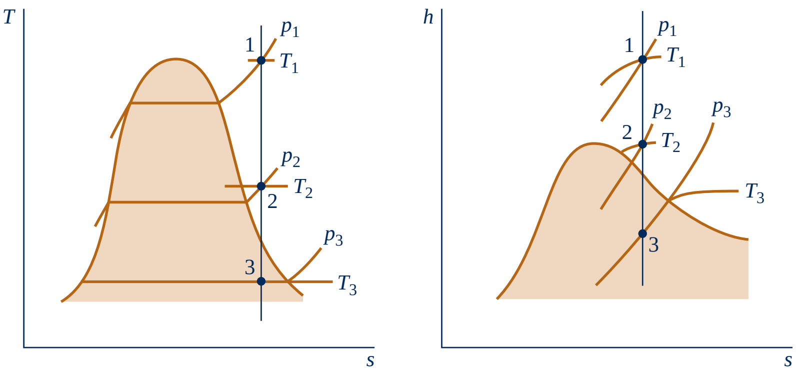


Fig. 6.8 ***T–s* and *h–s* diagrams showing states having the same value of specific entropy.**

6.11.1 General Considerations

The properties at states having the same specific entropy can be related using the graphical and tabular property data discussed in Sec. 6.2. For example, as illustrated by Fig. 6.8, temperature–entropy and enthalpy–entropy diagrams are particularly convenient for determining properties at states having the same value of specific entropy. All states on a vertical line passing through a given state have the same entropy. If state 1 on Fig. 6.8 is fixed by pressure p_1 and temperature T_1, states 2 and 3 are readily located once one additional property, such as pressure or temperature, is specified. The values of several other properties at states 2 and 3 can then be read directly from the figures.

Tabular data also can be used to relate two states having the same specific entropy. For the case shown in Fig. 6.8, the specific entropy at state 1 could be determined from the superheated vapor table. Then, with $s_2 = s_1$ and one other property value, such as p_2 or T_2, state 2 could be located in the superheated vapor table. The values of the properties v, u, and h at state 2 can then be read from the table. (An illustration of this procedure is given in Sec. 6.2.1.) Note that state 3 falls in the two-phase liquid–vapor regions of Fig. 6.8. Since $s_3 = s_1$, the quality at state 3 could be determined using Eq. 6.4. With the quality known, other properties such as v, u, and h could then be evaluated. Computer retrieval of entropy data provides an alternative to tabular data.

6.11.2 Using the Ideal Gas Model

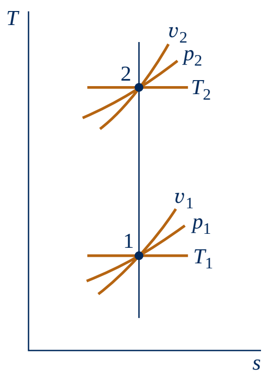


Fig. 6.9 **Two states of an ideal gas where $s_2 = s_1$.**

Figure 6.9 shows two states of an ideal gas having the same value of specific entropy. Let us consider relations among pressure, specific volume, and temperature at these states, first using the ideal gas tables and then assuming specific heats are constant.

Ideal Gas Tables

For two states having the same specific entropy, Eq. 6.20a reduces to

$$0 = s°(T_2) - s°(T_1) - R \ln \frac{p_2}{p_1} \tag{6.40a}$$

Equation 6.40a involves four property values: p_1, T_1, p_2, and T_2. If any three are known, the fourth can be determined. If, for example, the temperature at state 1 and the pressure ratio p_2/p_1 are known, the temperature at state 2 can be determined from

$$s°(T_2) = s°(T_1) + R \ln \frac{p_2}{p_1} \tag{6.40b}$$

Since T_1 is known, $s°(T_1)$ would be obtained from the appropriate table, the value of $s°(T_2)$ would be calculated, and temperature T_2 would then be determined by interpolation. If p_1, T_1, and T_2 are specified and the pressure at state 2 is the unknown, Eq. 6.40a would be solved to obtain

$$p_2 = p_1 \exp\left[\frac{s°(T_2) - s°(T_1)}{R}\right] \tag{6.40c}$$

Equations 6.40 can be used when $s°$ (or $\bar{s}°$) data are known, as for the gases of Tables A-22 and A-23.

AIR. For the special case of *air* modeled as an ideal gas, Eq. 6.40c provides the basis for an alternative tabular approach for relating the temperatures and pressures at two states having the same specific entropy. To introduce this, rewrite the equation as

$$\frac{p_2}{p_1} = \frac{\exp[s°(T_2)/R]}{\exp[s°(T_1)/R]}$$

The quantity $\exp[s°(T)/R]$ appearing in this expression is solely a function of temperature, and is given the symbol $p_r(T)$. A tabulation of p_r versus temperature for *air* is provided in Tables A-22.[1] In terms of the function p_r, the last equation becomes

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \qquad (s_1 = s_2, \text{air only}) \tag{6.41}$$

where $p_{r1} = p_r(T_1)$ and $p_{r2} = p_r(T_2)$. The function p_r is sometimes called the *relative pressure*. Observe that p_r is not truly a pressure, so the name relative pressure has no physical significance. Also, be careful not to confuse p_r with the reduced pressure of the compressibility diagram.

A relation between specific volumes and temperatures for two states of air having the same specific entropy can also be developed. With the ideal gas equation of state, $v = RT/p$, the ratio of the specific volumes is

$$\frac{v_2}{v_1} = \left(\frac{RT_2}{p_2}\right)\left(\frac{p_1}{RT_1}\right)$$

Then, since the two states have the same specific entropy, Eq. 6.41 can be introduced to give

$$\frac{v_2}{v_1} = \left[\frac{RT_2}{p_r(T_2)}\right]\left[\frac{p_r(T_1)}{RT_1}\right]$$

The ratio $RT/p_r(T)$ appearing on the right side of the last equation is solely a function of temperature, and is given the symbol $v_r(T)$. Values of v_r for *air* are tabulated

TAKE NOTE...
When applying the software *IT* to relate two states of an ideal gas having the same value of specific entropy, *IT* returns specific entropy *directly* and does not employ the special functions $s°$, p_r, and v_r.

[1]The values of p_r determined with this definition are inconveniently large, so they are divided by a scale factor before tabulating to give a convenient range of numbers.

versus temperature in Tables A-22. In terms of the function υ_r, the last equation becomes

$$\frac{\upsilon_2}{\upsilon_1} = \frac{\upsilon_{r2}}{\upsilon_{r1}} \qquad (s_1 = s_2, \text{ air only}) \tag{6.42}$$

where $\upsilon_{r1} = \upsilon_r(T_1)$ and $\upsilon_{r2} = \upsilon_r(T_2)$. The function υ_r is sometimes called the *relative volume*. Despite the name given to it, $\upsilon_r(T)$ is not truly a volume. Also, be careful not to confuse it with the pseudoreduced specific volume of the compressibility diagram.

Assuming Constant Specific Heats

Let us consider next how properties are related for isentropic processes of an ideal gas when the specific heats are constants. For any such case, Eqs. 6.21 and 6.22 reduce to the equations

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$0 = c_\upsilon \ln \frac{T_2}{T_1} + R \ln \frac{\upsilon_2}{\upsilon_1}$$

Introducing the ideal gas relations

$$c_p = \frac{kR}{k-1}, \qquad c_\upsilon = \frac{R}{k-1} \tag{3.47}$$

where k is the specific heat ratio and R is the gas constant, these equations can be solved, respectively, to give

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (s_1 = s_2, \text{ constant } k) \tag{6.43}$$

$$\frac{T_2}{T_1} = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{k-1} \qquad (s_1 = s_2, \text{ constant } k) \tag{6.44}$$

The following relation can be obtained by eliminating the temperature ratio from Eqs. 6.43 and 6.44:

$$\frac{p_2}{p_1} = \left(\frac{\upsilon_1}{\upsilon_2}\right)^{k} \qquad (s_1 = s_2, \text{ constant } k) \tag{6.45}$$

Previously, we have identified an internally reversible process described by $p\upsilon^n = constant$, where n is a constant, as a *polytropic process.* From the form of Eq. 6.45, it can be concluded that the polytropic process $p\upsilon^k = constant$ of an ideal gas with constant specific heat ratio k is an isentropic process. We observed in Sec. 3.15 that a polytropic process of an ideal gas for which $n = 1$ is an isothermal (constant-temperature) process. For *any* fluid, $n = 0$ corresponds to an isobaric (constant-pressure) process and $n = \pm\infty$ corresponds to an isometric (constant-volume) process. Polytropic processes corresponding to these values of n are shown in Fig. 6.10 on p–υ and T–s diagrams.

6.11.3 Illustrations: Isentropic Processes of Air

Means for evaluating data for isentropic processes of air modeled as an ideal gas are illustrated in the next two examples. In Example 6.9, we consider three alternative methods.

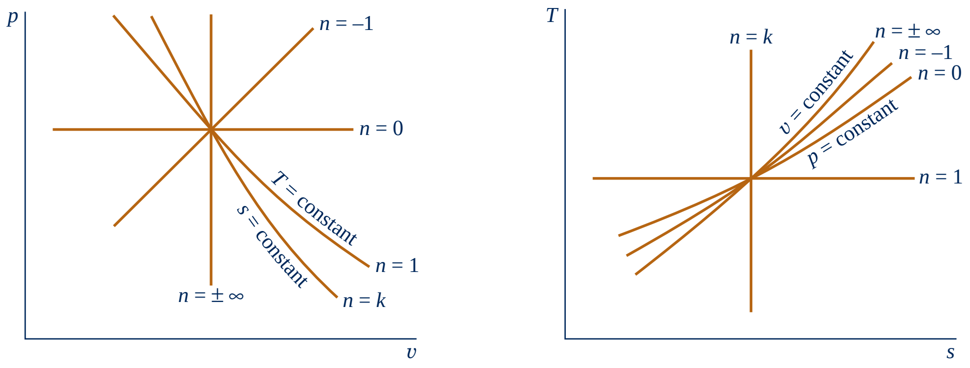


Fig. 6.10 **Polytropic processes on p–v and T–s diagrams.**

EXAMPLE 6.9

Analyzing an Isentropic Process of Air

Air undergoes an isentropic process from $p_1 = 1$ atm, $T_1 = 540°R$ to a final state where the temperature is $T_2 = 1160°R$. Employing the ideal gas model, determine the final pressure p_2, in atm. Solve using **(a)** p_r data from Table A-22E, **(b)** *Interactive Thermodynamics: IT*, and **(c)** a constant specific heat ratio k evaluated at the mean temperature, 850°R, from Table A-20E.

SOLUTION

Known: Air undergoes an isentropic process from a state where pressure and temperature are known to a state where the temperature is specified.

Find: Determine the final pressure using (a) p_r data, (b) *IT*, and (c) a constant value for the specific heat ratio k.

Schematic and Given Data:

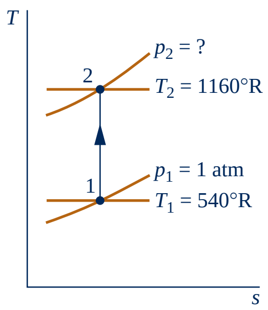


Fig. E6.9

Engineering Model:

1. A quantity of air as the system undergoes an isentropic process.
2. The air can be modeled as an ideal gas.
3. In part (c) the specific heat ratio is constant.

Analysis:

(a) The pressures and temperatures at two states of an ideal gas having the same specific entropy are related by Eq. 6.41

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}}$$

Solving

$$p_2 = p_1 \frac{p_{r2}}{p_{r1}}$$

With p_r values from Table A-22E

$$p = (1 \text{ atm})\frac{21.18}{1.3860} = 15.28 \text{ atm}$$

(b) The *IT* solution follows:

```
T1 = 540 // °R
p1 = 1 // atm
T2 = 1160 // °R
s_TP("Air", T1,p1) = s_TP("Air",T2,p2)

// Result: p2 = 15.28 atm
```

1 (marker on the s_TP line)

(c) When the specific heat ratio k is assumed constant, the temperatures and pressures at two states of an ideal gas having the same specific entropy are related by Eq. 6.43. Thus

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{k/(k-1)}$$

From Table A-20E at the mean temperature, 390°F (850°R), k = 1.39. Inserting values into the above expression

2 $$p_2 = (1 \text{ atm})\left(\frac{1160}{540}\right)^{1.39/0.39} = 15.26 \text{ atm}$$

1 *IT* returns a value for p_2 even though it is an implicit variable in the specific entropy function. Also note that *IT* returns values for specific entropy *directly* and does not employ special functions such as $s°$, p_r, and v_r.

2 The close agreement between the answer obtained in part (c) and that of parts (a), (b) can be attributed to the use of an appropriate value for the specific heat ratio k.

Skills Developed

Ability to...

- analyze an isentropic process using Table A-22E data,
- *Interactive Thermodynamics*, and
- a constant specific heat ratio k.

QuickQUIZ Determine the final pressure, in atm, using a constant specific heat ratio k evaluated at T_1 = 540°R. Expressed as a percent, how much does this pressure value differ from that of part (c)? Ans. 14.53 atm, −5%.

Another illustration of an isentropic process of an ideal gas is provided in Example 6.10 dealing with air leaking from a tank.

EXAMPLE 6.10

Considering Air Leaking from a Tank

A rigid, well-insulated tank is filled initially with 5 kg of air at a pressure of 5 bar and a temperature of 500 K. A leak develops, and air slowly escapes until the pressure of the air remaining in the tank is 1 bar. Employing the ideal gas model, determine the amount of mass remaining in the tank and its temperature.

SOLUTION

Known: A leak develops in a rigid, insulated tank initially containing air at a known state. Air slowly escapes until the pressure in the tank is reduced to a specified value.

Find: Determine the amount of mass remaining in the tank and its temperature.

Schematic and Given Data:

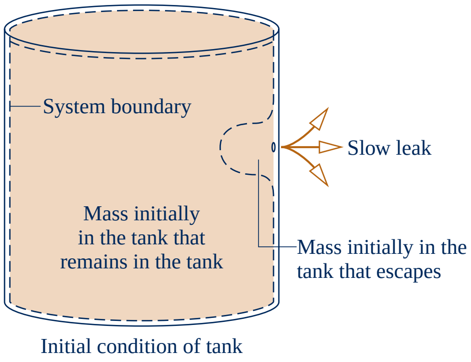


Fig. E6.10

Engineering Model:

1. As shown on the accompanying sketch, the closed system is the mass initially in the tank that remains in the tank.
2. There is no significant heat transfer between the system and its surroundings.
3. Irreversibilities within the tank can be ignored as the air slowly escapes.
4. The air is modeled as an ideal gas.

Analysis: With the ideal gas equation of state, the mass initially in the tank that *remains* in the tank at the end of the process is

$$m_2 = \frac{p_2 V}{(\bar{R}/M)T_2}$$

where p_2 and T_2 are the final pressure and temperature, respectively. Similarly, the initial amount of mass within the tank, m_1 is

$$m_1 = \frac{p_1 V}{(\bar{R}/M)T_1}$$

where p_1 and T_1 are the initial pressure and temperature, respectively. Eliminating volume between these two expressions, the mass of the system is

$$m_2 = \left(\frac{p_2}{p_1}\right)\left(\frac{T_1}{T_2}\right)m_1$$

Except for the final temperature of the air remaining in the tank, T_2, all required values are known. The remainder of the solution mainly concerns the evaluation of T_2.

For the closed system under consideration, there are no significant irreversibilities (assumption 3), and no heat transfer occurs (assumption 2). Accordingly, the entropy balance reduces to

$$\Delta S = \int_1^2 \cancel{\left(\frac{\delta Q}{T}\right)_b}^{\,0} + \cancel{\sigma}^{\,0} = 0$$

Since the system mass remains constant, $\Delta S = m_2\, \Delta s$, so

$$\Delta s = 0$$

That is, the initial and final states of the system have the same value of *specific* entropy.

Using Eq. 6.41

$$p_{r2} = \left(\frac{p_2}{p_1}\right)p_{r1}$$

where $p_1 = 5$ bar and $p_2 = 1$ bar. With $p_{r1} = 8.411$ from Table A-22 at 500 K, the previous equation gives $p_{r2} = 1.6822$. Using this to interpolate in Table A-22, $T_2 = 317$ K.

Finally, inserting values into the expression for system mass

$$m_2 = \left(\frac{1\ \text{bar}}{5\ \text{bar}}\right)\left(\frac{500\ \text{K}}{317\ \text{K}}\right)(5\ \text{kg}) = 1.58\ \text{kg}$$

Skills Developed

Ability to...

- ❑ develop an engineering model.
- ❑ apply the closed system entropy balance.
- ❑ analyze an isentropic process.

QuickQUIZ Evaluate the tank volume, in m^3. Ans. 1.43 m^3.

6.12 Isentropic Efficiencies of Turbines, Nozzles, Compressors, and Pumps

Engineers make frequent use of efficiencies and many different efficiency definitions are employed. In the present section, *isentropic* efficiencies for turbines, nozzles, compressors, and pumps are introduced. Isentropic efficiencies involve a comparison between the actual performance of a device and the performance that would be achieved under idealized circumstances for the same inlet state and the same exit pressure. These efficiencies are frequently used in subsequent sections of the book.

6.12.1 Isentropic Turbine Efficiency

To introduce the isentropic turbine efficiency, refer to Fig. 6.11, which shows a turbine expansion on a Mollier diagram. The state of the matter entering the turbine and the exit pressure are fixed. Heat transfer between the turbine and its surroundings is ignored, as are kinetic and potential energy effects. With these assumptions, the mass and energy rate balances reduce, at steady state, to give the work developed per unit of mass flowing through the turbine

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

Since state 1 is fixed, the specific enthalpy h_1 is known. Accordingly, the value of the work depends on the specific enthalpy h_2 only, and increases as h_2 is reduced. The *maximum* value for the turbine work corresponds to the smallest *allowed* value for the specific enthalpy at the turbine exit. This can be determined using the second law as follows.

Since there is no heat transfer, the allowed exit states are constrained by Eq. 6.39

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geq 0$$

Because the entropy production $\dot{\sigma}_{cv}/\dot{m}$ cannot be negative, states with $s_2 < s_1$ are not accessible in an adiabatic expansion. The only states that actually can be attained *adiabatically* are those with $s_2 > s_1$. The state labeled "2s"on Fig. 6.11 would be attained only in the limit of no internal irreversibilities. This corresponds

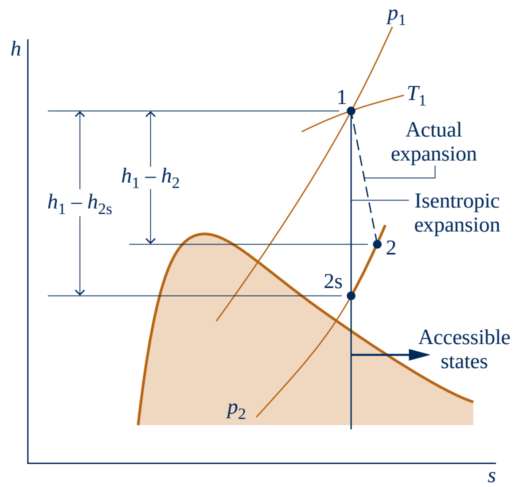


Fig. 6.11 **Comparison of actual and isentropic expansions through a turbine.**

to an isentropic expansion through the turbine. For fixed exit pressure, the specific enthalpy h_2 decreases as the specific entropy s_2 decreases. Therefore, the *smallest allowed* value for h_2 corresponds to state 2s, and the *maximum* value for the turbine work is

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_1 - h_{2s}$$

In an actual expansion through the turbine $h_2 > h_{2s}$, and thus less work than the maximum would be developed. This difference can be gauged by the **isentropic turbine efficiency** defined by

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \tag{6.46}$$

isentropic turbine efficiency

TAKE NOTE... The subscript *s* denotes a quantity evaluated for an isentropic process from a specified inlet state to a specified exit pressure.

Both the numerator and denominator of this expression are evaluated for the same inlet state and the same exit pressure. The value of η_t is typically 0.7 to 0.9 (70–90%).

The two examples to follow illustrate the isentropic turbine efficiency concept. In Example 6.11 the isentropic efficiency of a steam turbine is known and the objective is to determine the turbine work.

Turbine
A.19 – Tab e
A

EXAMPLE 6.11

Determining Turbine Work Using the Isentropic Efficiency

A steam turbine operates at steady state with inlet conditions of $p_1 = 5$ bar, $T_1 = 320°C$. Steam leaves the turbine at a pressure of 1 bar. There is no significant heat transfer between the turbine and its surroundings, and kinetic and potential energy changes between inlet and exit are negligible. If the isentropic turbine efficiency is 75%, determine the work developed per unit mass of steam flowing through the turbine, in kJ/kg.

SOLUTION

Known: Steam expands through a turbine operating at steady state from a specified inlet state to a specified exit pressure. The turbine efficiency is known.

Find: Determine the work developed per unit mass of steam flowing through the turbine.

Schematic and Given Data:

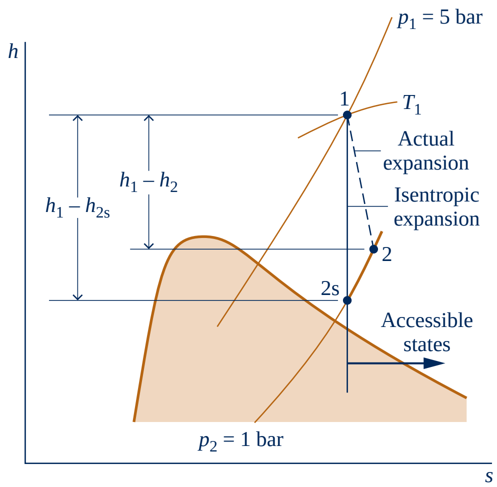


Fig. E6.11

Engineering Model:

1. A control volume enclosing the turbine is at steady state.
2. The expansion is adiabatic and changes in kinetic and potential energy between the inlet and exit can be neglected.

Analysis: The work developed can be determined using the isentropic turbine efficiency, Eq. 6.46, which on rearrangement gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = \eta_t\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = \eta_t(h_1 - h_{2s})$$

From Table A-4, $h_1 = 3105.6$ kJ/kg and $s_1 = 7.5308$ kJ/kg · K. The exit state for an isentropic expansion is
1 fixed by $p_2 = 1$ and $s_{2s} = s_1$. Interpolating with specific entropy in Table A-4 at 1 bar gives $h_{2s} = 2743.0$ kJ/kg. Substituting values

2 $$\frac{\dot{W}_{cv}}{\dot{m}} = 0.75(3105.6 - 2743.0) = 271.95 \text{ kJ/kg}$$

---

1 At 2s, the temperature is about 133°C.

2 The effect of irreversibilities is to exact a penalty on the work output of the turbine. The work is only 75% of what it would be for an isentropic expansion between the given inlet state and the turbine exhaust pressure. This is clearly illustrated in terms of enthalpy differences on the accompanying h–s diagram.

Skills Developed

Ability to...

- ❑ apply the isentropic turbine efficiency, Eq. 6.46.
- ❑ retrieve steam table data.

QuickQUIZ Determine the temperature of the steam at the turbine exit, in °C. Ans. 179°C.

Example 6.12 is similar to Example 6.11, but here the working substance is air as an ideal gas. Moreover, in this case the turbine work is known and the objective is to determine the isentropic turbine efficiency.

EXAMPLE 6.12

Evaluating Isentropic Turbine Efficiency

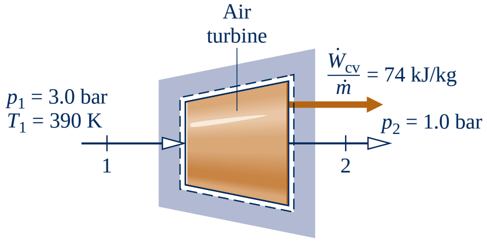
A turbine operating at steady state receives air at a pressure of $p_1 = 3.0$ bar and a temperature of $T_1 = 390$ K. Air exits the turbine at a pressure of $p_2 = 1.0$ bar. The work developed is measured as 74 kJ per kg of air flowing through the turbine. The turbine operates adiabatically, and changes in kinetic and potential energy between inlet and exit can be neglected. Using the ideal gas model for air, determine the isentropic turbine efficiency.

SOLUTION

Known: Air expands adiabatically through a turbine at steady state from a specified inlet state to a specified exit pressure. The work developed per kg of air flowing through the turbine is known.

Find: Determine the turbine efficiency.

Schematic and Given Data:



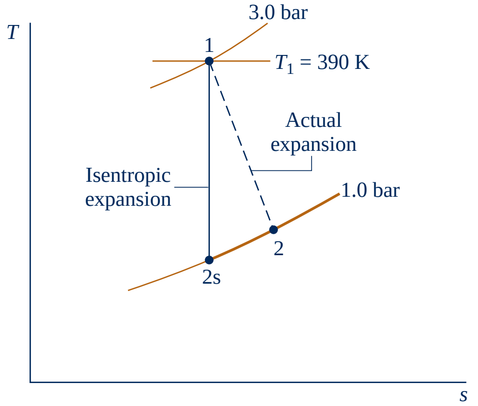


Fig. E6.12

Engineering Model:

1. The control volume shown on the accompanying sketch is at steady state.
2. The expansion is adiabatic and changes in kinetic and potential energy between inlet and exit can be neglected.
3. The air is modeled as an ideal gas.

Analysis: The numerator of the isentropic turbine efficiency, Eq. 6.46, is known. The denominator is evaluated as follows.

The work developed in an isentropic expansion from the given inlet state to the specified exit pressure is

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_1 - h_{2s}$$

From Table A-22 at 390 K, $h_1 = 390.88$ kJ/kg. To determine h_{2s}, use Eq. 6.41

$$p_r(T_{2s}) = \left(\frac{p_2}{p_1}\right)p_r(T_1)$$

With $p_1 = 3.0$ bar, $p_2 = 1.0$ bar, and $p_{r1} = 3.481$ from Table A-22 at 390 K

$$p_r(T_{2s}) = \left(\frac{1.0}{3.0}\right)(3.481) = 1.1603$$

Interpolation in Table A-22 gives $h_{2s} = 285.27$ kJ/kg. Thus

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = 390.88 - 285.27 = 105.6 \text{ kJ/kg}$$

Substituting values into Eq. 6.46

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{74 \text{ kJ/kg}}{105.6 \text{ kJ/kg}} = 0.70(70\%)$$

Skills Developed

Ability to...

- apply the isentropic turbine efficiency, Eq. 6.46.
- retrieve data for air as an ideal gas.

QuickQUIZ Determine the rate of entropy production, in kJ/K per kg of air flowing through the turbine. **Ans.** 0.105 kJ/kg · K.

6.12.2 Isentropic Nozzle Efficiency

A similar approach to that for turbines can be used to introduce the isentropic efficiency of nozzles operating at steady state. The **isentropic nozzle efficiency** is defined as the ratio of the actual specific kinetic energy of the gas leaving the nozzle, $V_2^2/2$, to the kinetic energy at the exit that would be achieved in an isentropic expansion between the same inlet state and the same exit pressure, $(V_2^2/2)_s$. That is,

$$\eta_{nozzle} = \frac{V_2^2/2}{(V_2^2/2)_s} \quad (6.47)$$

isentropic nozzle efficiency

Nozzle efficiencies of 95% or more are common, indicating that well-designed nozzles are nearly free of internal irreversibilities.

In Example 6.13, the objective is to determine the isentropic efficiency of a steam nozzle.

Nozzle
A.17 – Tab e

EXAMPLE 6.13

Evaluating Isentropic Nozzle Efficiency

Steam enters a nozzle operating at steady state at $p_1 = 140$ lbf/in.^{2} and $T_1 = 600$°F with a velocity of 100 ft/s. The pressure and temperature at the exit are $p_2 = 40$ lbf/in.^{2} and $T_2 = 350$°F. There is no significant heat transfer between the nozzle and its surroundings, and changes in potential energy between inlet and exit can be neglected. Determine the nozzle efficiency.

SOLUTION

Known: Steam expands through a nozzle at steady state from a specified inlet state to a specified exit state. The velocity at the inlet is known.

Find: Determine the nozzle efficiency.

Schematic and Given Data:

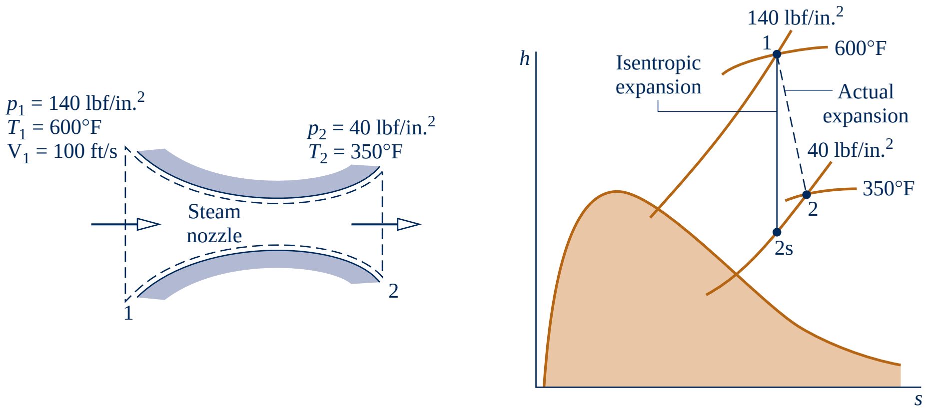


Fig. E6.13

Engineering Model:

1. The control volume shown on the accompanying sketch operates adiabatically at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and the change in potential energy between inlet and exit can be neglected.

Analysis: The nozzle efficiency given by Eq. 6.47 requires the actual specific kinetic energy at the nozzle exit and the specific kinetic energy that would be achieved at the exit in an isentropic expansion from the given inlet state to the given exit pressure. The energy rate balance for a one-inlet, one-exit control volume at steady state enclosing the nozzle reduces to give Eq. 4.21, which on rearrangement reads

$$\frac{V_2^2}{2} = h_1 - h_2 + \frac{V_1^2}{2}$$

This equation applies for both the actual expansion and the isentropic expansion.

From Table A-4E at $T_1 = 600$°F and $p_1 = 140$ lbf/in.^{2}, $h_1 = 1326.4$ Btu/lb, $s_1 = 1.7191$ Btu/lb · °R. Also, with $T_2 = 350$°F and $p_2 = 40$ lbf/in.^{2}, $h_2 = 1211.8$ Btu/lb. Thus, the actual specific kinetic energy at the exit in Btu/lb is

$$\frac{V_2^2}{2} = 1326.4\frac{\text{Btu}}{\text{lb}} - 1211.8\frac{\text{Btu}}{\text{lb}} + \frac{(100 \text{ ft/s})^2}{(2)\left|\frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}}\right|\left|\frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}}\right|}$$

$$= 114.8\frac{\text{Btu}}{\text{lb}}$$

Interpolating in Table A-4E at 40 lbf/in.^{2}, with $s_{2s} = s_1 = 1.7191$ Btu/lb · °R, results in $h_{2s} = 1202.3$ Btu/lb. Accordingly, the specific kinetic energy at the exit for an isentropic expansion is

$$\left(\frac{V_2^2}{2}\right)_s = 1326.4 - 1202.3 + \frac{(100)^2}{(2)|32.2||778|} = 124.3 \text{ Btu/lb}$$

Substituting values into Eq. 6.47

1
$$\eta_{\text{nozzle}} = \frac{(V_2^2/2)}{(V_2^2/2)_s} = \frac{114.8}{124.3} = 0.924\ (92.4\%)$$

1 The principal irreversibility in the nozzle is friction, in particular friction between the flowing steam and the nozzle wall. The effect of friction is that a smaller exit kinetic energy, and thus a smaller exit velocity, is realized than would have been obtained in an isentropic expansion to the same pressure.

Skills Developed

Ability to...

- ❑ apply the control volume energy rate balance.
- ❑ apply the isentropic nozzle efficiency, Eq. 6.47.
- ❑ retrieve steam table data.

QuickQUIZ Determine the temperature, in °F, corresponding to state 2s in Fig. E6.13. **Ans.** 331°F.

6.12.3 Isentropic Compressor and Pump Efficiencies

The form of the isentropic efficiency for compressors and pumps is taken up next. Refer to Fig. 6.12, which shows a compression process on a Mollier diagram. The state of the matter entering the compressor and the exit pressure are fixed. For negligible heat transfer with the surroundings and no appreciable kinetic and potential energy effects, the work *input* per unit of mass flowing through the compressor is

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_2 - h_1$$

Since state 1 is fixed, the specific enthalpy h_1 is known. Accordingly, the value of the work input depends on the specific enthalpy at the exit, h_2. The above expression shows that the magnitude of the work input decreases as h_2 decreases. The *minimum* work input corresponds to the smallest *allowed* value for the specific enthalpy at the compressor exit. With similar reasoning as for the turbine, the smallest allowed enthalpy at the exit state would be achieved in an isentropic compression from the specified inlet state to the specified exit pressure. The minimum work *input* is given, therefore, by

Compressor
A.20 – Tab e
Pump
A.21 – Tab e

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_{2s} - h_1$$

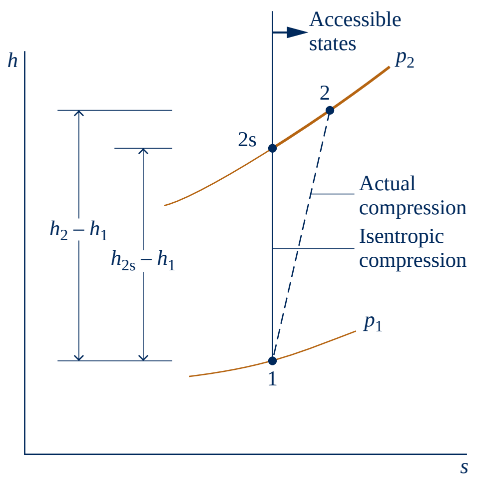


Fig. 6.12 Comparison of actual and isentropic compressions.

In an actual compression, $h_2 > h_{2s}$, and thus more work than the minimum would be required. This difference can be gauged by the **isentropic compressor efficiency** defined by

isentropic compressor efficiency

$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1} \tag{6.48}$$

Both the numerator and denominator of this expression are evaluated for the same inlet state and the same exit pressure. The value of η_c is typically 75 to 85% for compressors. An **isentropic pump efficiency**, η_p, is defined similarly.

isentropic pump efficiency

In Example 6.14, the isentropic efficiency of a refrigerant compressor is evaluated, first using data from property tables and then using *IT*.

EXAMPLE 6.14

Evaluating Isentropic Compressor Efficiency

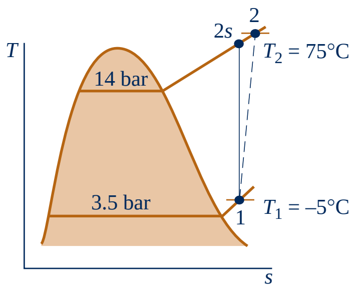
For the compressor of the heat pump system in Example 6.8, determine the power, in kW, and the isentropic efficiency using **(a)** data from property tables, **(b)** *Interactive Thermodynamics: IT*.

SOLUTION

Known: Refrigerant 22 is compressed adiabatically at steady state from a specified inlet state to a specified exit state. The mass flow rate is known.

Find: Determine the compressor power and the isentropic efficiency using (a) property tables, (b) *IT*.

Schematic and Given Data:



Engineering Model:

1. A control volume enclosing the compressor is at steady state.
2. The compression is adiabatic, and changes in kinetic and potential energy between the inlet and the exit can be neglected.

Fig. E6.14

Analysis: (a) By assumptions 1 and 2, the mass and energy rate balances reduce to give

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

From Table A-9, $h_1 = 249.75$ kJ/kg and $h_2 = 294.17$ kJ/kg. Thus, with the mass flow rate determined in Example 6.8

$$\dot{W}_{cv} = (0.07 \text{ kg/s})(249.75 - 294.17) \text{ kJ/kg} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = -3.11 \text{ kW}$$

The isentropic compressor efficiency is determined using Eq. 6.48

$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{(h_{2s} - h_1)}{(h_2 - h_1)}$$

In this expression, the denominator represents the work input per unit mass of refrigerant flowing for the actual compression process, as considered above. The numerator is the work input for an isentropic compression between the initial state and the same exit pressure. The isentropic exit state is denoted as state 2s on the accompanying T–s diagram.

From Table A-9, $s_1 = 0.9572$ kJ/kg · K. With $s_{2s} = s_1$, interpolation in Table A-9 at 14 bar gives $h_{2s} = 285.58$ kJ/kg. Substituting values

$$\eta_c = \frac{(285.58 - 249.75)}{(294.17 - 249.75)} = 0.81(81\%)$$

(b) The *IT* program follows. In the program, $\dot{W}_{cv}$ is denoted as Wdot, $\dot{m}$ as mdot, and η_c as eta_c.

```
// Given Data:
T1 = −5 // °C
p1 = 3.5 // bar
T2 = 75 // °C
p2 = 14 // bar
mdot = 0.07 // kg/s

// Determine the specific enthalpies.
h1 = h_PT("R22",p1,T1)
h2 = h_PT("R22",p2,T2)

// Calculate the power.
Wdot = mdot * (h1 − h2)
// Find h2s:
s1 = s_PT("R22",p1,T1)
1 s2s = s_Ph("R22",p2,h2s)
s2s = s1

// Determine the isentropic compressor efficiency.
eta_c = (h2s − h1)/(h2 − h1)
```

Use the **Solve** button to obtain: $\dot{W}_{cv} = -3.111$ kW and $\eta_c = 80.58\%$, which, as expected, agree closely with the values obtained above.

1 Note that *IT* solves for the value of h_{2s} even though it is an implicit variable in the specific entropy function.

Skills Developed

Ability to...

- apply the control volume energy rate balance.
- apply the isentropic compressor efficiency, Eq. 6.48.
- retrieve data for Refrigerant 22.

QuickQUIZ Determine the minimum theoretical work input, in kJ per kg flowing, for an adiabatic compression from state 1 to the exit pressure of 14 bar. Ans. 35.83 kJ/kg.

6.13 Heat Transfer and Work in Internally Reversible, Steady-State Flow Processes

This section concerns one-inlet, one-exit control volumes at steady state. The objective is to introduce expressions for the heat transfer and the work in the absence of internal irreversibilities. The resulting expressions have several important applications.

6.13.1 Heat Transfer

For a control volume at steady state in which the flow is both *isothermal* at temperature T and *internally reversible,* the appropriate form of the entropy rate balance is

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}}^{0}$$

where 1 and 2 denote the inlet and exit, respectively, and $\dot{m}$ is the mass flow rate. Solving this equation, the heat transfer per unit of mass passing through the control volume is

$$\frac{\dot{Q}_{cv}}{\dot{m}} = T(s_2 - s_1)$$

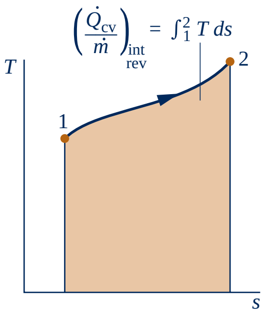


Fig. 6.13 Area representation of heat transfer for an internally reversible flow process.

More generally, temperature varies as the gas or liquid flows through the control volume. We can consider such a temperature variation to consist of a series of infinitesimal steps. Then, the heat transfer per unit of mass is given as

$$\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = \int_1^2 T\,ds \tag{6.49}$$

The subscript "int rev" serves to remind us that the expression applies only to control volumes in which there are no internal irreversibilities. The integral of Eq. 6.49 is performed from inlet to exit. When the states visited by a unit mass as it passes reversibly from inlet to exit are described by a curve on a T–s diagram, the magnitude of the heat transfer per unit of mass flowing can be represented as the area *under* the curve, as shown in Fig. 6.13.

6.13.2 Work

The work per unit of mass passing through a one-inlet, one-exit control volume can be found from an energy rate balance, which reduces at steady state to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

This equation is a statement of the conservation of energy principle that applies when irreversibilities are present within the control volume as well as when they are absent. However, if consideration is restricted to the internally reversible case, Eq. 6.49 can be introduced to obtain

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = \int_1^2 T\,ds + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2) \tag{6.50}$$

where the subscript "int rev" has the same significance as before.

Since internal irreversibilities are absent, a unit of mass traverses a sequence of equilibrium states as it passes from inlet to exit. Entropy, enthalpy, and pressure changes are therefore related by Eq. 6.10b

$$T\,ds = dh - v\,dp$$

which on integration gives

$$\int_1^2 T\,ds = (h_2 - h_1) - \int_1^2 v\,dp$$

Introducing this relation, Eq. 6.50 becomes

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\int_1^2 v\,dp + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2) \tag{6.51a}$$

When the states visited by a unit of mass as it passes reversibly from inlet to exit are described by a curve on a p–v diagram as shown in Fig. 6.14, the magnitude of the integral $\int v\,dp$ is represented by the shaded area *behind* the curve.

Equation 6.51a is applicable to devices such as turbines, compressors, and pumps. In many of these cases, there is no significant change in kinetic or potential energy from inlet to exit, so

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\int_1^2 v\,dp \qquad (\Delta ke = \Delta pe = 0) \tag{6.51b}$$

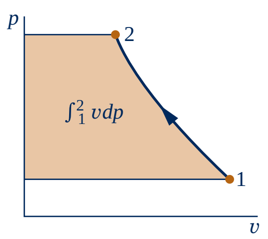


Fig. 6.14 Area representation of $\int_1^2 v\,dp$.

This expression shows that the work value is related to the magnitude of the specific volume of the gas or liquid as it flows from inlet to exit.

▸ **FOR EXAMPLE** consider two devices: a pump through which liquid water passes and a compressor through which water vapor passes. For the *same pressure rise,* the pump requires a much smaller work *input* per unit of mass flowing than the compressor because the liquid specific volume is much smaller than that of vapor. This conclusion is also qualitatively correct for actual pumps and compressors, where irreversibilities are present during operation. ◂ ◂ ◂ ◂ ◂

If the specific volume remains approximately constant, as in many applications with liquids, Eq. 6.51b becomes

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ \text{rev}}} = -v(p_2 - p_1) \qquad (v = \text{constant}, \Delta\text{ke} = \Delta\text{pe} = 0) \tag{6.51c}$$

Equation 6.51a also can be applied to study the performance of control volumes at steady state in which $\dot{W}_{cv}$ is zero, as in the case of nozzles and diffusers. For any such case, the equation becomes

$$\int_1^2 v\,dp + \left(\frac{V_2^2 - V_1^2}{2}\right) + g(z_2 - z_1) = 0 \tag{6.52}$$

Bernoulli equation

which is a form of the **Bernoulli equation** frequently used in fluid mechanics.

BIOCONNECTIONS Bats, the only mammals that can fly, play several important ecological roles, including feeding on crop-damaging insects. Currently, nearly one-quarter of bat species is listed as endangered or threatened. For unknown reasons, bats are attracted to large wind turbines, where some perish by impact and others from *hemorrhaging*. Near rapidly moving turbine blades there is a drop in air pressure that expands the lungs of bats, causing fine capillaries to burst and their lungs to fill with fluid, killing them.

The relationship between air velocity and pressure in these instances is captured by the following differential form of Eq. 6.52, the *Bernoulli equation:*

$$v\,dp = -V\,dV$$

which shows that as the *local* velocity V increases, the *local* pressure p decreases. The pressure reduction near the moving turbine blades is the source of peril to bats.

Some say *migrating* bats experience most of the fatalities, so the harm may be decreased at existing wind farms by reducing turbine operation during peak migration periods. New wind farms should be located away from known migratory routes.

6.13.3 Work In Polytropic Processes

We have identified an internally reversible process described by $pv^n = constant$, where n is a constant, as a *polytropic process* (see Sec. 3.15 and the discussion of Fig. 6.10). If each unit of mass passing through a one-inlet, one-exit control volume undergoes a polytropic process, then introducing $pv^n = constant$ in Eq. 6.51b, and performing the integration, gives the work per unit of mass in the absence of internal irreversibilities and significant changes in kinetic and potential energy. That is,

$$\begin{aligned}\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\ \text{rev}}} &= -\int_1^2 v\,dp = -(constant)^{1/n}\int_1^2 \frac{dp}{p^{1/n}}\\ &= -\frac{n}{n-1}(p_2v_2 - p_1v_1) \qquad (\text{polytropic}, n \neq 1)\end{aligned} \tag{6.53}$$

Equation 6.53 applies for any value of n except $n = 1$. When $n = 1$, $pv = constant$, and the work is

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\int_1^2 v\,dp = -constant\int_1^2 \frac{dp}{p}$$

$$= -(p_1v_1)\ln(p_2/p_1) \qquad (\text{polytropic}, n = 1) \tag{6.54}$$

Equations 6.53 and 6.54 apply generally to polytropic processes of *any* gas (or liquid).

IDEAL GAS CASE. For the special case of an ideal gas, Eq. 6.53 becomes

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\frac{nR}{n-1}(T_2 - T_1) \qquad (\text{ideal gas}, n \neq 1) \tag{6.55a}$$

For a polytropic process of an ideal gas, Eq. 3.56 applies:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

Thus, Eq. 6.55a can be expressed alternatively as

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\frac{nRT_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right] \qquad (\text{ideal gas}, n \neq 1) \tag{6.55b}$$

For the case of an ideal gas, Eq. 6.54 becomes

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -RT\ln(p_2/p_1) \qquad (\text{ideal gas}, n = 1) \tag{6.56}$$

In Example 6.15, we consider air modeled as an ideal gas undergoing a polytropic compression process at steady state.

EXAMPLE 6.15

Determining Work and Heat Transfer for a Polytropic Compression of Air

An air compressor operates at steady state with air entering at $p_1 = 1$ bar, $T_1 = 20°C$, and exiting at $p_2 = 5$ bar. Determine the work and heat transfer per unit of mass passing through the device, in kJ/kg, if the air undergoes a polytropic process with $n = 1.3$. Neglect changes in kinetic and potential energy between the inlet and the exit. Use the ideal gas model for air.

SOLUTION

Known: Air is compressed in a polytropic process from a specified inlet state to a specified exit pressure.

Find: Determine the work and heat transfer per unit of mass passing through the device.

Schematic and Given Data:

1

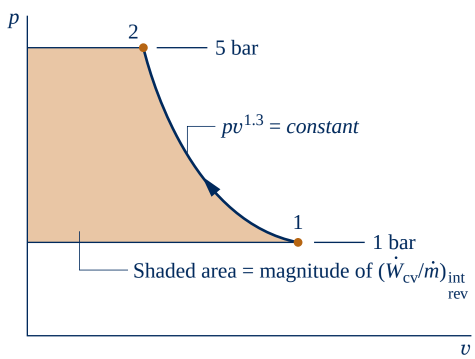


Fig. E6.15

Engineering Model:

1. A control volume enclosing the compressor is at steady state.
2. The air undergoes a polytropic process with $n = 1.3$.
3. The air behaves as an ideal gas.
4. Changes in kinetic and potential energy from inlet to exit can be neglected.

Analysis: The work is obtained using Eq. 6.55a, which requires the temperature at the exit, T_2. The temperature T_2 can be found using Eq. 3.56

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 293\left(\frac{5}{1}\right)^{(1.3-1)/1.3} = 425 \text{ K}$$

Substituting known values into Eq. 6.55a then gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nR}{n-1}(T_2 - T_1) = -\frac{1.3}{1.3-1}\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(425 - 293)\text{ K}$$
$$= -164.2 \text{ kJ/kg}$$

The heat transfer is evaluated by reducing the mass and energy rate balances with the appropriate assumptions to obtain

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1$$

Using the temperatures T_1 and T_2, the required specific enthalpy values are obtained from Table A-22 as $h_1 = 293.17$ kJ/kg and $h_2 = 426.35$ kJ/kg. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -164.15 + (426.35 - 293.17) = -31 \text{ kJ/kg}$$

1 The states visited in the polytropic compression process are shown by the curve on the accompanying p–v diagram. The magnitude of the work per unit of mass passing through the compressor is represented by the shaded area *behind* the curve.

Skills Developed

Ability to...

- analyze a polytropic process of an ideal gas.
- apply the control volume energy rate balance.

QuickQUIZ If the air were to undergo a polytropic process with $n = 1.0$, determine the work and heat transfer, each in kJ per kg of air flowing, keeping all other given data the same. Ans. -135.3 kJ/kg.

▶ CHAPTER SUMMARY AND STUDY GUIDE

In this chapter, we have introduced the property entropy and illustrated its use for thermodynamic analysis. Like mass and energy, entropy is an extensive property that can be transferred across system boundaries. Entropy transfer accompanies both heat transfer and mass flow. Unlike mass and energy, entropy is not conserved but is *produced* within systems whenever internal irreversibilities are present.

The use of entropy balances is featured in this chapter. Entropy balances are expressions of the second law that account for the entropy of systems in terms of entropy transfers and entropy production. For processes of closed systems, the entropy balance is Eq. 6.24, and a corresponding rate form is Eq. 6.28. For control volumes, rate forms include Eq. 6.34 and the companion steady-state expression given by Eq. 6.36.

The following checklist provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed you should be able to

- write out meanings of the terms listed in the margins throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important in subsequent chapters.
- apply entropy balances in each of several alternative forms, appropriately modeling the case at hand, correctly observing sign conventions, and carefully applying SI and English units.
- use entropy data appropriately, to include
  - –retrieving data from Tables A-2 through A-18, using Eq. 6.4 to evaluate the specific entropy of two-phase liquid–vapor mixtures, sketching T–s and h–s diagrams and locating states on such diagrams, and appropriately using Eqs. 6.5 and 6.13.
  - –determining Δs of ideal gases using Eq. 6.20 for variable specific heats together with Tables A-22 and A-23, and using Eqs. 6.21 and 6.22 for constant specific heats.

–evaluating isentropic efficiencies for turbines, nozzles, compressors, and pumps from Eqs. 6.46, 6.47, and 6.48, respectively, including for ideal gases the appropriate use of Eqs. 6.41–6.42 for variable specific heats and Eqs. 6.43–6.45 for constant specific heats.

▶ apply Eq. 6.23 for closed systems and Eqs. 6.49 and 6.51 for one-inlet, one-exit control volumes at steady state, correctly observing the restriction to internally reversible processes.

▶ KEY ENGINEERING CONCEPTS

entropy change, p. 282
***T–s* diagram, p. 285**
Mollier diagram, p. 286
***T ds* equations, p. 287**
isentropic process, p. 292
entropy transfer, pp. 292, 307
entropy balance, p. 295
entropy production, p. 296
entropy rate balance, pp. 301, 307
increase in entropy principle, p. 303
isentropic efficiencies, pp. 323, 325, 328

▶ KEY EQUATIONS

$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$	**(6.24) p. 295**	Closed system entropy balance.
$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$	**(6.28) p. 301**	Closed system entropy rate balance.
$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$	**(6.34) p. 307**	Control volume entropy rate balance.
$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$	**(6.36) p. 308**	Steady-state control volume entropy rate balance.
$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$	**(6.46) p. 323**	Isentropic turbine efficiency.
$\eta_{nozzle} = \frac{V_2^2/2}{(V_2^2/2)_s}$	**(6.47) p. 325**	Isentropic nozzle efficiency.
$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$	**(6.48) p. 328**	Isentropic compressor (and pump) efficiency.

Ideal Gas Model Relations

$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R \ln \frac{v_2}{v_1}$	**(6.17) p. 289**	Change in specific entropy; general form for T and v as independent properties.
$s(T_2, v_2) - s(T_1, v_1) = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$	**(6.21) p. 291**	Constant specific heat, c_v.
$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R \ln \frac{p_2}{p_1}$	**(6.18) p. 289**	Change in specific entropy; general form for T and p as independent properties.
$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}$	**(6.20a) p. 290**	s° for air from Table A-22. ($\bar{s}^\circ$ for other gases from Table A-23).
$s(T_2, p_2) - s(T_1, p_1) = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$	**(6.22) p. 291**	Constant specific heat, c_p.

$\dfrac{p_2}{p_1} = \dfrac{p_{r2}}{p_{r1}}$	**(6.41) p. 317**	$s_1 = s_2$ **(air only),** p_r **and** v_r **from Table A-22.**
$\dfrac{v_2}{v_1} = \dfrac{v_{r2}}{v_{r1}}$	**(6.42) p. 318**	
$\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{(k-1)/k}$	**(6.43) p. 318**	$s_1 = s_2$**, constant specific heat ratio** k**.**
$\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{k-1}$	**(6.44) p. 318**	
$\dfrac{p_2}{p_1} = \left(\dfrac{v_1}{v_2}\right)^{k}$	**(6.45) p. 318**	

▶ EXERCISES: THINGS ENGINEERS THINK ABOUT

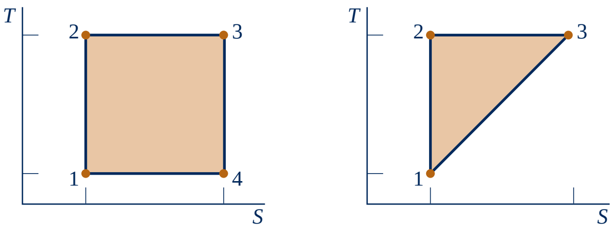
1. Is it possible for entropy *change* to be negative? For entropy *production* to be negative?

2. By what means can entropy be transferred across the boundary of a closed system? Across the boundary of a control volume?

3. Is it possible for the entropy of *both* a closed system and its surroundings to *decrease* during a process? Both to *increase* during a process?

4. What happens to the entropy produced within an insulated, one-inlet, one-exit control volume operating at steady state?

5. The two power cycles shown to the same scale in the figure are composed of internally reversible processes of a closed system. Compare the net work developed by these cycles. Which cycle has the greater thermal efficiency? Explain.



6. Can adiabatic mixing of two substances result in decreased entropy? Explain.

7. Is entropy produced within a system undergoing a *Carnot* cycle? Explain.

8. When a mixture of olive oil and vinegar *spontaneously* separates into two liquid phases, is the second law violated? Explain.

9. A magician claims that simply with a wave of her magic wand a cup of water, initially at room temperature, will be raised in temperature several degrees by quickly picking up energy from its surroundings. Is this possible? Explain.

10. How does the *Bernoulli* equation reduce to give the form used in the bat BIOCONNECTIONS discussion of Sec. 6.13.2?

11. Is Eq. 6.51a restricted to adiabatic processes and thus to isentropic processes? Explain.

12. Using Eq. 6.51c, what data are required to determine the *actual* power input of a basement sump pump?

13. What *is* the ENERGY STAR® program?

▶ PROBLEMS: DEVELOPING ENGINEERING SKILLS

Using Entropy Data and Concepts

6.1 Using the tables for water, determine the specific entropy at the indicated states, in kJ/kg · K. In each case, locate the state by hand on a sketch of the T–s diagram.

(a) $p = 5.0$ MPa, $T = 400$°C.
(b) $p = 5.0$ MPa, $T = 100$°C.
(c) $p = 5.0$ MPa, $u = 1872.5$ kJ/kg.
(d) $p = 5.0$ MPa, saturated vapor.

6.2 Using the tables for water, determine the specific entropy at the indicated states, in Btu/lb · °R In each case, locate the state by hand on a sketch of the T–s diagram.

(a) $p = 1000$ lbf/in.^{2}, $T = 750$°F.
(b) $p = 1000$ lbf/in.^{2}, $T = 300$°F.
(c) $p = 1000$ lbf/in.^{2}, $h = 932.4$ Btu/lb.
(d) $p = 1000$ lbf/in.^{2}, saturated vapor.

6.3 Using the appropriate table, determine the indicated property. In each case, locate the state by hand on sketches of the T–v and T–s diagrams.

(a) water at p = 0.20 bar, s = 4.3703 kJ/kg · K. Find h, in kJ/kg.
(b) water at p = 10 bar, u = 3124.4 kJ/kg. Find s, in kJ/kg · K.
(c) Refrigerant 134a at T = −28°C, x = 0.8. Find s, in kJ/kg · K.
(d) ammonia at T = 20°C, s = 5.0849 kJ/kg · K. Find u, in kJ/kg.

6.4 Using the appropriate table, determine the change in specific entropy between the specified states, in Btu/lb · °R.

(a) water, p_1 = 1000 lbf/in.², T_1 = 800°F, p_2 = 1000 lbf/in.², T_2 = 100°F.
(b) Refrigerant 134a, h_1 = 47.91 Btu/lb, T_1 = −40°F, saturated vapor at p_2 = 40 lbf/in.²
(c) air as an ideal gas, T_1 = 40°F, p_1 = 2 atm, T_2 = 420°F, p_2 = 1 atm.
(d) carbon dioxide as an ideal gas, T_1 = 820°F, p_1 = 1 atm, T_2 = 77°F, p_2 = 3 atm.

6.5 Using *IT*, determine the specific entropy of water at the indicated states. Compare with results obtained from the appropriate table.

(a) Specific entropy, in kJ/kg · K, for the cases of Problem 6.1.
(b) Specific entropy, in Btu/lb · °R, for the cases of Problem 6.2.

6.6 Using *IT*, repeat Prob. 6.4. Compare the results obtained using *IT* with those obtained using the appropriate table.

6.7 Using *steam table* data, determine the indicated property data for a process in which there is no change in specific entropy between state 1 and state 2. In each case, locate the states on a sketch of the T–s diagram.

(a) T_1 = 40°C, x_1 = 100%, p_2 = 150 kPa. Find T_2, in °C, and Δh, in kJ/kg.
(b) T_1 = 10°C, x_1 = 75%, p_2 = 1 MPa. Find T_2, in °C, and Δu, in kJ/kg.

6.8 Using the appropriate table, determine the indicated property for a process in which there is no change in specific entropy between state 1 and state 2.

(a) water, p_1 = 14.7 lbf/in.², T_1 = 500°F, p_2 = 100 lbf/in.² Find T_2 in °F.
(b) water, T_1 = 10°C, x_1 = 0.75, saturated vapor at state 2. Find p_2 in bar.
(c) air as an ideal gas, T_1 = 27°C, p_1 = 1.5 bar, T_2 = 127°C. Find p_2 in bar.
(d) air as an ideal gas, T_1 = 100°F, p_1 = 3 atm, p_2 = 2 atm. Find T_2 in °F.
(e) Refrigerant 134a, T_1 = 20°C, p_1 = 5 bar, p_2 = 1 bar. Find v_2 in m³/kg.

6.9 Using *IT*, obtain the property data requested in (a) Problem 6.7, (b) Problem 6.8, and compare with data obtained from the appropriate table.

6.10 Propane undergoes a process from state 1, where p_1 = 1.4 MPa, T_1 = 60°C, to state 2, where p_2 = 1.0 MPa, during which the change in specific entropy is $s_2 - s_1$ = −0.035 kJ/kg · K. At state 2, determine the temperature, in °C, and the specific enthalpy, in kJ/kg.

6.11 Air in a piston–cylinder assembly undergoes a process from state 1, where T_1 = 300 K, p_1 = 100 kPa, to state 2, where T_2 = 500 K, p_2 = 650 kPa. Using the ideal gas model for air, determine the change in specific entropy between these states, in kJ/kg · K, if the process occurs (a) without internal irreversibilities, (b) with internal irreversibilities.

6.12 Water contained in a closed, rigid tank, initially at 100 lbf/in.², 800°F, is cooled to a final state where the pressure is 20 lbf/in.² Determine the change in specific entropy, in Btu/lb · °R, and show the process on sketches of the T–v and T–s diagrams.

6.13 One-quarter lbmol of nitrogen gas (N_2) undergoes a process from p_1 = 20 lbf/in.², T_1 = 500°R to p_2 = 150 lbf/in.² For the process W = −500 Btu and Q = −125.9 Btu. Employing the ideal gas model, determine

(a) T_2, in °R.
(b) the change in entropy, in Btu/°R.

Show the initial and final states on a T–s diagram.

6.14 One kilogram of water contained in a piston–cylinder assembly, initially at 160°C, 150 kPa, undergoes an isothermal compression process to saturated liquid. For the process, W = −471.5 kJ. Determine for the process,

(a) the heat transfer, in kJ.
(b) the change in entropy, in kJ/K.

Show the process on a sketch of the T–s diagram.

6.15 One-tenth kmol of carbon monoxide (CO) in a piston–cylinder assembly undergoes a process from p_1 = 150 kPa, T_1 = 300 K to p_2 = 500 kPa, T_2 = 370 K. For the process, W = −300 kJ. Employing the ideal gas model, determine

(a) the heat transfer, in kJ.
(b) the change in entropy, in kJ/K.

Show the process on a sketch of the T–s diagram.

6.16 Argon in a piston–cylinder assembly is compressed from state 1, where T_1 = 300 K, V_1 = 1 m³, to state 2, where T_2 = 200 K. If the change in specific entropy is $s_2 - s_1$ = −0.27 kJ/kg · K, determine the final volume, in m³. Assume the ideal gas model with k = 1.67.

6.17 Steam enters a turbine operating at steady state at 1 MPa, 200°C and exits at 40°C with a quality of 83%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the power developed by the turbine, in kJ per kg of steam flowing, (b) the change in specific entropy from inlet to exit, in kJ/K per kg of steam flowing.

6.18 Answer the following true or false. Explain.

(a) The change of entropy of a closed system is the same for every process between two specified states.
(b) The entropy of a fixed amount of an ideal gas increases in every isothermal compression.

(c) The specific internal energy and enthalpy of an ideal gas are each functions of temperature alone but its specific entropy depends on two independent intensive properties.
(d) One of the $T\,ds$ equations has the form $T\,ds = du - p\,dv$.
(e) The entropy of a fixed amount of an incompressible substance increases in every process in which temperature decreases.

6.19 Showing all steps, derive Eqs. 6.43, 6.44, and 6.45.

Analyzing Internally Reversible Processes

6.20 One kilogram of water in a piston–cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.20. For each process, determine, in kJ, the heat transfer and the work.

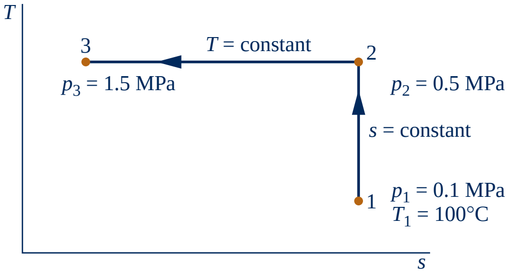


Fig. P6.20

6.21 One kilogram of water in a piston–cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.21. For each process, determine, in kJ, the heat transfer and the work.

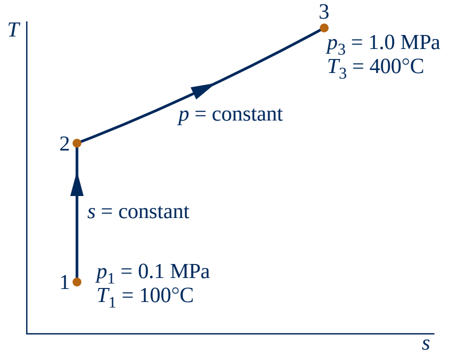


Fig. P6.21

6.22 One kilogram of water in a piston–cylinder assembly, initially at 160°C, 1.5 bar, undergoes an isothermal, internally reversible compression process to the saturated liquid state. Determine the work and heat transfer, each in kJ. Sketch the process on p–v and T–s coordinates. Associate the work and heat transfer with areas on these diagrams.

6.23 One pound mass of water in a piston–cylinder assembly, initially a saturated liquid at 1 atm, undergoes a constant-pressure, internally reversible expansion to $x = 90\%$. Determine the work and heat transfer, each in Btu. Sketch the process on p–v and T–s coordinates. Associate the work and heat transfer with areas on these diagrams.

6.24 A gas within a piston–cylinder assembly undergoes an isothermal process at 400 K during which the change in entropy is -0.3 kJ/K. Assuming the ideal gas model for the gas and negligible kinetic and potential energy effects, evaluate the work, in kJ.

6.25 Water within a piston–cylinder assembly, initially at 10 lbf/in.^{2}, 500°F, undergoes an internally reversible process to 80 lbf/in.^{2}, 800°F, during which the temperature varies linearly with specific entropy. For the water, determine the work and heat transfer, each in Btu/lb. Neglect kinetic and potential energy effects.

6.26 Nitrogen (N_2) initially occupying 0.1 m^3 at 6 bar, 247°C undergoes an internally reversible expansion during which $pV^{1.20} = constant$ to a final state where the temperature is 37°C. Assuming the ideal gas model, determine

(a) the pressure at the final state, in bar.
(b) the work and heat transfer, each in kJ.
(c) the entropy change, in kJ/K.

6.27 Air in a piston–cylinder assembly and modeled as an ideal gas undergoes two internally reversible processes in series from state 1, where $T_1 = 290$ K, $p_1 = 1$ bar.

Process 1–2: Compression to $p_2 = 5$ bar during which $pV^{1.19} = constant$.

Process 2–3: Isentropic expansion to $p_3 = 1$ bar.

(a) Sketch the two processes in series on T–s coordinates.
(b) Determine the temperature at state 2, in K.
(c) Determine the net work, in kJ/kg.

6.28 One lb of oxygen, O_2, in a piston–cylinder assembly undergoes a cycle consisting of the following processes:

Process 1–2: Constant-pressure expansion from $T_1 = 450$°R, $p_1 = 30$ lbf/in.^{2} to $T_2 = 1120$°R.

Process 2–3: Compression to $T_3 = 800$°R and $p_3 = 53.3$ lbf/in.^{2} with $Q_{23} = -60$ Btu.

Process 3–1: Constant-volume cooling to state 1.

Employing the ideal gas model with c_p evaluated at T_1, determine the change in specific entropy, in Btu/lb · °R, for each process. Sketch the cycle on p–v and T–s coordinates.

6.29 One-tenth kilogram of a gas in a piston–cylinder assembly undergoes a Carnot power cycle for which the isothermal expansion occurs at 800 K. The change in specific entropy of the gas during the isothermal compression, which occurs at 400 K, is -25 kJ/kg · K. Determine (a) the net work developed per cycle, in kJ, and (b) the thermal efficiency.

6.30 Figure P6.30 provides the T–s diagram of a Carnot refrigeration cycle for which the substance is Refrigerant 134a. Determine the coefficient of performance.

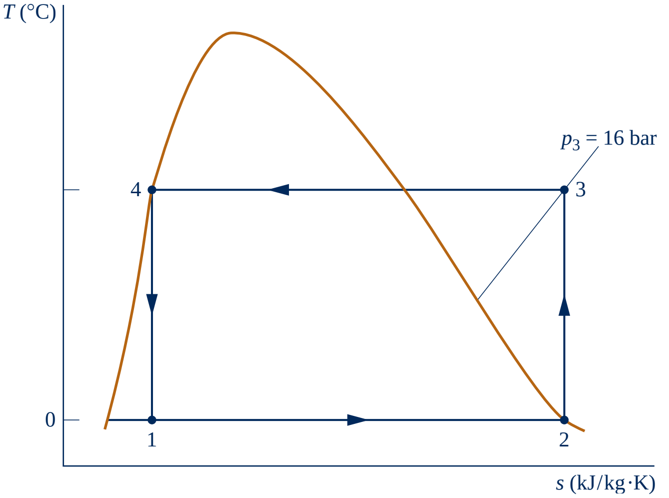


Fig. P6.30

6.31 Figure P6.31 provides the T–s diagram of a Carnot heat pump cycle for which the substance is ammonia. Determine the net work input required, in kJ, for 50 cycles of operation and 0.1 kg of substance.

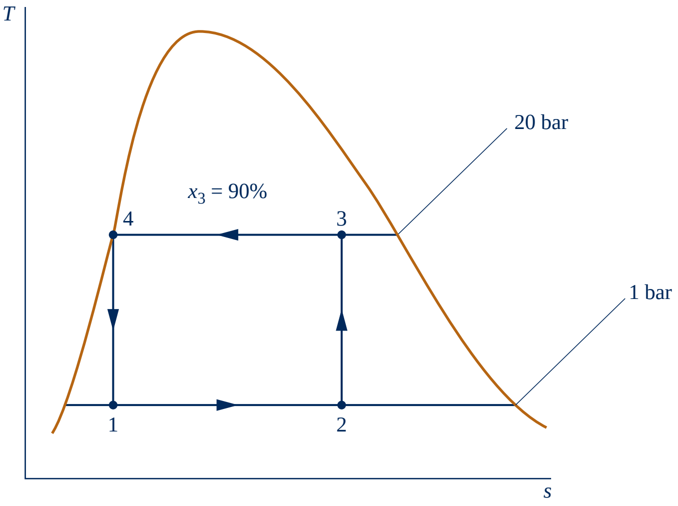


Fig. P6.31

6.32 Air in a piston–cylinder assembly undergoes a Carnot power cycle. The isothermal expansion and compression processes occur at 1400 K and 350 K, respectively. The pressures at the beginning and end of the isothermal compression are 100 kPa and 500 kPa, respectively. Assuming the ideal gas model with $c_p = 1.005$ kJ/kg · K, determine

(a) the pressures at the beginning and end of the isothermal expansion, each in kPa.
(b) the heat transfer and work, in kJ/kg, for each process.
(c) the thermal efficiency.

6.33 Water in a piston–cylinder assembly undergoes a Carnot power cycle. At the beginning of the isothermal expansion, the temperature is 250°C and the quality is 80%. The isothermal expansion continues until the pressure is 2 MPa. The adiabatic expansion then occurs to a final temperature of 175°C.

(a) Sketch the cycle on T–s coordinates.
(b) Determine the heat transfer and work, in kJ/kg, for each process.
(c) Evaluate the thermal efficiency.

6.34 A Carnot power cycle operates at steady state as shown in Fig. 5.15 with water as the working fluid. The boiler pressure is 200 lbf/in.^{2}, with saturated liquid entering and saturated vapor exiting. The condenser pressure is 20 lbf/in.^{2}
(a) Sketch the cycle on T–s coordinates.
(b) Determine the heat transfer and work for each process, in Btu per lb of water flowing.
(c) Evaluate the thermal efficiency.

6.35 Figure P6.35 shows a Carnot heat pump cycle operating at steady state with ammonia as the working fluid. The condenser temperature is 120°F, with saturated vapor entering and saturated liquid exiting. The evaporator temperature is 10°F.

(a) Determine the heat transfer and work for each process, in Btu per lb of ammonia flowing.
(b) Evaluate the coefficient of performance for the heat pump.
(c) Evaluate the coefficient of performance for a Carnot refrigeration cycle operating as shown in the figure.

Applying the Entropy Balance: Closed Systems

6.36 A closed system undergoes a process in which work is done on the system and the heat transfer Q occurs only at temperature T_b. For each case, determine whether the entropy change of the system is positive, negative, zero, or indeterminate.

(a) internally reversible process, $Q > 0$.
(b) internally reversible process, $Q = 0$.
(c) internally reversible process, $Q < 0$.
(d) internal irreversibilities present, $Q > 0$.
(e) internal irreversibilities present, $Q = 0$.
(f) internal irreversibilities present, $Q < 0$.

6.37 Answer the following true or false. Explain.

(a) A process that violates the second law of thermodynamics violates the first law of thermodynamics.
(b) When a net amount of work is done on a closed system undergoing an internally reversible process, a net heat transfer of energy from the system also occurs.
(c) One corollary of the second law of thermodynamics states that the change in entropy of a closed system must be greater than zero or equal to zero.
(d) A closed system can experience an increase in entropy only when irreversibilities are present within the system during the process.
(e) Entropy is produced in every internally reversible process of a closed system.
(f) In an adiabatic and internally reversible process of a closed system, the entropy remains constant.
(g) The energy of an isolated system must remain constant, but the entropy can only decrease.

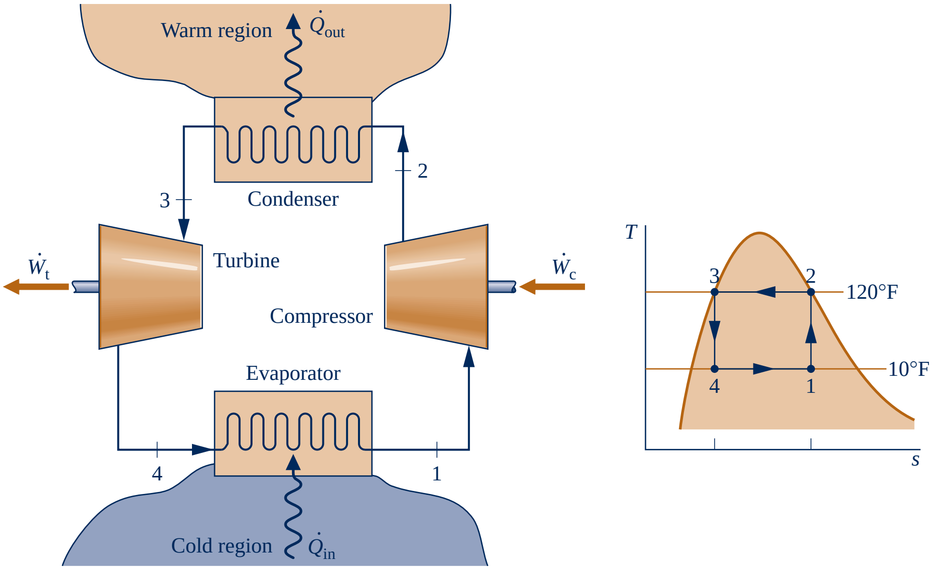


Fig. P6.35

6.38 One lb of water contained in a piston–cylinder assembly, initially saturated vapor at 1 atm, is condensed at constant pressure to saturated liquid. Evaluate the heat transfer, in Btu, and the entropy production, in Btu/°R, for

(a) the water as the system.
(b) an enlarged system consisting of the water and enough of the nearby surroundings that heat transfer occurs only at the ambient temperature, 80°F.

Assume the state of the nearby surroundings does not change during the process of the water, and ignore kinetic and potential energy.

6.39 Five kg of water contained in a piston–cylinder assembly expand from an initial state where $T_1 = 400°C$, $p_1 = 700$ kPa to a final state where $T_2 = 200°C$, $p_2 = 300$ kPa, with no significant effects of kinetic and potential energy. The accompanying table provides additional data at the two states. It is claimed that the water undergoes an adiabatic process between these states, while developing work. Evaluate this claim.

State	T(°C)	p(kPa)	v(m³/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg · K)
1	400	700	0.4397	2960.9	3268.7	7.6350
2	200	300	0.7160	2650.7	2865.5	7.3115

6.40 Two m³ of air in a rigid, insulated container fitted with a paddle wheel is initially at 293 K, 200 kPa. The air receives 710 kJ by work from the paddle wheel. Assuming the ideal gas model with $c_v = 0.72$ kJ/kg · K, determine for the air (a) the mass, in kg, (b) final temperature, in K, and (c) the amount of entropy produced, in kJ/K.

6.41 Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 1 bar, 330 K and a volume of 1.93 m³, receives an energy transfer by work from the paddle wheel in an amount of 400 kJ. Assuming the ideal gas model for the air, determine (a) the final temperature, in K, (b) the final pressure, in bar, and (c) the amount of entropy produced, in kJ/K. Ignore kinetic and potential energy.

6.42 Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 4 bar, 40°C and a volume of 0.2 m³, is stirred until its temperature is 353°C. Assuming the ideal gas model with $k = 1.4$ for the air, determine (a) the final pressure, in bar, (b) the work, in kJ, and (c) the amount of entropy produced, in kJ/K. Ignore kinetic and potential energy.

6.43 Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 300 K, 2 bar, and a volume of 2 m³, is stirred until its temperature is 500 K. Assuming the ideal gas model for the air, and ignoring kinetic and potential energy, determine (a) the final pressure, in bar, (b) the work, in kJ, and (c) the amount of entropy produced, in kJ/K. Solve using

(a) data from Table A-22.
(b) constant c_v read from Table A-20 at 400 K.

Compare the results of parts (a) and (b).

6.44 A rigid, insulated container fitted with a paddle wheel contains 5 lb of water, initially at 260°F and a quality of 60%. The water is stirred until the temperature is 350°F. For the water, determine (a) the work, in Btu, and (b) the amount of entropy produced, in Btu/°R.

6.45 Two kilograms of air contained in a piston–cylinder assembly are initially at 1.5 bar and 400 K. Can a final state at 6 bar and 500 K be attained in an adiabatic process?

6.46 One pound mass of Refrigerant 134a contained within a piston–cylinder assembly undergoes a process from a state where the temperature is 60°F and the refrigerant is saturated liquid to a state where the pressure is 140 lbf/in.² and quality is 50%. Determine the change in specific entropy of the refrigerant, in Btu/lb · °R. Can this process be accomplished adiabatically?

6.47 Refrigerant 134a contained in a piston–cylinder assembly rapidly expands from an initial state where $T_1 = 140°F$, $p_1 = 200$ lbf/in.^{2} to a final state where $p_2 = 5$ lbf/in.^{2} and the quality, x_2, is (a) 99%, (b) 95%. In each case, determine if the process can occur adiabatically. If yes, determine the work, in Btu/lb, for an adiabatic expansion between these states. If no, determine the direction of the heat transfer.

6.48 One kg of air contained in a piston–cylinder assembly undergoes a process from an initial state where $T_1 = 300$ K, $v_1 = 0.8$ m^3/kg to a final state where $T_2 = 420$ K, $v_2 = 0.2$ m^3/kg. Can this process occur adiabatically? If yes, determine the work, in kJ, for an adiabatic process between these states. If no, determine the direction of the heat transfer. Assume the ideal gas model for air.

6.49 Air as an ideal gas contained within a piston–cylinder assembly is compressed between two specified states. In each of the following cases, can the process occur adiabatically? If yes, determine the work in appropriate units for an adiabatic process between these states. If no, determine the direction of the heat transfer.

(a) State 1: $p_1 = 0.1$ MPa, $T_1 = 27°C$. State 2: $p_2 = 0.5$ MPa, $T_2 = 207°C$. Use Table A-22 data.
(b) State 1: $p_1 = 3$ atm, $T_1 = 80°F$ State 2: $p_2 = 10$ atm, $T_2 = 240°F$. Assume $c_p = 0.241$ Btu/lb°R.

6.50 One kilogram of propane initially at 8 bar and 50°C undergoes a process to 3 bar, 20°C while being rapidly expanded in a piston–cylinder assembly. Heat transfer between the propane and its surroundings occurs at an average temperature of 35°C. The work done by the propane is measured as 42.4 kJ. Kinetic and potential energy effects can be ignored. Determine whether it is possible for the work measurement to be correct.

6.51 As shown in Fig. P6.51, a divider separates 1 lb mass of carbon monoxide (CO) from a thermal reservoir at 150°F. The carbon monoxide, initially at 60°F and 150 lbf/in.^{2}, expands isothermally to a final pressure of 10 lbf/in.^{2} while receiving heat transfer through the divider from the reservoir. The carbon monoxide can be modeled as an ideal gas.

(a) For the carbon monoxide as the system, evaluate the work and heat transfer, each in Btu, and the amount of entropy produced, in Btu/°R.

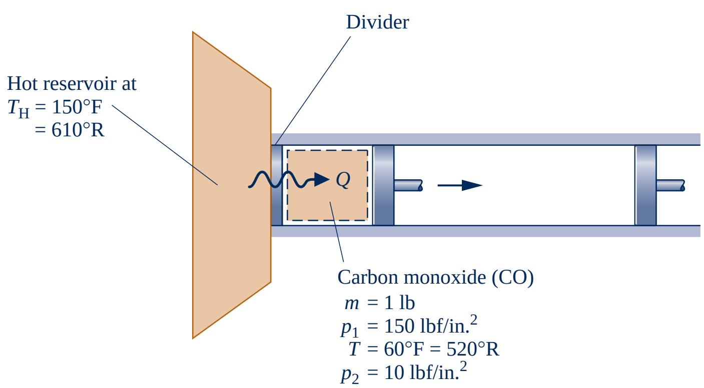


Fig. P6.51

(b) Evaluate the entropy production, in Btu/°R, for an enlarged system that includes the carbon monoxide and the divider, assuming the state of the divider remains unchanged. Compare with the entropy production of part (a) and comment on the difference.

6.52 Three kilograms of Refrigerant 134a initially a saturated vapor at 20°C expand to 3.2 bar, 20°C. During this process, the temperature of the refrigerant departs by no more than 0.01°C from 20°C. Determine the maximum theoretical heat transfer to the refrigerant during the process, in kJ.

6.53 An inventor claims that the device shown in Fig. P6.53 generates electricity while receiving a heat transfer at the rate of 250 Btu/s at a temperature of 500°R, a second heat transfer at the rate of 350 Btu/s at 700°R, and a third at the rate of 500 Btu/s at 1000°R. For operation at steady state, evaluate this claim.

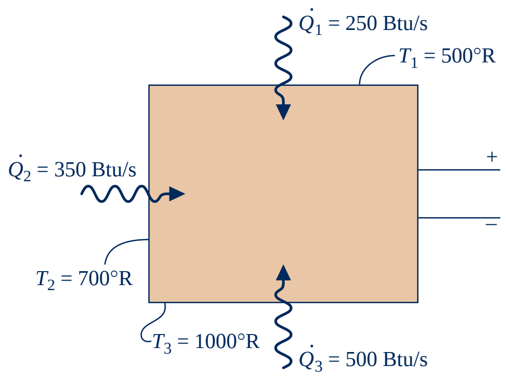


Fig. P6.53

6.54 For the silicon chip of Example 2.5, determine the rate of entropy production, in kW/K. What is the cause of entropy production in this case?

6.55 At steady state, the 20-W curling iron shown in Fig. P6.55 has an outer surface temperature of 180°F. For the curling iron, determine the rate of heat transfer, in Btu/h, and the rate of entropy production, in Btu/h · °R.

6.56 A rigid, insulated vessel is divided into two compartments connected by a valve. Initially, one compartment, occupying one-third of the total volume, contains air at 500°R, and the other is evacuated. The valve is opened and the air is allowed to fill the entire volume. Assuming the ideal gas model, determine the final temperature of the air, in °R, and the amount of entropy produced, in Btu/°R per lb of air.

6.57 A rigid, insulated vessel is divided into two equal-volume compartments connected by a valve. Initially, one compartment contains 1 m^3 of water at 20°C, $x = 50\%$, and the other is evacuated. The valve is opened and the water is allowed to fill the entire volume. For the water, determine the final temperature, in °C, and the amount of entropy produced, in kJ/K.

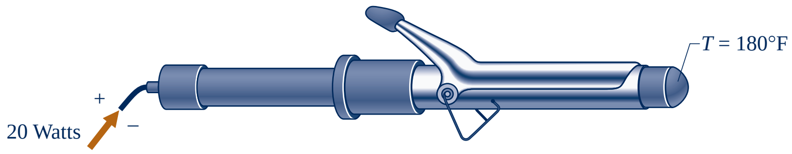


Fig. P6.55

6.58 An electric motor at steady state draws a current of 10 amp with a voltage of 110 V. The output shaft develops a torque of 10.2 N · m and a rotational speed of 1000 RPM.

(a) If the outer surface of the motor is at 42°C, determine the rate of entropy production within the motor, in kW/K.
(b) Evaluate the rate of entropy production, in kW/K, for an enlarged system that includes the motor and enough of the nearby surroundings that heat transfer occurs at the ambient temperature, 22°C.

6.59 A power plant has a turbogenerator, shown in Fig. P6.59, operating at steady state with an input shaft rotating at 1800 RPM with a torque of 16,700 N · m. The turbogenerator produces current at 230 amp with a voltage of 13,000 V. The rate of heat transfer between the turbogenerator and its surroundings is related to the surface temperature T_b and the lower ambient temperature T_0, and is given by $\dot{Q} = -hA(T_b - T_0)$, where h = 110 W/m^2 · K, A = 32 m^2, and T_0 = 298 K.

(a) Determine the temperature T_b, in K.
(b) For the turbogenerator as the system, determine the rate of entropy production, in kW/K.
(c) If the system boundary is located to take in enough of the nearby surroundings for heat transfer to take place at temperature T_0, determine the rate of entropy production, in kW/K, for the enlarged system.

6.60 At steady state, work is done by a paddle wheel on a *slurry* contained within a closed, rigid tank whose outer surface temperature is 245°C. Heat transfer from the tank and its contents occurs at a rate of 50 kW to surroundings that, away from the immediate vicinity of the tank, are at 27°C. Determine the rate of entropy production, in kW/K,

(a) for the tank and its contents as the system.
(b) for an enlarged system including the tank and enough of the nearby surroundings for the heat transfer to occur at 27°C.

6.61 A 33.8-lb aluminum bar, initially at 200°F, is placed in a tank together with 249 lb of liquid water, initially at 70°F, and allowed to achieve thermal equilibrium. The aluminum bar and water can be modeled as incompressible with specific heats 0.216 Btu/lb · °R and 0.998 Btu/lb · °R, respectively. For the aluminum bar and water as the system, determine (a) the final temperature, in °F, and (b) the amount of entropy produced within the tank, in Btu/°R. Ignore heat transfer between the system and its surroundings.

6.62 In a heat-treating process, a 1-kg metal part, initially at 1075 K, is quenched in a tank containing 100 kg of water, initially at 295 K. There is negligible heat transfer between the contents of the tank and their surroundings. The metal part and water can be modeled as incompressible with specific heats 0.5 kJ/kg · K and 4.2 kJ/kg · K, respectively. Determine (a) the final equilibrium temperature after quenching, in K, and (b) the amount of entropy produced within the tank, in kJ/K.

6.63 A 50-lb iron casting, initially at 700°F, is quenched in a tank filled with 2121 lb of oil, initially at 80°F. The iron casting and oil can be modeled as incompressible with specific heats 0.10 Btu/lb · °R, and 0.45 Btu/lb · °R, respectively. For the iron casting and oil as the system, determine (a) the final equilibrium temperature, in °F, and (b) the amount of entropy produced within the tank, in Btu/°R. Ignore heat transfer between the system and its surroundings.

6.64 A 2.64-kg copper part, initially at 400 K, is plunged into a tank containing 4 kg of liquid water, initially at 300 K. The copper part and water can be modeled as incompressible with specific heats 0.385 kJ/kg · K and 4.2 kJ/kg · K, respectively. For the copper part and water as the system, determine (a) the final equilibrium temperature, in K, and (b) the amount of entropy produced within the tank, in kJ/K. Ignore heat transfer between the system and its surroundings.

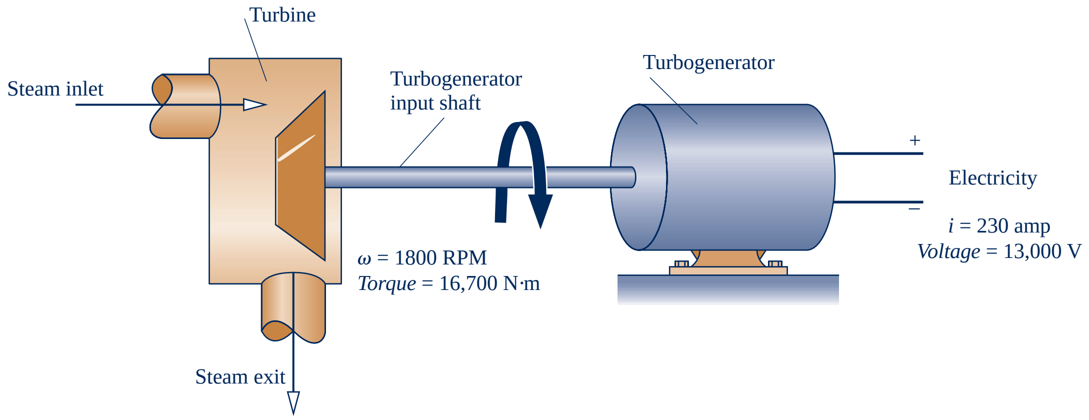


Fig. P6.59

6.65 Two insulated tanks are connected by a valve. One tank initially contains 1.2 lb of air at 240°F, 30 psia, and the other contains 1.5 lb of air at 60°F, 14.7 psia. The valve is opened and the two quantities of air are allowed to mix until equilibrium is attained. Employing the ideal gas model with $c_v = 0.18$ Btu/lb · °R determine

(a) the final temperature, in °F.
(b) the final pressure, in psia.
(c) the amount of entropy produced, in Btu/°R.

6.66 As shown in Fig. P6.66, an insulated box is initially divided into halves by a frictionless, thermally conducting piston. On one side of the piston is 1.5 m^3 of air at 400 K, 4 bar. On the other side is 1.5 m^3 of air at 400 K, 2 bar. The piston is released and equilibrium is attained, with the piston experiencing no change of state. Employing the ideal gas model for the air, determine

(a) the final temperature, in K.
(b) the final pressure, in bar.
(c) the amount of entropy produced, in kJ/kg.

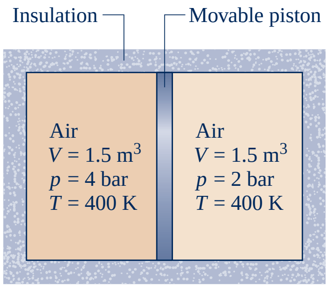


Fig. P6.66

6.67 An insulated vessel is divided into two equal-sized compartments connected by a valve. Initially, one compartment contains steam at 50 lbf/in.^{2} and 700°F, and the other is evacuated. The valve is opened and the steam is allowed to fill the entire volume. Determine

(a) the final temperature, in °F.
(b) the amount of entropy produced, in Btu/lb · °R.

6.68 An insulated, rigid tank is divided into two compartments by a frictionless, thermally conducting piston. One compartment initially contains 1 m^3 of saturated water vapor at 4 MPa and the other compartment contains 1 m^3 of water vapor at 20 MPa, 800°C. The piston is released and equilibrium is attained, with the piston experiencing no change of state. For the water as the system, determine

(a) the final pressure, in MPa.
(b) the final temperature, in °C.
(c) the amount of entropy produced, in kJ/K.

6.69 A system consisting of air initially at 300 K and 1 bar experiences the two different types of interactions described below. In each case, the system is brought from the initial state to a state where the temperature is 500 K, while volume remains constant.

(a) The temperature rise is brought about adiabatically by stirring the air with a paddle wheel. Determine the amount of entropy produced, in kJ/kg · K.
(b) The temperature rise is brought about by heat transfer from a reservoir at temperature T. The temperature at the system boundary where heat transfer occurs is also T. Plot the amount of entropy produced, in kJ/kg · K, versus T for $T \geq 500$ K. Compare with the result of (a) and discuss.

6.70 A cylindrical copper rod of base area A and length L is insulated on its lateral surface. One end of the rod is in contact with a wall at temperature T_H. The other end is in contact with a wall at a lower temperature T_C. At steady state, the rate at which energy is conducted into the rod from the hot wall is

$$\dot{Q}_H = \frac{\kappa A(T_H - T_C)}{L}$$

where κ is the thermal conductivity of the copper rod.

(a) For the rod as the system, obtain an expression for the time rate of entropy production in terms of A, L, T_H, T_C, and κ.
(b) If $T_H = 327°C$, $T_C = 77°C$, $\kappa = 0.4$ kW/m · K, A = 0.1 m^2, plot the heat transfer rate $\dot{Q}_H$, in kW, and the time rate of entropy production, in kW/K, each versus L ranging from 0.01 to 1.0 m. Discuss.

6.71 Figure P6.71 shows a system consisting of air in a rigid container fitted with a paddle wheel and in contact with a thermal energy reservoir. By heating and/or stirring, the air can achieve a specified increase in temperature from T_1 to T_2 in alternative ways. Discuss how the temperature increase of the air might be achieved with (a) minimum entropy production, and (b) maximum entropy production. Assume that the temperature on the boundary where heat transfer to the air occurs, T_b, is the same as the reservoir temperature. Let $T_1 < T_b < T_2$. The ideal gas model applies to the air.

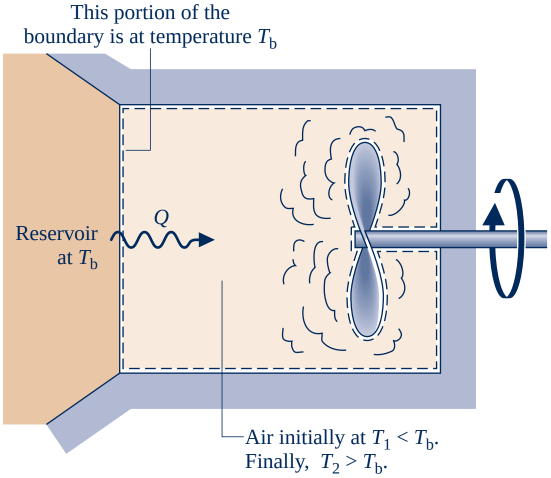


Fig. P6.71

6.72 An isolated system of total mass m is formed by mixing two equal masses of the same liquid initially at the temperatures T_1 and T_2. Eventually, the system attains an equilibrium state. Each mass is incompressible with constant specific heat c.

(a) Show that the amount of entropy produced is

$$\sigma = mc \ln\left[\frac{T_1 + T_2}{2(T_1T_2)^{1/2}}\right]$$

(b) Demonstrate that σ must be positive.

6.73 A cylindrical rod of length L insulated on its lateral surface is initially in contact at one end with a wall at temperature T_H and at the other end with a wall at a lower temperature T_C. The temperature within the rod initially varies linearly with position z according to

$$T(z) = T_H - \left(\frac{T_H - T_C}{L}\right)z$$

The rod is then insulated on its ends and eventually comes to a final equilibrium state where the temperature is T_f. Evaluate T_f in terms of T_H and T_C and show that the amount of entropy produced is

$$\sigma = mc\left(1 + \ln T_f + \frac{T_C}{T_H - T_C}\ln T_C - \frac{T_H}{T_H - T_C}\ln T_H\right)$$

where c is the specific heat of the rod.

6.74 A system undergoing a thermodynamic cycle receives Q_H at temperature T'_H and discharges Q_C at temperature T'_C. There are no other heat transfers.

(a) Show that the net work developed per cycle is given by

$$W_{cycle} = Q_H\left(1 - \frac{T'_C}{T'_H}\right) - T'_C\sigma$$

where σ is the amount of entropy produced per cycle owing to irreversibilities within the system.
(b) If the heat transfers Q_H and Q_C are with hot and cold reservoirs, respectively, what is the relationship of T'_H to the temperature of the hot reservoir T_H and the relationship of T'_C to the temperature of the cold reservoir T_C?
(c) Obtain an expression for W_{cycle} if there are (i) no internal irreversibilities, (ii) no internal *or* external irreversibilities.

6.75 A thermodynamic power cycle receives energy by heat transfer from an incompressible body of mass m and specific heat c initially at temperature T_H. The cycle discharges energy by heat transfer to another incompressible body of mass m and specific heat c initially at a lower temperature T_C. There are no other heat transfers. Work is developed by the cycle until the temperature of each of the two bodies is the same. Develop an expression for the maximum theoretical amount of work that can be developed, W_{max}, in terms of m, c, T_H, and T_C.

6.76 At steady state, an insulated mixing chamber receives two liquid streams of the same substance at temperatures T_1 and T_2 and mass flow rates $\dot{m}_1$ and $\dot{m}_2$, respectively. A single stream exits at T_3 and $\dot{m}_3$. Using the incompressible substance model with constant specific heat c, obtain an expression for

(a) T_3 in terms of T_1, T_2, and the ratio of mass flow rates $\dot{m}_1/\dot{m}_3$.
(b) the rate of entropy production per unit of mass exiting the chamber in terms of c, T_1/T_2 and $\dot{m}_1/\dot{m}_3$.
(c) For fixed values of c and T_1/T_2, determine the value of $\dot{m}_1/\dot{m}_3$ for which the rate of entropy production is a maximum.

6.77 The temperature of an incompressible substance of mass m and specific heat c is reduced from T_0 to T ($< T_0$) by a refrigeration cycle. The cycle receives energy by heat transfer at T from the substance and discharges energy by heat transfer at T_0 to the surroundings. There are no other heat transfers. Plot (W_{min}/mcT_0) versus T/T_0 ranging from 0.8 to 1.0, where W_{min} is the minimum theoretical work *input* required.

6.78 The temperature of a 12-oz (0.354-L) can of soft drink is reduced from 20 to 5°C by a refrigeration cycle. The cycle receives energy by heat transfer from the soft drink and discharges energy by heat transfer at 20°C to the surroundings. There are no other heat transfers. Determine the minimum theoretical work input required, in kJ, assuming the soft drink is an incompressible liquid with the properties of liquid water. Ignore the aluminum can.

6.79 As shown in Fig. P6.79, a turbine is located between two tanks. Initially, the smaller tank contains steam at 3.0 MPa, 280°C and the larger tank is evacuated. Steam is allowed to flow from the smaller tank, through the turbine, and into the larger tank until equilibrium is attained. If heat transfer with the surroundings is negligible, determine the maximum theoretical work that can be developed, in kJ.

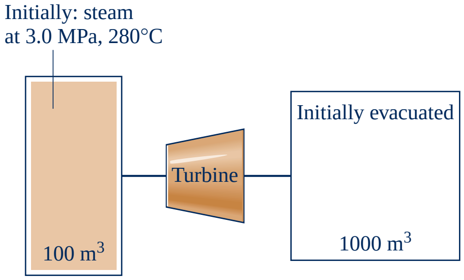


Fig. P6.79

Applying the Entropy Balance: Control Volumes

6.80 A gas flows through a one-inlet, one-exit control volume operating at steady state. Heat transfer at the rate $\dot{Q}_{cv}$ takes place only at a location on the boundary where the temperature is T_b. For each of the following cases, determine whether the specific entropy of the gas at the exit is greater than, equal to, or less than the specific entropy of the gas at the inlet:

(a) no internal irreversibilities, $\dot{Q}_{cv} = 0$.
(b) no internal irreversibilities, $\dot{Q}_{cv} < 0$.
(c) no internal irreversibilities, $\dot{Q}_{cv} > 0$.
(d) internal irreversibilities, $\dot{Q}_{cv} \geq 0$.

6.81 Steam at 15 bar, 540°C, 60 m/s enters an insulated turbine operating at steady state and exits at 1.5 bar, 89.4 m/s. The work developed per kg of steam flowing is claimed to be (a) 606.0 kJ/kg, (b) 765.9 kJ/kg. Can either claim be correct? Explain.

6.82 Air enters an insulated turbine operating at steady state at 8 bar, 1127°C and exits at 1.5 bar, 347°C. Neglecting kinetic and potential energy changes and assuming the ideal gas model for the air, determine

(a) the work developed, in kJ per kg of air flowing through the turbine.
(b) whether the expansion is internally reversible, irreversible, or impossible.

6.83 Water at 20 bar, 400°C enters a turbine operating at steady state and exits at 1.5 bar. Stray heat transfer and kinetic and potential energy effects are negligible. A hard-to-read data sheet indicates that the quality at the turbine exit is 98%. Can this quality value be correct? If no, explain. If yes, determine the power developed by the turbine, in kJ per kg of water flowing.

6.84 Air enters a compressor operating at steady state at 15 lbf/in.^{2}, 80°F and exits at 400°F. Stray heat transfer and kinetic and potential energy effects are negligible. Assuming the ideal gas model for the air, determine the maximum theoretical pressure at the exit, in lbf/in.^{2}

6.85 Propane at 0.1 MPa, 20°C enters an insulated compressor operating at steady state and exits at 0.4 MPa, 90°C Neglecting kinetic and potential energy effects, determine

(a) the power required by the compressor, in kJ per kg of propane flowing.
(b) the rate of entropy production within the compressor, in kJ/K per kg of propane flowing.

6.86 By injecting liquid water into superheated steam, the *desuperheater* shown in Fig. P6.86 has a saturated vapor stream at its exit. Steady-state operating data are provided in the accompanying table. Stray heat transfer and all kinetic and potential energy effects are negligible. (a) Locate states 1, 2, and 3 on a sketch of the T–s diagram. (b) Determine the rate of entropy production within the desuperheater, in kW/K.

State	p(MPa)	T(°C)	$v \times 10^3$(m^3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg · K)
1	2.7	40	1.0066	167.2	169.9	0.5714
2	2.7	300	91.01	2757.0	3002.8	6.6001
3	2.5	sat. vap.	79.98	2603.1	2803.1	6.2575

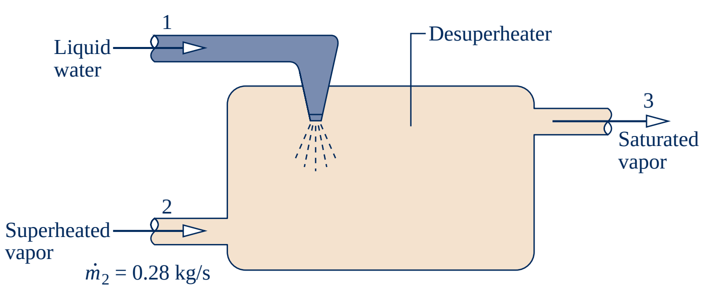


Fig. P6.86

6.87 An inventor claims that at steady state the device shown in Fig. P6.87 develops power from entering and exiting streams of water at a rate of 1174.9 kW. The accompanying table provides data for inlet 1 and exits 3 and 4. The pressure at inlet 2 is 1 bar. Stray heat transfer and kinetic and potential energy effects are negligible. Evaluate the inventor's claim.

State	$\dot{m}$(kg/s)	p(bar)	T(°C)	v(m^3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg · K)
1	4	1	450	3.334	3049.0	3382.4	8.6926
3	5	2	200	1.080	2654.4	2870.5	7.5066
4	3	4	400	0.773	2964.4	3273.4	7.8985

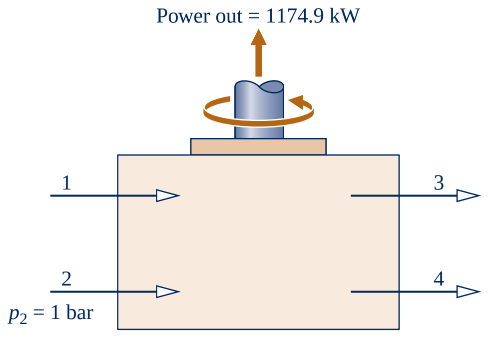


Fig. P6.87

6.88 Figure P6.88 provides steady-state operating data for a well-insulated device having steam entering at one location and exiting at another. Neglecting kinetic and potential energy effects, determine (a) the direction of flow and (b) the power output or input, as appropriate, in kJ per kg of steam flowing.

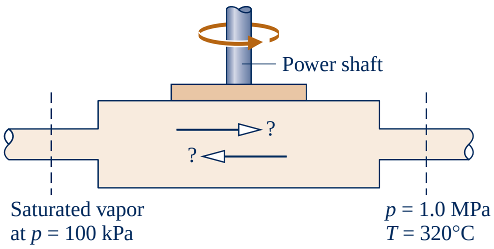


Fig. P6.88

6.89 Steam enters a well-insulated nozzle operating at steady state at 1000°F, 500 lbf/in.^{2} and a velocity of 10 ft/s. At the nozzle exit, the pressure is 14.7 lbf/in.^{2} and the velocity is 4055 ft/s. Determine the rate of entropy production, in Btu/°R per lb of steam flowing.

6.90 Air at 400 kPa, 970 K enters a turbine operating at steady state and exits at 100 kPa, 670 K. Heat transfer from the turbine occurs at an average outer surface temperature of 315 K at the rate of 30 kJ per kg of air flowing. Kinetic and potential energy effects are negligible. For air as an ideal gas with c_p = 1.1 kJ/kg · K, determine (a) the rate power is developed, in kJ per kg of air flowing, and (b) the rate of entropy production within the turbine, in kJ/K per kg of air flowing.

6.91 Steam at 240°C, 700 kPa enters an open feedwater heater operating at steady state with a mass flow rate of 0.5 kg/s. A separate stream of liquid water enters at 45°C, 700 kPa with a mass flow rate of 4 kg/s. A single mixed stream exits

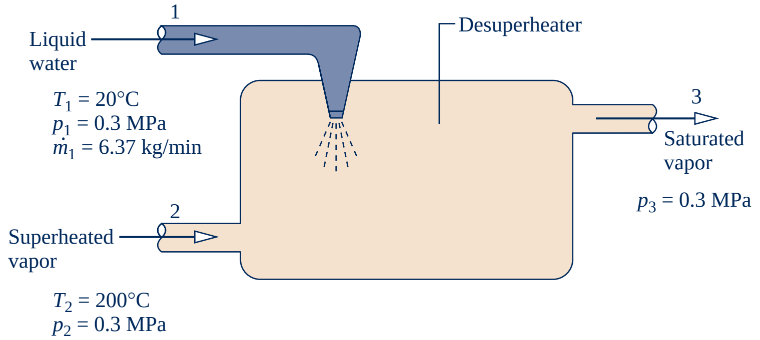


Fig. P6.92

at 700 kPa and temperature T. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine (a) T, in °C, and (b) the rate of entropy production within the feedwater heater, in kW/K. (c) Locate the three principal states on a sketch of the T–s diagram.

6.92 By injecting liquid water into superheated vapor, the *desuperheater* shown in Fig. P6.92 has a saturated vapor stream at its exit. Steady-state operating data are shown on the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine (a) the mass flow rate of the superheated vapor stream, in kg/min, and (b) the rate of entropy production within the desuperheater, in kW/K.

6.93 Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, (b) the mass flow rate, in kg/s, and (c) the rate of entropy production, in kW/K.

6.94 At steady state, air at 200 kPa, 52°C and a mass flow rate of 0.5 kg/s enters an insulated duct having differing inlet and exit cross-sectional areas. At the duct exit, the pressure of the air is 100 kPa, the velocity is 255 m/s, and the cross-sectional area is 2×10^{-3} m^2. Assuming the ideal gas model, determine

(a) the temperature of the air at the exit, in °C.
(b) the velocity of the air at the inlet, in m/s.
(c) the inlet cross-sectional area, in m^2.
(d) the rate of entropy production within the duct, in kW/K.

6.95 For the computer of Example 4.8, determine the rate of entropy production, in W/K, when air exits at 32°C. Ignore the change in pressure between the inlet and exit.

6.96 Electronic components are mounted on the inner surface of a horizontal cylindrical duct whose inner diameter is 0.2 m, as shown in Fig. P6.96. To prevent overheating of the electronics, the cylinder is cooled by a stream of air flowing through it and by convection from its outer surface. Air enters the duct at 25°C, 1 bar and a velocity of 0.3 m/s and exits at 40°C with negligible changes in kinetic energy and pressure. Convective cooling occurs on the outer surface to the surroundings, which are at 25°C, in accord with hA = 3.4 W/K, where h is the heat transfer coefficient and A is the surface area. The electronic components require 0.20 kW of electric power. For a control volume enclosing the cylinder, determine at steady state (a) the mass flow rate of the air, in kg/s, (b) the temperature on the outer surface of the duct, in °C, and (c) the rate of entropy production, in W/K. Assume the ideal gas model for air.

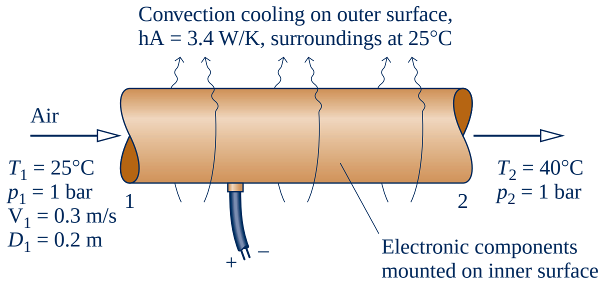


Fig. P6.96

6.97 Air enters a turbine operating at steady state at 500 kPa, 860 K and exits at 100 kPa. A temperature sensor indicates that the exit air temperature is 460 K. Stray heat transfer and kinetic and potential energy effects are negligible, and the air can be modeled as an ideal gas. Determine if the exit temperature reading can be correct. It yes, determine the power developed by the turbine for an expansion between these states, in kJ per kg of air flowing. If no, provide an explanation with supporting calculations.

6.98 Figure P6.98 provides steady-state test data for a control volume in which two entering streams of air mix to form a single exiting stream. Stray heat transfer and kinetic and potential energy effects are negligible. A hard-to-read photocopy of the data sheet indicates that the pressure of the exiting stream is either 1.0 MPa or 1.8 MPa. Assuming the ideal gas model for air with c_p = 1.02 kJ/kg · K, determine if either or both of these pressure values can be correct.

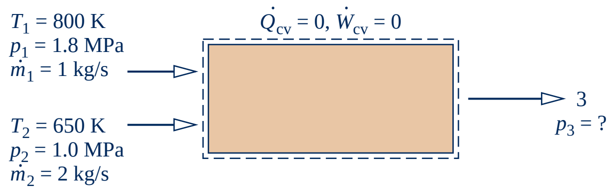


Fig. P6.98

6.99 Hydrogen gas (H_2) at 35°C and pressure p enters an insulated control volume operating at steady state for which $\dot{W}_{cv} = 0$. Half of the hydrogen exits the device at 2 bar and 90°C and the other half exits at 2 bar and −20°C. The effects of kinetic and potential energy are negligible. Employing the ideal gas model with constant c_p = 14.3 kJ/kg · K, determine the minimum possible value for the inlet pressure p, in bar.

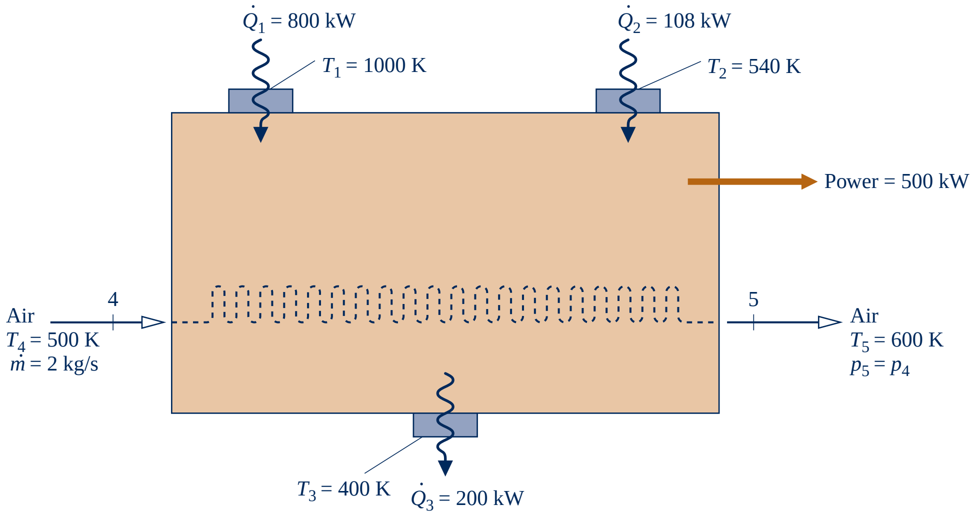


Fig. P6.101

6.100 An engine takes in streams of water at 120°C, 5 bar and 240°C, 5 bar. The mass flow rate of the higher temperature stream is three times that of the other. A single stream exits at 5 bar with a mass flow rate of 4 kg/s. There is no significant heat transfer between the engine and its surroundings, and kinetic and potential energy effects are negligible. For operation at steady state, determine the rate at which power is developed in the absence of internal irreversibilities, in kW.

6.101 An inventor has provided the steady-state operating data shown in Fig. P6.101 for a *cogeneration* system producing power and increasing the temperature of a stream of air. The system receives and discharges energy by heat transfer at the rates and temperatures indicated on the figure. All heat transfers are in the directions of the accompanying arrows. The ideal gas model applies to the air. Kinetic and potential energy effects are negligible. Using energy and entropy rate balances, evaluate the thermodynamic performance of the system.

6.102 Steam at 550 lbf/in.^{2}, 700°F enters an insulated turbine operating at steady state with a mass flow rate of 1 lb/s. A two-phase liquid–vapor mixture exits the turbine at 14.7 lbf/in.^{2} with quality x. Plot the power developed, in Btu/s, and the rate of entropy production, in Btu/°R · s, each versus x.

6.103 Refrigerant 134a at 30 lbf/in.^{2}, 40°F enters a compressor operating at steady state with a mass flow rate of 150 lb/h and exits as saturated vapor at 160 lbf/in.^{2} Heat transfer occurs from the compressor to its surroundings, which are at 40°F. Changes in kinetic and potential energy can be ignored. A power input of 0.5 hp is claimed for the compressor. Determine whether this claim can be correct.

6.104 Ammonia enters a horizontal 0.2-m-diameter pipe at 2 bar with a quality of 90% and velocity of 5 m/s and exits at 1.75 bar as saturated vapor. Heat transfer to the pipe from the surroundings at 300 K takes place at an average outer surface temperature of 253 K. For operation at steady state, determine

(a) the velocity at the exit, in m/s.
(b) the rate of heat transfer to the pipe, in kW.
(c) the rate of entropy production, in kW/K, for a control volume comprising only the pipe and its contents.
(d) the rate of entropy production, in kW/K, for an enlarged control volume that includes the pipe and enough of its immediate surroundings so that heat transfer from the control volume occurs at 300 K.

6.105 Air at 500 kPa, 500 K and a mass flow of 600 kg/h enters a pipe passing overhead in a factory space. At the pipe exit, the pressure and temperature of the air are 475 kPa and 450 K, respectively. Air can be modeled as an ideal gas with $k = 1.39$. Kinetic and potential energy effects can be ignored. Determine at steady state, (a) the rate of heat transfer, in kW, for a control volume comprising the pipe and its contents, and (b) the rate of entropy production, in kW/K, for an enlarged control volume that includes the pipe and enough of its surroundings that heat transfer occurs at the ambient temperature, 300 K.

6.106 Steam enters a turbine operating at steady state at 6 MPa, 600°C with a mass flow rate of 125 kg/min and exits as saturated vapor at 20 kPa, producing power at a rate of 2 MW. Kinetic and potential energy effects can be ignored. Determine (a) the rate of heat transfer, in kW, for a control volume including the turbine and its contents, and (b) the rate of entropy production, in kW/K, for an enlarged control volume that includes the turbine and enough of its surroundings that heat transfer occurs at the ambient temperature, 27°C.

6.107 Air enters a compressor operating at steady state at 1 bar, 22°C with a volumetric flow rate of 1 m^3/min and is compressed to 4 bar, 177°C. The power input is 3.5 kW. Employing the ideal gas model and ignoring kinetic and potential energy effects, obtain the following results:

(a) For a control volume enclosing the compressor only, determine the heat transfer rate, in kW, and the change in specific entropy from inlet to exit, in kJ/kg · K. What additional information would be required to evaluate the rate of entropy production?
(b) Calculate the rate of entropy production, in kW/K, for an enlarged control volume enclosing the compressor and a portion of its immediate surroundings so that heat transfer occurs at the ambient temperature, 22°C.

6.108 Carbon monoxide (CO) enters a nozzle operating at steady state at 25 bar, 257°C, and 45 m/s. At the nozzle exit, the conditions are 2 bar, 57°C, 560 m/s, respectively. The carbon monoxide can be modeled as an ideal gas.

(a) For a control volume enclosing the nozzle only, determine the heat transfer, in kJ, and the change in specific entropy, in kJ/K, each per kg of carbon monoxide flowing through the nozzle. What additional information would be required to evaluate the rate of entropy production?
(b) Evaluate the rate of entropy production, in kJ/K per kg of carbon monoxide flowing, for an enlarged control volume enclosing the nozzle and a portion of its immediate surroundings so that the heat transfer occurs at the ambient temperature, 27°C.

6.109 A counterflow heat exchanger operates at steady state with negligible kinetic and potential energy effects. In one stream, liquid water enters at 10°C and exits at 20°C with a negligible change in pressure. In the other stream, Refrigerant 134a enters at 10 bar, 80°C with a mass flow rate of 135 kg/h and exits at 10 bar, 20°C. The liquid water can be modeled as incompressible with c = 4.179 kJ/kg · K. Heat transfer from the outer surface of the heat exchanger can be ignored. Determine

(a) the mass flow rate of the liquid water, in kg/h.
(b) the rate of entropy production within the heat exchanger, in kW/K.

6.110 Saturated water vapor at 100 kPa enters a counterflow heat exchanger operating at steady state and exits at 20°C with a negligible change in pressure. Ambient air at 275 K, 1 atm enters in a separate stream and exits at 290 K, 1 atm. The air mass flow rate is 170 times that of the water. The air can be modeled as an ideal gas with c_p = 1.005 kJ/kg · K. Kinetic and potential energy effects can be ignored.

(a) For a control volume enclosing the heat exchanger, evaluate the rate of heat transfer, in kJ per kg of water flowing.
(b) For an enlarged control volume that includes the heat exchanger and enough of its immediate surroundings that heat transfer from the control volume occurs at the ambient temperature, 275 K, determine the rate of entropy production, in kJ/K per kg of water flowing.

6.111 Figure P6.111 shows data for a portion of the ducting in a ventilation system operating at steady state. The ducts are well insulated and the pressure is very nearly 1 atm throughout. Assuming the ideal gas model for air with c_p = 0.24 Btu/lb · °R, and ignoring kinetic and potential energy effects, determine (a) the temperature of the air at the exit, in °F, (b) the exit diameter, in ft, and (c) the rate of entropy production within the duct, in Btu/min · °R.

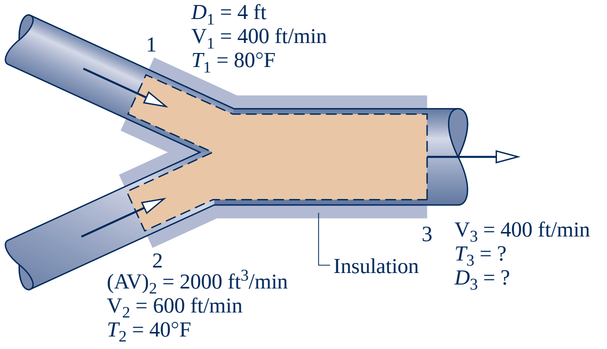


Fig. P6.111

6.112 Air flows through an insulated circular duct having a diameter of 2 cm. Steady-state pressure and temperature data obtained by measurements at two locations, denoted as 1 and 2, are given in the accompanying table. Modeling air as an ideal gas with c_p = 1.005 kJ/kg · K, determine (a) the direction of the flow, (b) the velocity of the air, in m/s, at each of the two locations, and (c) the mass flow rate of the air, in kg/s.

Measurement location	1	2
Pressure (kPa)	100	500
Temperature (°C)	20	50

6.113 Determine the rates of entropy production, in Btu/min · °R, for the steam generator and turbine of Example 4.10. Identify the component that contributes more to inefficient operation of the overall system.

6.114 Air as an ideal gas flows through the compressor and heat exchanger shown in Fig. P6.114. A separate liquid water stream also flows through the heat exchanger. The data given are for operation at steady state. Stray heat transfer to the surroundings can be neglected, as can all kinetic and potential energy changes. Determine

(a) the compressor power, in kW, and the mass flow rate of the cooling water, in kg/s.
(b) the rates of entropy production, each in kW/K, for the compressor and heat exchanger.

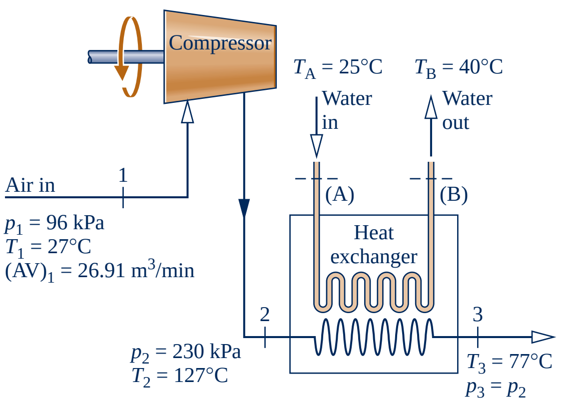


Fig. P6.114

6.115 Figure P6.115 shows several components in series, all operating at steady state. Liquid water enters the boiler at 60 bar. Steam exits the boiler at 60 bar, 540°C and undergoes a throttling process to 40 bar before entering the turbine. Steam expands adiabatically through the turbine to 5 bar, 240°C, and then undergoes a throttling process to 1 bar before entering the condenser. Kinetic and potential energy effects can be ignored.

(a) Locate each of the states 2–5 on a sketch of the T–s diagram.

(b) Determine the power developed by the turbine, in kJ per kg of steam flowing.
(c) For the valves and the turbine, evaluate the rate of entropy production, each in kJ/K per kg of steam flowing.
(d) Using the result of part (c), place the components in rank order, beginning with the component contributing the most to inefficient operation of the overall system.
(e) If the goal is to increase the power developed per kg of steam flowing, which of the components (if any) might be eliminated? Explain.

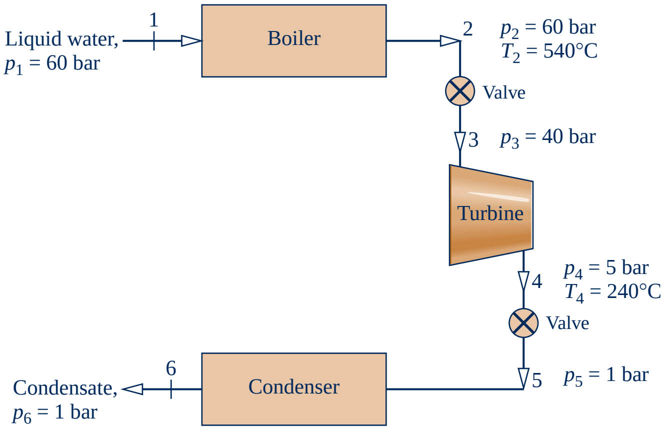


Fig. P6.115

6.116 Air as an ideal gas flows through the turbine and heat exchanger arrangement shown in Fig. P6.116. Steady-state data are given on the figure. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

(a) temperature T_3, in K.
(b) the power output of the second turbine, in kW.
(c) the rates of entropy production, each in kW/K, for the turbines and heat exchanger.
(d) Using the result of part (c), place the components in rank order, beginning with the component contributing most to inefficient operation of the overall system.

6.117 A rigid, insulated tank whose volume is 10 L is initially evacuated. A pinhole leak develops and air from the surroundings at 1 bar, 25°C enters the tank until the pressure in the tank becomes 1 bar. Assuming the ideal gas model with $k = 1.4$ for the air, determine (a) the final temperature in the tank, in °C, (b) the amount of air that leaks into the tank, in g, and (c) the amount entropy produced, in J/K.

6.118 An insulated, rigid tank whose volume is 0.5 m^3 is connected by a valve to a large vessel holding steam at 40 bar, 500°C. The tank is initially evacuated. The valve is opened only as long as required to fill the tank with steam to a pressure of 20 bar. Determine (a) the final temperature of the steam in the tank, in °C, (b) the final mass of the steam in the tank, in kg, and (c) the amount of entropy produced, in kJ/K.

6.119 For the control volume of Example 4.12, determine the amount of entropy produced during filling, in kJ/K. Repeat for the case where no work is developed by the turbine.

6.120 A well-insulated rigid tank of volume 10 m^3 is connected by a valve to a large-diameter supply line carrying air at 227°C and 10 bar. The tank is initially evacuated. Air is allowed to flow into the tank until the tank pressure is p . Using the ideal gas model with constant specific heat ratio k, plot tank temperature, in K, the mass of air in the tank, in kg, and the amount of entropy produced, in kJ/K, versus p in bar.

6.121 A 180-ft^3 tank initially filled with air at 1 atm and 70°F is evacuated by a device known as a *vacuum pump*, while the tank contents are maintained at 70°F by heat transfer through the tank walls. The vacuum pump discharges air to the surroundings at the temperature and pressure of the surroundings, which are 1 atm and 70°F, respectively. Determine the *minimum* theoretical work required, in Btu.

Using Isentropic Processes/Efficiencies

6.122 Air in a piston–cylinder assembly is compressed isentropically from T_1 = 60°F, p_1 = 20 lbf/in.^{2} to p_2 = 2000 lbf/in.^{2} Assuming the ideal gas model, determine the temperature at state 2, in °R using (a) data from Table A-22E, and (b) a constant specific heat ratio, k = 1.4. Compare the values obtained in parts (a) and (b) and comment.

6.123 Air in a piston–cylinder assembly is compressed isentropically from state 1, where T_1 = 35°C, to state 2, where the specific volume is one-tenth of the specific volume at state 1. Applying the ideal gas model with k = 1.4, determine (a) T_2, in °C and (b) the work, in kJ/kg.

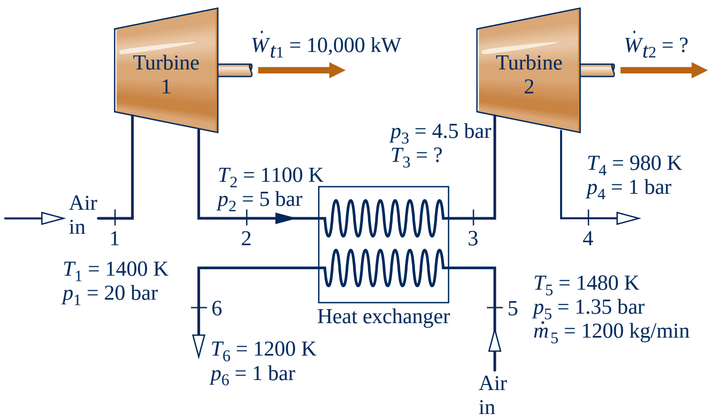


Fig. P6.116

6.124 Propane undergoes an isentropic expansion from an initial state where $T_1 = 40°C$, $p_1 = 1$ MPa to a final state where the temperature and pressure are T_2, p_2, respectively. Determine

(a) p_2, in kPa, when $T_2 = -40°C$.
(b) T_2, in °C, when $p_2 = 0.8$ MPa.

6.125 Argon in a piston–cylinder assembly is compressed isentropically from state 1, where $p_1 = 150$ kPa, $T_1 = 35°C$, to state 2, where $p_2 = 300$ kPa. Assuming the ideal gas model with $k = 1.67$, determine (a) T_2, in °C, and (b) the work, in kJ per kg of argon.

6.126 Air within a piston–cylinder assembly, initially at 12 bar, 620 K, undergoes an isentropic expansion to 1.4 bar. Assuming the ideal gas model for the air, determine the final temperature, in K, and the work, in kJ/kg. Solve two ways: using (a) data from Table A-22 and (b) $k = 1.4$.

6.127 Air within a piston–cylinder assembly, initially at 30 lbf/in.^{2}, 510°R, and a volume of 6 ft^3, is compressed isentropically to a final volume of 1.2 ft^3. Assuming the ideal gas model with $k = 1.4$ for the air, determine the (a) mass, in lb, (b) final pressure, in lbf/in.^{2}, (c) final temperature, in °R, and (d) work, in Btu.

6.128 Air contained in a piston–cylinder assembly, initially at 4 bar, 600 K and a volume of 0.43 m^3, expands isentropically to a pressure of 1.5 bar. Assuming the ideal gas model for the air, determine the (a) mass, in kg, (b) final temperature, in K, and (c) work, in kJ.

6.129 Air in a piston–cylinder assembly is compressed isentropically from an initial state where $T_1 = 340$ K to a final state where the pressure is 90% greater than at state 1. Assuming the ideal gas model, determine (a) T_2, in K, and (b) the work, in kJ/kg.

6.130 A rigid, insulated tank with a volume of 20 m^3 is filled initially with air at 10 bar, 500 K. A leak develops, and air slowly escapes until the pressure of the air remaining in the tank is 5 bar. Employing the ideal gas model with $k = 1.4$ for the air, determine the amount of mass remaining in the tank, in kg, and its temperature, in K.

6.131 A rigid, insulated tank with a volume of 21.61 ft^3 is filled initially with air at 110 lbf/in.^{2}, 535°R. A leak develops, and air slowly escapes until the pressure of the air remaining in the tank is 15 lbf/in.^{2} Employing the ideal gas model with $k = 1.4$ for the air, determine the amount of mass remaining in the tank, in lb, and its temperature, in °R.

6.132 The accompanying table provides steady-state data for an isentropic expansion of steam through a turbine. For a mass flow rate of 2.55 kg/s, determine the power developed by the turbine, in MW. Ignore the effects of potential energy.

	p(bar)	T(°C)	V(m/s)	h(kJ/kg)	s(kJ/kg · K)
Inlet	10	300	25	3051.1	7.1214
Exit	1.5	—	100		7.1214

6.133 Water vapor enters a turbine operating at steady state at 1000°F, 140 lbf/in.^{2}, with a volumetric flow rate of 21.6 ft^3/s, and expands isentropically to 2 lbf/in.^{2} Determine the power developed by the turbine, in hp. Ignore kinetic and potential energy effects.

6.134 Air enters a turbine operating at steady state at 6 bar and 1100 K and expands isentropically to a state where the temperature is 700 K. Employing the ideal gas model with data from Table A-22, and ignoring kinetic and potential energy changes, determine the pressure at the exit, in bar, and the work, in kJ per kg of air flowing.

6.135 Figure P6.135 shows a simple vapor power cycle operating at steady state with water as the working fluid. Data at key locations are given on the figure. Flow through the turbine and pump occurs isentropically. Flow through the steam generator and condenser occurs at constant pressure. Stray heat transfer and kinetic and potential energy effects are negligible. Sketch the four processes of this cycle in series on a T–s diagram. Determine the thermal efficiency.

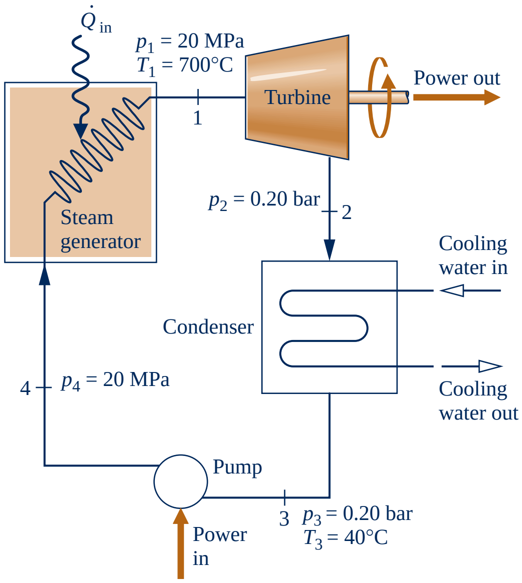


Fig. P6.135

6.136 The accompanying table provides steady-state data for steam expanding adiabatically though a turbine. The states are numbered as in Fig. 6.11. Kinetic and potential energy effects can be ignored. Determine for the turbine (a) the work developed per unit mass of steam flowing, in kJ/kg, (b) the amount of entropy produced per unit mass of steam flowing, in kJ/kg · K, and (c) the isentropic turbine efficiency.

State	p(bar)	T(°C)	x(%)	h(kJ/kg)	s(kJ/kg · K)
1	10	300	—	3051	7.121
2s	0.10	45.81	86.3	—	7.121
2	0.10	45.81	90.0	—	7.400

6.137 The accompanying table provides steady-state data for steam expanding adiabatically with a mass flow rate of 4 lb/s through a turbine. Kinetic and potential energy effects can be ignored. Determine for the turbine (a) the power developed, in hp, (b) the rate of entropy production, in hp/°R, and (c) the isentropic turbine efficiency.

	p(lbf/in.^{2})	T(°F)	u(Btu/lb)	h(Btu/lb)	s(Btu/lb · °R)
Inlet	140	1000	1371.0	1531.0	1.8827
Exit	2	270	1101.4	1181.7	2.0199

6.138 Water vapor at 800 lbf/in.^{2}, 1000°F enters a turbine operating at steady state and expands adiabatically to 2 lbf/in.^{2}, developing work at a rate of 490 Btu per lb of vapor flowing. Determine the condition at the turbine exit: two-phase liquid–vapor or superheated vapor? Also, evaluate the isentropic turbine efficiency. Kinetic and potential energy effects are negligible.

6.139 Air at 1600 K, 30 bar enters a turbine operating at steady state and expands adiabatically to the exit, where the temperature is 830 K. If the isentropic turbine efficiency is 90%, determine (a) the pressure at the exit, in bar, and (b) the work developed, in kJ per kg of air flowing. Assume ideal gas behavior for the air and ignore kinetic and potential energy effects.

6.140 Water vapor at 5 bar, 320°C enters a turbine operating at steady state with a volumetric flow rate of 0.65 m^3/s and expands adiabatically to an exit state of 1 bar, 160°C. Kinetic and potential energy effects are negligible. Determine for the turbine (a) the power developed, in kW, (b) the rate of entropy production, in kW/K, and (c) the isentropic turbine efficiency.

6.141 Air at 1175 K, 8 bar enters a turbine operating at steady state and expands adiabatically to 1 bar. The isentropic turbine efficiency is 92%. Employing the ideal gas model with $k = 1.4$ for the air, determine (a) the work developed by the turbine, in kJ per kg of air flowing, and (b) the temperature at the exit, in K. Ignore kinetic and potential energy effects.

6.142 Water vapor at 10 MPa, 600°C enters a turbine operating at steady state with a volumetric flow rate of 0.36 m^3 and exits at 0.1 bar and a quality of 92%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine for the turbine (a) the mass flow rate, in kg/s, (b) the power developed by the turbine, in MW, (c) the rate at which entropy is produced, in kW/K, and (d) the isentropic turbine efficiency.

6.143 Air modeled as an ideal gas enters a turbine operating at steady state at 1040 K, 278 kPa and exits at 120 kPa. The mass flow rate is 5.5 kg/s, and the power developed is 1120 kW. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the temperature of the air at the turbine exit, in K, and (b) the isentropic turbine efficiency.

6.144 Water vapor at 1000°F, 140 lbf/in.^{2} enters a turbine operating at steady state and expands to 2 lbf/in.^{2} The mass flow rate is 4 lb/s and the power developed is 1600 Btu/s. Stray heat transfer and kinetic and potential energy effects are negligible. Determine the isentropic turbine efficiency.

6.145 Water vapor at 6 MPa, 600°C enters a turbine operating at steady state and expands to 10 kPa. The mass flow rate is 2 kg/s, and the power developed is 2626 kW. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the isentropic turbine efficiency and (b) the rate of entropy production within the turbine, in kW/K.

6.146 Water vapor at 800 lbf/in.^{2}, 1000°F enters a turbine operating at steady state and expands to 2 lbf/in.^{2} The mass flow rate is 5.56 lb/s, and the isentropic turbine efficiency is 92%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine the power developed by the turbine, in hp.

6.147 Air enters the compressor of a gas turbine power plant operating at steady state at 290 K, 100 kPa and exits at 420 K, 330 kPa. Stray heat transfer and kinetic and potential energy effects are negligible. Using the ideal gas model for air, determine the isentropic compressor efficiency.

6.148 Air at 25°C, 100 kPa enters a compressor operating at steady state and exits at 260°C, 650 kPa. Stray heat transfer and kinetic and potential energy effects are negligible. Modeling air as an ideal gas with $k = 1.4$, determine the isentropic compressor efficiency.

6.149 Air at 290 K, 100 kPa enters a compressor operating at steady state and is compressed adiabatically to an exit state of 420 K, 330 kPa. The air is modeled as an ideal gas, and kinetic and potential energy effects are negligible. For the compressor, (a) determine the rate of entropy production, in kJ/K per kg of air flowing, and (b) the isentropic compressor efficiency.

6.150 Carbon dioxide (CO_2) at 1 bar, 300 K enters a compressor operating at steady state and is compressed adiabatically to an exit state of 10 bar, 520 K. The CO_2 is modeled as an ideal gas, and kinetic and potential energy effects are negligible. For the compressor, determine (a) the work input, in kJ per kg of CO_2 flowing, (b) the rate of entropy production, in kJ/K per kg of CO_2 flowing, and (c) the isentropic compressor efficiency.

6.151 Air at 300 K, 1 bar enters a compressor operating at steady state and is compressed adiabatically to 1.5 bar. The power input is 42 kJ per kg of air flowing. Employing the ideal gas model with $k = 1.4$ for the air, determine for the compressor (a) the rate of entropy production, in kJ/K per kg of air flowing, and (b) the isentropic compressor efficiency. Ignore kinetic and potential energy effects.

6.152 Air at 1 atm, 520°R enters a compressor operating at steady state and is compressed adiabatically to 3 atm. The isentropic compressor efficiency is 80%. Employing the ideal gas model with $k = 1.4$ for the air, determine for the compressor (a) the power input, in Btu per lb of air flowing, and (b) the amount of entropy produced, in Btu/°R per lb of air flowing. Ignore kinetic and potential energy effects.

6.153 Nitrogen (N_2) enters an insulated compressor operating at steady state at 1 bar, 37°C with a mass flow rate of 1000 kg/h and exits at 10 bar. Kinetic and potential energy effects are negligible. The nitrogen can be modeled as an ideal gas with $k = 1.391$.

(a) Determine the minimum theoretical power input required, in kW, and the corresponding exit temperature, in °C.
(b) If the exit temperature is 397°C, determine the power input, in kW, and the isentropic compressor efficiency.

6.154 Saturated water vapor at 300°F enters a compressor operating at steady state with a mass flow rate of 5 lb/s and is compressed adiabatically to 800 lbf/in.^{2} If the power input is 2150 hp, determine for the compressor (a) the isentropic compressor efficiency and (b) the rate of entropy production, in hp/°R. Ignore kinetic and potential energy effects.

6.155 Refrigerant 134a at a rate of 0.8 lb/s enters a compressor operating at steady state as saturated vapor at 30 psia and exits at a pressure of 160 psia. There is no significant heat transfer with the surroundings, and kinetic and potential energy effects can be ignored.

(a) Determine the minimum theoretical power input required, in Btu/s, and the corresponding exit temperature, in °F.
(b) If the refrigerant exits at a temperature of 130°F, determine the actual power, in Btu/s, and the isentropic compressor efficiency.

6.156 Air at 1.3 bar, 423 K and a velocity of 40 m/s enters a nozzle operating at steady state and expands adiabatically to the exit, where the pressure is 0.85 bar and velocity is 307 m/s. For air modeled as an ideal gas with $k = 1.4$, determine for the nozzle (a) the temperature at the exit, in K, and (b) the isentropic nozzle efficiency.

6.157 Water vapor at 100 lbf/in.^{2}, 500°F and a velocity of 100 ft/s enters a nozzle operating at steady state and expands adiabatically to the exit, where the pressure is 40 lbf/in.^{2} If the isentropic nozzle efficiency is 95%, determine for the nozzle (a) the velocity of the steam at the exit, in ft/s, and (b) the amount of entropy produced, in Btu/°R per lb of steam flowing.

6.158 Helium gas at 810°R, 45 lbf/in.^{2}, and a velocity of 10 ft/s enters an insulated nozzle operating at steady state and exits at 670°R, 25 lbf/in.^{2} Modeling helium as an ideal gas with $k = 1.67$, determine (a) the velocity at the nozzle exit, in ft/s, (b) the isentropic nozzle efficiency, and (c) the rate of entropy production within the nozzle, in Btu/°R per lb of helium flowing.

6.159 Air modeled as an ideal gas enters a one-inlet, one-exit control volume operating at steady state at 100 lbf/in.^{2}, 900°R and expands adiabatically to 25 lbf/in.^{2} Kinetic and potential energy effects are negligible. Determine the rate of entropy production, in Btu/°R per lb of air flowing,

(a) if the control volume encloses a turbine having an isentropic turbine efficiency of 89.1%.
(b) if the control volume encloses a throttling valve.

6.160 Ammonia enters a valve as saturated liquid at 9 bar and undergoes a throttling process to a pressure of 2 bar. Determine the rate of entropy production per unit mass of ammonia flowing, in kJ/kg · K. If the valve were replaced by a power-recovery turbine operating at steady state, determine the maximum theoretical power that could be developed per unit mass of ammonia flowing, in kJ/kg, and comment. In each case, ignore heat transfer with the surroundings and changes in kinetic and potential energy.

6.161 Figure P6.161 provides the schematic of a heat pump using Refrigerant 134a as the working fluid, together with steady-state data at key points. The mass flow rate of the refrigerant is 7 kg/min, and the power input to the compressor is 5.17 kW. (a) Determine the coefficient of performance for the heat pump. (b) If the valve were replaced by a turbine, power could be produced, thereby reducing the power requirement of the heat pump system. Would you recommend this *power-saving* measure? Explain.

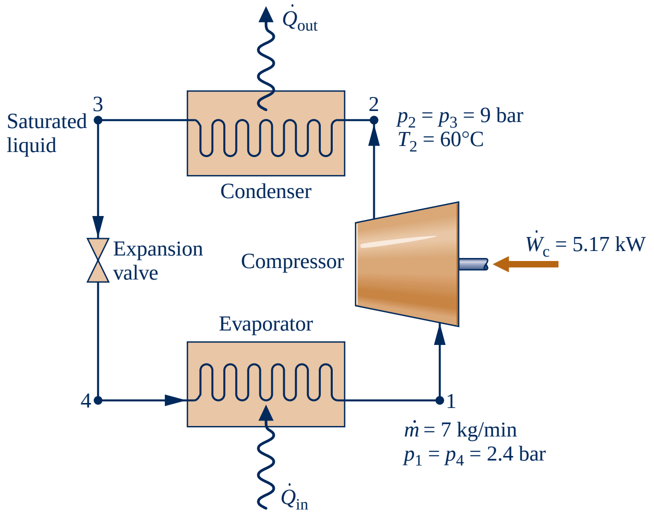


Fig. P6.161

6.162 Air enters an insulated diffuser operating at steady state at 1 bar, −3°C, and 260 m/s and exits with a velocity of 130 m/s. Employing the ideal gas model and ignoring potential energy, determine

(a) the temperature of the air at the exit, in °C.
(b) The maximum attainable exit pressure, in bar.

6.163 As shown in Fig. P6.163, air enters the diffuser of a jet engine at 18 kPa, 216 K with a velocity of 265 m/s, all data corresponding to high-altitude flight. The air flows adiabatically through the diffuser, decelerating to a velocity of 50 m/s at the diffuser exit. Assume steady-state operation, the ideal gas model for air, and negligible potential energy effects.

(a) Determine the temperature of the air at the exit of the diffuser, in K.
(b) If the air would undergo an isentropic process as it flows through the diffuser, determine the pressure of the air at the diffuser exit, in kPa.
(c) If friction were present, would the pressure of the air at the diffuser exit be greater than, less than, or equal to the value found in part (b)? Explain.

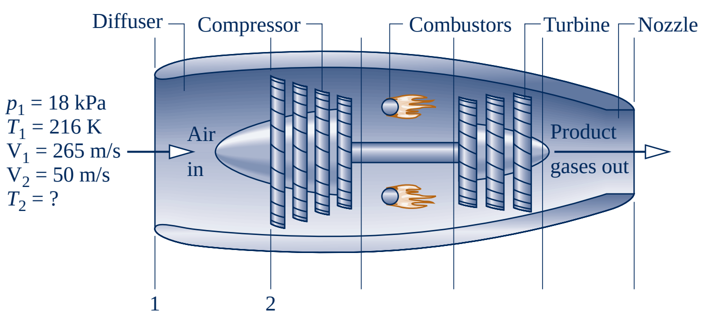


Fig. P6.163

6.164 As shown in Fig. P6.164, a steam turbine having an isentropic turbine efficiency of 90% drives an air compressor having an isentropic compressor efficiency of 85%. Steady-state operating data are provided on the figure. Assume the ideal gas model for air, and ignore stray heat transfer and kinetic and potential energy effects.

(a) Determine the mass flow rate of the steam entering the turbine, in kg of steam per kg of air exiting the compressor.
(b) Repeat part (a) if $\eta_t = \eta_c = 100\%$

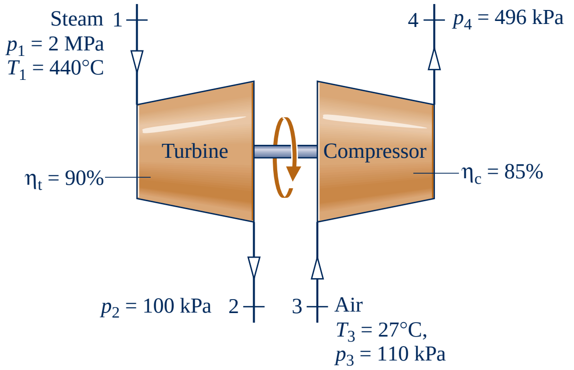


Fig. P6.164

6.165 Figure P6.165 shows a simple vapor power plant operating at steady state with water as the working fluid. Data at key locations are given on the figure. The mass flow rate of the water circulating through the components is 109 kg/s. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

(a) the net power developed, in MW.
(b) the thermal efficiency.
(c) the isentropic turbine efficiency.
(d) the isentropic pump efficiency.
(e) the mass flow rate of the cooling water, in kg/s.
(f) the rates of entropy production, each in kW/K, for the turbine, condenser, and pump.

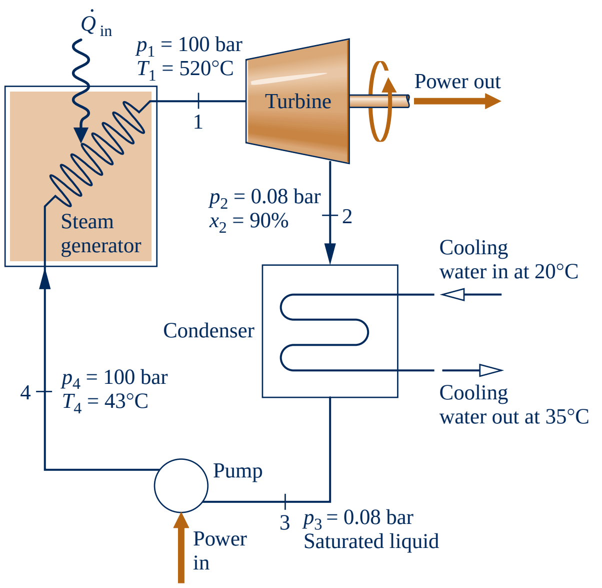


Fig. P6.165

6.166 Figure P6.166 shows a power system operating at steady state consisting of three components in series: an air compressor having an isentropic compressor efficiency of 80%, a heat exchanger, and a turbine having an isentropic turbine efficiency of 90%. Air enters the compressor at 1 bar, 300 K with a mass flow rate of 5.8 kg/s and exits at a pressure of 10 bar. Air enters the turbine at 10 bar, 1400 K and exits at a pressure of 1 bar. Air can be modeled as an ideal gas. Stray heat transfer and kinetic and potential energy effects are negligible. Determine, in kW, (a) the power required by the compressor, (b) the power developed by the turbine, and (c) the *net* power output of the overall power system.

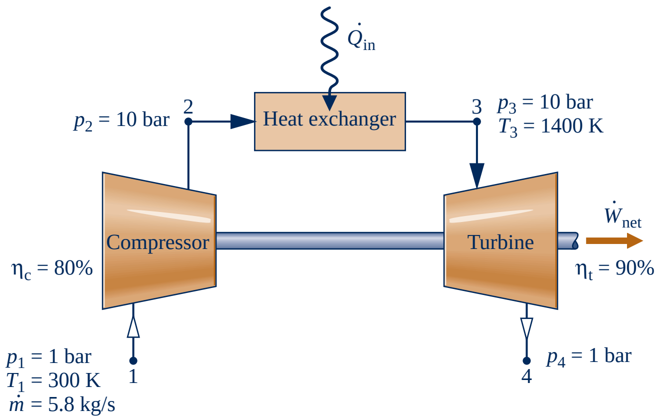


Fig. P6.166

6.167 As shown in Fig. P6.167, a well-insulated turbine operating at steady state has two stages in series. Steam enters the first stage at 800°F, 600 lbf/in.² and exits at 250 lbf/in.² The steam then enters the second stage and exits at 14.7 lbf/in.² The isentropic efficiencies of the stages are 85% and 91%, respectively. Show the principal states on a *T*–*s* diagram. At the exit of the second stage, determine the temperature, in °F, if superheated vapor exits or the quality if a two-phase liquid–vapor mixture exits. Also determine the work developed by each stage, in Btu per lb of steam flowing.

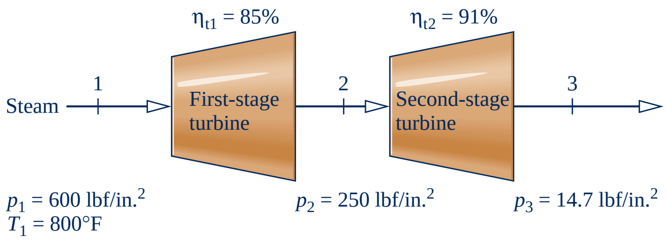


Fig. P6.167

6.168 A rigid tank is filled initially with 5.0 kg of air at a pressure of 0.5 MPa and a temperature of 500 K. The air is allowed to discharge through a turbine into the atmosphere, developing work until the pressure in the tank has fallen to the atmospheric level of 0.1 MPa. Employing the ideal gas model for the air, determine the *maximum* theoretical amount of work that could be developed, in kJ. Ignore heat transfer with the atmosphere and changes in kinetic and potential energy.

6.169 A tank initially containing air at 30 atm and 540°F is connected to a small turbine. Air discharges from the tank through the turbine, which produces work in the amount of 100 Btu. The pressure in the tank falls to 3 atm during the process and the turbine exhausts to the atmosphere at 1 atm. Employing the ideal gas model for the air and ignoring irreversibilities within the tank and the turbine, determine the volume of the tank, in ft^3. Heat transfer with the atmosphere and changes in kinetic and potential energy are negligible.

6.170 Air enters a 3600-kW turbine operating at steady state with a mass flow rate of 18 kg/s at 800°C, 3 bar and a velocity of 100 m/s. The air expands adiabatically through the turbine and exits at a velocity of 150 m/s. The air then enters a diffuser where it is decelerated isentropically to a velocity of 10 m/s and a pressure of 1 bar. Employing the ideal gas model, determine

(a) the pressure and temperature of the air at the turbine exit, in bar and °C, respectively.
(b) the rate of entropy production in the turbine, in kW/K.

Show the processes on a T–s diagram.

Analyzing Internally Reversible Flow Processes

6.171 Air enters a compressor operating at steady state with a volumetric flow rate of 0.2 m^3/s, at 20°C, 1 bar. The air is compressed isothermally without internal irreversibilities, exiting at 8 bar. The air is modeled as an ideal gas, and kinetic and potential energy effects can be ignored. Evaluate the power required and the heat transfer rate, each in kW.

6.172 Refrigerant 134a enters a compressor operating at steady state at 1 bar, -15°C with a volumetric flow rate of 3×10^{-2} m^3/s. The refrigerant is compressed to a pressure of 8 bar in an internally reversible process according to $pv^{1.06} =$ *constant*. Neglecting kinetic and potential energy effects, determine

(a) the power required, in kW.
(b) the rate of heat transfer, in kW.

6.173 An air compressor operates at steady state with air entering at $p_1 = 15$ lbf/in.^{2}, $T_1 = 60$°F. The air undergoes a polytropic process, and exits at $p_2 = 75$ lbf/in.^{2}, $T_2 = 294$°F. (a) Evaluate the work and heat transfer, each in Btu per lb of air flowing. (b) Sketch the process on p–v and T–s diagrams and associate areas on the diagrams with work and heat transfer, respectively. Assume the ideal gas model for air and neglect changes in kinetic and potential energy.

6.174 An air compressor operates at steady state with air entering at $p_1 = 1$ bar, $T_1 = 17$°C and exiting at $p_2 = 5$ bar. The air undergoes a polytropic process for which the compressor work input is 162.2 kJ per kg of air flowing. Determine (a) the temperature of the air at the compressor exit, in °C, and (b) the heat transfer, in kJ per kg of air flowing. (c) Sketch the process on p–v and T–s diagrams and associate areas on the diagrams with work and heat transfer, respectively. Assume the ideal gas model for air and neglect changes in kinetic and potential energy.

6.175 Water as saturated liquid at 1 bar enters a pump operating at steady state and is pumped isentropically to a pressure of 50 bar. Kinetic and potential energy effects are negligible. Determine the pump work input, in kJ per kg of water flowing, using (a) Eq. 6.51c, (b) an energy balance. Obtain data from Table A-3 and A-5, as appropriate. Compare the results of parts (a) and (b), and comment.

6.176 Compare the work required at steady state to compress *water vapor* isentropically to 3 MPa from the saturated vapor state at 0.1 MPa to the work required to pump *liquid water* isentropically to 3 MPa from the saturated liquid state at 0.1 MPa, each in kJ per kg of water flowing through the device. Kinetic and potential energy effects can be ignored.

6.177 A pump operating at steady state receives saturated liquid water at 50°C with a mass flow rate of 20 kg/s. The pressure of the water at the pump exit is 1 MPa. If the pump operates with negligible internal irreversibilities and negligible changes in kinetic and potential energy, determine the power required in kW.

6.178 A pump operating at steady state receives liquid water at 20°C 100 kPa with a mass flow rate of 53 kg/min. The pressure of the water at the pump exit is 5 MPa. The isentropic pump efficiency is 70%. Stray heat transfer and changes in kinetic and potential energy are negligible. Determine the power required by the pump, in kW.

6.179 A pump operating at steady state receives liquid water at 50°C, 1.5 MPa. The pressure of the water at the pump exit is 15 MPa. The magnitude of the work required by the pump is 18 kJ per kg of water flowing. Stray heat transfer and changes in kinetic and potential energy are negligible. Determine the isentropic pump efficiency.

6.180 Liquid water at 70°F, 14.7 lbf/in.^{2} and a velocity of 30 ft/s enters a system at steady state consisting of a pump and attached piping and exits at a point 30 ft above the inlet at 250 lbf/in.^{2}, a velocity of 15 ft/s, and no significant change in temperature. (a) In the absence of internal irreversibilities, determine the power input required by the system, in Btu per lb of liquid water flowing. (b) For the same inlet and exit states, in the presence of friction would the power input be greater, or less, than determined in part (a)? Explain. Let $g = 32.2$ ft/s^2.

6.181 A 3-hp pump operating at steady state draws in liquid water at 1 atm, 60°F and delivers it at 5 atm at an elevation 20 ft above the inlet. There is no significant change in velocity between the inlet and exit, and the local acceleration of gravity is 32.2 ft/s^2. Would it be possible to pump 1000 gal in 10 min or less? Explain.

6.182 An electrically driven pump operating at steady state draws water from a pond at a pressure of 1 bar and a rate of 50 kg/s and delivers the water at a pressure of 4 bar. There is no significant heat transfer with the surroundings, and changes in kinetic and potential energy can be neglected. The isentropic pump efficiency is 75%. Evaluating electricity at 8.5 cents per kW · h, estimate the hourly cost of running the pump.

6.183 As shown in Fig. P6.183, water behind a dam enters an intake pipe at a pressure of 24 psia and velocity of 5 ft/s, flows through a hydraulic turbine-generator, and exits at a point 200 ft below the intake at 19 psia, 45 ft/s, and a specific

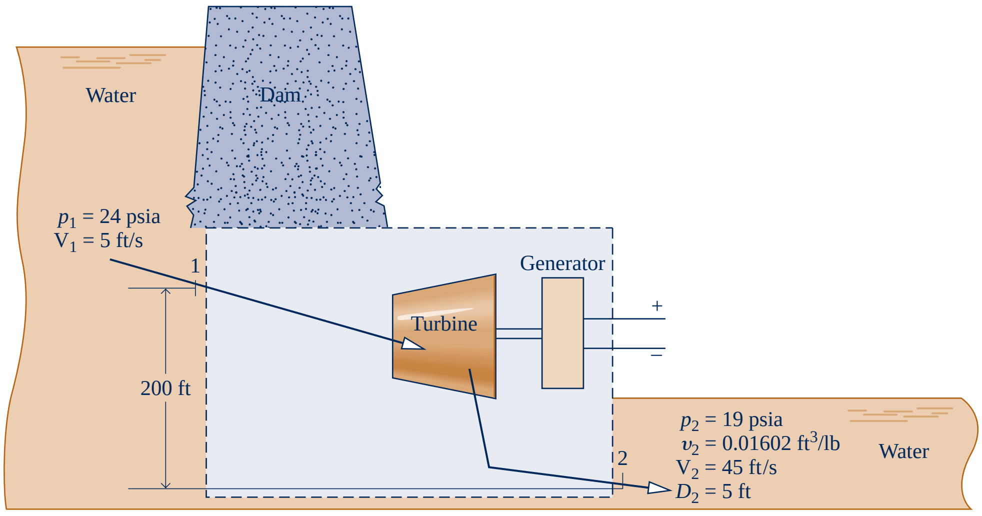


Fig. P6.183

volume of 0.01602 ft^3/lb. The diameter of the exit pipe is 5 ft and the local acceleration of gravity is 32.2 ft/s^2. Evaluating the electricity generated at 8.5 cents per kW · h, determine the value of the power produced, in \$/day, for operation at steady state and in the absence of internal irreversibilities.

6.184 As shown in Figure P6.184, water flows from an elevated reservoir through a hydraulic turbine operating at steady state. Determine the maximum power output, in MW, associated with a mass flow rate of 950 kg/s. The inlet and exit diameters are equal. The water can be modeled as incompressible with $v = 10^{-3}$ m^3/kg. The local acceleration of gravity is 9.8 m/s^2.

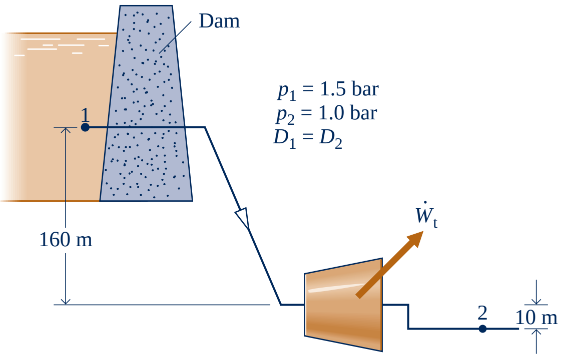


Fig. P6.184

6.185 Nitrogen (N_2) enters a nozzle operating at steady state at 0.2 MPa, 550 K with a velocity of 1 m/s and undergoes a polytropic expansion with $n = 1.3$ to 0.15 MPa. Using the ideal gas model with $k = 1.4$, and ignoring potential energy effects, determine (a) the exit velocity, in m/s, and (b) the rate of heat transfer, in kJ per kg of gas flowing.

6.186 Carbon monoxide enters a nozzle operating at steady state at 5 bar, 200°C with a velocity of 1 m/s and undergoes a polytropic expansion to 1 bar and an exit velocity of 630 m/s. Using the ideal gas model and ignoring potential energy effects, determine

(a) the exit temperature, in °C.
(b) the rate of heat transfer, in kJ per kg of gas flowing.

Reviewing Concepts

6.187 Answer the following true or false. Explain.

(a) For closed systems undergoing processes involving internal irreversibilities, both entropy change and entropy production are positive in value.
(b) The *Carnot* cycle is represented on a *Mollier* diagram by a rectangle.
(c) Entropy *change* of a closed system during a process can be greater than, equal to, or less than zero.
(d) For specified inlet state, exit pressure, and mass flow rate, the power *input* required by a compressor operating at steady state is less than that if compression occurred isentropically.
(e) The $T\,dS$ equations are fundamentally important in thermodynamics because of their use in deriving important property relations for pure, simple compressible systems.
(f) At liquid states, the following approximation is reasonable for many engineering applications $s(T, p) = s_f(T)$.

6.188 Answer the following true or false. Explain

(a) The steady-state form of the control volume entropy balance requires that the total rate at which entropy is transferred out of the control volume be *less* than the total rate at which entropy enters.
(b) In *statistical* thermodynamics, entropy is associated with the notion of microscopic *disorder*.
(c) For a gas modeled as an ideal gas, the specific internal energy, enthalpy, and entropy all depend on temperature only.
(d) The entropy change between two states of water can be read *directly* from the *steam tables*.
(e) The *increase of entropy principle* states that the only processes of an isolated system are those for which its entropy increases.
(f) Equation 6.52, the *Bernoulli* equation, applies generally to one-inlet, one-exit control volumes at steady state, whether internal irreversibilities are present or not.

6.189 Answer the following true or false. Explain

(a) The only entropy transfer to, or from, control volumes is that accompanying heat transfer.

(b) Heat transfer for internally reversible processes of closed systems can be represented on a temperature–entropy diagram as an area.
(c) For a specified inlet state, exit pressure, and mass flow rate, the power developed by a turbine operating at steady state is less than if expansion occurred isentropically.
(d) The entropy change between two states of air modeled as an ideal gas can be *directly* read from Table A-22 only when pressure at these states is the same.
(e) The term *isothermal* means constant temperature, whereas *isentropic* means constant specific volume.
(f) When a system undergoes a *Carnot* cycle, entropy is produced within the system.

▶ DESIGN & OPEN-ENDED PROBLEMS: EXPLORING ENGINEERING PRACTICE

6.1D Using the ENERGY STAR® home improvement tool box, obtain a rank-ordered list of the top three cost-effective improvements that would enhance the overall energy efficiency of your home. Develop a plan to implement the improvements. Write a report, including at least three references.

6.2D Ocean thermal energy conversion (OTEC) power plants generate electricity on ships or platforms at sea by exploiting the naturally occurring decrease of the temperature of ocean water with depth. One proposal for the use of OTEC-generated electricity is to produce and commercialize ammonia in three steps: Hydrogen (H_2) would first be obtained by electrolysis of desalted sea water. The hydrogen then would be reacted with nitrogen (N_2) from the atmosphere to obtain ammonia (NH_3). Finally, liquid ammonia would be shipped to shore, where it would be reprocessed into hydrogen or used as a feedstock. Some say a major drawback with the proposal is whether *current technology* can be integrated to provide cost-competitive end products. Investigate this issue and summarize your findings in a report with at least three references.

6.3D Natural gas is currently playing a significant role in meeting our energy needs and hydrogen may be just as important in years ahead. For natural gas *and* hydrogen, energy is required at every stage of distribution between production and end use: for storage, transportation by pipelines, trucks, trains and ships, and liquefaction, if needed. According to some observers, distribution energy requirements will weigh more heavily on hydrogen not only because it has special attributes but also because means for distributing it are less developed than for natural gas. Investigate the energy requirements for distributing hydrogen relative to that for natural gas. Write a report with at least three references.

6.4D For a compressor or pump located at your campus or workplace, take data sufficient for evaluating the isentropic compressor or pump efficiency. Compare the experimentally determined isentropic efficiency with data provided by the manufacturer. Rationalize any significant discrepancy between experimental and manufacturer values. Prepare a technical report including a full description of instrumentation, recorded data, results and conclusions, and at least three references.

6.5D Classical economics was developed largely in analogy to the notion of mechanical equilibrium. Some observers are now saying that a macroeconomic system is more like a thermodynamic system than a mechanical one. Further, they say the failure of traditional economic theories to account for recent economic behavior may be partially due to not recognizing the role of entropy in controlling economic change and equilibrium, similar to the role of entropy in thermodynamics. Write a report, including at least three references, on how the second law and entropy are used in economics.

6.6D Design and execute an experiment to obtain measured property data required to evaluate the change in entropy of a common gas, liquid, or solid undergoing a process of your choice. Compare the experimentally determined entropy change with a value obtained from published engineering data, including property software. Rationalize any significant discrepancy between values. Prepare a technical report including a full description of the experimental set-up and instrumentation, recorded data, sample calculations, results and conclusions, and at least three references.

6.7D The *maximum entropy method* is widely used in the field of astronomical data analysis. Over the last three decades, considerable work has been done using the method for data filtering and removing features in an image that are caused by the telescope itself rather than from light coming from the sky (called deconvolution). To further such aims, refinements of the method have evolved over the years. Investigate the maximum entropy method as it is used today in astronomy, and summarize the state-of-the-art in a memorandum.

6.8D The performance of turbines, compressors, and pumps decreases with use, reducing isentropic efficiency. Select one of these three types of components and develop a detailed understanding of how the component functions. Contact a manufacturer's representative to learn what measurements are typically recorded during operation, causes of degraded performance with use, and maintenance actions that can be taken to extend service life. Visit an industrial site where the selected component can be observed in operation and discuss the same points with personnel there. Prepare a poster presentation of your findings suitable for classroom use.

6.9D Elementary thermodynamic modeling, including the use of the temperature–entropy diagram for water and a form of the *Bernoulli equation* has been employed to study certain types of volcanic eruptions. (See L. G. Mastin, "Thermodynamics of Gas and Steam-Blast Eruptions," *Bull. Volcanol.*, 57, 85–98, 1995.) Write a report *critically evaluating* the underlying assumptions and application of thermodynamic principles, as reported in the article. Include at least three references.

6.10D In recent decades, many have written about the relationship between life in the biosphere and the second law of thermodynamics. Among these are Nobel Prize winners Erwin Schrodinger (Physics, 1933) and Ilya Prigogine (Chemistry, 1977). Contemporary observers such as Eric Schneider also have weighed in. Survey and *critically evaluate* such contributions to the literature. Summarize your conclusions in a report having at least three references.

6.11D Figure P6.11D shows an air compressor fitted with a water jacket fed from an existing water line accessible at a location 50 feet horizontally and 10 feet below the connection port on the water jacket. The compressor is a single-stage, double-acting, horizontal reciprocating compressor with a discharge pressure of 50 psig when compressing ambient air. Water at 45°F experiences a 10°F temperature rise as it flows through the jacket at a flow rate of 300 gal per hour. Design a cooling water piping system to meet these needs. Use standard pipe sizes and fittings and an appropriate off-the-shelf pump with a single-phase electric motor. Prepare a technical report including a diagram of the piping system, a full parts list, the pump specifications, an estimate of installed cost, and sample calculations.

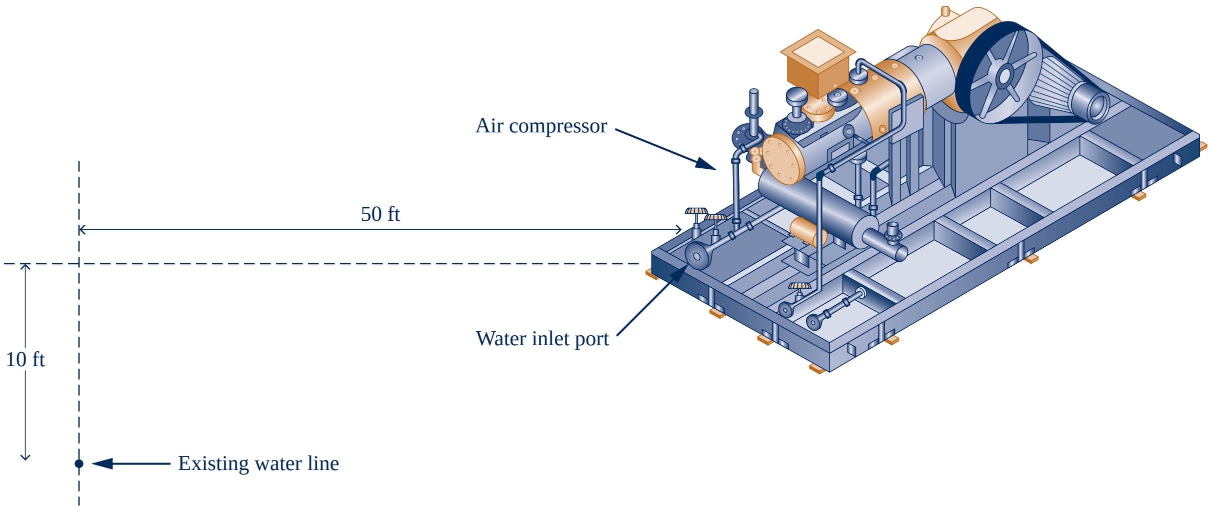


Fig. P6.11D

Exergy expresses energy transfer by work, heat, and mass flow in terms of a *common measure*: work *fully* available for lifting a weight; see Secs. 7.2.2, 7.4.1, and 7.5.1.

ENGINEERING CONTEXT The **objective** of this chapter is to introduce *exergy analysis*, which uses the conservation of mass and conservation of energy principles together with the second law of thermodynamics for the design and analysis of thermal systems.

The importance of developing thermal systems that make effective use of nonrenewable resources such as oil, natural gas, and coal is apparent. Exergy analysis is particularly suited for furthering the goal of more efficient resource use, since it enables the locations, types, and true magnitudes of waste and loss to be determined. This information can be used to design thermal systems, guide efforts to reduce sources of inefficiency in existing systems, and evaluate system economics.

7

Exergy Analysis

LEARNING OUTCOMES

When you complete your study of this chapter, you will be able to...

- demonstrate understanding of key concepts related to exergy analysis . . . including the exergy reference environment, the dead state, exergy transfer, and exergy destruction.
- evaluate exergy at a state and exergy change between two states, using appropriate property data.
- apply exergy balances to closed systems and to control volumes at steady state.
- define and evaluate exergetic efficiencies.
- apply exergy costing to heat loss and simple cogeneration systems.

7.1 Introducing Exergy

Energy is conserved in every device or process. It cannot be destroyed. Energy entering a system with fuel, electricity, flowing streams of matter, and so on can be accounted for in the products and by-products. However, the energy conservation idea alone is inadequate for depicting some important aspects of resource utilization.

▶ FOR EXAMPLE Figure 7.1*a* shows an *isolated system* consisting initially of a small container of fuel surrounded by air in abundance. Suppose the fuel burns (Fig. 7.1*b*) so that finally there is a slightly warm mixture of combustion products and air as shown in Fig. 7.1*c*. The total *quantity* of energy associated with the system is constant because no energy transfers take place across the boundary of an isolated system. Still, the initial fuel–air combination is intrinsically more useful than the final warm mixture. For instance, the fuel might be used in some device to generate electricity or produce superheated steam, whereas the uses of the final slightly warm mixture are far more limited in scope. We can say that the system has a greater *potential for use* initially than it has finally. Since nothing but a final warm mixture is achieved in the process, this potential is largely wasted. More precisely, the initial potential is largely *destroyed* because of the irreversible nature of the process. ◀ ◀ ◀ ◀ ◀

Anticipating the main results of this chapter, *exergy* is the property that quantifies *potential for use.* The foregoing example illustrates that, unlike energy, exergy is not conserved but is destroyed by irreversibilities.

Subsequent discussion shows that exergy not only can be destroyed by irreversibilities but also can be transferred *to* and *from* systems. Exergy transferred from a system to its surroundings without use typically represents a *loss.* Improved energy resource utilization can be realized by reducing exergy destruction within a system and/or reducing losses. An objective in exergy analysis is to identify sites where exergy destructions and losses occur and rank order them for significance. This allows attention to be centered on aspects of system operation that offer the greatest opportunities for cost-effective improvements.

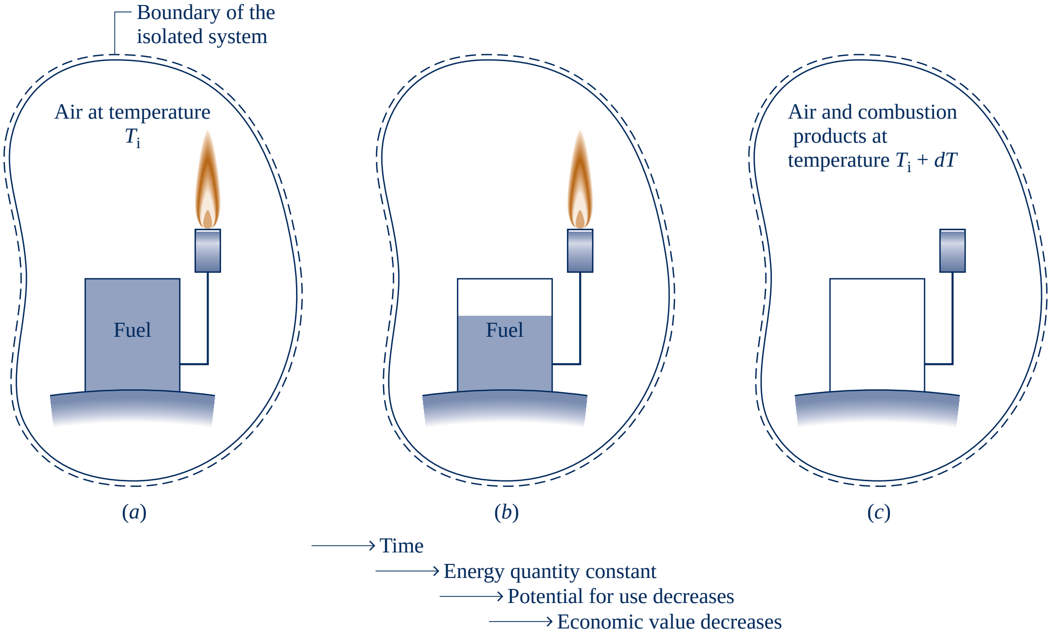


Fig. 7.1 Illustration used to introduce exergy.

Returning to Fig. 7.1, note that the fuel present initially has economic value while the final slightly warm mixture has little value. Accordingly, economic value decreases in this process. From such considerations we might infer there is a link between exergy and economic value, and this is the case as we will see in subsequent discussions.

7.2 Conceptualizing Exergy

The introduction to the second law in Chap. 5 also provides a basis for the exergy concept, as considered next.

Principal conclusions of the discussion of Fig. 5.1 given on p. 238 are that

- a potential for developing work exists whenever two systems at different states are brought into communication, and
- work can be developed as the two systems are allowed to come into equilibrium.

In Fig. 5.1*a*, for example, a body initially at an elevated temperature T_i placed in contact with the atmosphere at temperature T_0 cools spontaneously. To conceptualize how work might be developed in this case, see Fig. 7.2. The figure shows an *overall* system with three elements: the body, a power cycle, and the atmosphere at T_0 and p_0. The atmosphere is presumed to be large enough that its temperature and pressure remain constant. W_c denotes the work of the overall system.

Instead of the body cooling spontaneously as considered in Fig. 5.1*a*, Fig. 7.2 shows that if the heat transfer Q during cooling is passed to the power cycle, work W_c can be developed, while Q_0 is discharged to the atmosphere. These are the only energy transfers. The work W_c is *fully available* for lifting a weight or, equivalently, as shaft work or electrical work. Ultimately the body cools to T_0, and no more work would be developed. At equilibrium, the body and atmosphere each possess energy, but there no longer is any potential for developing work from the two because no further interaction can occur between them.

Note that work W_c also could be developed by the system of Fig. 7.2 if the initial temperature of the body were *less* than that of the atmosphere: $T_i < T_0$. In such a case, the directions of the heat transfers Q and Q_0 shown on Fig. 7.2 would each reverse. Work could be developed as the body *warms* to equilibrium with the atmosphere.

Since there is no net change of state for the power cycle of Fig. 7.2, we conclude that the work W_c is realized solely because the initial state of the body differs from that of the atmosphere. *Exergy is the maximum theoretical value of such work.*

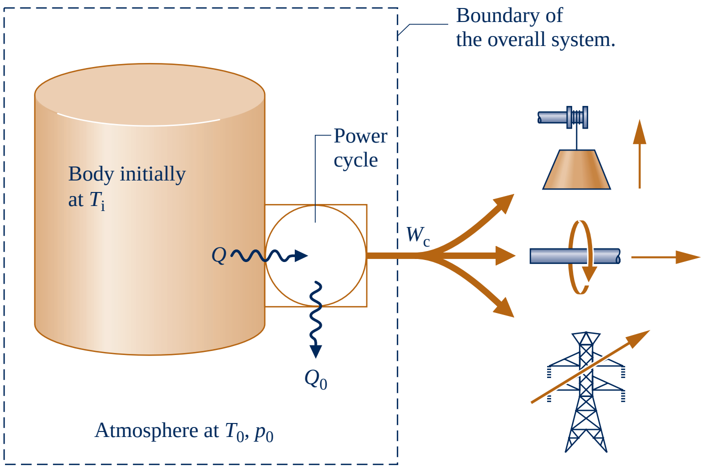


Fig. 7.2 Overall system of body, power cycle, and atmosphere used to conceptualize exergy.

7.2.1 Environment and Dead State

For thermodynamic analysis involving the exergy concept, it is necessary to model the atmosphere used in the foregoing discussion. The resulting model is called the **exergy reference environment**, or simply the **environment**.

environment

In this book the environment is regarded to be a simple compressible system that is *large* in extent and *uniform* in temperature, T_0, and pressure, p_0. In keeping with the idea that the environment represents a portion of the physical world, the values for both p_0 and T_0 used throughout a particular analysis are normally taken as typical ambient conditions, such as 1 atm and 25°C (77°F). Additionally, the intensive properties of the environment do not change significantly as a result of any process under consideration, and the environment is free of irreversibilities.

When a system of interest is at T_0 and p_0 and *at rest* relative to the environment, we say the system is at the **dead state**. At the dead state there can be no interaction between system and environment, and thus no potential for developing work.

dead state

7.2.2 Defining Exergy

The discussion to this point of the current section can be summarized by the following **definition of exergy**:

> Exergy is the maximum theoretical work obtainable from an *overall* system consisting of a system and the environment as the system comes into equilibrium with the environment (passes to the dead state).

definition of exergy

Interactions between the system and the environment may involve auxiliary devices, such as the power cycle of Fig. 7.2, that at least in principle allow the realization of the work. The work developed is fully available for lifting a weight or, equivalently, as shaft work or electrical work. We might expect that the maximum theoretical work would be obtained when there are no irreversibilities. As considered in the next section, this *is* the case.

7.3 Exergy of a System

exergy of a system

The **exergy of a system**, E, at a specified state is given by the expression

$$\mathsf{E} = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) + \text{KE} + \text{PE} \qquad \textbf{(7.1)}$$

TAKE NOTE...
In this book, E and e are used for exergy and specific exergy, respectively, while *E* and *e* denote energy and specific energy, respectively. Such notation is in keeping with standard practice. The appropriate concept, exergy or energy, will be clear in context. Still, care is required to avoid mistaking the symbols for these concepts.

where U, KE, PE, V, and S denote, respectively, internal energy, kinetic energy, potential energy, volume, and entropy of the system at the specified state. U_0, V_0, and S_0 denote internal energy, volume, and entropy, respectively, of the system when at the dead state. In this chapter kinetic and potential energy are evaluated relative to the environment. Thus, when the system is at the dead state, it is at rest relative the environment and the values of its kinetic and potential energies are zero: $\text{KE}_0 = \text{PE}_0 = 0$. By inspection of Eq. 7.1, the units of exergy are seen to be the same as those of energy.

Equation 7.1 can be derived by applying energy and entropy balances to the overall system shown in Fig. 7.3 consisting of a closed system and an environment. See the box for the derivation of Eq. 7.1.

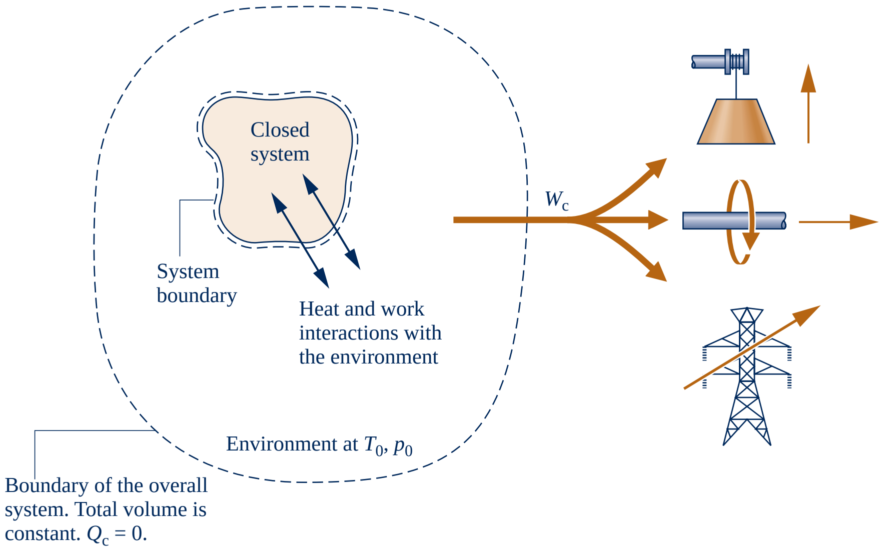


Fig. 7.3 Overall system of system and environment used to evaluate exergy.

Evaluating Exergy of a System

Referring to Fig. 7.3, exergy is the maximum theoretical value of the work W_c obtainable from the overall system as the closed system comes into equilibrium with the environment—that is, as the closed system passes to the dead state.

In keeping with the discussion of Fig. 7.2, the closed system plus the environment is referred to as the *overall* system. The boundary of the overall system is located so there is no energy transfer across it by heat transfer: $Q_c = 0$. Moreover, the boundary of the overall system is located so that the total volume remains constant, even though the volumes of the system and environment can vary. Accordingly, the work W_c shown on the figure is the *only* energy transfer across the boundary of the overall system and is *fully available* for lifting a weight, turning a shaft, or producing electricity in the surroundings. Next, we apply the energy and entropy balances to determine the maximum theoretical value for W_c.

Energy Balance

Consider a process where the system and the environment come to equilibrium. The energy balance for the overall system is

$$\Delta E_c = \cancel{Q_c}^{\,0} - W_c \tag{a}$$

where W_c is the work developed by the overall system and ΔE_c is the change in energy of the overall system: the sum of the energy changes of the system and the environment. The energy of the system initially is denoted by E, which includes the kinetic, potential, and internal energies of the system. Since the kinetic and potential energies are evaluated relative to the environment, the energy of the system at the dead state is just its internal energy, U_o. Accordingly, ΔE_c can be expressed as

$$\Delta E_c = (U_o - E) + \Delta U_e \tag{b}$$

where ΔU_e is the change in internal energy of the environment.

Since T_o and p_o are constant, changes in internal energy U_e, entropy S_e, and volume V_e of the environment are related through Eq. 6.8, the *first T dS* equation, as

$$\Delta U_e = T_o \Delta S_e - p_o \Delta V_e \tag{c}$$

Introducing Eq. (c) into Eq. (b) gives

$$\Delta E_c = (U_o - E) + (T_o \Delta S_e - p_o \Delta V_e) \tag{d}$$

Substituting Eq. (d) into Eq. (a) and solving for W_c gives

$$W_c = (E - U_o) - (T_o \Delta S_e - p_o \Delta V_e)$$

The total volume is constant. Hence, the change in volume of the environment is equal in magnitude and opposite in sign to the volume change of the system: $\Delta V_e = -(V_o - V)$. With this substitution, the above expression for work becomes

$$W_c = (E - U_o) + p_o(V - V_o) - (T_o \Delta S_e) \tag{e}$$

This equation gives the work for the overall system as the system passes to the dead state. The maximum theoretical work is determined using the entropy balance as follows.

Entropy Balance

The entropy balance for the overall system reduces to

$$\Delta S_c = \sigma_c$$

where the entropy transfer term is omitted because no heat transfer takes place across the boundary of the overall system. The term σ_c accounts for entropy production due to irreversibilities as the system comes into equilibrium with the environment. The entropy change ΔS_c is the sum of the entropy changes for the system and environment, respectively. That is

$$\Delta S_c = (S_o - S) + \Delta S_e$$

where S and S_o denote the entropy of the system at the given state and the dead state, respectively. Combining the last two equations

$$(S_o - S) + \Delta S_e = \sigma_c \tag{f}$$

Eliminating ΔS_e between Eqs. (e) and (f) results in

$$W_c = (E - U_o) + p_o(V - V_o) - T_o(S - S_o) - T_o\sigma_c \tag{g}$$

With $E = U + \text{KE} + \text{PE}$, Eq. (g) becomes

$$W_c = \underline{(U - U_o) + p_o(V - V_o) - T_o(S - S_o) + \text{KE} + \text{PE}} - T_o\sigma_c \tag{h}$$

The value of the underlined term in Eq. (h) is determined by the two end states of the system—the given state and the dead state—and is independent of the details of the process linking these states. However, the value of the term $T_o\sigma_c$ depends on the nature of the process as the system passes to the dead state. In accordance with the second law, $T_o\sigma_c$ is positive when irreversibilities are present and vanishes in the limiting case where there are no irreversibilities. The value of $T_o\sigma_c$ cannot be negative. Hence, the *maximum* theoretical value for the work of the overall system W_c is obtained by setting $T_o\sigma_c$ to zero in Eq. (h). By definition, this maximum value is the exergy, E. Accordingly, Eq. 7.1 is seen to be the appropriate expression for evaluating exergy.

7.3.1 Exergy Aspects

In this section, we list five important aspects of the exergy concept:

1. Exergy is a measure of the departure of the state of a system from that of the environment. It is therefore an attribute of the system and environment together. However, once the environment is specified, a value can be assigned to exergy in terms of property values for the system only, so exergy can be regarded as a property of the system. Exergy is an extensive property.

2. The value of exergy cannot be negative. If a system were at any state other than the dead state, the system would be able to change its condition *spontaneously* toward the dead state; this tendency would cease when the dead state was reached. No work must be done to effect such a spontaneous change. Accordingly, any change in state of the system to the dead state can be accomplished with *at least zero* work being developed, and thus the *maximum* work (exergy) cannot be negative.

3. Exergy is not conserved but is destroyed by irreversibilities. A limiting case is when exergy is completely destroyed, as would occur if a system were permitted to undergo a spontaneous change to the dead state with no provision to obtain work. The potential to develop work that existed originally would be completely wasted in such a spontaneous process.

4. Exergy has been viewed thus far as the *maximum* theoretical work obtainable from an *overall* system of system plus environment as the system passes *from* a given state *to* the dead state. Alternatively, exergy can be regarded as the magnitude of the *minimum* theoretical work *input* required to bring the system *from* the dead state *to* the given state. Using energy and entropy balances as above, we can readily develop Eq. 7.1 from this viewpoint. This is left as an exercise.

5. When a system is at the dead state, it is in *thermal* and *mechanical* equilibrium with the environment, and the value of exergy is zero. More precisely, the *thermomechanical* contribution to exergy is zero. This modifying term distinguishes the exergy concept of the present chapter from another contribution to exergy introduced in Sec. 13.6, where the contents of a system at the dead state are permitted to enter into chemical reaction with environmental components and in so doing develop additional work. This contribution to exergy is called *chemical exergy.* The chemical exergy concept is important in the second law analysis of many types of systems, in particular systems involving combustion. Still, as shown in this chapter, the thermomechanical exergy concept suffices for a wide range of thermodynamic evaluations.

BIOCONNECTIONS The U.S. poultry industry produces billions of pounds of meat annually, with chicken production accounting for over 80% of the total. The annual amount of waste produced by these birds also reaches into the billions of pounds. The waste may be more than can be managed by disposal over cropland as fertilizer. Some of the excess can be used to manufacture fertilizer pellets for commercial and domestic use. Despite its relatively low chemical exergy, poultry waste also can be used to produce methane through *anaerobic* digestion. The methane can be burned in power plants to make electricity or process steam. Digester systems are available for use right on the farm. These are positive developments for an important sector of the U.S. agricultural economy that has received adverse publicity for concerns over arsenic content of poultry waste, run-off of waste into streams and rivers, and excessive odor and fly infestation in the vicinity of huge farming operations.

7.3.2 Specific Exergy

Although exergy is an extensive property, it is often convenient to work with it on a unit mass or molar basis. Expressing Eq. 7.1 on a unit mass basis, the **specific exergy**, e, is

specific exergy

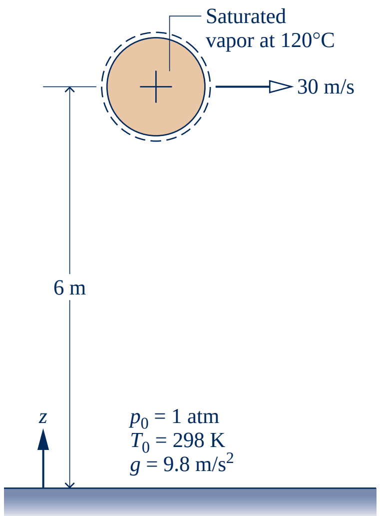
$$\mathsf{e} = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \mathrm{V}^2/2 + gz \qquad (7.2)$$

TAKE NOTE...
Kinetic and potential energy are rightfully considered as exergy. But for simplicity of expression in the present chapter, we refer to these terms—whether viewed as energy or exergy—as accounting for the effects of motion and gravity. The meaning will be clear in context.

where u, v, s, $\mathrm{V}^2/2$, and gz are the specific internal energy, volume, entropy, kinetic energy, and potential energy, respectively, at the state of interest; u_0, v_0, and s_0 are specific properties at the dead state: at T_0, p_0. In Eq. 7.2, the kinetic and potential energies are measured relative to the environment and thus contribute their full values to the exergy magnitude because, in principle, each could be fully converted to work were the system brought to rest at zero elevation relative to the environment. Finally, by inspection of Eq. 7.2, note that the units of specific exergy are the same as for specific energy, kJ/kg or Btu/lb.

The specific exergy at a specified state requires properties at that state and at the dead state.

▸ FOR EXAMPLE let us use Eq. 7.2 to determine the specific exergy of saturated water vapor at 120°C, having a velocity of 30 m/s and an elevation of 6 m, each relative to an exergy reference environment where $T_0 = 298$ K (25°C), $p_0 = 1$ atm, and $g = 9.8$ m/s^2. For water as saturated vapor at 120°C, Table A-2 gives $v = 0.8919$ m^3/kg, $u = 2529.3$ kJ/kg, $s = 7.1296$ kJ/kg · K. At the dead state, where $T_0 = 298$ K (25°C) and $p_0 = 1$ atm, water is a liquid. Thus, with Eqs. 3.11, 3.12, and 6.5 and values from Table A-2, we get $v_0 = 1.0029 \times 10^{-3}$ m^3/kg, $u_0 = 104.88$ kJ/kg, $s_0 = 0.3674$ kJ/kg · K. Substituting values



$$\begin{aligned}
\mathsf{e} &= (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{\mathrm{V}^2}{2} + gz \\
&= \left[(2529.3 - 104.88)\frac{\text{kJ}}{\text{kg}}\right] \\
&\quad + \left[\left(1.01325 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)(0.8919 - 1.0029 \times 10^{-3})\frac{\text{m}^3}{\text{kg}}\right]\left|\frac{1\ \text{kJ}}{10^3\ \text{N} \cdot \text{m}}\right| \\
&\quad - \left[(298\ \text{K})(7.1296 - 0.3674)\frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right] \\
&\quad + \left[\frac{(30\ \text{m/s})^2}{2} + \left(9.8\frac{\text{m}}{\text{s}^2}\right)(6\ \text{m})\right]\left|\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N} \cdot \text{m}}\right| \\
&= (2424.42 + 90.27 - 2015.14 + 0.45 + 0.06)\frac{\text{kJ}}{\text{kg}} = 500\frac{\text{kJ}}{\text{kg}}
\end{aligned}$$

The following example illustrates the use of Eq. 7.2 together with ideal gas property data.

EXAMPLE 7.1

Evaluating the Exergy of Exhaust Gas

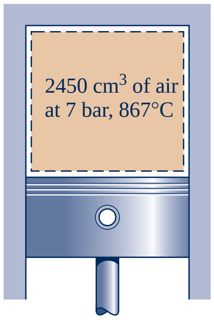
A cylinder of an internal combustion engine contains 2450 cm^3 of gaseous combustion products at a pressure of 7 bar and a temperature of 867°C just before the exhaust valve opens. Determine the specific exergy of the gas, in kJ/kg. Ignore the effects of motion and gravity, and model the combustion products as air behaving as an ideal gas. Take $T_0 = 300$ K (27°C) and $p_0 = 1.013$ bar.

SOLUTION

Known: Gaseous combustion products at a specified state are contained in the cylinder of an internal combustion engine.

Find: Determine the specific exergy.

Schematic and Given Data:



Engineering Model:

1. The gaseous combustion products are a closed system.

2. The combustion products are modeled as air behaving as an ideal gas.

3. The effects of motion and gravity can be ignored.

4. $T_0 = 300$ K (27°C) and $p_0 = 1.013$ bar.

Fig. E7.1

Analysis: With assumption 3, Eq. 7.2 becomes

$$\mathsf{e} = u - u_0 + p_0(v - v_0) - T_0(s - s_0)$$

The internal energy and entropy terms are evaluated using data from Table A-22, as follows:

$$\begin{aligned} u - u_0 &= (880.35 - 214.07) \text{ kJ/kg} \\ &= 666.28 \text{ kJ/kg} \\ s - s_0 &= s°(T) - s°(T_0) - \frac{\overline{R}}{M} \ln \frac{p}{p_0} \\ &= \left(3.11883 - 1.70203 - \left(\frac{8.314}{28.97}\right) \ln \left(\frac{7}{1.013}\right)\right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ &= 0.8621 \text{ kJ/kg} \cdot \text{K} \\ T_0(s - s_0) &= (300 \text{ K})(0.8621 \text{ kJ/kg} \cdot \text{K}) \\ &= 258.62 \text{ kJ/kg} \end{aligned}$$

The $p_0(v - v_0)$ term is evaluated using the ideal gas equation of state: $v = (\overline{R}/M)T/p$ and $v_0 = (\overline{R}/M)T_0/p_0$, so

$$\begin{aligned} p_0(v - v_0) &= \frac{\overline{R}}{M}\left(\frac{p_0 T}{p} - T_0\right) \\ &= \frac{8.314}{28.97}\left(\frac{(1.013)(1140)}{7} - 300\right)\frac{\text{kJ}}{\text{kg}} \\ &= -38.75 \text{ kJ/kg} \end{aligned}$$

Substituting values into the above expression for the specific exergy

$$\begin{aligned} \mathsf{e} &= (666.28 + (-38.75) - 258.62) \text{ kJ/kg} \\ &= 368.91 \text{ kJ/kg} \end{aligned}$$

1

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1 If the gases are discharged directly to the surroundings, the potential for developing work quantified by the exergy value determined in the solution is wasted. However, by venting the gases through a turbine, some work could be developed. This principle is utilized by the *turbochargers* added to some internal combustion engines.

Skills Developed

Ability to...

- ❑ evaluate specific exergy.
- ❑ apply the ideal gas model.

QuickQUIZ To what elevation, in m, would a 1-kg mass have to be raised from zero elevation with respect to the reference environment for its exergy to equal that of the gas in the cylinder? Assume $g = 9.81$ m/s². **Ans.** 197 m.

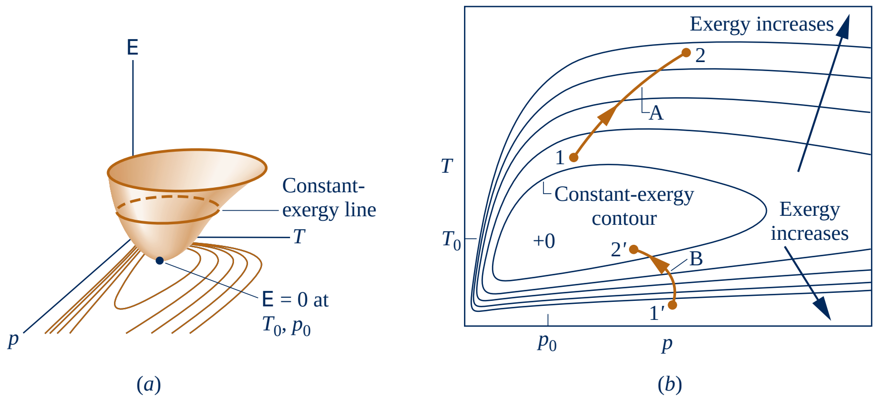


Fig. 7.4 Exergy-temperature-pressure surface for a gas. (*a*) Three-dimensional view (*b*) Constant exergy contours on a *T-p* diagram.

7.3.3 Exergy Change

A closed system at a given state can attain new states by various means, including work and heat interactions with its surroundings. The exergy value associated with a new state generally differs from the exergy value at the initial state. Using Eq. 7.1, we can determine the change in exergy between the two states. At the initial state

$$\mathsf{E}_1 = (U_1 - U_0) + p_0(V_1 - V_0) - T_0(S_1 - S_0) + \mathrm{KE}_1 + \mathrm{PE}_1$$

At the final state

$$\mathsf{E}_2 = (U_2 - U_0) + p_0(V_2 - V_0) - T_0(S_2 - S_0) + \mathrm{KE}_2 + \mathrm{PE}_2$$

Subtracting these we get the **exergy change**

exergy change

$$\mathsf{E}_2 - \mathsf{E}_1 = (U_2 - U_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1) + (\mathrm{KE}_2 - \mathrm{KE}_1) + (\mathrm{PE}_2 - \mathrm{PE}_1) \tag{7.3}$$

Note that the dead state values U_0, V_0, S_0 cancel when we subtract the expressions for E_1 and E_2.

Exergy *change* can be illustrated using Fig. 7.4, which shows an exergy-temperature-pressure surface for a gas together with constant-exergy contours projected on temperature-pressure coordinates. For a system undergoing Process A, exergy increases as its state *moves away* from the dead state (from 1 to 2). In process B, exergy decreases as the state moves *toward* the dead state (from 1′ to 2′.)

7.4 Closed System Exergy Balance

Like energy, exergy can be transferred across the boundary of a closed system. The change in exergy of a system during a process would not necessarily equal the net exergy transferred because exergy would be destroyed if irreversibilities were present within the system during the process. The concepts of exergy change, exergy transfer, and exergy destruction are related by the closed system exergy balance introduced in this section. The exergy balance concept is extended to control volumes in Sec. 7.5. Exergy balances are expressions of the second law of thermodynamics and provide the basis for exergy analysis.

7.4.1 Introducing the Closed System Exergy Balance

The **closed system exergy balance** is given by Eq. 7.4a. See the box for its development.

$$\underbrace{\mathsf{E}_2 - \mathsf{E}_1}_{\substack{\text{exergy}\\ \text{change}}} = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q - \left[W - p_0(V_2 - V_1)\right]}_{\substack{\text{exergy}\\ \text{transfers}}} - \underbrace{T_0\sigma}_{\substack{\text{exergy}\\ \text{destruction}}} \tag{7.4a}$$

closed system exergy balance

For specified end states and given values of p_0 and T_0, the exergy change $\mathsf{E}_2 - \mathsf{E}_1$ on the left side of Eq. 7.4a can be evaluated from Eq. 7.3. The underlined terms on the right depend explicitly on the nature of the process, however, and cannot be determined by knowing only the end states and the values of p_0 and T_0. These terms are interpreted in the discussions of Eqs. 7.5–7.7, respectively.

Developing the Exergy Balance

The exergy balance for a closed system is developed by combining the closed system energy and entropy balances. The forms of the energy and entropy balances used are, respectively

$$\Delta U + \Delta \mathrm{KE} + \Delta \mathrm{PE} = \left(\int_1^2 \delta Q\right) - W$$

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

where W and Q represent, respectively, work and heat transfer between the system and its surroundings. In the entropy balance, T_b denotes the temperature on the system boundary where δQ occurs. The term σ accounts for entropy produced within the system by internal irreversibilities.

As the first step in deriving the exergy balance, multiply the entropy balance by the temperature T_0 and subtract the resulting expression from the energy balance to obtain

$$(\Delta U + \Delta \mathrm{KE} + \Delta \mathrm{PE}) - T_0 \Delta S = \left(\int_1^2 \delta Q\right) - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_b - W - T_0\sigma$$

Collecting the terms involving δQ on the right side and introducing Eq. 7.3 on the left side, we get

$$(\mathsf{E}_2 - \mathsf{E}_1) - p_0(V_2 - V_1) = \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q - W - T_0\sigma$$

On rearrangement, this expression gives Eq. 7.4a, the closed system exergy balance.

Since Eq. 7.4a is obtained by deduction from the energy and entropy balances, it is not an independent result, but can be used in place of the entropy balance as an expression of the second law.

The first underlined term on the right side of Eq. 7.4a is associated with heat transfer to or from the system during the process. It is interpreted as the **exergy transfer accompanying heat transfer**. That is

exergy transfer accompanying heat transfer

$$\mathsf{E}_q = \left[\begin{array}{c}\textit{exergy}\text{ transfer}\\ \text{accompanying heat}\\ \text{transfer}\end{array}\right] = \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q \tag{7.5}$$

where T_b denotes the temperature on the boundary where heat transfer occurs.

The second underlined term on the right side of Eq. 7.4a is associated with work. It is interpreted as the **exergy transfer accompanying work**. That is

exergy transfer accompanying work

$$\mathsf{E}_w = \left[\begin{array}{c}\textit{exergy}\text{ transfer}\\ \text{accompanying work}\end{array}\right] = [W - p_0(V_2 - V_1)] \tag{7.6}$$

The third underlined term on the right side of Eq. 7.4a accounts for the **destruction of exergy** due to irreversibilities within the system. It is symbolized by E_d. That is

exergy destruction

$$\mathsf{E}_d = T_0\sigma \tag{7.7}$$

With Eqs. 7.5, 7.6 and 7.7, Eq. 7.4a is expressed alternatively as

$$\mathsf{E}_2 - \mathsf{E}_1 = \mathsf{E}_q - \mathsf{E}_w - \mathsf{E}_d \tag{7.4b}$$

Although not required for the practical application of the exergy balance in *any* of its forms, exergy transfer terms can be conceptualized in terms of work, as for the exergy concept itself. See the box for discussion.

Conceptualizing Exergy Transfer

In exergy analysis, heat transfer and work are expressed in terms of a *common measure:* work *fully available* for lifting a weight or, equivalently, as shaft or electrical work. This is the significance of the exergy transfer expressions given by Eqs. 7.5 and 7.6, respectively.

Without regard for the nature of the surroundings with which the system is *actually* interacting, we interpret the *magnitudes* of these exergy transfers as the maximum theoretical work that *could* be developed *were* the system interacting with the environment, as follows:

- On recognizing the term $(1 - T_o/T_b)$ as the Carnot efficiency (Eq. 5.9), the quantity $(1 - T_o/T_b)\delta Q$ appearing in Eq. 7.5 is interpreted as the work developed by a reversible power cycle receiving energy δQ by heat transfer at temperature T_b and discharging energy by heat transfer to the environment at temperature $T_o < T_b$. When T_b, is less than T_o, we also think of the work of a reversible cycle. But in this instance, E_q takes on a negative value signaling that heat transfer and the accompanying exergy transfer are *oppositely* directed.
- The exergy transfer given by Eq. 7.6 is the work W of the system less the work required to displace the environment whose pressure is p_o, namely $p_o(V_2 - V_1)$.

See Example 7.2 for an illustration of these interpretations.

To summarize, in each of its forms Eq. 7.4 states that the change in exergy of a closed system can be accounted for in terms of exergy transfers and the destruction of exergy due to irreversibilities within the system.

When applying the exergy balance, it is essential to observe the requirements imposed by the second law on the exergy destruction: In accordance with the second law, the exergy destruction is positive when irreversibilities are present within the system during the process and vanishes in the limiting case where there are no irreversibilities. That is

$$\mathsf{E}_d : \begin{cases} > 0 & \text{irreversibilities present within the system} \\ = 0 & \text{no irreversibilities present within the system} \end{cases} \tag{7.8}$$

The value of the exergy destruction cannot be negative. Moreover, exergy destruction is *not* a property. On the other hand, exergy *is* a property, and like other properties, the *change* in exergy of a system can be positive, negative, or zero

$$\mathsf{E}_2 - \mathsf{E}_1 : \begin{cases} >0 \\ =0 \\ <0 \end{cases}$$

For an *isolated* system, no heat or work interactions with the surroundings occur, and thus there are no transfers of exergy between the system and its surroundings. Accordingly, the exergy balance reduces to give

$$\Delta \mathsf{E}\big]_{\text{isol}} = -\mathsf{E}_d\big]_{\text{isol}} \tag{7.9}$$

Since the exergy destruction must be positive in any actual process, the only processes of an isolated system that occur are those for which the exergy of the isolated system *decreases*. For exergy, this conclusion is the counterpart of the increase of entropy principle (Sec. 6.8.1) and, like the increase of entropy principle, can be regarded as an alternative statement of the second law.

In Example 7.2, we consider exergy change, exergy transfer, and exergy destruction for the process of water considered in Example 6.1, which should be quickly reviewed before studying the current example.

EXAMPLE 7.2

Exploring Exergy Change, Transfer, and Destruction

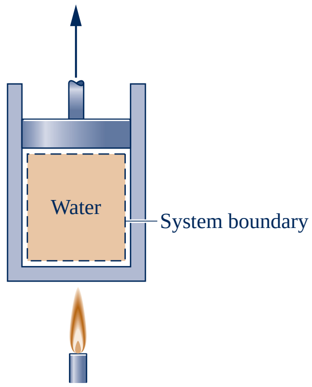
Water initially a saturated liquid at 150°C (423.15 K) is contained in a piston–cylinder assembly. The water is heated to the corresponding saturated vapor state in an internally reversible process at constant temperature and pressure. For $T_0 = 20°\text{C}$ (293.15 K), $p_0 = 1$ bar, and ignoring the effects of motion and gravity, determine per unit of mass, each in kJ/kg, **(a)** the change in exergy, **(b)** the exergy transfer accompanying heat transfer, **(c)** the exergy transfer accompanying work, and **(d)** the exergy destruction.

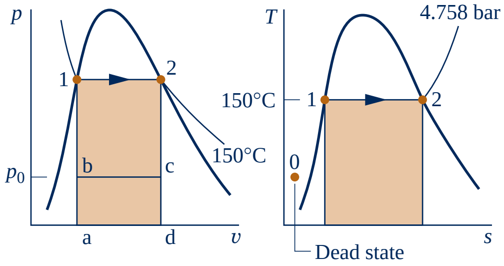
SOLUTION

Known: Water contained in a piston–cylinder assembly undergoes an internally reversible process at 150°C from saturated liquid to saturated vapor.

Find: Determine the change in exergy, the exergy transfers accompanying heat transfer and work, and the exergy destruction.

Schematic and Given Data:





Data from Example 6.1:

W/m = 186.38 kJ/kg, Q/m = 2114.1 kJ/kg

State	v (m^3/kg)	u (kJ/kg)	s (kJ/kg·K)
1	1.0905×10^{-3}	631.68	1.8418
2	0.3928	2559.5	6.8379

Engineering Model:

1. The water in the piston–cylinder assembly is a closed system.
2. The process is internally reversible.
3. Temperature and pressure are constant during the process.
4. Ignore the effects of motion and gravity.
5. $T_0 = 293.15$ K, $p_0 = 1$ bar.

Fig. E7.2

Analysis:

(a) Using Eq. 7.3 together with assumption 4, we have per unit of mass

$$\mathsf{e}_2 - \mathsf{e}_1 = u_2 - u_1 + p_0(v_2 - v_1) - T_0(s_2 - s_1) \quad \textbf{(a)}$$

With data from Fig. E7.2

$$\mathsf{e}_2 - \mathsf{e}_1 = (2559.5 - 631.68)\frac{\text{kJ}}{\text{kg}} + \left(1.0 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)(0.3928 - (1.0905 \times 10^{-3}))\frac{\text{m}^3}{\text{kg}}\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$-293.15\text{ K}\,(6.8379 - 1.8418)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= (1927.82 + 39.17 - 1464.61)\frac{\text{kJ}}{\text{kg}} = 502.38\frac{\text{kJ}}{\text{kg}}$$

(b) Noting that temperature remains constant, Eq. 7.5, on a per unit of mass basis, reads

1 $$\frac{\mathsf{E}_\text{q}}{m} = \left(1 - \frac{T_0}{T}\right)\frac{Q}{m} \quad \textbf{(b)}$$

With $Q/m = 2114.1$ kJ/kg from Fig. E7.2

$$\frac{\mathsf{E}_\text{q}}{m} = \left(1 - \frac{293.15\text{ K}}{423.15\text{ K}}\right)\left(2114.1\frac{\text{kJ}}{\text{kg}}\right) = 649.49\frac{\text{kJ}}{\text{kg}}$$

(c) With $W/m = 186.38$ kJ/kg from Fig. E7.2 and $p_0(v_2 - v_1) = 39.17$ kJ/kg from part (a), Eq. 7.6 gives, per unit of mass

2 $$\frac{\mathsf{E}_\text{w}}{m} = \frac{W}{m} - p_0(v_2 - v_1) \quad \textbf{(c)}$$

$$= (186.38 - 39.17)\frac{\text{kJ}}{\text{kg}} = 147.21\frac{\text{kJ}}{\text{kg}}$$

(d) Since the process is internally reversible, the exergy destruction is necessarily zero. This can be checked by inserting the results of parts (a)–(c) into an exergy balance. Thus, solving Eq. 7.4b for the exergy destruction per unit of mass, evaluating terms, and allowing for roundoff, we get

$$\frac{\mathsf{E}_\text{d}}{m} = -(\mathsf{e}_2 - \mathsf{e}_1) + \frac{\mathsf{E}_\text{q}}{m} - \frac{\mathsf{E}_\text{w}}{m}$$

$$= (-502.38 + 649.49 - 147.21)\frac{\text{kJ}}{\text{kg}} = 0$$

Alternatively, the exergy destruction can be evaluated using Eq. 7.7 together with the entropy production obtained from an entropy balance. This is left as an exercise.

❶ Recognizing the term $(1 - T_0/T)$ as the Carnot efficiency (Eq. 5.9), the right side of Eq. (b) can be interpreted as the work that *could be* developed by a reversible power cycle *were* it to receive energy Q/m at temperature T and discharge energy to the environment by heat transfer at T_0.

❷ The right side of Eq. (c) shows that *if* the system were interacting with the environment, all of the work W/m, represented by area 1-2-d-a-1 on the p–v diagram of Fig. E7.2, would not be fully available for lifting a weight. A portion would be spent in pushing aside the environment at pressure p_0. This portion is given by $p_0(v_2 - v_1)$, and is represented by area a-b-c-d-a on the p–v diagram of Fig. E7.2.

Skills Developed

Ability to...

- ❑ evaluate exergy change.
- ❑ evaluate exergy transfer accompanying heat transfer and work.
- ❑ evaluate exergy destruction.

QuickQUIZ If heating from saturated liquid to saturated vapor would occur at 100°C (373.15 K), evaluate the exergy transfers accompanying heat transfer and work, each in kJ/kg. **Ans.** 484, 0.

7.4.2 Closed System Exergy Rate Balance

As in the case of the mass, energy, and entropy balances, the exergy balance can be expressed in various forms that may be more suitable for particular analyses. A convenient form is the *closed system exergy rate balance* given by

$$\frac{d\mathsf{E}}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W} - p_0\frac{dV}{dt}\right) - \dot{\mathsf{E}}_\mathrm{d} \tag{7.10}$$

where $d\mathsf{E}/dt$ is the time rate of change of exergy. The term $(1 - T_0/T_j)\dot{Q}_j$ represents the time rate of exergy transfer accompanying heat transfer at the rate $\dot{Q}_j$ occurring where the instantaneous temperature on the boundary is T_j. The term $\dot{W}$ represents the time rate of energy transfer by work. The accompanying rate of exergy transfer is given by $(\dot{W} - p_0 dV/dt)$ where dV/dt is the time rate of change of system volume. The term $\dot{\mathsf{E}}_\mathrm{d}$ accounts for the time rate of exergy destruction due to irreversibilities within the system.

At steady state, $d\mathsf{E}/dt = dV/dt = 0$ and Eq. 7.10 reduces to give the **steady-state form of the exergy rate balance**.

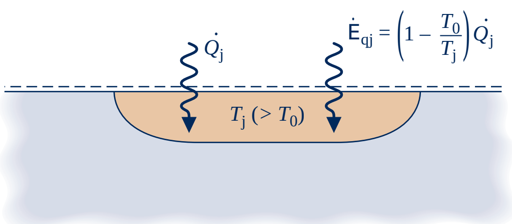
steady-state form of the closed system exergy rate balance

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W} - \dot{\mathsf{E}}_\mathrm{d} \tag{7.11a}$$

Note that for a system at steady state, the rate of exergy transfer accompanying the power $\dot{W}$ is simply the power.

The rate of exergy transfer accompanying heat transfer at the rate $\dot{Q}_j$ occurring where the temperature is T_j is compactly expressed as

$$\dot{\mathsf{E}}_{\mathrm{q}j} = \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j \tag{7.12}$$



As shown in the adjacent figure, heat transfer and the accompanying exergy transfer are in the same direction when $T_j > T_0$.

Using Eq. 7.12, Eq. 7.11a reads

$$0 = \sum_j \dot{\mathsf{E}}_{qj} - \dot{W} - \dot{\mathsf{E}}_d \tag{7.11b}$$

In Eqs. 7.11, the rate of exergy destruction within the system, $\dot{\mathsf{E}}_d$, is related to the rate of entropy production within the system by $\dot{\mathsf{E}}_d = T_0\dot{\sigma}$.

7.4.3 Exergy Destruction and Loss

Most thermal systems are supplied with exergy inputs derived directly or indirectly from the consumption of fossil fuels. Accordingly, *avoidable* destructions and losses of exergy represent the waste of these resources. By devising ways to reduce such inefficiencies, better use can be made of fuels. The exergy balance can be applied to determine the locations, types, and true magnitudes of energy resource waste, and thus can play an important part in developing strategies for more effective fuel use.

In Example 7.3, the steady-state form of the closed system energy and exergy rate balances are applied to an oven wall to evaluate exergy destruction and exergy loss, which are interpreted in terms of fossil fuel use.

EXAMPLE 7.3

Evaluating Exergy Destruction in an Oven Wall

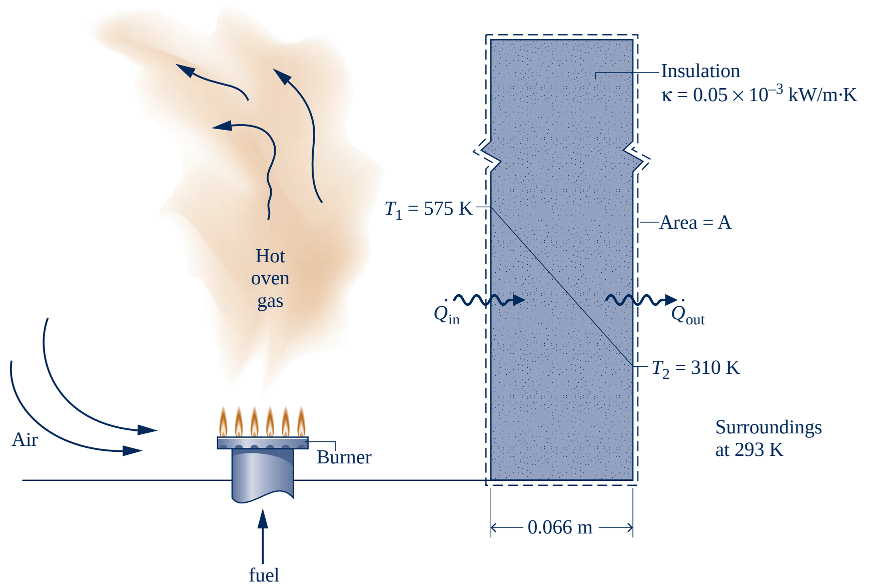
The wall of an industrial drying oven is constructed by sandwiching 0.066m-thick insulation, having a thermal conductivity $\kappa = 0.05 \times 10^{-3}$ kW/m · K, between thin metal sheets. At steady state, the inner metal sheet is at $T_1 = 575$ K and the outer sheet is at $T_2 = 310$ K. Temperature varies linearly through the wall. The temperature of the surroundings away from the oven is 293 K. Determine, in kW per m^2 of wall surface area, **(a)** the rate of heat transfer through the wall, **(b)** the rates of exergy transfer accompanying heat transfer at the inner and outer wall surfaces, and **(c)** the rate of exergy destruction within the wall. Let $T_0 = 293$ K.

SOLUTION

Known: Temperature, thermal conductivity, and wall-thickness data are provided for a plane wall at steady state.

Find: For the wall, determine (a) the rate of heat transfer through the wall, (b) the rates of exergy transfer accompanying heat transfer at the inner and outer surfaces, and (c) the rate of exergy destruction, each per m^2 of wall surface area.

Schematic and Given Data:



Engineering Model:

1. The closed system shown in the accompanying sketch is at steady state.
2. Temperature varies linearly through the wall.
3. $T_0 = 293$ K.

Fig. E7.3

Analysis:

(a) At steady state, an energy rate balance for the system reduces to give $\dot{Q}_{in} = \dot{Q}_{out}$—namely, the rates of heat transfer into and out of the wall are equal. Let $\dot{Q}$ denote the common heat transfer rate. Using Eq. 2.3.1 with assumption 2, the heat transfer rate is given by

$$(\dot{Q}/A) = -\kappa\left[\frac{T_2 - T_1}{L}\right]$$
$$= -\left(0.05 \times 10^{-3}\frac{\text{kW}}{\text{m}\cdot\text{K}}\right)\left[\frac{(310 - 575)\ \text{K}}{0.066\ \text{m}}\right] = 0.2\frac{\text{kW}}{\text{m}^2}$$

(b) The rates of exergy transfer accompanying heat transfer are evaluated using Eq. 7.12. At the inner surface

$$(\dot{\mathsf{E}}_{q1}/A) = \left[1 - \frac{T_0}{T_1}\right](\dot{Q}/A)$$
$$= \left[1 - \frac{293}{575}\right]\left(0.2\frac{\text{kW}}{\text{m}^2}\right) = 0.1\frac{\text{kW}}{\text{m}^2}$$

At the outer surface

$$(\dot{\mathsf{E}}_{q2}/A) = \left[1 - \frac{T_0}{T_2}\right](\dot{Q}/A)$$

1

$$= \left[1 - \frac{293}{310}\right]\left(0.2\frac{\text{kW}}{\text{m}^2}\right) = 0.01\frac{\text{kW}}{\text{m}^2}$$

(c) The rate of exergy destruction within the wall is evaluated using the exergy rate balance. Since $\dot{W} = 0$, Eq. 7.11b gives

2

$$(\dot{\mathsf{E}}_d/A) = (\dot{\mathsf{E}}_{q1}/A) - (\dot{\mathsf{E}}_{q2}/A)$$

3

$$= (0.1 - 0.01)\frac{\text{kW}}{\text{m}^2} = 0.09\frac{\text{kW}}{\text{m}^2}$$

---

1 The rates of heat transfer are the same at the inner and outer walls, but the rates of exergy transfer at these locations are much different. The rate of exergy transfer at the high-temperature inner wall is 10 times the rate of exergy transfer at the low-temperature outer wall. At each of these locations the exergy transfers provide a truer measure of thermodynamic value than the heat transfer rate. This is clearly seen at the outer wall, where the small exergy transfer indicates minimal potential for use, and thus minimal thermodynamic value.

2 The exergy transferred into the wall at $T_1 = 575$ K is either destroyed within the wall owing to spontaneous heat transfer or transferred out of the wall at $T_2 = 310$ K where it is *lost* to the surroundings. Exergy transferred to the surroundings accompanying stray heat transfer, as in the present case, is ultimately destroyed in the surroundings. Thicker insulation and/or insulation having a lower thermal conductivity value would reduce the heat transfer rate and thus lower the exergy destruction and loss.

3 In this example, the exergy destroyed and lost has its origin in the fuel supplied. Thus, cost-effective measures to reduce exergy destruction and loss have benefits in terms of better fuel use.

Skills Developed

Ability to...

- ❑ apply the energy and exergy rate balances.
- ❑ evaluate exergy transfer accompanying heat transfer.
- ❑ evaluate exergy destruction.

QuickQUIZ If the thermal conductivity were reduced to 0.04 × 10^{-3} kW/m · K, owing to a different selection of insulation material, while the insulation thickness were increased to 0.076 m, determine the rate of exergy destruction in the wall, in kW per m² of wall surface area, keeping the same inner and outer wall temperatures and the same temperature of the surroundings. **Ans.** 0.06 kW/m².

Horizons

Superconducting Power Cable Overcoming All Barriers?

According to industry sources, more than 7% of the electric power conducted through present-day transmission and distribution lines is forfeited en route owing to electrical resistance. They also say *superconducting* cable can nearly eliminate resistance to electricity flow, and thereby the accompanying reduction in power.

For superconducting cable to be effective, however, it must be cooled to about −200°C (−330°F). Cooling is achieved by a refrigeration system using liquid nitrogen. Since the refrigerator requires power to operate, such cooling reduces the power *saved* in superconducting electrical transmission. Moreover, the cost of superconducting cable is much greater today than conventional cable. Factors such as these impose barriers to rapid deployment of superconducting technology.

Still, electrical utilities have partnered with the government to develop and demonstrate superconducting technology that someday may increase U.S. power system efficiency and reliability.

7.4.4 Exergy Accounting

exergy accounting

In the next example, we reconsider the gearbox of Examples 2.4 and 6.4 from an exergy perspective to introduce **exergy accounting**, in which the various terms of an exergy balance for a system are systematically evaluated and compared.

EXAMPLE 7.4

Exergy Accounting of a Gearbox

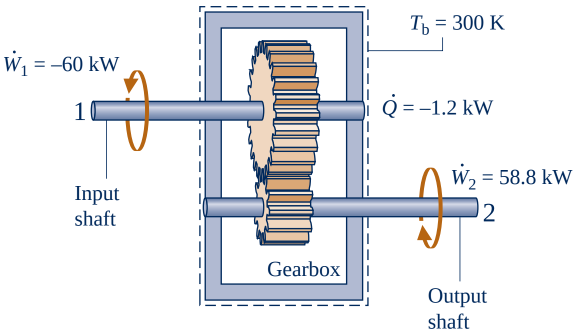
For the gearbox of Examples 2.4 and 6.4(a), develop a full exergy accounting of the power input. Let $T_0 = 293$ K.

SOLUTION

Known: A gearbox operates at steady state with known values for the power input, power output, and heat transfer rate. The temperature on the outer surface of the gearbox is also known.

Find: Develop a full exergy accounting of the input power.

Schematic and Given Data:



Engineering Model:

1. The gearbox is taken as a closed system operating at steady state.
2. The temperature at the outer surface does not vary.
3. $T_0 = 293$ K.

Fig. E7.4

Analysis: Since the gearbox is at steady state, the rate of exergy transfer accompanying power is simply the power. Accordingly, exergy is transferred *into* the gearbox via the high-speed shaft at a rate equal to the power *input*, 60 kW, and exergy is transferred *out* via the low-speed shaft at a rate equal to the power *output*, 58.8 kW. Additionally, exergy is transferred out accompanying stray heat transfer and destroyed by irreversibilities within the gearbox.

The rate of exergy transfer accompanying heat transfer is evaluated using Eq. 7.12. That is

$$\dot{\mathsf{E}}_q = \left(1 - \frac{T_0}{T_b}\right)\dot{Q}$$

With $\dot{Q} = -1.2$ kW and $T_b = 300$ K from Fig. E7.4, we get

$$\dot{\mathsf{E}}_q = \left(1 - \frac{293}{300}\right)(-1.2\text{ kW}) = -0.03\text{ kW}$$

where the minus sign denotes exergy transfer *from* the system.

The rate of exergy destruction is evaluated using the exergy rate balance. On rearrangement, and noting that $\dot{W} = \dot{W}_1 + \dot{W}_2 = -1.2$ kW, Eq. 7.11b gives

1 $$\dot{\mathsf{E}}_d = \dot{\mathsf{E}}_q - \dot{W} = -0.03\text{ kW} - (-1.2\text{ kW}) = 1.17\text{ kW}$$

The analysis is summarized by the following exergy *balance sheet* in terms of exergy magnitudes on a rate basis:

Rate of exergy in:	
high-speed shaft	60.00 kW (100%)
Disposition of the exergy:	
• Rate of exergy out	
low-speed shaft	58.80 kW (98%)
stray heat transfer	0.03 kW (0.05%)
• Rate of exergy destruction	1.17 kW (1.95%)
	60.00 kW (100%)

2

1 Alternatively, the rate of exergy destruction is calculated from $\dot{\mathsf{E}}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. From the solution to Example 6.4(a), $\dot{\sigma} = 4 \times 10^{-3}$ kW/K. Then

$$\begin{aligned}\dot{\mathsf{E}}_d &= T_0\dot{\sigma} \\ &= (293\text{ K})(4 \times 10^{-3}\text{ kW/K}) \\ &= 1.17\text{ kW}\end{aligned}$$

2 The difference between the input and output power is accounted for primarily by the rate of exergy destruction and only secondarily by the exergy transfer accompanying heat transfer, which is small by comparison. The exergy balance sheet provides a sharper picture of performance than the energy balance sheet of Example 2.4, which does not explicitly consider the effect of irreversibilities within the system.

Skills Developed

Ability to...

- ❑ apply the exergy rate balance.
- ❑ develop an exergy accounting.

QuickQUIZ By inspection of the exergy balance sheet, specify an exergy-based efficiency for the gear box. Ans. 98%.

7.5 Exergy Rate Balance for Control Volumes at Steady State

In this section, the exergy balance is extended to a form applicable to control volumes at steady state. The control volume form is generally the most useful for engineering analysis.

The exergy rate balance for a control volume can be derived using an approach like that employed in the box of Sec. 4.1, where the control volume form of the mass rate balance is obtained by transforming the closed system form. However, as in the developments of the energy and entropy rate balances for control volumes (Secs. 4.4.1 and 6.9, respectively), the present derivation is conducted less formally by modifying the closed system rate form, Eq. 7.10, to account for the exergy transfers at the inlets and exits. The result is

$$\frac{d\mathsf{E}_{cv}}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W}_{cv} - p_0 \frac{dV_{cv}}{dt}\right) + \underline{\sum_i \dot{m}_i \mathsf{e}_{fi} - \sum_e \dot{m}_e \mathsf{e}_{fe}} - \dot{\mathsf{E}}_d$$

where the underlined terms account for exergy transfer where mass enters and exits the control volume, respectively.

At steady state, $d\mathsf{E}_{cv}/dt = dV_{cv}/dt = 0$, giving the **steady-state exergy rate balance**

steady-state exergy rate balance: control volumes

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W}_{cv} + \sum_i \dot{m}_i \mathsf{e}_{fi} - \sum_e \dot{m}_e \mathsf{e}_{fe} - \dot{\mathsf{E}}_d \tag{7.13a}$$

where e_{fi} accounts for the exergy per unit of mass entering at inlet i and e_{fe} accounts for the exergy per unit of mass exiting at exit e. These terms, known as the **specific flow exergy**, are expressed as

specific flow exergy

$$\mathsf{e}_f = h - h_0 - T_0(s - s_0) + \frac{V^2}{2} + gz \tag{7.14}$$

where h and s represent the specific enthalpy and entropy, respectively, at the inlet or exit under consideration; h_0 and s_0 represent the respective values of these properties when evaluated at T_0, p_0. See the box for a derivation of Eq. 7.14 and discussion of the flow exergy concept.

Conceptualizing Specific Flow Exergy

To evaluate the exergy associated with a flowing stream of matter at a state given by h, s, V, and z, let us think of the stream being fed to the control volume operating at steady state shown in Fig. 7.5. At the exit of the control volume, the respective properties are those corresponding to the dead state: h_0, s_0, $V_0 = 0$, $z_0 = 0$. Heat transfer occurs only with the environment at $T_b = T_0$.

For the control volume of Fig. 7.5, energy and entropy balances read, respectively,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h - h_0) + \frac{(V^2 - (0)^2)}{2} + g(z - 0)\right] \tag{a}$$

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s - s_0) + \dot{\sigma}_{cv} \tag{b}$$

Eliminating $\dot{Q}_{cv}$ between Eqs. (a) and (b), the work developed per unit of mass flowing is

$$\frac{\dot{W}_{cv}}{\dot{m}} = \left[\underline{(h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz}\right] - T_0\left(\frac{\dot{\sigma}_{cv}}{\dot{m}}\right) \tag{c}$$

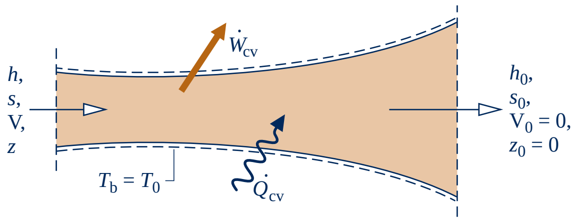


Fig. 7.5 **Control volume used to evaluate the specific flow exergy of a stream.**

The value of the underlined term in Eq. (c) is determined by two states: the given state and the dead state. However, the value of the entropy production term, which cannot be negative, depends on the nature of the flow. Hence, the maximum theoretical work that could be developed, per unit of mass flowing, corresponds to a zero value for the entropy production—that is, when the flow through the control volume of Fig. 7.5 is internally reversible. The specific flow exergy, e_f, is this work value, and thus Eq. 7.14 is seen to be the appropriate expression for the specific flow exergy.

Subtracting Eq. 7.2 from Eq. 7.14 gives the following relationship between specific flow exergy e_f and specific exergy e,

$$\mathsf{e}_f = \mathsf{e} + \underline{v(p - p_0)} \quad \text{(d)}$$

The underlined term of Eq. (d) has the significance of exergy transfer accompanying *flow work*. Thus, at a control volume inlet or exit, flow exergy e_f accounts for the sum of the exergy accompanying mass flow e and the exergy accompanying flow work. When pressure p at a control volume inlet or exit is less than the dead state pressure p_0, the flow work contribution of Eq. (d) is negative, signaling that the exergy transfer accompanying flow work is opposite to the direction of exergy transfer accompanying mass flow. Flow exergy aspects are also explored in end-of-chapter Problems 7.7 and 7.8.

TAKE NOTE...
Observe that the approach used here to evaluate flow exergy parallels that used in Sec. 7.3 to evaluate exergy of a system. In each case, energy and entropy balances are applied to evaluate maximum theoretical work in the limit as entropy production tends to zero. This approach is also used in Sec. 13.6 to evaluate chemical exergy.

The steady-state exergy rate balance, Eq. 7.13a, can be expressed more compactly as Eq. 7.13b

$$0 = \sum_j \dot{\mathsf{E}}_{qj} - \dot{W}_{cv} + \sum_i \dot{\mathsf{E}}_{fi} - \sum_e \dot{\mathsf{E}}_{fe} - \dot{\mathsf{E}}_d \quad \text{(7.13b)}$$

where

$$\dot{\mathsf{E}}_{qj} = \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j \quad \text{(7.15)}$$

$$\dot{\mathsf{E}}_{fi} = \dot{m}_i \mathsf{e}_{fi} \quad \text{(7.16a)}$$

$$\dot{\mathsf{E}}_{fe} = \dot{m}_e \mathsf{e}_{fe} \quad \text{(7.16b)}$$

are exergy transfer rates. Equation 7.15 has the same interpretation as given for Eq 7.5 in the box on p. 370, only on a time rate basis. Also note that at steady state the rate of exergy transfer accompanying the power $\dot{W}_{cv}$ is simply the power. Finally, the rate of exergy destruction within the control volume, $\dot{\mathsf{E}}_d$ is related to the rate of entropy production by $T_0\dot{\sigma}_{cv}$.

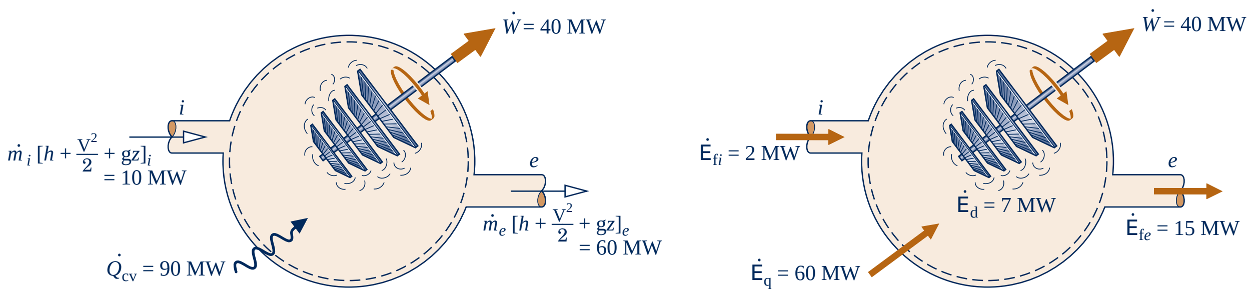
If there is a single inlet and a single exit, denoted by 1 and 2, respectively, the steady-state exergy rate balance, Eq. 7.13a, reduces to

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W}_{cv} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{\mathsf{E}}_d \quad \text{(7.17)}$$

where $\dot{m}$ is the mass flow rate. The term $(\mathsf{e}_{f1} - \mathsf{e}_{f2})$ is evaluated using Eq. 7.14 as

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \quad \text{(7.18)}$$

TAKE NOTE...
When the rate of exergy destruction $\dot{\mathsf{E}}_d$ is the objective, it can be determined either from an exergy rate balance or from $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production evaluated from an entropy rate balance. The second of these procedures normally requires fewer property evaluations and less computation.



Energy In	**Energy Out**
90 MW (heat transfer)	40 MW (power)
10 MW (at inlet i)	60 MW (at exit e)
100 MW	100 MW

(*a*)

Exergy In	**Exergy Out**
60 MW (heat transfer)	40 MW (power)
2 MW (at inlet i)	15 MW (at exit e)
62 MW	55 MW

Exergy Destroyed = 62 MW – 55 MW = 7 MW

(*b*)

Fig. 7.6 Comparing energy and exergy for a control volume at steady state. (*a*) Energy analysis. (*b*) Exergy analysis.

7.5.1 Comparing Energy and Exergy for Control Volumes at Steady State

Although energy and exergy share common units and exergy transfer accompanies energy transfer, energy and exergy are *fundamentally different* concepts. Energy and exergy relate, respectively, to the first and second laws of thermodynamics:

- ▶ Energy is *conserved*. Exergy is *destroyed* by irreversibilities.
- ▶ Exergy expresses energy transfer by work, heat, and mass flow in terms of a *common measure*—namely, work that is *fully available* for lifting a weight or, equivalently, as shaft or electrical work.

▶ FOR EXAMPLE Figure 7.6*a* shows energy transfer rates for a one-inlet, one-exit control volume at steady state. This includes energy transfers by work and heat and the energy transfers in and out where mass flows across the boundary. Figure 7.6*b* shows the same control volume, but now labeled with exergy transfer rates. Be sure to note that the *magnitudes* of the exergy transfers accompanying heat transfer and mass flow *differ* from the corresponding energy transfer magnitudes. These exergy transfer rates are calculated using Eqs. 7.15 and 7.16, respectively. At steady state, the rate of exergy transfer accompanying the power $\dot{W}_{cv}$ is simply the power. In accord with the conservation of energy principle, the total rate energy enters the control volume *equals* the total rate energy exits. However, the total rate exergy enters the control volume *exceeds* the total rate exergy exits. The difference between these exergy values is the rate at which exergy is destroyed by irreversibilities, in accord with the second law. ◀ ◀ ◀ ◀ ◀

To summarize, exergy gives a sharper picture of performance than energy because exergy expresses all energy transfers on a common basis and accounts explicitly for the effect of irreversibilities through the exergy destruction concept.

7.5.2 Evaluating Exergy Destruction in Control Volumes at Steady State

The following examples illustrate the use of mass, energy, and exergy rate balances for the evaluation of exergy destruction in control volumes at steady state. Property data also play an important role in arriving at solutions. The first example involves the expansion of steam through a valve (a throttling process, Sec. 4.10). From an energy perspective, the expansion occurs without loss. Yet, as shown in Example 7.5, such a valve is a site of inefficiency quantified thermodynamically in terms of exergy destruction.

EXAMPLE 7.5

Determining Exergy Destruction in a Throttling Valve

Superheated water vapor enters a valve at 500 lbf/in.^{2}, 500°F and exits at a pressure of 80 lbf/in.^{2} The expansion is a throttling process. Determine the exergy destruction per unit of mass flowing, in Btu/lb. Let T_0 = 77°F, p_0 = 1 atm.

Solution

Known: Water vapor expands in a throttling process through a valve from a specified inlet state to a specified exit pressure.

Find: Determine the exergy destruction per unit of mass flowing.

Schematic and Given Data:

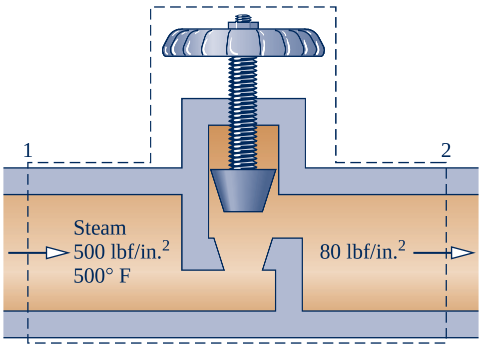


Fig. E7.5

Engineering Model:

1. The control volume shown in the accompanying figure is at steady state.
2. For the throttling process, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of motion and gravity can be ignored.
3. T_0 = 77°F, p_0 = 1 atm.

Analysis: The state at the inlet is specified. The state at the exit can be fixed by reducing the steady-state mass and energy rate balances to obtain Eq. 4.22:

$$h_2 = h_1 \tag{a}$$

Thus, the exit state is fixed by p_2 and h_2. From Table A-4E, h_1 = 1231.5 Btu/lb, s_1 = 1.4923 Btu/lb · °R. Interpolating at a pressure of 80 lbf/in.^{2} with $h_2 = h_1$, the specific entropy at the exit is s_2 = 1.680 Btu/lb · °R.

With assumptions listed, the steady-state form of the exergy rate balance, Eq. 7.17, reduces to

$$0 = \sum_j \cancelto{0}{\left(1 - \frac{T_0}{T_j}\right)} \dot{Q}_j - \cancelto{0}{\dot{W}_{cv}} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{\mathsf{E}}_d$$

Dividing by the mass flow rate $\dot{m}$ and solving, the exergy destruction per unit of mass flowing is

$$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = (\mathsf{e}_{f1} - \mathsf{e}_{f2}) \tag{b}$$

Introducing Eq. 7.18, using Eq. (a), and ignoring the effects of motion and gravity

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = \cancelto{0}{(h_1 - h_2)} - T_0(s_1 - s_2) + \cancelto{0}{\frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2}} + \cancelto{0}{g(z_1 - z_2)}$$

Eq. (b) becomes

1 $$\frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) \tag{c}$$

Inserting values,

2 $$\frac{\dot{E}_d}{\dot{m}} = 537°R\,(1.680 - 1.4923)\frac{\text{Btu}}{\text{lb} \cdot °\text{R}} = 100.8\ \text{Btu/lb}$$

1 Equation (c) can be obtained alternatively beginning with the relationship $\dot{E}_d = T_0\dot{\sigma}_{cv}$ and then evaluating the rate of entropy production $\dot{\sigma}_{cv}$ from an entropy balance. The details are left as an exercise.

2 Energy is conserved in the throttling process, but exergy is destroyed. The source of the exergy destruction is the uncontrolled expansion that occurs.

Skills Developed

Ability to...

- apply the energy and exergy rate balances.
- evaluate exergy destruction.

QuickQUIZ If air modeled as an ideal gas were to undergo a throttling process, evaluate the exergy destruction, in Btu per lb of air flowing, for the same inlet conditions and exit pressure as in this example. Ans. 67.5 Btu/lb.

Although heat exchangers appear from an energy perspective to operate without loss when stray heat transfer is ignored, they are a site of thermodynamic inefficiency quantified by exergy destruction. This is illustrated in Example 7.6.

EXAMPLE 7.6

Evaluating Exergy Destruction in a Heat Exchanger

1 Compressed air enters a counterflow heat exchanger operating at steady state at 610 K, 10 bar and exits at 860 K, 9.7 bar. Hot combustion gas enters as a separate stream at 1020 K, 1.1 bar and exits at 1 bar. Each stream has a mass flow rate of 90 kg/s. Heat transfer between the outer surface of the heat exchanger and the surroundings can be ignored. The effects of motion and gravity are negligible. Assuming the combustion gas stream has the properties of air, and using the ideal gas model for both streams, determine for the heat exchanger

(a) the exit temperature of the combustion gas, in K.

(b) the net change in the flow exergy rate from inlet to exit of each stream, in MW.

(c) the rate exergy is destroyed, in MW.

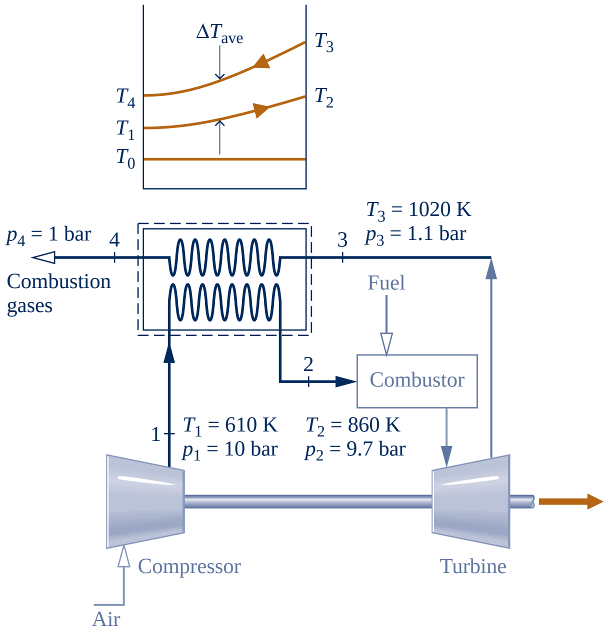
Let $T_0 = 300$ K, $p_0 = 1$ bar.

SOLUTION

Known: Steady-state operating data are provided for a counterflow heat exchanger.

Find: For the heat exchanger, determine the exit temperature of the combustion gas, the change in the flow exergy rate from inlet to exit of each stream, and the rate exergy is destroyed.

Schematic and Given Data:



Engineering Model:

1. The control volume shown in the accompanying figure is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and the effects of motion and gravity are negligible.
3. Each stream has the properties of air modeled as an ideal gas.
4. $T_0 = 300$ K, $p_0 = 1$ bar.

Fig. E7.6

Analysis:

(a) The temperature T_4 of the exiting combustion gases can be found by reducing the mass and energy rate balances for the control volume at steady state to obtain

$$0 = \underline{\dot{Q}_{cv}} - \underline{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \underline{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \underline{g(z_1 - z_2)}\right] + \dot{m}\left[(h_3 - h_4) + \underline{\left(\frac{V_3^2 - V_4^2}{2}\right)} + \underline{g(z_3 - z_4)}\right]$$

where $\dot{m}$ is the common mass flow rate of the two streams. The underlined terms drop out by listed assumptions, leaving

$$0 = \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4)$$

Dividing by $\dot{m}$ and solving for h_4

$$h_4 = h_3 + h_1 - h_2$$

From Table A-22, $h_1 = 617.53$ kJ/kg, $h_2 = 888.27$ kJ/kg, $h_3 = 1068.89$ kJ/kg. Inserting values

$$h_4 = 1068.89 + 617.53 - 888.27 = 798.15 \text{ kJ/kg}$$

Interpolating in Table A-22 gives $T_4 = 778$ K (505°C).

(b) The net change in the flow exergy rate from inlet to exit for the air stream flowing from 1 to 2 can be evaluated using Eq. 7.18, neglecting the effects of motion and gravity. With Eq. 6.20a and data from Table A-22

$$\begin{aligned}
\dot{m}(\mathsf{e}_{f2} - \mathsf{e}_{f1}) &= \dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)] \\
&= \dot{m}\left[(h_2 - h_1) - T_0\left(s_2^\circ - s_1^\circ - R \ln \frac{p_2}{p_1}\right)\right] \\
&= 90\frac{\text{kg}}{\text{s}}\left[(888.27 - 617.53)\frac{\text{kJ}}{\text{kg}} - 300 \text{ K}\left(2.79783 - 2.42644 - \frac{8.314}{28.97} \ln \frac{9.7}{10}\right)\frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right] \\
&= 14{,}103 \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 14.1 \text{ MW}
\end{aligned}$$

2 As the air flows from 1 to 2, its temperature *increases* relative to T_0 and the flow exergy *increases*.

Similarly, the change in the flow exergy rate from inlet to exit for the combustion gas is

$$\begin{aligned}\dot{m}(\mathsf{e}_{f4} - \mathsf{e}_{f3}) &= \dot{m}\left[(h_4 - h_3) - T_0\left(s_4^\circ - s_3^\circ - R\ln\frac{p_4}{p_3}\right)\right] \\ &= 90\left[(798.15 - 1068.89) - 300\left(2.68769 - 2.99034 - \frac{8.314}{28.97}\ln\frac{1}{1.1}\right)\right] \\ &= -16{,}934\frac{\text{kJ}}{\text{s}}\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right| = -16.93\text{ MW}\end{aligned}$$

As the combustion gas flows from 3 to 4, its temperature *decreases* relative to T_0 and the flow exergy *decreases*.

3 **(c)** The rate of exergy destruction within the control volume can be determined from an exergy rate balance, Eq. 7.13a,

$$0 = \sum_j \cancel{\left(1 - \frac{T_0}{T_j}\right)}^{0} \dot{Q}_j - \cancel{\dot{W}_{cv}}^{0} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}(\mathsf{e}_{f3} - \mathsf{e}_{f4}) - \dot{\mathsf{E}}_d$$

Solving for $\dot{\mathsf{E}}_d$ and inserting known values

$$\begin{aligned}\dot{\mathsf{E}}_d &= \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}(\mathsf{e}_{f3} - \mathsf{e}_{f4}) \\ &= (-14.1\text{ MW}) + (16.93\text{ MW}) = 2.83\text{ MW}\end{aligned}$$

4

Comparing results, the exergy increase of the compressed air: 14.1 MW is less than the magnitude of the exergy decrease of the combustion gas: 16.93 MW, even though the energy changes of the two streams are equal in magnitude. The difference in these exergy values is the exergy destroyed: 2.83 MW. Thus, energy is conserved but exergy is not.

---

1 Heat exchangers of this type are known as *regenerators* (see Sec. 9.7).

2 The variation in temperature of each stream passing through the heat exchanger is sketched in the schematic. The dead state temperature T_0 also is shown on the schematic for reference.

3 Alternatively, the rate of exergy destruction can be determined using $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production evaluated from an entropy rate balance. This is left as an exercise.

4 Exergy is destroyed by irreversibilities associated with fluid friction and stream-to-stream heat transfer. The pressure drops for the streams are indicators of frictional irreversibility. The average temperature difference between the streams, ΔT_{ave}, is an indicator of heat transfer irreversibility.

Skills Developed

Ability to...

❑ apply the energy and exergy rate balances.

❑ evaluate exergy destruction.

QuickQUIZ If the mass flow rate of each stream were 105 kg/s, what would be the rate of exergy destruction, in MW? Ans. 3.3 MW.

In previous discussions we have noted the effect of irreversibilities on *thermodynamic* performance. Some *economic* consequences of irreversibilities are considered in the next example.

EXAMPLE 7.7

Determining Cost of Exergy Destruction

For the heat pump of Examples 6.8 and 6.14, determine the exergy destruction rates, each in kW, for the compressor, condenser, and throttling valve. If exergy is valued at \$0.08 per kW · h, determine the daily cost of electricity to operate the compressor and the daily cost of exergy destruction in each component. Let $T_0 = 273$ K (0°C), which corresponds to the temperature of the outside air.

SOLUTION

Known: Refrigerant 22 is compressed adiabatically, condensed by heat transfer to air passing through a heat exchanger, and then expanded through a throttling valve. Data for the refrigerant and air are known.

Find: Determine the daily cost to operate the compressor. Also determine the exergy destruction rates and associated daily costs for the compressor, condenser, and throttling valve.

Schematic and Given Data:

See Examples 6.8 and 6.14.

Engineering Model:

1. See Examples 6.8 and 6.14.
2. $T_0 = 273$ K (0°C).

Analysis: The rates of exergy destruction can be calculated using

$$\dot{\mathsf{E}}_d = T_0\dot{\sigma}$$

together with data for the entropy production rates from Example 6.8. That is,

$$(\dot{\mathsf{E}}_d)_{comp} = (273\text{ K})(17.5 \times 10^{-4})\left(\frac{\text{kW}}{\text{K}}\right) = 0.478\text{ kW}$$

$$(\dot{\mathsf{E}}_d)_{valve} = (273)(9.94 \times 10^{-4}) = 0.271\text{ kW}$$

$$(\dot{\mathsf{E}}_d)_{cond} = (273)(7.95 \times 10^{-4}) = 0.217\text{ kW}$$

The costs of exergy destruction are, respectively

$$\begin{pmatrix}\text{daily cost of exergy destruction due}\\ \text{to compressor irreversibilities}\end{pmatrix} = (0.478\text{ kW})\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)\left|\frac{24\text{ h}}{\text{day}}\right| = \$0.92$$

1 $$\begin{pmatrix}\text{daily cost of exergy destruction due to}\\ \text{irreversibilities in the throttling value}\end{pmatrix} = (0.271)(0.08)|24| = \$0.52$$

$$\begin{pmatrix}\text{daily cost of exergy destruction due to}\\ \text{irreversibilities in the condenser}\end{pmatrix} = (0.217)(0.08)|24| = \$0.42$$

From the solution to Example 6.14, the magnitude of the compressor power is 3.11 kW. Thus, the daily cost is

$$\begin{pmatrix}\text{daily cost of electricity}\\ \text{to operate compressor}\end{pmatrix} = (3.11\text{ kW})\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)\left|\frac{24\text{ h}}{\text{day}}\right| = \$5.97$$

1 Associating exergy destruction with operating costs provides a rational basis for seeking cost-effective design improvements. Although it may be possible to select components that would destroy less exergy, the trade-off between any resulting reduction in operating cost and the potential increase in equipment cost must be carefully considered.

Skills Developed

Ability to...

- ❑ evaluate exergy destruction.
- ❑ conduct an elementary economic evaluation using exergy.

QuickQUIZ Expressed as a percent, how much of the cost of electricity to operate the compressor is attributable to exergy destruction in the three components? Ans. 31%.

7.5.3 Exergy Accounting in Control Volumes at Steady State

For a control volume, the location, types, and true magnitudes of inefficiency and loss can be pinpointed by systematically evaluating and comparing the various terms of the exergy balance for the control volume. This is an extension of *exergy accounting,* introduced in Sec. 7.4.4.

The next two examples provide illustrations of exergy accounting in control volumes. The first involves the steam turbine with stray heat transfer considered previously in Example 6.6, which should be quickly reviewed before studying the current example.

EXAMPLE 7.8

Exergy Accounting of a Steam Turbine

Steam enters a turbine with a pressure of 30 bar, a temperature of 400°C, and a velocity of 160 m/s. Steam exits as saturated vapor at 100°C with a velocity of 100 m/s. At steady state, the turbine develops work at a rate of 540 kJ per kg of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 350 K. Develop a full accounting of the *net exergy carried in* by the steam, in kJ per unit mass of steam flowing. Let T_0 = 25°C, p_0 = 1 atm.

SOLUTION

Known: Steam expands through a turbine for which steady-state data are provided.

Find: Develop a full exergy accounting of the *net exergy carried in* by the steam, in kJ per unit mass of steam flowing.

Schematic and Given Data: See Fig. E6.6. From Example 6.6, $\dot{W}_{cv}/\dot{m}$ = 540 kJ/kg, $\dot{Q}_{cv}/\dot{m} = -22.6$ kJ/kg.

Engineering Model:

1. See the solution to Example 6.6.
2. T_0 = 25°C, p_0 = 1 atm.

Analysis: The *net exergy carried in* per unit mass of steam flowing is obtained using Eq. 7.18

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = (h_1 - h_2) - T_0(s_1 - s_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1 - z_2)}$$

From Table A-4, h_1 = 3230.9 kJ/kg, s_1 = 6.9212 kJ/kg · K. From Table A-2, h_2 = 2676.1 kJ/kg, s_2 = 7.3549 kJ/kg · K. Hence, the net rate exergy is carried in is

$$\begin{aligned}\mathsf{e}_{f1} - \mathsf{e}_{f2} &= \left[(3230.9 - 2676.1)\frac{\text{kJ}}{\text{kg}} - 298(6.9212 - 7.3549)\frac{\text{kJ}}{\text{kg}} + \left[\frac{(160)^2 - (100)^2}{2}\right]\left(\frac{\text{m}}{\text{s}}\right)^2\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\right] \\ &= 691.84\text{ kJ/kg}\end{aligned}$$

The net exergy carried in can be accounted for in terms of the exergy transfers accompanying work and heat transfer and the exergy destruction within the control volume. At steady state, the exergy transfer accompanying work is simply the work, or $\dot{W}_{cv}/\dot{m}$ = 540 kJ/kg. The quantity $\dot{Q}_{cv}/\dot{m}$ is evaluated in the solution to Example 6.6 using the steady-state forms of the mass and energy rate balances: $\dot{Q}_{cv}/\dot{m} = -22.6$ kJ/kg. The accompanying exergy transfer is

$$\begin{aligned}\frac{\dot{\mathsf{E}}_q}{\dot{m}} &= \left(1 - \frac{T_0}{T_b}\right)\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right) \\ &= \left(1 - \frac{298}{350}\right)\left(-22.6\frac{\text{kJ}}{\text{kg}}\right) \\ &= -3.36\frac{\text{kJ}}{\text{kg}}\end{aligned}$$

where T_b denotes the temperature on the boundary where heat transfer occurs.

The exergy destruction can be determined by rearranging the steady-state form of the exergy rate balance, Eq. 7.17, to give

1
$$\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}} = \left(1 - \frac{T_0}{T_\mathrm{b}}\right)\left(\frac{\dot{Q}_\mathrm{cv}}{\dot{m}}\right) - \frac{\dot{W}_\mathrm{cv}}{\dot{m}} + (\mathrm{e}_{\mathrm{f}1} - \mathrm{e}_{\mathrm{f}2})$$

Substituting values

$$\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}} = -3.36 - 540 + 691.84 = 148.48 \text{ kJ/kg}$$

The analysis is summarized by the following exergy *balance sheet* in terms of exergy magnitudes on a rate basis:

Net rate of exergy in:	691.84 kJ/kg (100%)
Disposition of the exergy:	
• Rate of exergy out	
work	540.00 kJ/kg (78.05%)
heat transfer	3.36 kJ/kg (0.49%)
• Rate of exergy destruction	148.48 kJ/kg (21.46%)
	691.84 kJ/kg (100%)

Note that the exergy transfer accompanying heat transfer is small relative to the other terms.

1 The exergy destruction can be determined alternatively using $\dot{\mathrm{E}}_\mathrm{d} = T_0\dot{\sigma}_\mathrm{cv}$, where $\dot{\sigma}_\mathrm{cv}$ is the rate of entropy production from an entropy balance. The solution to Example 6.6 provides $\dot{\sigma}_\mathrm{cv}/\dot{m} = 0.4983$ kJ/kg · K.

Skills Developed

Ability to...

- ❑ evaluate exergy quantities for an exergy accounting.
- ❑ develop an exergy accounting.

QuickQUIZ By inspection of the exergy balance sheet, specify an exergy-based efficiency for the turbine. Ans. 78.05%.

The next example illustrates the use of exergy accounting to identify opportunities for improving thermodynamic performance of the waste heat recovery system considered in Example 4.10, which should be quickly reviewed before studying the current example.

EXAMPLE 7.9

Exergy Accounting of a Waste Heat Recovery System

Suppose the system of Example 4.10 is one option under consideration for utilizing the combustion products discharged from an industrial process.

(a) Develop a full accounting of the *net* exergy carried in by the combustion products.

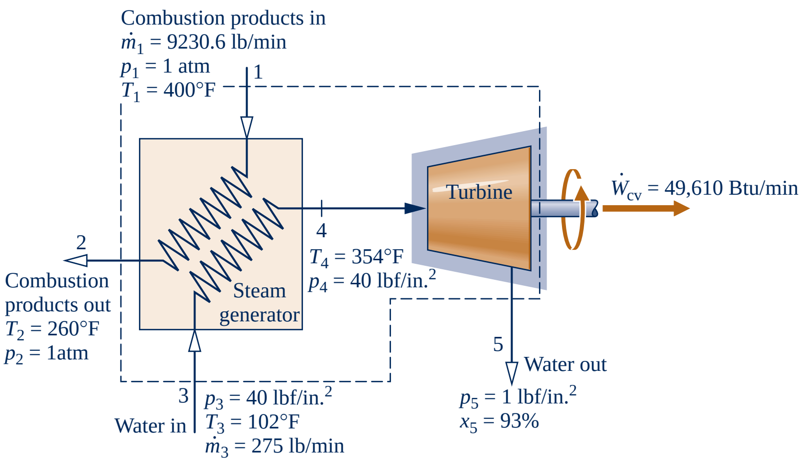
(b) Use the results of (a) to identify opportunities for improving thermodynamic performance.

SOLUTION

Known: Steady-state operating data are provided for a heat-recovery steam generator and a turbine.

Find: Develop a full accounting of the *net* rate exergy is carried in by the combustion products and use the results to identify opportunities for improving thermodynamic performance.

Schematic and Given Data:



Engineering Model:

1. See solution to Example 4.10.
2. $T_0 = 537°R$.

Fig. E7.9

Analysis:

(a) We begin by determining the *net rate exergy is carried into* the control volume. Modeling the combustion products as an ideal gas, the net rate is determined using Eq. 7.18 together with Eq. 6.20a as

$$\dot{m}_1[\mathsf{e}_{f1} - \mathsf{e}_{f2}] = \dot{m}_1[h_1 - h_2 - T_0(s_1 - s_2)]$$

$$= \dot{m}_1\left[h_1 - h_2 - T_0\left(s_1^\circ - s_2^\circ - R\ln\frac{p_1}{p_2}\right)\right]$$

With data from Table A-22E, h_1 = 206.46 Btu/lb, h_2 = 172.39 Btu/lb, s_1° = 0.71323 Btu/lb · °R, s_2° = 0.67002 Btu/lb · °R, and $p_2 = p_1$, we have

$$\dot{m}_1[\mathsf{e}_{f1} - \mathsf{e}_{f2}] = 9230.6\frac{\text{lb}}{\text{min}}\left[(206.46 - 172.39)\frac{\text{Btu}}{\text{lb}} - 537°\text{R}(0.71323 - 0.67002)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right]$$

$$= 100{,}300\ \text{Btu/min}$$

Next, we determine the rate exergy is carried *out* of the control volume. Exergy is carried out of the control volume by work at a rate of 49,610 Btu/min, as shown on the schematic. Additionally, the *net* rate exergy is carried *out* by the water stream is

$$\dot{m}_3[\mathsf{e}_{f5} - \mathsf{e}_{f3}] = \dot{m}_3[h_5 - h_3 - T_0(s_5 - s_3)]$$

From Table A-2E, $h_3 \approx h_f(102°\text{F})$ = 70 Btu/lb, $s_3 \approx s_f(102°\text{F})$ = 0.1331 Btu/lb · °R. Using saturation data at 1 lbf/in.^{2} from Table A-3E with x_5 = 0.93 gives h_5 = 1033.2 Btu/lb and s_5 = 1.8488 Btu/lb · °R. Substituting values

$$\dot{m}_3[\mathsf{e}_{f5} - \mathsf{e}_{f3}] = 275\frac{\text{lb}}{\text{min}}\left[(1033.2 - 70)\frac{\text{Btu}}{\text{lb}} - 537°\text{R}(1.8488 - 0.1331)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right]$$

$$= 11{,}510\ \text{Btu/min}$$

Next, the rate exergy is destroyed in the heat-recovery steam generator can be obtained from an exergy rate balance applied to a control volume enclosing the steam generator. That is, Eq. 7.13a takes the form

$$0 = \sum_j \cancelto{0}{\left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}_3(\mathsf{e}_{f3} - \mathsf{e}_{f4}) - \dot{\mathsf{E}}_d$$

Evaluating $(\mathsf{e}_{f3} - \mathsf{e}_{f4})$ with Eq. 7.18 and solving for $\dot{\mathsf{E}}_d$

$$\dot{\mathsf{E}}_d = \dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}_3[h_3 - h_4 - T_0(s_3 - s_4)]$$

The first term on the right is evaluated above. Then, with h_4 = 1213.8 Btu/lb, s_4 = 1.7336 Btu/lb · °R at 354°F, 40 lbf/in.^{2} from Table A-4E, and previously determined values for h_3 and s_3

$$\dot{\mathsf{E}}_d = 100{,}300\frac{\text{Btu}}{\text{min}} + 275\frac{\text{lb}}{\text{min}}\left[(70 - 1213.8)\frac{\text{Btu}}{\text{lb}} - 537°\text{R}(0.1331 - 1.7336)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right]$$

$$= 22{,}110\ \text{Btu/min}$$

Finally, the rate exergy is destroyed in the turbine can be obtained from an exergy rate balance applied to a control volume enclosing the turbine. That is, Eq. 7.17 takes the form

$$0 = \sum_j \overbrace{\left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j}^{0} - \dot{W}_{cv} + \dot{m}_4(\mathsf{e}_{f4} - \mathsf{e}_{f5}) - \dot{\mathsf{E}}_d$$

Solving for $\dot{\mathsf{E}}_d$, evaluating $(\mathsf{e}_{f4} - \mathsf{e}_{f5})$ with Eq. 7.18, and using previously determined values

$$\dot{\mathsf{E}}_d = -\dot{W}_{cv} + \dot{m}_4[h_4 - h_5 - T_0(s_4 - s_5)]$$

1 $$= -49{,}610\frac{\text{Btu}}{\text{min}} + 275\frac{\text{lb}}{\text{min}}\left[(1213.8 - 1033.2)\frac{\text{Btu}}{\text{lb}} - 537°\text{R}(1.7336 - 1.8488)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right]$$

$$= 17{,}070 \text{ Btu/min}$$

The analysis is summarized by the following exergy *balance sheet* in terms of exergy magnitudes on a rate basis:

Net rate of exergy in:	100,300 Btu/min (100%)
Disposition of the exergy:	
• Rate of exergy out	
power developed	49,610 Btu/min (49.46%)
water stream	11,510 Btu/min (11.48%)
• Rate of exergy destruction	
heat-recovery steam generator	22,110 Btu/min (22.04%)
turbine	17,070 Btu/min (17.02%)
	100,300 Btu/min (100%)

(b) The exergy balance sheet suggests an opportunity for improved *thermodynamic* performance because only about 50% of the net exergy carried in is obtained as power developed. The remaining nearly 50% of the net exergy carried in is either destroyed by irreversibilities or carried out by the water stream. Better thermodynamic performance might be achieved by modifying the design. For example, we might reduce the heat transfer irreversibility by specifying a heat-recovery steam generator with a smaller stream-to-stream temperature difference, and/or reduce the effects of friction by specifying a turbine with a higher isentropic efficiency. Thermodynamic performance alone would not determine the *preferred* system embodiment, however, for other factors such as cost must be considered, and can be overriding. Further discussion of the use of exergy analysis in design is provided in Sec. 7.7.2.

1 Alternatively, the rates of exergy destruction in control volumes enclosing the heat-recovery steam generator and turbine can be determined using $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production for the respective control volume evaluated from an entropy rate balance. This is left as an exercise.

Skills Developed

Ability to...

- ❑ evaluate exergy quantities for an exergy accounting.
- ❑ develop an exergy accounting.

QuickQUIZ For the turbine of the waste heat recovery system, evaluate the isentropic turbine efficiency and comment. Ans. 74%. This isentropic turbine efficiency value is at the low end of the range for steam turbines today, indicating scope for improved performance of the heat recovery system.

7.6 Exergetic (Second Law) Efficiency

The objective of this section is to show the use of the exergy concept in assessing the effectiveness of energy resource utilization. As part of the presentation, the **exergetic efficiency** concept is introduced and illustrated. Such efficiencies are also known as *second law* efficiencies.

exergetic efficiency

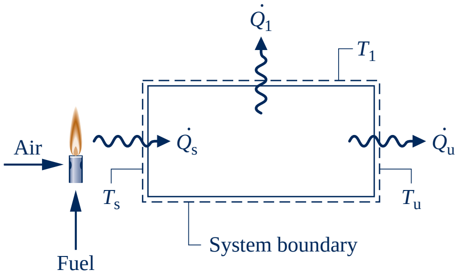


Fig. 7.7 Schematic used to discuss the efficient use of fuel.

7.6.1 Matching End Use to Source

Tasks such as space heating, heating in industrial furnaces, and process steam generation commonly involve the combustion of coal, oil, or natural gas. When the products of combustion are at a temperature significantly greater than required by a given task, the end use is not well matched to the source and the result is inefficient use of the fuel burned. To illustrate this simply, refer to Fig. 7.7, which shows a closed system receiving a heat transfer at the rate $\dot{Q}_s$ at a *source* temperature T_s and delivering $\dot{Q}_u$ at a *use* temperature T_u. Energy is lost to the surroundings by heat transfer at a rate $\dot{Q}_l$ across a portion of the surface at T_l. All energy transfers shown on the figure are in the directions indicated by the arrows.

Assuming that the system of Fig. 7.7 operates at steady state and there is no work, the closed system energy and exergy rate balances Eqs. 2.37 and 7.10 reduce, respectively, to

$$\cancel{\frac{dE}{dt}}^{0} = (\dot{Q}_s - \dot{Q}_u - \dot{Q}_l) - \cancel{\dot{W}}^{0}$$

$$\cancel{\frac{dE}{dt}}^{0} = \left[\left(1 - \frac{T_0}{T_s}\right)\dot{Q}_s - \left(1 - \frac{T_0}{T_u}\right)\dot{Q}_u - \left(1 - \frac{T_0}{T_l}\right)\dot{Q}_l\right] - \left[\cancel{\dot{W}}^{0} - p_0\cancel{\frac{dV}{dt}}^{0}\right] - \dot{\mathsf{E}}_d$$

These equations can be rewritten as follows

$$\dot{Q}_s = \dot{Q}_u + \dot{Q}_l \qquad \textbf{(7.19a)}$$

$$\left(1 - \frac{T_0}{T_s}\right)\dot{Q}_s = \left(1 - \frac{T_0}{T_u}\right)\dot{Q}_u + \left(1 - \frac{T_0}{T_l}\right)\dot{Q}_l + \dot{\mathsf{E}}_d \qquad \textbf{(7.19b)}$$

Equation 7.19a indicates that the energy carried in by heat transfer, $\dot{Q}_s$, is either used, $\dot{Q}_u$, or lost to the surroundings, $\dot{Q}_l$. This can be described by an efficiency in terms of energy rates in the form product/input as

$$\eta = \frac{\dot{Q}_u}{\dot{Q}_s} \qquad \textbf{(7.20)}$$

In principle, the value of η can be increased by applying insulation to reduce the loss. The limiting value, when $\dot{Q}_l = 0$, is $\eta = 1$ (100%).

Equation 7.19b shows that the exergy carried into the system accompanying the heat transfer $\dot{Q}_s$ is either transferred from the system accompanying the heat transfers $\dot{Q}_u$ and $\dot{Q}_l$ or destroyed by irreversibilities within the system. This can be described by an efficiency in terms of exergy rates in the form product/input as

$$\varepsilon = \frac{(1 - T_0/T_u)\dot{Q}_u}{(1 - T_0/T_s)\dot{Q}_s} \qquad \textbf{(7.21a)}$$

Introducing Eq. 7.20 into Eq. 7.21a results in

$$\boxed{\varepsilon = \eta\left(\frac{1 - T_0/T_u}{1 - T_0/T_s}\right)} \qquad \textbf{(7.21b)}$$

The parameter ε, defined with reference to the exergy concept, may be called an *exergetic* efficiency. Note that η and ε each gauge how effectively the input is converted to the product. The parameter η does this on an energy basis, whereas ε does it on an exergy basis. As discussed next, the value of ε is generally less than unity even when $\eta = 1$.

Equation 7.21b indicates that a value for η as close to unity as practical is important for proper utilization of the exergy transferred from the hot combustion gas to the system. However, this alone would not ensure effective utilization. The temperatures

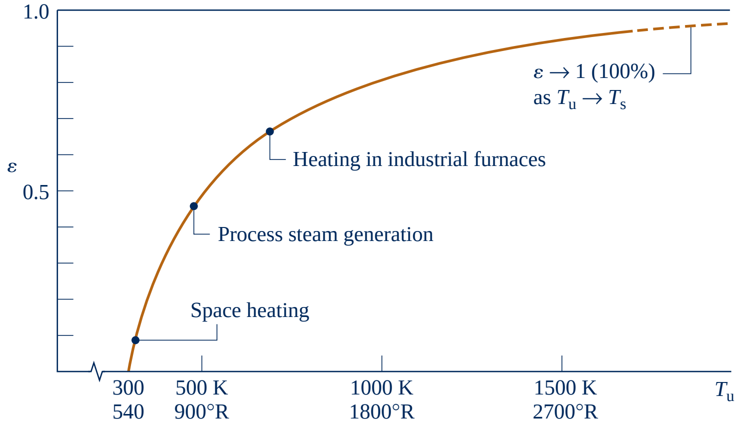


Fig. 7.8 Effect of use temperature T_u on the exergetic efficiency ε (T_s = 2200 K, η = 100%).

T_s and T_u are also important, with exergy utilization improving as the use temperature T_u approaches the source temperature T_s. For proper utilization of exergy, therefore, it is desirable to have a value for η as close to unity as practical and also a good *match* between the source and use temperatures.

To emphasize further the central role of the use temperature, a graph of Eq. 7.21b is provided in Fig. 7.8. The figure gives the exergetic efficiency ε versus the use temperature T_u for an assumed source temperature T_s = 2200 K (3960°R). Figure 7.8 shows that ε tends to unity (100%) as the use temperature approaches T_s. In most cases, however, the use temperature is substantially below T_s. Indicated on the graph are efficiencies for three applications: space heating at T_u = 320 K (576°R), process steam generation at T_u = 480 K (864°R), and heating in industrial furnaces at T_u = 700 K (1260°R). These efficiency values suggest that fuel is used far more effectively in higher-temperature industrial applications than in lower-temperature space heating. The especially low exergetic efficiency for space heating reflects the fact that fuel is consumed to produce only slightly warm air, which from an exergy perspective has little utility. The efficiencies given on Fig. 7.8 are actually on the *high* side, for in constructing the figure we have assumed η to be unity (100%). Moreover, as additional destruction and loss of exergy is associated with combustion, the overall efficiency from fuel input to end use would be much less than indicated by the values shown on the figure.

Costing Heat Loss

For the system in Fig. 7.7, it is instructive to consider further the rate of exergy loss accompanying the heat loss $\dot{Q}_1$, that is $(1 - T_0/T_1)\dot{Q}_1$. This expression measures the *true* thermodynamic value of the heat loss and is graphed in Fig. 7.9. The figure shows that the value of the heat loss in terms of exergy depends *significantly* on the temperature at which the heat loss occurs. We might expect that the economic value of such a loss varies similarly with temperature, and this is the case.

▶ **FOR EXAMPLE** since the source of the exergy loss by heat transfer is the fuel input (see Fig. 7.7), the economic value of the loss can be accounted for in terms of the *unit cost* of fuel based on exergy, c_F (in \$/kW · h, for example), as follows

$$\begin{bmatrix} \text{cost rate of heat loss} \\ \dot{Q}_1 \text{ at temperature } T_1 \end{bmatrix} = c_F(1 - T_0/T_1)\dot{Q}_1 \tag{7.22}$$

Equation 7.22 shows that the cost of such a loss is less at lower temperatures than at higher temperatures. ◀ ◀ ◀ ◀ ◀

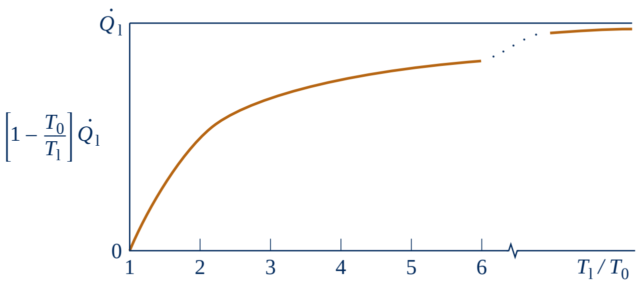


Fig. 7.9 **Effect of the temperature ratio T_1/T_0 on the exergy loss associated with heat transfer.**

The previous example illustrates what we would expect of a rational costing method. It would not be rational to assign the same economic value for a heat transfer occurring near ambient temperature, where the thermodynamic value is negligible, as for an equal heat transfer occurring at a higher temperature, where the thermodynamic value is significant. Indeed, it would be incorrect to assign the *same cost* to heat loss independent of the temperature at which the loss is occurring. For further discussion of exergy costing, see Sec. 7.7.3.

Horizons

Oil from Shale and Sand Deposits—The Jury Is Still Out

Traditional oil reserves are now widely anticipated to decline in years ahead. But the impact could be lessened if cost-effective and environmentally-benign technologies can be developed to recover oil-like substances from abundant oil shale and oil sand deposits in the United States and Canada.

Production means available today are both costly and inefficient in terms of exergy demands for the blasting, digging, transporting, crushing, and heating of the materials rendered into oil. Current production means not only use natural gas and large amounts of water, but also cause wide-scale environmental devastation, including air and water pollution and huge amounts of toxic waste.

Although significant rewards await developers of improved technologies, the challenges also are significant. Some say efforts are better directed to using traditional oil reserves more efficiently and to developing alternatives to oil-based fuels such as *cellulosic* ethanol produced with relatively low-cost biomass from urban, agricultural, and forestry sources.

7.6.2 Exergetic Efficiencies of Common Components

Exergetic efficiency expressions can take many different forms. Several examples are given in the current section for thermal system components of practical interest. In every instance, the efficiency is derived by the use of the exergy rate balance. The approach used here serves as a model for the development of exergetic efficiency expressions for other components. Each of the cases considered involves a control volume at steady state, and we assume no heat transfer between the control volume and its surroundings. The current presentation is not exhaustive. Many other exergetic efficiency expressions can be written.

Turbines

For a turbine operating at steady state with no heat transfer with its surroundings, the steady-state form of the exergy rate balance, Eq. 7.17, reduces as follows:

$$0 = \cancelto{0}{\sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j} - \dot{W}_{cv} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{\mathsf{E}}_d$$

This equation can be rearranged to read

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = \frac{\dot{W}_{cv}}{\dot{m}} + \frac{\dot{\mathsf{E}}_d}{\dot{m}} \tag{7.23}$$

The term on the left of Eq. 7.23 is the decrease in flow exergy from turbine inlet to exit. The equation shows that the flow exergy decrease is accounted for by the turbine work developed, $\dot{W}_{cv}/\dot{m}$, and the exergy destroyed, $\dot{E}_d/\dot{m}$. A parameter that gauges how effectively the flow exergy decrease is converted to the desired product is the *exergetic turbine efficiency*

$$\varepsilon = \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}} \tag{7.24}$$

This particular exergetic efficiency is sometimes referred to as the *turbine effectiveness.* Carefully note that the exergetic turbine efficiency is defined differently from the isentropic turbine efficiency introduced in Sec. 6.12.

▸ **FOR EXAMPLE** the exergetic efficiency of the turbine considered in Example 6.11 is 81.2% when $T_0 = 298$ K. It is left as an exercise to verify this value. ◂ ◂ ◂ ◂ ◂

Compressors and Pumps

For a compressor or pump operating at steady state with no heat transfer with its surroundings, the exergy rate balance, Eq. 7.17, can be placed in the form

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = e_{f2} - e_{f1} + \frac{\dot{E}_d}{\dot{m}}$$

Thus, the exergy *input* to the device, $-\dot{W}_{cv}/\dot{m}$, is accounted for by the increase in the flow exergy between inlet and exit and the exergy destroyed. The effectiveness of the conversion from work input to flow exergy increase is gauged by the *exergetic compressor* (or pump) *efficiency*

$$\varepsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} \tag{7.25}$$

▸ **FOR EXAMPLE** the exergetic efficiency of the compressor considered in Example 6.14 is 84.6% when $T_0 = 273$ K. It is left as an exercise to verify this value. ◂ ◂ ◂ ◂ ◂

Heat Exchanger Without Mixing

The heat exchanger shown in Fig. 7.10 operates at steady state with no heat transfer with its surroundings and both streams at temperatures above T_0. The exergy rate balance, Eq. 7.13a, reduces to

$$0 = \cancel{\sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + (\dot{m}_h e_{f1} + \dot{m}_c e_{f3}) - (\dot{m}_h e_{f2} + \dot{m}_c e_{f4}) - \dot{E}_d$$

where $\dot{m}_h$ is the mass flow rate of the hot stream and $\dot{m}_c$ is the mass flow rate of the cold stream. This can be rearranged to read

$$\dot{m}_h(e_{f1} - e_{f2}) = \dot{m}_c(e_{f4} - e_{f3}) + \dot{E}_d \tag{7.26}$$

The term on the left of Eq. 7.26 accounts for the decrease in the exergy of the hot stream. The first term on the right accounts for the increase in exergy of the cold stream. Regarding the hot stream as supplying the exergy increase of the cold stream as well as the exergy destroyed, we can write an *exergetic heat exchanger efficiency* as

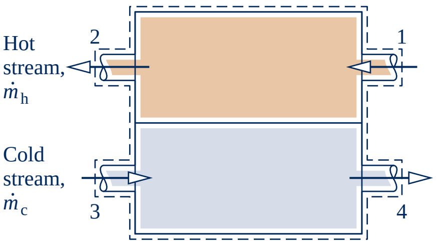


Fig. 7.10 **Counterflow heat exchanger.**

$$\varepsilon = \frac{\dot{m}_c(e_{f4} - e_{f3})}{\dot{m}_h(e_{f1} - e_{f2})} \tag{7.27}$$

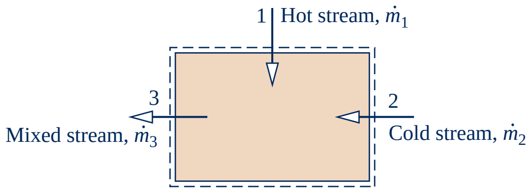


Fig. 7.11 Direct contact heat exchanger.

▶ **FOR EXAMPLE** the exergetic efficiency of the heat exchanger of Example 7.6 is 83.3%. It is left as an exercise to verify this value. ◀ ◀ ◀ ◀ ◀

Direct Contact Heat Exchanger

The direct contact heat exchanger shown in Fig. 7.11 operates at steady state with no heat transfer with its surroundings. The exergy rate balance, Eq. 7.13a, reduces to

$$0 = \cancelto{0}{\sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1 e_{f1} + \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} - \dot{E}_d$$

With $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ from a mass rate balance, this can be written as

$$\dot{m}_1(e_{f1} - e_{f3}) = \dot{m}_2(e_{f3} - e_{f2}) + \dot{E}_d \tag{7.28}$$

The term on the left of Eq. 7.28 accounts for the decrease in the exergy of the hot stream between inlet and exit. The first term on the right accounts for the increase in the exergy of the cold stream between inlet and exit. Regarding the hot stream as supplying the exergy increase of the cold stream as well as the exergy destroyed by irreversibilities, we can write an *exergetic efficiency* for a direct contact heat exchanger as

$$\varepsilon = \frac{\dot{m}_2(e_{f3} - e_{f2})}{\dot{m}_1(e_{f1} - e_{f3})} \tag{7.29}$$

7.6.3 Using Exergetic Efficiencies

Exergetic efficiencies are useful for distinguishing means for utilizing fossil fuels that are thermodynamically effective from those that are less so. Exergetic efficiencies also can be used to evaluate the effectiveness of engineering measures taken to improve the performance of systems. This is done by comparing the efficiency values determined before and after modifications have been made to show how much improvement has been achieved. Moreover, exergetic efficiencies can be used to gauge the potential for improvement in the performance of a given system by comparing the efficiency of the system to the efficiency of like systems. A significant difference between these values signals that improved performance is possible.

It is important to recognize that the limit of 100% exergetic efficiency should not be regarded as a practical objective. This theoretical limit could be attained only if there were no exergy destructions or losses. To achieve such idealized processes might require extremely long times to execute processes and/or complex devices, both of which are at odds with the objective of cost-effective operation. In practice, decisions are chiefly made on the basis of *total* costs. An increase in efficiency to reduce fuel consumption, or otherwise utilize fuels better, often requires additional expenditures for facilities and operations. Accordingly, an improvement might not be implemented if an increase in total cost would result. The trade-off between fuel savings and additional investment invariably dictates a lower efficiency than might be achieved *theoretically* and even a lower efficiency than could be achieved using the *best available* technology.

ENERGY & ENVIRONMENT A type of exergetic efficiency known as the *well-to-wheel efficiency* is used to compare different options for powering vehicles. The calculation of this efficiency begins at the well where the oil or natural gas feedstock is extracted from the ground and ends with the power delivered to a vehicle's wheels. The efficiency accounts separately for how effectively the vehicle's fuel is produced from feedstock, called the *well-to-fuel tank efficiency,* and how effectively the vehicle's power plant converts its fuel to power, called the *fuel tank-to-wheel efficiency.* The product of these gives the *overall* well-to-wheel efficiency.

The table below gives sample well-to-wheel efficiency values for three power plant options as reported by an automobile manufacturer:

	Well-to-Tank (Fuel Production Efficiency) (%)	×	Tank-to-Wheel (Vehicle Efficiency) (%)	=	Well-to-Wheel (Overall Efficiency) (%)
Conventional gasoline-fueled engine	88	×	16	=	14
Hydrogen-fueled fuel cell[a]	58	×	38	=	22
Gasoline-fueled hybrid electric	88	×	32	=	28

[a]Hydrogen produced from natural gas.

These data show that vehicles using conventional internal combustion engines do not fare well in terms of the well-to-wheel efficiency. The data also show that fuel-cell vehicles operating on hydrogen have the best tank-to-wheel efficiency of the three options, but lose out on an overall basis to hybrid vehicles, which enjoy a higher well-to-tank efficiency. Still, the well-to-wheel efficiency is just one consideration when making policy decisions concerning different options for powering vehicles. With increasing concern over global atmospheric CO_2 concentrations, another consideration is the well-to-wheel *total* production of CO_2 in kg per km driven (lb per mile driven).

7.7 Thermoeconomics

Thermal systems typically experience significant work and/or heat interactions with their surroundings, and they can exchange mass with their surroundings in the form of hot and cold streams, including chemically reactive mixtures. Thermal systems appear in almost every industry, and numerous examples are found in our everyday lives. Their design and operation involve the application of principles from thermodynamics, fluid mechanics, and heat transfer, as well as such fields as materials, manufacturing, and mechanical design. The design and operation of thermal systems also require explicit consideration of engineering economics, for cost is always a consideration. The term **thermoeconomics** may be applied to this general area of application, although it is often applied more narrowly to methodologies combining exergy and economics for optimization studies during design of new systems and process improvement of existing systems. **thermoeconomics**

7.7.1 Costing

Is costing an art or a science? The answer is a little of both. *Cost engineering* is an important engineering subdiscipline aimed at objectively applying real-world costing experience in engineering design and project management. Costing services are provided by practitioners skilled in the use of specialized methodologies, cost models, and databases, together with costing expertise and judgment garnered from years of professional practice. Depending on need, cost engineers provide services ranging from rough and rapid estimates to in-depth analyses. Ideally, cost engineers are involved with projects from the formative stages, for the *output* of cost engineering is an essential *input* to decision making. Such input can be instrumental in identifying feasible options from a set of alternatives and even pinpointing the best option.

Costing of thermal systems considers costs of owning and operating them. Some observers voice concerns that costs related to the environment often are only weakly taken into consideration in such evaluations. They say companies pay for the right to extract natural resources used in the production of goods and services but rarely pay fully for depleting nonrenewable resources and mitigating accompanying environmental degradation and loss of wildlife habitat, in many cases leaving the cost burden to future generations. Another concern is who pays for the costs of controlling air and water pollution, cleaning up hazardous wastes, and the impacts of pollution and waste on human health—industry, government, the public, or some combination of each? Yet when agreement about environmental costs is achieved among interested business, governmental, and advocacy groups, such costs are readily integrated in costing of thermal systems, including costing on an exergy basis, which is the present focus.

7.7.2 Using Exergy in Design

To illustrate the use of exergy reasoning in design, consider Fig. 7.12 showing a boiler at steady state. Fuel and air enter the boiler and react to form hot combustion gases.

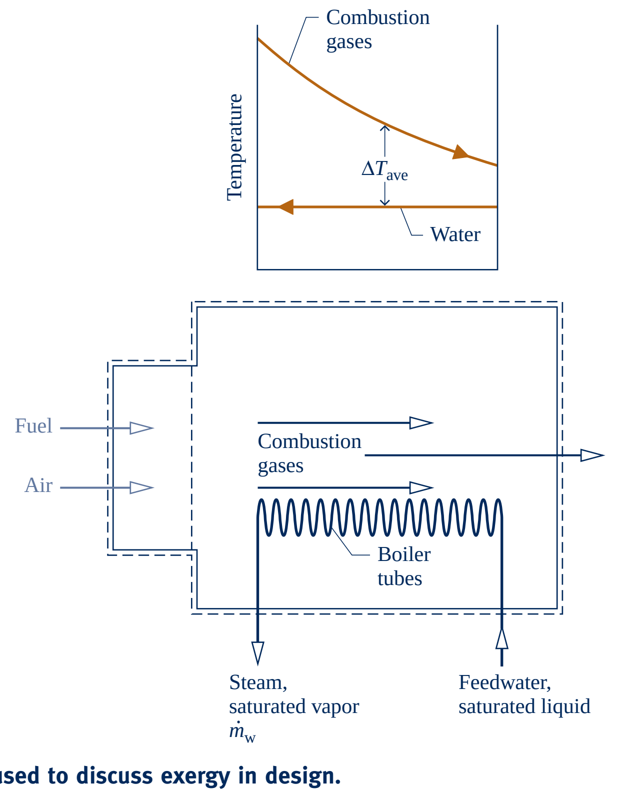


Fig. 7.12 **Boiler used to discuss exergy in design.**

Feedwater also enters as saturated liquid, receives exergy by heat transfer from the combustion gases, and exits without temperature change as saturated vapor at a specified condition for use elsewhere. Temperatures of the hot gas and water streams are also shown on the figure.

There are two main sources of exergy destruction in the boiler: (1) irreversible heat transfer occurring between the hot combustion gases and the water flowing through the boiler tubes, and (2) the combustion process itself. To simplify the present discussion, the boiler is considered to consist of a combustor unit in which fuel and air are burned to produce hot combustion gases, followed by a heat exchanger unit where water is vaporized as the hot gases cool.

The present discussion centers on the heat exchanger unit. Let us think about its total cost as the sum of fuel-related and capital costs. We will also take the average temperature difference between the two streams, ΔT_{ave}, as the *design variable*. From our study of the second law of thermodynamics, we know that the average temperature difference between the two streams is a measure of exergy destruction associated with heat transfer between them. The exergy destroyed owing to heat transfer originates in the fuel entering the boiler. Accordingly, a cost related to fuel consumption can be attributed to this source of irreversibility. Since exergy destruction increases with temperature difference between the streams, the fuel-related cost increases with *increasing* ΔT_{ave}. This variation is shown in Fig. 7.13 on an *annualized* basis, in dollars per year.

From our study of heat transfer, we know an inverse relation exits between ΔT_{ave} and the boiler tube surface area required for a desired heat transfer rate between the streams. For example, if we design for a small average temperature difference to reduce exergy destruction within the heat exchanger, this dictates a large surface area and typically a more costly boiler. From such considerations, we infer that boiler capital cost increases with *decreasing* ΔT_{ave}. This variation is shown in Fig. 7.13, again on an annualized basis.

The *total cost* is the sum of the capital cost and the fuel cost. The total cost curve shown in Fig. 7.13 exhibits a minimum at the point labeled a. Notice, however, that the curve is relatively flat in the neighborhood of the minimum, so there is a range of ΔT_{ave} values that could be considered *nearly optimal* from the standpoint of minimum total cost. If reducing the fuel cost were deemed more important than minimizing the

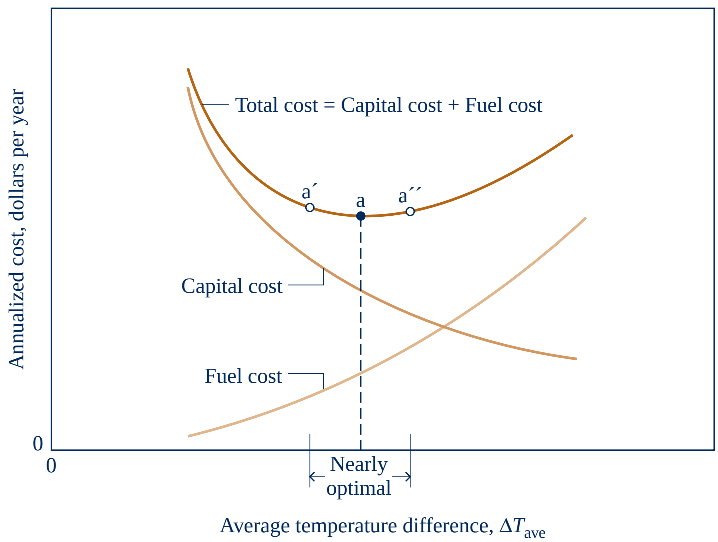


Fig. 7.13 Cost curves for the heat exchanger unit of the boiler of Fig. 7.12.

capital cost, we might choose a design that would operate at point a′. Point a″ would be a more desirable operating point if capital cost were of greater concern. Such trade-offs are common in design situations.

The actual design process differs significantly from the simple case considered here. For one thing, costs cannot be determined as precisely as implied by the curves in Fig. 7.13. Fuel prices vary widely over time, and equipment costs may be difficult to predict as they often depend on a bidding procedure. Equipment is manufactured in discrete sizes, so the cost also would not vary continuously as shown in the figure. Furthermore, thermal systems usually consist of several components that interact with one another. Optimization of components individually, as considered for the heat exchanger unit of the boiler, does not guarantee an optimum for the overall system. Finally, the example involves only ΔT_{ave} as a design variable. Often, several design variables must be considered and optimized simultaneously.

7.7.3 Exergy Costing of a Cogeneration System

Another important aspect of thermoeconomics is the use of exergy for *allocating* costs to the products of a thermal system. This involves assigning to each product the total cost to produce it, namely the cost of fuel and other inputs plus the cost of owning and operating the system (e.g., capital cost, operating and maintenance costs). Such costing is a common problem in plants where utilities such as electrical power, chilled water, compressed air, and steam are generated in one department and used in others. The plant operator needs to know the cost of generating each utility to ensure that the other departments are charged properly according to the type and amount of each utility used. Common to all such considerations are fundamentals from engineering economics, including procedures for annualizing costs, appropriate means for allocating costs, and reliable cost data.

To explore further the costing of thermal systems, consider the simple *cogeneration system* operating at steady state shown in Fig. 7.14. The system consists of a boiler and a turbine, with each having no significant heat transfer to its surroundings. The figure is labeled with exergy transfer rates associated with the flowing streams, where the subscripts F, a, P, and w denote fuel, combustion air, combustion products, and feedwater, respectively. The subscripts 1 and 2 denote high- and low-pressure steam, respectively. Means for evaluating the exergies of the fuel and combustion products are introduced in Chap. 13. The cogeneration system has two principal products: electricity, denoted by $\dot{W}_e$, and low-pressure steam for use in some process. The objective is to determine the cost at which each product is generated.

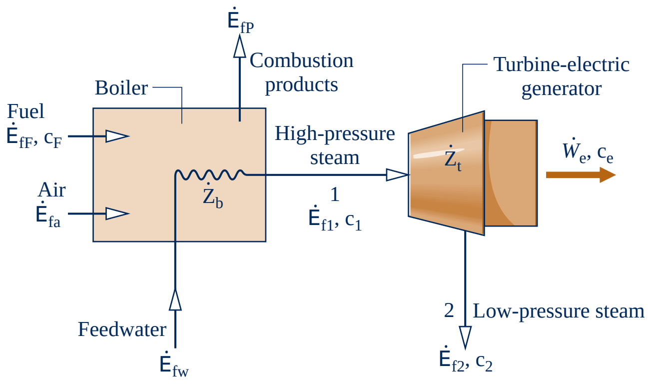


Fig. 7.14 Simple cogeneration system.

Boiler Analysis

Let us begin by evaluating the cost of the high-pressure steam produced by the boiler. For this, we consider a control volume enclosing the boiler. Fuel and air enter the boiler separately and combustion products exit. Feedwater enters and high-pressure steam exits. The total cost to produce the exiting high-pressure steam equals the total cost of the entering streams plus the cost of owning and operating the boiler. This is expressed by the following **cost rate balance** for the boiler

$$\dot{C}_1 = \dot{C}_F + \dot{C}_a + \dot{C}_w + \dot{Z}_b \tag{7.30}$$

cost rate balance

where $\dot{C}$ is the cost rate of the respective stream (in \$ per hour, for instance). $\dot{Z}_b$ accounts for the cost rate associated with owning and operating the boiler, including expenses related to proper disposal of the combustion products. In the present discussion, the cost rate $\dot{Z}_b$ is presumed known from a previous economic analysis.

Although the cost rates denoted by $\dot{C}$ in Eq. 7.30 are evaluated by various means in practice, the present discussion features the use of exergy for this purpose. Since exergy measures the true thermodynamic values of the work, heat, and other interactions between a system and its surroundings as well as the effect of irreversibilities within the system, exergy is a rational basis for assigning costs. With exergy costing, each of the cost rates is evaluated in terms of the associated rate of exergy transfer and a *unit cost.* Thus, for an entering or exiting stream, we write

$$\dot{C} = c\dot{E}_f \tag{7.31}$$

where c denotes the **cost per unit of exergy** (in \$ or cents per kW · h, for example) and $\dot{E}_f$ is the associated exergy transfer rate.

exergy unit cost

For simplicity, we assume the feedwater and combustion air enter the boiler with negligible exergy and cost. Thus Eq. 7.30 reduces as follows

$$\dot{C}_1 = \dot{C}_F + \cancel{\dot{C}_a}^{0} + \cancel{\dot{C}_w}^{0} + \dot{Z}_b$$

Then, with Eq. 7.31 we get

$$c_1\dot{E}_{f1} = c_F\dot{E}_{fF} + \dot{Z}_b \tag{7.32a}$$

Solving for c_1, the unit cost of the high-pressure steam is

$$\boxed{c_1 = c_F\left(\frac{\dot{E}_{fF}}{\dot{E}_{f1}}\right) + \frac{\dot{Z}_b}{\dot{E}_{f1}}} \tag{7.32b}$$

This equation shows that the unit cost of the high-pressure steam is determined by two contributions related, respectively, to the cost of the fuel and the cost of owning and operating the boiler. Due to exergy destruction and loss, less exergy exits the boiler with the high-pressure steam than enters with the fuel. Thus, $\dot{E}_{fF}/\dot{E}_{f1}$ is invariably greater than one, and the unit cost of the high-pressure steam is invariably greater than the unit cost of the fuel.

Turbine Analysis

Next, consider a control volume enclosing the turbine. The total cost to produce the electricity and low-pressure steam equals the cost of the entering high-pressure steam plus the cost of owning and operating the device. This is expressed by the *cost rate balance* for the turbine

$$\dot{C}_e + \dot{C}_2 = \dot{C}_1 + \dot{Z}_t \tag{7.33}$$

where $\dot{C}_e$ is the cost rate associated with the electricity, $\dot{C}_1$ and $\dot{C}_2$ are the cost rates associated with the entering and exiting steam, respectively, and $\dot{Z}_t$ accounts for the cost rate associated with owning and operating the turbine. With exergy costing, each of the cost rates $\dot{C}_e$, $\dot{C}_1$, and $\dot{C}_2$ is evaluated in terms of the associated rate of exergy transfer and a unit cost. Equation 7.33 then appears as

$$c_e \dot{W}_e + c_2 \dot{E}_{f2} = c_1 \dot{E}_{f1} + \dot{Z}_t \tag{7.34a}$$

The unit cost c_1 in Eq. 7.34a is given by Eq. 7.32b. In the present discussion, the same unit cost is assigned to the low-pressure steam; that is, $c_2 = c_1$. This is done on the basis that the purpose of the turbine is to generate electricity, and thus all costs associated with owning and operating the turbine should be charged to the power generated. We can regard this decision as a part of the *cost accounting* considerations that accompany the thermoeconomic analysis of thermal systems. With $c_2 = c_1$, Eq. 7.34a becomes

$$c_e \dot{W}_e = c_1(\dot{E}_{f1} - \dot{E}_{f2}) + \dot{Z}_t \tag{7.34b}$$

The first term on the right side accounts for the cost of the exergy used and the second term accounts for the cost of owning and operating the system.

Solving Eq. 7.34b for c_e, and introducing the exergetic turbine efficiency ε from Eq. 7.24

$$c_e = \frac{c_1}{\varepsilon} + \frac{\dot{Z}_t}{\dot{W}_e} \tag{7.34c}$$

This equation shows that the unit cost of the electricity is determined by the cost of the high-pressure steam and the cost of owning and operating the turbine. Because of exergy destruction within the turbine, the exergetic efficiency is invariably less than one, and therefore the unit cost of electricity is invariably greater than the unit cost of the high-pressure steam.

Summary

By applying cost rate balances to the boiler and the turbine, we are able to determine the cost of each product of the cogeneration system. The unit cost of the electricity is determined by Eq. 7.34c and the unit cost of the low-pressure steam is determined by the expression $c_2 = c_1$ together with Eq. 7.32b. The example to follow provides a detailed illustration. The same general approach is applicable for costing the products of a wide-ranging class of thermal systems.[1]

[1]See A. Bejan, G. Tsatsaronis, and M. J. Moran, *Thermal Design and Optimization,* John Wiley & Sons, New York, 1996.

EXAMPLE 7.10

Exergy Costing of a Cogeneration System

A cogeneration system consists of a natural gas-fueled boiler and a steam turbine that develops power and provides steam for an industrial process. At steady state, fuel enters the boiler with an exergy rate of 100 MW. Steam exits the boiler at 50 bar, 466°C with an exergy rate of 35 MW. Steam exits the turbine at 5 bar, 205°C and a mass flow rate of 26.15 kg/s. The unit cost of the fuel is 1.44 cents per kW · h of exergy. The costs of owning and operating the boiler and turbine are, respectively, \$1080/h and \$92/h. The feedwater and combustion air enter with negligible exergy and cost. Expenses related to proper disposal of the combustion products are included with the cost of owning and operating the boiler. Heat transfer with the surroundings and the effects of motion and gravity are negligible. Let $T_0 = 298$ K.

(a) For the turbine, determine the power and the rate exergy exits with the steam, each in MW.

(b) Determine the unit costs of the steam exiting the boiler, the steam exiting the turbine, and the power, each in cents per kW · h of exergy.

(c) Determine the cost rates of the steam exiting the turbine and the power, each in \$/h.

SOLUTION

Known: Steady-state operating data are known for a cogeneration system that produces both electricity and low-pressure steam for an industrial process.

Find: For the turbine, determine the power and the rate exergy exits with the steam. Determine the unit costs of the steam exiting the boiler, the steam exiting the turbine, and the power developed. Also determine the cost rates of the low-pressure steam and power.

Schematic and Given Data:

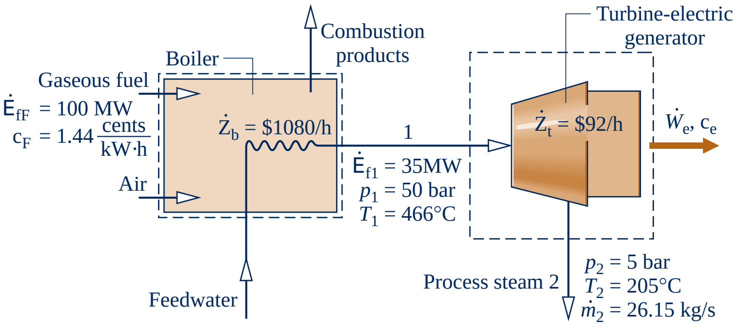


Fig. E7.10

Engineering Model:

1. Each control volume shown in the accompanying figure is at steady state.
2. For each control volume, $\dot{Q}_{cv} = 0$ and the effects of motion and gravity are negligible.
3. The feedwater and combustion air enter the boiler with negligible exergy and cost.
4. Expenses related to proper disposal of the combustion products are included with the cost of owning and operating the boiler.
5. The unit costs based on exergy of the high- and low-pressure steam are equal: $c_1 = c_2$.
6. For the environment, $T_0 = 298$ K.

Analysis:

(a) With assumption 2, the mass and energy rate balances for a control volume enclosing the turbine reduce at steady state to give

$$\dot{W}_e = \dot{m}(h_1 - h_2)$$

From Table A-4, $h_1 = 3353.54$ kJ/kg and $h_2 = 2865.96$ kJ/kg. Thus

$$\dot{W}_e = \left(26.15\frac{\text{kg}}{\text{s}}\right)(3353.54 - 2865.96)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\ \text{MW}}{10^3\ \text{kJ/s}}\right|$$
$$= 12.75\ \text{MW}$$

Using Eq. 7.18, the difference in the rates at which exergy enters and exits the turbine with the steam is

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m}(e_{f2} - e_{f1})$$
$$= \dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

Solving for $\dot{E}_{f2}$

$$\dot{E}_{f2} = \dot{E}_{f1} + \dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

With known values for $\dot{E}_{f1}$ and $\dot{m}$, and data from Table A-4: $s_1 = 6.8773$ kJ/kg · K and $s_2 = 7.0806$ kJ/kg · K, the rate exergy exits with the steam is

$$\dot{E}_{f2} = 35\ \text{MW} + \left(26.15\frac{\text{kg}}{\text{s}}\right)\left[(2865.96 - 3353.54)\frac{\text{kJ}}{\text{kg}} - 298\ \text{K}(7.0806 - 6.8773)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\ \text{MW}}{10^3\ \text{kJ/s}}\right|$$
$$= 20.67\ \text{MW}$$

(b) For a control volume enclosing the boiler, the cost rate balance reduces with assumptions 3 and 4 to give

$$c_1\dot{E}_{f1} = c_F\dot{E}_{fF} + \dot{Z}_b$$

where $\dot{E}_{fF}$ is the exergy rate of the entering fuel, c_F and c_1 are the unit costs of the fuel and exiting steam, respectively, and $\dot{Z}_b$ is the cost rate associated with owning and operating the boiler. Solving for c_1 we get Eq. 7.32b; then, inserting known values c_1 is determined:

$$c_1 = c_F\left(\frac{\dot{E}_{fF}}{\dot{E}_{f1}}\right) + \frac{\dot{Z}_b}{\dot{E}_{f1}}$$

$$= \left(1.44\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(\frac{100\text{ MW}}{35\text{ MW}}\right) + \left(\frac{1080\text{ \$/h}}{35\text{ MW}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{1\$}\right|$$

$$= (4.11 + 3.09)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

The cost rate balance for the control volume enclosing the turbine is given by Eq. 7.34a

$$c_e\dot{W}_e + c_2\dot{E}_{f2} = c_1\dot{E}_{f1} + \dot{Z}_t$$

where c_e and c_2 are the unit costs of the power and the exiting steam, respectively, and $\dot{Z}_t$ is the cost rate associated with owning and operating the turbine. Assigning the same unit cost to the steam entering and

1 exiting the turbine, $c_2 = c_1 = 7.2$ cents/kW · h, and solving for c_e

$$c_e = c_1\left[\frac{\dot{E}_{f1} - \dot{E}_{f2}}{\dot{W}_e}\right] + \frac{\dot{Z}_t}{\dot{W}_e}$$

Inserting known values

$$c_e = \left(7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left[\frac{(35 - 20.67)\text{ MW}}{12.75\text{ MW}}\right] + \left(\frac{92\text{\$/h}}{12.75\text{ MW}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{1\$}\right|$$

2

$$= (8.09 + 0.72)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 8.81\frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

(c) For the low-pressure steam and power, the cost rates are, respectively,

$$\dot{C}_2 = c_2\dot{E}_{f2}$$

$$= \left(7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(20.67\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right|$$

$$= \$1488/\text{h}$$

3

$$\dot{C}_e = c_e\dot{W}_e$$

$$= \left(8.81\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(12.75\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right|$$

$$= \$1123/\text{h}$$

---

1 The purpose of the turbine is to generate power, and thus all costs associated with owning and operating the turbine are charged to the power generated.

2 Observe that the unit costs c_1 and c_e are significantly greater than the unit cost of the fuel.

3 Although the unit cost of the steam is less than the unit cost of the power, the steam *cost* rate is greater because the associated exergy rate is much greater.

Skills Developed

Ability to...

- ❑ evaluate exergy quantities required for exergy costing.
- ❑ apply exergy costing.

QuickQUIZ If the unit cost of the fuel were to double to 2.88 cents/kW · h, what would be the change in the unit cost of power, expressed as a percent, keeping all other given data the same? Ans. +53%.

▶ CHAPTER SUMMARY AND STUDY GUIDE

In this chapter, we have introduced the property exergy and illustrated its use for thermodynamic analysis. Like mass, energy, and entropy, exergy is an extensive property that can be transferred across system boundaries. Exergy transfer accompanies heat transfer, work, and mass flow. Like entropy, exergy is not conserved. Exergy is destroyed within systems whenever internal irreversibilities are present. Entropy production corresponds to exergy destruction.

The use of exergy balances is featured in this chapter. Exergy balances are expressions of the second law that account for exergy in terms of exergy transfers and exergy destruction. For processes of closed systems, the exergy balance is given by Eqs. 7.4 and the companion steady-state forms are Eqs. 7.11. For control volumes, the steady-state expressions are given by Eqs. 7.13. Control volume analyses account for exergy transfer at inlets and exits in terms of flow exergy.

The following checklist provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed you should be able to

- ▶ write out meanings of the terms listed in the margins throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important.
- ▶ evaluate specific exergy at a given state using Eq. 7.2 and exergy change between two states using Eq. 7.3, each relative to a specified reference environment.
- ▶ apply exergy balances in each of several alternative forms, appropriately modeling the case at hand, correctly observing sign conventions, and carefully applying SI and English units.
- ▶ evaluate the specific flow exergy relative to a specified reference environment using Eq. 7.14.
- ▶ define and evaluate exergetic efficiencies for thermal system components of practical interest.
- ▶ apply exergy costing to heat loss and simple cogeneration systems.

▶ KEY ENGINEERING CONCEPTS

exergy, p. 362
exergy reference environment, p. 362
dead state, p. 362
specific exergy, p. 366
exergy change, p. 368
closed system exergy balance, p. 369
exergy transfer, p. 370
exergy destruction, p. 370
flow exergy, p. 378
control volume exergy rate balance, p. 378
exergy accounting, p. 385
exergetic efficiency, p. 389
thermoeconomics, p. 395
cost rate balance, p. 399
exergy unit cost, p. 399

▶ KEY EQUATIONS

$\mathsf{E} = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) + \mathrm{KE} + \mathrm{PE}$	**(7.1) p. 362**	**Exergy of a system.**
$\mathsf{e} = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \mathrm{V}^2/2 + gz$	**(7.2) p. 366**	**Specific exergy.**
$\mathsf{E}_2 - \mathsf{E}_1 = (U_2 - U_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1) + (\mathrm{KE}_2 - \mathrm{KE}_1) + (\mathrm{PE}_2 - \mathrm{PE}_1)$	**(7.3) p. 368**	**Exergy change.**
$\mathsf{E}_2 - \mathsf{E}_1 = \mathsf{E}_\mathrm{q} - \mathsf{E}_\mathrm{w} - \mathsf{E}_\mathrm{d}$	**(7.4b) p. 370**	**Closed system exergy balance. See Eqs. 7.5–7.7 for E_q, E_w, E_d, respectively.**
$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W} - \dot{\mathsf{E}}_\mathrm{d}$	**(7.11a) p. 373**	**Steady-state closed system exergy rate balance.**
$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W}_\mathrm{cv} + \sum_i \dot{m}_i \mathsf{e}_{\mathrm{f}i} - \sum_e \dot{m}_e \mathsf{e}_{\mathrm{f}e} - \dot{\mathsf{E}}_\mathrm{d}$	**(7.13a) p. 378**	**Steady-state control volume exergy rate balance.**
$\mathsf{e}_\mathrm{f} = h - h_0 - T_0(s - s_0) + \frac{\mathrm{V}^2}{2} + gz$	**(7.14) p. 378**	**Specific flow exergy.**

▶ EXERCISES: THINGS ENGINEERS THINK ABOUT

1. Is it possible for exergy to be negative? For exergy *change* to be negative? For exergy *destruction* to be negative?

2. When an automobile brakes to rest, what happens to the exergy associated with its motion?

3. A block of ice melts when left in a sunny location. Does its exergy increase or decrease? Explain.

4. When evaluating exergy destruction, is it *necessary* to use an exergy balance? Explain.

5. A gasoline-fueled electrical generator is claimed by its inventor to produce electricity at a lower unit cost than the unit cost of the fuel used, where each cost is based on exergy. Comment.

6. Can the exergetic efficiency of a power cycle ever be greater than the thermal efficiency of the same cycle? Explain.

7. After a vehicle receives an oil change and lube job, does the exergy destruction within a control volume enclosing the idling vehicle change? Explain.

8. How is exergy *destroyed* and *lost* in electrical transmission and distribution?

9. Is there a difference between practicing exergy *conservation* and exergy *efficiency*? Explain.

10. When installed on the engine of an automobile, which accessory, *supercharger* or *turbocharger*, will result in an engine with the higher exergetic efficiency? Explain.

11. How does the concept of exergy destruction relate to a cell phone or an iPod?

12. In terms of exergy, how does the flight of a bird compare with the flight of a baseball going over the centerfield fence?

13. What is the exergetic efficiency of the control volume of Fig. 7.6? Explain.

14. While exergy stored in the oceans is immense, we have exploited this exergy far less than that of fossil-fuel deposits. Why?

▶ PROBLEMS: DEVELOPING ENGINEERING SKILLS

Exploring Energy Concepts

7.1 By inspection of Fig. P7.1 giving a T–v diagram for water, indicate whether exergy would increase, decrease, or remain the same in (a) Process 1–2, (b) Process 3–4, (c) Process 5–6. Explain.

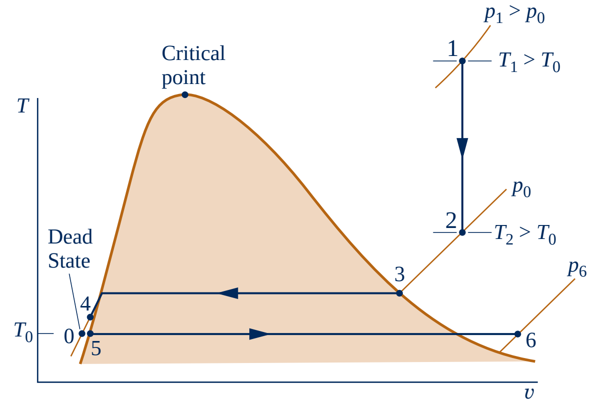


Fig. P7.1

7.2 An ideal gas is stored in a closed vessel at pressure p and temperature T.

(a) If $T = T_0$, derive an expression for the specific exergy in terms of p, p_0, T_0, and the gas constant R.
(b) If $p = p_0$, derive an expression for the specific exergy in terms of T, T_0, and the specific heat c_p, which can be taken as constant.

Ignore the effects of motion and gravity.

7.3 Consider an evacuated tank of volume V. For the space inside the tank as the system, show that the exergy is given by $\mathsf{E} = p_0 V$. Discuss.

7.4 Equal molar amounts of carbon dioxide and helium are maintained at the same temperature and pressure. Which has the greater value for exergy relative to the same reference environment? Assume the ideal gas model with constant c_v for each gas. There are no significant effects of motion and gravity.

7.5 Two solid blocks, each having mass m and specific heat c, and initially at temperatures T_1 and T_2, respectively, are brought into contact, insulated on their outer surfaces, and allowed to come into thermal equilibrium.

(a) Derive an expression for the exergy destruction in terms of m, c, T_1, T_2, and the temperature of the environment, T_0.
(b) Demonstrate that the exergy destruction cannot be negative.
(c) What is the source of exergy destruction in this case?

7.6 A system undergoes a refrigeration cycle while receiving Q_C by heat transfer at temperature T_C and discharging energy Q_H by heat transfer at a higher temperature T_H. There are no other heat transfers.

(a) Using energy and exergy balances, show that the net work input to the cycle cannot be zero.
(b) Show that the coefficient of performance of the cycle can be expressed as

$$\beta = \left(\frac{T_C}{T_H - T_C}\right)\left(1 - \frac{T_H \mathsf{E}_d}{T_0(Q_H - Q_C)}\right)$$

where E_d is the exergy destruction and T_0 is the temperature of the exergy reference environment.
(c) Using the result of part (b), obtain an expression for the maximum theoretical value for the coefficient of performance.

7.7 When matter flows across the boundary of a control volume, an energy transfer by work, called *flow work,* occurs. The rate is $\dot{m}(pv)$ where $\dot{m}$, p, and v denote the mass flow rate, pressure, and specific volume, respectively, of the matter crossing the boundary (see Sec. 4.4.2). Show that the *exergy transfer accompanying flow work* is given by $\dot{m}(pv - p_0 v)$, where p_0 is the pressure at the dead state.

7.8 When matter flows across the boundary of a control volume, an exergy transfer accompanying mass flow occurs, which is given by $\dot{m}\mathsf{e}$ where e is the specific exergy (Eq. 7.2) and $\dot{m}$ is the mass flow rate. An exergy transfer accompanying flow work, which is given by the result of Problem 7.7, also occurs at the boundary. Show that the sum of these exergy transfers is given by $\dot{m}\mathsf{e}_f$, where e_f is the specific flow exergy (Eq. 7.14).

7.9 For an ideal gas with constant specific heat ratio k, show that in the absence of significant effects of motion and gravity the specific flow exergy can be expressed as

$$\frac{\mathsf{e}_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \ln\left(\frac{p}{p_0}\right)^{(k-1)/k}$$

(a) For $k = 1.2$ develop plots of $\mathsf{e}_f/c_p T_0$ versus for T/T_0 for $p/p_0 = 0.25, 0.5, 1, 2, 4$. Repeat for $k = 1.3$ and 1.4.
(b) The specific flow exergy can take on negative values when $p/p_0 < 1$. What does a negative value mean physically?

7.10 An ideal gas with constant specific heat ratio k enters a turbine operating at steady state at T_1 and p_1 and expands adiabatically to T_2 and p_2. When would the value of the exergetic turbine efficiency exceed the value of the isentropic turbine efficiency? Discuss. Ignore the effects of motion and gravity.

Evaluating Exergy

7.11 A system consists of 2 kg of water at 100°C and 1 bar. Determine the exergy, in kJ, if the system is at rest and zero elevation relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar.

7.12 A domestic water heater holds 189 L of water at 60°C, 1 atm. Determine the exergy of the hot water, in kJ. To what elevation, in m, would a 1000-kg mass have to be raised from zero elevation relative to the reference environment for its exergy to equal that of the hot water? Let $T_0 = 298$ K, $p_0 = 1$ atm, $g = 9.81$ m/s^2.

7.13 Determine the specific exergy of argon at (a) $p = 2\,p_0$, $T = 2\,T_0$, (b) $p = p_0/2$, $T = T_0/2$. Locate each state relative to the dead state on temperature–pressure coordinates. Assume ideal gas behavior with $k = 1.67$. Let $T_0 = 537°R$, $p_0 = 1$ atm.

7.14 Determine the specific exergy, in Btu, of one pound mass of

(a) saturated liquid Refrigerant 134a at −5°F.
(b) saturated vapor Refrigerant 134a at 140°F.
(c) Refrigerant 134a at 60°F, 20 lbf/in.^{2}
(d) Refrigerant 134a at 60°F, 10 lbf/in.^{2}

In each case, consider a fixed mass at rest and zero elevation relative to an exergy reference environment for which $T_0 =$ 60°F, $p_0 = 15$ lbf/in.^{2}

7.15 A balloon filled with helium at 20°C, 1 bar and a volume of 0.5 m^3 is moving with a velocity of 15 m/s at an elevation of 0.5 km relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar. Using the ideal gas model with $k = 1.67$, determine the specific exergy of the helium, in kJ.

7.16 A vessel contains carbon dioxide. Using the ideal gas model

(a) determine the specific exergy of the gas, in Btu/lb, at $p = 90$ lbf/in.^{2} and $T = 200°F$.
(b) plot the specific exergy of the gas, in Btu/lb, versus pressure ranging from 15 to 90 lbf/in.^{2}, for $T = 80°F$.
(c) plot the specific exergy of the gas, in Btu/lb, versus temperature ranging from 80 to 200°F, for $p = 15$ lbf/in.^{2}

The gas is at rest and zero elevation relative to an exergy reference environment for which $T_0 = 80°F$, $p_0 = 15$ lbf/in.^{2}

7.17 Oxygen (O_2) at temperature T and 1 atm fills a balloon at rest on the surface of the earth at a location where the ambient temperature is 40°F and the ambient pressure is 1 atm. Using the ideal gas model with $c_p = 0.22$ Btu/lb · °R, plot the specific exergy of the oxygen, in Btu/lb, relative to the earth and its atmosphere at this location versus T ranging from 500 to 600°R.

7.18 A vessel contains 1 lb of air at pressure p and 200°F. Using the ideal gas model, plot the specific exergy of the air, in Btu/lb, for p ranging from 0.5 to 2 atm. The air is at rest and negligible elevation relative to an exergy reference environment for which $T_0 = 60°F$, $p_0 = 1$ atm.

7.19 Determine the exergy, in Btu, of a sample of water as saturated solid at 10°F, measuring 2.25 in. × 0.75 in. × 0.75 in. Let $T_0 = 537°R$ and $p_0 = 1$ atm.

7.20 Determine the exergy, in kJ, of the contents of a 1.5-m^3 storage tank, if the tank is filled with

(a) air as an ideal gas at 440°C and 0.70 bar.
(b) water vapor at 440°C and 0.70 bar.

Ignore the effects of motion and gravity and let $T_0 = 22°C$, $p_0 = 1$ bar.

7.21 A concrete slab measuring 0.3 m × 4 m × 6 m, initially at 298 K, is exposed to the sun for several hours, after which its temperature is 301 K. The density of the concrete is 2300 kg/m^3 and its specific heat is $c = 0.88$ kJ/kg · K. (a) Determine the increase in exergy of the slab, in kJ. (b) To what elevation, in m, would a 1000-kg mass have to be raised from zero elevation relative to the reference environment for its exergy to equal the exergy increase of the slab? Let $T_0 = 298$ K, $p_0 = 1$ atm, $g = 9.81$ m/s^2.

7.22 Refrigerant 134a vapor initially at 1 bar and 20°C fills a rigid vessel. The vapor is cooled until the temperature becomes −32°C. There is no work during the process. For the refrigerant, determine the heat transfer per unit mass and the change in specific exergy, each in kJ/kg. Comment. Let $T_0 = 20°C$, $p_0 =$ 0.1 MPa and ignore the effects of motion and gravity.

7.23 As shown in Fig. P7.23, two kilograms of water undergo a process from an initial state where the water is saturated vapor at 120°C, the velocity is 30 m/s, and the elevation is 6 m to a final state where the water is saturated liquid at 10°C, the velocity is 25 m/s, and the elevation is 3 m. Determine in kJ, (a) the exergy at the initial state, (b) the exergy at the final state, and (c) the change in exergy. Let $T_0 = 25°C$, $p_0 =$ 1 atm, and $g = 9.8$ m/s^2.

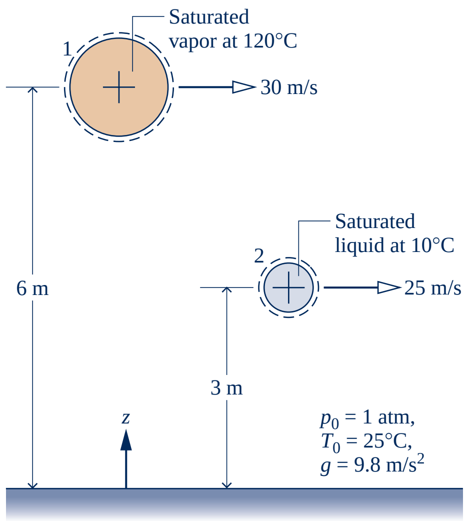


Fig. P7.23

7.24 Two pounds of air initially at 200°F and 50 lbf/in.^{2} undergo two processes in series:

Process 1–2: Isothermal to p_2 = 10 lbf/in.^{2}

Process 2–3: Constant pressure to T_3 = −10°F

Employing the ideal gas model

(a) represent each process on a p–v diagram and indicate the dead state.
(b) determine the change in exergy for each process, in Btu.

Let T_0 = 77°F, p_0 = 14.7 lbf/in.^{2} and ignore the effects of motion and gravity.

7.25 Twenty pounds of air initially at 1560°R, 3 atm fills a rigid tank. The air is cooled to 1040°R, 2 atm. For the air modeled as an ideal gas

(a) indicate the initial state, final state, and dead state on a T–v diagram.
(b) determine the heat transfer, in Btu.
(c) determine the change in exergy, in Btu, and interpret the sign using the T–v diagram of part (a).

Let T_0 = 520°R, p_0 = 1 atm and ignore the effects of motion and gravity.

7.26 Consider 100 kg of steam initially at 20 bar and 240°C as the system. Determine the change in exergy, in kJ, for each of the following processes:

(a) The system is heated at constant pressure until its volume doubles.
(b) The system expands isothermally until its volume doubles.

Let T_0 = 20°C, p_0 = 1 bar and ignore the effects of motion and gravity.

Applying the Exergy Balance: Closed Systems

7.27 Two kilograms of water in a piston–cylinder assembly, initially at 2 bar and 120°C, are heated at constant pressure with no internal irreversibilities to a final state where the water is a saturated vapor. For the water as the system, determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ. Let T_0 = 20°C, p_0 = 1 bar and ignore the effects of motion and gravity.

7.28 Two kilograms of carbon monoxide in a piston–cylinder assembly, initially at 1 bar and 27°C, is heated at constant pressure with no internal irreversibilities to a final temperature of 227°C. Employing the ideal gas model, determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ. Let T_0 = 300 K, p_0 = 1 bar and ignore the effects of motion and gravity.

7.29 As shown in Fig. P7.29, 1.11 kg of Refrigerant 134a is contained in a rigid, insulated vessel. The refrigerant is initially saturated vapor at −28°C. The vessel is fitted with a paddle wheel from which a mass is suspended. As the mass descends a certain distance, the refrigerant is stirred until it attains a final equilibrium state at a pressure of 1.4 bar. The only significant changes in state are experienced by the refrigerant and the suspended mass. Determine, in kJ,

(a) the change in exergy of the refrigerant.
(b) the change in exergy of the suspended mass.
(c) the change in exergy of an isolated system of the vessel and pulley–mass assembly.
(d) the destruction of exergy within the isolated system.

Let T_0 = 293 K(20°C), p_0 = 1 bar.

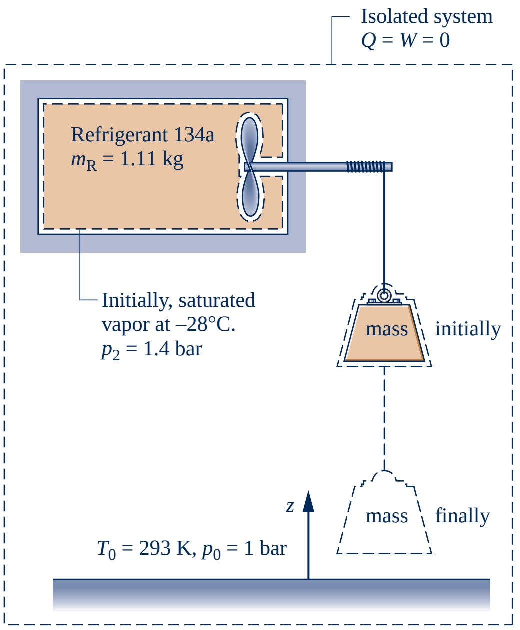


Fig. P7.29

7.30 A rigid, insulated tank contains 0.6 kg of air, initially at 200 kPa, 20°C. The air is stirred by a paddle wheel until its pressure is 250 kPa. Using the ideal gas model with c_v = 0.72 kJ/kg · K, determine, in kJ, (a) the work, (b) the change in exergy of the air, and (c) the amount of exergy destroyed. Ignore the effects of motion and gravity, and let T_0 = 20°C, p_0 = 100 kPa.

7.31 As shown in Fig. P7.31, two lb of ammonia is contained in a well-insulated piston–cylinder assembly fitted with an electrical resistor of negligible mass. The ammonia is initially at 20 lbf/in.^{2} and a quality of 80%. The resistor is activated until the volume of the ammonia increases by 25%, while its pressure varies negligibly. Determine, in Btu,

(a) the amount of energy transfer by electrical work and the accompanying exergy transfer.
(b) the amount of energy transfer by work to the piston and the accompanying exergy transfer.
(c) the change in exergy of the ammonia.
(d) the amount of exergy destruction.

Ignore the effects of motion and gravity and let T_0 = 60°F, p_0 = 1 atm.

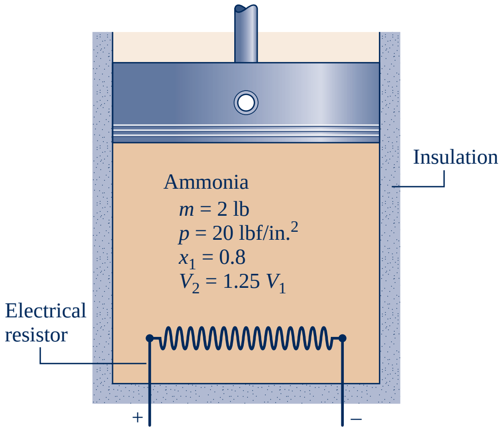


Fig. P7.31

7.32 One-half pound of air is contained in a closed, rigid, insulated tank. Initially the temperature is 520°R and the pressure is 14.7 psia. The air is stirred by a paddle wheel until its temperature is 600°R. Using the ideal gas model, determine for the air the change in exergy, the transfer of exergy accompanying work, and the exergy destruction, all in Btu. Ignore the effects of motion and gravity and let T_0 = 537°R, p_0 = 14.7 psia.

7.33 Three kilograms of nitrogen (N_2) initially at 47°C and 2 bar is contained within a rigid, insulated tank. The nitrogen is stirred by a paddle wheel until its pressure doubles. Employing the ideal gas model with constant specific heat evaluated at 300 K, determine the work and exergy destruction for the nitrogen, each in kJ. Ignore the effects of motion and gravity and let T_0 = 300 K, p_0 = 1 bar.

7.34 One lbmol of carbon dioxide gas is contained in a 100-ft^3 rigid, insulated vessel initially at 4 atm. An electric resistor of negligible mass transfers energy to the gas at a constant rate of 12 Btu/s for 1 min. Employing the ideal gas model and ignoring the effects of motion and gravity, determine (a) the change in exergy of the gas, (b) the electrical work, and (c) the exergy destruction, each in Btu. Let T_0 = 70°F, p_0 = 1 atm.

7.35 A rigid, well-insulated tank consists of two compartments, each having the same volume, separated by a valve. Initially, one of the compartments is evacuated and the other contains 0.25 lbmol of a gas at 50 lbf/in.^{2} and 100°F. The valve is opened and the gas expands to fill the total volume, eventually achieving an equilibrium state. Using the ideal gas model

(a) determine the final temperature, in °F, and final pressure, in lbf/in.^{2}
(b) evaluate the exergy destruction, in Btu.
(c) What is the cause of exergy destruction in this case?

Let T_0 = 70°F, p_0 = 1 atm.

7.36 As shown in Fig. P7.36, a 0.8-lb metal bar initially at 1900°R is removed from an oven and quenched by immersing it in a closed tank containing 20 lb of water initially at 530°R. Each substance can be modeled as incompressible. An appropriate constant specific heat for the water is c_w = 1.0 Btu/lb · °R, and an appropriate value for the metal is c_m = 0.1 Btu/lb · °R. Heat transfer from the tank contents can be neglected. Determine the exergy destruction, in Btu. Let T_0 = 77°F.

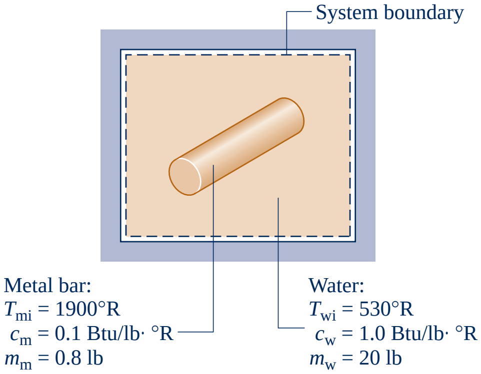


Fig. P7.36

7.37 Figure P7.37 provides steady-state data for a composite of a hot plate and two solid layers. Perform a full exergy accounting, in kW, of the electrical power provided to the composite, including the exergy transfer accompanying heat transfer from the composite and the destruction of exergy in the hot plate and each of the two layers. Let T_0 = 300 K.

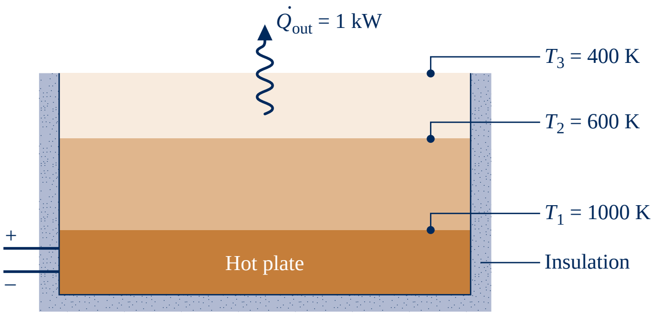


Fig. P7.37

7.38 As shown in Fig. P7.38, heat transfer at a rate of 1000 Btu/h takes place through the inner surface of a wall. Measurements made during steady-state operation reveal temperatures of T_1 = 2500°R and T_2 = 500°R at the inner and outer surfaces, respectively. Determine, in Btu/h

(a) the rates of exergy transfer accompanying heat at the inner and outer surfaces of the wall.
(b) the rate of exergy destruction.
(c) What is the cause of exergy destruction in this case?

Let T_0 = 500°R.

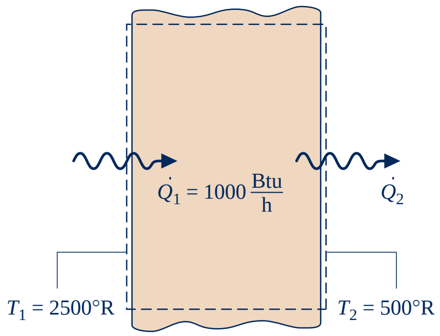


Fig. P7.38

7.39 Figure P7.39 provides steady-state data for the outer wall of a dwelling on a day when the indoor temperature is maintained at 20°C and the outdoor temperature is 0°C. The heat transfer rate through the wall is 1100 W. Determine, in W, the rate of exergy destruction (a) within the wall, and (b) within the enlarged system shown on the figure by the dashed line. Comment. Let $T_0 = 0°C$.

7.40 The sun shines on a 300-ft^2 south-facing wall, maintaining that surface at 98°F. Temperature varies linearly through the wall and is 77°F at its other surface. The wall thickness is 6 inches and its thermal conductivity is 0.04 Btu/h · ft · R. Assuming steady state, determine the rate of exergy destruction within the wall, in Btu/h. Let $T_0 = 77°F$.

7.41 A gearbox operating at steady state receives 2 hp along the input shaft and delivers 1.89 hp along the output shaft. The outer surface of the gearbox is at 110°F. For the gearbox, (a) determine, in Btu/s, the rate of heat transfer and (b) perform a full exergy accounting, in Btu/s, of the input power. Let $T_0 = 70°F$.

7.42 A gearbox operating at steady state receives 20 horsepower along its input shaft, delivers power along its output shaft, and is cooled on its outer surface according to $hA(T_b - T_0)$, where $T_b = 110°F$ is the temperature of the outer surface and $T_0 = 40°F$ is the temperature of the surroundings far from the gearbox. The product of the heat transfer coefficient h and outer surface area A is 35 Btu/h · °R. For the gearbox, determine, in hp, a full exergy accounting of the input power. Let $T_0 = 40°F$.

7.43 At steady state, an electric motor develops power along its output shaft of 0.5 hp while drawing 4 amps at 120 V. The outer surface of the motor is at 120°F. For the motor, (a) determine, in Btu/h, the rate of heat transfer and (b) perform a full exergy accounting, in Btu/h, of the electrical power input. Let $T_0 = 60°F$.

7.44 As shown in Fig. P7.44, a silicon chip measuring 5 mm on a side and 1 mm in thickness is embedded in a ceramic substrate. At steady state, the chip has an electrical power input of 0.225 W. The top surface of the chip is exposed to a coolant whose temperature is 20°C. The heat transfer coefficient for convection between the chip and the coolant is 150 W/m^2 · K. Heat transfer by conduction between the chip and the substrate is negligible. Determine (a) the surface temperature of the chip, in °C, and (b) the rate of exergy destruction within the chip, in W. What causes the exergy destruction in this case? Let $T_0 = 293$ K.

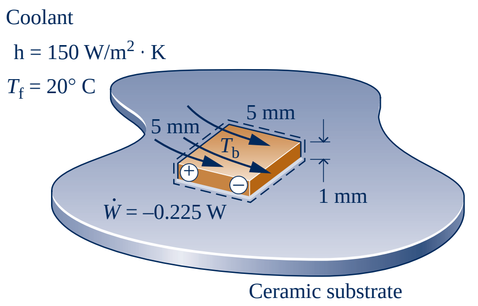


Fig. P7.44

7.45 An electric water heater having a 200-L capacity heats water from 23 to 55°C. Heat transfer from the outside of the water heater is negligible, and the states of the electrical heating element and the tank holding the water do not change significantly. Perform a full exergy accounting, in kJ, of the electricity supplied to the water heater. Model the water as incompressible with a specific heat $c = 4.18$ kJ/kg · K. Let $T_0 = 23°C$.

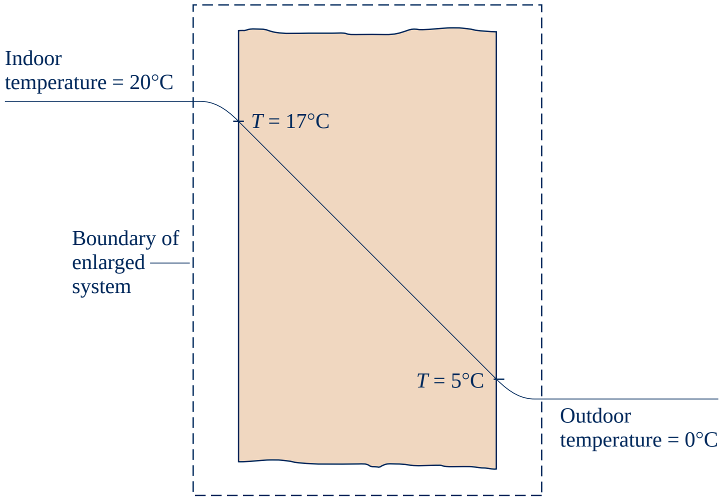


Fig. P7.39

7.46 A thermal reservoir at 1200 K is separated from another thermal reservoir at 300 K by a cylindrical rod insulated on its lateral surfaces. At steady state, energy transfer by conduction takes place through the rod. The rod diameter is 2 cm, the length is L, and the thermal conductivity is 0.4 kW/m · K. Plot the following quantities, each in kW, versus L ranging from 0.01 to 1 m: the rate of conduction through the rod, the rates of exergy transfer accompanying heat transfer into and out of the rod, and the rate of exergy destruction. Let $T_0 = 300$ K.

7.47 Four kilograms of a two-phase liquid–vapor mixture of water initially at 300°C and $x_1 = 0.5$ undergo the two different processes described below. In each case, the mixture is brought from the initial state to a saturated vapor state, while the volume remains constant. For each process, determine the change in exergy of the water, the net amounts of exergy transfer by work and heat, and the amount of exergy destruction, each in kJ. Let $T_0 = 300$ K, $p_0 = 1$ bar, and ignore the effects of motion and gravity. Comment on the difference between the exergy destruction values.

(a) The process is brought about adiabatically by stirring the mixture with a paddle wheel.
(b) The process is brought about by heat transfer from a thermal reservoir at 610 K. The temperature of the water at the location where the heat transfer occurs is 610 K.

7.48 As shown in Fig. P7.48, one-half pound of nitrogen (N_2), in a piston–cylinder assembly, initially at 80°F, 20 lbf/in.^{2}, is compressed isothermally to a final pressure of 100 lbf/in.^{2} During compression, the nitrogen rejects energy by heat transfer through the cylinder's end wall, which has inner and outer temperatures of 80°F and 70°F, respectively.

(a) For the nitrogen as the system, evaluate the work, heat transfer, exergy transfers accompanying work and heat transfer, and amount of exergy destruction, each in Btu.
(b) Evaluate the amount of exergy destruction, in Btu, for an enlarged system that includes the nitrogen and the wall, assuming the state of the wall remains unchanged. Comment.

Use the ideal gas model for the nitrogen and let $T_0 = 70$°F, $p_0 = 14.7$ lbf/in.^{2}

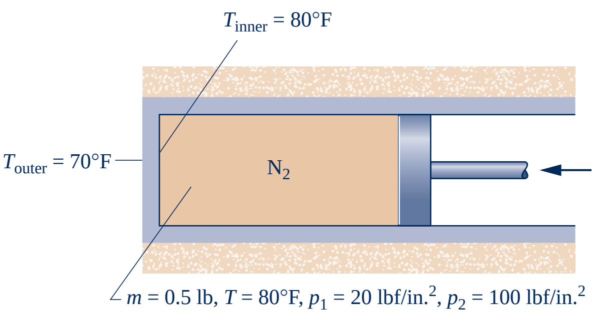


Fig. P7.48

7.49 Air initially at 1 atm and 500°R with a mass of 2.5 lb is contained within a closed, rigid tank. The air is slowly warmed, receiving 100 Btu by heat transfer through a wall separating the gas from a thermal reservoir at 800°R. This is the only energy transfer. Assuming the air undergoes an internally reversible process and using the ideal gas model,

(a) determine the change in exergy and the exergy transfer accompanying heat, each in Btu, for the air as the system.
(b) determine the exergy transfer accompanying heat and the exergy destruction, each in Btu, for an enlarged system that includes the air and the wall, assuming that the state of the wall remains unchanged. Compare with part (a) and comment.

Let $T_0 = 90$°F, $p_0 = 1$ atm.

Applying the Exergy Balance: Control Volumes at Steady State

7.50 Determine the specific flow exergy, in Btu/lbmol and Btu/lb, at 440°F, 73.5 lbf/in.^{2} for (a) nitrogen (N_2) and (b) carbon dioxide (CO_2), each modeled as an ideal gas, and relative to an exergy reference environment for which $T_0 = 77$°F, $p_0 = 14.7$ lbf/in.^{2} Ignore the effects of motion and gravity.

7.51 Determine the specific exergy and the specific flow exergy, each in Btu/lb, for steam at 350 lbf/in.^{2}, 700°F, with V = 120 ft/s and z = 80 ft. The velocity and elevation are relative to an exergy reference environment for which $T_0 = 70$°F, $p_0 = 14.7$ lbf/in.^{2}, and $g = 32.2$ ft/s^2.

7.52 Water at 24°C, 1 bar is drawn from a reservoir 1.25 km above a valley and allowed to flow through a hydraulic turbine-generator into a lake on the valley floor. For operation at steady state, determine the maximum theoretical rate at which electricity is generated, in MW, for a mass flow rate of 110 kg/s. Let $T_0 = 24$°C, $p_0 = 1$ bar and ignore the effects of motion.

7.53 At steady state, hot gaseous products of combustion cool from 2800°F to 260°F as they flow through a pipe. Owing to negligible fluid friction, the flow occurs at nearly constant pressure. Applying the ideal gas model with $c_p = 0.25$ Btu/lb · °R, determine the exergy transfer accompanying heat transfer from the gas, in Btu per lb of gas flowing. Let $T_0 = 60$°F and ignore the effects of motion and gravity.

7.54 For the simple vapor power plant of Problem 6.165, evaluate, in MW, (a) the *net* rate energy exits the plant with the cooling water and (b) the *net* rate exergy exits the plant with the cooling water. Comment. Let $T_0 = 20$°C, $p_0 = 1$ atm and ignore the effects of motion and gravity.

7.55 Water vapor enters a valve with a mass flow rate of 2 kg/s at a temperature of 320°C and a pressure of 60 bar and undergoes a throttling process to 40 bar.

(a) Determine the flow exergy rates at the valve inlet and exit and the rate of exergy destruction, each in kW.
(b) Evaluating exergy at 8.5 cents per kW · h, determine the annual cost, in $/year, associated with the exergy destruction, assuming 8400 hours of operation annually.

Let $T_0 = 25$°C, $p_0 = 1$ bar.

7.56 Steam at 1000 lbf/in.^{2}, 600°F enters a valve operating at steady state and undergoes a throttling process.

(a) Determine the exit temperature, in °F, and the exergy destruction rate, in Btu per lb of steam flowing, for an exit pressure of 500 lbf/in.^{2}

(b) Plot the exit temperature, in °F, and the exergy destruction rate, in Btu per lb of steam flowing, each versus exit pressure ranging from 500 to 1000 lbf/in.^{2}

Let T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

7.57 Air at 200 lbf/in.^{2}, 800°R, and a volumetric flow rate of 100 ft^3/min enters a valve operating at steady state and undergoes a throttling process. Assuming ideal gas behavior

(a) determine the rate of exergy destruction, in Btu/min, for an exit pressure of 15 lbf/in.^{2}

(b) plot the exergy destruction rate, in Btu/min, versus exit pressure ranging from 15 to 200 lbf/in.^{2}

Let T_0 = 530°R, p_0 = 15 lbf/in.^{2}

7.58 Water vapor at 4.0 MPa and 400°C enters an insulated turbine operating at steady state and expands to saturated vapor at 0.1 MPa. The effects of motion and gravity can be neglected. Determine the work developed and the exergy destruction, each in kJ per kg of water vapor passing through the turbine. Let T_0 = 27°C, p_0 = 0.1 MPa.

7.59 Air enters an insulated turbine operating at steady state at 8 bar, 500 K, and 150 m/s. At the exit the conditions are 1 bar, 320 K, and 10 m/s. There is no significant change in elevation. Determine the work developed and the exergy destruction, each in kJ per kg of air flowing. Let T_0 = 300 K, p_0 = 1 bar.

7.60 Air enters a turbine operating at steady state with a pressure of 75 lbf/in.^{2}, a temperature of 800°R, and a velocity of 400 ft/s. At the turbine exit, the conditions are 15 lbf/in.^{2}, 600°R, and 100 ft/s. Heat transfer from the turbine to its surroundings takes place at an average surface temperature of 620°R. The rate of heat transfer is 2 Btu per lb of air passing through the turbine. For the turbine, determine the work developed and the exergy destruction, each in Btu per lb of air flowing. Let T_0 = 40°F, p_0 = 15 lbf/in.^{2}

7.61 Steam enters a turbine operating at steady state at 4 MPa, 500°C with a mass flow rate of 50 kg/s. Saturated vapor exits at 10 kPa and the corresponding power developed is 42 MW. The effects of motion and gravity are negligible.

(a) For a control volume enclosing the turbine, determine the rate of heat transfer, in MW, from the turbine to its surroundings. Assuming an average turbine outer surface temperature of 50°C, determine the rate of exergy destruction, in MW.
(b) If the turbine is located in a facility where the ambient temperature is 27°C, determine the rate of exergy destruction for an enlarged control volume including the turbine and its immediate surroundings so heat transfer takes place at the ambient temperature. Explain why the exergy destruction values of parts (a) and (b) differ.

Let T_0 = 300 K, p_0 = 100 kPa.

7.62 An insulated turbine operating at steady state receives steam at 400 lbf/in.^{2}, 600°F and exhausts at 1 lbf/in.^{2} Plot the exergy destruction rate, in Btu per lb of steam flowing, versus turbine isentropic efficiency ranging from 70 to 100%. The effects of motion and gravity are negligible and T_0 = 60°F, p_0 = 1 atm.

7.63 Air enters a compressor operating at steady state at T_1 = 300 K, p_1 = 1 bar with a velocity of 70 m/s. At the exit, T_2 = 540 K, p_2 = 5 bar and the velocity is 150 m/s. The air can be modeled as an ideal gas with c_p = 1.01 kJ/kg · K. Stray heat transfer can be ignored. Determine, in kJ per kg of air flowing, (a) the power required by the compressor and (b) the rate of exergy destruction within the compressor. Let T_0 = 300 K, p_0 = 1 bar. Ignore the effects of motion and gravity.

7.64 Carbon monoxide (CO) enters an insulated compressor operating at steady state at 10 bar, 227°C, and a mass flow rate of 0.1 kg/s and exits at 15 bar, 327°C. Determine the power required by the compressor and the rate of exergy destruction, each in kW. Ignore the effects of motion and gravity. Let T_0 = 17°C, p_0 = 1 bar.

7.65 Refrigerant 134a at 10°C, 1.8 bar, and a mass flow rate of 5 kg/min enters an insulated compressor operating at steady state and exits at 5 bar. The isentropic compressor efficiency is 76.04%. Determine

(a) the temperature of the refrigerant exiting the compressor, in °C.
(b) the power input to the compressor, in kW.
(c) the rate of exergy destruction, in kW.

Ignore the effects of motion and gravity and let T_0 = 20°C, p_0 = 1 bar.

7.66 Air is compressed in an axial-flow compressor operating at steady state from 27°C, 1 bar to a pressure of 2.1 bar. The work required is 94.6 kJ per kg of air flowing. Heat transfer from the compressor occurs at an average surface temperature of 40°C at the rate of 14 kJ per kg of air flowing. The effects of motion and gravity can be ignored. Let T_0 = 20°C, p_0 = 1 bar. Assuming ideal gas behavior, determine (a) the temperature of the air at the exit, in °C, and (b) the rate of exergy destruction within the compressor, in kJ per kg of air flowing.

7.67 Figure P7.67 shows a device to develop power using a heat transfer from a high-temperature industrial process together with a steam input. The figure provides data for steady-state operation. All surfaces are well insulated, except for the one at 527°C, across which heat transfer occurs at a rate of 4.21 kW. The device develops power at a rate of 6 kW. Determine, in kW,

(a) the rate exergy enters accompanying heat transfer.
(b) the *net* rate exergy is carried in by the steam, $(\dot{E}_{f1} - \dot{E}_{f2})$.
(c) the rate of exergy destruction within the device.

Ignore the effects of motion and gravity and let T_0 = 293 K, p_0 = 1 bar.

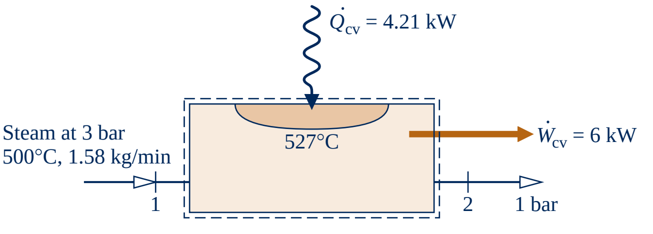


Fig. P7.67

7.68 Determine the rate of exergy destruction, in Btu/min, for the duct system of Problem 6.111. Let T_0 = 500°R, p_0 = 1 atm.

7.69 For the *vortex tube* of Example 6.7, determine the rate of exergy destruction, in Btu per lb of air entering. Referring to this value for exergy destruction, comment on the inventor's claim. Let $T_0 = 530°R$, $p_0 = 1$ atm.

7.70 Steam at 1.4 MPa and 350°C with a mass flow rate of 0.125 kg/s enters an insulated turbine operating at steady state and exhausts at 100 kPa. Plot the temperature of the exhaust steam, in °C, the power developed by the turbine, in kW, and the rate of exergy destruction within the turbine, in kW, each versus the isentropic turbine efficiency ranging from 0 to 100%. Ignore the effects of motion and gravity. Let $T_0 = 20°C$, $p_0 = 0.1$ Mpa.

7.71 Steam enters an insulated turbine operating at steady state at 100 lbf/in.^{2}, 500°F, with a mass flow rate of 3×10^5 lb/h and expands to a pressure of 1 atm. The isentropic turbine efficiency is 80%. If exergy is valued at 8 cents per kW · h, determine

(a) the value of the power produced, in \$/h.
(b) the cost of the exergy destroyed, in \$/h.
(c) Plot the values of the power produced and the exergy destroyed, each in \$/h, versus isentropic efficiency ranging from 80 to 100%.

Ignore the effects of motion and gravity. Let $T_0 = 70°F$, $p_0 =$ 1 atm.

7.72 Consider the parallel flow heat exchanger of Prob. 4.86. Verify that the stream of air exits at 780°R and the stream of carbon dioxide exits at 760°R. Pressure is constant for each stream. For the heat exchanger, determine the rate of exergy destruction, in Btu/s. Let $T_0 = 537°R$, $p_0 = 1$ atm.

7.73 Air at $T_1 = 1300°R$, $p_1 = 16$ lbf/in.^{2} enters a counterflow heat exchanger operating at steady state and exits at $p_2 =$ 14.7 lb/in.^{2} A separate stream of air enters at $T_3 = 850°R$, $p_3 = 60$ lbf/in.^{2} and exits at $T_4 = 1000°R$, $p_4 = 50$ lbf/in.^{2} The mass flow rates of the streams are equal. Stray heat transfer and the effects of motion and gravity can be ignored. Assuming the ideal gas model with $c_p = 0.24$ Btu/lb · °R, determine (a) T_2, in °R, and (b) the rate of exergy destruction within the heat exchanger, in Btu per lb of air flowing. Let $T_0 = 520°R$, $p_0 = 1$ atm.

7.74 A counterflow heat exchanger operating at steady state has water entering as saturated vapor at 1 bar with a mass flow rate of 2 kg/s and exiting as saturated liquid at 1 bar. Air enters in a separate stream at 300 K, 1 bar and exits at 335 K with a negligible change in pressure. Heat transfer between the heat exchanger and its surroundings is negligible. Determine

(a) the change in the flow exergy rate of each stream, in kW.
(b) the rate of exergy destruction in the heat exchanger, in kW.

Ignore the effects of motion and gravity. Let $T_0 = 300$ K, $p_0 =$ 1 bar.

7.75 Water at $T_1 = 100°F$, $p_1 = 30$ lbf/in.^{2} enters a counterflow heat exchanger operating at steady state with a mass flow rate of 100 lb/s and exits at $T_2 = 200°F$ with closely the same pressure. Air enters in a separate stream at $T_3 = 540°F$ and exits at $T_4 = 140°F$ with no significant change in pressure. Air can be modeled as an ideal gas and stray heat transfer can be ignored. Determine (a) the mass flow rate of the air, in lb/s, and (b) the rate of exergy destruction within the heat exchanger, in Btu/s. Ignore the effects of motion and gravity and let $T_0 = 60°F$, $p_0 = 1$ atm.

7.76 Air enters a counterflow heat exchanger operating at steady state at 22°C, 0.1 MPa and exits at 7°C. Refrigerant 134a enters at 0.2 MPa, a quality of 0.2, and a mass flow rate of 30 kg/h. Refrigerant exits at 0°C. Stray heat transfer is negligible and there is no significant change in pressure for either stream.

(a) For the Refrigerant 134a stream, determine the rate of heat transfer, in kJ/h.
(b) For each of the streams, evaluate the change in flow exergy rate, in kJ/h, and interpret its value and sign.

Let $T_0 = 22°C$, $p_0 = 0.1$ MPa, and ignore the effects of motion and gravity.

7.77 Liquid water enters a heat exchanger operating at steady state at $T_1 = 60°F$, $p_1 = 1$ atm and exits at $T_2 = 160°F$ with a negligible change in pressure. In a separate stream, steam enters at $T_3 = 20$ lbf/in.^{2}, $x_3 = 92\%$ and exits at $T_4 = 140°F$, $p_4 = 18$ lbf/in.^{2} Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 60°F$, $p_0 = 1$ atm. Determine (a) the ratio of the mass flow rates of the two streams and (b) the rate of exergy destruction, in Btu per lb of steam entering the heat exchanger.

7.78 Argon enters a nozzle operating at steady state at 1300 K, 360 kPa with a velocity of 10 m/s and exits the nozzle at 900 K, 130 kPa. Stray heat transfer can be ignored. Modeling argon as an ideal gas with $k = 1.67$, determine (a) the velocity at the exit, in m/s, and (b) the rate of exergy destruction, in kJ per kg of argon flowing. Let $T_0 = 293$ K, $p_0 = 1$ bar.

7.79 Nitrogen (N_2) enters a well-insulated nozzle operating at steady state at 75 lbf/in.^{2}, 1200°R, 80 ft/s. At the nozzle exit, the pressure is 20 lbf/in.^{2} The isentropic nozzle efficiency is 90%. For the nozzle, determine the exit velocity, in m/s, and the exergy destruction rate, in Btu per lb of nitrogen flowing. Let $T_0 = 70°F$, $p_0 = 14.7$ lbf/in.^{2}

7.80 Steady-state operating data are shown in Fig. P7.80 for an open feedwater heater. Heat transfer from the feedwater heater to its surroundings occurs at an average outer surface temperature of 50°C at a rate of 100 kW. Ignore the effects of motion and gravity and let $T_0 = 25°C$, $p_0 = 1$ bar. Determine

(a) the ratio of the incoming mass flow rates, $\dot{m}_1/\dot{m}_2$.
(b) the rate of exergy destruction, in kW.

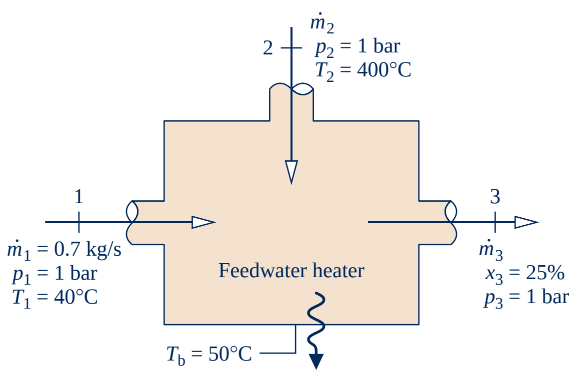


Fig. P7.80

7.81 An open feedwater heater operates at steady state with liquid water entering inlet 1 at 10 bar, 50°C, and a mass flow rate of 10 kg/s. A separate stream of steam enters inlet 2 at 10 bar and 200°C. Saturated liquid at 10 bar exits the feedwater heater. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 20°C$, $p_0 = 1$ bar. Determine (a) the mass flow rate of the streams at inlet 2 and the exit, each in kg/s, (b) the rate of exergy destruction, in kW, and (c) the cost of the exergy destroyed, in \$/year, for 8400 hours of operation annually. Evaluate exergy at 8.5 cents per kW · h.

7.82 Figure P7.82 and the accompanying table provide a schematic and steady-state operating data for a mixer that combines two streams of air. The stream entering at 1500 K has a mass flow rate of 2 kg/s. Stray heat transfer and the effects of motion and gravity are negligible. Assuming the ideal gas model for the air, determine the rate of exergy destruction, in kW. Let $T_0 = 300$ K, $p_0 = 1$ bar.

State	T(K)	p(bar)	h(kJ/kg)	$s°$(kJ/kg · K)[a]
1	1500	2	1635.97	3.4452
2	300	2	300.19	1.7020
3	—	1.9	968.08	2.8869

[a]$s°$ is the variable appearing in Eq. 6.20a and Table A-22.

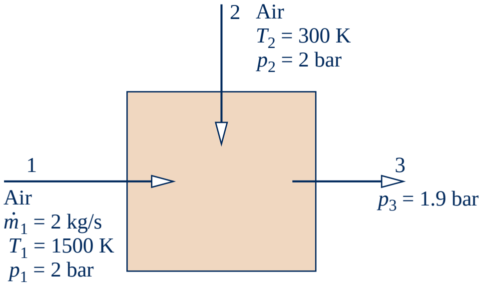


Fig. P7.82

7.83 Figure P7.83 provides steady-state operating data for a mixing chamber in which entering liquid and vapor streams of water mix to form an exiting saturated liquid stream. Heat transfer from the mixing chamber to its surroundings occurs at an average surface temperature of 100°F. The effects of motion and gravity are negligible. Let $T_0 = 70°F$, $p_0 = 1$ atm. For the mixing chamber, determine, each in Btu/s, (a) the rate of heat transfer and the accompanying rate of exergy transfer and (b) the rate of exergy destruction.

7.84 Liquid water at 20 lbf/in.^{2}, 50°F enters a mixing chamber operating at steady state with a mass flow rate of 5 lb/s and mixes with a separate stream of steam entering at 20 lbf/in.^{2}, 250°F with a mass flow rate of 0.38 lb/s. A single mixed stream exits at 20 lbf/in.^{2}, 130°F. Heat transfer from the mixing chamber occurs to its surroundings. Neglect the effects of motion and gravity and let $T_0 = 70°F$, $p_0 = 1$ atm. Determine the rate of exergy destruction, in Btu/s, for a control volume including the mixing chamber and enough of its immediate surroundings that heat transfer occurs at 70°F.

7.85 At steady state, steam with a mass flow rate of 10 lb/s enters a turbine at 800°F and 600 lbf/in.^{2} and expands to 60 lbf/in.^{2} The power developed by the turbine is 2852 horsepower. The steam then passes through a counterflow heat exchanger with a negligible change in pressure, exiting at 800°F. Air enters the heat exchanger in a separate stream at 1.1 atm, 1020°F and exits at 1 atm, 620°F. The effects of motion and gravity can be ignored and there is no significant heat transfer between either component and its surroundings. Determine

(a) the mass flow rate of air, in lb/s.
(b) the rates of exergy destruction in the turbine and heat exchanger, each in Btu/s.

Evaluating exergy at 8 cents per kW · h, determine the hourly cost of each of the exergy destructions found in part (b). Let $T_0 = 40°F$, $p_0 = 1$ atm.

7.86 A gas turbine operating at steady state is shown in Fig. P7.86. Air enters the compressor with a mass flow rate of 5 kg/s at 0.95 bar and 22°C and exits at 5.7 bar. The air then passes through a heat exchanger before entering the turbine at 1100 K, 5.7 bar. Air exits the turbine at 0.95 bar. The compressor and turbine operate adiabatically and the

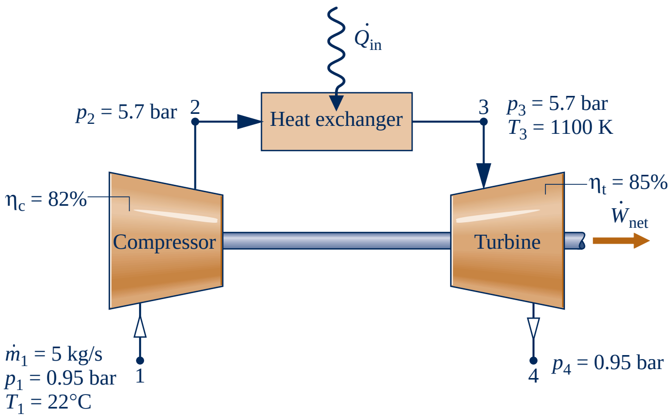


Fig. P7.86

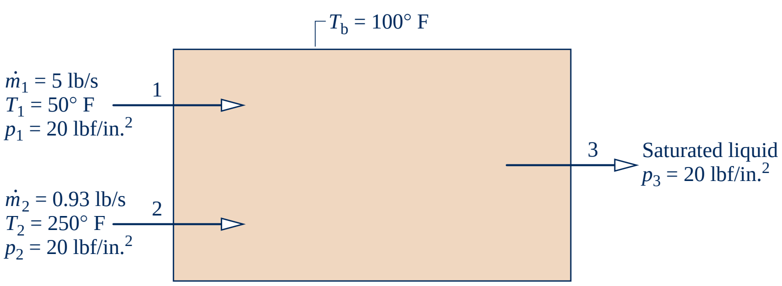


Fig. P7.83

effects of motion and gravity can be ignored. The compressor and turbine isentropic efficiencies are 82 and 85%, respectively. Using the ideal gas model for air, determine, each in kW,

(a) the *net* power developed.
(b) the rates of exergy destruction for the compressor and turbine.
(c) the *net* rate exergy is carried out of the plant at the turbine exit, $(\dot{E}_{f4} - \dot{E}_{f1})$.

Let $T_0 = 22°C$, $p_0 = 0.95$ bar.

7.87 Consider the *flash chamber* and turbine of Problem 4.105. Verify that the mass flow rate of saturated vapor entering the turbine is 0.371 kg/s and the power developed by the turbine is 149 kW. Determine the total rate of exergy destruction within the flash chamber and turbine, in kW. Comment. Let $T_0 = 298$ K.

7.88 Figure P7.88 and the accompanying table provide the schematic and steady-state operating data for a *flash chamber* fitted with an inlet valve that produces saturated vapor and saturated liquid streams from a single entering stream of liquid water. Stray heat transfer and the effects of motion and gravity are negligible. Determine (a) the mass flow rate, in lb/s, for each of the streams exiting the flash chamber and (b) the total rate of exergy destruction, in Btu/s. Let $T_0 = 77°F$, $p_0 = 1$ atm.

State	Condition	T(°F)	p(lbf/in.^{2})	h(Btu/lb)	s(Btu/lb · R)
1	liquid	300	80	269.7	0.4372
2	sat. vapor	—	30	1164.3	1.6996
3	sat. liquid	—	30	218.9	0.3682

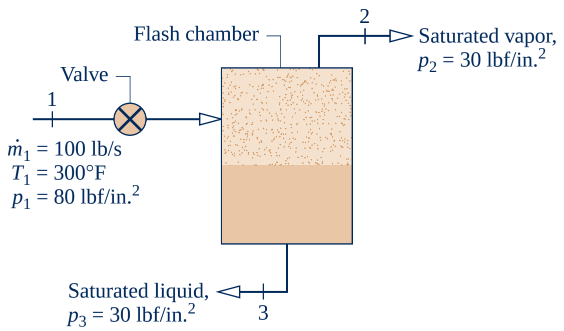


Fig. P7.88

7.89 Figure P7.89 shows a gas turbine power plant operating at steady state consisting of a compressor, a heat exchanger, and a turbine. Air enters the compressor with a mass flow rate of 3.9 kg/s at 0.95 bar, 22°C and exits the turbine at 0.95 bar, 421°C. Heat transfer to the air as it flows through the heat exchanger occurs at an average temperature of 488°C. The compressor and turbine operate adiabatically. Using the ideal gas model for the air, and neglecting the effects of motion and gravity, determine, in MW,

(a) the rate of exergy transfer accompanying heat transfer to the air flowing through the heat exchanger.
(b) the *net* rate exergy is carried out of the plant at the turbine exit, $(\dot{E}_{f2} - \dot{E}_{f1})$.
(c) the rate of exergy destruction within the power plant.
(d) Using the results of parts (a)–(c), perform a full exergy accounting of the exergy supplied to the power plant accompanying heat transfer. Comment.

Let $T_0 = 295$ K(22°C), $p_0 = 0.95$ bar.

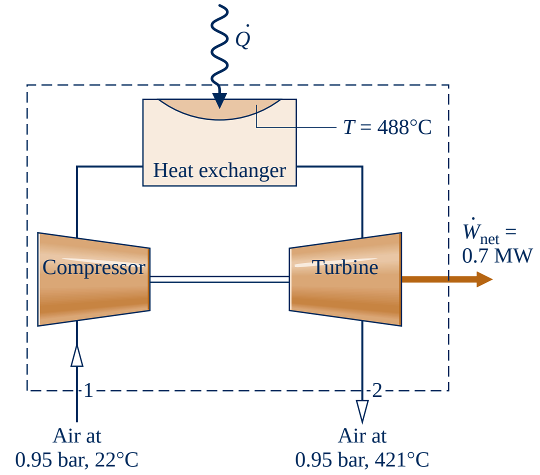


Fig. P7.89

7.90 Figure P7.90 shows a power-generating system at steady state. Saturated liquid water enters at 80 bar with a mass flow rate of 94 kg/s. Saturated liquid exits at 0.08 bar with the same mass flow rate. As indicated by arrows, three heat transfers occur, each at a specified temperature in the direction of the arrow: The first adds 135 MW at 295°C, the second adds 55 MW at 375°C, and the third removes energy at 20°C. The system generates power at the rate of 80 MW. The effects of motion and gravity can be ignored. Let $T_0 = 20°C$, $p_0 = 1$ atm. Determine, in MW, (a) the rate of heat transfer $\dot{Q}_3$ and the accompanying rate of exergy transfer and (b) a full exergy accounting of the *total* exergy supplied to the system with the two heat additions and with the *net* exergy, $(\dot{E}_{f1} - \dot{E}_{f2})$, carried in by the water stream as it passes from inlet to exit.

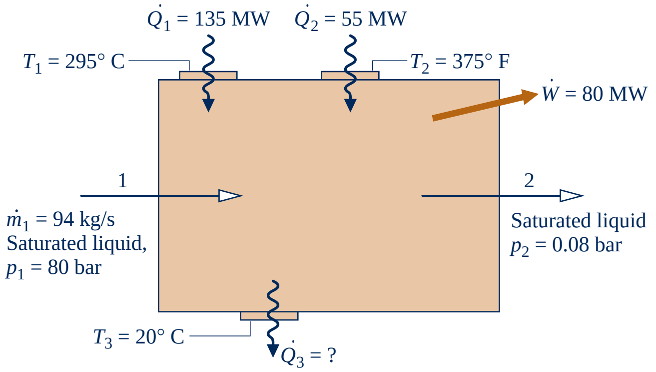


Fig. P7.90

7.91 Figure P7.91 shows a gas turbine power plant using air as the working fluid. The accompanying table gives steady-state operating data. Air can be modeled as an ideal gas. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 290$ K, $p_0 = 100$ kPa. Determine, each in kJ per kg of air flowing, (a) the *net* power developed, (b) the *net* exergy increase of the air passing through the heat exchanger, $(e_{f3} - e_{f2})$, and (c) a full exergy accounting based on the exergy supplied to the plant found in part (b). Comment.

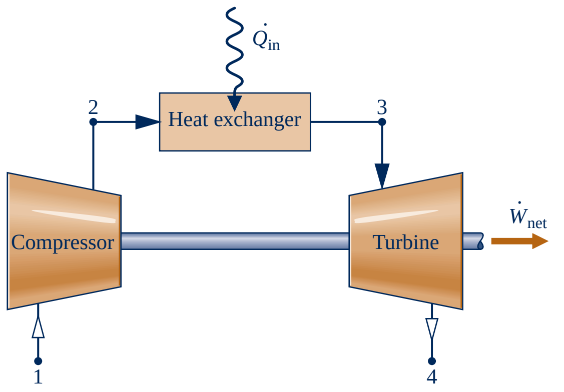


Fig. P7.91

State	p(kPa)	T(K)	h(kJ/kg)	$s°$(kJ/kg K)[a]
1	100	290	290.16	1.6680
2	500	505	508.17	2.2297
3	500	875	904.99	2.8170
4	100	635	643.93	2.4688

[a]$s°$ is the variable appearing in Eq. 6.20a and Table A-22.

7.92 Carbon dioxide (CO_2) gas enters a turbine operating at steady state at 50 bar, 500 K with a velocity of 50 m/s. The inlet area is 0.02 m^2. At the exit, the pressure is 20 bar, the temperature is 440 K, and the velocity is 10 m/s. The power developed by the turbine is 3 MW, and heat transfer occurs across a portion of the surface where the average temperature is 462 K. Assume ideal gas behavior for the carbon dioxide and neglect the effect of gravity. Let $T_0 = 298$ K, $p_0 = 1$ bar.

(a) Determine the rate of heat transfer, in kW.
(b) Perform a full exergy accounting, in kW, based on the *net* rate exergy is carried into the turbine by the carbon dioxide.

7.93 For the air compressor of Problem 7.66, perform a full exergy accounting, in kJ per kg of air flowing, based on the work input.

7.94 A compressor fitted with a water jacket and operating at steady state takes in air with a volumetric flow rate of 900 m^3/h at 22°C, 0.95 bar and discharges air at 317°C, 8 bar. Cooling water enters the water jacket at 20°C, 100 kPa with a mass flow rate of 1400 kg/h and exits at 30°C and essentially the same pressure. There is no significant heat transfer from the outer surface of the water jacket to its surroundings, and the effects of motion and gravity can be ignored. For the water-jacketed compressor, perform a full exergy accounting of the power input. Let $T_0 = 20$°C, $p_0 = 1$ atm.

7.95 For the compressor and heat exchanger of Problem 6.114, perform a full exergy accounting, in kW, based on the compressor power input. Let $T_0 = 300$ K, $p_0 = 96$ kPa.

7.96 Figure P7.96 shows liquid water at 80 lbf/in.^{2}, 300°F entering a flash chamber though a valve at the rate of 22 lb/s. At the valve exit, the pressure is 42 lbf/in.^{2} Saturated liquid at 40 lbf/in.^{2} exits from the bottom of the flash chamber and saturated vapor at 40 lbf/in.^{2} exits from near the top. The vapor stream is fed to a steam turbine having an isentropic efficiency of 90% and an exit pressure of 2 lbf/in.^{2} For steady-state operation, negligible heat transfer with the surroundings, and no significant effects of motion and gravity, perform a full exergy accounting, in Btu/s, of the *net* rate at which exergy is supplied: $(\dot{E}_{f1} - \dot{E}_{f3} - \dot{E}_{f5})$. Let $T_0 = 500$°R, $p_0 = 1$ atm.

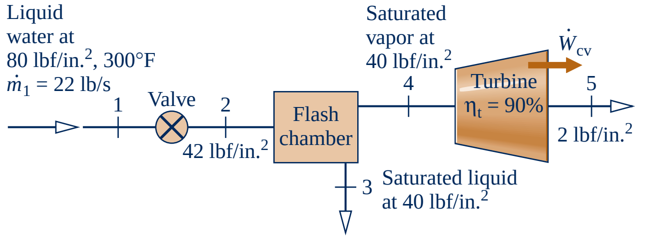


Fig. P7.96

7.97 Figure P7.97 provides steady-state operating data for a throttling valve in parallel with a steam turbine having an isentropic turbine efficiency of 90%. The streams exiting the valve and the turbine mix in a mixing chamber. Heat transfer with the surroundings and the effects of motion and gravity can be neglected. Determine

(a) the power developed by the turbine, in Btu/s.
(b) the mass flow rates through the turbine and valve, each in lb/s.
(c) a full exergy accounting, in Btu/s, of the *net* rate at which exergy is supplied: $(\dot{E}_{f1} - \dot{E}_{f4})$.

Let $T_0 = 500$°R, $p_0 = 1$ atm.

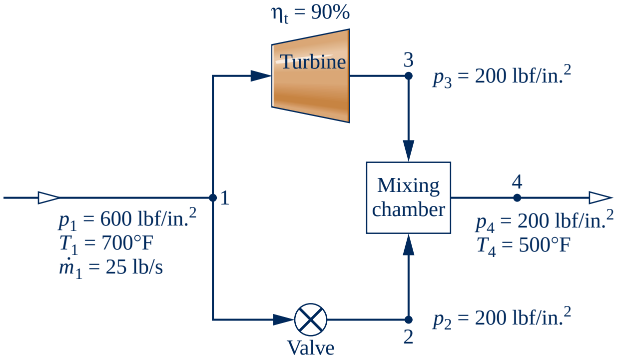


Fig. P7.97

Using Exergetic Efficiencies

7.98 For the water heater of Problem 7.45, devise and evaluate an exergetic efficiency.

7.99 For the compressor of Example 6.14, evaluate the exergetic efficiency given by Eq. 7.25. Let $T_0 = 20$°C, $p_0 = 1$ atm.

7.100 For the heat exchanger of Example 7.6, evaluate the exergetic efficiency given by Eq. 7.27 with states numbered for the case at hand.

7.101 Referring to the discussion of Sec. 7.6.2 as required, evaluate the exergetic efficiency for each of the following cases, assuming steady-state operation with negligible effects of heat transfer with the surroundings:

(a) Turbine: $\dot{W}_{cv} = 1200$ hp, $e_{f1} = 250$ Btu//lb, $e_{f2} = 15$ Btu/lb, $\dot{m} = 240$ lb/min.
(b) Compressor: $\dot{W}_{cv}/\dot{m} = -105$ kJ/kg, $e_{f1} = 5$ kJ/kg, $e_{f2} = 90$ kJ/kg, $\dot{m} = 2$ kg/s.
(c) Counterflow heat exchanger: $\dot{m}_h = 3$ kg/s, $\dot{m}_c = 10$ kg/s, $e_{f1} = 2100$ kJ/kg, $e_{f2} = 300$ kJ/kg, $\dot{E}_d = 3.4$ MW.
(d) Direct contact heat exchanger: $\dot{m}_1 = 10$ lb/s, $\dot{m}_3 = 15$ lb/s, $e_{f1} = 1000$ Btu/lb, $e_{f2} = 50$ Btu/lb, $e_{f3} = 400$ Btu/lb.

7.102 Plot the exergetic efficiency given by Eq. 7.21b versus T_u/T_0 for $T_s/T_0 = 8.0$ and $\eta = 0.4, 0.6, 0.8, 1.0$. What can be learned from the plot when T_u/T_0 is fixed? When ε is fixed? Discuss.

7.103 From an input of electricity, an electric resistance furnace operating at steady state delivers energy by heat transfer to a process at the rate $\dot{Q}_u$ at a *use* temperature T_u. There are no other significant energy transfers.

(a) Devise an exergetic efficiency for the furnace.
(b) Plot the efficiency obtained in part (a) versus the use temperature ranging from 300 to 900 K. Let $T_0 = 20°C$.

7.104 Figure P7.104 provides two options for generating hot water at steady state. In (a), water heating is achieved by utilizing *industrial waste heat* supplied at a temperature of 500 K. In (b), water heating is achieved by an electrical resistor. For each case, devise and evaluate an exergetic efficiency. Compare the calculated efficiency values and comment. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 20°C$, $p_0 = 1$ bar.

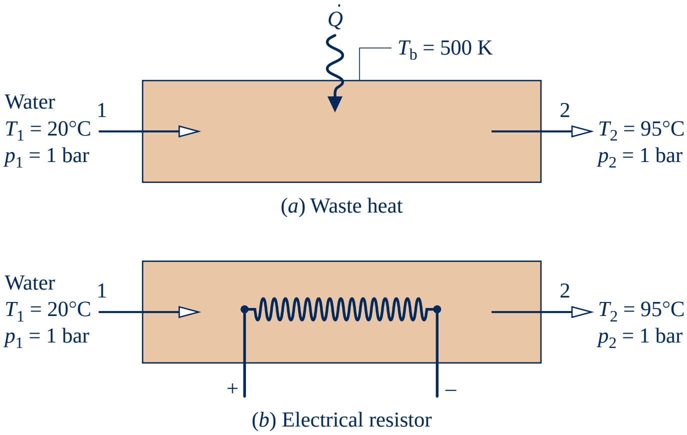


Fig. P7.104

7.105 Steam enters a turbine operating at steady state at $p_1 = 12$ MPa, $T_1 = 700°C$ and exits at $p_2 = 0.6$ MPa. The isentropic turbine efficiency is 88%. Property data are provided in the accompanying table. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 300$ K, $p_0 = 100$ kPa. Determine (a) the power developed and the rate of exergy destruction, each in kJ per kg of steam flowing, and (b) the exergetic turbine efficiency.

State	p(MPa)	T(°C)	h(kJ/kg)	s(kJ/kg · K)
Turbine inlet	12	700	3858.4	7.0749
Turbine exit	0.6	(η_t = 88%)	3017.5	7.2938

7.106 Saturated liquid water at 0.01 MPa enters a power plant pump operating at a steady state. Liquid water exits the pump at 10 MPa. The isentropic pump efficiency is 90%. Property data are provided in the accompanying table. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 300$ K, $p_0 = 100$ kPa. Determine (a) the power required by the pump and the rate of exergy destruction, each in kJ per kg of water flowing, and (b) the exergetic pump efficiency.

State	p(MPa)	h(kJ/kg)	s(kJ/kg · K)
Pump inlet	0.01	191.8	0.6493
Pump exit	10	204.5	0.6531

7.107 At steady state, an insulated steam turbine develops work at a rate of 389.1 Btu per lb of steam flowing through the turbine. Steam enters at 1200 psia and 1100°F and exits at 14.7 psia. Evaluate the isentropic turbine efficiency and the exergetic turbine efficiency. Ignore the effects of motion and gravity. Let $T_0 = 70°F$, $p_0 = 14.7$ psia.

7.108 Nitrogen (N_2) at 25 bar, 450 K enters a turbine and expands to 2 bar, 250 K with a mass flow rate of 0.2 kg/s. The turbine operates at steady state with negligible heat transfer with its surroundings. Assuming the ideal gas model with $k = 1.399$ and ignoring the effects of motion and gravity, determine

(a) the isentropic turbine efficiency.
(b) the exergetic turbine efficiency.

Let $T_0 = 25°C$, $p_0 = 1$ atm.

7.109 Steam enters a turbine operating at steady state at 5 MPa, 600°C and exits at 50 kPa. Stray heat transfer and the effects of motion and gravity can be ignored. If the rate of exergy destruction is 95.4 kJ per kg of steam flowing, determine (a) the isentropic turbine efficiency and (b) the exergetic turbine efficiency. Let $T_0 = 293$ K, $p_0 = 1$ bar.

7.110 Air enters an insulated turbine operating at steady state with a pressure of 5 bar, a temperature of 500 K, and a volumetric flow rate of 3 m^3/s. At the exit, the pressure is 1 bar. The isentropic turbine efficiency is 76.7%. Assuming the ideal gas model and ignoring the effects of motion and gravity, determine

(a) the power developed and the exergy destruction rate, each in kW.
(b) the exergetic turbine efficiency.

Let $T_0 = 20°C$, $p_0 = 1$ bar.

7.111 Steam at 200 lbf/in.^{2}, 660°F enters a turbine operating at steady state with a mass flow rate of 16.5 lb/min and exits at 14.7 lbf/in.^{2}, 238°F. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 537°R$, $p_0 = 14.7$ lbf/in.^{2} Determine for the turbine (a) the power developed and the rate of exergy destruction, each in Btu/min, and (b) the isentropic and exergetic turbine efficiencies.

7.112 Water vapor at 6 MPa, 600°C enters a turbine operating at steady state and expands adiabatically to 10 kPa. The mass flow rate is 2 kg/s and the isentropic turbine efficiency is 94.7%. Kinetic and potential energy effects are negligible. Determine

(a) the power developed by the turbine, in kW.
(b) the rate at which exergy is destroyed within the turbine, in kW.
(c) the exergetic turbine efficiency.

Let T_0 = 298 K, p_0 = 1 atm.

7.113 Figure P7.113 shows a turbine operating at steady state with steam entering at p_1 = 30 bar, T_1 = 350°C and a mass flow rate of 30 kg/s. Process steam is extracted at p_2 = 5 bar, T_2 = 200°C. The remaining steam exits at p_3 = 0.15 bar, x_3 = 90%, and a mass flow rate of 25 kg/s. Stray heat transfer and the effects of motion and gravity are negligible. Let T_0 = 25°C, p_0 = 1 bar. The accompanying table provides property data at key states. For the turbine, determine the power developed and rate of exergy destruction, each in MW. Also devise and evaluate an exergetic efficiency for the turbine.

State	p(bar)	T(°C)	h(kJ/kg)	s(kJ/kg · K)
1	30	350	3115.3	6.7428
2	5	200	2855.4	7.0592
3	0.15	(x = 90%)	2361.7	7.2831

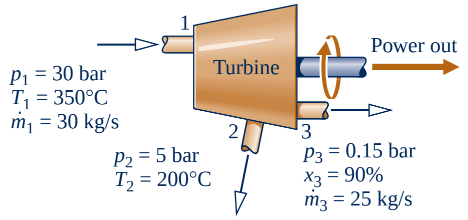


Fig. P7.113

7.114 For the turbine and heat exchanger arrangement of Problem 6.116, evaluate an exergetic efficiency for (a) each turbine, (b) the heat exchanger, and (c) an overall control volume enclosing the turbines and heat exchanger. Comment. Let T_0 = 300 K, p_0 = 1 bar.

7.115 Figure P7.115 and the accompanying table provide steady-state operating data for a two-stage steam turbine. Stray heat transfer and the effects of motion and gravity are negligible. For each turbine stage, determine the work developed, in kJ per kg of steam flowing, and the exergetic turbine efficiency. For the overall two-stage turbine, devise and evaluate an exergetic efficiency. Let T_0 = 298 K, p_0 = 1 atm.

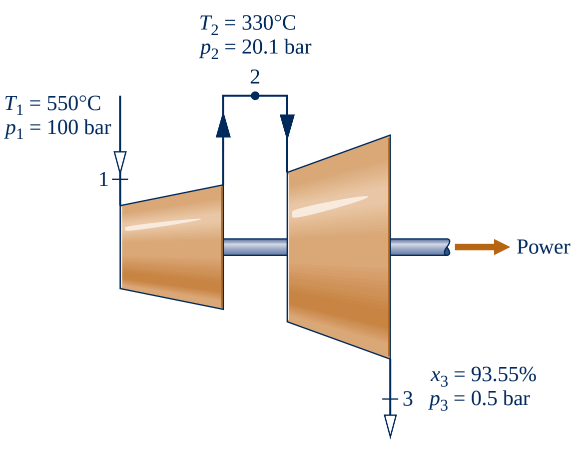


Fig. P7.115

State	T(°C)	p(bar)	h(kJ/kg)	s(kJ/kg · K)
1	550	100	3500	6.755
2	330	20.1	3090	6.878
3	(x = 93.55%)	0.5	2497	7.174

7.116 Steam at 400 lbf/in.², 600°F enters a well-insulated turbine operating at steady state and exits as saturated vapor at a pressure p.

(a) For p = 50 lbf/in.², determine the exergy destruction rate, in Btu per lb of steam expanding through the turbine, and the turbine exergetic and isentropic efficiencies.
(b) Plot the exergy destruction rate, in Btu per lb of steam flowing, and the exergetic efficiency and isentropic efficiency, each versus pressure p ranging from 1 to 50 lbf/in.²

Ignore the effects of motion and gravity and let T_0 = 60°F, p_0 = 1 atm.

7.117 Saturated water vapor at 400 lbf/in.² enters an insulated turbine operating at steady state. A two-phase liquid–vapor mixture exits at 0.6 lbf/in.² Plot each of the following versus the steam quality at the turbine exit ranging from 75 to 100%:

(a) the power developed and the rate of exergy destruction, each in Btu per lb of steam flowing.
(b) the isentropic turbine efficiency.
(c) the exergetic turbine efficiency.

Let T_0 = 60°F, p_0 = 1 atm. Ignore the effects of motion and gravity.

7.118 Argon enters an insulated turbine operating at steady state at 1000°C and 2 MPa and exhausts at 350 kPa. The mass flow rate is 0.5 kg/s. Plot each of the following versus the turbine exit temperature, in °C:

(a) the power developed, in kW.
(b) the rate of exergy destruction in the turbine, in kW.
(c) the exergetic turbine efficiency.

For argon, use the ideal gas model with k = 1.67. Ignore the effects of motion and gravity. Let T_0 = 20°C, p_0 = 1 bar.

7.119 An insulated steam turbine at steady state can be operated at part-load conditions by throttling the steam to a lower pressure before it enters the turbine. Before throttling, the steam is at 200 lbf/in.², 600°F. After throttling, the pressure is 150 lbf/in.² At the turbine exit, the steam is at 1 lbf/in.² and a quality x. For the turbine, plot the rate of exergy destruction, in kJ per kg of steam flowing, and exergetic efficiency, each versus x ranging from 90 to 100%. Ignore the effects of motion and gravity and let T_0 = 60°F, p_0 = 1 atm.

7.120 A pump operating at steady state takes in saturated liquid water at 65 lbf/in.² at a rate of 10 lb/s and discharges water at 1000 lbf/in.² The isentropic pump efficiency is 80.22%. Heat transfer with the surroundings and the effects of motion and gravity can be neglected. If T_0 = 75°F, determine for the pump

(a) the exergy destruction rate, in Btu/s.
(b) the exergetic efficiency.

7.121 Refrigerant 134a as saturated vapor at $-10°C$ enters a compressor operating at steady state with a mass flow rate of 0.3 kg/s. At the compressor exit the pressure of the refrigerant is 5 bar. Stray heat transfer and the effects of motion and gravity can be ignored. If the rate of exergy destruction within the compressor must be kept less than 2.4 kW, determine the allowed ranges for (a) the power required by the compressor, in kW, and (b) the exergetic compressor efficiency. Let $T_0 = 298$ K, $p_0 = 1$ bar.

7.122 For the turbine-compressor arrangement of Problem 6.164, determine the exergetic efficiency for (a) the turbine, (b) the compressor, (c) an overall control volume enclosing the turbine and compressor. Let $T_0 = 300$ K.

7.123 Figure P7.123 shows an insulated counterflow heat exchanger with carbon dioxide (CO_2) and air flowing through the inner and outer channels, respectively. The figure provides data for operation at steady state. The heat exchanger is a component of an overall system operating in an arctic region where the average annual ambient temperature is 20°F. Heat transfer between the heat exchanger and its surroundings can be ignored, as can effects of motion and gravity. Evaluate for the heat exchanger

(a) the rate of exergy destruction, in Btu/s.
(b) the exergetic efficiency given by Eq. 7.27.

Let $T_0 = 20°F$, $p_0 = 1$ atm.

7.124 A counterflow heat exchanger operating at steady state has oil and liquid water flowing in separate streams. The oil is cooled from 700 to 580°R, while the water temperature increases from 530 to 560°R. Neither stream experiences a significant pressure change. The mass flow rate of the water is 3 lb/s. The oil and water can be regarded as incompressible with constant specific heats of 0.51 and 1.00 Btu/lb · °R, respectively. Heat transfer between the heat exchanger and its surroundings can be ignored, as can the effects of motion and gravity. Determine

(a) the mass flow rate of the oil, in lb/s.
(b) the exergetic efficiency given by Eq. 7.27.
(c) the hourly cost of exergy destruction if exergy is valued at 8.5 cents per kW · h.

Let $T_0 = 50°F$, $p_0 = 1$ atm.

7.125 In the boiler of a power plant are tubes through which water flows as it is brought from 0.6 MPa, 130°C to 200°C at essentially constant pressure. Combustion gases with a mass flow rate of 400 kg/s pass over the tubes and cool from 827°C to 327°C at essentially constant pressure. The combustion gases can be modeled as air behaving as an ideal gas. There is no significant heat transfer from the boiler to its surroundings. Assuming steady state and neglecting the effects of motion and gravity, determine

(a) the mass flow rate of the water, in kg/s.
(b) the rate of exergy destruction, in kJ/s.
(c) the exergetic efficiency given by Eq. 7.27.

Let $T_0 = 25°C$, $p_0 = 1$ atm.

7.126 In the boiler of a power plant are tubes through which water flows as it is brought from a saturated liquid condition at 1000 lbf/in.^{2} to 1300°F at essentially constant pressure. Combustion gases passing over the tubes cool from 1740°F to temperature T at essentially constant pressure. The mass flow rates of the water and combustion gases are 400 and 2995.1 lb/s, respectively. The combustion gases can be modeled as air behaving as an ideal gas. There is no significant heat transfer from the boiler to its surroundings. Assuming steady state and neglecting the effects of motion and gravity, determine

(a) the exit temperature T of the combustion gases, in °F.
(b) the exergy destruction rate, in Btu/s.
(c) the exergetic efficiency given by Eq. 7.27.

Let $T_0 = 50°F$, $p_0 = 1$ atm.

7.127 Refrigerant 134a enters a counterflow heat exchanger operating at steady state at $-20°C$ and a quality of 35% and exits as saturated vapor at $-20°C$. Air enters as a separate stream with a mass flow rate of 4 kg/s and is cooled at a constant pressure of 1 bar from 300 to 260 K. Heat transfer between the heat exchanger and its surroundings can be ignored, as can the effects of motion and gravity.

(a) As in Fig. E7.6, sketch the variation with position of the temperature of each stream. Locate T_0 on the sketch.
(b) Determine the rate of exergy destruction within the heat exchanger, in kW.
(c) Devise and evaluate an exergetic efficiency for the heat exchanger.

Let $T_0 = 300$ K, $p_0 = 1$ bar.

7.128 Saturated water vapor at 1 bar enters a direct-contact heat exchanger operating at steady state and mixes with a stream of liquid water entering at 25°C, 1 bar. A two-phase liquid–vapor mixture exits at 1 bar. The entering streams have equal mass flow rates. Neglecting heat transfer with the surroundings and the effects of motion and gravity, determine for the heat exchanger

(a) the rate of exergy destruction, in kJ per kg of mixture exiting.
(b) the exergetic efficiency given by Eq. 7.29.

Let $T_0 = 20°C$, $p_0 = 1$ bar.

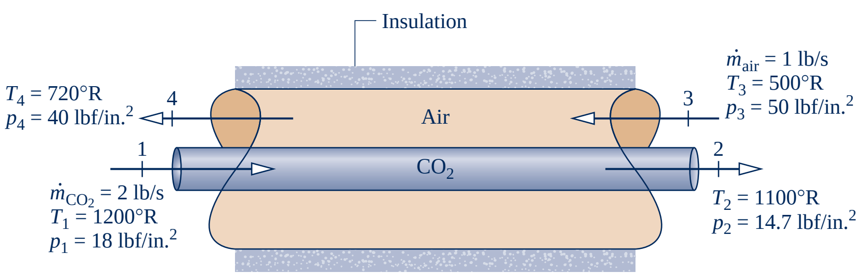


Fig. P7.123

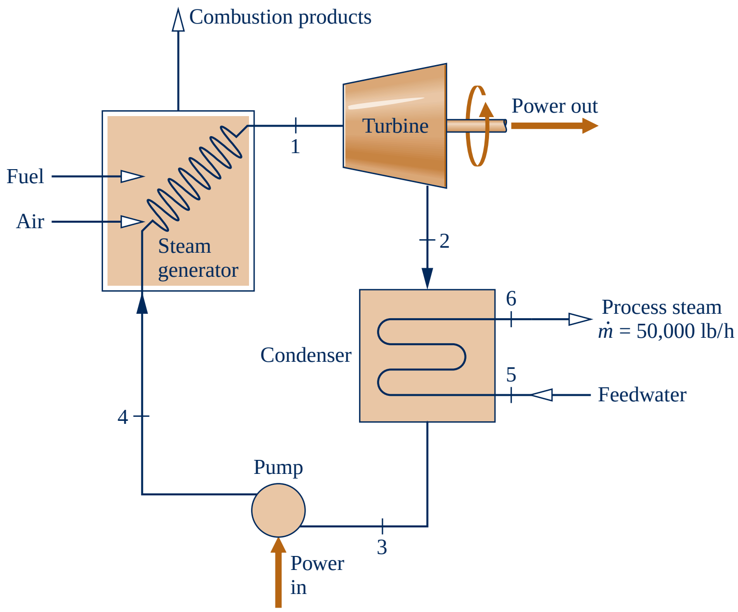


Fig. P7.131

7.129 Figure P7.129 and the accompanying table provide steady-state operating data for a direct-contact heat exchanger fitted with a valve. Water is the substance. The mass flow rate of the exiting stream is 20 lb/s. Stray heat transfer and the effects of motion and gravity are negligible. For an overall control volume, (a) evaluate the rate of exergy destruction, in Btu/s, and (b) devise and evaluate an exergetic efficiency. Let T_0 = 60°F, p_0 = 14.7 lbf/in.^{2}

State	T(°F)	p(lbf/in.^{2})	h(Btu/lb)	s(Btu/lb · R)
1	60	14.7	28.1	0.0556
2	500	20.0	1286.8	1.8919
3	320	14.7	1202.1	1.8274

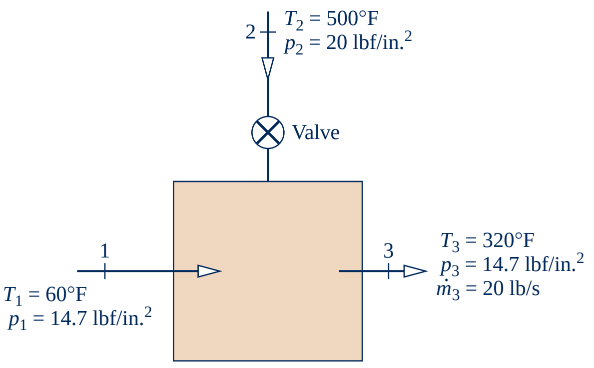


Fig. P7.129

7.130 For the compressed-air energy storage system of Problem 4.114, determine the amount of exergy destruction associated with filling the cavern, in GJ. Devise and evaluate the accompanying exergetic efficiency. Comment. Let T_0 = 290 K, p_0 = 1 bar.

7.131 Figure P7.131 and the accompanying table provide steady-state operating data for a *cogeneration system* that produces power and 50,000 lb/h of process steam. Stray heat transfer and the effects of motion and gravity are negligible. The isentropic pump efficiency is 100%. Determine

(a) the net power developed, in Btu/h.
(b) the net exergy increase of the water passing through the steam generator, $\dot{m}(e_{f1} - e_{f4})$, in Btu/h.
(c) a full exergy accounting based on the net exergy supplied to the system found in part (b).
(d) Using the result of part (c), devise and evaluate an exergetic efficiency for the overall cogeneration system. Comment.

Let T_0 = 70°F, p_0 = 1 atm.

State	T(°F)	p(lbf/in.^{2})	h(Btu/lb)	s(Btu/lb · R)
1	700	800	1338	1.5471
2	—	180	1221	1.5818
3	(x_3 = 0%)	180	346	0.5329
4	—	800	348	0.5329
5	250	140	219	0.3677
6	(x_6 = 100%)	140	1194	1.5761

7.132 Figure P7.132 shows a *cogeneration* system producing two useful products: net power and process steam. The accompanying table provides steady-state mass flow rate, temperature, pressure, and flow exergy data at the ten numbered states on the figure. Stray heat transfer and the effects of motion and gravity can be ignored. Let T_0 = 298.15 K, p_0 = 1.013 bar. Determine, in MW,

(a) the *net* rate exergy is carried out with the process steam, $(\dot{E}_{f9} - \dot{E}_{f8})$.
(b) the *net* rate exergy is carried out with the combustion products, $(\dot{E}_{f7} - \dot{E}_{f1})$.
(c) the rates of exergy destruction in the air preheater, heat-recovery steam generator, and the combustion chamber.

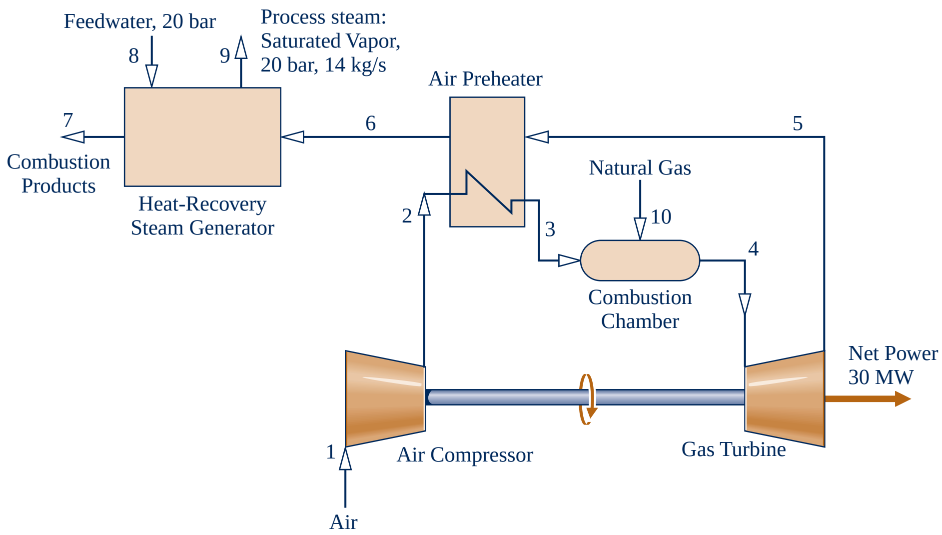


Fig. P7.132

Devise and evaluate an exergetic efficiency for the overall cogeneration system.

State	Substance	Mass Flow Rate (kg/s)	Temperature (K)	Pressure (bar)	Flow Exergy Rate, $\dot{E}_f$(MW)
1	Air	91.28	298.15	1.013	0.00
2	Air	91.28	603.74	10.130	27.54
3	Air	91.28	850.00	9.623	41.94
4	Combustion products	92.92	1520.00	9.142	101.45
5	Combustion products	92.92	1006.16	1.099	38.78
6	Combustion products	92.92	779.78	1.066	21.75
7	Combustion products	92.92	426.90	1.013	2.77
8	Water	14.00	298.15	20.000	0.06
9	Water	14.00	485.57	20.000	12.81
10	Methane	1.64	298.15	12.000	84.99

7.133 Figure P7.133 shows a *combined* gas turbine-vapor power plant operating at steady state. The gas turbine is numbered 1–5. The vapor power plant is numbered 6–9. The accompanying table gives data at these numbered states. The total *net* power output is 45 MW and the mass flow rate of the water flowing through the vapor power plant is 15.6 kg/s. Air flows through the gas turbine power plant, and the ideal gas model applies to the air. Stray heat transfer and the effects of motion and gravity can be ignored. Let T_0 = 300 K, p_0 = 100 kPa. Determine

(a) the mass flow rate of the air flowing through the gas turbine, in kg/s.
(b) the *net* rate exergy is carried out with the exhaust air stream, $(\dot{E}_{f5} - \dot{E}_{f1})$ in MW.
(c) the rate of exergy destruction in the compressor and pump, each in MW.
(d) the *net* rate of exergy increase of the air flowing through the combustor, $(\dot{E}_{f3} - \dot{E}_{f2})$, in MW.

Devise and evaluate an exergetic efficiency for the overall combined power plant.

Gas Turbine			Vapor Cycle		
State	h(kJ/kg)	$s°$(kJ/kg · K)[a]	State	h(kJ/kg)	s(kJ/kg · K)
1	300.19	1.7020	6	183.96	0.5975
2	669.79	2.5088	7	3138.30	6.3634
3	1515.42	3.3620	8	2104.74	6.7282
4	858.02	2.7620	9	173.88	0.5926
5	400.98	1.9919			

[a] $s°$ is the variable appearing in Eq. 6.20a and Table A-22.

Considering Thermoeconomics

7.134 A high-pressure (HP) boiler and a low-pressure (LP) boiler will be added to a plant's steam-generating system. Both boilers use the same fuel and at steady state have approximately the same rate of energy loss by heat transfer. The average temperature of the combustion gases is less in the LP boiler than in the HP boiler. In comparison to the LP boiler, might you spend more, the same, or less to insulate the HP boiler? Explain.

7.135 Reconsider Example 7.10 for a turbine exit state fixed by p_2 = 2 bar, h_2 = 2723.7 kJ/kg, s_2 = 7.1699 kJ/kg · K. The cost of owning and operating the turbine is $\dot{Z}_t = 7.2\,\dot{W}_e$, in \$/h, where $\dot{W}_e$ is in MW. All other data remain unchanged. Determine

(a) the power developed by the turbine, in MW.
(b) the exergy destroyed within the turbine, in MW.
(c) the exergetic turbine efficiency.
(d) the unit cost of the turbine power, in cents per kW · h of exergy.

7.136 At steady state, a turbine with an exergetic efficiency of 90% develops 7 × 10^7 kW · h of work annually (8000 operating hours). The annual cost of owning and operating the turbine is \$2.5 × 10^5. The steam entering the turbine has a specific flow exergy of 559 Btu/lb, a mass flow rate of 12.55 × 10^4 lb/h, and is valued at \$0.0165 per kW · h of exergy.

(a) Using Eq. 7.34c, evaluate the unit cost of the power developed, in \$ per kW · h.

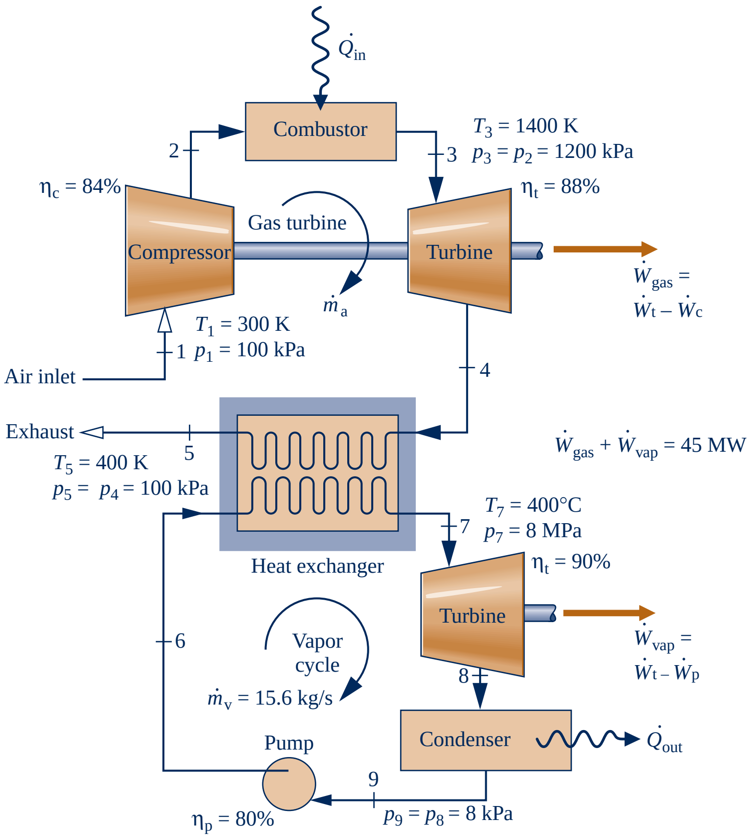


Fig. P7.133

(b) Evaluate the unit cost based on exergy of the steam entering and exiting the turbine, each in cents per lb of steam flowing through the turbine.

7.137 Figure P7.137 shows a boiler at steady state. Steam having a specific flow exergy of 1300 kJ/kg exits the boiler at a mass flow rate of 5.69×10^4 kg/h. The cost of owning and operating the boiler is \$91/h. The ratio of the exiting steam exergy to the entering fuel exergy is 0.45. The unit cost of the fuel based on exergy is \$1.50 per 10^6 kJ. If the cost rates of the combustion air, feedwater, heat transfer with the surroundings, and exiting combustion products are ignored, develop

(a) an expression in terms of exergetic efficiency and other pertinent quantities for the unit cost based on exergy of the steam exiting the boiler.

(b) Using the result of part (a), determine the unit cost of the steam, in cents per kg of steam flowing.

7.138 Consider an *overall* control volume comprising the boiler and steam turbine of the cogeneration system of Example 7.10. Assuming the power and process steam each have the same unit cost based on exergy: $c_e = c_2$, evaluate the unit cost, in cents per kW · h. Compare with the respective values obtained in Example 7.10 and comment.

7.139 A cogeneration system operating at steady state is shown schematically in Fig. P7.139. The exergy transfer rates of the entering and exiting streams are shown on the figure, in MW. The fuel, produced by reacting coal with steam, has a unit cost of 5.85 cents per kW · h of exergy. The cost of owning and operating the system is \$1800/h. The feedwater and combustion air enter with negligible exergy and cost. Expenses related to proper disposal of the combustion products are included with the cost of owning and operating the system.

(a) Determine the rate of exergy destruction within the cogeneration system, in MW.

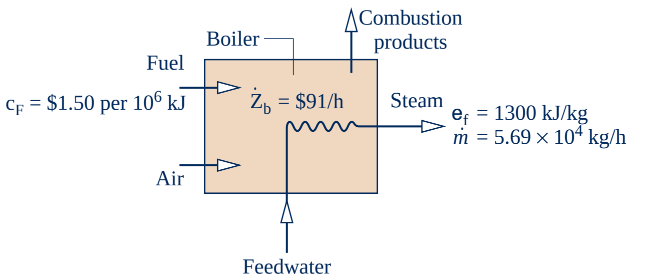


Fig. P7.137

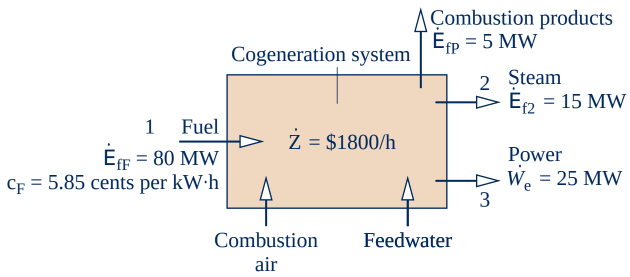


Fig. P7.139

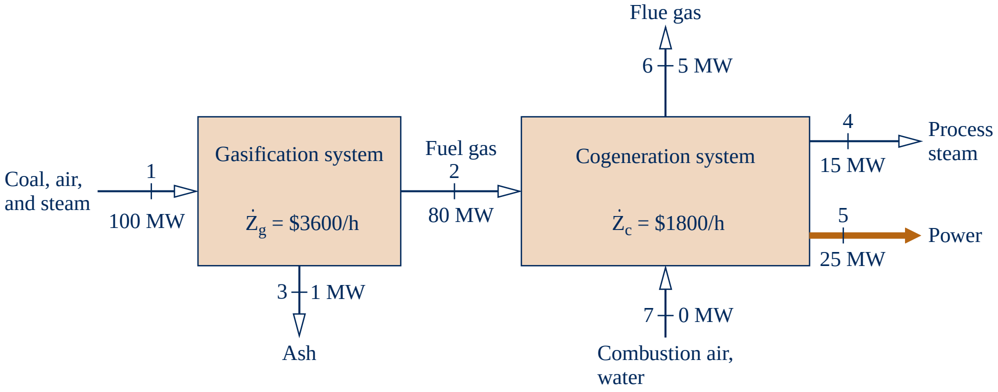


Fig. P7.140

(b) Devise and evaluate an exergetic efficiency for the system.
(c) Assuming the power and steam each have the same unit cost based on exergy, evaluate the unit cost, in cents per kW · h. Also evaluate the cost rates of the power and steam, each in \$/h.

7.140 Figure P7.140 provides steady-state operating data for a coal gasification system fueling a cogeneration system that produces power and process steam. The numbers given for each of the seven streams, in MW, represent rates of exergy flow. The unit cost based on exergy of stream 1 is $c_1 = 1.08$ cents per kW · h. On the advice of a *cost engineer*, the unit costs based on exergy of the process steam (stream 4) and power (stream 5) are assumed to be equal, and no cost is associated with combustion air and water (stream 7). The costs of owning and operating the gasification and cogeneration systems are \$3600/h and \$1800/h, respectively. These figures include expenses related to discharge of ash (stream 3) and flue gas (stream 6) to the surroundings. Determine the

(a) rate of exergy destruction, in MW, for each system.
(b) exergetic efficiency for each system and for an overall system formed by the two systems.
(c) unit cost based on exergy, in cents/kW · h, for each of streams 2, 4, and 5.
(d) cost rate, in \$/h, associated with each of streams 1, 2, 4 and 5.

7.141 Figure P7.141 provides steady-state operating data for an air compressor–intercooler system. The numbers given for each of the six streams, in MW, represent rates of exergy flow. The unit cost of the power input is $c_2 = 3.6$ cents per kW · h. On the advice of a *cost engineer*, the unit costs based on exergy of the compressed air (stream 3) and cooled compressed air (stream 4) are assumed to be equal, and no costs are associated with the incoming air (stream 1) and feedwater (stream 5). The costs of owning and operating the air compressor and intercooler are \$36/h and \$72/h, respectively. Determine the

(a) rate of exergy destruction for the air compressor and intercooler, each in MW.
(b) exergetic efficiency for the air compressor, the intercooler, and an overall system formed from the two components.
(c) unit cost based on exergy, in cents/kW · h, for each of streams 3, 4, and 6.
(d) cost rate, in \$/h, associated with each of streams 1, 3, 4, and 6, and comment.

7.142 Repeat parts (c) and (d) of Problem 7.141 as follows: On the advice of a *cost engineer*, assume $c_4 = c_6$. That is, the unit cost based on exergy of the cooled-compressed air is the same as the unit cost based on exergy of the heated feedwater.

Reviewing Concepts

7.143 Answer the following true or false. Explain.

(a) The *well-to-wheel* efficiency compares different options for generating electricity used in industry, business, and the home.
(b) *Exergy accounting* allows the location, type, and true magnitudes of inefficiency and loss to be identified and quantified.

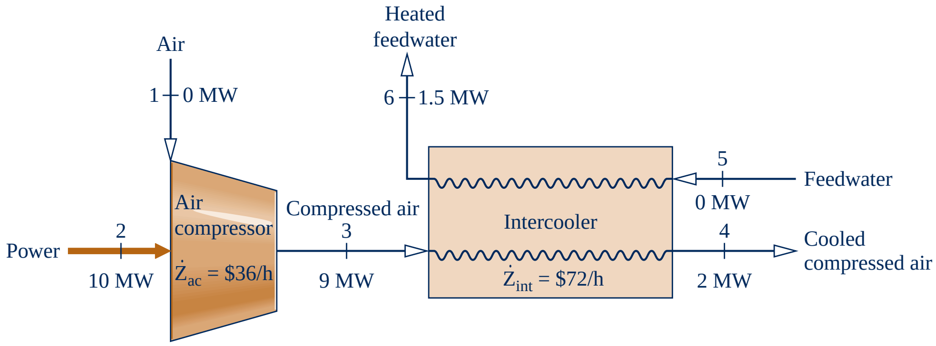


Fig. P7.141

(c) Like entropy, exergy is produced by action of irreversibilities.
(d) At every state, exergy cannot be negative; yet exergy *change* between two states can be positive, negative, or zero.
(e) To define exergy, we think of two systems: a system of interest and an *exergy reference environment*.
(f) The specific *flow exergy* cannot be negative.

7.144 Answer the following true or false. Explain.

(a) In a *throttling* process, energy *and* exergy are conserved.
(b) If unit costs are based on exergy, we expect the unit cost of the electricity generated by a turbine to be greater than the unit cost of the high-pressure steam provided to the turbine.
(c) When a closed system is at the *dead state*, it is in thermal and mechanical equilibrium with the exergy reference environment, and the values of the system's energy and thermomechanical exergy are each zero.
(d) The thermomechanical exergy at a state of a system can be thought of as the magnitude of the minimum theoretical work required to bring the system from the dead state to the given state.
(e) The exergy transfer accompanying heat transfer occurring at 1000 K is greater than the exergy transfer accompanying an equivalent heat transfer occurring at $T_0 = 300$ K.
(f) When products of combustion are at a temperature significantly greater than required by a specified task, we say the task is well matched to the fuel source.

7.145 Answer the following true or false. Explain.

(a) Exergy is a measure of the departure of the state of a system from that of the *exergy reference environment*.
(b) The energy of an isolated system must remain constant, but its exergy can only increase.
(c) When a system is at T_0 and p_0, the value of its *thermomechanical* contribution to exergy is zero but its *chemical* contribution does not necessarily have a zero value.
(d) Mass, volume, energy, entropy, and exergy are all intensive properties.
(e) Exergy destruction is proportional to entropy production.
(f) Exergy can be transferred to, and from, closed systems accompanying heat transfer, work, and mass flow.

▶ DESIGN & OPEN-ENDED PROBLEMS: EXPLORING ENGINEERING PRACTICE

7.1D Ways to run cars on water are frequently touted on the Internet. For each of two different such proposals, write a three-page evaluation. In each evaluation, clearly state the claims made in the proposal. Then, using principles of thermodynamics, including exergy principles, discuss fully the merit of the claims. Conclude with a statement in which you agree, or disagree, that the proposal is both feasible *and* worthy of use by consumers. For each evaluation, provide at least three references.

7.2D Many appliances, including ovens, stoves, clothes dryers, and hot-water heaters, offer a choice between electric and gas operation. Select an appliance that offers this choice and perform a detailed comparison between the two options, including but not necessarily limited to a *life-cycle* exergy analysis and an economic analysis accounting for purchase, installation, operating, maintenance, and disposal costs. Present your finding in a poster presentation.

7.3D Buying a light bulb today involves choosing between three different product options including incandescent, compact fluorescent (CFL), and light-emitting diode (LED), as illustrated in Fig. P7.3D. Using a 100-W incandescent bulb and its lighting level in lumens as the baseline, compare the three types of bulbs on the basis of life span, lighting level, product cost, and environmental impact related to manufacturing and disposal. For an operational period of 20,000 hours, compare the costs for electricity and bulbs. Present your findings in an executive summary including a prediction about the type of bulb that will be used most in 2020.

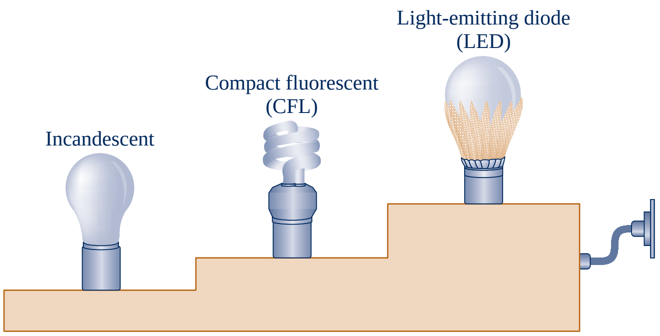


Fig. P7.3D

7.4D You have been invited to testify before a committee of your state legislature that is crafting regulations pertaining to the production of electricity using poultry waste as fuel. Develop a slide presentation providing a balanced assessment, including engineering, public health, and economic considerations.

7.5D Tankless microwave water heating systems have been introduced that not only quickly provide hot water but also significantly reduce the exergy destruction inherent in domestic water heating with conventional electrical and gas-fueled water heaters. For a 2500-ft^2 dwelling in your locale, investigate the feasibility of using a microwave water heating system. Include a detailed economic evaluation accounting for equipment, installation, and operating costs. Place your findings in a memorandum.

7.6D *Anaerobic digestion* is a proven means of producing methane from livestock waste. To provide for the space heating, water heating, and cooking needs of a typical farm dwelling in your locale, determine the size of the anaerobic digester and the number of waste-producing animals required. Select animals from poultry, swine, and cattle, as appropriate. Place your findings in a report, including an economic evaluation and at least three references.

7.7D Complete one of the following projects involving methods for *storing* electricity considered thus far in this book (see Secs. 2.7, 4.8.3). Report your findings in a report providing a full rationale together with supporting documentation.

(a) For each storage method, identify its principal sources of exergy destruction and develop an exergetic efficiency for it. Use principles of this chapter with assistance from the technical literature as required.
(b) From these storage methods, identify a subset of them well suited for storage duty associated with a 300-MW wind farm. On the basis of cost and other pertinent factors for such duty, place the subset in rank order.

7.8D The objective of this project is to design a low-cost, electric-powered, portable or wearable consumer product that meets a need you have identified. In performing every function, the electricity required must come fully from *human motion*. No electricity from batteries and/or wall sockets is allowed. Additionally, the product must not be intrusive or interfere with the normal activities of the user, alter his/her gait or range of motion, lead to possible physical disability, or induce accidents leading to injury. The product cannot resemble any existing product unless it has a valuable new feature, significantly reduces cost, or provides some other meaningful advantage. The final report will include schematics, circuit diagrams, a parts list, and a suggested retail cost based on comprehensive costing.

7.9D In the 1840s, British engineers developed *atmospheric railways* that featured a large-diameter tube located between the tracks and stretching the entire length of the railroad. Pistons attached by struts to the rail cars moved inside the tube. As shown in Fig. P7.9D, piston motion was achieved by maintaining a vacuum ahead of the piston while the atmosphere was allowed to act behind it. Although several such railways came into use, limitations of the technology then available eventually ended this mode of transportation. Investigate the feasibility of combining the atmospheric railway concept with today's technology to develop rail service for commuting within urban areas. Write a report, including at least three references.

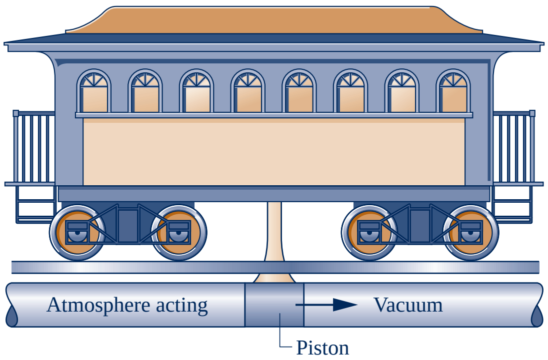


Fig. P7.9D

7.10D *Pinch analysis* (or *pinch technology*) is a popular methodology for optimizing the design of heat exchanger networks in complex thermal systems. Pinch analysis uses a primarily graphical approach to implement second-law reasoning. Write a paper, including at least three references, in which the role of pinch analysis within *thermoeconomics* is discussed.

Major ways for generating *electricity* are considered in the chapter introduction.
George Doyle/Getty Images, Inc.

ENGINEERING CONTEXT In the twenty-first century we will be challenged to responsibly provide for our growing power needs. The scope of the challenge and how we will address it are discussed in the introduction to power generation beginning on p. 426. You are encouraged to study this introduction before considering the several types of power-generating systems discussed in the present chapter and the next. In these chapters, we describe some of the practical arrangements employed for power production and illustrate how such power plants can be modeled thermodynamically. The discussion is organized into three main areas of application: vapor power plants, gas turbine power plants, and internal combustion engines. These power systems produce much of the electrical and mechanical power used worldwide. The **objective** of this chapter is to study *vapor* power plants in which the *working fluid* is alternately vaporized and condensed. Chapter 9 is concerned with gas turbines and internal combustion engines in which the working fluid remains a gas.

8

Vapor Power Systems

▶ LEARNING OUTCOMES

When you complete your study of this chapter, you will be able to...

- Demonstrate an understanding of the basic principles of vapor power plants.
- Develop and analyze thermodynamic models of vapor power plants based on the Rankine cycle and its modifications, including:
  - sketching schematic and accompanying *T–s* diagrams.
  - evaluating property data at principal states in the cycle.
  - applying mass, energy, and entropy balances for the basic processes.
  - determining power cycle performance, thermal efficiency, net power output, and mass flow rates.
- Explain the effects on Rankine cycle performance of varying key parameters.
- Discuss the principal sources of exergy destruction and loss in vapor power plants.

Introducing Power Generation

An exciting and urgent engineering challenge in the decades immediately ahead is to responsibly meet our national power needs. The challenge has its roots in the declining economically recoverable supplies of nonrenewable energy resources, effects of global climate change, and burgeoning population. In this introduction, we consider both conventional and emerging means for generating power. The present discussion also serves to introduce Chaps 8 and 9, which detail vapor and gas power systems, respectively.

Today

TABLE 8.1

Current U.S. Electricity Generation by Source

Coal	48.5%
Natural gas	21.6%
Nuclear	19.4%
Hydroelectric	5.8%
Other renewables[a]	2.5%
Oil	1.6%
Others	0.6%

[a]Wind, solar, geothermal, others
Source: Summary Statistics for the United States, Energy Information Administration, 2009, http://www.eia.doe.gov/cneaf/electricity/epa/epates.html.

An important feature of a prudent national energy posture is a wide range of sources for the power generation mix, thus avoiding vulnerabilities that can accompany over-reliance on too few energy sources. This feature is seen in Table 8.1, which gives a snapshot of the sources for nearly all U.S. electricity today. The table shows heavy dependance on coal for electricity generation. Natural gas and nuclear are also significant sources. All three are nonrenewable.

The United States has abundant coal reserves and a rail system allowing for smooth distribution of coal to electricity producers. This good news is tempered by significant human-health and environmental-impact issues associated with coal (see Energy & Environment, Sec. 8.2.1). Coal use in power generation is discussed further in Secs. 8.3 and 8.5.3.

Use of natural gas has been growing in the United States because it is competitive cost-wise with coal and has fewer adverse environmental effects related to combustion. Natural gas not only provides for home heating needs but also supports a broad deployment of natural gas–fueled power plants. Natural gas proponents stress its value as a transitional fuel as we move away from coal and toward greater reliance on renewables. Some advocate greater use of natural gas in transportation. North American natural gas supplies seem ample for years to come. This includes natural gas from deep-water ocean sites and shale deposits, each of which has environmental-impact issues associated with gas extraction. For instance, the hydraulic drilling technique known as *fracking* used to obtain gas from shale deposits produces huge amounts of briny, chemically laden wastewater that can affect human health and the environment if not properly managed. Despite increasing domestic natural gas supplies, natural gas in liquid form (LNG) is imported by ship to the United States (see Energy & Environment, Sec. 9.5).

The share of nuclear power in U.S. electricity generation is currently about the same as that of natural gas. In the 1950s, nuclear power was widely expected to be a dominant source of electricity by the year 2000. However, persistent concerns over reactor safety, an unresolved radioactive waste–disposal issue, and construction costs in the billions of dollars have resulted in a much smaller deployment of nuclear power than many had anticipated.

In some regions of the United States, hydroelectric power plants contribute significantly to meeting electricity needs. Although hydropower is a renewable source, it is not free of environmental impacts—for example, adverse effects on aquatic life in dammed rivers. The current share of wind, solar, geothermal, and other renewable sources in electricity generation is small but growing. Oil currently contributes only modestly.

Oil, natural gas, coal, and fissionable material are all in danger of reaching global production peaks in the foreseeable future and then entering periods of decline. Diminishing supplies will make these nonrenewable energy resources ever more costly. Increasing global demand for oil and fissionable material also pose national security concerns owing to the need for their importation to the United States.

Table 8.1 shows the United States currently has a range of sources for electricity generation and does not err by relying on too few. But in years ahead a gradual shift to a mix more reliant on renewable resources will be necessary.

TABLE 8.2

Large-Scale Electric Power Generation through 2050 from Renewable and Nonrenewable Sources[a]

Power Plant Type	Nonrenewable Source	Renewable Source	Thermodynamic Cycle
Coal-fueled	Yes		Rankine
Natural gas–fueled	Yes		Brayton[b]
Nuclear-fueled	Yes		Rankine
Oil-fueled	Yes		Rankine[c]
Biomass-fueled		Yes	Rankine
Geothermal		Yes	Rankine
Solar-concentrating		Yes	Rankine
Hydroelectric		Yes	None
Wind		Yes	None
Solar-photovoltaic		Yes	None
Fuel cells	Yes		None
Currents, tides, and waves		Yes	None

[a]For current information about these power plant types, visit www.energy.gov/energysources. The Rankine cycle is the subject of the current chapter.
[b]Brayton cycle applications are considered in Chap. 9. For electricity generation, natural gas is primarily used with gas turbine power plants based on the Brayton cycle.
[c]Petroleum-fueled *reciprocating internal combustion* engines, discussed in Chap. 9, also generate electricity.

Tomorrow

Looming scarcities of nonrenewable energy resources and their adverse effects on human health and the environment have sparked interest in broadening the ways in which we provide for our electricity needs—especially increasing use of renewable resources. Yet power production in the first half of the twenty-first century will rely primarily on means already available. Analysts say there are no technologies just over the horizon that will make much impact. Moreover, new technologies typically require decades and vast expenditures to establish.

Table 8.2 summarizes the types of power plants that will provide the electricity needed by our population up to mid-century, when electric power is expected to play an even larger role than now and new patterns of behavior affecting energy are likely (see Table 1.2).

There are several noteworthy features of Table 8.2. Seven of the twelve power plant types listed use renewable sources of energy. The five using nonrenewable sources include the three contributing most significantly to our current power mix (coal, natural gas, and nuclear). Four power plant types involve combustion—coal, natural gas, oil, and biomass—and thus require effective means for controlling gaseous emissions and power plant waste.

The twelve power plant types of Table 8.2 are unlikely to share equally in meeting national needs. In the immediate future coal, natural gas, and nuclear will continue to be major contributors while renewables will continue to lag. Gradually, the respective contributions are expected to shift to greater deployment of plants using renewables. This will be driven by state and national mandates requiring as much as 20% of electricity from renewables by 2020.

Of the emerging large-scale power plant types using a renewable source, wind is currently the most promising. Excellent wind resources exist in several places in the United States, both on land and offshore. The cost of wind-generated electricity is becoming competitive with coal-generated power. Other nations with active wind-power programs have goals of supplying up to 30% of their total electricity needs by wind within a few years, providing models for what might be possible in the United States. Still, wind turbines are not without environmental concerns. They are considered noisy by some and unsightly by others. Another concern is fatalities of birds and bats at wind-turbine sites (see Energy & Environment, Sec. 6.13.2).

Owing to higher costs, solar power is currently lagging behind wind power. Yet promising sites for solar power plants exist in many locations, especially in the Southwest. Active research and development efforts are focused on ways to reduce costs.

Geothermal plants use steam and hot water from deep hydrothermal reservoirs to generate electricity. Geothermal power plants exist in several states, including California, Nevada, Utah, and Hawaii. While geothermal power has considerable potential, its deployment has been inhibited by exploration, drilling, and extraction costs. The relatively low temperature of geothermal water also limits the extent to which electricity can be generated economically.

Although fuel cells are the subject of active research and development programs worldwide for stationary power generation and transportation, they are not yet widely deployed owing to costs. For more on fuel cells, see Sec. 13.4.

Power plants using current, tides, and waves are included in Table 8.2 because their potential for power generation is so vast. But thus far engineering embodiments are few, and this technology is not expected to mature in time to help much in the decades immediately ahead.

TAKE NOTE...
Here we provide a guide for navigating the parts of the book devoted to power generation.

The discussion of Table 8.2 concludes with a guide for navigating the parts of this book devoted to power generation. In Table 8.2, seven of the power plant types are identified with thermodynamic cycles. Those based on the Rankine cycle are considered in this chapter. Natural gas–fueled gas turbines based on the Brayton cycle are considered in Chap. 9, together with power generation by *reciprocating internal combustion* engines. Fuel cells are discussed in Sec. 13.4. Hydroelectric, wind, solar-photovoltaic, and currents, tides, and waves are also included in several end-of-chapter *Design and Open-Ended Problems*.

Power Plant Policy Making

Power plants not only require huge investments but also have useful lives measured in decades. Accordingly, decisions about constructing power plants must consider the present *and* look to the future.

Thinking about power plants is best done on a *life-cycle* basis, not with a narrow focus on the plant operation phase alone. The life cycle begins with extracting resources required by the plant from the earth and ends with the plant's eventual retirement from service. See Table 8.3.

To account accurately for total power plant cost, costs incurred in *all* phases should be considered, including costs of acquiring natural resources, plant construction, power plant furnishing, remediation of effects on the environment and human health, and eventual retirement. The extent of governmental subsidies should be carefully weighed in making an equitable assessment of cost.

TABLE 8.3

Power Plant Life-Cycle Snapshot

1. Mining, pumping, processing, and transporting
   (a) energy resources: coal, natural gas, oil, fissionable material, as appropriate.
   (b) commodities required for fabricating plant components and plant construction.
2. Remediation of environmental impacts related to the above.
3. Fabrication of plant components: boilers, pumps, reactors, solar arrays, steam and wind turbines, interconnections between components, and others.
4. Plant construction and connection to the power grid.
5. Plant operation: power production over several decades.
6. Capture, treatment, and disposal of effluents and waste products, including long-term storage when needed.
7. Retirement from service and site restoration when the useful life is over.

Capture, treatment, and proper disposal of effluents and waste products, including long-term storage where needed, must be a focus in power plant planning. None of the power plants listed in Table 8.2 are exempt from such scrutiny. While carbon dioxide production is particularly significant for power plants involving combustion, every plant type listed has carbon dioxide production in at least some of its life-cycle phases. The same can be said for other environmental and human-health impacts ranging from adverse land use to contamination of drinking water.

Public policy makers today have to consider not only the best ways to provide a reliable power *supply* but also how to do so judiciously. They should revisit entrenched regulations and practices suited to power generation and use in twentieth-century power generation but that now may stifle innovation. They also should be prepared to innovate when opportunities arise. See the *Horizons* feature that follows.

Policy makers must think critically about how to promote increased efficiency. Yet they must be watchful of a *rebound* effect sometimes observed when a resource, coal for instance, is used more efficiently to develop a product, electricity for instance. Efficiency-induced cost reductions can spur such demand for the product that little or no reduction in consumption of the resource occurs. With exceptional product demand, resource consumption can even rebound to a greater level than before.

Decision making in such a constrained social and technical environment is clearly a balancing act. Still, wise planning, including rationally decreasing waste and increasing efficiency, will allow us to stretch diminishing stores of nonrenewable energy resources, gain time to deploy renewable energy technologies, avoid construction of many new power plants, and reduce our contribution to global climate change, all while maintaining the lifestyle we enjoy.

Horizons

Reducing Carbon Dioxide Through Emissions Trading

Policy makers in 10 northeastern states (Connecticut, Delaware, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Rhode Island, and Vermont) with a total population approaching 50 million have boldly established the nation's first *cap-and-trade* program to harness the economic forces of the marketplace to reduce carbon dioxide emitted from power plants. The aim of these states is to spur a shift in the region's electricity supply toward more efficient generation and greater use of renewable energy technology.

The group of 10 states agreed to cap the total level of CO_2 emitted annually from power plants in the region starting in 2009 and continuing through 2014. To encourage innovation, the total CO_2 level then will be reduced 2.5% annually over the next four years to achieve a 10% decrease by 2019. Power plant operators have agreed to purchase *allowances* (or credits), which represent a permit to emit a specific amount of CO_2, to cover their expected CO_2 emissions. Proceeds of the sale of allowances are intended to support efforts in the region to foster energy efficiency and renewable energy technology. A utility that emits less than its projected allotment can sell unneeded allowances to utilities unable to meet their obligations. This is called a trade. In effect, the buyer pays a charge for polluting while the seller is rewarded for polluting less.

Costing carbon dioxide creates an economic incentive for decreasing such emissions. Accordingly, cap-and-trade provides a pathway for utilities to reduce carbon dioxide emitted from their power plants cost effectively. If the cap-and-trade program of this group of states is as successful as many expect, it could be a model for other regions and the nation as a whole.

Power Transmission and Distribution

Our society must not only generate the electricity required for myriad uses but also provide it to consumers. The interface between these closely linked activities has not always been smooth. The U.S. power grid transmitting and distributing electricity to consumers has changed little for several decades, while the number of consumers and their power needs have changed greatly. This has induced significant systemic issues. The current grid is increasingly a twentieth-century relic, susceptible to power outages threatening safety and security and costing the economy billions annually.

The principal difference between the current grid and the grid of the future is a change from an electricity transmission and distribution focus to an electricity management focus that accommodates multiple power generation technologies and fosters more efficient electricity use. A twenty-first-century grid will be equipped for real-time information, conducive to lightning decision making and response, and capable of providing consumers with quality, reliable, and affordable electricity anywhere and anytime.

This is a tall order, yet it has driven utilities and government to think deeply about how to bring electricity generation, transmission, and distribution into the twenty-first century and the digital age. The result is an electricity *superhighway* called the *smart grid*. See the *Horizons* feature that follows.

Horizons

Our Electricity Superhighway

The smart grid is envisioned as an intelligent system that accepts electricity from any source—renewable and nonrenewable, centralized and distributed (decentralized)—and delivers it locally, regionally, or across the nation. A robust and dynamic communications network will be at its core, enabling high-speed two-way data flow between power provider and end user. The grid will provide consumers at every level—industry, business, and the home—with information needed for decisions on when, where, and how to use electricity. Using *smart* meters and programmable controls, consumers will manage power use in keeping with individual requirements and lifestyle choices, yet in harmony with community, regional, and national priorities.

Other smart grid features include the ability to

- Respond to and manage *peak loads* responsibly
- Identify outages and their causes promptly
- Reroute power to meet changing demand automatically
- Use a mix of available power sources, including *distributed* generation, amicably and cost-effectively

And all the while it will foster exceptional performance in electricity generation, transmission, distribution, and end use.

The smart grid will accommodate emerging power technologies such as wind and solar, emerging large-scale power consumers such as plug-in and all-electric vehicles, and technologies yet to be invented. A more efficient and better-managed grid will meet the increase in electricity demand anticipated by 2050 without the need to build as many new fossil-fueled or nuclear power plants along the way. Fewer plants mean less carbon dioxide, other emissions, and solid waste.

Considering Vapor Power Systems

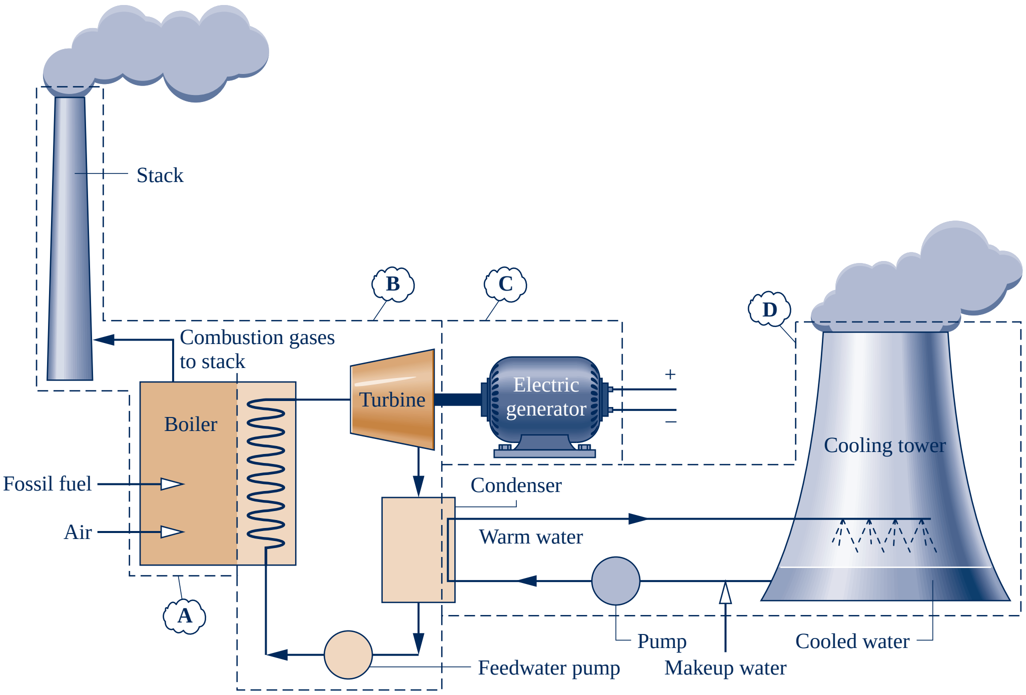
8.1 Introducing Vapor Power Plants

Referring again to Table 8.2, seven of the power plant types listed require a thermodynamic cycle, and six of these are identified with the *Rankine cycle*. The Rankine cycle is the basic building block of vapor power plants, which are the focus of this chapter.

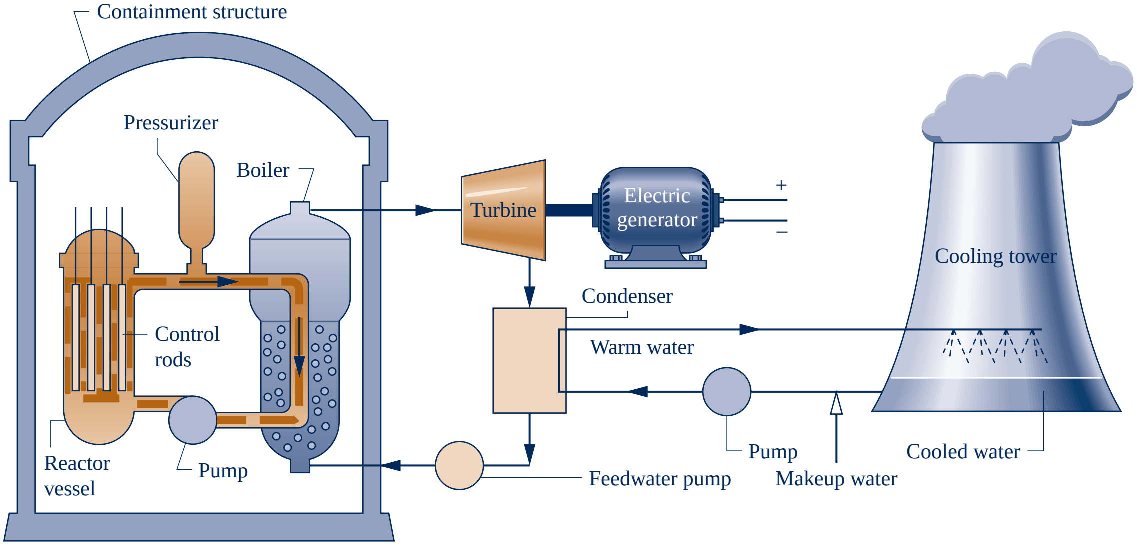
The components of four alternative vapor power plant configurations are shown schematically in Fig. 8.1. In order, these plants are (a) fossil-fueled, (b) nuclear-fueled, (c) solar thermal, and (d) geothermal. In Fig. 8.1*a*, the overall plant is broken into four major subsystems identified by the letters **A** through **D**. These letters have been omitted in the other three configurations for simplicity. The discussions of this chapter focus on subsystem **B**, where the energy conversion from *heat* to *work* occurs. The function of subsystem **A** is to supply the energy needed to vaporize the power plant *working fluid* into the vapor required by the turbine of subsystem **B**. The principal difference in the four power plant configurations shown in Fig. 8.1 is the way working fluid vaporization is accomplished by action of subsystem **A**:

▶ Vaporization is accomplished in fossil-fueled plants by heat transfer *to* water passing through the boiler tubes *from* hot gases produced in the combustion of the fuel, as shown in Fig. 8.1*a*. This is also seen in plants fueled by biomass, municipal waste (trash), and mixtures of coal and biomass.

▶ In nuclear plants, energy required for vaporizing the cycle working fluid originates in a controlled nuclear reaction occurring in a reactor-containment structure. The *pressurized-water* reactor shown in Fig. 8.1*b* has two water loops: One loop circulates



(*a*) Fossil-fueled vapor power plant.



(*b*) Pressurized-water reactor nuclear vapor power plant.

Fig. 8.1 Components of alternative vapor power plants (not to scale).

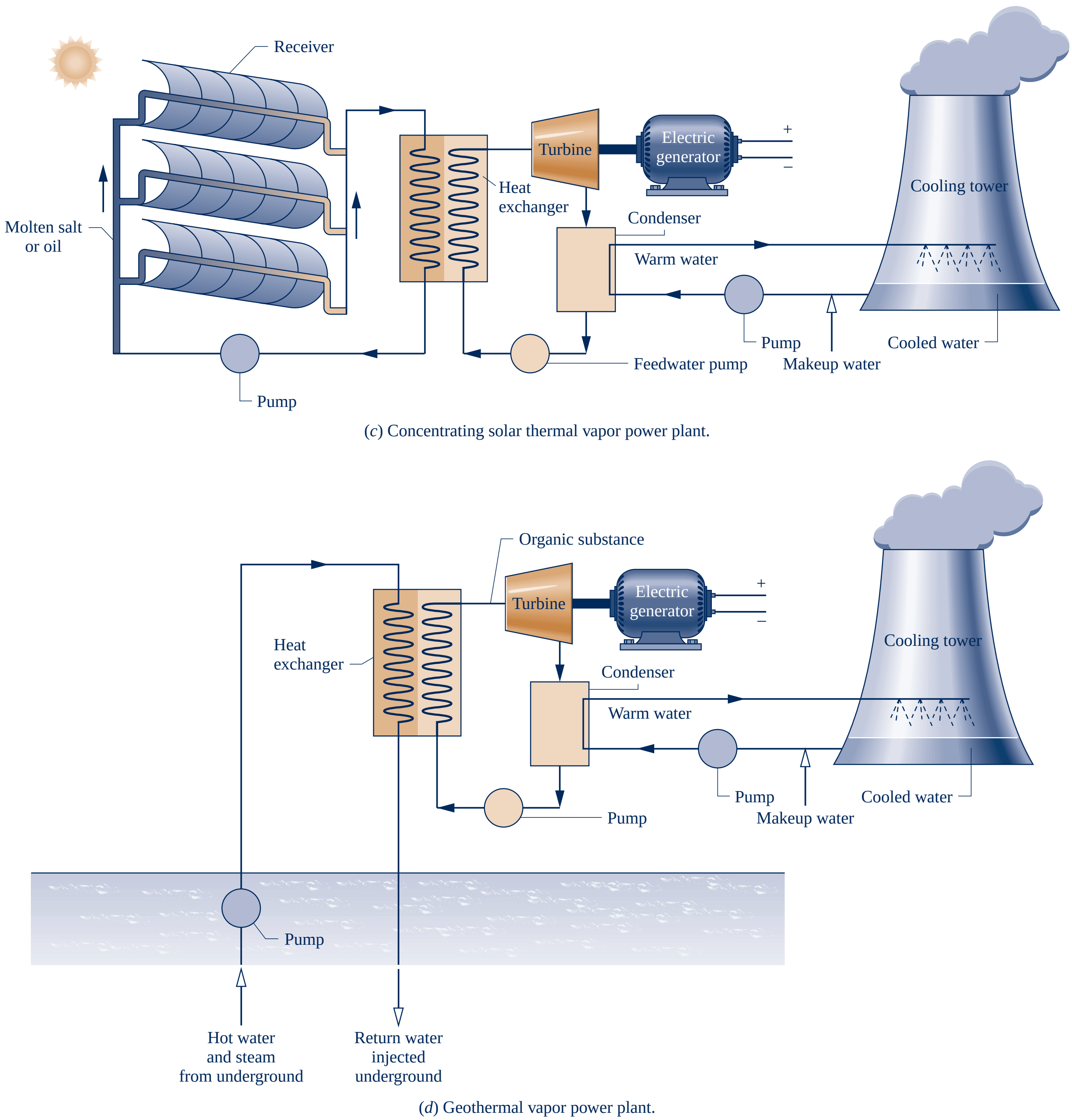


Fig. 8.1 ***(Continued)***

water through the reactor core and a boiler within the containment structure; this water is kept under pressure so it heats but does not boil. A separate loop carries steam from the boiler to the turbine. *Boiling-water* reactors (not shown in Fig. 8.1) have a single loop that boils water flowing through the core and carries steam directly to the turbine.

▶ Solar power plants have receivers for collecting and concentrating solar radiation. As shown in Fig. 8.1*c*, a suitable substance, molten salt or oil, flows through the receiver, where it is heated, directed to an interconnecting heat exchanger that replaces the boiler of the fossil- and nuclear-fueled plants, and finally returned to the receiver. The heated molten salt or oil provides energy required to vaporize water flowing in the other stream of the heat exchanger. This steam is provided to the turbine.

- The geothermal power plant shown in Fig. 8.1*d* also uses an interconnecting heat exchanger. In this case hot water and steam from deep below earth's surface flows on one side of the heat exchanger. A *secondary* working fluid having a lower boiling point than the water, such as isobutane or another organic substance, vaporizes on the other side of the heat exchanger. The secondary working fluid vapor is provided to the turbine.

Referring again to Fig. 8.1*a* as representative, let's consider the other subsystems, beginning with system **B**. Regardless of the source of the energy required to vaporize the working fluid and the type of working fluid, the vapor produced passes through the turbine, where it expands to lower pressure, developing power. The turbine power shaft is connected to an electric generator (subsystem **C**). The vapor exiting the turbine passes through the condenser, where it condenses on the outside of tubes carrying cooling water.

The cooling water circuit comprises subsystem **D**. For the plant shown, cooling water is sent to a cooling tower, where energy received from steam condensing in the condenser is rejected into the atmosphere. Cooling water then returns to the condenser.

Concern for the environment governs what is allowable in the interactions between subsystem **D** and its surroundings. One of the major difficulties in finding a site for a vapor power plant is access to sufficient quantities of condenser cooling water. To reduce cooling-water needs, harm to aquatic life in the vicinity of the plant, and other *thermal pollution* effects, large-scale power plants typically employ cooling towers (see Energy & Environment, Sec. 2.6.2).

Fuel processing and handling are significant issues for both fossil-fueled and nuclear-fueled plants because of human-health and environmental-impact considerations. Fossil-fueled plants must observe increasingly stringent limits on smokestack emissions and disposal of toxic solid waste. Nuclear-fueled plants are saddled with a significant radioactive waste–disposal problem. Still, all four of the power plant configurations considered in Fig. 8.1 have environmental, health, and land-use issues related to various stages of their life cycles, including how they are manufactured, installed, operated, and ultimately disposed.

8.2 The Rankine Cycle

Referring to subsystem **B** of Fig. 8.1*a* again, observe that each unit of mass of working fluid periodically undergoes a thermodynamic cycle as it circulates through the series of interconnected components. This cycle is the **Rankine cycle**.

Rankine cycle

Important concepts introduced in previous chapters for thermodynamic *power* cycles generally also apply to the Rankine cycle:

- The first law of thermodynamics requires that the *net* work developed by a system undergoing a power cycle must equal the *net* energy added by heat transfer to the system (Sec. 2.6.2).
- The second law of thermodynamics requires that the *thermal efficiency* of a power cycle to be less than 100% (Sec. 5.6.1).

It is recommended that you review this material as needed.

Discussions in previous chapters also have shown that improved thermodynamic performance accompanies the reduction of irreversibilities and losses. The extent to which irreversibilities and losses can be reduced in vapor power plants depends on several factors, however, including limits imposed by thermodynamics *and* by economics.

Power_Cycle
A.9 – All Tabs
A

8.2.1 Modeling the Rankine Cycle

TAKE NOTE...
When analyzing vapor power cycles, we take energy transfers as positive in the directions of arrows on system schematics and write energy balances accordingly.

The processes taking place in a vapor power plant are sufficiently complicated that idealizations are required to develop thermodynamic models of plant components and the overall plant. Depending on the objective, models can range from highly detailed computer models to very simple models requiring a hand calculator at most.

Study of such models, even simplified ones, can lead to valuable conclusions about the performance of the corresponding actual plants. Thermodynamic models allow at least *qualitative* deductions about how changes in major operating parameters affect actual performance. They also provide uncomplicated settings in which to investigate the functions and benefits of features intended to improve overall performance.

Whether the aim is a detailed or simplified model of a vapor power plant adhering to the Rankine cycle, all of the fundamentals required for thermodynamic analysis have been introduced in previous chapters. They include the conservation of mass and conservation of energy principles, the second law of thermodynamics, and use of thermodynamic data. These principles apply to individual plant components such as turbines, pumps, and heat exchangers as well as to the overall cycle.

Let us now turn to the thermodynamic modeling of subsystem **B** of Fig. 8.1*a*. The development begins by considering, in turn, the four principal components: turbine, condenser, pump, and boiler. Then we consider important performance parameters. Since the vast majority of large-scale vapor power plants use water as the working fluid, water is featured in the following discussions. For ease of presentation, we also focus on fossil-fuel plants, recognizing that major findings apply to the other types of power plants shown in Fig. 8.1.

The principal work and heat transfers of subsystem **B** are illustrated in Fig. 8.2. In subsequent discussions, these energy transfers are taken to be *positive in the directions of the arrows*. The unavoidable stray heat transfer that takes place between the plant components and their surroundings is neglected here for simplicity. Kinetic and potential energy changes are also ignored. Each component is regarded as operating at steady state. Using the conservation of mass and conservation of energy principles together with these idealizations, we develop expressions for the energy transfers shown on Fig. 8.2 beginning at state 1 and proceeding through each component in turn.

Turbine
A A.19 – Tabs a, b & c

Turbine

Vapor from the boiler at state 1, having an elevated temperature and pressure, expands through the turbine to produce work and then is discharged to the condenser at state 2 with relatively low pressure. Neglecting heat transfer with the surroundings, the mass and energy rate balances for a control volume around the turbine reduce at steady state to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_t + \dot{m}\left[h_1 - h_2 + \cancelto{0}{\frac{V_1^2 - V_2^2}{2}} + \cancelto{0}{g(z_1 - z_2)}\right]$$

or

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 \tag{8.1}$$

where $\dot{m}$ denotes the mass flow rate of the cycle working fluid, and $\dot{W}_t/\dot{m}$ is the rate at which work is developed per unit of mass of vapor passing through the turbine. As noted above, kinetic and potential energy changes are ignored.

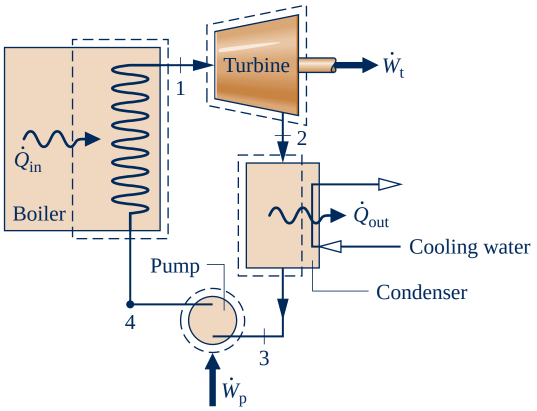


Fig. 8.2 Principal work and heat transfers of subsystem B.

Condenser

In the condenser there is heat transfer from the working fluid to cooling water flowing in a separate stream. The working fluid condenses and the temperature of the cooling water increases. At steady state, mass and energy rate balances for a control volume enclosing the condensing side of the heat exchanger give

$$\frac{\dot{Q}_{\text{out}}}{\dot{m}} = h_2 - h_3 \tag{8.2}$$

where $\dot{Q}_{\text{out}}/\dot{m}$ is the rate at which energy is transferred by heat *from* the working fluid to the cooling water per unit mass of working fluid passing through the condenser. This energy transfer is positive in the direction of the arrow on Fig. 8.2.

Pump

Pump
A.21 – Tabs a, b, & c A

The liquid condensate leaving the condenser at 3 is pumped from the condenser into the higher pressure boiler. Taking a control volume around the pump and assuming no heat transfer with the surroundings, mass and energy rate balances give

$$\frac{\dot{W}_{\text{p}}}{\dot{m}} = h_4 - h_3 \tag{8.3}$$

where $\dot{W}_{\text{p}}/\dot{m}$ is the rate of power *input* per unit of mass passing through the pump. This energy transfer is positive in the direction of the arrow on Fig. 8.2.

Boiler

The working fluid completes a cycle as the liquid leaving the pump at 4, called the boiler **feedwater**, is heated to saturation and evaporated in the boiler. Taking a control volume enclosing the boiler tubes and drums carrying the feedwater from state 4 to state 1, mass and energy rate balances give

feedwater

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_2 \tag{8.4}$$

where $\dot{Q}_{\text{in}}/\dot{m}$ is the rate of heat transfer from the energy source into the working fluid per unit mass passing through the boiler.

Performance Parameters

The thermal efficiency gauges the extent to which the energy input to the working fluid passing through the boiler is converted to the *net* work output. Using the quantities and expressions just introduced, the **thermal efficiency** of the power cycle of Fig. 8.2 is

$$\eta = \frac{\dot{W}_{\text{t}}/\dot{m} - \dot{W}_{\text{p}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} \tag{8.5a}$$

thermal efficiency

The net work output equals the net heat input. Thus, the thermal efficiency can be expressed alternatively as

$$\begin{aligned} \eta &= \frac{\dot{Q}_{\text{in}}/\dot{m} - \dot{Q}_{\text{out}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = 1 - \frac{\dot{Q}_{\text{out}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} \\ &= 1 - \frac{(h_2 - h_3)}{(h_1 - h_4)} \end{aligned} \tag{8.5b}$$

The **heat rate** is the amount of energy added by heat transfer to the cycle, usually in Btu, to produce a unit of net work output, usually in kW · h. Accordingly, the heat rate, which is inversely proportional to the thermal efficiency, has units of Btu/kW · h.

heat rate

Another parameter used to describe power plant performance is the **back work ratio**, or bwr, defined as the ratio of the pump work input to the work developed by the turbine. With Eqs. 8.1 and 8.3, the back work ratio for the power cycle of Fig. 8.2 is

$$\text{bwr} = \frac{\dot{W}_{\text{p}}/\dot{m}}{\dot{W}_{\text{t}}/\dot{m}} = \frac{(h_4 - h_3)}{(h_1 - h_2)} \tag{8.6}$$

back work ratio

Examples to follow illustrate that the change in specific enthalpy for the expansion of vapor through the turbine is normally many times greater than the increase in enthalpy for the liquid passing through the pump. Hence, the back work ratio is characteristically quite low for vapor power plants.

Provided states 1 through 4 are fixed, Eqs. 8.1 through 8.6 can be applied to determine the thermodynamic performance of a simple vapor power plant. Since these equations have been developed from mass and energy rate balances, they apply equally for actual performance when irreversibilities are present and for idealized performance in the absence of such effects. It might be surmised that the irreversibilities of the various power plant components can affect overall performance, and this is the case. Accordingly, it is instructive to consider an idealized cycle in which irreversibilities are assumed absent, for such a cycle establishes an *upper limit* on the performance of the Rankine cycle. The ideal cycle also provides a simple setting in which to study various aspects of vapor power plant performance. The ideal Rankine cycle is the subject of Sec. 8.2.2.

ENERGY & ENVIRONMENT The United States currently relies on relatively abundant coal supplies to generate half of its electricity (Table 8.1). A large fleet of coal-burning vapor power plants reliably provides comparatively inexpensive electricity to homes, businesses, and industry. Yet this good news is eroded by human-health and environmental-impact problems linked to coal combustion. Impacts accompany coal extraction, power generation, and waste disposal. Analysts say the cost of coal-derived electricity would be much higher if the full costs related to these adverse aspects of coal use were included.

Coal extraction practices such as mountaintop mining, where tops of mountains are sheared off to get at underlying coal, are a particular concern when removed rock, soil, and mining debris are discarded into streams and valleys below, marring natural beauty, affecting water quality, and devastating CO_2-trapping forests. Moreover, fatalities and critical injuries of coal miners while working to extract coal are widely seen as deplorable.

Gases formed in coal combustion include sulfur dioxide and oxides of nitrogen, which contribute to acid rain and smog. Fine particles and mercury, which more directly affect human health, are other unwanted outcomes of coal use. Coal combustion is also a major contributor to global climate change, primarily through carbon dioxide emissions. At the national level, controls are required for sulfur dioxide, nitric oxides, and fine particles but not currently required for mercury and carbon dioxide.

Solid waste is another major problem area. Solid waste from coal combustion is one of the largest waste streams produced in the United States. Solid waste includes sludge from smokestack scrubbers and fly ash, a by-product of pulverized coal combustion. While some of this waste is diverted to make commercial products, including cement, road de-icer, and synthetic gypsum used for drywall and as a fertilizer, vast amounts of waste are stored in landfills and pools containing watery slurries. Leakage from these impoundments can contaminate drinking water supplies. Watery waste accidentally released from holding pools causes widespread devastation and elevated levels of dangerous substances in surrounding areas. Some observers contend much more should be done to regulate health- and environment-endangering gas emissions and solid waste from coal-fired power plants and other industrial sites.

The more efficiently each ton of coal is utilized to generate power, the less CO_2, other combustion gases, and solid waste will be produced. Accordingly, improving efficiency is a well-timed pathway for continued coal use in the twenty-first century. Gradual replacement of existing power plants, beginning with those several decades old, by more efficient plants will reduce to some extent gas emissions and solid waste related to coal use.

Various advanced technologies also aim to foster coal use—but used more responsibly. They include *supercritical* vapor power plants (Sec. 8.3), *carbon capture and storage* (Sec. 8.5.3), and *integrated gasification combined cycle* (IGCC) power plants (Sec. 9.10). Owing to our large coal reserves and the critical importance of electricity to our society, major governmental and private-sector initiatives are in progress to develop additional technologies that promote responsible coal use.

8.2.2 Ideal Rankine Cycle

If the working fluid passes through the various components of the simple vapor power cycle without irreversibilities, frictional pressure drops would be absent from the boiler and condenser, and the working fluid would flow through these components at constant pressure. Also, in the absence of irreversibilities and heat transfer with the surroundings, the processes through the turbine and pump would be isentropic. A cycle adhering to these idealizations is the **ideal Rankine cycle** shown in Fig. 8.3.

ideal Rankine cycle

Referring to Fig. 8.3, we see that the working fluid undergoes the following series of internally reversible processes:

Process 1–2: Isentropic expansion of the working fluid through the turbine from saturated vapor at state 1 to the condenser pressure.
Process 2–3: Heat transfer *from* the working fluid as it flows at constant pressure through the condenser with saturated liquid at state 3.
Process 3–4: Isentropic compression in the pump to state 4 in the compressed liquid region.
Process 4–1: Heat transfer *to* the working fluid as it flows at constant pressure through the boiler to complete the cycle.

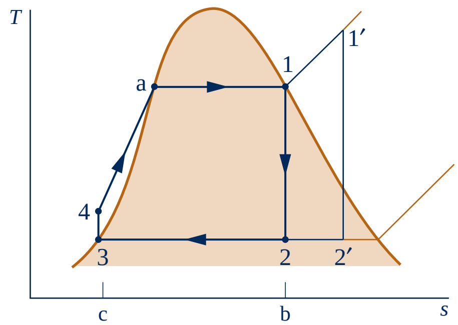


Fig. 8.3 Temperature–entropy diagram of the ideal Rankine cycle.

The ideal Rankine cycle also includes the possibility of superheating the vapor, as in cycle 1′–2′–3–4–1′. The importance of superheating is discussed in Sec. 8.3.

Since the ideal Rankine cycle consists of internally reversible processes, areas under the process lines of Fig. 8.3 can be interpreted as heat transfers per unit of mass flowing. Applying Eq. 6.49, area 1–b–c–4–a–1 represents the heat transfer to the working fluid passing through the boiler and area 2–b–c–3–2 is the heat transfer from the working fluid passing through the condenser, each per unit of mass flowing. The enclosed area 1–2–3–4–a–1 can be interpreted as the net heat input or, equivalently, the net work output, each per unit of mass flowing.

Because the pump is idealized as operating without irreversibilities, Eq. 6.51b can be invoked as an alternative to Eq. 8.3 for evaluating the pump work. That is

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \int_3^4 v\,dp \qquad \textbf{(8.7a)}$$

Rankine_Cycle
A.26 – Tabs a & b
A

where the minus sign has been dropped for consistency with the positive value for pump work in Eq. 8.3. The subscript "int rev" signals that this expression is restricted to an internally reversible process through the pump. An "int rev" designation is not required by Eq. 8.3, however, because it is obtained with the conservation of mass and energy principles and thus is generally applicable.

Evaluation of the integral of Eq. 8.7a requires a relationship between the specific volume and pressure for Process 3–4. Because the specific volume of the liquid normally varies only slightly as the liquid flows from the inlet to the exit of the pump, a plausible approximation to the value of the integral can be had by taking the specific volume at the pump inlet, v_3, as constant for the process. Then

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_s \approx v_3(p_4 - p_3) \qquad \textbf{(8.7b)}$$

TAKE NOTE...
For cycles, we modify the problem-solving methodology: The **Analysis** begins with a systematic evaluation of required property data at each numbered state. This reinforces what we know about the components, since given information and assumptions are required to fix the states.

where the subscript s signals the *isentropic*—internally reversible *and* adiabatic—process of the liquid flowing through the pump.

The next example illustrates the analysis of an ideal Rankine cycle.

EXAMPLE 8.1

Analyzing an Ideal Rankine Cycle

Steam is the working fluid in an ideal Rankine cycle. Saturated vapor enters the turbine at 8.0 MPa and saturated liquid exits the condenser at a pressure of 0.008 MPa. The *net* power output of the cycle is 100 MW. Determine for the cycle **(a)** the thermal efficiency, **(b)** the back work ratio, **(c)** the mass flow rate of the steam, in kg/h, **(d)** the rate of heat transfer, $\dot{Q}_{in}$, into the working fluid as it passes through the boiler, in MW, **(e)** the rate of heat transfer, $\dot{Q}_{out}$, from the condensing steam as it passes through the condenser, in MW, **(f)** the mass flow rate of the condenser cooling water, in kg/h, if cooling water enters the condenser at 15°C and exits at 35°C.

SOLUTION

Known: An ideal Rankine cycle operates with steam as the working fluid. The boiler and condenser pressures are specified, and the net power output is given.

Find: Determine the thermal efficiency, the back work ratio, the mass flow rate of the steam, in kg/h, the rate of heat transfer to the working fluid as it passes through the boiler, in MW, the rate of heat transfer from the condensing steam as it passes through the condenser, in MW, the mass flow rate of the condenser cooling water, which enters at 15°C and exits at 35°C.

Schematic and Given Data:

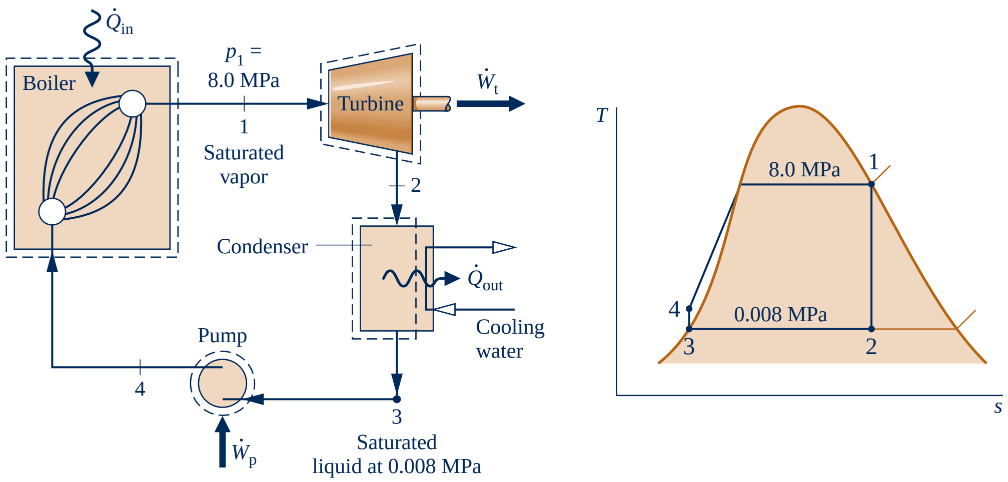


Fig. E8.1

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated vapor enters the turbine. Condensate exits the condenser as saturated liquid.

1 **Analysis:** To begin the analysis, we fix each of the principal states located on the accompanying schematic and T–s diagrams. Starting at the inlet to the turbine, the pressure is 8.0 MPa and the steam is a saturated vapor, so from Table A-3, $h_1 = 2758.0$ kJ/kg and $s_1 = 5.7432$ kJ/kg · K.

State 2 is fixed by $p_2 = 0.008$ MPa and the fact that the specific entropy is constant for the adiabatic, internally reversible expansion through the turbine. Using saturated liquid and saturated vapor data from Table A-3, we find that the quality at state 2 is

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{5.7432 - 0.5926}{7.6361} = 0.6745$$

The enthalpy is then

$$\begin{aligned} h_2 &= h_f + x_2 h_{fg} = 173.88 + (0.6745)2403.1 \\ &= 1794.8 \text{ kJ/kg} \end{aligned}$$

State 3 is saturated liquid at 0.008 MPa, so $h_3 = 173.88$ kJ/kg.

State 4 is fixed by the boiler pressure p_4 and the specific entropy $s_4 = s_3$. The specific enthalpy h_4 can be found by interpolation in the compressed liquid tables. However, because compressed liquid data are relatively sparse, it is more convenient to solve Eq. 8.3 for h_4, using Eq. 8.7b to approximate the pump work. With this approach

$$h_4 = h_3 + \dot{W}_p/\dot{m} = h_3 + v_3(p_4 - p_3)$$

By inserting property values from Table A-3

$$\begin{aligned} h_4 &= 173.88 \text{ kJ/kg} + (1.0084 \times 10^{-3} \text{ m}^3/\text{kg})(8.0 - 0.008)\text{MPa} \left| \frac{10^6 \text{ N/m}^2}{1 \text{ MPa}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \\ &= 173.88 + 8.06 = 181.94 \text{ kJ/kg} \end{aligned}$$

(a) The *net* power developed by the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p$$

Mass and energy rate balances for control volumes around the turbine and pump give, respectively,

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 \qquad \text{and} \qquad \frac{\dot{W}_p}{\dot{m}} = h_4 - h_3$$

where $\dot{m}$ is the mass flow rate of the steam. The rate of heat transfer to the working fluid as it passes through the boiler is determined using mass and energy rate balances as

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

The thermal efficiency is then

$$\begin{aligned} \eta &= \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} \\ &= \frac{[(2758.0 - 1794.8) - (181.94 - 173.88)] \text{ kJ/kg}}{(2758.0 - 181.94) \text{ kJ/kg}} \\ &= 0.371 \ (37.1\%) \end{aligned}$$

(b) The back work ratio is

2
$$\begin{aligned} \text{bwr} &= \frac{\dot{W}_p}{\dot{W}_t} = \frac{h_4 - h_3}{h_1 - h_2} = \frac{(181.94 - 173.88) \text{ kJ/kg}}{(2758.0 - 1794.8) \text{ kJ/kg}} \\ &= \frac{8.06}{963.2} = 8.37 \times 10^{-3} \ (0.84\%) \end{aligned}$$

(c) The mass flow rate of the steam can be obtained from the expression for the net power given in part (a). Thus

$$\begin{aligned} \dot{m} &= \frac{\dot{W}_{cycle}}{(h_1 - h_2) - (h_4 - h_3)} \\ &= \frac{(100 \text{ MW})|10^3 \text{ kW/MW}||3600 \text{ s/h}|}{(963.2 - 8.06) \text{ kJ/kg}} \\ &= 3.77 \times 10^5 \text{ kg/h} \end{aligned}$$

(d) With the expression for $\dot{Q}_{in}$ from part (a) and previously determined specific enthalpy values

$$\begin{aligned}\dot{Q}_{in} &= \dot{m}(h_1 - h_4)\\ &= \frac{(3.77 \times 10^5 \text{ kg/h})(2758.0 - 181.94) \text{ kJ/kg}}{|3600 \text{ s/h}|\,|10^3 \text{ kW/MW}|}\\ &= 269.77 \text{ MW}\end{aligned}$$

(e) Mass and energy rate balances applied to a control volume enclosing the steam side of the condenser give

$$\begin{aligned}\dot{Q}_{out} &= \dot{m}(h_2 - h_3)\\ &= \frac{(3.77 \times 10^5 \text{ kg/h})(1794.8 - 173.88) \text{ kJ/kg}}{|3600 \text{ s/h}|\,|10^3 \text{ kW/MW}|}\\ &= 169.75 \text{ MW}\end{aligned}$$

3 Note that the ratio of $\dot{Q}_{out}$ to $\dot{Q}_{in}$ is 0.629 (62.9%).

Alternatively, $\dot{Q}_{out}$ can be determined from an energy rate balance on the *overall* vapor power plant. At steady state, the net power developed equals the net rate of heat transfer to the plant

$$\dot{W}_{cycle} = \dot{Q}_{in} - \dot{Q}_{out}$$

Rearranging this expression and inserting values

$$\dot{Q}_{out} = \dot{Q}_{in} - \dot{W}_{cycle} = 269.77 \text{ MW} - 100 \text{ MW} = 169.77 \text{ MW}$$

The slight difference from the above value is due to round-off.

(f) Taking a control volume around the condenser, the mass and energy rate balances give at steady state

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_{cw}(h_{cw,in} - h_{cw,out}) + \dot{m}(h_2 - h_3)$$

where $\dot{m}_{cw}$ is the mass flow rate of the cooling water. Solving for $\dot{m}_{cw}$

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{(h_{cw,out} - h_{cw,in})}$$

The numerator in this expression is evaluated in part (e). For the cooling water, $h \approx h_f(T)$, so with saturated liquid enthalpy values from Table A-2 at the entering and exiting temperatures of the cooling water

$$\dot{m}_{cw} = \frac{(169.75 \text{ MW})|10^3 \text{ kW/MW}||3600 \text{ s/h}|}{(146.68 - 62.99) \text{ kJ/kg}} = 7.3 \times 10^6 \text{ kg/h}$$

1 Note that a slightly revised problem-solving methodology is used in this example problem: We begin with a systematic evaluation of the specific enthalpy at each numbered state.

2 Note that the back work ratio is relatively low for the Rankine cycle. In the present case, the work required to operate the pump is less than 1% of the turbine output.

3 In this example, 62.9% of the energy added to the working fluid by heat transfer is subsequently discharged to the cooling water. Although considerable energy is carried away by the cooling water, its exergy is small because the water exits at a temperature only a few degrees greater than that of the surroundings. See Sec. 8.6 for further discussion.

Skills Developed

Ability to...

- ❑ sketch the *T–s* diagram of the basic Rankine cycle.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ apply mass and energy balances.
- ❑ calculate performance parameters for the cycle.

QuickQUIZ If the mass flow rate of steam were 150 kg/s, what would be the net power, in MW, and the thermal efficiency? **Ans.** 143.2 MW, 37.1%.

8.2.3 Effects of Boiler and Condenser Pressures on the Rankine Cycle

In discussing Fig. 5.12 (Sec. 5.9.1), we observed that the thermal efficiency of power cycles tends to increase as the average temperature at which energy is added by heat transfer increases and/or the average temperature at which energy is rejected by heat transfer decreases. (For elaboration, see box.) Let us apply this idea to study the effects on performance of the ideal Rankine cycle of changes in the boiler and condenser pressures. Although these findings are obtained with reference to the ideal Rankine cycle, they also hold qualitatively for actual vapor power plants.

Figure 8.4*a* shows two ideal cycles having the same condenser pressure but different boiler pressures. By inspection, the average temperature of heat addition is seen to be greater for the higher-pressure cycle 1′–2′–3′–4′–1′ than for cycle 1–2–3–4–1. It follows that increasing the boiler pressure of the ideal Rankine cycle tends to increase the thermal efficiency.

Considering the Effect of Temperature on Thermal Efficiency

Since the ideal Rankine cycle consists entirely of internally reversible processes, an expression for thermal efficiency can be obtained in terms of *average* temperatures during the heat interaction processes. Let us begin the development of this expression by recalling that areas under the process lines of Fig. 8.3 can be interpreted as the heat transfer per unit of mass flowing through the respective components. For example, the total area 1–b–c–4–a–1 represents the heat transfer into the working fluid per unit of mass passing through the boiler. In symbols,

$$\left(\frac{\dot{Q}_{\text{in}}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \int_4^1 T\,ds = \text{area } 1\text{–b–c–4–a–1}$$

The integral can be written in terms of an average temperature of heat addition, $\overline{T}_{\text{in}}$, as follows:

$$\left(\frac{\dot{Q}_{\text{in}}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \overline{T}_{\text{in}}(s_1 - s_4)$$

where the overbar denotes *average*. Similarly, area 2–b–c–3–2 represents the heat transfer from the condensing steam per unit of mass passing through the condenser

$$\begin{aligned}\left(\frac{\dot{Q}_{\text{out}}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} &= T_{\text{out}}(s_2 - s_3) = \text{area } 2\text{–b–c–3–2}\\ &= T_{\text{out}}(s_1 - s_4)\end{aligned}$$

where T_{out} denotes the temperature on the steam side of the condenser of the ideal Rankine cycle pictured in Fig. 8.3. The thermal efficiency of the ideal Rankine cycle can be expressed in terms of these heat transfers as

$$\eta_{\text{ideal}} = 1 - \frac{(\dot{Q}_{\text{out}}/\dot{m})_{\substack{\text{int}\\\text{rev}}}}{(\dot{Q}_{\text{in}}/\dot{m})_{\substack{\text{int}\\\text{rev}}}} = 1 - \frac{T_{\text{out}}}{\overline{T}_{\text{in}}} \tag{8.8}$$

By the study of Eq. 8.8, we conclude that the thermal efficiency of the ideal cycle tends to increase as the average temperature at which energy is added by heat transfer increases and/or the temperature at which energy is rejected decreases. With similar reasoning, these conclusions can be shown to apply to the other ideal cycles considered in this chapter and the next.

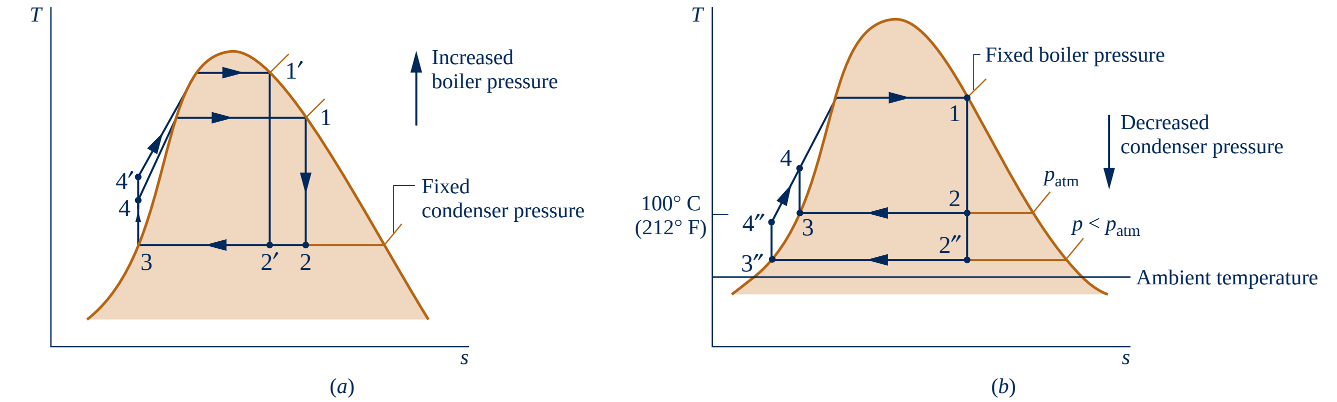


Fig. 8.4 **Effects of varying operating pressures on the ideal Rankine cycle. (*a*) Effect of boiler pressure. (*b*) Effect of condenser pressure.**

Figure 8.4*b* shows two cycles with the same boiler pressure but two different condenser pressures. One condenser operates at atmospheric pressure and the other at *less than* atmospheric pressure. The temperature of heat rejection for cycle 1–2–3–4–1 condensing at atmospheric pressure is 100°C (212°F). The temperature of heat rejection for the lower-pressure cycle 1–2″–3″–4″–1 is correspondingly lower, so this cycle has the greater thermal efficiency. It follows that decreasing the condenser pressure tends to increase the thermal efficiency.

The lowest feasible condenser pressure is the saturation pressure corresponding to the ambient temperature, for this is the lowest possible temperature for heat rejection to the surroundings. The goal of maintaining the lowest practical turbine exhaust (condenser) pressure is a primary reason for including the condenser in a power plant. Liquid water at atmospheric pressure could be drawn into the boiler by a pump, and steam could be discharged directly to the atmosphere at the turbine exit. However, by including a condenser in which the steam side is operated at a pressure *below atmospheric,* the turbine has a lower-pressure region in which to discharge, resulting in a significant increase in net work and thermal efficiency. The addition of a condenser also allows the working fluid to flow in a closed loop. This arrangement permits continual circulation of the working fluid, so purified water that is less corrosive than tap water can be used economically.

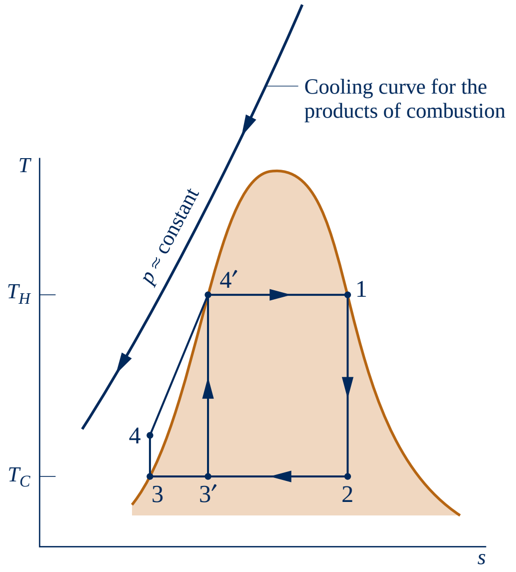


Fig. 8.5 **Illustration used to compare the ideal Rankine cycle with the Carnot cycle.**

Comparison with Carnot Cycle

Referring to Fig. 8.5, the ideal Rankine cycle 1–2–3–4–4′–1 has a lower thermal efficiency than the Carnot cycle 1–2–3′–4′–1 having the same maximum temperature T_H and minimum temperature T_C because the average temperature between 4 and 4′ is less than T_H. Despite the greater thermal efficiency of the Carnot cycle, it has shortcomings as a model for the simple fossil-fueled vapor power cycle. First, heat transfer to the working fluid of such vapor power plants is obtained from hot products of combustion cooling at approximately constant pressure. To exploit fully the energy released on combustion, the hot products should be cooled as much as possible. The first portion of the heating process of the Rankine cycle shown in Fig. 8.5, Process 4–4′, is achieved by cooling the combustion products *below* the maximum temperature T_H. With the Carnot cycle, however, the combustion products would be cooled *at the most* to T_H. Thus, a smaller portion of the energy released on combustion would be used. Another shortcoming of the Carnot vapor power cycle involves the pumping process. Note that state 3′ of Fig. 8.5 is a two-phase liquid–vapor mixture. Significant practical problems are encountered in developing pumps that handle two-phase mixtures, as would be required by Carnot cycle

1–2–3′–4′–1. It is better to condense the vapor completely and handle only liquid in the pump, as is done in the Rankine cycle. Pumping from 3 to 4 and heating without work from 4 to 4′ are processes that can be readily achieved in practice.

8.2.4 Principal Irreversibilities and Losses

Irreversibilities and losses are associated with each of the four subsystems designated in Fig. 8.1*a* by **A**, **B**, **C**, and **D**. Some of these effects have much greater influence on overall power plant performance than others. In this section, we consider irreversibilities and losses associated with the working fluid as it flows around the closed loop of subsystem **B**: the Rankine cycle. These effects are broadly classified as *internal* or *external* depending on whether they occur within subsystem **B** or its surroundings.

Internal Effects

TURBINE. The principal internal irreversibility experienced by the working fluid is associated with expansion through the turbine. Heat transfer from the turbine to its surroundings is a loss; but since it is of secondary importance, this loss is ignored in subsequent discussions. As illustrated by Process 1–2 of Fig. 8.6, actual adiabatic expansion through the turbine is accompanied by an increase in entropy. The work developed in this process per unit of mass flowing is *less* than that for the corresponding isentropic expansion 1–2s. Isentropic turbine efficiency, η_t, introduced in Sec. 6.12.1, accounts for the effect of irreversibilities within the turbine in terms of actual and isentropic work amounts. Designating states as in Fig. 8.6, the isentropic turbine efficiency is

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \tag{8.9}$$

Turbine
A.19 – Tab e

where the numerator is the actual work developed per unit of mass flowing through the turbine and the denominator is the work per unit of mass flowing for an isentropic expansion from the turbine inlet state to the turbine exhaust pressure. Irreversibilities within the turbine reduce the net power output of the plant and thus thermal efficiency.

PUMP. The work input to the pump required to overcome irreversibilities also reduces the net power output of the plant. As illustrated by Process 3–4 of Fig. 8.6, the actual pumping process is accompanied by an increase in entropy. For this process; the work *input* per unit of mass flowing is *greater* than that for the corresponding isentropic process 3–4s. As for the turbine, heat transfer is ignored as secondary. Isentropic pump efficiency, η_p, introduced in Sec. 6.12.3, accounts for the effect of irreversibilities within the pump in terms of actual and isentropic work amounts. Designating states as in Sec. 8.6, the isentropic pump efficiency is

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3} \tag{8.10a}$$

In Eq. 8.10a the pump work for the isentropic process appears in the numerator. The actual pump work, being the larger magnitude, is the denominator.

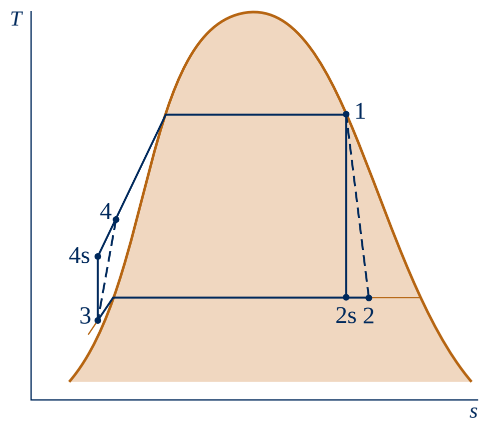


Fig. 8.6 **Temperature–entropy diagram showing the effects of turbine and pump irreversibilities.**

The pump work for the isentropic process can be evaluated using Eq. 8.7b to give an alternative expression for the isentropic pump efficiency:

Pump
A.21 – Tab e

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{v_3(p_4 - p_3)}{h_4 - h_3} \quad \textbf{(8.10b)}$$

Because pump work is much less than turbine work, irreversibilities in the pump have a much smaller impact on thermal efficiency than irreversibilities in the turbine.

OTHER EFFECTS. Frictional effects resulting in pressure reductions are additional sources of internal irreversibility as the working fluid flows through the boiler, condenser, and piping connecting the several components. Detailed thermodynamic analyses account for these effects. For simplicity, they are ignored in subsequent discussions. In keeping with this, Fig. 8.6 shows no pressure drops for flow through the boiler and condenser or between plant components.

Another detrimental effect on plant performance can be noted by comparing the ideal cycle of Fig. 8.6 with the counterpart ideal cycle of Fig. 8.3. In Fig. 8.6, pump inlet state 3 falls in the liquid region and is not saturated liquid as in Fig. 8.3, giving lower average temperatures of heat addition and rejection. The overall effect typically is a *lower* thermal efficiency in the case of Fig. 8.6 compared to that of Fig. 8.3.

External Effects

The turbine and pump irreversibilities considered above are *internal* irreversibilities experienced by the working fluid flowing around the closed loop of the Rankine cycle. They have detrimental effects on power plant performance. Yet the most significant source of irreversibility *by far* for a fossil-fueled vapor power plant is associated with combustion of the fuel and subsequent heat transfer from hot combustion gases to the cycle working fluid. As combustion and subsequent heat transfer occur in the surroundings of subsystem **B** of Fig. 8.1*a*, they are classified here as *external*. These effects are considered quantitatively in Sec. 8.6 and Chap. 13 using the exergy concept.

Another effect occurring in the surroundings of subsystem **B** is energy discharged by heat transfer to cooling water as the working fluid condenses. The significance of this loss is *far less* than suggested by the magnitude of the energy discharged. Although cooling water carries away considerable energy, the *utility* of this energy is extremely limited when condensation occurs near ambient temperature and the temperature of the cooling water increases only by a few degrees above the ambient during flow through the condenser. Such cooling water has little thermodynamic or economic value. Instead, the slightly warmed cooling water is normally *disadvantageous* for plant operators in terms of cost because operators must provide responsible means for disposing of the energy gained by cooling water in flow through the condenser—they provide a cooling tower, for instance. The limited utility of condenser cooling water is demonstrated quantitatively in Sec. 8.6 using the exergy concept.

Rankine_Cycle
A.26 – Tab c

Finally, stray heat transfers from the outer surfaces of plant components have detrimental effects on performance since they reduce the extent of conversion from heat to work. Such heat transfers are secondary effects ignored in subsequent discussions.

In the next example, the ideal Rankine cycle of Example 8.1 is modified to show the effects of turbine and pump isentropic efficiencies on performance.

EXAMPLE 8.2

Analyzing a Rankine Cycle with Irreversibilities

Reconsider the vapor power cycle of Example 8.1, but include in the analysis that the turbine and the pump each have an isentropic efficiency of 85%. Determine for the modified cycle **(a)** the thermal efficiency, **(b)** the mass flow rate of steam, in kg/h, for a net power output of 100 MW, **(c)** the rate of heat transfer $\dot{Q}_{in}$ into the working fluid as it passes through the boiler, in MW, **(d)** the rate of heat transfer $\dot{Q}_{out}$ from the condensing steam as it passes through the condenser, in MW, **(e)** the mass flow rate of the condenser cooling water, in kg/h, if cooling water enters the condenser at 15°C and exits as 35°C.

SOLUTION

Known: A vapor power cycle operates with steam as the working fluid. The turbine and pump both have efficiencies of 85%.

Find: Determine the thermal efficiency, the mass flow rate, in kg/h, the rate of heat transfer to the working fluid as it passes through the boiler, in MW, the heat transfer rate from the condensing steam as it passes through the condenser, in MW, and the mass flow rate of the condenser cooling water, in kg/h.

Schematic and Given Data:

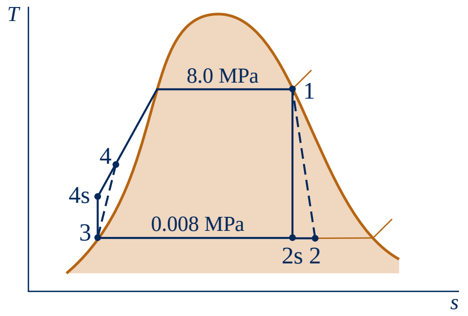


Fig. E8.2

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state.
2. The working fluid passes through the boiler and condenser at constant pressure. Saturated vapor enters the turbine. The condensate is saturated at the condenser exit.
3. The turbine and pump each operate adiabatically with an efficiency of 85%.
4. Kinetic and potential energy effects are negligible.

Analysis: Owing to the presence of irreversibilities during the expansion of the steam through the turbine, there is an increase in specific entropy from turbine inlet to exit, as shown on the accompanying *T–s* diagram. Similarly, there is an increase in specific entropy from pump inlet to exit. Let us begin the analysis by fixing each of the principal states. State 1 is the same as in Example 8.1, so $h_1 = 2758.0$ kJ/kg and $s_1 = 5.7432$ kJ/kg · K.

The specific enthalpy at the turbine exit, state 2, can be determined using the isentropic turbine efficiency, Eq. 8.9,

$$\eta_t = \frac{\dot{W}_t/\dot{m}}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

where h_{2s} is the specific enthalpy at state 2s on the accompanying *T–s* diagram. From the solution to Example 8.1, $h_{2s} = 1794.8$ kJ/kg. Solving for h_2 and inserting known values

$$\begin{aligned} h_2 &= h_1 - \eta_t(h_1 - h_{2s}) \\ &= 2758 - 0.85(2758 - 1794.8) = 1939.3 \text{ kJ/kg} \end{aligned}$$

State 3 is the same as in Example 8.1, so $h_3 = 173.88$ kJ/kg.

To determine the specific enthalpy at the pump exit, state 4, reduce mass and energy rate balances for a control volume around the pump to obtain $\dot{W}_p/\dot{m} = h_4 - h_3$. On rearrangement, the specific enthalpy at state 4 is

$$h_4 = h_3 + \dot{W}_p/\dot{m}$$

To determine h_4 from this expression requires the pump work. Pump work can be evaluated using the isentropic pump efficiency in the form of Eq. 8.10b: solving for $\dot{W}_p/\dot{m}$ results in

$$\frac{\dot{W}_p}{\dot{m}} = \frac{v_3(p_4 - p_3)}{\eta_p}$$

The numerator of this expression was determined in the solution to Example 8.1. Accordingly,

$$\frac{\dot{W}_p}{\dot{m}} = \frac{8.06 \text{ kJ/kg}}{0.85} = 9.48 \text{ kJ/kg}$$

The specific enthalpy at the pump exit is then

$$h_4 = h_3 + \dot{W}_p/\dot{m} = 173.88 + 9.48 = 183.36 \text{ kJ/kg}$$

(a) The net power developed by the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_\text{t} - \dot{W}_\text{p} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

The rate of heat transfer to the working fluid as it passes through the boiler is

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4)$$

Thus, the thermal efficiency is

$$\eta = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$

Inserting values

$$\eta = \frac{(2758 - 1939.3) - 9.48}{2758 - 183.36} = 0.314\ (31.4\%)$$

(b) With the net power expression of part (a), the mass flow rate of the steam is

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{(h_1 - h_2) - (h_4 - h_3)}$$

$$= \frac{(100\text{ MW})|3600\text{ s/h}||10^3\text{ kW/MW}|}{(818.7 - 9.48)\text{ kJ/kg}} = 4.449 \times 10^5\text{ kg/h}$$

(c) With the expression for $\dot{Q}_{\text{in}}$ from part (a) and previously determined specific enthalpy values

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4)$$

$$= \frac{(4.449 \times 10^5\text{ kg/h})(2758 - 183.36)\text{ kJ/kg}}{|3600\text{ s/h}||10^3\text{ kW/MW}|} = 318.2\text{ MW}$$

(d) The rate of heat transfer from the condensing steam to the cooling water is

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3)$$

$$= \frac{(4.449 \times 10^5\text{ kg/h})(1939.3 - 173.88)\text{ kJ/kg}}{|3600\text{ s/h}||10^3\text{ kW/MW}|} = 218.2\text{ MW}$$

(e) The mass flow rate of the cooling water can be determined from

$$\dot{m}_{\text{cw}} = \frac{\dot{m}(h_2 - h_3)}{(h_{\text{cw,out}} - h_{\text{cw,in}})}$$

$$= \frac{(218.2\text{ MW})|10^3\text{ kW/MW}||3600\text{ s/h}|}{(146.68 - 62.99)\text{ kJ/kg}} = 9.39 \times 10^6\text{ kg/h}$$

Skills Developed

Ability to...

- ❑ sketch the *T–s* diagram of the Rankine cycle with turbine and pump irreversibilities.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ apply mass, energy, and entropy principles.
- ❑ calculate performance parameters for the cycle.

QuickQUIZ If the mass flow rate of steam were 150 kg/s, what would be the pump power required, in kW, and the back work ratio? Ans. 1422 kW, 0.0116.

Discussion of Examples 8.1 and 8.2

The effect of irreversibilities within the turbine and pump can be gauged by comparing values from Example 8.2 with their counterparts in Example 8.1. In Example 8.2, the turbine work per unit of mass is *less* and the pump work per unit of mass is *greater* than in Example 8.1, as can be confirmed using data from these examples. The thermal efficiency in Example 8.2 is *less* than in the ideal case of Example 8.1. For a fixed net power output (100 MW), the smaller net work output per unit mass in Example 8.2 dictates a greater mass flow rate of steam than in Example 8.1. The magnitude of the heat transfer to cooling water is also greater in Example 8.2 than in Example 8.1; consequently, a greater mass flow rate of cooling water is required.

8.3 Improving Performance—Superheat, Reheat, and Supercritical

Rankine_Cycle
A.26 – Tab b
A

The representations of the vapor power cycle considered thus far do not depict actual vapor power plants faithfully, for various modifications are usually incorporated to improve overall performance. In this section we consider cycle modifications known as *superheat* and *reheat*. Both features are normally incorporated into vapor power plants. We also consider supercritical steam generation.

Let us begin the discussion by noting that an increase in the boiler pressure or a decrease in the condenser pressure may result in a reduction of the steam quality at the exit of the turbine. This can be seen by comparing states 2′ and 2″ of Figs. 8.4*a* and 8.4*b* to the corresponding state 2 of each diagram. If the quality of the mixture passing through the turbine becomes too low, the impact of liquid droplets in the flowing liquid–vapor mixture can erode the turbine blades, causing a decrease in the turbine efficiency and an increased need for maintenance. Accordingly, common practice is to maintain at least 90% quality ($x \geqq 0.9$) at the turbine exit. The cycle modifications known as *superheat* and *reheat* permit advantageous operating pressures in the boiler and condenser and yet avoid the problem of low quality of the turbine exhaust.

Superheat

superheat

First, let us consider **superheat**. As we are not limited to having saturated vapor at the turbine inlet, further energy can be added by heat transfer to the steam, bringing it to a superheated vapor condition at the turbine inlet. This is accomplished in a separate heat exchanger called a superheater. The combination of boiler and superheater is referred to as a *steam generator*. Figure 8.3 shows an ideal Rankine cycle with superheated vapor at the turbine inlet: cycle 1′–2′–3–4–1′. The cycle with superheat has a higher average temperature of heat addition than the cycle without superheating (cycle 1–2–3–4–1), so the thermal efficiency is higher. Moreover, the quality at turbine exhaust state 2′ is greater than at state 2, which would be the turbine exhaust state without superheating. Accordingly, superheating also tends to alleviate the problem of low steam quality at the turbine exhaust. With sufficient superheating, the turbine exhaust state may even fall in the superheated vapor region.

Reheat

reheat

A further modification normally employed in vapor power plants is **reheat**. With reheat, a power plant can take advantage of the increased efficiency that results with higher boiler pressures and yet avoid low-quality steam at the turbine exhaust. In the ideal reheat cycle shown in Fig. 8.7, steam does not expand to the condenser pressure in a single stage. Instead, steam expands through a first-stage turbine (Process 1–2) to some pressure between the steam generator and condenser pressures. Steam is then reheated in the steam generator (Process 2–3). Ideally, there would be no pressure drop as the steam is reheated. After reheating, the steam expands in a second-stage turbine to the condenser pressure (Process 3–4). Observe that with reheat the quality of the steam at the turbine exhaust is increased. This can be seen from the T–s diagram of Fig. 8.7 by comparing state 4 with state 4′, the turbine exhaust state without reheating.

TAKE NOTE...
When computing the thermal efficiency of a reheat cycle, it is necessary to account for the work output of both turbine stages as well as the total heat addition occurring in the vaporization/superheating and reheating processes. This calculation is illustrated in Example 8.3.

Supercritical

The temperature of the steam entering the turbine is restricted by metallurgical limitations imposed by materials used to fabricate the superheater, reheater, and turbine.

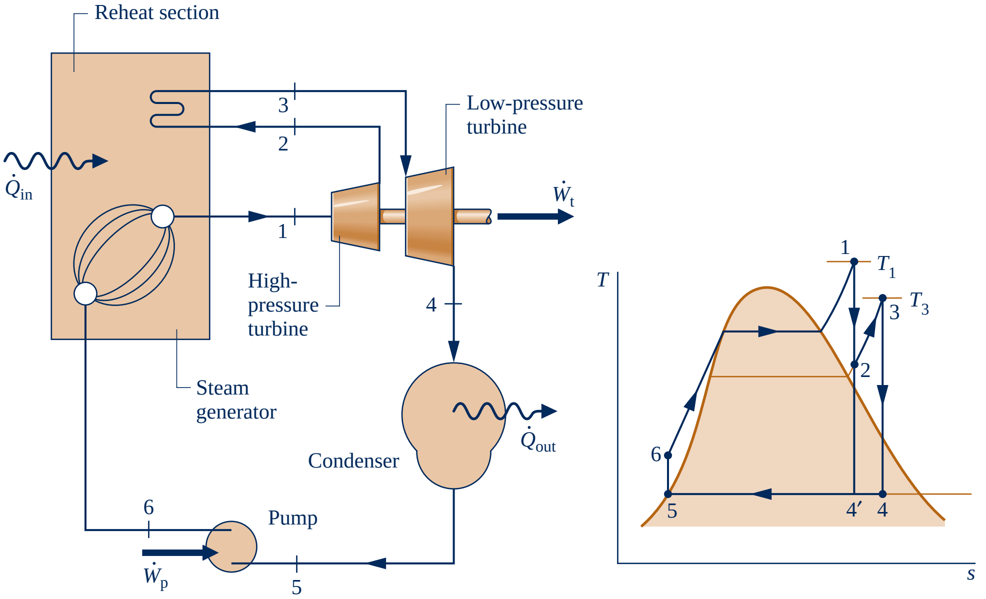


Fig. 8.7 Ideal reheat cycle.

High pressure in the steam generator also requires piping that can withstand great stresses at elevated temperatures. Still, improved materials and fabrication methods have gradually permitted significant increases in maximum allowed cycle temperature and steam generator pressure with corresponding increases in thermal efficiency that save fuel and reduce environmental impact. This progress now allows vapor power plants to operate with steam generator pressures exceeding the critical pressure of water (22.1 MPa, 3203.6 lbf/in.^{2}). These plants are known as **supercritical** vapor power plants.

supercritical

Figure 8.8 shows a supercritical ideal reheat cycle. As indicated by Process 6–1, steam generation occurs at a pressure above the critical pressure. No pronounced phase change occurs during this process, and a conventional boiler is not used. Instead, water flowing through tubes is gradually heated from liquid to vapor without the bubbling associated with boiling. In such cycles, heating is provided by combustion of pulverized coal with air.

Today's supercritical vapor power plants produce steam at pressures and temperatures near 30 MPa (4350 lbf/in.^{2}) and 600°C (1110°F), respectively, permitting thermal efficiencies up to 47%. As superalloys with improved high-temperature limit and corrosion resistance become commercially available, *ultra*-supercritical plants may produce steam at 35 MPa (5075 lbf/in.^{2}) and 700°C (1290°F) with thermal efficiencies exceeding 50%. Subcritical plants have efficiencies only up to about 40%.

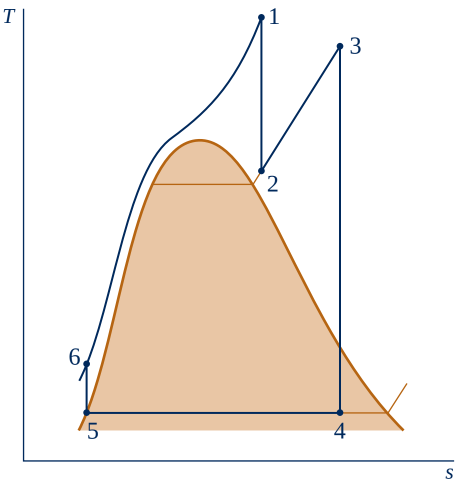


Fig. 8.8 Supercritical ideal reheat cycle.

While installation costs of supercritical plants are somewhat higher per unit of power generated than subcritical plants, fuel costs of supercritical plants are considerably lower owing to increased thermal efficiency. Since less fuel is used for a given power output, supercritical plants produce less carbon dioxide, other combustion gases, and solid waste than subcritical plants. The evolution of supercritical power plants from subcritical counterparts provides a case study on how advances in technology enable increases in thermodynamic efficiency with accompanying fuel savings and reduced environmental impact, and all cost-effectively.

In the next example, the ideal Rankine cycle of Example 8.1 is modified to include superheat and reheat.

EXAMPLE 8.3

Evaluating Performance of an Ideal Reheat Cycle

Steam is the working fluid in an ideal Rankine cycle with superheat and reheat. Steam enters the first-stage turbine at 8.0 MPa, 480°C, and expands to 0.7 MPa. It is then reheated to 440°C before entering the second-stage turbine, where it expands to the condenser pressure of 0.008 MPa. The *net* power output is 100 MW. Determine **(a)** the thermal efficiency of the cycle, **(b)** the mass flow rate of steam, in kg/h, **(c)** the rate of heat transfer $\dot{Q}_{out}$ from the condensing steam as it passes through the condenser, in MW. Discuss the effects of reheat on the vapor power cycle.

SOLUTION

Known: An ideal reheat cycle operates with steam as the working fluid. Operating pressures and temperatures are specified, and the net power output is given.

Find: Determine the thermal efficiency, the mass flow rate of the steam, in kg/h, and the heat transfer rate from the condensing steam as it passes through the condenser, in MW. Discuss.

Schematic and Given Data:

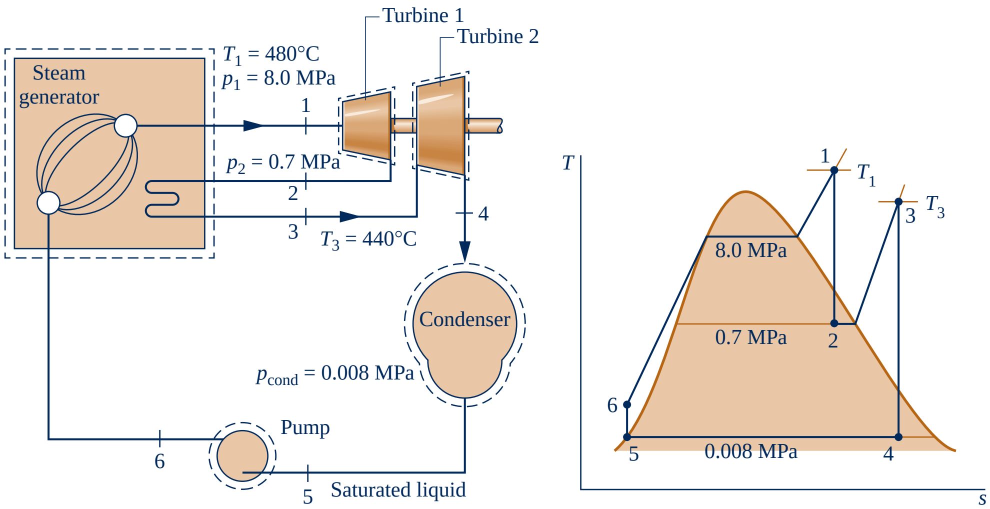


Fig. E8.3

Engineering Model:

1. Each component in the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Condensate exits the condenser as saturated liquid.
5. Kinetic and potential energy effects are negligible.

Analysis: To begin, we fix each of the principal states. Starting at the inlet to the first turbine stage, the pressure is 8.0 MPa and the temperature is 480°C, so the steam is a superheated vapor. From Table A-4, $h_1 = 3348.4$ kJ/kg and $s_1 = 6.6586$ kJ/kg · K.

State 2 is fixed by $p_2 = 0.7$ MPa and $s_2 = s_1$ for the isentropic expansion through the first-stage turbine. Using saturated liquid and saturated vapor data from Table A-3, the quality at state 2 is

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{6.6586 - 1.9922}{6.708 - 1.9922} = 0.9895$$

The specific enthalpy is then

$$\begin{aligned} h_2 &= h_f + x_2 h_{fg} \\ &= 697.22 + (0.9895)2066.3 = 2741.8 \text{ kJ/kg} \end{aligned}$$

State 3 is superheated vapor with $p_3 = 0.7$ MPa and $T_3 = 440$°C, so from Table A-4, $h_3 = 3353.3$ kJ/kg and $s_3 = 7.7571$ kJ/kg · K.

To fix state 4, use $p_4 = 0.008$ MPa and $s_4 = s_3$ for the isentropic expansion through the second-stage turbine. With data from Table A-3, the quality at state 4 is

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{7.7571 - 0.5926}{8.2287 - 0.5926} = 0.9382$$

The specific enthalpy is

$$h_4 = 173.88 + (0.9382)2403.1 = 2428.5 \text{ kJ/kg}$$

State 5 is saturated liquid at 0.008 MPa, so $h_5 = 173.88$ kJ/kg. Finally, the state at the pump exit is the same as in Example 8.1, so $h_6 = 181.94$ kJ/kg.

(a) The *net* power developed by the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_p$$

Mass and energy rate balances for the two turbine stages and the pump reduce to give, respectively

$$\begin{aligned} &\text{Turbine 1:} && \dot{W}_{t1}/\dot{m} = h_1 - h_2 \\ &\text{Turbine 2:} && \dot{W}_{t2}/\dot{m} = h_3 - h_4 \\ &\text{Pump:} && \dot{W}_p/\dot{m} = h_6 - h_5 \end{aligned}$$

where $\dot{m}$ is the mass flow rate of the steam.

The total rate of heat transfer to the working fluid as it passes through the boiler–superheater and reheater is

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_1 - h_6) + (h_3 - h_2)$$

Using these expressions, the thermal efficiency is

$$\begin{aligned} \eta &= \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)} \\ &= \frac{(3348.4 - 2741.8) + (3353.3 - 2428.5) - (181.94 - 173.88)}{(3348.4 - 181.94) + (3353.3 - 2741.8)} \\ &= \frac{606.6 + 924.8 - 8.06}{3166.5 + 611.5} = \frac{1523.3 \text{ kJ/kg}}{3778 \text{ kJ/kg}} = 0.403(40.3\%) \end{aligned}$$

(b) The mass flow rate of the steam can be obtained with the expression for net power given in part (a).

$$\begin{aligned} \dot{m} &= \frac{\dot{W}_{\text{cycle}}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)} \\ &= \frac{(100 \text{ MW})|3600 \text{ s/h}||10^3 \text{ kW/MW}|}{(606.6 + 924.8 - 8.06) \text{ kJ/kg}} = 2.363 \times 10^5 \text{ kg/h} \end{aligned}$$

(c) The rate of heat transfer from the condensing steam to the cooling water is

$$\dot{Q}_{out} = \dot{m}(h_4 - h_5)$$

$$= \frac{2.363 \times 10^5 \text{ kg/h } (2428.5 - 173.88) \text{ kJ/kg}}{|3600 \text{ s/h}||10^3 \text{ kW/MW}|} = 148 \text{ MW}$$

To see the effects of reheat, we compare the present values with their counterparts in Example 8.1. With superheat and reheat, the thermal efficiency is increased over that of the cycle of Example 8.1. For a specified net power output (100 MW), a larger thermal efficiency means that a smaller mass flow rate of steam is required. Moreover, with a greater thermal efficiency the rate of heat transfer to the cooling water is also less, resulting in a reduced demand for cooling water. With reheating, the steam quality at the turbine exhaust is substantially increased over the value for the cycle of Example 8.1.

Skills Developed

Ability to...

- ❑ sketch the *T–s* diagram of the ideal Rankine cycle with reheat.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ apply mass and energy balances.
- ❑ calculate performance parameters for the cycle.

QuickQUIZ What is the rate of heat addition for the reheat process, in MW, and what percent is that value of the total heat addition to the cycle? Ans. 40.1 MW, 16.2%.

The following example illustrates the effect of turbine irreversibilities on the ideal reheat cycle of Example 8.3.

EXAMPLE 8.4

Evaluating Performance of a Reheat Cycle with Turbine Irreversibility

Reconsider the reheat cycle of Example 8.3, but include in the analysis that each turbine stage has the same isentropic efficiency. **(a)** If η_t = 85%, determine the thermal efficiency. **(b)** Plot the thermal efficiency versus turbine stage isentropic efficiency ranging from 85 to 100%.

SOLUTION

Known: A reheat cycle operates with steam as the working fluid. Operating pressures and temperatures are specified. Each turbine stage has the same isentropic efficiency.

Find: If η_t = 85%, determine the thermal efficiency. Also, plot the thermal efficiency versus turbine stage isentropic efficiency ranging from 85 to 100%.

Schematic and Given Data:

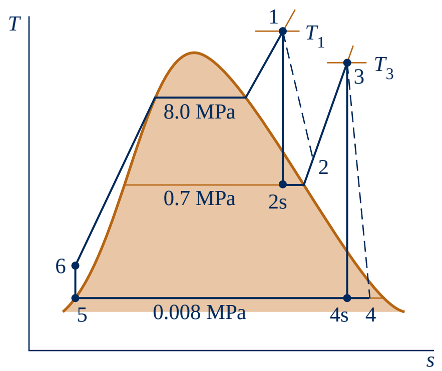


Fig. E8.4a

Engineering Model:

1. As in Example 8.3, each component is analyzed as a control volume at steady state.
2. Except for the two turbine stages, all processes are internally reversible.
3. The turbine and pump operate adiabatically.
4. The condensate exits the condenser as saturated liquid.
5. Kinetic and potential energy effects are negligible.

Analysis:

(a) From the solution to Example 8.3, the following specific enthalpy values are known, in kJ/kg: $h_1 = 3348.4$, $h_{2s} = 2741.8$, $h_3 = 3353.3$, $h_{4s} = 2428.5$, $h_5 = 173.88$, $h_6 = 181.94$.

The specific enthalpy at the exit of the first-stage turbine, h_2, can be determined by solving the expression for the turbine isentropic efficiency, Eq. 8.9, to obtain

$$\begin{aligned} h_2 &= h_1 - \eta_t(h_1 - h_{2s}) \\ &= 3348.4 - 0.85(3348.4 - 2741.8) = 2832.8 \text{ kJ/kg} \end{aligned}$$

The specific enthalpy at the exit of the second-stage turbine can be found similarly:

$$\begin{aligned} h_4 &= h_3 - \eta_t(h_3 - h_{4s}) \\ &= 3353.3 - 0.85(3353.3 - 2428.5) = 2567.2 \text{ kJ/kg} \end{aligned}$$

The thermal efficiency is then

1

$$\begin{aligned} \eta &= \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)} \\ &= \frac{(3348.4 - 2832.8) + (3353.3 - 2567.2) - (181.94 - 173.88)}{(3348.4 - 181.94) + (3353.3 - 2832.8)} \\ &= \frac{1293.6 \text{ kJ/kg}}{3687.0 \text{ kJ/kg}} = 0.351\ (35.1\%) \end{aligned}$$

(b) The *IT* code for the solution follows, where etat1 is η_{t1}, etat2 is η_{t2}, eta is η, Wnet $= \dot{W}_{net}/\dot{m}$, and Qin $= \dot{Q}_{in}/\dot{m}$.

```
// Fix the states

T1 = 480// °C
p1 = 80 // bar
h1 = h_PT ("Water/Steam", p1, T1)
s1 = s_PT ("Water/Steam", p1, T1)

p2 = 7 // bar
h2s = h_Ps ("Water/Steam", p2, s1)
etat1 = 0.85
h2 = h1 − etat1 * (h1 − h2s)

T3 = 440 // °C
p3 = p2
h3 = h_PT ("Water/Steam", p3, T3)
s3 = s_PT ("Water/Steam", p3, T3)

p4 = 0.08//bar
h4s = h_Ps ("Water/Steam", p4, s3)
etat2 = etat1
h4 = h3 − etat2 * (h3 − h4s)

p5 = p4
h5 = hsat_Px ("Water/Steam", p5, 0) // kJ/kg
v5 = vsat_Px ("Water/Steam", p5, 0) // m³/kg

p6 = p1
h6 = h5 + v5 * (p6 − p5) * 100// The 100 in this expression is a unit conversion factor.

// Calculate thermal efficiency
Wnet = (h1 − h2) + (h3 − h4) − (h6 − h5)
Qin = (h1 − h6) + (h3 − h2)
eta = Wnet/Qin
```

Using the **Explore** button, sweep eta from 0.85 to 1.0 in steps of 0.01. Then, using the **Graph** button, obtain the following plot:

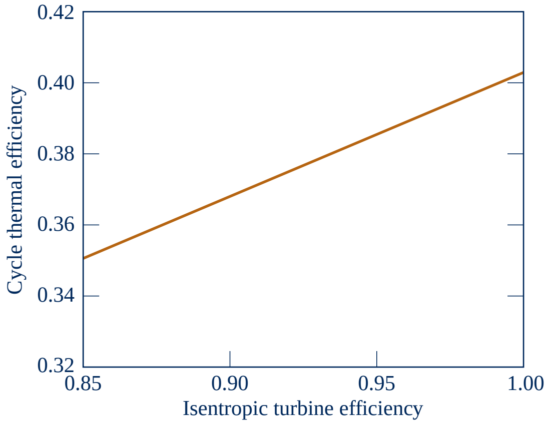


Fig. E8.4b

From Fig. E8.4b, we see that the cycle thermal efficiency increases from 0.351 to 0.403 as turbine stage isentropic efficiency increases from 0.85 to 1.00, as expected based on the results of Example 8.3 and part (a) of the present example. Turbine isentropic efficiency is seen to have a significant effect on cycle thermal efficiency.

1 Owing to the irreversibilities present in the turbine stages, the net work per unit of mass developed in the present case is significantly less than in the case of Example 8.3. The thermal efficiency is also considerably less.

Skills Developed

Ability to...

- ❑ sketch the *T*–*s* diagram of the Rankine cycle with reheat, including turbine and pump irreversibilities.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ apply mass, energy, and entropy principles.
- ❑ calculate performance parameters for the cycle.

QuickQUIZ If the temperature T_3 were increased to 480°C, would you expect the thermal efficiency to increase, decrease, or stay the same? **Ans.** Increase.

8.4 Improving Performance—Regenerative Vapor Power Cycle

Another commonly used method for increasing the thermal efficiency of vapor power plants is *regenerative feedwater heating,* or simply **regeneration**. This is the subject of the present section.

regeneration

To introduce the principle underlying regenerative feedwater heating, consider Fig. 8.3 once again. In cycle 1–2–3–4–a–1, the working fluid enters the boiler as a compressed liquid at state 4 and is heated while in the liquid phase to state a. With regenerative feedwater heating, the working fluid enters the boiler at a state *between* 4 and a. As a result, the average temperature of heat addition is increased, thereby tending to increase the thermal efficiency.

8.4.1 Open Feedwater Heaters

Let us consider how regeneration can be accomplished using an **open feedwater heater**, a direct contact-type heat exchanger in which streams at different temperatures mix

open feedwater heater

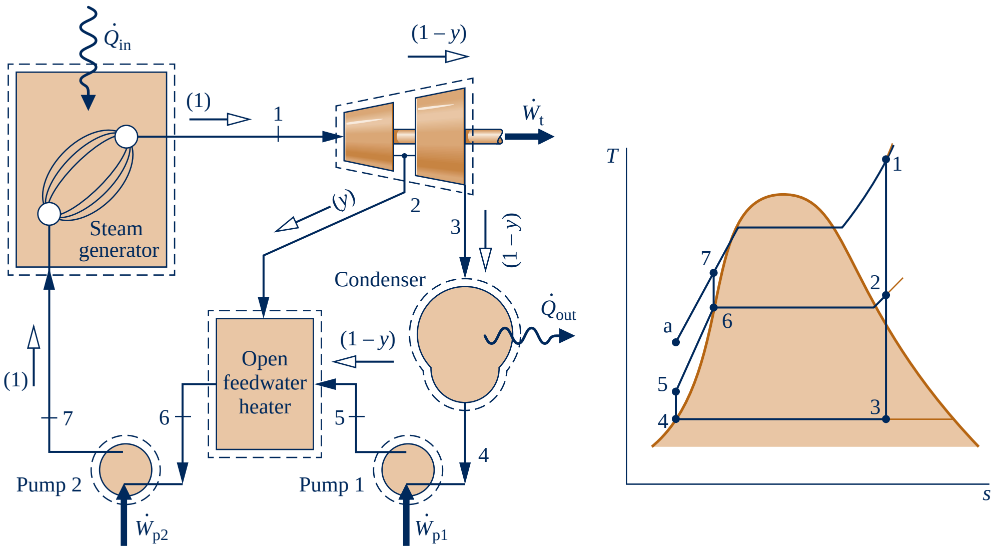


Fig. 8.9 **Regenerative vapor power cycle with one open feedwater heater.**

to form a stream at an intermediate temperature. Shown in Fig. 8.9 are the schematic diagram and the associated T–s diagram for a regenerative vapor power cycle having one open feedwater heater. For this cycle, the working fluid passes isentropically through the turbine stages and pumps, and flow through the steam generator, condenser, and feedwater heater takes place with no pressure drop in any of these components. Still, there is a source of irreversibility owing to mixing within the feedwater heater.

Steam enters the first-stage turbine at state 1 and expands to state 2, where a fraction of the total flow is *extracted,* or *bled,* into an open feedwater heater operating at the extraction pressure, p_2. The rest of the steam expands through the second-stage turbine to state 3. This portion of the total flow is condensed to saturated liquid, state 4, and then pumped to the extraction pressure and introduced into the feedwater heater at state 5. A single mixed stream exits the feedwater heater at state 6. For the case shown in Fig. 8.9, the mass flow rates of the streams entering the feedwater heater are such that state 6 is saturated liquid at the extraction pressure. The liquid at state 6 is then pumped to the steam generator pressure and enters the steam generator at state 7. Finally, the working fluid is heated from state 7 to state 1 in the steam generator.

Referring to the T–s diagram of the cycle, note that the heat addition would take place from state 7 to state 1, rather than from state a to state 1, as would be the case without regeneration. Accordingly, the amount of energy that must be supplied from the combustion of a fossil fuel, or another source, to vaporize and superheat the steam would be reduced. This is the desired outcome. Only a portion of the total flow expands through the second-stage turbine (Process 2–3), however, so less work would be developed as well. In practice, operating conditions are such that the reduction in heat added more than offsets the decrease in net work developed, resulting in an increased thermal efficiency in regenerative power plants.

Cycle Analysis

Consider next the thermodynamic analysis of the regenerative cycle illustrated in Fig. 8.9. An important initial step in analyzing any regenerative vapor cycle is the evaluation of the mass flow rates through each of the components. Taking a single control volume enclosing both turbine stages, the mass rate balance reduces at steady state to

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_1$$

where $\dot{m}_1$ is the rate at which mass enters the first-stage turbine at state 1, $\dot{m}_2$ is the rate at which mass is extracted and exits at state 2, and $\dot{m}_3$ is the rate at which mass exits the second-stage turbine at state 3. Dividing by $\dot{m}_1$ places this on the basis of a *unit of mass* passing through the first-stage turbine

$$\frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_3}{\dot{m}_1} = 1$$

Denoting the fraction of the total flow extracted at state 2 by y ($y = \dot{m}_2/\dot{m}_1$), the fraction of the total flow passing through the second-stage turbine is

$$\frac{\dot{m}_3}{\dot{m}_1} = 1 - y \tag{8.11}$$

The fractions of the total flow at various locations are indicated in parentheses on Fig. 8.9.

The fraction y can be determined by applying the conservation of mass and conservation of energy principles to a control volume around the feedwater heater. Assuming no heat transfer between the feedwater heater and its surroundings and ignoring kinetic and potential energy effects, the mass and energy rate balances reduce at steady state to give

$$0 = yh_2 + (1 - y)h_5 - h_6$$

Solving for y

$$y = \frac{h_6 - h_5}{h_2 - h_5} \tag{8.12}$$

Equation 8.12 allows the fraction y to be determined when states 2, 5, and 6 are fixed.

Expressions for the principal work and heat transfers of the regenerative cycle can be determined by applying mass and energy rate balances to control volumes around the individual components. Beginning with the turbine, the total work is the sum of the work developed by each turbine stage. Neglecting kinetic and potential energy effects and assuming no heat transfer with the surroundings, we can express the total turbine work on the basis of a unit of mass passing through the first-stage turbine as

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3) \tag{8.13}$$

The total pump work is the sum of the work required to operate each pump individually. On the basis of a unit of mass passing through the first-stage turbine, the total pump work is

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1 - y)(h_5 - h_4) \tag{8.14}$$

The energy added by heat transfer to the working fluid passing through the steam generator, per unit of mass expanding through the first-stage turbine, is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_7 \tag{8.15}$$

and the energy rejected by heat transfer to the cooling water is

$$\frac{\dot{Q}_{out}}{\dot{m}_1} = (1 - y)(h_3 - h_4) \tag{8.16}$$

The following example illustrates the analysis of a regenerative cycle with one open feedwater heater, including the evaluation of properties at state points around the cycle and the determination of the fractions of the total flow at various locations.

EXAMPLE 8.5

Considering a Regenerative Cycle with Open Feedwater Heater

Consider a regenerative vapor power cycle with one open feedwater heater. Steam enters the turbine at 8.0 MPa, 480°C and expands to 0.7 MPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 0.7 MPa. The remaining steam expands through the second-stage turbine to the condenser pressure of 0.008 MPa. Saturated liquid exits the open feedwater heater at 0.7 MPa. The isentropic efficiency of each turbine stage is 85% and each pump operates isentropically. If the net power output of the cycle is 100 MW, determine **(a)** the thermal efficiency and **(b)** the mass flow rate of steam entering the first turbine stage, in kg/h.

SOLUTION

Known: A regenerative vapor power cycle operates with steam as the working fluid. Operating pressures and temperatures are specified; the isentropic efficiency of each turbine stage and the net power output are also given.

Find: Determine the thermal efficiency and the mass flow rate into the turbine, in kg/h.

Schematic and Given Data:

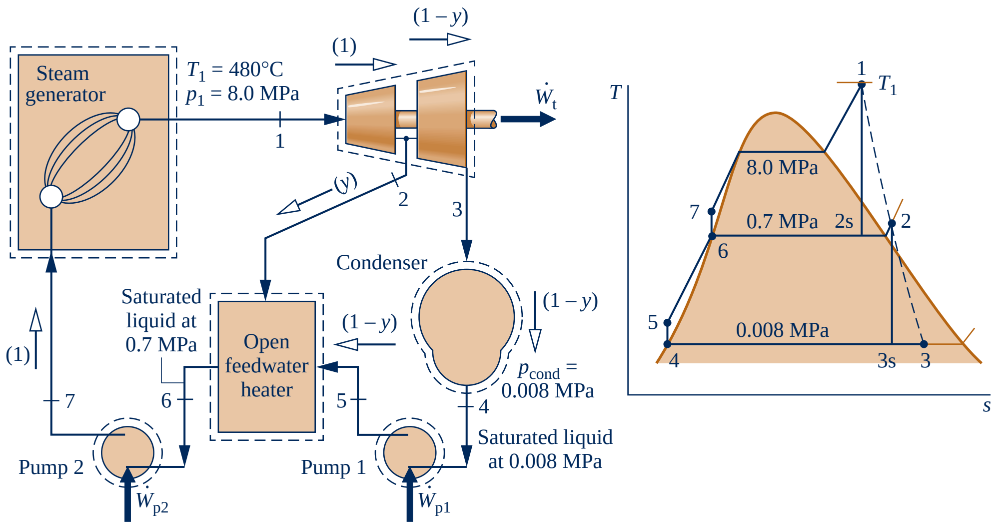


Fig. E8.5

Engineering Model:

1. Each component in the cycle is analyzed as a steady-state control volume. The control volumes are shown in the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible, except for the expansions through the two turbine stages and mixing in the open feedwater heater.
3. The turbines, pumps, and feedwater heater operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the open feedwater heater, and saturated liquid exits the condenser.

Analysis: The specific enthalpy at states 1 and 4 can be read from the steam tables. The specific enthalpy at state 2 is evaluated in the solution to Example 8.4. The specific entropy at state 2 can be obtained from the steam tables using the known values of enthalpy and pressure at this state. In summary, $h_1 = 3348.4$ kJ/kg, $h_2 = 2832.8$ kJ/kg, $s_2 = 6.8606$ kJ/kg · K, $h_4 = 173.88$ kJ/kg.

The specific enthalpy at state 3 can be determined using the isentropic efficiency of the second-stage turbine

$$h_3 = h_2 - \eta_t(h_2 - h_{3s})$$

With $s_{3s} = s_2$, the quality at state 3s is $x_{3s} = 0.8208$; using this, we get $h_{3s} = 2146.3$ kJ/kg. Hence

$$h_3 = 2832.8 - 0.85(2832.8 - 2146.3) = 2249.3 \text{ kJ/kg}$$

State 6 is saturated liquid at 0.7 MPa. Thus, $h_6 = 697.22$ kJ/kg.
Since the pumps operate isentropically, the specific enthalpy values at states 5 and 7 can be determined as

$$h_5 = h_4 + v_4(p_5 - p_4)$$
$$= 173.88 + (1.0084 \times 10^{-3})(\text{m}^3/\text{kg})(0.7 - 0.008)\ \text{MPa} \left|\frac{10^6\ \text{N/m}^2}{1\ \text{MPa}}\right| \left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 174.6\ \text{kJ/kg}$$
$$h_7 = h_6 + v_6(p_7 - p_6)$$
$$= 697.22 + (1.1080 \times 10^{-3})(8.0 - 0.7)|10^3| = 705.3\ \text{kJ/kg}$$

Applying mass and energy rate balances to a control volume enclosing the open heater, we find the fraction y of the flow extracted at state 2 from

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 174.6}{2832.8 - 174.6} = 0.1966$$

(a) On the basis of a unit of mass passing through the first-stage turbine, the total turbine work output is

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$
$$= (3348.4 - 2832.8) + (0.8034)(2832.8 - 2249.3) = 984.4\ \text{kJ/kg}$$

The total pump work per unit of mass passing through the first-stage turbine is

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1 - y)(h_5 - h_4)$$
$$= (705.3 - 697.22) + (0.8034)(174.6 - 173.88) = 8.7\ \text{kJ/kg}$$

The heat added in the steam generator per unit of mass passing through the first-stage turbine is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_7 = 3348.4 - 705.3 = 2643.1\ \text{kJ/kg}$$

The thermal efficiency is then

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{984.4 - 8.7}{2643.1} = 0.369\ (36.9\%)$$

(b) The mass flow rate of the steam entering the turbine, $\dot{m}_1$, can be determined using the given value for the net power output, 100 MW. Since

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p$$

and

$$\frac{\dot{W}_t}{\dot{m}_1} = 984.4\ \text{kJ/kg} \qquad \text{and} \qquad \frac{\dot{W}_p}{\dot{m}_1} = 8.7\ \text{kJ/kg}$$

it follows that

$$\dot{m}_1 = \frac{(100\ \text{MW})|3600\ \text{s/h}|}{(984.4 - 8.7)\ \text{kJ/kg}} \left|\frac{10^3\ \text{kJ/s}}{1\ \text{MW}}\right| = 3.69 \times 10^5\ \text{kg/h}$$

1 Note that the fractions of the total flow at various locations are labeled on the figure.

Skills Developed

Ability to...

- ❑ sketch the T–s diagram of the regenerative vapor power cycle with one open feedwater heater.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ apply mass, energy, and entropy principles.
- ❑ calculate performance parameters for the cycle.

QuickQUIZ If the mass flow rate of steam entering the first-stage turbine were 150 kg/s, what would be the net power, in MW, and the fraction of steam extracted, y? **Ans.** 146.4 MW, 0.1966.

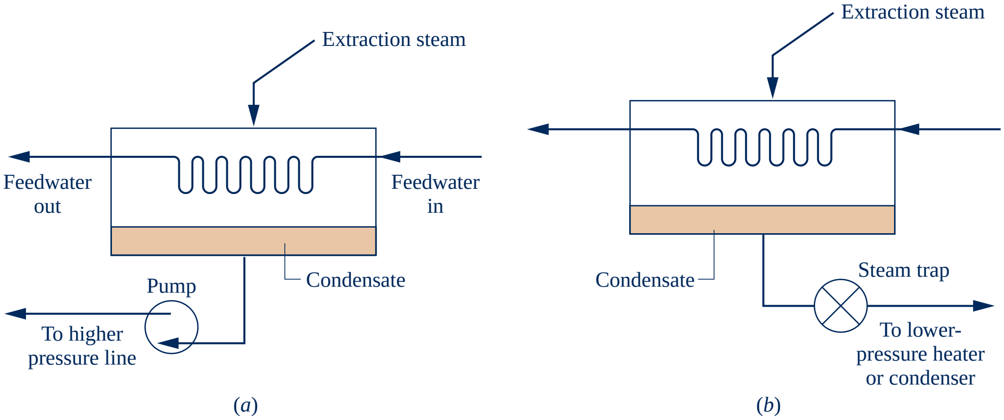


Fig. 8.10 Examples of closed feedwater heaters.

8.4.2 Closed Feedwater Heaters

closed feedwater heater

Regenerative feedwater heating also can be accomplished with **closed feedwater heaters**. Closed heaters are shell-and-tube-type recuperators in which the feedwater temperature increases as the extracted steam condenses on the outside of the tubes carrying the feedwater. Since the two streams do not mix, they can be at different pressures.

The diagrams of Fig. 8.10 show two different schemes for removing the condensate from closed feedwater heaters. In Fig. 8.10*a*, this is accomplished by means of a pump whose function is to pump the condensate forward to a higher-pressure point in the cycle. In Fig. 8.10*b*, the condensate is allowed to expand through a *trap* into a feedwater heater operating at a lower pressure or into the condenser. A trap is a type of valve that permits only liquid to pass through to a region of lower pressure.

A regenerative vapor power cycle having one closed feedwater heater with the condensate trapped into the condenser is shown schematically in Fig. 8.11. For this cycle, the working fluid passes isentropically through the turbine stages and pumps. Except for expansion through the trap, there are no pressure drops accompanying

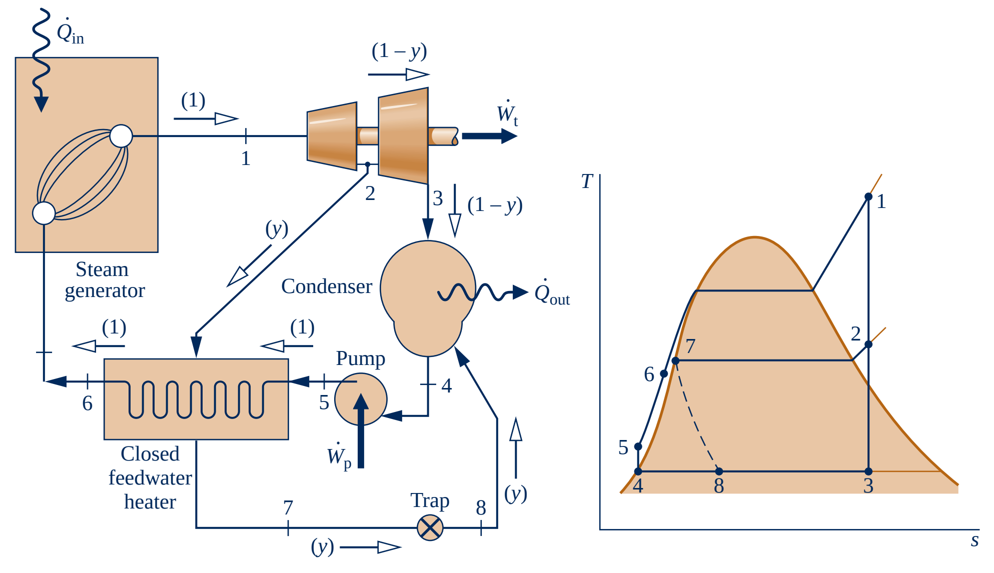


Fig. 8.11 Regenerative vapor power cycle with one closed feedwater heater.

flow through other components. The T–s diagram shows the principal states of the cycle.

The total steam flow expands through the first-stage turbine from state 1 to state 2. At this location, a fraction of the flow is bled into the closed feedwater heater, where it condenses. Saturated liquid at the extraction pressure exits the feedwater heater at state 7. The condensate is then trapped into the condenser, where it is reunited with the portion of the total flow passing through the second-stage turbine. The expansion from state 7 to state 8 through the trap is irreversible, so it is shown by a dashed line on the T–s diagram. The total flow exiting the condenser as saturated liquid at state 4 is pumped to the steam generator pressure and enters the feedwater heater at state 5. The temperature of the feedwater is increased in passing through the feedwater heater. The feedwater then exits at state 6. The cycle is completed as the working fluid is heated in the steam generator at constant pressure from state 6 to state 1. Although the closed heater shown on the figure operates with no pressure drop in either stream, there is a source of irreversibility due to the stream-to-stream temperature difference.

Cycle Analysis

The schematic diagram of the cycle shown in Fig. 8.11 is labeled with the fractions of the total flow at various locations. This is usually helpful in analyzing such cycles. The fraction of the total flow extracted, y, can be determined by applying the conservation of mass and conservation of energy principles to a control volume around the closed heater. Assuming no heat transfer between the feedwater heater and its surroundings and neglecting kinetic and potential energy effects, the mass and energy rate balances reduce at steady state to give

$$0 = y(h_2 - h_7) + (h_5 - h_6)$$

Solving for y

$$y = \frac{h_6 - h_5}{h_2 - h_7} \tag{8.17}$$

The principal work and heat transfers are evaluated as discussed previously.

8.4.3 Multiple Feedwater Heaters

The thermal efficiency of the regenerative cycle can be increased by incorporating several feedwater heaters at suitably chosen pressures. The number of feedwater heaters used is based on economic considerations, since incremental increases in thermal efficiency achieved with each additional heater must justify the added capital costs (heater, piping, pumps, etc.). Power plant designers use computer programs to simulate the thermodynamic and economic performance of different designs to help them decide on the number of heaters to use, the types of heaters, and the pressures at which they should operate.

Figure 8.12 shows the layout of a power plant with three closed feedwater heaters and one open heater. Power plants with multiple feedwater heaters ordinarily have at least one open feedwater heater operating at a pressure greater than atmospheric pressure so that oxygen and other dissolved gases can be vented from the cycle. This procedure, known as **deaeration**, is needed to maintain the purity of the working fluid in order to minimize corrosion. Actual power plants have many of the same basic features as the one shown in the figure.

deaeration

In analyzing regenerative vapor power cycles with multiple feedwater heaters, it is good practice to base the analysis on a unit of mass entering the first-stage turbine. To clarify the quantities of matter flowing through the various plant components, the

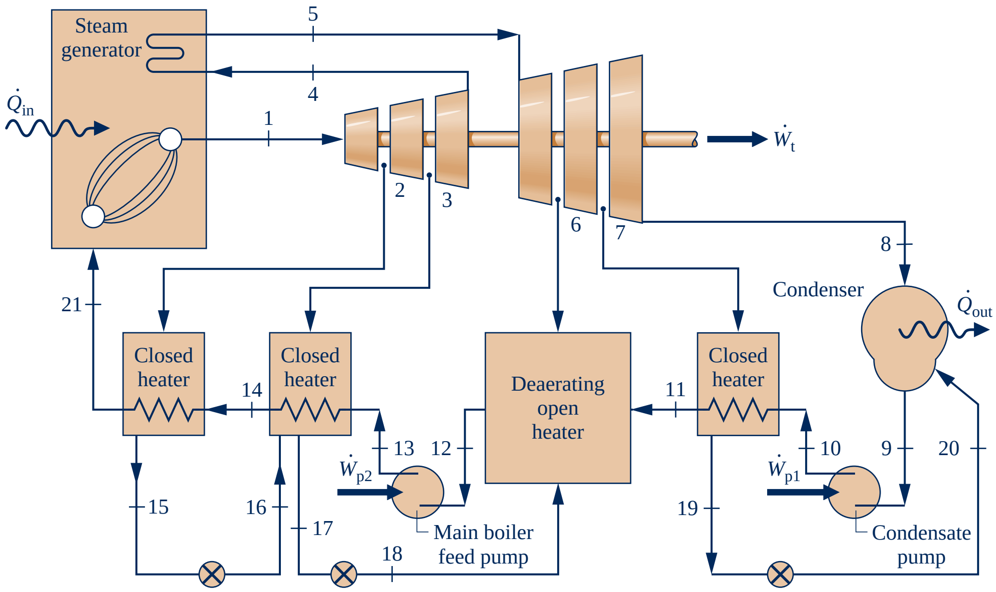


Fig. 8.12 **Example of a power plant layout.**

fractions of the total flow removed at each extraction point and the fraction of the total flow remaining at each state point in the cycle should be labeled on a schematic diagram of the cycle. The fractions extracted are determined from mass and energy rate balances for control volumes around each of the feedwater heaters, starting with the highest-pressure heater and proceeding to each lower-pressure heater in turn. This procedure is used in the next example that involves a reheat–regenerative vapor power cycle with two feedwater heaters, one open feedwater heater and one closed feedwater heater.

EXAMPLE 8.6

Considering a Reheat–Regenerative Cycle with Two Feedwater Heaters

Consider a reheat–regenerative vapor power cycle with two feedwater heaters, a closed feedwater heater and an open feedwater heater. Steam enters the first turbine at 8.0 MPa, 480°C and expands to 0.7 MPa. The steam is reheated to 440°C before entering the second turbine, where it expands to the condenser pressure of 0.008 MPa. Steam is extracted from the first turbine at 2 MPa and fed to the closed feedwater heater. Feedwater leaves the closed heater at 205°C and 8.0 MPa, and condensate exits as saturated liquid at 2 MPa. The condensate is trapped into the open feedwater heater. Steam extracted from the second turbine at 0.3 MPa is also fed into the open feedwater heater, which operates at 0.3 MPa. The stream exiting the open feedwater heater is saturated liquid at 0.3 MPa. The *net* power output of the cycle is 100 MW. There is no stray heat transfer from any component to its surroundings. If the working fluid experiences no irreversibilities as it passes through the turbines, pumps, steam generator, reheater, and condenser, determine **(a)** the thermal efficiency, **(b)** the mass flow rate of the steam entering the first turbine, in kg/h.

SOLUTION

Known: A reheat–regenerative vapor power cycle operates with steam as the working fluid. Operating pressures and temperatures are specified, and the net power output is given.

Find: Determine the thermal efficiency and the mass flow rate entering the first turbine, in kg/h.

Schematic and Given Data:

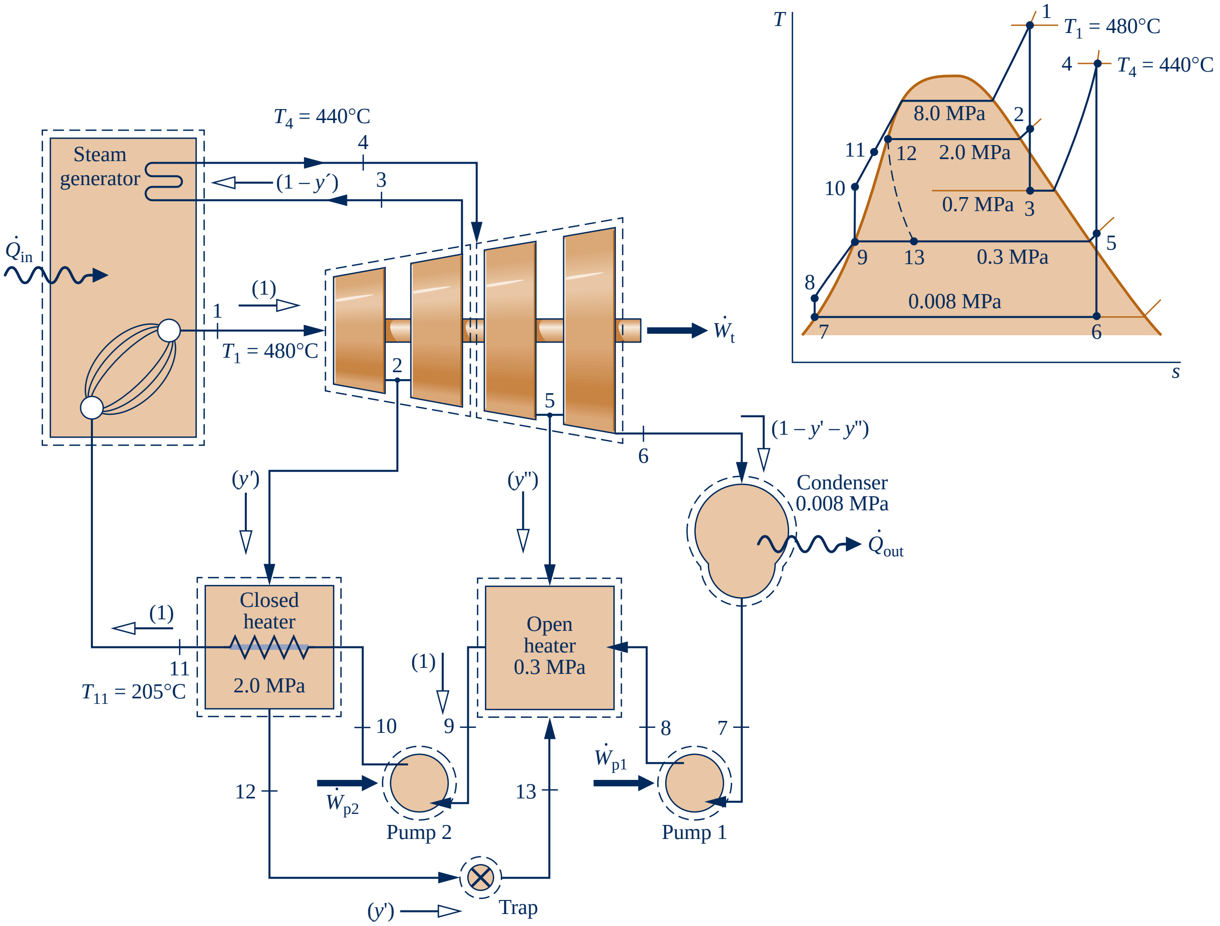


Fig. E8.6

Engineering Model:

1. Each component in the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.

2. There is no stray heat transfer from any component to its surroundings.

3. The working fluid undergoes internally reversible processes as it passes through the turbines, pumps, steam generator, reheater, and condenser.

4. The expansion through the trap is a *throttling* process.

5. Kinetic and potential energy effects are negligible.

6. Condensate exits the closed heater as a saturated liquid at 2 MPa. Feedwater exits the open heater as a saturated liquid at 0.3 MPa. Condensate exits the condenser as a saturated liquid.

Analysis: Let us determine the specific enthalpies at the principal states of the cycle. State 1 is the same as in Example 8.3, so $h_1 = 3348.4$ kJ/kg and $s_1 = 6.6586$ kJ/kg · K.

State 2 is fixed by $p_2 = 2.0$ MPa and the specific entropy s_2, which is the same as that of state 1. Interpolating in Table A-4, we get $h_2 = 2963.5$ kJ/kg. The state at the exit of the first turbine is the same as at the exit of the first turbine of Example 8.3, so $h_3 = 2741.8$ kJ/kg.

State 4 is superheated vapor at 0.7 MPa, 440°C. From Table A-4, $h_4 = 3353.3$ kJ/kg and $s_4 = 7.7571$ kJ/kg · K. Interpolating in Table A-4 at $p_5 = 0.3$ MPa and $s_5 = s_4 = 7.7571$ kJ/kg · K, the enthalpy at state 5 is $h_5 = 3101.5$ kJ/kg.

Using $s_6 = s_4$, the quality at state 6 is found to be $x_6 = 0.9382$. So

$$\begin{aligned} h_6 &= h_f + x_6 h_{fg} \\ &= 173.88 + (0.9382)2403.1 = 2428.5 \text{ kJ/kg} \end{aligned}$$

At the condenser exit, $h_7 = 173.88$ kJ/kg. The specific enthalpy at the exit of the first pump is

$$\begin{aligned} h_8 &= h_7 + v_7(p_8 - p_7) \\ &= 173.88 + (1.0084)(0.3 - 0.008) = 174.17 \text{ kJ/kg} \end{aligned}$$

The required unit conversions were considered in previous examples.

The liquid leaving the open feedwater heater at state 9 is saturated liquid at 0.3 MPa. The specific enthalpy is $h_9 = 561.47$ kJ/kg. The specific enthalpy at the exit of the second pump is

$$\begin{aligned} h_{10} &= h_9 + v_9(p_{10} - p_9) \\ &= 561.47 + (1.0732)(8.0 - 0.3) = 569.73 \text{ kJ/kg} \end{aligned}$$

The condensate leaving the closed heater is saturated at 2 MPa. From Table A-3, $h_{12} = 908.79$ kJ/kg. The fluid passing through the trap undergoes a throttling process, so $h_{13} = 908.79$ kJ/kg.

The specific enthalpy of the feedwater exiting the closed heater at 8.0 MPa and 205°C is found using Eq. 3.13 as

$$\begin{aligned} h_{11} &= h_f + v_f(p_{11} - p_{sat}) \\ &= 875.1 + (1.1646)(8.0 - 1.73) = 882.4 \text{ kJ/kg} \end{aligned}$$

where h_f and v_f are the saturated liquid specific enthalpy and specific volume at 205°C, respectively, and p_{sat} is the saturation pressure in MPa at this temperature. Alternatively, h_{11} can be found from Table A-5.

The schematic diagram of the cycle is labeled with the fractions of the total flow into the turbine that remain at various locations. The fractions of the total flow diverted to the closed heater and open heater, respectively, are $y' = \dot{m}_2/\dot{m}_1$ and $y'' = \dot{m}_5/\dot{m}_1$, where $\dot{m}_1$ denotes the mass flow rate entering the first turbine.

The fraction y' can be determined by application of mass and energy rate balances to a control volume enclosing the closed heater. The result is

$$y' = \frac{h_{11} - h_{10}}{h_2 - h_{12}} = \frac{882.4 - 569.73}{2963.5 - 908.79} = 0.1522$$

The fraction y'' can be determined by application of mass and energy rate balances to a control volume enclosing the open heater, resulting in

$$0 = y''h_5 + (1 - y' - y'')h_8 + y'h_{13} - h_9$$

Solving for y''

$$\begin{aligned} y'' &= \frac{(1 - y')h_8 + y'h_{13} - h_9}{h_8 - h_5} \\ &= \frac{(0.8478)174.17 + (0.1522)908.79 - 561.47}{174.17 - 3101.5} \\ &= 0.0941 \end{aligned}$$

(a) The following work and heat transfer values are expressed on the basis of a unit mass entering the first turbine. The work developed by the first turbine per unit of mass entering is the sum

$$\begin{aligned} \frac{\dot{W}_{t1}}{\dot{m}_1} &= (h_1 - h_2) + (1 - y')(h_2 - h_3) \\ &= (3348.4 - 2963.5) + (0.8478)(2963.5 - 2741.8) \\ &= 572.9 \text{ kJ/kg} \end{aligned}$$

Similarly, for the second turbine

$$\begin{aligned} \frac{\dot{W}_{t2}}{\dot{m}_1} &= (1 - y')(h_4 - h_5) + (1 - y' - y'')(h_5 - h_6) \\ &= (0.8478)(3353.3 - 3101.5) + (0.7537)(3101.5 - 2428.5) \\ &= 720.7 \text{ kJ/kg} \end{aligned}$$

For the first pump

$$\frac{\dot{W}_{p1}}{\dot{m}_1} = (1 - y' - y'')(h_8 - h_7)$$
$$= (0.7537)(174.17 - 173.88) = 0.22 \text{ kJ/kg}$$

and for the second pump

$$\frac{\dot{W}_{p2}}{\dot{m}_1} = (h_{10} - h_9)$$
$$= 569.73 - 561.47 = 8.26 \text{ kJ/kg}$$

The total heat added is the sum of the energy added by heat transfer during boiling/superheating and reheating. When expressed on the basis of a unit of mass entering the first turbine, this is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = (h_1 - h_{11}) + (1 - y')(h_4 - h_3)$$
$$= (3348.4 - 882.4) + (0.8478)(3353.3 - 2741.8)$$
$$= 2984.4 \text{ kJ/kg}$$

With the foregoing values, the thermal efficiency is

$$\eta = \frac{\dot{W}_{t1}/\dot{m}_1 + \dot{W}_{t2}/\dot{m}_1 - \dot{W}_{p1}/\dot{m}_1 - \dot{W}_{p2}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1}$$
$$= \frac{572.9 + 720.7 - 0.22 - 8.26}{2984.4} = 0.431 \; (43.1\%)$$

(b) The mass flow rate entering the first turbine can be determined using the given value of the net power output. Thus

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{\dot{W}_{t1}/\dot{m}_1 + \dot{W}_{t2}/\dot{m}_1 - \dot{W}_{p1}/\dot{m}_1 - \dot{W}_{p2}/\dot{m}_1}$$

1
$$= \frac{(100 \text{ MW})|3600 \text{ s/h}|\,|10^3 \text{ kW/MW}|}{1285.1 \text{ kJ/kg}} = 2.8 \times 10^5 \text{ kg/h}$$

1 Compared to the corresponding values determined for the simple Rankine cycle of Example 8.1, the thermal efficiency of the present regenerative cycle is substantially greater and the mass flow rate is considerably less.

Skills Developed

Ability to...

- ❑ sketch the *T–s* diagram of the reheat–regenerative vapor power cycle with one closed and one open feedwater heater.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ apply mass, energy, and entropy principles.
- ❑ calculate performance parameters for the cycle.

QuickQUIZ If each turbine stage had an isentropic efficiency of 85%, at which numbered states would the specific enthalpy values change? Ans. 2, 3, 5, and 6.

8.5 Other Vapor Power Cycle Aspects

In this section we consider vapor power cycle aspects related to working fluids, cogeneration systems, and carbon capture and storage.

8.5.1 Working Fluids

Demineralized water is used as the working fluid in the vast majority of vapor power systems because it is plentiful, low cost, nontoxic, chemically stable, and relatively noncorrosive. Water also has a large change in specific enthalpy as it vaporizes at typical steam generator pressures, which tends to limit the mass flow rate for a desired power output. With water, the pumping power is characteristically low, and the techniques of superheat, reheat, and regeneration are effective for improving power plant performance.

The high critical pressure of water (22.1 MPa, 3204 lbf/in.^{2}) has posed a challenge to engineers seeking to improve thermal efficiency by increasing steam generator pressure and thus average temperature of heat addition. See the discussion of supercritical cycles in Sec. 8.3.

Although water has some shortcomings as a working fluid, no other single substance is more satisfactory for large electrical generating plants. Still, vapor cycles intended for special applications employ working fluids more in tune with the application at hand than water.

organic cycle

Organic Rankine cycles employ organic substances as working fluids, including pentane, mixtures of hydrocarbons, commonly used refrigerants, ammonia, and silicon oil. The organic working fluid is typically selected to meet the requirements of the particular application. For instance, the relatively low boiling point of these substances allows the Rankine cycle to produce power from low-temperature sources, including industrial *waste heat,* geothermal hot water, and fluids heated by concentrating-solar collectors.

binary cycle

A **binary vapor cycle** couples two vapor cycles so the energy discharged by heat transfer from one cycle is the input for the other. Different working fluids are used in these cycles, one having advantageous high-temperature characteristics and another with complementary characteristics at the low-temperature end of the overall operating range. Depending on the application, these working fluids might include water and organic substances. The result is a combined cycle having a high average temperature of heat addition and a low average temperature of heat rejection, and thus a thermal efficiency greater than either cycle has individually.

Figure 8.13 shows the schematic and accompanying *T–s* diagram of a binary vapor cycle. In this arrangement, two ideal Rankine cycles are combined using an interconnecting heat exchanger that serves as the condenser for the higher-temperature cycle (topping cycle) and boiler for the lower-temperature cycle (bottoming cycle). Heat rejected from the topping cycle provides the heat input for the bottoming cycle.

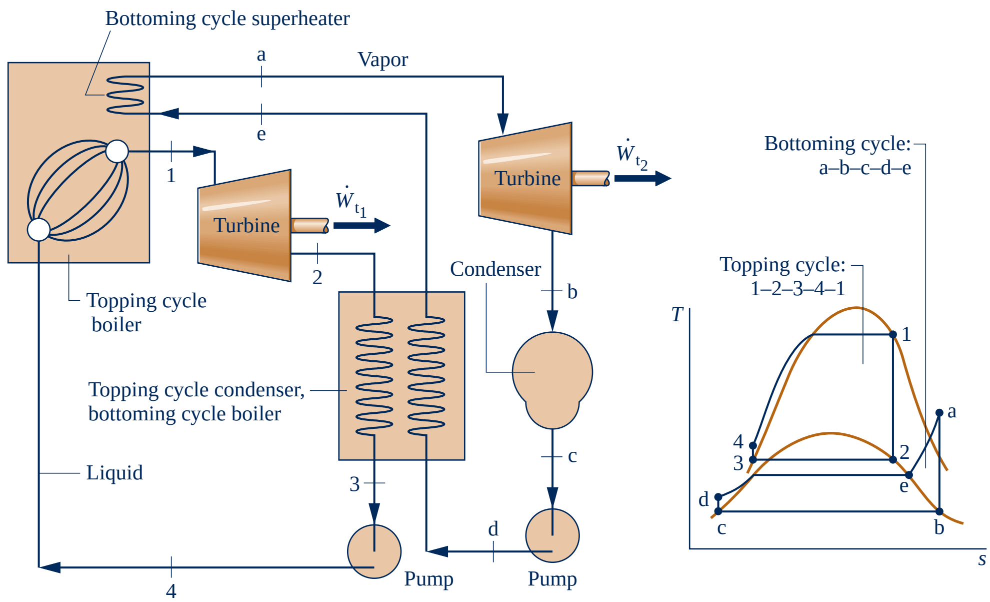


Fig. 8.13 **Binary vapor cycle.**

8.5.2 Cogeneration

Our society can use fuel more effectively through greater use of **cogeneration** systems, also known as combined heat and power systems. Cogeneration systems are integrated systems that simultaneously yield two valuable products, electricity and steam (or hot water), from a single fuel input. Cogeneration systems typically provide cost savings relative to producing power and steam (or hot water) in separate systems. Costing of cogeneration systems is introduced in Sec. 7.7.3.

cogeneration

Cogeneration systems are widely deployed in industrial plants, refineries, paper mills, food processing plants, and other facilities requiring process steam, hot water, and electricity for machines, lighting, and other purposes. **District heating** is another important cogeneration application. District heating plants are located within communities to provide steam or hot water for space heating and other thermal needs together with electricity for domestic, commercial, and industrial use. For instance, in New York City, district heating plants provide heating to Manhattan buildings while also generating electricity for various uses.

district heating

Cogeneration systems can be based on vapor power plants, gas turbine power plants, reciprocating internal combustion engines, and fuel cells. In this section, we consider vapor power–based cogeneration and, for simplicity, only district heating plants. The particular district heating systems considered have been selected because they are well suited for introducing the subject. Gas turbine–based cogeneration is considered in Sec. 9.9.2. The possibility of fuel cell–based cogeneration is considered in Sec. 13.4.

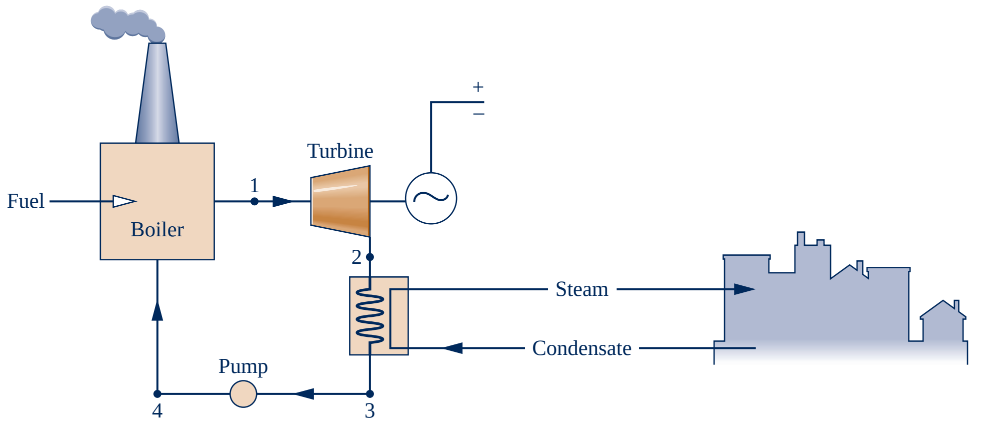
BACK-PRESSURE PLANTS. A *back-pressure* district heating plant is shown in Fig. 8.14*a*. The plant resembles the simple Rankine cycle plant considered in Sec. 8.2 but with an important difference: In this case, energy released when the cycle working fluid condenses during flow through the condenser is harnessed to produce steam for export to the nearby community for various uses. The steam comes at the expense of the potential for power, however.

The power generated by the plant is linked to the district heating need for steam and is determined by the pressure at which the cycle working fluid condenses, called the *back pressure*. For instance, if steam as saturated vapor at 100°C is needed by the community, the cycle working fluid, assumed here to be demineralized water, must condense at a temperature greater than 100°C and thus at a back pressure greater than 1 atm. Accordingly, for fixed turbine inlet conditions and mass flow rate, the power produced in district heating is necessarily less than when condensation occurs well below 1 atm as it does in a plant fully dedicated to power generation.

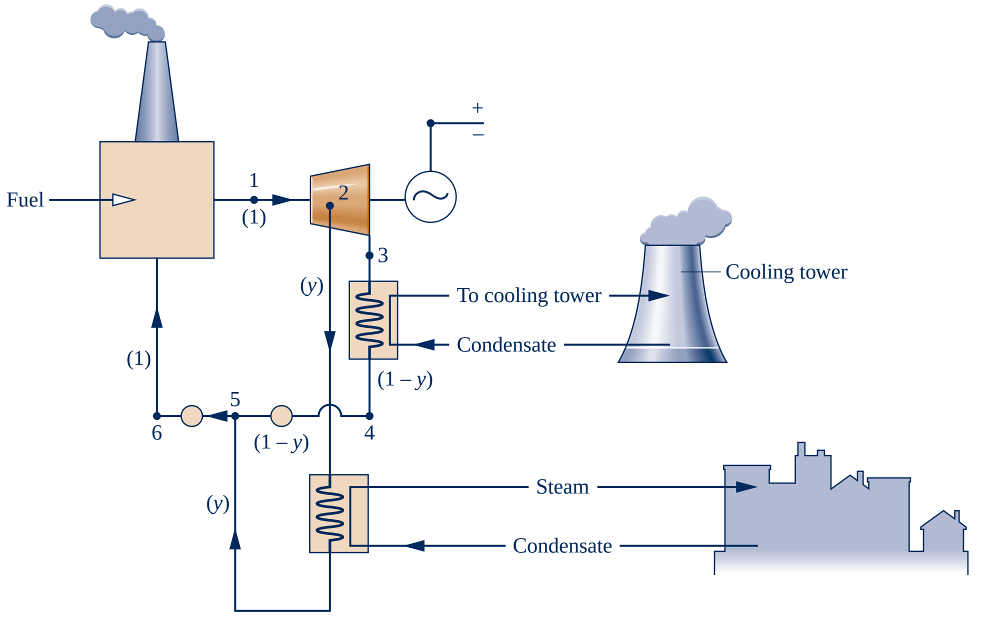
EXTRACTION PLANTS. An extraction district heating plant is shown in Fig. 8.14*b*. The figure is labeled (in parentheses) with fractions of the total flow entering the turbine remaining at various locations; in this respect the plant resembles the regenerative vapor power cycles considered in Sec. 8.4. Steam extracted from the turbine is used to service the district heating need. Differing heating needs can be flexibly met by varying the fraction of the steam extracted, denoted by y. For fixed turbine inlet conditions and mass flow rate, an increase in the fraction y to meet a greater district heating need is met by a reduction in power generated. When there is no demand for district heating, the full amount of steam generated in the boiler expands through the turbine, producing greatest power under the specified conditions. The plant then resembles the simple Rankine cycle of Sec. 8.2.

8.5.3 Carbon Capture and Storage

The concentration of carbon dioxide in the atmosphere has increased significantly since preindustrial times. Some of the increase is traceable to burning fossil fuels. Coal-fired vapor power plants are major sources. Evidence is mounting that excessive



(*a*) Back-pressure plant.



(*b*) Extraction plant.

Fig. 8.14 Vapor cycle district heating plants.

CO_2 in the atmosphere contributes to global climate change, and there is growing agreement that measures must be taken to reduce such emissions.

Carbon dioxide emissions can be reduced by using fossil fuels more efficiently and avoiding wasteful practices. Moreover, if utilities use fewer fossil-fueled plants and more wind, hydropower, and solar plants, less carbon dioxide will come from this sector. Practicing greater efficiency, eliminating wasteful practices, and using more renewable energy are important pathways for controlling CO_2. Yet these strategies are insufficient.

Since fossil fuels will be plentiful for several decades, they will continue to be used for generating electricity and meeting industrial needs. Accordingly, reducing CO_2 emissions at the *plant level* is imperative. One option is greater use of low-carbon fuels—more natural gas and less coal, for example. Another option involves removal of carbon dioxide from the exhaust gas of power plants, oil and gas refineries, and other industrial sources followed by storage (sequestration) of captured CO_2.

Figure 8.15 illustrates a type of carbon dioxide storage method actively under consideration today. Captured CO_2 is injected into depleted oil and gas reservoirs, unminable coal seams, deep salty aquifers, and other geological structures. Storage in oceans by injecting CO_2 to great depths from offshore pumping stations is another method under consideration.

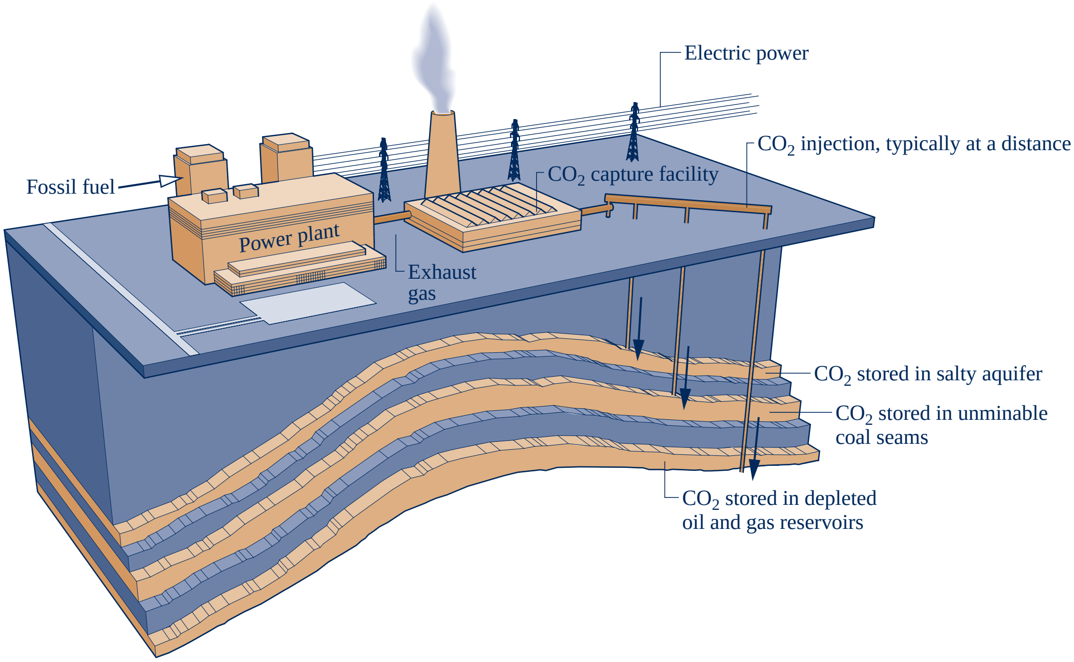


Fig. 8.15 Carbon capture and storage: power plant application.

Deployment of CO_2 capture and storage technology faces major hurdles, including uncertainty over how long injected gas will remain stored and possible collateral environmental impact when so much gas is stored in nature. Another technical challenge is the development of effective means for separating CO_2 from voluminous power plant and industrial gas streams.

Expenditures of energy resources and money required to capture CO_2, transport it to storage sites, and place it into storage will be significant. Yet with our current knowledge, carbon capture and storage is the principal strategy available today for reducing carbon dioxide emissions at the plant level. This area clearly is ripe for innovation. For more, see the following Horizons.

Horizons

What to Do About That CO_2?

The race is on to find alternatives to storage of carbon dioxide captured from power plant exhaust gas streams and other sources. Analysts say there may be no better alternative, but storage does not have to be the fate of *all* of the captured CO_2 if there are commercial applications for *some* of it.

One use of carbon dioxide is for *enhanced oil recovery*—namely, to increase the amount of oil extractable from oil fields. By injecting CO_2 at high pressure into an oil-bearing underground layer, difficult-to-extract oil is forced to the surface. Proponents contend that widespread application of captured carbon dioxide for oil recovery will provide a source of income instead of cost, which is the case when the CO_2 is simply stored underground. Some envision a lively commerce involving export of liquefied carbon dioxide by ship *from* industrialized, oil-importing nations *to* oil-producing nations.

Another possible commercial use proposed for captured carbon dioxide is to produce algae, a tiny single-cell plant. When supplied with carbon dioxide, algae held in bioreactors absorb the carbon dioxide via photosynthesis, spurring algae growth. Carbon-enriched algae can be processed into transportation fuels, providing substitutes for gasoline and a source of income.

Researchers are also working on other ways to turn captured carbon dioxide into fuel. One approach attempts to mimic processes occurring in living things, whereby carbon atoms, extracted from carbon dioxide, and hydrogen atoms, extracted from water, are combined to create hydrocarbon molecules. Another approach uses solar radiation to split carbon dioxide into carbon monoxide and oxygen, and split water into hydrogen and oxygen. These building blocks can be combined into liquid fuels, researchers say.

Algae growth and fuel production using carbon dioxide are each in early stages of development. Still, such concepts suggest potential commercial uses for captured carbon dioxide and inspire hope of others yet to be imagined.

8.6 Case Study: Exergy Accounting of a Vapor Power Plant

TAKE NOTE...
Chapter 7 is a prerequisite for the study of this section.

The discussions to this point show that a useful picture of power plant performance can be obtained with the conservation of mass and conservation of energy principles. However, these principles provide only the *quantities* of energy transferred to and from the plant and do not consider the *utility* of the different types of energy transfer. For example, with the conservation principles alone, a unit of energy exiting as generated electricity is regarded as equivalent to a unit of energy exiting in relatively low-temperature cooling water, even though the electricity has greater utility and economic value. Also, nothing can be learned with the conservation principles alone about the relative significance of the irreversibilities present in the various plant components and the losses associated with those components. The method of exergy analysis introduced in Chap. 7 allows issues such as these to be dealt with quantitatively.

Exergy Accounting

In this section we account for the exergy entering a power plant with the fuel. (Means for evaluating the fuel exergy are introduced in Sec. 13.6.) A portion of the fuel exergy is ultimately returned to the plant surroundings as the net work developed. However, the largest part is either destroyed by irreversibilities within the various plant components or carried from the plant by cooling water, stack gases, and unavoidable heat transfers with the surroundings. These considerations are illustrated in the present section by three solved examples, treating respectively the boiler, turbine and pump, and condenser of a simple vapor power plant.

The irreversibilities present in each power plant component exact a tariff on the exergy supplied to the plant, as measured by the exergy destroyed in that component. The component levying the greatest tariff is the boiler, for a significant portion of the exergy entering the plant with the fuel is destroyed by irreversibilities within it. There are two main sources of irreversibility in the boiler: (1) the irreversible heat transfer occurring between the hot combustion gases and the working fluid of the vapor power cycle flowing through the boiler tubes, and (2) the combustion process itself. To simplify the present discussion, the boiler is considered to consist of a combustor unit in which fuel and air are burned to produce hot combustion gases, followed by a heat exchanger unit where the cycle working fluid is vaporized as the hot gases cool. This idealization is illustrated in Fig. 8.16.

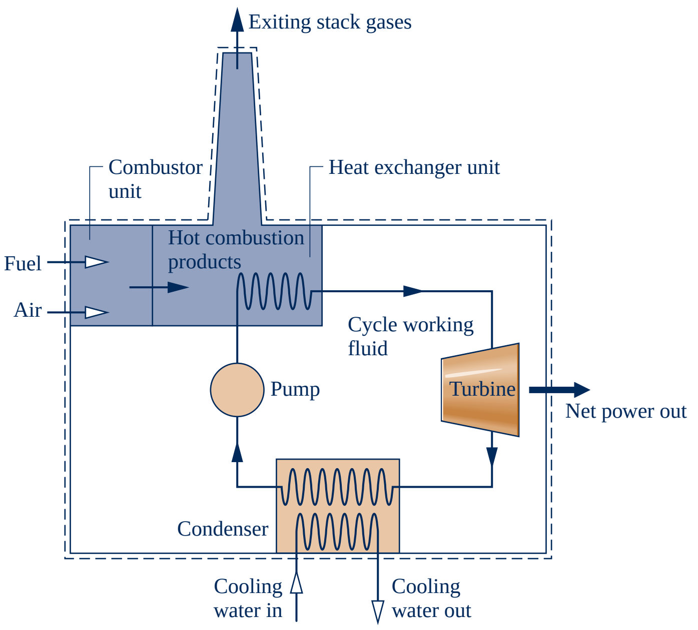


Fig. 8.16 **Power plant schematic for the exergy analysis case study.**

TABLE 8.4

Vapor Power Plant Exergy Accounting[a]

Outputs	
Net power out[b]	30%
Losses	
Condenser cooling water[c]	1%
Stack gases (assumed)	1%
Exergy destruction	
Boiler	
Combustion unit (assumed)	30%
Heat exchanger unit[d]	30%
Turbine[e]	5%
Pump[f]	—
Condenser[g]	3%
Total	100%

[a]All values are expressed as a percentage of the exergy carried into the plant with the fuel. Values are rounded to the nearest full percent. Exergy losses associated with stray heat transfer from plant components are ignored.

[b]Example 8.8
[c]Example 8.9.
[d]Example 8.7.
[e]Example 8.8.
[f]Example 8.8.
[g]Example 8.9.

For purposes of illustration, let us assume that 30% of the exergy entering the combustion unit with the fuel is destroyed by the combustion irreversibility and 1% of the fuel exergy exits the heat exchanger unit with the stack gases. The corresponding values for an actual power plant might differ from these nominal values. However, they provide characteristic values for discussion. (Means for evaluating the combustion exergy destruction and the exergy accompanying the exiting stack gases are introduced in Chap. 13.)

Using the foregoing values for the combustion exergy destruction and stack gas loss, it follows that a *maximum* of 69% of the fuel exergy remains for transfer from the hot combustion gases to the cycle working fluid. It is from this portion of the fuel exergy that the net work developed by the plant is obtained. In Examples 8.7 through 8.9, we account for the exergy supplied by the hot combustion gases passing through the heat exchanger unit. The principal results of this series of examples are reported in Table 8.4. Carefully note that the values of Table 8.4 are keyed to the vapor power plant of Example 8.2 and thus have only qualitative significance for vapor power plants in general.

Case Study Conclusions

The entries of Table 8.4 suggest some general observations about vapor power plant performance. First, the table shows that the exergy destructions are more significant than the plant losses. The largest portion of the exergy entering the plant with the fuel is destroyed, with exergy destruction in the boiler overshadowing all others. By contrast, the loss associated with heat transfer to the cooling water is relatively unimportant. The cycle thermal efficiency (calculated in the solution to Example 8.2) is 31.4%, so over two-thirds (68.6%) of the *energy* supplied to the cycle working fluid is subsequently carried out by the condenser cooling water. By comparison, the amount of *exergy* carried out is virtually negligible because the temperature of the cooling water is raised only a few degrees over that of the surroundings and thus has limited utility. The loss amounts to only 1% of the exergy entering the plant with the fuel. Similarly, losses accompanying unavoidable heat transfer with the surroundings and the exiting stack gases typically amount only to a few percent of the exergy entering the plant with the fuel and are generally overstated when considered from the perspective of energy alone.

An exergy analysis allows the sites where destructions or losses occur to be identified and rank ordered for significance. This knowledge is useful in directing attention to aspects of plant performance that offer the greatest opportunities for improvement through the application of practical engineering measures. However, the decision to adopt any particular modification is governed by economic considerations that take into account both economies in fuel use and the costs incurred to achieve those economies.

The calculations presented in the following examples illustrate the application of exergy principles through the analysis of a simple vapor power plant. There are no fundamental difficulties, however, in applying the methodology to actual power plants, including consideration of the combustion process. The same procedures also can be used for exergy accounting of the gas turbine power plants considered in Chap. 9 and other types of thermal systems.

The following example illustrates the exergy analysis of the heat exchanger unit of the boiler of the case study vapor power plant.

EXAMPLE 8.7

Vapor Cycle Exergy Analysis—Heat Exchanger Unit

The heat exchanger unit of the boiler of Example 8.2 has a stream of water entering as a liquid at 8.0 MPa and exiting as a saturated vapor at 8.0 MPa. In a separate stream, gaseous products of combustion cool at a constant pressure of 1 atm from 1107 to 547°C. The gaseous stream can be modeled as air as an ideal gas. Let $T_0 = 22°C$, $p_0 = 1$ atm. Determine **(a)** the net rate at which exergy is carried into the heat exchanger unit by the gas stream, in MW, **(b)** the net rate at which exergy is carried from the heat exchanger by the water stream, in MW, **(c)** the rate of exergy destruction, in MW, **(d)** the exergetic efficiency given by Eq. 7.27.

SOLUTION

Known: A heat exchanger at steady state has a water stream entering and exiting at known states and a separate gas stream entering and exiting at known states.

Find: Determine the net rate at which exergy is carried into the heat exchanger by the gas stream, in MW, the net rate at which exergy is carried from the heat exchanger by the water stream, in MW, the rate of exergy destruction, in MW, and the exergetic efficiency.

Schematic and Given Data:

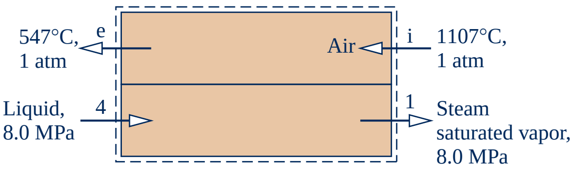


Fig. E8.7

Engineering Model:

1. The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$.
2. Kinetic and potential energy effects can be ignored.
3. The gaseous combustion products are modeled as air as an ideal gas.
4. The air and the water each pass through the steam generator at constant pressure.
5. Only 69% of the exergy entering the plant with the fuel remains after accounting for the stack loss and combustion exergy destruction.
6. $T_0 = 22°C$, $p_0 = 1$ atm.

Analysis: The analysis begins by evaluating the mass flow rate of the air in terms of the mass flow rate of the water. The air and water pass through the boiler in separate streams. Hence, at steady state the conservation of mass principle requires

$$\dot{m}_i = \dot{m}_e \qquad \text{(air)}$$
$$\dot{m}_4 = \dot{m}_1 \qquad \text{(water)}$$

Using these relations, an energy rate balance for the overall control volume reduces at steady state to

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_a(h_i - h_e) + \dot{m}(h_4 - h_1)$$

where $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ by assumption 1, and the kinetic and potential energy terms are dropped by assumption 2. In this equation $\dot{m}_a$ and $\dot{m}$ denote, respectively, the mass flow rates of the air and water. On solving

$$\frac{\dot{m}_a}{\dot{m}} = \frac{h_1 - h_4}{h_i - h_e}$$

The solution to Example 8.2 gives $h_1 = 2758$ kJ/kg and $h_4 = 183.36$ kJ/kg. From Table A-22, $h_i = 1491.44$ kJ/kg and $h_e = 843.98$ kJ/kg. Hence

$$\frac{\dot{m}_a}{\dot{m}} = \frac{2758 - 183.36}{1491.44 - 843.98} = 3.977 \frac{\text{kg (air)}}{\text{kg (steam)}}$$

From Example 8.2, $\dot{m} = 4.449 \times 10^5$ kg/h. Thus, $\dot{m}_a = 17.694 \times 10^5$ kg/h.

(a) The net rate at which exergy is carried into the heat exchanger unit by the gaseous stream can be evaluated using Eq. 7.18

$$\begin{bmatrix} \text{net rate at which exergy} \\ \text{is carried in by the} \\ \text{gaseous stream} \end{bmatrix} = \dot{m}_a(\mathsf{e}_{fi} - \mathsf{e}_{fe})$$

$$= \dot{m}_a[h_i - h_e - T_0(s_i - s_e)]$$

Since the gas pressure remains constant, Eq. 6.20a giving the change in specific entropy of an ideal gas reduces to $s_i - s_e = s_i^\circ - s_e^\circ$. Thus, with h and s° values from Table A-22

$$\dot{m}_a(\mathsf{e}_{fi} - \mathsf{e}_{fe}) = (17.694 \times 10^5 \text{ kg/h})[(1491.44 - 843.98) \text{ kJ/kg} - (295 \text{ K})(3.34474 - 2.74504) \text{ kJ/kg} \cdot \text{K}]$$

$$= \frac{8.326 \times 10^8 \text{ kJ/h}}{|3600 \text{ s/h}|}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 231.28 \text{ MW}$$

(b) The net rate at which exergy is carried out of the boiler by the water stream is determined similarly

$$\begin{bmatrix} \text{net rate at which exergy} \\ \text{is carried out by the} \\ \text{water stream} \end{bmatrix} = \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f4})$$

$$= \dot{m}[h_1 - h_4 - T_0(s_1 - s_4)]$$

From Table A-3, $s_1 = 5.7432$ kJ/kg · K. Double interpolation in Table A-5 at 8.0 MPa and $h_4 = 183.36$ kJ/kg gives $s_4 = 0.5957$ kJ/kg · K. Substituting known values

$$\dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f4}) = (4.449 \times 10^5)[(2758 - 183.36) - 295(5.7432 - 0.5957)]$$

1
$$= \frac{4.699 \times 10^8 \text{ kJ/h}}{|3600 \text{ s/h}|}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 130.53 \text{ MW}$$

(c) The rate of exergy destruction can be evaluated by reducing the exergy rate balance to obtain

2
$$\dot{\mathsf{E}}_d = \dot{m}_a(\mathsf{e}_{fi} - \mathsf{e}_{fe}) + \dot{m}(\mathsf{e}_{f4} - \mathsf{e}_{f1})$$

With the results of parts (a) and (b)

3
$$\dot{\mathsf{E}}_d = 231.28 \text{ MW} - 130.53 \text{ MW} = 100.75 \text{ MW}$$

(d) The exergetic efficiency given by Eq. 7.27 is

$$\varepsilon = \frac{\dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f4})}{\dot{m}_a(\mathsf{e}_{fi} - \mathsf{e}_{fe})} = \frac{130.53 \text{ MW}}{231.28 \text{ MW}} = 0.564 \; (56.4\%)$$

This calculation indicates that 43.6% of the exergy supplied to the heat exchanger unit by the cooling combustion products is destroyed. However, since only 69% of the exergy entering the plant with the fuel is assumed to remain after the stack loss and combustion exergy destruction are accounted for (assumption 5), it can be concluded that 0.69 × 43.6% = 30% of the exergy entering the plant with the fuel is destroyed within the heat exchanger. This is the value listed in Table 8.4.

1. Since energy is conserved, the rate at which energy is transferred *to* the water as it flows through the heat exchanger *equals* the rate at which energy is transferred *from* the gas passing through the heat exchanger. By contrast, the rate at which exergy is transferred *to* the water is *less* than the rate at which exergy is transferred *from* the gas by the rate at which exergy is *destroyed* within the heat exchanger.

2. The rate of exergy destruction can be determined alternatively by evaluating the rate of entropy production, $\dot{\sigma}_{cv}$, from an entropy rate balance and multiplying by T_0 to obtain $\dot{E}_d = T_0 \dot{\sigma}_{cv}$.

3. Underlying the assumption that each stream passes through the heat exchanger at constant pressure is the neglect of friction as an irreversibility. Thus, the only contributor to exergy destruction in this case is heat transfer from the higher-temperature combustion products to the vaporizing water.

Skills Developed

Ability to...

- ❑ perform exergy analysis of a power plant steam generator.

QuickQUIZ If the gaseous products of combustion are cooled to 517°C (h_e = 810.99 kJ/kg), what is the mass flow rate of the gaseous products, in kg/h? **Ans.** 16.83×10^5 kg/h.

In the next example, we determine the exergy destruction rates in the turbine and pump of the case study vapor power plant.

EXAMPLE 8.8

Vapor Cycle Exergy Analysis—Turbine and Pump

Reconsider the turbine and pump of Example 8.2. Determine for each of these components the rate at which exergy is destroyed, in MW. Express each result, and the net power output of the plant, as a percentage of the exergy entering the plant with the fuel. Let T_0 = 22°C, p_0 =1 atm.

SOLUTION

Known: A vapor power cycle operates with steam as the working fluid. The turbine and pump each have an isentropic efficiency of 85%.

Find: For the turbine and the pump individually, determine the rate at which exergy is destroyed, in MW. Express each result, and the net power output, as a percentage of the exergy entering the plant with the fuel.

Schematic and Given Data:

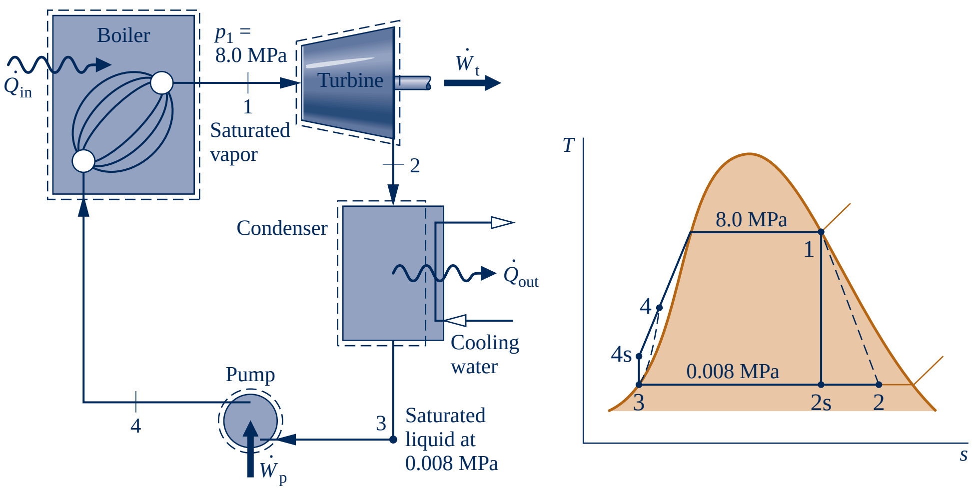


Fig. E8.8

Engineering Model:

1. The turbine and the pump can each be analyzed as a control volume at steady state.
2. The turbine and pump operate adiabatically and each has an isentropic efficiency of 85%.
3. Kinetic and potential energy effects are negligible.
4. Only 69% of the exergy entering the plant with the fuel remains after accounting for the stack loss and combustion exergy destruction.
5. $T_0 = 22°\text{C}$, $p_0 = 1$ atm.

Analysis: The rate of exergy destruction can be found by reducing the exergy rate balance or by use of the relationship $\dot{\text{E}}_\text{d} = T_0\dot{\sigma}_\text{cv}$, where $\dot{\sigma}_\text{cv}$ is the rate of entropy production from an entropy rate balance. With either approach, the rate of exergy destruction for the turbine can be expressed as

$$\dot{\text{E}}_\text{d} = \dot{m}T_0(s_2 - s_1)$$

From Table A-3, $s_1 = 5.7432$ kJ/kg · K. Using $h_2 = 1939.3$ kJ/kg from the solution to Example 8.2, the value of s_2 can be determined from Table A-3 as $s_2 = 6.2021$ kJ/kg · K. Substituting values

$$\begin{aligned}\dot{\text{E}}_\text{d} &= (4.449 \times 10^5 \text{ kg/h})(295 \text{ K})(6.2021 - 5.7432)(\text{kJ/kg} \cdot \text{K}) \\ &= \left(0.602 \times 10^8 \frac{\text{kJ}}{\text{h}}\right)\left|\frac{1 \text{ h}}{3600 \text{ s}}\right|\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 16.72 \text{ MW}\end{aligned}$$

From the solution to Example 8.7, the net rate at which exergy is supplied by the cooling combustion gases is 231.28 MW. The turbine rate of exergy destruction expressed as a percentage of this is (16.72/231.28)(100%) = 7.23%. However, since only 69% of the entering fuel exergy remains after the stack loss and combustion exergy destruction are accounted for, it can be concluded that 0.69 × 7.23% = 5% of the exergy entering the plant with the fuel is destroyed within the turbine. This is the value listed in Table 8.4.

Similarly, the exergy destruction rate for the pump is

$$\dot{\text{E}}_\text{d} = \dot{m}T_0(s_4 - s_3)$$

With s_3 from Table A-3 and s_4 from the solution to Example 8.7

$$\begin{aligned}\dot{\text{E}}_\text{d} &= (4.449 \times 10^5 \text{ kg/h})(295 \text{ K})(0.5957 - 0.5926)(\text{kJ/kg} \cdot \text{K}) \\ &= \left(4.07 \times 10^5 \frac{\text{kJ}}{\text{h}}\right)\left|\frac{1 \text{ h}}{3600 \text{ s}}\right|\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 0.11 \text{ MW}\end{aligned}$$

Expressing this as a percentage of the exergy entering the plant as calculated above, we have (0.11/231.28)(69%) = 0.03%. This value is rounded to zero in Table 8.4.

The net power output of the vapor power plant of Example 8.2 is 100 MW. Expressing this as a percentage of the rate at which exergy is carried into the plant with the fuel, (100/231.28)(69%) = 30%, as shown in Table 8.4.

Skills Developed

Ability to...

- ❑ perform exergy analysis of a power plant turbine and pump.

QuickQUIZ What is the exergetic efficiency of the power plant? Ans. 30%.

The following example illustrates the exergy analysis of the condenser of the case study vapor power plant.

EXAMPLE 8.9

Vapor Cycle Exergy Analysis—Condenser

The condenser of Example 8.2 involves two separate water streams. In one stream a two-phase liquid–vapor mixture enters at 0.008 MPa and exits as a saturated liquid at 0.008 MPa. In the other stream, cooling water enters at 15°C and exits at 35°C. **(a)** Determine the net rate at which exergy is carried from the condenser by the cooling water, in MW. Express this result as a percentage of the exergy entering the plant with the fuel. **(b)** Determine for the condenser the rate of exergy destruction, in MW. Express this result as a percentage of the exergy entering the plant with the fuel. Let $T_0 = 22°\text{C}$ and $p_0 = 1$ atm.

SOLUTION

Known: A condenser at steady state has two streams: (1) a two-phase liquid–vapor mixture entering and condensate exiting at known states and (2) a separate cooling water stream entering and exiting at known temperatures.

Find: Determine the net rate at which exergy is carried from the condenser by the cooling water stream and the rate of exergy destruction for the condenser. Express both quantities in MW and as percentages of the exergy entering the plant with the fuel.

Schematic and Given Data:

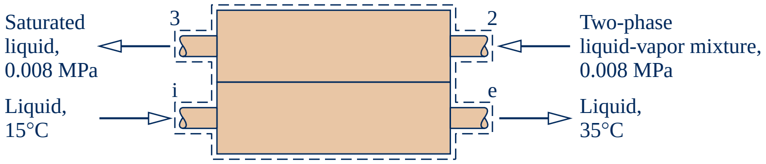


Fig. E8.9

Engineering Model:

1. The control volume shown on the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$.
2. Kinetic and potential energy effects can be ignored.
3. Only 69% of the fuel exergy remains after accounting for the stack loss and combustion exergy destruction.
4. $T_0 = 22°C$, $p_0 = 1$ atm.

Analysis:

(a) The net rate at which exergy is carried out of the condenser can be evaluated by using Eq. 7.18:

$$\begin{bmatrix} \text{net rate at which exergy} \\ \text{is carried out by the} \\ \text{cooling water} \end{bmatrix} = \dot{m}_{cw}(\mathsf{e}_{fe} - \mathsf{e}_{fi})$$

$$= \dot{m}_{cw}[h_e - h_i - T_0(s_e - s_i)]$$

where $\dot{m}_{cw}$ is the mass flow rate of the cooling water from the solution to Example 8.2. With saturated liquid values for specific enthalpy and entropy from Table A-2 at the specified inlet and exit temperatures of the cooling water

$$\dot{m}_{cw}(\mathsf{e}_{fe} - \mathsf{e}_{fi}) = (9.39 \times 10^6 \text{ kg/h})[(146.68 - 62.99) \text{ kJ/kg} - (295 \text{ K})(0.5053 - 0.2245) \text{ kJ/kg} \cdot \text{K}]$$

$$= \frac{8.019 \times 10^6 \text{ kJ/h}}{|3600 \text{ s/h}|}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 2.23 \text{ MW}$$

Expressing this as a percentage of the exergy entering the plant with the fuel, we get (2.23/231.28)(69%) = 1%. This is the value listed in Table 8.4.

(b) The rate of exergy destruction for the condenser can be evaluated by reducing the exergy rate balance. Alternatively, the relationship $\dot{\mathsf{E}}_d = T_0 \dot{\sigma}_{cv}$ can be employed, where $\dot{\sigma}_{cv}$ is the time rate of entropy production for the condenser determined from an entropy rate balance. With either approach, the rate of exergy destruction can be expressed as

$$\dot{\mathsf{E}}_d = T_0[\dot{m}(s_3 - s_2) + \dot{m}_{cw}(s_e - s_i)]$$

Substituting values

$$\dot{\mathsf{E}}_d = 295[(4.449 \times 10^5)(0.5926 - 6.2021) + (9.39 \times 10^6)(0.5053 - 0.2245)]$$

$$= \frac{416.1 \times 10^5 \text{ kJ/h}}{|3600 \text{ s/h}|}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 11.56 \text{ MW}$$

Expressing this as a percentage of the exergy entering the plant with the fuel, we get (11.56/231.28)(69%) = 3%. This is the value listed in Table 8.4.

Skills Developed

Ability to...

❑ perform exergy analysis of a power plant condenser.

QuickQUIZ Referring to data from Example 8.2, what percent of the *energy* supplied to the steam passing through the steam generator is carried out by the cooling water? Ans. 68.6%.

▶ CHAPTER SUMMARY AND STUDY GUIDE

This chapter begins with an introduction to power generation that surveys current U.S. power generation by source and looks ahead to power generation needs in decades to come. These discussions provide context for the study of vapor power plants in this chapter and gas power plants in Chap. 9.

In Chap. 8 we have considered practical arrangements for vapor power plants, illustrated how vapor power plants are modeled thermodynamically, and considered the principal irreversibilities and losses associated with such plants. The main components of *simple* vapor power plants are modeled by the Rankine cycle.

In this chapter, we also have introduced modifications to the simple vapor power cycle aimed at improving overall performance. These include superheat, reheat, regeneration, supercritical operation, cogeneration, and binary cycles. We have also included a case study to illustrate the application of exergy analysis to vapor power plants.

The following checklist provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed you should be able to

- ▶ write out the meanings of the terms listed in the margin throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important.
- ▶ sketch schematic diagrams and accompanying *T–s* diagrams of Rankine, reheat, and regenerative vapor power cycles.
- ▶ apply conservation of mass and energy, the second law, and property data to determine power cycle performance, including thermal efficiency, net power output, and mass flow rates.
- ▶ discuss the effects on Rankine cycle performance of varying steam generator pressure, condenser pressure, and turbine inlet temperature.
- ▶ discuss the principal sources of exergy destruction and loss in vapor power plants.

▶ KEY ENGINEERING CONCEPTS

Rankine cycle, p. 433
thermal efficiency, p. 435
back work ratio, p. 435
ideal Rankine cycle, p. 437
superheat, p. 447
reheat, p. 447
supercritical, p. 448
regeneration, p. 453
open feedwater heater, p. 453
closed feedwater heater, p. 458
organic cycle, p. 464
binary vapor cycle, p. 464
cogeneration, p. 465
district heating, p. 465
exergy accounting, p. 468

▶ KEY EQUATIONS

$\eta = \dfrac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \dfrac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$	**(8.5a) p. 435**	**Thermal efficiency for the Rankine cycle of Fig. 8.2**
$\text{bwr} = \dfrac{\dot{W}_p/\dot{m}}{\dot{W}_t/\dot{m}} = \dfrac{(h_4 - h_3)}{(h_1 - h_2)}$	**(8.6) p. 435**	**Back work ratio for the Rankine cycle of Fig. 8.2**
$\left(\dfrac{\dot{W}_p}{\dot{m}}\right)_s \approx v_3(p_4 - p_3)$	**(8.7b) p. 437**	**Approximation for the pump work of the ideal Rankine cycle of Fig. 8.3**

▶ EXERCISES: THINGS ENGINEERS THINK ABOUT

1. Many utility companies offer special rates for "green power." What does that mean?

2. Brainstorm some ways to use the cooling water exiting the condenser of a large power plant.

3. What effects on a river's ecology might result from a power plant's use of river water for condenser cooling?

4. Referring to Fig. 8.1*a*, what environmental impacts might result from the two plumes shown on the figure?

5. What is a *baseload* power plant?

6. If Iceland completes its planned transition to using only renewable energy throughout its society by mid-century, what significant changes in lifestyle will Icelanders have to tolerate?

7. What type of power plant produces the electricity used in your residence?

8. What is the relationship between global climate change and the *hole* in Earth's ozone layer?

9. Why is it important for power plant operators to keep pipes circulating water through plant components free from *fouling*?

10. What is the difference between solar-*concentrating* and solar-*photovoltaic* electricity generation?

11. Decades of coal mining have left piles of waste coal, or *culm*, in many locations through the United States. What effects does culm have on human health and the environment?

12. How do operators of electricity-generating plants detect and respond to changes in consumer demand throughout the day?

12. What is an energy *orb*?

▶ PROBLEMS: DEVELOPING ENGINEERING SKILLS

Analyzing Rankine Cycles

8.1 Water is the working fluid in an ideal Rankine cycle. The condenser pressure is 6 kPa, and saturated vapor enters the turbine at 10 MPa. Determine the heat transfer rates, in kJ per kg of steam flowing, for the working fluid passing through the boiler and condenser and calculate the thermal efficiency.

8.2 Water is the working fluid in an ideal Rankine cycle. Superheated vapor enters the turbine at 10 MPa, 480°C, and the condenser pressure is 6 kPa. Determine for the cycle

(a) the rate of heat transfer to the working fluid passing through the steam generator, in kJ per kg of steam flowing.
(b) the thermal efficiency.
(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam flowing.

8.3 Water is the working fluid in a Carnot vapor power cycle. Saturated liquid enters the boiler at a pressure of 10 MPa, and saturated vapor enters the turbine. The condenser pressure is 6 kPa. Determine

(a) the thermal efficiency.
(b) the back work ratio.
(c) the heat transfer to the working fluid per unit mass passing through the boiler, in kJ/kg.
(d) the heat transfer from the working fluid per unit mass passing through the condenser, in kJ/kg.

8.4 Plot each of the quantities calculated in Problem 8.2 versus condenser pressure ranging from 6 kPa to 0.1 MPa. Discuss.

8.5 Plot each of the quantities calculated in Problem 8.2 versus steam generator pressure ranging from 4 MPa to 20 MPa. Maintain the turbine inlet temperature at 480°C. Discuss.

8.6 A Carnot vapor power cycle operates with water as the working fluid. Saturated liquid enters the boiler at 1800 lbf/in.^{2}, and saturated vapor enters the turbine (state 1). The condenser pressure is 1.2 lbf/in.^{2} The mass flow rate of steam is 1×10^6 lb/h. Data at key points in the cycle are provided in the accompanying table. Determine

(a) the thermal efficiency.
(b) the back work ratio.
(c) the net power developed, in Btu/h.
(d) the rate of heat transfer to the working fluid passing through the boiler.

State	p(lbf/in.^{2})	h(Btu/lb)
1	1800	1150.4
2	1.2	735.7
3	1.2	472.0
4	1800	648.3

8.7 Water is the working fluid in an ideal Rankine cycle. Saturated vapor enters the turbine at 16 MPa, and the condenser pressure is 8 kPa. The mass flow rate of steam entering the turbine is 120 kg/s. Determine

(a) the net power developed, in kW.
(b) the rate of heat transfer to the steam passing through the boiler, in kW.
(c) the thermal efficiency.
(d) the mass flow rate of condenser cooling water, in kg/s, if the cooling water undergoes a temperature increase of 18°C with negligible pressure change in passing through the condenser.

8.8 Water is the working fluid in a Carnot vapor power cycle. Saturated liquid enters the boiler at 16 MPa, and saturated vapor enters the turbine. The condenser pressure is 8 kPa. The mass flow rate of steam entering the turbine is 120 kg/s. Determine

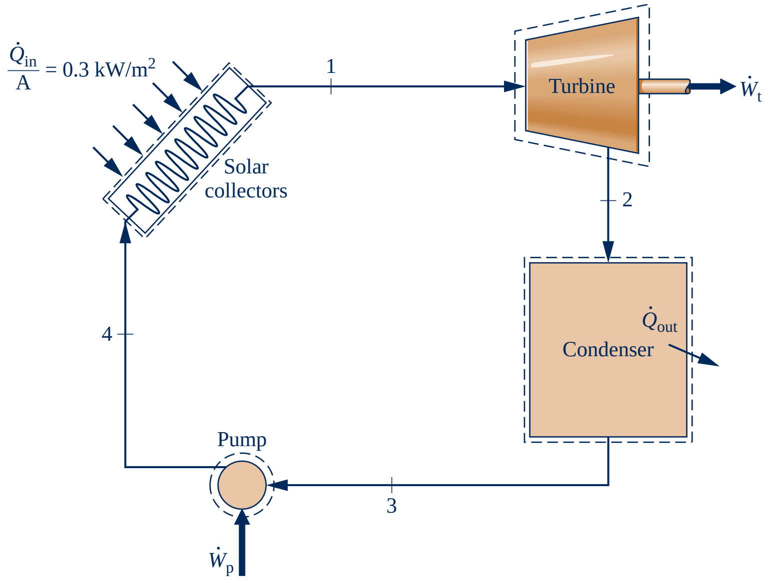
(a) the thermal efficiency.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rate of heat transfer from the working fluid passing through the condenser, in kW.

8.9 Plot each of the quantities calculated in Problem 8.7 versus turbine inlet temperature ranging from the saturation temperature at 16 MPa to 560°C Discuss.

8.10 Water is the working fluid in an ideal Rankine cycle. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F. The condenser pressure is 2 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Cooling water experiences a temperature increase from 60°F to 76°F, with negligible pressure drop, as it passes through the condenser. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.
(d) the mass flow rate of cooling water, in lb/h.

8.11 Plot each of the quantities calculated in Problem 8.10 versus condenser pressure ranging from 0.3 lbf/in.^{2} to 14.7 lbf/in.^{2} Maintain constant net power output. Discuss.



State	p (bar)	h (kJ/kg)	x
1	18	276.83	1
2	7	261.01	0.9952
3	7	86.78	0
4	18	87.93	—

Fig. P8.13

8.12 A power plant based on the Rankine cycle is under development to provide a net power output of 10 MW. Solar collectors are to be used to generate Refrigerant 22 vapor at 1.6 MPa, 50°C, for expansion through the turbine. Cooling water is available at 20°C. Specify the preliminary design of the cycle and estimate the thermal efficiency and the refrigerant and cooling water flow rates, in kg/h.

8.13 Figure P8.13 provides steady-state operating data for a solar power plant that operates on a Rankine cycle with Refrigerant 134a as its working fluid. The turbine and pump operate adiabatically. The rate of energy input to the collectors from solar radiation is 0.3 kW per m^2 of collector surface area, with 60% of the solar input to the collectors absorbed by the refrigerant as it passes through the collectors. Determine the solar collector surface area, in m^2 per kW of power developed by the plant. Discuss possible operational improvements that could reduce the required collector surface area.

8.14 On the south coast of the island of Hawaii, lava flows continuously into the ocean. It is proposed to anchor a floating power plant offshore of the lava flow that uses ammonia as the working fluid. The plant would exploit the temperature variation between the warm water near the surface at 130°F and seawater at 50°F from a depth of 500 ft to produce power. Figure P8.14 shows the configuration and provides additional data. Using the properties of pure water for the seawater and modeling the power plant as a Rankine cycle, determine

(a) the thermal efficiency.
(b) the mass flow rate of ammonia, in lb/min, for a net power output of 300 hp.

8.15 The cycle of Problem 8.3 is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes. If the states at the turbine and pump inlets remain unchanged, repeat parts (a)–(d) of Problem 8.3 for the modified Carnot cycle with $\eta_t = 0.80$ and $\eta_p = 0.70$.

8.16 Steam enters the turbine of a simple vapor power plant with a pressure of 10 MPa and temperature T, and expands adiabatically to 6 kPa. The isentropic turbine efficiency is 85%. Saturated liquid exits the condenser at 6 kPa and the isentropic pump efficiency is 82%.

(a) For T = 580°C, determine the turbine exit quality and the cycle thermal efficiency.
(b) Plot the quantities of part (a) versus T ranging from 580 to 700°C.

8.17 Water is the working fluid in a Rankine cycle. Superheated vapor enters the turbine at 10 MPa, 480°C, and the condenser pressure is 6 kPa. The turbine and pump have isentropic efficiencies of 80 and 70%, respectively. Determine for the cycle

(a) the rate of heat transfer to the working fluid passing through the steam generator, in kJ per kg of steam flowing.

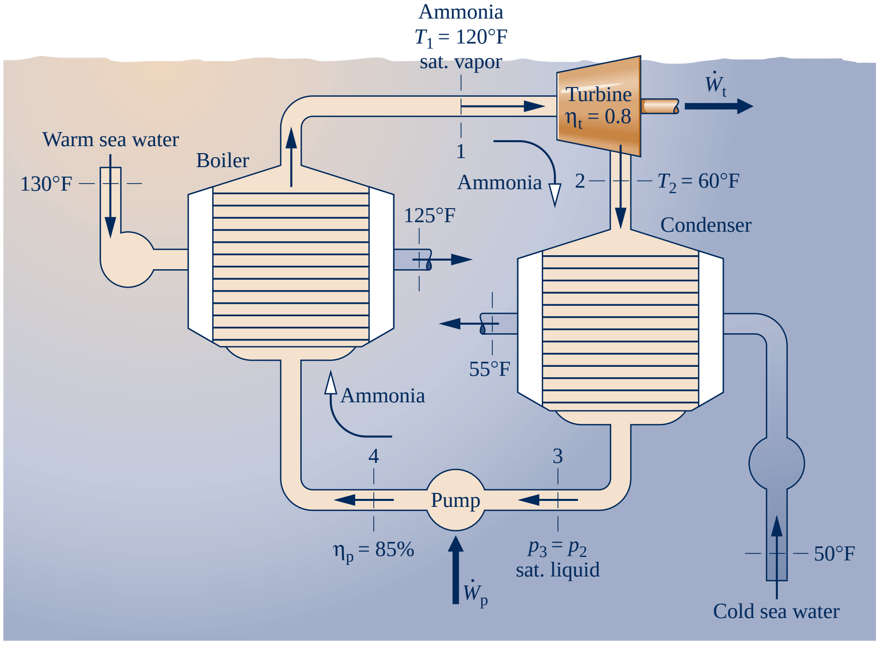


Fig. P8.14

(b) the thermal efficiency.
(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam flowing.

8.18 Steam enters the turbine of a Rankine cycle at 16 MPa, 560°C. The condenser pressure is 8 kPa. The turbine and pump each have isentropic efficiencies of 85%, and the mass flow rate of steam entering the turbine is 120 kg/s. Determine

(a) the net power developed, in kW.
(b) the rate of heat transfer to the steam passing through the boiler, in kW.
(c) the thermal efficiency.

Plot each of the quantities in parts (a)–(c) if the turbine and pump isentropic efficiencies remain equal to each other but vary from 80 to 100%.

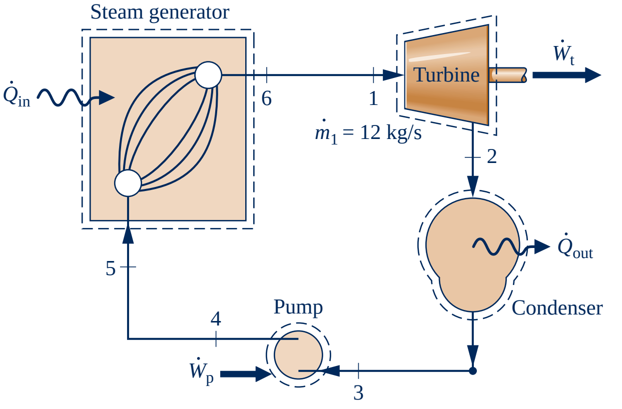
8.19 Water is the working fluid in a Rankine cycle. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F. The condenser pressure is 2 lbf/in.^{2} Both the turbine and pump have isentropic efficiencies of 85%. The working fluid has negligible pressure drop in passing through the steam generator. The net power output of the cycle is 1×10^9 Btu/h. Cooling water experiences a temperature increase from 60°F to 76°F, with negligible pressure drop, as it passes through the condenser. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.
(d) the mass flow rate of cooling water, in lb/h.

8.20 Water is the working fluid in a Rankine cycle. Superheated vapor enters the turbine at 8 MPa, 560°C with a mass flow rate of 7.8 kg/s and exits at 8 kPa. Saturated liquid enters the pump at 8 kPa. The isentropic turbine efficiency is 88%, and the isentropic pump efficiency is 82%. Cooling water enters the condenser at 18°C and exits at 36°C with no significant change in pressure. Determine

(a) the net power developed, in kW.
(b) the thermal efficiency.
(c) the mass flow rate of cooling water, in kg/s.

8.21 Figure P8.21 provides steady-state operating data for a vapor power plant using water as the working fluid. The



State	p	T (°C)	h (kJ/kg)
1	6 MPa	500	3422.2
2	10 kPa	- - -	1633.3
3	10 kPa	Sat.	191.83
4	7.5 MPa	- - -	199.4
5	7 MPa	40	167.57
6	6 MPa	550	3545.3

Fig. P8.21

mass flow rate of water is 12 kg/s. The turbine and pump operate adiabatically but not reversibly. Determine

(a) the thermal efficiency.
(b) the rates of heat transfer $\dot{Q}_{in}$ and $\dot{Q}_{out}$, each in kW.

8.22 Superheated steam at 8 MPa and 480°C leaves the steam generator of a vapor power plant. Heat transfer and frictional effects in the line connecting the steam generator and the turbine reduce the pressure and temperature at the turbine inlet to 7.6 MPa and 440°C, respectively. The pressure at the exit of the turbine is 10 kPa, and the turbine operates adiabatically. Liquid leaves the condenser at 8 kPa, 36°C. The pressure is increased to 8.6 MPa across the pump. The turbine and pump isentropic efficiencies are 88%. The mass flow rate of steam is 79.53 kg/s. Determine

(a) the net power output, in kW.
(b) the thermal efficiency.
(c) the rate of heat transfer from the line connecting the steam generator and the turbine, in kW.
(d) the mass flow rate of condenser cooling water, in kg/s, if the cooling water enters at 15°C and exits at 35°C with negligible pressure change.

8.23 Water is the working fluid in a Rankine cycle. Steam exits the steam generator at 1500 lbf/in.^{2} and 1100°F. Due to heat transfer and frictional effects in the line connecting the steam generator and turbine, the pressure and temperature at the turbine inlet are reduced to 1400 lbf/in.^{2} and 1000°F, respectively. Both the turbine and pump have isentropic efficiencies of 85%. Pressure at the condenser inlet is 2 lbf/in.^{2}, but due to frictional effects the condensate exits the condenser at a pressure of 1.5 lbf/in.^{2} and a temperature of 110°F. The condensate is pumped to 1600 lbf/in.^{2} before entering the steam generator. The net power output of the cycle is 1×10^9 Btu/h. Cooling water experiences a temperature increase from 60°F to 76°F, with negligible pressure drop, as it passes through the condenser. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.
(d) the mass flow rate of cooling water, in lb/h.

8.24 Steam enters the turbine of a vapor power plant at 600 lbf/in.^{2}, 1000°F and exits as a two-phase liquid–vapor mixture at temperature T. Condensate exits the condenser at a temperature 5°F lower than T and is pumped to 600 lbf/in.^{2} The turbine and pump isentropic efficiencies are 90 and 80%, respectively. The net power developed is 1 MW.

(a) For $T = 80$°F, determine the steam quality at the turbine exit, the steam mass flow rate, in lb/h, and the thermal efficiency.
(b) Plot the quantities of part (a) versus T ranging from 80 to 105°F.

8.25 Superheated steam at 18 MPa, 560°C, enters the turbine of a vapor power plant. The pressure at the exit of the turbine is 0.06 bar, and liquid leaves the condenser at 0.045 bar, 26°C. The pressure is increased to 18.2 MPa across the pump. The turbine and pump have isentropic efficiencies of 82 and 77%, respectively. For the cycle, determine

(a) the net work per unit mass of steam flow, in kJ/kg.
(b) the heat transfer to steam passing through the boiler, in kJ per kg of steam flowing.
(c) the thermal efficiency.
(d) the heat transfer to cooling water passing through the condenser, in kJ per kg of steam condensed.

8.26 In the preliminary design of a power plant, water is chosen as the working fluid and it is determined that the turbine inlet temperature may not exceed 520°C. Based on expected cooling water temperatures, the condenser is to operate at a pressure of 0.06 bar. Determine the steam generator pressure required if the isentropic turbine efficiency is 80% and the quality of steam at the turbine exit must be at least 90%.

Considering Reheat and Supercritical Cycles

8.27 Steam at 10 MPa, 600°C enters the first-stage turbine of an ideal Rankine cycle with reheat. The steam leaving the reheat section of the steam generator is at 500°C, and the condenser pressure is 6 kPa. If the quality at the exit of the second-stage turbine is 90%, determine the cycle thermal efficiency.

8.28 Water is the working fluid in an ideal Rankine cycle with superheat and reheat. Steam enters the first-stage turbine at 1400 lbf/in.^{2} and 1000°F, expands to a pressure of 350 lbf/in.^{2}, and is reheated to 900°F before entering the second-stage turbine. The condenser pressure is 2 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the rate of heat transfer, in Btu/h, to the working fluid passing through the reheater.
(d) the thermal efficiency.

8.29 Water is the working fluid in an ideal Rankine cycle with reheat. Superheated vapor enters the turbine at 10 MPa, 480°C, and the condenser pressure is 6 kPa. Steam expands through the first-stage turbine to 0.7 MPa and then is reheated to 480°C. Determine for the cycle

(a) the rate of heat addition, in kJ per kg of steam entering the first-stage turbine.
(b) the thermal efficiency.
(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam entering the first-stage turbine.

8.30 For the cycle of Problem 8.29, reconsider the analysis assuming the pump and each turbine stage has an isentropic efficiency of 80%. Answer the same questions as in Problem 8.29 for the modified cycle.

8.31 Investigate the effects on cycle performance as the reheat pressure and final reheat temperature take on other values. Construct suitable plots and discuss for the cycle of

(a) Problem 8.29.
(b) Problem 8.30.

8.32 An ideal Rankine cycle with reheat uses water as the working fluid. The conditions at the inlet to the first-stage turbine are $p_1 = 2500$ lbf/in.^{2}, $T_1 = 1000$°F. The steam is reheated at constant pressure p between the turbine stages to 1000°F. The condenser pressure is 1 lbf/in.^{2}

(a) If $p/p_1 = 0.2$, determine the cycle thermal efficiency and the steam quality at the exit of the second-stage turbine.
(b) Plot the quantities of part (a) versus the pressure ratio p/p_1 ranging from 0.05 to 1.0.

8.33 Steam at 32 MPa, 520°C enters the first stage of a supercritical reheat cycle including three turbine stages. Steam exiting the first-stage turbine at pressure p is reheated at constant pressure to 440°C, and steam exiting the second-stage turbine at 0.5 MPa is reheated at constant pressure to 360°C. Each turbine stage and the pump has an isentropic efficiency of 85%. The condenser pressure is 8 kPa.

(a) For $p = 4$ MPa, determine the net work per unit mass of steam flowing, in kJ/kg, and the thermal efficiency.
(b) Plot the quantities of part (a) versus p ranging from 0.5 to 10 MPa.

8.34 Steam at 4800 lbf/in.^{2}, 1000°F enters the first stage of a supercritical reheat cycle including two turbine stages. The steam exiting the first-stage turbine at 600 lbf/in.^{2} is reheated at constant pressure to 1000°F. Each turbine stage and the pump has an isentropic efficiency of 85%. The condenser pressure is 1 lbf/in.^{2} If the net power output of the cycle is 100 MW, determine

(a) the rate of heat transfer to the working fluid passing through the steam generator, in MW.
(b) the rate of heat transfer from the working fluid passing through the condenser, in MW.
(c) the cycle thermal efficiency.

8.35 An ideal Rankine cycle with reheat uses water as the working fluid. The conditions at the inlet to the first-stage turbine are 14 MPa, 600°C and the steam is reheated between the turbine stages to 600°C. For a condenser pressure of 6 kPa, plot the cycle thermal efficiency versus reheat pressure for pressures ranging from 2 to 12 MPa.

8.36 An ideal Rankine cycle with reheat uses water as the working fluid. The conditions at the inlet to the first turbine stage are 1600 lbf/in.^{2}, 1200°F and the steam is reheated between the turbine stages to 1200°F. For a condenser pressure of 1 lbf/in.^{2}, plot the cycle thermal efficiency versus reheat pressure for pressures ranging from 60 to 1200 lbf/in.^{2}.

Analyzing Regenerative Cycles

8.37 Water is the working fluid in an ideal regenerative Rankine cycle. Superheated vapor enters the turbine at 10 MPa, 480°C, and the condenser pressure is 6 kPa. Steam expands through the first-stage turbine to 0.7 MPa where some of the steam is extracted and diverted to an open feedwater heater operating at 0.7 MPa. The remaining steam expands through the second-stage turbine to the condenser pressure of 6 kPa. Saturated liquid exits the feedwater heater at 0.7 MPa. Determine for the cycle

(a) the rate of heat addition, in kJ per kg of steam entering the first-stage turbine.
(b) the thermal efficiency.
(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam entering the first-stage turbine.

8.38 For the cycle of Problem 8.37, reconsider the analysis assuming the pump and each turbine stage has an isentropic efficiency of 80%. Answer the same questions as in Problem 8.37 for the modified cycle.

8.39 Investigate the effects on cycle performance as the feedwater heater pressure takes on other values. Construct suitable plots and discuss for the cycle of

(a) Problem 8.37.
(b) Problem 8.38.

8.40 A power plant operates on a regenerative vapor power cycle with one open feedwater heater. Steam enters the first turbine stage at 12 MPa, 520°C and expands to 1 MPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 1 MPa. The remaining steam expands through the second turbine stage to the condenser pressure of 6 kPa. Saturated liquid exits the open feedwater heater at 1 MPa. For isentropic processes in the turbines and pumps, determine for the cycle (a) the thermal efficiency and (b) the mass flow rate into the first turbine stage, in kg/h, for a net power output of 330 MW.

8.41 Reconsider the cycle of Problem 8.40 as the feedwater heater pressure takes on other values. Plot the thermal efficiency and the rate of exergy destruction within the feedwater heater, in kW, versus the feedwater heater pressure ranging from 0.5 to 10 MPa. Let $T_0 = 293$ K.

8.42 Compare the results of Problem 8.40 with those for an ideal Rankine cycle having the same turbine inlet conditions and condenser pressure, but no regenerator.

8.43 For the cycle of Problem 8.40, investigate the effects on cycle performance as the feedwater heater pressure takes on other values. Construct suitable plots and discuss. Assume that each turbine stage and each pump has an isentropic efficiency of 80%.

8.44 Water is the working fluid in an ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands it to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

8.45 Water is the working fluid in a regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Each turbine stage and both pumps have isentropic efficiencies of 85%. Flow through the condenser, open feedwater heater, and steam generator is at constant pressure. Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

8.46 Water is the working fluid in an ideal regenerative Rankine cycle with one open feedwater heater. Superheated vapor enters the first-stage turbine at 16 MPa, 560°C, and the condenser pressure is 8 kPa. The mass flow rate of steam entering the first-stage turbine is 120 kg/s. Steam expands through the first-stage turbine to 1 MPa where some of the steam is extracted and diverted to an open feedwater heater operating at 1 MPa. The remainder expands through the second-stage turbine to the condenser pressure of 8 kPa. Saturated liquid exits the feedwater heater at 1 MPa. Determine

(a) the net power developed, in kW.
(b) the rate of heat transfer to the steam passing through the boiler, in kW.
(c) the thermal efficiency.
(d) the mass flow rate of condenser cooling water, in kg/s, if the cooling water undergoes a temperature increase of 18°C with negligible pressure change in passing through the condenser.

8.47 Reconsider the cycle of Problem 8.46, but include in the analysis that each turbine stage and the pump has an isentropic efficiency of 85%.

8.48 For the cycle of Problem 8.47, investigate the effects on cycle performance as the feedwater heater pressure takes on other values. Construct suitable plots and discuss.

8.49 Water is the working fluid in an ideal regenerative Rankine cycle with one closed feedwater heater. Superheated vapor enters the turbine at 10 MPa, 480°C, and the condenser pressure is 6 kPa. Steam expands through the first-stage turbine where some is extracted and diverted to a closed feedwater heater at 0.7 MPa. Condensate drains from the feedwater heater as saturated liquid at 0.7 MPa and is trapped into the condenser. The feedwater leaves the heater at 10 MPa and a temperature equal to the saturation temperature at 0.7 MPa. Determine for the cycle

(a) the rate of heat transfer to the working fluid passing through the steam generator, in kJ per kg of steam entering the first-stage turbine.
(b) the thermal efficiency.
(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam entering the first-stage turbine.

8.50 For the cycle of Problem 8.49, reconsider the analysis assuming the pump and each turbine stage have isentropic efficiencies of 80%. Answer the same questions as in Problem 8.49 for the modified cycle.

8.51 For the cycle of Problem 8.50, investigate the effects on cycle performance as the extraction pressure takes on other values. Assume that condensate drains from the closed feedwater heater as saturated liquid at the extraction pressure. Also, feedwater leaves the heater at 10 MPa and a temperature equal to the saturation temperature at the extraction pressure. Construct suitable plots and discuss.

8.52 A power plant operates on a regenerative vapor power cycle with one closed feedwater heater. Steam enters the first turbine stage at 120 bar, 520°C and expands to 10 bar, where some of the steam is extracted and diverted to a closed feedwater heater. Condensate exiting the feedwater heater as saturated liquid at 10 bar passes through a trap into the condenser. The feedwater exits the heater at 120 bar with a temperature of 170°C. The condenser pressure is 0.06 bar. For isentropic processes in each turbine stage and the pump, determine for the cycle (a) the thermal efficiency and (b) the mass flow rate into the first-stage turbine, in kg/h, if the net power developed is 320 MW.

8.53 Reconsider the cycle of Problem 8.52, but include in the analysis that each turbine stage has an isentropic efficiency of 82%. The pump efficiency remains 100%.

8.54 Modify the cycle of Problem 8.49 such that the saturated liquid condensate from the feedwater heater at 0.7 MPa is pumped into the feedwater line rather than being trapped into the condenser. Answer the same questions about the modified cycle as in Problem 8.49. List advantages and disadvantages of each scheme for removing condensate from the closed feedwater heater.

8.55 Water is the working fluid in an ideal regenerative Rankine cycle with one closed feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Condensate exiting the feedwater heater as saturated liquid at 120 lbf/in.^{2} undergoes a throttling process as it passes through a trap into the condenser. The feedwater leaves the heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

8.56 Water is the working fluid in a regenerative Rankine cycle with one closed feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Each turbine stage and the pump have isentropic efficiencies of 85%. Flow through the condenser, closed feedwater heater, and steam generator is at constant pressure. Condensate exiting the feedwater heater as saturated liquid at 120 lbf/in.^{2} undergoes a throttling process as it passes through a trap into the condenser. The feedwater leaves the heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 120 lbf/in.[2] The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

8.57 Water is the working fluid in an ideal regenerative Rankine cycle with one closed feedwater heater. Superheated vapor enters the turbine at 16 MPa, 560°C, and the condenser pressure is 8 kPa. The cycle has a closed feedwater heater using extracted steam at 1 MPa. Condensate drains from the feedwater heater as saturated liquid at 1 MPa and is trapped into the condenser. The feedwater leaves the heater at 16 MPa and a temperature equal to the saturation temperature at 1 MPa. The mass flow rate of steam entering the first-stage turbine is 120 kg/s. Determine

(a) the net power developed, in kW.
(b) the rate of heat transfer to the steam passing through the boiler, in kW.
(c) the thermal efficiency.
(d) the mass flow rate of condenser cooling water, in kg/s, if the cooling water undergoes a temperature increase of 18°C with negligible pressure change in passing through the condenser.

8.58 Reconsider the cycle of Problem 8.57, but include in the analysis that the isentropic efficiencies of the turbine stages and the pump are 85%.

8.59 Referring to Fig. 8.12, if the fractions of the total flow entering the first turbine stage (state 1) extracted at states 2, 3, 6, and 7 are y_2, y_3, y_6, and y_7, respectively, what are the fractions of the total flow at states 8, 11, and 17?

8.60 Consider a regenerative vapor power cycle with two feedwater heaters, a closed one and an open one, as shown in Figure P8.60. Steam enters the first turbine stage at 12 MPa, 480°C, and expands to 2 MPa. Some steam is extracted at 2 MPa and fed to the closed feedwater heater. The remainder expands through the second-stage turbine to 0.3 MPa, where an additional amount is extracted and fed into the open feedwater heater operating at 0.3 MPa. The steam expanding through the third-stage turbine exits at the condenser pressure of 6 kPa.

Feedwater leaves the closed heater at 210°C, 12 MPa, and condensate exiting as saturated liquid at 2 MPa is trapped into the open feedwater heater. Saturated liquid at 0.3 MPa leaves the open feedwater heater. Assume all pumps and turbine stages operate isentropically. Determine for the cycle

(a) the rate of heat transfer to the working fluid passing through the steam generator, in kJ per kg of steam entering the first-stage turbine.
(b) the thermal efficiency.
(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam entering the first-stage turbine.

8.61 For the cycle of Problem 8.60, reconsider the analysis assuming the pump and each turbine stage has an isentropic efficiency of 80%. Answer the same questions as in Problem 8.60 for the modified cycle.

8.62 For the cycle of Problem 8.60, investigate the effects on cycle performance as the higher extraction pressure takes on other values. The operating conditions for the open feedwater heater are unchanged from those in Problem 8.60. Assume that condensate drains from the closed feedwater heater as saturated liquid at the higher extraction pressure. Also,

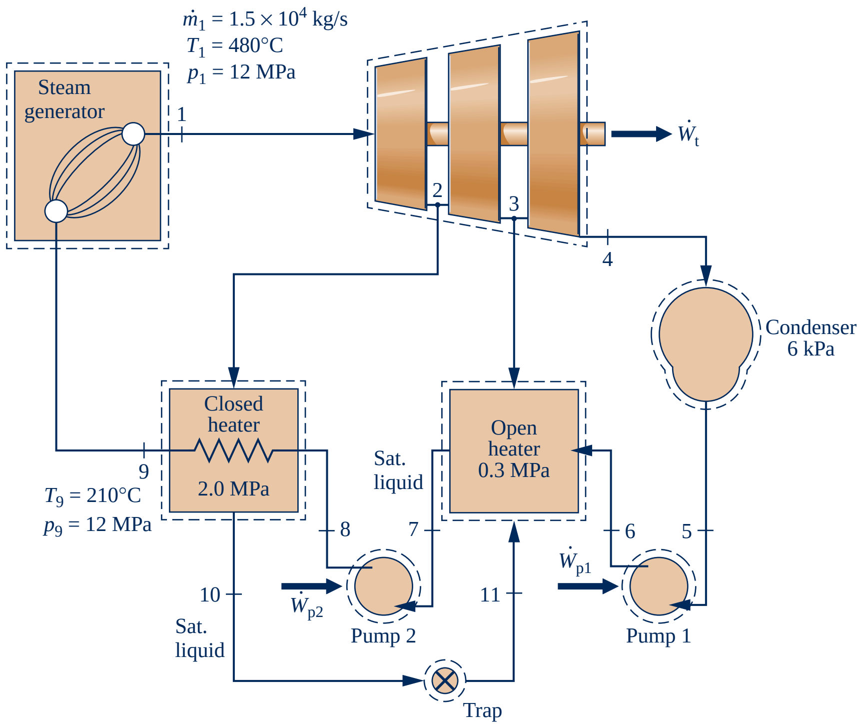


Fig. P8.60

feedwater leaves the heater at 12 MPa and a temperature equal to the saturation temperature at the extraction pressure. Construct suitable plots and discuss.

8.63 A power plant operates on a regenerative vapor power cycle with two feedwater heaters. Steam enters the first turbine stage as 12 MPa, 520°C and expands in three stages to the condenser pressure of 6 kPa. Between the first and second stages, some steam is diverted to a closed feedwater heater at 1 MPa, with saturated liquid condensate being pumped ahead into the boiler feedwater line. The feedwater leaves the closed heater at 12 MPa, 170°C. Steam is extracted between the second and third turbine stages at 0.15 MPa and fed into an open feedwater heater operating at that pressure. Saturated liquid at 0.15 MPa leaves the open feedwater heater. For isentropic processes in the pumps and turbines, determine for the cycle (a) the thermal efficiency and (b) the mass flow rate into the first-stage turbine, in kg/h, if the net power developed is 320 MW.

8.64 Reconsider the cycle of Problem 8.63, but include in the analysis that each turbine stage has an isentropic efficiency of 82% and each pump an efficiency of 100%.

8.65 Water is the working fluid in a regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam expands through the second-stage turbine to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The remaining steam expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} All processes of the working fluid in the turbine stages and pumps are internally reversible. Flow through the condenser, closed feedwater heater, open feedwater heater, and steam generator is at constant pressure. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

8.66 Water is the working fluid in a regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam expands through the second-stage turbine to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The remaining steam expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} The turbine stages and the pumps each operate adiabatically with isentropic efficiencies of 85%. Flow through the condenser, closed feedwater heater, open feedwater heater, and steam generator is at constant pressure. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

8.67 Water is the working fluid in a Rankine cycle modified to include one closed feedwater heater and one open feedwater heater. Superheated vapor enters the turbine at 16 MPa, 560°C, and the condenser pressure is 8 kPa. The mass flow rate of steam entering the first-stage turbine is 120 kg/s. The closed feedwater heater uses extracted steam at 4 MPa, and the open feedwater heater uses extracted steam at 0.3 MPa. Saturated liquid condensate drains from the closed feedwater heater at 4 MPa and is trapped into the open feedwater heater. The feedwater leaves the closed heater at 16 MPa and a temperature equal to the saturation temperature at 4 MPa. Saturated liquid leaves the open heater at 0.3 MPa. Assume all turbine stages and pumps operate isentropically. Determine

(a) the net power developed, in kW.
(b) the rate of heat transfer to the steam passing through the steam generator, in kW.
(c) the thermal efficiency.
(d) the mass flow rate of condenser cooling water, in kg/s, if the cooling water undergoes a temperature increase of 18°C with negligible pressure change in passing through the condenser.

8.68 Reconsider the cycle of Problem 8.67, but include in the analysis that the isentropic efficiencies of the turbine stages and pumps are 85%.

8.69 Consider a regenerative vapor power cycle with two feedwater heaters, a closed one and an open one, and reheat. Steam enters the first turbine stage at 12 MPa, 480°C, and expands to 2 MPa. Some steam is extracted at 2 MPa and fed to the closed feedwater heater. The remainder is reheated at 2 MPa to 440°C and then expands through the second-stage turbine to 0.3 MPa, where an additional amount is extracted and fed into the open feedwater heater operating at 0.3 MPa. The steam expanding through the third-stage turbine exits at the condenser pressure of 6 kPa. Feedwater leaves the closed heater at 210°C, 12 MPa, and condensate exiting as saturated liquid at 2 MPa is trapped into the open feedwater heater. Saturated liquid at 0.3 MPa leaves the open feedwater heater. Assume all pumps and turbine stages operate isentropically. Determine for the cycle

(a) the rate of heat transfer to the working fluid passing through the steam generator, in kJ per kg of steam entering the first-stage turbine.
(b) the thermal efficiency.

(c) the rate of heat transfer from the working fluid passing through the condenser to the cooling water, in kJ per kg of steam entering the first-stage turbine.

8.70 Reconsider the cycle of Problem 8.69, but include in the analysis that the turbine stage and pumps all have isentropic efficiencies of 80%. Answer the same questions about the modified cycle as in Problem 8.69.

8.71 For the cycle of Problem 8.70, plot thermal efficiency versus turbine stage and pump isentropic efficiencies for values ranging from 80 to 100%. Discuss.

8.72 Water is the working fluid in a reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam is reheated to 900°F before entering the second-stage turbine, where it expands to 120 lbf/in.^{2} Some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The rest expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} All processes of the working fluid in the turbine stages and pumps are internally reversible. Flow through the condenser, closed feedwater heater, open feedwater heater, steam generator, and reheater is at constant pressure. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator, including the reheat section.
(c) the thermal efficiency.

8.73 Water is the working fluid in a reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam is reheated to 900°F before entering the second-stage turbine, where it expands to 120 lbf/in.^{2} Some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The remaining steam expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} The turbine stages and the pumps each operate adiabatically with isentropic efficiencies of 85%. Flow through the condenser, closed feedwater heater, open feedwater heater, steam generator, and reheater is at constant pressure. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator, including the reheat section.
(c) the thermal efficiency.

8.74 Steam enters the first turbine stage of a vapor power cycle with reheat and regeneration at 32 MPa, 600°C, and expands to 8 MPa. A portion of the flow is diverted to a closed feedwater heater at 8 MPa, and the remainder is reheated to 560°C before entering the second turbine stage. Expansion through the second turbine stage occurs to 1 MPa, where another portion of the flow is diverted to a second closed feedwater heater at 1 MPa. The remainder of the flow expands through the third turbine stage to 0.15 MPa, where a portion of the flow is diverted to an open feedwater heater operating at 0.15 MPa, and the rest expands through the fourth turbine stage to the condenser pressure of 6 kPa. Condensate leaves each closed feedwater heater as saturated liquid at the respective extraction pressure. The feedwater streams leave each closed feedwater heater at a temperature equal to the saturation temperature at the respective extraction pressure. The condensate streams from the closed heaters each pass through traps into the next lower-pressure feedwater heater. Saturated liquid exiting the open heater is pumped to the steam generator pressure. If each turbine stage has an isentropic efficiency of 85% and the pumps operate isentropically

(a) sketch the layout of the cycle and number the principal state points.
(b) determine the thermal efficiency of the cycle.
(c) calculate the mass flow rate into the first turbine stage, in kg/h, for a net power output of 500 MW.

8.75 Steam enters the first turbine stage of a vapor power plant with reheat and regeneration at 1800 lbf/in.^{2}, 1100°F and expands in five stages to a condenser pressure of 1 lbf/in.^{2} Reheat is at 100 lbf/in.^{2} to 1000°F. The cycle includes three feedwater heaters. Closed heaters operate at 600 and 160 lbf/in.^{2}, with the drains from each trapped into the next lower-pressure feedwater heater. The feedwater leaving each closed heater is at the saturation temperature corresponding to the extraction pressure. An open feedwater heater operates at 20 lbf/in.^{2} The pumps operate isentropically, and each turbine stage has an isentropic efficiency of 88%.

(a) Sketch the layout of the cycle and number the principal state points.
(b) Determine the thermal efficiency of the cycle.
(c) Determine the heat rate, in Btu/kW · h.
(d) Calculate the mass flow rate into the first turbine stage, in lb/h, for a net power output of 3×10^9 Btu/h.

Considering Other Vapor Cycle Aspects

8.76 A binary vapor power cycle consists of two ideal Rankine cycles with steam and ammonia as the working fluids. In the steam cycle, superheated vapor enters the turbine at 6 MPa, 640°C, and saturated liquid exits the condenser at 60°C. The heat rejected from the steam cycle is provided to the ammonia cycle, producing saturated vapor at 50°C, which enters the ammonia turbine. Saturated liquid leaves the ammonia condenser at 1 MPa. For a net power output of 20 MW from the binary cycle, determine

(a) the power output of the steam and ammonia turbines, respectively, in MW.
(b) the rate of heat addition to the binary cycle, in MW.
(c) the thermal efficiency.

8.77 A binary vapor cycle consists of two Rankine cycles with steam and ammonia as the working fluids. In the steam cycle, superheated vapor enters the turbine at 900 lbf/in.², 1100°F, and saturated liquid exits the condenser at 140°F. The heat rejected from the steam cycle is provided to the ammonia cycle, producing saturated vapor at 120°F, which enters the ammonia turbine. Saturated liquid leaves the ammonia condenser at 75°F. Each turbine has an isentropic efficiency of 90% and the pumps operate isentropically. The net power output of the binary cycle is 7×10^7 Btu/h.

(a) Determine the quality at the exit of each turbine, the mass flow rate of each working fluid, in lb/h, and the overall thermal efficiency of the binary cycle.
(b) Compare the binary cycle performance to that of a single Rankine cycle using water as the working fluid and condensing at 75°F. The turbine inlet state, isentropic turbine efficiency, and net power output all remain the same.

8.78 Figure P8.78 shows a vapor power cycle with reheat and regeneration. The steam generator produces vapor at 1000 lbf/in.², 800°F. Some of this steam expands through the first turbine stage to 100 lbf/in.² and the remainder is directed to the heat exchanger. The steam exiting the first turbine stage enters the flash chamber. Saturated vapor and saturated liquid at 100 lbf/in.² exit the flash chamber as separate streams. The vapor is reheated in the heat exchanger to 530°F before entering the second turbine stage. The open feedwater heater operates at 100 lbf/in.², and the condenser pressure is 1 lbf/in.² Each turbine stage has an isentropic efficiency of 88% and the pumps operate isentropically. For a net power output of 5×10^9 Btu/h, determine

(a) the mass flow rate through the steam generator, in lb/h.
(b) the thermal efficiency of the cycle.
(c) the rate of heat transfer to the cooling water passing through the condenser, in Btu/h.

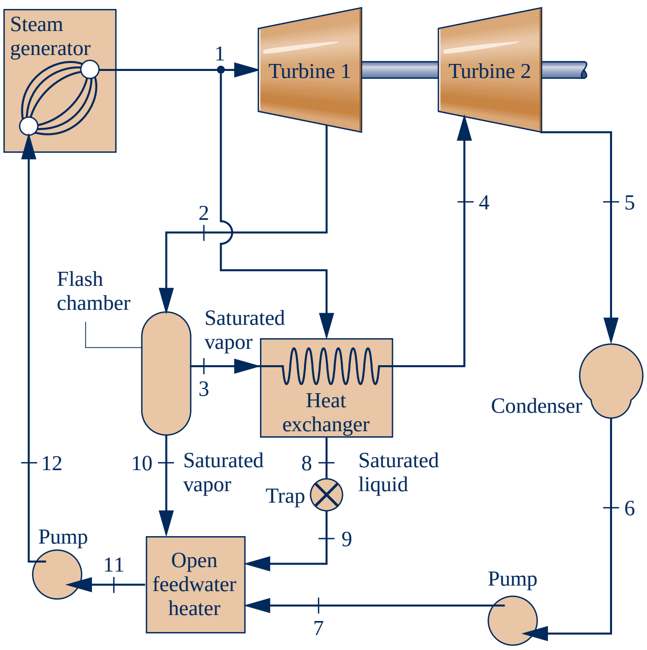


Fig. P8.78

8.79 Figure P8.79 provides steady-state operating data for a cogeneration cycle that generates electricity and provides heat for campus buildings. Steam at 1.5 MPa, 280°C, enters a two-stage turbine with a mass flow rate of 1 kg/s. A fraction of the total flow, 0.15, is extracted between the two stages at 0.2 MPa to provide for building heating, and the remainder expands through the second stage to the condenser pressure of 0.1 bar. Condensate returns from the campus buildings at 0.1 MPa, 60°C and passes through a trap into the condenser, where it is reunited with the main feedwater flow. Saturated liquid leaves the condenser at 0.1 bar. Determine

(a) the rate of heat transfer to the working fluid passing through the boiler, in kW.
(b) the net power developed, in kW.
(c) the rate of heat transfer for building heating, in kW.
(d) the rate of heat transfer to the cooling water passing through the condenser, in kW.

8.80 Consider a cogeneration system operating as illustrated in Fig. 8.14*b*. The steam generator provides 10^6 kg/h of steam at 8 MPa, 480°C, of which 4×10^5 kg/h is extracted between the first and second turbine stages at 1 MPa and diverted to a process heating load. Condensate returns from the process heating load at 0.95 MPa, 120°C and is mixed with liquid exiting the lower-pressure pump at 0.95 MPa. The entire flow is then pumped to the steam generator pressure. Saturated liquid at 8 kPa leaves the condenser. The turbine stages and the pumps operate with isentropic efficiencies of 86 and 80%, respectively. Determine

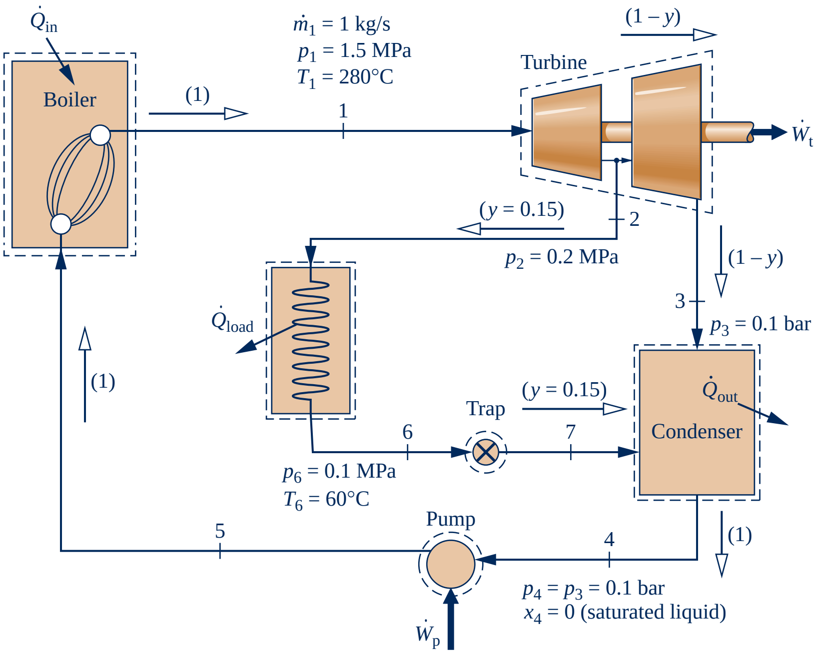
(a) the heating load, in kJ/h.
(b) the power developed by the turbine, in kW.
(c) the rate of heat transfer to the working fluid passing through the steam generator, in kJ/h.

8.81 Figure P8.81 shows a combined heat and power system (CHP) providing turbine power, process steam, and steam for a factory space heating load. Operating data are given on the figure at key states in the cycle. For the system, determine

(a) the rates that steam is extracted as process steam and for the heating load, each in lb/h.
(b) the rates of heat transfer for the process steam and the heating load, each in Btu/h.
(c) the net power developed, in Btu/h.

Devise and evaluate an overall energy-based efficiency for the combined heat and power system.

8.82 Figure P8.82 shows the schematic diagram of a cogeneration cycle. In the steam cycle, superheated vapor enters the turbine with a mass flow rate of 5 kg/s at 40 bar, 440°C and expands isentropically to 1.5 bar. Half of the flow is extracted at 1.5 bar and used for industrial process heating. The rest of the steam passes through a heat exchanger, which serves as the boiler of the Refrigerant 134a cycle and the



State	p	T (°C)	h (kJ/kg)
1	1.5 MPa	280	2992.7
2	0.2 MPa	sat	2652.9
3	0.1 bar	sat	2280.4
4	0.1 bar	sat	191.83
5	1.5 MPa	- - -	193.34
6	0.1 MPa	60	251.13
7	0.1 bar	- - -	251.13

Fig. P8.79

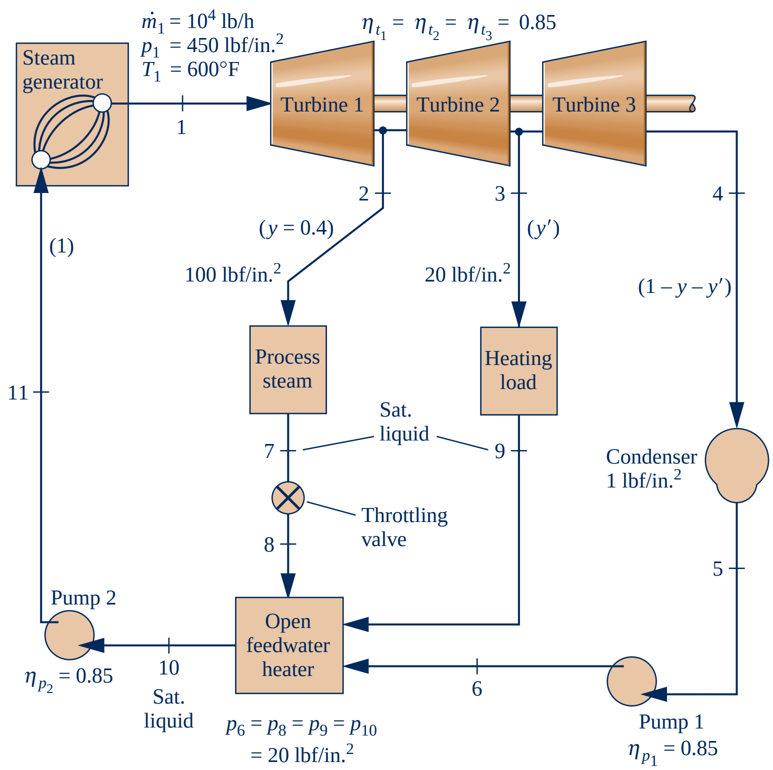


Fig. P8.81

condenser of the steam cycle. The condensate leaves the heat exchanger as saturated liquid at 1 bar, where it is combined with the return flow from the process, at 60°C and 1 bar, before being pumped isentropically to the steam generator pressure. The Refrigerant 134a cycle is an ideal Rankine cycle with refrigerant entering the turbine at 16 bar, 100°C and saturated liquid leaving the condenser at 9 bar. Determine, in kW,

(a) the rate of heat transfer to the working fluid passing through the steam generator of the steam cycle.
(b) the net power output of the binary cycle.
(c) the rate of heat transfer to the industrial process.

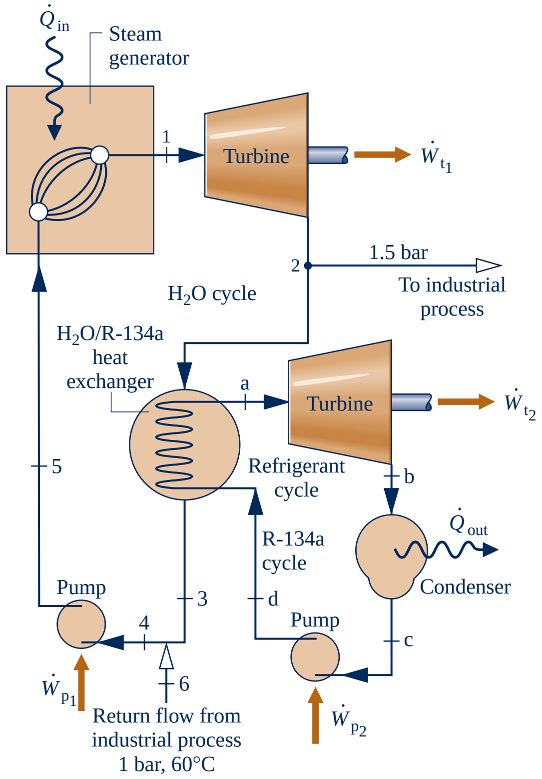


Fig. P8.82

Vapor Cycle Exergy Analysis

8.83 In a *cogeneration* system, a Rankine cycle operates with steam entering the turbine at 800 lbf/in.^{2}, 700°F, and a condenser pressure of 180 lbf/in.^{2} The isentropic turbine efficiency is 80%. Energy rejected by the condensing steam is transferred to a separate process stream of water entering at 250°F, 140 lbf/in.^{2} and exiting as saturated vapor at 140 lbf/in.^{2} Determine the mass flow rate, in lb/h, for the working fluid of the Rankine cycle if the mass flow rate of the process stream is 50,000 lb/h. Devise and evaluate an exergetic efficiency for the overall cogeneration system. Let T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

8.84 The steam generator of a vapor power plant can be considered for simplicity to consist of a combustor unit in which fuel and air are burned to produce hot combustion gases, followed by a heat exchanger unit where the cycle working fluid is vaporized and superheated as the hot gases cool. Consider water as the working fluid undergoing the cycle of Problem 8.17. Hot combustion gases, which are assumed to have the properties of air, enter the heat exchanger portion of the steam generator at 1200 K and exit at 600 K with a negligible change in pressure. Determine for the heat exchanger unit

(a) the net rate at which exergy is carried in by the gas stream, in kJ per kg of steam flowing.
(b) the net rate at which exergy is carried out by the water stream, in kJ per kg of steam flowing.
(c) the rate of exergy destruction, in kW.
(d) the exergetic efficiency given by Eq. 7.27.

Let T_0 = 15°C, p_0 = 0.1 MPa

8.85 Determine the rate of exergy input, in kJ per kg of steam flowing, to the working fluid passing through the steam generator in Problem 8.17. Perform calculations to account for all outputs, losses, and destructions of this exergy. Let T_0 = 15°C, p_0 = 0.1 MPa.

8.86 In the steam generator of the cycle of Problem 8.19, the energy input to the working fluid is provided by heat transfer from hot gaseous products of combustion, which cool as a separate stream from 1490 to 380°F with a negligible pressure drop. The gas stream can be modeled as air as an ideal gas. Determine, in Btu/h, the rate of exergy destruction in the

(a) heat exchanger unit of the steam generator.
(b) turbine and pump.
(c) condenser.

Also, calculate the net rate at which exergy is carried away by the cooling water passing through the condenser, in Btu/h. Let T_0 = 60°F, p_0 = 14.7 lbf/in.^{2}

8.87 For the regenerative vapor power cycle of Problem 8.67, calculate the rates of exergy destruction in the feedwater heaters in kW. Express each as a fraction of the flow exergy increase of the working fluid passing through the steam generator. Let T_0 = 16°C, p_0 = 1 bar.

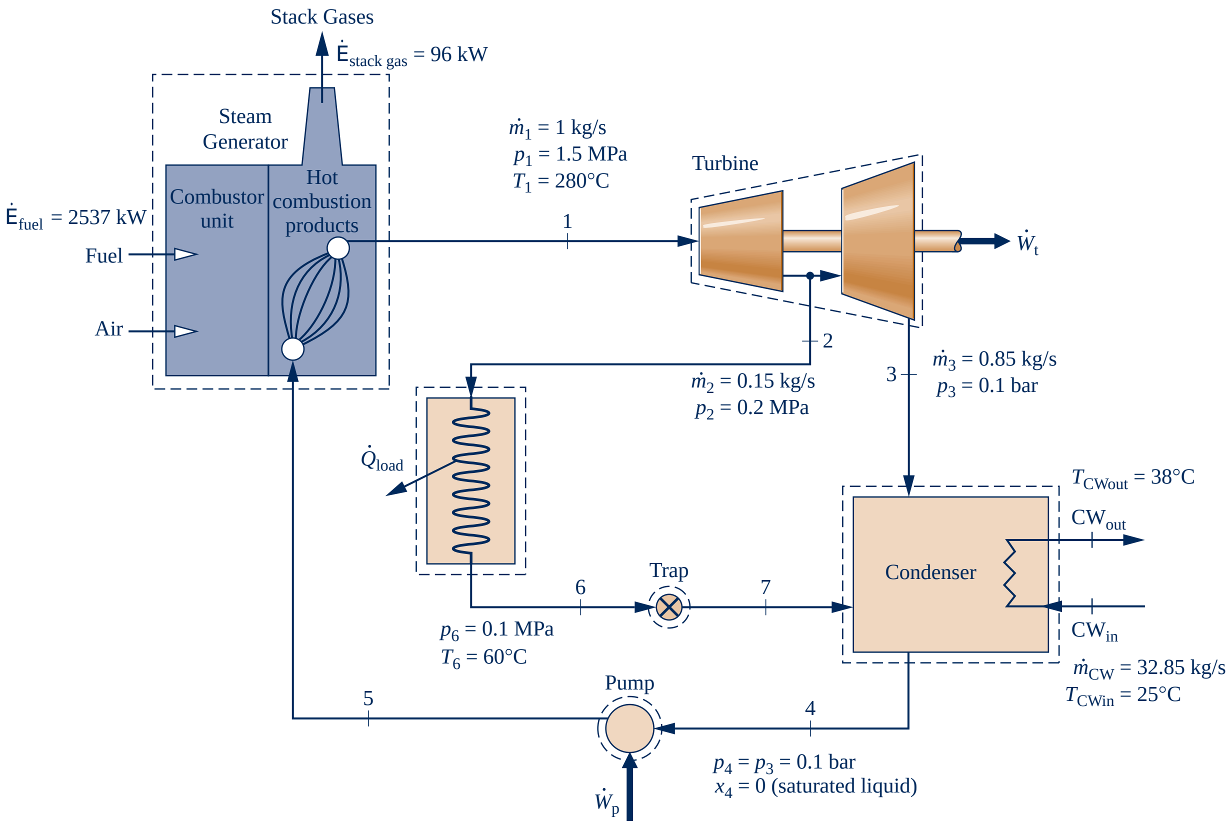
8.88 Determine the rate of exergy input, in MW, to the working fluid passing through the steam generator in Problem 8.74. Perform calculations to account for all outputs, losses, and destructions of this exergy. Let T_0 =15°C, p_0 = 1 bar.

8.89 Determine the rate of exergy input, in Btu/h, to the working fluid passing through the steam generator in Problem 8.75. Perform calculations to account for all outputs, losses, and destructions of this exergy. Let T_0 = 60°F, p_0 = 14.7 lbf/in.^{2}

8.90 Determine the rate of exergy transfer, in Btu/h, to the working fluid passing through the steam generator in Problem 8.78. Perform calculations to account for all outputs, losses, and destructions of this exergy. Let T_0 = 60°F, p_0 = 14.7 lbf/in.^{2}

8.91 Determine the rate of exergy transfer, in kJ per kg of steam entering the first-stage turbine, to the working fluid passing through the steam generator in Problem 8.46. Perform calculations to account for all outputs, losses, and destructions of this exergy. Let T_0 = 15°C, p_0 = 0.1 MPa.

8.92 Figure P8.92 provides steady-state operating data for a cogeneration cycle that generates electricity and provides



State	p	T (°C)	h (kJ/kg)	s (kJ/kg·K)
1	1.5 MPa	280	2992.7	6.8381
2	0.2 MPa	sat	2652.9	6.9906
3	0.1 bar	sat	2280.4	7.1965
4	0.1 bar	sat	191.83	0.6493
5	1.5 MPa	- - -	193.34	0.6539
6	0.1 MPa	60	251.13	0.8312
7	0.1 bar	- - -	251.13	0.8352
Cooling water$_{in}$	- - -	25	104.89	0.3674
Cooling water$_{out}$	- - -	38	159.21	0.5458

Fig. P8.92

heat for campus buildings. Steam at 1.5 MPa, 280°C, enters a two-stage turbine with a mass flow rate of 1 kg/s. Steam is extracted between the two stages at 0.2 MPa with a mass flow rate of 0.15 kg/s to provide for building heating, while the remainder expands through the second turbine stage to the condenser pressure of 0.1 bar with mass flow rate of 0.85 kg/s. The campus load heat exchanger in the schematic represents all of the heat transfer to the campus buildings. For the purposes of this analysis, assume that the heat transfer in the campus load heat exchanger occurs at an average boundary temperature of 110°C. Condensate returns from the campus buildings at 0.1 MPa, 60°C and passes through a trap into the condenser, where it is reunited with the main feedwater flow. The cooling water has a mass flow rate of 32.85 kg/s entering the condenser at 25°C and exiting the condenser at 38°C. The working fluid leaves the condenser as saturated liquid at 0.1 bar. The rate of exergy input with fuel entering the combustor unit of the steam generator is 2537 kW, and no exergy is carried in by the combustion air. The rate of exergy loss with the stack gases exiting the steam generator is 96 kW. Let T_0 = 25°C, p_0 = 0.1 MPa. Determine, as percentages of the rate of exergy input with fuel entering the combustor unit, all outputs, losses, and destructions of this exergy for the cogeneration cycle.

8.93 Steam enters the turbine of a simple vapor power plant at 100 bar, 520°C and expands adiabatically, exiting at 0.08 bar with a quality of 90%. Condensate leaves the condenser as saturated liquid at 0.08 bar. Liquid exits the pump at 100 bar, 43°C. The specific exergy of the fuel entering the combustor unit of the steam generator is estimated to be 14,700 kJ/kg. No exergy is carried in by the combustion air. The exergy of the stack gases leaving the steam generator is estimated to be 150 kJ per kg of fuel. The mass flow rate of the steam is 3.92 kg per kg of fuel. Cooling water enters the condenser at $T_0 = 20°C$, $p_0 = 1$ atm and exits at 35°C, 1 atm. Develop a full accounting of the exergy entering the plant with the fuel.

▶ DESIGN & OPEN-ENDED PROBLEMS: EXPLORING ENGINEERING PRACTICE

8.1D Use a web-based resource (such as http://www.eia.doe.gov) to locate the three largest electricity-generating plants in your home state. For each, determine the fuel type, plant's age, and reported safety issues. Determine how each plant contributes to global climate change and identify its likely effects on human health and the environment. For one of the plants, propose ways to reduce health and environmental impacts associated with the plant. Write a report, including at least three references.

8.2D Vast quantities of water circulate through the condensers of large vapor power plants, exiting at temperatures a few degrees above ambient temperature. For a 500-MW power plant, investigate possible uses for the warm condenser water. For one such use, estimate the annual economic benefit provided by the condenser water, in $. Report your findings in a PowerPoint presentation.

8.3D Write an *op-ed* (opinion-editorial) article on a significant issue relevant to providing electric power to U.S. consumers in the next 20 years. While op-eds are aimed at a general audience, they should be thoroughly researched and supported with evidence. Observe established practices for preparing op-ed articles and avoid technical jargon. Early in the writing process, consult with the publication, print or online, for which the article is intended to determine publication policies, procedures, and interest in your proposed topic. With your op-ed article submittal, provide the name of the publication for which it is intended and a file of your correspondence with its staff.

8.4D Study the feasibility of installing a 500-MW field of wind turbines off the coast of Chicago in Lake Michigan. Determine the number and type of turbines needed, estimate the cost of owning and operating the system, and perform an economic analysis. Investigate the availability of federal and state tax credits as part of your economic analysis. Compare the cost per kilowatt-hour for your proposed system with the average cost of electricity in Chicago from conventional power plants in the region. Write a report of your findings including at least three references.

8.5D Consider the feasibility of using *biomass* to fuel a 200-MW electric power plant in a rural area in your locale. Analyze the advantages and disadvantages of biomass in comparison with coal and natural gas. Include in your analysis material handling issues, plant operations, environmental considerations, and costs. Prepare a PowerPoint presentation of your recommendations.

8.6D A 5000-ft^2 research outpost is being designed for studying climate change in Antarctica. The outpost will house five scientists along with their communication and research equipment. Develop the preliminary design of an array of *photovoltaic cells* to provide all necessary power from solar energy during months of continuous sunlight. Specify the number and type of cells needed and present schematics of the system you propose.

8.7D Owing to strong local winds and large elevation differences on the Hawaiian island of Maui, it may be a suitable place to combine a wind farm with *pumped hydro* energy storage. At times when the wind turbines produce excess power, water is pumped to reservoirs at higher elevations. The water is released during periods of peak electric demand through hydraulic turbines to produce electricity. Develop a proposal to meet 30% of the island's power needs by the year 2020 using this renewable energy concept. In your report, list advantages and disadvantages of the proposed system. Include at least three references.

8.8D Critically evaluate carbon dioxide capture and underground storage for fossil-fueled power plants, including technical aspects and related costs. Consider ways to separate CO_2 from gas streams, issues related to injecting CO_2 at great depths, consequences of CO_2 migration from storage, and the expected increase in electricity cost per kW · h with the deployment of this technology. Formulate a position in favor of, or in opposition to, large-scale carbon dioxide capture and storage. Write a report, including at least three references.

8.9D Most electricity in the United States is generated today by large *centralized* power plants and distributed to end users via long-distance transmission lines. Some experts expect a gradual shift to a *distributed* (decentralized) power system where electricity is generated locally by smaller-scale plants primarily using locally available resources, including wind, solar, biomass, hydropower, and geothermal. Other experts agree on the distributed, smaller-scale approach but contend that the model for the future is the tightly integrated *industrial ecosystem,* already seen in Denmark. Critically evaluate these two visions technically and economically, together with hybrids obtained by combining them, each relative to the current centralized approach. Rank order all approaches considered, in descending order from the most likely to least likely future scenario. Write a report, including at least three references.

8.10D *Geoengineering* is an area of study focused on managing earth's environment to reduce effects of global climate change. For three such concepts, obtained from print or online sources, research each in terms of feasibility, including technical issues, costs, and risks. Determine if any of the three is a truly viable candidate for implementation. Report your findings in a PowerPoint presentation.

8.11D *Concurrent engineering design* considers all phases of a product's *life cycle* holistically with the aim of arriving at an acceptable final design more quickly and with less cost than achievable in a sequential approach. A tenet of concurrent design is the use of a multitalented design team having technical *and* nontechnical expertise. For power plant design, technical expertise necessarily includes the skills of several engineering disciplines. Determine the makeup of the design team and the skill set each member provides for the concurrent design of a power plant selected from those listed in Table 8.2. Summarize your findings in a poster presentation suitable for presentation at a technical conference.

8.12D Some observers contend that *enhanced oil recovery* is a viable commercial use for carbon dioxide captured from the exhaust gas of coal-fired power plants and other industrial sources. Proponents envision that this will foster transport of carbon dioxide by ship from industrialized, oil-importing nations to less industrialized, oil-producing nations. They say such commerce will require innovations in ship design. Develop a conceptual design of a carbon dioxide transport ship. Consider only major issues, including but not limited to: power plant type, cargo volume, means for loading and unloading carbon dioxide, minimization of carbon dioxide loss to the atmosphere, and costs. Include figures and sample calculations as appropriate. Explain how your carbon dioxide transport ship differs from ships transporting natural gas.

8.13D Silicon is one of earth's most abundant elements. Yet demand for the pricey high-purity form of silicon required to make solar cells has risen with the growth of the solar-photovoltaic industry. This, together with limitations of the energy-intensive technology customarily used to produce solar-grade silicon, has led many to think about the development of improved technologies for producing solar-grade silicon and using materials other than silicon for solar cells. Investigate means for producing solar cells using silicon, including conventional and improved methods, and for producing cells using other materials. Compare and critically evaluate all methods discovered based on energy use, environmental impact, and cost. Prepare a poster presentation of your findings.

8.14D Power plant planning is best done on a *life-cycle* basis (Table 8.3). The life cycle begins with extracting resources from the earth required by the plant and ends with eventual retirement of the plant after decades of operation. To obtain an accurate picture of cost, costs should be considered in *all* phases of the life cycle, including remediation of environmental impacts, effects on human health, waste disposal, and government subsidies, rather than just narrowly focusing on costs related to the plant construction and operation phase. For one of the locales listed below, and considering only major cost elements, determine on a life-cycle-cost basis the power plant option that better meets expected regional electricity needs up to 2050. Write a report fully documenting your findings.

(a) Locale: Midwest and Great Plains. Options: coal-fired plants, wind-power plants, or a combination.
(b) Locale: Northeast and Atlantic seaboard. Options: nuclear power plants, natural gas–fired plants, or a combination.
(c) Locale: South and Southwest. Options: natural gas–fired plants, concentrating-solar plants, or a combination.
(d) Locale: California and Northwest. Options: concentrating-solar plants, wind-power plants, hydropower plants, or a combination.

8.15D With another project team, conduct a formal debate on one of the propositions listed below or one assigned to you. Observe rules of formal debate, including but not limited to use of a traditional format: Each constructive speech (first affirmative and first negative, second affirmative and second negative) is given eight minutes, and each rebuttal speech (first negative and first affirmative, second negative and second affirmative) is given four minutes.

Proposition (a): As a national policy, *cost-benefit analysis* should be used when evaluating proposed environmental regulations. Alternative proposition: Should not be used.

Proposition (b): As a national policy, electricity production using nuclear technology should be greatly expanded. Alternative proposition: Should not be greatly expanded.

Proposition (c): As a national policy, the United States should strongly encourage developing nations to reduce their contributions to global climate change. Alternative proposition: Should not strongly encourage.

***Gas turbines*, introduced in Sec. 9.5, power aircraft and generate electricity for many ground-based uses.** **© Nicosan/Alamy**

ENGINEERING CONTEXT An introduction to power generation systems is provided in Chap. 8; see pp. 426–430. The introduction surveys current U.S. power generation by source and looks ahead to power generation needs in the next few decades. Because the introduction provides context for study of power systems generally, we recommend that you review it before continuing with the present chapter dealing with gas power systems.

While the focus of Chap. 8 is on vapor power systems in which the working fluids are alternatively vaporized and condensed, the **objective** of this chapter is to study power systems utilizing working fluids that are always a gas. Included in this group are gas turbines and internal combustion engines of the spark-ignition and compression-ignition types. In the first part of the chapter, internal combustion engines are considered. Gas turbine power plants are discussed in the second part of the chapter. The chapter concludes with a brief study of compressible flow in nozzles and diffusers, which are components in gas turbines for aircraft propulsion and other devices of practical importance.

9

Gas Power Systems

▶ LEARNING OUTCOMES

When you complete your study of this chapter, you will be able to...

- Perform air-standard analyses of internal combustion engines based on the Otto, Diesel, and dual cycles, including:
  - sketching p–v and T–s diagrams and evaluating property data at principal states.
  - applying energy, entropy, and exergy balances.
  - determining net power output, thermal efficiency, and mean effective pressure.
- Perform air-standard analyses of gas turbine power plants based on the Brayton cycle and its modifications, including:
  - sketching T–s diagrams and evaluating property data at principal states.
  - applying mass, energy, entropy, and exergy balances.
  - determining net power output, thermal efficiency, back work ratio, and the effects of compressor pressure ratio.
- For subsonic and supersonic flows through nozzles and diffusers:
  - demonstrate understanding of the effects of area change, the effects of back pressure on mass flow rate, and the occurrence of choking and normal shocks.
  - analyze the flow of ideal gases with constant specific heats.

Considering Internal Combustion Engines

This part of the chapter deals with *internal* combustion engines. Although most gas turbines are also internal combustion engines, the name is usually applied to *reciprocating* internal combustion engines of the type commonly used in automobiles, trucks, and buses. These engines differ from the power plants considered in Chap. 8 because the processes occur within reciprocating piston–cylinder arrangements and not in interconnected series of different components.

spark-ignition
compression-ignition

Two principal types of reciprocating internal combustion engines are the **spark-ignition** engine and the **compression-ignition** engine. In a spark-ignition engine, a mixture of fuel and air is ignited by a spark plug. In a compression-ignition engine, air is compressed to a high enough pressure and temperature that combustion occurs spontaneously when fuel is injected. Spark-ignition engines have advantages for applications requiring power up to about 225 kW (300 horsepower). Because they are relatively light and lower in cost, spark-ignition engines are particularly suited for use in automobiles. Compression-ignition engines are normally preferred for applications when fuel economy and relatively large amounts of power are required (heavy trucks and buses, locomotives and ships, auxiliary power units). In the middle range, spark-ignition and compression-ignition engines are used.

9.1 Introducing Engine Terminology

compression ratio

Figure 9.1 is a sketch of a reciprocating internal combustion engine consisting of a piston that moves within a cylinder fitted with two valves. The sketch is labeled with some special terms. The *bore* of the cylinder is its diameter. The *stroke* is the distance the piston moves in one direction. The piston is said to be at *top dead center* when it has moved to a position where the cylinder volume is a minimum. This minimum volume is known as the *clearance* volume. When the piston has moved to the position of maximum cylinder volume, the piston is at *bottom dead center*. The volume swept out by the piston as it moves from the top dead center to the bottom dead center position is called the *displacement volume*. The **compression ratio** r is defined as the volume at bottom dead center divided by the volume at top dead center. The reciprocating motion of the piston is converted to rotary motion by a crank mechanism.

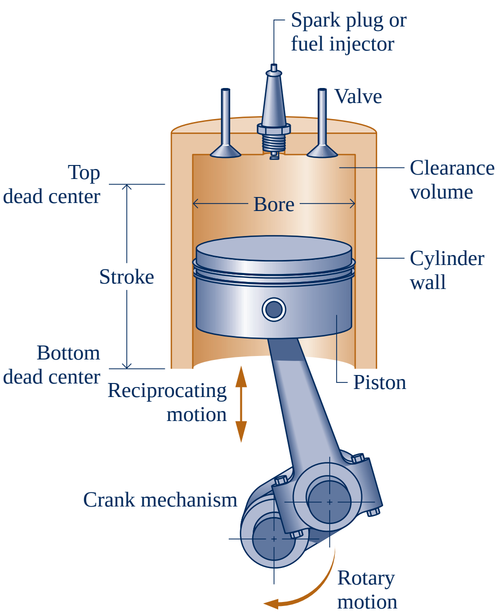


Fig. 9.1 Nomenclature for reciprocating piston–cylinder engines.

In a *four-stroke* internal combustion engine, the piston executes four distinct strokes within the cylinder for every two revolutions of the crankshaft. Fig. 9.2 gives a pressure–volume diagram such as might be displayed electronically.

1. With the intake valve open, the piston makes an *intake stroke* to draw a fresh charge into the cylinder. For spark-ignition engines, the charge is a combustible mixture of fuel and air. Air alone is the charge in compression-ignition engines.

2. With both valves closed, the piston undergoes a *compression stroke,* raising the temperature and pressure of the charge. This requires work input from the piston to the cylinder contents. A combustion process is then initiated, resulting in a high-pressure, high-temperature gas mixture. Combustion is induced near the end of the compression stroke in spark-ignition engines by the spark plug. In compression-ignition engines, combustion is initiated by injecting fuel into the hot compressed air, beginning near the end of the compression stroke and continuing through the first part of the expansion.

3. A *power* stroke follows the compression stroke, during which the gas mixture expands and work is done on the piston as it returns to bottom dead center.
4. The piston then executes an *exhaust stroke* in which the burned gases are purged from the cylinder through the open exhaust valve.

Smaller engines operate on *two-stroke* cycles. In two-stroke engines, the intake, compression, expansion, and exhaust operations are accomplished in one revolution of the crankshaft. Although internal combustion engines undergo *mechanical* cycles, the cylinder contents do not execute a *thermodynamic* cycle, for matter is introduced with one composition and is later discharged at a different composition.

A parameter used to describe the performance of reciprocating piston engines is the *mean effective pressure,* or mep. The **mean effective pressure** is the theoretical constant pressure that, if it acted on the piston during the power stroke, would produce the same *net* work as actually developed in one cycle. That is

$$\text{mep} = \frac{\text{net work for one cycle}}{\text{displacement volume}} \tag{9.1}$$

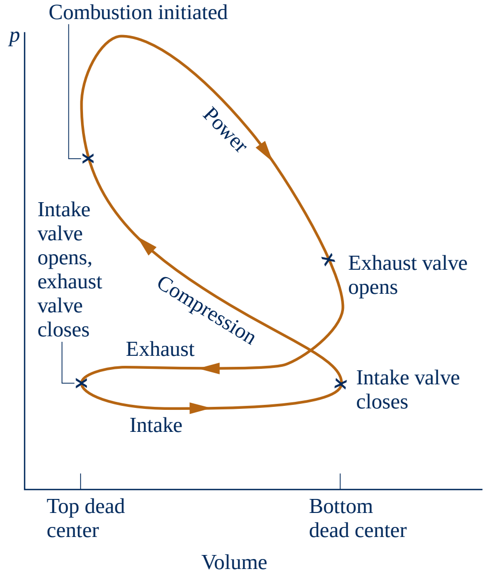


Fig. 9.2 Pressure–volume diagram for a reciprocating internal combustion engine.

mean effective pressure

For two engines of equal displacement volume, the one with a higher mean effective pressure would produce the greater net work and, if the engines run at the same speed, greater power.

AIR-STANDARD ANALYSIS. A detailed study of the performance of a reciprocating internal combustion engine would take into account many features. These would include the combustion process occurring within the cylinder and the effects of irreversibilities associated with friction and with pressure and temperature gradients. Heat transfer between the gases in the cylinder and the cylinder walls and the work required to charge the cylinder and exhaust the products of combustion also would be considered. Owing to these complexities, accurate modeling of reciprocating internal combustion engines normally involves computer simulation. To conduct *elementary* thermodynamic analyses of internal combustion engines, considerable simplification is required. One procedure is to employ an **air-standard analysis** having the following elements:

air-standard analysis: internal combustion engines

- A fixed amount of air modeled as an ideal gas is the working fluid. See Table 9.1 for a review of ideal gas relations.
- The combustion process is replaced by a heat transfer from an external source.
- There are no exhaust and intake processes as in an actual engine. The cycle is completed by a constant-volume heat transfer process taking place while the piston is at the bottom dead center position.
- All processes are internally reversible.

In addition, in a **cold air-standard analysis**, the specific heats are assumed constant at their ambient temperature values. With an air-standard analysis, we avoid dealing with the complexities of the combustion process and the change of composition during combustion. A comprehensive analysis requires that such complexities be considered, however. For a discussion of combustion, see Chap. 13.

cold air-standard analysis

Although an air-standard analysis simplifies the study of internal combustion engines considerably, values for the mean effective pressure and operating temperatures and pressures calculated on this basis may depart significantly from those of actual engines. Accordingly, air-standard analysis allows internal combustion engines to be examined only qualitatively. Still, insights concerning actual performance can result with such an approach.

TABLE 9.1

Ideal Gas Model Review

Equations of state:

$$pv = RT \quad (3.32)$$
$$pV = mRT \quad (3.33)$$

Changes in u and h:

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)\, dT \quad (3.40)$$

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)\, dT \quad (3.43)$$

Constant Specific Heats	Variable Specific Heats
$u(T_2) - u(T_1) = c_v(T_2 - T_1)$ (3.50) $h(T_2) - h(T_1) = c_p(T_2 - T_1)$ (3.51) See Tables A-20, 21 for data.	$u(T)$ and $h(T)$ are evaluated from appropriate tables: Tables A-22 for air (mass basis) and A-23 for other gases (molar basis).

Changes in s:

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1} \quad (6.17)$$

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R\ln\frac{p_2}{p_1} \quad (6.18)$$

Constant Specific Heats	Variable Specific Heats
$s(T_2, v_2) - s(T_1, v_1) = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$ (6.21) $s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$ (6.22) See Tables A-20, 21 for data.	$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R\ln\frac{p_2}{p_1}$ (6.20a) where $s^\circ(T)$ is evaluated from appropriate tables: Tables A-22 for air (mass basis) and A-23 for other gases (molar basis).

Relating states of equal specific entropy: $\Delta s = 0$:

Constant Specific Heats	Variable Specific Heats — Air Only
$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$ (6.43) $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$ (6.44) $\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k}$ (6.45) where $k = c_p/c_v$ is given in Tables A-20 for several gases.	$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}}$ (air only) (6.41) $\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$ (air only) (6.42) where p_r and v_r are provided for air in Tables A-22.

ENERGY & ENVIRONMENT Nature needed 500 million years to create the world's stock of readily accessible oil, but some observers predict we will consume much of what's left within the next 50 years. Still, the important issue, they say, is not when the world runs out of oil, but when production will peak. After that, unless demand is reduced, oil prices will rise. This will end the era of cheap oil we have enjoyed for decades and pose challenges for society.

The rate at which any well can produce oil typically rises to a maximum and then, when about half the oil has been pumped out, begins to fall as the remaining oil becomes increasingly difficult to extract. Using this model for the world oil supply as a whole, economists predict a peak in oil production by 2020, or even sooner. If we don't plan for the future, this could lead to shortages, higher fuel prices at the pump, and political repercussions.

The reality is that oil has become our Achilles heel with regard to energy. Transportation accounts for about 70% of current U.S. oil consumption, while nearly 60% of the oil we use is imported. Domestic oil production peaked in the 1970s and has declined since then. There have been calls for greater production from our offshore and wilderness areas, but analysts say this will not be a solution, for oil from many such locations will meet only a few months of current demand. Moreover, ecologists warn about environmental damage owing to huge oil spills; collateral soil, water, and air pollution; and other adverse effects related to oil extraction, delivery, and use. Many think a better way forward is to wean ourselves from relying on oil for transportation by using strategies for economizing fuel (see www.fueleconomy.gov), using biofuels such as *cellulosic* ethanol, and driving hybrid-electric or all-electric vehicles.

9.2 Air-Standard Otto Cycle

In the remainder of this part of the chapter, we consider three cycles that adhere to air-standard cycle idealizations: the Otto, Diesel, and dual cycles. These cycles differ from each other only in the way the heat addition process that replaces combustion in the actual cycle is modeled.

The air-standard Otto cycle is an ideal cycle that assumes heat addition occurs instantaneously while the piston is at top dead center. The **Otto cycle** is shown on the p–v and T–s diagrams of Fig. 9.3. The cycle consists of four internally reversible processes in series:

Otto cycle

- Process 1–2 is an isentropic compression of the air as the piston moves from bottom dead center to top dead center.
- Process 2–3 is a constant-volume heat transfer to the air from an external source while the piston is at top dead center. This process is intended to represent the ignition of the fuel–air mixture and the subsequent rapid burning.
- Process 3–4 is an isentropic expansion (power stroke).
- Process 4–1 completes the cycle by a constant-volume process in which heat is rejected from the air while the piston is at bottom dead center.

Since the air-standard Otto cycle is composed of internally reversible processes, areas on the T–s and p–v diagrams of Fig. 9.3 can be interpreted as heat and work,

TAKE NOTE...
For internally reversible processes of closed systems, see Secs. 2.2.5 and 6.6.1 for discussions of area interpretations of work and heat transfer on p–v and T–s diagrams, respectively.

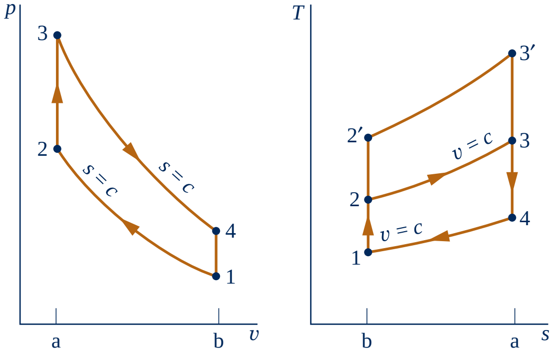


Fig. 9.3 p–v **and** T–s **diagrams of the air-standard Otto cycle.**

Otto_Cycle
A.27 – Tabs a & b

respectively. On the T–s diagram, area 2–3–a–b–2 represents heat added per unit of mass and area 1–4–a–b–1 the heat rejected per unit of mass. On the p–v diagram, area 1–2–a–b–1 represents work input per unit of mass during the compression process and area 3–4–b–a–3 is work done per unit of mass in the expansion process. The enclosed area of each figure can be interpreted as the net work output or, equivalently, the net heat added.

CYCLE ANALYSIS. The air-standard Otto cycle consists of two processes in which there is work but no heat transfer, Processes 1–2 and 3–4, and two processes in which there is heat transfer but no work, Processes 2–3 and 4–1. Expressions for these energy transfers are obtained by reducing the closed system energy balance assuming that changes in kinetic and potential energy can be ignored. The results are

$$\frac{W_{12}}{m} = u_2 - u_1, \qquad \frac{W_{34}}{m} = u_3 - u_4$$
$$\frac{Q_{23}}{m} = u_3 - u_2, \qquad \frac{Q_{41}}{m} = u_4 - u_1 \tag{9.2}$$

TAKE NOTE...
When analyzing air-standard cycles, it is frequently convenient to regard all work and heat transfers as positive quantities and write the energy balance accordingly.

Carefully note that in writing Eqs. 9.2, we have departed from our usual sign convention for heat and work. Thus, W_{12}/m is a positive number representing the work *input* during compression and Q_{41}/m is a positive number representing the heat *rejected* in Process 4–1. The *net work* of the cycle is expressed as

$$\frac{W_{\text{cycle}}}{m} = \frac{W_{34}}{m} - \frac{W_{12}}{m} = (u_3 - u_4) - (u_2 - u_1)$$

Alternatively, the net work can be evaluated as the *net heat added*

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (u_3 - u_2) - (u_4 - u_1)$$

which, on rearrangement, can be placed in the same form as the previous expression for net work.

The thermal efficiency is the ratio of the net work of the cycle to the heat added.

$$\eta = \frac{(u_3 - u_2) - (u_4 - u_1)}{u_3 - u_2} = 1 - \frac{u_4 - u_1}{u_3 - u_2} \tag{9.3}$$

When air table data are used to conduct an analysis involving an air-standard Otto cycle, the specific internal energy values required by Eq. 9.3 can be obtained from Table A-22 or A-22E as appropriate. The following relationships based on Eq. 6.42 apply for the isentropic processes 1–2 and 3–4

$$v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = \frac{v_{r1}}{r} \tag{9.4}$$

$$v_{r4} = v_{r3}\left(\frac{V_4}{V_3}\right) = r v_{r3} \tag{9.5}$$

compression ratio

where r denotes the **compression ratio**. Note that since $V_3 = V_2$ and $V_4 = V_1$, $r = V_1/V_2 = V_4/V_3$ The parameter v_r is tabulated versus temperature for air in Tables A-22.

When the Otto cycle is analyzed on a cold air-standard basis, the following expressions based on Eq. 6.44 would be used for the isentropic processes in place of Eqs. 9.4 and 9.5, respectively

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1} \qquad \text{(constant } k\text{)} \tag{9.6}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} = \frac{1}{r^{k-1}} \qquad \text{(constant } k\text{)} \tag{9.7}$$

where k is the specific heat ratio, $k = c_p/c_v$.

EFFECT OF COMPRESSION RATIO ON PERFORMANCE. By referring to the T–s diagram of Fig. 9.3, we can conclude that the Otto cycle thermal efficiency increases as the compression ratio increases. An increase in the compression ratio changes the cycle from 1–2–3–4–1 to 1–2′–3′–4–1. Since the average temperature of heat addition is greater in the latter cycle and both cycles have the same heat rejection process, cycle 1–2′–3′–4–1 would have the greater thermal efficiency. The increase in thermal efficiency with compression ratio is also brought out simply by the following development on a cold air-standard basis. For constant c_v, Eq. 9.3 becomes

$$\eta = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)}$$

On rearrangement

$$\eta = 1 - \frac{T_1}{T_2}\left(\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right)$$

From Eqs. 9.6 and 9.7 above, $T_4/T_1 = T_3/T_2$, so

$$\eta = 1 - \frac{T_1}{T_2}$$

Finally, introducing Eq. 9.6

$$\eta = 1 - \frac{1}{r^{k-1}} \quad \text{(cold-air standard basis)} \tag{9.8}$$

Equation 9.8 indicates that the cold air-standard Otto cycle thermal efficiency is a function of compression ratio and k. This relationship is shown in Fig. 9.4 for $k = 1.4$, representing ambient air.

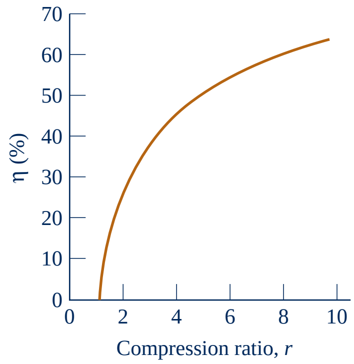


Fig. 9.4 Thermal efficiency of the cold air-standard Otto cycle, $k = 1.4$.

The foregoing discussion suggests that it is advantageous for internal combustion engines to have high compression ratios, and this is the case. The possibility of autoignition, or "knock," places an upper limit on the compression ratio of spark-ignition engines, however. After the spark has ignited a portion of the fuel–air mixture, the rise in pressure accompanying combustion compresses the remaining charge. Autoignition can occur if the temperature of the unburned mixture becomes too high before the mixture is consumed by the flame front. Since the temperature attained by the air–fuel mixture during the compression stroke increases as the compression ratio increases, the likelihood of autoignition occurring increases with the compression ratio. Autoignition may result in high-pressure waves in the cylinder (manifested by a knocking or pinging sound) that can lead to loss of power as well as engine damage.

Owing to performance limitations, such as autoignition, the compression ratios of spark-ignition engines using the *unleaded* fuel required today because of air pollution concerns are in the range 9.5 to 11.5, approximately. Higher compression ratios can be achieved in compression-ignition engines because only air is compressed. Compression ratios in the range 12 to 20 are typical. Compression-ignition engines also can use less refined fuels having higher ignition temperatures than the volatile fuels required by spark-ignition engines.

In the next example, we illustrate the analysis of the air-standard Otto cycle. Results are compared with those obtained on a cold air-standard basis.

EXAMPLE 9.1

Analyzing the Otto Cycle

The temperature at the beginning of the compression process of an air-standard Otto cycle with a compression ratio of 8 is 540°R, the pressure is 1 atm, and the cylinder volume is 0.02 ft^3. The maximum temperature during the cycle is 3600°R. Determine **(a)** the temperature and pressure at the end of each process of the cycle, **(b)** the thermal efficiency, and **(c)** the mean effective pressure, in atm.

SOLUTION

Known: An air-standard Otto cycle with a given value of compression ratio is executed with specified conditions at the beginning of the compression stroke and a specified maximum temperature during the cycle.

Find: Determine the temperature and pressure at the end of each process, the thermal efficiency, and mean effective pressure, in atm.

Schematic and Given Data:

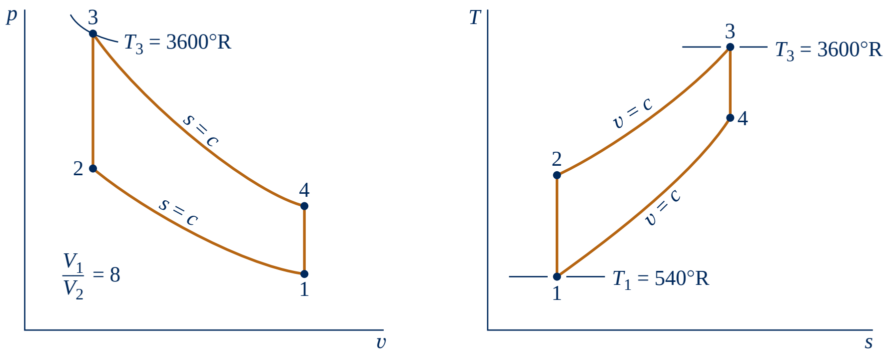


Fig. E9.1

Engineering Model:

1. The air in the piston–cylinder assembly is the closed system.
2. The compression and expansion processes are adiabatic.
3. All processes are internally reversible.
4. The air is modeled as an ideal gas.
5. Kinetic and potential energy effects are negligible.

Analysis:

(a) The analysis begins by determining the temperature, pressure, and specific internal energy at each principal state of the cycle. At $T_1 = 540°\text{R}$, Table A-22E gives $u_1 = 92.04$ Btu/lb and $v_{r1} = 144.32$.

For the isentropic compression Process 1–2

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{v_{r1}}{r} = \frac{144.32}{8} = 18.04$$

Interpolating with v_{r2} in Table A-22E, we get $T_2 = 1212°\text{R}$ and $u_2 = 211.3$ Btu/lb. With the ideal gas equation of state

$$p_2 = p_1 \frac{T_2}{T_1} \frac{V_1}{V_2} = (1 \text{ atm}) \left(\frac{1212°\text{R}}{540°\text{R}} \right) 8 = 17.96 \text{ atm}$$

The pressure at state 2 can be evaluated alternatively by using the isentropic relationship, $p_2 = p_1(p_{r2}/p_{r1})$.

Since Process 2–3 occurs at constant volume, the ideal gas equation of state gives

$$p_3 = p_2 \frac{T_3}{T_2} = (17.96 \text{ atm}) \left(\frac{3600°\text{R}}{1212°\text{R}} \right) = 53.3 \text{ atm}$$

At $T_3 = 3600°\text{R}$, Table A-22E gives $u_3 = 721.44$ Btu/lb and $v_{r3} = 0.6449$.

For the isentropic expansion process 3–4

$$v_{r4} = v_{r3} \frac{V_4}{V_3} = v_{r3} \frac{V_1}{V_2} = 0.6449(8) = 5.16$$

Interpolating in Table A-22E with v_{r4} gives $T_4 = 1878°R$, $u_4 = 342.2$ Btu/lb. The pressure at state 4 can be found using the isentropic relationship $p_4 = p_3(p_{r4}/p_{r3})$ or the ideal gas equation of state applied at states 1 and 4. With $V_4 = V_1$, the ideal gas equation of state gives

$$p_4 = p_1 \frac{T_4}{T_1} = (1 \text{ atm})\left(\frac{1878°\text{R}}{540°\text{R}}\right) = 3.48 \text{ atm}$$

(b) The thermal efficiency is

$$\eta = 1 - \frac{Q_{41}/m}{Q_{23}/m} = 1 - \frac{u_4 - u_1}{u_3 - u_2}$$
$$= 1 - \frac{342.2 - 92.04}{721.44 - 211.3} = 0.51\ (51\%)$$

(c) To evaluate the mean effective pressure requires the net work per cycle. That is

$$W_{\text{cycle}} = m[(u_3 - u_4) - (u_2 - u_1)]$$

where m is the mass of the air, evaluated from the ideal gas equation of state as follows:

$$m = \frac{p_1 V_1}{(\bar{R}/M)T_1}$$
$$= \frac{(14.696 \text{ lbf/in.}^2)|144 \text{ in.}^2/\text{ft}^2|(0.02 \text{ ft}^3)}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R})}$$
$$= 1.47 \times 10^{-3} \text{ lb}$$

Inserting values into the expression for W_{cycle}

$$W_{\text{cycle}} = (1.47 \times 10^{-3} \text{ lb})[(721.44 - 342.2) - (211.3 - 92.04)] \text{ Btu/lb}$$
$$= 0.382 \text{ Btu}$$

The displacement volume is $V_1 - V_2$, so the mean effective pressure is given by

$$\text{mep} = \frac{W_{\text{cycle}}}{V_1 - V_2} = \frac{W_{\text{cycle}}}{V_1(1 - V_2/V_1)}$$

1
$$= \frac{0.382 \text{ Btu}}{(0.02 \text{ ft}^3)(1 - 1/8)}\left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|\left|\frac{1 \text{ ft}^2}{144 \text{ in.}^2}\right|$$
$$= 118 \text{ lbf/in.}^2 = 8.03 \text{ atm}$$

1 This solution utilizes Table A-22E for air, which accounts explicitly for the variation of the specific heats with temperature. A solution also can be developed on a cold air-standard basis in which constant specific heats are assumed. This solution is left as an exercise, but for comparison the results are presented for the case $k = 1.4$ in the following table:

Parameter	Air-Standard Analysis	Cold Air-Standard Analysis, $k = 1.4$
T_2	1212°R	1241°R
T_3	3600°R	3600°R
T_4	1878°R	1567°R
η	0.51 (51%)	0.565 (56.5%)
mep	8.03 atm	7.05 atm

Skills Developed

Ability to...

- sketch the Otto cycle p–v and T–s diagrams.
- evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- calculate thermal efficiency and mean effective pressure.

QuickQUIZ Determine the heat addition and the heat rejection for the cycle, each in Btu. **Ans.** $Q_{23} = 0.750$ Btu, $Q_{41} = 0.368$ Btu.

9.3 Air-Standard Diesel Cycle

Diesel cycle

The air-standard Diesel cycle is an ideal cycle that assumes heat addition occurs during a constant-pressure process that starts with the piston at top dead center. The **Diesel cycle** is shown on p–v and T–s diagrams in Fig. 9.5. The cycle consists of four internally reversible processes in series. The first process from state 1 to state 2 is the same as in the Otto cycle: an isentropic compression. Heat is not transferred to the working fluid at constant volume as in the Otto cycle, however. In the Diesel cycle, heat is transferred to the working fluid at *constant pressure.* Process 2–3 also makes up the first part of the power stroke. The isentropic expansion from state 3 to state 4 is the remainder of the power stroke. As in the Otto cycle, the cycle is completed by constant-volume Process 4–1 in which heat is rejected from the air while the piston is at bottom dead center. This process replaces the exhaust and intake processes of the actual engine.

Since the air-standard Diesel cycle is composed of internally reversible processes, areas on the T–s and p–v diagrams of Fig. 9.5 can be interpreted as heat and work, respectively. On the T–s diagram, area 2–3–a–b–2 represents heat added per unit of mass and area 1–4–a–b–1 is heat rejected per unit of mass. On the p–v diagram, area 1–2–a–b–1 is work input per unit of mass during the compression process. Area 2–3–4–b–a–2 is the work done per unit of mass as the piston moves from top dead center to bottom dead center. The enclosed area of each figure is the net work output, which equals the net heat added.

A Diesel_Cycle A.28 – Tabs a & b

CYCLE ANALYSIS. In the Diesel cycle the heat addition takes place at constant pressure. Accordingly, Process 2–3 involves both work and heat. The work is given by

$$\frac{W_{23}}{m} = \int_2^3 p\,dv = p_2(v_3 - v_2) \tag{9.9}$$

The heat added in Process 2–3 can be found by applying the closed system energy balance

$$m(u_3 - u_2) = Q_{23} - W_{23}$$

Introducing Eq. 9.9 and solving for the heat transfer

$$\begin{aligned}\frac{Q_{23}}{m} &= (u_3 - u_2) + p(v_3 - v_2) = (u_3 + pv_3) - (u_2 + pv_2) \\ &= h_3 - h_2 \end{aligned} \tag{9.10}$$

where the specific enthalpy is introduced to simplify the expression. As in the Otto cycle, the heat rejected in Process 4–1 is given by

$$\frac{Q_{41}}{m} = u_4 - u_1$$

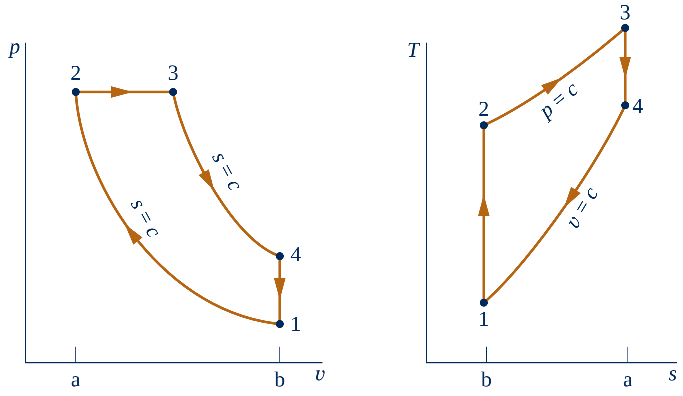


Fig. 9.5 ***p*–*v* and *T*–*s* diagrams of the air-standard Diesel cycle.**

The thermal efficiency is the ratio of the net work of the cycle to the heat added

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = 1 - \frac{Q_{41}/m}{Q_{23}/m} = 1 - \frac{u_4 - u_1}{h_3 - h_2} \tag{9.11}$$

As for the Otto cycle, the thermal efficiency of the Diesel cycle increases with the compression ratio.

To evaluate the thermal efficiency from Eq. 9.11 requires values for u_1, u_4, h_2, and h_3 or equivalently the temperatures at the principal states of the cycle. Let us consider next how these temperatures are evaluated. For a given initial temperature T_1 and compression ratio r, the temperature at state 2 can be found using the following isentropic relationship and v_r data

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{1}{r} v_{r1}$$

To find T_3, note that the ideal gas equation of state reduces with $p_3 = p_2$ to give

$$T_3 = \frac{V_3}{V_2} T_2 = r_c T_2$$

where $r_c = V_3/V_2$, called the **cutoff ratio**, has been introduced.

cutoff ratio

Since $V_4 = V_1$, the volume ratio for the isentropic process 3–4 can be expressed as

$$\frac{V_4}{V_3} = \frac{V_4}{V_2}\frac{V_2}{V_3} = \frac{V_1}{V_2}\frac{V_2}{V_3} = \frac{r}{r_c} \tag{9.12}$$

where the compression ratio r and cutoff ratio r_c have been introduced for conciseness.

Using Eq. 9.12 together with v_{r3} at T_3, the temperature T_4 can be determined by interpolation once v_{r4} is found from the isentropic relationship

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{r}{r_c} v_{r3}$$

In a *cold air-standard analysis*, the appropriate expression for evaluating T_2 is provided by

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1} \qquad \text{(constant } k\text{)}$$

The temperature T_4 is found similarly from

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} = \left(\frac{r_c}{r}\right)^{k-1} \qquad \text{(constant } k\text{)}$$

where Eq. 9.12 has been used to replace the volume ratio.

EFFECT OF COMPRESSION RATIO ON PERFORMANCE. As for the Otto cycle, the thermal efficiency of the Diesel cycle increases with increasing compression ratio. This can be brought out simply using a *cold* air-standard analysis. On a cold air-standard basis, the thermal efficiency of the Diesel cycle can be expressed as

$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right] \qquad \text{(cold-air standard basis)} \tag{9.13}$$

where r is the compression ratio and r_c is the cutoff ratio. The derivation is left as an exercise. This relationship is shown in Fig. 9.6 for $k = 1.4$ Equation 9.13 for the Diesel cycle differs from Eq. 9.8 for the Otto cycle only by the term in brackets, which for $r_c > 1$ is greater than unity. Thus, when the compression

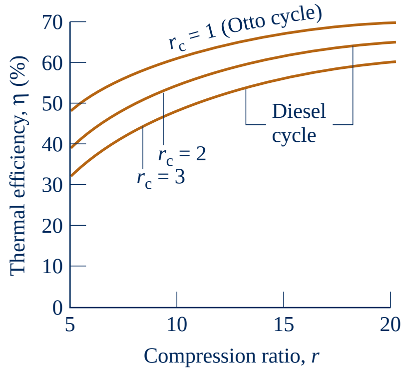


Fig. 9.6 Thermal efficiency of the cold air-standard Diesel cycle, $k = 1.4$.

ratio is the same, the thermal efficiency of the cold air-standard Diesel cycle is less than that of the cold air-standard Otto cycle.

In the next example, we illustrate the analysis of the air-standard Diesel cycle.

EXAMPLE 9.2

Analyzing the Diesel Cycle

At the beginning of the compression process of an air-standard Diesel cycle operating with a compression ratio of 18, the temperature is 300 K and the pressure is 0.1 MPa. The cutoff ratio for the cycle is 2. Determine **(a)** the temperature and pressure at the end of each process of the cycle, **(b)** the thermal efficiency, **(c)** the mean effective pressure, in MPa.

SOLUTION

Known: An air-standard Diesel cycle is executed with specified conditions at the beginning of the compression stroke. The compression and cutoff ratios are given.

Find: Determine the temperature and pressure at the end of each process, the thermal efficiency, and mean effective pressure.

Schematic and Given Data:

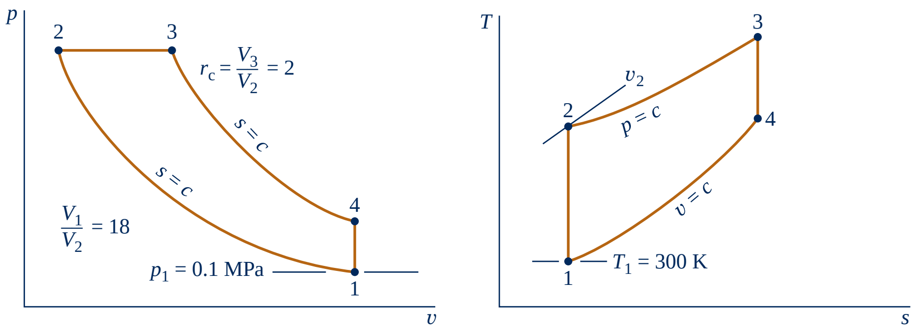


Fig. E9.2

Engineering Model:

1. The air in the piston–cylinder assembly is the closed system.
2. The compression and expansion processes are adiabatic.
3. All processes are internally reversible.
4. The air is modeled as an ideal gas.
5. Kinetic and potential energy effects are negligible.

Analysis:

(a) The analysis begins by determining properties at each principal state of the cycle. With $T_1 = 300$ K, Table A-22 gives $u_1 = 214.07$ kJ/kg and $v_{r1} = 621.2$. For the isentropic compression process 1–2

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{v_{r1}}{r} = \frac{621.2}{18} = 34.51$$

Interpolating in Table A-22, we get $T_2 = 898.3$ K and $h_2 = 930.98$ kJ/kg. With the ideal gas equation of state

$$p_2 = p_1 \frac{T_2}{T_1}\frac{V_1}{V_2} = (0.1)\left(\frac{898.3}{300}\right)(18) = 5.39 \text{ MPa}$$

The pressure at state 2 can be evaluated alternatively using the isentropic relationship, $p_2 = p_1\,(p_{r2}/p_{r1})$.

Since Process 2–3 occurs at constant pressure, the ideal gas equation of state gives

$$T_3 = \frac{V_3}{V_2} T_2$$

Introducing the cutoff ratio, $r_c = V_3/V_2$

$$T_3 = r_c T_2 = 2(898.3) = 1796.6 \text{ K}$$

From Table A-22, $h_3 = 1999.1$ kJ/kg and $v_{r3} = 3.97$.
For the isentropic expansion process 3–4

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{V_4}{V_2}\frac{V_2}{V_3} v_{r3}$$

Introducing $V_4 = V_1$, the compression ratio r, and the cutoff ratio r_c, we have

$$v_{r4} = \frac{r}{r_c} v_{r3} = \frac{18}{2}(3.97) = 35.73$$

Interpolating in Table A-22 with v_{r4}, we get $u_4 = 664.3$ kJ/kg and $T_4 = 887.7$ K. The pressure at state 4 can be found using the isentropic relationship $p_4 = p_3(p_{r4}/p_{r3})$ or the ideal gas equation of state applied at states 1 and 4. With $V_4 = V_1$, the ideal gas equation of state gives

$$p_4 = p_1 \frac{T_4}{T_1} = (0.1\text{ MPa})\left(\frac{887.7\text{ K}}{300\text{ K}}\right) = 0.3\text{ MPa}$$

(b) The thermal efficiency is found using

1
$$\eta = 1 - \frac{Q_{41}/m}{Q_{23}/m} = 1 - \frac{u_4 - u_1}{h_3 - h_2}$$
$$= 1 - \frac{664.3 - 214.07}{1999.1 - 930.98} = 0.578\ (57.8\%)$$

(c) The mean effective pressure written in terms of specific volumes is

$$\text{mep} = \frac{W_{\text{cycle}}/m}{v_1 - v_2} = \frac{W_{\text{cycle}}/m}{v_1(1 - 1/r)}$$

The net work of the cycle equals the net heat added

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (h_3 - h_2) - (u_4 - u_1)$$
$$= (1999.1 - 930.98) - (664.3 - 214.07)$$
$$= 617.9\text{ kJ/kg}$$

The specific volume at state 1 is

$$v_1 = \frac{(\bar{R}/M)T_1}{p_1} = \frac{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\text{ K})}{10^5\text{ N/m}^2} = 0.861\text{ m}^3/\text{kg}$$

Inserting values

$$\text{mep} = \frac{617.9\text{ kJ/kg}}{0.861(1 - 1/18)\text{ m}^3/\text{kg}}\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ MPa}}{10^6\text{ N/m}^2}\right|$$
$$= 0.76\text{ MPa}$$

1 This solution uses the air tables, which account explicitly for the variation of the specific heats with temperature. Note that Eq. 9.13 based on the assumption of *constant* specific heats has not been used to determine the thermal efficiency. The cold air-standard solution of this example is left as an exercise.

Skills Developed

Ability to...

- ❑ sketch the Diesel cycle p–v and T–s diagrams.
- ❑ evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- ❑ calculate the thermal efficiency and mean effective pressure.

QuickQUIZ If the mass of air is 0.0123 kg, what is the *displacement volume*, in L? Ans. 10 L.

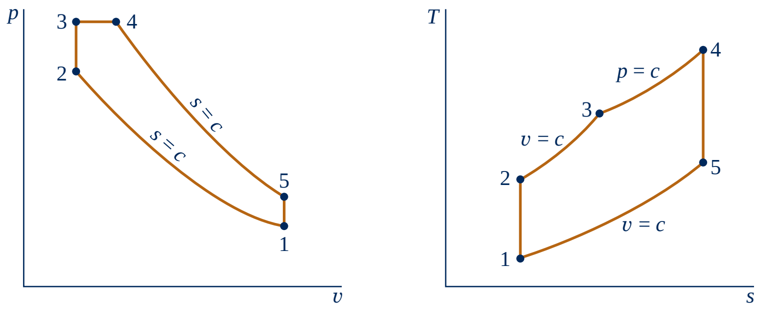


Fig. 9.7 ***p–v* and *T–s* diagrams of the air-standard dual cycle.**

9.4 Air-Standard Dual Cycle

dual cycle

The pressure–volume diagrams of actual internal combustion engines are not described well by the Otto and Diesel cycles. An air-standard cycle that can be made to approximate the pressure variations more closely is the *air-standard dual cycle*. The **dual cycle** is shown in Fig. 9.7. As in the Otto and Diesel cycles, Process 1–2 is an isentropic compression. The heat addition occurs in two steps, however: Process 2–3 is a constant-volume heat addition; Process 3–4 is a constant-pressure heat addition. Process 3–4 also makes up the first part of the power stroke. The isentropic expansion from state 4 to state 5 is the remainder of the power stroke. As in the Otto and Diesel cycles, the cycle is completed by a constant-volume heat rejection process, Process 5–1. Areas on the T–s and p–v diagrams can be interpreted as heat and work, respectively, as in the cases of the Otto and Diesel cycles.

Cycle Analysis

Since the dual cycle is composed of the same types of processes as the Otto and Diesel cycles, we can simply write down the appropriate work and heat transfer expressions by reference to the corresponding earlier developments. Thus, during the isentropic compression process 1–2 there is no heat transfer, and the work is

$$\frac{W_{12}}{m} = u_2 - u_1$$

As for the corresponding process of the Otto cycle, in the constant-volume portion of the heat addition process, Process 2–3, there is no work, and the heat transfer is

$$\frac{Q_{23}}{m} = u_3 - u_2$$

In the constant-pressure portion of the heat addition process, Process 3–4, there is both work and heat transfer, as for the corresponding process of the Diesel cycle

$$\frac{W_{34}}{m} = p(v_4 - v_3) \qquad \text{and} \qquad \frac{Q_{34}}{m} = h_4 - h_3$$

During the isentropic expansion process 4–5 there is no heat transfer, and the work is

$$\frac{W_{45}}{m} = u_4 - u_5$$

Finally, the constant-volume heat rejection process 5–1 that completes the cycle involves heat transfer but no work

$$\frac{Q_{51}}{m} = u_5 - u_1$$

The thermal efficiency is the ratio of the net work of the cycle to the *total* heat added

$$\eta = \frac{W_{cycle}/m}{(Q_{23}/m + Q_{34}/m)} = 1 - \frac{Q_{51}/m}{(Q_{23}/m + Q_{34}/m)}$$
$$= 1 - \frac{(u_5 - u_1)}{(u_3 - u_2) + (h_4 - h_3)} \quad (9.14)$$

The example to follow provides an illustration of the analysis of an air-standard dual cycle. The analysis exhibits many of the features found in the Otto and Diesel cycle examples considered previously.

EXAMPLE 9.3

Analyzing the Dual Cycle

At the beginning of the compression process of an air-standard dual cycle with a compression ratio of 18, the temperature is 300 K and the pressure is 0.1 MPa. The pressure ratio for the constant volume part of the heating process is 1.5:1. The volume ratio for the constant pressure part of the heating process is 1.2:1. Determine **(a)** the thermal efficiency and **(b)** the mean effective pressure, in MPa.

SOLUTION

Known: An air-standard dual cycle is executed in a piston–cylinder assembly. Conditions are known at the beginning of the compression process, and necessary volume and pressure ratios are specified.

Find: Determine the thermal efficiency and the mep, in MPa.

Schematic and Given Data:

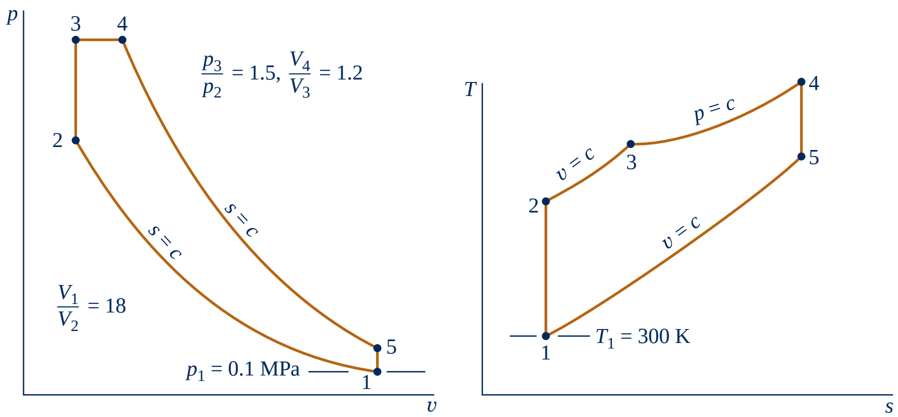


Fig. E9.3

Engineering Model:

1. The air in the piston–cylinder assembly is the closed system.
2. The compression and expansion processes are adiabatic.
3. All processes are internally reversible.
4. The air is modeled as an ideal gas.
5. Kinetic and potential energy effects are negligible.

Analysis: The analysis begins by determining properties at each principal state of the cycle. States 1 and 2 are the same as in Example 9.2, so $u_1 = 214.07$ kJ/kg, $T_2 = 898.3$ K, $u_2 = 673.2$ kJ/kg. Since Process 2–3 occurs at constant volume, the ideal gas equation of state reduces to give

$$T_3 = \frac{p_3}{p_2} T_2 = (1.5)(898.3) = 1347.5 \text{ K}$$

Interpolating in Table A-22, we get $h_3 = 1452.6$ kJ/kg and $u_3 = 1065.8$ kJ/kg.

Since Process 3–4 occurs at constant pressure, the ideal gas equation of state reduces to give

$$T_4 = \frac{V_4}{V_3} T_3 = (1.2)(1347.5) = 1617 \text{ K}$$

From Table A-22, $h_4 = 1778.3$ kJ/kg and $v_{r4} = 5.609$.

Process 4–5 is an isentropic expansion, so

$$v_{r5} = v_{r4} \frac{V_5}{V_4}$$

The volume ratio V_5/V_4 required by this equation can be expressed as

$$\frac{V_5}{V_4} = \frac{V_5}{V_3}\frac{V_3}{V_4}$$

With $V_5 = V_1$, $V_2 = V_3$, and given volume ratios

$$\frac{V_5}{V_4} = \frac{V_1}{V_2}\frac{V_3}{V_4} = 18\left(\frac{1}{1.2}\right) = 15$$

Inserting this in the above expression for v_{r5}

$$v_{r5} = (5.609)(15) = 84.135$$

Interpolating in Table A-22, we get $u_5 = 475.96$ kJ/kg.

(a) The thermal efficiency is

$$\eta = 1 - \frac{Q_{51}/m}{(Q_{23}/m + Q_{34}/m)} = 1 - \frac{(u_5 - u_1)}{(u_3 - u_2) + (h_4 - h_3)}$$

$$= 1 - \frac{(475.96 - 214.07)}{(1065.8 - 673.2) + (1778.3 - 1452.6)}$$

$$= 0.635(63.5\%)$$

(b) The mean effective pressure is

$$\text{mep} = \frac{W_{\text{cycle}}/m}{v_1 - v_2} = \frac{W_{\text{cycle}}/m}{v_1(1 - 1/r)}$$

The net work of the cycle equals the net heat added, so

$$\text{mep} = \frac{(u_3 - u_2) + (h_4 - h_3) - (u_5 - u_1)}{v_1(1 - 1/r)}$$

The specific volume at state 1 is evaluated in Example 9.2 as $v_1 = 0.861$ m^3/kg. Inserting values into the above expression for mep

$$\text{mep} = \frac{[(1065.8 - 673.2) + (1778.3 - 1452.6) - (475.96 - 214.07)]\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|\left|\frac{1 \text{ MPa}}{10^6 \text{ N/m}^2}\right|}{0.861(1 - 1/18) \text{ m}^3/\text{kg}} = 0.56 \text{ MPa}$$

Skills Developed

Ability to...

- sketch the Dual cycle p–v and T–s diagrams.
- evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- calculate the thermal efficiency and mean effective pressure.

QuickQUIZ Evaluate the total heat addition and the net work of the cycle, each in kJ per kg of air. **Ans.** $Q_{\text{in}}/m = 718$ kJ/kg, $W_{\text{cycle}}/m = 456$ kJ/kg.

Horizons

More Diesel-Powered Vehicles on the Way?

Diesel-powered vehicles are likely to be more prevalent in the United States in coming years. Diesel engines are more powerful and about one-third more fuel-efficient than similar-sized gasoline engines currently dominating the U.S. market. Increased fuel efficiency is achieved by diesels through advanced engine control and fuel-injection technology. Diesel-powered vehicles must meet the same emissions standards as gasoline vehicles. Ultra-low sulfur diesel fuel, commonly available today at U.S. fuel pumps, and improved exhaust treatment make this possible.

Biodiesel also can be used as fuel. Biodiesel is domestically produced from nonpetroleum, renewable sources, including vegetable oils (from soybeans, rapeseeds, sunflower seeds, and jatropha seeds), animal fats, and algae. Waste vegetable oil from industrial deep fryers, snack food factories, and restaurants can be converted to biodiesel. Biodiesel is biodegradable and safer to handle than petroleum diesel. The news about biodiesel is not all good, however. When compared to petroleum diesel, some say biodiesel is more expensive, has more nitrogen oxide (NO_x) emissions, and may have greater impact on engine durability.

Considering Gas Turbine Power Plants

This part of the chapter deals with gas turbine power plants. Gas turbines tend to be lighter and more compact than the vapor power plants studied in Chap. 8. The favorable power-output-to-weight ratio of gas turbines makes them well suited for transportation applications (aircraft propulsion, marine power plants, and so on). In recent decades, gas turbines also have contributed an increasing share of U.S. electric power needs, and they now provide about 22% of the total (see Table 8.1).

Today's electric power-producing gas turbines are almost exclusively fueled by natural gas. However, depending on the application, other fuels can be used by gas turbines, including distillate fuel oil; propane; gases produced from landfills, sewage treatment plants, and animal waste (see BIOCONNECTIONS, Sec. 7.3); and *syngas* (synthesis gas) obtained by gasification of coal (see Sec. 9.10).

9.5 Modeling Gas Turbine Power Plants

Gas turbine power plants may operate on either an open or closed basis. The *open* mode pictured in Fig. 9.8*a* is more common. This is an engine in which atmospheric air is continuously drawn into the compressor, where it is compressed to a high

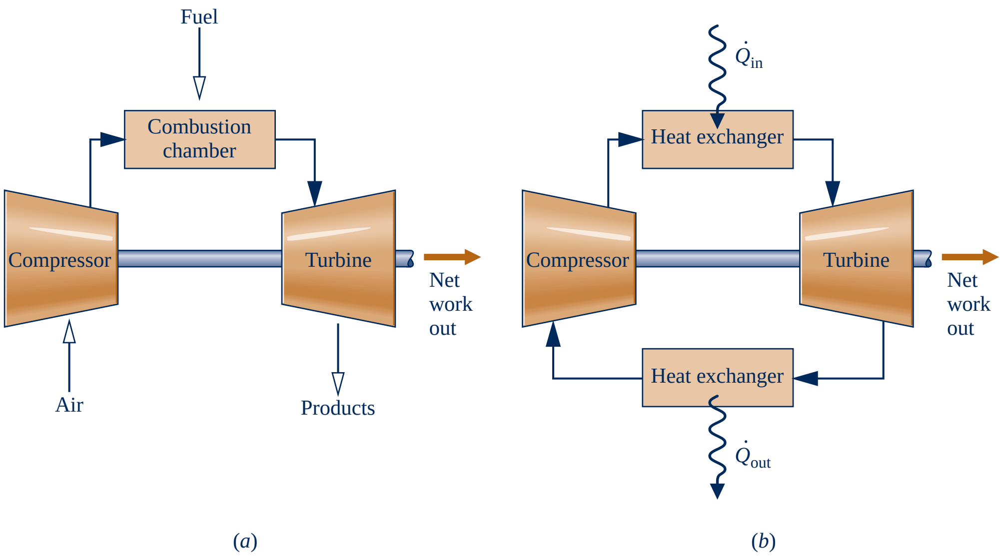


Fig. 9.8 Simple gas turbine. (*a*) Open to the atmosphere. (*b*) Closed.

pressure. The air then enters a combustion chamber, or combustor, where it is mixed with fuel and combustion occurs, resulting in combustion products at an elevated temperature. The combustion products expand through the turbine and are subsequently discharged to the surroundings. Part of the turbine work developed is used to drive the compressor; the remainder is available to generate electricity, to propel a vehicle, or for other purposes.

In the *closed* mode pictured in Fig. 9.8*b*, the working fluid receives an energy input by heat transfer from an external source, for example a gas-cooled nuclear reactor. The gas exiting the turbine is passed through a heat exchanger, where it is cooled prior to reentering the compressor.

air-standard analysis: gas turbines

An idealization often used in the study of open gas turbine power plants is that of an **air-standard analysis**. In an air-standard analysis two assumptions are always made:

- The working fluid is air, which behaves as an ideal gas.
- The temperature rise that would be brought about by combustion is accomplished by a heat transfer from an external source.

With an air-standard analysis, we avoid dealing with the complexities of the combustion process and the change of composition during combustion. Accordingly, an air-standard analysis simplifies study of gas turbine power plants considerably, but numerical values calculated on this basis may provide only qualitative indications of power plant performance. Still, we can learn important aspects of gas turbine operation using an air-standard analysis; see Sec. 9.6 for further discussion supported by solved examples.

ENERGY & ENVIRONMENT Natural gas is widely used for power generation by gas turbines, industrial and home heating, and chemical processing. The versatility of natural gas is matched by its relative abundance in North America, including natural gas extracted from deep-water ocean sites and shale deposits. Pipeline delivery of natural gas across the nation and from Canada has occurred for decades. Importation by ship from countries such as Trinidad, Algeria, and Norway is a more recent development.

Turning natural gas into liquid is the only practical way to import supplies from overseas. The liquefied natural gas (LNG) is stored in tanks onboard ships at about $-163°C$ ($-260°F$). To reduce heat transfer to the cargo LNG from outside sources, the tanks are insulated and the ships have double hulls with ample space between them. Still, a fraction of the cargo evaporates during long voyages. Such *boil-off* gas is commonly used to fuel the ship's propulsion system and meet other onboard energy needs. When tankers arrive at their destinations, LNG is converted to gas by heating it. The gas is then sent via pipeline to storage tanks onshore for distribution to customers.

Shipboard delivery of LNG has some minuses. Owing to cumulative effects during the LNG delivery chain, considerable exergy is destroyed and lost in liquefying gas at the beginning of the chain, transporting LNG by ship, and regasifying it when port is reached. If comparatively warm seawater is used to regasify LNG, environmentalists worry about the effect on aquatic life nearby. Many observers are also concerned about safety, especially when huge quantities of gas are stored at ports in major urban areas. Some say it would be better to use domestic supplies more efficiently than run such risks.

9.6 Air-Standard Brayton Cycle

A schematic diagram of an air-standard gas turbine is shown in Fig. 9.9. The directions of the principal energy transfers are indicated on this figure by arrows. In accordance with the assumptions of an air-standard analysis, the temperature rise that would be achieved in the combustion process is brought about by a heat transfer to the working fluid from an external source and the working fluid is considered to be air as an ideal gas. With the air-standard idealizations, air would be drawn into the compressor at state 1 from the surroundings and later returned to the surroundings at state 4 with a temperature greater than the ambient temperature. After interacting with the surroundings, each unit mass of discharged air would eventually return to the same state as the air entering the compressor, so we may think of the air passing through the components of the gas turbine as undergoing a thermodynamic cycle. A simplified representation of the states visited by the air in such a cycle can be devised by regarding the turbine exhaust air as restored to the compressor inlet state by passing through a heat exchanger where heat rejection to the surroundings occurs. The cycle that results with this further idealization is called the air-standard **Brayton cycle**.

Brayton cycle

9.6.1 Evaluating Principal Work and Heat Transfers

The following expressions for the work and heat transfers of energy that occur at steady state are readily derived by reduction of the control volume mass and energy rate balances. These energy transfers are positive in the directions of the arrows in Fig. 9.9. Assuming the turbine operates adiabatically and with negligible effects of kinetic and potential energy, the work developed per unit of mass flowing is

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4 \tag{9.15}$$

where $\dot{m}$ denotes the mass flow rate. With the same assumptions, the compressor work per unit of mass flowing is

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1 \tag{9.16}$$

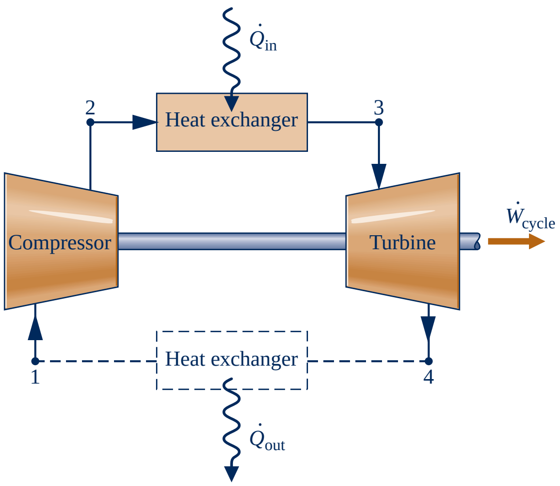


Fig. 9.9 Air-standard gas turbine cycle.

The symbol $\dot{W}_c$ denotes work *input* and takes on a positive value. The heat added to the cycle per unit of mass is

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 \tag{9.17}$$

The heat rejected per unit of mass is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1 \tag{9.18}$$

where $\dot{Q}_{out}$ is positive in value.

The thermal efficiency of the cycle in Fig. 9.9 is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} \tag{9.19}$$

The **back work ratio** for the cycle is

back work ratio

$$\text{bwr} = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{h_2 - h_1}{h_3 - h_4} \tag{9.20}$$

For the same pressure rise, a gas turbine compressor would require a much greater work input per unit of mass flow than the pump of a vapor power plant because the average specific volume of the gas flowing through the compressor would be many times greater than that of the liquid passing through the pump (see discussion of Eq. 6.51b in Sec. 6.13). Hence, a relatively large portion of the work developed by the turbine is required to drive the compressor. Typical back work ratios of gas turbines range from 40 to 80%. In comparison, the back work ratios of vapor power plants are normally only 1 or 2%.

If the temperatures at the numbered states of the cycle are known, the specific enthalpies required by the foregoing equations are readily obtained from the ideal gas table for air, Table A-22 or Table A-22E. Alternatively, with the sacrifice of some accuracy, the variation of the specific heats with temperature can be ignored and the specific heats taken as constant. The air-standard analysis is then referred to as a *cold air-standard analysis*. As illustrated by the discussion of internal combustion engines given previously, the chief advantage of the assumption of constant specific heats is that simple expressions for quantities such as thermal efficiency can be derived, and these can be used to deduce qualitative indications of cycle performance without involving tabular data.

Since Eqs. 9.15 through 9.20 have been developed from mass and energy rate balances, they apply equally when irreversibilities are present and in the absence of irreversibilities. Although irreversibilities and losses associated with the various power plant components have a pronounced effect on overall performance, it is instructive to consider an idealized cycle in which they are assumed absent. Such a cycle establishes an upper limit on the performance of the air-standard Brayton cycle. This is considered next.

9.6.2 Ideal Air-Standard Brayton Cycle

Ignoring irreversibilities as the air circulates through the various components of the Brayton cycle, there are no frictional pressure drops, and the air flows at constant pressure through the heat exchangers. If stray heat transfers to the surroundings are also ignored, the processes through the turbine and compressor are isentropic. The ideal cycle shown on the p–v and T–s diagrams in Fig. 9.10 adheres to these idealizations.

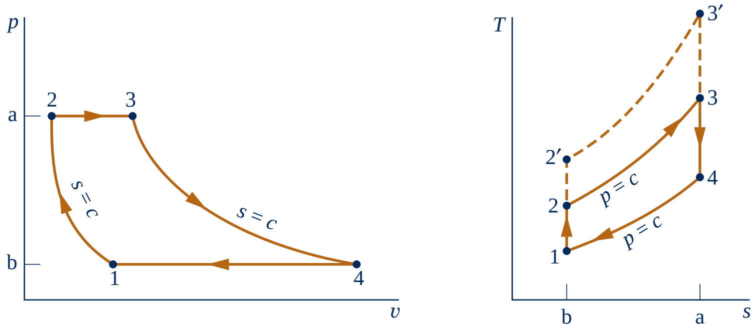


Fig. 9.10 **Air-standard ideal Brayton cycle.**

Areas on the T–s and p–v diagrams of Fig. 9.10 can be interpreted as heat and work, respectively, per unit of mass flowing. On the T–s diagram, area 2–3–a–b–2 represents the heat added per unit of mass and area 1–4–a–b–1 is the heat rejected per unit of mass. On the p–v diagram, area 1–2–a–b–1 represents the compressor work input per unit of mass and area 3–4–b–a–3 is the turbine work output per unit of mass. The enclosed area on each figure can be interpreted as the net work output or, equivalently, the net heat added.

When air table data are used to conduct an analysis involving the ideal Brayton cycle, the following relationships, based on Eq. 6.41, apply for the isentropic processes 1–2 and 3–4

TAKE NOTE...

For internally reversible flows through control volumes at steady state, see Sec. 6.13 for area interpretations of work and heat transfer on p–v and T–s diagrams, respectively.

$$p_{r2} = p_{r1}\frac{p_2}{p_1} \tag{9.21}$$

$$p_{r4} = p_{r3}\frac{p_4}{p_3} = p_{r3}\frac{p_1}{p_2} \tag{9.22}$$

where p_2/p_1 is the *compressor pressure ratio*. Recall that p_r is tabulated versus temperature in Tables A-22. Since air flows through the heat exchangers of the ideal cycle at constant pressure, it follows that $p_4/p_3 = p_1/p_2$. This relationship has been used in writing Eq. 9.22.

When an ideal Brayton cycle is analyzed on a cold air-standard basis, the specific heats are taken as constant. Equations 9.21 and 9.22 are then replaced, respectively, by the following expressions, based on Eq. 6.43

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} \tag{9.23}$$

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{(k-1)/k} = T_3\left(\frac{p_1}{p_2}\right)^{(k-1)/k} \tag{9.24}$$

Brayton_Cycle
A.29 – Tab a

where k is the specific heat ratio, $k = c_p/c_v$.

In the next example, we illustrate the analysis of an ideal air-standard Brayton cycle and compare results with those obtained on a cold air-standard basis.

EXAMPLE 9.4

Analyzing the Ideal Brayton Cycle

Air enters the compressor of an ideal air-standard Brayton cycle at 100 kPa, 300 K, with a volumetric flow rate of 5 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1400 K. Determine **(a)** the thermal efficiency of the cycle, **(b)** the back work ratio, **(c)** the *net* power developed, in kW.

SOLUTION

Known: An ideal air-standard Brayton cycle operates with given compressor inlet conditions, given turbine inlet temperature, and a known compressor pressure ratio.

Find: Determine the thermal efficiency, the back work ratio, and the *net* power developed, in kW.

Schematic and Given Data:

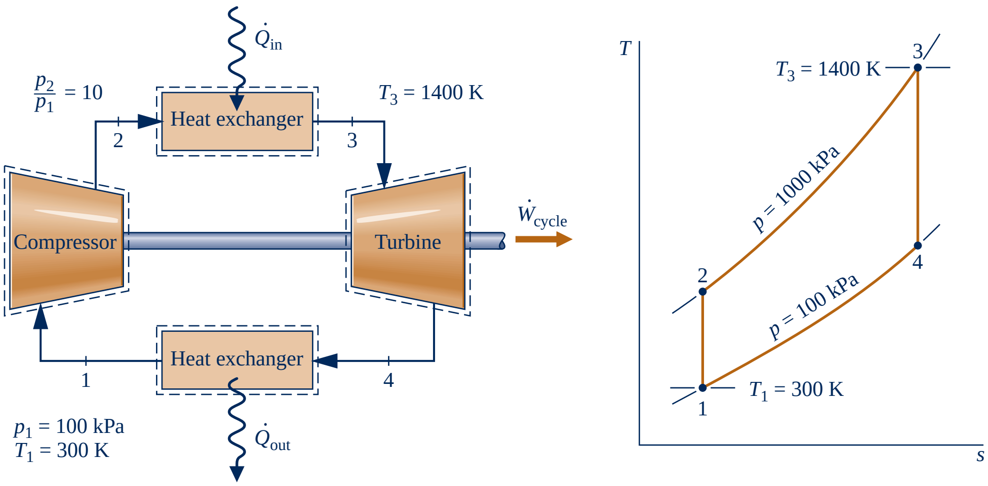


Fig. E9.4

Engineering Model:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and compressor processes are isentropic.
3. There are no pressure drops for flow through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

1 **Analysis:** The analysis begins by determining the specific enthalpy at each numbered state of the cycle. At state 1, the temperature is 300 K. From Table A-22, $h_1 = 300.19$ kJ/kg and $p_{r1} = 1.386$.

Since the compressor process is isentropic, the following relationship can be used to determine h_2

$$p_{r2} = \frac{p_2}{p_1} p_{r1} = (10)(1.386) = 13.86$$

Then, interpolating in Table A-22, we obtain $h_2 = 579.9$ kJ/kg.

The temperature at state 3 is given as $T_3 = 1400$ K. With this temperature, the specific enthalpy at state 3 from Table A-22 is $h_3 = 1515.4$ kJ/kg. Also, $p_{r3} = 450.5$.

The specific enthalpy at state 4 is found by using the isentropic relation

$$p_{r4} = p_{r3} \frac{p_4}{p_3} = (450.5)(1/10) = 45.05$$

Interpolating in Table A-22, we get $h_4 = 808.5$ kJ/kg.

(a) The thermal efficiency is

$$\eta = \frac{(\dot{W}_t/\dot{m}) - (\dot{W}_c/\dot{m})}{\dot{Q}_{in}/\dot{m}}$$

$$= \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = \frac{(1515.4 - 808.5) - (579.9 - 300.19)}{1515.4 - 579.9}$$

$$= \frac{706.9 - 279.7}{935.5} = 0.457\ (45.7\%)$$

(b) The back work ratio is

2

$$\text{bwr} = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{h_2 - h_1}{h_3 - h_4} = \frac{279.7}{706.9} = 0.396\ (39.6\%)$$

(c) The net power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

To evaluate the net power requires the mass flow rate $\dot{m}$, which can be determined from the volumetric flow rate and specific volume at the compressor inlet as follows

$$\dot{m} = \frac{(\text{AV})_1}{v_1}$$

Since $v_1 = (\overline{R}/M)T_1/p_1$, this becomes

$$\dot{m} = \frac{(\text{AV})_1 p_1}{(\overline{R}/M)T_1} = \frac{(5\ \text{m}^3/\text{s})(100 \times 10^3\ \text{N/m}^2)}{\left(\dfrac{8314}{28.97}\dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\ \text{K})}$$
$$= 5.807\ \text{kg/s}$$

Finally,

$$\dot{W}_{\text{cycle}} = (5.807\ \text{kg/s})(706.9 - 279.7)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 2481\ \text{kW}$$

1 The use of the ideal gas table for air is featured in this solution. A solution also can be developed on a cold air-standard basis in which constant specific heats are assumed. The details are left as an exercise, but for comparison the results are presented for the case $k = 1.4$ in the following table:

Parameter	Air-Standard Analysis	Cold Air-Standard Analysis, $k = 1.4$
T_2	574.1 K	579.2 K
T_4	787.7 K	725.1 K
η	0.457	0.482
bwr	0.396	0.414
$\dot{W}_{\text{cycle}}$	2481 kW	2308 kW

2 The value of the back work ratio in the present gas turbine case is significantly greater than the back work ratio of the simple vapor power cycle of Example 8.1.

Skills Developed

Ability to...

- ❑ sketch the schematic of the basic air-standard gas turbine and the T–s diagram for the corresponding ideal Brayton cycle.
- ❑ evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- ❑ calculate the thermal efficiency and back work ratio.

QuickQUIZ Determine the rate of heat transfer to the air passing through the combustor, in kW. **Ans.** 5432 kW.

EFFECT OF COMPRESSOR PRESSURE RATIO ON PERFORMANCE. Conclusions that are qualitatively correct for actual gas turbines can be drawn from a study of the ideal Brayton cycle. The first of these conclusions is that the thermal efficiency increases with increasing pressure ratio across the compressor.

▶ FOR EXAMPLE referring again to the T–s diagram of Fig. 9.10, we see that an increase in the compressor pressure ratio changes the cycle from 1–2–3–4–1 to 1–2′–3′–4–1 Since the average temperature of heat addition is greater in the latter cycle and both cycles have the same heat rejection process, cycle 1–2′–3′–4–1 would have the greater thermal efficiency.

The increase in thermal efficiency with the pressure ratio across the compressor is also brought out simply by the following development, in which the specific heat c_p, and thus the specific heat ratio k, is assumed constant. For constant c_p, Eq. 9.19 becomes

$$\eta = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Or, on further rearrangement

$$\eta = 1 - \frac{T_1}{T_2}\left(\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right)$$

From Eqs. 9.23 and 9.24, $T_4/T_1 = T_3/T_2$, so

$$\eta = 1 - \frac{T_1}{T_2}$$

Finally, introducing Eq. 9.23

$$\eta = 1 - \frac{1}{(p_2/p_1)^{(k-1)/k}} \quad \text{(cold-air standard basis)} \tag{9.25}$$

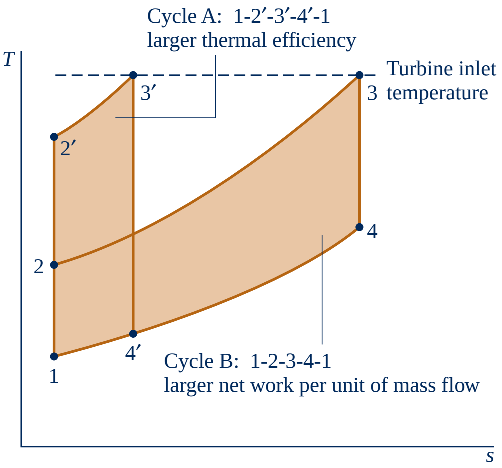


Fig. 9.11 Ideal Brayton cycles with different compressor pressure ratios and the same turbine inlet temperature.

By inspection of Eq. 9.25, it can be seen that the cold air-standard ideal Brayton cycle thermal efficiency increases with increasing pressure ratio across the compressor.

As there is a limit imposed by metallurgical considerations on the maximum allowed temperature at the turbine inlet, it is instructive to consider the effect of increasing compressor pressure ratio on thermal efficiency when the turbine inlet temperature is restricted to the maximum allowable temperature. We do this using Figs. 9.11 and 9.12.

The T–s diagrams of two ideal Brayton cycles having the same turbine inlet temperature but different compressor pressure ratios are shown in Fig. 9.11. Cycle A has a greater compressor pressure ratio than cycle B and thus the greater thermal efficiency. However, cycle B has a larger enclosed area and thus the greater net work developed per unit of mass flow. Accordingly, for cycle A to develop the same net *power* output as cycle B, a larger mass flow rate would be required, and this might dictate a larger system.

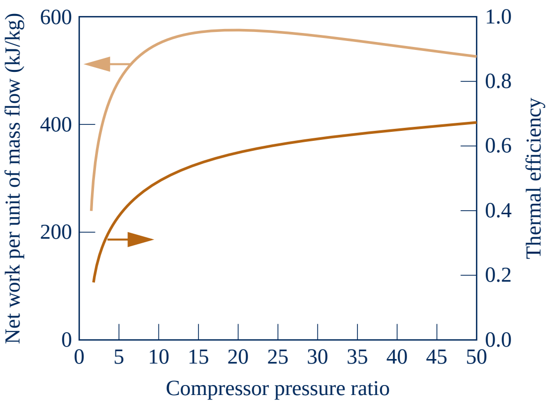


Fig. 9.12 Ideal Brayton cycle thermal efficiency and net work per unit of mass flow versus compressor pressure ratio for $k = 1.4$ and a turbine inlet temperature of 1700 K.

These considerations are important for gas turbines intended for use in vehicles where engine weight must be kept small. For such applications, it is desirable to operate near the compressor pressure ratio that yields the most work per unit of mass flow and not the pressure ratio for the greatest thermal efficiency. To quantify this, see Fig. 9.12 showing the variations with increasing compressor pressure ratio of thermal efficiency and net work per unit of mass flow for $k = 1.4$ and a turbine inlet temperature of 1700 K. While thermal efficiency increases with pressure ratio, the net work per unit of mass curve has a maximum value at a pressure ratio of about 21. Also observe that the curve is relatively flat in the vicinity of the maximum. Thus, for vehicle design purposes a wide range of compressor pressure ratio values may be considered as *nearly optimal* from the standpoint of maximum work per unit of mass.

Example 9.5 provides an illustration of the determination of the compressor pressure ratio for maximum net work per unit of mass flow for the cold air-standard Brayton cycle.

EXAMPLE 9.5

Determining Compressor Pressure Ratio for Maximum Net Work

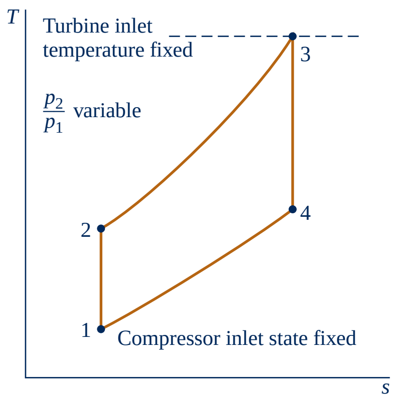
Determine the pressure ratio across the compressor of an ideal Brayton cycle for the maximum net work output per unit of mass flow if the state at the compressor inlet and the temperature at the turbine inlet are fixed. Use a cold air-standard analysis and ignore kinetic and potential energy effects. Discuss.

SOLUTION

Known: An ideal Brayton cycle operates with a specified state at the inlet to the compressor and a specified turbine inlet temperature.

Find: Determine the pressure ratio across the compressor for the maximum net work output per unit of mass flow, and discuss the result.

Schematic and Given Data:



Engineering Model:

1. Each component is analyzed as a control volume at steady state.
2. The turbine and compressor processes are isentropic.
3. There are no pressure drops for flow through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.
6. The specific heat c_p and thus the specific heat ratio k are constant.

Fig. E9.5

Analysis: The net work of the cycle per unit of mass flow is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1)$$

Since c_p is constant (assumption 6)

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = c_p[(T_3 - T_4) - (T_2 - T_1)]$$

Or on rearrangement

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = c_p T_1 \left(\frac{T_3}{T_1} - \frac{T_4}{T_3}\frac{T_3}{T_1} - \frac{T_2}{T_1} + 1 \right)$$

Replacing the temperature ratios T_2/T_1 and T_4/T_3 by using Eqs. 9.23 and 9.24, respectively, gives

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = c_p T_1 \left[\frac{T_3}{T_1} - \frac{T_3}{T_1}\left(\frac{p_1}{p_2}\right)^{(k-1)/k} - \left(\frac{p_2}{p_1}\right)^{(k-1)/k} + 1 \right]$$

From this expression it can be concluded that for specified values of T_1, T_3, and c_p, the value of the net work output per unit of mass flow varies with the pressure ratio p_2/p_1 only.

To determine the pressure ratio that maximizes the net work output per unit of mass flow, first form the derivative

$$\frac{\partial(\dot{W}_{\text{cycle}}/\dot{m})}{\partial(p_2/p_1)} = \frac{\partial}{\partial(p_2/p_1)}\left\{c_p T_1\left[\frac{T_3}{T_1} - \frac{T_3}{T_1}\left(\frac{p_1}{p_2}\right)^{(k-1)/k} - \left(\frac{p_2}{p_1}\right)^{(k-1)/k} + 1\right]\right\}$$

$$= c_p T_1\left(\frac{k-1}{k}\right)\left[\left(\frac{T_3}{T_1}\right)\left(\frac{p_1}{p_2}\right)^{-1/k}\left(\frac{p_1}{p_2}\right)^2 - \left(\frac{p_2}{p_1}\right)^{-1/k}\right]$$

$$= c_p T_1\left(\frac{k-1}{k}\right)\left[\left(\frac{T_3}{T_1}\right)\left(\frac{p_1}{p_2}\right)^{(2k-1)/k} - \left(\frac{p_2}{p_1}\right)^{-1/k}\right]$$

When the partial derivative is set to zero, the following relationship is obtained

$$\frac{p_2}{p_1} = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]} \qquad \textbf{(a)}$$

By checking the sign of the second derivative, we can verify that the net work per unit of mass flow is a maximum when this relationship is satisfied.

For gas turbines intended for transportation, it is desirable to keep engine size small. Thus, such gas turbines should operate near the compressor pressure ratio that yields the most work per unit of mass flow. The present example shows how the maximum net work per unit of mass flow is determined on a cold-air standard basis when the state at the compressor inlet and turbine inlet temperature are fixed.

Skills Developed

Ability to...

- ❑ complete the detailed derivation of a thermodynamic expression.
- ❑ use calculus to maximize a function.

QuickQUIZ For an ideal cold air-standard Brayton cycle with a compressor inlet temperature of 300 K and a maximum cycle temperature of 1700 K, use Eq. (a) above to find the compressor pressure ratio that maximizes the net power output per unit mass flow. Assume $k = 1.4$. **Ans.** 21. (Value agrees with Fig. 9.12.)

9.6.3 Considering Gas Turbine Irreversibilities and Losses

The principal state points of an air-standard gas turbine might be shown more realistically as in Fig. 9.13*a*. Because of frictional effects within the compressor and turbine, the working fluid would experience increases in specific entropy across these components. Owing to friction, there also would be pressure drops as the working fluid passes through the heat exchangers. However, because frictional pressure drops in the heat exchangers are less significant sources of irreversibility, we ignore them in subsequent discussions and for simplicity show the flow through the heat exchangers as occurring at constant pressure. This is illustrated by Fig. 9.13*b*. Stray heat transfers from the power plant

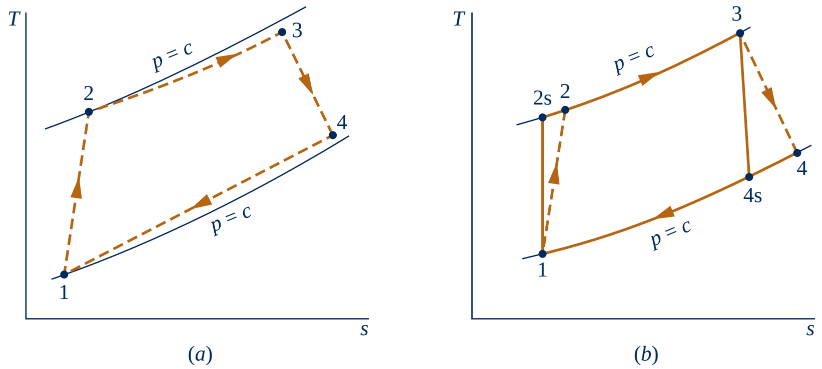


Fig. 9.13 Effects of irreversibilities on the air-standard gas turbine.

components to the surroundings represent losses, but these effects are usually of secondary importance and are also ignored in subsequent discussions.

As the effect of irreversibilities in the turbine and compressor becomes more pronounced, the work developed by the turbine decreases and the work input to the compressor increases, resulting in a marked decrease in the net work of the power plant. Accordingly, if appreciable net work is to be developed by the plant, relatively high isentropic turbine and compressor efficiencies are required.

After decades of developmental effort, efficiencies of 80 to 90% can now be achieved for the turbines and compressors in gas turbine power plants. Designating the states as in Fig. 9.13*b*, the isentropic turbine and compressor efficiencies are given by

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Among the irreversibilities of actual gas turbine power plants, irreversibilities within the turbine and compressor *are* important, but the most significant *by far* is combustion irreversibility. An air-standard analysis does not allow combustion irreversibility to be evaluated, however, and means introduced in Chap. 13 must be applied.

Example 9.6 brings out the effect of turbine and compressor irreversibilities on plant performance.

Brayton_Cycle
A.29 – Tab b

TAKE NOTE...
Isentropic turbine and compressor efficiencies are introduced in Sec. 6.12. See discussions of Eqs. 6.46 and 6.48, respectively.

EXAMPLE 9.6

Evaluating Performance of a Brayton Cycle with Irreversibilities

Reconsider Example 9.4, but include in the analysis that the turbine and compressor each have an isentropic efficiency of 80%. Determine for the modified cycle **(a)** the thermal efficiency of the cycle, **(b)** the back work ratio, **(c)** the *net* power developed, in kW.

SOLUTION

Known: An air-standard Brayton cycle operates with given compressor inlet conditions, given turbine inlet temperature, and known compressor pressure ratio. The compressor and turbine each have an isentropic efficiency of 80%.

Find: Determine the thermal efficiency, the back work ratio, and the net power developed, in kW.

Schematic and Given Data:

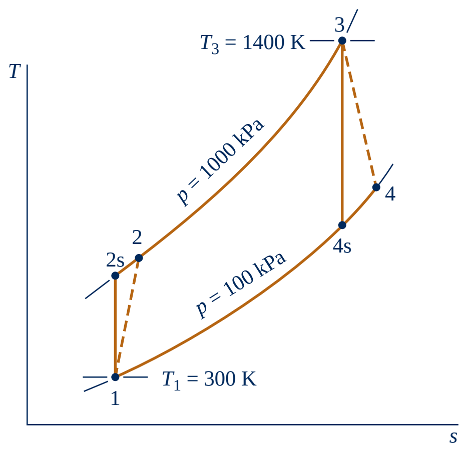


Fig. E9.6

Engineering Model:

1. Each component is analyzed as a control volume at steady state.
2. The compressor and turbine are adiabatic.
3. There are no pressure drops for flow through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

Analysis:

(a) The thermal efficiency is given by

$$\eta = \frac{(\dot{W}_t/\dot{m}) - (\dot{W}_c/\dot{m})}{\dot{Q}_{in}/\dot{m}}$$

The work terms in the numerator of this expression are evaluated using the given values of the compressor and turbine isentropic efficiencies as follows:

The turbine work per unit of mass is

$$\frac{\dot{W}_t}{\dot{m}} = \eta_t\left(\frac{\dot{W}_t}{\dot{m}}\right)_s$$

where η_t is the turbine efficiency. The value of $(\dot{W}_t/\dot{m})_s$ is determined in the solution to Example 9.4 as 706.9 kJ/kg. Thus

1 $$\frac{\dot{W}_t}{\dot{m}} = 0.8(706.9) = 565.5 \text{ kJ/kg}$$

For the compressor, the work per unit of mass is

$$\frac{\dot{W}_c}{\dot{m}} = \frac{(\dot{W}_c/\dot{m})_s}{\eta_c}$$

where η_c is the compressor efficiency. The value of $(\dot{W}_c/\dot{m})_s$ is determined in the solution to Example 9.4 as 279.7 kJ/kg, so

$$\frac{\dot{W}_c}{\dot{m}} = \frac{279.7}{0.8} = 349.6 \text{ kJ/kg}$$

The specific enthalpy at the compressor exit, h_2, is required to evaluate the denominator of the thermal efficiency expression. This enthalpy can be determined by solving

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1$$

to obtain

$$h_2 = h_1 + \dot{W}_c/\dot{m}$$

Inserting known values

$$h_2 = 300.19 + 349.6 = 649.8 \text{ kJ/kg}$$

The heat transfer to the working fluid per unit of mass flow is then

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 1515.4 - 649.8 = 865.6 \text{ kJ/kg}$$

where h_3 is from the solution to Example 9.4.

Finally, the thermal efficiency is

$$\eta = \frac{565.5 - 349.6}{865.6} = 0.249\ (24.9\%)$$

(b) The back work ratio is

$$\text{bwr} = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{349.6}{565.5} = 0.618\ (61.8\%)$$

(c) The mass flow rate is the same as in Example 9.4. The net power developed by the cycle is then

2 $$\dot{W}_{cycle} = \left(5.807\frac{\text{kg}}{\text{s}}\right)(565.5 - 349.6)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 1254 \text{ kW}$$

1 The solution to this example on a cold air-standard basis is left as an exercise.

2 Irreversibilities within the turbine and compressor have a significant impact on the performance of gas turbines. This is brought out by comparing the results of the present example with those of Example 9.4. Irreversibilities result in an increase in the work of compression and a reduction in work output of the turbine. The back work ratio is greatly increased and the thermal efficiency significantly decreased. Still, we should recognize that the most significant irreversibility of gas turbines *by far* is combustion irreversibility.

Skills Developed

Ability to...

- ❑ sketch the schematic of the basic air-standard gas turbine and the *T*–*s* diagram for the corresponding Brayton cycle with compressor and turbine irreversibilities.
- ❑ evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- ❑ calculate the thermal efficiency and back work ratio.

QuickQUIZ What would be the thermal efficiency and back work ratio if the isentropic turbine efficiency were 70% keeping isentropic compressor efficiency and other given data the same? Ans. η = 16.8%, bwr = 70.65%.

9.7 Regenerative Gas Turbines

The turbine exhaust temperature of a simple gas turbine is normally well above the ambient temperature. Accordingly, the hot turbine exhaust gas has significant thermodynamic utility (exergy) that would be irrevocably lost were the gas discarded directly to the surroundings. One way of utilizing this potential is by means of a heat exchanger called a **regenerator**, which allows the air exiting the compressor to be *preheated* before entering the combustor, thereby reducing the amount of fuel that must be burned in the combustor. The combined cycle arrangement considered in Sec. 9.9 is another way to utilize the hot turbine exhaust gas.

regenerator

An air-standard Brayton cycle modified to include a regenerator is illustrated in Fig. 9.14. The regenerator shown is a counterflow heat exchanger through which the hot turbine exhaust gas and the cooler air leaving the compressor pass in opposite directions. Ideally, no frictional pressure drop occurs in either stream. The turbine exhaust gas is cooled from state 4 to state y, while the air exiting the compressor is heated from state 2 to state x. Hence, a heat transfer from a source external to the

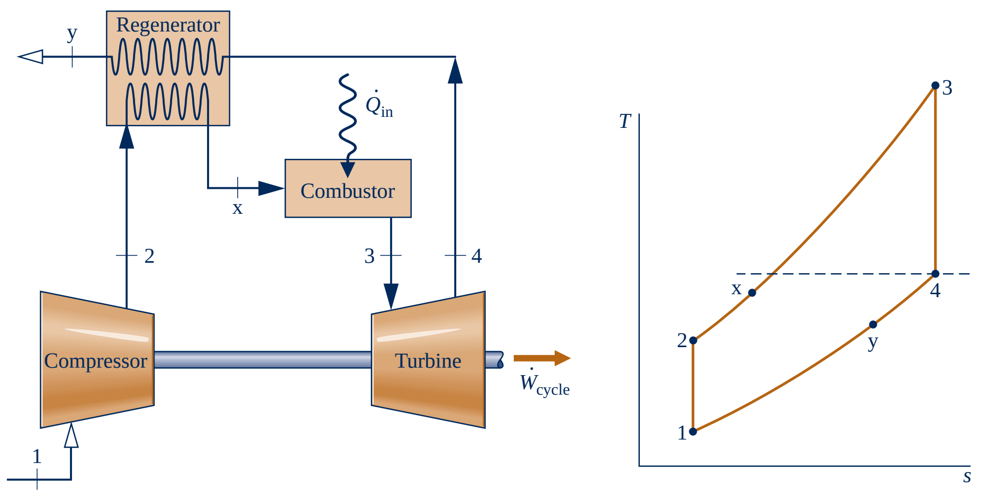


Fig. 9.14 **Regenerative air-standard gas turbine cycle.**

cycle is required only to increase the air temperature from state x to state 3, rather than from state 2 to state 3, as would be the case without regeneration. The heat added per unit of mass is then given by

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_3 - h_{\text{x}} \tag{9.26}$$

The net work developed per unit of mass flow is not altered by the addition of a regenerator. Thus, since the heat added is reduced, the thermal efficiency increases.

REGENERATOR EFFECTIVENESS. From Eq. 9.26 it can be concluded that the external heat transfer required by a gas turbine power plant decreases as the specific enthalpy h_{x} increases and thus as the temperature T_{x} increases. Evidently, there is an incentive in terms of fuel saved for selecting a regenerator that provides the greatest practical value for this temperature. To consider the *maximum* theoretical value for T_{x}, refer to Fig. 9.15, which shows temperature variations of the hot and cold streams of a counterflow heat exchanger.

- First, refer to Fig. 9.15*a*. Since a finite temperature difference between the streams is required for heat transfer to occur, the temperature of the cold stream at each location, denoted by the coordinate z, is less than that of the hot stream. In particular, the temperature of the colder stream as it exits the heat exchanger is less than the temperature of the incoming hot stream. If the heat transfer area were increased, providing more opportunity for heat transfer between the two streams, there would be a smaller temperature difference at each location.
- In the limiting case of infinite heat transfer area, the temperature difference would approach zero at all locations, as illustrated in Fig. 9.15*b*, and the heat transfer would approach reversibility. In this limit, the exit temperature of the colder stream would approach the temperature of the incoming hot stream. Thus, the highest possible temperature that could be achieved by the colder stream is the temperature of the incoming hot gas.

Referring again to the regenerator of Fig. 9.14, we can conclude from the discussion of Fig. 9.15 that the maximum theoretical value for the temperature T_{x} is the turbine exhaust temperature T_4, obtained if the regenerator were operating reversibly. The *regenerator effectiveness,* η_{reg}, is a parameter that gauges the departure of an actual

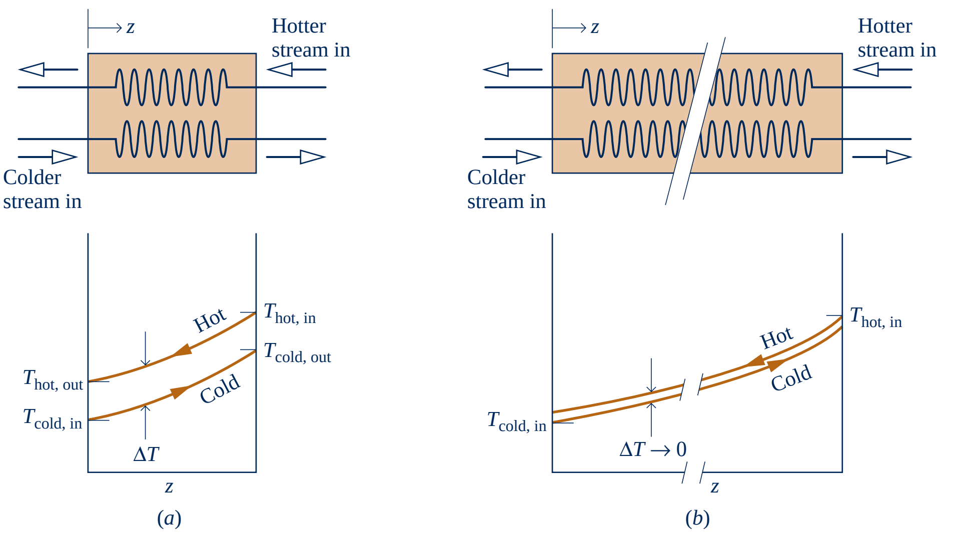


Fig. 9.15 Temperature distributions in counterflow heat exchangers. (*a*) Actual. (*b*) Reversible.

regenerator from such an ideal regenerator. The **regenerator effectiveness** is defined as the ratio of the actual enthalpy increase of the air flowing through the compressor side of the regenerator to the maximum theoretical enthalpy increase. That is,

regenerator effectiveness

$$\eta_{\text{reg}} = \frac{h_x - h_2}{h_4 - h_2} \tag{9.27}$$

As heat transfer approaches reversibility, h_x approaches h_4 and η_{reg} tends to unity (100%).

In practice, regenerator effectiveness values typically range from 60 to 80%, and thus the temperature T_x of the air exiting on the compressor side of the regenerator is normally well below the turbine exhaust temperature. To increase the effectiveness above this range would require greater heat transfer area, resulting in equipment costs that might cancel any advantage due to fuel savings. Moreover, the greater heat transfer area that would be required for a larger effectiveness can result in a significant frictional pressure drop for flow through the regenerator, thereby affecting overall performance. The decision to add a regenerator is influenced by considerations such as these, and the final decision is primarily an economic one.

In Example 9.7, we analyze an air-standard Brayton cycle with regeneration and explore the effect on thermal efficiency as the regenerator effectiveness varies.

EXAMPLE 9.7

Evaluating Thermal Efficiency of a Brayton Cycle with Regeneration

A regenerator is incorporated in the cycle of Example 9.4. **(a)** Determine the thermal efficiency for a regenerator effectiveness of 80%. **(b)** Plot the thermal efficiency versus regenerator effectiveness ranging from 0 to 80%.

SOLUTION

Known: A regenerative gas turbine operates with air as the working fluid. The compressor inlet state, turbine inlet temperature, and compressor pressure ratio are known.

Find: For a regenerator effectiveness of 80%, determine the thermal efficiency. Also plot the thermal efficiency versus the regenerator effectiveness ranging from 0 to 80%.

Schematic and Given Data:

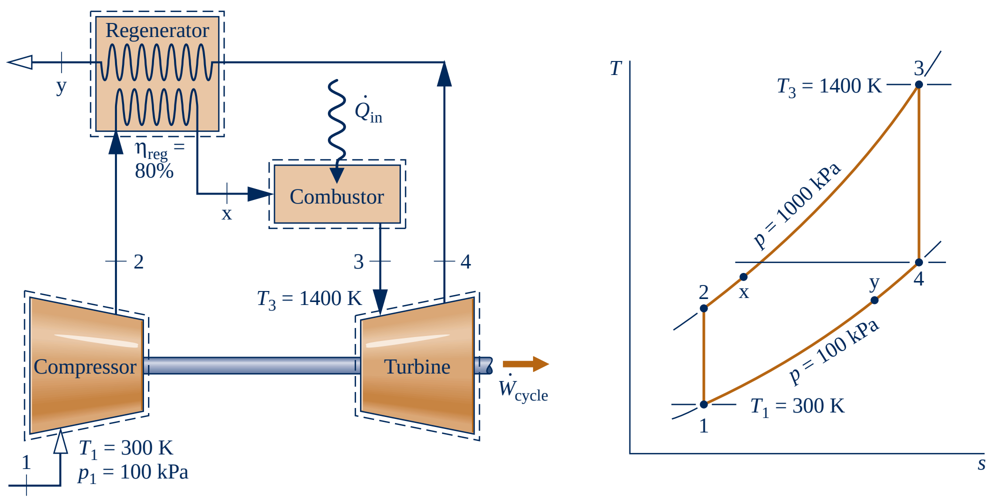


Fig. E9.7a

Engineering Model:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The compressor and turbine processes are isentropic.
3. There are no pressure drops for flow through the heat exchangers.
4. The regenerator effectiveness is 80% in part (a).
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

Analysis:

(a) The specific enthalpy values at the numbered states on the T–s diagram are the same as those in Example 9.4: $h_1 = 300.19$ kJ/kg, $h_2 = 579.9$ kJ/kg, $h_3 = 1515.4$ kJ/kg, $h_4 = 808.5$ kJ/kg.

To find the specific enthalpy h_x, the regenerator effectiveness is used as follows: By definition

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2}$$

Solving for h_x

$$\begin{aligned} h_x &= \eta_{reg}(h_4 - h_2) + h_2 \\ &= (0.8)(808.5 - 579.9) + 579.9 = 762.8 \text{ kJ/kg} \end{aligned}$$

With the specific enthalpy values determined above, the thermal efficiency is

1
$$\eta = \frac{(\dot{W}_t/\dot{m}) - (\dot{W}_c/\dot{m})}{(\dot{Q}_{in}/\dot{m})} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_x)}$$

$$= \frac{(1515.4 - 808.5) - (579.9 - 300.19)}{(1515.4 - 762.8)}$$

2
$$= 0.568\ (56.8\%)$$

(b) The *IT* code for the solution follows, where η_{reg} is denoted as etareg, η is eta, $\dot{W}_{comp}/\dot{m}$ is Wcomp, and so on.

```
// Fix the states

T1 = 300//K
p1 = 100//kPa
h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)

p2 = 1000//kPa
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)

T3 = 1400//K
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)

p4 = p1
s4 = s_TP("Air", T4, p4)
s4 = s3
h4 = h_T("Air", T4)
etareg = 0.8
hx = etareg*(h4 – h2) + h2

// Thermal efficiency
Wcomp = h2 – h1
```

```
Wturb = h3 – h4
Qin = h3 – hx
eta = (Wturb – Wcomp) / Qin
```

Using the **Explore** button, sweep etareg from 0 to 0.8 in steps of 0.01. Then, using the **Graph** button, obtain the following plot:

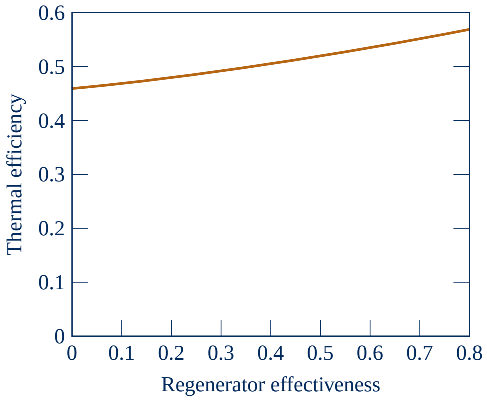


Fig. E9.7b

3 From the computer data, we see that the cycle thermal efficiency increases from 0.456, which agrees closely with the result of Example 9.4 (no regenerator), to 0.567 for a regenerator effectiveness of 80%, which agrees closely with the result of part (a). This trend is also seen in the accompanying graph. Regenerator effectiveness is seen to have a significant effect on cycle thermal efficiency.

Skills Developed

Ability to...

- ❑ sketch the schematic of the regenerative gas turbine and the T–s diagram for the corresponding air-standard cycle.
- ❑ evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- ❑ calculate the thermal efficiency.

---

1 The values for work per unit of mass flow of the compressor and turbine are unchanged by the addition of the regenerator. Thus, the back work ratio and net work output are not affected by this modification.

2 Comparing the present thermal efficiency value with the one determined in Example 9.4, it should be evident that the thermal efficiency can be increased significantly by means of regeneration.

3 The regenerator allows improved fuel utilization to be achieved by transferring a portion of the exergy in the hot turbine exhaust gas to the cooler air flowing on the other side of the regenerator.

QuickQUIZ What would be the thermal efficiency if the regenerator effectiveness were 100%? Ans. 60.4%.

9.8 Regenerative Gas Turbines with Reheat and Intercooling

Two modifications of the basic gas turbine that increase the net work developed are multistage expansion with *reheat* and multistage compression with *intercooling*. When used in conjunction with regeneration, these modifications can result in substantial increases in thermal efficiency. The concepts of reheat and intercooling are introduced in this section.

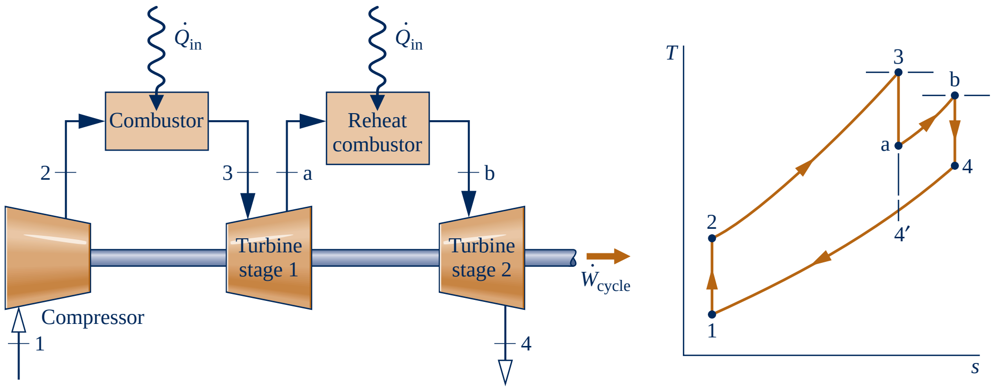


Fig. 9.16 **Ideal gas turbine with reheat.**

9.8.1 Gas Turbines with Reheat

For metallurgical reasons, the temperature of the gaseous combustion products entering the turbine must be limited. This temperature can be controlled by providing air in excess of the amount required to burn the fuel in the combustor (see Chap. 13). As a consequence, the gases exiting the combustor contain sufficient air to support the combustion of additional fuel. Some gas turbine power plants take advantage of the excess air by means of a multistage turbine with a **reheat combustor** between the stages. With this arrangement the net work per unit of mass flow can be increased. Let us consider reheat from the vantage point of an air-standard analysis.

reheat

The basic features of a two-stage gas turbine with reheat are brought out by considering an ideal air-standard Brayton cycle modified as shown in Fig. 9.16. After expansion from state 3 to state a in the first turbine, the gas is reheated at constant pressure from state a to state b. The expansion is then completed in the second turbine from state b to state 4. The ideal Brayton cycle without reheat, 1–2–3–4′–1, is shown on the same *T–s* diagram for comparison. Because lines of constant pressure on a *T–s* diagram diverge slightly with increasing entropy, the total work of the two-stage turbine is greater than that of a single expansion from state 3 to state 4′. Thus, the *net* work for the reheat cycle is greater than that of the cycle without reheat. Despite the increase in net work with reheat, the cycle thermal efficiency would not necessarily increase because a greater total heat addition would be required. However, the temperature at the exit of the turbine is higher with reheat than without reheat, so the potential for regeneration is enhanced.

When reheat and regeneration are used together, the thermal efficiency can increase significantly. The following example provides an illustration.

EXAMPLE 9.8

Determining Thermal Efficiency of a Brayton Cycle with Reheat and Regeneration

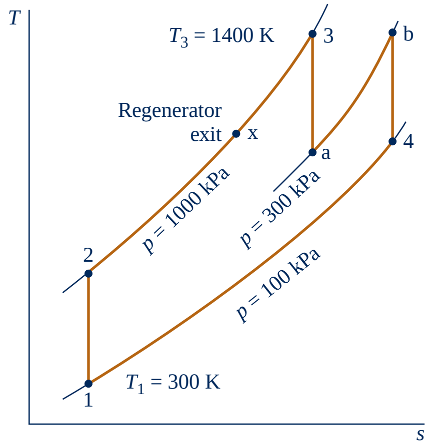
Consider a modification of the cycle of Example 9.4 involving reheat and regeneration. Air enters the compressor at 100 kPa, 300 K and is compressed to 1000 kPa. The temperature at the inlet to the first turbine stage is 1400 K. The expansion takes place isentropically in two stages, with reheat to 1400 K between the stages at a constant pressure of 300 kPa. A regenerator having an effectiveness of 100% is also incorporated in the cycle. Determine the thermal efficiency.

SOLUTION

Known: An ideal air-standard gas turbine cycle operates with reheat and regeneration. Temperatures and pressures at principal states are specified.

Find: Determine the thermal efficiency.

Schematic and Given Data:



Engineering Model:

1. Each component of the power plant is analyzed as a control volume at steady state.
2. The compressor and turbine processes are isentropic.
3. There are no pressure drops for flow through the heat exchangers.
4. The regenerator effectiveness is 100%.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

Fig. E9.8

Analysis: We begin by determining the specific enthalpies at each principal state of the cycle. States 1, 2, and 3 are the same as in Example 9.4: $h_1 = 300.19$ kJ/kg, $h_2 = 579.9$ kJ/kg, $h_3 = 1515.4$ kJ/kg. The temperature at state b is the same as at state 3, so $h_b = h_3$.

Since the first turbine process is isentropic, the enthalpy at state a can be determined using p_r data from Table A-22 and the relationship

$$p_{ra} = p_{r3}\frac{p_a}{p_3} = (450.5)\frac{300}{1000} = 135.15$$

Interpolating in Table A-22, we get $h_a = 1095.9$ kJ/kg.

The second turbine process is also isentropic, so the enthalpy at state 4 can be determined similarly. Thus

$$p_{r4} = p_{rb}\frac{p_4}{p_b} = (450.5)\frac{100}{300} = 150.17$$

Interpolating in Table A-22, we obtain $h_4 = 1127.6$ kJ/kg. Since the regenerator effectiveness is 100%, $h_x = h_4 = 1127.6$ kJ/kg.

The thermal efficiency calculation must take into account the compressor work, the work of *each* turbine, and the *total* heat added. Thus, on a unit mass basis

$$\eta = \frac{(h_3 - h_a) + (h_b - h_4) - (h_2 - h_1)}{(h_3 - h_x) + (h_b - h_a)}$$

$$= \frac{(1515.4 - 1095.9) + (1515.4 - 1127.6) - (579.9 - 300.19)}{(1515.4 - 1127.6) + (1515.4 - 1095.9)}$$

1 $$= 0.654\ (65.4\%)$$

1 Comparing the present value with the thermal efficiency determined in part (a) of Example 9.4, we can conclude that the use of reheat coupled with regeneration can result in a substantial increase in thermal efficiency.

Skills Developed

Ability to...

- sketch the schematic of the regenerative gas turbine with reheat and the *T–s* diagram for the corresponding air-standard cycle.
- evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- calculate the thermal efficiency.

QuickQUIZ What percentage of the total heat addition occurs in the reheat process? Ans. 52%.

9.8.2 Compression with Intercooling

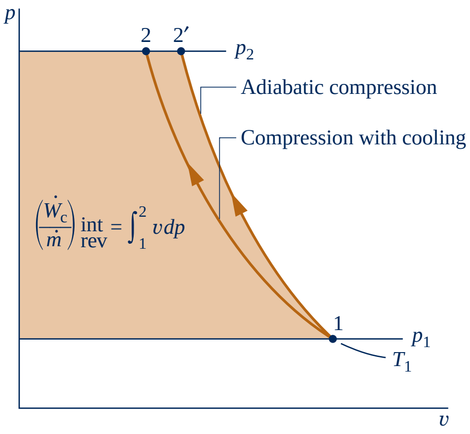


Fig. 9.17 **Internally reversible compression processes between two fixed pressures.**

The net work output of a gas turbine also can be increased by reducing the compressor work input. This can be accomplished by means of multistage compression with intercooling. The present discussion provides an introduction to this subject.

Let us first consider the work input to compressors at steady state, assuming that irreversibilities are absent and changes in kinetic and potential energy from inlet to exit are negligible. The p–v diagram of Fig. 9.17 shows two alternative compression paths from a specified state 1 to a specified final pressure p_2. Path 1–2′ is for an adiabatic compression. Path 1–2 corresponds to a compression with heat transfer *from* the working fluid to the surroundings. The area to the left of each curve equals the *magnitude* of the work per unit mass of the respective process (see Sec. 6.13.2). The smaller area to the left of Process 1–2 indicates that the work of this process is less than for the adiabatic compression from 1 to 2′. This suggests that cooling a gas *during* compression is advantageous in terms of the work-input requirement.

Although cooling a gas *as it is compressed* would reduce the work, a heat transfer rate high enough to effect a significant reduction in work is difficult to achieve in practice. A practical alternative is to separate the work and heat interactions into separate processes by letting compression take place in stages with heat exchangers, called **intercoolers**, cooling the gas between stages. Figure 9.18 illustrates a two-stage compressor with an intercooler. The accompanying p–v and T–s diagrams show the states for internally reversible processes:

intercooler

- ▶ Process 1–c is an isentropic compression from state 1 to state c where the pressure is p_i.
- ▶ Process c–d is constant-pressure cooling from temperature T_c to T_d.
- ▶ Process d–2 is an isentropic compression to state 2.

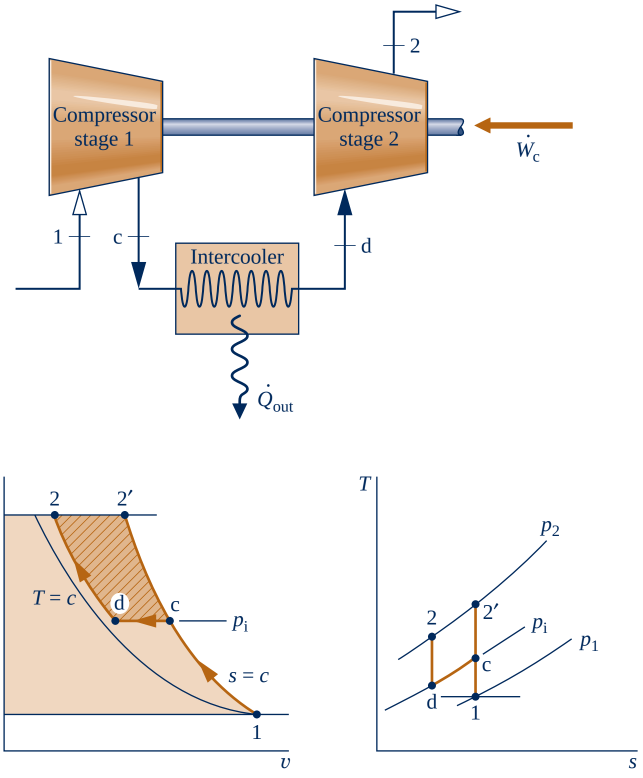


Fig. 9.18 **Two-stage compression with intercooling.**

The work input per unit of mass flow is represented on the p–v diagram by shaded area 1–c–d–2–a–b–1. Without intercooling the gas would be compressed isentropically in a single stage from state 1 to state 2′ and the work would be represented by enclosed area 1–2′–a–b–1. The crosshatched area on the p–v diagram represents the reduction in work that would be achieved with intercooling.

Some large compressors have several stages of compression with intercooling between stages. The determination of the number of stages and the conditions at which to operate the various intercoolers is a problem in optimization. The use of multistage compression with intercooling in a gas turbine power plant increases the net work developed by reducing the compression work. By itself, though, compression with intercooling would not necessarily increase the thermal efficiency of a gas turbine because the temperature of the air entering the combustor would be reduced (compare temperatures at states 2′ and 2 on the T–s diagram of Fig. 9.18). A lower temperature at the combustor inlet would require additional heat transfer to achieve the desired turbine inlet temperature. The lower temperature at the compressor exit enhances the potential for regeneration, however, so when intercooling is used in conjunction with regeneration, an appreciable increase in thermal efficiency can result.

In the next example, we analyze a two-stage compressor with intercooling between the stages. Results are compared with those for a single stage of compression.

EXAMPLE 9.9

Evaluating a Two-Stage Compressor with Intercooling

Air is compressed from 100 kPa, 300 K to 1000 kPa in a two-stage compressor with intercooling between stages. The intercooler pressure is 300 kPa. The air is cooled back to 300 K in the intercooler before entering the second compressor stage. Each compressor stage is isentropic. For steady-state operation and negligible changes in kinetic and potential energy from inlet to exit, determine **(a)** the temperature at the exit of the second compressor stage and **(b)** the total compressor work input per unit of mass flow. **(c)** Repeat for a single stage of compression from the given inlet state to the final pressure.

SOLUTION

Known: Air is compressed at steady state in a two-stage compressor with intercooling between stages. Operating pressures and temperatures are given.

Find: Determine the temperature at the exit of the second compressor stage and the total work input per unit of mass flow. Repeat for a single stage of compression.

Schematic and Given Data:

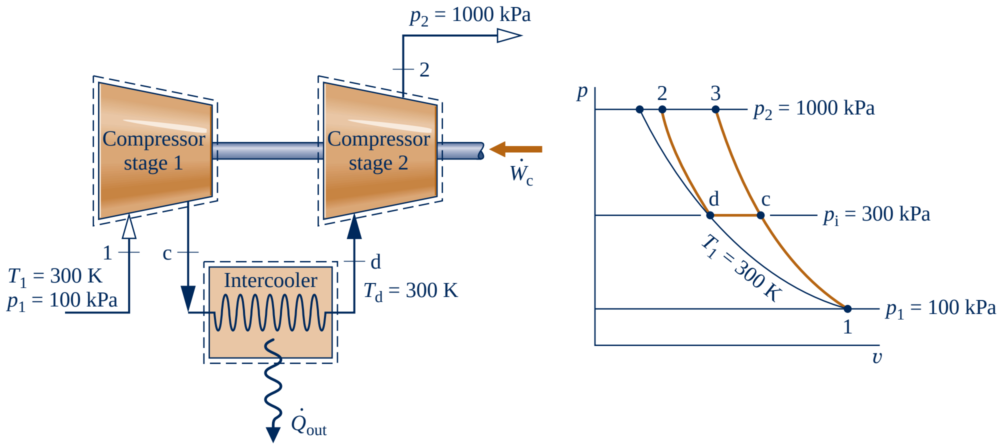


Fig. E9.9

Engineering Model:

1. The compressor stages and intercooler are analyzed as control volumes at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The compression processes are isentropic.
3. There is no pressure drop for flow through the intercooler.
4. Kinetic and potential energy effects are negligible.
5. The air is modeled as an ideal gas.

Analysis:

(a) The temperature at the exit of the second compressor stage, T_2, can be found using the following relationship for the isentropic process d–2

$$p_{r2} = p_{rd}\frac{p_2}{p_d}$$

With p_{rd} at $T_d = 300$ K from Table A-22, $p_2 = 1000$ kPa, and $p_d = 300$ kPa,

$$p_{r2} = (1.386)\frac{1000}{300} = 4.62$$

Interpolating in Table A-22, we get $T_2 = 422$ K and $h_2 = 423.8$ kJ/kg.

(b) The total compressor work input per unit of mass is the sum of the work inputs for the two stages. That is

$$\frac{\dot{W}_c}{\dot{m}} = (h_c - h_1) + (h_2 - h_d)$$

From Table A-22 at $T_1 = 300$ K, $h_1 = 300.19$ kJ/kg. Since $T_d = T_1$, $h_d = 300.19$ kJ/kg. To find h_c, use p_r data from Table A-22 together with $p_1 = 100$ kPa and $p_c = 300$ kPa to write

$$p_{rc} = p_{r1}\frac{p_c}{p_1} = (1.386)\frac{300}{100} = 4.158$$

Interpolating in Table A-22, we obtain $h_c = 411.3$ kJ/kg. Hence, the total compressor work per unit of mass is

$$\frac{\dot{W}_c}{\dot{m}} = (411.3 - 300.19) + (423.8 - 300.19) = 234.7 \text{ kJ/kg}$$

(c) For a single isentropic stage of compression, the exit state would be state 3 located on the accompanying p–v diagram. The temperature at this state can be determined using

$$p_{r3} = p_{r1}\frac{p_3}{p_1} = (1.386)\frac{1000}{100} = 13.86$$

Interpolating in Table A-22, we get $T_3 = 574$ K and $h_3 = 579.9$ kJ/kg.

The work input for a single stage of compression is then

$$\frac{\dot{W}_c}{\dot{m}} = h_3 - h_1 = 579.9 - 300.19 = 279.7 \text{ kJ/kg}$$

This calculation confirms that a smaller work input is required with two-stage compression and intercooling than with a single stage of compression. With intercooling, however, a much lower gas temperature is achieved at the compressor exit.

Skills Developed

Ability to...

- ❑ sketch the schematic of a two-stage compressor with intercooling between the stages and the corresponding T–s diagram.
- ❑ evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- ❑ apply energy and entropy balances.

QuickQUIZ In this case, what is the percentage reduction in compressor work with two-stage compression and intercooling compared to a single stage of compression? Ans. 16.1%.

Referring again to Fig. 9.18, the size of the crosshatched area on the p–v diagram representing the reduction in work with intercooling depends on both the temperature T_d at the exit of the intercooler and the intercooler pressure p_i. By properly selecting T_d and p_i, the total work input to the compressor can be minimized. For example, if the pressure p_i is specified, the work input would decrease (crosshatched area would increase) as the temperature T_d approaches T_1, the temperature at the inlet to the compressor. For air entering the compressor from the surroundings, T_1 would be the limiting temperature that could be achieved at state d through heat transfer with the surroundings only. Also, for a specified value of the temperature T_d, the pressure p_i can be selected so that the total work input is a minimum (crosshatched area is a maximum).

Example 9.10 provides an illustration of the determination of the intercooler pressure for minimum total work using a cold air-standard analysis.

EXAMPLE 9.10

Determining Intercooler Pressure for Minimum Compressor Work

For fixed inlet state and exit pressure, use a cold-air standard analysis to show that minimum total work input for a two-stage compressor is required when the pressure ratio is the same across each stage. Assume steady-state operation and the following idealizations: Each compression process is isentropic, there is no pressure drop through the intercooler, temperature at the inlet to each compressor stage is the same, and kinetic and potential energy effects can be ignored.

SOLUTION

Known: A two-stage compressor with intercooling operates at steady state under specified conditions.

Find: Show that the minimum total work input is required when the pressure ratio is the same across each stage.

Schematic and Given Data:

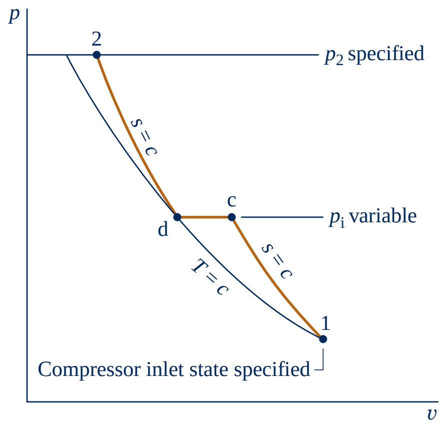


Fig. E9.10

Engineering Model:

1. The compressor stages and intercooler are analyzed as control volumes at steady state.
2. The compression processes are isentropic.
3. There is no pressure drop for flow through the intercooler.
4. The temperature at the inlet to both compressor stages is the same.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.
7. The specific heat c_p and thus the specific heat ratio k are constant.

Analysis: The total compressor work input per unit of mass flow is

$$\frac{\dot{W}_c}{\dot{m}} = (h_c - h_1) + (h_2 - h_d)$$

Since c_p is constant

$$\frac{\dot{W}_c}{\dot{m}} = c_p(T_c - T_1) + c_p(T_2 - T_d)$$

With $T_d = T_1$ (assumption 4), this becomes on rearrangement

$$\frac{\dot{W}_c}{\dot{m}} = c_p T_1 \left(\frac{T_c}{T_1} + \frac{T_2}{T_1} - 2 \right)$$

Since the compression processes are isentropic and the specific heat ratio k is constant, the pressure and temperature ratios across the compressor stages are related, respectively, by

$$\frac{T_c}{T_1} = \left(\frac{p_i}{p_1} \right)^{(k-1)/k} \qquad \text{and} \qquad \frac{T_2}{T_d} = \left(\frac{p_2}{p_i} \right)^{(k-1)/k}$$

In the second of these equations, $T_d = T_1$ by assumption 4.

Collecting results

$$\frac{\dot{W}_c}{\dot{m}} = c_p T_1 \left[\left(\frac{p_i}{p_1} \right)^{(k-1)/k} + \left(\frac{p_2}{p_i} \right)^{(k-1)/k} - 2 \right]$$

Hence, for specified values of T_1, p_1, p_2, and c_p, the value of the total compressor work input varies with the intercooler pressure only. To determine the pressure p_i that minimizes the total work, form the derivative

$$\begin{aligned}
\frac{\partial(\dot{W}_c/\dot{m})}{\partial p_i} &= \frac{\partial}{\partial p_i} \left\{ c_p T_1 \left[\left(\frac{p_i}{p_1} \right)^{(k-1)/k} + \left(\frac{p_2}{p_i} \right)^{(k-1)/k} - 2 \right] \right\} \\
&= c_p T_1 \left(\frac{k-1}{k} \right) \left[\left(\frac{p_i}{p_1} \right)^{-1/k} \left(\frac{1}{p_1} \right) + \left(\frac{p_2}{p_i} \right)^{-1/k} \left(-\frac{p_2}{p_i^2} \right) \right] \\
&= c_p T_1 \left(\frac{k-1}{k} \right) \frac{1}{p_i} \left[\left(\frac{p_i}{p_1} \right)^{(k-1)/k} - \left(\frac{p_2}{p_i} \right)^{(k-1)/k} \right]
\end{aligned}$$

When the partial derivative is set to zero, the desired relationship is obtained

1 $$\frac{p_i}{p_1} = \frac{p_2}{p_i}$$

By checking the sign of the second derivative, it can be verified that the total compressor work is a minimum.

---

1 This relationship is for a two-stage compressor. Appropriate relations can be obtained similarly for multistage compressors.

Skills Developed

Ability to...

- complete the detailed derivation of a thermodynamic expression.
- use calculus to minimize a function.

QuickQUIZ Air enters an ideal two-stage compressor with intercooling at 100 kPa. The overall compressor pressure ratio is 12. What pressure, in bar, would minimize the total work input required? **Ans.** 3.464 bar.

9.8.3 Reheat and Intercooling

Reheat between turbine stages and intercooling between compressor stages provide two important advantages: The net work output is increased, and the potential for regeneration is enhanced. Accordingly, when reheat and intercooling are used together with regeneration, a substantial improvement in performance can be realized. One arrangement incorporating reheat, intercooling, and regeneration is shown in Fig. 9.19. This gas turbine has two stages of compression and two turbine stages. The accompanying T–s diagram is drawn to indicate irreversibilities in the compressor

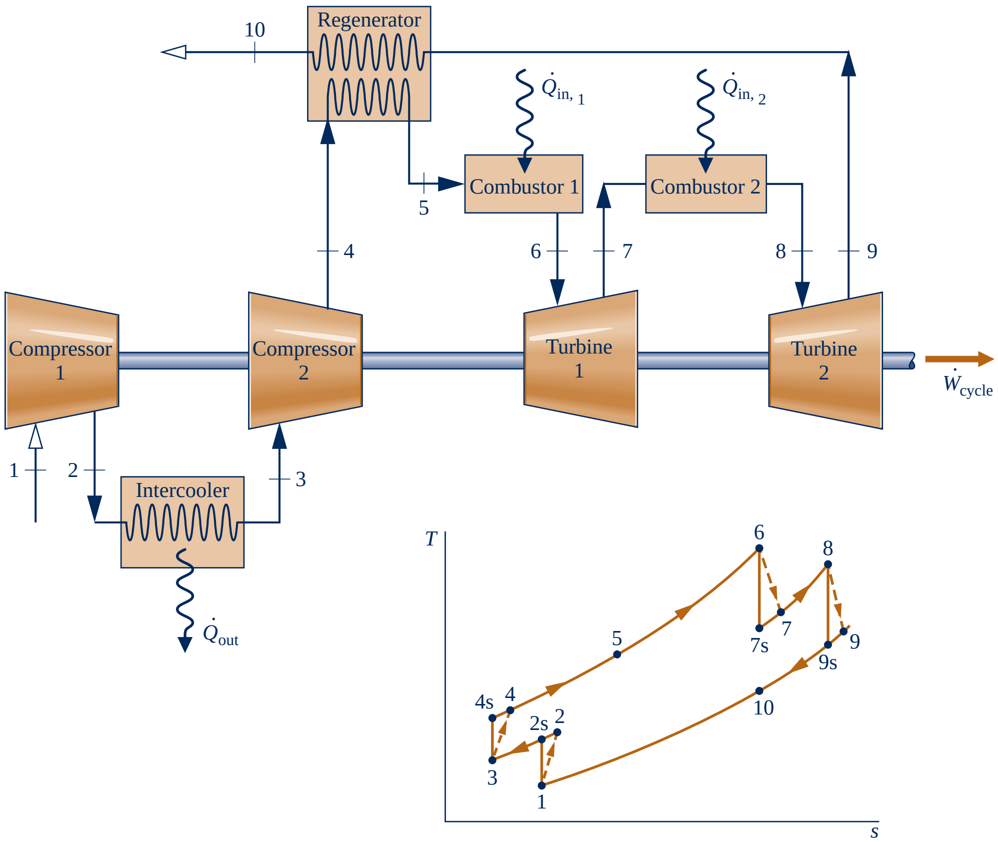


Fig. 9.19 **Regenerative gas turbine with intercooling and reheat.**

and turbine stages. The pressure drops that would occur as the working fluid passes through the intercooler, regenerator, and combustors are not shown.

Example 9.11 illustrates the analysis of a regenerative gas turbine with intercooling and reheat.

EXAMPLE 9.11

Analyzing a Regenerative Gas Turbine with Intercooling and Reheat

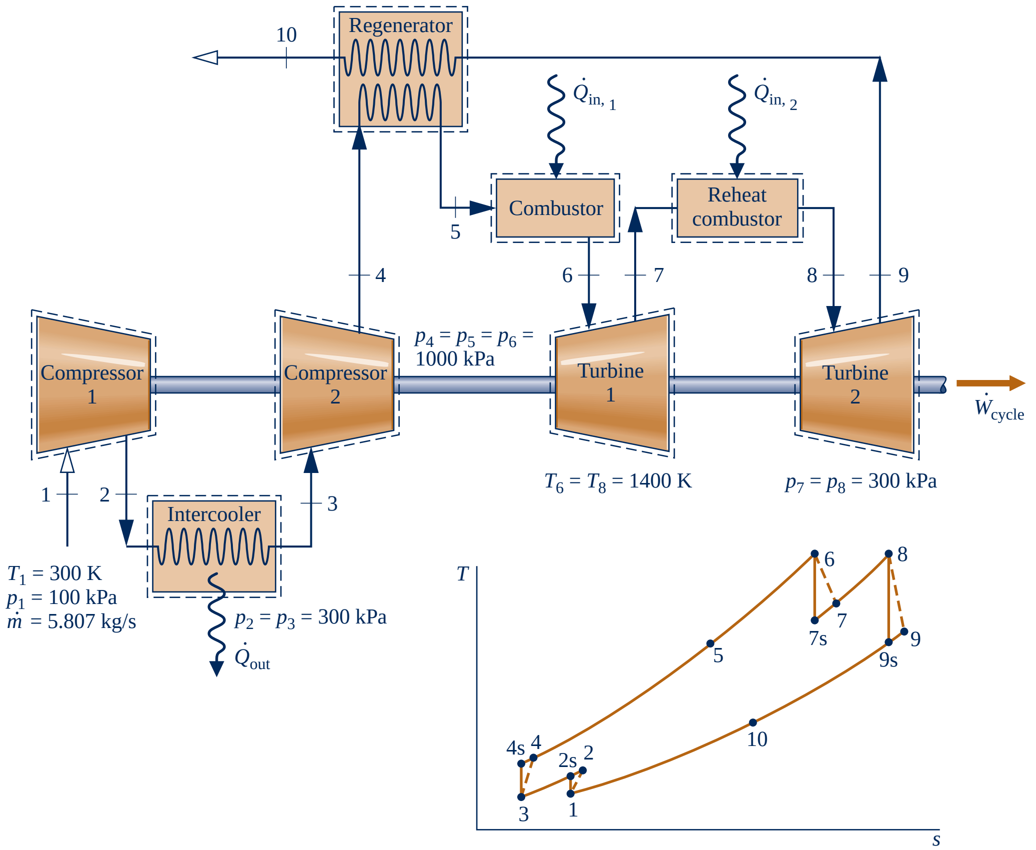
A regenerative gas turbine with intercooling and reheat operates at steady state. Air enters the compressor at 100 kPa, 300 K with a mass flow rate of 5.807 kg/s. The pressure ratio across the two-stage compressor is 10. The pressure ratio across the two-stage turbine is also 10. The intercooler and reheater each operate at 300 kPa. At the inlets to the turbine stages, the temperature is 1400 K. The temperature at the inlet to the second compressor stage is 300 K. The isentropic efficiency of each compressor and turbine stage is 80%. The regenerator effectiveness is 80%. Determine **(a)** the thermal efficiency, **(b)** the back work ratio, **(c)** the net power developed, in kW.

SOLUTION

Known: An air-standard regenerative gas turbine with intercooling and reheat operates at steady state. Operating pressures and temperatures are specified. Turbine and compressor isentropic efficiencies are given and the regenerator effectiveness is known.

Find: Determine the thermal efficiency, back work ratio, and net power developed, in kW.

Schematic and Given Data:



Engineering Model:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. There are no pressure drops for flow through the heat exchangers.
3. The compressor and turbine are adiabatic.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

Fig. E9.11

Analysis: We begin by listing the specific enthalpies at the principal states of this cycle. The enthalpies at states 1, 2s, 3, and 4s are obtained from the solution to Example 9.9 where these states are designated as 1, c, d, and 2, respectively. Thus, $h_1 = h_3 = 300.19$ kJ/kg, $h_{2s} = 411.3$ kJ/kg, $h_{4s} = 423.8$ kJ/kg.

The specific enthalpies at states 6, 7s, 8, and 9s are obtained from the solution to Example 9.8, where these states are designated as 3, a, b, and 4, respectively. Thus, $h_6 = h_8 = 1515.4$ kJ/kg, $h_{7s} = 1095.9$ kJ/kg, $h_{9s} = 1127.6$ kJ/kg.

The specific enthalpy at state 4 can be determined using the isentropic efficiency of the second compressor stage

$$\eta_c = \frac{h_{4s} - h_3}{h_4 - h_3}$$

Solving for h_4

$$\begin{aligned} h_4 &= h_3 + \frac{h_{4s} - h_3}{\eta_c} = 300.19 + \left(\frac{423.8 - 300.19}{0.8}\right) \\ &= 454.7 \text{ kJ/kg} \end{aligned}$$

Similarly, the specific enthalpy at state 2 is $h_2 = 439.1$ kJ/kg.

The specific enthalpy at state 9 can be determined using the isentropic efficiency of the second turbine stage

$$\eta_t = \frac{h_8 - h_9}{h_8 - h_{9s}}$$

Solving for h_9

$$\begin{aligned} h_9 &= h_8 - \eta_t(h_8 - h_{9s}) = 1515.4 - 0.8(1515.4 - 1127.6) \\ &= 1205.2 \text{ kJ/kg} \end{aligned}$$

Similarly, the specific enthalpy at state 7 is $h_7 = 1179.8$ kJ/kg.

The specific enthalpy at state 5 can be determined using the regenerator effectiveness

$$\eta_{reg} = \frac{h_5 - h_4}{h_9 - h_4}$$

Solving for h_5

$$\begin{aligned} h_5 &= h_4 + \eta_{reg}(h_9 - h_4) = 454.7 + 0.8(1205.2 - 454.7) \\ &= 1055.1 \text{ kJ/kg} \end{aligned}$$

(a) The thermal efficiency must take into account the work of both turbine stages, the work of both compressor stages, and the total heat added. The total turbine work per unit of mass flow is

$$\begin{aligned} \frac{\dot{W}_t}{\dot{m}} &= (h_6 - h_7) + (h_8 - h_9) \\ &= (1515.4 - 1179.8) + (1515.4 - 1205.2) = 645.8 \text{ kJ/kg} \end{aligned}$$

The total compressor work input per unit of mass flow is

$$\begin{aligned} \frac{\dot{W}_c}{\dot{m}} &= (h_2 - h_1) + (h_4 - h_3) \\ &= (439.1 - 300.19) + (454.7 - 300.19) = 293.4 \text{ kJ/kg} \end{aligned}$$

The total heat added per unit of mass flow is

$$\begin{aligned} \frac{\dot{Q}_{in}}{\dot{m}} &= (h_6 - h_5) + (h_8 - h_7) \\ &= (1515.4 - 1055.1) + (1515.4 - 1179.8) = 795.9 \text{ kJ/kg} \end{aligned}$$

Calculating the thermal efficiency

$$\eta = \frac{645.8 - 293.4}{795.9} = 0.443\ (44.3\%)$$

(b) The back work ratio is

$$\text{bwr} = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{293.4}{645.8} = 0.454\ (45.4\%)$$

(c) The net power developed is

$$\begin{aligned} \dot{W}_{cycle} &= \dot{m}(\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}) \\ &= \left(5.807 \frac{\text{kg}}{\text{s}}\right)(645.8 - 293.4)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 2046 \text{ kW} \end{aligned} \quad (1)$$

(1) Comparing the thermal efficiency, back work ratio, and net power values of the current example with the corresponding values of Example 9.6, it should be evident that gas turbine power plant performance can be increased significantly by coupling reheat and intercooling with regeneration.

Skills Developed

Ability to...

- ❑ sketch the schematic of the regenerative gas turbine with intercooling and reheat and the T–s diagram for the corresponding air-standard cycle.
- ❑ evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- ❑ calculate the thermal efficiency, back work ratio, and net power developed.

QuickQUIZ Using the results of parts (a) and (c), determine the total rate of heat addition to the cycle, in kW. **Ans.** 4619 kW.

9.8.4 Ericsson and Stirling Cycles

As illustrated by Example 9.11, significant increases in the thermal efficiency of gas turbine power plants can be achieved through intercooling, reheat, and regeneration. There is an economic limit to the number of stages that can be employed, and normally there would be no more than two or three. Nonetheless, it is instructive to consider the situation where the number of stages of both intercooling and reheat becomes indefinitely large.

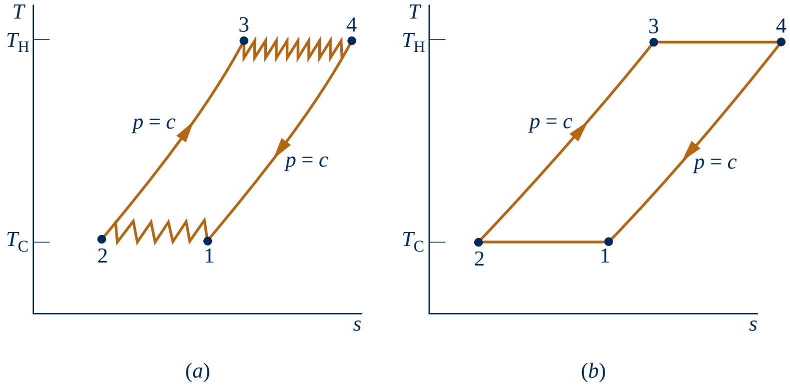


Fig. 9.20 Ericsson cycle as a limit of ideal gas turbine operation using multistage compression with intercooling, multistage expansion with reheating, and regeneration.

ERICSSON CYCLE. Figure 9.20*a* shows an *ideal* regenerative gas turbine cycle with several stages of compression and expansion and a regenerator whose effectiveness is 100%. As in Fig. 9.8*b*, this is a *closed* gas turbine cycle. Each intercooler is assumed to return the working fluid to the temperature T_C at the inlet to the first compression stage and each reheater restores the working fluid to the temperature T_H at the inlet to the first turbine stage. The regenerator allows the heat input for Process 2–3 to be obtained from the heat rejected in Process 4–1. Accordingly, all the heat added *externally* occurs in the reheaters, and all the heat rejected to the surroundings takes place in the intercoolers.

Ericsson cycle

In the limit, as an infinite number of reheat and intercooler stages are employed, all heat added occurs when the working fluid is at its highest temperature, T_H, and all heat rejected takes place when the working fluid is at its lowest temperature, T_C. The limiting cycle, shown in Fig. 9.20*b*, is called the **Ericsson cycle**.

Since irreversibilities are presumed absent and all heat is supplied and rejected isothermally, the thermal efficiency of the Ericsson cycle equals that of *any* reversible power cycle operating with heat addition at the temperature T_H and heat rejection at the temperature T_C: $\eta_{max} = 1 - T_C/T_H$. This expression is applied in Secs. 5.10 and 6.6 to evaluate the thermal efficiency of Carnot power cycles. Although the details of the Ericsson cycle differ from those of the Carnot cycle, both cycles have the same value of thermal efficiency when operating between the temperatures T_H and T_C.

STIRLING CYCLE. Another cycle that employs a regenerator is the *Stirling* cycle, shown on the p–v and T–s diagrams of Fig. 9.21. The cycle consists of four internally reversible processes in series: isothermal compression from state 1 to state 2 at temperature T_C, constant-volume heating from state 2 to state 3, isothermal expansion

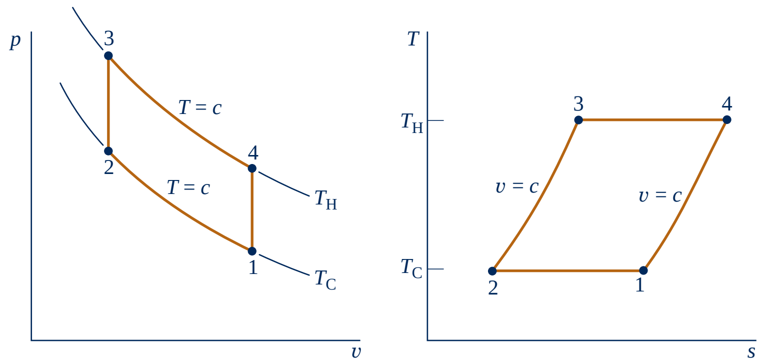


Fig. 9.21 ***p*–*v* and *T*–*s* diagrams of the Stirling cycle.**

from state 3 to state 4 at temperature T_H, and constant-volume cooling from state 4 to state 1 to complete the cycle.

A regenerator whose effectiveness is 100% allows the heat rejected during Process 4–1 to provide the heat input in Process 2–3. Accordingly, all the heat added to the working fluid externally takes place in the isothermal process 3–4 and all the heat rejected to the surroundings occurs in the isothermal process 1–2.

It can be concluded, therefore, that the thermal efficiency of the Stirling cycle is given by the same expression as for the Carnot and Ericsson cycles. Since all three cycles are *reversible,* we can imagine them as being executed in various ways, including use of gas turbines and piston–cylinder engines. In each embodiment, however, practical issues prevent it from actually being realized.

STIRLING ENGINE. The Ericsson and Stirling cycles are principally of theoretical interest as examples of cycles that exhibit the same thermal efficiency as the Carnot cycle. However, a practical engine of the piston–cylinder type that operates on a *closed* regenerative cycle having features in common with the Stirling cycle has been under study for years. This engine is known as a **Stirling engine**. The Stirling engine offers the opportunity for high efficiency together with reduced emissions from combustion products because combustion takes place externally and not within the cylinder as for internal combustion engines. In the Stirling engine, energy is transferred to the working fluid from products of combustion, which are kept separate. It is an *external* combustion engine.

Stirling engine

9.9 Gas Turbine–Based Combined Cycles

In this section, gas turbine–based combined cycles are considered for power generation. Cogeneration, including district heating, is also considered. These discussions complement those of Sec. 8.5, where vapor power system performing similar functions are introduced.

The present applications build on recognizing that the exhaust gas temperature of a simple gas turbine is typically well above ambient temperature and thus hot gas exiting the turbine has significant thermodynamic utility that might be harnessed economically. This observation provides the basis for the regenerative gas turbine cycle introduced in Sec. 9.7 and for the current applications.

9.9.1 Combined Gas Turbine–Vapor Power Cycle

A combined cycle couples two power cycles such that the energy discharged by heat transfer from one cycle is used partly or wholly as the heat input for the other cycle. This is illustrated by the combined cycle involving gas and vapor power turbines shown in Fig. 9.22. The gas and vapor power cycles are combined using an interconnecting heat-recovery steam generator that serves as the boiler for the vapor power cycle.

The combined cycle has the gas turbine's high average temperature of heat addition and the vapor cycle's low average temperature of heat rejection, and thus a thermal efficiency greater than either cycle would have individually. For many applications combined cycles are a good choice, and they are increasingly being used worldwide for electric power generation.

With reference to Fig. 9.22, the thermal efficiency of the combined cycle is

$$\eta = \frac{\dot{W}_{\text{gas}} + \dot{W}_{\text{vap}}}{\dot{Q}_{\text{in}}} \tag{9.28}$$

where $\dot{W}_{\text{gas}}$ is the *net* power developed by the gas turbine and $\dot{W}_{\text{vap}}$ is the *net* power developed by the vapor cycle. $\dot{Q}_{\text{in}}$ denotes the *total* rate of heat transfer to the combined cycle, including additional heat transfer, if any, to superheat the vapor entering

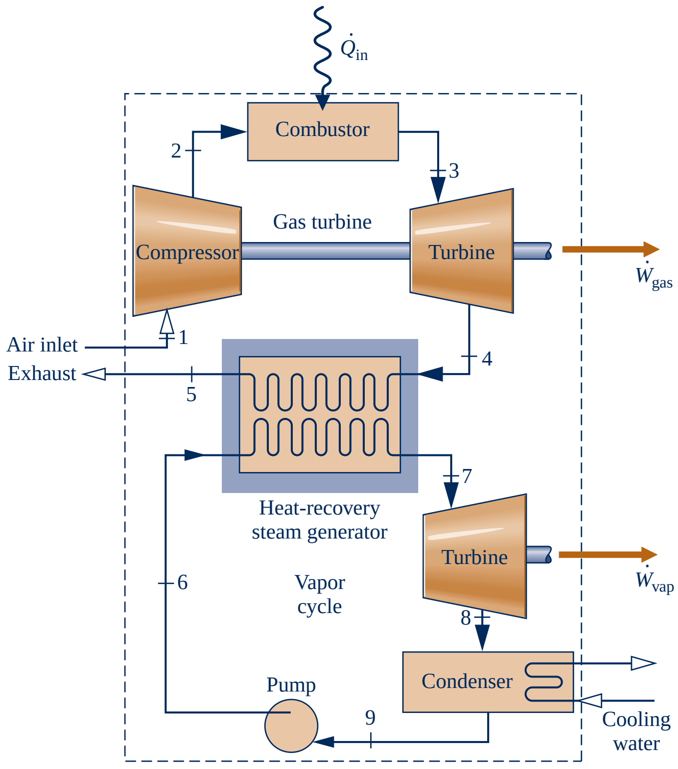


Fig. 9.22 Combined gas turbine–vapor power plant.

the vapor turbine. The evaluation of the quantities appearing in Eq. 9.28 follows the procedures described in the sections on vapor cycles and gas turbines.

The relation for the energy transferred from the gas cycle to the vapor cycle for the system of Fig. 9.22 is obtained by applying the mass and energy rate balances to a control volume enclosing the heat-recovery steam generator. For steady-state operation, negligible heat transfer with the surroundings, and no significant changes in kinetic and potential energy, the result is

$$\dot{m}_v(h_7 - h_6) = \dot{m}_g(h_4 - h_5) \tag{9.29}$$

where $\dot{m}_g$ and $\dot{m}_v$ are the mass flow rates of the gas and vapor, respectively.

As witnessed by relations such as Eqs. 9.28 and 9.29, combined cycle performance can be analyzed using mass and energy balances. To complete the analysis, however, the second law is required to assess the impact of irreversibilities and the true magnitudes of losses. Among the irreversibilities, the most significant is the exergy destroyed by combustion. About 30% of the exergy entering the combustor with the fuel is destroyed by combustion irreversibility. An analysis of the gas turbine on an air-standard basis does not allow this exergy destruction to be evaluated, however, and means introduced in Chap. 13 must be applied for this purpose.

ENERGY & ENVIRONMENT Advanced combined-cycle H-class power plants capable of achieving the long-elusive 60% combined-cycle thermal efficiency level are a reality. Here *H* denotes the largest electric power gas turbines. H-class power plants integrate a gas turbine, steam turbine, steam generator, and heat-recovery steam generator. They are capable of net power outputs reaching nearly 600 MW, while significantly saving fuel, reducing carbon dioxide emissions, and adhering to low nitric oxides standards.

Before the H-class breakthrough, gas turbine manufacturers had struggled against a temperature-imposed barrier that limited thermal efficiency for gas turbine-based power systems. For years, the barrier was a gas turbine inlet temperature of about 1260°C (2300°F). Above that level, available cooling technologies were unable to protect turbine blades and other key components from thermal degradation. Since higher temperatures go hand-in-hand with higher thermal efficiencies, the perceived temperature barrier limited the efficiency achievable.

Two developments were instrumental in allowing combined-cycle thermal efficiencies of 60% or more: steam cooling of both stationary and rotating blades and blades made from single crystals.

- In steam cooling, relatively low-temperature steam generated in the companion vapor power plant is fed to channels in the blades of the high-temperature stages of the gas turbine, thereby cooling the blades while producing superheated steam for use in the vapor plant, adding to overall cycle efficiency. Innovative coatings, typically ceramic composites, also help components withstand very high gas temperatures.
- H-class gas turbines also have *single-crystal* blades. Conventionally cast blades are *poly*crystalline. They consist of a multitude of small *grains* (crystals) with interfaces between the grains called grain boundaries. Adverse physical events such as corrosion and *creep* originating at grain boundaries greatly shorten blade life and impose limits on allowed turbine temperatures. Having no grain boundaries, single-crystal blades are far more durable and less prone to thermal degradation.

The next example illustrates the use of mass and energy balances, the second law, and property data to analyze combined cycle performance.

EXAMPLE 9.12

Energy and Exergy Analyses of a Combined Gas Turbine-Vapor Power Plant

A combined gas turbine–vapor power plant has a net power output of 45 MW. Air enters the compressor of the gas turbine at 100 kPa, 300 K, and is compressed to 1200 kPa. The isentropic efficiency of the compressor is 84%. The condition at the inlet to the turbine is 1200 kPa, 1400 K. Air expands through the turbine, which has an isentropic efficiency of 88%, to a pressure of 100 kPa. The air then passes through the interconnecting heat-recovery steam generator and is finally discharged at 400 K. Steam enters the turbine of the vapor power cycle at 8 MPa, 400°C, and expands to the condenser pressure of 8 kPa. Water enters the pump as saturated liquid at 8 kPa. The turbine and pump of the vapor cycle have isentropic efficiencies of 90 and 80%, respectively.

(a) Determine the mass flow rates of the air and the steam, each in kg/s; the net power developed by the gas turbine and vapor power cycle, each in MW; and the thermal efficiency.

(b) Develop a full accounting of the *net* rate of exergy increase as the air passes through the gas turbine combustor. Discuss.

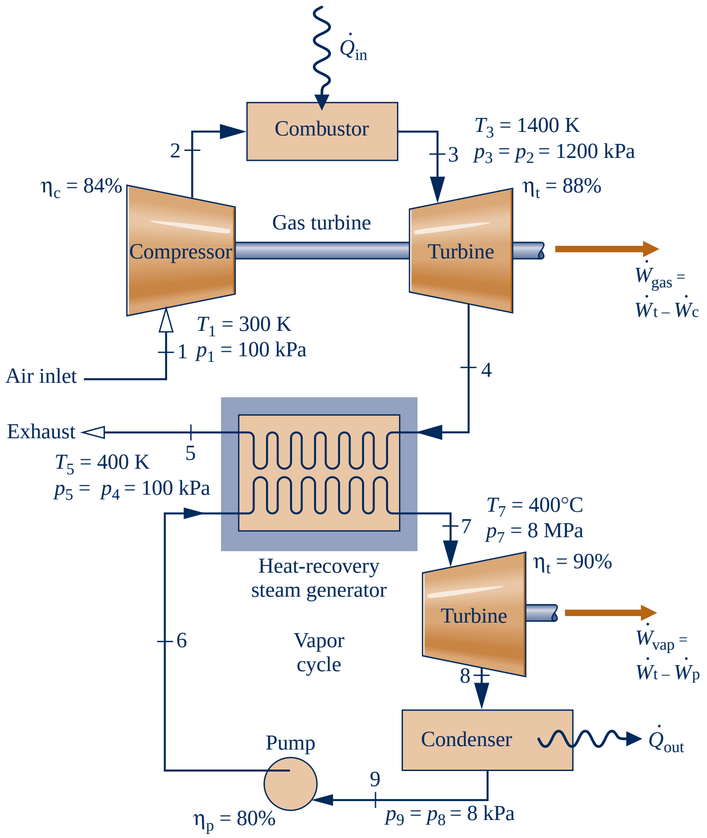
Let $T_0 = 300$ K, $p_0 = 100$ kPa.

SOLUTION

Known: A combined gas turbine–vapor power plant operates at steady state with a known net power output. Operating pressures and temperatures are specified. Turbine, compressor, and pump efficiencies are also given.

Find: Determine the mass flow rate of each working fluid, in kg/s; the net power developed by each cycle, in MW; and the thermal efficiency. Develop a full accounting of the exergy increase of the air passing through the gas turbine combustor and discuss the results.

Schematic and Given Data:



Engineering Model:

1. Each component on the accompanying sketch is analyzed as a control volume at steady state.
2. The turbines, compressor, pump, and interconnecting heat-recovery steam generator operate adiabatically.
3. Kinetic and potential energy effects are negligible.
4. There are no pressure drops for flow through the combustor, heat-recovery steam generator, and condenser.
5. An air-standard analysis is used for the gas turbine.
6. T_0 = 300 K, p_0 = 100 kPa.

Fig. E9.12

Analysis: The property data given in the table below are determined using procedures illustrated in previous solved examples of Chaps. 8 and 9. The details are left as an exercise.

Gas Turbine			Vapor Cycle		
State	h (kJ/kg)	$s°$ (kJ/kg · K)	State	h (kJ/kg)	s (kJ/kg · K)
1	300.19	1.7020	6	183.96	0.5975
2	669.79	2.5088	7	3138.30	6.3634
3	1515.42	3.3620	8	2104.74	6.7282
4	858.02	2.7620	9	173.88	0.5926
5	400.98	1.9919			

Energy Analysis

(a) To determine the mass flow rates of the vapor, $\dot{m}_v$, and the air, $\dot{m}_g$, begin by applying mass and energy rate balances to the interconnecting heat-recovery steam generator to obtain

$$0 = \dot{m}_g(h_4 - h_5) + \dot{m}_v(h_6 - h_7)$$

or

$$\frac{\dot{m}_v}{\dot{m}_g} = \frac{h_4 - h_5}{h_7 - h_6} = \frac{858.02 - 400.98}{3138.3 - 183.96} = 0.1547$$

Mass and energy rate balances applied to the gas turbine and vapor power cycles give the net power developed by each, respectively

$$\dot{W}_{\text{gas}} = \dot{m}_{\text{g}}[(h_3 - h_4) - (h_2 - h_1)]$$
$$\dot{W}_{\text{vap}} = \dot{m}_{\text{v}}[(h_7 - h_8) - (h_6 - h_9)]$$

With $\dot{W}_{\text{net}} = \dot{W}_{\text{gas}} + \dot{W}_{\text{vap}}$

$$\dot{W}_{\text{net}} = \dot{m}_{\text{g}}\left\{[(h_3 - h_4) - (h_2 - h_1)] + \frac{\dot{m}_{\text{v}}}{\dot{m}_{\text{g}}}[(h_7 - h_8) - (h_6 - h_9)]\right\}$$

Solving for $\dot{m}_{\text{g}}$, and inserting $\dot{W}_{\text{net}} = 45 \text{ MW} = 45{,}000 \text{ kJ/s}$ and $\dot{m}_{\text{v}}/\dot{m}_{\text{a}} = 0.1547$, we get

$$\dot{m}_{\text{g}} = \frac{45{,}000 \text{ kJ/s}}{\{[(1515.42 - 858.02) - (669.79 - 300.19)] + 0.1547[(3138.3 - 2104.74) - (183.96 - 173.88)]\} \text{ kJ/kg}}$$
$$= 100.87 \text{ kg/s}$$

and

$$\dot{m}_{\text{v}} = (0.1547)\dot{m}_{\text{g}} = 15.6 \text{ kg/s}$$

Using these mass flow rate values and specific enthalpies from the table above, the net power developed by the gas turbine and vapor power cycles, respectively, is

$$\dot{W}_{\text{gas}} = \left(100.87\frac{\text{kg}}{\text{s}}\right)\left(287.8\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 29.03 \text{ MW}$$

$$\dot{W}_{\text{vap}} = \left(15.6\frac{\text{kg}}{\text{s}}\right)\left(1023.5\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 15.97 \text{ MW}$$

The thermal efficiency is given by Eq. 9.28. The net power output is specified in the problem statement as 45 MW. Thus, only $\dot{Q}_{\text{in}}$ must be determined. Applying mass and energy rate balances to the combustor, we get

$$\dot{Q}_{\text{in}} = \dot{m}_{\text{g}}(h_3 - h_2)$$
$$= \left(100.87\frac{\text{kg}}{\text{s}}\right)(1515.42 - 669.79)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right|$$
$$= 85.3 \text{ MW}$$

1 Finally, thermal efficiency is

$$\eta = \frac{45 \text{ MW}}{85.3 \text{ MW}} = 0.528 \; (52.8\%)$$

Exergy Analysis

(b) The *net* rate of exergy increase of the air passing through the combustor is (Eq. 7.18)

$$\dot{\mathsf{E}}_{\text{f3}} - \dot{\mathsf{E}}_{\text{f2}} = \dot{m}_{\text{g}}[h_3 - h_2 - T_0(s_3 - s_2)]$$
$$= \dot{m}_g[h_3 - h_2 - T_0(s_3^\circ - s_2^\circ - R \ln p_3/p_2)]$$

With assumption 4, we have

$$\dot{\mathsf{E}}_{\text{f3}} - \dot{\mathsf{E}}_{\text{f2}} = \dot{m}_{\text{g}}\left[h_3 - h_2 - T_0\left(s_3^\circ - s_2^\circ - R \ln\cancelto{0}{\frac{p_3}{p_2}}\right)\right]$$
$$= \left(100.87\frac{\text{kJ}}{\text{s}}\right)\left[(1515.42 - 669.79)\frac{\text{kJ}}{\text{kg}} - 300 \text{ K}(3.3620 - 2.5088)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$
$$= 59{,}480\frac{\text{kJ}}{\text{s}}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 59.48 \text{ MW}$$

The *net* rate exergy is carried out of the plant by the exhaust air stream at 5 is

$$\dot{\mathsf{E}}_{f5} - \dot{\mathsf{E}}_{f1} = \dot{m}_g\left[h_5 - h_1 - T_0\left(s_5^\circ - s_1^\circ - R\ln\frac{\cancelto{0}{p_5}}{p_1}\right)\right]$$

$$= \left(100.87\,\frac{\text{kg}}{\text{s}}\right)[(400.98 - 300.19) - 300(1.9919 - 1.7020)]\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$

$$= 1.39\text{ MW}$$

The *net* rate exergy is carried out of the plant as water passes through the condenser is

$$\dot{\mathsf{E}}_{f8} - \dot{\mathsf{E}}_{f9} = \dot{m}_v[h_8 - h_9 - T_0(s_8 - s_9)]$$

$$= \left(15.6\,\frac{\text{kg}}{\text{s}}\right)\left[(2104.74 - 173.88)\frac{\text{kJ}}{\text{kg}} - 300\text{ K}(6.7282 - 0.5926)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$

$$= 1.41\text{ MW}$$

The rates of exergy destruction for the air turbine, compressor, steam turbine, pump, and heat-recovery steam generator are evaluated using $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, respectively, as follows:
Air turbine:

2

$$\dot{\mathsf{E}}_d = \dot{m}_g T_0(s_4 - s_3)$$

$$= \dot{m}_g T_0(s_4^\circ - s_3^\circ - R\ln p_4/p_3)$$

$$= \left(100.87\frac{\text{kg}}{\text{s}}\right)(300\text{ K})\left[(2.7620 - 3.3620)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{100}{1200}\right)\right]\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$

$$= 3.42\text{ MW}$$

Compressor:

$$\dot{\mathsf{E}}_d = \dot{m}_g T_0(s_2 - s_1)$$

$$= \dot{m}_g T_0(s_2^\circ - s_1^\circ - R\ln p_2/p_1)$$

$$= (100.87)(300)\left[(2.5088 - 1.7020) - \frac{8.314}{28.97}\ln\left(\frac{1200}{100}\right)\right]\left|\frac{1}{10^3}\right|$$

$$= 2.83\text{ MW}$$

Steam turbine:

$$\dot{\mathsf{E}}_d = \dot{m}_v T_0(s_8 - s_7)$$

$$= (15.6)(300)(6.7282 - 6.3634)\left|\frac{1}{10^3}\right|$$

$$= 1.71\text{ MW}$$

Pump:

$$\dot{\mathsf{E}}_d = \dot{m}_v T_0(s_6 - s_9)$$

$$= (15.6)(300)(0.5975 - 0.5926)\left|\frac{1}{10^3}\right|$$

$$= 0.02\text{ MW}$$

Heat-recovery steam generator:

$$\dot{\mathsf{E}}_d = T_0[\dot{m}_g(s_5 - s_4) + \dot{m}_v(s_7 - s_6)]$$

$$= (300\text{ K})\left[\left(100.87\frac{\text{kg}}{\text{s}}\right)(1.9919 - 2.7620)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} + \left(15.6\,\frac{\text{kg}}{\text{s}}\right)(6.3634 - 0.5975)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$

$$= 3.68\text{ MW}$$

3 The results are summarized by the following exergy rate *balance sheet* in terms of exergy magnitudes on a rate basis:

Net exergy increase of the gas passing through the combustor:	59.48 MW	100%	(70%)*
Disposition of the exergy:			
• Net power developed			
gas turbine cycle	29.03 MW	48.8%	(34.2%)
vapor cycle	15.97 MW	26.8%	(18.8%)
Subtotal	45.00 MW	75.6%	(53.0%)
• Net exergy lost			
with exhaust gas at state 5	1.39 MW	2.3%	(1.6%)
from water passing through condenser	1.41 MW	2.4%	(1.7%)
• Exergy destruction			
air turbine	3.42 MW	5.7%	(4.0%)
compressor	2.83 MW	4.8%	(3.4%)
steam turbine	1.71 MW	2.9%	(2.0%)
pump	0.02 MW	—	—
heat-recovery steam generator	3.68 MW	6.2%	(4.3%)

*Estimation based on fuel exergy. For discussion, see note 3.

The subtotals given in the table under the *net power developed* heading indicate that the combined cycle is effective in generating power from the exergy supplied. The table also indicates the relative significance of the exergy destructions in the turbines, compressor, pump, and heat-recovery steam generator, as well as the relative significance of the exergy losses. Finally, the table indicates that the total of the exergy destructions overshadows the losses. While the energy analysis of part (a) yields valuable results about combined-cycle performance, the exergy analysis of part (b) provides insights about the effects of irreversibilities and true magnitudes of the losses that cannot be obtained using just energy.

1 For comparison, note that the combined-cycle thermal efficiency in this case is much greater than those of the stand-alone regenerative vapor and gas cycles considered in Examples 8.5 and 9.11, respectively.

2 The development of the appropriate expressions for the rates of entropy production in the turbines, compressor, pump, and heat-recovery steam generator is left as an exercise.

3 In this exergy balance sheet, the percentages shown in parentheses are estimates based on the fuel exergy. Although combustion is the most significant source of irreversibility, the exergy destruction due to combustion cannot be evaluated using an air-standard analysis. Calculations of exergy destruction due to combustion (Chap. 13) reveal that approximately 30% of the exergy entering the combustor with the fuel would be destroyed, leaving about 70% of the fuel exergy for subsequent use. Accordingly, the value 59.48 MW for the net exergy increase of the air passing through the combustor is assumed to be 70% of the fuel exergy supplied. All other percentages in parentheses are obtained by multiplying the corresponding percentages, based on the exergy increase of the air passing through the combustor, by the factor 0.7. Since they account for combustion irreversibility, the table values in parentheses give the more accurate picture of combined cycle performance.

Skills Developed

Ability to...

- ❑ apply mass and energy balances.
- ❑ determine thermal efficiency.
- ❑ evaluate exergy quantities.
- ❑ develop an exergy accounting.

QuickQUIZ Determine the *net* rate energy is carried out of the plant as water passes through the condenser, in MW, and comment. Ans. 30.12 MW. The significance of this energy loss is *far less* than indicated by the answer. In terms of exergy, the loss at the condenser is 1.41 MW [see part (b)], which better measures the limited utility of the relatively low-temperature water flowing through the condenser.

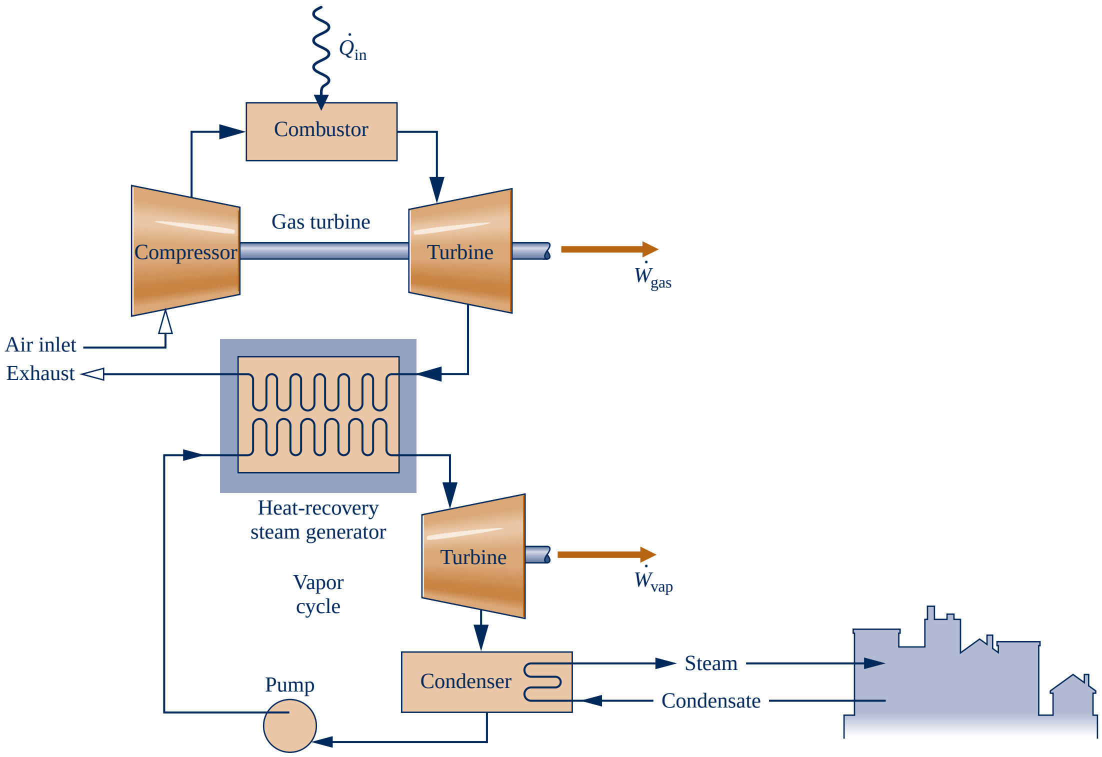


Fig. 9.23 Combined-cycle district heating plant.

9.9.2 Cogeneration

Cogeneration systems are integrated systems that yield two valuable products simultaneously from a single fuel input, electricity and steam (or hot water), achieving cost savings. Cogeneration systems have numerous industrial and commercial applications. District heating is one of these.

District heating plants are located within communities to provide steam or hot water for space heating and other needs together with electricity for domestic, commercial, and industrial use. Vapor cycle–based district heating plants are considered in Sec. 8.5.

Building on the combined gas turbine–vapor power cycle introduced in Sec. 9.9.1, Fig. 9.23 illustrates a district heating system consisting of a gas turbine cycle partnered with a vapor power cycle operating in the *back-pressure* mode discussed in Sec. 8.5.3. In this embodiment, steam (or hot water) is provided from the condenser to service the district heating load.

Referring again to Fig. 9.23, if the condenser is omitted, steam is supplied directly from the steam turbine to service the district heating load; condensate is returned to the heat-recovery steam generator. If the steam turbine is also omitted, steam passes directly from the heat-recovery unit to the community and back again; power is generated by the gas turbine alone.

9.10 Integrated Gasification Combined-Cycle Power Plants

For decades vapor power plants fueled by coal have been the workhorses of U.S. electricity generation (see Chap. 8). However, human-health and environmental-impact issues linked to coal combustion have placed this type of power generation under a cloud. In light of our large coal reserves and the critical importance of electricity to our society, major governmental and private-sector efforts are aimed at developing

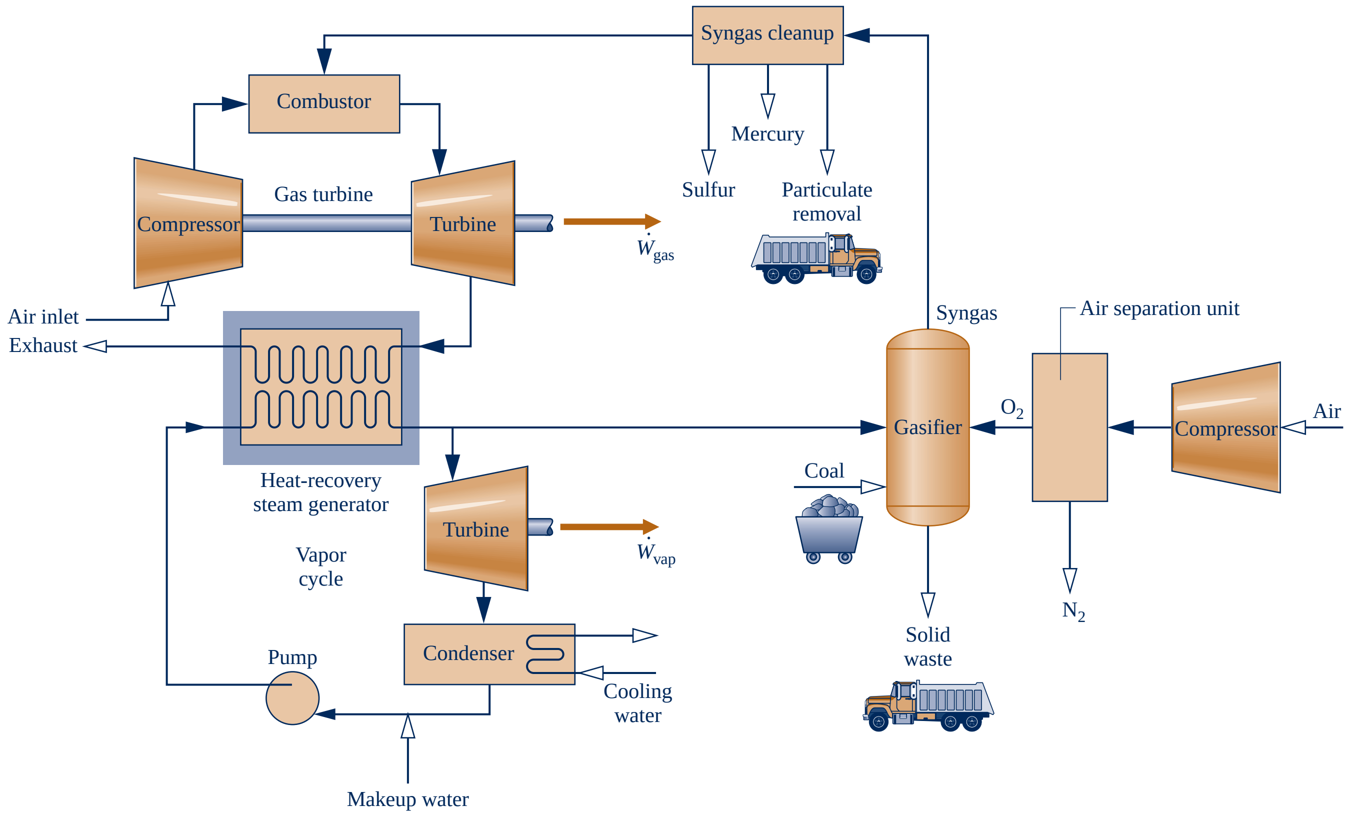


Fig. 9.24 Integrated gasification combined-cycle power plant.

alternative power generation technologies using coal, but with fewer adverse effects. In this section, we consider one such technology: integrated gasification combined-cycle (IGCC) power plants.

An IGCC power plant integrates a coal gasifier with a combined gas turbine–vapor power plant like that considered in Sec. 9.9. Key elements of an IGCC plant are shown in Fig. 9.24. Gasification is achieved through controlled combustion of coal with oxygen in the presence of steam to produce *syngas* (synthesis gas) and solid waste. Oxygen is provided to the gasifier by the companion air separation unit. Syngas exiting the gasifier is mainly composed of carbon monoxide and hydrogen. The syngas is cleaned of pollutants and then fired in the gas turbine combustor. The performance of the combined cycle follows the discussion provided in Sec. 9.9.

In IGCC plants, pollutants (sulfur compounds, mercury, and particulates) are removed *before* combustion when it is more effective to do so, rather than after combustion as in conventional coal-fueled power plants. While IGCC plants emit fewer sulfur dioxide, nitric oxide, mercury, and particulate emissions than comparable conventional coal plants, abundant solid waste is still produced that must be responsibly managed.

Taking a closer look at Fig. 9.24, better IGCC plant performance can be realized through tighter integration between the air-separation unit and combined cycle. For instance, by providing compressed air from the gas turbine compressor to the air-separation unit, the compressor feeding ambient air to the air-separation unit can be eliminated or reduced in size. Also, by injecting nitrogen produced by the separation unit into the air stream entering the combustor, mass flow rate through the turbine increases and therefore greater power is developed.

Only a few IGCC plants have been constructed worldwide thus far. Accordingly, only time will tell if this technology will make significant inroads against coal-fired vapor power plants, including the newest generation of supercritical plants. Proponents point to increased combined cycle thermal efficiency as a way to extend the viability of U.S. coal reserves. Others say investment might be better directed to technologies fostering

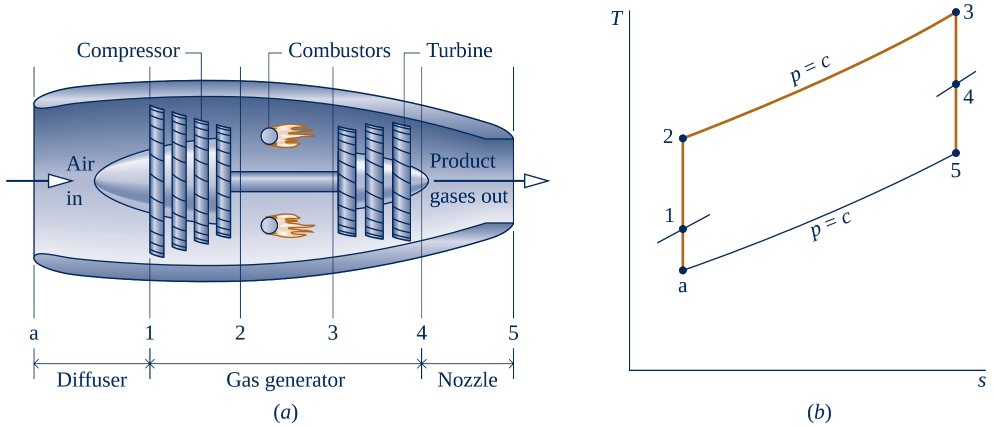


Fig. 9.25 **Turbojet engine schematic and accompanying ideal *T–s* diagram.**

use of renewable sources of energy for power generation than to technologies fostering use of coal, which has so many adverse effects related to its utilization.

9.11 Gas Turbines for Aircraft Propulsion

turbojet engine

Gas turbines are particularly suited for aircraft propulsion because of their favorable power-to-weight ratios. The **turbojet engine** is commonly used for this purpose. As illustrated in Fig. 9.25*a*, this type of engine consists of three main sections: the diffuser, the gas generator, and the nozzle.

ram effect

The diffuser placed before the compressor decelerates the incoming air relative to the engine. A pressure rise known as the **ram effect** is associated with this deceleration. The gas generator section consists of a compressor, combustor, and turbine, with the same functions as the corresponding components of a stationary gas turbine power plant. In a turbojet engine, the turbine power output need only be sufficient to drive the compressor and auxiliary equipment, however.

Combustion gases leave the turbine at a pressure significantly greater than atmospheric and expand through the nozzle to a high velocity before being discharged to the surroundings. The overall change in the velocity of the gases relative to the engine gives rise to the propulsive force, or thrust.

afterburner

Some turbojets are equipped with an **afterburner**, as shown in Fig. 9.26. This is essentially a reheat device in which additional fuel is injected into the gas exiting the turbine and burned, producing a higher temperature at the nozzle inlet than would be achieved otherwise. As a consequence, a greater nozzle exit velocity is attained, resulting in increased thrust.

TURBOJET ANALYSIS. The *T–s* diagram of the processes in an ideal turbojet engine is shown in Fig. 9.25*b*. In accordance with the assumptions of an air-standard analysis, the working fluid is air modeled as an ideal gas. The diffuser, compressor, turbine, and nozzle processes are isentropic, and the combustor operates at constant pressure.

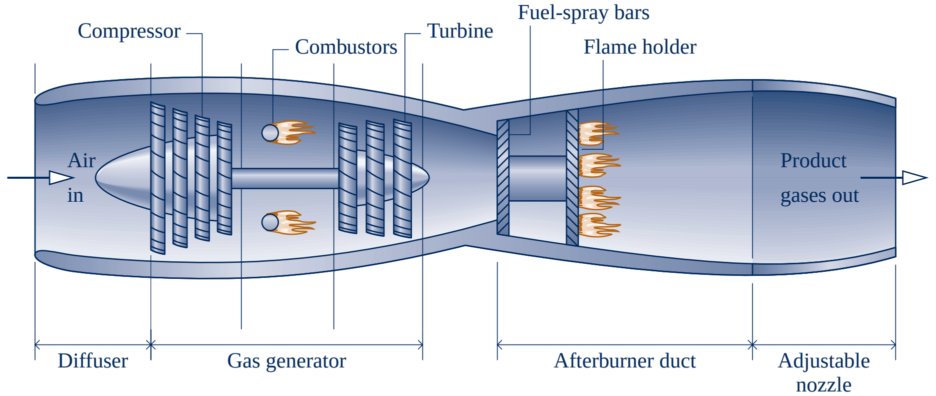


Fig. 9.26 **Schematic of a turbojet engine with afterburner.**

- Process a–1 shows the pressure rise that occurs in the diffuser as the air decelerates isentropically through this component.
- Process 1–2 is an isentropic compression.
- Process 2–3 is a constant-pressure heat addition.
- Process 3–4 is an isentropic expansion through the turbine during which work is developed.
- Process 4–5 is an isentropic expansion through the nozzle in which the air accelerates and the pressure decreases.

Owing to irreversibilities in an actual engine, there would be increases in specific entropy across the diffuser, compressor, turbine, and nozzle. In addition, there would be a combustion irreversibility and a pressure drop through the combustor of the actual engine. Further details regarding flow through nozzles and diffusers are provided in Secs. 9.13 and 9.14. The subject of combustion is discussed in Chap. 13.

In a typical thermodynamic analysis of a turbojet on an air-standard basis, the following quantities might be known: the velocity at the diffuser inlet, the compressor pressure ratio, and the turbine inlet temperature. The objective of the analysis might then be to determine the velocity at the nozzle exit. Once the nozzle exit velocity is known, the thrust can be determined by applying Newton's second law of motion in a form suitable for a control volume (Sec. 9.12). All principles required for the thermodynamic analysis of turbojet engines on an air-standard basis have been introduced. Example 9.13 provides an illustration.

EXAMPLE 9.13

Analyzing a Turbojet Engine

Air enters a turbojet engine at 11.8 lbf/in.^{2}, 430°R, and an inlet velocity of 620 miles/h (909.3 ft/s). The pressure ratio across the compressor is 8. The turbine inlet temperature is 2150°R and the pressure at the nozzle exit is 11.8 lbf/in.^{2} The work developed by the turbine equals the compressor work input. The diffuser, compressor, turbine, and nozzle processes are isentropic, and there is no pressure drop for flow through the combustor. For operation at steady state, determine the velocity at the nozzle exit and the pressure at each principal state. Neglect kinetic energy except at the inlet and exit of the engine, and neglect potential energy throughout.

SOLUTION

Known: An ideal turbojet engine operates at steady state. Key operating conditions are specified.

Find: Determine the velocity at the nozzle exit, in ft/s, and the pressure, in lbf/in.^{2}, at each principal state.

Schematic and Given Data:

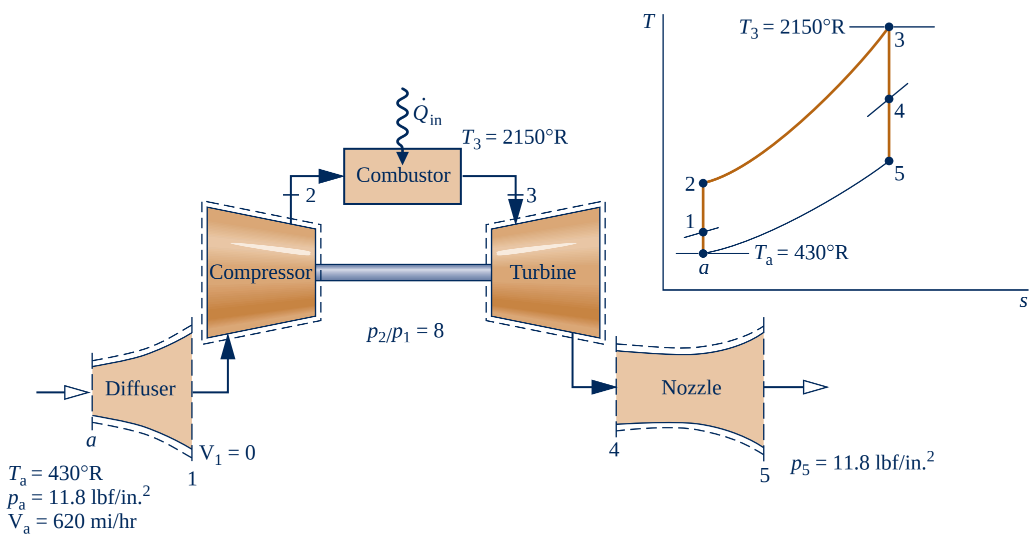


Fig. E9.13

Engineering Model:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The diffuser, compressor, turbine, and nozzle processes are isentropic.
3. There is no pressure drop for flow through the combustor.
4. The turbine work output equals the work required to drive the compressor.
5. Except at the inlet and exit of the engine, kinetic energy effects can be ignored. Potential energy effects are negligible throughout.
6. The working fluid is air modeled as an ideal gas.

Analysis: To determine the velocity at the exit to the nozzle, the mass and energy rate balances for a control volume enclosing this component reduce at steady state to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_4 - h_5) + \left(\frac{\cancelto{0}{V_4^2} - V_5^2}{2}\right) + g(\cancelto{0}{z_4 - z_5})\right]$$

where $\dot{m}$ is the mass flow rate. The inlet kinetic energy is dropped by assumption 5. Solving for V_5

$$V_5 = \sqrt{2(h_4 - h_5)}$$

This expression requires values for the specific enthalpies h_4 and h_5 at the nozzle inlet and exit, respectively. With the operating parameters specified, the determination of these enthalpy values is accomplished by analyzing each component in turn, beginning with the diffuser. The pressure at each principal state can be evaluated as a part of the analyses required to find the enthalpies h_4 and h_5.

Mass and energy rate balances for a control volume enclosing the diffuser reduce to give

$$h_1 = h_a + \frac{V_a^2}{2}$$

With h_a from Table A-22E and the given value of V_a

1

$$h_1 = 102.7 \text{ Btu/lb} + \left[\frac{(909.3)^2}{2}\right]\left(\frac{\text{ft}^2}{\text{s}^2}\right)\left|\frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right|$$
$$= 119.2 \text{ Btu/lb}$$

Interpolating in Table A-22E gives $p_{r1} = 1.051$. The flow through the diffuser is isentropic, so pressure p_1 is

$$p_1 = \frac{p_{r1}}{p_{ra}} p_a$$

With p_r data from Table A-22E and the known value of p_a

$$p_1 = \frac{1.051}{0.6268}(11.8 \text{ lbf/in.}^2) = 19.79 \text{ lbf/in.}^2$$

Using the given compressor pressure ratio, the pressure at state 2 is $p_2 = 8(19.79 \text{ lbf/in.}^2) = 158.3 \text{ lbf/in.}^2$

The flow through the compressor is also isentropic. Thus

$$p_{r2} = p_{r1}\frac{p_2}{p_1} = 1.051(8) = 8.408$$

Interpolating in Table A-22E, we get $h_2 = 216.2$ Btu/lb.

At state 3 the temperature is given as $T_3 = 2150°\text{R}$. From Table A-22E, $h_3 = 546.54$ Btu/lb. By assumption 3, $p_3 = p_2$. The work developed by the turbine is just sufficient to drive the compressor (assumption 4). That is

$$\frac{\dot{W}_t}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}}$$

or

$$h_3 - h_4 = h_2 - h_1$$

Solving for h_4

$$h_4 = h_3 + h_1 - h_2 = 546.54 + 119.2 - 216.2$$
$$= 449.5 \text{ Btu/lb}$$

Interpolating in Table A-22E with h_4 gives $p_{r4} = 113.8$.

The expansion through the turbine is isentropic, so

$$p_4 = p_3 \frac{p_{r4}}{p_{r3}}$$

With $p_3 = p_2$ and p_r data from Table A-22E

$$p_4 = (158.3 \text{ lbf/in.}^2)\frac{113.8}{233.5} = 77.2 \text{ lbf/in.}^2$$

The expansion through the nozzle is isentropic to $p_5 = 11.8$ lbf/in.^{2} Thus

$$p_{r5} = p_{r4}\frac{p_5}{p_4} = (113.8)\frac{11.8}{77.2} = 17.39$$

From Table A-22E, $h_5 = 265.8$ Btu/lb, which is the remaining specific enthalpy value required to determine the velocity at the nozzle exit.

Using the values for h_4 and h_5 determined above, the velocity at the nozzle exit is

2

$$\begin{aligned} V_5 &= \sqrt{2(h_4 - h_5)} \\ &= \sqrt{2(449.5 - 265.8)\frac{\text{Btu}}{\text{lb}}\left|\frac{32.2 \text{ lb} \cdot \text{ft/s}^2}{1 \text{ lbf}}\right|\left|\frac{778 \text{ ft} \cdot \text{lbf}}{1 \text{ Btu}}\right|} \\ &= 3034 \text{ ft/s } (2069 \text{ mi/h}) \end{aligned}$$

1 Note the unit conversions required here and in the calculation of V_5.

2 The increase in the velocity of the air as it passes through the engine gives rise to the thrust produced by the engine. A detailed analysis of the forces acting on the engine requires Newton's second law of motion in a form suitable for control volumes (see Sec. 9.12.1).

Skills Developed

Ability to...

- sketch the schematic of the turbojet engine and the T–s diagram for the corresponding air-standard cycle.
- evaluate temperatures and pressures at each principal state and retrieve necessary property data.
- apply mass, energy, and entropy principles.
- calculate the nozzle exit velocity.

QuickQUIZ Using Eq. 6.47, the isentropic nozzle efficiency, what is the nozzle exit velocity, in ft/s, if the efficiency is 90%? Ans. 2878 ft/s.

OTHER APPLICATIONS. Other related applications of the gas turbine include *turboprop* and *turbofan* engines. The turboprop engine shown in Fig. 9.27*a* consists of a gas turbine in which the gases are allowed to expand through the turbine to atmospheric pressure. The net power developed is directed to a propeller, which provides thrust to the aircraft. Turboprops are able to achieve speeds up to about 850 km/h (530 miles/h). In the turbofan shown in Fig. 9.27*b*, the core of the engine is much like a turbojet, and some thrust is obtained from expansion

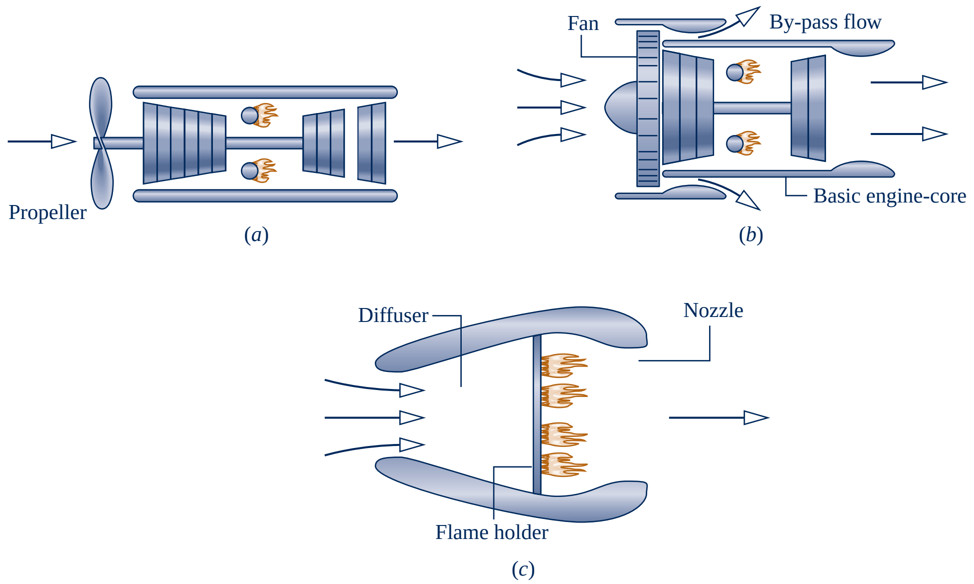


Fig. 9.27 **Other examples of aircraft engines. (*a*) Turboprop. (*b*) Turbofan. (*c*) Ramjet.**

through the nozzle. However, a set of large-diameter blades attached to the front of the engine accelerates air around the core. This *bypass flow* provides additional thrust for takeoff, whereas the core of the engine provides the primary thrust for cruising. Turbofan engines are commonly used for commercial aircraft with flight speeds of up to about 1000 km/h (600 miles/h). A particularly simple type of engine known as a ramjet is shown in Fig. 9.27*c*. This engine requires neither a compressor nor a turbine. A sufficient pressure rise is obtained by decelerating the high-speed incoming air in the diffuser (ram effect). For the ramjet to operate, therefore, the aircraft must already be in flight at high speed. The combustion products exiting the combustor are expanded through the nozzle to produce the thrust.

In each of the engines mentioned thus far, combustion of the fuel is supported by air brought into the engines from the atmosphere. For very high-altitude flight and space travel, where this is no longer possible, *rockets* may be employed. In these applications, both fuel and an oxidizer (such as liquid oxygen) are carried on board the craft. Thrust is developed when the high-pressure gases obtained on combustion are expanded through a nozzle and discharged from the rocket.

Considering Compressible Flow Through Nozzles and Diffusers

In many applications of engineering interest, gases move at relatively high velocities and exhibit appreciable changes in specific volume (density). The flows through the nozzles and diffusers of jet engines discussed in Sec. 9.11 are important examples. Other examples are the flows through wind tunnels, shock tubes, and steam ejectors. These flows are known as **compressible flows**. In this part of the chapter, we introduce some of the principles involved in analyzing compressible flows.

compressible flow

9.12 Compressible Flow Preliminaries

Concepts introduced in this section play important roles in the study of compressible flows. The momentum equation is introduced in a form applicable to the analysis of control volumes at steady state. The velocity of sound is also defined, and the concepts of Mach number and stagnation state are discussed.

9.12.1 Momentum Equation for Steady One-Dimensional Flow

The analysis of compressible flows requires the principles of conservation of mass and energy, the second law of thermodynamics, and relations among the thermodynamic properties of the flowing gas. In addition, Newton's second law of motion is required. Application of Newton's second law of motion to systems of fixed mass (closed systems) involves the familiar form

$$\mathbf{F} = m\mathbf{a}$$

where $\mathbf{F}$ is the resultant force acting *on* a system of mass m and $\mathbf{a}$ is the acceleration. The object of the present discussion is to introduce Newton's second law of motion in a form appropriate for the study of the control volumes considered in subsequent discussions.

Consider the control volume shown in Fig. 9.28, which has a single inlet, designated by 1, and a single exit, designated by 2. The flow is assumed to be one-dimensional at these locations. The energy and entropy rate equations for such a control volume have terms that account for energy and entropy transfers, respectively, at the inlets and exits. Momentum also can be carried into or out of the control volume at the inlets and exits, and such transfers can be accounted for as

$$\begin{bmatrix} \text{time rate of momentum} \\ \text{transfer into or} \\ \text{out of a control volume} \\ \text{accompanying mass flow} \end{bmatrix} = \dot{m}\mathbf{V} \tag{9.30}$$

In this expression, the momentum per unit of mass flowing across the boundary of the control volume is given by the velocity vector $\mathbf{V}$. In accordance with the one-dimensional flow model, the vector is normal to the inlet or exit and oriented in the direction of flow.

In words, Newton's second law of motion for control volumes is

$$\begin{bmatrix} \text{time rate of change} \\ \text{of momentum contained} \\ \text{within the control volume} \end{bmatrix} = \begin{bmatrix} \text{resultant force} \\ \text{acting } on \text{ the} \\ \text{control volume} \end{bmatrix} + \begin{bmatrix} \text{net rate at which momentum is} \\ \text{transferred into the control} \\ \text{volume accompanying mass flow} \end{bmatrix}$$

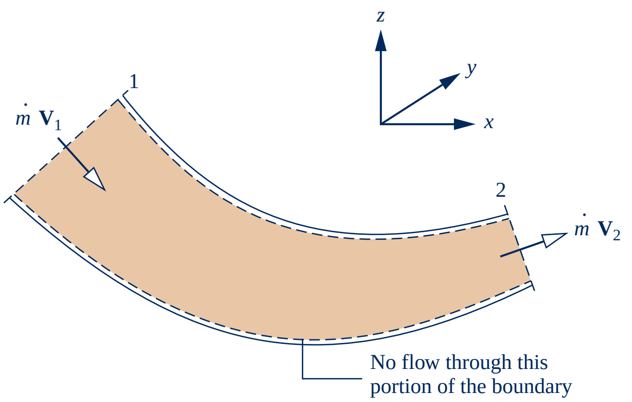


Fig. 9.28 **One-inlet, one-exit control volume at steady state labeled with momentum transfers accompanying mass flow.**

At steady state, the total amount of momentum contained in the control volume is constant with time. Accordingly, when applying Newton's second law of motion to control volumes at steady state, it is necessary to consider only the momentum accompanying the incoming and outgoing streams of matter and the forces acting on the control volume. Newton's law then states that the resultant force **F** acting *on* the control volume equals the difference between the rates of momentum exiting and entering the control volume accompanying mass flow. This is expressed by the following **steady-state form of the momentum equation**

steady-state momentum equation

$$\mathbf{F} = \dot{m}_2\mathbf{V}_2 - \dot{m}_1\mathbf{V}_1 = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1) \tag{9.31}$$

TAKE NOTE...
The resultant force **F** includes the forces due to pressure acting at the inlet and exit, forces acting on the portion of the boundary through which there is no mass flow, and the force of gravity.

Since $\dot{m}_1 = \dot{m}_2$ at steady state, the common mass flow is designated in this expression simply as $\dot{m}$. The expression of Newton's second law of motion given by Eq. 9.31 suffices for subsequent discussions. More general control volume formulations are normally provided in fluid mechanics texts.

9.12.2 Velocity of Sound and Mach Number

A sound wave is a small pressure disturbance that propagates through a gas, liquid, or solid at a velocity c that depends on the properties of the medium. In this section we obtain an expression that relates the *velocity of sound,* or sonic velocity, to other properties. The velocity of sound is an important property in the study of compressible flows.

MODELING PRESSURE WAVES. Let us begin by referring to Fig. 9.29*a*, which shows a pressure wave moving to the right with a velocity of magnitude c. The wave is generated by a small displacement of the piston. As shown on the figure, the pressure, density, and temperature in the region to the left of the wave depart from the respective values of the undisturbed fluid to the right of the wave, which are designated simply p, ρ, and T. After the wave has passed, the fluid to its left is in steady motion with a velocity of magnitude ΔV.

Figure 9.29*a* shows the wave from the point of view of a stationary observer. It is easier to analyze this situation from the point of view of an observer at rest relative to the wave, as shown in Fig. 9.29*b*. By adopting this viewpoint, a steady-state analysis can be applied to the control volume identified on the figure. To an observer at rest relative to the wave, it appears as though the fluid is moving toward the stationary wave from the right with velocity c, pressure p, density ρ, and temperature T and

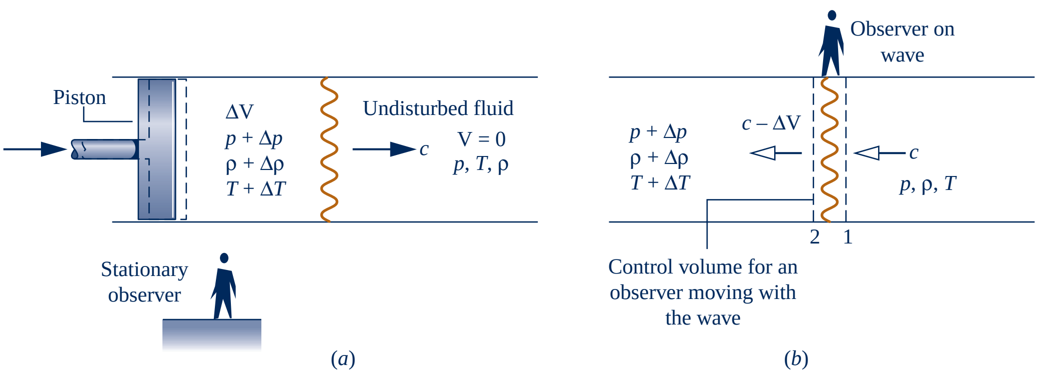


Fig. 9.29 **Illustrations used to analyze the propagation of a sound wave. (*a*) Propagation of a pressure wave through a quiescent fluid relative to a stationary observer. (*b*) Observer at rest relative to the wave.**

moving away on the left with velocity $c - \Delta V$, pressure $p + \Delta p$, density $\rho + \Delta\rho$, and temperature $T + \Delta T$.

At steady state, the conservation of mass principle for the control volume reduces to $\dot{m}_1 = \dot{m}_2$, or

$$\rho A c = (\rho + \Delta\rho)A(c - \Delta V)$$

On rearrangement

$$0 = c\Delta\rho - \rho\Delta V - \cancel{\Delta\rho\Delta V}^{0} \tag{9.32}$$

If the disturbance is *weak,* the third term on the right of Eq. 9.32 can be neglected, leaving

$$\Delta V = (c/\rho)\Delta\rho \tag{9.33}$$

Next, the momentum equation, Eq. 9.31, is applied to the control volume under consideration. Since the thickness of the wave is small, shear forces at the wall are negligible. The effect of gravity is also ignored. Hence, the only significant forces acting on the control volume in the direction of flow are the forces due to pressure at the inlet and exit. With these idealizations, the component of the momentum equation in the direction of flow reduces to

$$\begin{aligned} pA - (p + \Delta p)A &= \dot{m}(c - \Delta V) - \dot{m}c \\ &= \dot{m}(c - \Delta V - c) \\ &= (\rho A c)(-\Delta V) \end{aligned}$$

or

$$\Delta p = \rho c\, \Delta V \tag{9.34}$$

Combining Eqs. 9.33 and 9.34 and solving for c

$$c = \sqrt{\frac{\Delta p}{\Delta\rho}} \tag{9.35}$$

SOUND WAVES. For sound waves the differences in pressure, density, and temperature across the wave are quite small. In particular, $\Delta\rho << \rho$, justifying the neglect of the third term of Eq. 9.32. Furthermore, the ratio $\Delta p/\Delta\rho$ in Eq. 9.35 can be interpreted as the derivative of pressure with respect to density across the wave. Experiments also indicate that the relation between pressure and density across a sound wave is nearly *isentropic*. The expression for the **velocity of sound** then becomes

velocity of sound

$$\boxed{c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}} \tag{9.36a}$$

or in terms of specific volume

$$\boxed{c = \sqrt{-v^2\left(\frac{\partial p}{\partial v}\right)_s}} \tag{9.36b}$$

The velocity of sound is an intensive property whose value depends on the state of the medium through which sound propagates. Although we have assumed that sound propagates isentropically, the medium itself may be undergoing any process.

Means for evaluating the velocity of sound c for gases, liquids, and solids are introduced in Sec. 11.5. The special case of an ideal gas will be considered here because it is used extensively later in the chapter. For this case, the relationship between pressure and specific volume of an ideal gas at fixed entropy is pv^k = constant, where k is the specific heat ratio (Sec. 6.11.2). Thus, $(\partial p/\partial v)_s = -kp/v$, and Eq. 9.36b gives $c = \sqrt{kpv}$. Or, with the ideal gas equation of state

$$c = \sqrt{kRT} \qquad \text{(ideal gas)} \tag{9.37}$$

▶ **FOR EXAMPLE** to illustrate the use of Eq. 9.37, let us calculate the velocity of sound in air at 300 K (540°R) and 650 K (1170°R) From Table A-20 at 300 K, $k = 1.4$. Thus

$$c = \sqrt{1.4\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\text{ K})\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|} = 347\frac{\text{m}}{\text{s}}\left(1138\frac{\text{ft}}{\text{s}}\right)$$

At 650 K, $k = 1.37$, and $c = 506$ m/s (1660 ft/s), as can be verified. As examples in English units, consider next helium at 495°R (275 K) and 1080°R (600 K). For a monatomic gas, the specific heat ratio is essentially independent of temperature and has the value $k = 1.67$. Thus, at 495°R

$$c = \sqrt{1.67\left(\frac{1545}{4}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(495^\circ\text{R})\left|\frac{32.2\text{ lb}\cdot\text{ft/s}^2}{1\text{ lbf}}\right|} = 3206\frac{\text{ft}}{\text{s}}\left(977\frac{\text{m}}{\text{s}}\right)$$

At 1080°R, $c = 4736$ ft/s (1444 m/s), as can be verified. ◀ ◀ ◀ ◀ ◀

Mach Number

In subsequent discussions, the ratio of the velocity V at a state in a flowing fluid to the value of the sonic velocity c at the same state plays an important role. This ratio

Mach number

is called the **Mach number** M

$$M = \frac{\text{V}}{c} \tag{9.38}$$

supersonic
subsonic

When $M > 1$, the flow is said to be **supersonic**; when $M < 1$, the flow is **subsonic**; and when $M = 1$, the flow is *sonic*. The term *hypersonic* is used for flows with Mach numbers much greater than one, and the term *transonic* refers to flows where the Mach number is close to unity.

BIOCONNECTIONS For centuries, physicians have used sounds emanating from the human body to aid diagnosis. Since the early nineteenth century, stethoscopes have enabled more effective sound detection. Sounds within the human body have also interested researchers. For instance, in the 1600s an observer described a phenomenon that now mainly serves as a diversion: Place your thumbs over your ear openings so the ear canals are completely covered. Then, with elbows raised, tighten your hands into fists and hear muscular noise as a soft distant rumbling.

Today's medical researchers and practitioners use sound in various ways. Researchers have found that during strong and repeated activity, the wrist muscles of people with untreated Parkinson's disease produce sound at much lower frequencies than those of healthy individuals. This knowledge may prove useful in monitoring muscle degeneration in Parkinson's patients or aid in early diagnosis of this debilitating disease.

A more commonly encountered sound-related method in medical practice is the use of sound at frequencies above that audible by the human ear, known as *ultrasound*. *Ultrasound imaging* allows physicians to peer inside the body and evaluate solid structures in the abdominal cavity. Ultrasound devices beam sound waves into the body and collect return echoes as the beam encounters regions of differing density. The reflected sound waves produce shadow pictures of structures below the skin on a monitor screen. Pictures show the shape, size, and movement of target objects in the path of the beam.

Obstetricians commonly use ultrasound to assess the fetus during pregnancy. Emergency physicians use ultrasound to assess abdominal pain or other concerns. Ultrasound is also used to break up small kidney stones.

Cardiologists use an ultrasound application known as *echocardiography* to evaluate the heart and its valve function, measure the amount of blood pumped with each stroke, and detect blood clots in veins and artery blockage. Among several uses of echocardiography are the *stress* test, where the echocardiogram is done both before and after exercise, and the *transesophageal* test, where the probe is passed down the esophagus to locate it closer to the heart, thereby enabling clearer pictures of the heart.

9.12.3 Determining Stagnation State Properties

When dealing with compressible flows, it is often convenient to work with properties evaluated at a reference state known as the **stagnation state**. The stagnation state is the state a flowing fluid would attain if it were decelerated to zero velocity isentropically. We might imagine this as taking place in a diffuser operating at steady state. By reducing an energy balance for such a diffuser, it can be concluded that the enthalpy at the stagnation state associated with an actual state in the flow where the specific enthalpy is h and the velocity is V is given by

stagnation state

$$h_o = h + \frac{V^2}{2} \tag{9.39}$$

stagnation enthalpy

The enthalpy designated here as h_o is called the **stagnation enthalpy**. The pressure p_o and temperature T_o at a stagnation state are called the **stagnation pressure** and **stagnation temperature**, respectively.

stagnation pressure and temperature

9.13 Analyzing One-Dimensional Steady Flow in Nozzles and Diffusers

Although the subject of compressible flow arises in a great many important areas of engineering application, the remainder of this presentation is concerned only with flow through nozzles and diffusers. Texts dealing with compressible flow should be consulted for discussion of other areas of application.

In the present section we determine the shapes required by nozzles and diffusers for subsonic and supersonic flow. This is accomplished using mass, energy, entropy, and momentum principles, together with property relationships. In addition, we study how the flow through nozzles is affected as conditions at the nozzle exit are changed. The presentation concludes with an analysis of normal shocks, which can exist in supersonic flows.

9.13.1 Exploring the Effects of Area Change in Subsonic and Supersonic Flows

The objective of the present discussion is to establish criteria for determining whether a nozzle or diffuser should have a converging, diverging, or converging–diverging shape. This is accomplished using differential equations relating the principal variables that are obtained using mass and energy balances together with property relations, as considered next.

GOVERNING DIFFERENTIAL EQUATIONS. Let us begin by considering a control volume enclosing a nozzle or diffuser. At steady state, the mass flow rate is constant, so

$$\rho \mathrm{AV} = \text{constant}$$

In differential form

$$d(\rho \mathrm{AV}) = 0$$
$$\mathrm{AV}\, d\rho + \rho \mathrm{A}\, d\mathrm{V} + \rho \mathrm{V}\, d\mathrm{A} = 0$$

or on dividing each term by ρAV

$$\frac{d\rho}{\rho} + \frac{d\mathrm{V}}{\mathrm{V}} + \frac{d\mathrm{A}}{\mathrm{A}} = 0 \tag{9.40}$$

Assuming $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible potential energy effects, an energy rate balance reduces to give

$$h_2 + \frac{\mathrm{V}_2^2}{2} = h_1 + \frac{\mathrm{V}_1^2}{2}$$

Introducing Eq. 9.39, it follows that the stagnation enthalpies at states 1 and 2 are equal: $h_{o2} = h_{o1}$. Since any state downstream of the inlet can be regarded as state 2, the following relationship between the specific enthalpy and kinetic energy must be satisfied at each state

$$h + \frac{\mathrm{V}^2}{2} = h_{o1} \qquad \text{(constant)}$$

In differential form this becomes

$$dh = -\mathrm{V}\, d\mathrm{V} \tag{9.41}$$

This equation shows that if the velocity increases (decreases) in the direction of flow, the specific enthalpy must decrease (increase) in the direction of flow.

In addition to Eqs. 9.40 and 9.41 expressing conservation of mass and energy, relationships among properties must be taken into consideration. Assuming the flow occurs isentropically, the property relation (Eq. 6.10b)

$$T\, ds = dh - \frac{dp}{\rho}$$

reduces to give

$$dh = \frac{1}{\rho}\, dp \tag{9.42}$$

This equation shows that when pressure increases or decreases in the direction of flow, the specific enthalpy changes in the same way.

Forming the differential of the property relation $p = p(\rho, s)$

$$dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho ds$$

The second term vanishes in isentropic flow. Introducing Eq. 9.36a, we have

$$dp = c^2\, d\rho \tag{9.43}$$

which shows that when pressure increases or decreases in the direction of flow, density changes in the same way.

Additional conclusions can be drawn by combining the above differential equations. Combining Eqs. 9.41 and 9.42 results in

$$\frac{1}{\rho}\, dp = -\mathrm{V}\, d\mathrm{V} \tag{9.44}$$

which shows that if the velocity increases (decreases) in the direction of flow, the pressure must decrease (increase) in the direction of flow.

Eliminating dp between Eqs. 9.43 and 9.44 and combining the result with Eq. 9.40 gives

$$\frac{dA}{A} = -\frac{dV}{V}\left[1 - \left(\frac{V}{c}\right)^2\right]$$

or with the *Mach number M*

$$\frac{dA}{A} = -\frac{dV}{V}(1 - M^2) \tag{9.45}$$

VARIATION OF AREA WITH VELOCITY. Equation 9.45 shows how area must vary with velocity. The following four cases can be identified:

Case 1: Subsonic nozzle. $dV > 0, M < 1 \Rightarrow dA < 0$: The duct *converges* in the direction of flow.

Case 2: Supersonic nozzle. $dV > 0, M > 1 \Rightarrow dA > 0$: The duct *diverges* in the direction of flow.

Case 3: Supersonic diffuser. $dV < 0, M > 1 \Rightarrow dA < 0$: The duct *converges* in the direction of flow.

Case 4: Subsonic diffuser. $dV < 0, M < 1 \Rightarrow dA > 0$: The duct *diverges* in the direction of flow.

The conclusions reached above concerning the nature of the flow in subsonic and supersonic nozzles and diffusers are summarized in Fig. 9.30. From Fig. 9.30*a*, we see that to accelerate a fluid flowing subsonically, a converging nozzle must be used, but once $M = 1$ is achieved, further acceleration can occur only in a diverging nozzle. From Fig. 9.30*b*, we see that a converging diffuser is required to decelerate a fluid flowing supersonically, but once $M = 1$ is achieved, further deceleration can occur only in a diverging diffuser. These findings suggest that a Mach number of unity can occur only at the location in a nozzle or diffuser where the cross-sectional area is a minimum. This location of minimum area is called the **throat**. **throat**

The developments of this section have not required the specification of an equation of state; thus, the conclusions hold for all gases. Moreover, although the conclusions have been drawn under the restriction of isentropic flow through nozzles and diffusers, they are at least qualitatively valid for actual flows because the flow through well-designed nozzles and diffusers is nearly isentropic. Isentropic nozzle efficiencies (Sec. 6.12) in excess of 95% can be attained in practice.

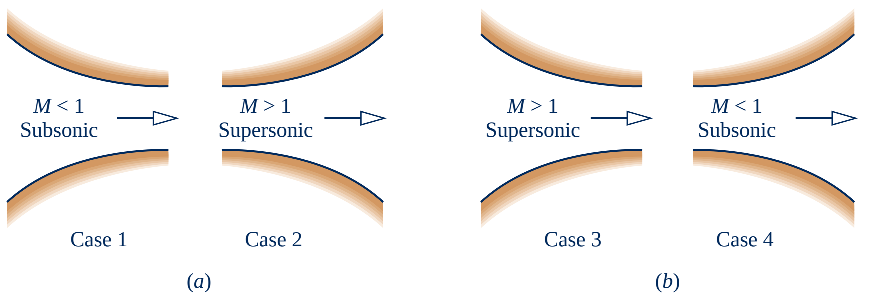


Fig. 9.30 **Effects of area change in subsonic and supersonic flows. (*a*) Nozzles: *V* increases; *h*, *p*, and *ρ* decrease. (*b*) Diffusers: *V* decreases; *h*, *p*, and *ρ* increase.**

9.13.2 Effects of Back Pressure on Mass Flow Rate

back pressure

In the present discussion we consider the effect of varying the *back pressure* on the rate of mass flow through nozzles. The **back pressure** is the pressure in the exhaust region outside the nozzle. The case of converging nozzles is taken up first and then converging–diverging nozzles are considered.

CONVERGING NOZZLES. Figure 9.31 shows a converging duct with stagnation conditions at the inlet, discharging into a region in which the back pressure p_B can be varied. For the series of cases labeled a through e, let us consider how the mass flow rate $\dot{m}$ and nozzle exit pressure p_E vary as the back pressure is decreased while keeping the inlet conditions fixed.

When $p_B = p_E = p_o$, there is no flow, so $\dot{m} = 0$. This corresponds to case a of Fig. 9.31. If the back pressure p_B is decreased, as in cases b and c, there will be flow through the nozzle. As long as the flow is subsonic at the exit, information about changing conditions in the exhaust region can be transmitted upstream. Decreases in back pressure thus result in greater mass flow rates and new pressure variations within the nozzle. In each instance, the velocity is subsonic throughout the nozzle and the exit pressure equals the back pressure. The exit Mach number increases as p_B decreases, however, and eventually a Mach number of unity will be attained at the nozzle exit. The corresponding pressure is denoted by p^*, called the *critical pressure*. This case is represented by d on Fig. 9.31.

Recalling that the Mach number cannot increase beyond unity in a converging section, let us consider next what happens when the back pressure is reduced further to a value less than p^*, such as represented by case e. Since the velocity at the exit equals the velocity of sound, information about changing conditions in the exhaust region no longer can be transmitted upstream past the exit plane. Accordingly, reductions in p_B below p^* have no effect on flow conditions in the nozzle. Neither the pressure variation within the nozzle nor the mass flow rate is affected. Under these

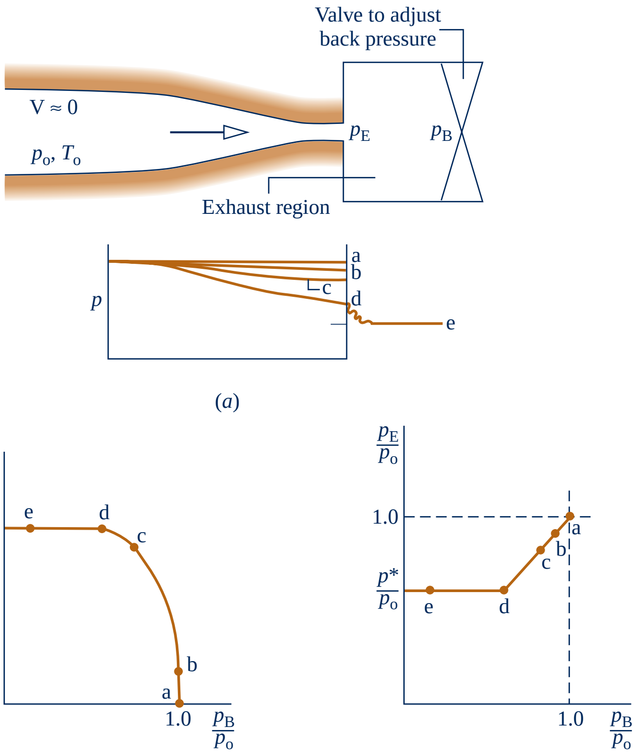


Fig. 9.31 Effect of back pressure on the operation of a converging nozzle.

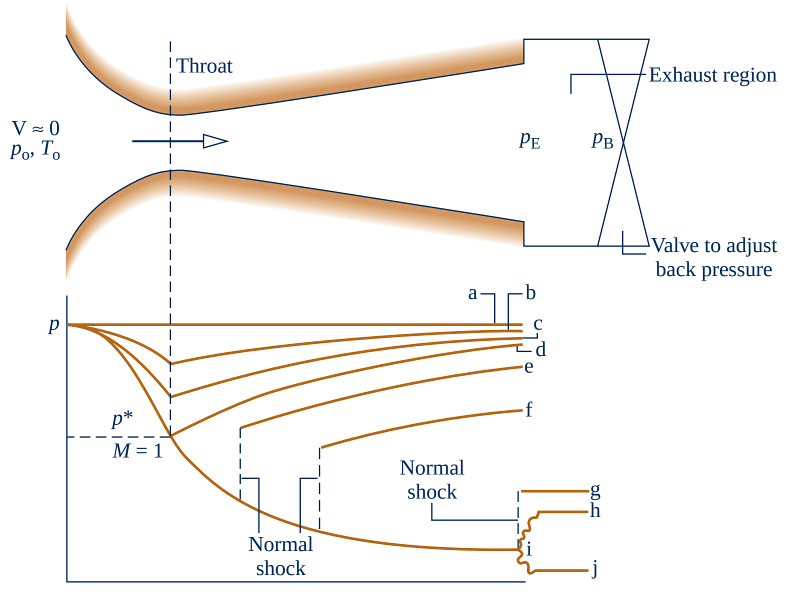


Fig. 9.32 **Effect of back pressure on the operation of a converging–diverging nozzle.**

conditions, the nozzle is said to be **choked**. When a nozzle is choked, the mass flow rate is the *maximum possible for the given stagnation conditions*. For p_B less than p^*, the flow expands outside the nozzle to match the lower back pressure, as shown by case e of Fig. 9.31. The pressure variation outside the nozzle cannot be predicted using the one-dimensional flow model.

choked flow: converging nozzle

CONVERGING–DIVERGING NOZZLES. Figure 9.32 illustrates the effects of varying back pressure on a *converging–diverging* nozzle. The series of cases labeled a through j is considered next.

- Let us first discuss the cases designated a, b, c, and d. Case a corresponds to $p_B = p_E = p_o$ for which there is no flow. When the back pressure is slightly less than p_o (case b), there is some flow, and the flow is subsonic throughout the nozzle. In accordance with the discussion of Fig. 9.30, the greatest velocity and lowest pressure occur at the throat, and the diverging portion acts as a diffuser in which pressure increases and velocity decreases in the direction of flow. If the back pressure is reduced further, corresponding to case c, the mass flow rate and velocity at the throat are greater than before. Still, the flow remains subsonic throughout and qualitatively the same as case b. As the back pressure is reduced, the Mach number at the throat increases, and eventually a Mach number of unity is attained there (case d). As before, the greatest velocity and lowest pressure occur at the throat, and the diverging portion remains a subsonic diffuser. However, because the throat velocity is sonic, the nozzle is now **choked**: The *maximum mass flow rate has been attained for the given stagnation conditions*. Further reductions in back pressure cannot result in an increase in the mass flow rate.

choked flow: converging–diverging nozzle

- When the back pressure is reduced below that corresponding to case d, the flow through the converging portion and at the throat remains unchanged. Conditions within the diverging portion can be altered, however, as illustrated by cases e, f, and g. In case e, the fluid passing the throat continues to expand and becomes supersonic in the diverging portion just downstream of the throat; but at a certain location an abrupt change in properties occurs. This is called a **normal shock**. Across the shock, there is a rapid and irreversible increase in pressure, accompanied by a rapid decrease from supersonic to subsonic flow. Downstream of the shock, the diverging duct acts as a subsonic diffuser in which the fluid continues to decelerate and the pressure

normal shock

increases to match the back pressure imposed at the exit. If the back pressure is reduced further (case f), the location of the shock moves downstream, but the flow remains qualitatively the same as in case e. With further reductions in back pressure, the shock location moves farther downstream of the throat until it stands at the exit (case g). In this case, the flow throughout the nozzle is isentropic, with subsonic flow in the converging portion, $M = 1$ at the throat, and supersonic flow in the diverging portion. Since the fluid leaving the nozzle passes through a shock, it is subsonic just downstream of the exit plane.

- Finally, let us consider cases h, i, and j where the back pressure is less than that corresponding to case g. In each of these cases, the flow through the nozzle is not affected. The adjustment to changing back pressure occurs outside the nozzle. In case h, the pressure decreases continuously as the fluid expands isentropically through the nozzle and then increases to the back pressure outside the nozzle. The compression that occurs outside the nozzle involves *oblique shock waves*. In case i, the fluid expands isentropically to the back pressure and no shocks occur within or outside the nozzle. In case j, the fluid expands isentropically through the nozzle and then expands outside the nozzle to the back pressure through *oblique expansion* waves. Once $M = 1$ is achieved at the throat, the mass flow rate is fixed at the maximum value for the given stagnation conditions, so the mass flow rate is the same for back pressures corresponding to cases d through j. The pressure variations outside the nozzle involving oblique waves cannot be predicted using the one-dimensional flow model.

9.13.3 Flow Across a Normal Shock

We have seen that under certain conditions a rapid and abrupt change of state called a shock takes place in the diverging portion of a supersonic nozzle. In a *normal* shock, this change of state occurs across a plane normal to the direction of flow. The object of the present discussion is to develop means for determining the change of state across a normal shock.

MODELING NORMAL SHOCKS. A control volume enclosing a normal shock is shown in Fig. 9.33. The control volume is assumed to be at steady state with $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$ and negligible effects of potential energy. The thickness of the shock is very small (on the order of 10^{-5} cm). Thus, there is no significant change in flow area across the shock, even though it may occur in a diverging passage, and the forces acting at the wall can be neglected relative to the pressure forces acting at the upstream and downstream locations denoted by x and y, respectively.

The upstream and downstream states are related by the following equations:

Mass:

$$\rho_x V_x = \rho_y V_y \tag{9.46}$$

Energy:

$$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} \tag{9.47a}$$

or

$$h_{ox} = h_{oy} \tag{9.47b}$$

Momentum:

$$p_x - p_y = \rho_y V_y^2 - \rho_x V_x^2 \tag{9.48}$$

Entropy:

$$s_y - s_x = \dot{\sigma}_{cv}/\dot{m} \tag{9.49}$$

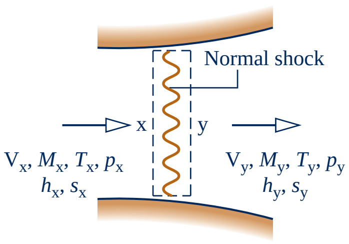


Fig. 9.33 Control volume enclosing a normal shock.

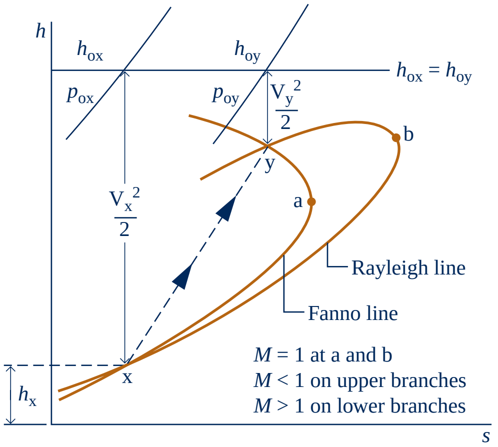


Fig. 9.34 Intersection of Fanno and Rayleigh lines as a solution to the normal shock equations.

When combined with property relations for the particular fluid under consideration, Eqs. 9.46, 9.47, and 9.48 allow the downstream conditions to be determined for specified upstream conditions. Equation 9.49, which corresponds to Eq. 6.39, leads to the important conclusion that the downstream state *must* have greater specific entropy than the upstream state, or $s_y > s_x$.

FANNO AND RAYLEIGH LINES. The mass and energy equations, Eqs. 9.46 and 9.47, can be combined with property relations for the particular fluid to give an equation that when plotted on an *h–s* diagram is called a **Fanno line**. Similarly, the mass and momentum equations, Eqs. 9.46 and 9.48, can be combined to give an equation that when plotted on an *h–s* diagram is called a **Rayleigh line**. Fanno and Rayleigh lines are sketched on *h–s* coordinates in Fig. 9.34. It can be shown that the point of maximum entropy on each line, points a and b, corresponds to $M = 1$. It also can be shown that the upper and lower branches of each line correspond, respectively, to subsonic and supersonic velocities.

Fanno line

Rayleigh line

The downstream state y must satisfy the mass, energy, and momentum equations simultaneously, so state y is fixed by the intersection of the Fanno and Rayleigh lines passing through state x. Since $s_y > s_x$, it can be concluded that the flow across the shock can only pass *from* x *to* y. Accordingly, the velocity changes from supersonic before the shock ($M_x > 1$) to subsonic after the shock ($M_y < 1$). This conclusion is consistent with the discussion of cases e, f, and g in Fig. 9.32. A significant increase in pressure across the shock accompanies the decrease in velocity. Figure 9.34 also locates the stagnation states corresponding to the states upstream and downstream of the shock. The stagnation enthalpy does not change across the shock, but there is a marked decrease in stagnation pressure associated with the irreversible process occurring in the normal shock region.

9.14 Flow in Nozzles and Diffusers of Ideal Gases with Constant Specific Heats

The discussion of flow in nozzles and diffusers presented in Sec. 9.13 requires no assumption regarding the equation of state, and therefore the results obtained hold generally. Attention is now restricted to ideal gases with constant specific heats. This case is appropriate for many practical problems involving flow through nozzles and diffusers. The assumption of constant specific heats also allows the derivation of relatively simple closed-form equations.

9.14.1 Isentropic Flow Functions

Let us begin by developing equations relating a state in a compressible flow to the corresponding stagnation state. For the case of an ideal gas with constant c_p, Eq. 9.39 becomes

$$T_o = T + \frac{V^2}{2c_p}$$

where T_o is the stagnation temperature. Using Eq. 3.47a, $c_p = kR/(k-1)$, together with Eqs. 9.37 and 9.38, the relation between the temperature T and the Mach number M of the flowing gas and the corresponding stagnation temperature T_o is

$$\frac{T_o}{T} = 1 + \frac{k-1}{2}M^2 \tag{9.50}$$

With Eq. 6.43, a relationship between the temperature T and pressure p of the flowing gas and the corresponding stagnation temperature T_o and the stagnation pressure p_o is

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{k/(k-1)}$$

Introducing Eq. 9.50 into this expression gives

$$\frac{p_o}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)} \tag{9.51}$$

Although sonic conditions may not actually be attained in a particular flow, it is convenient to have an expression relating the area A at a given section to the area A* that *would be* required for sonic flow ($M = 1$) at the same mass flow rate and stagnation state. These areas are related through

$$\rho AV = \rho^* A^* V^*$$

where ρ^* and V* are the density and velocity, respectively, when $M = 1$. Introducing the ideal gas equation of state, together with Eqs. 9.37 and 9.38, and solving for A/A*

$$\frac{A}{A^*} = \frac{1}{M}\frac{p^*}{p}\left(\frac{T}{T^*}\right)^{1/2} = \frac{1}{M}\frac{p^*/p_o}{p/p_o}\left(\frac{T/T_o}{T^*/T_o}\right)^{1/2}$$

where T^* and p^* are the temperature and pressure, respectively, when $M = 1$. Then with Eqs. 9.50 and 9.51

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}M^2\right)\right]^{(k+1)/2(k-1)} \tag{9.52}$$

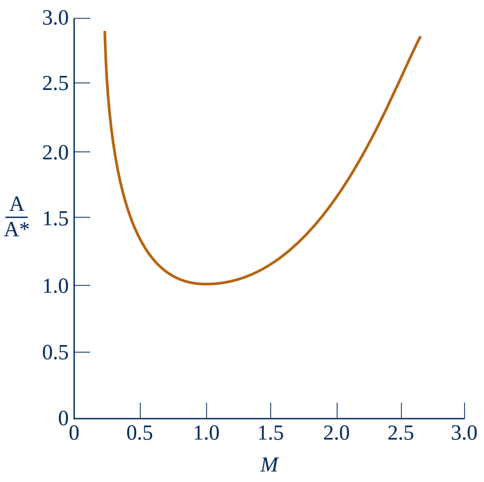


Fig. 9.35 Variation of A/A* with Mach number in isentropic flow for $k = 1.4$.

The variation of A/A* with M is given in Fig. 9.35 for $k = 1.4$. The figure shows that a unique value of A/A* corresponds to any choice of M. However, for a given value of A/A* other than unity, there are two possible values for the Mach number, one subsonic and one supersonic. This is consistent with the discussion of Fig. 9.30, where it was found that a converging–diverging passage with a section of minimum area is required to accelerate a flow from subsonic to supersonic velocity.

Equations 9.50, 9.51, and 9.52 allow the ratios T/T_o, p/p_o, and A/A* to be computed and tabulated with the Mach number as the single independent variable for a specified value of k. Table 9.2 provides a tabulation of this kind for $k = 1.4$. Such a table facilitates the analysis of flow through nozzles and diffusers. Equations 9.50, 9.51, and 9.52 also can be readily evaluated using calculators and computer software such as *Interactive Thermodynamics: IT*.

TABLE 9.2

Isentropic Flow Functions for an Ideal Gas with $k = 1.4$

M	T/T_o	p/p_o	A/A^*
0	1.000 00	1.000 00	∞
0.10	0.998 00	0.993 03	5.8218
0.20	0.992 06	0.972 50	2.9635
0.30	0.982 32	0.939 47	2.0351
0.40	0.968 99	0.895 62	1.5901
0.50	0.952 38	0.843 02	1.3398
0.60	0.932 84	0.784 00	1.1882
0.70	0.910 75	0.720 92	1.094 37
0.80	0.886 52	0.656 02	1.038 23
0.90	0.860 58	0.591 26	1.008 86
1.00	0.833 33	0.528 28	1.000 00
1.10	0.805 15	0.468 35	1.007 93
1.20	0.776 40	0.412 38	1.030 44
1.30	0.747 38	0.360 92	1.066 31
1.40	0.718 39	0.314 24	1.1149
1.50	0.689 65	0.272 40	1.1762
1.60	0.661 38	0.235 27	1.2502
1.70	0.633 72	0.202 59	1.3376
1.80	0.606 80	0.174 04	1.4390
1.90	0.580 72	0.149 24	1.5552
2.00	0.555 56	0.127 80	1.6875
2.10	0.531 35	0.109 35	1.8369
2.20	0.508 13	0.093 52	2.0050
2.30	0.485 91	0.079 97	2.1931
2.40	0.464 68	0.068 40	2.4031

In Example 9.14, we consider the effect of back pressure on flow in a converging nozzle. The *first step* of the analysis is to check whether the flow is choked.

EXAMPLE 9.14

Determining the Effect of Back Pressure: Converging Nozzle

A converging nozzle has an exit area of 0.001 m^2. Air enters the nozzle with negligible velocity at a pressure of 1.0 MPa and a temperature of 360 K. For isentropic flow of an ideal gas with $k = 1.4$, determine the mass flow rate, in kg/s, and the exit Mach number for back pressures of **(a)** 500 kPa and **(b)** 784 kPa.

SOLUTION

Known: Air flows isentropically from specified stagnation conditions through a converging nozzle with a known exit area.

Find: For back pressures of 500 and 784 kPa, determine the mass flow rate, in kg/s, and the exit Mach number.

Schematic and Given Data:

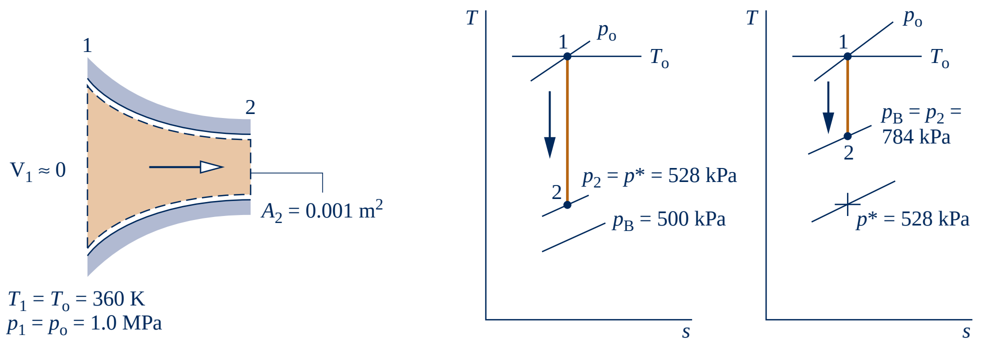


Fig. E9.14

Engineering Model:

1. The control volume shown in the accompanying sketch operates at steady state.

2. The air is modeled as an ideal gas with $k = 1.4$.

3. Flow through the nozzle is isentropic.

Analysis: The *first step* is to check whether the flow is choked. With $k = 1.4$ and $M = 1.0$, Eq. 9.51 gives
1 $p^*/p_o = 0.528$. Since $p_o = 1.0$ MPa, the critical pressure is $p^* = 528$ kPa. Thus, for back pressures of 528 kPa or less, the Mach number is unity at the exit and the nozzle is choked.

(a) From the above discussion, it follows that for a back pressure of 500 kPa, the nozzle is choked. At the exit, $M_2 = 1.0$ and the exit pressure equals the critical pressure, $p_2 = 528$ kPa. The mass flow rate is the maximum value that can be attained for the given stagnation properties. With the ideal gas equation of state, the mass flow rate is

$$\dot{m} = \rho_2 A_2 V_2 = \frac{p_2}{RT_2} A_2 V_2$$

The exit area A_2 required by this expression is specified as 10^{-3} m^2. Since $M = 1$ at the exit, the exit temperature T_2 can be found from Eq. 9.50, which on rearrangement gives

$$T_2 = \frac{T_o}{1 + \dfrac{k-1}{2}M^2} = \frac{360 \text{ K}}{1 + \left(\dfrac{1.4-1}{2}\right)(1)^2} = 300 \text{ K}$$

Then, with Eq. 9.37, the exit velocity V_2 is

$$\begin{aligned} V_2 &= \sqrt{kRT_2} \\ &= \sqrt{1.4\left(\frac{8314}{28.97}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300 \text{ K})\left|\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right|} = 347.2 \text{ m/s} \end{aligned}$$

Finally

$$\dot{m} = \frac{(528 \times 10^3 \text{ N/m}^2)(10^{-3} \text{ m}^2)(347.2 \text{ m/s})}{\left(\dfrac{8314}{28.97}\,\dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300 \text{ K})} = 2.13 \text{ Kg/s}$$

(b) Since the back pressure of 784 kPa is greater than the critical pressure determined above, the flow throughout the nozzle is subsonic and the exit pressure equals the back pressure, $p_2 = 784$ kPa. The exit Mach number can be found by solving Eq. 9.51 to obtain

$$M_2 = \left\{\frac{2}{k-1}\left[\left(\frac{p_o}{p_2}\right)^{(k-1)/k} - 1\right]\right\}^{1/2}$$

Inserting values

$$M_2 = \left\{\frac{2}{1.4-1}\left[\left(\frac{1 \times 10^6}{7.84 \times 10^5}\right)^{0.286} - 1\right]\right\}^{1/2} = 0.6$$

With the exit Mach number known, the exit temperature T_2 can be found from Eq. 9.50 as 336 K. The exit velocity is then

$$\begin{aligned} V_2 &= M_2 c_2 = M_2\sqrt{kRT_2} = 0.6\sqrt{1.4\left(\frac{8314}{28.97}\right)(336)} \\ &= 220.5 \text{ m/s} \end{aligned}$$

The mass flow rate is

$$\begin{aligned} \dot{m} &= \rho_2 A_2 V_2 = \frac{p_2}{RT_2} A_2 V_2 = \frac{(784 \times 10^3)(10^{-3})(220.5)}{(8314/28.97)(336)} \\ &= 1.79 \text{ kg/s} \end{aligned}$$

Skills Developed

Ability to...

- apply the ideal gas model with constant k in the analysis of isentropic flow through a converging nozzle.
- understand when choked flow occurs in a converging nozzle for different back pressures.
- determine conditions at the throat and the mass flow rate for different back pressures and a fixed stagnation state.

1 The use of Table 9.2 reduces some of the computation required in the solution. It is left as an exercise to develop a solution using this table. Also, observe that the first step of the analysis is to check whether the flow is choked.

QuickQUIZ Using the isentropic flow functions in Table 9.2, determine the exit temperature and Mach number for a back pressure of 843 kPa. Ans. 342.9 K, 0.5.

9.14.2 Normal Shock Functions

Next, let us develop closed-form equations for normal shocks for the case of an ideal gas with constant specific heats. For this case, it follows from the energy equation, Eq. 9.47b, that there is no change in stagnation temperature across the shock, $T_{ox} = T_{oy}$. Then, with Eq. 9.50, the following expression for the ratio of temperatures across the shock is obtained

$$\frac{T_y}{T_x} = \frac{1 + \frac{k-1}{2} M_x^2}{1 + \frac{k-1}{2} M_y^2} \tag{9.53}$$

Rearranging Eq. 9.48

$$p_x + \rho_x V_x^2 = p_y + \rho_y V_y^2$$

Introducing the ideal gas equation of state, together with Eqs. 9.37 and 9.38, the ratio of the pressure downstream of the shock to the pressure upstream is

$$\frac{p_y}{p_x} = \frac{1 + kM_x^2}{1 + kM_y^2} \tag{9.54}$$

Similarly, Eq. 9.46 becomes

$$\frac{p_y}{p_x} = \sqrt{\frac{T_y}{T_x}} \frac{M_x}{M_y}$$

The following equation relating the Mach numbers M_x and M_y across the shock can be obtained when Eqs. 9.53 and 9.54 are introduced in this expression

$$M_y^2 = \frac{M_x^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_x^2 - 1} \tag{9.55}$$

The ratio of stagnation pressures across a shock p_{oy}/p_{ox} is often useful. It is left as an exercise to show that

$$\frac{p_{oy}}{p_{ox}} = \frac{M_x}{M_y} \left(\frac{1 + \frac{k-1}{2} M_y^2}{1 + \frac{k-1}{2} M_x^2} \right)^{(k+1)/2(k-1)} \tag{9.56}$$

TABLE 9.3

Normal Shock Functions for an Ideal Gas with $k = 1.4$

M_x	M_y	p_y/p_x	T_y/T_x	p_{oy}/p_{ox}
1.00	1.000 00	1.0000	1.0000	1.000 00
1.10	0.911 77	1.2450	1.0649	0.998 92
1.20	0.842 17	1.5133	1.1280	0.992 80
1.30	0.785 96	1.8050	1.1909	0.979 35
1.40	0.739 71	2.1200	1.2547	0.958 19
1.50	0.701 09	2.4583	1.3202	0.929 78
1.60	0.668 44	2.8201	1.3880	0.895 20
1.70	0.640 55	3.2050	1.4583	0.855 73
1.80	0.616 50	3.6133	1.5316	0.812 68
1.90	0.595 62	4.0450	1.6079	0.767 35
2.00	0.577 35	4.5000	1.6875	0.720 88
2.10	0.561 28	4.9784	1.7704	0.674 22
2.20	0.547 06	5.4800	1.8569	0.628 12
2.30	0.534 41	6.0050	1.9468	0.583 31
2.40	0.523 12	6.5533	2.0403	0.540 15
2.50	0.512 99	7.1250	2.1375	0.499 02
2.60	0.503 87	7.7200	2.2383	0.460 12
2.70	0.495 63	8.3383	2.3429	0.423 59
2.80	0.488 17	8.9800	2.4512	0.389 46
2.90	0.481 38	9.6450	2.5632	0.357 73
3.00	0.475 19	10.333	2.6790	0.328 34
4.00	0.434 96	18.500	4.0469	0.138 76
5.00	0.415 23	29.000	5.8000	0.061 72
10.00	0.387 57	116.50	20.388	0.003 04
∞	0.377 96	∞	∞	0.0

Since there is no area change across a shock, Eqs. 9.52 and 9.56 combine to give

$$\frac{A_x^*}{A_y^*} = \frac{p_{oy}}{p_{ox}} \tag{9.57}$$

For specified values of M_x and specific heat ratio k, the Mach number downstream of a shock can be found from Eq. 9.55. Then, with M_x, M_y, and k known, the ratios T_y/T_x, p_y/p_x, and p_{oy}/p_{ox} can be determined from Eqs. 9.53, 9.54, and 9.56. Accordingly, tables can be set up giving M_y, T_y/T_x, p_y/p_x, and p_{oy}/p_{ox} versus the Mach number M_x as the single independent variable for a specified value of k. Table 9.3 is a tabulation of this kind for $k = 1.4$.

In the next example, we consider the effect of back pressure on flow in a converging–diverging nozzle. Key elements of the analysis include determining whether the flow is choked and if a normal shock exists.

EXAMPLE 9.15

Determining the Effect of Back Pressure: Converging–Diverging Nozzle

A converging–diverging nozzle operating at steady state has a throat area of 1.0 in.^{2} and an exit area of 2.4 in.^{2} Air enters the nozzle with a negligible velocity at a pressure of 100 lbf/in.^{2} and a temperature of 500°R. For air as an ideal gas with $k = 1.4$, determine the mass flow rate, in lb/s, the exit pressure, in lbf/in.^{2}, and exit Mach number for each of the five following cases. **(a)** Isentropic flow with $M = 0.7$ at the throat. **(b)** Isentropic flow with $M = 1$ at the throat and the diverging portion acting as a diffuser. **(c)** Isentropic flow with $M = 1$ at the throat and the diverging portion acting as a nozzle. **(d)** Isentropic flow through the nozzle with a normal shock standing at the exit. **(e)** A normal shock stands in the diverging section at a location where the area is 2.0 in.^{2} Elsewhere in the nozzle, the flow is isentropic.

SOLUTION

Known: Air flows from specified stagnation conditions through a converging–diverging nozzle having a known throat and exit area.

Find: The mass flow rate, exit pressure, and exit Mach number are to be determined for each of five cases.

Schematic and Given Data:

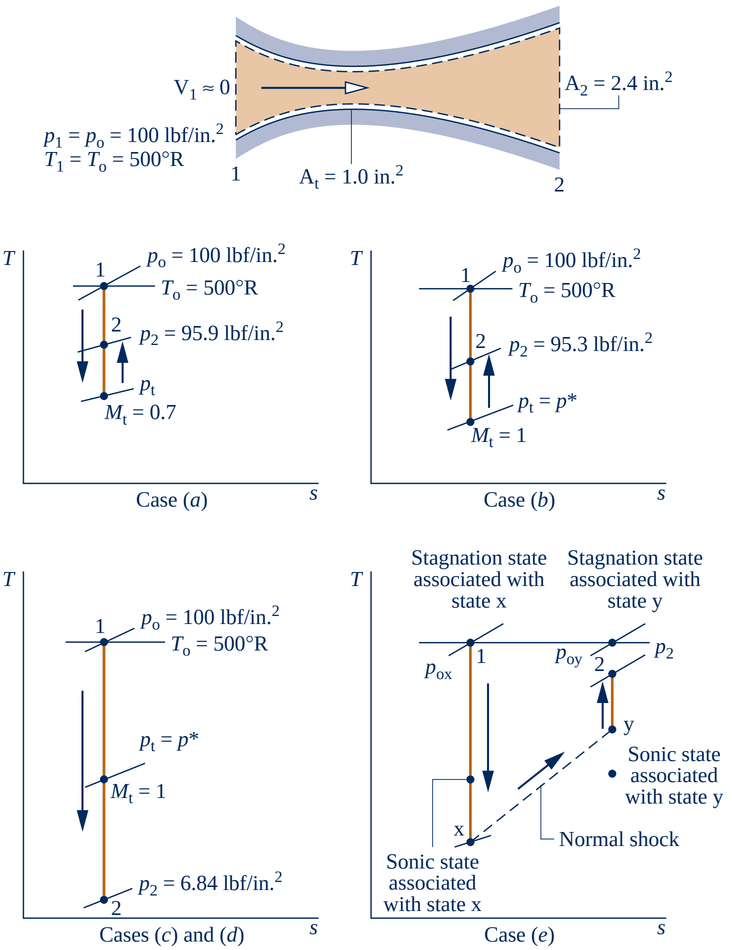


Fig. E9.15

Engineering Model:

1. The control volume shown in the accompanying sketch operates at steady state. The *T–s* diagrams provided locate states within the nozzle.
2. The air is modeled as an ideal gas with $k = 1.4$.
3. Flow through the nozzle is isentropic throughout, except for case (*e*), where a shock stands in the diverging section.

Analysis:

(a) The accompanying *T–s* diagram shows the states visited by the gas in this case. The following are known: the Mach number at the throat, $M_t = 0.7$, the throat area, $A_t = 1.0$ in.^{2}, and the exit area, $A_2 = 2.4$ in.^{2} The exit Mach number M_2, exit temperature T_2, and exit pressure p_2 can be determined using the identity

$$\frac{A_2}{A^*} = \frac{A_2}{A_t}\frac{A_t}{A^*}$$

With $M_t = 0.7$, Table 9.2 gives $A_t/A^* = 1.09437$. Thus

$$\frac{A_2}{A^*} = \left(\frac{2.4 \text{ in.}^2}{1.0 \text{ in.}^2}\right)(1.09437) = 2.6265$$

The flow throughout the nozzle, including the exit, is subsonic. Accordingly, with this value for A_2/A^*, Table 9.2 gives $M_2 \approx 0.24$. For $M_2 = 0.24$, $T_2/T_o = 0.988$, and $p_2/p_o = 0.959$. Since the stagnation temperature and pressure are 500°R and 100 lbf/in.^{2}, respectively, it follows that $T_2 = 494$°R and $p_2 = 95.9$ lbf/in.^{2}

The velocity at the exit is

$$\begin{aligned} V_2 &= M_2 c_2 = M_2\sqrt{kRT_2} \\ &= 0.24\sqrt{1.4\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(494^\circ\text{R})\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|} \\ &= 262\ \text{ft/s} \end{aligned}$$

The mass flow rate is

$$\begin{aligned} \dot{m} &= \rho_2 A_2 V_2 = \frac{p_2}{RT_2} A_2 V_2 \\ &= \frac{(95.9\ \text{lbf/in.}^2)(2.4\ \text{in.}^2)(262\ \text{ft/s})}{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(494^\circ\text{R})} = 2.29\ \text{lb/s} \end{aligned}$$

(b) The accompanying T–s diagram shows the states visited by the gas in this case. Since $M = 1$ at the throat, we have $A_t = A^*$, and thus $A_2/A^* = 2.4$. Table 9.2 gives two Mach numbers for this ratio: $M \approx 0.26$ and $M \approx 2.4$. The diverging portion acts as a diffuser in the present part of the example; accordingly, the subsonic value is appropriate. The supersonic value is appropriate in part (c).

Thus, from Table 9.2 we have at $M_2 = 0.26$, $T_2/T_o = 0.986$, and $p_2/p_o = 0.953$. Since $T_o = 500$°R and $p_o = 100$ lbf/in.^{2}, it follows that $T_2 = 493$°R and $p_2 = 95.3$ lbf/in.^{2}

The velocity at the exit is

$$\begin{aligned} V_2 &= M_2 c_2 = M_2\sqrt{kRT_2} \\ &= 0.26\sqrt{(1.4)\left(\frac{1545}{28.97}\right)(493)|32.2|} = 283\ \text{ft/s} \end{aligned}$$

The mass flow rate is

$$\dot{m} = \frac{p_2}{RT_2} A_2 V_2 = \frac{(95.3)(2.4)(283)}{\left(\frac{1545}{28.97}\right)(493)} = 2.46\ \text{lb/s}$$

This is the maximum mass flow rate for the specified geometry and stagnation conditions: the flow is choked.

(c) The accompanying T–s diagram shows the states visited by the gas in this case. As discussed in part (b), the exit Mach number in the present part of the example is $M_2 = 2.4$. Using this, Table 9.2 gives $p_2/p_0 = 0.0684$. With $p_o = 100$ lbf/in.^{2}, the pressure at the exit is $p_2 = 6.84$ lbf/in.^{2} Since the nozzle is choked, the mass flow rate is the same as found in part (b).

(d) Since a normal shock stands at the exit and the flow upstream of the shock is isentropic, the Mach number M_x and the pressure p_x correspond to the values found in part (c), $M_x = 2.4$, $p_x = 6.84$ lbf/in.^{2} Then, from Table 9.3, $M_y \approx 0.52$ and $p_y/p_x = 6.5533$. The pressure downstream of the shock is thus 44.82 lbf/in.^{2} This is the exit pressure. The mass flow is the same as found in part (b).

(e) The accompanying T–s diagram shows the states visited by the gas. It is known that a shock stands in the diverging portion where the area is $A_x = 2.0$ in.^{2} Since a shock occurs, the flow is sonic at the throat, so $A_x^* = A_t = 1.0$ in.^{2} The Mach number M_x can then be found from Table 9.2, by using $A_x/A_x^* = 2$, as $M_x = 2.2$.

The Mach number at the exit can be determined using the identity

$$\frac{A_2}{A_y^*} = \left(\frac{A_2}{A_x^*}\right)\left(\frac{A_x^*}{A_y^*}\right)$$

Introducing Eq. 9.57 to replace A_x^*/A_y^*, this becomes

$$\frac{A_2}{A_y^*} = \left(\frac{A_2}{A_x^*}\right)\left(\frac{p_{oy}}{p_{ox}}\right)$$

where p_{ox} and p_{oy} are the stagnation pressures before and after the shock, respectively. With $M_x = 2.2$, the ratio of stagnation pressures is obtained from Table 9.3 as $p_{oy}/p_{ox} = 0.62812$. Thus

$$\frac{A_2}{A_y^*} = \left(\frac{2.4 \text{ in.}^2}{1.0 \text{ in.}^2}\right)(0.62812) = 1.51$$

Using this ratio and noting that the flow is subsonic after the shock, Table 9.2 gives $M_2 \approx 0.43$, for which $p_2/p_{oy} = 0.88$.

The pressure at the exit can be determined using the identity

$$p_2 = \left(\frac{p_2}{p_{oy}}\right)\left(\frac{p_{oy}}{p_{ox}}\right)p_{ox} = (0.88)(0.628)\left(100 \frac{\text{lbf}}{\text{in.}^2}\right) = 55.3 \text{ lbf/in.}^2$$

Since the flow is choked, the mass flow rate is the same as that found in part (b).

1 With reference to cases labeled on Fig. 9.32, part (a) of the present example corresponds to case c on the figure, part (b) corresponds to case d, part (c) corresponds to case i, part (d) corresponds to case g, and part (e) corresponds to case f.

Skills Developed

Ability to...

- ❑ analyze isentropic flow through a converging–diverging nozzle for an ideal gas with constant *k*.
- ❑ understand the occurrence of choked flow and normal shocks in a converging–diverging nozzle for different back pressures.
- ❑ analyze the flow through a converging–diverging nozzle when normal shocks are present for an ideal gas with constant *k*.

QuickQUIZ What is the stagnation temperature, in °R, corresponding to the exit state for case (e)? **Ans.** 500°R.

▶ CHAPTER SUMMARY AND STUDY GUIDE

In this chapter, we have studied the thermodynamic modeling of internal combustion engines, gas turbine power plants, and compressible flow in nozzles and diffusers. The modeling of cycles is based on the use of air-standard analysis, where the working fluid is considered to be air as an ideal gas.

The processes in internal combustion engines are described in terms of three air-standard cycles: the Otto, Diesel, and dual cycles, which differ from each other only in the way the heat addition process is modeled. For these cycles, we have evaluated the principal work and heat transfers along with two important performance parameters: the mean effective pressure and the thermal efficiency. The effect of varying compression ratio on cycle performance is also investigated.

The performance of simple gas turbine power plants is described in terms of the air-standard Brayton cycle. For this cycle, we evaluate the principal work and heat transfers along with two important performance parameters: the back work ratio and the thermal efficiency. We also consider the effects on performance of irreversibilities and of varying compressor pressure ratio. Three modifications of the simple cycle to improve performance are introduced: regeneration, reheat, and compression with intercooling. Applications related to gas turbines are also considered, including combined gas turbine–vapor power cycles, integrated gasification combined-cycle (IGCC) power plants, and gas turbines for aircraft propulsion. In addition, the Ericsson and Stirling cycles are introduced.

The chapter concludes with the study of compressible flow through nozzles and diffusers. We begin by introducing the momentum equation for steady, one-dimensional flow, the velocity of sound, and the stagnation state. We then consider the effects of area change and back pressure on performance in both subsonic and supersonic flows. Choked flow and the presence of normal shocks in such flows are investigated. Tables are introduced to facilitate analysis for the case of ideal gases with constant specific heat ratio, $k = 1.4$.

The following list provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed, you should be able to

- ▶ write out the meanings of the terms listed in the margin throughout the chapter and understand each of the related concepts. The subset of key concepts listed on p. 570 is particularly important.
- ▶ sketch p–v and T–s diagrams of the Otto, Diesel, and dual cycles. Apply the closed system energy balance and the second law along with property data to determine the performance of these cycles, including mean effective pressure, thermal efficiency, and the effects of varying compression ratio.

- ▶ sketch schematic diagrams and accompanying T–s diagrams of the Brayton cycle and modifications involving regeneration, reheat, and compression with intercooling. In each case, be able to apply mass and energy balances, the second law, and property data to determine gas turbine power cycle performance, including thermal efficiency, back work ratio, net power output, and the effects of varying compressor pressure ratio.
- ▶ analyze the performance of gas turbine–related applications involving combined gas turbine–vapor power plants, IGCC power plants, and aircraft propulsion. You also should be able to apply the principles of this chapter to Ericsson and Stirling cycles.
- ▶ discuss for nozzles and diffusers the effects of area change in subsonic and supersonic flows, the effects of back pressure on mass flow rate, and the appearance and consequences of choking and normal shocks.
- ▶ analyze the flow in nozzles and diffusers of ideal gases with constant specific heats, as in Examples 9.14 and 9.15.

▶ KEY ENGINEERING CONCEPTS

▶ KEY EQUATIONS

$\text{mep} = \dfrac{\text{net work for one cycle}}{\text{displacement volume}}$	**(9.1) p. 495**	**Mean effective pressure for reciprocating piston engines**
	Otto Cycle	
$\eta = \dfrac{(u_3 - u_2) - (u_4 - u_1)}{u_3 - u_2} = 1 - \dfrac{u_4 - u_1}{u_3 - u_2}$	**(9.3) p. 498**	**Thermal efficiency (Figure 9.3)**
$\eta = 1 - \dfrac{1}{r^{k-1}}$	**(9.8) p. 499**	**Thermal efficiency (cold-air standard basis)**
	Diesel Cycle	
$\eta = \dfrac{W_{\text{cycle}}/m}{Q_{23}/m} = 1 - \dfrac{Q_{41}/m}{Q_{23}/m} = 1 - \dfrac{u_4 - u_1}{h_3 - h_2}$	**(9.11) p. 503**	**Thermal efficiency (Figure 9.5)**
$\eta = 1 - \dfrac{1}{r^{k-1}}\left[\dfrac{r_c^k - 1}{k(r_c - 1)}\right]$	**(9.13) p. 503**	**Thermal efficiency (cold-air standard basis)**
	Brayton Cycle	
$\eta = \dfrac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \dfrac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$	**(9.19) p. 512**	**Thermal efficiency (Figure 9.9)**
$\text{bwr} = \dfrac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \dfrac{h_2 - h_1}{h_3 - h_4}$	**(9.20) p. 512**	**Back work ratio (Figure 9.9)**
$\eta = 1 - \dfrac{1}{(p_2/p_1)^{(k-1)/k}}$	**(9.25) p. 516**	**Thermal efficiency (cold-air standard basis)**
$\eta_{\text{reg}} = \dfrac{h_x - h_2}{h_4 - h_2}$	**(9.27) p. 523**	**Regenerator effectiveness for the regenerative gas turbine cycle (Figure 9.14)**

Compressible Flow in Nozzles and Diffusers

$\mathbf{F} = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1)$	(9.31) p. 552	Momentum equation for steady-state one-dimensional flow
$c = \sqrt{kRT}$	(9.37) p. 554	Ideal gas velocity of sound
$M = \mathrm{V}/c$	(9.38) p. 554	Mach number
$h_o = h + \mathrm{V}^2/2$	(9.39) p. 555	Stagnation enthalpy
$\dfrac{T_o}{T} = 1 + \dfrac{k-1}{2}M^2$	(9.50) p. 562	Isentropic flow function relating temperature and stagnation temperature (constant k)
$\dfrac{p_o}{p} = \left(\dfrac{T_o}{T}\right)^{k/(k-1)} = \left(1 + \dfrac{k-1}{2}M^2\right)^{k/(k-1)}$	(9.51) p. 562	Isentropic flow function relating pressure and stagnation pressure (constant k)

▶ EXERCISES: THINGS ENGINEERS THINK ABOUT

1. Diesel engines are said to produce higher *torque* than gasoline engines. What does that mean?

2. Formula One race cars have 2.4 liter engines. What does that signify? How is your car's engine sized in liters?

3. The ideal Brayton and Rankine cycles are composed of the same four processes, yet look different when represented on a T–s diagram. Explain.

4. The term *regeneration* is used to describe the use of regenerative feedwater heaters in vapor power plants and regenerative heat exchangers in gas turbines. In what ways are the purposes of these devices similar? How do they differ?

5. You jump off a raft into the water in the middle of a lake. What direction does the raft move? Explain.

6. What is the purpose of a *rear diffuser* on a race car?

7. What is the meaning of the *octane rating* that you see posted on gas pumps? Why is it important to consumers?

8. Why aren't jet engines of airliners fitted with screens to avoid birds being pulled into the intake?

9. When did the main power plant providing electricity to your residence begin generating power? How long is it expected to continue operating?

10. What is the purpose of the gas turbine–powered *auxiliary power units* commonly seen at airports near commercial aircraft?

11. A nine-year-old camper is suddenly awakened by a metallic *click* coming from the direction of a railroad track passing close to her camping area; soon afterward, she hears the deep growling of a diesel locomotive pulling an approaching train. How would you interpret these different sounds to her?

12. Automakers have developed prototype gas turbine–powered vehicles, but the vehicles have not been generally marketed to consumers. Why?

13. In making a quick stop at a friend's home, is it better to let your car's engine idle or turn it off and restart when you leave?

14. How do today's more effective diesel engine exhaust treatment systems work?

15. What is the range of fuel efficiencies, in miles per gallon, you get with you car? At what speeds, in miles per hour, is the peak achieved?

16. Where is *Marcellus* shale and why is it significant?

17. Does your state regulate the practice of venting high-pressure natural gas to clean debris from pipelines leading to power plant gas turbines? What hazards are associated with this practice?

▶ PROBLEMS: DEVELOPING ENGINEERING SKILLS

Otto, Diesel, and Dual Cycles

9.1 An air-standard Otto cycle has a compression ratio of 9. At the beginning of compression, $p_1 = 100$ kPa and $T_1 = 300$ K. The heat addition per unit mass of air is 1350 kJ/kg. Determine

(a) the net work, in kJ per kg of air.
(b) the thermal efficiency of the cycle.
(c) the mean effective pressure, in kPa.
(d) the maximum temperature in the cycle, in K.
(e) To investigate the effects of varying compression ratio, plot each of the quantities calculated in parts (a) through (d) for compression ratios ranging from 1 to 12.

9.2 Solve Problem 9.1 on a cold air-standard basis with specific heats evaluated at 300 K.

9.3 At the beginning of the compression process of an air-standard Otto cycle, $p_1 = 1$ bar, $T_1 = 290$ K, $V_1 = 400$ cm^3.

The maximum temperature in the cycle is 2200 K and the compression ratio is 8. Determine

(a) the heat addition, in kJ.
(b) the net work, in kJ.
(c) the thermal efficiency.
(d) the mean effective pressure, in bar.
(e) Develop a full accounting of the exergy transferred to the air during the heat addition, in kJ.
(f) Devise and evaluate an exergetic efficiency for the cycle.

Let $T_0 = 290$ K, $p_0 = 1$ bar.

9.4 Plot each of the quantities specified in parts (a) through (d) of Problem 9.3 versus the compression ratio ranging from 2 to 12.

9.5 Solve Problem 9.3 on a cold air-standard basis with specific heats evaluated at 300 K.

9.6 A four-cylinder, four-stroke internal combustion engine operates at 2800 RPM. The processes within each cylinder are modeled as an air-standard Otto cycle with a pressure of 14.7 lbf/in.^{2}, a temperature of 80°F, and a volume of 0.0196 ft^3 at the beginning of compression. The compression ratio is 10, and maximum pressure in the cycle is 1080 lbf/in.^{2} Determine, using a cold air-standard analysis with $k = 1.4$, the power developed by the engine, in horsepower, and the mean effective pressure, in lbf/in.^{2}

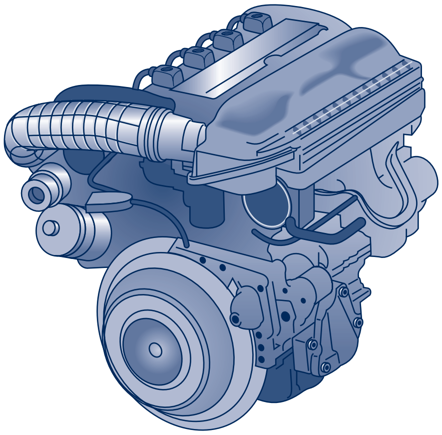


Fig. P9.6

9.7 An air-standard Otto cycle has a compression ratio of 8 and the temperature and pressure at the beginning of the compression process are 520°R and 14.2 lbf/in.^{2}, respectively. The mass of air is 0.0015 lb. The heat addition is 0.9 Btu. Determine

(a) the maximum temperature, in °R.
(b) the maximum pressure, in lbf/in.^{2}
(c) the thermal efficiency.
(d) To investigate the effects of varying compression ratio, plot each of the quantities calculated in parts (a) through (c) for compression ratios ranging from 2 to 12.

9.8 Solve Problem 9.7 on a cold air-standard basis with specific heats evaluated at 520°R.

9.9 At the beginning of the compression process in an air-standard Otto cycle, $p_1 = 14.7$ lbf/in.^{2} and $T_1 = 530$°R. Plot the thermal efficiency and mean effective pressure, in lbf/in.^{2}, for maximum cycle temperatures ranging from 2000 to 5000°R and compression ratios of 6, 8, and 10.

9.10 Solve Problem 9.9 on a cold air-standard basis using $k = 1.4$.

9.11 Consider an air-standard Otto cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.3. The mass of air is 0.002 kg. Determine

(a) the heat addition and the heat rejection, each in kJ.
(b) the net work, in kJ.
(c) the thermal efficiency.
(d) the mean effective pressure, in kPa.

State	*T* (K)	*p* (kPa)	*u* (kJ/kg)
1	305	85	217.67
2	367.4	767.9	486.77
3	960	2006	725.02
4	458.7	127.8	329.01

9.12 Consider a cold air-standard Otto cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.3. The heat rejection from the cycle is 86 Btu per lb of air. Assuming c_v = 0.172 Btu/lb · °R, determine

(a) the compression ratio.
(b) the net work per unit mass of air, in Btu/lb.
(c) the thermal efficiency.
(d) the mean effective pressure, in lbf/in.^{2}

State	*T* (°R)	*p* (lbf/in.^{2})
1	500	47.50
2	1204.1	1030
3	2408.2	2060
4	1000	95

9.13 Consider a modification of the air-standard Otto cycle in which the isentropic compression and expansion processes are each replaced with polytropic processes having $n = 1.3$. The compression ratio is 9 for the modified cycle. At the beginning of compression, $p_1 = 1$ bar and $T_1 = 300$ K and $V_1 = 2270$ cm^3. The maximum temperature during the cycle is 2000 K. Determine

(a) the heat transfer and work in kJ, for each process in the modified cycle.
(b) the thermal efficiency.
(c) the mean effective pressure, in bar.

9.14 A four-cylinder, four-stroke internal combustion engine has a bore of 2.55 in. and a stroke of 2.10 in. The clearance volume is 12% of the cylinder volume at bottom dead center and the crankshaft rotates at 3600 RPM. The processes within each cylinder are modeled as an air-standard Otto cycle with a pressure of 14.6 lbf/in.^{2} and a temperature of 100°F at the beginning of compression. The maximum temperature in the cycle is 5200°R. Based on this model, calculate the net work per cycle, in Btu, and the power developed by the engine, in horsepower.

9.15 At the beginning of the compression process in an air-standard Otto cycle, $p_1 = 1$ bar and $T_1 = 300$ K. The maximum cycle temperature is 2000 K. Plot the net work per unit of mass, in kJ/kg, the thermal efficiency, and the mean effective pressure, in bar, versus the compression ratio ranging from 2 to 14.

9.16 Investigate the effect of maximum cycle temperature on the net work per unit mass of air for air-standard Otto cycles with compression ratios of 5, 8, and 11. At the beginning of the compression process, $p_1 = 1$ bar and $T_1 = 295$ K. Let the maximum temperature in each case vary from 1000 to 2200 K.

9.17 The pressure-specific volume diagram of the air-standard *Lenoir* cycle is shown in Fig. P9.17. The cycle consists of constant volume heat addition, isentropic expansion, and constant pressure compression. For the cycle, $p_1 = 14.7$ lbf/in.^{2} and $T_1 = 540$°R. The mass of air is 4.24×10^{-3} lb, and the maximum cycle temperature is 1600°R. Assuming $c_v = 0.171$ Btu/lb · °R, determine for the cycle

(a) the net work, in Btu.
(b) the thermal efficiency.

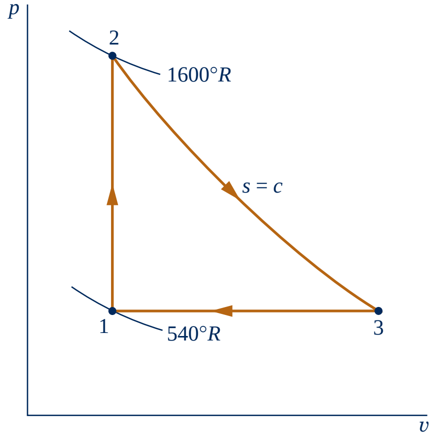


Fig. P9.17

9.18 The pressure-specific volume diagram of the air-standard *Atkinson* cycle is shown in Fig. P9.18. The cycle consists of isentropic compression, constant volume heat addition, isentropic expansion, and constant pressure compression. For a particular Atkinson cycle, the compression ratio during isentropic compression is 8.5. At the beginning of this compression process, $p_1 = 100$ kPa and $T_1 = 300$ K. The constant volume heat addition per unit mass of air is 1400 kJ/kg. (a) Sketch the cycle on T–s coordinates. Determine (b) the net work, in kJ per kg of air, (c) the thermal efficiency of the cycle, and (d) the mean effective pressure, in kPa.

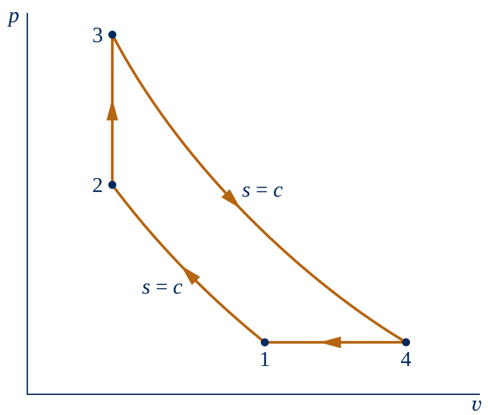


Fig. P9.18

9.19 On a cold air-standard basis, derive an expression for the thermal efficiency of the Atkinson cycle (see Fig. P9.18) in terms of the volume ratio during the isentropic compression, the pressure ratio for the constant volume process, and the specific heat ratio. Compare the thermal efficiencies of the cold air-standard Atkinson and Otto cycles, each having the same compression ratio and maximum temperature. Discuss.

9.20 The pressure and temperature at the beginning of compression of an air-standard Diesel cycle are 95 kPa and 300 K, respectively. At the end of the heat addition, the pressure is 7.2 MPa and the temperature is 2150 K. Determine

(a) the compression ratio.
(b) the cutoff ratio.
(c) the thermal efficiency of the cycle.
(d) the mean effective pressure, in kPa.

9.21 Solve Problem 9.20 on a cold air-standard basis with specific heats evaluated at 300 K.

9.22 Consider an air-standard Diesel cycle. At the beginning of compression, $p_1 = 14.0$ lbf/in.^{2} and $T_1 = 520$°R. The mass of air is 0.145 lb and the compression ratio is 17. The maximum temperature in the cycle is 4000°R. Determine

(a) the heat addition, in Btu.
(b) the thermal efficiency.
(c) the cutoff ratio.

9.23 Solve Problem 9.22 on a cold air-standard basis with specific heats evaluated at 520°R.

9.24 Consider an air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. Determine

(a) the cutoff ratio.
(b) the heat addition per unit mass, in kJ/kg.
(c) the net work per unit mass, in kJ/kg.
(d) the thermal efficiency.

State	T (K)	p (kPa)	u (kJ/kg)	h (kJ/kg)
1	380	100	271.69	380.77
2	1096.6	5197.6	842.40	1157.18
3	1864.2	5197.6	1548.47	2082.96
4	875.2	230.1	654.02	905.26

9.25 Consider a cold air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. For $k = 1.4$, $c_v = 0.718$ kJ/(kg · K), and $c_p = 1.005$ kJ/(kg · K), determine

(a) the heat transfer per unit mass and work per unit mass for each process, in kJ/kg, and the cycle thermal efficiency.
(b) the exergy transfers accompanying heat and work for each process, in kJ/kg. Devise and evaluate an exergetic efficiency for the cycle. Let $T_0 = 300$ K and $p_0 = 100$ kPa.

State	T (K)	p (kPa)	v (m^3/kg)
1	340	100	0.9758
2	1030.7	4850.3	0.06098
3	2061.4	4850.3	0.1220
4	897.3	263.9	0.9758

9.26 Consider an air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. Determine

(a) the cutoff ratio.
(b) the heat addition per unit mass, in Btu/lb.
(c) the net work per unit mass, in Btu/lb.
(d) the thermal efficiency.

State	T (°R)	p (lbf/in.^{2})	u (Btu/lb)	h (Btu/lb)
1	520	14.2	88.62	124.27
2	1502.5	657.8	266.84	369.84
3	3000	657.8	585.04	790.68
4	1527.1	41.8	271.66	376.36

9.27 Consider a cold air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. For $k = 1.4$, $c_v = 0.172$ Btu/(lb · °R), and $c_p = 0.240$ Btu/(lb · °R), determine

(a) heat transfer per unit mass and work per unit mass for each process, in Btu/lb, and the cycle thermal efficiency.
(b) exergy transfers accompanying heat and work for each process, in Btu/lb. Devise and evaluate an exergetic efficiency for the cycle. Let $T_0 = 540$°R and $p_0 = 14.7$ lbf/in.^{2}

State	T (°R)	p (lbf/in.^{2})	v (ft^3/lb)
1	540	14.7	13.60
2	1637	713.0	0.85
3	3274	713.0	1.70
4	1425.1	38.8	13.60

9.28 The displacement volume of an internal combustion engine is 5.6 liters. The processes within each cylinder of the engine are modeled as an air-standard Diesel cycle with a cutoff ratio of 2.4. The state of the air at the beginning of compression is fixed by $p_1 = 95$ kPa, $T_1 = 27$°C, and $V_1 = 6.0$ liters. Determine the net work per cycle, in kJ, the power developed by the engine, in kW, and the thermal efficiency, if the cycle is executed 1500 times per min.

9.29 The state at the beginning of compression of an air-standard Diesel cycle is fixed by $p_1 = 100$ kPa and $T_1 = 310$ K. The compression ratio is 15. For cutoff ratios ranging from 1.5 to 2.5, plot

(a) the maximum temperature, in K.
(b) the pressure at the end of the expansion, in kPa.
(c) the net work per unit mass of air, in kJ/kg.
(d) the thermal efficiency.

9.30 An air-standard Diesel cycle has a maximum temperature of 1800 K. At the beginning of compression, $p_1 = 95$ kPa and $T_1 = 300$ K. The mass of air is 12 g. For compression ratios ranging from 15 to 25, plot

(a) the net work of the cycle, in kJ.
(b) the thermal efficiency.
(c) the mean effective pressure, in kPa.

9.31 At the beginning of compression in an air-standard Diesel cycle, $p_1 = 170$ kPa, $V_1 = 0.016$ m^3, and $T_1 = 315$ K. The compression ratio is 15 and the maximum cycle temperature is 1400 K. Determine

(a) the mass of air, in kg.
(b) the heat addition and heat rejection per cycle, each in kJ.
(c) the net work, in kJ, and the thermal efficiency.

9.32 The thermal efficiency, η, of a cold air-standard diesel cycle can be expressed by Eq. 9.13:

$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

where r is compression ratio and r_c is cutoff ratio. Derive this expression.

9.33 At the beginning of the compression process in an air-standard Diesel cycle, $p_1 = 1$ bar and $T_1 = 300$ K. For maximum cycle temperatures of 1200, 1500, 1800, and 2100 K, plot the heat addition per unit of mass, in kJ/kg, the net work per unit of mass, in kJ/kg, the mean effective pressure, in bar, and the thermal efficiency, each versus compression ratio ranging from 5 to 20.

9.34 An air-standard dual cycle has a compression ratio of 9. At the beginning of compression, $p_1 = 100$ kPa, $T_1 = 300$ K, and $V_1 = 14$ L. The heat addition is 22.7 kJ, with one half added at constant volume and one half added at constant pressure. Determine

(a) the temperatures at the end of each heat addition process, in K.
(b) the net work of the cycle per unit mass of air, in kJ/kg.
(c) the thermal efficiency.
(d) the mean effective pressure, in kPa.

9.35 For the cycle in Problem 9.34, plot each of the quantities calculated in parts (a) through (d) versus the ratio of constant-volume heat addition to total heat addition varying from 0 to 1. Discuss.

9.36 Solve Problem 9.34 on a cold air-standard basis with specific heats evaluated at 300 K.

9.37 The thermal efficiency, η, of a cold air-standard dual cycle can be expressed as

$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{r_p r_c^k - 1}{(r_p - 1) + k r_p (r_c - 1)}\right]$$

where r is compression ratio, r_c is cutoff ratio, and r_p is the pressure ratio for the constant volume heat addition. Derive this expression.

9.38 Consider an air-standard dual cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.7. If the mass of air is 0.05 kg, determine

(a) the cutoff ratio.
(b) the heat addition to the cycle, in kJ.
(c) the heat rejection from the cycle, in kJ.
(d) the net work, in kJ.
(e) the thermal efficiency.

State	T (K)	p (kPa)	u (kJ/kg)	h (kJ/kg)
1	300	95	214.07	300.19
2	862.4	4372.8	643.35	890.89
3	1800	9126.9	1487.2	2003.3
4	1980	9126.9	1659.5	2227.1
5	840.3	265.7	625.19	866.41

9.39 The pressure and temperature at the beginning of compression in an air-standard dual cycle are 14.0 lbf/in.^{2} and 520°R, respectively. The compression ratio is 15 and the heat addition per unit mass of air is 800 Btu/lb. At the end of the constant volume heat addition process, the pressure is 1200 lbf/in.^{2} Determine

(a) the net work of the cycle per unit mass of air, in Btu/lb.
(b) the heat rejection for the cycle per unit mass of air, in Btu/lb.
(c) the thermal efficiency.
(d) the cutoff ratio.
(e) To investigate the effects of varying compression ratio, plot each of the quantities calculated in parts (a) through (d) for compression ratios ranging from 10 to 28.

9.40 An air-standard dual cycle has a compression ratio of 16. At the beginning of compression, $p_1 = 14.5$ lbf/in.^{2}, $V_1 = 0.5$ ft^3, and $T_1 = 50$°F. The pressure doubles during the constant volume heat addition process. For a maximum cycle temperature of 3000°R, determine

(a) the heat addition to the cycle, in Btu.
(b) the net work of the cycle, in Btu.
(c) the thermal efficiency.
(d) the mean effective pressure, in lbf/in.^{2}
(e) To investigate the effects of varying maximum cycle temperature, plot each of the quantities calculated in parts (a) through (d) for maximum cycle temperatures ranging from 3000 to 4000°R.

9.41 At the beginning of the compression process in an air-standard dual cycle, $p_1 = 1$ bar and $T_1 = 300$ K. The total heat addition is 1000 kJ/kg. Plot the net work per unit of mass, in kJ/kg, the mean effective pressure, in bar, and the thermal efficiency versus compression ratio for different fractions of constant volume and constant pressure heat addition. Consider compression ratio ranging from 10 to 20.

Brayton Cycle

9.42 An ideal air-standard Brayton cycle operating at steady state produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.9. Sketch the T–s diagram for the cycle and determine

(a) the mass flow rate of air, in kg/s.
(b) the rate of heat transfer, in kW, to the working fluid passing through the heat exchanger.
(c) the thermal efficiency.

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2	1200	603.5	610.65
3	1200	1450	1575.57
4	100	780.7	800.78

9.43 An ideal cold air-standard Brayton cycle operates at steady state with compressor inlet conditions of 300 K and 100 kPa, fixed turbine inlet temperature of 1700 K, and $k = 1.4$. For the cycle,

(a) determine the net power developed per unit mass flowing, in kJ/kg, and the thermal efficiency for a compressor pressure ratio of 8.
(b) plot the net power developed per unit mass flowing, in kJ/kg, and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

9.44 An ideal air-standard Brayton cycle operates at steady state with compressor inlet conditions of 300 K and 100 kPa and a fixed turbine inlet temperature of 1700 K. For the cycle,

(a) determine the net power developed per unit mass flowing, in kJ/kg, and the thermal efficiency for a compressor pressure ratio of 8.
(b) plot the net power developed per unit mass flowing, in kJ/kg, and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

9.45 Air enters the compressor of an ideal cold air-standard Brayton cycle at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10, and the turbine inlet temperature is 1400 K. For $k = 1.4$, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.

9.46 For the Brayton cycle of Problem 9.45, investigate the effects of varying compressor pressure ratio and turbine inlet temperature. Plot the same quantities calculated in Problem 9.45 for

(a) a compressor pressure ratio of 10 and turbine inlet temperatures ranging from 1000 to 1600 K.
(b) a turbine inlet temperature of 1400 K and compressor pressure ratios ranging from 2 to 20.

Discuss.

9.47 The rate of heat addition to an ideal air-standard Brayton cycle is 3.4×10^9 Btu/h. The pressure ratio for the cycle is 14 and the minimum and maximum temperatures are 520°R and 3000°R, respectively. Determine

(a) the thermal efficiency of the cycle.
(b) the mass flow rate of air, in lb/h.
(c) the net power developed by the cycle, in Btu/h.

9.48 Solve Problem 9.47 on a cold air-standard basis with specific heats evaluated at 520°R.

9.49 On the basis of a cold air-standard analysis, show that the back work ratio of an ideal air-standard Brayton cycle equals the ratio of absolute temperatures at the compressor inlet and the turbine outlet.

9.50 The compressor inlet temperature of an ideal air-standard Brayton cycle is 520°R and the maximum allowable turbine inlet temperature is 2600°R. Plot the net work developed per unit mass of air flow, in Btu/lb, and the thermal efficiency versus compressor pressure ratio for pressure ratios ranging from 12 to 24. Using your plots, estimate the pressure ratio for maximum net work and the corresponding value of thermal efficiency. Compare the results to those obtained in analyzing the cycle on a cold air-standard basis.

9.51 The compressor inlet temperature for an ideal Brayton cycle is T_1 and the turbine inlet temperature is T_3. Using a cold air-standard analysis, show that the temperature T_2 at the compressor exit that maximizes the net work developed per unit mass of air flow is $T_2 = (T_1T_3)^{1/2}$.

9.52 Air enters the compressor of a cold air-standard Brayton cycle at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10, and the turbine inlet temperature is 1400 K. The turbine and compressor each have isentropic efficiencies of 80%. For $k = 1.4$, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rates of exergy destruction in the compressor and turbine, respectively, each in kW, for $T_0 = 300$ K.

Plot the quantities calculated in parts (a) through (d) versus isentropic efficiency for equal compressor and turbine isentropic efficiencies ranging from 70 to 100%. Discuss.

9.53 The cycle of Problem 9.42 is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes. If the states at the compressor and turbine inlets remain unchanged, the cycle produces 10 MW of power, and the compressor and turbine isentropic efficiencies are both 80%, determine

(a) the pressure, in kPa, temperature, in K, and specific enthalpy, in kJ/kg, at each principal state of the cycle and sketch the T–s diagram.
(b) the mass flow rate of air, in kg/s.
(c) the rate of heat transfer, in kW, to the working fluid passing through the heat exchanger.
(d) the thermal efficiency.

9.54 Air enters the compressor of an air-standard Brayton cycle with a volumetric flow rate of 60 m^3/s at 0.8 bar, 280 K. The compressor pressure ratio is 20, and the maximum cycle temperature is 2100 K. For the compressor, the isentropic efficiency is 92% and for the turbine the isentropic efficiency is 95%. Determine

(a) the net power developed, in MW.
(b) the rate of heat addition in the combustor, in MW.
(c) the thermal efficiency of the cycle.

9.55 Air enters the compressor of a simple gas turbine at $p_1 = 14$ lbf/in.^{2}, $T_1 = 520$°R. The isentropic efficiencies of the compressor and turbine are 83 and 87%, respectively. The compressor pressure ratio is 14 and the temperature at the turbine inlet is 2500°R. The net power developed is 5×10^6 Btu/h. On the basis of an air-standard analysis, calculate

(a) the volumetric flow rate of the air entering the compressor, in ft^3/min.
(b) the temperatures at the compressor and turbine exits, each in °R.
(c) the thermal efficiency of the cycle.

9.56 Solve Problem 9.55 on a cold air-standard basis with specific heats evaluated at 520°R.

9.57 Air enters the compressor of a simple gas turbine at 100 kPa, 300 K, with a volumetric flow rate of 5 m^3/s. The compressor pressure ratio is 10 and its isentropic efficiency is 85%. At the inlet to the turbine, the pressure is 950 kPa, and the temperature is 1400 K. The turbine has an isentropic efficiency of 88% and the exit pressure is 100 kPa. On the basis of an air-standard analysis,

(a) develop a full accounting of the *net* exergy increase of the air passing through the gas turbine combustor, in kW.
(b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

Let $T_0 = 300$ K, $p_0 = 100$ kPa.

9.58 Air enters the compressor of a simple gas turbine at 14.5 lbf/in.^{2}, 80°F, and exits at 87 lbf/in.^{2}, 514°F. The air enters the turbine at 1540°F, 87 lbf/in.^{2} and expands to 917°F, 14.5 lbf/in.^{2} The compressor and turbine operate adiabatically, and kinetic and potential energy effects are negligible. On the basis of an air-standard analysis,

(a) develop a full accounting of the *net* exergy increase of the air passing through the gas turbine combustor, in Btu/lb.
(b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

Let $T_0 = 80$°F, $p_0 = 14.5$ lbf/in.^{2}

Regeneration, Reheat, and Compression with Intercooling

9.59 An ideal air-standard regenerative Brayton cycle produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.14. Sketch the T–s diagram and determine

(a) the mass flow rate of air, in kg/s.
(b) the rate of heat transfer, in kW, to the working fluid passing through the combustor.
(c) the thermal efficiency.

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2	1200	603.5	610.65
x	1200	780.7	800.78
3	1200	1450	1575.57
4	100	780.7	800.78
y	100	603.5	610.65

9.60 The cycle of Problem 9.59 is modified to include the effects of irreversibilities in the adiabatic expansion and compression process. The regenerator effectiveness is 100%. If the states at the compressor and turbine inlets remain unchanged, the cycle produces 10 MW of power, and the compressor and turbine isentropic efficiencies are both 80%, determine

(a) the pressure, in kPa, temperature, in K, and enthalpy, in kJ/kg, at each principal state of the cycle and sketch the T–s diagram.
(b) the mass flow rate of air, in kg/s.
(c) the rate of heat transfer, in kW, to the working fluid passing through the combustor.
(d) the thermal efficiency.

9.61 The cycle of Problem 9.60 is modified to include a regenerator with an effectiveness of 70%. Determine

(a) the specific enthalpy, in kJ/kg, and the temperature, in K, for each stream exiting the regenerator and sketch the T–s diagram.

(b) the mass flow rate of air, in kg/s.
(c) the rate of heat transfer, in kW, to the working fluid passing through the combustor.
(d) the thermal efficiency.

9.62 Air enters the compressor of a cold air-standard Brayton cycle with regeneration at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10, and the turbine inlet temperature is 1400 K. The turbine and compressor each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For $k = 1.4$, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rate of exergy destruction in the regenerator, in kW, for $T_0 = 300$ K.

9.63 Air enters the compressor of a regenerative air-standard Brayton cycle with a volumetric flow rate of 60 m^3/s at 0.8 bar, 280 K. The compressor pressure ratio is 20, and the maximum cycle temperature is 2100 K. For the compressor, the isentropic efficiency is 92% and for the turbine the isentropic efficiency is 95%. For a regenerator effectiveness of 85%, determine

(a) the net power developed, in MW.
(b) the rate of heat addition in the combustor, in MW.
(c) the thermal efficiency of the cycle.

Plot the quantities calculated in parts (a) through (c) for regenerator effectiveness values ranging from 0 to 100%. Discuss.

9.64 Reconsider Problem 9.55, but include a regenerator in the cycle. For regenerator effectiveness values ranging from 0 to 100%, plot

(a) the thermal efficiency.
(b) the percent decrease in heat addition to the air.

9.65 On the basis of a cold air-standard analysis, show that the thermal efficiency of an ideal regenerative gas turbine can be expressed as

$$\eta = 1 - \left(\frac{T_1}{T_3}\right)(r)^{(k-1)/k}$$

where r is the compressor pressure ratio, and T_1 and T_3 denote the temperatures at the compressor and turbine inlets, respectively.

9.66 An air-standard Brayton cycle has a compressor pressure ratio of 10. Air enters the compressor at $p_1 = 14.7$ lbf/in.^{2}, $T_1 = 70$°F with a mass flow rate of 90,000 lb/h. The turbine inlet temperature is 2200°R. Calculate the thermal efficiency and the net power developed, in horsepower, if

(a) the turbine and compressor isentropic efficiencies are each 100%.
(b) the turbine and compressor isentropic efficiencies are 88 and 84%, respectively.
(c) the turbine and compressor isentropic efficiencies are 88 and 84%, respectively, and a regenerator with an effectiveness of 80% is incorporated.

9.67 Fig. P9.67 illustrates a gas turbine power plant that uses solar energy as the source of heat addition (see U.S. Patent 4,262,484). Operating data are given on the figure. Modeling the cycle as a Brayton cycle, and assuming no pressure drops in the heat exchanger or interconnecting piping, determine

(a) the thermal efficiency.
(b) the air mass flow rate, in kg/s, for a net power output of 500 kW.

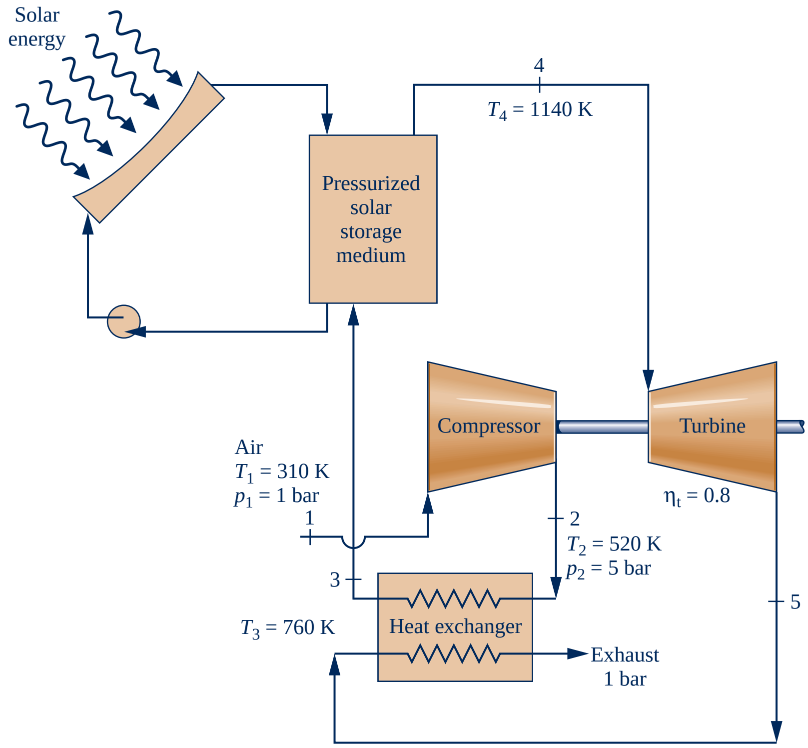


Fig. P9.67

9.68 Air enters the compressor of a regenerative gas turbine with a volumetric flow rate of 3.2×10^5 ft^3/min at 14.5 lbf/in.^{2}, 77°F, and is compressed to 60 lbf/in.^{2} The air then passes through the regenerator and exits at 1120°R. The temperature at the turbine inlet is 1700°R. The compressor and turbine each has an isentropic efficiency of 84%. Using an air-standard analysis, calculate

(a) the thermal efficiency of the cycle.
(b) the regenerator effectiveness.
(c) the net power output, in Btu/h.

9.69 Air enters the turbine of a gas turbine at 1200 kPa, 1200 K, and expands to 100 kPa in two stages. Between the stages, the air is reheated at a constant pressure of 350 kPa to 1200 K. The expansion through each turbine stage is isentropic. Determine, in kJ per kg of air flowing,

(a) the work developed by each stage.
(b) the heat transfer for the reheat process.
(c) the increase in net work as compared to a single stage of expansion with no reheat.

9.70 Reconsider Problem 9.69 and include in the analysis that each turbine stage might have an isentropic efficiency less than 100%. Plot each of the quantities calculated in parts (a) through (c) of Problem 9.69 for values of the interstage pressure ranging from 100 to 1200 kPa and for isentropic efficiencies of 100%, 80%, and 60%.

9.71 Consider a two-stage turbine operating at steady state with reheat at constant pressure between the stages. Show that the maximum work is developed when the pressure ratio is the same across each stage. Use a cold air-standard analysis, assuming the inlet state and the exit pressure are specified, each expansion process is isentropic, and the temperature at the inlet to each turbine stage is the same. Kinetic and potential energy effects can be ignored.

9.72 Air enters the compressor of a cold air-standard Brayton cycle with regeneration and reheat at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10, and the inlet temperature for each turbine stage is 1400 K. The pressure ratios across each turbine stage are equal. The turbine stages and compressor each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For $k = 1.4$, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rates of exergy destruction in the compressor and each turbine stage as well as the regenerator, in kW, for $T_0 = 300$ K.

9.73 Air enters a two-stage compressor operating at steady state at 520°R, 14 lbf/in.^{2} The overall pressure ratio across the stages is 12 and each stage operates isentropically. Intercooling occurs at constant pressure at the value that minimizes compressor work input as determined in Example 9.10, with air exiting the intercooler at 520°R. Assuming ideal gas behavior, with $k = 1.4$, determine the work per unit mass of air flowing for the two-stage compressor. Kinetic and potential energy effects can be ignored.

9.74 A two-stage air compressor operates at steady state, compressing 10 m^3/min of air from 100 kPa, 300 K, to 1200 kPa. An intercooler between the two stages cools the air to 300 K at a constant pressure of 350 kPa. The compression processes are isentropic. Calculate the power required to run the compressor, in kW, and compare the result to the power required for isentropic compression from the same inlet state to the same final pressure.

9.75 Reconsider Problem 9.74 and include in the analysis that each compressor stage might have an isentropic efficiency less than 100%. Plot, in kW, (a) the power input to each stage, (b) the heat transfer rate for the intercooler, and (c) the decrease in power input as compared to a single stage of compression with no intercooling, for values of interstage pressure ranging from 100 to 1200 kPa and for isentropic efficiencies of 100%, 80%, and 60%.

9.76 Air enters a compressor operating at steady state at 14 lbf/in.^{2}, 60°F, with a volumetric flow rate of 6000 ft^3/min. The compression occurs in two stages, with each stage being a polytropic process with $n = 1.27$. The air is cooled to 80°F between the stages by an intercooler operating at 45 lbf/in.^{2} Air exits the compressor at 150 lbf/in.^{2} Determine, in Btu per min

(a) the power and heat transfer rate for each compressor stage.
(b) the heat transfer rate for the intercooler.

9.77 Air enters the first compressor stage of a cold air-standard Brayton cycle with regeneration and intercooling at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The overall compressor pressure ratio is 10, and the pressure ratios are the same across each compressor stage. The temperature at the inlet to the second compressor stage is 300 K. The compressor stages and turbine each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For $k = 1.4$, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rates of exergy destruction in each compressor stage and the turbine stage as well as the regenerator, in kW, for $T_0 = 300$ K.

9.78 Referring to Example 9.10, show that if $T_d > T_1$ the pressure ratios across the two compressor stages are related by

$$\frac{p_i}{p_1} = \left(\frac{p_2}{p_i}\right)\left(\frac{T_d}{T_1}\right)^{k/(k-1)}$$

9.79 Rework Example 9.10 for the case of a three-stage compressor with intercooling between stages.

9.80 An air-standard regenerative Brayton cycle operating at steady state with intercooling and reheat produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.19. Sketch the T–s diagram for the cycle and determine

(a) the mass flow rate of air, in kg/s.
(b) the rate of heat transfer, in kW, to the working fluid passing through each combustor.
(c) the thermal efficiency.

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2	300	410.1	411.22
3	300	300	300.19
4	1200	444.8	446.50
5	1200	1111.0	1173.84
6	1200	1450	1575.57
7	300	1034.3	1085.31
8	300	1450	1575.57
9	100	1111.0	1173.84
10	100	444.8	446.50

9.81 Air enters the compressor of a cold air-standard Brayton cycle with regeneration, intercooling, and reheat at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10, and the pressure ratios are the same across each compressor stage. The intercooler and reheater both operate at the same pressure. The temperature at the inlet to the second compressor stage is 300 K, and the inlet temperature for each turbine stage is 1400 K. The compressor and turbine stages each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For $k = 1.4$, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rates of exergy destruction in the compressor and turbine stages as well as the regenerator, in kW, for $T_0 = 300$ K.

9.82 An air-standard Brayton cycle produces 10 MW of power. The compressor and turbine isentropic efficiencies are both 80%. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.9.

(a) Fill in the missing data in the table and sketch the T–s diagram for the cycle.
(b) Determine the mass flow rate of air, in kg/s.
(c) Perform a full accounting for the net rate of exergy increase as the air passes through the combustor.

Let $T_0 = 300$ K, $p_0 = 100$ kPa.

State	p (kPa)	T (K)	h (kJ/kg)	s° [kJ/(kg · K)]	p_r
1	100	300	300.19	1.70203	1.3860
2	1200				
3	1200	1450	1575.57	3.40417	522
4	100				

9.83 For each of the following modifications of the cycle of part (c) of Problem 9.66, determine the thermal efficiency and net power developed, in horsepower.

(a) Introduce a two-stage turbine expansion with reheat between the stages at a constant pressure of 50 lbf/in.^{2} Each turbine stage has an isentropic efficiency of 88% and the temperature of the air entering the second stage is 2000°R.
(b) Introduce two-stage compression, with intercooling between the stages at a pressure of 50 lbf/in.^{2} Each compressor stage has an isentropic efficiency of 84% and the temperature of the air entering the second stage is 70°F.
(c) Introduce both compression with intercooling and reheat between turbine stages. Compression occurs in two stages, with intercooling to 70°F between the stages at 50 lbf/in.^{2} The turbine expansion also occurs in two stages, with reheat to 2000°R between the stages at 50 lbf/in.^{2} The isentropic efficiencies of the turbine and compressor stages are 88 and 84%, respectively.

Other Gas Power System Applications

9.84 Air at 26 kPa, 230 K, and 220 m/s enters a turbojet engine in flight. The air mass flow rate is 25 kg/s. The compressor pressure ratio is 11, the turbine inlet temperature is 1400 K, and air exits the nozzle at 26 kPa. The diffuser and nozzle processes are isentropic, the compressor and turbine have isentropic efficiencies of 85% and 90%, respectively, and there is no pressure drop for flow through the combustor. Kinetic energy is negligible everywhere except at the diffuser inlet and the nozzle exit. On the basis of air-standard analysis, determine

(a) the pressures, in kPa, and temperatures, in K, at each principal state.
(b) the rate of heat addition to the air passing through the combustor, in kJ/s.
(c) the velocity at the nozzle exit, in m/s.

9.85 For the turbojet in Problem 9.84, plot the velocity at the nozzle exit, in m/s, the pressure at the turbine exit, in kPa, and the rate of heat input to the combustor, in kW, each as a function of compressor pressure ratio in the range of 6 to 14. Repeat for turbine inlet temperatures of 1200 K and 1000 K.

9.86 Air enters the diffuser of a turbojet engine with a mass flow rate of 85 lb/s at 9 lbf/in.^{2}, 420°R, and a velocity of 750 ft/s. The pressure ratio for the compressor is 12, and its isentropic efficiency is 88%. Air enters the turbine at 2400°R with the same pressure as at the exit of the compressor. Air exits the nozzle at 9 lbf/in.^{2} The diffuser operates isentropically and the nozzle and turbine have isentropic efficiencies of 92% and 90%, respectively. On the basis of an air-standard analysis, calculate

(a) the rate of heat addition, in Btu/h.
(b) the pressure at the turbine exit, in lbf/in.^{2}
(c) the compressor power input, in Btu/h.
(d) the velocity at the nozzle exit, in ft/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

9.87 Consider the addition of an afterburner to the turbojet in Problem 9.84 that raises the temperature at the inlet of the nozzle to 1300 K. Determine the velocity at the nozzle exit, in m/s.

9.88 Consider the addition of an afterburner to the turbojet in Problem 9.86 that raises the temperature at the inlet of the nozzle to 2200°R. Determine the velocity at the nozzle exit, in ft/s.

9.89 Air enters the diffuser of a ramjet engine at 6 lbf/in.^{2}, 420°R, with a velocity of 1600 ft/s, and decelerates essentially to zero velocity. After combustion, the gases reach a temperature of 2000°R before being discharged through the nozzle at 6 lbf/in.^{2} On the basis of an air-standard analysis, determine

(a) the pressure at the diffuser exit, in lbf/in.^{2}
(b) the velocity at the nozzle exit, in ft/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

9.90 Air enters the diffuser of a ramjet engine at 40 kPa, 240 K, with a velocity of 2500 km/h and decelerates to negligible velocity. On the basis of an air-standard analysis, the heat addition is 1080 kJ per kg of air passing through the engine. Air exits the nozzle at 40 kPa. Determine

(a) the pressure at the diffuser exit, in kPa.
(b) the velocity at the nozzle exit, in m/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

9.91 A turboprop engine consists of a diffuser, compressor, combustor, turbine, and nozzle. The turbine drives a propeller as well as the compressor. Air enters the diffuser with a volumetric flow rate of 83.7 m^3/s at 40 kPa, 240 K, and a velocity of 180 m/s, and decelerates essentially to zero velocity. The compressor pressure ratio is 10 and the compressor has an isentropic efficiency of 85%. The turbine inlet temperature is 1140 K, and its isentropic efficiency is 85%. The turbine exit pressure is 50 kPa. Flow through the diffuser and nozzle is isentropic. Using an air-standard analysis, determine

(a) the power delivered to the propeller, in MW.
(b) the velocity at the nozzle exit, in m/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

9.92 A turboprop engine consists of a diffuser, compressor, combustor, turbine, and nozzle. The turbine drives a propeller as well as the compressor. Air enters the diffuser at 12 $lbf/in.^2$, 460°R, with a volumetric flow rate of 30,000 ft^3/min and a velocity of 520 ft/s. In the diffuser, the air decelerates isentropically to negligible velocity. The compressor pressure ratio is 9, and the turbine inlet temperature is 2100°R. The turbine exit pressure is 25 $lbf/in.^2$, and the air expands to 12 $lbf/in.^2$ through a nozzle. The compressor and turbine each has an isentropic efficiency of 87%, and the nozzle has an isentropic efficiency of 95%. Using an air-standard analysis, determine

(a) the power delivered to the propeller, in hp.
(b) the velocity at the nozzle exit, in ft/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

9.93 Helium is used in a combined cycle power plant as the working fluid in a simple closed gas turbine serving as the topping cycle for a vapor power cycle. A nuclear reactor is the source of energy input to the helium. Figure P9.93 provides steady-state operating data. Helium enters the compressor of the gas turbine at 200 $lbf/in.^2$, 180°F with a mass flow rate of 8×10^5 lb/h and is compressed to 800 $lbf/in.^2$ The isentropic efficiency of the compressor is 80%. The helium then passes through the reactor with a negligible decrease in pressure, exiting at 1400°F. Next, the helium expands through the turbine, which has an isentropic efficiency of 80%, to a pressure of 200 $lbf/in.^2$ The helium then passes through the interconnecting heat exchanger. A separate stream of liquid water enters the heat exchanger and exits as saturated vapor at 1200 $lbf/in.^2$ The vapor is superheated before entering the turbine at 800°F, 1200 $lbf/in.^2$ The steam expands through the turbine to 1 $lbf/in.^2$ and a quality of 0.9. Saturated liquid exits the condenser at 1 $lbf/in.^2$ Cooling water passing through the condenser experiences a temperature rise from 60 to 90°F. The isentropic pump efficiency is 100%. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

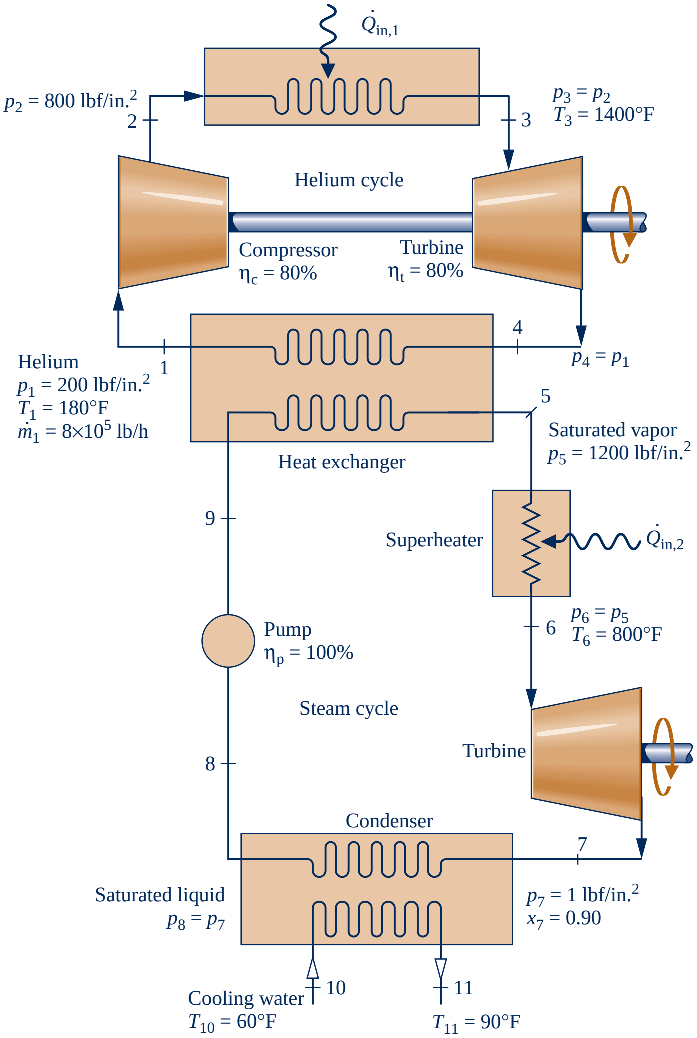


Fig. P9.93

(a) the mass flow rates of the steam and the cooling water, each in lb/h.
(b) the net power developed by the gas turbine and vapor cycles, each in Btu/h.
(c) the thermal efficiency of the combined cycle.

9.94 A combined gas turbine-vapor power plant operates as shown in Fig. P9.94. Pressure and temperature data are given at principal states, and the net power developed by the gas turbine is 147 MW. Using air-standard analysis for the gas turbine, determine

(a) the net power, in MW, developed by the power plant.
(b) the overall thermal efficiency of the plant.

Develop a full accounting of the *net* rate of exergy increase of the air passing through the gas turbine combustor. Let T_0 = 300 K, p_0 = 1 bar.

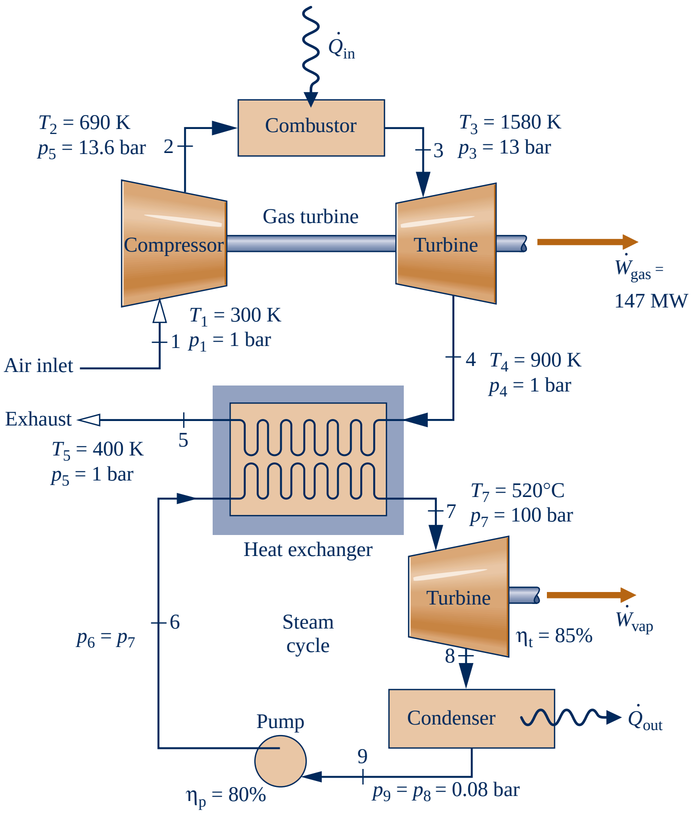


Fig. P9.94

9.95 Air enters the compressor of a combined gas turbine–vapor power plant (Fig. 9.22) at 1 bar, 25°C. The isentropic compressor efficiency is 85% and the compressor pressure ratio is 14. The air passing through the combustor receives energy by heat transfer at a rate of 50 MW with no significant decrease in pressure. At the inlet to the turbine the air is at 1250°C. The air expands through the turbine, which has an isentropic efficiency of 87%, to a pressure of 1 bar. Then, the air passes through the interconnecting heat exchanger and is finally discharged at 200°C, 1 bar. Steam enters the turbine of the vapor cycle at 12.5 MPa, 500°C, and expands to a condenser pressure of 0.1 bar. Water enters the pump as a saturated liquid at 0.1 bar. The turbine and pump have isentropic efficiencies of 90 and 100%, respectively. Cooling water enters the condenser at 20°C and exits at 35°C. Determine

(a) the mass flow rates of the air, steam, and cooling water, each in kg/s.
(b) the net power developed by the gas turbine cycle and the vapor cycle, respectively, each in MW.
(c) the thermal efficiency of the combined cycle.
(d) the *net* rate at which exergy is carried out with the exhaust air, $\dot{m}_{air}[e_{f5} - e_{f1}]$, in MW.
(e) the *net* rate at which exergy is carried out with the cooling water, in MW.

Let $T_0 = 20°C$, $p_0 = 1$ bar.

9.96 A combined gas turbine–vapor power plant (Fig. 9.22) has a net power output of 100 MW. Air enters the compressor of the gas turbine at 100 kPa, 300 K, and is compressed to 1200 kPa. The isentropic efficiency of the compressor is 84%. The conditions at the inlet to the turbine are 1200 kPa and 1400 K. Air expands through the turbine, which has an isentropic efficiency of 88%, to a pressure of 100 kPa. The air then passes through the interconnecting heat exchanger, and is finally discharged at 480 K. Steam enters the turbine of the vapor power cycle at 8 MPa, 400°C, and expands to the condenser pressure of 8 kPa. Water enters the pump as saturated liquid at 8 kPa. The turbine and pump have isentropic efficiencies of 90 and 80%, respectively. Determine

(a) the mass flow rates of air and steam, each in kg/s.
(b) the thermal efficiency of the combined cycle.
(c) a full accounting of the *net* exergy increase of the air passing through the combustor of the gas turbine, $\dot{m}_{air}[e_{f3} - e_{f2}]$, in MW. Discuss.

Let $T_0 = 300$ K, $p_0 = 100$ kPa.

9.97 A simple gas turbine is the topping cycle for a simple vapor power cycle (Fig. 9.22). Air enters the compressor of the gas turbine at 60°F, 14.7 lbf/in.^{2}, with a volumetric flow rate of 40,000 ft^3/min. The compressor pressure ratio is 12 and the turbine inlet temperature is 2600°R. The compressor and turbine each have isentropic efficiencies of 88%. The air leaves the interconnecting heat exchanger at 840°R, 14.7 lbf/in.^{2} Steam enters the turbine of the vapor cycle at 1000 lbf/in.^{2}, 900°F, and expands to the condenser pressure of 1 lbf/in.^{2} Water enters the pump as saturated liquid at 1 lbf/in.^{2} The turbine and pump efficiencies are 90 and 70%, respectively. Cooling water passing through the condenser experiences a temperature rise from 60 to 80°F with a negligible change in pressure. Determine

(a) the mass flow rates of the air, steam, and cooling water, each in lb/h.
(b) the net power developed by the gas turbine cycle and the vapor cycle, respectively, each in Btu/h.
(c) the thermal efficiency of the combined cycle.
(d) a full accounting of the *net* exergy increase of the air passing through the combustor of the gas turbine, $\dot{m}_{air}[e_{f3} - e_{f2}]$, in Btu/h. Discuss.

Let $T_0 = 520°R$, $p_0 = 14.7$ lbf/in.^{2}

9.98 Air enters the compressor of an Ericsson cycle at 300 K, 1 bar, with a mass flow rate of 5 kg/s. The pressure and temperature at the inlet to the turbine are 10 bar and 1400 K, respectively. Determine

(a) the net power developed, in kW.
(b) the thermal efficiency.
(c) the back work ratio.

9.99 For the cycle in Problem 9.98, plot the net power developed, in kW, for compressor pressure ratios ranging from 2 to 15. Repeat for turbine inlet temperatures of 1200 K and 1000 K.

9.100 Air is the working fluid in an Ericsson cycle. Expansion through the turbine takes place at a constant temperature of 2250°R. Heat transfer from the compressor occurs at 560°R. The compressor pressure ratio is 12. Determine

(a) the net work, in Btu per lb of air flowing.
(b) the thermal efficiency.

9.101 Nitrogen (N_2) is the working fluid of a Stirling cycle with a compression ratio of nine. At the beginning of the isothermal compression, the temperature, pressure, and volume are 310 K, 1 bar, and 0.008 m^3, respectively. The temperature during the isothermal expansion is 1000 K. Determine

(a) the net work, in kJ.
(b) the thermal efficiency.
(c) the mean effective pressure, in bar.

9.102 Helium is the working fluid in a Stirling cycle. In the isothermal compression, the helium is compressed from 15 lbf/in.^{2}, 100°F, to 150 lbf/in.^{2} The isothermal expansion occurs at 1500°F. Determine

(a) the work and heat transfer, in Btu per lb of helium, for each process in the cycle.
(b) the thermal efficiency.

Compressible Flow

9.103 Calculate the thrust developed by the turbojet engine in Problem 9.84, in kN.

9.104 Calculate the thrust developed by the turbojet engine in Problem 9.86, in lbf.

9.105 Calculate the thrust developed by the turbojet engine with afterburner in Problem 9.87, in kN.

9.106 Referring to the turbojet in Problem 9.86 and the modified turbojet in Problem 9.88, calculate the thrust developed by each engine, in lbf. Discuss.

9.107 Air enters the diffuser of a turbojet engine at 18 kPa, 216 K, with a volumetric flow rate of 230 m^3/s and a velocity of 265 m/s. The compressor pressure ratio is 15, and its isentropic efficiency is 87%. Air enters the turbine at 1360 K and the same pressure as at the exit of the compressor. The turbine isentropic efficiency is 89%, and the nozzle isentropic efficiency is 97%. The pressure at the nozzle exit is 18 kPa. On the basis of an air-standard analysis, calculate the thrust, in kN.

9.108 Calculate the ratio of the thrust developed to the mass flow rate of air, in N per kg/s, for the ramjet engine in Problem 9.90.

9.109 Air flows at steady state through a horizontal, well-insulated, constant-area duct of diameter 0.25 m. At the inlet, $p_1 = 2.4$ bar, $T_1 = 430$ K. The temperature of the air leaving the duct is 370 K. The mass flow rate is 600 kg/min. Determine the magnitude, in N, of the net horizontal force exerted by the duct wall on the air. In which direction does the force act?

9.110 Liquid water at 70°F flows at steady state through a 2-in.-diameter horizontal pipe. The mass flow rate is 25 lb/s. The pressure decreases by 2 lbf/in.^{2} from inlet to exit of the pipe. Determine the magnitude, in lbf, and direction of the horizontal force required to hold the pipe in place.

9.111 Air enters a horizontal, well-insulated nozzle operating at steady state at 12 bar, 500K, with a velocity of 50 m/s and exits at 7 bar, 440 K. The mass flow rate is 1 kg/s. Determine the net force, in N, exerted by the air on the duct in the direction of flow.

9.112 Using the ideal gas model, determine the sonic velocity of

(a) air at 60°F.
(b) oxygen (O_2) at 900°R.
(c) argon at 540°R.

9.113 A flash of lightning is sighted and 3 seconds later thunder is heard. Approximately how far away was the lightning strike?

9.114 Using data from Table A-4, estimate the sonic velocity, in m/s, of steam of 60 bar, 360°C. Compare the result with the value predicted by the ideal gas model.

9.115 Plot the Mach number of carbon dioxide at 1 bar, 460 m/s, as a function of temperature in the range 250 to 1000 K.

9.116 An ideal gas flows through a duct. At a particular location, the temperature, pressure, and velocity are known. Determine the Mach number, stagnation temperature, in °R, and the stagnation pressure, in lbf/in.^{2}, for

(a) air at 310°F, 100 lbf/in.^{2}, and a velocity of 1400 ft/s.
(b) helium at 520°R, 20 lbf/in.^{2}, and a velocity of 900 ft/s.
(c) nitrogen at 600°R, 50 lbf/in.^{2}, and a velocity of 500 ft/s.

9.117 For Problem 9.111, determine the values of the Mach number, the stagnation temperature, in K, and the stagnation pressure, in bar, at the inlet and exit of the duct, respectively.

9.118 Using the Mollier diagram, Fig. A-8E, determine for water vapor at 500 lbf/in.^{2}, 600°F, and 1000 ft/s

(a) the stagnation enthalpy, in Btu/lb.
(b) the stagnation temperature, in °F.
(c) the stagnation pressure, in lbf/in.^{2}

9.119 Steam flows through a passageway, and at a particular location the pressure is 3 bar, the temperature is 281.4°C, and the velocity is 688.8 m/s. Determine the corresponding specific stagnation enthalpy, in kJ/kg, and stagnation temperature, in °C, if the stagnation pressure is 7 bar.

9.120 For the isentropic flow of an ideal gas with constant specific heat ratio k, the ratio of the temperature T^* to the stagnation temperature T_o is $T^*/T_o = 2/(k + 1)$. Develop this relationship.

9.121 A gas expands isentropically through a converging nozzle from a large tank at 8 bar, 500 K. Assuming ideal gas behavior, determine the critical pressure p^*, in bar, and the corresponding temperature, in K, if the gas is

(a) air.
(b) carbon dioxide (CO_2).
(c) water vapor.

9.122 Carbon dioxide is contained in a large tank, initially at 100 lbf/in.^{2}, 800°R. The gas discharges through a converging nozzle to the surroundings, which are at 14.7 lbf/in.^{2}, and the pressure in the tank drops. Estimate the pressure in the tank, in lbf/in.^{2}, when the flow first ceases to be choked.

9.123 Steam expands isentropically through a converging nozzle operating at steady state from a large tank at 1.83 bar, 280°C. The mass flow rate is 2 kg/s, the flow is choked, and the exit plane pressure is 1 bar. Determine the diameter of the nozzle, in cm, at locations where the pressure is 1.5 bar, and 1 bar, respectively.

9.124 An ideal gas mixture with $k = 1.31$ and a molecular weight of 23 is supplied to a converging nozzle at $p_o = 5$ bar, $T_o = 700$ K, which discharges into a region where the pressure is 1 bar. The exit area is 30 cm^2. For steady isentropic flow through the nozzle, determine

(a) the exit temperature of the gas, in K.
(b) the exit velocity of the gas, in m/s.
(c) the mass flow rate, in kg/s.

9.125 An ideal gas expands isentropically through a converging nozzle from a large tank at 120 lbf/in.^{2}, 600°R, and discharges into a region at 60 lbf/in.^{2} Determine the mass flow rate, in lb/s, for an exit flow area of 1 in.^{2}, if the gas is

(a) air, with $k = 1.4$.
(b) carbon dioxide, with $k = 1.26$.
(c) argon, with $k = 1.667$.

9.126 Air at $p_o = 1.4$ bar, $T_o = 280$ K expands isentropically through a converging nozzle and discharges to the atmosphere at 1 bar. The exit plane area is 0.0013 m^2.

(a) Determine the mass flow rate, in kg/s.
(b) If the supply region pressure, p_o, were increased to 2 bar, what would be the mass flow rate, in kg/s?

9.127 Air enters a nozzle operating at steady state at 45 lbf/in.^{2}, 800°R, with a velocity of 480 ft/s, and expands isentropically to an exit velocity of 1500 ft/s. Determine

(a) the exit pressure, in lbf/in.^{2}
(b) the ratio of the exit area to the inlet area.
(c) whether the nozzle is diverging only, converging only, or converging–diverging in cross section.

9.128 Air as an ideal gas with $k = 1.4$ enters a converging–diverging nozzle operating at steady state and expands isentropically as shown in Fig. P9.128. Using data from the figure and from Table 9.2 as needed, determine

(a) the stagnation pressure, in lbf/in.^{2}, and the stagnation temperature, in °R.
(b) the throat area, in in.^{2}
(c) the exit area, in in.^{2}

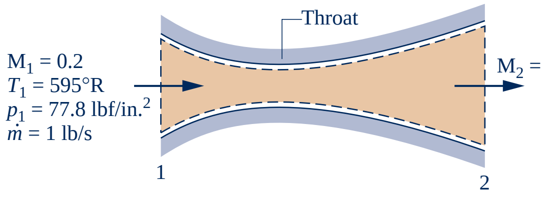


Fig. P9.128

9.129 Air as an ideal gas with $k = 1.4$ enters a diffuser operating at steady state at 4 bar, 290 K, with a velocity of 512 m/s. Assuming isentropic flow, plot the velocity, in m/s, the Mach number, and the area ratio A/A* for locations in the flow corresponding to pressures ranging from 4 to 14 bar.

9.130 A converging–diverging nozzle operating at steady state has a throat area of 3 cm^2 and an exit area of 6 cm^2. Air as an ideal gas with $k = 1.4$ enters the nozzle at 8 bar, 400 K, and a Mach number of 0.2, and flows isentropically throughout. If the nozzle is choked, and the diverging portion acts as a supersonic nozzle, determine the mass flow rate, in kg/s, and the Mach number, pressure, in bar, and temperature, in K, at the exit. Repeat if the diverging portion acts as a supersonic diffuser.

9.131 A converging–diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ enters the nozzle at 500 K, 6 bar, and a Mach number of 0.3. The air flows isentropically to the exit plane, where a normal shock stands. The temperature just upstream of the shock is 380.416 K. Determine the back pressure, in bar.

9.132 A converging–diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ enters the nozzle at 500 K, 6 bar, and a Mach number of 0.3. A normal shock stands in the diverging section at a location where the Mach number 1.40. The cross-sectional areas of the throat and the exit plane are 4 cm^2 and 6 cm^2, respectively. The flow is isentropic, except where the shock stands. Determine the exit pressure, in bar, and the mass flow rate, in kg/s.

9.133 Air as an ideal gas with $k = 1.4$ enters a converging-diverging duct with a Mach number of 2. At the inlet, the pressure is 26 lbf/in.^{2} and the temperature is 445°R. A normal shock stands at a location in the converging section of the duct, with $M_x = 1.5$. At the exit of the duct, the pressure is 150 lbf/in.^{2} The flow is isentropic everywhere except in the immediate vicinity of the shock. Determine temperature, in °R, and the Mach number at the exit.

9.134 Air as an ideal gas with $k = 1.4$ undergoes a normal shock. The upstream conditions are $p_x = 0.5$ bar, $T_x = 280$ K, and $M_x = 1.8$. Determine

(a) the pressure p_y, in bar.
(b) the stagnation pressure p_{ox}, in bar.
(c) the stagnation temperature T_{ox}, in K.
(d) the change in specific entropy across the shock, in kJ/kg · K.
(e) Plot the quantities of parts (a)–(d) versus M_x ranging from 1.0 to 2.0. All other upstream conditions remain the same.

9.135 A converging–diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ flows through the nozzle, discharging to the atmosphere at 14.7 lbf/in.^{2} and 520°R. A normal shock stands at the exit plane with $M_x = 1.5$. The exit plane area is 1.8 in.^{2} Upstream of the shock, the flow is isentropic. Determine

(a) the stagnation pressure p_{ox}, in lbf/in.^{2}
(b) the stagnation temperature T_{ox}, in °R.
(c) the mass flow rate, in lb/s.

9.136 A converging–diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ flows through the nozzle, discharging to the atmosphere at 14.7 lbf/in.^{2} and 510°R. A normal shock stands at the exit plane with $p_x = 9.714$ lbf/in.^{2} The exit plane area is 2 in.^{2} Upstream of the shock, the flow is isentropic. Determine

(a) the throat area, in in.^{2}
(b) the entropy produced in the nozzle, in Btu/°R per lb of air flowing.

9.137 Air at 3.4 bar, 530 K, and a Mach number of 0.4 enters a converging–diverging nozzle operating at steady state. A normal shock stands in the diverging section at a location where the Mach number is $M_x = 1.8$. The flow is isentropic, except where the shock stands. If the air behaves as an ideal gas with $k = 1.4$, determine

(a) the stagnation temperature T_{ox}, in K.
(b) the stagnation pressure p_{ox}, in bar.
(c) the pressure p_x, in bar.
(d) the pressure p_y, in bar.
(e) the stagnation pressure p_{oy}, in bar.
(f) the stagnation temperature T_{oy}, in K.

If the throat area is 7.6×10^{-4} m^2, and the exit plane pressure is 2.4 bar, determine the mass flow rate, in kg/s, and the exit area, in m^2.

9.138 Air as an ideal gas with $k = 1.4$ enters a converging–diverging channel at a Mach number of 1.6. A normal shock stands at the inlet to the channel. Downstream of the shock the flow is isentropic; the Mach number is unity at the throat; and the air exits at 20 lbf/in.^{2}, 700°R, with negligible velocity. If the mass flow rate is 45 lb/s, determine the inlet and throat areas, in ft^2.

9.139 Derive the following expressions: (a) Eq. 9.55, (b) Eq. 9.56, (c) Eq. 9.57.

9.140 Using *Interactive Thermodynamics: IT*, generate tables of the same isentropic flow functions as in Table 9.2 for specific heat ratios of 1.2, 1.3, 1.4, and 1.67 and Mach numbers ranging from 0 to 5.

9.141 Using *Interactive Thermodynamics: IT*, generate tables of the same normal shock functions as in Table 9.3 for specific heat ratios of 1.2, 1.3, 1.4, and 1.67 and Mach numbers ranging from 1 to 5.

▶ DESIGN & OPEN-ENDED PROBLEMS: EXPLORING ENGINEERING PRACTICE

9.1D Congress has mandated the average fuel economy for passenger cars sold in the United States to be 35 miles per gallon beginning in 2020. In Europe, the goal is 47 miles per gallon by 2012. In each case, identify the major factors spurring legislative action, including, as appropriate, technical, economic, societal, and political factors. Analyze the disparity between these goals. Comment on their likely effectiveness in achieving the respective legislative aims. Report your findings in a PowerPoint presentation.

9.2D Automotive gas turbines have been under development for decades but have not been commonly used in automobiles. Yet helicopters routinely use gas turbines. Explore why different types of engines are used in these respective applications. Compare selection factors such as performance, power-to-weight ratio, space requirements, fuel availability, and environmental impact. Summarize your findings in a report with at least three references.

9.3D The *Annual Energy Outlook with Projections* report released by the U.S. Energy Information Administration projects annual consumption estimates for various fuel types through the next 25 years. According to the report, *biofuels* will play an increasing role in the liquid fuel supply over that time period. Based on technologies commercially available or reasonably expected to become available in the next decade, identify the most viable options for producing biofuels. Compare several options based on energy return on energy invested (EROEI), water and land requirements, and effects on global climate change. Draw conclusions based on your study, and present your findings in a report with at least three references.

9.4D Investigate the following technologies: plug-in hybrid vehicles, all-electric vehicles, hydrogen fuel cell vehicles, diesel-powered vehicles, natural gas–fueled vehicles, and ethanol-fueled vehicles, and make recommendations on which of these technologies should receive federal research, development, and deployment support over the next decade. Base your recommendation on the result of a decision matrix method such as the *Pugh method* to compare the various technologies. Clearly identify and justify the criteria used for the comparison and the logic behind the scoring process. Prepare a 15-minute briefing and an executive summary suitable for a conference with your local congressperson.

9.5D Figure P9.5D shows a wheeled platform propelled by thrust generated using an onboard water tank discharging water through an elbow to which a nozzle is attached. Design and construct such an apparatus using easily obtained materials like a skateboard and one-gallon milk jug. Investigate the effects of elbow angle and nozzle exit area on volumetric flow and thrust. Prepare a report including results and conclusions together with an explanation of the measurement techniques and experimental procedures.

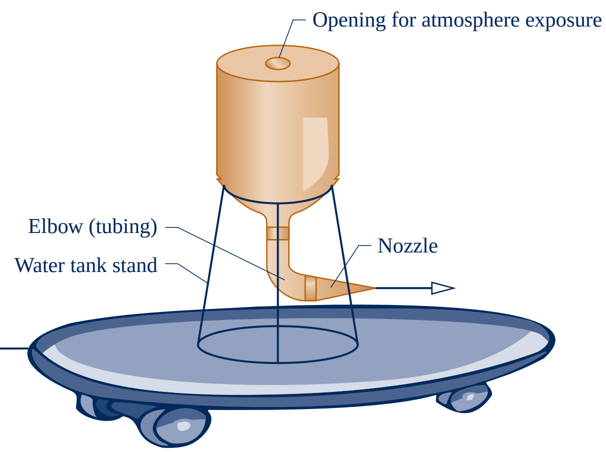


Fig. P9.5D

9.6D Owing to its very low temperature relative to seawater, liquid natural gas (LNG) arriving at U.S. ports by ship has considerable thermomechanical exergy. Yet when LNG is regasified in heat exchangers where seawater is the other stream, that exergy is largely destroyed. Conduct a search of the patent literature for methods to recover a substantial portion of LNG exergy during regasification. Consider patents both granted and pending. Critically evaluate the technical merit and economic feasibility of two different methods found in your search. Report your conclusions in an executive summary and PowerPoint presentation.

9.7D Develop the preliminary specifications for a 160-MW closed-cycle gas turbine power plant. Consider carbon dioxide and helium as possible gas turbine working fluids that circulate through a nuclear power unit where they absorb energy by heat transfer. Sketch the schematic of your proposed cycle. For each of the two working fluids, determine key operating pressures and temperatures of the gas turbine cycle and estimate the expected performance of the cycle. Write a report that includes your analysis and design, and recommend a working fluid. Include at least three references.

9.8D As an engineer you must recommend whether or not to purchase 2 MW of electric power from the local utility company at a cost of \$0.06 per kW · h for a plant expansion or to purchase and operate a diesel engine generator set that runs on natural gas. Assume natural gas is priced at 6 dollars per thousand cubic feet, each cubic foot of gas has a *heating value* of 1000 Btu, and the plant operates 7800 h per year. Specify diesel-generator equipment that would supply the required power and perform an economic analysis to determine which alternative is best. Present your recommendation in a memorandum with supporting analysis and calculations, including at least three references.

9.9D A financial services company has a computer server facility that requires very reliable electric power. The power demand is 3000 kW. The company has hired you as a consultant to study the feasibility of using 250-kW *microturbines* for this application. Write a report discussing the pros and cons of such an arrangement compared to purchasing power from the local utility.

9.10D Figure P9.10D shows a combined cycle formed by a topping gas turbine and an organic Rankine bottoming cycle. Steady-state operating data are labeled on the figure. Owing to internal irreversibilities, the generator electricity output is 95% of the input shaft power. The regenerator preheats air entering the combustor. In the evaporator, hot exhaust gas from the regenerator vaporizes the bottoming

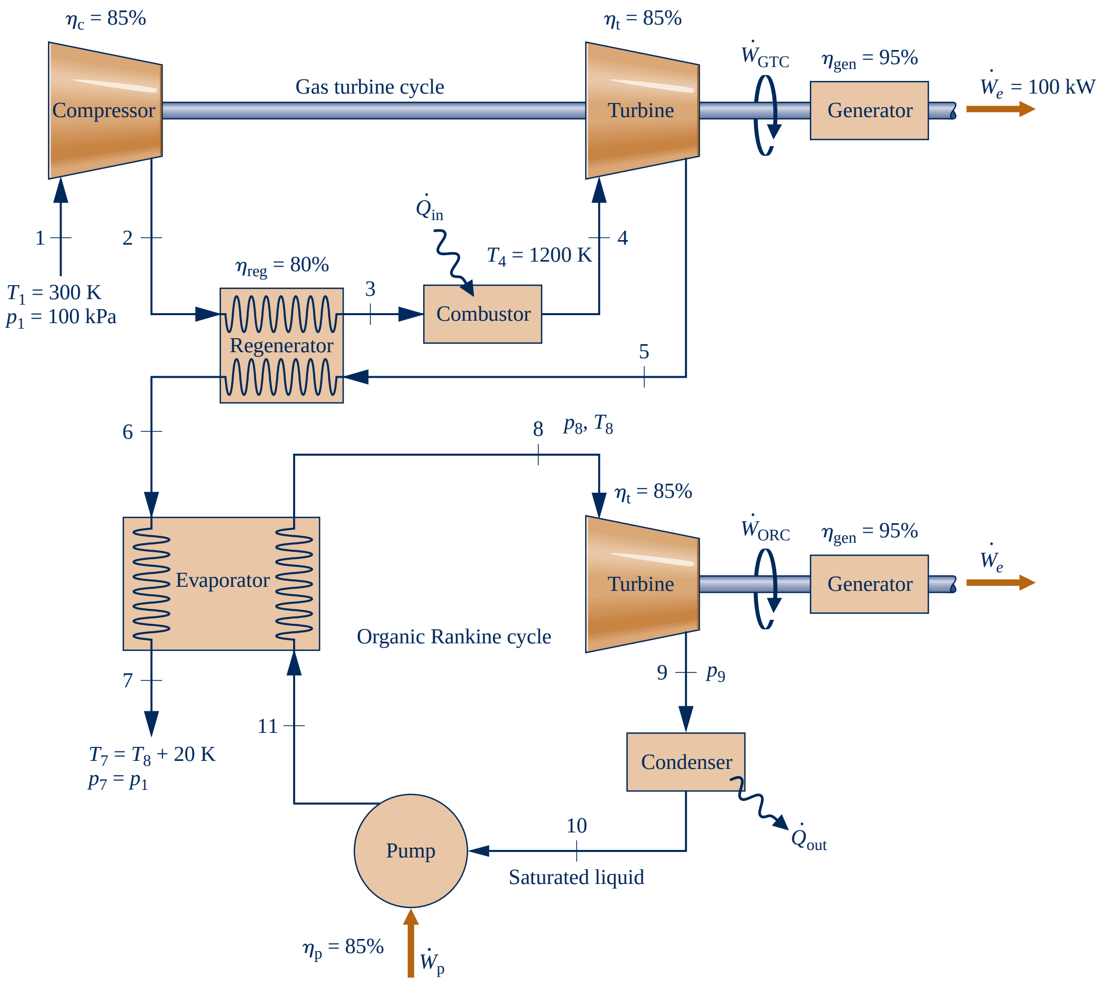


Fig. P9.10D

cycle working fluid. For each of three such working fluids—propane, Refrigerant 22, and Refrigerant 134a—specify appropriate ranges for p_8, turbine inlet pressure, and T_8, turbine inlet temperature; also determine turbine exit pressure p_9. For each working fluid, investigate the influence on net combined-cycle electricity production and on combined-cycle thermal efficiency of varying p_8, T_8, and compressor pressure ratio. Identify the bottoming working fluid and operating conditions with greatest net combined-cycle electricity production. Repeat for greatest combined-cycle thermal efficiency. Apply engineering modeling compatible with that used in the text for Rankine cycles and air-standard analysis of gas turbines. Present your analyses, results, and recommendations in a technical article adhering to ASME standards with at least three references.

9.11D Micro-CHP (combined heat and power) units capable of producing up to 1.8 kW of electric power are now commercially available for use in the home. Such units contribute to domestic space or water heating needs while providing electricity as a by-product. They operate on an internal combustion engine fueled by natural gas. By hybridizing a micro-CHP unit with a gas furnace, *all* domestic heating needs can be met while generating a substantial portion of the annual electric power requirement. Evaluate this hybrid form for application to a typical single-family dwelling in your locale having natural gas service. Consider on-site battery

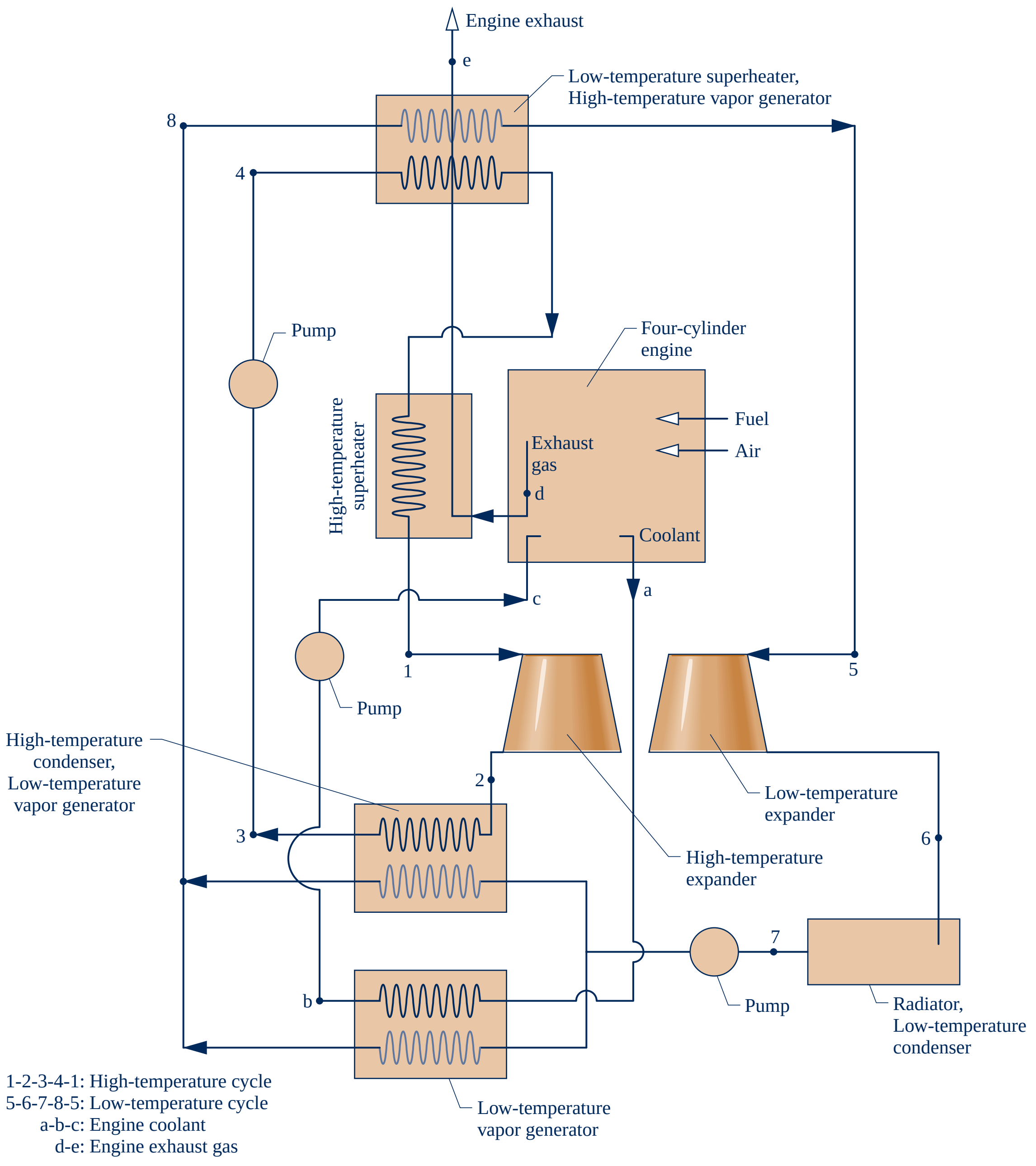


Fig. P9.12D

storage of excess electricity generated and the possibility of *net metering*. Specify equipment and determine costs, including initial cost and installation cost. Estimate the annual cost for heating and power using the hybrid and compare it to the annual cost for heating and power with a stand-alone gas furnace and grid power. Using your findings, recommend the better approach for the dwelling.

9.12D Figure P9.12D provides the schematic of an internal combustion automobile engine fitted with two Rankine vapor power bottoming cycles: a high-temperature cycle 1-2-3-4-1 and a low-temperature cycle 5-6-7-8-5. These cycles develop additional power using *waste heat* derived from exhaust gas and engine coolant. Using operating data for a commercially available car having a conventional four-cylinder internal combustion engine with a size of 2.5 liters or less, specify cycle working fluids and state data at key points sufficient to produce at least 15 hp more power. Apply engineering modeling compatible with that used in the text for Rankine cycles and air-standard analysis of internal combustion engines. Write a final report justifying your specifications together with supporting calculations. Provide a critique of the use of such bottoming cycles on car engines and a recommendation about whether such technology should be actively pursued by automakers.

Refrigeration systems **commonly used for preserving food are introduced in Sec. 10.1.**
© Mike Kemp RubberBall/Age Fotostock America, Inc.

ENGINEERING CONTEXT Refrigeration systems for food preservation and air conditioning play prominent roles in our everyday lives. Heat pumps also are used for heating buildings and for producing industrial process heat. There are many other examples of commercial and industrial uses of refrigeration, including air separation to obtain liquid oxygen and liquid nitrogen, liquefaction of natural gas, and production of ice.

To achieve refrigeration by most conventional means requires an electric power input. Heat pumps also require power to operate. Referring again to Table 8.1, we see that in the United States electricity is obtained today primarily from coal, natural gas, and nuclear, all of which are nonrenewable. These nonrenewables have significant adverse effects on human health and the environment associated with their use. Depending on the type of resource, such effects are related to extraction from the earth, processing and distribution, emissions during power production, and waste products.

Ineffective refrigeration and heat pump systems, excessive building cooling and heating, and other wasteful practices and lifestyle choices not only misuse increasingly scarce nonrenewable resources but also endanger our health and burden the environment. Accordingly, refrigeration and heat pump systems is an area of application where more effective systems and practices can significantly improve our national energy posture.

The **objective** of this chapter is to describe some of the common types of refrigeration and heat pump systems presently in use and to illustrate how such systems can be modeled thermodynamically. The three principal types described are the vapor-compression, absorption, and reversed Brayton cycles. As for the power systems studied in Chaps. 8 and 9, both vapor and gas systems are considered. In vapor systems, the refrigerant is alternately vaporized and condensed. In gas refrigeration systems, the refrigerant remains a gas.

10

Refrigeration and Heat Pump Systems

▶ LEARNING OUTCOMES

When you complete your study of this chapter, you will be able to...

- Demonstrate understanding of basic vapor-compression refrigeration and heat pump systems.
- Develop and analyze thermodynamic models of vapor-compression systems and their modifications, including
  - sketching schematic and accompanying *T–s* diagrams.
  - evaluating property data at principal states of the systems.
  - applying mass, energy, entropy, and exergy balances for the basic processes.
  - determining refrigeration and heat pump system performance, coefficient of performance and capacity.
- Explain the effects on vapor-compression system performance of varying key parameters.
- Demonstrate understanding of the operating principles of absorption and gas refrigeration systems, and perform thermodynamic analysis of gas systems.

Refrig_Cycle
A.10 – All Tabs

10.1 Vapor Refrigeration Systems

The purpose of a refrigeration system is to maintain a *cold* region at a temperature below the temperature of its surroundings. This is commonly achieved using the vapor refrigeration systems that are the subject of the present section.

10.1.1 Carnot Refrigeration Cycle

To introduce some important aspects of vapor refrigeration, let us begin by considering a Carnot vapor refrigeration cycle. This cycle is obtained by reversing the Carnot vapor power cycle introduced in Sec. 5.10. Figure 10.1 shows the schematic and accompanying T–s diagram of a Carnot refrigeration cycle operating between a region at temperature T_C and another region at a higher temperature T_H. The cycle is executed by a refrigerant circulating steadily through a series of components. All processes are internally reversible. Also, since heat transfers between the refrigerant and each region occur with no temperature differences, there are no external irreversibilities. The energy transfers shown on the diagram are positive in the directions indicated by the arrows.

Let us follow the refrigerant as it passes steadily through each of the components in the cycle, beginning at the inlet to the evaporator. The refrigerant enters the evaporator as a two-phase liquid–vapor mixture at state 4. In the evaporator some of the refrigerant changes phase from liquid to vapor as a result of heat transfer from the region at temperature T_C to the refrigerant. The temperature and pressure of the refrigerant remain constant during the process from state 4 to state 1. The refrigerant is then compressed adiabatically from state 1, where it is a two-phase liquid–vapor mixture, to state 2, where it is a saturated vapor. During this process, the temperature of the refrigerant increases from T_C to T_H, and the pressure also increases. The refrigerant passes from the compressor into the condenser, where it changes phase from saturated vapor to saturated liquid as a result of heat transfer to the region at temperature T_H. The temperature and pressure remain constant in the process from state 2 to state 3. The refrigerant returns to the state at the inlet of the evaporator by expanding adiabatically through a turbine. In this process, from state 3 to state 4, the temperature decreases from T_H to T_C, and there is a decrease in pressure.

TAKE NOTE...
See Sec. 6.13.1 for the area interpretation of heat transfer on a T–s diagram for the case of internally reversible flow though a control volume at steady state.

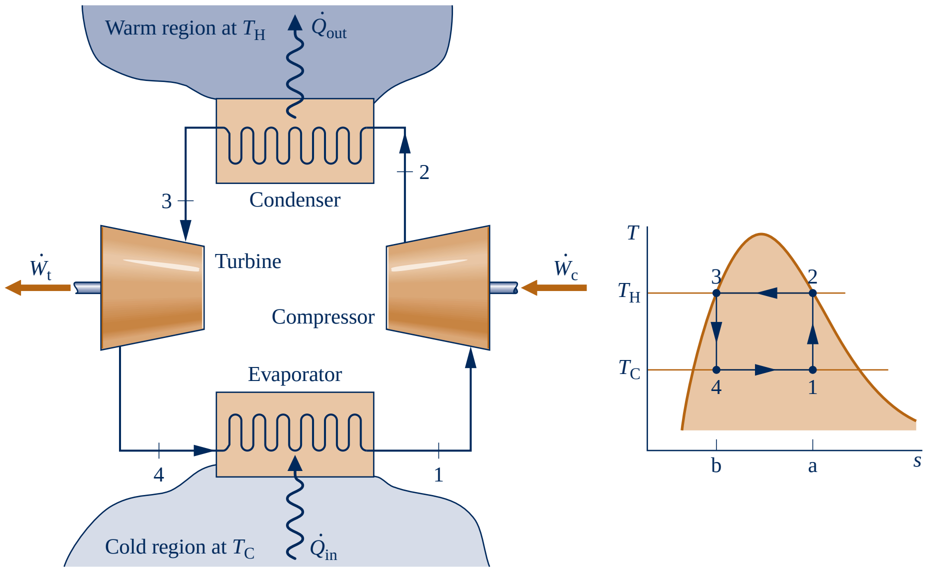


Fig. 10.1 **Carnot vapor refrigeration cycle.**

Since the Carnot vapor refrigeration cycle is made up of internally reversible processes, areas on the *T–s* diagram can be interpreted as heat transfers. Area 1–a–b–4–1 is the heat added to the refrigerant from the cold region per unit mass of refrigerant flowing. Area 2–a–b–3–2 is the heat rejected from the refrigerant to the warm region per unit mass of refrigerant flowing. The enclosed area 1–2–3–4–1 is the *net* heat transfer *from* the refrigerant. The net heat transfer *from* the refrigerant equals the net work done *on* the refrigerant. The net work is the difference between the compressor work input and the turbine work output.

The *coefficient of performance* β of *any* refrigeration cycle is the ratio of the refrigeration effect to the net work input required to achieve that effect. For the Carnot vapor refrigeration cycle shown in Fig. 10.1, the coefficient of performance is

$$\beta_{\max} = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_{\text{c}}/\dot{m} - \dot{W}_{\text{t}}/\dot{m}} = \frac{\text{area 1–a–b–4–1}}{\text{area 1–2–3–4–1}} = \frac{T_{\text{C}}(s_{\text{a}} - s_{\text{b}})}{(T_{\text{H}} - T_{\text{C}})(s_{\text{a}} - s_{\text{b}})}$$

which reduces to

$$\boxed{\beta_{\max} = \frac{T_{\text{C}}}{T_{\text{H}} - T_{\text{C}}}} \tag{10.1}$$

This equation, which corresponds to Eq. 5.10, represents the *maximum* theoretical coefficient of performance of any refrigeration cycle operating between regions at T_{C} and T_{H}.

10.1.2 Departures from the Carnot Cycle

Actual vapor refrigeration systems depart significantly from the Carnot cycle considered above and have coefficients of performance lower than would be calculated from Eq. 10.1. Three ways actual systems depart from the Carnot cycle are considered next.

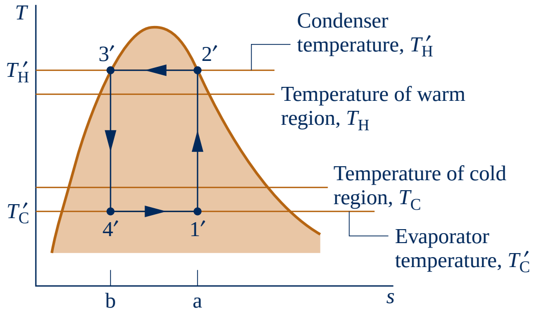


Fig. 10.2 Comparison of the condenser and evaporator temperatures with those of the warm and cold regions.

▶ One of the most significant departures is related to the heat transfers between the refrigerant and the two regions. In actual systems, these heat transfers are not accomplished reversibly as presumed above. In particular, to achieve a rate of heat transfer sufficient to maintain the temperature of the cold region at T_{C} with a practical-sized evaporator requires the temperature of the refrigerant in the evaporator, T'_{C}, to be several degrees *below* T_{C}. This is illustrated by the placement of the temperature T'_{C} on the *T–s* diagram of Fig. 10.2. Similarly, to obtain a sufficient heat transfer rate from the refrigerant to the warm region requires that the refrigerant temperature in the condenser, T'_{H}, be several degrees *above* T_{H}. This is illustrated by the placement of the temperature T'_{H} on the *T–s* diagram of Fig. 10.2.

Maintaining the refrigerant temperatures in the heat exchangers at T'_{C} and T'_{H} rather than at T_{C} and T_{H}, respectively, has the effect of reducing the coefficient of performance. This can be seen by expressing the coefficient of performance of the refrigeration cycle designated by 1′–2′–3′–4′–1′ on Fig. 10.2 as

$$\beta' = \frac{\text{area 1′–a–b–4′–1}}{\text{area 1′–2′–3′–4′–1′}} = \frac{T'_{\text{C}}}{T'_{\text{H}} - T'_{\text{C}}} \tag{10.2}$$

Comparing the areas underlying the expressions for $\beta_{\max}$ and β' given above, we conclude that the value of β' is less than $\beta_{\max}$. This conclusion about the effect of refrigerant temperature on the coefficient of performance also applies to other refrigeration cycles considered in the chapter.

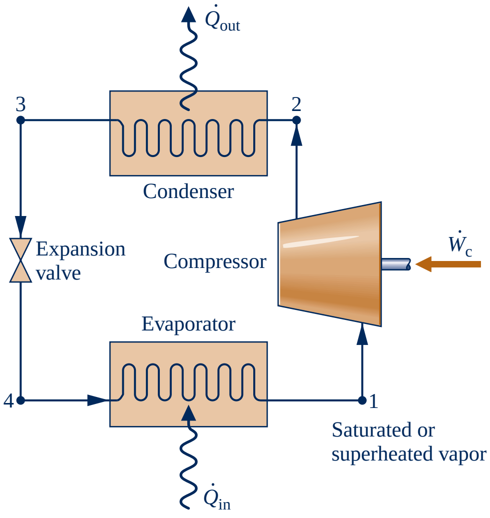


Fig. 10.3 Components of a vapor-compression refrigeration system.

► Even when the temperature differences between the refrigerant and warm and cold regions are taken into consideration, there are other features that make the vapor refrigeration cycle of Fig. 10.2 impractical as a prototype. Referring again to the figure, note that the compression process from state 1′ to state 2′ occurs with the refrigerant as a two-phase liquid–vapor mixture. This is commonly referred to as *wet compression*. Wet compression is normally avoided because the presence of liquid droplets in the flowing liquid–vapor mixture can damage the compressor. In actual systems, the compressor handles vapor only. This is known as *dry compression*.

► Another feature that makes the cycle of Fig. 10.2 impractical is the expansion process from the saturated liquid state 3′ to the low-quality, two-phase liquid–vapor mixture state 4′. This expansion typically produces a relatively small amount of work compared to the work input in the compression process. The work developed by an actual turbine would be smaller yet because turbines operating under these conditions have low isentropic efficiencies. Accordingly, the work output of the turbine is normally sacrificed by substituting a simple throttling valve for the expansion turbine, with consequent savings in initial and maintenance costs. The components of the resulting cycle are illustrated in Fig. 10.3, where dry compression is presumed. This cycle, known as the *vapor-compression refrigeration cycle,* is the subject of the section to follow.

10.2 Analyzing Vapor-Compression Refrigeration Systems

vapor-compression refrigeration

Vapor-compression refrigeration systems are the most common refrigeration systems in use today. The object of this section is to introduce some important features of systems of this type and to illustrate how they are modeled thermodynamically.

10.2.1 Evaluating Principal Work and Heat Transfers

Let us consider the steady-state operation of the vapor-compression system illustrated in Fig. 10.3. Shown on the figure are the principal work and heat transfers, which are positive in the directions of the arrows. Kinetic and potential energy changes are neglected in the following analyses of the components. We begin with the evaporator, where the desired refrigeration effect is achieved.

► As the refrigerant passes through the evaporator, heat transfer from the refrigerated space results in the vaporization of the refrigerant. For a control volume enclosing the refrigerant side of the evaporator, the mass and energy rate balances reduce to give the rate of heat transfer per unit mass of refrigerant flowing as

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 \tag{10.3}$$

refrigeration capacity

ton of refrigeration

where $\dot{m}$ is the mass flow rate of the refrigerant. The heat transfer rate $\dot{Q}_{in}$ is referred to as the **refrigeration capacity**. In the SI unit system, the capacity is normally expressed in kW. In the English unit system, the refrigeration capacity may be expressed in Btu/h. Another commonly used unit for the refrigeration capacity is the **ton of refrigeration**, which is equal to 200 Btu/min or about 211 kJ/min.

► The refrigerant leaving the evaporator is compressed to a relatively high pressure and temperature by the compressor. Assuming no heat transfer to or from the compressor, the mass and energy rate balances for a control volume enclosing the compressor give

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1 \tag{10.4}$$

where $\dot{W}_c/\dot{m}$ is the rate of power *input* per unit mass of refrigerant flowing.

▶ Next, the refrigerant passes through the condenser, where the refrigerant condenses and there is heat transfer from the refrigerant to the cooler surroundings. For a control volume enclosing the refrigerant side of the condenser, the rate of heat transfer from the refrigerant per unit mass of refrigerant flowing is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 \tag{10.5}$$

▶ Finally, the refrigerant at state 3 enters the expansion valve and expands to the evaporator pressure. This process is usually modeled as a *throttling* process for which

$$h_4 = h_3 \tag{10.6}$$

The refrigerant pressure decreases in the irreversible adiabatic expansion, and there is an accompanying increase in specific entropy. The refrigerant exits the valve at state 4 as a two-phase liquid–vapor mixture.

In the vapor-compression system, the net power input is equal to the compressor power, since the expansion valve involves no power input or output. Using the quantities and expressions introduced above, the coefficient of performance of the vapor-compression refrigeration system of Fig. 10.3 is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{h_1 - h_4}{h_2 - h_1} \tag{10.7}$$

Provided states 1 through 4 are fixed, Eqs. 10.3 through 10.7 can be used to evaluate the principal work and heat transfers and the coefficient of performance of the vapor-compression system shown in Fig. 10.3. Since these equations have been developed by reducing mass and energy rate balances, they apply equally for actual performance when irreversibilities are present in the evaporator, compressor, and condenser and for idealized performance in the absence of such effects. Although irreversibilities in the evaporator, compressor, and condenser can have a pronounced effect on overall performance, it is instructive to consider an idealized cycle in which they are assumed absent. Such a cycle establishes an upper limit on the performance of the vapor-compression refrigeration cycle. It is considered next.

10.2.2 Performance of Ideal Vapor-Compression Systems

If irreversibilities within the evaporator and condenser are ignored, there are no frictional pressure drops, and the refrigerant flows at constant pressure through the two heat exchangers. If compression occurs without irreversibilities, and stray heat transfer to the surroundings is also ignored, the compression process is isentropic. With these considerations, the vapor-compression refrigeration cycle labeled 1–2s–3–4–1 on the *T–s* diagram of Fig. 10.4 results. The cycle consists of the following series of processes:

Process 1–2s: *Isentropic* compression of the refrigerant from state 1 to the condenser pressure at state 2s.

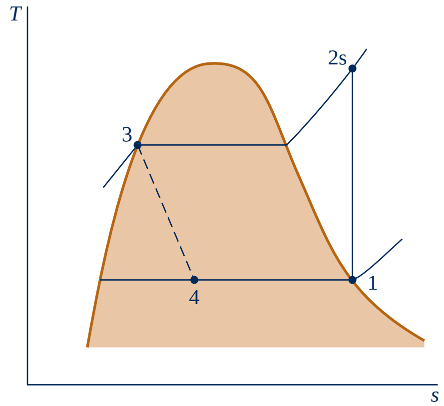


Fig. 10.4 ***T–s*** **diagram of an ideal vapor-compression cycle.**

VCRC
A.30 – Tab a

Process 2s–3: Heat transfer *from* the refrigerant as it flows at constant pressure through the condenser. The refrigerant exits as a liquid at state 3.
Process 3–4: *Throttling* process from state 3 to a two-phase liquid–vapor mixture at 4.
Process 4–1: Heat transfer *to* the refrigerant as it flows at constant pressure through the evaporator to complete the cycle.

ideal vapor-compression cycle

All processes of the cycle shown in Fig. 10.4 are internally reversible except for the throttling process. Despite the inclusion of this irreversible process, the cycle is commonly referred to as the **ideal vapor-compression cycle**.

The following example illustrates the application of the first and second laws of thermodynamics along with property data to analyze an ideal vapor-compression cycle.

EXAMPLE 10.1

Analyzing an Ideal Vapor-Compression Refrigeration Cycle

Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle that communicates thermally with a cold region at 0°C and a warm region at 26°C. Saturated vapor enters the compressor at 0°C and saturated liquid leaves the condenser at 26°C. The mass flow rate of the refrigerant is 0.08 kg/s. Determine **(a)** the compressor power, in kW, **(b)** the refrigeration capacity, in tons, **(c)** the coefficient of performance, and **(d)** the coefficient of performance of a Carnot refrigeration cycle operating between warm and cold regions at 26 and 0°C, respectively.

SOLUTION

Known: An ideal vapor-compression refrigeration cycle operates with Refrigerant 134a. The states of the refrigerant entering the compressor and leaving the condenser are specified, and the mass flow rate is given.

Find: Determine the compressor power, in kW, the refrigeration capacity, in tons, coefficient of performance, and the coefficient of performance of a Carnot vapor refrigeration cycle operating between warm and cold regions at the specified temperatures.

Schematic and Given Data:

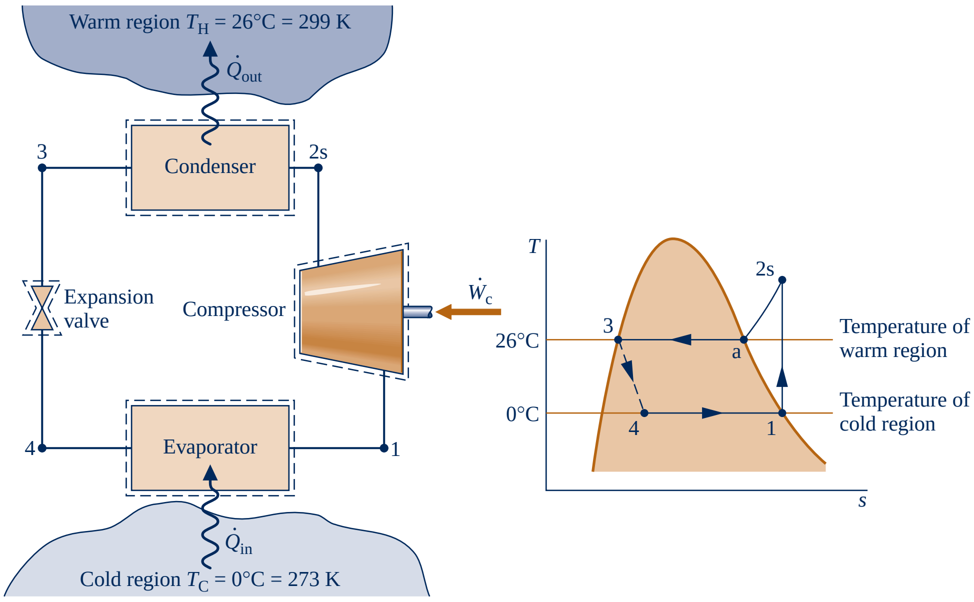


Fig. E10.1

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying sketch.

2. Except for the expansion through the valve, which is a throttling process, all processes of the refrigerant are internally reversible.

3. The compressor and expansion valve operate adiabatically.

4. Kinetic and potential energy effects are negligible.

5. Saturated vapor enters the compressor, and saturated liquid leaves the condenser.

Analysis: Let us begin by fixing each of the principal states located on the accompanying schematic and T–s diagrams. At the inlet to the compressor, the refrigerant is a saturated vapor at 0°C, so from Table A-10, $h_1 = 247.23$ kJ/kg and $s_1 = 0.9190$ kJ/kg · K.

The pressure at state 2s is the saturation pressure corresponding to 26°C, or $p_2 = 6.853$ bar. State 2s is fixed by p_2 and the fact that the specific entropy is constant for the adiabatic, internally reversible compres-
1 sion process. The refrigerant at state 2s is a superheated vapor with $h_{2s} = 264.7$ kJ/kg.

State 3 is saturated liquid at 26°C, so $h_3 = 85.75$ kJ/kg. The expansion through the valve is a throttling process (assumption 2), so $h_4 = h_3$.

(a) The compressor work input is

$$\dot{W}_c = \dot{m}(h_{2s} - h_1)$$

where $\dot{m}$ is the mass flow rate of refrigerant. Inserting values

$$\dot{W}_c = (0.08 \text{ kg/s})(264.7 - 247.23) \text{ kJ/kg} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 1.4 \text{ kW}$$

(b) The refrigeration capacity is the heat transfer rate to the refrigerant passing through the evaporator. This is given by

$$\begin{aligned} \dot{Q}_{in} &= \dot{m}(h_1 - h_4) \\ &= (0.08 \text{ kg/s})|60 \text{ s/min}|(247.23 - 85.75) \text{ kJ/kg} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| \\ &= 3.67 \text{ ton} \end{aligned}$$

(c) The coefficient of performance β is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{h_1 - h_4}{h_{2s} - h_1} = \frac{247.23 - 85.75}{264.7 - 247.23} = 9.24$$

(d) For a Carnot vapor refrigeration cycle operating at $T_H = 299$ K and $T_C = 273$ K, the coefficient of performance determined from Eq. 10.1 is

2 $$\beta_{max} = \frac{T_C}{T_H - T_C} = 10.5$$

1 The value for h_{2s} can be obtained by double interpolation in Table A-12 or by using *Interactive Thermodynamics: IT*.

2 As expected, the ideal vapor-compression cycle has a lower coefficient of performance than a Carnot cycle operating between the temperatures of the warm and cold regions. The smaller value can be attributed to the effects of the external irreversibility associated with desuperheating the refrigerant in the condenser (Process 2s–a on the T–s diagram) and the internal irreversibility of the throttling process.

Skills Developed

Ability to...

- ❑ sketch the T–s diagram of the ideal vapor compression refrigeration cycle.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ calculate refrigeration capacity and coefficient of performance.
- ❑ compare with the corresponding Carnot refrigeration cycle.

QuickQUIZ Keeping all other given data the same, determine the mass flow rate of refrigerant, in kg/s, for a 10-ton refrigeration capacity. Ans. 0.218 kg/s.

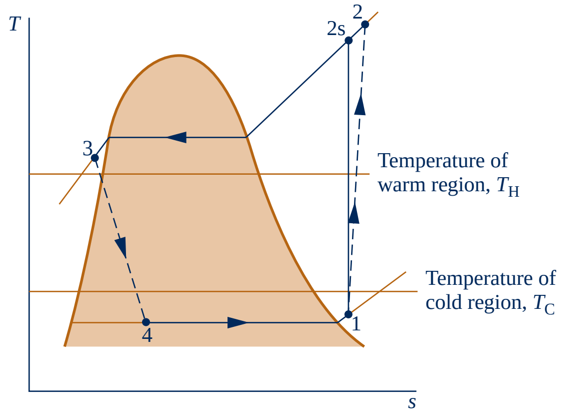


Fig. 10.5 ***T–s* diagram of an actual vapor-compression cycle.**

10.2.3 Performance of Actual Vapor-Compression Systems

VCRC
A.30 – Tab b

Figure 10.5 illustrates several features exhibited by *actual* vapor-compression systems. As shown in the figure, the heat transfers between the refrigerant and the warm and cold regions are not accomplished reversibly: the refrigerant temperature in the evaporator is less than the cold region temperature, T_C, and the refrigerant temperature in the condenser is greater than the warm region temperature, T_H. Such irreversible heat transfers have a significant effect on performance. In particular, the coefficient of performance decreases as the average temperature of the refrigerant in the evaporator decreases and as the average temperature of the refrigerant in the condenser increases. Example 10.2 provides an illustration.

EXAMPLE 10.2

Considering the Effect of Irreversible Heat Transfer on Performance

Modify Example 10.1 to allow for temperature differences between the refrigerant and the warm and cold regions as follows. Saturated vapor enters the compressor at −10°C. Saturated liquid leaves the condenser at a pressure of 9 bar. Determine for the modified vapor-compression refrigeration cycle **(a)** the compressor power, in kW, **(b)** the refrigeration capacity, in tons, **(c)** the coefficient of performance. Compare results with those of Example 10.1.

SOLUTION

Known: An ideal vapor-compression refrigeration cycle operates with Refrigerant 134a as the working fluid. The evaporator temperature and condenser pressure are specified, and the mass flow rate is given.

Find: Determine the compressor power, in kW, the refrigeration capacity, in tons, and the coefficient of performance. Compare results with those of Example 10.1.

Schematic and Given Data:

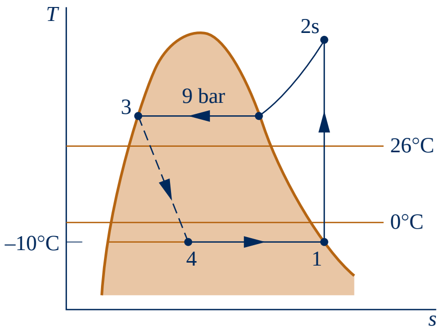


Fig. E10.2

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the sketch accompanying Example 10.1.
2. Except for the process through the expansion valve, which is a throttling process, all processes of the refrigerant are internally reversible.
3. The compressor and expansion valve operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated vapor enters the compressor, and saturated liquid exits the condenser.

Analysis: Let us begin by fixing each of the principal states located on the accompanying T–s diagram. Starting at the inlet to the compressor, the refrigerant is a saturated vapor at $-10°C$, so from Table A-10, $h_1 = 241.35$ kJ/kg and $s_1 = 0.9253$ kJ/kg · K.

The superheated vapor at state 2s is fixed by $p_2 = 9$ bar and the fact that the specific entropy is constant for the adiabatic, internally reversible compression process. Interpolating in Table A-12 gives $h_{2s} = 272.39$ kJ/kg.

State 3 is a saturated liquid at 9 bar, so $h_3 = 99.56$ kJ/kg. The expansion through the valve is a throttling process; thus, $h_4 = h_3$.

(a) The compressor power input is

$$\dot{W}_c = \dot{m}(h_{2s} - h_1)$$

where $\dot{m}$ is the mass flow rate of refrigerant. Inserting values

$$\dot{W}_c = (0.08 \text{ kg/s})(272.39 - 241.35) \text{ kJ/kg} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$
$$= 2.48 \text{ kW}$$

(b) The refrigeration capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$
$$= (0.08 \text{ kg/s})|60 \text{ s/min}|(241.35 - 99.56) \text{ kJ/kg} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right|$$
$$= 3.23 \text{ ton}$$

(c) The coefficient of performance β is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{h_1 - h_4}{h_{2s} - h_1} = \frac{241.35 - 99.56}{272.39 - 241.35} = 4.57$$

Comparing the results of the present example with those of Example 10.1, we see that the power input required by the compressor is greater in the present case. Furthermore, the refrigeration capacity and coefficient of performance are smaller in this example than in Example 10.1. This illustrates the considerable influence on performance of irreversible heat transfer between the refrigerant and the cold and warm regions.

Skills Developed

Ability to...

- sketch the T–s diagram of the ideal vapor compression refrigeration cycle.
- fix each of the principal states and retrieve necessary property data.
- calculate compressor power, refrigeration capacity, and coefficient of performance.

QuickQUIZ Determine the rate of heat transfer from the refrigerant passing through the condenser to the surroundings, in kW. **Ans.** 13.83 kW.

Referring again to Fig. 10.5, we can identify another key feature of actual vapor-compression system performance. This is the effect of irreversibilities during compression, suggested by the use of a dashed line for the compression process from state 1 to state 2. The dashed line is drawn to show the increase in specific entropy that accompanies an *adiabatic irreversible* compression. Comparing cycle 1–2–3–4–1 with cycle 1–2s–3–4–1, the refrigeration capacity would be the same for each, but the work input would be greater in the case of irreversible compression than in the ideal cycle. Accordingly, the coefficient of performance of cycle 1–2–3–4–1 is less than that of cycle 1–2s–3–4–1. The effect of irreversible compression can be accounted for by using the isentropic compressor efficiency, which for states designated as in Fig. 10.5 is given by

TAKE NOTE... The isentropic compressor efficiency is introduced in Sec. 6.12.3. See Eq. 6.48.

$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

VCRC
A.30 – Tabs c & d

Additional departures from ideality stem from frictional effects that result in pressure drops as the refrigerant flows through the evaporator, condenser, and piping connecting the various components. These pressure drops are not shown on the T–s diagram of Fig. 10.5 and are ignored in subsequent discussions for simplicity.

Finally, two additional features exhibited by actual vapor-compression systems are shown in Fig. 10.5. One is the superheated vapor condition at the evaporator exit (state 1), which differs from the saturated vapor condition shown in Fig. 10.4. Another is the subcooling of the condenser exit state (state 3), which differs from the saturated liquid condition shown in Fig. 10.4.

Example 10.3 illustrates the effects of irreversible compression and condenser exit subcooling on the performance of the vapor-compression refrigeration system.

EXAMPLE 10.3

Analyzing an Actual Vapor-Compression Refrigeration Cycle

Reconsider the vapor-compression refrigeration cycle of Example 10.2, but include in the analysis that the compressor has an isentropic efficiency of 80%. Also, let the temperature of the liquid leaving the condenser be 30°C. Determine for the modified cycle **(a)** the compressor power, in kW, **(b)** the refrigeration capacity, in tons, **(c)** the coefficient of performance, and **(d)** the rates of exergy destruction within the compressor and expansion valve, in kW, for $T_0 = 299$ K (26°C).

SOLUTION

Known: A vapor-compression refrigeration cycle has an isentropic compressor efficiency of 80%.

Find: Determine the compressor power, in kW, the refrigeration capacity, in tons, the coefficient of performance, and the rates of exergy destruction within the compressor and expansion valve, in kW.

Schematic and Given Data:

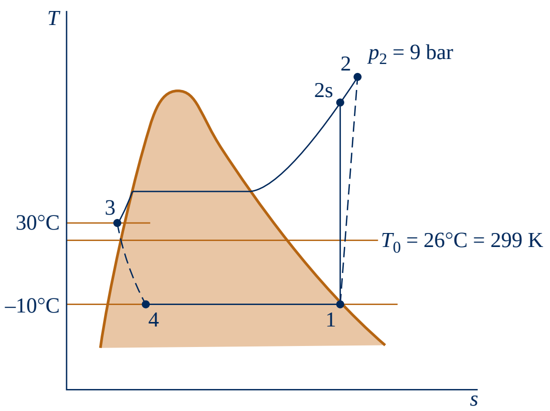


Fig. E10.3

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state.
2. There are no pressure drops through the evaporator and condenser.
3. The compressor operates adiabatically with an isentropic efficiency of 80%. The expansion through the valve is a throttling process.
4. Kinetic and potential energy effects are negligible.
5. Saturated vapor at −10°C enters the compressor, and liquid at 30°C leaves the condenser.
6. The environment temperature for calculating exergy is $T_0 = 299$ K (26°C).

Analysis: Let us begin by fixing the principal states. State 1 is the same as in Example 10.2, so $h_1 = 241.35$ kJ/kg and $s_1 = 0.9253$ kJ/kg · K.

Owing to the presence of irreversibilities during the adiabatic compression process, there is an increase in specific entropy from compressor inlet to exit. The state at the compressor exit, state 2, can be fixed using the isentropic compressor efficiency

$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{\dot{W}_c/\dot{m}} = \frac{(h_{2s} - h_1)}{(h_2 - h_1)}$$

where h_{2s} is the specific enthalpy at state 2s, as indicated on the accompanying T–s diagram. From the solution to Example 10.2, $h_{2s} = 272.39$ kJ/kg. Solving for h_2 and inserting known values

$$h_2 = \frac{h_{2s} - h_1}{\eta_c} + h_1 = \frac{(272.39 - 241.35)}{(0.80)} + 241.35 = 280.15 \text{ kJ/kg}$$

State 2 is fixed by the value of specific enthalpy h_2 and the pressure, $p_2 = 9$ bar. Interpolating in Table A-12, the specific entropy is $s_2 = 0.9497$ kJ/kg · K.

The state at the condenser exit, state 3, is in the liquid region. The specific enthalpy is approximated using Eq. 3.14, together with saturated liquid data at 30°C, as follows: $h_3 \approx h_f = 91.49$ kJ/kg. Similarly, with Eq. 6.5, $s_3 \approx s_f = 0.3396$ kJ/kg · K.

The expansion through the valve is a throttling process; thus, $h_4 = h_3$. The quality and specific entropy at state 4 are, respectively

$$x_4 = \frac{h_4 - h_{f4}}{h_{g4} - h_{f4}} = \frac{91.49 - 36.97}{204.39} = 0.2667$$

and

$$\begin{aligned} s_4 &= s_{f4} + x_4(s_{g4} - s_{f4}) \\ &= 0.1486 + (0.2667)(0.9253 - 0.1486) = 0.3557 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

(a) The compressor power is

$$\begin{aligned} \dot{W}_c &= \dot{m}(h_2 - h_1) \\ &= (0.08 \text{ kg/s})(280.15 - 241.35) \text{ kJ/kg} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 3.1 \text{ kW} \end{aligned}$$

(b) The refrigeration capacity is

$$\begin{aligned} \dot{Q}_{in} &= \dot{m}(h_1 - h_4) \\ &= (0.08 \text{ kg/s})|60 \text{ s/min}|(241.35 - 91.49) \text{ kJ/kg} \left| \frac{1 \text{ ton}}{211 \text{ kJ/min}} \right| \\ &= 3.41 \text{ ton} \end{aligned}$$

(c) The coefficient of performance is

1 $$\beta = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{(241.35 - 91.49)}{(280.15 - 241.35)} = 3.86$$

(d) The rates of exergy destruction in the compressor and expansion valve can be found by reducing the exergy rate balance or using the relationship $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance. With either approach, the rates or exergy destruction for the compressor and valve are, respectively

$$(\dot{\mathsf{E}}_d)_c = \dot{m}T_0(s_2 - s_1) \qquad \text{and} \qquad (\dot{\mathsf{E}}_d)_{valve} = \dot{m}T_0(s_4 - s_3)$$

Substituting values

2 $$(\dot{\mathsf{E}}_d)_c = \left(0.08\frac{\text{kg}}{\text{s}}\right)(299 \text{ K})(0.9497 - 0.9253)\frac{\text{kJ}}{\text{kg} \cdot \text{K}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 0.58 \text{ kW}$$

and

$$(\dot{\mathsf{E}}_d)_{valve} = (0.08)(299)(0.3557 - 0.3396) = 0.39 \text{ kW}$$

1 While the refrigeration capacity is greater than in Example 10.2, irreversibilities in the compressor result in an increase in compressor power compared to isentropic compression. The overall effect is a lower coefficient of performance than in Example 10.2.

Skills Developed

Ability to...

- ❑ sketch the T–s diagram of the vapor compression refrigeration cycle with irreversibilities in the compressor and subcooled liquid exiting the condenser.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ calculate compressor power, refrigeration capacity, and coefficient of performance.
- ❑ calculate exergy destruction in the compressor and expansion valve.

❷ The exergy destruction rates calculated in part (d) measure the effect of irreversibilities as the refrigerant flows through the compressor and valve. The percentages of the power input (exergy input) to the compressor destroyed in the compressor and valve are 18.7 and 12.6%, respectively.

QuickQUIZ What would be the coefficient of performance if the isentropic compressor efficiency were 100%? Ans. 4.83.

10.2.4 The *p–h* Diagram

***p–h* diagram**

A thermodynamic property diagram widely used in the refrigeration field is the pressure–enthalpy or ***p–h* diagram**. Figure 10.6 shows the main features of such a property diagram. The principal states of the vapor-compression cycles of Fig. 10.5 are located on this *p–h* diagram. It is left as an exercise to sketch the cycles of Examples 10.1, 10.2, and 10.3 on *p–h* diagrams. Property tables and *p–h* diagrams for many refrigerants are given in handbooks dealing with refrigeration.

10.3 Selecting Refrigerants

Refrigerant selection for a wide range of refrigeration and air-conditioning applications is generally based on three factors: performance, safety, and environmental impact. The term *performance* refers to providing the required cooling or heating capacity reliably and cost effectively. Safety refers to avoiding hazards such as toxicity and flammability. Finally, environmental impact primarily refers to using refrigerants that do not harm the stratospheric ozone layer or contribute significantly to global climate change. We begin by considering some performance aspects.

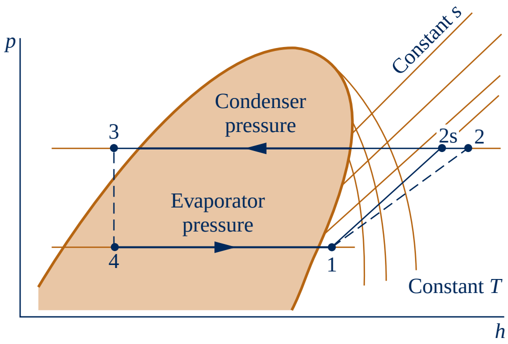


Fig. 10.6 Principal features of the pressure–enthalpy diagram for a typical refrigerant, with vapor-compression cycles superimposed.

The temperatures of the refrigerant in the evaporator and condenser of vapor-compression cycles are governed by the temperatures of the cold and warm regions, respectively, with which the system interacts thermally. This, in turn, determines the operating pressures in the evaporator and condenser. Consequently, the selection of a refrigerant is based partly on the suitability of its pressure–temperature relationship in the range of the particular application. It is generally desirable to avoid excessively low pressures in the evaporator and excessively high pressures in the condenser. Other considerations in refrigerant selection include chemical stability, corrosiveness, and cost. The type of compressor also affects the choice of refrigerant. Centrifugal compressors are best suited for low evaporator pressures and refrigerants with large specific volumes at low pressure. Reciprocating compressors perform better over large pressure ranges and are better able to handle low specific volume refrigerants.

Refrigerant Types and Characteristics

Prior to the 1930s, accidents were prevalent among those who worked closely with refrigerants due to the toxicity and flammability of most refrigerants at the time. Because of such hazards, two classes of synthetic refrigerants were developed, each containing chlorine and possessing highly stable molecular structures: CFCs (chlorofluorocarbons) and HCFCs (hydrochlorofluorocarbons). These refrigerants were widely known as "freons," the common trade name.

In the early 1930s, CFC production began with R-11, R-12, R-113, and R-114. In 1936, the first HCFC refrigerant, R-22, was introduced. Over the next several decades, nearly all of the synthetic refrigerants used in the U.S. were either CFCs or HCFCs, with R-12 being most commonly used.

To keep order with so many new refrigerants having complicated names, the "R" numbering system was established in 1956 by DuPont and persists today as the industry standard. Table 10.1 lists information, including refrigerant number, chemical composition, and global warming potential for selected refrigerants.

TAKE NOTE...
Global warming refers to an increase in global average temperature due to a combination of natural phenomena and human industrial, agricultural, and lifestyle activities.

The Global Warming Potential (GWP) is a simplified index that aims to estimate the potential future influence on global warming of different gases when released to the atmosphere. The GWP of a gas refers to how much that gas contributes to global warming in comparison to the same amount of carbon dioxide. The GWP of carbon dioxide is taken to be 1.

Environmental Considerations

After decades of use, compelling scientific data indicating that release of chlorine-containing refrigerants into the atmosphere is harmful became widely recognized. Concerns focused on released refrigerants depleting the stratospheric ozone layer and contributing to global climate change. Because of the molecular stability of the CFC and HCFC molecules, their adverse effects are long-lasting.

In 1987, an international agreement was adopted to ban production of certain chlorine-containing refrigerants. In response, a new class of chlorine-free refrigerants was developed: the HFCs (hydrofluorocarbons). One of these, R-134a, has been used for over 20 years as the primary replacement for R-12. Although R-134a and other HFC refrigerants do not contribute to atmospheric ozone depletion, they do contribute to global climate change. Owing to a relatively high Global Warming Potential of about 1430 for R-134a, we may soon see reductions in its use in the United States despite widespread deployment in refrigeration and air-conditioning systems, including automotive air conditioning. Carbon dioxide (R-744) and R-1234yf are potential replacements for R-134a in automotive systems. See Sec. 10.7.3 for discussion of CO_2-charged automotive air-conditioning systems.

Another refrigerant that has been used extensively in air-conditioning and refrigeration systems for decades, R-22, is being phased out under a 1995 amendment to the international agreement on refrigerants because of its chlorine content. Effective in 2010, R-22 cannot be installed in new systems. However, recovered and recycled R-22 can be used to service existing systems until supplies are no longer available. As R-22 is phased out, replacement refrigerants are being introduced, including R-410A and R-407C, both HFC blends.

Natural Refrigerants

Nonsynthetic, naturally occurring substances also can be used as refrigerants. Called *natural* refrigerants, they include carbon dioxide, ammonia, and hydrocarbons. As

TABLE 10.1

Refrigerant Data Including Global Warming Potential (GWP)

Refrigerant Number	Type	Chemical Formula	Approx. GWP[a]
R-12	CFC	CCl_2F_2	10900
R-11	CFC	CCl_3F	4750
R-114	CFC	$CClF_2CClF_2$	10000
R-113	CFC	CCl_2FCClF_2	6130
R-22	HCFC	$CHClF_2$	1810
R-134a	HFC	CH_2FCF_3	1430
R-1234yf	HFC	$CF_3CF{=}CH_2$	4
R-410A	HFC blend	R-32, R-125 (50/50 Weight %)	1725
R-407C	HFC blend	R-32, R-125, R-134a (23/25/52 Weight %)	1526
R-744 (carbon dioxide)	Natural	CO_2	1
R-717 (ammonia)	Natural	NH_3	0
R-290 (propane)	Natural	C_3H_8	10
R-50 (methane)	Natural	CH_4	25
R-600 (butane)	Natural	C_4H_{10}	10

[a]The Global Warming Potential (GWP) depends on the time period over which the potential influence on global warming is estimated. The values listed are based on a 100 year time period, which is an interval favored by some regulators.

indicated by Table 10.1, natural refrigerants typically have low Global Warming Potentials.

Ammonia (R-717), which was widely used in the early development of vapor-compression refrigeration, continues to serve today as a refrigerant for large systems used by the food industry and in other industrial applications. In the past two decades, ammonia has been increasingly used because of the R-12 phase out and is receiving even greater interest today due to the R-22 phase out. Ammonia is also used in the absorption systems discussed in Sec. 10.5.

Hydrocarbons, such as propane (R-290), are used worldwide in various refrigeration and air-conditioning applications including commercial and household appliances. In the United States, safety concerns limit propane use to niche markets like industrial process refrigeration. Other hydrocarbons—methane (R-50) and butane (R-600)—are also under consideration for use as refrigerants.

Refrigeration with No Refrigerant Needed

Alternative cooling technologies aim to achieve a refrigerating effect without use of refrigerants, thereby avoiding adverse effects associated with release of refrigerants to the atmosphere. One such technology is thermoelectric cooling. See the box.

New Materials May Improve Thermoelectric Cooling

You can buy a thermoelectric cooler powered from the cigarette lighter outlet of your car. The same technology is used in space flight applications and in power amplifiers and microprocessors.

Figure 10.7 shows a thermoelectric cooler separating a cold region at temperature T_C and a warm region at temperature T_H. The cooler is formed from two *n-type* and two *p-type* semiconductors with low thermal conductivity, five metallic interconnects with high electrical conductivity and high thermal conductivity, two electrically insulating ceramic substrates, and a power source. When power is provided by the source, current flows through the resulting electric circuit, giving a refrigeration effect: a heat transfer of energy *from* the cold region. This is known as the *Peltier* effect.

The p-type semiconductor material in the right leg of the cooler shown in Fig. 10.7 has electron vacancies, called *holes*. Electrons move through this material by filling individual holes, slowing electron motion. In the adjacent n-type semiconductor, no holes exist in its

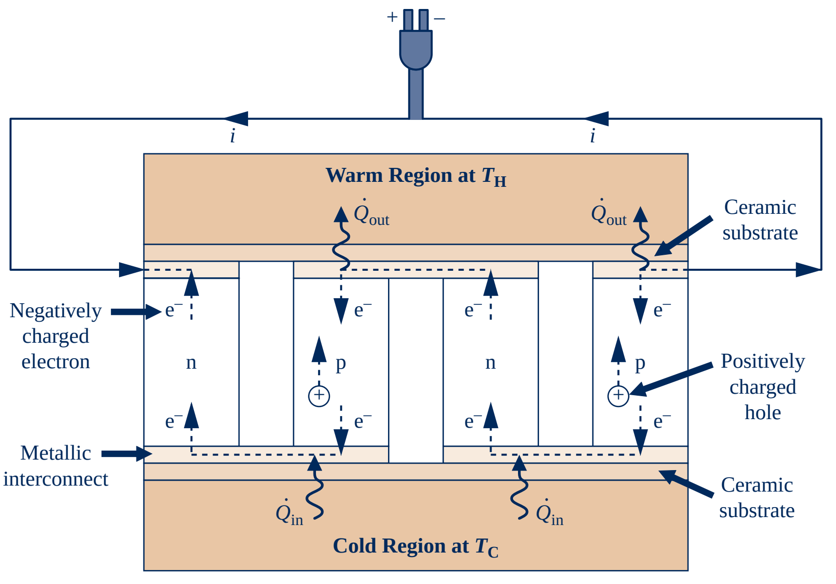


Fig. 10.7 Schematic of a thermoelectric cooler.

material structure, so electrons move freely and more rapidly through that material. When power is provided by the power source, positively charged holes move in the direction of current while negatively charged electrons move opposite to the current; each transfers energy from the cold region to the warm region.

The process of Peltier refrigeration may be understood by following the journey of an electron as it travels from the negative terminal of the power source to the positive terminal. On flowing through the metallic interconnect and into the p-type material, the electron slows and loses energy, causing the surrounding material to warm. At the other end of the p-type material, the electron accelerates as it enters the metallic interconnect and then the n-type material. The accelerating electron acquires energy from the surrounding material and causes the end of the p-type leg to cool. While the electron traverses the p-type material from the hot end to the cold end, holes are moving from the cold end to the hot end, transferring energy away from the cold end. While traversing the n-type material from the cold end to the hot end, the electron also transfers energy away from the cold end to the hot end. When it reaches the warm end of the n-type leg, the electron flows through the metallic interconnect and enters the next p-type material, where it slows and again loses energy. This scenario repeats itself at each pair of p-type and n-type legs, resulting in more removal of energy from the cold end and its deposit at the hot end. Thus, the overall effect of the thermoelectric cooler is heat transfer from the cold region to the warm region.

These simple coolers have no moving parts at a macroscopic level and are compact. They are reliable and quiet. They also use no refrigerants that harm the ozone layer or contribute to global climate change. Despite such advantages, thermoelectric coolers have found only specialized application because of low coefficients of performance compared to vapor-compression systems. However new materials and production methods may make this type of cooler more effective, material scientists report.

As shown in Fig. 10.7, at the core of a thermoelectric cooler are two dissimilar materials, in this case n-type and p-type semiconductors. To be effective for thermoelectric cooling, the materials must have low thermal conductivity and high electrical conductivity, a rare combination in nature. However, new materials with novel microscopic structures at the *nanometer* level may lead to improved cooler performance. With nanotechnology and other advanced techniques, material scientists are striving to find materials with the favorable characteristics needed to improve the performance of thermoelectric cooling devices.

10.4 Other Vapor-Compression Applications

The basic vapor-compression refrigeration cycle can be adapted for special applications. Three are presented in this section. The first is *cold storage,* which is a thermal energy storage approach that involves chilling water or making ice. The second is a *combined-cycle* arrangement where refrigeration at relatively low temperature is achieved through a series of vapor-compression systems, with each normally employing a different refrigerant. In the third, compression work is reduced through *multistage compression* with *intercooling* between stages. The second and third applications considered are analogous to power cycle applications considered in Chaps. 8 and 9.

10.4.1 Cold Storage

Chilling water or making ice during *off-peak* periods, usually overnight or over weekends, and storing chilled water/ice in tanks until needed for cooling is known as *cold storage*. Cold storage is an aspect of thermal energy storage considered in the box on p. 111. Applications of cold storage include cooling of office and commercial buildings, medical centers, college campus buildings, and shopping malls.

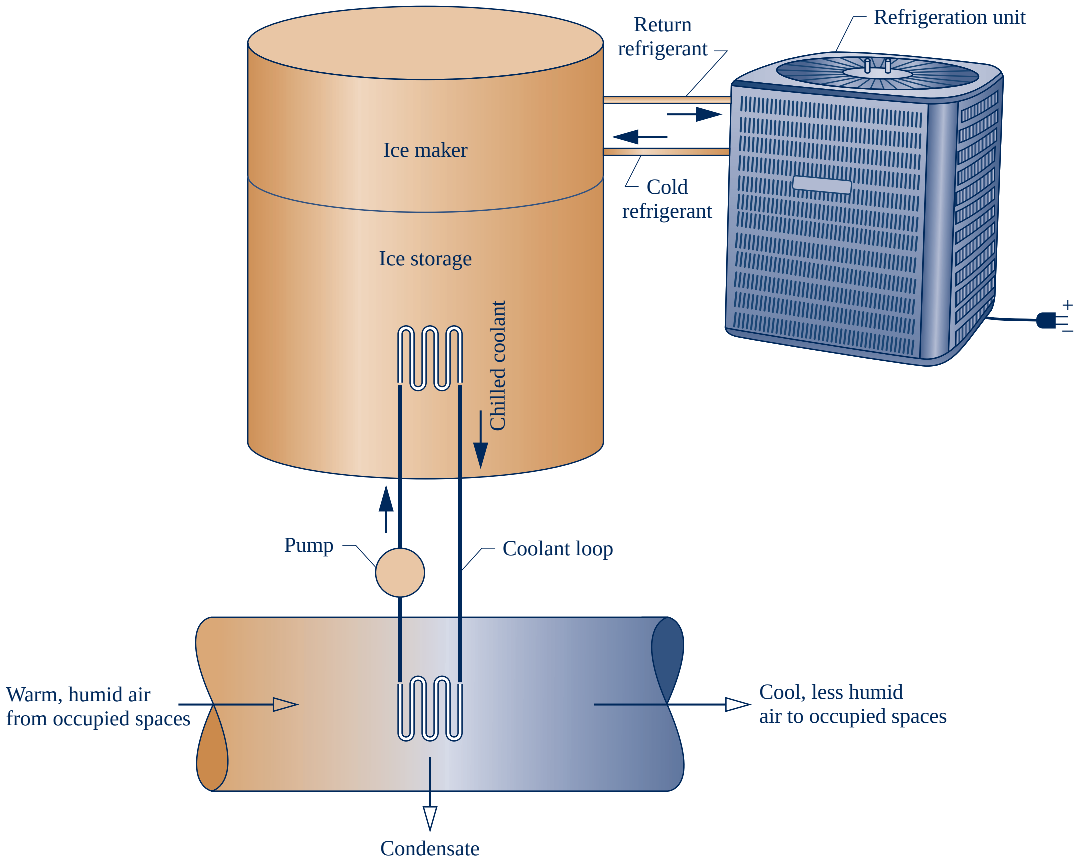


Fig. 10.8 Cold storage applied to comfort cooling.

Figure 10.8 illustrates a cold storage system intended for the comfort cooling of an occupied space. It consists of a vapor-compression refrigeration unit, ice maker and ice storage tank, and coolant loop. Running at night, when less power is required for its operation due to cooler ambient temperatures and when electricity rates are lowest, the refrigeration unit freezes water. The ice produced is stored in the accompanying tank. When cooling is required by building occupants during the day, the temperature of circulating building air is reduced as it passes over coils carrying chilled coolant flowing from the ice storage tank. Depending on local climate, some moisture also may be removed or added (see Secs. 12.8.3 and 12.8.4). Cool storage can provide all cooling required by the occupants or work in tandem with vapor-compression or other comfort cooling systems to meet needs.

10.4.2 Cascade Cycles

Combined cycle arrangements for refrigeration are called *cascade* cycles. In Fig. 10.9 a cascade cycle is shown in which *two* vapor-compression refrigeration cycles, labeled A and B, are arranged in series with a counterflow heat exchanger linking them. In the intermediate heat exchanger, the energy rejected during condensation of the refrigerant in the lower-temperature cycle A is used to evaporate the refrigerant in the higher-temperature cycle B. The desired refrigeration effect occurs in the low-temperature evaporator, and heat rejection from the overall cycle occurs in the high-temperature condenser. The coefficient of performance is the ratio of the refrigeration effect to the *total* work input

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cA}} + \dot{W}_{\text{cB}}}$$

The mass flow rates in cycles A and B are normally different. However, the mass flow rates are related by mass and energy rate balances on the interconnecting counterflow heat exchanger serving as the condenser for cycle A and the evaporator for cycle B. Although only two cycles are shown in Fig. 10.9, cascade cycles may employ three or more individual cycles.

A significant feature of the cascade system illustrated in Fig. 10.9 is that the refrigerants in the two or more stages can be selected to have advantageous evaporator and condenser pressures in the two or more temperature ranges. In a double cascade system, a refrigerant would be selected for cycle A that has a saturation pressure–temperature relationship that allows refrigeration at a relatively low temperature without excessively low evaporator pressures. The refrigerant for cycle B would have saturation characteristics that permit condensation at the required temperature without excessively high condenser pressures.

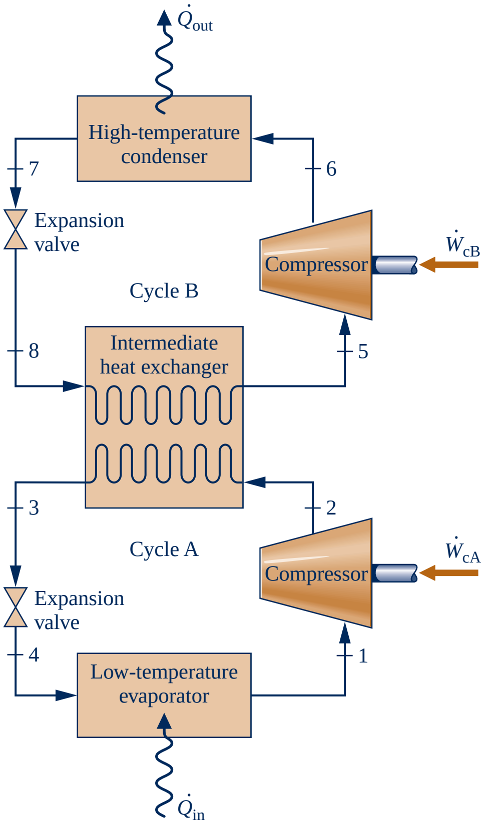


Fig. 10.9 Example of a cascade vapor-compression refrigeration cycle.

10.4.3 Multistage Compression with Intercooling

The advantages of multistage compression with intercooling between stages have been cited in Sec. 9.8, dealing with gas power systems. Intercooling is achieved in gas power systems by heat transfer to the lower-temperature surroundings. In refrigeration systems, the refrigerant temperature is below that of the surroundings for much of the cycle, so other means must be employed to accomplish intercooling and achieve the attendant savings in the required compressor work input. An arrangement for two-stage compression using the refrigerant itself for intercooling is shown in Fig. 10.10. The principal states of the refrigerant for an ideal cycle are shown on the accompanying T–s diagram.

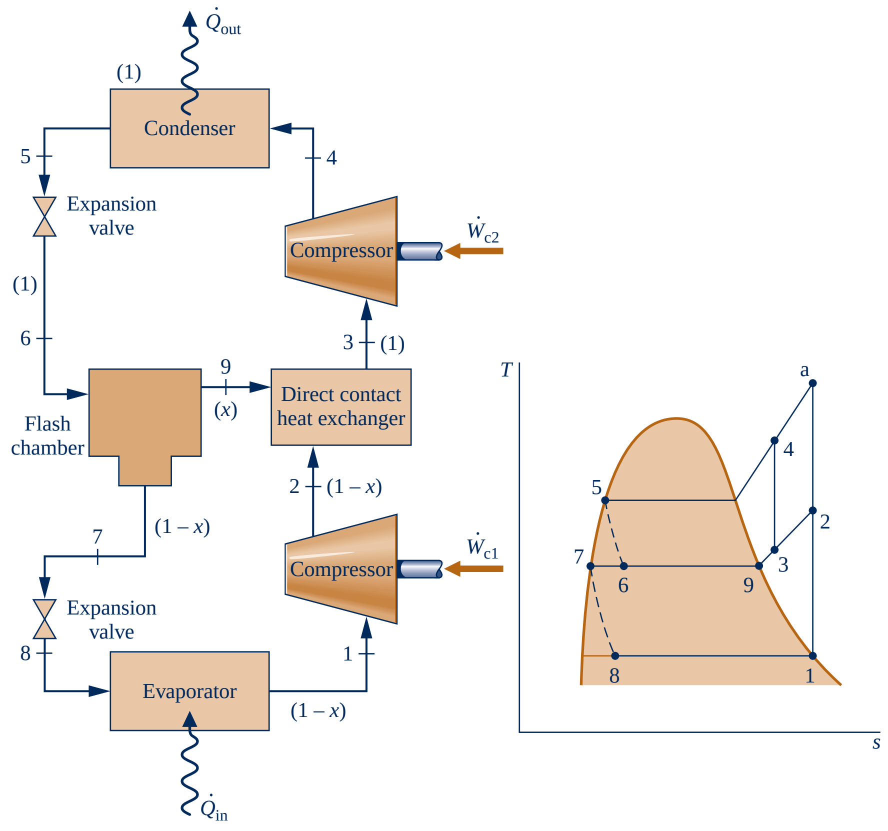


Fig. 10.10 Refrigeration cycle with two stages of compression and flash intercooling.

Intercooling is accomplished in this cycle by means of a direct contact heat exchanger. Relatively low-temperature saturated vapor enters the heat exchanger at state 9, where it mixes with higher-temperature refrigerant leaving the first compression stage at state 2. A single mixed stream exits the heat exchanger at an intermediate temperature at state 3 and is compressed in the second compressor stage to the condenser pressure at state 4. Less work is required per unit of mass flow for compression from 1 to 2 followed by compression from 3 to 4 than for a single stage of compression 1–2–a. Since the refrigerant temperature entering the condenser at state 4 is lower than for a single stage of compression in which the refrigerant would enter the condenser at state a, the external irreversibility associated with heat transfer in the condenser is also reduced.

flash chamber

A central role is played in the cycle of Fig. 10.10 by a liquid–vapor separator, called a **flash chamber**. Refrigerant exiting the condenser at state 5 expands through a valve and enters the flash chamber at state 6 as a two-phase liquid–vapor mixture with quality x. In the flash chamber, the liquid and vapor components separate into two streams. Saturated vapor exiting the flash chamber enters the heat exchanger at state 9, where intercooling is achieved as discussed above. Saturated liquid exiting the flash chamber at state 7 expands through a second valve into the evaporator. On the basis of a unit of mass flowing through the condenser, the fraction of the vapor formed in the flash chamber equals the quality x of the refrigerant at state 6. The fraction of the liquid formed is then $(1 - x)$. The fractions of the total flow at various locations are shown in parentheses on Fig. 10.10.

10.5 Absorption Refrigeration

absorption refrigeration

Absorption refrigeration cycles are the subject of this section. These cycles have some features in common with the vapor-compression cycles considered previously but differ in two important respects:

- One is the nature of the compression process. Instead of compressing a vapor between the evaporator and the condenser, the refrigerant of an absorption system is absorbed by a secondary substance, called an absorbent, to form a *liquid solution*. The liquid solution is then *pumped* to the higher pressure. Because the average specific volume of the liquid solution is much less than that of the refrigerant vapor, significantly less work is required (see the discussion of Eq. 6.51b in Sec. 6.13.2). Accordingly, absorption refrigeration systems have the advantage of relatively small work input compared to vapor-compression systems.
- The other main difference between absorption and vapor-compression systems is that some means must be introduced in absorption systems to retrieve the refrigerant vapor from the liquid solution before the refrigerant enters the condenser. This involves heat transfer from a relatively high-temperature source. Steam or waste heat that otherwise would be discharged to the surroundings without use is particularly economical for this purpose. Natural gas or some other fuel can be burned to provide the heat source, and there have been practical applications of absorption refrigeration using alternative energy sources such as solar and geothermal energy.

The principal components of an absorption refrigeration system are shown schematically in Fig. 10.11. In this case, ammonia is the refrigerant and water is the absorbent. Ammonia circulates through the condenser, expansion valve, and evaporator as in a vapor-compression system. However, the compressor is replaced by the absorber, pump, generator, and valve shown on the right side of the diagram.

- In the *absorber*, ammonia vapor coming from the evaporator at state 1 is absorbed by liquid water. The formation of this liquid solution is exothermic. Since the amount of ammonia that can be dissolved in water increases as the solution temperature

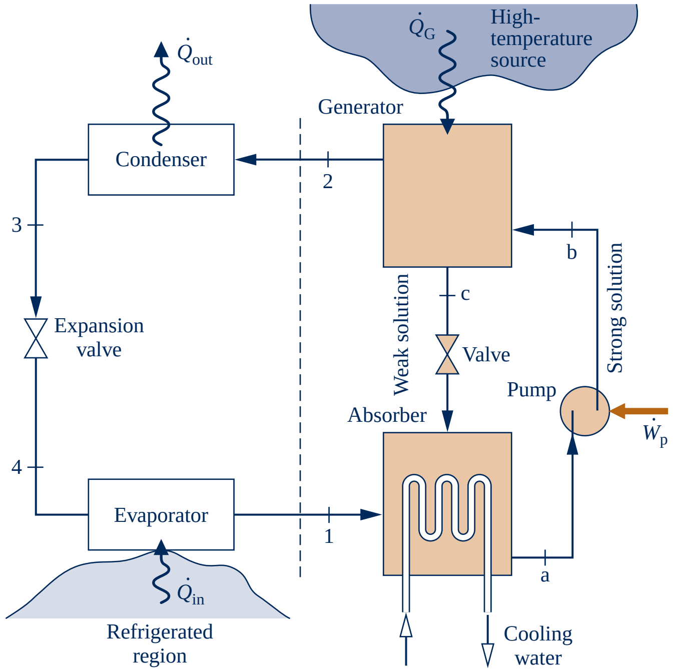


Fig. 10.11 Simple ammonia–water absorption refrigeration system.

decreases, cooling water is circulated around the absorber to remove the energy released as ammonia goes into solution and maintain the temperature in the absorber as low as possible. The strong ammonia–water solution leaves the absorber at point a and enters the *pump,* where its pressure is increased to that of the generator.

▶ In the *generator,* heat transfer from a high-temperature source drives ammonia vapor out of the solution (an endothermic process), leaving a weak ammonia–water solution in the generator. The vapor liberated passes to the condenser at state 2, and the remaining weak solution at c flows back to the absorber through a *valve*. The only work input is the power required to operate the pump, and this is small in comparison to the work that would be required to compress refrigerant vapor between the same pressure levels. However, costs associated with the heat source and extra equipment not required by vapor-compressor systems can cancel the advantage of a smaller work input.

Ammonia–water systems normally employ several modifications of the simple absorption cycle considered above. Two common modifications are illustrated in Fig. 10.12. In this cycle, a heat exchanger is included between the generator and the absorber that allows the strong water–ammonia solution entering the generator to be preheated by the weak solution returning from the generator to the absorber, thereby reducing the heat transfer to the generator, $\dot{Q}_G$. The other modification shown on the figure is the *rectifier* placed between the generator and the condenser. The function of the rectifier is to remove any traces of water from the refrigerant before it enters the condenser. This eliminates the possibility of ice formation in the expansion valve and the evaporator.

Another type of absorption system uses *lithium bromide* as the absorbent and *water* as the refrigerant. The basic principle of operation is the same as for ammonia–water systems. To achieve refrigeration at lower temperatures than are possible with water as the refrigerant, a lithium bromide–water absorption system may be combined with another cycle

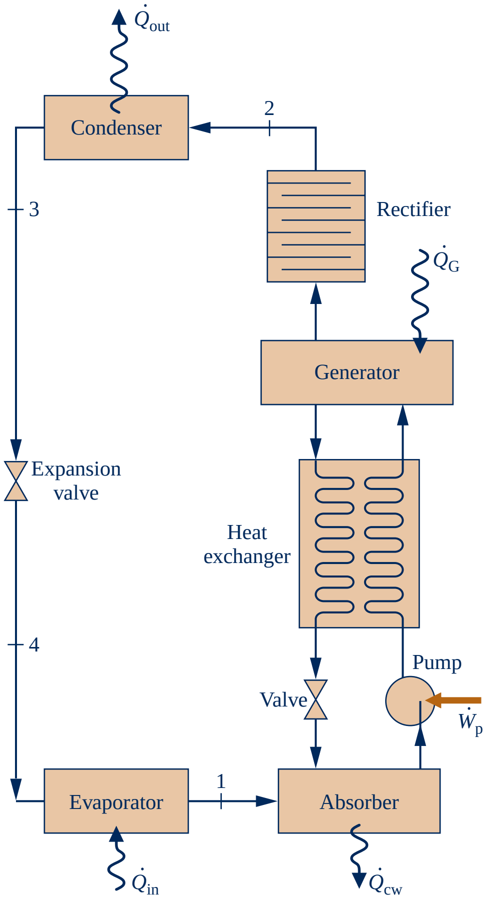


Fig. 10.12 Modified ammonia–water absorption system.

using a refrigerant with good low-temperature characteristics, such as ammonia, to form a cascade refrigeration system.

10.6 Heat Pump Systems

The objective of a heat pump is to maintain the temperature within a dwelling or other building above the temperature of the surroundings or to provide a heat transfer for certain industrial processes that occur at elevated temperatures. Heat pump systems have many features in common with the refrigeration systems considered thus far and may be of the vapor-compression or absorption type. Vapor-compression heat pumps are well suited for space heating applications and are commonly used for this purpose. Absorption heat pumps have been developed for industrial applications and are also increasingly being used for space heating. To introduce some aspects of heat pump operation, let us begin by considering the Carnot heat pump cycle.

Heat_Pump_Cycle
A.11 – All Tabs

10.6.1 Carnot Heat Pump Cycle

By simply changing our viewpoint, we can regard the cycle shown in Fig. 10.1 as a *heat pump*. The objective of the cycle now, however, is to deliver the heat transfer $\dot{Q}_{\text{out}}$ to the warm region, which is the space to be heated. At steady state, the rate at which energy is supplied to the warm region by heat transfer is the sum of the energy supplied to the working fluid from the cold region, $\dot{Q}_{\text{in}}$, and the net rate of work input to the cycle, $\dot{W}_{\text{net}}$. That is

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} + \dot{W}_{\text{net}} \tag{10.8}$$

The *coefficient of performance* of *any* heat pump cycle is defined as the ratio of the heating effect to the net work required to achieve that effect. For the Carnot heat pump cycle of Fig. 10.1

$$\gamma_{\max} = \frac{\dot{Q}_{\text{out}}/\dot{m}}{\dot{W}_{\text{c}}/\dot{m} - \dot{W}_{\text{t}}/\dot{m}} = \frac{\text{area 2–a–b–3–2}}{\text{area 1–2–3–4–1}}$$

which reduces to

$$\boxed{\gamma_{\max} = \frac{T_{\text{H}}(s_{\text{a}} - s_{\text{b}})}{(T_{\text{H}} - T_{\text{C}})(s_{\text{a}} - s_{\text{b}})} = \frac{T_{\text{H}}}{T_{\text{H}} - T_{\text{C}}}} \tag{10.9}$$

This equation, which corresponds to Eq. 5.11, represents the *maximum* theoretical coefficient of performance for any heat pump cycle operating between two regions at temperatures T_{C} and T_{H}. Actual heat pump systems have coefficients of performance that are lower than calculated from Eq. 10.9.

A study of Eq. 10.9 shows that as the temperature T_{C} of the cold region decreases, the coefficient of performance of the Carnot heat pump decreases. This trait is also exhibited by actual heat pump systems and suggests why heat pumps in which the role of the cold region is played by the local atmosphere (air-source heat pumps) normally require backup systems to provide heating on days when the ambient temperature becomes very low. If sources such as well water or the ground itself are used, relatively high coefficients of performance can be achieved despite low ambient air temperatures, and backup systems may not be required.

10.6.2 Vapor-Compression Heat Pumps

Actual heat pump systems depart significantly from the Carnot cycle model. Most systems in common use today are of the vapor-compression type. The method of analysis of *vapor-compression heat pumps* is the same as that of vapor-compression

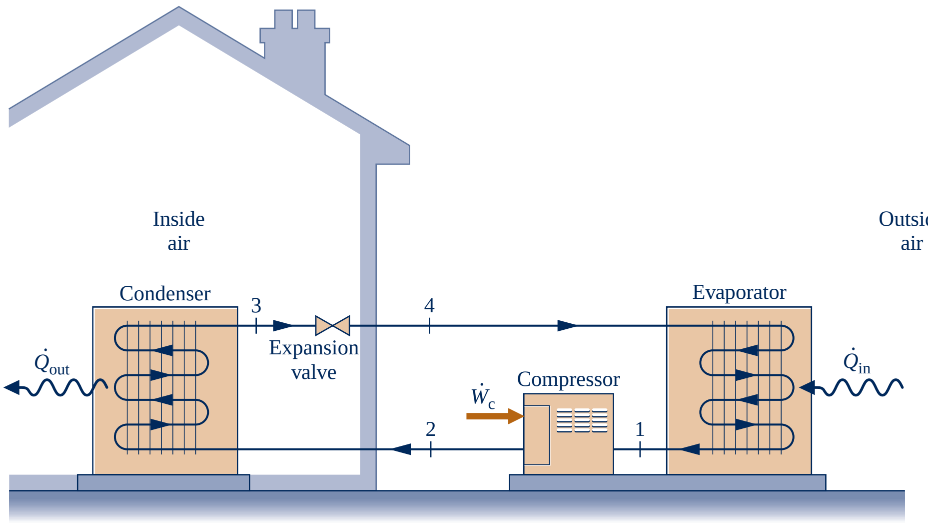


Fig. 10.13 Air-source vapor-compression heat pump system.

refrigeration cycles considered previously. Also, the previous discussions concerning the departure of actual systems from ideality apply for vapor-compression heat pump systems as for vapor-compression refrigeration cycles.

As illustrated by Fig. 10.13, a typical **vapor-compression heat pump** for space heating has the same basic components as the vapor-compression refrigeration system: compressor, condenser, expansion valve, and evaporator. The objective of the system is different, however. In a heat pump system, $\dot{Q}_{in}$ comes from the surroundings, and $\dot{Q}_{out}$ is directed to the dwelling as the desired effect. A net work input is required to accomplish this effect.

vapor-compression heat pump

The coefficient of performance of a simple vapor-compression heat pump with states as designated on Fig. 10.13 is

$$\gamma = \frac{\dot{Q}_{out}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{h_2 - h_3}{h_2 - h_1} \qquad \textbf{(10.10)}$$

The value of γ can never be less than unity.

Many possible sources are available for heat transfer to the refrigerant passing through the evaporator, including outside air; the ground; and lake, river, or well water. Liquid circulated through a solar collector and stored in an insulated tank also can be used as a source for a heat pump. Industrial heat pumps employ waste heat or warm liquid or gas streams as the low-temperature source and are capable of achieving relatively high condenser temperatures.

In the most common type of vapor-compression heat pump for space heating, the evaporator communicates thermally with the outside air. Such **air-source heat pumps** also can be used to provide cooling in the summer with the use of a reversing valve, as illustrated in Fig. 10.14. The solid lines show the flow path of the refrigerant in the heating mode, as described previously. To use the same components as an air conditioner, the valve is actuated, and the refrigerant follows the path indicated by the dashed line. In the cooling mode, the outside heat exchanger becomes the condenser, and the inside heat exchanger becomes the evaporator. Although heat pumps can be more costly to install and operate than other direct heating systems, they can be competitive when the potential for dual use is considered.

air-source heat pump

Example 10.4 illustrates use of the first and second laws of thermodynamics together with property data to analyze the performance of an actual heat pump cycle, including cost of operation.

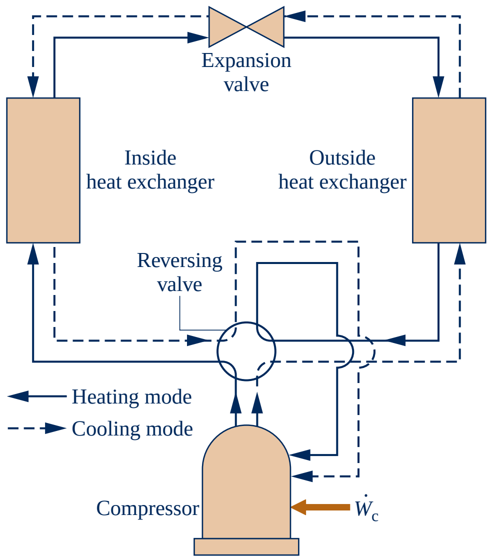


Fig. 10.14 **Example of an air-to-air reversing heat pump.**

EXAMPLE 10.4

Analyzing an Actual Vapor-Compression Heat Pump Cycle

Refrigerant 134a is the working fluid in an electric-powered, air-source heat pump that maintains the inside temperature of a building at 22°C for a week when the average outside temperature is 5°C. Saturated vapor enters the compressor at −8°C and exits at 50°C, 10 bar. Saturated liquid exits the condenser at 10 bar. The refrigerant mass flow rate is 0.2 kg/s for steady-state operation. Determine **(a)** the compressor power, in kW, **(b)** the isentropic compressor efficiency, **(c)** the heat transfer rate provided to the building, in kW, **(d)** the coefficient of performance, and **(e)** the total cost of electricity, in $, for 80 hours of operation during that week, evaluating electricity at 15 cents per kW · h.

SOLUTION

Known: A heat pump cycle operates with Refrigerant 134a. The states of the refrigerant entering and exiting the compressor and leaving the condenser are specified. The refrigerant mass flow rate and interior and exterior temperatures are given.

Find: Determine the compressor power, the isentropic compressor efficiency, the heat transfer rate to the building, the coefficient of performance, and the cost to operate the electric heat pump for 80 hours of operation.

Schematic and Given Data:

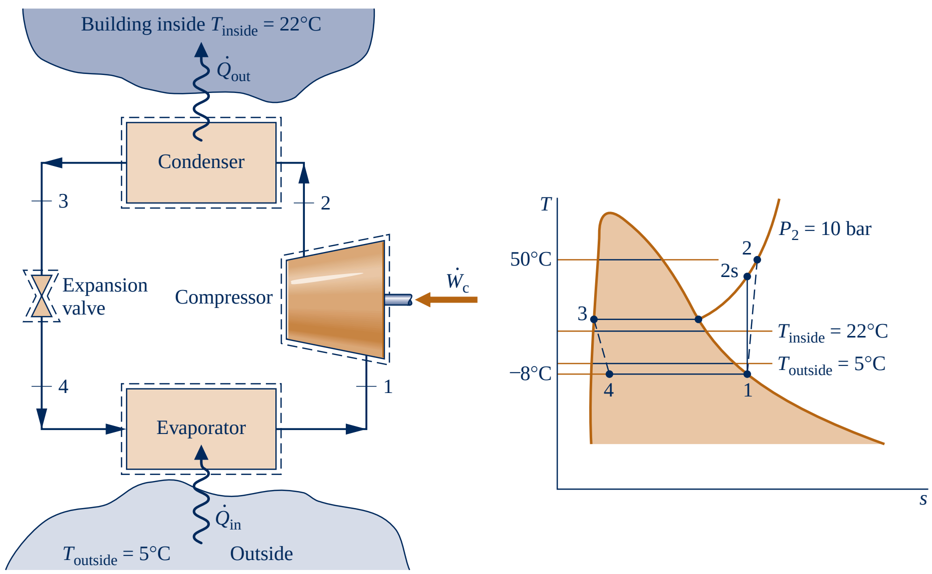


Fig. E10.4

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state.

2. There are no pressure drops through the evaporator and condenser.

3. The compressor operates adiabatically. The expansion through the valve is a throttling process.

4. Kinetic and potential energy effects are negligible.

5. Saturated vapor enters the compressor and saturated liquid exits the condenser.

6. For costing purposes, conditions provided are representative of the entire week of operation and the value of electricity is 15 cents per kW · h.

Analysis: Let us begin by fixing the principal states located on the accompanying schematic and T–s diagram. State 1 is saturated vapor at −8°C; thus h_1 and s_1 are obtained directly from Table A-10. State 2 is superheated vapor; knowing T_2 and p_2, h_2 is obtained from Table A-12. State 3 is saturated liquid at 10 bar and h_3 is obtained from Table A-11. Finally, expansion through the valve is a throttling process; therefore, $h_4 = h_3$. A summary of property values at these states is provided in the following table:

State	T (°C)	p (bar)	h (kJ/kg)	s (kJ/kg · K)
1	−8	2.1704	242.54	0.9239
2	50	10	280.19	—
3	—	10	105.29	—
4	—	2.1704	105.29	—

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 0.2\frac{\text{kg}}{\text{s}}(280.19 - 242.54)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 7.53\text{ kW}$$

(b) The isentropic compressor efficiency is

$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{(h_{2s} - h_1)}{(h_2 - h_1)}$$

where h_{2s} is the specific entropy at state 2s, as indicated on the accompanying T–s diagram. State 2s is fixed using p_2 and $s_{2s} = s_1$. Interpolating in Table A-12, $h_{2s} = 274.18$ kJ/kg. Solving for compressor efficiency

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} = \frac{(274.18 - 242.54)}{(280.19 - 242.54)} = 0.84\ (84\%)$$

(c) The heat transfer rate provided to the building is

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) = \left(0.2\frac{\text{kg}}{\text{s}}\right)(280.19 - 105.29)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 34.98\text{ kW}$$

(d) The heat pump coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{34.98\text{ kW}}{7.53\text{ kW}} = 4.65$$

(e) Using the result from part (a) together with the given cost and use data

$$[\text{electricity cost for 80 hours of operation}] = (7.53\text{ kW})(80\text{ h})\left(0.15\frac{\$}{\text{kW}\cdot\text{h}}\right) = \$90.36$$

Skills Developed

Ability to...

- ❑ sketch the T–s diagram of the vapor-compression heat pump cycle with irreversibilities in the compressor.
- ❑ fix each of the principal states and retrieve necessary property data.
- ❑ calculate the compressor power, heat transfer rate delivered, and coefficient of performance.
- ❑ calculate isentropic compressor efficiency.
- ❑ conduct an elementary economic evaluation.

QuickQUIZ If the cost of electricity is 10 cents per kW · h, which is the U. S. average for the period under consideration, evaluate the cost to operate the heat pump, in \$, keeping all other data the same. Ans. \$60.24.

10.7 Gas Refrigeration Systems

gas refrigeration systems

All refrigeration systems considered thus far involve changes in phase. Let us now turn to **gas refrigeration systems** in which the working fluid remains a gas throughout. Gas refrigeration systems have a number of important applications. They are used to achieve very low temperatures for the liquefaction of air and other gases and for other specialized applications such as aircraft cabin cooling. The Brayton refrigeration cycle illustrates an important type of gas refrigeration system.

10.7.1 Brayton Refrigeration Cycle

Brayton refrigeration cycle

The **Brayton refrigeration cycle** is the reverse of the closed Brayton power cycle introduced in Sec. 9.6. A schematic of the reversed Brayton cycle is provided in Fig. 10.15*a*. The refrigerant gas, which may be air, enters the compressor at state 1, where the temperature is somewhat below the temperature of the cold region, T_C, and is compressed to state 2. The gas is then cooled to state 3, where the gas temperature approaches the temperature of the warm region, T_H. Next, the gas is expanded to state 4, where the temperature, T_4, is well below that of the cold region. Refrigeration is achieved through heat transfer from the cold region to the gas as it passes from state 4 to state 1, completing the cycle. The T–s diagram in Fig. 10.15*b* shows an *ideal* Brayton refrigeration cycle, denoted by 1–2s–3–4s–1, in which all processes are assumed to be internally reversible and the processes in the turbine and compressor are adiabatic. Also shown is the cycle 1–2–3–4–1, which suggests the effects of irreversibilities during adiabatic compression and expansion. Frictional pressure drops have been ignored.

CYCLE ANALYSIS. The method of analysis of the Brayton refrigeration cycle is similar to that of the Brayton power cycle. Thus, at steady state the work of the compressor and the turbine per unit of mass flow are, respectively

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1 \qquad \text{and} \qquad \frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

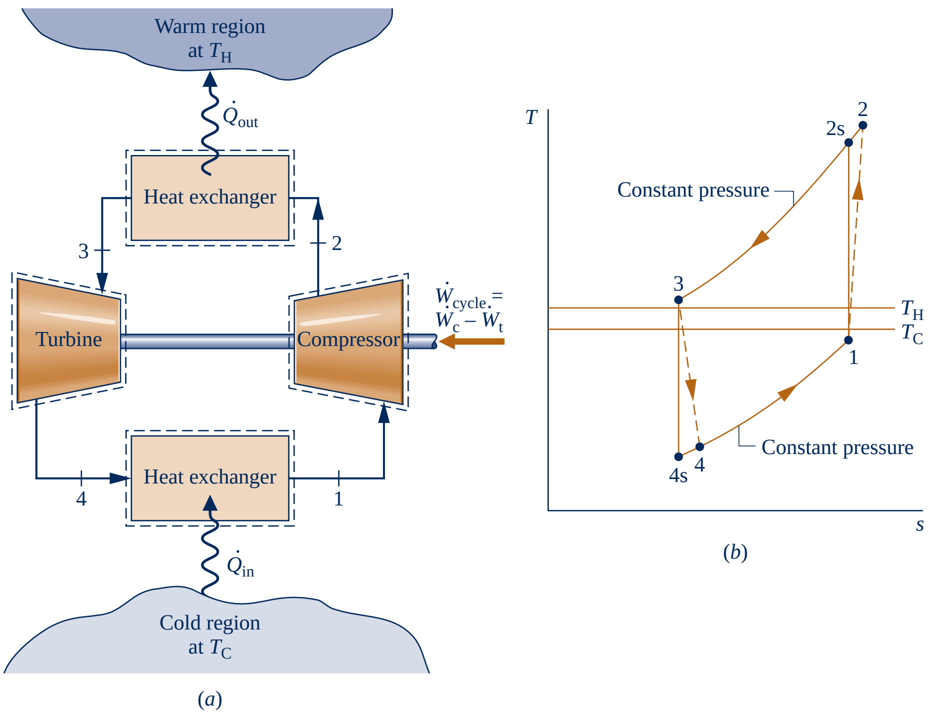


Fig. 10.15 **Brayton refrigeration cycle.**

In obtaining these expressions, heat transfer with the surroundings and changes in kinetic and potential energy have been ignored. The magnitude of the work developed by the turbine of a Brayton refrigeration cycle is typically significant relative to the compressor work input.

Heat transfer from the cold region to the refrigerant gas circulating through the low-pressure heat exchanger, the refrigeration effect, is

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_4$$

The coefficient of performance is the ratio of the refrigeration effect to the net work input:

$$\beta = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_{\text{c}}/\dot{m} - \dot{W}_{\text{t}}/\dot{m}} = \frac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)} \tag{10.11}$$

In the next example, we illustrate the analysis of an ideal Brayton refrigeration cycle.

EXAMPLE 10.5

Analyzing an Ideal Brayton Refrigeration Cycle

Air enters the compressor of an ideal Brayton refrigeration cycle at 1 atm, 480°R, with a volumetric flow rate of 50 ft^3/s. If the compressor pressure ratio is 3 and the turbine inlet temperature is 540°R, determine **(a)** the *net* power input, in Btu/min, **(b)** the refrigeration capacity, in Btu/min, **(c)** the coefficient of performance.

SOLUTION

Known: An ideal Brayton refrigeration cycle operates with air. Compressor inlet conditions, the turbine inlet temperature, and the compressor pressure ratio are given.

Find: Determine the *net* power input, in Btu/min, the refrigeration capacity, in Btu/min, and the coefficient of performance.

Schematic and Given Data:

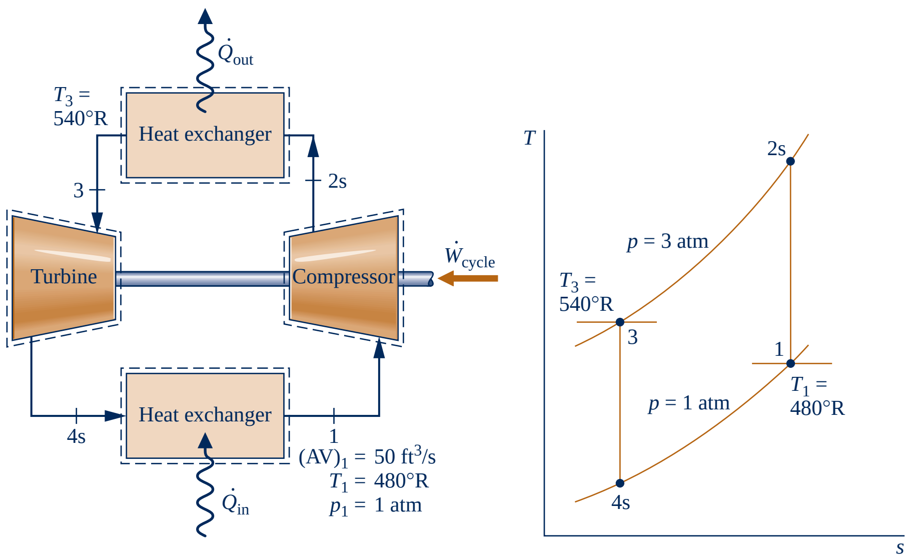


Fig. E10.5

Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying sketch.
2. The turbine and compressor processes are isentropic.
3. There are no pressure drops through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

Analysis: The analysis begins by determining the specific enthalpy at each numbered state of the cycle. At state 1, the temperature is 480°R. From Table A-22E, $h_1 = 114.69$ Btu/lb, $p_{r1} = 0.9182$. Since the compressor process is isentropic, h_{2s} can be determined by first evaluating p_r at state 2s. That is

$$p_{r2} = \frac{p_2}{p_1} p_{r1} = (3)(0.9182) = 2.755$$

Then, interpolating in Table A-22E, we get $h_{2s} = 157.1$ Btu/lb.

The temperature at state 3 is given as $T_3 = 540$°R. From Table A-22E, $h_3 = 129.06$ Btu/lb, $p_{r3} = 1.3860$. The specific enthalpy at state 4s is found by using the isentropic relation

$$p_{r4} = p_{r3} \frac{p_4}{p_3} = (1.3860)(1/3) = 0.462$$

Interpolating in Table A-22E, we obtain $h_{4s} = 94.1$ Btu/lb.

(a) The net power input is

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_{2s} - h_1) - (h_3 - h_{4s})]$$

This requires the mass flow rate $\dot{m}$, which can be determined from the volumetric flow rate and the specific volume at the compressor inlet:

$$\dot{m} = \frac{(\text{AV})_1}{v_1}$$

Since $v_1 = (\overline{R}/M)T_1/p_1$

$$\begin{aligned}\dot{m} &= \frac{(\text{AV})_1 p_1}{(\overline{R}/M)T_1} \\ &= \frac{(50 \text{ ft}^3/\text{s})|60 \text{ s/min}|(14.7 \text{ lbf/in.}^2)|144 \text{ in.}^2/\text{ft}^2|}{\left(\dfrac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot °\text{R}}\right)(480°\text{R})} \\ &= 248 \text{ lb/min}\end{aligned}$$

Finally

$$\begin{aligned}\dot{W}_{\text{cycle}} &= (248 \text{ lb/min})[(157.1 - 114.69) - (129.06 - 94.1)] \text{ Btu/lb} \\ &= 1848 \text{ Btu/min}\end{aligned}$$

(b) The refrigeration capacity is

$$\begin{aligned}\dot{Q}_{\text{in}} &= \dot{m}(h_1 - h_{4s}) \\ &= (248 \text{ lb/min})(114.69 - 94.1) \text{ Btu/lb} \\ &= 5106 \text{ Btu/min}\end{aligned}$$

(c) The coefficient of performance is

1 $$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cycle}}} = \frac{5106}{1848} = 2.76$$

---

1 Irreversibilities within the compressor and turbine serve to decrease the coefficient of performance significantly from that of the corresponding ideal cycle because the compressor work requirement is increased and the turbine work output is decreased. This is illustrated in Example 10.6.

Skills Developed

Ability to...

- sketch the *T–s* diagram of the ideal Brayton refrigeration cycle.
- fix each of the principal states and retrieve necessary property data.
- calculate net power input, refrigeration capacity, and coefficient of performance.

QuickQUIZ Determine the refrigeration capacity in tons of refrigeration. Ans. 25.53 ton.

Example 10.6 illustrates the effects of irreversible compression and turbine expansion Building on Example 10.5, on the performance of Brayton cycle refrigeration. For this, we apply the isentropic compressor and turbine efficiencies introduced in Sec. 6.12.

EXAMPLE 10.6

Evaluating Performance of a Brayton Refrigeration Cycle with Irreversibilities

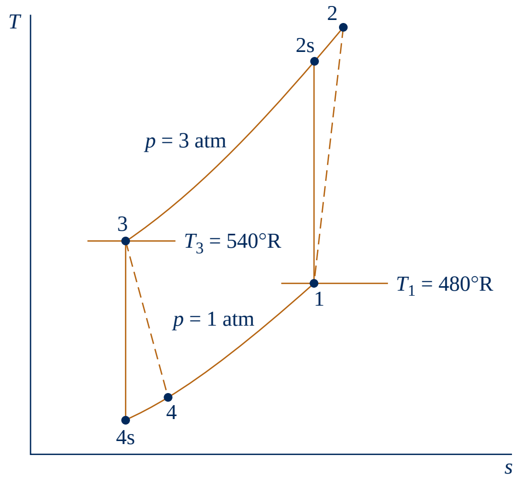
Reconsider Example 10.5, but include in the analysis that the compressor and turbine each have an isentropic efficiency of 80%. Determine for the modified cycle **(a)** the *net* power input, in Btu/min, **(b)** the refrigeration capacity, in Btu/min, **(c)** the coefficient of performance, and discuss its value.

SOLUTION

Known: A Brayton refrigeration cycle operates with air. Compressor inlet conditions, the turbine inlet temperature, and the compressor pressure ratio are given. The compressor and turbine each have an isentropic efficiency of 80%.

Find: Determine the *net* power input and the refrigeration capacity, each in Btu/min. Also, determine the coefficient of performance and discuss its value.

Schematic and Given Data:



Engineering Model:

1. Each component of the cycle is analyzed as a control volume at steady state.
2. The compressor and turbine are adiabatic.
3. There are no pressure drops through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

Fig. E10.6

Analysis:

(a) The power input to the compressor is evaluated using the isentropic compressor efficiency, η_c. That is

$$\frac{\dot{W}_c}{\dot{m}} = \frac{(\dot{W}_c/\dot{m})_s}{\eta_c}$$

The value of the work per unit mass for the isentropic compression, $(\dot{W}_c/\dot{m})_s$, is determined with data from the solution in Example 10.5 as 42.41 Btu/lb. The actual power required is then

$$\dot{W}_c = \frac{\dot{m}(\dot{W}_c/\dot{m})_s}{\eta_c} = \frac{(248 \text{ lb/min})(42.41 \text{ Btu/lb})}{(0.8)}$$
$$= 13{,}147 \text{ Btu/min}$$

The turbine power output is determined using the turbine isentropic efficiency η_t. Thus, $\dot{W}_t/\dot{m} = \eta_t(\dot{W}_t/\dot{m})_s$. Using data from the solution to Example 10.5 gives $(\dot{W}_t/\dot{m})_s = 34.96$ Btu/lb. The actual turbine work is then

$$\dot{W}_t = \dot{m}\eta_t(\dot{W}_t/\dot{m})_s = (248 \text{ lb/min})(0.8)(34.96 \text{ Btu/lb})$$
$$= 6936 \text{ Btu/min}$$

The *net* power input to the cycle is

$$\dot{W}_{\text{cycle}} = 13{,}147 - 6936 = 6211 \text{ Btu/min}$$

(b) The specific enthalpy at the turbine exit, h_4, is required to evaluate the refrigeration capacity. This enthalpy can be determined by solving $\dot{W}_t = \dot{m}(h_3 - h_4)$ to obtain $h_4 = h_3 - \dot{W}_t/\dot{m}$. Inserting known values

$$h_4 = 129.06 - \left(\frac{6936}{248}\right) = 101.1 \text{ Btu/lb}$$

The refrigeration capacity is then

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = (248)(114.69 - 101.1) = 3370 \text{ Btu/min}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{cycle}}} = \frac{3370}{6211} = 0.543$$

The value of the coefficient of performance in this case is less than unity. This means that the refrigeration effect is smaller than the net work required to achieve it. Additionally, note that irreversibilities in the compressor and turbine have a significant effect on the performance of gas refrigeration systems. This is brought out by comparing the results of the present example with those of Example 10.5. Irreversibilities result in an increase in the work of compression and a reduction in the work output of the turbine. The refrigeration capacity is also reduced. The overall effect is that the coefficient of performance is decreased significantly.

Skills Developed

Ability to...

- sketch the *T–s* diagram of the Brayton refrigeration cycle with irreversibilities in the turbine and compressor.
- fix each of the principal states and retrieve necessary property data.
- calculate net power input, refrigeration capacity, and coefficient of performance.

QuickQUIZ Determine the coefficient of performance for a Carnot refrigeration cycle operating between reservoirs at 480°R and 540°R. **Ans.** 8.

10.7.2 Additional Gas Refrigeration Applications

To obtain even moderate refrigeration capacities with the Brayton refrigeration cycle, equipment capable of achieving relatively high pressures and volumetric flow rates is needed. For most applications involving air conditioning and for ordinary refrigeration processes, vapor-compression systems can be built more cheaply and can operate with higher coefficients of performance than gas refrigeration systems. With suitable modifications, however, gas refrigeration systems can be used to achieve temperatures of about −150°C (−240°F), which are well below the temperatures normally obtained with vapor systems.

Figure 10.16 shows the schematic and *T–s* diagram of an ideal Brayton cycle modified by introduction of a heat exchanger. The heat exchanger allows the air exiting the compressor at state 2 to cool *below* the warm region temperature T_H, giving a low turbine inlet temperature, T_3. Without the heat exchanger, air could be cooled only close to T_H, as represented on the figure by state a. In the subsequent expansion through the turbine, the air achieves a much lower temperature at state 4 than would have been possible without the heat exchanger. Accordingly, the refrigeration effect, achieved from state 4 to state b, occurs at a correspondingly lower average temperature.

An example of the application of gas refrigeration to cabin cooling in an aircraft is illustrated in Fig. 10.17. As shown in the figure, a small amount of high-pressure air is extracted from the main jet engine compressor and cooled by heat transfer to the ambient. The high-pressure air is then expanded through an auxiliary turbine to the pressure maintained in the cabin. The air temperature is reduced in the expansion and thus is able to fulfill its cabin cooling function. As an additional benefit, the turbine expansion can provide some of the auxiliary power needs of the aircraft.

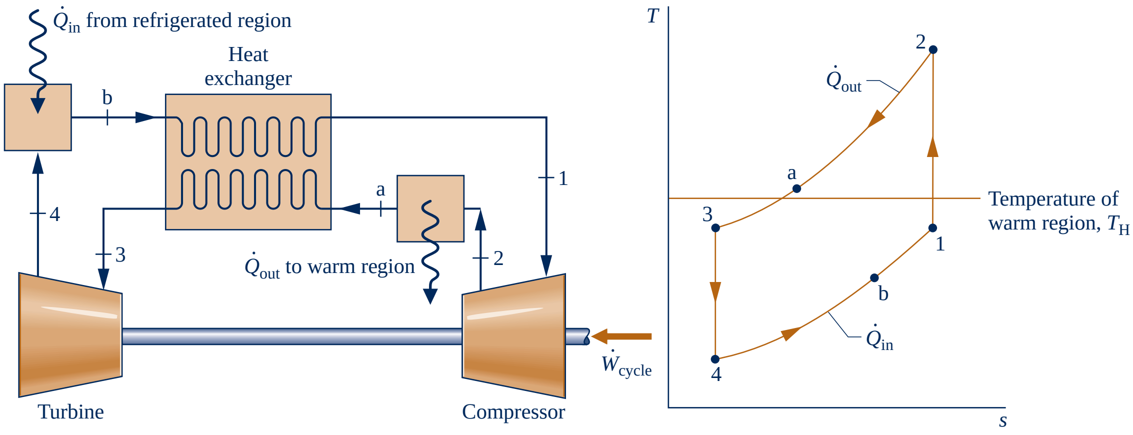


Fig. 10.16 Brayton refrigeration cycle modified with a heat exchanger.

Size and weight are important considerations in the selection of equipment for use in aircraft. Open-cycle systems, like the example given here, utilize *compact* high-speed rotary turbines and compressors. Furthermore, since the air for cooling comes directly from the surroundings, there are fewer heat exchangers than would be needed if a separate refrigerant were circulated in a closed vapor-compression cycle.

10.7.3 Automotive Air Conditioning Using Carbon Dioxide

Owing primarily to environmental concerns, automotive air-conditioning systems using CO_2 are currently under active consideration. Carbon dioxide causes no harm to the ozone layer, and its Global Warming Potential of 1 is small compared to that of R-134a, commonly used in automotive air-conditioning systems. Carbon dioxide is nontoxic and nonflammable. As it is abundant in the atmosphere and the exhaust gas of coal-burning power and industrial plants, CO_2 is a relatively inexpensive choice as a refrigerant. Still, automakers considering a shift to CO_2 away from R-134a must

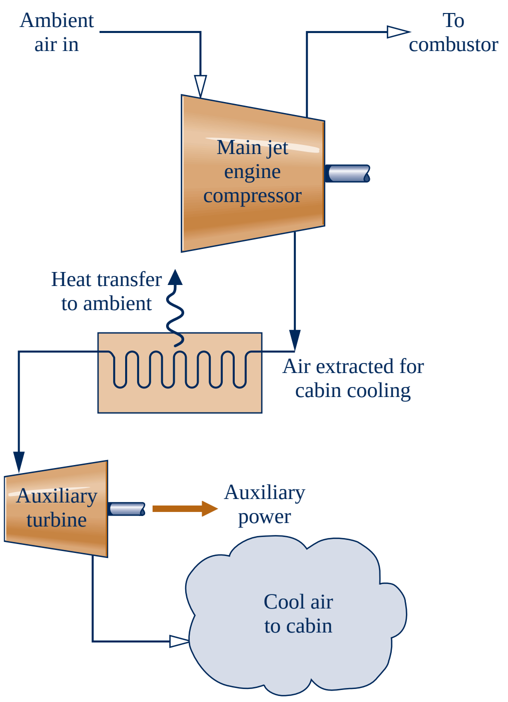


Fig. 10.17 An application of gas refrigeration to aircraft cabin cooling.

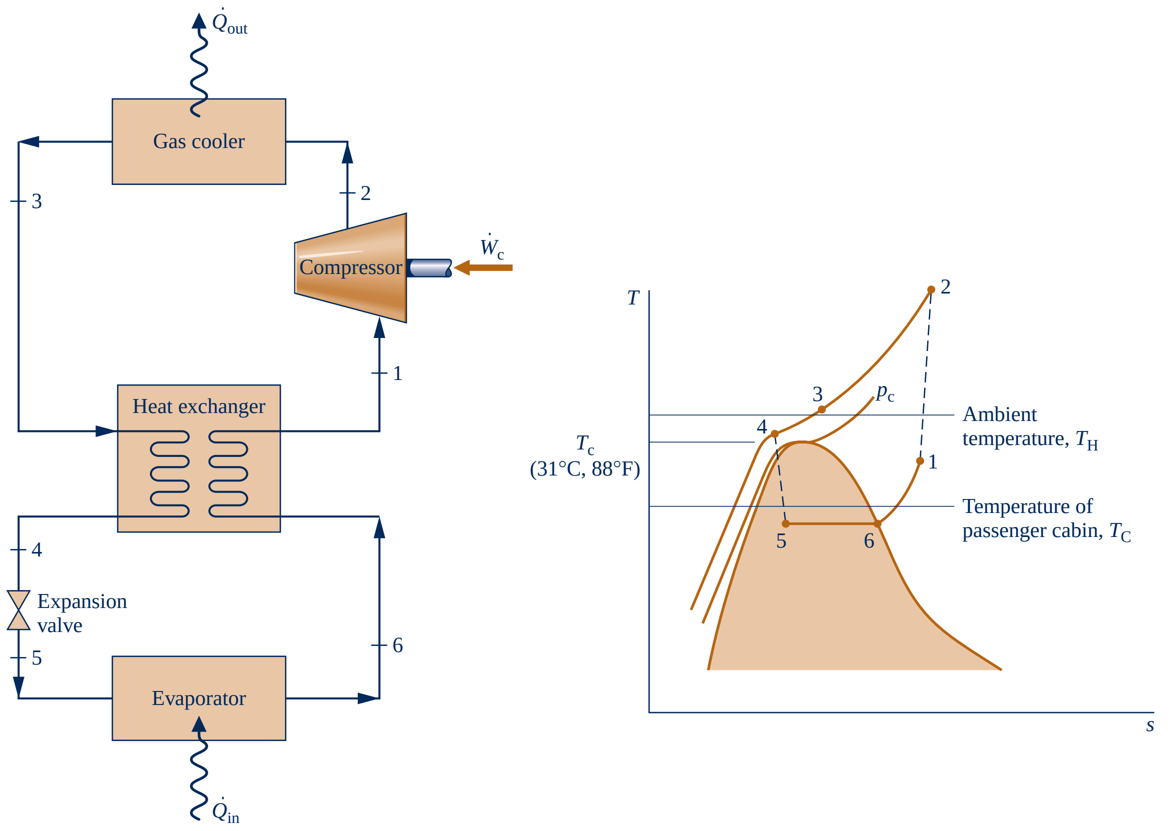


Fig. 10.18 Carbon dioxide automotive air-conditioning system.

weigh system performance, equipment costs, and other key issues before embracing such a change in longstanding practice.

Figure 10.18 shows the schematic of a CO_2-charged automotive air-conditioning system with an accompanying T–s diagram labeled with the critical temperature T_c and critical pressure p_c of CO_2: 31°C (88°F) and 72.9 atm, respectively. The system combines aspects of gas refrigeration with aspects of vapor-compression refrigeration. Let us follow the CO_2 as it passes steadily through each of the components, beginning with the inlet to the compressor.

Carbon dioxide enters the compressor as superheated vapor at state 1 and is compressed to a much higher temperature and pressure at state 2. The CO_2 passes from the compressor into the gas cooler, where it cools at constant pressure to state 3 as a result of heat transfer to the ambient. The temperature at state 3 approaches that of the ambient, denoted on the figure by T_H. CO_2 is further cooled in the interconnecting heat exchanger at constant pressure to state 4, where the temperature is below that of ambient. Cooling is provided by low-temperature CO_2 in the other stream of the heat exchanger. During this portion of the refrigeration cycle, the processes are like those seen in gas refrigeration.

This similarity ends abruptly as the CO_2 next expands through the valve to state 5 in the liquid–vapor region and then enters the evaporator, where it is vaporized to state 6 by heat transfer from the passenger cabin at temperature T_C, thereby cooling the passenger cabin. These processes are like those seen in vapor-compression refrigeration systems. Finally, at state 6 the CO_2 enters the heat exchanger, exiting at state 1. The heat exchanger increases the cycle's performance in two ways: by delivering lower-quality two-phase mixture at state 5, increasing the refrigerating effect through the evaporator, and by producing higher-temperature superheated vapor at state 1, reducing the compressor power required.

▶ CHAPTER SUMMARY AND STUDY GUIDE

In this chapter we have considered refrigeration and heat pump systems, including vapor systems where the refrigerant is alternately vaporized and condensed, and gas systems where the refrigerant remains a gas. The three principal types of systems discussed are the vapor-compression, absorption, and reversed Brayton cycles.

The performance of simple vapor refrigeration systems is described in terms of the vapor-compression cycle. For this cycle, we have evaluated the principal work and heat transfers along with two important performance parameters: the coefficient of performance and the refrigeration capacity. We have considered the effect on performance of irreversibilities during the compression process and in the expansion across the valve, as well as the effect of irreversible heat transfer between the refrigerant and the warm and cold regions. Variations of the basic vapor-compression refrigeration cycle also have been considered, including cold storage, cascade cycles, and multistage compression with intercooling. A discussion of vapor-compression heat pump systems is also provided.

Qualitative discussions are presented of refrigerant properties and of considerations in selecting refrigerants. Absorption refrigeration and heat pump systems are also discussed qualitatively. The chapter concludes with a study of gas refrigeration systems.

The following list provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed, you should be able to

- write out the meanings of the terms listed in the margin throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important.
- sketch the *T–s* diagrams of vapor-compression refrigeration and heat pump cycles and of Brayton refrigeration cycles, correctly showing the relationship of the refrigerant temperature to the temperatures of the warm and cold regions.
- apply the first and second laws along with property data to determine the performance of vapor-compression refrigeration and heat pump cycles and of Brayton refrigeration cycles, including evaluation of the power required, the coefficient of performance, and the capacity.
- sketch schematic diagrams of vapor-compression cycle modifications, including cascade cycles and multistage compression with intercooling between the stages. In each case be able to apply mass and energy balances, the second law, and property data to determine performance.
- explain the operation of absorption refrigeration systems.

▶ KEY ENGINEERING CONCEPTS

vapor-compression refrigeration, p. 592
refrigeration capacity, p. 592
ton of refrigeration, p. 592
absorption refrigeration, p. 606
vapor-compression heat pump, p. 609
Brayton refrigeration cycle, p. 612

▶ KEY EQUATIONS

$\beta_{\max} = \dfrac{T_C}{T_H - T_C}$	**(10.1) p. 591**	**Coefficient of performance of the Carnot refrigeration cycle (Fig. 10.1)**
$\beta = \dfrac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m}} = \dfrac{h_1 - h_4}{h_2 - h_1}$	**(10.7) p. 593**	**Coefficient of performance of the vapor-compression refrigeration cycle (Fig. 10.3)**
$\gamma_{\max} = \dfrac{T_H}{T_H - T_C}$	**(10.9) p. 608**	**Coefficient of performance of the Carnot heat pump cycle (Fig. 10.1)**
$\gamma = \dfrac{\dot{Q}_{out}/\dot{m}}{\dot{W}_c/\dot{m}} = \dfrac{h_2 - h_3}{h_2 - h_1}$	**(10.10) p. 609**	**Coefficient of performance of the vapor-compression heat pump cycle (Fig. 10.13)**
$\beta = \dfrac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \dfrac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)}$	**(10.11) p. 613**	**Coefficient of performance of the Brayton refrigeration cycle (Fig. 10.15)**

▶ EXERCISES: THINGS ENGINEERS THINK ABOUT

1. What are the temperatures inside the fresh food and freezer compartments of your refrigerator? Do you know what values are *recommended* for these temperatures?

2. You have a refrigerator located in your garage. Does it perform differently in the summer than in the winter? Explain.

3. Abbe installs a dehumidifier to dry the walls of a small, closed basement room. When she enters the room later, it feels warm. Why?

4. Why does the indoor unit of a central air conditioning system have a drain hose?

5. Your air conditioner has a label that lists an *EER* of 10 Btu/h per watt. What does that mean?

6. Can the coefficient of performance of a heat pump have a value less than one?

7. You see an advertisement for a natural gas–fired absorption refrigeration system. How can *burning* natural gas play a role in achieving *cooling*?

8. What are *n-type* and *p-type* semiconductors?

9. What qualifies a refrigerator to be an *Energy Star*® appliance?

10. You see an advertisement claiming that heat pumps are particularly effective in Atlanta, Georgia. Why might that be true?

11. If your car's air conditioner discharges only warm air while operating, what might be wrong with it?

12. Large office buildings often use air conditioning to cool interior areas even in winter in cold climates. Why?

13. In what North American locations are heat pumps not a good choice for heating dwellings? Explain.

14. If the heat exchanger is omitted from the system of Fig. 10.16, what is the effect on the coefficient of performance?

▶ PROBLEMS: DEVELOPING ENGINEERING SKILLS

Vapor Refrigeration Systems

10.1 Refrigerant 22 is the working fluid in a Carnot refrigeration cycle operating at steady state. The refrigerant enters the condenser as saturated vapor at 32°C and exits as saturated liquid. The evaporator operates at 0°C. What is the coefficient of performance of the cycle? Determine, in kJ per kg of refrigerant flowing

(a) the work input to the compressor.
(b) the work developed by the turbine.
(c) the heat transfer to the refrigerant passing through the evaporator.

10.2 Refrigerant 22 is the working fluid in a Carnot vapor refrigeration cycle for which the evaporator temperature is −30°C. Saturated vapor enters the condenser at 36°C, and saturated liquid exits at the same temperature. The mass flow rate of refrigerant is 10 kg/min. Determine

(a) the rate of heat transfer to the refrigerant passing through the evaporator, in kW.
(b) the net power input to the cycle, in kW.
(c) the coefficient of performance.
(d) the refrigeration capacity, in tons.

10.3 A Carnot vapor refrigeration cycle operates between thermal reservoirs at 40°F and 90°F. For (a) Refrigerant 134a, (b) propane, (c) water, (d) Refrigerant 22, and (e) ammonia as the working fluid, determine the operating pressures in the condenser and evaporator, in lbf/in.^{2}, and the coefficient of performance.

10.4 Consider a Carnot vapor refrigeration cycle with Refrigerant 134a as the working fluid. The cycle maintains a cold region at 40°F when the ambient temperature is 90°F. Data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 10.1. Sketch the T–s diagram for the cycle and determine the

(a) temperatures in the evaporator and condenser, each in °R.
(b) compressor and turbine work, each in Btu per lb of refrigerant flowing.
(c) coefficient of performance.
(d) coefficient of performance for a Carnot cycle operating at the reservoir temperatures.

Compare the coefficients of performance determined in (c) and (d), and comment.

State	p (lbf/in.^{2})	h (Btu/lb)	s (Btu/lb · °R)
1	40	104.12	0.2161
2	140	114.95	0.2161
3	140	44.43	0.0902
4	40	42.57	0.0902

10.5 For the cycle in Problem 10.4, determine

(a) the rates of heat transfer, in Btu per lb of refrigerant flowing, for the refrigerant flowing through the evaporator and condenser, respectively.
(b) the rates and directions of exergy transfer accompanying each of these heat transfers, in Btu per lb of refrigerant flowing. Let $T_0 = 90°F$.

10.6 An ideal vapor-compression refrigeration cycle operates at steady state with Refrigerant 134a as the working fluid. Saturated vapor enters the compressor at 2 bar, and saturated liquid exits the condenser at 8 bar. The mass flow rate of refrigerant is 7 kg/min. Determine

(a) the compressor power, in kW.
(b) the refrigerating capacity, in tons.
(c) the coefficient of performance.

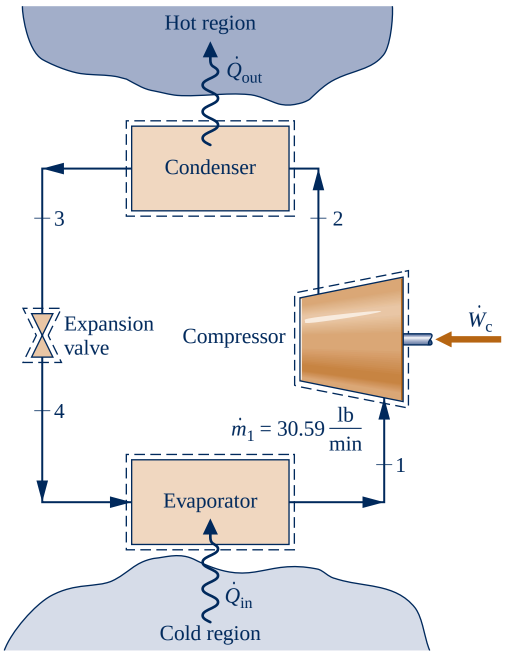
10.7 Plot each of the quantities in Problem 10.6 versus evaporator temperature for evaporator pressures ranging from 0.6 to 4 bar, while the condenser pressure remains fixed at 8 bar.

10.8 Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle operating at steady state. Refrigerant enters the compressor at 1.4 bar, −12°C, and the condenser pressure is 9 bar. Liquid exits the condenser at 32°C. The mass flow rate of refrigerant is 7 kg/min. Determine

(a) the compressor power, in kW.
(b) the refrigeration capacity, in tons.
(c) the coefficient of performance.

10.9 Figure P10.9 provides steady-state operating data for an ideal vapor-compression refrigeration cycle with Refrigerant 134a as the working fluid. The mass flow rate of refrigerant is 30.59 lb/min. Sketch the *T–s* diagram for the cycle and determine

(a) the compressor power, in horsepower.
(b) the rate of heat transfer, from the working fluid passing through the condenser, in Btu/min.
(c) the coefficient of performance.



State	p (lbf/in.^{2})	T (°F)	h (Btu/lb)	s (Btu/lb·°R)
1	10	0	102.94	0.2391
2	180	---	131.04	0.2391
3	180	Sat.	50.64	0.1009
4	10	Sat.	50.64	---

Fig. P10.9

10.10 Refrigerant 22 enters the compressor of an ideal vapor-compression refrigeration system as saturated vapor at −40°C with a volumetric flow rate of 15 m^3/min. The refrigerant leaves the condenser at 19°C, 9 bar. Determine

(a) the compressor power, in kW.
(b) the refrigerating capacity, in tons.
(c) the coefficient of performance.

10.11 An ideal vapor-compression refrigeration cycle, with ammonia as the working fluid, has an evaporator temperature of −20°C and a condenser pressure of 12 bar. Saturated vapor enters the compressor, and saturated liquid exits the condenser. The mass flow rate of the refrigerant is 3 kg/min. Determine

(a) the coefficient of performance.
(b) the refrigerating capacity, in tons.

10.12 Refrigerant 134a enters the compressor of an ideal vapor-compression refrigeration cycle as saturated vapor at −10°F. The condenser pressure is 160 lbf/in.^{2} The mass flow rate of refrigerant is 6 lb/min. Plot the coefficient of performance and the refrigerating capacity, in tons, versus the condenser exit temperature ranging from the saturation temperature at 160 lbf/in.^{2} to 90°F.

10.13 To determine the effect of changing the evaporator temperature on the performance of an ideal vapor-compression refrigeration cycle, plot the coefficient of performance and the refrigerating capacity, in tons, for the cycle in Problem 10.11 for saturated vapor entering the compressor at temperatures ranging from −40 to −10°C. All other conditions are the same as in Problem 10.11.

10.14 To determine the effect of changing condenser pressure on the performance of an ideal vapor-compression refrigeration cycle, plot the coefficient of performance and the refrigerating capacity, in tons, for the cycle in Problem 10.11 for condenser pressures ranging from 8 to 16 bar. All other conditions are the same as in Problem 10.11.

10.15 A vapor-compression refrigeration cycle operates at steady state with Refrigerant 134a as the working fluid. Saturated vapor enters the compressor at 2 bar, and saturated liquid exits the condenser at 8 bar. The isentropic compressor efficiency is 80%. The mass flow rate of refrigerant is 7 kg/min. Determine

(a) the compressor power, in kW.
(b) the refrigeration capacity, in tons.
(c) the coefficient of performance.

10.16 Modify the cycle in Problem 10.9 to have an isentropic compressor efficiency of 83% and let the temperature of the liquid leaving the condenser be 100°F. Determine, for the modified cycle,

(a) the compressor power, in horsepower.
(b) the rate of heat transfer from the working fluid passing through the condenser, in Btu/min.
(c) the coefficient of performance.
(d) the rates of exergy destruction in the compressor and expansion valve, each in Btu/min, for $T_0 = 90°F$.

10.17 Data for steady-state operation of a vapor-compression refrigeration cycle with Refrigerant 134a as the working fluid are given in the table below. The states are numbered as in Fig. 10.3. The refrigeration capacity is 4.6 tons. Ignoring heat transfer between the compressor and its surroundings, sketch the *T–s* diagram of the cycle and determine

(a) the mass flow rate of the refrigerant, in kg/min.
(b) the isentropic compressor efficiency.
(c) the coefficient of performance.
(d) the rates of exergy destruction in the compressor and expansion valve, each in kW.
(e) the net changes in flow exergy rate of the refrigerant passing through the evaporator and condenser, respectively, each in kW.

Let $T_0 = 21°C$, $p_0 = 1$ bar.

State	*p* (bar)	*T* (°C)	*h* (kJ/kg)	*s* (kJ/kg · K)
1	1.4	−10	243.40	0.9606
2	7	58.5	295.13	1.0135
3	7	24	82.90	0.3113
4	1.4	−18.8	82.90	0.33011

10.18 A vapor-compression refrigeration system, using ammonia as the working fluid, has evaporator and condenser pressures of 30 and 200 lbf/in.^{2}, respectively. The refrigerant passes through each heat exchanger with a negligible pressure drop. At the inlet and exit of the compressor, the temperatures are 10°F and 300°F, respectively. The heat transfer rate from the working fluid passing through the condenser is 50,000 Btu/h, and liquid exits at 200 lbf/in.^{2}, 90°F. If the compressor operates adiabatically, determine

(a) the compressor power input, in hp.
(b) the coefficient of performance.

10.19 If the minimum and maximum allowed refrigerant pressures are 1 and 10 bar, respectively, which of the following can be used as the working fluid in a vapor-compression refrigeration system that maintains a cold region at 0°C, while discharging energy by heat transfer to the surrounding air at 30°C: Refrigerant 22, Refrigerant 134a, ammonia, propane?

10.20 Consider the following vapor-compression refrigeration cycle used to maintain a cold region at temperature T_C when the ambient temperature is 80°F: Saturated vapor enters the compressor at 15°F below T_C, and the compressor operates adiabatically with an isentropic efficiency of 80%. Saturated liquid exits the condenser at 95°F. There are no pressure drops through the evaporator or condenser, and the refrigerating capacity is 1 ton. Plot refrigerant mass flow rate, in lb/min, coefficient of performance, and *refrigerating efficiency,* versus T_C ranging from 40°F to −25°F if the refrigerant is

(a) Refrigerant 134a.
(b) propane.
(c) Refrigerant 22.
(d) ammonia.

The refrigerating efficiency is defined as the ratio of the cycle coefficient of performance to the coefficient of performance of a Carnot refrigeration cycle operating between thermal reservoirs at the ambient temperature and the temperature of the cold region.

10.21 In a vapor-compression refrigeration cycle, ammonia exits the evaporator as saturated vapor at −22°C. The refrigerant enters the condenser at 16 bar and 160°C, and saturated liquid exits at 16 bar. There is no significant heat transfer between the compressor and its surroundings, and the refrigerant passes through the evaporator with a negligible change in pressure. If the refrigerating capacity is 150 kW, determine

(a) the mass flow rate of refrigerant, in kg/s.
(b) the power input to the compressor, in kW.
(c) the coefficient of performance.
(d) the isentropic compressor efficiency.

10.22 A vapor-compression refrigeration system with a capacity of 10 tons has superheated Refrigerant 134a vapor entering the compressor at 15°C, 4 bar, and exiting at 12 bar. The compression process can be modeled by $pv^{1.01} = constant$. At the condenser exit, the pressure is 11.6 bar, and the temperature is 44°C. The condenser is water-cooled, with water entering at 20°C and leaving at 30°C with a negligible change in pressure. Heat transfer from the outside of the condenser can be neglected. Determine

(a) the mass flow rate of the refrigerant, in kg/s.
(b) the power input and the heat transfer rate for the compressor, each in kW.
(c) the coefficient of performance.
(d) the mass flow rate of the cooling water, in kg/s.
(e) the rates of exergy destruction in the condenser and expansion valve, each expressed as a percentage of the power input. Let $T_0 = 20°C$.

10.23 Data for steady-state operation of a vapor-compression refrigeration cycle with propane as the working fluid are given in the table below. The states are numbered as in Fig. 10.3. The mass flow rate of refrigerant is 8.42 lb/min. Heat transfer from the compressor to its surroundings occurs at a rate of 3.5 Btu per lb of refrigerant passing through the compressor. The condenser is water-cooled, with water entering at 65°F and leaving at 80°F with negligible change in pressure. Sketch the *T–s* diagram of the cycle and determine

(a) the refrigeration capacity, in tons.
(b) the compressor power, in horsepower.
(c) the mass flow rate of the condenser cooling water, in lb/min.
(d) the coefficient of performance.

State	*p* (lbf/in.^{2})	*T* (°F)	*h* (Btu/lb)
1	38.4	0	193.2
2	180	120	229.8
3	180	85	74.41
4	38.4	0	74.41

10.24 A window-mounted air conditioner supplies 19 m^3/min of air at 15°C, 1 bar to a room. Air returns from the room to the evaporator of the unit at 22°C. The air conditioner operates at steady state on a vapor-compression refrigeration cycle with Refrigerant-22 entering the compressor at 4 bar, 10°C. Saturated liquid refrigerant at 9 bar leaves the condenser. The compressor has an isentropic efficiency of 70%, and refrigerant exits the compressor at 9 bar. Determine the compressor power, in kW, the refrigeration capacity, in tons, and the coefficient of performance.

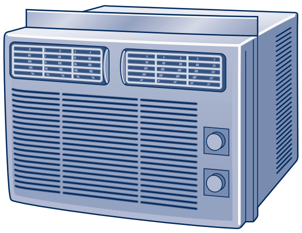


Fig. P10.24

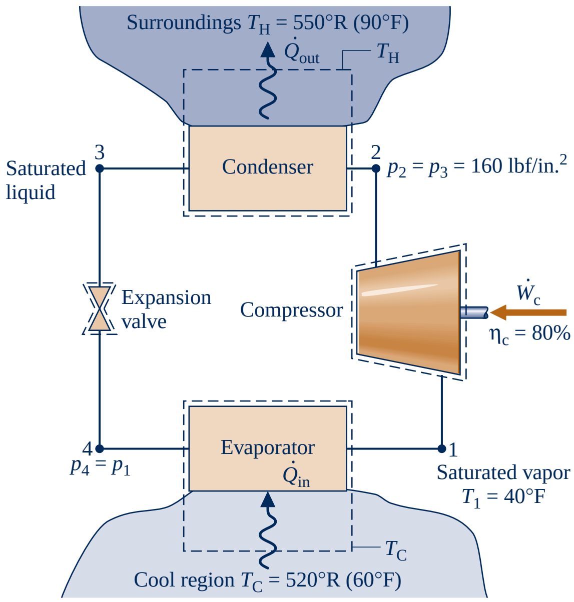


Fig. P10.26

10.25 A vapor-compression refrigeration system for a household refrigerator has a refrigerating capacity of 1000 Btu/h. Refrigerant enters the evaporator at −10°F and exits at 0°F. The isentropic compressor efficiency is 80%. The refrigerant condenses at 95°F and exits the condenser subcooled at 90°F. There are no significant pressure drops in the flows through the evaporator and condenser. Determine the evaporator and condenser pressures, each in lbf/in.^{2}, the mass flow rate of refrigerant, in lb/min, the compressor power input, in horsepower, and the coefficient of performance for (a) Refrigerant 134a and (b) propane as the working fluid.

10.26 A vapor-compression air conditioning system operates at steady state as shown in Fig. P10.26. The system maintains a cool region at 60°F and discharges energy by heat transfer to the surroundings at 90°F. Refrigerant 134a enters the compressor as a saturated vapor at 40°F and is compressed adiabatically to 160 lbf/in.^{2} The isentropic compressor efficiency is 80%. Refrigerant exits the condenser as a saturated liquid at 160 lbf/in.^{2} The mass flow rate of the refrigerant is 0.15 lb/s. Kinetic and potential energy changes are negligible as are changes in pressure for flow through the evaporator and condenser. Determine

(a) the power required by the compressor, in Btu/s.
(b) the coefficient of performance.
(c) the rates of exergy destruction in the compressor and expansion valve, each in Btu/s.
(d) the rates of exergy destruction and exergy transfer accompanying heat transfer, each in Btu/s, for a control volume comprising the evaporator and a portion of the cool region such that heat transfer takes place at T_C = 520°R (60°F).
(e) the rates of exergy destruction and exergy transfer accompanying heat transfer, each in Btu/s, for a control volume enclosing the condenser and a portion of the surroundings such that heat transfer takes place at T_H = 550°R (90°F).

Let T_0 = 550°R.

10.27 A vapor-compression refrigeration cycle with Refrigerant 134a as the working fluid operates with an evaporator temperature of 50°F and a condenser pressure of 180 lbf/in.^{2} Saturated vapor enters the compressor. Refrigerant enters the condenser at 140°F and exits as saturated liquid. The cycle has a refrigeration capacity of 5 tons. Determine

(a) the refrigerant mass flow rate, in lb/min.
(b) the compressor isentropic efficiency.
(c) the compressor power, in horsepower.
(d) the coefficient of performance.

Plot each of the quantities calculated in parts (b) through (d) for compressor exit temperatures varying from 130°F to 140°F.

Cascade and Multistage Systems

10.28 A vapor-compression refrigeration system operates with the cascade arrangement of Fig. 10.9. Refrigerant 22 is the working fluid in the high-temperature cycle and Refrigerant 134a is used in the low-temperature cycle. For the Refrigerant 134a cycle, the working fluid enters the compressor as saturated vapor at −30°F and is compressed isentropically to 50 lbf/in.^{2} Saturated liquid leaves the intermediate heat exchanger at 50 lbf/in.^{2} and enters the expansion valve. For the Refrigerant 22 cycle, the working fluid enters the compressor as saturated vapor at a temperature 5°F below that of the condensing temperature of the Refrigerant 134a in the intermediate heat exchanger. The Refrigerant 22 is compressed isentropically to 250 lbf/in.^{2} Saturated liquid then enters the expansion valve at 250 lbf/in.^{2} The refrigerating capacity of the cascade system is 20 tons. Determine

(a) the power input to each compressor, in Btu/min.
(b) the overall coefficient of performance of the cascade cycle.
(c) the rate of exergy destruction in the intermediate heat exchanger, in Btu/min. Let T_0 = 80°F, p_0 = 14.7 lbf/in.^{2}

10.29 A vapor-compression refrigeration system uses the arrangement shown in Fig. 10.10 for two-stage compression with intercooling between the stages. Refrigerant 134a is the working fluid. Saturated vapor at $-30°C$ enters the first compressor stage. The flash chamber and direct contact heat exchanger operate at 4 bar, and the condenser pressure is 12 bar. Saturated liquid streams at 12 and 4 bar enter the high- and low-pressure expansion valves, respectively. If each compressor operates isentropically and the refrigerating capacity of the system is 10 tons, determine

(a) the power input to each compressor, in kW.
(b) the coefficient of performance.

10.30 Figure P10.30 shows a two-stage vapor-compression refrigeration system with ammonia as the working fluid. The system uses a direct contact heat exchanger to achieve intercooling. The evaporator has a refrigerating capacity of 30 tons and produces $-20°F$ saturated vapor at its exit. In the first compressor stage, the refrigerant is compressed adiabatically to 80 lbf/in.^{2}, which is the pressure in the direct contact heat exchanger. Saturated vapor at 80 lbf/in.^{2} enters the second compressor stage and is compressed adiabatically to 250 lbf/in.^{2} Each compressor stage has an isentropic efficiency of 85%. There are no significant pressure drops as the refrigerant passes through the heat exchangers. Saturated liquid enters each expansion valve. Determine

(a) the ratio of mass flow rates, $\dot{m}_3/\dot{m}_1$.
(b) the power input to each compressor stage, in horsepower.
(c) the coefficient of performance.
(d) Plot each of the quantities calculated in parts (a)–(c) versus the direct-contact heat exchanger pressure ranging from 20 to 200 lbf/in.^{2} Discuss.

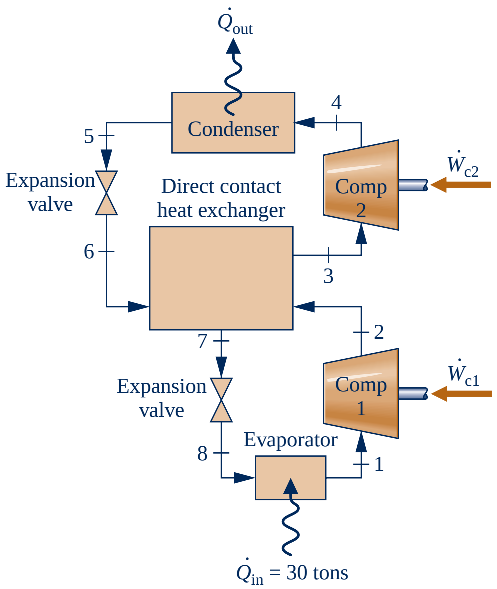


Fig. P10.30

10.31 Figure P10.31 shows a two-stage, vapor-compression refrigeration system with two evaporators and a direct contact heat exchanger. Saturated vapor ammonia from evaporator 1 enters compressor 1 at 18 lbf/in.^{2} and exits at 70 lbf/in.^{2} Evaporator 2 operates at 70 lbf/in.^{2}, with saturated vapor exiting at state 8. The condenser pressure is 200 lbf/in.^{2}, and saturated liquid refrigerant exits the condenser. Each compressor stage has an isentropic efficiency of 80%. The refrigeration capacity of each evaporator is shown on the figure. Sketch the T–s diagram of the cycle and determine

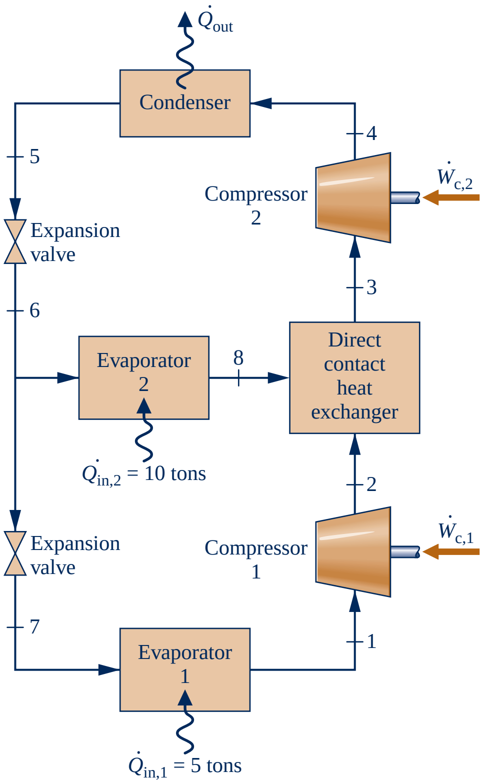


Fig. P10.31

(a) the temperatures, in °F, of the refrigerant in each evaporator.
(b) the power input to each compressor stage, in horsepower.
(c) the overall coefficient of performance.

10.32 Figure P10.32 shows the schematic diagram of a vapor-compression refrigeration system with two evaporators using Refrigerant 134a as the working fluid. This arrangement is used to achieve refrigeration at two different temperatures with a single compressor and a single condenser. The low-temperature evaporator operates at $-18°C$ with saturated vapor at its exit and has a refrigerating capacity of 3 tons. The higher-temperature evaporator produces saturated vapor at 3.2 bar at its exit and has a refrigerating capacity of 2 tons. Compression is isentropic to the condenser pressure of 10 bar. There are no significant pressure drops in the flows through the condenser and the two evaporators, and the refrigerant leaves the condenser as saturated liquid at 10 bar. Calculate

(a) the mass flow rate of refrigerant through each evaporator, in kg/min.
(b) the compressor power input, in kW.
(c) the rate of heat transfer from the refrigerant passing through the condenser, in kW.

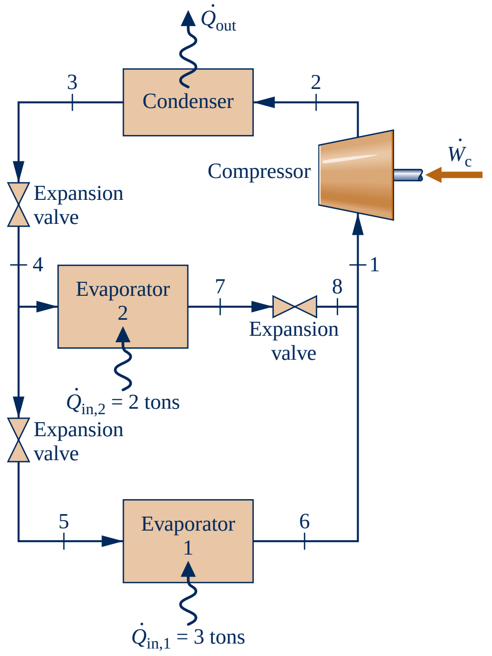


Fig. P10.32

10.33 An ideal vapor-compression refrigeration cycle is modified to include a counterflow heat exchanger, as shown in Fig. P10.33. Ammonia leaves the evaporator as saturated vapor at 1.0 bar and is heated at constant pressure to 5°C before entering the compressor. Following isentropic compression to 18 bar, the refrigerant passes through the condenser, exiting at 40°C, 18 bar. The liquid then passes through the heat exchanger, entering the expansion valve at 18 bar. If the mass flow rate of refrigerant is 12 kg/min, determine

(a) the refrigeration capacity, in tons of refrigeration.
(b) the compressor power input, in kW.
(c) the coefficient of performance.

Discuss possible advantages and disadvantages of this arrangement.

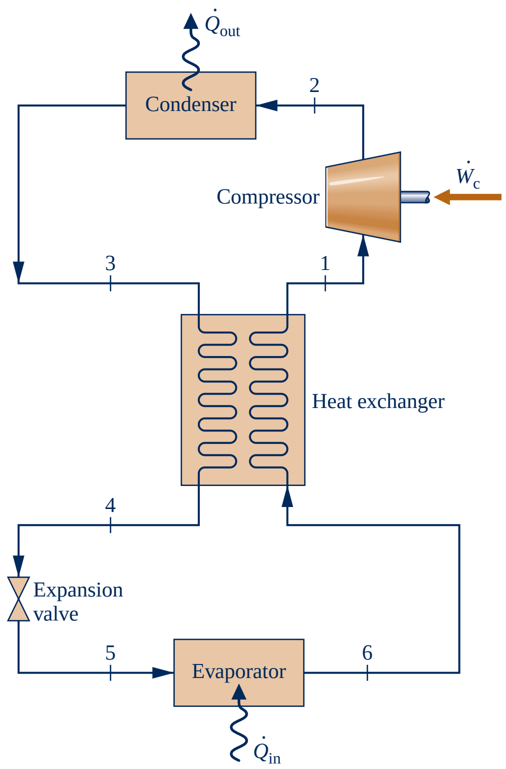


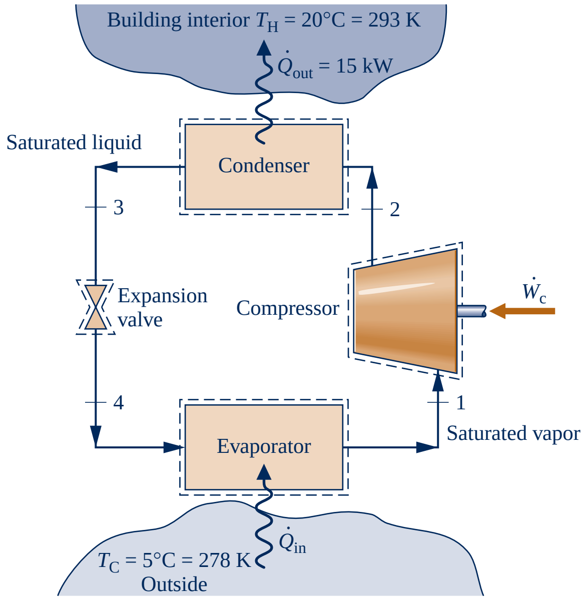
Fig. P10.33

Vapor-Compression Heat Pump Systems

10.34 Figure P10.34 gives data for an ideal vapor-compression heat pump cycle operating at steady state with Refrigerant 134a as the working fluid. The heat pump provides heating at a rate of 15 kW to maintain the interior of a building at 20°C when the outside temperature is 5°C. Sketch the *T–s* diagram for the cycle and determine the

(a) temperatures at the principal states of the cycle, each in °C.
(b) the power input to the compressor, in kW.
(c) the coefficient of performance.
(d) the coefficient of performance for a Carnot heat pump cycle operating between reservoirs at the building interior and outside temperatures, respectively.

Compare the coefficients of performance determined in (c) and (d). Discuss



State	p (bar)	h (kJ/kg)
1	2.4	244.09
2	8	268.97
3	8	93.42
4	2.4	93.42

Fig. P10.34

10.35 Refrigerant 134a is the working fluid in a vapor-compression heat pump system with a heating capacity of 60,000 Btu/h. The condenser operates at 200 lbf/in.^{2}, and the evaporator temperature is 0°F. The refrigerant is a saturated vapor at the evaporator exit and a liquid at 110°F at the condenser exit. Pressure drops in the flows through the evaporator and condenser are negligible. The compression process is adiabatic, and the temperature at the compressor exit is 180°F. Determine

(a) the mass flow rate of refrigerant, in lb/min.
(b) the compressor power input, in horsepower.
(c) the isentropic compressor efficiency.
(d) the coefficient of performance.

10.36 Refrigerant 134a is the working fluid in a vapor-compression heat pump that provides 35 kW to heat a dwelling on a day when the outside temperature is below freezing. Saturated vapor enters the compressor at 1.6 bar, and saturated liquid exits the condenser, which operates at 8 bar. Determine, for isentropic compression

(a) the refrigerant mass flow rate, in kg/s.
(b) the compressor power, in kW.
(c) the coefficient of performance.

Recalculate the quantities in parts (b) and (c) for an isentropic compressor efficiency of 75%.

10.37 An office building requires a heat transfer rate of 20 kW to maintain the inside temperature at 21°C when the outside temperature is 0°C. A vapor-compression heat pump with Refrigerant 134a as the working fluid is to be used to provide the necessary heating. The compressor operates adiabatically with an isentropic efficiency of 82%. Specify appropriate evaporator and condenser pressures of a cycle for this purpose assuming $\Delta T_{cond} = \Delta T_{evap} = 10°C$, as shown in Figure P10.37. The states are numbered as in Fig. 10.13. The refrigerant exits the evaporator as saturated vapor and exits the condenser as saturated liquid at the respective pressures. Determine the

(a) mass flow rate of refrigerant, in kg/s.
(b) compressor power, in kW.
(c) coefficient of performance and compare with the coefficient of performance for a Carnot heat pump cycle operating between reservoirs at the inside and outside temperatures, respectively.

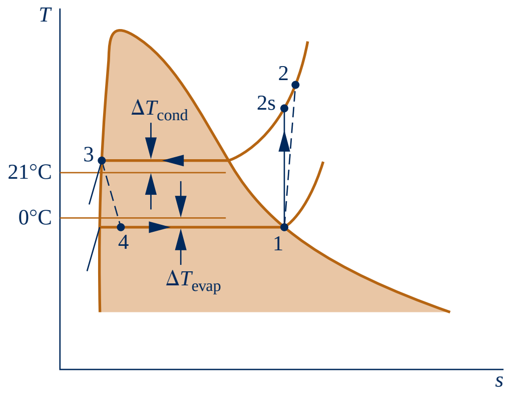
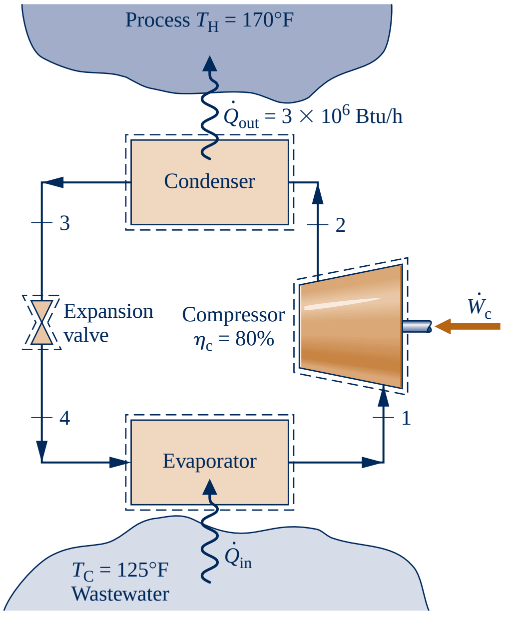


Fig. P10.37

10.38 Repeat the calculations of Problem 10.37 for Refrigerant 22 as the working fluid. Compare the results with those of Problem 10.37 and discuss.

10.39 A process requires a heat transfer rate of 3×10^6 Btu/h at 170°F. It is proposed that a Refrigerant 134a vapor-compression heat pump be used to develop the process heating using a wastewater stream at 125°F as the lower-temperature source. Figure P10.39 provides data for this cycle operating at steady state. The compressor isentropic efficiency is 80%. Sketch the T–s diagram for the cycle and determine the

(a) specific enthalpy at the compressor exit, in Btu/lb.
(b) temperatures at each of the principal states, in °F.
(c) mass flow rate of the refrigerant, in lb/h.
(d) compressor power, in Btu/h.
(e) coefficient of performance and compare with the coefficient of performance for a Carnot heat pump cycle operating between reservoirs at the process temperature and the wastewater temperature, respectively.



State	p (lbf/in.^{2})	h (Btu/lb)
1	180	116.74
2	400	?
3	400	76.11
4	180	76.11

Fig. P10.39

10.40 A vapor-compression heat pump with a heating capacity of 500 kJ/min is driven by a power cycle with a thermal efficiency of 25%. For the heat pump, Refrigerant 134a is compressed from saturated vapor at −10°C to the condenser pressure of 10 bar. The isentropic compressor efficiency is 80%. Liquid enters the expansion valve at 9.6 bar, 34°C. For the power cycle, 80% of the heat rejected is transferred to the heated space.

(a) Determine the power input to the heat pump compressor, in kW.
(b) Evaluate the ratio of the total rate that heat is delivered to the heated space to the rate of heat input to the power cycle. Discuss.