

SEVENTH EDITION

ENGINEERING ECONOMY



LELAND BLANK • ANTHONY TARQUIN

This seventh edition of *Engineering Economy* presents a crisp, bold new design using color to highlight important concepts, terms, and equations. There are new features, new topics, and new online tools; yet no compromise on coverage, examples, or the well-accepted writing style of previous editions. Check out the Preface for an overview of What's New and What's Best. Examples, problems, and case studies target many engineering challenges including energy, ethics, the environment, and the world's changing economics.

Use ONLINE PRESENTATIONS to review key topics
Many sections can be reviewed by viewing abbreviated, downloadable modules that feature animated, voice-over PowerPoint slides. Presentations are keyed to text sections via an icon in the margin.

Apply ETHICS to engineering economy problems
The increasingly important role of ethics and ethical practices in engineering is treated in three different chapters throughout the text—definitions and fundamental concepts of professional ethics, the dilemmas of ethical practices in public projects, and ethical challenges in cost estimation and operating cost analysis.

Utilize our SPREADSHEETS

The integration of spreadsheets has earned this text high praise. Screen shots and function labels lead a student through the use of Microsoft Excel™ for improved understanding of spreadsheet-based problem-solving techniques. The use of spreadsheets is balanced with hand solutions in examples and problems.

Learn more with PROGRESSIVE EXAMPLES

Several chapters feature an in-depth problem statement at the beginning of the chapter with follow-up examples throughout the sections of the chapter. Expanded information and solution approaches help the learner understand different techniques of decision making on real-world problems.

Increase engagement through NEW TOPICS

Several new and timely topics are treated in this edition—for example, service-sector projects, real options analysis for staged decisions, engineering ethics, and value-added taxes.

Promote teamwork with our CASE STUDIES

Integrally tied to the learning outcomes of a chapter, improved case studies challenge students to look at real-world issues in areas such as energy, ethics, financing, and investing. Each case includes a background statement, an information section, and several exercises.

Count on an extensive variety of END-OF-CHAPTER EXERCISES

Approximately 90% of the end-of-chapter problems have been revised or are new to this edition. Concept, numerical, hand, and spreadsheet problems are available. The Fundamentals of Engineering (FE) Exam review questions can double as additional short problems or as a review of topics and techniques in the chapter.

Rely on flexible topic ORGANIZATION

Teach the chapters in sequential order or rearrange the chapters to suit your needs. While the chapters are progressive and the flow between topics is smooth, a majority of the chapters can stand alone, allowing great flexibility. Organization of the material provides adaptability to a wide range of teaching pedagogies and depth of coverage.

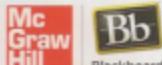
Make the most of ONLINE RESOURCES

Visit the website at www.mhhe.com/blank for a rich array of resources for students and instructors. The website contains general book information, downloadable, animated voice-over slide presentations, sample PowerPoint lecture slides, spreadsheet exercises, additional FE Exam review questions, and solutions to end-of-chapter problems (for instructors only)—with more to come.

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Format for Spreadsheet Functions on Excel®

	Contents of ()
Present worth:	
= PV(<i>i%</i> , <i>n</i> , <i>A</i> , <i>F</i>)	for constant <i>A</i> series; single <i>F</i> value
= NPV(<i>i%</i> ,second_cell:last_cell) + first_cell	for varying cash flow series
Future worth:	
= FV(<i>i%</i> , <i>n</i> , <i>A</i> , <i>P</i>)	for constant <i>A</i> series; single <i>P</i> value
Annual worth:	
= PMT(<i>i%</i> , <i>n</i> , <i>P</i> , <i>F</i>)	for single amounts with no <i>A</i> series
= PMT(<i>i%</i> , <i>n</i> ,NPV)	to find AW from NPV; embed NPV function
Number of periods (years):	
= NPER(<i>i%</i> , <i>A</i> , <i>P</i> , <i>F</i>)	for constant <i>A</i> series; single <i>P</i> and <i>F</i>
(Note: The PV, FV, and PMT functions change the sense of the sign. Place a minus in front of the function to retain the same sign.)	
Rate of return:	
= RATE(<i>n</i> , <i>A</i> , <i>P</i> , <i>F</i>)	for constant <i>A</i> series; single <i>P</i> and <i>F</i>
= IRR(first_cell:last_cell)	for varying cash flow series
Interest rate:	
= EFFECT(<i>r%</i> , <i>m</i>)	for nominal <i>r</i> , compounded <i>m</i> times per period
= NOMINAL(<i>i%</i> , <i>m</i>)	for effective annual <i>i</i> , compounded <i>m</i> times per year
Depreciation:	
= SLN(<i>P</i> , <i>S</i> , <i>n</i>)	straight line depreciation for each period
= DDB(<i>P</i> , <i>S</i> , <i>n</i> , <i>t</i> , <i>d</i>)	double declining balance depreciation for period <i>t</i> at rate <i>d</i> (optional)
= DB(<i>P</i> , <i>S</i> , <i>n</i> , <i>t</i>)	declining balance, rate determined by the function
= VBD(<i>P</i> ,0, <i>n</i> ,MAX(0, <i>t</i> -1.5), MIN(<i>n</i> , <i>t</i> -0.5), <i>d</i>)	MACRS depreciation for year <i>t</i> at rate <i>d</i> for DDB or DB method
Logical IF function:	
= IF(logical_test,value_if_true,value_if_false)	for logical two-branch operations

Relations for Discrete Cash Flows with End-of-Period Compounding

Type	Find/Given	Factor Notation and Formula	Relation	Sample Cash Flow Diagram
Single Amount	F/P Compound amount	$(F/P,i,n) = (1 + i)^n$	$F = P(F/P,i,n)$	
	P/F Present worth	$(P/F,i,n) = \frac{1}{(1 + i)^n}$	$P = F(P/F,i,n)$ (Sec. 2.1)	
Uniform Series	P/A Present worth	$(P/A,i,n) = \frac{(1 + i)^n - 1}{i(1 + i)^n}$	$P = A(P/A,i,n)$	
	A/P Capital recovery	$(A/P,i,n) = \frac{i(1 + i)^n}{(1 + i)^n - 1}$	$A = P(A/P,i,n)$ (Sec. 2.2)	
	F/A Compound amount	$(F/A,i,n) = \frac{(1 + i)^n - 1}{i}$	$F = A(F/A,i,n)$	
	A/F Sinking fund	$(A/F,i,n) = \frac{i}{(1 + i)^n - 1}$	$A = F(A/F,i,n)$ (Sec. 2.3)	
Arithmetic Gradient	P_G/G Present worth	$(P/G,i,n) = \frac{(1 + i)^n - in - 1}{i^2(1 + i)^n}$	$P_G = G(P/G,i,n)$	
	A_G/G Uniform series	$(A/G,i,n) = \frac{1}{i} - \frac{n}{(1 + i)^n - 1}$ (Gradient only)	$A_G = G(A/G,i,n)$ (Sec. 2.5)	
Geometric Gradient	P_g/A_1 and g Present worth	$P_g = \begin{cases} \frac{A_1 [1 - (1 + g)^n]}{i - g} & g \neq i \\ A_1 \frac{n}{1 + i} & g = i \end{cases}$ (Gradient and base A_1)	$(Sec. 2.6)$	

Seventh Edition



ENGINEERING ECONOMY



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This book is dedicated to Dr. Frank W. Sheppard, Jr. His lifelong commitment to education, fair financial practices, international outreach, and family values has been an inspiration to many—one person at a time.

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PREFACE TO SEVENTH EDITION

This edition includes the time-tested approach and topics of previous editions and introduces significantly new print and electronic features useful to learning about and successfully applying the exciting field of engineering economics. Money makes a huge difference in the life of a corporation, an individual, and a government. Learning to understand, analyze, and manage the money side of any project is vital to its success. To be professionally successful, every engineer must be able to deal with the time value of money, economic facts, inflation, cost estimation, tax considerations, as well as spreadsheet and calculator use. This book is a great help to the learner and the instructor in accomplishing these goals by using easy-to-understand language, simple graphics, and online features.

What's New and What's Best ● ● ●

This seventh edition is full of new information and features. Plus the supporting online materials are new and updated to enhance the teaching and learning experience.

New topics:

- Ethics and the economics of engineering
- Service sector projects and their evaluation
- Real options development and analysis
- Value-added taxes and how they work
- Multiple rates of return and ways to eliminate them using spreadsheets
- No tabulated factors needed for equivalence computations (Appendix D)

New features in print and online:

- Totally new design to highlight important terms, concepts, and decision guidelines
- Progressive examples that continue throughout a chapter
- Downloadable online presentations featuring voice-over slides and animation
- Vital concepts and guidelines identified in margins; brief descriptions available (Appendix E)
- Fresh spreadsheet displays with on-image comments and function details
- Case studies (21 of them) ranging in topics from ethics to energy to simulation

Retained features:

- Many end-of-chapter problems (over 90% are new or revised)
- Easy-to-read language to enhance understanding in a variety of course environments
- Fundamentals of Engineering (FE) Exam review questions that double as additional or review problems for quizzes and tests
- Hand and spreadsheet solutions presented for many examples
- Flexible chapter ordering after fundamental topics are understood
- Complete solutions manual available online (with access approval for instructors)

How to Use This Text ● ● ●

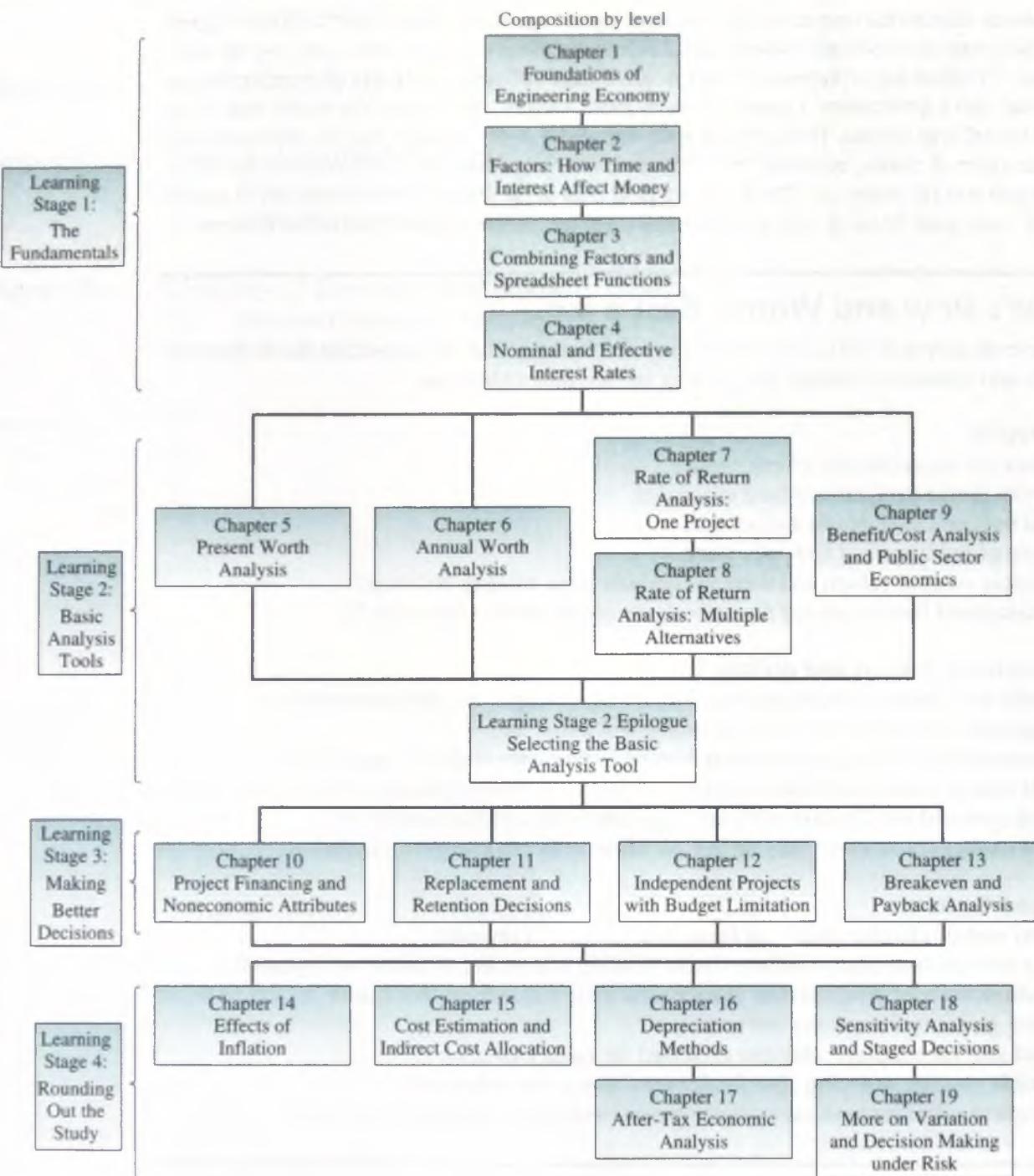
This textbook is best suited for a one-semester or one-quarter undergraduate course. Students should be at the sophomore level or above with a basic understanding of engineering concepts and terminology. A course in calculus is not necessary; however, knowledge of the concepts in advanced mathematics and elementary probability will make the topics more meaningful.

Practitioners and professional engineers who need a refresher in economic analysis and cost estimation will find this book very useful as a reference document as well as a learning medium.

Chapter Organization and Coverage Options ● ● ●

The textbook contains 19 chapters arranged into four learning stages, *as indicated in the flowchart* on the next page, and five appendices. Each chapter starts with a statement of purpose and a specific learning outcome (ABET style) for each section. Chapters include a summary, numerous

CHAPTERS IN EACH LEARNING STAGE



end-of-chapter problems (essay and numerical), multiple-choice problems useful for course review and FE Exam preparation, and a case study.

The appendices are important elements of learning for this text:

Appendix A Spreadsheet layout and functions (Excel is featured)

Appendix B Accounting reports and business ratios

Appendix C Code of Ethics for Engineers (from NSPE)

Appendix D Equivalence computations using calculators and geometric series; no tables

Appendix E Concepts, guidelines, terms, and symbols for engineering economics

There is considerable flexibility in the sequencing of topics and chapters once the first six chapters are covered, *as shown in the progression graphic* on the next page. If the course is designed to emphasize sensitivity and risk analysis, Chapters 18 and 19 can be covered immediately

CHAPTER AND TOPIC PROGRESSION OPTIONS

Topics may be introduced at the point indicated or any point thereafter
 (Alternative entry points are indicated by)

Numerical progression through chapters	Inflation	Cost Estimation	Taxes and Depreciation	Sensitivity, Staged Decisions, and Risk
--	-----------	-----------------	------------------------	---

1. Foundations
2. Factors
3. More Factors
4. Effective i
5. Present Worth
6. Annual Worth

7. Rate of Return
8. More ROR
9. Benefit/Cost

10. Financing and Noneconomic Attributes
11. Replacement
12. Capital Budgeting
13. Breakeven and Payback

14. Inflation

15. Estimation

16. Depreciation

17. After-Tax

18. Sensitivity, Decision Trees, and Real Options

19. Risk and Simulation

after Learning Stage 2 (Chapter 9) is completed. If depreciation and tax emphasis are vitally important to the goals of the course, Chapters 16 and 17 can be covered once Chapter 6 (annual worth) is completed. The progression graphic can help in the design of the course content and topic ordering.

SAMPLE OF RESOURCES FOR

LEARNING OUTCOMES

Each chapter begins with a purpose, list of topics, and learning outcomes (ABET style) for each corresponding section. This behavioral-based approach sensitizes the reader to what is ahead, leading to improved understanding and learning.

LEARNING OUTCOMES		
Purpose: Use multiple factors and spreadsheet functions to find equivalent amounts for cash flows that have nonstandard placement.		
SECTION	TOPIC	LEARNING OUTCOME
3.1	Shifted series	<ul style="list-style-type: none">Determine the P, F or A values of a series starting at a time other than period 1.
3.2	Shifted series and single cash flows	<ul style="list-style-type: none">Determine the P, F, or A values of a shifted series and randomly placed single cash flows.
3.3	Shifted gradients	<ul style="list-style-type: none">Make equivalence calculations for shifted arithmetic or geometric gradient series that increase or decrease in size of cash flows.

 Time value of money

It is a well-known fact that money makes money. The time value of money explains the change in the amount of money over time for funds that are owned (invested) or owed (borrowed). This is the most important concept in engineering economy.

CONCEPTS AND GUIDELINES

To highlight the fundamental building blocks of the course, a checkmark and title in the margin call attention to particularly important concepts and decision-making guidelines. Appendix E includes a brief description of each fundamental concept.

IN-CHAPTER EXAMPLES

Numerous in-chapter examples throughout the book reinforce the basic concepts and make understanding easier. Where appropriate, the example is solved using separately marked hand and spreadsheet solutions.

EXAMPLE 4.6

A dot-com company plans to place money in a new venture capital fund that currently returns 18% per year, compounded daily. What effective rate is this (a) yearly and (b) semiannually?

Solution

(a) Use Equation [4.7], with $r = 0.18$ and $m = 365$.

$$\text{Effective } i\% \text{ per year} = \left(1 + \frac{0.18}{365}\right)^{365} - 1 = 19.716\%$$

(b) Here $r = 0.09$ per 6 months and $m = 182$ days.

$$\text{Effective } i\% \text{ per 6 months} = \left(1 + \frac{0.09}{182}\right)^{182} - 1 = 9.415\%$$

PROGRESSIVE EXAMPLES

Several chapters include a progressive example—a more detailed problem statement introduced at the beginning of the chapter and expanded upon throughout the chapter in specially marked examples. This approach illustrates different techniques and some increasingly complex aspects of a real-world problem.

PE

Water for Semiconductor Manufacturing Case: The worldwide contribution of semiconductor sales is about \$250 billion per year, or about 10% of the world's GDP (gross domestic product). This industry produces the microchips used in many of the communication, entertainment, transportation, and computing devices we use every day. Depending upon the type and size of fabrication plant (fab), the need for ultrapure water (UPW) to manufacture these tiny integrated circuits is high, ranging from 500 to 2000 gpm (gallons per minute). Ultrapure water is obtained by special processes that commonly include reverse osmosis/deionizing resin bed technologies. Potable water obtained from purifying seawater or brackish groundwater may cost from \$2 to \$3 per 1000 gallons, but to obtain UPW on-site for semiconductor manufacturing may cost an additional \$1 to \$3 per 1000 gallons.

A fab costs upward of \$2.5 billion to construct, with approximately 1% of this total, or \$25 million, required to provide the ultrapure water needed, including the necessary wastewater and recycling equipment.

A newcomer to the industry, Angular Enterprises, has estimated the cost profiles for two options to supply its anticipated fab with water. It is fortunate to

have the option of desalinated seawater or purified groundwater sources in the location chosen for its new fab. The initial cost estimates for the UPW system are given below.

Source	Seawater (\$)	Groundwater (\$)
Equipment first cost, \$M	-20	-22
AOC, \$M per year	-0.5	-0.3
Salvage value, % of first cost	5	10
Cost of UPW, \$ per 1000 gallons	4	5

Angular has made some initial estimates for the UPW system.

Life of UPW equipment	10 years
UPW needs	1500 gpm
Operating time	16 hours per day for 250 days per year

This case is used in the following topics (Sections) and problems of this chapter:

- PW analysis of equal-life alternatives (Section 5.2)
- PW analysis of different-life alternatives (Section 5.3)
- Capitalized cost analysis (Section 5.5)
- Problems 5.20 and 5.34

INSTRUCTORS AND STUDENTS

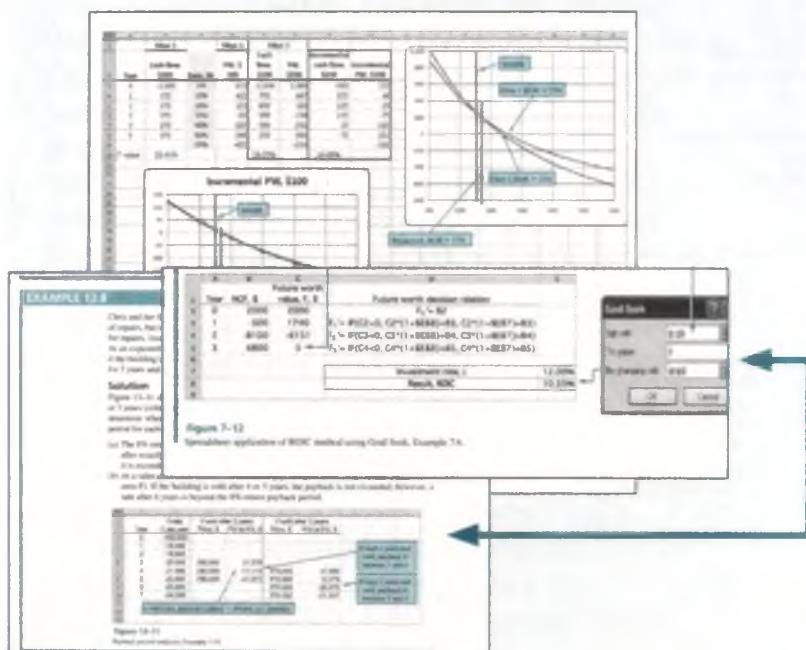
ONLINE PRESENTATIONS

An icon in the margin indicates the availability of an animated voice-over slide presentation that summarizes the material in the section and provides a brief example for learners who need a review or prefer video-based materials. Presentations are keyed to the sections of the text.

3.1 Calculations for Uniform Series That Are Shifted ● ● ●

When a uniform series begins at a time other than at the end of period 1, it is called a **shifted series**. In this case several methods can be used to find the equivalent present worth P . For example, P of the uniform series shown in Figure 3–1 could be determined by any of the following methods:

- Use the P/F factor to find the present worth of each disbursement at year 0 and add them.
- Use the F/P factor to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total, using $P = F(P/F,i,13)$.
- Use the F/A factor to find the future amount $F = A(F/A,i,10)$, and then compute the present worth, using $P = F(P/F,i,13)$.
- Use the P/A factor to compute the "present worth" $P_3 = A(P/A,i,10)$ (which will be located in year 3, not year 0), and then find the present worth in year 0 by using the $(P/F,i,3)$ factor.



SPREADSHEETS

The text integrates spreadsheets to show how easy they are to use in solving virtually any type of engineering economic analysis problem. Cell tags or full cells detail built-in functions and relations developed to solve a specific problem.

FE EXAM AND COURSE REVIEWS

Each chapter concludes with several multiple-choice, FE Exam–style problems that provide a simplified review of chapter material. Additionally, these problems cover topics for test reviews and homework assignments.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

8.38 When conducting a rate of return (ROR) analysis involving multiple mutually exclusive alternatives, the first step is to:

- (a) Rank the alternatives according to decreasing initial investment cost
- (b) Rank the alternatives according to increasing initial investment cost
- (c) Calculate the present worth of each alternative using the MARR
- (d) Find the LCM between all of the alternatives

8.39 In comparing mutually exclusive alternatives by the ROR method, you should:

- (a) Find the ROR of each alternative and pick the one with the *highest* ROR
- (b) \$2500
- (c) \$3500
- (d) \$6000
- (e) \$8000

8.43 For these alternatives, the sum of the incremental cash flows is:

Year	A	B
0	−10,000	−14,000
1	+2,500	+4,000
2	+2,500	+4,000
3	+2,500	+4,000
4	+2,500	+4,000
5	+2,500	+4,000

CASE STUDY

RENEWABLE ENERGY SOURCES FOR ELECTRICITY GENERATION

Background

Pedernales Electric Cooperative (PEC) is the largest member-owned electric co-op in the United States with over 232,000 meters in 12 Central Texas counties. PEC has a capacity of approximately 1300 MW (megawatts) of power, of which 277 MW, or about 21%, is from renewable sources. The latest addition is 60 MW of power from a wind farm in south Texas close to the city of Corpus Christi. A constant question is how much of PEC's generation capacity should be from renewable sources, especially given the environmental issues with coal-generated electricity and the rising costs of hydrocarbon fuels.

Wind and nuclear sources are the current consideration for the PEC leadership as Texas is increasing its generation by nuclear power and the state is the national leader in wind farm-produced electricity.

Consider yourself a member of the board of directors of PEC. You are an engineer who has been newly elected by the PEC membership to serve a 3-year term as a director-at-large. As such, you do not represent a specific district within the entire service area; all other directors do represent a specific district. You have many questions about the operations of PEC, plus you are interested in the economic and societal benefits of pursuing more renewable source generation capacity.

Information

Here are some data that you have obtained. The information is sketchy, as this point, and the numbers are very approximate. Electricity generation cost estimates are national in scope, not PEC-specific, and are provided in cents per kilowatt-hour (¢/kWh).

Fuel Source	Generation Cost, ¢/kWh	
	Likely Range	Reasonable Average
Coal	4 to 9	7.4
Natural gas	4 to 10.5	8.6
Wind	4.8 to 9.1	8.2
Solar	4.5 to 15.5	8.8

National average cost of electricity to residential customers: 11¢/kWh

PEC average cost to residential customers: 10.27 ¢/kWh (from primary sources) and 10.92 ¢/kWh (renewable sources)

Expected life of a generation facility: 20 to 40 years (it is likely closer to 20 than 40)

Time to construct a facility: 2 to 5 years

Capital cost to build a generation facility: \$900 to \$1500 per kW

You have also learned that the PEC staff uses the well-recognized *levelized energy cost* (LEC) method to determine the price of electricity that must be charged to customers to break even. The formula takes into account the capital cost of the generation facilities, the cost of capital of borrowed money, annual maintenance and operation (M&O) costs, and the expected life of the facility. The LEC formula, expressed in dollars per kWh for ($t = 1, 2, \dots, n$), is

$$\text{LEC} = \frac{\sum_{t=1}^n \frac{P_t + A_t + C_t}{(1+i)^t}}{\sum_{t=1}^n \frac{E_t}{(1+i)^t}}$$

where P_t = capital investments made in year t

A_t = annual maintenance and operating (M&O) costs for year t

C_t = fuel costs for year t

E_t = amount of electricity generated in year t

n = expected life of facility

i = discount rate (cost of capital)

Case Study Exercises

- If you wanted to know more about the new arrangement with the wind farm in south Texas for the additional 60 MW per year, what types of questions would you ask of a staff member in your first meeting with him or her?
- Much of the current generation capacity of PEC facilities utilizes coal and natural gas as the primary fuel source. What about the ethical aspects of the government's allowance for these plants to continue polluting the atmosphere with the emissions that may cause health problems for citizens and further the effects of global warming? What types of regulations, if any, should be developed for PEC (and other generators) to follow in the future?

New and updated case studies at the end of most chapters present real-world, in-depth treatments and exercises in the engineering profession. Each case includes a background, relevant information, and an exercise section.

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If you discover errors that require correction in the next printing of the textbook or in updates of the online resources, please contact us. We hope you find the contents of this edition helpful in your academic and professional activities.

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The Fundamentals

LEARNING STAGE 1 The Fundamentals

CHAPTER 1

Foundations of
Engineering Economy

CHAPTER 2

Factors: How Time
and Interest Affect
Money

CHAPTER 3

Combining Factors
and Spreadsheet
Functions

CHAPTER 4

Nominal and Effective
Interest Rates

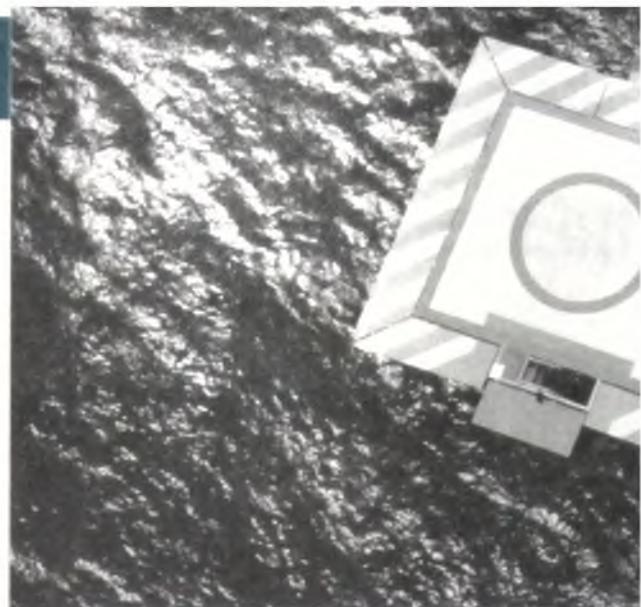
The fundamentals of engineering economy are introduced in these chapters. When you have completed stage 1, you will be able to understand and work problems that account for the **time value of money**, **cash flows** occurring at different times with different amounts, and **equivalence** at different interest rates. The techniques you master here form the basis of how an engineer in any discipline can take **economic value** into account in virtually any project environment.

The factors commonly used in all engineering economy computations are introduced and applied here. Combinations of these factors assist in moving monetary values forward and backward through time and at different interest rates. Also, after these chapters, you should be comfortable using many of the spreadsheet functions.

Many of the terms common to economic decision making are introduced in learning stage 1 and used in later chapters. A checkmark icon in the margin indicates that a new **concept or guideline** is introduced at this point.



Foundations of Engineering Economy



LEARNING OUTCOMES

Purpose: Understand and apply fundamental concepts and use the terminology of engineering economics.

SECTION	TOPIC	LEARNING OUTCOME
1.1	Description and role	<ul style="list-style-type: none">Define engineering economics and describe its role in decision making.
1.2	Engineering economy study approach	<ul style="list-style-type: none">Understand and identify the steps in an engineering economy study.
1.3	Ethics and economics	<ul style="list-style-type: none">Identify areas in which economic decisions can present questionable ethics.
1.4	Interest rate	<ul style="list-style-type: none">Perform calculations for interest rates and rates of return.
1.5	Terms and symbols	<ul style="list-style-type: none">Identify and use engineering economic terminology and symbols.
1.6	Cash flows	<ul style="list-style-type: none">Understand cash flows and how to graphically represent them.
1.7	Economic equivalence	<ul style="list-style-type: none">Describe and calculate economic equivalence.
1.8	Simple and compound interest	<ul style="list-style-type: none">Calculate simple and compound interest amounts for one or more time periods.
1.9	MARR and opportunity cost	<ul style="list-style-type: none">State the meaning and role of Minimum Attractive Rate of Return (MARR) and opportunity costs.
1.10	Spreadsheet functions	<ul style="list-style-type: none">Identify and use some Excel functions commonly applied in engineering economics.

The need for engineering economy is primarily motivated by the work that engineers do in performing analyses, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of making decisions. These decisions involve the fundamental elements of cash flows of money, time, and interest rates. This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

1.1 Engineering Economics: Description and Role in Decision Making ● ● ●



Decisions are made routinely to choose one alternative over another by individuals in everyday life; by engineers on the job; by managers who supervise the activities of others; by corporate presidents who operate a business; and by government officials who work for the public good. Most decisions involve money, called **capital** or **capital funds**, which is usually limited in amount. The decision of where and how to invest this limited capital is motivated by a primary goal of **adding value** as future, anticipated results of the selected alternative are realized. Engineers play a vital role in capital investment decisions based upon their ability and experience to design, analyze, and synthesize. The factors upon which a decision is based are commonly a combination of economic and noneconomic elements. Engineering economy deals with the economic factors. By definition,

Engineering economy involves formulating, estimating, and evaluating the expected economic outcomes of alternatives designed to accomplish a defined purpose. Mathematical techniques simplify the economic evaluation of alternatives.

Because the formulas and techniques used in engineering economics are applicable to all types of money matters, they are equally useful in business and government, as well as for individuals. Therefore, besides applications to projects in your future jobs, what you learn from this book and in this course may well offer you an economic analysis tool for making personal decisions such as car purchases, house purchases, major purchases on credit, e.g., furniture, appliances, and electronics.

Other terms that mean the same as *engineering economy* are *engineering economic analysis*, *capital allocation study*, *economic analysis*, and similar descriptors.

People make decisions; computers, mathematics, concepts, and guidelines assist people in their decision-making process. Since most decisions affect what will be done, the time frame of engineering economy is primarily the **future**. Therefore, the numbers used in engineering economy are **best estimates of what is expected to occur**. The estimates and the decision usually involve four essential elements:

Cash flows

Times of occurrence of cash flows

Interest rates for time value of money

Measure of economic worth for selecting an alternative

Since the estimates of cash flow amounts and timing are about the future, they will be somewhat different than what is actually observed, due to changing circumstances and unplanned events. In short, the variation between an amount or time estimated now and that observed in the future is caused by the stochastic (random) nature of all economic events. **Sensitivity analysis** is utilized to determine how a decision might change according to varying estimates, especially those expected to vary widely. Example 1.1 illustrates the fundamental nature of variation in estimates and how this variation may be included in the analysis at a very basic level.

EXAMPLE 1.1

An engineer is performing an analysis of warranty costs for drive train repairs within the first year of ownership of luxury cars purchased in the United States. He found the average cost (to the nearest dollar) to be \$570 per repair from data taken over a 5-year period.

Year	2006	2007	2008	2009	2010
Average Cost, \$/repair	525	430	619	650	625

What range of repair costs should the engineer use to ensure that the analysis is sensitive to changing warranty costs?

Solution

At first glance the range should be approximately -25% to $+15\%$ of the \$570 average cost to include the low of \$430 and high of \$650. However, the last 3 years of costs are higher and more consistent with an average of \$631. The observed values are approximately $\pm 3\%$ of this more recent average.

If the analysis is to use the most recent data and trends, a range of, say, $\pm 5\%$ of \$630 is recommended. If, however, the analysis is to be more inclusive of historical data and trends, a range of, say, $\pm 20\%$ or $\pm 25\%$ of \$570 is recommended.

The criterion used to select an alternative in engineering economy for a specific set of estimates is called a **measure of worth**. The measures developed and used in this text are

Present worth (PW)	Future worth (FW)	Annual worth (AW)
Rate of return (ROR)	Benefit/cost (B/C)	Capitalized cost (CC)
Payback period	Economic value added (EVA)	Cost Effectiveness

All these measures of worth account for the fact that money makes money over time. This is the concept of the **time value of money**.



Time value of money

It is a well-known fact that money **makes** money. The time value of money explains the change in the amount of money **over time** for funds that are owned (invested) or owed (borrowed). This is the most important concept in engineering economy.

The time value of money is very obvious in the world of economics. If we decide to invest capital (money) in a project today, we inherently expect to have more money in the future than we invested. If we borrow money today, in one form or another, we expect to return the original amount plus some additional amount of money.

Engineering economics is equally well suited for the future and for the **analysis of past cash flows** in order to determine if a specific criterion (measure of worth) was attained. For example, assume you invested \$4975 exactly 3 years ago in 53 shares of IBM stock as traded on the New York Stock Exchange (NYSE) at \$93.86 per share. You expect to make 8% per year appreciation, not considering any dividends that IBM may declare. A quick check of the share value shows it is currently worth \$127.25 per share for a total of \$6744.25. This increase in value represents a rate of return of 10.67% per year. (These type of calculations are explained later.) This past investment has well exceeded the 8% per year criterion over the last 3 years.

1.2 Performing an Engineering Economy Study ● ● ●

An engineering economy study involves many elements: problem identification, definition of the objective, cash flow estimation, financial analysis, and decision making. Implementing a structured procedure is the best approach to select the best solution to the problem.

The steps in an engineering economy study are as follows:

1. Identify and understand the problem; identify the objective of the project.
2. Collect relevant, available data and define viable solution alternatives.
3. Make realistic cash flow estimates.
4. Identify an economic measure of worth criterion for decision making.

5. Evaluate each alternative; consider noneconomic factors; use sensitivity analysis as needed.
6. Select the best alternative.
7. Implement the solution and monitor the results.

Technically, the last step is not part of the economy study, but it is, of course, a step needed to meet the project objective. There may be occasions when the best economic alternative requires more capital funds than are available, or significant noneconomic factors preclude the most economic alternative from being chosen. Accordingly, steps 5 and 6 may result in selection of an alternative different from the economically best one. Also, sometimes more than one project may be selected and implemented. This occurs when projects are independent of one another. In this case, steps 5 through 7 vary from those above. Figure 1–1 illustrates the steps above for one alternative. Descriptions of several of the elements in the steps are important to understand.

Problem Description and Objective Statement A succinct statement of the problem and primary objective(s) is very important to the formation of an alternative solution. As an illustration, assume the problem is that a coal-fueled power plant must be shut down by 2015 due to the production of excessive sulfur dioxide. The objectives may be to generate the forecasted electricity

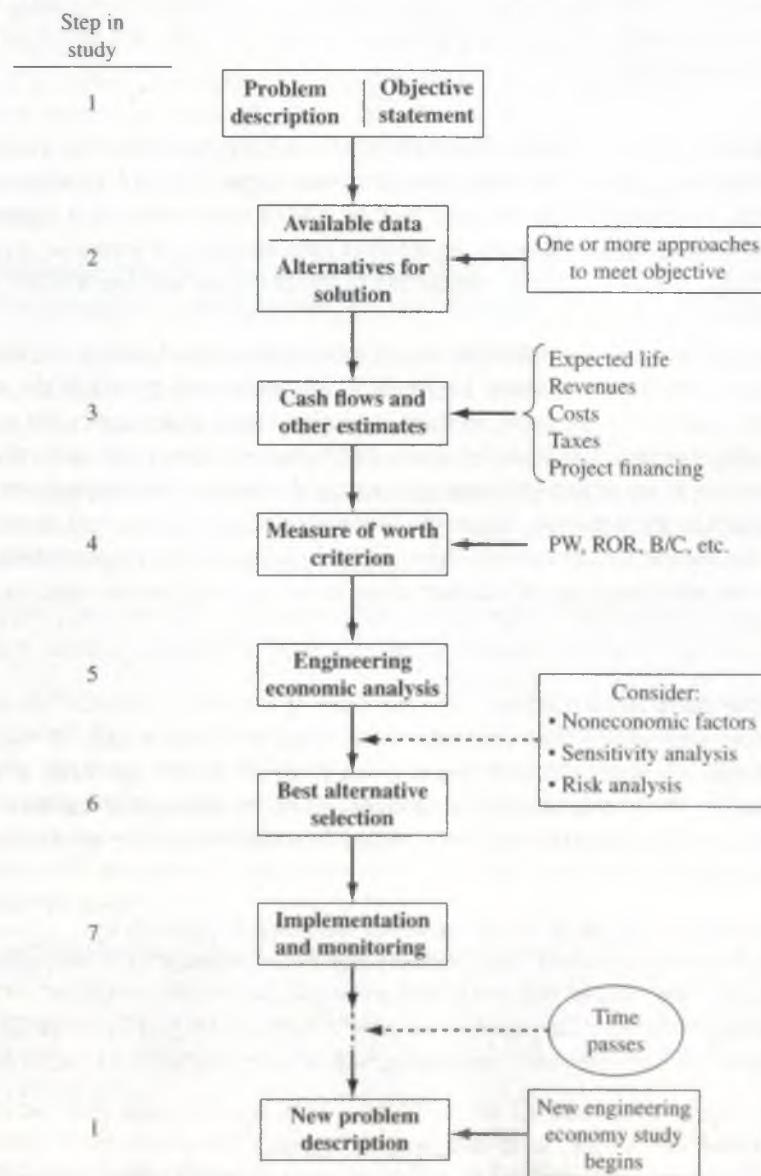


Figure 1–1
Steps in an engineering economy study.

needed for 2015 and beyond, plus to not exceed all the projected emission allowances in these future years.

Alternatives These are stand-alone descriptions of viable solutions to problems that can meet the objectives. Words, pictures, graphs, equipment and service descriptions, simulations, etc. define each alternative. The best estimates for parameters are also part of the alternative. Some parameters include equipment first cost, expected life, salvage value (estimated trade-in, resale, or market value), and annual operating cost (AOC), which can also be termed *maintenance and operating (M&O) cost*, and subcontract cost for specific services. If changes in income (revenue) may occur, this parameter must be estimated.

Detailing all viable alternatives at this stage is crucial. For example, if two alternatives are described and analyzed, one will likely be selected and implementation initiated. If a third, more attractive method that was available is later recognized, a wrong decision was made.

Cash Flows All cash flows are estimated for each alternative. Since these are future expenditures and revenues, the results of step 3 usually prove to be inaccurate when an alternative is actually in place and operating. When cash flow estimates for specific parameters are expected to vary significantly from a *point estimate* made now, risk and sensitivity analyses (step 5) are needed to improve the chances of selecting the best alternative. Sizable variation is usually expected in estimates of revenues, AOC, salvage values, and subcontractor costs. Estimation of costs is discussed in Chapter 15, and the elements of variation (risk) and sensitivity analysis are included throughout the text.

Engineering Economy Analysis The techniques and computations that you will learn and use throughout this text utilize the cash flow estimates, time value of money, and a selected measure of worth. The result of the analysis will be one or more numerical values; this can be in one of several terms, such as money, an interest rate, number of years, or a probability. In the end, a selected measure of worth mentioned in the previous section will be used to select the best alternative.

Before an economic analysis technique is applied to the cash flows, some decisions about what to include in the analysis must be made. Two important possibilities are taxes and inflation. Federal, state or provincial, county, and city taxes will impact the costs of every alternative. An after-tax analysis includes some additional estimates and methods compared to a before-tax analysis. If taxes and inflation are expected to impact all alternatives equally, they may be disregarded in the analysis. However, if the size of these projected costs is important, taxes and inflation should be considered. Also, if the impact of inflation over time is important to the decision, an additional set of computations must be added to the analysis; Chapter 14 covers the details.

Selection of the Best Alternative The measure of worth is a primary basis for selecting the best economic alternative. For example, if alternative A has a rate of return (ROR) of 15.2% per year and alternative B will result in an ROR of 16.9% per year, B is better economically. However, there can always be **noneconomic or intangible factors** that must be considered and that may alter the decision. There are many possible noneconomic factors; some typical ones are

- Market pressures, such as need for an increased international presence
- Availability of certain resources, e.g., skilled labor force, water, power, tax incentives
- Government laws that dictate safety, environmental, legal, or other aspects
- Corporate management's or the board of director's interest in a particular alternative
- Goodwill offered by an alternative toward a group: employees, union, county, etc.

As indicated in Figure 1–1, once all the economic, noneconomic, and risk factors have been evaluated, a final decision of the “best” alternative is made.

At times, only one viable alternative is identified. In this case, the **do-nothing (DN) alternative** may be chosen provided the measure of worth and other factors result in the alternative being a poor choice. The do-nothing alternative maintains the status quo.

Whether we are aware of it or not, we use criteria every day to choose between alternatives. For example, when you drive to campus, you decide to take the “best” route. But how did you define *best*? Was the best route the safest, shortest, fastest, cheapest, most scenic, or what? Obviously, depending upon which criterion or combination of criteria is used to identify the best, a different route might be selected each time. In economic analysis, **financial units (dollars or other currency)** are generally used as the tangible basis for evaluation. Thus, when there are several ways of accomplishing a stated objective, the alternative with the lowest overall cost or highest overall net income is selected.

1.3 Professional Ethics and Economic Decisions ● ● ●

Many of the fundamentals of engineering ethics are intertwined with the roles of money and economics-based decisions in the making of professionally ethical judgments. Some of these integral connections are discussed here, plus sections in later chapters discuss additional aspects of ethics and economics. For example, Chapter 9, Benefit/Cost Analysis and Public Sector Economics, includes material on the ethics of public project contracts and public policy. Although it is very limited in scope and space, it is anticipated that this coverage of the important role of economics in engineering ethics will prompt further interest on the part of students and instructors of engineering economy.

The terms **morals** and **ethics** are commonly used interchangeably, yet they have slightly different interpretations. Morals usually relate to the underlying tenets that form the character and conduct of a person in judging right and wrong. Ethical practices can be evaluated by using a code of morals or **code of ethics** that forms the standards to guide decisions and actions of individuals and organizations in a profession, for example, electrical, chemical, mechanical, industrial, or civil engineering. There are several different levels and types of morals and ethics.

Universal or common morals These are fundamental moral beliefs held by virtually all people. Most people agree that to steal, murder, lie, or physically harm someone is wrong.

It is possible for **actions** and **intentions** to come into conflict concerning a common moral. Consider the World Trade Center buildings in New York City. After their collapse on September 11, 2001, it was apparent that the design was not sufficient to withstand the heat generated by the firestorm caused by the impact of an aircraft. The structural engineers who worked on the design surely did not have the intent to harm or kill occupants in the buildings. However, their design actions did not foresee this outcome as a measurable possibility. Did they violate the common moral belief of not doing harm to others or murdering?

Individual or personal morals These are the moral beliefs that a person has and maintains over time. These usually parallel the common morals in that stealing, lying, murdering, etc. are immoral acts.

It is quite possible that an individual strongly supports the common morals and has excellent personal morals, but these may conflict from time to time when decisions must be made. Consider the engineering student who genuinely believes that cheating is wrong. If he or she does not know how to work some test problems, but must make a certain minimum grade on the final exam to graduate, the decision to cheat or not on the final exam is an exercise in following or violating a personal moral.

Professional or engineering ethics Professionals in a specific discipline are guided in their decision making and performance of work activities by a formal standard or code. The code states the commonly accepted standards of honesty and integrity that each individual is expected to demonstrate in her or his practice. There are codes of ethics for medical doctors, attorneys, and, of course, engineers.

Although each engineering profession has its own code of ethics, the **Code of Ethics for Engineers** published by the National Society of Professional Engineers (NSPE) is very commonly used and quoted. This code, reprinted in its entirety in Appendix C, includes numerous sections that have direct or indirect economic and financial impact upon the designs, actions,

and decisions that engineers make in their professional dealings. Here are three examples from the Code:

“Engineers, in the fulfillment of their duties, shall hold paramount the *safety, health, and welfare of the public.*” (section I.1)

“Engineers shall *not accept financial or other considerations*, including free engineering designs, from material or equipment suppliers for specifying their product.” (section III.5.a)

“Engineers using designs supplied by a client recognize that the *designs remain the property of the client* and may not be duplicated by the engineer for others without express permission.” (section III.9.b)

As with common and personal morals, conflicts can easily rise in the mind of an engineer between his or her own ethics and that of the employing corporation. Consider a manufacturing engineer who has recently come to firmly disagree morally with war and its negative effects on human beings. Suppose the engineer has worked for years in a military defense contractor’s facility and does the detailed cost estimations and economic evaluations of producing fighter jets for the Air Force. The Code of Ethics for Engineers is silent on the ethics of producing and using war materiel. Although the employer and the engineer are not violating any ethics code, the engineer, as an individual, is stressed in this position. Like many people during a declining national economy, retention of this job is of paramount importance to the family and the engineer. Conflicts such as this can place individuals in real dilemmas with no or mostly unsatisfactory alternatives.

At first thought, it may not be apparent how activities related to engineering economics may present an ethical challenge to an individual, a company, or a public servant in government service. Many money-related situations, such as those that follow, can have ethical dimensions.

In the design stage:

- Safety factors are compromised to ensure that a price bid comes in as low as possible.
- Family or personal connections with individuals in a company offer unfair or insider information that allows costs to be cut in strategic areas of a project.
- A potential vendor offers specifications for company-specific equipment, and the design engineer does not have sufficient time to determine if this equipment will meet the needs of the project being designed and costed.

While the system is operating:

- Delayed or below-standard maintenance can be performed to save money when cost overruns exist in other segments of a project.
- Opportunities to purchase cheaper repair parts can save money for a subcontractor working on a fixed-price contract.
- Safety margins are compromised because of cost, personal inconvenience to workers, tight time schedules, etc.

A good example of the last item—safety is compromised while operating the system—is the situation that arose in 1984 in Bhopal, India (Martin and Schinzinger 2005, pp. 245–8). A Union Carbide plant manufacturing the highly toxic pesticide chemical methyl isocyanate (MIC) experienced a large gas leak from high-pressure tanks. Some 500,000 persons were exposed to inhalation of this deadly gas that burns moist parts of the body. There were 2500 to 3000 deaths within days, and over the following 10-year period, some 12,000 death claims and 870,000 personal injury claims were recorded. Although Union Carbide owned the facility, the Indian government had only Indian workers in the plant. Safety practices clearly eroded due to cost-cutting measures, insufficient repair parts, and reduction in personnel to save salary money. However, one of the surprising practices that caused unnecessary harm to workers was the fact that masks, gloves, and other protective gear were not worn by workers in close proximity to the tanks containing MIC. Why? Unlike in plants in the United States and other countries, there was no air conditioning in the Indian plant, resulting in high ambient temperatures in the facility.

Many ethical questions arise when corporations operate in international settings where the corporate rules, worker incentives, cultural practices, and costs in the home country differ from those in the host country. Often these ethical dilemmas are fundamentally based in the economics that provide cheaper labor, reduced raw material costs, less government oversight, and a host of

other cost-reducing factors. When an engineering economy study is performed, it is important for the engineer performing the study to consider all ethically related matters to ensure that the cost and revenue estimates reflect what is likely to happen once the project or system is operating.

It is important to understand that the translation from universal morals to personal morals and professional ethics does vary from one culture and country to another. As an example, consider the common belief (universal moral) that the awarding of contracts and financial arrangements for services to be performed (for government or business) should be accomplished in a fair and transparent fashion. In some societies and cultures, corruption in the process of contract making is common and often “overlooked” by the local authorities, who may also be involved in the affairs. Are these immoral or unethical practices? Most would say, “Yes, this should not be allowed. Find and punish the individuals involved.” Yet, such practices do continue, thus indicating the differences in interpretation of common morals as they are translated into the ethics of individuals and professionals.

EXAMPLE 1.2

Jamie is an engineer employed by Burris, a United States-based company that develops subway and surface transportation systems for medium-sized municipalities in the United States and Canada. He has been a registered professional engineer (PE) for the last 15 years. Last year, Carol, an engineer friend from university days who works as an individual consultant, asked Jamie to help her with some cost estimates on a metro train job. Carol offered to pay for his time and talent, but Jamie saw no reason to take money for helping with data commonly used by him in performing his job at Burris. The estimates took one weekend to complete, and once Jamie delivered them to Carol, he did not hear from her again; nor did he learn the identity of the company for which Carol was preparing the estimates.

Yesterday, Jamie was called into his supervisor’s office and told that Burris had not received the contract award in Sharpstown, where a metro system is to be installed. The project estimates were prepared by Jamie and others at Burris over the past several months. This job was greatly needed by Burris, as the country and most municipalities were in a real economic slump, so much so that Burris was considering furloughing several engineers if the Sharpstown bid was not accepted. Jamie was told he was to be laid off immediately, not because the bid was rejected, but because he had been secretly working without management approval for a prime consultant of Burris’ main competitor. Jamie was astounded and angry. He knew he had done nothing to warrant firing, but the evidence was clearly there. The numbers used by the competitor to win the Sharpstown award were the same numbers that Jamie had prepared for Burris on this bid, and they closely matched the values that he gave Carol when he helped her.

Jamie was told he was fortunate, because Burris’ president had decided to not legally charge Jamie with unethical behavior and to not request that his PE license be rescinded. As a result, Jamie was escorted out of his office and the building within one hour and told to not ask anyone at Burris for a reference letter if he attempted to get another engineering job.

Discuss the ethical dimensions of this situation for Jamie, Carol, and Burris’ management. Refer to the NSPE Code of Ethics for Engineers (Appendix C) for specific points of concern.

Solution

There are several obvious errors and omissions present in the actions of Jamie, Carol, and Burris’ management in this situation. Some of these mistakes, oversights, and possible code violations are summarized here.

Jamie

- Did not learn identity of company Carol was working for and whether the company was to be a bidder on the Sharpstown project
- Helped a friend with confidential data, probably innocently, without the knowledge or approval of his employer
- Assisted a competitor, probably unknowingly, without the knowledge or approval of his employer
- Likely violated, at least, Code of Ethics for Engineers section II.1.c, which reads, “Engineers shall not reveal facts, data, or information without the prior consent of the client or employer except as authorized or required by law or this Code.”

Carol

- Did not share the intended use of Jamie's work
- Did not seek information from Jamie concerning his employer's intention to bid on the same project as her client
- Misled Jamie in that she did not seek approval from Jamie to use and quote his information and assistance
- Did not inform her client that portions of her work originated from a source employed by a possible bid competitor
- Likely violated, at least, Code of Ethics for Engineers section III.9.a, which reads, "Engineers shall, whenever possible, name the person or persons who may be individually responsible for designs, inventions, writings, or other accomplishments."

Burris' management

- Acted too fast in dismissing Jamie; they should have listened to Jamie and conducted an investigation
- Did not put him on administrative leave during a review
- Possibly did not take Jamie's previous good work record into account

These are not all ethical considerations; some are just plain good business practices for Jamie, Carol, and Burris.



1.4 Interest Rate and Rate of Return ● ● ●

Interest is the manifestation of the time value of money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an amount of interest—interest paid and interest earned. These are illustrated in Figure 1–2. Interest is **paid** when a person or organization borrowed money (obtained a loan) and repays a larger amount over time. Interest is **earned** when a person or organization saved, invested, or lent money and obtains a return of a larger amount over time. The numerical values and formulas used are the same for both perspectives, but the interpretations are different.

Interest paid on borrowed funds (a loan) is determined using the original amount, also called the *principal*,

$$\text{Interest} = \text{amount owed now} - \text{principal} \quad [1.1]$$

When interest paid over a *specific time unit* is expressed as a percentage of the principal, the result is called the **interest rate**.

$$\text{Interest rate (\%)} = \frac{\text{interest accrued per time unit}}{\text{principal}} \times 100\% \quad [1.2]$$

The time unit of the rate is called the **interest period**. By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as 1% per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, 8.5%, a 1-year interest period is assumed.

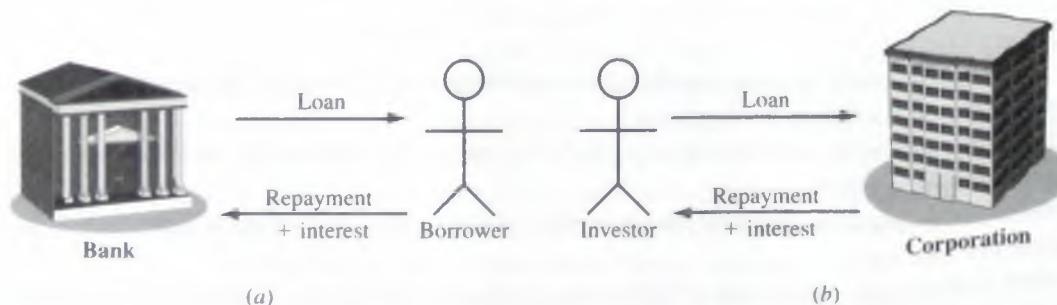


Figure 1-2

(a) Interest paid over time to lender. (b) Interest earned over time by investor.

EXAMPLE 1.3

An employee at LaserKinetics.com borrows \$10,000 on May 1 and must repay a total of \$10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

Solution

The perspective here is that of the borrower since \$10,700 repays a loan. Apply Equation [1.1] to determine the interest paid.

$$\text{Interest paid} = \$10,700 - 10,000 = \$700$$

Equation [1.2] determines the interest rate paid for 1 year.

$$\text{Percent interest rate} = \frac{\$700}{\$10,000} \times 100\% = 7\% \text{ per year}$$

EXAMPLE 1.4

Stereophonics, Inc., plans to borrow \$20,000 from a bank for 1 year at 9% interest for new recording equipment. (a) Compute the interest and the total amount due after 1 year. (b) Construct a column graph that shows the original loan amount and total amount due after 1 year used to compute the loan interest rate of 9% per year.

Solution

(a) Compute the total interest accrued by solving Equation [1.2] for interest accrued.

$$\text{Interest} = \$20,000(0.09) = \$1800$$

The total amount due is the sum of principal and interest.

$$\text{Total due} = \$20,000 + 1800 = \$21,800$$

(b) Figure 1–3 shows the values used in Equation [1.2]: \$1800 interest, \$20,000 original loan principal, 1-year interest period.

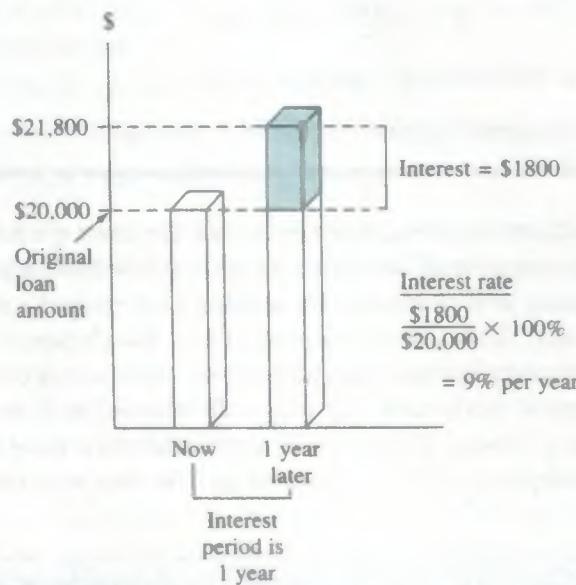


Figure 1–3

Values used to compute an interest rate of 9% per year. Example 1.4.

Comment

Note that in part (a), the total amount due may also be computed as

$$\text{Total due} = \text{principal}(1 + \text{interest rate}) = \$20,000(1.09) = \$21,800$$

Later we will use this method to determine future amounts for times longer than one interest period.

From the perspective of a saver, a lender, or an investor, **interest earned** (Figure 1–2b) is the final amount minus the initial amount, or principal.

$$\text{Interest earned} = \text{total amount now} - \text{principal} \quad [1.3]$$

Interest earned over a specific period of time is expressed as a percentage of the original amount and is called **rate of return (ROR)**.

$$\text{Rate of return (\%)} = \frac{\text{interest accrued per time unit}}{\text{principal}} \times 100\% \quad [1.4]$$

The time unit for rate of return is called the **interest period**, just as for the borrower's perspective. Again, the most common period is 1 year.

The term **return on investment (ROI)** is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs.

The numerical values in Equations [1.2] and [1.4] are the same, but the term *interest rate paid* is more appropriate for the borrower's perspective, while the *rate of return earned* is better for the investor's perspective.

EXAMPLE 1.5

- (a) Calculate the amount deposited 1 year ago to have \$1000 now at an interest rate of 5% per year.
- (b) Calculate the amount of interest earned during this time period.

Solution

- (a) The total amount accrued (\$1000) is the sum of the original deposit and the earned interest. If X is the original deposit,

$$\text{Total accrued} = \text{deposit} + \text{deposit(interest rate)}$$

$$\$1000 = X + X(0.05) = X(1 + 0.05) = 1.05X$$

The original deposit is

$$X = \frac{1000}{1.05} = \$952.38$$

- (b) Apply Equation [1.3] to determine the interest earned.

$$\text{Interest} = \$1000 - 952.38 = \$47.62$$

In Examples 1.3 to 1.5 the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved, e.g., the amount of interest after 3 years, it is necessary to state whether the interest is accrued on a *simple* or *compound* basis from one period to the next. This topic is covered later in this chapter.

Since **inflation** can significantly increase an interest rate, some comments about the fundamentals of inflation are warranted at this early stage. By definition, inflation represents a decrease in the value of a given currency. That is, \$10 now will not purchase the same amount of gasoline for your car (or most other things) as \$10 did 10 years ago. The changing value of the currency affects market interest rates.



Inflation

In simple terms, interest rates reflect two things: a so-called real rate of return *plus* the expected inflation rate. The real rate of return allows the investor to purchase more than he or she could have purchased before the investment, while inflation raises the real rate to the market rate that we use on a daily basis.

The safest investments (such as government bonds) typically have a 3% to 4% real rate of return built into their overall interest rates. Thus, a market interest rate of, say, 8% per year on a bond means that investors expect the inflation rate to be in the range of 4% to 5% per year. Clearly, inflation causes interest rates to rise.

From the borrower's perspective, the rate of inflation is another interest rate *tacked on to the real interest rate*. And from the vantage point of the saver or investor in a fixed-interest account,

inflation reduces the real rate of return on the investment. Inflation means that cost and revenue cash flow estimates increase over time. This increase is due to the changing value of money that is forced upon a country's currency by inflation, thus making a unit of currency (such as the dollar) worth less relative to its value at a previous time. We see the effect of inflation in that money purchases less now than it did at a previous time. Inflation contributes to

- A reduction in purchasing power of the currency
- An increase in the CPI (consumer price index)
- An increase in the cost of equipment and its maintenance
- An increase in the cost of salaried professionals and hourly employees
- A reduction in the real rate of return on personal savings and certain corporate investments

In other words, inflation can materially contribute to changes in corporate and personal economic analysis.

Commonly, engineering economy studies assume that inflation affects all estimated values equally. Accordingly, an interest rate or rate of return, such as 8% per year, is applied throughout the analysis without accounting for an additional inflation rate. However, if inflation were explicitly taken into account, and it was reducing the value of money at, say, an average of 4% per year, then it would be necessary to perform the economic analysis using an inflated interest rate. (The rate is 12.32% per year using the relations derived in Chapter 14.)

1.5 Terminology and Symbols ● ● ●

The equations and procedures of engineering economy utilize the following terms and symbols. Sample units are indicated.

P = value or amount of money at a time designated as the present or time 0. Also P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); monetary units, such as dollars

F = value or amount of money at some future time. Also F is called future worth (FW) and future value (FV); dollars

A = series of consecutive, equal, end-of-period amounts of money. Also A is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, euros per month

n = number of interest periods; years, months, days

i = interest rate per time period; percent per year, percent per month

t = time, stated in periods; years, months, days

The symbols P and F represent one-time occurrences: A occurs with the same value in each interest period for a specified number of periods. It should be clear that a present value P represents a single sum of money at some time prior to a future value F or prior to the first occurrence of an equivalent series amount A .

It is important to note that the symbol A always represents a uniform amount (i.e., the same amount each period) that extends through consecutive interest periods. Both conditions must exist before the series can be represented by A .

The interest rate i is expressed in percent per interest period, for example, 12% per year. Unless stated otherwise, assume that the rate applies throughout the entire n years or interest periods. The decimal equivalent for i is always used in formulas and equations in engineering economy computations.

All engineering economy problems involve the element of time expressed as n and interest rate i . In general, every problem will involve at least four of the symbols P , F , A , n , and i , with at least three of them estimated or known.

Additional symbols used in engineering economy are defined in Appendix E.

EXAMPLE 1.6

Today, Julie borrowed \$5000 to purchase furniture for her new house. She can repay the loan in either of the two ways described below. Determine the engineering economy symbols and their value for each option.

- (a) Five equal annual installments with interest based on 5% per year.
 (b) One payment 3 years from now with interest based on 7% per year.

Solution

(a) The repayment schedule requires an equivalent annual amount A , which is unknown.

$$P = \$5000 \quad i = 5\% \text{ per year} \quad n = 5 \text{ years} \quad A = ?$$

(b) Repayment requires a single future amount F , which is unknown.

$$P = \$5000 \quad i = 7\% \text{ per year} \quad n = 3 \text{ years} \quad F = ?$$

EXAMPLE 1.7

You plan to make a lump-sum deposit of \$5000 now into an investment account that pays 6% per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

Solution

All five symbols are present, but the future value in year 6 is the unknown.

$$P = \$5000$$

$$A = \$1000 \text{ per year for 5 years}$$

$$F = ? \text{ at end of year 6}$$

$$i = 6\% \text{ per year}$$

$$n = 5 \text{ years for the } A \text{ series and 6 for the } F \text{ value}$$

EXAMPLE 1.8

Last year Jane's grandmother offered to put enough money into a savings account to generate \$5000 in interest this year to help pay Jane's expenses at college. (a) Identify the symbols, and (b) calculate the amount that had to be deposited exactly 1 year ago to earn \$5000 in interest now, if the rate of return is 6% per year.

Solution

(a) Symbols P (last year is -1) and F (this year) are needed.

$$P = ?$$

$$i = 6\% \text{ per year}$$

$$n = 1 \text{ year}$$

$$F = P + \text{interest} = ? + \$5000$$

(b) Let $F =$ total amount now and $P =$ original amount. We know that $F - P = \$5000$ is accrued interest. Now we can determine P . Refer to Equations [1.1] through [1.4].

$$F = P + Pi$$

The \$5000 interest can be expressed as

$$\text{Interest} = F - P = (P + Pi) - P$$

$$= Pi$$

$$\$5000 = P(0.06)$$

$$P = \frac{\$5000}{0.06} = \$83,333.33$$

1.6 Cash Flows: Estimation and Diagramming ● ● ●

As mentioned in earlier sections, cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. All cash flows occur during specific time periods, such as 1 month, every 6 months, or 1 year. Annual is the most common time period. For example, a payment of \$10,000 once every year in December for 5 years is a series of 5 outgoing cash flows. And an estimated receipt of \$500 every month for 2 years is a series of 24 incoming cash flows. Engineering economy bases its computations on the timing, size, and direction of cash flows.

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A **plus sign** indicates a cash inflow.



Cash flow

Cash outflows are costs, disbursements, expenses, and taxes caused by projects and business activity. A **negative or minus sign** indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

Of all the steps in Figure 1–1 that outline the engineering economy study, estimating cash flows (step 3) is the most difficult, primarily because it is an attempt to predict the future. Some examples of cash flow estimates are shown here. As you scan these, consider how the cash inflow or outflow may be estimated most accurately.

Cash Inflow Estimates

- Income: +\$150,000 per year from sales of solar-powered watches
- Savings: +\$24,500 tax savings from capital loss on equipment salvage
- Receipt: +\$750,000 received on large business loan plus accrued interest
- Savings: +\$150,000 per year saved by installing more efficient air conditioning
- Revenue: +\$50,000 to +\$75,000 per month in sales for extended battery life iPhones

Cash Outflow Estimates

- Operating costs: -\$230,000 per year annual operating costs for software services
- First cost: -\$800,000 next year to purchase replacement earthmoving equipment
- Expense: -\$20,000 per year for loan interest payment to bank
- Initial cost: -\$1 to -\$1.2 million in capital expenditures for a water recycling unit

All of these are **point estimates**, that is, *single-value estimates* for cash flow elements of an alternative, except for the last revenue and cost estimates listed above. They provide a **range estimate**, because the persons estimating the revenue and cost do not have enough knowledge or experience with the systems to be more accurate. For the initial chapters, we will utilize point estimates. The use of risk and sensitivity analysis for range estimates is covered in the later chapters of this book.

Once all cash inflows and outflows are estimated (or determined for a completed project), the **net cash flow** for each time period is calculated.

$$\begin{aligned} \text{Net cash flow} &= \text{cash inflows} - \text{cash outflows} & [1.5] \\ \text{NCF} &= R - D & [1.6] \end{aligned}$$

where NCF is net cash flow, R is receipts, and D is disbursements.

At the beginning of this section, the *timing, size, and direction of cash flows* were mentioned as important. Because cash flows may take place at any time during an interest period, as a matter of convention, all cash flows are assumed to occur at the end of an interest period.

The end-of-period convention means that all cash inflows and all cash outflows are assumed to take place at the **end of the interest period** in which they actually occur. When several inflows and outflows occur within the same period, the **net cash flow** is assumed to occur at the **end of the period**.



End-of-period convention

Figure 1–4
A typical cash flow time scale for 5 years.

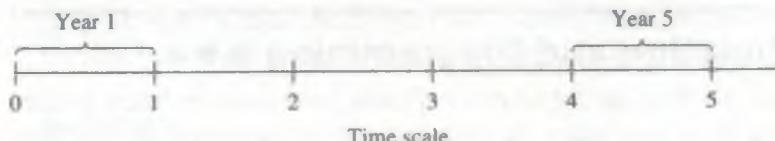
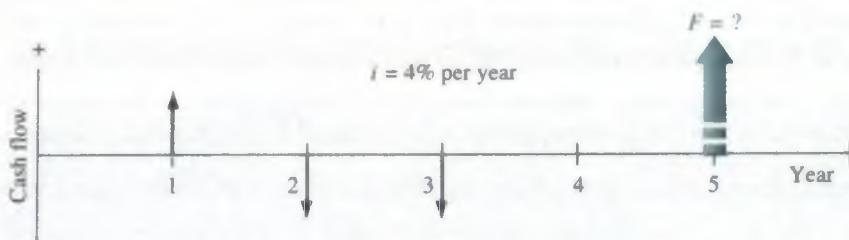


Figure 1–5
Example of positive and negative cash flows.



In assuming end-of-period cash flows, it is important to understand that future (F) and uniform annual (A) amounts are located at the end of the interest period, which is not necessarily December 31. If in Example 1.7 the lump-sum deposit took place on July 1, 2011, the withdrawals will take place on July 1 of each succeeding year for 6 years. Remember, end of the period means end of interest period, not end of calendar year.

The **cash flow diagram** is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on the y axis with a time scale on the x axis. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

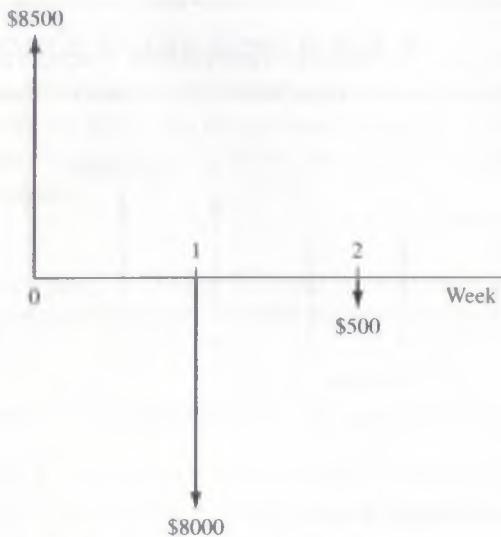
Cash flow diagram time $t = 0$ is the present, and $t = 1$ is the end of time period 1. We assume that the periods are in years for now. The time scale of Figure 1–4 is set up for 5 years. Since the end-of-year convention places cash flows at the ends of years, the “1” marks the end of year 1.

While it is not necessary to use an exact scale on the cash flow diagram, you will probably avoid errors if you make a neat diagram to approximate scale for both time and relative cash flow magnitudes.

The direction of the arrows on the diagram is important to differentiate income from outgo. A vertical arrow pointing up indicates a positive cash flow. Conversely, a down-pointing arrow indicates a negative cash flow. **We will use a bold, colored arrow to indicate what is unknown and to be determined.** For example, if a future value F is to be determined in year 5, a wide, colored arrow with $F = ?$ is shown in year 5. The interest rate is also indicated on the diagram. Figure 1–5 illustrates a cash inflow at the end of year 1, equal cash outflows at the end of years 2 and 3, an interest rate of 4% per year, and the unknown future value F after 5 years. The arrow for the unknown value is generally drawn in the opposite direction from the other cash flows; however, the engineering economy computations will determine the actual sign on the F value.

Before the diagramming of cash flows, a perspective or vantage point must be determined so that + or – signs can be assigned and the economic analysis performed correctly. Assume you borrow \$8500 from a bank today to purchase an \$8000 used car for cash next week, and you plan to spend the remaining \$500 on a new paint job for the car two weeks from now. There are several perspectives possible when developing the cash flow diagram—those of the borrower (that's you), the banker, the car dealer, or the paint shop owner. The cash flow signs and amounts for these perspectives are as follows.

Perspective	Activity	Cash flow with Sign, \$	Time, week
You	Borrow	+8500	0
	Buy car	-8000	1
	Paint job	-500	2
Banker	Lender	-8500	0
Car dealer	Car sale	+8000	1
Painter	Paint job	+500	2

**Figure 1–6**

Cash flows from perspective of borrower for loan and purchases.

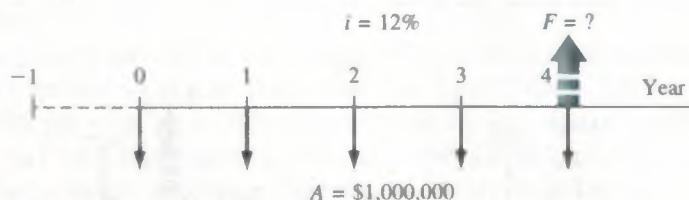
One, and only one, of the perspectives is selected to develop the diagram. For your perspective, all three cash flows are involved and the diagram appears as shown in Figure 1–6 with a time scale of weeks. Applying the end-of-period convention, you have a receipt of +\$8500 now (time 0) and cash outflows of -\$8000 at the end of week 1, followed by -\$500 at the end of week 2.

EXAMPLE 1.9

Each year Exxon-Mobil expends large amounts of funds for mechanical safety features throughout its worldwide operations. Carla Ramos, a lead engineer for Mexico and Central American operations, plans expenditures of \$1 million now and each of the next 4 years just for the improvement of field-based pressure-release valves. Construct the cash flow diagram to find the equivalent value of these expenditures at the end of year 4, using a cost of capital estimate for safety-related funds of 12% per year.

Solution

Figure 1–7 indicates the uniform and negative cash flow series (expenditures) for five periods, and the unknown F value (positive cash flow equivalent) at exactly the same time as the fifth expenditure. Since the expenditures start immediately, the first \$1 million is shown at time 0, not time 1. Therefore, the last negative cash flow occurs at the end of the fourth year, when F also occurs. To make this diagram have a full 5 years on the time scale, the addition of the year -1 completes the diagram. This addition demonstrates that year 0 is the end-of-period point for the year -1.

**Figure 1–7**

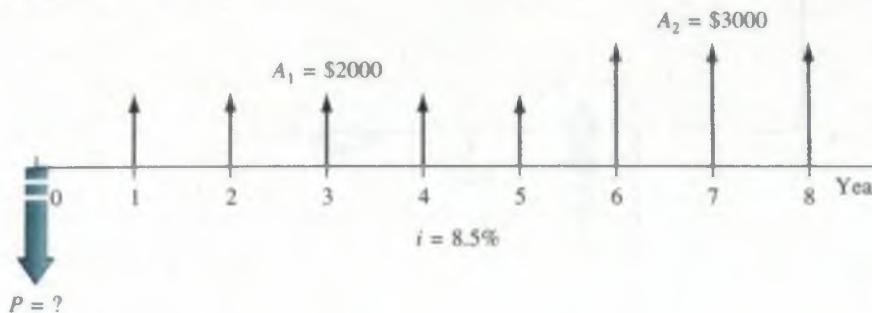
Cash flow diagram, Example 1.9.

EXAMPLE 1.10

An electrical engineer wants to deposit an amount P now such that she can withdraw an equal annual amount of $A_1 = \$2000$ per year for the first 5 years, starting 1 year after the deposit, and a different annual withdrawal of $A_2 = \$3000$ per year for the following 3 years. How would the cash flow diagram appear if $i = 8.5\%$ per year?

Solution

The cash flows are shown in Figure 1–8. The negative cash outflow P occurs now. The withdrawals (positive cash inflow) for the A_1 series occur at the end of years 1 through 5, and A_2 occurs in years 6 through 8.

**Figure 1–8**

Cash flow diagram with two different A series, Example 1.10.

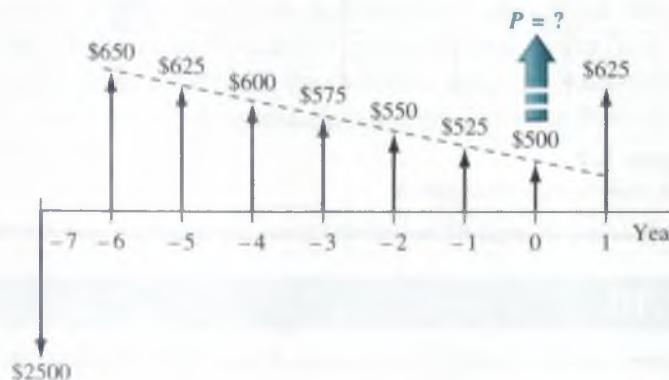
EXAMPLE 1.11

A rental company spent \$2500 on a new air compressor 7 years ago. The annual rental income from the compressor has been \$750. The \$100 spent on maintenance the first year has increased each year by \$25. The company plans to sell the compressor at the end of next year for \$150. Construct the cash flow diagram from the company's perspective and indicate where the present worth now is located.

Solution

Let now be time $t = 0$. The incomes and costs for years -7 through 1 (next year) are tabulated below with net cash flow computed using Equation [1.5]. The net cash flows (one negative, eight positive) are diagrammed in Figure 1–9. Present worth P is located at year 0.

End of Year	Income	Cost	Net Cash Flow
-7	\$ 0	\$2500	\$-2500
-6	750	100	650
-5	750	125	625
-4	750	150	600
-3	750	175	575
-2	750	200	550
-1	750	225	525
0	750	250	500
1	750 + 150	275	625

**Figure 1–9**

Cash flow diagram, Example 1.11.

1.7 Economic Equivalence ● ● ●

Economic equivalence is a fundamental concept upon which engineering economy computations are based. Before we delve into the economic aspects, think of the many types of equivalency we may utilize daily by transferring from one scale to another. Some example transfers between scales are as follows:

Length:

$$\begin{array}{lll} 12 \text{ inches} = 1 \text{ foot} & 3 \text{ feet} = 1 \text{ yard} & 39.370 \text{ inches} = 1 \text{ meter} \\ 100 \text{ centimeters} = 1 \text{ meter} & 1000 \text{ meters} = 1 \text{ kilometer} & 1 \text{ kilometer} = 0.621 \text{ mile} \end{array}$$

Pressure:

$$1 \text{ atmosphere} = 1 \text{ newton/meter}^2 = 10^3 \text{ pascal} = 1 \text{ kilopascal}$$

Often equivalency involves two or more scales. Consider the equivalency of a *speed* of 110 kilometers per hour (kph) into miles per minute using conversions between distance and time scales with three-decimal accuracy.

Speed:

$$\begin{array}{ll} 1 \text{ mile} = 1.609 \text{ kilometers} & 1 \text{ hour} = 60 \text{ minutes} \\ 110 \text{ kph} = 68.365 \text{ miles per hour (mph)} & 68.365 \text{ mph} = 1.139 \text{ miles per minute} \end{array}$$

Four scales—time in minutes, time in hours, length in miles, and length in kilometers—are combined to develop these equivalent statements on speed. Note that throughout these statements, the fundamental relations of 1 mile = 1.609 kilometers and 1 hour = 60 minutes are applied. If a fundamental relation changes, the entire equivalency is in error.

Now we consider economic equivalency.

Economic equivalence is a combination of **interest rate** and **time value of money** to determine the different amounts of money at different points in time that are equal in economic value.



Economic equivalence

As an illustration, if the interest rate is 6% per year, \$100 today (present time) is equivalent to \$106 one year from today.

$$\text{Amount accrued} = 100 + 100(0.06) = 100(1 + 0.06) = \$106$$

If someone offered you a gift of \$100 today or \$106 one year from today, it would make no difference which offer you accepted from an economic perspective. In either case you have \$106 one year from today. However, the two sums of money are equivalent to each other *only* when the interest rate is 6% per year. At a higher or lower interest rate, \$100 today is not equivalent to \$106 one year from today.

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of \$100 now is equivalent to $\$100 / 1.06 = \94.34 one year ago at an interest rate of 6% per year. From these illustrations, we can state the following: \$94.34 last year, \$100 now, and \$106 one year from now are equivalent at an interest rate of 6% per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1-year interest periods.

$$\frac{\$6}{\$100} \times 100\% = 6\% \text{ per year}$$

and

$$\frac{\$5.66}{\$94.34} \times 100\% = 6\% \text{ per year}$$

The cash flow diagram in Figure 1–10 indicates the amount of interest needed each year to make these three different amounts equivalent at 6% per year.

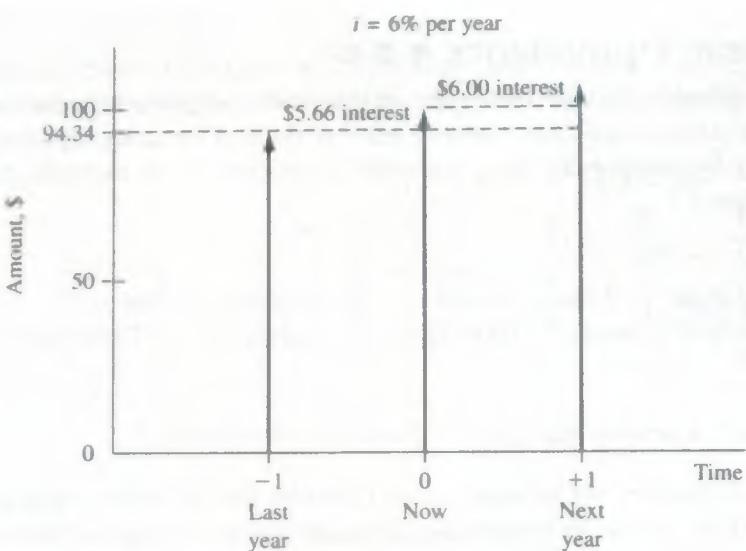


Figure 1–10
Equivalence of money at 6% per year interest.

EXAMPLE 1.12

Manufacturers make backup batteries for computer systems available to Batteries+ dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Batteries+ outlet. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.

- The amount of \$98 now is equivalent to a cost of \$105.60 one year from now.
- A truck battery cost of \$200 one year ago is equivalent to \$205 now.
- A \$38 cost now is equivalent to \$39.90 one year from now.
- A \$3000 cost now is equivalent to \$2887.14 one year earlier.
- The carrying charge accumulated in 1 year on an investment of \$20,000 worth of batteries is \$1000.

Solution

- Total amount accrued = $98(1.05) = \$102.90 \neq \105.60 ; therefore, it is false. Another way to solve this is as follows: Required original cost is $105.60/1.05 = \$100.57 \neq \98 .
- Equivalent cost 1 year ago is $205.00/1.05 = \$195.24 \neq \200 ; therefore, it is false.
- The cost 1 year from now is $38(1.05) = \$39.90$; true.
- Cost now is $2887.14(1.05) = \$3031.50 \neq \3000 ; false.
- The charge is 5% per year interest, or $\$20,000(0.05) = \1000 ; true.

Comparison of alternative cash flow series requires the use of equivalence to determine when the series are economically equal or if one is economically preferable to another. The keys to the analysis are the interest rate and the timing of the cash flows. Example 1.13 demonstrates how easy it is to be misled by the size and timing of cash flows.

EXAMPLE 1.13

Howard owns a small electronics repair shop. He wants to borrow \$10,000 now and repay it over the next 1 or 2 years. He believes that new diagnostic test equipment will allow him to work on a wider variety of electronic items and increase his annual revenue. Howard received 2-year repayment options from banks A and B.

Year	Amount to pay, \$ per year	
	Bank A	Bank B
1	−5,378.05	−5,000.00
2	−5,378.05	−5,775.00
Total paid	−10,756.10	−10,775.00

After reviewing these plans, Howard decided that he wants to repay the \$10,000 after only 1 year based on the expected increased revenue. During a family conversation, Howard's brother-in-law offered to lend him the \$10,000 now and take \$10,600 after exactly 1 year. Now Howard has three options and wonders which one to take. Which one is economically the best?

Solution

The repayment plans for both banks are economically equivalent at the interest rate of 5% per year. (This is determined by using computations that you will learn in Chapter 2.) Therefore, Howard can choose either plan even though the bank B plan requires a slightly larger sum of money over the 2 years.

The brother-in-law repayment plan requires a total of \$600 in interest 1 year later plus the principal of \$10,000, which makes the interest rate 6% per year. Given the two 5% per year options from the banks, this 6% plan should not be chosen as it is not economically better than the other two. Even though the sum of money repaid is smaller, the timing of the cash flows and the interest rate make it less desirable. The point here is that cash flows themselves, or their sums, cannot be relied upon as the primary basis for an economic decision. The interest rate, timing, and economic equivalence must be considered.

1.8 Simple and Compound Interest ● ● ●

The terms *interest*, *interest period*, and *interest rate* (introduced in Section 1.4) are useful in calculating equivalent sums of money for one interest period in the past and one period in the future. However, for more than one interest period, the terms *simple interest* and *compound interest* become important.

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as

$$\begin{aligned} \text{Simple interest} &= (\text{principal})(\text{number of periods})(\text{interest rate}) \\ I &= Pni \end{aligned} \quad [1.7]$$

where I is the amount of interest earned or paid and the interest rate i is expressed in decimal form.

EXAMPLE 1.14

GreenTree Financing lent an engineering company \$100,000 to retrofit an environmentally unfriendly building. The loan is for 3 years at 10% per year simple interest. How much money will the firm repay at the end of 3 years?

Solution

The interest for each of the 3 years is

$$\text{Interest per year} = \$100,000(0.10) = \$10,000$$

Total interest for 3 years from Equation [1.7] is

$$\text{Total interest} = \$100,000(3)(0.10) = \$30,000$$

The amount due after 3 years is

$$\text{Total due} = \$100,000 + 30,000 = \$130,000$$

The interest accrued in the first year and in the second year does not earn interest. The interest due each year is \$10,000 calculated only on the \$100,000 loan principal.

In most financial and economic analyses, we use **compound interest** calculations.

For *compound interest*, the interest accrued for each interest period is calculated on the **principal plus the total amount of interest accumulated in all previous periods**. Thus, compound interest means interest on top of interest.

Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as

$$\text{Compound interest} = (\text{principal} + \text{all accrued interest})(\text{interest rate}) \quad [1.8]$$

In mathematical terms, the interest I_t for time period t may be calculated using the relation.

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j \right) (i) \quad [1.9]$$

EXAMPLE 1.15

Assume an engineering company borrows \$100,000 at 10% per year compound interest and will pay the principal and all the interest after 3 years. Compute the annual interest and total amount due after 3 years. Graph the interest and total owed for each year, and compare with the previous example that involved simple interest.

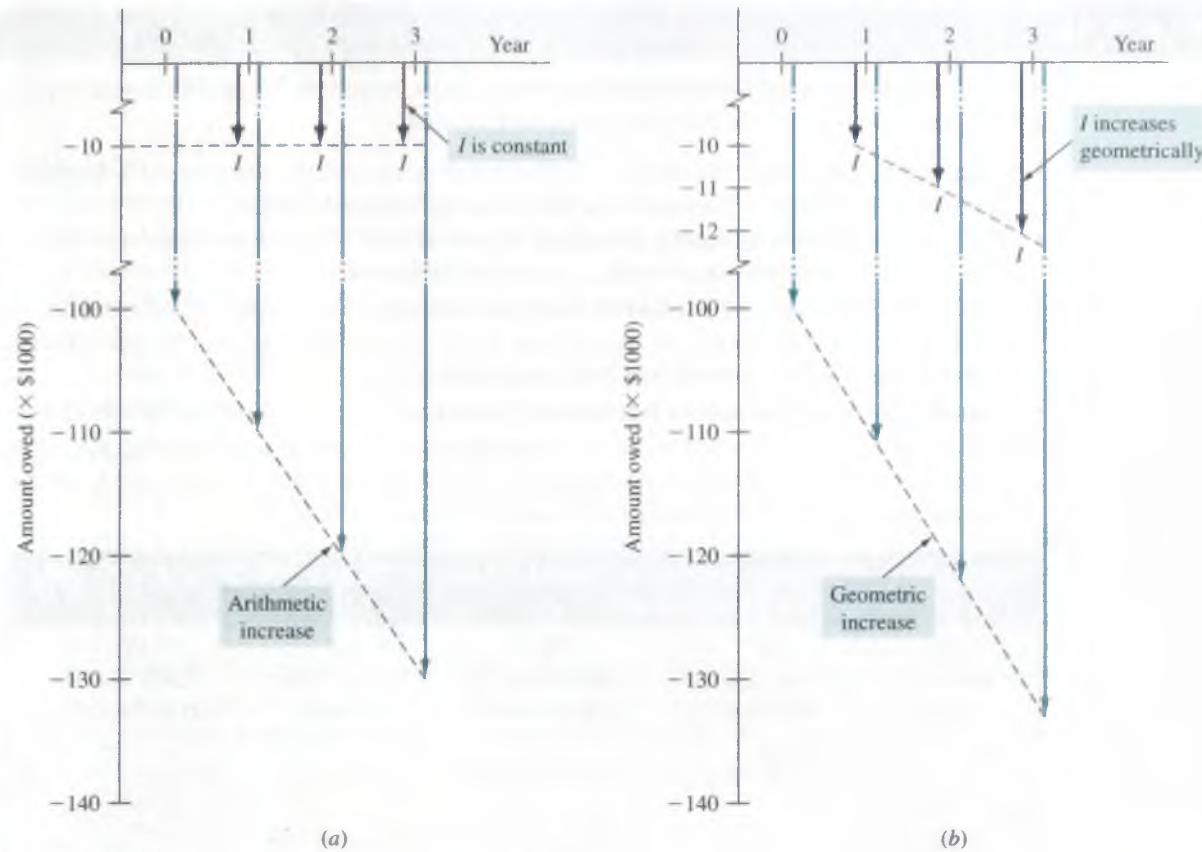
Solution

To include compounding of interest, the annual interest and total owed each year are calculated by Equation [1.8].

Interest, year 1:	$100,000(0.10) = \$10,000$
Total due, year 1:	$100,000 + 10,000 = \$110,000$
Interest, year 2:	$110,000(0.10) = \$11,000$
Total due, year 2:	$110,000 + 11,000 = \$121,000$
Interest, year 3:	$121,000(0.10) = \$12,100$
Total due, year 3:	$121,000 + 12,100 = \$133,100$

The repayment plan requires no payment until year 3 when all interest and the principal, a total of \$133,100, are due. Figure 1–11 uses a cash flow diagram format to compare end-of-year (a) simple and (b) compound interest and total amounts owed. The differences due to compounding are clear. An extra $\$133,100 - \$130,000 = \$3100$ in interest is due for the compounded interest loan.

Note that while simple interest due each year is constant, the compounded interest due grows geometrically. Due to this geometric growth of compound interest, the difference between simple and compound interest accumulation increases rapidly as the time frame increases. For example, if the loan is for 10 years, not 3, the extra paid for compounding interest may be calculated to be \$59,374.

**Figure 1-11**

Interest I owed and total amount owed for (a) simple interest (Example 1.14) and (b) compound interest (Example 1.15).

A more efficient way to calculate the total amount due after a number of years in Example 1.15 is to utilize the fact that compound interest increases geometrically. This allows us to skip the year-by-year computation of interest. In this case, the **total amount due at the end of each year** is

$$\text{Year 1: } \$100,000(1.10)^1 = \$110,000$$

$$\text{Year 2: } \$100,000(1.10)^2 = \$121,000$$

$$\text{Year 3: } \$100,000(1.10)^3 = \$133,100$$

This allows future totals owed to be calculated directly without intermediate steps. The general form of the equation is

$$\begin{aligned} \text{Total due after } n \text{ years} &= \text{principal}(1 + \text{interest rate})^n \text{ years} \\ &= P(1 + i)^n \end{aligned} \quad [1.10]$$

where i is expressed in decimal form. Equation [1.10] was applied above to obtain the \$133,100 due after 3 years. This fundamental relation will be used many times in the upcoming chapters.

We can combine the concepts of interest rate, compound interest, and equivalence to demonstrate that different loan repayment plans may be equivalent, but differ substantially in amounts paid from one year to another and in the total repayment amount. This also shows that there are many ways to take into account the time value of money.

EXAMPLE 1.16

Table 1–1 details four different loan repayment plans described below. Each plan repays a \$5000 loan in 5 years at 8% per year compound interest.

- Plan 1: Pay all at end.** No interest or principal is paid until the end of year 5. Interest accumulates each year on the total of principal and all accrued interest.
- Plan 2: Pay interest annually, principal repaid at end.** The accrued interest is paid each year, and the entire principal is repaid at the end of year 5.
- Plan 3: Pay interest and portion of principal annually.** The accrued interest and one-fifth of the principal (or \$1000) are repaid each year. The outstanding loan balance decreases each year, so the interest (column 2) for each year decreases.
- Plan 4: Pay equal amount of interest and principal.** Equal payments are made each year with a portion going toward principal repayment and the remainder covering the accrued interest. Since the loan balance decreases at a rate slower than that in plan 3 due to the equal end-of-year payments, the interest decreases, but at a slower rate.

TABLE 1–1 Different Repayment Schedules Over 5 Years for \$5000 at 8% Per Year Compound Interest

(1) End of Year	(2) Interest Owed for Year	(3) Total Owed at End of Year	(4) End-of-Year Payment	(5) Total Owed After Payment
<i>Plan 1: Pay All at End</i>				
0				\$5000.00
1	\$400.00	\$5400.00	—	5400.00
2	432.00	5832.00	—	5832.00
3	466.56	6298.56	—	6298.56
4	503.88	6802.44	—	6802.44
5	544.20	7346.64	\$–7346.64	
Total				\$–7346.64
<i>Plan 2: Pay Interest Annually; Principal Repaid at End</i>				
0				\$5000.00
1	\$400.00	\$5400.00	\$–400.00	5000.00
2	400.00	5400.00	–400.00	5000.00
3	400.00	5400.00	–400.00	5000.00
4	400.00	5400.00	–400.00	5000.00
5	400.00	5400.00	–5400.00	
Total				\$–7000.00
<i>Plan 3: Pay Interest and Portion of Principal Annually</i>				
0				\$5000.00
1	\$400.00	\$5400.00	\$–1400.00	4000.00
2	320.00	4320.00	–1320.00	3000.00
3	240.00	3240.00	–1240.00	2000.00
4	160.00	2160.00	–1160.00	1000.00
5	80.00	1080.00	–1080.00	
Total				\$–6200.00
<i>Plan 4: Pay Equal Annual Amount of Interest and Principal</i>				
0				\$5000.00
1	\$400.00	\$5400.00	\$–1252.28	4147.72
2	331.82	4479.54	–1252.28	3227.25
3	258.18	3485.43	–1252.28	2233.15
4	178.65	2411.80	–1252.28	1159.52
5	92.76	1252.28	–1252.28	
Total				\$–6261.40

- (a) Make a statement about the *equivalence* of each plan at 8% compound interest.
 (b) Develop an 8% per year *simple* interest repayment plan for this loan using the same approach as plan 2. Comment on the total amounts repaid for the two plans.

Solution

(a) The amounts of the annual payments are different for each repayment schedule, and the total amounts repaid for most plans are different, even though each repayment plan requires exactly 5 years. The difference in the total amounts repaid can be explained by the time value of money and by the partial repayment of principal prior to year 5.

A loan of \$5000 at time 0 made at 8% per year compound interest is equivalent to each of the following:

Plan 1 \$7346.64 at the end of year 5

Plan 2 \$400 per year for 4 years and \$5400 at the end of year 5

Plan 3 Decreasing payments of interest and partial principal in years 1 (\$1400) through 5 (\$1080)

Plan 4 \$1252.28 per year for 5 years

An engineering economy study typically uses plan 4; interest is compounded, and a constant amount is paid each period. This amount covers accrued interest and a partial amount of principal repayment.

(b) The repayment schedule for 8% per year simple interest is detailed in Table 1–2. Since the annual accrued interest of \$400 is paid each year and the principal of \$5000 is repaid in year 5, the schedule is exactly the same as that for 8% per year compound interest, and the total amount repaid is the same at \$7000. In this unusual case, simple and compound interest result in the same total repayment amount. Any deviation from this schedule will cause the two plans and amounts to differ.

TABLE 1–2 A 5-Year Repayment Schedule of \$5000 at 8% per Year Simple Interest

End of Year	Interest Owed for Year	Total Owed at End of Year	End-of-Year Payment	Total Owed After Payment
0				\$5000
1	\$400	\$5400	-\$400	5000
2	400	5400	-400	5000
3	400	5400	-400	5000
4	400	5400	-400	5000
5	400	5400	<u>-5400</u>	0
Total			-\$7000	

1.9 Minimum Attractive Rate of Return ● ● ●

For any investment to be profitable, the investor (corporate or individual) expects to receive more money than the amount of capital invested. In other words, a fair *rate of return*, or *return on investment*, must be realizable. The definition of ROR in Equation [1.4] is used in this discussion, that is, amount earned divided by the principal.

Engineering alternatives are evaluated upon the prognosis that a reasonable ROR can be expected. Therefore, some reasonable rate must be established for the selection criteria (step 4) of the engineering economy study (Figure 1–1).



Minimum Attractive Rate of Return (MARR)

The Minimum Attractive Rate of Return (MARR) is a reasonable rate of return established for the evaluation and selection of alternatives. A project is not economically viable unless it is **expected to return at least the MARR**. MARR is also referred to as the *hurdle rate*, *cutoff rate*, *benchmark rate*, and *minimum acceptable rate of return*.

Figure 1–12 indicates the relations between different rate of return values. In the United States, the current U.S. Treasury Bill return is sometimes used as the benchmark safe rate. The MARR will always be higher than this, or a similar, safe rate. The MARR is not a rate that is calculated as a ROR. The MARR is established by (financial) managers and is used as a criterion against which an alternative's ROR is measured, when making the accept/reject investment decision.



Cost of capital

To develop a foundation-level understanding of how a MARR value is established and used to make investment decisions, we return to the term **capital** introduced in Section 1.1. Although the MARR is used as a criterion to decide on investing in a project, the size of MARR is fundamentally connected to how much it costs to obtain the needed capital funds. It always costs money in the form of interest to raise capital. The interest, expressed as a percentage rate per year, is called the **cost of capital**. As an example on a personal level, if you want to purchase a new widescreen HDTV, but do not have sufficient money (capital), you could obtain a bank loan for, say, a cost of capital of 9% per year and pay for the TV in cash now. Alternatively, you might choose to use your credit card and pay off the balance on a monthly basis. This approach will probably cost you at least 15% per year. Or, you could use funds from your savings account that earns 5% per year and pay cash. This approach means that you also forgo future returns from these funds. The 9%, 15%, and 5% rates are your cost of capital estimates to raise the capital for the system by different methods of capital financing. In analogous ways, corporations estimate the **cost of capital** from different sources to raise funds for engineering projects and other types of projects.

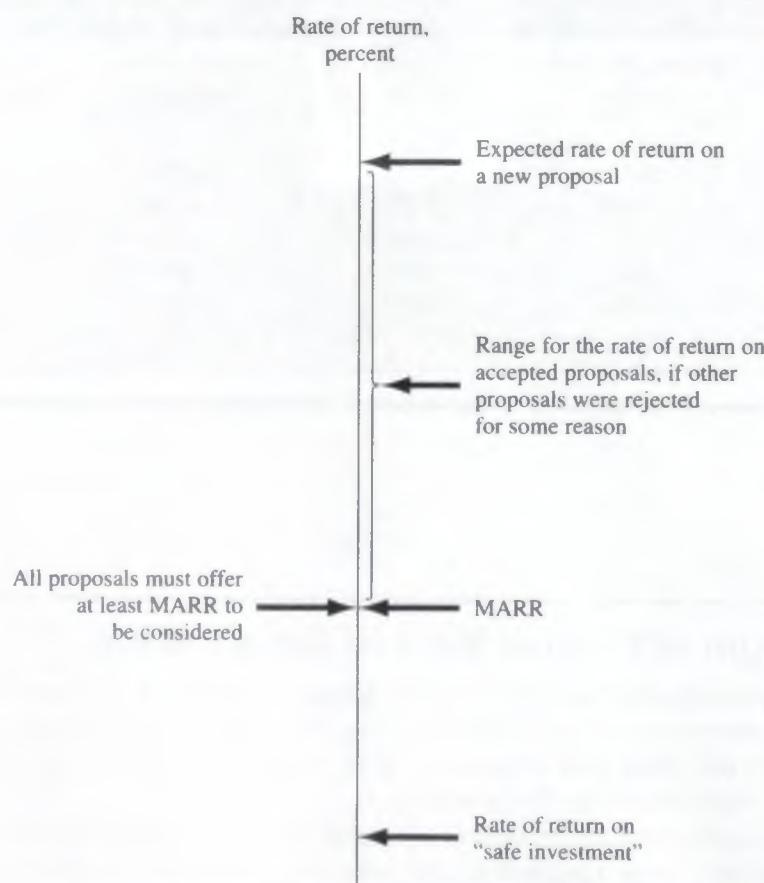


Figure 1–12
Size of MAAR relative to other rate of return values.

In general, capital is developed in two ways—equity financing and debt financing. A combination of these two is very common for most projects. Chapter 10 covers these in greater detail, but a snapshot description follows.

Equity financing The corporation uses its own funds from cash on hand, stock sales, or retained earnings. Individuals can use their own cash, savings, or investments. In the example above, using money from the 5% savings account is equity financing.

Debt financing The corporation borrows from outside sources and repays the principal and interest according to some schedule, much like the plans in Table 1–1. Sources of debt capital may be bonds, loans, mortgages, venture capital pools, and many others. Individuals, too, can utilize debt sources, such as the credit card (15% rate) and bank options (9% rate) described above.

Combinations of debt-equity financing mean that a **weighted average cost of capital (WACC)** results. If the HDTV is purchased with 40% credit card money at 15% per year and 60% savings account funds earning 5% per year, the weighted average cost of capital is $0.4(15) + 0.6(5) = 9\%$ per year.

For a corporation, the *established MARR* used as a criterion to accept or reject an investment alternative will usually be *equal to or higher than the WACC* that the corporation must bear to obtain the necessary capital funds. So the inequality

$$\text{ROR} \geq \text{MARR} > \text{WACC} \quad [1.11]$$

must be correct for an accepted project. Exceptions may be government-regulated requirements (safety, security, environmental, legal, etc.), economically lucrative ventures expected to lead to other opportunities, etc.

Often there are many alternatives that are expected to yield a ROR that exceeds the MARR as indicated in Figure 1–12, but there may not be sufficient capital available for all, or the project's risk may be estimated as too high to take the investment chance. Therefore, new projects that are undertaken usually have an expected return at least as great as the return on another alternative that is not funded. The expected rate of return on the unfunded project is called the **opportunity cost**.

The opportunity cost is the rate of return of a forgone opportunity caused by the inability to pursue a project. Numerically, it is the **largest rate of return of all the projects not accepted (forgone) due to the lack of capital funds or other resources**. When no specific MARR is established, the de facto MARR is the opportunity cost, i.e., the ROR of the first project not undertaken due to unavailability of capital funds.



Opportunity cost

As an illustration of opportunity cost, refer to Figure 1–12 and assume a MARR of 12% per year. Further, assume that a proposal, call it A, with an expected ROR = 13% is not funded due to a lack of capital. Meanwhile, proposal B has a ROR = 14.5% and is funded from available capital. Since proposal A is not undertaken due to the lack of capital, its estimated ROR of 13% is the **opportunity cost**; that is, the opportunity to make an additional 13% return is forgone.

1.10 Introduction to Spreadsheet Use ● ● ●

The functions on a computer spreadsheet can greatly reduce the amount of hand work for equivalency computations involving *compound interest* and the terms P , F , A , i , and n . The use of a calculator to solve most simple problems is preferred by many students and professors as described in Appendix D. However, as cash flow series become more complex, the spreadsheet offers a good alternative. Microsoft Excel is used throughout this book because it is readily available and easy to use. Appendix A is a primer on using spreadsheets and Excel. The functions used in engineering economy are described there in detail, with explanations of all the

parameters. Appendix A also includes a section on spreadsheet layout that is useful when the economic analysis is presented to someone else—a coworker, a boss, or a professor.

A total of seven Excel functions can perform most of the fundamental engineering economy calculations. The functions are great supplemental tools, but they do not replace the understanding of engineering economy relations, assumptions, and techniques. Using the symbols P , F , A , i , and n defined in the previous section, the functions most used in engineering economic analysis are formulated as follows.

To find the present value P : = **PV(i%,n,A,F)**

To find the future value F : = **FV(i%,n,A,P)**

To find the equal, periodic value A : = **PMT(i%,n,P,F)**

To find the number of periods n : = **NPER(i%,A,P,F)**

To find the compound interest rate i : = RATE(n,A,P,F)

To find the compound interest rate i : = IRR(first_cell:last_cell)

To find the present value P of any series: = NPV(i%, second_cell:last_cell) + first_cell

If some of the parameters don't apply to a particular problem, they can be omitted and zero is assumed. For readability, spaces can be inserted between parameters within parentheses. If the parameter omitted is an interior one, the comma must be entered. The last two functions require that a series of numbers be entered into contiguous spreadsheet cells, but the first five can be used with no supporting data. In all cases, the function must be preceded by an equals sign (=) in the cell where the answer is to be displayed.

To understand how the spreadsheet functions work, look back at Example 1.6a, where the equivalent annual amount A is unknown, as indicated by $A = ?$. (In Chapter 2, we learn how engineering economy factors calculate A , given P , i , and n .) To find A using a spreadsheet function, simply enter the PMT function = PMT(5%,5,5000). Figure 1–13 is a screen image of a spreadsheet with the PMT function entered into cell B4. The answer (\$1154.87) is displayed. The answer may appear in red and in parentheses, or with a minus sign on your screen to indicate a negative amount from the perspective of a reduction in the account balance. The right side of Figure 1–13 presents the solution to Example 1.6b. The future value F is determined by using the FV function. The FV function appears in the formula bar; and many examples throughout this text will include cell tags, as shown here, to indicate the format of important entries.

The following example demonstrates the use of a spreadsheet to develop relations (not built-in functions) to calculate interest and cash flows. Once set up, the spreadsheet can be used to perform sensitivity analysis for estimates that are subject to change. We will illustrate the use of spreadsheets throughout the chapters. (*Note:* The spreadsheet examples may be omitted, if spreadsheets are not used in the course. A solution by hand is included in virtually all examples.)

Figure 1-13

Use of spreadsheet functions PMT and FV: Example 1.6

EXAMPLE 1.17

A Japan-based architectural firm has asked a United States-based software engineering group to infuse GPS sensing capability via satellite into monitoring software for high-rise structures in order to detect greater than expected horizontal movements. This software could be very beneficial as an advance warning of serious tremors in earthquake-prone areas in Japan and the United States. The inclusion of accurate GPS data is estimated to increase annual revenue over that for the current software system by \$200,000 for each of the next 2 years, and by \$300,000 for each of years 3 and 4. The planning horizon is only 4 years due to the rapid advances made internationally in building-monitoring software. Develop spreadsheets to answer the questions below.

- Determine the total interest and total revenue after 4 years, using a compound rate of return of 8% per year.
- Repeat part (a) if estimated revenue increases from \$300,000 to \$600,000 in years 3 and 4.
- Repeat part (a) if inflation is estimated to be 4% per year. This will decrease the *real rate of return* from 8% to 3.85% per year (Chapter 14 shows why).

Solution by Spreadsheet

Refer to Figure 1–14a to d for the solutions. All the spreadsheets contain the same information, but some cell values are altered as required by the question. (Actually, all the questions can be answered on one spreadsheet by changing the numbers. Separate spreadsheets are shown here for explanation purposes only.)

The Excel functions are constructed with reference to the cells, not the values themselves, so that sensitivity analysis can be performed without function changes. This approach treats the value in a cell as a *global variable* for the spreadsheet. For example, the 8% rate in cell B2 will be referenced in all functions as B2, not 8%. Thus, a change in the rate requires only one alteration in the cell B2 entry, not in every relation where 8% is used. See Appendix A for additional information about using cell referencing and building spreadsheet relations.

- Figure 1–14a shows the results, and Figure 1–14b presents all spreadsheet relations for estimated interest and revenue (yearly in columns C and E, cumulative in columns D and F). As an illustration, for year 3 the interest I_3 and revenue plus interest R_3 are

$$\begin{aligned} I_3 &= (\text{cumulative revenue through year 2})(\text{rate of return}) \\ &= \$416,000(0.08) \\ &= \$33,280 \end{aligned}$$

$$\begin{aligned} R_3 &= \text{revenue in year 3} + I_3 \\ &= \$300,000 + 33,280 \\ &= \$333,280 \end{aligned}$$

The detailed relations shown in Figure 1–14b calculate these values in cells C8 and E8.

Cell C8 relation for I_3 : = F7*B2

Cell E8 relation for CF_3 : = B8 + C8

The equivalent amount after 4 years is \$1,109,022, which is comprised of \$1,000,000 in total revenue and \$109,022 in interest compounded at 8% per year. The shaded cells in Figure 1–14a and b indicate that the sum of the annual values and the last entry in the cumulative columns must be equal.

- To determine the effect of increasing estimated revenue for years 3 and 4 to \$600,000, use the same spreadsheet and change the entries in cells B8 and B9 as shown in Figure 1–14c. Total interest increases 22%, or \$24,000, from \$109,222 to \$133,222.
- Figure 1–14d shows the effect of changing the original i value from 8% to an inflation-adjusted rate of 3.85% in cell B2 on the first spreadsheet. [Remember to return to the \$300,000 revenue estimates for years 3 and 4 after working part (b).] Inflation has now reduced total interest by 53% from \$109,222 to \$51,247, as shown in cell C10.

Part (a) - Find totals in year 4					
End of Year	Revenue at end of year, \$	Interest earned during year, \$	Cumulative interest, \$	Revenue during year with interest, \$	Cumulative revenue with interest, \$
0					
1	200,000	0	0	200,000	200,000
2	200,000	16,000	16,000	216,000	416,000
3	300,000	33,280	49,280	333,280	749,280
4	300,000	59,942	109,222	359,942	1,109,222
		109,222		1,109,222	

(a) Total interest and revenue for base case, year 4

Part (a) - Find totals in year 4					
End of Year	Revenue at end of year, \$	Interest earned during year, \$	Cumulative interest, \$	Revenue during year with interest, \$	Cumulative revenue with interest, \$
0					
1	200000	0	-C6	-B6 + C6	-E6
2	200000	-F6*\$B\$1	-C7 + D6	-B7 + C7	-E7 + F6
3	300000	-F7*\$B\$1	-C8 + D7	-B8 + C8	-E8 + F7
4	300000	-F8*\$B\$1	-C9 + D8	-B9 + C9	-E9 + F8
				-SUM(E6:E9)	

(b) Spreadsheet relations for base case

Part (b) - Find totals in year 4 with increased revenues					
End of Year	Revenue at end of year, \$	Interest earned during year, \$	Cumulative interest, \$	Revenue during year with interest, \$	Cumulative revenue with interest, \$
0					
1	200,000	0	0	200,000	200,000
2	200,000	16,000	16,000	216,000	416,000
3	600,000	33,280	49,280	633,280	1,049,280
4	600,000	83,942	133,222	683,942	1,733,222
		133,222		1,733,222	

(c) Totals with increased revenue in years 3 and 4

Part (c) - Find totals in year 4 considering 4% inflation					
End of Year	Revenue at end of year, \$	Interest earned during year, \$	Cumulative interest, \$	Revenue during year with interest, \$	Cumulative revenue with interest, \$
0					
1	200,000	0	0	200,000	200,000
2	200,000	7,700	7,700	207,700	407,700
3	300,000	15,696	23,396	315,696	723,396
4	300,000	27,851	51,247	327,851	1,051,247
		51,247		1,051,247	

(d) Totals with inflation of 4% per year considered

Figure 1-14

Spreadsheet solutions with sensitivity analysis, Example 1.17a to c.

Comment

Later we will learn how to utilize the NPV and FV Excel financial functions to obtain the same answers determined in Figure 1-14, where we developed each basic relation.

When you are working with an Excel spreadsheet, it is possible to display all of the entries and functions on the screen as shown in Figure 1-14b by simultaneously touching the **<Ctrl>** and **<~>** keys, which may be in the upper left of the keyboard on the key with **<~>**.

CHAPTER SUMMARY

Engineering economy is the application of economic factors and criteria to evaluate alternatives, considering the time value of money. The engineering economy study involves computing a specific economic measure of worth for estimated cash flows over a specific period of time.

The concept of *equivalence* helps in understanding how different sums of money at different times are equal in economic terms. The differences between simple interest (based on principal only) and compound interest (based on principal and interest upon interest) have been described in formulas, tables, and graphs. This power of compounding is very noticeable, especially over extended periods of time, and for larger sums of money.

The MARR is a reasonable rate of return established as a hurdle rate to determine if an alternative is economically viable. The MARR is always higher than the return from a safe investment and the cost to acquire needed capital.

Also, we learned a lot about cash flows:

- End-of-year convention for cash flow location
- Net cash flow computation
- Different perspectives in determining the cash flow sign
- Construction of a cash flow diagram
- Difficulties with estimating future cash flows accurately

PROBLEMS

Basic Concepts

- 1.1 List the four essential elements involved in decision making in engineering economic analysis.
- 1.2 What is meant by (a) limited capital funds and (b) sensitivity analysis?
- 1.3 List three measures of worth that are used in engineering economic analysis.
- 1.4 Identify the following factors as either economic (tangible) or noneconomic (intangible): first cost, leadership, taxes, salvage value, morale, dependability, inflation, profit, acceptance, ethics, interest rate.

Ethics

- 1.5 Stefanie is a design engineer with an international railroad locomotive manufacturing company in Illinois. Management wants to return some of the engineering design work to the United States rather than export all of it to India, where the primary design work has been accomplished for the last decade. This transfer will employ more people locally and could improve the economic conditions for families in and around Illinois.

Stefanie and her design team were selected as a test case to determine the quality and speed of the design work they could demonstrate on a more fuel-efficient diesel locomotive. Neither she nor

any of her team members have done such a significant design job, because their jobs had previously entailed only the interface with the subcontracted engineers in India. One of her team members had a great design idea on a key element that will improve fuel efficiency by approximately 15%. She told Stefanie it came from one of the Indian-generated documents, but that it would probably be okay for the team to use it and remain silent as to its origin, since it was quite clear the U.S. management was about to cancel the foreign contract. Although reluctant at first, Stefanie did go forward with a design that included the efficiency improvement, and no mention of the origin of the idea was made at the time of the oral presentation or documentation delivery. As a result, the Indian contract was canceled and full design responsibility was transferred to Stefanie's group.

Consult the NSPE Code of Ethics for Engineers (Appendix C) and identify sections that are points of concern about Stefanie's decisions and actions.

- 1.6 Consider the common moral precept that stealing is wrong. Hector is with a group of friends in a local supermarket. One of Hector's buddies takes a high-energy drink from a six-pack on the shelf, opens it, drinks it, and returns the empty can to the package, with no intention of paying for it. He then invites the others to do the same, saying, "It's only

- one drink. Others do it all the time." All the others, except Hector, have now consumed a drink of their choice. Personally, Hector believes this is a form of stealing. State three actions that Hector can take, and evaluate them from the personal moral perspective.
- 1.7 While going to work this morning off site from his office, an engineer accidentally ran a stop sign and was in a car accident that resulted in the death of a 5-year-old child. He has a strong belief in the universal moral that it is wrong to do serious harm to another person. Explain the conflict that can arise for him between the universal moral and his personal moral about doing serious harm, given the accident was deemed his fault.
- 1.8 Claude is a fourth-year engineering university student who has just been informed by his instructor that he made a very low grade on his Spanish language final test for the year. Although he had a passing score prior to the final, his final grade was so low that he has now flunked the entire year and will likely have to extend his graduation another semester or two.
- Throughout the year, Claude, who hated the course and his instructor, has copied homework, cheated on tests, and never seriously studied for anything in the course. He did realize during the semester that he was doing something that even he considered wrong morally and ethically. He knew he had done badly on the final. The classroom was reconfigured for the final exam in a way that he could not get any answers from classmates, and cell phones were collected prior to the exam, thus removing texting possibilities to friends outside the classroom who might help him on the final exam. Claude is now face to face with the instructor in her office. The question to Claude is, "What have you been doing throughout this year to make passing scores repeatedly, but demonstrate such a poor command of Spanish on the final exam?"
- From an ethical viewpoint, what options does Claude have in his answer to this question? Also, discuss some of the possible effects that this experience may have upon Claude's future actions and moral dilemmas.
- ### Interest Rate and Rate of Return
- 1.9 RKI Instruments borrowed \$3,500,000 from a private equity firm for expansion of its manufacturing facility for making carbon monoxide monitors/controllers. The company repaid the loan after 1 year with a single payment of \$3,885,000. What was the interest rate on the loan?
- 1.10 Emerson Processing borrowed \$900,000 for installing energy-efficient lighting and safety equipment in its La Grange manufacturing facility. The terms of the loan were such that the company could pay *interest only* at the end of each year for up to 5 years, after which the company would have to pay the entire amount due. If the interest rate on the loan was 12% per year and the company paid only the interest for 4 years, determine the following:
- The amount of each of the four interest payments
 - The amount of the final payment at the end of year 5
- 1.11 Which of the following 1-year investments has the highest rate of return?
- \$12,500 that yields \$1125 in interest,
 - \$56,000 that yields \$6160 in interest, or
 - \$95,000 that yields \$7600 in interest.
- 1.12 A new engineering graduate who started a consulting business borrowed money for 1 year to furnish the office. The amount of the loan was \$23,800, and it had an interest rate of 10% per year. However, because the new graduate had not built up a credit history, the bank made him buy loan-default insurance that cost 5% of the loan amount. In addition, the bank charged a loan setup fee of \$300. What was the effective interest rate the engineer paid for the loan?
- 1.13 When the inflation rate is expected to be 8% per year, what is the market interest rate likely to be?
- ### Terms and Symbols
- 1.14 The symbol *P* represents an amount of money at a time designated as present. The following symbols also represent a present amount of money and require similar calculations. Explain what each symbol stands for: PW, PV, NPV, DCF, and CC.
- 1.15 Identify the four engineering economy symbols and their values from the following problem statement. Use a question mark with the symbol whose value is to be determined.
- Thompson Mechanical Products is planning to set aside \$150,000 now for possibly replacing its large synchronous refiner motors whenever it becomes necessary. If the replacement is not needed for 7 years, how much will the company have in its investment set-aside account, provided it achieves a rate of return of 11% per year?
- 1.16 Identify the four engineering economy symbols and their values from the following problem statement.

Use a question mark with the symbol whose value is to be determined.

Atlas Long-Haul Transportation is considering installing Valutemp temperature loggers in all of its refrigerated trucks for monitoring temperatures during transit. If the systems will reduce insurance claims by \$100,000 two years from now, how much should the company be willing to spend now, if it uses an interest rate of 12% per year?

- 1.17 Identify the four engineering economy symbols and their values from the following problem statement. Use a question mark with the symbol whose value is to be determined.

A green algae, *Chlamydomonas reinhardtii*, can produce hydrogen when temporarily deprived of sulfur for up to 2 days at a time. A small company needs to purchase equipment costing \$3.4 million to commercialize the process. If the company wants to earn a rate of return of 10% per year and recover its investment in 8 years, what must be the net value of the hydrogen produced each year?

- 1.18 Identify the four engineering economy symbols and their values from the following problem statement. Use a question mark with the symbol whose value is to be determined.

Vision Technologies, Inc., is a small company that uses ultra-wideband technology to develop devices that can detect objects (including people) inside of buildings, behind walls, or below ground. The company expects to spend \$100,000 per year for labor and \$125,000 per year for supplies before a product can be marketed. At an interest rate of 15% per year, what is the total equivalent future amount of the company's expenses at the end of 3 years?

Cash Flows

- 1.19 What is meant by end-of-period convention?

- 1.20 Identify the following as cash inflows or outflows to commercial air carriers: fuel cost, pension plan contributions, fares, maintenance, freight revenue, cargo revenue, extra-bag charges, water and sodas, advertising, landing fees, seat preference fees.

- 1.21 Many credit unions use semiannual interest periods to pay interest on customer savings accounts. For a credit union that uses June 30 and December 31 as its semiannual interest periods, determine the end-of-period amounts that will be recorded for the deposits shown in the table.

Month	Deposit, \$
Jan	50
Feb	70
Mar	—
Apr	120
May	20
June	—
July	150
Aug	90
Sept	—
Oct	—
Nov	40
Dec	110

- 1.22 For a company that uses a year as its interest period, determine the *net cash flow* that will be recorded at the *end of the year* from the cash flows shown.

Month	Receipts, \$1000	Disbursements, \$1000
Jan	500	300
Feb	800	500
Mar	200	400
Apr	120	400
May	600	500
June	900	600
July	800	300
Aug	700	300
Sept	900	500
Oct	500	400
Nov	400	400
Dec	1800	700

- 1.23 Construct a cash flow diagram for the following cash flows: \$25,000 outflow at time 0, \$9000 per year inflow in years 1 through 5 at an interest rate of 10% per year, and an unknown future amount in year 5.

- 1.24 Construct a cash flow diagram to find the present worth in year 0 at an interest rate of 15% per year for the following situation.

Year	Cash Flow, \$
0	-19,000
1-4	+8,100

- 1.25 Construct a cash flow diagram that represents the amount of money that will be accumulated in 15 years from an investment of \$40,000 now at an interest rate of 8% per year.

Equivalence

- 1.26 At an interest rate of 15% per year, an investment of \$100,000 one year ago is equivalent to how much now?

- 1.27 During a recession, the price of goods and services goes down because of low demand. A company that makes Ethernet adapters is planning to expand its production facility at a cost of \$1,000,000 one year from now. However, a contractor who needs work has offered to do the job for \$790,000 if the company will do the expansion now instead of 1 year from now. If the interest rate is 15% per year, how much of a discount is the company getting?
- 1.28 As a principal in the consulting firm where you have worked for 20 years, you have accumulated 5000 shares of company stock. One year ago, each share of stock was worth \$40. The company has offered to buy back your shares for \$225,000. At what interest rate would the firm's offer be equivalent to the worth of the stock last year?
- 1.29 A design/build engineering company that usually gives year-end bonuses in the amount of \$8000 to each of its engineers is having cash flow problems. The company said that although it could not give bonuses this year, it would give each engineer two bonuses next year, the regular one of \$8000 plus an amount equivalent to the \$8000 that each engineer should have gotten this year. If the interest rate is 8% per year, what will be the total amount of bonus money the engineers should get next year?
- 1.30 University tuition and fees can be paid by using one of two plans.
 Early-bird: Pay total amount due *1 year in advance* and get a 10% discount.
 On-time: Pay total amount due when classes start.
 The cost of tuition and fees is \$10,000 per year.
 (a) How much is paid in the early-bird plan?
 (b) What is the equivalent amount of the savings compared to the on-time payment at the time that the on-time payment is made?

Simple and Compound Interest

- 1.31 If a company sets aside \$1,000,000 now into a contingency fund, how much will the company have in 2 years, if it does not use any of the money and the account grows at a rate of 10% per year?
- 1.32 Iselt Welding has extra funds to invest for future capital expansion. If the selected investment pays simple interest, what interest rate would be required for the amount to grow from \$60,000 to \$90,000 in 5 years?
- 1.33 To finance a new product line, a company that makes high-temperature ball bearings borrowed \$1.8 million at 10% per year interest. If the com-

pany repaid the loan in a lump sum amount after 2 years, what was (a) the amount of the payment and (b) the amount of interest?

- 1.34 Because market interest rates were near all-time lows at 4% per year, a hand tool company decided to call (i.e., pay off) the high-interest bonds that it issued 3 years ago. If the interest rate on the bonds was 9% per year, how much does the company have to pay the bond holders? The face value (principal) of the bonds is \$6,000,000.
- 1.35 A solid waste disposal company borrowed money at 10% per year interest to purchase new haulers and other equipment needed at the company-owned landfill site. If the company got the loan 2 years ago and paid it off with a single payment of \$4,600,000, what was the principal amount P of the loan?
- 1.36 If interest is compounded at 20% per year, how long will it take for \$50,000 to accumulate to \$86,400?
- 1.37 To make CDs look more attractive than they really are, some banks advertise that their rates are higher than their competitors' rates; however, the fine print says that the rate is a simple interest rate. If a person deposits \$10,000 at 10% per year simple interest, what compound interest rate would yield the same amount of money in 3 years?

MARR and Opportunity Cost

- 1.38 Give three other names for minimum attractive rate of return.
- 1.39 Identify the following as either equity or debt financing: bonds, stock sales, retained earnings, venture capital, short-term loan, capital advance from friend, cash on hand, credit card, home equity loan.
- 1.40 What is the weighted average cost of capital for a corporation that finances an expansion project using 30% retained earnings and 70% venture capital? Assume the interest rates are 8% for the equity financing and 13% for the debt financing.
- 1.41 Managers from different departments in Zenith Trading, a large multinational corporation, have offered six projects for consideration by the corporate office. A staff member for the chief financial officer used key words to identify the projects and then listed them in order of projected rate of return as shown below. If the company wants to grow rapidly through high leverage and uses only 10% equity financing that has a cost of equity capital of 9% and 90% debt financing with a cost of debt

capital of 16%, which projects should the company undertake?

Project ID	Projected ROR, % per year
Inventory	30
Technology	28.4
Warehouse	19
Products	13.1
Energy	9.6
Shipping	8.2

Spreadsheet Functions

- 1.42 State the purpose for each of the following built-in spreadsheet functions.

- (a) PV($i\%,n,A,F$)
- (b) FV($i\%,n,A,P$)
- (c) RATE(n,A,P,F)
- (d) IRR(first_cell:last_cell)
- (e) PMT($i\%,n,P,F$)
- (f) NPER($i\%,A,P,F$)

- 1.43 What are the values of the engineering economy symbols P , F , A , i , and n in the following functions? Use a question mark for the symbol that is to be

determined.

- (a) NPER(8%, -1500, 8000, 2000)
- (b) FV(7%, 102000, -9000)
- (c) RATE(10, 1000, -12000, 2000)
- (d) PMT(11%, 20, , 14000)
- (e) PV(8%, 15, -1000, 800)

- 1.44 Write the engineering economy symbol that corresponds to each of the following spreadsheet functions.
 (a) PMT (b) FV (c) NPER (d) PV (e) IRR

- 1.45 In a built-in spreadsheet function, if a certain parameter is not present, (a) under what circumstances can it be left blank and (b) when must a comma be entered in its place?

- 1.46 Sheryl and Marcella both invest \$1000 at 10% per year for 4 years. Sheryl receives simple interest and Marcella gets compound interest. Use a spreadsheet and cell reference formats to develop relations that show a total of \$64 more interest for Marcella at the end of the 4 years. Assume no withdrawals or further deposits are made during the 4 years.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 1.47 The concept that different sums of money at different points in time can be said to be equal to each other is known as:

- (a) Evaluation criterion
- (b) Equivalence
- (c) Cash flow
- (d) Intangible factors

- 1.48 The evaluation criterion that is usually used in an economic analysis is:

- (a) Time to completion
- (b) Technical feasibility
- (c) Sustainability
- (d) Financial units (dollars or other currency)

- 1.49 All of the following are examples of cash outflows, except:

- (a) Asset salvage value
- (b) Income taxes
- (c) Operating cost of asset
- (d) First cost of asset

- 1.50 In most engineering economy studies, the best alternative is the one that:

- (a) Will last the longest time
- (b) Is most politically correct
- (c) Is easiest to implement
- (d) Has the lowest cost

- 1.51 The following annual maintenance and operation (M&O) costs for a piece of equipment were collected over a 5-year period: \$12,300, \$8900, \$9200, \$11,000, and \$12,100. The average is \$10,700. In conducting a sensitivity analysis, the most reasonable range of costs to use (i.e., percent from the average) is:

- (a) $\pm 5\%$ (b) $\pm 11\%$ (c) $\pm 17\%$ (d) $\pm 25\%$

- 1.52 At an interest rate of 10% per year, the equivalent amount of \$10,000 *one year ago* is closest to:

- (a) \$8264 (b) \$9091 (c) \$11,000 (d) \$12,000

- 1.53 Assume that you and your best friend each have \$1000 to invest. You invest your money in a fund that pays 10% per year *compound* interest. Your friend invests her money at a bank that pays 10% per year *simple* interest. At the end of 1 year, the difference in the total amount for each of you is:

- (a) You have \$10 more than she does
- (b) You have \$100 more than she does
- (c) You both have the same amount of money
- (d) She has \$10 more than you do

- 1.54 The time it would take for a given sum of money to double at 4% per year *simple* interest is closest to:

- (a) 30 years (b) 25 years
- (c) 20 years (d) 10 years

- 1.55 All of the following are examples of equity financing, *except*:
- Mortgage
 - Money from savings
 - Cash on hand
 - Retained earnings

- 1.56 To finance a new project costing \$30 million, a company borrowed \$21 million at 16% per year interest and used retained earnings valued at 12% per year for the remainder of the investment. The company's weighted average cost of capital for the project was closest to:
- 12.5%
 - 13.6%
 - 14.8%
 - 15.6%

CASE STUDY

RENEWABLE ENERGY SOURCES FOR ELECTRICITY GENERATION

Background

Pedernales Electric Cooperative (PEC) is the largest member-owned electric co-op in the United States with over 232,000 meters in 12 Central Texas counties. PEC has a capacity of approximately 1300 MW (megawatts) of power, of which 277 MW, or about 21%, is from renewable sources. The latest addition is 60 MW of power from a wind farm in south Texas close to the city of Corpus Christi. A constant question is how much of PEC's generation capacity should be from renewable sources, especially given the environmental issues with coal-generated electricity and the rising costs of hydrocarbon fuels.

Wind and nuclear sources are the current consideration for the PEC leadership as Texas is increasing its generation by nuclear power and the state is the national leader in wind farm-produced electricity.

Consider yourself a member of the board of directors of PEC. You are an engineer who has been newly elected by the PEC membership to serve a 3-year term as a director-at-large. As such, you do not represent a specific district within the entire service area; all other directors do represent a specific district. You have many questions about the operations of PEC, plus you are interested in the economic and societal benefits of pursuing more renewable source generation capacity.

Information

Here are some data that you have obtained. The information is sketchy, as this point, and the numbers are very approximate. Electricity generation cost estimates are national in scope, not PEC-specific, and are provided in cents per kilowatt-hour ($\text{¢}/\text{kWh}$).

Generation Cost, $\text{¢}/\text{kWh}$

Fuel Source	Likely Range	Reasonable Average
Coal	4 to 9	7.4
Natural gas	4 to 10.5	8.6
Wind	4.8 to 9.1	8.2
Solar	4.5 to 15.5	8.8

National average cost of electricity to residential customers: 11¢/kWh

PEC average cost to residential customers: 10.27 ¢/kWh (from primary sources) and 10.92 ¢/kWh (renewable sources)

Expected life of a generation facility: 20 to 40 years (it is likely closer to 20 than 40)

Time to construct a facility: 2 to 5 years

Capital cost to build a generation facility: \$900 to \$1500 per kW

You have also learned that the PEC staff uses the well-recognized *levelized energy cost* (LEC) method to determine the price of electricity that must be charged to customers to break even. The formula takes into account the capital cost of the generation facilities, the cost of capital of borrowed money, annual maintenance and operation (M&O) costs, and the expected life of the facility. The LEC formula, expressed in dollars per kWh for $(t = 1, 2, \dots, n)$, is

$$\text{LEC} = \frac{\sum_{t=1}^n P_t + A_t + C_t}{\sum_{t=1}^n \frac{E_t}{(1+i)^t}}$$

where P_t = capital investments made in year t

A_t = annual maintenance and operating (M&O) costs for year t

C_t = fuel costs for year t

E_t = amount of electricity generated in year t

n = expected life of facility

i = discount rate (cost of capital)

Case Study Exercises

- If you wanted to know more about the new arrangement with the wind farm in south Texas for the additional 60 MW per year, what types of questions would you ask of a staff member in your first meeting with him or her?
- Much of the current generation capacity of PEC facilities utilizes coal and natural gas as the primary fuel source. What about the ethical aspects of the government's allowance for these plants to continue polluting the atmosphere with the emissions that may cause health problems for citizens and further the effects of global warming? What types of regulations, if any, should be developed for PEC (and other generators) to follow in the future?

3. You developed an interest in the LEC relation and the publicized cost of electricity of 10.27¢/kWh for this year. You wonder if the addition of 60 MW of wind-sourced electricity will make any difference in the LEC value for this next year. You did learn the following:

This is year $t = 11$ for LEC computation purposes

$n = 25$ years

$i = 5\%$ per year

$$E_{11} = 5.052 \text{ billion kWh}$$

LEC last year was 10.22 ¢/kWh (last year's breakeven cost to customers)

From these sketchy data, can you determine the value of unknowns in the LEC relation for this year? Is it possible to determine if the wind farm addition of 60 MW makes any difference in the electricity rate charged to customers? If not, what additional information is necessary to determine the LEC with the wind source included?

CASE STUDY

REFRIGERATOR SHELLS

Background

Large refrigerator manufacturers such as Whirlpool, General Electric, Frigidaire, and others may subcontract the molding of their plastic liners and door panels. One prime national subcontractor is Innovations Plastics. Because of improvements in mechanical properties, the molded plastic can sustain increased vertical and horizontal loading, thus significantly reducing the need for attached metal anchors for some shelving. However, improved molding equipment is needed to enter this market now. The company president wants a recommendation on whether Innovations should offer the new technology to the major manufacturers and an estimate of the necessary capital investment to enter this market.

You work as an engineer for Innovations. At this stage, you are not expected to perform a complete engineering economic analysis, for not enough information is available. You are asked to formulate reasonable alternatives, determine what data and estimates are needed for each one, and ascertain what criteria (economic and noneconomic) should be utilized to make the final decision.

Information

Some information useful at this time is as follows:

- The technology and equipment are expected to last about 10 years before new methods are developed.
- Inflation and income taxes will not be considered in the analysis.
- The expected returns on capital investment used for the last three new technology projects were compound rates of 15%, 5%, and 18%. The 5% rate was the criterion for

enhancing an employee-safety system on an existing chemical-mixing process.

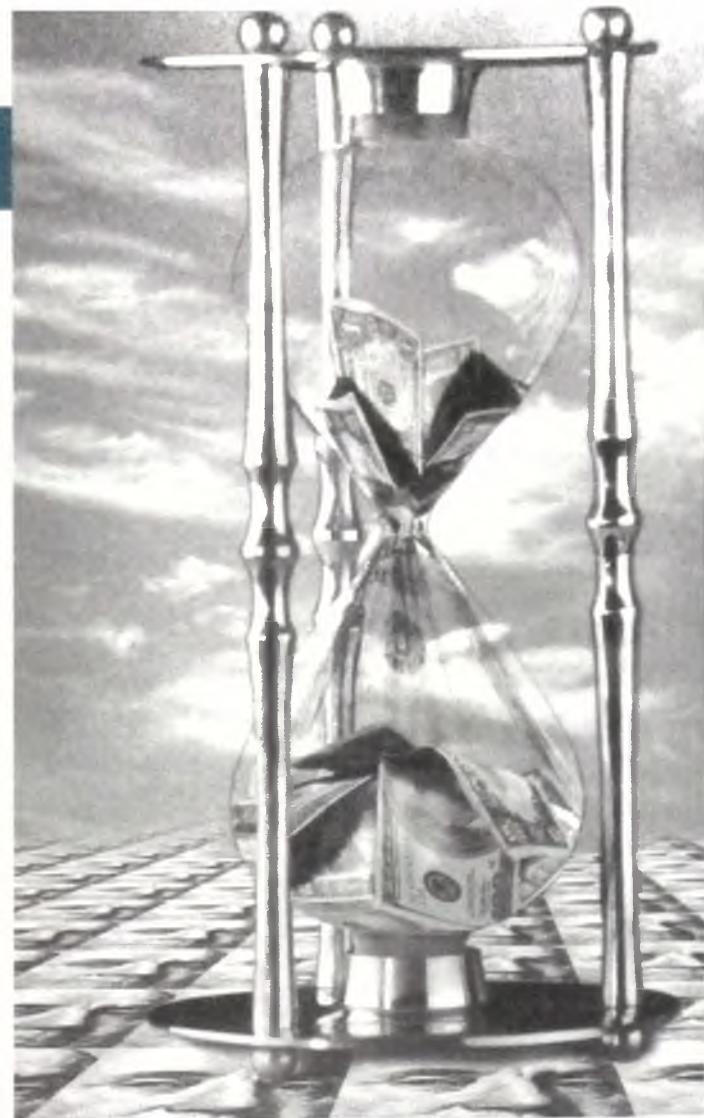
- Equity capital financing beyond \$5 million is not possible. The amount of debt financing and its cost are unknown.
- Annual operating costs have been averaging 8% of first cost for major equipment.
- Increased annual training costs and salary requirements for handling the new plastics and operating new equipment can range from \$800,000 to \$1.2 million.

There are two manufacturers working on the new-generation equipment. You label these options as alternatives A and B.

Case Study Exercises

- Use the first four steps of the decision-making process to generally describe the alternatives and identify what economic-related estimates you will need to complete an engineering economy analysis for the president.
- Identify any noneconomic factors and criteria to be considered in making the alternative selection.
- During your inquiries about alternative B from its manufacturer, you learn that this company has already produced a prototype molding machine and has sold it to a company in Germany for \$3 million (U.S. dollars). Upon inquiry, you further discover that the German company already has unused capacity on the equipment for manufacturing plastic shells. The company is willing to sell time on the equipment to Innovations immediately to produce its own shells for U.S. delivery. This could allow immediate market entry into the United States. Consider this as alternative C, and develop the estimates necessary to evaluate C at the same time as alternatives A and B.

Factors: How Time and Interest Affect Money



LEARNING OUTCOMES

Purpose: Derive and use the engineering economy factors to account for the time value of money.

SECTION	TOPIC	LEARNING OUTCOME
2.1	F/P and P/F factors	<ul style="list-style-type: none">Derive and use factors for single amounts—compound amount (F/P) and present worth (P/F) factor.
2.2	P/A and A/P factors	<ul style="list-style-type: none">Derive and use factors for uniform series—present worth (P/A) and capital recovery (A/P) factors.
2.3	F/A and A/F factors	<ul style="list-style-type: none">Derive and use factors for uniform series—compound amount (F/A) and sinking fund (A/F) factors.
2.4	Factor values	<ul style="list-style-type: none">Use linear interpolation in factor tables or spreadsheet functions to determine factor values.
2.5	Arithmetic gradient	<ul style="list-style-type: none">Use the present worth (P/G) and uniform annual series (A/G) factors for arithmetic gradients.
2.6	Geometric gradient	<ul style="list-style-type: none">Use the geometric gradient series factor ($P/A,g$) to find present worth.
2.7	Find i or n	<ul style="list-style-type: none">Use equivalence relations to determine i (interest rate or rate of return) or n for a cash flow series.



The cash flow is fundamental to every economic study. Cash flows occur in many configurations and amounts—isolated single values, series that are uniform, and series that increase or decrease by constant amounts or constant percentages.

This chapter develops derivations for all the commonly used engineering economy factors that take the time value of money into account.

The application of factors is illustrated using their mathematical forms and a standard notation format. Spreadsheet functions are used in order to rapidly work with cash flow series and to perform sensitivity analysis.

If the derivation and use of factors are not covered in the course, alternate ways to perform time value of money calculations are summarized in Appendix D.

PE

The Cement Factory Case: Votorantim Cimentos North America, Inc., is a subsidiary of a Brazil-based company that recently announced plans to develop a new cement factory in Houston County in the state of Georgia. The plant will be called Houston American Cement, or HAC. The location is ideal for cement making because of the large deposit of limestone in the area.

The plant investment, expected to amount to \$200 million, has been planned for 2012; however, it is currently delayed due to the economic downturn in construction. When the plant is completed and operating at full capacity,

based upon the projected needs and cost per metric ton, it is possible that the plant could generate as much as \$50,000,000 annually in revenue. All analysis will use a planning horizon of 5 years commencing when the plant begins operation.

This case is used in the following topics (and sections) of this chapter:

- Single-amount factors (2.1)
- Uniform series factors (2.2 and 2.3)
- Arithmetic gradient factors (2.5)
- Geometric gradient factors (2.6)
- Determining unknown n values (2.7)

2.1 Single-Amount Factors (F/P and P/F) ● ● ●

The **most fundamental factor in engineering economy** is the one that determines the amount of money F accumulated after n years (or periods) from a *single* present worth P , with interest compounded one time per year (or period). Recall that compound interest refers to interest paid on top of interest. Therefore, if an amount P is invested at time $t = 0$, the amount F_1 accumulated 1 year hence at an interest rate of i percent per year will be

$$\begin{aligned} F_1 &= P + Pi \\ &= P(1 + i) \end{aligned}$$

where the interest rate is expressed in decimal form. At the end of the second year, the amount accumulated F_2 is the amount after year 1 plus the interest from the end of year 1 to the end of year 2 on the entire F_1 .

$$\begin{aligned} F_2 &= F_1 + F_1i \\ &= P(1 + i) + P(1 + i)i \\ &= P(1 + i)^2 \end{aligned} \tag{2.1}$$

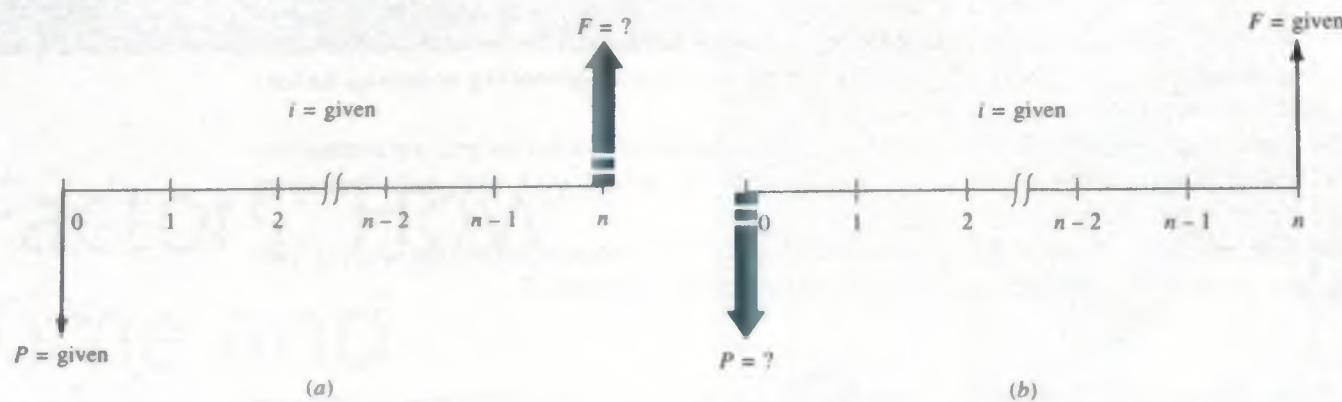
The amount F_2 can be expressed as

$$\begin{aligned} F_2 &= P(1 + i + i + i^2) \\ &= P(1 + 2i + i^2) \\ &= P(1 + i)^2 \end{aligned}$$

Similarly, the amount of money accumulated at the end of year 3, using Equation [2.1], will be

$$F_3 = F_2 + F_2i$$



**Figure 2-1**

Cash flow diagrams for single-payment factors: (a) find F , given P , and (b) find P , given F .

Substituting $P(1 + i)^2$ for F_2 and simplifying, we get

$$F_3 = P(1 + i)^3$$

From the preceding values, it is evident by mathematical induction that the formula can be generalized for n years. To find F , given P ,

$$F = P(1 + i)^n \quad [2.2]$$

The factor $(1 + i)^n$ is called the *single-payment compound amount factor* (SPCAF), but it is usually referred to as the **F/P factor**. This is the conversion factor that, when multiplied by P , yields the future amount F of an initial amount P after n years at interest rate i . The cash flow diagram is seen in Figure 2-1a.

Reverse the situation to determine the **P value for a stated amount F** that occurs n periods in the future. Simply solve Equation [2.2] for P .

$$P = F \left[\frac{1}{(1 + i)^n} \right] = F(1 + i)^{-n} \quad [2.3]$$

The expression $(1 + i)^{-n}$ is known as the *single-payment present worth factor* (SPPWF), or the **P/F factor**. This expression determines the present worth P of a given future amount F after n years at interest rate i . The cash flow diagram is shown in Figure 2-1b.

Note that the two factors derived here are for *single payments*; that is, they are used to find the present or future amount when only one payment or receipt is involved.

A standard notation has been adopted for all factors. The notation includes two cash flow symbols, the interest rate, and the number of periods. It is always in the general form $(X/Y,i,n)$. The letter X represents what is sought, while the letter Y represents what is given. For example, F/P means *find F when given P*. The i is the interest rate in percent, and n represents the number of periods involved.

Using this notation, $(F/P, 6\%, 20)$ represents the factor that is used to calculate the future amount F accumulated in 20 periods if the interest rate is 6% per period. The P is given. The standard notation, simpler to use than formulas and factor names, will be used hereafter.

Table 2-1 summarizes the standard notation and equations for the F/P and P/F factors. This information is also included inside the front cover.

TABLE 2-1 F/P and P/F Factors: Notation and Equations

Notation	Factor	Find/Given	Standard Notation Equation	Equation with Factor Formula	Excel Function
$(F/P,i,n)$	Single-payment compound amount	F/P	$F = P(F/P,i,n)$	$F = P(1 + i)^n$	$= FV(i\%,n,,P)$
$(P/F,i,n)$	Single-payment present worth	P/F	$P = F(P/F,i,n)$	$P = F(1 + i)^{-n}$	$= PV(i\%,n,,F)$

To simplify routine engineering economy calculations, tables of factor values have been prepared for interest rates from 0.25% to 50% and time periods from 1 to large n values, depending on the i value. These tables, found at the rear of the book, have a colored edge for easy identification. They are arranged with factors across the top and the number of periods n down the left side. The word *discrete* in the title of each table emphasizes that these tables utilize the end-of-period convention and that interest is compounded once each interest period. For a given factor, interest rate, and time, the correct factor value is found at the intersection of the factor name and n . For example, the value of the factor ($P/F, 5\%, 10$) is found in the P/F column of Table 10 at period 10 as 0.6139. This value is determined by using Equation [2.3].

$$\begin{aligned}(P/F, 5\%, 10) &= \frac{1}{(1+i)^n} \\ &= \frac{1}{(1.05)^{10}} \\ &= \frac{1}{1.6289} = 0.6139\end{aligned}$$

For spreadsheets, a future value F is calculated by the FV function using the format

$$= FV(i\%, n, P) \quad [2.4]$$

A present amount P is determined using the PV function with the format

$$= PV(i\%, n, F) \quad [2.5]$$

These functions are included in Table 2-1. Refer to Appendix A or Excel online help for more information on the use of FV and PV functions.

EXAMPLE 2.1

Sandy, a manufacturing engineer, just received a year-end bonus of \$10,000 that will be invested immediately. With the expectation of earning at the rate of 8% per year, Sandy hopes to take the entire amount out in exactly 20 years to pay for a family vacation when the oldest daughter is due to graduate from college. Find the amount of funds that will be available in 20 years by using (a) hand solution by applying the factor formula and tabulated value and (b) a spreadsheet function.

Solution

The cash flow diagram is the same as Figure 2-1a. The symbols and values are

$$P = \$10,000 \quad F = ? \quad i = 8\% \text{ per year} \quad n = 20 \text{ years}$$

(a) *Factor formula:* Apply Equation [2.2] to find the future value F . Rounding to four decimals, we have

$$\begin{aligned}F &= P(1 + i)^n = 10,000(1.08)^{20} = 10,000(4.6610) \\ &= \$46,610\end{aligned}$$

Standard notation and tabulated value: Notation for the F/P factor is ($F/P, i\%, n$).

$$\begin{aligned}F &= P(F/P, 8\%, 20) = 10,000(4.6610) \\ &= \$46,610\end{aligned}$$

Table 13 provides the tabulated value. Round-off errors can cause a slight difference in the final answer between these two methods.

(b) *Spreadsheet:* Use the FV function to find the amount 20 years in the future. The format is that shown in Equation [2.4]; the numerical entry is = FV(8%, 20, , 10000). The spreadsheet will appear similar to that in the right side of Figure 1-13, with the answer (\$46,609.57) displayed. (You should try it on your own computer now.) The FV function has performed the computation in part (a) and displayed the result.

The equivalency statement is: If Sandy invests \$10,000 now and earns 8% per year every year for 20 years, \$46,610 will be available for the family vacation.

EXAMPLE 2.2 The Cement Factory Case

PE

As discussed in the introduction to this chapter, the Houston American Cement factory will require an investment of \$200 million to construct. Delays beyond the anticipated implementation year of 2012 will require additional money to construct the factory. Assuming that the cost of money is 10% per year, compound interest, use both **tabulated factor values** and **spreadsheet functions** to determine the following for the board of directors of the Brazilian company that plans to develop the plant.

- The equivalent investment needed if the plant is built in 2015.
- The equivalent investment needed had the plant been constructed in the year 2008.

Solution

Figure 2–2 is a cash flow diagram showing the expected investment of \$200 million (\$200 M) in 2012, which we will identify as time $t = 0$. The required investments 3 years in the future and 4 years in the past are indicated by $F_3 = ?$ and $P_{-4} = ?$, respectively.

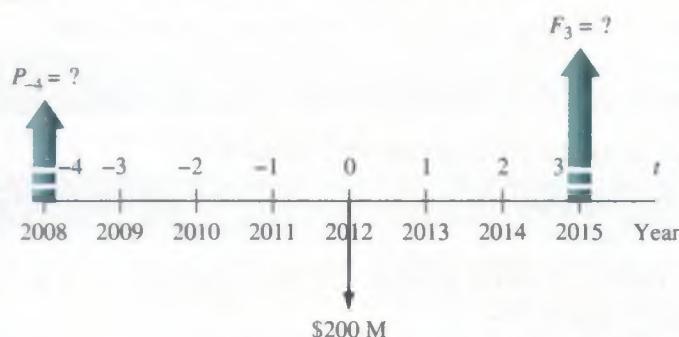


Figure 2–2
Cash flow diagram for
Example 2.2a and b.

- To find the equivalent investment required in 3 years, apply the F/P factor. Use \$1 million units and the tabulated value for 10% interest (Table 15).

$$\begin{aligned} F_3 &= P(F/P,i,n) = 200(F/P,10\%,3) = 200(1.3310) \\ &= \$266.2 \quad (\$266,200,000) \end{aligned}$$

Now, use the FV function on a spreadsheet to find the same answer, $F_3 = \$266.20$ million. (Refer to Figure 2–3, left side.)

A	B	C	D	E	F	G	H	I	J	K
1										
2	$P = \$200$	$i = 10\%$		$n = 3$		$F = \$200$		$i = 10\%$		$n = 4$
3										
4										
5	$F_3 = (\$266.20)$	\leftarrow	$= FV(10\%,3,,200)$			$P_{-4} = (\$136.60)$	\leftarrow	$= PV(10\%,4,,200)$		
6										
7										

Figure 2–3
Spreadsheet functions for Example 2.2.

- The year 2008 is 4 years prior to the planned construction date of 2012. To determine the equivalent cost 4 years earlier, consider the \$200 M in 2012 ($t = 0$) as the future value F and apply the P/F factor for $n = 4$ to find P_{-4} . (Refer to Figure 2–2.) Table 15 supplies the tabulated value.

$$\begin{aligned} P_{-4} &= F(P/F,i,n) = 200(P/F,10\%,4) = 200(0.6830) \\ &= \$136.6 \quad (\$136,600,000) \end{aligned}$$

The PV function = $PV(10\%,4,,200)$ will display the same amount as shown in Figure 2–3, right side.

This equivalence analysis indicates that at \$136.6 M in 2008, the plant would have cost about 68% as much as in 2012, and that waiting until 2015 will cause the price tag to increase about 33% to \$266 M.

2.2 Uniform Series Present Worth Factor and Capital Recovery Factor (P/A and A/P) ●●●

The equivalent present worth P of a uniform series A of end-of-period cash flows (investments) is shown in Figure 2–4a. An expression for the present worth can be determined by considering each A value as a future worth F , calculating its present worth with the P/F factor, Equation [2.3], and summing the results.

$$\begin{aligned} P &= A\left[\frac{1}{(1+i)^1}\right] + A\left[\frac{1}{(1+i)^2}\right] + A\left[\frac{1}{(1+i)^3}\right] + \dots \\ &\quad + A\left[\frac{1}{(1+i)^{n-1}}\right] + A\left[\frac{1}{(1+i)^n}\right] \end{aligned}$$

The terms in brackets are the P/F factors for years 1 through n , respectively. Factor out A .

$$P = A\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] \quad [2.6]$$

To simplify Equation [2.6] and obtain the P/A factor, multiply the n -term geometric progression in brackets by the $(P/F, i\%, 1)$ factor, which is $1/(1+i)$. This results in Equation [2.7]. Now subtract the two equations, [2.6] from [2.7], and simplify to obtain the expression for P when $i \neq 0$ (Equation [2.8]).

$$\frac{P}{1+i} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4} + \dots + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}}\right] \quad [2.7]$$

$$\begin{aligned} \frac{1}{1+i}P &= A\left[\cancel{\frac{1}{(1+i)^2}} + \cancel{\frac{1}{(1+i)^3}} + \dots + \cancel{\frac{1}{(1+i)^n}} + \frac{1}{(1+i)^{n+1}}\right] \\ - & \quad P = A\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] \\ \frac{-i}{1+i}P &= A\left[\frac{1}{(1+i)^{n+1}} - \frac{1}{(1+i)^1}\right] \\ P &= \frac{A}{-i}\left[\frac{1}{(1+i)^n} - 1\right] \end{aligned}$$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad i \neq 0 \quad [2.8]$$

The term in brackets in Equation [2.8] is the conversion factor referred to as the *uniform series present worth factor* (USPWF). It is the P/A factor used to calculate the *equivalent P value in year 0* for a uniform end-of-period series of A values beginning at the end of period 1 and extending for n periods. The cash flow diagram is Figure 2–4a.

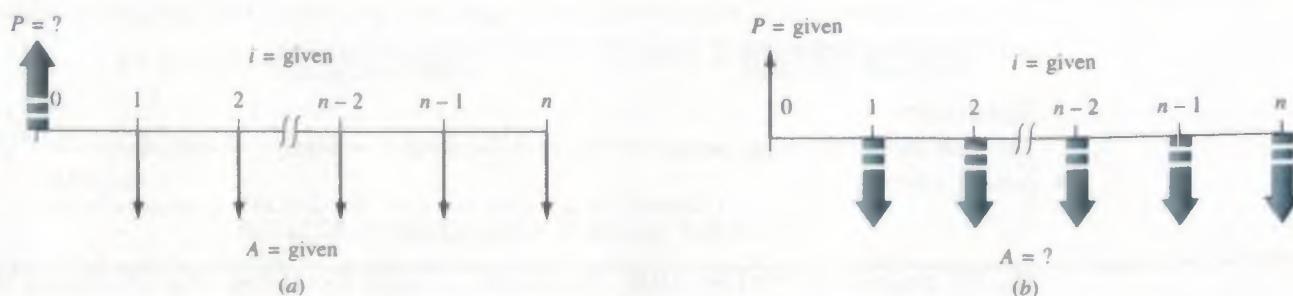


Figure 2–4

Cash flow diagrams used to determine (a) P , given a uniform series A , and (b) A , given a present worth P .



TABLE 2–2 P/A and A/P Factors: Notation and Equations

Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Function
(P/A, i, n)	Uniform series present worth	P/A	$\frac{(1 + i)^n - 1}{i(1 + i)^n}$	$P = A(P/A, i, n)$	= PV(i%, n, A)
(A/P, i, n)	Capital recovery	A/P	$\frac{i(1 + i)^n}{(1 + i)^n - 1}$	$A = P(A/P, i, n)$	= PMT(i%, n, P)

To reverse the situation, the present worth P is known and the equivalent uniform series amount A is sought (Figure 2–4b). The first A value occurs at the end of period 1, that is, one period after P occurs. Solve Equation [2.8] for A to obtain

$$A = P \left[\frac{i(1 + i)^n}{(1 + i)^n - 1} \right] \quad [2.9]$$

The term in brackets is called the *capital recovery factor* (CRF), or **A/P factor**. It calculates the *equivalent uniform annual worth* A over n years for a given P in year 0, when the interest rate is i .



The P/A and A/P factors are derived with the present worth P and the first uniform annual amount A *one year (period) apart*. That is, the present worth P *must always be located one period prior to the first A*.

The factors and their use to find P and A are summarized in Table 2–2 and inside the front cover. The standard notations for these two factors are $(P/A, i\%, n)$ and $(A/P, i\%, n)$. Tables at the end of the text include the factor values. As an example, if $i = 15\%$ and $n = 25$ years, the P/A factor value from Table 19 is $(P/A, 15\%, 25) = 6.4641$. This will find the equivalent present worth at 15% per year for any amount A that occurs uniformly from years 1 through 25.

Spreadsheet functions can determine both P and A values in lieu of applying the P/A and A/P factors. The PV function calculates the P value for a given A over n years and a separate F value in year n , if it is given. The format, is

$$= PV(i\%, n, A, F) \quad [2.10]$$

Similarly, the A value is determined by using the PMT function for a given P value in year 0 and a separate F , if given. The format is

$$= PMT(i\%, n, P, F) \quad [2.11]$$

Table 2–2 includes the PV and PMT functions.

EXAMPLE 2.3

How much money should you be willing to pay now for a guaranteed \$600 per year for 9 years starting next year, at a rate of return of 16% per year?

Solution

The cash flows follow the pattern of Figure 2–4a, with $A = \$600$, $i = 16\%$, and $n = 9$. The present worth is

$$P = 600(P/A, 16\%, 9) = 600(4.6065) = \$2763.90$$

The PV function = $PV(16\%, 9, 600)$ entered into a single spreadsheet cell will display the answer $P = (\$2763.93)$.

EXAMPLE 2.4 The Cement Factory Case

PE

As mentioned in the chapter introduction of this case, the Houston American Cement plant may generate a revenue base of \$50 million per year. The president of the Brazilian parent company Votorantim Cimentos may have reason to be quite pleased with this projection for the simple reason that over the 5-year planning horizon, the expected revenue would total \$250 million, which is \$50 million more than the initial investment. With money worth 10% per year, address the following question from the president: Will the initial investment be recovered over the 5-year horizon with the time value of money considered? If so, by how much extra in present worth funds? If not, what is the equivalent annual revenue base required for the recovery plus the 10% return on money? Use both tabulated factor values and spreadsheet functions.

Solution

Tabulated value: Use the P/A factor to determine whether $A = \$50$ million per year for $n = 5$ years starting 1 year after the plant's completion ($t = 0$) at $i = 10\%$ per year is equivalently less or greater than \$200 M. The cash flow diagram is similar to Figure 2-4a, where the first A value occurs 1 year after P . Using \$1 million units and Table 15 values,

$$\begin{aligned} P &= 50(P/A, 10\%, 5) = 50(3.7908) \\ &= \$189.54 \quad (\$189,540,000) \end{aligned}$$

The present worth value is less than the investment plus a 10% per year return, so the president should not be satisfied with the projected annual revenue.

To determine the minimum required to realize a 10% per year return, use the A/P factor. The cash flow diagram is the same as Figure 2-4b, where A starts 1 year after P at $t = 0$ and $n = 5$.

$$\begin{aligned} A &= 200(A/P, 10\%, 5) = 200(0.26380) \\ &= \$52.76 \text{ per year} \end{aligned}$$

The plant needs to generate \$52,760,000 per year to realize a 10% per year return over 5 years.

Spreadsheet: Apply the PV and PMT functions to answer the question. Figure 2-5 shows the use of $= PV(i\%, n, A, F)$ on the left side to find the present worth and the use of $= PMT(i\%, n, P, F)$ on the right side to determine the minimum A of \$52,760,000 per year. Because there are no F values, it is omitted from the functions. The minus sign placed before each function name forces the answer to be positive, since these two functions always display the answer with the opposite sign entered on the estimated cash flows.

A	B	C	D	E	F	G	H	I	J	K	L
1											
2	A = \$50		i = 10%		n = 5		P = \$200		i = 10%		n = 5
3											
4											
5	P = \$189.54	←	= -PV(10%, 5, 50)				A = \$52.76	←	= -PMT(10%, 5, 200)		
6											
7											

Figure 2-5
Spreadsheet functions to find P and A for the cement factory case, Example 2.4.



2.3 Sinking Fund Factor and Uniform Series Compound Amount Factor (A/F and F/A) ●●●

The simplest way to derive the A/F factor is to substitute into factors already developed. If P from Equation [2.3] is substituted into Equation [2.9], the following formula results.

$$A = F \left[\frac{1}{(1+i)^n} \right] \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$A = F \left[\frac{i}{(1+i)^n - 1} \right] \quad [2.12]$$

The expression in brackets in Equation [2.12] is the **A/F or sinking fund factor**. It determines the **uniform annual series A** that is equivalent to a given future amount F . This is shown graphically in Figure 2–6a, where A is a uniform annual investment.



Placement of F

The uniform series A begins at the **end of year (period) 1** and continues **through the year of the given F** . The last A value and F occur at the same time.

Equation [2.12] can be rearranged to find F for a stated A series in periods 1 through n (Figure 2–6b).

$$F = A \left[\frac{(1+i)^n - 1}{i} \right] \quad [2.13]$$

The term in brackets is called the **uniform series compound amount factor (USCAF)**, or **F/A factor**. When multiplied by the given uniform annual amount A , it yields the **future worth of the uniform series**. It is important to remember that the future amount F occurs in the same period as the last A .

Standard notation follows the same form as that of other factors. They are $(F/A,i,n)$ and $(A/F,i,n)$. Table 2–3 summarizes the notations and equations, as does the inside front cover.

As a matter of interest, the uniform series factors can be symbolically determined by using an abbreviated factor form. For example, $F/A = (F/P)(P/A)$, where cancellation of the P is correct. Using the factor formulas, we have

$$(F/A,i,n) = [(1+i)^n] \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] = \frac{(1+i)^n - 1}{i}$$

For solution by spreadsheet, the **FV** function calculates F for a stated A series over n years. The format is

$$= \text{FV}(i\%, n, A, P) \quad [2.14]$$

The P may be omitted when no separate present worth value is given. The **PMT** function determines the A value for n years, given F in year n and possibly a separate P value in year 0. The format is

$$= \text{PMT}(i\%, n, P, F) \quad [2.15]$$

If P is omitted, the comma must be entered so the function knows the last entry is an F value.

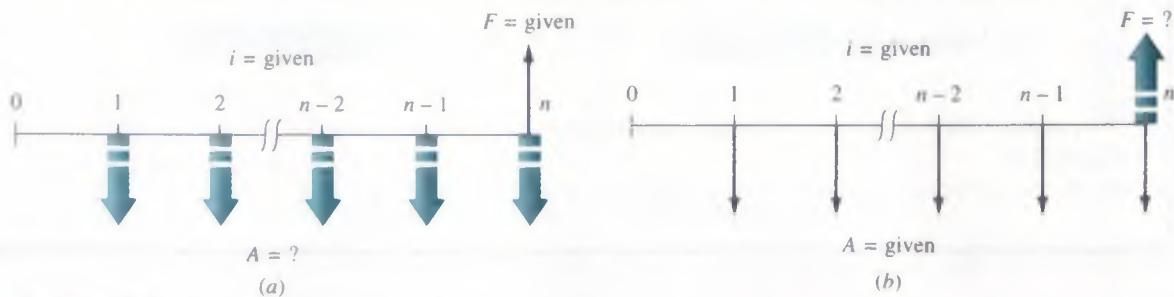


Figure 2–6

Cash flow diagrams to (a) find A , given F , and (b) find F , given A .

TABLE 2-3 F/A and A/F Factors: Notation and Equations

Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Functions
$(F/A, i, n)$	Uniform series compound amount	F/A	$\frac{(1 + i)^n - 1}{i}$	$F = A(F/A, i, n)$	$= \text{FV}(i\%, n, A)$
$(A/F, i, n)$	Sinking fund	A/F	$\frac{i}{(1 + i)^n - 1}$	$A = F(A/F, i, n)$	$= \text{PMT}(i\%, n, F)$

EXAMPLE 2.5

The president of Ford Motor Company wants to know the equivalent future worth of a \$1 million capital investment each year for 8 years, starting 1 year from now. Ford capital earns at a rate of 14% per year.

Solution

The cash flow diagram (Figure 2-7) shows the annual investments starting at the end of year 1 and ending in the year the future worth is desired. In \$1000 units, the F value in year 8 is found by using the F/A factor.

$$F = 1000(F/A, 14\%, 8) = 1000(13.2328) = \$13,232.80$$

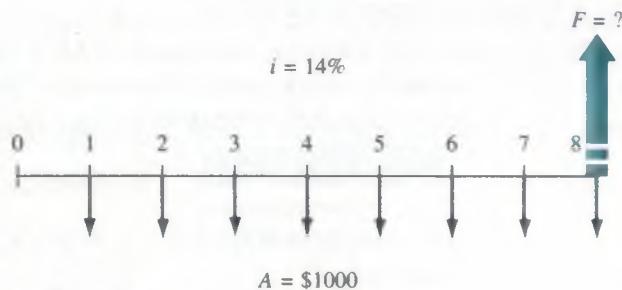


Figure 2-7
Diagram to find F for a uniform series, Example 2.5.

EXAMPLE 2.6 The Cement Factory Case

PE

Once again, consider the HAC case presented at the outset of this chapter, in which a projected \$200 million investment can generate \$50 million per year in revenue for 5 years starting 1 year after start-up. A 10% per year time value of money has been used previously to determine P , F , and A values. Now the president would like the answers to a couple of new questions about the estimated annual revenues. Use tabulated values, factor formulas, or spreadsheet functions to provide the answers.

- (a) What is the equivalent future worth of the estimated revenues after 5 years at 10% per year?
- (b) Assume that, due to the economic downturn, the president predicts that the corporation will earn only 4.5% per year on its money, not the previously anticipated 10% per year. What is the required amount of the annual revenue series over the 5-year period to be economically equivalent to the amount calculated in (a)?

Solution

- (a) Figure 2-6b is the cash flow diagram with $A = \$50$ million. Note that the last A value and $F = ?$ both occur at the end of year $n = 5$. We use tabulated values and the spreadsheet function to find F in year 5.

Tabulated value: Use the F/A factor and 10% interest factor table. In \$1 million units, the future worth of the revenue series is

$$\begin{aligned} F &= 50(F/A, 10\%, 5) = 50(6.1051) \\ &= \$305.255 \quad (\$305,255,000) \end{aligned}$$

<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>1</td></tr> <tr><td>2 A = \$50</td><td>i = 10%</td><td>n = 5</td></tr> <tr><td>3</td></tr> <tr><td>4</td></tr> <tr><td>5 F = \$305.255</td><td colspan="2" style="text-align: center;">$= -FV(10\%, 5, 50)$</td></tr> <tr><td>6</td></tr> <tr><td>7</td></tr> <tr><td>8</td></tr> <tr><td>9</td></tr> </table>	1	2 A = \$50	i = 10%	n = 5	3	4	5 F = \$305.255	$= -FV(10\%, 5, 50)$		6	7	8	9	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>F = value in cell B5</td><td>i = 4.5%</td><td>n = 5</td></tr> <tr><td>A = \$55.798</td><td colspan="2" style="text-align: center;">$= -PMT(4.5\%, 5, , B5)$</td></tr> </table>	F = value in cell B5	i = 4.5%	n = 5	A = \$55.798	$= -PMT(4.5\%, 5, , B5)$	
1																				
2 A = \$50	i = 10%	n = 5																		
3																				
4																				
5 F = \$305.255	$= -FV(10\%, 5, 50)$																			
6																				
7																				
8																				
9																				
F = value in cell B5	i = 4.5%	n = 5																		
A = \$55.798	$= -PMT(4.5\%, 5, , B5)$																			

Figure 2–8Spreadsheet functions to find F and A at $i = 4.5\%$ for the cement factory case.

Example 2.6.

If the rate of return on the annual revenues were 0%, the total amount after 5 years would be \$250,000,000. The 10% per year return is projected to grow this value by 22%.

Spreadsheet: Apply the FV factor in the format $= -FV(10\%, 5, 50)$ to determine $F = \$305.255$ million. Because there is no present amount in this computation, P is omitted from the factor. See Figure 2–8, left side. (As before, the minus sign forces the FV function to result in a positive value.)

- (b) The president of the Brazilian company planning to develop the cement plant in Georgia is getting worried about the international economy. He wants the revenue stream to generate the equivalent that it would at a 10% per year return, that is, \$305.255 million, but thinks that only a 4.5% per year return is achievable.

Factor formula: The A/F factor will determine the required A for 5 years. Since the factor tables do not include 4.5%, use the formula to answer the question. In \$1 million units,

$$A = 305.255(A/F, 4.5\%, 5) = 305.255 \left[\frac{0.045}{(1.045)^5 - 1} \right] = 305.255(0.18279) \\ = \$55.798$$

The annual revenue requirement grows from \$50 million to nearly \$55,800,000. This is a significant increase of 11.6% each year.

Spreadsheet: It is easy to answer this question by using the $= PMT(i\%, n, , F)$ function with $i = 4.5\%$ and $F = \$305.255$ found in part (a). We can use the cell reference method (described in Appendix A) for the future amount F . Figure 2–8, right side, displays the required A of \$55.798 per year (in \$1 million units).

2.4 Factor Values for Untabulated i or n Values ● ● ●

Often it is necessary to know the correct numerical value of a factor with an i or n value that is not listed in the compound interest tables in the rear of the book. Given specific values of i and n , there are several ways to obtain any factor value.

- Use the formula listed in this chapter or the front cover of the book,
- Use an Excel function with the corresponding P , F , or A value set to 1.
- Use linear interpolation in the interest tables.

When the **formula** is applied, the factor value is accurate since the specific i and n values are input. However, it is possible to make mistakes since the formulas are similar to each other, especially when uniform series are involved. Additionally, the formulas become more complex when gradients are introduced, as you will see in the following sections.

A **spreadsheet function** determines the factor value if the corresponding P , A , or F argument in the function is set to 1 and the other parameters are omitted or set to zero. For example, the P/F factor is determined using the PV function with A omitted (or set to 0) and $F = 1$, that is, $PV(i\%, n, , 1)$ or $PV(i\%, n, 0, 1)$. A minus sign preceding the function identifier causes the factor to have a positive value. Functions to determine the six common factors are as follows.

Factor	To Do This	Excel Function
P/F	Find P , given F .	= -PV(i%,n,,1)
F/P	Find F , given P .	= -FV(i%,n,,1)
P/A	Find P , given A .	= -PV(i%,n,1)
A/P	Find A , given P .	= -PMT(i%,n,1)
F/A	Find F , given A .	= -FV(i%,n,1)
A/F	Find A , given F .	= -PMT(i%,n,,1)

Figure 2–9 shows a spreadsheet developed explicitly to determine these factor values. When it is made live in Excel, entering any combination of i and n displays the exact value for all six factors. The values for $i = 3.25\%$ and $n = 25$ years are shown here. As we already know, these same functions will determine a final P , A , or F value when actual or estimated cash flow amounts are entered.

Linear interpolation for an untabulated interest rate i or number of years n takes more time to complete than using the formula or spreadsheet function. Also interpolation introduces some level of inaccuracy, depending upon the distance between the two boundary values selected for i or n , as the formulas themselves are nonlinear functions. Interpolation is included here for individuals who wish to utilize it in solving problems. Refer to Figure 2–10 for a graphical description of the following explanation. First, select two tabulated values (x_1 and x_2) of the parameter for which the factor is requested, that is, i or n , ensuring that the two values surround and are not too distant from the required value x . Second, find the corresponding tabulated factor values (f_1 and f_2). Third, solve for the unknown, linearly interpolated value f using the formulas below, where the differences in parentheses are indicated in Figure 2–10 as a through c .

A	B	C	D
1 i =	n =		
2 3.25%	25.00	Enter requested i and n	
Value obtained with this function			
4 Factor	Value		
5 P/F	0.44952	= -PV(\$A\$2,\$B\$2,,1)	
6 P/A	16.93786	= -PV(\$A\$2,\$B\$2,1)	
7			
8 F/P	2.22460	= -FV(\$A\$2,\$B\$2,,1)	
9 F/A	37.67993	= -FV(\$A\$2,\$B\$2,1)	
10			
11 A/F	0.02654	= -PMT(\$A\$2,\$B\$2,,1)	
12 A/P	0.05904	= -PMT(\$A\$2,\$B\$2,1)	
13			

Figure 2–9

Use of Excel functions to display factor values for any i and n values.

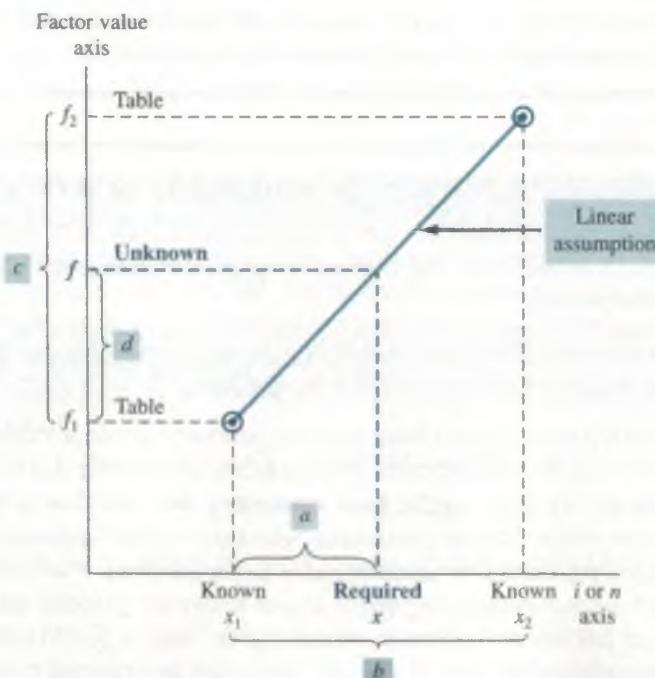


Figure 2–10

Linear interpolation in factor value tables.

$$f = f_1 + \frac{(x - x_1)}{(x_2 - x_1)}(f_2 - f_1) \quad [2.16]$$

$$f = f_1 + \frac{a}{b}c = f_1 + d \quad [2.17]$$

The value of d will be positive or negative if the factor is increasing or decreasing, respectively, in value between x_1 and x_2 .

EXAMPLE 2.7

Determine the P/A factor value for $i = 7.75\%$ and $n = 10$ years, using the three methods described previously.

Solution

Factor formula: Apply the formula from inside the front cover of the book for the P/A factor. Showing 5-decimal accuracy,

$$(P/A, 7.75\%, 10) = \frac{(1 + i)^n - 1}{i(1 + i)^n} = \frac{(1.0775)^{10} - 1}{0.0775(1.0775)^{10}} = \frac{1.10947}{0.16348} \\ = 6.78641$$

Spreadsheet: Utilize the spreadsheet function in Figure 2–9, that is, $= -PV(7.75\%, 10, 1)$, to display 6.78641.

Linear interpolation: Use Figure 2–10 as a reference for this solution. Apply the Equation [2.16] and [2.17] sequence, where x is the interest rate i , the bounding interest rates are $i_1 = 7\%$ and $i_2 = 8\%$, and the corresponding P/A factor values are $f_1 = (P/A, 7\%, 10) = 7.0236$ and $f_2 = (P/A, 8\%, 10) = 6.7101$. With 4-place accuracy,

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1) = 7.0236 + \frac{(7.75 - 7)}{(8 - 7)}(6.7101 - 7.0236) \\ = 7.0236 + (0.75)(-0.3135) = 7.0236 - 0.2351 \\ = 6.7885$$

Comment

Note that since the P/A factor value decreases as i increases, the linear adjustment is negative at -0.2351 . As is apparent, linear interpolation provides an approximation to the correct factor value for 7.75% and 10 years, plus it takes more calculations than using the formula or spreadsheet function. It is possible to perform two-way linear interpolation for untabulated i and n values; however, the use of a spreadsheet or factor formula is recommended.



2.5 Arithmetic Gradient Factors (P/G and A/G) ● ● ●

Assume a manufacturing engineer predicts that the cost of maintaining a robot will increase by \$5000 per year until the machine is retired. The cash flow series of maintenance costs involves a constant gradient, which is \$5000 per year.

An **arithmetic gradient** series is a cash flow series that either increases or decreases by a **constant amount** each period. The amount of change is called the **gradient**.

Formulas previously developed for an A series have year-end amounts of equal value. In the case of a gradient, each year-end cash flow is different, so new formulas must be derived. First, assume that the cash flow at the end of year 1 is the **base amount** of the cash flow series and, therefore, not part of the gradient series. This is convenient because in actual applications, the base amount is usually significantly different in size compared to the gradient. For example, if you purchase a used car with a 1-year warranty, you might expect to pay the gasoline and insurance costs during the first year of operation. Assume these cost \$2500; that is, \$2500 is the base amount. After the first year, you absorb the cost of repairs, which can be expected to increase

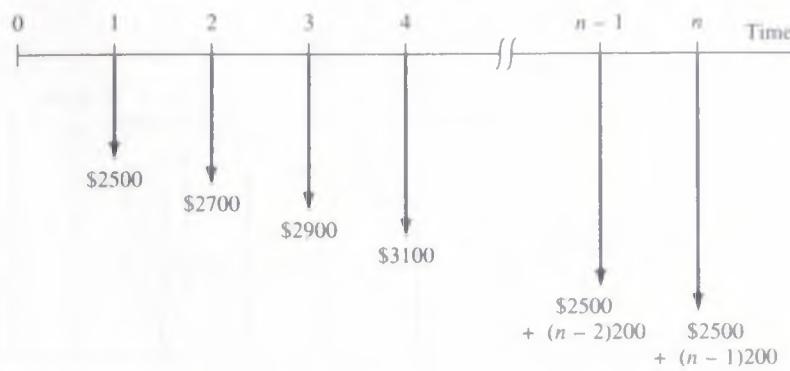


Figure 2-11
Cash flow diagram of an arithmetic gradient series.

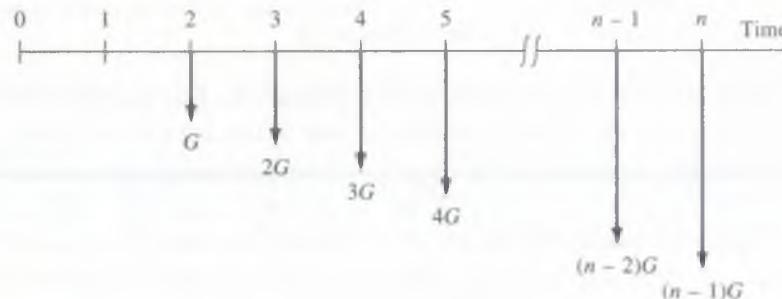


Figure 2-12
Conventional arithmetic gradient series without the base amount.

each year. If you estimate that total costs will increase by \$200 each year, the amount the second year is \$2700, the third \$2900, and so on to year n , when the total cost is $2500 + (n - 1)200$. The cash flow diagram is shown in Figure 2–11. Note that the gradient (\$200) is first observed between year 1 and year 2, and the base amount (\$2500 in year 1) is not equal to the gradient.

Define the symbols G for gradient and CF_n for cash flow in year n as follows.

G = constant arithmetic change in cash flows from one time period to the next; G may be positive or negative.

$$CF_n = \text{base amount} + (n - 1)G \quad [2.18]$$

It is important to realize that the base amount defines a uniform cash flow series of the size A that occurs each time period. We will use this fact when calculating equivalent amounts that involve arithmetic gradients. If the base amount is ignored, a generalized arithmetic (increasing) gradient cash flow diagram is as shown in Figure 2–12. Note that the gradient begins between years 1 and 2. This is called a **conventional gradient**.

EXAMPLE 2.8

A local university has initiated a logo-licensing program with the clothier Holister, Inc. Estimated fees (revenues) are \$80,000 for the first year with uniform increases to a total of \$200,000 by the end of year 9. Determine the gradient and construct a cash flow diagram that identifies the base amount and the gradient series.

Solution

The year 1 base amount is $CF_1 = \$80,000$, and the total increase over 9 years is

$$CF_9 - CF_1 = 200,000 - 80,000 = \$120,000$$

Equation [2.18], solved for G , determines the arithmetic gradient.

$$\begin{aligned} G &= \frac{(CF_9 - CF_1)}{n - 1} = \frac{120,000}{9 - 1} \\ &= \$15,000 \text{ per year} \end{aligned}$$

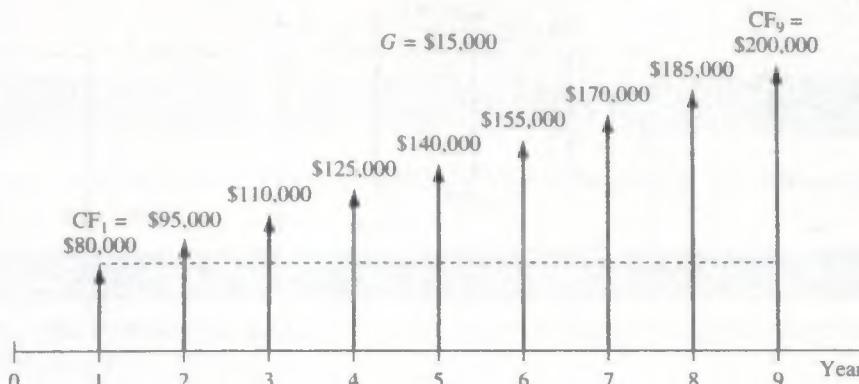


Figure 2-13
Diagram for gradient series, Example 2.8.

The cash flow diagram (Figure 2-13) shows the base amount of \$80,000 in years 1 through 9 and the \$15,000 gradient starting in year 2 and continuing through year 9.

The total present worth P_T for a series that includes a base amount A and conventional arithmetic gradient must consider the present worth of both the uniform series defined by A and the arithmetic gradient series. The addition of the two results in P_T .

$$P_T = P_A \pm P_G \quad [2.19]$$

where P_A is the present worth of the uniform series only, P_G is the present worth of the gradient series only, and the + or - sign is used for an increasing (+G) or decreasing (-G) gradient, respectively.

The corresponding equivalent annual worth A_T is the sum of the base amount series annual worth A_A and gradient series annual worth A_G , that is,

$$A_T = A_A \pm A_G \quad [2.20]$$

Three factors are derived for arithmetic gradients: the P/G factor for present worth, the A/G factor for annual series, and the F/G factor for future worth. There are several ways to derive them. We use the single-payment present worth factor ($P/F, i, n$), but the same result can be obtained by using the F/P , F/A , or P/A factor.

In Figure 2-12, the present worth at year 0 of only the gradient is equal to the sum of the present worths of the individual cash flows, where each value is considered a future amount.

$$\begin{aligned} P &= G(P/F, i, 2) + 2G(P/F, i, 3) + 3G(P/F, i, 4) + \dots \\ &\quad + [(n-2)G](P/F, i, n-1) + [(n-1)G](P/F, i, n) \end{aligned}$$

Factor out G and use the P/F formula.

$$P = G \left[\frac{1}{(1+i)^2} + \frac{2}{(1+i)^3} + \frac{3}{(1+i)^4} + \dots + \frac{n-2}{(1+i)^{n-1}} + \frac{n-1}{(1+i)^n} \right] \quad [2.21]$$

Multiplying both sides of Equation [2.21] by $(1+i)^1$ yields

$$P(1+i)^1 = G \left[\frac{1}{(1+i)^1} + \frac{2}{(1+i)^2} + \frac{3}{(1+i)^3} + \dots + \frac{n-2}{(1+i)^{n-2}} + \frac{n-1}{(1+i)^{n-1}} \right] \quad [2.22]$$

Subtract Equation [2.21] from Equation [2.22] and simplify.

$$iP = G \left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n} \right] - G \left[\frac{n}{(1+i)^n} \right] \quad [2.23]$$

The left bracketed expression is the same as that contained in Equation [2.6], where the P/A factor was derived. Substitute the closed-end form of the P/A factor from Equation [2.8]

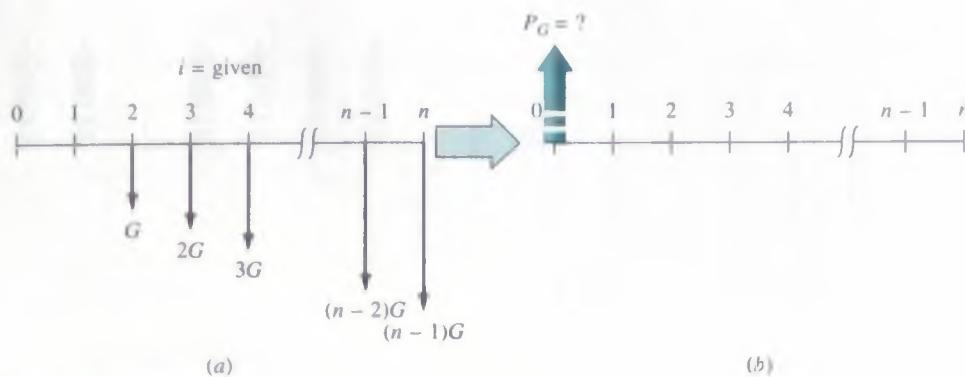


Figure 2-14

FIGURE 7-1
Conversion diagram from an arithmetic gradient to a present worth.

into Equation [2.23] and simplify to solve for P_G , the present worth of the gradient series only.

$$P_G = \frac{G}{i} \left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n} \right] \quad [2.24]$$

Equation [2.24] is the general relation to convert an arithmetic gradient G (not including the base amount) for n years into a present worth at year 0. Figure 2-14a is converted into the equivalent cash flow in Figure 2-14b. The *arithmetic gradient present worth factor*, or P/G factor, may be expressed in two forms:

$$(P/G,i,n) = \frac{1}{i} \left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n} \right]$$

or

$$(P/G, i, n) = \frac{(1+i)^n - in - 1}{i^2(1+i)^n} \quad [2.25]$$

Remember: The conventional arithmetic gradient starts in year 2, and P is located in year 0.

Equation [2.24] expressed as an engineering economy relation is

$$P_G = G(P/G,i,n) \quad [2.26]$$

which is the rightmost term in Equation [2.19] to calculate total present worth. The G carries a minus sign for decreasing gradients.

The equivalent uniform annual series A_G for an arithmetic gradient G is found by multiplying the present worth in Equation [2.26] by the $(A/P, i, n)$ formula. In standard notation form, the equivalent of algebraic cancellation of P can be used.

$$A_G = G(P/G,i,n)(A/P,i,n) \\ = G(A/G,i,n)$$

In equation form,

$$A_G = \frac{G}{i} \left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n} \right] \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

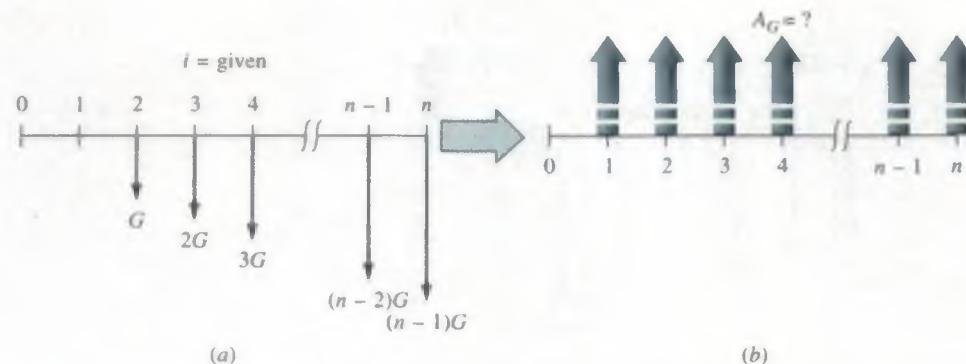
$$A_6 = G \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1} \right] \quad [2.27]$$

which is the rightmost term in Equation [2.20]. The expression in brackets in Equation [2.27] is called the *arithmetic gradient uniform series factor* and is identified by $(A/G,i,n)$. This factor converts Figure 2-15a into Figure 2-15b.

The P/G and A/G factors and relations are summarized inside the front cover. Factor values are tabulated in the two rightmost columns of factor values at the rear of this text.



Placement of gradient P_G

**Figure 2–15**

Conversion diagram of an arithmetic gradient series to an equivalent uniform annual series.

There is no direct, single-cell spreadsheet function to calculate P_G or A_G for an arithmetic gradient. Use the NPV function to display P_G and the PMT function to display A_G after entering all cash flows (base and gradient amounts) into contiguous cells. General formats for these functions are

$$= \text{NPV}(i\%, \text{second_cell:last_cell}) + \text{first_cell} \quad [2.28]$$

$$= \text{PMT}(i\%, n, \text{cell_with_}P_G) \quad [2.29]$$

The word entries in italic are cell references, not the actual numerical values. (See Appendix A, Section A.2, for a description of cell reference formatting.) These functions are demonstrated in Example 2.10.

An **F/G factor** (*arithmetic gradient future worth factor*) to calculate the future worth F_G of a gradient series can be derived by multiplying the P/G and F/P factors. The resulting factor, $(F/G,i,n)$, in brackets, and engineering economy relation is

$$F_G = G \left[\left(\frac{1}{i} \right) \left(\frac{(1+i)^n - 1}{i} \right) - n \right]$$

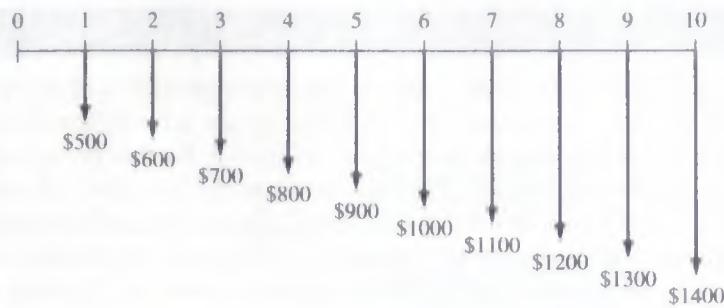
EXAMPLE 2.9

Neighboring parishes in Louisiana have agreed to pool road tax resources already designated for bridge refurbishment. At a recent meeting, the engineers estimated that a total of \$500,000 will be deposited at the end of next year into an account for the repair of old and safety-questionable bridges throughout the area. Further, they estimate that the deposits will increase by \$100,000 per year for only 9 years thereafter, then cease. Determine the equivalent (a) present worth and (b) annual series amounts, if public funds earn at a rate of 5% per year.

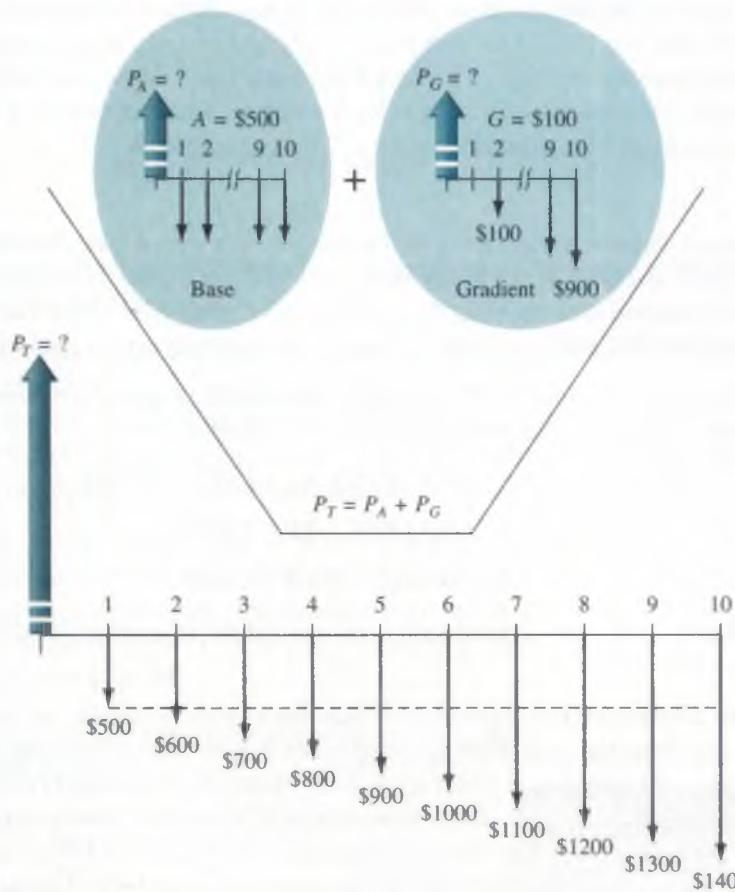
Solution

- (a) The cash flow diagram of this conventional arithmetic gradient series from the perspective of the parishes is shown in Figure 2–16. According to Equation [2.19], two computations must be made and added: the first for the present worth of the base amount P_A and the second for the present worth of the gradient P_G . The total present worth P_T occurs in year 0. This is illustrated by the partitioned cash flow diagram in Figure 2–17. In \$1000 units, the total present worth is

$$\begin{aligned} P_T &= 500(P/A, 5\%, 10) + 100(P/G, 5\%, 10) \\ &= 500(7.7217) + 100(31.6520) \\ &= \$7026.05 \quad (\$7,026,050) \end{aligned}$$

**Figure 2-16**

Cash flow series with a conventional arithmetic gradient (in \$1000 units).
Example 2.9.

**Figure 2-17**

Partitioned cash flow diagram (in \$1000 units), Example 2.9.

- (b) Here, too, it is necessary to consider the gradient and the base amount separately. The total annual series A_T is found by Equation [2.20] and occurs in years 1 through 10.

$$\begin{aligned} A_T &= 500 + 100(A/G, 5\%, 10) = 500 + 100(4.0991) \\ &= \$909.91 \text{ per year } (\$909,910) \end{aligned}$$

Comment

Remember: The P/G and A/G factors determine the present worth and annual series of the gradient only. Any other cash flows must be considered separately.

If the present worth is already calculated [as in part (a)], P_T can be multiplied by an A/P factor to get A_T . In this case, considering round-off error,

$$\begin{aligned} A_T &= P_T(A/P, 5\%, 10) = 7026.05(0.12950) \\ &= \$909.873 \quad (\$909,873) \end{aligned}$$

EXAMPLE 2.10 The Cement Factory Case

PE

The announcement of the HAC cement factory states that the \$200 million (M) investment is planned for 2012. Most large investment commitments are actually spread out over several years as the plant is constructed and production is initiated. Further investigation may determine, for example, that the \$200 M is a present worth in the year 2012 of anticipated investments during the next 4 years (2013 through 2016). Assume the amount planned for 2013 is \$100 M with constant decreases of \$25 M each year thereafter. As before, assume the time value of money for investment capital is 10% per year to answer the following questions using tabulated factors and spreadsheet functions, as requested below.

- In equivalent present worth values, does the planned decreasing investment series equal the announced \$200 M in 2012? Use both tabulated factors and spreadsheet functions.
- Given the planned investment series, what is the equivalent annual amount that will be invested from 2013 to 2016? Use both tabulated factors and spreadsheet functions.
- (This optional question introduces Excel's Goal Seek tool.) What must be the amount of yearly constant decrease through 2016 to have a present worth of exactly \$200 M in 2012, provided \$100 M is expended in 2013? Use a spreadsheet.

Solution

- The investment series is a decreasing arithmetic gradient with a base amount of \$100 M in year 1 (2013) and $G = -\$25$ M through year 4 (2016). Figure 2–18 diagrams the cash flows with the shaded area showing the constantly declining investment each year. The P_T value at time 0 at 10% per year is determined by using tables and a spreadsheet.

Tabulated factors: Equation [2.19] with the minus sign for negative gradients determines the total present worth P_T . Money is expressed in \$1 million units.

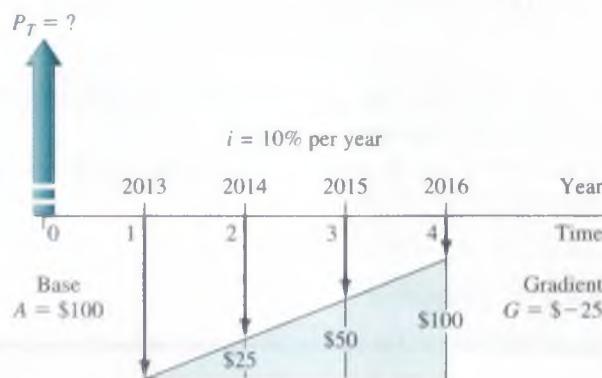
$$\begin{aligned} P_T &= P_A - P_G = 100(P/A, 10\%, 4) - 25(P/G, 10\%, 4) \quad [2.30] \\ &= 100(3.1699) - 25(4.3781) \\ &= \$207.537 \quad (\$207,537,000) \end{aligned}$$

In present worth terms, the planned series will exceed the equivalent of \$200 M in 2012 by approximately \$7.5 M.

Spreadsheet: Since there is no spreadsheet function to directly display present worth for a gradient series, enter the cash flows in a sequence of cells (rows or columns) and use the NPV function to find present worth. Figure 2–19 shows the entries and function $NPV(i\%, \text{second_cell:last_cell})$. There is no first_cell entry here, because there is no investment per se in year 0. The result displayed in cell C9, \$207.534, is the total P_T for the planned series. (Note that the NPV function does not consider two separate series of cash flows as is necessary when using tabulated factors.)

The interpretation is the same as in part (a); the planned investment series exceeds the \$200 M in present worth terms by approximately \$7.5 M.

- Tabulated factors:* There are two equally correct ways to find A_T . First, apply Equation [2.20] that utilizes the A/G factor, and second, use the P_T value obtained above and the A/P factor. Both relations are illustrated here, in \$1 million units,

**Figure 2–18**

Cash flow diagram for decreasing gradient in \$1 million units, Example 2.10.

A	B	C	D	E	F	G
1 In \$1 M units	G =	-25.000				
2	Year	Time, t	Investment, \$1 M			
3	2012	0				
4	2013	1	100.000			
5	2014	2	75.000			
6	2015	3	50.000			
7	2016	4	25.000			
8	P @ 10%		207.534			
9	A of inv @ 10%			65.471		
10						
11						
12						
13						

Figure 2-19

Spreadsheet solution for Example 2.10a and b.

Use Equation [2.20]:

$$A_T = 100 - 25(A/G, 10\%, 4) = 100 - 25(1.3812)$$

$$= \$65.471 \quad (\$65,471,000 \text{ per year})$$

Use P_T :

$$A_T = 207.537(A/P, 10\%, 4) = 207.537(0.31547)$$

$$= \$65.471 \text{ per year}$$

Spreadsheet: Apply the PMT function in Equation [2.29] to obtain the same $A_T = \$65.471$ per year (Figure 2-19).

- (c) (Optional) The Goal Seek tool is described in Appendix A. It is an excellent tool to apply when one cell entry must equal a specific value and only one other cell can change. This is the case here; the NPV function (cell C9 in Figure 2-19) must equal \$200, and the gradient G (cell C1) is unknown. This is the same as stating $P_T = 200$ in Equation [2.30] and solving for G . All other parameters retain their current value.

Figure 2-20 (top) pictures the same spreadsheet used previously with the Goal Seek template added and loaded. When OK is clicked, the solution is displayed; $G = \$-26.721$. Refer to Figure 2-20 again. This means that if the investment is decreased by a constant annual amount of \$26.721 M, the equivalent total present worth invested over the 4 years will be exactly \$200 M.

A	B	C	D
1 in \$1 M units	G =	-25.000	←
2	Year	Time, t	Investment, \$1 M
3	2012	0	
4	2013	1	100.000
5	2014	2	75.000
6	2015	3	50.000
7	2016	4	25.000
8	P @ 10%		207.534
9	A of inv @ 10%		65.471
10			
11			
12			
13			
14			

Present worth of investments:
 $= \text{NPV}(10\%, \text{C5:C8})$

Goal Seek
 Set cell:
 To value:
 By changing cell:

OK Cancel

Set up Goal Seek template

A	B	C	
1 in \$1 M units	G =	-26.721	
2	Year	Time, t	Investment, \$1 M
3	2012	0	
4	2013	1	100.000
5	2014	2	73.279
6	2015	3	46.559
7	2016	4	19.838
8	P @ 10%		200.000
9			
10			

Solution for $G = \$-26.721$ to make present worth exactly \$200

Figure 2-20

Solution for arithmetic gradient using Goal Seek, Example 2.10c.



2.6 Geometric Gradient Series Factors ● ● ●

It is common for annual revenues and annual costs such as maintenance, operations, and labor to go up or down by a constant percentage, for example, +5% or -3% per year. This change occurs every year on top of a starting amount in the first year of the project. A definition and description of new terms follow.

A **geometric gradient series** is a cash flow series that either increases or decreases by a **constant percentage** each period. The uniform change is called the **rate of change**.

g = **constant rate of change**, in decimal form, by which cash flow values increase or decrease from one period to the next. The gradient g can be + or -.

A_1 = **initial cash flow in year 1** of the geometric series

P_g = **present worth** of the entire geometric gradient series, including the initial amount A_1

Note that the initial cash flow A_1 is **not considered separately** when working with geometric gradients.

Figure 2-21 shows increasing and decreasing geometric gradients starting at an amount A_1 in time period 1 with present worth P_g located at time 0. The relation to determine the total present worth P_g for the entire cash flow series may be derived by multiplying each cash flow in Figure 2-21a by the P/F factor $1/(1+i)^n$.

$$\begin{aligned} P_g &= \frac{A_1}{(1+i)^1} + \frac{A_1(1+g)}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \dots + \frac{A_1(1+g)^{n-1}}{(1+i)^n} \\ &= A_1 \left[\frac{1}{1+i} + \frac{1+g}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \dots + \frac{(1+g)^{n-1}}{(1+i)^n} \right] \end{aligned} \quad [2.31]$$

Multiply both sides by $(1+g)/(1+i)$, subtract Equation [2.31] from the result, factor out P_g , and obtain

$$P_g \left(\frac{1+g}{1+i} - 1 \right) = A_1 \left[\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{1+i} \right]$$

Solve for P_g and simplify.

$$P_g = A_1 \left[\frac{1 - \left(\frac{1+g}{1+i} \right)^n}{i-g} \right] \quad g \neq i \quad [2.32]$$

The term in brackets in Equation [2.32] is the $(P/A, g, i, n)$ or *geometric gradient series present worth factor* for values of g not equal to the interest rate i . When $g = i$, substitute i for g in Equation [2.31] and observe that the term $1/(1+i)$ appears n times.

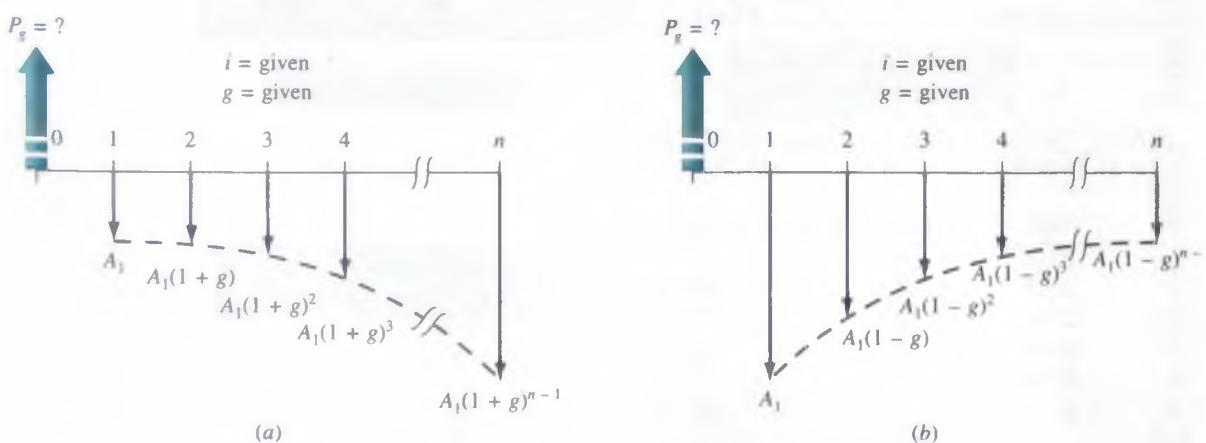


Figure 2-21

Cash flow diagram of (a) increasing and (b) decreasing geometric gradient series and present worth P_g .

$$\begin{aligned} P_g &= A_1 \left(\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots + \frac{1}{(1+i)^n} \right) \\ P_g &= \frac{nA_1}{(1+i)} \end{aligned} \quad [2.33]$$

The $(P/A, g, i, n)$ factor calculates P_g in period $t = 0$ for a geometric gradient series starting in period 1 in the amount A_1 and increasing by a constant rate of g each period.



The equation for P_g and the $(P/A, g, i, n)$ factor formula are

$$P_g = A_1(P/A, g, i, n) \quad [2.34]$$

$$(P/A, g, i, n) = \begin{cases} \frac{1 - (1+g)^n}{i-g} & g \neq i \\ \frac{n}{1+i} & g = i \end{cases} \quad [2.35]$$

It is possible to derive factors for the equivalent A and F values; however, it is easier to determine the P_g amount and then multiply by the A/P or F/P factor.

As with the arithmetic gradient series, there are no direct spreadsheet functions for geometric gradient series. Once the cash flows are entered, P and A are determined using the NPV and PMT functions, respectively.

EXAMPLE 2.11

A coal-fired power plant has upgraded an emission control valve. The modification costs only \$8000 and is expected to last 6 years with a \$200 salvage value. The maintenance cost is expected to be high at \$1700 the first year, increasing by 11% per year thereafter. Determine the equivalent present worth of the modification and maintenance cost by hand and by spreadsheet at 8% per year.

Solution by Hand

The cash flow diagram (Figure 2–22) shows the salvage value as a positive cash flow and all costs as negative. Use Equation [2.35] for $g \neq i$ to calculate P_g . Total P_T is the sum of three present worth components.

$$\begin{aligned} P_T &= -8000 - P_g + 200(P/F, 8\%, 6) \\ &= -8000 - 1700 \left[\frac{1 - (1.11/1.08)^6}{0.08 - 0.11} \right] + 200(P/F, 8\%, 6) \\ &= -8000 - 1700(5.9559) + 126 = \$-17,999 \end{aligned}$$

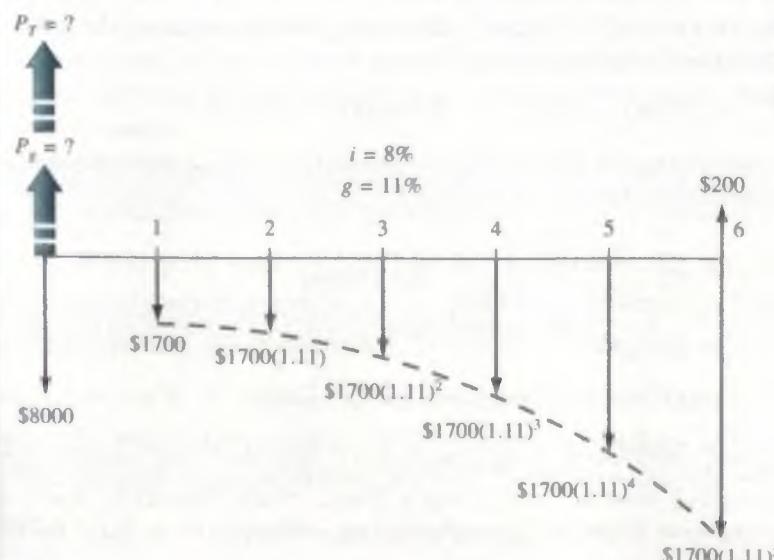


Figure 2–22
Cash flow diagram of a geometric gradient, Example 2.11.

Solution by Spreadsheet

Figure 2–23 details the spreadsheet operations to find the geometric gradient present worth P_g and total present worth P_T . To obtain $P_T = \$-17,999$, three components are summed—first cost, present worth of estimated salvage in year 6, and P_g . Cell tags detail the relations for the second and third components; the first cost occurs at time 0.

Comment

The relation that calculates the $(P/A, g, i\%, n)$ factor is rather complex, as shown in the cell tag and formula bar for C9. If this factor is used repeatedly, it is worthwhile using cell reference formatting so that A_1 , i , g , and n values can be changed and the correct value is always obtained. Try to write the relation for cell C9 in this format.

CB		
A	B	C
Information provided	Estimates	P value, \$
Interest rate, $i\%$	8%	
First cost, \$	-8000	-8000
Life, n , years	6	
Salvage, \$	200	128
Maintenance cost, year 1, \$	-1,700	-10,125
Cost gradient, $g\%$	11%	
Total, \$		-17,999

$= -1700 * ((1 - ((1.11) / (1.08))^6) / (0.08 - 0.11))$

$= -PV(8\%, 6, 200)$

$= -1700 * ((1 - ((1.11) / (1.08))^6) / (0.08 - 0.11))$

Figure 2–23

Geometric gradient and total present worth calculated via spreadsheet, Example 2.11.

EXAMPLE 2.12 The Cement Factory Case

PE

Now let's go back to the proposed Houston American Cement plant in Georgia. The revenue series estimate of \$50 million annually is quite optimistic, especially since there are many other cement product plants operating in Florida and Georgia on the same limestone deposit. (The website for the HAC plant shows where they are located currently; it is clear that keen competition will be present.) Therefore, it is important to be sensitive in our analysis to possibly declining and increasing revenue series, depending upon the longer-term success of the plant's marketing, quality, and reputation. Assume that revenue may start at \$50 million by the end of the first year, but then decreases geometrically by 12% per year through year 5. Determine the **present worth** and **future worth** equivalents of all revenues during this 5-year time frame at the same rate used previously, that is, 10% per year.

Solution

The cash flow diagram appears much like Figure 2–21b, except that the arrows go up for revenues. In year 1, $A_1 = \$50$ M and revenues decrease in year 5 to

$$A_1(1 - g)^{n-1} = 50 \text{ M}(1 - 0.12)^{5-1} = 50 \text{ M}(0.88)^4 = \$29.98 \text{ M}$$

First, we determine P_g in year 0 using Eq. [2.35] with $i = 0.10$ and $g = -0.12$, then we calculate F in year 5. In \$1 million units,

$$P_g = 50 \left[\frac{1 - \left(\frac{0.88}{1.10} \right)^5}{0.10 - (-0.12)} \right] = 50[3.0560]$$

$$= \$152.80$$

$$F = 152.80(F/P, 10\%, 5) = 152.80(1.6105)$$

$$= \$246.08$$

This means that the decreasing revenue stream has a 5-year future equivalent worth of \$246.080 M. If you look back to Example 2.6, we determined that the F in year 5 for the

uniform revenue series of \$50 M annually is \$305.255 M. In conclusion, the 12% declining geometric gradient has lowered the future worth of revenue by \$59.175 M, which is a sizable amount from the perspective of the owners of Votorantim Cimentos North America, Inc.

2.7 Determining i or n for Known Cash Flow Values ● ● ●

When all the cash flow values are known or have been estimated, the i value (interest rate or rate of return) or n value (number of years) is often the unknown. An example for which i is sought may be stated as follows: A company invested money to develop a new product. After the net annual income series is known following several years on the market, determine the rate of return i on the investment. There are several ways to find an unknown i or n value, depending upon the nature of the cash flow series and the method chosen to find the unknown. The simplest case involves only single amounts (P and F) and solution utilizing a spreadsheet function. The most difficult and complex involves finding i or n for irregular cash flows mixed with uniform and gradient series utilizing solution by hand and calculator. The solution approaches are summarized below, followed by examples.

Single Amounts— P and F Only

Hand or Calculator Solution Set up the equivalence relation and (1) solve for the variable using the factor formula, or (2) find the factor value and interpolate in the tables.

Spreadsheet Solution Use the IRR or RATE function to find i or the NPER function to find n . (See below and Appendix A for details.)

Uniform Series— A Series

Hand or Calculator Solution Set up the equivalence relation using the appropriate factor (P/A , A/P , F/A , or A/F), and use the second method mentioned above.

Spreadsheet Solution Use the IRR or RATE function to find i or the NPER function to find n .

Mixed A Series, Gradients, and/or Isolated Values

Hand or Calculator Solution Set up the equivalence relation and use (1) trial and error or (2) the calculator functions.

Spreadsheet Solution Use the IRR or RATE function to find i or the NPER function to find n . (This is the recommended approach.)

Besides the PV, FV, and NPV functions, other spreadsheet functions useful in determining i are IRR (internal rate of return) and RATE, and NPER (number of periods) to find n . The formats are shown here and the inside front cover with a detailed explanation in Appendix A. In all three of these functions, at least one cash flow entry must have a sign opposite that of others in order to find a solution.

$$= \text{IRR(first_cell:last_cell)} \quad [2.36]$$

To use IRR to find i , enter all cash flows into contiguous cells, including zero values.

$$= \text{RATE}(n,A,P,F) \quad [2.37]$$

The single-cell RATE function finds i when an A series and single P and/or F values are involved.

$$= \text{NPER}(i\%,A,P,F) \quad [2.38]$$

NPER is a single-cell function to find n for single P and F values, or with an A series.

EXAMPLE 2.13

If Laurel made a \$30,000 investment in a friend's business and received \$50,000 5 years later, determine the rate of return.

Solution

Since only single amounts are involved, i can be determined directly from the P/F factor.

$$P = F(P/F, i, n) = F \frac{1}{(1+i)^n}$$

$$30,000 = 50,000 \frac{1}{(1+i)^5}$$

$$0.600 = \frac{1}{(1+i)^5}$$

$$i = \left(\frac{1}{0.6}\right)^{0.2} - 1 = 0.1076 \quad (10.76\%)$$

Alternatively, the interest rate can be found by setting up the standard P/F relation, solving for the factor value, and interpolating in the tables.

$$P = F(P/F, i, n)$$

$$30,000 = 50,000(P/F, i, 5)$$

$$(P/F, i, 5) = 0.60$$

From the interest tables, a P/F factor of 0.6000 for $n = 5$ lies between 10% and 11%. Interpolate between these two values to obtain $i = 10.76\%$.

EXAMPLE 2.14

Pyramid Energy requires that for each of its offshore wind power generators \$5000 per year be placed into a capital reserve fund to cover unexpected major rework on field equipment. In one case, \$5000 was deposited for 15 years and covered a rework costing \$100,000 in year 15. What rate of return did this practice provide to the company? Solve by hand and spreadsheet.

Solution by Hand

The cash flow diagram is shown in Figure 2–24. Either the A/F or F/A factor can be used. Using A/F ,

$$A = F(A/F, i, n)$$

$$5000 = 100,000(A/F, i, 15)$$

$$(A/F, i, 15) = 0.0500$$

From the A/F interest tables for 15 years, the value 0.0500 lies between 3% and 4%. By interpolation, $i = 3.98\%$.

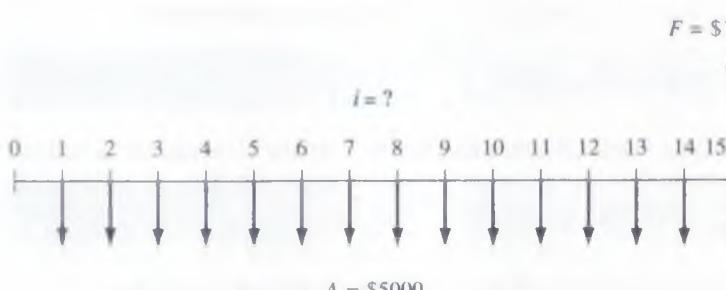


Figure 2–24
Diagram to determine the rate of return, Example 2.14.

Solution by Spreadsheet

Refer to the cash flow diagram (Figure 2–24) while completing the spreadsheet (Figure 2–25). A single-cell solution using the RATE function can be applied since $A = -\$5000$ occurs each year and $F = \$100,000$ takes place in the last year of the series. The function $= \text{RATE}(15, -5000, , 100000)$ displays the value $i = 3.98\%$. This function is fast, but it allows only limited sensitivity analysis because all the A values have to change by the same amount. The IRR function is much better for answering “what if” questions.

To apply the IRR function, enter the value 0 in a cell (for year 0), followed by -5000 for 14 years and in year 15 enter $+95,000$ (Figure 2–25). In any cell enter the IRR function. The answer $i = 3.98\%$ is displayed. It is advisable to enter the year numbers 0 through n (15 in this example) in the column immediately to the left of the cash flow entries. The IRR function does not need these numbers, but it makes the cash flow entry activity easier and more accurate. Now any cash flow can be changed, and a new rate will be displayed immediately via IRR.

	A	B	C	D	E
1				Year	Cash flow, \$
2	Using RATE			0	0
3	3.98%			1	-5000
4				2	-5000
5				3	-5000
6				4	-5000
7				5	-5000
8				6	-5000
9	Using IRR			7	-5000
10	3.98%			8	-5000
11				9	-5000
12				10	-5000
13				11	-5000
14				12	-5000
15				13	-5000
16				14	-5000
17				15	95000
18					

Figure 2–25

Use of RATE and IRR functions to determine i value for a uniform series, Example 2.14.

EXAMPLE 2.15 The Cement Factory Case

PE

From the introductory comments about the HAC plant, the annual revenue is planned to be \$50 million. All analysis thus far has taken place at 10% per year; however, the parent company has made it clear that its other international plants are able to show a 20% per year return on the initial investment. Determine the number of years required to generate 10%, 15%, and 20% per year returns on the \$200 million investment at the Georgia site.

Solution

If hand solution is utilized, the present worth relation can be established and the n values interpolated in the tables for each of the three rate of return values. In \$1 million units, the relation is

$$P = -200 + 50(P/A, i\%, n) \quad (i = 10\%, 15\%, 20\%)$$

$$(P/A, i\%, n) = 4.00$$

This is a good opportunity to utilize a spreadsheet and repeated NPER functions from Equation [2.38], since several i values are involved. Figure 2–26 shows the single-cell $= \text{NPER}(i\%, 50, -200)$ function for each rate of return. The number of years (rounded up) to produce at least the required returns are

Return, $i\%$	Years
10	6
15	7
20	9

A	B	C	D	
1				
2	i value	10%	15%	20%
3				
4	NPER function	=NPER(10%,50,-200)	=NPER(15%,50,-200)	=NPER(20%,50,-200)
5				
6	Result, years	5.36	6.56	8.83
7				

Figure 2–26

Use of NPER function to find n values for various rate of return requirements.
Example 2.15.

CHAPTER SUMMARY

Formulas and factors derived and applied in this chapter perform equivalence calculations for present, future, annual, and gradient cash flows. Capability in using these formulas and their standard notation manually and with spreadsheets is critical to complete an engineering economy study. Using these formulas and spreadsheet functions, you can convert single cash flows into uniform cash flows, gradients into present worths, and much more. Additionally, you can solve for rate of return i or time n .

PROBLEMS

Use of Interest Tables

- 2.1 Look up the numerical value for the following factors from the interest tables.
1. $(P/F, 6\%, 8)$
 2. $(A/P, 10\%, 10)$
 3. $(A/G, 15\%, 20)$
 4. $(A/F, 2\%, 30)$
 5. $(P/G, 35\%, 15)$

Determination of F , P , and A

- 2.2 How much can Haydon Rheosystems, Inc., afford to spend now on an energy management system if the software will save the company \$21,300 per year for the next 5 years? Use an interest rate of 10% per year.
- 2.3 A manufacturer of off-road vehicles is considering the purchase of dual-axis inclinometers for installation in a new line of tractors. The distributor of the inclinometers is temporarily overstocked and is offering them at a 40% discount from the regular cost of \$142. If the purchaser gets them now instead of 2 years from now, which is when they will be needed, what is the present worth of the savings per unit? The company would pay the regular price, if purchased in 2 years. Assume the interest rate is 10% per year.
- 2.4 The Moller Skycar M400 is a flying car known as a personal air vehicle (PAV) that is expected

to be FAA-certified by December 31, 2011. The cost is \$985,000, and a \$100,000 deposit will hold one of the first 100 “cars.” Assume a buyer pays the \$885,000 balance 3 years after making the \$100,000 deposit. At an interest rate of 10% per year, what is the effective total cost of the PAV in year 3?

- 2.5 A family that won a \$100,000 prize on America’s Funniest Home Videos decided to put one-half of the money in a college fund for their child who was responsible for the prize. If the fund earned interest at 6% per year, how much was in the account 14 years after it was started?
- 2.6 One of the biggest vulnerabilities in a control system is network devices, such as Ethernet-based network switches that are located in unsecured locations and accessible to everyone. DeltaX switches, manufactured by Dahne Security, allow the user to automatically lock and unlock the port access to all switches in the network. The company is considering expanding its manufacturing lines now or doing it in 3 years. If the cost now would be \$1.9 million, what equivalent amount could the company afford to spend in 3 years? The interest rate is 15% per year.

- 2.7 A company that sells high-purity laboratory chemicals is considering investing in new equipment

- that will reduce cardboard costs by better matching the size of the products to be shipped to the size of the shipping container. If the new equipment will cost \$220,000 to purchase and install, how much must the company save each year for 3 years in order to justify the investment, if the interest rate is 10% per year?
- 2.8 Red Valve Co. of Carnegie, Pennsylvania, makes a control pinch valve that provides accurate, repeatable control of abrasive and corrosive slurries, out-lasting gate, plug, ball, and even satellite coated valves. How much can the company afford to spend now on new equipment in lieu of spending \$75,000 four years from now? The company's rate of return is 12% per year.
- 2.9 If GHD Plastics purchases a new building now for \$1.3 million for its corporate headquarters, what must the building be worth in 10 years? The company expects all expenditures to earn a rate of return of at least 18% per year.
- 2.10 CGK Rheosystems makes high-performance rotational viscometers capable of steady shear and yield stress testing in a rugged, compact footprint. How much could the company afford to spend now on new equipment in lieu of spending \$200,000 one year from now and \$300,000 three years from now, if the company uses an interest rate of 15% per year?
- 2.11 Five years ago a consulting engineer purchased a building for company offices constructed of bricks that were not properly fired. As a result, some of the bricks were deteriorated from their exposure to rain and snow. Because of the problem with the bricks, the selling price of the building was 25% below the price of comparable, structurally sound buildings. The engineer repaired the damaged bricks and arrested further deterioration by applying an extra-strength solvent-based RTV elastomeric sealant. This resulted in restoring the building to its fair market value. If the depressed purchase price of the building was \$600,000 and the cost of getting it repaired was \$25,000, what is the equivalent value of the "forced appreciation" today, if the interest rate is 8% per year?
- 2.12 Metso Automation, which manufactures addressable quarter-turn electric actuators, is planning to set aside \$100,000 now and \$150,000 one year from now for possible replacement of the heating and cooling systems in three of its larger manufacturing plants. If the replacement won't be needed for 4 years, how much will the company have in the account, if it earns interest at a rate of 8% per year?
- 2.13 Syringe pumps often fail because reagents adhere to the ceramic piston and deteriorate the seal. Trident Chemical developed an integrated polymer dynamic seal that provides a higher sealing force on the sealing lip, resulting in extended seal life. One of Trident's customers expects to reduce downtime by 30% as a result of the new seal design. If lost production would have cost the company \$110,000 per year for the next 4 years, how much could the company afford to spend now on the new seals, if it uses an interest rate of 12% per year?
- 2.14 China spends an estimated \$100,000 per year on cloud seeding efforts, which includes using anti-aircraft guns and rocket launchers to fill the sky with silver iodide. In the United States, utilities that run hydroelectric dams are among the most active cloud seeders, because they believe it is a cost-effective way to increase limited water supplies by 10% or more. If the yields of cash crops will increase by 4% each year for the next 3 years because of extra irrigation water captured behind dams during cloud seeding, what is the maximum amount the farmers should spend now on the cloud seeding activity? The value of the cash crops without the extra irrigation water would be \$600,000 per year. Use an interest rate of 10% per year.
- 2.15 The Public Service Board (PSB) awarded two contracts worth a combined \$1.07 million to improve (i.e., deepen) a retention basin and reconstruct the spillway that was severely damaged in a flood 2 years ago. The PSB said that, because of the weak economy, the bids came in \$950,000 lower than engineers expected. If the projects are assumed to have a 20-year life, what is the annual worth of the savings at an interest rate of 6% per year?
- 2.16 The National Highway Traffic Safety Administration raised the average fuel efficiency standard to 35.5 miles per gallon for cars and light trucks by the year 2016. The rules will cost consumers an average of \$434 extra per vehicle in the 2012 model year. If a person purchases a new car in 2012 and keeps it for 5 years, how much must be saved in fuel costs each year to justify the extra cost? Use an interest rate of 8% per year.
- 2.17 In an effort to reduce childhood obesity by reducing the consumption of sugared beverages, some states have imposed taxes on soda and other soft drinks. A survey by Roland Sturm of 7300 fifth-graders revealed that if taxes averaged 4 cents on each dollar's worth of soda, no real difference in overall consumption was noticed. However, if taxes were increased to 18 cents on the dollar, Sturm calculated they would make a

- significant difference. For a student who consumes 100 sodas per year, what is the future worth of the extra cost from 4 cents to 18 cents per soda? Assume the student consumes sodas from grade 5 through graduation in grade 12. Use an interest rate of 6% per year.
- 2.18 The Texas Tomorrow Fund (TTF) is a program started in 1996 in Texas wherein parents could prepay their child's college tuition when the child was young. Actuaries set the price based on costs and investment earnings at that time. Later, the Texas legislature allowed universities to set their own tuition rates; tuition costs jumped dramatically. The cost for entering a newborn in 1996 was \$10,500. If the TTF fund grew at a rate of 4% per year, while tuition costs increased at 7% per year, determine the state's shortfall when a newborn enters college 18 years later.
- 2.19 Henry Mueller Supply Co. sells tamperproof, normally open thermostats (i.e., thermostat closes as temperature rises). Annual cash flows are shown in the table below. Determine the future worth of the net cash flows at an interest rate of 10% per year.
- | Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Income, \$1000 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 |
| Cost, \$1000 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 |
- 2.20 A company that makes self-clinching fasteners expects to purchase new production-line equipment in 3 years. If the new units will cost \$350,000, how much should the company set aside each year, if the account earns 10% per year?
- ### Factor Values
- 2.21 Find the numerical value of the following factors using (a) interpolation and (b) the formula.
1. $(A/P, 13\%, 15)$
 2. $(P/G, 27\%, 10)$
- 2.22 Find the numerical value of the following factors using (a) interpolation, (b) the formula, and (c) a spreadsheet function.
1. $(F/P, 14\%, 62)$
 2. $(A/F, 1\%, 45)$
- 2.23 For the factor $(F/P, 10\%, 43)$, find the percent difference between the interpolated and formula-calculated values, assuming the formula-calculated value is the correct one.
- 2.24 For the factor $(F/A, 15\%, 52)$, find the percent difference between the interpolated and formula-calculated values, assuming the formula-calculated value is the correct one.
- 2.25 Profits from recycling paper, cardboard, aluminum, and glass at a liberal arts college have increased at a constant rate of \$1100 in each of the last 3 years. If this year's profit (end of year 1) is expected to be \$6000 and the profit trend continues through year 5, (a) what will the profit be at the end of year 5 and (b) what is the present worth of the profit at an interest rate of 8% per year?
- 2.26 A report by the Government Accountability Office (GAO) shows that the GAO expects the U.S. Postal Service to lose a record \$7 billion at the end of this year, and if the business model is not changed, the losses will total \$241 billion by the end of year 10. If the losses increase uniformly over the 10-year period, determine the following:
- (a) The expected increase in losses each year
 - (b) The loss 5 years from now
 - (c) The equivalent uniform worth of the losses at an interest rate of 8% per year
- 2.27 Rolled ball screws are suitable for high-precision applications such as water jet cutting. Their total manufacturing cost is expected to decrease because of increased productivity, as shown in the table. Determine the equivalent annual cost at an interest rate of 8% per year.
- | Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Cost, \$1000 | 200 | 195 | 190 | 185 | 180 | 175 | 170 | 165 |
- 2.28 Western Hydra Systems makes a panel milling machine with a 2.7-m-diameter milling head that emits low vibration and processes stress-relieved aluminum panels measuring up to 6000 mm long. The company wants to borrow money for a new production/warehouse facility. If the company offers to repay the loan with \$60,000 in year 1 and amounts increasing by \$10,000 each year through year 5, how much can the company borrow at an interest rate of 10% per year?
- 2.29 GKX Industries expects sales of its hydraulic seals (in inch and metric sizes) to increase according to the cash flow sequence $\$70 + 4k$, where k is in years and cash flow is in \$1000.
- (a) What is the amount of the cash flow in year 3?
 - (b) What is the future worth of the entire cash flow series in year 10? Let $i = 10\%$ per year.

calculated values, assuming the formula-calculated value is the correct one.

Arithmetic Gradient

- 2.30 For the cash flows below, determine the amount in year 1, if the annual worth in years 1 through 9 is \$601.17 and the interest rate is 10% per year.

Year	1	2	3	4	5	6	7	8	9
Cost, \$1000	A	$A + 30$	$A + 60$	$A + 90$	$A + 120$	$A + 150$	$A + 180$	$A + 210$	$A + 240$

- 2.31 Apple Computer wants to have \$2.1 billion available 5 years from now to finance production of a handheld "electronic brain" that, based on your behavior, will learn how to control nearly all the electronic devices in your home, such as the thermostat, coffee pot, TV, and sprinkler system. The company expects to set aside uniformly increasing amounts of money each year to meet its goal. If the amount set aside at the end of year 1 is \$50 million, how much will the constant increase G have to be each year? Assume the investment account grows at a rate of 18% per year.
- 2.32 Tacozza Electric, which manufactures brush dc servomotors, budgeted \$75,000 per year to pay for certain components over the next 5 years. If the company expects to spend \$15,000 in year 1, how much of a uniform (arithmetic) increase each year is the company expecting in the cost of this part? Assume the company uses an interest rate of 10% per year.

Geometric Gradient

- 2.33 There are no tables in the back of your book for the geometric gradient series factors. Calculate the first two annual worth factor values, that is, A values for $n = 1$ and 2, that would be in a 10% interest table for a growth rate of 4% per year.
- 2.34 Determine the present worth of a geometric gradient series with a cash flow of \$50,000 in year 1 and increases of 6% each year through year 8. The interest rate is 10% per year.
- 2.35 Determine the difference in the present worth values of the following two commodity contracts at an interest rate of 8% per year.
Contract 1 has a cost of \$10,000 in year 1; costs will escalate at a rate of 4% per year for 10 years. Contract 2 has the same cost in year 1, but costs will escalate at 6% per year for 11 years.
- 2.36 El Paso Water Utilities (EPWU) purchases surface water for treatment and distribution to EPWU customers from El Paso County Water Improvement District during the irrigation season. A new contract between the two entities resulted in a reduction in future price increases in the cost of the water from

8% per year to 4% per year for the next 20 years. If the cost of water next year (which is year 1 of the new contract) will be \$260 per acre-foot, what is the present worth of the savings (in \$/acre-ft) to the utility between the old and the new contracts? Let the interest rate equal 6% per year.

- 2.37 Determine the present worth of a maintenance contract that has a cost of \$30,000 in year 1 and annual increases of 6% per year for 10 years. Use an interest rate of 6% per year.

Interest Rate and Rate of Return

- 2.38 Gesky Industrial Products manufactures brushless blowers for boilers, food service equipment, kilns, and fuel cells. The company borrowed \$18,000,000 for a plant expansion and repaid the loan in seven annual payments of \$3,576,420, with the first payment made 1 year after the company received the money. What was the interest rate on the loan? Use hand and spreadsheet solutions.
- 2.39 If the value of Jane's retirement portfolio increased from \$170,000 to \$813,000 over a 15-year period, with no deposits made to the account over that period, what annual rate of return did she make?
- 2.40 A person's credit score is important in determining the interest rate on a home mortgage. According to Consumer Credit Counseling Service, a homeowner with a \$100,000 mortgage and a 520 credit score will pay \$110,325 more in interest charges over the life of a 30-year loan than a homeowner with the same mortgage and a credit score of 720. How much higher would the interest rate per year have to be in order to account for this much difference in interest charges, if the \$100,000 loan is repaid in a single lump-sum payment at the end of 30 years?
- 2.41 During a period when the real estate market in Phoenix, Arizona, was undergoing a significant downturn, CSM Consulting Engineers made an agreement with a distressed seller to purchase an office building under the following terms: total price of \$1.2 million with a down payment of \$200,000 now and no payments for 4 years, after which the remaining balance of \$1 million would

be paid. CSM was able to make this deal because of poor market conditions at the time of purchase, and, at the same time, planning to sell the building in 4 years (when market conditions would probably be better) and move to a larger office building in Scottsdale, Arizona. If CSM was able to sell the building in exactly 4 years for \$1.9 million, what rate of return per year did the company make on the investment?

- 2.42 A start-up company that makes hydraulic seals borrowed \$800,000 to expand its packaging and shipping facility. The contract required the company to repay the investors through an innovative mechanism called *faux dividends*, a series of uniform annual payments over a fixed period of time. If the company paid \$250,000 per year for 5 years, what was the interest rate on the loan?
- 2.43 Bessimer Electronics manufactures addressable actuators in one of its Maquiladora plants in Mexico. The company believes that by investing \$24,000 each year in years 1, 2, and 3, it will avoid spending \$87,360 in year 3. If the company does make the annual investments, what rate of return will it realize?
- 2.44 UV curable epoxy resins are used in sealing, in gap filling, and as a clear coating. Your boss saw a report submitted by the chief financial officer (CFO) that said the equivalent annual worth of maintaining the equipment used in producing the resins was \$48,436 over the last 5 years. The report showed that the cost in year 1 was \$42,000, and it increased arithmetically by \$4000 each year. Your boss thought \$48,436 was too high, so she asked you to determine what interest rate the CFO used in making the calculations. What was the interest rate?

Number of Years

- 2.45 Acme Bricks, a masonry products company, wants to have \$600,000 on hand before it invests in new conveyors, trucks, and other equipment. If the company sets aside \$80,000 per year in an account that increases in value at a rate of 15% per year, how many years will it be before Acme can purchase the equipment?
- 2.46 An engineer who was contemplating retirement had \$1.6 million in his investment portfolio. However, a severe recession caused his portfolio to decrease to only 55% of the original amount, so he kept working. If he was able to invest his money at a rate of return of 9% per year after the recession ended, how many years did it take for his account to get back to the \$1.6 million value?

- 2.47 You own a small engineering consulting company. If you invest \$200,000 of the company's money in a natural gas well that is expected to provide income of \$29,000 per year, how long must the well produce at that rate in order to get the money back plus a rate of return of 10% per year?
- 2.48 A perceptive engineer started saving for her retirement 15 years ago by diligently saving \$18,000 each year through the present time. She invested in a stock fund that averaged a 12% rate of return over that period. If she makes the same annual investment and gets the same rate of return in the future, how long will it be from now (time zero) before she has \$1,500,000 in her retirement fund?
- 2.49 A mechanical engineering graduate who wanted to have his own business borrowed \$350,000 from his father as start-up money. Because he was family, his father charged interest at only 4% per year. If the engineer was able to pay his father \$15,000 in year 1, \$36,700 in year 2, and amounts increasing by \$21,700 each year, how many years did it take for the engineer to repay the loan?
- 2.50 The energy costs of a company involved in powder coating of outdoor furniture are expected to increase at a rate of \$400 per year. The cost at the end of the next year (year 1) is expected to be \$13,000. How many years will it be from now before the equivalent annual cost is \$16,000 per year, if interest is 8% per year?
- 2.51 In cleaning out some files that were left behind by the engineer who preceded you in your current job, you found an old report that had a calculation for the present worth of certain maintenance costs for state highways. The report contained the following equation (with cost in \$1 million):
- $$12(1 - [(1 + 0.03)/(1 + 0.06)]^x) / (0.06 - 0.03) = 140$$
- The value of x that was used in the calculation was illegible. What is its value?
- 2.52 The equivalent annual worth of an increasing arithmetic gradient is \$135,300. If the cash flow in year 1 is \$35,000 and the gradient amount is \$19,000, what is the value of n at an interest rate of 10% per year?
- 2.53 You are told that the present worth of an increasing geometric gradient is \$88,146. If the cash flow in year 1 is \$25,000 and the gradient increase is 18% per year, what is the value of n ? The interest rate is 10% per year.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 2.54** The amount of money that Diamond Systems can spend now for improving productivity in lieu of spending \$30,000 three years from now at an interest rate of 12% per year is closest to:
- \$15,700
 - \$17,800
 - \$19,300
 - \$21,350
- 2.55** A manufacturing company spent \$30,000 on a new conveyor belt. If the conveyor belt resulted in cost savings of \$4200 per year, the length of time it would take for the company to recover its investment at 8% per year is closest to:
- Less than 9 years
 - 9 to 10 years
 - 11 to 12 years
 - Over 12 years
- 2.56** Levi Strauss has some of its jeans stone-washed under a contract with independent U.S. Garment Corp. If U.S. Garment's operating cost per machine is \$22,000 for year 1 and increases by a constant \$1000 per year through year 5, what is the equivalent uniform annual cost per machine for the 5 years at an interest rate of 8% per year?
- \$23,850
 - \$24,650
 - \$25,930
 - Over \$26,000
- 2.57** The F/G factor values can be derived by multiplying:
- (P/F) and (A/G) factor values
 - (F/P) and (A/G) factor values
 - (P/F) and (P/G) factor values
 - (F/P) and (P/G) factor values
- 2.58** At $i = 4\%$ per year, A for years 1 through 6 of the cash flows shown below is closest to:
- \$300
 - \$560
 - \$800
 - \$1040
- Years

0	1	2	3	4	5	6
---	---	---	---	---	---	---
- 2.59** The value of the factor $(P/F, i, 10)$ can be found by getting the factor values for $(P/F, i, 4)$ and $(P/F, i, 6)$ and:
- Adding the values for $(P/F, i, 4)$ and $(P/F, i, 6)$
 - Multiplying the values for $(P/F, i, 4)$ and $(P/F, i, 6)$
 - Dividing the value for $(P/F, i, 6)$ by the value for $(P/F, i, 4)$
 - None of the above
- 2.60** A small construction company is considering the purchase of a used bulldozer for \$61,000. If the company purchases the dozer now, the equivalent future amount in year 4 that the company is paying for the dozer at 4% per year interest is closest to:
- \$52,143
 - \$65,461
 - \$71,365
 - Over \$72,000
- 2.61** The cost of lighting and maintaining the tallest smokestack in the United States (at a shuttered ASARCO refinery) is \$90,000 per year. At an interest rate of 10% per year, the present worth of maintaining the smokestack for 10 years is closest to:
- \$1,015,000
 - \$894,000
 - \$712,000
 - \$553,000
- 2.62** An enthusiastic new engineering graduate plans to start a consulting firm by borrowing \$100,000 at 10% per year interest. The loan payment each year to pay off the loan in 7 years is closest to:
- \$18,745
 - \$20,540
 - \$22,960
 - \$23,450
- 2.63** An engineer who believed in "save now and play later" wanted to retire in 20 years with \$1.5 million. At 10% per year interest, to reach the \$1.5 million goal, starting 1 year from now, the engineer must annually invest:
- \$26,190
 - \$28,190
 - \$49,350
 - \$89,680
- 2.64** The cost of a border fence is \$3 million per mile. If the life of such a fence is assumed to be 10 years, the equivalent annual cost of a

- 10-mile-long fence at an interest rate of 10% per year is closest to:
- \$3.6 million
 - \$4.2 million
 - \$4.9 million
 - Over \$5.0 million
- 2.65 An investment of \$75,000 in equipment that will reduce the time for machining self-locking fasteners will save \$20,000 per year. At an interest rate of 10% per year, the number of years required to recover the initial investment is closest to:
- 6 years
 - 5 years
 - 4 years
 - 3 years
- 2.66 The number of years required for an account to accumulate \$650,000 if Ralph deposits \$50,000 each year and the account earns interest at a rate of 6% per year is closest to:
- 13 years
 - 12 years
 - 11 years
 - 10 years
- 2.67 Aero Serve, Inc., manufactures cleaning nozzles for reverse-pulse jet dust collectors. The company spent \$40,000 on a production control system that will increase profits by \$13,400 per year for 5 years. The rate of return per year on the investment is closest to:
- 20%
 - 18%
 - 16%
 - Less than 15%
- 2.68 Energy costs for a green chemical treatment have been increasing uniformly for 5 years. If the cost in year 1 was \$26,000 and it increased by \$2000 per year through year 5, the present worth of the costs at an interest rate of 10% per year is closest to:
- \$102,900
 - \$112,300
 - \$122,100
 - \$195,800
- 2.69 In planning for your retirement, you expect to save \$5000 in year 1, \$6000 in year 2, and amounts increasing by \$1000 each year through year 20. If your investments earn 10% per year, the amount you will have at the end of year 20 is closest to:
- \$242,568
 - \$355,407
 - \$597,975
 - \$659,125
- 2.70 Income from a precious metals mining operation has been decreasing uniformly for 5 years. If income in year 1 was \$300,000 and it decreased by \$30,000 per year through year 4, the annual worth of the income at 10% per year is closest to:
- \$310,500
 - \$258,600
 - \$203,900
 - \$164,800
- 2.71 If you are able to save \$5000 in year 1, \$5150 in year 2, and amounts increasing by 3% each year through year 20, the amount you will have at the end of year 20 at 10% per year interest is closest to:
- \$60,810
 - \$102,250
 - \$351,500
 - Over \$410,000

CASE STUDY

TIME MARCHES ON; SO DOES THE INTEREST RATE

Background

During the last week, Sundara has read about different situations that involve money, interest rate, and different amounts of time. She has gotten interested in the major effects that time and interest rates have on the amount of money necessary to do things and the significant growth in the amount of money when a large number of years are considered. In all cases, the interest focuses on the amount of money at the end of the time period.

Information

The four situations are described here.

- Manhattan Island was purchased in 1626 for \$24. After 385 years in 2011, at 6% per year compounded interest, the current value must be very large.
- At the age of 22, if she saved only \$2000 per year for the next 10 years (starting next year) and made a return of 6% per year, by today's standards, she would have accumulated a nice sum at the age of 70.

- C. A corporation invested \$2 million in developing and marketing a new product in 1945 (just after World War II, this was a lot of money) and has made a steady net cash flow of \$300,000 per year for some 65 years. Sundara estimated the annual rate of return must be quite good, especially given that she is lucky to earn 4% per year on her own investments these days.
- D. A friend who is not good with money, went to a pawn shop and borrowed \$200 for one week and paid \$30 in interest. Sundara thought this might be a pretty good deal, in case she ever ran low on cash. However, she did not know whether the interest was simple or compounded monthly, and how much may be owed were this loan not paid off for 1 year.

Case Study Exercises

1. What is the annual interest rate for each situation? Include both the annual simple and the compound rates for situation D.
2. Calculate and observe the total amount of money involved in each situation at the end of the time periods compared to the starting amount. Is the ending amount larger or smaller than you would expect it to be prior to making any computations?
3. Think of a situation for yourself that may be similar to any of those above. Determine the interest rate, the time period, and the starting and ending amounts of money.

CHAPTER 3

Combining Factors and Spreadsheet Functions



LEARNING OUTCOMES

Purpose: Use multiple factors and spreadsheet functions to find equivalent amounts for cash flows that have nonstandard placement.

SECTION	TOPIC	LEARNING OUTCOME
3.1	Shifted series	<ul style="list-style-type: none">Determine the P, F or A values of a series starting at a time other than period 1.
3.2	Shifted series and single cash flows	<ul style="list-style-type: none">Determine the P, F, or A values of a shifted series and randomly placed single cash flows.
3.3	Shifted gradients	<ul style="list-style-type: none">Make equivalence calculations for shifted arithmetic or geometric gradient series that increase or decrease in size of cash flows.

M

ost estimated cash flow series do not fit exactly the series for which the factors, equations, and spreadsheet functions in Chapter 2 were developed. For a given sequence of cash flows, there are usually several correct ways to determine the equivalent present worth P , future worth F , or annual worth A . This chapter explains how to combine engineering economy factors and spreadsheet functions to address more complex situations involving shifted uniform series, gradient series, and single cash flows.

3.1 Calculations for Uniform Series That Are Shifted ● ● ●

When a uniform series begins at a time other than at the end of period 1, it is called a **shifted series**. In this case several methods can be used to find the equivalent present worth P . For example, P of the uniform series shown in Figure 3–1 could be determined by any of the following methods:

- Use the P/F factor to find the present worth of each disbursement at year 0 and add them.
- Use the F/P factor to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total, using $P = F(P/F,i,13)$.
- Use the F/A factor to find the future amount $F = A(F/A,i,10)$, and then compute the present worth, using $P = F(P/F,i,13)$.
- Use the P/A factor to compute the “present worth” $P_3 = A(P/A,i,10)$ (which will be located in year 3, not year 0), and then find the present worth in year 0 by using the $(P/F,i,3)$ factor.

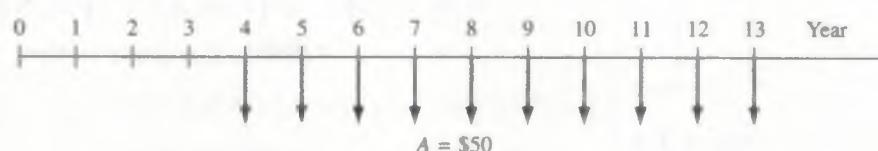
Typically the last method is used for calculating the present worth of a uniform series that does not begin at the end of period 1. For Figure 3–1, the “present worth” obtained using the P/A factor is located in year 3. This is shown as P_3 in Figure 3–2. Note that a P value is always located *1 year or period prior* to the beginning of the first series amount. Why? Because the P/A factor was derived with P in time period 0 and A beginning at the end of period 1. The most common mistake made in working problems of this type is improper placement of P . Therefore, it is extremely important to remember:

The present worth is always located **one period prior** to the first uniform series amount when using the P/A factor.

To determine a future worth or F value, recall that the F/A factor derived in Section 2.3 had the F located in the *same* period as the last uniform series amount. Figure 3–3 shows the location of the future worth when F/A is used for Figure 3–1 cash flows.

The future worth is always located in the **same period as the last** uniform series amount when using the F/A factor.

It is also important to remember that the number of periods n in the P/A or F/A factor is equal to the number of uniform series values. It may be helpful to *relabel* the cash flow diagram to avoid errors in counting. Figures 3–2 and 3–3 show Figure 3–1 relabeled to determine $n = 10$.



Placement of P

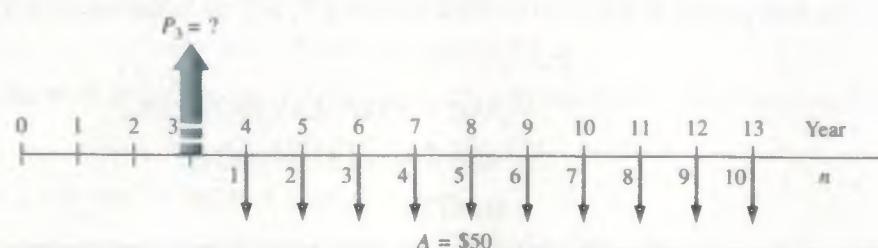
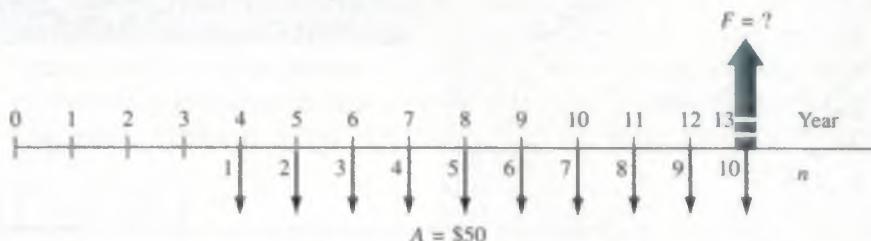


Figure 3–1
A uniform series that is shifted.

Figure 3–2
Location of present worth and renumbering for n for the shifted uniform series in Figure 3–1.

Figure 3–3
Placement of F and renumbering for n for the shifted uniform series of Figure 3–1.



As stated above, several methods can be used to solve problems containing a uniform series that is shifted. However, it is generally more convenient to use the uniform series factors than the single-amount factors. Specific steps should be followed to avoid errors:

1. Draw a diagram of the positive and negative cash flows.
2. Locate the present worth or future worth of each series on the cash flow diagram.
3. Determine n for each series by renumbering the cash flow diagram.
4. Draw another cash flow diagram representing the desired equivalent cash flow.
5. Set up and solve the equations.

These steps are illustrated below.

EXAMPLE 3.1

The offshore design group at Bechtel just purchased upgraded CAD software for \$5000 now and annual payments of \$500 per year for 6 years starting 3 years from now for annual upgrades. What is the present worth in year 0 of the payments if the interest rate is 8% per year?

Solution

The cash flow diagram is shown in Figure 3–4. The symbol P_A is used throughout this chapter to represent the present worth of a uniform annual series A , and P'_A represents the present worth at a time other than period 0. Similarly, P_T represents the total present worth at time 0. The correct placement of P'_A and the diagram renumbering to obtain n are also indicated. Note that P'_A is located in actual year 2, not year 3. Also, $n = 6$, not 8, for the P/A factor. First find the value of P'_A of the shifted series.

$$P'_A = \$500(P/A, 8\%, 6)$$

Since P'_A is located in year 2, now find P_A in year 0.

$$P_A = P'_A(P/F, 8\%, 2)$$

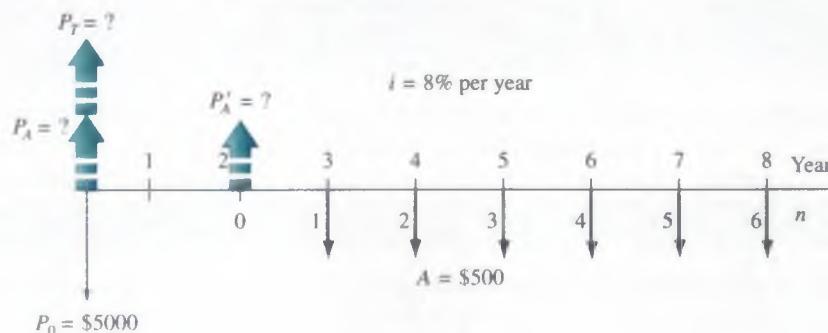


Figure 3–4
Cash flow diagram with placement of P values, Example 3.1.

The total present worth is determined by adding P_A and the initial payment P_0 in year 0.

$$\begin{aligned} P_T &= P_0 + P_A \\ &= 5000 + 500(P/A, 8\%, 6)(P/F, 8\%, 2) \\ &= 5000 + 500(4.6229)(0.8573) \\ &= \$6981.60 \end{aligned}$$

The more complex that cash flow series become, the more useful are the spreadsheet functions. When the uniform series A is shifted, the NPV function is used to determine P , and the PMT function finds the equivalent A value. The NPV function, like the PV function, determines the P values, but NPV can handle any combination of cash flows directly from the cells. As we learned in Chapter 2, enter the net cash flows in contiguous cells (column or row), making sure to enter "0" for all zero cash flows. Use the format

$$\text{NPV}(i\%, \text{second_cell:last_cell}) + \text{first_cell}$$

First_cell contains the cash flow for year 0 and must be listed separately for NPV to correctly account for the time value of money. The cash flow in year 0 may be 0.

The easiest way to find an equivalent A over n years for a shifted series is with the PMT function, where the P value is from the NPV function above. The format is the same as we learned earlier; the entry for P is a cell reference, not a number.

$$\text{PMT}(i\%, n, \text{cell_with_}P, F)$$

Alternatively, the same technique can be used when an F value was obtained using the FV function. Now the last entry in PMT is "cell_with_F."

It is very fortunate that any parameter in a spreadsheet function can itself be a function. Thus, it is possible to write the PMT function in a single cell by embedding the NPV function (and FV function, if needed). The format is

$$\text{PMT}(i\%, n, \text{NPV}(i\%, \text{second_cell:last_cell}) + \text{first_cell}, F) \quad [3.1]$$

Of course, the answer for A is the same for the two-cell operation or a single-cell, embedded function. All three of these functions are illustrated in Example 3.2.

EXAMPLE 3.2

Recalibration of sensitive measuring devices costs \$8000 per year. If the machine will be recalibrated for each of 6 years starting 3 years after purchase, calculate the 8-year equivalent uniform series at 16% per year. Show hand and spreadsheet solutions.

Solution by Hand

Figure 3–5a and b shows the original cash flows and the desired equivalent diagram. To convert the \$8000 shifted series to an equivalent uniform series over all periods, first convert the uniform series into a present worth or future worth amount. Then either the A/P factor or the A/F factor can be used. Both methods are illustrated here.

Present worth method. (Refer to Figure 3–5a.) Calculate P'_A for the shifted series in year 2, followed by P_T in year 0. There are 6 years in the A series.

$$P'_A = 8000(P/A, 16\%, 6)$$

$$\begin{aligned} P_T &= P'_A(P/F, 16\%, 2) = 8000(P/A, 16\%, 6)(P/F, 16\%, 2) \\ &= 8000(3.6847)(0.7432) = \$21,907.75 \end{aligned}$$

The equivalent series A' for 8 years can now be determined via the A/P factor.

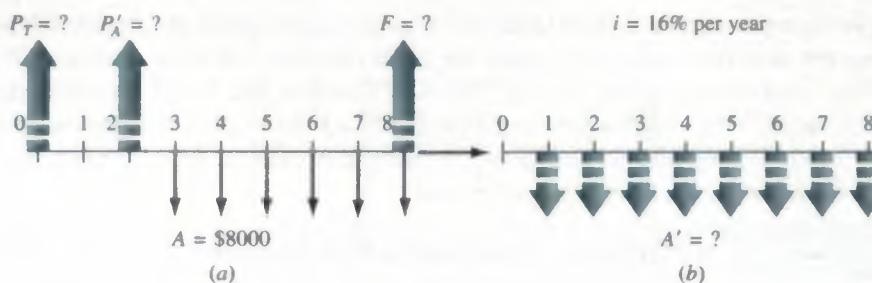
$$A' = P_T(A/P, 16\%, 8) = \$5043.60$$

Future worth method. (Refer to Figure 3–5a.) First calculate the future worth F in year 8.

$$F = 8000(F/A, 16\%, 6) = \$71,820$$

The A/F factor is now used to obtain A' over all 8 years.

$$A' = F(A/F, 16\%, 8) = \$5043.20$$



A	B	C	D	E	F	G
1						
2	Year	Cash flow				
3	0	0				
4	1	0				
5	2	0				
6	3	-\$8,000				
7	4	-\$8,000				
8	5	-\$8,000				
9	6	-\$8,000				
10	7	-\$8,000				
11	8	-\$8,000				
12	P value	-\$21,906.87				
13						
14						
15						
16						

(c)

Figure 3–5

(a) Original and (b) equivalent cash flow diagrams; and (c) spreadsheet functions to determine P and A , Example 3.2.

Solution by Spreadsheet

(Refer to Figure 3–5c.) Enter the cash flows in B3 through B11 with entries of “0” in the first three cells. Use the NPV function to display $P = \$21,906.87$.

There are two ways to obtain the equivalent A over 8 years. Of course, only one of these PMT functions needs to be entered. (1) Enter the PMT function making direct reference to the P value (see cell tag for D/E5), or (2) use Equation [3.1] to embed the NPV function into the PMT function (see cell tag for D/E8).



3.2 Calculations Involving Uniform Series and Randomly Placed Single Amounts ● ● ●

When a cash flow includes both a uniform series and randomly placed single amounts, the procedures of Section 3.1 are applied to the uniform series and the single-amount formulas are applied to the one-time cash flows. This approach, illustrated in Examples 3.3 and 3.4, is merely a combination of previous ones. For spreadsheet solutions, it is necessary to enter the net cash flows before using the NPV and other functions.

EXAMPLE 3.3

An engineering company in Wyoming that owns 50 hectares of valuable land has decided to lease the mineral rights to a mining company. The primary objective is to obtain long-term income to finance ongoing projects 6 and 16 years from the present time. The engineering company makes a proposal to the mining company that it pay \$20,000 per year for 20 years beginning 1 year from now, plus \$10,000 six years from now and \$15,000 sixteen years from now. If the mining company wants to pay off its lease immediately, how much should it pay now if the investment is to make 16% per year?

Solution

The cash flow diagram is shown in Figure 3–6 from the owner's perspective. Find the present worth of the 20-year uniform series and add it to the present worth of the two one-time amounts to determine P_T .

$$\begin{aligned} P_T &= 20,000(P/A, 16\%, 20) + 10,000(P/F, 16\%, 6) + 15,000(P/F, 16\%, 16) \\ &= \$124,075 \end{aligned}$$

Note that the \$20,000 uniform series starts at the end of year 1, so the P/A factor determines the present worth at year 0.

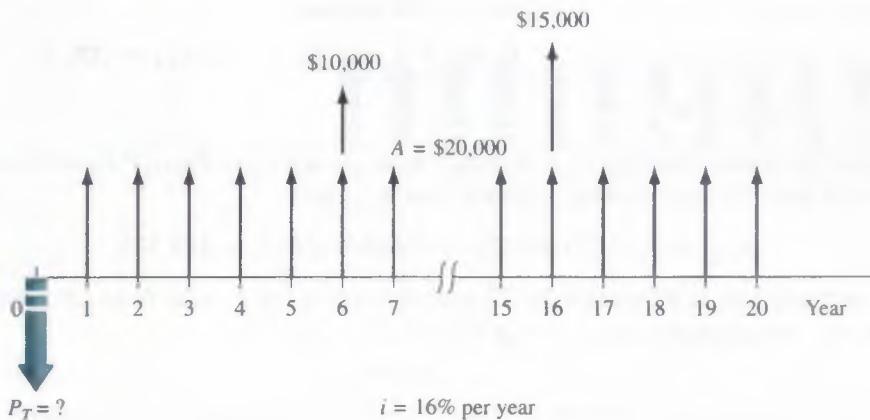


Figure 3–6
Diagram including a uniform series and single amounts, Example 3.3.

When you calculate the A value for a cash flow series that includes randomly placed single amounts and uniform series, **first convert everything to a present worth or a future worth**. Then you obtain the A value by multiplying P or F by the appropriate A/P or A/F factor. Example 3.4 illustrates this procedure.

EXAMPLE 3.4

A design-build-operate engineering company in Texas that owns a sizable amount of land plans to lease the drilling rights (oil and gas only) to a mining and exploration company. The contract calls for the mining company to pay \$20,000 per year for 20 years beginning 3 years from now (i.e., beginning at the end of year 3 and continuing through year 22) plus \$10,000 six years from now and \$15,000 sixteen years from now. Utilize engineering economy relations by hand and by spreadsheet to determine the five *equivalent values* listed below at 16% per year.

1. Total present worth P_T in year 0
2. Future worth F in year 22
3. Annual series over all 22 years
4. Annual series over the first 10 years
5. Annual series over the last 12 years

Solution by Hand

Figure 3–7 presents the cash flows with equivalent P and F values indicated in the correct years for the P/A , P/F , and F/A factors.

1. P_T in year 0: First determine P'_A of the series in year 2. Then P_T is the sum of three P values: the series present worth value moved back to $t = 0$ with the P/F factor, and the two P values at $t = 0$ for the two single amounts in years 6 and 16.

$$\begin{aligned}
 P'_A &= 20,000(P/A, 16\%, 20) \\
 P_T &= P'_A(P/F, 16\%, 2) + 10,000(P/F, 16\%, 6) + 15,000(P/F, 16\%, 16) \\
 &= 20,000(P/A, 16\%, 20)(P/F, 16\%, 2) + 10,000(P/F, 16\%, 6) \\
 &\quad + 15,000(P/F, 16\%, 16) \\
 &= \$93,625
 \end{aligned} \tag{3.2}$$

2. ***F* in year 22:** To determine *F* in year 22 from the original cash flows (Figure 3–7), find *F* for the 20-year series and add the two *F* values for the two single amounts. Be sure to carefully determine the *n* values for the single amounts: *n* = 22 – 6 = 16 for the \$10,000 amount and *n* = 22 – 16 = 6 for the \$15,000 amount.

$$\begin{aligned}
 F &= 20,000(F/A, 16\%, 20) + 10,000(F/P, 16\%, 16) + 15,000(F/P, 16\%, 6) \\
 &= \$2,451,626
 \end{aligned} \tag{3.3}$$

3. ***A* over 22 years:** Multiply $P_T = \$93,625$ from (1) above by the *A/P* factor to determine an equivalent 22-year *A* series, referred to as A_{1-22} here.

$$A_{1-22} = P_T(A/P, 16\%, 22) = 93,625(0.16635) = \$15,575 \tag{3.4}$$

An alternate way to determine the 22-year series uses the *F* value from (2) above. In this case, the computation is $A_{1-22} = F(A/F, 16\%, 22) = \$15,575$.

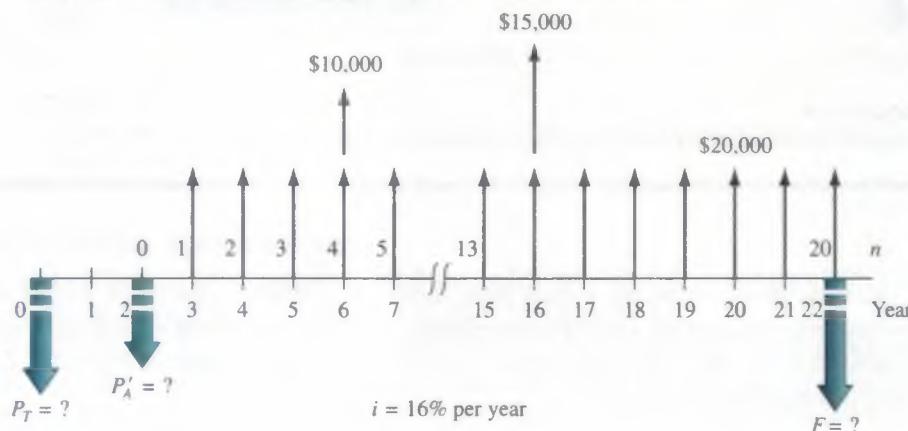


Figure 3–7
Diagram for Example 3.4.

4. ***A* over years 1 to 10:** This and (5), which follows, are special cases that often occur in engineering economy studies. The equivalent *A* series is calculated for a number of years different from that covered by the original cash flows. This occurs when a defined *study period* or *planning horizon* is preset for the analysis. (More is mentioned about study periods later.) To determine the equivalent *A* series for years 1 through 10 only (call it A_{1-10}), the P_T value must be used with the *A/P* factor for *n* = 10. This computation transforms Figure 3–7 into the equivalent series A_{1-10} in Figure 3–8a.

$$A_{1-10} = P_T(A/P, 16\%, 10) = 93,625(0.20690) = \$19,371 \tag{3.5}$$

5. ***A* over years 11 to 22:** For the equivalent 12-year series for years 11 through 22 (call it A_{11-22}), the *F* value must be used with the *A/F* factor for 12 years. This transforms Figure 3–7 into the 12-year series A_{11-22} in Figure 3–8b.

$$A_{11-22} = F(A/F, 16\%, 12) = 2,451,626(0.03241) = \$79,457 \tag{3.6}$$

Notice the huge difference of more than \$60,000 in equivalent annual amounts that occurs when the present worth of \$93,625 is allowed to compound at 16% per year for the first 10 years. This is another demonstration of the time value of money.

Solution by Spreadsheet

Figure 3–9 is a spreadsheet image with answers for all five questions. The \$20,000 series and the two single amounts have been entered into separate columns, B and C. The zero cash flow

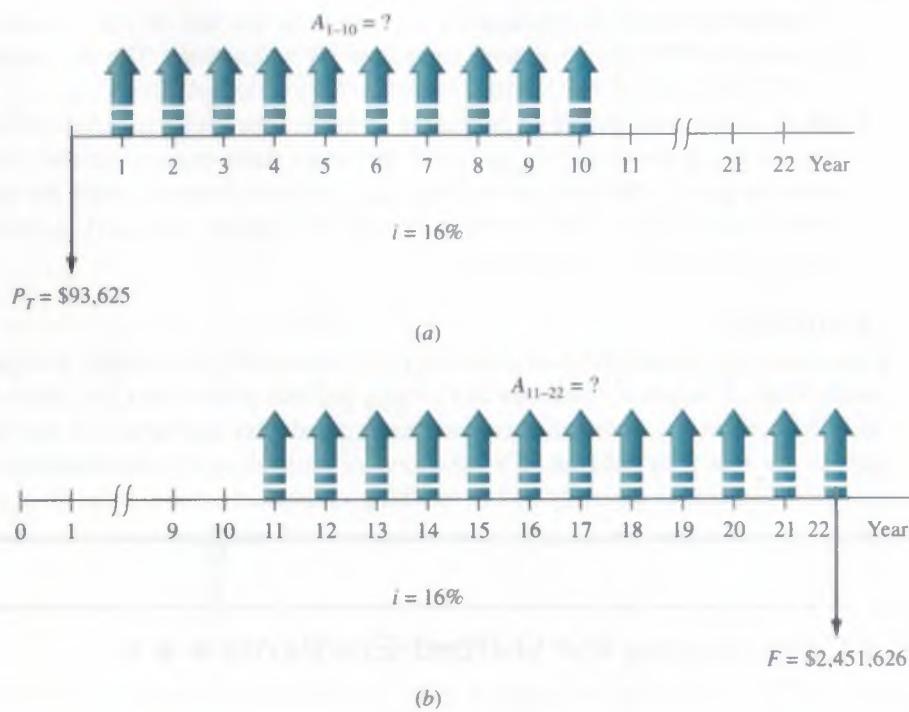


Figure 3-8
Cash flows of Figure 3-7 converted to equivalent uniform series for
(a) years 1 to 10 and
(b) years 11 to 22.

		Interest rate		F	
		16.00%			
		Cash flows			
Year	Series	Single		Value calculated	Spreadsheet function used to obtain result in column F
5	0	0	0	Present worth of series	= NPV(D1,B6:B27) + B5
6	1	0	0		\$88,122
7	2	0	0		
8	3	\$ 20,000	0	Present worth of singles	= NPV(D1,C6:C27) + C5
9	4	\$ 20,000	0		\$5,500
10	5	\$ 20,000	0		
11	6	\$ 20,000	\$ 10,000	1. Present worth total	= F6 + F10 or = NPV(D1,B6:B27) + NPV(D1,C6:C27)
12	7	\$ 20,000	0		\$93,622
13	8	\$ 20,000	0	2. Future worth total	= FV(D1,22,0,-F14)
14	9	\$ 20,000	0		\$2,451,621
15	10	\$ 20,000	0	3. Annual series over 22 years	= PMT(D1,22,-F14)
16	11	\$ 20,000	0		\$15,574
17	12	\$ 20,000	0	4. Annual series for first 10 years	= PMT(D1,10,-F14)
18	13	\$ 20,000	0		\$10,370
19	14	\$ 20,000	0	5. Annual series for last 12 years	= PMT(D1,12,0,-F16)
20	15	\$ 20,000	0		\$79,400
21	16	\$ 20,000	\$ 15,000		
22	17	\$ 20,000	0		
23	18	\$ 20,000	0		
24	19	\$ 20,000	0		
25	20	\$ 20,000	0		
26	21	\$ 20,000	0		
27	22	\$ 20,000	0		

Figure 3-9
Spreadsheet using cell reference format.
Example 3.4.

values are all entered so that the functions will work correctly. This is an excellent example demonstrating the versatility of the NPV, FV, and PMT functions. To prepare for sensitivity analysis, the functions are developed using cell reference format or global variables, as indicated in the column E function. This means that virtually any number—the interest rate, any cash flow estimate in the series or the single amounts, and the timing within the 22-year time frame—can be changed and the new answers will be immediately displayed.

1. Present worth values for the series and single amounts are determined in cells F6 and F10, respectively, using the NPV function. The sum of these in F14 is $P_T = \$93,622$, which corresponds to the value in Equation [3.2]. Alternatively, P_T can be determined directly via the sum of two NPV functions, shown in row 15.
2. The FV function in row 18 uses the P value in F14 (preceded by a minus sign) to determine F twenty-two years later. This is significantly easier than Equation [3.3].
3. To find the 22-year A series starting in year 1, the PMT function in row 21 references the P value in cell F14. This is effectively the same procedure used in Equation [3.4] to obtain A_{1-22} .

For the spreadsheet enthusiast, it is possible to find the 22-year A series value directly by using the PMT function with embedded NPV functions. The cell reference format is $= \text{PMT}(\text{D1}, 22, -(\text{NPV}(\text{D1}, \text{B6:B27}) + \text{B5} + \text{NPV}(\text{D1}, \text{C6:C27}) + \text{C5}))$.

4. and 5. It is quite simple to determine an equivalent uniform series over any number of periods using a spreadsheet, provided the series starts one period after the P value is located or ends in the same period that the F value is located. These are both true for the series requested here. The results in F24 and F27 are the same as A_{1-10} and A_{11-22} in Equations [3.5] and [3.6], respectively.

Comment

Remember that **round-off error** will always be present when comparing hand and spreadsheet results. The spreadsheet functions carry more decimal places than the tables during calculations. Also, be very careful when constructing spreadsheet functions. It is easy to miss a value, such as the P or F in PMT and FV functions, or a minus sign between entries. Always check your function entries carefully before touching <Enter>.



Placement of gradient P

3.3 Calculations for Shifted Gradients ● ● ●

In Section 2.5, we derived the relation $P = G(P/G, i, n)$ to determine the present worth of the **arithmetic gradient** series. The P/G factor, Equation [2.25], was derived for a present worth in year 0 with the gradient first appearing in year 2.

The present worth of an **arithmetic gradient** will always be located **two periods before the gradient starts**.

Refer to Figure 2-14 as a refresher for the cash flow diagrams.

The relation $A = G(A/G, i, n)$ was also derived in Section 2.5. The A/G factor in Equation [2.27] performs the equivalence transformation of a **gradient only** into an A series from years 1 through n (Figure 2-15). Recall that the base amount must be treated separately. Then the equivalent P or A values can be summed to obtain the equivalent total present worth P_T and total annual series A_T .

A conventional gradient series starts between periods 1 and 2 of the cash flow sequence. A gradient starting at any other time is called a **shifted gradient**. The n value in the P/G and A/G factors for a shifted gradient is determined by renumbering the time scale. The period in which the *gradient first appears* is labeled *period 2*. The n value for the gradient factor is determined by the renumbered period where the last gradient increase occurs.

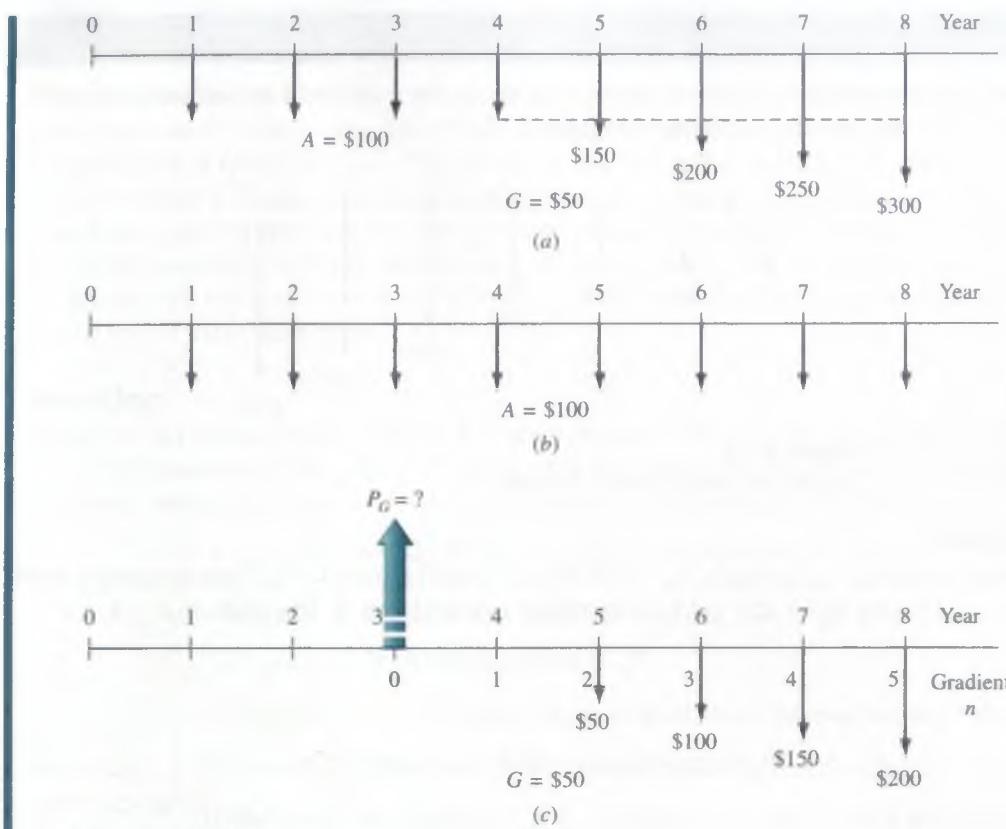
Partitioning a cash flow series into the arithmetic gradient series and the remainder of the cash flows can make very clear what the gradient n value should be. Example 3.5 illustrates this partitioning.

EXAMPLE 3.5

Fujitsu, Inc. has tracked the average inspection cost on a robotics manufacturing line for 8 years. Cost averages were steady at \$100 per completed unit for the first 4 years, but have increased consistently by \$50 per unit for each of the last 4 years. Analyze the gradient increase, using the P/G factor. Where is the present worth located for the gradient? What is the general relation used to calculate total present worth in year 0?

Solution

The cash flow diagram in Figure 3-10a shows the base amount $A = \$100$ and the arithmetic gradient $G = \$50$ starting between periods 4 and 5. Figure 3-10b and c partitions these two series. Gradient year 2 is placed in actual year 5 of the entire sequence in Figure 3-10c. It is clear that $n = 5$ for the P/G factor. The $P_G = ?$ arrow is correctly placed in gradient year 0, which is year 3 in the cash flow series.

**Figure 3–10**

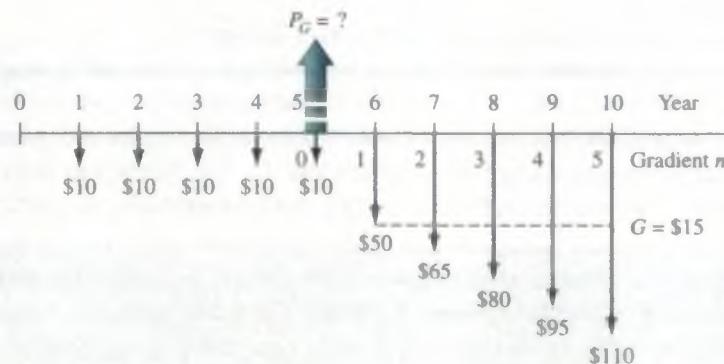
Partitioned cash flow, (a) = (b) + (c), Example 3.5.

The general relation for P_T is taken from Equation [2.19]. The uniform series $A = \$100$ occurs for all 8 years, and the $G = \$50$ gradient present worth P_G appears in year 3.

$$P_T = P_A + P_G = 100(P/A,i,8) + 50(P/G,i,5)(P/F,i,3)$$

It is important to note that the A/G factor *cannot* be used to find an equivalent A value in periods 1 through n for cash flows involving a shifted gradient. Consider the cash flow diagram of Figure 3–11. To find the equivalent annual series in years 1 through 10 for the gradient series only, first find the present worth P_G of the gradient in actual year 5, take this present worth back to year 0, and annualize the present worth for 10 years with the A/P factor. If you apply the annual series gradient factor ($A/G,i,5$) directly, the gradient is converted into an equivalent annual series over years 6 through 10 only, not years 1 through 10, as requested.

To find the equivalent A series of a shifted gradient through all the n periods, first find the present worth of the gradient at actual time 0, then apply the $(A/P,i,n)$ factor.

**Figure 3–11**Determination of G and n values used in factors for a shifted gradient.

EXAMPLE 3.6

Set up the engineering economy relations to compute the equivalent annual series in years 1 through 7 for the cash flow estimates in Figure 3–12.

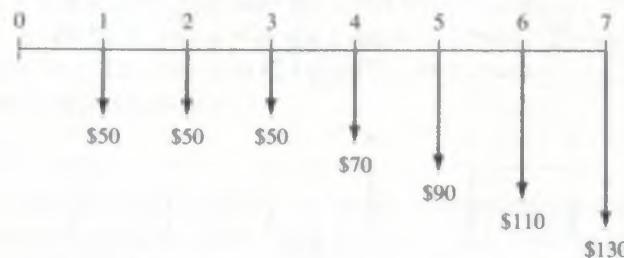


Figure 3–12
Diagram of a shifted gradient, Example 3.6.

Solution

The base amount annual series is $A_B = \$50$ for all 7 years (Figure 3–13). Find the present worth P_G in actual year 2 of the \$20 gradient that starts in actual year 4. The gradient n is 5.

$$P_G = 20(P/G,i,5)$$

Bring the gradient present worth back to actual year 0.

$$P_0 = P_G(P/F,i,2) = 20(P/G,i,5)(P/F,i,2)$$

Annualize the gradient present worth from year 1 through year 7 to obtain A_G .

$$A_G = P_0(A/P,i,7)$$

Finally, add the base amount to the gradient annual series.

$$A_T = 20(P/G,i,5)(P/F,i,2)(A/P,i,7) + 50$$

For a spreadsheet solution, enter the original cash flows into adjacent cells, say, B3 through B10, and use an embedded NPV function in the PMT function. The single-cell function is $= \text{PMT}(i\%,7,-\text{NPV}(i\%, B3:B10))$.

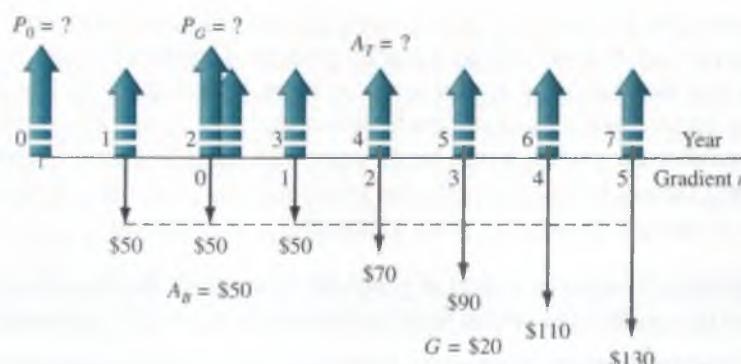


Figure 3–13
Diagram used to determine A for a shifted gradient, Example 3.6.

In Section 2.6, we derived the relation $P_g = A_1(P/A,g,i,n)$ to determine the present worth of a **geometric gradient** series, including the initial amount A_1 . The factor was derived to find the present worth in year 0, with A_1 in year 1 and the first gradient appearing in year 2.



The present worth of a **geometric gradient series** will always be located **two periods before the gradient starts**, and the **initial amount is included** in the resulting present worth. Refer to Figure 2–21 as a refresher for the cash flows.

Equation [2.35] is the formula used for the factor. It is not tabulated.

EXAMPLE 3.7

Chemical engineers at a Coleman Industries plant in the Midwest have determined that a small amount of a newly available chemical additive will increase the water repellency of Coleman's tent fabric by 20%. The plant superintendent has arranged to purchase the additive through a 5-year contract at \$7000 per year, starting 1 year from now. He expects the annual price to increase by 12% per year thereafter for the next 8 years. Additionally, an initial investment of \$35,000 was made now to prepare a site suitable for the contractor to deliver the additive. Use $i = 15\%$ per year to determine the equivalent total present worth for all these cash flows.

Solution

Figure 3–14 presents the cash flows. The total present worth P_T is found using $g = 0.12$ and $i = 0.15$. Equations [2.34] and [2.35] are used to determine the present worth P_g for the entire geometric series at actual year 4, which is moved to year 0 using $(P/F, 15\%, 4)$.

$$\begin{aligned} P_T &= 35,000 + A(P/A, 15\%, 4) + A_1(P/A, 12\%, 15\%, 9)(P/F, 15\%, 4) \\ &= 35,000 + 7000(2.8550) + \left[7000 \frac{1 - (1.12/1.15)^9}{0.15 - 0.12} \right] (0.5718) \\ &= 35,000 + 19,985 + 28,247 \\ &= \$83,232 \end{aligned}$$

Note that $n = 4$ in the $(P/A, 15\%, 4)$ factor because the \$7000 in year 5 is the cash flow of the initial amount A_1 .

For solution by spreadsheet, enter the cash flows of Figure 3–14. If cells B1 through B14 are used, the function to find $P = \$83,230$ is

$$\text{NPV}(15\%, \text{B2:B14}) + \text{B1}$$

The fastest way to enter the geometric series is to enter \$7000 for year 5 (into cell B6) and set up each succeeding cell multiplied by 1.12 for the 12% increase.

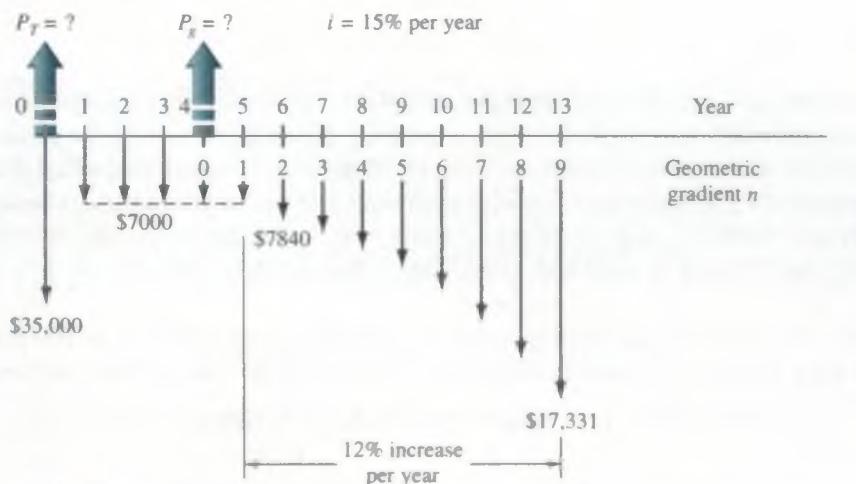
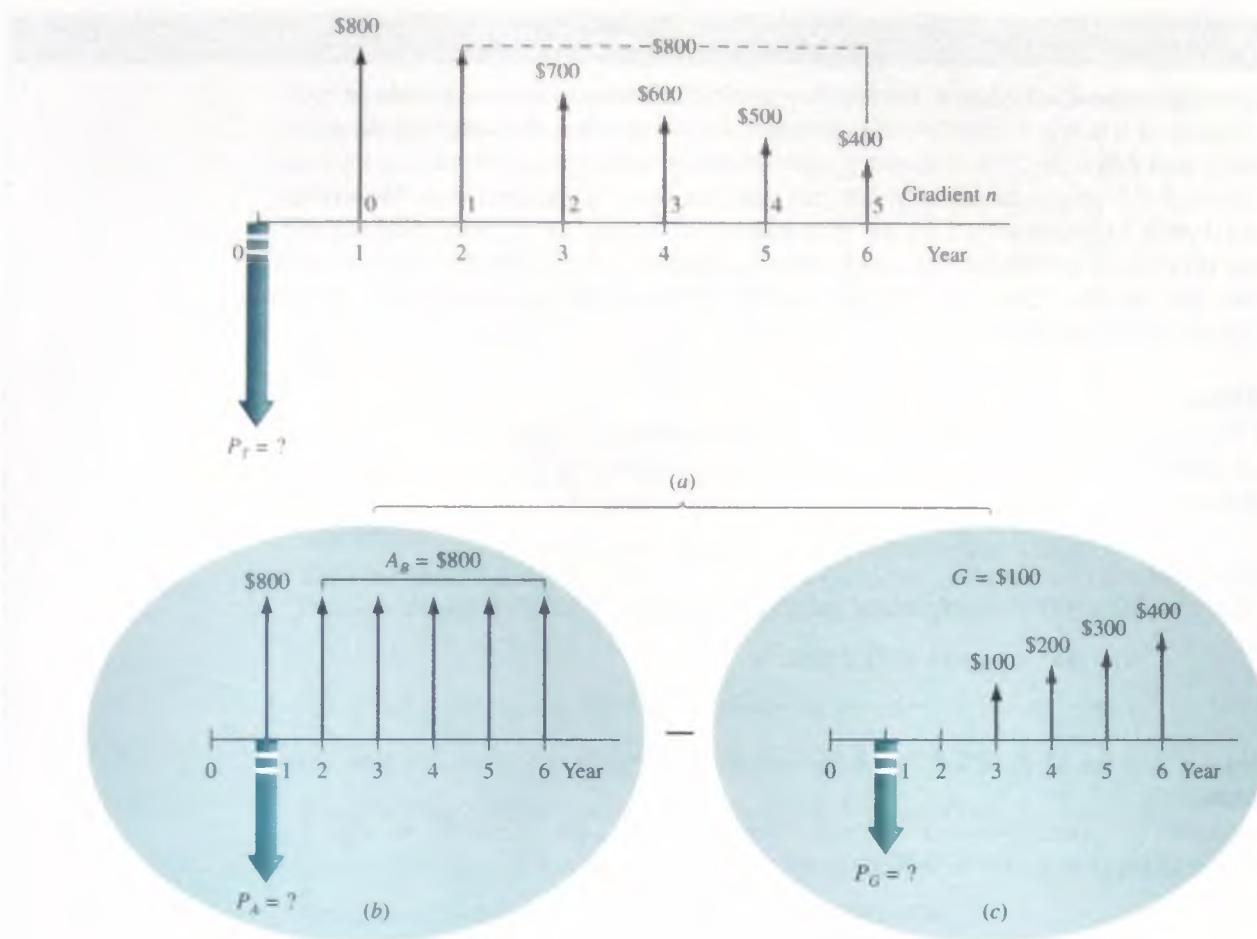


Figure 3–14

Cash flow diagram including a geometric gradient with $g = 12\%$, Example 3.7.

Decreasing arithmetic and geometric gradients are common, and they are often **shifted gradient series**. That is, the constant gradient is $-G$ or the percentage change is $-g$ from one period to the next, and the first appearance of the gradient is at some time period (year) other than year 2 of the series. Equivalence computations for present worth P and annual worth A are basically the same as discussed in Chapter 2, except for the following.

**Figure 3-15**

Partitioned cash flow of a shifted arithmetic gradient. (a) = (b) – (c).

For shifted, decreasing gradients:

- The base amount A (arithmetic) or initial amount A_1 (geometric) is the *largest amount* in the first year of the series.
- The gradient amount is *subtracted from* the previous year's amount, not added to it.
- The amount used in the factors is $-G$ for arithmetic and $-g$ for geometric gradient series.
- The present worth P_G or P_g is located 2 years prior to the appearance of the first gradient; however, a P/F factor is necessary to find the present worth in year 0.

Figure 3-15 partitions a decreasing arithmetic gradient series with $G = \$-100$ that is shifted 1 year forward in time. P_G occurs in actual year 1, and P_T is the sum of three components.

$$P_T = \$800(P/F,i,1) + 800(P/A,i,5)(P/F,i,1) - 100(P/G,i,5)(P/F,i,1)$$

EXAMPLE 3.8

Morris Glass Company has decided to invest funds for the next 5 years so that development of “smart” glass is well funded in the future. This type of new-technology glass uses electro-chrome coating to allow rapid adjustment to sun and dark in building glass, as well as assisting with internal heating and cooling cost reduction. The financial plan is to invest first, allow appreciation to occur, and then use the available funds in the future. All cash flow estimates are in \$1000 units, and the interest rate expectation is 8% per year.

Years 1 through 5: Invest \$7000 in year 1, decreasing by \$1000 per year through year 5.

Years 6 through 10: No new investment and no withdrawals.

Years 11 through 15: Withdraw \$20,000 in year 11, decreasing 20% per year through year 15.

Determine if the anticipated withdrawals will be covered by the investment and appreciation plans. If the withdrawal series is over- or underfunded, what is the exact amount available in year 11, provided all other estimates remain the same?

Solution by Hand

Figure 3–16 presents the cash flow diagram and the placement of the equivalent P values used in the solution. Calculate the present worth of both series in actual year 0 and add them to determine if the investment series is adequate to fund the anticipated withdrawals.

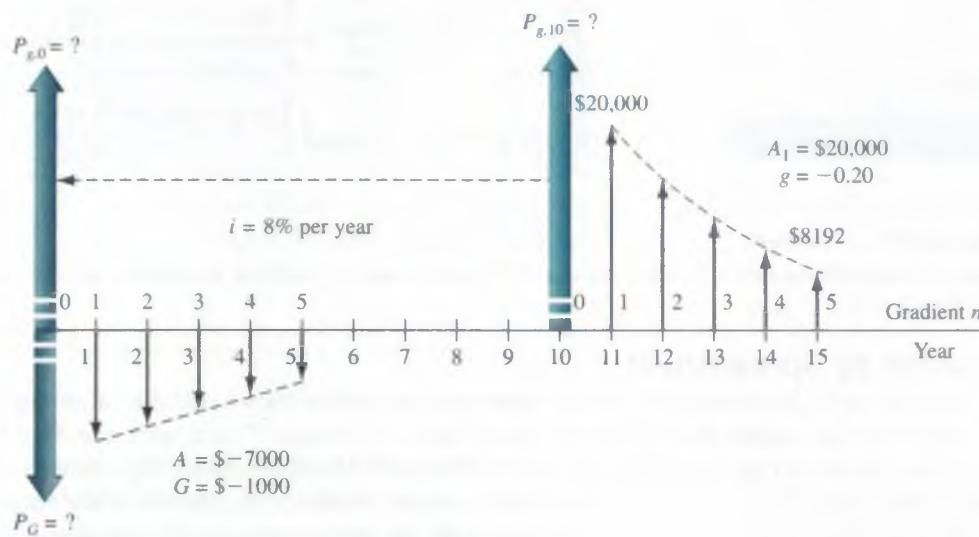


Figure 3-16

Investment and withdrawal series. Example 3.8.

Investment series: Decreasing, conventional arithmetic series starting in year 2 with $A = \$-7000$, $G = \$-1000$, and gradient $n = 5$ years. The present worth in year 0 is

$$\begin{aligned} P_G &= -[7000(P/A, 8\%, 5) - 1000(P/G, 8\%, 5)] \\ &= \$-20,577 \end{aligned}$$

Withdrawal series: Decreasing, shifted geometric series starting in year 12 with $A_1 = \$20,000$, $g = -0.20$, and gradient $n = 5$ years. If the present worth in year 10 is identified as $P_{g,10}$, the present worth in year 0 is $P_{g,0}$. Use Equation [2.35] for the $(P/A, -20\%, 8\%, 5)$ factor.

$$\begin{aligned} P_{g,0} &= P_{g,10} (P/F, i, n) = A_1 (P/A, g, i, n) (P/F, i, n) \quad [3.7] \\ &= 20,000 \left\{ \frac{1 - \left[\frac{1 + (-0.20)}{1 + 0.08} \right]^5}{0.08 - (-0.20)} \right\} (0.4632) \\ &= 20,000(2.7750)(0.4632) \\ &= \$25,707 \end{aligned}$$

The net total present worth is

$$P_T = -20,577 + 25,707 = \$+5130$$

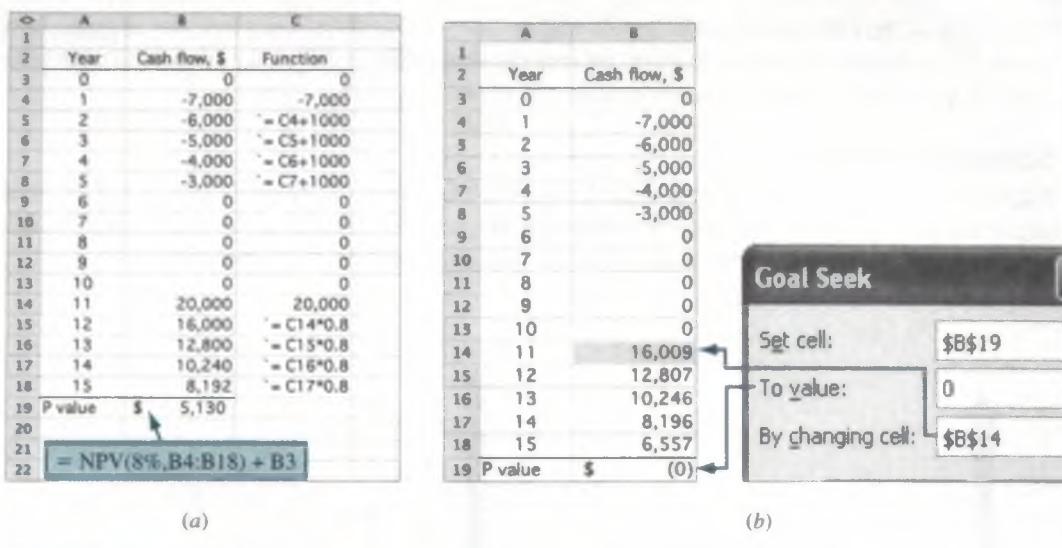
This means that more is withdrawn than the investment series earns. Either additional funds must be invested or less must be withdrawn to make the series equivalent at 8% per year.

To find the exact amount of the initial withdrawal series to result in $P_T = 0$, let A_1 be an unknown in Equation [3.7] and set $P_{g,0} = -P_G = 20,577$.

$$20,577 = A_1(2.7750)(0.4632)$$

$$A_1 = \$16,009 \text{ in year 11}$$

The geometric series withdrawal would be 20% less each year.

**Figure 3-17**

Spreadsheet solution, Example 3.8. (a) Cash flows and NPV function and (b) Goal Seek to determine initial withdrawal amount in year 11.

Solution by Spreadsheet

See Figure 3-17a. To determine if the investment series will cover the withdrawal series, enter the cash flows (in column B using the functions shown in column C) and apply the NPV function shown in the cell tag to display $P_T = \$+5130$ directly. As above, the + sign indicates that, from a time value of money perspective, there is more withdrawn than invested and earned.

The Goal Seek tool is very handy in determining the initial withdrawal amount that results in $P_T = 0$ (cell B19). Figure 3-17b shows the template and result $A_1 = \$16,009$ in year 11. Each succeeding withdrawal is 80% of the previous one.

Comment

If the withdrawal series is fixed as estimated initially and the investment series base amount A can be increased, the Goal Seek tool can be used to, again, set $P_T = 0$ (cell B19). However, now establish the entry in B4 as the changing cell. The response is $A = \$-8285$, and, as before, succeeding investments are \$1000 less.

CHAPTER SUMMARY

In Chapter 2, we derived the equations to calculate the present, future, or annual worth of specific cash flow series. In this chapter, we learned how to use these equations on cash flow series that are shifted in time from those for which the basic relations are derived. For example, when a uniform series does not begin in period 1, we still use the P/A factor to find the “present worth” of the series, except the P value is located one period ahead of the first A value, not at time 0. For arithmetic and geometric gradients, the P value is two periods ahead of where the gradient starts. With this information, it is possible to solve for P , A , or F for any conceivable cash flow series.

We have used the power of spreadsheet functions in determining P , F , and A values by single-cell entries and using cash flow estimates entered into a series of spreadsheet cells. Though spreadsheet solutions are fast, they do remove some of the understanding of how the time value of money and the factors change the equivalent value of money.

PROBLEMS

Present Worth Calculations

- 3.1 Industrial Electric Services has a contract with the U.S. Embassy in Mexico to provide maintenance for scanners and other devices in the building. If

the first payment of \$12,000 is received now, what is the present worth of the contract, provided the company will receive a total of 10 payments (i.e., years 0 through 9) and the interest rate is 10% per year?

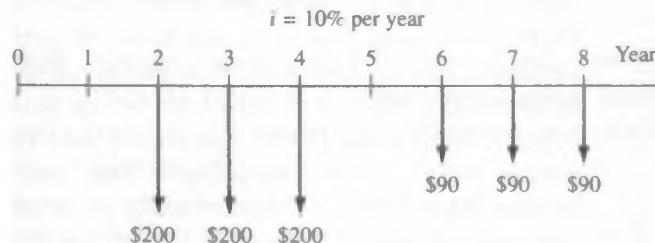
- 3.2 Civil engineering consulting firms that provide services to outlying communities are vulnerable to a number of factors that affect the financial condition of the communities, such as bond issues and real estate developments. A small consulting firm entered into a fixed-price contract with a large developer, resulting in a stable income of \$260,000 per year in years 1 through 3. At the end of that time, a mild recession slowed the development, so the parties signed another contract for \$190,000 per year for 2 more years. Determine the present worth of the two contracts at an interest rate of 10% per year.

- 3.3 The net cash flow associated with development and sale of a new product is shown. Determine the present worth at an interest rate of 12% per year. The cash flow is in \$1000 units. Show (a) hand and (b) spreadsheet solutions.

Year	1	2	3	4	5	6	7	8	9
Cash Flow, \$	-120	-100	-40	+50	+50	+80	+80	+80	+80

- 3.4 Standby power for water utility pumps and other electrical devices is provided by diesel-powered generators. As an alternative, the utility can use natural gas to power the generators, but it will be a few years before the gas is available at remote sites. The utility estimates that by switching to gas, it will save \$22,000 per year, starting 3 years from now. At an interest rate of 8% per year, determine the present worth in year 0 of the projected savings that will occur in years 3 through 10.

- 3.5 Find the present worth at $i = 10\%$ per year for the cash flow series shown below.



- 3.6 The subsidized cost of producing water at the El Paso Water Utilities (EPWU) Kay Bailey Hutchison (KBH) desalting plant is \$1.56 per 1000 gallons. Under a contract that EPWU has with Fort Bliss, the utility sells water to the Army post at a discounted price of \$1.28 per 1000 gallons. (The KBH plant was built on Army property.) If Fort Bliss uses 2 billion gallons of water each year, what is the present worth of the discount for a 20-year period at an interest rate of 6% per year?

- 3.7 The rising cost of athletic programs at major universities has induced college presidents and athletic

directors to devise innovative ways to finance cash-strapped sports programs. One of the latest schemes for big-time athletics is the "sports mortgage." At the University of Kansas, Jayhawk fans can sign up to pay \$105,000 now, or over a 10-year period, for the right to buy top seats for football games during the next 30 years. In return, the seats themselves will stay locked in at current-year prices. Season tickets in tier 1 are currently selling for \$350 each. A fan plans to purchase the sports mortgage along with a current-season ticket and pay for both *now*, then buy a ticket each year for the next 30 years. What is the total present worth of the pricing plan at an interest rate of 10% per year?

- 3.8 The cash flow associated with making self-locking fasteners is shown below. Determine the net present worth (year 0) at an interest rate of 10% per year.

Year	0	1	2	3	4	5	6	7	8	9
Income, \$1000	20	20	20	20	30	30	30	30	30	30
Cost, \$1000	8	8	8	8	12	12	12	12	12	25

- 3.9 Beckman Technologies, a relatively small manufacturer of precision laboratory equipment, borrowed \$2 million to renovate one of its testing labs. In an effort to pay off the loan quickly, the company made four payments in years 1 through 4, with each payment being twice as large as the preceding one. At an interest rate of 10% per year, what was the size of the first payment?

Annual Worth Calculations

- 3.10 Revenue from the sale of ergonomic hand tools was \$300,000 in years 1 through 4 and \$465,000 in years 5 through 9. Determine the equivalent annual revenue in years 1 through 9 at an interest rate of 10% per year.

- 3.11 Two engineering graduates who recently got married are planning for their early retirement 20 years from now. They believe that they will need \$2,000,000 in year 20. Their plan is to live on one of their salaries and invest the other. They already have \$25,000 in their investment account. (a) How much will they have to invest each year if the account grows at a rate of 10% per year? (b) If the maximum they have available to invest each year is \$40,000, will they reach their goal of \$2 million by year 20?

- 3.12 Costs associated with the manufacture of miniature high-sensitivity piezoresistive pressure transducers are \$73,000 per year. A clever industrial engineer found that by spending \$16,000 now to reconfigure the production line and reprogram

two of the robotic arms, the cost will go down to \$58,000 next year and \$52,000 in years 2 through 5. Using an interest rate of 10% per year, determine (a) the equivalent annual *cost* of the manufacturing operations and (b) the equivalent annual *savings* in years 1 through 5.

- 3.13 Calculate the equivalent annual cost in years 1 through 9 of the following series of disbursements. Use an interest rate of 10% per year. Show (a) hand and (b) spreadsheet solutions.

Year	Disbursement, \$	Year	Disbursement, \$
0	8000	5	4000
1	4000	6	5000
2	4000	7	5000
3	4000	8	5000
4	4000	9	5000

- 3.14 For the cash flows below, find the value of x that makes the equivalent annual worth in *years 1 through 7* equal to \$300 per year. Use an interest rate of 10% per year. Show solutions (a) by hand and (b) using the Goal Seek tool.

Year	Cash Flow, \$	Year	Cash Flow, \$
0	200	4	x
1	200	5	200
2	200	6	200
3	200	7	200

- 3.15 Precision Instruments, Inc. manufactures high-sensitivity mini accelerometers designed for modal analysis testing. The company borrowed \$10,000,000 with the understanding that it would make a \$2,000,000 payment at the end of year 1 and then make equal annual payments in years 2 through 5 to pay off the loan. If the interest rate on the loan was 9% per year, how much was each payment in years 2 through 5?

- 3.16 A construction management company is examining its cash flow requirements for the next 7 years. The company expects to replace office machines and computer equipment at various times over the 7-year planning period. Specifically, the company expects to spend \$6000 one year from now, \$9000 three years from now, and \$10,000 six years from now. What is the annual worth (in years 1 through 7) of the planned expenditures, at an interest rate of 10% per year?

- 3.17 Find the equivalent annual worth for the cash flows shown, using an interest rate of 12% per year. Monetary units are \$1000.

Year	1	2	3	4	5	6	7	8	9
Cash Flow, \$	20	20	20	20	60	60	60	60	60

- 3.18 Calculate the net annual worth in years 1 through 10 of the following series of incomes and expenses, if the interest rate is 10% per year.

Year	Income, \$/Year	Expense, \$/Year
0	0	-2500
1–4	700	-200
5–10	2000	-300

- 3.19 The city of El Paso gave the El Paso Tennis and Swim Club a 20-year lease to continue using a 10-acre arroyo park for its facilities. The club will pay \$1000 per year plus make \$350,000 in improvements at the park. In addition, the club will open its tennis courts to the public from 1 to 5 p.m. Monday through Thursday. If the club makes \$100,000 worth of improvements now and then \$50,000 worth each year for the next 5 years, what is the equivalent annual cost of the lease to the club at an interest rate of 10% per year?

- 3.20 Stadium Capital Financing Group is a Chicago company that conceived the so-called sports mortgage wherein fans agree to pay a relatively large amount of money over a 10- to 30-year period for the right to buy top seats for football games for up to the next 50 years. In return, season ticket prices stay locked in at current-year prices, and the package can be sold in the secondary market, while taking a tax write-off for donating to a school. Assume a fan buys a sports mortgage at West Virginia University for \$150,000 that is to be paid over a 10-year period with the right to buy two season tickets for \$300 each for the next 30 years. The first payment is made now (i.e., beginning-of-year payment), and an additional nine payments are to be made at the end of each year for the next 9 years. Assume the fan also purchases the two season tickets (also beginning-of-year payments). What is the total amount of the payment each year in years 0 through 9? Use an interest rate of 10% per year.

Future Worth Calculations

- 3.21 The expansion plans of Acme Granite, Stone & Brick call for the company to add capacity for a new product in 5 years. The company wants to have \$360,000 available before it announces the product. If the company sets aside \$55,000 now and \$90,000 in year 2, what uniform annual amount will it have to put in an account in years 3 through 5 to have the \$360,000? Assume the account earns interest at 8% per year.

- 3.22 For the cash flows shown, calculate the future worth in year 8 using $i = 10\%$ per year.

Year	0	1	2	3	4	5	6
Cash Flow, \$	100	100	100	100	100	300	300

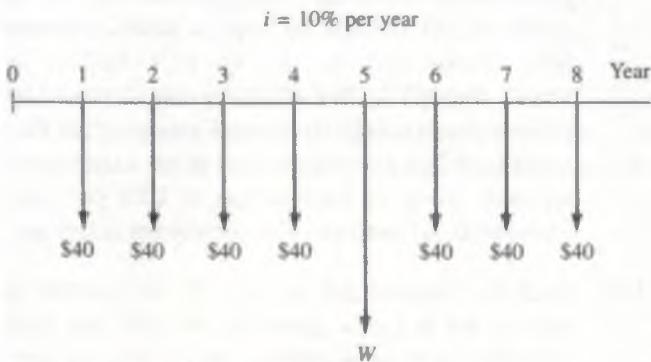
- 3.23 Assume you plan to start an annuity plan by making your first deposit *now*. If you make annual deposits of a uniform amount A into the account that earns interest at a rate of 7% per year, how many years from now will it be until the value of the account is equal to 10 times the value of a single deposit?

- 3.24 New actuator element technology enables engineers to simulate complex computer-controlled movements in any direction. If the technology results in cost savings in the design of amusement park rides, what is the future worth in year 5 of savings of \$70,000 now and \$20,000 per year in years 1 through 3 at an interest rate of 10% per year?

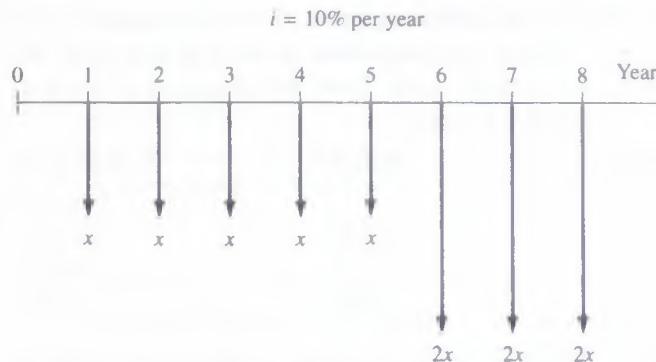
- 3.25 Austin Utilities is planning to install solar panels to provide some of the electricity for its groundwater desalting plant. The project would be done in two phases. The first phase will cost \$4 million in year 1 and \$5 million in year 2. This investment will result in energy savings (phase 2) of \$540,000 in year 3, \$546,000 in year 4, and amounts increasing by \$6000 each year through year 10. Let $i = 10\%$ per year.

- (a) What is the future worth of the *savings*?
 (b) Is the cost of the solar project justified by the savings? (Hint: Calculate the difference between savings and cost).

- 3.26 For the cash flow diagram shown, determine the value of W that will render the equivalent future worth in year 8 equal to \$-500 at an interest rate of 10% per year.



- 3.27 For the cash flows shown in the diagram, determine the value of x that will make the future worth in year 8 equal to \$-70,000.



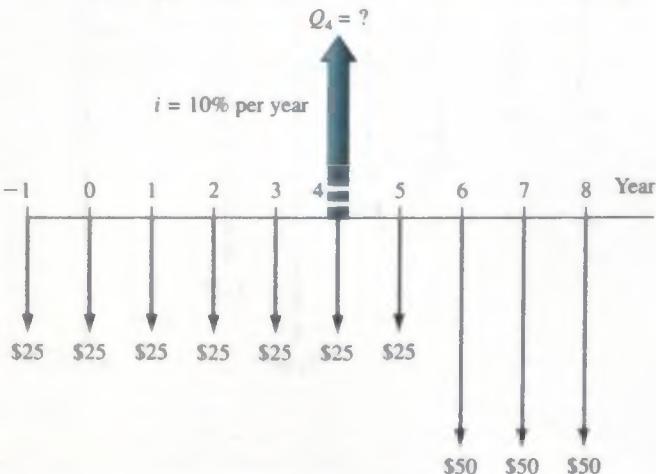
Random Single Amounts and Uniform Series

- 3.28 A small oil company is planning to replace its Coriolis flowmeters with Emerson Hastelloy flowmeters. The replacement process will cost the company \$50,000 three years from now. How much money must the company set aside each year beginning *now* (year 0) in order to have the total amount available immediately after making the last deposit at the end of year 3? Assume the company can invest its funds at 15% per year.

- 3.29 A company that manufactures air-operated drain valve assemblies budgeted \$74,000 per year to pay for plastic components over a 5-year period. If the company spent only \$42,000 in year 1, what uniform annual amount should the company expect to spend in each of the next 4 years to expend the entire budget? Assume the company uses an interest rate of 10% per year.

- 3.30 A recently hired chief executive officer wants to reduce future production costs to improve the company's earnings, thereby increasing the value of the company's stock. The plan is to invest \$40,000 now and \$40,000 in each of the next 2 years to improve productivity. By how much must annual costs decrease in years 3 through 7 to recover the investment plus a return of 12% per year?

- 3.31 Use the cash flow diagram to determine the single amount of money Q_4 in year 4 that is equivalent to all of the cash flows shown. Use $i = 10\%$ per year.



- 3.32 For the following series of income and expenses, find the equivalent value in year 9 at an interest rate of 12% per year. Show (a) hand and (b) spreadsheet solutions.

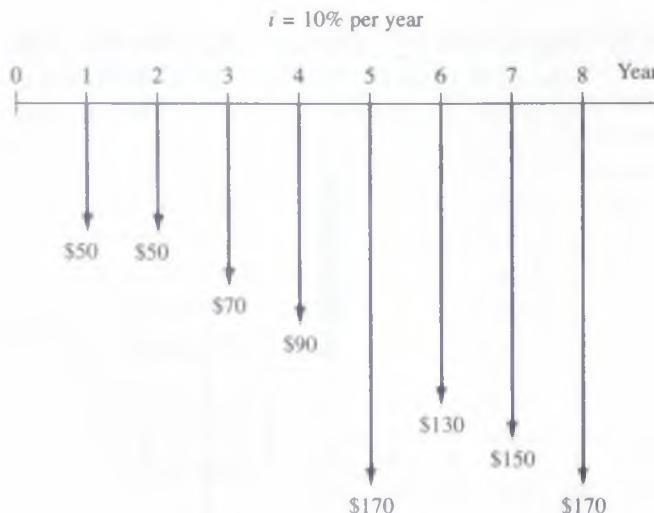
Years	Income, \$	Expense, \$
0	0	-70,000
1–6	9,000	-13,000
7–9	28,000	-14,000
10–16	38,000	-19,000

- 3.33 An investor just purchased property under a unique financing agreement with the seller. The contract price is \$1.6 million. The payment plan is Z dollars now, $2Z$ dollars in year 2, and $3Z$ dollars in years 3 through 5. If the interest rate on the transaction is 10% per year, how much is the payment in year 2?

- 3.34 A foursome of entrepreneurial electrical engineering graduates has a plan to start a new solar power equipment company based on STE (solar thermal electric) technology. They have recently approached a group of investors with their idea and asked for a loan of \$5 million. Within the agreement, the loan is to be repaid by allocating 80% of the company's profits each year for the first 4 years to the investors. In the fifth year, the company will pay the balance remaining on the loan in cash. The company's business plan indicates that they expect to make no profit for the first year, but in years 2 through 4, they anticipate profits to be \$1.5 million per year. If the investors accept the deal at an interest rate of 15% per year, and the business plan works to perfection, what is the expected amount of the last loan payment (in year 5)?

Shifted Gradients

- 3.35 Find the present worth in year 0 for the cash flows shown. Let $i = 10\%$ per year.



- 3.36 For the cash flows shown, determine the present worth in year 0, if the interest rate is 12% per year.

Year	1	2	3	4	5	6	7	8	9	10
Cash Flow, \$	13	13	13	13	16	19	22	25	28	31

- 3.37 A low-cost noncontact temperature measuring tool may be able to identify railroad car wheels that are in need of repair long before a costly structural failure occurs. If BNF Railroad saves \$100,000 in years 1 through 5, \$110,000 in year 6, and constant amounts increasing by \$10,000 each year through year 20, what is the equivalent annual worth over the 20 years of the savings? The interest rate is 10% per year.

- 3.38 Determine the present worth in year 0 of the cash flows shown at an interest rate of 15% per year.

Year	1	2	3	4	5	6	7	8	9	10
Cash Flow, \$	90	90	90	85	80	75	70	65	60	55

- 3.39 The Pedernales Electric Cooperative estimates that the present worth *now* of income from an investment in renewable energy sources is \$12,475,000. There will be no income in years 1 and 2, but in year 3 income will be \$250,000, and thereafter it will increase according to an arithmetic gradient through year 15. What is the required gradient, if the interest rate is 15% per year? Solve (a) by hand and (b) using a spreadsheet.

- 3.40 Calculate the annual cost in years 1 through 9 of the following series of disbursements. Use an interest rate of 10% per year.

Year	Disbursement, \$	Year	Disbursement, \$
0	5000	5	7500
1	5500	6	8000
2	6000	7	8500
3	6500	8	9000
4	7000	9	9500

- 3.41 The cost associated with manufacturing high-performance lubricants closely follows the cost of crude oil. For the last 10 years, a small, independent refiner had a cost of \$3.4 million in years 1 through 3, after which the cost increased by 3% per year through this year. Determine the current equivalent worth (i.e., *now*) of the manufacturing cost, using an interest rate of 10% per year. Show both (a) hand and (b) spreadsheet solutions.

- 3.42 Find the future worth in year 10 of \$50,000 in year 0 and amounts increasing by 15% per year through year 10 at an interest rate of 10% per year.

- 3.43 The cost of tuition at public universities has been steadily increasing for many years. One Midwestern university pledged to keep the tuition constant for

4 years for all students who finished in the top 3% of their class. One such student who liked research planned to enroll at the university and continue there until earning a PhD degree (a total time of 9 years). If the tuition for the first 4 years will be \$7200 per year and it increases by 5% per year for the next 5 years, what is the present worth of the tuition cost at an interest rate of 8% per year?

- 3.44 A private equity firm purchased a cable company and assumed the company's debt as part of the transaction. The deal was structured such that the private equity firm received \$3 million immediately after the deal was closed (in year 0) through the sale of some assets. This year (year 1) income was \$3.36 million, and it is projected to increase by 12% each year through expansion of the customer base. What was the present worth in the year of purchase of the income stream over a 10-year period? The firm's expected rate of return for any purchase is 15% per year.
- 3.45 Calculate the present worth in year 0 of a series of cash flows that starts at \$150,000 in year 0 and increases by 10% per year through year 5. Assume $i = 10\%$ per year.

Shifted Decreasing Gradients

- 3.46 For the cash flows shown, determine the value of G such that the present worth in year 0 equals \$16,000 at an interest rate of 10% per year.

Year	0	1	2	3	4	5
Cash Flow, \$	0	8000	8000	$8000 - G$	$8000 - 2G$	$8000 - 3G$

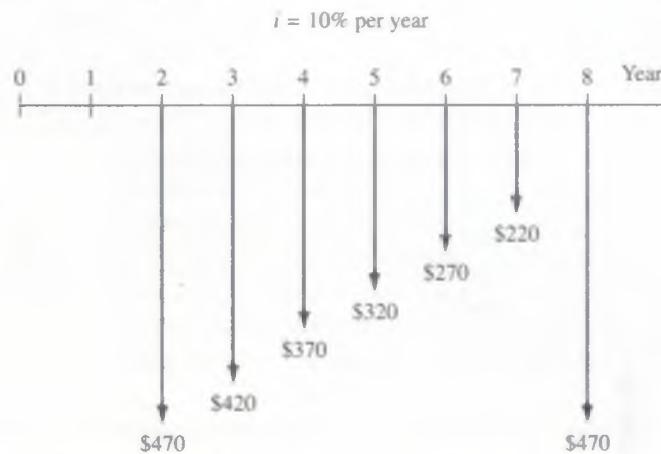
- 3.47 For these cash flows, find the equivalent annual worth in years 1 through 7 at an interest rate of 10% per year.

Year	Cash Flow, \$	Year	Cash Flow, \$
0	850	4	650
1	800	5	600
2	750	6	550
3	700	7	500

- 3.48 The pumping cost for delivering water from the Ohio River to Wheeling Steel for cooling hot-

rolled steel was \$1.8 million for the first 3 years. An effective energy conservation program resulted in a reduced cost to \$1.77 million in year 4, \$1.74 million in year 5, and amounts decreasing by \$30,000 each year through year 9. What is the equivalent annual worth of the pumping costs over the 9 years at an interest rate of 12% per year?

- 3.49 Income from the mining of mineral deposits usually decreases as the resource becomes more difficult to extract. Determine the future worth in year 10 of a mineral lease that yielded income of \$14,000 in years 1 through 4 and then amounts that decreased by 5% per year through year 10. Use an interest rate of 18% per year. Show (a) hand and (b) spreadsheet solutions.
- 3.50 For the cash flows shown in the diagram, determine the future worth in year 8 at an interest rate of 10% per year.



- 3.51 Income from the sale of application software (apps) is usually constant for several years and then decreases quite rapidly as the market gets close to saturation. Income from one smart phone app was \$38,000 in years 1 through 3 and then decreased geometrically by 15% per year through year 7. Determine the equivalent annual income in years 1 through 7, using an interest rate of 10% per year.
- 3.52 Determine the future worth in year 10 of a cash flow series that starts in year 0 at \$100,000 and decreases by 12% per year. Use an interest rate of 12% per year.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 3.53 A manufacturer of toilet flush valves wants to have \$1,900,000 available 3 years from now so that a new product line can be initiated. If the company plans to deposit money each year, starting *now*, the equation that represents the deposit each year at 8% per year interest is:

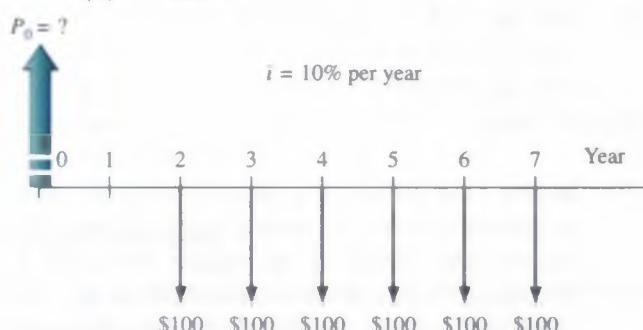
- (a) $1,900,000(A/F, 8\%, 3)$
- (b) $1,900,000(A/F, 8\%, 4)$
- (c) $1,900,000 + 1,900,000(A/F, 8\%, 3)$
- (d) $1,900,000 + 1,900,000(A/F, 8\%, 2)$

- 3.54 The present worth in year 0 of a lease that requires a payment of \$9000 *now* and amounts increasing by 5% per year through year 10 at 8% per year interest is closest to:
- (a) \$73,652
 - (b) \$79,939
 - (c) \$86,335
 - (d) Over \$87,000

- 3.55 For the diagram shown, the respective values of n for the following equation are:

$$P_0 = 100(P/A, 10\%, n)(P/F, 10\%, n)$$

- (a) 6 and 1
- (b) 6 and 2
- (c) 7 and 1
- (d) 7 and 2



- 3.56 Summit Metals is planning to expand its Wichita, Kansas, manufacturing operation 5 years from now at a cost of \$10,000,000. If the company plans to deposit money into an account each year for 4 years beginning 2 years from now (first deposit is in year 2) to pay for the expansion, the equation that represents the amount of the deposit at 9% per year interest is:
- (a) $A = 10,000,000(A/F, 9\%, 5)$
 - (b) $A = 10,000,000(A/F, 9\%, 4)$
 - (c) $A = 10,000,000(A/P, 9\%, 4)$
 - (d) $A = 10,000,000(A/F, 9\%, 4)(P/F, 9\%, 1)$

- 3.57 The amount of money a person must deposit 3 years from now in order to withdraw \$10,000 per year for 10 years beginning 15 years from now at an interest rate of 10% per year is closest to:
- (a) \$15,500
 - (b) \$17,200
 - (c) \$19,300
 - (d) \$21,500

- 3.58 Cindy wants to deposit money for 4 consecutive years starting 3 years from now so she can withdraw \$50,000 twelve years from now. Assume the interest rate is 8% per year. The annual deposit is closest to:

- (a) \$6990
- (b) \$7670
- (c) \$8530
- (d) \$10,490

- 3.59 The net present worth in year 0 of the following series of incomes and expenses at 8% per year is closest to:

Years	Income, \$	Expenses, \$
0	12,000	-1000
1-6	700	-100
7-11	900	-200

- (a) \$14,300
- (b) \$15,500
- (c) \$16,100
- (d) \$16,500

- 3.60 For the cash flows shown, the equivalent annual worth in periods 1 through 5 at an interest rate of 10% per year is closest to:

Annual Period	Amount, \$
0	1000
1	1000
2	1000
3	1000
4	1000
5	1500

- (a) \$1120
- (b) \$1240
- (c) \$1350
- (d) \$1490

- 3.61 For the cash flows shown, the value of X that will make the present worth in year 0 equal to \$5000 at an interest rate of 10% per year is closest to:

Year	0	1	2	3	4	5	6	7	8	9
Cash Flow, \$	200	300	400	500	600	700	800	900	1000	X

- (a) \$2895
- (b) \$3125
- (c) \$3305
- (d) \$3765

- 3.62 In order to have cash available for unforeseen emergencies, Baring Systems, a military contractor, wants to have \$2,000,000 in a contingency fund 4 years from now. The amount the company must deposit each year in years 0 through 4 at an interest rate of 10% per year is closest to:

- (a) \$420,100
- (b) \$327,600
- (c) \$284,600
- (d) \$206,900

CASE STUDY

PRESERVING LAND FOR PUBLIC USE

Background and Information

The Trust for Public Land (TPL) is a national organization that purchases and oversees the improvement of large land sites for government agencies at all levels. Its mission is to ensure the preservation of the natural resources, while providing necessary, but minimal, development for recreational use by the public. All TPL projects are evaluated at 7% per year, and TPL reserve funds earn 7% per year.

A southern U.S. state, which has long-term groundwater problems, has asked the TPL to manage the purchase of 10,000 acres of aquifer recharge land and the development of three parks of different use types on the land. The 10,000 acres will be acquired in increments over the next 5 years with \$4 million expended immediately on purchases. Total annual purchase amounts are expected to decrease 25% each year through year 5 and then cease for this particular project.

A city with 1.5 million citizens immediately to the southeast of this acreage relies heavily on the aquifer's water. Its citizens passed a bond issue last year, and the city government now has available \$3 million for the purchase of land. The bond interest rate is an effective 7% per year.

The engineers working on the park plan intend to complete all the development over a 3-year period starting in year 4, when the amount budgeted is \$550,000. Increases in construction costs are expected to be \$100,000 each year through year 6.

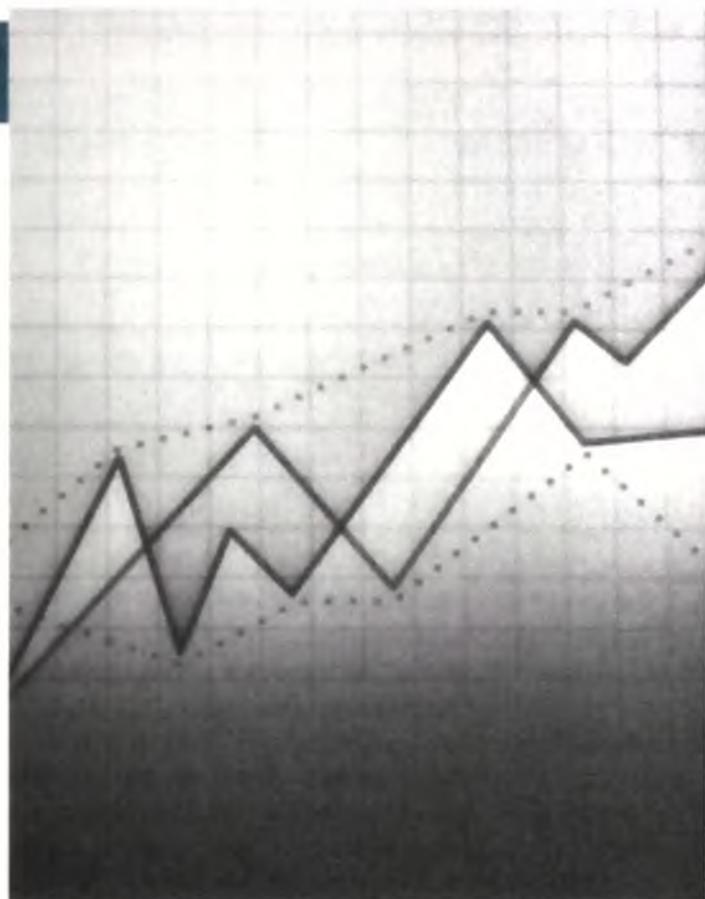
At a recent meeting, the following agreements were made:

- Purchase the initial land increment now. Use the bond issue funds to assist with this purchase. Take the remaining amount from TPL reserves.
- Raise the remaining project funds over the next 2 years in equal annual amounts.
- Evaluate a financing alternative (suggested informally by one individual at the meeting) in which the TPL provides all funds, except the \$3 million available now, until the parks development is initiated in year 4.

Case Study Exercises

1. For each of the 2 years, what is the equivalent annual amount necessary to supply the remaining project funds?
2. If the TPL did agree to fund all costs except the \$3 million bond proceeds now available, determine the equivalent annual amount that must be raised in years 4 through 6 to supply all remaining project funds. Assume the TPL will not charge any extra interest over the 7% to the state or city on the borrowed funds.
3. Review the TPL website (www.tpl.org). Identify some economic and noneconomic factors that you believe must be considered when the TPL is deciding to purchase land to protect it from real estate development.

Nominal and Effective Interest Rates



LEARNING OUTCOMES

Purpose: Make computations for interest rates and cash flows that are on a time basis other than a year.

SECTION	TOPIC	LEARNING OUTCOME
4.1	Statements	<ul style="list-style-type: none">Understand interest rate statements that include nominal and effective rates.
4.2	Effective annual rate	<ul style="list-style-type: none">Derive and use the formula for an effective annual interest rate.
4.3	Effective rate	<ul style="list-style-type: none">Determine the effective interest rate for any stated time period.
4.4	Payment period and compounding period	<ul style="list-style-type: none">Determine the payment period (PP) and compounding period (CP) for equivalence computations.
4.5	Single cash flows with $PP \geq CP$	<ul style="list-style-type: none">Perform equivalence calculations for single-amount cash flows and $PP \geq CP$.
4.6	Series cash flows with $PP \geq CP$	<ul style="list-style-type: none">Perform equivalence calculations for series and gradient cash flows and $PP \geq CP$.
4.7	Single amounts and series with $PP < CP$	<ul style="list-style-type: none">Perform equivalence calculations for cash flows with $PP < CP$.
4.8	Continuous compounding	<ul style="list-style-type: none">Derive and use the effective interest rate formula for interest rates that are compounded continuously.
4.9	Varying rates	<ul style="list-style-type: none">Perform equivalency calculations for interest rates that vary from one time period to another.

In all engineering economy relations developed thus far, the interest rate has been a constant, annual value. For a substantial percentage of the projects evaluated by professional engineers in practice, the interest rate is compounded more frequently than once a year; frequencies such as semiannually, quarterly, and monthly are common. In fact, weekly, daily, and even continuous compounding may be experienced in some project evaluations. Also, in our own personal lives, many of our financial considerations—loans of all types (home mortgages, credit cards, automobiles, boats), checking and savings accounts, investments, stock option plans, etc.—have interest rates compounded for a time period shorter than 1 year. This requires the introduction of two new terms—**nominal and effective interest rates**.

This chapter explains how to understand and use nominal and effective interest rates in engineering practice and in daily life situations. Equivalence calculations for any compounding frequency in combination with any cash flow frequency are presented.

PE

The Credit Card Offer Case: Today, Dave received a special offer of a new credit card from Chase Bank linked with the major airline that he flies frequently. It offers a generous bonus package for signing up by a specific date about 60 days from now. The bonus package includes extra airline points (once the first purchase is made), priority airport check-in services (for 1 year), several free checked-bag allowances (for up to 10 check-ins), extra frequent-flyer points on the airline, access to airline lounges (provided he uses the card on a set time basis), plus several other rewards (rental car discounts, cruise trip amenities, and floral order discounts).

The annual fee of \$85 for membership does not start until the second year, and balance transfers from other credit cards have a low transfer fee, provided they are made at the time of initial membership.

Dave has a credit card currently with a bank that he is planning to leave due to its poor customer service and high monthly fees. If he enrolls, he will transfer the \$1000 balance on the current card to the new Chase Bank card.

In the page that accompanies the offer letter, "pricing information" is included. This includes interest rates, interest charges, and fees. A summary of several of these rates and fees follows.

APR (annual percentage rate) for purchases and balance transfers*

14.24% per year (sum of the current U.S. Government prime rate of 3.25% and 10.99%, which is the APR added to determine the balance transfer APR for Chase Bank)

APR for cash and overdraft advances*
Penalty APR for late minimum payment, exceeding credit limit, and returned unpaid payments†

19.24% per year

29.99% per year (maximum penalty APR)

Fees are listed as follows:

Annual membership	\$85; free the first year
Balance transfers	\$5 or 3% of each transfer, whichever is greater
Cash advances	\$10 or 3% of each advance, whichever is greater
Late payment	\$39 each occurrence, if balance exceeds \$250
Over the credit limit	\$39 each occurrence
Returned check or payment	\$39 each occurrence

*All APR rates are variable, based on a current 3.25% prime rate with 10.99% added to determine purchase/balance transfer APR; with 15.99% added to determine cash/overdraft APR; and with 26.99% added to determine penalty APR.

†The penalty APR applies indefinitely to future transactions. If no minimum payment is received within 60 days, the penalty APR applies to all outstanding balances and all future transactions on the account.

(Continued)

This case is used in the following topics (and sections) of this chapter:
Nominal and effective interest rate statements (4.1)

Effective annual interest rates (4.2)
Equivalence relations: Series with $PP \geq CP$ (4.6)



4.1 Nominal and Effective Interest Rate Statements ● ● ●

In Chapter 1, we learned that the primary difference between simple interest and compound interest is that compound interest includes interest on the interest earned in the previous period, while simple interest does not. Here we discuss *nominal and effective interest rates*, which have the same basic relationship. The difference here is that the concepts of nominal and effective must be used when interest is compounded more than once each year. For example, if an interest rate is expressed as 1% per month, the terms *nominal* and *effective* interest rates must be considered.

To understand and correctly handle effective interest rates is important in engineering practice as well as for individual finances. The interest amounts for loans, mortgages, bonds, and stocks are commonly based upon interest rates compounded more frequently than annually. The engineering economy study must account for these effects. In our own personal finances, we manage most cash disbursements and receipts on a nonannual time basis. Again, the effect of compounding more frequently than once per year is present. First, consider a **nominal interest rate**.



Nominal interest rate r

A nominal interest rate r is an interest rate that **does not account** for compounding. By definition,

$$r = \text{interest rate per time period} \times \text{number of periods} \quad [4.1]$$

A nominal rate may be calculated for *any time period longer than the time period stated* by using Equation [4.1]. For example, the interest rate of 1.5% per month is the same as each of the following nominal rates.

Time Period	Nominal Rate by Equation [4.1]	What This Is
24 months	$1.5 \times 24 = 36\%$	Nominal rate per 2 years
12 months	$1.5 \times 12 = 18\%$	Nominal rate per 1 year
6 months	$1.5 \times 6 = 9\%$	Nominal rate per 6 months
3 months	$1.5 \times 3 = 4.5\%$	Nominal rate per 3 months

Note that none of these rates mention anything about compounding of interest; they are all of the form “ $r\%$ per time period.” These nominal rates are calculated in the same way that simple rates are calculated using Equation [1.7], that is, interest rate *times* number of periods.

After the nominal rate has been calculated, the **compounding period (CP)** must be included in the interest rate statement. As an illustration, again consider the nominal rate of 1.5% per month. If we define the CP as 1 month, the nominal rate statement is 18% per year, *compounded monthly*, or 4.5% per quarter, *compounded monthly*. Now we can consider an **effective interest rate**.



Effective interest rate i

An effective interest rate i is a rate wherein the **compounding of interest is taken into account**. Effective rates are commonly expressed on an annual basis as an effective annual rate; however, any time basis may be used.

The most common form of interest rate statement when compounding occurs over time periods shorter than 1 year is “% per time period, compounded CP-ly,” for example, 10% per year, compounded monthly, or 12% per year, compounded weekly. An effective rate may not always include the compounding period in the statement. If the CP is not mentioned, it is understood to

be the same as the time period mentioned with the interest rate. For example, an interest rate of "1.5% per month" means that interest is compounded each month; that is, CP is 1 month. An equivalent effective rate statement, therefore, is 1.5% per month, compounded monthly.

All of the following are effective interest rate statements because either **they state they are effective** or the **compounding period is not mentioned**. In the latter case, the CP is the same as the time period of the interest rate.

Statement	CP	What This Is
$i = 10\%$ per year	CP not stated; CP = year	Effective rate per year
$i = \text{effective } 10\%$ per year, compounded monthly	CP stated; CP = month	Effective rate per year
$i = 1\frac{1}{2}\%$ per month	CP not stated; CP = month	Effective rate per month
$i = \text{effective } 1\frac{1}{2}\%$ per month, compounded monthly	CP stated; CP = month	Effective rate per month; terms <i>effective</i> and <i>compounded monthly</i> are redundant
$i = \text{effective } 3\%$ per quarter, compounded daily	CP stated; CP = day	Effective rate per quarter

All nominal interest rates can be converted to effective rates. The formula to do this is discussed in the next section.

All interest formulas, factors, tabulated values, and spreadsheet functions must use an effective interest rate to properly account for the time value of money.

The term **APR (Annual Percentage Rate)** is often stated as the annual interest rate for credit cards, loans, and house mortgages. This is the same as the **nominal rate**. An APR of 15% is the same as a nominal 15% per year or a nominal 1.25% on a monthly basis. Also the term **APY (Annual Percentage Yield)** is a commonly stated annual rate of return for investments, certificates of deposit, and saving accounts. This is the same as an **effective rate**. The names are different, but the interpretations are identical. As we will learn in the following sections, the effective rate is always greater than or equal to the nominal rate, and similarly $\text{APY} \geq \text{APR}$.

Based on these descriptions, there are always three time-based units associated with an interest rate statement.

Interest period (t)—The period of time over which the interest is expressed. This is the t in the statement of $r\%$ per time period t , for example, $1\% \text{ per month}$. The time unit of 1 year is by far the most common. It is assumed when not stated otherwise.

Compounding period (CP)—The shortest time unit over which interest is charged or earned. This is defined by the compounding term in the interest rate statement, for example, 8% per year, *compounded monthly*. If CP is not stated, it is assumed to be the same as the interest period.

Compounding frequency (m)—The number of times that compounding occurs within the interest period t . If the compounding period CP and the time period t are the same, the compounding frequency is 1, for example, $1\% \text{ per month, compounded monthly}$.

Consider the (nominal) rate of 8% per year, compounded monthly. It has an interest period t of 1 year, a compounding period CP of 1 month, and a compounding frequency m of 12 times per year. A rate of 6% per year, compounded weekly, has $t = 1$ year, CP = 1 week, and $m = 52$, based on the standard of 52 weeks per year.

In previous chapters, all interest rates had t and CP values of 1 year, so the compounding frequency was always $m = 1$. This made them all effective rates, because the interest period and compounding period were the same. Now, it will be necessary to express a nominal rate as an effective rate on the same time base as the compounding period.

An effective rate can be determined from a nominal rate by using the relation

$$\text{Effective rate per CP} = \frac{r\% \text{ per time period } t}{m \text{ compounding periods per } t} = \frac{r}{m} \quad [4.2]$$

As an illustration, assume $r = 9\%$ per year, compounded monthly; then $m = 12$. Equation [4.2] is used to obtain the effective rate of $9\%/12 = 0.75\%$ per month, compounded monthly.

Note that changing the interest period t does not alter the compounding period, which is 1 month in this illustration. Therefore, $r = 9\%$ per year, compounded monthly, and $r = 4.5\%$ per 6 months, compounded monthly, are two expression of the same interest rate.

EXAMPLE 4.1

Three different bank loan rates for electric generation equipment are listed below. Determine the effective rate on the basis of the compounding period for each rate.

- (a) 9% per year, compounded quarterly.
 (b) 9% per year, compounded monthly.
 (c) 4.5% per 6 months, compounded weekly.

Solution

Apply Equation [4.2] to determine the effective rate per CP for different compounding periods. The graphic in Figure 4-1 indicates the effective rate per CP and how the interest rate is distributed over time.

Figure 4-1

Relations between interest period t , compounding period CP, and effective interest rate per CP.

Sometimes it is not obvious whether a stated rate is a nominal or an effective rate. Basically there are three ways to express interest rates, as detailed in Table 4–1. The right column includes a statement about the effective rate. For the first format, a nominal interest rate is given and the compounding period is stated. The effective rate must be calculated (discussed in the next sections). In the second format, the stated rate is identified as effective (or APY could also be used), so the rate is used directly in computations.

In the third format, no compounding period is identified, for example, 8% per year. This rate is effective over a compounding period equal to the stated interest period of 1 year in this case. The effective rate for any other time period must be calculated.

TABLE 4-1 Various Ways to Express Nominal and Effective Interest Rates

Format of Rate Statement	Example of Statement	What about the Effective Rate?
Nominal rate stated, compounding period stated	8% per year, compounded quarterly	Find effective rate for any time period (next two sections)
Effective rate stated	Effective 8.243% per year, compounded quarterly	Use effective rate of 8.243% per year directly for annual cash flows
Interest rate stated, no compounding period stated	8% per year	Rate is effective for CP equal to stated interest period of 1 year; find effec- tive rate for all other time periods

EXAMPLE 4.2 The Credit Card Offer Case

As described in the introduction to this case, Dave has been offered what is described as a credit card deal that should not be refused—at least that is what the Chase Bank offer letter implies. The balance transfer APR interest rate of 14.24% is an annual rate, with no compounding period mentioned. Therefore, it follows the format of the third entry in Table 4–1, that is, interest rate stated, no CP stated. Therefore, we should conclude that the CP is 1 year, the same as the annual interest period of the APR. However, as Dave and we all know, credit card payments are required monthly.

- First, determine the effective interest rates for *compounding periods of 1 year and 1 month* so Dave knows some effective rates he might be paying when he transfers the \$1000 balance from his current card.
- Second, assume that immediately after he accepts the card and completes the \$1000 transfer, Dave gets a bill that is due 1 month later. What is the amount of the total balance he owes?

Now, Dave looks a little closer at the fine print of the “pricing information” sheet and discovers a small-print statement that Chase Bank uses the daily balance method (including new transactions) to determine the balance used to calculate the interest due at payment time.

- We will reserve the implication of this new finding until later, but for now help Dave by determining the *effective daily interest rate* that may be used to calculate interest due at the end of 1 month, provided the CP is 1 day.

Solution

- The interest period is 1 year. Apply Equation [4.2] for both CP values of 1 year ($m = 1$ compounding period per year) and 1 month ($m = 12$ compounding periods per year).

$$\text{CP of year: } \text{Effective rate per year} = 14.24/1 = 14.24\%$$

$$\text{CP of month: } \text{Effective rate per month} = 14.24/12 = 1.187\%$$

- The interest will be at the monthly effective rate, plus the balance transfer fee of 3%.

$$\begin{aligned}\text{Amount owed after 1 month} &= 1000 + 1000(0.01187) + 0.03(1000) \\ &= 1000 + 11.87 + 30 \\ &= \$1041.87\end{aligned}$$

Including the \$30 fee, this represents an interest rate of $(41.87/1000)(100\%) = 4.187\%$ for only the 1-month period.

- Again apply Equation [4.2], now with $m = 365$ compounding periods per year.

$$\text{CP of day: } \text{Effective rate per day} = 14.24/365 = 0.039\%$$

4.2 Effective Annual Interest Rates ● ● ●

In this section, effective **annual interest rates** are calculated. Therefore, the year is used as the interest period t , and the compounding period CP can be any time unit less than 1 year. For example, we will learn that a *nominal 18% per year*, compounded quarterly is the same as an *effective rate of 19.252% per year*.



The symbols used for nominal and effective interest rates are

r = nominal interest rate per year

CP = time period for each compounding

m = number of compounding periods per year

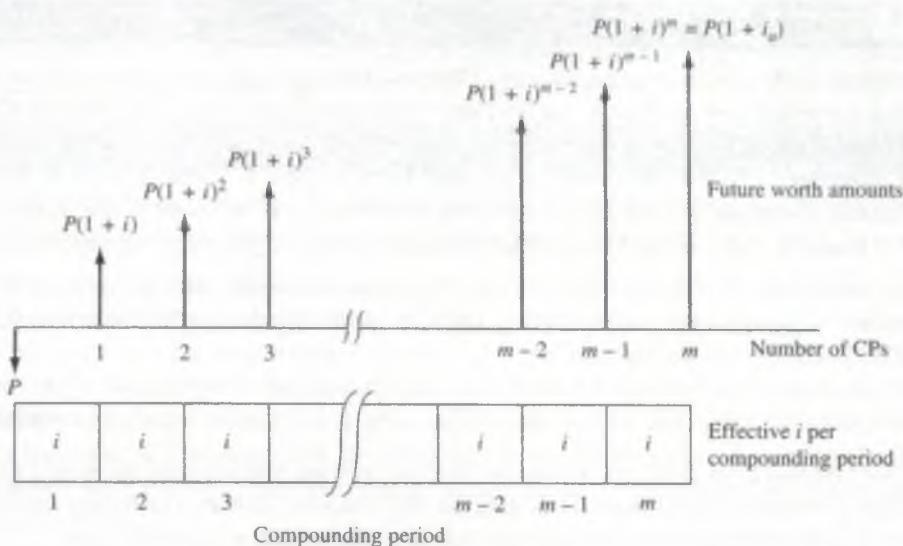
i = effective interest rate per compounding period = r/m

i_a = effective interest rate per year

The relation $i = r/m$ is exactly the same as Equation [4.2].

Figure 4–2

Future worth calculation at a rate i , compounded m times in a year.



As mentioned earlier, treatment for nominal and effective interest rates parallels that of simple and compound interest. Like compound interest, an effective interest rate at any point during the year includes (compounds) the interest rate for all previous compounding periods during the year. Therefore, the derivation of an effective interest rate formula directly parallels the logic used to develop the future worth relation $F = P(1 + i)^n$. We set $P = \$1$ for simplification.

The future worth F at the end of 1 year is the principal P plus the interest $P(i)$ through the year. Since interest may be compounded several times during the year, use the effective annual rate symbol i_a to write the relation for F with $P = \$1$.

$$F = P + Pi_a = 1(1 + i_a)$$

Now consider Figure 4–2. The effective rate i per CP must be compounded through all m periods to obtain the total effect of compounding by the end of the year. This means that F can also be written as

$$F = 1(1 + i)^m$$

Equate the two expressions for F and solve for i_a . The **effective annual interest rate formula** for i_a is

$$i_a = (1 + i)^m - 1 \quad [4.3]$$

Equation [4.3] calculates the effective annual interest rate i_a for any number of compounding periods per year when i is the rate for one compounding period.

If the effective annual rate i_a and compounding frequency m are known, Equation [4.3] can be solved for i to determine the *effective interest rate per compounding period*.

$$i = (1 + i_a)^{1/m} - 1 \quad [4.4]$$

As an illustration, Table 4–2 utilizes the nominal rate of 18% per year for different compounding periods (year to week) to determine the effective annual interest rate. In each case, the effective rate i per CP is applied m times during the year. Table 4–3 summarizes the effective annual rate for frequently quoted nominal rates using Equation [4.3]. A standard of 52 weeks and 365 days per year is used throughout. The values in the continuous-compounding column are discussed in Section 4.8.

TABLE 4–2 Effective Annual Interest Rates Using Equation [4.3]

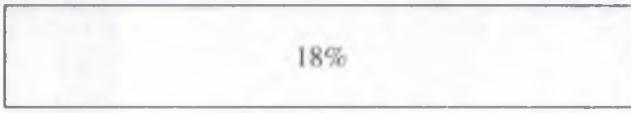
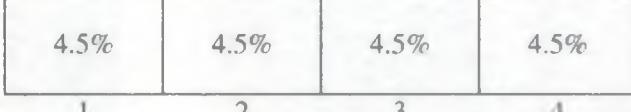
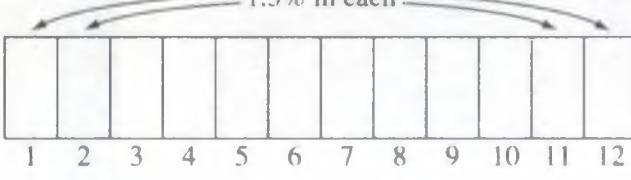
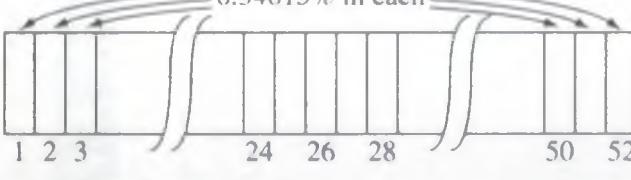
Compounding Period, CP	Times Compounded per Year, m	Rate per Compound Period, $i\%$	Distribution of i over the Year of Compounding Periods	Effective Annual Rate, $i_a = (1 + i)^m - 1$
Year	1	18		$(1.18)^1 - 1 = 18\%$
6 months	2	9		$(1.09)^2 - 1 = 18.81\%$
Quarter	4	4.5		$(1.045)^4 - 1 = 19.252\%$
Month	12	1.5		$(1.015)^{12} - 1 = 19.562\%$
Week	52	0.34615		$(1.0034615)^{52} - 1 = 19.684\%$

TABLE 4-3 Effective Annual Interest Rates for Selected Nominal Rates

Nominal Rate $r\%$	Semiannually ($m = 2$)	Quarterly ($m = 4$)	Monthly ($m = 12$)	Weekly ($m = 52$)	Daily ($m = 365$)	Continuously ($m = \infty; e^r - 1$)
0.25	0.250	0.250	0.250	0.250	0.250	0.250
0.50	0.501	0.501	0.501	0.501	0.501	0.501
1.00	1.003	1.004	1.005	1.005	1.005	1.005
1.50	1.506	1.508	1.510	1.511	1.511	1.511
2	2.010	2.015	2.018	2.020	2.020	2.020
3	3.023	3.034	3.042	3.044	3.045	3.046
4	4.040	4.060	4.074	4.079	4.081	4.081
5	5.063	5.095	5.116	5.124	5.126	5.127
6	6.090	6.136	6.168	6.180	6.180	6.184
7	7.123	7.186	7.229	7.246	7.247	7.251
8	8.160	8.243	8.300	8.322	8.328	8.329
9	9.203	9.308	9.381	9.409	9.417	9.417
10	10.250	10.381	10.471	10.506	10.516	10.517
12	12.360	12.551	12.683	12.734	12.745	12.750
15	15.563	15.865	16.076	16.158	16.177	16.183
18	18.810	19.252	19.562	19.684	19.714	19.722
20	21.000	21.551	21.939	22.093	22.132	22.140
25	26.563	27.443	28.073	28.325	28.390	28.403
30	32.250	33.547	34.489	34.869	34.968	34.986
40	44.000	46.410	48.213	48.954	49.150	49.182
50	56.250	60.181	63.209	64.479	64.816	64.872

EXAMPLE 4.3

Janice is an engineer with Southwest Airlines. She purchased Southwest stock for \$6.90 per share and sold it exactly 1 year later for \$13.14 per share. She was very pleased with her investment earnings. Help Janice understand exactly what she earned in terms of (a) effective annual rate and (b) effective rate for quarterly compounding, and for monthly compounding. Neglect any commission fees for purchase and selling of stock and any quarterly dividends paid to stockholders.

Solution

- (a) The effective annual rate of return i_a has a compounding period of 1 year, since the stock purchase and sales dates are exactly 1 year apart. Based on the purchase price of \$6.90 per share and using the definition of interest rate in Equation [1.2],

$$i_a = \frac{\text{amount of increase per 1 year}}{\text{original price}} \times 100\% = \frac{6.24}{6.90} \times 100\% = 90.43\% \text{ per year}$$

- (b) For the effective annual rates of 90.43% per year, compounded quarterly, and 90.43%, compounded monthly, apply Equation [4.4] to find corresponding effective rates on the basis of each compounding period.

$$\text{Quarter: } m = 4 \text{ times per year} \quad i = (1.9043)^{1/4} - 1 = 1.17472 - 1 = 0.17472$$

This is 17.472% per quarter, compounded quarterly.

$$\text{Month: } m = 12 \text{ times per year} \quad i = (1.9043)^{1/12} - 1 = 1.05514 - 1 = 0.05514$$

This is 5.514% per month, compounded monthly.

Comment

Note that these quarterly and monthly rates are less than the effective annual rate divided by the number of quarters or months per year. In the case of months, this would be $90.43\%/12 = 7.54\%$ per month. This computation is incorrect because it neglects the fact that compounding takes place 12 times during the year to result in the effective annual rate of 90.43%.

The spreadsheet function that displays the result of Equation [4.3], that is, the **effective annual rate** i_a , is the EFFECT function. The format is

= EFFECT(nominal_rate_per_year, compounding_frequency)

= EFFECT($r\%$, m)

[4.5]

Note that the rate entered in the EFFECT function is the **nominal annual rate $r\%$ per year**, not the effective rate $i\%$ per compounding period. The function automatically finds i for use in Equation [4.3]. As an example, assume the nominal annual rate is $r = 5.25\%$ per year, compounded quarterly, and you want to find the effective annual rate i_a . The correct input is = EFFECT(5.25%,4) to display $i_a = 5.354\%$ per year. This is the spreadsheet equivalent of Equation [4.3] with $i = 5.25/4 = 1.3125\%$ per quarter with $m = 4$.

$$i_a = (1 + 0.013125)^4 - 1 = 0.05354 \quad (5.354\%)$$

The thing to remember about using the EFFECT function is that the nominal rate r entered must be expressed over the same period of time as that of the effective rate required, which is 1 year here.

The NOMINAL spreadsheet function finds the **nominal annual rate r** . The format is

= NOMINAL(effective_rate, compounding_frequency_per_year)

= NOMINAL($i_a\%$, m)

[4.6]

This function is designed to display **only nominal annual rates**. Accordingly, the m entered must be the number of times interest is compounded per year. For example, if the effective annual rate is 10.381% per year, compounded quarterly, and the nominal annual rate is requested, the function is = NOMINAL(10.381%,4) to display $r = 10\%$ per year, compounded quarterly. The nominal rates for shorter time periods than 1 year are determined by using Equation [4.1]. For example, the quarterly rate is $10\%/4 = 2.5\%$.

The things to remember when using the NOMINAL function are that the answer is always a nominal annual rate, the rate entered must be an effective annual rate, and the m must equal the number of times interest is compounded annually.

EXAMPLE 4.4 The Credit Card Offer Case

PE

In our Progressive Example, Dave is planning to accept the offer for a Chase Bank credit card that carries an APR (nominal rate) of 14.24% per year, or 1.187% per month. He will transfer a balance of \$1000 and plans to pay it and the transfer fee of \$30, due at the end of the first month. Let's assume that Dave makes the transfer, and only days later his employer has a 1-year assignment for him in the country of the Cameroon in northwestern Africa. Dave accepts the employment offer, and in his hurried, excited departure, he forgets to send the credit card service company a change of address. Since he is now out of mail touch, he does not pay his monthly balance due, which we calculated in Example 4.2 to be \$1041.87.

- (a) If this situation continues for a total of 12 months, determine the total due after 12 months and the effective annual rate of interest Dave has accumulated. Remember, the fine print on the card's interest and fee information states a penalty APR of 29.99% per year after one late payment of the minimum payment amount, plus a late payment fee of \$39 per occurrence.
- (b) If there were no penalty APR and no late-payment fee, what effective annual interest rate would be charged for this year? Compare this rate with the answer in part (a).

Solution

- (a) Because Dave did not pay the first month's amount, the new balance of \$1041.87 and all future monthly balances will accumulate interest at the higher monthly rate of

$$29.99\%/12 = 2.499\% \text{ per month}$$

Additionally, the \$39 late-payment fee will be added each month, starting with the second month, and interest will be charged on these fees also each month thereafter. The first 3 months and last 2 months are detailed below. Figure 4-3 details the interest and fees for all 12 months.

	Amount of credit	Monthly interest	Fees	Amount due	Interest rates
Month					Regular Penalty
3	0	1000.00	0	1000.00	
4	1	1000.00	11.87	30	1041.87
5	2	1041.87	26.04	39	1106.90
6	3	1106.90	27.66	39	1173.57
7	4	1173.57	29.33	39	1241.90
8	5	1241.90	31.04	39	1311.93
9	6	1311.93	32.79	39	1383.72
10	7	1383.72	34.58	39	1457.30
11	8	1457.30	36.42	39	1532.72
12	9	1532.72	38.31	39	1610.03
13	10	1610.03	40.24	39	1689.27
14	11	1689.27	42.22	39	1770.48
15	12	1770.48	44.25	39	1853.73
16	Month 1: transfer fee Months 2-12: late-payment fee				
17	Month 1: 1.187% Months 2-12: penalty 2.499%				
18					
19					
20					

Figure 4–3

Monthly amounts due for a credit card, Example 4.4.

$$\text{Month 1: } 1000 + 1000(0.01187) + 30 = \$1041.87$$

$$\text{Month 2: } 1041.87 + 1041.87(0.02499) + 39 = \$1106.91$$

$$\text{Month 3: } 1106.91 + 1106.91(0.02499) + 39 = \$1173.57$$

$$\text{Month 11: } 1689.25 + 1689.25(0.02499) + 39 = \$1770.46$$

$$\text{Month 12: } 1770.46 + 1770.46(0.02499) + 39 = \$1853.71$$

The *effective monthly rate* is determined by using the F/P factor to find the i value at which \$1000 now is equivalent to \$1853.71 after 12 periods.

$$1853.71 = 1000(F/P, i, 12) = 1000(1 + i)^{12}$$

$$1 + i = (1.85371)^{1/12} = 1.05278$$

$$i = 5.278\% \text{ per month}$$

Since the compounding period is 1 month, use Equation [4.3] to determine the *effective annual rate* of 85.375% per year, compounded monthly.

$$i_a = (1 + i)^m - 1 = (1.05278)^{12} - 1$$

$$= 0.85375 \quad (85.375\%)$$

- (b) If there were no penalty fees for late payments and the nominal annual rate of 14.24% (or 1.187% per month) were applied throughout the 12 months, the effective annual rate would be 15.207% per year, compounded monthly. By Equation [4.3], with a small rounding error included,

$$i_a = (1 + i)^m - 1 = (1.01187)^{12} - 1 = 0.15207$$

First, Dave will not pay at the stated rate of 14.24%, because this is the APR (nominal rate), not the APY (effective rate) of 15.207%. Second, and much more important, is the huge difference made by (1) the increase in rate to an APR of 29.99% and (2) the monthly fees of \$39 for not making a payment. These large fees become part of the credit balance and accumulate interest at the penalty rate of 29.99% per year. The result is an effective annual rate jump from 15.207% to 85.375% per year, compounded monthly.

Comment

This is but one illustration of why the best advice to an individual or company in debt is to spend down the debt. The quoted APR by credit card, loan, and mortgage institutions can be quite deceiving; plus, the addition of penalty fees increases the effective rate very rapidly.

When Equation [4.3] is applied to find i_a the result is usually not an integer. Therefore, the engineering economy factor cannot be obtained directly from the interest factor tables. There are alternative ways to find the factor value.

- Use the factor formula with the i_a rate substituted for i .
- Use the spreadsheet function with i_a (as discussed in Section 2.4).
- Linearly interpolate between two tabulated rates (as discussed in Section 2.4).

4.3 Effective Interest Rates for Any Time Period ● ● ●

Equation [4.3] in Section 4.2 calculates an effective interest rate per year from any effective rate over a shorter time period. We can generalize this equation to determine the **effective interest rate for any time period** (shorter or longer than 1 year).



$$\text{Effective } i \text{ per time period} = \left(1 + \frac{r}{m}\right)^m - 1 \quad [4.7]$$

where i = effective rate for specified time period (say, semiannual)

r = nominal interest rate for same time period (semiannual)

m = number of times interest is compounded per stated time period (times per 6 months)

The term r/m is always the effective interest rate over a compounding period CP, and m is always the number of times that interest is compounded per the time period on the left of the equals sign in Equation [4.7]. Instead of i_a , this general expression uses i as the symbol for the effective interest rate, which conforms to the use of i throughout the remainder of this text. Examples 4.5 and 4.6 illustrate the use of this equation.

EXAMPLE 4.5

Tesla Motors manufactures high-performance battery electric vehicles. An engineer is on a Tesla committee to evaluate bids for new-generation coordinate-measuring machinery to be directly linked to the automated manufacturing of high-precision vehicle components. Three bids include the interest rates that vendors will charge on unpaid balances. To get a clear understanding of finance costs, Tesla management asked the engineer to determine the effective semiannual and annual interest rates for each bid. The bids are as follows:

- Bid 1:** 9% per year, compounded quarterly
Bid 2: 3% per quarter, compounded quarterly
Bid 3: 8.8% per year, compounded monthly

- (a) Determine the effective rate for each bid on the basis of semiannual periods.
(b) What are the effective annual rates? These are to be a part of the final bid selection.
(c) Which bid has the lowest effective annual rate?

Solution

- (a) Convert the nominal rates to a semiannual basis, determine m , then use Equation [4.7] to calculate the effective semiannual interest rate i . For bid 1,

$$r = 9\% \text{ per year} = 4.5\% \text{ per 6 months}$$

$$m = 2 \text{ quarters per 6 months}$$

$$\text{Effective } i\% \text{ per 6 months} = \left(1 + \frac{0.045}{2}\right)^2 - 1 = 1.0455 - 1 = 4.55\%$$

Table 4-4 (left section) summarizes the effective semiannual rates for all three bids.

TABLE 4-4 Effective Semiannual and Annual Interest Rates for Three Bid Rates, Example 4.5

Bid	Semiannual Rates			Annual Rates		
	Nominal r per 6 Months, %	CP per 6 Months, m	Equation [4.7], Effective i , %	Nominal r per Year, %	CP per Year, m	Equation [4.7], Effective i , %
1	4.5	2	4.55	9	4	9.31
2	6.0	2	6.09	12	4	12.55
3	4.4	6	4.48	8.8	12	9.16

(b) For the effective annual rate, the time basis in Equation [4.7] is 1 year. For bid 1,

$$r = 9\% \text{ per year} \quad m = 4 \text{ quarters per year}$$

$$\text{Effective } i\% \text{ per year} = \left(1 + \frac{0.09}{4}\right)^4 - 1 = 1.0931 - 1 = 9.31\%$$

The right section of Table 4-4 includes a summary of the effective annual rates.

(c) Bid 3 includes the lowest effective annual rate of 9.16%, which is equivalent to an effective semiannual rate of 4.48% when interest is compounded monthly.

EXAMPLE 4.6

A dot-com company plans to place money in a new venture capital fund that currently returns 18% per year, compounded daily. What effective rate is this (a) yearly and (b) semiannually?

Solution

(a) Use Equation [4.7], with $r = 0.18$ and $m = 365$.

$$\text{Effective } i\% \text{ per year} = \left(1 + \frac{0.18}{365}\right)^{365} - 1 = 19.716\%$$

(b) Here $r = 0.09$ per 6 months and $m = 182$ days.

$$\text{Effective } i\% \text{ per 6 months} = \left(1 + \frac{0.09}{182}\right)^{182} - 1 = 9.415\%$$



4.4 Equivalence Relations: Payment Period and Compounding Period ● ● ●

Now that the procedures and formulas for determining effective interest rates with consideration of the compounding period are developed, it is necessary to consider the **payment period**.

The payment period (PP) is the length of time between cash flows (inflows or outflows). It is common that the lengths of the payment period and the compounding period (CP) do not coincide. It is important to determine if $PP = CP$, $PP > CP$, or $PP < CP$.

If a company deposits money each month into an account that earns at the nominal rate of 8% per year, compounded semiannually, the cash flow deposits define a payment period of 1 month and the nominal interest rate defines a compounding period of 6 months. These time periods are shown in Figure 4-4. Similarly, if a person deposits a bonus check once a year into an account that compounds interest quarterly, $PP = 1$ year and $CP = 3$ months.

$r = \text{nominal } 8\% \text{ per year, compounded semiannually}$

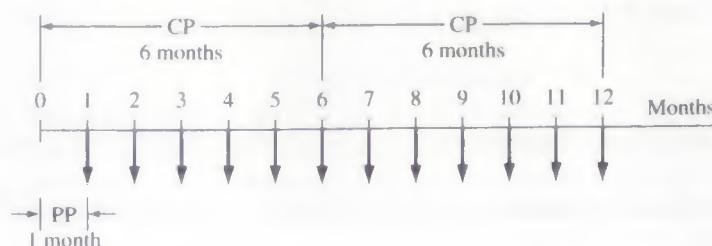


Figure 4-4

One-year cash flow diagram for a monthly payment period (PP) and semiannual compounding period (CP).

TABLE 4-5 Section References for Equivalence Calculations Based on Payment Period and Compounding Period Comparison

Length of Time	Involves Single Amounts (P and F Only)	Involves Uniform Series or Gradient Series (A , G , or g)
$PP = CP$	Section 4.5	Section 4.6
$PP > CP$	Section 4.5	Section 4.6
$PP < CP$	Section 4.7	Section 4.7

As we learned earlier, to correctly perform equivalence calculations, an effective interest rate is needed in the factors and spreadsheet functions. Therefore, it is essential that the time periods of the interest rate and the payment period be on the same time basis. The next three sections (4.5 to 4.7) describe procedures to determine correct i and n values for engineering economy factors and spreadsheet functions. First, compare the length of PP and CP, then identify the cash flows as only single amounts (P and F) or as a series (A , G , or g). Table 4-5 provides the section reference. The section references are the same when $PP = CP$ and $PP > CP$, because the procedures to determine i and n are the same, as discussed in Sections 4.5 and 4.6.

A general principle to remember throughout these equivalence computations is that when cash actually flows, it is necessary to account for the time value of money. For example, assume that cash flows occur every 6 months and that interest is compounded quarterly. After 3 months there is no cash flow and no need to determine the effect of quarterly compounding. However, at the 6-month time point, it is necessary to consider the interest accrued during the previous two quarters.

4.5 Equivalence Relations: Single Amounts with $PP \geq CP$ ● ● ●

With only P and F estimates defined, the payment period is not specifically identified. In virtually all situations, PP will be equal to or greater than CP. The length of the PP is defined by the interest period in the stated interest rate. If the rate is 8% per year, for example, $PP = CP = 1$ year. However, if the rate is 10% per year, compounded quarterly, then PP is 1 year, CP is 1 quarter or 3 months, and $PP > CP$. The procedures to perform equivalence computations are the same for both situations, as explained here.

When only single-amount cash flows are involved, there are two equally correct ways to determine i and n for P/F and F/P factors. Method 1 is easier to apply, because the interest tables in the back of the text can usually provide the factor value. Method 2 likely requires a factor formula calculation, because the resulting effective interest rate is not an integer. For spreadsheets, either method is acceptable; however, method 1 is usually easier.



Method 1: Determine the effective interest rate over the **compounding period CP**, and set n equal to the number of compounding periods between P and F . The relations to calculate P and F are

$$P = F(P/F, \text{effective } i\% \text{ per CP, total number of periods } n) \quad [4.8]$$

$$F = P(F/P, \text{effective } i\% \text{ per CP, total number of periods } n) \quad [4.9]$$

For example, assume that the stated credit card rate is nominal 15% per year, compounded monthly. Here CP is 1 month. To find P or F over a 2-year span, calculate the effective monthly rate of $15\%/12 = 1.25\%$ and the total months of $2(12) = 24$. Then 1.25% and 24 are used in the P/F and F/P factors.

Any time period can be used to determine the effective interest rate; however, the interest rate that is associated with the CP is typically the best because it is usually a whole number. Therefore, the factor tables in the back of the text can be used.

Method 2: Determine the **effective interest rate for the time period t of the nominal rate**, and set n equal to the total number of periods, using this same time period.

The P and F relations are the same as in Equations [4.8] and [4.9] with the term *effective $i\%$ per year* substituted for the interest rate. For a credit card rate of 15% per year, compounded monthly, the time period t is 1 year. The effective rate over 1 year and the n values are

$$\text{Effective } i\% \text{ per year} = \left(1 + \frac{0.15}{12}\right)^{12} - 1 = 16.076\%$$

$$n = 2 \text{ years}$$

The P/F factor is the same by both methods: $(P/F, 1.25\%, 24) = 0.7422$ using Table 5 in the rear of the text; and $(P/F, 16.076\%, 2) = 0.7422$ using the P/F factor formula.

EXAMPLE 4.7

Over the past 10 years, Gentrack has placed varying sums of money into a special capital accumulation fund. The company sells compost produced by garbage-to-compost plants in the United States and Vietnam. Figure 4–5 is the cash flow diagram in \$1000 units. Find the amount in the account now (after 10 years) at an interest rate of 12% per year, compounded semiannually.

Solution

Only P and F values are involved. Both methods are illustrated to find F in year 10.

Method 1: Use the semiannual CP to express the effective semiannual rate of 6% per 6-month period. There are $n = (2)(\text{number of years})$ semiannual periods for each cash flow. Using tabulated factor values, the future worth by Equation [4.9] is

$$\begin{aligned} F &= 1000(F/P, 6\%, 20) + 3000(F/P, 6\%, 12) + 1500(F/P, 6\%, 8) \\ &= 1000(3.2071) + 3000(2.0122) + 1500(1.5938) \\ &= \$11,634 \quad (\$11.634 \text{ million}) \end{aligned}$$

Method 2: Express the effective annual rate, based on semiannual compounding.

$$\text{Effective } i\% \text{ per year} = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 12.36\%$$

The n value is the actual number of years. Use the factor formula $(F/P, i, n) = (1.1236)^n$ and Equation [4.9] to obtain the same answer as above.

$$\begin{aligned} F &= 1000(F/P, 12.36\%, 10) + 3000(F/P, 12.36\%, 6) + 1500(F/P, 12.36\%, 4) \\ &= 1000(3.2071) + 3000(2.0122) + 1500(1.5938) \\ &= \$11,634 \quad (\$11.634 \text{ million}) \end{aligned}$$

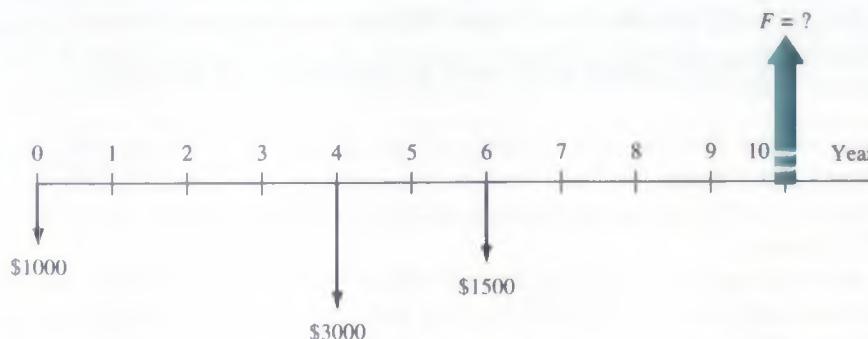


Figure 4–5
Cash flow diagram, Example 4.7.

TABLE 4–6 Examples of n and i Values Where PP = CP or PP > CP

Cash Flow Series	Interest Rate	What to Find; What Is Given	Standard Notation
\$500 semiannually for 5 years	16% per year, compounded semiannually	Find P ; given A	$P = 500(P/A, 8\%, 10)$
\$75 monthly for 3 years	24% per year, compounded monthly	Find F ; given A	$F = 75(F/A, 2\%, 36)$
\$180 quarterly for 15 years	5% per quarter	Find F ; given A	$F = 180(F/A, 5\%, 60)$
\$25 per month increase for 4 years	1% per month	Find P ; given G	$P = 25(P/G, 1\%, 48)$
\$5000 per quarter for 6 years	1% per month	Find A ; given P	$A = 5000(A/P, 3.03\%, 24)$

4.6 Equivalence Relations: Series with PP \geq CP ● ● ●

When uniform or gradient series are included in the cash flow sequence, the procedure is basically the same as method 2 above, except that PP is now defined by the length of time between cash flows. This also establishes the time unit of the effective interest rate. For example, if cash flows occur on a *quarterly* basis, PP is 1 *quarter* and the effective *quarterly* rate is necessary. The n value is the total number of *quarters*. If PP is a quarter, 5 years translates to an n value of 20 quarters. This is a direct application of the following general guideline:



When cash flows involve a series (i.e., A , G , g) and the payment period equals or exceeds the compounding period in length:

- Find the effective i per payment period.
- Determine n as the total number of payment periods.

In performing equivalence computations for series, *only* these values of i and n can be used in interest tables, factor formulas, and spreadsheet functions. In other words, there are no other combinations that give the correct answers, as there are for *single-amount cash flows*.

Table 4–6 shows the correct formulation for several cash flow series and interest rates. Note that n is always equal to the total number of payment periods and i is an effective rate expressed over the same time period as n .

EXAMPLE 4.8

For the past 7 years, Exelon Energy has paid \$500 every 6 months for a software maintenance contract. What is the equivalent total amount after the last payment, if these funds are taken from a pool that has been returning 8% per year, compounded quarterly?

Solution

The cash flow diagram is shown in Figure 4–6. The payment period (6 months) is longer than the compounding period (quarter); that is, PP > CP. Applying the guideline, we need to determine an **effective semiannual interest rate**. Use Equation [4.7] with $r = 4\%$ per 6-month period and $m = 2$ quarters per semiannual period.

$$\text{Effective } i\% \text{ per 6 months} = \left(1 + \frac{0.04}{2}\right)^2 - 1 = 4.04\%$$

The effective semiannual interest rate can also be obtained from Table 4–3 by using the r value of 4% and $m = 2$ to get $i = 4.04\%$.

The value $i = 4.04\%$ seems reasonable, since we expect the effective rate to be slightly higher than the nominal rate of 4% per 6-month period. The total number of semiannual payment periods is $n = 2(7) = 14$. The relation for F is

$$\begin{aligned} F &= A(F/A, 4.04\%, 14) \\ &= 500(18.3422) \\ &= \$9171.09 \end{aligned}$$

To determine the F/A factor value 18.3422 using a spreadsheet, enter the FV function from Figure 2–9, that is, $= -FV(4.04\%, 14, 1)$. Alternatively, the final answer of \$9171.09 can be displayed directly using the function $= -FV(4.04\%, 14, 500)$.

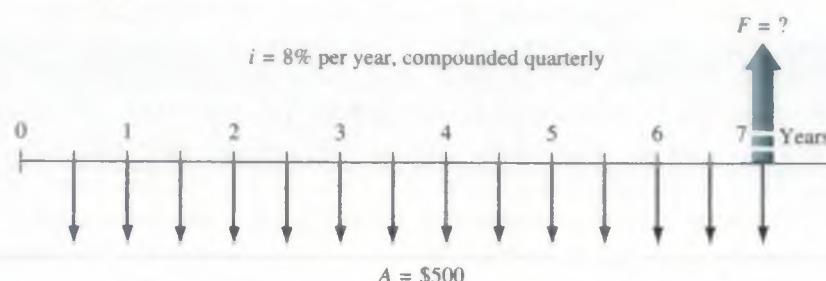


Figure 4–6

Diagram of semiannual deposits used to determine F , Example 4.8.

EXAMPLE 4.9 Credit Card Offer Case

PE

In our continuing credit card saga of Dave and his job transfer to Africa, let's assume he did remember that the total balance is \$1030, including the \$30 balance transfer fee, and he wants to set up a monthly automatic checking account transfer to pay off the entire amount in 2 years. Once he learned that the minimum payment is \$25 per month, Dave decided to be sure the monthly transfer exceeds this amount to avoid any further penalty fees and the penalty APR of 29.99% per year. What amount should he ask to be transferred by the due date each month? What is the APY he will pay, if this plan is followed exactly and Chase Bank does not change the APR during the 2-year period? Also, assume he left the credit card at home and will charge no more to it.

Solution

The monthly A series is needed for a total of $n = 2(12) = 24$ payments. In this case, $PP = CP = 1$ month, and the effective monthly rate is $i = 14.24\%/12 = 1.187\%$ per month.

Solution by hand: Use a calculator or hand computation to determine the A/P factor value.

$$\begin{aligned} A &= P(A/P, i, n) = 1030(A/P, 1.187\%, 24) = 1030(0.04813) \\ &= \$49.57 \text{ per month for 24 months} \end{aligned}$$

Solution by Spreadsheet: Use the function $= -PMT(1.187\%, 24, 1)$ to determine the factor value 0.04813 to determine A for $n = 24$ payments. Alternatively use the function $= PMT(1.187\%, 24, 1030)$ to directly display the required monthly payment of $A = -\$49.57$.

The *effective annual interest rate or APY* is computed using Equation [4.7] with $r = 14.24\%$ per year, compounded monthly, and $m = 12$ times per year.

$$\begin{aligned} \text{Effective } i \text{ per year} &= \left(1 + \frac{0.1424}{12}\right)^{12} - 1 = 1.15207 - 1 \\ &= 15.207\% \text{ per year} \end{aligned}$$

This is the same effective annual rate i_a determined in Example 4.4b.

EXAMPLE 4.10

The Scott and White Health Plan (SWHP) has purchased a robotized prescription fulfillment system for faster and more accurate delivery to patients with stable, pill-form medication for chronic health problems, such as diabetes, thyroid, and high blood pressure. Assume this high-volume system costs \$3 million to install and an estimated \$200,000 per year for all materials, operating, personnel, and maintenance costs. The expected life is 10 years. An SWHP biomedical engineer wants to estimate the total revenue requirement for each 6-month period that is necessary to recover the investment, interest, and annual costs. Find this semiannual A value both by hand and by spreadsheet, if capital funds are evaluated at 8% per year, using two different compounding periods:

Rate 1. 8% per year, compounded *semiannually*.

Rate 2. 8% per year, compounded *monthly*.

Solution

Figure 4–7 shows the cash flow diagram. Throughout the 20 semiannual periods, the annual cost occurs every other period, and the capital recovery series is sought for every 6-month period. This pattern makes the solution by hand quite involved if the P/F factor, not the P/A factor, is used to find P for the 10 annual \$200,000 costs. The spreadsheet solution is recommended in cases such as this.

Solution by hand—rate 1: Steps to find the semiannual A value are summarized below:

PP = CP at 6 months; find the effective rate per semiannual period.

Effective semiannual $i = 8\%/2 = 4\%$ per 6 months, compounded semiannually.

Number of semiannual periods $n = 2(10) = 20$.

Calculate P , using the P/F factor for $n = 2, 4, \dots, 20$ periods because the costs are annual, not semiannual. Then use the A/P factor over 20 periods to find the semiannual A .

$$\begin{aligned} P &= 3,000,000 + 200,000 \left[\sum_{k=2,4}^{20} (P/F, 4\%, k) \right] \\ &= 3,000,000 + 200,000(6.6620) = \$4,332,400 \\ A &= \$4,332,400(A/P, 4\%, 20) = \$318,778 \end{aligned}$$

Conclusion: Revenue of \$318,778 is necessary every 6 months to cover all costs and interest at 8% per year, compounded semiannually.

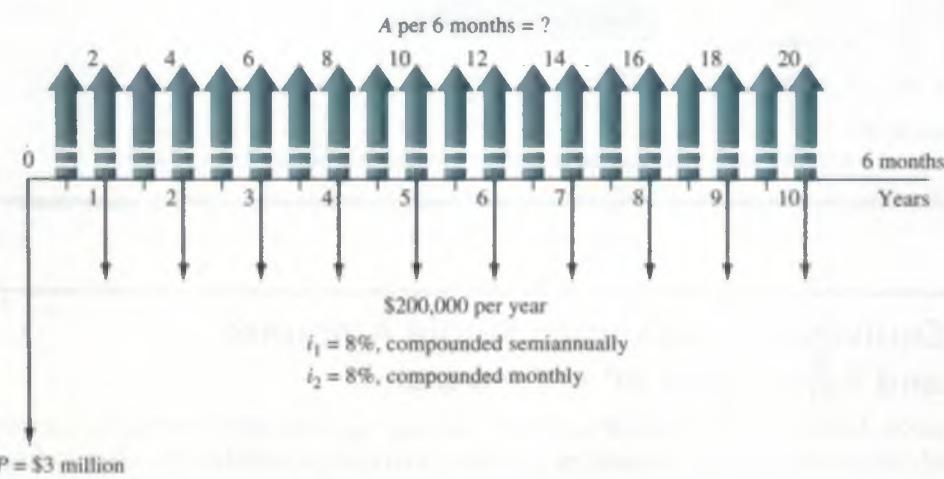


Figure 4–7

Cash flow diagram with two different compounding periods, Example 4.10.

Solution by hand—rate 2: The PP is 6 months, but the CP is now monthly; therefore, PP > CP. To find the effective semiannual rate, the effective interest rate Equation [4.7] is applied with $r = 4\%$ and $m = 6$ months per semiannual period.

$$\text{Effective semiannual } i = \left(1 + \frac{0.04}{6}\right)^6 - 1 = 4.067\%$$

$$P = 3,000,000 + 200,000 \left[\sum_{k=2,4}^{20} (P/F, 4.067\%, k) \right]$$

$$= 3,000,000 + 200,000(6.6204) = \$4,324,080$$

$$A = \$4,324,080(A/P, 4.067\%, 20) = \$320,064$$

Now, \$320,064, or \$1286 more semiannually, is required to cover the more frequent compounding of the 8% per year interest. Note that all P/F and A/P factors must be calculated with factor formulas at 4.067%. This method is usually more calculation-intensive and error-prone than the spreadsheet solution.

Solution by spreadsheet—rates 1 and 2: Figure 4–8 presents a general solution for the problem at both rates. (Several rows at the bottom of the spreadsheet are not printed. They continue the cash flow pattern of \$200,000 every other 6 months through cell B32.) The functions in C8 and E8 are general expressions for the effective rate per PP, expressed in months. This allows some sensitivity analysis to be performed for different PP and CP values. Note the functions in C7 and E7 to determine m for the effective rate relations. This technique works well for spreadsheets once PP and CP are entered in the time unit of the CP.

Each 6-month period is included in the cash flows, including the \$0 entries, so the NPV and PMT functions work correctly. The final A values in D14 (\$318,784) and F14 (\$320,069) are the same (except for rounding) as those above.

	A	B	C	D	E	F	G	H	I	J	K
1			Rate #1		Rate #2						
2	Nominal rate per year		8%		8%			= C5/C6			
3	Compounding period		Semiannual		Month			= E5/E6			
4	Number of periods, n	20		20		6					
5	PP in months	6		6		1					
6	CP in months	6		6		6					
7	Number of CP per PP, m	1		1		1					
8	Effective rate per PP	4.00%		4.07%							
9			Present worth	\$4,332,385							
10	Semiannual										
11	period		Cash flow	A, \$1/6-month	\$ 318,784						
12	0										
13	1										
14	2			\$ 200,000							
15	3			0							
16	4			\$ 200,000							
17	5			0							
18	6			\$ 200,000							
19	7			0							
20	8			\$ 200,000							
21	9			0							
22	10			\$ 200,000							
23	11			0							
24	12			\$ 200,000							
25	13			0							
26	14			\$ 200,000							
27	15			0							
28	16			\$ 200,000							
29	17			0							
30	18			\$ 200,000							
31	19			0							
32	20			\$ 200,000							

Figure 4–8

Spreadsheet solution for semiannual A series for different compounding periods, Example 4.10.

4.7 Equivalence Relations: Single Amounts and Series with PP < CP ● ● ●

If a person deposits money each month into a savings account where interest is compounded quarterly, do all the monthly deposits earn interest before the next quarterly compounding time? If a person's credit card payment is due with interest on the 15th of the month, and if the full payment is made on the 1st, does the financial institution reduce the interest owed, based on early payment? The usual answers are no. However, if a monthly payment on a \$10 million, quarterly

compounded, bank loan were made early by a large corporation, the corporate financial officer would likely insist that the bank reduce the amount of interest due, based on early payment. These are examples of $PP < CP$. The timing of cash flow transactions between compounding points introduces the question of how **interperiod compounding** is handled. Fundamentally, there are two policies: interperiod cash flows earn *no interest*, or they earn *compound interest*.

For a no-interperiod-interest policy, negative cash flows (deposits or payments, depending on the perspective used for cash flows) are all regarded as *made at the end of the compounding period*, and positive cash flows (receipts or withdrawals) are all regarded as *made at the beginning*.

As an illustration, when interest is compounded quarterly, all monthly deposits are moved to the end of the quarter (no interperiod interest is earned), and all withdrawals are moved to the beginning (no interest is paid for the entire quarter). This procedure can significantly alter the distribution of cash flows before the effective quarterly rate is applied to find P , F , or A . This effectively forces the cash flows into a $PP = CP$ situation, as discussed in Sections 4.5 and 4.6. Example 4.11 illustrates this procedure and the economic fact that, within a one-compounding-period time frame, there is no interest advantage to making payments early. Of course, noneconomic factors may be present.

EXAMPLE 4.11

Last year AllStar Venture Capital agreed to invest funds in Clean Air Now (CAN), a start-up company in Las Vegas that is an outgrowth of research conducted in mechanical engineering at the University of Nevada–Las Vegas. The product is a new filtration system used in the process of carbon capture and sequestration (CCS) for coal-fired power plants. The venture fund manager generated the cash flow diagram in Figure 4–9a in \$1000 units from AllStar's perspective. Included are payments (outflows) to CAN made over the first year and receipts (inflows) from CAN to AllStar. The receipts were unexpected this first year; however, the product has great promise, and advance orders have come from eastern U.S. plants anxious to become zero-emission coal-fueled plants. The interest rate is 12% per year, compounded quarterly, and AllStar uses the no-interperiod-interest policy. How much is AllStar in the “red” at the end of the year?

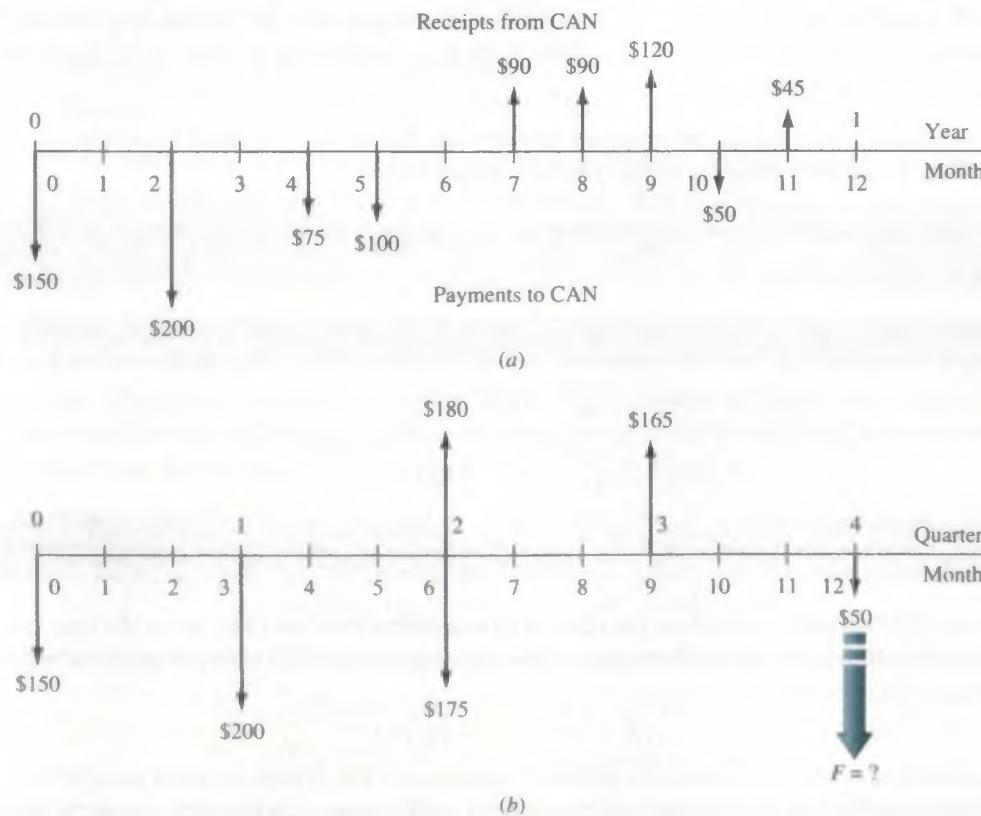


Figure 4–9

(a) Actual and (b) moved cash flows (in \$1000) for quarterly compounding periods using no interperiod interest, Example 4.11.

Solution

With no interperiod interest considered, Figure 4–9b reflects the moved cash flows. All negative cash flows (payments to CAN) are moved to the end of the respective quarter, and all positive cash flows (receipts) are moved to the beginning of the respective quarter. Calculate the F value at $12\%/4 = 3\%$ per quarter.

$$\begin{aligned} F &= 1000[-150(F/P, 3\%, 4) - 200(F/P, 3\%, 3) + (-175 + 180)(F/P, 3\%, 2) \\ &\quad + 165(F/P, 3\%, 1) - 50] \\ &= \$-262,111 \end{aligned}$$

AllStar has a net investment of \$262,111 in CAN at the end of the year.

If $PP < CP$ and interperiod compounding is earned, then the cash flows are not moved, and the equivalent P , F , or A values are determined using the effective interest rate per payment period. The engineering economy relations are determined in the same way as in the previous two sections for $PP \geq CP$. The effective interest rate formula will have an m value less than 1, because there is only a fractional part of the CP within one PP . For example, weekly cash flows and quarterly compounding require that $m = 1/13$ of a quarter. When the nominal rate is 12% per year, compounded quarterly (the same as 3% per quarter, compounded quarterly), the effective rate per PP is

$$\text{Effective weekly } i\% = (1.03)^{1/13} - 1 = 0.228\% \text{ per week}$$

4.8 Effective Interest Rate for Continuous Compounding ● ● ●

If we allow compounding to occur more and more frequently, the compounding period becomes shorter and shorter and m , the number of compounding periods per payment period, increases.

Continuous compounding is present when the duration of CP , the compounding period, becomes infinitely small and m , the number of times interest is compounded per period, becomes infinite. Businesses with large numbers of cash flows each day consider the interest to be continuously compounded for all transactions.

As m approaches infinity, the effective interest rate Equation [4.7] must be written in a new form. First, recall the definition of the natural logarithm base.

$$\lim_{h \rightarrow \infty} \left(1 + \frac{1}{h}\right)^h = e = 2.71828 + \quad [4.10]$$

The limit of Equation [4.7] as m approaches infinity is found by using $r/m = 1/h$, which makes $m = hr$.

$$\begin{aligned} \lim_{m \rightarrow \infty} i &= \lim_{m \rightarrow \infty} \left(1 + \frac{r}{m}\right)^m - 1 \\ &= \lim_{h \rightarrow \infty} \left(1 + \frac{1}{h}\right)^{hr} - 1 = \lim_{h \rightarrow \infty} \left[\left(1 + \frac{1}{h}\right)^h\right]^r - 1 \end{aligned}$$

$$i = e^r - 1 \quad [4.11]$$

Equation [4.11] is used to compute the **effective continuous interest rate**, when the time periods on i and r are the same. As an illustration, if the nominal annual $r = 15\%$ per year, the effective continuous rate per year is

$$i\% = e^{0.15} - 1 = 16.183\%$$

For convenience, Table 4–3 includes effective continuous rates for the nominal rates listed.

To find an effective or nominal interest rate for continuous compounding using the spreadsheet functions **EFFECT** or **NOMINAL**, enter a very large value for the compounding frequency m in Equation [4.5] or [4.6], respectively. A value of 10,000 or higher provides sufficient accuracy. Both functions are illustrated in Example 4.12.

EXAMPLE 4.12

- (a) For an interest rate of 18% per year, compounded continuously, calculate the effective monthly and annual interest rates.
- (b) An investor requires an effective return of at least 15%. What is the minimum annual nominal rate that is acceptable for continuous compounding?

Solution by Hand

- (a) The nominal monthly rate is $r = 18\%/12 = 1.5\%$, or 0.015 per month. By Equation [4.11], the effective monthly rate is

$$i\% \text{ per month} = e^r - 1 = e^{0.015} - 1 = 1.511\%$$

Similarly, the effective annual rate using $r = 0.18$ per year is

$$i\% \text{ per year} = e^r - 1 = e^{0.18} - 1 = 19.722\%$$

- (b) Solve Equation [4.11] for r by taking the natural logarithm.

$$e^r - 1 = 0.15$$

$$e^r = 1.15$$

$$\ln e^r = \ln 1.15$$

$$r = 0.13976$$

Therefore, a rate of 13.976% per year, compounded continuously, will generate an effective 15% per year return. The general formula to find the nominal rate, given the effective continuous rate i , is $r = \ln(1 + i)$.

Solution by Spreadsheet

- (a) Use the EFFECT function with the nominal monthly rate $r = 1.5\%$ and annual rate $r = 18\%$ with a large m to display effective i values. The functions to enter on a spreadsheet and the responses are as follows:

Monthly: = EFFECT(1.5%,10000) effective $i = 1.511\%$ per month

Annual: = EFFECT(18%,10000) effective $i = 19.722\%$ per year

- (b) Use the function in Equation [4.6] in the format = NOMINAL(15%,10000) to display the nominal rate of 13.976% per year, compounded continuously.

EXAMPLE 4.13

Engineers Marci and Suzanne both invest \$5000 for 10 years at 10% per year. Compute the future worth for both individuals if Marci receives annual compounding and Suzanne receives continuous compounding.

Solution

Marci: For annual compounding the future worth is

$$F = P(F/P, 10\%, 10) = 5000(2.5937) = \$12,969$$

Suzanne: Using Equation [4.11], first find the effective i per year for use in the F/P factor.

$$\text{Effective } i\% = e^{0.10} - 1 = 10.517\%$$

$$F = P(F/P, 10.517\%, 10) = 5000(2.7183) = \$13,591$$

Continuous compounding causes a \$622 increase in earnings. For comparison, daily compounding yields an effective rate of 10.516% ($F = \$13,590$), only slightly less than the 10.517% for continuous compounding.

For some business activities, cash flows occur throughout the day. Examples of costs are energy and water costs, inventory costs, and labor costs. A realistic model for these activities is to increase the frequency of the cash flows to become continuous. In these cases, the economic analysis can be performed for **continuous cash flow** (also called continuous funds flow) and the continuous compounding of interest as discussed above. Different expressions must be derived for the factors for these cases. In fact, the monetary differences for continuous cash flows relative to the discrete cash flow and discrete compounding assumptions are usually not large. Accordingly, most engineering economy studies do not require the analyst to utilize these mathematical forms to make a sound economic decision.

4.9 Interest Rates That Vary over Time ● ● ●

Real-world interest rates for a corporation vary from year to year, depending upon the financial health of the corporation, its market sector, the national and international economies, forces of inflation, and many other elements. Loan rates may increase from one year to another. Home mortgages financed using ARM (adjustable-rate mortgage) interest is a good example. The mortgage rate is slightly adjusted annually to reflect the age of the loan, the current cost of mortgage money, etc.

When P , F , and A values are calculated using a constant or average interest rate over the life of a project, rises and falls in i are neglected. If the variation in i is large, the equivalent values will vary considerably from those calculated using the constant rate. Although an engineering economy study can accommodate varying i values mathematically, it is more involved computationally to do so.

To determine the P value for future cash flow values (F_t) at different i values (i_t) for each year t , we will assume *annual compounding*. Define

$$i_t = \text{effective annual interest rate for year } t \quad (t = \text{years 1 to } n)$$

To determine the present worth, calculate the P of each F_t value, using the applicable i_t , and sum the results. Using standard notation and the P/F factor,

$$\begin{aligned} P &= F_1(P/F, i_1, 1) + F_2(P/F, i_1, 1)(P/F, i_2, 1) + \dots \\ &\quad + F_n(P/F, i_1, 1)(P/F, i_2, 1) \cdots (P/F, i_n, 1) \end{aligned} \quad [4.12]$$

When only single amounts are involved, that is, one P and one F in the final year n , the last term in Equation [4.12] is the expression for the present worth of the future cash flow.

$$P = F_n(P/F, i_1, 1)(P/F, i_2, 1) \cdots (P/F, i_n, 1) \quad [4.13]$$

If the equivalent uniform series A over all n years is needed, first find P , using either of the last two equations; then substitute the symbol A for each F_t symbol. Since the equivalent P has been determined numerically using the varying rates, this new equation will have only one unknown, namely, A . Example 4.14 illustrates this procedure.

EXAMPLE 4.14

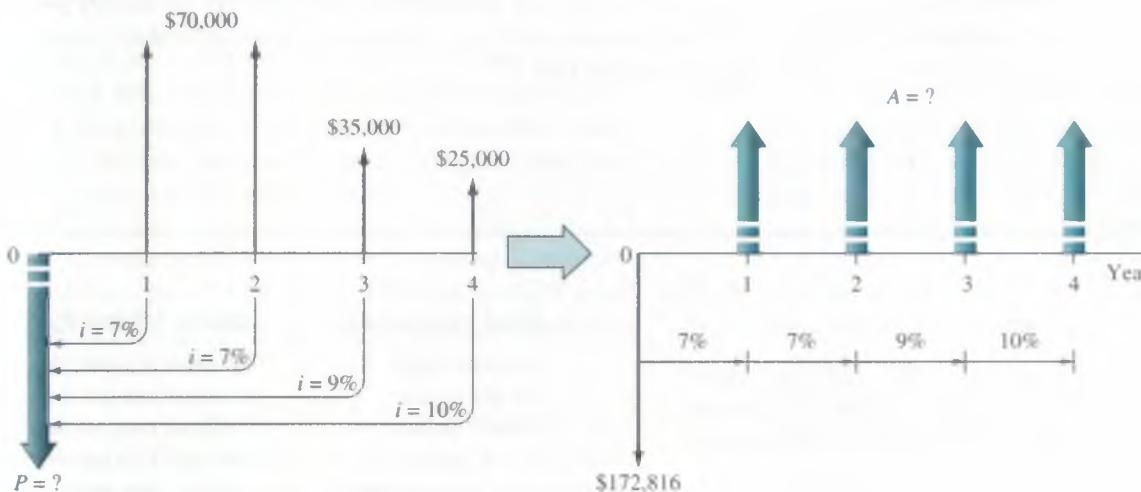
CE, Inc., leases large earth tunneling equipment. The net profit from the equipment for each of the last 4 years has been decreasing, as shown below. Also shown are the annual rates of return on invested capital. The return has been increasing. Determine the present worth P and equivalent uniform series A of the net profit series. Take the annual variation of rates of return into account.

Year	1	2	3	4
Net Profit	\$70,000	\$70,000	\$35,000	\$25,000
Annual Rate	7%	7%	9%	10%

Solution

Figure 4–10 shows the cash flows, rates for each year, and the equivalent P and A . Equation [4.12] is used to calculate P . Since for both years 1 and 2 the net profit is \$70,000 and the annual rate is 7%, the P/A factor can be used for these 2 years only.

$$\begin{aligned} P &= [70(P/A, 7\%, 2) + 35(P/F, 7\%, 2)(P/F, 9\%, 1) \\ &\quad + 25(P/F, 7\%, 2)(P/F, 9\%, 1)(P/F, 10\%, 1)](1000) \\ &= [70(1.8080) + 35(0.8013) + 25(0.7284)](1000) \\ &= \$172,816 \end{aligned} \quad [4.14]$$

**Figure 4–10**

Equivalent P and A values for varying interest rates, Example 4.14.

To determine an equivalent annual series, substitute the symbol A for all net profit values on the right side of Equation [4.14], set it equal to $P = \$172,816$, and solve for A . This equation accounts for the varying i values each year. See Figure 4–10 for the cash flow diagram transformation.

$$\$172,816 = A[(1.8080) + (0.8013) + (0.7284)] = A[3.3377]$$

$$A = \$51,777 \text{ per year}$$

Comment

If the average of the four annual rates, that is, 8.25% is used, the result is $A = \$52,467$. This is a \$690 per year overestimate of the equivalent annual net profit.

When there is a cash flow in year 0 and interest rates vary annually, this cash flow must be included to determine P . In the computation for the equivalent uniform series A over all years, including year 0, it is important to include this initial cash flow at $t = 0$. This is accomplished by inserting the factor value for $(P/F, i_0, 0)$ into the relation for A . This factor value is always 1.00. It is equally correct to find the A value using a future worth relation for F in year n . In this case, the A value is determined using the F/P factor, and the cash flow in year n is accounted for by including the factor $(F/P, i_n, 0) = 1.00$.

CHAPTER SUMMARY

Since many real-world situations involve cash flow frequencies and compounding periods other than 1 year, it is necessary to use nominal and effective interest rates. When a nominal rate r is stated, the effective interest rate i per payment period is determined by using the effective interest rate equation.

$$\text{Effective } i = \left(1 + \frac{r}{m}\right)^m - 1$$

The m is the number of compounding periods (CP) per interest period. If interest compounding becomes more and more frequent, then the length of a CP approaches zero, continuous compounding results, and the effective i is $e^r - 1$.

All engineering economy factors require the use of an effective interest rate. The i and n values placed in a factor depend upon the type of cash flow series. If only single amounts (P and F) are present, there are several ways to perform equivalence calculations using the factors. However, when series cash flows (A , G , and g) are present, only one combination of the effective rate i and number of periods n is correct for the factors. This requires that the relative lengths of PP and CP be considered as i and n are determined. The interest rate and payment periods must have the same time unit for the factors to correctly account for the time value of money.

From one year (or interest period) to the next, interest rates will vary. To accurately perform equivalence calculations for P and A when rates vary significantly, the applicable interest rate should be used, not an average or constant rate.

PROBLEMS

Nominal and Effective Rates

- 4.1 From the interest statement 18% per year, compounded monthly, determine the values for interest period, compounding period, and compounding frequency.
- 4.2 From the interest statement 1% per month, determine the values for interest period, compounding period, and compounding frequency.
- 4.3 Determine the number of times interest would be compounded in 6 months from the interest statements (a) 18% per year, compounded monthly, (b) 1% per month, and (c) 2% per quarter.
- 4.4 For an interest rate of 1% per 2 months, determine the number of times interest would be compounded in (a) 2 months, (b) two semiannual periods, and (c) 3 years.
- 4.5 Identify the compounding period for the following interest statements: (a) 3% per quarter; (b) 10% per year, compounded semiannually; (c) nominal 7.2% per year, compounded monthly; (d) effective 3.4% per quarter, compounded weekly; and (e) 2% per month, compounded continuously.
- 4.6 Identify the following interest rate statements as either nominal or effective: (a) 1.5% per month, compounded daily; (b) 17% per year, compounded quarterly; (c) effective 15% per year, compounded monthly; (d) nominal 0.6% per month, compounded weekly; (e) 0.3% per week, compounded weekly; and (f) 8% per year.
- 4.7 Convert the given interest rates in the left-hand column into the nominal rates listed in the right-hand column. (Assume 4 weeks/month.)

Given Interest Rate	Desired Interest Rate
1% per month	Nominal rate per year
3% per quarter	Nominal rate per 6 months
2% per quarter	Nominal rate per year
0.28% per week	Nominal rate per quarter
6.1% per 6 months	Nominal rate per 2 years

- 4.8 What nominal interest rate per year is equivalent to 11.5% per year, compounded monthly?
- 4.9 For a Federal Credit Union that offers an interest rate of 8% per year, compounded quarterly, determine the nominal rate per 6 months.
- 4.10 The Second National Bank of Fullertum advertises an APR of 14% compounded monthly for student loans. Determine the APY. Show hand and spreadsheet solutions.
- 4.11 For an effective annual rate i_a of 15.87% compounded quarterly, determine (a) the effective quarterly rate and (b) the nominal annual rate. (c) What is the spreadsheet function to find the nominal annual rate above?
- 4.12 High-tech companies such as IBM, AMD, and Intel have been using nanotechnology for several years to make microchips with faster speeds while using less power. A less well-known company in the chip business has been growing fast enough that the company uses a minimum attractive rate of return of 60% per year. If this MARR is an effective annual rate compounded monthly, determine the effective monthly rate.
- 4.13 An interest rate of 21% per year, compounded every 4 months, is equivalent to what effective rate per year? Show hand and spreadsheet solutions.

- 4.14 An interest rate of 8% per 6 months, compounded monthly, is equivalent to what effective rate per quarter?
- 4.15 A small company that makes modular bevel gear drives with a tight swing ratio for optimizing pallet truck design was told that the interest rate on a mortgage loan would be an *effective* 4% per quarter, compounded monthly. The owner was confused by the terminology and asked you to help. What are (a) the APR and (b) the APY?
- 4.16 In 'N Out Payday Loans advertises that for a fee of only \$10, you can immediately borrow up to \$200 for one month. If a person accepts the offer, what are (a) the nominal interest rate per year and (b) the effective rate per year?
- 4.17 A government-required truth-in-lending document showed that the APR was 21% and the APY was 22.71%. Determine the compounding frequency at which the two rates are equivalent.
- 4.18 Julie has a low credit rating, plus she was furloughed from her job 2 months ago. She has a new job starting next week and expects a salary to start again in a couple of weeks. Since she is a little short on money to pay her rent, she decided to borrow \$100 from a loan company, which will charge her only \$10 interest if the \$110 is paid no more than 1 week after the loan is made. What are the (a) nominal annual and (b) effective annual interest rates that she will pay on this loan?

Equivalence When $PP \geq CP$

- 4.19 Assume you deposit 25% of your monthly check of \$5500 into a savings account at a credit union that compounds interest semiannually. (a) What are the payment and compounding periods? (b) Is the payment period greater than or less than the compounding period?
- 4.20 Interest is compounded quarterly, and single-payment cash flows (that is, F and P) are separated by 5 years. What must the *compounding period* be on the interest rate, if the value of n in the P/F or F/P equation is (a) $n = 5$, (b) $n = 20$, or (c) $n = 60$?
- 4.21 When interest is compounded quarterly and a uniform series cash flow of \$4000 occurs every 6 months, what time periods on i and n must be used?
- 4.22 Pinpoint Laser Systems is planning to set aside \$260,000 now for possibly replacing its 4-pole, 38-MW synchronous motors when it becomes necessary. If the replacement is expected to take place in 3 years, how much will the company have in its investment set-aside account? Assume the company can achieve a rate of return of 12% per year, compounded quarterly.
- 4.23 Wheeling-Pittsburgh Steel is investigating whether it should replace some of its basic oxygen furnace equipment now or wait to do it later. The cost later (i.e., 3 years from now) is estimated to be \$1,700,000. How much can the company afford to spend now, if its minimum attractive rate of return is 1.5% per month?
- 4.24 How much can Wells Fargo lend to a developer who will repay the loan by selling 6 view lots at \$190,000 each 2 years from now? Assume the bank will lend at a nominal 14% per year, compounded semiannually.
- 4.25 How much will be in a high-yield account at the National Bank of Arizona 12 years from now if you deposit \$5000 now and \$7000 five years from now? The account earns interest at a rate of 8% per year, compounded quarterly.
- 4.26 Loadstar Sensors is a company that makes load/force sensors based on capacitive sensing technology. The company wants to have \$28 million for a plant expansion 4 years from now. If the company has already set aside \$12 million in an investment account for the expansion, how much more must the company add to the account next year (i.e., 1 year from now) so that it will have the \$28 million 4 years from now? The account earns interest at 12% per year, compounded quarterly.
- 4.27 A structural engineering consulting company is examining its cash flow requirements for the next 6 years. The company expects to replace office machines and computer equipment at various times over the 6-year planning period. Specifically, the company expects to spend \$21,000 two years from now, \$24,000 three years from now, and \$10,000 five years from now. What is the present worth of the planned expenditures at an interest rate of 10% per year, compounded semiannually?
- 4.28 Irvin Aerospace of Santa Ana, California, was awarded a 5-year contract to develop an advanced space capsule airbag landing attenuation system for NASA's Langley Research Center. The company's computer system uses fluid structure interaction modeling to test and analyze each airbag design concept. What is the present worth of the contract at 16% per year, compounded quarterly, if the quarterly cost in years 1 through 5 is \$2 million per quarter?

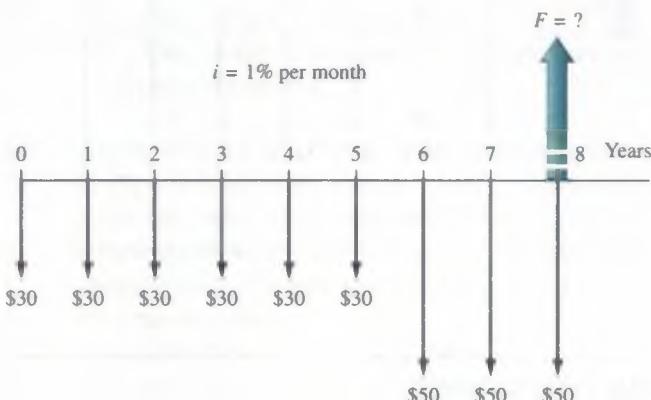
- 4.29 Heyden Motion Solutions ordered \$7 million worth of seamless tubes for its drill collars from the Timken Company of Canton, Ohio. (A drill collar is the heavy tubular connection between a drill pipe and a drill bit.) At 12% per year, compounded semiannually, what is the equivalent uniform cost per semiannual period over a 5-year amortization period?
- 4.30 In 2010, the National Highway Traffic Safety Administration raised the average fuel efficiency standard to 35.5 miles per gallon (mpg) for cars and light trucks by the year 2016. The rules will cost consumers an average of \$926 extra per vehicle in the 2016 model year. Assume Yolanda will purchase a new car in 2016 and plans to keep it for 5 years. How much will the monthly savings in the cost of gasoline have to be to recover Yolanda's extra cost? Use an interest rate of 0.75% per month.
- 4.31 Fort Bliss, a U.S. Army military base, contributed \$3.3 million of the \$87 million capital cost for a desalting plant constructed and operated by El Paso Water Utilities (EPWU). In return, EPWU agreed to sell water to Fort Bliss at \$0.85 per 1000 gallons for 20 years. If the army base uses 200 million gallons of water per month, what is the Army's cost per month for water? The \$3.3 million capital cost is amortized at an interest rate of 6% per year, compounded monthly.
- 4.32 Beginning in 2011, city hall, administrative offices, and municipal courts in the city of El Paso, Texas, will go on a 10 hour/day, 4-day workweek from the beginning of May through the end of September. The shortened workweek will affect 25% of the city's 6100 employees and will save \$42,600 per month for those 5 months, because of reduced fuel, utility, and janitorial expenses. If this work schedule continues for the next 10 years, what is the future worth of the savings at the end of that time (i.e., end of year 2020)? Use an interest rate of 0.5% per month.
- 4.33 In October 2009, Wal-Mart started selling caskets on its website that undercut many funeral homes. Prices ranged from \$999 for steel models such as *Dad Remembered* to \$3199 for the *Sienna Bronze* casket. Part of the business model is to get people to plan ahead, so the company is allowing people to pay for the caskets over a 12-month period with no interest. An individual purchased a *Sienna Bronze* casket and made 12 equal monthly payments (in months 1 through 12) at no interest. How much did this person save each month compared to another person who paid an interest rate of 6% per year, compounded monthly?
- 4.34 NRG Energy plans to construct a giant solar plant in Santa Teresa, New Mexico, to supply electricity to West Texas Electric. The plant will have 390,000 heliostats to concentrate sunlight onto 32 water towers to generate steam. It will provide enough power for 30,000 homes. If the company spends \$28 million per month for 2 years in constructing the plant, how much will the company have to make each month in years 3 through 22 (that is, 20 years) to recover its investment plus 18% per year, compounded monthly?
- 4.35 Many college students have Visa credit cards that carry an interest rate of "simple 24% per year" (that is, 2% per month). When the balance on such a card is \$5000, the minimum payment is \$110.25.
- What is the amount of interest in the first payment?
 - How long will it take, in months, to pay off the balance, if the cardholder continues to make payments of \$110.25 per month and adds no other charges to the card?
- 4.36 Bart is an engineering graduate who did not take the engineering economy elective during his B.S. degree course work. After working for a year or so, he found himself in financial trouble, and he borrowed \$500 from a friend in the finance department at his office. Bart agreed to repay the loan principal plus \$75 interest 1 month later. The two got separated doing different jobs, and 1 year went by. The friend e-mailed Bart after exactly 1 year and asked for the loan repayment plus the interest with monthly compounding, since Bart had made no effort to repay the loan during the year.
- What does Bart now owe his friend?
 - What effective annual interest rate did Bart pay on this \$500 loan?
- 4.37 AT&T announced that the early termination fee for smart phones will jump from \$175 to \$375. The fee will be reduced by \$10 each month of the 2-year contract. Wireless carriers justify the fees by pointing out that the cost of a new phone is heavily discounted from what the carrier pays the manufacturer. Assume AT&T pays \$499 for an iPhone that it sells for \$199. How much profit would the company have to make each month (i.e., prior to the termination), if it wanted to make a rate of return of 1.5% per month on its \$300 investment in a customer who terminates the contract after 12 months?
- 4.38 What is the future worth of a present cost of \$285,000 to Monsanto, Inc. 5 years from now at an interest rate of 2% per month?

- 4.39 Environmental recovery company RexChem Partners plans to finance a site reclamation project that will require a 4-year cleanup period. The company plans to borrow \$3.6 million now. How much will the company have to get in a lump-sum payment when the project is over in order to earn 24% per year, compounded quarterly, on its investment?

- 4.40 For the cash flows shown below, determine the equivalent uniform worth in years 1 through 5 at an interest rate of 18% per year, compounded monthly.

Year	1	2	3	4	5
Cash Flow, \$	200,000	0	350,000	0	400,000

- 4.41 For the cash flow diagram shown, solve for F , using an interest rate of 1% per month.



- 4.42 According to the Government Accountability Office (GAO), if the U.S. Postal Service does not change its business model, it will lose \$480 million next month and \$500 million the month after that, and the losses will increase by \$20 million per month for the next 10 years. At an interest rate of 0.25% per month, what is the equivalent uniform amount per month of the losses through year 10?

- 4.43 Equipment maintenance costs for manufacturing explosion-proof pressure switches are projected to be \$100,000 in year 1 and increase by 4% each year through year 5. What is the present worth of the maintenance costs at an interest rate of 10% per year, compounded quarterly?

Equivalence When PP < CP

- 4.44 If you deposit \$1000 per month into an investment account that pays interest at a rate of 6% per year, compounded quarterly, how much will be in the account at the end of 5 years? There is no interperiod compounding.

- 4.45 Freeport McMoran purchased two model MTVS peristaltic pumps (to inject sulfuric acid and King

Lee antiscalant) for use at its nanofiltration water conditioning plant. The cost of the pumps was \$950 each. If the chemical cost is \$11 per day, determine the equivalent cost *per month* at an interest rate of 12% per year, compounded monthly. Assume 30 days per month and a 3-year pump life.

- 4.46 Income from recycling paper and cardboard at the U.S. Army's Fort Benning Maneuver Center has averaged \$3000 per month for 2½ years. What is the future worth of the income (after the 2½ years) at an interest rate of 6% per year, compounded quarterly? Assume there is no interperiod compounding.

- 4.47 The Autocar E3 refuse truck has an energy recovery system developed by Parker Hannifin LLC that is expected to reduce fuel consumption by 50%. Pressurized fluid flows from carbon fiber-reinforced accumulator tanks to two hydrostatic motors that propel the vehicle forward. (The truck recharges the accumulators when it brakes.) The fuel cost for a regular refuse truck is \$900 per month. How much can a private waste-hauling company afford to spend now on the recovery system, if it wants to recover its investment in 3 years plus a return of 14% per year, compounded semiannually? Assume no interperiod compounding.

Continuous Compounding

- 4.48 What effective interest rate per year is equal to 1.2% per month, compounded continuously? Show hand and spreadsheet solutions.

- 4.49 What effective interest rate per quarter is equal to a nominal 1.6% per month, compounded continuously?

- 4.50 What nominal rate per month is equivalent to an effective 1.3% per month, compounded continuously?

- 4.51 Companies such as GE that have huge amounts of cash flow every day base their financial calculations on continuous compounding. If the company wants to make an effective 25% per year, compounded continuously, what nominal daily rate of return has to be realized? Assume 365 days per year.

- 4.52 U.S. Steel is planning a plant expansion that is expected to cost \$13 million. How much money must the company set aside now in a lump-sum investment to have the money in 2 years? Capital funds earn interest at a rate of 12% per year, compounded continuously.

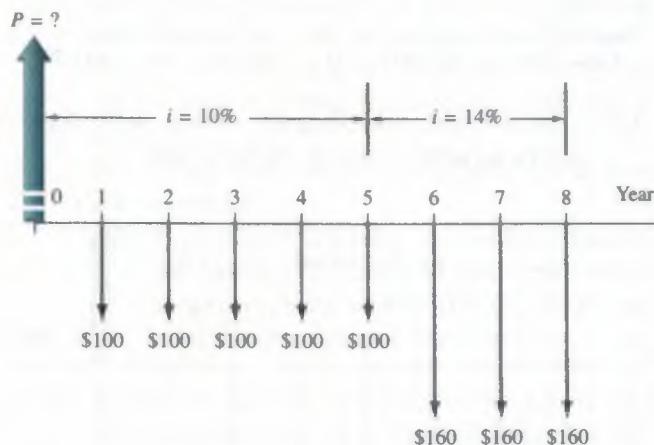
- 4.53 Periodic outlays for inventory control software at Baron Chemicals are expected to be \$150,000 immediately, \$200,000 in 1 year, and \$350,000 in 2 years. What is the present worth of the costs at an interest rate of 10% per year, compounded continuously?

Varying Interest Rates

- 4.54 Many small companies use accounts receivable as collateral to borrow money for continuing operations and meeting payrolls. If a company borrows \$300,000 now at an interest rate of 1% per month, but the rate changes to 1.25% per month after 4 months, how much will the company owe at the end of 1 year?
- 4.55 The maintenance cost for furnaces at a copper smelting plant have been constant at \$140,000 per year for the past 5 years. If the interest rate was 8% per year for the first 3 years and then it increased to 10% in years 4 and 5, what is the equivalent future worth (in year 5) of the maintenance cost? Show hand and spreadsheet solutions.
- 4.56 By filling carbon nanotubes with minuscule wires made of iron and iron carbide, incredibly thin

nanowires can be extruded by blasting the carbon nanotubes with an electron beam. If Gentech Technologies plans to spend \$1.7 million in year 1, \$2.1 million in year 2, and \$3.4 million in year 3 to develop the technology, determine the present worth of the investments in year 0, if the interest rate in year 1 is 10% and in years 2 and 3 it is 12% per year.

- 4.57 Find (a) the present worth P , and (b) the equivalent uniform annual worth A for the cash flows shown below.



ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 4.58 The term *annual percentage rate* is the same as:
- Effective rate
 - Nominal rate
 - Annual percentage yield
 - All of the above
- 4.59 An interest rate is an effective rate under all of the following conditions, *except when*:
- The compounding period is not stated
 - The interest period and compounding period are the same
 - The interest statement says that the interest rate is effective
 - The interest period is shorter than the compounding period
- 4.60 An interest rate of nominal 12% per year, compounded weekly, is:
- An effective rate per year
 - An effective rate per week
 - A nominal rate per year
 - A nominal rate per week
- 4.61 An interest rate of 1.5% per month, compounded continuously, is the same as:
- An effective 1.5% per month
 - 4.5% per quarter, compounded continuously
- (c) 6.0% per quarter, compounded continuously
(d) 9% per 6 months
- 4.62 An effective 12.68% per year, compounded monthly, is the closest to:
- 12% per year
 - 12% per year, compounded annually
 - 1% per month
 - 1% per month, compounded annually
- 4.63 For an interest rate of 2% per month, the effective semiannual interest rate is:
- 2.02%
 - 12.005%
 - 12.31%
 - 12.62%
- 4.64 If you make quarterly deposits for 3 years into an account that compounds interest at 1% per month, the value of n in the F/A factor that will determine F at the end of the 3-year period is:
- 3
 - 12
 - 36
 - None of these

- 4.65 Identify the following interest rates as nominal or effective.

- Rate 1: 1.5% per quarter
 Rate 2: 1.5% per quarter, compounded monthly
- Both are nominal rates.
 - Rate 1 is nominal and rate 2 is effective.
 - Rate 1 is effective and rate 2 is nominal.
 - Both are effective.

- 4.66 In solving uniform series problems, n in the standard factor notation equation is equal to the number of arrows (i.e., cash flows) in the original cash flow diagram when:

- The payment period (PP) is longer than the compounding period (CP).
- The compounding period is equal to the payment period.
- Both (a) and (b) are correct.
- The compounding period is longer than the payment period.

- 4.67 An engineer who is saving for her retirement plans to deposit \$500 every quarter, starting one quarter from now, into an investment account. If the account pays interest at 6% per year, compounded semiannually, the total she will have at the end of 25 years is closest to:

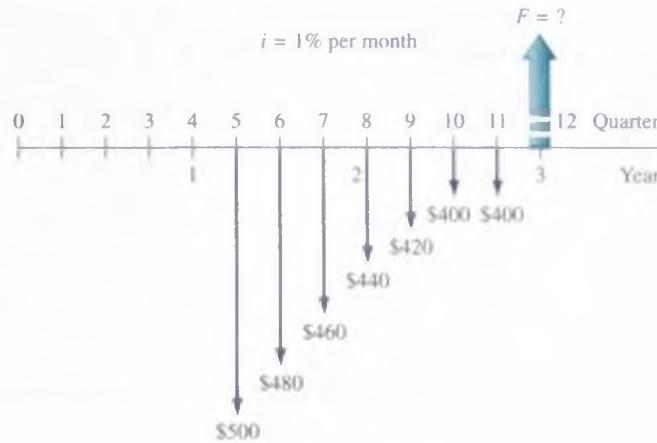
- \$50,000
- \$56,400
- \$79,700
- \$112,800

- 4.68 A company that makes flange-mount, motorized rotary potentiometers expects to spend \$50,000 for a certain machine 4 years from now. At an interest rate of 12% per year, compounded quarterly, the present worth of the machine's cost is represented by the following equation:

- $P = 50,000(P/F, 3\%, 16)$
- $P = 50,000(P/F, \text{effective } i/6 \text{ months}, 8)$
- $P = 50,000(P/F, \text{effective } i/\text{year}, 4)$
- Any of the above

- 4.69 For the cash flow diagram shown, the unit of the payment period (PP) is:

- Months
- Quarters
- Semiannual
- Years



- 4.70 A small company plans to spend \$10,000 in year 2 and \$10,000 in year 5. At an interest rate of *effective* 10% per year, compounded semiannually, the equation that represents the equivalent annual worth A in years 1 through 5 is:

- $A = 10,000(P/F, 10\%, 2)(A/P, 10\%, 5)$
+ $10,000(A/F, 10\%, 5)$
- $A = 10,000(A/P, 10\%, 4) + 10,000(A/F, 10\%, 5)$
- $A = 10,000(P/F, 5\%, 2)(A/P, 5\%, 10)$
+ $10,000(A/F, 5\%, 10)$
- $A = [10,000(F/P, 10\%, 5)$
+ $10,000](A/F, 10\%, 5)$

- 4.71 Assume you make *monthly* deposits of \$200 starting one month from now into an account that pays 6% per year, compounded semiannually. If you want to know the total after 4 years, the value of n you should use in the F/A factor is:

- 2
- 4
- 8
- 12

- 4.72 The cost of replacing part of a cell phone video-chip production line in 6 years is estimated to be \$500,000. At an interest rate of 14% per year, compounded semiannually, the uniform amount that must be deposited into a sinking fund every 6 months is closest to:

- \$21,335
- \$24,825
- \$27,950
- \$97,995

CASE STUDY

IS OWNING A HOME A NET GAIN OR NET LOSS OVER TIME?

Background

The Carroltons are deliberating whether to purchase a house or continue to rent for the next 10 years. They are assured by both of their employers that no transfers to new locations will occur for at least this number of years. Plus, the high school that their children attend is very good for their college prep education, and they all like the neighborhood where they live now.

They have a total of \$40,000 available now and estimate that they can afford up to \$2850 per month for the total house payment.

If the Carroltons do not buy a house, they will continue to rent the house they currently occupy for \$2700 per month. They will also place the \$40,000 into an investment instrument that is expected to earn at the rate of 6% per year. Additionally, they will add to this investment *at the end of each year* the same amount as the monthly 15-year mortgage payments. This alternative is called the rent-don't buy plan.

Information

Two financing plans using fixed-rate mortgages are currently available. The details are as follows.

Plan	Description
A	30-year fixed rate of 5.25% per year interest; 10% down payment
B	15-year fixed rate of 5.0% per year interest; 10% down payment

Other information:

- Price of the house is \$330,000.
- Taxes and insurance (T&I) are \$500 per month.
- Up-front fees (origination fee, survey fee, attorney's fee, etc.) are \$3000.

Any money not spent on the down payment or monthly payment will be invested and return at a rate of 6% per year (0.5% per month).

The Carroltons anticipate selling the house after 10 years and plan for a 10% increase in price, that is, \$363,000 (after all selling expenses are paid)

Case Study Exercises

1. The 30-year fixed-rate mortgage (plan A) is analyzed below. No taxes are considered on proceeds from the savings or investments.

Perform a similar analysis for the 15-year loan (plan B) and the rent-don't buy plan. The Carroltons decided to use the *largest future worth after 10 years* to select the best of the plans. Do the analysis for them and select the best plan.

Plan A analysis: 30-year fixed-rate loan

Amount of money required for closing costs:

Down payment (10% of \$330,000)	\$33,000
Up-front fees (origination fee, attorney's fee, survey, filing fee, etc.)	3,000
Total	\$36,000

The amount of the loan is \$297,000, and equivalent monthly principal and interest (P&I) is determined at $5.25\%/12 = 0.4375\%$ per month for $30(12) = 360$ months.

$$A = 297,000(A/P, 0.4375\%, 360) = 297,000(0.005522) \\ = \$1640$$

Add the T&I of \$500 for a total monthly payment of

$$\text{Payment}_A = \$2140 \text{ per month}$$

The future worth of plan A is the sum of three future worth components: remainder of the \$40,000 available for the closing costs (F_{1A}); left-over money from that available for monthly payments (F_{2A}); and increase in the house value when it is sold after 10 years (F_{3A}). These are calculated here.

$$F_{1A} = (40,000 - 36,000)(F/P, 0.5\%, 120) \\ = \$7278$$

Money available each month to invest after the mortgage payment, and the future worth after 10 years is

$$2850 - 2140 = \$710 \\ F_{2A} = 710(F/A, 0.5\%, 120) \\ = \$116,354$$

Net money from the sale in 10 years (F_{3A}) is the difference between the net selling price (\$363,000) and the remaining balance on the loan.

Loan balance

$$\begin{aligned} &= 297,000(F/P, 0.4375\%, 120) \\ &\quad - 1640(F/A, 0.4375\%, 120) \\ &= 297,000(1.6885) - 1640(157.3770) \\ &= \$243,386 \end{aligned}$$

$$F_{3A} = 363,000 - 243,386 = \$119,614$$

Total future worth of plan A is

$$\begin{aligned} F_A &= F_{1A} + F_{2A} + F_{3A} \\ &= 7278 + 116,354 + 119,614 \\ &= \$243,246 \end{aligned}$$

2. Perform this analysis if all estimates remain the same, except that when the house sells 10 years after purchase, the bottom has fallen out of the housing market and the net selling price is only 70% of the purchase price, that is, \$231,000.

LEARNING STAGE 2

Basic Analysis Tools

LEARNING STAGE 2 Basic Analysis Tools

CHAPTER 5

Present Worth
Analysis

CHAPTER 6

Annual Worth
Analysis

CHAPTER 7

Rate of Return
Analysis: One Project

CHAPTER 8

Rate of Return
Analysis: Multiple
Alternatives

CHAPTER 9

Benefit/Cost Analysis
and Public Sector
Economics

An engineering project or alternative is formulated to make or purchase a **product**, to develop a **process**, or to provide a **service** with specified results. An engineering economic analysis evaluates cash flow estimates for parameters such as initial cost, annual costs and revenues, nonrecurring costs, and possible salvage value over an estimated useful life of the product; process, or service. The chapters in this Learning Stage develop and demonstrate the basic tools and techniques to evaluate one or more alternatives using the factors, formulas, and spreadsheet functions learned in Stage 1.

After completing these chapters, you will be able to evaluate most engineering project proposals using a well-accepted economic analysis technique, such as present worth, future worth, capitalized cost, life-cycle costing, annual worth, rate of return, or benefit/cost analysis.

The epilogue to this stage provides an approach useful in selecting the engineering economic method that will provide the best analysis for the estimates and conditions present once the mutually exclusive alternatives are defined.

Important note: If depreciation and/or after-tax analysis is to be considered along with the evaluation methods in Chapters 5 through 9, Chapter 16 and/or Chapter 17 should be covered, preferably after Chapter 6.

CHAPTER 5

Present Worth Analysis



LEARNING OUTCOMES

Purpose: Utilize different present worth techniques to evaluate and select alternatives.

SECTION	TOPIC	LEARNING OUTCOME
5.1	Formulate alternatives	<ul style="list-style-type: none">Identify mutually exclusive and independent projects; define revenue and cost alternatives.
5.2	PW of equal-life alternatives	<ul style="list-style-type: none">Select the best of equal-life alternatives using present worth analysis.
5.3	PW of different-life alternatives	<ul style="list-style-type: none">Select the best of different-life alternatives using present worth analysis.
5.4	FW analysis	<ul style="list-style-type: none">Select the best alternative using future worth analysis.
5.5	CC analysis	<ul style="list-style-type: none">Select the best alternative using capitalized cost (CC) analysis.



future amount of money converted to its equivalent value now has a present worth (PW) that is always less than that of the future cash flow, because all P/F factors have a value less than 1.0 for any interest rate greater than zero. For this reason, present worth values are often referred to as *discounted cash flows (DCF)*, and the interest rate is referred to as the *discount rate*. Besides PW, two other terms frequently used are *present value (PV)* and *net present value (NPV)*. Up to this point, present worth computations have been made for one project or alternative. In this chapter, techniques for comparing two or more mutually exclusive alternatives by the present worth method are treated. Two additional applications are covered here—**future worth and capitalized cost**. Capitalized costs are used for projects with very long expected lives or long planning horizons.

To understand how to organize an economic analysis, this chapter begins with a description of independent and mutually exclusive projects as well as revenue and cost alternatives.

PE

Water for Semiconductor Manufacturing Case: The worldwide contribution of semiconductor sales is about \$250 billion per year, or about 10% of the world's GDP (gross domestic product). This industry produces the microchips used in many of the communication, entertainment, transportation, and computing devices we use every day. Depending upon the type and size of fabrication plant (fab), the need for ultrapure water (UPW) to manufacture these tiny integrated circuits is high, ranging from 500 to 2000 gpm (gallons per minute). Ultrapure water is obtained by special processes that commonly include reverse osmosis/deionizing resin bed technologies. Potable water obtained from purifying seawater or brackish groundwater may cost from \$2 to \$3 per 1000 gallons, but to obtain UPW on-site for semiconductor manufacturing may cost an additional \$1 to \$3 per 1000 gallons.

A fab costs upward of \$2.5 billion to construct, with approximately 1% of this total, or \$25 million, required to provide the ultrapure water needed, including the necessary wastewater and recycling equipment.

A newcomer to the industry, Angular Enterprises, has estimated the cost profiles for two options to supply its anticipated fab with water. It is fortunate to

have the option of desalinated seawater or purified groundwater sources in the location chosen for its new fab. The initial cost estimates for the UPW system are given below.

Source	Seawater (\$)	Groundwater (G)
Equipment first cost, \$M	-20	-22
AOC, \$M per year	-0.5	-0.3
Salvage value, % of first cost	5	10
Cost of UPW, \$ per 1000 gallons	4	5

Angular has made some initial estimates for the UPW system.

Life of UPW equipment	10 years
UPW needs	1500 gpm
Operating time	16 hours per day for 250 days per year

This case is used in the following topics (Sections) and problems of this chapter:

PW analysis of equal-life alternatives (Section 5.2)

PW analysis of different-life alternatives (Section 5.3)

Capitalized cost analysis (Section 5.5)

Problems 5.20 and 5.34

5.1 Formulating Alternatives ● ● ●

The evaluation and selection of economic proposals require **cash flow estimates** over a stated period of time, mathematical techniques to calculate the **measure of worth** (review Example 1.2 for possible measures), and a **guideline for selecting the best proposal**. From all the

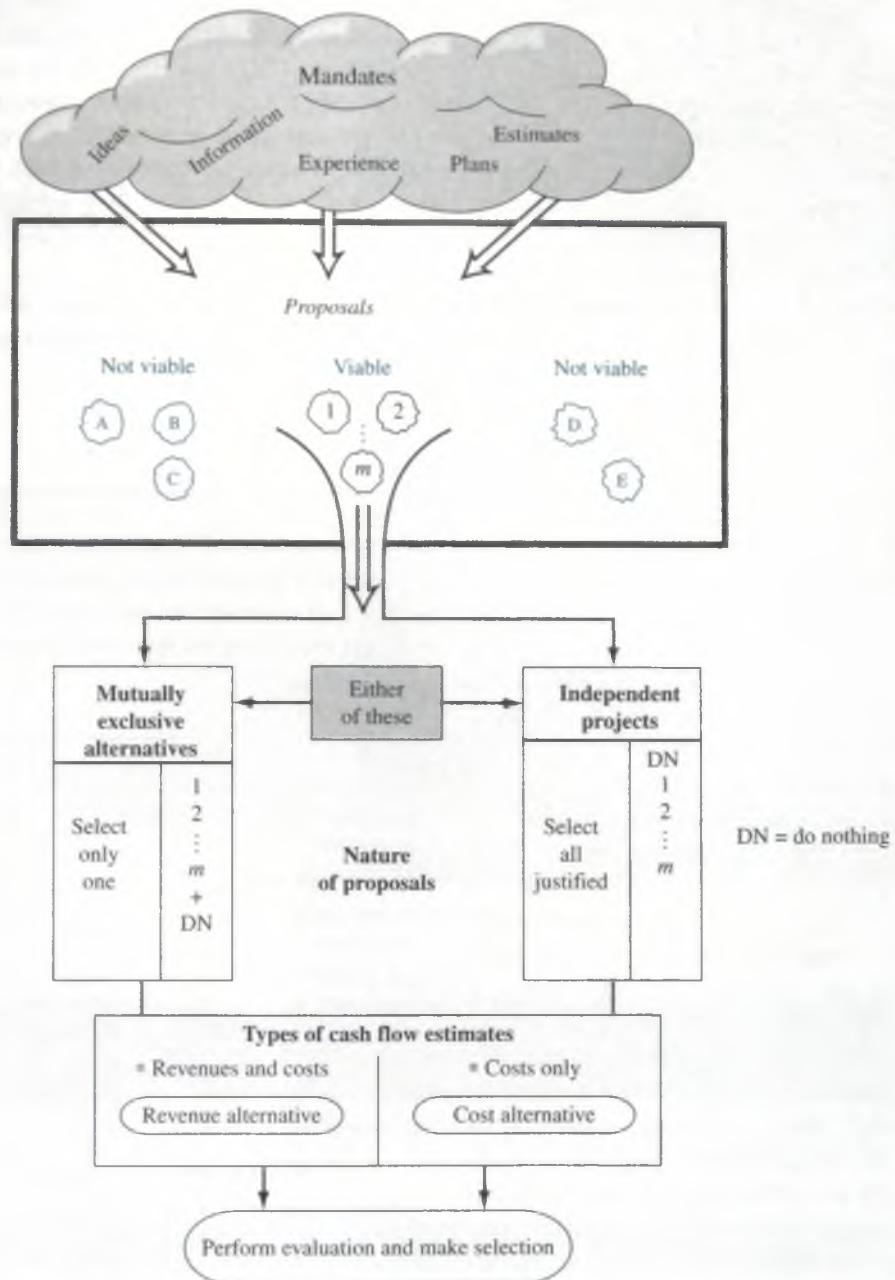


Figure 5–1
Progression from proposals to economic evaluation to selection.

proposals that may accomplish a stated purpose, the alternatives are formulated. This progression is detailed in Figure 5–1. Up front, some proposals are viable from technological, economic, and/or legal perspectives; others are not viable. Once the obviously nonviable ideas are eliminated, the remaining viable proposals are fleshed out to form the alternatives to be evaluated. Economic evaluation is one of the primary means used to select the best alternative(s) for implementation.

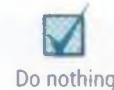
The nature of the economic proposals is always one of two types:

Mutually exclusive alternatives: Only one of the proposals can be selected. For terminology purposes, each viable proposal is called an *alternative*.

Independent projects: More than one proposal can be selected. Each viable proposal is called a *project*.

The **do-nothing (DN)** proposal is usually understood to be an option when the evaluation is performed.

The DN alternative or project means that the **current approach is maintained**; nothing new is initiated. No new costs, revenues, or savings are generated.



Do nothing

If it is absolutely required that one or more of the defined alternatives be selected, do nothing is not considered. This may occur when a mandated function must be installed for safety, legal, government, or other purposes.

Mutually exclusive alternatives and independent projects are selected in completely different ways. A mutually exclusive selection takes place, for example, when an engineer must select *the best* diesel-powered engine from several available models. Only one is chosen, and the rest are rejected. If none of the alternatives are economically justified, then all can be rejected and, by default, the DN alternative is selected. For independent projects one, two or more, in fact, all of the projects that are economically justified can be accepted, provided capital funds are available. This leads to the two following fundamentally different evaluation bases:

Mutually exclusive alternatives **compete with one another** and are compared pairwise.

Independent projects are evaluated one at a time and **compete only with the DN project**.

Any of the techniques in Chapters 5 through 9 can be used to evaluate either type of proposal—mutually exclusive or independent. When performed correctly as described in each chapter, any of the techniques will reach the same conclusion of which alternative or alternatives to select. This chapter covers the present worth method.

A parallel can be developed between independent and mutually exclusive evaluation. Assume there are m independent projects. Zero, one, two, or more may be selected. Since each project may be in or out of the selected group of projects, there are a total of 2^m **mutually exclusive alternatives**. This number includes the DN alternative, as shown in Figure 5–1. For example, if the engineer has three diesel engine models (A, B, and C) and may select any number of them, there are $2^3 = 8$ alternatives: DN, A, B, C, AB, AC, BC, ABC. Commonly, in real-world applications, there are restrictions, such as an upper budgetary limit, that eliminate many of the 2^m alternatives. Independent project analysis without budget limits is discussed in this chapter and through Chapter 9. Chapter 12 treats independent projects with a budget limitation; this is called capital budgeting.

Finally, it is important to recognize the nature of the cash flow estimates before starting the computation of a measure of worth that leads to the final selection. Cash flow estimates determine whether the alternatives are revenue- or cost-based. All the alternatives or projects must be of the same type when the economic study is performed. Definitions for these types follow:

Revenue: Each alternative generates cost (cash outflow) and revenue (cash inflow) estimates, and possibly savings, also considered cash inflows. Revenues can vary for each alternative.

Cost: Each alternative has only cost cash flow estimates. Revenues or savings are assumed equal for all alternatives; thus they are not dependent upon the alternative selected. These are also referred to as **service alternatives**.



Revenue or cost alternative

Although the exact procedures vary slightly for revenue and cost cash flows, all techniques and guidelines covered through Chapter 9 apply to both. Differences in evaluation methodology are detailed in each chapter.

5.2 Present Worth Analysis of Equal-Life Alternatives ● ● ●

The PW comparison of alternatives with equal lives is straightforward. The present worth P is renamed PW of the alternative. The present worth method is quite popular in industry because all future costs and revenues are transformed to **equivalent monetary units NOW**; that is, all future cash flows are converted (discounted) to present amounts (e.g., dollars) at a specific rate of return, which is the MARR. This makes it very simple to determine which alternative has the best economic advantage. The required conditions and evaluation procedure are as follows:

If the alternatives have the same capacities for the same time period (life), the **equal-service requirement** is met. Calculate the PW value at the stated MARR for each alternative.



Equal-service requirement

For **mutually exclusive (ME)** alternatives, whether they are revenue or cost alternatives, the following guidelines are applied to justify a single project or to select one from several alternatives.



Project evaluation

ME alternative selection

One alternative: If $PW \geq 0$, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the PW that is **numerically largest**, that is, less negative or more positive. This indicates a lower PW of cost for cost alternatives or a larger PW of net cash flows for revenue alternatives.

Note that the guideline to select one alternative with the lowest cost or highest revenue uses the criterion of **numerically largest**. This is not the absolute value of the PW amount, because the sign matters. The selections below correctly apply the guideline for two alternatives A and B.

PW_A	PW_B	Selected Alternative
\$-2300	\$-1500	B
-500	+1000	B
+2500	+2000	A
+4800	-400	A



Independent project selection

For **independent** projects, each PW is considered separately, that is, compared with the DN project, which always has $PW = 0$. The selection guideline is as follows:

One or more independent projects: Select all projects with $PW \geq 0$ at the MARR.

The independent projects must have positive and negative cash flows to obtain a PW value that can exceed zero; that is, they must be revenue projects.

All PW analyses require a MARR for use as the i value in the PW relations. The bases used to establish a realistic MARR were summarized in Chapter 1 and are discussed in detail in Chapter 10.

EXAMPLE 5.1

A university lab is a research contractor to NASA for in-space fuel cell systems that are hydrogen- and methanol-based. During lab research, three equal-service machines need to be evaluated economically. Perform the present worth analysis with the costs shown below. The MARR is 10% per year.

	Electric-Powered	Gas-Powered	Solar-Powered
First cost, \$	-4500	3500	-6000
Annual operating cost (AOC), \$/year	-900	-700	-50
Salvage value S , \$	200	350	100
Life, years	8	8	8

Solution

These are cost alternatives. The salvage values are considered a “negative” cost, so a + sign precedes them. (If it costs money to dispose of an asset, the estimated disposal cost has a - sign.) The PW of each machine is calculated at $i = 10\%$ for $n = 8$ years. Use subscripts E, G, and S.

$$PW_E = -4500 - 900(P/A, 10\%, 8) + 200(P/F, 10\%, 8) = \$-9208$$

$$PW_G = -3500 - 700(P/A, 10\%, 8) + 350(P/F, 10\%, 8) = \$-7071$$

$$PW_S = -6000 - 50(P/A, 10\%, 8) + 100(P/F, 10\%, 8) = \$-6220$$

The solar-powered machine is selected since the PW of its costs is the lowest; it has the numerically largest PW value.

EXAMPLE 5.2 Water for Semiconductor Manufacturing Case

PE

As discussed in the introduction to this chapter, ultrapure water (UPW) is an expensive commodity for the semiconductor industry. With the options of seawater or groundwater sources, it is a good idea to determine if one system is more economical than the other. Use a MARR of 12% per year and the present worth method to select one of the systems.

Solution

An important first calculation is the cost of UPW per year. The general relation and estimated costs for the two options are as follows:

$$\text{UPW cost relation: } \frac{\$}{\text{year}} = \left(\frac{\text{cost in \$}}{1000 \text{ gallons}} \right) \left(\frac{\text{gallons}}{\text{minute}} \right) \left(\frac{\text{minutes}}{\text{hour}} \right) \left(\frac{\text{hours}}{\text{day}} \right) \left(\frac{\text{days}}{\text{year}} \right)$$

$$\text{Seawater: } (4/1000)(1500)(60)(16)(250) = \$1.44 \text{ M per year}$$

$$\text{Groundwater: } (5/1000)(1500)(60)(16)(250) = \$1.80 \text{ M per year}$$

Calculate the PW at $i = 12\%$ per year and select the option with the lower cost (larger PW value). In \$1 million units:

PW relation: $\text{PW} = \text{first cost} - \text{PW of AOC} - \text{PW of UPW} + \text{PW of salvage value}$

$$\begin{aligned} \text{PW}_S &= -20 - 0.5(P/A, 12\%, 10) - 1.44(P/A, 12\%, 10) + 0.05(20)(P/F, 12\%, 10) \\ &= -20 - 0.5(5.6502) - 1.44(5.6502) + 1(0.3220) \\ &= \$-30.64 \end{aligned}$$

$$\begin{aligned} \text{PW}_G &= -22 - 0.3(P/A, 12\%, 10) - 1.80(P/A, 12\%, 10) + 0.10(22)(P/F, 12\%, 10) \\ &= -22 - 0.3(5.6502) - 1.80(5.6502) + 2.2(0.3220) \\ &= \$-33.16 \end{aligned}$$

Based on this present worth analysis, the seawater option is cheaper by \$2.52 M.

5.3 Present Worth Analysis of Different-Life Alternatives ● ● ●

When the present worth method is used to compare mutually exclusive alternatives that have different lives, the equal-service requirement must be met. The procedure of Section 5.2 is followed, with one exception:

The PW of the alternatives must be compared over the **same number of years** and must end at the same time to satisfy the equal-service requirement.



Equal-service requirement

This is necessary, since the present worth comparison involves calculating the equivalent PW of all future cash flows for each alternative. A fair comparison requires that PW values represent cash flows associated with equal service. For cost alternatives, failure to compare equal service will always favor the shorter-lived mutually exclusive alternative, even if it is not the more economical choice, because fewer periods of costs are involved. The equal-service requirement is satisfied by using either of two approaches:

LCM: Compare the PW of alternatives over a period of time equal to the **least common multiple (LCM)** of their estimated lives.

Study period: Compare the PW of alternatives using a **specified study period of n years**. This approach does not necessarily consider the useful life of an alternative. The study period is also called the *planning horizon*.



LCM or study period

For either approach, calculate the PW at the MARR and use the same selection guideline as that for equal-life alternatives. The LCM approach makes the cash flow estimates extend to the same period, as required. For example, lives of 3 and 4 years are compared over a 12-year period.

The first cost of an alternative is reinvested at the beginning of each life cycle, and the estimated salvage value is accounted for at the end of each life cycle when calculating the PW values over the LCM period. Additionally, the LCM approach requires that some assumptions be made about subsequent life cycles.

The assumptions when using the LCM approach are that

1. The service provided will be needed over the entire LCM years or more.
2. The selected alternative can be repeated over each life cycle of the LCM in exactly the same manner.
3. Cash flow estimates are the same for each life cycle.

As will be shown in Chapter 14, the third assumption is valid only when the cash flows are expected to change by exactly the inflation (or deflation) rate that is applicable through the LCM time period. If the cash flows are expected to change by any other rate, then the PW analysis must be conducted using constant-value dollars, which considers inflation (Chapter 14).

A study period analysis is necessary if the first assumption about the length of time the alternatives are needed cannot be made. For the study period approach, a time horizon is chosen over which the economic analysis is conducted, and only those cash flows which occur during that time period are considered relevant to the analysis. All cash flows occurring beyond the study period are ignored. An estimated market value at the end of the study period must be made. The time horizon chosen might be relatively short, especially when short-term business goals are very important. The study period approach is often used in replacement analysis (Chapter 11). It is also useful when the LCM of alternatives yields an unrealistic evaluation period, for example, 5 and 9 years.

EXAMPLE 5.3

National Homebuilders, Inc., plans to purchase new cut-and-finish equipment. Two manufacturers offered the estimates below.

	Vendor A	Vendor B
First cost, \$	−15,000	−18,000
Annual M&O cost, \$ per year	−3,500	−3,100
Salvage value, \$	1,000	2,000
Life, years	6	9

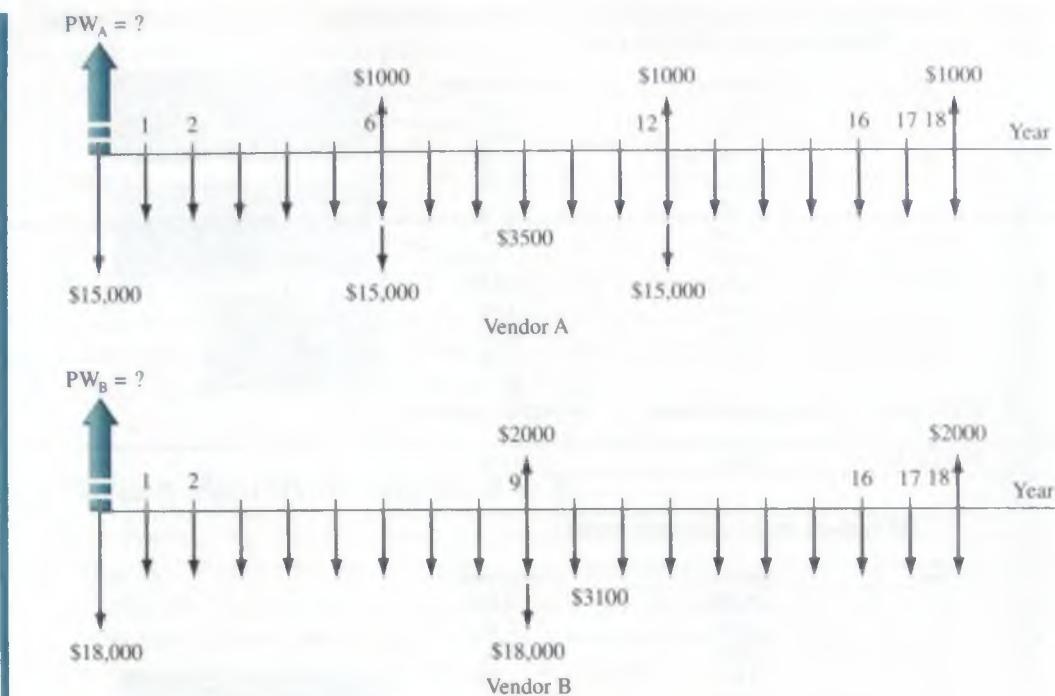
- Determine which vendor should be selected on the basis of a present worth comparison, if the MARR is 15% per year.
- National Homebuilders has a standard practice of evaluating all options over a 5-year period. If a study period of 5 years is used and the salvage values are not expected to change, which vendor should be selected?

Solution

- Since the equipment has different lives, compare them over the LCM of 18 years. For life cycles after the first, the first cost is repeated in year 0 of each new cycle, which is the last year of the previous cycle. These are years 6 and 12 for vendor A and year 9 for B. The cash flow diagram is shown in Figure 5–2. Calculate PW at 15% over 18 years.

$$\begin{aligned}
 PW_A &= -15,000 - 15,000(P/F, 15\%, 6) + 1000(P/F, 15\%, 6) \\
 &\quad - 15,000(P/F, 15\%, 12) + 1000(P/F, 15\%, 12) + 1000(P/F, 15\%, 18) \\
 &\quad - 3,500(P/A, 15\%, 18) \\
 &= \$-45,036
 \end{aligned}$$

$$\begin{aligned}
 PW_B &= -18,000 - 18,000(P/F, 15\%, 9) + 2000(P/F, 15\%, 9) \\
 &\quad + 2000(P/F, 15\%, 18) - 3100(P/A, 15\%, 18) \\
 &= \$-41,384
 \end{aligned}$$

**Figure 5-2**

Cash flow diagram for different-life alternatives, Example 5.3a.

Vendor B is selected, since it costs less in PW terms; that is, the PW_B value is numerically larger than PW_A .

(b) For a 5-year study period, no cycle repeats are necessary. The PW analysis is

$$\begin{aligned} PW_A &= -15,000 - 3500(P/A, 15\%, 5) + 1000(P/F, 15\%, 5) \\ &= \$-26,236 \end{aligned}$$

$$\begin{aligned} PW_B &= -18,000 - 3100(P/A, 15\%, 5) + 2000(P/F, 15\%, 5) \\ &= \$-27,397 \end{aligned}$$

Vendor A is now selected based on its smaller PW value. This means that the shortened study period of 5 years has caused a switch in the economic decision. In situations such as this, the standard practice of using a fixed study period should be carefully examined to ensure that the appropriate approach, that is, LCM or fixed study period, is used to satisfy the equal-service requirement.

EXAMPLE 5.4 Water for Semiconductor Manufacturing Case

PE

When we discussed this case in the introduction, we learned that the initial estimates of equipment life were 10 years for both options of UPW (ultrapure water)—seawater and groundwater. As you might guess, a little research indicates that seawater is more corrosive and the equipment life is shorter—5 years rather than 10. However, it is expected that, instead of complete replacement, a total refurbishment of the equipment for \$10 M after 5 years will extend the life through the anticipated 10th year of service.

With all other estimates remaining the same, it is important to determine if this 50% reduction in expected usable life and the refurbishment expense may alter the decision to go with the seawater option, as determined in Example 5.2. For a complete analysis, consider both a 10-year and a 5-year option for the expected use of the equipment, regardless of the source of UPW.

PW analysis over LCM of 10 years					
Year	Seawater	Groundwater			
0	-20.00	-22.00			
1	-1.94	-2.10			
2	-1.94	-2.10			
3	-1.94	-2.10			
4	-1.94	-2.10			
5	-11.94	-2.10			
6	-1.94	-2.10			
7	-1.94	-2.10			
8	-1.94	-2.10			
9	-1.94	-2.10			
10	-0.94	0.10			
11					
12					
13					
14					
15					
16	NPV function	=NPV(12%,B5:B14)+B4	=NPV(12%,C5:C14)+C4		
17					
18	NPV (\$1 M)	(36.31)	(33.16)		
19					
20					
21	PW analysis over 5-year study period				
22					
23	Year	Seawater	Groundwater		
24	0	-20.00	-22.00		
25	1	-1.94	-2.10		
26	2	-1.94	-2.10		
27	3	-1.94	-2.10		
28	4	-1.94	-2.10		
29	5	-0.94	0.10		
30					
31	NPV function	=NPV(12%,B25:B29)+B24	=NPV(12%,C25:C29)+C24		
32					
33	NPV (\$1 M)	(26.43)	(28.32)		

Seawater option
= AOC + UPW cost/year
= -0.5 - 1.44

Groundwater option
= AOC + UPW cost/year
= -0.3 - 1.80

Both options
Salvage values included here

Salvage values are the same after 5 and 10 years

Figure 5–3

PW analyses using LCM and study period approaches for water for semiconductor manufacturing case, Example 5.4.

Solution

A spreadsheet and the NPV function are a quick and easy way to perform this dual analysis. The details are presented in Figure 5–3.

LCM of 10 years: In the top part of the spreadsheet, the LCM of 10 years is necessary to satisfy the equal-service requirement; however, the first cost in year 5 is the refurbishment cost of \$–10 M, not the \$–20 M expended in year 0. Each year's cash flow is entered in consecutive cells; the \$–11.94 M in year 5 accounts for the continuing AOC and annual UPW cost of \$–1.94 M, plus the \$–10 M refurbishment cost. The NPV functions shown on the spreadsheet determine the 12% per year PW values in \$1 million units.

$$PW_S = \$ - 36.31 \quad PW_G = \$ - 33.16$$

Now, the *groundwater option is cheaper*; the economic decision is reversed with this new estimate of life and year 5 refurbishment expense.

Study period of 5 years: The lower portion of Figure 5–3 details a PW analysis using the second approach to evaluating different-life alternatives, that is, a specific study period, which is 5 years in this case study. Therefore, all cash flows after 5 years are neglected.

Again the economic decision is reversed as the 12% per year PW values *favor the seawater option*.

$$PW_S = \$ - 26.43 \quad PW_G = \$ - 28.32$$

Comments

The decision switched between the LCM and study period approaches. Both are correct answers given the decision of how the equal-service requirement is met. This analysis demonstrates how important it is to compare mutually exclusive alternatives over time periods that are believable and to take the time necessary to make the most accurate cost, life, and MARR estimates when the evaluation is performed.

If the PW evaluation is incorrectly performed using the respective lives of the two options, the equal-service requirement is violated, and PW values favor the shorter-lived option, that is, seawater. The PW values are

- Option S: $n = 5$ years, $PW_S = \$-26.43$ M, from the bottom left calculation in Figure 5–3.
 Option G: $n = 10$ years, $PW_G = \$-33.16$ M, from the top right calculation in Figure 5–3.

For independent projects, use of the LCM approach is unnecessary since each project is compared to the do-nothing alternative, not to each other, and satisfying the equal-service requirement is not a problem. Simply use the MARR to determine the PW over the respective life of each project, and select all projects with a $PW \geq 0$.

5.4 Future Worth Analysis ● ● ●

The future worth (FW) of an alternative may be determined directly from the cash flows, or by multiplying the PW value by the F/P factor, at the established MARR. The n value in the F/P factor is either the LCM value or a specified study period. Analysis of alternatives using FW values is especially applicable to large capital investment decisions when a prime goal is to maximize the *future wealth* of a corporation's stockholders.

Future worth analysis over a specified study period is often utilized if the asset (equipment, a building, etc.) might be sold or traded at some time before the expected life is reached. Suppose an entrepreneur is planning to buy a company and expects to trade it within 3 years. FW analysis is the best method to help with the decision to sell or keep it 3 years hence. Example 5.5 illustrates this use of FW analysis. Another excellent application of FW analysis is for projects that will come online at the end of a multiyear investment period, such as electric generation facilities, toll roads, airports, and the like. They are analyzed using the FW value of investment commitments made during construction.

The selection guidelines for FW analysis are the same as for PW analysis; $FW \geq 0$ means the MARR is met or exceeded. For two or more mutually exclusive alternatives, select the one with the numerically largest FW value.



ME alternative selection

EXAMPLE 5.5

A British food distribution conglomerate purchased a Canadian food store chain for £75 million 3 years ago. There was a net loss of £10 million at the end of year 1 of ownership. Net cash flow is increasing with an arithmetic gradient of £+5 million per year starting the second year, and this pattern is expected to continue for the foreseeable future. This means that breakeven net cash flow was achieved this year. Because of the heavy debt financing used to purchase the Canadian chain, the international board of directors expects a MARR of 25% per year from any sale.

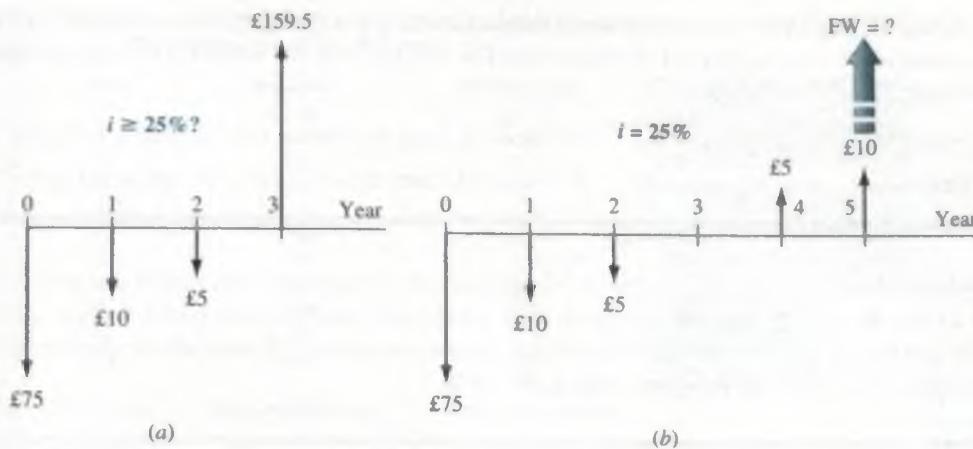
- The British conglomerate has just been offered £159.5 million by a French company wishing to get a foothold in Canada. Use FW analysis to determine if the MARR will be realized at this selling price.
- If the British conglomerate continues to own the chain, what selling price must be obtained at the end of 5 years of ownership to just make the MARR?

Solution

- Set up the future worth relation in year 3 (FW_3) at $i = 25\%$ per year and an offer price of £159.5 million. Figure 5–4a presents the cash flow diagram in million £ units.

$$\begin{aligned} FW_3 &= -75(F/P, 25\%, 3) - 10(F/P, 25\%, 2) - 5(F/P, 25\%, 1) + 159.5 \\ &= -168.36 + 159.5 = \text{£} -8.86 \text{ million} \end{aligned}$$

No, the MARR of 25% will not be realized if the £159.5 million offer is accepted.

**Figure 5-4**Cash flow diagrams for Example 5.5. (a) $i = ?$; (b) $FW = ?$.

- (b) Determine the future worth 5 years from now at 25% per year. Figure 5-4b presents the cash flow diagram. The A/G and F/A factors are applied to the arithmetic gradient.

$$\begin{aligned} FW_5 &= -75(F/P, 25\%, 5) - 10(F/A, 25\%, 5) + 5(A/G, 25\%, 5)(F/A, 25\%, 5) \\ &= \text{£}-246.81 \text{ million} \end{aligned}$$

The offer must be for at least £246.81 million to make the MARR. This is approximately 3.3 times the purchase price only 5 years earlier, in large part based on the required MARR of 25%.

5.5 Capitalized Cost Analysis ● ● ●

Many public sector projects such as bridges, dams, highways and toll roads, railroads, and hydroelectric and other power generation facilities have very long expected useful lives. A **perpetual or infinite life** is the effective planning horizon. Permanent endowments for charitable organizations and universities also have perpetual lives. The economic worth of these types of projects or endowments is evaluated using the present worth of the cash flows.

Capitalized Cost (CC) is the present worth of a project that has a very long life (more than, say, 35 or 40 years) or when the planning horizon is considered very long or infinite.

The formula to calculate CC is derived from the PW relation $P = A(P/A, i\%, n)$, where $n = \infty$ time periods. Take the equation for P using the P/A factor and divide the numerator and denominator by $(1 + i)^n$ to obtain

$$P = A \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

As n approaches ∞ , the bracketed term becomes $1/i$. We replace the symbols P and PW with CC as a reminder that this is a capitalized cost equivalence. Since the A value can also be termed AW for annual worth, the capitalized cost formula is simply

$$CC = \frac{A}{i} \quad \text{or} \quad CC = \frac{AW}{i} \quad [5.1]$$

Solving for A or AW , the amount of new money that is generated each year by a capitalization of an amount CC is

$$AW = CC(i) \quad [5.2]$$

This is the same as the calculation $A = P(i)$ for an infinite number of time periods. Equation [5.2] can be explained by considering the time value of money. If \$20,000 is invested now (this is the

capitalization) at 10% per year, the maximum amount of money that can be withdrawn at the end of every year for *eternity* is \$2000, which is the interest accumulated each year. This leaves the original \$20,000 to earn interest so that another \$2000 will be accumulated the next year.

The cash flows (costs, revenues, and savings) in a capitalized cost calculation are usually of two types: *recurring*, also called periodic, and *nonrecurring*. An annual operating cost of \$50,000 and a rework cost estimated at \$40,000 every 12 years are examples of recurring cash flows. Examples of nonrecurring cash flows are the initial investment amount in year 0 and one-time cash flow estimates at future times, for example, \$500,000 in fees 2 years hence.

The procedure to determine the CC for an infinite sequence of cash flows is as follows:

1. Draw a cash flow diagram showing all nonrecurring (one-time) cash flows and at least two cycles of all recurring (periodic) cash flows.
2. Find the present worth of all nonrecurring amounts. This is their CC value.
3. Find the *A* value through *one life cycle* of all recurring amounts. (This is the same value in all succeeding life cycles, as explained in Chapter 6.) Add this to all other uniform amounts (*A*) occurring in years 1 through infinity. The result is the total equivalent uniform annual worth (AW).
4. Divide the AW obtained in step 3 by the interest rate *i* to obtain a CC value. This is an application of Equation [5.1].
5. Add the CC values obtained in steps 2 and 4.

Drawing the cash flow diagram (step 1) is more important in CC calculations than elsewhere, because it helps separate nonrecurring and recurring amounts. In step 5 the present worths of all component cash flows have been obtained; the total capitalized cost is simply their sum.

EXAMPLE 5.6

The Haverty County Transportation Authority (HCTA) has just installed new software to charge and track toll fees. The director wants to know the total equivalent cost of all future costs incurred to purchase the software system. If the new system will be used for the indefinite future, find the equivalent cost (a) now, a CC value, and (b) for each year hereafter, an AW value.

The system has an installed cost of \$150,000 and an additional cost of \$50,000 after 10 years. The annual software maintenance contract cost is \$5000 for the first 4 years and \$8000 thereafter. In addition, there is expected to be a recurring major upgrade cost of \$15,000 every 13 years. Assume that *i* = 5% per year for county funds.

Solution

(a) The five-step procedure to find CC now is applied.

1. Draw a cash flow diagram for two cycles (Figure 5–5).
2. Find the present worth of the nonrecurring costs of \$150,000 now and \$50,000 in year 10 at *i* = 5%. Label this CC_1 .

$$CC_1 = -150,000 - 50,000(P/F, 5\%, 10) = -\$180,695$$

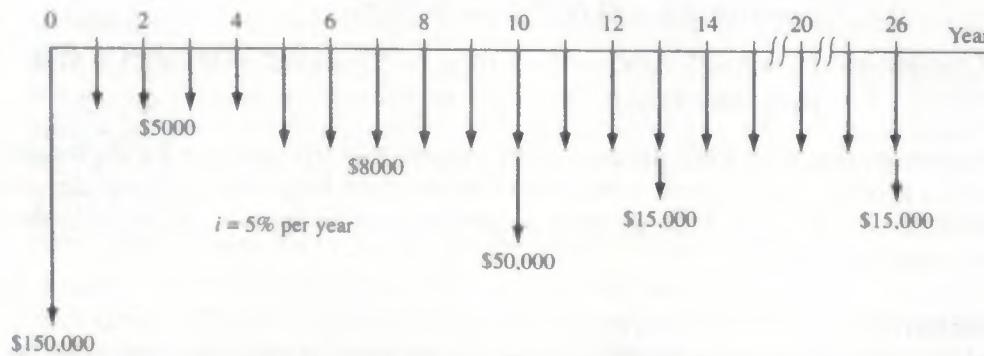


Figure 5–5

Cash flows for two cycles of recurring costs and all nonrecurring amounts, Example 5.6.

- 3 and 4.** Convert the \$15,000 recurring cost to an A value over the first cycle of 13 years, and find the capitalized cost CC_2 at 5% per year using Equation [5.1].

$$A = -15,000(A/F, 5\%, 13) = \$-847$$

$$CC_2 = -847/0.05 = \$-16,940$$

There are several ways to convert the annual software maintenance cost series to A and CC values. A straightforward method is to, first, consider the \$-5000 an A series with a capitalized cost of

$$CC_3 = -5000/0.05 = \$-100,000$$

Second, convert the step-up maintenance cost series of \$-3000 to a capitalized cost CC_4 in year 4, and find the present worth in year 0. (Refer to Figure 5-5 for cash flow timings.)

$$CC_4 = \frac{-3,000}{0.05} (P/F, 5\%, 4) = \$-49,362$$

- 5.** The total capitalized cost CC_T for Haverty County Transportation Authority is the sum of the four component CC values.

$$\begin{aligned} CC_T &= -180,695 - 16,940 - 100,000 - 49,362 \\ &= \$-346,997 \end{aligned}$$

(b) Equation [5.2] determines the AW value forever.

$$AW = Pi = CC_T(i) = \$346,997(0.05) = \$17,350$$

Correctly interpreted, this means Haverty County officials have committed the equivalent of \$17,350 forever to operate and maintain the toll management software.

For the comparison of two alternatives on the basis of capitalized cost, use the procedure above to find the A value and CC_T for each alternative. Since the capitalized cost represents the total present worth of financing and maintaining a given alternative forever, the alternatives will automatically be compared for the same number of years (i.e., infinity). The alternative with the smaller capitalized cost will represent the more economical one. This evaluation is illustrated in Example 5.7 using the progressive example for this chapter.

EXAMPLE 5.7 Water for Semiconductor Manufacturing Case

PE

Our case study has progressed (in Example 5.4) to the point that the life of the seawater option can be extended to 10 years with a major refurbishment cost after 5 years. This extension is possible only one time, after which a new life cycle would commence. In \$1 million units, the estimates and PW values (from Figure 5-3) are as follows:

Seawater: $P_S = \$-20$; $AOC_S = \$-1.94$; $n_S = 10$ years; refurbishment, year 5 = \$-10;
 $S_S = 0.05(20) = \$1.00$; $PW_S = \$-36.31$

Groundwater: $P_G = \$-22$; $AOC_G = \$-2.10$; $n_G = 10$ years; $S_G = 0.10(22) = \$2.2$;
 $PW_G = \$-33.16$

If we assume that the UPW (ultrapure water) requirement will continue for the foreseeable future, a good number to know is the present worth of the long-term options at the selected MARR of 12% per year. What are these capitalized costs for the two options using the estimates made thus far?

Solution

Find the equivalent A value for each option over its respective life, then determine the CC value using the relation $CC = A/i$. Select the option with the lower CC . This approach satisfies the equal-service requirement because the time horizon is infinity when the CC is determined.

$$\text{Seawater: } A_S = PW_S(A/P, 12\%, 10) = -36.31(0.17698) = \$-6.43$$

$$CC_S = -6.43/0.12 = \$-53.58$$

$$\text{Groundwater: } A_G = PW_G(A/P, 12\%, 10) = -33.16(0.17698) = \$-5.87$$

$$CC_G = -5.87/0.12 = \$-48.91$$

In terms of capitalized cost, the *groundwater alternative is cheaper*.

Comment

If the seawater-life extension is not considered a viable option, the original alternative of 5 years could be used in this analysis. In this case, the equivalent A value and CC computations in \$1 million units are as follows.

$$A_{S,5 \text{ years}} = -20(A/P, 12\%, 5) - 1.94 + 0.05(20)(A/F, 12\%, 5)$$

$$= \$-7.33$$

$$CC_{S,5 \text{ years}} = -7.33/0.12 = \$-61.08$$

Now, the economic advantage of the groundwater option is even larger.

If a finite-life alternative (for example, 5 years) is compared to one with an indefinite or very long life, capitalized costs can be used. To determine capitalized cost for the finite life alternative, calculate the equivalent A value for one life cycle and divide by the interest rate (Equation [5.1]). This procedure is illustrated in Example 5.8 using a spreadsheet.

EXAMPLE 5.8

The State Legislature has mandated a statewide recycling program to include all types of plastic, paper, metal, and glass refuse. The goal is zero landfill by 2020. Two options for the materials separation equipment are outlined below. The interest rate for state-mandated projects is 5% per year.

Contractor option (C): \$8 million now and \$25,000 per year will provide separation services at a maximum of 15 sites. No contract period is stated; thus the contract and services are offered for as long as the State needs them.

Purchase option (P): Purchase equipment at each site for \$275,000 per site and expend an estimated \$12,000 in annual operating costs (AOC). Expected life of the equipment is 5 years with no salvage value.

- (a) Perform a capitalized cost analysis for a total of 10 recycling sites.
- (b) Determine the maximum number of sites at which the equipment can be purchased and still have a capitalized cost less than that of the contractor option.

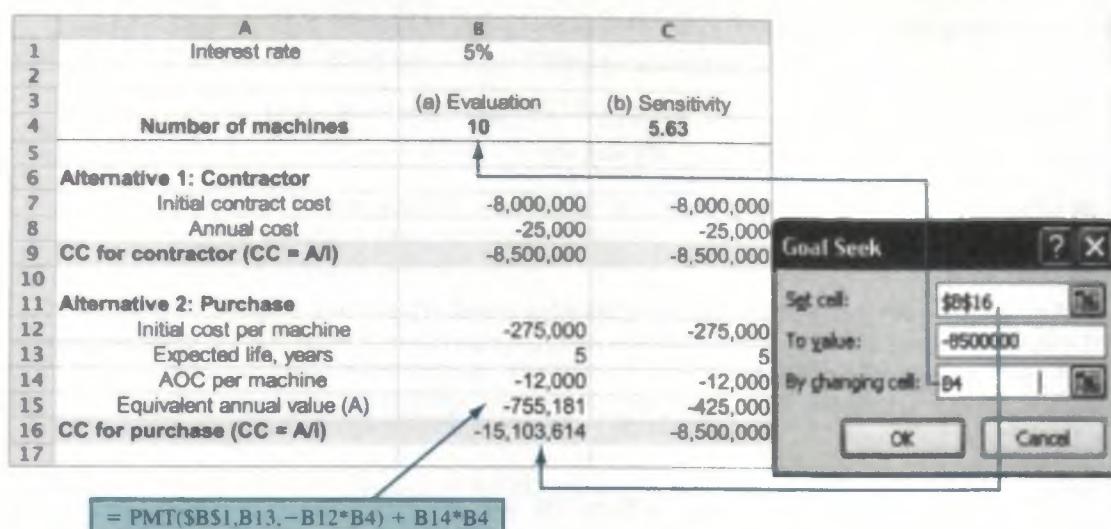
Solution

(a) Figure 5–6, column B, details the solution. The contract, as proposed, has a long life. Therefore, the \$8 million is already a capitalized cost. The annual charge of $A = \$25,000$ is divided by $i = 0.05$ to determine its CC value. Summing the two values results in $CC_C = \$-8.5$ million.

For the finite, 5-year purchase alternative, column B shows the first cost (\$–275,000 per site), AOC (\$–12,000), and equivalent A value of \$–755,181, which is determined via the PMT function (cell tag). Divide A by the interest rate of 5% to determine $CC_P = \$-15.1$ million.

The *contractor option is by far more economical* for the anticipated 10 sites.

- (b) A quick way to find the maximum number of sites for which $CC_P < CC_C$ is to use Excel's **Goal Seek** tool, introduced in Chapter 2, Example 2.10. (See Appendix A for details on how to use this tool.) The template is set up in Figure 5–6 to make the two CC values equal as the number of sites is altered (decreased). The result, shown in column C, indicates that 5.63 sites make the options economically equivalent. Since the number of sites

**Figure 5–6**

Spreadsheet solution of Example 5.8 using capitalized cost (a) for 10 recycling sites and (b) to determine the number of sites to make the alternatives economically equal.

must be an integer, 5 or fewer sites will favor purchasing the equipment and 6 or more sites will favor contracting the separation services.

This approach to problem solution will be called *break-even analysis* in later chapters of the text. By the way, another way to determine the number of sites is by trial and error. Enter different values in cell B4 until the CC values favor the purchase alternative.

CHAPTER SUMMARY

The present worth method of comparing alternatives involves converting all cash flows to present dollars at the MARR. The alternative with the numerically larger (or largest) PW value is selected. When the alternatives have different lives, the comparison must be made for equal-service periods. This is done by performing the comparison over either the LCM of lives or a specific study period. Both approaches compare alternatives in accordance with the equal-service requirement. When a study period is used, any remaining value in an alternative is recognized through the estimated future market value.

If the life of the alternatives is considered to be very long or infinite, capitalized cost is the comparison method. The CC value is calculated as A/i , because the P/A factor reduces to $1/i$ in the limit of $n = \infty$.

PROBLEMS

Types of Projects

- 5.1 What is the difference between mutually exclusive alternatives and independent projects?
- 5.2 (a) What is meant by the do-nothing alternative?
(b) When is the do-nothing alternative *not* an option?

- 5.3 (a) How many alternatives are possible from four independent projects identified as W, X, Y, and Z?
(b) List all of the possibilities.

- 5.4 What is the difference between a revenue and a cost alternative?

- 5.5 What is meant by the term *equal service*?
- 5.6 What two approaches can be used to satisfy the equal-service requirement?

Alternative Comparison—Equal Lives

- 5.7 A company that manufactures magnetic membrane switches is investigating two production options that have the estimated cash flows shown (\$1 million units). Which one should be selected on the basis of a present worth analysis at 10% per year?

	In-house	Contract
First cost, \$	-30	0
Annual cost, \$ per year	-5	-2
Annual income, \$ per year	14	3.1
Salvage value, \$	2	—
Life, years	5	5

- 5.8 The manager of a canned food processing plant must decide between two different labeling machines. Machine A will have a first cost of \$42,000, an annual operating cost of \$28,000, and a service life of 4 years. Machine B will cost \$51,000 to buy and will have an annual operating cost of \$17,000 during its 4-year life. At an interest rate of 10% per year, which should be selected on the basis of a present worth analysis?
- 5.9 A metallurgical engineer is considering two materials for use in a space vehicle. All estimates are made. (a) Which should be selected on the basis of a present worth comparison at an interest rate of 12% per year? (b) At what first cost for the material *not selected above* will it become the more economic alternative?

	Material X	Material Y
First cost, \$	-15,000	-35,000
Maintenance cost, \$ per year	-9,000	-7,000
Salvage value, \$	2,000	20,000
Life, years	5	5

- 5.10 To retain high-performing engineers, a large semiconductor company provides corporate stock as part of the compensation package. In one particular year, the company offered 1000 shares of either class A or class B stock. The class A stock was selling for \$30 per share at the time, and stock market analysts predicted that it would increase at a rate of 6% per year for the next 5 years. Class B stock was selling for \$20 per share, but its price was expected to increase by 12% per year. At an interest rate of 8% per year, which stock should the engineers select on the basis of a present worth analysis and a 5-year planning horizon?

- 5.11 The Murphy County Fire Department is considering two options for upgrading its aging physical facilities. Plan A involves remodeling the fire stations on Alameda Avenue and Trowbridge Boulevard that are 57 and 61 years old, respectively. (The industry standard is about 50 years of use for a station.) The cost for remodeling the Alameda station is estimated at \$952,000 while the cost of redoing the Trowbridge station is \$1.3 million. Plan B calls for buying 5 acres of land somewhere between the two stations, building a new fire station, and selling the land and structures at the previous sites. The cost of land in that area is estimated to be \$366,000 per acre. The size of the new fire station would be 9000 square feet with a construction cost of \$151.18 per square foot. Contractor fees for overhead, profit, etc. are expected to be \$340,000, and architect fees will be \$81,500. (Assume all of the costs for plan B occur at time 0.) If plan A is adopted, the extra cost for personnel and equipment will be \$126,000 per year. Under plan B, the sale of the old sites is anticipated to net a positive \$500,000 five years in the future. Use an interest rate of 6% per year and a 50-year useful life for the remodeled and new stations to determine which plan is better on the basis of a present worth analysis.

- 5.12 Delcon Properties is a commercial developer of shopping centers and malls in various places around the country. The company needs to analyze the economic feasibility of rainwater drains in a 60-acre area that it plans to develop. Since the development won't be started for 3 years, this large open space will be subject to damage from heavy thunderstorms that cause soil erosion and heavy rutting. If no drains are installed, the cost of refilling and grading the washed out area is expected to be \$1500 per thunderstorm. Alternatively, a temporary corrugated steel drainage pipe could be installed that will prevent the soil erosion. The cost of the pipe will be \$3 per foot for the total length of 7000 feet required. Some of the pipe will be salvageable for \$4000 at the end of the 3-year period between now and when the construction begins. Assuming that thunderstorms occur regularly at 3-month intervals, starting 3 months from now, which alternative should be selected on the basis of a present worth comparison using an interest rate of 4% per quarter?

- 5.13 A public water utility is trying to decide between two different sizes of pipe for a new water main. A 250-mm line will have an initial cost of \$155,000, whereas a 300-mm line will cost \$210,000. Since there is more head loss through the 250-mm pipe, the pumping cost is expected to be \$3000 more per year than for the 250-mm line. If the lines are

expected to last for 30 years, which size should be selected on the basis of a present worth analysis using an interest rate of 10% per year?

- 5.14 The supervisor of a community swimming pool has developed two methods for chlorinating the pool. If gaseous chlorine is added, a chlorinator will be required that has an initial cost of \$8000 and a useful life of 5 years. The chlorine will cost \$650 per year, and the labor cost will be \$800 per year. Alternatively, dry chlorine can be added manually at a cost of \$1000 per year for chlorine and \$1900 per year for labor. Which method should be used on the basis of a present worth analysis if the interest rate is 10% per year?
- 5.15 Anion, an environmental engineering consulting firm, is trying to be eco-friendly in acquiring an automobile for general office use. It is considering a gasoline-electric hybrid and a gasoline-free all-electric hatchback. The hybrid under consideration is GM's Volt, which will cost \$35,000 and have a range of 40 miles on the electric battery and several hundred more miles when the gasoline engine kicks in. Nissan's Leaf, on the other hand, is a pure electric that will have a range of only 100 miles, after which its lithium-ion battery will have to be recharged. The Leaf's relatively limited range creates a psychological effect known as *range anxiety*. This fact alone has caused the company to lean toward purchasing the Volt, which is assumed to have a salvage value of \$15,000 in 5 years. The Leaf could be leased for \$349 per month (end-of-month payments) for 5 years after an initial \$1500 down payment for "account activation." If the consulting company plans to ignore the range anxiety effect in making its decision, which automobile is the better option on the basis of a present worth analysis at an interest rate of 0.75% per month? Assume the operating cost will be the same for both vehicles.
- 5.16 A pipeline engineer working in Kuwait for the oil giant BP wants to perform a present worth analysis on alternative pipeline routings—the first predominately by land and the second primarily undersea. The undersea route is more expensive initially due to extra corrosion protection and installation costs, but cheaper security and maintenance reduces annual costs. Perform the analysis for the engineer at 15% per year.

	Land	Undersea
Installation cost, \$ million	-215	-350
Pumping, operating, security, \$ million per year	-22	-2
Replacement of valves and appurtenances in year 25, \$ million	-30	-70
Expected life, years	50	50

Alternative Comparison—Different Lives

- 5.17 An electric switch manufacturing company has to choose one of three different assembly methods. Method A will have a first cost of \$40,000, an annual operating cost of \$9000, and a service life of 2 years. Method B will cost \$80,000 to buy and will have an annual operating cost of \$6000 over its 4-year service life. Method C will cost \$130,000 initially with an annual operating cost of \$4000 over its 8-year life. Methods A and B will have no salvage value, but method C will have some equipment worth an estimated \$12,000. Which method should be selected? Use present worth analysis at an interest rate of 10% per year.
- 5.18 Midwest Power and Light operates 14 coal-fired power plants in several states around the United States. The company recently settled a lawsuit by agreeing to pay \$60 million in mitigation costs related to acid rain. The settlement included \$21 million to reduce emissions from barges and trucks in the Ohio River Valley, \$24 million for projects to conserve energy and produce alternative energy, \$3 million for Chesapeake Bay, \$2 million for Shenandoah National Park, and \$10 million to acquire ecologically sensitive lands in Appalachia. The question of how to distribute the money over time has been posed. Plan A involves spending \$5 million now and the remaining \$55 million equally over a 10-year period (that is, \$5.5 million in each of years 1 through 10). Plan B requires expenditures of \$5 million now, \$25 million 2 years from now, and \$30 million 7 years from now. Determine which plan is more economical on the basis of a present worth analysis over a 10-year period at an interest rate of 10% per year.
- 5.19 Machines that have the following costs are under consideration for a robotized welding process. Using an interest rate of 10% per year, determine which alternative should be selected on the basis of a present worth analysis. Show (a) hand and (b) spreadsheet solutions.

	Machine X	Machine Y
First cost, \$	-250,000	-430,000
Annual operating cost, \$ per year	-60,000	-40,000
Salvage value, \$	70,000	95,000
Life, years	3	6

- 5.20 Water for Semiconductor Manufacturing Case 

Throughout the present worth analyses, the decision between seawater and groundwater switched multiple times in Examples 5.2 and 5.4. A summary is given here in \$1 million units.

Seawater (S)			
Life n, years	First cost, \$	PW at 12%, \$	Selected
10	-20	-30.64	Yes
5	-20, plus -10 after 5 years	-36.31	No
5 (study period)	-20	-26.43	Yes

The confusion about the recommended source for UPW has not gone unnoticed by the general manager. Yesterday, you were asked to settle the issue by determining the first cost X_S of the seawater option to ensure that it is the economic choice over groundwater. The study period is set by the manager as 10 years, simply because that is the time period on the lease agreement for the building where the fab will be located. Since the seawater equipment must be refurbished or replaced after 5 years, the general manager told you to assume that the equipment will be purchased anew after 5 years of use. What is the maximum first cost that Angular Enterprises should pay for the seawater option?

- 5.21 Accurate airflow measurement requires straight unobstructed pipe for a minimum of 10 diameters upstream and 5 diameters downstream of the measuring device. In a field application, physical constraints compromise the pipe layout, so the engineer is considering installing the airflow probes in an elbow, knowing that flow measurement will be less accurate but good enough for process control. This is plan 1, which will be in place for only 3 years, after which a more accurate flow measurement system with the same costs as plan 1 will be available. This plan will have a first cost of \$26,000 with an annual maintenance cost estimated at \$5000.

Plan 2 involves installation of a recently designed submersible airflow probe. The stainless steel probe can be installed in a drop pipe with the transmitter located in a waterproof enclosure on the handrail. The first cost of this system is \$83,000, but because it is accurate and more durable, it will not have to be replaced for at least 6 years. Its maintenance cost is estimated to be \$1400 per year plus \$2500 in year 3 for replacement of signal processing software. Neither system will have a salvage value. At an interest rate of 10% per year, which one should be selected on the basis of a present worth comparison?

- 5.22 An engineer is considering two different liners for an evaporation pond that will receive salty concentrate from a brackish water desalting plant. A plastic liner will cost \$0.90 per square foot initially and will

Groundwater (G)			
Life n, years	First cost, \$	PW at 12%, \$	Selected
10	-22	-33.16	No
10	-22	-33.16	Yes
5 (study period)	-22	-28.32	No

require replacement in 15 years when precipitated solids will have to be removed from the pond using heavy equipment. This removal will cost \$500,000. A rubberized elastomeric liner is tougher and, therefore, is expected to last 30 years, but it will cost \$2.20 per square foot. If the size of the pond is 110 acres (1 acre = 43,560 square feet), which liner is more cost effective on the basis of a present worth comparison at an interest rate of 8% per year?

- 5.23 A sports mortgage is the brainchild of Stadium Capital Financing Group, a company headquartered in Chicago, Illinois. It is an innovative way to finance cash-strapped sports programs by allowing fans to sign up to pay a "mortgage" over a certain number of years for the right to buy good seats at football games for several decades with season ticket prices locked in at current prices. In California, the locked-in price period is 50 years. Assume UCLA fan X purchases a \$130,000 mortgage and pays for it now to get season tickets for \$290 each for 50 years, while fan Y buys season tickets at \$290 in year 1, with prices increasing by \$20 per year for 50 years. (a) Which fan made the better deal if the interest rate is 8% per year? (b) What should fan X be willing to pay up front for the mortgage to make the two plans exactly equivalent economically? (Assume he has no reason to give extra money to UCLA at this point.)

- 5.24 A chemical processing corporation is considering three methods to dispose of a non-hazardous chemical sludge: land application, fluidized-bed incineration, and private disposal contract. The estimates for each method are shown. Determine which has the least cost on the basis of a present worth comparison at 10% per year for the following scenarios:

- (a) The estimates as shown
(b) The contract award cost increases by 20% every 2-year renewal

	Land Application	Incineration	Contract
First cost, \$	-130,000	-900,000	0
Annual operating cost, \$ per year	-95,000	-60,000	-120,000
Salvage value, \$	25,000	300,000	0
Life, years	3	6	2

- 5.25 An assistant to Stacy gave her the PW values for four alternatives they are comparing for the development of a remote control vibration control system for offshore platform application. The results in the table use a MARR of 14% per year. Determine which alternative(s) should be selected (a) if the alternatives are exclusive, and (b) if the projects are independent.

	I	J	K	L
Life n , years	3	4	12	6
PW over n years, \$	16.08	31.12	-257.46	140.46
PW over 6 years, \$	26.94	15.78	-653.29	140.46
PW over 12 years, \$	39.21	60.45	-257.46	204.46

Future Worth Comparison

- 5.26 An industrial engineer is considering two robots for purchase by a fiber-optic manufacturing company. Robot X will have a first cost of \$80,000, an annual maintenance and operation (M&O) cost of \$30,000, and a \$40,000 salvage value. Robot Y will have a first cost of \$97,000, an annual M&O cost of \$27,000, and a \$50,000 salvage value. Which should be selected on the basis of a future worth comparison at an interest rate of 15% per year? Use a 3-year study period.
- 5.27 Two processes can be used for producing a polymer that reduces friction loss in engines. Process T will have a first cost of \$750,000, an operating cost of \$60,000 per year, and a salvage value of \$80,000 after its 2-year life. Process W will have a first cost of \$1,350,000, an operating cost of \$25,000 per year, and a \$120,000 salvage value after its 4-year life. Process W will also require updating at the end of year 2 at a cost of \$90,000. Which process should be selected on the basis of a future worth analysis at an interest rate of 12% per year?
- 5.28 Compare the alternatives shown below on the basis of a future worth analysis, using an interest rate of 8% per year.

	P	Q
First cost, \$	-23,000	-30,000
Annual operating cost, \$ per year	-4,000	-2,500
Salvage value, \$	3,000	1,000
Life, years	3	6

- 5.29 Two manufacturers supply MRI systems for medical imaging. St. Jude's Hospital wishes to replace its current MRI equipment that was purchased 8 years ago with the newer technology and clarity of a state-of-the-art system. System K will have a first cost of \$1,600,000, an operating cost of \$70,000 per year, and a salvage value of \$400,000 after its 4-year life. System L will have a first cost of \$2,100,000, an

operating cost of \$50,000 the first year with an expected increase of \$3000 per year thereafter, and no salvage value after its 8-year life. Which system should be selected on the basis of a future worth analysis at an interest rate of 12% per year?

- 5.30 A retail shopping center developer signed a contract to build a \$100 million high-end shopping center in City Center, because the city and county governments agreed to sales and tax rebates totaling \$18.7 million over 10 years. The contract called for the developer to raze existing buildings 2 years from the date the contract was signed and to have the shopping center built by the end of year 3. However, due to a real estate-induced recession in the United States, the developer sought and was granted a new contract. The new contract required the developer to raze the existing buildings at the end of year 1, but the shopping center would not have to be completed for 7 years from the date the contract was signed. Assume that the cost for razing the existing buildings is \$1.3 million and the developer does not build the shopping center until 7 years from now (at a cost of \$100 million). Determine the difference in the future worth cost in year 7 of the two contracts at an interest rate of 10% per year.

Capitalized Cost

- 5.31 A wealthy businessman wants to start a permanent fund for supporting research directed toward sustainability. The donor plans to give equal amounts of money for each of the next 5 years, plus one now (i.e., six donations) so that \$100,000 per year can be withdrawn each year forever, beginning in year 6. If the fund earns interest at a rate of 8% per year, how much money must be donated each time?
- 5.32 Bob, a philanthropist, is not sure what rate of return his gifts may realize once donated to his favorite charity. Determine the capitalized cost of \$10,000 every 5 years *forever*, starting 5 years from now at an interest rate of (a) 3% and (b) 8% per year. (c) Explain the significant difference between the two capitalized costs.
- 5.33 Find the capitalized cost of a present cost of \$300,000, annual costs of \$35,000, and periodic costs every 5 years of \$75,000. Use an interest rate of 12% per year.

- 5.34 Water for Semiconductor Manufacturing Case 
- It is anticipated that the needs for UPW (ultrapure water) at the new Angular Enterprises site will continue for a long time, as long as 50 years. This is the rationale for using capitalized cost as a basis for the economic decision between desalinated

seawater (S) and purified groundwater (G). These costs were determined (Example 5.7) to be $CC_S = -\$53.58$ million and $CC_G = -\$48.91$ million. Groundwater is the clear economic choice.

Yesterday, the general manager had lunch with the president of Brissa Water, who offered to supply the needed UPW at a cost of \$5 million per year for the indefinite future. It would mean a dependence upon a contractor to supply the water, but the equipment, treatment, and other costly activities to obtain UPW on-site would be eliminated. The manager asks you to make a recommendation about this seemingly attractive alternative under the following conditions at the same MARR of 12% per year as used for the other analyses:

- (a) The annual cost of \$5 million remains constant throughout the time it is needed.
- (b) The annual cost starts at \$5 million for the first year only, and then it increases 2% per year. (This increase is above the cost of providing UPW by either of the other two methods.)

- 5.35 Compare the alternatives shown on the basis of their capitalized costs using an interest rate of 10% per year.

	Alternative M	Alternative N
First cost, \$	-150,000	-800,000
Annual operating cost, \$ per year	-50,000	-12,000
Salvage value, \$	8,000	1,000,000
Life, years	5	∞

- 5.36 The cost of maintaining a certain permanent monument in Washington, DC occurs as periodic outlays of \$1000 every year and \$5000 every 4 years. Calculate the capitalized cost of the maintenance using an interest rate of 10% per year.

- 5.37 Because you are thankful for what you learned in engineering economy, you plan to start a permanent scholarship fund in the name of the professor who taught the course. You plan to deposit money now with the stipulation that the scholarships be awarded beginning 12 years from now (which happens to be the exact time that your daughter plans to begin college). The interest that is accumulated between now and year 12 is to be added to the principal of the endowment. After that, the interest that is earned each year will be awarded as scholarship money. If you want the amount of the scholarships to be \$40,000 per year, how much must you donate now if the fund earns interest at a rate of 8% per year?

- 5.38 A patriotic group of firefighters is raising money to erect a permanent (i.e., infinite life) monument in New York City to honor those killed in the line of duty. The initial cost of the monument will be \$150,000, and the annual maintenance will cost \$5000. There will be an additional one-time cost of \$20,000 in 2 years to add names of those who were missed initially. At an interest rate of 6% per year, how much money must they raise now in order to construct and maintain the monument forever?

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 5.39 One assumption inherent in the present worth method of analysis is that:

- (a) The alternatives will be used only through the life of the *shortest-lived* alternative.
- (b) The alternatives will be used only through the life of the *longest-lived* alternative.
- (c) The cash flows of each alternative will change only by the inflation or deflation rate in succeeding life cycles.
- (d) At least one of the alternatives will have a finite life.

- 5.40 When only one alternative can be selected from two or more, the alternatives are said to be:

- (a) Mutually exclusive
- (b) Independent alternatives
- (c) Cost alternatives
- (d) Revenue alternatives

- 5.41 For the mutually exclusive alternatives shown, the one(s) that should be selected are:

Alternative	PW, \$
A	-25,000
B	-12,000
C	10,000
D	15,000

- (a) Only C
- (b) Only A
- (c) C and D
- (d) Only D

- 5.42 The alternatives shown are to be compared on the basis of their present worth values. At an interest rate of 10% per year, the values of n that you should use in the uniform series factors to make a

correct comparison by the present worth method are:

	Alternative A	Alternative B
First cost, \$	-50,000	-90,000
Annual operating cost, \$ per year	-10,000	-4000
Salvage value, \$	13,000	15,000
Life, years	3	6

- (a) $n = 3$ years for A and $n = 3$ years for B
- (b) $n = 3$ years for A and $n = 6$ years for B
- (c) $n = 6$ years for A and $n = 6$ years for B
- (d) None of the above

- 5.43 The value of the *future worth* for alternative P at an interest rate of 8% per year is closest to:

	P	Q
First cost, \$	-23,000	-30,000
Annual operating cost, \$ per year	-4,000	-2,500
Salvage value, \$	3,000	1,000
Life, years	3	6

- (a) $FW_P = \$ -88,036$
- (b) $FW_P = \$ -86,026$
- (c) $FW_P = \$ -81,274$
- (d) $FW_P = \$ -70,178$

- 5.44 The present worth of \$50,000 now, \$10,000 per year in years 1 through 15, and \$20,000 per year in years 16 through infinity at 10% per year is closest to:

- (a) Less than \$-169,000
- (b) \$-169,580
- (c) \$-173,940
- (d) \$-195,730

- 5.45 A donor (you) wishes to start an endowment that will provide scholarship money of \$40,000 per year beginning in year 5 and continuing indefinitely. If the university earns 10% per year on the endowment, the amount you must donate now is closest to:

- (a) \$-225,470
- (b) \$-248,360
- (c) \$-273,200
- (d) \$-293,820

Problems 5.46 through 5.48 are based on the following information.

	Alternative I	Alternative J
Initial cost, \$	-150,000	-250,000
Annual income, \$ per year	20,000	40,000
Annual expenses, \$ per year	-9,000	-14,000
Salvage value, \$	25,000	35,000
Life, years	3	6

The interest rate is 15% per year.

- 5.46 In comparing alternatives I and J by the present worth method, the value of n that must be used in $11,000(P/A,i,n)$ for alternative I is:

- (a) 3
- (b) 6
- (c) 18
- (d) 36

- 5.47 In comparing alternatives I and J by the present worth method, the equation that yields the present worth of alternative J is:

- (a) $PW_J = -250,000 + 40,000(P/A,15\%,6) + 35,000(P/F,15\%,6)$
- (b) $PW_J = -250,000 + 26,000(P/A,15\%,6) + 35,000(P/F,15\%,6)$
- (c) $PW_J = -250,000 - 26,000(P/A,15\%,6) + 35,000(P/F,15\%,6)$
- (d) $PW_J = -250,000 - 26,000(P/A,15\%,6) - 35,000(P/F,15\%,6)$

- 5.48 In comparing alternatives I and J by the present worth method, the equation that yields the present worth of alternative I is:

- (a) $PW_I = -150,000 + 11,000(P/A,15\%,3) + 25,000(P/F,15\%,3)$
- (b) $PW_I = -150,000 + 11,000(P/A,15\%,6) + 25,000(P/F,15\%,6)$
- (c) $PW_I = -150,000 + 11,000(P/A,15\%,6) + 175,000(P/F,15\%,3) + 25,000(P/F,15\%,6)$
- (d) $PW_I = -150,000 + 11,000(P/A,15\%,6) - 125,000(P/F,15\%,3) + 25,000(P/F,15\%,6)$

Problems 5.49 and 5.50 are based on the following information.

	Machine X	Machine Y
Initial cost, \$	-80,000	-95,000
Annual operating cost, \$ per year	-20,000	-15,000
Salvage value, \$	10,000	30,000
Life, years	2	4

The interest rate is 10% per year.

- 5.49 The equation that will calculate the present worth of machine X is:

- (a) $PW_X = -80,000 - 15,000(P/A,10\%,4) + 30,000(P/F,10\%,4)$
- (b) $PW_X = -80,000 - 20,000(P/A,10\%,4) - 80,000(P/F,10\%,2) + 10,000(P/F,10\%,4)$
- (c) $PW_X = -80,000 - 20,000(P/A,10\%,2) + 10,000(P/F,10\%,2)$
- (d) $PW_X = -80,000 - 20,000(P/A,10\%,4) - 70,000(P/F,10\%,2) + 10,000(P/F,10\%,4)$

- 5.50 In comparing the machines on a present worth basis, the present worth of machine Y is closest to:

- (a) \$-112,320
- (b) \$-122,060
- (c) \$-163,040
- (d) \$-175,980

- 5.51 The capitalized cost of \$10,000 every 5 years *forever*, starting now at an interest rate of 10% per year, is closest to:
- (a) \$-13,520
 - (b) \$-16,380
 - (c) \$-26,380
 - (d) \$-32,590
- 5.52 At an interest rate of 10% per year, the capitalized cost of \$10,000 in year 0, \$5000 per year in years 1 through 5, and \$1000 per year thereafter forever is closest to:
- (a) -\$29,652
 - (b) -\$35,163
 - (c) -\$38,954
 - (d) -\$43,221

CASE STUDY

COMPARING SOCIAL SECURITY BENEFITS

Background

When Sheryl graduated from Northeastern University in 2000 and went to work for BAE Systems, she did not pay much attention to the monthly payroll deduction for social security. It was a “necessary evil” that may be helpful in retirement years. However, this was so far in the future that she fully expected this government retirement benefit system to be broke and gone by the time she could reap any benefits from her years of contributions.

This year, Sheryl and Brad, another engineer at BAE, got married. Recently, they both received notices from the Social Security Administration of their potential retirement amounts, were they to retire and start social security benefits at preset ages. Since both of them hope to retire a few years early, they decided to pay closer attention to the predicted amount of retirement benefits and to do some analysis on the numbers.

Information

They found that their projected benefits are substantially the same, which makes sense since their salaries are very close to each other. Although the numbers were slightly different in their two mailings, the similar messages to Brad and Sheryl can be summarized as follows:

If you stop working and start receiving benefits . . .

At age 62, your payment would be about	\$1400 per month
At you full retirement age (67 years), your payment would be about	\$2000 per month
At age 70, your payment would be about	\$2480 per month

These numbers represent a reduction of 30% for early retirement (age 62) and an increase of 24% for delayed retirement (age 70).

This couple also learned that it is possible for a spouse to take spousal benefits at the time that one of them is at full retirement age. In other words, if Sheryl starts her \$2000 benefit at age 67, Brad can receive a benefit equal to 50% of hers. Then, when Brad reaches 70 years of age, he can discontinue

spousal benefits and start his own. In the meantime, his benefits will have increased by 24%. Of course, this strategy could be switched with Brad taking his benefits and Sheryl receiving spousal benefits until age 70.

All these options led them to define four alternative plans.

- A: Each takes early benefits at age 62 with a 30% reduction to \$1400 per month.
- B: Each takes full benefits at full retirement age of 67 and receives \$2000 per month.
- C: Each delays benefits until age 70 with a 24% increase to \$2480 per month.
- D: One person takes full benefits of \$2000 per month at age 67, and the other person receives spousal benefits (\$1000 per month at age 67) and switches to delayed benefits of \$2480 at age 70.

They realize, of course, that the numbers will change over time, based on their respective salaries and number of years of contribution to the social security system by them and by their employers.

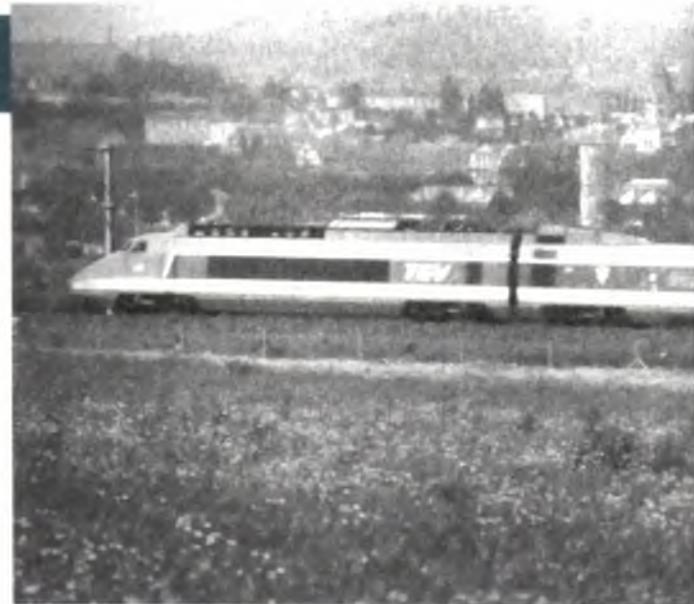
Case Study Exercises

Brad and Sheryl are the same age. Brad determined that most of their investments make an average of 6% per year. With this as the interest rate, the analysis for the four alternatives is possible. Sheryl and Brad plan to answer the following questions, but don’t have time this week. Can you please help them? (Do the analysis for one person at a time, not the couple, and stop at the age of 85.)

1. How much in total (without the time value of money considered) will each plan A through D pay through age 85?
2. What is the future worth at 6% per year of each plan at age 85?
3. Plot the future worth values for all four plans on one spreadsheet graph.
4. Economically, what is the best combination of plans for Brad and Sheryl, assuming they both live to be 85 years old?
5. Develop at least one additional question that you think Sheryl and Brad may have. Answer the question.

CHAPTER 6

Annual Worth Analysis



LEARNING OUTCOMES

Purpose: Utilize different annual worth techniques to evaluate and select alternatives.

SECTION	TOPIC	LEARNING OUTCOME
6.1	Advantages of AW	<ul style="list-style-type: none">Demonstrate that the AW value is the same for each life cycle.
6.2	CR and AW values	<ul style="list-style-type: none">Calculate and interpret the capital recovery (CR) and AW amounts.
6.3	AW analysis	<ul style="list-style-type: none">Select the best alternative using an annual worth analysis.
6.4	Perpetual life	<ul style="list-style-type: none">Evaluate alternatives with very long lives using AW analysis.
6.5	LCC analysis	<ul style="list-style-type: none">Perform a life-cycle cost (LCC) analysis using AW methods.

In this chapter, we add to our repertoire of alternative comparison tools. In Chapter 5 we learned the PW method. Here we learn the equivalent annual worth, or AW, method. AW analysis is commonly considered the more desirable of the two methods because the AW value is easy to calculate; the measure of worth—AW in monetary units per year—is understood by most individuals; and its assumptions are essentially identical to those of the PW method.

Annual worth is also known by other titles. Some are *equivalent annual worth* (EAW), *equivalent annual cost* (EAC), *annual equivalent* (AE), and *equivalent uniform annual cost* (EUAC). The alternative selected by the AW method will always be the same as that selected by the PW method, and all other alternative evaluation methods, provided they are performed correctly.

An additional application of AW analysis treated here is **life-cycle cost (LCC) analysis**. This method considers all costs of a product, process, or system from concept to phaseout.

6.1 Advantages and Uses of Annual Worth Analysis ● ● ●

For many engineering economic studies, the AW method is the best to use, when compared to PW, FW, and rate of return (Chapters 7 and 8). Since the AW value is the equivalent uniform annual worth of all estimated receipts and disbursements during the life cycle of the project or alternative, AW is easy to understand by any individual acquainted with annual amounts, for example, dollars per year. The AW value, which has the same interpretation as A used thus far, is the economic equivalent of the PW and FW values at the MARR for n years. All three can be easily determined from each other by the relation

$$AW = PW(A/P,i,n) = FW(A/F,i,n) \quad [6.1]$$



The n in the factors is the number of years for equal-service comparison. This is the LCM or the stated study period of the PW or FW analysis.

When all cash flow estimates are converted to an AW value, this value applies for every year of the life cycle and for *each additional life cycle*.

The annual worth method offers a prime computational and interpretation advantage because the AW value needs to be calculated for **only one life cycle**. The AW value determined over one life cycle is the AW for all future life cycles. Therefore, it is **not necessary to use the LCM of lives** to satisfy the equal-service requirement.



As with the PW method, there are three fundamental assumptions of the AW method that should be understood. When alternatives being compared have different lives, the AW method makes the assumptions that

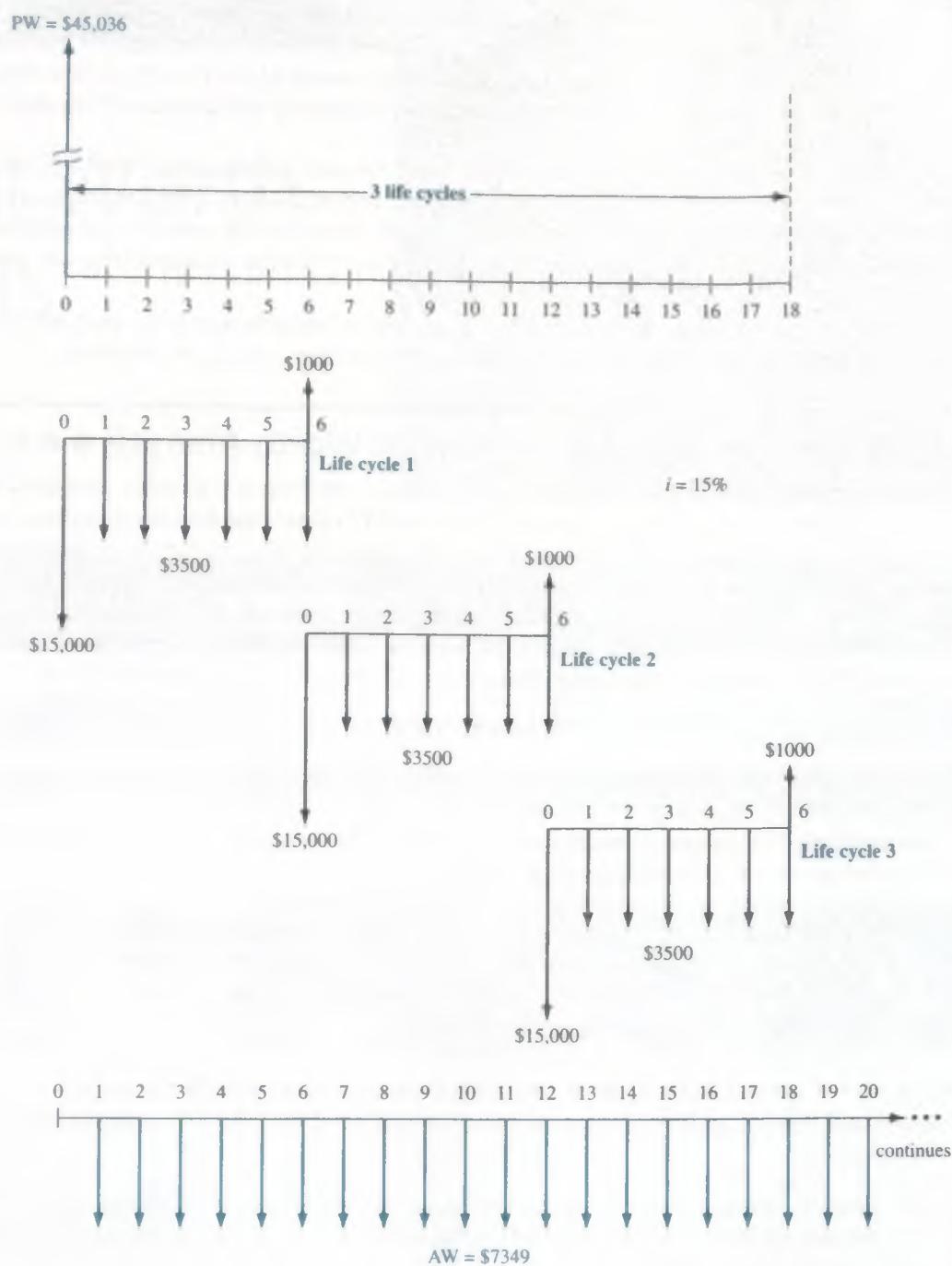
1. The services provided are needed for at least the LCM of the lives of the alternatives.
2. The selected alternative will be repeated for succeeding life cycles in exactly the same manner as for the first life cycle.
3. All cash flows will have the same estimated values in every life cycle.

In practice, no assumption is precisely correct. If, in a particular evaluation, the first two assumptions are not reasonable, a study period must be established for the analysis. Note that for assumption 1, the length of time may be the indefinite future (forever). In the third assumption, all cash flows are expected to change exactly with the inflation (or deflation) rate. If this is not a reasonable assumption, new cash flow estimates must be made for each life cycle, and again a study period must be used. AW analysis for a stated study period is discussed in Section 6.3.

EXAMPLE 6.1

In Example 5.3, National Homebuilders, Inc. evaluated cut-and-finish equipment from vendor A (6-year life) and vendor B (9-year life). The PW analysis used the LCM of 18 years. Consider only the vendor A option now. The diagram in Figure 6–1 shows the cash flows for all three life cycles (first cost \$–15,000; annual M&O costs \$–3500; salvage value \$1000). Demonstrate the equivalence at $i = 15\%$ of PW over three life cycles and AW over one cycle. In Example 5.3, present worth for vendor A was calculated as $PW = \$-45,036$.

Figure 6–1
PW and AW values
for three life cycles,
Example 6.1.



Solution

Calculate the equivalent uniform annual worth value for all cash flows in the first life cycle.

$$AW = -15,000(A/P, 15\%, 6) + 1000(A/F, 15\%, 6) - 3500 = -\$7349$$

When the same computation is performed on each succeeding life cycle, the AW value is $-\$7349$. Now Equation [6.1] is applied to the PW value for 18 years.

$$AW = -45,036(A/P, 15\%, 18) = -\$7349$$

The one-life-cycle AW value and the AW value based on 18 years are equal.

Not only is annual worth an excellent method for performing engineering economy studies, but also it is applicable in any situation where PW (and FW and benefit/cost) analysis can be utilized. The AW method is especially useful in certain types of studies: asset replacement and retention time studies to minimize overall annual costs (both covered in Chapter 11), breakeven studies and make-or-buy decisions (Chapter 13), and all studies dealing with production or manufacturing costs where a cost/unit or profit/unit measure is the focus.

If income taxes are considered, a slightly different approach to the AW method is used by some large corporations and financial institutions. It is termed *economic value added*, or EVA. This approach, covered in Chapter 17, concentrates upon the wealth-increasing potential that an alternative offers a corporation. The resulting EVA values are the equivalent of an AW analysis of after-tax cash flows.

6.2 Calculation of Capital Recovery and AW Values ● ● ●

An alternative should have the following cash flow estimates:

Initial investment P . This is the total first cost of all assets and services required to initiate the alternative. When portions of these investments take place over several years, their present worth is an equivalent initial investment. Use this amount as P .

Salvage value S . This is the terminal estimated value of assets at the end of their useful life. The S is zero if no salvage is anticipated; S is negative when it will cost money to dispose of the assets. For study periods shorter than the useful life, S is the estimated market value or trade-in value at the end of the study period.

Annual amount A . This is the equivalent annual amount (costs only for cost alternatives; costs and receipts for revenue alternatives). Often this is the annual operating cost (AOC) or M&O cost, so the estimate is already an equivalent A value.

The annual worth (AW) value for an alternative is comprised of two components: **capital recovery** for the initial investment P at a stated interest rate (usually the MARR) and the equivalent annual amount A . The symbol CR is used for the capital recovery component. In equation form,

$$\text{AW} = \text{CR} + A \quad [6.2]$$

Both CR and A represent costs. The total annual amount A is determined from uniform recurring costs (and possibly receipts) and nonrecurring amounts. The P/A and P/F factors may be necessary to first obtain a present worth amount; then the A/P factor converts this amount to the A value in Equation [6.2]. (If the alternative is a revenue project, there will be positive cash flow estimates present in the calculation of the A value.)

The recovery of an amount of capital P committed to an asset, plus the time value of the capital at a particular interest rate, is a **fundamental principle of economic analysis**.

Capital recovery (CR) is the equivalent annual amount that the asset, process, or system must earn (new revenue) each year to just **recover the initial investment plus a stated rate of return** over its expected life. Any expected salvage value is considered in the computation of CR.

The A/P factor is used to convert P to an equivalent annual cost. If there is some anticipated positive salvage value S at the end of the asset's useful life, its equivalent annual value is recovered using the A/F factor. This action reduces the equivalent annual cost of the asset. Accordingly, CR is calculated as

$$\text{CR} = -P(A/P,i,n) + S(A/F,i,n) \quad [6.3]$$

EXAMPLE 6.2

Lockheed Martin is increasing its booster thrust power in order to win more satellite launch contracts from European companies interested in opening up new global communications markets. A piece of earth-based tracking equipment is expected to require an investment of \$13 million, with \$8 million committed now and the remaining \$5 million expended at the end of year 1 of the project. Annual operating costs for the system are expected to start the first year and continue at \$0.9 million per year. The useful life of the tracker is 8 years with a salvage value of \$0.5 million. Calculate the CR and AW values for the system, if the corporate MARR is 12% per year.

Solution

Capital recovery: Determine P in year 0 of the two initial investment amounts, followed by the use of Equation [6.3] to calculate the capital recovery. In \$1 million units,

$$P = 8 + 5(P/F, 12\%, 1) = \$12.46$$



Salvage/market value



Capital recovery

$$\begin{aligned} CR &= -12.46(A/P, 12\%, 8) + 0.5(A/F, 12\%, 8) \\ &= -12.46(0.20130) + 0.5(0.08130) \\ &= \$-2.47 \end{aligned}$$

The correct interpretation of this result is very important to Lockheed Martin. It means that each and every year for 8 years, the equivalent total net revenue from the tracker must be at least \$2,470,000 just to recover the initial present worth investment plus the required return of 12% per year. This does not include the AOC of \$0.9 million each year.

Annual worth: To determine AW, the cash flows in Figure 6-2a must be converted to an equivalent AW series over 8 years (Figure 6-2b). Since $CR = \$-2.47$ million is an *equivalent annual cost*, as indicated by the minus sign, total AW is determined.

$$AW = -2.47 - 0.9 = \$-3.37 \text{ million per year}$$

This is the AW for all future life cycles of 8 years, provided the costs rise at the same rate as inflation, and the same costs and services are expected to apply for each succeeding life cycle.

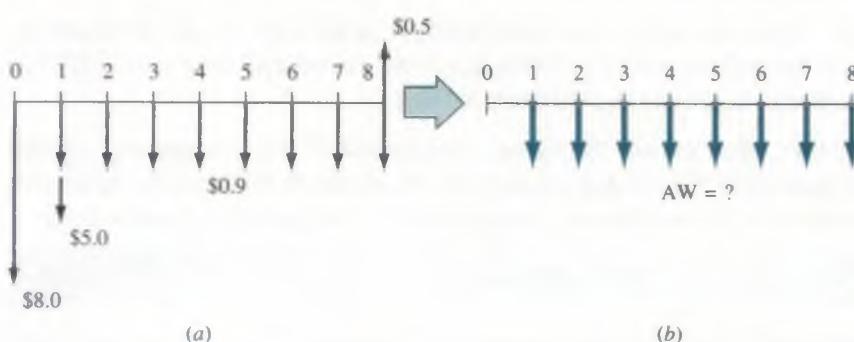


Figure 6-2
(a) Cash flow diagram for satellite tracker costs, and (b) conversion to an equivalent AW (in \$1 million), Example 6.2.

There is a second, equally correct way to determine CR. Either method results in the same value. There is a relation between the A/P and A/F factors.

$$(A/F, i, n) = (A/P, i, n) - i$$

Both factors are present in the CR Equation [6.3]. Substitute for the A/F factor to obtain

$$\begin{aligned} CR &= -P(A/P, i, n) + S[(A/P, i, n) - i] \\ &= -[(P - S)(A/P, i, n) + S(i)] \end{aligned}$$

There is a basic logic to this formula. Subtracting S from the initial investment P before applying the A/P factor recognizes that the salvage value will be recovered. This reduces CR, the annual cost of asset ownership. However, the fact that S is not recovered until year n of ownership is compensated for by charging the annual interest $S(i)$ against the CR. Although either CR relation results in the same amount, it is better to consistently use the same method. The first method, Equation [6.3], will be used in this text.

For solution by spreadsheet, use the PMT function to determine CR only in a single spreadsheet cell. The general function = PMT($i\%, n, P, F$) is rewritten using the initial investment as P and $-S$ for the salvage value. The format is

$$= \text{PMT}(i\%, n, P, -S) \quad [6.4]$$

As an illustration, determine only the CR in Example 6.2. The equivalent initial investment in year 0 is \$12.46 million. The complete function for the CR amount (in \$1 million units) is = PMT(12%, 8, 12.46, -0.5). The answer of \$-2.47 (million) will be displayed in the spreadsheet cell.

As we learned in Section 3.1, one spreadsheet function can be embedded in another function. In the case of Example 6.2, the initial investment is distributed over a 2-year period. The one-cell PMT function, with the PV function embedded (in bold), can be written as = PMT(12%, 8, 8+PV(12%, 1, -5), -0.5) to display the same value $CR = \$-2.47$.

6.3 Evaluating Alternatives by Annual Worth Analysis ●●●

The annual worth method is typically the easiest to apply of the evaluation techniques when the MARR is specified. The AW is calculated over the respective life of each alternative, and the selection guidelines are the same as those used for the PW method. For **mutually exclusive** alternatives, whether cost- or revenue-based, the guidelines are as follows:

One alternative: If $AW \geq 0$, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the AW that is **numerically largest**, that is, less negative or more positive. This indicates a lower AW of cost for cost alternatives or a larger AW of net cash flows for revenue alternatives.



Project evaluation

ME alternative selection

If any of the three assumptions in Section 6.1 is not acceptable for an alternative, a study period analysis must be used. Then the cash flow estimates over the study period are converted to AW amounts. The following two examples illustrate the AW method for one project and two alternatives.

EXAMPLE 6.3

Heavenly Pizza, which is located in Toronto, fares very well with its competition in offering fast delivery. Many students at the area universities and community colleges work part-time delivering orders made via the web. The owner, Jerry, a software engineering graduate, plans to purchase and install five portable, in-car systems to increase delivery speed and accuracy. The systems provide a link between the web order-placement software and the On-Star system for satellite-generated directions to any address in the area. The expected result is faster, friendlier service to customers and larger income.

Each system costs \$4600, has a 5-year useful life, and may be salvaged for an estimated \$300. Total operating cost for all systems is \$1000 for the first year, increasing by \$100 per year thereafter. The MARR is 10%. Perform an annual worth evaluation for the owner that answers the following questions. Perform the solution by hand and by spreadsheet.

- How much new annual net income is necessary to recover the investment at the MARR of 10% per year?
- Jerry estimates increased net income of \$6000 per year for all five systems. Is this project financially viable at the MARR?
- Based on the answer in part (b), determine how much new net income Heavenly Pizza must have to economically justify the project. Operating costs remain as estimated.

Solution by Hand

- The capital recovery amount calculated by Equation [6.3] answers the first question.

$$\begin{aligned} CR &= -5[4600(A/P, 10\%, 5)] + 5[300(A/F, 10\%, 5)] \\ &= -5[4600(0.26380)] + 5[300(0.16380)] \\ &= \$-5822 \end{aligned}$$

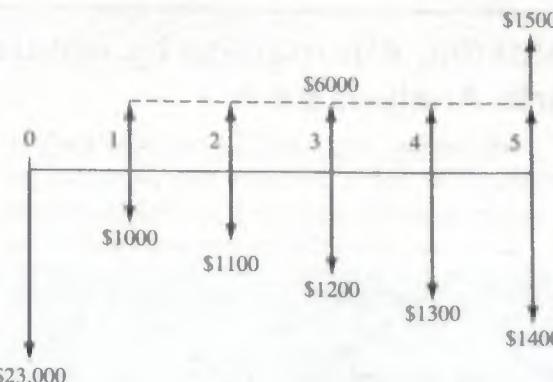
The five systems must generate an equivalent annual new revenue of \$5822 to recover the initial investment plus a 10% per year return.

- Figure 6–3 presents the cash flows over 5 years. The annual operating cost series, combined with the estimated \$6000 annual income, forms an arithmetic gradient series with a base amount of \$5000 and $G = \$-100$. The project is financially viable if $AW \geq 0$ at $i = 10\%$ per year. Apply Equation [6.2], where A is the equivalent annual net income series.

$$\begin{aligned} AW &= CR + A = -5822 + 5000 - 100(A/G, 10\%, 5) \\ &= \$-1003 \end{aligned}$$

The system is not financially justified at the net income level of \$6000 per year.

Figure 6–3
Cash flow diagram used to compute AW, Example 6.3.



- (c) Let the required income equal R , and set the AW relation equal to zero to find the minimum income to justify the system.

$$0 = -5822 + (R - 1000) - 100(A/G, 10\%, 5)$$

$$\begin{aligned} R &= -5822 - 1000 - 100(1.8101) \\ &= \$7003 \text{ per year} \end{aligned}$$

Solution by Spreadsheet

The spreadsheet in Figure 6–4 summarizes the estimates and answers the questions posed for Heavenly Pizza with the same values that were determined in the hand solution.

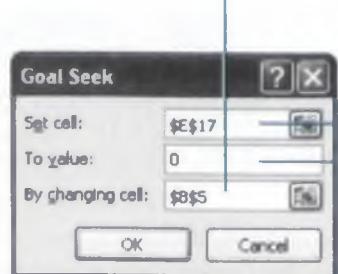
Figure 6–4
Spreadsheet solution of Example 6.3.
(a) Capital recovery in cell B16, (b) AW in cell E17, and (c) Goal Seek template and outcome in cell B5.

	A	B	C	D	E
1					
2	MARR	10%			
3	# of systems	5			
4	Cost/system	-4,600			
5	Income/year	\$ 6,000			
6					
7					
8	Year	Investment and salvage	Annual costs	Annual income	Net income
9	0	-23,000	0	0	-23,000
10	1	0	-1,000	6,000	5,000
11	2	0	-1,100	6,000	4,900
12	3	0	-1,200	6,000	4,800
13	4	0	-1,300	6,000	4,700
14	5	1,500	-1,400	6,000	6,100
15					
16	(a) CR value	-5,822			
17	(b) AW value				-1,003
18		= -PMT(\$B\$2, 5, NPV(\$B\$2,B10:B14)+B9))			
19					
20					
21					= -PMT(\$B\$2, 5, NPV(\$B\$2,E10:E14)+E9))
22					

(a) and (b)

	A	B	C	D	E
1					
2	MARR	10%			
3	# of systems	5			
4	Cost/system	-4,600			
5	Income/year	\$ 7,003			
6					
7					
8	Year	Investment and salvage	Annual costs	Annual income	Net income
9	0	-23,000	0	0	-23,000
10	1	0	-1,000	7,003	6,003
11	2	0	-1,100	7,003	5,903
12	3	0	-1,200	7,003	5,803
13	4	0	-1,300	7,003	5,703
14	5	1,500	-1,400	7,003	7,103
15					
16	(a) CR value	-5,822			
17	(b) AW value				0
18					

(c)



Cell references are used in the spreadsheet functions to accommodate changes in estimated values.

- (a) The capital recovery $CR = \$-5822$ is displayed in column B using the PMT function with an embedded NPV function, as shown in the cell tag.
- (b) The annual worth $AW = \$-1003$ is displayed in column E using the PMT function shown. The arithmetic gradient series of costs and estimated income of \$6000 in columns C and D, respectively, are added to obtain the net income necessary for the PMT function.
- (c) The minimum required income is determined in the lower part of Figure 6–4. This is easily accomplished by setting $AW = 0$ (column E) in the Goal Seek tool and letting it find the income per year of \$7003 to balance the AW equation.

EXAMPLE 6.4

Luby's Cafeterias is in the process of forming a separate business unit that provides meals to facilities for the elderly, such as assisted care and long-term care centers. Since the meals are prepared in one central location and distributed by trucks throughout the city, the equipment that keeps food and drink cold and hot is very important. Michele is the general manager of this unit, and she wishes to choose between two manufacturers of temperature retention units that are mobile and easy to sterilize after each use. Use the cost estimates below to select the more economic unit at a MARR of 8% per year.

	Hamilton (H)	Infinity Care (IC)
Initial cost P , \$	−15,000	−20,000
Annual M&O, \$/year	−6,000	−9,000
Refurbishment cost, \$	0	−2,000 every 4 years
Trade-in value S , % of P	20	40
Life, years	4	12

Solution

The best evaluation technique for these different-life alternatives is the annual worth method, where AW is taken at 8% per year over the respective lives of 4 and 12 years.

$$\begin{aligned}
 AW_H &= \text{annual equivalent of } P - \text{annual M&O} + \text{annual equivalent of } S \\
 &= -15,000(A/P, 8\%, 4) - 6000 + 0.2(15,000)(A/F, 8\%, 4) \\
 &= -15,000(0.30192) - 6000 + 3000(0.22192) \\
 &= \$-9,863
 \end{aligned}$$

$$\begin{aligned}
 AW_{IC} &= \text{annual equivalent of } P - \text{annual M&O} - \text{annual equivalent of refurbishment} \\
 &\quad + \text{annual equivalent of } S \\
 &= -20,000(A/P, 8\%, 12) - 9000 - 2000[(P/F, 8\%, 4) + (P/F, 8\%, 8)](A/P, 8\%, 12) \\
 &\quad + 0.4(20,000)(A/F, 8\%, 12) \\
 &= -20,000(0.13270) - 9000 - 2000[0.7350 + 0.5403](0.13270) + 8000(0.05270) \\
 &= \$-11,571
 \end{aligned}$$

The Hamilton unit is considerably less costly on an annual equivalent basis.

If the projects are **independent**, the AW at the MARR is calculated. All projects with $AW \geq 0$ are acceptable.

6.4 AW of a Permanent Investment ● ● ●

This section discusses the annual worth equivalent of the capitalized cost introduced in Section 5.5. Evaluation of public sector projects, such as flood control dams, irrigation canals, bridges, or other large-scale projects, requires the comparison of alternatives that have such long



Independent project selection

lives that they may be considered infinite in economic analysis terms. For this type of analysis, the annual worth (and capital recovery amount) of the initial investment is the perpetual annual interest on the initial investment, that is, $A = Pi = (CC)i$. This is Equation [5.2].

Cash flows recurring at regular or irregular intervals are handled exactly as in conventional AW computations; convert them to equivalent uniform annual amounts A for one cycle. This automatically annualizes them for each succeeding life cycle. Add all the A values to the CR amount to find total AW, as in Equation [6.2].

EXAMPLE 6.5

The U.S. Bureau of Reclamation is considering three proposals for increasing the capacity of the main drainage canal in an agricultural region of Nebraska. Proposal A requires dredging the canal to remove sediment and weeds that have accumulated during previous years' operation. The capacity of the canal will have to be maintained in the future near its design peak flow because of increased water demand. The Bureau is planning to purchase the dredging equipment and accessories for \$650,000. The equipment is expected to have a 10-year life with a \$17,000 salvage value. The annual operating costs are estimated to total \$50,000. To control weeds in the canal itself and along the banks, environmentally safe herbicides will be sprayed during the irrigation season. The yearly cost of the weed control program is expected to be \$120,000.

Proposal B is to line the canal with concrete at an initial cost of \$4 million. The lining is assumed to be permanent, but minor maintenance will be required every year at a cost of \$5000. In addition, lining repairs will have to be made every 5 years at a cost of \$30,000.

Proposal C is to construct a new pipeline along a different route. Estimates are an initial cost of \$6 million, annual maintenance of \$3000 for right-of-way, and a life of 50 years.

Compare the alternatives on the basis of annual worth, using an interest rate of 5% per year.

Solution

Since this is an investment for a permanent project, compute the AW for one cycle of all recurring costs. For proposals A and C, the CR values are found using Equation [6.3], with $n_A = 10$ and $n_C = 50$, respectively. For proposal B, the CR is simply $P(i)$.

Proposal A

CR of dredging equipment:	
$-650,000(A/P,5\%,10) + 17,000(A/F,5\%,10)$	\$ -82,824
Annual cost of dredging	-50,000
Annual cost of weed control	-120,000
	<hr/>
	\$ -252,824

Proposal B

CR of initial investment: $-4,000,000(0.05)$	\$ -200,000
Annual maintenance cost	-5,000
Lining repair cost: $-30,000(A/F,5\%,5)$	-5,429
	<hr/>
	\$ -210,429

Proposal C

CR of pipeline: $-6,000,000(A/P,5\%,50)$	\$ -328,680
Annual maintenance cost	-3,000
	<hr/>
	\$ -331,680

Proposal B, which is a permanent solution, is selected due to its lowest AW of costs.

Comment

Note the use of the A/F factor for the lining repair cost in proposal B. The A/F factor is used instead of A/P , because the lining repair cost begins in year 5, not year 0, and continues indefinitely at 5-year intervals.

If the 50-year life of proposal C is considered infinite, $CR = P(i) = \$ -300,000$, instead of $\$ -328,680$ for $n = 50$. This is a small economic difference. How long lives of 40 or more years are treated economically is a matter of "local" practice.

EXAMPLE 6.6

At the end of each year, all owners and employees at Bell County Utility Cooperative are given a bonus check based on the net profit of the Coop for the previous year. Bart just received his bonus in the amount of \$8530. He plans to invest it in an annuity program that returns 7% per year. Bart's long-term plans are to quit the Coop job some years in the future when he is still young enough to start his own business. Part of his future living expenses will be paid from the proceeds that this year's bonus accumulates over his remaining years at the Coop.

- Use a spreadsheet to determine the amount of annual year-end withdrawal that he can anticipate (starting 1 year after he quits) that will continue forever. He is thinking of working 15 or 20 more years.
- Determine the amount Bart must accumulate after 15 and 20 years to generate \$3000 per year forever.

Solution by Spreadsheet

- Figure 6–5 presents the cash flow diagram for $n = 15$ years of accumulation at 7% per year on the \$8530 deposited now. The accumulated amount after $n = 15$ years is indicated as $F_{\text{after } 15} = ?$ and the withdrawal series starts at the end of year 16. The diagram for $n = 20$ would appear the same, except there is a 20-year accumulation period.

The spreadsheet in Figure 6–6 shows the functions and answers for $n = 15$ years in columns C and D. The FV function displays the total accumulated at 7% after 15 years as \$23,535. The perpetual withdrawal is determined by viewing this accumulated amount as a P value and by applying the formula

$$A = P(i) = 23,535(0.07) = \$1647 \text{ per year}$$

The spreadsheet function =D9*B7 performs this same computation in cell reference format in column D.

Answers for $n = 20$ years are displayed in column E. At a consistent 7% per year return, Bart's perpetual income is estimated as \$1647 after 15 years, or \$2311 per year if he waits for 20 years.

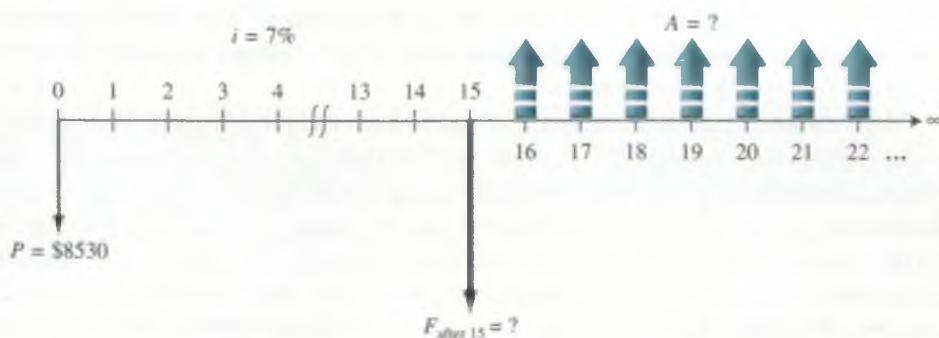


Figure 6–5
Diagram for a perpetual series starting after 15 years of accumulation, Example 6.6.

	A	B	C	D	E
3			Functions for $n = 15$ years		Number of years, n
4				15	20
7 Interest rate, % per year		7%			
8 Amount deposited today		\$8,530			
9 Amount accumulated after n years			=FV(B7,D4,B8)	\$ 23,535	\$ 33,008
10 (a) Withdrawal forever, \$ per year			=D9*B7	\$ 1,647	\$ 2,311
13 (b) Required accumulation for \$3000/year	7%		=3000/B13	\$ 42,857	\$ 42,857
15 Accumulation years for \$3000 per year	23.86		=NPER(B7,-B13,42857)		
16					

Figure 6–6
Spreadsheet solution, Example 6.6.

- (b) To obtain a perpetual annual withdrawal of \$3000, it is necessary to determine how much must be accumulated 1 year before the first withdrawal of \$3000. This is an application of the relation $A = P(i)$ solved for P , or

$$P = \frac{A}{i} = \frac{3000}{0.07} = \$42,857$$

This P value is independent of how long Bart works at the Coop, because he must accumulate this amount to achieve his goal. Figure 6–6, row 13, shows the function and result. Note that the number of years n does not enter into the function = 3000/B13.

Comment

The NPER function can be used to determine how many years it will take for the current amount of \$8530 to accumulate the required \$42,857 at 7% per year. The row 15 function indicates that Bart will have to work at the Coop for just under 24 additional years.

6.5 Life-Cycle Cost Analysis ● ● ●

The PW and AW analysis techniques discussed thus far have concentrated on estimates for first cost P , annual operating and maintenance costs (AOC or M&O), salvage value S , and predictable periodic repair and upgrade costs, plus any revenue estimates that may favor one alternative over another. There are usually a host of additional costs involved when the complete project life costs are evaluated. A life-cycle cost analysis includes these additional estimates to the extent that they can be reliably determined.

Life-cycle cost (LCC) analysis utilizes AW or PW methods to evaluate cost estimates for the entire life cycle of one or more projects. Estimates will cover the **entire life span** from the early conceptual stage, through the design and development stages, throughout the operating stage, and even the phaseout and disposal stages. Both **direct and indirect costs** are included to the extent possible, and differences in revenue and savings projections between alternatives are included.

Some typical LCC applications are life-span analysis for military and commercial aircraft, new manufacturing plants, new automobile models, new and expanded product lines, and government systems at federal and state levels. For example, the U.S. Department of Defense requires that a government contractor include an LCC budget and analysis in the originating proposal for most defense systems.

Most commonly the LCC analysis includes costs, and the AW method is used for the analysis, especially if only one alternative is evaluated. If there are expected revenue or other benefit differences between alternatives, a PW analysis is recommended. Public sector projects are usually evaluated using a benefit/cost analysis (Chapter 9), rather than LCC analysis, because estimates to the citizenry are difficult to make with much accuracy. The *direct costs* mentioned above include material, human labor, equipment, supplies, and other costs directly related to a product, process, or system. Some examples of *indirect cost components* are taxes, management, legal, warranty, quality, human resources, insurance, software, purchasing, etc. Direct and indirect costs are discussed further in Chapter 15.

LCC analysis is most effectively applied when a substantial percentage of the life span (post-purchase) costs, relative to the initial investment, will be expended in direct and indirect operating, maintenance, and similar costs once the system is operational. For example, the evaluation of two equipment purchase alternatives with expected useful lives of 5 years and M&O costs of 5% to 10% of the initial investment does not require an LCC analysis. However, let's assume that Exxon-Mobil wants to evaluate the design, construction, operation, and support of a new type and style of tanker that can transport oil over long distances of ocean. If the initial costs are in the \$100 millions with support and operating costs ranging from 25% to 35% of this amount over a 25-year life, the logic of an LCC analysis will offer a better understanding of the economic viability of the project.

To understand how a LCC analysis works, first we must understand the phases and stages of systems engineering or systems development. Many books and manuals are available on systems development and analysis. Generally, the LCC estimates may be categorized into a simplified

format for the major phases of *acquisition*, *operation*, and *phaseout/disposal*, and their respective stages.

Acquisition phase: all activities prior to the delivery of products and services.

- Requirements definition stage—Includes determination of user/customer needs, assessing them relative to the anticipated system, and preparation of the system requirements documentation.
- Preliminary design stage—Includes feasibility study, conceptual, and early-stage plans; final go–no go decision is probably made here.
- Detailed design stage—Includes detailed plans for resources—capital, human, facilities, information systems, marketing, etc.; there is some acquisition of assets, if economically justifiable.

Operation phase: all activities are functioning, products and services are available.

- Construction and implementation stage—Includes purchases, construction, and implementation of system components; testing; preparation, etc.
- Usage stage—Uses the system to generate products and services; the largest portion of the life cycle.

Phaseout and disposal phase: covers all activities to transition to a new system; removal/recycling/disposal of old system.

EXAMPLE 6.7

In the 1860s, General Mills Inc. and Pillsbury Inc. both started in the flour business in the Twin Cities of Minneapolis–St. Paul, Minnesota. In the decade of 2000 to 2010, General Mills purchased Pillsbury for a combination cash and stock deal worth more than \$10 billion and integrated the product lines. Food engineers, food designers, and food safety experts made many cost estimates as they determined the needs of consumers and the combined company's ability to technologically and safely produce and market new food products. At this point only cost estimates have been addressed—no revenues or profits.

Assume that the major cost estimates below have been made based on a 6-month study about two new products that could have a 10-year life span for the company. Use LCC analysis at the industry MARR of 18% to determine the size of the commitment in AW terms. (Time is indicated in product-years. Since all estimates are for costs, they are not preceded by a minus sign.)

Consumer habits study (year 0)	\$0.5 million
Preliminary food product design (year 1)	0.9 million
Preliminary equipment/plant design (year 1)	0.5 million
Detail product designs and test marketing (years 1, 2)	1.5 million each year
Detail equipment/plant design (year 2)	1.0 million
Equipment acquisition (years 1 and 2)	\$2.0 million each year
Current equipment upgrades (year 2)	1.75 million
New equipment purchases (years 4 and 8)	2.0 million (year 4) + 10% per purchase thereafter
Annual equipment operating cost (AOC) (years 3–10)	200,000 (year 3) + 4% per year thereafter
Marketing, year 2	\$8.0 million
years 3–10	5.0 million (year 3) and –0.2 million per year thereafter
year 5 only	3.0 million extra
Human resources, 100 new employees for 2000 hours per year (years 3–10)	\$20 per hour (year 3) + 5% per year
Phaseout and disposal (years 9 and 10)	\$1.0 million each year

Solution

LCC analysis can get complicated rapidly due to the number of elements involved. Calculate the PW by phase and stage, add all PW values, then find the AW over 10 years. Values are in \$1 million units.

Acquisition phase:

Requirements definition: consumer study

$$PW = \$0.5$$

Preliminary design: product and equipment

$$PW = 1.4(P/F, 18\%, 1) = \$1.187$$

Detailed design: product and test marketing, and equipment

$$PW = 1.5(P/A, 18\%, 2) + 1.0(P/F, 18\%, 2) = \$3.067$$

Operation phase:

Construction and implementation: equipment and AOC

$$PW = 2.0(P/A, 18\%, 2) + 1.75(P/F, 18\%, 2) + 2.0(P/F, 18\%, 4) + 2.2(P/F, 18\%, 8)$$

$$+ 0.2 \left[\frac{1 - \left(\frac{1.04}{1.18} \right)^8}{0.14} \right] (P/F, 18\%, 2) = \$6.512$$

Use: marketing

$$\begin{aligned} PW &= 8.0(P/F, 18\%, 2) + [5.0(P/A, 18\%, 8) - 0.2(P/G, 18\%, 8)](P/F, 18\%, 2) \\ &\quad + 3.0(P/F, 18\%, 5) \\ &= \$20.144 \end{aligned}$$

Use: human resources: (100 employees)(2000 h/yr)(\$20/h) = \$4.0 million in year 3

$$PW = 4.0 \left[\frac{1 - \left(\frac{1.05}{1.18} \right)^8}{0.13} \right] (P/F, 18\%, 2) = \$13.412$$

Phaseout phase:

$$PW = 1.0(P/A, 18\%, 2)(P/F, 18\%, 8) = \$0.416$$

The sum of all PW of costs is $PW = \$45.238$ million. Finally, determine the AW over the expected 10-year life span.

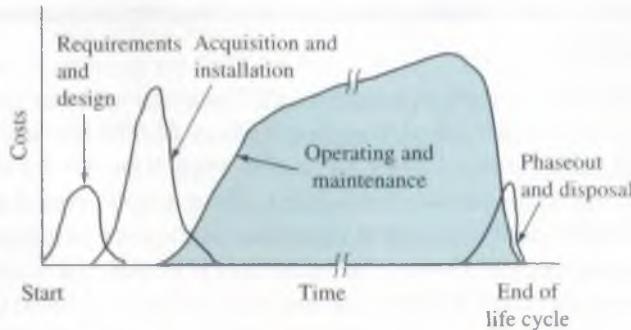
$$AW = 45.238 \text{ million}(A/P, 18\%, 10) = \$10.066 \text{ million per year}$$

This is the LCC estimate of the equivalent annual commitment to the two proposed products.

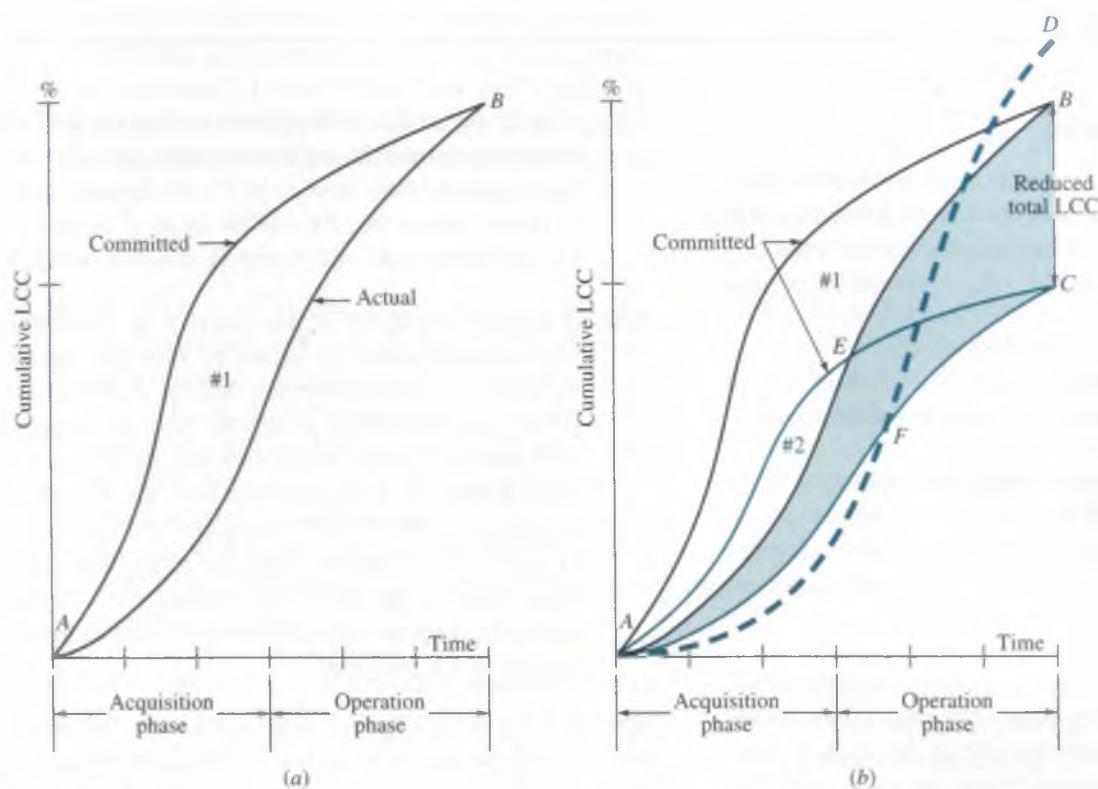
Often the alternatives compared by LCC do not have the same level of output or amount of usage. For example, if one alternative will produce 20 million units per year and a second alternative will operate at 35 million per year, the AW values should be compared on a currency unit/unit produced basis, such as dollar/unit or euro/hour operated.

Figure 6–7 presents an overview of how costs may be distributed over an entire life cycle. For some systems, typically defense systems, operating and maintenance costs rise fast after acquisition and remain high until phaseout occurs.

The total LCC for a system is established or locked in early in the life cycle. It is not unusual to have 75% to 85% of the entire life span LCC committed during the preliminary and detail

**Figure 6-7**

Typical distribution of life-cycle costs of the phases for one life cycle.

**Figure 6-8**

LCC envelopes for committed and actual costs: (a) design 1, (b) improved design 2.

design stages. As shown in Figure 6-8a, the actual or observed LCC (bottom curve AB) will trail the committed LCC throughout the life span (unless some major design flaw increases the total LCC of design #1 above point B).

The potential for significantly reducing total LCC occurs primarily during the early stages. A more effective design and more efficient equipment can reposition the envelope to design #2 in Figure 6-8b. Now the committed LCC curve AEC is below AB at all points, as is the actual LCC curve AFC. It is this lower envelope #2 we seek. The shaded area represents the reduction in actual LCC.

Even though an effective LCC envelope may be established early in the acquisition phase, it is not uncommon that unplanned cost-saving measures are introduced during the acquisition phase and early operation phase. These apparent “savings” may actually increase the total LCC, as shown by curve AFD. This style of ad hoc cost savings, often imposed by management early in the design stage and/or construction stage, can substantially increase costs later, especially in the after-sale portion of the use stage. For example, the use of inferior-strength concrete and steel has been the cause of structural failures many times, thus increasing the overall life span LCC.

CHAPTER SUMMARY

The annual worth method of comparing alternatives is often preferred to the present worth method, because the AW comparison is performed for only one life cycle. This is a distinct advantage when comparing different-life alternatives. AW for the first life cycle is the AW for the second, third, and all succeeding life cycles, under certain assumptions. When a study period is specified, the AW calculation is determined for that time period, regardless of the lives of the alternatives.

For infinite-life (perpetual) alternatives, the initial cost is annualized simply by multiplying P by i . For finite-life alternatives, the AW through one life cycle is equal to the perpetual equivalent annual worth.

Life-cycle cost analysis is appropriate for systems that have a large percentage of costs in operating and maintenance. LCC analysis helps in the analysis of all costs from design to operation to disposal phases.

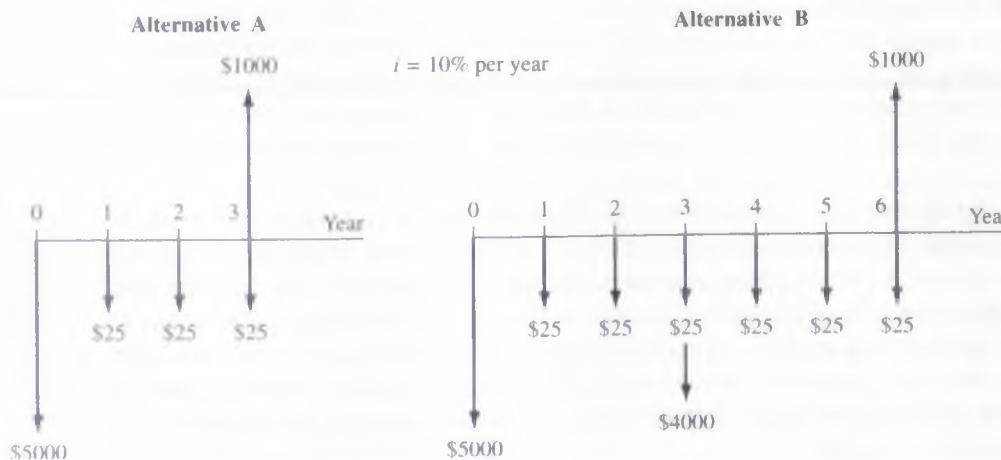
PROBLEMS

Annual Worth Calculations

- 6.1 If you are asked to provide an annual worth (AW) comparison of alternatives after a present worth (PW) comparison has already been done, what factor multiplied by the PW values provide the correct AW values?
- 6.2 List three assumptions that are inherent in the annual worth method of comparing alternatives.
- 6.3 In the annual worth method of comparing alternatives that have different lives, why do you calculate the AW of the alternatives *over their respective life cycles* instead of over the least common multiple of their lives?
- 6.4 James developed the two cash flow diagrams shown at the bottom of this page. The cash flows for alternative B represent two life cycles of A. Calculate the annual worth value of each over the respective life cycles to demonstrate that they are the same. Use an interest rate of 10% per year.
- 6.5 An asset with a first cost of \$20,000 has an annual operating cost of \$12,000 and a \$4000 salvage value

after its 4-year life. If the project will be needed for 6 years, what would the market (salvage) value of the 2-year-old asset have to be for the annual worth to be the same as it is for one life cycle of the asset? Use an interest rate of 10% per year.

- 6.6 A sports mortgage is an innovative way to finance cash-strapped sports programs by allowing fans to sign up to pay a “mortgage” for the right to buy good seats at football games for several decades with season tickets locked in at current prices. At Notre Dame, the locked-in price period is 50 years. If a fan pays a \$130,000 “mortgage” fee now (i.e., in year 0) when season tickets are selling for \$290 each, what is the equivalent annual cost of the football tickets over the 50-year period at an interest rate of 8% per year?
- 6.7 If a fan buys a sports mortgage to USC football games by paying \$130,000 in 10 equal payments *starting now*, and then pays a fixed price of \$290 per year for 50 years (starting 1 year from now) for season tickets, what is the AW in years 1 through 50 of the season tickets at 8% per year interest?



- 6.8 Eight years ago, Ohio Valley Trucking purchased a large-capacity dump truck for \$115,000 to provide short-haul earthmoving services. The company sold it today for \$45,000. Operating and maintenance costs averaged \$10,500 per year. A complete overhaul at the end of year 4 cost an extra \$3600. Calculate the annual cost of the truck at 8% per year interest.
- 6.9 A major repair on the suspension system of a 5-year-old car cost \$2000 because the warranty expired after 3 years of ownership. The cost of periodic maintenance has been \$800 every 2 years. If the owner donates the car to charity after 8 years of ownership, what is the equivalent annual cost of the repair and maintenance in the 8-year period of ownership? Use an interest rate of 8% per year, and assume that the owner paid the \$800 maintenance cost immediately before donating the car in year 8.

Capital Recovery

- 6.10 Ten years ago, Jacobson Recovery purchased a wrecker for \$285,000 to move disabled 18-wheelers. He anticipated a salvage value of \$50,000 after 10 years. During this time his average annual revenue totaled \$52,000. (a) Did he recover his investment and a 12% per year return? (b) If the annual M&O cost was \$10,000 the first year and increased by a constant \$1000 per year, was the AW positive or negative at 12% per year? Assume the \$50,000 salvage was realized.
- 6.11 Sylvia has received a \$500,000 inheritance from her favorite, recently deceased aunt in Hawaii. Sylvia is planning to purchase a condo in Hawaii in the same area where her aunt lived all her life and to rent it to vacationers. She hopes to make 8% per year on this purchase over an ownership period of 20 years. The condo's total first cost is \$500,000, and she conservatively expects to sell it for 90% of the purchase price. No annual M&O costs are considered in the analysis. (a) What is the capital recovery amount? (b) If there is a real boom in rental real estate 10 years in the future, what sales price (as a percentage of original purchase price) is necessary at that time (year 10) to realize the same amount as the 8% return expected over the 20-year ownership period?
- 6.12 Humana Hospital Corporation installed a new MRI machine at a cost of \$750,000 this year in its medical professional clinic in Cedar Park. This state-of-the-art system is expected to be used for 5 years and then sold for \$75,000. Humana uses a return requirement of 24% per year for all of its medical diagnostic equipment. As a bioengineering student

currently serving a coop semester on the management staff of Humana Corporation in Louisville, Kentucky, you are asked to determine the minimum revenue required each year to realize the expected recovery and return. Also, you are asked to draw two cash flow diagrams, one showing the MRI purchase and sale cash flow and a second depicting the required capital recovery each year.

Alternative Comparison

- 6.13 Polypropylene wall caps, used for covering exterior vents for kitchen cooktops, bathroom fans, dryers, and other building air exhausts, can be made by two different methods. Method X will have a first cost of \$75,000, an operating cost of \$32,000 per year, and a \$9000 salvage value after 4 years. Method Y will have a first cost of \$140,000, an operating cost of \$24,000 per year, and a \$19,000 salvage value after its 4-year life. At an interest rate of 10% per year, which method should be used on the basis of an annual worth analysis?
- 6.14 Nissan's all-electric car, the Leaf, has a base price of \$32,780 in the United States, but it is eligible for a \$7500 federal tax credit. A consulting engineering company wants to evaluate the purchase or lease of one of the vehicles for use by its employees traveling to job sites in the local area. The cost for leasing the vehicle will be \$4200 per year (payable at the end of each year) after an initialization charge of \$2500 paid now. If the company purchases the vehicle, it will also purchase a home charging station for \$2200 that will be partially offset by a 50% tax credit. If the company expects to be able to sell the car and charging station for 40% of the base price of the car alone at the end of 3 years, should the company purchase or lease the car? Use an interest rate of 10% per year and annual worth analysis.
- 6.15 A new structural design software package is available for analyzing and designing three-sided guyed towers and three- and four-sided self-supporting towers. A single-user license will cost \$6000 per year. A site license has a one-time cost of \$22,000. A structural engineering consulting company is trying to decide between two alternatives: buy a single-user license *now* and one each year for the next 3 years (which will provide 4 years of service), or buy a site license *now*. Determine which strategy should be adopted at an interest rate of 10% per year for a 4-year planning period using the annual worth method of evaluation.
- 6.16 The city council in a certain southwestern city is considering whether to construct permanent restrooms in 22 of its smaller parks (i.e., parks of less

than 12 acres) or pay for portable toilets on a year-round basis. The cost of constructing the 22 permanent restrooms will be \$3.8 million. The 22 portable restrooms can be rented for \$7500 each for 1 year. The service life of a permanent restroom is 20 years. Using an interest rate of 6% per year and an annual worth analysis, determine if the city should build the permanent restrooms or lease the portable ones.

- 6.17 A remotely located air sampling station can be powered by solar cells or by running an above ground electric line to the site and using conventional power. Solar cells will cost \$16,600 to install and will have a useful life of 5 years with no salvage value. Annual costs for inspection, cleaning, etc., are expected to be \$2400. A new power line will cost \$31,000 to install, with power costs expected to be \$1000 per year. Since the air sampling project will end in 5 years, the salvage value of the line is considered to be zero. At an interest rate of 10% per year, (a) which alternative should be selected on the basis of an annual worth analysis and (b) what must be the first cost of the above ground line to make the two alternatives equally attractive economically?

- 6.18 The cash flows for two small raw water treatment systems are shown. Determine which should be selected on the basis of an annual worth analysis at 10% per year interest.

	MF	UF
First cost, \$	-33,000	-51,000
Annual cost, \$ per year	-8,000	-3,500
Salvage value, \$	4,000	11,000
Life, years	3	6

- 6.19 PGM Consulting is under contract to Montgomery County for evaluating alternatives that use a robotic, liquid-propelled "pig" to periodically inspect the interior of buried potable water pipes for leakage, corrosion, weld strength, movement over time, and a variety of other parameters. Two equivalent robot instruments are available. Robot Joeboy will have a first cost of \$85,000, annual M&O costs of \$30,000, and a \$40,000 salvage value after 3 years. Robot Watcheye will have a first cost of \$125,000, annual M&O costs of \$27,000, and a \$33,000 salvage value after its 5-year life. Assume an interest rate of 8% per year.
 (a) Which robot is the better economic option?
 (b) Using the spreadsheet Goal Seek tool, determine the first cost of the robot *not* selected in (a) so that it will be the economic selection.

- 6.20 TT Racing and Performance Motor Corporation wishes to evaluate two alternative CNC machines

for NHRA engine building. Use the AW method at 10% per year to select the better alternative.

	Machine R	Machine S
First cost, \$	-250,000	-370,500
Annual operating cost, \$ per year	-40,000	-50,000
Salvage value, \$	20,000	30,000
Life, years	3	5

- 6.21 The maintenance and operation (M&O) cost of front-end loaders working under harsh environmental conditions tends to increase by a constant \$1200 per year for the first 5 years of operation. For a loader that has a first cost of \$39,000 and first-year M&O cost of \$17,000, compare the equivalent annual worth of a loader kept for 4 years with one kept for 5 years at an interest rate of 12% per year. The salvage value of a used loader is \$23,000 after 4 years and \$18,000 after 5 years.

- 6.22 You work for Midstates Solar Power. A manager asked you to determine which of the following two machines will have the lower (a) capital recovery and (b) equivalent annual total cost. Machine Semi2 has a first cost of \$80,000 and an operating cost of \$21,000 in year 1, increasing by \$500 per year through year 5, after which time it will have a salvage value of \$13,000. Machine Auto1 has a first cost of \$62,000 and an operating cost of \$21,000 in year 1, increasing by 8% per year through year 5, after which time it will have a scavenge value of \$2000. Utilize an interest rate of 10% per year to determine both estimates.

Permanent Investments

- 6.23 The State of Chiapas, Mexico, decided to fund a program for literacy. The first cost is \$200,000 now, and an update budget of \$100,000 every 7 years *forever* is requested. Determine the perpetual equivalent annual cost at an interest rate of 10% per year.

- 6.24 Calculate the perpetual equivalent annual cost (years 1 through infinity) of \$5 million in year 0, \$2 million in year 10, and \$100,000 in years 11 through infinity. Use an interest rate of 10% per year.

- 6.25 A Pennsylvania coal mining operation has installed an in-shaft monitoring system for oxygen tank and gear readiness for emergencies. Based on maintenance patterns for previous systems, there are no maintenance costs for the first 2 years, they

increase for a time period, and then they level off. Maintenance costs are expected to be \$150,000 in year 3, \$175,000 in year 4, and amounts increasing by \$25,000 per year through year 6 and remain constant thereafter for the expected 10-year life of the system. If similar systems with similar costs will replace the current one, determine the perpetual equivalent annual maintenance cost at $i = 10\%$ per year.

- 6.26 Compare two alternatives for a security system surrounding a power distribution substation using annual worth analysis and an interest rate of 10% per year.

	Condi	Torro
First cost, \$	-25,000	-130,000
Annual cost, \$ per year	-9,000	-2,500
Salvage value, \$	3,000	150,000
Life, years	3	∞

- 6.27 A new bridge across the Allegheny River in Pittsburgh is expected to be permanent and will have an initial cost of \$30 million. This bridge must be resurfaced every 5 years at a cost of \$1 million. The annual inspection and operating costs are estimated to be \$50,000. Determine its equivalent annual worth at an interest rate of 10% per year.
- 6.28 For the cash flows shown, use an annual worth comparison and an interest rate of 10% per year.
- Determine the alternative that is economically best.
 - Determine the first cost required for each of the two alternatives not selected in (a) so that all alternatives are equally acceptable. Use a spreadsheet to answer this question.

	X	Y	Z
First cost, \$	-90,000	-400,000	-650,000
Annual cost, \$ per year	-40,000	-20,000	-13,000
Overhaul every 10 years, \$	—	—	-80,000
Salvage value, \$	7,000	25,000	200,000
Life, years	3	10	∞

Life-Cycle Cost

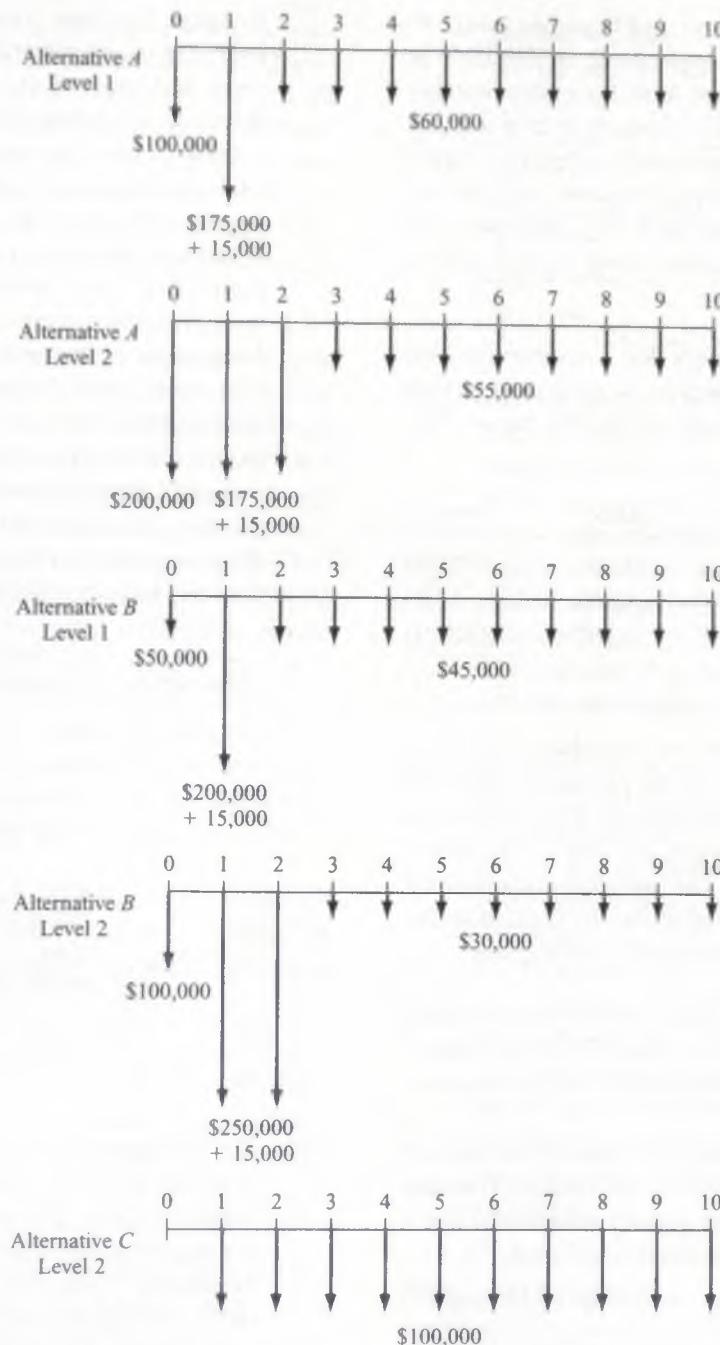
- 6.29 Blanton Agriculture of Santa Monica, California, offers different types and levels of irrigation water conservation systems for use in areas where groundwater depletion is a serious concern. A large farming corporation in India, where depletion is occurring at an alarming rate of 1.6 inches (4 centimeters) per year due to exponential growth

in usage for crop irrigation, is considering the purchase of one of the Blanton systems. There are three options with two levels of automation for the first two options. The estimated costs and associated cash flow diagrams over a 10-year period are summarized below and on the next page, respectively, for each of the five alternatives. Costs are categorized as design (Des), development (Dev), and operation (Oper). For alternatives A and B, there is an extra cost of \$15,000 per installation year to maintain the manual system in place now. Level 2 development costs are distributed equally over a 2-year period. Alternative C is a retrofit of the current manual system with no design or development costs, and there is no level 1 option. At an interest rate of 10% per year and a 10-year study period, determine which alternative and level has the lowest LCC.

Alternative	Cost Component	Cost	Level
		Level 1	Level 2
A	Des	\$100,000	\$200,000
	Dev	175,000	350,000
	Oper	60,000	55,000
	Install time	1 year	2 years
B	Des	\$ 50,000	\$100,000
	Dev	200,000	500,000
	Oper	45,000	30,000
	Install time	1 year	2 years
C	Oper		\$100,000

- 6.30 The Pentagon asked a defense contractor to estimate the life-cycle cost for a proposed light-duty support vehicle. The list of items included the following categories: R&D costs (R&D), nonrecurring investment costs (NRI), recurring upgrade costs (RU), scheduled and unscheduled maintenance costs (Maint), equipment usage costs (Equip), and phaseout/disposal costs (Po/D). Use the cost estimates (shown in \$1 million) for the 20-year life cycle to calculate the annual LCC at an interest rate of 7% per year.

Year	R&D	NRI	RU	Maint	Equip	Po/D
0	5.5	1.1				
1	3.5					
2	2.5					
3	0.5	5.2	1.3	0.6	1.5	
4		10.5	3.1	1.4	3.6	
5		10.5	4.2	1.6	5.3	
6–10			6.5	2.7	7.8	
11 on				2.2	3.5	8.5
18–20						2.7



Cash flow diagrams for Problem 6.29

- 6.31 A manufacturing software engineer at a major aerospace corporation has been assigned the management responsibility of a project to design, build, test, and implement AREMSS, a new-generation automated scheduling system for routine and expedited maintenance. Reports on the disposition of each service will also be entered by field personnel, then filed and archived by the system. The initial application will be on existing Air Force in-flight refueling aircraft. The system is expected to be widely used

over time for other aircraft maintenance scheduling. Once it is fully implemented, enhancements will have to be made, but the system is expected to serve as a worldwide scheduler for up to 15,000 separate aircraft. The engineer, who must make a presentation next week of the best estimates of costs over a 20-year life period, has decided to use the life-cycle cost approach of cost estimations. Use the following information to determine the current annual LCC at 6% per year for the AREMSS scheduling system.

Cost Category	Cost in Year (\$ millions)							
	1	2	3	4	5	6 on	10	18
Field study	0.5							
Design of system	2.1	1.2	0.5					
Software design		0.6	0.9					
Hardware purchases			5.1					
Beta testing		0.1	0.2					
Users manual development		0.1	0.1	0.2	0.2	0.06		
System implementation			1.3	0.7				
Field hardware		0.4	6.0	2.9				
Training trainers			0.3	2.5	2.5	0.7		
Software upgrades				0.6	3.0	3.7		

6.32 The U.S. Army received two proposals for a turnkey design-build project for barracks for infantry unit soldiers in training. Proposal A involves an off-the-shelf "bare-bones" design and standard grade construction of walls, windows, doors, and other features. With this option, heating and cooling costs will be greater, maintenance costs will be higher, and replacement will be sooner than for proposal B. The initial cost for A will be \$750,000. Heating and cooling costs will average \$72,000 per year, with maintenance costs averaging \$24,000 per year. Minor remodeling will be required in years 5, 10, and 15 at a cost of \$150,000 each time in order to render the units usable for 20 years. They will have no salvage value.

Proposal B will include tailored design and construction costs of \$1.1 million initially, with

estimated heating and cooling costs of \$36,000 per year and maintenance costs of \$12,000 per year. There will be no salvage value at the end of the 20-year life. Which proposal should be accepted on the basis of an annual life-cycle cost analysis, if the interest rate is 6% per year?

6.33 A medium-size municipality plans to develop a software system to assist in project selection during the next 10 years. A life-cycle cost approach has been used to categorize costs into development, programming, operating, and support costs for each alternative. There are three alternatives under consideration, identified as M, N, and O. The costs are summarized below. Use an annual life-cycle cost approach to identify the best alternative at 8% per year.

Alternative	Component	Estimated Cost
M	Development	\$250,000 now, \$150,000 years 1–4
	Programming	\$45,000 now, \$35,000 years 1, 2
	Operation	\$50,000 years 1–10
	Support	\$30,000 years 1–5
N	Development	\$10,000 now
	Programming	\$45,000 year 0, \$30,000 years 1–3
	Operation	\$80,000 years 1–10
	Support	\$40,000 years 1–10
O	Operation	\$175,000 years 1–10

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

6.34 All of the following are fundamental assumptions for the annual worth method of analysis *except*:

- (a) The alternatives will be needed for only one life cycle.
- (b) The services provided are needed for at least the LCM of the lives of the alternatives.
- (c) The selected alternative will be repeated for the succeeding life cycles in exactly the same manner as for the first life cycle.
- (d) All cash flows will have the same estimated values in every life cycle.

6.35 When comparing five alternatives that have different lives by the AW method, you must:

- (a) Find the AW of each over the life of the longest-lived alternative.
- (b) Find the AW of each over the life of the shortest-lived alternative.
- (c) Find the AW of each over the LCM of all of the alternatives.
- (d) Find the AW of each alternative over its life without considering the life of the other alternatives.

6.36 The annual worth of an alternative can be calculated from the alternative's:

- (a) Present worth by multiplying by $(A/P,i,n)$
- (b) Future worth by multiplying by $(F/A,i,n)$
- (c) Either (a) or (b)
- (d) Neither (a) nor (b)

6.37 The alternatives shown are to be compared on the basis of annual worth. At an interest rate of 10% per year, the values of n that you could use in the $(A/P,i,n)$ factors to make a correct comparison by the annual worth method are:

	A	B
First cost, \$	-50,000	-90,000
Annual cost, \$ per year	-10,000	-4,000
Salvage value, \$	13,000	15,000
Life, years	3	6

- (a) $n = 3$ years for A and 3 years for B
- (b) $n = 3$ years for A and 6 years for B
- (c) Either (a) or (b)
- (d) Neither (a) nor (b)

- 6.38 The alternatives shown are to be compared on the basis of a perpetual (i.e., forever) equivalent annual worth. At an interest rate of 10% per year, the equation that represents the perpetual AW of X1 is:

	X1	Y1
First cost, \$	-50,000	-90,000
Annual cost, \$ per year	-10,000	-4,000
Salvage value, \$	13,000	15,000
Life, years	3	6

- (a) $AW_{X1} = -50,000(0.10) - 10,000 + 13,000(0.10)$
- (b) $AW_{X1} = -50,000(0.10) - 10,000 + 13,000(A/F, 10\%, 3)$
- (c) $AW_{X1} = -50,000(0.10) - 10,000 - 37,000(P/F, 10\%, 3)(0.10) + 13,000(0.10)$
- (d) $AW_{X1} = -50,000(A/P, 10\%, 3) - 10,000 + 13,000(A/F, 10\%, 3)$

- 6.39 To get the AW of a cash flow of \$10,000 that occurs every 10 years forever, with the first one occurring 10 years from now, you should:
- (a) Multiply \$10,000 by $(A/P, i, 10)$.
 - (b) Multiply \$10,000 by $(A/F, i, 10)$.
 - (c) Multiply \$10,000 by i .
 - (d) Multiply \$10,000 by $(A/F, i, n)$ and then multiply by i .

Problems 6.40 through 6.43 refer to the following estimates.

The alternatives are mutually exclusive and the MARR is 6% per year.

	Vendor 1	Vendor 2	Vendor 3
First cost, \$	-200,000	-550,000	-1,000,000
Annual cost, \$ per year	-50,000	-20,000	-10,000
Revenue, \$ per year	120,000	120,000	110,000
Salvage value, \$	25,000	0	500,000
Life, years	10	15	∞

- 6.40 The annual worth of vendor 2 cash flow estimates is closest to:
- (a) \$-63,370
 - (b) \$43,370
 - (c) \$-43,370
 - (d) \$63,370

- 6.41 Of the following three relations, the correct one or ones to calculate the annual worth of vendor 1 cash flow estimates is (note: all dollar values are in thousands):

$$\text{Relation 1: } AW_1 = -200(A/P, 6\%, 10) + 70 + 25(A/F, 6\%, 10)$$

$$\text{Relation 2: } AW_1 = [-200 - 50(P/A, 6\%, 10) + 120(P/A, 6\%, 10) + 25(P/F, 6\%, 10)](A/P, 6\%, 10)$$

$$\text{Relation 3: } AW_1 = -200(F/P, 6\%, 10) + 25 + (-50 + 120)(A/P, 6\%, 10)$$

- (a) 1 and 3
- (b) Only 1
- (c) 1 and 2
- (d) Only 3

- 6.42 The AW values for the alternatives are listed below. The vendor or vendors that should be recommended is:

$$AW_1 = \$44,723 \quad AW_2 = \$43,370 \\ AW_3 = \$40,000$$

- (a) 1 and 2
- (b) 3
- (c) 2
- (d) 1

- 6.43 The capital recovery amount for vendor 3 is:

- (a) \$40,000 per year
- (b) \$60,000 per year
- (c) \$43,370 per year
- (d) \$100,000 per year

- 6.44 If a revenue alternative has a negative AW value and it was correctly calculated, it means the following:

- (a) The equivalent annual worth of revenues does not exceed that of the costs.
- (b) The estimates are wrong somewhere.
- (c) A minus or plus sign of a cash flow was entered incorrectly into the PMT spreadsheet function.
- (d) The alternative should have a longer life so revenues will exceed costs.

- 6.45 Estimates for one of two process upgrades are as follows: first cost of \$40,000, annual cost of \$5000 per year, market value that decreases by \$2000 per year to the salvage value of \$20,000 after the expected life of 10 years. If a 4-year study period is used for AW analysis at 15% per year, the correct AW value is closest to:

- (a) \$-15,000
- (b) \$-11,900
- (c) \$-7600
- (d) \$-12,600

- 6.46 The perpetual annual worth of investing \$50,000 now and \$20,000 per year starting in year 16 and continuing forever at 12% per year is closest to:

- (a) \$-4200
- (b) \$-8650
- (c) \$-9655
- (d) \$-10,655

- 6.47 All the following statements about the capital recovery amount for an alternative are false *except*:
- Annual revenue can be no more than this amount, if the alternative is selected.
 - A monetary estimate of new capital funds required each year for the life of the alternative.
 - An amount of revenue required to recover the first cost plus a stated return over the life of the alternative.
 - Does not consider the salvage value, since it is returned at the end of the alternative's life.

CASE STUDY

THE CHANGING SCENE OF AN ANNUAL WORTH ANALYSIS

Background and Information

Harry, owner of an automobile battery distributorship in Atlanta, Georgia, performed an economic analysis 3 years ago when he decided to place surge protectors in-line for all his major pieces of testing equipment. The estimates used and the annual worth analysis at $MARR = 15\%$ are summarized below. Two different manufacturers' protectors were compared.

	PowrUp	Lloyd's
Cost and installation, \$	-26,000	-36,000
Annual maintenance cost, \$ per year	-800	-300
Salvage value, \$	2,000	3,000
Equipment repair savings, \$	25,000	35,000
Useful life, years	6	10

The spreadsheet in Figure 6–9 is the one Harry used to make the decision. Lloyd's was the clear choice due to its substantially larger AW value. The Lloyd's protectors were installed.

During a quick review this last year (year 3 of operation), it was obvious that the maintenance costs and repair savings

have not followed (and will not follow) the estimates made 3 years ago. In fact, the maintenance contract cost (which includes quarterly inspection) is going from \$300 to \$1200 per year next year and will then increase 10% per year for the next 10 years. Also, the repair savings for the last 3 years were \$35,000, \$32,000, and \$28,000, as best as Harry can determine. He believes savings will decrease by \$2000 per year hereafter. Finally, these 3-year-old protectors are worth nothing on the market now, so the salvage in 7 years is zero, not \$3000.

Case Study Exercises

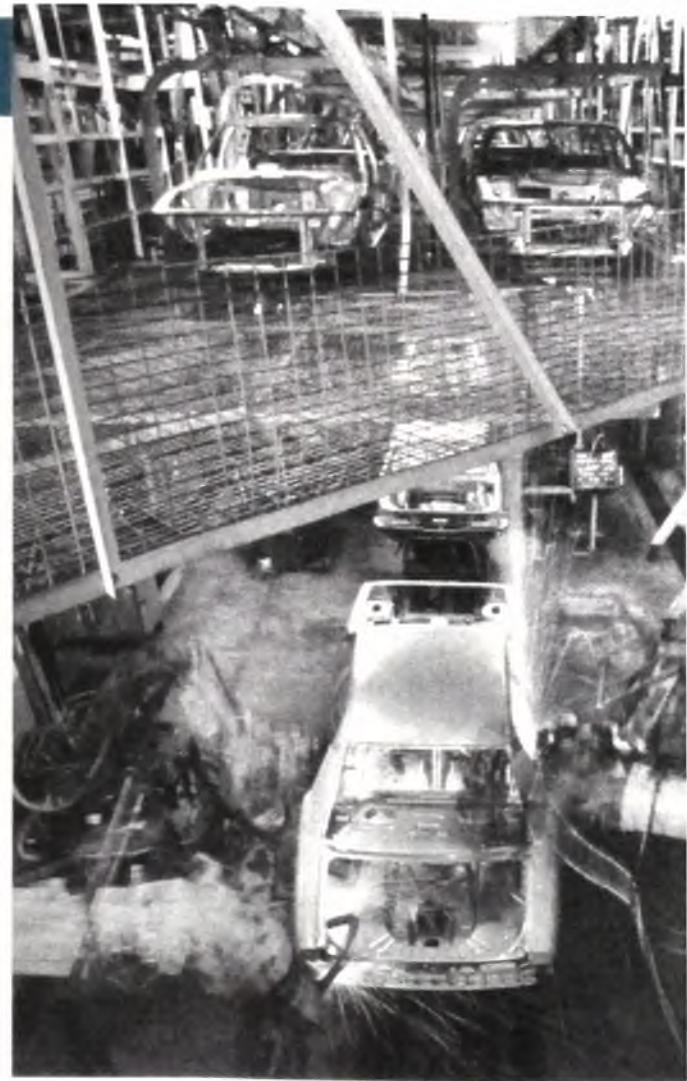
- Plot a graph of the newly estimated maintenance costs and repair savings projections, assuming the protectors last for 7 more years.
- With these new estimates, what is the recalculated AW for the Lloyd's protectors? Use the old first cost and maintenance cost estimates for the first 3 years. If these estimates had been made 3 years ago, would Lloyd's still have been the economic choice?
- How has the capital recovery amount changed for the Lloyd's protectors with these new estimates?

MARR = 15%		PowrUp			Lloyd's		
Year	Investment and salvage	Annual maintenance	Repair savings	Investment and salvage	Annual maintenance	Repair savings	
0	-26,000	0	0	-36,000	0	0	
1	0	-800	25,000	0	-300	35,000	
2	0	-800	25,000	0	-300	35,000	
3	0	-800	25,000	0	-300	35,000	
4	0	-800	25,000	0	-300	35,000	
5	0	-800	25,000	0	-300	35,000	
6	2,000	-800	25,000	0	-300	35,000	
7				0	-300	35,000	
8				0	-300	35,000	
9				0	-300	35,000	
10				0	-300	35,000	
11				0	-300	35,000	
12				0	-300	35,000	
13				0	-300	35,000	
14				0	-300	35,000	
15				0	-300	35,000	
16				3,000	-300	35,000	
17	AW element	-8,642	-800	25,000	-7,025	-300	35,000
18	Total AW			\$ 17,555			\$ 27,675

Figure 6–9

Annual worth analysis of surge protector alternatives, case study.

Rate of Return Analysis: One Project



LEARNING OUTCOMES

Purpose: Understand the meaning of rate of return and perform an ROR evaluation of a single project.

SECTION	TOPIC	LEARNING OUTCOME
7.1	Definition	<ul style="list-style-type: none">State and understand the meaning of rate of return.
7.2	Calculate ROR	<ul style="list-style-type: none">Use a PW or AW relation to determine the ROR of a series of cash flows.
7.3	Cautions	<ul style="list-style-type: none">State the difficulties of using the ROR method, relative to the PW and AW methods.
7.4	Multiple RORs	<ul style="list-style-type: none">Determine the maximum number of possible ROR values and their values for a specific cash flow series.
7.5	Calculate EROR	<ul style="list-style-type: none">Determine the external rate of return using the techniques of modified ROR and return on invested capital.
7.6	Bonds	<ul style="list-style-type: none">Calculate the nominal and effective rate of return for a bond investment.



The most commonly quoted measure of economic worth for a project or alternative is its rate of return (ROR). Whether it is an engineering project with cash flow estimates or an investment in a stock or bond, the rate of return is a well-accepted way of determining if the project or investment is economically acceptable. Compared to the PW or AW value, the ROR is a generically different type of measure of worth, as is discussed in this chapter. Correct procedures to calculate a rate of return using a PW or AW relation are explained here, as are some cautions necessary when the ROR technique is applied to a single project's cash flows.

The ROR is known by other names such as the *internal rate of return* (IROR), which is the technically correct term, and *return on investment* (ROI). We will discuss the computation of ROI in the latter part of this chapter.

In some cases, more than one ROR value may satisfy the PW or AW equation. This chapter describes how to recognize this possibility and an approach to find the multiple values. Alternatively, one reliable ROR value can be obtained by using additional information established separately from the project cash flows. Two of the techniques are covered: the modified ROR technique and the ROIC (return on invested capital) technique.

Only one alternative is considered here; Chapter 8 applies these same principles to multiple alternatives. Finally, the rate of return for a bond investment is discussed.

7.1 Interpretation of a Rate of Return Value ● ● ●

From the perspective of someone who has borrowed money, the interest rate is applied to the *unpaid balance* so that the total loan amount and interest are paid in full exactly with the last loan payment. From the perspective of a lender of money, there is an *unrecovered balance* at each time period. The interest rate is the return on this unrecovered balance so that the total amount lent and the interest are recovered exactly with the last receipt. *Rate of return* describes both of these perspectives.

Rate of return (ROR) is the rate paid on the **unpaid balance of borrowed money**, or the rate earned on the **unrecovered balance of an investment**, so that the final payment or receipt brings the **balance to exactly zero** with interest considered.



Rate of return

The rate of return is expressed as a percent per period, for example, $i = 10\%$ per year. It is stated as a positive percentage; the fact that interest paid on a loan is actually a negative rate of return from the borrower's perspective is not considered. The numerical value of i can range from -100% to infinity, that is, $-100\% \leq i < \infty$. In terms of an investment, a return of $i = -100\%$ means the entire amount is lost.

The definition above does not state that the rate of return is on the initial amount of the investment; rather it is on the **unrecovered balance**, which changes each time period. Example 7.1 illustrates this difference.

EXAMPLE 7.1

To get started in a new telecommuting position with AB Hammond Engineers, Jane took out a \$1000 loan at $i = 10\%$ per year for 4 years to buy home office equipment. From the lender's perspective, the investment in this young engineer is expected to produce an equivalent net cash flow of \$315.47 for each of 4 years.

$$A = \$1000(A/P, 10\%, 4) = \$315.47$$

This represents a 10% per year rate of return on the unrecovered balance. Compute the amount of the unrecovered investment for each of the 4 years using (a) the rate of return on the unrecovered balance (the correct basis) and (b) the return on the initial \$1000 investment. (c) Explain why all of the initial \$1000 amount is not recovered by the final payment in part (b).

Solution

- (a) Table 7-1 shows the unrecovered balance at the end of each year in column 6 using the 10% rate on the *unrecovered balance at the beginning of the year*. After 4 years the total \$1000 is recovered, and the balance in column 6 is exactly zero.

TABLE 7-1 Unrecovered Balances Using a Rate of Return of 10% on the Unrecovered Balance

(1)	(2)	(3) = 0.10 × (2)	(4)	(5) = (4) – (3)	(6) = (2) + (5)
Year	Beginning Unrecovered Balance	Interest on Unrecovered Balance	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0	—	—	\$–1000.00	—	\$–1000.00
1	\$–1000.00	\$100.00	+315.47	\$215.47	–784.53
2	–784.53	78.45	+315.47	237.02	–547.51
3	–547.51	54.75	+315.47	260.72	–286.79
4	–286.79	28.68	+315.47	286.79	0
		\$261.88		\$1000.00	

TABLE 7-2 Unrecovered Balances Using a 10% Return on the Initial Amount

(1)	(2)	(3) = 0.10 × (2)	(4)	(5) = (4) – (3)	(6) = (2) + (5)
Year	Beginning Unrecovered Balance	Interest on Initial Amount	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0	—	—	\$–1000.00	—	\$–1000.00
1	\$–1000.00	\$100	+315.47	\$215.47	–784.53
2	–784.53	100	+315.47	215.47	–569.06
3	–569.06	100	+315.47	215.47	–353.59
4	–353.59	100	+315.47	215.47	–138.12
		\$400		\$861.88	

- (b) Table 7-2 shows the unrecovered balance if the 10% return is always figured on the *initial \$1000*. Column 6 in year 4 shows a remaining unrecovered amount of \$138.12, because only \$861.88 is recovered in the 4 years (column 5).
- (c) As shown in column 3, a total of \$400 in interest must be earned if the 10% return each year is based on the initial amount of \$1000. However, only \$261.88 in interest must be earned if a 10% return on the unrecovered balance is used. There is more of the annual cash flow available to reduce the remaining loan when the rate is applied to the unrecovered balance as in part (a) and Table 7-1. Figure 7-1 illustrates the correct interpretation of rate of return in Table 7-1.

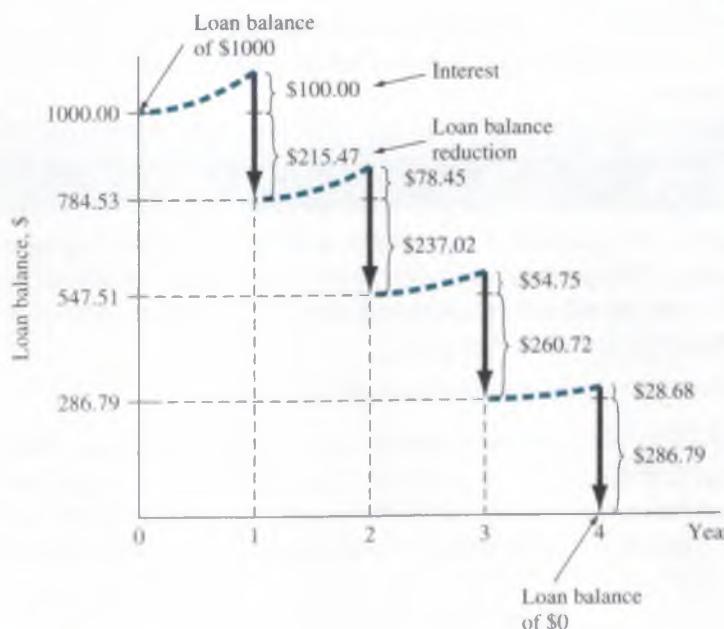


Figure 7-1
Plot of unrecovered balances and 10% per year rate of return on a \$1000 amount, Table 7-1.

Each year the \$315.47 receipt represents 10% interest on the unrecovered balance in column 2 plus the recovered amount in column 5.

Because rate of return is the interest rate on the unrecovered balance, the computations in *Table 7-1 for part (a)* present a correct interpretation of a 10% rate of return. Clearly, an interest rate applied only to the principal represents a higher rate than is stated. In practice, a so-called add-on interest rate is frequently based on principal only, as in part (b). This is sometimes referred to as the *installment financing* problem.

Installment financing can be discovered in many forms in everyday finances. One popular example is a “no-interest program” offered by retail stores on the sale of major appliances, audio and video equipment, furniture, and other consumer items. Many variations are possible, but in most cases, if the purchase is not paid for in full by the time the promotion is over, usually 6 months to 1 year later, *finance charges are assessed from the original date of purchase*. Further, the program’s fine print may stipulate that the purchaser use a credit card issued by the retail company, which often has a higher interest rate than that of a regular credit card, for example, 24% per year compared to 15% per year. In all these types of programs, the one common theme is more interest paid over time by the consumer. Usually, the correct definition of i as interest on the unpaid balance does not apply directly; i has often been manipulated to the financial disadvantage of the purchaser. This was demonstrated by Example 4.4 using the Credit Card Case in Chapter 4.

7.2 Rate of Return Calculation Using a PW or AW Relation ● ● ●

The ROR value is determined in a generically different way compared to the PW or AW value for a series of cash flows. For a moment, consider only the present worth relation for a cash flow series. Using the MARR, which is established independent of any particular project’s cash flows, a mathematical relation determines the PW value in actual monetary units, say, dollars or euros. For the ROR values calculated in this and later sections, **only the cash flows themselves** are used to determine an interest rate that balances the present worth relation. Therefore, ROR may be considered a relative measure, while PW and AW are absolute measures. Since the resulting interest rate depends only on the cash flows themselves, the correct term is **internal rate of return (IROR)**; however, the term *ROR* is used interchangeably. Another definition of rate of return is based on our previous interpretations of PW and AW.



The *rate of return* is the interest rate that makes the present worth or annual worth of a cash flow series exactly equal to 0.



Rate of return

To determine the rate of return, develop the ROR equation using either a PW or AW relation, set it equal to 0, and solve for the interest rate. Alternatively, the present worth of cash outflows (costs and disbursements) PW_O may be equated to the present worth of cash inflows (revenues and savings) PW_I . That is, solve for i using either of the relations

$$0 = PW \quad [7.1]$$

or

$$PW_O = PW_I$$

The annual worth approach utilizes the AW values in the same fashion to solve for i .

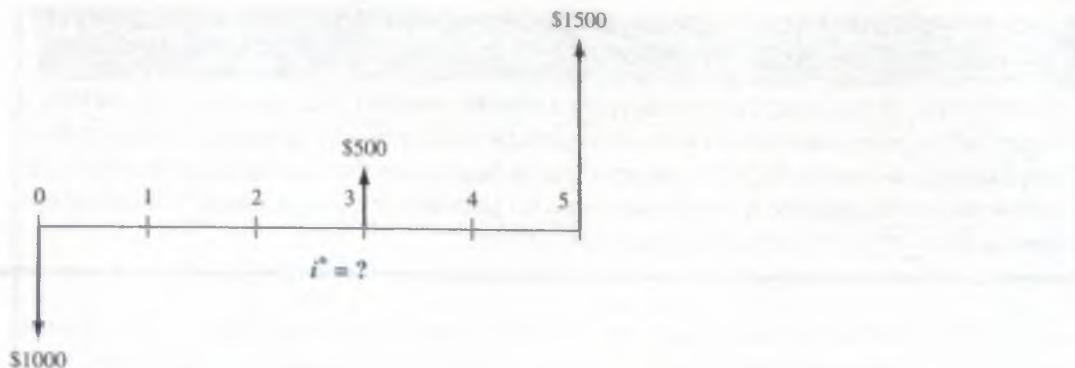
$$0 = AW \quad [7.2]$$

or

$$AW_O = AW_I$$

The i value that makes these equations numerically correct is called i^* . It is the root of the ROR relation. To determine if the investment project’s cash flow series is viable, compare i^* with the established MARR.

Figure 7–2
Cash flow for which
a value of i is to be
determined.



The guideline is as follows:



Project evaluation

If $i^* \geq \text{MARR}$, accept the project as economically viable.

If $i^* < \text{MARR}$, the project is not economically viable.

The purpose of engineering economy calculations is *equivalence* in PW or AW terms for a stated $i \geq 0\%$. In rate of return calculations, the objective is to *find the interest rate i^** at which the cash flows are equivalent. The calculations are the reverse of those made in previous chapters, where the interest rate was known. For example, if you deposit \$1000 now and are promised payments of \$500 three years from now and \$1500 five years from now, the rate of return relation using PW factors and Equation [7.1] is

$$1000 = 500(P/F, i^*, 3) + 1500(P/F, i^*, 5) \quad [7.3]$$

The value of i^* that makes the equality correct is to be determined (see Figure 7–2). If the \$1000 is moved to the right side of Equation [7.3], we have the form $0 = \text{PW}$.

$$0 = -1000 + 500(P/F, i^*, 3) + 1500(P/F, i^*, 5)$$

The equation is solved for $i^* = 16.9\%$ by hand using trial and error or using a spreadsheet function. The rate of return will always be greater than zero if the total amount of cash inflow is greater than the total amount of outflow, when the time value of money is considered. Using $i^* = 16.9\%$, a graph similar to Figure 7–1 can be constructed. It will show that the unrecovered balances each year, starting with \$–1000 in year 1, are exactly recovered by the \$500 and \$1500 receipts in years 3 and 5.

It should be evident that rate of return relations are merely a rearrangement of a present worth equation. That is, if the above interest rate is known to be 16.9%, and it is used to find the present worth of \$500 three years from now and \$1500 five years from now, the PW relation is

$$\text{PW} = 500(P/F, 16.9\%, 3) + 1500(P/F, 16.9\%, 5) = \$1000$$

This illustrates that rate of return and present worth equations are set up in exactly the same fashion. The only differences are what is given and what is sought.

There are several ways to determine i^* once the PW relation is established: solution via trial and error by hand, using a programmable calculator, and solution by spreadsheet function. The spreadsheet is faster; the first helps in understanding how ROR computations work. We summarize two methods here and in Example 7.2. Refer to Appendix D for the discussion about solution by calculator.

***i** Using Trial and Error** The general procedure of using a PW-based equation is as follows:

1. Draw a cash flow diagram.
2. Set up the rate of return equation in the form of Equation [7.1].
3. Select values of i by trial and error until the equation is balanced.

When the trial-and-error method is applied to determine i^* , it is advantageous in step 3 to get fairly close to the correct answer on the first trial. If the cash flows are combined in such a manner that the income and disbursements can be represented by a *single factor* such as P/F or P/A , it is

possible to look up the interest rate (in the tables) corresponding to the value of that factor for n years. The problem, then, is to combine the cash flows into the format of only one of the factors. This may be done through the following procedure:

1. Convert all *disbursements* into either single amounts (P or F) or uniform amounts (A) by neglecting the time value of money. For example, if it is desired to convert an A to an F value, simply multiply the A by the number of years n . The scheme selected for movement of cash flows should be the one that minimizes the error caused by neglecting the time value of money. That is, if most of the cash flow is an A and a small amount is an F , convert the F to an A rather than the other way around.
2. Convert all *receipts* to either single or uniform values.
3. Having combined the disbursements and receipts so that a P/F , P/A , or A/F format applies, use the interest tables to find the approximate interest rate at which the P/F , P/A , or A/F value is satisfied. The rate obtained is a good estimate for the first trial.

It is important to recognize that this first-trial rate is only an *estimate* of the actual rate of return, because the time value of money is neglected. The procedure is illustrated in Example 7.2.

i^* by Spreadsheet The fastest way to determine an i^* value when there is a series of equal cash flows (A series) is to apply the RATE function. This is a powerful one-cell function, where it is acceptable to have a separate P value in year 0 and a separate F value in year n . The format is

$$= \text{RATE}(n, A, P, F) \quad [7.4]$$

When cash flows vary from year to year (period to period), the best way to find i^* is to enter the net cash flows into contiguous cells (including any \$0 amounts) and apply the IRR function in any cell. The format is

$$= \text{IRR(first_cell:last_cell,guess)} \quad [7.5]$$

where "guess" is the i value at which the function starts searching for i^* .

The PW-based procedure for sensitivity analysis and a graphical estimation of the i^* value is as follows:

1. Draw the cash flow diagram.
2. Set up the ROR relation in the form of Equation [7.1], $PW = 0$.
3. Enter the cash flows onto the spreadsheet in contiguous cells.
4. Develop the IRR function to display i^* .
5. Use the NPV function to develop a PW graph (PW versus i values). This graphically shows the i^* value at which $PW = 0$.

EXAMPLE 7.2

Applications of green, lean manufacturing techniques coupled with value stream mapping can make large financial differences over future years while placing greater emphasis on environmental factors. Engineers with Monarch Paints have recommended to management an investment of \$200,000 now in novel methods that will reduce the amount of wastewater, packaging materials, and other solid waste in their consumer paint manufacturing facility. Estimated savings are \$15,000 per year for each of the next 10 years and an additional savings of \$300,000 at the end of 10 years in facility and equipment upgrade costs. Determine the rate of return using hand and spreadsheet solutions.

Solution by Hand

Use the trial-and-error procedure based on a PW equation.

1. Figure 7–3 shows the cash flow diagram.
2. Use Equation [7.1] format for the ROR equation.

$$0 = -200,000 + 15,000(P/A, i^*, 10) + 300,000(P/F, i^*, 10) \quad [7.6]$$

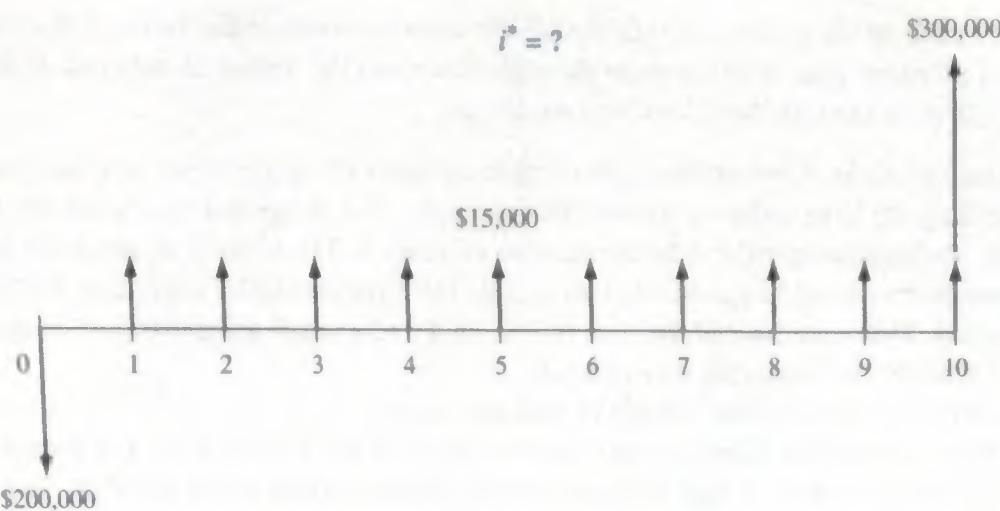


Figure 7-3
Cash flow diagram, Example 7.2.

3. Use the estimation procedure to determine i for the first trial. All income will be regarded as a single F in year 10 so that the P/F factor can be used. The P/F factor is selected because most of the cash flow (\$300,000) already fits this factor and errors created by neglecting the time value of the remaining money will be minimized. Only for the first estimate of i , define $P = \$200,000$, $n = 10$, and $F = 10(15,000) + 300,000 = \$450,000$. Now we can state that

$$200,000 = 450,000(P/F, i, 10)$$

$$(P/F, i, 10) = 0.444$$

The roughly estimated i is between 8% and 9%. Use 9% as the first trial because this approximate rate for the P/F factor will be lower than the true value when the time value of money is considered.

$$0 = -200,000 + 15,000(P/A, 9\%, 10) + 300,000(P/F, 9\%, 10)$$

$$0 < \$22,986$$

The result is positive, indicating that the return is more than 9%. Try $i = 11\%$.

$$0 = -200,000 + 15,000(P/A, 11\%, 10) + 300,000(P/F, 11\%, 10)$$

$$0 > \$-6002$$

Since the interest rate of 11% is too high, linearly interpolate between 9% and 11%.

$$i^* = 9.00 + \frac{22,986 - 0}{22,986 - (-6002)} (2.0)$$

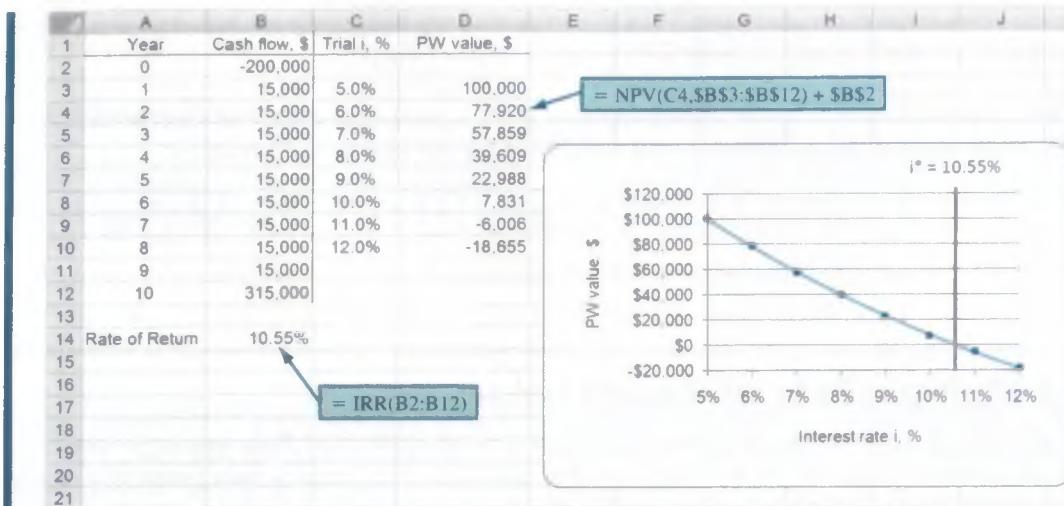
$$= 9.00 + 1.58 = 10.58\%$$

Solution by Spreadsheet

The fastest way to find i^* is to use the RATE function (Equation [7.4]). The entry $= RATE(10, 15000, -200000, 300000)$ displays $i^* = 10.55\%$ per year. It is equally correct to use the IRR function. Figure 7-4, column B, shows the cash flows and $= IRR(B2:B12)$ function to obtain i^* .

For a complete spreadsheet analysis, use the procedure outlined above.

1. Figure 7-3 shows cash flows.
2. Equation [7.6] is the ROR relation.
3. Figure 7-4 shows the net cash flows in column B.
4. The IRR function in cell B14 displays $i^* = 10.55\%$.
5. To graphically observe $i^* = 10.55\%$, column D displays the PW graph for different i values. The NPV function is used repeatedly to calculate PW for the xy scatter chart.

**Figure 7–4**Spreadsheet to determine i^* and develop a PW graph, Example 7.2.

Just as i^* can be found using a PW equation, it may equivalently be determined using an AW relation. This method is preferred when uniform annual cash flows are involved. Solution by hand is the same as the procedure for a PW-based relation, except Equation [7.2] is used. In the case of Example 7.2, $i^* = 10.55\%$ is determined using the AW-based relation.

$$0 = -200,000(A/P, i^*, 10) + 15,000 + 300,000(A/F, i^*, 10)$$

The procedure for solution by spreadsheet is exactly the same as outlined above using the IRR function. Internally, IRR calculates the NPV function at different i values until $NPV = 0$. (There is no equivalent way to utilize the PMT function, since it requires a fixed value of i to calculate an A value.)

7.3 Special Considerations When Using the ROR Method ● ● ●

The rate of return method is commonly used in engineering and business settings to evaluate one project, as discussed in this chapter, and to select one alternative from two or more, as explained in the next chapter. As mentioned earlier, an ROR analysis is performed using a different basis than PW and AW analyses. The cash flows themselves determine the (internal) rate of return. As a result, there are some assumptions and special considerations with ROR analysis that must be made when calculating i^* and in interpreting its real-world meaning. A summary is provided below.

- **Multiple i^* values.** Depending upon the sequence of net cash inflows and outflows, there may be more than one real-number root to the ROR equation, resulting in *more than one i^* value*. This possibility is discussed in Section 7.4.
- **Reinvestment at i^* .** Both the PW and AW methods assume that any net positive investment (i.e., net positive cash flows once the time value of money is considered) is reinvested at the MARR. However, the ROR method assumes reinvestment at the i^* rate. When i^* is not close to the MARR (e.g., if i^* is substantially larger than MARR), this is an unrealistic assumption. In such cases, the i^* value is not a good basis for decision making. This situation is discussed in Section 7.5.
- **Different procedure for multiple alternative evaluations.** To correctly use the ROR method to choose from two or more mutually exclusive alternatives requires an *incremental analysis* procedure that is significantly more involved than PW and AW analysis. Chapter 8 explains this procedure.

If possible, from an engineering economic study perspective, the **AW or PW method at a stated MARR should be used in lieu of the ROR method**. However, there is a strong appeal

for the ROR method because rate of return values are very commonly quoted. And it is easy to compare a proposed project's return with that of in-place projects.

When it is important to know the exact value of i^* , a good approach is to determine PW or AW at the MARR, then determine the specific i^* for the selected alternative.

As an illustration, if a project is evaluated at $MARR = 15\%$ and has $PW < 0$, there is no need to calculate i^* , because $i^* < 15\%$. However, if PW is positive, but close to 0, calculate the exact i^* and report it along with the conclusion that the project is financially justified.



7.4 Multiple Rate of Return Values ●●●

In Section 7.2 a unique rate of return i^* was determined. In the cash flow series presented thus far, the algebraic signs on the *net cash flows* changed only once, usually from minus in year 0 to plus at some time during the series. This is called a *conventional (or simple) cash flow series*. However, for some series the net cash flows switch between positive and negative from one year to another, so there is more than one sign change. Such a series is called *nonconventional (non-simple)*. As shown in the examples of Table 7-3, each series of positive or negative signs may be one or more in length. Relatively large net cash flow (NCF) changes in amount and sign can occur in projects that require significant spending at the end of the expected life. Nuclear plants, open-pit mines, petroleum well sites, refineries, and the like often require environmental restoration, waste disposal, and other expensive phaseout costs. The cash flow diagram will appear similar to Figure 7-5a. Plants and systems that have anticipated major refurbishment costs or upgrade investments in future years may have considerable swings in cash flow and sign changes over the years, as shown by the pattern in Figure 7-5b.

TABLE 7-3 Examples of Conventional and Nonconventional Net Cash Flow for a 6-year Project

Type of Series	Sign on Net Cash Flow by Year							Number of Sign Changes
	0	1	2	3	4	5	6	
Conventional	-	+	+	+	+	+	+	1
Conventional	-	-	-	+	+	+	+	1
Conventional	+	+	+	+	+	-	-	1
Nonconventional	-	+	+	+	-	-	-	2
Nonconventional	+	+	-	-	-	+	+	2
Nonconventional	-	+	-	-	+	+	+	3

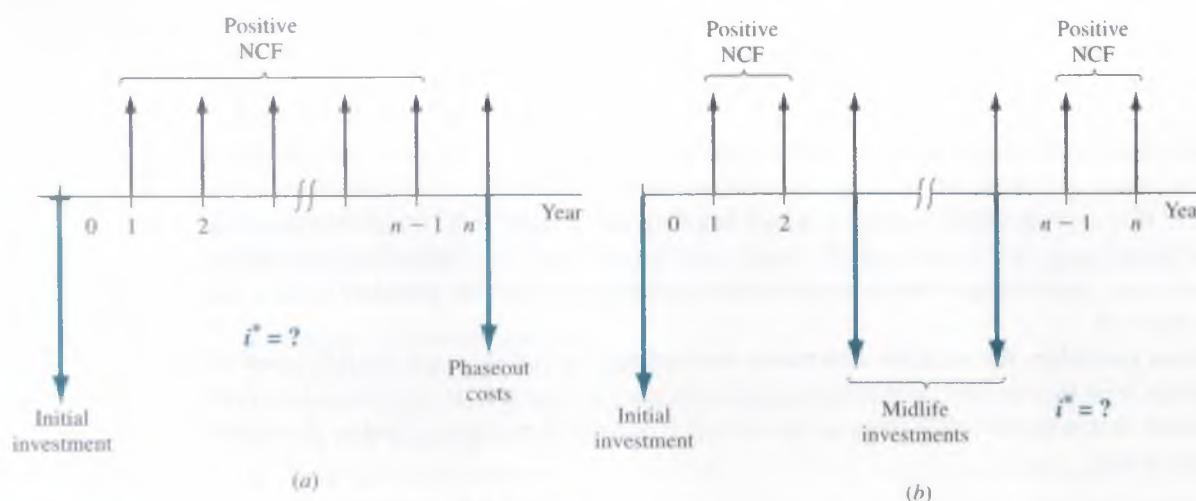


Figure 7-5

Typical cash flow diagrams for projects with (a) large restoration or remediation costs, and (b) upgrade or refurbishment costs.

When there is more than one sign change in the net cash flows, it is possible that there will be multiple i^* values in the -100% to plus infinity range. There are two tests to perform in sequence on the nonconventional series to determine if there is one unique value or possibly multiple i^* values that are real numbers.

Test 1: (Descartes') rule of signs states that the total number of real-number roots is always less than or equal to the number of sign changes in the series.

This rule is derived from the fact that the relation set up by Equation [7.1] or [7.2] to find i^* is an n th-order polynomial. (It is possible that imaginary values or infinity may also satisfy the equation.)

Test #2: Cumulative cash flow sign test, also known as *Norstrom's criterion*, states that only one sign change in a series of cumulative cash flows which *starts negatively* indicates that there is **one positive root** to the polynomial relation.

Zero values in the series are neglected when applying Norstrom's criterion. This is a more discriminating test that determines if there is one, real-number, positive i^* value. There may be negative roots that satisfy the ROR relation, but these are not useful i^* values. To perform the test, determine the series

$$S_t = \text{cumulative cash flows through period } t$$

Observe the sign of S_0 and count the sign changes in the series S_0, S_1, \dots, S_n . Only if $S_0 < 0$ and signs change one time in the series is there a single, real-number, positive i^* .

With the results of these two tests, the ROR relation is solved for either the unique i^* or the multiple i^* values, using trial and error by hand, using a programmable calculator, or by spreadsheet using an IRR function that incorporates the "guess" option. Development of the PW graph is recommended, especially when using a spreadsheet. Examples 7.3 and 7.4 illustrate the tests and solution for i^* .

EXAMPLE 7.3

Sept-Îles Aluminum Company operates a bauxite mine to supply its aluminum smelter located about 2 km from the current open pit. A new branch for the pit is proposed that will supply an additional 10% of the bauxite currently available over the next 10-year period. The lease for the land will cost \$400,000 immediately. The contract calls for the restoration of the land and development as part of a state park and wildlife area at the end of the 10 years. This is expected to cost \$300,000. The increased production capacity is estimated to net an additional \$75,000 per year for the company. Perform an ROR analysis that will provide the following information:

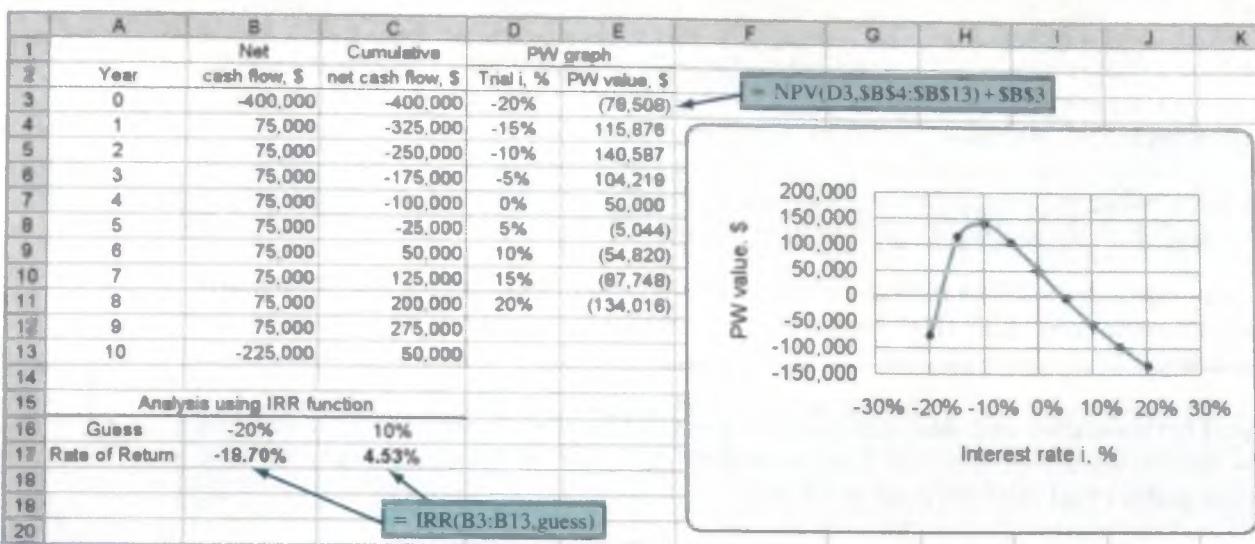
- (a) Type of cash flow series and possible number of ROR values
- (b) PW graph showing all i^* values
- (c) Actual i^* values determined using the ROR relation and spreadsheet function
- (d) Conclusions that can be drawn about the correct rate of return from this analysis

Solution

- (a) The net cash flows will appear like those in Figure 7-5a with an initial investment of \$-400,000, annual net cash flow (NCF) of \$75,000 for years 1 through 10, and a phase-out cost of \$-300,000 in year 10. Figure 7-6 details the NCF series (column B) and cumulative NCF (column C) for use in the two tests for unique and multiple i^* values. The series is **nonconventional** based on the sign changes throughout the series.

Test #1: There are two sign changes in the NCF series, which indicates a **possible maximum of two roots** to the polynomial equation or i^* values for the ROR equation.

Test #2: There is one sign change in the cumulative NCF series, which indicates a unique positive root or **one positive i^* value**.

**Figure 7–6**Spreadsheet determination of multiple i^* values and PW graph. Example 7.3.

(b) Columns D and E of the spreadsheet in Figure 7–6 use i values ranging from -20% to $+20\%$ per year to plot the PW vs i curve via the NPV function. There are two times that the parabolic-shaped curve crosses the PW = 0 line; these are approximately $i_1^* = -18\%$ and $i_2^* = 5\%$.

(c) The ROR equation based on PW computations is

$$0 = -400,000 + 75,000(P/A, i^*, 10) - 300,000(P/F, i^*, 10) \quad [7.7]$$

i^ values by hand* If hand solution is chosen, the same procedure used in Example 7.2 can be applied here. However, the technique to estimate the initial i value will not work as well in this case since the majority of the cash flows do not fit either the P/F or the F/P factor. In fact, using the P/F factor, the initial i value is indicated to be 1.25% . Trial-and-error solution of Equation [7.7] with various i values will approximate the correct answer of about 4.5% per year. This complies with the test results of one positive i^* value.

i^ by spreadsheet function* Use the =IRR(B3:B13,guess) function to determine the i^* value for the NCF series in column B, Figure 7–6. Entering different values in the optional “guess” field will force the function to find multiple i^* values, if they exist. As shown in row 17, two are found.

$$i_1^* = -18.70\% \quad i_2^* = +4.53\%$$

This result does not conflict with test results, as there is one positive value, but a negative value also balances the ROR equation.

(d) The positive $i^* = 4.53\%$ is accepted as the correct internal rate of return (IROR) for the project. The negative value is not useful in economic conclusions about the project.

EXAMPLE 7.4

The engineering design and testing group for Honda Motor Corp. does contract-based work for automobile manufacturers throughout the world. During the last 3 years, the net cash flows for contract payments have varied widely, as shown below, primarily due to a large manufacturer's inability to pay its contract fee.

Year	0	1	2	3
Cash Flow (\$1000)	+2000	-500	-8100	+6800

- (a) Determine the maximum number of i^* values that may satisfy the ROR equation.
 (b) Write the PW equation and approximate the i^* value(s) by plotting PW vs i .
 (c) What do the i^* values mean?

Solution

(a) Table 7–4 shows the annual cash flows and cumulative cash flows. Since there are two sign changes in the cash flow sequence, the rule of signs indicates a maximum of two i^* values. The cumulative cash flow sequence starts with a positive number $S_0 = +2000$, indicating that test #2 is inconclusive. As many as two i^* values can be found.

TABLE 7–4 Cash Flow and Cumulative Cash Flow Sequences, Example 7.4

Year	Cash Flow (\$1000)	Sequence Number	Cumulative Cash Flow (\$1000)
0	+2000	S_0	+2000
1	-500	S_1	+1500
2	-8100	S_2	-6600
3	+6800	S_3	+200

- (b) The PW relation is

$$PW = 2000 - 500(P/F,i,1) - 8100(P/F,i,2) + 6800(P/F,i,3)$$

The PW values are shown below and plotted in Figure 7–7 for several i values. The characteristic parabolic shape for a second-degree polynomial is obtained, with PW crossing the i axis at approximately $i_1^* = 8\%$ and $i_2^* = 41\%$.

$i\%$	5	10	20	30	40	50
PW (\$1000)	+51.44	-39.55	-106.13	-82.01	-11.83	+81.85

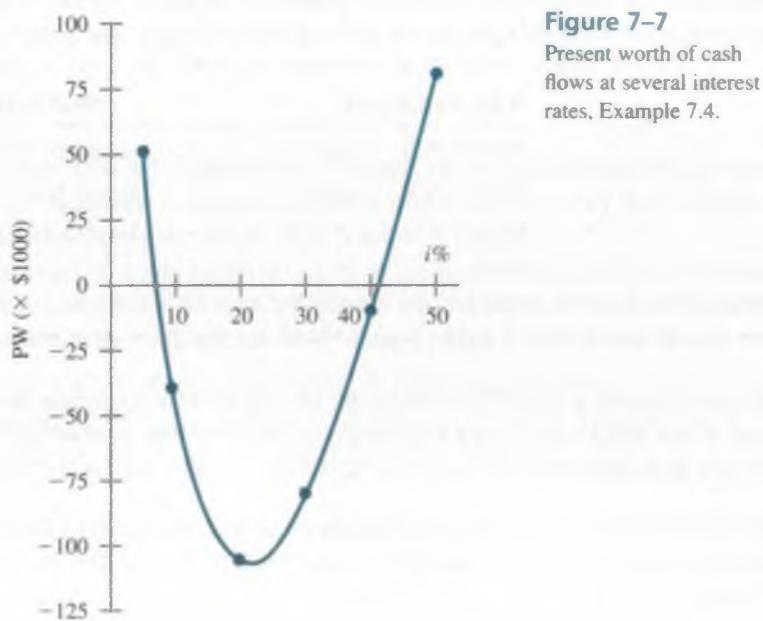


Figure 7–7
Present worth of cash flows at several interest rates, Example 7.4.

Figure 7–8 presents the spreadsheet PW graph with the PW curve crossing the x axis at $PW = 0$ two times. Also, the solution for two positive i^* values using the IRR function with different guess values is displayed. The values are

$$i_1^* = 7.47\% \quad i_2^* = 41.35\%$$

- (c) Since both i^* values are positive, they are not of much value, because neither can be considered the true ROR of the cash flow series. This result indicates that additional

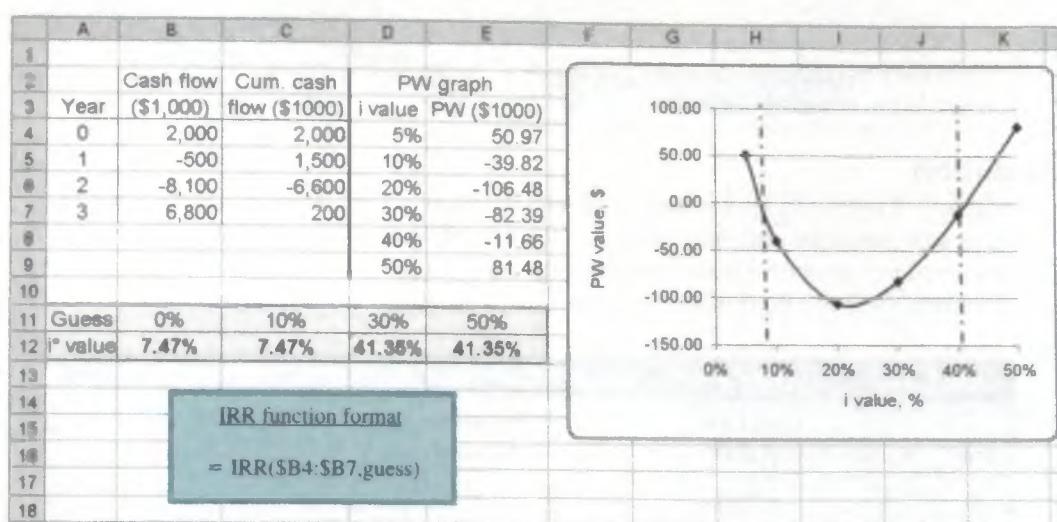


Figure 7-8
Spreadsheet solution, Example 7.4.

information is needed to calculate a more useful project ROR, namely, some information about the anticipated return on funds invested external to the project and the cost of capital to borrow money to continue the project. This problem is a good example of when an approach discussed in the next section should be taken.

If the guess option is not used in the IRR function, the starting point is 10% per year. The function will find the one ROR closest to 10% that satisfies the PW relation. Entering various guess values will allow IRR to find multiple i^* values in the range of -100% to ∞ , if they exist. Often, the results are unbelievable or unacceptable values that are rejected. Some helpful guidelines can be developed. Assume there are two i^* values for a particular cash flow series.

If the Results Are	What to Do
Both $i^* < 0$	Discard both values.
Both $i^* > 0$	Discard both values.
One $i^* > 0$; one $i^* < 0$	Use $i^* > 0$ as ROR.

If both i^* values are discarded, proceed to the approach discussed in the next section to determine one rate of return value for the project. However, remember the prime recommendation.

Always determine the PW or AW at the MARR first for a reliable measure of economic justification. If the PW or AW is greater than zero and the ROR is needed, then find the actual i^* of the project cash flows.

This recommendation is not to dissuade you from using the ROR method. Rather it is a recommendation that the use of the ROR technique be reserved for times when the actual i^* value is essential to the decision-making process.

7.5 Techniques to Remove Multiple Rates of Return ● ● ●

The techniques developed here are used under the following conditions:

- The PW or AW value at the MARR is determined and could be used to make the decision, but information on the ROR is deemed necessary to finalize the economic decision, and
- The two tests of cash flow sign changes (Descartes' and Norstrom's) indicate multiple roots (i^* values) are possible, and



- More than one positive i^* value or all negative i^* values are obtained when the PW graph and IRR function are developed, and
- A single, reliable rate of return value is required by management or engineers to make a clear economic decision.

We will present a couple of ways to remove multiple i^* values. The selected approach depends upon what estimates are the most reliable for the project being evaluated. An important fact to remember is the following.

The result of follow-up analysis to obtain a single ROR value when multiple, nonuseful i^* values are present does **not determine the internal rate of return (IROR)** for nonconventional net cash flow series. The resulting rate is a function of the additional information provided to make the selected technique work, and the accuracy is further dependent upon the reliability of this information.

We will refer to the resulting value as the **external rate of return (EROR)** as a reminder that it is different from the IROR obtained in all previous sections. First, it is necessary to identify the perspective about the annual net cash flows of a project. Take the following view: You are the project manager and the project generates cash flows each year. Some years produce positive NCF, and you want to invest the excess money at a good rate of return. We will call this the **investment rate i_i** . This can also be called the *reinvestment rate*. Other years, the net cash flow will be negative and you must borrow funds from some source to continue. The interest rate you pay should be as small as possible; we will call this the **borrowing rate i_b** , also referred to as the *finance rate*. Each year, you must consider the time value of money, which must utilize either the investment rate or the borrowing rate, depending upon the sign on the NCF of the preceding year. With this perspective, it is now possible to outline two approaches that rectify the multiple i^* situation. The resulting ROR value will not be the same for each method, because slightly different additional information is necessary and the cash flows are treated in slightly different fashions from the time value of money viewpoint.

Modified ROR (MIRR) Approach This is the easier approach to apply, and it has a spreadsheet function that can find the single EROR value quickly. However, the investment and borrowing rates must be reliably estimated, since the results may be quite sensitive to them. The symbol i' will identify the result.

Return on Invested Capital (ROIC) Approach Though more mathematically rigorous, this technique provides a more reliable estimate of the EROR and it requires only the investment rate i_i . The symbol i'' is used to identify the result.

Before covering the techniques, it would be good to review the material in Section 7.1, including Example 7.1. Though the i' or i'' value determined here is not the ROR defined earlier in the chapter, the concepts used to make the ending cash flow balance equal to zero are used.

Modified ROR Approach

The technique requires that two rates external to the project net cash flows be estimated.

- **Investment rate i_i** is the rate at which extra funds are invested in some source external to the project. This applies to all positive annual NCF. It is reasonable that the MARR is used for this rate.
- **Borrowing rate i_b** is the rate at which funds are borrowed from an external source to provide funds to the project. This applies to all negative annual NCF. The weighted average cost of capital (WACC) can be used for this rate.

It is possible to make all rates the same, that is, $i_i = i_b = \text{MARR} = \text{WACC}$. However, this is not a good idea as it implies that the company is willing to borrow funds and invest in projects at the same rate. This implies no profit margin over time, so the company can't survive for long using this strategy. Commonly $\text{MARR} > \text{WACC}$, so usually $i_i > i_b$. (See Section 1.9 for a quick review of MARR and WACC and Chapter 10 for a more detailed discussion of WACC.)

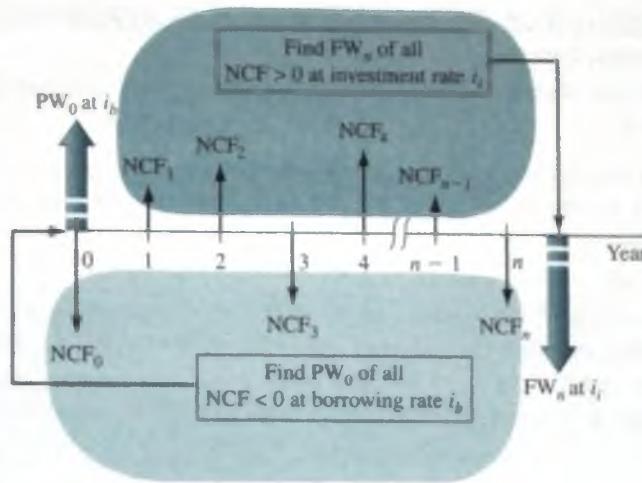
**Figure 7-9**Typical cash flow diagram to determine modified rate of return i' .

Figure 7-9 is a reference diagram that has multiple i^* values, since the net cash flows change sign multiple times. The modified ROR method uses the following procedure to determine a single external rate of return i' and to evaluate the economic viability of the project.

1. Determine the PW value in year 0 of *all negative NCF at the borrowing rate i_b* (lightly shaded area and resulting PW_0 value in Figure 7-9).
2. Determine the FW value in year n of *all positive NCF at the investment rate i_i* (darker shaded area and resulting FW_n value in Figure 7-9).
3. Calculate the modified rate of return i' at which the *PW and FW values are equivalent* over the n years using the following relation, where i' is to be determined.

$$FW_n = PW_0 (F/P, i', n) \quad [7.8]$$

If using a spreadsheet rather than hand computation, the MIRR function displays i' directly with the format

$$= MIRR(first_cell:last_cell, i_b, i_i) \quad [7.9]$$

4. The guideline for economic decision making compares the EROR or i' to MARR.

If $i' > \text{MARR}$, the project is economically justified.

If $i' < \text{MARR}$, the project is not economically justified.

As in other situations, on the rare occasion that $i' = \text{MARR}$, there is indifference to the project's economic acceptability; however, acceptance is the usual decision.

EXAMPLE 7.5

The cash flows experienced by Honda Motors in Example 7.4 are repeated below. There are two positive i^* values that satisfy the PW relation, 7.47% and 41.35% per year. Use the modified ROR method to determine the EROR value. Studies indicate that Honda has a WACC of 8.5% per year and that projects with an estimated return of less than 9% per year are routinely rejected. Due to the nature of this contract business, any excess funds generated are expected to earn at a rate of 12% per year.

Year	0	1	2	3
Net Cash Flow (\$1000)	+2000	-500	-8100	+6800

Solution by Spreadsheet

Using the information in the problem statement, the rate estimates are as follows:

MARR:	9% per year
Investment rate, i_i :	12% per year
Borrowing rate, i_b :	8.5% per year

The fast way to find i' is with the MIRR function. Figure 7–10 shows the result of $i' = 9.39\%$ per year. Since $9.39\% > \text{MARR}$, the project is economically justified.

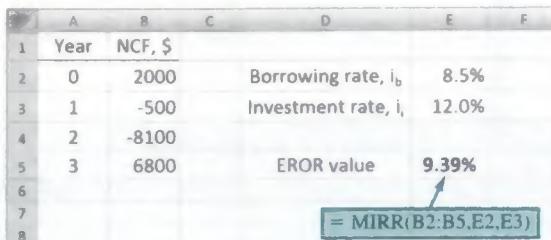


Figure 7–10

Spreadsheet application of MIRR function, Example 7.5.

It is vital that the interpretation be correct. The 9.39% is not the internal rate of return (IROR); it is the external ROR (EROR) based on the two external rates for investing and borrowing money.

Solution by Hand

Figure 7–9 can serve as a reference as the procedure to find i' manually is applied.

Step 1. Find PW_0 of all negative NCF at $i_b = 8.5\%$.

$$\begin{aligned} PW_0 &= -500(P/F, 8.5\%, 1) - 8100(P/F, 8.5\%, 2) \\ &= \$-7342 \end{aligned}$$

Step 2. Find FW_3 of all positive NCF at $i_i = 12\%$.

$$\begin{aligned} FW_3 &= 2000(F/P, 12\%, 3) + 6800 \\ &= \$9610 \end{aligned}$$

Step 3. Find the rate i' at which the PW and FW are equivalent.

$$\begin{aligned} PW_0(F/P, i', 3) + FW_3 &= 0 \\ -7342(1 + i')^3 + 9610 &= 0 \\ i' &= \left(\frac{9610}{7342}\right)^{1/3} - 1 \\ &= 0.939 \quad (9.39\%) \end{aligned}$$

Step 4. Since $i' > \text{MARR}$ of 9%, the project is economically justified using this EROR approach.

Return on Invested Capital Approach

The definition of ROIC should be understood before we discuss the approach.

Return on invested capital (ROIC) is a rate-of-return measure of how effectively a project utilizes the funds invested in it, that is, funds that remain internal to the project. For a corporation, ROIC is a measure of how effectively it utilizes the funds invested in its operations, including facilities, equipment, people, systems, processes, and all other assets used to conduct business.

The technique requires that the investment rate i_i be estimated for excess funds generated in any year that they are not needed by the project. The ROIC rate, which has the symbol i'' , is determined using an approach called the **net-investment procedure**. It involves developing a series of future worth (F) relations moving forward 1 year at a time. In those years that the net balance of the project cash flows is positive (extra funds generated by the project), the funds are invested at the i_i rate. Usually, i_i is set equal to the MARR. When the net balance is negative, the ROIC rate is used, since the project keeps all of its funds internal to itself. The ROIC method uses the following procedure to determine a single external rate of return i'' and to evaluate the economic viability of the project. Remember that the perspective is that you are the project manager and when the project generates extra cash flows, they are invested external to the project at the investment rate i_i .

1. Develop a series of future worth relations by setting up the following relation for each year t ($t = 1, 2, \dots, n$ years).

$$F_t = F_{t-1}(1 + k) + NCF_t \quad [7.10]$$

where F_t = future worth in year t based on previous year and time value of money

NCF_t = net cash flow in year t

$$k = \begin{cases} i_i & \text{if } F_{t-1} > 0 \\ i'' & \text{if } F_{t-1} < 0 \end{cases} \quad \begin{array}{l} (\text{extra funds available}) \\ (\text{project uses all available funds}) \end{array}$$

2. Set the future worth relation for the last year n equal to 0, that is, $F_n = 0$, and solve for i'' to balance the equation. The i'' value is the ROIC for the specified investment rate i_i .

The F_t series and solution for i'' in the $F_n = 0$ relation can become involved mathematically. Fortunately, the Goal Seek spreadsheet tool can assist in the determination of i'' because there is only one unknown in the F_n relation and the target value is zero. (Both the hand and spreadsheet solutions are demonstrated in Example 7.6.)

3. The guideline for economic decision making is the same as above, namely,

If $\text{ROIC} \geq \text{MARR}$, the project is economically justified.

If $\text{ROIC} < \text{MARR}$, the project is not economically justified.

It is important to remember that the **ROIC is an external rate of return dependent upon the investment rate choice**. It is not the same as the internal rate of return discussed at the beginning of this chapter, nor is it the multiple rates, nor is it the MIRR rate found by the previous method. This is a separate technique to find a single rate for the project.

EXAMPLE 7.6

Once again, we will use the cash flows experienced by Honda Motors in Example 7.4 (repeated below). Use the ROIC method to determine the EROR value. The MARR is 9% per year, and any excess funds generated by the project can earn at a rate of 12% per year.

Year	0	1	2	3
Net Cash Flow (\$1000)	+2000	-500	-8100	+6800

Solution by Hand

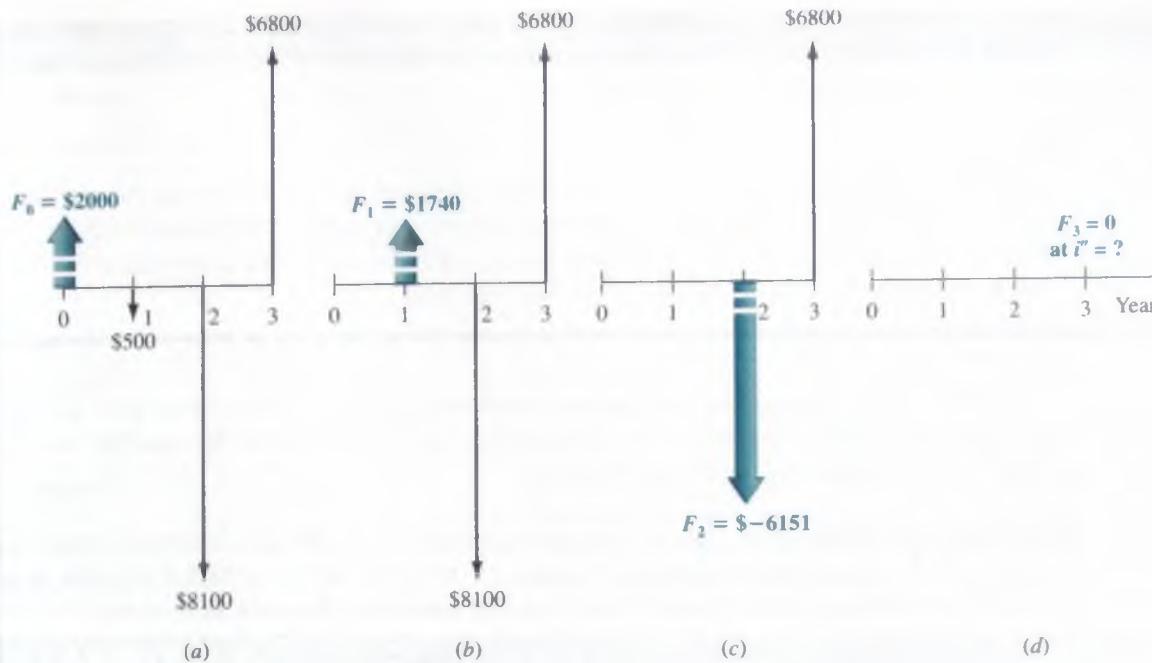
The hand solution is presented first to provide the logic of the ROIC method. Use $\text{MARR} = 9\%$ and $i_i = 12\%$ per year in the procedure to determine i'' , which is the ROIC. Figure 7-11 details the cash flows and tracks the progress as each F_t is developed. Equation [7.10] is applied to develop each F_t .

Step 1. Year 0: $F_0 = \$+2000$

Since $F_0 > 0$, externally invest in year 1 at $i_i = 12\%$.

Year 1: $F_1 = 2000(1.12) - 500 = \$+1740$

Since $F_1 > 0$, use $i_i = 12\%$ for year 2.

**Figure 7-11**

Application of ROIC method at $i_i = 12\%$ per year: (a) original cash flow; equivalent form in (b) year 1, (c) year 2, and (d) year 3.

$$\text{Year 2: } F_2 = 1740(1.12) - 8100 = \$-6151$$

Now $F_2 < 0$, use i'' for year 3, according to Equation [7.10].

$$\text{Year 3: } F_3 = -6151(1 + i'') + 6800$$

This is the last year. See Figure 7-11 for equivalent net cash flow diagrams.

Go to step 2.

Step 2. Solve for $i'' = \text{ROIC}$ from $F_3 = 0$.

$$-6151(1 + i'') + 6800 = 0$$

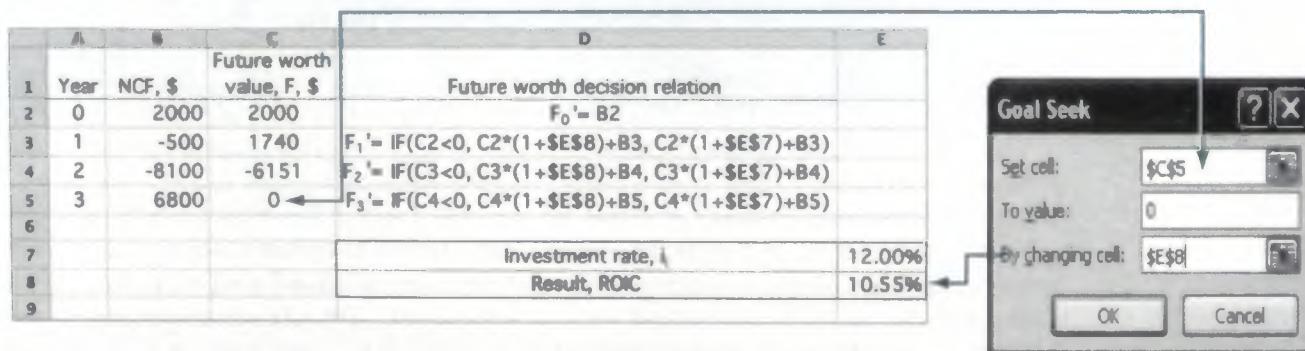
$$i'' = 6800/6151 - 1$$

$$= 0.1055 \quad (10.55\%)$$

Step 3. Since $\text{ROIC} > \text{MARR} = 9\%$, the project is economically justified.

Solution by Spreadsheet

Figure 7-12 provides a spreadsheet solution. The future worth values F_1 through F_3 are determined by the conditional IF statements in rows 3 through 5. The functions are shown in column D. In each year, Equation [7.10] is applied. If there are surplus funds generated by the project, $F_{t-1} > 0$ and the investment rate i_i (in cell E7) is used to find F_t . For example, because the F_1 value (in cell C3) of $1740 > 0$, the time value of money for the next year is calculated at the investment rate of 12% per year, as shown in the hand solution above for year 2.

**Figure 7-12**

Spreadsheet application of ROIC method using Goal Seek, Example 7.6.

The Goal Seek template sets the F_3 value to zero by changing the ROIC value (cell E8). The result is $i'' = \text{ROIC} = 10.55\%$ per year. As before, since $10.55\% > 9\%$, the MARR, the project is economically justified.

Comment

Note that the rate by the ROIC method (10.55%) is different than the MIRR rate (9.39%). Also these are both different than the multiple rates determined earlier (7.47% and 41.35%). This shows how dependent the different methods are upon the additional information provided when multiple i^* rates are indicated by the two sign tests.

Now that we have learned two techniques to remove multiple i^* values, there are some connections between the multiple i^* values, the external rate estimates, and the resulting external rates (i' and i'') obtained by the two methods.

Modified ROR technique When both the borrowing rate i_b and the investment rate i_i are exactly equal to any one of the multiple i^* values, the rate i' found by the MIRR function, or by hand solution, will equal the i^* value. That is, all four parameters have the same value.

$$\text{If any } i^* = i_b = i_i, \quad \text{then } i' = i^*$$

ROIC technique Similarly, if the investment rate i_i is exactly equal to any one of the multiple i^* values, the rate found by the Goal Seek tool, or by hand when the equation $F_n = 0$ is solved, will be $i'' = i^*$ value.

Finally, it is very important to remember the following fact.

None of the details of the modified ROR (MIRR) technique or the return on invested capital (ROIC) technique are necessary if the PW or AW method of project evaluation is applied at a specific MARR. When the MARR is established, this is, in effect, fixing the i^* value. Therefore, a definitive economic decision can be made directly from the PW or AW value.

7.6 Rate of Return of a Bond Investment ● ● ●

A time-tested method of raising capital funds is through the issuance of an IOU, which is financing through debt, not equity (see Chapter 1). One very common form of IOU is a bond—a long-term note issued by a corporation or a government entity (the borrower) to finance major projects. The borrower receives money now in return for a promise to pay the *face value* V of the bond on a stated maturity date. Bonds are usually issued in face value amounts of \$1000, \$5000, or \$10,000. **Bond dividend** I , also called *bond interest*, is paid periodically between the time the money is borrowed and the time the face value is repaid. The bond dividend is paid c times per year. Expected payment periods are usually semiannually or quarterly. The amount of interest is determined using the stated dividend or interest rate, called the *bond coupon rate* b .

$$I = \frac{\text{(face value)} \cdot (\text{bond coupon rate})}{\text{number of payment periods per year}}$$

$$I = \frac{Vb}{c} \quad [7.11]$$

There are many types or classifications of bonds. Four general classifications are summarized in Table 7–5 according to their issuing entity, fundamental characteristics, and example names or purposes. For example, *Treasury securities* are issued in different monetary amounts (\$1000 and up) with varying periods of time to the maturity date (Bills up to 1 year; Notes for 2 to 10 years). In the United States, Treasury securities are considered a very safe bond purchase because they are backed with the “full faith and credit of the U.S. government.” The safe investment rate indicated in Figure 1–6 as the lowest level for establishing a MARR is the coupon rate on a U.S. Treasury security. Funds obtained through corporate bond issues are used for new product development, facilities upgrade, expansion into international markets, and similar business ventures.

TABLE 7-5 Classification and Characteristics of Bonds

Classification	Issued by	Characteristics	Examples
Treasury securities	Federal government	Backed by faith and credit of the federal government	Bills (\leq 1 year) Notes (2–10 years) Bonds (10–30 years)
Municipal	Local governments	Federal tax-exempt Issued against taxes received	General obligation Revenue Zero coupon Put
Mortgage	Corporation	Backed by specified assets or mortgage Low rate/low risk on first mortgage Foreclosure, if not repaid	First mortgage Second mortgage Equipment trust
Debenture	Corporation	Not backed by collateral, but by reputation of corporation Bond rate may "float" Higher interest rates and higher risks	Convertible Subordinated Junk or high yield

EXAMPLE 7.7

General Electric just released \$10 million worth of \$10,000 ten-year bonds. Each bond pays dividends semiannually at a rate of 6% per year. (a) Determine the amount a purchaser will receive each 6 months and after 10 years. (b) Suppose a bond is purchased at a time when it is discounted by 2% to \$9800. What are the dividend amounts and the final payment amount at the maturity date?

Solution

(a) Use Equation [7.11] for the dividend amount.

$$I = \frac{10,000(0.06)}{2} = \$300 \text{ per 6 months}$$

The face value of \$10,000 is repaid after 10 years.

(b) Purchasing the bond at a discount from face value does not change the dividend or final repayment amounts. Therefore, \$300 per 6 months and \$10,000 after 10 years remain the amounts.

The cash flow series for a bond investment is conventional and has one unique i^* , which is best determined by solving a PW-based rate of return equation in the form of Equation [7.1], that is, $0 = PW$.

EXAMPLE 7.8

Allied Materials needs \$3 million in debt capital for expanded composites manufacturing. It is offering small-denomination bonds at a discount price of \$800 for a 4% \$1000 bond that matures in 20 years with a dividend payable semiannually. What nominal and effective interest rates per year, compounded semiannually, will Allied Materials pay an investor?

Solution

The income that a purchaser will receive from the bond purchase is the bond dividend $I = \$20$ every 6 months plus the face value in 20 years. The PW-based equation for calculating the rate of return is

$$0 = -800 + 20(P/A, i^*, 40) + 1000(P/F, i^*, 40)$$

Solve by the IRR function or by hand to obtain $i^* = 2.8435\%$ semiannually. The nominal interest rate per year is computed by multiplying i^* by 2.

Nominal $i = (2.8435)(2) = 5.6870\%$ per year, compounded semiannually

Using Equation [4.5], the effective annual rate is

$$i_a = (1.028435)^2 - 1 = 5.7678\%$$

EXAMPLE 7.9

Gerry is a project engineer. He took a financial risk and bought a bond from a corporation that had defaulted on its interest payments. He paid \$4240 for an 8% \$10,000 bond with dividends payable quarterly. The bond paid no interest for the first 3 years after Gerry bought it. If interest was paid for the next 7 years and then Gerry was able to resell the bond for \$11,000, what rate of return did he make on the investment? Assume the bond is scheduled to mature 18 years after he bought it. Perform hand and spreadsheet analysis.

Solution by Hand

The bond interest received in years 4 through 10 was

$$I = \frac{(10,000)(0.08)}{4} = \$200 \text{ per quarter}$$

The effective rate of return *per quarter* can be determined by solving the PW equation developed on a per quarter basis.

$$0 = -4240 + 200(P/A, i^* \text{ per quarter}, 28)(P/F, i^* \text{ per quarter}, 12) + 11,000(P/F, i^* \text{ per quarter}, 40)$$

The equation is correct for $i^* = 4.1\%$ per quarter, which is a nominal 16.4% per year, compounded quarterly.

Solution by Spreadsheet

Once all the cash flows are entered into contiguous cells, the function = IRR(B2:B42) is used in Figure 7-13, row 43, to display the answer of a nominal rate of return of 4.10% per quarter. (Note that many of the row entries have been hidden to conserve space.) This is the same as the nominal annual rate of

$$i^* = 4.10\%(4) = 16.4\% \text{ per year, compounded quarterly}$$

Gerry did well on his bond investment.

A	B	
1	Quarter	Cash flow, \$
2	0	-4,240
3	1	0
4	2	0
	6	0
13	11	0
14	12	0
15	13	200
21	19	200
22	20	200
29	27	200
41	39	200
42	40	11,200
43	i [*] /qtr	4.10%

Figure 7-13
Spreadsheet solution for
a bond investment.
Example 7.9.

If a bond investment is being considered and a required rate of return is stated, the same PW-based relation used to find i^* can be used to determine the maximum amount to pay for the bond now to ensure that the rate is realized. The stated rate is the MARR, and the PW computations are performed exactly as they were in Chapter 5. As an illustration, in the last example, if 12% per year, compounded quarterly, is the target MARR, the PW relation is used to find the maximum that Gerry should pay now; P is determined to be \$6004. The quarterly MARR is $12\%/4 = 3\%$.

$$0 = -P + 200(P/A, 3\%, 28)(P/F, 3\%, 12) + 11,000(P/F, 3\%, 40)$$

$$P = \$6004$$

CHAPTER SUMMARY

The rate of return of a cash flow series is determined by setting a PW-based or AW-based relation equal to zero and solving for the value of i^* . The ROR is a term used and understood by almost everybody. Most people, however, can have considerable difficulty in calculating a rate of return correctly for anything other than a conventional cash flow series. For some types of series, more than one ROR possibility exists. The maximum number of i^* values is equal to the number of changes in the sign of the net cash flow series (Descartes' rule of signs). Also, a single positive rate can be found if the cumulative net cash flow series starts negatively and has only one sign change (Norstrom's criterion).

When multiple i^* values are indicated, either of the two techniques covered in this chapter can be applied to find a single, reliable rate for the nonconventional net cash flow series. In the case of the ROIC technique, additional information is necessary about the *investment rate* that excess project funds will realize, while the modified ROR technique requires this same information, plus the *borrowing rate* for the organization considering the project. Usually, the investment rate is set equal to the MARR, and the borrowing rate takes on the historical WACC rate. Each technique will result in slightly different rates, but they are reliable for making the economic decision, whereas the multiple rates are often not useful to decision making.

If an exact ROR is not necessary, it is strongly recommended that the PW or AW method at the MARR be used to decide upon economic justification.

PROBLEMS

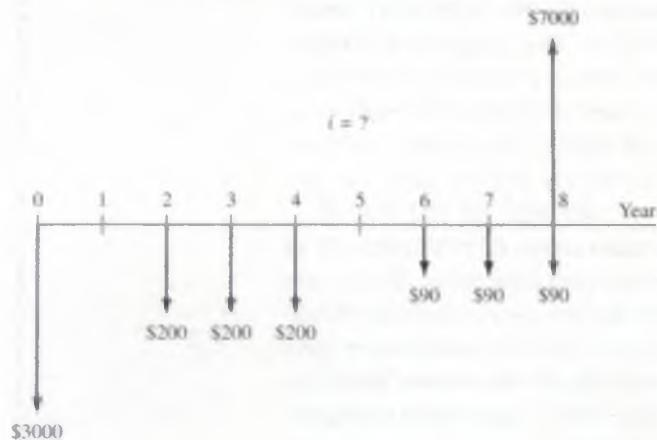
Understanding ROR

- 7.1 Under what circumstances would the rate of return be (a) -100% , and (b) infinite?
- 7.2 A shrewd investor loaned \$1,000,000 to a start-up company at 10% per year interest for 3 years, but the terms of the agreement were such that interest would be charged on the principal rather than on the unpaid balance. How much extra interest did the company pay?
- 7.3 What is the *nominal* rate of return per year on an investment that increases in value by 8% every 3 months?
- 7.4 Assume you borrow \$50,000 at 10% per year interest and you agree to repay the loan in five equal annual payments. What is the amount of the unrecovered balance immediately after you make the third payment?
- 7.5 International Potash got a \$50 million loan amortized over a 10-year period at 10% per year interest. The loan agreement stipulates that the loan will be repaid in 10 equal annual payments with interest charged on the principal amount of the loan (not on the unrecovered balance).
 - (a) What is the amount of each payment?
 - (b) What is the total amount of interest paid? How does the total interest paid compare with the principal of the loan?

Determination of ROR

- 7.6 In 2010, the city of Houston, Texas, collected \$24,112,054 in fines from motorists because of traffic violations caught by red-light cameras. The cost of operating the system was \$8,432,372. The net profit, that is, profit after operating costs, is split equally (that is, 50% each) between the city and the operator of the camera system. What will be the rate of return over a 3-year period to the contractor that paid for, installed, and operates the system, if its initial cost was \$9,000,000 and the profit for each of the 3 years is the same as it was in 2010?
- 7.7 P&G sold its prescription drug business to Warner-Chilcott, Ltd. for \$3.1 billion. If income from product sales is \$2 billion per year and net profit is 20% of sales, what rate of return will the company make over a 10-year planning horizon?
- 7.8 Water damage from a major flood in a Midwestern city resulted in damages estimated at \$108 million. As a result of the claimant payouts, insurance companies raised homeowners' insurance rates by an average of \$59 per year for each of the 160,000 households in the affected city. If a 20-year study period is considered, what was the rate of return on the \$108 million paid by the insurance companies?

- 7.9 Determine the rate of return for the cash flows shown in the diagram. (If requested by your instructor, show both hand and spreadsheet solutions.)



- 7.10 The Office of Naval Research sponsors a contest for college students to build underwater robots that can perform a series of tasks without human intervention. The University of Florida, with its Subjuggator robot, won the \$7000 first prize (and serious bragging rights) over 21 other universities. If the team spent \$2000 for parts (at time 0) and the project took 2 years, what annual rate of return did the team make?

- 7.11 For the cash flows shown, determine the rate of return.

Year	0	1	2	3	4	5
Expense, \$	-17,000	-2,500	-2,500	-2,500	-2,500	-2,500
Revenue, \$	0	5,000	6,000	7,000	8,000	12,000

- 7.12 In an effort to avoid foreclosure proceedings on struggling mortgage customers, Bank of America proposed an allowance that a jobless customer make no payment on their mortgage for up to 9 months. If the customer did not find a job within that time period, they would have to sign over their house to the bank. The bank would give them \$2000 for moving expenses.

Assume John and his family had a mortgage payment of \$2900 per month and he was not able to find a job within the 9-month period. If the bank saved \$40,000 in foreclosure costs, what rate of return per month did the bank make on the allowance? Assume the first payment that was skipped was due at the end of month 1 and the \$40,000 foreclosure savings and \$2000 moving expense occurred at the end of the 9-month forbearance period.

- 7.13 The Closing the Gaps initiative by the Texas Higher Education Coordinating Board established the goal of increasing the number of students in higher education in Texas from 1,064,247 in 2000

to 1,694,247 in 2015. If the increase were to occur uniformly, what rate of increase would be required each year to meet the goal?

- 7.14 U.S. Census Bureau statistics show that the annual earnings for persons with a high school diploma are \$35,220 versus \$57,925 for someone with a bachelor's degree. If the cost of attending college is assumed to be \$30,000 per year for 4 years and the forgone earnings during those years are assumed to be \$35,220 per year, what rate of return does earning a bachelor's degree represent? Use a 35-year study period. (*Hint:* The investment in years 1 through 4 is the cost of college plus the foregone earnings, and the income in years 5 through 35 is the difference in income between a high school diploma and a bachelor's degree.)

- 7.15 The Ester Municipal Water Utility issued 20-year bonds in the amount of \$53 million for several high-priority flood control improvement projects. The bonds carried a 5.38% dividend rate with the dividend payable annually. The U.S. economy was in a recession at that time, so as part of the federal stimulus program, the Utility gets a 35% reimbursement on the dividend it pays.

- (a) What is the effective dividend rate that the Utility is paying on the bonds?
- (b) What is the total dollar amount the Utility will save in dividends over the life of the bonds?
- (c) What is the future worth in year 20 of the dividend savings, if the interest rate is 6% per year?

- 7.16 A contract between BF Goodrich and the Steelworkers Union of America called for the company to spend \$100 million in capital investment to keep the facilities competitive. The contract also required the company to provide buyout packages for 400 workers. If the average buyout package is \$100,000 and the company is able to reduce costs by \$20 million per year, what rate of return will the company make over a 10-year period? Assume all of the company's expenditures occur at time 0 and the savings begin 1 year later.

- 7.17 Rubber sidewalks made from ground up tires are said to be environmentally friendly and easier on people's knees. Rubbersidewalks, Inc. of Gardena, California, manufactures the small rubberized squares that are being installed where tree roots, freezing weather, and snow removal have required sidewalk replacement or major repairs every 3 years. The District of Columbia spent \$60,000 for a rubber sidewalk to replace broken concrete in a residential neighborhood lined with towering willow oaks. If a concrete sidewalk costs \$28,000 and

- lasts only 3 years versus a 9-year life for the rubber sidewalks, what rate of return does this represent?
- 7.18 Efficient light jets (ELJs) are smaller aircraft that may revolutionize the way people travel by plane. They cost between \$1.5 and \$3 million, seat 5 to 7 people, and can fly up to 1100 miles at cruising speeds approaching 425 mph. Eclipse Aerospace was founded in 2009, and its sole business is making ELJs. The company invested \$500 million (time 0) and began taking orders 2 years later. If the company accepted orders for 2500 planes and received 10% down (in year 2) on planes having an average cost of \$1.8 million, what rate of return will the company make over a 10-year planning period? Assume 500 of the planes are delivered each year in years 6 through 10 and that the company's M&O costs average \$10 million per year in years 1 through 10. (If requested by your instructor, show both hand and spreadsheet solutions.)
- 7.19 Betson Enterprises distributes and markets the Big Buck video game which allows players to "hunt" for elk, antelope, moose, and bucks without shivering outside in the cold. E-sports entertainment in New York City purchased five machines for \$6000 each and took in an average of \$600 total per week in sales. What rate of return does this represent (a) per week and (b) per year (nominal)? Use a 3-year study period with 52 weeks per year.
- 7.20 A 473-foot, 7000-ton World War II troop carrier (once commissioned as the *USS Excambion*) was sunk in the Gulf of Mexico to serve as an underwater habitat and diving destination. The project took 10 years of planning and cost \$4 million. Assume the \$4 million was expended equally in years 1 through 10. What rate of return does the venture represent, if increased fishing and recreation activities are valued at \$270,000 per year beginning in year 11 and they continue in perpetuity? (If assigned by your instructor, show both hand and spreadsheet solutions.)
- 7.24 According to Descartes' rule of signs, what is the maximum number of real-number values that will balance a rate of return equation?
- 7.25 According to Descartes' rule of signs, how many possible i^* values are there for net cash flows that have the following signs?
 (a) + - + + + - +
 (b) - - - + + + +
 (c) + + + + - - - + - - + - - -
- 7.26 According to Norstrom's criterion, there are two requirements regarding the cumulative cash flows that must be satisfied to ensure that there is only one positive root in a rate of return equation. What are they?
- 7.27 According to Descartes' rule of signs, how many possible i^* values are there for the cash flows shown?

Year	1	2	3	4	5	6
Net Cash	+4100	-2000	-7000	+12,000	-700	+800
Flow, \$						

- 7.28 According to Descartes' rule of signs, how many i^* values are possible for the cash flows shown?

Year	1	2	3	4
Revenue, \$	25,000	13,000	4,000	70,000
Costs, \$	-30,000	-7,000	-6,000	-12,000

- 7.29 According to Descartes' rule and Norstrom's criterion, how many i^* values are possible for the cash flow (CF) sequence shown?

Year	1	2	3	4	5
Net Cash	+16,000	-32,000	-25,000	+50,000	-8,000
Flow, \$					
Cumulative	+16,000	-16,000	-41,000	+9,000	+1,000
CF, \$					

- 7.30 For the cash flows shown, determine the sum of the cumulative cash flows.

Year	0	1	2	3	4
Revenue, \$		25,000	15,000	4,000	18,000
Costs, \$	-6,000	-30,000	-7,000	-6,000	-12,000

- 7.31 Stan-Rite Corp of Manitowoc, Wisconsin, is a B to B company that manufactures many types of industrial products, including portable measuring arms with absolute encoders, designed to perform 3D inspections of industrial parts. If the company's cash flow (in millions) for one of its product divisions is as shown on the next page, determine (a) the number of possible i^* values and (b) all rate of return values between 0% and 100%.

Multiple ROR Values

- 7.21 What is meant by a *nonconventional* cash flow series?
- 7.22 Explain at least three types of projects in which large net cash flow changes may cause sign changes during the life of the project, thus indicating the possible presence of multiple ROR values.
- 7.23 Explain a situation with which you are personally familiar for which the net cash flows have changed signs in a fashion similar to those in Figure 7-5.

Year	Expenses	Revenues
0	\$-30	\$ 0
1	-20	18
2	-25	19
3	-15	36
4	-22	52
5	-20	38
6	-30	70

- 7.32 Julie received a \$50 bill for her birthday at the end of January. At the end of February, she spent this \$50 and an additional \$150 to buy clothes. Her parents then gave her \$50 and \$125 at the end of March and April, respectively, as she prepared to go to summer school and needed the clothes. Her conclusion was that over the 4 months, she had received \$25 more than she spent. Determine if Julie has a multiple rate of return situation for these cash flows. If so, determine the multiple rates and comment on their validity. The cash flow values are as follows:

Month	Jan (0)	Feb (1)	Mar (2)	Apr (3)
Cash Flow, \$	50	-200	50	125

- 7.33 Veggie Burger Boy sells franchises to individuals who want to start small in the sandwiches-for-vegetarians business and grow in net cash flow over the years. A franchisee in Mississippi provided the \$5000 up-front money, but did very poorly the first year. He was allowed to borrow at the end of his first year from the corporation's capital incentive fund with a promise to repay the loan in addition to the annual share that the corporation contractually receives from annual sales. The net cash flows from the corporation's perspective are shown below.

Year	0	1	2	3	4	5	6
NCF, \$	5000	-10,100	500	2000	2000	2000	2000

The corporate chief financial officer (CFO) has some questions concerning this NCF series. Help her by doing the following, using a spreadsheet.

- (a) Plot the PW versus i graph and estimate the rate of return for this franchise.
- (b) Use the IRR function on a spreadsheet to find the corresponding return.
- (c) Basing your conclusions on Descartes' and Norstrom's rules, provide the CFO with some advice on what ROR value is the most reliable for this franchise over the 6-year period. The normal corporate MARR used for franchisee evaluation is 30% per year.

- 7.34 In 2011, Vaught Industries closed its plant in Marionsville following labor, environmental, and safety problems. The plant was built in 2005

based on an older technology to produce meat products. It had positive NCF until 2010 and discontinued operation in 2011 due to labor and safety problems. In 2012, prior to the sale of the facility and property, Vaught spent \$1 million to make the site environmentally acceptable to a potential buyer. The net cash flows in \$100,000 over the years are listed below. Use a spreadsheet to do the following.

- (a) Check for multiple rates of return.
- (b) Find all rates that are real numbers between -25% and +50%, and calculate the PW value for interest rates in this range.
- (c) Indicate which is the best and correct i^* value to use in a PW analysis.

Year	2005	2006	2007	2008	2009	2010	2011	2012
NCF, \$	-25	+10	+10	+15	+15	-5	-6	-10

- 7.35 Five years ago, VistaCare spent \$5 million to develop and introduce a new service in home health care for people who require frequent blood dialysis treatments. The service was not well received after the first year and was removed from the market. When reintroduced 4 years after its initial launch, it was much more profitable. Now, in year 5, VistaCare has spent a large sum on research to broaden the application of this service. Use the NCF series below to plot the PW versus i graph and estimate the ROR over the 5 years. NCF values are in \$1 million units.

Year	0	1	2	3	4	5
Net Cash Flow, \$	-5000	5000	0	0	15,000	-15,000

Removing Multiple i^* Values

- 7.36 In calculating the external rate of return by the modified rate of return approach, it is necessary to use two different rates of return, the investment rate i_i and the borrowing rate i_b . When is each used?

- 7.37 In the modified rate of return approach for determining a single interest rate from net cash flows, state which interest rate is usually higher, the investment rate i_i or the borrowing rate i_b . State why.

- 7.38 Use the modified rate of return approach with an investment rate of 18% per year and a borrowing rate of 10% to find the external rate of return for the following cash flows.

Year	0	1	2	3
Net Cash Flow, \$	+16,000	-32,000	-25,000	+70,000

- 7.39 Harley worked for many years to save enough money to start his own residential landscape design business. The cash flows shown are those he recorded for the first 6 years as his own boss. Find the external rate of

return using the modified rate of return approach, an investment rate of 15% per year, and a borrowing rate of 8%. (After using the procedure, use the MIRR function to confirm your answer.)

Year	0	1	2	3	4	5	6
NCF, \$	-9000	+4100	-2000	-7000	+12,000	+700	+800

- 7.40 Samara, an engineer working for GE, invested her bonus money each year in company stock. Her bonus has been \$8000 each year for the past 6 years (i.e., at the end of years 1 to 6). At the end of year 7, she sold the stock for \$52,000 to buy a condo; she purchased no stock that year. In years 8 to 10, she again invested the \$8000 bonus. Samara sold all of the remaining stock for \$28,000 immediately after the investment at the end of year 10.
- (a) Determine the number of possible rate of return values in the net cash flow series using the two sign tests.
 - (b) Determine the external rate of return by hand, using the modified rate of return approach with an investment rate of 12% per year and a borrowing rate of 8%.
 - (c) Find the external rate of return by spreadsheet using the ROIC approach with an investment rate of 12% per year.
 - (d) Enter the cash flows into a spreadsheet, and use the IRR function to find the i^* value. You should get the same value as the ROIC in part (c). Explain why this is so, given that the investment rate is 12% per year. (Hint: Look carefully at the column labeled "Future worth, F, \$" when you solved part (c) using the spreadsheet.)

- 7.41 Swagelok Co. of Solon, Ohio, makes variable area flowmeters (VAFs) that measure liquid and gas flow rates by means of a tapered tube and float. If tooling and setup costs were \$400,000 in year 0 and an additional \$190,000 in year 3, determine the external rate of return using the modified rate of return approach. The revenue was \$160,000 per year in years 1 through 10. Assume the company's MARR is 20% per year and its cost of capital is 9% per year.

- 7.42 A company that makes clutch disks for race cars has the cash flows shown for one department.

Year	Cash Flow, \$1000
0	-65
1	30
2	84
3	-10
4	-12

- (a) Determine the number of positive roots to the rate of return relation.
- (b) Calculate the internal rate of return.

- (c) Calculate the external rate of return using the return on invested capital (ROIC) approach with an investment rate of 15% per year. (As assigned by your instructor, solve by hand and/or spreadsheet.)

- 7.43 For the cash flow series below, calculate the external rate of return, using the return on invested capital approach with an investment rate of 14% per year.

Year	Cash Flow, \$
0	3000
1	-2000
2	1000
3	-6000
4	3800

- 7.44 Five years ago, a company made a \$500,000 investment in a new high-temperature material. The product did poorly after only 1 year on the market. However, with a new name and advertising campaign 4 years later it did much better. New development funds have been expended this year (year 5) at a cost of \$1.5 million. Determine the external rate of return using the ROIC approach and an investment rate of 15% per year. The i^* rate is 44.1% per year.

Year	Cash Flow, \$
0	-500,000
1	400,000
2	0
3	0
4	2,000,000
5	-1,500,000

Bonds

- 7.45 What is the bond coupon rate on a \$25,000 mortgage bond that has semiannual interest payments of \$1250 and a 20-year maturity date?
- 7.46 An equipment trust bond with a face value of \$10,000 has a bond coupon rate of 8% per year, payable quarterly. What are the amount and frequency of the dividend payments?
- 7.47 What is the face value of a municipal bond that matures in 20 years and has a bond coupon rate of 6% per year with semiannual payments of \$900?
- 7.48 What is the present worth of a \$50,000 debenture bond that has a bond coupon rate of 8% per year, payable quarterly? The bond matures in 15 years. The interest rate in the marketplace is 6% per year, compounded quarterly.

- 7.49 Best Buy issued collateral bonds 4 years ago that have a face value of \$20,000 each and a coupon rate of 8% per year, payable semiannually. If the bond maturity date is 20 years from the date they were issued and the interest rate in the marketplace is now 12% per year, compounded semiannually, what is the present worth (now) of one bond?
- 7.50 In 2011, El Paso Water Utilities (EPWU) issued bonds worth \$9.125 million to improve the Van Buren dam in central El Paso and to finance three other drainage projects. The bonds were purchased by the Texas Water Development Board under the federal stimulus program wherein EPWU did not have to pay any dividend on the bonds. If the bond dividend rate would have been 4% per year, payable quarterly, with a bond maturity date 18 years after issuance, what is the present worth of the dividend savings to EPWU rate payers? Assume the market interest rate is 6% per year.
- 7.51 A recently issued industrial bond with a face value of \$10,000 has a coupon rate of 8% per year, payable annually. The bond matures 20 years from now. Jeremy is interested in buying one bond. If he pays \$10,000 for the bond and plans to hold it to maturity, what rate of return per year will he realize?
- 7.52 Due to a significant troop buildup at the local military base, a school district issued \$10,000,000 in bonds to build new schools. The bond coupon rate is 6% per year, payable semiannually, with a maturity date of 20 years. If an investor is able to purchase one of the bonds that has a face value of \$5000 for \$4800, what rate of return per 6 months will the investor realize? Assume the bond is kept to maturity.
- 7.53 As the name implies, a zero-coupon bond pays no dividend, only the face value when it matures. If a zero coupon bond that has a face value of \$10,000 and a maturity date of 15 years is for sale for \$2000, what rate of return will the purchaser make, provided the bond is held to maturity?
- 7.54 To provide infrastructure in the outlying areas of Morgantown, West Virginia, the city council issued 30-year bonds with a face value of \$25 million. The bond coupon rate was set at 5% per year, payable semiannually. Because the market interest rate increased immediately before the bonds were sold, the city received only \$23.5 million from the bond sale. What was the semiannual interest rate when the bonds were sold?
- 7.55 An investor who purchased a \$10,000 mortgage bond today paid only \$6000 for it. The bond coupon rate is 8% per year, payable quarterly, and the maturity date is 18 years from the year of issuance. Because the bond is in default, it will pay no dividend for the next 2 years. If the bond dividend is in fact paid for the following 5 years (after the 2 years) and the investor then sells the bond for \$7000, what rate of return will be realized (a) per quarter and (b) per year (nominal)?
- 7.56 Five years ago, GSI, an oil services company headquartered in Texas, issued \$10 million worth of 12% 30-year bonds with the dividend payable quarterly. The bonds have a call date of this year if GSI decides to take advantage of it. The interest rate in the marketplace decreased enough that the company is considering calling the bonds since the coupon rate is relatively high. If the company buys the bonds back now for \$11 million, determine the rate of return that the company will make (a) per quarter and (b) per year (nominal). (Hint: The "call" option means the following: By spending \$11 million now, the company will not have to make the quarterly bond dividend payments or pay the face value of the bonds when they come due 25 years from now.)
- 7.57 Four years ago, Chevron issued \$5 million worth of debenture bonds with a coupon rate of 10% per year, payable semiannually. Market interest rates dropped, and the company called the bonds (i.e., paid them off in advance) at a 10% premium on the face value. Therefore, it cost the corporation \$5.5 million to retire the bonds. What semiannual rate of return did an investor make who purchased a \$5000 bond 4 years ago and held it until it was called 4 years later?

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 7.58 All of the following mean the same as *rate of return* except:
- Internal rate of return
 - Time for return of capital
 - Interest rate
 - Return on investment
- 7.59 The numerical value of i in a rate of return equation can range from:
- 0% to 100%
 - 0 to ∞
 - 100% to +100%
 - 100% to ∞

- 7.60 The internal rate of return on an investment refers to the interest rate earned on the:

- (a) Initial investment
- (b) Unrecovered balance of the investment
- (c) Money recovered from an investment
- (d) Income from an investment

- 7.61 A conventional (or simple) cash flow series is one wherein:

- (a) The algebraic signs on the net cash flows change only once.
- (b) The interest rate you get is a simple interest rate.
- (c) The total of the net cash flows is equal to 0.
- (d) The total of the cumulative cash flows is equal to 0.

- 7.62 According to Descartes' rule of signs, for a net cash flow sequence of $- - + + - +$, the number of possible i values is:

- (a) 2
- (b) 3
- (c) 4
- (d) 5

- 7.63 According to Norstrom's criterion, the one statement below that is correct is:

- (a) The cumulative cash flow must start out positively.
- (b) The cumulative cash flow must start out negatively.
- (c) The cumulative cash flow must equal 0.
- (d) The net cash flow must start out positively.

- 7.64 According to Descartes' rule and Norstrom's criterion, the number of positive i^* values for the following cash flow sequence is:

Year	1	2	3	4
Revenue, \$	25,000	15,000	4,000	18,000
Cost, \$	30,000	7,000	6,000	12,000

- (a) 1
- (b) 2
- (c) 3
- (d) 4

- 7.65 For the net cash flows and cumulative cash flows shown, the value of x is:

Year	1	2	3	4	5
NCF, \$	+13,000	-29,000	-25,000	x	-8000
Cumulative	+13,000	-16,000	-41,000	+9000	+1000
NCF, \$					

- (a) \$7000
- (b) \$16,000
- (c) \$36,000
- (d) \$50,000

- 7.66 A company that uses a minimum attractive rate of return of 10% per year is evaluating new processes to improve operational efficiency. The estimates associated with candidate processes are shown.

	Alternative I	Alternative J
First cost, \$	-40,000	-50,000
Annual cost, \$ per year	-15,000	-12,000
Salvage value, \$	5,000	5,000
Life, years	3	6

The statement that is most correct is:

- (a) The alternatives are revenue alternatives.
- (b) The alternatives are cost alternatives.
- (c) The alternatives are revenue alternatives and DN is an option.
- (d) The alternatives are cost alternatives and DN is an option.

- 7.67 Scientific Instruments, Inc. uses a MARR of 8% per year. The company is evaluating a new process to reduce water effluents from its manufacturing processes. The estimate associated with the process follows. In evaluating the process on the basis of a rate of return analysis, the correct equation to use is:

New Process
First cost, \$
NCF, \$ per year
Salvage value, \$
Life, years

- (a) $0 = -40,000 + 13,000(P/A,i,3) + 5000(P/F,i,3)$
- (b) $0 = -40,000(A/P,i,3) + 13,000 + 5000(A/F,i,3)$
- (c) $0 = -40,000(F/P,i,3) + 13,000(F/A,i,3) + 5000$
- (d) Any of the above

- 7.68 When one is using the modified ROR method to remove multiple ROR values, an additional estimate needed besides the cash flows and their timings is:

- (a) The ROIC value
- (b) External rate of return
- (c) Investment rate
- (d) Internal rate of return

- 7.69 For the following cash flows, the modified rate of return method uses a borrowing rate of 10%, and an investment rate is 12% per year. The correct computation for the present worth in year 0 is:

Year	1	2	3	4	5
NCF, \$	-10,000	0	0	-19,000	+25,000

- (a) $-10,000 - 19,000(P/F,12\%,4)$
- (b) $-10,000 - 19,000(P/F,12\%,4) + 25,000(P/F,10\%,5)$

- (c) $25,000(P/F, 10\%, 5)$
 (d) $-10,000 - 19,000(P/F, 10\%, 4)$
- 7.70 The return on invested capital (ROIC) method removes multiple ROR values from a cash flow sequence. If the future worth computation in year t is $F_t < 0$, the ROIC rate i^* is used. The interpretation of $F_t < 0$ most closely means:
- The net balance of project cash flows in year t is negative.
 - The resulting external rate of return will be positive.
 - The net balance of project cash flows is positive in year t .
 - The sequence has nonremovable negative ROR values.
- 7.71 The meaning of *return on invested capital* for a corporation is best stated as:
- A rate-of-return measure that equates the internal and external ROR
 - A measure of how effectively the corporation uses capital funds invested in it
 - The value at which borrowing ROR and investing ROR are equal
 - The external rate of return value is based on total capital invested
- 7.72 A corporate \$10,000 bond has a coupon rate of 8% per year, payable semiannually. The bond matures 20 years from now. At an interest rate of 6% per year, compounded semiannually, the amount and frequency of the bond dividend payments are:
- \$600 every 6 months
 - \$800 every 6 months
 - \$300 every 6 months
 - \$400 every 6 months
- 7.73 A \$20,000 mortgage bond that is due in 1 year pays interest of \$500 every 3 months. The bond's coupon rate is:
- 2.5% per year, payable quarterly
 - 5% per year, payable quarterly
 - 5% per year, payable semiannually
 - 10% per year, payable quarterly
- 7.74 A \$10,000 municipal bond due in 10 years pays interest of \$400 every 6 months. If an investor purchases the bond now for \$9000 and holds it to maturity, the rate of return received can be determined by the following equation:
- $0 = -9000 + 400(P/A, i, 10)$
 $+ 10,000(P/F, i, 10)$
 - $0 = -9000 + 400(P/A, i, 20)$
 $+ 10,000(P/F, i, 20)$
 - $0 = -10,000 + 400(P/A, i, 20)$
 $+ 10,000(P/F, i, 20)$
 - $0 = -9000 + 800(P/A, i, 10)$
 $+ 10,000(P/F, i, 10)$
- 7.75 A debenture bond issued 3 years ago has a face value of \$5000, a coupon rate of 4% per year, payable semiannually, and a maturity date of 20 years *from the date it was issued*. The bond is for sale now for \$4500. If the interest rate in the marketplace is compounded quarterly, the value of n that must be used in the P/A factor to calculate the rate of return for the bond is:
- 34
 - 40
 - 68
 - 80

CASE STUDY

DEVELOPING AND SELLING AN INNOVATIVE IDEA

Background

Three engineers who worked for Mitchell Engineering, a company specializing in public housing development, went to lunch together several times a week. Over time they decided to work on solar energy production ideas. After a lot of weekend time over several years, they had designed and developed a prototype of a low-cost, scalable solar energy plant for use in multifamily dwellings on the low end and medium-sized manufacturing facilities on the upper end. For residential applications, the collector could be mounted along side a TV dish and be programmed to track the sun. The generator and additional equipment are installed in a closet-sized area in an apartment or on a floor for multiple-apartment supply.

The system serves as a supplement to the electricity provided by the local power company. After some 6 months of testing, it was agreed that the system was ready to market and reliably state that an electricity bill in high-rises could be reduced by approximately 40% per month. This was great news for low-income dwellers on government subsidy that are required to pay their own utility bills.

Information

With a hefty bank loan and \$200,000 of their own capital, they were able to install demonstration sites in three cities in the sunbelt. Net cash flow after all expenses, loan repayment, and taxes for the first 4 years was acceptable; \$55,000 at the

end of the first year, increasing by 5% each year thereafter. A business acquaintance introduced them to a potential buyer of the patent rights and current subscriber base with an estimated \$500,000 net cashout after only these 4 years of ownership. However, after serious discussion replaced the initial excitement of the sales offer, the trio decided to not sell at this time. They wanted to stay in the business for a while longer to develop some enhancement ideas and to see how much revenue may increase over the next few years.

During the next year, the fifth year of the partnership, the engineer who had received the patents upon which the collector and generator designs were based became very displeased with the partnering arrangements and left the trio to go into partnership with an international firm in the energy business. With new research and development funds and the patent rights, a competing design was soon on the market and took much of the business away from the original two developers. Net cash flow dropped to \$40,000 in year 5 and continued to decrease by \$5000 per year. Another offer to sell in year 8 was presented, but it was only for \$100,000 net cash. This was considered too much of a loss, so the two owners did not accept. Instead, they decided to put \$200,000 more of their own savings into the company to develop additional applications in the housing market.

It is now 12 years since the system was publicly launched. With increased advertising and development, net cash flow has been positive the last 4 years, starting at \$5000 in year 9 and increasing by \$5000 each year until now.

Case Study Exercises

It is now 12 years after the products were developed, and the engineers invested most of their savings in an innovative idea. However, the question of "When do we sell?" is always present in these situations. To help with the analysis, determine the following:

1. The rate of return at the end of year 4 for two situations:
(a) The business is sold for the net cash amount of \$500,000 and (b) no sale.
2. The rate of return at the end of year 8 for two situations:
(a) The business is sold for the net cash amount of \$100,000 and (b) no sale.
3. The rate of return now at the end of year 12.
4. Consider the cash flow series over the 12 years. Is there any indication that multiple rates of return may be present? If so, use the spreadsheet already developed to search for ROR values in the range $\pm 100\%$ other than the one determined in exercise 3 above.
5. Assume you are an investor with a large amount of ready cash, looking for an innovative solar energy product. What amount would you be willing to offer for the business at this point (end of year 12) if you require a 12% per year return on all your investments and, if purchased, you plan to own the business for 12 additional years? To help make the decision, assume the current NCF series continues increasing at \$5000 per year for the years you would own it. **Explain your logic for offering this amount.**



8 Rate of Return Analysis: Multiple Alternatives



LEARNING OUTCOMES

Purpose: Select the best alternative on the basis of incremental rate of return analysis.

SECTION	TOPIC	LEARNING OUTCOME
8.1	Incremental analysis	<ul style="list-style-type: none">State why the ROR method of comparing alternatives requires an incremental cash flow analysis.
8.2	Incremental cash flows	<ul style="list-style-type: none">Calculate the incremental cash flow series for two alternatives.
8.3	Meaning of Δi^*	<ul style="list-style-type: none">Interpret the meaning of the incremental ROR (Δi^*) determined from the incremental cash flow series.
8.4	Δi^* from PW relation	<ul style="list-style-type: none">Based on a PW relation, select the better of two alternatives using incremental ROR analysis or a breakeven ROR value.
8.5	Δi^* from AW relation	<ul style="list-style-type: none">Select the better of two alternatives using incremental ROR analysis based on an AW relation.
8.6	More than two alternatives	<ul style="list-style-type: none">Select the best from several alternatives using incremental ROR analysis.
8.7	All-in-one spreadsheet	<ul style="list-style-type: none">Use a single spreadsheet to perform PW, AW, ROR, and incremental ROR analyses for mutually exclusive and independent alternatives.



This chapter presents the methods by which two or more alternatives can be evaluated using a rate of return (ROR) comparison based on the methods of the previous chapter. The ROR evaluation, correctly performed, will result in the same selection as the PW and AW analyses, but the computational procedure is considerably different for ROR evaluations. The ROR analysis evaluates the increments between two alternatives in pairwise comparisons. As the cash flow series becomes more complex, spreadsheet functions help speed computations.

8.1 Why Incremental Analysis Is Necessary ● ● ●

When two or more mutually exclusive alternatives are evaluated, engineering economy can identify the one alternative that is the best economically. As we have learned, the PW and AW techniques can be used to do so, and are the recommended methods. Now the procedure using ROR to identify the best is presented.

Let's assume that a company uses a MARR of 16% per year, that the company has \$90,000 available for investment, and that two alternatives (A and B) are being evaluated. Alternative A requires an investment of \$50,000 and has an internal rate of return i_A^* of 35% per year. Alternative B requires \$85,000 and has an i_B^* of 29% per year. Intuitively we may conclude that the better alternative is the one that has the larger return, A in this case. However, this is not necessarily so. While A has the higher projected return, its initial investment (\$50,000) is much less than the total money available (\$90,000). What happens to the investment capital that is left over? It is generally assumed that excess funds will be invested at the company's MARR, as we learned in previous chapters. Using this assumption, it is possible to determine the consequences of the two alternative investments. If alternative A is selected, \$50,000 will return 35% per year. The \$40,000 left over will be invested at the MARR of 16% per year. The rate of return on the total capital available, then, will be the weighted average. Thus, if alternative A is selected,

$$\text{Overall ROR}_A = \frac{50,000(0.35) + 40,000(0.16)}{90,000} = 26.6\%$$

If alternative B is selected, \$85,000 will be invested at 29% per year, and the remaining \$5000 will earn 16% per year. Now the weighted average is

$$\text{Overall ROR}_B = \frac{85,000(0.29) + 5000(0.16)}{90,000} = 28.3\%$$

These calculations show that even though the i^* for alternative A is higher, alternative B presents the better overall ROR for the \$90,000. If either a PW or AW comparison is conducted using the MARR of 16% per year as i , alternative B will be chosen.

This simple example illustrates a major fact about the rate of return method for ranking and comparing alternatives:

Under some circumstances, project ROR values do not provide the same ranking of alternatives as do PW and AW analyses. This situation does not occur if we conduct an **incremental ROR analysis** (discussed below).

When **independent projects** are evaluated, no incremental analysis is necessary between projects. Each project is evaluated separately from others, and more than one can be selected. Therefore, the only comparison is with the do-nothing alternative for each project. The project ROR can be used to accept or reject each one.



8.2 Calculation of Incremental Cash Flows for ROR Analysis ● ● ●

To conduct an incremental ROR analysis, it is necessary to calculate the **incremental cash flow** series over the lives of the alternatives. Based upon the equivalence relations (PW and AW), ROR evaluation makes the equal-service assumption.



Independent project selection

TABLE 8-1 Format for Incremental Cash Flow Tabulation

Year	Cash Flow		Incremental Cash Flow (3) = (2) - (1)
	Alternative A (1)	Alternative B (2)	
0			
1			
.			
.			
.			



Equal-service requirement

The incremental ROR method requires that the equal-service requirement be met. Therefore, the LCM (least common multiple) of lives for each pairwise comparison must be used. All the assumptions of equal service present for PW analysis are necessary for the incremental ROR method.

A format for hand or spreadsheet solutions is helpful (Table 8-1). Equal-life alternatives have n years of incremental cash flows, while unequal-life alternatives require the LCM of lives for analysis. At the end of each life cycle, the salvage value and initial investment for the next cycle must be included for the LCM case.

When a **study period** is established, only this number of years is used for the evaluation. All incremental cash flows outside the period are neglected. As we learned earlier, using a study period, especially one shorter than the life of either alternative, can change the economic decision from that rendered when the full lives are considered.

Only for the purpose of simplification, use the convention that between two alternatives, the one with the *larger initial investment* will be regarded as *alternative B*. Then, for each year in Table 8-1,

$$\text{Incremental cash flow} = \text{cash flow}_B - \text{cash flow}_A \quad [8.1]$$

The initial investment and annual cash flows for each alternative (excluding the salvage value) are one of the types identified in Chapter 5:

Revenue alternative, where there are both negative and positive cash flows

Cost alternative, where all cash flow estimates are negative

In either case, Equation [8.1] is used to determine the incremental cash flow series with the sign of each cash flow carefully determined.



Revenue or cost alternative

EXAMPLE 8.1

A tool and die company in Hanover is considering the purchase of a drill press with fuzzy-logic software to improve accuracy and reduce tool wear. The company has the opportunity to buy a slightly used machine for \$15,000 or a new one for \$21,000. Because the new machine is a more sophisticated model, its operating cost is expected to be \$7000 per year, while the used machine is expected to require \$8200 per year. Each machine is expected to have a 25-year life with a 5% salvage value. Tabulate the incremental cash flow.

Solution

Incremental cash flow is tabulated in Table 8-2. The subtraction performed is (new – used) since the new machine has a larger initial cost. The salvage values in year 25 are separated from ordinary cash flow for clarity. When disbursements are the same for a number of consecutive years, *for hand solution only*, it saves time to make a single cash flow listing, as is done for years 1 to 25. However, remember that several years were combined when performing the analysis. This approach *cannot be used for spreadsheets*, when the IRR or NPV function is used, as each year must be entered separately.

TABLE 8-2 Cash Flow Tabulation for Example 8.1

Year	Cash Flow		Incremental Cash Flow (New – Used)
	Used Press	New Press	
0	\$ -15,000	\$ -21,000	\$ -6,000
1–25	-8,200	-7,000	+1,200
25	+750	+1,050	+300

EXAMPLE 8.2

A sole-source vendor can supply a new industrial park with large transformers suitable for underground utilities and vault-type installation. Type A has an initial cost of \$70,000 and a life of 8 years. Type B has an initial cost of \$95,000 and a life expectancy of 12 years. The annual operating cost for type A is expected to be \$9000, while the AOC for type B is expected to be \$7000. If the salvage values are \$5000 and \$10,000 for type A and type B, respectively, tabulate the incremental cash flow using their LCM for hand and spreadsheet solutions.

Solution by Hand

The LCM of 8 and 12 is 24 years. In the incremental cash flow tabulation for 24 years (Table 8-3), note that the reinvestment and salvage values are shown in years 8 and 16 for type A and in year 12 for type B.

Solution by Spreadsheet

Figure 8-1 shows the incremental cash flows for the LCM of 24 years. As in the hand tabulation, reinvestment is made in the last year of each intermediate life cycle. The incremental values in column D are the result of subtractions of column B from C.

Note that the final row includes a summation check. The total incremental cash flow should agree in both the column D total and the subtraction C29 – B29. Also note that the incremental values change signs three times, indicating the possibility of multiple i^* values, per Descartes' rule of signs. This possible dilemma is discussed later in the chapter.

TABLE 8-3 Incremental Cash Flow Tabulation, Example 8.2

Year	Cash Flow		Incremental Cash Flow (B – A)
	Type A	Type B	
0	\$ -70,000	\$ -95,000	\$ -25,000
1–7	-9,000	-7,000	+2,000
8	{ -70,000 -9,000 +5,000}	-7,000	+67,000
9–11	-9,000	-7,000	+2,000
12	-9,000	{ -95,000 -7,000 +10,000}	-83,000
13–15	-9,000	-7,000	+2,000
16	{ -70,000 -9,000 +5,000}	-7,000	+67,000
17–23	-9,000	-7,000	+2,000
24	{ -9,000 +5,000}	{ -7,000 +10,000}	+7,000
	\$ -411,000	\$ -338,000	\$ +73,000

Figure 8-1
Spreadsheet computation of incremental cash flows for unequal-life alternatives, Example 8.2.

	A	B	C	D	E	F	G
1				Incremental			
2		Net cash flows		cash flow			
3	Year	Type A	Type B	(B - A)			
4	0	70,000	-95,000	-25,000			
5	1	-9,000	-7,000	2,000			
6	2	-9,000	-7,000	2,000			
7	3	-9,000	-7,000	2,000			
8	4	-9,000	-7,000	2,000			
9	5	-9,000	-7,000	2,000			
10	6	-9,000	-7,000	2,000			
11	7	-9,000	7,000	2,000	Starting new life cycle for A = initial cost + AOC + salvage = - 70,000 - 9,000 + 5,000		
12	8	74,000	-7,000	67,000			
13	9	9,000	-7,000	2,000			
14	10	-9,000	-7,000	2,000			
15	11	-9,000	-7,000	2,000			
16	12	9,000	-92,000	-83,000			
17	13	9,000	-7,000	2,000			
18	14	9,000	-7,000	2,000			
19	15	-9,000	-7,000	2,000			
20	16	74,000	-7,000	67,000			
21	17	-9,000	-7,000	2,000			
22	18	-9,000	-7,000	2,000			
23	19	9,000	-7,000	2,000			
24	20	9,000	-7,000	2,000			
25	21	9,000	-7,000	2,000			
26	22	-9,000	-7,000	2,000			
27	23	-9,000	-7,000	2,000			
28	24	-4,000	3,000	7,000			
29	Totals	-411,000	-338,000	73,000	Check on summations Incremental column should equal difference of columns		
30							
31							



8.3 Interpretation of Rate of Return on the Extra Investment ● ● ●

The incremental cash flows in year 0 of Tables 8-2 and 8-3 reflect the *extra investment or cost* required if the alternative with the larger first cost is selected. This is important in an incremental ROR analysis in order to determine the ROR earned on the extra funds expended for the larger-investment alternative. If the incremental cash flows of the larger investment don't justify it, we must select the cheaper one. In Example 8.1 the new drill press requires an extra investment of \$6000 (Table 8-2). If the new machine is purchased, there will be a "savings" of \$1200 per year for 25 years, plus an extra \$300 in year 25. The decision to buy the used or new machine can be made on the basis of the profitability of investing the extra \$6000 in the new machine. If the equivalent worth of the savings is greater than the equivalent worth of the extra investment at the MARR, the extra investment should be made (i.e., the larger first-cost proposal should be accepted). On the other hand, if the extra investment is not justified by the savings, select the lower-investment proposal.

It is important to recognize that the rationale for making the selection decision is the same as if only *one alternative* were under consideration, that alternative being the one represented by the incremental cash flow series. When viewed in this manner, it is obvious that unless this investment yields a rate of return equal to or greater than the MARR, the extra investment should not be made. As further clarification of this extra investment rationale, consider the following: The rate of return attainable through the incremental cash flow is an alternative to investing at the MARR. Section 8.1 states that any excess funds not invested in the alternative are assumed to be invested at the MARR. The conclusion is clear:

If the rate of return available through the incremental cash flow equals or exceeds the MARR, the alternative associated with the extra investment should be selected.



ME alternative selection

Not only must the return on the extra investment meet or exceed the MARR, but also the return on the investment that is common to both alternatives must meet or exceed the MARR. Accordingly, prior to performing an incremental ROR analysis, it is advisable to determine the internal rate of return i^* for each alternative. This can be done only for *revenue alternatives*, because cost alternatives have only cost (negative) cash flows and no i^* can be determined. The guideline is as follows:

For multiple revenue alternatives, calculate the internal rate of return i^* for each alternative, and eliminate all alternatives that have an $i^* < \text{MARR}$. Compare the remaining alternatives incrementally.

As an illustration, if the MARR = 15% and two alternatives have i^* values of 12% and 21%, the 12% alternative can be eliminated from further consideration. With only two alternatives, it is obvious that the second one is selected. If both alternatives have $i^* < \text{MARR}$, no alternative is justified and the do-nothing alternative is the best economically. When three or more alternatives are evaluated, it is usually worthwhile, but not required, to calculate i^* for each alternative for preliminary screening. Alternatives that cannot meet the MARR may be eliminated from further evaluation using this option. This option is especially useful when performing the analysis by spreadsheet. The IRR function applied to each alternative's cash flow estimates can quickly indicate unacceptable alternatives, as demonstrated in Section 8.6.

When **independent projects** are evaluated, there is no comparison on the extra investment. The ROR value is used to accept all projects with $i^* \geq \text{MARR}$, assuming there is no budget limitation. For example, assume MARR = 10%, and three independent projects are available with ROR values of

$$i_A^* = 12\% \quad i_B^* = 9\% \quad i_C^* = 23\%$$

Projects A and C are selected, but B is not because $i_B^* < \text{MARR}$.



Independent project selection

8.4 Rate of Return Evaluation Using PW: Incremental and Breakeven ● ● ●

In this section we discuss the primary approach to making mutually exclusive alternative selections by the incremental ROR method. A PW-based relation is developed for the incremental cash flows and set equal to zero. Use hand solution or spreadsheet functions to find Δi_{B-A}^* , the internal ROR for the series. Placing Δ (delta) before i_{B-A}^* distinguishes it from the overall ROR values i_A^* and i_B^* . (Δi^* may replace Δi_{B-A}^* when only two alternatives are present.)

Since incremental ROR requires equal-service comparison, the LCM of lives must be used in the PW formulation. Because of the reinvestment requirement for PW analysis for different-life assets, the incremental cash flow series may contain several sign changes, indicating multiple Δi^* values. Though incorrect, this indication is usually neglected in actual practice. The correct approach is to follow one of the techniques of Section 7.5. This means that the single external ROR ($\Delta i'$ or $\Delta i''$) for the incremental cash flow series is determined.

These three elements—**incremental cash flow series, LCM, and multiple roots**—are the primary reasons that the ROR method is often applied incorrectly in engineering economy analyses of multiple alternatives. As stated earlier, it is always possible, and generally advisable, to use a PW or AW analysis *at an established MARR* in lieu of the ROR method when multiple rates are indicated.



The complete procedure for hand or spreadsheet solution for an incremental ROR analysis for two alternatives is as follows:

1. Order the alternatives by initial investment or cost, starting with the smaller one, called A. The one with the larger initial investment is in the column labeled B in Table 8-1.
2. Develop the cash flow and incremental cash flow series using the LCM of years, assuming reinvestment in alternatives.
3. Draw an incremental cash flow diagram, if needed.

4. Count the number of sign changes in the incremental cash flow series to determine if multiple rates of return may be present. If necessary, use Norstrom's criterion to determine if a single positive root exists.
5. Set up the $PW = 0$ equation and determine Δi_{B-A}^* .
6. Select the economically better alternative as follows:



If $\Delta i_{B-A}^* < MARR$, select alternative A.

If $\Delta i_{B-A}^* \geq MARR$, the extra investment is justified; select alternative B.

If Δi^* is exactly equal to or very near the MARR, noneconomic considerations help in the selection of the "better" alternative.

In step 5, if trial and error by hand is used, time may be saved if the Δi_{B-A}^* value is bracketed, rather than approximated by a point value using linear interpolation, provided that a single ROR value is not needed. For example, if the MARR is 15% per year and you have established that Δi_{B-A}^* is in the 15% to 20% range, an exact value is not necessary to accept B since you already know that $\Delta i_{B-A}^* \geq MARR$.

The IRR function on a spreadsheet will normally determine one Δi^* value. Multiple guess values can be input to find multiple roots in the range $-100\% \text{ to } \infty$ for a nonconventional series, as illustrated in Example 7.4. If this is not the case, to be correct, the indication of multiple roots in step 4 requires that one of the techniques of Section 7.5 be applied to find an EROR.

EXAMPLE 8.3

As Ford Motor Company retools an old truck assembly plant in Michigan to produce a fuel-efficient economy model, the Ford Focus. Ford and its suppliers are seeking additional sources for light, long-life transmissions. Automatic transmission component manufacturers use highly finished dies for precision forming of internal gears and other moving parts. Two United States-based vendors make the required dies. Use the per unit estimates below and a MARR of 12% per year to select the more economical vendor bid. Show both hand and spreadsheet solutions.

	A	B
Initial cost, \$	-8,000	-13,000
Annual costs, \$ per year	-3,500	-1,600
Salvage value, \$	0	2,000
Life, years	10	5

Solution by Hand

These are cost alternatives, since all cash flows are costs. Use the procedure described above to determine Δi_{B-A}^* .

1. Alternatives A and B are correctly ordered with the higher first-cost alternative in column 2 of Table 8-4.
2. The cash flows for the LCM of 10 years are tabulated.

TABLE 8-4 Incremental Cash Flow Tabulation, Example 8.3

Year	Cash Flow A (1)	Cash Flow B (2)	Incremental Cash Flow (3) = (2) - (1)
0	\$ -8,000	\$ -13,000	\$ -5,000
1-5	-3,500	-1,600	+1,900
5	—	{ +2,000 -13,000	-11,000
6-10	-3,500	-1,600	+1,900
10	—	+2,000	+2,000
	\$-43,000	\$-38,000	\$+5,000

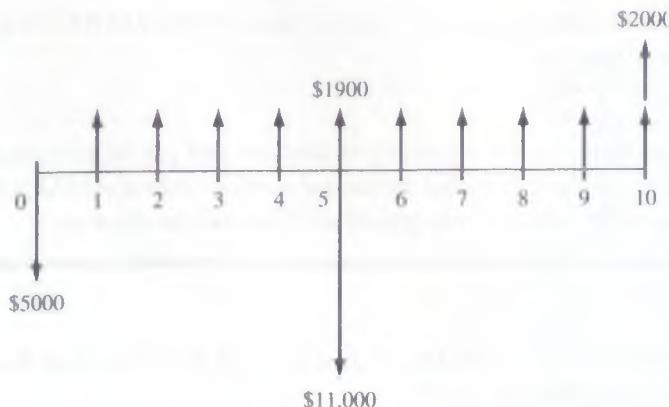


Figure 8-2
Diagram of incremental cash flows. Example 8.3.

3. The incremental cash flow diagram is shown in Figure 8-2.
4. There are three sign changes in the incremental cash flow series, indicating as many as three roots. There are also three sign changes in the cumulative incremental series, which starts negatively at $S_0 = \$-5000$ and continues to $S_{10} = \$+5000$, indicating that more than one positive root may exist.
5. The rate of return equation based on the PW of incremental cash flows is

$$0 = -5000 + 1900(P/A, \Delta i^*, 10) - 11,000(P/F, \Delta i^*, 5) + 2000(P/F, \Delta i^*, 10) \quad [8.2]$$

In order to resolve any multiple-root problem, we can assume that the investment rate i_i in the ROIC technique will equal the Δi^* found by trial and error. Solution of Equation [8.2] for the first root discovered results in Δi^* between 12% and 15%. By interpolation $\Delta i^* = 12.65\%$.

6. Since the rate of return of 12.65% on the extra investment is greater than the 12% MARR, the higher-cost vendor B is selected.

Comment

In step 4, the presence of up to three i^* values is indicated. The preceding analysis finds one of the roots at 12.65%. When we state that the incremental ROR is 12.65%, we assume that any positive net cash flows are reinvested at 12.65%. If this is not a reasonable assumption, the ROIC or modified ROR technique (Section 7.5) must be applied to find a different single $\Delta i'$ or $\Delta i''$ to compare with MARR = 12%.

The other two roots are very large positive and negative numbers, as the IRR function of Excel reveals. So they are not useful to the analysis.

Solution by Spreadsheet

Steps 1 through 4 are the same as above.

5. Figure 8-3 includes the same incremental net cash flows from Table 8-4 calculated in column D. Cell D15 displays the Δi^* value of 12.65% using the IRR function.

	A	B	C	D	E
1					
2					
3	Year	Vendor A cash flow, \$	Vendor B cash flow, \$	Incremental cash flow, \$	
4	0	-8,000	-13,000	-5,000	
5	1	-3,500	-1,600	1,900	
6	2	-3,500	-1,600	1,900	
7	3	-3,500	-1,600	1,900	
8	4	-3,500	-1,600	1,900	
9	5	-3,500	-12,600	-9,100	
10	6	-3,500	-1,600	1,900	
11	7	-3,500	-1,600	1,900	
12	8	-3,500	-1,600	1,900	
13	9	-3,500	-1,600	1,900	
14	10	-3,500	400	3,900	
15	Incremental i^*			12.65%	
16					
17	Check: PW @ 12%		\$137.67		
18					
19					
20				= NPV(12%,D5:D14) + D4	

Figure 8-3
Spreadsheet solution using LCM of lives and IRR function. Example 8.3.

6. Since the rate of return on the extra investment is greater than the 12% MARR, the higher-cost vendor B is selected.

Comment

Once the spreadsheet is set up, there are a wide variety of analyses that can be performed. For example, row 17 uses the NPV function to verify that the present worth is positive at $MARR = 12\%$. Charts such as PW versus Δi and PW versus i help graphically interpret the situation.



Break-even ROR

The rate of return determined for the incremental cash flow series or the actual cash flows can be interpreted as a **break-even rate of return** value.

The **break-even rate of return** is the incremental i^* value, Δi^* , at which the PW (or AW) value of the *incremental cash flows* is exactly zero. Equivalently, the break-even ROR is the i value, i^* , at which the PW (or AW) values of two alternatives' actual cash flows are exactly equal to each other.

If the incremental cash flow ROR (Δi^*) is greater than the MARR, the larger-investment alternative is selected. For example, if the PW versus Δi graph for the incremental cash flows in Table 8-4 (and spreadsheet Figure 8-3) is plotted for various interest rates, the graph shown in Figure 8-4 is obtained. It shows the Δi^* breakeven at 12.65%. The conclusions are that

- For $MARR < 12.65\%$, the extra investment for B is justified.
- For $MARR > 12.65\%$, the opposite is true—the extra investment in B should not be made, and vendor A is selected.
- If $MARR$ is exactly 12.65%, the alternatives are equally attractive.

Figure 8-5, which is a breakeven graph of PW versus i for the cash flows (not incremental) of each alternative in Example 8.3, provides the same results. Since all net cash flows are negative

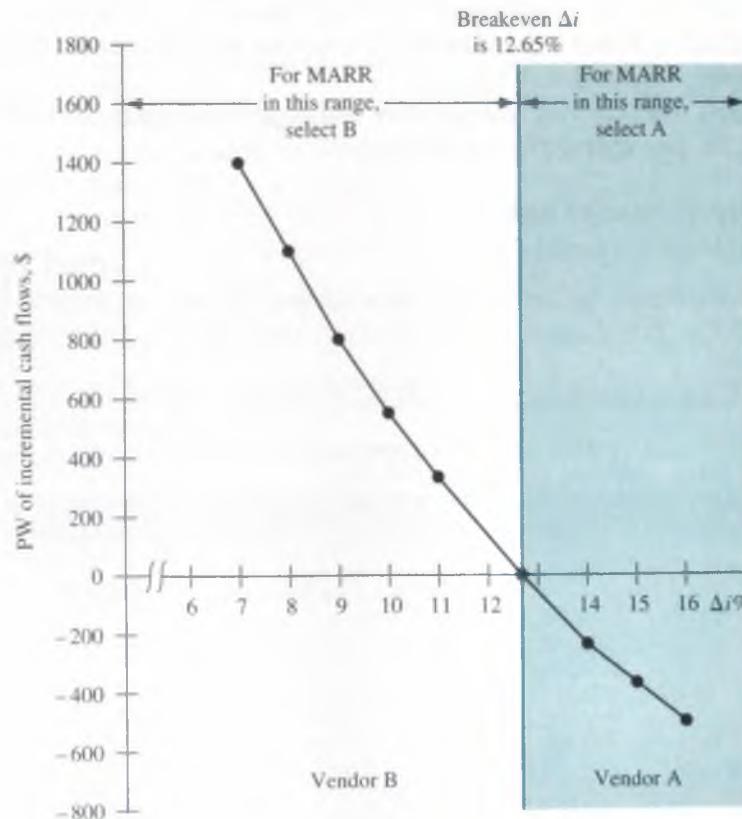


Figure 8-4

Plot of present worth of incremental cash flows for Example 8.3 at various Δi values.

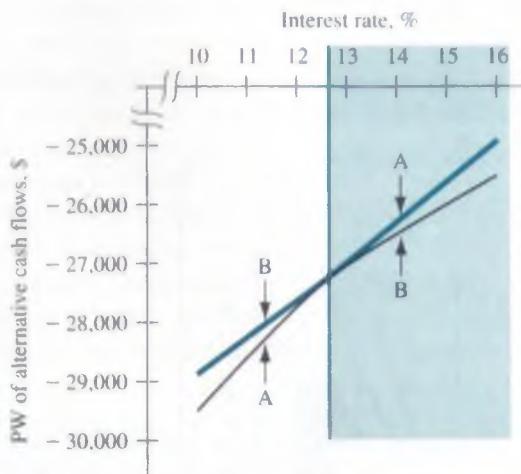


Figure 8-5
Breakeven graph of Example 8.3 cash flows (not incremental).

(cost alternatives), the PW values are negative. Now, the same conclusions are reached using the following logic:

- If MARR < 12.65%, select B since its PW of cost cash flows is smaller (numerically larger).
- If MARR > 12.65%, select A since its PW of costs is smaller.
- If MARR is exactly 12.65%, either alternative is equally attractive.

Example 8.4 illustrates incremental ROR evaluation and breakeven rate of return graphs for revenue alternatives. More of breakeven analysis is covered in Chapter 13.

EXAMPLE 8.4

New filtration systems for commercial airliners are available that use an electric field to remove up to 99.9% of infectious diseases and pollutants from aircraft air. This is vitally important, as many of the flu germs, viruses, and other contagious diseases are transmitted through the systems that recirculate aircraft air many times per hour. Investments in the new filtration equipment can cost from \$100,000 to \$150,000 per aircraft, but savings in fuel, customer complaints, legal actions, etc., can also be sizable. Use the estimates below (in \$100 units) from two suppliers provided to an international carrier to do the following, using a spreadsheet and an MARR of 15% per year.

- Plot two graphs: PW versus i values for both alternatives' cash flows and PW versus Δi values for incremental cash flows.
- Estimate the breakeven ROR values from both graphs, and use this estimate to select one alternative.

	Air Cleanser (Filter 1)	Purely Heaven (Filter 2)
Initial cost per aircraft, \$	-1000	-1500
Estimated savings, \$ per year	375	700 in year 1, decreasing by 100 per year thereafter
Estimated life, years	5	5

Solution by Spreadsheet

Refer to Figure 8-6 as the solution is explained. For information, row 10 shows the ROR values calculated using the IRR function for filter 1 and filter 2 cash flows and the incremental cash flow series (filter 2 – filter 1).

The cash flow sign tests for each filter indicate no multiple rates. The incremental cash flow sign test does not indicate the presence of a unique positive root; however, the second rate is an extremely large and useless value. The PW values of filter 1 and filter 2 cash flows are plotted on the right side for i values ranging from 0% to 60%. Since the PW curves cross each other at approximately $i^* = 17\%$, the rate does exceed the MARR of 15%. The higher-cost filter 2 (Purely Heaven) is selected.

The PW curve for the incremental cash flows series (column G) is plotted at the bottom left of Figure 8-6. As expected, the curve crosses the PW = 0 line at approximately 17%, indicating the same economic conclusion of filter 2.

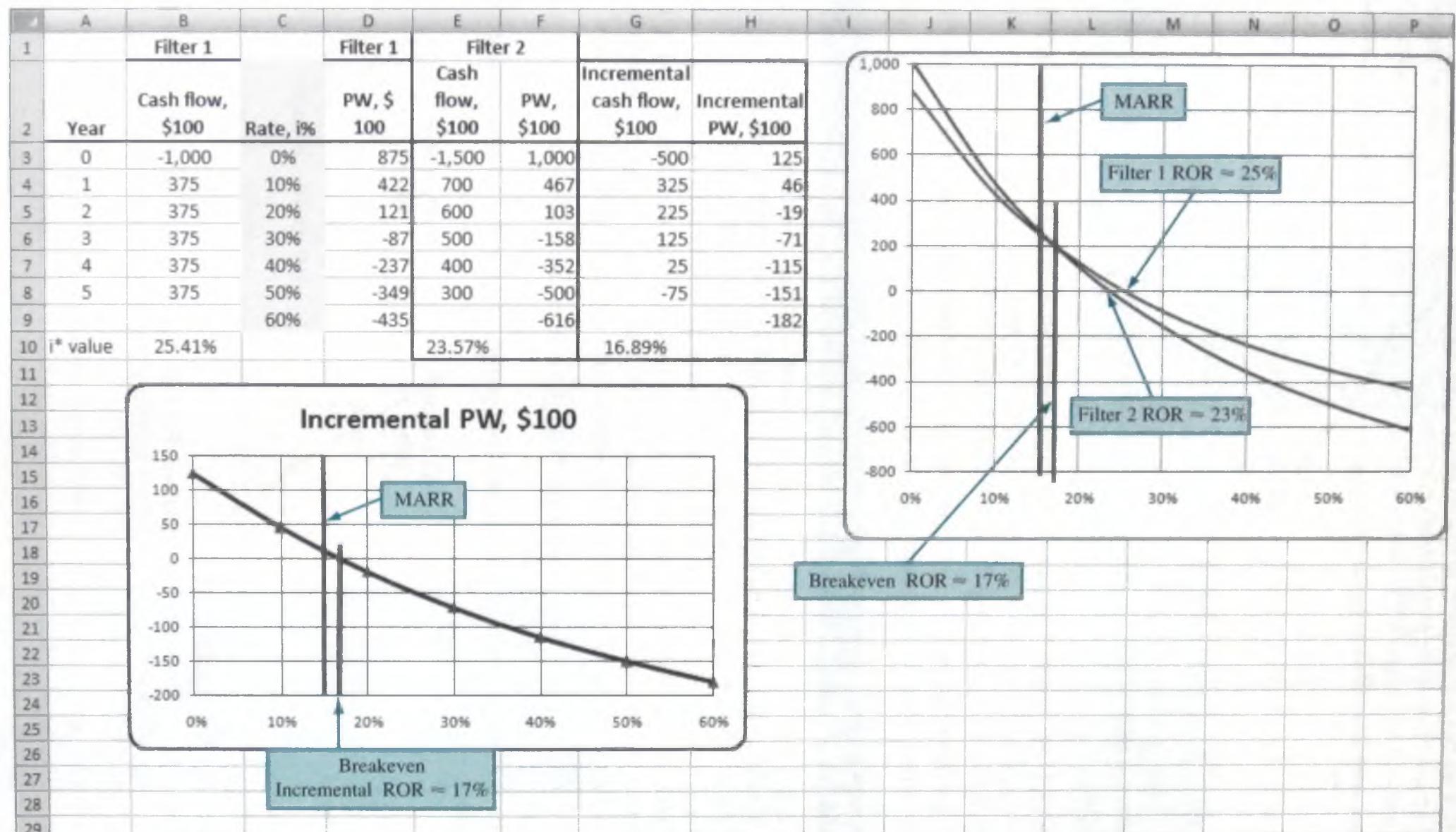


Figure 8–6
PW versus i graph and PW versus incremental i graph, Example 8.4.

Figure 8–6 provides an excellent opportunity to see why the ROR method can result in selecting the wrong alternative when only i^* values are used to select between two alternatives. This is sometimes called the **ranking inconsistency problem** of the ROR method. *The inconsistency occurs when the MARR is set less than the breakeven rate between two revenue alternatives.* Since the MARR is established based on conditions of the economy and market, MARR is established external to any particular alternative evaluation. In Figure 8–6 the incremental breakeven rate is 16.89%, and the MARR is 15%. The MARR is lower than breakeven; therefore, the *incremental ROR analysis* results in correctly selecting filter 2. But if only the i^* values were used, filter 1 would be wrongly chosen, because its i^* exceeds that of filter 2 ($25.41\% > 23.57\%$). This error occurs because the rate of return method assumes reinvestment at the alternative's ROR value, while PW and AW analyses use the MARR as the reinvestment rate. The conclusion is simple:

If the ROR method is used to evaluate two or more alternatives, use the **incremental cash flows** and Δi^* to make the decision between alternatives.

8.5 Rate of Return Evaluation Using AW ● ● ●

Comparing alternatives by the ROR method (correctly performed) always leads to the same selection as PW and AW analyses, whether the ROR is determined using a PW-based or an AW-based relation. However, for the AW-based technique, there are two equivalent ways to perform the evaluation: (1) using the *incremental cash flows* over the LCM of alternative lives, just as for the PW-based relation (Section 8.4), or (2) finding the AW for each alternative's *actual cash flows* and setting the difference of the two equal to zero to find the Δi^* value. There is no difference between the two approaches if the alternative lives are equal. Both methods are summarized here.

Since the ROR method requires comparison for equal service, the **incremental cash flows must be evaluated over the LCM of lives**. There may be no real computational advantage to using AW, as was found in Chapter 6. The same six-step procedure of the previous section (for PW-based calculation) is used, except in step 5 the AW-based relation is developed.



Equal-service requirement

The second AW-based method mentioned above takes advantage of the AW technique's assumption that the equivalent AW value is the same for each year of the first and all succeeding life cycles. Whether the lives are equal or unequal, set up the *AW relation for the cash flows of each alternative*, form the relation below, and solve for i^* .

$$0 = AW_B - AW_A \quad [8.3]$$

For both methods, all equivalent values are on an AW basis, so the i^* that results from Equation [8.3] is the same as the Δi^* found using the first approach. Example 8.5 illustrates ROR analysis using AW-based relations for unequal lives.

EXAMPLE 8.5

Compare the alternatives of vendors A and B for Ford in Example 8.3, using an AW-based incremental ROR method and the same MARR of 12% per year.

Solution

For reference, the PW-based ROR relation, Equation [8.2], for the incremental cash flow in Example 8.3 shows that vendor B should be selected with $\Delta i^* = 12.65\%$.

For the AW relation, there are two equivalent solution approaches. Write an AW-based relation on the *incremental cash flow series over the LCM of 10 years*, or write Equation [8.3] for the *two actual cash flow series over one life cycle* of each alternative.

For the incremental method, the AW equation is

$$0 = -5000(A/P, \Delta i^*, 10) - 11,000(P/F, \Delta i^*, 5)(A/P, \Delta i^*, 10) + 2000(A/F, \Delta i^*, 10) + 1900$$

It is easy to enter the incremental cash flows onto a spreadsheet, as in Figure 8–3, column D, and use the =IRR(D4:D14) function to display $\Delta i^* = 12.65\%$.

For the second method, the ROR is found using the actual cash flows and the respective lives of 10 years for A and 5 years for B.

$$AW_A = -8000(A/P,i,10) - 3500$$

$$AW_B = -13,000(A/P,i,5) + 2000(A/F,i,5) - 1600$$

Now develop $0 = AW_B - AW_A$.

$$0 = -13,000(A/P,i^*,5) + 2000(A/F,i^*,5) + 8000(A/P,i^*,10) + 1900$$

Solution again yields $i^* = 12.65\%$.

Comment

It is very important to remember that when an incremental ROR analysis using an AW-based equation is made on the *incremental cash flows*, the LCM must be used.



8.6 Incremental ROR Analysis of Multiple Alternatives ● ● ●

This section treats selection from multiple alternatives that are mutually exclusive, using the incremental ROR method. Acceptance of one alternative automatically precludes acceptance of any others. The analysis is based upon PW (or AW) relations for incremental cash flows between two alternatives at a time.

When the incremental ROR method is applied, the entire investment must return at least the MARR. When the i^* values on several alternatives exceed the MARR, incremental ROR evaluation is required. (For revenue alternatives, if not even one $i^* \geq MARR$, the do-nothing alternative is selected.) For all alternatives (revenue or cost), the incremental investment must be separately justified. If the return on the extra investment equals or exceeds the MARR, then the extra investment should be made in order to maximize the total return on the money available, as discussed in Section 8.1.

For ROR analysis of multiple, mutually exclusive alternatives, the following criteria are used.



ME alternative selection

Select the one alternative

That requires the **largest investment**, and

Indicates that the **extra investment over another acceptable alternative is justified**.

An important rule to apply when evaluating multiple alternatives by the incremental ROR method is that *an alternative should never be compared with one for which the incremental investment is not justified*.

The incremental ROR evaluation procedure for multiple, equal-life alternatives is summarized below. Step 2 applies only to revenue alternatives, because the first alternative is compared to DN only when revenue cash flows are estimated. The terms **defender** and **challenger** are dynamic in that they refer, respectively, to the alternative that is currently selected (the defender) and the one that is challenging it for acceptance based on Δi^* . In every pairwise evaluation, there is one of each. The steps for solution by hand or by spreadsheet are as follows:

1. Order the alternatives from **smallest to largest initial investment**. Record the annual cash flow estimates for each equal-life alternative.
2. *Revenue alternatives only:* Calculate i^* for the first alternative. In effect, this makes DN the defender and the first alternative the challenger. If $i^* < MARR$, eliminate the alternative and go to the next one. Repeat this until $i^* \geq MARR$ for the first time, and define that alternative as the defender. The next alternative is now the challenger. Go to step 3. (*Note:* This is where solution by spreadsheet can be a quick assist. Calculate the i^* for all alternatives first using the IRR function, and select as the defender the first one for which $i^* \geq MARR$. Label it the defender and go to step 3.)
3. Determine the incremental cash flow between the challenger and defender, using the relation

$$\text{Incremental cash flow} = \text{challenger cash flow} - \text{defender cash flow}$$

Set up the ROR relation.

4. Calculate Δi^* for the incremental cash flow series using a PW- or AW-based equation. (PW is most commonly used.)
5. If $\Delta i^* \geq \text{MARR}$, the challenger becomes the defender and the previous defender is eliminated. Conversely, if $\Delta i^* < \text{MARR}$, the challenger is removed, and the defender remains against the next challenger.
6. Repeat steps 3 to 5 until only one alternative remains. It is the selected one.

Note that only two alternatives are compared at any one time. It is vital that the correct alternatives be compared, or the wrong alternative may be selected.

EXAMPLE 8.6

Caterpillar Corporation wants to build a spare parts storage facility in the Phoenix, Arizona, vicinity. A plant engineer has identified four different location options. The initial cost of earth-work and prefab building and the annual net cash flow estimates are detailed in Table 8–5. The annual net cash flow series vary due to differences in maintenance, labor costs, transportation charges, etc. If the MARR is 10%, use incremental ROR analysis to select the one economically best location.

TABLE 8–5 Estimates for Four Alternative Building Locations, Example 8.6

	A	B	C	D
Initial cost, \$	−200,000	−275,000	−190,000	−350,000
Annual cash flow, \$ per year	+22,000	+35,000	+19,500	+42,000
Life, years	30	30	30	30

Solution

All sites have a 30-year life, and they are revenue alternatives. The procedure outlined above is applied.

1. The alternatives are ordered by increasing initial cost in Table 8–6.
2. Compare location C with the do-nothing alternative. The ROR relation includes only the P/A factor.

$$0 = -190,000 + 19,500(P/A, i^*, 30)$$

Table 8–6, column 1, presents the calculated $(P/A, \Delta i^*, 30)$ factor value of 9.7436 and $\Delta i^* = 9.63\%$. Since $9.63\% < 10\%$, location C is eliminated. Now the comparison is A to DN, and column 2 shows that $\Delta i^* = 10.49\%$. This eliminates the do-nothing alternative; the defender is now A and the challenger is B.

TABLE 8–6 Computation of Incremental Rate of Return for Four Alternatives, Example 8.6

	C (1)	A (2)	B (3)	D (4)
Initial cost, \$	−190,000	−200,000	−275,000	−350,000
Cash flow, \$ per year	+19,500	+22,000	+35,000	+42,000
Alternatives compared	C to DN	A to DN	B to A	D to B
Incremental cost, \$	−190,000	−200,000	−75,000	−75,000
Incremental cash flow, \$	+19,500	+22,000	+13,000	+7,000
Calculated $(P/A, \Delta i^*, 30)$	9.7436	9.0909	5.7692	10.7143
$\Delta i^*, \%$	9.63	10.49	17.28	8.55
Increment justified?	No	Yes	Yes	No
Alternative selected	DN	A	B	B

3. The incremental cash flow series, column 3, and Δi^* for *B-to-A comparison* are determined from

$$\begin{aligned} 0 &= -275,000 - (-200,000) + (35,000 - 22,000)(P/A, \Delta i^*, 30) \\ &= -75,000 + 13,000(P/A, \Delta i^*, 30) \end{aligned}$$

4. From the interest tables, look up the *P/A* factor at the MARR, which is $(P/A, 10\%, 30) = 9.4269$. Now, any *P/A* value greater than 9.4269 indicates that the Δi^* will be less than 10% and is unacceptable. The *P/A* factor is 5.7692, so *B* is acceptable. For reference purposes, $\Delta i^* = 17.28\%$.
5. Alternative *B* is justified incrementally (new defender), thereby eliminating *A*.
6. Comparison *D-to-B* (steps 3 and 4) results in the PW relation $0 = -75,000 + 7000(P/A, \Delta i^*, 30)$ and a *P/A* value of 10.7143 ($\Delta i^* = 8.55\%$). Location *D* is eliminated, and **only alternative *B* remains; it is selected.**

Comment

An alternative must *always* be incrementally compared with an acceptable alternative, and the do-nothing alternative can end up being the only acceptable one. Since *C* was not justified in this example, location *A* was not compared with *C*. Thus, if the *B-to-A* comparison had not indicated that *B* was incrementally justified, then the *D-to-A* comparison would be correct instead of *D-to-B*.

To demonstrate how important it is to apply the ROR method correctly, consider the following. If the i^* of each alternative is computed initially, the results by ordered alternatives are

Location	C	A	B	D
$i^*, \%$	9.63	10.49	12.35	11.56

Now apply *only* the first criterion stated earlier; that is, make the largest investment that has a MARR of 10% or more. Location *D* is selected. But, as shown above, this is the wrong selection, because the extra investment of \$75,000 over location *B* will not earn the MARR. In fact, it will earn only 8.55%. This is another example of the *ranking inconsistency problem of the ROR method* mentioned in Section 8.4.

For cost alternatives, the incremental cash flow is the difference between costs for two alternatives. There is no do-nothing alternative and no step 2 in the solution procedure. Therefore, the lowest-investment alternative is the initial defender against the next-lowest investment (challenger). This procedure is illustrated in Example 8.7 using a spreadsheet solution.

EXAMPLE 8.7

The complete failure of an offshore platform and the resulting spillage of up to 800,000 to 1,000,000 gallons per day into the Gulf of Mexico in the spring of 2010 have made major oil producers and transporters very conscious of the harm done to people's livelihood and all forms of aquatic life by spills of this magnitude. To address the specific danger to birds that are shoreline feeders and dwellers, environmental engineers from several international petroleum corporations and transport companies—Exxon-Mobil, BP, Shell, and some transporters for OPEC producers—have developed a plan to strategically locate throughout the world newly developed equipment that is substantially more effective than manual procedures in cleaning crude oil residue from bird feathers. The Sierra Club, Greenpeace, and other international environmental interest groups are in favor of the initiative. Alternative machines from manufacturers in Asia, America, Europe, and Africa are available with the cost estimates in Table 8-7. Annual cost estimates are expected to be high to ensure readiness at any time. The company representatives have agreed to use the average of the corporate MARR values, which results in $MARR = 13.5\%$. Use a spreadsheet and incremental ROR analysis to determine which manufacturer offers the best economic choice.

TABLE 8-7 Costs for Four Alternative Machines, Example 8.7

	Machine 1	Machine 2	Machine 3	Machine 4
First cost, \$	−5,000	−6,500	−10,000	−15,000
Annual operating cost, \$	−3,500	−3,200	−3,000	−1,400
Salvage value, \$	+500	+900	+700	+1,000
Life, years	8	8	8	8

Solution by Spreadsheet

Follow the procedure for incremental ROR analysis. The spreadsheet in Figure 8-7 contains the complete solution.

1. The alternatives are already ordered by increasing first costs.
2. These are cost alternatives, so there is no comparison to DN, since i^* values cannot be calculated.
3. Machine 2 is the first challenger to machine 1; the incremental cash flows for the 2-to-1 comparison are in column D.
4. The 2-to-1 comparison results in $\Delta i^* = 14.57\%$ by applying the IRR function.
5. This return exceeds MARR = 13.5%, so machine 2 is the new defender (cell D17).

The comparison continues for 3-to-2 in column E, where the return is negative at $\Delta i^* = -18.77\%$; machine 2 is retained as the defender. Finally the 4-to-2 comparison has an incremental ROR of 13.60%, which is slightly larger than MARR = 13.5%. The conclusion is to purchase machine 4 because the extra investment is (marginally) justified.

	A	B	C	D	E	F
1		Year	Machine 1	Machine 2	Machine 3	Machine 4
2	Initial investment, \$		-5,000	-6,500	-10,000	-15,000
3	Annual cost, \$ per year		-3,500	-3,200	-3,000	-1,400
4	Salvage value, \$		500	900	700	1,000
5	Incremental comparison			2 to 1	3 to 2	4 to 2
6	Incremental investment, \$	0		-1,500	-3,500	-8,500
7	Incremental cash flow, \$ per year	1		300	200	1,800
8		2		300	200	1,800
9		3		300	200	1,800
10		4		300	200	1,800
11		5		300	200	1,800
12		6		300	200	1,800
13		7		300	200	1,800
14		8		700	0	1,900
15	Incremental i^* (Δi^*)			14.57%	-18.77%	13.60%
16	Increment justified?			Yes	No	Yes
17	Alternative selected			2	2	4
18			= IRR(D6:D14)			
19						

Figure 8-7
Spreadsheet solution to select from multiple cost alternatives, Example 8.7.

Comment

As mentioned earlier, it is not possible to generate a PW versus i graph for each cost alternative because all cash flows are negative. However, it is possible to generate PW versus Δi graphs for the incremental series in the same fashion as we have done previously. The curves will cross the PW = 0 line at the Δi^* values determined by the IRR functions.

Selection from multiple, mutually exclusive alternatives with **unequal lives** using Δi^* values requires that the incremental cash flows be evaluated over the LCM of the two alternatives being compared. This is another application of the principle of equal-service comparison. The spreadsheet application in the next section illustrates the computations.

It is always possible to rely on PW or AW analysis of the incremental cash flows at the MARR to make the selection. In other words, don't find Δi^* for each pairwise comparison; find PW or

AW at the MARR instead. However, it is still necessary to make the comparison over the LCM number of years for an incremental analysis to be performed correctly.

8.7 All-in-One Spreadsheet Analysis (Optional) ● ● ●

For professors and students who like to pack a spreadsheet, Example 8.8 combines many of the economic analysis techniques we have learned so far—(internal) ROR analysis, incremental ROR analysis, PW analysis, and AW analysis. Now that the IRR, NPV, and PV functions are mastered, it is possible to perform a wide variety of evaluations for multiple alternatives on a single spreadsheet. No cell tags are provided in this example. A nonconventional cash flow series for which multiple ROR values may be found, and selection from both mutually exclusive alternatives and independent projects, are included in this example.

EXAMPLE 8.8

In-flight texting, phone, and Internet connections provided at airline passenger seats are an expected service by many customers. Singapore Airlines knows it will have to replace 15,000 to 24,000 units in the next few years on its Boeing 757, 777, and A300 aircraft. Four optional data handling features that build upon one another are available from the manufacturer, but at an added cost per unit. Besides costing more, the higher-end options (e.g., satellite-based plug-in video service) are estimated to have longer lives before the next replacement is forced by new, advanced features expected by flyers. All four options are expected to boost annual revenues by varying amounts. Figure 8–8 spreadsheet rows 2 through 6 include all the estimates for the four options.

- Using MARR = 15%, perform ROR, PW, and AW evaluations to select the one level of options that is the most promising economically.
- If more than one level of options can be selected, consider the four that are described as independent projects. If no budget limitations are considered at this time, which options are acceptable if the MARR is increased to 20% when more than one option may be implemented?

	A	B	C	D		
Alternative	A	B	C	D		
Initial cost, \$	-6,000	-7,000	-9,000	-17,000		
Annual cash flow, \$ per year	2,000	3,000	3,000	3,500		
Salvage value, \$	0	200	300	1,000		
Life, years	Year	3	4	6	12	
Incr. ROR comparison						
Incremental investment, \$	0	-6,000	-7,000	-9,000	-2,000	-8,000
Incremental cash flow over the LCM, \$ per year	1	2,000	3,000	3,000	0	3,500
	2	2,000	3,000	3,000	0	3,500
	3	2,000	3,000	3,000	0	3,500
	4		3,200	3,000	6,800	3,500
	5			3,000	0	3,500
	6			3,300	-8,700	3,500
	7				0	3,500
	8				6,800	3,500
	9				0	3,500
	10				0	3,500
	11				0	3,500
	12				100	4,500
Overall i*		0.00%	26.32%	24.68%		17.87%
Retain or eliminate?		Eliminate	Retain	Retain		Retain
Incremental i* (Δi^*)					19.42%	11.23%
Increment justified?					Yes	No
Alternative selected					C	C
AW at MARR	-628	588	656		398	
PW at MARR	-3,403	3,188	3,557		2,159	
Alternative selected	No	No	Yes		No	

Figure 8–8

Spreadsheet analysis using ROR, PW, and AW methods for unequal-life, revenue alternatives, Example 8.8.

Solution by Spreadsheet

(a) The spreadsheet (Figure 8–8) is divided into six sections:

Section 1 (rows 1, 2): MARR value and the alternative names (A through D) are in increasing order of initial cost.

Section 2 (rows 3 to 6): Per-unit net cash flow estimates for each alternative. These are revenue alternatives with unequal lives.

Section 3 (rows 7 to 20): Actual and incremental cash flows are displayed here.

Section 4 (rows 21, 22): Because these are all revenue alternatives, i^* values are determined by the IRR function. If an alternative passes the MARR test ($i^* > 15\%$), it is retained and a column is added to the right of its actual cash flows so the incremental cash flows can be determined. Columns F and H were inserted to make space for the incremental evaluations. Alternative A does not pass the i^* test.

Section 5 (rows 23 to 25): The IRR functions display the Δi^* values in columns F and H.

Comparison of C to B takes place over the LCM of 12 years. Since $\Delta i_{C-B}^* = 19.42\% > 15\%$, eliminate B; alternative C is the new defender and D is the next challenger. The final comparison of D to C over 12 years results in $\Delta i_{D-C}^* = 11.23\% < 15\%$, so D is eliminated. Alternative C is the chosen one.

Section 6 (rows 26 to 28): These include the AW and PW analyses. The AW value over the life of each alternative is calculated using the PMT function at the MARR with an embedded NPV function. Also, the PW value is determined from the AW value for 12 years using the PV function. For both measures, alternative C has the numerically largest value, as expected.

Conclusion: All methods result in the same, correct choice of alternative C.

(b) Since each option is independent of the others, and there is no budget limitation at this time, each i^* value in row 21 of Figure 8–8 is compared to MARR = 20%. This is a comparison of each option with the do-nothing alternative. Of the four, options B and C have $i^* > 20\%$. They are acceptable; the other two are not.

Comment

In part (a), we should have applied the two multiple-root sign tests to the incremental cash flow series for the C-to-B comparison. The series itself has three sign changes, and the cumulative cash flow series starts negatively and also has three sign changes. Therefore, up to three real-number roots may exist. The IRR function is applied in cell F23 to obtain $\Delta i_{C-B}^* = 19.42\%$ without using a supplemental (Section 7.5) procedure. This means that the investment assumption of 19.42% for positive cash flows is a reasonable one. If the MARR = 15%, or some other earning rate were more appropriate, the ROIC procedure could be applied to determine a single rate, which would be different from 19.42%. Depending upon the investment rate chosen, alternative C may or may not be incrementally justified against B. Here, the assumption is made that the Δi^* value is reasonable, so C is justified.

CHAPTER SUMMARY

Just as present worth and annual worth methods find the best alternative from among several, incremental rate of return calculations can be used for the same purpose. In using the ROR technique, it is necessary to consider the **incremental cash flows** when selecting between mutually exclusive alternatives. The incremental investment evaluation is conducted between only two alternatives at a time, beginning with the lowest initial investment alternative. Once an alternative has been eliminated, it is not considered further.

Rate of return values have a natural appeal to management, but the ROR analysis is often more difficult to set up and complete than the PW or AW analysis using an established MARR. Care must be taken to perform a ROR analysis correctly on the incremental cash flows; otherwise it may give incorrect results.

If there is no budget limitation when independent projects are evaluated, the ROR value of each project is compared to the MARR. Any number, or none, of the projects can be accepted.

PROBLEMS

Understanding Incremental ROR

- 8.1 In a tabulation of cash flow, the column entitled "Rate of return on the incremental cash flow" represents the rate of return on what?
- 8.2 If the rate of return on the incremental cash flow between two alternatives is less than the minimum attractive rate of return, which alternative should be selected, if any?
- 8.3 An engineer is comparing three projects by the incremental ROR method. There are revenue and cost cash flow estimates. He used the IRR function of Excel to determine the ROR values for each project. The first one is 3.5% above the MARR, the second is 1.2% below the MARR, and the third is 2.4% above the MARR. Which alternatives, if any, must he include in the incremental ROR analysis?
- 8.4 The rates of return on alternatives X and Y are 15% and 12%, respectively. Alternative Y requires a larger investment than alternative X.
- What is known about the rate of return on the increment of investment between the two alternatives?
 - If the MARR is 12%, which alternative should be selected and why?
- 8.5 Victoria is comparing two mutually exclusive alternatives, A and B. The overall ROR on alternative A is greater than the MARR, and the overall ROR on alternative B, which requires the larger investment, is exactly equal to the MARR.
- What is known about the ROR on the increment between A and B?
 - Which alternative should be selected?
- 8.6 A food processing company is considering two types of moisture analyzers. Only one can be selected. The company expects an infrared model to yield a rate of return of 27% per year. A more expensive microwave model will yield a rate of return of 22% per year. If the company's MARR is 19% per year, can you determine which model should be purchased solely on the basis of the overall rate of return information provided? Why or why not?
- 8.7 If \$80,000 is invested at 30% and another \$50,000 is invested at 20% per year, what is the overall rate of return on the entire \$130,000?
- 8.8 A total of \$100,000 was invested in two different projects identified as Z1 and Z2. If the overall rate

of return on the \$100,000 was 30% and the rate of return on the \$30,000 invested in Z1 was 15%, what was the rate of return on Z2?

- 8.9 Tuggle, Inc., which manufactures rigid shaft couplings, has \$600,000 to invest. The company is considering three different projects that will yield the following rates of return.

Project X	$i_X = 24\%$
Project Y	$i_Y = 18\%$
Project Z	$i_Z = 30\%$

The initial investment required for each project is \$100,000, \$300,000, and \$200,000, respectively. If Tuggle's MARR is 15% per year and it invests in all three projects, what rate of return will the company make?

- 8.10 Two options are available for setting up a wireless meter scanner and controller. A simple setup is good for 2 years and has an initial cost of \$12,000, no salvage value, and an operating cost of \$27,000 per year. A more permanent system has a higher first cost of \$73,000, but it has an estimated life of 6 years and a salvage value of \$15,000. It costs only \$14,000 per year to operate and maintain. If the two options are compared using an incremental rate of return, what are the incremental cash flows in (a) year 0 and (b) year 2?

- 8.11 Prepare a tabulation of incremental cash flows for the two machine alternatives below.

	Machine X	Machine Y
First cost, \$	−35,000	−90,000
Annual operating cost, \$ per year	−31,600	−19,400
Salvage value, \$	0	8,000
Life, years	2	4

- 8.12 For the alternatives shown, determine the sum of the incremental cash flows for Q – P.

	Alternative P	Alternative Q
First cost, \$	−50,000	−85,000
Annual operating cost, \$ per year	−8,600	−2,000
Annual revenue, \$ per year	22,000	45,000
Salvage value, \$	3,000	8,000
Life, years	3	6

- 8.13 The tabulation of the incremental cash flows between alternatives A and B is shown on the next page. Alternative A has a 3-year life and alternative B a 6-year life. If neither alternative has a salvage value, what is (a) the first cost of alternative A and (b) the first cost of alternative B?

Year	Incremental Cash Flow (B - A), \$
0	-20,000
1	5,000
2	5,000
3	12,000
4	5,000
5	5,000
6	5,000

- 8.14 Standby power for pumps at water distribution booster stations can be provided by either gasoline- or diesel-powered engines. The costs for the gasoline engines are as follows:

Gasoline	
First cost, \$	-150,000
Annual M&O, \$ per year	-41,000
Salvage value, \$	23,000
Life, years	15

The incremental PW cash flow equation associated with (diesel – gasoline) is

$$0 = -40,000 + 11,000(P/A,i,15) + 16,000(P/F,i,15)$$

Determine the following:

- (a) First cost of the diesel engines
- (b) Annual M&O cost of the diesel engines
- (c) Salvage value of the diesel engines

- 8.15 Several high-value parts for NASA's reusable space exploration vehicle can be either anodized or powder-coated. Some of the costs for each process are shown below:

	Anodize	Powder Coat
First cost, \$?	-65,000
Annual cost, \$ per year	-21,000	?
Resale value, \$?	6,000
Life, years	3	3

The incremental AW cash flow equation associated with (powder coat – anodize) is

$$0 = -14,000(A/P,i,3) + 5000 + 2000(A/F,i,3)$$

What is (a) the first cost for anodizing, (b) the annual cost for powder coating, and (c) the resale (salvage) value of the anodized parts?

Incremental ROR Comparison (Two Alternatives)

- 8.16 Specialty Gases & Chemicals manufactures nitrogen trifluoride, a highly specialized gas used as an industrial cleansing agent for flat panels installed in laptop computers, televisions, and desktop monitors. The incremental cash flow associated with two alternatives for chemical storage and handling systems

(identified as P3 and X3) has been calculated in \$1000 units. Determine (a) the rate of return on the incremental cash flows and (b) which one should be selected if the company's MARR is 25% per year. Alternative X3 requires the larger initial investment.

Year	Incremental Cash Flow (X3 – P3), \$1000
0	\$-4600
1–9	1100
10	2000

- 8.17 As groundwater wells age, they sometimes begin to pump sand (and they become known as "sandlers"), and this can cause damage to downstream desalting equipment. This situation can be dealt with by drilling a new well at a cost of \$1,000,000 or by installing a tank and self-cleaning screen ahead of the desalting equipment. The tank and screen will cost \$230,000 to install and \$61,000 per year to operate and maintain. A new well will have a pump that is more efficient than the old one, and it will require almost no maintenance, so its operating cost will be only \$18,000 per year. If the salvage values are estimated at 10% of the first cost, use a present worth relation to (a) calculate the incremental rate of return and (b) determine which alternative is better at a MARR of 6% per year over a 20-year study period.

- 8.18 Konica Minolta plans to sell a copier that prints documents on both sides simultaneously, cutting in half the time it takes to complete big commercial jobs. The costs associated with producing chemically-treated vinyl rollers and fiber-impregnated rubber rollers are shown below. Determine which of the two types should be selected by calculating the rate of return on the incremental investment. Assume the company's MARR is 21% per year. (Solve by hand and/or spreadsheet, as instructed.)

	Treated	Impregnated
First cost, \$	-50,000	-95,000
Annual cost, \$ per year	-100,000	-85,000
Salvage value, \$	5,000	11,000
Life, years	3	6

- 8.19 The Texas Department of Transportation (TxDOT) is considering two designs for crash barriers along a reconstructed portion of I-10. Design 2B will cost \$3 million to install and \$135,000 per year to maintain. Design 4R will cost \$3.7 million to install and \$70,000 per year to maintain. Calculate the rate of return and determine which design is preferred if TxDOT uses a MARR of 6% per year and a 20-year project period.

8.20 Chem-Tex Chemical is considering two additives for improving the dry-weather stability of its low-cost acrylic paint. Additive A has a first cost of \$110,000 and an annual operating cost of \$60,000. Additive B has a first cost of \$175,000 and an annual operating cost of \$35,000. If the company uses a 3-year recovery period for paint products and a MARR of 20% per year, which process is economically favored? Use an incremental ROR analysis.

8.21 The manager of Liquid Sleeve, Inc., a company that makes a sealing solution for machine shaft surfaces that have been compromised by abrasion, high pressures, or inadequate lubrication, is considering adding metal-based nanoparticles of either type Al or Fe to its solution to increase the product's performance at high temperatures. The costs associated with each are shown below. The company's MARR is 20% per year. Do the following using a PW-based rate of return analysis and a spreadsheet:

- Determine which nanoparticle type the company should select using the Δi^* value.
- On the same graph, plot the PW versus different i values for each alternative. Indicate the breakeven i^* value and the MARR value on the plot.
- Use the plot of PW versus Δi values to select the better alternative with MARR = 20% per year. Is the answer the same as in part (a)?

	Type Fe	Type Al
First cost, \$	−150,000	−280,000
Annual operating cost, \$ per year	−92,000	−74,000
Salvage value, \$	30,000	70,000
Life, years	2	4

8.22 A chemical company is considering two processes for isolating DNA material. The incremental cash flows between the two alternatives, J and S, have an incremental rate of return that is less than 40%, which is the MARR of the company. However, the company CEO prefers the more expensive process S. She believes the company can implement cost controls to reduce the annual cost of the more expensive process. By how much would she have to reduce the annual operating cost of alternative S (in \$ per year) for it to have an incremental rate of return of exactly 40%?

Year	Incremental Cash Flow	
	(S − J), \$	
0	−900,000	
1	400,000	
2	400,000	
3	400,000	

8.23 The incremental cash flows for two alternative electrode setups are shown. The MARR is 12% per year, and alternative Dryloc requires a larger initial investment compared to NPT.

- Determine which should be selected using an AW-based rate of return analysis.
- Use a graph of incremental values to determine the largest MARR value that will justify the NPT alternative.

Incremental Cash Flow (Dryloc − NPT), \$	
Year	
0	−56,000
1–8	+8,900
9	+12,000

8.24 Hewett Electronics manufactures amplified pressure transducers. It must decide between two machines for a finishing operation. Select one for them on the basis of AW-based rate of return analysis. The company's MARR is 18% per year.

	Variable Speed	Dual Speed
First cost, \$	−270,000	−245,000
Annual operating cost, \$ per year	−135,000	−139,000
Salvage value, \$	75,000	35,000
Life, years	6	6

8.25 A manufacturer of hydraulic equipment is trying to determine whether it should use monoflange double block and bleed (DBB) valves or a multi-valve system (MVS) for chemical injection. The costs are shown below. Use an AW-based rate of return analysis and a MARR of 18% per year to determine the better of the two options.

	DBB	MVS
First cost, \$	−40,000	−71,000
Annual cost, \$ per year	−60,000	−65,000
Salvage value, \$	0	18,000
Life, years	2	4

8.26 Poly-Chem Plastics is considering two types of injection molding machines—hydraulic and electric. The hydraulic press (HP) will have a first cost of \$600,000, annual costs of \$200,000, and a salvage value of \$70,000 after 5 years. Electric machine technology (EMT) will have a first cost of \$800,000, annual costs of \$150,000, and a salvage value of \$130,000 after 5 years.

- Use an AW-based rate of return equation to determine the ROR on the increment of investment between the two.

- (b) Determine which machine the company should select, if the MARR = 16% per year.
 (c) Plot the AW versus i graph for each alternative's cash flows, and utilize it to determine the largest MARR that will justify the EMT extra investment of \$200,000.

- 8.27 Last week Eduardo calculated the overall project ROR values for two alternatives A and B using the estimates below. He calculated $i_A^* = 34.2\%$ and $i_B^* = 31.2\%$ and recommended acceptance of A since its rate of return exceeded the established MARR of 30% by a greater amount than project B. Yesterday, the general manager of the company announced a major capital investment program, which includes a large drop in the MARR from 30% to 20% per year. Do the following to help Eduardo better understand the rate of return method and what this reduction in MARR means.
 (a) Explain the error that Eduardo made in performing the rate of return analysis.
 (b) Perform the correct analysis using each MARR value.
 (c) Illustrate the ranking inconsistency problem using the two MARR values, and determine the maximum MARR that will justify alternative B.

	Alternative A	Alternative B
First cost, \$	-40,000	-85,000
Annual operating cost, \$ per year	-5,500	-15,000
Annual revenue, \$ per year	22,000	45,000
Salvage value, \$	0	20,000
Life, years	6	6
$i^*, \%$	34.2	29.2

Multiple Alternative Comparison

- 8.28 Four mutually exclusive revenue alternatives are under consideration to automate a baking and packaging process at Able Bakery Products. The alternatives are ranked in order of increasing initial investment and compared by incremental rate of return analysis. The rate of return on each increment of investment was less than the MARR. Which alternative should be selected?
 8.29 A WiMAX wireless network integrated with a satellite network can provide connectivity to any location within 10 km of the base station. The number of sectors per base station can be varied to increase the bandwidth. An independent cable operator is considering three bandwidth alternatives. Assume a life of 20 years and

a MARR of 10% per year to determine which alternative is best using an incremental rate of return analysis.

Bandwidth, Mbps	First Cost, \$1000	Operating Cost, \$1000 per Year	Annual Income, \$1000 per Year
44	-40,000	-2000	+5000
55	-46,000	-1000	+5000
88	-61,000	-500	+8000

- 8.30 Xerox's iGenX high-speed commercial printers cost \$1.5 billion to develop. The machines cost \$500,000 to \$750,000 depending on what options the client selects. Spectrum Imaging Systems is considering the purchase of a new printer based on recent contracts it received for printing weekly magazine and mail-out advertising materials. The operating costs and revenues generated are related to a large extent to the speed and other capabilities of the copier. Spectrum is considering the four machines shown below. The company uses a 3-year planning period and a MARR of 15% per year. Determine which copier the company should acquire on the basis of an incremental rate of return analysis.

Copier	Initial Investment, \$	Operating Cost, \$ per Year	Annual Revenue, \$ per Year	Salvage Value, \$
iGen-1	-500,000	-350,000	+450,000	+70,000
iGen-2	-600,000	-300,000	+460,000	+85,000
iGen-3	-650,000	-275,000	+480,000	+95,000
iGen-4	-750,000	-200,000	+510,000	+120,000

- 8.31 Ashley Foods, Inc. has determined that any one of five machines can be used in one phase of its chili canning operation. The costs of the machines are estimated below, and all machines are estimated to have a 4-year useful life. If the minimum attractive rate of return is 20% per year, determine which machine should be selected on the basis of a rate of return analysis.

Machine	First Cost, \$	Annual Operating Cost, \$ per Year
1	-31,000	-16,000
2	-29,000	-19,300
3	-34,500	-17,000
4	-49,000	-12,200
5	-41,000	-15,500

- 8.32 Five revenue projects are under consideration by General Dynamics for improving material flow through an assembly line. The initial cost in

\$1000 and the life of each project are as follows (revenue estimates are not shown):

	Project				
	A	B	C	D	E
Initial cost, \$1000	-700	-2300	-900	-300	-1600
Life, years	5	8	5	5	6

An engineer made the comparisons shown below. From the calculations, determine which project, if any, should be undertaken if the company's MARR is (a) 11.5% per year and (b) 13.5% per year. If other calculations are necessary to make a decision, state which ones.

Comparison	Incremental Rate of Return, %
B vs DN	13%
A vs B	19%
D vs DN	11%
E vs B	15%
E vs D	24%
E vs A	21%
C vs DN	7%
C vs A	19%
E vs DN	12%
A vs DN	10%
E vs C	33%
D vs C	33%
D vs B	29%

- 8.33 The five alternatives shown here are being evaluated by the rate of return method.

Alternative	Initial Investment, \$	ROR versus DN, %	Incremental Rate of Return, %				
			A	B	C	D	E
A	-25,000	9.6	—	27.3	9.4	35.3	25.0
B	-35,000	15.1	—	0	38.5	24.4	—
C	-40,000	13.4	—	—	46.5	27.3	—
D	-60,000	25.4	—	—	—	6.8	—
E	-75,000	20.2	—	—	—	—	—

- (a) If the alternatives are *mutually exclusive* and the MARR is 26% per year, which alternative should be selected?
- (b) If the alternatives are *mutually exclusive* and the MARR is 15% per year, which alternative should be selected?
- (c) If the alternatives are *independent* and the MARR is 15% per year, which alternative(s) should be selected?

- 8.34 Five mutually exclusive revenue alternatives that have *infinite lives* are under consideration for increasing productivity in a manufacturing operation. The initial costs and cash flows of each project are shown. If the MARR is 14.9% per year, which alternative should be selected?

	Alternative				
	A	B	C	D	E
Initial cost, \$	-7,000	-23,000	-9,000	-3,000	-16,000
Cash flow, \$ per year	1,000	3,500	1,400	500	2,200
Rate of return (vs DN), %	14.3	15.2	15.6	16.7	13.8

- 8.35 The plant manager at Automaton Robotics is looking at the summarized incremental rate of return information shown below for five mutually exclusive alternatives, one of which must be chosen. The table includes the overall ROR and the incremental comparison of alternatives. Which alternative is best if the minimum attractive rate of return is (a) 15% per year and (b) 12% per year?

Alternative	First Cost, \$	Overall ROR, %	Incremental Rate of Return, %				
			A	B	C	D	E
A	-80,000	14	—	12	11	17	24
B	-60,000	16	12	—	14	23	21
C	-40,000	17	11	14	—	35	29
D	-30,000	12	17	23	35	—	17
E	-20,000	8	24	21	29	17	—

- 8.36 Only one of four different machines can be purchased for Glass Act Products. An engineer performed the following analysis to select the best machine, all of which have a 10-year life. Which machine, if any, should the company select at a MARR of 22% per year?

		Machine			
		1	2	3	4
Initial cost, \$		-44,000	-60,000	-72,000	-98,000
Annual cost, \$ per year		-70,000	-64,000	-61,000	-58,000
Annual savings, \$ per year		+80,000	+80,000	+80,000	+82,000
Overall ROR, %		18.6	23.4	23.1	20.8
Machines compared		2 to 1	3 to 2	4 to 3	
Incremental investment, \$		-16,000	-12,000	-26,000	
Incremental cash flow, \$ per year		+6,000	+3,000	+5,000	
ROR on increment, %		35.7	21.4	14.1	

- 8.37 The U.S. Bureau of Reclamation is considering five national park projects shown below, all of which can be considered to last indefinitely. At a MARR of 7.5% per year, determine which should be selected, if they are (a) independent and (b) mutually exclusive.

Project ID	First Cost, \$1000	Annual Income, \$1000	Rate of Return, %
A	-20,000	+2000	10.0
B	-10,000	+1300	13.0
C	-15,000	+1000	6.6
D	-70,000	+4000	5.7
E	-50,000	+2600	5.2

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 8.38** When conducting a rate of return (ROR) analysis involving multiple mutually exclusive alternatives, the first step is to:
- Rank the alternatives according to decreasing initial investment cost
 - Rank the alternatives according to increasing initial investment cost
 - Calculate the present worth of each alternative using the MARR
 - Find the LCM between all of the alternatives
- 8.39** In comparing mutually exclusive alternatives by the ROR method, you should:
- Find the ROR of each alternative and pick the one with the *highest* ROR
 - Select the alternative whose *incremental* ROR is the highest
 - Select the alternative with $ROR \geq MARR$ that has the lowest initial investment cost
 - Select the alternative with the largest initial investment that has been incrementally justified
- 8.40** When comparing independent projects by the ROR method, you should:
- Find the ROR of each project and pick the ones with the highest ROR
 - Select all projects that have an overall $ROR \geq MARR$
 - Select the project with an overall $ROR \geq MARR$ that involves the lowest initial investment cost
 - Select the project with the largest initial investment that has been incrementally justified
- 8.41** Of the following scenarios, alternative Y requires a higher initial investment than alternative X, and the MARR is 20% per year. The only scenario that requires an incremental investment analysis to select an alternative is that:
- X has an overall ROR of 22% per year, and Y has an overall ROR of 24% per year
 - X has an overall ROR of 19% per year, and Y has an overall ROR of 23% per year
 - X has an overall ROR of 18% per year, and Y has an overall ROR of 19% per year
 - X has an overall ROR of 28% per year, and Y has an overall ROR of 26% per year
- 8.42** Alternatives whose cash flows (excluding the salvage value) are all negative are called:
- Revenue alternatives
 - Nonconventional alternatives
 - Cost alternatives
 - Independent alternatives
- 8.43** For these alternatives, the sum of the incremental cash flows is:
- | Year | A | B |
|------|---------|---------|
| 0 | −10,000 | −14,000 |
| 1 | +2,500 | +4,000 |
| 2 | +2,500 | +4,000 |
| 3 | +2,500 | +4,000 |
| 4 | +2,500 | +4,000 |
| 5 | +2,500 | +4,000 |
- \$2500
 - \$3500
 - \$6000
 - \$8000
- 8.44** Helical Systems, Inc. uses a minimum attractive rate of return of 8% per year, compounded annually. The company is evaluating two new processes for improving the efficiency of its manufacturing operation. The cash flow estimates associated with each process are shown below. A correct equation to use for an incremental rate of return analysis is:
- | | Process I | Process J |
|--------------------------|-----------|-----------|
| First cost, \$ | −40,000 | −50,000 |
| Annual cost, \$ per year | −15,000 | −12,000 |
| Salvage value, \$ | 5,000 | 6,000 |
| Life, years | 3 | 3 |
- $0 = -10,000 + 3000(P/A, \Delta i^*, 3) + 1000(P/F, \Delta i^*, 3)$
 - $0 = -40,000(A/P, \Delta i^*, 3) - 15,000 + 5000(A/F, \Delta i^*, 3)$
 - $0 = -50,000(A/P, \Delta i^*, 3) - 12,000 + 6000(A/F, \Delta i^*, 3)$
 - $0 = -10,000 - 3000(P/A, \Delta i^*, 3) + 1000(P/F, \Delta i^*, 3)$
- 8.45** For the four independent projects shown, the one or ones to select using a MARR of 14% per year are:
- | Project | Rate of Return,
% per Year |
|---------|-------------------------------|
| A | 14 |
| B | 12 |
| C | 15 |
| D | 10 |
- Only C
 - Only A and C
 - Only A
 - Can't tell; need to conduct incremental analysis

Problems 8.46 through 8.48 are based on the following information.

Five alternatives are being evaluated by the incremental rate of return method.

Alternative	Initial Investment, \$	Overall ROR versus DN, %	Incremental Rate of Return, %				
			A	B	C	D	E
A	-25,000	9.6	—	27.3	9.4	35.3	25.0
B	-35,000	15.1	—	0	38.5	24.4	—
C	-40,000	13.4	—	—	46.5	27.3	—
D	-60,000	25.4	—	—	—	6.8	—
E	-75,000	20.2	—	—	—	—	—

- 8.46 If the projects are mutually exclusive and the minimum attractive rate of return is 14% per year, the best alternative is:

- (a) B (b) C (c) D (d) E

- 8.47 If the projects are mutually exclusive and the MARR is 20% per year, the best alternative is:

- (a) B
(b) C
(c) D
(d) E

- 8.48 If the projects are independent, instead of mutually exclusive, the one or ones to select at an MARR of 18% per year are:

- (a) B and C
(b) B, D, and E
(c) D and E
(d) B, C, and E

CASE STUDY

ROR ANALYSIS WITH ESTIMATED LIVES THAT VARY

Background

Make-to-Specs is a software system under development by ABC Corporation. It will be able to translate digital versions of three-dimensional computer models, containing a wide variety of part shapes with machined and highly finished (ultra-smooth) surfaces. The product of the system is the numerically controlled (NC) machine code for the part's manufacturing. Additionally, Make-to-Specs will build the code for superfine finishing of surfaces with continuous control of the finishing machines.

Information

There are two alternative computers that can provide the server function for the software interfaces and shared database updates on the manufacturing floor while Make-to-Specs is operating in parallel mode. The server first cost and estimated contribution to annual net cash flow are summarized below.

	Server 1	Server 2
First cost, \$	100,000	200,000
Net cash flow, \$/year	35,000	50,000 year 1, plus 5000 per year for years 2, 3, and 4 (gradient) 70,000 maximum for years 5 on, even if the server is replaced
Life, years	3 or 4	5 or 8

The life estimates were developed by two different individuals: a design engineer and a manufacturing manager. They have asked that, at this stage of the project, all analyses be performed using both life estimates for each system.

Case Study Exercises

Use spreadsheet analysis to determine the following:

1. If the MARR = 12%, which server should be selected? Use the PW or AW method to make the selection.
2. Use incremental ROR analysis to decide between the servers at MARR = 12%.
3. Use any method of economic analysis to display on the spreadsheet the value of the incremental ROR between server 2 with a life estimate of 5 years and a life estimate of 8 years.

CASE STUDY

HOW A NEW ENGINEERING GRADUATE CAN HELP HIS FATHER¹

Background

"I don't know whether to sell it, expand it, lease it, or what. But I don't think we can keep doing the same thing for many more years. What I really want to do is to keep it for 5 more years, then sell it for a bundle," Elmer Kettler said to his wife, Janise, their son, John Kettler, and new daughter-in-law, Suzanne Gestory, as they were gathered around the dinner table. Elmer was sharing thoughts on Gulf Coast Wholesale Auto Parts, a company he has owned and operated for 25 years on the southern outskirts of Houston, Texas. The business has excellent contracts for parts supply with several national retailers operating in the area—NAPA, AutoZone, O'Reilly, and Advance. Additionally, Gulf Coast operates a rebuild shop serving these same retailers for major automobile components, such as carburetors, transmissions, and air conditioning compressors.

At his home after dinner, John decided to help his father with an important and difficult decision: What to do with his business? John graduated just last year with an engineering degree from a major state university in Texas, where he completed a course in engineering economy. Part of his job at Energcon Industries is to perform basic rate of return and present worth analyses on energy management proposals.

Information

Over the next few weeks, John outlined five options, including his dad's favorite of selling in 5 years. John summarized all the estimates over a 10-year horizon. The options and estimates were given to Elmer, and he agreed with them.

Option 1: Remove rebuild. Stop operating the rebuild shop and concentrate on selling wholesale parts. The removal of the rebuild operations and the switch to an "all-parts house" are expected to cost \$750,000 in the first year. Overall revenues will drop to \$1 million the first year with an expected 4% increase per year thereafter. Expenses are projected at \$0.8 million the first year, increasing 6% per year thereafter.

Option 2: Contract rebuild operations. To get the rebuild shop ready for an operations contractor to take over will cost \$400,000 immediately. If expenses stay the same for 5 years, they will average \$1.4 million per year, but they can be expected to rise to \$2 million per year in year 6 and thereafter. Elmer thinks revenues under a contract arrangement can be \$1.4 million the first year and can rise 5% per year for the duration of a 10-year contract.

Option 3: Maintain status quo and sell out after 5 years (Elmer's personal favorite). There is no cost now, but

the current trend of negative net profit will probably continue. Projections are \$1.25 million per year for expenses and \$1.15 million per year in revenue. Elmer had an appraisal last year, and the report indicated Gulf Coast Wholesale Auto Parts is worth a net \$2 million. Elmer's wish is to sell out completely after 5 more years at this price, and to make a deal that the new owner pay \$500,000 per year at the end of year 5 (sale time) and the same amount for the next 3 years.

Option 4: Trade-out. Elmer has a close friend in the antique auto parts business who is making a "killing," so he says, with e-commerce. Although the possibility is risky, it is enticing to Elmer to consider a whole new line of parts, but still in the basic business that he already understands. The trade-out would cost an estimated \$1 million for Elmer immediately. The 10-year horizon of annual expenses and revenues is considerably higher than for his current business. Expenses are estimated at \$3 million per year and revenues at \$3.5 million each year.

Option 5: Lease arrangement. Gulf Coast could be leased to some turnkey company with Elmer remaining the owner and bearing part of the expenses for building, delivery trucks, insurance, etc. The first-cut estimates for this option are \$1.5 million to get the business ready now, with annual expenses at \$500,000 per year and revenues at \$1 million per year for a 10-year contract.

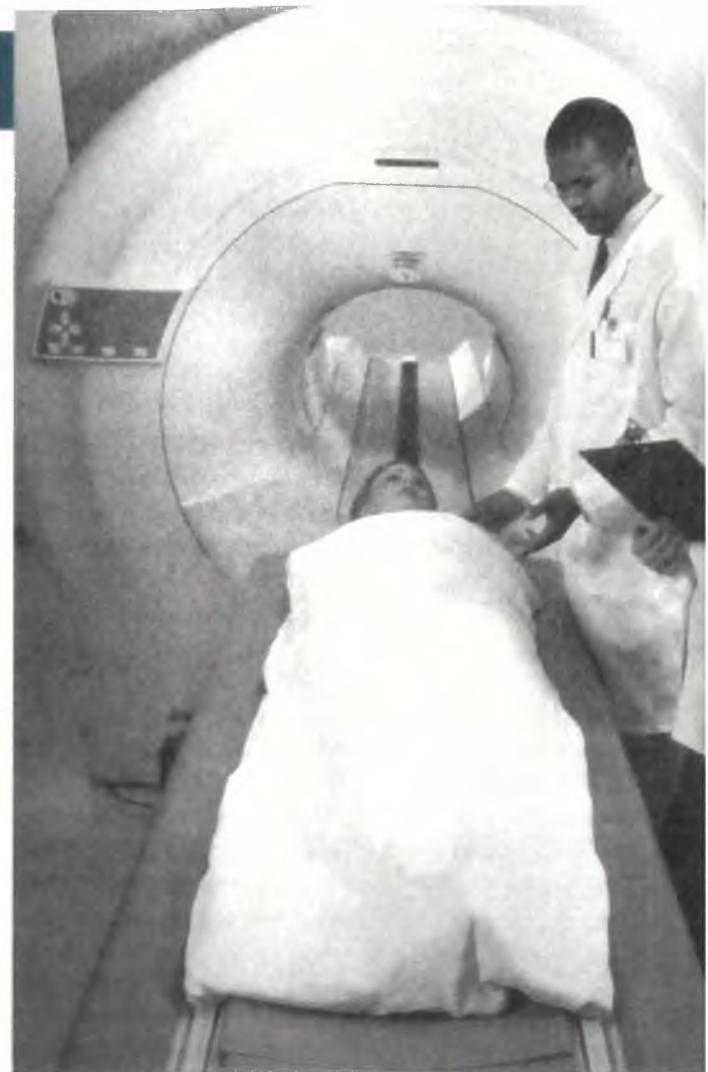
Case Study Exercises

Help John with the analysis by doing the following:

1. Develop the actual cash flow series and incremental cash flow series (in \$1000 units) for all five options in preparation for an incremental ROR analysis.
2. Discuss the possibility of multiple rate of return values for all the actual and incremental cash flow series. Find any multiple rates in the range of 0% to 100%.
3. If John's father insists that he make 25% per year or more on the selected option over the next 10 years, what should he do? Use all the methods of economic analysis you have learned so far (PW, AW, ROR) so John's father can understand the recommendation in one way or another.
4. Prepare plots of the PW versus i for each of the five options. Estimate the breakeven rate of return between options.
5. What is the minimum amount that must be received in each of years 5 through 8 for option 3 (the one Elmer wants) to be best economically? Given this amount, what does the sale price have to be, assuming the same payment arrangement as presented above?

¹Based upon a study by Alan C. Stewart, Consultant, Communications and High Tech Solutions Engineering, Accenture LLP.

Benefit/Cost Analysis and Public Sector Economics



LEARNING OUTCOMES

Purpose: Understand public sector projects and select the best alternative on the basis of incremental benefit/cost analysis.

SECTION	TOPIC	LEARNING OUTCOME
9.1	Public sector	<ul style="list-style-type: none">Explain some of the fundamental differences between private and public sector projects.
9.2	B/C for single project	<ul style="list-style-type: none">Calculate the benefit/cost ratio and use it to evaluate a single project.
9.3	Incremental B/C	<ul style="list-style-type: none">Select the better of two alternatives using the incremental B/C ratio method.
9.4	More than two alternatives	<ul style="list-style-type: none">Based on the incremental B/C ratios, select the best of multiple alternatives.
9.5	Service projects and CEA	<ul style="list-style-type: none">Explain service sector projects and use cost-effectiveness analysis (CEA) to evaluate projects.
9.6	Ethical considerations	<ul style="list-style-type: none">Explain the major aspects of public project activities, and describe how ethical compromise may enter public sector project analysis.



The evaluation methods of previous chapters are usually applied to alternatives in the private sector, that is, for-profit and not-for-profit corporations and businesses. This chapter introduces **public sector and service sector alternatives** and their economic consideration. In the case of public projects, the owners and users (beneficiaries) are the citizens and residents of a government unit—city, county, state, province, or nation. Government units provide the mechanisms to raise capital and operating funds. Public-private partnerships have become increasingly common, especially for large infrastructure projects such as major highways, power generation plants, water resource developments, and the like.

The benefit/cost (B/C) ratio introduces objectivity into the economic analysis of public sector evaluation, thus reducing the effects of politics and special interests. The different formats of B/C analysis, and associated disbenefits of an alternative, are discussed here. The B/C analysis can use equivalency computations based on PW, AW, or FW values. Performed correctly, the benefit/cost method will always select the same alternative as PW, AW, and ROR analyses.

This chapter also introduces **service sector projects** and discusses how their economic evaluation is different from that for other projects. Finally, there is a discussion on **professional ethics** and ethical dilemmas in the public sector.

PE

Water Treatment Facility #3 Case: Allen Water Utilities has planned for the last 25 years to construct a new drinking water treatment facility that will supply the rapidly growing north and northwest areas of the city. An expectation of over 100,000 new residents in the next several years and 500,000 by 2040 prompted the development of the plant starting in 2012. The supply is from a large surface lake currently used to provide water to all of Allen and the surrounding communities. The project is termed WTF3, and its initial capital investment is \$540 million for the treatment plant and two large steel-pipe transmission mains (84- and 48-inch) that will be installed via tunneling approximately 100 to 120 feet under suburban areas of the city to reach current reservoirs.

Tunneling was selected after geotechnical borings indicated that open trenching was not supportable by the soil and based upon a large public outcry against trenching in the living areas along the selected transmission routes. Besides the treatment plant construction on the 95-acre site, there must be at least three large vertical shafts (25 to 50 feet in diameter) bored along each transmission main to gain underground access for equipment and debris removal during the tunneling operations.

The stated criteria used to make decisions for WTF3 and the transmission mains were economics, environment, community impact, and constructability.

There are major long-term benefits for the new facility. These are some mentioned by city engineers:

- It will meet projected water needs of the city for the next 50 years.
- The new treatment plant is at a higher elevation than the current two plants, allowing gravity flow to replenish reservoirs, thereby using little or no electric pumping.
- There will be an increase in the diversity and reliability of supply as other plants age.
- It will provide a water quality that is more consistent due to the location of the raw water intakes.
- The facility uses water supplies already purchased; therefore, there is no need to negotiate additional allowances.

The disbenefits are mostly short-term during the construction of WTF3 and transmission mains. Some of these are mentioned by citizen groups and one retired city engineer:

- There will be disruption of habitat for some endangered species of birds, lizards, and trees not

- found in any other parts of the country.
- Large amounts of dust and smoke will enter the atmosphere in a residential area during the 3½ years of construction, tunneling, and transmission main completion.
 - Noise pollution and traffic congestion will result during an estimated 26,000 truck trips to remove debris from the plant site and tunnel shafts, in addition to the problems from regular construction traffic.
 - Natural landscape in plant and tunnel shaft sites will be destroyed.
 - Safety will be compromised for children in a school where large trucks will pass about every 5 minutes for approximately 12 hours per day, 6 days per week for 2½ years.
 - There may be delays in fire and ambulance services in emergencies, since many neighborhood streets are country-road width and offer only single ingress/egress streets for neighborhoods along the indicated routes.

- The need for the facility has not been proved, as the water will be sold to developers outside the city limits, not provided to residences within Allen.
- Newly generated revenues will be used to pay off the capital funding bonds approved for the plant's construction.

Last year, the city engineers did a benefit/cost analysis for this massive public sector project; none of the results were publicized. Public and elected official intervention has now caused some of the conclusions using the criteria mentioned above to be questioned by the general manager of Allen Water Utilities.

This case is used in the following topics of this chapter:

- Public sector projects (Section 9.1)
- Incremental B/C analysis, two alternatives (Section 9.3)
- Incremental B/C analysis, more than two alternatives (Section 9.4)



9.1 Public Sector Projects ● ● ●

Virtually all the examples and problems of previous chapters have involved the private sector, where products, systems, and services are developed and offered by corporations and businesses for use by individual customers and clients, the government, or other companies. (Notable exceptions are the long-life alternatives discussed in Chapters 5 (PW) and 6 (AW) where capitalized cost analysis was applied.) Now we will explore projects that concentrate on government units and the citizens they serve. These are called public sector projects.

A **public sector project** is a product, service, or system used, financed, and owned by the citizens of any government level. The primary purpose is to **provide service to the citizenry for the public good at no profit**. Areas such as public health, criminal justice, safety, transportation, welfare, and utilities are publically owned and require economic evaluation.

Upon reflection, it is surprising how much of what we use on a daily or as-needed basis is publicly owned and financed to serve us—the citizenry. These are some public sector examples:

- Hospitals and clinics
- Parks and recreation
- Utilities: water, electricity, gas, sewer, sanitation
- Schools: primary, secondary, community colleges, universities

- Economic development projects
- Convention centers
- Sports arenas
- Transportation: highways, bridges, waterways

Police and fire protection	Public housing
Courts and prisons	Emergency relief
Food stamp and rent relief programs	Codes and standards
Job training	

There are significant differences in the characteristics of private and public sector alternatives. They are summarized here.

Characteristic	Public sector	Private sector
Size of investment	Large	Some large; more medium to small

Often alternatives developed to serve public needs require large initial investments, possibly distributed over several years. Modern highways, public transportation systems, universities, airports, and flood control systems are examples.

Characteristic	Public sector	Private sector
Life estimates	Longer (30–50+ years)	Shorter (2–25 years)

The long lives of public projects often prompt the use of the capitalized cost method, where infinity is used for n and annual costs are calculated as $A = P(i)$. As n gets larger, especially over 30 years, the differences in calculated A values become small. For example, at $i = 7\%$, there will be a very small difference in 30 and 50 years, because $(A/P, 7\%, 30) = 0.08059$ and $(A/P, 7\%, 50) = 0.07246$.

Characteristic	Public sector	Private sector
Annual cash flow estimates	No profit; costs, benefits, and disbenefits are estimated	Revenues contribute to profits; costs are estimated

Public sector projects (also called publicly owned) do not have profits; they do have costs that are paid by the appropriate government unit; and they benefit the citizenry. Public sector projects often have undesirable consequences, as interpreted by some sectors of the public. It is these consequences that can cause public controversy about the projects. The economic analysis should consider these consequences in monetary terms to the degree estimable. (Often in private sector analysis, undesirable consequences are not considered, or they may be directly addressed as costs.) To perform a benefit/cost economic analysis of public alternatives, the costs (initial and annual), the benefits, and the disbenefits, if considered, must be estimated as accurately as possible in *monetary units*.

Costs—estimated expenditures to the government entity for construction, operation, and maintenance of the project, less any expected salvage value.

Benefits—advantages to be experienced by the owners, the public.

Disbenefits—expected undesirable or negative consequences to the owners if the alternative is implemented. Disbenefits may be indirect economic disadvantages of the alternative.

It is difficult to estimate and agree upon the economic impact of benefits and disbenefits for a public sector alternative. For example, assume a short bypass around a congested area in town is recommended. How much will it benefit a driver in *dollars per driving minute* to be able to bypass five traffic lights while averaging 35 miles per hour, as compared to currently driving through the lights averaging 20 miles per hour and stopping at an average of two lights for an average of 45 seconds each? The bases and standards for benefits estimation are always difficult to establish and verify. Relative to revenue cash flow estimates in the private sector, benefit estimates are much harder to make, and vary more widely around uncertain averages. (The inability to make economic estimates for benefits may be overcome by using the evaluation technique discussed in Section 9.5.) And the disbenefits that accrue from an alternative are even harder to estimate. In fact, the disbenefit itself may not be known at the time the evaluation is performed.

Characteristic	Public sector	Private sector
Funding	Taxes, fees, bonds, private funds	Stocks, bonds, loans, individual owners
Interest rate	Lower	Higher, based on cost of capital

The capital used to finance public sector projects is commonly acquired from taxes, bonds, and fees. Taxes are collected from those who are the owners—the citizens (e.g., federal gasoline taxes for highways are paid by all gasoline users, and health care costs are covered by insurance premiums). This is also the case for fees, such as toll road fees for drivers. Bonds are often issued: U.S. Treasury bonds, municipal bond issues, and special-purpose bonds, such as utility district bonds. Private lenders can provide up-front financing. Also, private donors may provide funding for museums, memorials, parks, and garden areas through gifts.

Characteristic	Public sector	Private sector
Interest rate	Lower	Higher, based on cost of capital

Because many of the financing methods for public sector projects are classified as *low-interest*, the interest rate is virtually always lower than for private sector alternatives. Government agencies are exempt from taxes levied by higher-level units. For example, municipal projects do not have to pay state taxes. (Private corporations and individual citizens do pay taxes.) Many loans are very low-interest, and grants with no repayment requirement from federal programs may share project costs. This results in interest rates in the 4% to 8% range. It is common that a government agency will direct that all projects be evaluated at a specific rate. As a matter of standardization, directives to use a specific interest rate are beneficial because different government agencies are able to obtain varying types of funding at different rates. This can result in projects of the same type being rejected in one state or city but accepted in another. Standardized rates tend to increase the consistency of economic decisions and to reduce gamesmanship.

The determination of the interest rate for public sector evaluation is as important as the determination of the MARR for a private sector analysis. The public sector interest rate is identified as i ; however, it is referred to by other names to distinguish it from the private sector rate. The most common terms are *discount rate* and *social discount rate*.

Characteristic	Public sector	Private sector
Alternative selection criteria	Multiple criteria	Primarily based on rate of return

Multiple categories of users, economic as well as noneconomic interests, and special-interest political and citizen groups make the selection of one alternative over another much more difficult in public sector economics. Seldom is it possible to select an alternative on the sole basis of a criterion such as PW or ROR. It is important to describe and itemize the criteria and selection method prior to the analysis. This helps determine the perspective or viewpoint when the evaluation is performed. Viewpoint is discussed below.

Characteristic	Public sector	Private sector
Environment of the evaluation	Politically inclined	Primarily economic

There are often public meetings and debates associated with public sector projects to accommodate the various interests of citizens (owners). Elected officials commonly assist with the selection, especially when pressure is brought to bear by voters, developers, environmentalists, and others. The selection process is not as “clean” as in private sector evaluation.

The **viewpoint of the public sector analysis** must be determined before cost, benefit, and dis-benefit estimates are made and before the evaluation is formulated and performed. There are several viewpoints for any situation, and the different perspectives may alter how a cash flow estimate is classified.

Some example perspectives are the citizen; the city tax base; number of students in the school district; creation and retention of jobs; economic development potential; a particular industry interest (agriculture, banking, electronics manufacturing); even the reelection of a public officeholder (often termed *pork projects*). In general, the viewpoint of the analysis should be as broadly defined as those who will bear the costs of the project and reap its benefits. Once established, the viewpoint assists in categorizing the costs, benefits, and disbenefits of each alternative, as illustrated in Example 9.1.

EXAMPLE 9.1 Water Treatment Facility #3 Case

PE

The situation with the location and construction of the new WTF3 and associated transmission mains described in the chapter's introduction has reached a serious level because of recent questions posed by some city council members and citizen groups. Before going public to the city council with the analysis performed last year, the director of Allen Water Utilities has asked an engineering management consultant to review it and determine if it was an acceptable analysis and correct economic decision, then and now. The lead consultant, Joel Whiterson, took engineering economy as a part of his B.S. education and has previously worked on economic studies in the government sector, but never as the lead person.

Within the first hour of checking background notes, Joel found several initial estimates (shown below) from last year for expected consequences if WTF3 were built. He realized that no viewpoint of the study was defined, and, in fact, the estimates were never classified as costs, benefits, or disbenefits. He did determine that disbenefits were considered at some point in the analysis, though the estimates for them are very sketchy.

Joel defined two viewpoints: a *citizen* of Allen and the Allen Water Utilities *budget*. He wants to identify each of the estimates as a cost, benefit, or disbenefit from each viewpoint. Please help with this classification.

Economic Dimension	Monetary Estimate
1. Cost of water: 10% annual increase to Allen households	Average of \$29.7 million (years 1–5, steady thereafter)
2. Bonds: Annual debt service at 3% per year on \$540 million	\$16.2 million (years 1–19); \$516.2 million (year 20)
3. Use of land: Payment to Parks and Recreation for shaft sites and construction areas	\$300,000 (years 1–4)
4. Property values: Loss in value, sales price, and property taxes	\$4 million (years 1–5)
5. Water sales: Increases in sales to surrounding communities	\$5 million (year 4) plus 5% per year (years 5–20)
6. M&O: Annual maintenance and operations costs	\$300,000 plus 4% per year increase (years 1–20)
7. Peak load purchases: Savings in purchases of treated water from secondary sources	\$500,000 (years 5–20)

Solution

The perspective of each viewpoint is identified and estimates are classified. (How this classification is done will vary depending upon who does the analysis. This solution offers only one logical answer.)

Viewpoint 1: Citizen of the city of Allen. Goal: Maximize the quality of life and wellness of citizens with family and neighborhood as prime concerns.

Costs: 1, 2, 4, 6 Benefits: 5, 7 Disbenefits: 3

Viewpoint 2: Allen Water Utilities budget. Goal: Ensure the budget is balanced and of sufficient size to fund rapidly growing city service demands.

Costs: 2, 3, 6 Benefits: 1, 5, 7 Disbenefits: 4

Citizens view costs in a different light than a city budget employee does. For example, the loss of property values (item 4) is considered a real cost to a citizen, but is an unfortunate disbenefit

from the city budget perspective. Similarly, the Allen Water Utilities budget interprets estimate 3 (payment for use of land to Parks and Recreation) as a real cost; but a citizen might interpret this as merely a movement of funds between two municipal budgets—therefore, it is a disbenefit, not a real cost.

Comment

The inclusion of disbenefits can easily change the economic decision. However, agreement on the disbenefits and their monetary estimates is difficult (to impossible) to develop, often resulting in the exclusion of any disbenefits from the economic analysis. Unfortunately, this usually transfers the consideration of disbenefits to the noneconomic (i.e., political) realm of public project decision making.

Most of the large public sector projects are developed through public-private partnerships (PPPs). A partnership is advantageous in part because of the greater efficiency of the private sector and in part because of the sizable cost to design, construct, and operate such projects. Full funding by the government unit may not be possible using traditional means—fees, taxes, and bonds. Some examples of the projects are as follows:

Project	Some Purposes of the Project
Mass transportation	Reduce transit time; reduce congestion; improve environment; decrease road accidents
Bridges and tunnels	Speed traffic flows; reduce congestion; improve safety
Ports and harbors	Increase cargo capacity; support industrial development; increase tourism
Airports	Increase capacity; improve passenger safety; support development
Water resources	Desalination for drinking water; meet irrigation and industrial needs; improve wastewater treatment

In these joint ventures, the public sector (government) is responsible for the funding and service to the citizenry, and the private sector partner (corporation) is responsible for varying aspects of the projects as detailed below. The government unit cannot make a profit, but the corporation(s) involved can realize a reasonable profit; in fact, the profit margin is usually written into the contract that governs the design, construction, and operation of the project.

Traditional methods of contracting were *fixed-price* (historically called lump-sum) and *cost reimbursable* (also called cost-plus). In these formats, a government unit took responsibility for funding and possibly some of the design elements, and later all operation activities, while the contractor did not share in the risks involved—liability, natural disasters, funding shortfalls, etc. More recently, the PPP has become the arrangement of choice for most large public projects. Commonly these are called **design-build contracts**, under which contractors take on more and more of the functions from design to operation. Details are explained in publications such as *Design-Build Contracting Handbook* (Cushman and Loulakis). The most reliance is placed upon a contractor or contractors with a DBOMF contract, as described below.

The **Design-Build-Operate-Maintain-Finance (DBOMF) contract** is considered a turnkey approach to a project. It requires the contractor(s) to perform all the DBOMF activities with collaboration and approval of the owner (the government unit). The activity of **financing** is the *management of cash flow to support project implementation* by a contracting firm. Although a contractor may assist in some instances, the **funding** (obtaining the capital funds) remains the government's responsibility through bonding, commercial loans, taxation, grants, and gifts.

When the financing activity is not managed by a contractor, the contract is a DBOM; it is also common to develop a design-build contract. In virtually all cases, some forms of design-build arrangements for public projects are made because they offer several advantages to the government and citizens served:

- Cost and time savings in the design, build, and operate phases
- Earlier and more reliable (less variable) cost estimates

- Reduced administrative responsibilities for the owner
- Better efficiency of resource allocation of private enterprise
- Environmental, liability, and safety issues addressed by the private sector, where there usually is greater expertise

Many of the projects in international settings and in developing countries utilize the public-private partnership. There are, of course, disadvantages to this arrangement. One risk is that the amount of funding committed to the project may not cover the actual build cost because it is considerably higher than estimated. Another risk is that a reasonable profit may not be realized by the private corporation due to low usage of the facility during the operate phase. To prevent such problems, the original contract may provide for special subsidies and loans guaranteed by the government unit. The subsidy may cover costs plus (contractually agreed-to) profit if usage is lower than a specified level. The level used may be the breakeven point with the agreed-to profit margin considered.

9.2 Benefit/Cost Analysis of a Single Project ● ● ●

The benefit/cost ratio is relied upon as a fundamental analysis method for public sector projects. The B/C analysis was developed to introduce greater objectivity into public sector economics, and as one response to the U.S. Congress approving the Flood Control Act of 1936. There are several variations of the B/C ratio; however, the fundamental approach is the same. All cost and benefit estimates must be converted to a common equivalent monetary unit (PW, AW, or FW) at the discount rate (interest rate). The B/C ratio is then calculated using one of these relations:

$$\text{B/C} = \frac{\text{PW of benefits}}{\text{PW of costs}} = \frac{\text{AW of benefits}}{\text{AW of costs}} = \frac{\text{FW of benefits}}{\text{FW of costs}} \quad [9.1]$$



Present worth and annual worth equivalencies are preferred to future worth values. The sign convention for B/C analysis is positive signs; **costs are preceded by a + sign**. Salvage values and additional revenues to the government, when they are estimated, are subtracted from costs in the denominator. Disbenefits are considered in different ways depending upon the model used. Most commonly, **disbenefits are subtracted from benefits** and placed in the numerator. The different formats are discussed below.

The decision guideline is simple:

If $\text{B/C} \geq 1.0$, accept the project as economically justified for the estimates and discount rate applied.

If $\text{B/C} < 1.0$, the project is not economically acceptable.



Project evaluation

If the B/C value is exactly or very near 1.0, noneconomic factors will help make the decision.

The *conventional B/C ratio*, probably the most widely used, is calculated as follows:

$$\text{B/C} = \frac{\text{benefits} - \text{disbenefits}}{\text{costs}} = \frac{B - D}{C} \quad [9.2]$$

In Equation [9.2] disbenefits are subtracted from benefits, not added to costs. The B/C value could change considerably if disbenefits are regarded as costs. For example, if the numbers 10, 8, and 5 are used to represent the PW of benefits, disbenefits, and costs, respectively, the correct procedure results in $\text{B/C} = (10 - 8)/5 = 0.40$. The incorrect placement of disbenefits in the denominator results in $\text{B/C} = 10/(8 + 5) = 0.77$, which is approximately twice the correct B/C value of 0.40. Clearly, then, the method by which disbenefits are handled affects the magnitude of the B/C ratio. However, regardless of whether disbenefits are (correctly) subtracted from the numerator or (incorrectly) added to costs in the denominator, a B/C ratio of less than 1.0 by the first method will always yield a B/C ratio less than 1.0 by the second method, and vice versa.

The *modified B/C ratio* includes all the estimates associated with the project, once operational. Maintenance and operation (M&O) costs are placed in the numerator and treated in a manner

similar to disbenefits. The denominator includes only the initial investment. Once all amounts are expressed in PW, AW, or FW terms, the modified B/C ratio is calculated as

$$\text{Modified B/C} = \frac{\text{benefits} - \text{disbenefits} - \text{M&O costs}}{\text{initial investment}}$$

[9.3]

Salvage value is usually included in the denominator as a negative cost. The modified B/C ratio will obviously yield a different value than the conventional B/C method. However, as with disbenefits, *the modified procedure can change the magnitude of the ratio but not the decision to accept or reject the project.*

The **benefit and cost difference** measure of worth, which does not involve a ratio, is based on the difference between the PW, AW, or FW of benefits and costs, that is, $B - C$. If $(B - C) \geq 0$, the project is acceptable. This method has the advantage of eliminating the discrepancies noted above when disbenefits are regarded as costs, because B represents *net benefits*. Thus, for the numbers 10, 8, and 5, the same result is obtained regardless of how disbenefits are treated.

Subtracting disbenefits from benefits: $B - C = (10 - 8) - 5 = -3$

Adding disbenefits to costs: $B - C = 10 - (8 + 5) = -3$

Before calculating the B/C ratio by any formula, check whether the alternative with the larger AW or PW of costs also yields a larger AW or PW of benefits. It is possible for one alternative with larger costs to generate lower benefits than other alternatives, thus making it unnecessary to further consider the larger-cost alternative.

By the very nature of benefits and especially disbenefits, monetary estimates are difficult to make and will vary over a wide range. The extensive use of **sensitivity analysis** on the more questionable parameters helps determine how sensitive the economic decision is to estimate variation. This approach assists in determining the **economic and public acceptance risk** associated with a defined project. Also, the use of sensitivity analysis can alleviate some of the public's concerns commonly expressed that people (managers, engineers, consultants, contractors, and elected officials) designing (and promoting) the public project are narrowly receptive to different approaches to serving the public's interest.

EXAMPLE 9.2

In the past, the Afram Foundation has awarded many grants to improve the living and medical conditions of people in war-torn and poverty-stricken countries throughout the world. In a proposal for the foundation's board of directors to construct a new hospital and medical clinic complex in a deprived central African country, the project manager has developed some estimates. These are developed, so she states, in a manner that does not have a major negative effect on prime agricultural land or living areas for citizens.

Award amount: \$20 million (end of) first year, decreasing by \$5 million per year for 3 additional years; local government will fund during the first year only

Annual costs: \$2 million per year for 10 years, as proposed

Benefits: Reduction of \$8 million per year in health-related expenses for citizens

Disbenefits: \$0.1 to \$0.6 million per year for removal of arable land and commercial districts

Use the conventional and modified B/C methods to determine if this grant proposal is economically justified over a 10-year study period. The foundation's discount rate is 6% per year.

Solution

Initially, determine the AW for each parameter over 10 years. In \$1 million units,

Award: $20 - 5(A/G, 6\%, 4) = \$12.864$ per year

Annual costs: \$2 per year

Benefits: \$8 per year

Disbenefits: Use \$0.6 for the first analysis

The conventional B/C analysis applies Equation [9.2].

$$B/C = \frac{8.0 - 0.6}{12.864 + 2.0} = 0.50$$

The modified B/C analysis uses Equation [9.3].

$$\text{Modified } B/C = \frac{8.0 - 0.6 - 2.0}{12.864} = 0.42$$

The proposal is not justified economically since both measures are less than 1.0. If the low disbenefits estimate of \$0.1 million per year is used, the measures increase slightly, but not enough to justify the proposal.

It is possible to develop a direct formula connection between the B/C of a public sector and B/C of a private sector project that is a **revenue alternative**; that is, both revenues and costs are estimated. Further, we can identify a direct correspondence between the modified B/C relation in Equation [9.3] and the PW method we have used repeatedly. (The following development also applies to AW or FW values.) Let's neglect the initial investment in year 0 for a moment, and concentrate on the cash flows of the project for year 1 through its expected life. For the private sector, the PW for project cash flows is

$$\text{PW of project} = \text{PW of revenue} - \text{PW of costs}$$

Since private sector revenues are approximately the same as public sector benefits minus disbenefits ($B - D$), the modified B/C relation in Equation [9.3] may be written as

$$\text{Modified } B/C = \frac{\text{PW of } (B - D) - \text{PW of } C}{\text{PW of initial investment}}$$

This relation can be slightly rewritten to form the **profitability index (PI)**, which can be used to evaluate revenue projects in the public or private sector. For years $t = 1, 2, \dots, n$,

$$\text{PI} = \frac{\text{PW of NCF}_t}{\text{PW of initial investment}} \quad [9.4]$$

Note that the denominator includes only first cost (initial investment) items, while the numerator has only cash flows that result from the project for years 1 through its life. The PI measure of worth provides a sense of getting the most for the investment dollar (euro, yen, etc.). That is, the result is in PW units per PW of money invested at the beginning. This is a "bang for the buck" measure. When used solely for a private sector project, the disbenefits are usually omitted, whereas they should be estimated and included in the modified B/C version of this measure for a public project.

The evaluation guideline for a single project using the PI is the same as for the conventional B/C or modified B/C.

If $\text{PI} \geq 1.0$, the project is economically acceptable at the discount rate.

If $\text{PI} < 1.0$, the project is not economically acceptable at the discount rate.



Project evaluation

Remember, the computations for PI and modified B/C are essentially the same, except the PI is usually applied without disbenefits estimated. The PI has another name: the **present worth index (PWI)**. It is often used to rank and assist in the selection of independent projects when the capital budget is limited. This application is discussed in Chapter 12, Section 12.5.

EXAMPLE 9.3

The Georgia Transportation Directorate is considering a public-private partnership with Young Construction as the prime contractor using a DBOMF contract for a new 22.51-mile toll road on the outskirts of Atlanta's suburban area. The design includes three 4-mile-long commercial/retail corridors on both sides of the toll road. Highway construction is expected to require

5 years at an average cost of \$3.91 million per mile. The discount rate is 4% per year, and the study period is 30 years. Evaluate the economics of the proposal using (a) the modified B/C analysis from the State of Georgia perspective and (b) the profitability index from the Young corporate viewpoint in which disbenefits are not included.

Initial investment: \$88 million distributed over 5 years; \$4 million now and in year 5 and \$20 million in each of years 1 through 4.

Annual M&O cost: \$1 million per year, plus an additional \$3 million each fifth year, including year 30.

Annual revenue/benefits: Include tolls and retail/commercial growth; start at \$2 million in year 1, increasing by a constant \$0.5 million annually through year 10, and then increasing by a constant \$1 million per year through year 20 and remaining constant thereafter.

Estimable disbenefits: Include loss of business income, taxes, and property value in surrounding areas; start at \$10 million in year 1, decrease by \$0.5 million per year through year 21, and remain at zero thereafter.

Solution

The PW values in year 0 for all estimates must be developed initially usually by hand, calculator, or spreadsheet computations. If the 30 years of estimates are entered into a spreadsheet and NPV functions at 4% are applied, the results in \$1 million units are obtained. All values are positive because of the sign convention for B/C and PI measures.

$$\begin{array}{ll} \text{PW of investment} = \$71.89 & \text{PW of benefits} = \$167.41 \\ \text{PW of costs} = \$26.87 & \text{PW of disbenefits} = \$80.12 \end{array}$$

(a) From the public project perspective, the State will apply Equation [9.3].

$$\text{Modified B/C} = \frac{167.41 - 80.12 - 26.87}{71.89} = 0.84$$

The toll road proposal is *not economically acceptable*, since $B/C < 1.0$.

(b) From the private corporation viewpoint, Young Construction will apply Equation [9.4].

$$PI = \frac{167.41 - 26.87}{71.89} = 1.95$$

The proposal is clearly justified without the disbenefits, since $PI > 1.0$. The private project perspective predicts that every investment dollar will return an equivalent of \$1.95 over 30 years at a 4% per year discount rate.

Comment

The obvious question that arises concerns the correct measure to use. When PI is used in the private project setting, there is no problem, since disbenefits are virtually never considered in the economic analysis. The public project setting will commonly use some form of the B/C ratio with the disbenefit considered. When a public-private partnership is initiated, there should be some agreement beforehand that establishes the economic measure acceptable for analysis and decision making throughout the project. Then the numerical dilemma presented above should not occur.



ME alternative selection

9.3 Alternative Selection Using Incremental B/C Analysis ● ● ●

The technique to compare two mutually exclusive alternatives using benefit/cost analysis is virtually the same as that for incremental ROR in Chapter 8. The incremental (conventional) B/C ratio, which is identified as $\Delta B/C$, is determined using PW, AW, or FW calculations. The higher-cost alternative is justified if $\Delta B/C$ is equal to or larger than 1.0. The selection rule is as follows:

If $\Delta B/C \geq 1.0$, choose the higher-cost alternative, because its extra cost is economically justified.

If $\Delta B/C < 1.0$, choose the lower-cost alternative.

To perform a correct incremental B/C analysis, it is required that each alternative be compared only with another alternative for which the incremental cost is already justified. This same rule was used for incremental ROR analysis.

There are two dimensions of an incremental B/C analysis that differ from the incremental ROR method in Chapter 8. We already know the first, all costs have a positive sign in the B/C ratio. The second, and significantly more important, concerns the ordering of alternatives prior to incremental analysis.

Alternatives are ordered by increasing equivalent total costs, that is, PW or AW of all cost estimates that will be utilized in the denominator of the B/C ratio. When not done correctly, the incremental B/C analysis may reject a justified higher-cost alternative.

If two alternatives, A and B, have equal initial investments and lives, but B has a larger equivalent annual cost, then B must be incrementally justified against A. (This is illustrated in Example 9.4 below.) If this convention is not correctly followed, it is possible to get a negative cost value in the denominator, which can incorrectly make $B/C < 1$ and reject a higher-cost alternative that is actually justified.

Follow these steps to correctly perform a conventional B/C ratio analysis of two alternatives. Equivalent values can be expressed in PW, AW, or FW terms.

1. Determine the equivalent total costs for both alternatives.
2. Order the alternatives by equivalent total cost: first smaller, then larger. Calculate the incremental cost (ΔC) for the larger-cost alternative. This is the denominator in $\Delta B/C$.
3. Calculate the equivalent total benefits and any disbenefits estimated for both alternatives. Calculate the incremental benefits (ΔB) for the larger-cost alternative. This is $\Delta(B - D)$ if disbenefits are considered.
4. Calculate the $\Delta B/C$ ratio using Equation [9.2], $(B - D)/C$.
5. Use the selection guideline to select the higher-cost alternative if $\Delta B/C \geq 1.0$.

When the B/C ratio is determined for the lower-cost alternative, it is a comparison with the do-nothing (DN) alternative. If $B/C < 1.0$, then DN should be selected and compared to the second alternative. If neither alternative has an acceptable B/C value and one of the alternatives does not have to be selected, the DN alternative must be selected. In public sector analysis, the DN alternative is usually the current condition.

EXAMPLE 9.4

The city of Garden Ridge, Florida, has received designs for a new patient room wing to the municipal hospital from two architectural consultants. One of the two designs must be accepted in order to announce it for construction bids. The costs and benefits are the same in most categories, but the city financial manager decided that the estimates below should be considered to determine which design to recommend at the city council meeting next week and to present to the citizenry in preparation for an upcoming bond referendum next month.

	Design A	Design B
Construction cost, \$	10,000,000	15,000,000
Building maintenance cost, \$/year	35,000	55,000
Patient usage copay, \$/year	450,000	200,000

The patient usage copay is an estimate of the amount paid by patients over the insurance coverage generally allowed for a hospital room. The discount rate is 5%, and the life of the building is estimated at 30 years.

- Use incremental B/C analysis to select design A or B.
- Once the two designs were publicized, the privately owned hospital in the directly adjacent city of Forest Glen lodged a complaint that design A will reduce its own municipal hospital's income by an estimated \$500,000 per year because some of the day-surgery features of design A duplicate its services. Subsequently, the Garden Ridge merchants' association argued that design B could reduce its annual revenue by an estimated \$400,000, because it

will eliminate an entire parking lot used by their patrons for short-term parking. The city financial manager stated that these concerns would be entered into the evaluation as disbenefits of the respective designs. Redo the B/C analysis to determine if the economic decision is still the same as when disbenefits were not considered.

Solution

- (a) Since most of the cash flows are already annualized, the incremental B/C ratio will use AW values. No disbenefit estimates are considered. Follow the steps of the procedure above:

1. The AW of costs is the sum of construction and maintenance costs.

$$AW_A = 10,000,000(A/P, 5\%, 30) + 35,000 = \$685,500$$

$$AW_B = 15,000,000(A/P, 5\%, 30) + 55,000 = \$1,030,750$$

2. Design B has the larger AW of costs, so it is the alternative to be incrementally justified. The incremental cost is

$$\Delta C = AW_B - AW_A = \$345,250 \text{ per year}$$

3. The AW of benefits is derived from the patient usage copays, since these are consequences to the public. The benefits for the $\Delta B/C$ analysis are not the estimates themselves, but the *difference* if design B is selected. The lower usage copay is a positive benefit for design B.

$$\Delta B = usage_A - usage_B = \$450,000 - \$200,000 = \$250,000 \text{ per year}$$

4. The incremental B/C ratio is calculated by Equation [9.2].

$$\Delta B/C = \frac{\$250,000}{\$345,250} = 0.72$$

5. The B/C ratio is less than 1.0, indicating that the extra costs associated with design B are not justified. Therefore, design A is selected for the construction bid.

- (b) The revenue loss estimates are considered disbenefits. Since the disbenefits of design B are \$100,000 less than those of A, this positive difference is added to the \$250,000 benefits of B to give it a total benefit of \$350,000. Now

$$\Delta B/C = \frac{\$350,000}{\$345,250} = 1.01$$

Design B is slightly favored. In this case the inclusion of disbenefits has reversed the previous economic decision. This has probably made the situation more difficult politically. New disbenefits will surely be claimed in the near future by other special-interest groups.

Like other methods, incremental B/C analysis requires **equal-service comparison** of alternatives. Usually, the expected useful life of a public project is long (25 or 30 or more years), so alternatives generally have equal lives. However, when alternatives do have unequal lives, the use of PW or AW to determine the equivalent costs and benefits requires that the LCM of lives be used to calculate $\Delta B/C$. As with ROR analysis of two alternatives, this is an excellent opportunity to use the AW equivalency of estimated (not incremental) costs and benefits, if the implied assumption that the project could be repeated is reasonable. Therefore, use AW-based analysis of actual costs and benefits for B/C ratios when different-life alternatives are compared.

EXAMPLE 9.5 Water Treatment Facility #3 Case

As our case unfolds, the consultant, Joel Whiterson, has pieced together some of the B/C analysis estimates for the 84-inch Jolleyville transmission main study completed last year. The two options for constructing this main were open trench (OT) for the entire 6.8-mile distance or a combination of trenching and bore tunneling (TT) for a shorter route of 6.3 miles. One of the two options had to be selected to transport approximately 300 million gallons per day (gpd) of treated water from the new WTF3 to an existing aboveground reservoir.

The general manager of Allen Water Utilities has stated publicly several times that the trench-tunnel combination option was selected over the open-trench alternative based on analysis of both quantitative and nonquantitative data. He stated the equivalent annual costs in an internal e-mail some months ago, based on the expected construction periods of 24 and 36 months, respectively, as equivalent to

$$AW_{OT} = \$1.20 \text{ million per year}$$

$$AW_{TT} = \$2.37 \text{ million per year}$$

This analysis indicated that the open-trench option was economically better, at that time. The planning horizon for the transmission mains is 50 years; this is a reasonable study period, Joel concluded. Use the estimates below that Joel has unearthed to perform a correct incremental B/C analysis and comment on the results. The interest (discount) rate is 3% per year, compounded annually, and 1 mile is 5280 feet.

	Open trench (OT)	Trench-tunnel (TT)
Distance, miles	6.8	6.3
First cost, \$ per foot	700	Trench for 2.0 miles: 700 Tunnel for 4.3 miles: 2100
Time to complete, months	24	36
Construction support costs, \$ per month	250,000	175,000
Ancillary expenses, \$ per month:		
Environmental	150,000	20,000
Safety	140,000	60,000
Community interface	20,000	5,000

Solution

One of the alternatives must be selected, and the construction lives are unequal. Since it is not reasonable to assume that this construction project will be repeated many cycles in the future, it is incorrect to conduct an AW analysis over the respective completion periods of 24 and 36 months, or the LCM of these time periods. However, the study period of 50 years is a reasonable evaluation time frame, since the mains are considered permanent installations. We can assume that the construction first costs are a present worth value in year 0, but the equivalent PW and 50-year AW of other monthly costs must be determined.

$$\begin{aligned} PW_{OT} &= PW \text{ of construction} + PW \text{ of construction support costs} \\ &= 700(6.8)(5280) + 250,000(12)(P/A, 3\%, 2) \\ &= \$30,873,300 \end{aligned}$$

$$\begin{aligned} AW_{OT} &= 30,873,300(A/P, 3\%, 50) \\ &= \$1.20 \text{ million per year} \end{aligned}$$

$$\begin{aligned} PW_{TT} &= [700(2.0) + 2100(4.3)](5280) + 175,000(12)(P/A, 3\%, 3) \\ &= \$61,010,460 \end{aligned}$$

$$\begin{aligned} AW_{TT} &= 61,010,460(A/P, 3\%, 50) \\ &= \$2.37 \text{ million per year} \end{aligned}$$

The *trench-tunnel (TT) alternative has a larger equivalent cost*; it must be justified against the OT alternative. The incremental cost is

$$\Delta C = AW_{TT} - AW_{OT} = 2.37 - 1.20 = \$1.17 \text{ million per year}$$

The difference between ancillary expenses defines the incremental benefit for TT.

$$\begin{aligned} PW_{OT-anc} &= 310,000(12)(P/A, 3\%, 2) \\ &= \$7,118,220 \end{aligned}$$

$$\begin{aligned} AW_{OT-anc} &= 7,118,220(A/P, 3\%, 50) \\ &= \$276,685 \text{ per year} \end{aligned}$$

$$\begin{aligned} PW_{TT-\text{anc}} &= 85,000(12)(P/A, 3\%, 3) \\ &= \$2,885,172 \end{aligned}$$

$$\begin{aligned} AW_{TT-\text{anc}} &= 2,885,172(A/P, 3\%, 50) \\ &= \$112,147 \text{ per year} \end{aligned}$$

$$\Delta B = AW_{OT-\text{anc}} - AW_{TT-\text{anc}} = 276,685 - 112,147 = \$164,538 \text{ per year } (\$0.16 \text{ million})$$

Calculate the incremental B/C ratio.

$$\Delta B/C = 0.16/1.17 = 0.14$$

Since $\Delta B/C \ll 1.0$, the trench-tunnel option is not economically justified. Joel can now conclude that the general manager's earlier comment that the TT option was selected based on quantitative *and* nonquantitative data must have had heavy dependence on nonquantitative information not yet discovered.



9.4 Incremental B/C Analysis of Multiple, Mutually Exclusive Alternatives ● ● ●

The procedure necessary to select one from three or more mutually exclusive alternatives using incremental B/C analysis is essentially the same as that of Section 9.3. The procedure also parallels that for incremental ROR analysis in Section 8.6. The selection guideline is as follows:

Choose the largest-cost alternative that is justified with an incremental $B/C \geq 1.0$ when this selected alternative has been compared with another justified alternative.

There are two types of benefit estimates—estimation of **direct benefits**, and **implied benefits based on usage cost** estimates. The previous two examples (9.4 and 9.5) are good illustrations of the second type of implied benefit estimation. When *direct benefits are estimated*, the B/C ratio for each alternative may be calculated first as an initial screening mechanism to eliminate unacceptable alternatives. At least one alternative must have $B/C \geq 1.0$ to perform the incremental B/C analysis. If all alternatives are unacceptable, the DN alternative is the choice. (This is the same approach as that of step 2 for “revenue alternatives only” in the ROR procedure of Section 8.6. However, the term *revenue alternative* is not applicable to public sector projects.)

As in the previous section when comparing two alternatives, selection from multiple alternatives by incremental B/C ratio utilizes equivalent total costs to initially order alternatives from smallest to largest. Pairwise comparison is then undertaken. Also, remember that all costs are considered positive in B/C calculations. The terms *defender* and *challenger alternative* are used in this procedure, as in a ROR-based analysis. The procedure for incremental B/C analysis of multiple alternatives is as follows:

1. Determine the equivalent total cost for all alternatives. Use AW, PW, or FW equivalencies.
2. Order the alternatives by equivalent total cost, smallest first.
3. Determine the equivalent total benefits (and any disbenefits estimated) for each alternative.
4. *Direct benefits estimation only:* Calculate the B/C for the first ordered alternative. If $B/C < 1.0$, eliminate it. By comparing each alternative to DN in order, we eliminate all that have $B/C < 1.0$. The lowest-cost alternative with $B/C \geq 1.0$ becomes the defender and the next higher-cost alternative is the challenger in the next step. (For analysis by spreadsheet, determine the B/C for all alternatives initially and retain only acceptable ones.)
5. Calculate incremental costs (ΔC) and benefits (ΔB) using the relations

$$\Delta C = \text{challenger cost} - \text{defender cost}$$

$$\Delta B = \text{challenger benefits} - \text{defender benefits}$$

If relative **usage costs** are estimated for each alternative, rather than direct benefits, ΔB may be found using the relation

$$\Delta B = \text{defender usage costs} - \text{challenger usage costs}$$

[9.5]

6. Calculate the $\Delta B/C$ for the first challenger compared to the defender.

$$\Delta B/C = \Delta B/\Delta C \quad [9.6]$$

If $\Delta B/C \geq 1.0$ in Equation [9.6], the challenger becomes the defender and the previous defender is eliminated. Conversely, if $\Delta B/C < 1.0$, remove the challenger and the defender remains against the next challenger.

7. Repeat steps 5 and 6 until only one alternative remains. It is the selected one.

In all the steps above, incremental disbenefits may be considered by replacing ΔB with $\Delta(B - D)$.

EXAMPLE 9.6

Schlitterbahn Waterparks of Texas, a very popular water and entertainment park headquartered in New Braunfels, has been asked by four different cities outside of Texas to consider building a park in their area. All the offers include some version of the following incentives:

- Immediate cash incentive (year 0)
- A 10% of first-year incentive as a direct property tax reduction for 8 years
- Sales tax rebate sharing plan for 8 years
- Reduced entrance (usage) fees for area residents for 8 years

Table 9–1 (top section) summarizes the estimates for each proposal, including the present worth of the initial construction cost and anticipated annual revenue. The annual M&O costs are expected to be the same for all locations. Use incremental B/C analysis at 7% per year and an 8-year study period to advise the board of directors if they should consider any of the offers to be economically attractive.

Solution

The viewpoint is that of Schlitterbahn, and the benefits are direct estimates. Develop the AW equivalents over 8 years, and use the procedure detailed above. The results are presented in Table 9–1.

1. AW of total costs and an example for city 1 are determined in \$1 million units.

$$\begin{aligned} \text{AW of costs} &= \text{first cost}(A/P, 7\%, 8) + \text{entrance fee reduction to residents} \\ &= 38.5(0.16747) + 0.5 \\ &= \$6.948 \quad (\$6,948,000 \text{ per year}) \end{aligned}$$

2. The four alternatives are correctly ordered by increasing equivalent total cost in Table 9–1.

TABLE 9–1 Incremental B/C Analysis of Water Park Proposals, Example 9.6

	City 1	City 2	City 3	City 4
First cost, \$ million	38.5	40.1	45.9	60.3
Entrance fee costs, \$/year	500,000	450,000	425,000	250,000
Annual revenue, \$ million/year	7.0	6.2	10.0	10.4
Initial cash incentive, \$	250,000	350,000	500,000	800,000
Property tax reduction, \$/year	25,000	35,000	50,000	80,000
Sales tax sharing, \$/year	310,000	320,000	320,000	340,000
AW of total costs, \$ million/year	6.948	7.166	8.112	10.348
AW of total benefits, \$ million/year	7.377	6.614	10.454	10.954
Overall B/C	1.06	0.92	1.29	1.06
Alternatives compared	1-to-DN	B/C < 1.0	3-to-1	4-to-3
Incremental costs ΔC , \$/year	6.948		1.164	2.236
Incremental benefits ΔB , \$/year	7.377		3.077	0.50
$\Delta B/C$	1.06		2.64	0.22
Increment justified?	Yes	Eliminated	Yes	No
City selected	1		3	3

3. AW of total benefits and an example for city 1 are also determined in \$1 million units.

$$\begin{aligned}\text{AW of benefits} &= \text{revenue} + \text{initial incentive}(A/P, 7\%, 8) \\ &\quad + \text{property tax reduction} + \text{sales tax sharing} \\ &= 7.0 + 0.25(0.16747) + 0.025 + 0.31 \\ &= \$7.377 \quad (\$7,377,000 \text{ per year})\end{aligned}$$

4. Since benefits are directly estimated (and no disbenefits are included), determine the overall B/C for each alternative using Equation [9.1]. In the case of city 1,

$$B/C_1 = 7.377/6.948 = 1.06$$

City 2 is eliminated with $B/C_2 = 0.92$; the rest are initially acceptable.

5. The ΔC and ΔB values are the actual estimates for the 1-to-DN comparison.
 6. The overall B/C is the same as $\Delta B/C = 1.06$, using Equation [9.6]. City 1 is economically justified and becomes the defender.
 7. Repeat steps 5 and 6. Since city 2 is eliminated, the 3-to-1 comparison results in

$$\begin{aligned}\Delta C &= 8.112 - 6.948 = 1.164 \\ \Delta B &= 10.454 - 7.377 = 3.077 \\ \Delta B/C &= 3.077/1.164 = 2.64\end{aligned}$$

City 3 is well justified and becomes the defender against city 4. From Table 9–1, $\Delta B/C = 0.22$ for the 4-to-3 comparison. City 4 falls out easily, and city 3 is the one to recommend to the board. Note that the DN alternative could have been selected had no proposal met the B/C or $\Delta B/C$ requirements.



Independent project selection

When two or more *independent projects* are evaluated using B/C analysis and there is no budget limitation, no incremental comparison is necessary. The only comparison is between each project separately with the do-nothing alternative. The project B/C values are calculated, and those with $B/C \geq 1.0$ are accepted.

This is the same procedure as that used to select from independent projects using the ROR method (Chapter 8). When a budget limitation is imposed, the capital budgeting procedure discussed in Chapter 12 must be applied.

When the lives of mutually exclusive alternatives are so long that they can be considered infinite, the capitalized cost is used to calculate the equivalent PW or AW values for costs and benefits. Equation [5.3], $A = P(i)$, is used to determine the equivalent AW values in the incremental B/C analysis. Example 9.7 illustrates this using the progressive example and a spreadsheet.

EXAMPLE 9.7 Water Treatment Facility #3 Case

PE

Land for Water Treatment Facility #3 was initially purchased in the year 2010 for \$19.3 million; however, when it was publicized, influential people around Allen spoke strongly against the location. We will call this location 1. Some of the plant design had already been completed when the general manager announced that this site was not the best choice anyway, and that it would be sold and a different, better site (location 2) would be purchased for \$28.5 million. This was well over the budget amount of \$22.0 million previously set for land acquisition. As it turns out, there was a third site (location 3) available for \$35.0 million that was never seriously considered.

In his review and after much resistance from Allen Water Utilities staff, the consultant, Joel, received a copy of the estimated costs and benefits for the three plant location options. The revenues, savings, and sale of bulk water rights to other communities are estimated as increments from a base amount for all three locations. Using the assumption of a very long life for the WTF3 facility and the established discount rate of 3% per year, determine what Joel discovered when he did the B/C analysis. Was the general manager correct in concluding that location 2 was the best, all said and done?

	Location 1	Location 2	Location 3
Land cost, \$ million	19.3	28.5	35.0
Facility first cost, \$ million	460.0	446.0	446.0
Benefits, \$ per year:			
Pumping cost savings	5	3	0
Sales to area communities	12	10	8
Added revenue from Allen	6	6	6
Total benefits, \$ per year	23	19	14

Solution

A spreadsheet can be very useful when performing an incremental B/C analysis of three or more alternatives. Figure 9–1a presents the analysis with the preliminary input of AW values for costs using the relation $A = P(i)$ and annual benefits. Figure 9–1b details all the functions used in the analysis. Logical IF statements indicate alternative elimination and selection decisions. In \$1 million units,

$$\begin{aligned} \text{Location 1: } & \text{AW of costs} = A \text{ of land cost} + A \text{ of facility first cost} \\ & = (19.3 + 460.0)(0.03) \\ & = \$14.379 \text{ per year} \end{aligned}$$

$$\text{AW of benefits} = \$23$$

$$\text{Location 2: } \text{AW of costs} = \$14.235 \quad \text{AW of benefits} = \$19$$

$$\text{Location 3: } \text{AW of costs} = \$14.430 \quad \text{AW of benefits} = \$14$$

Though the AW of cost values are close to one another, the increasing order is locations 2, 1, and 3 to determine $\Delta B/C$ values. The benefits are direct estimates; therefore, the overall B/C ratios indicate that location 3 (row 5; $B/C_3 = 0.97$) is not economically justified at the outset. It is eliminated, and one of the remaining locations must be selected. Location 2 is justified against the DN alternative ($B/C_2 = 1.33$); the only remaining comparison is 1-to-2 as detailed in column C of Figure 9–1. **Location 1** is a clear winner with $\Delta B/C = 27.78$.

In conclusion, Joel has learned that location 1 is indeed the best and that, from the economic perspective, the general manager was incorrect in stating that location 2 was better. However, given the original evaluation criteria listed in the introduction—*economics, environment, community impact, and constructability*—location 2 is likely a good compromise selection.

A	B	C	D
1			
2	Order of analysis	Location 2	Location 1
3	AW of cost, \$M/year	14.235	14.379
4	Annual benefits, \$M/year	19.0	23.0
5	Overall B/C	1.33	1.60
6	Acceptable	Yes	No
7			
8	Comparison	1-to-2	Eliminated
9	ΔC \$/year	0.144	
10	ΔB , \$/year	4.0	
11	$\Delta B/C$	27.78	
12	Increment justified?	Yes	
13	Selection	Location 1	

(a)

A	B	C	D
1			
2	Order of analysis	Location 2	Location 1
3	AW of cost, \$M/year	14.235	14.379
4	Annual benefits, \$M/year	19	23
5	Overall B/C	= B4/B3	= C4/C3
6	Acceptable	= IF(B5<1, "No", "Yes")	= IF(C5<1, "No", "Yes")
7			
8	Comparison		1-to-2
9	ΔC \$/year		= C3-B3
10	ΔB , \$/year		= C4-B4
11	$\Delta B/C$		= C10/C9
12	Increment justified?		= IF(C11<1, "No", "Yes")
13	Selection		= IF(C12="Yes", C2, "No")

(b)

Figure 9–1

Incremental B/C analysis for WTF3 case: (a) numerical results and (b) functions developed for the analysis.

Comment

This is an actual situation with changed names and values. Location 1 was initially purchased and planned for WTF3. However, the presence of political, community, and environmental stress factors changed the decision to location 2, when all was said and done.

9.5 Service Sector Projects and Cost-Effectiveness Analysis ●●●

Much of the GDP of the United States and some countries in Europe and Asia is generated by what has become known as the service sector of the economy. A large percentage of service sector projects are generated by and dependent upon the private sector (corporations, businesses, and other for-profit institutions). However, many projects in the public sector are also service sector projects.

A **service sector project** is a process or system that provides services to individuals, businesses, or government units. The economic value is developed primarily by the **intangibles** of the process or system, **not the physical entities**, such as buildings, machines, and equipment. Manufacturing and construction activities are commonly not considered a service sector project, though they may support the theme of the service provided.

Service projects have a tremendous range of variety and purpose; to name a few: health care systems, health and life insurance, airline reservation systems, credit card services, police and court systems, security programs, safety training programs, and all types of consulting projects. The intangible and intellectual work done by engineers and other professionals is often a part of a service sector project.

The economic evaluation of a service project is difficult to a great degree because the cost and benefit estimates are not accurate and often not within an acceptable degree of error. In other words, undue risk may be introduced into the decision because of poor monetary estimates. For example, consider the decision to place red-light cameras at stop lights to ticket drivers who run the red light. This is a public and a service project, but its (*economic*) benefits are quite difficult to estimate. Depending upon the viewpoint, benefits could be in terms of accidents averted, deaths prevented, police personnel released from patrolling the intersection, or, from a more mercenary viewpoint, amount of fines collected. In all but the last case, benefits in monetary terms will be poor estimates. These are examples where B/C analysis does not work well and a different form of analysis is needed.

In service and public sector projects, as expected, it is the benefits that are the more difficult to estimate. An evaluation method that combines monetary cost estimates with nonmonetary benefit estimates is **cost-effectiveness analysis (CEA)**. The CEA approach utilizes a **cost-effectiveness measure** or the **cost-effectiveness ratio (CER)** as a basis of ranking projects and selecting the best of independent projects or mutually exclusive alternatives. The CER ratio is defined as

$$\text{CER} = \frac{\text{equivalent total costs}}{\text{total of effectiveness measure}} = \frac{C}{E} \quad [9.7]$$

In the red-light camera example, the effectiveness measure (the benefit) may be one of the samples mentioned earlier, accidents averted or deaths prevented. Different from the B/C ratio of costs to benefits, CER places the PW or AW of total costs in the numerator and the effectiveness measure in the denominator. (The reciprocal of Equation [9.7] can also be used as the measure of worth, but we will use CER as defined above.) With costs in the numerator, smaller ratio values are more desirable for the same value of the denominator, since smaller ratio values indicate a lower cost for the same level of effectiveness.

Like ROR and B/C analysis, cost-effectiveness analysis requires the ordering (ranking) of alternatives prior to selection and the use of incremental analysis for mutually exclusive

alternative selection. Cost-effectiveness analysis utilizes a different ranking criterion than ROR or BC analysis. The *ordering criteria* are as follows:

Independent projects: Initially rank projects by **CER value**.

Mutually exclusive alternatives: Initially rank alternatives by **effectiveness measure**, then perform an incremental CER analysis.

Return again to the public/service project of red-light cameras. If the CER is defined as “*cost per total accidents averted*” and the projects are independent, increasing CER value is the ranking basis. If the projects are mutually exclusive, “*total accidents averted*” is the correct ranking basis and an incremental analysis is necessary.

There are significantly different analysis procedures for independent and mutually exclusive proposals. To select some from several (independent) projects, a budget limit, termed b , is inherently necessary once ordering is complete. However, for selecting one from several (mutually exclusive) alternatives, a pairwise incremental analysis is necessary and selection is made on the basis of $\Delta C/E$ ratios. The procedures and examples follow.

For **independent projects**, the procedure is as follows:

1. Determine the equivalent total cost C and effectiveness measure E , and calculate the CER measure for each project.
2. Order projects from the *smallest to the largest CER value*.
3. Determine cumulative cost for each project and compare with the budget limit b .
4. The selection criterion is to fund all projects such that b is not exceeded.



Independent project selection

EXAMPLE 9.8

Recent research indicates that corporations throughout the world need employees who demonstrate creativity and innovation for new processes and products. One measure of these talents is the number of patents approved each year through the R&D efforts of a company. Rollings Foundation for Innovative Thinking has allocated \$1 million in grant funds to award to corporations that enroll their top R&D personnel in a 1- to 2-month professional training program in their home state that has a historically proven track record over the last 5 years in helping individuals earn patents.

Table 9–2 summarizes data for six corporations that submitted proposals. Columns 2 and 3 give the proposed number of attendees and cost per person, respectively, and column 4 provides the historical track record of program graduates in patents per year. Use cost-effectiveness analysis to select the corporations and programs to fund.

Solution

We assume that across all programs and all patent awards there is equal quality. Use the procedure for independent projects and $b = \$1$ million to select from the proposals.

TABLE 9–2 Data for Programs to Increase Patents Used for CEA

Program (1)	Total Personnel (2)	Cost/Person, \$ (3)	5-Year History, Patents/Graduate/Year (4)
1	50	5000	0.5
2	35	4500	3.1
3	57	8000	1.9
4	24	2500	2.1
5	12	5500	2.9
6	87	3800	0.6

- Using Equation [9.7], the effectiveness measure E is patents per year, and the CER is

$$\text{CER} = \frac{\text{program cost per person}}{\text{patents per graduate}} = \frac{C}{E}$$

The program cost C is a PW value, and the E values are obtained from the proposals.

- The CER values are shown in Table 9–3 in increasing order, column 5.
- Cost per course, column 6, and cumulative costs, column 7, are determined.
- Programs 4, 2, 5, 3, and 6 (68 of the 87 people) are selected to not exceed \$1 million.

TABLE 9–3 Programs Ordered by CER Value, Example 9.8

Program (1)	Total Personnel (2)	Cost/Person $C, \$$ (3)	Patents per Year E (4)	CER, \$ per Patent (5) = (3)/(4)	Program Cost, \$ (6) = (2)(3)	Cumulative Cost, \$ (7) = $\Sigma(6)$
4	24	2,500	2.1	1,190	60,000	60,000
2	35	4,500	3.1	1,452	157,500	217,500
5	12	5,500	2.9	1,897	66,000	283,500
3	57	8,000	1.9	4,211	456,000	739,500
6	87	3,800	0.6	6,333	330,600	1,070,100
1	50	5,000	0.5	10,000	250,000	1,320,100

Comment

This is the first time that a budget limit has been imposed for the selection among independent projects. This is often referred to as *capital budgeting*, which is discussed further in Chapter 12.

For **mutually exclusive** alternatives and no budget limit, the alternative with the highest effectiveness measure E is selected without further analysis. Otherwise, an incremental CER analysis is necessary and the budget limit is applied to the selected alternative(s). The analysis is based on the incremental ratio $\Delta C/E$, and the procedure is similar to that we have applied for incremental ROR and B/C, except now the concept of *dominance* is utilized.



ME alternative selection

Dominance occurs when the incremental analysis indicates that the challenger alternative offers an improved incremental CER measure compared to the defender's CER, that is,

$$(\Delta C/E)_{\text{challenger}} < (C/E)_{\text{defender}}$$

Otherwise, no dominance is present, and both alternatives remain in the analysis.

For **mutually exclusive alternatives**, the selection procedure is as follows:

- Order the alternatives from *smallest to largest effectiveness measure E* . Record the cost for each alternative.
- Calculate the CER measure for the first alternative. This, in effect, makes DN the defender and the first alternative the challenger. This CER is a baseline for the next incremental comparison, and the first alternative becomes the new defender.
- Calculate incremental costs (ΔC) and effectiveness (ΔE) and the incremental measure $\Delta C/E$ for the new challenger using the relation

$$\Delta C/E = \frac{\text{cost of challenger} - \text{cost of defender}}{\text{effectiveness of challenger} - \text{effectiveness of defender}} = \frac{\Delta C}{\Delta E}$$

- If $\Delta C/E < C/E_{\text{defender}}$, the challenger dominates the defender and it becomes the new defender; the previous defender is eliminated. Otherwise, no dominance is present and both alternatives are retained for the next incremental evaluation.
- Dominance present:* Repeat steps 3 and 4 to compare the next ordered alternative (challenger) and new defender. Determine if dominance is present.

Dominance not present: The current challenger becomes the new defender, and the next alternative is the new challenger. Repeat steps 3 and 4 to compare the new challenger and new defender. Determine if dominance is present.

6. Continue steps 3 through 5 until only one alternative or only nondominated alternatives remain.
7. Apply the budget limit (or other criteria) to determine which of the remaining alternative(s) can be funded.

EXAMPLE 9.9

One of the corporations not selected for funding in Example 9.8 decided to fund its 50 R&D personnel to attend one of the innovation and creativity programs at its own expense. One criterion is that the program must have a historical average for a graduate of at least 2.0 patents per year. Use the data in Table 9–2 to select the best program.

TABLE 9–4 Mutually Exclusive Alternatives Evaluated by Cost-Effectiveness Analysis, Example 9.9

Program (1)	Total Personnel (2)	Cost/Person C, \$ (3)	Patents per Year E (4)	CER, \$ per Patent (5) = (3)/(4)	Program Cost, \$ (6) = (2)(3)
4	50	2,500	2.1	1190	125,000
5	50	5,500	2.9	1897	275,000
2	50	4,500	3.1	1452	225,000

Solution

From Table 9–2, three programs—2, 4, and 5—have a historical record of at least two patents per graduate per year. Since only one program will be selected, these are now *mutually exclusive alternatives*. Use the procedure to perform the incremental analysis.

1. The alternatives are ranked by increasing patents per year in Table 9–4, column 4.
2. The CER measure for program 4 is compared to the DN alternative.

$$C/E_4 = \frac{\text{program cost per person}}{\text{patents per graduate}} = \frac{2500}{2.1} = 1190$$

3. Program 5 is now the challenger.

$$\text{5-to-4 comparison: } \Delta C/E = \frac{\Delta C}{\Delta E} = \frac{5500 - 2500}{2.9 - 2.1} = 3750$$

4. In comparison to $C/E_4 = 1190$, it costs \$3750 per additional patent if program 5 is chosen over 4. Program 5 is more expensive for more patents; however, clear dominance is not present; both programs are retained for further evaluation.
5. *Dominance not present:* Program 5 becomes the new defender, and program 2 is the new challenger. Perform the 2-to-5 comparison.

$$\text{2-to-5 comparison: } \Delta C/E = \frac{\Delta C}{\Delta E} = \frac{4500 - 5500}{3.1 - 2.9} = -5000$$

Compared to $C/E_5 = 1897$, this increment is much cheaper—more patents for less money per person. Dominance is present; eliminate program 5 and compare 2 to 4.

6. Repeat steps 3 through 5 and compare $\Delta C/E$ to $C/E_4 = 1190$.

$$\text{2-to-4 comparison: } \Delta C/E = \frac{\Delta C}{\Delta E} = \frac{4500 - 2500}{3.1 - 2.1} = 2000$$

This does not represent dominance of program 2 over 4. The conclusion is that both programs are eligible for funding, that is, CEA in this case does not indicate only one program. This occurs when there is not lower cost and higher effectiveness of one alternative over another; that is, one alternative does not dominate all the others.

7. Now the budget and other considerations (probably noneconomic) are brought to bear to make the final decision. The fact that program 4 costs \$125,000—significantly less than program 2 at \$225,000—will likely enter into the decision.

Cost-effectiveness analysis is a form of multiattribute decision-making in which economic and noneconomic dimensions are integrated to evaluate alternatives from several perspectives by different decision makers. See Chapter 10 for further discussion of alternative analysis using multiple attributes.

9.6 Ethical Considerations in the Public Sector ● ● ●

Usual expectations by citizens of their elected officials—locally, nationally, and internationally—are that they make decisions for the good of the public, ensuring safety and minimizing risk and cost to the public. Above these is the long-standing expectation that public servants have **integrity**.

Similarly, the expectations of engineers employed by government departments, and those serving as consultants to government agencies, are held to high standards. **Impartiality**, consideration of a **wide range of circumstances**, and the use of **realistic assumptions** are but three of the foundation elements upon which engineers should base their recommendations to decision makers. This implies that engineers in public service avoid

- Self-serving, often greedy individuals and clients with goals of excessive profits and future contract awards
- Using a politically favorable perspective that compromises the results of a study
- Narrowly-defined assumptions that serve special interest groups and subcommunities potentially affected by the findings

Many people are disappointed and discouraged with government when elected officials and public employees (engineers and others, alike) do not have real commitment to integrity and unbiasedness in their work.

Engineers are routinely involved in two of the major aspects of public sector activities:

Public policy making—the **development of strategy** for public service, behavior, fairness, and justice. This may involve literature study, background discovery, data collection, opinion giving, and hypothesis testing. An example is *transportation management*. Engineers make virtually all the recommendations based on data and long-standing decision algorithms for policy items such as capacity of roads, expansion of highways, planning and zoning rules, traffic signal usage, speed limit corridors, and many related topics in transportation policy. Public officials use these findings to establish public transportation policy.

Public planning—the **development of projects** that implement strategy and affect people, the environment, and financial resources in a variety of ways. Consider *traffic control*, where the use and placement of traffic control signs, signals, speed limits, parking restrictions, etc. are detailed based upon established policy and current data. (In effect, this is systems engineering, that is, an application of the life-cycle phases and stages explained in Section 6.5.)

Whether in the arena of policy making or public planning, engineers can find ethical compromise a possibility when working with the public sector. A few circumstances are summarized here.

- *Use of technology* Many public projects involve the use of new technology. The public risks and safety factors are not always known for these new advances. It is common and expected that engineers make every attempt to apply the latest technology while ensuring that the public is not exposed to undue risk.
- *Scope of study* A client may pressure the engineers to limit the range of options, the assumption base, or the breadth of alternative solutions. These restrictions may be based on financial reasons, politically-charged topics, client-favored options, or a wide variety of other reasons. To remain impartial, it is the responsibility of the engineer to submit a fully unbiased analysis, report, and recommendation, even though it may jeopardize future contract possibilities, promote public disfavor, or generate other negative consequences.
- *Negative community impact* It is inevitable that public projects will adversely affect some groups of people, or the environment, or businesses. The intentional silencing of these projected effects is often the cause of strong public outcries against what may be a project that is in the best interest of the community at large. Engineers who find (stumble onto) such negative impacts may be pressured by clients, managers, or public figures to overlook them, though

TABLE 9–5 Some Ethical Considerations When Performing B/C and CEA Analysis

What the Study Includes	Ethical Dimension	Example
Audience for study	Is it ethical to select a specific group of people affected by the project and neglect possible effects on other groups?	Construct children's health care clinics for city dwellers, but neglect rural families with poor transportation means.
Impact time of decision	Is it ethical to decide now for future generations who may be adversely and economically affected by the current project decision?	Accomplish financial bailouts of corporations when future generations' taxes will be significantly higher to recover the costs, plus interest and inflation effects.
Greater good for community as a whole	Vulnerable minority groups, especially economically deprived ones, may be disproportionately affected. Is this ethical if the impact is predictable?	Allow a chemical plant that is vital to the community's employment to pollute a waterway when a minority group is known to eat fish from the water that is predictably contaminated.
Reliance on economic measures only	Is it acceptable to reduce all costs and benefits to monetary estimates for a decision, then subjectively impute nonquantified factors in the final decision?	Softening of building codes can improve the financial outlook for home builders; however, increased risks of fire loss, storm and water damage to structures, and reduced future resale values are considered only in passing as a new subdivision is approved by the planning and zoning committee.
Scope of disbenefits estimated and evaluated	Is it ethical to disregard any disbenefits in the B/C study or use indirect effectiveness measures in a CEA study based on the difficulty to estimate some of them?	Noise and air pollution caused by a planned open-pit quarry will have a negative effect on area ranchers, residents, wildlife, and plant life; but the effectiveness measure considers only suburban residents due to estimation difficulty of effects on other constituencies.

the Code of Ethics for Engineers dictates a full and fair analysis and report. For example, a planned rerouting of a city street may effectively cut off a section of international citizens' businesses, thus resulting in a clearly predictable economic downturn. Considering this outcome in the recommendation to the transportation department should be a goal of the analyzing engineers, yet pressure to bias the results may be quite high.

The results of a B/C or CEA analysis are routinely depended upon by public officials and government staff members to assist in making public planning decisions. As discussed earlier, estimations for benefits, disbenefits, effectiveness measures, and costs can be difficult and inaccurate, but these analysis tools are often the best available to structure a study. Some example ethically-oriented challenges that may be confronted during B/C and CEA analyses are summarized in Table 9–5.

CHAPTER SUMMARY

The benefit/cost method is used primarily to evaluate alternatives in the public sector. When one is comparing mutually exclusive alternatives, the incremental B/C ratio must be greater than or equal to 1.0 for the incremental equivalent total cost to be economically justified. The PW, AW, or FW of the initial costs and estimated benefits can be used to perform an incremental B/C analysis. For independent projects, no incremental B/C analysis is necessary. All projects with $B/C \geq 1.0$ are selected provided there is no budget limitation. It is usually quite *difficult to make accurate estimates of benefits* for public sector projects. The characteristics of public sector projects are substantially different from those of the private sector: initial costs are larger; expected life is longer; additional sources of capital funds include taxation, user fees, and government grants; and interest (discount) rates are lower.

Service projects develop economic value largely based on the intangibles of the services provided to users, not the physical items associated with the process or system. Evaluation by B/C analysis can be difficult with no good way to make monetary estimates of benefits. Cost-effectiveness analysis (CEA) combines cost estimates and a nonmonetary effectiveness measure (the benefit) to evaluate independent or mutually exclusive projects using procedures that are similar to incremental ROR and B/C analysis. The concept of dominance is incorporated into the procedure for comparing mutually exclusive alternatives.

As a complement to the discussion on professional ethics in Chapter 1, some potential ethical challenges in the public sector for engineers, elected officials, and government consultants are discussed here. Examples are included.

PROBLEMS

Public Projects and B/C Concepts

- 9.1 What is the difference between disbenefits and costs?
- 9.2 Identify the following as primarily public or private sector undertakings: eBay, farmer's market, state police department, car racing facility, social security, EMS, ATM, travel agency, amusement park, gambling casino, swap meet, football stadium.
- 9.3 State whether the following characteristics are primarily associated with public or private sector projects: large initial investment, park user fees, short-life projects, profit, disbenefits, tax-free bonds, subsidized loans, low interest rate, income tax, water quality regulations.
- 9.4 Identify the following cash flows as a benefit, disbenefit, or cost.
 - (a) Loss of income to local businesses because of a new freeway
 - (b) Less travel time because of a loop bypass
 - (c) \$400,000 annual income to local businesses because of tourism created by a national park
 - (d) Cost of fish from a hatchery to stock a lake at the state park
 - (e) Less tire wear because of smoother road surfaces
 - (f) Decrease in property values due to the closure of a government research lab
 - (g) School overcrowding because of a military base expansion
 - (h) Revenue to local motels because of an extended weekend holiday
- 9.5 What is a fundamental difference between DBOM and DBOMF contracts?
- 9.6 Buster County has proposed a strict water conservation policy for all industrial plants within the county limits. Enforcement is proposed to be a

public-private partnership between the sheriff's office and a private security company. In preparing for a B/C analysis of the proposal, estimates must be categorized as a benefit, cost, or disbenefit. The categories chosen will vary depending upon a person's viewpoint. Assume that possible viewpoints include sales revenues, politics, service to the public, the environment, contract obligations, future generations of residents, legal matters, revenue and budget (in general), and customers.

- (a) Select the *top two viewpoints* (in your opinion) for each of the following individuals as they would categorize estimates as a cost, benefit, or disbenefit.
 1. An industrial plant manager in the county
 2. County sheriff's deputy (appointed office)
 3. County commissioner (elected office)
 4. Security company president
- (b) Explain your answers by writing a short description of why you selected as you did.

Project B/C Value

- 9.7 If an alternative has a salvage value, how is it handled in the calculation of a B/C ratio relative to benefits, disbenefits, costs, or savings?
- 9.8 The cost of grading and spreading gravel on a short rural road is expected to be \$300,000. The road will have to be maintained at a cost of \$25,000 per year. Even though the new road is not very smooth, it allows access to an area that previously could only be reached with off-road vehicles. The improved accessibility has led to a 150% increase in the property values along the road. If the previous market value of a property was \$900,000, calculate the B/C ratio using an interest rate of 6% per year and a 20-year study period.
- 9.9 Arsenic enters drinking water supplies from natural deposits in the earth or from agricultural and industrial practices. Since it has been linked to

cancer of the bladder, kidney, and other internal organs, the EPA has lowered the arsenic standard for drinking water from 0.050 parts per million to 0.010 parts per million (10 parts per billion). The annual cost to public water utilities to meet the new standard is estimated to be \$200 per household. If it is estimated that there are 90 million households in the United States and that the lower standard can save 50 lives per year valued at \$4,000,000 per life, what is the benefit/cost ratio of the regulation?

- 9.10 From the following estimates, determine the B/C ratio for a project that has a 20-year life. Use an interest rate of 8% per year.

Consequences to the People		Consequences to the Government	
Annual benefits	\$90,000 per year	First cost	\$750,000
Annual disbenefits	\$10,000 per year	Annual cost	\$50,000 per year
		Annual savings	\$30,000 per year

- 9.11 A project to extend irrigation canals into an area that was recently cleared of mesquite trees (a nuisance tree in Texas) and large weeds is projected to have a capital cost of \$2,000,000. Annual maintenance and operation costs will be \$100,000 per year. Annual favorable consequences to the general public of \$820,000 per year will be offset to some extent by annual adverse consequences of \$400,000 to a portion of the general public. If the project is assumed to have a 20-year life, what is the B/C ratio at an interest rate of 8% per year?
- 9.12 Calculate the B/C ratio for the following cash flow estimates at a discount rate of 7% per year.

Item	Cash Flow
FW of benefits, \$	30,800,000
AW of disbenefits, \$ per year	105,000
First cost, \$	1,200,000
M&O costs, \$ per year	400,000
Life of project, years	20

- 9.13 The benefits associated with a nuclear power plant cooling water filtration project located on the Ohio River are \$10,000 per year forever, starting in year 1. The costs are \$50,000 in year 0 and \$50,000 at the end of year 2. Calculate the B/C ratio at $i = 10\%$ per year.
- 9.14 A privately funded wind-based electric power generation company in the southern part of the country has developed the following estimates

(in \$1000) for a new turbine farm. The MARR is 10% per year, and the project life is 25 years. Calculate (a) the profitability index and (b) the modified B/C ratio.

Benefits:	\$20,000 in year 0 and \$30,000 in year 5
Savings:	\$2000 in years 1–20
Cost:	\$50,000 in year 0
Disbenefits:	\$3000 in years 1–10

- 9.15 For the values shown, calculate the conventional B/C ratio at $i = 10\%$ per year.

	PW, \$	AW, \$/Year	FW, \$
First cost	100,000	—	259,370
M&O cost	61,446	10,000	159,374
Benefits	—	40,000	637,496
Disbenefits	30,723	5,000	—

- 9.16 A proposal to reduce traffic congestion on I-5 has a B/C ratio of 1.4. The annual worth of benefits minus disbenefits is \$560,000. What is the first cost of the project if the interest rate is 6% per year and the project is expected to have a 20-year life?

- 9.17 Oil spills in the Gulf of Mexico have been known to cause extensive damage to both public and private oyster grounds along the Louisiana and Mississippi shores. One way to protect shellfish along the shoreline is to release large volumes of freshwater from the Mississippi River to flush oil out to sea. This procedure inevitably results in death to some of the saltwater shellfish while preventing more widespread destruction to public reefs. Oil containment booms and other temporary structures can also be used to intercept floating oil before it damages sensitive fishing grounds. If the Fish and Wildlife Service spent \$110 million in year 0 and \$50 million in years 1 and 2 to minimize environmental damage from one particular oil spill, what is the benefit-to-cost ratio provided the efforts resulted in saving 3000 jobs valued at a total of \$175 million per year? Assume disbenefits associated with oyster deaths amounted to \$30 million in year 0. Use a 5-year study period and an interest rate of 8% per year.

- 9.18 On-site granular ferric hydroxide (GFH) systems can be used to remove arsenic from water when daily flow rates are relatively low. If the operating cost is \$600,000 per year and the public health benefits are assumed to be \$800,000 per year, what initial investment in the GFH system is necessary to guarantee a modified B/C ratio of at least 1.0? Assume that the equipment life is 10 years and the interest rate is 6% per year.

- 9.19 The Parks and Recreation Department of Burkett County has estimated that the initial cost of a "bare-bones" permanent river park will be \$2.3 million. Annual upkeep costs are estimated at \$120,000. Benefits of \$340,000 per year and disbenefits of \$40,000 per year have also been identified. Using a discount rate of 6% per year, calculate (a) the conventional B/C ratio and (b) the modified B/C ratio.

- 9.20 From the following data, calculate the (a) conventional and (b) modified benefit/cost ratios using an interest rate of 6% per year and an infinite project period.

	To the People	To the Government
Benefits:	\$300,000 now and \$100,000 per year thereafter	Costs: \$1.5 million now and \$200,000 three years from now
Disbenefits: \$40,000 per year		Savings: \$70,000 per year

- 9.21 In 2010, Brazil began construction of the Belo Monte hydroelectric dam on the Xingu River (which feeds the Amazon River). The project is funded by a consortium of investors and is expected to cost \$11 billion. It will begin producing electricity in 2015. Even though the dam will provide clean energy for millions of people, environmentalists are sharply opposed. They say it will devastate wildlife and the livelihoods of 40,000 people who live in the area to be flooded.

Assume that the funding will occur evenly over the 5-year period from 2010 through 2014 at \$2.2 billion per year. The disbenefits are estimated to be \$100,000 for each displaced person and \$1 billion for wildlife destruction. Assume that the disbenefits will occur evenly through the 5-year construction period and anticipated benefits will begin at the end of 2015 and continue indefinitely. Use an interest rate of 8% per year to determine what the equivalent annual benefits must be to ensure a B/C ratio of at least 1.0.

- 9.22 In the United States, the average number of airplanes in the sky on an average morning is 4000. There are another 16,000 planes on the ground. Aerospace company Rockwell Collins developed what it calls a *digital parachute*—a panic-button technology that will land any plane in a pinch at the closest airport, no matter what the weather or geography and without the help of a pilot. The technology can be applied if a pilot is no longer capable of flying the plane or is panicked and confused about what to do in an emergency. Assume that the cost of retrofitting 20,000 commercial air-

planes is \$100,000 each and the plane stays in service for 15 years. If the technology saves an average of 30 lives per year, with the value of a human life placed at \$4,000,000, what is the B/C ratio? Use an interest rate of 10% per year.

- 9.23 Although the lower Rio Grande is regulated by the Elephant Butte Dam and the Caballo Reservoir, serious flooding has occurred in El Paso and other cities located along the river. This has required homeowners living in valley areas near the river to purchase flood insurance costing between \$145 and \$2766 per year. To alleviate the possibility of flooding, the International Boundary and Water Commission undertook a project costing \$220 million to raise the levees along flood-prone portions of the river. As a result, 13,000 properties were freed of the federal mandate to purchase flood insurance. In addition, historical records indicate that damage to infrastructure will be avoided, which amounts to an average benefit of \$8,200,000 per year. If the average cost of flood insurance is \$460 per household per year, calculate the benefit-to-cost ratio of the levee-raising project. Use an interest rate of 6% per year and a 30-year study period.

- 9.24 For the data shown, calculate the *conventional* B/C ratio at $i = 6\%$ per year.

Benefits:	\$20,000 in year 0 and \$30,000 in year 5
Disbenefits:	\$7000 in year 3
Savings (to government):	\$25,000 in years 1–4
Cost:	\$100,000 in year 0
Project life:	5 years

- 9.25 Explain a fundamental difference between the modified B/C ratio and the profitability index.

- 9.26 Gerald Corporation entered a public-private partnership using a DBOM contract with the state of Massachusetts 10 years ago for railroad system upgrades. Determine the profitability index for the financial results listed below using a MARR of 8% per year.

Investments:	Year 0	\$-4.2 million
	Year 5	\$-3.5 million
Net savings:	Years 1–5	\$1.2 million per year
	Years 6–10	\$2.5 million per year

- 9.27 A project had a staged investment distributed over the 6-year contract period. For the cash flows shown (next page) and an interest rate of 10% per year, determine the profitability index and determine if the project was economically justified.

Year	0	1	2	3	4	5	6
Investment, \$1000	-25	0	-10	0	-5	0	0
NCF, \$1000 per year	0	5	7	9	11	13	15

Two Alternative Comparison

- 9.28 In comparing two alternatives by the B/C method, if the overall B/C ratio for both alternatives is calculated to be exactly 1.0, which alternative should you select?
- 9.29 In comparing alternatives X and Y by the B/C method, if $B/C_X = 1.6$ and $B/C_Y = 1.8$, what is known about the B/C ratio on the increment of investment between X and Y?
- 9.30 Logan Well Services Group is considering two sites for storage and recovery of reclaimed water. The mountain site (MS) will use injection wells that cost \$4.2 million to develop and \$280,000 per year for M&O costs. This site will be able to accommodate 150 million gallons per year. The valley site (VS) will involve recharge basins that cost \$11 million to construct and \$400,000 to operate and maintain. At this site, 890 million gallons can be injected each year. If the value of the injected water is \$3.00 per 1000 gallons, which alternative, if either, should be selected according to the B/C ratio method? Use an interest rate of 8% per year and a 20-year study period.
- 9.31 The estimates shown are for a bridge under consideration for a river crossing in Wheeling, West Virginia. Use the B/C ratio method at an interest rate of 6% per year to determine which bridge, if either, should be built.

	East Location	West Location
Initial cost, \$	11×10^6	27×10^6
Annual M&O, \$/year	100,000	90,000
Benefits, \$/year	990,000	2,400,000
Disbenefits, \$/year	120,000	100,000
Life, years	∞	∞

- 9.32 Select the better of two proposals to improve street safety and lighting in a colonia in south central New Mexico. Use a B/C analysis and an interest rate of 8% per year.

	Proposal 1	Proposal 2
Initial cost, \$	900,000	1,700,000
Annual M&O cost, \$/year	120,000	60,000
Annual benefits, \$/year	530,000	650,000
Annual disbenefits, \$/year	300,000	195,000
Life, years	10	20

- 9.33 Conventional and solar alternatives are available for providing energy at a remote radar site. Use the B/C ratio to determine which method should be

selected at an interest rate of 8% per year and a 5-year study period.

	Conventional	Solar
Initial cost, \$	300,000	2,500,000
Annual cost, \$ per year	700,000	5,000

- 9.34 The two alternatives shown are under consideration for improving security at a county jail in Travis County, New York. Determine which one should be selected, based on a B/C analysis, an interest rate of 7% per year and a 10-year study period.

	Extra Cameras (EC)	New Sensors (NS)
First cost, \$	38,000	87,000
Annual M&O, \$/year	49,000	64,000
Benefits, \$/year	110,000	160,000
Disbenefits, \$/year	26,000	—

- 9.35 The U.S. government recently released an RFP to construct a second-story floor on an existing building at the Pentagon Complex. Separate contractors proposed two methods. Method 1 will use lightweight expanded shale on a metal deck with open web joists and steel beams. For this method, the costs will be \$14,100 for concrete, \$6000 for metal decking, \$4300 for joists, and \$2600 for beams. Method 2 will construct a reinforced concrete slab costing \$5200 for concrete, \$1400 for rebar, \$2600 for equipment rental, and \$1200 for expendable supplies. Special additives will be included in the lightweight concrete that will improve the heat-transfer properties of the floor. If the energy costs for method 1 will be \$600 per year lower than for method 2, which one is more attractive? Use an interest rate of 7% per year, a 20-year study period, and the B/C method.

- 9.36 A project to control flooding from rare, but sometimes heavy rainfalls in the arid southwest will have the cash flows shown below. Determine which project should be selected on the basis of a B/C analysis at $i = 8\%$ per year and a 20-year study period.

	Sanitary Sewers	Open Channels
First cost, \$	26 million	53 million
M&O cost, \$ per year	400,000	30,000
Homeowner cleanup costs, \$ per year	60,000	0

- 9.37 Two routes are under consideration for a new interstate highway. The long intervalley route would be 25 kilometers in length and would have an initial cost of \$25 million. The short transmountain route

would be 10 kilometers long and would have an initial cost of \$45 million. Maintenance costs are estimated at \$150,000 per year for the long route and \$35,000 per year for the short route. Regardless of which route is selected, the volume of traffic is expected to be 400,000 vehicles per year. If the vehicle operating expenses are assumed to be \$0.30 per kilometer, determine which route should be selected on the basis of (a) conventional B/C analysis and (b) modified B/C analysis. Assume an infinite life for each road, and use an interest rate of 8% per year.

- 9.38 The Idaho Department of Fish and Wildlife (IDFW) is considering two locations for a new state park. Location E would require an investment of \$3 million and \$50,000 per year to maintain. Location W would cost \$7 million to construct, but the IDFW would receive an additional \$25,000 per year in park fees. The operating cost of location W will be \$65,000 per year. The revenue to park concessionaires will be \$500,000 per year at location E and \$700,000 at location W. The disbenefits associated with each location are \$30,000 per year for location E and \$40,000 per year for location W. Assume the park will be maintained indefinitely. Use an interest rate of 12% per year to determine which location, if either, should be selected on the basis of (a) the B/C method and (b) the modified B/C method.

- 9.39 Three engineers made the estimates shown below for two optional methods by which new construction technology would be implemented at a site for public housing. Either one of the two options or the current method may be selected. Set up a spreadsheet for B/C sensitivity analysis and determine if option 1, option 2 or the do-nothing option is selected by each of the three engineers. Use a life of 5 years and a discount rate of 10% per year for all analyses.

	Engineer Bob		Engineer Judy		Engineer Chen	
	Option 1	Option 2	Option 1	Option 2	Option 1	Option 2
Initial cost, \$	50,000	90,000	75,000	90,000	60,000	70,000
Cost, \$/year	3,000	4,000	3,800	3,000	6,000	3,000
Benefits, \$/year	20,000	29,000	30,000	35,000	30,000	35,000
Disbenefits, \$/year	500	1,500	1,000	0	5,000	1,000

Multiple Alternatives

- 9.40 A group of engineers responsible for developing advanced missile detection and tracking technologies, such as shortwave infrared, thermal infrared detection, target tracking radar, etc., recently came up with six proposals for consideration. The present worth (in \$ billions) of the capital requirements and benefits is shown for each alternative in the table. Determine which one(s) should be undertaken, if they are (a) independent and (b) mutually exclusive.

Alternative	A	B	C	D	E	F
PW of capital, \$	80	50	72	43	89	81
PW of benefits, \$	70	55	76	52	85	84

- 9.41 Comparison of five mutually exclusive alternatives is shown. One *must* be accepted. According to the B/C ratio, which alternative should be selected (costs increase from A to E).

Comparison	$\Delta B/C$ Ratio
A versus B	0.75
B versus C	1.4
C versus D	1.3
A versus C	1.1
A versus D	0.2
B versus D	1.9
C versus E	1.2
D versus E	0.9

- 9.42 A consulting engineer is currently evaluating four different projects for the Department of Housing and Urban Development. The future worth of costs, benefits, disbenefits, and cost savings is shown. The interest rate is 10% per year, compounded continuously. Determine which of the projects, if any, should be selected, if the projects are (a) independent and (b) mutually exclusive.

	Project ID			
	Good	Better	Best	Best of All
FW of first costs, \$	10,000	8,000	20,000	14,000
FW of benefits, \$	15,000	11,000	25,000	42,000
FW of disbenefits, \$	6,000	1,000	20,000	32,000
FW of cost savings, \$	1,500	2,000	16,000	3,000

- 9.43 From the data shown below for six mutually exclusive projects, determine which project, if any, should be selected.

	Project ID					
	A	B	C	D	E	F
Annual cost, \$ per year	8000	25,000	15,000	32,000	17,000	20,000
Annual benefits, \$ per year	?	?	?	?	?	?
B/C ratio (alternative vs. DN)	1.23	1.12	0.87	0.97	0.71	1.10

Selected Incremental B/C Ratios

- A versus B = 1.07
- A versus C = 0.46
- A versus F = 1.02
- B versus D = 0.43
- B versus E = 2.00
- B versus F = 1.20
- C versus D = 1.06
- C versus F = 1.80

- 9.44 Four mutually exclusive revenue alternatives are being compared using the B/C method. Which alternative, if any, should be selected?

Alternative	Initial Cost, \$ Millions	B/C Ratio vs. DN	Incremental B/C When Compared with Alternative			
			A	B	C	D
A	30	0.87	—	2.38	1.30	1.38
B	38	1.18	—	0.58	1.13	—
C	52	1.04	—	—	1.45	—
D	81	1.16	—	—	—	—

- 9.45 The city of St. Louis, Missouri, is considering various proposals regarding the disposal of used tires. All of the proposals involve shredding, but the charges for the service and the handling of the tire shreds differ in each plan. An incremental B/C analysis was initiated but never completed.
- Fill in all the missing blanks in the table.
 - Determine which alternative should be selected.

Alternative	PW of Costs, \$	PW of Benefits, \$	PW of Disbenefits, \$	B/C Ratio	Incremental B/C When Compared with Alternative			
					J	K	L	M
J	20	?	1	1.05	—	?	?	?
K	23	28	?	1.13	—	?	?	?
L	28	35	3	?	—	—	?	—
M	?	51	4	1.34	—	—	—	—

Cost-Effectiveness Analysis

- 9.46 In an effort to improve productivity in a large semiconductor manufacturing plant, the plant manager decided to undertake on a trial basis a series of actions directed toward improving employee morale. Six different strategies were implemented, such as increased employee autonomy, flexible work schedules, improved training, company picnics, electronic suggestion box, and better work environment. Periodically, the company surveyed the employees to measure the change in morale. The measure of effectiveness is the difference between the number of employees who rate their job satisfaction as very high and those who rate it very low. The per-employee cost of each strategy (identified as A through F) and the resultant measurement score are shown in the next column.

The manager has a maximum of \$50 per employee to spend on the permanent implementation of as many of the strategies as are justified from both the effectiveness and economic viewpoints. Determine which strategies are the best to implement. (Hand or spreadsheet solution is acceptable, as you are instructed.)

Strategy	Cost/Employee, \$	Measurement Score
A	5.20	50
B	23.40	182
C	3.75	40
D	10.80	75
E	8.65	53
F	15.10	96

- 9.47 There are a number of techniques to help people stop smoking, but their cost and effectiveness vary widely. One accepted measure of effectiveness of a program is the percentage of enrollees quitting. The table below shows several techniques touted as effective stop-smoking methods, some historical data on the approximate cost of each program per person, and the percentage of people smoke-free 3 months after the program ended.

The Cancer Society provides annual cost-offset funding to cancer patients so more people can afford these programs. A large clinic in St. Louis has the capacity to treat each year the number of people shown. If the clinic plans to place a proposal with the Cancer Society to treat a specified number of people annually, estimate the amount of money the clinic should ask for in its proposal to do the following:

- Conduct programs at the capacity level for the technique with the lowest cost-effectiveness ratio.
- Offer programs using as many techniques as possible to treat up to 1300 people per year using the most cost-effective techniques.

Technique	Cost, \$/ Enrollee	Treatment Capacity per Year	
		% Quitting	
Acupuncture	700	9	250
Subliminal message	150	1	500
Aversion therapy	1700	10	200
Outpatient clinic	2500	39	400
In-patient clinic	1800	41	550
Nicotine replacement therapy (NRT)	1300	20	100

- 9.48 The cost in \$ per year and effectiveness measure in items salvaged per year for four mutually exclusive service sector alternatives have been collected. (a) Calculate the cost effectiveness ratio for each alternative, and (b) use the CER to identify the best alternative.

Alternative	Cost C, \$/Year	Salvaged Items/Year E
W	355	20
X	208	17
Y	660	41
Z	102	7

- 9.49 An engineering student has only 30 minutes before the final exam in statics and dynamics. He wants to

get help solving a type of problem that he knows will be on the test from the professor's review during the previous class. There is time for using only one method of assistance before the exam; he must select well. In a rapid process of estimation, he determines how many minutes it would take for each method of assistance and how many points it might gain for him on the final. The method and estimates follow. Where should he seek help to be most effective?

Assistance from	Minutes for Assistance	Points Gained
Teaching assistant	15	15
Course slides on Web	20	10
Friend in class	10	5
Professor	20	15

Public Sector Ethics

9.50 During the design and specifications development stages of a remote meter reading system for residential electricity use (a system that allows monthly usage to be transmitted via phone lines with no need to physically view meters), the two engineers working on the project for the city of Forest Ridge noted something different from what they expected. The first, an electrical/software engineer, noted that the city liaison staff member provided all the information on the software options, but only one option, the one from Lorier Software, was ever discussed and detailed. The second designer, an industrial/systems engineer, further noted that all the hardware specifications provided to them by

this same liaison came from the same distributor, namely, Delsey Enterprises. Coincidentally, at a weekend family picnic for city employees, to which the engineers had been invited, they met a couple named Don Delsey and Susan Lorier. Upon review, they learned that Don is the son-in-law of the city liaison and Susan is his stepdaughter. Based on these observations and before they complete the system design and specifications, what should the two engineers do, if anything, about their suspicions that the city liaison person is trying to bias the design to favor of the use of his relatives' software and hardware businesses?

- 9.51 Explain the difference between public policy making and public planning.
- 9.52 Since transportation via automobile was introduced, drivers throughout the country of Yalturia in eastern Europe have driven on the left side of the road. Recently the Yalturian National Congress passed a law that within 3 years, a right-hand driving convention will be adopted and implemented throughout the country. This is a major policy change for the country and will require significant public planning and project development to implement successfully and safely. Assume you are the lead engineering consultant from Halcrow Engineers, responsible for developing and describing many of these major projects. Identify six of the projects you deem necessary. State a name and provide a one- or two-sentence description of each project.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 9.53 All of the following are examples of public sector projects, except:
- (a) Bridges (b) Emergency relief
 (c) Prisons (d) Oil wells
- 9.54 All of the following are usually associated with public sector projects except:
- (a) Funding from taxes (b) Profit
 (c) Disbenefits (d) Infinite life
- 9.55 All of the following would be examples of public projects except:
- (a) Air traffic control system
 (b) Establishing a dot.com company
 (c) Dam with irrigation canals
 (d) Mass transit system
- 9.56 In a conventional B/C ratio, revenue received by the government from admission to national parks should be:
- (a) Added to benefits in the numerator
 (b) Subtracted from benefits in the numerator

- (c) Subtracted from costs in the denominator
 (d) Added to costs in the denominator
- 9.57 In a conventional B/C ratio:
- (a) Disbenefits and M&O costs are subtracted from benefits.
 (b) Disbenefits are subtracted from benefits and M&O costs are included in costs.
 (c) Disbenefits and M&O costs are added to costs.
 (d) Disbenefits are added to costs and M&O costs are subtracted from benefits.
- 9.58 In a modified B/C ratio:
- (a) Disbenefits are put in the denominator.
 (b) Benefits are subtracted from costs.
 (c) M&O costs are put in the denominator.
 (d) M&O costs are put in the numerator.
- 9.59 If two mutually exclusive alternatives have B/C ratios of 1.4 and 1.5 for the lower- and higher-cost alternatives, respectively, the following is correct:

CASE STUDY

COMPARING B/C ANALYSIS AND CEA OF TRAFFIC ACCIDENT REDUCTION

Background

This case study compares benefit/cost analysis and cost-effectiveness analysis on the same information about highway lighting and its role in accident reduction.

Poor highway lighting may be one reason that proportionately more traffic accidents occur at night. Traffic accidents are categorized into six types by severity and value. For example, an accident with a *fatality* is valued at

approximately \$4 million, while an accident in which there is *property damage* (to the car and contents) is valued at \$6000. One method by which the impact of lighting is measured compares day and night accident rates for lighted and unlighted highway sections with similar characteristics. Observed reductions in accidents seemingly caused by too low lighting can be translated into either monetary estimates of the benefits *B* of lighting or used as the effectiveness measure *E* of lighting.

Information

Freeway accident data were collected in a 5-year study. The property damage category is commonly the largest based on the accident rate. The number of accidents recorded on a section of highway is presented here.

Accident Type	Number of Accidents Recorded ¹			
	Unlighted		Lighted	
	Day	Night	Day	Night
Property damage	379	199	2069	839

The ratios of night to day accidents involving property damage for the unlighted and lighted freeway sections are $199/379 = 0.525$ and $839/2069 = 0.406$, respectively. These results indicate that the lighting was beneficial. To quantify the benefit, the accident rate ratio from the unlighted section will be applied to the lighted section. This will yield the number of accidents that were prevented. Thus, there would have been $(2069)(0.525) = 1086$ accidents instead of 839 if there had not been lights on the freeway. This is a difference of 247 accidents. At a cost of \$6000 per accident, this results in a net annual benefit of

$$B = (247)(\$6000) = \$1,482,000$$

For an effectiveness measure of number of accidents prevented, this results in $E = 247$.

To determine the cost of the lighting, it will be assumed that the light poles are center poles 67 meters apart with 2 bulbs each. The bulb size is 400 watts, and the installation cost is \$3500 per pole. Since these data were collected over 87.8 kilometers of lighted freeway, the installed cost of the lighting is (with number of poles rounded off):

$$\begin{aligned} \text{Installation cost} &= \$3500 \left(\frac{87.8}{0.067} \right) \\ &= 3500(1310) \\ &= \$4,585,000 \end{aligned}$$

There are a total of $87.8/0.067 = 1310$ poles, and electricity costs \$0.10 per kWh. Therefore, the annual power cost is

$$\begin{aligned} \text{Annual power cost} &= 1310 \text{ poles}(2 \text{ bulbs/pole})(0.4 \text{ kilowatt/bulb}) \\ &\quad \times (12 \text{ hours/day})(365 \text{ days/year}) \\ &\quad \times (\$0.10/\text{kilowatt-hour}) \\ &= \$459,024 \text{ per year} \end{aligned}$$

¹Portion of data reported in Michael Griffin, "Comparison of the Safety of Lighting on Urban Freeways," *Public Roads*, vol. 58, pp. 8–15, 1994.

The data were collected over a 5-year period. Therefore, the annualized cost *C* at $i = 6\%$ per year is

$$\begin{aligned} \text{Total annual cost} &= \$4,585,000(A/P, 6\%, 5) \\ &\quad + 459,024 \\ &= \$1,547,503 \end{aligned}$$

If a benefit/cost analysis is the basis for a decision on additional lighting, the *B/C* ratio is

$$B/C = \frac{1,482,000}{1,547,503} = 0.96$$

Since $B/C < 1.0$, the lighting is not justified. Consideration of other categories of accidents is necessary to obtain a better basis for decisions. If a cost-effectiveness analysis (CEA) is applied, due to a judgment that the monetary estimates for lighting's benefit is not accurate, the *C/E* ratio is

$$C/E = \frac{1,547,503}{247} = 6265$$

This can serve as a base ratio for comparison when an incremental CEA is performed for additional accident reduction proposals.

These preliminary *B/C* and *C/E* analyses prompted the development of four lighting options:

- W) Implement the plan as detailed above; light poles every 67 meters at a cost of \$3500 per pole.
- X) Install poles at twice the distance apart (134 meters). This is estimated to cause the accident prevention benefit to decrease by 40%.
- Y) Install cheaper poles and surrounding safety guards, plus slightly lowered lumen bulbs (350 watts) at a cost of \$2500 per pole; place the poles 67 meters apart. This is estimated to reduce the benefit by 25%.
- Z) Install cheaper equipment for \$2500 per pole with 350-watt lightbulbs and place them 134 meters apart. This plan is estimated to reduce the accident prevention measure by 50% from 247 to 124.

Case Study Exercises

Determine if a definitive decision on lighting can be determined by doing the following:

1. Use a *benefit/cost analysis* to compare the four alternatives to determine if any are economically justified.
2. Use a cost-effectiveness analysis to compare the four alternatives.

From an understanding viewpoint, consider the following:

3. How many property-damage accidents could be prevented on the unlighted portion if it were lighted?
4. What would the lighted, night-to-day accident ratio have to be to make alternative Z economically justified by the *B/C* ratio?
5. Discuss the analysis approaches of *B/C* and *C/E*. Does one seem more appropriate in this type of situation than the other? Why? Can you think of other bases that might be better for decisions for public projects such as this one?

LEARNING STAGE 2: EPILOGUE

Selecting the Basic Analysis Tool

In the previous five chapters, several equivalent evaluation techniques have been discussed. Any method—PW, AW, FW, ROR, or B/C—can be used to select one alternative from two or more and obtain the same, correct answer. Only one method is needed to perform the engineering economy analysis, because any method, correctly performed, will select the same alternative. Yet different information about an alternative is available with each different method. The selection of a method and its correct application can be confusing.

Table LS2-1 gives a recommended evaluation method for different situations, if it is not specified by the instructor in a course or by corporate practice in professional work. The primary criteria for selecting a method are speed and ease of performing the analysis. Interpretation of the entries in each column follows.

Evaluation period: Most private sector alternatives (revenue and cost) are compared over their equal or unequal estimated lives, or over a specific period of time. Public sector projects are commonly evaluated using the B/C ratio and usually have long lives that may be considered infinite for economic computation purposes.

Type of alternatives: Private sector alternatives have cash flow estimates that are revenue-based (includes income and cost estimates) or cost-based (cost estimates only). For cost alternatives, the revenue cash flow series is assumed to be equal for all alternatives. For public sector projects, the difference between costs and timing is used to select one alternative over another. *Service sector projects* for which benefits are estimated using a nonmonetary effectiveness measure are usually evaluated with a method such as cost-effectiveness analysis. This applies to all evaluation periods.

Recommended method: Whether an analysis is performed by hand, calculator, or spreadsheet, the method(s) recommended in Table LS2-1 will correctly select one alternative from two or more as

TABLE LS2-1 Recommended Method to Compare Mutually Exclusive Alternatives, Provided the Method Is Not Preselected

Evaluation Period	Type of Alternatives	Recommended Method	Series to Evaluate
Equal lives of alternatives	Revenue or cost	AW or PW	Cash flows
	Public sector	B/C, based on AW or PW	Incremental cash flows
Unequal lives of alternatives	Revenue or cost	AW	Cash flows
	Public sector	B/C, based on AW	Incremental cash flows
Study period	Revenue or cost	AW or PW	Updated cash flows
	Public sector	B/C, based on AW or PW	Updated incremental cash flows
Long to infinite	Revenue or cost	AW or PW	Cash flows
	Public sector	B/C, based on AW	Incremental cash flows

rapidly as possible. Any other method can be applied subsequently to obtain additional information and, if needed, verification of the selection. For example, if lives are unequal and the rate of return is needed, it is best to first apply the AW method at the MARR and then determine the selected alternative's i^* using the same AW relation with i as the unknown.

Series to evaluate: The estimated cash flow series for one alternative and the incremental series between two alternatives are the only two options for present worth or annual worth evaluation. For spreadsheet analyses, this means that the NPV or PV functions (for present worth) or the PMT function (for annual worth) is applied. The word *updated* is added as a reminder that a study period analysis requires that cash flow estimates (especially salvage/market values) be reexamined and updated before the analysis is performed.

Once the evaluation method is selected, a specific procedure must be followed. These procedures were the primary topics of the last five chapters. Table LS2-2 summarizes the important

TABLE LS2-2 Characteristics of an Economic Analysis of Mutually Exclusive Alternatives Once the Evaluation Method Is Determined						
Evaluation Method	Equivalence Relation	Lives of Alternatives	Time Period for Analysis	Series to Evaluate	Rate of Return; Interest Rate	Decision Guideline: Select ¹
Present worth	PW	Equal	Lives	Cash flows	MARR	Numerically largest PW
	PW	Unequal	LCM	Cash flows	MARR	Numerically largest PW
	PW	Study period	Study period	Updated cash flows	MARR	Numerically largest PW
	CC	Long to infinite	Infinity	Cash flows	MARR	Numerically largest CC
Future worth	FW	Same as present worth for equal lives, unequal lives, and study period				Numerically largest FW
Annual worth	AW	Equal or unequal	Lives	Cash flows	MARR	Numerically largest AW
	AW	Study period	Study period	Updated cash flows	MARR	Numerically largest AW
	AW	Long to infinite	Infinity	Cash flows	MARR	Numerically largest AW
Rate of return	PW or AW	Equal	Lives	Incremental cash flows	Find Δi^*	Last $\Delta i^* \geq$ MARR
	PW or AW	Unequal	LCM of pair	Incremental cash flows	Find Δi^*	Last $\Delta i^* \geq$ MARR
	AW	Unequal	Lives	Cash flows	Find Δi^*	Last $\Delta i^* \geq$ MARR
	PW or AW	Study period	Study period	Updated incremental cash flows	Find Δi^*	Last $\Delta i^* \geq$ MARR
Benefit/cost	PW	Equal or unequal	LCM of pairs	Incremental cash flows	Discount rate	Last $\Delta B/C \geq 1.0$
	AW	Equal or unequal	Lives	Incremental cash flows	Discount rate	Last $\Delta B/C \geq 1.0$
	AW or PW	Long to infinite	Infinity	Incremental cash flows	Discount rate	Last $\Delta B/C \geq 1.0$

¹Lowest equivalent cost or largest equivalent income.

elements of the procedure for each method—PW, AW, ROR, and B/C. FW is included as an extension of PW. The meaning of the entries in Table LS2–2 follows.

Equivalence relation The basic equation written to perform any analysis is either a PW or an AW relation. The capitalized cost (CC) relation is a PW relation for infinite life, and the FW relation is likely determined from the PW equivalent value. Additionally, as we learned in Chapter 6, AW is simply PW times the A/P factor over the LCM or study period.

Lives of alternatives and time period for analysis The length of time for an evaluation (the n value) will always be one of the following: equal lives of the alternatives, LCM of unequal lives, specified study period, or infinity because the lives are very long.

- PW analysis always requires the LCM of compared alternatives.
- Incremental ROR and B/C methods require the LCM of the two alternatives being compared.
- The AW method allows analysis over the respective alternative lives.
- CC analysis has an infinite time line and uses the relation $P = A/i$.

The one exception is for the incremental ROR method for unequal-life alternatives using an AW relation for *incremental cash flows*. The LCM of the two alternatives compared must be used. This is equivalent to using an AW relation for the *actual cash flows* over the respective lives. Both approaches find the incremental rate of return Δi^* .

Series to evaluate Either the estimated cash flow series or the incremental series is used to determine the PW value, the AW value, the i^* value, or the B/C ratio.

Rate of return (interest rate) The MARR value must be stated to complete the PW, FW, or AW method. This is also correct for the discount rate for public sector alternatives analyzed by the B/C ratio. The ROR method requires that the incremental rate be found in order to select one alternative. It is here that the dilemma of multiple rates appears, if the sign tests indicate that a unique, real number root does not necessarily exist for a nonconventional series.

Decision guideline The selection of one alternative is accomplished using the general guideline in the rightmost column. Always select the alternative with the **numerically largest PW, FW, or AW value**. This is correct for both revenue and cost alternatives. The incremental cash flow methods—ROR and B/C—require that the largest initial cost and incrementally justified alternative be selected, provided it is justified against an alternative that is itself justified. This means that the Δi^* exceeds MARR, or the $\Delta B/C$ exceeds 1.0.

EXAMPLE LS2–1

Read through the problem statement of the following examples, neglecting the evaluation method used in the example. Determine which evaluation method is probably the fastest and easiest to apply. Is this the method used in the example? (a) 8.6, (b) 6.5, (c) 5.8, (d) 5.4.

Solution

Referring to the contents of Table LS2–1, the following methods should be applied first.

- (a) Example 8.6 involves four revenue alternatives with equal lives. Use the AW or PW value at the MARR of 10%. The incremental ROR method was applied in the example.
- (b) Example 6.5 requires selection between three public sector alternatives with unequal lives, one of which is 50 years and another is infinite. The B/C ratio of AW values is the best choice. This is how the problem was solved.

- (c) Since Example 5.8 involves two cost alternatives with one having a long life, either AW or PW can be used. Since one life is long, capitalized cost, based on $P = A/i$, is best in this case. This is the method applied in the example.
- (d) Example 5.4 is in the series of progressive examples. It involves 5-year and 10-year cost alternatives. The AW method is the best to apply in this case. The PW method for the LCM of 10 years and a study period of 5 years were both presented in the example.

LEARNING STAGE 3

Making Better Decisions

LEARNING STAGE 3 Making Better Decisions

CHAPTER 10
Project Financing and
Noneconomic
Attributes

CHAPTER 11
Replacement and
Retention Decisions

CHAPTER 12
Independent
Projects with
Budget Limitation

CHAPTER 13
Breakeven and
Payback Analysis

Most of the evaluations in the real world involve more than a simple economic selection of new assets or projects. The chapters in this stage introduce information-gathering and techniques that make decisions better. For example, **noneconomic parameters** can be introduced into the project analysis study through multiple attribute evaluation, and the **appropriate MARR** for a corporation or type of alternative can tailor and improve the economic decision.

The future is certainly not exact. However, techniques such as **replacement/retention** studies, **breakeven analysis**, and **payback analysis** help make informed decisions about future uses of existing assets and systems.

After completing these chapters, you will be able to go beyond the basic alternative analysis tools of the previous chapters. The techniques covered in this learning stage take into consideration the moving targets of change over time.

Important note: If asset depreciation and taxes are to be considered by an *after-tax analysis*, Chapters 16 and 17 should be covered before or in conjunction with these chapters.

Project Financing and Noneconomic Attributes



LEARNING OUTCOMES

Purpose: Explain debt and equity financing, select the appropriate MARR, and consider multiple attributes when comparing alternatives.

SECTION	TOPIC	LEARNING OUTCOME
10.1	COC and MARR	<ul style="list-style-type: none">Explain the relation between cost of capital and the MARR; explain why MARR values vary.
10.2	D-E mix and WACC	<ul style="list-style-type: none">Understand debt-to-equity mix and calculate the weighted average cost of capital.
10.3	Cost of debt capital	<ul style="list-style-type: none">Estimate the cost of debt capital, considering tax advantages.
10.4	Cost of equity capital	<ul style="list-style-type: none">Estimate the cost of equity capital and describe its relation to MARR and WACC.
10.5	High D-E mixes	<ul style="list-style-type: none">Demonstrate the connection between high D-E mixes and corporate (or personal) financial risk.
10.6	Multiple attributes	<ul style="list-style-type: none">Develop weights for multiple attributes used in alternative evaluation and selection.
10.7	Additive weights	<ul style="list-style-type: none">Apply the weighted attribute method to alternative evaluations that include noneconomic attributes.



This chapter discusses the different ways to finance a project through debt and equity sources and explains how the MARR is established. The descriptions here complement the introductory material of Chapter 1 on the same topics. Some of the parameters specified earlier are unspecified here, and in future chapters. As a result, some of the textbook aspects apparent in previous chapters are removed, thus coming closer to treating the more complex, real-world situations in which professional practice and decision making occur.

Until now, only one dimension—the economic one—has been the basis for judging the economic viability of one project, or the selection basis from two or more alternatives. In this chapter, guidelines and techniques explain the determination and use of multiple (noneconomic) attributes helpful in selecting between alternatives.

10.1 MARR Relative to the Cost of Capital ● ● ●

The MARR value used in alternative evaluation is one of the most important parameters of a study. In Chapter 1 the MARR was described relative to the weighted costs of debt and equity capital. This and the next four sections explain how to establish a MARR under varying conditions.

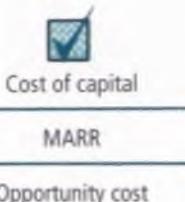
To form the basis for a realistic MARR, the types and cost of each source of project financing should be understood and estimated. There is a strong connection between the costs of debt and equity capital and the MARR used to evaluate one or more alternatives, whether they are mutually exclusive or independent. There are several terms and relationships important to the understanding of project financing and the MARR that is specified to evaluate projects using PW, AW, FW, or B/C methods. (Reference to Section 1.9 will complement the following material.)



The **cost of capital** is the weighted average interest rate paid based on the proportion of investment capital from *debt* and *equity sources*.

The **MARR** is then set relative to the cost of capital. The MARR can be set for one project, a series of projects, a division of a corporation, or the entire company. MARR values *change over time* due to changing circumstances.

When no specific MARR is established, the estimated net cash flows and available capital establish an inherent MARR. This rate is determined by finding the ROR (i^*) value of the project cash flows. This rate is utilized as the **opportunity cost**, which is the ROR of the first project not funded due to the lack of capital funds.



Before we discuss cost of capital, we review the two primary sources of capital.

Debt capital represents borrowing from outside the company, with the principal repaid at a stated interest rate following a specified time schedule. Debt financing includes borrowing via *bonds*, *loans*, and *mortgages*. The lender does not share in the profits made using the debt funds, but there is **risk** in that the borrower could default on part of or all the borrowed funds. The amount of outstanding debt financing is indicated in the liabilities section of the corporate balance sheet.

Equity capital is corporate money comprised of the *funds of owners* and *retained earnings*. Owners' funds are further classified as common and preferred stock proceeds or owners' capital for a private (non-stock-issuing) company. Retained earnings are funds previously retained in the corporation for capital investment. The amount of equity is indicated in the net worth section of the corporate balance sheet.

To illustrate the relation between cost of capital and MARR, assume a new greenhouse gas emission control system will be completely financed by a \$25,000,000 bond issue (100% debt financing), and assume the dividend rate on the bonds is 8%. Therefore, the cost of debt capital is 8% as shown in Figure 10–1. This 8% is the minimum for MARR. Management may increase this MARR in increments that reflect its desire for added return and its perception of risk. For example, management may add an amount for all capital commitments in this area. Suppose this amount is 2%. This increases the expected return to 10% (Figure 10–1). Also, if the risk associated with the investment is considered substantial enough to warrant an additional 1% return requirement, the final MARR is 11%.

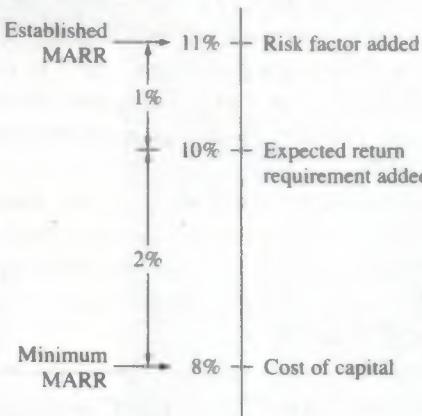


Figure 10–1
A fundamental relation between cost of capital and MARR used in practice.

The recommended approach does not follow the logic presented above. Rather, the cost of capital (8% here) should be the established MARR. Then the i^* value is determined from the estimated net cash flows. Using this approach, suppose the control system is estimated to return 11%. Now, additional return requirements and risk factors are considered to determine if 3% above the MARR of 8% is sufficient to justify the capital investment. After these considerations, if the project is not funded, the effective MARR is now 11%. This is the opportunity cost discussed previously—the unfunded project i^* has established the effective MARR for emission control system alternatives at 11%, not 8%.

The setting of the MARR for an economy study is not an exact process. The debt and equity capital mix changes over time and between projects. Also, the MARR is not a fixed value established corporatewide. It is altered for different opportunities and types of projects. For example, a corporation may use a MARR of 10% for evaluating the purchase of assets (equipment, cars) and a MARR of 20% for expansion investments, such as acquiring smaller companies.

The effective MARR varies from one project to another and through time because of factors such as the following:

Project risk. Where there is greater risk (perceived or actual) associated with proposed projects, the tendency is to set a higher MARR. This is encouraged by the higher cost of debt capital for projects considered risky. This usually means that there is some concern that the project will not realize its projected revenue requirements.

Investment opportunity. If management is determined to expand in a certain area, the MARR may be lowered to encourage investment with the hope of recovering lost revenue in other areas. This common reaction to investment opportunity can create havoc when the guidelines for setting a MARR are too strictly applied. Flexibility becomes very important.

Government intervention. Depending upon the state of the economy, international relations, and a host of other factors, the federal government (and possibly lower levels) can dictate the forces and direction of the free market. This may occur through price limits, subsidies, import tariffs, and limitation on availability. Both short-term and long-term government interventions are commonly present in different areas of the economy. Examples are steel imports, foreign capital investment, car imports, and agricultural product exports. During the time that such government actions are in force, there is a strong impact to increase or decrease taxes, prices, etc., thus tending to move the MARR up or down.

Tax structure. If corporate taxes are rising (due to increased profits, capital gains, local taxes, etc.), pressure to increase the MARR is present. Use of after-tax analysis may assist in eliminating this reason for a fluctuating MARR, since accompanying business expenses will tend to decrease taxes and after-tax costs.

Limited capital. As debt and equity capital become limited, the MARR is increased. If the demand for limited capital exceeds supply, the MARR may tend to be set even higher. The opportunity cost has a large role in determining the MARR actually used.

Market rates at other corporations. If the MARR increases at other corporations, especially competitors, a company may alter its MARR upward in response. These variations are often based on changes in interest rates for loans, which directly impact the cost of capital.

If the details of after-tax analysis are not of interest, but the effects of income taxes are important, the MARR may be increased by incorporating an effective tax rate using the formula

$$\text{Before-tax MARR} = \frac{\text{after-tax MARR}}{1 - \text{tax rate}} \quad [10.1]$$

The total or effective tax rate, including federal, state, and local taxes, for most corporations is in the range of 30% to 50%. If an after-tax rate of return of 10% is required and the effective tax rate is 35%, the MARR for the before-tax economic analysis is $10\%/(1 - 0.35) = 15.4\%$.

EXAMPLE 10.1

Twin brother and sister, Carl and Christy graduated several years ago from college. Carl, an architect, has worked in home design with Bulte Homes since graduation. Christy, a civil engineer, works with Butler Industries in structural components and analysis. They both reside in Richmond, Virginia. They have started a creative e-commerce network through which Virginia-based builders can buy their “spec home” plans and construction materials much more cheaply. Carl and Christy want to expand into a regional e-business corporation. They have gone to the Bank of America (BA) in Richmond for a business development loan. Identify some factors that might cause the loan rate to vary when BA provides the quote. Also, indicate any impact on the established MARR when Carl and Christy make economic decisions for their business.

Solution

In all cases the direction of the loan rate and the MARR will be the same. Using the six factors mentioned above, some loan rate considerations are as follows:

Project risk: The loan rate may increase if there has been a noticeable downturn in housing starts, thus reducing the need for the e-commerce connection.

Investment opportunity: The rate could increase if other companies offering similar services have already applied for a loan at other BA branches regionally or nationwide.

Government intervention: The loan rate may decrease if the federal government has recently offered Federal Reserve loan money at low rates to banks. The intervention may be designed to boost the housing economic sector in an effort to offset a significant slowdown in new home construction.

Taxes: If the state recently removed house construction materials from the list of items subject to sales tax, the rate might be lowered slightly.

Capital limitation: Assume the computer equipment and software rights held by Carl and Christy were bought with their own funds and there are no outstanding loans. If additional equity capital is not available for this expansion, the rate for the loan (debt capital) should be lowered.

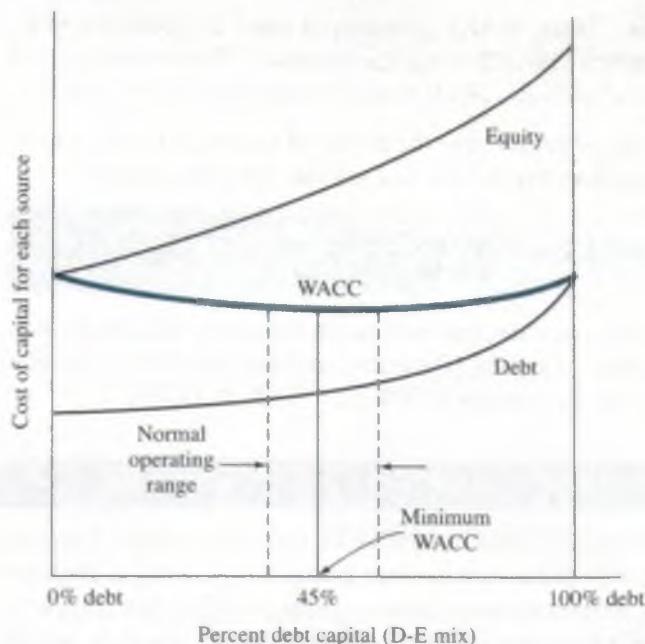
Market loan rates: The local BA branch probably obtains its development loan money from a large national pool. If market loan rates to this BA branch have increased, the rate for this loan will likely increase, because money is becoming “tighter.”

10.2 Debt-Equity Mix and Weighted Average Cost of Capital ● ● ●

The **debt-to-equity (D-E) mix** identifies the percentages of debt and equity financing for a corporation. A company with a 40–60 D-E mix has 40% of its capital originating from debt capital sources (bonds, loans, and mortgages) and 60% derived from equity sources (stocks and retained



Figure 10–2
General shape of different cost of capital curves.



earnings). Most projects are funded with a combination of debt and equity capital made available specifically for the project or taken from a corporate *pool of capital*. The **weighted average cost of capital (WACC)** of the pool is estimated by the relative fractions from debt and equity sources. If known exactly, these fractions are used to estimate WACC; otherwise the historical fractions for each source are used in the relation

$$\text{WACC} = (\text{equity fraction})(\text{cost of equity capital}) + (\text{debt fraction})(\text{cost of debt capital}) \quad [10.2]$$

The two *cost* terms are expressed as percentage interest rates.

Since virtually all corporations have a mixture of capital sources, the WACC is a value between the debt and equity costs of capital. If the fraction of each type of equity financing—common stock, preferred stock, and retained earnings—is known, Equation [10.2] is expanded.

$$\begin{aligned} \text{WACC} = & (\text{common stock fraction})(\text{cost of common stock capital}) \\ & + (\text{preferred stock fraction})(\text{cost of preferred stock capital}) \\ & + (\text{retained earnings fraction})(\text{cost of retained earnings capital}) \\ & + (\text{debt fraction})(\text{cost of debt capital}) \end{aligned} \quad [10.3]$$

Figure 10–2 indicates the usual shape of cost of capital curves. If 100% of the capital is derived from equity or 100% is from debt sources, the WACC equals the cost of capital of that source of funds. There is virtually always a mixture of capital sources involved for any capitalization program. As an illustration only, Figure 10–2 indicates a minimum WACC at about 45% debt capital. Most firms operate over a range of D-E mixes. For example, a range of 30% to 50% debt financing for some companies may be very acceptable to lenders, with no increases in risk or MARR. However, another company may be considered “risky” with only 20% debt capital. It takes knowledge about management ability, current projects, and the economic health of the specific industry to determine a reasonable operating range of the D-E mix for a particular company.

EXAMPLE 10.2

Historically, Hong Kong has imported over 95% of its fresh vegetables each day. In an effort to develop sustainable and renewable vegetable sources, a new commercial vertical crop technology is being installed through a public-private partnership with Valcent Products.¹ For

¹ “Valcent Announces Agreement to Supply Verticrop™ Vertical Farming Technology to Hong Kong’s VF Innovations Ltd.” www.valcent.net, June 16, 2010 news release.

For illustration purposes, assume that the present worth of the total system cost is \$20 million with financing sources and costs as follows.

Commercial loan for debt financing	\$10 million at 6.8% per year
Retained earnings from partnering corporations	\$4 million at 5.2% per year
Sale of stock (common and preferred)	\$6 million at 5.9% per year

There are three existing international vertical farming projects with capitalization and WACC values as follows:

- Project 1: \$5 million with $\text{WACC}_1 = 7.9\%$
- Project 2: \$30 million with $\text{WACC}_2 = 10.2\%$
- Project 3: \$7 million with $\text{WACC}_3 = 4.8\%$

Compare the WACC for the Hong Kong (HK) project with the WACC of the existing projects.

Solution

To apply Equation [10.3] to this new project, the fraction of equity (stock and retained earnings) and debt financing is needed. These are 0.3 for stock (\$6 out of \$20 million), 0.2 for retained earnings, and 0.5 for debt (\$10 out of \$20 million).

$$\text{WACC}_{\text{HK}} = 0.3(5.9\%) + 0.2(5.2\%) + 0.5(6.8\%) = 6.210\%$$

To correctly weight the other three project WACCs by size, determine the fraction in each one of the \$42 million in total capital: project 1 has \$5 million/\$42 million = 0.119; project 2 has 0.714; project 3 has 0.167. The WACC weighted by project size is WACC_w .

$$\text{WACC}_w = 0.119(7.9\%) + 0.714(10.2\%) + 0.167(4.8\%) = 9.025\%$$

The Hong Kong project has a considerably lower cost of capital than the weighted average of other projects, considering all sources of funding.

The WACC value can be computed using before-tax or after-tax values for cost of capital. The after-tax method is the correct one since debt financing has a distinct tax advantage, as discussed in Section 10.3 below. Approximations of after-tax or before-tax cost of capital are made using the effective tax rate T_e in the relation

$$\text{After-tax cost of debt capital} = (\text{before-tax cost})(1 - T_e) \quad [10.4]$$

The effective tax rate is a combination of federal, state, and local tax rates. They are reduced to a single number T_e to simplify computations. Equation [10.4] may be used to approximate the cost of debt capital separately or inserted into Equation [10.2] for an after-tax WACC rate. Chapter 17 treats taxes and after-tax economic analysis in detail.

10.3 Determination of the Cost of Debt Capital ● ● ●

Debt financing includes borrowing, primarily via bonds and loans. (We learned about bonds in Section 7.6.) In most industrialized countries, bond dividends and loan interest payments are tax-deductible as a corporate expense. This reduces the taxable income base upon which taxes are calculated, with the end result of less taxes paid. The cost of debt capital is, therefore, reduced because there is an annual *tax savings* of the expense cash flow times the effective tax rate T_e . This tax savings is subtracted from the debt-capital expense in order to calculate the cost of debt capital. In formula form,

$$\text{Tax savings} = (\text{expenses}) (\text{effective tax rate}) = \text{expenses} (T_e) \quad [10.5]$$

$$\text{Net cash flow} = \text{expenses} - \text{tax savings} = \text{expenses} (1 - T_e) \quad [10.6]$$

To find the cost of debt capital, develop a PW- or AW-based relation of the net cash flow (NCF) series with i^* as the unknown. Find i^* by trial and error, by calculator, or by the RATE

or IRR function on a spreadsheet. This is the cost of debt capital used in the WACC computation, Equation [10.2].

EXAMPLE 10.3

AT&T will generate \$5 million in debt capital by issuing five thousand \$1000 8% per year 10-year bonds. If the effective tax rate of the company is 30% and the bonds are discounted 2%, compute the cost of debt capital (a) before taxes and (b) after taxes from the company perspective. Obtain the answers by hand and spreadsheet.

Solution by Hand

- (a) The annual bond dividend is $\$1000(0.08) = \80 , and the 2% discounted sales price is \$980 now. Using the company perspective, find the i^* in the PW relation

$$0 = 980 - 80(P/A, i^*, 10) - 1000(P/F, i^*, 10)$$

$$i^* = 8.3\%$$

The before-tax cost of debt capital is $i^* = 8.3\%$, which is slightly higher than the 8% bond interest rate, because of the 2% sales discount.

- (b) With the allowance to reduce taxes by deducting the bond dividend, Equation [10.5] shows a tax savings of $\$80(0.3) = \24 per year. The bond dividend amount for the PW relation is now $\$80 - 24 = \56 . Solving for i^* after taxes reduces the cost of debt capital to 5.87%.

Solution by Spreadsheet

Figure 10–3 is a spreadsheet image for both before-tax (column B) and after-tax (column C) analysis using the IRR function. The after-tax net cash flow is calculated using Equation [10.6] with $T_e = 0.3$.

	A	B	C	D	E	F	G
1							
2	Year		Before-tax cash flow	After-tax cash flow			
3	0	980	980				
4	1	-80	-56				
5	2	-80	-56				
6	3	-80	-56				
7	4	-80	-56				
8	5	-80	-56				
9	6	-80	-56				
10	7	-80	-56				
11	8	-80	-56				
12	9	-80	-56				
13	10	-1,080	-1,056				
14	Cost of debt capital	8.30%	5.87%				
15							
16							
17							

Figure 10–3

Use of IRR function to determine cost of debt capital before taxes and after taxes, Example 10.3.

EXAMPLE 10.4

LST Trading Company will purchase a \$20,000 ten-year-life asset. Company managers have decided to put \$10,000 down now from retained earnings and borrow \$10,000 at an interest rate of 6%. The simplified loan repayment plan is \$600 in interest each year, with the entire \$10,000 principal paid in year 10. (a) What is the after-tax cost of debt capital if the effective tax rate is 42%? (b) How are the interest rate and cost of debt capital used to calculate WACC?

Solution

(a) The after-tax net cash flow for interest on the \$10,000 loan is an annual amount of $600(1 - 0.42) = \$348$ by Equation [10.6]. The loan repayment is \$10,000 in year 10. PW is used to estimate a cost of debt capital of 3.48%.

$$0 = 10,000 - 348(P/A, i^*, 10) - 10,000(P/F, i^*, 10)$$

(b) The 6% annual interest on the \$10,000 loan is not the WACC because 6% is paid only on the borrowed funds. Nor is 3.48% the WACC, since it is only the cost of debt capital. The cost of the \$10,000 equity capital is needed to determine the WACC.

10.4 Determination of the Cost of Equity Capital and the MARR ●●●

Equity capital is usually obtained from the following sources:

- Sale of preferred stock
- Sale of common stock
- Use of retained earnings
- Use of owner's private capital

The cost of each type of financing is estimated separately and entered into the WACC computation. A summary of one commonly accepted way to estimate each source's cost of capital is presented here. One additional method for estimating the cost of equity capital via common stock is presented. *There are no tax savings for equity capital, because dividends paid to stockholders and owners are not tax-deductible.*

Issuance of **preferred stock** carries with it a commitment to pay a stated dividend annually. The cost of capital is the stated dividend percentage, for example, 10%, or the dividend amount divided by the price of the stock. Preferred stock may be sold at a discount to speed the sale, in which case the actual proceeds from the stock should be used as the denominator. For example, if a 10% dividend preferred stock with a value of \$200 is sold at a 5% discount for \$190 per share, there is a cost of equity capital of $(\$20/\$190) \times 100\% = 10.53\%$.

Estimating the cost of equity capital for **common stock** is more involved. The dividends paid are not a true indication of what the stock issue will actually cost in the future. Usually a valuation of the common stock is used to estimate the cost. If R_e is the cost of equity capital (in decimal form),

$$R_e = \frac{\text{first-year dividend}}{\text{price of stock}} + \text{expected dividend growth rate}$$

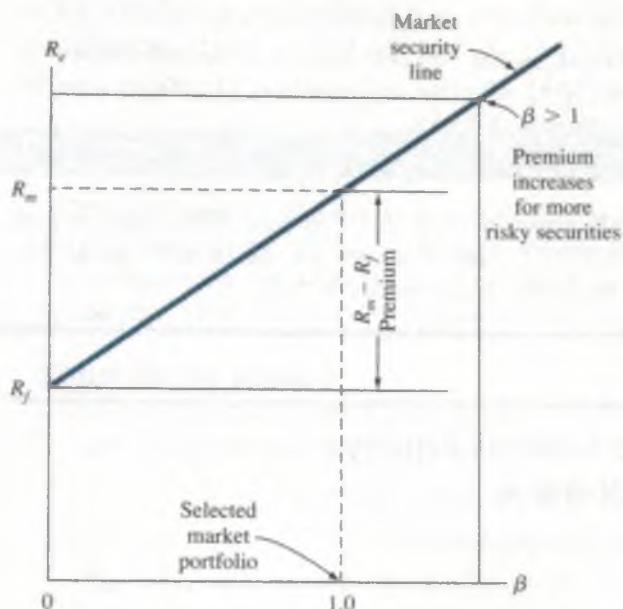
$$= \frac{DV_1}{P} + g \quad [10.7]$$

The growth rate g is an estimate of the annual increase in returns that the shareholders receive. Stated another way, it is the compound growth rate on dividends that the company believes is required to attract stockholders. For example, assume a multinational corporation plans to raise capital through its U.S. subsidiary for a new plant in South America by selling \$2,500,000 worth of common stock valued at \$20 each. If a 5% or \$1 dividend is planned for the first year and an appreciation of 4% per year is anticipated for future dividends, the cost of capital for this common stock issue from Equation [10.7] is 9%.

$$R_e = \frac{1}{20} + 0.04 = 0.09$$

The **retained earnings and owner's funds** cost of equity capital is usually set equal to the common stock cost, since it is the shareholders and owners who will realize any returns from projects in which these funds are invested.

Figure 10–4
Expected return on common stock issue using CAPM.



Once the cost of capital for all planned equity sources is estimated, the WACC is calculated using Equation [10.3].

A second method used to estimate the **cost of common stock capital** is the **capital asset pricing model (CAPM)**. Because of the fluctuations in stock prices and the higher return demanded by some corporations' stocks compared to others, this valuation technique is commonly applied. The cost of equity capital from common stock R_e , using CAPM, is

$$\begin{aligned} R_e &= \text{risk-free return} + \text{premium above risk-free return} \\ &= R_f + \beta(R_m - R_f) \end{aligned} \quad [10.8]$$

where β = volatility of a company's stock relative to other stocks in the market ($\beta = 1.0$ is the norm)

R_m = return on stocks in a defined market portfolio measured by a prescribed index

R_f = return on a "safe investment," which is usually the U.S. Treasury bill rate

The term $(R_m - R_f)$ is the premium paid above the safe or risk-free rate. The coefficient β (beta) indicates how the stock is expected to vary compared to a selected portfolio of stocks in the same general market area, often the Standard and Poor's 500 stock index. If $\beta < 1.0$, the stock is less volatile, so the resulting premium can be smaller; when $\beta > 1.0$, larger price movements are expected, so the premium is increased.

Security is a word that identifies a stock, bond, or any other instrument used to develop capital. To better understand how CAPM works, consider Figure 10–4. This is a plot of a market security line, which is a linear fit by regression analysis to indicate the expected return for different β values. When $\beta = 0$, the risk-free return R_f is acceptable (no premium). As β increases, the premium return requirement grows. Beta values are published periodically for most stock-issuing corporations. Once complete, this estimated cost of common stock equity capital can be included in the WACC computation in Equation [10.3].

EXAMPLE 10.5

The lead software engineer at SafeSoft, a food industry service corporation, has convinced the president to develop new software technology for the meat and food safety industry. It is envisioned that processes for prepared meats can be completed more safely and faster using this automated control software. A common stock issue is a possibility to raise capital if the cost of equity capital is below 9%. SafeSoft, which has a historical beta value of 1.09, uses CAPM to determine the premium of its stock compared to other software corporations. The security market line indicates that a 5% premium above the risk-free rate is desirable. If U.S. Treasury bills are paying 2%, estimate the cost of common stock capital.

Solution

The premium of 5% represents the term $R_m - R_f$ in Equation [10.8].

$$R_e = 2.0 + 1.09(5.0) = 7.45\%$$

Since this cost is lower than 9%, SafeSoft should issue common stock to finance this new venture.

In theory, a correctly performed engineering economy study uses a MARR equal to the cost of the capital committed to the specific alternatives in the study. Of course, such detail is not known. For a combination of debt and equity capital, the calculated WACC sets the minimum for the MARR. The most rational approach is to set MARR between the cost of equity capital and the corporation's WACC. The risks associated with an alternative should be treated separately from the MARR determination, as stated earlier. This supports the guideline that the MARR should not be arbitrarily increased to account for the various types of risk associated with the cash flow estimates. Unfortunately, the MARR is often set above the WACC because management does want to account for risk by increasing the MARR.

EXAMPLE 10.6

The Engineering Products Division of 4M Corporation has two mutually exclusive alternatives A and B with ROR values of $i_A^* = 9.2\%$ and $i_B^* = 5.9\%$. The financing scenario is yet unsettled, but it will be one of the following: plan 1—use all equity funds, which are currently earning 8% for the corporation; plan 2—use funds from the corporate capital pool which is 25% debt capital costing 14.5% and the remainder from the same equity funds mentioned above. The cost of debt capital is currently high because the company has narrowly missed its projected revenue on common stock for the last two quarters, and banks have increased the borrowing rate for 4M. Make the economic decision on alternative A versus B under each financing scenario. The MARR is set by the calculated WACC.

Solution

The capital is available for one of the two mutually exclusive alternatives. For plan 1, 100% equity, the financing is specifically known, so the cost of equity capital is the MARR, that is, 8%. Only alternative A is acceptable; alternative B is not since the estimated return of 5.9% does not exceed this MARR.

Under financing plan 2, with a D-E mix of 25–75,

$$\text{WACC} = 0.25(14.5) + 0.75(8.0) = 9.625\%$$

Now, neither alternative is acceptable since both ROR values are less than MARR = WACC = 9.625%. The selected alternative should be to do nothing, unless one alternative absolutely must be selected, in which case noneconomic attributes must be considered.

10.5 Effect of Debt-Equity Mix on Investment Risk ● ● ●

The D-E mix was introduced in Section 10.2. As the proportion of debt capital increases, the calculated cost of capital decreases due to the tax advantages of debt capital.

The leverage offered by larger debt capital percentages increases the riskiness of projects undertaken by the company. When large debts are already present, additional financing using debt (or equity) sources gets more difficult to justify, and the corporation can be placed in a situation where it owns a smaller and smaller portion of itself. This is sometimes referred to as a **highly leveraged corporation**.

Inability to obtain operating and investment capital means increased difficulty for the company and its projects. Thus, a reasonable balance between debt and equity financing is important for



the financial health of a corporation. Example 10.7 illustrates the disadvantages of unbalanced D-E mixes.

EXAMPLE 10.7

Three auto parts manufacturing companies have the following debt and equity capital amounts and D-E mixes. Assume all equity capital is in the form of common stock.

Company	Amount of Capital		
	Debt (\$ in Millions)	Equity (\$ in Millions)	D-E Mix (%-%)
A	10	40	20-80
B	20	20	50-50
C	40	10	80-20

Assume the annual revenue is \$15 million for each one and that after interest on debt is considered, the net incomes are \$14.4, \$13.4, and \$10.0 million, respectively. Compute the return on common stock for each company, and comment on the return relative to the D-E mixes.

Solution

Divide the net income by the stock (equity) amount to compute the common stock return. In million dollars,

$$\text{Return}_A = \frac{14.4}{40} = 0.36 \quad (36\%)$$

$$\text{Return}_B = \frac{13.4}{20} = 0.67 \quad (67\%)$$

$$\text{Return}_C = \frac{10.0}{10} = 1.00 \quad (100\%)$$

As expected, the return is by far the largest for highly leveraged C, where only 20% of the company is in the hands of the ownership. The return is excellent, but the risk associated with this firm is high compared to A, where the D-E mix is only 20% debt.

The use of *large percentages of debt financing greatly increases the risk taken by lenders and stock owners*. Long-term confidence in the corporation diminishes, no matter how large the short-term return on stock.

The leverage of large D-E mixes does increase the return on *equity capital*, as shown in previous examples; but it can also work against the owners and investors. A decrease in asset value will more negatively affect a highly debt-leveraged company compared to one with small leveraging. Example 10.8 illustrates this fact.

EXAMPLE 10.8

During the last several years, the U.S. airline industry has had financial problems, in part due to high fuel costs, fewer customers, security problems, government regulations, aging aircraft, and union dissatisfaction. As a consequence, the D-E mixes of the so-called traditional companies (American, United, Delta, and others) have become larger on the debt side than is historically acceptable. Meanwhile, the D-E mixes of so-called low-cost airlines (Southwest, JetBlue, and others) have suffered, but not to the same degree. In an effort to reduce costs, assume that three airlines joined forces to cooperate on a range of services (baggage handling, onboard food preparation, ticket services, and software development) by forming a new company called FullServe, Inc. This required \$5 billion (\$5 B) up-front funding from each airline.

Table 10–1 summarizes the D-E mixes and the total equity capitalization for each airline *after* its share of \$5 B was removed from available equity funds. The percentage of the \$5 B obtained as debt capital was the same proportion as the debt in the company's D-E mix. For

TABLE 10-1 Debt and Equity Statistics, Example 10.8

Airline Company	Corporate D-E Mix, %	Amount Borrowed, \$ B	Equity Capital Available, \$ B
National	30–70	1.50	5.0
Global	65–35	3.25	3.7
PanAm	91–9	4.55	6.7

example, National had 30% of its capitalization in debt capital; therefore, 30% of \$5 B was borrowed, and 70% was provided from National's equity fund.

Unfortunately, after a short time, it was clear that the three-way collaborative effort was a complete failure, and FullServe was dissolved and its assets were distributed or sold for a total of \$3.0 billion, only 20% of its original value. A total of \$1.0 billion in equity capital was returned to each airline. The commercial banks that provided the original loans then required that the airlines each pay back the entire borrowed amount now, since FullServe was dissolved and no profit from the venture could be realized. Assuming the loan and equity amounts are the same as shown in Table 10-1, determine the resulting equity capital situation for each airline after it pays off the loan from its own equity funds. Also, describe one impact on each company as a result of this expensive and risky venture that failed.

Solution

Determine the level of post-FullServe equity capital using the following relation, in \$ billions.

$$\text{Equity capital} = \text{pre-FullServe level} + \text{returned capital} - \text{loan repayment}$$

$$\text{National: } \text{Equity capital} = 5.0 + 1.0 - 1.50 = \$4.50$$

$$\text{Global: } \text{Equity capital} = 3.7 + 1.0 - 3.25 = \$1.45$$

$$\text{PanAm: } \text{Equity capital} = 6.7 + 1.0 - 4.55 = \$3.15$$

Comparing the equity capital levels (Table 10-1) with the levels above indicates that the FullServe effort *reduced equity amounts* by 10% for National, 60% for Global, and 53% for PanAm. The debt capital to fund the failed FullServe effort has affected National airlines the least, in large part due to its low D-E mix of 30%–70%. However, Global and PanAm are in much worse shape financially, and they now must maintain business with a significantly lower ownership level and much reduced ability to obtain future capital—debt or equity.

The same principles discussed above for corporations are applicable to individuals. The person who is highly leveraged has large debts in terms of credit card balances, personal loans, and house mortgages. As an example, assume two engineers each have a take-home amount of \$40,000 after all income tax, social security, and insurance premiums are deducted from their annual salaries. Further, assume that the cost of the debt (money borrowed via credit cards and loans) averages 15% per year and that the total debt is being repaid in equal amounts over 20 years. If Jamal has a total debt of \$25,000 and Barry owes \$100,000, the remaining amount of the annual take-home pay may be calculated as follows:

Person	Total Debt, \$	Amount Paid, \$ per Year		
		Cost of Debt at 15%, \$	Repayment of Debt, \$	Amount Remaining from \$40,000, \$
Jamal	25,000	3,750	1,250	35,000
Barry	100,000	15,000	5,000	20,000

Jamal has 87.5% of his base available while Barry has only 50% available.



10.6 Multiple Attribute Analysis: Identification and Importance of Each Attribute ●●●

In Chapter 1 the role and scope of engineering economy in decision making were outlined. The decision-making process explained in that chapter (Figure 1–1) included the seven steps listed on the right side of Figure 10–5. Step 4 is to identify the one or multiple attributes (criteria) upon which the selection will be based. In all prior evaluations, only one attribute—the economic one—has been identified and used to select the best alternative. The criterion has been the maximization of the equivalent value of PW, AW, FW, ROR, B/C ratio, or the CER for service projects. As we are all aware, most evaluations do and should take into account multiple attributes in decision making. These are the factors labeled as noneconomic in step 5 of Figure 1–1. However, these noneconomic dimensions tend to be intangible and often difficult, if not impossible, to quantify with economic and other scales. Nonetheless, among the many attributes that can be identified, there are key ones that must be considered in earnest before the alternative selection process is complete. This section and the next describe some of the techniques that accommodate multiple attributes in an engineering study.

Multiple attributes enter into the decision-making process in many studies. Public and service sector projects are excellent examples of multiple-attribute problem solving. For example, the proposal to construct a dam to form a lake in a low-lying area or to widen the catch basin of a river usually has several purposes, such as flood control; drinking water; industrial use; commercial development; recreation; nature conservation for fish, plants, and birds; and possibly other less obvious purposes. High levels of complexity are introduced into the selection process by the multiple attributes thought to be important in selecting an alternative for the dam's location, design, environmental impact, etc.

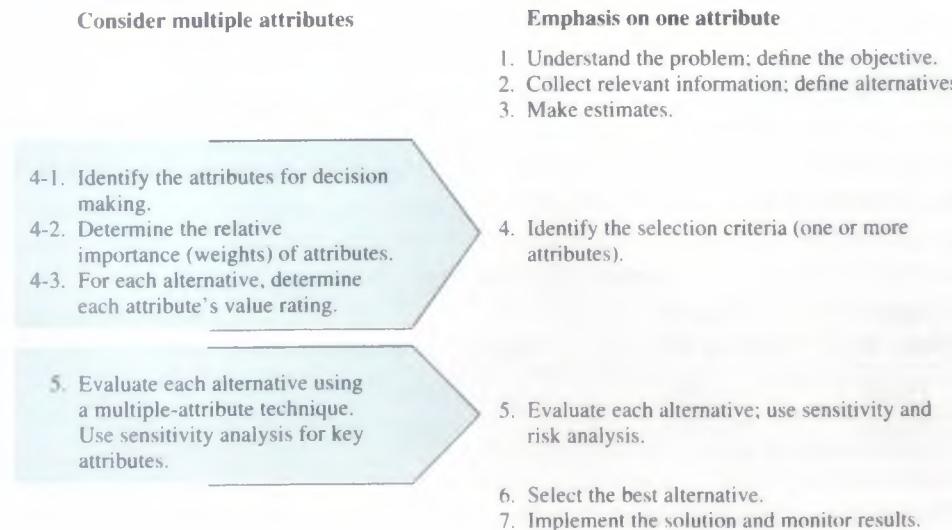
The left side of Figure 10–5 expands steps 4 and 5 to consider multiple attributes. The discussion below concentrates on the expanded step 4 and the next section focuses on the evaluation measure and alternative selection of step 5.

4-1 Attribute Identification Attributes to be considered in the evaluation methodology can be identified and defined by several methods, some much better than others depending upon the situation surrounding the study itself. To seek input from individuals other than the analyst is important; it helps focus the study on key attributes. The following is an incomplete listing of ways in which key attributes are identified.

- Comparison with similar studies that include multiple attributes
- Input from experts with relevant past experience
- Surveys of constituencies (customers, employees, managers) impacted by the alternatives
- Small group discussions using approaches such as focus groups, brainstorming, or nominal group technique
- Delphi method, which is a progressive procedure to develop reasoned consensus from different perspectives and opinions

Figure 10–5

Expansion of the decision-making process to include multiple attributes.



As an illustration, assume that Delta Airlines has decided to purchase five new Boeing 787s for overseas flights, primarily between the North American west coast and Asian cities, principally Hong Kong, Tokyo, and Singapore. There are approximately 8000 options for each plane that must be decided upon by Delta's engineering, purchasing, maintenance, and marketing personnel before the order to Boeing is placed. Options range *in scope* from the material and color of the plane's interior to the type of latching devices used on the engine cowlings, and *in function* from maximum engine thrust to pilot instrument design. An economic study based on the equivalent AW of the estimated passenger income per trip has determined that 150 of these options are clearly advantageous. But other noneconomic attributes are to be considered before some of the more expensive options are specified. A Delphi study was performed using input from 25 individuals. Concurrently, option choices for another, unidentified airline's recent order were shared with Delta personnel. From these two studies it was determined that there are 10 key attributes for options selection. Four of the most important attributes are

- *Repair time*: mean time to repair or replace (MTTR) if the option is or affects a flight-critical component.
- *Safety*: mean time to failure (MTTF) of flight-critical components.
- *Economic*: estimated extra revenue for the option. (Basically, this is the attribute evaluated by the economic study already performed.)
- *Crewmember needs*: some measure of the necessity and/or benefits of the option as judged by representative crewmembers—pilots and attendants.

The economic attribute of extra revenue may be considered an indirect measure of customer satisfaction, one that is more quantitative than customer opinion/satisfaction survey results. Of course, there are many other attributes that can be, and are, used. However, the point is that the economic study may directly address only one or a few of the key attributes vital to alternative decision making.

An attribute routinely identified by individuals and groups is **risk**.

Risk is a possible **variation** in a parameter from an expected, desired, or predicted value that may be detrimental to observing the intended outcome(s) of the product, process, or system. It represents the absence of or deviation from certainty. Risk is present when there are two or more observable values of a parameter and it is possible to assume or estimate the **chance** that each value may occur.



Risk

Actually, risk is not a stand-alone attribute, because it is a part of every attribute in one form or another. Considerations of variation, probabilistic estimates, etc., in the decision-making process are treated in Chapters 18 and 19. Formalized sensitivity analysis, expected values, simulation, and decision trees are some of the techniques useful in handling risk.

4-2 Importance (Weights) for the Attributes Determination of the **extent of importance** for each attribute i results in a weight W_i that is incorporated into the final evaluation measure. The weight, a number between 0 and 1, is based upon the experienced opinion of one individual or a group of persons familiar with the attributes, and possibly the alternatives. If a group is utilized to determine the weights, there must be consensus among the members for each weight. Otherwise, some averaging technique must be applied to arrive at one weight value for each attribute.

Table 10–2 is a tabular layout of attributes and alternatives used to perform a multiple attribute evaluation. Weights W_i for each attribute are entered on the left side. The remainder of the table is discussed as we proceed through steps 4 and 5 of the expanded decision-making process.

Attribute weights are usually normalized such that their sum over all the alternatives is 1.0. This normalizing implies that each attribute's importance score is divided by the sum S over all attributes. Expressed in formula form, these two properties of weights for attribute i ($i = 1, 2, \dots, m$) are

$$\text{Normalized weights: } \sum_{i=1}^m W_i = 1.0 \quad [10.9]$$

$$\text{Weight calculation: } W_i = \frac{\text{importance score}_i}{\sum_{i=1}^m \text{importance score}_i} = \frac{\text{importance score}_i}{S} \quad [10.10]$$

Attributes	Weights	Alternatives				
		1	2	3	...	n
1	w_1					
2	w_2					
3	w_3					
\vdots	\vdots					
m	w_m					
					Value ratings V_i	

Of the many procedures developed to assign weights to an attribute, an analyst is likely to rely upon one that is relatively simple, such as equal weighting, rank order, or weighted rank order. Pairwise comparison is another technique. Each is briefly presented below.

Equal Weighting All attributes are considered to be of approximately the **same importance**, or there is no rationale to distinguish the more important from the less important attribute. This is the default approach. Each weight in Table 10–2 will be $1/m$, according to Equation [10.10]. Alternatively, the normalizing can be omitted, in which case each weight is 1 and their sum is m . In this case, the final evaluation measure for an alternative will be the sum over all attributes.

Rank Order The m attributes are ordered (ranked) by increasing importance with a score of 1 assigned to the least important and m assigned to the most important. By Equation [10.10], the weights follow the pattern $1/S$, $2/S$, \dots , m/S . With this method, the difference in weights between attributes of **increasing importance is constant**.

Weighted Rank Order The m attributes are again placed in the order of increasing importance. However, now **differentiation between attributes** is possible. The most important attribute is assigned a score, usually 100, and all other attributes are scored relative to it between 100 and 0. Now, define the score for each attribute as s_i , and Equation [10.10] takes the form

$$w_i = \frac{s_i}{\sum_{i=1}^m s_i} \quad [10.11]$$

This is a very practical method to determine weights because one or more attributes can be heavily weighted if they are significantly more important than the remaining ones, and Equation [10.11] automatically normalizes the weights. For example, suppose the four key attributes in the previous aircraft purchase example are ordered: safety, repair time, crew-member needs, and economic. If repair time is only one-half as important as safety, and the last two attributes are each one-half as important as repair time, the scores and weights are as follows.

Attribute	Score	Weights
Safety	100	$100/200 = 0.50$
Repair time	50	$50/200 = 0.25$
Crewmember needs	25	$25/200 = 0.125$
Economic	25	$25/200 = 0.125$
Sum of scores and weights	200	1.000

Pairwise Comparison Each attribute is compared to each other attribute in a pairwise fashion using a rating scale that indicates the importance of one attribute over the other. Assume the

TABLE 10-3 Pairwise Comparison of Three Attributes to Determine Weights

Attribute <i>i</i>	1 = Cost	2 = Constructability	3 = Environment
Cost	—	0	1
Constructability	2	—	1
Environment	1	1	—
Sum of scores	3	1	2
Weight W_i	0.500	0.167	0.333

three criteria (attributes) upon which a public works project decision is based are *cost*, *constructability*, and *environmental impact*. Define the importance comparison scale as follows:

- 0 if attribute is *less important* than one compared to
- 1 if attribute is *equally important* as one compared to
- 2 if attribute is *more important* than one compared to

Set up a table listing attributes across the top and down the side, and perform the pairwise comparison for each column attribute with each row attribute. Table 10–3 presents a comparison with importance scores included. The arrow to the right of the table indicates the direction of comparison, i.e., column with row attribute. For example, cost is judged more important than constructability, thus a score of 2. The complement score of 0 is placed in the reverse comparison of constructability with cost. The weights are determined by normalizing the scores using Equation [10.11], where the sum for each column is s_i . For the first attribute, cost $i = 1$.

$$\begin{aligned}s_1 &= 3 \\ \sum s_i &= 3 + 1 + 2 = 6 \\ \text{Cost weight } W_1 &= 3/6 = 0.500\end{aligned}$$

Similarly, the other weights are $W_2 = 1/6 = 0.167$ and $W_3 = 2/6 = 0.333$.

There are other attribute weighting techniques, especially for group processes, such as utility functions, and the Dunn-Rankin procedure. These become increasingly sophisticated, but they are able to provide an advantage that these simple methods do not afford the analyst: *consistency of ranks and scores* between attributes and between individuals. If this consistency is important in that several decision makers with diverse opinions about attribute importance are involved in a study, a more sophisticated technique may be warranted. There is substantial literature on this topic.

4-3 Value Rating of Each Alternative by Attribute This is the final step prior to calculating the evaluation measure. Each alternative j is awarded a value rating V_{ij} for each attribute i . These are the entries within the cells in Table 10–2. The ratings are appraisals by decision makers of how well an alternative will perform as each attribute is considered.

The scale for the value rating can vary depending upon what is easiest to understand for those who do the valuation. A scale of 0 to 100 can be used for attribute importance scoring. However, the most popular is a scale of 4 or 5 gradations about the perceived ability of an alternative to accomplish the intent of the attribute. This is called a *Likert scale*, which can have descriptions for the gradations (e.g., very poor, poor, good, very good), or numbers assigned between 0 and 10, or -1 to $+1$, or -2 to $+2$. The last two scales can give a negative impact to the evaluation measure for poor alternatives. An example numerical scale of 0 to 10 is as follows:

If You Value the Alternative as	Give It a Rating between the Numbers
Very poor	0–2
Poor	3–5
Good	6–8
Very good	9–10

It is preferable to have a Likert scale with four choices (an even number) so that the central tendency of “fair” is not overrated.

TABLE 10-4 Completed Layout for Four Attributes and Three Alternatives for Multiple Attribute Evaluation

Attributes	Weights	Alternatives		
		1	2	3
Safety	0.50	6	4	8
Repair	0.25	9	3	1
Crew needs	0.125	5	6	6
Economic	0.125	5	9	7

If we now build upon the aircraft purchase illustration to include value ratings, the cells are filled with ratings awarded by a decision maker. Table 10-4 includes example ratings V_{ij} and the weights W_i determined above. Initially, there will be one such table for each decision maker. Prior to calculating the final evaluation measure R_j , the ratings can be combined in some fashion; or a different R_j can be calculated using each decision maker's ratings. Determination of this evaluation measure is discussed below.



10.7 Evaluation Measure for Multiple Attributes ● ● ●

The need for an evaluation measure that accommodates multiple attributes is indicated in step 5 of Figure 10-5. The measure can be one that attempts to retain all of the ratings, values, and complexity of previous assessments by multiple decision makers, or the measure can reduce these inputs to a single-dimension measure. This section introduces a single-dimension measure that is widely accepted.

A **single-dimension measure** effectively combines the different aspects addressed by the attribute importance weights W_i and the alternative value ratings V_{ij} . The resulting evaluation measure is a formula that calculates an aggregated measure for use in selecting from two or more alternatives. The approach applied in this process is called the *rank-and-rate method*.

This reduction process removes much of the complexity of trying to balance the different attributes; however, it also eliminates much of the robust information captured by the process of ranking attributes for their importance and rating each alternative's performance against each attribute.

There are additive, multiplicative, and exponential measures, but by far the most commonly applied is the additive model. The most used additive model is the **weighted attribute method**, also called the **additive weight technique**. The evaluation measure, symbolized by R_j for each alternative j , is defined as

$$R_j = \text{sum of (weight} \times \text{value rating}) \\ = \sum_{i=1}^m W_i \times V_{ij} \quad [10.12]$$

The W_i numbers are the attribute importance weights, and V_{ij} is the value rating by attribute i for each alternative j . If the attributes are of equal weight (also called *unweighted*), all $W_i = 1/m$, as determined by Equation [10.10]. This means that W_i can be moved outside of the summation in the formula for R_j . (If an equal weight of $W_i = 1.0$ is used for all attributes, in lieu of $1/m$, then the R_j value is simply the sum of all ratings for the alternative.)

The selection guideline is as follows:



ME alternative selection

Choose the alternative with the **largest R_j value**. This measure assumes that increasing weights W_i mean more important attributes and increasing ratings V_{ij} mean better performance of an alternative.

Sensitivity analysis for any score, weight, or value rating is used to determine sensitivity of the decision to it. The Chapter 18 case study includes an example of multiple attribute sensitivity analysis.

EXAMPLE 10.9

The Island of Niue in the South Pacific Ocean (www.niueisland.com) released a request for proposal (RFP)¹ for a new or reconditioned workboat as it upgrades the infrastructure of its port services. The spreadsheet in Figure 10–6, left two columns, presents the attributes and normalized weights W_i published in the RFP for use in selecting one of the tenders presenting proposals. Four acceptable proposals were received. The next four columns (C through F) include value ratings between 0 and 100 developed by a group of decision makers when the details of each proposal were evaluated against each attribute. For example, proposal 2 received a perfect score of 100 on delivery time, but lifetime costs were considered too high (rating of 20) and the price was considered relatively high (rating of 55). Use these weights and ratings to determine which proposal to pursue first.

Solution

Assume an additive weighting model is appropriate and apply the weighted attribute method. Equation [10.12] determines the R_i measure for the four alternatives. As an illustration, for proposal 3,

$$\begin{aligned} R_3 &= 0.30(95) + 0.20(60) + 0.05(90) + 0.35(85) + 0.10(100) \\ &= 28.5 + 12.0 + 4.5 + 29.8 + 10.0 \\ &= 84.8 \end{aligned}$$

The four totals in Figure 10–6 (columns G through J, row 8) indicate that proposal 3 is the overall best choice for the attributes and weights published in the RFP.

Comment

Any economic measure can be incorporated into a multiple attribute evaluation using this method. All measures of worth—PW, AW, ROR, B/C, and C/E—can be included; however, their impact on the final selection will vary relative to the importance placed on the noneconomic attributes.

Attribute i	Normalized weight, W_i	Value rating V_{ij} (0 to 100 basis)				Evaluation measure R_i			
		Proposal 1	Proposal 2	Proposal 3	Proposal 4	Proposal 1	Proposal 2	Proposal 3	Proposal 4
Construction*	0.30	95	60	95	95	28.5	18.0	28.5	28.5
Delivery time	0.20	60	100	60	60	12.0	20.0	12.0	12.0
Contract form†	0.05	100	80	90	100	5.0	4.0	4.5	5.0
Price	0.35	20	55	85	65	7.0	18.3	29.8	22.8
Lifetime costs	0.10	100	20	100	100	10.0	2.0	10.0	10.0
Total	1.00					82.5	63.3	84.8	78.3

10 * Includes skills and experience of designer and builder, previous projects, and references.
11 † Includes warranties and payment schedule based on stated milestones.

Figure 10–6

Attributes, weights, ratings, and evaluation measure for Niue workboat proposals, Example 10.9.

¹Used with permission of Government of Niue, Infrastructure Department, “Request for Proposal: Supply of Workboat,” released March 30, 2010, www.gov.nu/Documents/workboattender3022.pdf.

CHAPTER SUMMARY

The interest rate at which the MARR is established depends principally upon the cost of capital and the mix between debt and equity financing. The MARR is strongly influenced by the *weighted average* cost of capital (WACC). Risk, profit, and other factors can be considered after the AW, PW, or ROR analysis is completed and prior to final alternative selection. A high debt-to-equity mix can significantly increase the riskiness of a project and make further debt financing difficult to acquire for the corporation.

If multiple attributes, which include more than the economic dimension of a study, are to be considered in making the alternative decision, first the attributes must be identified and their relative importance assessed. Then each alternative can be value-rated for each attribute. The evaluation measure is determined using a model such as the weighted attribute method, where the measure is calculated by Equation [10.12]. The largest value indicates the best alternative.

PROBLEMS

Working with MARR

- 10.1 List at least three factors that affect the MARR, and discuss how each one affects it.
- 10.2 State whether each of the following involves debt financing or equity financing.
- \$10,000 taken from one partner's savings account to pay for equipment repair
 - Issuance of preferred stock worth \$1.3 million
 - Short-term loan of \$75,000 from a local bank
 - Issuance of \$3 million worth of 20-year bonds
 - Del Engineering buyback of \$8 million of its own stock using internal funds
- 10.3 Helical Products, Inc. uses an after-tax MARR of 12% per year. If the company's effective tax rate (federal, state, and local taxes) is 40%, determine the company's before-tax MARR.
- 10.4 The owner of a small pipeline construction company is trying to figure out how much he should bid in his attempt to win his first "big" contract. He estimates that his cost to complete the project will be \$7.2 million. He wants to bid an amount that will give him an after-tax rate of return of 15% per year if he gets the job, but he doesn't know how much he should bid on a before-tax basis. He told you that his effective state tax rate is 7% and his effective federal tax rate is 22%.
- The expression for determining the overall effective tax rate is

$$\text{state rate} + (1 - \text{state rate})(\text{federal rate})$$

What should his before-tax MARR be in order for him to make an after-tax MARR of 15% per year?
 - How much should he bid on the job?
- 10.5 A group of private equity investors provided \$16 million to a start-up company involved in making high-technology detection systems for drugs and other types of contraband. Immediately after the investment was made, another investment opportunity came up for which the investors didn't have enough capital. That project would have yielded an estimated rate of return of 29% per year before taxes. If the group's effective tax rate is 32%, what after-tax rate of return would the forgone project have yielded?
- 10.6 Five projects were ranked in decreasing order by two measures—rate of return (ROR) and present worth (PW)—to determine which ones should be

funded, with the total initial investment not to exceed \$18 million. Use the results below to determine the opportunity cost for each measure.

Project	ROR, %	PW at 15%, \$1000	Initial Investment, \$1000	Ranking by ROR		Ranking by PW	
				Project	Cumulative Investment, \$1000	Project	Cumulative Investment, \$1000
A	44.5	7,138	8,000	A	8,000	A	8,000
B	12.8	-1,162	15,000	E	13,000	C	16,000
C	20.4	1,051	8,000	C	21,000	E	21,000
D	9.6	-863	8,000	B	36,000	D	29,000
E	26.0	936	5,000	D	44,000	B	44,000

- 10.7 Tom, the owner of Burger Palace, determined that his weighted average cost of capital is 8%. He expects a return of 4% per year on all of his investments. A proposal presented by the owner of the Dairy Choice next door seems quite risky to Tom, but is an intriguing partnership opportunity. Tom has determined that the proposal's "risk factor" will require an additional 3% per year return for him to accept it.
- Use the recommended approach to determine the MARR that Tom should use, and explain how the 3% risk factor is compensated for in this MARR.
 - Determine the effective MARR for his business if Tom turns down the proposal.

D-E Mix and WACC

- 10.8 Electrical generators produce not only electricity, but also heat from conductor resistance and from friction losses in bearings. A company that manufactures generator coolers for nuclear and gas turbine power plants undertook a plant expansion through financing that had a debt-equity mix of 45–55. If \$18 million came from mortgages and bond sales, what was the total amount of the financing?
- 10.9 Determine the debt-to-equity mix when Applied Technology bought out Southwest Semiconductor using financing as follows: \$12 million from mortgages, \$5 million from retained earnings, \$10 million from cash on hand, and \$20 million from bonds.
- 10.10 Business and engineering seniors are comparing methods of financing their college education during their senior year. The business student has \$30,000 in student loans that come due at graduation. Interest is an effective 4% per year. The engineering senior owes \$50,000, 50% from his parents with no interest due and 50% from a

credit union loan. This latter amount is also due at graduation with an effective rate of 7% per year.

- (a) What is the D-E mix for each student?
- (b) If their grandparents pay the loans in full at graduation, what are the amounts on the checks they write for each graduate?
- (c) When grandparents pay the full amount at graduation, what percent of the principal does the interest represent?

10.11 Two public corporations, First Engineering and Midwest Development, each show capitalization of \$175 million in their annual reports. The balance sheet for First Engineering indicates a total debt of \$87 million, and that of Midwest Development indicates a net worth of \$62 million. Determine the D-E mix for each company.

10.12 Forest Products, Inc. invested \$50 million. The company's overall D-E mix is 60–40. What is the return on the company's equity, if the net income is \$5 million on a revenue base of \$6 million?

10.13 Determine the weighted average cost of capital for a company that manufactures miniature triaxial accelerometers for space-restricted applications. The financing profile, with interest rates, is as follows: \$3 million in stock sales at 15% per year, \$4 million in bonds at 9%, and \$6 million in retained earnings at 7% per year.

10.14 Growth Transgenics Enterprises (GTE) is contemplating the purchase of its rival. One of GTE's genetics engineers got interested in the financing strategy of the buyout. He learned there are two plans being considered. Plan 1 requires 50% equity funds from GTE's retained earnings that currently earn 9% per year, with the balance borrowed externally at 6%, based on the company's excellent stock rating. Plan 2 requires only 20% equity funds with the balance borrowed at a higher rate of 8% per year.

- (a) Which plan has the lower average cost of capital?
- (b) If GTE's owners decide that the current corporate WACC of 8.2% will not be exceeded, what is the maximum cost of debt capital allowed for each plan? Are these rates higher or lower than the current estimates?

10.15 Midac Corporation wants to arrange for \$50 million in capital to finance the manufacturing of a new consumer product. The current plan is 60% equity capital and 40% debt financing. Calculate the WACC for the following scenario:

Equity capital: 60%, or \$35 million, via common stock sales for 40% of this amount that will pay

dividends at a rate of 5% per year, and the remaining 60% from retained earnings, which currently earn 9% per year.

Debt capital: 40%, or \$15 million, obtained through two sources—bank loans for \$10 million borrowed at 8% per year, and the remainder in convertible bonds at an estimated 10% per year bond dividend rate.

10.16 A public corporation in which you own common stock reported a WACC of 11.1% for the year in its report to stockholders. The common stock that you own has averaged a total return of 7% per year over the last 3 years. The annual report also mentions that projects within the corporation are 75% funded by its own capital. Estimate the company's cost of debt capital.

10.17 BASF will invest \$14 million this year to upgrade its ethylene glycol processes. This chemical is used to produce polyester resins to manufacture products varying from construction materials to aircraft, and from luggage to home appliances. Equity capital costs 14.5% per year and will supply 65% of the capital funds. Debt capital costs 10% per year before taxes. The effective tax rate for BASF is 36%.

- (a) Determine the amount of annual revenue after taxes that is consumed in covering the interest on the project's initial cost.
- (b) If the corporation does not want to use 65% of its own funds, the financing plan may include 75% debt capital. Determine the amount of annual revenue needed to cover the interest with this plan, and explain the effect it may have on the corporation's ability to borrow in the future.

10.18 A couple planning for their child's college education can fund part of or all the expected \$100,000 tuition cost from their own funds (through an education IRA) or borrow all or part of it. The average return for their own funds is 7% per year, but the loan is expected to have a higher interest rate as the loan amount increases. Use a spreadsheet to generate a plot of the WACC curve with the estimated loan interest rates below and determine the best D-E mix for the couple.

Loan Amount, \$	Interest Rate, % per year
10,000	5.0
30,000	6.0
50,000	8.0
60,000	9.0
75,000	11.0
100,000	13.0

10.19 Over the last few years, Carol's Fashion Store, a statewide franchise, has experienced the D-E mixes and costs of debt and equity capital on several projects summarized below.

- (a) Plot debt, equity, and weighted average cost of capital.
- (b) Determine what mix of debt and equity capital provided the lowest WACC.

Project	Debt capital		Equity capital	
	Percent	Rate	Percent	Rate
A	100%	14.5%		
B	70	13.0	30%	7.8%
C	65	12.0	35	7.8
D	50	11.5	50	7.9
E	35	9.9	65	9.8
F	20	12.4	80	12.5
G			100	12.5

10.20 For Problem 10.19, use a spreadsheet to (a) determine the best D-E mix and (b) determine the best D-E mix if the cost of debt capital increases by 10% per year, for example, 13.0% increases to 14.3%.

Cost of Debt Capital

10.21 The cash flow plan associated with a debt financing transaction allowed a company to receive \$2,800,000 now in lieu of future interest payments of \$196,000 per year for 10 years plus a lump sum of \$2,800,000 in year 10. If the company's effective tax rate is 33%, determine the company's cost of debt capital (a) before taxes and (b) after taxes.

10.22 A company that makes several different types of skateboards, Jennings Outdoors, incurred interest expenses of \$1,200,000 per year from various types of debt financing. The company borrowed \$19,000,000 in year 0 and repaid the principal of the loans in year 15 in a lump-sum payment of \$20,000,000. If the company's effective tax rate is 29%, what was the company's cost of debt capital (a) before taxes and (b) after taxes?

10.23 Molex Inc., a manufacturer of cable assemblies for polycrystalline photovoltaic solar modules, requires \$3.1 million in debt capital. The company plans to sell 15-year bonds that carry a dividend of 6% per year, payable semiannually. Molex has an effective tax rate of 32% per year. Determine (a) the nominal annual after-tax cost of debt capital and (b) the effective annual after-tax cost of debt capital.

10.24 Tri-States Gas Producers expects to borrow \$800,000 for field engineering improvements. Two

methods of debt financing are possible—borrow it all from a bank or issue debenture bonds. The company will pay an effective 8% per year to the bank for 8 years. The principal on the loan will be reduced uniformly over the 8 years, with the remainder of each annual payment going toward interest. The bond issue will be for 800 10-year bonds of \$1000 each that require a 6% per year dividend payment.

- (a) Which method of financing is cheaper after an effective tax rate of 40% is considered?
- (b) Which is the cheaper method using a before-tax analysis?

10.25 The Sullivan Family Partnership plans to purchase a refurbished condo in their hometown for investment purposes. The negotiated \$200,000 purchase price will be financed with 20% of savings (retained earnings) which consistently makes 6.5% per year after all relevant income taxes are paid. Eighty percent will be borrowed at a before-tax rate of 9% per year for 15 years with the principal repaid in equal annual installments. If the effective tax rate is 22% per year, based only on these data, answer the following.

- (a) What is the partnership's annual loan payment for each of the 15 years?
- (b) What is the net present worth difference between the \$200,000 now and the PW of the cost of the 80–20 D-E mix series of cash flows necessary to finance the purchase? What does this PW value mean?
- (c) What is the after-tax WACC for this purchase?

Cost of Equity Capital

10.26 Determine the cost of equity capital to Hy-Lok USA if the company sells 500,000 shares of its preferred stock at a 5% discount from its price of \$130. The stock carries a \$10 per year dividend.

10.27 The initial public offering price for the common stock of SW Refining is \$23 per share, and it will pay a first-year dividend of \$0.92 per share. If the appreciation rate in dividends is anticipated to be 3.2% per year, determine the cost of equity capital for the stock offering.

10.28 The cost of debt capital is lower after taxes than before taxes. The cost of equity capital is more difficult to estimate using the dividend method or the CAPM model, for example, yet the after-tax and before-tax cost of equity capital is the same. Why are the after-tax rates not the same for both types of financing?

10.29 H2W Technologies is considering raising capital to expand its offerings of 2-phase and 4-phase

linear stepper motors. The beta value for its stock is 1.41. Use the capital asset pricing model and a 3.8% premium above the risk-free return to determine the cost of equity capital if the risk-free return is 3.2%.

- 10.30 Management at Hirschman Engineering has asked you to determine the cost of equity capital based on the company's common stock. The company wants you to use two methods: the dividend method and the CAPM. Last year, the first year for dividends, the stock paid \$0.75 per share on the average of \$11.50 on the New York Stock Exchange. Management hopes to grow the dividend rate at 3% per year. Hirschman Engineering stock has a volatility that is higher than the norm at 1.3. If safe investments are returning 5.5% and the 3% growth on common stocks is also the premium above the risk-free investments that Hirschman Engineering plans to pay, calculate the cost of equity capital using the two methods.
- 10.31 Common stocks issued by Meggitt Sensing Systems paid stockholders \$0.93 per share on an average price of \$18.80 last year. The company expects to grow the dividend rate at a maximum of 1.5% per year. The stock volatility is 1.19, and other stocks in the same industry are paying an average of 4.95% per year dividend. U.S Treasury bills are returning 4.5%. Determine the company's cost of equity capital last year using (a) the dividend method and (b) the CAPM.
- 10.32 Last year a Japanese engineering materials corporation, Yamachi Inc., purchased some U.S. Treasury bonds that return an average of 4% per year. Now, Euro bonds are being purchased with a realized average return of 3.9% per year. The volatility factor of Yamachi stock last year was 1.10 and has increased this year to 1.18. Other publicly traded stocks in this same business are paying an average of 5.1% dividends per year. Determine the cost of equity capital for each year, and explain why the increase or decrease seems to have occurred.
- 10.33 The engineering manager at FXO Plastics wants to complete an alternative evaluation study. She asked the finance manager for the corporate MARR. The finance manager gave her some data on the project and stated that all projects must clear their average (pooled) cost by at least 4%.

Funds Source	Amount, \$	Average Cost, %
Retained earnings	4,000,000	7.4
Stock sales	6,000,000	4.8
Long-term loans	5,000,000	9.8

- (a) Use the data to determine the minimum MARR.
- (b) The study is after taxes and part (a) provided the before-tax MARR. Determine the correct MARR to use if the tax rate was 32% last year and the finance manager meant that the 4% above the cost is for after-tax evaluations.

Different D-E Mixes

- 10.34 Why is it financially unhealthy for an individual to maintain a large percentage of debt financing over a long time, that is, to be highly leveraged?
- 10.35 In a leveraged buyout of one company by another, the purchasing company usually obtains borrowed money and inserts as little of its own equity as possible into the purchase. Explain some circumstances under which such a buyout may put the purchasing company at economic risk.
- 10.36 Grainger and Company has an opportunity to invest \$500,000 in a new line of direct-drive rotary screw compressors. Financing will be equally split between common stock (\$250,000) and a loan with an 8% after-tax interest rate. The estimated annual NCF after taxes is \$48,000 for the next 7 years. The effective tax rate is 50%. Grainger uses the capital asset pricing model for evaluation of its common stock. Recent analysis shows that it has a volatility rating of 0.95 and is paying a premium of 5% above a safe return on its common stock. Nationally, the safest investment is currently paying 3% per year. Is the investment financially attractive if Grainger uses as the MARR its (a) equity cost of capital and (b) WACC?
- 10.37 Fairmont Industries primarily relies on 100% equity financing to fund projects. A good opportunity is available that will require \$250,000 in capital. The Fairmont owner can supply the money from personal investments that currently earn an average of 7.5% per year. The annual net cash flow from the project is estimated at \$30,000 for the next 15 years. Alternatively, 60% of the required amount can be borrowed for 15 years at 7% per year. If the MARR is the WACC, determine which plan, if either, should be undertaken. This is a before-tax analysis.
- 10.38 Omega Engineering Inc. has an opportunity to invest \$10,000,000 in a new engineering remote control system for offshore drilling platforms. Financing will be split between common stock sales (\$5,000,000) and a loan with an 8% per year interest rate. Omega's share of the annual net cash flow is estimated to be \$1.35 million for each of the next 5 years. Omega is about to initiate CAPM as

its common stock evaluation model. Recent analysis shows that it has a volatility rating of 1.22 and is paying a premium of 5% on its common stock dividend. The U.S. Treasury bills are currently paying 4% per year. Is the venture financially attractive if the MARR equals (a) the cost of equity capital and (b) the WACC?

- 10.39 A new annular die process is to be installed for extruding pipes, tubes, and tubular films. The phase I installed price for the dies and machinery is \$2,000,000. The manufacturer has not decided how to finance the system. The WACC over the last 5 years has averaged 10% per year.

- (a) Two financing alternatives have been defined. The first requires an investment of 40% equity funds at 9% and a loan for the balance at an interest rate of 10% per year. The second alternative requires only 25% equity funds and the balance borrowed at 10.5% per year. Which approach will result in the smaller average cost of capital?
- (b) Yesterday, the corporate finance committee decided that the WACC for all new projects must not exceed the 5-year historical average of 10% per year. With this restriction, what is the maximum loan interest rate that can be incurred for each of the financing alternatives?

- 10.40 Shadowland, a manufacturer of air-freightable pet crates, has identified two projects that, though having a relatively high risk, are expected to move the company into new revenue markets. Utilize a spreadsheet solution to (a) select any combination of the projects if the MARR is equal to the after-tax WACC and (b) determine if the same projects should be selected if the risk factors are enough to require an additional 2% per year for the investment to be made.

Project	Initial Investment, \$	After-Tax Cash Flow, \$/Year	Life, Years
Wildlife (W)	−250,000	48,000	10
Reptiles (R)	−125,000	30,000	5

Financing will be developed using a D-E mix of 60–40 with equity funds costing 7.5% per year. Debt financing will be developed from \$10,000, 5% per year, paid quarterly, 10-year bonds. The effective tax rate is 30% per year.

- 10.41 Two friends each invested \$20,000 of their own (equity) funds. Stan, being more conservative, purchased utility and manufacturing corporation stocks. Theresa, being a risk taker, leveraged the \$20,000 and purchased a \$100,000 condo for rental

property. Considering no taxes, dividends, or revenues, analyze these two purchases by doing the following for one year after the funds were invested.

- (a) Determine the year-end values of their equity funds if there was a 10% *increase* in the value of the stocks and the condo.
- (b) Determine the year-end values of their equity funds if there was a 10% *decrease* in the value of the stocks and the condo.
- (c) Use your results to explain why *leverage* can be financially risky.

Multiple, Noneconomic Attributes

- 10.42 In multiple attribute analysis, if three different alternatives are to be evaluated on the basis of eight attributes that are considered of equal importance, what is the weight of each attribute?

- 10.43 A consulting engineer asked a company manager to assign importance values (0 to 100) to five attributes that will be included in an alternative evaluation process. Determine the weight of each attribute using the importance scores.

Attribute	Importance Score
1. Safety	60
2. Cost	40
3. Impact	80
4. Environmental	30
5. Acceptability	20

- 10.44 Ten attributes were rank-ordered in terms of increasing importance and were identified as A, B, C, . . . , and J. Determine the weight of (a) attribute C and (b) attribute J.

- 10.45 A team of three people submitted the following statements about the attributes to be used in a weighted attribute evaluation. Use the statements to determine the normalized weights if assigned scores are between 0 and 100.

Attribute	Comment
1. Flexibility (F)	The most important factor
2. Safety (S)	70% as important as uptime
3. Uptime (U)	One-half as important as flexibility
4. Rate of return (R)	Twice as important as safety

- 10.46 Different types and capacities of crawler hoes are being considered for use in a major excavation on a pipe-laying project. Several supervisors who served on similar projects in the past have identified some of the attributes and their view of relative importance. For the information that follows, determine the weighted rank order, using a 0-to-10 scale and the normalized weights.

Attribute	Comment
1. Truck versus hoe height	90% as important as trenching speed
2. Type of topsoil	Only 10% of most important attribute
3. Type of subsoil	30% as important as trenching speed
4. Hoe cycle time	Twice as important as type of subsoil
5. Hoe trenching speed	Most important attribute
6. Pipe laying speed	80% as important as hoe cycle time

- 10.47 John, who works at Swatch, has decided to use the weighted attribute method to compare three systems for manufacturing a watchband. The vice president and her assistant VP have evaluated each of three attributes in terms of importance to them, and John has placed an evaluation from 0 to 100 on each alternative for the three attributes. John's ratings for each alternative are as follows:

Attribute	Alternative		
	1	2	3
Economic return > MARR	50	70	100
High throughput	100	60	30
Low scrap rate	100	40	50

Use the weights below to evaluate the alternatives. Are the results the same for both persons' weights? Why?

Importance Score	VP	Assistant VP
Economic return > MARR	20	100
High throughput	80	80
Low scrap rate	100	20

- 10.48 The Athlete's Shop has evaluated two proposals for weight lifting and exercise equipment. A present worth analysis at $i = 15\%$ per year of estimated revenues and costs resulted in $PW_A = \$440,000$ and $PW_B = \$390,000$. In addition to this economic measure, three more attributes were independently assigned a relative importance score from 0 to 100 by the shop manager and the lead trainer.

Attribute	Importance Score	
	Manager	Trainer
Economics	80	80
Durability	35	80
Flexibility	30	100
Maintainability	20	50

Separately, you have used the four attributes to rate the two equipment proposals on a scale of 0 to 1.0 as shown in the following table. The economic attribute was rated using the PW values.

Attribute	Proposal A	Proposal B
Economics	1.00	0.90
Durability	0.35	1.00
Flexibility	1.00	0.90
Maintainability	0.25	1.00

Select the better proposal using each of the following methods.

- (a) Present worth
- (b) Weighted evaluations of the shop manager
- (c) Weighted evaluations of the lead trainer

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

- 10.49 The term *opportunity cost* refers to:
- (a) The first cost of an alternative that has been accepted for funding
 - (b) The total cost of an alternative that has been accepted for funding
 - (c) The rate of return or profit available on the next-best alternative that had to be forgone due to lack of capital funds
 - (d) The cost of an alternative that was not recognized as an alternative that actually represented a good opportunity
- 10.50 The cost of capital is established on the basis of:
- (a) The cost of debt financing
 - (b) The weighted average of debt and equity financing
- (c) The cost of equity financing
- (d) The cost of debt financing plus the expected inflation rate
- 10.51 All of the following are examples of debt capital except:
- (a) Retained earnings
 - (b) Long-term bonds
 - (c) Loan from a local bank
 - (d) Purchase of equipment using a credit card
- 10.52 All of the following are examples of equity capital except:
- (a) Sale of preferred stock
 - (b) Long-term bonds
 - (c) Company cash on hand
 - (d) Use of retained earnings

- 10.53** If a public utility expands its capacity to generate electricity by obtaining \$41 million from retained earnings and \$30 million from municipal bond sales, the utilities' debt-to-equity mix is closest to:
- 58% debt and 42% equity
 - 73% debt and 27% equity
 - 27% debt and 73% equity
 - 42% debt and 58% equity
- 10.54** Gentech, Inc. financed a new product as follows: \$5 million in stock sales at 13.7% per year, \$2 million in retained earnings at 8.9% per year, and \$3 million through convertible bonds at 7.8% per year. The company's WACC is closest to:
- 9% per year
 - 10% per year
 - 11% per year
 - 12% per year
- 10.55** If the after-tax rate of return for a cash flow series is 11.2% and the corporate effective tax rate is 39%, the approximated before-tax rate of return is closest to:
- 6.8%
 - 5.4%
 - 18.4%
 - 28.7%
- 10.56** Medzyme Pharmaceuticals has maintained a 50-50 D-E mix for capital investments. Equity capital has cost 11%; however, debt capital that historically cost 9% has now increased by 20% per year.

If Medzyme does not want to exceed its historical weighted average cost of capital (WACC), and it is forced to go to a D-E mix of 75–25, the maximum acceptable cost of equity capital is closest to:

- 7.6%
- 9.2%
- 9.8%
- 10.9%

- 10.57** The importance values (0 to 100) for five attributes are shown below. The weight to assign to attribute 1 is:

- 0.16
- 0.20
- 0.22
- 0.25

Attribute	Importance Score
1	55
2	45
3	85
4	30
5	60

- 10.58** For eight attributes rank-ordered in terms of increasing importance, the weighting of the sixth attribute is closest to:

- 0.17
- 0.14
- 0.08
- 0.03

CASE STUDY

WHICH IS BETTER—DEBT OR EQUITY FINANCING?

Background

Pizza Hut Corporation has decided to enter the catering business in three states within its Southeastern U.S. Division, using the name Pizza Hut At-Your-Place. To deliver the meals and serving personnel, it is about to purchase 200 vans with custom interiors for a total of \$1.5 million. Each van is expected to be used for 10 years and have a \$1000 salvage value.

A feasibility study completed last year indicated that the At-Your-Place business venture could realize an estimated annual net cash flow of \$300,000 before taxes in the three states. After-tax considerations would have to take into account the effective tax rate of 35% paid by Pizza Hut.

An engineer with Pizza Hut's Distribution Division has worked with the corporate finance office to determine how to best develop the \$1.5 million capital needed for the purchase of vans. There are two viable financing plans.

Information

Plan A is debt financing for 50% of the capital (\$750,000) with the 8% per year compound interest loan repaid over 10 years with uniform year-end payments. (A simplifying assumption that \$75,000 of the principal is repaid with each annual payment can be made.)

Plan B is 100% equity capital raised from the sale of \$15 per share common stock. The financial manager informed the engineer that stock is paying \$0.50 per share in dividends and that this dividend rate has been increasing at an average of 5% each year. This dividend pattern is expected to continue, based on the current financial environment.

Case Study Exercises

- What values of MARR should the engineer use to determine the better financing plan?

2. The engineer must make a recommendation on the financing plan by the end of the day. He does not know how to consider all the tax angles for the debt financing in plan A. However, he does have a handbook that gives these relations for equity and debt capital about taxes and cash flows:

Equity capital: no income tax advantages

$$\text{After-tax net cash flow} = (\text{before-tax net cash flow})(1 - \text{tax rate})$$

Debt capital: income tax advantage comes from interest paid on loans

$$\begin{aligned}\text{After-tax net cash flow} &= \text{before-tax net cash flow} \\ &\quad - \text{loan principal} \\ &\quad - \text{loan interest} - \text{taxes}\end{aligned}$$

$$\text{Taxes} = (\text{taxable income})(\text{tax rate})$$

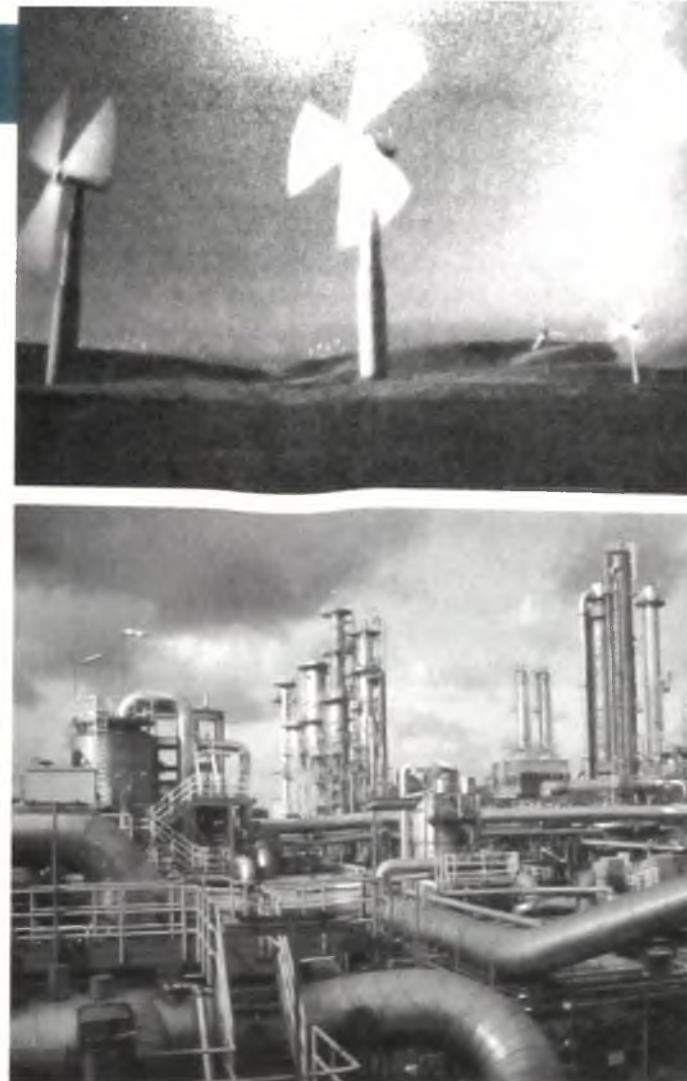
$$\begin{aligned}\text{Taxable income} &= \text{net cash flow} \\ &\quad - \text{loan interest}\end{aligned}$$

He decides to forget any other tax consequences and use this information to prepare a recommendation. Is A or B the better plan?

3. The division manager would like to know how much the WACC varies for different D-E mixes, especially about 15% to 20% on either side of the 50% debt financing option in plan A. Plot the WACC curve and compare its shape with that of Figure 10-2.



Replacement and Retention Decisions



LEARNING OUTCOMES

Purpose: Perform a replacement/retention study between an in-place asset, process, or system and one that could replace it.

SECTION	TOPIC	LEARNING OUTCOME
11.1	Replacement study basics	<ul style="list-style-type: none">Explain the fundamental approach and terminology of replacement analysis.
11.2	Economic service life	<ul style="list-style-type: none">Determine the ESL that minimizes the total AW for estimated costs and salvage value.
11.3	Replacement analysis	<ul style="list-style-type: none">Perform a replacement/retention study between a defender and the best challenger.
11.4	Additional considerations	<ul style="list-style-type: none">Understand the approach to special situations in a replacement study.
11.5	Study period analysis	<ul style="list-style-type: none">Perform a replacement/retention study over a specified number of years.
11.6	Replacement value	<ul style="list-style-type: none">Calculate the minimum market (trade-in) value required to make the challenger economically attractive.

O

ne of the most common and important issues in industrial practice is that of replacement or retention of an asset, process, or system that is currently installed.

This differs from previous situations where all the alternatives were new. The fundamental question answered by a replacement study (also called a replacement/retention study) about a currently installed system is, *Should it be replaced now or later?* When an asset is currently in use and its function is needed in the future, it will be replaced at some time. In reality, a replacement study answers the question of *when, not if, to replace*.

A replacement study is usually designed to first make the economic decision to retain or replace *now*. If the decision is to replace, the study is complete. If the decision is to retain, the cost estimates and decision can be revisited each year to ensure that the decision to retain is still economically correct. This chapter explains how to perform the initial-year and follow-on year replacement studies.

A replacement study is an application of the AW method of comparing unequal-life alternatives, first introduced in Chapter 6. In a replacement study with no specified study period, the AW values are determined by a technique called the **economic service life (ESL)** analysis. If a study period is specified, the replacement study procedure is different from that used when no study period is set.

If asset depreciation and taxes are to be considered in an *after-tax replacement analysis*, Chapters 16 and 17 should be covered before or in conjunction with this chapter. After-tax replacement analysis is included in Chapter 17.

PE

Keep or Replace the Kiln Case: B&T

Enterprises manufactures and sells high-melting-temperature ceramics and high-performance metals to other corporations. The products are sold to a wide range of industries from the nuclear and solar power industry to sports equipment manufacturers of specialty golf and tennis gear, where kiln temperatures up to approximately 1700°C are needed. For years, B&T has owned and been very satisfied with Harper International pusher-plate tunnel kilns. Two are in use currently at plant locations on each coast of the country; one kiln is 10 years old, and the second was purchased only 2 years ago and serves, primarily, the ceramics industry needs on the west coast. This newer kiln can reach temperatures of 2800°C.

During the last two or three quarterly maintenance visits, the Harper team leader and the head of B&T quality have discussed the ceramic and metal industry needs for higher temperatures. In some cases the temperatures are as high as 3000°C for emerging nitride, boride, and carbide transition metals that form very high-melting-temperature oxides. These may find use in hypersonic vehicles, engines, plasma arc electrodes, cutting tools, and high-temperature shielding.

A looming question on the mind of the senior management and financial officers of B&T revolves around the need to seriously consider a new graphite hearth kiln, which can meet higher temperature and other needs of the current and projected

customer base. This unit will have lower operating costs and significantly greater furnace efficiency in heat time, transit, and other crucial parameters. Since virtually all of this business is on the west coast, the graphite hearth kiln would replace the newer of the two kilns currently in use.

For identification, let

PT identify the currently installed

pusher-plate tunnel kiln (defender)

GH identify the proposed new graphite hearth kiln (challenger)

Relevant estimates follow in \$ millions for monetary units.

	PT	GH
First cost, \$ M	\$25; 2 years ago	\$38; with no trade-in
AOC, \$ M per year	year 1: \$5.2; year 2: \$6.4	starts at \$3.4, increases 10%/year
Life, years	6 (remaining)	12 (estimated)
Heating element, \$ M	—	\$2.0 every 6 years

This case is used in the following topics of this chapter:

Economic service life (Section 11.2)

Replacement study (Section 11.3)

Replacement study with study period (Section 11.5)

Replacement value (Section 11.6)

Problems 11.18 and 11.41

11.1 Basics of a Replacement Study ●●●

The need for a replacement study can develop from several sources:

Reduced performance. Because of physical deterioration, the ability to perform at an expected level of *reliability* (being available and performing correctly when needed) or *productivity* (performing at a given level of quality and quantity) is not present. This usually results in increased costs of operation, higher scrap and rework costs, lost sales, reduced quality, diminished safety, and larger maintenance expenses.

Altered requirements. New requirements of accuracy, speed, or other specifications cannot be met by the existing equipment or system. Often the choice is between complete replacement or enhancement through retrofitting or augmentation.

Obsolescence. International competition and rapidly changing technology make currently-used systems and assets perform acceptably but less productively than equipment coming available. The ever-decreasing development cycle time to bring new products to market is often the reason for *premature replacement* studies, that is, studies performed before the estimated useful or economic life is reached.

Replacement studies use some terminology that is closely related to terms in previous chapters.

Defender and **challenger** are the names for two mutually exclusive alternatives. The defender is the currently installed asset, and the challenger is the potential replacement. A replacement study compares these two alternatives. The challenger is the “best” challenger because it has been selected as the best one to possibly replace the defender. (This is the same terminology used earlier for incremental ROR and B/C analysis, but both alternatives were new).



Salvage/market value

Market value is the current value of the installed asset if it were sold or traded on the open market. Also called *trade-in value*, this estimate is obtained from professional appraisers, resellers, or liquidators familiar with the industry. As in previous chapters, **salvage value** is the estimated value at the end of the expected life. In replacement analysis, the salvage value at the end of one year is used as the market value at the beginning of the next year.

AW values are used as the primary economic measure of comparison between the defender and challenger. The term *equivalent uniform annual cost (EUAC)* may be used in lieu of AW, because often only costs are included in the evaluation; revenues generated by the defender or challenger are assumed to be equal. (Since EUAC calculations are exactly the same as for AW, we use the term AW.) Therefore, all values will be negative when only costs are involved. Salvage or market value is an exception; it is a cash inflow and carries a plus sign.



Economic service life

Economic service life (ESL) for an alternative is the *number of years at which the lowest AW of cost occurs*. The equivalency calculations to determine ESL establish the life *n* for the best challenger and the lowest cost life for the defender in a replacement study. The next section explains how to find the ESL.

Defender first cost is the initial investment amount *P* used for the defender. The *current market value (MV)* is the correct estimate to use for *P* for the defender in a replacement study. The estimated salvage value at the end of one year becomes the market value at the beginning of the next year, provided the estimates remain correct as the years pass. It is incorrect to use the following as MV for the defender first cost: trade-in value that *does not represent a fair market value*, or the depreciated book value taken from accounting records. If the defender must be upgraded or augmented to make it equivalent to the challenger (in speed, capacity, etc.), this cost is added to the MV to obtain the estimated defender first cost.

Challenger first cost is the amount of capital that must be recovered (amortized) when replacing a defender with a challenger. This amount is almost always equal to *P*, the first cost of the challenger.

If an unrealistically high trade-in value is offered for the defender compared to its fair market value, the *net* cash flow required for the challenger is reduced, and this fact should be considered in the analysis. The correct amount to recover and use in the economic analysis for the challenger is its first cost minus the difference between the trade-in value (TIV) and market value (MV) of the defender. In equation form, this is $P - (TIV - MV)$. This amount represents the actual cost to the company because it includes both the opportunity cost (i.e., market value of the defender) and the out-of-pocket cost (i.e., first cost – trade-in) to acquire the challenger. Of course, when the trade-in and market values are the same, the challenger P value is used in all computations.

The challenger first cost is the estimated initial investment necessary to acquire and install it. Sometimes, an analyst or manager will attempt to *increase* this first cost by an amount equal to the *unrecovered capital* remaining in the defender, as shown on the accounting records for the asset. This incorrect treatment of capital recovery is observed most often when the defender is working well and in the early stages of its life, but technological obsolescence, or some other reason, has forced consideration of a replacement. This leads us to identify two additional characteristics of replacement analysis, in fact, of any economic analysis: *sunk costs* and *nonowner's viewpoint*.

A **sunk cost** is a prior expenditure or loss of capital (money) that cannot be recovered by a decision about the future. The replacement alternative for an asset, system, or process that has incurred a nonrecoverable cost should not include this cost in any direct fashion; sunk costs should be handled in a realistic way using tax laws and write-off allowances.



Sunk cost

A sunk cost should never be added to the challenger's first cost, because it will make the *challenger appear to be more costly than it actually is*. For example, assume an asset costing \$100,000 two years ago has a depreciated value of \$80,000 on the corporate books. It must be replaced prematurely due to rapidly advancing technology. If the replacement alternative (challenger) has a first cost of \$150,000, the \$80,000 from the current asset is a sunk cost were the challenger purchased. For the purposes of an economic analysis, it is *incorrect to increase* the challenger's first cost to \$230,000 or any number between this and \$150,000.

The second characteristic is the perspective taken when conducting a replacement study. You, the analyst, are a consultant from outside the company.

The **nonowner's viewpoint**, also called the *outsider's viewpoint* or *consultant's viewpoint*, provides the greatest objectivity in a replacement study. This viewpoint performs the analysis without bias; it means the analyst owns neither the defender nor the challenger. Additionally, it assumes the services provided by the defender can be purchased now by making an "initial investment" equal to the market value of the defender.

Besides being unbiased, this perspective is correct because the defender's market value is a forgone opportunity of cash inflow were the replacement not selected, and the defender chosen.

As mentioned in the introduction, a replacement study is an application of the annual worth method. As such, the fundamental assumptions for a replacement study parallel those of an AW analysis. If the *planning horizon is unlimited*, that is, a study period is not specified, the assumptions are as follows:

1. The services provided are needed for the indefinite future.
2. The challenger is the best challenger available now and in the future to replace the defender. When this challenger replaces the defender (now or later), it will be repeated for succeeding life cycles.
3. Cost estimates for every life cycle of the defender and challenger will be the same as in their first cycle.

As expected, none of these assumptions is precisely correct. We discussed this previously for the AW method (and the PW method). When the intent of one or more of the assumptions becomes incorrect, the estimates for the alternatives must be updated and a new replacement study conducted. The replacement procedure discussed in Section 11.3 explains how to do this. When the planning horizon is limited to a specified study period, the assumptions above do not hold. The procedure of Section 11.5 discusses replacement analysis over a fixed study period.

EXAMPLE 11.1

Only 2 years ago, Techtron purchased for \$275,000 a fully loaded SCADA (supervisory control and data acquisition) system including hardware and software for a processing plant operating on the Houston ship channel. When it was purchased, a life of 5 years and salvage of 20% of first cost were estimated. Actual M&O costs have been \$25,000 per year, and the book value is \$187,000. There has been a series of insidious malware infections targeting Techtron's command and control software, plus next-generation hardware marketed only recently could greatly reduce the competitiveness of the company in several of its product lines. Given these factors, the system is likely worth nothing if kept in use for the final 3 years of its anticipated useful life.

Model K2-A1, a new replacement turnkey system, can be purchased for \$300,000 net cash, that is, \$400,000 first cost and a \$100,000 trade-in for the current system. A 5-year life, salvage value of 15% of stated first cost or \$60,000, and an M&O cost of \$15,000 per year are good estimates for the new system. The current system was appraised this morning, and a market value of \$100,000 was confirmed for today; however, with the current virus discovery, the appraiser anticipates that the market value will fall rapidly to the \$80,000 range once the virus problem and new model are publicized.

Using the above values as the best possible today, state the correct defender and challenger estimates for P , M&O, S , and n in a replacement study to be performed today.

Solution

Defender: Use the current market value of \$100,000 as the first cost for the defender. All others—original cost of \$275,000, book value of \$187,000, and trade-in value of \$100,000—are irrelevant to a replacement study conducted today. The estimates are as follows:

First cost	$P = \$-100,000$
M&O cost	$A = \$-25,000 \text{ per year}$
Expected life	$n = 3 \text{ years}$
Salvage value	$S = 0$

Challenger: The \$400,000 stated first cost is the correct one to use for P , because the trade-in and market values are equal.

First cost	$P = \$-400,000$
M&O cost	$A = \$-15,000 \text{ per year}$
Expected life	$n = 5 \text{ years}$
Salvage value	$S = \$60,000$

Comment

If the replacement study is conducted next week when estimates will have changed, the defender's first cost will be \$80,000, the new market value according to the appraiser. The challenger's first cost will be \$380,000, that is, $P - (\text{TIV} - \text{MV}) = 400,000 - (100,000 - 80,000)$.



11.2 Economic Service Life ● ● ●

Until now the estimated life n of an alternative or asset has been stated. In reality, the best life estimate to use in the economic analysis is not known initially. When a replacement study or an analysis between new alternatives is performed, the best value for n should be determined using current cost estimates. The best life estimate is called the *economic service life*.



Economic service life

The **economic service life (ESL)** is the number of years n at which the equivalent uniform annual worth (AW) of costs is the minimum, considering the most current cost estimates over all possible years that the asset may provide a needed service.

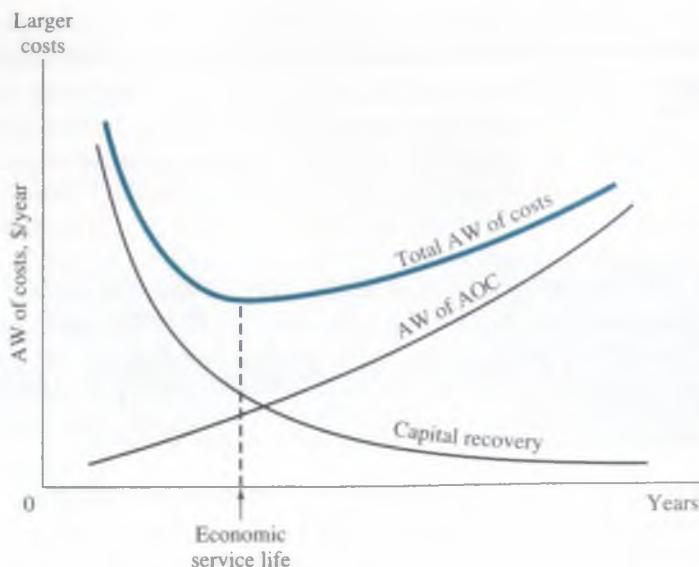


Figure 11-1
Annual worth curves of cost elements that determine the economic service life.

The ESL is also referred to as the *economic life* or *minimum cost life*. Once determined, the ESL should be the estimated life for the asset used in an engineering economy study, if only economics are considered. When n years have passed, the ESL indicates that the asset should be replaced to minimize overall costs. To perform a replacement study correctly, it is important that the ESL of the challenger and the ESL of the defender be determined, since their n values are usually not preestablished.

The ESL is determined by calculating the total AW of costs if the asset is in service 1 year, 2 years, 3 years, and so on, up to the last year the asset is considered useful. Total AW of costs is the sum of capital recovery (CR), which is the AW of the initial investment and any salvage value, and the AW of the estimated annual operating cost (AOC), that is,

$$\begin{aligned}\text{Total AW} &= \text{capital recovery} - \text{AW of annual operating costs} \\ &= CR - AW \text{ of AOC}\end{aligned}\quad [11.1]$$

The ESL is the n value for the smallest total AW of costs. (Remember: These AW values are cost estimates, so the AW values are negative numbers. Therefore, $-$200$ is a lower cost than $-$500$.) Figure 11-1 shows the characteristic shape of a total AW of cost curve. The CR component of total AW decreases, while the AOC component increases, thus forming the concave shape. The two AW components are calculated as follows.

Decreasing cost of capital recovery. The capital recovery is the AW of investment; it decreases with each year of ownership. Capital recovery is calculated by Equation [6.3], which is repeated here. The salvage value S , which usually decreases with time, is the estimated market value (MV) in that year.

$$\text{Capital recovery} = -P(A/P,i,n) + S(A/F,i,n)\quad [11.2]$$



Increasing cost of AW of AOC. Since the AOC (or M&O) estimates usually increase over the years, the AW of AOC increases. To calculate the AW of the AOC series for 1, 2, 3, ... years, determine the present worth of each AOC value with the P/F factor, then redistribute this P value over the years of ownership, using the A/P factor.

The complete equation for total AW of costs over k years ($k = 1, 2, 3, \dots$) is

$$\text{Total AW}_k = -P(A/P,i,k) + S_k(A/F,i,k) - \left[\sum_{j=1}^{j=k} AOC_j(P/F,i,j) \right] (A/P,i,k) \quad [11.3]$$

where P = initial investment or current market value
 S_k = salvage value or market value after k years
 AOC_j = annual operating cost for year j ($j = 1$ to k)

The current MV is used for P when the asset is the defender, and the estimated future MV values are substituted for the S values in years 1, 2, 3, Plotting the AW_k series as in Figure 11–1 clearly indicates where the ESL is located and the trend of the AW_k curve on each side of the ESL.

To determine ESL by spreadsheet, the PMT function (with embedded NPV functions as needed) is used repeatedly for each year to calculate capital recovery and the AW of AOC. Their sum is the total AW for k years of ownership. The PMT function formats for the capital recovery and AOC components for each year k ($k = 1, 2, 3, \dots$) are as follows:

Capital recovery for the challenger: Capital recovery for the defender: AW of AOC:	$\text{PMT}(i\%, \text{years}, P, -\text{MV_in_year_k})$ $\text{PMT}(i\%, \text{years}, \text{current_MV}, -\text{MV_in_}$ $\text{year_k})$ $- \text{PMT}(i\%, \text{years}, \text{NPV}(i\%, \text{year_1_AOC:}$ $\text{year_k_AOC}) + 0)$
---	---

[11.4]

When the spreadsheet is developed, it is recommended that the PMT functions in year 1 be developed using cell-reference format; then drag down the function through each column. A final column summing the two PMT results displays total AW. Augmenting the table with an Excel xy scatter chart graphically displays the cost curves in the general form of Figure 11–1, and the ESL is easily identified. Example 11.2 illustrates ESL determination by hand and by spreadsheet.

EXAMPLE 11.2

A 3-year-old backup power system is being considered for early replacement. Its current market value is \$20,000. Estimated future market values and annual operating costs for the next 5 years are given in Table 11–1, columns 2 and 3. What is the economic service life of this defender if the interest rate is 10% per year? Solve by hand and by spreadsheet.

Solution by Hand

Equation [11.3] is used to calculate total AW_k for $k = 1, 2, \dots, 5$. Table 11–1, column 4, shows the capital recovery for the \$20,000 current market value ($j = 0$) plus 10% return. Column 5 gives the equivalent AW of AOC for k years. As an illustration, the computation of total AW for $k = 3$ from Equation [11.3] is

$$\begin{aligned} \text{Total } AW_3 &= -P(A/P,i,3) + MV_3(A/F,i,3) - [\text{PW of AOC}_1, \text{AOC}_2, \text{ and AOC}_3](A/P,i,3) \\ &= -20,000(A/P,10\%,3) + 6000(A/F,10\%,3) - [5000(P/F,10\%,1) \\ &\quad + 6500(P/F,10\%,2) + 8000(P/F,10\%,3)](A/P,10\%,3) \\ &= -6230 - 6405 = -\$12,635 \end{aligned}$$

A similar computation is performed for each year 1 through 5. The lowest equivalent cost (numerically largest AW value) occurs at $k = 3$. Therefore, the defender ESL is $n = 3$ years, and the AW value is $-\$12,635$. In the replacement study, this AW will be compared with the best challenger AW determined by a similar ESL analysis.

Solution by Spreadsheet

See Figure 11–2 for the spreadsheet screen shot and chart that shows the ESL is $n = 3$ years and $AW = -\$12,634$. (This format is a template for any ESL analysis; simply change the

TABLE 11–1 Computation of Economic Service Life

Year j (1)	MV_j , \$ (2)	AOC_j , \$ (3)	Capital Recovery, \$ (4)	AW of AOC, \$ (5)	Total AW_k , \$ (6) = (4) + (5)
1	10,000	-5,000	-12,000	-5,000	-17,000
2	8,000	-6,500	-7,714	-5,714	-13,428
3	6,000	-8,000	-6,230	-6,405	-12,635
4	2,000	-9,500	-5,878	-7,072	-12,950
5	0	-12,500	-5,276	-7,961	-13,237

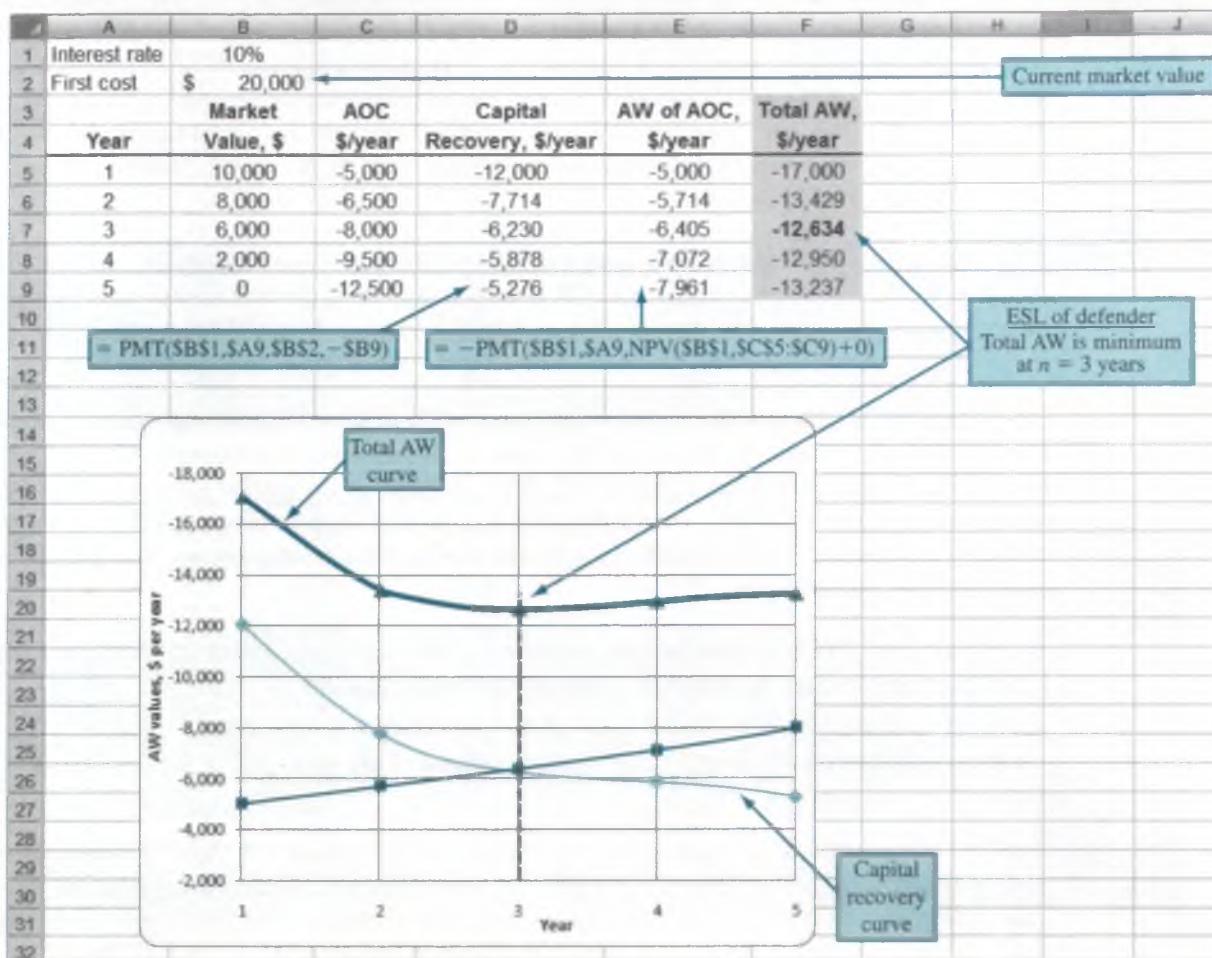


Figure 11-2

Determination of ESL and plot of curves, Example 11.2

estimates and add rows for more years.) Contents of columns D and E are described below. The PMT functions apply the formats as described in Equation [11.4]. Cell tags show detailed cell-reference format for year 5. The \$ symbols are included for absolute cell referencing, needed when the entry is dragged down through the column.

Column D: Capital recovery is the AW of the \$20,000 investment in year 0 for each year 1 through 5 with the estimated MV in that year. For example, in actual numbers, the cell-reference PMT function in year 5 shown on the spreadsheet reads = PMT(10%,5,20000, -0), resulting in \$-5276. This series is plotted in Figure 11-2.

Column E: The NPV function embedded in the PMT function obtains the present worth in year 0 of all AOC estimates through year k . Then PMT calculates the AW of AOC over k years. For example, in year 5, the PMT in numbers is = -PMT(10%,5,NPV(10%,C5:C9)+0). The 0 is the AOC in year 0; it is optional. The graph plots the AW of AOC curve, which constantly increases in cost because the AOC estimates increase each year.

Comment

The *capital recovery curve* in Figure 11-2 (middle curve) is not the expected shape (see year 4) because the estimated market value changes each year. If the same MV were estimated for each year, the curve would appear like Figure 11-1. When several total AW values are approximately equal, the curve will be flat over several periods. This indicates that the ESL is relatively insensitive to costs.

It is reasonable to ask about the difference between the ESL analysis above and the AW analyses performed in previous chapters. Previously we had a *specific life estimated to be n years* with associated other estimates: first cost in year 0, possibly a salvage value in year n , and an AOC that remained constant or varied each year. For all previous analyses, the calculation of AW using these estimates determined the AW over n years. This is the economic service life when n is fixed. Also, in all previous cases, there were no year-by-year market value estimates. Therefore, we can conclude the following:

When the expected life n is known and specified for the challenger or defender, no ESL computations are necessary. Determine the AW over n years, using the first cost or current market value, estimated salvage value after n years, and AOC estimates. This AW value is the correct one to use in the replacement study.

However, when n is not fixed, the following is useful. First the market/salvage series is needed. It is not difficult to estimate this series for a new or current asset. For example, an asset with a first cost of P can lose market value of, say, 20% per year, so the market value series for years 0, 1, 2, . . . is $P, 0.8P, 0.64P, \dots$, respectively. If it is reasonable to predict the MV series on a year-by-year basis, it can be combined with the AOC estimates to produce what is called the *marginal costs* for the asset.

Marginal costs (MC) are year-by-year estimates of the costs to own and operate an asset for that year. Three components are added to determine the marginal cost:

- Cost of ownership (loss in market value is the best estimate of this cost)
- Forgone interest on the market value at the beginning of the year
- AOC for each year

Once the marginal costs are estimated for each year, their equivalent AW value is calculated. *The sum of the AW values of the first two of these components is the capital recovery amount.* Now, it should be clear that the total AW of all three marginal cost components over k years is the same value as the total annual worth for k years calculated in Equation [11.3]. That is, the following relation is correct.

$$\text{AW of marginal costs} = \text{total AW of costs}$$

[11.5]

Therefore, there is no need to perform a separate, detailed marginal cost analysis when yearly market values are estimated. The ESL analysis presented in Example 11.2 is sufficient in that it results in the same numerical values. This is demonstrated in Example 11.3 using the progressive example.

EXAMPLE 11.3 Keep or Replace the Kiln Case

PE

In our progressive example, B&T Enterprises is considering the replacement of a 2-year-old kiln with a new one to meet emerging market needs. When the current tunnel kiln was purchased 2 years ago for \$25 million, an ESL study indicated that the minimum cost life was between 3 and 5 years of the expected 8-year life. The analysis was not very conclusive because the total AW cost curve was flat for most years between 2 and 6, indicating insensitivity of the ESL to changing costs. Now, the same type of question arises for the proposed graphite hearth model that costs \$38 million new: What are the ESL and the estimated total AW of costs? The Manager of Critical Equipment at B&T estimates that the market value after only 1 year will drop to \$25 million and then retain 75% of the previous year's value over the 12-year expected life. Use this market value series and $i = 15\%$ per year to illustrate that an ESL analysis and marginal cost analysis result in exactly the same total AW of cost series.

Solution

Figure 11–3 is a spreadsheet screen shot of the two analyses in \$ million units. The market value series is detailed in column B starting at \$25 (million) and decreasing by 25% per year. A brief description of each analysis follows.

A	B	C	D	E	F	G
1	Interest rate	15%	ESL analysis		First cost, \$	38.00
2	Market	AOC	Capital	AW of AOC,	Total AW, \$/year	
3	Year	Value, \$	\$/year	\$/year	\$/year	
4	1	25.00	-3.40	-18.70	-3.40	-22.10
5	2	18.75	-3.74	-14.65	-3.56	-18.21
6	3	14.06	-4.11	-12.59	-3.72	-16.31
7	4	10.55	-4.53	-11.20	-3.88	-15.08
8	5	7.91	-4.98	-10.16	-4.04	-14.21
9	6	5.93	-7.48	-9.36	-4.43	-13.80
10	7	4.45	-6.02	-8.73	-4.58	-13.31
11	8	3.34	-6.63	-8.23	-4.73	-12.95
12	9	2.50	-7.29	-7.81	-4.88	-12.69
13	10	1.88	-8.02	-7.48	-5.03	-12.51
14	11	1.41	-8.82	-7.20	-5.19	-12.39
15	12	1.06	-9.70	-6.97	-5.35	-12.32
16	Formulas, year	=B1*0.75	=C14*1.1	=PMT(\$B\$1,\$A15,\$A1-\$B15)	=-PMT(\$B\$1,\$A15,NPV(\$B\$1,\$C\$4:\$C\$15)+0)	=E15+D15
17						
18	Interest rate	15%	Marginal cost analysis		Current MV, \$	38.00
19	Market	Loss in Market Value, \$	Lost Interest on MV for Year, \$	AOC, \$	Marginal Cost for the Year, \$	AW of Marginal Cost, \$/year
20	Year	Value, \$				
21	1	25.00	-13.00	-5.70	-3.40	-22.10
22	2	18.75	-6.25	-3.75	-3.74	-18.21
23	3	14.06	-4.69	-2.81	-4.11	-16.31
24	4	10.55	-3.52	-2.11	-4.53	-15.08
25	5	7.91	-2.64	-1.58	-4.98	-14.21
26	6	5.93	-1.98	-1.19	-7.48	-13.80
27	7	4.45	-1.48	-0.89	-6.02	-13.31
28	8	3.34	-1.11	-0.67	-6.63	-12.95
29	9	2.50	-0.83	-0.50	-7.29	-12.69
30	10	1.88	-0.63	-0.38	-8.02	-12.51
31	11	1.41	-0.47	-0.28	-8.82	-12.39
32	12	1.06	-0.35	-0.21	-9.70	-12.32
33	Formulas, year	=B31*0.75	=B32-B31	=-\$B\$1*\$B31	=E31*1.1	=-PMT(\$B\$1,\$A32,NPV(\$B\$1,\$F\$21:\$F\$32)+0)
34						

Figure 11-3

Comparison of annual worth series resulting from ESL analysis and marginal cost analysis, Example 11.3.

ESL analysis: Equation [11.4] is applied repeatedly for $k = 1, 2, \dots, 12$ years (columns C, D, and E) in the top of Figure 11-3. Row 16 details the spreadsheet functions for year 12. The result in column F is the total AW series that is of interest now.

Marginal cost (MC): The functions in the bottom of Figure 11-3 (columns C, D, and E) develop the three components added to obtain the MC series. Row 33 details the functions for year 12. The resulting AW of marginal costs (column G) is the series to compare with the corresponding ESL series above (column F).

The two AW series are identical, thus demonstrating that Equation [11.5] is correct. Therefore, either an ESL or a marginal cost analysis will provide the same information for a replacement study. In this case, the results show that the new kiln will have a minimum AW of costs of \$-12.32 million at its full 12-year life.

We can draw two important conclusions about the n and AW values to be used in a replacement study. These conclusions are based on the extent to which detailed annual estimates are made for the market value.

- 1. Year-by-year market value estimates are made.** Use them to perform an ESL analysis, and determine the n value with the lowest total AW of costs. These are the best n and AW values for the replacement study.
- 2. Yearly market value estimates are not available.** The only estimate available is market value (salvage value) in year n . Use it to calculate the AW over n years. These are the n and AW values to use; however, they may not be the “best” values in that they may not represent the best equivalent total AW of cost value found if an ESL analysis were performed.

Upon completion of the ESL analysis (item 1 above), the replacement study procedure in Section 11.3 is applied using the values

Challenger alternative (C): AW_C for n_C years

Defender alternative (D): AW_D for n_D years



11.3 Performing a Replacement Study ● ● ●

Replacement studies are performed in one of two ways: without a study period specified or with one defined. Figure 11–4 gives an overview of the approach taken for each situation. The procedure discussed in this section applies when no study period (planning horizon) is specified. If a specific number of years is identified for the replacement study, for example, over the next 5 years, with no continuation considered after this time period in the economic analysis, the procedure in Section 11.5 is applied.

A replacement study determines when a challenger replaces the in-place defender. The complete study is finished if the challenger (C) is selected to replace the defender (D) now. However, if the defender is retained now, the study may extend over a number of years equal to the life of the defender n_D , after which a challenger replaces the defender. Use the annual worth and life values for C and D determined in the ESL analysis in the following procedure. Assume the services provided by the defender could be obtained at the AW_D amount.

The replacement study procedure is:

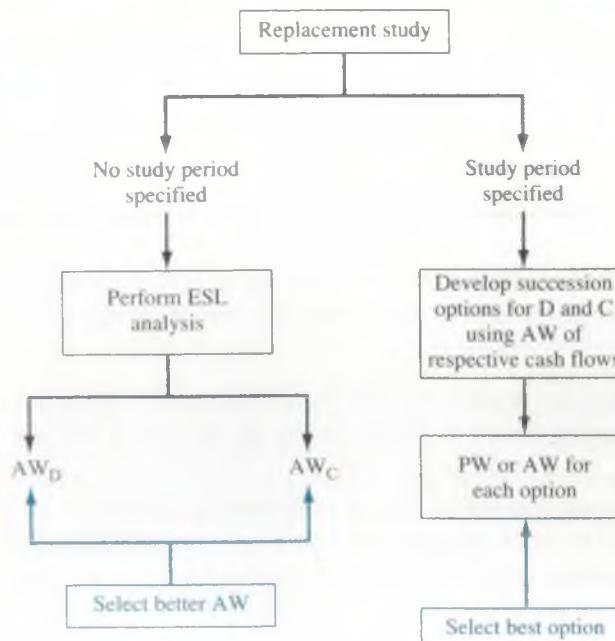
New replacement study:

1. On the basis of the better AW_C or AW_D value, select the challenger C or defender D. When the challenger is selected, replace the defender now, and expect to keep the challenger for n_C years. This replacement study is complete. If the defender is selected, plan to retain it for up to n_D more years. (This is the leftmost branch of Figure 11–4.) Next year, perform the following steps.

One-year-later analysis:

2. Determine if all estimates are still current for both alternatives, especially first cost, market value, and AOC. If not, proceed to step 3. If yes and this is year n_D , replace the defender. If this is not year n_D , retain the defender for another year and repeat this same step. This step may be repeated several times.
3. Whenever the estimates have changed, update them and determine new AW_C and AW_D values. Initiate a new replacement study (step 1).

Figure 11–4
Overview of replacement study approaches.



If the defender is selected initially (step 1), estimates may need updating after 1 year of retention (step 2). Possibly there is a new best challenger to compare with D. Either significant changes in defender estimates or availability of a new challenger indicates that a new replacement study is to be performed. In actuality, a replacement study can be performed each year or more frequently to determine the advisability of replacing or retaining any defender, provided a competitive challenger is available.

Example 11.4 illustrates the application of ESL analysis for a challenger and defender, followed by the use of the replacement study procedure. The planning horizon is unspecified in this example.

EXAMPLE 11.4

Two years ago, Toshiba Electronics made a \$15 million investment in new assembly line machinery. It purchased approximately 200 units at \$70,000 each and placed them in plants in 10 different countries. The equipment sorts, tests, and performs insertion-order kitting on electronic components in preparation for special-purpose circuit boards. This year, new international industry standards will require a \$16,000 retrofit on each unit, in addition to the expected operating cost. Due to the new standards, coupled with rapidly changing technology, a new system is challenging the retention of these 2-year-old machines. The chief engineer at Toshiba USA realizes that the economics must be considered, so he has asked that a replacement study be performed this year and each year in the future, if need be. The i is 10% and the estimates are below.

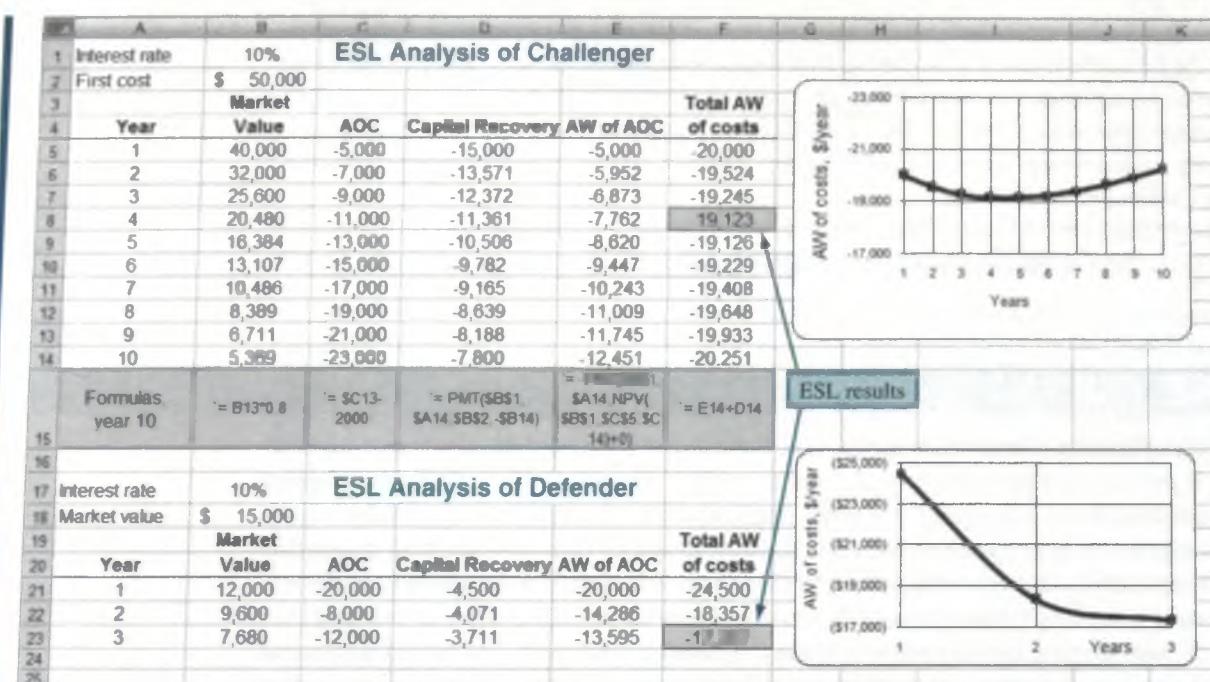
Challenger:	First cost: \$50,000 Future market values: decreasing by 20% per year Estimated retention period: no more than 10 years AOC estimates: \$5000 in year 1 with increases of \$2000 per year thereafter
Defender:	Current international market value: \$15,000 Future market values: decreasing by 20% per year Estimated retention period: no more than 3 more years AOC estimates: \$4000 next year, increasing by \$4000 per year thereafter, plus the \$16,000 retrofit next year

- (a) Determine the AW values and economic service lives necessary to perform the replacement study.
- (b) Perform the replacement study now.
- (c) After 1 year, it is time to perform the follow-up analysis. The challenger is making large inroads to the market for electronic components assembly equipment, especially with the new international standards features built in. The expected market value for the defender is still \$12,000 this year, but it is expected to drop to virtually nothing in the future—\$2000 next year on the worldwide market and zero after that. Also, this prematurely outdated equipment is more costly to keep serviced, so the estimated AOC next year has been increased from \$8000 to \$12,000 and to \$16,000 two years out. Perform the follow-up replacement study analysis.

Solution

- (a) The results of the ESL analysis, shown in Figure 11–5, include all the MV and AOC estimates in columns B and C. For the challenger, note that $P = \$50,000$ is also the MV in year 0. The total AW of costs is for each year, should the challenger be placed into service for that number of years. As an example, the year $k = 4$ amount of $-\$19,123$ is determined using Equation [11.3], where the A/G factor accommodates the arithmetic gradient series in the AOC.

$$\begin{aligned} \text{Total AW}_4 &= -50,000(A/P, 10\%, 4) + 20,480(A/F, 10\%, 4) \\ &\quad - [5000 + 2000(A/G, 10\%, 4)] \\ &= -\$19,123 \end{aligned}$$

**Figure 11-5**

ESL analysis of challenger and defender, Example 11.4.

For spreadsheet-based ESL analysis, this same result is achieved in cell F8 using Equation [11.4]. The functions are

$$\begin{aligned} \text{Total AW}_4 &= \text{PMT}(10\%, 4, 50000, -20480) - \text{PMT}(10\%, 4, \text{NPV}(10\%, C5:C8) + 0) \\ &= -11,361 - 7,762 \\ &= -19,123 \end{aligned}$$

The defender costs are analyzed in the same way up to the maximum retention period of 3 years.

The lowest AW cost (numerically largest) values for the replacement study are as follows:

$$\begin{array}{lll} \text{Challenger: } & \text{AW}_C = \$-19,123 & \text{for } n_C = 4 \text{ years} \\ \text{Defender: } & \text{AW}_D = \$-17,307 & \text{for } n_D = 3 \text{ years} \end{array}$$

The challenger total AW of cost curve (Figure 11-5) is classically shaped and relatively flat between years 3 and 6; there is virtually no difference in the total AW for years 4 and 5. For the defender, note that the estimated AOC values change substantially over 3 years, and they do not constantly increase or decrease.

- (b) To perform the replacement study now, apply only the first step of the procedure. Select the defender because it has the better AW of costs (\$-17,307), and expect to retain it for 3 more years. Prepare to perform the one-year-later analysis 1 year from now.
- (c) One year later, the situation has changed significantly for the equipment Toshiba retained last year. Apply the steps for the one-year-later analysis:
 2. After 1 year of defender retention, the challenger estimates are still reasonable, but the defender market value and AOC estimates are substantially different. Go to step 3 to perform a new ESL analysis for the defender.
 3. The defender estimates in Figure 11-5 are updated below for the ESL analysis. New AW values are calculated using Equation [11.3]. There is now a maximum of 2 more years of retention, 1 year less than the 3 years determined last year.

Year <i>k</i>	Market Value, \$	AOC, \$	Total AW If Retained <i>k</i> More Years, \$
0	12,000	—	—
1	2,000	-12,000	-23,200
2	0	-16,000	-20,819

The AW and n values for the new replacement study are as follows:

- Challenger: unchanged at $AW_C = \$-19,123$ for $n_C = 4$ years
 Defender: new $AW_D = \$-20,819$ for $n_D = 2$ more years

Now select the challenger based on its favorable AW value. Therefore, replace the defender now, not 2 years from now. Expect to keep the challenger for 4 years, or until a better challenger appears on the scene.

EXAMPLE 11.5 Keep or Replace the Kiln Case

PE

We continue with the progressive example of possibly replacing a kiln at B&T Enterprises. A marketing study revealed that the improving business activity on the west coast implies that the revenue profile between the installed kiln (PT) and the proposed new one (GH) would be the same, with the new kiln possibly bringing in new revenue within the next couple of years. The president of B&T decided it was time to do a replacement study. Assume you are the lead engineer and that you previously completed the ESL analysis on the challenger (Example 11.3). It indicates that for the GH system the ESL is its expected useful life.

Challenger: ESL $n_{GH} = 12$ years with total equivalent annual cost $AW_{GH} = \$-12.32$ million

The president asked you to complete the replacement study, stipulating that, due to the rapidly rising annual operating costs (AOC), the defender would be retained a maximum of 6 years. You are expected to make the necessary estimates for the defender (PT) and perform the study at a 15% per year return.

Solution

After some data collection, you have good evidence that the market value for the PT system will stay high, but that the increasing AOC is expected to continue rising about \$1.2 million per year. The best estimates for the next 6 years in \$ million units are these:

Year	1	2	3	4	5	6
Market Value, \$ M	22.0	22.0	22.0	20.0	18.0	18.0
AOC, \$ M per year	-5.2	-6.4	-7.6	-8.8	-10.0	-11.2

You developed a spreadsheet and performed the analysis in Figure 11–6. As an illustration, total AW computation for 3 years of retention, in \$ million units, is

$$\begin{aligned} \text{Total AW}_3 &= -22.0(A/P, 15\%, 3) + 22.0(A/F, 15\%, 3) - [5.2(P/F, 15\%, 1) + 6.4(P/F, 15\%, 2) \\ &\quad + 7.6(P/F, 15\%, 3)](A/P, 15\%, 3) \\ &= -9.63 + 6.34 - [14.36](0.43798) \\ &= \$-9.59 \text{ per year} \end{aligned}$$

Though the system could be retained up to 6 years, the ESL is much shorter at 1 year.

Defender: ESL $n_{PT} = 1$ year with total equivalent annual cost $AW_{PT} = \$-8.50$ million

ESL Analysis of PT					
Interest rate	15%			Market value	\$ 22.00
Year	Market Value	AOC	Capital Recovery	AW of AOC	Total AW of costs
1	22.00	-5.20	-3.30	-5.20	-8.50
2	22.00	-6.40	-3.30	-5.76	-9.06
3	22.00	-7.60	-3.30	-6.29	-9.59
4	20.00	-8.80	-3.70	-6.79	-10.49
5	18.00	-10.00	-3.89	-7.27	-11.16
6	18.00	-11.20	-3.76	-7.72	-11.47
Formula, year 3	22.00	-7.60	= PMT(\$B\$2,\$A\$3,-\$B\$8)	= -PMT(\$B\$2,\$A\$3,\$C\$8)*0	= D8+E8

Figure 11–6
ESL analysis of defender kiln PT for progressive example, Example 11.5.

To make the replacement/retention decision, apply step 1 of the procedure. Since $AW_{PT} = -\$8.50$ million per year is considerably less than $AW = -\$12.32$ million, you should recommend keeping the current kiln only 1 more year and doing another study during the year to determine if the current estimates are still reliable.

Comment

An observation of the trends of the two final AW series in this problem is important. A comparison of Figure 11–3 (top), column F, and Figure 11–6, column F, shows us that the largest total AW of the current system ($-\$11.47$ M for 6 years) is still below the smallest total AW of the proposed system ($-\$12.32$ M for 12 years). This indicates that the graphite hearth system (the challenger) will not be chosen on an economic basis, if the decision to consider the installed kiln as a defender in the future were made. It would take some significant estimate changes to justify the challenger.

11.4 Additional Considerations in a Replacement Study ● ● ●

There are several additional aspects of a replacement study that may be introduced. Three of these are identified and discussed in turn.

- Future-year replacement decisions at the time of the initial replacement study
- Opportunity cost versus cash flow approaches to alternative comparison
- Anticipation of improved future challengers

In most cases when management initiates a replacement study, the question is best framed as, “Replace now, 1 year from now, 2 years from now, etc.?” The procedure above does answer this question provided the estimates for C and D do not change as each year passes. In other words, *at the time it is performed, step 1 of the procedure does answer the replacement question for multiple years*. It is only when estimates change over time that the decision to retain the defender may be prematurely reversed (prior to n_D years) in favor of the then-best challenger.

The first costs (P values) for the challenger and defender have been correctly taken as the initial investment for the challenger C and current market value for the defender D. This is called the **opportunity cost approach** because it recognizes that a cash inflow of funds equal to the market value is forgone if the defender is selected. This approach, also called the conventional approach, is correct for every replacement study. A second approach, called the **cash flow approach**, recognizes that when C is selected, the market value cash inflow for the defender is received and, in effect, immediately reduces the capital needed to invest in the challenger. *Use of the cash flow approach is strongly discouraged* for at least two reasons: possible violation of the equal-service requirement and incorrect capital recovery value for C. As we are aware, all economic evaluations must compare alternatives with equal service. Therefore, the cash flow approach can work only when challenger and defender lives are exactly equal. This is commonly not the case; in fact, the ESL analysis and the replacement study procedure are designed to compare two mutually exclusive, *unequal-life* alternatives via the annual worth method. If this equal-service comparison reason is not enough to avoid the cash flow approach, consider what happens to the challenger’s capital recovery amount when its first cost is decreased by the market value of the defender. The capital recovery (CR) terms in Equation [11.3] will decrease, resulting in a falsely low value of CR for the challenger, were it selected. From the vantage point of the economic study itself, the decision for C or D will not change; but when C is selected and implemented, this CR value is not reliable. The conclusion is simple:

Use the initial investment of C and the market value of D as the first costs in the ESL analysis and in the replacement study.

A basic premise of a replacement study is that some challenger will replace the defender at a future time, provided the service continues to be needed and a worthy challenger is available. The expectation of ever-improving challengers can offer strong encouragement to retain the defender until some situational elements—technology, costs, market fluctuations, contract negotiations, etc.—

stabilize. This was the case in the previous two examples. A large expenditure on equipment when the standards changed soon after purchase forced an early replacement consideration and a large loss of invested capital. The replacement study is no substitute for forecasting challenger availability. *It is important to understand trends, new advances, and competitive pressures that can complement the economic outcome of a good replacement study.* It is often better to compare a challenger with an augmented defender in the replacement study. Adding needed features to a currently installed defender may prolong its useful life and productivity until challenger choices are more appealing.

It is possible that a significant tax impact may occur when a defender is traded early in its expected life. If taxes should be considered, proceed now, or after the next section, to Chapter 17 and the after-tax replacement analysis in Section 17.7.

11.5 Replacement Study over a Specified Study Period ● ● ●

When the time period for the replacement study is limited to a specified study period or planning horizon, for example, 6 years, the ESL analysis is not performed.

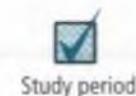


The AW values for the challenger and for the remaining life of the defender are not based on the economic service life; the AW is calculated over the study period only. What happens to the alternatives after the study period is not considered in the replacement analysis.

This means that the defender or challenger is not needed beyond the study period. In fact, a study period of fixed duration does not comply with the three assumptions stated in Section 11.1—service needed for indefinite future, best challenger available now, and estimates will be identical for future life cycles.

When performing a replacement study over a fixed study period, it is crucial that the estimates used to determine the AW values be accurate and used in the study. This is especially important for the defender. Failure to do the following violates the requirement of equal-service comparison.

When the defender's remaining life is **shorter than the study period**, the cost of providing the defender's services from the end of its expected remaining life to the end of the study period must be estimated as accurately as possible and included in the replacement study.



The right branch of Figure 11–4 presents an overview of the replacement study procedure for a stated study period.

1. *Succession options and AW values.* Develop all the viable ways to use the defender and challenger during the study period. There may be only one option or many options; the longer the study period, the more complex this analysis becomes. The AW values for the challenger and defender cash flows are used to build the equivalent cash flow values for each option.
2. *Selection of the best option.* The PW or AW for each option is calculated over the study period. Select the option with the lowest cost, or highest income if revenues are estimated. (As before, the best option will have the numerically largest PW or AW value.)

The following examples use this procedure and illustrate the importance of making cost estimates for the defender alternative when its remaining life is less than the study period.

EXAMPLE 11.6

Claudia works with Lockheed-Martin (LMCO) in the aircraft maintenance division. She is preparing for what she and her boss, the division chief, hope to be a new 10-year defense contract with the U.S. Air Force on C-5A cargo aircraft. A key piece of equipment for maintenance operations is an avionics circuit diagnostics system. The current system was purchased 7 years ago on an earlier contract. It has no capital recovery costs remaining, and the following are reliable estimates: current market value = \$70,000, remaining life of 3 more years, no salvage value, and AOC = \$30,000 per year. The only options for this system are to replace it now or retain it for the full 3 additional years.

Claudia has found that there is only one good challenger system. Its cost estimates are: first cost = \$750,000, life = 10 years, S = 0, and AOC = \$50,000 per year.

Realizing the importance of accurate defender alternative cost estimates, Claudia asked the division chief what system would be a logical follow-on to the current one 3 years hence, if LMCO wins the contract. The chief predicted LMCO would purchase the very system she had identified as the challenger, because it is the best on the market. The company would keep it for the entire 10 additional years for use on an extension of this contract or some other application that could recover the remaining 3 years of invested capital. Claudia interpreted the response to mean that the last 3 years would also be capital recovery years, but on some project other than this one. Claudia's estimate of the first cost of this same system 3 years from now is \$900,000. Additionally, the \$50,000 per year AOC is the best estimate at this time.

The division chief mentioned any study had to be conducted using the interest rate of 10%, as mandated by the U.S. Office of Management and Budget (OMB). Perform a replacement study for the fixed contract period of 10 years.

Solution

The study period is fixed at 10 years, so the intent of the replacement study assumptions is not present. This means the defender follow-on estimates are very important to the analysis. Further, *any analyses to determine the ESL values are unnecessary* since alternative lives are already set and no projected annual market values are available. The first step of the replacement study procedure is to define the options. Since the defender will be replaced now or in 3 years, there are only two options:

1. Challenger for all 10 years.
2. Defender for 3 years, followed by the challenger for 7 years.

Cash flows are diagrammed in Figure 11–7. For option 1, the challenger is used for all 10 years. Equation [11.3] is applied to calculate AW using the following estimates:

$$\begin{array}{ll} \text{Challenger:} & P = \$-750,000 \quad \text{AOC} = \$-50,000 \\ & n = 10 \text{ years} \quad S = 0 \end{array}$$

$$AW_C = -750,000(A/P, 10\%, 10) - 50,000 = \$-172,063$$

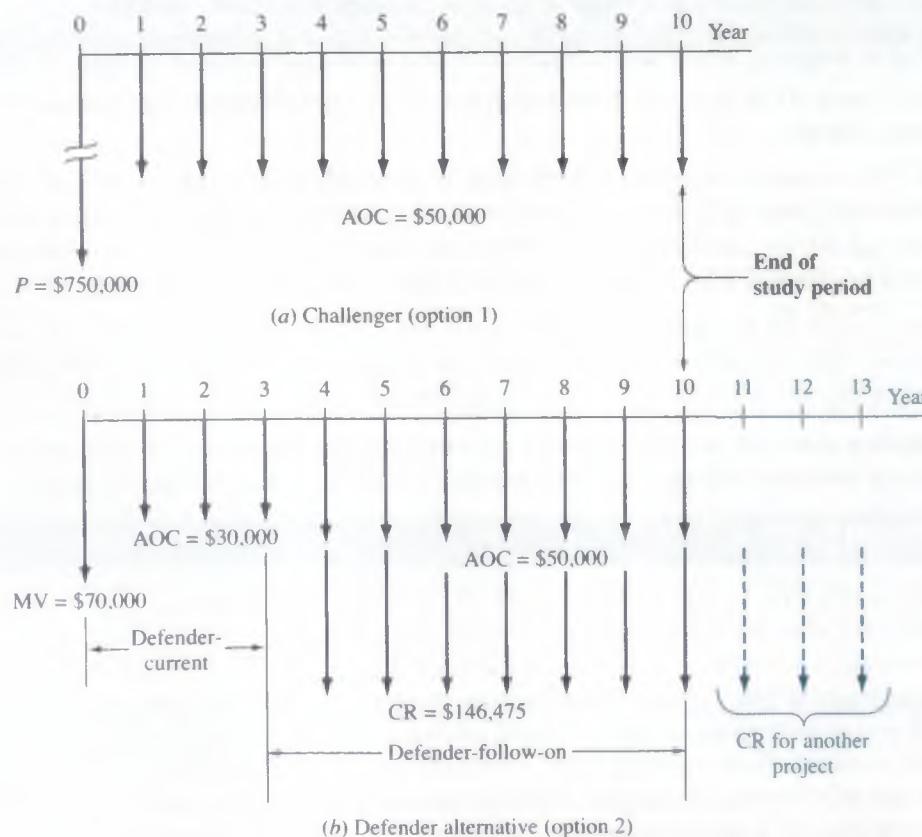


Figure 11–7
Cash flow diagrams for a 10-year study period replacement study, Example 11.6.

The second option has more complex cost estimates. The AW for the in-place system is calculated over the first 3 years. Added to this is the capital recovery for the defender follow-on for the next 7 years. *However in this case, the CR amount is determined over its full 10-year life.* (It is not unusual for the recovery of invested capital to be moved between projects, especially for contract work.) Refer to the AW components as AW_{DC} (subscript DC for defender current) and AW_{DF} (subscript DF for defender follow-on). The final cash flows are shown in Figure 11-7b.

$$\begin{array}{lll} \text{Defender current:} & \text{market value} = \$-70,000 & \text{AOC} = \$-30,000 \\ & n = 3 \text{ years} & S = 0 \end{array}$$

$$AW_{DC} = [-70,000 - 30,000(P/A, 10\%, 3)](A/P, 10\%, 10) = \$-23,534$$

Defender follow-on: $P = \$-900,000$, $n = 10$ years for capital recovery calculation only, AOC = $\$-50,000$ for years 4 through 10, $S = 0$.

The CR and AW for all 10 years are

$$CR_{DF} = -900,000(A/P, 10\%, 10) = \$-146,475 \quad [11.6]$$

$$AW_{DF} = (-146,475 - 50,000)(F/A, 10\%, 7)(A/F, 10\%, 10) = \$-116,966$$

Total AW_D for the defender is the sum of the two annual worth values above. This is the AW for option 2.

$$AW_D = AW_{DC} + AW_{DF} = -23,534 - 116,966 = \$-140,500$$

Option 2 has a lower cost ($\$-140,500$ versus $\$-172,063$). Retain the defender now and expect to purchase the follow-on system 3 years hence.

Comment

The capital recovery cost for the defender follow-on will be borne by some yet-to-be-identified project for years 11 through 13. If this assumption were not made, its capital recovery cost would be calculated over 7 years, not 10, in Equation [11.6], increasing CR to $\$-184,869$. This raises the annual worth to $AW_D = \$-163,357$. The defender alternative (option 2) is still selected.

EXAMPLE 11.7

Three years ago Chicago's O'Hare Airport purchased a new fire truck. Because of flight increases, new fire-fighting capacity is needed once again. An additional truck of the same capacity can be purchased now, or a double-capacity truck can replace the current fire truck. Estimates are presented below. Compare the options at 12% per year using (a) a 12-year study period and (b) a 9-year study period.

	Presently Owned	New Purchase	Double Capacity
First cost P , \$	$-151,000$ (3 years ago)	$-175,000$	$-190,000$
AOC, \$	$-1,500$	$-1,500$	$-2,500$
Market value, \$	70,000	—	—
Salvage value, \$	10% of P	12% of P	10% of P
Life, years	12	12	12

Solution

Identify option 1 as retention of the presently owned truck and augmentation with a new same-capacity vehicle. Define option 2 as replacement with the double-capacity truck.

	Option 1		Option 2
	Presently Owned	Augmentation	Double Capacity
P , \$	$-70,000$	$-175,000$	$-190,000$
AOC, \$	$-1,500$	$-1,500$	$-2,500$
S , \$	15,100	21,000	19,000
n , years	9	12	12

(a) For a full-life 12-year study period of option 1,

$$\begin{aligned} AW_1 &= (AW \text{ of presently owned}) + (AW \text{ of augmentation}) \\ &= [-70,000(A/P, 12\%, 9) + 15,100(A/F, 12\%, 9) - 1500] \\ &\quad + [-175,000(A/P, 12\%, 12) + 21,000(A/F, 12\%, 12) - 1500] \\ &= -13,616 - 28,882 \\ &= \$-42,498 \end{aligned}$$

This computation assumes the equivalent services provided by the current fire truck can be purchased at \$-13,616 per year for years 10 through 12.

$$\begin{aligned} AW_2 &= -190,000(A/P, 12\%, 12) + 19,000(A/F, 12\%, 12) - 2500 \\ &= \$-32,386 \end{aligned}$$

Replace now with the double-capacity truck (option 2) at an advantage of \$10,112 per year.

(b) The analysis for an abbreviated 9-year study period is identical, except that $n = 9$ in each factor; that is, 3 fewer years are allowed for the augmentation and double-capacity trucks to recover the capital investment plus a 12% per year return. The salvage values remain the same since they are quoted as a percentage of P for all years.

$$AW_1 = \$-46,539 \quad AW_2 = \$-36,873$$

Option 2 is again selected.

The previous two examples indicate an important consideration for setting the length of the study period for a replacement analysis. It involves the capital recovery amount for the challenger, when the strict definition of a study period is applied.



Study period

Capital recovery

When a study period shorter than the life of the challenger is defined, the challenger's capital recovery amount **increases** in order to recover the initial investment plus a return in this **shortened time period**. Highly abbreviated study periods tend to disadvantage the challenger because no consideration of time beyond the end of the study period is made in calculating the challenger's capital recovery amount.

If there are several options for the number of years that the defender may be retained before replacement with the challenger, the first step of the replacement study—succession options and AW values—must include all the viable options. For example, if the study period is 5 years and the defender will remain in service 1 year, or 2 years, or 3 years, cost estimates must be made to determine AW values for each defender retention period. In this case, there are four options; call them W, X, Y, and Z.

Option	Defender Retained, Years	Challenger Serves, Years
W	3	2
X	2	3
Y	1	4
Z	0	5

The respective AW values for defender retention and challenger use define the cash flows for each option. Example 11.8 illustrates the procedure using the progressive example.

EXAMPLE 11.8 | Keep or Replace the Kiln Case

PE

We have progressed to the point that the replacement study between the defender PT and challenger GH was completed (Example 11.5). The defender was the clear choice with a much smaller AW value (\$-8.50 M) than that of the challenger (\$-12.32 M). Now the management of B&T is in a dilemma. They know the current tunnel kiln is much cheaper than the new graphite hearth, but the prospect of future new business should not be dismissed.

TABLE 11–2 Replacement Study Options and Total AW Values, Example 11.8

Option	Defender PT			Challenger GH		
	Years Retained	AW, \$ M/Year		Years Retained	AW, \$ M/Year	
A	1	-8.50		5	-14.21	
B	2	-9.06		4	-15.08	
C	3	-9.59		3	-16.31	
D	4	-10.49		2	-18.21	
E	5	-11.16		1	-22.10	
F	6	-11.47		0	—	

The president asked, "Is it possible to determine when it is economically the cheapest to purchase the new kiln, provided the current one is kept at least 1 year, but no more than 6 years, its remaining expected life?" The chief financial officer answered, yes, of course. (a) Determine the answer for the president. (b) Discuss the next step in the analysis based on the conclusion reached here.

Solution

(a) Actually, this is a quite easy question to answer, because all the information has been determined previously. We know the MARR is 15% per year, the study period has been established at 6 years, and the defender PT will stay in place between 1 and 6 years. Therefore, the challenger GH will be considered for 0 to 5 years of service. The total AW values were determined for the defender in Example 11.5 (Figure 11–6) and for the challenger in Example 11.3 (Figure 11–3). They are repeated in Table 11–2 for convenience. Use the procedure for a replacement study with a fixed study period.

Step 1: Succession options and AW values. There are six options in this case; the defender is retained from 1 to 6 years while the challenger is installed from 0 to 5 years. We will label them A through F. Figure 11–8 presents the options and the AW series for each option from Table 11–2. No consideration of the fact that the challenger has an expected life of 12 years is made since the study period is fixed at 6 years.

Step 2: Selection of the best option. The PW value for each option is determined over the 6-year study period in column J of Figure 11–8. The conclusion is clearly to **keep the defender** in place for 6 more years.

(b) Every replacement analysis has indicated that the defender should be retained for the near future. If the analysis is to be carried further, the possibility of increased revenue based on services of the challenger's high-temperature and operating efficiency should be considered next. In the introductory material, new business opportunities were mentioned. A revenue increase for the challenger will reduce its AW of costs and possibly make it more economically viable.

Option	Time in Service, Years		AW Cash Flows for Each Option, \$M per year						Option PW at 15%, \$M
	Defender	Challenger	1	2	3	4	5	6	
	PT	GH	-8.50	-14.21	-14.21	-14.21	-14.21	-14.21	
A	1	5	-8.50	-14.21	-14.21	-14.21	-14.21	-14.21	48.81
B	2	4	-9.06	-9.06	-15.08	-15.08	-15.08	-15.08	47.28
C	3	3	-9.59	-9.59	-9.59	-16.31	-16.31	-16.31	46.38
D	4	2	-10.49	-10.49	-10.49	-10.49	-18.21	-18.21	46.87
E	5	1	-11.16	-11.16	-11.16	-11.16	-22.10	-22.10	46.96
F	6	0	-11.47	-11.47	-11.47	-11.47	-11.47	-11.47	43.41

Conclusion: Keep defender all 6 years

Figure 11–8

PW values for 6-year study period replacement analysis, Example 11.8.

11.6 Replacement Value ● ● ●

Often it is helpful to know the minimum market value of the defender necessary to make the challenger economically attractive. If a realizable market value or trade-in of at least this amount can be obtained, from an economic perspective the challenger should be selected immediately. This is a *breakeven value* between AW_C and AW_D ; it is referred to as the **replacement value (RV)**. Set up the relation $AW_C = AW_D$ with the market value for the defender identified as RV , which is the unknown. The AW_C is known, so RV can be determined. The selection guideline is as follows:

If the actual market trade-in exceeds the breakeven replacement value, the challenger is the better alternative and should replace the defender now.

Determination of the RV for a defender is an excellent opportunity to utilize the Goal Seek tool in Excel. The target cell is the current market value, and the AW_D value is forced to equal the AW_C amount. Example 11.9 discusses a replacement value of the progressive example.

EXAMPLE 11.9 Keep or Replace the Kiln Case

As one final consideration of the challenger kiln, you decide to determine what the trade-in amount would have to be so that the challenger is the economic choice next year. This is based on the ESL analysis that concluded the following (Examples 11.3 and 11.5):

$$\begin{array}{ll} \text{Defender:} & ESL \ n_{PT} = 1 \text{ year with } AW_{PT} = \$-8.50 \text{ million} \\ \text{Challenger:} & ESL \ n_{GH} = 12 \text{ years with } AW_{GH} = \$-12.32 \text{ million} \end{array}$$

The original defender price was \$25 million, and a current market value of \$22 million was estimated earlier (Figure 11–6). Since the installed kiln is known for retention of its market value (MV), you are hopeful the difference between RV and estimated MV may not be so significant. What will you discover RV to be? The MARR is 15% per year.

Solution

Set the AW relation for the defender for the ESL time of 1 year equal to $AW_{GH} = \$-12.32$ and solve for RV . The estimates for AOC and MV next year are in Figure 11–6; they are, in \$ million,

$$\begin{aligned} \text{Year 1:} \quad & AOC = \$-5.20 \quad MV = \$22.0 \\ & -12.32 = -RV(A/P, 15\%, 1) + 22.00(A/F, 15\%, 1) - 5.20 \\ & 1.15RV = 12.32 + 22.00 - 5.20 \\ & RV = \$25.32 \end{aligned}$$

Though the RV is larger than the defender's estimated MV of \$22 million, some flexibility in the trade-in offer or the challenger's first cost may cause the challenger to be economically justifiable.

Comment

To find RV using a spreadsheet, return to Figure 11–6. In the Goal Seek template, the "set" cell is the AW for 1 year (currently \$-8.50), and the required value is \$-12.32, the AW_{GH} for its ESL of 12 years. The "changing" cell is the current market value (cell F2), currently \$22.00. When "OK" is touched, \$25.32 is displayed as the breakeven market value. This is the RV .

CHAPTER SUMMARY

It is important in a replacement study to compare the best challenger with the defender. *Best (economic) challenger is described as the one with the lowest annual worth (AW) of costs for some number of years.* If the expected remaining life of the defender and the estimated life of the challenger are specified, the AW values over these years are determined and the replacement study proceeds. However, if reasonable estimates of the expected market value (MV) and AOC

for each year of ownership can be made, these year-by-year (marginal) costs help determine the best challenger.

The economic service life (ESL) analysis is designed to determine the best challenger's years of service and the resulting lowest total AW of costs. The resulting n_C and AW_C values are used in the replacement study. The same analysis can be performed for the ESL of the defender.

Replacement studies in which no study period (planning horizon) is specified utilize the annual worth method of comparing two unequal-life alternatives. The better AW value determines how long the defender is retained before replacement.

When a study period is specified for the replacement study, it is vital that the market value and cost estimates for the defender be as accurate as possible. When the defender's remaining life is shorter than the study period, it is critical that the cost for continuing service be estimated carefully. All the viable options for using the defender and challenger are enumerated, and their AW equivalent cash flows are determined. For each option, the PW or AW value is used to select the best option. This option determines how long the defender is retained before replacement.

PROBLEMS

Foundations of Replacement

11.1 In a replacement study, what is meant by "taking the nonowner's viewpoint"?

11.2 An asset that was purchased 3 years ago for \$100,000 is becoming obsolete faster than expected. The company thought the asset would last 5 years and that its book value would decrease by \$20,000 each year and, therefore, be worthless at the end of year 5. In considering a more versatile, more reliable high-tech replacement, the company discovered that the presently owned asset has a market value of only \$15,000. If the replacement is purchased immediately at a first cost of \$75,000 and if it will have a lower annual worth, what is the amount of the sunk cost? Assume the company's MARR is 15% per year.

11.3 As a muscle car aficionado, a friend of yours likes to restore cars of the 60s and 70s and sell them for a profit. He started his latest project (a 1965 Shelby GT350) four months ago and has a total of \$126,000 invested so far. Another opportunity has come up (a 1969 Dodge Charger) that he is thinking of buying because he believes he could sell it for a profit of \$60,000 after it is completely restored. To do so, however, he would have to sell the unfinished Shelby first. He thought that the completely restored Shelby would be worth \$195,000, resulting in a tidy profit of \$22,000, but in its half-restored condition, the most he could get now is \$115,000. In discussing the situation with you, he stated that if he could sell the Shelby now and buy the Charger at a reduced price, he would make up for the money he will lose in selling the Shelby at a lower-than-desired price.

- (a) What is wrong with this thinking?
- (b) What is his sunk cost in the Shelby?

11.4 In conducting a replacement study wherein the planning horizon is unspecified, list three assumptions that are inherent in an annual worth analysis of the defender and challenger.

11.5 A civil engineer who owns his own design/build/operate company purchased a small crane 3 years ago at a cost of \$60,000. At that time, it was expected to be used for 10 years and then traded in for its salvage value of \$10,000. Due to increased construction activities, the company would prefer to trade for a new, larger crane now that will cost \$80,000. The company estimates that the old crane can be used, if necessary, for another 3 years, at which time it would have a \$23,000 estimated market value. Its current market value is estimated to be \$39,000, and if it is used for another 3 years, it will have M&O costs (exclusive of operator costs) of \$17,000 per year. Determine the values of P , n , S , and AOC that should be used for the existing crane in a replacement analysis.

11.6 Equipment that was purchased by Newport Corporation for making pneumatic vibration isolators cost \$90,000 two years ago. It has a market value that can be described by the relation $$90,000 - 8000k$, where k is the years from time of purchase. Experience with this type of equipment has shown that the operating cost for the first 4 years is \$65,000 per year, after which it increases by \$6300 per year. The asset's salvage value was originally estimated to be \$7000 after a predicted 10-year useful life. Determine the values of P , S , and AOC if a replacement study is done (a) now and (b) 1 year from now.

11.7 A piece of equipment that was purchased 2 years ago by Toshiba Imaging for \$50,000 was expected to have a useful life of 5 years with a \$5000 salvage

value. Its performance was less than expected, and it was upgraded for \$20,000 one year ago. Increased demand now requires that the equipment be upgraded again for another \$17,000 so that it can be used for 3 more years. If upgraded, its annual operating cost will be \$27,000 and it will have a \$12,000 salvage after 3 years. Alternatively, it can be replaced with new equipment priced at \$65,000 with operating costs of \$14,000 per year and a salvage value of \$23,000 after 6 years. If replaced now, the existing equipment will be sold for \$7000. Determine the values of P , S , AOC, and n for the defender in a replacement study.

Economic Service Life

- 11.8 For equipment that has a first cost of \$10,000 and the estimated operating costs and year-end salvage values shown below, determine the economic service life at $i = 10\%$ per year.

Year	Operating Cost, \$ per Year	Salvage Value, \$
1	-1000	7000
2	-1200	5000
3	-1300	4500
4	-2000	3000
5	-3000	2000

- 11.9 To improve package tracking at a UPS transfer facility, conveyor equipment was upgraded with RFID sensors at a cost of \$345,000. The operating cost is expected to be \$148,000 per year for the first 3 years and \$210,000 for the next 3 years. The salvage value of the equipment is expected to be \$140,000 for the first 3 years, but due to obsolescence, it won't have a significant value after that. At an interest rate of 10% per year, determine
- (a) The economic service life of the equipment and associated annual worth
 - (b) The percentage increase in the AW of cost if the equipment is retained 2 years longer than the ESL

- 11.10 Economic service life calculations for an asset are shown below. If an interest rate of 10% per year was used in making the calculations, determine the values of P and S that were used in calculating the AW for year 3.

Years Retained	AW of First Cost, \$	AW of Operating Cost, \$ per Year	AW of Salvage Value, \$
1	-51,700	-15,000	35,000
2	-27,091	-17,000	13,810
3	-18,899	-19,000	6,648
4	-14,827	-21,000	4,309
5	-12,398	-23,000	2,457

- 11.11 The initial cost of a bridge that is expected to be in place forever is \$70 million. Maintenance can be done at 1-, 2-, 3-, or 4-year intervals, but the longer the interval between servicing, the higher the cost. The costs of servicing are estimated at \$83,000, \$91,000, \$125,000, and \$183,000 for intervals of 1 through 4 years, respectively. What interval should be scheduled for maintenance to minimize the overall equivalent annual cost? The interest rate is 8% per year.

- 11.12 A construction company bought a 180,000 metric ton earth sifter at a cost of \$65,000. The company expects to keep the equipment a maximum of 7 years. The operating cost is expected to follow the series described by $40,000 + 10,000k$, where k is the number of years since it was purchased ($k = 1, 2, \dots, 7$). The salvage value is estimated to be \$30,000 for years 1 and 2 and \$20,000 for years 3 through 7. At an interest rate of 10% per year, determine the economic service life and the associated equivalent annual cost of the sifter.

- 11.13 An engineer determined the ESL of a new \$80,000 piece of equipment and recorded the calculations shown below. [Note that the numbers are annual worth values associated with various years of retention; that is, if the equipment is kept for, say, 3 years, the AW (years 1 through 3) of the first cost is \$32,169, the AW of the operating cost is \$51,000, and the AW of the salvage value is \$6042.] The engineer forgot to enter the AW of the salvage value for 2 years of retention. From the information available, determine the following:

- (a) The interest rate used in the ESL calculations.
- (b) The salvage value after 2 years, if the total AW of the equipment in year 2 was \$78,762. Use the interest rate determined in part (a).

Years Retained	AW of First Cost, \$	AW of Operating Cost, \$	AW of Salvage Value, \$
1	-88,000	-45,000	50,000
2	-46,095	-46,000	?
3	-32,169	-51,000	6,042

- 11.14 A large, standby electricity generator in a hospital operating room has a first cost of \$70,000 and may be used for a maximum of 6 years. Its salvage value, which decreases by 15% per year, is described by the equation $S = 70,000(1 - 0.15)^n$, where n is the number of years after purchase. The operating cost of the generator will be constant at \$75,000 per year. At an interest rate of 12% per year, what are the economic service life and the associated AW value?

- 11.15 A piece of equipment has a first cost of \$150,000, a maximum useful life of 7 years, and a market (salvage) value described by the relation $S = 120,000 - 20,000k$, where k is the number of years since it was purchased. The salvage value cannot go below zero. The AOC series is estimated using $AOC = 60,000 + 10,000k$. The interest rate is 15% per year. Determine the economic service life (a) by hand solution, using regular AW computations, and (b) by spreadsheet, using annual marginal cost estimates.

- 11.16 Determine the economic service life and corresponding AW value for a machine that has the following cash flow estimates. Use an interest rate of 14% per year and hand solution.

Year	Salvage Value, \$	Operating Cost, \$ per Year
0	100,000	—
1	75,000	-28,000
2	60,000	-31,000
3	50,000	-34,000
4	40,000	-34,000
5	25,000	-34,000
6	15,000	-45,000
7	0	-49,000

- 11.17 Use the annual marginal costs to find the economic service life for Problem 11.16 on a spreadsheet. Assume the salvage values are the best estimates of future market value. Develop an Excel chart of annual marginal costs (MC) and AW of MC over 7 years.

11.18 Keep or Replace the Kiln Case



In Example 11.3, the market value (salvage value) series of the proposed \$38 million replacement kiln (GH) dropped to \$25 million in only 1 year and then retained 75% of the previous year's market value through the remainder of its 12-year expected life. Based on the experience with the current kiln, and the higher-temperature capability of the replacement, the new kiln is actually expected to retain only 50% of the previous year's value starting in year 5. Additionally, the heating element replacement in year 6 will probably cost \$4 million, not \$2 million. And finally, the maintenance costs will be considerably higher as the kiln ages. Starting in year 5, the AOC is expected to increase by 25% per year, not 10% as predicted earlier. The Manager of Critical Equipment is now very concerned that the ESL will be significantly decreased from the 12 years calculated earlier (Example 11.3).

- (a) Determine the new ESL and associated AW value.
- (b) In percentage changes, estimate how much these new cost estimates may affect the minimum-cost life and AW of cost estimate.

Replacement Study

- 11.19 In a one-year-later analysis, what action should be taken if (a) all estimates are still current and the year is n_D , (b) all estimates are still current and the year is not n_D , and (c) the estimates have changed?

- 11.20 Based on company records of similar equipment, a consulting aerospace engineer at Aerospatiale estimated AW values for a presently owned, highly accurate steel rivet inserter.

If Retained This Number of Years	AW Value, \$ per Year
1	-62,000
2	-51,000
3	-49,000
4	-53,000
5	-70,000

A challenger has $ESL = 2$ years and $AW_C = \$-48,000$ per year. (a) If the consultant must recommend a replace/retain decision today, should the company keep the defender or purchase the challenger? Why? The MARR is 15% per year. (b) When should the next replacement evaluation take place?

- 11.21 In planning a plant expansion, MedImmune has an economic decision to make—upgrade the existing controlled-environment rooms or purchase new ones. The presently owned ones were purchased 4 years ago for \$250,000. They have a current “quick sale” value of \$20,000, but for an investment of \$100,000 now, they would be adequate for another 4 years, after which they would be sold for \$40,000. Alternatively, new controlled-environment rooms could be purchased at a cost of \$270,000. They are expected to have a 10-year life with a \$50,000 salvage value at that time. Determine whether the company should upgrade or replace. Use a MARR of 20% per year.

- 11.22 Three years ago, Witt Gas Controls purchased equipment for \$80,000 that was expected to have a useful life of 5 years with a \$9000 salvage value. Increased demand necessitated an upgrade costing \$30,000 one year ago. Technology changes now require that the equipment be upgraded again for another \$25,000 so that it can be used for 3 more years. Its annual operating cost will be \$47,000, and it will have a \$22,000 salvage after 3 years. Alternatively, it can be replaced with new equipment that will cost \$68,000 with operating costs of \$35,000 per year and a salvage value of \$21,000 after 3 years. If replaced now, the existing equipment will be sold for \$9000. Calculate the annual worth of the defender at an interest rate of 10% per year.

- 11.23 A recent environmental engineering graduate is trying to decide whether he should keep his presently owned car or purchase a more environmentally friendly hybrid. A new car will cost \$26,000 and have annual operation and maintenance costs of \$1200 per year with an \$8000 salvage value in 5 years (which is its estimated economic service life).

The presently owned car has a resale value *now* of \$5000; one year from now it will be \$3000, two years from now \$2500, and 3 years from now \$2200. Its operating cost is expected to be \$1900 this year, with costs increasing by \$200 per year. The presently owned car will definitely not be kept longer than 3 more years. Assuming used cars like the one presently owned *will always* be available, should the presently owned car be sold now, 1 year from now, 2 years from now, or 3 years from now? Use annual worth calculations at $i = 10\%$ per year and show your work.

- 11.24 A pulp and paper company is evaluating whether it should retain the current bleaching process that uses chlorine dioxide or replace it with a proprietary “oxypure” process. The relevant information for each process is shown. Use an interest rate of 15% per year to perform the replacement study.

	Current Process	Oxypure Process
Original cost 6 years ago, \$	−450,000	—
Investment cost now, \$	—	−700,000
Current market value, \$	25,000	—
Annual operating cost, \$/year	−180,000	−70,000
Salvage value, \$	0	50,000
Remaining life, years	5	10

- 11.25 A machine that is critical to the Phelps-Dodge copper refining operation was purchased 7 years ago for \$160,000. Last year a replacement study was performed with the decision to retain it for 3 more years. The situation has changed. The equipment is estimated to have a value of \$8000 if “scavenged” for parts now or anytime in the future. If kept in service, it can be minimally upgraded at a cost of \$43,000, which will make it usable for up to 2 more years. Its operating cost is expected to be \$22,000 the first year and \$25,000 the second year. Alternatively, the company can purchase a new system that will have an equivalent annual worth of \$−47,063 per year over its ESL. The company uses a MARR of 10% per year. Calculate the relevant annual worth values, and determine when the company should replace the machine.

- 11.26 A crushing machine that is a basic component of a metal recycling operation is wearing out faster than expected. The machine was purchased 2 years ago for \$400,000. At that time, the buyer thought the machine would serve its needs for at least 5 years, at which time the machine would be sold to a smaller independent recycler for \$80,000. Now, however, the company thinks the market value of the diminished machine is only \$50,000. If it is kept, the machine’s operating cost will be \$37,000 per year for the next 2 years, after which it will be scrapped for \$1000. If it is kept for only 1 year, the market value is estimated to be \$10,000. Alternatively, the company can outsource the process now for a fixed cost of \$56,000 per year. At an interest rate of 10% per year, should the company continue with the current machine or outsource the process?

- 11.27 The data associated with operating and maintaining an asset are shown below. The company manager has already decided to keep the machine for 1 more year (i.e., until the end of year 1), but you have been asked to determine the cost of keeping it 1 more year after that. At an interest rate of 10% per year, estimate the AW of keeping the machine from year 1 to year 2.

Year	Market Value, \$	Operating Cost, \$ per Year
0	30,000	−15,000
1	25,000	−15,000
2	14,000	−15,000
3	10,000	−15,000

- 11.28 A machine that cost \$120,000 three years ago can be sold now for \$54,000. Its market value for the next 2 years is expected to be \$40,000 and \$20,000 one year and 2 years from now, respectively. Its operating cost was \$18,000 for the first 3 years of its life, but the M&O cost is expected to be \$23,000 for the next 2 years. A new improved machine that can be purchased for \$138,000 will have an economic life of 5 years, an operating cost of \$9000 per year, and a salvage value of \$32,000 after 5 years. At an interest rate of 10% per year, determine if the presently owned machine should be replaced now, 1 year from now, or 2 years from now.

- 11.29 The projected market value and M&O costs associated with a presently owned machine are shown (next page). An outside vendor of services has offered to provide the service of the existing machine at a fixed price per year. If the presently owned machine is replaced now, the cost of the fixed-price contract will be \$33,000 for each of the next 3 years. If the presently owned machine is

replaced next year or the year after that, the contract price will be \$35,000 per year. Determine if and when the defender should be replaced with the outside vendor using an interest rate of 10% per year. Assume used equipment similar to the defender will always be available.

Year	Market Value, \$	M&O Cost, \$ per Year
0	30,000	—
1	25,000	−24,000
2	14,000	−25,000
3	10,000	−26,000
4	8,000	—

- 11.30 BioHealth, a biodevice systems leasing company, is considering a new equipment purchase to replace a currently owned asset that was purchased 2 years ago for \$250,000. It is appraised at a current market value of only \$50,000. An upgrade is possible for \$200,000 now that would be adequate for another 3 years of lease rights, after which the entire system could be sold on the international circuit for an estimated \$40,000. The challenger, which can be purchased for \$300,000, has an expected life of 10 years and a \$50,000 salvage value. Determine whether the company should upgrade or replace at a MARR of 12% per year. Assume the AOC estimates are the same for both alternatives.

- 11.31 For the estimates in Problem 11.30, use a spreadsheet-based analysis to determine the first cost for the augmentation of the current system that will make the defender and challenger break even. Is this a maximum or minimum for the upgrade, if the current system is to be retained?

- 11.32 Herald Richter and Associates, 5 years ago, purchased for \$45,000 a microwave signal graphical plotter for corrosion detection in concrete structures. It is expected to have the market values and annual operating costs shown below for its remaining useful life of up to 3 years. It could be traded now at an appraised market value of \$8000.

Year	Market Value at End of Year, \$	AOC, \$ per Year
1	6000	−50,000
2	4000	−53,000
3	1000	−60,000

A replacement plotter with new Internet-based, digital technology costing \$125,000 has an estimated \$10,000 salvage value after its 5-year life and an AOC of \$31,000 per year. At an interest rate of 15% per year, determine how many more years

Richter should retain the present plotter. Solve (a) by hand and (b) by using a spreadsheet.

- 11.33 In the opportunity cost approach to replacement analysis, what does the *opportunity* refer to?

- 11.34 State what is meant by the *cash flow approach* to replacement analysis, and list two reasons why it is not a good idea to use this method.

Replacement Study over a Specified Study Period

- 11.35 ABB Communications is considering replacing equipment that had a first cost of \$300,000 five years ago. The company CEO wants to know if the equipment should be replaced now or at any other time over the next 3 years to minimize the cost of producing miniature background suppression sensors. Since the present equipment or the proposed equipment can be used for any or all of the 3-year period, one of the company's industrial engineers produced AW cost information for the defender and challenger as shown below. The values represent the annual costs of the respective equipment if used for the indicated number of years. Determine when the defender should be replaced to minimize the cost to ABB for the 3-year study period using an interest rate of 10% per year.

Number of Years Kept	AW If Kept Stated	
	Defender	Challenger
1	−22,000	−29,000
2	−24,000	−26,000
3	−27,000	−25,000

- 11.36 The table below shows present worth calculations of the costs associated with using a presently owned machine (defender) and a possible replacement (challenger) for different numbers of years. Determine when the defender should be replaced using an interest rate of 10% per year and a 5-year study period. Show solutions (a) by hand and (b) by spreadsheet.

Number of Years Kept/Used	PW If Kept/Used Stated	
	Defender	Challenger
1	−36,000	−89,000
2	−75,000	−96,000
3	−125,000	−102,000
4	−166,000	−113,000
5	−217,000	−149,000

- 11.37 Nano Technologies intends to use the newest and finest equipment in its labs. Accordingly, a senior engineer has recommended that a 2-year-old piece

of precision measurement equipment be replaced immediately. This engineer believes it can be shown that the proposed equipment is economically advantageous at a 10% per year return and a planning horizon of 3 years.

- Perform the replacement analysis using the annual worth method for a specified 3-year study period.
- Determine the challenger's capital recovery amount for the 3-year study period and the expected full life. Comment on the effect made by the 3-year study period.

	Current	Proposed
Original purchase price, \$	−30,000	−40,000
Current market value, \$	17,000	—
Remaining life, years	5	15
Estimated value in 3 years, \$	9,000	20,000
Salvage value after 15 years, \$	—	5,000
Annual operating cost, \$ per year	−8,000	−3,000

- 11.38 An industrial engineer at a fiber-optic manufacturing company is considering two robots to reduce costs in a production line. Robot X will have a first cost of \$82,000, an annual maintenance and operation (M&O) cost of \$30,000, and salvage values of \$50,000, \$42,000, and \$35,000 after 1, 2, and 3 years, respectively. Robot Y will have a first cost of \$97,000, an annual M&O cost of \$27,000, and salvage values of \$60,000, \$51,000, and \$42,000 after 1, 2, and 3 years, respectively. Which robot should be selected if a 2-year study period is specified at an interest rate of 15% per year and replacement after 1 year is not an option?

- 11.39 A 3-year-old machine purchased for \$140,000 is not able to meet today's market demands. The machine can be upgraded now for \$70,000 or sold to a subcontracting company for \$40,000. The current machine will have an annual operating cost of \$85,000 per year and a \$30,000 salvage value in 3 years. If upgraded, the presently owned machine will be retained for only 3 more years, then replaced with a machine to be used in the manufacture of several other product lines. The replacement machine, which will serve the company now and for at least 8 years, will cost \$220,000. Its salvage value will be \$50,000 for years 1 through 4; \$20,000 after 5 years; and \$10,000 thereafter. It will have an estimated operating cost of \$65,000 per year. You want to perform an economic analysis at 15% per year using a 3-year planning horizon.

- Should the company replace the presently owned machine now, or do it 3 years from now?
- Compare the capital recovery requirements for the replacement machine (challenger) over the study period and an expected life of 8 years.

- 11.40 Two processes can be used for producing a polymer that reduces friction loss in engines. Process K, which is currently in place, has a market value of \$165,000 now, an operating cost of \$69,000 per year, and a salvage value of \$50,000 after 1 more year and \$40,000 after its maximum 2-year remaining life. Process L, the challenger, will have a first cost of \$230,000, an operating cost of \$65,000 per year, and salvage values of \$100,000 after 1 year, \$70,000 after 2 years, \$45,000 after 3 years, and \$26,000 after its maximum expected 4-year life. The company's MARR is 12% per year. You have been asked to determine which process to select when (a) a 2-year study period is used and (b) a 3-year study period is used.

11.41 Keep or Replace the Kiln Case



In Example 11.8, the in-place kiln and replacement kiln (GH) were evaluated using a fixed study period of 6 years. This is a significantly shortened period compared to the expected 12-year life of the challenger. Use the best estimates available throughout this case to determine the impact on the capital recovery amount for the GH kiln of shortening the evaluation time from 12 to 6 years.

- 11.42 Nabisco Bakers currently employs staff to operate the equipment used to sterilize much of the mixing, baking, and packaging facilities in a large cookie and cracker manufacturing plant in Iowa. The plant manager, who is dedicated to cutting costs but not sacrificing quality and hygiene, has the projected data shown in the table below if the current system were retained for up to its maximum expected life of 5 years. A contract company has proposed a turnkey sanitation system for \$5.0 million per year if Nabisco signs on for 4 to 10 years, and \$5.5 million per year for a shorter number of years.

Years Retained	AW, \$ per Year	Close-Down Expense, \$
0	—	−3,000,000
1	−2,300,000	−2,500,000
2	−2,300,000	−2,000,000
3	−3,000,000	−1,000,000
4	−3,000,000	−1,000,000
5	−3,500,000	−500,000

- At a MARR = 8% per year, perform a replacement study for the plant manager with a fixed study period of 5 years, when it is anticipated that the plant will be shut down due to the age of the facility and projected technological obsolescence. As you perform the study, take into account that regardless of the number of years that the current sanitation system is retained, a one-time close-down

cost will be incurred for personnel and equipment during the last year of operation. (*Hint:* Calculate AW values for all combinations of defender/challenger options.)

- (b) What is the percentage change in the AW amount each year for the 5-year period? If the decision to retain the current sanitation system for all 5 years is made, what is the economic disadvantage in AW compared to that of the most economic retention period?

Replacement Value

11.43 In 2008, Amphenol Industrial purchased a new quality inspection system for \$550,000. The estimated salvage value was \$50,000 after 8 years. Currently the expected remaining life is 3 years with an AOC of \$27,000 per year and an estimated salvage value of \$30,000. The new president has recommended early replacement of the system with one that costs \$400,000 and has a 5-year economic service life, a \$45,000 salvage value, and an estimated AOC of \$50,000 per year. If the MARR for the corporation is 12% per year, find the minimum trade-in value now necessary to make the president's replacement economically advantageous.

11.44 A CNC milling machine purchased by Proto Tool and Die 10 years ago for \$75,000 can be used for 3 more years. Estimates are an annual operating cost of \$63,000 and a salvage value of \$25,000. A challenger will cost \$130,000 with an economic life of 6 years and an operating cost of \$32,000 per year. Its salvage value will be \$45,000. On the basis of

these estimates, what market value for the existing asset will render the challenger equally attractive? Use an interest rate of 12% per year.

11.45 Hydrochloric acid, which fumes at room temperatures, creates a very corrosive work environment, causing steel tools to rust and equipment to fail prematurely. A distillation system purchased 4 years ago for \$80,000 can be used for only 2 more years, at which time it will be scrapped with no value. Its operating cost is \$75,000 per year. A more corrosion-resistant challenger will cost \$220,000 with an operating cost of \$49,000 per year. It is expected to have a \$30,000 salvage value after its 6-year ESL. At an interest rate of 10% per year, what minimum replacement value now for the present system will render the challenger attractive?

11.46 Engine oil purifier machines can effectively remove acid, pitch, particles, water, and gas from used oil. Purifier A was purchased 5 years ago for \$90,000. Its operating cost is higher than expected, so if it is not replaced now, it will likely be used for only 3 more years. Its operating cost this year will be \$140,000, increasing by \$2000 per year through the end of its useful life, at which time it will be donated for its recyclable scrap value. A more efficient challenger, purifier B, will cost \$150,000 with a \$50,000 salvage value after its 8-year ESL. Its operating cost is expected to be \$82,000 for year 1, increasing by \$500 per year thereafter. What is the market value for A that will make the two purifiers equally attractive at an interest rate of 12% per year?

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

11.47 In conducting a replacement study, all of the following are correct viewpoints for the analyst except:

- (a) Consultant's
- (b) Owner's
- (c) Outsider's
- (d) Nonowner's

11.48 A sunk cost is the difference between:

- (a) The first cost and the salvage value
- (b) The market value and the salvage value
- (c) The first cost and the market value
- (d) The book value and the market value

11.49 A truck was purchased 3 years ago for \$45,000 and can be sold today for \$24,000. The operating costs are \$9000 per year, and it is expected to last 4 more years with a \$5000 salvage value. A new truck,

which will perform that same service, can be purchased for \$50,000, and it will have a life of 10 years with operating costs of \$28,000 per year and a \$10,000 salvage value. The value that should be used as P for the presently owned vehicle in a replacement study is:

- (a) \$45,000
- (b) \$5000
- (c) \$50,000
- (d) \$24,000

11.50 The economic service life of an asset with the PW and AW values on the next page is:

- (a) 1 year
- (b) 2 years
- (c) 3 years
- (d) 4 years

Years <i>n</i>	Present Worth If Kept <i>n</i> Years, \$	Annual Worth If Kept <i>n</i> Years, \$/Year
1	-15,455	-17,000
2	-28,646	-16,286
3	-45,019	-18,102
4	-57,655	-18,188
5	-72,867	-19,222

11.51 In looking for ways to cut costs and increase profit (to make the company's stock go up), one of the company's industrial engineers (IEs) determined that the equivalent annual worth of an existing machine over its remaining useful life of 3 years will be \$-70,000 per year. The IE also determined that a replacement with more advanced features will have an AW of \$-80,000 per year if it is kept for 2 years or less, \$-75,000 if it is kept between 3 and 4 years, and \$-65,000 if it is kept for 5 to 10 years. If the engineer uses a 3-year study period and an interest rate of 15% per year, she should recommend that the existing machine be:

- (a) Replaced now
- (b) Replaced 1 year from now
- (c) Replaced 2 years from now
- (d) Not replaced

11.52 The equivalent annual worth of an existing machine at American Semiconductor is estimated to be \$-70,000 per year over its remaining useful life of 3 years. It can be replaced now or later with a machine that will have an AW of \$-90,000 per year if it is kept for 2 years or less, -\$65,000 if it is kept between 3 and 5 years, and \$-110,000 if it is kept for 6 to 8 years. The company wants an analysis of what it should do for a 3-year study period at an interest rate of 15% per year. The replacement must be made now or 3 years from now, according to the department supervisor. You should recommend that the existing machine be replaced:

- (a) Now
- (b) 1 year from now
- (c) 2 years from now
- (d) Can't tell; must find AW of different retention combinations for the two machines

11.53 The cost characteristics of a CO testing machine at Dytran Instruments are shown below. The cost of a new tester is \$100,000. The equation for determining the AW of keeping the tester for 2 years is:

- (a) $AW = -100,000(A/P,i,8) - [42,000(P/F,i,1) + 47,000(P/F,i,2)](A/P,i,8) + 40,000(A/F,i,8)$
- (b) $AW = -100,000(A/P,i,2) - [42,000(P/F,i,1) + 47,000(P/F,i,2)](A/P,i,2) + 40,000(A/F,i,2)$

$$(c) \quad AW = -100,000(A/P,i,2) - 47,000 \\ + 40,000(A/F,i,2)$$

$$(d) \quad AW = -100,000(A/P,i,2) - 42,000 \\ + 40,000(A/F,i,2)$$

Machine Age, Years	M&O Costs, \$ per Year	Salvage Value at End of Year, \$
1	-42,000	60,000
2	-47,000	40,000
3	-49,000	31,000
4	-50,000	24,000
5	-52,000	15,000
6	-54,000	10,000
7	-63,000	10,000
8	-67,000	10,000

11.54 An engineer with Haliburton calculated the AW values shown for a presently owned machine by using estimates he obtained from the vendor and company records.

Retention Period, Years	AW Value, \$ per Year
1	-92,000
2	-81,000
3	-87,000
4	-89,000
5	-95,000

A challenger has an economic service life of 7 years with an AW of \$-86,000 per year. Assume that used machines like the one presently owned will always be available and that the MARR is 12% per year. If all future costs remain as estimated for the analysis, the company should purchase the challenger:

- (a) Now
- (b) After 2 years
- (c) After 3 years
- (d) Never

11.55 The annual worth values for a defender, which can be replaced with a similar used asset, and a challenger are estimated. The defender should be replaced:

- (a) Now
- (b) 1 year from now
- (c) 2 years from now
- (d) 3 years from now

Number of Years Retained	AW Value, \$ per Year	
	Defender	Challenger
1	-14,000	-21,000
2	-13,700	-18,000
3	-16,900	-13,100
4	-17,000	-15,600
5	-18,000	-17,500

CASE STUDY

WILL THE CORRECT ESL PLEASE STAND?

Background

New pumper system equipment is under consideration by a Gulf Coast chemical processing plant. One crucial pump moves highly corrosive liquids from specially lined tanks on intercoastal barges into storage and preliminary refining facilities dockside. Because of the variable quality of the raw chemical and the high pressures imposed on the pump chassis and impellers, a close log is maintained on the number of hours per year that the pump operates. Safety records and pump component deterioration are considered critical control points for this system. As currently planned, rebuild and M&O cost estimates are increased accordingly when cumulative operating time reaches the 6000-hour mark.

Information

You are the safety engineer at the plant. Estimates made for this pump are as follows:

First cost:	\$-800,000
Rebuild cost:	\$-150,000 whenever 6000 cumulative hours are logged. Each rework will cost 20% more than the previous one. A maximum of 3 rebuilds is allowed.
M&O costs:	\$-25,000 for each year 1 through 4 \$-40,000 per year starting the year after the first rebuild, plus 15% per year thereafter
MARR:	10% per year

Based on previous logbook data, the current estimates for number of operating hours per year are as follows:

Year	Hours per Year
1	500
2	1500
3 on	2000

Case Study Questions

1. Determine the economic service life of the pump. How does the ESL compare with the maximum allowed rebuilds?
2. The *plant superintendent* told you, the safety engineer, that only one rebuild should be planned for, because these types of pumps usually have their minimum-cost life before the second rebuild. Determine a market value for this pump that will force the ESL to be 6 years. Comment on the practicality of ESL = 6 years, given the MV calculated.
3. In a separate conversation, the *line manager* told you to not plan for a rebuild after 6000 hours, because the pump will be replaced after a total of 10,000 hours of operation. The line manager wants to know what the base AOC in year 1 can be to make the ESL 6 years. He also told you to assume now that the 15% growth rate applies from year 1 forward. How does this base AOC value compare with the rebuild cost after 6000 hours?
4. What do you think of these suggestions from the plant superintendent and the line manager?

Independent Projects with Budget Limitation



LEARNING OUTCOMES

Purpose: Select independent projects for funding when there is a limitation on the amount of capital available for investment.

SECTION	TOPIC	LEARNING OUTCOME
12.1	Capital rationing	<ul style="list-style-type: none">Explain how a capital budgeting problem is approached.
12.2	Equal-life projects	<ul style="list-style-type: none">Use PW-based capital budgeting to select from several equal-life independent projects.
12.3	Unequal-life projects	<ul style="list-style-type: none">Use PW-based capital budgeting to select from several unequal-life independent projects.
12.4	Linear programming	<ul style="list-style-type: none">Set up the linear programming model and use the Solver spreadsheet tool to select projects.
12.5	Ranking options	<ul style="list-style-type: none">Use the internal rate of return (IROR) and profitability index (PI) to rank and select from independent projects.



In most of the previous economic comparisons, the alternatives have been mutually exclusive; only one could be selected. If the projects are not mutually exclusive, they are categorized as independent of one another, as discussed at the beginning of Chapter 5. Now we learn techniques to select from several independent projects. It is possible to select any number of projects from none (do nothing) to all viable projects.

There is virtually always some upper limit on the amount of capital available for investment in new projects. This limit is considered as each independent project is economically evaluated. The techniques applied are called *capital budgeting methods*, also referred to as capital rationing. They determine the economically best rationing of initial investment capital among independent projects based upon different measures, such as PW, ROR, and the profitability index. These three are discussed here.

12.1 An Overview of Capital Rationing among Projects ● ● ●



Investment capital is a scarce resource for all corporations; thus there is virtually always a limited amount to be distributed among competing investment opportunities. When a corporation has several options for placing investment capital, a “reject or accept” decision must be made for each project. Effectively, each option is independent of other options, so the evaluation is performed on a project-by-project basis. Selection of one project does not impact the selection decision for any other project. This is the fundamental difference between mutually exclusive alternatives and independent projects.

The term **project** is used to identify each independent option. We use the term **bundle** to identify a collection of independent projects. The term *mutually exclusive alternative* continues to identify a project when only one may be selected from several.

There are two exceptions to purely independent projects: A *contingent project* is one that has a condition placed upon its acceptance or rejection. Two examples of contingent projects A and B are as follows: A cannot be accepted unless B is accepted; and A can be accepted in lieu of B, but both are not needed. A *dependent project* is one that must be accepted or rejected based on the decision about another project(s). For example, B must be accepted if both A and C are accepted. In practice, these complicating conditions can be bypassed by forming packages of related projects that are economically evaluated themselves as independent projects along with the remaining, unconditioned projects.

A **capital budgeting study** has the following characteristics:

- Several independent projects are identified, and net cash flow estimates are available.
- Each project is either selected entirely or not selected; that is, partial investment in a project is not possible.
- A stated budgetary constraint restricts the total amount available for investment. Budget constraints may be present for the first year only or for several years. This investment limit is identified by the symbol b .
- The objective is to maximize the return on the investments using a measure of worth, such as the PW value.



Independent project selection

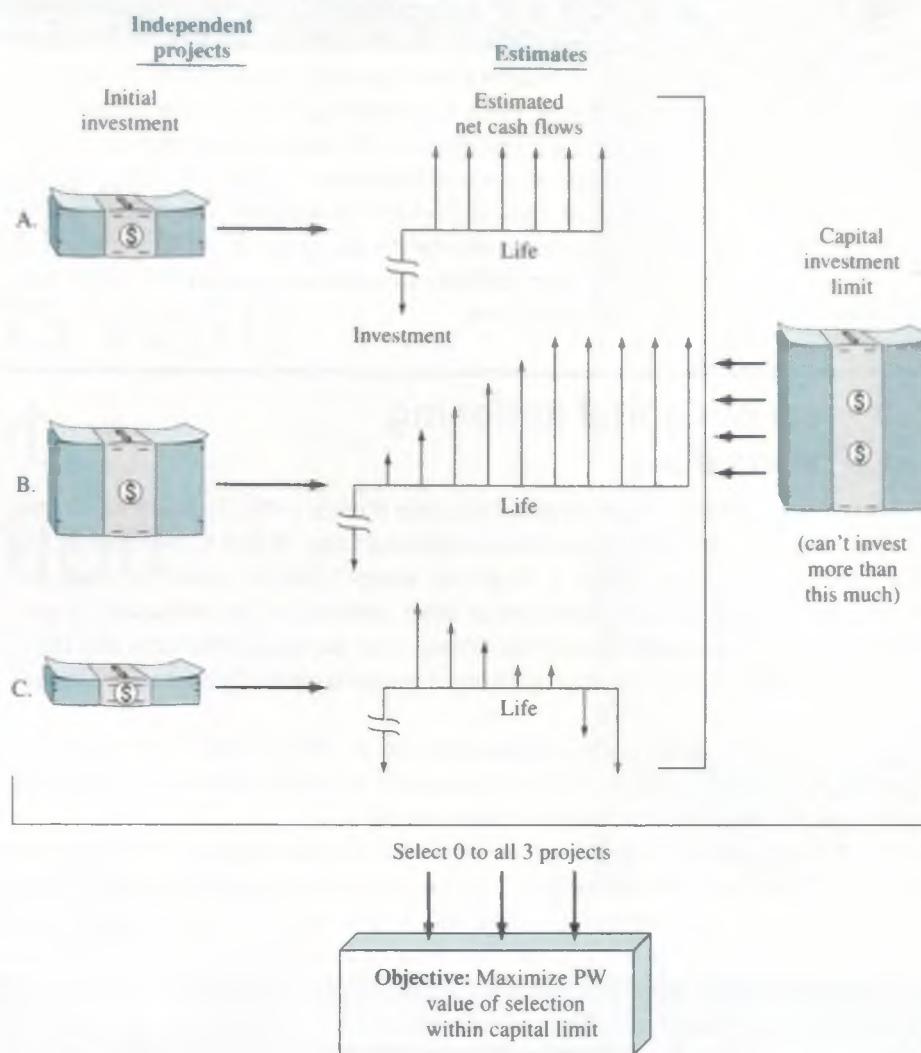
Limited budget

By nature, independent projects are usually quite different from one another. For example, in the public sector, a city government may develop several projects to choose from: drainage, city park, metro rail, and an upgraded public bus system. In the private sector, sample projects may be a new warehousing facility, expanded product base, improved quality program, upgraded information system, automation, and acquisition of another firm. The size of the study can range from only four or five projects to a complex study involving 50 to 100 projects. The typical capital budgeting problem is illustrated in Figure 12–1. For each independent project there is an initial investment, project life, and estimated net cash flows that can include a salvage value.

Present worth analysis using the capital budgeting process is the recommended method to select projects. The general selection guideline is as follows:

Accept projects with the **best PW values** determined at the MARR over the project life, provided the investment capital limit is not exceeded.

Figure 12–1
Basic characteristics of a capital budgeting study.



This guideline is not different from that used for selection in previous chapters for independent projects. As before, each project is compared with the do-nothing project; that is, incremental analysis between projects is not necessary. The primary difference now is that the amount of money available to invest is limited, thus the title *capital budgeting* or *rationing*. A specific solution procedure that incorporates this constraint is needed.

Previously, PW analysis had the requirement of equal service between alternatives. This assumption is not necessary for capital rationing, because there is no life cycle of a project beyond its estimated life. Rather, the selection guideline has the following implied assumption.

When the present worth at the MARR over the respective project life is used to select projects, the reinvestment assumption is that all positive net cash flows are reinvested at the MARR from the time they are realized until the **end of the longest-lived project**.

This fundamental assumption is demonstrated to be correct at the end of Section 12.3, which treats PW-based capital rationing for unequal-life projects.

Another dilemma of capital rationing among independent projects concerns the flexibility of the capital investment limit b . The limit may marginally disallow an acceptable project that is next in line for acceptance. For example, assume project A has a positive PW value at the MARR. If A will cause the capital limit of \$5,000,000 to be exceeded by only \$1000, should A be included in the PW analysis? Commonly, a capital investment limit is somewhat flexible, so project A would usually be included. However, in the examples here, we will not exceed a stated investment limit.

As we learned earlier (Sections 1.9 and 10.1), the rate of return on the first unfunded project is an **opportunity cost**. The lack of capital to fund the next project defines the ROR level that is forgone. The opportunity cost will vary with each set of independent projects evaluated, but over time it provides information to fine-tune the MARR and other measures used by the company in future evaluations.



Equal-service requirement



Opportunity cost

It is common to rank independent projects and select from them based on measures other than PW at the MARR. Two are the *internal rate of return (IROR)*, discussed in Chapter 7, and the *profitability index (PI)*, also called the *present worth index (PWI)*, introduced in Chapter 9. Neither of these measures guarantees an optimal PW-based selection. The capital budgeting process, covered in the next 3 sections, does find the optimal solution for PW values. We recommend use of this PW-based technique; however, it should be recognized that both PI and IROR usually provide excellent, near-optimal selections, and they both work very well when the number of projects is large. Application of these two measures is presented in Section 12.5.

12.2 Capital Rationing Using PW Analysis of Equal-Life Projects ● ● ●



To select from projects that have the same expected life while investing no more than the limit b , first formulate all **mutually exclusive bundles**—one project at a time, two at a time, etc. Each feasible bundle must have a total investment that does not exceed b . One of these bundles is the do-nothing (DN) project. The total number of bundles for m projects is 2^m . The number increases rapidly with m . For $m = 4$, there are $2^4 = 16$ bundles, and for $m = 16$, $2^{16} = 65,536$ bundles. Then the PW of each bundle is determined at the MARR. The bundle with the largest PW value is selected.

To illustrate the development of mutually exclusive bundles, consider these four projects with equal lives.

Project	Initial Investment, \$
A	-10,000
B	-5,000
C	-8,000
D	-15,000

If the investment limit is $b = \$25,000$, of the 16 bundles, there are 12 feasible ones to evaluate. The bundles ABD, ACD, BCD, and ABCD have investment totals that exceed \$25,000. The viable bundles are shown below.

Projects	Total Initial Investment, \$	Projects	Total Initial Investment, \$
A	-10,000	AD	-25,000
B	-5,000	BC	-13,000
C	-8,000	BD	-20,000
D	-15,000	CD	-23,000
AB	-15,000	ABC	-23,000
AC	-18,000	Do nothing	0

The procedure to conduct a capital budgeting study using PW analysis is as follows:

1. Develop all mutually exclusive bundles with a total initial investment that does not exceed the capital limit b .
2. Sum the net cash flows NCF_{jt} for all projects in each bundle j ($j = 1, 2, \dots, 2^m$) and each year t ($t = 1, 2, \dots, n_j$). Refer to the initial investment of bundle j at time $t = 0$ as NCF_{j0} .
3. Compute the present worth value PW_j for each bundle at the MARR.

$$PW_j = PW \text{ of bundle net cash flows} - \text{initial investment}$$

$$= \sum_{t=1}^{n_j} NCF_{jt}(P/F, i, t) - NCF_{j0} \quad [12.1]$$

4. Select the bundle with the (numerically) largest PW_j value.

Selecting the maximum PW_j means that this bundle has a PW value larger than any other bundle. Any bundle with $PW_j < 0$ is discarded, because it does not produce a return of at least the MARR.

EXAMPLE 12.1

The projects review committee of Microsoft has \$20 million to allocate next year to new software product development. Any or all of five projects in Table 12-1 may be accepted. All amounts are in \$1000 units. Each project has an expected life of 9 years. Select the project(s) if a 15% return is expected.

TABLE 12-1 Five Equal-Life Independent Projects (\$1000 Units)

Project	Initial Investment, \$	Annual Net Cash Flow, \$	Project Life, Years
A	-10,000	2870	9
B	-15,000	2930	9
C	-8,000	2680	9
D	-6,000	2540	9
E	-21,000	9500	9

Solution

Use the procedure above with $b = \$20,000$ to select one bundle that maximizes present worth. Remember the units are in \$1000.

1. There are $2^5 = 32$ possible bundles. The eight bundles that require no more than \$20,000 in initial investments are described in columns 2 and 3 of Table 12-2. The \$21,000 investment for E eliminates it from all bundles.

TABLE 12-2 Summary of Present Worth Analysis of Equal-Life Independent Projects (\$1000 Units)

Bundle j (1)	Projects Included (2)	Initial Investment NCF_{j0} , \$ (3)	Annual Net Cash Flow NCF_j , \$ (4)	Present Worth PW_j , \$ (5)
1	A	-10,000	2,870	+3,694
2	B	-15,000	2,930	-1,019
3	C	-8,000	2,680	+4,788
4	D	-6,000	2,540	+6,120
5	AC	-18,000	5,550	+8,482
6	AD	-16,000	5,410	+9,814
7	CD	-14,000	5,220	+10,908
8	Do nothing	0	0	0

2. The bundle net cash flows, column 4, are the sum of individual project net cash flows.
3. Use Equation [12.1] to compute the present worth for each bundle. Since the annual NCF and life estimates are the same for a bundle, PW_j reduces to

$$PW_j = NCF_j(P/A, 15\%, 9) - NCF_{j0}$$

4. Column 5 of Table 12-2 summarizes the PW_j values at $i = 15\%$. Bundle 2 does not return 15%, since $PW_2 < 0$. The largest is $PW_7 = \$10,908$; therefore, invest \$14 million in C and D. This leaves \$6 million uncommitted.

This analysis assumes that the \$6 million not used in this initial investment will return the MARR by placing it in some other, unspecified investment opportunity.

12.3 Capital Rationing Using PW Analysis of Unequal-Life Projects ● ● ●



Usually independent projects do not have the same expected life. As stated in Section 12.1, the PW method for solution of the capital budgeting problem assumes that each project will last for the period of the longest-lived project n_L . Additionally, reinvestment of any positive net cash flows is assumed to be at the MARR from the time they are realized until the end of the longest-lived project, that is, from year n_j through year n_L . Therefore, use of the **LCM of lives is not necessary**, and it is correct to use Equation [12.1] to select bundles of unequal-life projects by PW analysis using the procedure of the previous section.

EXAMPLE 12.2

For MARR = 15% per year and $b = \$20,000$, select from the following independent projects. Solve by hand and by spreadsheet.

Project	Initial Investment, \$	Annual Net Cash Flow, \$	Project Life, Years
A	-8,000	3870	6
B	-15,000	2930	9
C	-8,000	2680	5
D	-8,000	2540	4

Solution by Hand

The unequal-life values make the net cash flows vary over a bundle's life, but the selection procedure is the same as above. Of the $2^4 = 16$ bundles, 8 are economically feasible. Their PW values by Equation [12.1] are summarized in Table 12-3. As an illustration, for bundle 7:

$$PW_7 = -16,000 + 5220(P/A, 15\%, 4) + 2680(P/F, 15\%, 5) = \$235$$

Select bundle 5 (projects A and C) for a \$16,000 investment.

TABLE 12-3 Present Worth Analysis for Unequal-Life Independent Projects, Example 12.2

Bundle J (1)	Project (2)	Initial Investment, NCF_{j0} , \$ (3)	Net Cash Flows		Present Worth PW_j , \$ (6)
			Year t (4)	NCF_{jt} , \$ (5)	
1	A	-8,000	1-6	3,870	+6,646
2	B	-15,000	1-9	2,930	-1,019
3	C	-8,000	1-5	2,680	+984
4	D	-8,000	1-4	2,540	-748
5	AC	-16,000	1-5	6,550	+7,630
			6	3,870	
6	AD	-16,000	1-4	6,410	+5,898
			5-6	3,870	
7	CD	-16,000	1-4	5,220	+235
			5	2,680	
8	Do nothing	0		0	0

Solution by Spreadsheet

Figure 12-2 presents a spreadsheet with the same information as in Table 12-3. It is necessary to initially develop the mutually exclusive bundles manually and total net cash flows each year using each project's NCF. The NPV function is used to determine PW for each bundle j over its respective life. Bundle 5 (projects A and C) has the largest PW value (row 16).

A	B	C	D	E	F	G	H	I
1	MARR = 15%							= D7+E7
3	Bundle	1	2	3	4	5	6	
4	Projects	A	B	C	D	AC	AD	CD
5	Year			Net cash flows, NCF, \$				Do nothing
6	0	-8,000	-15,000	-8,000	-8,000	-16,000	-16,000	0
7	1	3,870	2,930	2,680	2,540	6,550	6,410	5,220
8	2	3,870	2,930	2,680	2,540	6,550	6,410	5,220
9	3	3,870	2,930	2,680	2,540	6,550	6,410	5,220
10	4	3,870	2,930	2,680	2,540	6,550	6,410	5,220
11	5	3,870	2,930	2,680		6,550	3,870	2,680
12	6	3,870	2,930			3,870	3,870	0
13	7		2,930					0
14	8		2,930					0
15	9		2,930					0
16	PW @ 15%	6,646	-1,019	984	-748	7,630	5,898	235
17								
18							= NPV(\$B\$1,F7:F15) + F6	
19								

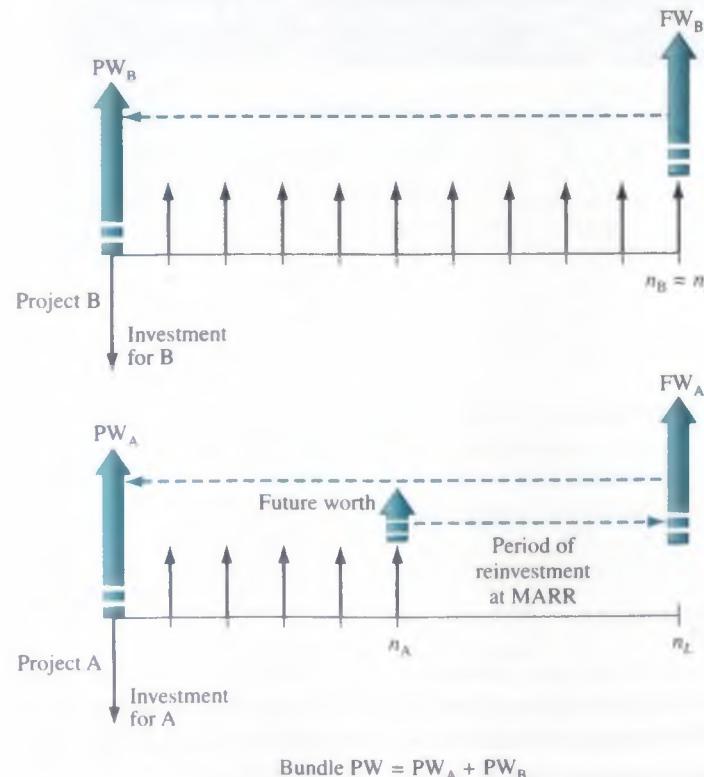
Figure 12-2

Computation of PW values for independent project selection, Example 12.2.

The rest of this section will help you understand why solution of the capital budgeting problem by PW evaluation using Equation [12.1] is correct. The following logic verifies the assumption of reinvestment at the MARR for all net positive cash flows when project lives are unequal. Refer to Figure 12-3, which uses the general layout of a two-project bundle. Assume each project has the same net cash flow each year. The P/A factor is used for PW computation. Define n_L as the life of the longer-lived project. At the end of the shorter-lived project, the bundle has a total future worth of $NCF_j(F/A, \text{MARR}, n_j)$ as determined for each project. Now, assume reinvestment at the MARR from year n_{j+1} through year n_L (a total of $n_L - n_j$ years). The assumption of the return at the MARR is important; this PW approach does not necessarily select the correct projects if the return is not at the MARR. The results are the two future worth arrows in year n_L in Figure 12-3. Finally,

Figure 12-3

Representative cash flows used to compute PW for a bundle of two independent unequal-life projects by Equation [12.1].



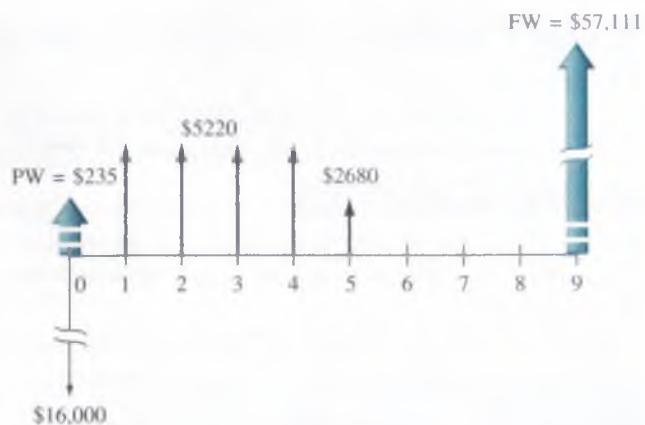


Figure 12-4
Initial investment and cash flows for bundle 7, projects C and D, Example 17.2.

compute the bundle PW value in the initial year. This is the bundle $PW = PW_A + PW_B$. In general form, the bundle j present worth is

$$PW_j = NCF_j(F/A, MARR, n_j)(F/P, MARR, n_L - n_j)(P/F, MARR, n_L) \quad [12.2]$$

Substitute the symbol i for the MARR, and use the factor formulas to simplify.

$$\begin{aligned} PW_j &= NCF_j \frac{(1+i)^{n_j} - 1}{i} (1+i)^{n_L - n_j} \frac{1}{(1+i)^{n_L}} \\ &= NCF_j \left[\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_L}} \right] \\ &= NCF_j (P/A, i, n_j) \end{aligned} \quad [12.3]$$

Since the bracketed expression in Equation [12.3] is the $(P/A, i, n_j)$ factor, computation of PW_j for n_j years assumes reinvestment at the MARR of all positive net cash flows until the longest-lived project is completed in year n_L .

To demonstrate numerically, consider bundle $j = 7$ in Example 12.2. The evaluation is in Table 12-3, and the net cash flow is pictured in Figure 12-4. Calculate the future worth in year 9, which is the life of the longest-lived project (B).

$$FW = 5220(F/A, 15\%, 4)(F/P, 15\%, 5) + 2680(F/P, 15\%, 4) = \$57,111$$

The present worth at the initial investment time is

$$PW = -16,000 + 57,111(P/F, 15\%, 9) = \$235$$

The PW value is the same as PW_7 in Table 12-3 and Figure 12-2. This demonstrates the reinvestment assumption for positive net cash flows. If this assumption is not realistic, the PW analysis must be conducted using the *LCM of all project lives*.

12.4 Capital Budgeting Problem Formulation Using Linear Programming ●●●

The procedure discussed above requires the development of mutually exclusive bundles one project at a time, two projects at a time, etc., until all 2^m bundles are developed and each one is compared with the capital limit b . As the number of independent projects increases, this process becomes prohibitively cumbersome and unworkable. Fortunately, the capital budgeting problem can be stated in the form of a linear programming model. The problem is formulated using the integer linear programming (ILP) model, which means simply that all relations are linear and that the variable x can take on only integer values. In this case, the variables can only take on the values 0 or 1, which makes it a special case called the 0-or-1 ILP model. The formulation in words follows.

Maximize: Sum of PW of net cash flows of independent projects.

Constraints:

- Capital investment constraint is that the sum of initial investments must not exceed a specified limit.
- Each project is completely selected or not selected.

For the math formulation, define b as the capital investment limit, and let x_k ($k = 1$ to m projects) be the variables to be determined. If $x_k = 1$, project k is completely selected; if $x_k = 0$, project k is not selected. Note that the subscript k represents each independent project, not a mutually exclusive bundle.

If the sum of PW of the net cash flows is Z , the math programming formulation is as follows:

$$\begin{aligned} \text{Maximize: } & \sum_{k=1}^{k=m} PW_k x_k = Z \\ \text{Constraints: } & \sum_{k=1}^{k=m} NCF_{k0} x_k \leq b \\ & x_k = 0 \text{ or } 1 \quad \text{for } k = 1, 2, \dots, m \end{aligned} \quad [12.4]$$

The PW_k of each project is calculated using Equation [12.1] at $MARR = i$.

$$\begin{aligned} PW_k &= \text{PW of project net cash flows for } n_k \text{ years} \\ &= \sum_{t=1}^{t=n_k} NCF_{kt} (P/F, i, t) - NCF_{k0} \end{aligned} \quad [12.5]$$

Computer solution is accomplished by a linear programming software package which treats the ILP model. Also, Excel and its optimizing tool **Solver** can be used to develop the formulation and select the projects. The Solver tool is similar to Goal Seek with significantly more capabilities. For example, Solver allows the target cell to be maximized, minimized, or set to a specific value. This means that the function Z in Equation [12.4] can be maximized. Also, multiple changing cells can be identified, so the 0 or 1 value of the unknowns can be determined. Additionally, with the added capability to include constraints, the investment limit b and 0-or-1 requirement on the unknowns in Equation [12.4] can be accommodated. Solver is explained in detail in Appendix A, and Example 12.3 illustrates its use.

EXAMPLE 12.3

Review Example 12.2. (a) Formulate the capital budgeting problem using the math programming model presented in Equation [12.4]. (b) Select the projects using Solver.

Solution

(a) Define the subscript $k = 1$ through 4 for the four projects, which are relabeled as 1, 2, 3, and 4. The capital investment limit is $b = \$20,000$ in Equation [12.4].

$$\begin{aligned} \text{Maximize: } & \sum_{k=1}^{k=4} PW_k x_k = Z \\ \text{Constraints: } & \sum_{k=1}^{k=4} NCF_{k0} x_k \leq 20,000 \\ & x_k = 0 \text{ or } 1 \quad \text{for } k = 1, 2, 3, 4 \end{aligned}$$

Now, substitute the PW_k and NCF_{k0} values from Table 12–3 into the model. Plus signs are used for all values in the budget constraint. We have the complete 0-or-1 ILP formulation.

$$\begin{aligned} \text{Maximize: } & 6646x_1 - 1019x_2 + 984x_3 - 748x_4 = Z \\ \text{Constraints: } & 8000x_1 + 15,000x_2 + 8000x_3 + 8000x_4 \leq 20,000 \\ & x_1, x_2, x_3, \text{ and } x_4 = 0 \text{ or } 1 \end{aligned}$$

The maximum PW is \$7630, and the solution from Example 12.2 is written as

$$x_1 = 1 \quad x_2 = 0 \quad x_3 = 1 \quad x_4 = 0$$

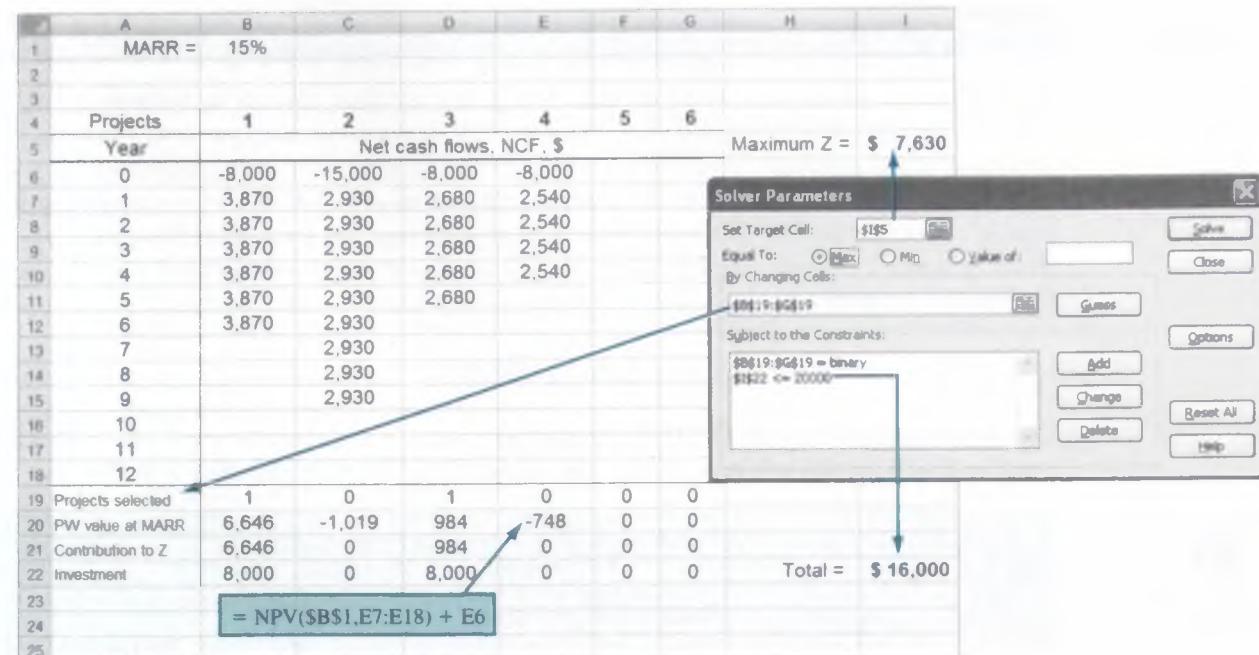


Figure 12–5

Spreadsheet and Solver template configured to solve a capital budgeting problem, Example 12.3.

- (b) Figure 12–5 presents a spreadsheet template developed to select from six or fewer independent projects with 12 years or less of net cash flow estimates per project. The spreadsheet template can be expanded in either direction if needed. Figure 12–5 (inset) shows the Solver parameters set to solve this example for four projects and an investment limit of \$20,000. The descriptions below and the cell tag identify the contents of the rows and cells in Figure 12–5, and their linkage to Solver parameters.

Rows 4 and 5: Projects are identified by numbers to distinguish them from spreadsheet column letters. Cell I5 is the expression for Z, the sum of the PW values for the projects. This is the target cell for Solver to maximize.

Rows 6 to 18: These are initial investments and net cash flow estimates for each project. Zero values that occur after the life of a project need not be entered; however, any \$0 estimates that occur during a project's life must be entered.

Row 19: The entry in each cell is 1 for a selected project and 0 if not selected. These are the changing cells for Solver. Since each entry must be 0 or 1, a binary constraint is placed on all row 19 cells in Solver, as shown in Figure 12–5. When a problem is to be solved, it is best to initialize the spreadsheet with 0s for all projects. Solver will find the solution to maximize Z.

Row 20: The NPV function is used to find the PW for each net cash flow series. The NPV functions are developed for any project with a life up to 12 years at the MARR entered in cell B1.

Row 21: When a project is selected, the contribution to the Z function is shown by multiplying rows 19 and 20.

Row 22: This row shows the initial investment for the selected projects. Cell I22 is the total investment. This cell has the budget limitation placed on it by the constraint in Solver. In this example, the constraint is I22 \leq \$20,000.

To solve the example, set all values in row 19 to 0, set up the Solver parameters as described above, and click on Solve. (Since this is a linear model, the Solver options choice "Assume Linear Model" may be checked, if desired.) If needed, further directions on saving the solution, making changes, etc., are available in Appendix A, Section A.5, and on the Excel help function.

For this problem, the selection is projects 1 and 3 with Z = \$7630, the same as determined previously, and \$16,000 of the \$20,000 limit is invested.

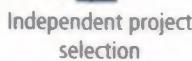


12.5 Additional Project Ranking Measures ● ● ●

The PW-based method of solving a capital budgeting problem covered in Sections 2.2 to 2.4 provides an optimal solution that maximizes the PW of the competing projects. However, it is very common in industrial, professional, and government settings to learn that the rate of return is the basis for ranking projects. The **internal rate of return (IROR)**, as we learned in Section 7.2, is determined by setting a PW or AW relation equal to zero and solving for i^* — the IROR. Using a PW basis and the estimated net cash flow (NCF) series for each project j , solve for i^* in the relation

$$0 = \sum_{t=1}^{n_j} NCF_{jt}(P/F, i^*, t) - NCF_{j0} \quad [12.6]$$

This is the same as Equation [12.1] set equal to zero with $i = i^*$ as the unknown. The spreadsheet function RATE or IRR will provide the same answer. The selection guideline is as follows:



Once the project ranking by IROR is complete, select all projects in order without exceeding the investment limit b .

If there is no budget limit, select all projects that have $IROR \geq MARR$.

The ordering of projects using *IROR ranking* may differ from the *PW-based ranking* we used in previous sections. This can occur because IROR ranking maximizes the overall rate of return, not necessarily the PW value. The use of IROR ranking is illustrated in the next example.

Another common ranking method is the **profitability index (PI)** that we learned in Section 9.2. This is a “bang for the buck” measure that provides a sense of getting the most for the investment dollar over the life of the project. (Refer to Section 9.2 for more details.) When utilized for project ranking, it is often called the **present worth index (PWI)**; however, we will use the PI term for consistency, and because there are other ways to mathematically define measures also referred to as a PW index. The PI measure is defined as

$$\frac{\text{PW of net cash flows}}{\text{PW of initial investment}} = \frac{\sum_{t=1}^{n_j} NCF_{jt}(P/F, i, t)}{|NCF_{j0}|} \quad [12.7]$$

Note that the denominator includes only the initial investment, and it is its *absolute value* that is used. The numerator has only cash flows that result from the project for years 1 through its life n_j . Salvage value, if there is one estimated, is incorporated into the numerator. Similar to the previous case, the selection guideline is as follows:



Once the project ranking by PI is complete, select all projects in order without exceeding the investment limit b .

If there is no budget limit, select all projects that have $PI \geq 1.0$.

Depending upon the project NCF estimates, the *PI ranking* can differ from the *IROR ranking*. Example 12.4 compares results using the different ranking methods—IROR, PI, and PW values. None of these results are incorrect; they simply maximize different measures, as you will see. The use of IROR, PI, or other measures is common when there are a large number of projects, because the PW basis (when solved by hand, Solver, or ILP software) becomes increasingly cumbersome as the number of possible mutually exclusive bundles grows using the formula 2^n . Additionally, greater complexity is introduced when dependent and contingent projects are involved.

EXAMPLE 12.4

Georgia works as a financial analyst in the Management Science Group of General Electronics. She has been asked to recommend which of the five projects detailed in Table 12-4 should be funded if the MARR is 15% per year and the investment budget limit for next year is a firm

TABLE 12-4 IROR, PI, and PW Values for Five Projects, Example 12.4

Projects	1	2	3	4	5
Investment, NCF_0 , \$1000	−8,000	−15,000	−8,000	−8,000	−5,000
NCF, \$1000 per year	4,000	2,900	2,700	2,500	2,600
Life n_j , years	6	9	5	4	3
IROR, %	44.5	12.8	20.4	9.6	26.0
PI	1.89	0.92	1.13	0.89	1.19
PW at 15%, \$1000	7,138	−1,162	1,051	−863	936

\$18 million. She has confirmed the computations and is ready to do the ranking and make the selection. Help her by doing the following:

- (a) Use the IROR measure to rank and select projects.
- (b) Use the PI measure to rank and select projects.
- (c) Use the PW measure to rank and select projects.
- (d) Compare the selected projects by the three methods and determine which one will maximize the overall ROR value of the \$18 million budget.

Solution

Refer to Table 12-5 for the ranking, cumulative investment for each project, and selection based on the ranking and budget limit $b = \$18$ million.

- (a) Ranking by overall IROR values indicates that projects 1 and 5 should be selected with \$13 million of the \$18 million budget expended. The remaining \$5 million is assumed to be invested at the MARR of 15% per year.
- (b) As an example, the PI for project 1 is calculated using Equation [12.7].

$$\begin{aligned} PI_1 &= \frac{4000(P/A, 15\%, 6)}{1 - 8000} \\ &= 15,138/8,000 \\ &= 1.89 \end{aligned}$$

Ranking and projects selected are the same as in the IROR-based analysis. Again, the remaining \$5 million is assumed to generate a return of MARR = 15% per year.

- (c) Ranking by PW value results in a different selection from that of IROR and PI ranking. Projects 1 and 3, rather than 1 and 5, are selected for a total PW = \$8.189 million and an investment of \$16 million. The remaining \$2 million is assumed to earn 15% per year.
- (d) IROR and PI rankings result in projects 1 and 5 being selected. PW ranking results in the selection of projects 1 and 3. With MARR = 15%, the \$18 million will earn at the following ROR values. In \$1000 units,

Projects 1 and 5	NCF, year 0:	\$−13,000
	NCF, years 1–3:	\$6,600
	NCF, years 4–6:	\$4,000

$$0 = -13,000 + 6600(P/A, i, 3) + 4000(P/A, i, 3)(P/F, i, 3)$$

TABLE 12-5 Ranking of Projects by Different Measures, Example 12.4 (Monetary units in \$1000)

Ranking by IROR			Ranking by PI			Ranking by PW		
IROR, % (1)	Project (2)	Cumulative Investment, \$ (3)	PI (4)	Project (5)	Cumulative Investment, \$ (6)	PW, \$ (7)	Project (8)	Cumulative Investment, \$ (9)
44.5	1	8,000	1.89	1	8,000	7,138	1	8,000
26.0	5	13,000	1.19	5	13,000	1,051	3	16,000
20.4	3	21,000	1.13	3	21,000	936	5	21,000
12.8	2		0.92	2		−863	4	
9.6	4		0.89	4		−1,162	2	

By IRR function, the rate of return is 39.1%. The overall return on the entire budget is

$$\begin{aligned} \text{ROR} &= [39.1(13,000) + 15.0(5000)]/18,000 \\ &= 32.4\% \end{aligned}$$

Projects 1 and 3	NCF, year 0:	\$-16,000
	NCF, years 1–5:	\$6,700
	NCF, year 6:	\$4,000

$$0 = -16,000 + 6700(P/A,i,5) + 4000(P/F,i,6)$$

By IRR function, the rate of return is 33.5%. The overall return on the entire budget is

$$\begin{aligned} \text{ROR} &= [33.5(16,000) + 15.0(2000)]/18,000 \\ &= 31.4\% \end{aligned}$$

In conclusion, the IROR and PI selections maximize the overall rate of return at 32.4%. The PW selection maximizes the PW value at \$8.189 (i.e., 7.138 + 1.051) million, as determined from Table 12–5, column 7.

CHAPTER SUMMARY

Investment capital is always a scarce resource, so it must be rationed among competing projects using economic and noneconomic criteria. Capital budgeting involves proposed projects, each with an initial investment and net cash flows estimated over the life of the project. The fundamental capital budgeting problem has specific characteristics (Figure 12–1).

- Selection is made from among independent projects.
- Each project must be accepted or rejected as a whole.
- Maximizing the present worth of the net cash flows is the objective.
- The total initial investment is limited to a specified maximum.

The present worth method is used for evaluation. To start the procedure, formulate all mutually exclusive bundles that do not exceed the investment limit, including the do-nothing bundle. There are a maximum of 2^m bundles for m projects. Calculate the PW at MARR for each bundle, and select the bundle with the largest PW value. Reinvestment of net positive cash flows at the MARR is assumed for all projects with lives shorter than that of the longest-lived project.

The capital budgeting problem may be formulated as a linear programming problem to select projects directly in order to maximize the total PW. Excel's Solver tool solves this problem by spreadsheet.

Projects can be ranked and selected on bases other than PW values. Two measures are the internal rate of return (IROR) and the profitability index (PI), also called the PW index. The ordering of projects may differ between the various ranking bases since different measures are optimized. When there are a large number of projects, the IROR basis is commonly applied in industrial settings.

PROBLEMS

Understanding Capital Rationing

- 12.1 Define the following terms: bundle, contingent project, dependent project.
- 12.2 State two assumptions made when doing capital rationing using a PW analysis for unequal-life projects.
- 12.3 For independent projects identified as A, B, C, D, E, F, and G, how many mutually exclusive bundles can be formed?

- 12.4 For independent projects identified as W, X, Y, and Z, develop all of the mutually exclusive bundles. Projects X and Y perform the same function with different processes; both should not be selected.

- 12.5 Five projects have been identified for possible implementation by a company that makes dry ice blasters—machines that propel tiny dry ice pellets at supersonic speeds so they flash-freeze and then lift grime, paint, rust, mold, asphalt, and

other contaminants off in-place machines and a wide range of surfaces. The total present worth of the initial investment for each project is shown. Determine which bundles are possible, provided the budget limitation is (a) \$34,000 and (b) \$45,000.

Project	PW of Investment at 15%, \$
L	28,000
M	11,000
N	43,000
O	38,000
P	6,000

- 12.6 Four independent projects (1, 2, 3, and 4) are proposed for investment by Perfect Manufacturing, Inc. List all the acceptable mutually exclusive bundles based on the following selection restrictions developed by the department of engineering production:

Project 2 can be selected only if project 3 is selected.
Projects 1 and 4 should not both be selected; they are essentially duplicates.

- 12.7 Develop all acceptable mutually exclusive bundles for the four independent projects described below if the investment limit is \$400 and the following project selection restriction applies: Project 1 can be selected only if both projects 3 and 4 are selected.

Project	Investment, \$
1	-100
2	-150
3	-75
4	-235

Selection from Independent Projects

- 12.8 Listed below are bundles, each comprised of three independent proposals for which the PW has been estimated. Select the best bundle if the capital budget limit is \$45,000 and the MARR is the cost of capital, which is 9% per year.

Proposal Bundle	Initial Investment, For Bundle, \$	PW at 9%, \$
1	-18,000	-1,400
2	-26,000	8,500
3	-34,000	7,100
4	-41,000	10,500

- 12.9 The general manager for Woodsome Appliance Company Plant #A14 in Mexico City has four independent projects that she can fund this year to

improve surface durability on stainless steel products. The project costs and 18% per year PW values are as shown. What projects should be accepted if the budget limit is (a) no limit and (b) \$55,000?

Project	Initial Investment, \$	PW at 18%, \$
1	-15,000	-400
2	-25,000	8500
3	-20,000	500
4	-40,000	9600

- 12.10 The capital fund for research project investment at SummaCorp is limited to \$100,000 for next year. Select any or all of the following proposals if a MARR of 15% per year is established by the board of directors.

Project	Initial Investment, \$	Annual Net Cash Flow, \$/Year	Life, Years	Salvage Value, \$
I	-25,000	6,000	4	4,000
II	-30,000	9,000	4	-1,000
III	-50,000	15,000	4	20,000

- 12.11 An electrical engineer at GE is assigned the responsibility to determine how to invest up to \$100,000 in none, some, or all of the following independent proposals. Use (a) hand and (b) spreadsheet-based PW analysis and a 15% per year return requirement to help this engineer make the best decision from a purely economic perspective.

Project	Initial Investment, \$	Annual Net Cash Flow, \$/Year	Life, Years	Salvage Value, \$
A	-25,000	6,000	4	4,000
B	-20,000	9,000	4	0
C	-50,000	15,000	4	20,000

- 12.12 Dwayne has four independent vendor proposals to contract the nationwide oil recycling services for Ford Corporation manufacturing plants. All combinations are acceptable, except that vendors B and C cannot both be chosen. Revenue sharing of recycled oil sales with Ford is a part of the requirement. Develop all possible mutually exclusive bundles under the additional following restrictions and select the best projects. The corporate MARR is 10% per year.

- (a) A maximum of \$4 million can be spent.
- (b) A larger budget of \$5.5 million is allowed, but no more than two vendors can be selected.
- (c) There is no limit on spending.

Vendor	Initial Investment, \$	Life, Years	Annual Net Revenue, \$ per Year
A	−1.5 million	8	360,000
B	−3.0 million	10	600,000
C	−1.8 million	5	620,000
D	−2.0 million	4	630,000

- 12.13 Use the PW method at 8% per year to select up to three projects from the four available ones if no more than \$20,000 can be invested. Estimated lives and annual net cash flows vary.

Project	Initial Investment, \$	Net Cash Flow, \$ per Year					
		1	2	3	4	5	6
W	−5,000	1,000	1,700	1,800	2,500	2,000	
X	−8,000	900	950	1,000	1,050	10,500	
Y	−8,000	4,000	3,000	1,000	500	500	2,000
Z	−10,000	0	0	0	17,000		

- 12.14 Charlie's Garage has \$70,000 to spend on new equipment that may increase revenue for his car repair shop. Use the PW method to determine which of these independent investments are financially acceptable at 6% per year, compounded monthly. All are expected to last 3 years.

Feature	Installed Cost, \$	Estimated Added Revenue, \$ per Month
Diagnostics system	−45,000	2200
Exhaust analyzer	−30,000	2000
Hybrid engine tester	−22,000	1500

- 12.15 (a) Determine which of the following independent projects should be selected for investment if \$315,000 is available and the MARR is 10% per year. Use the PW method to evaluate mutually exclusive bundles to make the selection. (Solve by hand or spreadsheet as instructed.)

Project	Initial Investment, \$	NCF, \$ per Year	Life, Years
A	−100,000	50,000	8
B	−125,000	24,000	8
C	−120,000	75,000	8
D	−220,000	39,000	8
E	−200,000	82,000	8

- (b) If the five projects are mutually exclusive alternatives, perform the present worth analysis and select the best alternative.

- 12.16 Use the following analysis of five independent projects to select the best, if the capital limitation is (a) \$30,000, (b) \$52,000, and (c) unlimited.

Project	Initial Investment, \$	Life, Years	PW at 12% per Year, \$
S	−15,000	6	8,540
A	−25,000	8	12,325
M	−10,000	6	3,000
E	−25,000	4	10
H	−40,000	12	15,350

- 12.17 The independent project estimates below have been developed by the engineering and finance managers. The corporate MARR is 8% per year, and the capital investment limit is \$4 million. Select the economically best projects using the PW method and (a) hand solution and (b) spreadsheet solution.

Project	Cost, \$ M	Life, Years	NCF, \$ per Year
1	−1.5	8	360,000
2	−3.0	10	600,000
3	−1.8	5	520,000
4	−2.0	4	820,000

- 12.18 Use the PW method to evaluate four independent projects. Select as many as three of the four projects. The MARR is 12% per year, and up to \$16,000 in capital investment funds are available.

Investment, \$	Life, years	Project			
		1	2	3	4
−5000	5	−8,000	−9,000	−10,000	
		5	5	3	4

Year	NCF Estimates, \$ per Year			
	1	2	3	4
1	1000	500	5000	0
2	1700	500	5000	0
3	2400	500	2000	0
4	3000	500		17,000
5	3800	10,500		

12.19 Work Problem 12.18 using a spreadsheet.

- 12.20 A capital rationing problem is defined for you as follows: Three projects are to be evaluated at a MARR of 12.5% per year. No more than \$3.0 million can be invested.

- (a) Use a spreadsheet to select from the independent projects.
 (b) If the life of project 3 can be increased from 5 to 10 years for the same \$1 million investment, use Goal Seek to determine the NCF in year 1 for project 3 alone to have the same PW as the best bundle in part (a). All other estimates remain the same. With these new NCF and life estimates, what are the best projects for investment?

Project	Investment, \$ M	Life, Years	Estimated NCF, \$ per Year	
			Year 1	Gradient after Year 1
1	-0.9	6	250,000	-5000
2	-2.1	10	485,000	+5000
3	-1.0	5	200,000	+20%

Linear Programming and Capital Budgeting

- 12.21 Formulate the linear programming model, develop a spreadsheet, and solve the capital rationing problem in Example 12.1 (a) as presented and (b) using an investment limit of \$13 million.
- 12.22 Use linear programming and a spreadsheet-based solution technique to select from the independent unequal-life projects in Problem 12.17.
- 12.23 Solve the capital budgeting problem in Problem 12.20(a), using the linear programming model and a spreadsheet.
- 12.24 Johnson and Johnson is expanding its first-aid products line for individuals allergic to latex. The research team comprised of doctors, engineers, and chemists has proposed the four projects estimated in Problem 12.18. Develop the linear programming model and use a spreadsheet to select the projects to be funded. The MARR is 12% per year, and the budget limit is \$16 million. All monetary values are in \$1000 units.
- 12.25 Using the estimates in Problem 12.18 and repeated spreadsheet solution of the capital budgeting problem for budget limits ranging from $b = \$5000$ to $b = \$25,000$, develop a spreadsheet chart that plots b versus the value of Z .

Other Ranking Measures

- 12.26 A newly proposed project has a first cost of \$325,000 and estimated annual income of \$60,000 per year for 8 years.
- Determine the IRR, PI, and PW values if the MARR is 15% per year.
 - Is the project economically justified?
- 12.27 An engineer at Delphi Systems is considering the projects below, all of which can be considered to last indefinitely. The company's MARR is 13% per year.
- Determine which projects should be selected on the basis of IRR if the budget limitation is \$39,000.
 - What is the overall rate of return if the money not invested in projects is assumed to earn a rate of return equal to the MARR?

Project	First Cost, \$	Net Income, \$ per Year	Rate of Return, %
A	-20,000	4,000	20.0
B	-10,000	1,900	19.0
C	-15,000	2,600	17.3
D	-70,000	10,000	14.3
E	-50,000	6,000	12.0

- 12.28 The five independent projects shown below are under consideration for implementation by KNF Neuberger, Inc. The company's MARR is 15% per year.
- Determine which projects should be undertaken on the basis of IRR if the budget limitation is \$97,000. (Solve by hand or spreadsheet as instructed.)
 - Determine the overall rate of return if the money not invested in projects is assumed to earn a rate of return equal to the MARR.

Project	First Cost, \$	Annual Income, \$ per Year	Project Life, Years
L	-30,000	9,000	10
A	-15,000	4,900	10
N	-45,000	11,100	10
D	-70,000	9,000	10
T	-40,000	10,000	10

- 12.29 An estimated 6 billion gallons of clean drinking water disappear each day across the United States due to aging, leaky pipes and water mains before it gets to the consumer or industrial user. The American Society of Civil Engineers (ASCE) has teamed with municipalities, counties, and several excavation companies to develop robots that can travel through mains, detect leaks, and repair many of them immediately. Four proposals have been received for funding. There is a \$100 million limit on capital funding, and the MARR is established at 12% per year.
- Use the IRR method to rank and determine which of the four independent projects should be funded. Solve by spreadsheet, unless instructed to use hand solution.
 - Determine the rate of return for the combination of projects selected in part (a). Is it economically justified?
 - Determine the overall rate of return for the \$100 million with the projects selected in part (a). Assume that excess funds are invested at the MARR.

Project	First Cost, \$ M	Estimated Annual Savings, \$M per Year	Project Life, Years
W	-12	5.0	3
X	-25	7.3	4
Y	-45	12.1	6
Z	-60	9.0	8

12.30 Determine the profitability index at 10% per year interest for a project that has a first cost of \$200,000 in year 0 and \$200,000 in year 2, an annual operating cost of \$80,000 per year, revenue of \$170,000 per year, and a salvage value of \$60,000 after its 5-year life.

12.31 The budget limit is \$120,000, and the interest rate is 10% per year. All projects have a 10-year life. Use (a) the PI method and (b) the IRR method to rank and select from the independent projects. (c) Are different projects selected using the two methods?

Project	First Cost, \$	Net Income, \$ per Year	IRR, %
A	-18,000	4,000	18.0
B	-15,000	2,800	13.3
C	-35,000	12,600	34.1
D	-60,000	13,000	17.3
E	-50,000	8,000	9.6

12.32 The six independent projects shown below are under consideration by Peyton Packing under budget-constrained conditions. The company always has more projects to engage in than it has capital to fund projects. Therefore, it uses a relatively high MARR of 25% per year. Since all projects are considered long-term ventures, the company uses an infinite period for their life. Determine which projects the company should fund and the total investment for a capital budget of \$700,000 if the capital budgeting method used is (a) the IRR method, (b) the PI method, and (c) the PW method.

Project	First Cost, \$	Estimated Annual Income, \$ per Year
F	-200,000	54,000
G	-120,000	21,000
H	-250,000	115,000
I	-370,000	205,000
J	-50,000	26,000
K	-9,000	2,100

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

12.33 A project that has a condition associated with its acceptance or rejection is known as:

- (a) A mutually exclusive alternative
- (b) A contingent project
- (c) A dependent project
- (d) Both (b) and (c)

12.34 An assumption inherent in capital budgeting is that all positive net cash flows are reinvested at the MARR from the time they are realized until:

- (a) The end of the shortest-lived project
- (b) The end of the longest-lived project
- (c) The end of the respective project that generated the cash flow
- (d) The average of the lives of the projects included in the bundle

12.35 All of the following are correct when a capital budgeting problem is solved using the 0-1 integer linear programming model except:

- (a) Partial investment in a project is acceptable.
- (b) The objective is to maximize the present worth of the investments.
- (c) Budget constraints may be present for the first year only or for several years.
- (d) Contingent and dependent project restrictions may be considered.

12.36 All of the following are true when formulating mutually exclusive bundles of independent projects except:

- (a) One of the bundles is the do-nothing project.
- (b) A bundle may consist of only one project.
- (c) The capital limit may be exceeded as long as it is exceeded by less than 3%.
- (d) A bundle may include contingent and dependent projects.

12.37 When there are 5 projects involved in a capital budgeting study, the maximum number of bundles that can be formulated is:

- (a) 6
- (b) 10
- (c) 31
- (d) 32

12.38 For a capital limit of \$25,000, the selected independent projects are:

- (a) P only
- (b) Q only
- (c) R only
- (d) P and R

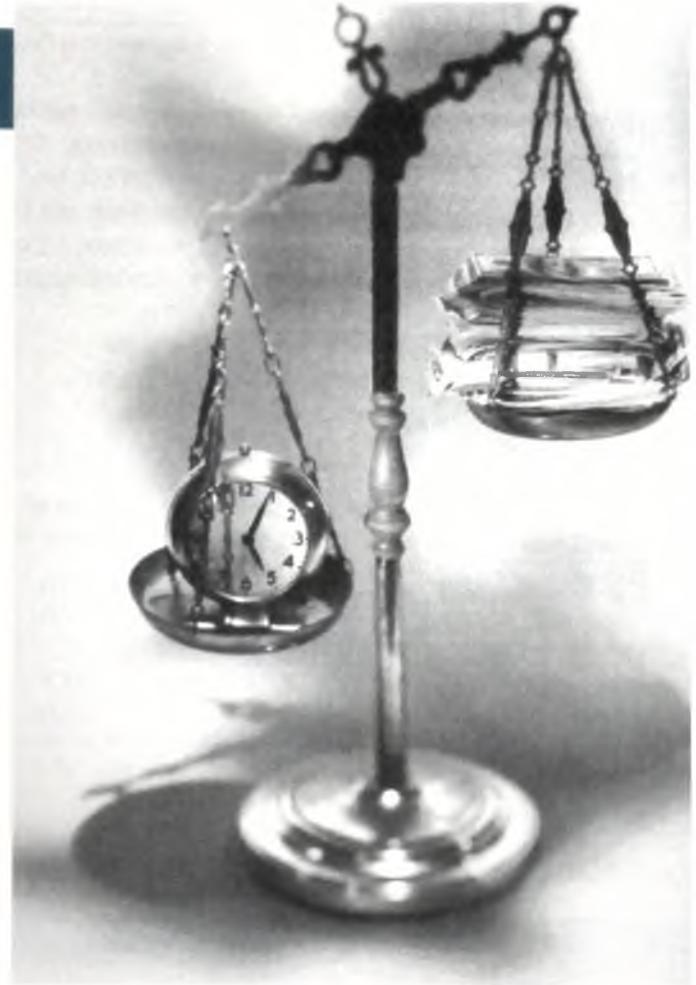
Project	Initial Investment, \$	Life, Years	PW at 12%, \$
P	-15,000	6	8,540
Q	-25,000	8	12,325
R	-10,000	6	3,000
S	-25,000	4	10
T	-40,000	12	15,350

- 12.39 The independent projects shown below are under consideration for possible implementation by Renishaw Inc. of Hoffman Estates, Rhode Island. If the company's MARR is 14% per year and it uses the IRR method of capital budgeting, the projects it should select under a budget limitation of \$105,000 are:
- A, B, and C
 - A, B, and D
 - B, C, and D
 - A, C, and D

Project	First Cost, \$	Annual Income, \$ per Year	Rate of Return, %
A	-20,000	4,000	20.0
B	-10,000	1,900	19.0
C	-15,000	2,600	17.3
D	-70,000	10,000	14.3
E	-50,000	6,000	12.0

- 12.40 For a project that requires an initial investment of \$26,000 and yields \$10,000 per year for 4 years, the PI at an interest rate of 10% per year is closest to:
- 1.03
 - 1.22
 - 1.38
 - 1.56

Breakeven and Payback Analysis



LEARNING OUTCOMES

Purpose: Determine the breakeven for one or two alternatives and calculate the payback period with and without a return required.

SECTION	TOPIC	LEARNING OUTCOME
13.1	Breakeven point	<ul style="list-style-type: none">Determine the breakeven point for one parameter.
13.2	Two alternatives	<ul style="list-style-type: none">Calculate the breakeven point of a parameter and use it to select between two alternatives.
13.3	Payback period	<ul style="list-style-type: none">Determine the payback period of a project at $i = 0\%$ and $i > 0\%$. Illustrate the cautions when using payback analysis.
13.4	Spreadsheet	<ul style="list-style-type: none">Answer breakeven and payback questions that are best resolved by spreadsheet and Goal Seek tools.

B

reakeven analysis is performed to determine the value of a variable or parameter of a project or alternative that makes two elements equal, for example, the sales volume that will equate revenues and costs. A breakeven study is performed for two alternatives to determine when either alternative is equally acceptable. Breakeven analysis is commonly applied in **make-or-buy decisions** when a decision is needed about the source for manufactured components, services, etc.

Payback analysis determines the required minimum life of an asset, process, or system to recover the initial investment. There are two types of payback: return ($i > 0\%$) and no return ($i = 0\%$). Payback analysis should not be considered the final decision maker; it is used as a **screening tool** or to provide **supplemental information** for a PW, AW, or other analysis. These aspects are discussed in depth in this chapter.

Breakeven and payback studies use estimates that are considered to be certain; that is, if the estimated values are expected to vary enough to possibly change the outcome, another study is necessary using different estimates. If the variable of interest is allowed to vary, the approaches of sensitivity analysis (Chapter 18) should be used. Additionally, if probability and risk assessment are considered, the tools of simulation (Chapter 19) can be used to supplement the static nature of a breakeven or payback study.

13.1 Breakeven Analysis for a Single Project ● ● ●

When one of the engineering economy symbols— P , F , A , i , or n —is not known or not estimated, a breakeven quantity can be determined by setting an equivalence relation for PW or AW equal to zero. This form of breakeven analysis has been used many times so far. For example, we have solved for the rate of return i^* , found the replacement value for a defender, and determined the P , F , A , or salvage value S at which a series of cash flow estimates return a specific MARR. Methods used to determine the quantity include

Direct solution by hand if only one factor is present (say, P/A) or only single amounts are estimated (for example, P and F)

Trial and error by hand or calculator when multiple factors are present

Spreadsheet when cash flow and other estimates are entered into cells and used in resident functions (PV, FV, RATE, IRR, NPV, PMT, and NPER) or tools (Goal Seek and Solver).

We now concentrate on the determination of the **breakeven quantity Q_{BE} for one parameter or decision variable**. For example, the variable may be a design element to minimize cost or the production level needed to realize revenues that exceed costs by 10%.

Breakeven analysis finds the value of a parameter that **makes two elements equal**. The breakeven point Q_{BE} is determined from mathematical relations, e.g., product revenue and costs or materials supply and demand or other parameters that involve the parameter Q . Breakeven analysis is fundamental to evaluations such as make-buy decisions.



The unit of the parameter Q may vary widely: units per year, cost per kilogram, hours per month, percentage of full plant capacity, etc.

Figure 13–1a presents different shapes of a revenue relation identified as R . A linear revenue relation is commonly assumed, but a nonlinear relation is often more realistic. It can model an increasing per unit revenue with larger volumes (curve 1 in Figure 13–1a) or a decreasing per unit revenue that usually prevails at higher quantities (curve 2).

Costs, which may be linear or nonlinear, usually include two components—fixed and variable—as indicated in Figure 13–1b.

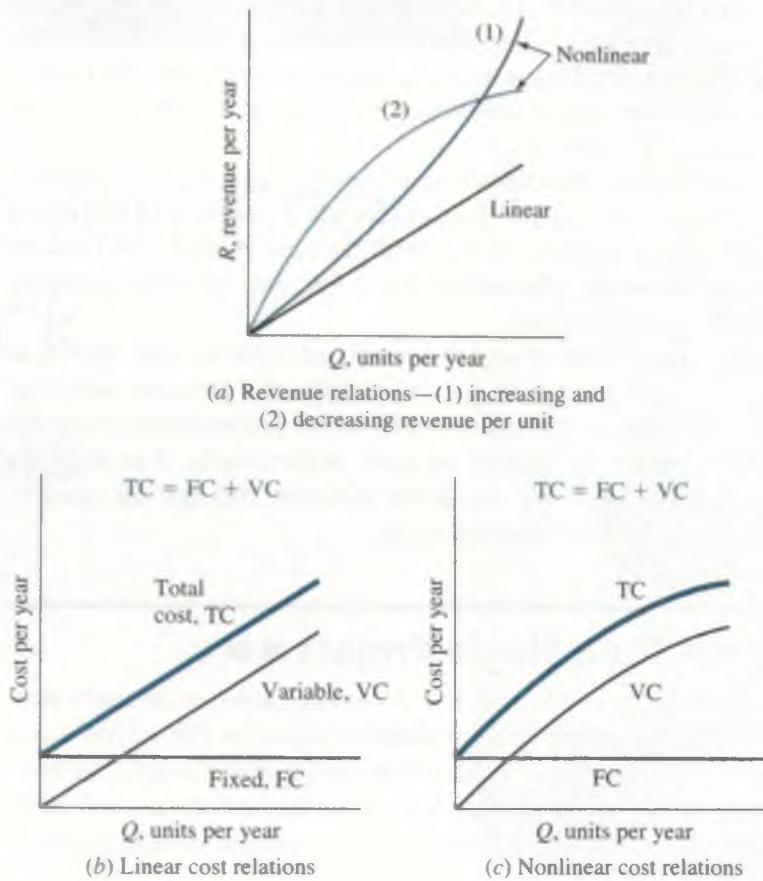
Fixed costs (FC). These include costs such as buildings, insurance, fixed overhead, some minimum level of labor, equipment capital recovery, and information systems.

Variable costs (VC). These include costs such as direct labor, materials, indirect costs, contractors, marketing, advertisement, and warranty.

The **fixed-cost** component is essentially constant for all values of the variable, so it does not vary for a large range of operating parameters, such as production level or workforce size. Even if no units are produced, fixed costs are incurred at some threshold level. Of course, this situation

Figure 13–1

Linear and nonlinear revenue and cost relations.



cannot last long before the plant must shut down to reduce fixed costs. Fixed costs are reduced through improved equipment, information systems, and workforce utilization; and less costly fringe benefit packages, subcontracting specific functions, and so on.

Variable costs change with production level, workforce size, and other parameters. It is usually possible to decrease variable costs through better product design, manufacturing efficiency, improved quality and safety, and higher sales volume.

When FC and VC are added, they form the **total cost relation TC**. Figure 13–1b illustrates the TC relation for linear fixed and variable costs. Figure 13–1c shows a general TC curve for a nonlinear VC in which unit variable costs decrease as the quantity level rises.

At a specific but unknown value Q of the decision variable, the revenue R and total cost TC relations will intersect to identify the breakeven point Q_{BE} (Figure 13–2). If $Q > Q_{BE}$, there is a predictable **profit**; but if $Q < Q_{BE}$, there is a loss. For linear models of R and VC, the greater the quantity, the larger the profit. Profit is calculated as

$$\begin{aligned} \text{Profit} &= \text{revenue} - \text{total cost} \\ &= R - TC \\ &= R - (FC + VC) \end{aligned} \quad [13.1]$$

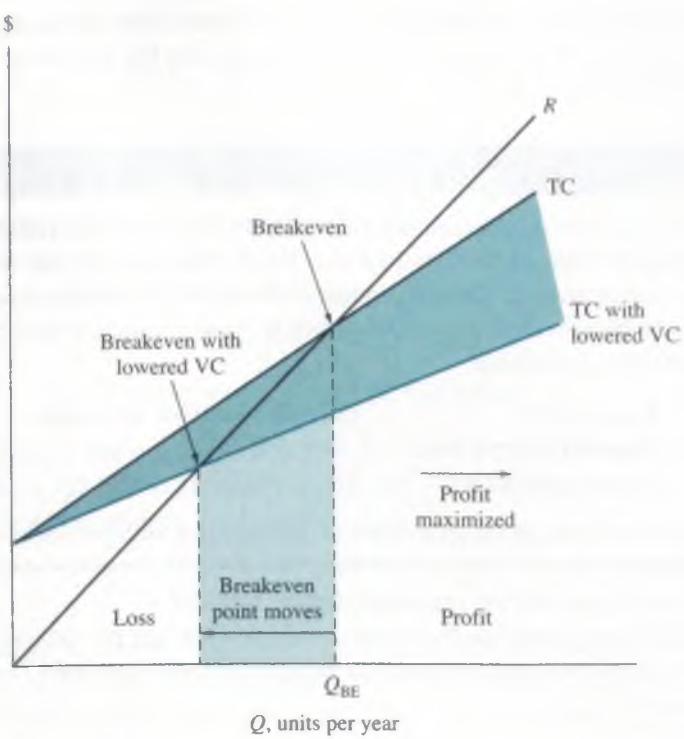
A relation for the breakeven point may be derived when revenue and total cost are *linear functions of quantity Q* by setting the relations for R and TC equal to each other, indicating a profit of zero.

$$\begin{aligned} R &= TC \\ rQ &= FC + vQ \end{aligned}$$

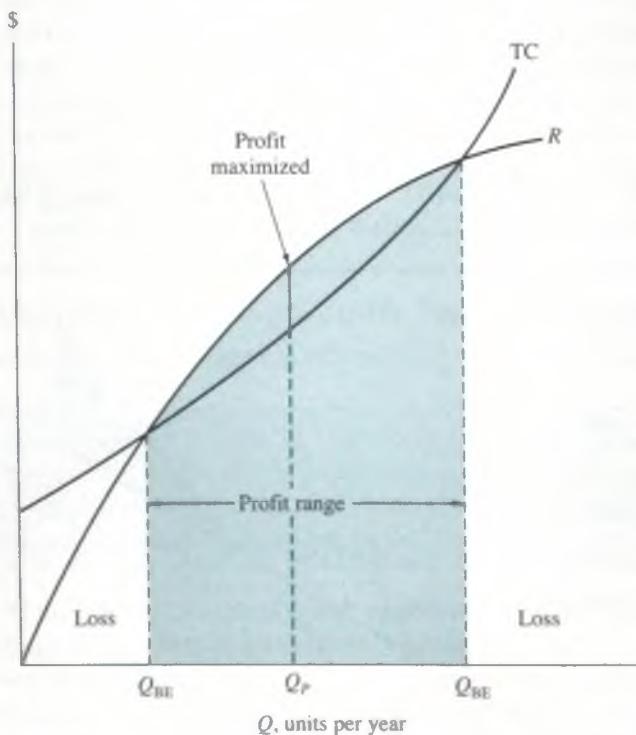
where r = revenue per unit
 v = variable cost per unit

Solve for the breakeven quantity $Q = Q_{BE}$ for linear R and TC functions.

$$Q_{BE} = \frac{FC}{r - v} \quad [13.2]$$

**Figure 13–2**

Effect on the breakeven point when the variable cost per unit is reduced.

**Figure 13–3**

Breakeven points and maximum profit point for a nonlinear analysis.

The breakeven graph is an important management tool because it is easy to understand and may be used in decision making in a variety of ways. For example, if the variable cost per unit is reduced, then the TC line has a smaller slope (Figure 13–2) and the breakeven point will decrease. This is an advantage because the smaller the value of Q_{BE} , the greater the profit for a given amount of revenue. A similar analysis is possible for fixed VC and increased levels of production, as shown in the next example.

If nonlinear R or TC models are used, there may be more than one breakeven point. Figure 13–3 presents this situation for two breakeven points. The **maximum profit** occurs at Q_P between the two breakeven points where the distance between the R and TC relations is greatest.

Of course, no static R and TC relations—linear or nonlinear—are able to estimate exactly the revenue and cost amounts over an extended period of time. But the breakeven point is an excellent target for planning purposes.

EXAMPLE 13.1

Indira Industries is a major producer of diverter dampers used in the gas turbine power industry to divert gas exhausts from the turbine to a side stack, thus reducing the noise to acceptable levels for human environments. Normal production level is 60 diverter systems per month, but due to significantly improved economic conditions in Asia, production is at 72 per month. The following information is available.

Fixed costs	$FC = \$2.4$ million per month
Variable cost per unit	$v = \$35,000$
Revenue per unit	$r = \$75,000$

- (a) How does the increased production level of 72 units per month compare with the current breakeven point?
- (b) What is the current profit level per month for the facility?
- (c) What is the difference between the revenue and variable cost per damper that is necessary to break even at a significantly reduced monthly production level of 45 units, if fixed costs remain constant?

Solution

- (a) Use Equation [13.2] to determine the breakeven number of units. All dollar amounts are in \$1000 units.

$$\begin{aligned} Q_{BE} &= \frac{FC}{r - v} \\ &= \frac{2400}{75 - 35} = 60 \text{ units per month} \end{aligned}$$

Figure 13–4 is a plot of R and TC lines. The breakeven value is 60 damper units. The increased production level of 72 units is above the breakeven value.

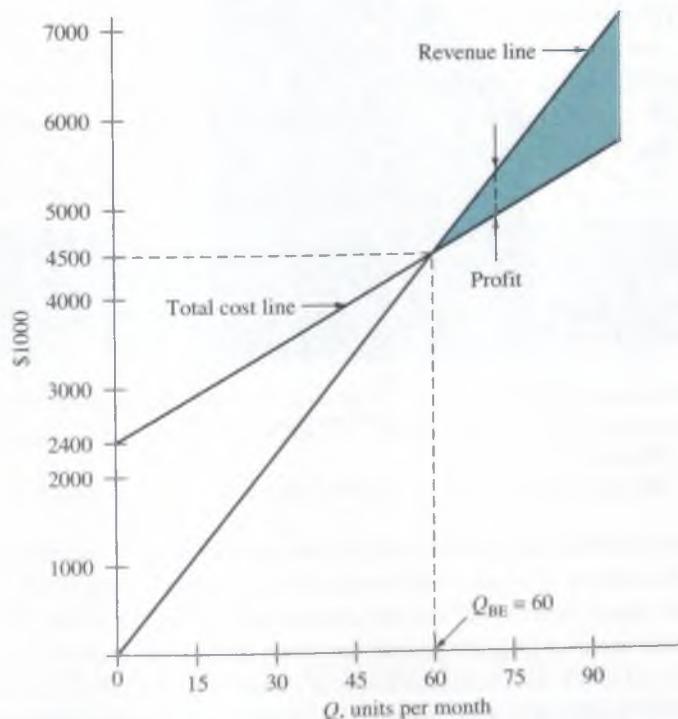


Figure 13–4
Breakeven graph, Example 13.1.

(b) To estimate profit (in \$1000 units) at $Q = 72$ units per month, use Equation [13.1].

$$\begin{aligned} \text{Profit} &= R - TC = rQ - (FC + vQ) \\ &= (r - v)Q - FC \\ &= (75 - 35)72 - 2400 \\ &= \$480 \end{aligned} \quad [13.3]$$

There is a profit of \$480,000 per month currently.

(c) To determine the required difference $r - v$, use Equation [13.3] with profit = 0, $Q = 45$, and $FC = \$2.4$ million. In \$1000 units,

$$\begin{aligned} 0 &= (r - v)(45) - 2400 \\ r - v &= \frac{2400}{45} = \$53.33 \text{ per unit} \end{aligned}$$

The spread between r and v must be \$53,330. If v stays at \$35,000, the revenue per damper must increase from \$75,000 to \$88,330 (i.e., \$35,000 + \$53,330) just to break even at a production level of $Q = 45$ per month.

In some circumstances, breakeven analysis performed on a per unit basis is more meaningful. The value of Q_{BE} is still calculated using Equation [13.2], but the TC relation is divided by Q to obtain an expression for cost per unit, also termed *average cost per unit* C_u .

$$C_u = \frac{TC}{Q} = \frac{FC + vQ}{Q} = \frac{FC}{Q} + v \quad [13.4]$$

At the breakeven quantity $Q = Q_{BE}$, the revenue per unit is exactly equal to the cost per unit. If graphed, the FC per unit term in Equation [13.4] takes on the shape of a hyperbola.

The breakeven point for a project with one unknown variable can always be determined by equating revenue and total cost. This is the same as setting profit equal to zero in Equation [13.1]. It may be necessary to perform some dimensional analysis initially to obtain the correct revenue and total cost relations in order to use the same dimension for both relations, for example, \$ per unit, miles per month, or units per year.

13.2 Breakeven Analysis Between Two Alternatives ● ● ●

Now we consider breakeven analysis between two mutually exclusive alternatives.

Breakeven analysis determines the value of a common variable or parameter between two alternatives. Equating the two PW or AW relations determines the breakeven point. Selection of the alternative is different depending upon two facts: **slope of the variable cost curve** and **the parameter value relative to the breakeven point**.

The parameter can be the interest rate i , first cost P , annual operating cost (AOC), or any parameter. We have already performed breakeven analysis between alternatives on several parameters. For example, the incremental ROR value (Δi^*) is the breakeven rate between alternatives. If the MARR is lower than Δi^* , the extra investment of the larger-investment alternative is justified. In Section 11.6, the replacement value (RV) of a defender was determined. If the market value is larger than RV, the decision should favor the challenger.

Often breakeven analysis involves revenue or cost variables common to both alternatives, such as price per unit, operating cost, cost of materials, or labor cost. Figure 13–5 illustrates the breakeven concept for two alternatives with linear cost relations. The fixed cost of alternative 2 is greater than that of alternative 1. However, alternative 2 has a smaller variable cost, as indicated by its lower slope. The intersection of the total cost lines locates the breakeven point, and the variable cost establishes the slope. Thus, if the number of units of the common variable is greater than the breakeven amount, alternative 2 is selected, since the total cost will be lower. Conversely, an anticipated level of operation below the breakeven point favors alternative 1.



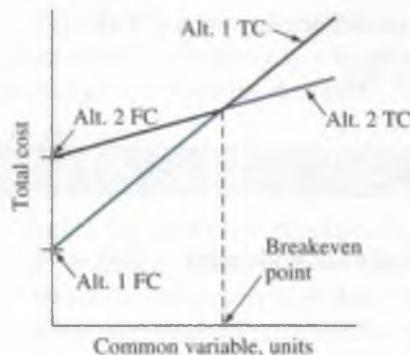
Online Presentation

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Breakeven

Figure 13–5

Breakeven between two alternatives with linear cost relations.



Instead of plotting the total costs of each alternative and estimating the breakeven point graphically, it may be easier to calculate the breakeven point numerically using engineering economy expressions for the PW or AW at the MARR. The AW is preferred when the variable units are expressed on a yearly basis, and AW calculations are simpler for alternatives with unequal lives. The following steps determine the breakeven point of the common variable and the slope of a linear total cost relation.

1. Define the common variable and its dimensional units.
2. Develop the PW or AW relation for each alternative as a function of the common variable.
3. Equate the two relations and solve for the breakeven value of the variable.

Selection between alternatives is based on this guideline:

If the anticipated level of the common variable is **below** the breakeven value, select the alternative with the higher variable cost (larger slope).

If the level is **above** the breakeven point, select the alternative with the lower variable cost. (Refer to Figure 13–5.)

EXAMPLE 13.2

A small aerospace company is evaluating two alternatives: the purchase of an automatic feed machine and a manual feed machine for a finishing process. The auto feed machine has an initial cost of \$23,000, an estimated salvage value of \$4000, and a predicted life of 10 years. One person will operate the machine at a rate of \$12 per hour. The expected output is 8 tons per hour. Annual maintenance and operating cost is expected to be \$3500.

The alternative manual feed machine has a first cost of \$8000, no expected salvage value, a 5-year life, and an output of 6 tons per hour. However, three workers will be required at \$8 per hour each. The machine will have an annual maintenance and operation cost of \$1500. All projects are expected to generate a return of 10% per year. How many tons per year must be finished to justify the higher purchase cost of the auto feed machine?

Solution

Use the steps above to calculate the breakeven point between the two alternatives.

1. Let x represent the number of tons per year.
2. For the auto feed machine, the annual variable cost is

$$\begin{aligned}\text{Annual VC} &= \frac{\$12}{\text{hour}} \frac{1 \text{ hour}}{8 \text{ tons}} \frac{x \text{ tons}}{\text{year}} \\ &= 1.5x\end{aligned}$$

The VC is developed in dollars per year. The AW expression for the auto feed machine is

$$\begin{aligned}\text{AW}_{\text{auto}} &= -23,000(A/P, 10\%, 10) + 4000(A/F, 10\%, 10) - 3500 - 1.5x \\ &= \$-6992 - 1.5x\end{aligned}$$

Similarly, the annual variable cost and AW for the manual feed machine are

$$\begin{aligned}\text{Annual VC} &= \frac{\$8}{\text{hour}} (3 \text{ operators}) \frac{1 \text{ hour}}{6 \text{ tons}} \frac{x \text{ tons}}{\text{year}} \\ &= 4x \\ \text{AW}_{\text{manual}} &= -8000(A/P, 10\%, 5) - 1500 - 4x \\ &= \$-3610 - 4x\end{aligned}$$

3. Equate the two cost relations and solve for x .

$$\begin{aligned}\text{AW}_{\text{auto}} &= \text{AW}_{\text{manual}} \\ -6992 - 1.5x &= -3610 - 4x \\ x &= 1353 \text{ tons per year}\end{aligned}$$

If the output is expected to exceed 1353 tons per year, purchase the auto feed machine, since its VC slope of 1.5 is smaller than the manual feed VC slope of 4.

The breakeven analysis approach is commonly used for **make-or-buy decisions**. This means the company contracts to *buy* the product or service from the *outside*, or *makes* it *within* the company. The alternative to buy usually has no fixed cost and a larger variable cost than the option to make. Where the two cost relations cross is the make-buy decision quantity. Amounts above this indicate that the item should be made, not purchased outside.

EXAMPLE 13.3

Guardian is a national manufacturing company of home health care appliances. It is faced with a make-or-buy decision. A newly engineered lift can be installed in a car trunk to raise and lower a wheelchair. The steel arm of the lift can be purchased internationally for \$3.50 per unit or made in-house. If manufactured on site, two machines will be required. Machine A is estimated to cost \$18,000, have a life of 6 years, and have a \$2000 salvage value; machine B will cost \$12,000, have a life of 4 years, and have a \$-500 salvage value (carry-away cost). Machine A will require an overhaul after 3 years costing \$3000. The annual operating cost for machine A is expected to be \$6000 per year and for machine B is \$5000 per year. A total of four operators will be required for the two machines at a rate of \$12.50 per hour per operator. In a normal 8-hour period, the operators and two machines can produce parts sufficient to manufacture 1000 units. Use a MARR of 15% per year to determine the following.

- (a) Number of units to manufacture each year to justify the in-house (make) option.
- (b) The maximum capital expense justifiable to purchase machine A, assuming all other estimates for machines A and B are as stated. The company expects to produce 10,000 units per year.

Solution

- (a) Use steps 1 to 3 stated previously to determine the breakeven point.

1. Define x as the number of lifts produced per year.
2. There are variable costs for the operators and fixed costs for the two machines for the make option.

$$\begin{aligned}\text{Annual VC} &= (\text{cost per unit})(\text{units per year}) \\ &= \frac{4 \text{ operators}}{1000 \text{ units}} \frac{\$12.50}{\text{hour}} (8 \text{ hours})x \\ &= 0.4x\end{aligned}$$

The annual fixed costs for machines A and B are the AW amounts.

$$\begin{aligned}\text{AW}_A &= -18,000(A/P, 15\%, 6) + 2000(A/F, 15\%, 6) \\ &\quad - 6000 - 3000(P/F, 15\%, 3)(A/P, 15\%, 6) \\ \text{AW}_B &= -12,000(A/P, 15\%, 4) - 500(A/F, 15\%, 4) - 5000\end{aligned}$$

Total cost is the sum of AW_A , AW_B , and VC.

3. Equating the annual costs of the buy option ($3.50x$) and the make option yields

$$\begin{aligned}
 -3.50x &= AW_A + AW_B - VC \\
 &= -18,000(A/P, 15\%, 6) + 2000(A/F, 15\%, 6) - 6000 \\
 &\quad - 3000(P/F, 15\%, 3)(A/P, 15\%, 6) - 12,000(A/P, 15\%, 4) \\
 &\quad - 500(A/F, 15\%, 4) - 5000 - 0.4x \\
 -3.10x &= -20,352 \\
 x &= 6565 \text{ units per year}
 \end{aligned} \tag{13.5}$$

A minimum of 6565 lifts must be produced each year to justify the make option, which has the lower variable cost of $0.4x$.

- (b) Substitute 10,000 for x and P_A for the to-be-determined first cost of machine A (currently \$18,000) in Equation [13.5]. Solution yields $P_A = \$58,295$. This is approximately three times the estimated first cost of \$18,000, because the production of 10,000 per year is considerably larger than the breakeven amount of 6565.

Even though the preceding examples treat only two alternatives, the same type of analysis can be performed for three or more alternatives. To do so, compare the alternatives in pairs to find their respective breakeven points. The results are the ranges through which each alternative is more economical. For example, in Figure 13–6, if the output is less than 40 units per hour, alternative 1 should be selected. Between 40 and 60, alternative 2 is more economical; and above 60, alternative 3 is favored.

If the variable cost relations are nonlinear, analysis is more complicated. If the costs increase or decrease uniformly, mathematical expressions that allow direct determination of the break-even point can be developed.



13.3 Payback Analysis ● ● ●

Payback analysis is another use of the present worth technique. It is used to determine the amount of time, usually expressed in years, required to recover the first cost of an asset or project. Payback is allied with breakeven analysis; this is illustrated later in the section. The **payback period**, also called *payback* or *payout period*, has the following definition and types.

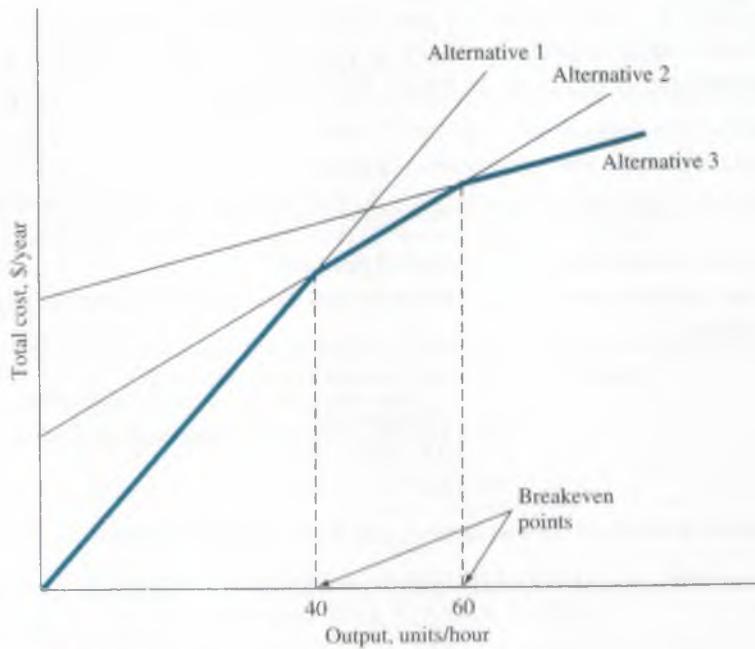


Figure 13–6
Breakeven points for three alternatives.

The **payback period** n_p is an estimated time for the revenues, savings, and any other monetary benefits to completely recover the **initial investment plus a stated rate of return i** .

There are two types of payback analysis as determined by the required return.

No return; $i=0\%$: Also called *simple payback*, this is the recovery of only the initial investment.

Discounted payback; $i > 0\%$: The time value of money is considered in that some return, for example, 10% per year, must be realized in addition to recovering the initial investment.



Payback period

An example application of payback may be a corporate senior manager who insists that every proposal return the initial cost and some stated return within 3 years. Using payback as an initial screening tool, no proposal with $n_p > 3$ years can become a viable alternative. The payback period should be determined using a required $i > 0\%$. Unfortunately in practice, no-return payback is used too often to make economic decisions. After the formulas are presented, a couple of cautions about payback usage are provided.

The equations used to determine n_p differ for each type of analysis. For both types, the terminology is P for the initial investment in the asset, project, contract, etc., and NCF for the estimated annual net cash flow. Using Equation [1.5], annual NCF is

$$\text{NCF} = \text{cash inflows} - \text{cash outflows}$$

To calculate the payback period for $i = 0\%$ or $i > 0\%$, determine the pattern of the NCF series. Note that n_p is usually not an integer. For $t = 1, 2, \dots, n_p$,

$$\text{No return, } i = 0\%; \text{ NCF}_t \text{ varies annually: } 0 = -P + \sum_{t=1}^{n_p} \text{NCF}_t \quad [13.6]$$

$$\text{No return, } i = 0\%; \text{ annual uniform NCF: } n_p = \frac{P}{\text{NCF}} \quad [13.7]$$

$$\text{Discounted, } i > 0\%; \text{ NCF}_t \text{ varies annually: } 0 = -P + \sum_{t=1}^{n_p} \text{NCF}_t(P/F, i, t) \quad [13.8]$$

$$\text{Discounted, } i > 0\%; \text{ annual uniform NCF: } 0 = -P + \text{NCF}(P/A, i, n_p) \quad [13.9]$$

After n_p years, the cash flows will recover the investment in year 0 plus the required return of $i\%$. If the alternative is used more than n_p years, with the same or similar cash flows, a larger return results. If the estimated life is less than n_p years, there is not enough time to recover the investment and $i\%$ return. It is important to understand that payback analysis *neglects all cash flows after the payback period of n_p years*. Consequently, it is preferable to use payback as an **initial screening method or supplemental tool** rather than as the primary means to select an alternative. The reasons for this caution are that

- No-return payback **neglects the time value of money**, since no return on an investment is required.
- Either type of payback **disregards all cash flows occurring after the payback period**. These cash flows may increase the return on the initial investment.

Payback analysis utilizes a significantly different approach to alternative evaluation than the primary methods of PW, AW, ROR, and B/C. It is possible for payback analysis to select a different alternative than these techniques. However, the information obtained from discounted payback analysis performed at an appropriate $i > 0\%$ can be very useful in that a sense of the **risk involved in undertaking an alternative is provided**. For example, if a company plans to utilize a machine for only 3 years and payback is 6 years, indication is that the equipment should not be obtained. Even here, the 6-year payback is considered supplemental information and does not replace a complete economic analysis.

EXAMPLE 13.4

The board of directors of Halliburton International has just approved an \$18 million worldwide engineering construction design contract. The services are expected to generate new annual net cash flows of \$3 million. The contract has a potentially lucrative repayment clause

to Halliburton of \$3 million at any time that the contract is canceled by either party during the 10 years of the contract period. (a) If $i = 15\%$, compute the payback period. (b) Determine the no-return payback period and compare it with the answer for $i = 15\%$. This is an initial check to determine if the board made a good economic decision. Show both hand and spreadsheet solutions.

Solution by Hand

- (a) The net cash flow each year is \$3 million. The single \$3 million payment (call it CV for cancellation value) could be received at any time within the 10-year contract period. Equation [13.9] is altered to include CV.

$$0 = -P + NCF(P/A,i,n) + CV(P/F,i,n)$$

In \$1,000,000 units,

$$0 = -18 + 3(P/A,15\%,n) + 3(P/F,15\%,n)$$

The 15% payback period is $n_p = 15.3$ years, found by trial and error. During the period of 10 years, the contract will not deliver the required return.

- (b) If Halliburton requires absolutely no return on its \$18 million investment, Equation [13.6] results in $n_p = 5$ years, as follows (in \$ million).

$$0 = -18 + 5(3) + 3$$

There is a very significant difference in n_p for 15% and 0%. At 15% this contract would have to be in force for 15.3 years, while the no-return payback period requires only 5 years. A longer time is always required for $i > 0\%$ for the obvious reason that the time value of money is considered.

Solution by Spreadsheet

Enter the function = NPER(15%,3,-18,3) to display 15.3 years. Change the rate from 15% to 0% to display the no-return payback period of 5 years.

If two or more alternatives are evaluated using payback periods to indicate that one may be better than the other(s), the second shortcoming of payback analysis (neglect of cash flows after n_p) may lead to an economically incorrect decision. When cash flows that occur after n_p are neglected, it is possible to favor short-lived assets even when longer-lived assets produce a higher return. In these cases, PW (or AW) analysis should always be the primary selection method. Comparison of short- and long-lived assets in Example 13.5 illustrates this situation.

EXAMPLE 13.5

Two equivalent pieces of quality inspection equipment are being considered for purchase by Square D Electric. Machine 2 is expected to be versatile and technologically advanced enough to provide net income longer than machine 1.

	Machine 1	Machine 2
First cost, \$	12,000	8,000
Annual NCF, \$	3,000	1,000 (years 1–5), 3,000 (years 6–14)
Maximum life, years	7	14

The quality manager used a return of 15% per year and software that incorporates Equations [13.8] and [13.9] to recommend machine 1 because it has a shorter payback period of 6.57 years at $i = 15\%$. The computations are summarized here.

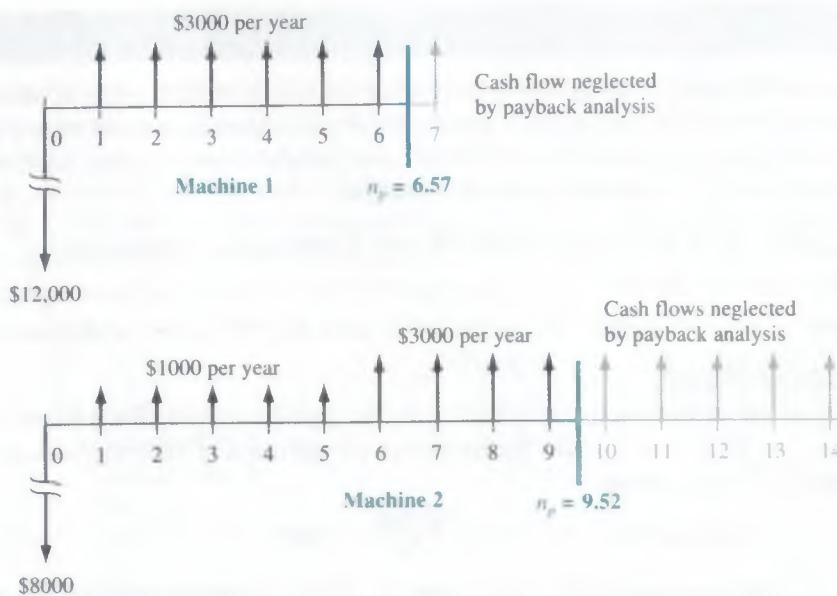
**Figure 13–7**

Illustration of payback periods and neglected net cash flows. Example 13.5.

Machine 1: $n_p = 6.57$ years, which is less than the 7-year life.

$$\text{Equation used: } 0 = -12,000 + 3000(P/A, 15\%, n_p)$$

Machine 2: $n_p = 9.52$ years, which is less than the 14-year life.

$$\begin{aligned} \text{Equation used: } 0 &= -8000 + 1000(P/A, 15\%, 5) \\ &\quad + 3000(P/A, 15\%, n_p - 5)(P/F, 15\%, 5) \end{aligned}$$

Recommendation: Select machine 1.

Now, use a 15% PW analysis to compare the machines and comment on any difference in the recommendation.

Solution

For each machine, consider the net cash flows for all years during the estimated (maximum) life. Compare them over the LCM of 14 years.

$$PW_1 = -12,000 - 12,000(P/F, 15\%, 7) + 3000(P/A, 15\%, 14) = \$663$$

$$\begin{aligned} PW_2 &= -8000 + 1000(P/A, 15\%, 5) + 3000(P/A, 15\%, 9)(P/F, 15\%, 5) \\ &= \$2470 \end{aligned}$$

Machine 2 is selected since its PW value is numerically larger than that of machine 1 at 15%. This result is the opposite of the payback period decision. The PW analysis accounts for the increased cash flows for machine 2 in the later years. As illustrated in Figure 13–7 (for one life cycle for each machine), payback analysis neglects all cash flow amounts that may occur after the payback time has been reached.

Comment

This is a good example of why payback analysis is best used for initial screening and supplemental risk assessment. Often a shorter-lived alternative evaluated by payback analysis may appear to be more attractive, when the longer-lived alternative has cash flows later in its life that make it more economically attractive.

As mentioned in the introduction to this section, breakeven and payback analyses are allied. They can be used in conjunction to determine the payback period when a desired level of break-even is specified. The reverse is also possible; when a desired payback period is established, the breakeven value with or without a return requirement can be determined. By working together in this fashion, better economic decisions can be made. Example 13.6 illustrates the second of the situations mentioned above.

EXAMPLE 13.6

The president of a local company expects a product to have a profitable life of between 1 and 5 years. Help her determine the breakeven number of units that must be sold annually (without any return) to realize payback for each of the time periods 1 year, 2 years, and so on up to 5 years. The cost and revenue estimates are as follows:

Fixed costs: Initial investment of \$80,000 with \$1000 annual operating cost.

Variable cost: \$8 per unit.

Revenue: Twice the variable cost for the first 5 years and 50% of the variable cost thereafter.

Solution by Hand

Define X_{BE} as the breakeven quantity and n_p as the payback period. Since values of X_{BE} are sought for $n_p = 1, 2, 3, 4, 5$, solve for breakeven by substituting each payback period. First develop the FC , r , and v terms.

Fixed cost, FC	$\frac{80,000}{n_p} + 1000$
Revenue per unit, r	\$16 (years 1 through 5 only)
Variable cost per unit, v	\$8

The breakeven relation from Equation [13.2] is

$$X_{BE} = \frac{80,000/n_p + 1000}{8} \quad [13.10]$$

Insert n_p values and solve for X_{BE} , the breakeven value.

n_p , payback years	1	2	3	4	5
X_{BE} , units per year	10,125	5125	3458	2625	2125

Solution by Spreadsheet

Figure 13–8 presents a spreadsheet solution for the breakeven values. Equation [13.10] is encoded to display the answers in column C. The breakeven values are the same as those above, e.g., sell 5125 units per year to pay back in 2 years. The breakeven curve rapidly flattens out as shown in the accompanying chart in Figure 13–8.

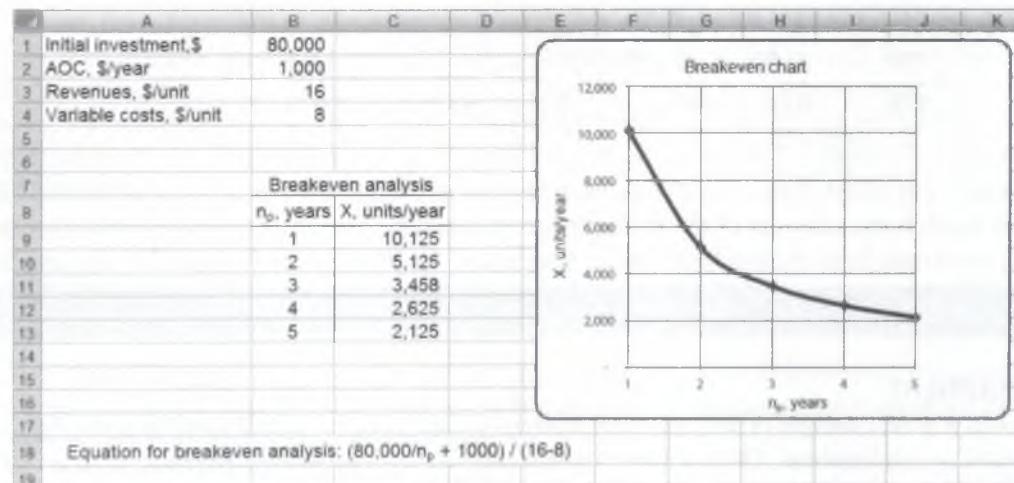


Figure 13–8

Breakeven number of units for different payback periods, Example 13.6.

13.4 More Breakeven and Payback Analysis on Spreadsheets

The Goal Seek tool that we have used previously is an excellent tool to perform breakeven and payback analysis. Examples 13.7 and 13.8 demonstrate the use of Goal Seek for both types of problems.

EXAMPLE 13.7

The Naruse brake-accelerator pedal (www.autoblog.com/tag/Masuyuki+Naruse) is designed to minimize the chances that a driver will accidentally step on the accelerator pedal of the car when the brake pedal is the intended target. The design is based on the fact that a person naturally steps downward on his or her foot when surprised, shocked, or struck with a medical emergency. In this pedal design, downward motion of the foot will always engage the brake, never the accelerator. Assume that for the manufacture of pedal components, two equally qualified machines have been identified and estimates made.

	Machine 1	Machine 2
First cost, \$	-80,000	-110,000
Net cash flow, \$ per year	25,000	22,000
Salvage value, \$	2,000	3,000
Life, years	4	6

Using an AW analysis at MARR = 10%, the spreadsheet screen shot in Figure 13–9 indicates that machine 1 is the economic choice with a positive AW value of \$193. However, the automated controls, safety features, and ergonomic design of machine 2 make it a better choice for the plant in the opinion of the project engineer. Use breakeven analysis to find the threshold values for each of several parameters that will make machine 2 equally qualified economically. The parameters to concentrate on are (a) first cost, (b) net cash flow, and (c) life of machine 2, if all other estimates remain the same.

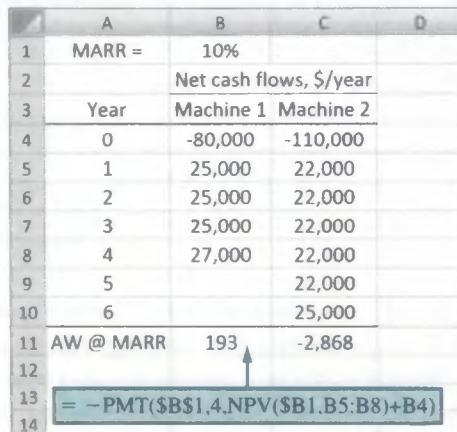


Figure 13–9
AW values for two machines. Example 13.7.

Solution

Figure 13–10 shows the spreadsheet and Goal Seek templates that determine breakeven values for first cost and NCF.

- (a) Figure 13–10a: By forcing the AW for machine 2 to equal \$193, Goal Seek finds a breakeven of \$96,669. If the first cost can be negotiated down to this cost from \$110,000, machine 2 will be economically equivalent to machine 1.
- (b) Figure 13–10b: (Remember to reset the first cost to \$-110,000 on the spreadsheet.) By setting all NCFs equal to the value in year 1 (using the function = \$C\$5), Goal Seek determines a breakeven of \$25,061 per year. Therefore, if the NCF estimate can realistically be increased from \$22,000 to \$25,061, again machine 2 will be economically equivalent.
- (c) Finding an extended life estimate for machine 2 is a payback question; Goal Seek is not needed. The easiest approach is to use the NPER function to find the payback period. Entering = NPER(10%,22000,-110000,3000) displays $n_p = 7.13$ years. Therefore, extending the estimated life from 6 to between 7 and 8 years and retaining the salvage value of \$3000 will select machine 2 over 1.

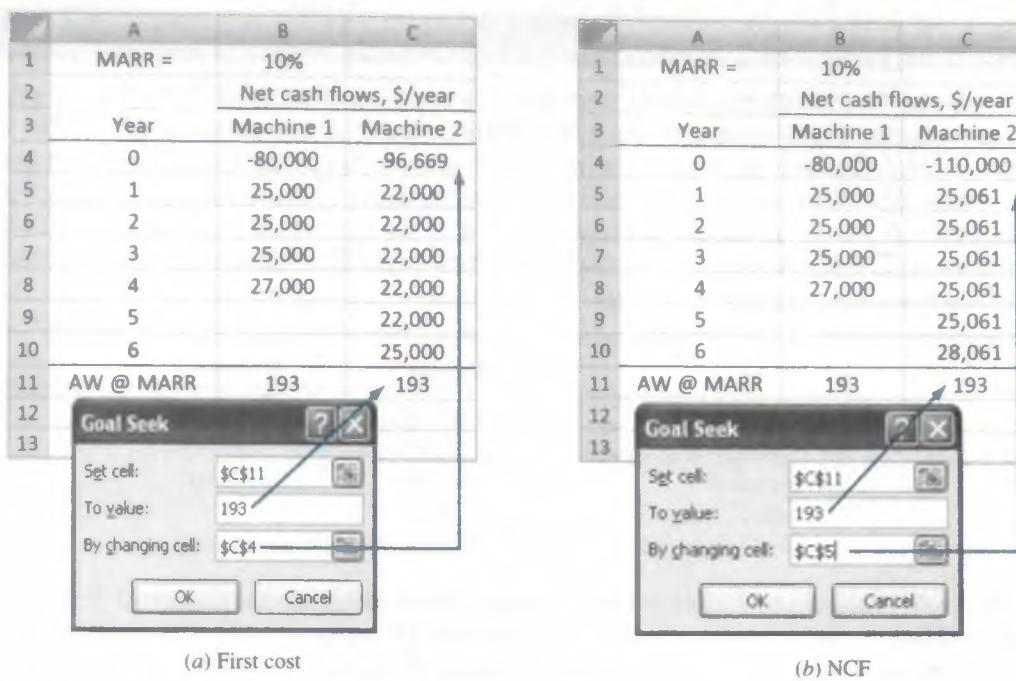


Figure 13-10

Breakeven values for (a) first cost and (b) annual net cash flow using Goal Seek. Example 13.7.

EXAMPLE 13.8

Chris and her father just purchased a small office building for \$160,000 that is in need of a lot of repairs, but is located in a prime commercial area of the city. The estimated costs each year for repairs, insurance, etc. are \$18,000 the first year, increasing by \$1000 per year thereafter. At an expected 8% per year return, use spreadsheet analysis to determine the payback period if the building is (a) kept for 2 years and sold for \$290,000 sometime beyond year 2 or (b) kept for 3 years and sold for \$370,000 sometime beyond 3 years.

Solution

Figure 13-11 shows the annual costs (column B) and the sales prices if the building is kept 2 or 3 years (columns C and E, respectively). The NPV function is applied (columns D and F) to determine when the PW changes sign from plus to minus. These results bracket the payback period for each retention period and sales price. When PW > 0, the 8% return is exceeded.

- (a) The 8% return payback period is between 3 and 4 years (column D). If the building is sold after exactly 3 years for \$290,000, the payback period was not exceeded; but after 4 years it is exceeded.
- (b) At a sales price of \$370,000, the 8% return payback period is between 5 and 6 years (column F). If the building is sold after 4 or 5 years, the payback is not exceeded; however, a sale after 6 years is beyond the 8%-return payback period.

A	B	C	D	E	F	G	H	I
1	Year	Costs, \$ per year	If sold after 2 years Price, \$	If sold after 2 years PW at 8%, \$	If sold after 3 years Price, \$	If sold after 3 years PW at 8%, \$		
2								
3	0	-160,000						
4	1	-18,000						
5	2	-19,000						
6	3	-20,000	290,000	21,379				
7	4	-21,000	290,000	-11,110	370,000	47,693		
8	5	-22,000	290,000	-41,872	370,000	12,575		
9	6	-23,000			370,000	-20,572		
10	7	-24,000			370,000	-51,847		
11			= NPV(8%, \$B\$4:\$B7) + \$B\$3 - PV(8%, A7, , 290000)					
12								
13								

Figure 13-11

Payback period analysis, Example 13.8

CHAPTER SUMMARY

The breakeven point for a variable for one project is expressed in terms such as units per year or hours per month. At the breakeven amount Q_{BE} , there is indifference to accept or reject the project. Use the following decision guideline:

Single Project (Refer to Figure 13–2.)

Estimated quantity is *larger* than Q_{BE} → accept project

Estimated quantity is *smaller* than Q_{BE} → reject project

For two or more alternatives, determine the breakeven value of the common variable. Use the following guideline to select an alternative:

Two Alternatives (Refer to Figure 13–5.)

Estimated level is *below* breakeven → select alternative with higher variable cost (larger slope)

Estimated level is *above* breakeven → select alternative with lower variable cost (smaller slope)

Payback analysis estimates the number of years necessary to recover the initial investment plus a stated rate of return. This is a supplemental analysis technique used primarily for initial screening prior to a full evaluation by PW or some other method. The technique has some drawbacks, especially for no-return payback analysis, where $i = 0\%$ is the stated return.

PROBLEMS

Breakeven Analysis for a Project

- 13.1 A design-to-cost approach to product pricing involves determining the selling price of the product and then figuring out if it can be made at a cost lower than that. Banner Engineering's QT50R radar-based sensor features frequency-modulated technology to accurately monitor or detect objects up to 15 miles away while resisting rain, wind, humidity, and extreme temperatures. It has a list price of \$589, and the variable cost of manufacturing the unit is \$340.

- (a) What could the company's fixed cost per year be in order for Banner to break even with sales of 9000 units per year?
- (b) If Banner's fixed cost is actually \$750,000 per year, what is the profit at a sales level of 7000 units per year?

- 13.2 Handheld fiber-optic meters with white light polarization interferometry are useful for measuring temperature, pressure, and strain in electrically noisy environments. The fixed costs associated with manufacturing are \$800,000 per year. If a base unit sells for \$2950 and its variable cost is \$2075, (a) how many units must be sold each year for breakeven and (b) what will the profit be for sales of 3000 units per year?

- 13.3 A metallurgical engineer has estimated that the capital investment cost for recovering valuable

metals (nickel, silver, platinum, gold, etc.) from the copper refinery's wastewater stream will be \$12 million. The equipment will have a useful life of 15 years with no salvage value. Its operating cost is represented by the relation $(\$2,600,000)E^{1/9}$, where E is the efficiency of the metal recovery operation (in decimal form). The amount of metal currently discharged is 2880 pounds per year prior to recovery operations, and the efficiency of recovery is estimated at 71%. What must the average selling price per pound be for the precious metals that are recovered and sold in order for the company to break even at its MARR of 15% per year?

Problems 13.4 through 13.7 are based on the following information.

Hambry Enterprises produces a component for recycling uranium used as a nuclear fuel in power plant generators in France and the United States. Use the following cost and revenue figures, quoted in U.S. dollars per hundred-weight (cwt), recorded for this year to calculate the answers for each plant.

Location	Fixed Cost, \$ million	Revenue, \$ per cwt	Cost, \$ per cwt
France	3.50	8,500	3,900
United States	2.65	12,500	9,900

- 13.4 Determine the breakeven point for each plant.
- 13.5 Estimate the minimum revenue per hundredweight required for next year if breakeven values and variable costs remain constant, but fixed costs increase by 10%.
- 13.6 During this year, the French plant sold 950 units in Europe, and the U.S. plant sold 850 units. Determine the year's profit (loss) for each plant.
- 13.7 Hambry's president has a goal of \$1 million profit next year at each plant with no revenue or fixed cost increases. Determine the decreases *in dollar amounts and percentages* in variable cost necessary to meet this goal, if the number of units sold is the same as this year.
-
- 13.8 The National Highway Traffic Safety Administration raised the average fuel efficiency standard to 35.5 miles per gallon (mpg) for cars and light trucks by the year 2016. The rules will cost consumers an average of \$926 extra per vehicle in the 2016 model year. Assume a person purchases a new car in 2016 that gets 35.5 mpg and keeps it for 5 years. If the person drives an average of 1000 miles per month and gets an extra 10 miles per gallon of gasoline, how much will the gasoline have to cost in order for the buyer to recover the extra investment in 5 years at an interest rate of 0.75% per month?
- 13.9 A call center in India used by U.S. and U.K. credit card holders has a capacity of 1,400,000 calls annually. The fixed cost of the center is \$775,000 with an average variable cost of \$1 and revenue of \$2.50 per call.
- Find the percentage of the call capacity that must be placed each year to break even.
 - The center manager expects to dedicate the equivalent of 500,000 of the 1,400,000 capacity to a new product line. This is expected to increase the center's fixed cost to \$900,000 of which 50% will be allocated to the new product line. Determine the average revenue per call necessary to make 500,000 calls the breakeven point for only the new product. How does this required revenue compare with the current center revenue of \$2.50 per call?
- 13.10 The addition of a turbocharger to a small V-6 engine that gets 18 miles per gallon of gasoline can boost its power to that of a V-8 engine and increase fuel efficiency at the same time. If Bill will pay \$800 to turbocharge his engine and his fuel efficiency increases by 3 miles per gallon, how many miles will he have to drive each month for 3 years in order to break even? Assume the cost of gasoline is \$3.25 per gallon and the interest rate is 1% per month.
- 13.11 Transporting extremely heavy patients (people who weigh more than 500 pounds) is much more difficult than transporting normal-weight patients. Various cities in Colorado, Nebraska, and Kansas charge \$1421 for an extremely obese patient compared to \$758 for a typical patient. The extra fees are justified by the ambulance companies on the basis of the specialty equipment required and the extra personnel involved. If it is assumed that 50 extremely obese patients are transported every year, how much could the ambulance companies afford to spend on the specialty equipment *now* and break even on the initial cost in 5 years just from the extra charges? Assume the extra costs are \$400 per patient and the company's MARR is 10% per year.
- 13.12 High-profile vehicles have poor fuel efficiency because of increased wind resistance from their large front area. A number of companies make devices that they claim will significantly increase a vehicle's fuel efficiency. One company claims that by spending \$560 on its friction-reducing device, the fuel efficiency of a pickup truck will increase by 25%. Assuming the device works as claimed, for a vehicle that currently gets 20 miles per gallon (mpg), how many miles would the owner have to drive each year to break even in 5 years? Assume the cost of gasoline is \$3.50 per gallon and the interest rate is 10% per year.
- 13.13 As the price of gasoline goes up, people are willing to drive farther to fill their tank in order to save money. Assume you had been buying gasoline for \$2.90 per gallon and that it went up to \$2.98 per gallon at the station where you usually go. If you drive an F-150 pickup that gets 18 miles per gallon, what is the round-trip distance you can drive to break even if it will take 20 gallons to fill your tank? Use an interest rate of 8% per year.
- 13.14 An automobile company is investigating the advisability of converting a plant that manufactures economy cars into one that will make retro sports cars. The initial cost for equipment conversion will be \$200 million with a 20% salvage value anytime within a 5-year period. The cost of producing a car will be \$21,000, but it is expected to have a selling price of \$33,000 to dealers. The production capacity for the first year will be 4000 units. At an interest rate of 12% per year, by what uniform amount will production have to increase each year in order for the company to recover its investment in 3 years?

13.15 For the last 2 years, The Health Company has experienced a fixed cost of \$850,000 per year and an $(r - v)$ value of \$1.25 per unit for its multivitamin line of products. International competition has become severe enough that some financial changes must be made to keep market share at the current level.

- (a) Perform a spreadsheet-based graphical analysis to estimate the effect on the breakeven point if the difference between revenue and variable cost per unit increases somewhere between 1% and 15% of its current value.
- (b) If fixed costs and revenue per unit remain at their current values, what type of change must take place to make the breakeven point go down?

13.16 (This is an extension of Problem 13.15) Expand the analysis performed in Problem 13.15 by changing the variable cost per unit. The financial manager estimates that fixed costs will fall to \$750,000 when the required production rate to break even is at or below 600,000 units. What happens to the breakeven points over the $(r - v)$ range of 1% to 15% increase as evaluated previously?

Breakeven Analysis Between Alternatives

13.17 Providing restrooms at parks, zoos, and other city-owned recreation facilities is a considerable expense for municipal governments. City councils usually opt for permanent restrooms in larger parks and portable restrooms in smaller ones. The cost of renting and servicing a portable restroom is \$7500 per year. In one northeastern municipality, the parks director informed the city council that the cost of constructing a permanent restroom is \$218,000 and the annual cost of maintaining it is \$12,000. He remarked that the rather high cost is due to the necessity to use expensive materials and construction techniques that are tailored to minimize damage from vandalism that often occurs in unattended public facilities. If the useful life of a permanent restroom is assumed to be 20 years, how many portable restrooms could the city afford to rent each year and break even with the cost of one permanent facility? Let the interest rate be 6% per year.

13.18 A consulting engineer is considering two methods for lining ponds used for evaporating concentrate generated during reverse osmosis treatment of brackish groundwater for the Clay County Industrial Park. A geosynthetic bentonite clay liner (GCL) will cost \$1.8 million to install, and if it is renovated after 4 years at a cost of \$375,000, its life can be extended another 2 years. Alternatively,

a high-density polyethylene (HDPE) geomembrane can be installed that will have a useful life of 12 years. At an interest rate of 6% per year, how much money can be spent on the HDPE liner for the two methods to break even?

13.19 An irrigation canal contractor wants to determine whether he should purchase a used Caterpillar mini excavator or a Toro powered rotary tiller for servicing irrigation ditches in an agricultural area of California. The initial cost of the excavator is \$26,500 with a \$9000 salvage value after 10 years. Fixed costs for insurance, license, etc. are expected to be \$18,000 per year. The excavator will require one operator at \$15 per hour and maintenance at \$1 per hour. In 1 hour, 0.15 mile of ditch can be prepared. Alternatively, the contractor can purchase a tiller and hire 2 workers at \$11 per hour each. The tiller costs \$1200 and has a useful life of 5 years with no salvage value. Its operating cost is expected to be \$1.20 per hour, and with the tiller, the two workers can prepare 0.04 mile of ditch in 1 hour. The contractor's MARR is 10% per year. Determine the number of miles of ditch per year the contractor would have to service for the two options to break even.

13.20 An effective method to recover water used for regeneration of ion exchange resins is to use a reverse osmosis system in a batch treatment mode. Such a system involves recirculation of the partially treated water back into the feed tank, causing the water to heat up. The water can be cooled using one of two systems: a single-pass heat exchanger or a closed-loop heat exchange system. The single-pass system, good for 3 years, requires a small chiller costing \$920 plus stainless steel tubing, connectors, valves, etc. costing \$360. The cost of water, treatment charges, electricity, etc. will be \$3.10 per hour. The closed-loop system will cost \$3850 to buy, will have a useful life of 5 years, and will cost \$1.28 per hour to operate. What is the minimum number of hours per year that the cooling system must be used in order to justify purchase of the closed-loop system? The MARR is 10% per year, and the salvage values are negligible.

13.21 Samsung Electronics is trying to reduce supply chain risk by making more responsible make-buy decisions through improved cost estimation. A high-use component (expected usage is 5000 units per year) can be purchased for \$25 per unit with delivery promised within a week. Alternatively, Samsung can make the component in-house and have it readily available at a cost of \$5 per unit, if equipment costing \$150,000 is purchased. Labor

and other operating costs are estimated to be \$35,000 per year over the study period of 5 years. Salvage is estimated at 10% of first cost and $i = 12\%$ per year. Neglect the element of availability (a) to determine the breakeven quantity and (b) to recommend making or buying at the expected usage level.

- 13.22 A partner in a medium-size A/E (architectural/engineering) design firm is evaluating two alternatives for improving the exterior appearance of the building they occupy. The building can be completely painted at a cost of \$6500. The paint is expected to remain attractive for 4 years, at which time repainting will be necessary. Every time the building is repainted, the cost will be 20% higher than the previous time. Alternatively, the building can be sandblasted now and every 6 years at a cost 40% greater than the previous time. If the company's MARR is 10% per year, what is the maximum amount that could be spent now on the sandblasting alternative that would render the two alternatives indifferent over a study period of 12 years?

- 13.23 A junior mechanical engineering student is co-oping this semester at Regency Aircraft, which customizes the interiors of private and corporate jets. Her first assignment is to develop the specifications for a new machine to cut, shape, and sew leather or vinyl covers and trims. The first cost is not easy to estimate due to many options, but the annual revenue and M&O costs should net out at \$ + 15,000 per year over a 10-year life. Salvage is expected to be 20% of the first cost. Determine the breakeven first cost of the machine to just recover its first cost and a return of 8% per year under two scenarios:

- I: No outside revenue will be developed by the machine.
- II: Outside contracting will occur with estimated revenue of \$10,000 the first year, increasing by \$5000 per year thereafter.

Solve using (a) hand and (b) spreadsheet solutions.

- 13.24 Ascarate Fishing Club (a nonprofit organization dedicated to teaching kids how to fish) is considering two options for providing a heavily stocked pond for kids who have never caught a fish before. Option 1 is an above-ground swimming pool made of heavy vinyl plastic that will be assembled and disassembled for each quarterly event. The purchase price will be \$400. Leaks from hooks piercing the fabric will be repaired with a vinyl repair kit at a cost of \$70 per year, but the pool will have to be replaced when too many repairs have been made.

Option 2 is an in-ground pond that will be excavated by club members at no cost and lined with fabric that costs \$1 per square foot. The pond will be 15 ft in diameter and 3 ft deep. Assume 300 ft² of liner will be purchased. A chain link fence at \$10 per lineal foot will be installed around the pond (100 ft of fence). Maintenance inside the fence is expected to cost \$20 per year. The park where the pond will be constructed has committed the land for only 10 years. At an interest rate of 6% per year, how long would the above-ground pool have to last to break even?

- 13.25 A rural subdivision has several miles of access roads that need a new surface treatment. Alternative 1 is a gravel base and pavement with an initial cost of \$500,000 that will last for 15 years and has an annual upkeep cost of \$100 per mile. Alternative 2 is to enhance the gravel base now at a cost of \$50,000 and immediately coat the surface with a durable hot oil mix, which costs \$130 per barrel applied. Annual reapplication of the mix is required. A barrel covers 0.05 mile. (a) If the discount rate is 6% per year, determine the number of miles at which the two alternatives break even. (b) A drive in a pickup indicates a total of 12.5 miles of road. Which is the more economical alternative?

- 13.26 A waste-holding lagoon situated near the main plant receives sludge daily. When the lagoon is full, it is necessary to remove the sludge to a site located 8.2 kilometers from the main plant. Currently, when the lagoon is full, the sludge is removed by pump into a tank truck and hauled away. This process requires the use of a portable pump that initially costs \$800 and has an 8-year life. The company pays a contract individual to operate the pump and oversee environmental and safety factors at a rate of \$100 per day, plus the truck and driver must be rented for \$200 per day. The company has the option to install a pump and pipeline to the remote site. The pump would have an initial cost of \$1600 and a life of 10 years and will cost \$3 per day to operate. The company's MARR is 10% per year.

- (a) If the pipeline will cost \$12 per meter to construct and will have a 10-year life, how many days per year must the lagoon require pumping to justify construction of the pipeline?
- (b) If the company expects to pump the lagoon once per week every week of the year, how much money can it afford to spend now on the 10-year life pipeline to just break even?

- 13.27 Lorraine can select from two nutrient injection systems for her cottage industry of hydroponic

tomato and lettuce greenhouses. (a) Use an AW relation to determine the minimum number of hours per year to operate the pumps that will justify the Auto Green system, if the MARR is 10% per year. (b) Which pump is economically better if it operates 7 hours per day, 365 days per year?

	Nutra Jet (N)	Auto Green (A)
Initial cost, \$	-4,000	-10,300
Life, years	3	6
Rebuild cost, \$	-1,000	-2,200
Time before rebuild, annually or minimum hours	2,000	8,000
Cost to operate, \$ per hour	1.00	0.90

- 13.28 An engineering practitioner can lease a fully equipped computer and color printer system for \$800 per month or purchase one for \$8500 now and pay a \$75 per month maintenance fee. If the nominal interest rate is 15% per year, determine the months of use necessary for the two to break even. Show both (a) hand and (b) spreadsheet solutions.

- 13.29 The office manager of an environmental engineering consulting firm was instructed to make an eco-friendly decision in acquiring an automobile for general office use. He is considering a gasoline-electric hybrid or a gasoline-free, all-electric hatchback. The hybrid under consideration is GM's Volt, which will cost \$35,000, will have a salvage value of \$15,000 after 5 years, and will have a range of 40 miles on the electric battery, plus several hundred more miles when the gasoline engine kicks in. Nissan's Leaf, on the other hand, is a pure electric that will have a range of only 100 miles, after which its lithium ion battery must be recharged. The Leaf's relatively limited range creates a psychological effect known as *range anxiety* (RA), which has the company leaning toward purchasing the Volt. The Leaf can be leased for \$349 per month after an initial \$500 down payment.

The accountant for the consulting firm told the office manager that the Leaf is the better economic option based on an evaluation she performed earlier. If the office manager purchases the Volt anyway (instead of leasing the Leaf), what is the monthly equivalent (AW value) of the extra amount of money the company will be paying to eliminate range anxiety? Assume the operating costs will be the same for both vehicles and the MARR is 0.75% per month.

Payback Analysis

- 13.30 How long will you have to sell a product that has an income of \$5000 per month and expenses of

\$1500 per month if your initial investment is \$28,000 and your MARR is (a) 0% and (b) 3% per month? Solve by formula. (c) Write the spreadsheet functions to display the payback period for both 0% and 3% per month.

- 13.31 (a) Determine the payback period at an interest rate of 8% per year for an asset that initially cost \$28,000, has a scrap value of \$1500 whenever it is sold, and generates cash flow of \$2900 per year.
 (b) If the asset will be in service for 12 years, should it be purchased?
- 13.32 ABB purchased fieldbus communication equipment for a project in South Africa for \$3.15 million. The net cash flow is estimated at \$500,000 per year, and a salvage value of \$400,000 is anticipated regardless of when it is sold. Determine the number of years the equipment must be used to obtain payback at MARR values of (a) 0% and 8% per year and (b) 15% and 16% per year. (c) Use a spreadsheet to plot the payback years for all four return values.
- 13.33 Sundance Detective Agency purchased new surveillance equipment with the following estimates. The year index is $k = 1, 2, 3, \dots$.
- | | |
|-------------------------|----------------------|
| First cost | \$1050 |
| Annual maintenance cost | \$70 + 5k per year |
| Extra annual revenue | \$200 + 50k per year |
| Salvage value | \$600 for all years |
- (a) Calculate the payback period to make a return of 10% per year.
 (b) For a preliminary conclusion, should the equipment be purchased if the actual useful life is 7 years?
- 13.34 Clarisa, an engineering manager, wants to purchase a resort accommodation to rent to skiers. She is considering the purchase of a three-bedroom lodge in upper Montana that will cost \$250,000. The property in the area is rapidly appreciating in value because people anxious to get away from urban developments are bidding up the prices. If Clarisa spends an average of \$500 per month for utilities and the investment increases at a rate of 2% per month, how long would it be before she could sell the property for \$100,000 more than she has invested in it?

- 13.35 Laura's grandparents helped her purchase a small self-serve laundry business to make extra money during her 5 college years. When she completed her engineering management degree, she sold the business and her grandparents told her to keep the

money as a graduation present. For the net cash flows listed below, determine the following:

- The percentage of the investment recovered during the 5 years
- The actual rate of return over the 5-year period
- How long it took to pay back the \$75,000 investment in year 0, plus a 7% per year return

Year	0	1	2	3	4	5
NCF, \$ per year	-75,000	-10,500	18,600	-2000	28,000	105,000

- 13.36 Buhler Tractor sold a tractor for \$45,000 to Tom Edwards 10 years ago. (a) What is the uniform net cash flow that Tom must make each year to realize payback and a return of 5% per year on his investment over a period of 3 years? 5 years? 8 years? All 10 years? (b) If the net cash flow was actually \$5000 per year, what is the amount Tom should have paid for the tractor to realize payback plus the 5% per year return over these 10 years?

- 13.37 National Parcel Service has historically owned and maintained its own delivery trucks. Leasing is an option being seriously considered because costs for maintenance, fuel, insurance, and some liability issues will be transferred to Pacific Leasing, the truck leasing company. The study period is no more than 24 months for either alternative. The annual lease cost is paid at the beginning of each year and is not refundable for partially used years. Use the first cost and net cash flow estimates to determine the payback in months with a nominal 9% per year return for the (a) purchase option and (b) lease option.

Purchase: $P = \$-30,000$ now
 Monthly cost = \$-1000
 Monthly revenue = \$4500

Lease: $P = \$-10,000$ at the beginning of each year (months 0 and 12)
 Monthly cost = \$-2500
 Monthly revenue = \$4500

- 13.38 Julian Browne, owner of Clear Interior Environments, purchased an air scrubber, HEPA vacuum, and other equipment for mold removal for \$15,000 eight months ago. Net cash flows were \$-2000 for each of the first 2 months, followed by \$1000 per month for months 3 and 4. For the last 4 months, a contract generated a net \$6000 per month. Julian sold the equipment yesterday for \$3000 to a friend. Determine (a) the no-return payback period and (b) the nominal 18%-per-year payback period.

- 13.39 Explain why payback analysis may favor an alternative with a shorter payback period when it is not the better choice economically.

- 13.40 When comparing two alternatives, why is it best to use no-return payback analysis as a preliminary screening tool prior to conducting a complete PW or AW evaluation?

Spreadsheet Problems

- 13.41 Benjamin used regression analysis to fit quadratic relations to monthly revenue and cost data with the following results:

$$R = -0.007Q^2 + 32Q$$

$$TC = 0.004Q^2 + 2.2Q + 8$$

- Plot R and TC . Estimate the quantity Q_p at which the maximum profit should occur. Estimate the amount of profit at this quantity.
- The profit relation $P = R - TC$ and calculus can be used to determine the quantity Q_p at which the maximum profit will occur and the amount of the profit. The equations are

$$\text{Profit} = aQ^2 + bQ + c$$

$$Q_p = \frac{-b}{2a}$$

$$\text{Maximum profit} = \frac{-b^2}{4a} + c$$

Use these relations to confirm the graphical estimates you made in (a). (Your instructor may ask you to derive the relations above.)

- 13.42 The National Potato Cooperative purchased a de-skiving machine last year for \$150,000. Revenue for the first year was \$50,000. Over the total estimated life of 8 years, what must the remaining equivalent annual revenues (years 2 through 8) equal to break even by recovering the investment and a return of 10% per year? Costs are expected to be constant at \$42,000 per year, and a salvage value of \$20,000 is anticipated.

Problems 13.43 and 13.44 are based on the following information.

Wilson Partners manufactures thermocouples for electronics applications. The current system has a fixed cost of \$300,000 per year and a variable cost of \$10 per unit. Wilson sells the units for \$14 each. A newly proposed process will add on-board features that allow the revenue to increase to \$16 per unit, but the fixed cost will now be \$500,000 per year. The variable cost of the new system will be based on a \$48 per hour rate with 0.2 hour required to produce each unit.

- 13.43 Determine the annual breakeven quantity for (a) the current system and (b) the new system.

- 13.44 Plot the two profit relations and estimate graphically the breakeven quantity between the two alternatives.
-

Problems 13.45 through 13.48 are based on the following information.

Mid-Valley Industrial Extension Service, a state-sponsored agency, provides water quality sampling services to all business and industrial firms in a 10-county region. Just last month, the service purchased all necessary lab equipment for full in-house testing and analysis. Now an outsourcing agency has offered to take over this function on a per sample basis. Data and quotes for the two options have been collected. The MARR for government projects is 5% per year, and a study period of 8 years is chosen.

In-house: Equipment and supplies initially cost \$125,000 for a life of 8 years, an AOC of \$15,000, and annual salaries of \$175,000. Sample costs average \$25 each. There is no significant market value for the equipment and supplies currently owned.

Outsourced: Contractors quote sample cost averages of \$100 for the first 5 years, increasing to \$125 per sample for years 6 through 8.

13.45 Determine the breakeven number of tests between the two options.

13.46 Use a spreadsheet to graph the AW curves for both options for test loads between 0 and 4000 per year in increments of 1000 tests. What is the estimated breakeven quantity?

13.47 The service director has asked the outsource company to reduce the per sample costs by 25% across the board over the 8-year study period. What will this do to the breakeven point? (*Hint:* Look carefully at your graph from Problem 13.46 before answering.)

13.48 Assume the Extension Service can reduce its annual salaries from \$175,000 to \$100,000 per year and the per sample cost from \$25 to \$20. What will this do to the breakeven point? (*Hint:* Again, look carefully at your graph from the previous problem before answering.) What is the new annual breakeven test quantity?

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

13.49 In linear breakeven analysis, if a company expects to operate at a point *above* the breakeven point, it should select the alternative:

- (a) With the lower fixed cost
- (b) With the higher fixed cost
- (c) With the lower variable cost
- (d) With the higher variable cost

13.50 A company is considering two alternatives to automate the pH of process liquids. Alternative A will have fixed costs of \$42,000 per year and will require 2 workers at \$48 per day each. Together, these workers can generate 100 units of product per day. Alternative B will have fixed costs of \$56,000 per year, but with this alternative, 3 workers will generate 200 units of product. If x is the number of units per year, the variable cost (VC) in \$ per year for alternative B is represented by:

- (a) $[2(48)/100]x$
- (b) $[3(48)/200]x$
- (c) $[3(48)/200]x + 56,000$
- (d) $[2(48)/100]x + 42,000$

13.51 When the variable cost is reduced for linear total cost and revenue lines, the breakeven point decreases. This is an economic advantage because:

- (a) The revenue per unit will increase.
- (b) The two lines will now cross at zero.
- (c) The profit will increase for the same revenue per unit.
- (d) The total cost line becomes nonlinear.

$$13.52 \quad AW_1 = -23,000(A/P, 10\%, 10) + 4000(A/F, 10\%, 10) \\ - 3000 - 3x$$

$$AW_2 = -8,000(A/P, 10\%, 4) - 2000 - 6x$$

For these two AW relations, the breakeven point x , in miles per year, is closest to:

- (a) 1130
- (b) 1224
- (c) 1590
- (d) 655

13.53 To make an item in-house, equipment costing \$250,000 must be purchased. It will have a life of 4 years, an annual cost of \$80,000, and each unit will cost \$40 to manufacture. Buying the item externally will cost \$100 per unit. At $i = 15\%$ per year, it is cheaper to make the item in-house if the number per year needed is:

- (a) Above 1047 units
- (b) Above 2793 units
- (c) Equal to 2793 units
- (d) Below 2793 units

13.54 A procedure at Mercy Hospital has fixed costs of \$10,000 per year and variable costs of \$50 per test. If the procedure is automated, its fixed cost will be \$21,500 per year, but its variable cost will be only \$10 per test. The number of tests that must be performed each year for the two operations to break even is closest to:

- (a) 290
- (b) 455
- (c) 750
- (d) Over 800

13.55 An assembly process can be completed using either alternative X or Y. Alternative X has fixed costs of \$10,000 per year with a variable cost of \$50 per unit. If the process is automated per alternative Y, its fixed cost will be \$5000 per year and its variable cost will be only \$10 per unit. The number of units that must be produced each year in order for alternative Y to be favored is closest to:

- (a) Y will be favored for any level of production
- (b) 125
- (c) 375
- (d) X will be favored for any level of production

13.56 Two different methods are under consideration for building a bypass road. Material C will cost \$100,000 per mile and last for 10 years. Its annual maintenance cost will be \$10,000 per year per mile. Material D will cost \$30,000 per mile and last for 5 years. At an interest rate of 6% per year, the annual maintenance cost for material D that will make the two methods cost the same is closest to:

- (a) Less than \$14,000
- (b) \$14,270
- (c) \$16,470
- (d) \$19,510

13.57 A construction company can purchase a piece of equipment for \$50,000 and spend \$100 per day in operating costs. The equipment will have a 5-year life with no salvage value. Alternatively, the company can lease the equipment for \$400 per day. The number of days per year the company must require the equipment to justify its purchase at an interest rate of 8% per year is closest to:

- (a) 10 days
- (b) 42 days
- (c) 51 days
- (d) 68 days

13.58 A tractor has a first cost of \$40,000, a monthly operating cost of \$1500, and a salvage value of \$12,000 in 10 years. The MARR is 12% per year. An identical tractor can be rented for \$3200 per month (operating cost not included). If n is the minimum number of months per year the tractor must be used in order to justify its purchase, the relation to find n is represented by:

- (a) $-40,000(A/P,1\%,10) - 1500n + 12,000(A/F,1\%,10) = 3200n$
- (b) $-40,000(A/P,12\%,10) - 1500n + 12,000(A/F,12\%,10) = 3200n$
- (c) $-40,000(A/P,1\%,120) - 1500n + 12,000(A/F,1\%,120) = 3200n$
- (d) $-40,000(A/P,11.4\%,10) - 1500n + 12,000(A/F,11.4\%,10) = 3200n$

13.59 An anticorrosive coating for a chemical storage tank will cost \$5000 and last 5 years if touched up at the end of 3 years at a cost of \$1000. If an

oil-base enamel coating could be used that will last 2 years, the amount the enamel coating can cost for the two to breakeven at $i = 8\%$ per year is closest to:

- | | |
|------------|------------|
| (a) \$2120 | (b) \$2390 |
| (c) \$2590 | (d) \$2725 |

13.60 The price of a car is \$50,000 today. Its price is expected to increase by \$2400 each year. You now have \$25,000 in an investment that is earning 20% per year. The number of years before you have enough money to buy the car, without borrowing any money, is closest to:

- | | |
|-------------|-------------|
| (a) 3 years | (b) 5 years |
| (c) 7 years | (d) 9 years |

13.61 Process A has a fixed cost of \$16,000 per year and a variable cost of \$40 per unit. For process B, 5 units can be produced in 1 day at a cost of \$125. If the company's MARR is 10% per year, the fixed cost of process B that will make the two alternatives have the same annual cost at a production rate of 1000 units per year is closest to:

- (a) Less than \$10,000
- (b) \$18,000
- (c) \$27,000
- (d) Over \$30,000

13.62 The profit relation for the following estimates at a quantity that is 20% above breakeven is:

$$\begin{aligned} \text{Fixed cost} &= \$500,000 \text{ per year} \\ \text{Variable cost per unit} &= \$200 \\ \text{Revenue per unit} &= \$250 \end{aligned}$$

- (a) Profit = $200(12,000) - 250(12,000) - 500,000$
- (b) Profit = $250(12,000) - 500,000 - 200(12,000)$
- (c) Profit = $250(12,000) - 200(12,000) + 500,000$
- (d) Profit = $250(10,000) - 200(10,000) - 500,000$

13.63 Two methods of weed control in an irrigation canal are under consideration. Method A involves lining at a cost of \$4000. The lining will last 20 years. The maintenance cost with this method will be \$3 per mile per year. Method B involves spraying a chemical that costs \$40 per gallon. One gallon will treat 8 miles, but the treatment must be applied 4 times per year. In determining the number of miles per year that would result in breakeven, the variable cost for method B is closest to:

- (a) \$5 per mile
- (b) \$15 per mile
- (c) \$20 per mile
- (d) \$40 per mile

13.64 How long will you have to maintain a business that has an income of \$5000 per year and expenses of \$1500 per year if your initial investment was \$28,000 and your MARR is 10% per year?

- | | |
|-----------------------|--------------|
| (a) Less than 6 years | (b) 8 years |
| (c) 12 years | (d) 17 years |

CASE STUDY

WATER TREATMENT PLANT PROCESS COSTS

Background

Aeration and sludge recirculation have been practiced for many years at municipal and industrial water treatment plants. Aeration is used primarily for the physical removal of gases or volatile compounds, while sludge recirculation can be beneficial for turbidity removal and hardness reduction.

When the advantages of aeration and sludge recirculation in water treatment were first recognized, energy costs were so low that such considerations were seldom of concern in treatment plant design and operation. With the huge increases in electricity cost that have occurred in some localities, however, it became necessary to review the cost-effectiveness of all water treatment processes that consume significant amounts of energy. This study was conducted at a municipal water treatment plant for evaluating the cost-effectiveness of the pre-aeration and sludge recirculation practices.

Information

This study was conducted at a 106 m³ per minute water-treatment plant where, under normal operating circumstances, sludge from the secondary clarifiers is returned to the aerator and subsequently removed in the primary clarifiers. Figure 13–12 is a schematic of the process.

To evaluate the effect of sludge recirculation, the sludge pump was turned off, but aeration was continued. Next, the sludge pump was turned back on, and aeration was discontinued. Finally, both processes were discontinued. Results obtained during the test periods were averaged and compared to the values obtained when both processes were operational.

The results obtained from the four operating modes showed that the hardness decreased by 4.7% when both processes were in operation (i.e., sludge recirculation and aeration). When only sludge was recirculated, the reduction was 3.8%. There was no reduction due to aeration only, or when

there was neither aeration nor recirculation. For turbidity, the reduction was 28% when both recirculation and aeration were used. The reduction was 18% when *neither* aeration nor recirculation was used. The reduction was also 18% when aeration alone was used, which means that aeration alone was of no benefit for turbidity reduction. With sludge recirculation alone, the turbidity reduction was only 6%, meaning that sludge recirculation alone actually resulted in an *increase* in turbidity—the difference between 18% and 6%.

Since aeration and sludge recirculation did cause readily identifiable effects on treated water quality (some good and others bad), the cost-effectiveness of each process for turbidity and hardness reduction was investigated. The calculations are based on the following data:

$$\text{Aerator motor} = 40 \text{ hp}$$

$$\text{Aerator motor efficiency} = 90\%$$

$$\text{Sludge recirculation motor} = 5 \text{ hp}$$

$$\text{Recirculation pump efficiency} = 90\%$$

$$\text{Electricity cost} = 9 \text{ ¢/kWh (previous analysis)}$$

$$\text{Lime cost} = 7.9 \text{ ¢/kg}$$

$$\text{Lime required} = 0.62 \text{ mg/L per mg/L hardness}$$

$$\text{Coagulant cost} = 16.5 \text{ ¢/kg}$$

$$\text{Days/month} = 30.5$$

As a first step, the costs associated with aeration and sludge recirculation were calculated. In each case, costs are independent of flow rate.

Aeration cost:

$$\begin{aligned} 40 \text{ hp} \times 0.75 \text{ kW/hp} \times 0.09 \text{ \$/kWh} \times 24 \text{ h/day} \\ + 0.90 = \$72 \text{ per day or } \$2196 \text{ per month} \end{aligned}$$

Sludge recirculation cost:

$$\begin{aligned} 5 \text{ hp} \times 0.75 \text{ kW/hp} \times 0.09 \text{ \$/kWh} \times 24 \text{ h/day} \\ + 0.90 = \$9 \text{ per day or } \$275 \text{ per month} \end{aligned}$$

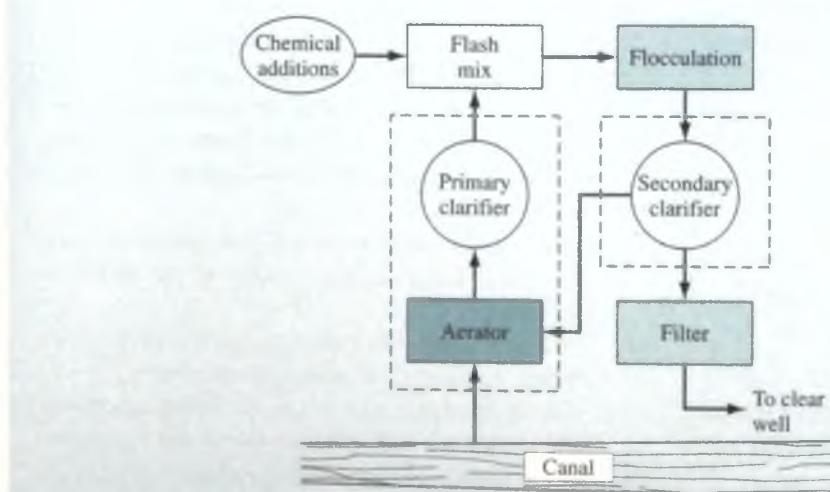


Figure 13–12
Schematic of water treatment plant.

TABLE 13-1 Cost Summary in Dollars per Month

Alt. I.D.	Alternative Description	Savings from Discontinuation of		Total Savings (3) = (1) + (2)	Extra Cost for Removal of		Total Extra Cost (6) = (4) + (5)	Net Savings (7) = (3) - (6)
		Aeration (1)	Recirculation (2)		Hardness (4)	Turbidity (5)		
1	Sludge recirculation and aeration	Normal operating condition						
2	Aeration only	—	275	275	1380	469	1849	-1574
3	Sludge recirculation only	2196	—	2196	262	845	1107	+1089
4	Neither aeration nor sludge recirculation	2196	275	2471	1380	469	1849	+622

The estimates appear in columns 1 and 2 of the cost summary in Table 13-1.

Costs associated with turbidity and hardness removal are a function of the chemical dosage required and the water flow rate. The calculations below are based on a design flow of 53 m³/minute.

As stated earlier, there was less turbidity reduction through the primary clarifier without aeration than there was with it (28% versus 6%). The extra turbidity reaching the flocculators could require further additions of the coagulating chemical. If it is assumed that, as a worst case, these chemical additions would be proportional to the extra turbidity, then 22 percent more coagulant would be required. Since the average dosage before discontinuation of aeration was 10 mg/L, the *incremental chemical cost* incurred because of the increased turbidity in the clarifier effluent would be

$$(10 \times 0.22) \text{ mg/L} \times 10^{-6} \text{ kg/mg} \times 53 \text{ m}^3/\text{min} \\ \times 1000 \text{ L/m}^3 \times 0.165 \text{ \$/kg} \times 60 \text{ min/h} \\ \times 24 \text{ h/day} = \$27.70/\text{day or } \$845/\text{month}$$

Similar calculations for the other operating conditions (i.e., aeration only, and neither aeration nor sludge recirculation) reveal that the additional cost for turbidity removal would be \$469 per month in each case, as shown in column 5 of Table 13-1.

Changes in hardness affect chemical costs by virtue of the direct effect on the amount of lime required for water softening. With aeration and sludge recirculation, the average hardness reduction was 12.1 mg/L (that is, 258 mg/L × 4.7%). However, with sludge recirculation only, the reduction was 9.8 mg/L, resulting in a difference of 2.3 mg/L attributed to aeration. The *extra cost of lime* incurred because of the discontinuation of aeration, therefore, was

$$2.3 \text{ mg/L} \times 0.62 \text{ mg/L lime} \times 10^{-6} \text{ kg/mg} \\ \times 53 \text{ m}^3/\text{min} \times 1000 \text{ L/m}^3 \times 0.079 \text{ \$/kg} \\ \times 60 \text{ min/h} \times 24 \text{ h/day} = \$8.60/\text{day or} \\ \$262/\text{month}$$

When sludge recirculation was discontinued, there was no hardness reduction through the clarifier, so that the extra lime cost would be \$1380 per month.

The total savings and total costs associated with changes in plant operating conditions are tabulated in columns 3 and 6 of Table 13-1, respectively, with the net savings shown in column 7. Obviously, the optimum condition is represented by "sludge recirculation only." This condition would result in a net savings of \$1089 per month, compared to a net savings of \$622 per month when both processes are discontinued and a net *cost* of \$1574 per month for aeration only. Since the calculations made here represent worst-case conditions, the actual savings that resulted from modifying the plant operating procedures were greater than those indicated.

In summary, the commonly applied water treatment practices of sludge recirculation and aeration can significantly affect the removal of some compounds in the primary clarifier. However, increasing energy and chemical costs warrant continued investigations on a case-by-case basis of the cost-effectiveness of such practices.

Case Study Exercises

- What will be the monthly savings in electricity from discontinuation of aeration if the cost of electricity is now 12 ¢/kWh?
- Does a decrease in the efficiency of the aerator motor make the selected alternative of sludge recirculation only more attractive, less attractive, or the same as before?
- If the cost of lime were to increase by 50%, would the cost difference between the best alternative and second-best alternative increase, decrease, or remain the same?
- If the efficiency of the sludge recirculation pump were reduced from 90% to 70%, would the net savings difference between alternatives 3 and 4 increase, decrease, or stay the same?
- If hardness removal were to be discontinued at the treatment plant, which alternative would be the most cost-effective?
- If the cost of electricity decreased to 8 ¢/kWh, which alternative would be the most cost-effective?
- At what electricity cost would the following alternatives just break even? (a) Alternatives 1 and 2, (b) alternatives 1 and 3, (c) alternatives 1 and 4.

Rounding Out the Study

LEARNING STAGE 4 Rounding Out the Study

CHAPTER 14 Effects of Inflation

CHAPTER 15 Cost Estimation and Indirect Cost Allocation

CHAPTER 16 Depreciation Methods

CHAPTER 17 After-Tax Economic Analysis

CHAPTER 18 Sensitivity Analysis and Staged Decisions

CHAPTER 19 More on Variation and Decision Making under Risk

This stage includes topics to enhance your ability to perform a thorough engineering economic study of one project or several alternatives. The effects of **inflation**, **depreciation**, **income taxes** in all types of studies, and **indirect costs** are incorporated into the methods of previous chapters. Techniques of **cost estimation** to better predict cash flows are treated in order to base alternative selection on more accurate estimates. The last two chapters include additional material on the use of engineering economics in decision making. An expanded version of **sensitivity analysis** is developed to examine parameters that vary over a predictable range of values. The use of **decision trees** and an introduction to **real options** are included. Finally, the elements of **risk** and **probability** are explicitly considered using expected values, probabilistic analysis, and spreadsheet-based Monte Carlo simulation.

Several of these topics can be covered earlier in the text, depending on the objectives of the course. Use the chart in the Preface to determine appropriate points at which to introduce the material in Learning Stage 4.



Effects of Inflation



LEARNING OUTCOMES

Purpose: Consider the effects of inflation when performing an engineering economy evaluation.

SECTION	TOPIC	LEARNING OUTCOME
14.1	Inflationary impact	<ul style="list-style-type: none">Demonstrate the difference that inflation makes on money now and money in the future; also, explain deflation.
14.2	PW with inflation	<ul style="list-style-type: none">Calculate the PW of cash flows with an adjustment made for inflation.
14.3	FW with inflation	<ul style="list-style-type: none">Determine the real interest rate and calculate the inflation-adjusted FW with different interpretations of future worth values.
14.4	CR with inflation	<ul style="list-style-type: none">Calculate capital recovery of an investment using the AW value with inflation considered.



This chapter concentrates upon understanding and calculating the effects of inflation in time value of money computations. Inflation is a reality that we deal with nearly everyday in our professional and personal lives.

The annual inflation rate is closely watched and historically analyzed by government units, businesses, and industrial corporations. An engineering economy study can have different outcomes in an environment in which inflation is a serious concern compared to one in which it is of minor consideration. In the first decade of the 21st century, inflation has not been a major concern in the United States or most industrialized nations. But the inflation rate is sensitive to real, as well as perceived, factors of the economy. Factors such as the cost of energy, interest rates, availability and cost of skilled people, scarcity of materials, political stability, and other, less tangible factors have short-term and long-term impacts on the inflation rate. In some industries, it is vital that the effects of inflation be integrated into an economic analysis. The basic techniques to do so are covered here.

14.1 Understanding the Impact of Inflation ● ● ●

We are all very well aware that \$20 now does not purchase the same amount as \$20 did in 2005 and purchases significantly less than in 2000. Why? Primarily this is due to inflation and the purchasing power of money.



Inflation is an increase in the amount of money necessary to obtain the **same amount** of goods or services before the inflated price was present.



Purchasing power, or buying power, measures the value of a currency in terms of the quantity and quality of goods or services that one unit of money will purchase. Inflation decreases the purchasing ability of money in that **less** goods or services **can be purchased** for the same one unit of money.

Inflation occurs because the value of the currency has changed—it has gone down in value. The value of money has decreased, and as a result, it takes more money for the same amount of goods or services. This is a **sign of inflation**. To make comparisons between monetary amounts that occur in different time periods, the different-valued money first must be converted to constant-value money in order to represent the same **purchasing power over time**. This is especially important when future sums of money are considered, as is the case with all alternative evaluations.

Money in one period of time t_1 can be brought to the same value as money in another period of time t_2 by using the equation

$$\text{Amount in period } t_1 = \frac{\text{amount in period } t_2}{\text{inflation rate between } t_1 \text{ and } t_2} \quad [14.1]$$

Using dollars as the currency, dollars in period t_1 are called **constant-value dollars** or **today's dollars**. Dollars in period t_2 are called **future dollars** or **then-current dollars** and have inflation taken into account. If f represents the inflation rate per period (year) and n is the number of time periods (years) between t_1 and t_2 , Equation [14.1] is

$$\text{Constant-value dollars} = \frac{\text{future dollars}}{(1 + f)^n} \quad [14.2]$$

$$\text{Future dollars} = \text{constant-value dollars}(1 + f)^n \quad [14.3]$$

We can express future dollars in terms of constant-value dollars, and vice versa, by applying the last two equations. This is how the Consumer Price Index (CPI) and cost estimation indices (of Chapter 15) are determined. As an illustration, use the price of a cheese pizza.

\$8.99 March 2011

If inflation on food prices averaged 5% during the last year, in *constant-value 2010 dollars*, this cost is last year's equivalent of

\$8.99 / 1.05 = \$8.56 March 2010

A predicted price in 2012, according to Equation [14.3], is

$$\$8.99(1.05) = \$9.44 \quad \text{March 2012}$$

The price of \$9.44 in 2012 buys exactly the same cheese pizza as \$8.56 did in 2010. If inflation averages 5% per year over the next 10 years, Equation [14.3] is used to predict a price in 2020 based on 2010.

$$\$8.56(1.05)^{10} = \$13.94 \quad \text{March 2020}$$

This is a 63% increase over the 2010 price at 5% inflation for prepared food prices, which is generally not considered excessive. In some areas of the world, hyperinflation may average 50% per year. In such an unfortunate economy, the cheese pizza in 10 years rises from the dollar equivalent of \$8.99 to \$518.44! This is why countries experiencing hyperinflation must devalue the currency by factors of 100 and 1000 when unacceptable inflation rates persist.

Placed into an industrial or business context, at a reasonably low inflation rate averaging 4% per year, equipment or services with a first cost of \$209,000 will increase by 48% to \$309,000 over a 10-year span. This is before any consideration of the rate of return requirement is placed upon the equipment's revenue-generating ability. **Make no mistake: Inflation is a formidable force in our economy.**

There are three different rates that are important to understanding inflation: the real interest rate (i), the market interest rate (i_f), and the inflation rate (f). Only the first two are interest rates.

Real or inflation-free interest rate i . This is the rate at which interest is earned when the effects of changes in the value of currency (inflation) have been removed. Thus, the real interest rate presents an actual gain in purchasing power. (The equation used to calculate i , with the influence of inflation removed, is derived later in Section 14.3.) The real rate of return that generally applies for individuals is approximately 3.5% per year. This is the "safe investment" rate. The required real rate for corporations (and many individuals) is set above this safe rate when a MARR is established without an adjustment for inflation.

Inflation-adjusted or market interest rate i_f . As its name implies, this is the interest rate that has been adjusted to take inflation into account. This is the interest rate we hear everyday. It is a combination of the real interest rate i and the inflation rate f , and, therefore, it changes as the inflation rate changes. It is also known as the *inflated interest rate*. A company's MARR adjusted for inflation is referred to as the inflation-adjusted or market MARR. The determination of this value is discussed in Section 14.3.

Inflation rate f . As described above, this is a measure of the rate of change in the value of the currency.

Deflation is the opposite of inflation in that when deflation is present, the purchasing power of the monetary unit is greater in the future than at present. That is, it will take fewer dollars in the future to buy the same amount of goods or services as it does today. Inflation occurs much more commonly than deflation, especially at the national economy level. In deflationary economic conditions, the market interest rate is always less than the real interest rate.

Temporary price deflation may occur in specific sectors of the economy due to the introduction of improved products, cheaper technology, or imported materials or products that force current prices down. In normal situations, prices equalize at a competitive level after a short time. However, deflation over a short time in a specific sector of an economy can be orchestrated through **dumping**. An example of dumping may be the importation of materials, such as steel, cement, or cars, into one country from international competitors at very low prices compared to current market prices in the targeted country. The prices will go down for the consumer, thus forcing domestic manufacturers to reduce their prices in order to compete for business. If domestic manufacturers are not in good financial condition, they may fail, and the imported items replace the domestic supply. Prices may then return to normal levels and, in fact, become inflated over time, if competition has been significantly reduced.

On the surface, having a moderate rate of deflation sounds good when inflation has been present in the economy over long periods. However, if deflation occurs at a more general

level, say nationally, it is likely to be accompanied by the lack of money for new capital. Another result is that individuals and families have less money to spend due to fewer jobs, less credit, and fewer loans available; an *overall "tighter" money* situation prevails. As money gets tighter, less is available to be committed to industrial growth and capital investment. In the extreme case, this can evolve over time into a deflationary spiral that disrupts the entire economy. This has happened on occasion, notably in the United States during the Great Depression of the 1930s.

Engineering economy computations that consider deflation use the same relations as those for inflation. For basic equivalence between constant-value dollars and future dollars, Equations [14.2] and [14.3] are used, except the deflation rate is a $-f$ value. For example, if deflation is estimated to be 2% per year, an asset that costs \$10,000 today would have a first cost 5 years from now determined by Equation [14.3].

$$10,000(1 - f)^n = 10,000(0.98)^5 = 10,000(0.9039) = \$9039$$

14.2 Present Worth Calculations Adjusted for Inflation ● ● ●

When the dollar amounts in different time periods are to be expressed in *constant-value dollars*, the equivalent present and future amounts must be determined using the *real interest rate i*. The calculations involved in this procedure are illustrated in Table 14-1, where the inflation rate is 4% per year. Column 2 shows the inflation-driven increase for each of the next 4 years for an item that has a cost of \$5000 today. Column 3 shows the cost in future dollars, and column 4 verifies the cost in constant-value dollars via Equation [14.2]. When the future dollars of column 3 are converted to constant-value dollars (column 4), the cost is always \$5000, the same as the cost at the start. This is predictably true when the costs are increasing by an amount *exactly equal* to the inflation rate. The actual cost (in inflated dollars) of the item 4 years from now will be \$5849, but in constant-value dollars the cost in 4 years will still amount to \$5000. Column 5 shows the present worth of future amounts of \$5000 at a real interest rate of $i = 10\%$ per year.

Two conclusions can be drawn. At $f = 4\%$, \$5000 today inflates to \$5849 in 4 years. And \$5000 four years in the future has a PW of only \$3415 now in constant-value dollars at a real interest rate of 10% per year.

Figure 14-1 graphs the differences over a 4-year period of the constant-value amount of \$5000, the future-dollar costs at 4% inflation, and the present worth at 10% real interest with inflation considered. The effect of compounded inflation and interest rates can be large, as you can see by the shaded area.

An alternative, less complicated method of accounting for inflation in a present worth analysis involves adjusting the interest formulas themselves to account for inflation. Consider the P/F formula, where i is the real interest rate.

$$P = F \frac{1}{(1 + i)^n}$$

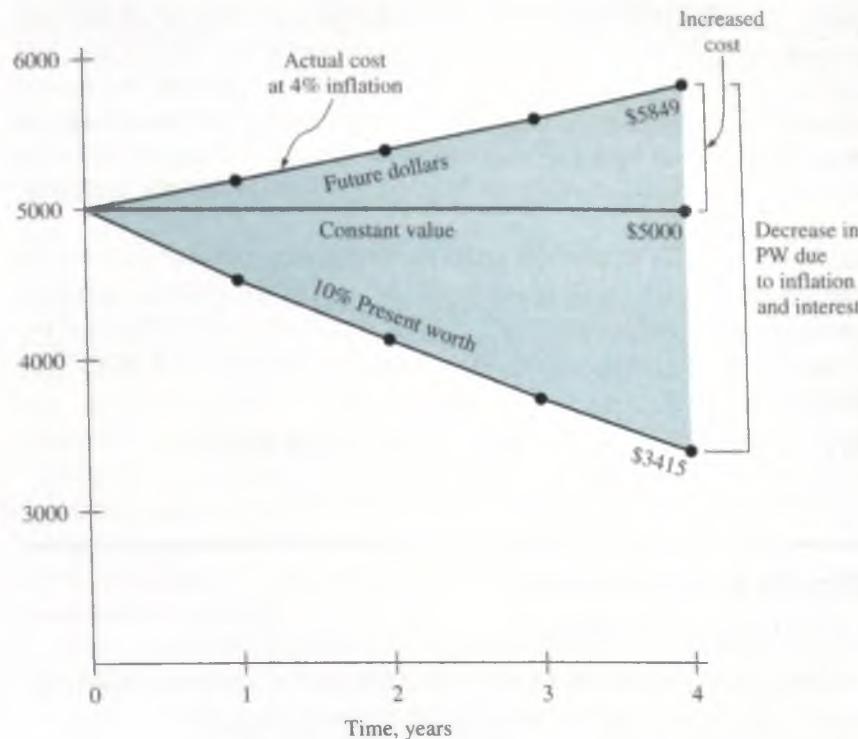


TABLE 14-1 Inflation Calculations Using Constant-Value Dollars ($f = 4\%$, $i = 10\%$)

Year <i>n</i>	Cost Increase due to 4% Inflation, \$	Cost in Future Dollars, \$	Future Cost in Constant-Value Dollars, \$ (4) = (3)/1.04 ^{<i>n</i>}	Present Worth at Real <i>i</i> = 10%, \$ (5) = (4)(P/F, 10%, <i>n</i>)
(1)	(2)	(3)	(4)	(5)
0		5000	5000	5000
1	$5000(0.04) = 200$	5200	$5200/(1.04)^1 = 5000$	4545
2	$5200(0.04) = 208$	5408	$5408/(1.04)^2 = 5000$	4132
3	$5408(0.04) = 216$	5624	$5624/(1.04)^3 = 5000$	3757
4	$5624(0.04) = 225$	5849	$5849/(1.04)^4 = 5000$	3415

Figure 14–1

Comparison of constant-value dollars, future dollars, and their present worth values.



The F , which is a future-dollar amount with inflation built in, can be converted to constant-value dollars by using Equation [14.2].

$$\begin{aligned} P &= \frac{F}{(1+f)^n} \frac{1}{(1+i)^n} \\ &= F \frac{1}{(1+i+f+if)^n} \end{aligned} \quad [14.4]$$

If the term $i + f + if$ is defined as i_f , the equation becomes

$$P = F \frac{1}{(1+i_f)^n} = F(P/F, i_f, n) \quad [14.5]$$

As described earlier, i_f is the **inflation-adjusted or market interest rate** and is defined as

$$i_f = i + f + if \quad [14.6]$$

where i = real interest rate

f = inflation rate

For a real interest rate of 10% per year and an inflation rate of 4% per year, Equation [14.6] yields a market interest rate of 14.4%.

$$i_f = 0.10 + 0.04 + 0.10(0.04) = 0.144$$

Table 14–2 illustrates the use of $i_f = 14.4\%$ in PW calculations for \$5000 now, which inflates to \$5849 in future dollars 4 years hence. As shown in column 4, the present worth for each year is the same as column 5 of Table 14–1.

The present worth of any series of cash flows—uniform, arithmetic gradient, or geometric gradient—can be found similarly. That is, either i or i_f is introduced into the P/A , P/G , or P_g factors, depending upon whether the cash flow is expressed in constant-value (today's) dollars or future dollars, respectively.

If a cash flow series is expressed in today's (constant-value) dollars, then its PW is the discounted value using the real interest rate i .

If the cash flow is expressed in future dollars, the PW value is obtained using i_f .

It is always acceptable to first convert all future dollars to constant-value dollars using Equation [14.2] and then find the PW at the real interest rate i .

TABLE 14–2 Present Worth Calculation Using an Inflated Interest Rate

Year <i>n</i> (1)	Cost in Future Dollars, \$ (2)	(P/F, 14.4%, <i>n</i>) (3)	PW, \$ (4) = (2)(3)
0	5000	1	5000
1	5200	0.8741	4545
2	5408	0.7641	4132
3	5624	0.6679	3757
4	5849	0.5838	3415

EXAMPLE 14.1

Glyphosate is the active ingredient in the herbicide Roundup® marketed by Monsanto Co. Roundup has been a dependable product used by farmers, municipalities, and suburbanites alike to control weeds in fields, yards, gardens, streets, and parks. Contributions to Monsanto's revenue have been reduced significantly by international dumping of generic glyphosate, as announced in mid-2010.¹ Monsanto's sales price was decreased from \$16 to \$12 per gallon to compete with the highly competitive pricing, and it is expected that the international price will settle at approximately \$10 per gallon. Assume when the price was set at \$16 per gallon, there was a prediction that in 5 years the price would inflate to \$19 per gallon. Perform the following analysis.

- (a) Determine the annual rate of inflation over 5 years to increase the price from \$16 to \$19.
- (b) Using the same annual rate determined above as the rate at which the price continues to *decline* from the new \$12 price, calculate the expected price in 5 years. Compare this result with \$10 per gallon that Monsanto predicted would be the longer-term price.
- (c) Provided Monsanto were somehow able to recover the same market share as it had previously, and the same inflation rate was applied to the reduced \$12 per gallon price, determine the price 5 years in the future and compare it with the pre-dumping price of \$16 per gallon.
- (d) Determine the market interest rate that must be used in economic equivalence computations, if inflation is considered and an 8% per year real return is expected by Monsanto.

Solution

The first three parts involve inflation only—no return on investments.

- (a) Solve Equation [14.2] for the annual inflation rate *f* with known constant-value and future amounts.

$$\begin{aligned} 16 &= 19(P/F, f, 5) = \frac{19}{(1+f)^5} \\ 1+f &= (1.1875)^{0.2} \\ f &= 0.035 \quad (3.5\% \text{ per year}) \end{aligned}$$

- (b) If the price deflation rate is 3.5% per year, find the *F* value in 5 years with *P* = \$12.

$$\begin{aligned} F &= P(F/P, -3.5\%, 5) = 12(1 - 0.035)^5 \\ &= 12(0.8368) \\ &= \$10.04 \end{aligned}$$

The price will fall to exactly \$10 per gallon after 5 years, as Monsanto predicted.

¹S. Kilman and I. Berry, "Monsanto Cuts Roundup Prices as Knockoffs Flood Farm Belt," *Wall Street Journal*, May 28, 2010.

(c) Five years in the future, at 3.5% per year inflation, the price will be

$$\begin{aligned} F &= P(F/P, 3.5\%, 5) = 12(1.035)^5 \\ &= 12(1.1877) \\ &= \$14.25 \end{aligned}$$

After 5 years of recovery at the same level as historically experienced, the price will still be considerably lower than it was at the pre-dumping point (\$14.25 versus \$16 per gallon).

(d) With inflation at 3.5% per year and a real return of 8% per year, Equation [14.6] results in a market rate of 11.78% per year.

$$\begin{aligned} i_f &= 0.08 + 0.035 + (0.08)(0.035) \\ &= 0.1178 \quad (11.78\% \text{ per year}) \end{aligned}$$

EXAMPLE 14.2

A 15-year \$50,000 bond that has a dividend rate of 10% per year, payable semiannually, is currently for sale. If the expected rate of return of the purchaser is 8% per year, compounded semiannually, and if the inflation rate is expected to be 2.5% each 6-month period, what is the bond worth now (a) without an adjustment for inflation and (b) when inflation is considered? Show both hand and spreadsheet solutions.

Solution by Hand

(a) *Without inflation adjustment:* The semiannual dividend is $I = [(50,000)(0.10)]/2 = \2500 . At a nominal 4% per 6 months for 30 periods,

$$PW = 2500(P/A, 4\%, 30) + 50,000(P/F, 4\%, 30) = \$58,645$$

(b) *With inflation:* Use the inflated rate i_f

$$i_f = 0.04 + 0.025 + (0.04)(0.025) = 0.066 \text{ per semiannual period}$$

$$\begin{aligned} PW &= 2500(P/A, 6.6\%, 30) + 50,000(P/F, 6.6\%, 30) \\ &= 2500(12.9244) + 50,000(0.1470) \\ &= \$39,660 \end{aligned}$$

Solution by Spreadsheet

Both (a) and (b) require simple, single-cell functions on a spreadsheet (Figure 14–2). Without an inflation adjustment, the PV function is developed at the nominal 4% rate for 30 periods; with inflation considered the rate is $i_f = 6.6\%$, as determined above.

Comment

The \$18,985 difference in PW values illustrates the tremendous negative impact made by only 2.5% inflation each 6 months (5.06% per year). Purchasing the \$50,000 bond means receiving \$75,000 in dividends over 15 years and the \$50,000 principal in year 15. Yet, this is worth only \$39,660 in constant-value dollars.

	A	Function	PW value, \$
1			
2	(a) PW without inflation	= PV(4%, 30, -2500, -50000)	58,646
3			
4	(b) PW with inflation adjustment	= PV(6.6%, 30, -2500, -50000)	39,660
5			
6			

Figure 14–2

PW computation of a bond purchase (a) without and (b) with an inflation adjustment, Example 14.2.

EXAMPLE 14.3

A self-employed chemical engineer is on contract with Dow Chemical, currently working in a relatively high-inflation country in Central America. She wishes to calculate a project's PW with estimated costs of \$35,000 now and \$7000 per year for 5 years beginning 1 year from now with increases of 12% per year thereafter for the next 8 years. Use a real interest rate of 15% per year to make the calculations (a) without an adjustment for inflation and (b) considering inflation at a rate of 11% per year.

Solution

(a) Figure 14–3 presents the cash flows. The PW without an adjustment for inflation is found using $i = 15\%$ and $g = 12\%$ in Equations [2.34] and [2.35] for the geometric series.

$$\begin{aligned} \text{PW} &= -35,000 - 7000(P/A, 15\%, 4) \\ &\quad - \left\{ \frac{7000 \left[1 - \left(\frac{1.12}{1.15} \right)^9 \right]}{0.15 - 0.12} \right\} (P/F, 15\%, 4) \\ &= -35,000 - 19,985 - 28,247 \\ &= \$-83,232 \end{aligned}$$

In the P/A factor, $n = 4$ because the \$7000 cost in year 5 is the A_1 term in Equation [2.34].

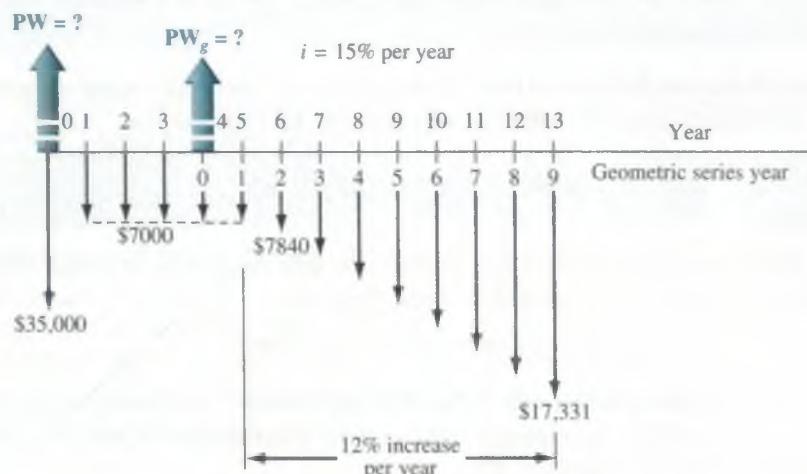


Figure 14–3
Cash flow diagram, Example 14.3.

(b) To adjust for inflation, calculate the inflated interest rate by Equation [14.6] and use it to calculate PW.

$$\begin{aligned} i_f &= 0.15 + 0.11 + (0.15)(0.11) = 0.2765 \\ \text{PW} &= -35,000 - 7000(P/A, 27.65\%, 4) \\ &\quad - \left\{ \frac{7000 \left[1 - \left(\frac{1.12}{1.2765} \right)^9 \right]}{0.2765 - 0.12} \right\} (P/F, 27.65\%, 4) \\ &= -35,000 - 7000(2.2545) - 30,945(0.3766) \\ &= \$-62,436 \end{aligned}$$

Comment

This result demonstrates that in a high-inflation economy, when negotiating the amount of the payments to repay a loan, it is economically advantageous for the borrower to use future (inflated) dollars whenever possible to make the payments. The present value of future inflated dollars is significantly less when the inflation adjustment is included. And the higher the inflation rate, the larger the discounting because the P/F and P/A factors decrease in size.

Examples 14.2 and 14.3 above add credence to the “buy now, pay later” philosophy. However, at some point, the debt-ridden company or individual will have to pay off the debts and the accrued interest with the inflated dollars. If cash is not readily available at that time, the debts cannot be repaid. This can happen, for example, when a company unsuccessfully launches a new product, when there is a serious downturn in the economy, or when an individual loses a salary. In the longer term, this buy now, pay later approach must be tempered with sound financial practices now, and in the future.



14.3 Future Worth Calculations Adjusted for Inflation ● ● ●

In future worth calculations, a future amount F can have any one of four different interpretations:

Case 1. The *actual amount* of money that will be *accumulated* at time n .

Case 2. The *purchasing power* of the actual amount accumulated at time n , but stated in today's (constant-value) dollars.

Case 3. The number of *future dollars required* at time n to maintain the *same purchasing power* as today; that is, inflation is considered, but interest is not.

Case 4. The amount of money required at time n to *maintain purchasing power and earn a stated real interest rate*.

Depending upon which interpretation is intended, the F value is calculated differently, as described below. Each case is illustrated.

Case 1: Actual Amount Accumulated It should be clear that F , the actual amount of money accumulated, is obtained using the inflation-adjusted (market) interest rate.

$$F = P(1 + i_f)^n = P(F/P, i_f, n) \quad [14.7]$$

For example, when we quote a market rate of 10%, the inflation rate is included. Over a 7-year period, \$1000 invested at 10% per year will accumulate to

$$F = 1000(F/P, 10\%, 7) = \$1948$$

Case 2: Constant-Value Dollars with Purchasing Power The purchasing power of future dollars is determined by first using the market rate i_f to calculate F and then deflating the future amount through division by $(1 + f)^n$.

$$F = \frac{P(1 + i_f)^n}{(1 + f)^n} = \frac{P(F/P, i_f, n)}{(1 + f)^n} \quad [14.8]$$

This relation, in effect, recognizes the fact that inflated prices mean \$1 in the future purchases less than \$1 now. The percentage loss in purchasing power is a measure of how much less. As an illustration, consider the same \$1000 now, and a 10% per year *market rate*, which includes an inflation rate of 4% per year. In 7 years, the purchasing power has risen, but only to \$1481.

$$F = \frac{1000(F/P, 10\%, 7)}{(1.04)^7} = \frac{\$1948}{1.3159} = \$1481$$

This is \$467 (or 24%) less than the \$1948 actually accumulated at 10% (case 1). Therefore, we conclude that 4% inflation over 7 years reduces the purchasing power of money by 24%.

Also for case 2, the future amount of money accumulated with today's buying power could equivalently be determined by calculating the real interest rate and using it in the F/P factor to compensate for the decreased purchasing power. This **real interest rate** is the i in Equation [14.6].

$$\begin{aligned} i_f &= i + f + if \\ &= i(1 + f) + f \end{aligned}$$

$$i = \frac{i_f - f}{1 + f} \quad [14.9]$$

The real interest rate i represents the rate at which today's dollars expand with their *same purchasing power* into equivalent future dollars. An inflation rate larger than the market interest rate leads to a negative real interest rate. The use of this interest rate is appropriate for calculating the future worth of an investment (such as a savings account or money market fund) when the effect of inflation must be removed. For the example of \$1000 in today's dollars from Equation [14.9]

$$i = \frac{0.10 - 0.04}{1 + 0.04} = 0.0577 \quad (5.77\%)$$

$$F = 1000(F/P, 5.77\%, 7) = \$1481$$

The market interest rate of 10% per year has been reduced to a real rate that is less than 6% per year because of the erosive effects of 4% per year inflation.

Case 3: Future Amount Required, No Interest This case recognizes that prices increase when inflation is present. Simply put, future dollars are worth less, so more are needed. No interest rate is considered in this case—only inflation. This is the situation if someone asks, “How much will a car cost in 5 years if its current cost is \$20,000 and its price will increase by the inflation rate of 6% per year?” (The answer is \$26,765.) No interest rate—only inflation—is involved. To find the future cost, substitute f for the interest rate in the F/P factor.

$$F = P(1 + f)^n = P(F/P, f, n)$$

[14.10]

Reconsider the \$1000 used previously. If it is escalating at exactly the inflation rate of 4% per year, the amount 7 years from now will be

$$F = 1000(F/P, 4\%, 7) = \$1316$$

Case 4: Inflation and Real Interest This is the case applied when a market MARR is established. Maintaining purchasing power and earning interest must account for both increasing prices (case 3) and the time value of money. If the growth of capital is to keep up, funds must grow at a rate equal to or above the real interest rate i plus the inflation rate f . Thus, to make a *real rate of return of 5.77%* when the inflation rate is 4%, i_f is the market (inflation-adjusted) rate that must be used. For the same \$1000 amount,

$$i_f = 0.0577 + 0.04 + 0.0577(0.04) = 0.10$$

$$F = 1000(F/P, 10\%, 7) = \$1948$$

This calculation shows that \$1948 seven years in the future will be equivalent to \$1000 now with a real return of $i = 5.77\%$ per year and inflation of $f = 4\%$ per year.

Table 14–3 summarizes which rate is used in the equivalence formulas for the different interpretations of F . The calculations made in this section explain the following:

- The amount of \$1000 now at a market rate of 10% per year will accumulate to \$1948 in 7 years.
- The \$1948 will have the purchasing power of \$1481 of today's dollars if $f = 4\%$ per year.
- An item with a cost of \$1000 now will cost \$1316 in 7 years at an inflation rate of 4% per year.
- It will take \$1948 of future dollars to be equivalent to \$1000 now at a real interest rate of 5.77% with inflation considered at 4% per year.

Most corporations evaluate alternatives at a MARR large enough to cover inflation plus some return greater than their cost of capital, and significantly higher than the safe investment return of approximately 3.5% mentioned earlier. Therefore, for case 4, the resulting MARR will normally be higher than the market rate i_f . Define the symbol MARR_f as the inflation-adjusted or market MARR, which is calculated in a fashion similar to i_f .

$$\text{MARR}_f = i + f + i(f)$$

[14.11]

TABLE 14-3 Calculation Methods for Various Future Worth Interpretations

Future Worth Desired	Method of Calculation	Example for $P = \$1000, n = 7,$ $i_f = 10\%, f = 4\%$
<i>Case 1:</i> Actual dollars accumulated	Use stated market rate i_f in equivalence formulas	$F = 1000(F/P, 10\%, 7)$
<i>Case 2:</i> Purchasing power of accumulated dollars in terms of constant-value dollars	Use market rate i_f in equivalence and divide by $(1 + f)^n$ or	$F = \frac{1000(F/P, 10\%, 7)}{(1.04)^7}$ or
	Use real i	$F = 1000(F/P, 5.77\%, 7)$
<i>Case 3:</i> Dollars required for same purchasing power	Use f in place of i in equivalence formulas	$F = 1000(F/P, 4\%, 7)$
<i>Case 4:</i> Future dollars to maintain purchasing power and to earn a return	Calculate i_f and use in equivalence formulas	$F = 1000(F/P, 10\%, 7)$

The real rate of return i used here is the required rate for the corporation relative to its cost of capital. Now the future worth F , or FW, is calculated as

$$F = P(1 + MARR_f)^n = P(F/P, MARR_f, n)$$

For example, if a company has a WACC (weighted average cost of capital) of 10% per year and requires that a project return 3% per year above its WACC, the real return is $i = 13\%$. The inflation-adjusted MARR is calculated by including the inflation rate of, say, 4% per year. Then the project PW, AW, or FW will be determined at the rate obtained from Equation [14.11].

$$MARR_f = 0.13 + 0.04 + 0.13(0.04) = 0.1752 \quad (17.52\%)$$

EXAMPLE 14.4

Abbott Mining Systems wants to determine whether it should upgrade a piece of equipment used in deep mining operations in one of its international operations now or later. If the company selects plan A, the upgrade will be purchased now for \$200,000. However, if the company selects plan I, the purchase will be deferred for 3 years when the cost is expected to rise to \$300,000. Abbott is ambitious; it expects a real MARR of 12% per year. The inflation rate in the country has averaged 3% per year. From only an economic perspective, determine whether the company should purchase now or later (a) when inflation is not considered and (b) when inflation is considered.

Solution

(a) *Inflation not considered:* The real rate, or MARR, is $i = 12\%$ per year. The cost of plan I is \$300,000 three years hence. Calculate the FW value for plan A three years from now and select the lower cost.

$$\begin{aligned} FW_A &= -200,000(F/P, 12\%, 3) = -\$280,986 \\ FW_I &= -\$300,000 \end{aligned}$$

Select plan A (purchase now).

(b) *Inflation considered:* This is case 4; the real rate (12%), and inflation of 3% must be accounted for. First, compute the inflation-adjusted MARR by Equation [14.11].

$$MARR_f = 0.12 + 0.03 + 0.12(0.03) = 0.1536$$

Use MARR_f to compute the FW value for plan A in future dollars.

$$FW_A = -200,000(F/P, 15.36\%, 3) = \$-307,040$$

$$FW_I = \$-300,000$$

Purchase later (plan I) is now selected, because it requires fewer equivalent future dollars. The inflation rate of 3% per year has raised the equivalent future worth of costs by 9.3% from \$280,986 to \$307,040. This is the same as an increase of 3% per year, compounded over 3 years, or $(1.03)^3 - 1 = 9.3\%$.

Most countries have inflation rates in the range of 2% to 8% per year, but **hyperinflation** is a problem in countries where political instability, overspending by the government, weak international trade balances, etc., are present. Hyperinflation rates may be very high—10% to 100% *per month*. In these cases, the government may take drastic actions: redefine the currency in terms of the currency of another country, control banks and corporations, and control the flow of capital into and out of the country in order to decrease inflation.

In a hyperinflated environment, people usually spend all their money immediately since the cost of goods and services will be so much higher the next month, week, or day. To appreciate the disastrous effect of hyperinflation on a company's ability to keep up, we can rework Example 14.4b using an inflation rate of 10% per month, that is, a nominal 120% per year (not considering the compounding effect of inflation). The FW_A amount skyrockets and plan I is a clear choice. Of course, in such an environment the \$300,000 purchase price for plan I three years hence would obviously not be guaranteed, so the entire economic analysis is unreliable. Good economic decisions in a hyperinflated economy are very difficult to make using traditional engineering economy methods, since the estimated future values are totally unreliable and the future availability of capital is uncertain.

14.4 Capital Recovery Calculations Adjusted for Inflation ● ● ●

It is particularly important in capital recovery (CR) calculations used for AW analysis to include inflation because current capital dollars must be recovered with future inflated dollars. Since future dollars have less buying power than today's dollars, it is obvious that more dollars will be required to recover the present investment. This suggests the use of the inflated interest rate in the A/P formula. For example, if **\$1000 is invested today** at a real interest rate of 10% per year when the inflation rate is 8% per year, the equivalent amount that must be recovered each year for 5 years in future dollars is

$$A = 1000(A/P, 18.8\%, 5) = \$325.59$$

On the other hand, the decreased value of dollars through time means that investors can spend fewer present (higher-value) dollars to accumulate a specified amount of future (inflated) dollars. This suggests the use of a higher interest rate, that is, the *i_f* rate, to produce a lower *A* value in the A/F formula. The annual equivalent (with adjustment for inflation) of **F = \$1000 five years from now** in future dollars is

$$A = 1000(A/F, 18.8\%, 5) = \$137.59$$

For comparison, the equivalent annual amount to accumulate **F = \$1000** at a real *i* = 10% (without adjustment for inflation) is $1000(A/F, 10\%, 5) = \$163.80$. When *F* is a future known cost, uniformly distributed payments should be spread over as long a time period as possible so that the leveraging effect of inflation will reduce the effective annual payment (\$137.59 versus \$163.80 **here**).



EXAMPLE 14.5

What annual deposit is required for 5 years to accumulate an amount of money with the same purchasing power as \$680.58 today, if the market interest rate is 10% per year and inflation is 8% per year?

Solution

First, find the actual number of inflated dollars required 5 years in the future that is equivalent to \$680.58 today. This is case 3; Equation [14.10] applies.

$$F = (\text{present purchasing power})(1 + f)^5 = 680.58(1.08)^5 = \$1000$$

The actual amount of the annual deposit is calculated using the market interest rate of 10%. This is case 4 where A is calculated for a given F .

$$A = 1000(A/F, 10\%, 5) = \$163.80$$

Comment

The real interest rate is $i = 1.85\%$ as determined using Equation [14.9]. To put these calculations into perspective, if the inflation rate is zero when the real interest rate is 1.85%, the future amount of money with the same purchasing power as \$680.58 today is obviously \$680.58. Then the annual amount required to accumulate this future amount in 5 years is $A = 680.58(A/F, 1.85\%, 5) = \131.17 . This is \$32.63 lower than the \$163.80 calculated above for $f = 8\%$. This difference is due to the fact that during inflationary periods, dollars deposited now have more purchasing power than the dollars returned at the end of the period. To make up the purchasing power difference, more higher-value dollars are required. That is, to maintain equivalent purchasing power at $f = 8\%$ per year, an extra \$32.63 per year is required.

The logic discussed here explains why, in times of increasing inflation, lenders of money (credit card companies, mortgage companies, and banks) tend to further increase their market interest rates. People tend to pay off less of their incurred debt at each payment because they use any excess money to purchase additional items before the price is further inflated. Also, the lending institutions must have more money in the future to cover the expected higher costs of lending money. All this is due to the spiraling effect of increasing inflation. Breaking this cycle is difficult to do at the individual level and much more difficult to alter at a national level.

CHAPTER SUMMARY

Inflation, treated computationally as an interest rate, makes the cost of the same product or service increase over time due to the decreased value of money. There are several ways to consider inflation in engineering economy computations in terms of today's (constant-value) dollars and in terms of future dollars. Some important relations are the following:

Inflated interest rate: $i_f = i + f + if$

Real interest rate: $i = (i_f - f)/(1 + f)$

PW of a future amount with inflation considered: $P = F(P/F, i_f, n)$

Future worth in constant-value dollars of a present amount with the same purchasing power:

$$F = P(F/P, i_f, n)$$

Future amount to cover a current amount with inflation only: $F = P(F/P, f, n)$

Future amount to cover a current amount with inflation and interest: $F = P(F/P, i_f, n)$

Annual equivalent of a future amount: $A = F(A/F, i_f, n)$

Annual equivalent of a present amount in future dollars: $A = P(A/P, i_f, n)$

Hyperinflation implies very high f values. Available funds are expended immediately because costs increase so rapidly that larger cash inflows cannot offset the fact that the currency is losing value. This can, and usually does, cause a national financial disaster when it continues over extended periods of time.

PROBLEMS

Interest Rate and Currency Considerations with Inflation

- 14.1 What is the difference between today's dollars and constant-value dollars (a) when using *today* as the reference point in time and (b) when using 2 years ago as the reference point?
- 14.2 State the conditions under which the market interest rate is (a) higher than, (b) lower than, and (c) the same as the real interest rate.
- 14.3 What annual inflation rate is implied from an inflation-adjusted interest rate of 10% per year, when the real interest rate is 4% per year?
- 14.4 Determine the inflation-adjusted interest rate for a growth company that wants to earn a real rate of return of 20% per year when the inflation rate is 5% per year.
- 14.5 For a high-growth company that wants to make a real rate of return of 30% per year, compounded monthly, determine the inflation-adjusted nominal interest rate per year. Assume the inflation rate is 1.5% per month.
- 14.6 A high-tech company whose stock trades on the NASDAQ stock exchange uses a MARR of 35% per year. If the chief financial officer (CFO) said the company expects to make a *real rate of return* of 25% per year on its investments over the next 3-year period, what is the company expecting the inflation rate per year to be over that time period?
- 14.7 Calculate the inflation-adjusted interest rate per quarter when the real interest rate is 4% per quarter and the inflation rate is 1% per quarter.
- 14.8 Calculate the real interest rate per month if the nominal inflation-adjusted interest rate per year, compounded monthly, is 18% and the inflation rate per month is 0.5%.
- 14.9 A southwestern city has a contract with Firestone Vehicle Fleet Maintenance to provide maintenance services on its fleet of city-owned vehicles such as street sweepers, garbage trucks, backhoes, and carpool vehicles. The contract price is fixed at \$45,000 per year for 4 years. If you were asked to convert the future amounts into constant-value amounts per today's dollars, what would the respective amounts be? Assume the current market interest rate of 10% per year and inflation rate of 5% per year are expected to remain the same over the next 4-year period.

14.10 When the inflation rate is 5% per year, how many inflated dollars will be required 10 years from now to buy the same things that \$10,000 buys now?

14.11 Assume that you want to retire 30 years from now with an amount of money that will have the same value (same purchasing power) as \$1.5 million today. If you estimate the inflation rate will be 4% per year, how many future (then-current) dollars will you need?

Adjusting for Inflation

- 14.12 If the inflation rate is 7% per year, how many years will it take for the cost of something to double when prices increase at exactly the same rate as inflation?
- 14.13 The inflation rate in a Central American country is 6% per year. What real rate of return will an investor make on a \$100,000 investment in a copper mine stock that yields an overall internal rate of return of 28% per year?
- 14.14 During periods of hyperinflation, prices increase rapidly over short periods of time. In 1993, the Consumer Price Index (CPI) in Brazil was 113.6 billion. In 1994, the CPI was 2472.4 billion.
(a) What was the inflation rate per year between 1993 and 1994?
(b) Assuming the inflation rate calculated in part (a) occurred uniformly throughout the year and represented a nominal rate, what were the monthly and daily inflation rates over that time period?
- 14.15 A trust was set up by your grandfather that states you are to receive \$250,000 exactly 5 years from today. Determine the buying power of the \$250,000 in terms of today's dollars if the market interest rate is 10% per year and the inflation rate is 4% per year.
- 14.16 Assume the inflation rate is 4% per year and the market interest rate is 5% above the inflation rate. Determine (a) the number of constant-value dollars 5 years in the future that is equivalent to \$30,000 now and (b) the number of future dollars that will be equivalent to \$30,000 now.
- 14.17 The Pell Grant program of the federal government provides financial aid to needy college students. If the average grant is slated to increase from \$4050 to \$5400 over the next 5 years "to keep up with inflation," what is the average inflation rate per year expected to be?

- 14.18 Ford Motor Company announced that the price of its F-150 pickup trucks is going to increase by only the inflation rate for the next 3 years. If the current price of a well-equipped truck is \$28,000 and the inflation rate averages 2.1% per year, what is the expected price of a comparably equipped truck next year? 3 years from now?
- 14.19 In an effort to reduce pipe breakage, water hammer, and product agitation, a chemical company plans to install several chemically resistant pulsation dampeners. The cost of the dampeners today is \$120,000, but the company has to wait until a permit is approved for its bidirectional port-to-plant product pipeline. The permit approval process will take at least 2 years because of the time required for preparation of an environmental impact statement. Because of intense foreign competition, the manufacturer plans to increase the price only by the inflation rate each year. If the inflation rate is 2.8% per year and the company's MARR is 20% per year, estimate the cost of the dampeners in 2 years in terms of (a) today's dollars and (b) future dollars.
- 14.20 A machine currently under consideration by Holzmann Industries has a cost of \$45,000. When the purchasing manager complained that a similar machine the company purchased 5 years ago was much cheaper, the salesman responded that the cost of the machine has increased solely in accordance with the inflation rate, which has averaged 3% per year. When the purchasing manager checked the invoice for the machine he purchased 5 years ago, he saw that the price was \$29,000. Was the salesman telling the truth about the increase in the cost of the machine? If not, what should the machine cost now, provided the price increased by only the inflation rate?
- 14.21 A report by the National Center for Public Policy and Higher Education stated that tuition and fees (T&F) at public colleges and universities increased by 439% over the last 25 years. During this same time period, the median family income (MFI) rose 147%. When the report was written, T&F at a 4-year public university constituted 28% of the MFI of \$52,000 (tuition and fees at a private university constituted 76% of MFI).
- What was the tuition and fee cost per year when the report was written?
 - What was the T&F cost 25 years ago?
 - What percentage was T&F of the MFI 25 years ago?
- 14.22 The headline on a *Chronicle of Higher Education* article reads "College Costs Rise Faster than Inflation." The article states that tuition at public colleges and universities increased by 58% over the past 5 years. (a) What was the average annual percentage increase over that period of time? (b) If the real increase in tuition (i.e., without inflation) was 5% per year, what was the inflation rate per year?
- 14.23 Stadium Capital Financing Group helps cash-strapped sports programs through the marketing of its "sports mortgage." At the University of Kansas, Jayhawks fans can sign up to pay \$105,000 over 10 years for the right to buy top seats for football during the next 30 years. In return, the season tickets will stay locked in at current-year prices. Season tickets in tier 1 are currently selling for \$350. A fan plans to purchase the sports mortgage along with season tickets now and each year for the next 30 years (31 seasons). What is the dollar amount of the savings on the tickets (with no interest considered), if ticket prices rise at a rate of 3% per year for the next 30 years?

Present Worth Calculations with Inflation

- 14.24 There are two ways to account for inflation in present worth calculations. What are they?
- 14.25 An environmental testing company needs to purchase equipment 2 years from now and expects to pay \$50,000 *at that time*. At a real interest rate of 10% per year and inflation rate of 4% per year, what is the present worth of the cost of the equipment?
- 14.26 Carlsbad Gas and Electric is planning to purchase a degassing tower for removing CO₂ from acidified saltwater. The supplier quoted a price of \$125,000 if the unit is purchased within the next 3 years. Your supervisor has asked you to calculate the present worth of the tower when considering inflation. Assuming the tower will not be purchased for 3 years, calculate the present worth at an interest rate of 10% per year and an inflation rate of 4% per year.
- 14.27 How much can the manufacturer of superconducting magnetic energy storage systems afford to spend now on new equipment in lieu of spending \$75,000 four years from now? The company's real MARR is 12% per year, and the inflation rate is 3% per year.
- 14.28 Find the present worth of the cash flows shown. Some are expressed as constant-value (CV) dollars, and others are inflated dollars. Assume a real interest rate of 8% per year and an inflation rate of 6% per year.

Year	1	2	3	4	5
Cash Flow, \$	3000	6000	8000	4000	5000
Stated as	CV	Inflated	Inflated	CV	CV

14.29 A doctor is on contract to a medium-sized oil company to provide medical services at remotely located, widely separated refineries. The doctor is considering the purchase of a private plane to reduce the total travel time between refineries. The doctor can buy a used Lear jet now or wait for a new very light jet (VLJ) that will be available 3 years from now. The cost of the VLJ will be \$1.9 million, payable when the plane is delivered in 3 years. The doctor has asked you, his friend, to determine the present worth of the VLJ so that he can decide whether to buy the used Lear now or wait for the VLJ. If the MARR is 15% per year and the inflation rate is projected to be 3% per year, what is the present worth of the VLJ with inflation considered?

14.30 A regional infrastructure building and maintenance contractor is trying to decide whether to buy a new compact horizontal directional drilling (HDD) machine now or wait to buy it 2 years from now (when a large pipeline contract will require the new equipment). The HDD machine will include an innovative pipe loader design and maneuverable undercarriage system. The cost of the system is \$68,000 if purchased now or \$81,000 if purchased 2 years from now. At a real MARR of 10% per year and an inflation rate of 5% per year, determine if the company should buy now or later (a) without any adjustment for inflation and (b) with inflation considered.

14.31 An engineer must recommend one of two rapid-prototyping machines for integration into an upgraded manufacturing line. She obtained estimates from salespeople from two companies. Salesman A gave her the estimates in constant-value (today's) dollars, while saleswoman B provided the estimates in future (then-current) dollars. The company's MARR is equal to the real rate of return of 20% per year, and inflation is estimated at 4% per year. Use PW analysis to determine which machine the engineer should recommend.

	A (in CV Dollars)	B (in Future Dollars)
First cost, \$	−140,000	−155,000
AOC, \$ per year	−25,000	−40,000
Life, years	10	10

14.32 A salesman from Industrial Water Services (IWS), who is trying to get his foot in the door for a large account in Fremont, offered water chlorination equipment for \$2.1 million. This is \$400,000 more than the price offered by a competing saleswoman from AG Enterprises. However, IWS said the company would not have to pay for the equipment until the warranty runs out. If the equipment has

a 2-year warranty, determine which offer is better. The company's real MARR is 12% per year, and the inflation rate is 4% per year.

14.33 A chemical engineer is considering two sizes of pipes, small (S) and large (L), for moving distillate from a refinery to the tank farm. A small pipeline will cost less to purchase (including valves and other appurtenances) but will have a high head loss and, therefore, a higher pumping cost. In writing the report, the engineer compared the alternatives based on future worth values, but the company president wants the costs expressed as present dollars. Determine present worth values if future worth values are $FW_S = \$2.3$ million and $FW_L = \$2.5$ million. The company uses a real interest rate of 1% per month and an inflation rate of 0.4% per month. Assume the future worth values were for a 10-year project period.

14.34 As an innovative way to pay for various software packages, a new high-tech service company has offered to pay your company, Custom Computer Services (CCS), in one of three ways: (1) pay \$480,000 now, (2) pay \$1.1 million 5 years from now, or (3) pay an amount of money 5 years from now that will have the same purchasing power as \$850,000 now. If you, as president of CCS, want to earn a real interest rate of 10% per year when the inflation rate is 6% per year, which offer should you accept?

Future Worth and Other Calculations with Inflation

14.35 If the inflation rate is 6% per year and a person wants to earn a true (real) interest rate of 10% per year, determine the number of future dollars she has to receive 10 years from now if the present investment is \$10,000.

14.36 How many future dollars would you need 5 years from now just to have the same buying power as \$50,000 now, if the *deflation* rate is 3% per year?

14.37 If a company deposits \$100,000 into an account that earns a market interest rate of 10% per year at a time when the *deflation* rate is 1% per year, what will be the purchasing power of the accumulated amount (with respect to today's dollars) at the end of 15 years?

14.38 Harmony Corporation plans to set aside \$60,000 per year beginning 1 year from now for replacing equipment 5 years from now. What will be the purchasing power (in terms of current-value dollars) of the amount accumulated, if the investment grows by 10% per year, but inflation averages 4% per year?

14.39 The strategic plan of a solar energy company that manufactures high-efficiency solar cells includes an expansion of its physical plant in 4 years. The engineer in charge of planning estimates the expenditure required now to be \$8 million, but in 4 years, the cost will be higher by an amount equal to the inflation rate. If the company sets aside \$7,000,000 now into an account that earns interest at 7% per year, what will the inflation rate have to be in order for the company to have exactly the right amount of money for the expansion?

14.40 Five years ago, an industrial engineer deposited \$10,000 into an account and left it undisturbed until now. The account is now worth \$25,000.

- (a) What was the overall ROR during the 5 years?
- (b) If the inflation over that period was 4% per year, what was the real ROR?
- (c) What is the buying power of the \$25,000 now compared to the buying power 5 years ago?

14.41 A Toyota Tundra can be purchased today for \$32,350. A civil engineering firm is going to need three more trucks in 2 years because of a land development contract it just won. If the price of the truck increases exactly in accordance with an estimated inflation rate of 3.5% per year, determine how much the three trucks will cost in 2 years.

14.42 The Nobel Prize is administered by The Nobel Foundation, a private institution that was founded in 1900 based on the will of Alfred Nobel, the inventor of dynamite. In part, his will stated: "The capital shall be invested by my executors in safe securities and shall constitute a fund, the interest on which shall be annually distributed in the form of prizes to those who, during the preceding year, shall have conferred the greatest benefit on mankind." The will further stated that the prizes were to be awarded in physics, chemistry, peace, physiology or medicine, and literature. In addition to a gold medal and a diploma, each recipient receives a substantial sum of money that depends on the Foundation's income that year. The first Nobel Prize was awarded in 1901 in the amount of \$150,000. In 1996, the award was \$653,000; it was \$1.4 million in 2009.

- (a) If the increase between 1996 and 2009 was strictly due to inflation, what was the average inflation rate per year during that 13-year period?
- (b) If the Foundation expects to invest money with a return of 5% above the inflation rate, how much will a laureate receive in 2020, provided the inflation rate averages 3% per year between 2009 and 2020?

14.43 Timken Roller Bearing is a manufacturer of seamless tubes for drill bit collars. The company is planning to add larger-capacity robotic arms to one of its assembly lines 3 years from now. If it is done now, the cost of the equipment with installation is \$2.4 million. If the company's real MARR is 15% per year, determine the equivalent amount the company can spend 3 years from now in then-current dollars if the inflation rate is 2.8% per year.

14.44 The data below show two patterns of inflation that are exactly the opposite of each other over a 20-year time period.

- (a) If each machine costs \$10,000 in year 0 and they both increase in cost exactly in accordance with the inflation rate, how much will each machine cost at the end of year 20?
- (b) What is the average inflation rate over the time period for machine A (that is, what single inflation rate would result in the same final cost for machine A)?
- (c) In which years will machine A cost more than machine B?

Year	Machine A, %	Machine B, %
1	10	2
2	10	2
3	2	10
4	2	10
5	10	2
6	10	2
7	2	10
8	2	10
:	:	:
19	2	10
20	2	10

14.45 Factors that increase costs and prices—especially for materials and manufacturing costs sensitive to market, technology, and labor availability—can be considered separately using the real interest rate i , the inflation rate f , and additional increases that grow at a geometric rate g . The future amount is calculated based on a current estimate by using the relation

$$F = P(1 + i)^n(1 + f)^n(1 + g)^n \\ = P[(1 + i)(1 + f)(1 + g)]^n$$

The product of the first two terms enclosed in parentheses results in the inflated interest rate i_f . The geometric rate is the same one used in the geometric series (Chapter 2). It commonly applies to maintenance and repair cost increases as machinery ages. This is over and above the inflation rate. If the current cost to manufacture an electronic subcomponent is \$300,000 per year, what is the

equivalent cost in 3 years, provided the average annual rates for i , f , and g are 10%, 3%, and 2%, respectively?

Capital Recovery with Inflation

- 14.46** An electric utility is considering two alternatives for satisfying state regulations regarding pollution control for one of its generating stations. This particular station is located at the outskirts of a major U.S. city and a short distance from a large city in a neighboring country. The station is currently producing excess VOCs and oxides of nitrogen. Two plans have been proposed for satisfying the regulators. Plan A involves replacing the burners and switching from fuel oil to natural gas. The cost of the option will be \$300,000 initially and an extra \$900,000 per year in fuel costs. Plan B involves going to the foreign city and running gas lines to many of the "backyard" brick-making sites that now use wood, tires, and other combustible waste materials for firing the bricks. The idea behind plan B is that by reducing the particulate pollution responsible for smog in the neighboring city, there would be greater benefit to U.S. citizens than would be achieved through plan A. The initial cost of plan B will be \$1.2 million for installation of the lines. Additionally, the electric company would subsidize the cost of gas for the brick makers to the extent of \$200,000 per year. Extra air monitoring associated with this plan will cost an additional \$150,000 per year. For a 10-year project period and no salvage value for either plan, which one should be selected on the basis of an annual worth analysis at a real interest rate of 7% per year and an inflation rate of 4% per year?
- 14.47** A chemical engineer who is smitten with wanderlust wants to build a reserve fund that will be sufficient to permit her to take time off from work to travel around the world. Her goal is to save enough money over the next 3 years so that when she begins her trip, the amount she has accumulated will have the same buying power as \$72,000 today. If she expects to earn a market rate of 12% per year on her investments and inflation averages 5% per year over the 3 years, how much must she save each year to reach her goal?
- 14.48** An entrepreneur engaged in wildcat oil well drilling is seeking investors who will put up \$500,000 for an opportunity to reap high returns if the venture is successful. The prospectus states that a return of at least 22% per year for 5 years is likely. How much will the investors have to receive each year to recover their money if an

inflation rate of 5% per year is to be included in the calculation?

- 14.49** Veri-Trol, Inc. manufactures in situ calibration verification systems that confirm flow measurement accuracies without removing the meters. The company has decided to modify the main assembly line with one of the enhancement processes shown below. If the company's real MARR is 15% per year, which process has the lower annual cost when inflation of 5% per year is considered?

	Process X	Process Y
First cost, \$	−65,000	−90,000
Operating cost, \$ per year	−40,000	−34,000
Salvage value, \$	0	10,000
Life, years	5	5

- 14.50** Johnson Thermal Products used austenitic nickel-chromium alloys to manufacture resistance heating wire. The company is considering a new annealing-drawing process to reduce costs. If the new process will cost \$3.7 million now, how much must be saved each year to recover the investment in 5 years if the company's MARR is a real 12% per year and the inflation rate is 3% per year?

- 14.51** The costs associated with a small X-ray inspection system are \$40,000 now and \$24,000 per year, with a \$6000 salvage value after 3 years. Determine the equivalent cost of the system if the real interest rate is 10% per year and the inflation rate is 4% per year.

- 14.52** Maintenance costs for pollution control equipment on a pulverized coal cyclone furnace are expected to be \$180,000 now and another \$70,000 three years from now. The CFO of Monongahela Power wants to know the equivalent annual cost of the equipment in years 1 through 5. If the company uses a real interest rate of 9% per year and the inflation rate averages 3% per year, what is the equivalent annual cost of the equipment?

- 14.53** MetroKlean LLC, a hazardous waste soil cleaning company, borrowed \$2.5 million for 5 years to finance start-up costs for a new project involving site reclamation. The company expects to earn a real rate of return of 20% per year. The average inflation rate is 5% per year.

- (a) Determine the capital recovery required each year with inflation considered.
- (b) Determine the capital recovery if the company is satisfied with accumulating \$2.5 million at the end of the 5 years with inflation considered.
- (c) Determine the capital recovery in part (b) without considering inflation.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

14.54 Inflation occurs when:

- (a) Productivity increases
- (b) The value of the currency decreases
- (c) The value of the currency increases
- (d) The price of gold decreases

14.55 "Then-current" (future) dollars can be converted into constant-value dollars by:

- (a) Multiplying them by $(1 + f)^n$
- (b) Multiplying them by $(1 + f)^n / (1 + i)^n$
- (c) Dividing them by $(1 + f)^n$
- (d) Dividing them by $(1 + i_f)^n$

14.56 To calculate how much something will cost if you expect its cost to increase by exactly the inflation rate, you should:

- (a) Multiply by $(1 + f)^n$
- (b) Multiply by $[(1 + f)^n / (1 + i)^n]$
- (c) Divide by $(1 + f)^n$
- (d) Multiply by $(1 + i_f)^n$

14.57 If the market interest rate is 16% per year when the inflation rate is 9% per year, the real interest rate is closest to:

- (a) 6.4%
- (b) 7%
- (c) 9%
- (d) 15.6%

14.58 The market interest rate i_f is 12% per year, compounded semiannually. For an inflation rate of 2% per 6 months, the effective semiannual interest rate is closest to:

- (a) 2%
- (b) 3%
- (c) 4%
- (d) 6%

14.59 Construction equipment has a cost today of \$40,000. If its cost has increased only by the inflation rate of 6% per year, when the market interest rate was 10% per year, its cost 10 years ago was closest to:

- (a) \$15,420
- (b) \$22,335
- (c) \$27,405
- (d) \$71,630

14.60 The amount of money that would be accumulated now from an investment of \$1000 25 years ago at

a market rate of 5% per year and an inflation rate averaging 2% per year over that time period is closest to:

- (a) \$1640
- (b) \$3385
- (c) \$5430
- (d) Over \$5500

14.61 If \$1000 is invested now, the number of future dollars required 10 years from now to earn a real interest rate of 6% per year, when the inflation rate is 4% per year, is closest to:

- (a) \$1480
- (b) \$1790
- (c) \$2650
- (d) Over \$2700

14.62 If you expect to receive a gift of \$50,000 six years from now, the present worth of the gift at a real interest rate of 4% per year and an inflation rate of 3% per year is closest to:

- (a) \$27,600
- (b) \$29,800
- (c) \$33,100
- (d) \$37,200

14.63 New state-mandated emission testing equipment for annual inspection of automobiles at Charlie's Garage has a first cost of \$30,000, an annual operating cost of \$7000, and a \$5000 salvage value after its 20-year life. For a real interest rate of 5% per year and an inflation rate of 4% per year, the annual capital recovery requirement for the equipment (in future dollars) is determined by:

- (a) $AW = -30,000(A/P,4\%,20) - 7000 + 5000(A/F,4\%,20)$
- (b) $AW = -30,000(A/P,5\%,20) - 7000 + 5000(A/F,5\%,20)$
- (c) $AW = -30,000(A/P,9\%,20) - 7000 + 5000(A/F,9\%,20)$
- (d) $AW = -30,000(A/P,9.2\%,20) - 7000 + 5000(A/F,9.2\%,20)$

14.64 Temporary price deflation may occur in specific sectors of the economy for all of the following reasons except:

- (a) Improved technology
- (b) Excessive demand
- (c) Dumping
- (d) Increased productivity

CASE STUDY

INFLATION VERSUS STOCK AND BOND INVESTMENTS

Background

The savings and investments that an individual maintains should have some balance between equity (corporate stocks that rely on market growth and dividend income) and fixed-income investments (bonds that pay dividends to the purchaser and a guaranteed amount upon maturity). When inflation is moderately high, bonds offer a low return relative to stocks, because the potential for market growth is not present with bonds. Additionally, the forces of inflation make the dividends worth less in future years, because there is no inflation adjustment made in the amount the dividend pays as time passes. However, bonds do offer a steady income that may be important to an individual, and they serve to preserve the principal invested in the bond, because the face value is returned at maturity.

Information

Earl is an engineer who wants a predictable flow of money for travel and vacations. He has a collection of stocks in his retirement portfolio, but no bonds. He has accumulated a total of \$50,000 of his own funds in low-yielding savings accounts and wants to improve his long-term return from this nonretirement program “nest egg.” He can choose additional stocks or bonds, but has decided to not split the \$50,000 between the two forms of investments. There are two choices he has outlined, with the best estimates he can make at this time. He assumes the effects of federal and state income taxes will be the same for both forms of investment.

Stock purchase: Stocks purchased through a mutual fund would pay an estimated 2% per year dividend and appreciate in value at 5% per year.

Bond purchase: If he purchased a bond, he would have a predictable income of 5% per year and the \$50,000 face value after the 12-year maturity period.

Case Study Questions

The analysis that Earl has laid out has the following questions. Can you answer them for him for both choices?

1. What is the overall rate of return after 12 years?
2. If he decided to sell the stock or bond immediately after the fifth annual dividend, what is his minimum selling price to realize a 7% real return? Include an adjustment of 4% per year for inflation.
3. If Earl needed some money in the future, say, immediately after the fifth dividend payment, what would be the minimum selling price in future dollars, if he were only interested in recovering an amount that maintained the purchasing power of the original price?
4. As a follow-on to question 3, what happens to the selling price (in future dollars) 5 years after purchase, if Earl is willing to remove (net out) the future purchasing power of each of the dividends in the computation to determine the required selling price 5 years hence?
5. Earl plans to keep the stocks or bonds for 12 years, that is, until the bond matures. However, he wants to make the 7% per year real return and make up for the expected 4% per year inflation. For what amount must he sell the stocks after 12 years, or buy the bonds now to ensure he realizes this return? Do these amounts seem reasonable to you, given your knowledge of the way that stocks and bonds are bought and sold?

Cost Estimation and Indirect Cost Allocation



LEARNING OUTCOMES

Purpose: Make cost estimates using different methods; demonstrate the allocation of indirect costs using traditional and activity-based costing rates.

SECTION	TOPIC	LEARNING OUTCOME
15.1	Approach	<ul style="list-style-type: none">Explain the bottom-up and design-to-cost (top-down) approaches to cost estimation.
15.2	Unit method	<ul style="list-style-type: none">Use the unit method to make a preliminary cost estimate.
15.3	Cost index	<ul style="list-style-type: none">Use a cost index to estimate a present cost based on historical data.
15.4	Cost capacity	<ul style="list-style-type: none">Use a cost-capacity equation to estimate component, system, or plant costs.
15.5	Factor method	<ul style="list-style-type: none">Estimate total plant cost using the factor method.
15.6	Indirect cost rates	<ul style="list-style-type: none">Allocate indirect costs using traditional indirect cost rates.
15.7	ABC allocation	<ul style="list-style-type: none">Use the Activity-Based Costing (ABC) method to allocate indirect costs.
15.8	Ethics and profit	<ul style="list-style-type: none">Describe how biased estimation can become an ethical dilemma.

U

p to this point, cost and revenue cash flow values have been stated or assumed as known. In reality, they are not; they must be estimated. This chapter explains what cost estimation involves and applies cost estimation techniques. **Cost estimation** is important in all aspects of a project, but especially in the stages of project conception, preliminary design, detailed design, and economic analysis. When a project is developed in the private or the public sector, questions about costs and revenues will be posed by individuals representing many different functions: management, engineering, construction, production, quality, finance, safety, environmental, legal, and marketing, to name some. In engineering practice, the estimation of costs receives much more attention than revenue estimation; costs are the topic of this chapter.

Unlike direct costs for labor and materials, indirect costs are not easily traced to a specific department, machine, or processing line. Therefore, allocation of indirect costs for functions such as utilities, safety, management and administration, purchasing, and quality is made using some rational basis. Both the traditional method of allocation and the Activity-Based Costing (ABC) method are covered in this chapter.

15.1 Understanding How Cost Estimation Is Accomplished ● ● ●



Cost estimation is a major activity performed in the initial stages of virtually every effort in industry, business, and government. In general, most cost estimates are developed for either a **project** or a **system**; however, combinations of these are very common. A **project** usually involves physical items, such as a building, bridge, manufacturing plant, or offshore drilling platform, to name just a few. A **system** is usually an operational design that involves processes, services, software, and other nonphysical items. Examples might be a purchase order system, a software package, an Internet-based remote-control system, or a health care delivery system. Of course, many projects will have major elements that are not physical, so estimates of both types must be developed. For example, consider a computer network system. There would be no operational system if only the costs of computer hardware plus wire and wireless connectors were estimated; it is equally important to estimate the software, personnel, and maintenance costs.

Thus far virtually all cash flow estimates in the examples, problems, progressive examples, and case studies were stated or assumed to be known. In real-world practice, the cash flows for costs and revenues must be estimated prior to the evaluation of a project or comparison of alternatives. We concentrate on cost estimation because costs are the primary values estimated for the economic analysis. Revenue estimates utilized by engineers are usually developed in marketing, sales, and other departments.

Costs are comprised of **direct costs** and **indirect costs**. Normally direct costs are estimated with some detail, then the indirect costs are added using standard rates and factors. However, direct costs in many industries, including manufacturing and assembly settings, have become a small percentage of overall product cost, while indirect costs have become much larger. Accordingly, many industrial settings require some estimating for indirect costs as well. Indirect cost allocation is discussed in detail in later sections of this chapter. Primarily, direct costs are discussed here.

Because cost estimation is a complex activity, the following questions form a structure for the discussion that follows.

- What cost *components* must be estimated?
- What *approach* to cost estimation will be applied?
- How *accurate* should the estimates be?
- What estimation *techniques* will be utilized?



Direct/Indirect costs

Costs to Estimate If a project revolves around a single piece of equipment, for example, an industrial robot, the **cost components** will be significantly simpler and fewer than the components for a complete system such as the manufacturing and testing line for a new product. Therefore, it is important to know up front how much the cost estimation task will involve. Examples of cost components are the first cost P and the annual operating cost (AOC), also called the M&O costs

(maintenance and operating) of equipment. Each component will have several *cost elements*. Listed below are sample elements of the first cost and AOC components.

First cost component P:

- Elements: Equipment cost
- Delivery charges
- Installation cost
- Insurance coverage
- Initial training of personnel for equipment use

Delivered-equipment cost is the sum of the first two elements in the list above; **installed-equipment cost** adds the third element.

AOC component (part of the equivalent annual cost A):

- Elements: Direct labor cost for operating personnel
- Direct materials
- Maintenance costs (daily, periodic, repairs, etc.)
- Rework and rebuild

Some of these elements, such as equipment cost, can be determined with high accuracy; others, such as maintenance costs, are harder to estimate. When costs for an entire system must be estimated, the number of cost components and elements is likely to be in the hundreds. It is then necessary to prioritize the estimation tasks.

For familiar projects (houses, office buildings, highways, and some chemical plants) there are standard cost estimation software packages available. For example, state highway departments utilize software that prompts for the cost components (bridges, pavement, cut-and-fill profiles, etc.) and estimates costs with time-proven, built-in relations. Once these components are estimated, exceptions for the specific project are added. However, there are no “canned” software packages for a large percentage of industrial, business, service and public sector projects.

Cost Estimation Approach Historically in industry, business, and the public sector, a **bottom-up** approach to cost estimation was applied. For a simple rendition of this approach, see Figure 15–1 (left side). The progression is as follows: cost components and their elements are identified, cost elements are estimated, and estimates are summed to obtain total direct cost. The price is then determined by adding indirect costs and the profit margin, which is usually a percentage of the total cost.

The **bottom-up approach** treats the required price as an **output variable** and the cost estimates as input variables. This approach works well when competition is not a dominant factor in pricing the product or service.

Figure 15–1 (right side) shows a simplistic progression for the **design-to-cost**, or top-down, approach. The competitive price establishes the target cost.

The **design-to-cost**, or **top-down**, approach treats the competitive price as an **input variable** and the cost estimates as output variables. This approach is useful in encouraging innovation, new design, manufacturing process improvement, and efficiency. These are some of the essentials of *value engineering* and value-added systems engineering.

This approach places greater emphasis on the accuracy of the price estimation activity. The target cost must be realistic, or else it can be a disincentive to design and engineering staff. The design-to-cost approach is best applied in the early stages of a new or enhanced product design. The detailed design and specific equipment options are not yet known, but the price estimates assist in establishing target costs for different components.

Usually, the resulting approach is some combination of these two philosophies. However, it is helpful to understand up front what approach is to be emphasized. Historically, the bottom-up approach was more predominant in Western engineering cultures, especially in North America. The design-to-cost approach is considered routine in Eastern engineering cultures; however,

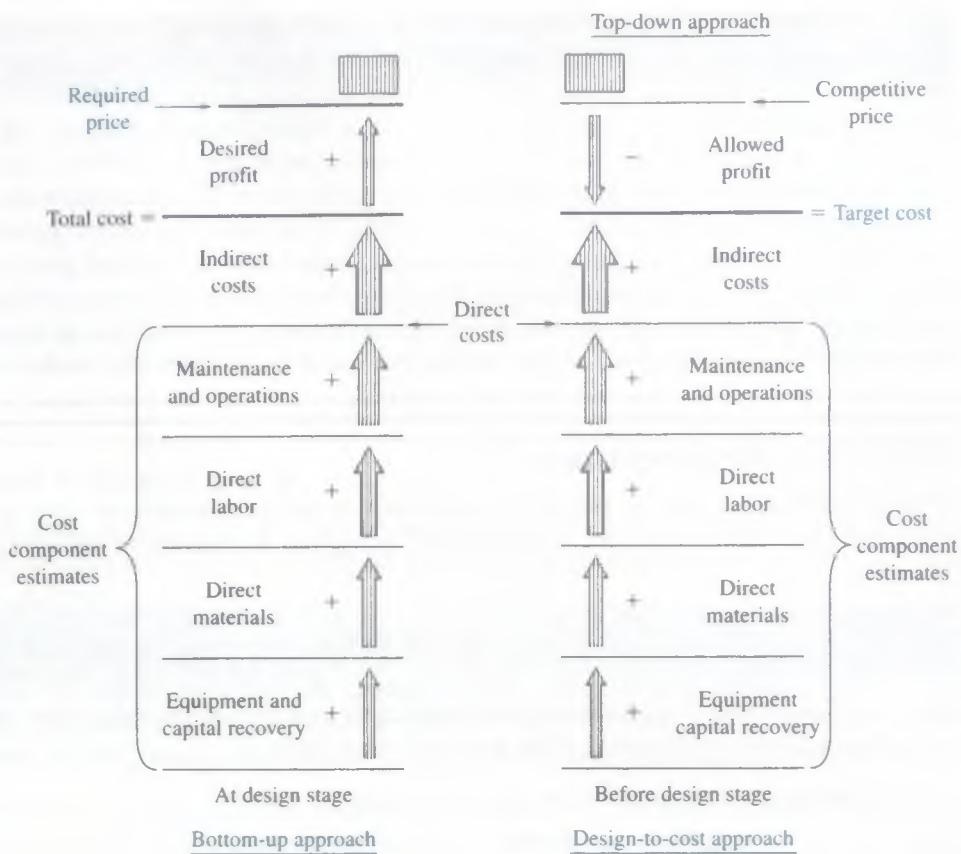


Figure 15-1
Simplified cost estimation processes for bottom-up and top-down approaches.

globalization of engineering design has speeded the adoption of the design-to-cost approach worldwide.

Accuracy of the Estimates No cost estimates are expected to be exact; however, they are expected to be reasonable and accurate enough to support economic scrutiny. The accuracy required increases as the project progresses from preliminary design to detailed design and on to economic evaluation. Cost estimates made before and during the preliminary design stage are expected to be good “first-cut” estimates that serve as input to the project budget.

When utilized at early and conceptual design stages, estimates are referred to as **order-of-magnitude** estimates and generally range within $\pm 20\%$ of actual cost. At the detailed design stage, cost estimates are expected to be accurate enough to support economic evaluation for a go–no go decision. Every project setting has its own characteristics, but a range of $\pm 5\%$ of actual costs is expected at the detailed design stage. Figure 15–2 shows the general range of estimate accuracy for the construction cost of a building versus time spent in preparing the estimate. Obviously, the desire for better accuracy has to be balanced against the cost of obtaining it.

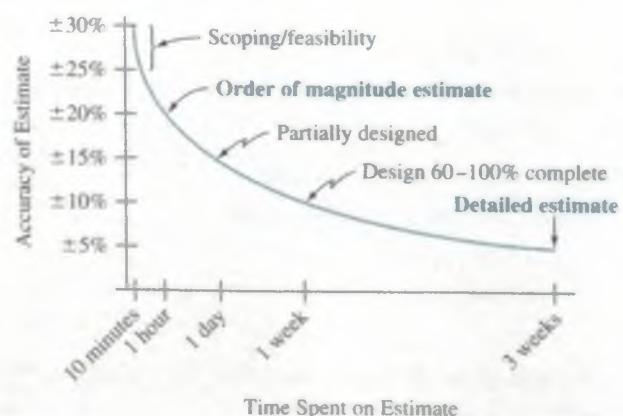


Figure 15-2
Characteristic curve of estimate accuracy versus time spent to estimate construction cost of a building.

Cost Estimation Techniques Methods such as expert opinion and comparison with comparable installations serve as excellent estimators. The use of the *unit method* and *cost indexes* base the present estimate on past cost experiences, with inflation considered. Models such as *cost-capacity equations* and the *factor method* are simple mathematical techniques applied at the preliminary design stage. They are called **cost estimating relationships (CERs)**. There are many additional methods discussed in the handbooks and publications of different industries.

Most cost estimates made in a professional setting are accomplished in part or wholly using software packages linked to updated databases that contain cost indexes and rates for the locations, products, or processes being studied. There are a wide variety of estimators, cost trackers, and cost compliance software systems, most of them developed for specific industries. Corporations usually standardize on one or two packages to ensure consistency over time and projects.



15.2 Unit Method ● ● ●

The **unit method** is a popular preliminary estimation technique applicable to virtually all professions. The total estimated cost C_T is obtained by multiplying the number of units N by a per unit cost factor u .

$$C_T = u \times N$$

[15.1]

Unit cost factors must be updated frequently to remain current with changing costs, areas, and inflation. Some sample unit cost factors (and values) are

- Total average cost of operating an automobile (52¢ per mile)
- Cost to bury fiber cable in a suburban area (\$30,000 per mile)
- Cost to construct a parking space in a parking garage (\$4500 per space)
- Cost of constructing interstate highway (\$6.2 million per mile)
- Cost of house construction per livable area (\$225 per square foot)

Applications of the unit method to estimate costs are easily found. If house construction costs average \$225 per square foot, a preliminary cost estimate for an 1800-square-foot house, using Equation [15.1], is \$405,000. Similarly, a 200-mile trip should cost about \$104 for the car only at 52¢ per mile.

When there are several components to a project or system, the unit cost factors for each component are multiplied by the amount of resources needed, and the results are summed to obtain the total cost C_T . This is illustrated in Example 15.1.

EXAMPLE 15.1

Justin, an ME with Dynamic Castings, has been asked to make a preliminary estimate of the total cost to manufacture 1500 sections of high-pressure gas pipe using an advanced centrifugal casting method. Since a $\pm 20\%$ estimate is acceptable at this preliminary stage, a unit method estimate is sufficient. Use the following resource and unit cost factor estimates to help Justin.

- Materials: 3000 tons at \$45.90 per ton
- Machinery and tooling: 1500 hours at \$120 per hour
- Direct labor in plant:
 - Casting and treating: 3000 hours at \$55 per hour
 - Finishing and shipping: 1200 hours at \$45 per hour
- Indirect labor: 400 hours at \$75 per hour

Solution

Apply Equation [15.1] to each of the five areas and sum the results to obtain the total cost estimate of \$566,700. Table 15–1 provides the details.

TABLE 15-1 Total Cost Estimate Using Unit Cost Factors for Several Resource Areas, Example 15.1

Resource	Amount N	Unit Cost Factor u , \$	Cost Estimate, $u \times N$, \$
Materials	3000 tons	45.90 per ton	137,700
Machinery, tooling	1500 hours	120 per hour	180,000
Labor, casting	3000 hours	55 per hour	165,000
Labor, finishing	1200 hours	45 per hour	54,000
Labor, indirect	400 hours	75 per hour	30,000
Total cost estimate			566,700

15.3 Cost Indexes ● ● ●

This section explains indexes and their use in cost estimation. An *index* is a ratio or other number based on observation and used as an indicator or measure. A preliminary cost estimate is often based on a cost index.



A **cost index** is a ratio of the cost of something today to its cost sometime in the past. The index is dimensionless and measures relative cost change over time. Because these indexes are sensitive to technological change, the predefined quantity and quality of elements used to define the index may be hard to retain over time, thus causing “index creep.” Timely updating of the index is very important.

One such index that most people are familiar with is the Consumer Price Index (CPI), which shows the relationship between present and past costs for many of the things that “typical” consumers must buy. This index includes such items as rent, food, transportation, and certain services. Other indexes track the costs of equipment, and goods and services that are more pertinent to the engineering disciplines. Table 15-2 is a listing of some of the more common indexes.

TABLE 15-2 Types and Sources of Various Cost Indexes

Type of Index	Source
Overall prices	
Consumer (CPI)	Bureau of Labor Statistics
Producer (wholesale)	U.S. Department of Labor
Construction	
Chemical plant overall	<i>Chemical Engineering</i>
Equipment, machinery, and supports	
Construction labor	
Buildings	
Engineering and supervision	
<i>Engineering News Record</i> overall	<i>Engineering News Record (ENR)</i>
Construction	
Building	
Common labor	
Skilled labor	
Materials	
EPA treatment plant indexes	Environmental Protection Agency
Large-city advanced treatment (LCAT)	and <i>ENR</i>
Small-city conventional treatment (SCCT)	
Federal highway	
Contractor cost	
Equipment	
Marshall and Swift (M&S) overall	Marshall & Swift
M&S specific industries	
Labor	
Output per worker-hour by industry	U.S. Department of Labor

TABLE 15-3 Values for Selected Indexes

Year	CE Plant Cost Index	ENR Construction Cost Index	M&S Equipment Cost Index
1995	381.1	5471	1027.5
1996	381.8	5620	1039.2
1997	386.5	5826	1056.8
1998	389.5	5920	1061.9
1999	390.6	6059	1068.3
2000	394.1	6221	1089.0
2001	394.3	6343	1093.9
2002	395.6	6538	1104.2
2003	401.7	6694	1123.6
2004	444.2	7115	1178.5
2005	468.2	7446	1244.5
2006	499.6	7751	1302.3
2007	525.4	7967	1373.3
2008	575.4	8310	1449.3
2009	521.9	8570	1468.6
2010 (midyear)	555.3	8837	1461.3

The general equation for updating costs through the use of a cost index over a period from time $t = 0$ (base) to another time t is

$$C_t = C_0 \left(\frac{I_t}{I_0} \right) \quad [15.2]$$

where C_t = estimated cost at present time t

C_0 = cost at previous time t_0

I_t = index value at time t

I_0 = index value at time t_0

Generally, the indexes for equipment and materials are made up of a mix of components that are assigned certain weights, with the components sometimes further subdivided into more basic items. For example, the equipment, machinery, and support component of the chemical plant cost index is subdivided into process machinery, pipes, valves and fittings, pumps and compressors, and so forth. These subcomponents, in turn, are built up from even more basic items such as pressure pipe, black pipe, and galvanized pipe. Table 15-3 shows the *Chemical Engineering* plant cost index (CEPCI), the *Engineering News Record* (ENR) construction cost index, and the Marshall and Swift (M&S) equipment cost index for several years. The base period of 1957 to 1959 is assigned a value of 100 for the CEPCI, 1913 = 100 for the ENR index, and 1926 = 100 for the M&S equipment cost index.

Current and past values of several of the indexes may be obtained from the Internet (usually for a fee). For example, the *CE* plant cost index is available at www.che.com/pci. The *ENR* construction cost index is found at www.construction.com. This latter site offers a comprehensive series of construction-related resources, including several *ENR* cost indexes and cost estimation systems. A website used by many engineering professionals in the form of a “technical chat room” for all types of topics, including estimation, is www.eng-tips.com.

EXAMPLE 15.2

In evaluating the feasibility of a major construction project, an engineer is interested in estimating the cost of skilled labor for the job. The engineer finds that a project of similar complexity and magnitude was completed 5 years ago at a skilled labor cost of \$360,000. The *ENR* skilled labor index was 3496 then and is now 5127. What is the estimated skilled labor cost for the new project?

Solution

The base time t_0 is 5 years ago. Using Equation [15.2], the present cost estimate is

$$\begin{aligned} C_r &= 360,000 \left(\frac{5127}{3496} \right) \\ &= \$527,952 \end{aligned}$$

In the manufacturing and service industries, tabulated cost indexes are not readily available. The cost index will vary, perhaps with the region of the country, the type of product or service, and many other factors. When estimating costs for a manufacturing system, for example, it is often necessary to develop the cost index for high-priority items such as subcontracted components, selected materials, and labor costs. The development of the cost index requires the actual cost at different times for a prescribed quantity and quality of the item. The **base period** is a selected time when the index is defined with a basis value of 100 (or 1). The index each year (period) is determined as the cost divided by the base-year cost and multiplied by 100 (or 1). Future index values may be forecast using simple extrapolation or more refined mathematical techniques, such as time-series analysis.

EXAMPLE 15.3

Sean, owner of Alamo Pictures, makes fact-based documentaries about the Old West and sells them in a variety of outlets, by mail, and online. He has decided to expand into new areas and wants to make cost estimates for three of the more significant labor costs involved in making these types of films. The Director of Finance of Alamo Pictures generated the annual average hourly costs (Table 15-4).

- (a) Make 2008 the base year, and determine the cost indexes using a basis of 1.00. Comment on the trend of each index over the years.
- (b) Sean expects to utilize a lot of graphics services in 2014 for a planned documentary; however, it is the most rapidly increasing cost component. The cost in 2010 was \$78 per hour; assume a worst-case scenario is that the graphics index continues the same arithmetic trend it had from 2010 to 2011. Determine the hourly cost that he should budget for in 2014.

Solution

- (a) For each type of service, calculate I_t/I_0 where $t = 2005, 2006, \dots$ with 2008 as the base year 0. Table 15-5 presents the indexes. Observations about trend are as follows:

Graphics labor cost: Constantly increasing over all years.

Stuntmen labor cost: Rising until 2009, then stable.

Actors labor cost: Higher in 2007 and 2008; comparatively lower and stable in other years.

TABLE 15-4 Average Hourly Costs for Three Services, Example 15.3

Type of Service	Cost of Service, Average \$ per Hour						
	2005	2006	2007	2008	2009	2010	2011
Graphics	50	56	65	67	70	78	90
Stuntmen	50	55	60	70	87	83	85
Actors	80	80	90	90	80	75	85

TABLE 15-5 Index Values with 2008 as Base Year, Example 15.3

Type of Service	Index I_t/I_0						
	2005	2006	2007	2008	2009	2010	2011
Graphics	0.75	0.84	0.97	1.00	1.04	1.16	1.34
Stuntmen	0.71	0.79	0.86	1.00	1.24	1.19	1.21
Actors	0.89	0.89	1.00	1.00	0.89	0.83	0.94

- (b) The index in 2010 is 1.16. The increase to 2011 is 0.18; the index value in 2014 will be $1.34 + 3(0.18) = 1.88$. Equation [15.2] finds the expected, worst-case cost in 2014.

$$\begin{aligned} C_{2014} &= C_{2010}(I_{2014}/I_{2010}) = 78(1.88/1.16) \\ &= 78(1.62) \\ &= \$126 \text{ per hour} \end{aligned}$$



15.4 Cost-Estimating Relationships: Cost-Capacity Equations ● ● ●

Design variables (speed, weight, thrust, physical size, etc.) for plants, equipment, and construction are determined in the early design stages. **Cost-estimating relationships (CERs)** use these design variables to predict costs. Thus, a CER is generically different from the cost index method, because the index is based on the cost history of a defined quantity and quality of a variable.

One of the most widely used CER models is a **cost-capacity equation**. As the name implies, an equation relates the cost of a component, system, or plant to its capacity. This is also known as the *power law and sizing model*. Since many cost-capacity equations plot as a straight line on log-log paper, a common form is

$$C_2 = C_1 \left(\frac{Q_2}{Q_1} \right)^x \quad [15.3]$$

where C_1 = cost at capacity Q_1

C_2 = cost at capacity Q_2

x = correlating exponent

The value of the exponent for various components, systems, or entire plants can be obtained or derived from a number of sources, including *Plant Design and Economics for Chemical Engineers*, *Preliminary Plant Design in Chemical Engineering*, *Chemical Engineers' Handbook*, technical journals (especially *Chemical Engineering*), the U.S. Environmental Protection Agency, professional or trade organizations, consulting firms, handbooks, and equipment companies. Table 15–6 is a partial listing of typical values of the exponent for various units. When an exponent value for a particular unit is not known, it is common practice to use the value of $x = 0.6$. In fact, in the chemical processing industry, Equation [15.3] is referred to as the six-tenths model.

The exponent x in the cost-capacity equation is commonly in the range $0 < x \leq 1$.

If $x < 1$, *economies of scale* provide a cost advantage for larger sizes.

If $x = 1$, a *linear* relationship is present.

If $x > 1$, there are *diseconomies of scale* present in that a larger size is more costly than that of a linear relation.

It is especially powerful to combine the time adjustment of the cost index (I_t/I_0) from Equation [15.2] with a cost-capacity equation to estimate costs that change over time. If the index is embedded into the cost-capacity computation in Equation [15.3], the cost at time t and capacity level 2 may be written as the product of two independent terms.

$$\begin{aligned} C_{2,t} &= (\text{cost at time 0 of level 2}) \times (\text{time adjustment cost index}) \\ &= \left[C_{1,0} \left(\frac{Q_2}{Q_1} \right)^x \right] \left(\frac{I_t}{I_0} \right) \end{aligned}$$

This is commonly expressed without the time subscripts. Thus,

$$C_2 = C_1 \left(\frac{Q_2}{Q_1} \right)^x \left(\frac{I_t}{I_0} \right) \quad [15.4]$$

The following example illustrates the use of this relation.

TABLE 15–6 Sample Exponent Values for Cost-Capacity Equations

Component/System/Plant	Size Range	Exponent
Activated sludge plant	1–100 MGD	0.84
Aerobic digester	0.2–40 MGD	0.14
Blower	1000–7000 ft/min	0.46
Centrifuge	40–60 in	0.71
Chlorine plant	3000–350,000 tons/year	0.44
Clarifier	0.1–100 MGD	0.98
Compressor, reciprocating (air service)	5–300 hp	0.90
Compressor	200–2100 hp	0.32
Cyclone separator	20–8000 ft ³ /min	0.64
Dryer	15–400 ft ²	0.71
Filter, sand	0.5–200 MGD	0.82
Heat exchanger	500–3000 ft ²	0.55
Hydrogen plant	500–20,000 scfd	0.56
Laboratory	0.05–50 MGD	1.02
Lagoon, aerated	0.05–20 MGD	1.13
Pump, centrifugal	10–200 hp	0.69
Reactor	50–4000 gal	0.74
Sludge drying beds	0.04–5 MGD	1.35
Stabilization pond	0.01–0.2 MGD	0.14
Tank, stainless	100–2000 gal	0.67

Note: MGD = million gallons per day; hp = horsepower; scfd = standard cubic feet per day.

EXAMPLE 15.4

The total design and construction cost for a digester to handle a flow rate of 0.5 million gallons per day (MGD) was \$1.7 million in 2010. Estimate the cost today for a flow rate of 2.0 MGD. The exponent from Table 15–6 for the MGD range of 0.2 to 40 is 0.14. The cost index in 2010 of 131 has been updated to 225 for this year.

Solution

Equation [15.3] can estimate the cost of the larger system in 2010, but it must be updated by the cost index to today's dollars. Equation [15.4] performs both operations at once. The estimated cost is

$$C_2 = 1,700,000 \left(\frac{2.0}{0.5} \right)^{0.14} \left(\frac{225}{131} \right)$$

$$= 1,700,000(1.214)(1.718) = \$3.546 \text{ million}$$

15.5 Cost-Estimating Relationships:

Factor Method ● ● ●

Another widely used model for preliminary cost estimates of process plants is called the **factor method**. While the methods discussed above can be used to estimate the costs of major items of equipment, processes, and the total plant costs, the factor method was developed specifically for **total plant costs**. The method is based on the premise that fairly reliable total plant costs can be obtained by multiplying the cost of the major equipment by certain factors. Since major equipment costs are readily available, rapid plant estimates are possible if the appropriate factors are known. These factors are commonly referred to as *Lang factors* after Hans J. Lang, who first proposed the method.

In its simplest form, the factor method is expressed in the same form as the unit method

$$C_T = hC_E \quad [15.5]$$

where C_T = total plant cost

h = overall cost factor or sum of individual cost factors

C_E = total cost of major equipment

The h may be one overall cost factor or, more realistically, the sum of individual cost components such as construction, maintenance, direct labor, materials, and indirect cost elements. This follows the cost estimation approaches presented in Figure 15–1.



In his original work, Lang showed that *direct cost factors* and *indirect cost factors* can be combined into one overall factor for some types of plants as follows: solid process plants, 3.10; solid-fluid process plants, 3.63; and fluid process plants, 4.74. These factors reveal that the total installed-plant cost is many times the first cost of the major equipment.

EXAMPLE 15.5

An engineer with Valero Petroleum has learned that an expansion of the solid-fluid process plant is expected to have a delivered equipment cost of \$2.08 million. If the overall cost factor for this type of plant is 3.63, estimate the plant's total cost.

Solution

The total plant cost is estimated by Equation [15.5].

$$\begin{aligned} C_T &= 3.63(2,080,000) \\ &= \$7,550,400 \end{aligned}$$

Subsequent refinements of the factor method have led to the development of separate factors for direct and indirect cost components. Direct costs as discussed in Section 15.1 are specifically identifiable with a product, function, or process. Indirect costs are not directly attributable to a single function, but are shared by several because they are necessary to perform the overall objective. Examples of indirect costs are general administration, computer services, quality, safety, taxes, security, and a variety of support functions. The factors for both direct and indirect costs are sometimes developed for use with *delivered-equipment costs* and other times for *installed-equipment costs*, as defined in Section 15.1. In this text, we assume that all factors apply to delivered-equipment costs, unless otherwise specified.

For indirect costs, some of the factors apply to equipment costs only, while others apply to the total direct cost. In the former case, the simplest procedure is to add the direct and indirect cost factors before multiplying by the delivered-equipment cost. The overall cost factor h can be written as

$$h = 1 + \sum_{i=1}^n f_i \quad [15.6]$$

where f_i = factor for each cost component

i = 1 to n components, including indirect cost

If the indirect cost factor is applied to the total direct cost, only the direct cost factors are added to obtain h . Therefore, Equation [15.5] is rewritten as

$$C_T = \left[C_E \left(1 + \sum_{i=1}^n f_i \right) \right] (1 + f_l) \quad [15.7]$$

where f_l = indirect cost factor

f_i = factors for direct cost components

Examples 15.6 and 15.7 illustrate these equations.

EXAMPLE 15.6

The delivered-equipment cost for a small chemical process plant is expected to be \$2 million. If the direct cost factor is 1.61 and the indirect cost factor is 0.25, determine the total plant cost.

Solution

Since all factors apply to the delivered-equipment cost, they are added to obtain h , the total cost factor in Equation [15.6].

$$h = 1 + 1.61 + 0.25 = 2.86$$

The total plant cost is

$$C_T = 2.86(2,000,000) = \$5,720,000$$

EXAMPLE 15.7

A new container-handling crane at the Port of Singapore is expected to have a delivered-equipment cost of \$875,000. The cost factor for the installation of tracks, concrete, steel, noise abatement, supports, etc., is 0.49. The construction factor is 0.53, and the indirect cost factor is 0.21. Determine the total cost if (a) all cost factors are applied to the cost of the delivered equipment and (b) the indirect cost factor is applied to the total direct cost.

Solution

(a) Total equipment cost is \$875,000. Since both the direct and indirect cost factors are applied to only the equipment cost, the overall cost factor from Equation [15.6] is

$$h = 1 + 0.49 + 0.53 + 0.21 = 2.23$$

The total cost is

$$C_T = 2.23(875,000) = \$1,951,250$$

(b) Now the total direct cost is calculated first, and Equation [15.7] is used to estimate total cost.

$$h = 1 + \sum_{i=1}^n f_i = 1 + 0.49 + 0.53 = 2.02$$

$$C_T = [875,000(2.02)](1.21) = \$2,138,675$$

Comment

Note the decrease in estimated cost when the indirect cost is applied to the equipment cost only in part (a). This illustrates the importance of determining exactly what the factors apply to before they are used.

15.6 Traditional Indirect Cost Rates and Allocation ● ● ●

Costs incurred in the production of an item or delivery of a service are tracked and assigned by a **cost accounting system**. For the manufacturing environment, it can be stated generally that the *statement of cost of goods sold* (discussed in Appendix B) is one end product of this system. The cost accounting system accumulates material costs, labor costs, and indirect costs (also called overhead costs or factory expenses) by using **cost centers**. All costs incurred in one department or process line are collected under a cost center title, for example, Department 3X. Since direct materials and direct labor are usually directly assignable to a cost center, the system need only identify and track these costs. Of course, this in itself is no easy chore, and the cost of the tracking system may prohibit collection of all direct cost data to the level of detail desired.

One of the primary and more difficult tasks of cost accounting is the allocation of **indirect costs** when it is necessary to allocate them separately to departments, processes, and product lines.

TABLE 15-7 Sample Indirect Cost Allocation Bases

Cost Category	Possible Allocation Basis
Taxes	Space occupied
Heat, light	Space, usage, number of outlets
Power	Space, direct labor hours, horsepower-hours, machine hours
Receiving, purchasing	Cost of materials, number of orders, number of items
Personnel, machine shop	Direct labor hours, direct labor cost
Building maintenance	Space occupied, direct labor cost
Software	Number of accesses
Quality control	Number of inspections



Indirect costs

Indirect costs are costs associated with property taxes, service and maintenance departments, personnel, legal, quality, supervision, purchasing, utilities, software development, etc. They must all be allocated to the using cost center. Detailed collection of these data is cost-prohibitive and often impossible; thus, allocation schemes are utilized to distribute the expenses on a reasonable basis.

A listing of possible allocation bases is included in Table 15-7. Historically, common bases have been direct labor cost, direct labor hours, machine-hours, number of employees, space, and direct materials.

Most allocation is accomplished utilizing a predetermined *indirect cost rate*, computed by using the general relation.

$$\text{Indirect cost rate} = \frac{\text{estimated total indirect costs}}{\text{estimated basis level}} \quad [15.8]$$

The estimated indirect cost is the amount allocated to a cost center. For example, if a division has two producing departments, the total indirect cost allocated to a department is used as the numerator in Equation [15.8] to determine the department rate. Example 15.8 illustrates allocation when the cost center is a machine.

EXAMPLE 15.8

The manager of beauty products at BestWay wants to determine allocation rates for \$150,000 of indirect costs for the three machines used to process beauty lotions. The following information was obtained from last year's budget for the three machines. Determine rates for each machine if the amount is equally distributed.

Cost Source	Allocation Basis	Estimated Activity Level
Machine 1	Direct labor cost	\$100,000
Machine 2	Direct labor hours	2000 hours
Machine 3	Direct material cost	\$250,000

Solution

Applying Equation [15.8] for each machine, the annual rates are

$$\begin{aligned} \text{Machine 1 rate} &= \frac{\text{indirect budget}}{\text{direct labor cost}} = \frac{50,000}{100,000} \\ &= \$0.50 \text{ per direct labor dollar} \end{aligned}$$

$$\begin{aligned} \text{Machine 2 rate} &= \frac{\text{indirect budget}}{\text{direct labor hours}} = \frac{50,000}{2000} \\ &= \$25 \text{ per direct labor hour} \end{aligned}$$

$$\begin{aligned} \text{Machine 3 rate} &= \frac{\text{indirect budget}}{\text{material cost}} = \frac{50,000}{250,000} \\ &= \$0.20 \text{ per direct material dollar} \end{aligned}$$

Now the actual direct labor costs and hours and material costs are determined for this year, and each dollar of direct labor cost spent on machine 1 implies that \$0.50 in indirect cost will be added to the cost of the product. Similarly, indirect costs are added for machines 2 and 3.

When the same allocation basis is used to distribute indirect costs to several cost centers, a **blanket rate** may be determined. For example, if direct materials are the basis for allocation to four separate processing lines, the blanket rate is

$$\text{Indirect cost rate} = \frac{\text{total indirect costs}}{\text{total direct materials cost}}$$

If \$500,000 in indirect costs and \$3 million in materials are estimated for the four lines, the blanket indirect rate is $500,000/3,000,000 = \$0.167$ per materials cost dollar. Blanket rates are easier to calculate and apply, but they do not account for differences in the type of activities accomplished in each cost center.

In most cases, machinery or processes add value to the end product at different rates per unit or hour of use. For example, light machinery may contribute less per hour than heavy, more expensive machinery. This is especially true when advanced technology processing, for example, an automated manufacturing cell, is used along with traditional methods, for example, nonautomated finishing equipment. The use of blanket rates in these cases is not recommended, as the indirect cost will be incorrectly allocated. The lower-value-contribution machinery will accumulate too much of the indirect cost. The approach should be the application of different bases for different machines, activities, etc., as discussed earlier and illustrated in Example 15.8. The use of different, appropriate bases is often called the **productive hour rate method** since the cost rate is determined based on the value added, not a uniform or blanket rate. Realization that more than one basis should be normally used in allocating indirect costs has led to the use of activity-based costing methods, as discussed in the next section.

Once a period of time (month, quarter, or year) has passed, the indirect cost rates are applied to determine the indirect cost *charge*, which is then added to direct costs. This results in the total cost of production, which is called the cost of goods sold, or **factory cost**. These costs are all accumulated by *cost center*.

If the total indirect cost budget is correct, the indirect costs charged to all cost centers for the period of time should equal this budget amount. However, since some error in budgeting always exists, there will be overallocation or underallocation relative to actual charges, which is termed **allocation variance**. Experience in indirect cost estimation assists in reducing the variance at the end of the accounting period.

EXAMPLE 15.9

Since the manager determined indirect cost rates for BestWay (Example 15.8), she can now compute the total factory cost for a month. Perform the computations using the data in Table 15–8. Also calculate the variance for indirect cost allocation for the month.

Solution

Start with the cost of goods sold (factory cost) relation given by Equation [B.1] in Appendix B, which is

$$\text{Cost of goods sold} = \text{direct materials} + \text{direct labor} + \text{indirect costs}$$

TABLE 15–8 Actual Monthly Data Used for Indirect Cost Allocation

Cost Source	Machine Number	Actual Cost, \$	Actual Hours
Material	1	3,800	650
	3	19,550	
Labor	1	2,500	750
	2	3,200	
	3	2,800	720

To determine indirect cost, the rates from Example 15.8 are applied:

$$\text{Machine 1 indirect} = (\text{labor cost})(\text{rate}) = 2500(0.50) = \$1250$$

$$\text{Machine 2 indirect} = (\text{labor hours})(\text{rate}) = 750(25.00) = \$18,750$$

$$\text{Machine 3 indirect} = (\text{material cost})(\text{rate}) = 19,550(0.20) = \$3910$$

$$\text{Total charged indirect cost} = \$23,910$$

Factory cost is the sum of actual material and labor costs from Table 15–8 of \$31,850 and the indirect cost charge for a total of \$55,760.

Based on the annual indirect cost budget of \$150,000, one month represents 1/12 of the total or

$$\begin{aligned}\text{Monthly budget} &= \frac{150,000}{12} \\ &= \$12,500\end{aligned}$$

The allocation variance for total indirect cost is

$$\text{Variance} = 12,500 - 23,910 = \$-11,410$$

This is a large budget underallocation, since much more was actually charged than allocated. The \$12,500 budgeted for the three machines represents a 91.3% underallocation of indirect costs. This analysis for only one month of a year will most likely prompt a rapid review of the rates and the indirect cost budget.

Once estimates of indirect costs are determined, it is possible to perform an economic analysis of the present operation versus a proposed operation. Such a study is described in Example 15.10.

EXAMPLE 15.10

For several years, Cuisinart Corporation has purchased the carafe assembly of its major coffeemaker line at an annual cost of \$2.2 million. The suggestion to make the component in-house has been made. For the three departments involved, the annual indirect cost rates, estimated material, labor, and hours are found in Table 15–9. The allocated hours column is the time necessary to produce the carafes for a year.

Equipment must be purchased with the following estimates: first cost of \$2 million, salvage value of \$50,000, and life of 10 years. Perform an economic analysis for the make alternative, assuming that a market rate of 15% per year is the MARR.

Solution

For making the components in-house, the AOC is comprised of direct labor, direct material, and indirect costs. Use the data of Table 15–9 to calculate the indirect cost allocation.

$$\text{Department A: } 25,000(10) = \$250,000$$

$$\text{Department B: } 25,000(5) = 125,000$$

$$\begin{array}{ll}\text{Department C: } & 10,000(15) = \underline{150,000} \\ & \$525,000\end{array}$$

$$\text{AOC} = 500,000 + 300,000 + 525,000 = \$1,325,000$$

TABLE 15–9 Production Cost Estimates for Example 15.10

Department	Basis, Hours	Indirect Costs			Direct Labor Cost, \$
		Rate per Hour, \$	Allocated Hours	Material Cost, \$	
A	Labor	10	25,000	200,000	200,000
B	Machine	5	25,000	50,000	200,000
C	Labor	15	10,000	50,000	100,000
				300,000	500,000

The make alternative annual worth is

$$\begin{aligned} AW_{\text{make}} &= -P(A/P,i,n) + S(A/F,i,n) - AOC \\ &= -2,000,000(A/P,15\%,10) + 50,000(A/F,15\%,10) - 1,325,000 \\ &= \$-1,721,037 \end{aligned}$$

Currently, the carafes are purchased with an AW of

$$AW_{\text{buy}} = \$-2,200,000$$

It is cheaper to make, because the AW of costs is less.

15.7 Activity-Based Costing (ABC) for Indirect Costs ● ● ●

As automation, software, and manufacturing technologies have advanced, the number of direct labor hours necessary to manufacture a product has decreased substantially. Where once as much as 35% to 45% of the final product cost was represented in labor, now the labor component is commonly 5% to 15% of total manufacturing cost. However, the indirect cost may represent as much as 35% to 45% of the total manufacturing cost. The use of bases, such as direct labor hours, to allocate indirect cost is not accurate enough for automated and technologically advanced environments. This has led to the development of methods that replace or supplement traditional cost allocations that rely upon one form or another of Equation [15.8]. Also, allocation bases different from traditional ones are commonly utilized.

A product that by traditional methods may have contributed a large portion to profit *may actually be a loser* when indirect costs are allocated more correctly. Companies that have a wide variety of products and produce some in small lots may find that traditional allocation methods have a tendency to underallocate the indirect cost to small-lot products. This may indicate that they are profitable, when in actuality they are losing money.

The best allocation method for high-overhead industries is **Activity-Based Costing (ABC)**. It is designed to identify *cost centers*, *activities*, and *cost drivers*. Descriptions of each follow.

Cost centers: The final products or services of the corporation are called cost centers or cost pools. They *receive* the allocated indirect costs.

Activities: These are usually support departments (purchasing, quality, IT, maintenance, engineering, supervision) that *generate* the indirect costs which are then distributed to the cost centers.

Cost drivers: Commonly expressed in volumes, these *drive* the consumption of a shared resource. Examples are the number of purchase orders, cost of engineering change orders, number of machine setups, number of safety violations, and the like.

Implementing ABC involves several steps.

1. Identify each *activity* and its total cost.
2. Identify the *cost drivers* and their usage volumes.
3. Calculate the indirect cost *rate* for each activity.

$$\text{ABC indirect cost rate} = \frac{\text{total cost of activity}}{\text{total volume of cost driver}}$$

[15.9]

4. Use the rate to *allocate* indirect cost to cost centers for each activity.

As an illustration, assume a company that produces two types of industrial lasers (cost centers) has three primary support departments (activities; step 1 in the procedure). The allocation for costs generated by the purchasing department, for example, is based on the number of purchase orders (step 2) to support laser production. The ABC rate (step 3) in dollars per purchase order is used to allocate indirect costs to the two laser products (step 4).

EXAMPLE 15.11

A multinational aerospace firm uses traditional methods to allocate manufacturing and management support costs for its European division. However, accounts such as business travel have historically been allocated on the basis of the number of employees at the plants in France, Italy, Germany, and Spain.

The president recently stated that some product lines are likely generating much more travel than others. The ABC system is chosen to augment the traditional method to more precisely allocate travel costs to major product lines at each plant.

- (a) First, assume that allocation of total observed travel expenses of \$500,000 to the plants using a traditional basis of workforce size is sufficient. If total employment of 29,100 is distributed as follows, allocate the \$500,000.

Paris, France plant	12,500 employees
Florence, Italy plant	8,600 employees
Hamburg, Germany plant	4,200 employees
Barcelona, Spain plant	3,800 employees

- (b) Now, assume that corporate management wants to know more about travel expenses based on product line, not merely plant location and workforce size. The ABC method will be applied to allocate travel costs to major product lines. Annual plant support budgets indicate that the following percentages are expended for travel:

Paris	5% of \$2 million
Florence	15% of \$500,000
Hamburg	17.5% of \$1 million
Barcelona	30% of \$500,000

Further, the study indicates that in 1 year a total of 500 travel vouchers were processed by the management of the major five product lines produced at the four plants. The distribution is as follows:

Paris	Product lines—1 and 2; number of vouchers—50 for line 1, 25 for 2.
Florence	Product lines—1, 3, and 5; vouchers—80 for line 1, 30 for 3, 30 for 5.
Hamburg	Product lines—1, 2 and 4; vouchers—100 for line 1, 25 for 2, 20 for 4.
Barcelona	Product line—5; vouchers—140 for line 5.

Use the ABC method to determine how the product lines drive travel costs at the plants.

Solution

- (a) In this case, Equation [15.8] takes the form of a blanket rate per employee.

$$\begin{aligned}\text{Indirect cost rate} &= \frac{\text{travel budget}}{\text{total workforce}} \\ &= \frac{\$500,000}{29,100} = \$17.1821 \text{ per employee}\end{aligned}$$

Using this traditional basis of rate times workforce size results in a plant-by-plant allocation.

Paris	\$17.1821(12,500) = \$214,777
Florence	\$147,766
Hamburg	\$72,165
Barcelona	\$65,292

- (b) The ABC method is more involved to apply, and the by-plant amounts will be different from those in part (a) since completely different bases are applied. Use the 4-step procedure to allocate travel costs to the five products.

Step 1. The total amount to be allocated is determined from the percentages of each plant's support budget devoted to travel. The number is determined from the percent-of-budget data as follows:

$$0.05(2,000,000) + \dots + 0.30(500,000) = \$500,000$$

TABLE 15-10 ABC Allocation of Travel Cost (\$ in Thousands), Example 15.11

	Product Line					Total
	1	2	3	4	5	
Paris	50	25				75
Florence	80		30		30	140
Hamburg	100	25		20		145
Barcelona					140	140
Total	\$230	\$50	\$30	\$20	\$170	\$500

Step 2. The cost driver is the number of travel vouchers submitted by the management unit responsible for each product line at each plant. The allocation will be to the products directly, not to the plants. However, the travel allocation to the plants can be determined afterward since we know what product lines are produced at each plant.

Step 3. Equation [15.9] determines an ABC allocation rate.

$$\begin{aligned} \text{ABC allocation rate} &= \frac{\text{total cost of travel}}{\text{total number of vouchers}} \\ &= \frac{\$500,000}{500} \\ &= \$1000 \text{ per voucher} \end{aligned}$$

Step 4. Table 15–10 summarizes the vouchers and allocation by product line and by city. Product 1 (\$230,000) and product 5 (\$170,000) drive the travel costs based on the ABC analysis. Comparison of the by-plant totals in Table 15–10 (far right column) with the respective totals in part (a) indicates a substantial difference in the amounts allocated, especially to Paris, Hamburg, and Barcelona. This comparison verifies the president's statement that product lines, not plants, drive travel requirements.

Comment

Let's assume that product 1 has been produced in small lots at the Hamburg plant for a number of years. This analysis, when compared to the traditional cost allocation method in part (a), reveals a very interesting fact. In the ABC analysis, Hamburg has a total of \$145,000 travel dollars allocated, \$100,000 from product 1. In the traditional analysis based on workforce size, Hamburg was allocated only \$72,165—about 50% of the more precise ABC analysis amount. This indicates to management the need to examine the manufacturing lot size practices at Hamburg and possibly other plants, especially when a product is currently manufactured at more than one plant.

Some proponents of the ABC method recommend discarding the traditional cost accounting methods of a company and utilizing ABC exclusively. This is not a good approach, since ABC is not a complete cost system. The ABC method provides information that assists in *cost control*, while the traditional method emphasizes cost allocation and cost estimation. The two systems work well together with the traditional methods allocating costs that have identifiable direct bases, for example, direct labor. ABC analysis is usually more expensive and time-consuming than a traditional cost allocation system, but in many cases it can assist in understanding the economic impact of management decisions and in controlling certain types of indirect costs.

15.8 Making Estimates and Maintaining Ethical Practices ● ● ●

Making estimates about the future of costs, revenues, cash flows, rates of return, and many other parameters is routine when one is engaged in any type of economic analysis. Public agencies, private corporations, and not-for-profit businesses all make economic decisions based on these

estimates, most of which are made by employees of the organizations or by outside consultants hired to perform specific activities under contract. The opportunities for bias, poor accuracy, deception, and profit-driven or other motives are always present. The personal morals and adherence to codes of professional ethics discussed in Section 1.3 guide individuals in their work to make fair and believable estimates for the analyses and decisions that follow.

The NSPE Code of Ethics for Engineers (Appendix C) referenced previously starts with a list of six Fundamental Canons. One very relevant to estimation integrity is “Avoid deceptive acts.” Acts that bias the results from experimental samples, previous cost data, or survey results for the purposes of personal gain, increased profits, or favoritism are examples of unethical behavior. Estimates of all types should be founded on practices such as the following:

Base estimates on sound information gathered over a range of situations representative of the current one.

Use accepted theory and techniques in taking statistical samples, building budget elements, and drawing conclusions that are included in proposals, applications, and recommendations.

As a consultant or contractor, keep personal and working relationships separate when making estimates and delivering the final documentation to a client or sponsor.

The second case study at the end of this chapter presents an example of some ethical challenges present when preparing estimates and proposals for contract work.

CHAPTER SUMMARY

Cost estimates are not expected to be exact, but they should be accurate enough to support a thorough economic analysis using an engineering economy approach. There are bottom-up and top-down approaches; each treats price and cost estimates differently.

Costs can be updated via a cost index, which is a ratio of costs for the same item at two separate times. The Consumer Price Index (CPI) is an often-quoted example of cost indexing. Cost estimating may also be accomplished with a variety of models called cost-estimating relationships. Two of them are

Cost-capacity equation—good for estimating costs from design variables for equipment, materials, and construction

Factor method—good for estimating total plant cost

Traditional cost allocation uses an indirect cost rate determined for a machine, department, product line, etc. Bases such as direct labor cost, direct material cost, and direct labor hours are used. With increased automation and information technology, different techniques of indirect cost allocation have been developed. The Activity-Based Costing method is an excellent technique to augment the traditional allocation method.

The ABC method allocates indirect costs on the rationale that purchase orders, inspections, machine setups, reworks, etc. *drive* the costs accumulated in departments or functions, such as quality, purchasing, accounting, and maintenance. Improved understanding of how the company or plant accumulates indirect costs is a major by-product of implementing the ABC method.

PROBLEMS

Understanding Cost Estimation

- 15.1 Rank the following estimate types in terms of time spent to carry out the estimate (most time to least time): partially designed, design 60% to 100% complete, order of magnitude, scoping/feasibility, detailed estimate.
- 15.2 Classify the following cost elements as first cost (FC) components or annual operating cost (AOC)

components for a piece of equipment on the shop floor: supplies, insurance, equipment cost, utility cost, installation, delivery charges, labor cost.

- 15.3 State whether actual (A) or estimated (E) costs are more likely to be used to carry out the following activities: calculate taxes, make bids, pay bonuses, determine profit or loss, predict sales, set prices,

evaluate proposals, distribute resources, plan production, and set goals.

- 15.4 Identify the output and input variables in both the bottom-up and top-down approaches to cost estimating.

- 15.5 Classify the following costs as typically direct (D) or indirect (I):

Project staff	Audit and legal
Utilities	Rent
Raw materials	Equipment training
Project supplies	Labor
Administrative staff	Miscellaneous office supplies

- 15.6 Identify each of the following costs associated with owning an automobile as direct or indirect (assume a direct cost is one that is directly attributable to the number of miles driven): license plate, driver's license, gasoline, highway toll fee, oil change, repairs after collision, gasoline tax, monthly loan payment, annual inspection fee, and garage rental.

- 15.7 In the early and conceptual design stages of a project, what are the cost estimates called? Approximately how close should they be to the actual cost?

Unit Costs

- 15.8 Use the unit cost method to determine the preliminary cost of a guardrail for a small bridge if a total of 120 linear feet will be required at a cost of \$58.19 per linear foot for material, equipment, and labor.

- 15.9 Sun-Metro Bus is planning to construct a 600-car parking garage on the outskirts of the city to encourage people to use the "Park 'n Ride" to work or for shopping downtown. Prepare a preliminary cost estimate for the garage if the cost per parking space is \$4700.

- 15.10 The Office of the Undersecretary of Defense periodically releases unit cost data for use in military construction programs. If the unit cost for a satellite communications center (with shielding and generator) is \$496 per square foot, what would be the estimated cost of a 6000-square-foot building?

- 15.11 The Department of Defense uses area cost factors (ACFs) to adjust for differences in construction costs in different parts of the country (and world). The area cost factor for Andros Island in the Bahamas is 1.70 while the ACF for Rapid City, South Dakota, is 0.93. If a cold storage processing warehouse costs \$1,350,000 in Rapid City, estimate the cost for Andros Island.

- 15.12 Preliminary cost estimates for jails can be made using costs based on either unit area (square feet) or unit volume (cubic feet). If the unit area cost is \$185 per square foot and the average height of the ceilings is 10 feet, what is the unit volume cost?

- 15.13 The unit area and unit volume total project costs for a library are \$114 per square foot and \$7.55 per cubic foot, respectively. Based on these numbers, what is the average height of library rooms?

- 15.14 The equipment required to place 160 cubic yards of concrete by experienced workers is two gasoline engine vibrators and one concrete pump. If the vibrators cost \$76 per day and the concrete pump costs \$580 per day, estimate (a) the cost of the equipment per cubic yard of concrete and (b) the equipment cost for placing 56 cubic yards of concrete.

- 15.15 A labor crew for placing concrete consists of 1 labor foreman at \$25.85 per hour, 1 cement finisher at \$28.60 per hour, 5 laborers at \$23.25 per hour each, and 1 equipment operator at \$31.45 per hour. Such a crew, called a C20 crew, can place 160 cubic yards of concrete per 8-hour day. Determine (a) the cost per day of labor for the C20 crew, (b) the cost of the C20 crew per cubic yard of concrete, and (c) the cost to place 250 cubic yards of concrete.

- 15.16 Site work activities associated with constructing a small bridge are shown in the table below. The table includes the quantity of each activity, the unit of measurement associated with each activity, and the unit cost of each activity. Use the data to determine (a) the total cost for structural excavation, (b) the total cost for the pile-driving rig, and (c) the total labor cost for the site work.

Activity	Quantity	Unit of Measurement	Labor	Equipment	Material
			Unit Cost, \$	Unit Cost, \$	Unit Cost, \$
Excavation, unclassified	1667	cy	1.35	1.43	0
Excavation, structural	120	cy	21.31	5.00	0
Backfill, compacted	340	cy	7.78	1.72	0
Pile-driving rig	job	ls	5688	6420	300
Piling, steel, driving	2240	lf	3.13	2.93	16.57

Legend: cy = cubic yard; ls = lump sum; lf = linear foot

Cost Indexes

- 15.17 From historical data, you discover that the national average construction cost of middle schools is \$10,500 per student. If the state index for Texas is 76.9 and for California it is 108.5, estimate the total construction cost of a middle school for 800 students in each state.

- 15.18 A consulting engineering firm is preparing a preliminary cost estimate for a design-construct project of a coal processing plant. The firm, which completed a similar project in 2001 with a construction cost of \$30 million, wants to use the *ENR* construction cost index to update the cost. Use the values in Table 15–3 to estimate the cost of construction for a similar-size plant in mid-2010.
- 15.19 If the editors at *ENR* decide to redo the construction cost index so that the year 2000 has a base value of 100, determine the value for the year (a) 1995 and (b) 2009.
- 15.20 (a) Estimate the value of the *ENR* construction cost index by using the average (compounded) percentage change in its value between 1995 and 2005 to predict the value in 2009.
 (b) How much difference (numerical) is there between the estimated and actual 2009 values? Is this an underestimate or overestimate?
- 15.21 An engineer who owns a construction company that specializes in large commercial projects noticed that material costs increased at a rate of 1% per month over the past 12 months. If a material cost index were created for each month of this year with the value of the index set at 100 for the beginning of the year, what would be the value of the index at the end of the year? Express your answer to two decimal places.
- 15.22 Using 1913 as the base year with a value of 100, the *ENR* construction cost index (CCI) for August 2009 was 8563.35. For August 2010, the CCI value was 8837.38. (a) What was the inflation rate for construction for that 1-year period? (b) What will be the CCI value in August 2013, provided the inflation rate remains the same as it was in the 2009–2010 period?
- 15.23 Electropneumatic general-purpose pressure transducers convert supply pressure to regulated output pressure in direct proportion to an electrical input signal. The cost of a certain transducer was \$194 in 1985 when the M&S equipment index was at 789.6. If the price increased exactly in proportion to the M&S equipment index, what would its cost be in 2010 (midyear) when the index value was 1461.3?
- 15.24 The *ENR* construction cost index for New York City had a value of 12,381.40 in February 2007. For Pittsburgh and Atlanta, the values were 7341.32 and 4874.06, respectively. If a general contractor in Atlanta won construction jobs totaling \$54.3 million, determine their equivalent total value in New York City.
- 15.25 The *ENR* construction cost index (CCI) for August 2010 had a value of 8837.37 when the base year was 1913 with a value of 100. If the base year is 1967 with a value of 1.0, the CCI for August 2010 would be 8.2272. In this case, what is the CCI value for 1967 when the base year is 1913?
- 15.26 The *ENR* materials cost index (MCI) had a value of 2708.51 in August 2010. In the same month, the cost for cement was \$96.55 per ton. If cement increased in price exactly in accordance with the MCI, what was the cost of cement per ton in 1913 when the MCI index value was 100?
- 15.27 A contractor purchased equipment costing \$40,000 in 2010 when the M&S equipment cost index was at 1461.3. He remembers purchasing the same equipment for \$21,771 many years ago, but he does not remember the year that he did so. If the M&S equipment cost index increased by 2.68% per year over that time period and the equipment increased in price exactly in proportion to the index, (a) in what year did he purchase the equipment and (b) what was the value of the index in that year?

Cost-Capacity and Factor Methods

- 15.28 If the materials cost index decreased by 2% over the same time period that the construction cost index increased by 2%, what does that suggest about the labor cost index, if labor represents 35% of the cost of construction?
- 15.29 Use the exponent values in Table 15–6 to estimate the cost for the following equipment to be placed on an offshore drilling platform.
 (a) The cost of a 125-hp centrifugal pump if a 200-hp pump costs \$28,000.
 (b) The cost of a 1700-gallon stainless steel tank if a 900-gallon tank costs \$4100.
- 15.30 A high-pressure stainless steel pump (1000 psi) with a variable-frequency drive (VFD) is installed in a seawater reverse-osmosis pilot plant that is recovering water from membrane concentrate at a rate of 4 gallons per minute (gpm). The cost of the pump was \$13,000. Because of favorable results from the pilot study, the city utility wants to go with a full-scale system that will produce 500 gpm. Determine the estimated cost of the larger pump, if the exponent in the cost-capacity equation has a value of 0.37.
- 15.31 A 0.75 million gallon per day (MGD) induced-draft packed tower for air-stripping trihalomethanes from drinking water costs \$58,890. Estimate the cost of a 2-MGD tower if the exponent in the cost-capacity equation is 0.58.

- 15.32 The on-site manager told you that the variable-frequency drive (VFD) for a 300-hp motor costs \$20,000. Make the best cost estimate possible for the VFD for a 100-hp motor. The exponent in the cost-capacity equation is not available currently where you are located.
- 15.33 The cost of a 68 m^2 falling-film evaporator was 1.52 times the cost of the 30 m^2 unit. What exponent value in the cost-capacity equation yielded these results?
- 15.34 Reinforced concrete pipe (RCP) that is 12 inches in diameter had a cost of \$12.54 per linear foot in Dallas, Texas in 2007. The cost for 24-inch RCP was \$27.23 per foot. If the cross-sectional area of the pipe is considered the “capacity” in the cost-capacity equation, determine the value of the exponent in the cost-capacity equation that exactly relates the two pipe sizes.
- 15.35 A 100,000 barrel per day (bpd) fractionation tower cost \$1.2 million in 2001 when the *Chemical Engineering* plant cost index value was 394.3. How much would a 450,000 bpd plant cost when the index value is 575.8, provided the exponent in the cost-capacity equation is 0.67?
- 15.36 A mini wind tunnel for calibrating vane or hot-wire anemometers cost \$3750 in 2002 when the M&S equipment index value was 1104.2. If the index value is now 1620.6, estimate the cost of a tunnel twice as large. The cost-capacity equation exponent is 0.89.
- 15.37 In 2008, a military engineer estimated the cost for a classified laser-guided device to be \$376,900. The engineer used the M&S equipment index for the years 1998 and 2008 and the cost-capacity equation with an exponent value of 0.61. If the original equipment had only one-fourth the capacity of the new equipment, what was the cost of the original equipment in 1998?
- 15.38 Instead of using a cost-capacity equation for relating project size and construction cost, the Department of Defense uses Size Adjustment Factors (SAFs) that are based on Size Relationship Ratios (SRRs). For example, a SRR of 2.00 has a SAF of 0.942. Determine the required exponent value in the cost-capacity equation in order for C_2 to be $0.942C_1$, when $Q_2/Q_1 = 2.00$.
- 15.39 The equipment cost for removing arsenic from a well that delivers 800 gallons per minute (gpm) is \$1.8 million. If the overall cost factor for this type of treatment system is 2.25, what is the total plant cost expected to be?
- 15.40 A closed-loop filtration system for waterjet cutting industries eliminates the cost of makeup water treatments (water softeners, reverse osmosis, etc.) while maximizing orifice life and machine performance. If the equipment cost is \$225,000 and the total plant cost is \$1.32 million, what is the overall cost factor for the system?
- 15.41 The equipment cost for a laboratory that plans to specialize in analyzing for endocrine disrupters, pharmaceuticals, and personal care products is \$870,000. If the direct cost factor is 1.32 and the indirect cost factor is 0.45 (applies to equipment only), determine the expected cost of the laboratory.
- 15.42 The equipment cost for a 10 gallon per minute farm-scale ethanol fuel production plant is \$243,000. The direct cost factor for construction is 1.28 and for installation is 0.23. The indirect cost factor for licenses, insurance, etc. is 0.84 (applied to total direct cost). Determine the estimated total plant cost.
- 15.43 A chemical engineer at Western Refining has estimated that the total cost for a diesel fuel desulfurization system will be \$2.3 million. If the direct cost factor is 1.35 and the indirect cost factor is 0.41, what is the total equipment cost? Both factors apply to delivered-equipment cost.
- 15.44 A mechanical engineer estimated that the equipment cost for a multitube cyclone system with a capacity of 60,000 cfm would be \$400,000. If the direct cost factor is 3.1 and the indirect cost factor is 0.38, what is the estimated total plant cost? The indirect cost factor applies to the total direct cost.
- 15.45 Nicole is an engineer on temporary assignment at a refinery operation in Seaside. She has reviewed a cost estimate for \$430,000, which covers some new processing equipment for the ethylene line. The equipment itself is estimated at \$250,000 with a construction cost factor of 0.30 and an installation cost factor of 0.30. No indirect cost factor is listed, but she knows from other sites that indirect cost is a sizable amount that increases the cost of the line's equipment. (a) If the indirect cost factor should be 0.40, determine whether the current estimate includes a factor comparable to this value (b) Determine the cost estimate if the 0.40 indirect cost factor is used.

Indirect Cost Allocation

- 15.46 The company you work for currently allocates insurance costs on the basis of cost per direct labor hour. This indirect cost component for the year is budgeted at \$36,000. If the direct labor hours for

departments A, B, and C are expected to be 2000, 8000, and 5000, respectively, this year, determine the allocation to each department.

- 15.47 The director of public works needs to distribute the indirect cost allocation of \$1.2 million to the three branches around the city. The recorded amounts for this year are as follows:

Branch	Records for This Year	
	Miles Driven	Direct Labor Hours
North	275,000	38,000
South	247,000	31,000
Midtown	395,000	55,500

The director plans to use the allocation and information from last year to determine the rates for this year. This information follows:

Branch	Miles Driven	Direct Labor		Indirect Cost Allocation
		Hours	Basis	Last Year, \$
North	350,000	40,000	Miles	300,000
South	200,000	20,000	Labor	200,000
Midtown	500,000	64,000	Labor	450,000

- (a) Determine the rates for this year for each branch.
- (b) Use the rate to distribute this year's total indirect cost. What percentage of this year's budget is now distributed?

- 15.48 A company has a processing department with 10 stations. Because of the nature and use of three of these stations, each is considered a separate cost center for indirect cost allocation. The remaining seven are grouped as one center, CC190. Machine operating hours are used as the allocation basis for all machines. A total of \$250,000 is allocated to the department for next year. Use the data collected this year to determine the indirect cost rate for each center.

Cost Center	Indirect Cost Allocated, \$	Estimated Machine Hours
CC100	25,000	800
CC110	50,000	200
CC120	75,000	1200
CC190	100,000	1600

- 15.49 All the indirect costs are allocated by accounting for a department. The manager has obtained records of allocation rates and actual charges for the prior 3 months and estimates for this month (May) and next month (see the table). The basis

of allocation is not indicated, and the company accountant has no record of the basis used. However, the accountant advises the manager to not be concerned because the allocation rates have decreased each month.

Month	Indirect Cost, \$		
	Rate	Allocated	Charged
February	1.40	2800	2600
March	1.33	3400	3800
April	1.37	3500	3500
May	1.03	3600	
June	0.92	6000	

During the evaluation, the following additional information from departmental and accounting records is obtained.

Month	Direct Labor		Material Cost, \$	Departmental Space, ft ²
	Hours	Cost, \$		
February	640	2560	5400	2000
March	640	2560	4600	2000
April	640	2560	5700	3500
May	640	2720	6300	3500
June	800	3320	6500	3500

- (a) With this information determine the allocation basis used each month.
- (b) Comment on the accountant's statement about decreasing allocation rates.

- 15.50 The mechanical components division manager asks you to recommend a make/buy decision on a major automotive subassembly that is currently purchased externally for a total of \$3.9 million this year. This cost is expected to continue rising at a rate of \$300,000 per year. Your manager asks that both direct and indirect costs be included when in-house manufacturing (make alternative) is evaluated. New equipment will cost \$3 million and will have a salvage of \$0.5 million and a life of 6 years. Estimates of materials, labor costs, and other direct costs are \$1.5 million per year. Typical indirect rates, bases, and expected usage are shown below. Perform the AW evaluation at MARR = 12% per year over a 6-year study period. Show both hand and spreadsheet solutions, as directed.

Department	Basis	Rate	Expected Usage
X	Direct labor cost	\$2.40 per \$	\$450,000
Y	Materials cost	\$0.50 per \$	\$850,000
Z	Number of inspections	\$20 per inspection	4,500

ABC Method

15.51 The municipal water and desalination utility in a California city currently allocates some costs for maintenance shop workers to pumping stations based on the number of pumps at each station. At the last director's semiannual meeting, a suggestion was made to change the allocation basis to the number of trips that pump service personnel make to each station, because some stations have old pumps that require more maintenance. Information about the stations is below. The indirect cost budget is \$20,000 per pump.

- (a) Allocate the budget to each station based on the number of service trips.
- (b) Determine the old allocation on the basis of the number of pumps, and comment on any significant differences in the amounts allocated to the stations.

Station ID	No. of Pumps	Service Trips per Year
Sylvester	5	190
Laurel	7	55
7 th St	3	38
Spicewood	4	104

15.52 Factory Direct manufactures and sells manufactured homes. Traditionally, it has distributed indirect costs to its three construction plants based on materials cost. Each plant builds different models and floor plans. Advances in weight and shape of plastic and wood composite components have decreased cost and time to produce a unit. Because of these advances, the CFO plans to use build-time per unit as the new basis. However, he initially wants to determine what the allocation would have been this year had build-time been the basis prior to incorporation of the new materials. The data shown represents average costs and times. Use this data and the three bases indicated to determine the allocation rates and indirect cost distribution of \$900,000 for this year.

Plant	Texas	Oklahoma	Kansas
Direct material cost, \$ per unit	20,000	12,700	18,600
Previous build-time per unit, work-hours	400	415	580
New build-time per unit, work-hours	425	355	480

Problems 15.53 through 15.55 use the following information.

Jet Green Airways historically distributes the indirect costs of lost and damaged baggage to its three major hubs using a basis of annual number of flights in and out of each hub. Last year \$667,500 was distributed as follows:

Hub Airport	Flights	Rate, \$ per Flight	Allocation, \$
DFW	55,000	6	330,000
YYZ	20,833	9	187,500
MEX	15,000	10	150,000

The airline's baggage management director suggests that an allocation on the basis of baggage traffic, not flights, will better represent the distribution, primarily based on the fact that the high fees now charged to passengers to check luggage have significantly changed the number of bags handled at the major hubs. Total number of bags handled during the year are 2,490,000 at DFW, 1,582,400 at YYZ, and 763,500 at MEX.

15.53 What are the activity and the cost driver for the suggested baggage-traffic basis?

15.54 Using the baggage-traffic basis, determine the allocation rate using last year's total of \$667,500, and distribute this amount to the hubs this year.

15.55 What are the percentage changes in allocation at each hub using the two different bases?

15.56 On-line Vacation distributes advertising costs to its four resort sites in the Caribbean on the basis of the size of the resort budget. For this year, in round numbers, the budgets and allocation of \$1 million advertising indirect costs are as follows:

	Site			
	A	B	C	D
Budget, \$	2 million	3 million	4 million	1 million
Allocation, \$	200,000	300,000	400,000	100,000

- (a) Determine the allocation if the ABC method is used with a new basis. Define the activity as the advertising department at each resort. The cost driver is the number of guests during the year.

	Site			
	A	B	C	D
Guests	3500	4000	8000	1000

- (b) Again use the ABC method, but now make the cost driver the total number of guest nights at each resort. The average number of lodging nights for guests at each site is as follows:

	Site			
	A	B	C	D
Length of stay, nights	3.0	2.5	1.25	4.75

- (c) Comment on the distribution of advertising costs using the two methods. Identify any other cost drivers that might be considered for the ABC approach that may reflect a realistic allocation of the costs.

ADDITIONAL PROBLEMS AND FE EXAM REVIEW QUESTIONS

15.57 In the bottom-up approach to cost estimating:

- (a) Required price is an input variable.
- (b) Cost estimates are an output variable.
- (c) Required price is an output variable.
- (d) Both (a) and (b) are correct.

15.58 A ratio of the cost of something today to its cost at some time in the past is called a:

- (a) Cost-capacity index
- (b) Cost index
- (c) Buyer's guide
- (d) Bluebook index

15.59 Index values could probably be obtained from all of the following places except:

- (a) Trade organizations
- (b) *Engineering News Record* magazine
- (c) Government organizations
- (d) Home Depot or Lowe's home improvement stores

15.60 A 50-hp turbine pump was purchased for \$2100. If the exponent in the cost-capacity equation has a value of 0.76, a 200-hp turbine pump could be expected to cost about:

- (a) \$6020
- (b) \$5320
- (c) \$4890
- (d) \$4260

15.61 The city built a recreation park in 1980 for \$500,000. The *ENR* construction cost index had a value of 3378.17 at that time. If the city is planning to construct a similar recreation park when the index value is 5542.16, the estimated cost is closest to:

- (a) \$695,800
- (b) \$750,700
- (c) \$820,300
- (d) \$910,500

15.62 A small company bought a 250-hp compressor in 1998 for \$3000 when the M&S equipment cost index had a value of 1061.9. If the exponent in the cost-capacity equation is 0.32 and the M&S index value was 1449.3 in 2008, the cost of a 500-hp compressor in 2008 was closest to:

- (a) \$3744
- (b) \$4094
- (c) \$4627
- (d) \$5110

15.63 The cost for implementing a manufacturing process that has a capacity of 6000 units per day was \$550,000. If the cost for a plant with a capacity of

100,000 units per day was \$3 million, the value of the exponent in the cost-capacity equation is:

- (a) 0.26
- (b) 0.39
- (c) 0.45
- (d) 0.60

15.64 The equipment for applying specialty coatings that provide a high angle of skid for the paper-board and corrugated box industries has a delivered cost of \$390,000. If the overall cost factor for the complete system is 2.96, the total plant cost is approximately:

- (a) \$954,400
- (b) \$1,054,400
- (c) \$1,154,400
- (d) \$1,544,400

15.65 The delivered-equipment cost for setting up a production and assembly line for high-sensitivity, gas-damped accelerometers is \$650,000. If the direct cost and indirect cost factors are 1.82 and 0.31, respectively, and both factors apply to delivered-equipment cost, the total plant cost estimate is approximately:

- (a) \$2,034,500
- (b) \$1,734,500
- (c) \$1,384,500
- (d) \$1,183,000

15.66 A police department wants to allocate the indirect cost of speed monitoring to the three toll roads around the city. An allocation basis that may not be reasonable is:

- (a) Miles of toll road monitored
- (b) Average number of cars patrolling per hour
- (c) Amount of car traffic per section of toll road
- (d) Cost to operate a patrol car

15.67 The IT department allocates indirect costs to user departments on the basis of CPU time at the rate of \$2000 per second. For the first quarter, the two heaviest-use departments logged 900 and 1300 seconds, respectively. If the IT indirect budget for the year is \$8.0 million, the percentage of this year's allocation consumed by these departments is closest to:

- (a) 32%
- (b) 22.5%
- (c) 55%
- (d) Not enough information to determine

15.68 If the engineering department is the activity to receive indirect cost allocation for the year, cost

drivers for the ABC method that seem reasonable may be:

1. Cost of engineering changes processed
 2. Size of the workforce
 3. Administrative cost to process a change order
- (a) 1
(b) 2
(c) 3
(d) 1 and 3

15.69 Advantages of the ABC method are:

1. It is an excellent replacement for a traditional cost accounting system.
 2. It is always cheaper to operate than a traditional allocation system.
 3. It can help explain the economic impact of management decisions.
- (a) 1
(b) 2
(c) 3
(d) 1 and 3

CASE STUDY

INDIRECT COST ANALYSIS OF MEDICAL EQUIPMENT MANUFACTURING COSTS

Background

Three years ago Medical Dynamics, a medical equipment unit of Johnson and Sons, Inc., initiated the manufacture and sales of a portable sterilization unit (Quik-Sterz) that can be placed in the hospital room of a patient. This unit sterilizes and makes available at the bedside some of the reusable instruments that nurses and doctors usually obtain by walking to or receiving delivery from a centralized area. This new unit makes the instruments available at the point and time of use for burn and severe wound patients who are in a regular patient room.

There are two models of Quik-Sterz sold. The standard version sells for \$10.75, and a premium version with customized trays and a battery backup system sells for \$29.75. The product has sold well to hospitals, convalescent units, and nursing homes at the level of about 1 million units per year.

Information

Medical Dynamics has historically used an indirect cost allocation system based upon direct hours to manufacture for

all its other product lines. The same was applied when Quik-Sterz was priced. However, Arnie, the person who performed the indirect cost analysis and set the sales price, is no longer at the company, and the detailed analysis is no longer available. Through e-mail and telephone conversations, Arnie said the current price was set at about 10% above the total manufacturing cost determined 2 years ago, and that some records were available in the design department files. A search of these files revealed the manufacturing and cost information in Table 15–11. It is clear from these and other records that Arnie used traditional indirect cost analysis based on direct labor hours to estimate the total manufacturing costs of \$9.73 per unit for the standard model and \$27.07 per unit for the premium model.

Last year management decided to place the entire plant on the ABC system of indirect cost allocation. The costs and sales figures collected for Quik-Sterz the year before were still accurate. Five activities and their cost drivers were identified for the Medical Dynamics manufacturing operations (Table 15–12). Also, the volumes for each model are summarized in this table.

TABLE 15–11 Historical Records of Direct and Indirect Cost Analyses for Quik-Sterz

Quik-Sterz Direct Cost (DC) Evaluation				
Model	Direct Labor, \$/Unit ¹	Direct Material, \$/Unit	Direct Labor, Hours/Unit	Total Direct Labor Hours
Standard	5.00	2.50	0.25	187,500
Premium	10.00	3.75	0.50	125,000
Quik-Sterz Indirect Cost (IDC) Evaluation				
Model	Direct Labor, Hours/Unit	Fraction IDC Allocated	Allocated IDC, \$	Sales, Units/Year
Standard	0.25	$\frac{1}{3}$	1.67 million	750,000
Premium	0.50	$\frac{2}{3}$	3.33 million	250,000

¹Average direct labor rate is \$20 per hour.

TABLE 15-12 Quik-Sterz Activities, Cost Drivers, and Volume Levels for ABC-Based Indirect Cost Allocation

Activity	Cost Driver	Volume/Year	Actual Cost, \$/Year
Quality	Inspections	20,000 inspections	800,000
Purchasing	Purchase orders	40,000 orders	1,200,000
Scheduling	Change orders	1,000 orders	800,000
Production setup	Setups	5,000 setups	1,000,000
Machine operations	Machine hours	10,000 hours	1,200,000
Volume Level for the Year			
Cost Driver	Standard	Premium	
Quality inspections	8,000	12,000	
Purchase orders	30,000	10,000	
Number of change orders	400	600	
Production setup	1,500	3,500	
Machine hours	7,000	3,000	

The ABC method will be used henceforth, with the intention of determining the total cost and price based on its results. The first impression of the production manager is that the new system will show that indirect costs for Quik-Sterz are about the same as they have been for other products over the last several years when a standard model and an upgrade (premium) model were sold. Predictably, they state, the standard model will receive about 1/3 of the indirect cost, and the premium will receive the remaining 2/3. Fundamentally, there are two reasons why the production manager does not like to produce premium versions: They are less profitable for the company, and they require significantly more time and operations to manufacture.

Case Study Exercises

1. Use traditional indirect cost allocation to verify Arnie's cost and price estimates.

2. Use the ABC method to estimate the indirect cost allocation and total cost for each model.
3. If the prices and number of units sold are the same next year (750,000 standard and 250,000 premium), and all other costs remain constant, compare the profit from Quik-Sterz under the ABC method with the profit using the traditional indirect cost allocation method.
4. What prices should Medical Dynamics charge next year based on the ABC method and a 10% markup over cost? What is the total profit from Quik-Sterz predicted to be if sales hold steady?
5. Using the results above, comment on the production manager's prediction of indirect costs using ABC (1/3 standard; 2/3 premium) and the two reasons given to not produce the premium version of Quik-Sterz.

CASE STUDY

DECEPTIVE ACTS CAN GET YOU IN TROUBLE

Contributed by Dr. Paul Askenasy, Agronomist, Texas Commission on Environmental Quality

Background

Surface mining of coal is the removal of soil and sediments from underlying strata that lie above the material to be mined. The law requires that land disturbed by these types of mining activities be returned to a productive capacity that is as good as or better than its productive capacity before mining.

The productive capacity of soils is directly correlated to the textural (sand, silt, and clay content) and chemical

characteristics of the soil (for example, pH). To this end, mining companies must sample the different soils found in the areas to be disturbed by mining activities. The purpose of the sampling is to establish a baseline characterizing the textural and chemical makeup of the soils prior to mining. Soils in a low pH range (pH values < 5) are indicative of low fertility. Once the natural resource, such as coal, is removed, the pit is backfilled with sediments and the terrain surface is contoured to reestablish the premine drainages. To meet the baseline for pH, the acreage of the mine soils with low pH should not exceed the acreage of the unmined soils with low pH.

Information

Yucatan Mining Company (not the actual name) planned to disturb 600 acres due to mining activities. The different soils within the 600 acres were depicted in the County Soil Survey where the mining activities were to take place. Prior to mining, the company obtained soil samples from 10 different locations within each soil type and had them analyzed for a number of parameters including pH. Assessment of the data indicated that 30% of the area (180 acres) occupied by the soils in the area to be disturbed had pH values between 4.0 and 4.9. The application for mining was approved by the State Department of Mining and Reclamation.

Six years later, 450 acres had been mined and the terrain surface had been leveled to reestablish premine slopes. Of the 450 acres leveled, 175 acres had pH values between 4.0 and 4.9. The president of Yucatan indicated that the company would submit a revised soil baseline based on new sampling in the remaining 150 acres of unmined soils because, in his opinion, the first soil baseline was biased.

The request to do more soil analyses to augment their existing soil baseline was approved. The company quickly hired a consultant to develop the new baseline, and subsequently Yucatan submitted the final report from its consultant to the State Department of Mining and Reclamation. This revised premine soil baseline indicated that 45% of the premine soils had pH values between 4.0 and 4.9. Comparative results between the old and new samples can be expressed as follows:

Percent and Acreage of Area		
Soil Baselines	Old Soil Baseline	Revised Soil Baseline
pH: 4.0 – 4.9	30%	45%
	180 acres	270 acres

A rough statistical check between the old and revised soil baselines indicated that the results were mixed. Based on this preliminary result and the fact that there was a significant increase in the percent of area with low-productivity soil, an in-depth analysis of the revised baseline sample study was performed. Contained in the submitted new-sample package was a letter from the Yucatan consultant. It indicated to Yucatan's management that 100 separate soil samples had been obtained and analyzed and that the revised premine soil baseline had been developed using the data from the 30 samples *with the lowest pH values*.

The State Department of Mining and Reclamation staff concluded that the revised soil pH sample data had been carefully "screened" to reduce the amount of remediation work that Yucatan Mining would have to complete. Within a week, Yucatan was notified that further review of the revised soil baseline could not be pursued, because it appeared the revised soil baseline was developed using a technique that skewed results in favor of lower pH values. It was also noted that should Yucatan Mining disagree with this response, the case would be filed with the legal staff as a contested case.

Within several days, the Yucatan president responded indicating that the company was withdrawing the new application from consideration by the department.

Case Study Questions

1. Assume you are the director of the State Department of Mining and Reclamation and were informed of the findings on the new samples versus the old samples. What actions would you direct your staff to take concerning this situation?
2. Suppose there had been several cases of deceptive acts similar to this one over the last few years. What type of "audit" procedures might you want implemented to identify these possibly unethical activities?
3. Yucatan clearly would state that both the old and new samples were randomly located about the entire mining area. When the 30 lowest pH samples were used to establish the new baseline, were the samples still random, according to experimental design standards? If so, why? If not, why not?
4. You and the president of Yucatan Mining have been acquaintances for some years. You have golfed together several times, your and his children are on the same soccer team at school, and your families are members of the same community swimming pool club. What effect would this event have upon your and your family's relationships with the family of the Yucatan president? How would you handle this situation?
5. As a matter of principle and practice, do you believe there is some amount of data-altering or bias-making that is allowed before an application (such as the one described here) should be considered the result of professionally unethical acts? How would you define such a threshold limit?

Depreciation Methods



LEARNING OUTCOMES

Purpose: Use depreciation or depletion methods to reduce the book value of a capital investment in an asset and natural resource.

SECTION	TOPIC	LEARNING OUTCOME
16.1	Terminology	<ul style="list-style-type: none">Define and use the basic terms of asset depreciation.
16.2	Straight line	<ul style="list-style-type: none">Apply the straight line (SL) method of depreciation.
16.3	Declining balance	<ul style="list-style-type: none">Apply the declining balance (DB) and double declining balance (DDB) methods of depreciation.
16.4	MACRS	<ul style="list-style-type: none">Apply the modified accelerated cost recovery system for tax depreciation purposes for U.S.-based corporations.
16.5	Recovery period	<ul style="list-style-type: none">Select the asset recovery period for MACRS depreciation.
16.6	Depletion	<ul style="list-style-type: none">Explain depletion; apply cost depletion and percentage depletion methods.
Chapter 16 Appendix		
16A.1	Historical methods	<ul style="list-style-type: none">Apply the sum-of-years-digits (SYD) and unit-of-production (UOP) methods of depreciation.
16A.2	Switching	<ul style="list-style-type: none">Switch between classical depreciation methods; explain how MACRS provides for switching.
16A.3	MACRS and switching	<ul style="list-style-type: none">Calculate MACRS rates using switching between classical methods and MACRS rules.

The capital investments of a corporation in tangible assets—equipment, computers, vehicles, buildings, and machinery—are commonly recovered on the books of the corporation through *depreciation*. Although the depreciation amount is **not an actual cash flow**, the process of depreciating an asset on the books of the corporation accounts for the decrease in an asset's value because of age, wear, and obsolescence. Even though an asset may be in excellent working condition, the fact that it is worth less through time is taken into account in after-tax economic evaluation studies. An introduction to depreciation types, terminology, and classical methods is followed by a discussion of the *Modified Accelerated Cost Recovery System (MACRS)*, which is the standard in the United States for tax purposes. Other countries commonly use the classical methods for tax computations.

Why is depreciation important to engineering economy? Depreciation is a **tax-allowed deduction** included in tax calculations in virtually all industrialized countries. Depreciation lowers income taxes via the general relation

$$\text{Taxes} = (\text{income} - \text{deductions})(\text{tax rate})$$

Income taxes are discussed further in Chapter 17.

This chapter concludes with an introduction to two methods of *depletion*, which are used to recover capital investments in deposits of natural resources such as oil, gas, minerals, ores, and timber.

The chapter appendix describes two historically useful methods of depreciation—sum-of-years-digits and unit-of-production. Additionally, the appendix includes an in-depth derivation of the MACRS depreciation rates from the straight line and declining balance rates. This is accomplished using a procedure called **switching** between classical depreciation methods.

16.1 Depreciation Terminology ● ● ●

The concept and types of depreciation are defined here. Most descriptions are applicable to corporations as well as individuals who own depreciable assets.

Depreciation is a book method (noncash) to represent the reduction in value of a tangible asset. The method used to depreciate an asset is a way to account for the decreasing value of the asset to the owner *and* to represent the diminishing value (amount) of the capital funds invested in it. The annual depreciation amount is **not an actual cash flow**, nor does it necessarily reflect the actual usage pattern of the asset during ownership.

Though the term **amortization** is sometimes used interchangeably with the term *depreciation*, they are different. Depreciation is applied to tangible assets, while amortization is used to reflect the decreasing value of intangibles, such as loans, mortgages, patents, trademarks, and goodwill. In addition, the term *capital recovery* is sometimes used to identify depreciation. This is clearly a different use of the term than what we learned in Chapter 5. The term *depreciation* is used throughout this book.

There are two different purposes for using the depreciation methods we will cover in this chapter:

Book depreciation Used by a corporation or business for *internal* financial accounting to track the value of an asset or property over its life.

Tax depreciation Used by a corporation or business to *determine taxes due* based on current tax laws of the government entity (country, state, province, etc.). Even though depreciation itself is not a cash flow, it can result in actual cash flow changes because the amount of tax depreciation is a deductible item when calculating annual income taxes for the corporation or business.

The methods applied for these two purposes may or may not utilize the same formulas. Book depreciation indicates the reduced investment in an asset based upon the usage pattern and expected useful life of the asset. There are classical, internationally accepted depreciation methods used to determine book depreciation: straight line, declining balance, and the historical

sum-of-years-digits method. The amount of tax depreciation is important in an after-tax engineering economy study and will vary among nations.

In most industrialized countries, the annual **tax depreciation is tax-deductible**; that is, it is subtracted from income when calculating the amount of taxes due each year. However, the tax depreciation amount must be calculated using a government-approved method.

Tax depreciation may be calculated and referred to differently in countries outside the United States. For example, in Canada the equivalent is CCA (capital cost allowance), which is calculated based on the undepreciated value of all corporate properties that form a particular class of assets, whereas in the United States depreciation may be determined for each asset separately.

Where allowed, tax depreciation is usually based on an **accelerated method**, whereby the depreciation for the first years of use is larger than that for later years. In the United States this method is called MACRS, as covered in later sections. In effect, accelerated methods defer some of the income tax burden to later in the asset's life; they do not reduce the total tax burden.

Common terms used in depreciation are explained here.

First cost P or unadjusted basis B is the delivered and installed cost of the asset including purchase price, delivery and installation fees, and other depreciable direct costs incurred to prepare the asset for use. The term *unadjusted basis*, or simply *basis*, is used when the asset is new, with the term *adjusted basis* used after some depreciation has been charged. When the first cost has no added, depreciable costs, the basis is the first cost, that is, $P = B$.

Book value BV , represents the remaining, undepreciated capital investment on the books after the total amount of depreciation charges to date has been subtracted from the basis. The book value is determined at the end of each year t ($t = 1, 2, \dots, n$), which is consistent with the end-of-year convention.

Recovery period n is the depreciable life of the asset in years. Often there are different n values for book and tax depreciation. Both of these values may be different from the asset's estimated productive life.

Market value MV , a term also used in replacement analysis, is the estimated amount realizable if the asset were sold on the open market. Because of the structure of depreciation laws, the book value and market value may be substantially different. For example, a commercial building tends to increase in market value, but the book value will decrease as depreciation charges are taken. However, a computer workstation may have a market value much lower than its book value due to rapidly changing technology.

Salvage value S is the estimated trade-in or market value at the end of the asset's useful life. The salvage value, expressed as an estimated dollar amount or as a percentage of the first cost, may be positive, zero, or negative due to dismantling and carry-away costs.

Depreciation rate or recovery rate d , is the fraction of the first cost removed by depreciation each year t . This rate may be the same each year, which is called the straight line rate d , or different for each year of the recovery period.

Personal property, one of the two types of property for which depreciation is allowed is the income-producing, tangible possessions of a corporation used to conduct business. Included is most manufacturing and service industry property—vehicles, manufacturing equipment, materials handling devices, computers and networking equipment, communications equipment, office furniture, refining process equipment, construction assets, and much more.

Real property includes real estate and all improvements—office buildings, manufacturing structures, test facilities, warehouses, apartments, and other structures. *Land itself is considered real property, but it is not depreciable.*

Half-year convention assumes that assets are placed in service or disposed of in midyear, regardless of when these events actually occur during the year. This convention is utilized in this text and in most U.S.-approved tax depreciation methods. There are also midquarter and midmonth conventions.