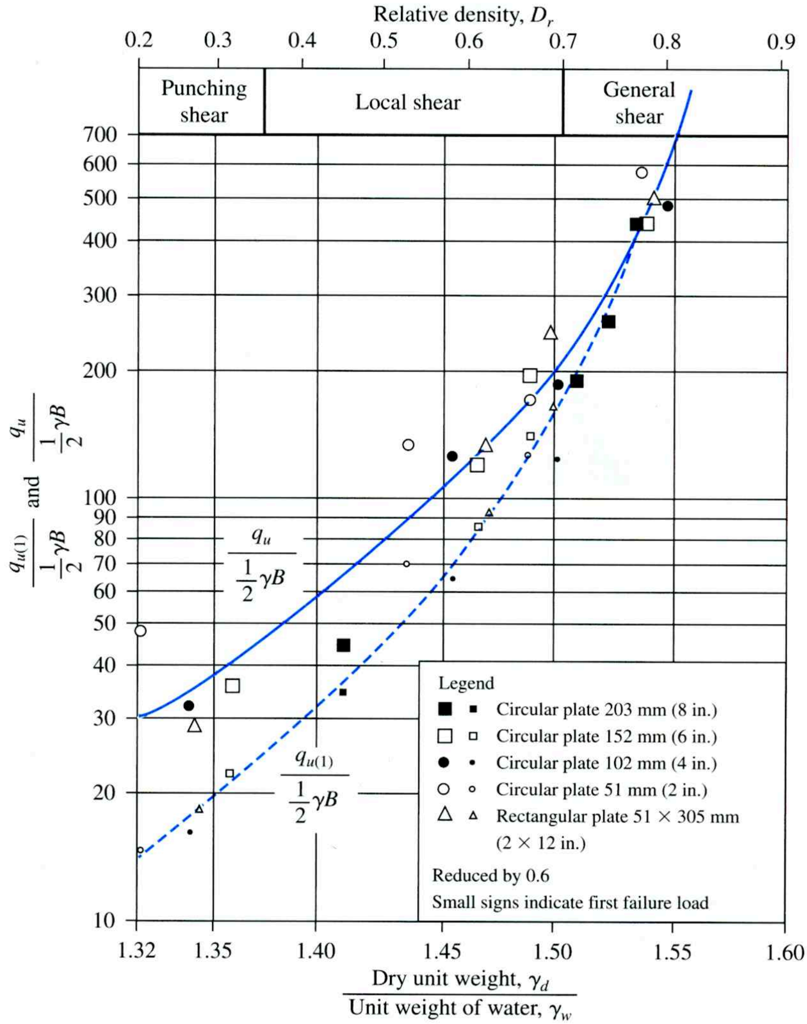


Figure 3.2 Variation of $q_{u(1)}/0.5\gamma B$ and $q_u/0.5\gamma B$ for circular and rectangular plates on the surface of a sand (Adapted from Vesic, 1963) (From Vesic, A. B. Bearing Capacity of Deep Foundations in Sand. In Highway Research Record 39, Highway Research Board, National Research Council, Washington, D.C., 1963, Figure 28, p. 137. Reproduced with permission of the Transportation Research Board.)

shows this relationship, which involves the notation

D_r = relative density of sand

D_f = depth of foundation measured from the ground surface

$$B^* = \frac{2BL}{B + L} \tag{3.1}$$

where

B = width of foundation
L = length of foundation

(*Note:* L is always greater than B.)

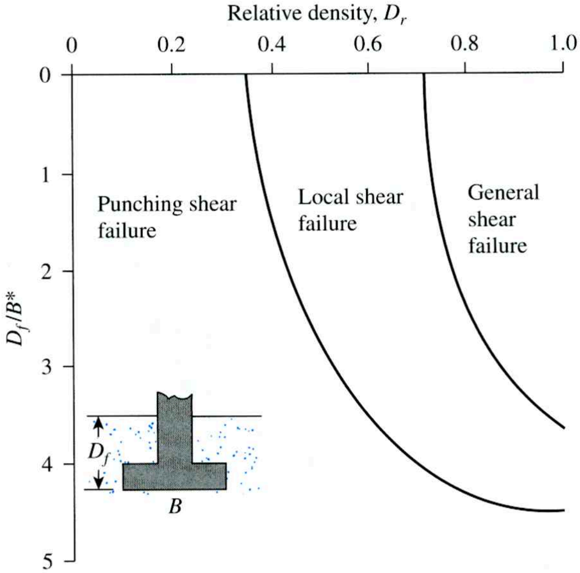


Figure 3.3 Modes of foundation failure in sand (After Vesic, 1973) (Vesic, A. S. (1973). "Analysis of Ultimate Loads of Shallow Foundations," *Journal of Soil Mechanics and Foundations Division*, American Society of Civil Engineers, Vol. 99, No. SM1, pp. 45–73. With permission from ASCE.)

For square foundations, $B = L$; for circular foundations, $B = L =$ diameter, so

$$B^* = B \tag{3.2}$$

Figure 3.4 shows the settlement S of the circular and rectangular plates on the surface of a sand at *ultimate load,* as described in Figure 3.2. The figure indicates a general range of S/B with the relative density of compaction of sand. So, in general, we can say that, for foundations at a shallow depth (i.e., small D_f/B^*), the ultimate load may occur at a foundation settlement of 4 to 10% of B. This condition arises together with general shear failure in soil; however, in the case of local or punching shear failure, the ultimate load may occur at settlements of 15 to 25% of the width of the foundation (B).

3.3 Terzaghi's Bearing Capacity Theory

Terzaghi (1943) was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations. According to this theory, a foundation is *shallow* if its depth, D_f (Figure 3.5), is less than or equal to its width. Later investigators, however, have suggested that foundations with D_f equal to 3 to 4 times their width may be defined as *shallow foundations.*

Terzaghi suggested that for a *continuous,* or *strip, foundation* (i.e., one whose width-to-length ratio approaches zero), the failure surface in soil at ultimate load may be assumed to be similar to that shown in Figure 3.5. (Note that this is the case of general shear failure, as defined in Figure 3.1a.) The effect of soil above the bottom of the foundation may also be assumed to be replaced by an equivalent surcharge, $q = \gamma D_f$ (where γ is a unit weight of soil). The failure zone under the foundation can be separated into three parts (see Figure 3.5):

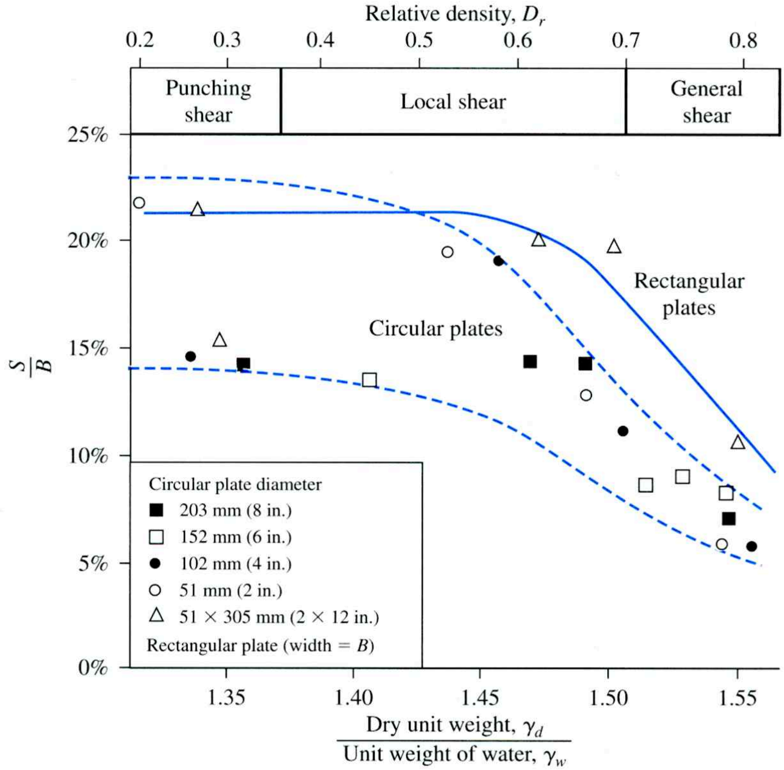


Figure 3.4 Range of settlement of circular and rectangular plates at ultimate load ($D_f/B = 0$) in sand (Modified from Vesic, 1963) (From Vesic, A. B. Bearing Capacity of Deep Foundations in Sand. In Highway Research Record 39, Highway Research Board, National Research Council, Washington, D.C., 1963, Figure 29, p. 138. Reproduced with permission of the Transportation Research Board.)

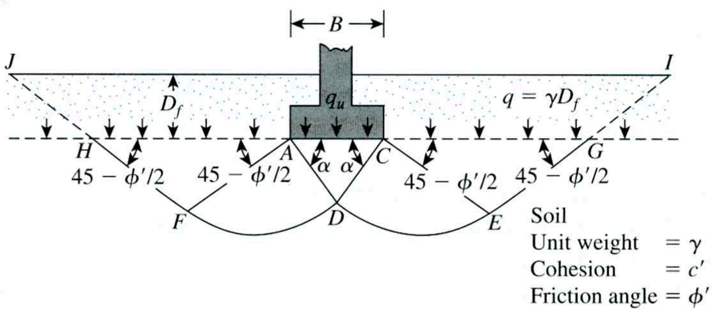


Figure 3.5 Bearing capacity failure in soil under a rough rigid continuous (strip) foundation

1. The *triangular zone ACD* immediately under the foundation
2. The *radial shear zones ADF* and *CDE,* with the curves *DE* and *DF* being arcs of a logarithmic spiral
3. Two triangular *Rankine passive zones AFH* and *CEG*

The angles *CAD* and *ACD* are assumed to be equal to the soil friction angle ϕ'. Note that, with the replacement of the soil above the bottom of the foundation by an equivalent surcharge q, the shear resistance of the soil along the failure surfaces *GI* and *HJ* was neglected.

Using equilibrium analysis, Terzaghi expressed the ultimate bearing capacity in the form

$$q_u = c'N_c + qN_q + \tfrac{1}{2}\gamma BN_\gamma \qquad \text{(continuous or strip foundation)} \tag{3.3}$$

where

c' = cohesion of soil
γ = unit weight of soil
$q = \gamma D_f$
N_c, N_q, N_γ = bearing capacity factors that are nondimensional and are functions only of the soil friction angle ϕ'

The bearing capacity factors N_c, N_q, and N_γ are defined by

$$N_c = \cot\phi'\left[\frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(\dfrac{\pi}{4}+\dfrac{\phi'}{2}\right)} - 1\right] = \cot\phi'(N_q - 1) \tag{3.4}$$

$$N_q = \frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(45+\dfrac{\phi'}{2}\right)} \tag{3.5}$$

and

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)\tan\phi' \tag{3.6}$$

where $K_{p\gamma}$ = passive pressure coefficient.

The variations of the bearing capacity factors defined by Eqs. (3.4), (3.5), and (3.6) are given in Table 3.1.

To estimate the ultimate bearing capacity of *square* and *circular foundations,* Eq. (3.1) may be respectively modified to

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma \quad \text{(square foundation)} \tag{3.7}$$

Table 3.1 Terzaghi's Bearing Capacity Factors—Eqs. (3.4), (3.5), and (3.6) a From Kumbhojkar (1993)

ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

and

$$q_u = 1.3c'N_c + qN_q + 0.3\gamma BN_\gamma \quad \text{(circular foundation)} \tag{3.8}$$

In Eq. (3.7), B equals the dimension of each side of the foundation; in Eq. (3.8), B equals the diameter of the foundation.

For foundations that exhibit the local shear failure mode in soils, Terzaghi suggested the following modifications to Eqs. (3.3), (3.7), and (3.8):

$$q_u = \tfrac{2}{3}c'N'_c + qN'_q + \tfrac{1}{2}\gamma BN'_\gamma \quad \text{(strip foundation)} \tag{3.9}$$

$$q_u = 0.867c'N'_c + qN'_q + 0.4\gamma BN'_\gamma \quad \text{(square foundation)} \tag{3.10}$$

$$q_u = 0.867c'N'_c + qN'_q + 0.3\gamma BN'_\gamma \quad \text{(circular foundation)} \tag{3.11}$$

Table 3.2 Terzaghi's Modified Bearing Capacity Factors N_c', N_q', and N_γ'

ϕ'	N_c'	N_q'	N_γ'	ϕ'	N_c'	N_q'	N_γ'
0	5.70	1.00	0.00	26	15.53	6.05	2.59
1	5.90	1.07	0.005	27	16.30	6.54	2.88
2	6.10	1.14	0.02	28	17.13	7.07	3.29
3	6.30	1.22	0.04	29	18.03	7.66	3.76
4	6.51	1.30	0.055	30	18.99	8.31	4.39
5	6.74	1.39	0.074	31	20.03	9.03	4.83
6	6.97	1.49	0.10	32	21.16	9.82	5.51
7	7.22	1.59	0.128	33	22.39	10.69	6.32
8	7.47	1.70	0.16	34	23.72	11.67	7.22
9	7.74	1.82	0.20	35	25.18	12.75	8.35
10	8.02	1.94	0.24	36	26.77	13.97	9.41
11	8.32	2.08	0.30	37	28.51	15.32	10.90
12	8.63	2.22	0.35	38	30.43	16.85	12.75
13	8.96	2.38	0.42	39	32.53	18.56	14.71
14	9.31	2.55	0.48	40	34.87	20.50	17.22
15	9.67	2.73	0.57	41	37.45	22.70	19.75
16	10.06	2.92	0.67	42	40.33	25.21	22.50
17	10.47	3.13	0.76	43	43.54	28.06	26.25
18	10.90	3.36	0.88	44	47.13	31.34	30.40
19	11.36	3.61	1.03	45	51.17	35.11	36.00
20	11.85	3.88	1.12	46	55.73	39.48	41.70
21	12.37	4.17	1.35	47	60.91	44.45	49.30
22	12.92	4.48	1.55	48	66.80	50.46	59.25
23	13.51	4.82	1.74	49	73.55	57.41	71.45
24	14.14	5.20	1.97	50	81.31	65.60	85.75
25	14.80	5.60	2.25				

N_c', N_q', and N_γ', the *modified bearing capacity factors,* can be calculated by using the bearing capacity factor equations (for N_c, N_q, and N_γ, respectively) by replacing ϕ' by $\overline{\phi}' = \tan^{-1}(\frac{2}{3}\tan\phi')$. The variation of N_c', N_q', and N_γ' with the soil friction angle ϕ' is given in Table 3.2.

Terzaghi's bearing capacity equations have now been modified to take into account the effects of the foundation shape (B/L), depth of embedment (D_f), and the load inclination. This is given in Section 3.6. Many design engineers, however, still use Terzaghi's equation, which provides fairly good results considering the uncertainty of the soil conditions at various sites.

3.4 Factor of Safety

Calculating the gross *allowable load-bearing capacity* of shallow foundations requires the application of a factor of safety (FS) to the gross ultimate bearing capacity, or

$$q_{\text{all}} = \frac{q_u}{\text{FS}} \tag{3.12}$$

However, some practicing engineers prefer to use a factor of safety such that

$$\text{Net stress increase on soil} = \frac{\text{net ultimate bearing capacity}}{\text{FS}} \tag{3.13}$$

The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the foundation that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level. If the difference between the unit weight of concrete used in the foundation and the unit weight of soil surrounding is assumed to be negligible, then

$$q_{\text{net}(u)} = q_u - q \tag{3.14}$$

where

$q_{\text{net}(u)}$ = net ultimate bearing capacity
$q = \gamma D_f$

So

$$q_{\text{all(net)}} = \frac{q_u - q}{\text{FS}} \tag{3.15}$$

The factor of safety as defined by Eq. (3.15) should be at least 3 in all cases.

Example 3.1

A square foundation is 2 m × 2 m in plan. The soil supporting the foundation has a friction angle of $\phi' = 25°$ and $c' = 20\ \text{kN/m}^2$. The unit weight of soil, γ, is $16.5\ \text{kN/m}^3$. Determine the allowable gross load on the foundation with a factor of safety (FS) of 3. Assume that the depth of the foundation (D_f) is 1.5 m and that general shear failure occurs in the soil.

Solution

From Eq. (3.7)

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

From Table 3.1, for $\phi' = 25°$,

$$N_c = 25.13$$
$$N_q = 12.72$$
$$N_\gamma = 8.34$$

Thus,

$$q_u = (1.3)(20)(25.13) + (1.5 \times 16.5)(12.72) + (0.4)(16.5)(2)(8.34)$$
$$= 653.38 + 314.82 + 110.09 = 1078.29\ \text{kN/m}^2$$

So, the allowable load per unit area of the foundation is

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{1078.29}{3} \approx 359.5 \text{ kN/m}^2$$

Thus, the total allowable gross load is

$$Q = (359.5)\ B^2 = (359.5)\ (2 \times 2) = \mathbf{1438\ kN}$$

3.5 Modification of Bearing Capacity Equations for Water Table

Equations (3.3) and (3.7) through (3.11) give the ultimate bearing capacity, based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, some modifications of the bearing capacity equations will be necessary. (See Figure 3.6.)

Case I. If the water table is located so that $0 \leq D_1 \leq D_f$, the factor q in the bearing capacity equations takes the form

$$q = \text{effective surcharge} = D_1\gamma + D_2(\gamma_{\text{sat}} - \gamma_w) \tag{3.16}$$

where

γ_{sat} = saturated unit weight of soil
γ_w = unit weight of water

Also, the value of γ in the last term of the equations has to be replaced by $\gamma' = \gamma_{\text{sat}} - \gamma_w$.

Case II. For a water table located so that $0 \leq d \leq B$,

$$q = \gamma D_f \tag{3.17}$$

In this case, the factor γ in the last term of the bearing capacity equations must be replaced by the factor

$$\overline{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma') \tag{3.18}$$

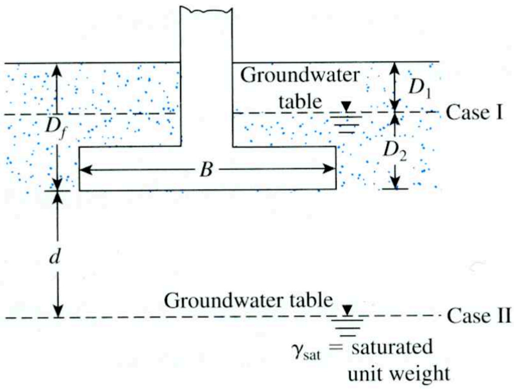


Figure 3.6 Modification of bearing capacity equations for water table

The preceding modifications are based on the assumption that there is no seepage force in the soil.

Case III. When the water table is located so that $d \geq B$, the water will have no effect on the ultimate bearing capacity.

3.6 The General Bearing Capacity Equation

The ultimate bearing capacity equations (3.3), (3.7), and (3.8) are for continuous, square, and circular foundations only; they do not address the case of rectangular foundations ($0 < B/L < 1$). Also, the equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation (the portion of the failure surface marked as *GI* and *HJ* in Figure 3.5). In addition, the load on the foundation may be inclined. To account for all these shortcomings, Meyerhof (1963) suggested the following form of the general bearing capacity equation:

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i} \tag{3.19}$$

In this equation:

c' = cohesion
q = effective stress at the level of the bottom of the foundation
γ = unit weight of soil
B = width of foundation (= diameter for a circular foundation)
$F_{cs}, F_{qs}, F_{\gamma s}$ = shape factors
$F_{cd}, F_{qd}, F_{\gamma d}$ = depth factors
$F_{ci}, F_{qi}, F_{\gamma i}$ = load inclination factors
N_c, N_q, N_γ = bearing capacity factors

The equations for determining the various factors given in Eq. (3.19) are described briefly in the sections that follow. Note that the original equation for ultimate bearing capacity is derived only for the plane-strain case (i.e., for continuous foundations). The shape, depth, and load inclination factors are empirical factors based on experimental data.

Bearing Capacity Factors

The basic nature of the failure surface in soil suggested by Terzaghi now appears to have been borne out by laboratory and field studies of bearing capacity (Vesic, 1973). However, the angle α shown in Figure 3.5 is closer to $45 + \phi'/2$ than to ϕ'. If this change is accepted, the values of N_c, N_q, and N_γ for a given soil friction angle will also change from those given in Table 3.1. With $\alpha = 45 + \phi'/2$, it can be shown that

$$N_q = \tan^2\left(45 + \frac{\phi'}{2}\right)e^{\pi \tan \phi'} \tag{3.20}$$

and

$$N_c = (N_q - 1)\cot\phi' \tag{3.21}$$

Equation (3.21) for N_c was originally derived by Prandtl (1921), and Eq. (3.20) for N_q was presented by Reissner (1924). Caquot and Kerisel (1953) and Vesic (1973) gave the relation for N_γ as

$$N_\gamma = 2(N_q + 1)\tan\phi' \tag{3.22}$$

Table 3.3 shows the variation of the preceding bearing capacity factors with soil friction angles.

Table 3.3 Bearing Capacity Factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

Shape, Depth, Inclination Factors

Commonly used shape, depth, and inclination factors are given in Table 3.4.

Table 3.4 Shape, Depth and Inclination Factors (DeBeer (1970); Hansen (1970); Meyerhof (1963); Meyerhof and Hanna (1981))

Factor	Relationship	Reference
Shape	$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$ $F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$ $F_{\gamma s} = 1-0.4\left(\frac{B}{L}\right)$	DeBeer (1970)
Depth	$\boldsymbol{\frac{D_f}{B} \leq 1}$ For $\phi = 0$: $F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$ $F_{qd} = 1$ $F_{\gamma d} = 1$ For $\phi' > 0$: $F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$ $F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$ $F_{\gamma d} = 1$ $\boldsymbol{\frac{D_f}{B} > 1}$ For $\phi = 0$: $F_{cd} = 1 + 0.4\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$ $F_{qd} = 1$ $F_{\gamma d} = 1$ For $\phi' > 0$: $F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$ $F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$ $F_{\gamma d} = 1$	Hansen (1970)
Inclination	$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$ $F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$ β = inclination of the load on the foundation with respect to the vertical	Meyerhof (1963); Hanna and Meyerhof (1981)

Example 3.2

Solve Example Problem 3.1 using Eq. (3.19).

Solution

From Eq. (3.19),

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Since the load is vertical, $F_{ci} = F_{qi} = F_{\gamma i} = 1$. From Table 3.3 for $\phi' = 25°$, $N_c = 20.72$, $N_q = 10.66$, and $N_\gamma = 10.88$.

Using Table 3.4,

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{2}{2}\right)\left(\frac{10.66}{20.72}\right) = 1.514$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1 + \left(\frac{2}{2}\right)\tan 25 = 1.466$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{2}{2}\right) = 0.6$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$$

$$= 1 + (2)(\tan 25)(1 - \sin 25)^2\left(\frac{1.5}{2}\right) = 1.233$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'} = 1.233 - \left[\frac{1 - 1.233}{(20.72)(\tan 25)}\right] = 1.257$$

$$F_{\gamma d} = 1$$

Hence,

$$\begin{aligned} q_u &= (20)(20.72)(1.514)(1.257)(1) \\ &\quad + (1.5 \times 16.5)(10.66)(1.466)(1.233)(1) \\ &\quad + \frac{1}{2}(16.5)(2)(10.88)(0.6)(1)(1) \\ &= 788.6 + 476.9 + 107.7 = 1373.2 \text{ kN/m}^2 \end{aligned}$$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{1373.2}{3} = 457.7 \text{ kN/m}^2$$

$$Q = (457.7)(2 \times 2) = \mathbf{1830.8\ kN}$$

■

Example 3.3

A square foundation ($B \times B$) has to be constructed as shown in Figure 3.7. Assume that $\gamma = 105$ lb/ft^3, $\gamma_{sat} = 118$ lb/ft^3, $\phi' = 34°$, $D_f = 4$ ft, and $D_1 = 2$ ft. The gross allowable load, Q_{all}, with FS = 3 is 150,000 lb. Determine the size of the footing. Use Eq. (3.19).

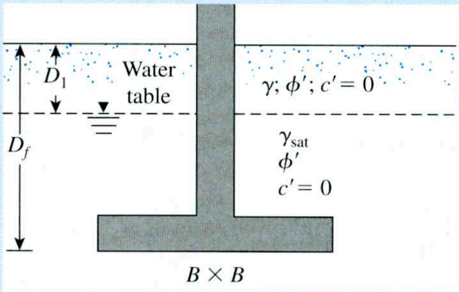


Figure 3.7 A square foundation

Solution

We have

$$q_{all} = \frac{Q_{all}}{B^2} = \frac{150{,}000}{B^2} \text{ lb/ft}^2 \tag{a}$$

From Eq. (3.19) (with $c' = 0$), for vertical loading, we obtain

$$q_{all} = \frac{q_u}{\text{FS}} = \frac{1}{3}\left(qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma' BN_\gamma F_{\gamma s}F_{\gamma d}\right)$$

For $\phi' = 34°$, from Table 3.3, $N_q = 29.44$ and $N_\gamma = 41.06$. Hence,

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' = 1 + \tan 34 = 1.67$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4 = 0.6$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 2\tan 34\,(1 - \sin 34)^2\frac{4}{B} = 1 + \frac{1.05}{B}$$

$$F_{\gamma d} = 1$$

and

$$q = (2)(105) + 2(118 - 62.4) = 321.2 \text{ lb/ft}^2$$

So

$$q_{\text{all}} = \frac{1}{3}\left[(321.2)(29.44)(1.67)\left(1 + \frac{1.05}{B}\right)\right.$$

$$\left. + \left(\frac{1}{2}\right)(118 - 62.4)(B)(41.06)(0.6)(1)\right] \quad \text{(b)}$$

$$= 5263.9 + \frac{5527.1}{B} + 228.3B$$

Combining Eqs. (a) and (b) results in

$$\frac{150{,}000}{B^2} = 5263.9 + \frac{5527.1}{B} + 228.3B$$

By trial and error, we find that $B \approx$ **4.5 ft.** ■

3.7 Case Studies on Ultimate Bearing Capacity

In this section, we will consider two field observations related to the ultimate bearing capacity of foundations on soft clay. The failure loads on the foundations in the field will be compared with those estimated from the theory presented in Section 3.6.

Foundation Failure of a Concrete Silo

An excellent case of bearing capacity failure of a 6-m (20-ft) diameter concrete silo was provided by Bozozuk (1972). The concrete tower silo was 21 m (70 ft) high and was constructed over soft clay on a ring foundation. Figure 3.8 shows the variation of the undrained shear strength (c_u) obtained from field vane shear tests at the site. The groundwater table was located at about 0.6 m (2 ft) below the ground surface.

On September 30, 1970, just after it was filled to capacity for the first time with corn silage, the concrete tower silo suddenly overturned due to bearing capacity failure. Figure 3.9 shows the approximate profile of the failure surface in soil. The failure surface extended to about 7 m (23 ft) below the ground surface. Bozozuk (1972) provided the following average parameters for the soil in the failure zone and the foundation:

- Load per unit area on the foundation when failure occurred $\approx 160 \text{ kN/m}^2$
- Average plasticity index of clay (PI) ≈ 36
- Average undrained shear strength (c_u) from 0.6 to 7 m depth obtained from field vane shear tests $\approx 27.1 \text{ kN/m}^2$
- From Figure 3.9, $B \approx 7.2$ m and $D_f \approx 1.52$ m.

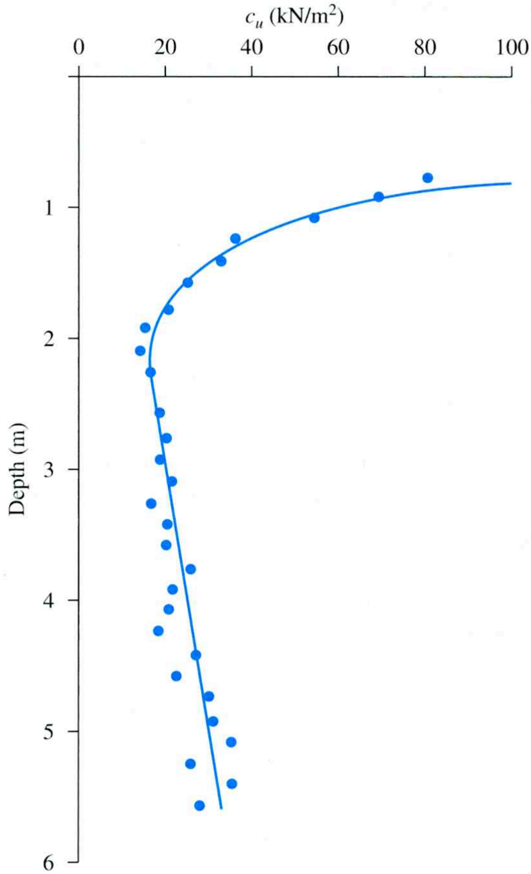


Figure 3.8 Variation of c_u with depth obtained from field vane shear test

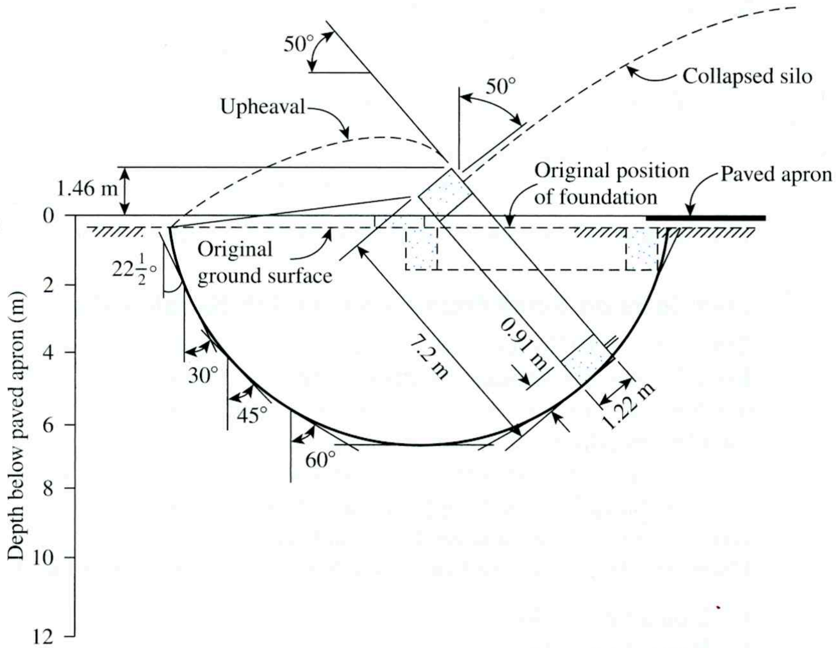


Figure 3.9 Approximate profile of silo failure (Adapted from Bozozuk, 1972)

We can now calculate the factor of safety against bearing capacity failure. From Eq. (3.19)

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_cF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B\, N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

For $\phi = 0$ condition and vertical loading, $c' = c_u$, $N_c = 5.14$, $N_q = 1$, $N_\gamma = 0$, and $F_{ci} = F_{qi} = F_{\gamma i} = 0$. Also, from Table 3.4,

$$F_{cs} = 1 + \left(\frac{7.2}{7.2}\right)\left(\frac{1}{5.14}\right) = 1.195$$

$$F_{qs} = 1$$

$$F_{cd} = 1 + (0.4)\left(\frac{1.52}{7.2}\right) = 1.08$$

$$F_{qd} = 1$$

Thus,

$$q_u = (c_u)(5.14)(1.195)(1.08)(1) + (\gamma)(1.52)$$

Assuming $\gamma \approx 18\ \text{kN/m}^3$,

$$q_u = 6.63c_u + 27.36 \tag{3.23}$$

According to Eqs. (2.34) and (2.35a),

$$c_{u(\text{corrected})} = \lambda\, c_{u(\text{VST})}$$

$$\lambda = 1.7 - 0.54 \log[\text{PI}(\%)]$$

For this case, PI ≈ 36 and $c_{u(\text{VST})} = 27.1\ \text{kN/m}^2$. So

$$c_{u(\text{corrected})} = \{1.7 - 0.54 \log[\text{PI}(\%)]\}c_{u(\text{VST})}$$

$$= (1.7 - 0.54 \log 36)(27.1) \approx 23.3\ \text{kN/m}^2$$

Substituting this value of c_u in Eq. (3.23)

$$q_u = (6.63)(23.3) + 27.36 = 181.8\ \text{kN/m}^2$$

The factor of safety against bearing capacity failure

$$\text{FS} = \frac{q_u}{\text{applied load per unit area}} = \frac{181.8}{160} = 1.14$$

This factor of safety is too low and approximately equals one, for which the failure occurred.

Load Tests on Small Foundations in Soft Bangkok Clay

Brand et al. (1972) reported load test results for five small square foundations in soft Bangkok clay in Rangsit, Thailand. The foundations were 0.6 m × 0.6 m, 0.675 m × 0.675 m, 0.75 m × 0.75 m, 0.9 m × 0.9 m, and 1.05 m × 1.05 m. The depth of of the foundations (D_f) was 1.5 m in all cases.

Figure 3.10 shows the vane shear test results for clay. Based on the variation of $c_{u(\text{VST})}$ with depth, it can be approximated that $c_{u(\text{VST})}$ is about 35 kN/m^2 for depths between zero to 1.5 m measured from the ground surface, and $c_{u(\text{VST})}$ is approximately equal to 24 kN/m^2 for depths varying from 1.5 to 8 m. Other properties of the clay are

- Liquid limit = 80
- Plastic limit = 40
- Sensitivity ≈ 5

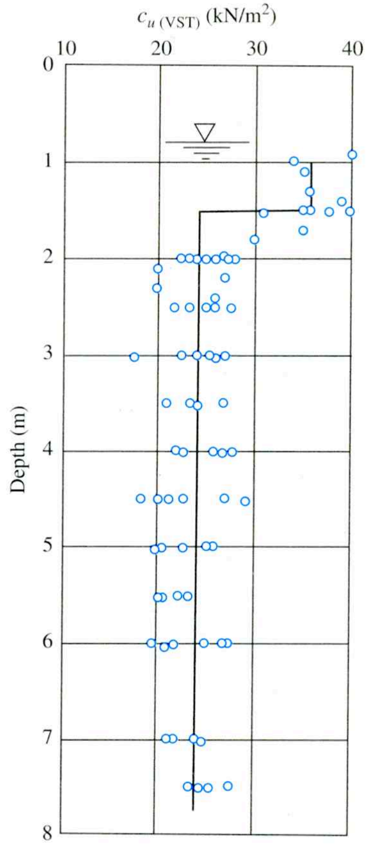


Figure 3.10 Variation of $c_{u(\mathrm{VST})}$ with depth for soft Bangkok clay

Figure 3.11 shows the load-settlement plots obtained from the bearing-capacity tests on all five foundations. The ultimate loads, Q_u, obtained from each test are shown in Figure 3.11 and given in Table 3.5. The ultimate load is defined as the point where the load-settlement plot becomes practically linear.

From Eq. (3.19),

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

For undrained condition and vertical loading (that is, $\phi = 0$) from Tables 3.3 and 3.4,

- $F_{ci} = F_{qi} = F_{\gamma i} = 1$
- $c' = c_u$, $N_c = 5.14$, $N_q = 1$, and $N_\gamma = 0$
- $F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + (1)\left(\frac{1}{5.14}\right) = 1.195$
- $F_{qs} = 1$
- $F_{qd} = 1$
- $F_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B}\right) = 1 + 0.4\tan^{-1}\left(\frac{1.5}{B}\right)$ (3.24)

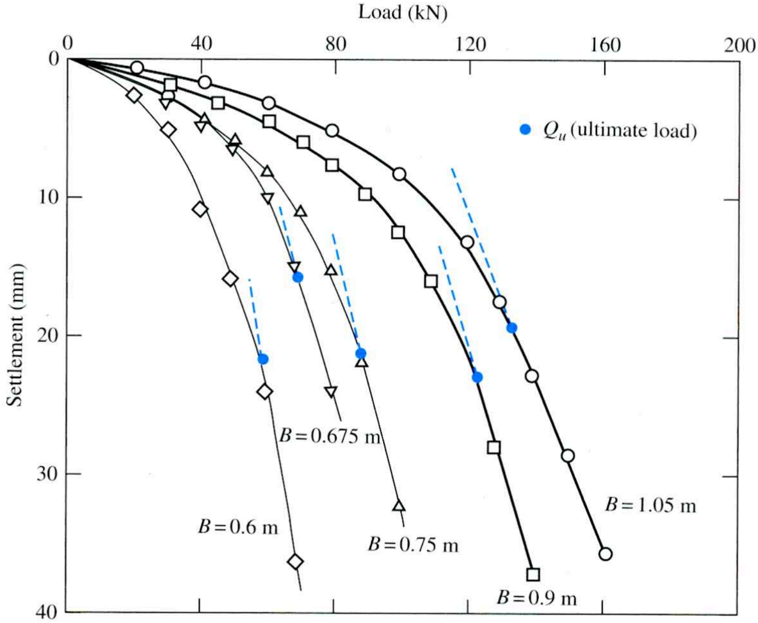


Figure 3.11 Load-settlement plots obtained from bearing capacity tests

(*Note*: $D_f/B > 1$ in all cases)

Thus,

$$q_u = (5.14)(c_u)(1.195)F_{cd} + q \tag{3.25}$$

The values of $c_{u(\text{VST})}$ need to be corrected for use in Eq. (3.25). From Eq. (2.34),

$$c_u = \lambda c_{u(\text{VST})}$$

From Eq. (2.35b),

$$\lambda = 1.18e^{-0.08(\text{PI})} + 0.57 = 1.18e^{-0.08(80-40)} + 0.57 = 0.62$$

From Eq. (2.35c),

$$\lambda = 7.01e^{-0.08(\text{LL})} + 0.57 = 7.01e^{-0.08(80)} + 0.57 = 0.58$$

Table 3.5 Comparison of Ultimate Bearing Capacity—Theory versus Field Test Results

B (m) (1)	D_f (m) (2)	F_{cd}[‡] (3)	$q_{u(\text{theory})}$[‡‡] (kN/m²) (4)	$Q_{u(\text{field})}$ (kN) (5)	$q_{u(\text{field})}$[‡‡‡] (kN/m²) (6)	$\frac{q_{u(\text{field})} - q_{u(\text{theory})}}{q_{u(\text{field})}}$ (%) (7)
0.600	1.5	1.476	158.3	60	166.6	4.98
0.675	1.5	1.459	156.8	71	155.8	−0.64
0.750	1.5	1.443	155.4	90	160.6	2.87
0.900	1.5	1.412	152.6	124	153.0	0.27
1.050	1.5	1.384	150.16	140	127.0	−18.24

[‡]Eq. (3.24); [‡‡]Eq. (3.26); [‡‡‡]$Q_{u(\text{field})}/B^2 = q_{u(\text{field})}$

So the average value of $\lambda \approx 0.6$. Hence,

$$c_u = \lambda c_{u(\mathrm{VST})} = (0.6)(24) = 14.4 \text{ kN/m}^2$$

Let us assume $\gamma = 18.5 \text{ kN/m}^2$. So

$$q = \gamma D_f = (18.5)(1.5) = 27.75 \text{ kN/m}^2$$

Substituting $c_u = 14.4 \text{ kN/m}^2$ and $q = 27.75 \text{ kN/m}^2$ into Eq. (3.25), we obtain

$$q_u(\text{kN/m}^2) = 88.4F_{cd} + 27.75 \tag{3.26}$$

The values of q_u calculated using Eq. (3.26) are given in column 4 of Table 3.5. Also, the q_u determined from the field tests are given in column 6. The theoretical and field values of q_u compare very well. The important lessons learned from this study are

1. The ultimate bearing capacity is a function of c_u. If Eq. (2.35a) would have been used to correct the undrained shear strength, the theoretical values of q_u would have varied between 200 kN/m^2 and 210 kN/m^2. These values are about 25% to 55% more than those obtained from the field and are on the unsafe side.
2. It is important to recognize that empirical correlations like those given in Eqs. (2.35a), (2.35b) and (2.35c) are sometimes site specific. Thus, proper engineering judgment and any record of past studies would be helpful in the evaluation of bearing capacity.

3.8 Effect of Soil Compressibility

In Section 3.3, Eqs. (3.3), (3.7), and (3.8), which apply to the case of general shear failure, were modified to Eqs. (3.9), (3.10), and (3.11) to take into account the change of failure mode in soil (i.e., local shear failure). The change of failure mode is due to soil compressibility, to account for which Vesic (1973) proposed the following modification of Eq. (3.19):

$$q_u = c'N_cF_{cs}F_{cd}F_{cc} + qN_qF_{qs}F_{qd}F_{qc} + \tfrac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c} \tag{3.27}$$

In this equation, F_{cc}, F_{qc}, and $F_{\gamma c}$ are soil compressibility factors.

The soil compressibility factors were derived by Vesic (1973) by analogy to the expansion of cavities. According to that theory, in order to calculate F_{cc}, F_{qc}, and $F_{\gamma c}$, the following steps should be taken:

Step 1. Calculate the *rigidity index*, I_r, of the soil at a depth approximately $B/2$ below the bottom of the foundation, or

$$I_r = \frac{G_s}{c' + q' \tan \phi'} \tag{3.28}$$

where

G_s = shear modulus of the soil
q = effective overburden pressure at a depth of $D_f + B/2$

Step 2. The critical rigidity index, $I_{r(\mathrm{cr})}$, can be expressed as

$$I_{r(\mathrm{cr})} = \frac{1}{2}\left\{\exp\left[\left(3.30 - 0.45\frac{B}{L}\right)\cot\left(45 - \frac{\phi'}{2}\right)\right]\right\} \tag{3.29}$$

The variations of $I_{r(\mathrm{cr})}$ with B/L are given in Table 3.6.

Step 3. If $I_r \geq I_{r(\mathrm{cr})}$, then

$$F_{cc} = F_{qc} = F_{\gamma c} = 1$$

However, if $I_r < I_{r(\mathrm{cr})}$, then

$$F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi' + \left[\frac{(3.07\sin\phi')(\log 2I_r)}{1 + \sin\phi'}\right]\right\} \tag{3.30}$$

Figure 3.12 shows the variation of $F_{\gamma c} = F_{qc}$ [see Eq. (3.30)] with ϕ' and I_r. For $\phi = 0$,

$$F_{cc} = 0.32 + 0.12\frac{B}{L} + 0.60\log I_r \tag{3.31}$$

For $\phi' > 0$,

$$F_{cc} = F_{qc} - \frac{1 - F_{qc}}{N_q \tan\phi'} \tag{3.32}$$

Table 3.6 Variation of $I_{r(\mathrm{cr})}$ with ϕ' and B/L

ϕ' (deg)	$I_{r(\mathrm{cr})}$					
	B/L = 0	**B/L = 0.2**	**B/L = 0.4**	**B/L = 0.6**	**B/L = 0.8**	**B/L = 1.0**
0	13.56	12.39	11.32	10.35	9.46	8.64
5	18.30	16.59	15.04	13.63	12.36	11.20
10	25.53	22.93	20.60	18.50	16.62	14.93
15	36.85	32.77	29.14	25.92	23.05	20.49
20	55.66	48.95	43.04	37.85	33.29	29.27
25	88.93	77.21	67.04	58.20	50.53	43.88
30	151.78	129.88	111.13	95.09	81.36	69.62
35	283.20	238.24	200.41	168.59	141.82	119.31
40	593.09	488.97	403.13	332.35	274.01	225.90
45	1440.94	1159.56	933.19	750.90	604.26	486.26

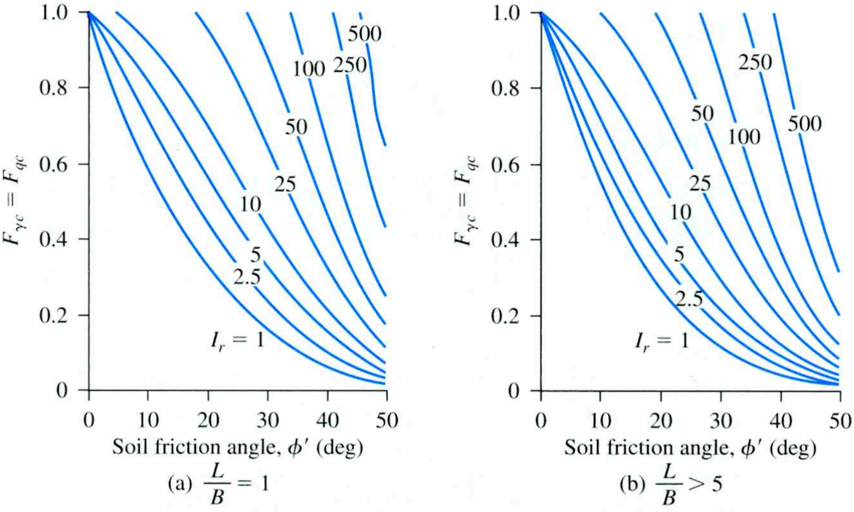


Figure 3.12 Variation of $F_{\gamma c} = F_{qc}$ with I_r and ϕ'

Example 3.4

For a shallow foundation, $B = 0.6$ m, $L = 1.2$ m, and $D_f = 0.6$ m. The known soil characteristics are as follows:

Soil:

$$\phi' = 25^\circ$$
$$c' = 48 \text{ kN/m}^2$$
$$\gamma = 18 \text{ kN/m}^3$$

Modulus of elasticity, $E_s = 620 \text{ kN/m}^2$

Poisson's ratio, $\mu_s = 0.3$

Calculate the ultimate bearing capacity.

Solution

From Eq. (3.28),

$$I_r = \frac{G_s}{c' + q' \tan \phi'}$$

However,

$$G_s = \frac{E_s}{2(1 + \mu_s)}$$

So

$$I_r = \frac{E_s}{2(1 + \mu_s)[c' + q' \tan \phi']}$$

Now,

$$q' = \gamma\left(D_f + \frac{B}{2}\right) = 18\left(0.6 + \frac{0.6}{2}\right) = 16.2 \text{ kN/m}^2$$

Thus,

$$I_r = \frac{620}{2(1 + 0.3)[48 + 16.2 \tan 25]} = 4.29$$

From Eq. (3.29),

$$I_{r(\mathrm{cr})} = \frac{1}{2}\left\{\exp\left[\left(3.3 - 0.45\frac{B}{L}\right)\cot\left(45 - \frac{\phi'}{2}\right)\right]\right\}$$

$$= \frac{1}{2}\left\{\exp\left[\left(3.3 - 0.45\frac{0.6}{1.2}\right)\cot\left(45 - \frac{25}{2}\right)\right]\right\} = 62.41$$

Since $I_{r(\mathrm{cr})} > I_r$, we use Eqs. (3.30) and (3.32) to obtain

$$F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi' + \left[\frac{(3.07\sin\phi')\log(2I_r)}{1 + \sin\phi'}\right]\right\}$$

$$= \exp\left\{\left(-4.4 + 0.6\frac{0.6}{1.2}\right)\tan 25 + \left[\frac{(3.07\sin 25)\log(2 \times 4.29)}{1 + \sin 25}\right]\right\} = 0.347$$

and

$$F_{cc} = F_{qc} - \frac{1 - F_{qc}}{N_c \tan\phi'}$$

For $\phi' = 25°$, $N_c = 20.72$ (see Table 3.3); therefore,

$$F_{cc} = 0.347 - \frac{1 - 0.347}{20.72\tan 25} = 0.279$$

Now, from Eq. (3.27),

$$q_u = c'N_cF_{cs}F_{cd}F_{cc} + qN_qF_{qs}F_{qd}F_{qc} + \tfrac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c}$$

From Table 3.3, for $\phi' = 25°$, $N_c = 20.72$, $N_q = 10.66$, and $N_\gamma = 10.88$. Consequently,

$$F_{cs} = 1 + \left(\frac{N_q}{N_c}\right)\left(\frac{B}{L}\right) = 1 + \left(\frac{10.66}{20.72}\right)\left(\frac{0.6}{1.2}\right) = 1.257$$

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' = 1 + \frac{0.6}{1.2}\tan 25 = 1.233$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\frac{0.6}{1.2} = 0.8$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 25(1 - \sin 25)^2\left(\frac{0.6}{0.6}\right) = 1.311$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c\tan\phi'} = 1.311 - \frac{1 - 1.311}{20.72\tan 25}$$

$$= 1.343$$

and

$$F_{\gamma d} = 1$$

Thus,

$$q_u = (48)(20.72)(1.257)(1.343)(0.279) + (0.6 \times 18)(10.66)(1.233)(1.311)(0.347) + (\tfrac{1}{2})(18)(0.6)(10.88)(0.8)(1)(0.347) = \mathbf{549.32\ kN/m^2}$$

3.9 Eccentrically Loaded Foundations

In several instances, as with the base of a retaining wall, foundations are subjected to moments in addition to the vertical load, as shown in Figure 3.13a. In such cases, the distribution of pressure by the foundation on the soil is not uniform. The nominal distribution of pressure is

$$q_{\max} = \frac{Q}{BL} + \frac{6M}{B^2L} \tag{3.33}$$

and

$$q_{\min} = \frac{Q}{BL} - \frac{6M}{B^2L} \tag{3.34}$$

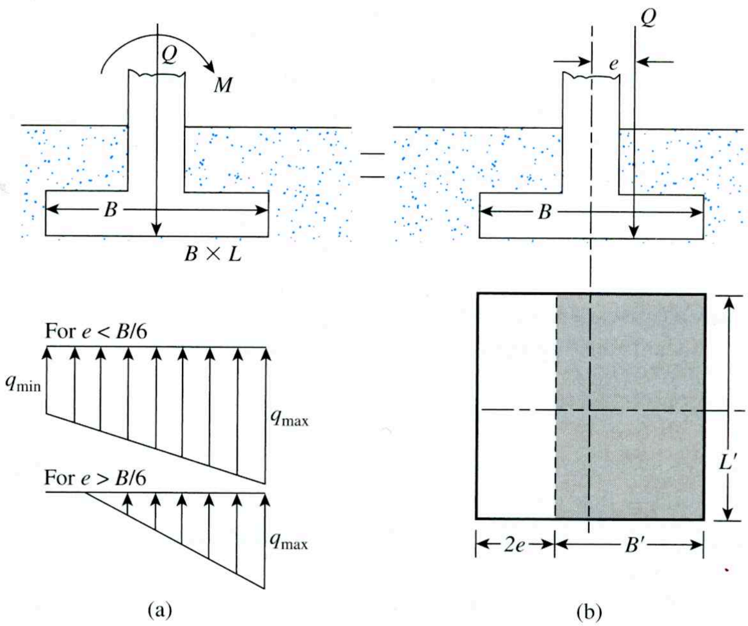


Figure 3.13 Eccentrically loaded foundations

where

Q = total vertical load
M = moment on the foundation

Figure 3.13b shows a force system equivalent to that shown in Figure 3.13a. The distance

$$e = \frac{M}{Q} \tag{3.35}$$

is the eccentricity. Substituting Eq. (3.35) into Eqs. (3.33) and (3.34) gives

$$q_{\max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right) \tag{3.36}$$

and

$$q_{\min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right) \tag{3.37}$$

Note that, in these equations, when the eccentricity e becomes $B/6$, $q_{\min}$ is zero. For $e > B/6$, $q_{\min}$ will be negative, which means that tension will develop. Because soil cannot take any tension, there will then be a separation between the foundation and the soil underlying it. The nature of the pressure distribution on the soil will be as shown in Figure 3.13a. The value of $q_{\max}$ is then

$$q_{\max} = \frac{4Q}{3L(B - 2e)} \tag{3.38}$$

The exact distribution of pressure is difficult to estimate.

Figure 3.14 shows the nature of failure surface in soil for a surface strip foundation subjected to an eccentric load. The factor of safety for such type of loading against bearing capacity failure can be evaluated as

$$\mathrm{FS} = \frac{Q_{\text{ult}}}{Q} \tag{3.39}$$

where Q_{ult} = ultimate load-carrying capacity.

The following sections describe several theories for determining Q_{ult}.

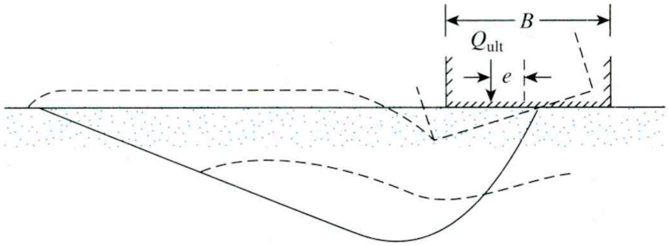


Figure 3.14 Nature of failure surface in soil supporting a strip foundation subjected to eccentric loading (*Note:* $D_f = 0$; Q_{ult} is ultimate load per unit length of foundation)

3.10 Ultimate Bearing Capacity under Eccentric Loading—One-Way Eccentricity

Effective Area Method (Meyerhoff, 1953)

In 1953, Meyerhof proposed a theory that is generally referred to as the *effective area method*.

The following is a step-by-step procedure for determining the ultimate load that the soil can support and the factor of safety against bearing capacity failure:

Step 1. Determine the effective dimensions of the foundation (Figure 3.13b):

$$B' = \text{effective width} = B - 2e$$
$$L' = \text{effective length} = L$$

Note that if the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to $L - 2e$. The value of B' would equal B. The smaller of the two dimensions (i.e., L' and B') is the effective width of the foundation.

Step 2. Use Eq. (3.19) for the ultimate bearing capacity:

$$q'_u = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \tag{3.40}$$

To evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$, use the relationships given in Table 3.4 with *effective length* and *effective width* dimensions instead of L and B, respectively. To determine F_{cd}, F_{qd}, and $F_{\gamma d}$, use the relationships given in Table 3.4. However, do not replace B with B'.

Step 3. The total ultimate load that the foundation can sustain is

$$Q_{\text{ult}} = q'_u \overbrace{(B')(L')}^{A'} \tag{3.41}$$

where A' = effective area.

Step 4. The factor of safety against bearing capacity failure is

$$\text{FS} = \frac{Q_{\text{ult}}}{Q}$$

Prakash and Saran Theory

Prakash and Saran (1971) analyzed the problem of ultimate bearing capacity of eccentrically and vertically loaded continuous (strip) foundations by using the one-sided failure surface in soil, as shown in Figure 3.14. According to this theory, the ultimate load *per unit length of a continuous foundation* can be estimated as

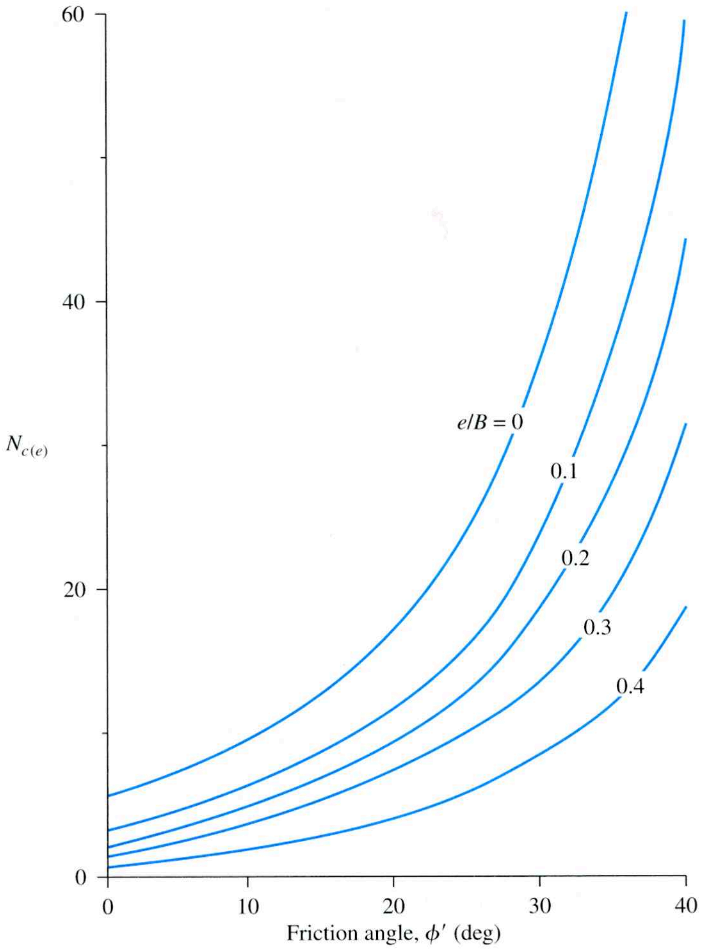
$$Q_{\text{ult}} = B\left[c'N_{c(e)} + qN_{q(e)} + \frac{1}{2}\gamma B N_{\gamma(e)}\right] \tag{3.42}$$

where $N_{c(e)}$, $N_{q(e)}$, $N_{\gamma(e)}$ = bearing capacity factors under eccentric loading.

The variations of $N_{c(e)}$, $N_{q(e)}$, and $N_{\gamma(e)}$ with soil friction angle ϕ' are given in Figures 3.15, 3.16, and 3.17. For rectangular foundations, the ultimate load can be given as

$$Q_{\text{ult}} = BL\left[c'N_{c(e)}F_{cs(e)} + qN_{q(e)}F_{qs(e)} + \frac{1}{2}\gamma B N_{\gamma(e)}F_{\gamma s(e)}\right] \tag{3.43}$$

where $F_{cs(e)}$, $F_{qs(e)}$, and $F_{\gamma s(e)}$ = shape factors.



$\phi' = 40°$

e/B	$N_{c(e)}$
0	94.83
0.1	66.60
0.2	54.45
0.3	36.3
0.4	18.15

Figure 3.15 Variation of $N_{c(e)}$ with ϕ'

Prakash and Saran (1971) also recommended the following for the shape factors:

$$F_{cs(e)} = 1.2 - 0.025\frac{L}{B} \text{ (with a minimum of 1.0)} \tag{3.44}$$

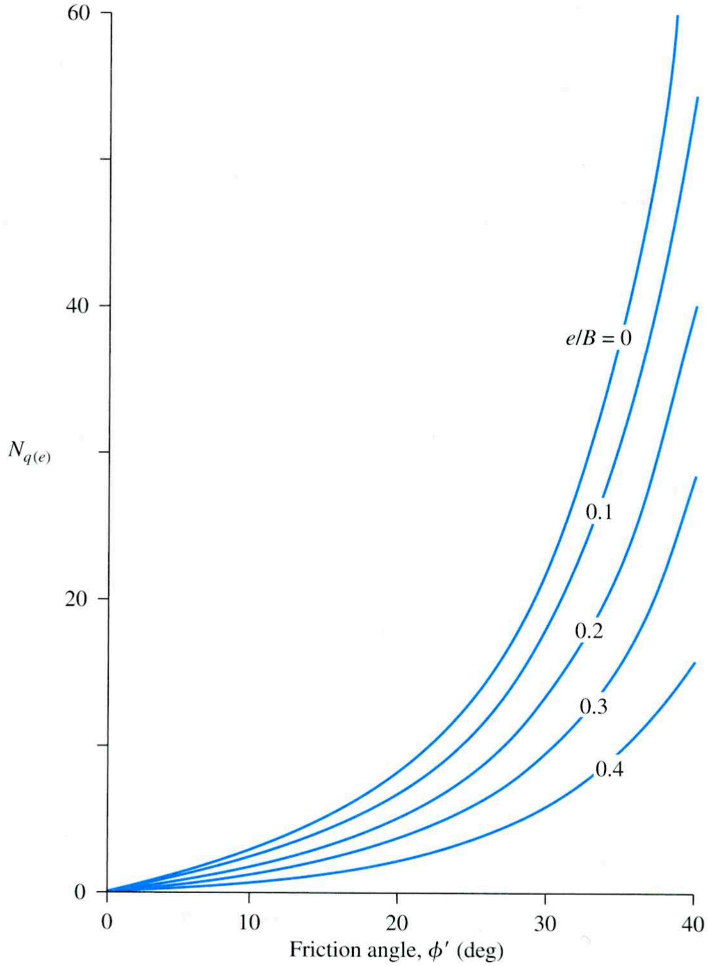
$$F_{qs(e)} = 1 \tag{3.45}$$

and

$$F_{\gamma s(e)} = 1.0 + \left(\frac{2e}{B} - 0.68\right)\frac{B}{L} + \left[0.43 - \left(\frac{3}{2}\right)\left(\frac{e}{B}\right)\right]\left(\frac{B}{L}\right)^2 \tag{3.46}$$

Reduction Factor Method (For Granular Soil)

Purkayastha and Char (1977) carried out stability analysis of eccentrically loaded *continuous foundations supported by a layer of sand* using the method of slices. Based on that analysis, they proposed



$\phi' = 40°$

e/B	$N_{q(e)}$
0	81.27
0.1	56.09
0.2	45.18
0.3	30.18
0.4	15.06

Figure 3.16 Variation of $N_{q(e)}$ with ϕ'

$$R_k = 1 - \frac{q_{u(\text{eccentric})}}{q_{u(\text{centric})}} \tag{3.47}$$

where

R_k = reduction factor

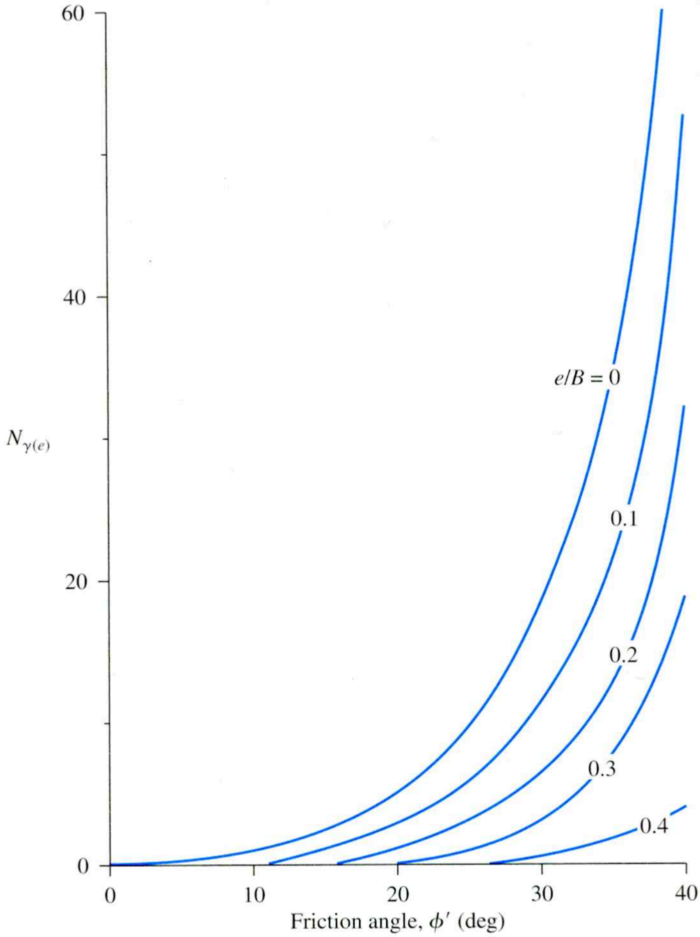
$q_{u(\text{eccentric})}$ = ultimate bearing capacity of eccentrically loaded continuous foundations

$q_{u(\text{centric})}$ = ultimate bearing capacity of centrally loaded continuous foundations

The magnitude of R_k can be expressed as

$$R_k = a\left(\frac{e}{B}\right)^k \tag{3.48}$$

where a and k are functions of the embedment ratio D_f/B (Table 3.7).



$\phi' = 40°$

e/B	$N_{\gamma(e)}$
0	115.80
0.1	71.80
0.2	41.60
0.3	18.50
0.4	4.62

Figure 3.17 Variation of $N_{\gamma(e)}$ with ϕ'

Table 3.7 Variations of a and k [Eq. (3.48)]

D_f/B	a	k
0.00	1.862	0.73
0.25	1.811	0.785
0.50	1.754	0.80
1.00	1.820	0.888

Hence, combining Eqs. (3.47) and (3.48)

$$q_{u(\text{eccentric})} = q_{u(\text{centric})}(1 - R_k) = q_{u(\text{centric})}\left[1 - a\left(\frac{e}{B}\right)^k\right] \tag{3.49}$$

where

$$q_{u(\text{centric})} = qN_qF_{qd} + \frac{1}{2}\gamma BN_\gamma F_{\gamma d} \tag{3.50}$$

The relationships for F_{qd}, and $F_{\gamma d}$ are given in Table 3.4.

The ultimate load *per unit length* of the foundation can then be given as

$$Q_u = Bq_{u(\text{eccentric})} \tag{3.51}$$

Example 3.5

A continuous foundation is shown in Figure 3.18. If the load eccentricity is 0.2 m, determine the ultimate load, Q_{ult}, per unit length of the foundation. Use Meyerhof's effective area method.

Solution

For $c' = 0$, Eq. (3.40) gives

$$q'_u = qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma' B' N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

where $q = (16.5)\,(1.5) = 24.75\ \text{kN/m}^2$.

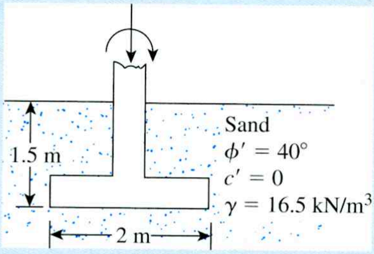


Figure 3.18 A continuous foundation with load eccentricity

For $\phi' = 40°$, from Table 3.3, $N_q = 64.2$ and $N_\gamma = 109.41$. Also,

$$B' = 2 - (2)(0.2) = 1.6\ \text{m}$$

Because the foundation in question is a continuous foundation, B'/L' is zero. Hence, $F_{qs} = 1$, $F_{\gamma s} = 1$. From Table 3.4,

$$F_{qi} = F_{\gamma i} = 1$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 0.214\left(\frac{1.5}{2}\right) = 1.16$$

$$F_{\gamma d} = 1$$

and

$$q'_u = (24.75)\,(64.2)\,(1)\,(1.16)\,(1) + \left(\frac{1}{2}\right)(16.5)\,(1.6)\,(109.41)\,(1)\,(1)\,(1) = 3287.39\ \text{kN/m}^2$$

Consequently,

$$Q_{\text{ult}} = (B')(1)(q'_u) = (1.6)(1)(3287.39) \approx \mathbf{5260\ kN}$$

■

Example 3.6

Solve Example 3.5 using Eq. (3.42).

Solution

Since $c' = 0$

$$Q_{ult} = B\left[qN_{q(e)} + \frac{1}{2}\gamma BN_{\gamma(e)}\right]$$

$$\frac{e}{B} = \frac{0.2}{2} = 0.1$$

For $\phi' = 40°$ and $e/B = 0.1$, Figures 3.16 and 3.17 give $N_{q(e)} = 56.09$ and $N_{\gamma(e)} \approx 71.8$. Hence,

$$Q_{\text{ult}} = 2[(24.75)(56.09) + (\tfrac{1}{2})(16.5)(2)(71.8)] = \mathbf{5146\ kN}$$ ■

Example 3.7

Solve Example 3.5 using Eq. (3.49).

Solution

With $c' = 0$,

$$q_{u(\text{centric})} = qN_qF_{qd} + \frac{1}{2}\gamma BN_\gamma F_{\gamma d}$$

For $\phi' = 40°$, $N_q = 64.2$ and $N_\gamma = 109.41$ (see Table 3.3). Hence,

$$F_{qd} = 1.16 \text{ and } F_{\gamma d} = 1 \text{ (see Example 3.5)}$$

$$q_{u(\text{centric})} = (24.75)(64.2)(1.16) + \frac{1}{2}(16.5)(2)(109.41)(1)$$
$$= 1843.18 + 1805.27 = 3648.45 \text{ kN/m}^2$$

From Eq. (3.48),

$$R_k = a\left(\frac{e}{B}\right)^k$$

For $D_f/B = 1.5/2 = 0.75$, Table 3.7 gives $a \approx 1.75$ and $k \approx 0.85$. Hence,

$$R_k = 1.79\left(\frac{0.2}{2}\right)^{0.85} = 0.253$$

$$Q_u = Bq_{u(\text{eccentric})} = Bq_{u(\text{centric})}(1 - R_k) = (2)(3648.45)(1 - 0.253) \approx \mathbf{5451\ kN}$$ ■

3.11 Bearing Capacity—Two-way Eccentricity

Consider a situation in which a foundation is subjected to a vertical ultimate load Q_{ult} and a moment M, as shown in Figures 3.19a and b. For this case, the components of the moment M about the x- and y-axes can be determined as M_x and M_y, respectively. (See Figure 3.19.) This condition is equivalent to a load Q_{ult} placed eccentrically on the foundation with $x = e_B$ and $y = e_L$ (Figure 3.19d). Note that

$$e_B = \frac{M_y}{Q_{\text{ult}}} \tag{3.52}$$

and

$$e_L = \frac{M_x}{Q_{\text{ult}}} \tag{3.53}$$

If Q_{ult} is needed, it can be obtained from Eq. (3.41); that is,

$$Q_{\text{ult}} = q'_u A'$$

where, from Eq. (3.40),

$$q'_u = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

and

$$A' = \text{effective area} = B'L'$$

As before, to evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$ (Table 3.4), we use the effective length L' and effective width B' instead of L and B, respectively. To calculate F_{cd}, F_{qd}, and $F_{\gamma d}$, we do

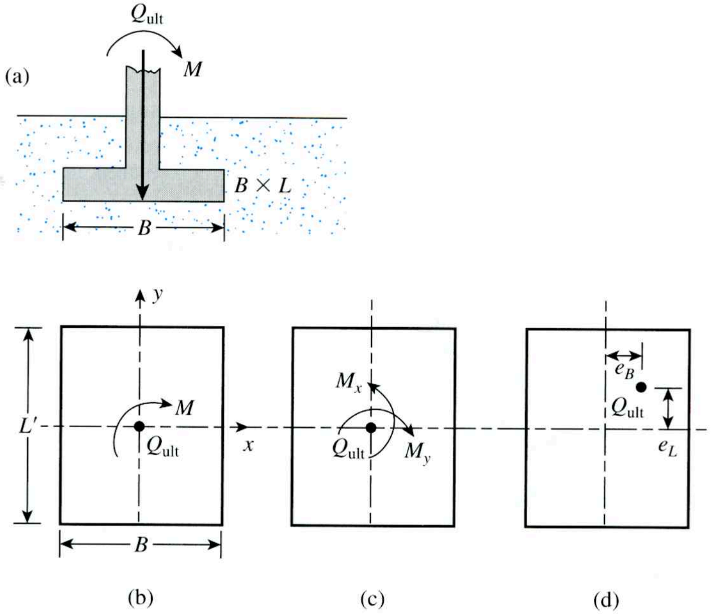


Figure 3.19 Analysis of foundation with two-way eccentricity

not replace B with B'. In determining the effective area A', effective width B', and effective length L', five possible cases may arise (Highter and Anders, 1985).

Case I. $e_L/L \geq \frac{1}{6}$ and $e_B/B \geq \frac{1}{6}$. The effective area for this condition is shown in Figure 3.20, or

$$A' = \tfrac{1}{2}B_1L_1 \tag{3.54}$$

where

$$B_1 = B\left(1.5 - \frac{3e_B}{B}\right) \tag{3.55}$$

and

$$L_1 = L\left(1.5 - \frac{3e_L}{L}\right) \tag{3.56}$$

The effective length L' is the larger of the two dimensions B_1 and L_1. So the effective width is

$$B' = \frac{A'}{L'} \tag{3.57}$$

Case II. $e_L/L < 0.5$ and $0 < e_B/B < \frac{1}{6}$. The effective area for this case, shown in Figure 3.21a, is

$$A' = \tfrac{1}{2}(L_1 + L_2)B \tag{3.58}$$

The magnitudes of L_1 and L_2 can be determined from Figure 3.21b. The effective width is

$$B' = \frac{A'}{L_1 \text{ or } L_2 \quad (\text{whichever is larger})} \tag{3.59}$$

The effective length is

$$L' = L_1 \text{ or } L_2 \quad (\text{whichever is larger}) \tag{3.60}$$

Case III. $e_L/L < \frac{1}{6}$ and $0 < e_B/B < 0.5$. The effective area, shown in Figure 3.22a, is

$$A' = \tfrac{1}{2}(B_1 + B_2)L \tag{3.61}$$

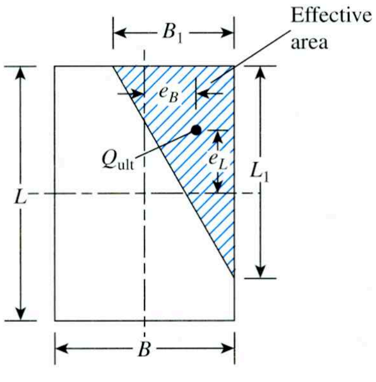


Figure 3.20 Effective area for the case of $e_L/L \geqslant \frac{1}{6}$ and $e_B/B \geqslant \frac{1}{6}$

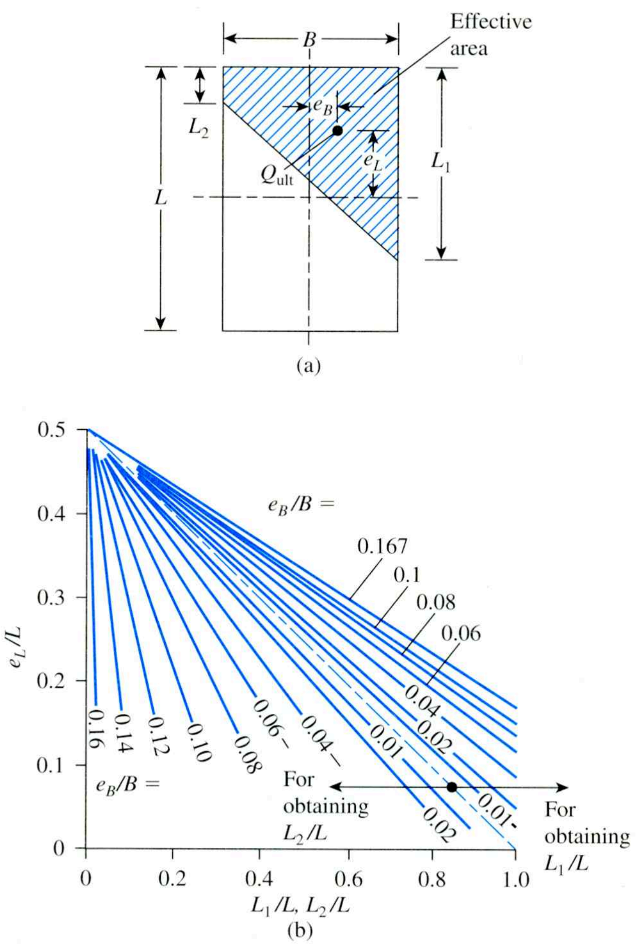


Figure 3.21 Effective area for the case of $e_L/L < 0.5$ and $0 < e_B/B < \frac{1}{6}$ (After Highter and Anders, 1985) (Highter, W. H. and Anders, J. C. (1985). "Dimensioning Footings Subjected to Eccentric Loads," *Journal of Geotechnical Engineering*, American Society of Civil Engineers, Vol. 111, No. GT5, pp. 659–665. With permission from ASCE.)

The effective width is

$$B' = \frac{A'}{L} \tag{3.62}$$

The effective length is

$$L' = L \tag{3.63}$$

The magnitudes of B_1 and B_2 can be determined from Figure 3.22b.

Case IV. $e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$. Figure 3.23a shows the effective area for this case. The ratio B_2/B, and thus B_2, can be determined by using the e_L/L curves that slope upward. Similarly, the ratio L_2/L, and thus L_2, can be determined by using the e_L/L curves that slope downward. The effective area is then

$$A' = L_2B + \tfrac{1}{2}(B + B_2)(L - L_2) \tag{3.64}$$

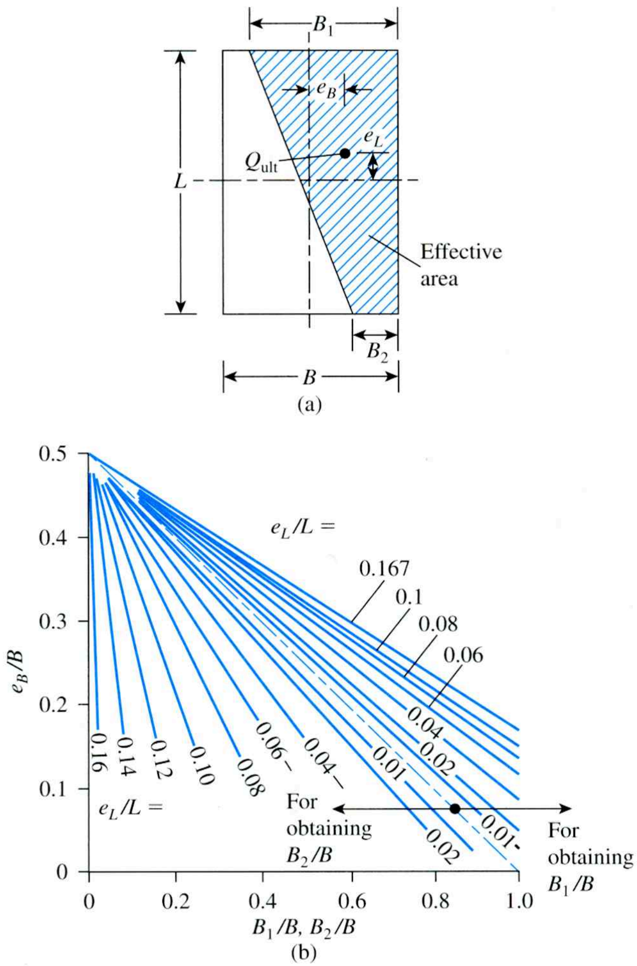


Figure 3.22 Effective area for the case of $e_L/L < \frac{1}{6}$ and $0 < e_B/B < 0.5$ (After Highter and Anders, 1985) Highter, W. H. and Anders, J. C. (1985). "Dimensioning Footings Subjected to Eccentric Loads," *Journal of Geotechnical Engineering*, American Society of Civil Engineers, Vol. 111, No. GT5, pp. 659–665. With permission from ASCE.)

The effective width is

$$B' = \frac{A'}{L} \tag{3.65}$$

The effective length is

$$L' = L \tag{3.66}$$

Case V. (Circular Foundation) In the case of circular foundations under eccentric loading (Figure 3.24a), the eccentricity is always one way. The effective area A' and the effective width B' for a circular foundation are given in a nondimensional form in Table 3.8. Once A' and B' are determined, the effective length can be obtained as

$$L' = \frac{A'}{B'}$$

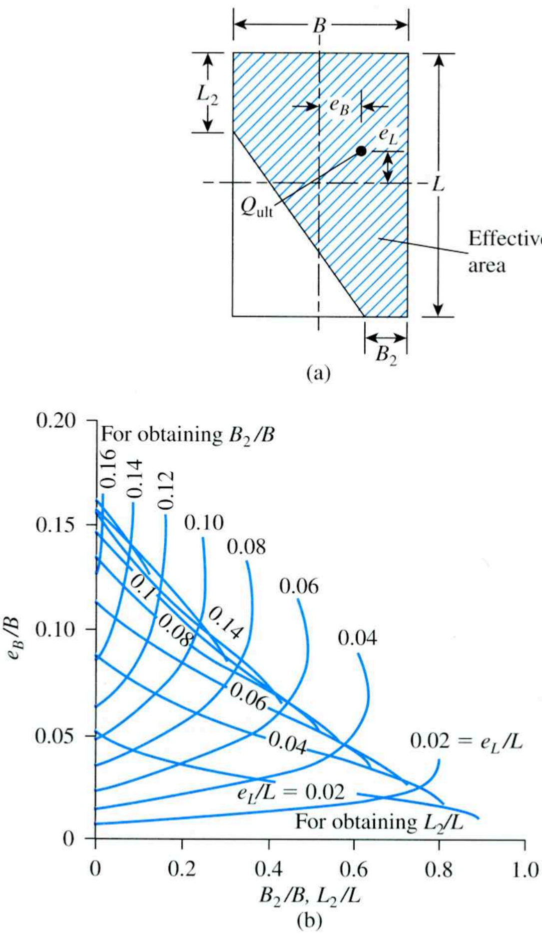


Figure 3.23 Effective area for the case of $e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$ (After Highter and Anders, 1985) (Highter, W. H. and Anders, J. C. (1985). "Dimensioning Footings Subjected to Eccentric Loads," *Journal of Geotechnical Engineering*, American Society of Civil Engineers, Vol. 111, No. GT5, pp. 659–665. With permission from ASCE.)

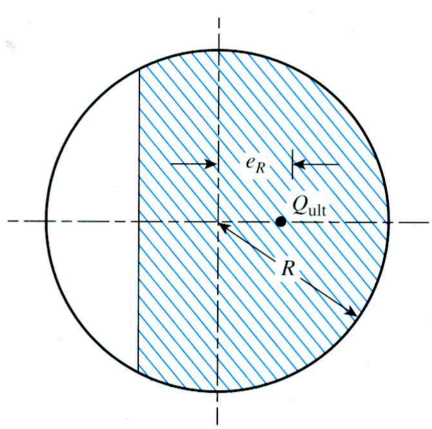


Figure 3.24 Effective area for circular foundation

Table 3.8 Variation of A'/R^2 and B'/R with e_R/R for Circular Foundations

e_R'/R	A'/R^2	B'/R
0.1	2.8	1.85
0.2	2.4	1.32
0.3	2.0	1.2
0.4	1.61	0.80
0.5	1.23	0.67
0.6	0.93	0.50
0.7	0.62	0.37
0.8	0.35	0.23
0.9	0.12	0.12
1.0	0	0

Example 3.8

A square foundation is shown in Figure 3.25, with $e_L = 0.3$ m and $e_B = 0.15$ m. Assume two-way eccentricity, and determine the ultimate load, Q_{ult}.

Solution

We have

$$\frac{e_L}{L} = \frac{0.3}{1.5} = 0.2$$

and

$$\frac{e_B}{B} = \frac{0.15}{1.5} = 0.1$$

This case is similar to that shown in Figure 3.21a. From Figure 3.21b, for $e_L/L = 0.2$ and $e_B/B = 0.1$,

$$\frac{L_1}{L} \approx 0.85; \qquad L_1 = (0.85)(1.5) = 1.275 \text{ m}$$

and

$$\frac{L_2}{L} \approx 0.21; \qquad L_2 = (0.21)(1.5) = 0.315 \text{ m}$$

From Eq. (3.58),

$$A' = \tfrac{1}{2}(L_1 + L_2)B = \tfrac{1}{2}(1.275 + 0.315)(1.5) = 1.193 \text{ m}^2$$

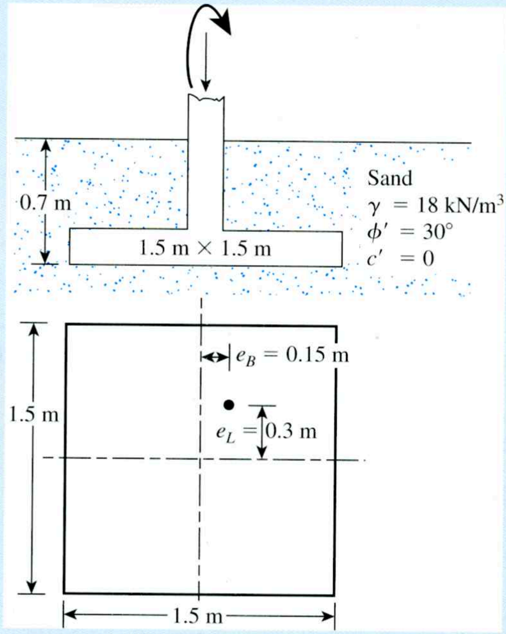


Figure 3.25 An eccentrically loaded foundation

From Eq. (3.60),

$$L' = L_1 = 1.275 \text{ m}$$

From Eq. (3.59),

$$B' = \frac{A'}{L'} = \frac{1.193}{1.275} = 0.936 \text{ m}$$

Note from Eq. (3.40) with $c' = 0$,

$$q'_u = qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

where $q = (0.7)(18) = 12.6 \text{ kN/m}^2$.

For $\phi' = 30°$, from Table 3.3, $N_q = 18.4$ and $N_\gamma = 22.4$. Thus from Table 3.4,

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{0.936}{1.275}\right)\tan 30° = 1.424$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{0.936}{1.275}\right) = 0.706$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\,\frac{D_f}{B} = 1 + \frac{(0.289)(0.7)}{1.5} = 1.135$$

and

$$F_{\gamma d} = 1$$

So

$$Q_{ult} = A'q'_u = A'(qN_qF_{qs}F_{qd} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d})$$
$$= (1.193)[(12.6)(18.4)(1.424)(1.135)$$
$$+(0.5)(18)(0.936)(22.4)(0.706)(1)] \approx \mathbf{606\ kN}$$

Example 3.9

Consider the foundation shown in Figure 3.25 with the following changes:

$$e_L = 0.18 \text{ m}$$
$$e_B = 0.12 \text{ m}$$

For the soil, $\gamma = 16.5 \text{ kN/m}^3$

$$\phi' = 25°$$
$$c' = 25 \text{ kN/m}^2$$

Determine the ultimate load, Q_{ult}.

Solution

$$\frac{e_L}{L} = \frac{0.18}{1.5} = 0.12; \qquad \frac{e_B}{B} = \frac{0.12}{1.5} = 0.08$$

This is the case shown in Figure 3.23a. From Figure 3.23b,

$$\frac{B_2}{B} \approx 0.1; \qquad \frac{L_2}{L} \approx 0.32$$

So

$$B_2 = (0.1)(1.5) = 0.15 \text{ m}$$
$$L_2 = (0.32)(1.5) = 0.48 \text{ m}$$

From Eq. (3.64),

$$A' = L_2B + \frac{1}{2}(B + B_2)(L - L_2) = (0.48)(1.5) + \frac{1}{2}(1.5 + 0.15)(1.5 - 0.48)$$
$$= 0.72 + 0.8415 = 1.5615 \text{ m}^2$$
$$B' = \frac{A'}{L} = \frac{1.5615}{1.5} = 1.041\text{m}$$
$$L' = 1.5 \text{ m}$$

From Eq. (3.40),

$$q'_u = c'N_cF_{cs}F_{cd} + qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}$$

For $\phi' = 25°$, Table 3.3 gives $N_c = 20.72$, $N_q = 10.66$ and $N_\gamma = 10.88$. From Table 3.4,

$$F_{cs} = 1 + \left(\frac{B'}{L'}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1.041}{1.5}\right)\left(\frac{10.66}{20.72}\right) = 1.357$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{1.041}{1.5}\right)\tan 25 = 1.324$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.041}{1.5}\right) = 0.722$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 25(1 - \sin 25)^2\left(\frac{0.7}{1.5}\right) = 1.145$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'} = 1.145 - \frac{1 - 1.145}{20.72\tan 25} = 1.16$$

$$F_{\gamma d} = 1$$

Hence,

$$q'_u = (25)(20.72)(1.357)(1.16) + (16.5 \times 0.7)(10.66)(1.324)(1.145)$$
$$+ \frac{1}{2}(16.5)(1.041)(10.88)(0.722)(1)$$
$$= 815.39 + 186.65 + 67.46 = 1069.5 \text{ kN/m}^2$$
$$Q_{ult} = A'q'_u = (1069.5)(1.5615) = \mathbf{1670\ kN}$$

■

3.12 Bearing Capacity of a Continuous Foundation Subjected to Eccentric Inclined Loading

The problem of ultimate bearing capacity of a *continuous foundation* subjected to an eccentric inclined load was studied by Saran and Agarwal (1991). If a continuous foundation is located at a depth D_f below the ground surface and is subjected to an eccentric load (load eccentricity $= e$) inclined at an angle β to the vertical, the ultimate capacity can be expressed as

$$Q_{ult} = B\left[c'N_{c(ei)} + qN_{q(ei)} + \frac{1}{2}\gamma B N_{\gamma(ei)}\right] \tag{3.67}$$

where $N_{c(ei)}$, $N_{q(ei)}$, and $N_{\gamma(ei)}$ = bearing capacity factors

$$q = \gamma D_f$$

The variations of the bearing capacity factors with e/B, ϕ', and β derived by Saran and Agarwal are given in Figures 3.26, 3.27, and 3.28.

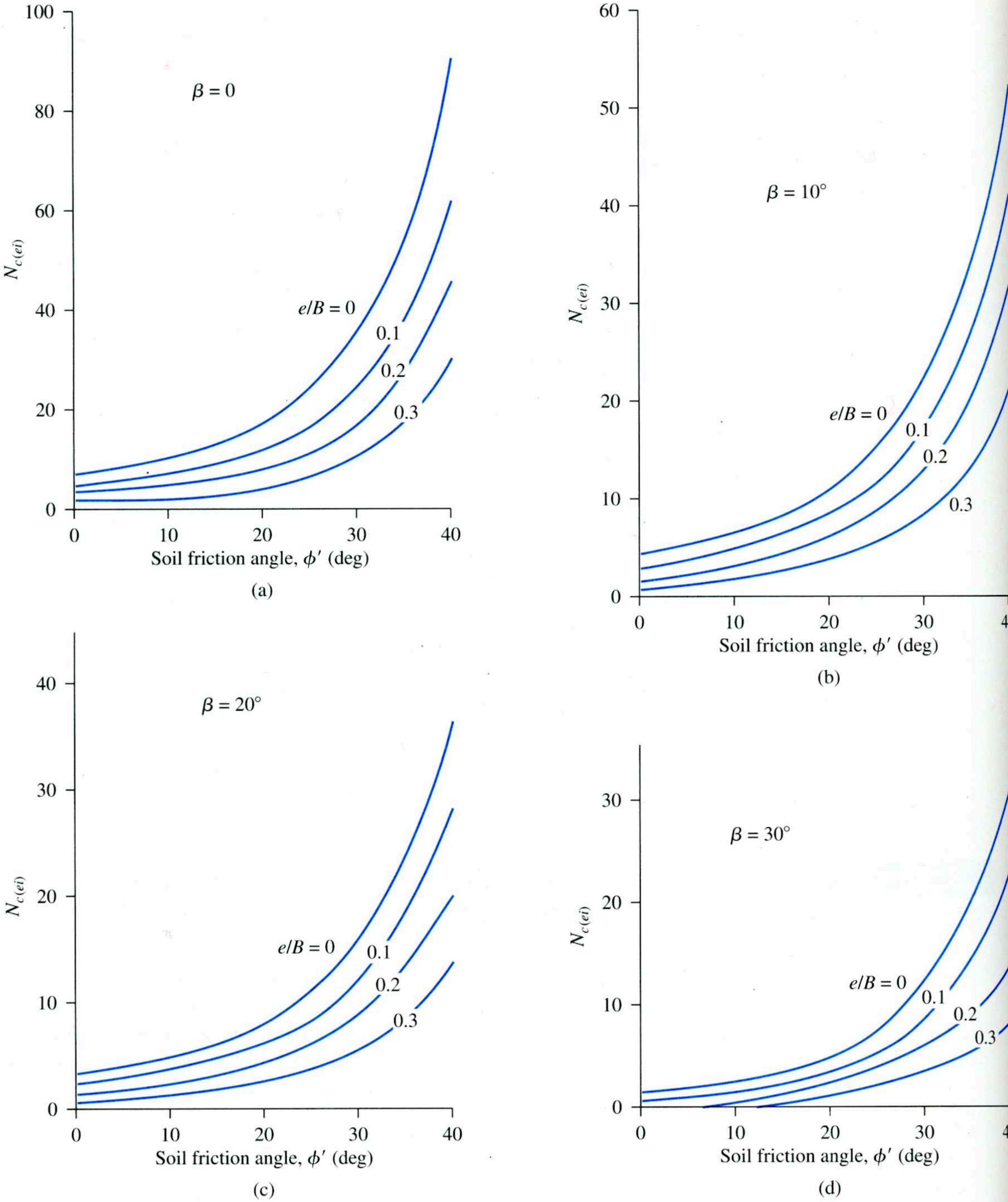


Figure 3.26 Variation of $N_{c(ei)}$ with ϕ', e/B, and β

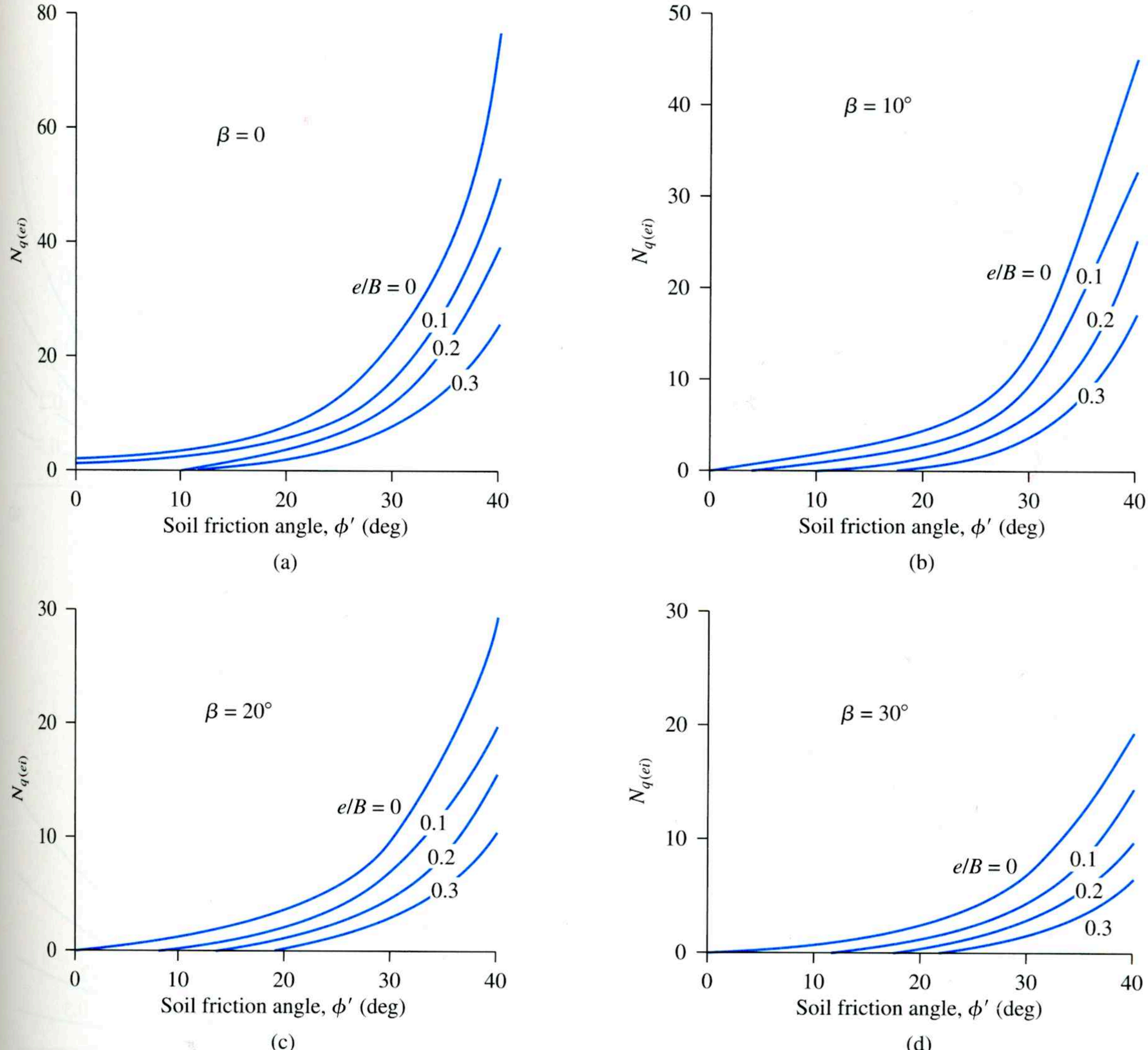


Figure 3.27 Variation of $N_{q(ei)}$ with ϕ', e/B, and β

Example 3.10

A continuous foundation is shown in Figure 3.29. Estimate the ultimate load, Q_{ult} per unit length of the foundation.

Solution

With $c' = 0$, from Eq. (3.67),

$$Q_{\text{ult}} = \left[qN_{q(ei)} + \frac{1}{2}\gamma B N_{\gamma(ei)} \right]$$

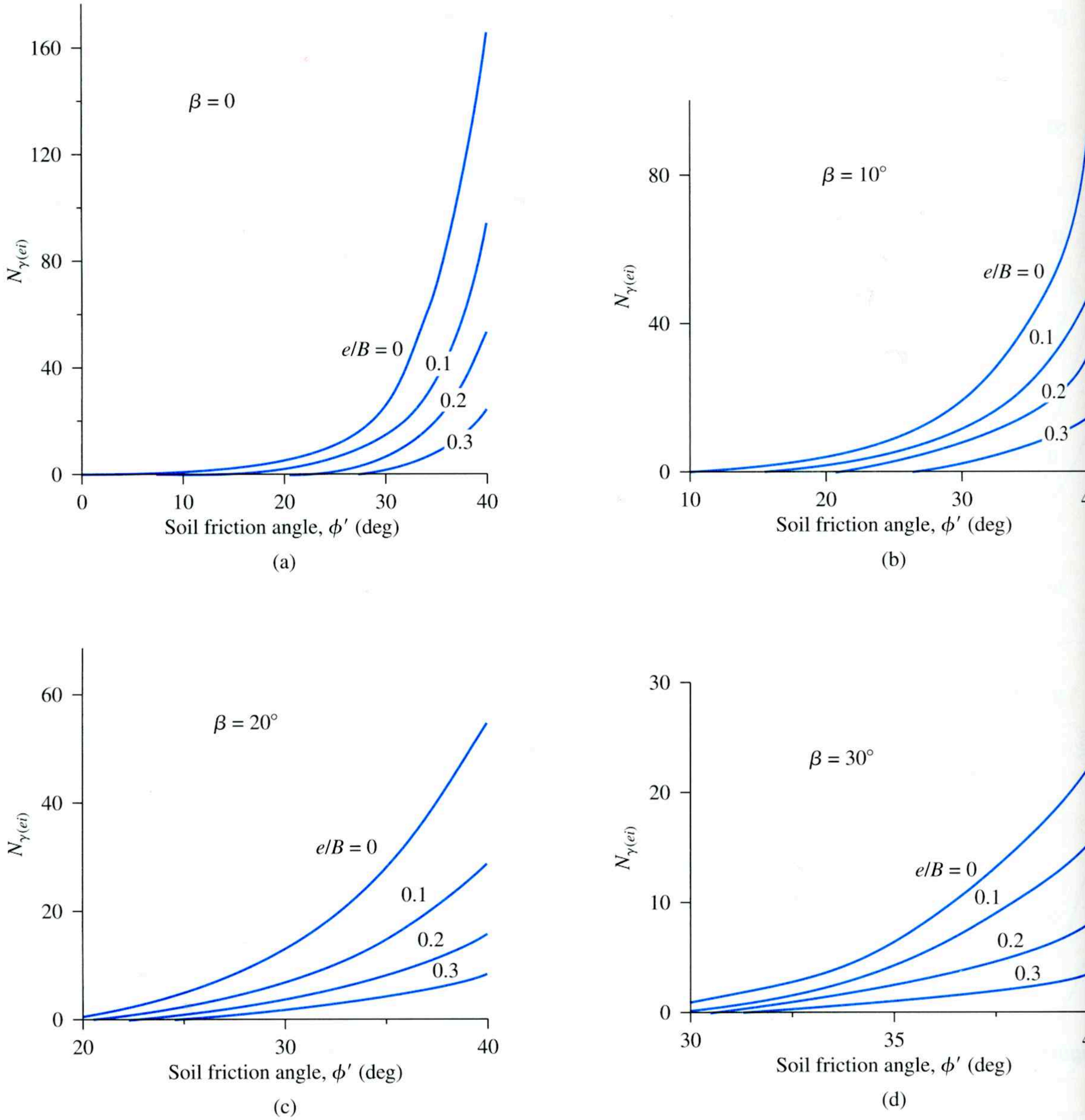


Figure 3.28 Variation of $N_{\gamma(ei)}$ with ϕ', e/B, and β

$B = 1.5$ m, $q = D_f\gamma = (1)(16) = 16$ kN/m^2, $e/B = 0.15/1.5 = 0.1$, and $\beta = 20°$. From Figures 3.27(c) and 3.28(c), $N_{q(ei)} = 14.2$ and $N_{\gamma(ei)} = 20$. Hence,

$$Q_{\text{ult}} = (1.5)[(16)(14.2) + (\tfrac{1}{2})(16)(1.5)(20)] = \mathbf{700.8\ kN/m}$$

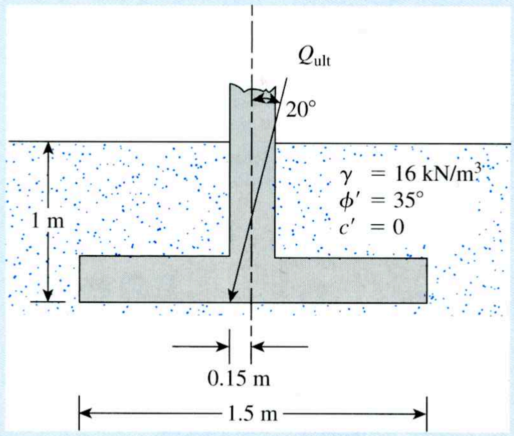


Figure 3.29 ■

Problems

3.1 For the following cases, determine the allowable gross vertical load-bearing capacity of the foundation. Use Terzaghi's equation and assume general shear failure in soil. Use FS = 4.

Part	B	D_f	ϕ'	c'	γ	Foundation type
a.	4 ft	3 ft	25°	600 lb/ft^2	110 lb/ft^3	Continuous
b.	2 m	1 m	30°	0	17 kN/m^3	Continuous
c.	3 m	2 m	30°	0	16.5 kN/m^3	Square

3.2 A square column foundation has to carry a gross allowable load of 1805 kN (FS = 3). Given: $D_f = 1.5$ m, $\gamma = 15.9$ kN/m^3, $\phi' = 34°$, and $c' = 0$. Use Terzaghi's equation to determine the size of the foundation (B). Assume general shear failure.

3.3 Use the general bearing capacity equation [Eq. (3.19)] to solve the following:

a. Problem 3.1a
b. Problem 3.1b
c. Problem 3.1c

3.4 The applied load on a shallow square foundation makes an angle of 15° with the vertical. Given: $B = 6$ ft, $D_f = 3$ ft, $\gamma = 115$ lb/ft^3, $\phi' = 25°$, and $c' = 500$ lb/ft^2. Use FS = 4 and determine the gross allowable load. Use Eq. (3.19).

3.5 A column foundation (Figure P3.5) is 3 m × 2 m in plan. Given: $D_f = 1.5$ m, $\phi' = 25°$, $c' = 70$ kN/m^2. Using Eq. (3.19) and FS = 3, determine the net allowable load [see Eq. (3.15)] the foundation could carry.

3.6 For a square foundation that is $B \times B$ in plan, $D_f = 2$ m; vertical gross allowable load, $Q_{all} = 3330$ kN, $\gamma = 16.5$ kN/m^3; $\phi' = 30°$; $c' = 0$; and FS = 4. Determine the size of the foundation. Use Eq. (3.19).

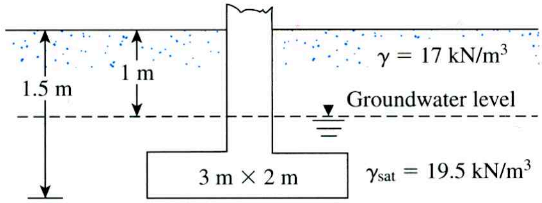


Figure P3.5

3.7 For the design of a shallow foundation, given the following:

Soil: $\phi' = 25°$
$c' = 50\ \text{kN/m}^2$
Unit weight, $\gamma = 17\ \text{kN/m}^3$
Modulus of elasticity, $E_s = 1020\ \text{kN/m}^2$
Poisson's ratio, $\mu_s = 0.35$

Foundation: $L = 1.5$ m
$B = 1$ m
$D_f = 1$ m

Calculate the ultimate bearing capacity. Use Eq. (3.27).

3.8 An eccentrically loaded foundation is shown in Figure P3.8. Use FS of 4 and determine the maximum allowable load that the foundation can carry. Use Meyerhof's effective area method.

3.9 Repeat Problem 3.8 using Prakash and Saran's method.

3.10 For an eccentrically loaded continuous foundation on sand, given $B = 1.8$ m, $D_f = 0.9$ m, $e/B = 0.12$ (one-way eccentricity), $\gamma = 16\ \text{kN/m}^3$, and $\phi' = 35°$. Using the reduction factor method, estimate the ultimate load per unit length of the foundation.

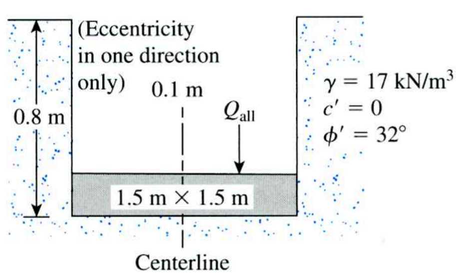


Figure P3.8

3.11 An eccentrically loaded continuous foundation is shown in Figure P3.11. Determine the ultimate load Q_u per unit length that the foundation can carry. Use the reduction factor method.

3.12 A square footing is shown in Figure P3.12. Use FS = 6, and determine the size of the footing. Use Prakash and Saran theory [Eq. (3.43)].

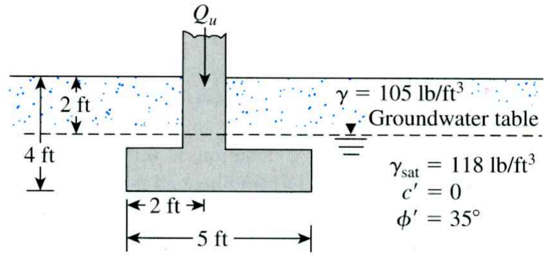


Figure P3.11

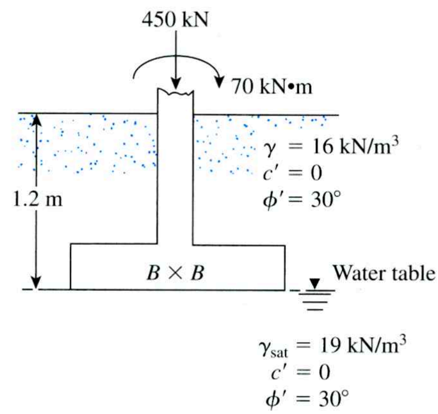


Figure P3.12

3.13 The shallow foundation shown in Figure 3.19 measures 1.2 m $\times$ 1.8 m and is subjected to a centric load and a moment. If $e_B = 0.12$ m, $e_L = 0.36$ m, and the depth of the foundation is 1 m, determine the allowable load the foundation can carry. Use a factor of safety of 3. For the soil, we are told that unit weight $\gamma = 17$ kN/m^3, friction angle $\phi' = 35°$, and cohesion $c' = 0$.

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4 Ultimate Bearing Capacity of Shallow Foundations: Special Cases

4.1 Introduction

The ultimate bearing capacity problems described in Chapter 3 assume that the soil supporting the foundation is homogeneous and extends to a great depth below the bottom of the foundation. They also assume that the ground surface is horizontal. However, that is not true in all cases: It is possible to encounter a rigid layer at a shallow depth, or the soil may be layered and have different shear strength parameters. In some instances, it may be necessary to construct foundations on or near a slope, or it may be required to design a foundation subjected to uplifting load.

This chapter discusses bearing capacity problems relating to these special cases.

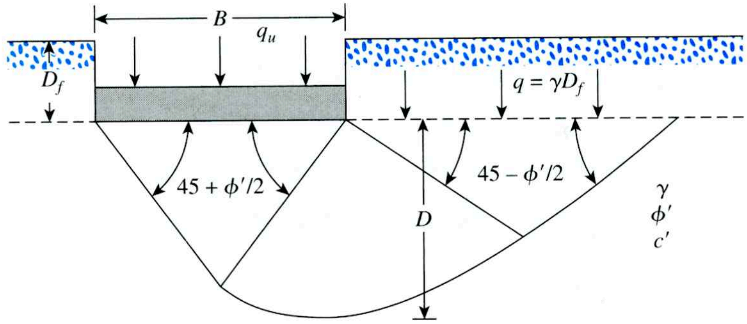
4.2 Foundation Supported by a Soil with a Rigid Base at Shallow Depth

Figure 4.1(a) shows a shallow, rough *continuous* foundation supported by a soil that extends to a great depth. Neglecting the depth factor, for vertical loading Eq. (3.19) will take the form

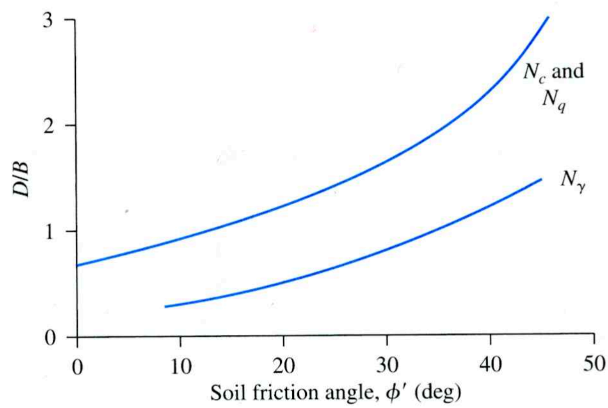
$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma \tag{4.1}$$

The general approach for obtaining expressions for N_c, N_q, and N_γ was outlined in Chapter 3. The extent of the failure zone in soil, D, at ultimate load obtained in the derivation of N_c and N_q by Prandtl (1921) and Reissner (1924) is given in Figure 4.1(b). Similarly, the magnitude of D obtained by Lundgren and Mortensen (1953) in evaluating N_γ is given in the figure.

Now, if a rigid, rough base is located at a depth of $H < D$ below the bottom of the foundation, full development of the failure surface in soil will be restricted. In such a case, the soil failure zone and the development of slip lines at ultimate load will be as shown in Figure 4.2.



(a)



(b)

Figure 4.1 (a) Failure surface under a rough continuous foundation; (b) variation of D/B with soil friction angle ϕ'

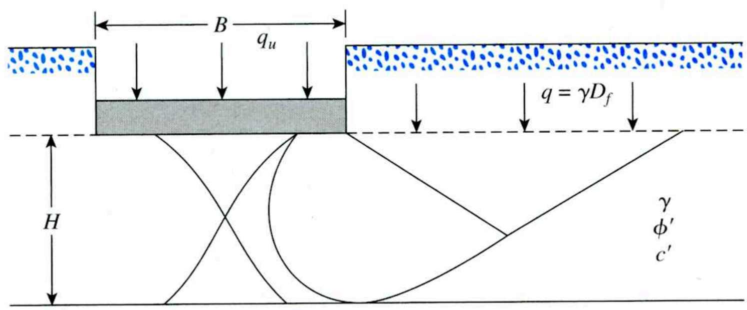


Figure 4.2 Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth

Mandel and Salencon (1972) determined the bearing capacity factors applicable to this case by numerical integration, using the theory of plasticity. According to their theory, the ultimate bearing capacity of a rough continuous foundation with a rigid, rough base located at a shallow depth can be given by the relation

$$q_u = c'N_c^* + qN_q^* + \frac{1}{2}\gamma BN_\gamma^* \tag{4.2}$$

where

N_c^*, N_q^*, N_γ^* = modified bearing capacity factors
B = width of foundation
γ = unit weight of soil

Note that, for $H \geqslant D$, $N_c^* = N_c$, $N_q^* = N_q$, and $N_\gamma^* = N_\gamma$ (Lundgren and Mortensen, 1953). The variations of N_c^*, N_q^*, and N_γ^* with H/B and the soil friction angle ϕ' are given in Figures 4.3, 4.4, and 4.5, respectively.

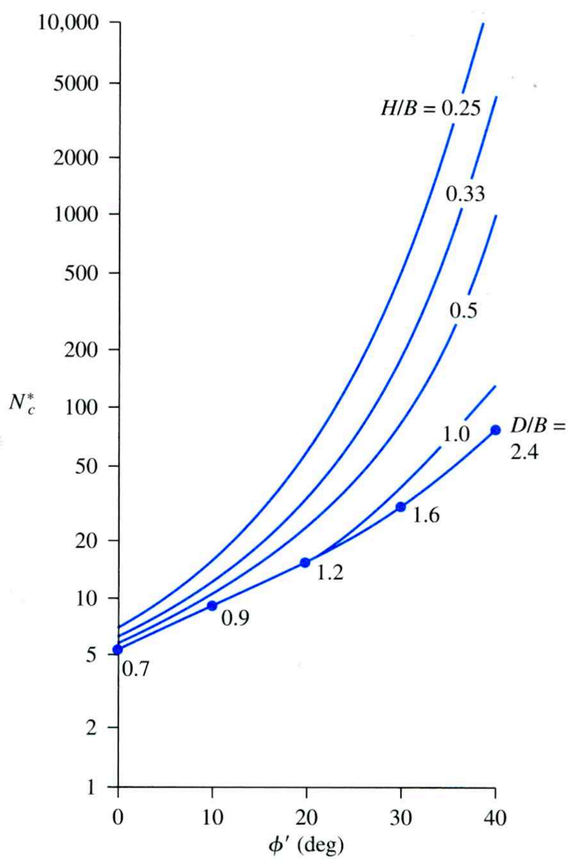


Figure 4.3 Mandel and Salencon's bearing capacity factor N_c^* [Eq. (4.2)]

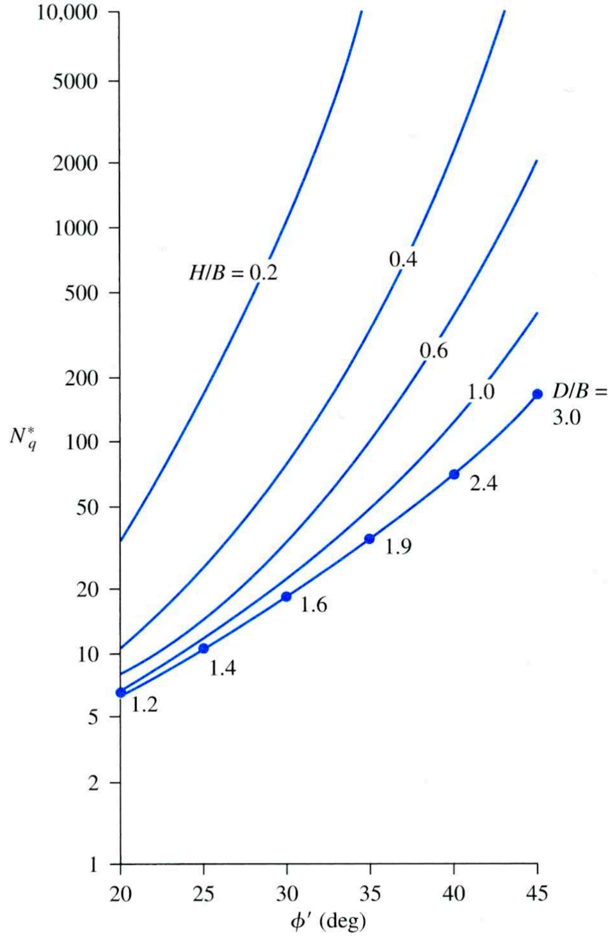


Figure 4.4 Mandel and Salencon's bearing capacity factor N_q^* [Eq. (4.2)]

Rectangular Foundation on Granular Soil

Neglecting the depth factors, the ultimate bearing capacity of rough circular and rectangular foundations on a sand layer ($c' = 0$) with a rough, rigid base located at a shallow depth can be given as

$$q_u = qN_q^* F_{qs}^* + \frac{1}{2}\gamma B N_\gamma^* F_{\gamma s}^* \tag{4.3}$$

where $F_{qs}^*, F_{\gamma s}^*$ = modified shape factors.

The shape factors F_{qs}^* and $F_{\gamma s}^*$ are functions of H/B and ϕ'. On the basis of the work of Meyerhof and Chaplin (1953), and simplifying the assumption that, in radial planes, the stresses and shear zones are identical to those in transverse planes, Meyerhof (1974) proposed that

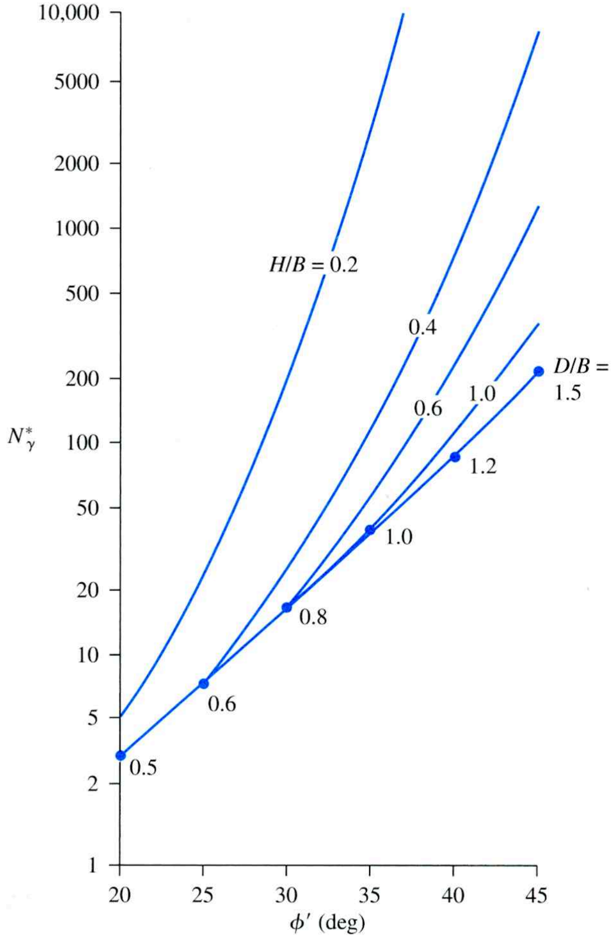


Figure 4.5 Mandel and Salencon's bearing capacity factor N_γ^* [Eq. (4.2)]

$$F_{qs}^* \approx 1 - m_1\left(\frac{B}{L}\right) \tag{4.4}$$

and

$$F_{\gamma s}^* \approx 1 - m_2\left(\frac{B}{L}\right) \tag{4.5}$$

where L = length of the foundation. The variations of m_1 and m_2 with H/B and ϕ' are shown in Figure 4.6.

More recently, Cerato and Lutenegger (2006) provided some test results for the bearing capacity factor, N_γ^*. These tests were conducted using *square* and *circular* plates with B varying from 0.152 m (6 in.) to 0.305 m (12 in.). It was assumed that Terzaghi's bearing-capacity equations for *square and circular foundations* can be used. Or, from Eqs. (3.10) and (3.11) with $c' = 0$,

$$q_u = qN_q^* + 0.4\gamma BN_\gamma^* \text{ (square foundation)} \tag{4.6}$$

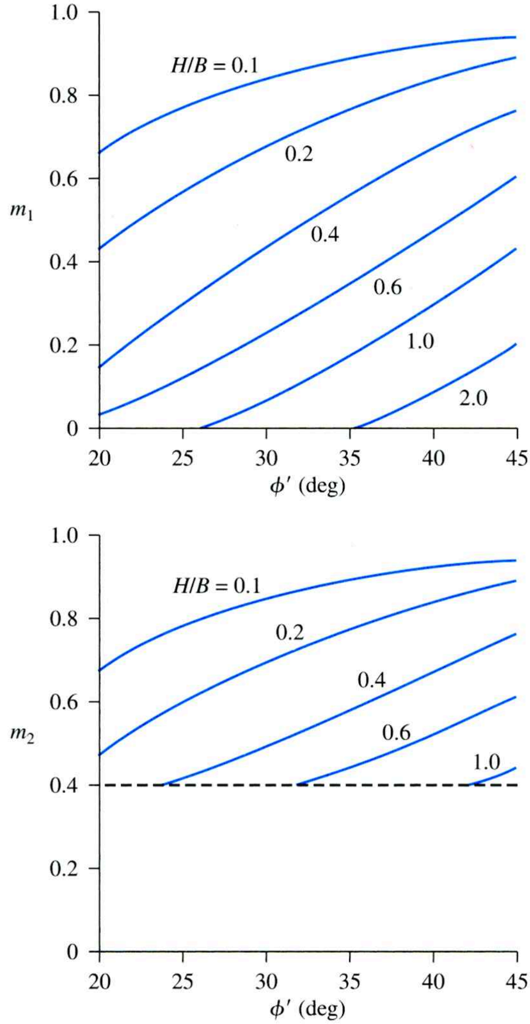


Figure 4.6 Variation of m_1 and m_2 with H/B and ϕ'

and

$$q_u = qN_q^* + 0.3\gamma BN_\gamma^* \text{ (circular foundation)} \tag{4.7}$$

The experimentally determined variation of N_γ^* is shown in Figure 4.7. It also was observed in this study that N_γ^* becomes equal to N_γ at $H/B \approx 3$ instead of D/B, as shown in Figure 4.5. For that reason, Figure 4.7 shows the variation of N_γ^* for $H/B = 0.5$ to 3.0.

Foundation on Saturated Clay

For saturated clay (i.e., under the undrained condition, or $\phi = 0$), Eq. (4.2) will simplify to the form

$$q_u = c_u N_c^* + q \tag{4.8}$$

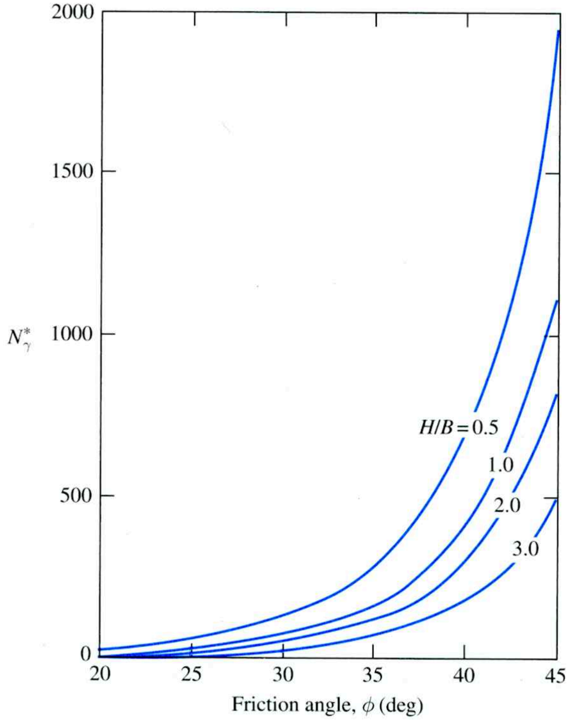


Figure 4.7 Cerato and Lutenegger's test results for N_γ^*

Mandel and Salencon (1972) performed calculations to evaluate N_c^* for *continuous foundations.* Similarly, Buisman (1940) gave the following relationship for obtaining the ultimate bearing capacity of square foundations:

$$q_{u(\text{square})} = \left(\pi + 2 + \frac{B}{2H} - \frac{\sqrt{2}}{2}\right)c_u + q \qquad \left(\text{for } \frac{B}{2H} - \frac{\sqrt{2}}{2} \geqslant 0\right) \tag{4.9}$$

In this equation, c_u is the undrained shear strength.

Equation (4.9) can be rewritten as

$$q_{u(\text{square})} = \underbrace{5.14\left(1 + \frac{0.5\dfrac{B}{H} - 0.707}{5.14}\right)}_{N^*_{c(\text{square})}} c_u + q \tag{4.10}$$

Table 4.1 gives the values of N_c^* for continuous and square foundations.

Table 4.1 Values of N_c^* for Continuous and Square Foundations ($\phi = 0$)

$\frac{B}{H}$	N_c^* Square[a]	N_c^* Continuous[b]
2	5.43	5.24
3	5.93	5.71
4	6.44	6.22
5	6.94	6.68
6	7.43	7.20
8	8.43	8.17
10	9.43	9.05

[a] Buisman's analysis (1940)
[b] Mandel and Salencon's analysis (1972)

Example 4.1

A square foundation measuring 2.5 ft × 2.5 ft is constructed on a layer of sand. We are given that $D_f = 2$ ft, $\gamma = 110$ lb/ft^3, $\phi' = 35°$, and $c' = 0$. A rock layer is located at a depth of 1.5 ft below the bottom of the foundation. Using a factor of safety of 4, determine the gross allowable load the foundation can carry.

Solution

From Eq. (4.3),

$$q_u = qN_q^*F_{qs}^* + \frac{1}{2}\gamma BN_\gamma^*F_{\gamma s}^*$$

and we also have

$$q = 110 \times 2 = 220 \text{ lb/ft}^2$$

For $\phi' = 35°$, $H/B = 1.5/2.5 = 0.6$, $N_q^* \approx 90$ (Figure 4.4), and $N_\gamma^* \approx 50$(Figure 4.5), and we have

$$F_{qs}^* = 1 - m_1(B/L)$$

From Figure 4.6(a), for $\phi' = 35°$, $H/B = 0.6$, and the value of $m_1 = 0.34$, so

$$F_{qs}^* = 1 - (0.34)(2.5/2.5) = 0.66$$

Similarly,

$$F_{\gamma s}^* = 1 - m_2(B/L)$$

From Figure 4.6(b), $m_2 = 0.45$, so

$$F_{\gamma s}^* = 1 - (0.45)(2.5/2.5) = 0.55$$

Hence,

$$q_u = (220)(90)(0.66) + (1/2)(110)(2.5)(50)(0.55) = 16{,}849 \text{ lb/ft}^2$$

and

$$Q_{\text{all}} = \frac{q_u B^2}{\text{FS}} = \frac{(16{,}849)(2.5 \times 2.5)}{4} = \mathbf{26{,}326\ lb}$$ ■

Example 4.2

Solve Example 4.1 using Eq. (4.6).

Solution

From Eq. (4.6),

$$q_u = qN_q^* + 0.4\gamma B N_\gamma^*$$

For $\phi' = 35°$ and $H/B = 0.6$, the value of $N_q^* \approx 90$ (Figure 4.4) and $N_\gamma^* \approx 230$ (Figure 4.7).

So

$$q_u = (220)(90) + (0.4)(110)(2.5)(230) = 19{,}800 + 25{,}300 = 45{,}100 \text{ lb/ft}^2$$

$$Q_{\text{all}} = \frac{q_u B^2}{\text{FS}} \approx \mathbf{70{,}469\ lb}$$ ■

Example 4.3

Consider a square foundation 1 m × 1 m in plan located on a saturated clay layer underlain by a layer of rock. Given:

Clay: $c_u = 72 \text{ kN/m}^2$

Unit weight: $\gamma = 18 \text{ kN/m}^3$

Distance between the bottom of foundation and the rock layer = 0.25 m

$D_f = 1$ m

Estimate the gross allowable bearing capacity of the foundation. Use FS = 3.

Solution

From Eq. (4.10),

$$q_u = 5.14\left(1 + \frac{0.5\dfrac{B}{H} - 0.707}{5.14}\right)c_u + q$$

For $B/H = 1/0.25 = 4$; $c_u = 72\ \text{kN/m}^2$; and $q = \gamma D_f = (18)(1) = 18\ \text{kN/m}^3$.

$$q_u = 5.14\left[1 + \frac{(0.5)(4) - 0.707}{5.14}\right]72 + 18 = 481.2\ \text{kN/m}^2$$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{481.2}{3} = \mathbf{160.4\ kN/m^2}$$

4.3 Bearing Capacity of Layered Soils: Stronger Soil Underlain by Weaker Soil

The bearing capacity equations presented in Chapter 3 involve cases in which the soil supporting the foundation is homogeneous and extends to a considerable depth. The cohesion, angle of friction, and unit weight of soil were assumed to remain constant for the bearing capacity analysis. However, in practice, layered soil profiles are often encountered. In such instances, the failure surface at ultimate load may extend through two or more soil layers, and a determination of the ultimate bearing capacity in layered soils can be made in only a limited number of cases. This section features the procedure for estimating the bearing capacity for layered soils proposed by Meyerhof and Hanna (1978) and Meyerhof (1974).

Figure 4.8 shows a shallow, continuous foundation supported by a *stronger soil layer*, underlain by a weaker soil that extends to a great depth. For the two soil layers, the physical parameters are as follows:

	Soil properties		
Layer	**Unit weight**	**Friction angle**	**Cohesion**
Top	γ_1	ϕ'_1	c'_1
Bottom	γ_2	ϕ'_2	c'_2

At ultimate load per unit area (q_u), the failure surface in soil will be as shown in the figure. If the depth H is relatively small compared with the foundation width B, a punching shear failure will occur in the top soil layer, followed by a general shear failure in the bottom soil layer. This is shown in Figure 4.8(a). However, if the depth H is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity. This is shown in Figure 4.8b.

The ultimate bearing capacity for this problem, as shown in Figure 4.8a, can be given as

$$q_u = q_b + \frac{2(C_a + P_p \sin \delta')}{B} - \gamma_1 H \tag{4.11}$$

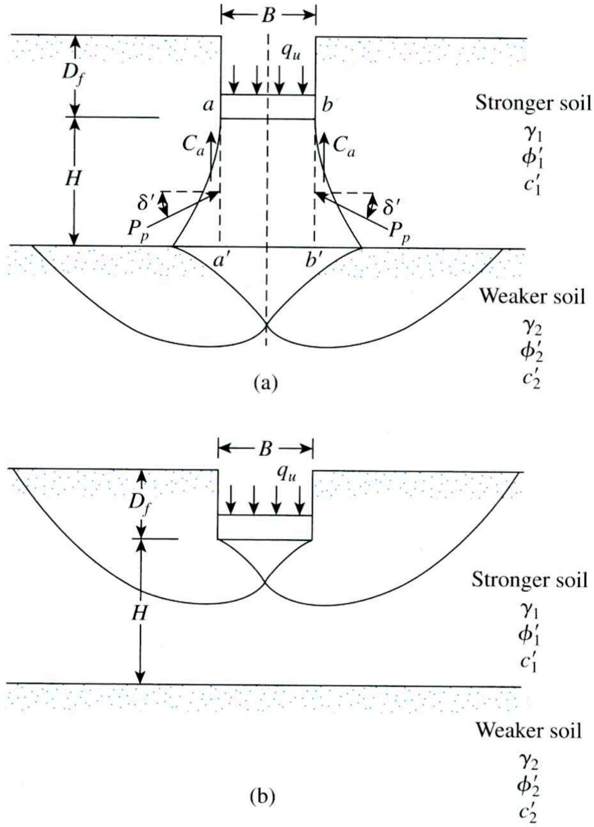


Figure 4.8 Bearing capacity of a continuous foundation on layered soil

where

B = width of the foundation
C_a = adhesive force
P_p = passive force per unit length of the faces aa' and bb'
q_b = bearing capacity of the bottom soil layer
δ' = inclination of the passive force P_p with the horizontal

Note that, in Eq. (4.11),

$$C_a = c'_a H$$

where c'_a = adhesion.

Equation (4.11) can be simplified to the form

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \frac{K_{pH} \tan \delta'}{B} - \gamma_1 H \tag{4.12}$$

where K_{pH} = horizontal component of passive earth pressure coefficient.

However, let

$$K_{pH} \tan \delta' = K_s \tan \phi'_1 \tag{4.13}$$

where K_s = punching shear coefficient. Then,

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi'_1}{B} - \gamma_1 H \tag{4.14}$$

The punching shear coefficient, K_s, is a function of q_2/q_1 and ϕ'_1, or, specifically,

$$K_s = f\left(\frac{q_2}{q_1}, \phi'_1\right)$$

Note that q_1 and q_2 are the ultimate bearing capacities of a continuous foundation of width B under vertical load on the surfaces of homogeneous thick beds of upper and lower soil, or

$$q_1 = c'_1 N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)} \tag{4.15}$$

and

$$q_2 = c'_2 N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)} \tag{4.16}$$

where

$N_{c(1)}, N_{\gamma(1)}$ = bearing capacity factors for friction angle ϕ'_1 (Table 3.3)
$N_{c(2)}, N_{\gamma(2)}$ = bearing capacity factors for friction angle ϕ'_2 (Table 3.3)

Observe that, for the top layer to be a stronger soil, q_2/q_1 should be less than unity.

The variation of K_s with q_2/q_1 and ϕ'_1 is shown in Figure 4.9. The variation of c'_a/c'_1 with q_2/q_1 is shown in Figure 4.10. If the height H is relatively large, then the

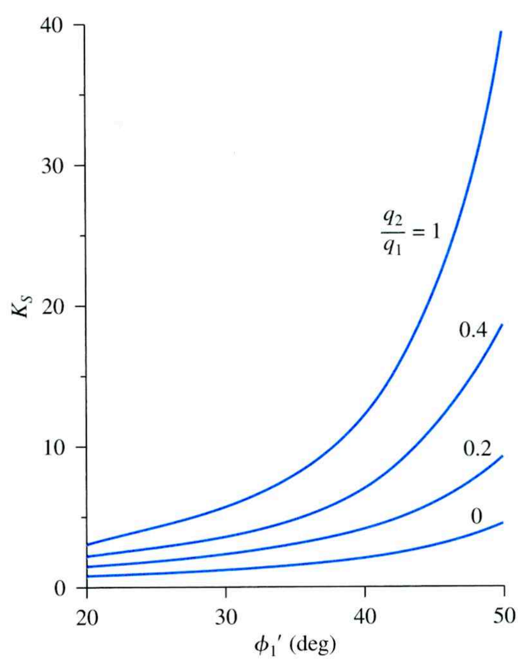


Figure 4.9 Meyerhof and Hanna's punching shear coefficient K_s

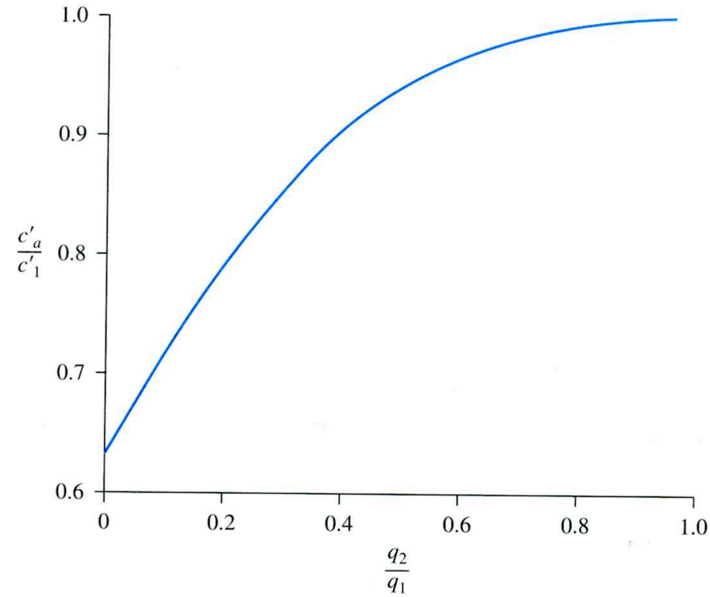


Figure 4.10 Variation of c'_a/c'_1 with q_2/q_1 based on the theory of Meyerhof and Hanna (1978)

failure surface in soil will be completely located in the stronger upper-soil layer (Figure 4.8b). For this case,

$$q_u = q_t = c'_1 N_{c(1)} + qN_{q(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}. \tag{4.17}$$

where $N_{c(1)}$, $N_{q(1)}$, and $N_{\gamma(1)}$ = bearing capacity factors for $\phi' = \phi'_1$ (Table 3.3) and $q = \gamma_1 D_f$.

Combining Eqs. (4.14) and (4.17) yields

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi'_1}{B} - \gamma_1 H \le q_t \tag{4.18}$$

For rectangular foundations, the preceding equation can be extended to the form

$$\begin{aligned} q_u = q_b &+ \left(1 + \frac{B}{L}\right)\left(\frac{2c'_a H}{B}\right) \\ &+ \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan\phi'_1}{B}\right) - \gamma_1 H \le q_t \end{aligned} \tag{4.19}$$

where

$$q_b = c'_2 N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)} \tag{4.20}$$

and

$$q_t = c_1' N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)} \tag{4.21}$$

in which

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors with respect to top soil layer (Table 3.4)
$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors with respect to bottom soil layer (Table 3.4)

Special Cases

1. *Top layer is strong sand and bottom layer is saturated soft clay* ($\phi_2 = 0$). From Eqs. (4.19), (4.20), and (4.21),

$$q_b = \left(1 + 0.2\frac{B}{L}\right)5.14c_2 + \gamma_1(D_f + H) \tag{4.22}$$

and

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)} \tag{4.23}$$

Hence,

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_2 + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} + \gamma_1 D_f \leq \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)} \tag{4.24}$$

where c_2 = undrained cohesion.

For a determination of K_s from Figure 4.9,

$$\frac{q_2}{q_1} = \frac{c_2 N_{c(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{5.14c_2}{0.5\gamma_1 B N_{\gamma(1)}} \tag{4.25}$$

2. *Top layer is stronger sand and bottom layer is weaker sand* ($c_1' = 0, c_2' = 0$). The ultimate bearing capacity can be given as

$$q_u = \left[\gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}\right] + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \leq q_t \tag{4.26}$$

where

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)} \tag{4.27}$$

Then

$$\frac{q_2}{q_1} = \frac{\frac{1}{2}\gamma_2 B N_{\gamma(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}} \tag{4.28}$$

3. *Top layer is stronger saturated clay* ($\phi_1 = 0$) *and bottom layer is weaker saturated clay* ($\phi_2 = 0$). The ultimate bearing capacity can be given as

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_2 + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f \leq q_t \tag{4.29}$$

where

$$q_t = \left(1 + 0.2\frac{B}{L}\right)5.14c_1 + \gamma_1 D_f \tag{4.30}$$

and c_1 and c_2 are undrained cohesions. For this case,

$$\frac{q_2}{q_1} = \frac{5.14c_2}{5.14c_1} = \frac{c_2}{c_1} \tag{4.31}$$

Example 4.4

Refer to Figure 4.8(a) and consider the case of a continuous foundation with $B = 2$ m, $D_f = 1.2$ m, and $H = 1.5$ m. The following are given for the two soil layers:

Top sand layer:

Unit weight $\gamma_1 = 17.5\ \text{kN/m}^3$
$\phi'_1 = 40°$
$c'_1 = 0$

Bottom clay layer:

Unit weight $\gamma_2 = 16.5\ \text{kN/m}^3$
$\phi'_2 = 0$
$c_2 = 30\ \text{kN/m}^2$

Determine the gross ultimate load per unit length of the foundation.

Solution

For this case, Eqs. (4.24) and (4.25) apply. For $\phi_1' = 40°$, from Table 3.3, $N_\gamma = 109.41$ and

$$\frac{q_2}{q_1} = \frac{c_2 N_{c(2)}}{0.5\gamma_1 B N_{\gamma(1)}} = \frac{(30)(5.14)}{(0.5)(17.5)(2)(109.41)} = 0.081$$

From Figure 4.9, for $c_2 N_{c(2)}/0.5\gamma_1 B N_{\gamma(1)} = 0.081$ and $\phi_1' = 40°$, the value of $K_s \approx 2.5$. Equation (4.24) then gives

$$q_u = \left[1 + (0.2)\left(\frac{B}{L}\right)\right]5.14c_2 + \left(1 + \frac{B}{L}\right)\gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)K_s\frac{\tan\phi_1'}{B} + \gamma_1 D_f$$

$$= [1 + (0.2)(0)](5.14)(30) + (1 + 0)(17.5)(1.5)^2$$

$$\times\left[1 + \frac{(2)(1.2)}{1.5}\right](2.5)\frac{\tan 40}{2.0} + (17.5)(1.2)$$

$$= 154.2 + 107.4 + 21 = 282.6 \text{ kN/m}^2$$

Again, from Eq. (4.27),

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

From Table 3.3, for $\phi_1' = 40°$, $N_\gamma = 109.4$ and $N_q = 64.20$.
From Table 3.4,

$$F_{qs(1)} = 1 + \left(\frac{B}{L}\right)\tan\phi_1' = 1 + (0)\tan 40 = 1$$

and

$$F_{\gamma s(1)} = 1 - 0.4\frac{B}{L} = 1 - (0.4)(0) = 1$$

so that

$$q_t = (17.5)(1.2)(64.20)(1) + (\tfrac{1}{2})(17.5)(2)(109.4)(1) = 3262.7 \text{ kN/m}^2$$

Hence,

$$q_u = 282.6 \text{ kN/m}^2$$

$$Q_u = (282.6)(B) = (282.6)(2) = \mathbf{565.2\ kN/m}$$ ■

Example 4.5

A foundation 5 ft × 3 ft is located at a depth D_f of 3 ft in a stronger clay. A softer clay layer is located at a depth H of 3 ft, measured from the bottom of the foundation. For the top clay layer,

$$\text{Undrained shear strength} = 2500 \text{ lb/ft}^2$$
$$\text{Unit weight} = 108 \text{ lb/ft}^3$$

and for the bottom clay layer,

$$\text{Undrained shear strength} = 1000 \text{ lb/ft}^2$$
$$\text{Unit weight} = 102 \text{ lb/ft}^3$$

Determine the gross allowable load for the foundation with an FS of 3.

Solution

For this problem, Eqs. (4.29), (4.30), and (4.31) will apply, or

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_2 + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f$$
$$\leq \left(1 + 0.2\frac{B}{L}\right)5.14c_1 + \gamma_1 D_f$$

We are given the following data:

$$B = 3 \text{ ft} \qquad H = 3 \text{ ft} \qquad D_f = 3 \text{ ft}$$
$$L = 5 \text{ ft} \qquad \gamma_1 = 108 \text{ lb/ft}^3$$

From Figure 4.10 for $c_2/c_1 = 1000/2500 = 0.4$, the value of $c_a/c_1 \approx 0.9$, so

$$c_a = (0.9)(2500) = 2250 \text{ lb/ft}^2$$

and

$$q_u = \left[1 + (0.2)\left(\frac{3}{5}\right)\right](5.14)(1000) + \left(1 + \frac{3}{5}\right)\left[\frac{(2)(2250)(3)}{3}\right] + (108)(3)$$
$$= 5756.8 + 7200 + 324 = 13{,}280.8 \text{ lb/ft}^2$$

As a check, we have, from Eq. (4.30),

$$q_t = \left[1 + (0.2)\left(\frac{3}{5}\right)\right](5.14)(2500) + (108)(3)$$
$$= 14{,}716 \text{ lb/ft}^2$$

Thus, $q_u = 13{,}280.8 \text{ lb/ft}^2$ (i.e., the smaller of the two values just calculated), and

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{13{,}280.8}{3} \approx 4427 \text{ lb/ft}^2$$

The total allowable load is therefore

$$(q_{\text{all}})(3 \times 5) = \mathbf{66{,}405 \text{ lb}}$$ ■

4.4 Bearing Capacity of Layered Soil: Weaker Soil Underlain by Stronger Soil

When a foundation is supported by a weaker soil layer underlain by a stronger layer (Figure 4.11a), the ratio of q_2/q_1 defined by Eqs. (4.15) and (4.16) will be greater than one. Also, if H/B is relatively small, as shown in the left-hand half of Figure 4.11a, the failure surface in soil at ultimate load will pass through both soil layers. However, for larger H/B ratios, the failure surface will be fully located in the top, weaker soil layer, as shown in the

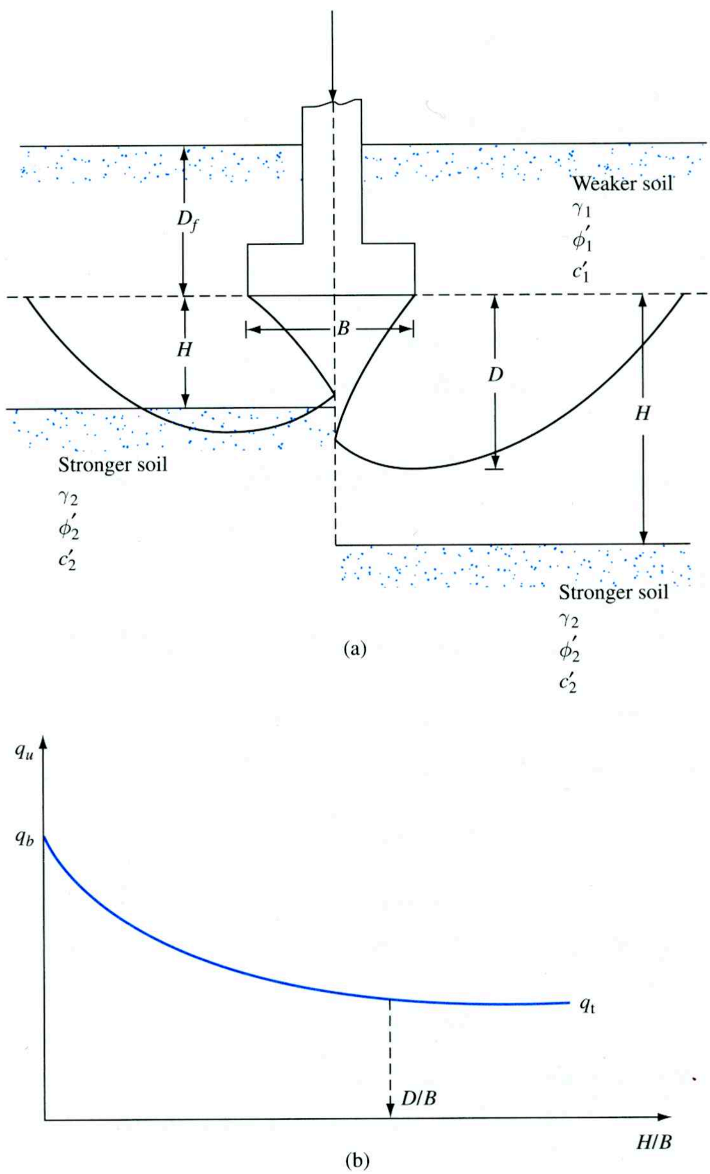


Figure 4.11 (a) Foundation on weaker soil layer underlain by stronger sand layer, (b) Nature of variation of q_u with H/B

right-hand half of Figure 4.11a. For this condition, the ultimate bearing capacity (Meyerhof, 1974; Meyerhof and Hanna, 1978) can be given by the empirical equation

$$q_u = q_t + (q_b - q_t)\left(\frac{H}{D}\right)^2 \geq q_t \tag{4.32}$$

where

D = depth of failure surface beneath the foundation in the thick bed of the upper weaker soil layer
q_t = ultimate bearing capacity in a thick bed of the upper soil layer
q_b = ultimate bearing capacity in a thick bed of the lower soil layer

So

$$q_t = c_1 N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)} \tag{4.33}$$

and

$$q_t = c_2 N_{c(2)} F_{cs(2)} + \gamma_2 D_f N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)} \tag{4.34}$$

where

$N_{c(1)}, N_{q(1)}, N_{\gamma(1)}$ = bearing capacity factors corresponding to the soil friction angle ϕ_1'
$N_{c(2)}, N_{q(2)}, N_{\gamma(2)}$ = bearing capacity factors corresponding to the soil friction angle ϕ_2'
$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors corresponding to the soil friction angle ϕ_1'
$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors corresponding to the soil friction angle ϕ_2'

Meyerhof and Hanna (1978) suggested that

- $D \approx B$ for loose sand and clay
- $D \approx 2B$ for dense sand

Equations (4.32), (4.33), and (4.34) imply that the maximum and minimum values of q_u will be q_b and q_t, respectively, as shown in Figure 4.11b.

Example 4.6

Refer to Figure 4.11a. For a layered saturated-clay profile, given: $L = 6$ ft, $B = 4$ ft, $D_f = 3$ ft, $H = 2$ ft, $\gamma_1 = 110$ lb/ft^3, $\phi_1 = 0$, $c_1 = 1200$ lb/ft^2, $\gamma_2 = 125$ lb/ft^3, $\phi_2 = 0$, and $c_2 = 2500$ lb/ft^2. Determine the ultimate bearing capacity of the foundation.

Solution

From Eqs. (4.15) and (4.16),

$$\frac{q_2}{q_1} = \frac{c_2 N_c}{c_1 N_c} = \frac{c_2}{c_1} = \frac{2500}{1200} = 2.08 > 1$$

So, Eq. (4.32) will apply.
From Eqs. (4.33) and (4.34) with $\phi_1 = \phi_2 = 0$,

$$q_t = \left(1 + 0.2\frac{B}{L}\right)N_c c_1 + \gamma_1 D_f$$

$$= \left[1 + (0.2)\left(\frac{4}{6}\right)\right](5.14)(1200) + (3)(110) = 6990.4 + 330 = 7320.4 \text{ lb/ft}^2$$

and

$$q_b = \left(1 + 0.2\frac{B}{L}\right)N_c c_2 + \gamma_2 D_f$$

$$= \left[1 + (0.2)\left(\frac{4}{6}\right)\right](5.14)(2500) + (3)(125)$$

$$= 14{,}563.3 + 375 = 14{,}938.3 \text{ lb/ft}^2$$

From Eq. (4.32),

$$q_u = q_t + (q_b - q_t)\left(\frac{H}{D}\right)^2$$

$$D \approx B$$

$$q_u = 7320.4 + (14{,}938.3 - 7320.4)\left(\frac{2}{4}\right)^2 \approx 9225 \text{ lb/ft}^2 > q_t$$

Hence,

$$\mathbf{q_u = 9225\ lb/ft^2}$$

■

4.5 Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

In Chapter 3, theories relating to the ultimate bearing capacity of single rough continuous foundations supported by a homogeneous soil extending to a great depth were discussed. However, if foundations are placed close to each other with similar soil conditions, the ultimate bearing capacity of each foundation may change due to the interference effect of the failure surface in the soil. This was theoretically investigated by Stuart (1962) for *granular soils*. It was assumed that the geometry of the rupture surface in the soil mass would be the same as that assumed by Terzaghi (Figure 3.5). According to Stuart, the following conditions may arise (Figure 4.12).

Case I. (Figure 4.12a) If the center-to-center spacing of the two foundations is $x \geq x_1$, the rupture surface in the soil under each foundation will not overlap. So the ultimate bearing capacity of each continuous foundation can be given by Terzaghi's equation [Eq. (3.3)]. For ($c' = 0$)

$$q_u = qN_q + \tfrac{1}{2}\gamma B N_\gamma \tag{4.35}$$

where N_q, N_γ = Terzaghi's bearing capacity factors (Table 3.1).

Case II. (Figure 4.12b) If the center-to-center spacing of the two foundations ($x = x_2 < x_1$) are such that the Rankine passive zones just overlap, then the magnitude of q_u will still be given by Eq. (4.35). However, the foundation settlement at ultimate load will change (compared to the case of an isolated foundation).

Case III. (Figure 4.12c) This is the case where the center-to-center spacing of the two continuous foundations is $x = x_3 < x_2$. Note that the triangular wedges in the soil

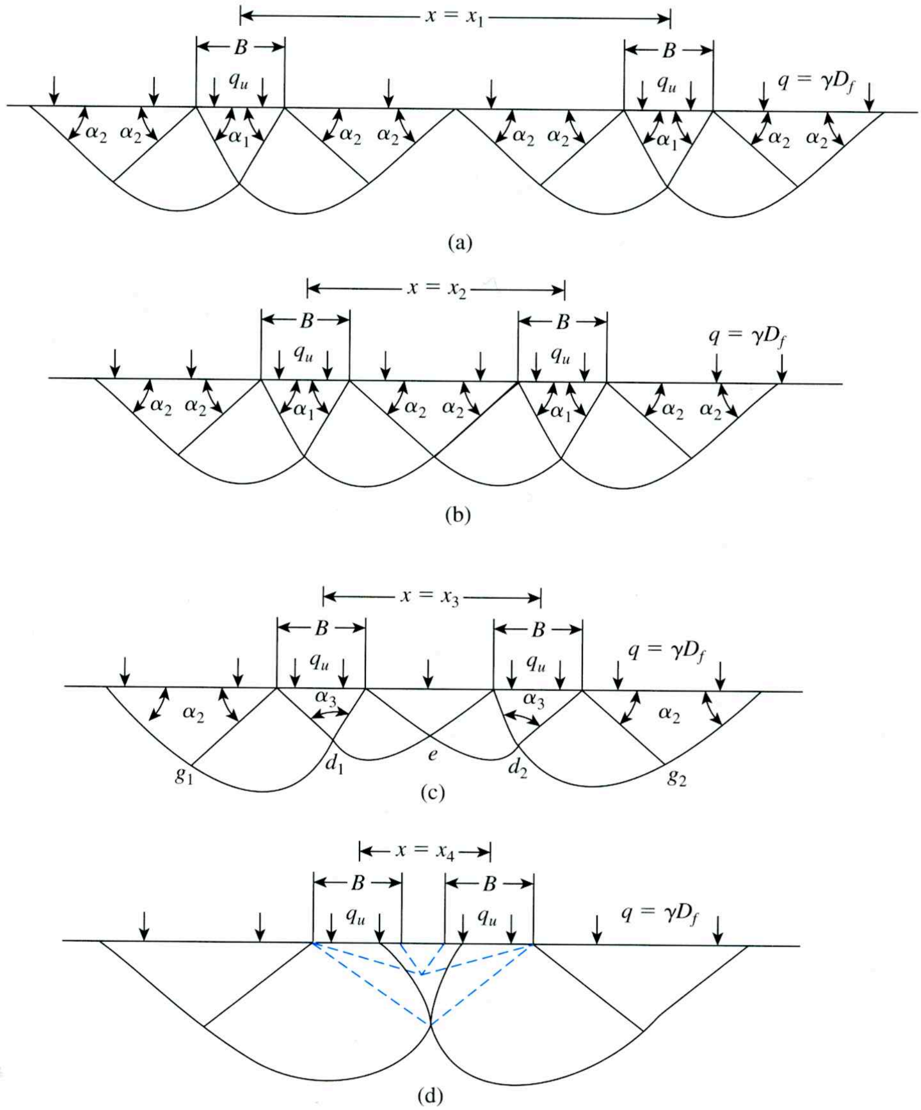


Figure 4.12 Assumptions for the failure surface in granular soil under two closely spaced rough continuous foundations
(*Note:* $\alpha_1 = \phi'$, $\alpha_2 = 45 - \phi'/2$, $\alpha_3 = 180 - 2\phi'$)

under the foundations make angles of $180° - 2\phi'$ at points d_1 and d_2. The arcs of the logarithmic spirals d_1g_1 and d_1e are tangent to each other at d_1. Similarly, the arcs of the logarithmic spirals d_2g_2 and d_2e are tangent to each other at d_2. For this case, the ultimate bearing capacity of each foundation can be given as ($c' = 0$)

$$q_u = qN_q\zeta_q + \tfrac{1}{2}\gamma BN_\gamma\zeta_\gamma \tag{4.36}$$

where ζ_q, ζ_γ = efficiency ratios.

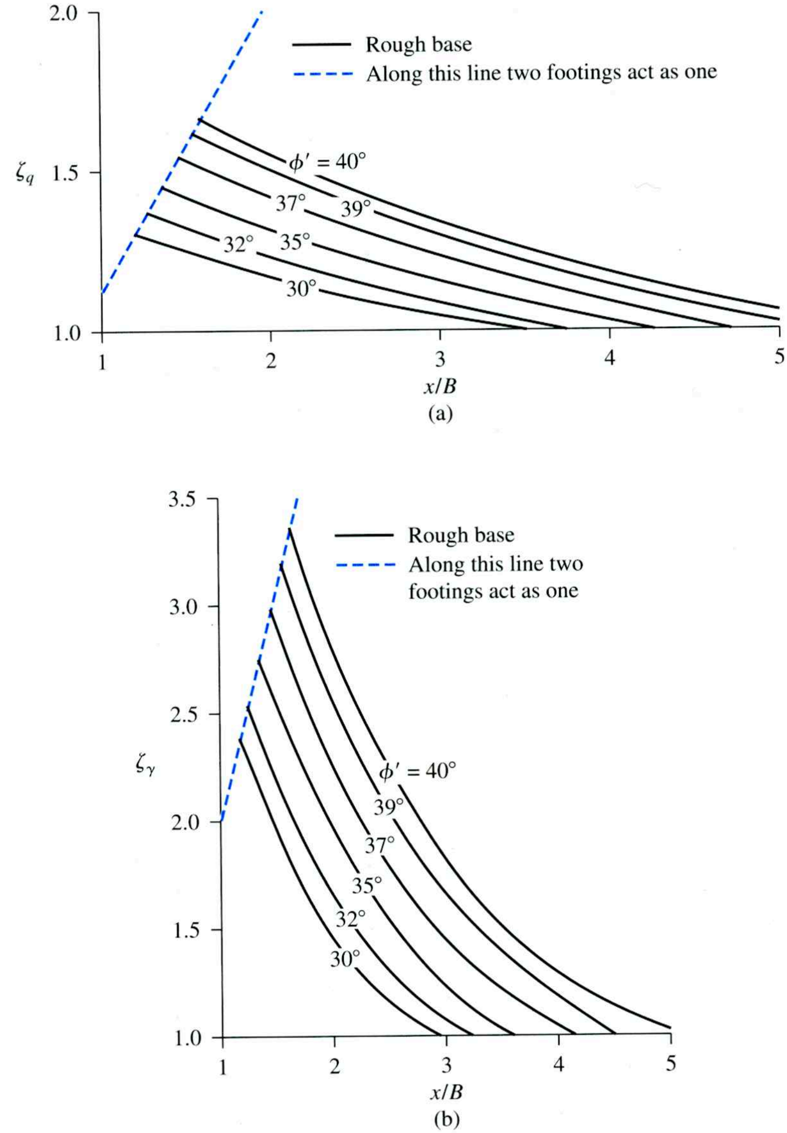


Figure 4.13 Variation of efficiency ratios with x/B and ϕ'

The efficiency ratios are functions of x/B and soil friction angle ϕ'. The theoretical variations of ζ_q and ζ_γ are given in Figure 4.13.

Case IV. (Figure 4.12d): If the spacing of the foundation is further reduced such that $x = x_4 < x_3$, blocking will occur and the pair of foundations will act as a single foundation. The soil between the individual units will form an inverted arch which travels down with the foundation as the load is applied. When the two foundations touch, the zone of arching disappears and the system behaves as a single foundation with a width equal to $2B$. The ultimate bearing capacity for this case can be given by Eq. (4.35), with B being replaced by $2B$ in the second term.

The ultimate bearing capacity of two continuous foundations spaced close to each other may increase since the efficiency ratios are greater than one. However, when the closely spaced foundations are subjected to a similar load per unit area, the settlement S_e will be larger when compared to that for an isolated foundation.

4.6 Bearing Capacity of Foundations on Top of a Slope

In some instances, shallow foundations need to be constructed on top of a slope. In Figure 4.14, the height of the slope is H, and the slope makes an angle β with the horizontal. The edge of the foundation is located at a distance b from the top of the slope. At ultimate load, q_u, the failure surface will be as shown in the figure.

Meyerhof (1957) developed the following theoretical relation for the ultimate bearing capacity for *continuous foundations:*

$$q_u = c'N_{cq} + \tfrac{1}{2}\gamma B N_{\gamma q} \tag{4.37}$$

For purely granular soil, $c' = 0$, thus,

$$q_u = \tfrac{1}{2}\gamma B N_{\gamma q} \tag{4.38}$$

Again, for purely cohesive soil, $\phi = 0$ (the undrained condition); hence,

$$q_u = cN_{cq} \tag{4.39}$$

where c = undrained cohesion.

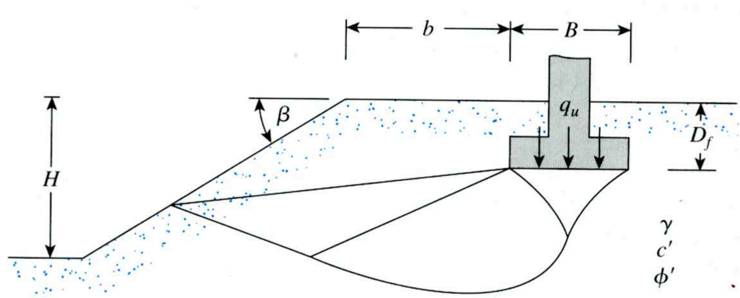


Figure 4.14 Shallow foundation on top of a slope

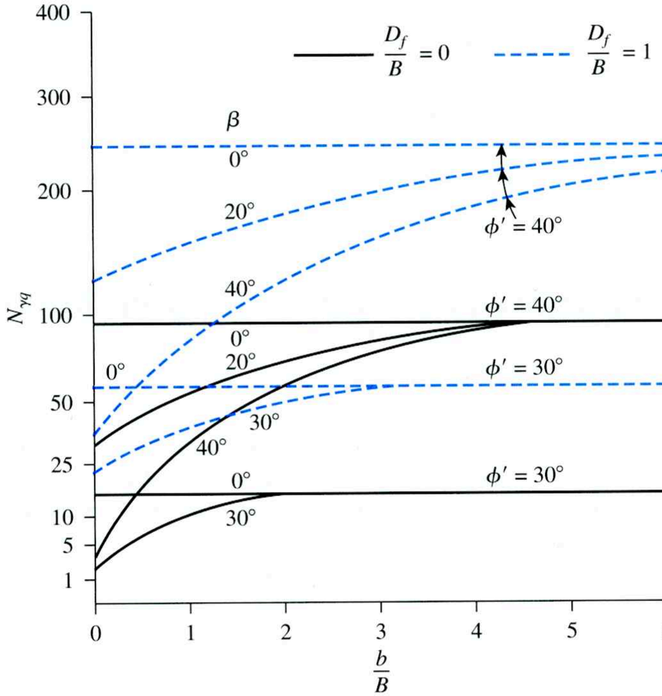


Figure 4.15 Meyerhof's bearing capacity factor $N_{\gamma q}$ for granular soil ($c' = 0$)

The variations of $N_{\gamma q}$ and N_{cq} defined by Eqs. (4.38) and (4.39) are shown in Figures 4.15 and 4.16, respectively. In using N_{cq} in Eq. (4.39) as given in Figure 4.16, the following points need to be kept in mind:

1. The term

$$N_s = \frac{\gamma H}{c} \tag{4.40}$$

   is defined as the stability number.
2. If $B < H$, use the curves for $N_s = 0$.
3. If $B \geqslant H$, use the curves for the calculated stability number N_s.

Stress Characteristics Solution for Granular Soil Slopes

For slopes in granular soils, the ultimate bearing capacity of a continuous foundation can be given by Eq. (4.38), or

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

On the basis of the method of stress characteristics, Graham, Andrews, and Shields (1988) provided a solution for the bearing capacity factor $N_{\gamma q}$ for a shallow continuous foundation on the top of a slope in *granular soil*. Figure 4.17 shows the schematics of the failure zone in the soil for embedment (D_f/B) and setback (b/B) assumed for those authors' analysis. The variations of $N_{\gamma q}$ obtained by this method are shown in Figures 4.18, 4.19, and 4.20.

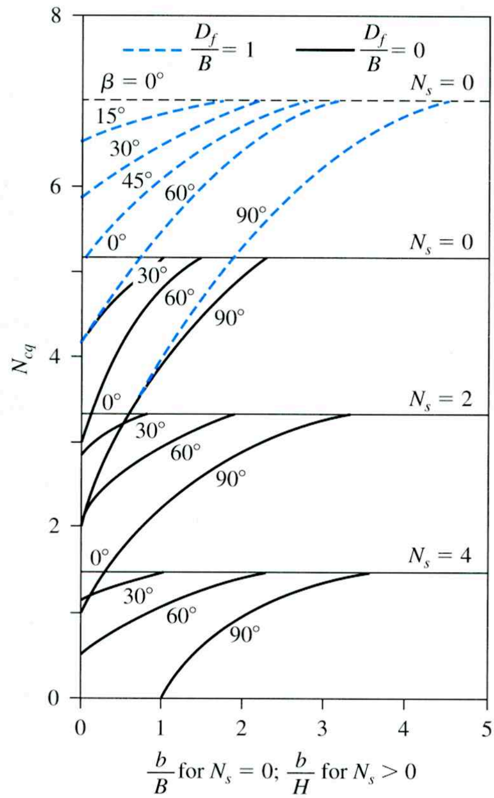


Figure 4.16 Meyerhof's bearing capacity factor N_{cq} for purely cohesive soil

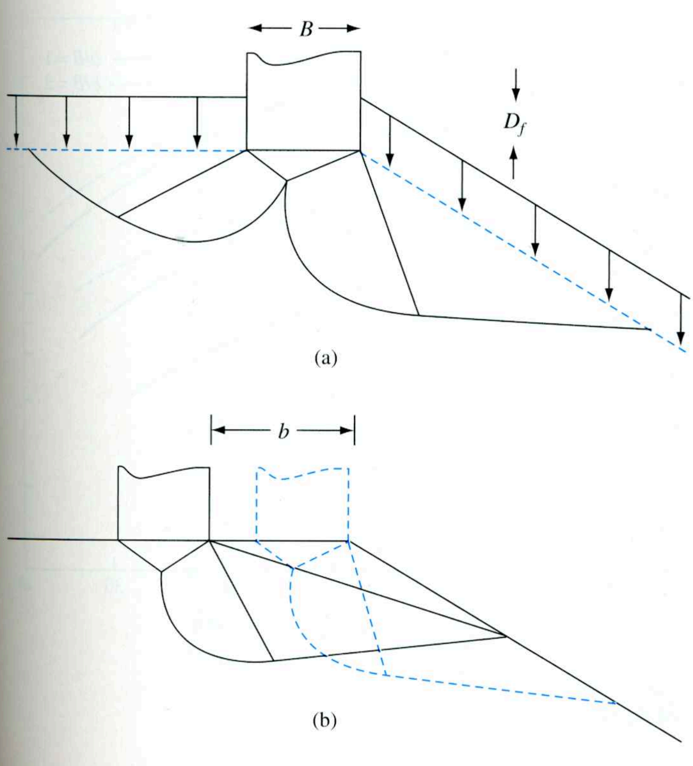


Figure 4.17 Schematic diagram of failure zones for embedment and setback: (a) $D_f/B > 0$; (b) $b/B > 0$

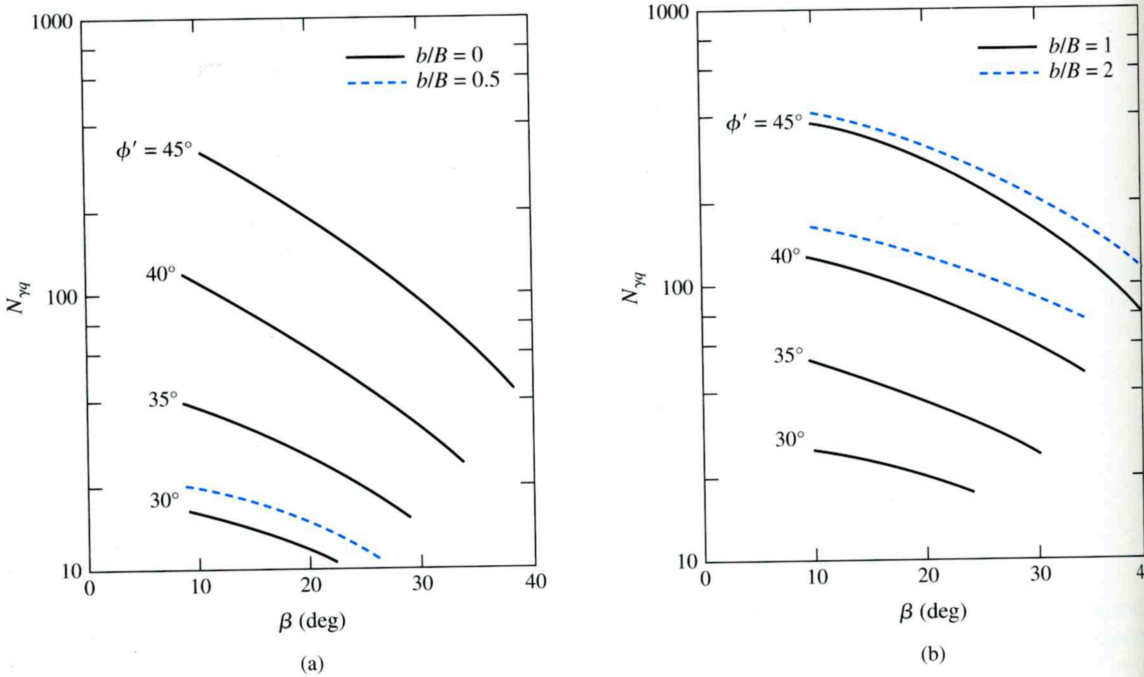


Figure 4.18 Graham et al.'s theoretical values of $N_{\gamma q}(D_f/B = 0)$

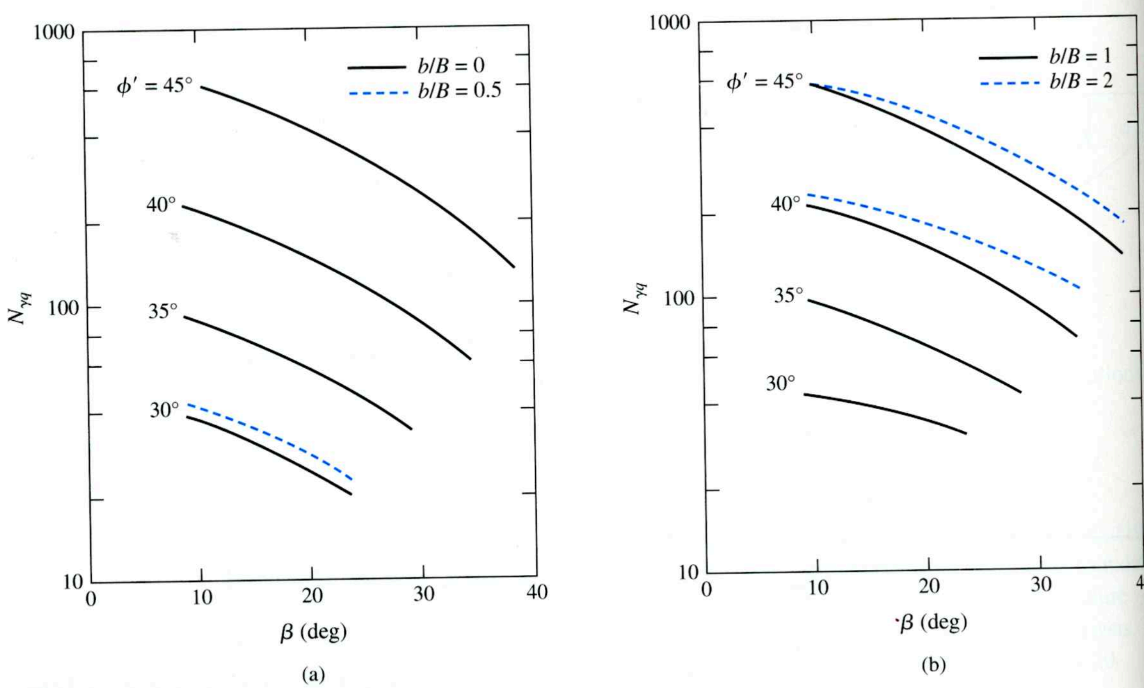
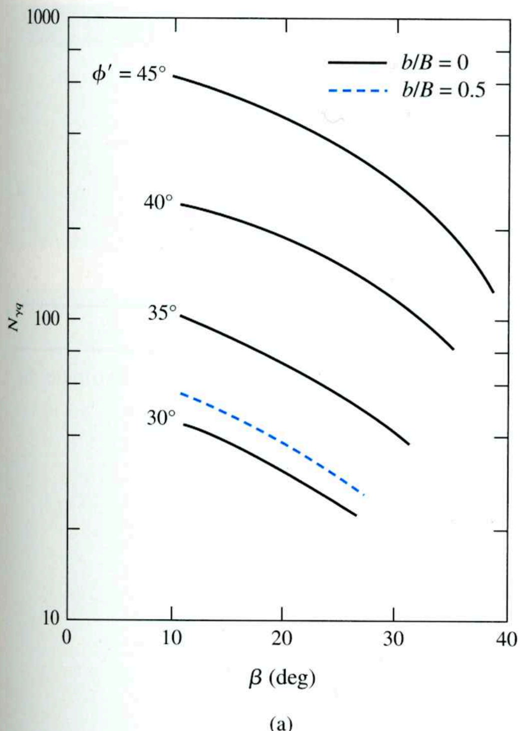


Figure 4.19 Graham et al.'s theoretical values of $N_{\gamma q}(D_f/B = 0.5)$



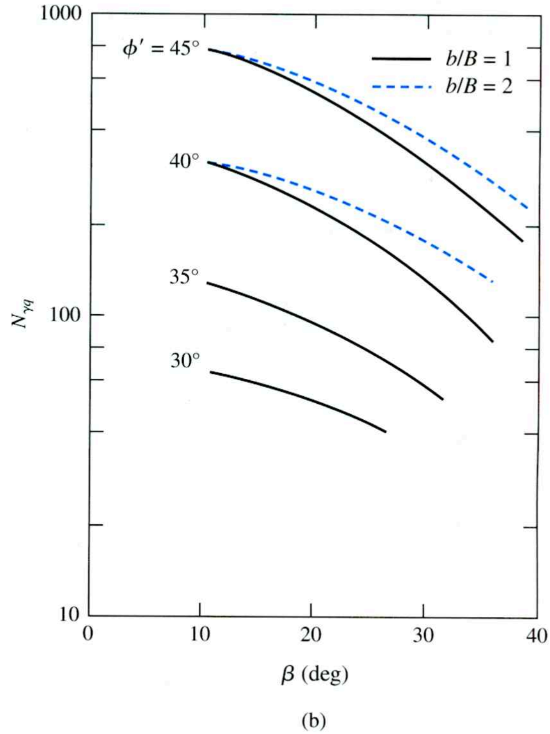


Figure 4.20 Graham et al.'s theoretical values of $N_{\gamma q}(D_f/B = 1)$

Example 4.7

In Figure 4.14, for a shallow continuous foundation in a clay, the following data are given: $B = 1.2$ m; $D_f = 1.2$ m; $b = 0.8$ m; $H = 6.2$ m; $\beta = 30°$; unit weight of soil $= 17.5$ kN/m^3; $\phi = 0$; and $c = 50$ kN/m^2. Determine the gross allowable bearing capacity with a factor of safety FS = 4.

Solution

Since $B < H$, we will assume the stability number $N_s = 0$. From Eq. (4.39),

$$q_u = cN_{cq}$$

We are given that

$$\frac{D_f}{B} = \frac{1.2}{1.2} = 1$$

and

$$\frac{b}{B} = \frac{0.8}{1.2} = 0.67$$

For $\beta = 30°$, $D_f/B = 1$ and $b/B = 0.67$, Figure 4.16 gives $N_{cq} = 6.3$. Hence,

$$q_u = (50)(6.3) = 315 \text{ kN/m}^2$$

and

$$q_{all} = \frac{q_u}{FS} = \frac{315}{4} = \mathbf{78.8\ kN/m^2}$$

Example 4.8

Figure 4.21 shows a continuous foundation on a slope of a granular soil. Estimate the ultimate bearing capacity.

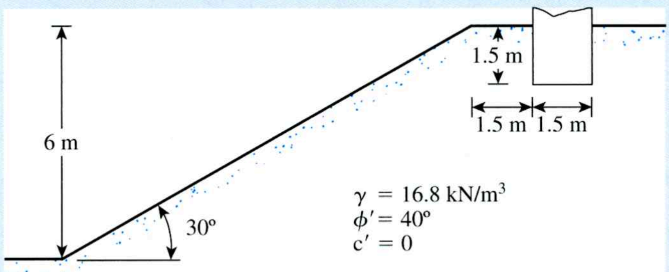


Figure 4.21 Foundation on a granular slope

Solution

For granular soil ($c' = 0$), from Eq. (4.38),

$$q_u = \tfrac{1}{2}\gamma B N_{\gamma q}$$

We are given that $b/B = 1.5/1.5 = 1$, $D_f/B = 1.5/1.5 = 1$, $\phi' = 40°$, and $\beta = 30°$.

From Figure 4.15, $N_{\gamma q} \approx 120$. So,

$$q_u = \tfrac{1}{2}(16.8)(1.5)(120) = \mathbf{1512\ kN/m^2}$$

Example 4.9

Solve Example 4.8 using the stress characteristics solution method.

Solution

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

From Figure 4.20b, $N_{\gamma q} \approx 110$. Hence,

$$q_u = \frac{1}{2}(16.8)(1.5)(110) = \mathbf{1386\ kN/m^2}$$

4.7 Seismic Bearing Capacity of a Foundation at the Edge of a Granular Soil Slope

Figure 4.22 shows a continuous surface foundation ($B/L = 0$, $D_f/B = 0$) at the edge of a granular slope. The foundation is subjected to a loading inclined at an angle α to the vertical. Let the foundation be subjected to seismic loading with a horizontal coefficient of acceleration, k_h. Based on their analysis of method of slices, Huang and Kang (2008) expressed the ultimate bearing capacity as

$$q_u = \frac{1}{2}\gamma B N_\gamma F_{\gamma i} F_{\gamma \beta} F_{\gamma e} \tag{4.41}$$

where

N_γ = bearing capacity factor (Table 3.3)
$F_{\gamma i}$ = load inclination factor
$F_{\gamma\beta}$ = slope inclination factor
$F_{\gamma e}$ = correction factor for the inertia force induced by seismic loading

The relationships for $F_{\gamma i}$, $F_{\gamma\beta}$, and $F_{\gamma e}$ are as follow:

$$F_{\gamma i} = \left[1 - \left(\frac{\alpha^\circ}{\phi'^\circ}\right)\right]^{(0.1\phi' - 1.21)} \tag{4.42}$$

$$F_{\gamma\beta} = [1 - (1.062 - 0.014\phi')\tan\phi']^{\left(\frac{\beta^\circ}{10^\circ}\right)} \tag{4.43}$$

and

$$F_{\gamma e} = 1 - [(2.57 - 0.043\phi')e^{1.45 \tan \beta}]k_h \tag{4.44}$$

In Eqs. (4.42) through (4.44), ϕ' is in degrees.

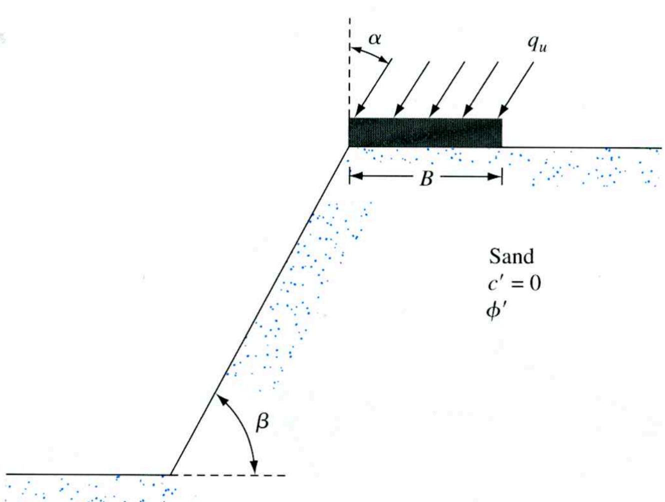


Figure 4.22 Continuous foundation at the edge of a granular slope subjected to seismic loading

Example 4.10

Consider a continuous surface foundation on a granular soil slope subjected to a seismic loading, as shown in Figure 4.22. Given: $B = 1.5$ m, $\gamma = 17.5$ kN/m^3, $\phi' = 35°$, $c' = 0$, $\beta = 30°$, $\alpha = 10°$, and $k_h = 0.2$. Calculate the ultimate bearing capacity, q_u.

Solution

From Eq. (4.41),

$$q_u = \frac{1}{2}\gamma B N_\gamma F_{\gamma i} F_{\gamma\beta} F_{\gamma e}$$

For $\phi' = 35°$, $N_\gamma = 48.03$ (Table 3.3). Thus,

$$F_{\gamma t} = \left[1 - \left(\frac{\alpha°}{\phi'°}\right)\right]^{(0.1\phi' - 1.21)} = \left[1 - \left(\frac{10}{35}\right)\right]^{[(0.1\times 35) - 1.21]} = 0.463$$

$$F_{\gamma\beta} = [1 - (1.062 - 0.014\phi')\tan\phi']^{\left(\frac{\beta°}{10°}\right)}$$

$$= [1 - (1.062 - 0.014 \times 35)\tan 35]^{\left(\frac{30}{10}\right)} = 0.215$$

$$F_{\gamma e} = 1 - [(2.57 - 0.043\phi')e^{1.45\tan\beta}]k_h$$

$$= 1 - [(2.57 - 0.043 \times 35)e^{1.45\tan 30}](0.2) = 0.508$$

So

$$q_u = \frac{1}{2}(17.5)(1.5)(48.03)(0.463)(0.215)(0.508) = \mathbf{31.9\ kN/m^2}$$

4.8 Bearing Capacity of Foundations on a Slope

A theoretical solution for the ultimate bearing capacity of a shallow foundation located on the face of a slope was developed by Meyerhof (1957). Figure 4.23 shows the nature of the plastic zone developed under a rough continuous foundation of width B. In Figure 4.23, abc is an elastic zone, acd is a radial shear zone, and ade is a mixed shear zone. Based on this solution, the ultimate bearing capacity can be expressed as

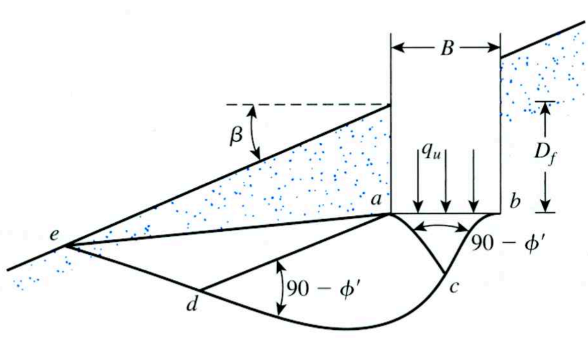


Figure 4.23 Nature of plastic zone under a rough continuous foundation on the face of a slope

$$q_u = cN_{cqs} \text{ (for purely cohesive soil, that is, } \phi = 0) \tag{4.45}$$

and

$$q_u = \tfrac{1}{2}\gamma B N_{\gamma qs} \text{ (for granular soil, that is } c' = 0) \tag{4.46}$$

The variations of N_{cqs} and $N_{\gamma qs}$ with slope angle β are given in Figures 4.24 and 4.25.

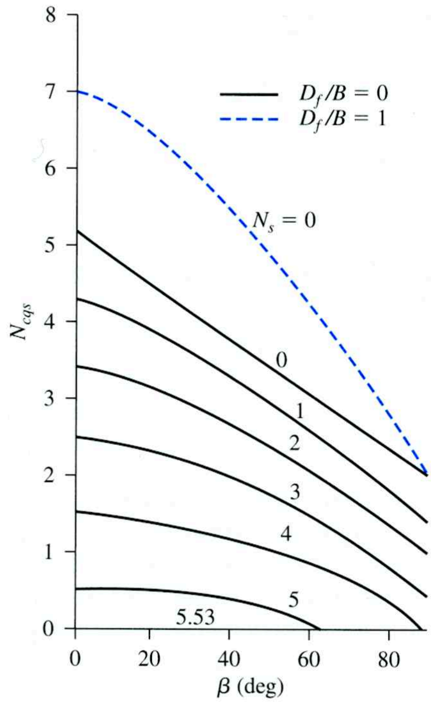


Figure 4.24 Variation of N_{cqs} with β. (*Note:* $N_s = \gamma H/c$)

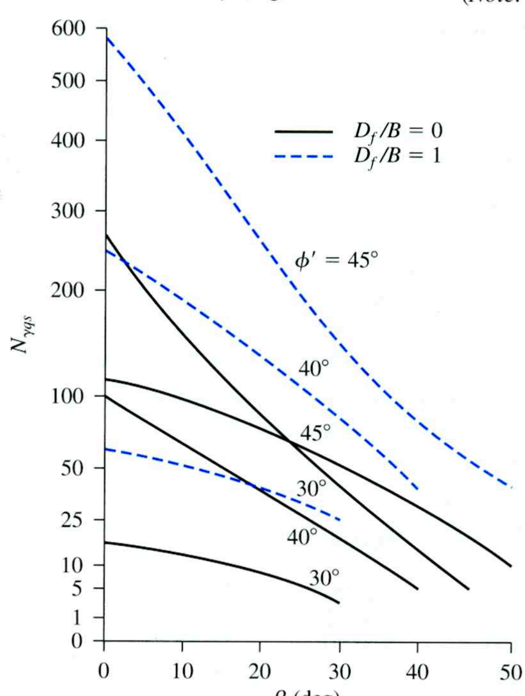


Figure 4.25 Variation of $N_{\gamma qs}$ with β

4.9 Foundations on Rock

On some occasions, shallow foundations may have to be built on rocks, as shown in Figure 4.26. For estimation of the ultimate bearing capacity of shallow foundations on rock, we may use Terzaghi's bearing capacity equations [Eqs. (3.3), (3.7) and (3.8)] with the bearing capacity factors given here (Stagg and Zienkiewicz, 1968; Bowles, 1996):

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right) \tag{4.47}$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right) \tag{4.48}$$

$$N_\gamma = N_q + 1 \tag{4.49}$$

For rocks, the magnitude of the cohesion intercept, c', can be expressed as

$$q_{uc} = 2c'\tan\left(45 + \frac{\phi'}{2}\right) \tag{4.50}$$

where

q_{uc} = unconfined compression strength of rock
ϕ' = angle of friction

The unconfined compression strength and the friction angle of rocks can vary widely. Table 4.2 gives a general range of q_{uc} for various types of rocks. It is important to keep in mind that the magnitude of q_{uc} and ϕ' (hence c') reported from laboratory tests are for intact rock specimens. It does not account for the effect of discontinuities. To account for discontinuities, Bowles (1996) suggested that the ultimate bearing capacity q_u should be modified as

$$q_{u(\text{modified})} = q_u(\text{RQD})^2 \tag{4.51}$$

where RQD = rock quality designation (see Chapter 2).

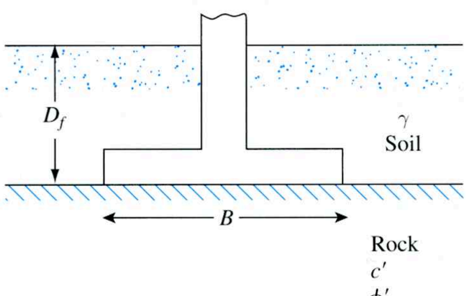


Figure 4.26 Foundation on rock

Table 4.2 Range of the Unconfined Compression Strength of Various Types of Rocks

Rock type	q_{uc} MN/m^2	q_{uc} kip/in^2	ϕ' (deg)
Granite	65–250	9.5–36	45–55
Limestone	30–150	4–22	35–45
Sandstone	25–130	3.5–19	30–45
Shale	5–40	0.75–6	15–30

In any case, the upper limit of the allowable bearing capacity should not exceed f_c' (28-day compressive strength of concrete).

Example 4.11

Refer to Figure 4.26. A square column foundation is to be constructed over siltstone. Given:

Foundation: $B \times B = 2.5 \text{ m} \times 2.5 \text{ m}$
$D_f = 2 \text{ m}$
Soil: $\gamma = 17 \text{ kN/m}^3$
Siltstone: $c' = 32 \text{ MN/m}^2$
$\phi' = 31°$
$\gamma = 25 \text{ kN/m}^3$
RDQ = 50%

Estimate the allowable load-bearing capacity. Use FS = 4. Also, for concrete, use $f_c' = 30 \text{ MN/m}^2$.

Solution

From Eq. (3.7),

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right) = 5\tan^4\left(45 + \frac{31}{2}\right) = 48.8$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right) = \tan^6\left(45 + \frac{31}{2}\right) = 30.5$$

$$N_\gamma = N_q + 1 = 30.5 + 1 = 31.5$$

Hence,

$$\begin{aligned} q_u &= (1.3)(32 \times 10^3 \text{ kN/m}^2)(48.8) + (17 \times 2)(30.5) + (0.4)(25)(2.5)(31.5) \\ &= 2030.08 \times 10^3 + 1.037 \times 10^3 + 0.788 \times 10^3 \\ &= 2031.9 \times 10^3 \text{ kN/m}^2 \approx 2032 \text{ MN/m}^2 \end{aligned}$$

$$q_{u(\text{modified})} = q_u(\text{RQD})^2 = (2032)(0.5)^2 = 508 \text{ MN/m}^2$$

$$q_{\text{all}} = \frac{508}{4} = 127 \text{ MN/m}^2$$

Since 127 MN/m² is greater than f_c', use $q_{\text{all}} = $ **30 MN/m²**. ■

4.10 Uplift Capacity of Foundations

Foundations may be subjected to uplift forces under special circumstances. During the design process for those foundations, it is desirable to provide a sufficient factor of safety against failure by uplift. This section will provide the relationships for the uplift capacity of foundations in granular and cohesive soils.

Foundations in Granular Soil ($c' = 0$)

Figure 4.27 shows a shallow continuous foundation that is being subjected to an uplift force. At ultimate load, Q_u, the failure surface in soil will be as shown in the figure. The ultimate load can be expressed in the form of a nondimensional breakout factor, F_q. Or

$$F_q = \frac{Q_u}{A\gamma D_f} \tag{4.52}$$

where A = area of the foundation.

The breakout factor is a function of the soil friction angle ϕ' and D_f/B. For a given soil friction angle, F_q increases with D_f/B to a maximum at $D_f/B = (D_f/B)_{cr}$ and remains constant thereafter. For foundations subjected to uplift, $D_f/B \le (D_f/B)_{cr}$ is considered a shallow foundation condition. When a foundation has an embedment ratio of $D_f/B > (D_f/B)_{cr}$, it is referred to as a deep foundation. Meyerhof and Adams (1968) provided relationships to estimate the ultimate uplifting load Q_u for shallow [that is, $D_f/B \le (D_f/B)_{cr}$], circular, and rectangular foundations. Using these relationships and Eq. (4.52), Das and Seeley (1975) expressed the breakout factor F_q in the following form

$$F_q = 1 + 2\left[1 + m\left(\frac{D_f}{B}\right)\right]\left(\frac{D_f}{B}\right)K_u \tan\phi' \tag{4.53}$$

(for shallow circular and square foundations)

$$F_q = 1 + \left\{\left[1 + 2m\left(\frac{D_f}{B}\right)\right]\left(\frac{B}{L}\right) + 1\right\}\left(\frac{D_f}{B}\right)K_u \tan\phi' \tag{4.54}$$

(for shallow rectangular foundations)

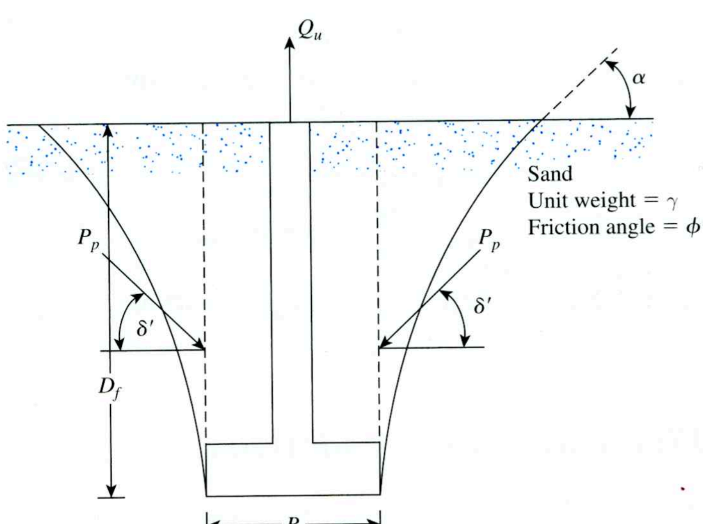


Figure 4.27 Shallow continuous foundation subjected to uplift

where

m = a coefficient which is a function of ϕ'

K_u = nominal uplift coefficient

The variations of K_u, m, and $(D_f/B)_{cr}$ for square and circular foundations are given in Table 4.3 (Meyerhof and Adams, 1968).

For rectangular foundations, Das and Jones (1982) recommended that

$$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}} = \left(\frac{D_f}{B}\right)_{\text{cr-square}}\left[0.133\left(\frac{L}{B}\right) + 0.867\right] \le 1.4\left(\frac{D_f}{B}\right)_{\text{cr-square}} \tag{4.55}$$

Using the values of K_u, m, and $(D_f/B)_{cr}$ in Eq. (4.53), the variations of F_q for square and circular foundations have been calculated and are shown in Figure 4.28. A step-by-step procedure to estimate the uplift capacity of foundations in granular soil follows.

Step 1. Determine, D_f, B, L, and ϕ'.
Step 2. CalculateD_f/B.
Step 3. Using Table 4.3 and Eq. (4.55), calculate $(D_f/B)_{cr}$.
Step 4. If D_f/B is less than or equal to $(D_f/B)_{cr}$, it is a shallow foundation.
Step 5. If $D_f/B > (D_f/B)_{cr}$, it is a deep foundation.
Step 6. For shallow foundations, use D_f/B calculated in Step 2 in Eq. (4.53) or (4.54) to estimate F_q. Thus, $Q_u = F_q A\gamma D_f$.
Step 7. For deep foundations, substitute $(D_f/B)_{cr}$ for D_f/B in Eq. (4.53) or (4.54) to obtain F_q, from which the ultimate load Q_u may be obtained.

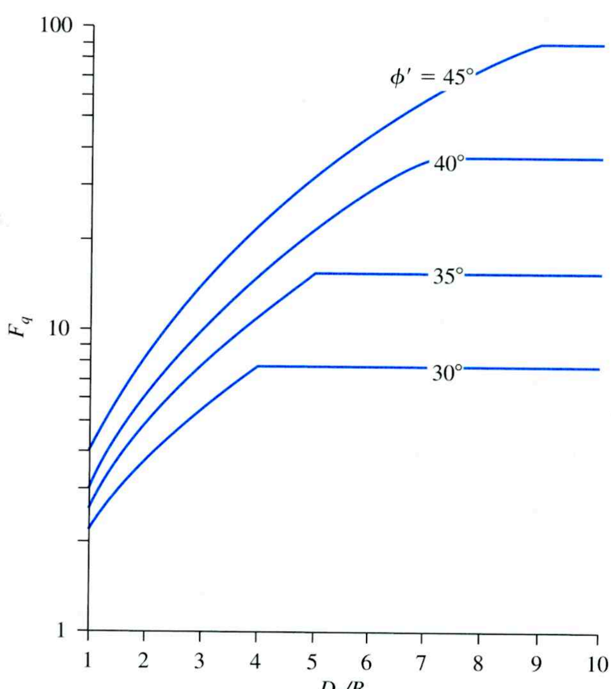


Figure 4.28 Variation of F_q with D_f/B and ϕ'

Table 4.3 Variation of K_u, m, and $(D_f/B)_{cr}$

Soil friction angle, ϕ' (deg)	K_u	m	$(D_f/B)_{cr}$ for square and circular foundations
20	0.856	0.05	2.5
25	0.888	0.10	3
30	0.920	0.15	4
35	0.936	0.25	5
40	0.960	0.35	7
45	0.960	0.50	9

Foundations in Cohesive Soil ($\left(\frac{D_f}{B}\right)_{\text{cr-square}}$ 0)

The ultimate uplift capacity, Q_u, of a foundation in a purely cohesive soil can be expressed as

$$Q_u = A(\gamma D_f + c_u F_c) \tag{4.56}$$

where

A = area of the foundation
c_u = undrained shear strength of soil
F_c = breakout factor

As in the case of foundations in granular soil, the breakout factor F_c increases with embedment ratio and reaches a maximum value of $F_c = F_c^*$ at $D_f/B = (D_f/B)_{cr}$ and remains constant thereafter.

Das (1978) also reported some model test results with square and rectangular foundations. Based on these test results, it was proposed that

$$\left(\frac{D_f}{B}\right)_{\text{cr-square}} = 0.107c_u + 2.5 \leq 7 \tag{4.57}$$

where

$\left(\frac{D_f}{B}\right)_{\text{cr-square}}$ = critical embedment ratio of square (or circular) foundations

c_u = undrained cohesion, in kN/m^2

It was also observed by Das (1980) that

$$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}} = \left(\frac{D_f}{B}\right)_{\text{cr-square}}\left[0.73 + 0.27\left(\frac{L}{B}\right)\right] \leq 1.55\left(\frac{D_f}{B}\right)_{\text{cr-square}} \tag{4.58}$$

where

$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}}$ = critical embedment ratio of rectangular foundations

L = length of foundation

Based on these findings, Das (1980) proposed an empirical procedure to obtain the breakout factors for shallow and deep foundations. According to this procedure, α' and β' are two nondimensional factors defined as

$$\alpha' = \frac{\dfrac{D_f}{B}}{\left(\dfrac{D_f}{B}\right)_{\text{cr}}} \tag{4.59}$$

and

$$\beta' = \frac{F_c}{F_c^*} \tag{4.60}$$

For a given foundation, the critical embedment ratio can be calculated using Eqs. (4.57) and (4.58). The magnitude of F_c^* can be given by the following empirical relationship

$$F^*_{c\text{-rectangular}} = 7.56 + 1.44\left(\frac{B}{L}\right) \tag{4.61}$$

where $F^*_{c\text{-rectangular}}$ = breakout factor for deep rectangular foundations

Figure 4.29 shows the experimentally derived plots (upper limit, lower limit, and average of β' and α'. Following is a step-by-step procedure to estimate the ultimate uplift capacity.

Step 1. Determine the representative value of the undrained cohesion, c_u.
Step 2. Determine the critical embedment ratio using Eqs. (4.57) and (4.58).
Step 3. Determine the D_f/B ratio for the foundation.
Step 4. If $D_f/B > (D_f/B)_{\text{cr}}$, as determined in Step 2, it is a deep foundation. However, if $D_f/B \le (D_f/B)_{\text{cr}}$, it is a shallow foundation.

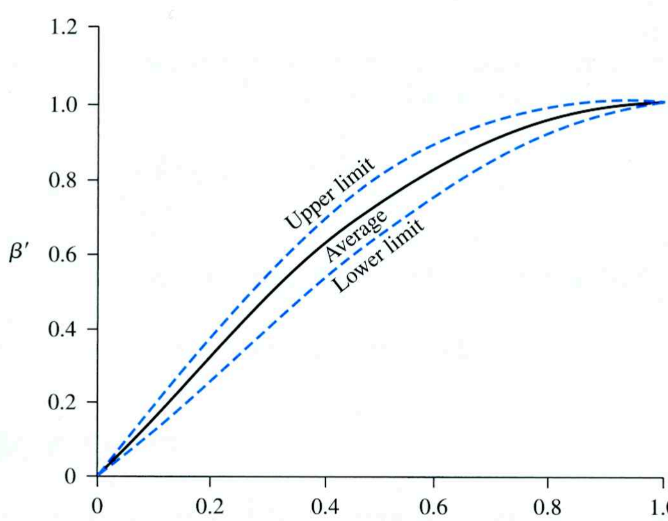


Figure 4.29 Plot of β' versus α'

Step 5. For $D_f/B > (D_f/B)_{cr}$

$$F_c = F_c^* = 7.56 + 1.44\left(\frac{B}{L}\right)$$

Thus,

$$Q_u = A\left\{\left[7.56 + 1.44\left(\frac{B}{L}\right)\right]c_u + \gamma D_f\right\} \tag{4.62}$$

where A = area of the foundation.

Step 6. For $D_f/B \leq (D_f/B)_{cr}$

$$Q_u = A(\beta' F_c^* c_u + \gamma D_f) = A\left\{\beta'\left[7.56 + 1.44\left(\frac{B}{L}\right)\right]c_u + \gamma D_f\right\} \tag{4.63}$$

The value of β' can be obtained from the average curve of Figure 4.29. The procedure outlined above gives fairly good results for estimating the net ultimate uplift capacity of foundations and agrees reasonably well with the theoretical solution of Merifield et al. (2003).

Example 4.12

Consider a circular foundation in sand. Given for the foundation: diameter, B = 1.5 m and depth of embedment, D_f = 1.5 m. Given for the sand: unit weight, γ = 17.4 kN/m^3, and friction angle, ϕ' = 35°. Calculate the ultimate bearing capacity.

Solution

$D_f/B = 1.5/1.5 = 1$ and $\phi' = 35°$. For circular foundation, $(D_f/B)_{cr} = 5$. Hence, it is a shallow foundation. From Eq. (4.53)

$$F_q = 1 + 2\left[1 + m\left(\frac{D_f}{B}\right)\right]\left(\frac{D_f}{B}\right)K_u \tan\phi'$$

For $\phi' = 35°$, $m = 0.25$, and $K_u = 0.936$ (Table 4.3). So

$$F_q = 1 + 2[1 + (0.25)(1)](1)(0.936)(\tan 35) = 2.638$$

So

$$Q_u = F_q \gamma A D_f = (2.638)(17.4)\left[\left(\frac{\pi}{4}\right)(1.5)^2\right](1.5) = \mathbf{121.7\ kN}$$ ■

Example 4.13

A rectangular foundation in a saturated clay measures 1.5 m × 3 m. Given: $D_f = 1.8$ m, $c_u = 52$ kN/m², and $\gamma = 18.9$ kN/m³. Estimate the ultimate uplift capacity.

Solution

From Eq. (4.57)

$$\left(\frac{D_f}{B}\right)_{\text{cr-square}} = 0.107c_u + 2.5 = (0.107)(52) + 2.5 = 8.06$$

So use $(D_f/B)_{\text{cr-square}} = 7$. Again from Eq. (4.58),

$$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}} = \left(\frac{D_f}{B}\right)_{\text{cr-square}}\left[0.73 + 0.27\left(\frac{L}{B}\right)\right]$$

$$= 7\left[0.73 + 0.27\left(\frac{3}{1.5}\right)\right] = 8.89$$

Check:
$$1.55\left(\frac{D_f}{B}\right)_{\text{cr-square}} = (1.55)(7) = 10.85$$

So use $(D_f/B)_{\text{cr-rectangular}} = 8.89$. The actual embedment ratio is $D_f/B = 1.8/1.5 = 1.2$. Hence, this is a shallow foundation.

$$\alpha' = \frac{\dfrac{D_f}{B}}{\left(\dfrac{D_f}{B}\right)_{\text{cr}}} = \frac{1.2}{8.89} = 0.135$$

Referring to the average curve of Figure 4.29, for $\alpha' = 0.135$, the magnitude of $\beta' = 0.2$. From Eq. (4.63),

$$Q_u = A\left\{\beta'\left[7.56 + 1.44\left(\frac{B}{L}\right)\right]c_u + \gamma D_f\right\}$$

$$= (1.5)(3)\left\{(0.2)\left[7.56 + 1.44\left(\frac{1.5}{3}\right)\right](52) + (18.9)(1.8)\right\} = \mathbf{540.6\ kN}$$ ■

Problems

4.1 Refer to Figure 4.2 and consider a rectangular foundation. Given: $B = 3$ ft, $L = 6$ ft, $D_f = 3$ ft, $H = 2$ ft, $\phi' = 40°$, $c' = 0$, and $\gamma = 115$ lb/ft³. Using a factor of safety of 4, determine the gross allowable load the foundation can carry. Use Eq. (4.3).

4.2 Repeat Problem 4.1 with the following data: $B = 1.5$ m, $L = 1.5$ m, $D_f = 1$ m, $H = 0.6$ m, $\phi' = 35°$, $c' = 0$, and $\gamma = 15$ kN/m^3. Use FS = 3.

4.3 Refer to Figure 4.2. Given: $B = L = 1.75$ m, $D_f = 1$ m, $H = 1.75$ m, $\gamma = 17$ kN/m^3, $c' = 0$, and $\phi' = 30°$. Using Eq. (4.6) and FS = 4, determine the gross allowable load the foundation can carry.

4.4 Refer to Figure 4.2. A square foundation measuring 4 ft × 4 ft is supported by a saturated clay layer of limited depth underlain by a rock layer. Given that $D_f = 3$ ft, $H = 2$ ft, $c_u = 2400$ lb/ft^2, and $\gamma = 120$ lb/ft^3, estimate the ultimate bearing capacity of the foundation.

4.5 Refer to Figure 4.8. For a strip foundation in two-layered clay, given:

- $\gamma_1 = 115$ lb/ft^3, $c_1 = 1200$ lb/ft^2, $\phi_1 = 0$
- $\gamma_2 = 110$ lb/ft^3, $c_2 = 600$ lb/ft^2, $\phi_2 = 0$
- $B = 3$ ft, $D_f = 2$ ft, $H = 2$ ft

Find the gross allowable bearing capacity. Use a factor of safety of 3.

4.6 Refer to Figure 4.8. For a strip foundation in two-layered clay, given:

- $B = 0.92$ m, $L = 1.22$ m, $D_f = 0.92$ m, $H = 0.76$ m
- $\gamma_1 = 17$ kN/m^3, $\phi_1 = 0$, $c_1 = 72$ kN/m^2
- $\gamma_2 = 17$ kN/m^3, $\phi_2 = 0$, $c_2 = 43$ kN/m^2

Determine the gross ultimate bearing capacity.

4.7 Refer to Figure 4.8. For a square foundation on layered sand, given:

- $B = 1.5$ m, $D_f = 1.5$ m, $H = 1$ m
- $\gamma_1 = 18$ kN/m^3, $\phi_1' = 40°$, $c_1' = 0$
- $\gamma_2 = 16.7$ kN/m^3, $\phi_2' = 32°$, $c_2' = 0$

Determine the net allowable load that the foundations can support. Use FS = 4.

4.8 Refer to Figure 4.11. For a rectangular foundation on layered sand, given:

- $B = 4$ ft, $L = 6$ ft, $H = 2$ ft, $D_f = 3$ ft
- $\gamma_1 = 98$ lb/ft^3, $\phi_1' = 30°$, $c_1' = 0$
- $\gamma_2 = 108$ lb/ft^3, $\phi_2' = 38°$, $c_2' = 0$

Using a factor of safety of 4, determine the gross allowable load the foundation can carry.

4.9 Two continuous shallow foundations are constructed alongside each other in a granular soil. Given, for the foundation: $B = 1.2$ m, $D_f = 1$ m, and center-to-center spacing = 2 m. The soil friction angle, $\phi' = 35°$. Estimate the net allowable bearing capacity of the foundations. Use a factor of safety of FS = 4 and a unit weight of soil $\gamma = 16.8$ kN/m^3.

4.10 A continuous foundation with a width of 1 m is located on a slope made of clay soil. Refer to Figure 4.14 and let $D_f = 1$ m, $H = 4$ m, $b = 2$ m, $\gamma = 16.8$ kN/m^3, $c = 68$ kN/m^2, $\phi = 0$, and $\beta = 60°$.

a. Determine the allowable bearing capacity of the foundation. Let FS = 3.

b. Plot a graph of the ultimate bearing capacity q_u if b is changed from 0 to 6 m.

4.11 A continuous foundation is to be constructed near a slope made of granular soil (see Figure 4.14). If $B = 4$ ft, $b = 6$ ft, $H = 15$ ft, $D_f = 4$ ft, $\beta = 30°$, $\phi' = 40°$, and $\gamma = 110$ lb/ft^3, estimate the ultimate bearing capacity of the foundation. Use Meyerhof's solution.

4.12 A square foundation in a sand deposit measures 4 ft × 4 ft in plan. Given: $D_f = 5$ ft, soil friction angle = 35°, and unit weight of soil = 112 lb/ft^3. Estimate the ultimate uplift capacity of the foundation.

4.13 A foundation measuring 1.2 m × 2.4 m in plan is constructed in a saturated clay. Given: depth of embedment of the foundation = 2 m, unit weight of soil = 18 kN/m^3, and undrained cohesion of clay = 74 kN/m^2. Estimate the ultimate uplift capacity of the foundation.

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5 Shallow Foundations: Allowable Bearing Capacity and Settlement

5.1 Introduction

It was mentioned in Chapter 3 that, in many cases, the allowable settlement of a shallow foundation may control the allowable bearing capacity. The allowable settlement itself may be controlled by local building codes. Thus, the allowable bearing capacity will be the smaller of the following two conditions:

$$q_{\text{all}} = \begin{cases} \dfrac{q_u}{\text{FS}} \\ \text{or} \\ q_{\text{allowable settlement}} \end{cases}$$

The settlement of a foundation can be divided into two major categories: (a) elastic, or immediate, settlement and (b) consolidation settlement. Immediate, or elastic, settlement of a foundation takes place during or immediately after the construction of the structure. Consolidation settlement occurs over time. Pore water is extruded from the void spaces of saturated clayey soils submerged in water. The total settlement of a foundation is the sum of the elastic settlement and the consolidation settlement.

Consolidation settlement comprises two phases: *primary* and *secondary.* The fundamentals of primary consolidation settlement were explained in detail in Chapter 1. Secondary consolidation settlement occurs after the completion of primary consolidation caused by slippage and reorientation of soil particles under a sustained load. Primary consolidation settlement is more significant than secondary settlement in inorganic clays and silty soils. However, in organic soils, secondary consolidation settlement is more significant.

For the calculation of foundation settlement (both elastic and consolidation), it is required that we estimate the vertical stress increase in the soil mass due to the net load applied on the foundation. Hence, this chapter is divided into the following three parts:

1. Procedure for calculation of vertical stress increase
2. Elastic settlement calculation
3. Consolidation settlement calculation

Vertical Stress Increase in a Soil Mass Caused by Foundation Load

5.2 Stress Due to a Concentrated Load

In 1885, Boussinesq developed the mathematical relationships for determining the normal and shear stresses at any point inside *homogeneous, elastic,* and *isotropic* mediums due to a *concentrated point load* located at the surface, as shown in Figure 5.1. According to his analysis, the *vertical stress increase* at point A caused by a point load of magnitude P is given by

$$\Delta\sigma = \frac{3P}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \tag{5.1}$$

where

$r = \sqrt{x^2 + y^2}$

x, y, z = coordinates of the point A

Note that Eq. (5.1) is not a function of Poisson's ratio of the soil.

5.3 Stress Due to a Circularly Loaded Area

The Boussinesq equation (5.1) can also be used to determine the vertical stress below the center of a flexible circularly loaded area, as shown in Figure 5.2. Let the radius of the loaded area be $B/2$, and let q_o be the uniformly distributed load per unit area. To determine

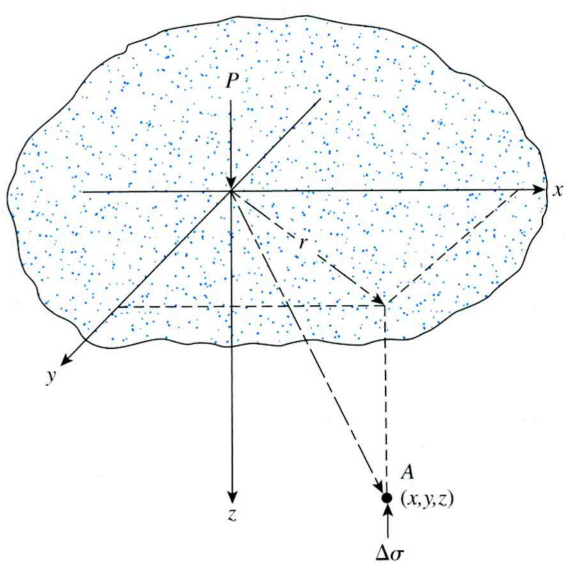

Figure 5.1 Vertical stress at a point A caused by a point load on the surface

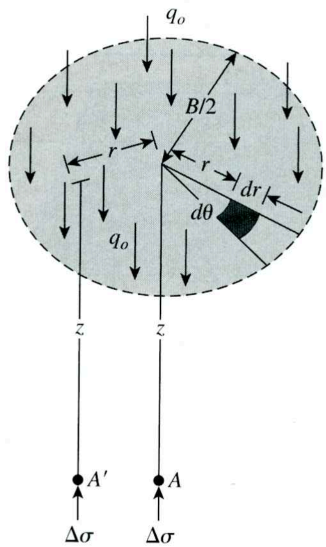

Figure 5.2 Increase in pressure under a uniformly loaded flexible circular area

the stress increase at a point A, located at a depth z below the center of the circular area, consider an elemental area on the circle. The load on this elemental area may be taken to be a point load and expressed as $q_o r\, d\theta\, dr$. The stress increase at A caused by this load can be determined from Eq. (5.1) as

$$d\sigma = \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1 + \left(\dfrac{r}{z}\right)^2\right]^{5/2}} \tag{5.2}$$

The total increase in stress caused by the entire loaded area may be obtained by integrating Eq. (5.2), or

$$\Delta\sigma = \int d\sigma = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=B/2} \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1 + \left(\dfrac{r}{z}\right)^2\right]^{5/2}}$$

$$= q_o\left\{1 - \frac{1}{\left[1 + \left(\dfrac{B}{2z}\right)^2\right]^{3/2}}\right\} \tag{5.3}$$

Similar integrations could be performed to obtain the vertical stress increase at A', located a distance r from the center of the loaded area at a depth z (Ahlvin and Ulery, 1962). Table 5.1 gives the variation of $\Delta\sigma/q_o$ with $r/(B/2)$ and $z/(B/2)$ [for $0 \le r/(B/2) \le 1$]. Note that the variation of $\Delta\sigma/q_o$ with depth at $r/(B/2) = 0$ can be obtained from Eq. (5.3).

Table 5.1 Variation of $\Delta\sigma/q_o$ for a Uniformly Loaded Flexible Circular Area

	$r/(B/2)$					
$z/(B/2)$	0	0.2	0.4	0.6	0.8	1.0
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.999	0.999	0.998	0.996	0.976	0.484
0.2	0.992	0.991	0.987	0.970	0.890	0.468
0.3	0.976	0.973	0.963	0.922	0.793	0.451
0.4	0.949	0.943	0.920	0.860	0.712	0.435
0.5	0.911	0.902	0.869	0.796	0.646	0.417
0.6	0.864	0.852	0.814	0.732	0.591	0.400
0.7	0.811	0.798	0.756	0.674	0.545	0.367
0.8	0.756	0.743	0.699	0.619	0.504	0.366
0.9	0.701	0.688	0.644	0.570	0.467	0.348
1.0	0.646	0.633	0.591	0.525	0.434	0.332
1.2	0.546	0.535	0.501	0.447	0.377	0.300
1.5	0.424	0.416	0.392	0.355	0.308	0.256
2.0	0.286	0.286	0.268	0.248	0.224	0.196
2.5	0.200	0.197	0.191	0.180	0.167	0.151
3.0	0.146	0.145	0.141	0.135	0.127	0.118
4.0	0.087	0.086	0.085	0.082	0.080	0.075

5.4 Stress below a Rectangular Area

The integration technique of Boussinesq's equation also allows the vertical stress at any point A below the corner of a flexible rectangular loaded area to be evaluated. (See Figure 5.3.) To do so, consider an elementary area $dA = dx\,dy$ on the flexible loaded area. If the load per unit area is q_o, the total load on the elemental area is

$$dP = q_o\,dx\,dy \tag{5.4}$$

This elemental load, dP, may be treated as a point load. The increase in vertical stress at point A caused by dP may be evaluated by using Eq. (5.1). Note, however, the need to substitute $dP = q_o\,dx\,dy$ for P and $x^2 + y^2$ for r^2 in that equation. Thus,

$$\text{The stress increase at } A \text{ caused by } dP = \frac{3q_o\,(dx\,dy)z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}}$$

The total stress increase $\Delta\sigma$ caused by the entire loaded area at point A may now be obtained by integrating the preceding equation:

$$\Delta\sigma = \int_{y=0}^{L}\int_{x=0}^{B}\frac{3q_o\,(dx\,dy)z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}} = q_o I \tag{5.5}$$

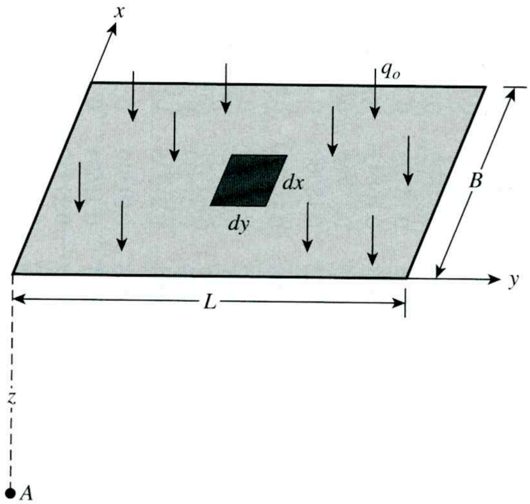

Figure 5.3 Determination of stress below the corner of a flexible rectangular loaded area

Here,

$$I = \text{influence factor} = \frac{1}{4\pi}\left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \cdot \frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} + \tan^{-1}\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + 1 - m^2n^2}\right) \tag{5.6}$$

where

$$m = \frac{B}{z} \tag{5.7}$$

and

$$n = \frac{L}{z} \tag{5.8}$$

The variations of the influence values with m and n are given in Table 5.2.

The stress increase at any point below a rectangular loaded area can also be found by using Eq. (5.5) in conjunction with Figure 5.4. To determine the stress at a depth z below point O, divide the loaded area into four rectangles, with O the corner common to each. Then use Eq. (5.5) to calculate the increase in stress at a depth z below O caused by each rectangular area. The total stress increase caused by the entire loaded area may now be expressed as

$$\Delta\sigma = q_o\,(I_1 + I_2 + I_3 + I_4) \tag{5.9}$$

where I_1, I_2, I_3, and I_4 = the influence values of rectangles 1, 2, 3, and 4, respectively.

In most cases, the vertical stress below the center of a rectangular area is of importance. This can be given by the relationship

$$\Delta\sigma = q_oI_c \tag{5.10}$$

Table 5.2 Variation of Influence Value I [Eq. (5.6)][a]

	n											
m	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**	**1.2**	**1.4**
0.1	0.00470	0.00917	0.01323	0.01678	0.01978	0.02223	0.02420	0.02576	0.02698	0.02794	0.02926	0.03007
0.2	0.00917	0.01790	0.02585	0.03280	0.03866	0.04348	0.04735	0.05042	0.05283	0.05471	0.05733	0.05894
0.3	0.01323	0.02585	0.03735	0.04742	0.05593	0.06294	0.06858	0.07308	0.07661	0.07938	0.08323	0.08561
0.4	0.01678	0.03280	0.04742	0.06024	0.07111	0.08009	0.08734	0.09314	0.09770	0.10129	0.10631	0.10941
0.5	0.01978	0.03866	0.05593	0.07111	0.08403	0.09473	0.10340	0.11035	0.11584	0.12018	0.12626	0.13003
0.6	0.02223	0.04348	0.06294	0.08009	0.09473	0.10688	0.11679	0.12474	0.13105	0.13605	0.14309	0.14749
0.7	0.02420	0.04735	0.06858	0.08734	0.10340	0.11679	0.12772	0.13653	0.14356	0.14914	0.15703	0.16199
0.8	0.02576	0.05042	0.07308	0.09314	0.11035	0.12474	0.13653	0.14607	0.15371	0.15978	0.16843	0.17389
0.9	0.02698	0.05283	0.07661	0.09770	0.11584	0.13105	0.14356	0.15371	0.16185	0.16835	0.17766	0.18357
1.0	0.02794	0.05471	0.07938	0.10129	0.12018	0.13605	0.14914	0.15978	0.16835	0.17522	0.18508	0.19139
1.2	0.02926	0.05733	0.08323	0.10631	0.12626	0.14309	0.15703	0.16843	0.17766	0.18508	0.19584	0.20278
1.4	0.03007	0.05894	0.08561	0.10941	0.13003	0.14749	0.16199	0.17389	0.18357	0.19139	0.20278	0.21020
1.6	0.03058	0.05994	0.08709	0.11135	0.13241	0.15028	0.16515	0.17739	0.18737	0.19546	0.20731	0.21510
1.8	0.03090	0.06058	0.08804	0.11260	0.13395	0.15207	0.16720	0.17967	0.18986	0.19814	0.21032	0.21836
2.0	0.03111	0.06100	0.08867	0.11342	0.13496	0.15326	0.16856	0.18119	0.19152	0.19994	0.21235	0.22058
2.5	0.03138	0.06155	0.08948	0.11450	0.13628	0.15483	0.17036	0.18321	0.19375	0.20236	0.21512	0.22364
3.0	0.03150	0.06178	0.08982	0.11495	0.13684	0.15550	0.17113	0.18407	0.19470	0.20341	0.21633	0.22499
4.0	0.03158	0.06194	0.09007	0.11527	0.13724	0.15598	0.17168	0.18469	0.19540	0.20417	0.21722	0.22600
5.0	0.03160	0.06199	0.09014	0.11537	0.13737	0.15612	0.17185	0.18488	0.19561	0.20440	0.21749	0.22632
6.0	0.03161	0.06201	0.09017	0.11541	0.13741	0.15617	0.17191	0.18496	0.19569	0.20449	0.21760	0.22644
8.0	0.03162	0.06202	0.09018	0.11543	0.13744	0.15621	0.17195	0.18500	0.19574	0.20455	0.21767	0.22652
10.0	0.03162	0.06202	0.09019	0.11544	0.13745	0.15622	0.17196	0.18502	0.19576	0.20457	0.21769	0.22654
∞	0.03162	0.06202	0.09019	0.11544	0.13745	0.15623	0.17197	0.18502	0.19577	0.20458	0.21770	0.22656