6. Set the instrument over D, backsight on F, and set nail 7. Plunge and set nail 8.
7. Set batter-board nails 9, 10, 11, 12, 13, and 14 by measurements from established points.
8. Stretch the string lines to create the building's outline and check all diagonals.

(*Note:* When each batter board is set, it must be placed with its top at the proper elevation.)

As an alternative to this building stakeout procedure, radial methods (described in Section 9.9) can be used. This can substantially reduce the number of instrument setups and stakeout time required. In the radial method, coordinates of all building corners are computed in the same coordinate system as the lot corners. Then the total station instrument is set on any convenient control point and oriented in azimuth by sighting another intervisible control point. Angles and distances, computed from coordinates, are then laid off to mark each building corner. Measuring the distances between adjacent points, and also the diagonals, checks the layout. (An example illustrating radial stakeout of a circular curve is given in Section 24.11.) After constructing the batter boards and setting the cross pieces at the desired elevations, the alignment nails on the batter boards can be set by pulling taut string lines across established corners. For example, in Figure 23.8, with corners D and F marked, a line stretched across these two points enables placing nails 7 and 8 on the boards. With the strings in place after setting batter-board nails, diagonals between corners and wall lengths should be checked.

Another method of laying out buildings is to stake two points on the building, occupy one of them with the total station instrument, take a backsight on the other, and stake all (or many) of the remaining points from that setup using precalculated angles and distances. In some cases, advantage can be taken of symmetrical layouts to save considerable time. Figure 23.9 shows an unusual symmetrical building shape, which was laid out rapidly using only two setups (at points A and O). With this choice of stations, half the corners could be set from each setup, and the same calculated angles and distances could be used (see the

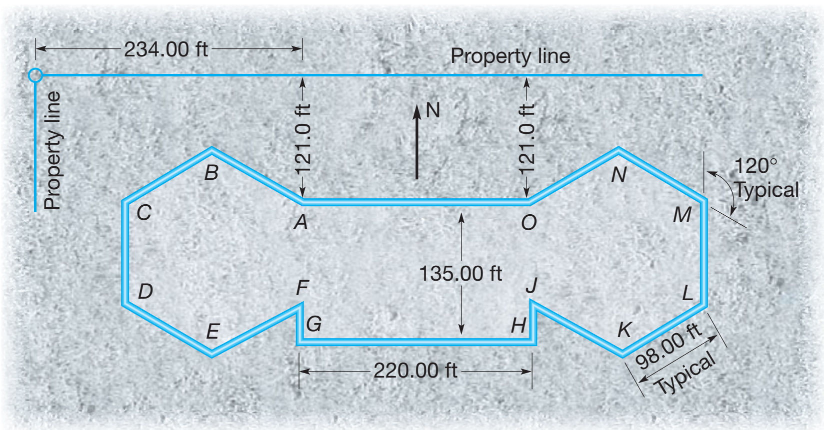

Figure 23.9
Building layout.

BUILDING LAYOUT

Sta.	Angle	To the	Distance
	⊼ @ Point *A*		
O	0°00′	Rt	220.00 ft
F	90°00′	Rt	98.00 ft
G	90°00′	Rt	135.00 ft
E	120°00′	Rt	169.74 ft
D	150°00′	Rt	196.00 ft
C	180°00′	Rt	169.74 ft
B	210°00′	Rt	98.00 ft
	⊼ @ Point *O*		
A	0°00′	Lt	220.00 ft
J	90°00′	Lt	98.00 ft
H	90°00′	Lt	135.00 ft
K	120°00′	Lt	169.74 ft
L	150°00′	Lt	196.00 ft
M	180°00′	Lt	169.74 ft
N	210°00′	Lt	98.00 ft

Figure 23.10 Precalculated angles and distances for building layout.

field notes of Figure 23.10). Again if this method is used, it is essential that enough dimensions be checked between marked corners to ensure that no large errors or mistakes were made.

To control elevations on a building construction site, a benchmark (two or more for large projects) should be set outside the construction limits but within easy sight distance. Rotating lasers (see Section 23.2.1) can be used to control elevations for the tops of footings, floors, and so on.

Permanent foresights are helpful in establishing the principal lines of the structure. Targets or marks on nearby existing buildings can be used if movement due to thermal effects or settlement is considered negligible. On formed concrete structures, such as retaining walls, offset lines are necessary because the outside wall face is obstructed. Two-by-two inch hubs with tacks can first mark the positions of such things as interior footings, anchor bolts for columns, and special piping or equipment. Survey disks, scratches on bolts or concrete surfaces, and steel pins can also be used. Batter boards set inside the building dimensions for column footings have to be removed as later construction develops.

On multistory buildings, care is required to ensure vertical alignment in the construction of walls, columns, elevator shafts, structural steel, and so on. One method of checking plumbness of constructed members is to carefully aim a total station's line of sight on a reference mark at the base of the member. The line of sight is then raised to its top. For an instrument that has been carefully leveled and that is in proper adjustment, the line of sight will define a vertical plane as it is raised. It should not be assumed that the instrument is in good adjustment; therefore, the line should be raised in both the direct and reversed positions. It is necessary to check plumbness in two perpendicular directions when using this procedure. To guide construction of vertical members in real time, two instruments can be set up with their lines of sight oriented perpendicular to each other, and verticality monitored as construction progresses. Alternatively, lasers can be used to guide and monitor vertical construction.

If the surveyor does not give sufficient forethought to the basic control points required, the best method to establish them, and the most efficient approach to staking out a building, the job can be a time-consuming and difficult process. The number of instrument setups should be minimized to conserve time and calculations made in the office if possible, rather than in the field while a survey party waits.

■ 23.7 STAKING OUT HIGHWAYS

Alignments for highways, railroads, and other transportation routes are designed after careful study of existing maps, air photos, and preliminary survey data of the area. From alternative routes, the one that best meets the overall objectives while minimizing costs and environmental impacts is selected. Before construction can begin, the surveyor must transfer that alignment (either the centerline or an offset reference line) to the ground.

Normally staking will commence at the initial point where the first straight segment (*tangent*) is run, placing stakes at *full stations* (100-ft intervals) if the English system of units is used, or at perhaps 30- or 40-m spacing if the metric system is employed. Stationing (this subject is described in Section 5.9.1) continues until the planned alignment changes direction at the first *point of intersection* (PI). The deflection angle is measured there and the second tangent stationed forward to the next PI, where the deflection angle there is measured. The process continues to the terminal point. Staking continuously from the initial point to the terminus may result in large amounts of accumulated error on long projects. Therefore, work should be checked by making frequent ties to intermediate horizontal control points and adjustments should be made as necessary. Alternatively, on smaller projects the alignments can be run from both ends to a point near the middle.

After tangents are established, horizontal curves (usually circular arcs) are inserted at all PIs according to plan. The subject of horizontal alignments, including methods for computing and laying out horizontal curves, is discussed in detail in Chapter 24. Vertical alignments are described in Chapter 25.

After the centerline or reference line (including curves) has been established, the PIs, intermediate *points on tangent* (POTs) on long tangents, and points where horizontal curves begin (PCs), and end (PTs), are referenced using procedures described in Section 9.5. Points used in referencing must be located safely outside the construction limits. Referencing is important because the centerline points will be destroyed during various phases of construction and will need to be replaced several times. Benchmarks are also established at regular spacing (usually not more than about 1000 ft apart) along the route. These are placed on the right of way, far enough from the centerline to be safe from destruction, but convenient for access.

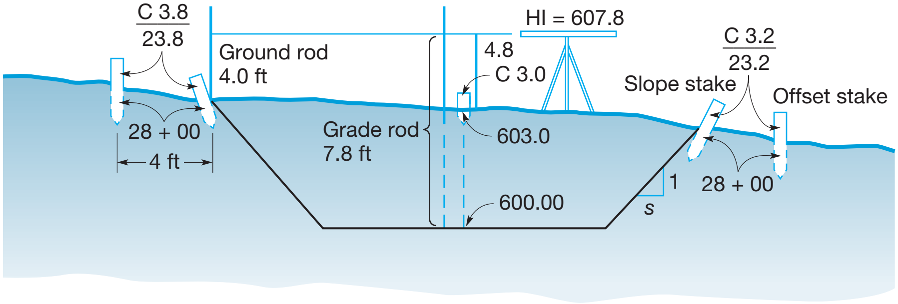

Figure 23.11 Slope stakes (shoulders and ditches not shown).

After the centerline or reference line has been established, stakes marking the right-of-way should be set. This is normally done by carefully measuring perpendicular offsets from the established reference line. The right-of-way is staked at every change in its width, at all changes in alignment, including each PC and PT, and at sufficient other intermediate points along the tangents so that it is clearly delineated.

When the reference line and right-of-way have been staked, the limits of actual construction are marked so that the contractor can clear the area of obstructions. Following this, some contractors want points set on the right-of-way with subgrade elevations, showing cut or fill to a given elevation, for use in performing rough grading and preliminary excavation of excess material.

To guide a contractor in making final excavations and embankments, *slope stakes* are driven at the *slope intercepts* (intersections of the original ground and each side slope), or offset a short distance, perhaps 4 ft (see Figure 23.11). The cut or fill at each location is marked on the slope stake. Note that there is no cut or fill at a slope stake—the value given is the vertical distance from the ground elevation at the slope stake to grade.

Grade stakes are set at points that have the same ground and grade elevation. This happens when a grade line changes from cut to fill, or vice versa. As shown in Figure 23.12, three *transition sections* normally occur in passing from cut to fill (or vice versa), and a grade stake is set at each one. A line connecting grade stakes, perhaps scratched out on the ground, defines the change from cut to fill, as we will see in line *ABC* in Figure 26.1.

Slope stakes can be set at slope intercept locations predetermined in the office from cross-sectional data. (Methods for determining slope intercepts from cross sections are described in Chapter 26.) If predetermined slope intercepts are used, the ground elevation at each stake must still be checked in the field to verify its agreement with the cross section. If a significant discrepancy in elevation exists, the stake's position must be adjusted by a trial-and-error method, as shown in Example 23.1. The amount of cut or fill marked on the stake is computed from the actual difference in elevation between the ground at the slope stake and grade elevation.

If slope intercepts have not been precalculated from cross-section data, slope stakes are located by a trial-and-error method based on mental calculations involving the *HI*, grade rod, ground rod, half roadway width, and side slopes. One

	Sta.	L	CL	R
(a)	61 + 20	$\frac{C\ 8.2}{28.2}$	C 3.9	$\frac{C\ 0.0}{20.0}$
(b)	61 + 70	$\frac{C\ 4.0}{24.0}$	C 0.0	$\frac{F\ 6.4}{29.6}$
(c)	61 + 95	$\frac{C\ 0.0}{20.0}$	F 3.3	$\frac{F\ 5.7}{28.5}$

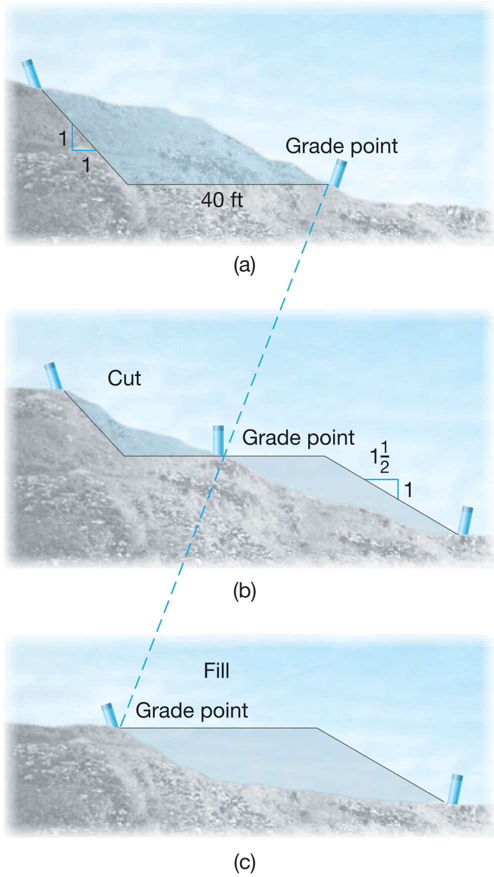

Figure 23.12 Grade points at transition sections.

or two trials are generally sufficient to fix the stake position within an allowable error of 0.3 to 0.5 ft for rough grading. The infinite number of ground variations prohibits use of a standard formula in slope staking. An experienced surveyor employs only mental arithmetic, without scratch paper or hand calculator. Whether using the method described in Example 23.1 or any other, systematic procedures must be followed to avoid confusion and mistakes.

Example 23.1 lists the sequential steps to be taken in slope staking, *assuming for simplicity, academic conditions of a level roadway*. In practice, travel lanes and shoulders of modern highways have lateral slopes for drainage, then a steeper slope to a ditch in cut, and another slope up the hillside to the slope intercept. These *design templates* of the roadbed can often be loaded into a data collector for field stakeout. Transition sections may have half-roadway widths in cuts different from those in fills to accommodate ditches, and flatter side slopes for fills that tend to be less stable than cuts. But the same basic steps still apply and can be extended by students after learning the fundamental approach.

Example 23.1

List the field procedures, including calculations, necessary to set slope stakes for a 40-ft wide level roadbed with side slopes of 1:1 in cut and 1-1/2:1 in fill (see Figures 23.11 and 23.12).

Solution

1. Compute the cut at the centerline stake from profile and grade elevations (603.0 − 600.0 = C 3.0 in Figure 23.11). Check in field by grade rod minus ground rod = 7.8 − 4.8 = C 3.0 ft. Mark the stake C 3.0/0.0. (On some jobs the center stake is omitted and stakes are set only at the slope intercepts.)
2. Estimate the difference in elevation between the left-side slope-stake point (20+ ft out) and the center stake. Apply the difference—say, +0.5 ft—to the center cut and get an estimated cut of 3.5 ft.
3. Mentally calculate the distance out to the slope stake, 20 + 1(3.5) = 23.5 ft, where 1 is the side slope.
4. Hold the zero end of a cloth tape at the center stake while the rodperson goes out at right angles with the other end and holds the rod at 23.5 ft. [The right angle can be established by prism (see Figure 16.10) or by using a total station instrument.]
5. *Forget all previous calculations to avoid confusion of too many numbers and remember only the grade-rod value.*
6. Read the rod with the level and get the cut from grade rod minus ground rod, perhaps 7.8 − 4.0 = C 3.8 ft.
7. Compute the required distance out for this cut, 20 + 1(3.8) = 23.8 ft.
8. Check the tape to see what is actually being held and find it is 23.5 ft.
9. The distance is within a few tenths of a foot and close enough. Move out to 23.8 ft if the ground is level and drive the stake. Move farther out if the ground slopes up, since a greater cut would result, and thus the slope stake must be beyond the computed distance, or not so far if the ground has begun to slope down, which gives a smaller cut.
10. If the distance has been missed badly, make a better estimate of the cut, compute a new distance out, and take a reading to repeat the procedure.
11. In going out on the other side, the rodperson lines up the center and left-hand slope stake to get the right-angle direction.
12. To locate grade stakes at the road edge, one person carries the zero end of the tape along the centerline while the rodperson walks parallel, holding the 20-ft mark until the required ground-rod reading is found by trial. Note that the grade rod changes during the movement but can be computed at 5- or 10-ft intervals. The notekeeper should have the grade rod listed in the field book for quick reference at full stations and other points where slope stakes are to be set.
13. Grade points on the centerline are located using a starting estimate determined by comparing cut and fill at back and forward stations.

Practice varies for different organizations, but often the slope stake is set 4 ft beyond the slope intercept. It is marked with the required cut or fill, distanced out from the centerline to the slope-stake point, side-slope ratio, and base half-width noted on the side facing the centerline. Stationing is given on the backside. A reference stake having the same information on it may also be placed 6 ft or farther out of the way of clearing and grading. On transition sections, grade-stake points are marked.

Total station instruments, with their ability to automatically reduce measured slope distances to horizontal and vertical components, speed slope staking significantly, especially in rugged terrain where slope intercept elevations differ greatly from centerline grade. When the data collector allows the user to input the design template (see Section 26.3), it can rapidly determine the positions of the slope stakes using field observed data. GNSS receivers operating in the real-time kinematic mode (see Chapter 15) can also be advantageously used in these types of terrain if satellite visibility exists. Combined with a machine control system (see Section 23.11), GNSS receivers allow heavy equipment operators to shape the design without the need for stakes.

Slope staking should be done with utmost care, for once cut and fill embankments are started, it is difficult and expensive to reshape them if a mistake is discovered.

After rough grading has shaped cuts and embankments to near final elevation, finished grade is constructed more accurately from *blue tops* (stakes whose tops are driven to grade elevation and then marked with a blue keel or spray paint). These are not normally offset, but rather driven directly on centerline or shoulder points. The procedure for setting blue tops at required grade elevation is described in Section 25.8.

Highway and railroad grades can often be rounded off to multiples of 0.05 or 0.10 percent without appreciably increasing earthwork costs or sacrificing good drainage. Streets need a minimum 0.50% grade for drainage from intersection to intersection or from midblock both ways to the corners. They are also crowned to provide for lateral flow to gutters. Drainage profiles, prepared to verify or construct drainage cross sections, can be used to locate drainage structures and easements accurately. An experienced engineer when asked a question regarding the three most important items in highway work, thoughtfully replied "drainage, drainage, and drainage." Good surveying and design must satisfy this requirement.

To ensure unobstructed drainage after construction, culverts must be placed in most fill sections so that water can continue to flow in its normal pattern from one side of the embankment to the other. In staking culverts, their locations, skew angles, if any, lengths, and invert elevations are taken from the plans. Required pipe alignments and grades are marked using stakes, offset from each end of the pipe's extended centerline. The invert elevation (or an even number of feet above or below it) is noted on the stake. This field procedure, like setting slope stakes, requires marking a point on the stake where a rod reading equals the difference between the required grade and the current *HI* of a leveling instrument.

After the subgrade has been completed, if the highway is being surfaced with rigid concrete pavement, paving pins will be necessary to guide this operation.

They are usually about 1/2-in. diameter steel rods, driven to mark an offset line parallel to one edge of the required pavement. This line is usually staked at 50-ft increments, but closer spacing may be used on sharp curves. The finished grade (or one parallel to it but offset vertically above) is marked on the pins using tape or affixing a special stringline holding device. Again in this operation, elevations are set by marking the stake where a rod reading equals the difference between the finished grade (or a vertically offset one) and a current *HI*. (The need for frequent project benchmarks at convenient locations is obvious.)

Utility relocation surveys may be necessary in connection with highway construction; for example, manhole or valve-box covers have to be set at correct grade before earthwork begins so they will conform to finished grade. Here differential elevation resulting from the transverse surface slope must be considered. Utilities are located by centerline station and offset distance.

Location staking for railroads, rapid-transit systems, and canals follows the same general methods outlined above for highways.

■ 23.8 OTHER CONSTRUCTION SURVEYS

For planning and constructing causeways, bridges, and offshore platforms, it is often necessary to perform hydrographic surveys (see Section 17.12). These types of projects require special procedures to solve the problem of establishing horizontal positions and depths where it is impossible to hold a rod or reflector. Modern surveying equipment and procedures, and sonar mapping devices, are used to plot dredging cross sections for underwater trenching and pipe layout. Today, more pipelines are crossing wider rivers, lakes, and bays than ever before. Mammoth pipeline projects now in progress to transport crude oil, natural gas, and water have introduced numerous new problems and solutions. Permafrost, extremely low temperatures, and the need to provide animal crossings are examples of special problems associated with the Alaskan pipeline construction projects.

Large earthwork projects such as dams and levees require widespread permanent control for quick setups and frequent replacement of slope slakes, all of which may disappear under fill in one day. Fixed signals for elevation and alignment painted or mounted on canyon walls or hillsides can mark important reference lines. Failures of some large structures demonstrate the need for monitoring them periodically so that any necessary remedial work can be done.

Underground surveys in tunnels and mines necessitate transferring lines and elevations from the ground above, often down shafts. Directions of lines in mine tunnels can be most conveniently established using north-seeking gyros. In another, and still practiced method, two heavy plumb bobs hung on wires (and damped in oil or water) from opposite sides of the surface opening can be aligned by total station there and in the tunnel. (A vertical collimator will also provide two points on line below ground.) A total station or laser is “wiggled-in” (see Section 8.16) on the short line defined by the two plumb-bob wires, a station mark set in the tunnel ceiling above the instrument, and the line extended. Later setups are made beneath spads (surveying nails with hooks) anchored in the ceiling. Elevations are brought down by taping or other means. Benchmarks and instrument stations are set on the ceiling, out of the way of equipment.

Surveys are run at intervals on all large jobs to check progress for periodic payments to the contractor. And finally, an *as-built* survey is made to determine compliance with plans, note changes, make terminal contract payment, and document the project for future reference.

Airplane and ship construction requires special equipment and methods as part of a unique branch of surveying called *optical tooling*. The precise location and erection of offshore oil drilling platforms many miles from a coast utilizes new surveying technology, principally the global navigation satellite systems.

■ 23.9 CONSTRUCTION SURVEYS USING TOTAL STATION INSTRUMENTS

The procedures described here apply to most total station instruments, although some may require interfaced data collectors to perform the operations described.

Before using a total station for stakeout, it is necessary to orient the instrument. Depending on the type of project, *horizontal* or both *horizontal and vertical* orientation may be needed. For example, if just the lot corners of a subdivision are being staked, then only horizontal orientation (establishing the instrument's position and direction of pointing) is needed. If grade stakes are to be set, then the instrument must also be oriented vertically (its *HI* determined).

With total station instruments, three methods are commonly used for horizontal orientation: (1) *azimuth*, (2) *coordinates*, and (3) *resection*. The first two apply where an existing control point is occupied, and the latter is used when the instrument is set up at a noncontrol point. In azimuth orientation, the coordinates of the occupied control station and the known azimuth to a backsight station are entered into the instrument. If the occupied station's coordinates have been downloaded into the instrument prior to going into the field, it is only necessary to input its point number. The backsight station is then sighted, and when completed, the azimuth of the line is transferred to the total station by a keyboard stroke, whereupon it appears in the display.

The coordinate method of orientation uses the same approach, except that the coordinates of both the occupied and the backsight station are entered. Again these data could have been downloaded previously so that it would only be necessary to key in the numbers identifying the two stations. The instrument computes the backsight line's azimuth from the coordinates, displays it, and prompts the operator to sight the backsight station. Upon completion of the backsight, the azimuth is transferred to the instrument with a keystroke and it appears on the display.

In the resection procedure, a station whose position is unknown is occupied and the instrument's position determined by sighting two or more control stations (see Sections 11.7 and 11.10). This is very convenient on projects where a certain point of high elevation in an open area gives good visibility to all (or most) points to be staked. As noted, two or more control points must be sighted. Observations of angles, or of angles and distances, are made to the control stations. The microprocessor then computes the instrument's position by the methods discussed in Sections 11.7 and 11.10.

Project conditions will normally dictate which orientation procedure to use. Regardless of the procedure selected, after orientation is completed, a check

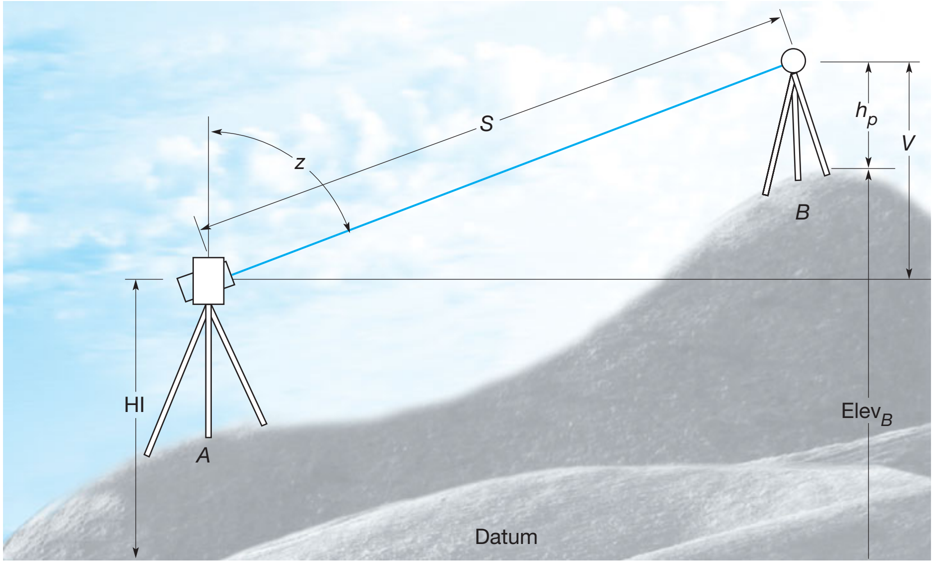

Figure 23.13 Vertical orientation of total station.

should be made by sighting another control point and comparing the observed azimuth and distance against their known values. If there is a discrepancy, the orientation procedure should be repeated. It is also a good idea to recheck orientation at regular intervals after stakeout has commenced, especially on large projects. In fact, if possible a reflector should be left on a control point just for that purpose.

Vertical orientation of a total station (i.e., determining its *HI*) can be achieved using one of two procedures. The simplest case occurs if the elevation of the occupied station is known, as then it is only necessary to *carefully* measure and add the *hi* (height of instrument above the point) to the elevation of the point. If the occupied station's elevation is unknown, then another station of known elevation must be sighted. The situation is illustrated in Figure 23.13, where the instrument is located at station *A* of unknown elevation, and station *B* whose elevation is known is sighted. From slope distance *S* and zenith angle *z* the instrument computes *V*. Then its *HI* is

$$HI = elev_B + h_r - V \quad \textbf{(23.1)}$$

where h_r is the reflector height above station *B*. As with horizontal orientation, it is a good practice to check the instrument's vertical orientation by sighting a second vertical control point.

Once orientation is completed, project stakeout can begin. In general, staking is either a two- or three-dimensional problem. Staking lots of a subdivision or layout of horizontal construction alignments is generally two dimensional. Slope staking, blue-top setting, pipeline layout, and batter-board placement require both horizontal position and elevation and are therefore three dimensional.

For two-dimensional stakeout, after the file of coordinates for all control stations and points to be staked is downloaded and the instrument is oriented horizontally, the identifying number of a point to be staked is entered into the instrument through the keyboard. The microprocessor immediately calculates the horizontal distance and azimuth required to stake the point. The difference between the instrument's current direction of pointing and that required is displayed. The operator turns the telescope until the difference becomes zero to achieve the required direction. With total stations having robotic capabilities, the instrument will swing in direction to the proper azimuth without any further operator intervention.

Following azimuth alignment, the distance to the point must be laid out. To do this, the reflector is directed onto the azimuth alignment and a horizontal distance reading taken, whereupon the difference between it and that required is displayed. The reflector is then directed inward or outward, as necessary, until the distance difference is zero and the stake placed there. A two-way radio is invaluable for communicating with the reflector person in this operation. Additionally, a small tape measure can often be used to speed the process of locating the rod in its correct position. This procedure for stakeout is discussed further in Section 24.13, and an example problem is presented.

Special tracking systems have been developed to aid the reflector person in getting on line with the station. For example, some total stations utilize "constraint" and "flashing" lights to indicate whether the reflector is left or right of the line of sight while others use lights of different colors. The prism person, upon seeing these lights, immediately knows what direction to move to get on line.

For three-dimensional staking, the total station must be oriented vertically as well as horizontally. The initial part of three-dimensional stakeout is exactly like that described for the two-dimensional procedure; that is, the horizontal position of the stake is set first. Then simultaneously with the observation of the stake's horizontal position, its vertical component, and thus its elevation is determined. The difference ΔZ between the required elevation and the stake's elevation is displayed with a plus or minus sign, the former indicating fill, the latter cut. This information is communicated to the reflector person for marking the stake, or incrementally driving it further down until the required grade, or if desired, some even number of feet above or below grade is reached.

With the high order of accuracy possible using total station instruments, stakes quite distant from the instrument can be laid out, and thus many points set from a single setup. Often, in fact, an entire project can be staked from one location. This is made possible in many cases because of the flexibility that resection orientation provides in instrument placement. It should be remembered that if long distances are involved in three-dimensional staking, Earth curvature and refraction should be considered (see Section 4.4). Also with total stations, each point is set independently of the others, and thus no inherent checks are available. Checks should therefore be made by either repeating the observations, checking placements from different control stations, or measuring between staked stations to ascertain their relative accuracies.

■ 23.10 CONSTRUCTION SURVEYS USING GNSS EQUIPMENT

Any of the GNSS surveying methods discussed in Chapters 14 and 15 could be used on construction projects. Specifically, static surveys can be used to establish project control and kinematic surveys can be used to produce maps for planning and design. Finally, real-time kinematic (RTK) surveys (see Chapter 15) can be used to locate construction stakes.

The base receiver does not have to occupy a station with known GNSS coordinates. Instead the application software can be instructed to determine its position in *autonomous* mode. This process determines the base station receiver coordinates using point-positioning methods yielding low-accuracy coordinates. However, all points determined from this station will have GNSS-type accuracies relative to the base station coordinates. The localization process discussed in Sections 15.8 and 19.6.6 transforms this set of low-accuracy GNSS coordinates into the project control reference frame eliminating the inaccuracies of the autonomous base station coordinates.

As discussed in Sections 15.8 and 19.6.6, care must be taken to ensure that points located using GNSS are placed in the same reference frame as the project coordinates. As discussed in Section 15.9, sufficient project control known in the local reference frame must be established at the perimeter of the construction project. Then prior to staking any points, the GNSS receiver must occupy this control and determine their coordinates in the WGS 84 reference frame; these are GNSS coordinates. Using the project coordinates and the GNSS coordinates, transformation parameters (see Section 19.6.6) are computed so that the GNSS-derived coordinates can be transformed into the local project reference frame. It is important that this transformation occur only once in a project and include important control in the transformation. That is, if a benchmark on a bridge abutment was used to design a replacement structure, then this benchmark should be included in the localization process regardless of its location in the project. The localization process should only occur once during a construction project to avoid the introduction of varying orientation parameters caused by random errors. Once the localization is accepted, the transformation parameters should be distributed amongst the various GNSS receivers involved in the project.

Additional control points must also be established at critical locations in the construction project to provide convenient location of base station receivers, total stations (in canopy conditions), and laser levels for finish work. While the type of radio and antenna will determine the range of the base station radio, base station radios typically have a range of about 10 km. Thus, sufficient horizontal control must be established to support the radio's range. However, vertical control is often limited by the range of the laser level, and thus benchmarks are often required every 1000 to 1500 ft.

In construction staking using RTK surveying methods, a minimum of two receivers are needed. Each is equipped with a radio. One receiver occupies a nearby control station and the other, called the "rover," is moved from one point (to be set) to another. The points set must have their project coordinates known before setting any stakes. The base radio broadcasts the raw satellite signals from the base receiver to the rover. At the rover, the application software processes the satellite signals from both receivers in real time using relative positioning techniques (see Section 13.9). This determines the location of the rover relative the base station. If the observed coordinates do not agree with the required values for the point being staked, the GNSS controller will indicate the direction and distance that the rover must be moved. The rover's position is adjusted until agreement is reached and the stake is set at this location.

Although excellent horizontal accuracies can be achieved using GNSS, elevations are less reliable. GNSS-determined ellipsoid heights are typically accurate to within a few centimeters, but to get an orthometric height (elevation related to datum), the geoidal height must be applied as discussed in Section 19.5. Geoidal heights are not precisely known, but models are available which yield values that are generally accurate to within two centimeters in flat regions. However, they can be off by several decimeters in mountainous regions. For this reason, if very precise elevations are required in construction staking, GNSS surveys are unsatisfactory. However, it does provide sufficient accuracy for lower-accuracy jobs, such as slope staking, assuming corrections are made for geoidal heights by applying the geoid model. As shown in Figure 23.14, string lines have traditionally been used to guide finishing work. One manufacturer has included a laser level in its GNSS construction package to provide millimeter accuracy in both horizontal and vertical positioning as a solution to this problem. Another has mounted a GNSS receiver on a robotic total station, which allows the robotic total station to be used in areas where previous control points were not established.

GNSS surveys are particularly useful in staking widely spaced points, especially in areas where terrain or vegetation makes it difficult to conduct traditional ground surveys. Staking subdivisions containing large parcels in rugged terrain, and setting slope stakes in rugged areas where deep cuts and fills exist, are examples of situations where GNSS surveys can be very convenient for construction surveying. Of course, GNSS surveys require an obstruction-free view of the satellites.

Figure 23.14
A string line guiding a paver. (Courtesy Topcon Positioning Systems.)

■ 23.11 MACHINE GUIDANCE AND CONTROL

In recent years, research has led to *stakeless* construction where GNSS receivers, robotic total stations, and laser levels are used to guide earth-moving equipment in real time. The major difference between machine control and machine guidance is that the machine control system actually controls the heavy equipment on the job site through the use of hydraulics, whereas machine guidance informs the operator to take action to either change the direction of the equipment or the level of the cutting edge to meet the desired design. Data necessary for this machine guidance and control include a *digital elevation model* (DEM) or *digital terrain model* (DTM) (see Section 17.8) and digital design plans with their alignments, grades, and design templates developed in the same three-dimensional coordinate system such as shown in Figure 18.10. With GNSS receivers, robotic total stations, sonic receptors, and lasers to guide the equipment operators, and an on-board computer that continually updates cut and fill information, the grading can be accomplished without the need for construction stakes and with limited assistance of grade foremen. Machine guidance and control has been implemented on dozers, hoes, pans, graders, and trucks.

Using machine guidance and control, the surveyor's role in construction surveying shifts to tasks such as establishing the project reference coordinate systems, creating a DTM (see Section 18.14) of the existing surface for the design and grading work, managing the electronic design on the job site, calibrating the surveying equipment with respect to the construction site, providing for the calibration of the cutting surfaces of the heavy equipment with respect to the surveying control, and developing the necessary digital data for the system operation.

As discussed in the previous section, the project design is normally performed in a project reference coordinate system. Thus, the GNSS receiver must be localized (see Sections 15.8 and 19.6.6) before any excavation is performed. It is important in this process to have control surrounding the job as well as including any control crucial to the design of any structures. Once localization is performed and accepted, the application software will convert the GNSS coordinates of latitude, longitude, and height to the project's horizontal and vertical reference frames. Additionally, the GNSS receivers must be referenced to the cutting edges of the construction equipment. Since the GNSS receiver is often placed on the blade of the construction vehicle, this often means measuring the height of the antenna reference point above the cutting edge of the blade on a regular basis to account for wear.

GNSS surveys can provide heights to a few centimeters. Thus, it is sufficiently accurate for rough grading. However, in finished grading, a robotic total station or laser level is required. One manufacturer has combined a laser level with the GNSS receiver to provide millimeter accuracies in both horizontal and vertical locations. The range of the vehicle from the laser level is often limited, and thus additional vertical control will be required to control the final grades.

Three-dimensional machine control is also possible with a robotic total station. In this system, a multifaceted, 360° prism replaces the GNSS receiver on the construction vehicle. Similar to using GNSS, a DTM and grading plan of the site are loaded into the system. The robotic total station then tracks the prism mounted on the construction vehicle and provides the system with the prism's position and elevation. This in turn guides the operator during the excavation and finishing process. Again, the offset from the prism to the cutting edge of the equipment must be measured and entered into the system on a regular basis to account for wear.

Since robotic total stations typically work in a local coordinate system, there is no need for localization using a robotic total station. However, the range of a robotic total station is generally limited to only 1000 ft (300 m). Additionally there must be continuous line of sight between the robotic total station and construction vehicle. Thus, many more control stations must be added to the site when using a robotic total station.[1] Another drawback is that a robotic total station must be dedicated to each piece of construction equipment. When using GNSS equipment for machine control, a single base station can serve as the control for many pieces of construction equipment and is only limited by the range of its radio. For example, to cover the typical range (10 km) of a GNSS base station radio using a robotic total station would require 33 control stations.[2] Another drawback of using robotic total stations in machine control is that each total station must be located and oriented for each construction vehicle. While a GNSS system requires an additional machine control system for each construction vehicle in the base station's range, it does require continuous line of sight from the base station or any additional base station receivers and radios. Thus, necessary control monumentation is reduced. Additionally, since the surveyor is no longer required to place stakes in the path of the construction vehicles, safety is also increased.

Similar to machine control, site communication applications on trucks allow managers to monitor quantities of excavation, hauling distances, and any surplus or fill earthwork. The system provides managers with daily reports on earthwork quantities, machine maintenance, and timetable gains or loses. Because of these features, many companies are finding that they can complete projects in a timely fashion often receiving bonuses for completing the project on or ahead of schedule.

■ 23.12 AS-BUILT SURVEYS WITH LASER SCANNING

As mentioned in Section 23.8, as-built surveys are performed at the completion of a construction project to ensure that project specifications are met and note any changes to plans. In many cases these surveys are performed with traditional surveying equipment. However, in projects that involve extensive detail, danger to instrument operator, or interruption of daily commerce, laser scanning can provide superior results in a fraction of the time. Figure 23.15 shows a rendered image of

[1]To reduce the need for physical monuments, one manufacturer has integrated a GNSS receiver with their robotic total station so that the total station can be located anywhere within range of the GNSS base station without the need of a physical monument.

[2]The range of a GNSS base station can be extended with data modems where cell coverage is available.

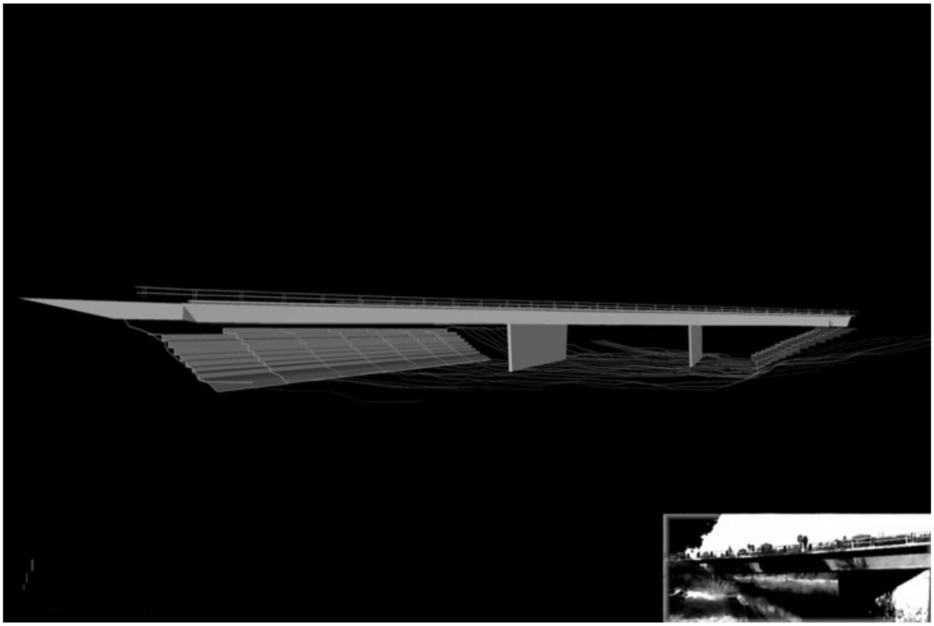

Figure 23.15 Rendered and rotated image of bridge shown in lower-right corner. (Courtesy of Christopher Gibbons, Leica Geosystems AG.)

a bridge that was surveyed for renovations. In the bridge survey, enormous quantities of data were collected from an on-shore location. The digital image of the bridge is shown in the lower-right inset. The rendered, three-dimensional image of the bridge allows designers to obtain accurate measurements between points in the image. Figure 17.12 depicts the point-cloud image of a refinery with the path of a new pipe shown in white. This three-dimensional image allowed engineers to design the new pipe alignment so that existing obstructions were cleared. A traditional survey would have either lacked the detail provided by the three-dimensional laser-scanned image or cost considerably more to locate all the existing elements. Using laser-scanning technology in these projects saved thousands of dollars and provided safe conditions for the field crews.

■ 23.13 SOURCES OF ERROR IN CONSTRUCTION SURVEYS

Important sources of error in construction surveys are:

1. Inadequate number and/or location of control points on the job site.
2. Errors in establishing control.
3. Observational errors in layout.
4. Failure to double-center in laying out angles or extending lines, and failure to check vertical members by plunging the instrument.
5. Careless referencing of key points.
6. Movement of stakes and marks.

23.14 MISTAKES

Typical mistakes often made in construction surveys are:

1. Lack of foresight as to where construction will destroy points.
2. Notation for cut (or fill) and stationing on stake not checked.
3. Wrong datum for cuts, whether cut is to finished grade or subgrade.
4. Arithmetic mistakes, generally due to lack of checking.
5. Use of incorrect elevations, grades, and stations.
6. Failure to check the diagonals of a building.
7. Carrying out computed values to too many decimal places (one good hundredth is better than all the bad thousandths).
8. Reading the rod on top of stakes instead of on the ground beside them in profiling and in slope staking.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

23.1 Discuss how machine control has changed the construction project site.

23.2 Discuss how line and grade can be set with a total station instrument.

23.3 How are rotating beam lasers used?

23.4 In what types of construction is a reflectorless EDM most advantageous?

23.5 Discuss how a laser is used in pipeline layout.

23.6 What is a story pole and how is it used in pipeline layout?

23.7 What is the purpose of localization when performing a GNSS stakeout survey?

23.8 What information is typically conveyed to the contractor on stakes for laying a pipeline?

23.9 A sewer pipe is to be laid from station 10 + 00 to station 12 + 50 on a 0.50% grade, starting with invert elevation of 83.64 ft at 10 + 00. Calculate invert elevations at each 50-ft station along the line.

23.10* A sewer pipe must be laid from a starting invert elevation of 650.73 ft at station 9 + 25 to an ending invert elevation of 653.81 ft at station 12 + 75. Determine the uniform grade needed, and calculate invert elevations at each 50-ft station.

23.11 Grade stakes for a pipeline running between stations 0 + 00 and 6 + 37 are to be set at each full station. Elevations of the pipe invert must be 843.95 ft at station 0 + 00 and 847.22 ft at 6 + 37, with a uniform grade between. After staking an offset centerline, an instrument is set up nearby, and a plus sight of 5.32 taken on BM *A* (elevation 853.63 ft). The following minus sights are taken with the rod held on ground at each stake: (0 + 00, 5.36); (1 + 00, 5.86); (2 + 00, 5.88); (3 + 00, 6.47); (4 + 00, 7.53); (5 + 00, 8.42); (6 + 00, 8.89); (6 + 37, 9.12); and (*A*, 5.36). Prepare a set of suitable field notes for this project (see Plate B.6 in Appendix B) and compute the cut required at each stake. Close the level circuit on the benchmark.

23.12 If batter boards are to be set exactly 6.00 ft above the pipe invert at each station on the project of Problem 23.11, calculate the necessary rod readings for placing the batter boards. Assume the instrument has the same HI as in Problem 23.11.

23.13 What are the requirements for the placement of horizontal and vertical control in a project?

23.14 By means of a sketch, show how and where batter boards should be located: (**a**) for an I-shaped building (**b**) For an L-shaped structure.

23.15 A building in the shape of an L must be staked. Corners $ABCDEF$ all have right angles. Proceeding clockwise around the building, the required outside dimensions are $AB = 80.00$ ft, $BC = 30.00$ ft, $CD = 40.00$ ft, $DE = 40.00$ ft, $EF = 40.00$ ft, and $FA = 70.00$ ft. After staking the batter boards for this building and stretching string lines taut, check measurements of the diagonals should be made. What should be the values of AC, AD, AE, FB, FC, FD, and BD?

23.16* Compute the floor area of the building in Problem 23.15.

23.17* The design floor elevation for a building to be constructed is 332.56 ft. An instrument is set up nearby, leveled, and a plus sight of 6.37 ft taken on BM A whose elevation is 330.05 ft. If batter boards are placed exactly 1.00 ft above floor elevation, what rod readings are necessary on the batter board tops to set them properly?

23.18 Compute the diagonals necessary to check the stakeout of the building in Figure 23.8.

23.19 Explain how the corner of a building can be plumbed using a total station?

23.20 Where is the invert of a pipe measured?

23.21 Discuss the importance of localizing a GNSS survey.

23.22 Explain why slope intercepts are placed an offset distance from the actual slope intercept.

23.23 What information is normally written on a slope stake?

23.24 Discuss the advantages of combining digital elevation models with design templates in staking out highway alignments with a data collector.

23.25 What are grade stakes?

23.26 What are spads and how are they used in mine surveys?

23.27 A highway centerline subgrade elevation is 985.20 ft at station 12 + 00 and 993.70 ft at 17 + 00 with a smooth grade in between. To set blue tops for this portion of the centerline, a level is setup in the area and a plus sight of 4.19 ft taken on a benchmark whose elevation is 992.05 ft. From that HI, what rod readings will be necessary to set the blue tops for the full stations from 12 + 00 through 17 + 00?

23.28 Similar to Problem 23.27, except the elevations at stations 12 + 00 and 17 + 00 are 985.20 and 993.70 ft, respectively, the BM elevation is 990.05 ft, and the backsight is 6.19 ft.

23.29 Discuss the checks that should be made when laying out a building using coordinates.

23.30 What are the jobs of a surveyor in a project using machine control?

23.31 Describe the procedure for localization of a GNSS survey.

23.32 Why is localization important in a GNSS survey?

23.33 How can finished grades be established in machine control projects?

23.34 What is the minimum number of control points needed to establish finished grades using a robotic total station on a machine-controlled project that is 8 mi in length?

23.35 What advantages does GNSS-supported machine control have over robotic total station methods? What are the disadvantages?

23.36 Discuss the advantages of using laser-scanning technology when planning for a new pipeline in a refinery.

23.37 Do an article review on an application of machine control.

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24 Horizontal Curves

■ 24.1 INTRODUCTION

Straight (tangent) sections of most types of transportation routes, such as highways, railroads, and pipelines, are connected by curves in both the horizontal and vertical planes. An exception is a transmission line, in which a series of straight lines is used with abrupt angular changes at tower locations if needed.

Curves used in horizontal planes to connect two straight tangent sections are called *horizontal curves*. Two types are used: *circular arcs* and *spirals*. Both are readily laid out in the field with surveying equipment. A *simple curve* [Figure 24.1(a)] is a circular arc connecting two tangents. It is the type most often used. A *compound curve* [Figure 24.1(b)] is composed of two or more circular arcs of different radii tangent to each other, with their centers on the same side of the alignment. The combination of a short length of tangent (less than 100 ft) connecting two circular arcs that have centers on the same side [Figure 24.1(c)] is called a *broken-back* curve. A *reverse curve* [Figure 24.1(d)] consists of two circular arcs tangent to each other, with their centers on opposite sides of the alignment. Compound, broken-back, and reverse curves are unsuitable for modern high-speed highway, rapid transit, and railroad traffic and should be avoided if possible. However, they are sometimes necessary in mountainous terrain to avoid excessive grades or very deep cuts and fills. Compound curves are often used on exit and entrance ramps of interstate highways and expressways, although easement curves are generally a better choice for these situations.

Easement curves are desirable, especially for railroads and rapid transit systems, to lessen the sudden change in curvature at the junction of a tangent and a circular curve. A *spiral* makes an excellent easement curve because its radius decreases uniformly from infinity at the tangent to that of the curve it meets. Spirals are used to connect a tangent with a circular curve, a tangent with a tangent

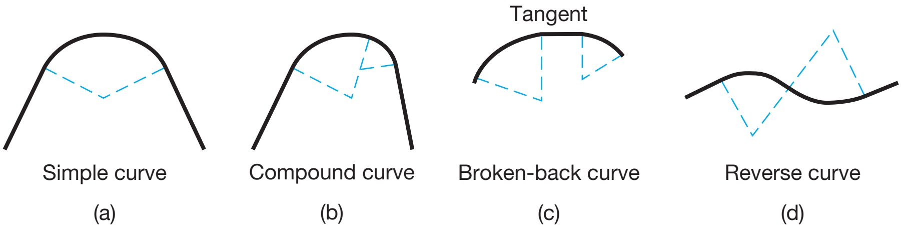

Figure 24.1
Circular curves.

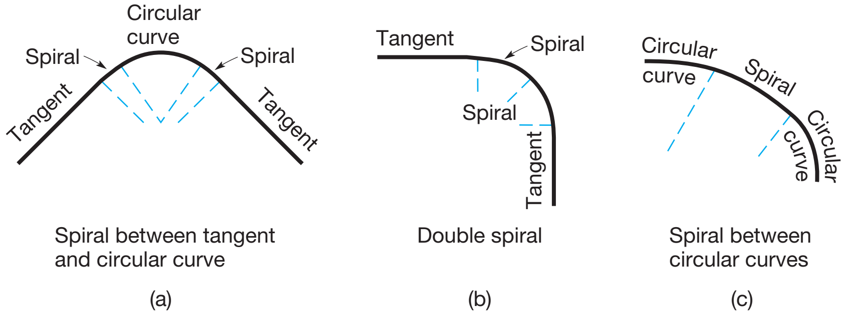

Figure 24.2
Use of spiral transition curves.

(double spiral), and a circular curve with a circular curve. Figure 24.2 illustrates these arrangements.

The effect of centrifugal force on a vehicle passing around a curve can be balanced by *superelevation*, which raises the outer rail of a track or outer edge of a highway pavement. Correct transition into superelevation on a spiral increases uniformly with the distance from the beginning of the spiral and is in inverse proportion to the radius at any point. Properly superelevated spirals ensure smooth and safe riding with less wear on equipment. As noted, spirals are used for railroads and rapid-transit systems. This is because trains are constrained to follow the tracks, and thus a smooth, safe, and comfortable ride can only be assured with properly constructed alignments that include easement curves. On highways, spirals are less frequently used because drivers are able to overcome abrupt directional changes at circular curves by steering a spiraled path as they enter and exit the curves.

Although this chapter concentrates on circular curves, methods of computing and laying out spirals are introduced in Section 24.18.

■ 24.2 DEGREE OF CIRCULAR CURVE

The rate of curvature of circular curves can be designated either by their *radius* (e.g., a 1500-m curve or a 1000-ft curve) or by their *degree of curve*. There are two different designations for degree of curve, the *arc definition* and the *chord definition*, both of which are defined using the English system of units. By the arc definition, degree of curve is the central angle subtended by a circular *arc* of 100 ft [see Figure 24.3(a)]. This definition is preferred for highway work. By the chord definition, degree of curve is the angle at the center of a circular arc subtended by a chord of 100 ft [see Figure 24.3(b)]. This definition is convenient for very gentle curves and hence is preferred for railroads. The formulas relating radius R and degree D of curves for both definitions are shown next to the illustrations.

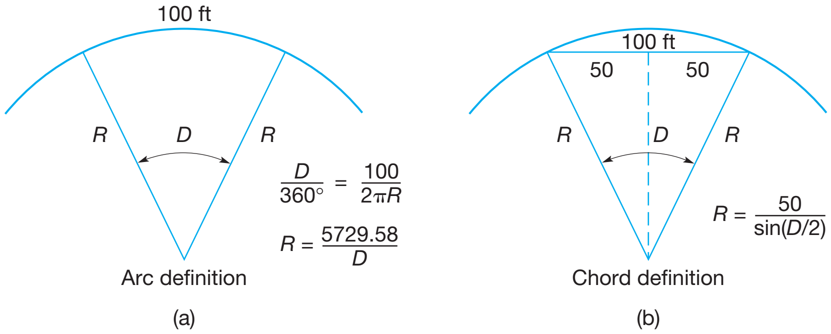

Figure 24.3 Degree of circular curve.

Using the equations given in Figure 24.3, radii of arc and chord definition curves for values of D from 1° to 10° have been computed and are given in columns (2) and (5) of Table 24.1. Although radius differences between the two definitions appear to be small in this range, they are significant.

When metric units are used, degree of curve can still be specified. As an example, a curve having a radius of exactly 700 m would have a degree of curve (arc definition) of

$$\frac{5729.58}{700(3.28083)} = 2°29'41''$$

Arc-definition curves have the advantage that computations are somewhat simplified as compared to the chord definition and, as will be shown later, the formula for curve length is exact, which simplifies preparing right-of-way descriptions. A disadvantage with the arc definition is that most measurements between full stations are shorter than a full 100-ft tape length, but this is of little significance

TABLE 24.1 FUNCTIONS OF CIRCULAR CURVES (LENGTHS IN FEET)

	Arc Definition		Chord Definition			
Degree of Curve D (1)	**Radius R (2)**	**True Chord Full Sta (3)**	**True Chord Half-Sta (4)**	**Radius R (5)**	**Arc Length Full Sta (6)**	**True Chord Half-Sta (7)**
1	5729.58	100.00	50.00	5729.65	100.00	50.00
2	2864.79	99.99	50.00	2864.93	100.01	50.00
3	1909.86	99.99	50.00	1910.08	100.01	50.00
4	1432.39	99.98	50.00	1432.69	100.02	50.01
5	1145.92	99.97	50.00	1146.28	100.03	50.01
6	954.93	99.95	50.00	955.37	100.05	50.02
7	818.51	99.94	50.00	819.02	100.06	50.02
8	716.20	99.92	49.99	716.78	100.08	50.03
9	636.62	99.90	49.99	637.27	100.10	50.04
10	572.96	99.88	49.98	573.69	100.13	50.05

if a total station instrument is used for stakeout. With the chord definition, full stations are separated by chords of exactly 100 ft regardless of the value of D.

For a given value of D, arc and chord definitions give practically the same result when applied to the flat curves common on modern highways, railroads, and rapid-transit systems. However, as degree of curve increases, the differences become greater.

■ 24.3 DEFINITIONS AND DERIVATION OF CIRCULAR CURVE FORMULAS

Circular curve elements are shown in Figure 24.4. The *point of intersection* PI, of the two tangents, is also called the *vertex V.* In stationing, the *back tangent* precedes the PI, the *forward tangent* follows it. The beginning of the curve, or *point of curvature* PC, and the end of the curve, or *point of tangency* PT, are also sometimes called BC and EC, respectively. Other expressions for these points are tangent to curve, TC, and curve to tangent, CT. The curve radius is R. Note that the radii at the PC and PT are perpendicular to the back tangent and forward tangent, respectively.

The distance from PC to PI and from PI to PT is called the *tangent distance*, T. The line connecting the PC and PT is the *long chord* LC. The *length of the curve, L*, is the distance from PC to PT, *measured along the curve for the arc definition, or by 100-ft chords for the chord definition.*

The external distance E is the length from the PI to the curve midpoint on a radial line. The middle ordinate M is the (radial) distance from the midpoint of the long chord to the curve's midpoint. Any *point on curve* is POC; any *point on tangent,* POT. The degree of any curve is D_a (arc definition) or D_c (chord definition). The change in direction of two tangents is the intersection angle I, which is also equal to the central angle subtended by the curve.

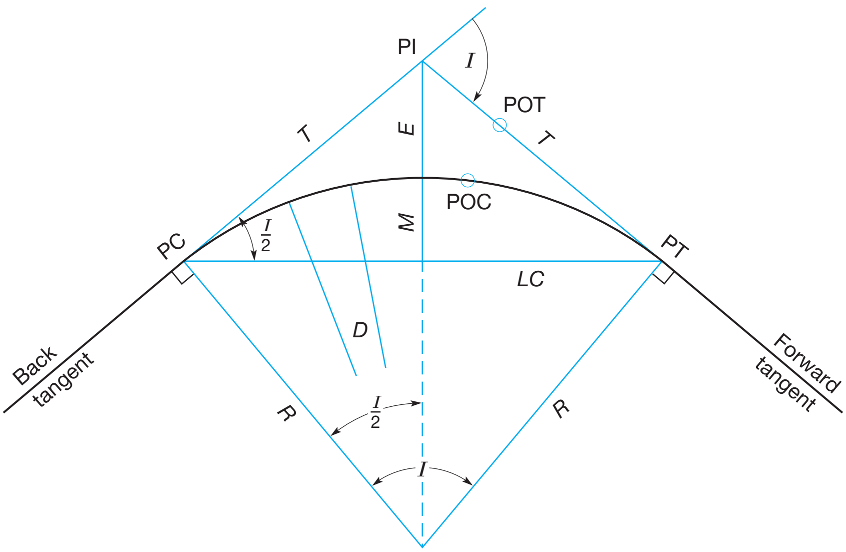

Figure 24.4
Circular curve elements.

By definition, and from inspection of Figure 24.4, relations for the arc definition follow:

$$L = RI\,(I \text{ in radians}) \tag{24.1}$$

$$L = 100\frac{I^\circ}{D^\circ}(\text{ft}) \tag{24.2a}$$

$$L = \frac{I^\circ}{D^\circ}(\text{sta}) \tag{24.2b}$$

$$R = \frac{5729.58}{D}(\text{ft}) \tag{24.3}$$

$$T = R\tan\frac{I}{2} \tag{24.4}$$

$$LC = 2R\sin\frac{I}{2} \tag{24.5}$$

$$\frac{R}{R + E} = \cos\frac{I}{2} \quad \text{and} \quad E = R\left[\frac{1}{\cos(I/2)} - 1\right] \tag{24.6}$$

$$\frac{R - M}{R} = \cos\frac{I}{2} \quad \text{and} \quad M = R\left(1 - \cos\frac{I}{2}\right) \tag{24.7}$$

Other convenient formulas that can be derived are

$$E = T\tan\frac{I}{4} \tag{24.8}$$

$$M = E\cos\frac{I}{2} \tag{24.9}$$

Although curves are normally calculated by computers, if a handheld calculator is used, it is expedient to compute R, T, E, and M in the sequence of Equations (24.3), (24.4), (24.8), and (24.9), because the previously computed value is in the calculator and available for each succeeding calculation.

The formulas for T, L, LC, E, and M also apply to a chord-definition curve. However, L calculated by Equation (24.2a) implies the total length as if observed along the 100-ft chords of an inscribed polygon. The following formula is used for relating R and D for a chord definition curve:

$$R = \frac{50}{\sin(D/2)}(\text{ft}) \tag{24.10}$$

Note that for the equations given above, Equations (24.2a), (24.2b), (24.3), and (24.10) involve degree of curve and thus assume distances in feet, while either metric or English units can be used in all others.

24.4 CIRCULAR CURVE STATIONING

Normally, an initial route survey consists of establishing the PIs according to plan, laying out the tangents, and establishing continuous *stationing* along them from the start of the project, through each PI, to the end of the job. (Stationing was described in Section 5.9.1.) The beginning point of any project is assigned a station value and all other points along the reference line are then related to it. If the beginning point is also the end point of a previous adjacent project, its station value may be retained and the new job referenced to that stationing. Otherwise, an arbitrary value such as 100 + 00 for English unit stationing, or 10 + 000 for metric stationing is assigned. Assigning a starting stationing of 0 + 00 is generally not done to avoid the possibility that future revisions to the project could extend it back beyond the starting point and hence result in negative stationing. In the English system, staking is usually in *full stations* (100 ft apart), although *half-stations* (50 ft apart) or even *quarter stations* (25 ft apart), can be set depending on conditions. In metric stationing, full stations are generally 1 km apart, but stakes may be set at 40, 30, 20, or even 10 m apart, depending on conditions. Staking at the closer spacing is usually done in urban situations, on sharp curves, or in rugged terrain, while the stakes are placed farther apart in relatively flat or gently rolling rural areas.

After the tangents have been staked and stationed, the intersection angle (I) is observed at each PI and curves computed and staked. The station locations of points on any curve are based upon the stationing of the curve's PI. To compute the PC station, tangent distance T is subtracted from the PI station, and to calculate the PT station, curve length L is added to the PC station.

Example 24.1

Assume that $I = 8°24'$, the station of the PI is 64 + 27.46, and terrain conditions require the minimum radius permitted by the specifications of, say, 2864.79 ft (arc definition). Calculate the PC and PT stationing and the external and middle ordinate distances for this curve.

Solution

By Equation (24.1) $$L = 2864.79(8°24')\frac{\pi}{180} = 420.00 \text{ ft}$$

By Equation (24.2a) $$D° = 100\frac{8°24'}{420} = 2°00'$$

By Equation (24.4) $$T = 2864.79 \tan\left(\frac{8°24'}{2}\right) = 210.38 \text{ ft}$$

Calculate stationing

$$\begin{aligned} \text{PI station} &= 64 + 27.46 \\ -T &= \underline{-2 + 10.38} \\ \text{PC station} &= 62 + 17.08 \\ +L &= \underline{4 + 20.00} \\ \text{PT station} &= 66 + 37.08 \end{aligned}$$

Also by Equation (24.5) $LC = 2 \times 2864.79 \sin\left(\frac{8°24'}{2}\right) = 419.62 \text{ ft}$

And by Equation (24.8) $E = 210.38 \tan\left(\frac{8°24'}{4}\right) = 7.71 \text{ ft}$

Finally by Equation (24.9) $M = 7.71 \cos\left(\frac{8°24'}{2}\right) = 7.69 \text{ ft}$

Calculation for the stations of the PC and PT should be arranged as shown. Note that the stationing of the PT cannot be obtained by adding the tangent distance to the station of the PI, although the location of the PT on the ground is determined by measuring the tangent distance from the PI. Points representing the PC and PT must be carefully marked and placed exactly on the tangent lines at the correct distance from the PI so other computed values will fit their fixed positions on the ground.

If route surveys are originally staked as a series of tangents having continuous stationing, as described above, then an adjustment has to be made at each PT after curves are inserted. This is necessary because the length around the curve from PC to PT is shorter than the distance along the tangents from the PC to the PI to the PT. Thus for final stationing at the PT, there is a "station equation," which relates the stationing back along the curve to that forward along the tangent. For Example 24.1, it would be $66 + 37.08$ back $= 66 + 37.84$ ahead, where $66 + 37.08 = \text{PI} - T + L$ and $66 + 37.84 = \text{PI} + T$. The difference between the ahead and back stations represents the amount the route was shortened by inserting the curve. If the curves are run in and stationed at the time of staking the original alignment, continuous stationing along the route results and station equations at PTs are avoided.

The curve used in a particular situation is selected to fit ground conditions and specification limitations of minimum R or maximum D. Normally the value of the intersection angle, I, and the station of the PI are available from field observations on the preliminary line. Then a value of R or D, suitable for the highway or railroad, is chosen. Most of today's highways are designed using a minimum value for the radius R. Sometimes the distance E or M required to miss a stream or steep slope outside or inside the PI is observed, and D or R computed holding that distance fixed. Tangent distance governs infrequently. (One exception is to make a railroad, bus, or subway station fall on a tangent rather than a superelevated curve.) The length of curve practically never governs.

■ 24.5 GENERAL PROCEDURE OF CIRCULAR CURVE LAYOUT BY DEFLECTION ANGLES

Except for unusual cases, the radii of curves on route surveys are too large to permit swinging an arc from the curve center. Circular curves are therefore laid out by more practical methods, including (1) deflection angles, (2) coordinates, (3) tangent offsets, (4) chord offsets, (5) middle ordinates, and (6) ordinates from the long chord. Layout by deflection angles has been the standard approach, although with the advent of total station instruments, the coordinate method is becoming increasingly popular.

Layout of a curve by deflection angles can be done by either the *incremental chord method* or the *total chord method.* In the past, the incremental chord method was almost always used as it could be readily accomplished with a theodolite and tape. The method can still be used when a total station instrument is employed, although then the distances are observed by taping rather than electronically. (Taping is still efficient in staking the stations along alignments because of the relatively short distance increments involved.) The total chord method was not practical until the advent of total stations, but with these instruments it is now conveniently employed even though longer distance measurements are involved.

The incremental chord method is illustrated in Figure 24.5. Assume that the instrument is set up over the PC (station 62 + 17.08 in Example 24.1). For this illustration, assume that each full station is to be marked along the curve, since cross sections are normally taken, construction stakes set, and computations of earthwork made at these points. (Half-stations or any other critical points can also be established, of course.) The first station to be set in this example is 63 + 00. To mark that point from the PC, a backsight is taken on the PI with zero set on the instrument's horizontal circle. Deflection angle δ_a to station 63 + 00 is then turned and two tapepersons measure chord c_a from the PC and set 63 + 00 at the end of the chord on the instrument's line of sight. With station 63 + 00 set, the tapepersons next measure the chord length c from it and stake station 64 + 00, where the line of sight of the instrument, now set to δ_{64}, intersects the end of that chord. This process is repeated until the entire curve is laid out. In this procedure it is seen that the accuracy in the placement of each succeeding station depends on the accuracies of all those stations previously set.

The total chord method can also be described with reference to Figure 24.5. In this procedure, a total station instrument is set up at the PC, a backsight taken on the PI, and zero indexed on the horizontal circle. To set station 63 + 00,

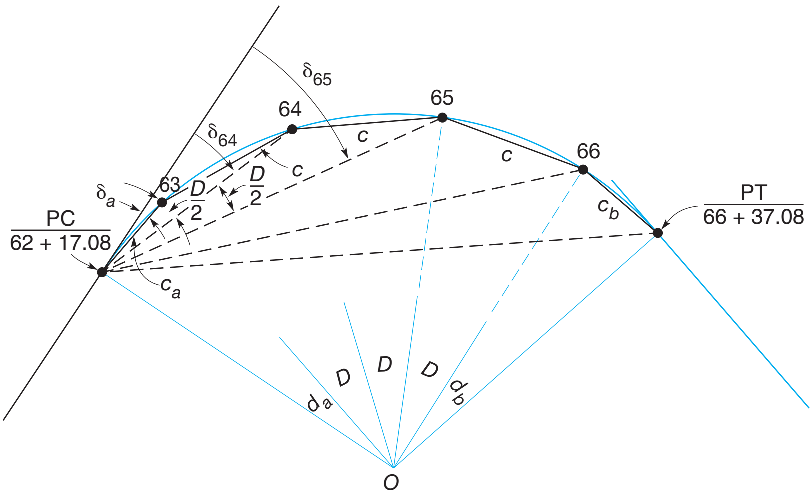

Figure 24.5 Circular curve layout by deflection angles.

deflection angle δ_a is turned with the instrument, the reflector placed on line and adjusted until its distance from the instrument is c_a, and the stake set. To set station 64 + 00, deflection angle δ_{64} is turned, the reflector placed on this line of sight and adjusted in position until the total chord from the PC to station 64 + 00 is obtained and the stake set.

This procedure is repeated, with each station being set independently of the others, until the entire curve is staked. This method of staking a curve has some drawbacks. One is that in some areas vegetation or other obstructions can block sight lines along the chords. Another is that each station is set independently and thus there is no check at the end of the curve. For these reasons, the incremental chord approach is often preferred over the total chord method.

■ 24.6 COMPUTING DEFLECTION ANGLES AND CHORDS

From the preceding discussion it is clear that deflection angles and chords are important values that must be calculated if a curve is to be run by the deflection-angle method. To stake the first station, which is normally an odd distance from the PC (shorter than a full-station increment), *subdeflection angle* δ_a and *subchord* c_a are needed. These are shown in Figure 24.6. In this figure, central angle d_a subtended by arc s_a from the PC to 63 + 00 is calculated by proportion according to the definition of D as

$$\frac{d_a}{s_a} = \frac{D}{100} \quad \text{from which} \quad d_a = \frac{s_a D}{100}\ \text{(degrees)} \tag{24.11a}$$

where s_a is the difference in stationing between the two points. Equation (24.11a) is based on curves defined in English units using the arc definition for degree of curvature. For curves computed in English or metric units, the equivalent expression is

$$\frac{d_a}{s_a} = \frac{I}{L} \quad \text{from which} \quad d_a = \frac{s_a I}{L}\ \text{(degrees)} \tag{24.11b}$$

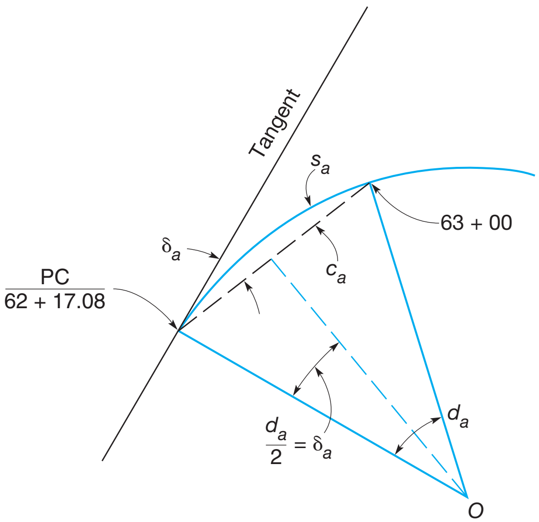

Figure 24.6 Subchords and subdeflections.

A fundamental theorem of geometry helpful in circular curve computation and stakeout is that *the angle at a point between a tangent and any chord is equal to half the central angle subtended by the chord.* Thus, subdeflection angle δ_a needed to stake station 63 + 00 is $d_a/2$, or

$$\delta_a = \frac{s_a D}{200} \text{ (degrees)} \tag{24.12a}$$

Also recognizing that I and L are constants for any particular curve, Equation (24.11b) can be rewritten as

$$\delta_a = s_a k \text{ (degrees)} \tag{24.12b}$$

where k is $I/(2L)$.

The length of the subchord c_a can be represented in terms of δ_a and the curve radius as

$$\sin \delta_a = \frac{c_a}{2R} \quad \text{from which} \quad c_a = 2R \sin \delta_a \tag{24.13}$$

Since the arc between full stations subtends the central angle D, from the earlier stated geometric theorem, deflection angles to each full station beyond 63 + 00 are found by adding $D/2$ to the previous deflection angle. Full chord c, which corresponds to 100 ft of curve length, is calculated using Equation (24.13), except that $D/2$ is substituted for δ_a. Equations (24.12) and (24.13) are also used to compute the last subdeflection angle δ_b and subchord c_b, but the difference in stationing s_b between the last full station and the PT replaces arc length s_a.

Equations (24.12) and (24.13) are also used for computing deflection angles and total chords for the total chord method of staking. Here s is simply the difference in stationing between the station being set and the PC.

For curves up to about 2°00′ (arc definition), the lengths of arcs and their corresponding chords are nearly the same. On sharper curves, chords are shorter than corresponding arc lengths. This is verified by the data in columns (3) and (4) of Table 24.1, which give true chord lengths for full- and half-station increments for varying values of D (arc definition).

Computations for deflection angles and chords on chord-definition curves use the same formulas, but R is calculated by Equation (24.10). Note that for a given degree of curve, R is longer for chord definition than for arc definition. Also arc lengths for full stations are longer than their nominal 100.00 ft value, and true subchords are longer than their *nominal values* (differences in stationing). A check of columns (6) and (7) in Table 24.1 verifies these facts.

Example 24.2

Compute subdeflection angles and subchords δ_a, c_a, δ_b, and c_b, and calculate chord c of Example 24.1.

Solution

By Equation (24.12a)

$$\delta_a = 82.92(0.0100^\circ) = 0.8292^\circ = 0^\circ 49' 45''$$

$$\delta_b = 37.08(0.0100^\circ) = 0.3708^\circ = 0^\circ 22' 15''$$

$$(\text{Note that } D/200 = 2^\circ/200 = 0.01^\circ)$$

By Equation (24.13)

$$c_a = 2(2864.79) \sin 0^\circ 49' 45'' = 82.92 \text{ ft}$$

$$c_b = 2(2864.79) \sin 0^\circ 22' 15'' = 37.08 \text{ ft}$$

$$c = 2(2864.79) \sin 1^\circ 00' 00'' = 99.99 \text{ ft}$$

24.7 NOTES FOR CIRCULAR CURVE LAYOUT BY DEFLECTION ANGLES AND INCREMENTAL CHORDS

Based on principles discussed, the deflection angle and incremental chord data for stakeout of the complete curve of Examples 24.1 and 24.2 have been computed and listed in Table 24.2. Normally, as has been done in this case, the data are prepared for stakeout from the PC, although field conditions may not allow the curve to be completely run from there. This problem is discussed in Section 24.9.

Values of deflection angles are normally carried out to several decimal places for checking purposes, and to avoid accumulating small errors when D is a noninteger number, such as, perhaps, $3^\circ 17' 24''$. Note in Table 24.2 that the deflection angle to the PT is $4^\circ 12'$, exactly half the I angle of $8^\circ 24'$. This comparison affords an important check on the calculations of all deflection angles.

Field notes for the curve of this example are recorded in Figure 24.7 as they would appear in a field book. Notes run up the page to simplify sketching while looking in a forward direction. Computers can conveniently perform all necessary calculations and list the notes for curve stakeout by deflection angles.

TABLE 24.2 DEFLECTION ANGLE AND INCREMENTAL CHORD DATA FOR EXAMPLE CURVE

Station	Incremental Chord	Deflection Increment	Deflection Angle
66 + 37.08 (PT)	37.08	0°22′15″	4°12′00″✓
66 + 00	99.99	1°00′00″	3°49′45″
65 + 00	99.99	1°00′00″	2°49′45″
64 + 00	99.99	1°00′00″	1°49′45″
63 + 00	82.92	0°49′45″	0°49′45″
62 + 17.08 (PC)			

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Station	Chord	Total Def.	Calc. Bearing	Mag. Bearing	Curve Data
P.O.T. 68	100.00				
67	62.92				
			N24°42′E	N24°45′E	
P.T. 66+37.08	37.08	4°12′00″			
					I=8°24′
66	99.99	3°49′~~48~~45″			R=2864.79′
					D=2°00′
P.O.C. 65	99.99	2°49′45″			L=420.00′
					T=210.38′
64	99.99	1°49′45″			E=7.71′
					M=7.69′
63	82.92	0°49′45″			LC =419.62′
P.C. 62+17.08	17.08	0°00′00″			
			N16°18′E	N16°30′E	
62	100.00				
61	100.00				
60	100.00				
P.O.T. 59					

Lafayette College Campus
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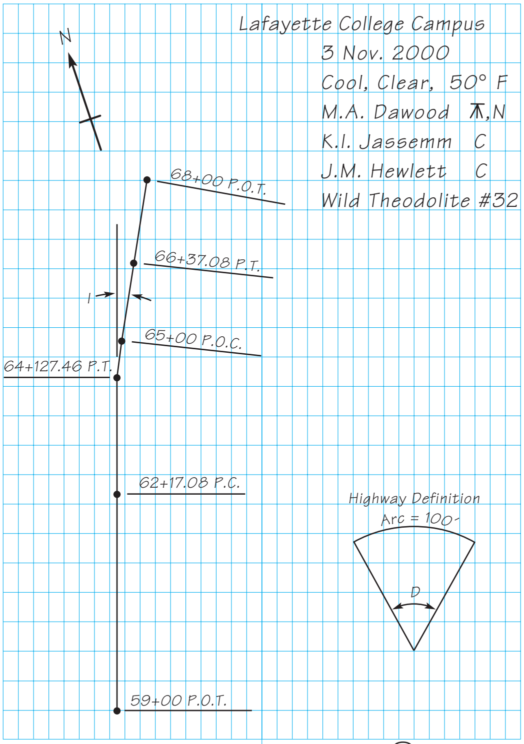
Cool, Clear, 50° F
M.A. Dawood ⊼,N
K.I. Jassemm C
J.M. Hewlett C
Wild Theodolite #32



M. A. Dawood

Figure 24.7 Field notes for horizontal curve in Examples 24.1 and 24.2.

In many cases, it is desirable to "back in" a curve by setting up over the PT instead of the PC. One setup is thereby eliminated and the long sights are taken on the first measurements. In precise work it is better to run in the curve from both ends to the center, where small errors can be adjusted more readily. On long or very sharp circular curves, or if obstacles block sights from the PC or PT, setups on the curve are necessary (see Section 24.9).

■ 24.8 DETAILED PROCEDURES FOR CIRCULAR CURVE LAYOUT BY DEFLECTION ANGLES AND INCREMENTAL CHORDS

Regardless of the method used to stake intermediate curve points, the first steps in curve layout are: (1) establishing the PC and PT, normally by measuring tangent distance T from the PI along both the back and forward tangents and (2) measuring the total deflection angle at the PC from PI to PT. This latter step should be performed whenever possible, since the observed angle must check $I/2$; if it does not, an error exists in either observation or computation, and time should not be wasted running an impossible curve.

It is also good practice to stake the midpoint of a curve before beginning to set intermediate points, especially on long curves. The midpoint can be set by bisecting the angle $180° - I$ at the PI and laying off the external distance from there. A check of the deflection angle from the PC to the midpoint should yield $I/4$. When staking intermediate points along the curve has reached the midpoint of the curve, a chord check measurement should be made to it.

The remaining steps in staking intermediate curve points by the deflection angle incremental chord method are presented with reference to the curve of Examples 24.1 and 24.2. With the instrument set up and leveled over the PC, it is oriented by backsighting on the PI, or on a point along the back tangent, with $0°00'$ on the circle. The subdeflection angle of $0°49'45''$ is then turned. Meanwhile, the 17-ft mark of the tape is held on the PC. The zero end of the (add) tape is swung until the line of sight hits a point 0.08 ft ahead from the zero mark. This is station 63 + 00. To stake station 64 + 00, the rear tapeperson next holds the 99-ft mark on station 63 and the forward tapeperson sets station 64 at distance 99.99 ft by direction from the instrument operator, who has placed an angle of $1°49'45''$ on the circle. An experienced forward tapeperson will walk along the extended first full chord, know or estimate the chord offset, and from an outside-the-chord position will be holding the tape end and a stake within a foot or so of the correct location when the instrument operator has the deflection angle ready.

After placing the final full station (66 + 00 in this example), to determine any misclosure in staking a curve, the closing PT should be staked using the final deflection angle and subchord. This will rarely agree with the PT established by measuring distance T along the forward tangent from the PI because of accumulated errors. Any misclosure ("falling") should be observed; then the field precision can be expressed as a numerical ratio like that used in traverse checks. The observed falling distance is the numerator, and $L + 2T$ the denominator. If the misclosure of this example was 0.25 ft, the precision would be $0.25/[420.00 + 2(210.38)] = 1/3300$.

■ 24.9 SETUPS ON CURVE

Obstacles that prevent visibility along curve chords and extremely long sight distances sometimes make it necessary to set up on the curve after it has been partially staked. The simplest procedure to follow is one that permits use of the same notes computed for running the curve from the PC. In this method the following rule applies: *The instrument is moved forward to the last staked point, backsighted on a station with the telescope inverted and the circle set to the deflection angle from the PC for the station sighted. The telescope is then plunged to the normal position, and deflection angles previously computed from the PC for the various stations are used.*

The example of the preceding sections is used to illustrate this rule. If a setup is required at station 65, place $0°00'$ on the instrument and sight the PC with the telescope inverted. Plunge, set the circle to read the deflection angle $3°49'45''$ and stake station 66.

To prove this geometric rule, assume the instrument were set on station 65 + 00 and the PC backsighted with $0°00'00''$ on the circle. If the telescope were plunged and turned 2°49′45″ in azimuth, the line of sight would then be tangent to the curve. To stake the next station (66 + 00), an additional angle of $D/2$ or 1°00′00″ would need to be turned. The sum of 2°49′45″ and 1°00′00″ is of course the deflection angle to station 66 + 00 from the PC. The rule also applies for backsights to any previously set stations, not just the PC. Thus, with the instrument at 65 + 00, station 63 + 00 could be sighted with 0°49′45″ on the circle and then turned to 3°49′45″ to set station 66 + 00. Further study of the geometry illustrated in Figure 24.5 should clarify this procedure.

■ 24.10 METRIC CIRCULAR CURVES BY DEFLECTION ANGLES AND INCREMENTAL CHORDS

Most foreign countries and many highway departments in the United States use metric units for observations and stationing on their projects. As noted earlier, in the metric system, circular curves are designated by their radius values rather than degree of curve. Otherwise, as illustrated by the following example, computations for laying out a curve using metric units by the deflection angle incremental chord method follow the same procedures as for the English system of units and stationing.

■ Example 24.3

Assume that a metric curve will be used at a PI where $I = 8°24'$. Assume also that the station of the PI is 6 + 427.464, and that terrain conditions require a minimum radius of 900 m. Calculate the PC and PT stationing, and other defining elements of the curve. Also compute notes for staking the curve using 20-m increments.

Solution

By Equation (24.1) $$L = 900\left(8°24'\frac{\pi}{180°}\right) = 131.947 \text{ m}$$

By Equation (24.4) $$T = 900 \tan\left(\frac{8°24'}{2}\right) = 66.092 \text{ m}$$

Calculate stationing

$$\begin{aligned} \text{PI station} &= 6 + 427.464 \\ -T &= \underline{066.092} \\ \text{PC station} &= 6 + 361.372 \\ +L &= \underline{131.947} \\ \text{PT station} &= 6 + 493.319 \end{aligned}$$

Also by Equation (24.5) $$LC = 2(900) \sin\left(\frac{8°24'}{2}\right) = 131.829 \text{ m}$$

TABLE 24.3 DEFLECTION ANGLE AND INCREMENTAL CHORD DATA FOR EXAMPLE CURVE

Station	Incremental Chord	Deflection Increment	Deflection Angle
6 + 493.319 (PT)	18.628	0°25′26.2″	4°12′00″✓
6 + 480	19.999	0°38′11.8″	3°46′34″
6 + 460	19.999	0°38′11.8″	3°08′22″
6 + 440	19.999	0°38′11.8″	2°30′10″
6 + 420	19.999	0°38′11.8″	1°51′58″
6 + 400	19.999	0°38′11.8″	1°13′46″
6 + 380	13.318	0°35′34.6″	0°35′35″
6 + 361.372 (PC)			

And by Equation (24.8) $E = 66.092 \tan\left(\frac{8°24'}{4}\right) = 2.423 \text{ m}$

Finally by Equation (24.9) $M = 2.423 \cos\left(\frac{8°24'}{2}\right) = 2.416 \text{ m}$

The arc distance from the PC to the station 6 + 380 is (6380 − 6361.372) = 18.628 m. The arc distance for the final stationing is 6493.319 − 6480 = 13.319 m. All other stations have 20-m stationing intervals. Table 24.3 and Figure 24.8 depict the curve data and field notes necessary to stake the curve in this example.

ALIGNMENT OF LAFAYETTE HIGHWAY

Station	Chord	Total Def.		Curve Data
(PT)		° ′ ″		
6+493.319		4 12 00		I = 8°24′
	13.318			R = 900 m
6+480		3 46 34		k = 0.03183096
	19.999			L = 131.947 m
6+460		3 08 22		T = 66.092 m
	19.999			LC = 131.829 m
6+440		2 30 10		E = 2.423 m
	19.999			M = 2.416 m
6+420		1 51 58		
	19.999			
6+400		1 13 46		
	19.999			
6+380		0 35 35		
	18.628			
6+361.372				
(PC)				

Figure 24.8 Field notes for horizontal curve in Example 24.3.

By Equation (24.12b)

$$
\begin{aligned}
k &= 8°24'/[2(131.947)] = 0.03183096° \\
\delta_a &= 0.03183096(18.628) = 0.59295° = 0°35'34.6'' \\
\delta &= 0.03183096(20) = 0.63662° = 0°38'11.8'' \\
\delta_b &= 0.03183096(13.319) = 0.42396° = 0°25'26.2''
\end{aligned}
$$

By Equation (24.13)

$$
\begin{aligned}
c_a &= 2(900)\sin 0°35'34.6'' = 18.628 \text{ m} \\
c &= 2(900)\sin 0°38'11.8'' = 19.999 \text{ m} \\
c_b &= 2(900)\sin 0°25'26.2'' = 13.318 \text{ m}
\end{aligned}
$$

24.11 CIRCULAR CURVE LAYOUT BY DEFLECTION ANGLES AND TOTAL CHORDS

If field conditions permit and a total station instrument is available, curves may be conveniently laid out by deflection angles and total chords. By using this method, the field party size is reduced from three to two, or possibly even a single person if a robotic total station instrument is available. Deflection angles are calculated and laid off as in the preceding example, but the chords are all measured electronically as radial distances (total chords) from the PC or other station where the instrument is placed. If stakeout is planned from the PC, total chords from there are the dashed lines of Figure 24.5. They are calculated by Equation (24.13), except that the deflection angle for each station is substituted for δ_α to obtain the corresponding chord. The total chords necessary to stake the curve of Example 24.2 using a total station instrument set up at PC are 82.92 ft for 63 + 00, 182.89 ft for 64 + 00, 282.80 ft for 65 + 00, 382.63 ft for 66 + 00, and 419.62 ft for the PT, which is the long chord LC given by Equation (24.5). The same deflection angles given in Table 24.2 apply.

To stake curves using a total station, the instrument is placed in its tracking mode. The deflection angle to each station is turned and the required chord to that station entered in the instrument. The instrument operator directs the person with the reflector to the proper alignment. The reflector is then moved forward or back as necessary, until the proper total chord distance is achieved, where the stake is set. It is often convenient to carry a short tape when staking out stations to quickly move to the final position from a nearby trial position. If intermediate setups are required on the curve using this method, the instrument is oriented as described in Section 24.9. New radial chords to be measured from the intermediate station would then have to be calculated.

Although curves can be staked rapidly with total stations using this method, as noted earlier an associated danger is that each stake is set individually, and therefore does not depend on previous stations. Thus, a check at the end of the curve is not achieved as in the incremental chord method and mistakes in angles or distances at intermediate stations could go undetected. Large blunders can usually be discovered by visual inspection of the curve stakes, but quickly taping the incremental chords between adjacent stations gives a better check.

■ 24.12 COMPUTATION OF COORDINATES ON A CIRCULAR CURVE

Today, because of the availability of total station instruments with data collectors, circular curves are often staked using the coordinate method. For this procedure, coordinates of the points on the curve to be staked must first be determined in some reference coordinate system. Although they are most often based upon an established map projection such as the State Plane Coordinate System or the Universal Transverse Mercator projection (see Chapter 20), often an arbitrary project coordinate system will suffice. This section describes the process of determining coordinates for stations on circular curves.

In Figure 24.9, assume that the azimuth of the back-tangent going from A to V is known, the coordinates of the PI (point V) are known, and that the defining parts of the curve have been computed using Equations (24.1) through (24.10). Using the tangent distance and azimuth of the back tangent, the departure and latitude are computed by Equations (10.1) and (10.2), where Az_{VA} is the back azimuth of line AV. The coordinates of A (the PC) are then

$$X_A = X_V + T \sin Az_{VA}$$
$$Y_A = Y_V + T \cos Az_{VA} \qquad \textbf{(24.14)}$$

With the coordinates of the PC known, coordinates of points on the curve can be computed using the same deflection angles and subchords used to stake out the curve by the total chord method. Deflection angles are added to the azimuth of AV to get azimuths of the chords to all stations to be set. Using the total chord length and chord azimuth for each station, departures and latitudes are

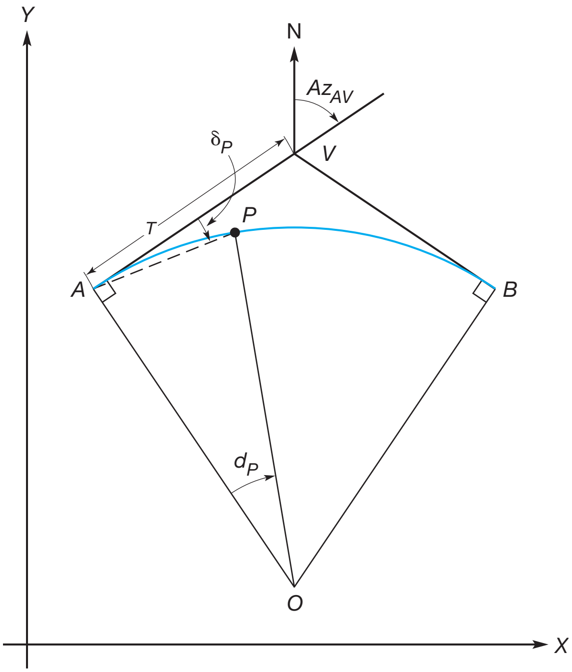


Figure 24.9 Geometry for computing coordinates of curve points.

calculated, and added to the coordinates of A (the PC) to get the station coordinates. With coordinates known for all curve points, they can be staked with the total station occupying any convenient point whose coordinates are also known in the same system. The PC, PT, PI, or curve midpoint are points that are often used.

It is sometimes convenient to stake a circular curve by placing the instrument at the center of the curve, that is, point O of Figure 24.9. In this case, the coordinates of the curve's center point are computed, and then the coordinates of the stations to be set can be conveniently computed using radial lines from that point. From Figure 24.9, the azimuth of the radius going from A to the center of the curve is

$$Az_{AO} = Az_{AV} + 90° \qquad \textbf{(24.15a)}$$

Equation (24.15a) is valid for a curve that lies right of the back tangent. For a curve that bends to the left, the proper expression is

$$Az_{AO} = Az_{AV} - 90° \qquad \textbf{(24.15b)}$$

Using the appropriate azimuth from Equations (24.15) and the radius of the curve, R, the coordinates of center point O of Figure 24.9 are

$$\begin{aligned} X_O &= X_A + R \sin Az_{AO} \\ Y_O &= Y_A + R \cos Az_{AO} \end{aligned} \qquad \textbf{(24.16)}$$

The azimuth of the radius line from O to any station P on the curve is

$$Az_{OP} = Az_{OA} + d_P \qquad \textbf{(24.17)}$$

where d_P is determined in Equation (24.11). Then the coordinates of P are

$$\begin{aligned} X_P &= X_O + R \sin Az_{OP} \\ Y_P &= Y_O + R \cos Az_{OP} \end{aligned} \qquad \textbf{(24.18)}$$

To stake the curve points, the total station instrument is set on the curve's center point, a backsight taken on point A, and the azimuth of line OA indexed on the horizontal circle. To stake any point such as P of Figure 24.9, the azimuth of OP is placed on the instrument's circle and the stake placed on the line of sight at a distance R from the instrument. Staking a curve from the center point has the advantage of providing an easy method of computing and laying out a line, which is offset from the reference line. This is done by simply using either a longer or shorter radius in Equations (24.18) to compute the coordinates of the offset line and then using that same radius value in staking. A disadvantage in staking a curve from its center is that radius values of the curves typically used on transportation routes are rather long which means that the instrument operator and prism person will usually be relatively far apart. Also with these long-radius values, obstructions that block lines of sight will often exist.

Example 24.4 in the following section demonstrates the method of computing coordinates of curve points using deflection angles and total chords.

24.13 CIRCULAR CURVE LAYOUT BY COORDINATES

The coordinate method can be used to advantage in staking circular curves, especially if a total station instrument or GNSS receivers are employed. In this procedure, the coordinates of each curve station to be staked are calculated as described in the preceding section. The total station instrument can then be placed at the PC, PT, curve midpoint, curve center point, or any other nearby control station, which gives a good vantage point of the entire area where the curve will be laid out. Azimuths and distances to each station are computed by inversing, using the coordinates of the occupied station and those of each curve station. The instrument is oriented by backsighting another visible control station. Then each curve point is staked by laying out its computed distance along its calculated azimuth.

Figure 24.10 illustrates a situation where a curve is being staked by the coordinate method. The total station instrument is placed at control station *B* because all curve points are visible from there. After a backsight on control station *A*, distances and directions are used to stake all curve points. The computations necessary for staking this curve by the coordinate method are illustrated with the following example.

Example 24.4

Two tangents intersect at a PI station of 4 + 545.500 whose coordinates are $X = 5723.183$ m and $Y = 3728.947$ m. The intersection angle is 24°32′ left, and the azimuth of the back tangent is 326°40′20″. A curve with radius, R, of 400 m will be used to join the tangents. Compute the data needed to stake the curve at 20 m increments by coordinates using a total station instrument. For staking, the instrument will be set at station B, whose coordinates are $X = 5735.270$ m and

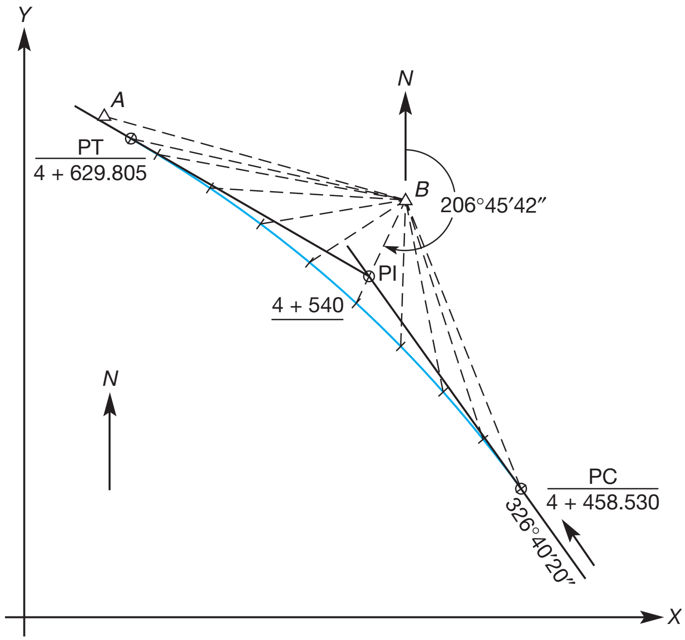


Figure 24.10 Layout of circular curve by coordinates with a total station instrument.

$Y = 3750.402$ m, and a backsight taken on station A, whose coordinates are $X = 5641.212$ m and $Y = 3778.748$ m.

Solution

By Equation (24.1) $L = 400 \times 24°32'\left(\frac{\pi}{180°}\right) = 171.275$ m

By Equation (24.4) $T = 400.000 \tan(12°16') = 86.970$ m

Curve stationing

$$\begin{aligned} \text{PI} &= 4 + 545.50 \\ -\text{T} &= \underline{86.970} \\ \text{PC} &= 4 + 458.530 \\ +\text{L} &= \underline{171.275} \\ \text{PT} &= 4 + 629.805 \end{aligned}$$

A tabular solution for curve point coordinates is given in Table 24.4. The differences in stationing from one curve point to the next are listed in column (2). Total deflection angles are calculated using Equation (24.12b) and tabulated in column (3). Total chords, computed from Equation (24.13) using these total deflection angles, are tabulated in column (4). From the azimuth of the back tangent and the deflection angles, the azimuth of each total chord is calculated and tabulated in column (5). The coordinates of the PC are computed using Equations (24.14) as

$$X_{PC} = 5723.183 + 86.970 \sin(326°40'20'' - 180°) = 5770.967 \text{ m}$$
$$Y_{PC} = 3728.947 + 86.970 \cos(326°40'20'' - 180°) = 3656.280 \text{ m}$$

TABLE 24.4 COMPUTATIONS FOR STAKING CURVE OF EXAMPLE 24.4 BY COORDINATES

Station (1)	Station Difference (2)	Total Deflection (3)	Total Chord (4)	Chord Azimuth (5)	Δ X(6)	Δ Y (7)	X (8)	Y (9)
4 + 458.530							5770.967	3656.280
4 + 460	1.470	0°06′19″	1.470	326°34′01″	−0.810	1.227	5770.157	3657.507
4 + 480	21.470	1°32′16″	21.468	325°08′04″	−12.272	17.614	5758.695	3673.894
4 + 500	41.470	2°58′12″	41.452	323°42′08″	−24.539	33.408	5746.428	3689.688
4 + 520	61.470	4°24′09″	61.410	322°16′11″	−37.580	48.569	5733.387	3704.849
4 + 540	81.470	5°50′06″	81.330	320°50′14″	−51.362	63.060	5719.605	3719.340
4 + 560	101.470	7°16′02″	101.199	319°24′18″	−65.851	76.843	5705.116	3733.123
4 + 580	121.470	8°41′59″	121.004	317°58′21″	−81.011	89.885	5689.956	3746.165
4 + 600	141.470	10°07′55″	140.734	316°32′25″	−96.803	102.153	5674.164	3758.433
4 + 620	161.470	11°33′52″	160.376	315°06′28″	−113.189	113.616	5657.777	3769.896
4 + 629.805	171.275	12°16′00″	169.970	314°24′20″	−121.427	118.933	5649.540	3775.213

Note: All lengths and coordinate values are in meters.

Using their lengths and azimuths, the departure ΔX and latitude ΔY of each total chord are calculated. These are added to the coordinates of the PC to obtain the coordinates of the curve points. Values of ΔX and ΔY are tabulated in columns (6) and (7), and the X and Y coordinates are listed in columns (8) and (9) of Table 24.4.

A check on the PT coordinates of Table 24.4 can be obtained by computing them independently using the azimuth and tangent length of the forward tangent. The azimuth of the forward tangent is calculated by subtracting the I angle from the back tangent's azimuth,

$$Az = 326°40'20'' - 24°32' = 302°08'20''$$

Then the X and Y coordinates of the PT are

$$X_{PT} = 5723.183 + 86.970 \sin(302°08'20'') = 5649.540 \text{ m} \checkmark$$

$$Y_{PT} = 3728.947 + 86.970 \cos(302°08'20'') = 3775.213 \text{ m} \checkmark$$

Computations for the lengths and azimuths of the radial lines needed to stake curve points from station B are listed in Table 24.5. Column (1) gives the curve stations, and columns (2) and (3) list the differences ΔX and ΔY between each curve point's X and Y coordinates and those of station B. Radial lengths L computed from Equation (11.4) and azimuths computed by Equation (11.5) are tabulated in columns (4) and (5). While the calculations needed to stake a curve by coordinates may seem rather extensive, they are routinely handled by computers.

TABLE 24.5 COMPUTATIONS FOR RADIAL LENGTHS AND AZIMUTHS FOR STAKING CURVE OF EXAMPLE 24.4 BY COORDINATES

Station (1)	ΔX (2)	ΔY (3)	L (4)	Az (5)
4 + 458.530 (PC)	−85.730	24.811	89.248	286°08′27″
4 + 460	−77.493	19.494	79.907	284°07′13″
4 + 480	−61.106	8.031	61.631	277°29′14″
4 + 500	−45.314	−4.238	45.512	264°39′25″
4 + 520	−30.154	−17.280	34.754	240°11′05″
4 + 540	−15.665	−31.063	34.789	206°45′42″
4 + 560	−1.883	−45.553	45.592	182°22′01″
4 + 580	11.158	−60.714	61.731	169°35′11″
4 + 600	23.425	76.508	80.014	162°58′36″
4 + 620	34.887	92.895	99.230	159°24′58″
4 + 629.805 (PT)	35.697	94.122	100.664	159°13′48″

Note: The units of ΔX, ΔY, and L are meters.

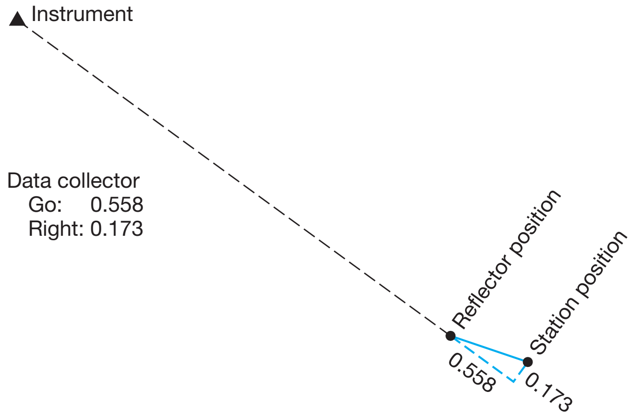


Figure 24.11
Setting a station with a tape from a "stake-out" position.

For orienting the instrument it is necessary to calculate the azimuth of line *BA*. This, also by Equation (11.5), is

$$Az_{BA} = \tan^{-1}\left(\frac{5641.212 - 5735.270}{3778.748 - 3750.402}\right) + 360° = 286°46'16''$$

After backsighting station *A*, 286°46′16″ is indexed on the total station's horizontal circle. Then each curve point is staked by measuring its radial distance and azimuth taken from Table 24.5. The radial lines are shown dashed in Figure 24.11. Note that to stake station 4 + 540, for example, a distance of 34.789 m is observed on an azimuth of 206°45′42″, as shown in the figure.

The process of staking a curve can be greatly simplified by using a data collector equipped with a "stakeout" option. When operating in this mode, before going into the field a file of point identifiers (IDs) and their corresponding coordinates for the job are downloaded into the data collector. In the field, the operator must input (1) the occupied station ID, (2) the backsight station ID (or the azimuth of the backsight line), and (3) the ID of the point to be staked. The backsight is then taken, which automatically orients the instrument. Next, the prism person walks to the estimated location of the station to be set, and the instrument operator sights the reflector. The stakeout software determines the coordinates of the reflector, and informs the operator of the distances and directions that the reflector must be moved, from the prism-person's perspective, to establish the station. For example, the software may say, "GO 0.558 RIGHT 0.173." This indicates that the rodperson should move the reflector 0.558 m from the instrument and 0.173 m to their right. Or the data collector may say "COME 0.558 LEFT 0.173." This means that the rod should be moved 0.558 m toward the instrument and 0.173 m to the left. A small tape is useful in quickly determining the location of the station to be set. Figure 24.11 shows the final measurements needed to set a station with a stakeout command of "GO 0.558 RIGHT 0.173." If a robotic total station and RPU (see Section 8.6) are available to set a curve, the instrument will automatically swing in azimuth to the desired line of sight. This equipment enables one person to lay out a curve.

As noted earlier, in staking circular curves by the coordinate method, any point can be selected for the instrument station as long as its coordinates are known. It can be a point on curve, another established control point, or a new point can be set by traversing (see Chapter 9). Alternatively the instrument can be set at any point of unknown location that provides good vantage and its position can quickly be determined by the resection method (see Sections 11.7 and 23.9). In any case, after the instrument has been set up and oriented, a check should be made to assure its accuracy by measuring to another station of known coordinates. Any significant discrepancy should be reconciled before the curve is laid out.

It is important to note once again that each curve point is staked independent of the others, and thus there is no misclosure at the end to verify the accuracy of the work. Thus, the work must be done very carefully and the layout checked. To check, chord distances between successive stations can be quickly measured with a tape or all stations could be checked from an instrument setup at a second station of known coordinates.

The WOLFPACK software, which can be found on the companion website at http://www.pearsonhighered.com/ghilani, can be used to establish curve-staking notes. The data entry screen for the horizontal curve computations option is shown in Figure 24.12. With this software the user can compute coordinates for the curve and have these coordinates saved to a coordinate file for uploading to a data collector. The figure shows a completed data entry screen for computing the curve in Example 24.4. Two *Computation Options* are selected: "Compute coordinates"

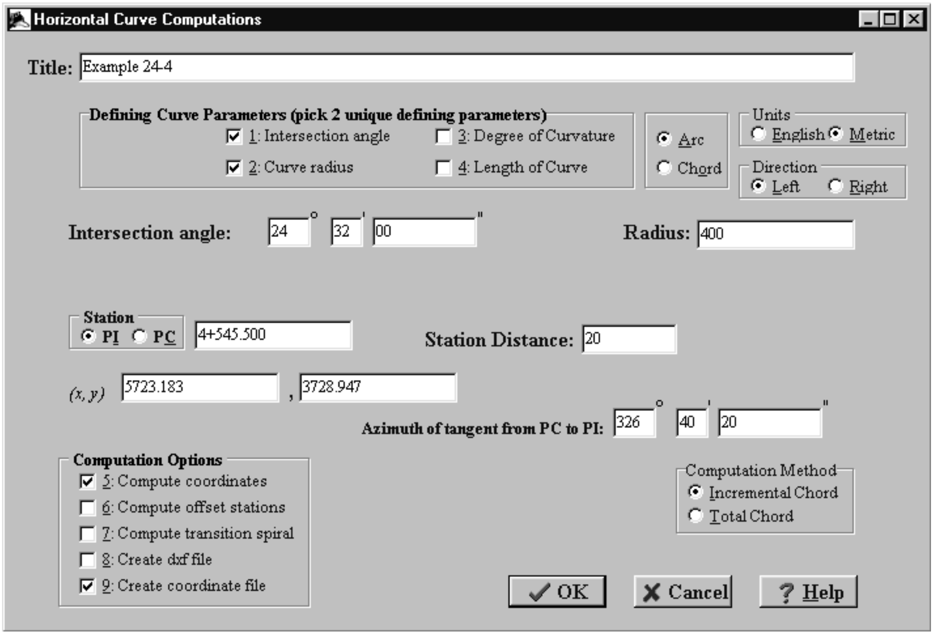


Figure 24.12 Data entry screen for horizontal curve computations in WOLFPACK.

and "Create coordinate file." As additional options are selected, additional data entry boxes will be displayed. The help file that comes with the software describes each option.

■ 24.14 CURVE STAKEOUT USING GNSS RECEIVERS AND ROBOTIC TOTAL STATIONS

As discussed in Sections 23.10 and 23.11, horizontal curves can also be staked out using real-time kinematic (RTK) GNSS surveying methods. However, when doing this, it is important to perform a localization procedure first as discussed in Sections 15.8 and 23.10 to place the satellite-derived coordinates into the project coordinate system. This procedure requires control points known in the project coordinate system that encompass the project area. As discussed in Section 23.10, the base station coordinates can be established initially using the autonomous mode or by a prior GNSS survey (see Chapters 14 and 15). Since only relative positions of stations are required, the transformation process will remove any inaccuracies in the base station coordinates. However, failure to apply or perform this transformation when using GNSS in stakeout, or extrapolation of stakeout points beyond the area encompassed by the project control, can result in serious errors. Additionally, this procedure should only be performed once for any project. Repeated localizations will result in varying solutions due to random errors, which, in turn, will propagate into positional errors. Using RTK surveying methods, the surveyor is guided by the GNSS survey controller to each stake location, where it is witnessed with a hub.

Horizontal layout is also possible using a GNSS machine guidance and control system (see Section 23.11). As discussed in Section 23.11, a GNSS receiver is located on the construction vehicle and used in conjunction with a DTM and finished grading plan to guide the vehicle through the construction process and control earthwork. A base station must be located within radio range of the rover, typically less than 10 km, to supply the rover with the base receiver observations.

Machine control systems are also available for robotic total stations. In this process, the total station is set up on a point with known coordinates and referenced to another. A DTM and grading plan are loaded into the machine control system and the robotic total station guides the construction equipment by tracking a multifaceted, 360° prism mounted on the construction vehicle.

The robotic total station requires sufficient control stations in the construction site to provide both location and orientation. Since the typical range of this system is about 1000 ft (300 m), and since there must be continuous line of sight between the total station and prism, this system requires more control stations in the project area than does a GNSS machine control system. However, it typically works in the project coordinate system and thus does not require either localization or control that surrounds the project area. Furthermore, it is unaffected by canopy conditions and typically provides the only solution to machine control in these situations. The robotic total station provides both horizontal and vertical positioning to the machine control system. Additionally, it offers the benefit of having sufficient accuracy to control finish grading without the need of a laser level. Machine guidance and control for this system is so accurate that it has provided guidance to curbing machines without the use of a stringline.

24.15 CIRCULAR CURVE LAYOUT BY OFFSETS

For short curves, when a total station instrument is not available, and for checking purposes, one of four offset-type methods can be used for laying out circular curves: *tangent offsets* (TO), *chord offsets* (CO), *middle ordinates* (MO), and *ordinates from the long chord.* Figure 24.13 shows the relationship of CO, TO, and MO. Visually and by formula comparison, the chord offset for full station is

$$\text{CO} = 2c \sin\frac{D}{2} = \frac{c^2}{R} = TO \approx 8\, MO \tag{24.19}$$

Since $\sin 1° = 0.0175$ (approx.), $\text{CO} = c(0.0175)D$, where D is in degrees and decimals.

The middle ordinate m for any subchord is $R(1 - \cos \delta)$, with δ being the deflection angle for the chord. A useful equation in the layout or checking of curves in place is

$$D\text{ (degrees)} = m\text{ (inches) for a 62-ft chord (approx.)} \tag{24.20}$$

The geometry of the tangent-offset method is shown in Figure 24.14. The figure illustrates that the curve is most conveniently laid out in both directions from PC and PT to a common point near the middle of the curve. This procedure avoids long observations and provides a check where small adjustments can be made more easily if necessary. To lay out a curve using this method, tangent distances are measured to established temporary points A, B, and C in Figure 24.14. From these points, right-angle observations (tangent offsets) are made to set the

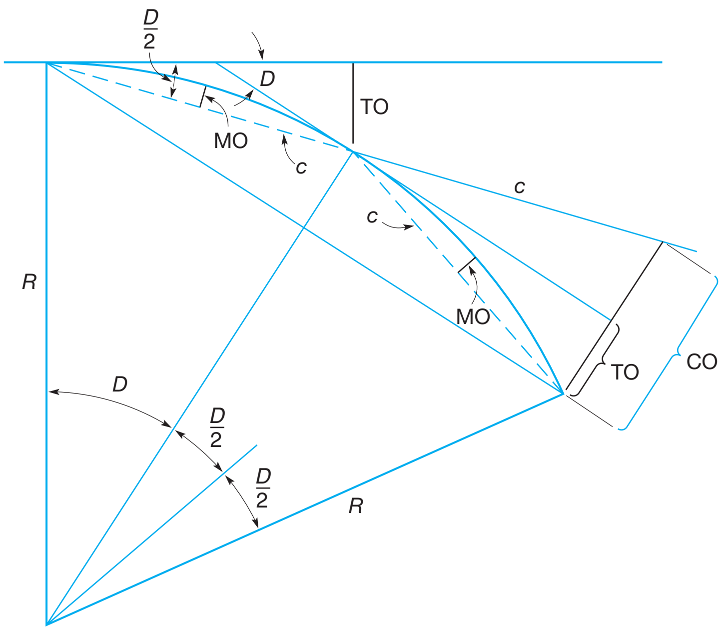


Figure 24.13 Circular curve offsets.

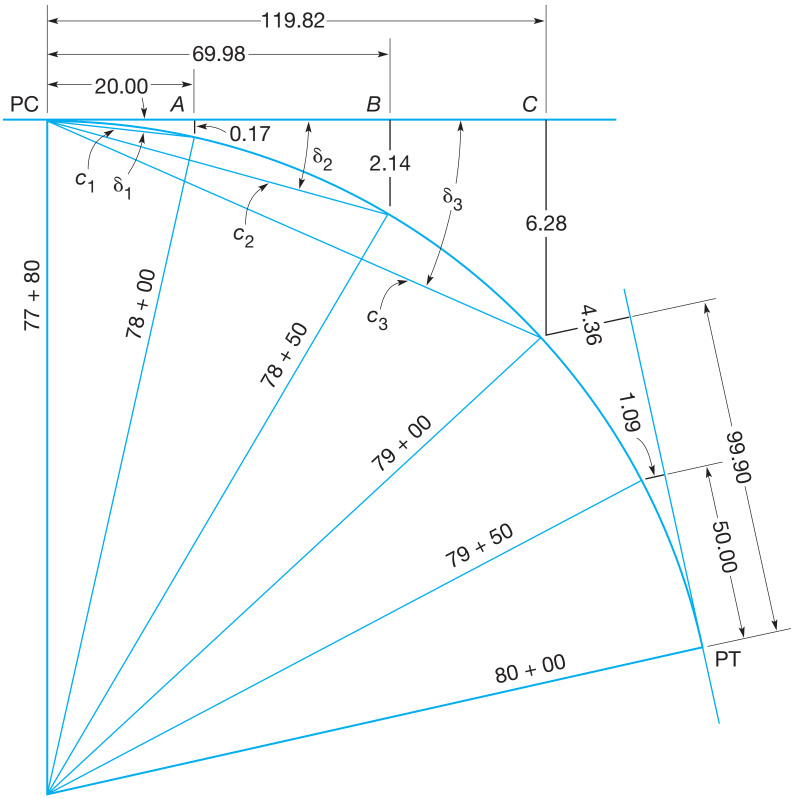


Figure 24.14 Circular curve layout by tangent offsets.

curve stakes. Tangent distances (TD) and tangent offsets (TO) are calculated using chords and angles in the following formulas:

$$\text{TD} = c \cos \delta \tag{24.21}$$

$$\text{TO} = c \sin \delta \tag{24.22}$$

where δ angles are calculated using either Equation (24.12a) or (24.12b), and chords c are determined from Equation (24.13). These procedures are seldom used by today's surveyors (geomaticians).

Example 24.5

Compute and tabulate the data necessary to stake, by tangent offsets, the half-stations of a circular curve having $I = 11°00'$, $D_c = 5°00'$ (chord definition), and PC = 77 + 80.00.

Solution

By Equation (24.2a), the curve length is $L = 100(11/5) = 220$ ft.

Therefore, the PT station is (77 + 80) + (2 + 20) = 80 + 00. Intermediate stations to be staked are 78 + 00, 78 + 50, 79 + 00, and 79 + 50, as shown in Figure 24.14.

TABLE 24.6 TANGENT OFFSET DATA FOR EXAMPLE 24.5

Station	Deflection Angle δ	Chord c	Tangent Offset $c \cos \delta$	Tangent Distance $c \sin \delta$
80 + 00 (PT)				
79 + 50	1°15′	50.01	50.00	1.09
79 + 00	2°30′	100.00	99.90	4.36
79 + 00	3°00	119.98	119.82	6.28
78 + 50	1°45′	70.01	69.98	2.14
78 + 00	0°30′	20.00	20.00	0.17
77 + 80 (PC)				

By Equation [(24.12(a)], δ angles from the PC are

$$\delta_1 = 0.025(20) = 0.50° = 0°30'$$

$$\delta_2 = 0.025(70) = 1.75° = 1°45'$$

$$\delta_3 = 0.025(120) = 3.00° = 3°00'$$

where $D/200 = 0.025$.

By Equation (24.10), the radius is

$$R = \frac{50}{\sin 2°30'} = 1146.28 \text{ ft}$$

By Equation (24.13), chords from the PC are

$$c_1 = 2(1146.28) \sin 0°30' = 20.00 \text{ ft}$$
$$c_2 = 2(1146.28) \sin 1°45' = 70.01 \text{ ft}$$
$$c_3 = 2(1146.28) \sin 3°00' = 119.98 \text{ ft}$$

Now, using Equations (24.21) and (24.22), tangent distances and tangent offsets are calculated. Chords, angles, tangent distances, and tangent offsets to stake points from the PT are computed in the same manner. All data for the problem are listed in Table 24.6. The tangent distances tabulated are lengths from the PC or PT that must be measured to establish points A, B, C, etc., and the tangent offsets are distances from these points needed to locate the curve stakes. Accurate curve layout by tangent offsets generally requires a total station to turn the right angles from the tangent. This involves more instrument setups and a greater amount of time than stakeout by deflection angles or coordinates. However, rough layouts can be done using a tape and right-angle prism.

24.16 SPECIAL CIRCULAR CURVE PROBLEMS

Many special problems arise in the design and computation of circular curves. Three of the more common ones are discussed here, and each can be solved using the coordinate geometry formulas given in Chapter 11.

24.16.1 Passing a Circular Curve Through a Fixed Point

One problem that often occurs in practice is to determine the radius of a curve connecting two established tangents and going through a fixed point such as an underpass, overpass, or existing bridge. The problem can be solved by establishing an XY coordinate system, as shown in Figure 24.15 where the origin occurs at V (the PI) and X coincides with the back tangent. Coordinates of the radius point in this system are $X_0 = -R\tan(I/2)$ and $Y_0 = -R$. From observations of distance PV and angle θ the coordinates X_P and Y_P of point P can be determined. Then the following equation for a circle, obtained by substitution into Equation (11.9), can be written as:

$$R^2 = \left(X_P + R\tan\frac{I}{2}\right)^2 + (Y_P + R)^2 \qquad \textbf{(24.23)}$$

With X_P, Y_P, and $I/2$ known, a solution for R can be found. The equation is quadratic and can be solved using Equation (11.3).

24.16.2 Intersection of a Circular Curve and a Straight Line

Another frequently encountered problem involving curve computation is the determination of the intersection point of a circular curve and a straight line. An

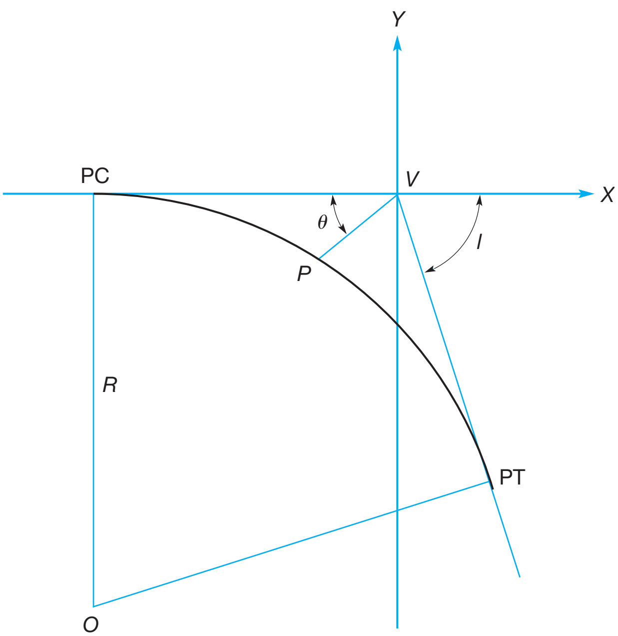


Figure 24.15 Passing a circular curve through a point.

example is illustrated in Figure 11.6. In typical cases, coordinates X_A, Y_A, X_B, and Y_B are known, as well as R. Procedures for solving the problem are outlined in Section 11.5, and a worked example is presented.

24.16.3 Intersection of Two Circular Curves

Figure 11.7 illustrates another common problem: computing the intersection point of two circular curves. This can be handled by coordinate geometry, as discussed in Section 11.6. Coordinates X_A, Y_A, X_B, and Y_B are typically determined through survey, and R_1 and R_2 are selected based on design or topographic constraints. A worked example is given in Section 11.6.

The problems described in this section and the preceding one arise most often in the design of subdivisions and interchanges, and in calculating right-of-way points along highways and railroads.

■ 24.17 COMPOUND AND REVERSE CURVES

Compound and reverse curves are combinations of two or more circular curves. They should be used only for low-speed traffic routes, and in terrain where simple curves cannot be fitted to the ground without excessive construction costs since the rapid change in curvature causes unsafe driving conditions. Special formulas have been derived to facilitate computations for such curves and are demonstrated in texts on route surveying. A compound curve can be staked with instrument setups at the beginning PC and ending PT, or perhaps with one setup at the *point of compound curvature* (PCC) where the two curves join. Reverse curves are handled in similar fashion.

■ 24.18 SIGHT DISTANCE ON HORIZONTAL CURVES

Highway safety requires certain minimum sight distances in zones where passing is permitted, and in nonpassing areas to assure a reasonable stopping distance if there is an object on the roadway. Specifications and tables list suitable values based on vehicular speeds, the perception and reaction times of an average individual, the braking distance for a given coefficient of friction during deceleration, and type and condition of the pavement.

A minimum stopping sight distance of 450 to 550 ft is desirable for a speed of 55 mi/h. An approximate formula for determining the available horizontal sight distance on a circular curve can be derived from Figure 24.16, in which the clear sight distance past an obstruction is the length of the long chord AS, denoted by C; and the required clearance is the middle ordinate PM, denoted by m. Then in similar triangles SPG and SOH

$$\frac{m}{SP} = \frac{SP/2}{R} \quad \text{and} \quad m = \frac{(SP)^2}{2R}$$

Usually m is small compared with R, and SP may be assumed equal to $C/2$. Then

$$C = \sqrt{8mR} \tag{24.24}$$

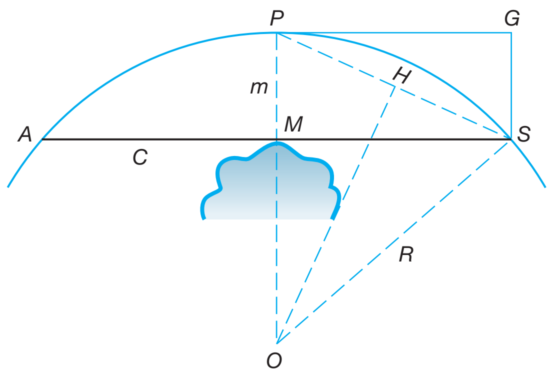


Figure 24.16 Horizontal sight distance on a circular curve.

If distance m from the centerline of a highway to the obstruction is known or can be measured, the available sight distance C is calculated from the formula. Actually cars travel on either the inside or the outside lane, so sight distance AS is not exactly the true stopping distance, but the computed length is on the safe side and satisfactory for practical use. If the calculation reveals a sight distance restriction, the obstruction might possibly be removed or a safe speed limit posted in the area.

■ 24.19 SPIRALS

As noted in Section 24.1, spirals are used to provide gradual transitions in horizontal curvature. Their most common use is to connect straight sections of alignment with circular curves, thereby lessening the sudden change in direction that would otherwise occur at the point of tangency. Since spirals introduce curvature gradually, they afford the logical location for introducing superelevation to offset the centrifugal force experienced by vehicles entering curves.

24.19.1 Spiral Geometry

Figure 24.17 illustrates the geometry of spirals connecting tangents with a circular curve of radius R and degree of curvature D. The entrance spiral at the left begins on the back tangent at the *TS* (*tangent to spiral*) and ends at the SC (*spiral to curve*). The circular curve runs from the *SC* to the beginning of the exit spiral at the *CS* (*curve to spiral*), and the exit spiral terminates on the forward tangent at the *ST* (*spiral to tangent*).

The entrance and exit spirals are geometrically identical. Their length L_S is the arc distance from the *TS* to the *SC* or *CS* to *ST*. The designer selects this length to provide sufficient distance for introducing the curve's superelevation.

If a tangent to the entrance spiral (and curve) at the SC is projected to the back tangent, it locates the *spiral point of intersection* SPI. The angle at the SPI between the two tangents is the spiral angle Δ_S. From the basic property of a spiral, that is, *its radius changes uniformly from infinity at the TS to the radius of the circular curve at the SC*, it follows that the spiral's degree of curve changes uniformly from 0° at the *TS* to D at the *SC*. Since the change is uniform, the average

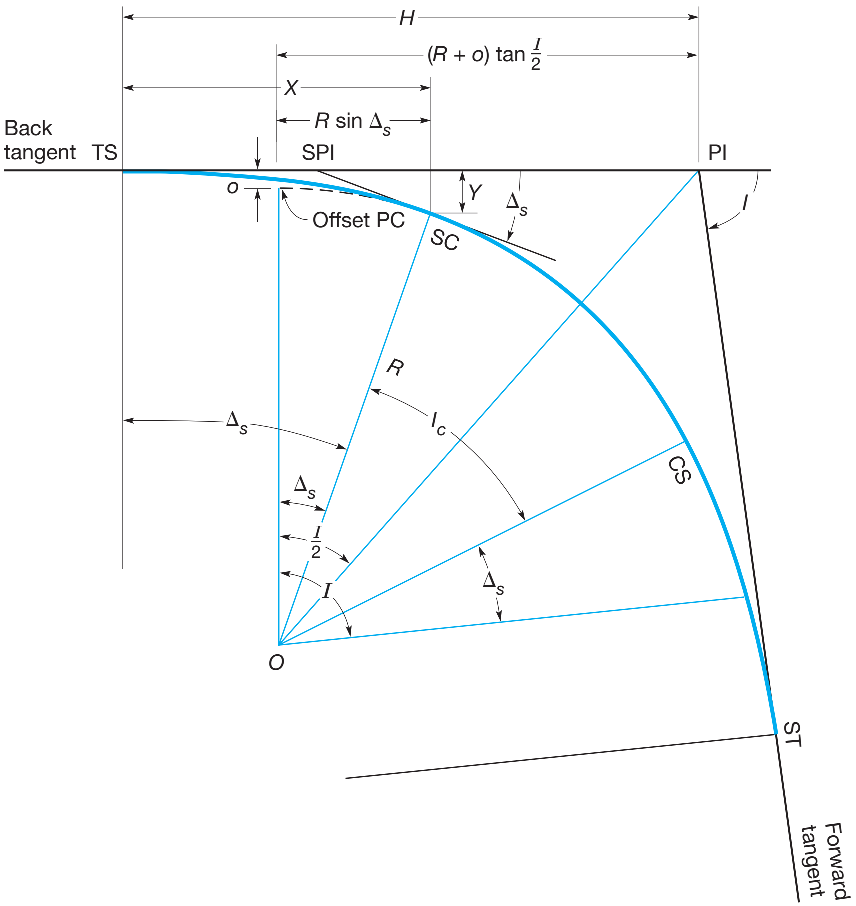


Figure 24.17
Spiral geometry.

degree of curve over the spiral's length is $D/2$. Thus, from the definition of degree of curve, spiral angle Δ_S is

$$\Delta_S = L_S \frac{D}{2} \qquad \textbf{(24.25)}$$

where L_S is in stations and Δ_S and D are in degrees.

Assume in Figure 24.18 that M is at the midpoint of the spiral, so its distance from the TS is $L_S/2$. If this reasoning is continued, the degree of curvature at M is $D/2$, the average degree of curvature from the TS to M is $(D/2)/2 = D/4$, and the spiral angle Δ_M is

$$\Delta_M = \frac{L_S}{2}\frac{D}{4} = \frac{L_S D}{8} \qquad \textbf{(a)}$$

Solving for D in Equation (24.25) and substituting into Equation (a) gives

$$\Delta_M = \frac{L_S}{8}\frac{2\Delta_S}{L_S} = \frac{\Delta_S}{4} \qquad \textbf{(b)}$$

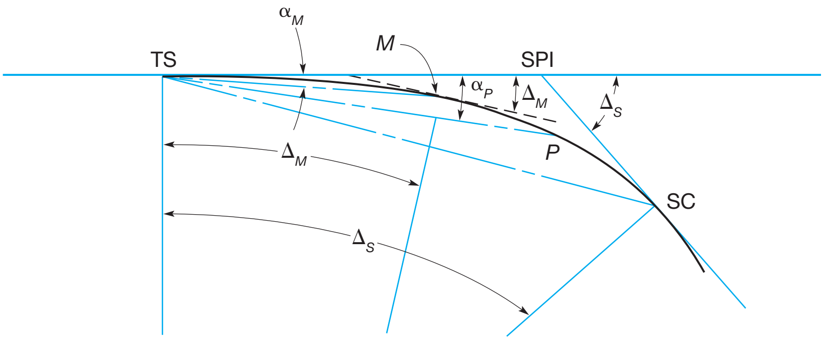


Figure 24.18 Spiral deflection angles.

According to Equation (b), at $L_S/2$, the spiral angle is $\Delta_S/4$. This demonstrates another basic property of a spiral: *spiral angles at any point are proportional to the square of the distance from TS to the point, or*

$$\Delta_P = \left(\frac{L_P}{L_S}\right)^2 \Delta_S \tag{24.26}$$

where Δ_P is the spiral angle at any point P whose distance from the TS is L_P.

24.19.2 Spiral Calculation and Layout

Deflection angles and chords in a manner similar to that used for circular curves can be used to lay out spirals. In Figure 24.18, α_P is the deflection angle from the *TS* to any point *P*. From calculus it can be shown that for the gradual spirals used on transportation routes, deflection angles are very nearly one third their corresponding spiral angles. Thus

$$\alpha_P = \left(\frac{L_P}{L_S}\right)^2 \frac{\Delta_S}{3} \tag{24.27}$$

In Equation (24.27), L_P is the distance from the *TS* to *P*, which is simply the difference in stationing from the *TS* to point *P*.

In Figure 24.17, coordinates *X* and *Y* give the position of the *SC*. In this coordinate system, the origin is at the *TS*, and the *X*-axis coincides with the back tangent. Approximate formulas for computing *X* and *Y* are

$$X = L_S(100 - 0.0030462\,\Delta_S^2)\,(\text{ft}) \tag{24.28}$$

$$Y = L_S(0.58178\Delta_S - 0.000012659\,\Delta_S^3)\,(\text{ft}) \tag{24.29}$$

where L_S is in stations and Δ_S is in degrees. More accurate formulas for computing *X* and *Y* coordinates of any station, *P*, which is a distance L_P along the spiral, are

$$\begin{aligned} X_P &= L_P\left(1 - \frac{\Delta_P^2}{10} + \frac{\Delta_P^4}{216} - \frac{\Delta_P^6}{9360} + \frac{\Delta_P^8}{685{,}440}\right) \\ Y_P &= L_P\left(\frac{\Delta_P}{3} - \frac{\Delta_P^3}{42} + \frac{\Delta_P^5}{1320} - \frac{\Delta_P^7}{75{,}600} + \frac{\Delta_P^9}{6{,}894{,}720}\right) \end{aligned} \tag{24.30}$$

where Δ_P, defined in Equation (24.26), is expressed in radian units and L_P is the stationing distance, in either feet or meters, from the *TS* to point *P*. If L_S and Δ_S are substituted for L_P and Δ_P, respectively in Equations 24.30, then coordinates *X* and *Y* of the *SC* will result. Equations 24.30 can be used to compute spiral coordinates in either metric or English units. The spiral could be staked by offsets using these coordinates, or they could be used with Equations (11.4) and (11.5) to calculate the deflection angles and chord distances necessary to stake out the spiral.

When a spiral is inserted ahead of a circular curve, as illustrated in Figure 24.17, the circular curve is shifted inward by an amount o, which is known as the *throw*. This can be visualized by constructing the circular curve back from the *SC* to the offset *PC* (the point where a tangent to the curve is parallel to the back tangent). The perpendicular distance from the offset *PC* to the back tangent is the throw, which from Figure 24.17 is

$$o = Y - R(1 - \cos \Delta_S) \tag{24.31}$$

To lay out a spiral in the field, distance *H* of Figure 24.17 from *PI* to *TS* is measured back along the tangent to locate the *TS*. Then the *SC* can be established by laying out *X* and *Y*. From the geometry of Figure 24.17 distance *H* is

$$H = X - R \sin \Delta_S + (R + o) \tan \frac{I}{2} \tag{24.32}$$

The following example will illustrate the computations required in laying out a spiral by the deflection-angle method.

Example 24.6

A spiral of 300-ft length is used for transition into a 3°00′ circular curve (arc definition). Angle *I* at the PI station of 20 + 00 is 60°00′. Compute and tabulate the deflection angles and chords necessary to stake out this spiral at half-stations.

Solution

By Equation (24.3) $\quad R = 5729.58/3.00 = 1909.86$ ft

By Equation (24.25) $\quad \Delta_S = 3(3.00)/2 = 4.5° = 4°30'$

By Equations (24.28) and (24.29)

$$X = 3[100 - 0.0030462(4.5)^2] = 299.81 \text{ ft}$$

$$Y = 3[0.58178(4.5) - 0.000012659(4.5)^3] = 7.86 \text{ ft}$$

[*Note:* These same results can be obtained using Equations 24.30, with $L_P =$ 300.00 ft, and $\Delta_P = (4.5° \times \pi/180°)$.]

By Equation (24.31)

$$o = 7.86 - 1909.86\,(1 - \cos 4°30') = 1.97 \text{ ft}$$

By Equation (24.32)

$$H = 299.81 - 1909.86 \sin 4°30' + (1909.86 + 1.97) \tan 30° = 1253.75 \text{ ft}$$

TABLE 24.7 DATA FOR STAKING SPIRAL OF EXAMPLE 24.6

Station (1)	Total Chord Distance from TS (ft) (2)	Deflection Angle (3)	Incremental Chord (ft) (4)
10 + 46.25 (SC)	300.00	1°30.0′	46.25
10 + 00	253.75	1°04.4′	50.00
9 + 50	203.75	0°41.5′	50.00
9 + 00	153.75	0°23.6′	50.00
8 + 50	103.75	0°10.8′	50.00
8 + 00	53.75	0°02.9′	50.00
7 + 50	3.75	0°00.0′	3.75
7 + 46.25 (TS)			

Calculate stationing

$$\begin{aligned} \text{PI station} &= 20 + 00.00 \\ -H &= \underline{-12 + 53.75} \\ \text{TS station} &= 7 + 46.25 \\ +L_S &= \underline{3 + 00.00} \\ \text{SC station} &= 10 + 46.25 \end{aligned}$$

The deflection angles calculated using Equation (24.27) are listed in column (3) of Table 24.7. The values for L_P used in the equation are given in column (2). If a total station is used for stakeout with a setup at the *TS*, the total chords of column (2) are used with the deflection angles of column (3). If a theodolite and tape were used, the deflection angles, and incremental chords listed in column (4) would be used. The chords in columns (2) and (4) are simply station differences between points on the spiral, and are nearly exact for the relatively flat curvature of highway and railroad spirals. After progressing through the spiral, the *SC* is finally staked. The falling between its position and the *SC* set by coordinates should be measured, the precision calculated, and a decision made to accept or repeat the work.

To continue staking the alignment, the instrument is moved forward to the *SC*. With $2\Delta_S/3$ set on the horizontal circle, a backsight is taken on the TS and the line of sight plunged.[1] Turning the instrument to 0°00′00″ orients the line of sight tangent to the circular curve, ready for laying off deflection angles. The circular curve is computed and laid out by methods given in earlier sections of this chapter, except that its intersection angle I_c, as illustrated in Figure 24.17, is given by

$$I_c = I - 2\Delta_S \tag{24.33}$$

[1]From Equation (24.27), the angle at the *TS* in triangle *TS-SPI-SC* of Figure 24.18 is $\Delta_S/3$. Also from Figure 24.18, the angle at the SPI in this triangle is $180° - \Delta_S$. It follows therefore that with the instrument at the *SC*, after backsighting the *TS*, the angle that must be turned to get the line-of-sight tangent to the spiral and circular curve is $2\Delta_S/3$.

When staking reaches the circular curve's end, the *CS* is set. The exit spiral is calculated by the same methods described for the entrance spiral and laid out by staking back from the *ST*.

Spirals can be computed and staked by several different methods. In this brief treatment, only one commonly applied procedure has been discussed. Students interested in further study of spirals can consult a text on route surveying listed in the Bibliography at the end of this chapter.

■ 24.20 COMPUTATION OF "AS-BUILT" CIRCULAR ALIGNMENTS

Most highways that exist today were carefully designed and then constructed according to plan. Therefore, their centerlines "as-built" are precisely known and coordinates for critical points on their alignments are on file for future use. However, some roads have their origins from "cartways" that through the years were periodically upgraded and improved in place. Thus, it is possible that no formal plans or records of their alignments exist. Yet the boundary lines of adjoining properties may be referenced to the centerline, and thus it becomes important for it to be precisely established. Also, it is sometimes desirable to determine the parameters of an as-built roadway to check adherence to contract specifications. In these cases, the coordinates of critical points on the approximate centerline of the facility, both on the curves and on the tangents, must be determined. One further important application of this type of problem relates to railroad abandonment programs. Here the rails have served for years as monuments for delineating right of way lines. Therefore, before their removal it is important to obtain the coordinates of important points along their alignments for use in future work related to establishing property lines adjoining the railroad right of way.

The procedure of establishing the coordinates of an existing or as-built alignment is illustrated in Figure 24.19. In the figure, assume that a traverse was used to establish coordinates for points A through F along the existing alignment, as shown. From points B through E, a least-squares fit of points to Equation (11.10) can be performed, which will establish the coordinates of the center point O and the radius R of the circle. In this example, the matrices of coefficients, unknowns, and observations are

$$\mathbf{A} = \begin{bmatrix} 2X_B & 2Y_B & -1 \\ 2X_C & 2Y_C & -1 \\ 2X_D & 2Y_D & -1 \\ 2X_E & 2Y_E & -1 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} X_O \\ Y_O \\ f \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} X_B^2 + Y_B^2 \\ X_C^2 + Y_C^2 \\ X_D^2 + Y_D^2 \\ X_E^2 + Y_E^2 \end{bmatrix} \tag{24.34}$$

Using Equations (15.6) or (15.7), the least-squares solution for X_O, Y_O, and f are determined. The radius R can then be computed from

$$R = \sqrt{X_O^2 + Y_O^2 - f} \tag{24.35}$$

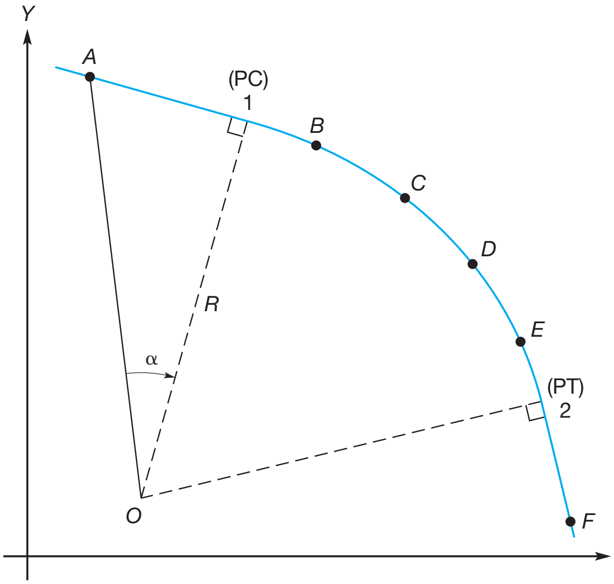


Figure 24.19 Geometry of an "as-built" survey.

Example 24.7

In reference to Figure 24.19, the following coordinates were determined during an "as-built" survey. What are the defining parameters for the curve, and the coordinates of the *PC* and *PT?*

Point	*X* (ft)	*Y* (ft)	Point	*X* (ft)	*Y* (ft)
A	5354.86	7444.14	*D*	8084.03	6071.29
B	6975.82	6947.93	*E*	8431.38	5542.00
C	7577.11	6572.60	*F*	8877.96	4268.90

Solution

Substituting the coordinate values for points *B* through *E* into Equation (24.34) yield the matrices *A* and *L* as

$$\boldsymbol{A} = \begin{bmatrix} 2(6975.82) & 2(6947.93) & -1 \\ 2(7577.11) & 2(6572.60) & -1 \\ 2(8084.03) & 2(6071.29) & -1 \\ 2(8431.38) & 2(5542.00) & -1 \end{bmatrix} = \begin{bmatrix} 13{,}951.64 & 13{,}895.86 & -1 \\ 15{,}154.22 & 13{,}145.20 & -1 \\ 16{,}168.06 & 12{,}142.58 & -1 \\ 16{,}862.76 & 11{,}084.00 & -1 \end{bmatrix}$$

$$\boldsymbol{L} = \begin{bmatrix} 6975.82^2 + 6947.93^2 \\ 7577.11^2 + 6572.60^2 \\ 8084.03^2 + 6071.29^2 \\ 8431.38^2 + 5542.00^2 \end{bmatrix} = \begin{bmatrix} 96{,}935{,}795.96 \\ 100{,}611{,}666.71 \\ 102{,}212{,}103.30 \\ 101{,}801{,}932.70 \end{bmatrix}$$

```
Data file for Example 24.7 using WOLFPACK
B 6975.82 6947.93 **Point id, X, Y
C 7577.11 6572.60
D 8084.03 6071.29
E 8431.38 5542.00

Results of Least Squares Adjustment of Points
*****  Least Squares Fit Of Circle  *****
Center of Circle at: X = 5,587.375
                     Y = 4,053.992
With A Radius of:        3,209.766

Misfit of Points to Best Fit Curve
Station    Distance From Curve
====================
B              0.009
C             -0.026
D              0.028
E             -0.010
```

Figure 24.20 Data file and adjustment results for Example 24.7 using WOLFPACK.

Substituting the A and L matrices into Equation (15.6) yields the solution

$$X_O = 5587.375$$
$$Y_O = 4053.992$$
$$f = 37{,}351{,}007.416$$

And finally by Equation (24.35), the radius of the circle is

$$R = \sqrt{(5587.375)^2 + (4053.992)^2 - 37{,}351{,}007.416} = 3209.766$$

Figure 24.20 shows the data file and the results of the least-squares fit of points to a circle from the software WOLFPACK, which is available on the companion website for this book at http://www.pearsonhighered.com/ghilani.

From this solution, the coordinates of the PC can be determined by computing the length and azimuth of AO of Figure 24.19 using Equations (11.4) and (11.5), and then solving right triangle $AO1$ for angle α using the cosine function. For this example, the values are

By Equations (11.4) and (11.5)

$$OA = \sqrt{(5354.86 - 5587.375)^2 + (7444.14 - 4053.992)^2} = 3398.11$$

$$Az_{OA} = \tan^{-1}\left(\frac{5354.86 - 5587.375}{7444.14 - 4053.992}\right) + 360° = 356°04'35''$$

From the cosine function

$$\alpha = \cos^{-1}\left(\frac{3209.766}{3398.11}\right) = 19°09'56''$$

From Figure 24.19, the azimuth of line $O1$ is

$$Az_{O1} = 356°04'35'' + 19°09'56'' - 360° = 15°14'31''$$

Now the coordinates of the PC can be determined using the following equations

$$X_{PC} = X_O + R \sin Az_{O1} = 5587.375 + 843.838 = 6431.21$$
$$Y_{PC} = Y_O + R \cos Az_{O1} = 4053.992 + 3096.859 = 7150.85$$

In similar fashion, the coordinates for the PT can be computed. From the azimuths of lines $O1$ and $O2$, the intersection angle can be determined using Equation (11.11). Additionally, the long chord can be determined using Equation (11.4). From these parameters, the tangent, external, and middle ordinate distances can be computed using Equations (24.4), (24.8), and (24.9), respectively. These steps are left as an exercise for the student.

24.21 SOURCES OF ERROR IN LAYING OUT CIRCULAR CURVES

Some sources of error in curve layout are:

1. Errors in setting up, leveling, and reading the instrument.
2. Total station (or theodolite) out of adjustment.
3. Bull's-eye bubble out of adjustment on prism pole used for stakeout with a total station.
4. Measurement errors in laying out angles and distances.

24.22 MISTAKES

Typical mistakes that occur in laying out a curve in the field are:

1. Failure to take equal numbers of direct and reversed measurements of the deflection angle at the PI before computing or laying out the curve.
2. Using 100.00-ft chords to lay out arc-definition curves having D greater than 2°.
3. Taping subchords of nominal length for chord-definition curves having D greater than 5° (a nominal 50-ft subchord for a 6° curve requires a measurement of 50.02 ft).
4. Failure to check curve points after staking them using either the total chord or the coordinate method.
5. A mistake in the backsight when staking by the coordinate method.
6. Failure to stake the PT independently by measuring the tangent distance forward from the PI.
7. Incorrect orientation of the instrument's horizontal circle.
8. Failure to properly establish the setup information in an automatic data collector.
9. Misidentification of a station, or stations.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

24.1 Why is a reverse curve objectionable for transportation alignments?

24.2 For the following circular curves having a radius R, what is their degree of curvature by (1) arc definition and (2) chord definition?

*(a) 500.00 ft
(b) 750.00 ft
(c) 2000.00 ft

Compute L, T, E, M, LC, R, and stations of the PC and PT for the circular curves in Problems 24.3 through 24.6. Use the chord definition for the railroad curve and the arc definition for the highway curves.

24.3* Railroad curve with $D_c = 4°00'$, $I = 24°00'$, and PI station $= 36 + 45.00$ ft.
24.4 Highway curve with $D_a = 2°40'$, $I = 14°20'$, and PI station $= 24 + 65.00$ ft.
24.5 Highway curve with $R = 500.000$ m, $I = 18°30'$, and PI station $= 6 + 517.500$ m.
24.6 Highway curve with $R = 750.000$ m, $I = 18°30'$, and PI station $= 12 + 324.800$ m.

Tabulate R or D, T, L, E, M, PC, PT, deflection angles, and incremental chords to lay out the circular curves at full stations (100 ft or 30 m) in Problems 24.7 through 24.14.

24.7 Highway curve with $D_a = 3°46'$, $I = 16°30'$, and PI station $= 29 + 64.20$ ft.
24.8 Railroad curve with $D_c = 2°30'$, $I = 15°00'$, and PI station $= 58 + 65.42$ ft.
24.9 Highway curve with $R = 800$ m, $I = 12°00'$, and PI station $= 3 + 281.615$ m.
24.10 Highway curve with $R = 700$ m, $I = 14°30'$, and PI station $= 1 + 632.723$ m.
24.11 Highway curve with $R = 850$ ft, $I = 40°00'$, and PI station $= 85 + 40.00$ ft.
24.12 Highway curve with $L = 350$ m, $R = 400$ m, and PI station $= 4 + 332.690$ m.
24.13 Highway curve with $T = 265.00$ ft, $R = 1250$ ft, and PI station $= 87 + 33.55$ ft.
24.14 Railroad curve with $T = 155.00$ ft, $D_c = 2°35'$, and PI station $= 48 + 10.00$ ft.

In Problems 24.15 through 24.18 tabulate the curve data, deflection angles, and incremental chords needed to lay out the following circular curves at full-station increments using a total station instrument set up at the PC.

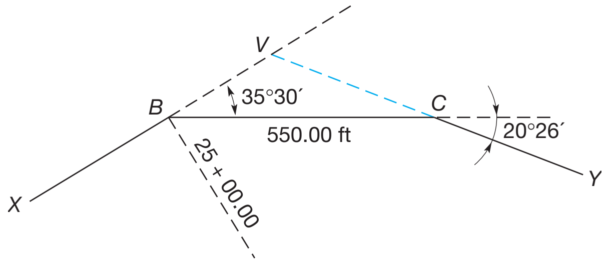
24.15 The curve of Problem 24.7
24.16 The curve of Problem 24.8
24.17 The curve of Problem 24.9
24.18 The curve of Problem 24.10
24.19 A rail line on the center of a 50-ft street makes a 55°24′ turn into another street of equal width. The corner curb line has $R = 10$ ft. What is the largest R that can be given a circular curve for the track centerline if the law requires it to be at least 10 ft from the curb?

Tabulate all data required to lay out by deflection angles and incremental chords, at the indicated stationing, for the circular curves of Problems 24.20 and 24.21.

24.20 The R for a highway curve (arc definition) will be rounded off to the nearest larger multiple of 100 ft. Field conditions require M to be approximately 20 ft to avoid an embankment. The PI $= 94 + 18.70$ and $I = 23°00'$ with stationing at 100 ft.
24.21 For a highway curve, R will be rounded off to the nearest multiple of 10 m. Field measurements show that T should be approximately 80 m to avoid an overpass. The PI $= 6 + 356.400$ and $I = 13°20'$ with stationing at 30 m.

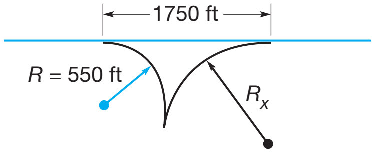
24.22 A highway survey PI falls in a pond, so a cut off line $AB = 275.21$ ft is run between the tangents. In the triangle formed by points A, B, and PI, the angle at $A = 16°28'$ and at $B = 22°16'$. The station of A is 54 + 92.30. Calculate and tabulate curve notes to run, by deflection angles and incremental chords, a 4°30′ (arc definition) circular curve at full-station increments to connect the tangents.

24.23 In the figure, a single circular highway curve (arc definition) will join tangents XV and VY and also be tangent to BC. Calculate R, L, and the stations of the PC and PT.



Problem 24.23

24.24* Compute R_x to fit requirements of the figure and make the tangent distances of the two curves equal.



Problem 24.24

24.25 After a backsight on the PC with 0°00′ set on the instrument, what is the deflection angle to the following circular curve points?
*(a) Setup at curve midpoint, deflection to the PT.
(b) Instrument at curve midpoint, deflection to 3/4 point.
(c) Setup at 1/4 point of curve, deflection to 3/4 point.

24.26 In surveying a construction alignment, why should the I angle be measured by repetition?

24.27 A highway curve (arc definition) to the right, having $R = 500$ m and $I = 18°30'$, will be laid out by coordinates with a total station instrument setup at the PI. The PI station is 3 + 855.200 m, and its coordinates are $X = 75{,}428.863$ m and $Y = 36{,}007.434$ m. The azimuth (from north) of the back tangent proceeding toward the PI is 48°17′12″. To orient the total station, a backsight will be made on a POT on the back tangent. Compute lengths and azimuths necessary to stake the curve at 30-m stations.

24.28 In Problem 24.27, compute the XY coordinates at 30-m stations.

24.29 An exercise track must consist of two semicircles and two tangents, and be exactly 1500 ft along its centerline. The two tangents are 200.00 ft each. Calculate L, R, and D_a for the curves.

24.30 Make the computations necessary to lay out the curve of Problem 24.8 by the tangent offset method. Approximately half the curve is to be laid out from the PC and the other half from the PT.

What sight distance is available if there is an obstruction on a radial line through the PI inside the curves in Problems 24.31 and 24.32?

24.31* For Problem 24.7, obstacle 10 ft from curve.
24.32 For Problem 24.12, obstacle 10 m from curve.
24.33 If the misclosure for the curve of Problem 24.7, computed as described in Section 24.8, is 0.12 ft, what is the field layout precision?
24.34 Assume that a 150-ft entry spiral will be used with the curve of Problem 24.7. Compute and tabulate curve notes to stake out the alignment from the TS to ST at full stations using a total station and the deflection-angle, total chord method.
24.35 Same as Problem 24.34, except use a 300-ft spiral for the curve of Problem 24.8.
24.36 Same as Problem 24.34, except for the curve of Problem 24.9, with a 50-m entry spiral using stationing of 30 m and a total station instrument.
24.37 Compute the area bounded by the two arcs and tangent in Problem 24.24.
24.38 In an as-built survey, the XY coordinates in meters of three points on the centerline of a highway curve are determined to be A: (295.338, 419.340); B: (312.183, 433.927); C: (326.969, 445.072). What are the radius, and coordinates for the center of the curve in meters?
24.39 In Problem 24.38, if the (x, y) coordinates in meters of two points on the centerline of the tangents are (262.066, 384.915) and (378.361, 476.370), what are the coordinates of the PC, PT, and the curve parameters L, T, and I?
24.40 Write a computational program to calculate circular curve notes for layout with a total station by deflection angles and total chords from a set up at the PC.
24.41 Develop a computational program to calculate the coordinates of the stations on a circular curve.

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25 Vertical Curves

25.1 INTRODUCTION

Curves are needed to provide smooth transitions between straight segments (tangents) of grade lines for highways and railroads. Because these curves exist in vertical planes, they are called vertical curves. An example is illustrated in Figure 25.1, which shows the profile view of a proposed section of highway to be constructed from A to B. A grade line consisting of three tangent sections has been designed to fit the ground profile. Two vertical curves are needed: curve *a* to join tangents 1 and 2, and curve *b* to connect tangents 2 and 3. The function of each curve is to provide a gradual change in grade from the initial (back) tangent to the grade of the second (forward) tangent. Because parabolas provide a constant rate of change of grade, they are ideal and almost always applied for vertical alignments used by vehicular traffic.

Two basic types of vertical curves exist, *crest* and *sag*. These are illustrated in Figure 25.1. Curve *a* is a crest type, which by definition undergoes a negative change in grade; that is, the curve turns downward. Curve *b* is a sag type, in which the change in grade is positive and the curve turns upward.

There are several factors that must be taken into account when designing a grade line of tangents and curves on any highway or railroad project. They include (1) providing a good fit with the existing ground profile, thereby minimizing the depths of cuts and fills, (2) balancing the volume of cut material against fill, (3) maintaining adequate drainage, (4) not exceeding maximum specified grades, and (5) meeting fixed elevations such as intersections with other roads. In addition, the curves must be designed to (a) fit the grade lines they connect, (b) have lengths sufficient to meet specifications covering a maximum rate of change of grade (which affects the comfort of vehicle occupants), and (c) provide sufficient sight distance for safe vehicle operation (see Section 25.11).

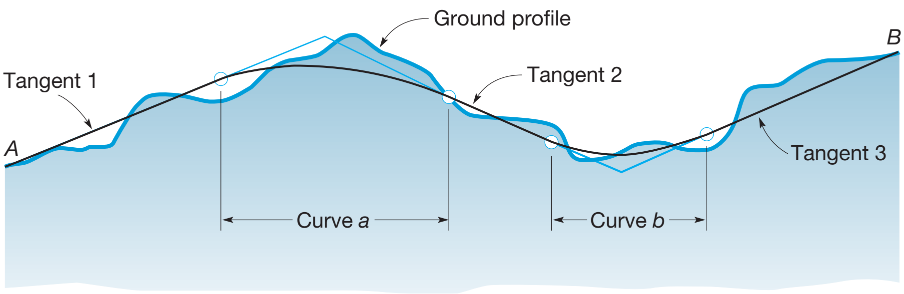

Figure 25.1 Grade line and ground profile of a proposed highway section.

Elevations at selected points (e.g., full or half stations in the English system of stationing, or 20, 30, or 40 m in the metric system) along vertical parabolic curves are usually computed by the *tangent-offset* method. It is simple, straightforward, conveniently performed with calculators and computers, and self-checking. After the elevations of curve points have been computed, they are staked in the field to guide construction operations so the route can be built according to plan.

25.2 GENERAL EQUATION OF A VERTICAL PARABOLIC CURVE

The general mathematical expression of a parabola, with respect to an XY rectangular coordinate system, is given by

$$Y_P = a + bX_P + cX_P^2 \tag{25.1}$$

where Y_P is the ordinate at any point p on the parabola located at distance X_P from the origin of the curve, and a, b, and c are constants. Figure 25.2 shows a

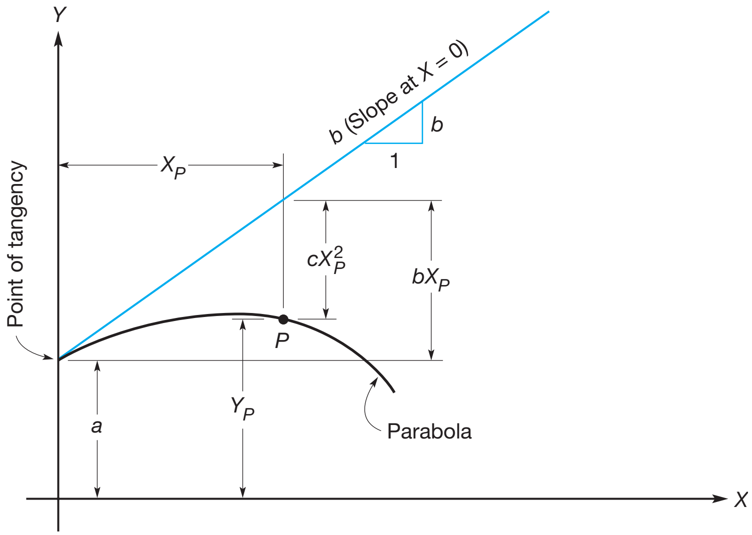

Figure 25.2 Terms for a parabola.

parabola in a XY rectangular coordinate system and illustrates the physical significance of the terms in Equation (25.1). Note from the figure that a is the ordinate at the beginning of the curve where $X = 0$, b is the slope of a tangent to the curve at $X = 0$, bX_P the change in the ordinate along the tangent over distance X_P, and cX_P^2 the parabola's departure from the tangent (*tangent offset*) in distance X_P. When the terms a, bX_P, and cX_P^2 are combined as in Equation (25.1) and shown in Figure 25.2, they produce Y_P, the elevation on curve at X_P. For the crest curve of Figure 25.2, b has positive algebraic sign and c is negative.

25.3 EQUATION OF AN EQUAL TANGENT VERTICAL PARABOLIC CURVE

Figure 25.3 shows a parabola that joins two intersecting tangents of a grade line. The parabola is essentially identical to that in Figure 25.2, except that the terms used are those commonly employed by surveyors and engineers. In the figure, *BVC* denotes the beginning of vertical curve, sometimes called the VPC (vertical point of curvature); V is the vertex, often called the VPI (vertical point of intersection); and *EVC* denotes the end of vertical curve, interchangeably called the VPT (vertical point of tangency). The percent grade of the back tangent (straight segment preceding V) is g_1, that of the forward tangent (straight segment following V) is g_2. The curve length L is the horizontal distance (in stations) from the *BVC* to the *EVC*. The curve of Figure 25.3 is called equal tangent because the horizontal distances from the *BVC* to V and from V to the *EVC* are equal, each being $L/2$. Proof of this is given in Section 25.5.

On the XY axis system in Figure 25.3, X values are horizontal distances measured from the *BVC*, and Y values are elevations measured from the vertical

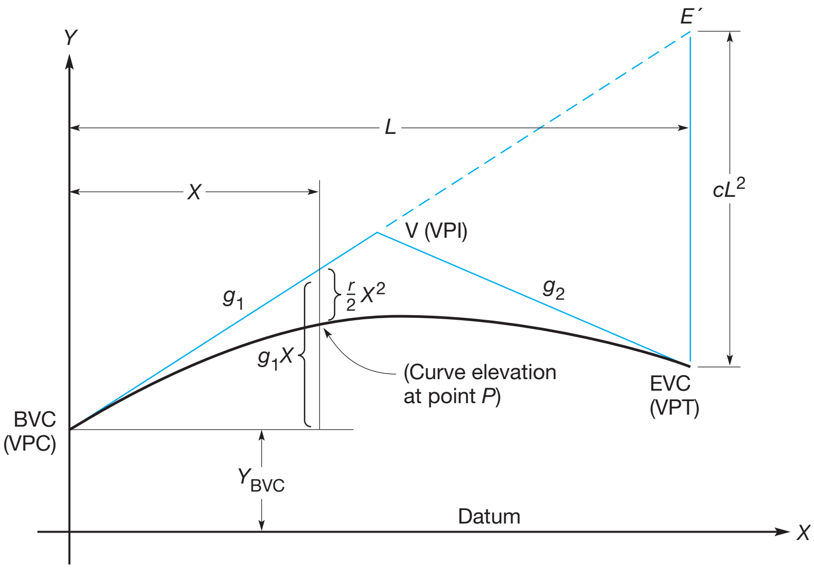

Figure 25.3
Vertical parabolic curve relationships.

datum of reference. By substituting this surveying terminology into Equation (25.1), the parabola can be expressed as

$$Y = Y_{BVC} + g_1X + cX^2 \tag{25.2}$$

In Equation (25.2), if the English system of units is used, Y is in ft, g_1 is percent grade, and for consistency of units, X must therefore be in 100 ft stations. If the metric system is used, Y is in meters, g_1 is again in percent grade, and thus X must be in units of 100 m or 1/10th stations, where full stations are 1 km apart. The correspondence of terms in Equation (25.2) to those of Equation (25.1) is $a = Y_{BVC}$ (elevation of BVC) and $bX_P = g_1X$ (change in elevation along the back tangent with increasing X). To express the constant c of Equation (25.2) in surveying terminology, consider the tangent offset from E' on the extended back tangent to the EVC (dashed line in Figure 25.3). Its value (which is negative for the crest curve shown) is cL^2 where L (horizontal distance from BVC to E') is substituted for X. From the figure, cL^2 can be expressed in terms of horizontal lengths (in stations) and percent grades as follows:

$$cL^2 = g_1\left(\frac{L}{2}\right) + g_2\left(\frac{L}{2}\right) - g_1L \tag{a}$$

Solving Equation (a) for constant c gives

$$c = \frac{g_2 - g_1}{2L} \tag{b}$$

Substituting Equation (b) into Equation (25.2) results in the following equation for an equal-tangent vertical curve in surveying terminology:

$$Y = Y_{BVC} + g_1X + \left(\frac{g_2 - g_1}{2L}\right)X^2 \tag{25.3}$$

The *rate of change of grade, r*, for an equal-tangent parabolic curve equals the total grade change from BVC to EVC divided by length L (in stations for the English system, or 1/10th stations for metric units), over which the change occurs, or

$$r = \frac{g_2 - g_1}{L} \tag{25.4}$$

As mentioned earlier, the value r (which is negative for a crest curve and positive for a sag type) is an important design parameter because it controls the rate of curvature, and hence rider comfort. To incorporate it in the equation for parabolic curves, Equation (25.4) is substituted into Equation (25.3)

$$Y = Y_{BVC} + g_1X + \left(\frac{r}{2}\right)X^2 \tag{25.5}$$

Figure 25.3 illustrates how the terms of Equation (25.5) combine to give the curve elevation at point P. Because the last term of the equation is the curve's offset from the back tangent, this formula is commonly called the *tangent offset equation*.

25.4 HIGH OR LOW POINT ON A VERTICAL CURVE

To investigate drainage conditions, clearance beneath overhead structures, cover over pipes, or sight distance, it may be necessary to determine the elevation and location of the low (or high) point on a vertical curve. At the low or high point, a tangent to the curve will be horizontal and its slope equal to zero. Based on this fact, by taking the derivative of Equation (25.3) and setting it equal to zero, the following formula is readily derived:

$$X = \frac{g_1 L}{g_1 - g_2} \tag{25.6}$$

where X is the distance from the BVC to the high or low point of the curve (in stations in the English system of units, and in 1/10th stations in the metric system), g_1 the tangent grade through the BVC, g_2 the tangent grade through the EVC, and L the curve length (in stations or 1/10th stations).

If g_2 is substituted for g_1 in the numerator of Equation (25.6), distance X is measured back from the EVC. By substituting Equation (25.4) into Equation (25.6), the following alternate formula for locating the high or low point results

$$X = \frac{-g_1}{r} \tag{25.7}$$

25.5 VERTICAL CURVE COMPUTATIONS USING THE TANGENT OFFSET EQUATION

In designing grade lines, the locations and individual grades of the tangents are normally selected first. This produces a series of intersection points V, each defined by its station and elevation. A curve is then chosen to join each pair of intersecting tangents. The parameter selected in vertical-curve design is length L. Having chosen it, the station of the BVC is obtained by subtracting $L/2$ from the vertex station. Adding L to the BVC station then determines the EVC station.

Stationing for the points on curve that are computed and staked are those that are *evenly divisible by the selected staking increment*. Thus, if a full station is the staking increment selected for a curve in the English system of units, each full station, that is, 10 + 00, 11 + 00, 12 + 00, etc. would be staked, but 10 + 50, 11 + 50, etc. would not be staked. For example, if the staking increment is half stations in the English system, then 15 + 00, 15 + 50, 16 + 00, 16 + 50, and so on, would be staked, and not 15 + 25, 15 + 75, 16 + 25, 16 + 75, etc. In the metric system, if 40 m is the staking increment, then 2 + 400, 2 + 440, 2 + 480, 2 + 520, and so on would be staked, but not 2 + 420, 2 + 460, 2 + 500, etc.

Computations for vertical parabolic curves are normally done in tabular form.

25.5.1 Example Computations Using the English System of Units

Following are example computations for an equal-tangent vertical curve in the English system of units. The curve is a crest type.

Example 25.1

A grade g_1 of +3.00% intersects grade g_2 of −2.40% at a vertex whose station and elevation are 46 + 70 and 853.48 ft, respectively. An equal-tangent parabolic curve 600 ft long has been selected to join the two tangents. Compute and tabulate the curve for stakeout at full stations. (Figure 25.4 shows the curve.)

Solution

By Equation (25.4)

$$r = \frac{-2.40 - 3.00}{6} = -0.90\% \text{ station}$$

Stationing

$$\begin{aligned} V &= 46 + 70 \\ -L/2 &= \underline{\ 3 + 00} \\ BVC &= 43 + 70 \\ +L &= \underline{\ 6 + 00} \\ EVC &= 49 + 70 \end{aligned}$$

$$\text{Elev}_{\text{BVC}} = 853.48 - 3.00(3) = 844.48 \text{ ft}$$

The remaining calculations utilize Equation (25.5) and are listed in Table 25.1.

A check on curve elevations is obtained by computing the first and second differences between the elevations of full stations, as shown in the right-hand columns of the table. Unless disturbed by rounding errors, all second differences (rate of change) should be equal. For curves in the English system of units at

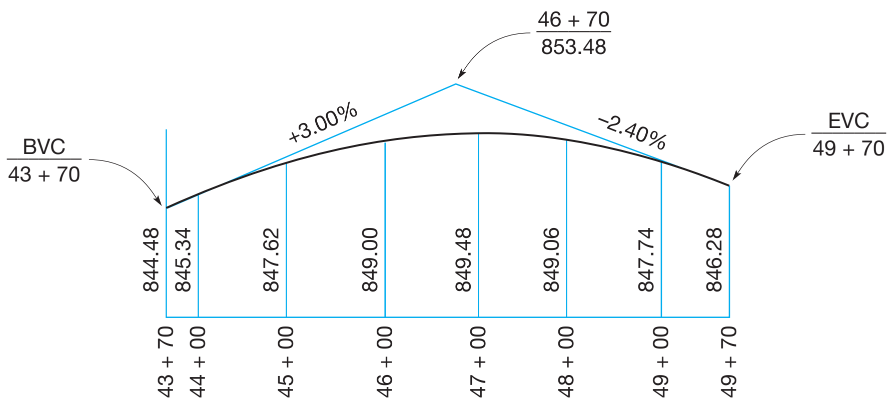

Figure 25.4 Crest curve of Example 25.1.

TABLE 25.1 NOTES FOR CURVE OF EXAMPLE 25.1

Station	X (sta)	g_1X	$\frac{rX^2}{2}$	Curve Elevation	First Difference	Second Difference
49 + 70 (EVC)	6.0	18.00	−16.20	846.28		
49 + 00	5.3	15.90	−12.64	847.74	−1.32	−0.90
48 + 00	4.3	12.90	−8.32	849.06	−0.42	−0.90
47 + 00	3.3	9.90	−4.90	849.48	0.48	−0.90
46 + 00	2.3	6.90	−2.38	849.00	1.38	−0.90
45 + 00	1.3	3.90	−0.76	847.62	2.28	
44 + 00	0.3	0.90	−0.04	845.34		
43 + 70 (BVC)	0.0	0.00	−0.00	844.48		

Check: $EVC = ELEV_V - g_2\left(\frac{L}{2}\right) = 853.48 - 2.40(3) = 846.28$ ✓

full-station increments, the second differences should equal r; for half-station increments they should be $r/4$.

It is sometimes desirable to calculate the elevation of the curve's center point. This can be done using $X = L/2$ in Equation (25.5). For Example 25.1, it is

$$Y_{\text{center}} = 844.48 + 3.00(3) - \left(\frac{0.90}{2}\right)(3)^2 + 849.43 \text{ ft}$$

This can be checked by employing the property of a parabolic curve, which is *the curve center falls halfway between the vertex and the midpoint of the long chord* (line from *BVC* to *EVC*). The elevation of the midpoint of the long chord (*LC*) is simply the average of the elevations of the *BVC* and *EVC*. For Example 25.1, it is

$$Y_{\text{midpoint } LC} = \frac{844.48 + 846.28}{2} = 845.38 \text{ ft}$$

By the property just stated, the elevation of the curve center for Example 25.1 is the average of the vertex elevation and that of the midpoint of the long chord, or

$$Y_{\text{center}} = \frac{845.38 + 853.48}{2} = 849.43 \text{ ft} \checkmark$$

25.5.2 Example Computations Using the Metric System

The following example illustrates the computations for an equal-tangent vertical curve when metric units are used. The curve is a sag type.

Example 25.2

A grade g_1 of -3.629% intersects grade g_2 of 0.151% at a vertex whose station and elevation are 5 + 265.000 and 350.520 m, respectively. An equal-tangent parabolic curve of 240 m length will be used to join the tangents. Compute and tabulate the curve for staking at 40 m increments.

Solution

By Equation (25.4)

$$r = \frac{0.151 + 3.629}{2.4} = 1.575$$

[Note that L used in Equation (25.4) is in units of m/100, or 1/10th stations.]

Stationing

$$\begin{aligned} VPI \text{ Station} &= 5 + 265 \\ -L/2 &= \quad\;\; 120 \\ BVC \text{ Station} &= 5 + 145 \\ +L &= \quad\;\; 240 \\ EVC \text{ Station} &= 5 + 385 \end{aligned}$$

$$\text{Elev}_{BVC} = 350.520 + 3.629(120/100) = 354.875$$

The remaining calculations employ Equation (25.5) and are listed in Table 25.2.

TABLE 25.2 NOTES FOR CURVE OF EXAMPLE 25.2

Station	$X\left(\frac{m}{100}\right)$	g_1X	$\frac{rX^2}{2}$	Curve Elevation (m)	First Difference	Second Difference
5 + 385.000 (EVC)	2.400	−8.710	4.536	350.701		
5 + 360.000	2.150	−7.802	3.640	350.713	−0.223	0.252
5 + 320.000	1.750	−6.351	2.412	350.936	−0.475	0.252
5 + 280.000	1.350	−4.899	1.435	351.411	−0.727	0.252
5 + 240.000	0.950	−3.448	0.711	352.138	−0.979	0.252
5 + 200.000	0.550	−1.996	0.238	353.117	−1.232	
5 + 160.000	0.150	−0.544	0.018	354.348		
5 + 145.000 (BVC)	0.000	−0.000	0.000	354.875		

Check: $\text{EVC} = \text{ELEV}_V - g_2\left(\frac{L}{2}\right) = 350.520 + (0.151 \times 120/100) = 350.701$

Note that the second differences are all equal, which checks the computations. [The value of 0.252 is $r/6.25$, where 6.25 is $(100 \text{ m}/40 \text{ m})^2$.]

Example 25.3

Compute the station and elevation of the curve's high point in Example 25.1.

Solution

By Equation (25.7), $X = -3.00/-0.90 = 3.3333$ stations
Then the station of the high point is

$$\text{station}_{\text{high}} = (43 + 70) + (3 + 33.33) = 47 + 03.33$$

By Equation (25.3), the elevation at this point is

$$844.48 + 3.00(3.333) + \frac{-2.40 - 3.00}{2(6)}(3.3333)^2 = 849.48$$

Note that in using Equation (25.7) and all other equations of this chapter, correct algebraic signs must be applied to grades g_1 and g_2.

By applying the same equations and procedures, the station and elevation of the low point of the curve of Example 25.2 are 5 + 375.413 and 350.694 m, respectively. The calculations are left as an exercise.

25.6 EQUAL TANGENT PROPERTY OF A PARABOLA

The curve defined by Equations (25.3) and (25.5) has been called an equal-tangent parabolic curve, which means the vertex occurs at a distance $X = L/2$ from the BVC. Proof of this property is readily made with reference to Figure 25.5, which illustrates a sag curve. In the figure, assume the horizontal distance from BVC to V is an unknown value X; thus, the remaining distance from V to EVC is $L - X$. Two expressions can be written for the elevation of the EVC. The first, using Equation (25.3) with $X = L$, yields

$$Y_{EVC} = Y_{BVC} + g_1 L + \left(\frac{g_2 - g_1}{2L}\right)L^2 \tag{c}$$

The second, using changes in elevation that occur along the tangents, gives

$$Y_{EVC} = Y_{BVC} + g_1 X + g_2(L - X) \tag{d}$$

Equating Equations (c) and (d) and solving, $X = L/2$. Thus, distances BVC to V and V to EVC are equal—hence the term *equal-tangent parabolic curve*.

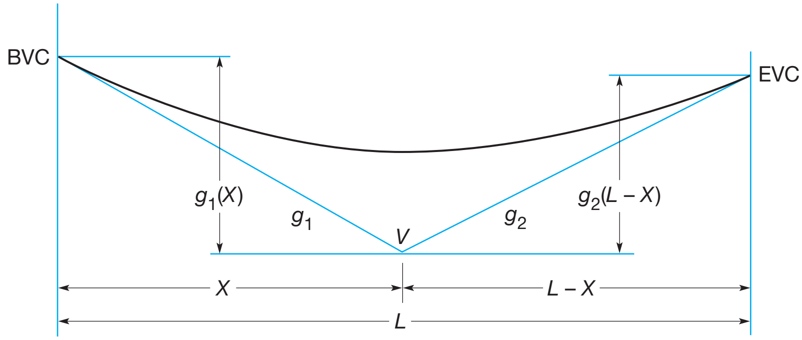

Figure 25.5 Proof of equal-tangent property of a parabola.

25.7 CURVE COMPUTATIONS BY PROPORTION

The following basic property of a parabola can be used to simplify vertical curve calculations: offsets from a tangent to a parabola are proportional to the squares of the distances from the point of tangency. Calculating the offset E at the midpoint of the curve, and then computing offsets at any other distance X from the BVC by proportion according to the following formula conveniently utilize this property.

$$\text{offset}_X = E\left(\frac{X}{L/2}\right)^2 \tag{25.8}$$

The value of E in Equation (25.8) is simply the difference in elevation from the curve's midpoint to the VPI. Computation of the curve's midpoint elevation was discussed in Section 25.4, and for Example 25.1 it was 849.43 ft. To illustrate the use of Equation (25.8), the offset from the tangent to the curve for station 47 + 00, where $X = 3.3$ stations, will be computed

$$\text{offset}_{47+00} = (849.43 - 853.48)\left(\frac{3.3}{3}\right)^2 = -4.90 \text{ ft}$$

In the above calculation, 3.3 in the numerator is X in stations, and 3 in the denominator is $L/2$ also in stations. The value -4.90 checks the tangent offset listed in Table 25.1 for station 47 + 00. This simplified procedure is convenient for computing vertical curves in the field with a handheld calculator.

25.8 STAKING A VERTICAL PARABOLIC CURVE

Prior to initiating construction on a route project, the planned centerline, or an offset one, will normally be staked at full or half stations, as well as other critical horizontal alignment points such as PCs and PTs. Then, as described in Section 23.7, slope stakes will be set out perpendicular to the centerline at or near the slope intercepts to guide rough grading. Excavation and embankment construction then proceed and continue until the grade is near plan elevation. The centerline stations are then staked again, this time using sharpened 2-in. square wooden hubs, usually about 10 in. long. These are known as "blue tops," so called because when their tops are driven to grade elevation, they are colored blue. Contractors request blue tops when excavated areas are still slightly high and embankments somewhat low. After blue tops are set to mark the precise grade, final grading is completed.

To set blue tops at grade, a circuit of differential levels is run from a nearby benchmark to establish the *HI* of a leveling instrument in the project area. The difference between the *HI* and any station's grade is the required rod reading on that stake. Assume, for example, a *HI* of 856.20 ft exists and station 45 + 00 of Example 25.1 is to be set. The required rod reading is 856.20 − 847.62 = 8.58 ft. With the stake initially driven firmly into the ground, and the rod held on its top, suppose a reading of 8.25 is obtained. The stake then must be driven down an additional 8.58 − 8.25 = 0.33 ft further and the notetaker so indicates. After the stake is driven, a distance estimated to be somewhat less than 0.33 ft, the rod reading is checked. This is repeated until the required reading of 8.58 is achieved, whereupon the stake is colored blue using keel or spray paint.

This process is continued until all stakes are set. The required rod reading at station 46 + 00, for an *HI* of 856.20, is 7.20 ft. If a rock is encountered and the stake cannot be driven to grade, a vertical offset of, say, 1.00 ft above grade can be marked and noted on the stake.

When the level is too far away from the station being set, a turning point is established and the instrument is brought forward to establish a new *HI*. Whenever possible, level circuit checks should be made by closing on nearby benchmarks as blue top work on the project progresses. Also, when quitting for the day or when the job is finished, the level circuit must always be closed to verify that no mistakes were made.

■ 25.9 MACHINE CONTROL IN GRADING OPERATIONS

As stated in Section 23.11, GNSS methods provide sufficient vertical accuracy for rough grading operations. However, it is not sufficient to provide final grades on most projects. Thus, a GNSS machine control project must be augmented with laser levels to provide vertical accuracies under 3 mm. The laser level requires a sensor on the construction vehicle and a rotating laser (see Figure 23.3). The level must be positioned over a known calibrated point within 1 ft horizontally. The laser covers a radius of about 1500 ft. Continuous line of sight must be maintained between the rotating laser level and the sensor on the construction vehicle. One manufacturer has included combined laser level with their GNSS unit to provide what they call "mm-GPS."

As stated in Section 23.11, a robotic total station with multifaceted 360° prism can be used also to establish horizontal and vertical positioning. This system is limited to a range of about 1000 ft from the total station to the construction vehicle. However, it can provide both horizontal and vertical accuracies at a sufficient level for most construction applications.

■ 25.10 COMPUTATIONS FOR AN UNEQUAL TANGENT VERTICAL CURVE

An unequal-tangent vertical curve is simply a pair of equal-tangent curves, where the *EVC* of the first curve is the *BVC* of the second. This point is called *CVC*, point of compound vertical curvature. In Figure 25.6, a −2.00% grade intersects a +1.60% grade at station 87 + 00 and elevation 743.24 ft. A vertical curve of

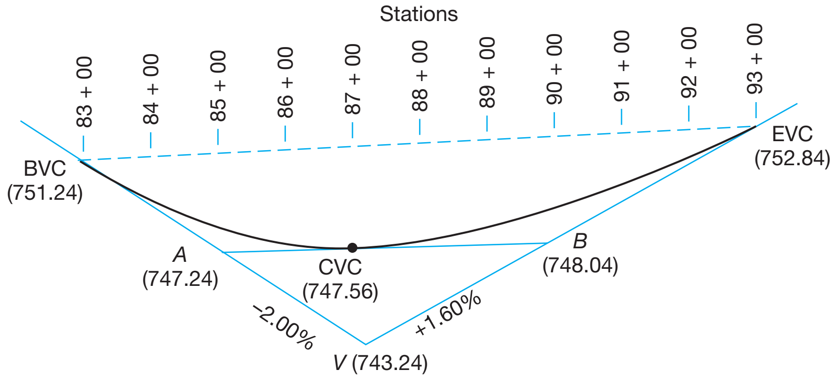

Figure 25.6
Unequal-tangent vertical curve.

length $L_1 = 400$ ft is to be extended back from the vertex, and a curve of length $L_2 = 600$ ft run forward to closely fit ground conditions.

To perform calculations for this type of curve, connect the midpoints of the tangents for the two curves, stations 85 + 00 and 90 + 00, to obtain line *AB*. Point *A* is the vertex of the first curve and is located at a distance of $L_1/2$ back from *V*. Point *B* is the vertex of the second curve which is $L_2/2$ forward from *V*. Compute elevations for *A* and *B* and, using them, calculate the grade of *AB* by dividing the difference in elevation between *B* and *A* by the distance in stations separating these two points. From grade *AB*, determine the *CVC* elevation.

Now compute two equal-tangent vertical curves, one from *BVC* to *CVC* and another from *CVC* to *EVC*, by the methods of Section 25.4. Since both curves are tangent to the same line *AB* at point *CVC*, they will be tangent to each other and form a smooth curve.

Example 25.4

For the configuration of Figure 25.6, compute and tabulate the notes necessary to stake the unequal-tangent vertical curve at full stations.

Solution

1. Calculate elevations of *BVC*, *EVC*, *A*, *B*, and *CVC*, and grade *AB*

$$\text{Elev}_{BVC} = 743.24 + 4(2.00) = 751.24 \text{ ft}$$
$$\text{Elev}_{A} = 743.24 + 2(2.00) = 747.24 \text{ ft}$$
$$\text{Elev}_{EVC} = 743.24 + 6(1.60) = 752.84 \text{ ft}$$
$$\text{Elev}_{B} = 743.24 + 3(1.60) = 748.04 \text{ ft}$$
$$\text{Grade}_{AB} = \left(\frac{748.04 - 747.24}{5}\right) = +0.16\%$$
$$\text{Elev}_{CVC} = 747.24 + 2(0.16) = 747.56 \text{ ft}$$

These elevations are shown in Figure 25.6.

TABLE 25.3 NOTES FOR CURVE OF EXAMPLE 25.3

Station	X (sta)	g_1X	$rX^2/2$	Curve Elevation	First Difference	Second Difference
93 + 00 (EVC)	6	0.96	4.32	752.84 ✓	1.48	
92 + 00	5	0.80	3.00	751.36	1.24	0.24
91 + 00	4	0.64	1.92	750.12	1.00	0.24
90 + 00	3	0.48	1.08	749.12	0.76	0.24
89 + 00	2	0.32	0.48	748.36	0.52	0.24
88 + 00	1	0.16	0.12	747.84	0.28	0.24
87 + 00 (CVC)	4	−8.00	4.32	747.56 ✓	−0.11	
86 + 00	3	−6.00	2.43	747.67	−0.65	0.54
85 + 00	2	−4.00	1.08	748.32	−1.19	0.54
84 + 00	1	−2.00	0.27	749.51	−1.73	0.54
83 + 00 (BVC)	0	0.00	0.00	751.24		

2. In computing the first curve, the grade of AB will be g_2 in the formulas, and for the second curve it will be g_1. The rates of change of grade for the two curves are, by Equation (25.4)

$$r_1 = \frac{0.16 - (-2.00)}{4} = +0.54\%/\text{station}$$

$$r_2 = \frac{1.60 - 0.16}{6} = +0.24\%/\text{station}$$

3. Equation (25.5) is now solved in tabular form and the results are listed in Table 25.3.

Vertical-curve computations by themselves are quite simple, hardly a challenge to a modern computer. But when vertical curves are combined with horizontal curves, spirals, and superelevation in complex highway designs, programming saves time. Figure 25.7 shows a completed data entry box for computing the first vertical curve of Example 25.3 using the WOLFPACK software that accompanies this book at http://www.pearsonhighered.com/ghilani. The software employs the equations discussed in this chapter to compute staking notes for a vertical curve. The computed results are written to a file in table form like those given in Tables 25.1, 25.2, and 25.3, so they can be printed and taken into the field for staking purposes.

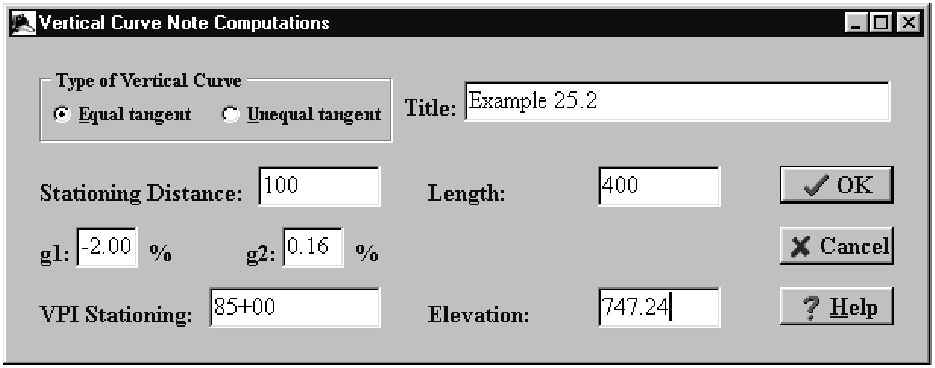

Figure 25.7 Data entry screen in WOLFPACK for computation of first vertical curve staking notes in Example 25.3.

25.11 DESIGNING A CURVE TO PASS THROUGH A FIXED POINT

The problem of designing a parabolic curve to pass through a point of fixed station and elevation is frequently encountered in practice. For example, it occurs where a new grade line must meet an existing railroad or highway crossings or when a minimum vertical distance must be maintained between the grade line and underground utilities or drainage structures.

Given the station and elevation of the VPI, and grades g_1 and g_2 of the back and forward tangents, respectively, the problem consists of calculating the curve length required to meet the fixed condition. It is solved by substituting known quantities into Equation (25.3) and reducing the equation to its quadratic form containing only L as an unknown. Two values will satisfy the quadratic equation, but the correct one will be obvious.

Example 25.5

In Figure 25.8, grades $g_1 = -4.00\%$ and $g_2 = +3.80\%$ meet at VPI station 52 + 00 and elevation 1261.50. Design a parabolic curve to meet a railroad crossing, which exists at station 53 + 50 and elevation 1271.20.

Solution

In referring to Figure 25.8 and substituting known quantities into Equation (25.3), the following equation is obtained:

$$1271.20 = \left[1261.50 + 4.00\left(\frac{L}{2}\right)\right] + \left[-4.00\left(\frac{L}{2} + 1.5\right)\right] + \left[\frac{3.80 + 4.00}{2L}\left(\frac{L}{2} + 1.5\right)^2\right]$$

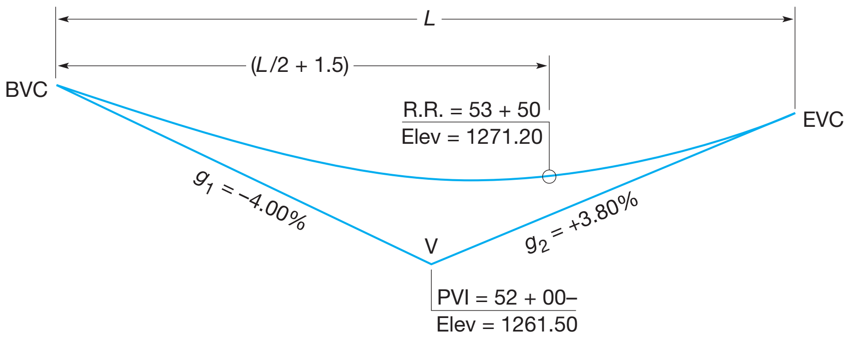

Figure 25.8 Designing a parabolic curve to pass through a fixed point.

In this expression, the value of X for the railroad crossing is $L/2 + 1.5$ and the terms within successive brackets are Y_{BVC}, g_1X, and $(r/2)X^2$, respectively. Reducing the equation to quadratic form gives

$$0.975L^2 - 9.85L + 8.775 = 0$$

Solving by use of Equation (11.3) for L gives 9.1152 stations. To check the solution, $L = 9.1152$ stations and $X = [9.1152/2) + 1.5]$ stations are used in Equation (25.3) to calculate the elevation at station 53 + 50. A value of 1271.20 checks the computations.

■ 25.12 SIGHT DISTANCE

The vertical alignments of highways should provide ample sight distance for safe vehicular operation. Two types of sight distances are involved: (1) stopping sight distance (the distance required, for a given "design speed,"[1] to safely stop a vehicle thus avoiding a collision with an unexpected stationary object in the roadway ahead) and (2) passing sight distance (the distance required for a given design speed, on two-lane two-way highways to safely overtake a slower moving vehicle, pass it, and return to the proper lane of travel leaving suitable clearance for an oncoming vehicle in the opposing lane). For either condition, as speed increases, required sight distance also increases. All highways should provide safe stopping sight distances for their entire extent at their given design speed, and if this cannot be achieved in certain sections, signs must be posted to reduce travel speeds to levels consistent with the available sight distances. Passing sight distance should be provided at frequent intervals along any section of highway

[1]Design speed is defined as the maximum safe speed that can be maintained over a specified section of highway when conditions are so favorable that the design features of the highway govern. Once selected, all of the pertinent features of the highway, especially those involving safety, should be related to that speed.

TABLE 25.4 MINIMUM SIGHT DISTANCES FOR VARYING DESIGN SPEEDS ON LEVEL SECTIONS

Design Speed (mph)	Stopping Sight Distance (ft)	Passing Sight Distance (ft)
30	200	1090
40	305	1470
50	425	1835
60	570	2135
70	730	2480

to allow faster moving vehicles to pass slower moving ones. In sections of highway that do not provide ample passing sight distances, appropriate centerline markings and signs are used to inform drivers of this condition. The American Association of State Highway and Transportation Officials (AASHTO) has recommended minimum sight distances for both stopping and passing for various design speeds. Table 25.4 lists these values for some commonly used design speeds.

Sight distances must be carefully considered in the design of vertical alignments of highway projects. Given the grades of two intersecting tangent sections, the length of vertical curve used to provide a transition from one to the other fixes the sight distance. A longer curve provides a greater sight distance.

The formula for length of curve L necessary to provide sight distance S on a *crest vertical curve, where S is less than L* is

$$L = \frac{S^2(g_1 - g_2)}{2(\sqrt{h_1} + \sqrt{h_2})^2} \tag{25.9}$$

In Equation (25.9), the units of S and L are stations if the English system is used, and one-tenth stations in the metric system. Also the units of h_1 (the height of the driver's eye) and h_2 (the height of an object sighted on the roadway ahead) are in feet for the English system and meters for the metric system. For design, AASHTO recommends 3.5 ft (1080 mm) for h_1. Recommended values for h_2 are 2.0 ft (600 mm) for stopping and 4.25 ft (1.300 m) for passing. The lower value for h_2 represents the size of an object that would damage a vehicle and the higher value represents the height of an oncoming car.

Then for a crest curve having grades of $g_1 = +1.40\%$ and $g_2 = -1.00\%$, by Equation (25.9) the length of curve needed to provide a 570-ft stopping sight distance is

$$L = \frac{(5.70)^2(1.40 + 1.00)}{2(\sqrt{3.50} + \sqrt{2.00})^2} = 3.61 \text{ stations}$$

Since S is greater than L, and thus not in agreement with the assumption used in deriving the formula, a different expression must be employed. If the vehicle is off the curve but on the tangent leading to it and *S is greater than L*, the applicable sight distance formula is

$$L = 2S - \frac{2(\sqrt{h_1} + \sqrt{h_2})^2}{g_1 - g_2} \tag{25.10}$$

Then in the preceding example, the length of curve necessary to provide 570 ft of stopping sight distance is

$$L = 2(5.70) - \frac{2(\sqrt{3.50} + \sqrt{2.0})^2}{1.40 + 1.00} = 2.41 \text{ stations}$$

In this solution, the sight distance of 570 feet is greater than the computed curve length of 2.41 stations (241 ft), and thus the conditions are met.

Sag vertical curves also limit sight distances because they reduce lengths ahead that can be illuminated by headlights during night driving. Equations that apply in computing lengths of sag vertical curves based upon headlight criteria are:

(a) S less than L

$$L = \frac{S^2(g_2 - g_1)}{4 + 3.5S} \tag{25.11}$$

(b) S greater than L

$$L = 2S - \frac{4 + 3.5S}{g_1 - g_2} \tag{25.12}$$

As discussed in Section 24.17, horizontal curves may also limit visibility and sight distances for them can be computed as well. For a combined horizontal and vertical curve, the sight distance that governs is the smaller of the two values computed independently for each curve.

For a complete discussion on sight distances for the design of highways and streets, the reader should refer to the AASHTO publication, *A Policy on Geometric Design of Highways and Streets*, which is cited in the bibliography at the end of this chapter.

■ 25.13 SOURCES OF ERROR IN LAYING OUT VERTICAL CURVES

Some sources of error in staking out parabolic curves are:

1. Making errors in measuring distances and angles when staking the centerline.

2. Not holding the level rod plumb when setting blue tops.
3. Using a leveling instrument that is out of adjustment.

25.14 MISTAKES

Some typical mistakes made in computations for vertical curves include the following:

1. Arithmetic mistakes.
2. Failure to properly account for the algebraic signs of g_1 and g_2.
3. Subtracting offsets from tangents for a sag curve or adding them for a crest curve.
4. Failure to make the second-difference check.
5. Not completing the level circuit back to a benchmark after setting blue tops.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

25.1 Why are vertical curves needed on the grade lines for highways and railroads?

25.2 What is meant by the "rate of grade change" on vertical curves and why is it important?

Tabulate station elevations for an equal-tangent parabolic curve for the data given in Problems 25.3 through 25.9. Check by second differences.

25.3 A +1.55% grade meets a −2.50% grade at station 44 + 25 and elevation 682.34 ft, 800-ft curve, stakeout at half stations.

25.4 A −2.50% grade meets a +2.50% grade at station 4 + 200 and elevation 293.585 m, 300-m curve, stakeout at 30-m increments.

25.5 A 375-ft curve, grades of $g_1 = -2.60\%$ and $g_2 = +0.90\%$, VPI at station 36 + 40, and elevation 605.35 ft, stakeout at full stations.

25.6 A 450-ft curve, grades of $g_1 = -4.00\%$ and $g_2 = -3.00\%$, VPI at station 66 + 50, and elevation 560.00 ft, stakeout at full stations.

25.7 A 150-m curve, $g_1 = +3.00\%$, $g_2 = -2.00\%$, VPI station = 2 + 175, VPI elevation = 157.830 m, stakeout at 30-m increments.

25.8 A 200-ft curve, $g_1 = -1.50\%$, $g_2 = +2.50\%$, VPI station = 46 + 00, VPI elevation = 895.00 ft, stakeout at quarter stations.

25.9 An 90-m curve, $g_1 = -1.50\%$, $g_2 = +0.75\%$, VPI station = 6 + 280, VPI elevation = 550.600 m, stakeout at 10-m increments.

Field conditions require a highway curve to pass through a fixed point. Compute a suitable equal-tangent vertical curve and full-station elevations for Problems 25.10 through 25.12.

25.10* Grades of $g_1 = -2.50\%$ and $g_2 = +1.00\%$, VPI elevation 750.00 ft at station 30 + 00. Fixed elevation 753.00 ft at station 30 + 00.

25.11 Grades of $g_1 = -2.50\%$ and $g_2 = +1.50\%$, VPI elevation 2560.00 ft at station 315 + 00. Fixed elevation 2567.00 ft at station 314 + 00.

25.12 Grades of $g_1 = +5.00\%$ and $g_2 = +1.50\%$, VPI station 6 + 300 and elevation 185.920 m. Fixed elevation 185.610 m at station 6 + 400. (Use 100-m stationing.)

25.13 A −1.10% grade meets a +0.90% grade at station 36 + 00 and elevation 800.00 ft. The +0.90% grade then joins a +1.50% grade at station 39 + 00. Compute and tabulate the notes for an equal-tangent vertical curve, at half stations, that passes through the midpoint of the 0.90% grade.

25.14 When is it advantageous to use an unequal-tangent vertical curve instead of an equal-tangent one?

Compute and tabulate full-station elevations for an unequal-tangent vertical curve to fit the requirements in Problems 25.15 through 25.18.

25.15 A +4.00% grade meets a −2.00% grade at station 60 + 00 and elevation 1086.00 ft. Length of first curve 500 ft, second curve 400 ft.

25.16 Grade $g_1 = +1.25\%$, $g_2 = +3.50\%$, VPI at station 62 + 00 and elevation 650.00 ft, $L_1 = 600$ ft and $L_2 = 500$ ft.

25.17 Grades g_1 of +4.00% and g_2 of −2.00% meet at the VPI at station 4 + 300 and elevation 154.960 m. Lengths of curves are 100 and 200 m. (Use 30-m stationing.)

25.18 A −1.80% grade meets a +3.00% grade at station 95 + 00 and elevation 320.64 ft. Length of first curve is 300 ft, of second curve, 200 ft.

25.19* A manhole is 12 ft from the centerline of a 30-ft wide street that has a 6-in. parabolic crown. The street center at the station of the manhole is at elevation 612.58 ft. What is the elevation of the manhole cover?

25.20 A 60-ft wide street has an average parabolic crown from the center to each edge of 1/4 in./ft. How much does the surface drop from the street center to a point 6 ft from the edge?

25.21 Determine the station and elevation at the high point of the curve in Problem 25.3.

25.22 Calculate the station and elevation at the low point of the curve in Problem 25.4.

25.23 Compute the station and elevation at the low point of the curve of Problem 25.5.

25.24 What are the station and elevation of the high point of the curve of Problem 25.7?

25.25 What additional factor must be considered in the design of crest vertical curves that is not of concern in sag curves?

25.26* Compute the sight distance available in Problem 25.3. (Assume $h_1 = 3.50$ ft and $h_2 = 4.25$ ft.)

25.27 Similar to Problem 25.26, except $h_2 = 2.00$ ft.

25.28 Similar to Problem 25.26, except for the data of Problem 25.7, where $h_1 = 1.0$ m and $h_2 = 0.5$ m.

25.29 In determining sight distances on vertical curves, how does the designer determine whether the cars or objects are on the curve or tangent?

What is the minimum length of a vertical curve to provide a required sight distance for the conditions given in Problems 25.30 through 25.32?

25.30* Grades of +3.00% and −2.50%, sight distance 600 ft, $h_1 = 3.50$ ft and $h_2 = 1.25$ ft.

25.31 A crest curve with grades of +4.50% and −3.00%, sight distance 500 ft, $h_1 = 4.25$ ft and $h_2 = 1.00$ ft.

25.32 Sight distance of 200 m, grades of +1.00% and −2.25%, $h_1 = 1.1$ m and $h_2 = 0.3$ m.

25.33* A backsight of 6.85 ft is taken on a benchmark whose elevation is 567.50 ft. What rod reading is needed at that *HI* to set a blue top at grade elevation of 572.55 ft?

25.34 A backsight of 6.92 ft is taken on a benchmark whose elevation is 867.50 ft. A foresight of 3.64 ft and a backsight of 7.04 ft are then taken in turn on TP_1 to establish a *HI*. What rod reading will be necessary to set a blue top at a grade elevation of 872.06 ft?

25.35 Develop a computational program that performs the vertical-curve computations.

BIBLIOGRAPHY

American Association of State Highway and Transportation Officials. 2001. *Guidelines for Geometric Design of Very-Low Volume Local Roads (ADT ≤ 400)*. Washington, DC: AASHTO.

American Association of State Highway and Transportation Officials. 2004. *A Policy on Geometric Design of Highways and Streets*. Washington, DC: AASHTO.

26 Volumes

■ 26.1 INTRODUCTION

Persons engaged in surveying (geomatics) are often called on to determine volumes of various types of material. Quantities of earthwork and concrete are needed, for example, on many types of construction projects. Volume computations are also required to determine the capacities of bins, tanks, reservoirs, and buildings, and to check stockpiles of coal, gravel, and other materials. The determination of quantities of water discharged by streams and rivers, per unit of time, is also important.

The most common unit of volume is a cube having edges of unit length. Cubic feet, cubic yards, and cubic meters are used in surveying calculations, with cubic yards and cubic meters being most common for earthwork. (*Note:* 1 $yd^3 = 27\ ft^3$; $1\ m^3 = 35.3144\ ft^3$). The *acre-foot* (the volume equivalent to an acre of area, 1-ft deep) is commonly used for large quantities of water, while cubic feet per second (ft^3/sec) and cubic meters per second (m^3/sec) are the usual units for water flow measurement.

■ 26.2 METHODS OF VOLUME MEASUREMENT

Direct measurement of volumes is rarely made in surveying, since it is difficult to actually apply a unit of measure to the material involved. Instead, indirect measurements are obtained by measuring lines and areas that have a relationship to the volume desired.

Three principal systems are used: (1) the cross-section method, (2) the unit-area (or borrow-pit) method, and (3) the contour-area method.

■ 26.3 THE CROSS-SECTION METHOD

The cross-section method is employed almost exclusively for computing volumes on linear construction projects such as highways, railroads, and canals. In this procedure, after the centerline has been staked, ground profiles called cross sections are taken (at right angles to the centerline), usually at intervals of full or half stations if the English system of units is being used, or at perhaps 10, 20, 30, or 40 m if the metric system is being employed. Cross-sectioning consists of observing ground elevations and their corresponding distances left and right perpendicular to the centerline. Readings must be taken at the centerline, at high and low points, and at locations where slope changes occur to determine the ground profile accurately. This can be done in the field using a level, level rod, and tape. Plate B.5 in Appendix B illustrates a set of field notes for cross-sectioning.

Much of the fieldwork formerly involved in running preliminary centerline, getting cross-section data, and making slope-stake and other measurements on long route surveys is now being done more efficiently by photogrammetry. Research has shown that earthwork quantities computed from photogrammetric cross-sectioning agree to within about 1 or 2% of those obtained from good-quality field cross sections. It is not intended to discuss photogrammetric methods in this chapter; rather, basic field and office procedures for determining and calculating volumes will be presented briefly. Chapter 27 discusses the subject of photogrammetry.

In Section 17.8, the subject of terrain representation by means of digital elevation models (DEMs) was introduced, and concepts for deriving triangulated irregular networks (TINs) from DEMs were presented. It was noted that once a TIN model is created for a region, profiles and cross sections anywhere within the area could be readily derived using a computer. This can be of significant advantage where the general location of a proposed road or railroad has been decided upon, but the final alignment is not yet fixed. In those situations, controlling points and breaklines can be surveyed in the region where the facility is expected to be located and a DEM for the area generated. Either ground or photogrammetric methods can be employed for deriving the terrain data. From the DEM information, a TIN model can be created and then the computer can provide cross sections for the analysis of any number of alternate alignments automatically.

After cross sections have been taken and plotted, *design templates* (outlines of base widths and side slopes of the planned excavation or embankment) are superimposed on each plot to define the excavation or embankment to be constructed at each cross-section location. Areas of these sections, called *end areas,* are obtained by computation or by planimeter (see Section 12.9.4). Nowadays, using computers, end areas are calculated directly from field cross-section data and design information. From the end areas, volumes are determined by the *average-end-area*, or *prismoidal* formula, discussed later in this chapter.

Figure 26.1 portrays a section of planned highway construction and illustrates some of the points just discussed. Centerline stakes are shown in place, with their stationing given in the English system of units. They mark locations where cross sections are taken, in this instance at full stations. End areas, based on the planned grade line, size of roadway, and selected embankment and excavation

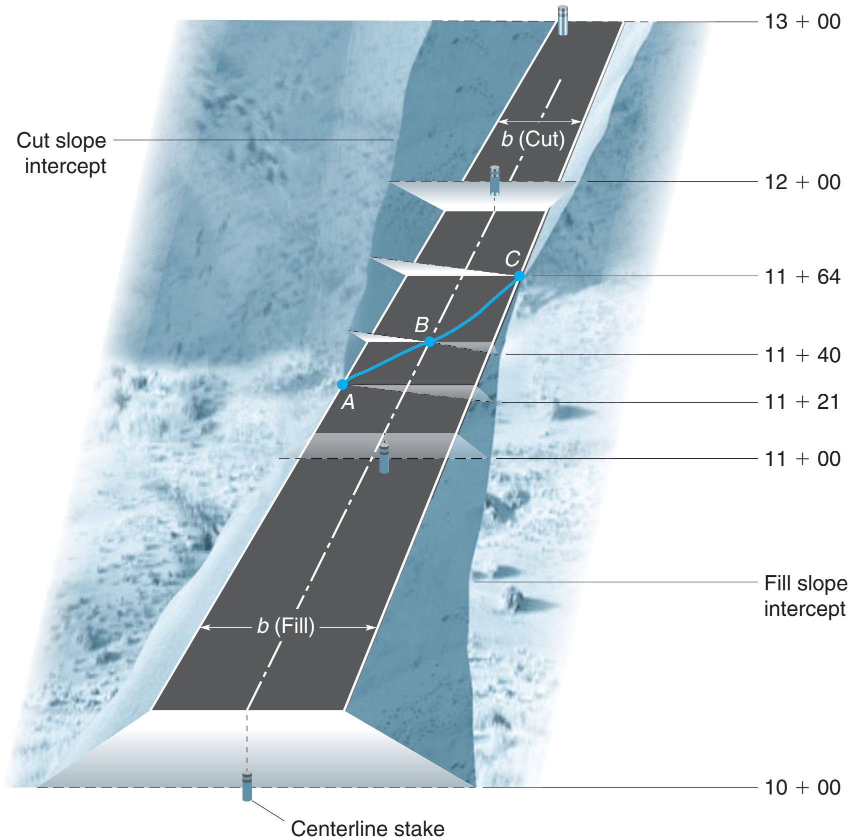

Figure 26.1 Section of roadway illustrating excavation (cut) and embankment (fill).

slopes, are superimposed at each station and are shown shaded. Areas of these shaded sections are determined, whereupon volumes are computed using formulas given in Section 26.5 or 26.8. Note in the figure that embankment, or *fill,* is planned from stations 10 + 00 through 11 + 21, a transition from fill to excavation, or *cut*, occurs from station 11 + 21 to 11 + 64, and cut is required from stations 11 + 64 through 13 + 00.

■ 26.4 TYPES OF CROSS SECTIONS

The types of cross sections commonly used on route surveys are shown in Figure 26.2. In flat terrain the *level section* (a) is suitable. The *three-level section* (b) is generally used where ordinary ground conditions prevail. Rough topography may require a *five-level section* (c), or more practically an *irregular section* (d). A *transition section* (e) and a *side-hill section* (f) occur when passing from cut to fill and on side-hill locations. In Figure 26.1, transition sections occur at stations 11 + 21 and 11 + 64, while a *side-hill section* exists at 11 + 40.

The width of base *b*, the finished roadway, is fixed by project requirements. As shown in Figure 26.1, it is usually wider in cuts than on fills to provide for drainage ditches. The side slope *s* [the horizontal dimension required for a unit vertical rise

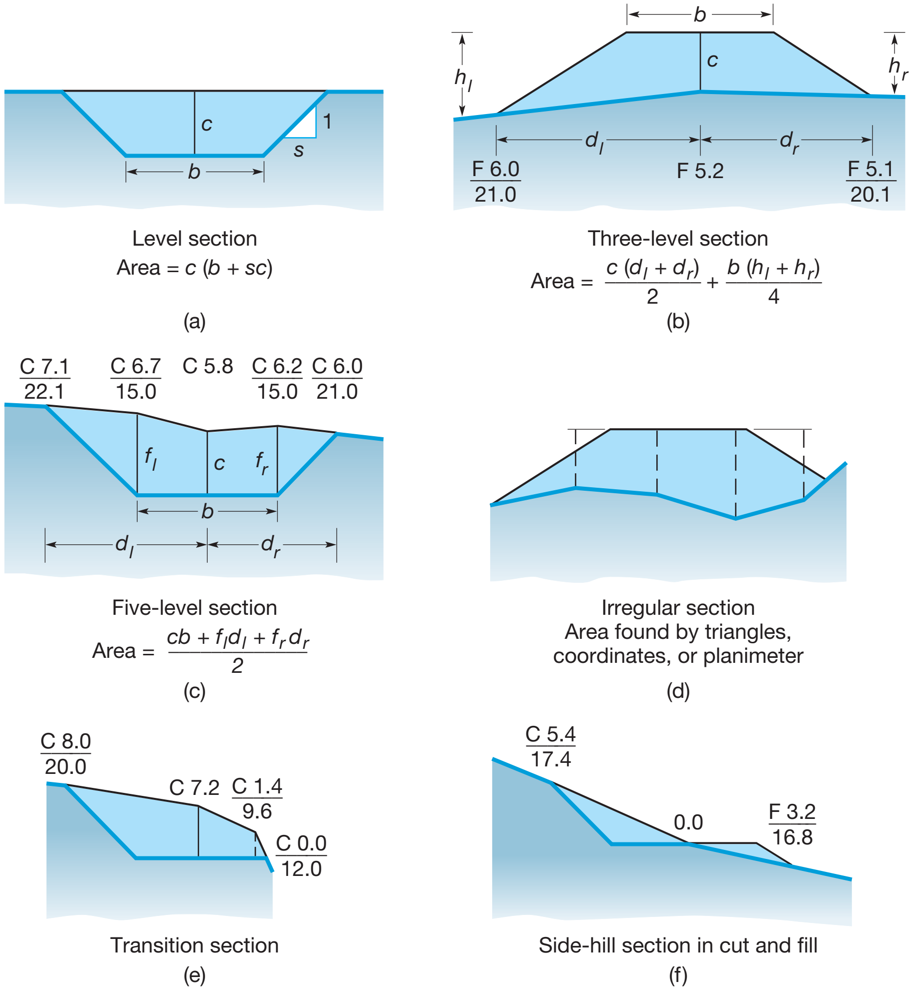

Figure 26.2
Earthwork sections.

and illustrated in Figure 26.2(a)] depends on the type of soil encountered. Side slopes in fills usually are flatter than those in cuts where the soil remains in its natural state.

Cut slopes of 1:1 (1 horizontal to 1 vertical) and fill slopes of 1-1/2:1 might be satisfactory for ordinary loam soils, but 1-1/2:1 in excavation and 2:1 in embankment are common. Even flatter proportions may be required—one cut in the Panama Canal area was 13:1—depending on soil type, rainfall, and other factors. Formulas for areas of sections are readily derived and listed with some of the sketches in Figure 26.2.

■ 26.5 AVERAGE-END-AREA FORMULA

Figure 26.3 illustrates the concept of computing volumes by the average-end-area method. In the figure, A_1 and A_2 are end areas at two stations separated by a horizontal distance L. The volume between the two stations is equal to the average of the end areas multiplied by the horizontal distance L between them.

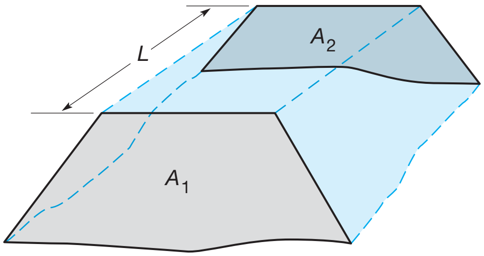

Figure 26.3 Volume by average-end-area-method.

Thus,

$$V_e = \frac{A_1 + A_2}{2} \times \frac{L}{27}(\text{yd}^3) \quad \textbf{(26.1a)}$$

or

$$V_e = \frac{A_1 + A_2}{2} \times L(\text{m}^3) \quad \textbf{(26.1b)}$$

In Equation (26.1a), V_e is the average-end-area volume in cubic yards, A_1 and A_2 are in square feet, and L is in feet. In Equation (26.1b), A_1 and A_2 are in m^2, L is in m, and V_e is in m^3. Equations (26.1b) also applies to computing volumes in acre-ft, where A_1 and A_2 are in acres, and L is in ft.

If L is 100 ft, as for full stations in the English system of units, Equation (26.1a) becomes

$$V_e = 1.852(A_1 + A_2)\text{yd}^3 \quad \textbf{(26.2)}$$

Equations (26.1) and (26.2) are approximate and give answers that generally are slightly larger than the true prismoidal volumes (see Section 26.8). They are used in practice because of their simplicity, and contractors are satisfied because pay quantities are generally slightly greater than true values. Increased accuracy is obtained by decreasing the distance L between sections. When the ground is irregular, cross sections must be taken closer together.

Example 26.1

Compute the volume of excavation between station 24 + 00, with an end area of 711 ft^2, and station 25 + 00, with an end area of 515 ft^2.

Solution

By Equation (26.2), $V = 1.852(A_1 + A_2) = 1.852(711 + 515) = 2270\ \text{yd}^3$.

26.6 DETERMINING END AREAS

End areas can be determined either graphically or by computation. In graphic methods, the cross section and template are plotted to scale on grid paper; then the number of small squares within the section can be counted and converted to area, or the area within the section can be measured using a planimeter (see

Section 12.9.4). Computational procedures consist of either dividing the section into simple figures such as triangles and trapezoids, and computing and summing these areas, or using the coordinate formula (see Section 12.5). These computational methods are discussed in the sections that follow. Most such calculations are now done by computer—usually by the coordinate method, which is general and readily programmed.

26.6.1 End Areas by Simple Figures

To illustrate the procedures of calculating end areas by simple figures such as triangles or trapezoids, assume the following excerpt of field notes (in the English system of units) applies to the cross section and end area, shown in Figure 26.4. In the notes, Lt indicates that the readings were started on the left side of the reference line as viewed facing in the direction of increasing stationing.

HI = 879.29 ft

	867.3	870.9	874.7	876.9	869.0	872.8
24 + 00 Lt	12.0	8.4	4.6	2.4	10.3	6.5
	50	36	20	CL	12	50

In this excerpt of field notes, the top numbers are elevations (in ft) obtained by subtracting rod readings (middle numbers) from the leveling instrument's HI. Bottom numbers are distances from centerline (in ft), beginning from the left. Assume the design calls for a level roadbed of 30-ft width, cut slopes of 1-1/2:1, and a subgrade elevation at station 24 + 00 of 858.9 ft. A corresponding *design template* is superimposed over the plotted cross section in Figure 26.4. Subtracting the subgrade elevation from cross-section elevations at C, D, and E yields the ordinates of cut required at those locations. Elevations and distances out from centerline to the slope intercepts at L and R must be either scaled from the plot or computed. Assuming they have been scaled (methods for computing them are

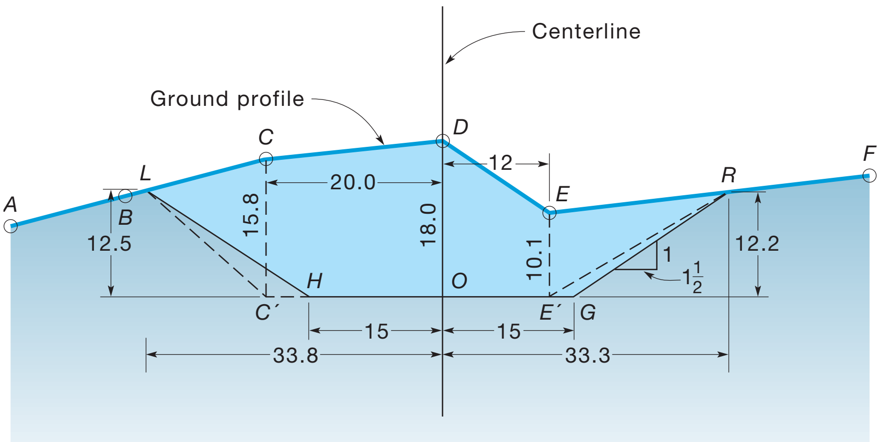

Figure 26.4 End-area computation.

given in Section 26.7), the following tabulation of distances from centerline and required cut ordinates at each point to subgrade elevation was made:

Station	*H*	*L*	*C*	*D*	*E*	*R*	*G*
24 + 00	$\frac{0}{15}$	$\frac{C12.5}{33.8}$	$\frac{C15.8}{20}$	$\frac{C18.0}{0}$	$\frac{C10.1}{12}$	$\frac{C12.2}{33.3}$	$\frac{0}{15}$

Numbers above the lines in the fractions (preceded by the letter *C*) are cut ordinates in feet; those below the lines are corresponding distances out from the centerline. Fills are denoted by the letter *F*. Using *C* instead of plus for cut and *F* instead of minus for fill eliminates confusion. From the cut ordinates and distances from centerline shown, the area of the cross section in Figure 26.4 is computed by summing the individual areas of triangles and trapezoids. A list of the calculations is given in Table 26.1. (Refer to Figure 26.4 for triangle and trapezoid designations.)

26.6.2 End Areas by Coordinates

The coordinate method for computing end areas can be used for any type of section, and has many engineering applications. The procedure was described in Section 12.5 as a way to determine the area contained within a closed polygon traverse.

To demonstrate the method in end-area calculation, the example of Figure 26.4 will be solved. Coordinates of each point of the section are calculated in an axis system having point *O* as its origin, using the earlier listed data on cuts and distances from centerline. In computing coordinates, distances to the right of centerline and cut values are considered plus; distances left and fill values are minus. Beginning with point *O* and proceeding clockwise around the figure, the coordinates of each point are listed in sequence. *Point O is repeated at the end* (see Table 26.2). Then Equation (12.7) is applied, with products of diagonals downward to the right (↘) considered minus, and diagonal products down to the left (↙) plus. Algebraic signs of the coordinates must be considered. Thus, a positive product (↙) having a negative coordinate will be minus. The total area is

TABLE 26.1 END AREA BY SIMPLE FIGURES

Figure	Computation	Area
ODCC′	[(18.0 + 15.8)20]/2	338.0
C′CL	[(15.8)13.8]/2	109.0
HLC′	[−(5)12.5]/2	−31.2
ODEE′	[(18.0 + 10.1)12]/2	168.6
EE′R	[(10.1)21.3]/2	107.6
E′RG	[(3)12.2]/2	18.3
		Area = 710 ft^2

TABLE 26.2 END AREAS BY COORDINATES

Point	X	Y	Plus +	Minus −
O	0	0	↙	↘
H	−15	0	0	0
L	−33.8	12.5	0	+188
C	−20	15.8	−250	+534
D	0	18.0	0	+360
E	12	10.1	216	0
R	33.3	12.2	336	−146
G	15	0	183	0
O	0	0	0	0
			+485	+936
			+936	
			Σ = 1421	
			Area = 1421 ÷ 2 = 710 ft²	(nearest ft²)

obtained by dividing the absolute value of the algebraic summation of all products by 2. The calculations are illustrated in Table 26.2.

It is necessary to make separate computations for cut and fill end areas when they occur in the same section (as at station 11 + 40 of Figure 26.1), since they must always be tabulated independently for pay purposes. Payment is normally made only for excavation (its unit price includes making and shaping the fills) except on projects consisting primarily of embankment such as levees, earth dams, some military fortifications, and highways built up by continuous fills in flat areas.

■ 26.7 COMPUTING SLOPE INTERCEPTS

The elevations and distances out from the centerline to the slope intercepts can be calculated using cross-section data and the cut or fill slope values. In Figure 26.4, for example, intercept *R* occurs between ground profile point *E* (distance 12 ft right and elevation 869.0) and point *F* (distance 50 ft right and elevation 872.8). The cut slope is 1-1/2:1, or 0.67 ft/ft. A more detailed diagram, illustrating the geometry for calculating slope intercept *R*, is given in Figure 26.5.

The slope along ground line *EF* is $(872.8 - 869.0)/38 = 0.10$ ft/ft, where 38 ft is the horizontal distance between the points. The elevation of G' (point vertically above *G*) is $869.0 + 0.10(3) = 869.3$; thus ordinate GG' is $(869.3 - 858.9) = 10.4$ ft. Lines *EF* and *GR* converge at a rate equal to the difference in their slopes (because they are both sloping upward), or $0.67 - 0.10 = 0.57$ ft/ft. Dividing ordinate GG' by this convergence yields horizontal distance *GR*, or $10.4/0.57 = 18.3$ ft. Adding 18.3 to distance *OG* yields $18.3 + 15 = 33.3$ ft, which is the distance from centerline to slope intercept *R*. Finally, to obtain the elevation of *R*, the increase in elevation from *E* to *R* is added to the elevation of *E*, or $0.10(21.3) + 869.0 = 871.1$.

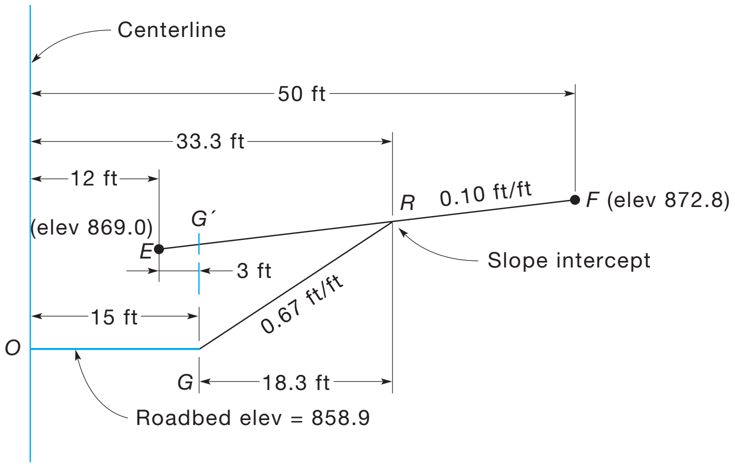

Figure 26.5 Computation of slope intercept *R* of Figure 26.4.

Thus, the cut ordinate at *R* equals 871.1 − 858.9 = 12.2 ft. Recall that 33.3 and 12.2 were the *X* and *Y* coordinates, respectively, used in the end-area calculations of Section 26.6.

The elevation and distance from centerline of the slope intercept *L* of Figure 26.4 are calculated in a similar manner, except the rate of convergence of lines *CB* and *HL* is the sum of their slopes because *CB* slopes downward and *HL* upward. Writing equations for the intersecting lines is another method used to compute slope intercepts. The equations are then set equal to each other and solved for *x*. This is demonstrated using the data in Figure 26.5 and Equation (11.6) in Example 26.2.

Example 26.2

Determine the coordinates of point *R* in Figure 26.5 using Equation (11.6).

Solution

The coordinates for the pertinent endpoints are

Point	*x*	*y*
E	12	869.0
F	50	872.8
G	15	858.9

The equation for line *EF* is

$$y = \left(\frac{872.8 - 869.0}{50 - 12}\right)x + b = 0.1x + b$$

Substituting in the coordinates for either *E* or *F*, we find that *b* is 867.8. Thus the equation for line *EF* is

$$y = 0.1x + 867.8 \tag{a}$$

The slope intercept for the side slope equation can be determined using the slope of the side slope, which is 2/3, and the coordinates of point G as

$$b = 858.9 - (2/3)15 = 848.9$$

Thus, the line equation for the side slope is

$$y = (2/3)x + 848.9 \tag{b}$$

Setting Equation (a) equal to (b) and solving for x yields

$$0.1x + 867.8 = \frac{2}{3}x + 848.9$$

$$18.9 = \left(\frac{2}{3} - 0.1\right)x$$

$$x = 33.3$$

Using either Equation (a) or (b), the elevation at 33.3 is

$$y = 0.1(33.3) + 867.8 = \left(\frac{2}{3}\right)33.3 + 848.9$$

$$= 871.1$$

Note that this procedure results in the same solution as that previously determined.

Calculations of slope intercepts are somewhat laborious, but routine when programmed for solution by computer. If a computer is not used for computing end areas and volumes, an alternate procedure is to plot the cross sections and templates, determine the end area by planimeter, and scale the slope intercepts from the plot. Slope intercepts are essential since the placement of slope stakes that guide construction operations is based on them.

26.8 PRISMOIDAL FORMULA

The prismoidal formula applies to volumes of all geometric solids that can be considered prismoids. A prismoid, illustrated in Figure 26.6, is a solid having ends that are parallel but not congruent and trapezoidal sides that are also not congruent. Most earthwork solids obtained from cross-section data fit this classification.

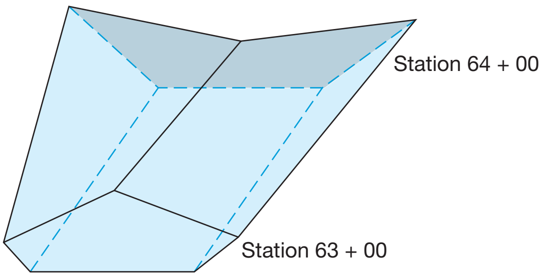

Figure 26.6 Sections for which the prismoidal correction is added to the end-area volume.

However, from a practical standpoint, the differences in volumes computed by the average-end-area method and the prismoidal formula are usually so small as to be negligible. Where extreme accuracy is needed, such as in expensive rock cuts, the prismoidal method can be used.

One arrangement of the prismoidal formula is

$$V_P = \frac{L(A_1 + 4A_m + A_2)}{6 \times 27} (\text{yd}^3) \tag{26.3}$$

where V_P is the prismoidal volume in cubic yards, A_1 and A_2 are areas of successive cross sections taken in the field, A_m is the area of a "computed" section midway between A_1 and A_2, and L is the horizontal distance between A_1 and A_2. Prismoidal volumes in m^3 can be obtained by using a slight modification of Equation (26.3), that is, the conversion factor 27 in the denominator is dropped, and L is in meters, A_1, A_m, and A_2 are in m^2.

To use the prismoidal formula, it is necessary to know area A_m of the section halfway between the stations of A_1 and A_2. This is found by the usual computation *after averaging the heights and widths of the two end sections*. Obviously, the middle area is not the average of the end areas, since there would then be no difference between the results of the end-area formula and the prismoidal formula.

The prismoidal formula generally gives a volume smaller than that found by the average-end-area formula. For example, the volume of a pyramid by the prismoidal formula is $Ah/3$ (the exact value), whereas by the average-end-area method it is $Ah/2$. An exception occurs when the center height is great but the width narrow at one station, and the center height small but the width large at the adjacent station. Figure 26.6 illustrates this condition.

The difference between the volumes obtained by the average-end-area formula and the prismoidal formula is called the *prismoidal correction* C_p. Various books on route surveying give formulas and tables for computing prismoidal corrections, which can be applied to average-end-area volumes to get prismoidal volumes. A prismoidal correction formula, which provides accurate results for three-level sections, is

$$C_P = \frac{L}{12 \times 27}(c_1 - c_2)(w_1 - w_2)(\text{yd}^3) \tag{26.4}$$

where C_P is the volume of the prismoidal correction in cubic yards, c_1 and c_2 are center heights in cut (or in fill), and w_1 and w_2 are widths of sections (from slope intercept to slope intercept) at adjacent sections. If the product of $(c_1 - c_2)(w_1 - w_2)$ is minus, as in Figure 26.6, the prismoidal correction is added rather than subtracted from the end-area volume. For sections other than three-level, Equation (26.4) may not be accurate enough, and therefore Equation (26.3) is recommended.

Example 26.3

Compute the volume using the prismoidal formula and by average end areas for the following three-level sections of a roadbed having a base of 24 ft and side slopes of $1\frac{1}{2}$:1.

Solution

Station	L	C	R	Area
12 + 00	$\frac{C7.8}{23.7}$	$\frac{C5.3}{0}$	$\frac{C7.4}{23.0}$	$\frac{5.3(23.7 + 23.0)}{2} + \frac{24(7.8 + 7.4)}{4} = 215.0 \text{ ft}^2$
12 + 50	$\frac{C6.5}{21.8}$	$\frac{C6.0}{0}$	$\frac{C7.5}{23.2}$	$\frac{6.0(21.8 + 23.2)}{2} + \frac{24(6.5 + 7.5)}{4} = 219.0 \text{ ft}^2$
13 + 00	$\frac{C5.8}{24.8}$	$\frac{C6.6}{0}$	$\frac{C7.0}{23.5}$	$\frac{6.6(24.8 + 23.5)}{2} + \frac{24(5.8 + 7.0)}{4} = 236.2 \text{ ft}^2$

Using Equation (26.3) yields a volume of

$$\frac{100(215.0 + 4(219.0) + 236.2)}{6(27)} = 819.2 \text{ yd}^3$$

Using Equation (26.1a) yields $\dfrac{100(215.0 + 236.2)}{2(27)} = 835.6 \text{ yd}^3$

Using Equation (26.5) yields a prismoidal correction of

$$\frac{100}{12(27)}(5.3 - 6.6)(46.7 - 48.3) = 0.6 \text{ yd}^3$$

Note that the difference between the volume computed by the prismoidal formula and the average end area is only 1.9%. The prismoidal correction applied to Equation (26.1a) yields a volume of 835 yd^3.

■ 26.9 VOLUME COMPUTATIONS

Volume calculations for route construction projects are usually done by computer and arranged in tabular form. To illustrate this procedure, assume that end areas listed in columns (2) and (3) of Table 26.3 apply to the section of roadway illustrated in Figure 26.1. By using Equation (26.1a), cut and fill volumes are computed and tabulated in columns (4) and (5).

The volume computations illustrated in Table 26.3 include the transition sections of Figure 26.1. This is normally not done when preliminary earthwork volumes are being estimated (during design and prior to construction) because the exact locations of the transition sections and their configurations are usually unknown until slope staking occurs. Thus, for calculating preliminary earthwork quantities, an end area of zero would be used at the station of the centerline grade point (station 11 + 40 of Figure 26.1), and transition sections (stations 11 + 21 and 11 + 64 of Figure 26.1) would not appear in the computations. After slope staking (procedures for slope staking are described in Section 23.7) the locations and end areas of transition sections are known, and they should be included in final volume computations, especially if they significantly affect the quantities for which payment is made.

TABLE 26.3 TABULAR FORM OF VOLUME COMPUTATION

Station (1)	End Area (ft^2) Cut (2)	Fill (3)	Volume (yd^3) Cut (4)	Fill (5)	Fill Volume +25% (yd^3) (6)	Cumulative Volume (yd^3) (7)
10 + 00		992				0
				2614	3268	
11 + 00		421				−3268
				190	238	
11 + 21	0	68				−3506
			12	29	37	
11 + 40	34	31				−3531
			79	14	17	
11 + 64	144	0				−3469
			553			
12 + 00	686					−3916
			2967			
13 + 00	918					+51

In highway and railroad construction, excavation or cut material is used to build embankments or fill sections. Unless there are other controlling factors, a well-designed grade line should nearly balance total cut volume against total fill volume. To accomplish a balance, either fill volumes must be expanded or cut volumes shrunk.[1] This is necessary because, except for rock cuts, embankments are compacted to a density greater than that of material excavated from its natural state, and to balance earthwork this must be considered. (Rock cut expands to occupy a greater fill volume; thus either the cut must be expanded or the fill shrunk to obtain a balance.) The rate of expansion depends on the type of material and can never be estimated exactly. However, samples and records of past projects in the immediate area are helpful in assigning reasonable factors. Column (6) of Table 26.3 lists expanded fills for the example of Figure 26.1, where a 25% factor was applied.

To investigate whether or not an earthwork balance is achieved, *cumulative volumes* are computed. This involves adding cut and expanded fill volumes algebraically from project beginning to end, with cuts considered positive and fills negative. Cumulative volumes are listed in column (7) of Table 26.3. In this example,

[1]Expansion of fill volumes is generally preferred, since payment is usually based on actual volumes of material excavated.

there is a cut volume excess of 51 yd^3 between stations 10 + 00 and 13 + 00 or, in other words, there is a surplus of that much excavation.

To analyze the movement of earthwork quantities on large projects, *mass diagrams* are constructed. These are plots of cumulative volumes for each station as the ordinate versus the stations on the abscissa. Horizontal (balance) lines on the mass diagram then determine the limit of economic haul and the direction of movement of material. Mass diagrams are described more thoroughly in books on route surveying. If there is insufficient material from cuts to make the required fills, the difference must be *borrowed* [obtained from borrow pits or other sources such as by "day-lighting" curves (flattening cut slopes to improve visibility)]. If there is excess cut, it is wasted or perhaps used to extend and flatten the fills.

For projects with more than a few cross sections, computer programs are available and are generally used for earthwork computations, but surveyors and engineers must still understand the basic methods.

■ 26.10 UNIT-AREA, OR BORROW-PIT, METHOD

On many projects, except long linear route constructions, the quantity of earth, gravel, rock, or other material excavated or filled can often best be determined by the borrow-pit method. The quantities computed form the basis for payment to the contractor or supplier of materials. The volume of coal or other loose materials in stockpiles can be found in the same way.

As an example, assume the area shown in Figure 26.7 is to be graded to an elevation of 358.0 ft for a building site. Notes for the fieldwork are shown in Plate B.2 of Appendix B. The area to be covered in this example is staked in squares of 20 ft, although 10, 50, 100, or more feet could be used, with the choice depending on project size and accuracy desired. A total station instrument and tape, or only a tape, may be used for the layout. A benchmark of known or assumed elevation is established outside the area in a place not likely to be disturbed.

After the area is laid out in squares, elevations are determined at all grid intersection points. For this, a level is set up at any convenient location, a plus sight

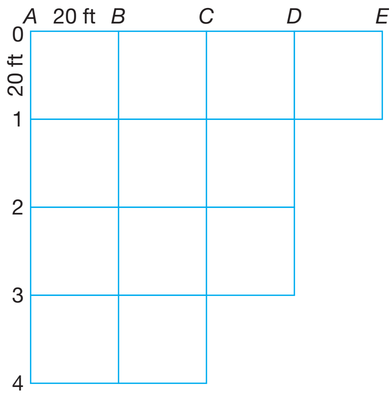

Figure 26.7
Borrow-pit leveling.

taken on the benchmark, and minus sights read on each grid intersection. If the terrain is not too rough, it may be possible to select a point near the area center and take sights on all grid intersections from the same setup, as in the example of Plate B.2. For rough terrain, it may be most convenient to determine the elevations by radial surveying from one well-chosen setup using a total station instrument (see Section 16.9.1).

Letters and numbers designate grid intersection points, such as *A*-1, *C*-4, and *D*-2. For site grading to a specified elevation, say 358.0 ft, the amount of cut or fill at each grid square corner is obtained by subtracting 358.0 from its ground elevation. For each square, then, the average height of the four corners of each prism of cut or fill is determined and multiplied by the base area, 20×20 ft $= 400$ ft^2, to get the volume. The total volume is found by adding the individual values for each block and dividing by 27 to obtain the result in cubic yards.

To simplify calculations, the cut at each corner multiplied by the number of times it enters the volume computation can be shown in a separate column. The column sum is divided by 4 and multiplied by the base area of one block to get the volume. In equation form, this procedure is given as

$$V = \sum(h_{i,j}n)\left(\frac{A}{4 \times 27}\right)(\text{yd}^3) \qquad \textbf{(26.5)}$$

where $h_{i,j}$ is the corner height in row i and column j, and n the number of squares to which that height is common. The corner at *C*-4, for example, is common to only one square, *D*-2 is common to two, *D*-1 is common to three, and *C*-1 is common to four. $\Sigma(h_{i,j}n)$ is the sum of the products of the height and the number of common squares, and A is the area of one square. An example illustrating the use of Equation (26.5) is given in the field notes of Plate B.2.

■ 26.11 CONTOUR-AREA METHOD

Volumes based on contours can be obtained from contour maps by using a planimeter to determine the area enclosed by each contour. Alternatively, CAD software can be used to determine these areas. Then the average area of the adjacent contours is obtained using Equation (26.1b) and the volume obtained by multiplying by the contour spacing (i.e., contour interval). Use of the prismoidal formula is seldom, if ever, justified in this type of computation. This procedure is the basis for volume computations in CAD software (see Section 17.14).

Instead of determining areas enclosed within contours by planimeter, they can be obtained using the coordinate formula [Equation (12.7) or (12.8)]. In this procedure a tablet digitizer like the one shown in Figure 28.8 is first used to measure the coordinates along each contour at enough points to define its configuration satisfactorily.

The contour-area method is suitable for determining volumes over large areas, for example, computing the amounts and locations of cut and fill in the grading for a proposed airport runway to be constructed at a given elevation. Another

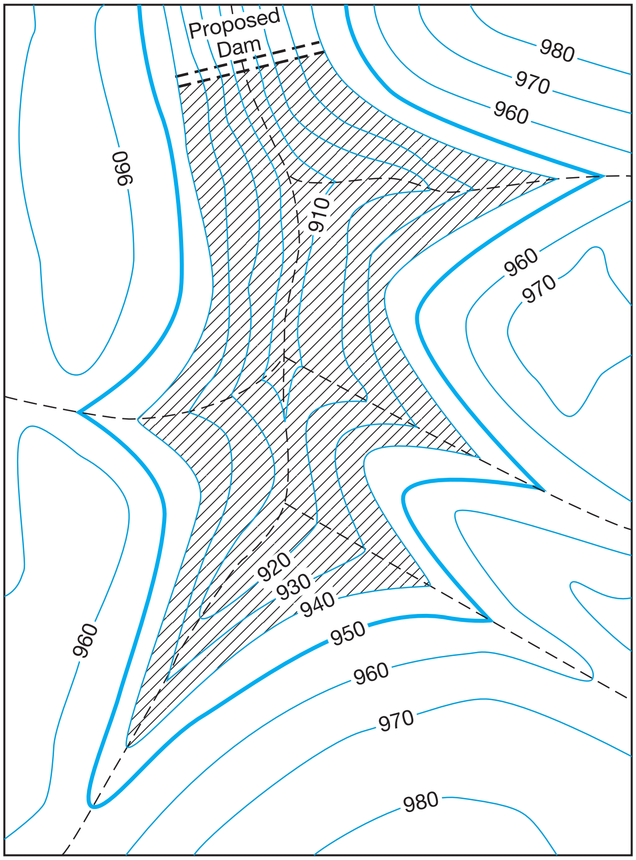

Figure 26.8 Determining the volume of water impounded in a reservoir by the contour-area method.

useful application of the contour-area method is in determining the volume of water that will be impounded in the reservoir created by a proposed dam.

Example 26.4

Compute the volume of water impounded by the proposed dam illustrated in Figure 26.8. Map scale is 500 ft/in. and the proposed spillway elevation 940 ft.

Solution

The crosshatched portion of Figure 26.8 represents the area that will be inundated with water when the reservoir is full. The solution is presented in Table 26.4. Column (2) gives the area enclosed within each contour (determined by using a tablet digitizer and the coordinate method) in square inches, and in column (3) these areas have been converted to acres based on map scale, that is, 1 in.^{2} = $[(500)^2]/43{,}560 = 5.739$ acres. Column (4) gives the volumes between adjacent contours, computed by Equation [26.1(b)]. The sum of column (4), 1544.3 acre-ft, is the volume of the reservoir.

TABLE 26.4 VOLUME COMPUTATION BY CONTOUR-AREA METHOD

Contour (1)	Area (in.^{2}) (2)	Area (acres) (3)	Volume (acre-ft) (4)
910	1.683	9.659	—
920	5.208	29.889	197.7
930	11.256	64.598	472.4
940	19.210	110.246	874.2
			Σ = 1544.3

26.12 MEASURING VOLUMES OF WATER DISCHARGE

Volumes of water discharge in streams and rivers are a matter of vital concern, and must be monitored regularly. In the usual procedure, the stream's cross section is broken into a series of uniformly spaced vertical sections, as illustrated in Figure 26.9. The U.S. Geological Survey recommends using from 25 to 30 sections, with not more than 5% of the total flow occurring in any particular section. Depths and current velocities are measured at each ordinate using a *current meter.* (There are various types available.) The discharge volume for each section is the product of its area and average current velocity. The sum of all section discharges is the total volume of water passing through the stream at the cross-section location. Units of section areas and current velocities can be either ft^2 and ft/sec, respectively, with the discharge in ft^3/sec; or in m^2 and m/sec, respectively, giving the volume in m^3/sec.

Current velocities can be measured at every 0.1 of the depth at each ordinate and the average taken. Alternatively, a good average results from the mean of the 0.2 and 0.8 depth velocities or a single measurement at the 0.6 depth point. For depths up to 2-1/2 ft, the U.S. Geological Survey uses the 0.6 method; for deeper sections the 0.2 and 0.8 procedure is employed.

The cross section should be taken at right angles to the stream, and in a straight reach with solid bottom and uniform flow. In shallow streams, measurements can be made by wading, in which case, the current meter is held upstream free from eddies caused by the wader's legs. In deeper streams and rivers, measurements are taken from boats, bridges, or overhead cable cars. In these situations, the current meter, with a heavy weight attached to its bottom, is suspended by a cable and thus doubles as a lead line for measuring depths.

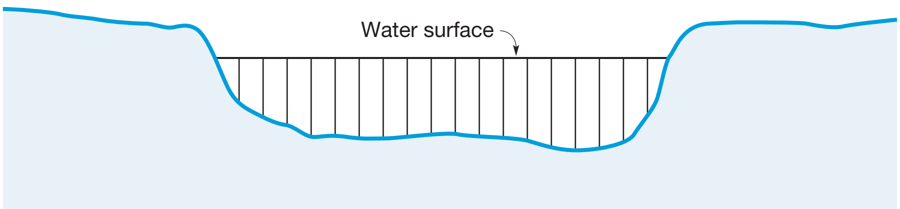

Figure 26.9 Vertical sections for making stream discharge measurements.

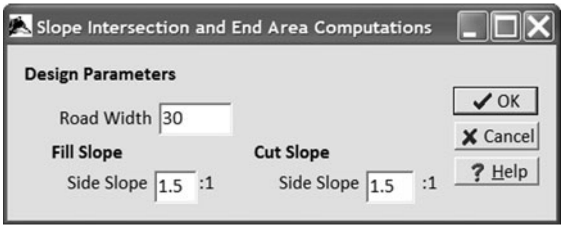

Figure 26.10 End-area computations in WOLFPACK.

■ 26.13 SOFTWARE

As discussed in Section 26.7, the intersection of two lines can be easily programmed to determine the slope intercepts of an end area. As shown in Figure 26.10, WOLFPACK, which can be found on the companion website at http://www.pearsonhighered.com/ghilani, has been programmed to perform this operation. As shown in this Figure, the user must supply the width of the bed, cut slope ratio, and fill slope ratio. In Section 26.7, the roadbed width was 30 ft and had a cut slope of 1-1/2:1. These values are entered in the dialog box as shown. Also note that only one set of design parameters may be used per file. The data file used by this option is shown in Figure 26.11. Each file begins with a title line, which can contain any information pertinent to the file. The title line is followed by the stationing of the

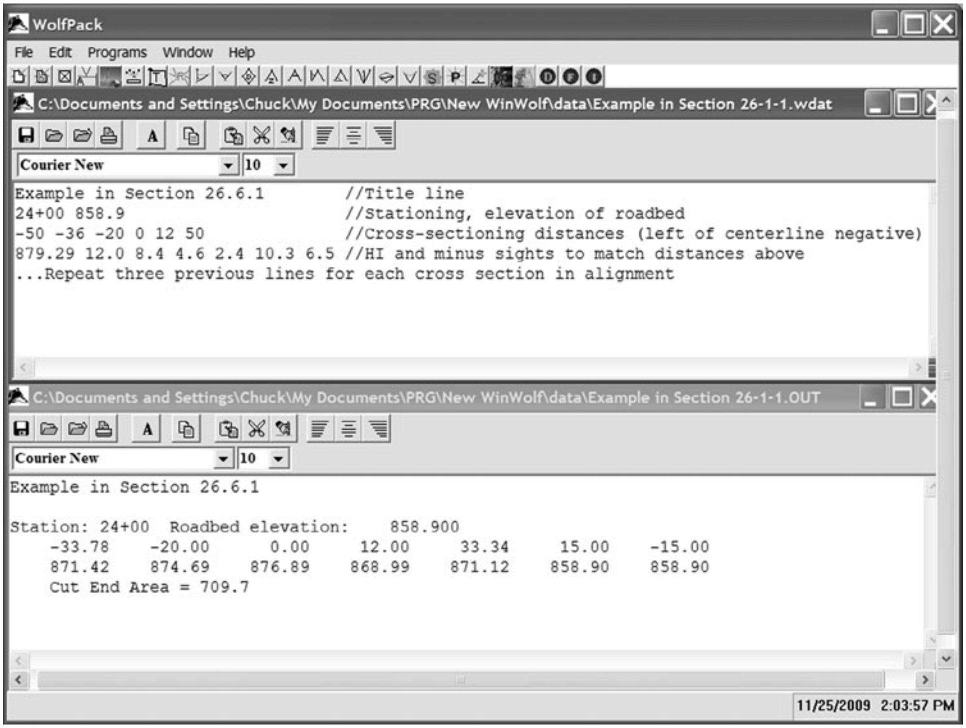

Figure 26.11 Data file and resultant end area computations file.

first station in the alignment and its elevation. This line is followed by the cross-sectioning information. Distances are entered from left to right as viewed looking forward on the alignment. Cross-sectioning distances to the left of the centerline should be entered as negative values. The distances are followed by a line containing the height of the instrument (HI), and the minus sights that should match their respective distances from the centerline given in the previous line. If more than one set of cross-sectioning notes is available for the alignment, the data for the cross sections can follow the first station's set of notes in succession. At the bottom of Figure 26.11 is the resultant output file. As shown, the left intercept occurs at –33.78 with an elevation of 871.42 ft. The right intercept occurs at 33.34 ft with an elevation of 871.12 ft. The end area is a cut section with an area of 709.7 ft^2.

26.14 SOURCES OF ERROR IN DETERMINING VOLUMES

Some common errors in determining areas of sections and volumes of earthwork are:

1. Making errors in measuring field cross sections, for example, not being perpendicular to the centerline.
2. Making errors in measuring end areas.
3. Failing to use the prismoidal formula where it is justified.
4. Carrying out areas of cross sections beyond the limit justified by the field data.

26.15 MISTAKES

Some typical mistakes made in earthwork calculations are:

1. Confusing algebraic signs in end-area computations using the coordinate method.
2. Using Equation (26.2) for full-station volume computation when partial stations are involved.
3. Using average end-area volumes for pyramidal or wedge-shaped solids.
4. Mixing cut and fill quantities.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

26.1 Why must cut and fill volumes be totaled separately?

26.2 Prepare a table of end areas versus depths of fill from 0 to 20 ft by increments of 4 ft for level sections, a 36-ft-wide level roadbed, and side slopes of 1-1/2:1.

26.3 Similar to Problem 26.2, except use side slopes of 2-1/2:1.

Draw the cross sections and compute V_e for the data given in Problems 26.4 through 26.7.

26.4* Two level sections 75 ft apart with center heights 4.8 and 7.2 ft in fill, base width 30 ft, side slopes 2:1.

26.5 Two level sections of 40-m stations with center heights of 2.04 and 2.53 m. in cut, base width 15 m, side slopes 3:1.

26.6 The end area at station 36 + 00 is 265 ft^2. Notes giving distance from centerline and cut ordinates for station 36 + 60 are C 4.8/17.2, C 5.9/0, and C 6.8/20.2. Base is 20 ft.

26.7 An irrigation ditch with $b = 12$ ft and side slopes of 2:1. Notes giving distances from centerline and cut ordinates for stations 52 + 00 and 53 + 00 are C 2.4/10.8; C 3.0; C 3.7/13.4; and C 3.1/14.2; C 3.8; C 4.1/14.2.

26.8 Why is a roadway in cut normally wider than the same roadway in fill?

26.9* For the data tabulated, calculate the volume of excavation in cubic yards between stations 10 + 00 and 15 + 00.

Station	Cut End Area (ft^2)
10 + 00	263
11 + 00	358
12 + 00	446
13 + 00	402
14 + 00	274
15 + 00	108

26.10 For the data listed, tabulate cut, fill, and cumulative volumes in cubic yards between stations 10 + 00 and 20 + 00. Use an expansion factor of 1.30 for fills.

	End Area (ft^2)	
Station	Cut	Fill
10 + 00	0	
11 + 00	168	
12 + 00	348	
13 + 00	371	
14 + 00	146	
14 + 60	0	0
15 + 00		142
16 + 00		238
17 + 00		305
18 + 00		247
19 + 00		138
20 + 00		106

26.11 Calculate the section areas in Problem 26.4 by the coordinate method.

26.12 Compute the section areas in Problem 26.5 by the coordinate method.

26.13 Determine the section areas in Problem 26.7 by the coordinate method.

26.14* Compute C_P and V_P for Problem 26.4. Is C_P significant?

26.15 Calculate C_P and V_P for Problem 26.7. Would C_P be significant in rock cut?

26.16 From the following excerpt of field notes, plot the cross section on graph paper and superimpose on it a design template for a 30-ft-wide level roadbed with fill slopes of 2-1/2:1 and a subgrade elevation at centerline of 970.30 ft. Determine the end area graphically by counting squares.

HI = 968.31 ft						
20 + 00 Lt	$\frac{5.2}{50}$	$\frac{4.8}{22}$	$\frac{6.6}{0}$	$\frac{5.9}{12}$	$\frac{7.0}{30}$	$\frac{8.1}{50}$

26.17 For the data of Problem 26.16, determine the end area by plotting the points in a CAD package, and listing the area.

26.18 For the data of Problem 26.16, calculate slope intercepts, and determine the end area by the coordinate method.

26.19 From the following excerpt of field notes, plot the cross section on graph paper and superimpose on it a design template for a 40-ft-wide level roadbed with cut slopes of 3:1 and a subgrade elevation of 1239.50 ft. Determine the end area graphically by counting squares.

HI = 1254.80 ft						
46 + 00 Lt	$\frac{8.0}{60}$	$\frac{7.9}{27}$	$\frac{5.5}{10}$	$\frac{4.9}{0}$	$\frac{6.6}{24}$	$\frac{7.5}{60}$

26.20 For the data of Problem 26.19, calculate slope intercepts and determine the end area by the coordinate method.

26.21* Complete the following notes and compute V_e and V_P. The roadbed is level, the base is 30 ft.

Station 89 + 00	$\frac{C3.1}{24.3}$	$\frac{C4.9}{0}$	$\frac{C4.3}{35.2}$
Station 88 + 00	$\frac{C6.4}{34.2}$	$\frac{C3.6}{0}$	$\frac{C5.7}{32.1}$

26.22 Similar to Problem 26.21, except the base is 24 ft.

26.23 Calculate V_e and V_P for the following notes. Base is 36 ft.

12 + 90	$\frac{C6.4}{43.6}$	$\frac{C3.6}{0}$	$\frac{C5.7}{40.8}$
12 + 30	$\frac{C3.1}{30.4}$	$\frac{C4.9}{0}$	$\frac{C4.3}{35.2}$

26.24 Calculate V_e, C_P, and V_P for the following notes. The base in fill is 20 ft and base in cut is 30 ft.

46 + 00	$\frac{C3.4}{20.1}$	$\frac{C2.0}{0}$	$\frac{C0.0}{6.0}$	$\frac{F2.0}{13.0}$
45 + 00	$\frac{C2.2}{18.3}$	$\frac{0.0}{0}$	$\frac{F3.0}{14.5}$	

For Problems 26.25 and 26.26, compute the reservoir capacity (in acre-ft) between highest and lowest contours for areas on a topographic map.

26.25*

Elevation (ft)	860	870	880	890	900	910
Area (ft^2)	1370	1660	2293	2950	3550	4850

26.26

Elevation (ft)	1015	1020	1025	1030	1035	1040
Area (ft^2)	1815	2097	2391	2246	2363	2649

26.27 State two situations where prismoidal corrections are most significant.

26.28 Write a computer program to calculate slope intercepts and end areas by the coordinate method, given cross-section notes and roadbed design information. Use the program to calculate the slope intercepts for the data of Problem 26.16.

26.29* Distances (ft) from the left bank, corresponding depths (ft), and velocities (ft/sec), respectively, are given for a river discharge measurement. What is the volume in ft^3/sec? 0, 1.0, 0; 10, 2.3, 1.30; 20, 3.0, 1.54; 30, 2.7, 1.90; 40, 2.4, 1.95; 50, 3.0, 1.60; 60, 3.1, 1.70; 74, 3.0, 1.70; 80, 2.8, 1.54; 90, 3.3, 1.24; 100, 2.0, 0.58; 108, 2.2, 0.28; 116, 1.5, 0.

26.30 Prepare a computational program that computes the volumes in Problem 26.9.

26.31 Prepare a computational program that computes the end-areas in Problem 26.20.

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27 Photogrammetry

■ 27.1 INTRODUCTION

Photogrammetry may be defined as the science, art, and technology of obtaining reliable information from photographs. It encompasses two major areas of specialization: *metrical* and *interpretative*. The first area is of principal interest to those involved in surveying (geomatics), since it is applied in determining spatial information including distances, elevations, areas, volumes, cross sections, and data for compiling topographic maps from measurements made on photographs. *Aerial* photographs (exposed from aircraft) are normally used, although in certain special applications, *terrestrial* photos (taken from Earth-based cameras) are employed.

Interpretative photogrammetry involves recognizing objects from their photographic images and judging their significance. Critical factors considered in identifying objects are the shapes, sizes, patterns, shadows, tones, and textures of their images. This area of photogrammetry was traditionally called *photographic interpretation* because initially it relied on aerial photos. Now other sensing and imaging devices such as multispectral scanners, thermal scanners, radiometers, and side-looking airborne radar are used, which aid greatly in interpretation. These instruments sense energy in wavelengths beyond those which the human eye can see, or standard photographic films can record. They are often carried in aircraft as remote as satellites; hence the term, *remote sensing*, is now generally applied to the interpretative area of photogrammetry.

In this chapter, metrical photogrammetry using aerial photographs will be emphasized because it is the area of specialization most frequently applied in surveying work. However, remote sensing has also become very important in small-scale mapping, and in monitoring our environment and managing our natural resources. This subject is discussed further in Section 27.20.

Metrical photogrammetry is accomplished in different ways depending upon project requirements and the type of equipment available. Simple analyses and computations can be made by making measurements on paper prints of aerial photos using engineer's scales, and assuming that the photos are "truly vertical," that is, the camera axis coincided with a plumb line at the time of photography. These methods produce results of lower order, but they are suitable for a variety of applications. Other more advanced techniques, including *analog*, *analytical*, and *softcopy* methods, do not assume vertical photos and provide more accurate determinations of the spatial locations of objects. The analog procedure relies on precise optical and mechanical devices to create models of the terrain that can be measured and mapped. The analytical method is based upon precise measurements of the photographic positions of the images of objects of interest, followed by a mathematical solution for their locations. Softcopy instruments utilize digital images in computerized procedures that are highly automated. While analog and analytical instruments may still exist in academic environments, softcopy instruments are most likely used exclusively in industry. For this reason, readers interested in the analog and analytical instruments should refer to earlier editions of this book or references listed in the bibliography.

■ 27.2 USES OF PHOTOGRAMMETRY

Photography dates back to 1839, and the first attempt to use photogrammetry in preparing a topographic map occurred a year later. Photogrammetry is now the principal method employed in topographic mapping and compiling other forms of spatial data. For example, the U.S. Geological Survey uses the procedure almost exclusively in compiling its maps. Cameras and other photogrammetric instruments and techniques have improved continually, so that spatial data collected by photogrammetry today meets very high accuracy standards. Other advantages of this method are the (1) speed of collecting spatial data in an area, (2) relatively low cost, (3) ease of obtaining topographic details, especially in inaccessible areas, and (4) reduced likelihood of omitting details in spatial data collection.

Photogrammetry presently has many applications in surveying and engineering. For example, it is used in land surveying to compute coordinates of section corners, boundary corners, or points of evidence that help locate these corners. Large-scale maps are made by photogrammetric procedures for many uses, one being subdivision design. Photogrammetry is used to map shorelines in hydrographic surveying, to determine precise ground coordinates of points in control surveying and to develop maps and cross sections for route and engineering surveys. Photogrammetry is playing an increasingly important role in developing the necessary data for modern Land and Geographic Information systems.

Photogrammetry is also being successfully applied in many nonengineering fields, for example, geology, forestry, agriculture, conservation, planning, archeology, military intelligence, traffic management, and accident investigation. It is beyond the scope of this chapter to describe all the varied applications of photogrammetry. Use of the science has increased dramatically in recent years, and its future growth for solving measurement and mapping problems is assured.

■ 27.3 AERIAL CAMERAS

Aerial mapping cameras are perhaps the most important photogrammetric instruments, since they expose the photographs on which the science depends. To understand photogrammetry, especially the geometrical foundation of its equations, it is essential to have a fundamental understanding of cameras and how they operate. Aerial cameras must be capable of exposing a large number of photographs in rapid succession while moving in an aircraft at high speed; so a short cycling time, fast lens, efficient shutter, and large-capacity magazine or digital storage are required.

Single-lens frame cameras are the type most often used in metrical photogrammetry. These cameras expose the entire frame or format simultaneously through a lens held at a fixed distance from the focal plane. Generally they have a format size of 9 × 9 in. (23 × 23 cm) and lenses with focal lengths of 6 in. (152.4 mm), although 3-1/2, 8-1/4, and 12 in. (90, 210, and 305 mm) focal lengths are also used. A single-lens frame camera, together with its viewfinder and electronic controls, is shown in Figure 27.1.

The principal components of a single-lens frame camera are shown in the diagram of Figure 27.2. These include the *lens* (the most important part), which gathers incoming light rays and brings them to focus on the focal plane; the *shutter* to control the interval of time that light passes through the lens; a diaphragm to

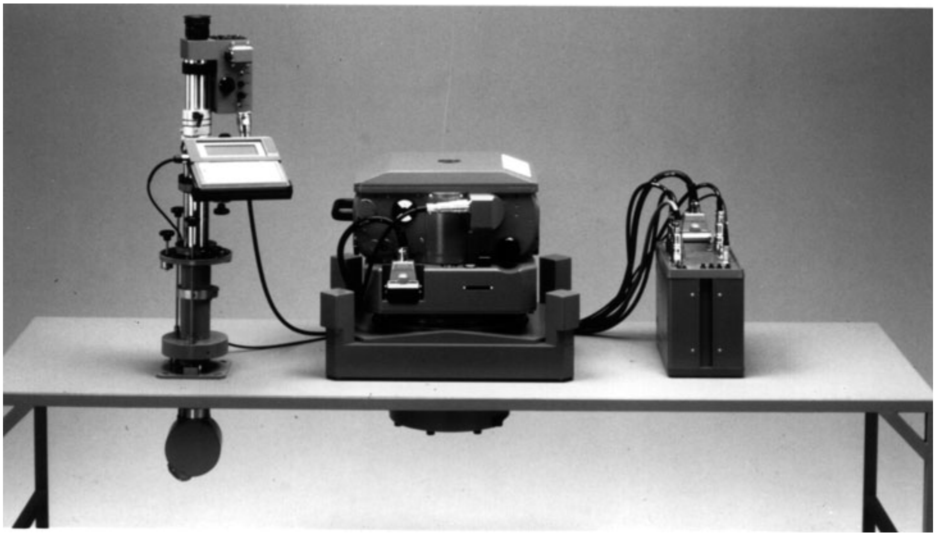

Figure 27.1
Aerial camera with viewfinder and electronic controls. (From *Elements of Photogrammetry: With Applications in GIS*, by Wolf & Dewitt, 2000; Courtesy Carl Zeiss, Inc. and McGraw-Hill Book Co., Inc.)

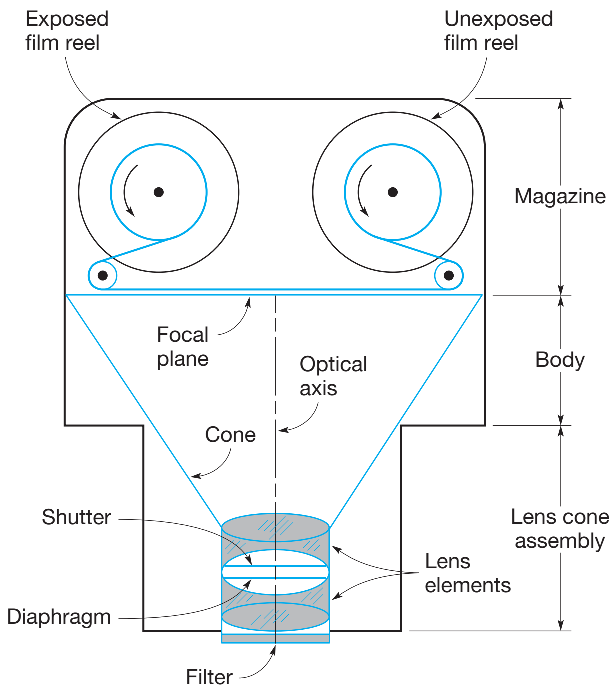

Figure 27.2 Principal components of a single lens frame aerial camera.

regulate the size of lens opening; a *filter* to reduce the effect of haze and distribute light uniformly over the format; a *camera cone* to support the lens-shutter-diaphragm assembly with respect to the focal plane and prevent stray light from striking the film; a *focal plane,* the surface on which the film lies when exposed; *fiducial marks* (not shown in Figure 27.2 but illustrated later), four or eight in number, which are essential to define the geometry of the photographs; a *camera body* to house the drive mechanism that cocks and trips the shutter, flattens the film, and advances it between exposures; and *a magazine,* which holds the supply of exposed and unexposed film or houses the digital storage device.

An aerial camera shutter can be operated manually by an operator, or automatically by the electronic control mechanism, so that photos are taken at specified intervals. A level vial attached to the camera enables an operator to keep the optical axis of the camera lens, which is perpendicular to the focal plane, nearly vertical in spite of any slight tip and tilt of the aircraft.

Images of the fiducial marks are printed on the photographs and lines joining opposite pairs intersect at or very near the *principal point*, defined as the point where a perpendicular from the emergent nodal point of the camera lens strikes the focal plane. Fiducial marks may be located in the corners, on the sides, or preferably in both places, as shown in Figures 27.4 and 27.5.

In digital cameras, an array of solid state detectors, which are placed in the focal plane capture the image from the lens. The most common type of detector is the *charge-coupled device* (CCD). The array is composed of tiny detectors arranged in contiguous rows and columns, as shown in Figure 27.3. Each detector senses the energy received from its corresponding ground scene, and this

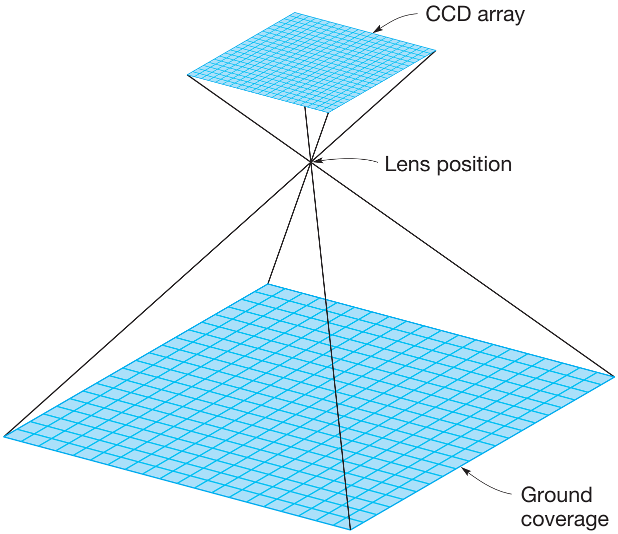

Figure 27.3 Geometry of a digital frame camera. (From *Elements of Photogrammetry: With Applications in GIS*, 3rd Ed., by Paul Wolf and Bon DeWitt. © 2000 by The McGraw-Hill Companies, Inc. Reprinted by permission.)

constitutes one "picture element" *(pixel)* within the overall image. The principle of operation of CCDs is fundamentally quite simple. At any specific pixel location, the CCD element there is exposed to incident light energy which builds up an electric charge proportional to the intensity of the incoming light. The electric charge is amplified, converted from analog to digital form and stored in a file together with its row and column location within the array. Currently, the sizes of the individual CCD elements being manufactured are in the range of from about 5 to 15 μm^2, and arrays may consist of from 500 rows and columns (250,000 pixels) for inexpensive cameras to more than 4000 rows and columns. Obviously, significant storage and data handling capabilities are necessary in acquiring and processing digital images.

Aerial mapping cameras, whether the film- or digital-type, are calibrated to get precise values for the focal length and lens distortions. Flatness of the focal plane, relative position of the principal point with respect to the fiducial marks, and fiducial mark locations are also specified. These calibration data are necessary for precise photogrammetric work.

■ 27.4 TYPES OF AERIAL PHOTOGRAPHS

Aerial photographs exposed with single-lens frame cameras are classified as *vertical* (taken with the camera axis aimed vertically downward or as nearly vertical as possible) and *oblique* (made with the camera axis intentionally inclined at an angle between the horizontal and vertical). Oblique photographs are further classified as *high* if the horizon shows on the picture and *low* if it does not. Figures 27.4 and 27.5 show examples of vertical and low oblique photographs, respectively. As illustrated by these examples, aerial photos clearly depict all natural and cultural features within the region covered such as roads, railroads, buildings, rivers, bridges, trees, and cultivated lands.

Figure 27.4 Vertical aerial photograph. (Courtesy Pennsylvania Department of Transportation.)

Vertical photographs are the principal mode of obtaining imagery for photogrammetric work. Oblique photographs are seldom used for metrical applications but are advantageous in interpretative work and for reconnaissance.

■ 27.5 VERTICAL AERIAL PHOTOGRAPHS

A *truly vertical photograph* results if the axis of the camera is exactly vertical when exposure is made. Despite all precautions, small tilts, generally less than 1° and rarely greater than 3° are invariably present, and the resulting photos are called *near-vertical* or *tilted* photographs. Although vertical photographs look like maps to laypersons, they are not true orthographic projections of the Earth's surface. Rather, they are perspective views and the principles of perspective geometry must be applied to prepare maps from them. Figure 27.6 illustrates the geometry of a vertical photograph taken at exposure station L. The photograph, considered a *contact print positive,* is a 180° exact reversal of the negative. The positive shown in Figure 27.6 is used to develop photogrammetric equations in subsequent sections.

Figure 27.5 Low oblique aerial photograph showing state capital and downtown Madison, Wisconsin. (Courtesy Department of Transportation, State of Wisconsin.)

Distance *oL* (Figure 27.6) is the camera focal length. The *x* and *y* reference axes system for measuring photographic coordinates of images is defined by straight lines joining opposite-side fiducial marks shown on the positive of Figure 27.6. The *x*-axis, arbitrarily designated as the line most nearly parallel with the direction of flight, is positive in the direction of flight. Positive *y* is 90° counterclockwise from positive *x*.

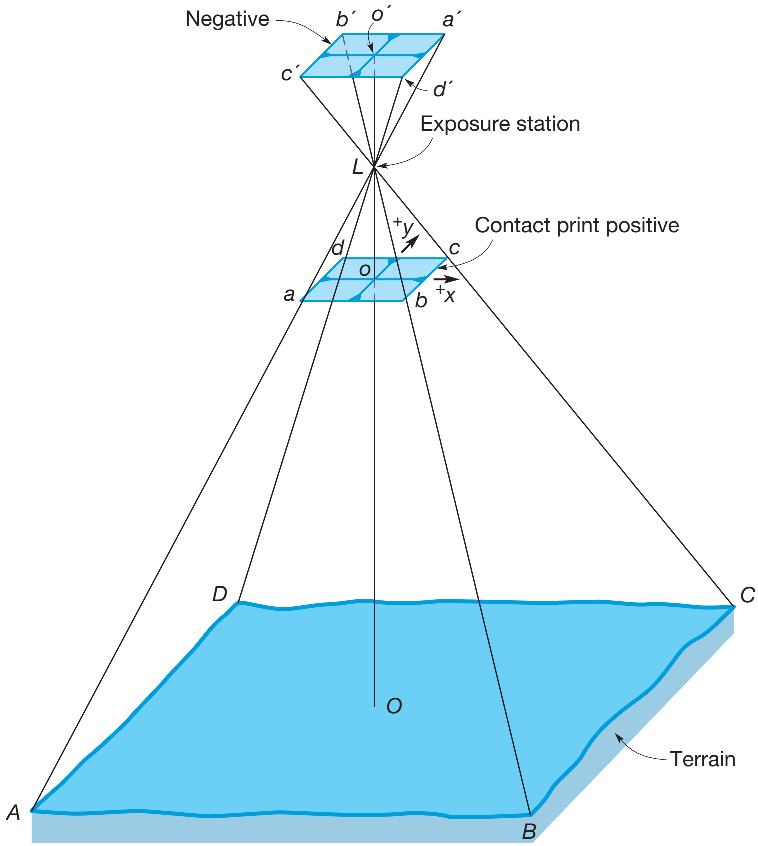

Figure 27.6 Geometry of a vertical aerial photograph.

Vertical photographs for topographic mapping are taken in strips, which normally run lengthwise over the area to be covered. The strips or *flight lines* generally have a *sidelap* (overlap of adjacent flight lines) of about 30%. *Endlap* (overlap of adjacent photographs in the same flight line) is usually about 60 ± 5%. Figures 27.18a and b illustrate endlap and sidelap. An endlap of 50% or greater is necessary to assure that all ground points will appear in at least two photographs and that some will show in three. Images common to three photographs permit *aerotriangulation* to extend or densify control through a strip or block of photographs using only minimal existing control.

■ 27.6 SCALE OF A VERTICAL PHOTOGRAPH

Scale is ordinarily interpreted as the ratio of a distance on a map to that same length on the ground. On a map it is uniform throughout because a map is an orthographic projection. The scale of a vertical photograph is the ratio of a photo distance to the corresponding ground distance. Since a photograph is a perspective view, scale varies from point to point with variations in terrain elevation.

In Figure 27.7, L is the exposure station of a vertical photograph taken at an altitude H above datum. The camera focal length is f and o is the photographic

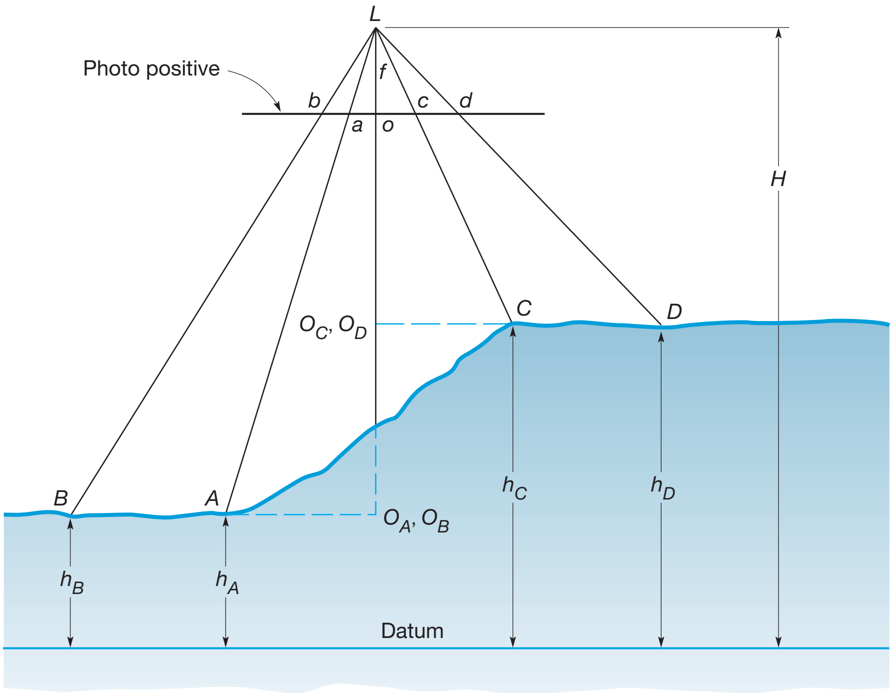

Figure 27.7 Scale of a vertical photograph.

principal point. Points A, B, C, and D, which lie at elevations above datum of h_A, h_B, h_C, and h_D, respectively, are imaged on the photograph at a, b, c, and d. The scale at any point can be expressed in terms of its elevation, the camera focal length, and the flying height above datum. From Figure 27.7, from similar triangles Lab and LAB, the following expression can be written:

$$\frac{ab}{AB} = \frac{La}{LA} \tag{a}$$

Also from similar triangles Loa and LOA, a similar expression results:

$$\frac{La}{LA} = \frac{f}{H - h_A} \tag{b}$$

Equating (a) and (b), recognizing that ab/AB equals photo scale at A and B, and considering AB to be infinitesimally short, the equation for the scale at A is

$$S_A = \frac{f}{H - h_A} \tag{c}$$

Scales at B, C, and D may be expressed similarly as $S_B = f/(H - h_B)$, $S_C = f/(H - h_C)$, and $S_D = f/(H - h_D)$.

It is apparent from these relationships that photo scale increases at higher elevations and decreases at lower ones. This concept is seen graphically in

Figure 27.7, where ground lengths AB and CD are equal, but photo distances ab and cd are not, cd being longer and at larger scale than ab because of the higher elevation of CD. In general, by dropping subscripts, the scale S at any point whose elevation above datum is h may be expressed as

$$S = \frac{f}{H - h} \tag{27.1}$$

where S is the scale at any point on a vertical photo, f is the camera focal length, H the flying height above datum, and h the elevation of the point.

Use of an average photographic scale is frequently desirable, but must be accepted with caution as an approximation. For any vertical photographs taken of terrain whose average elevation above datum is h_{avg}, the average scale S_{avg} is

$$S_{avg} = \frac{f}{H - h_{avg}} \tag{27.2}$$

Example 27.1

The vertical photograph of Figure 27.7 was exposed with a 6-in. focal length camera at a flying height of 10,000 ft above datum. (a) What is the photo scale at point a if the elevation of point A on the ground is 2500 ft above datum? (b) For this photo, if the average terrain is 4000 ft above datum, what is the average photo scale?

Solution

(a) From Equation (27.1),

$$S_A = \frac{f}{H - h_A} = \frac{6 \text{ in.}}{10{,}000 - 2500} = \frac{1 \text{ in.}}{1250 \text{ ft}} = 1{:}15{,}000$$

(b) From Equation (27.2),

$$S_{avg} = \frac{f}{H - h_{avg}} = \frac{6 \text{ in.}}{10{,}000 - 4000} = \frac{1 \text{ in.}}{1000 \text{ ft}} = 1{:}12{,}000$$

The scale of a photograph can be determined if a map is available of the same area. This method does not require the focal length and flying height to be known. Rather, it is necessary only to measure the photographic distance between two well-defined points also identifiable on the map. Photo scale is then calculated from the equation

$$\text{photo scale} = \frac{\text{photo distance}}{\text{map distance}} \times \text{map scale} \tag{27.3}$$

In using Equation (27.3), the distances must be in the same units, and the answer is the scale at the average elevation of the two points used.

Example 27.2

On a vertical photograph, the length of an airport runway measures 4.24 in. On a map plotted to a scale of 1:9600, it extends 7.92 in. What is the photo scale at the runway elevation?

Solution

From Equation (27.3),

$$S = \frac{4.24}{7.92}\left(\frac{1}{9600}\right) = \frac{1}{17{,}900} \text{ or } 1 \text{ in.} = 1490 \text{ ft}$$

The scale of a photograph can also be computed readily if lines whose lengths are common knowledge appear in the photograph. Section lines, a football or baseball field, and so on, can be measured on the photograph and an approximate scale at that elevation ascertained as the ratio of measured photo distance to known ground length. With an approximate photographic scale known, rough determinations of the lengths of lines appearing in the photo can be made.

Example 27.3

On a certain vertical aerial photo, a section line (assumed to be 5280 ft long) is imaged. Its photographic length is 3.32 in. On this same photo, a rectangular parcel of land measures 1.74 by 0.83 in. Calculate the approximate ground dimensions of the parcel and its acreage.

Solution

1. Approximate photo scale

$$\frac{3.32}{5280} = \frac{1 \text{ in.}}{1590 \text{ ft}} \text{ or } 1 \text{ in.} = 1590 \text{ ft}$$

2. Parcel dimensions and area:

$$\text{length} = 1590(1.74) = 2770 \text{ ft}$$

$$\text{width} = 1590(0.83) = 1320 \text{ ft}$$

$$\text{area} = \frac{2770(1320)}{43{,}560} = 84 \text{ acres}$$

■ 27.7 GROUND COORDINATES FROM A SINGLE VERTICAL PHOTOGRAPH

Ground coordinates of points whose images appear in a vertical photograph can be determined with respect to an arbitrary ground-axis system. The arbitrary X and Y ground axes are in the same vertical planes as photographic x and y, respectively, and the system's origin is in the datum plane vertically beneath the exposure station. Ground coordinates of points determined in this manner are used to calculate horizontal distances, horizontal angles, and areas.

Figure 27.8 illustrates a vertical photograph taken at flying height H above datum. Images a and b of ground points A and B appear on the photograph. The measured photographic coordinates are x_a, y_a, x_b, and y_b; the ground coordinates are X_A, Y_A, X_B, and Y_B. From similar triangles LO_AA' and Loa',

$$\frac{oa'}{O_AA'} = \frac{f}{H - h_A} = \frac{x_a}{X_A}$$

Then

$$X_A = \frac{(H - h_A)x_a}{f} \tag{27.4}$$

Also from similar triangles $LA'A$ and $La'a$,

$$\frac{a'a}{A'A} = \frac{f}{H - h_A} = \frac{y_a}{Y_A}$$

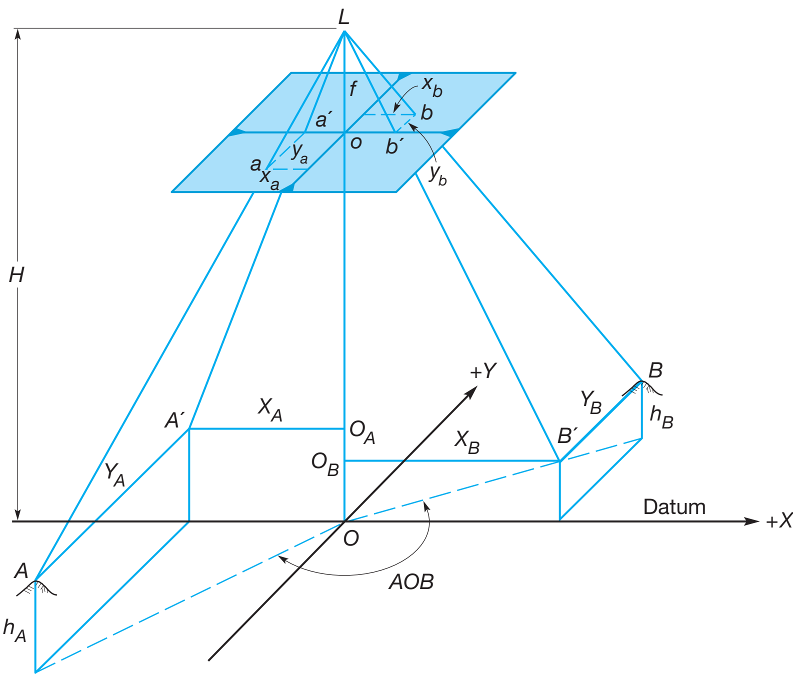

Figure 27.8 Ground coordinates from a vertical photograph.

and

$$Y_A = \frac{(H - h_A)y_a}{f} \tag{27.5}$$

Similarly,

$$X_B = \frac{(H - h_B)x_b}{f} \tag{27.6}$$

$$Y_B = \frac{(H - h_B)y_b}{f} \tag{27.7}$$

Note that Equations (27.4) through (27.7) require point elevations h_A and h_B for their solution. These are normally either taken from existing contour maps, or they can be obtained by differential or trigonometric leveling. From the X and Y coordinates of points A and B, the horizontal length of line AB can be calculated using Equation (14.4).

If X and Y coordinates of all corners of a parcel are computed in this way, the parcel area can be determined from those coordinates by the method discussed in Chapter 12. The advantage of calculating lengths and areas by the coordinate formulas, rather than by average scale as in Example 27.3, is better accuracy results because differences in elevation, which cause the photo scale to vary, are more rigorously taken into account.

■ 27.8 RELIEF DISPLACEMENT ON A VERTICAL PHOTOGRAPH

Relief displacement on a vertical photograph is the shift or movement of an image from its theoretical datum location caused by the object's relief—that is, its elevation above or below datum. Relief displacement on a vertical photograph occurs along radial lines from the principal point and increases in magnitude with greater distance from the principal point to the image.

The concept of relief displacement in a vertical photograph taken from a flying height H above datum is illustrated in Figure 27.9, where the camera focal length is f and o is the principal point. Points B and C are the base and top, respectively, of a pole with images at b and c on the photograph. A is an imaginary point on the datum plane vertically beneath B with corresponding imaginary position a on the photograph. Distance ab on the photograph is the image displacement due to h_B, the elevation of B above datum, and bc is the image displacement because of the height of the pole.

From similar triangles of Figure 27.9, an expression for relief displacement is formulated. First, from triangles LO_AA and Loa,

$$\frac{r_a}{R} = \frac{f}{H}$$

and rearranging,

$$r_aH = fR \tag{d}$$

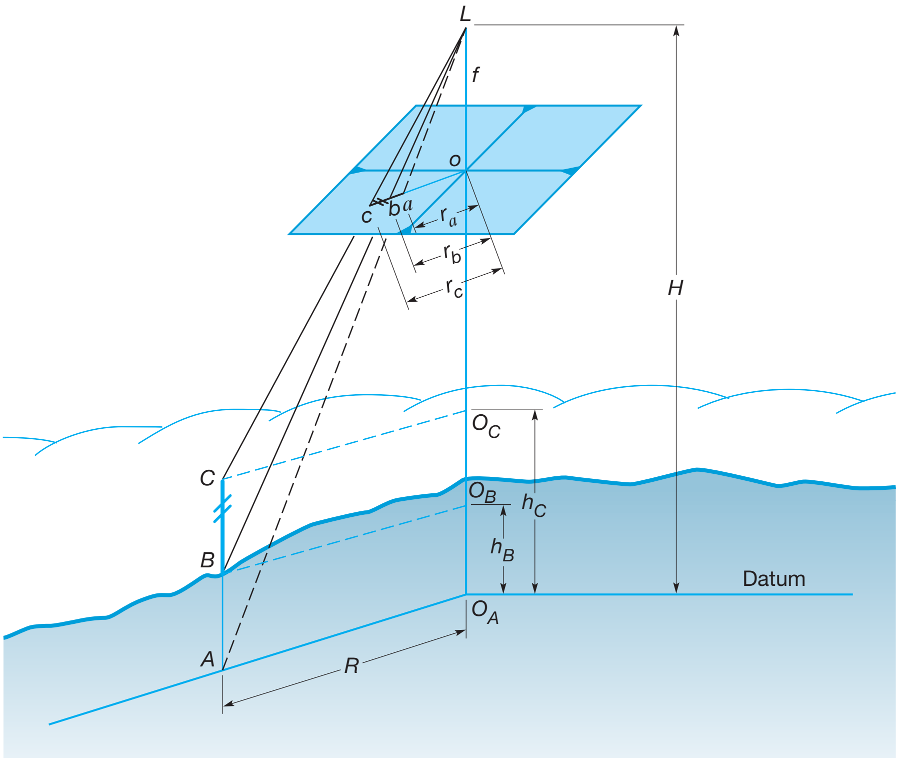

Figure 27.9
Relief displacement on a vertical photograph.

Also from similar triangles LO_BB and Lob,

$$\frac{r_b}{R} = \frac{f}{H - h_B} \quad \text{or} \quad r_b(H - h_B) = fR \tag{e}$$

Equating (d) and (e),

$$r_aH = r_b(H - h_B)$$

and rearranging,

$$r_b - r_a = \frac{r_bh_B}{H}$$

If $d_b = r_b - r_a$ is the relief displacement of image b, then $d_b = r_bh_b/H$. Dropping subscripts, the equation can be written in general terms as

$$d = \frac{rh}{H} \tag{27.8}$$

where d is the relief displacement, r the photo radial distance from the principal point to the image of the displaced point, h the height above datum of the displaced point, and H the flying height above that same datum.

Equation (27.8) can be used to locate the datum photographic positions of images on a vertical photograph. True horizontal angles may then be scaled directly from the datum images, and if the photo scale at datum is known, true horizontal lengths of the lines can be measured directly. The datum position is located by scaling the calculated relief displacement d of a point along a radial line to the principal point (inward for a point whose elevation is above datum).

Equation (27.8) can also be applied in computing heights of vertical objects such as buildings, church steeples, radio towers, trees, and power poles. To determine heights using the equation, images of both the top and bottom of an object must be visible.

Example 27.4

In Figure 27.9, radial distance r_b to the image of the base of the pole is 75.23 mm, and radial distance r_c to the image of its top is 76.45 mm. The flying height H is 4000 ft above datum, and the elevation of B is 450 ft. What is the height of the pole?

Solution

The relief displacement is $r_c - r_b = 76.45 - 75.23 = 1.22$ mm. Selecting a datum at the pole's base and applying Equation (27.8),

$$d = \frac{rh}{H} \text{ so } 1.22 = \frac{76.45h}{4000 - 450}$$

Then

$$h = \frac{3550 \times 1.22}{76.45} = 56.6 \text{ ft}$$

The relief displacement equation is particularly valuable to photo interpreters, who are usually interested in relative heights rather than absolute elevations.

Figure 27.4 vividly illustrates relief displacements. This vertical photo shows the relief displacement of a water tower in the center-right-hand portion of the format. This displacement, as well as that of other buildings throughout the photograph, occurs radially outward from the principal point.

27.9 FLYING HEIGHT OF A VERTICAL PHOTOGRAPH

From previous sections it is apparent that the flying height above datum is an important parameter in solving basic photogrammetry equations. For rough computations, flying heights can be taken from altimeter readings if available. An approximate H can also be obtained by using Equation (27.1) if a line of known length appears on a photograph.

Example 27.5

The length of a section line (known to be 5280 ft) is measured on a vertical photograph as 4.15 in. Find the approximate flying height above the terrain if $f = 6$ in.

Solution

Assuming the datum at the section line elevation, Equation (27.1) reduces to

$$\text{scale} = \frac{f}{H} \quad \text{and} \quad \frac{4.15}{5280} = \frac{6}{H}$$

from which

$$H = \frac{5280(6)}{4.15} = 7630 \text{ ft above the terrain}$$

If the images of two ground control points a and b appear on a vertical photograph, the flying height can be determined more precisely from the Pythagorean theorem,

$$L^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2$$

Substituting Equations (27.4) through (27.7) into this above expression,

$$L^2 = \left[\frac{(H - h_B)x_b - (H - h_A)x_a}{f}\right]^2 + \left[\frac{(H - h_B)y_b - (H - h_A)y_a}{f}\right]^2 \tag{27.9}$$

where L is the horizontal length of ground line AB, H the flying height above datum, h_A and h_B the elevations of the control points above datum, and x and y the measured photo coordinates of the control points.

In Equation (27.9) all variables except H are known. Hence a direct solution can be found for the unknown flying height. The equation is quadratic, so there are two solutions, but the incorrect one will be obvious and can be discarded.

27.10 STEREOSCOPIC PARALLAX

Parallax is defined as the apparent displacement of the position of an object with respect to a frame of reference due to a shift in the point of observation. For example, a person looking through the view finder of an aerial camera in an aircraft as it moves forward sees images of objects moving across the field of view. This apparent motion (parallax) is due to the changing location of the observer. By using the camera format as a frame of reference, it can be seen that parallax exists for all images appearing on successive photographs due to forward motion between exposures. Points closer to the camera (of higher elevation) will appear to move faster and have greater parallaxes than lower ones. For 60% endlap, the parallax of images on successive photographs should average approximately 40% of the focal plane width.

Parallax of a point is a function of its relief and consequently measuring it provides a means of calculating elevations. It is also possible to compute X and Y ground coordinates from parallax.

Movement of an image across the focal plane between successive exposures takes place in a line parallel with the direction of flight. Thus to measure parallax, that direction must first be established. For a pair of overlapping photos, this is done by locating positions of the principal points and *corresponding principal points* (that is, principal points transferred to their places in the overlap area of the other photo). A line on each print ruled through these points defines the direction of flight. It also serves as the photographic x-axis for parallax measurement. The y-axis for making parallax measurements is drawn perpendicular to the flight line passing through each photo's principal point. The x coordinate of a point is scaled on each photograph with respect to the axes so constructed and the parallax of the point is then calculated from the expression

$$p = x - x_1 \tag{27.10}$$

Photographic coordinates x and x_1 are measured on the left-hand and right-hand prints, respectively, with due regard given for algebraic signs.

Figure 27.10 illustrates an overlapping pair of vertical photographs exposed at equal flight heights H above datum. The distance between exposure stations L and L_1 is called B, the *air base*. The inset figure shows the two exposure stations

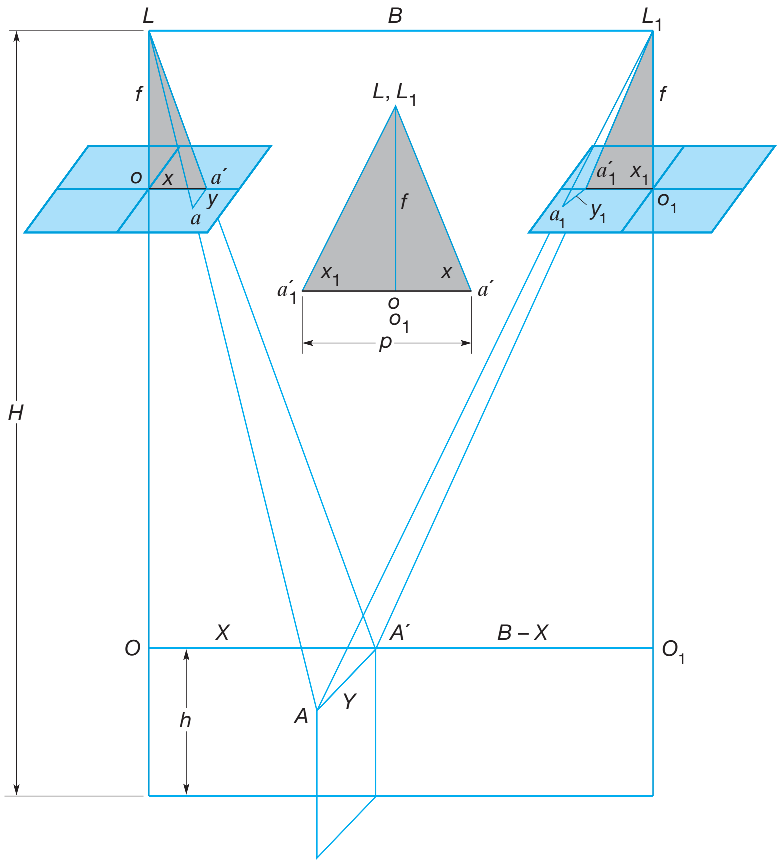

Figure 27.10 Stereoscopic parallax relationships.

L and L_1 in superposition to make the similarity of triangles $La_1'a'$ and $LA'L_1$ more easily recognized. When these two similar triangles are equated, there results

$$\frac{p}{f} = \frac{B}{H - h}$$

from which

$$H - h = \frac{Bf}{p} \tag{27.11}$$

Also from similar triangles LOA' and Loa',

$$X = \frac{x}{f}(H - h) \tag{f}$$

Substituting Equation (27.11) into (f) gives

$$X = \frac{B}{p}x \tag{27.12}$$

and from triangles LAA' and Laa', with substitution of Equation (27.11) yields

$$Y = \frac{B}{p}y \tag{27.13}$$

In Equations (27.12) and (27.13), X and Y are ground coordinates of a point with respect to an origin vertically beneath the exposure station of the left photograph, with positive X coinciding with the direction of flight. Positive Y is 90° counterclockwise to positive X. The parallax of the point is p, x and y the photographic coordinates of a point on the left-hand print, H the flying height above datum, h the point's elevation above the same datum, and f the camera lens focal length.

Equations (27.11) through (27.13), commonly called the *parallax equations*, are useful in calculating horizontal lengths of lines and elevations of points. They also provide the fundamental basis for the design and operation of stereoscopic plotting instruments.

Example 27.6

The length of line AB and elevations of points A and B, whose images appear on two overlapping vertical photographs, are needed. The flying height above datum was 4050 ft and the air base was 2410 ft. The camera had a 6-in. focal length. Measured photographic coordinates (in inches) on the left-hand image are $x_a = 2.10$, $x_b = 3.50$, $y_a = 2.00$, and $y_b = -1.05$; on the right-hand image, $x_{1a} = -2.25$ and $x_{1b} = -1.17$.

Solution

From Equation (27.10),

$$p_a = x_a - x_{1a} = 2.10 - (-2.25) = 4.35 \text{ in.}$$

$$p_b = x_b - x_{1b} = 3.50 - (-1.17) = 4.67 \text{ in.}$$

By Equations (27.12) and (27.13),

$$X_A = \frac{B}{p_a}x_a = \frac{2410(2.10)}{4.35} = 1160 \text{ ft}$$

$$X_B = \frac{2410(3.50)}{4.67} = 1810 \text{ ft}$$

$$Y_A = \frac{B}{p_a}y_a = \frac{2410(2.00)}{4.35} = 1110 \text{ ft}$$

$$Y_B = \frac{2410(-1.05)}{4.67} = -542 \text{ ft}$$

By Equation (11.4), length AB is

$$AB = \sqrt{(1810 - 1160)^2 + (-542 - 1110)^2} = 1780 \text{ ft}$$

By Equation (27.11), the elevations of A and B are

$$h_A = H - \frac{Bf}{p_a} = 4050 - \frac{2410(6)}{4.35} = 726 \text{ ft}$$

$$h_B = H - \frac{Bf}{p_b} = 4050 - \frac{2410(6)}{4.67} = 954 \text{ ft}$$

■ 27.11 STEREOSCOPIC VIEWING

The term *stereoscopic viewing* means seeing an object in three dimensions. This is a process that requires normal *binocular* (two-eyed) vision. In Figure 27.11, two eyes L and R are separated by a distance b called the *eye base.* When the eyes are focused on point A, their optical axes converge to form angle ϕ_1, and when sighting on B, ϕ_2 is produced. Angles ϕ_1 and ϕ_2 are called *parallactic angles* and the brain associates distances d_A and d_B with them. The depth $d_B - d_A$ of the object is perceived from the brain's unconscious comparison of these parallactic angles.

If two photographs of the same subject are taken from two different perspectives or camera stations, the left print viewed with the left eye and simultaneously the right print seen with the right eye, a mental impression of a three-dimensional model results. In normal stereoscopic viewing (not using photos), the eye base gives a true impression of parallactic angles. While looking at aerial photographs stereoscopically, the exposure station spacing simulates an eye base so the viewer actually sees parallactic angles comparable with having one eye at each of the two exposure stations. This creates a condition called *vertical exaggeration,* which

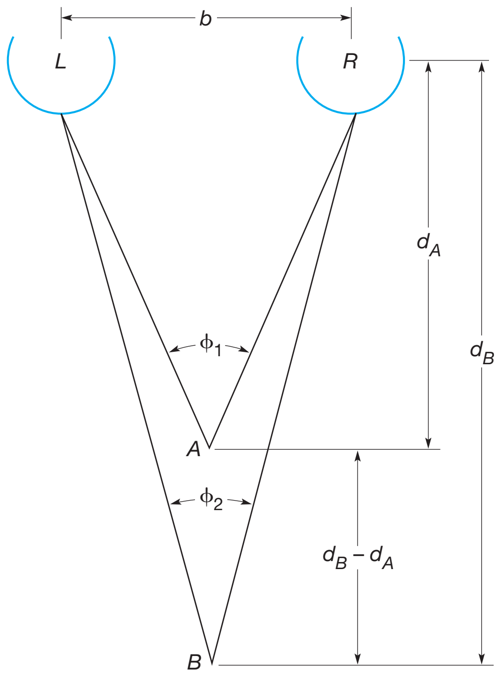

Figure 27.11 Parallactic angles in stereoscopic viewing.

causes the vertical scale of the three-dimensional model to appear greater than its horizontal scale; that is, objects are perceived to be too tall. The condition is of concern to photo interpreters who often estimate heights of objects and slopes of surfaces when viewing air photos stereoscopically. The amount of vertical exaggeration varies with percent endlap and the camera's format dimensions and focal length. A factor of about 4 results if endlap is 60% and the camera has a 9-in. (23-cm) format with 6-in. (152-mm) focal length.

The *stereoscope* shown in Figure 27.12 permits viewing photographs stereoscopically by enabling the left and right eyes to focus comfortably on the left and

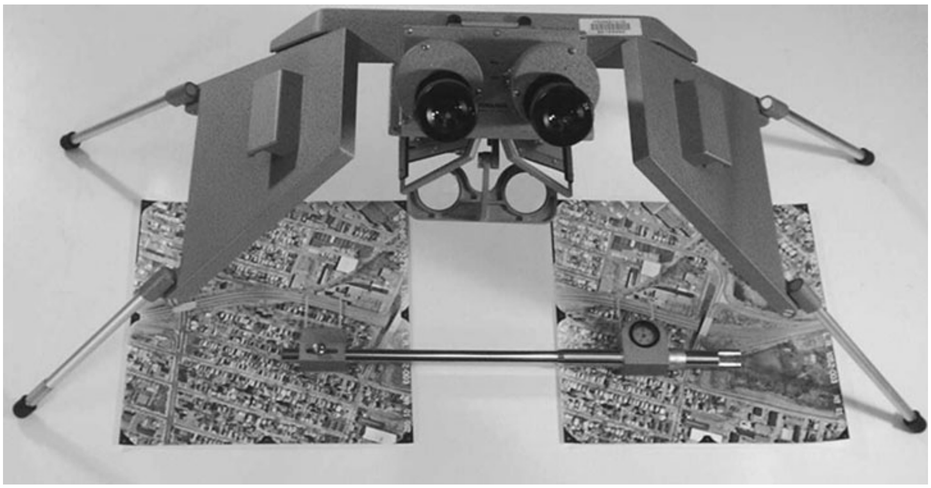

Figure 27.12 Folding mirror stereoscope with parallax bar.

right prints, respectively, assuming proper orientation has been made of the overlapping pair of photographs under the instrument. Correct orientation requires the two photographs to be laid out in the same order they were taken with the stereoscope so set that the line joining its lens centers is parallel with the direction of flight. The print spacing is varied, carefully maintaining this parallelism, until a clear three-dimensional view (stereoscopic model) is obtained.

■ 27.12 STEREOSCOPIC MEASUREMENT OF PARALLAX

The parallax of a point can be measured while viewing stereoscopically with the advantage of speed and, because binocular vision is used, greater accuracy. As the viewer looks through a stereoscope, imagine that two small identical marks etched on pieces of clear glass, called *half-marks,* are placed over each photograph. The viewer simultaneously sees one mark with the left eye and the other with the right eye. Then the positions of the marks are shifted until they seem to fuse together as one mark that appears to lie at a certain elevation. The height of the mark will vary or "float" as the spacing of the half-marks is varied; hence it is called the *floating mark.* Figure 27.13 demonstrates this principle and also illustrates that the floating mark can be set exactly on particular points such as A, B, and C by placing the half-marks (small black dots) at a and a', b and b', and c and c', respectively.

Based on the floating mark principle, the parallax of points is observed stereoscopically with a parallax bar, as shown beneath the stereoscope in Figure 27.12. It is simply a bar to which two half-marks are fastened. The right mark can be moved with respect to the left one by turning a micrometer screw to register the displacement on a dial. When the floating mark appears to rest on a point, a

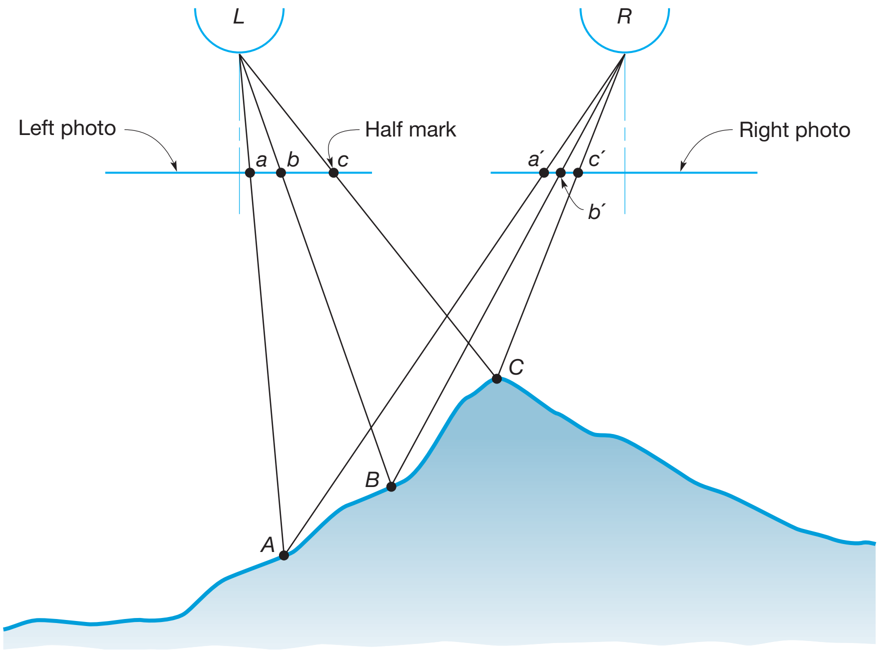

Figure 27.13
Principle of the floating mark.

micrometer reading is taken and added to the parallax bar *setup constant* to obtain the parallax.

When a parallax bar is used, two overlapping photographs are oriented properly for viewing under a mirror stereoscope and fastened securely with respect to each other using drafting tape. The parallax bar constant for the setup is determined by measuring the photo coordinates for a discrete point and applying Equation (27.10) to obtain its parallax.

The floating mark is placed on the same point, the micrometer read, and the constant for the setup found by

$$C = p - r \tag{27.14}$$

where C is the parallax bar setup constant, p the parallax of a point determined by Equation (27.10), and r the micrometer reading obtained with the floating mark set on that same point.

Once the constant has been determined, the parallax of any other point can be computed by adding its micrometer reading to the constant. Thus, a single measurement gives the parallax of a point. Each time another pair of photos is oriented for parallax measurements, a new parallax bar setup constant must be determined. A major advantage of the stereoscopic method is that parallaxes of nondiscrete points can be determined. Thus elevations of hilltops, depressions, and so on, in fields can be calculated using Equation (27.11), even though their x coordinates cannot be measured for use in Equation (27.10).

■ 27.13 ANALYTICAL PHOTOGRAMMETRY

Analytical photogrammetry involves the rigorous mathematical calculation of ground coordinates of points using computers. Input data consists of camera parameters (i.e., the lens focal length, its distortion characteristics, and the principal point location); observed photo coordinates of the images of all points whose ground coordinates are to be determined, as well as those of a limited number of well-distributed ground control points; and the ground coordinates of the control points. The photo coordinates are measured with respect to the coordinate system illustrated in Figure 27.6. Extremely precise instruments called *comparators* are used and values are recorded to the nearest micrometer. Unlike the elementary methods presented in earlier sections of this chapter that assume vertical photos and equal flying heights, analytical photogrammetry rigorously accounts for these variations.

Analytical photogrammetry generally involves the formation of large, rather complex, systems of redundant equations, which are then solved using the method of least squares. The concepts have existed for many years, but it was not until the advent of computers that the procedures became practical. The formation of the equations used in analytical photogrammetry is beyond the scope of this text, but interested students can find their derivations, and illustrations of their use, in textbooks that specialize in photogrammetry.[1]

[1]See Wolf, P. R. and B. A. Dewitt, "Elements of Photogrammetry: With Applications in GIS," 3rd Ed., 2000, McGraw-Hill Book Co., Inc., New York.

As noted above, accuracies attainable using analytical photogrammetry are very high and are frequently expressed as a ratio of the flying height of the photography used. Accuracies within about 1/10,000th to 1/15,000th of the flying height above ground are routinely obtained in computed X, Y, and Z coordinates. Thus, for photos taken from 6,000 ft above ground, coordinates accurate to within about $\pm$ 0.4 to 0.6 ft can be expected.

Analytical photogrammetry forms the basis for *softcopy stereoplotters*, which is discussed in the following section.

■ 27.14 STEREOSCOPIC PLOTTING INSTRUMENTS

Stereoscopic plotting instruments, also simply called *stereoplotters,* are devices designed to provide accurate solutions for X, Y, and Z object space coordinates of points from their corresponding image locations on overlapping pairs of photos. The fact that the photos may contain varying amounts of tilt and have differing flying heights is of no consequence, because these instruments rigorously account for the position and orientation of the camera for each exposure. Stereoplotters are used to take cross sections, record digital elevation models, compile topographic maps, and generate other types of spatially related topographic information from overlapping aerial photographs.

Stereoplotters can be classified into four different categories: (1) *optical projection*, (2) *mechanical projection*, (3) *analytical*, and (4) *digital* or "softcopy" systems. Regardless of the type, all instruments contain optical and mechanical elements, and the newer versions have either built-in or interfaced computers. Only analytical and softcopy stereoplotters are now used in industry. For this reason, only these two types of stereoplotters will be discussed herein. Readers can refer to previous editions of this book for information on the other type of plotters.

27.14.1 Basic Concepts in Stereoplotters

Figure 27.14 illustrates the basic design concepts of a stereoplotter. In Figure 27.14a, an overlapping pair of aerial photos is exposed. *Diapositives* (positives developed on film or glass plates) are prepared to exacting standards from the negatives and placed in the projectors of the stereoplotter, as shown in Figure 27.14b. With the diapositives in place, light rays are projected through them and the positions and orientations of the projectors are adjusted until the rays from corresponding images intersect below to form a model of the overlap area of the aerial photos. The model of the terrain is called a *stereomodel* and once formed, it can be viewed, measured, and mapped.

In order to accomplish the steps outlined in the preceding paragraph, it is necessary that stereoplotters incorporate the following components: (1) a projection system (to project the light rays that create a stereomodel), (2) a viewing system (which enables an operator to see the stereomodel in three dimensions), and (3) a measuring/tracing system (for measuring or mapping the stereomodel). Projectors used in optical projection stereoplotters function like ordinary slide projectors. However, they are much more precise and can be adjusted in angular orientation and position to recreate the spatial locations and attitudes that the aerial cameras

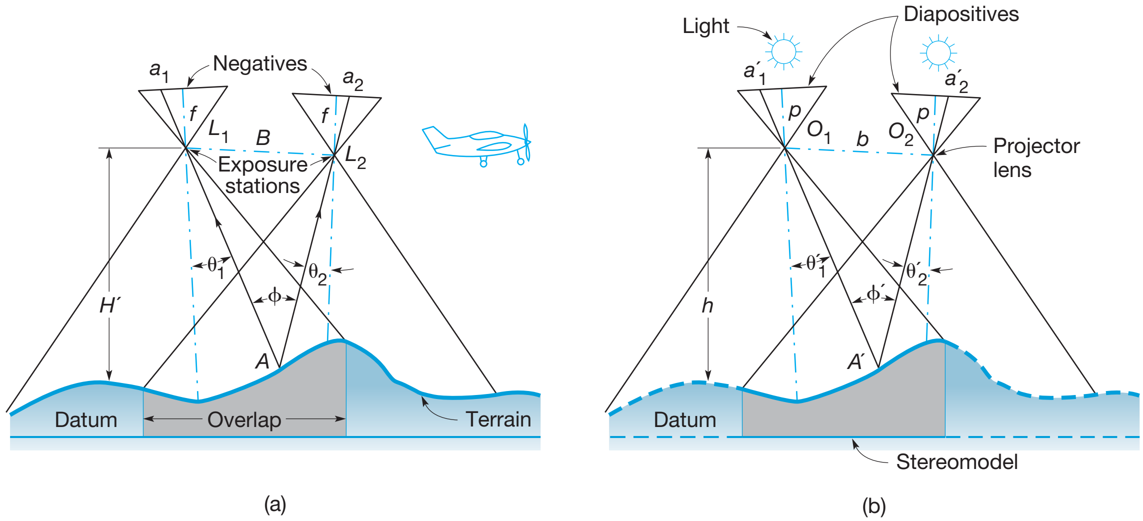

Figure 27.14 Fundamental concepts of stereoscopic plotting instrument design. (a) Aerial photography; (b) Stereoscopic plotter. (From *Elements of Photogrammetry: With Applications in GIS*, 3rd Ed., by Paul Wolf and Bon DeWitt. © 2000 by The McGraw-Hill Companies, Inc. Reprinted by permission.)

had when the overlapping photos were exposed. This produces a "true" model of the terrain in the overlap area. The scale of the stereomodel is, of course, greatly reduced, and is the ratio of the *model base* (distance b between projector lenses) to the *air base* (actual distance B between the two exposure stations).

Plotter viewing systems must provide a stereoscopic view and hence be designed so that the left and right eyes see only projected images of the corresponding left and right diapositives. One method of accomplishing this is to project one image with a blue filter and the other with a red filter. The operator wears a pair of spectacles with corresponding blue and red lenses. Another method of separating the left and right images is to project them on half of the screen. The operator must then look through a viewing system similar to a stereoscope to restrict the left eye to view the left image only and the right eye the right. These systems of viewing stereoscopically are called the *anaglyphic* method. Other viewing systems are known as *passive* and *active*. In the passive viewing system, left and right images are projected in opposite polarity. The operator wears polarized spectacles with corresponding polarity in the lenses so that the left eye can only see the left image and the right eye can only see the right image. As shown in Figure 27.15, the active viewing system also projects the left and right images in opposite polarity. However, in the active system, the images are rapidly swapped in the projection system also. The operator's viewing spectacles are synchronized so that the left and right eyes can see the corresponding left and right images only.

A stereoplotter operator, preparing to measure or map a stereomodel, must go through a three-stage orientation process consisting of *interior orientation*, *relative orientation*, and *absolute orientation*. Interior orientation ensures that the light rays are geometrically correct, that is, angles θ'_1 and θ'_2 of Figure 27.14(b) (i.e., the angles between the light rays and the axis of the lens) must be identical

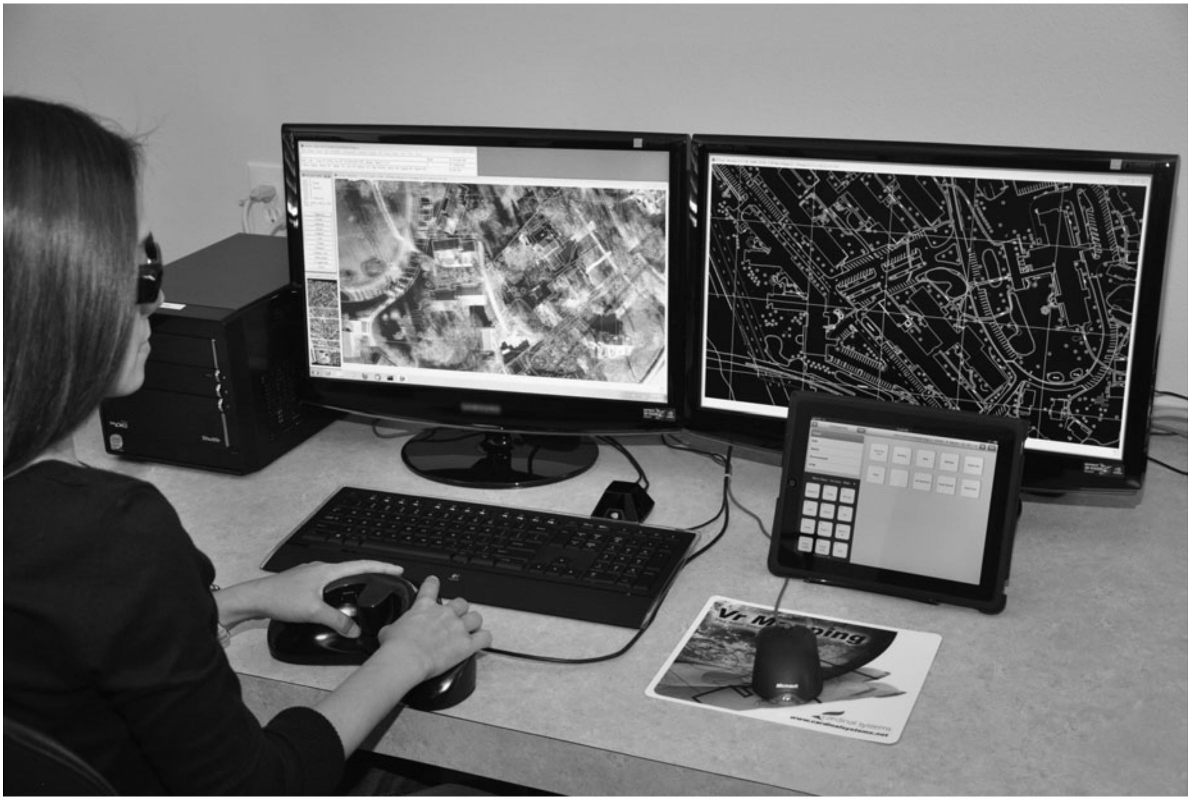

Figure 27.15 Vr Mapping Digital Photogrammetric Workstation. (Courtesy Cardinal Systems, LLC; www.cardinalsystems.net)

to corresponding angles θ_1 and θ_2, respectively, in Figure 27.14(a) (i.e., the angles between the incoming light rays and the camera axis). Preparing the digital images to exacting specifications and precisely observing the images of the fiducial marks will accomplish this. The process called *relative orientation* is accomplished when parallactic angle ϕ' of Figure 27.14(b) for each corresponding pair of light rays will be identical to its corresponding parallactic angle ϕ of Figure 27.14(a) and a perfect three-dimensional model is formed. The model is brought to required scale by making the rays of at least two, but preferably three, ground control points intersect at their known positions at a desired scale. It is leveled by measuring image coordinates on a minimum of three, but preferably four, corner ground control points when the floating mark is set on them. *Absolute orientation* is a term applied to the processes of scaling and leveling the model.

When orientation is completed, a map can be made from the model, or cross sections and other spatial information compiled. In mapping, planimetric details are located first by bringing the floating mark into contact with objects in the model and tracing them. The position of the details are digitally determined and recorded in a map file. Contours are traced by observing the positions and elevations of selected points throughout the model space. Then used procedures similar to those discussed in 18.14, a digital terrain model of the ground is created. Contours are determined from the triangulated model as discussed in 18.14. When a digital map is completed, it is examined for omissions and mistakes and field-checked.

27.14.2 Analytical Stereoplotters

Analytical plotters combine a precise stereoscopic system for measuring photo coordinates, a digital computer, and sophisticated analytical photogrammetry software. In using an analytical plotter, an operator looks through a binocular viewing system and sees the stereomodel formed from a pair of overlapping photos. The floating mark, which again consists of half-marks superimposed within the optics of the viewing system, is placed on points whose ground positions are desired. When the mark is precisely positioned, the x and y photo coordinates of the point from both photos are measured by means of encoders and fed directly to the computer. The computer uses these photo coordinates, together with camera parameters and ground coordinates of control points that have been input into the system, to calculate the point's X, Y, and Z ground coordinates in real time. For this calculation, the analytical procedures described in Section 27.13 are employed. These ground coordinates are then stored in a file within the system's computer. Of course, before extracting information from the stereomodel, analytical plotters must be oriented by following the same basic processes as previously described. Figure 27.16 shows an analytical plotter with a monitor screen on the right to give the operator a visual record of work performed and to permit review and editing of the digitized data.

27.14.3 Softcopy Stereoplotters

Softcopy plotters are the latest development in the evolution of stereoscopic plotting instruments. These systems utilize digital images. The images can be acquired by using a digital camera of the type described in Section 27.3, but more

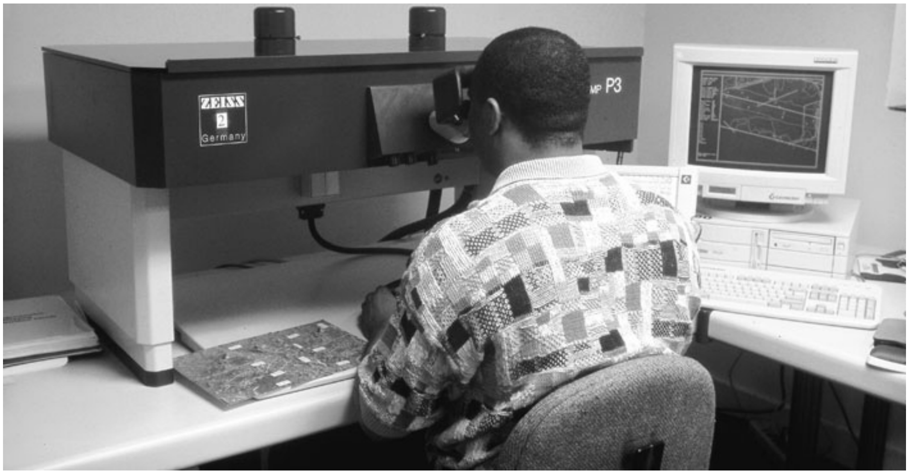

Figure 27.16 Zeiss P3 analytical plotter. (Courtesy Tom Pantages.)

Figure 27.17 Zeiss/Intergraph PS1 PhotoScan. (Courtesy Carl Zeiss.)

often they are obtained by scanning the negatives of aerial photos taken with film cameras. Scanners convert the contents of an aerial photo into an array of pixels arranged in rows and columns. Each pixel is identified by its row and column, and is assigned a digital number, which corresponds to its *gray level* or degree of darkness at that particular element. Figure 27.17 shows the PhotoScan, developed jointly by Carl Zeiss and Intergraph Corporation for digitizing photographs.

A softcopy photogrammetric system requires a computer with a high-resolution graphics display. The computer must be capable of efficiently manipulating large files of digital images and must also be able to display the left and right photos of a stereopair simultaneously. Also the equipment must include a stereoscopic viewing mechanism, that is, one that enables the operator to see only the left image with the left eye and only the right image with the right eye. The operator moves the floating mark about the stereomodel and places it on any point whose position is desired. Once the mark is set, the row and column locations of the pixels at its location identify the photo coordinates of the point to the computer, which can then calculate, in real time, its ground X, Y, and Z coordinates by solving equations of analytical photogrammetry.

One type of softcopy photogrammetric workstation, the digital video plotter (DVP), is shown in Figure 27.15. As seen in the figure, it is simply a standard personal computer with a stereoscopic viewing system attached. With this device, the left and right photos are displayed simultaneously on the left and right portions, respectively, of the computer's monitor. An operator looking through the stereoscope can view the stereomodel in three dimensions. Using the cursor of an interfaced digitizer or the keyboard's arrow keys, the left and right images can be shifted on the screen with respect to each other. The floating marks, made of two reference pixels, one on each image, can also be moved about the screen. While viewing stereoscopically, an operator shifts the imagery and moves the floating mark until it appears to rest exactly on the point of interest. This identifies to the computer the image pixels, which correspond to the point in both the left and right photos. The computer converts the pixel row and column locations to x and y photo coordinates and then calculates the point's ground coordinates by employing standard analytical photogrammetry equations.

Other softcopy photogrammetric systems are also available. Some rely on polarizing lenses for stereoviewing, while others, such as the Intergraph Image Station Z shown in Figure 1.6, use a system of electronically synchronized shutters. All are controlled by sophisticated software. Some advanced systems have become almost totally automated. They employ a process called *digital image correlation.* In this operation, the computer automatically matches points in the left photo of the stereopair to their corresponding or *conjugate* points in the right photo. This is done by comparing the patterns of image densities in the point's immediate area on both photos. Thus, an operator's process of making measurements while stereoviewing is eliminated.

The basic difference between the analytical and softcopy stereoplotters is minimal. Both rely on precise measurements of images, which are then processed using analytical photogrammetric equations. The primary difference is that the analytical stereoplotter requires a hardcopy photograph, whereas the softcopy system uses only digital images. Softcopy systems are gradually replacing analytical stereoplotters in the workplace. In Section 27.20, a simplified analytical photogrammetric system that is implemented in WOLFPACK, which is available on the book's companion website http://www.pearsonhighereed.com/ghilani, is discussed.

27.15 ORTHOPHOTOS

As implied by their name, orthophotos are orthographic representations of the terrain in picture form. They are derived from aerial photos in a process called *differential rectification*, which removes scale variations and image displacements due to relief and tilt. Thus, the imaged features are shown in their true planimetric positions.

Instruments used for differential rectification have varied considerably in their designs. The first-generation instruments were basically modified stereoscopic plotters with either optical or mechanical projection. Optical projection instruments derived an orthophoto by systematically *scanning* a stereomodel and photographing it in a series of adjacent narrow strips. *Rectification* (removal of tilt) was accomplished by leveling the model to ground control prior to scanning, and scale variations due to terrain relief were removed by varying the projection distance during scanning. As the instrument automatically traversed back and forth across the model, exposure was made through a narrow slit onto an orthonegative. An operator, viewing the model in three dimensions, continually monitored the scans and adjusted the projection distance to keep the exposure slit in contact with the stereomodel. Because the model itself had uniform scale throughout, the resulting *orthonegative* (from which the orthophoto was made) was also of uniform scale. Orthophoto systems based on modified mechanical projection stereoplotters functioned in a similar fashion. These mechanical instruments are seldom used today.

Contemporary orthophoto production is done using softcopy photogrammetric systems in a procedure called *digital image processing*. These systems employ digital images, which, as described in Section 27.14.4, may be obtained either by using digital cameras or by scanning negatives obtained with film cameras. As noted earlier, a digital image consists of a *raster* (grid) of tiny pixels, each of which is assigned a digital value corresponding to its gray level, and each having its photo location given in terms of its row and column within the raster. The digital image is input to the system's computer, which uses analytical photogrammetry equations to modify each pixel location according to the tilt in the photograph and the scale at that point in the stereomodel. Through this process, all pixels are modified to locations they would have on a truly vertical photo and all are brought to a common scale. The modified pixels are then printed electronically to produce an orthophoto.

Orthophotos combine the advantages of both aerial photos and line maps. Like photos, they show features by their actual images rather than as lines and symbols, thus making them more easily interpreted and understood. Like maps, orthophotos show the features in their true planimetric positions. Therefore true distances, angles, and areas can be scaled directly from them. *Orthophotomaps* (maps produced from orthophotos) are used for a variety of applications, including planning and engineering design. They have been particularly valuable in cadastral and tax mapping, because the identification of property boundaries is greatly aided through visual interpretation of fence lines, roads, and other evidence. Because they are in digital form, they are also ideal for use as base maps and for analyses in geographic information systems.

Orthophotos can generally be prepared more rapidly and economically than line or symbol planimetric maps. With their many significant advantages, orthophotos have superseded conventional maps for many uses.

■ 27.16 GROUND CONTROL FOR PHOTOGRAMMETRY

As pointed out in preceding sections, almost all phases of photogrammetry depend on ground control (points of known positions and elevations with identifiable images on the photograph). Ground control can be basic control—traverse, triangulation, trilateration or GNSS monuments already in existence and marked prior to photography to make them visible on the photos; or it can be *photo control*—natural points having images recognizable on the photographs, and positions that are subsequently determined by ground surveys originating from basic control. Instruments and procedures used in ground surveys were described in earlier chapters. Ordinarily, photo control points are selected after photography to ensure their satisfactory location and positive identification. Premarking points with artificial targets is sometimes necessary in areas that lack natural objects to provide definite images.

As discussed in Section 27.14, for mapping with stereoplotters scaling and leveling stereomodels require a practical minimum of three horizontal control points and four vertical points in each model. For large mapping jobs, therefore, the cost of establishing the required ground control is substantial. In these situations, *analytical aerotriangulation* (see Section 27.13) is used to establish many of the needed control points from only a sparse network of ground-surveyed points. This reduces costs significantly.

Currently GNSS survey methods are being used for real-time positioning of the camera at the instant each photograph is exposed. The kinematic GNSS surveying procedure is being employed (see Chapter 15), which requires two GNSS receivers. One unit is stationed at a ground control point, and the other is placed within the aircraft carrying the camera. The integer ambiguity problem is resolved using *on-the-fly* techniques (see Section 15.2). During the flight, camera positions are continuously determined at time intervals of a few seconds using the GNSS units, and precise timing of each photo exposure is also recorded. From this information, the precise location of each exposure station, in the ground coordinate system, can be calculated. Many projects have been completed using these methods, and they have produced highly accurate results, especially when supplemented with only a few ground control points. It is now possible to complete photogrammetric projects with only a few ground photo control points used for checking purposes.

■ 27.17 FLIGHT PLANNING

Certain factors, depending generally on the purpose of the photography, must be specified to guide a flight crew in executing its mission of taking aerial photographs. Some of them are (1) boundaries of the area to be covered, (2) required scale of the photography, (3) camera focal length and format size, (4) endlap, and (5) sidelap. Once these elements have been fixed, it is possible to compute the entire flight plan and prepare a flight map on which the required flight lines have been delineated. The pilot then flies the specified flight lines by choosing and correlating headings on existing natural features shown on the flight map. In the most modern systems, the flight planning is done using a computer and the coordinates of flight lines are calculated. Then the aircraft is automatically guided by an on-board GNSS system along the planned flight lines.

The purpose of the photography is the paramount consideration in flight planning. For example, in taking aerial photos for topographic mapping using a stereoplotter, endlap should optimally be 60% and sidelap 30%. The required scale and contour interval of the final map must be evaluated to settle flying height. Enlargement capability from photo scale to map compilation scale is restricted for stereoplotting, and generally should not exceed about 5 if satisfactory accuracy is to be achieved. By these criteria, if required map scale is 200 ft/in., photo scale becomes fixed at 1000 ft/in. If the camera focal length is 6 in., flying height is established by Equation (27.2) at $6(1000) = 6000$ ft above average terrain. Some organizations may push this factor higher than 5, but it should be done with caution.

The *C factor* (the ratio of flight height above ground to contour interval that is practical for any specific stereoplotter) is a criterion often used to select the flying height in relation to the required contour interval. To ensure that their maps meet required accuracy standards, many organizations employ a *C* factor of about 1200 to 1500. Other organizations may push the value somewhat higher, but again this should be done with caution. By this criterion, if a plotter has a C factor of, say, 1200, and a map is to be compiled with a 5-ft contour interval, a flight height of not more than $1200(5) = 6000$ ft above the terrain should be sustained.

Information ordinarily calculated in flight planning includes (1) flying height above mean sea leve1, (2) distance between exposures, (3) number of photographs per flight line, (4) distance between flight lines, (5) number of flight lines, and (6) total number of photographs. A flight plan is prepared based on these items.

Example 27.7

A flight plan for an area 10 mi wide and 15 mi long is required. The average terrain in the area is 1500 ft above datum. The camera has a 6 in. focal length with 9×9 in. format. Endlap is to be 60%, sidelap 25%. The required scale of the photography is 1:12,000 (1000 ft/in.).

Solution

1. Flying height above datum from Equation (27.2):

$$\text{scale} = \frac{f}{H - h_{\text{avg}}} \text{ so } \frac{1}{1000} = \frac{6}{H - 1500} \text{ and } H = 7500 \text{ ft}$$

2. Distance between exposures, d_e: endlap is 60%, so the linear advance per photograph is 40% of the total coverage of 9 in. $\times$ 1000 ft/in. = 9000 ft. Thus, the distance between exposures is $0.40 \times 9000 = 3600$ ft.
3. Total number of photographs per flight line:

$$\text{length of each flight line} = 15 \text{ mi}(5280 \text{ ft/mi}) = 79{,}200 \text{ ft}$$

$$\text{number of photos per flight line, } N_{\text{Photos/Line}} = \frac{79{,}200 \text{ ft}}{3600 \text{ ft/photo}} + 1 = 23$$

Adding two photos on each end to ensure complete coverage, the total is $23 + 2 + 2 = 27$ photos per flight line.

4. Distance between flight lines: sidelap is 25%, so the lateral advance per flight line is 75% of the total photographic coverage,

$$\text{distance between flight lines, } d_s = 0.75(9000 \text{ ft}) = 6750 \text{ ft}$$

5. Number of flight lines:

$$\text{width of area} = 10 \text{ mi}(5280 \text{ ft/mi}) = 52{,}800 \text{ ft}$$

$$\text{number of spaces between flight lines} = \frac{52{,}800 \text{ ft}}{6750 \text{ ft/line}} = 7.8 \text{ (say 8)}$$

$$\text{total flight lines, } N_{\text{Lines}} = 8 + 1 = 9$$

$$\text{planned spacing between flight lines} = \frac{52{,}800}{8} = 6600 \text{ ft}$$

(*Note:* The first and last flight lines should either coincide with or be near the edges of the area, thus providing a safety factor to ensure complete coverage.)

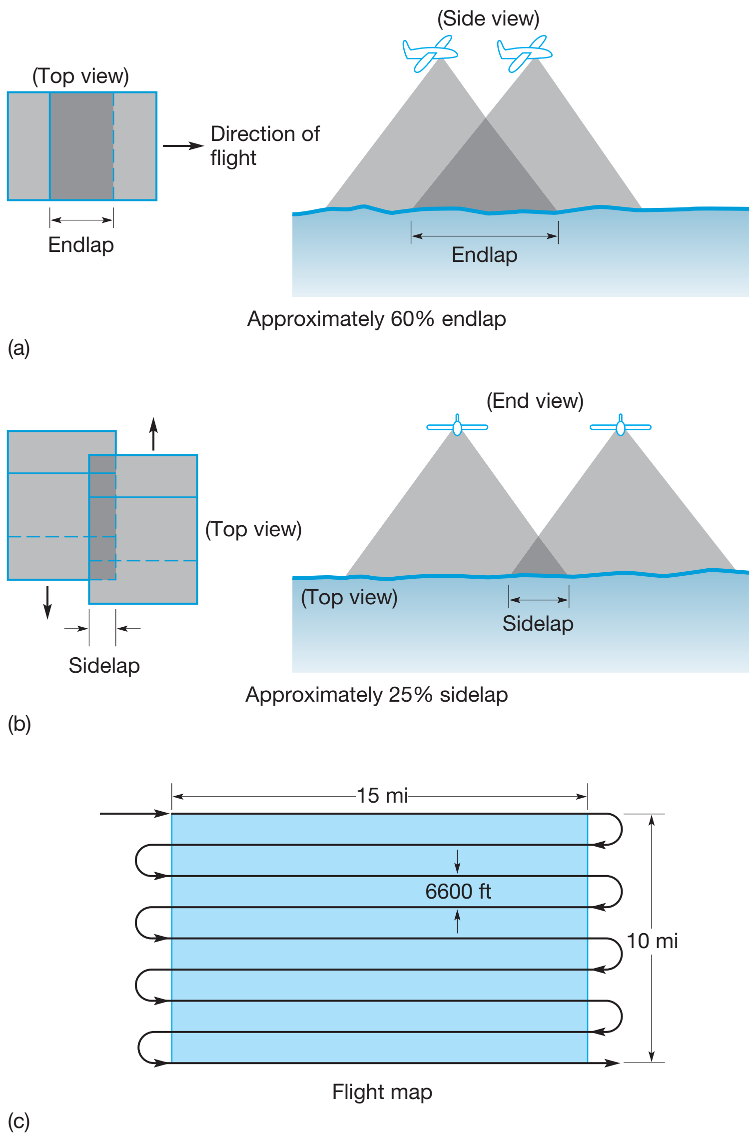

Figure 27.18
(a) Endlap,
(b) Sidelap, and
(c) Flight map

6. Total number of photos required:

$$\text{total photos} = 27 \text{ per flight line} \times 9 \text{ flight lines} = 243 \text{ photos}$$

Figures 27.18(a) and (b) illustrate endlap and sidelap, respectively, and (c) shows the flight map.

■ 27.18 AIRBORNE LASER-MAPPING SYSTEMS

Airborne laser-mapping systems called *LiDAR* (**Li**ght **D**etection **a**nd **R**anging) have recently been developed which show great promise for the future. These systems, which are carried in airborne vehicles, consist of a laser scanning device, an inertial navigation system, a GNSS receiver, and computer. As the aircraft flies along its trajectory, laser pulses are transmitted toward the terrain below, reflected from the ground or other objects, and detected nearly instantaneously. From these pulses, distances and angles to reflective objects are determined. Concurrently, the inertial navigation device records the aircraft's attitude angles (pitch, yaw, and roll), and the GNSS receiver determines the X, Y, and Z positions of the detector. The computer processes all of this information to determine vector displacements (distances and directions) from known positions in the air, to unknown positions on the ground, and as a result it is able to compute the X, Y, and Z positions of the ground points.

The laser transmitter can generate pulses at an extremely rapid rate, that is, thousands per second, so that the coordinates of a dense pattern of ground points can be determined. Not only are positions of ground points determined, but an image of the ground is also generated. The data can be used to produce digital elevation models (DEMs) and from them contour maps and other topographic products can be produced. Accuracies possible with LiDAR devices are currently in the range of 10–15 cm, but with continued research and development, this is expected to improve.

■ 27.19 REMOTE SENSING

In general, remote sensing can be defined as any methodology employed to study the characteristics of objects using data collected from a remote observation point. More specifically, and in the context of surveying and photogrammetry, it is the extraction of information about the earth and our environment from imagery obtained by various sensors carried in aircraft and satellites. Satellite imagery is unique because it affords a practical means of monitoring our entire planet on a regular basis.

Remote sensing imaging systems operate much the same as the human eye, but they can sense or "see" over a much broader range than humans. Cameras that expose various types of film are among the best types of remote sensing imaging systems. Nonphotographic systems such as *multispectral scanners* (MSS), *radiometers*, *side-looking airborne radar* (SLAR), and *passive microwave* are also employed. Their manner of operation, and some applications of the imagery, is briefly described below.

The sun and other sources emit a wide range of electromagnetic energy called the *electromagnetic spectrum*. X-rays, visible light rays, and radio waves are some familiar examples of energy variations within the electromagnetic spectrum. Energy is classified according to its wavelength (see Figure 27.19). Visible

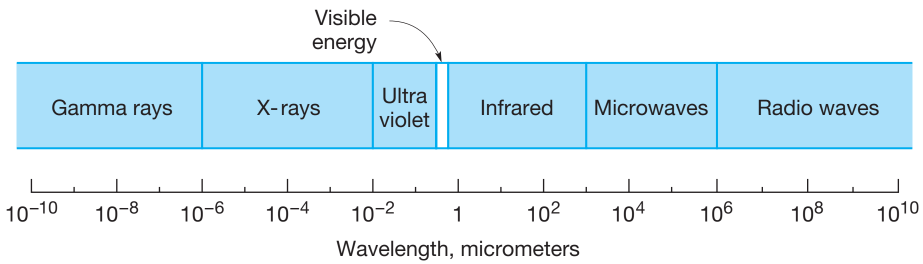

Figure 27.19 Classification of electromagnetic spectrum by wavelength.

light (that energy to which our eyes are sensitive) has wavelengths from about 0.4 to 0.7 μm and thus, as illustrated in the figure, comprises only a very small portion of the spectrum.

Within the wavelengths of visible light, the human eye is able to distinguish different colors. The primary colors (blue, green, and red) consist of wavelengths in the ranges of 0.4–0.5, 0.5–0.6, and 0.6–0.7 μm, respectively. All other hues are combinations of the primary colors. To the human eye, an object appears a certain color because it reflects energy of wavelengths producing that color. If an object reflects all wavelengths of energy in the visible range, it will appear white, and if it absorbs all wavelengths, it will be black. If an object absorbs all green and red energy but reflects blue, that object will appear blue.

Just as the retina of the human eye can detect variations in wavelengths, photographic films or *emulsions* are also manufactured to have wavelength sensitivity variations. Normal color emulsions are sensitive to blue, green, and red energy; others respond to energy in the near-infrared range. These are called *infrared* (IR) emulsions. They make it possible to photograph energy that is invisible to the human eye. An early application of IR film was in camouflage detection, where it was found that dead foliation or green netting reflected infrared energy differently than normal vegetation, even though both appeared green to the human eye. Infrared film is now widely used for a variety of applications, such as detection of crop stress, and for identification and mapping of tree species.

Nonphotographic imaging systems used in remote sensing are able to detect energy variations over a broad range of the electromagnetic spectrum. MSS systems, for example, are carried in satellites and can operate within wavelengths from about 0.3–14 μm. In a manner similar to the way humans detect colors, MSS units isolate incoming energy into discrete spectral categories or *bands*, and then convert them into electric signals that can be represented by digits. These devices capture a digital image (see Section 27.3), that is, the scanned scene below the satellite's path is recorded as a series of contiguous rows and columns of pixels. The digits associated with each pixel represent intensities of the various bands of energy within them. This digital format is ideal for computer processing and analysis and enables prints to be made by electronic processing. The bands of a scene can be analyzed separately, which is extremely useful in identifying and interpreting imaged objects. For certain applications, it is useful to combine two or more bands into a composite. The geometry of nonphotographic images differs from that of perspective photos, and thus methods for analyzing them also differ.[2]

Figure 27.20 shows an image obtained with the MSS system carried in an early Landsat satellite. It was taken at an altitude of 560 miles and shows a large portion of southeastern Wisconsin, including the city of Milwaukee. A portion of Lake Michigan is shown on the right side of the figure. Imagery of this type is useful for a variety of applications. As examples, geologic formations over large areas can be studied; the number of lakes in the area, their relative positions, shapes, and acreages can be determined readily; acreages of croplands and forests, with a

[2]Information on the geometry of nonphotographic imaging systems can be found in Lillesand and Kiefer, *Remote Sensing and Image Interpretation*. (See the bibliography at the end of this chapter.)

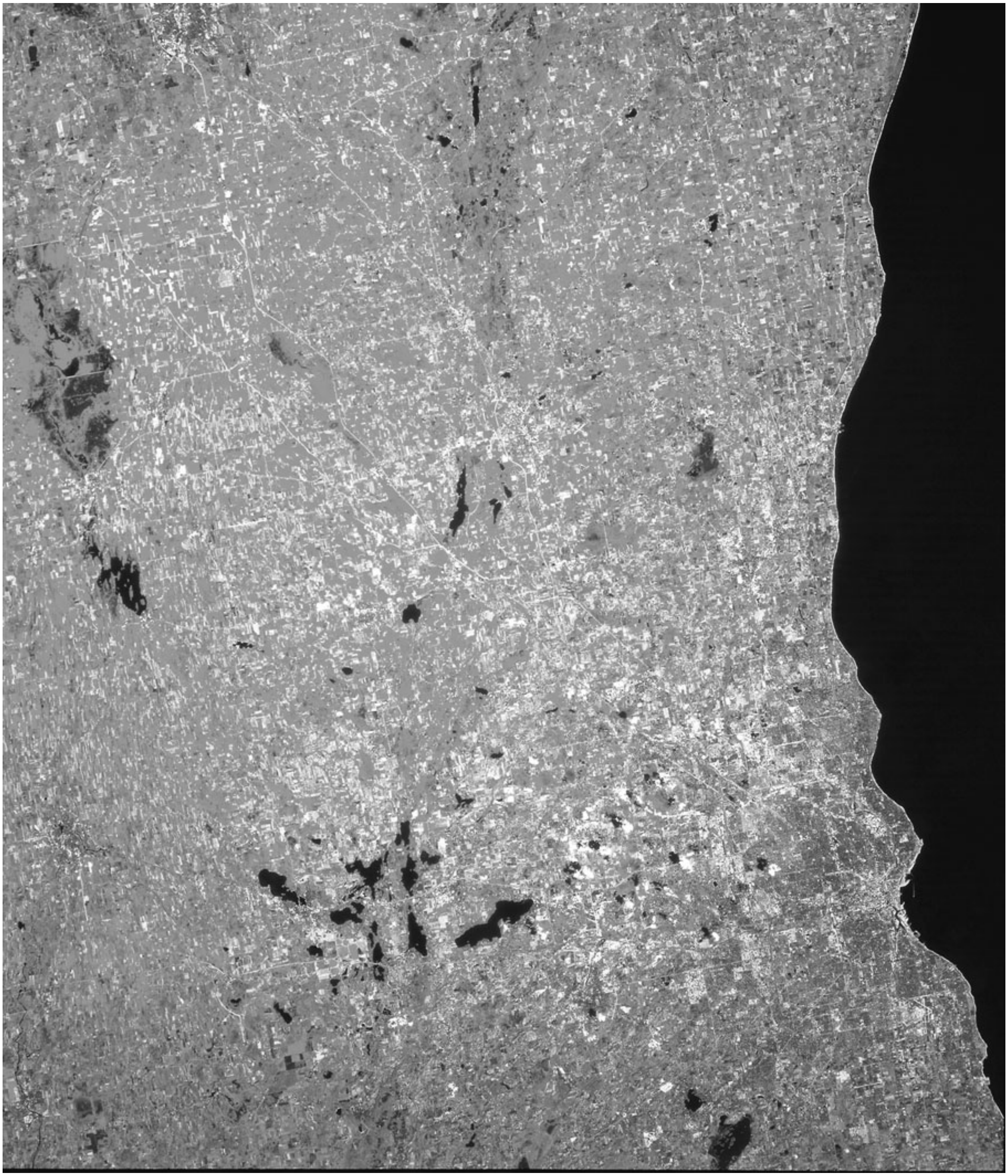

Figure 27.20 Multispectral scanner image taken from a first-generation Landsat satellite over Milwaukee, Wisconsin.

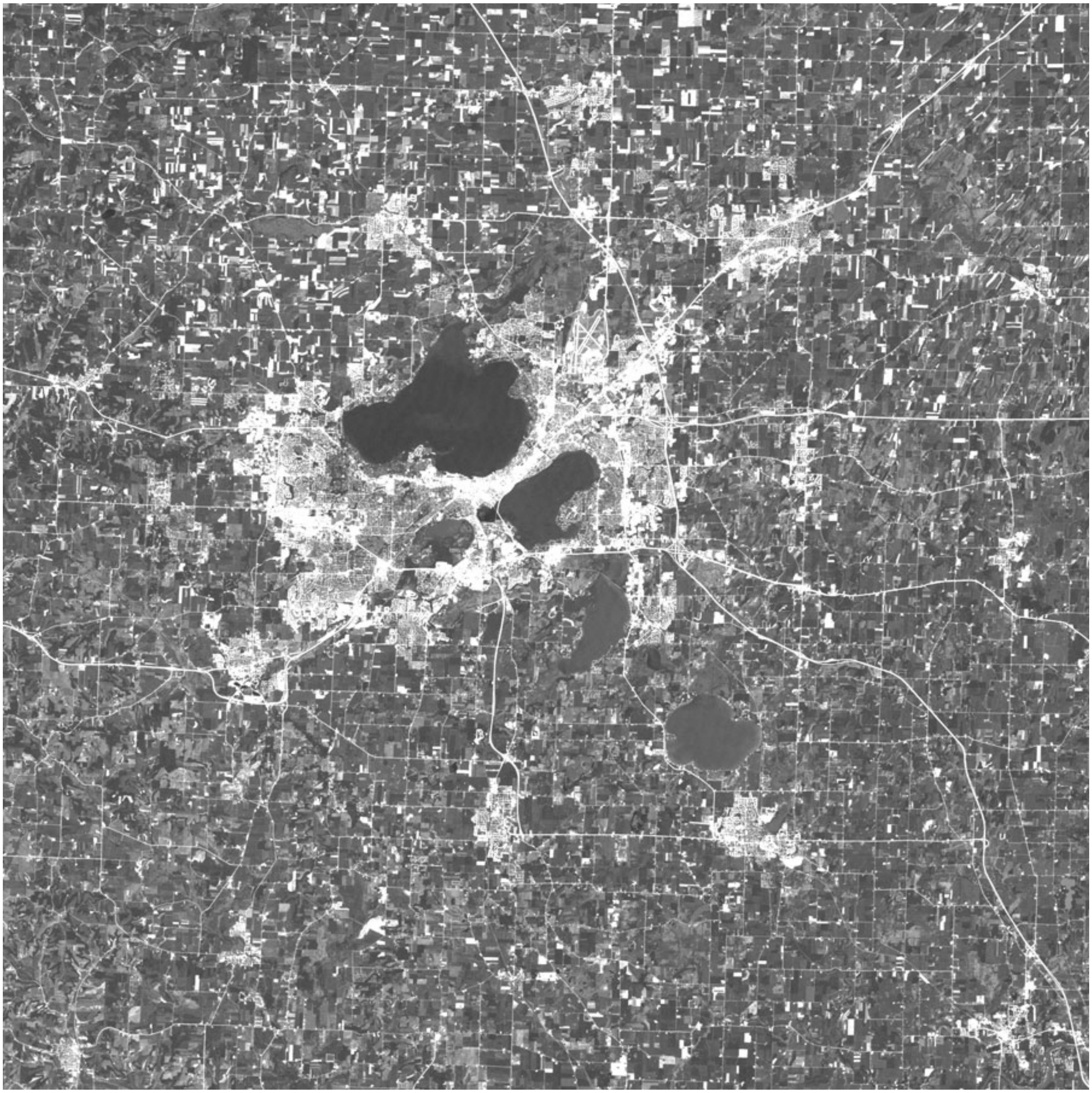

Figure 27.21 Landsat Thematic Mapper image taken near Madison, Wisconsin. (Note the improved resolution compared to the Landsat MSS image of Figure 27.20.) (Courtesy Environmental Remote Sensing Center, University of Wisconsin–Madison.)

breakdown of coniferous and deciduous tree types, can be obtained; and small-scale planimetric maps showing these different land-use classifications can be prepared. All of these tasks are amenable to computer processing.

The resolution of the earlier Landsat MSS imaging systems (like that used to acquire Figure 27.20) was 80 m, that is, each pixel represents a square of 80 m on the ground. This was improved to 30 m for the *Thematic Mapper* (TM) imaging systems carried in the second generation of Landsat satellites. Figure 27.21

shows an image taken by Landsat TM near Madison, Wisconsin. It clearly illustrates the improved resolution as compared to Figure 27.20. Note, for example, the clarity of the roads and the detail with which urban areas and agricultural crops are shown. Images of this type have been applied for land-use mapping; measuring and monitoring various agricultural crops; mapping soils; detecting diseased crops and trees; locating forest fires; studying wildlife; mapping the effects of natural disasters such as tornadoes, floods, and earthquakes; analyzing population growth and distribution; determining the locations and extent of oil spills; monitoring water quality and detecting the presence of pollutants; and accomplishing numerous other tasks over large areas for the benefit of humankind.

In recent years a significant amount of research and development has been directed toward the design of satellite imaging systems with improved resolution and geometric properties. The goals have been to enable smaller objects to be identified and analyzed, and to improve the mapping capabilities of the systems, thus making them useful for many additional types of applications. The *Enhanced Thematic Mapper Plus* (ETM+) imaging system aboard the most recent Landsat Satellite (Landsat 7), which was launched in April, 1999, has a resolution capability of 15 m. This satellite operates at an altitude of 438 mi, and the imaging system covers a ground swath beneath its orbit that is 115 mi wide.[3] The French *Systeme Pour d'Observation de la Terre* (SPOT) satellite has an imaging system with 10 m resolution and a nadir ground swath width of 37 miles. Its imaging system can be aimed at angles up to 27° off-nadir. With this feature, the same areas that were imaged on earlier passes can be covered again on subsequent passes from different orbits, thereby achieving stereoscopic coverage. This imagery is therefore suitable not only for small-scale planimetric mapping but also for determining elevations.[4]

In September, 1999, Space Imaging launched the first commercial imaging satellite, IKONOS.[5] The most remarkable characteristic of its imaging system is its resolution of 1 m. This satellite is in orbit at an altitude of 423 miles above Earth, and the width of its image at the nadir is approximately 7 miles. The imaging system can be aimed off-nadir, either from side to side or fore and aft. This not only enables pinpointing coverage on areas of interest but can also be used to obtain stereoscopic coverage. Thus, the imagery is suitable for both planimetric mapping and for determining elevations of ground points. Figure 27.22 is a 1-m image taken from IKONOS on October 11, 1999 over the city of San Francisco. It features an Aquatic Park and a Fisherman's Wharf. Note that the level of detail that can be resolved from this image is far superior to those of Figures 27.20 or 27.21. Individual houses, trees, small boats, and even automobiles can be identified. This imagery is useful for many types of applications. To mention just a few

[3]Images from Landsat 7 and all previous Landsat satellites are available from the U.S. Geological Survey, Earth Science Information Center (ESIC), 12201 Sunrise Valley Drive, Reston, VA 20192. Contact can also be made by telephone at (888)ASK-USGS [(888)275-8747], or at the following website: http://mapping.usgs.gov/esic/.

[4]Information on SPOT imagery can be obtained at the following website: http://www.spotimage.fr.

[5]Information about IKONOS satellite Imagery can be obtained from Space Imaging by telephone at (800)232-9037, or by visiting the following website: http://www.spaceimaging.com.

Figure 27.22
1-m resolution image obtained from the IKONOS satellite showing Aquatic Park and Fisherman's Wharf in San Francisco. (Courtesy NASA.)

possibilities of interest to surveyors and engineers, the imagery is suitable for (a) preparing site maps and preliminary plans for proposed construction projects; (b) planning for locations of GNSS stations in large-area control surveys, or planning aerial photography; (c) generating detailed information layers such as land cover, hydrography, transportation networks, etc., for use in geographic information systems; and (d) using the images as reference frames for performing GIS analyses. There are also a variety of additional applications in many other fields such as forestry, geology, agriculture, etc.

In the future, those engaged in surveying (geomatics) will be called on to prepare maps and to extract a variety of other positional types of information from satellite images. Remote sensing will play a significant future role in providing data to assess the impacts of human activities on our air, water, and land resources. It can provide valuable information to assist in making sound decisions and formulating policies related to resource management and land-use and land development activities.

■ 27.20 SOFTWARE

As previously discussed, softcopy photogrammetric stereoplotters are efficient, as well as versatile. Not only are they capable of producing maps, cross sections, digital elevation models, and other digital topographic files, but they can also be employed for a variety of image interpretation problems, and they can support the production of mosaics and orthophotos (see Section 27.15). Also, digital maps produced by softcopy systems are created in a computer environment, and are therefore in formats compatible for CAD applications, and for use in the databases of Geographic Information Systems. Softcopy systems have the added advantage that their major item of hardware is a computer rather than an expensive single-purpose stereoplotter, so it can be used for many other tasks in addition to stereoplotting. For these reasons, softcopy stereoplotters are taking over the industry.

As shown in Figure 27.23, a digital image viewing and measuring system has been incorporated in the WOLFPACK software, which is available on the book's companion website http://www.pearsonhighered.com/ghilani. While this software is far from being a softcopy stereoplotter, it helps demonstrate some of the basic principles in analytical photogrammetry. The software utilizes digital images in the bitmap (bmp) format. The resulting image coordinates can be placed into the photo coordinate system using the *interior orientation* option. This option requires the calibrated fiducial coordinates of the camera and transforms the digitized image coordinates into the photo coordinate system. Furthermore, by observing a minimum of

Figure 27.23 The observation of fiducial F3 in the WOLFPACK photogrammetric software.

three imaged points whose ground coordinates are known, an *exterior orientation* can be performed. The exterior orientation determines the camera location and orientation parameters at the time of exposure. Finally if this procedure is also performed on common points lying in the overlapping region of two photos, the ground coordinates of these points can be determined using the *space intersection* option. All the required points (fiducial points, ground control, and photo-identifiable points) for the analytical process should be observed in a single digitizing session so that repetition of the procedures is not necessary. This photogrammetric process is described in the help file that accompanies the WOLFPACK software.

While these functions only demonstrate the rudimentary operations of softcopy systems, it allows the reader to experience them. The companion website also contains suitable aerial imagery, a file of calibrated fiducial coordinates (camera.fid), and another of ground control (ground.crd). The fiducial coordinates on the image are labeled F1 through F8 and the ground control is also circled and labeled in the imagery to aid the user in identifying the points. The calibrated focal length of the aerial camera was 153.742 mm. Problems at the end of this chapter refer to this material. In Figure 27.23, fiducial F3, which is in the upper-left corner of the image, is being observed. Note that the user can monitor the image coordinates of the point and their standard deviations in the lower portion of the display. The thumbnail image in the lower portion of the screen allows the user to move rapidly around the image without using the scroll bars. The reset image button restores the image to its original position in the screen. In the lower-right corner of the display is an option box to modify the cursor to several different shapes. A circular cursor with a point at its center is being used to point on the fiducial mark.

■ 27.21 SOURCES OF ERROR IN PHOTOGRAMMETRY

Some sources of error in photogrammetric work are:

1. Measuring instruments not standardized or calibrated.
2. Inaccurate location of principal and corresponding principal points.
3. Failure to use camera calibration data.
4. Assumption of vertical exposures when photographs are actually tilted.
5. Presumption of equal flying heights when they were unequal.
6. Disregard of differential shrinkage or expansion of photographic prints.
7. Incorrect orientation of photographs under a stereoscope or in a stereoscopic plotter.
8. Faulty setting of the floating mark on a point.

■ 27.22 MISTAKES

Some mistakes that occur in photogrammetry are:

1. Incorrect reading of measuring scales.
2. Mistake of units—for example, inches instead of millimeters.
3. Confusion in identifying corresponding points on different photographs.

4. Failure to provide proper control or use of erroneous control coordinates.
5. Attachment of an incorrect sign (plus or minus) to a measured photographic coordinate.
6. Blunder in computations.
7. Misidentification of control-point images.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

27.1 Describe the difference between vertical, low oblique, and high oblique aerial photos.

27.2 Discuss the advantages of softcopy stereoplotters over optical stereoplotters.

27.3 Define the terms (a) metric photogrammetry and (b) interpretative photogrammetry.

27.4 Describe briefly how a digital camera operates.

27.5 The distance between two points on a vertical photograph is *ab* and the corresponding ground distance is *AB*. For the following data, compute the average photographic scale along the line *ab*.

*__(a)__ ab = 2.41 in.; AB = 4820 ft

(b) ab = 5.29 in.; AB = 13,218 ft

(c) ab = 107.389 mm; AB = 536.943 m

27.6 On a vertical photograph of flat terrain, section corners appear a distance d apart. If the camera focal length is f compute flying height above average ground in feet for the following data:

(a) d = 1.85 in.; f = 3-1/2 in.

(b) d = 82.184 mm; f = 153.20 mm

27.7 On a vertical photograph of flat terrain, the scaled distance between two points is *ab*. Find the average photographic scale along *ab* if the measured length between the same line is *AB* on a map plotted at a scale of S_{map} for the following data.

(a) ab = 1.47 in.; AB = 3.52 in.; S_{map} = 1:6000

(b) ab = 41.53 mm; AB = 6.23 mm; S_{map} = 1:20,000

27.8 What are the average scales of vertical photographs for the following data, given flying height above sea level H, camera focal length f, and average ground elevation h?

*__(a)__ H = 7300 ft; f = 152.4 mm; h = 1250 ft

(b) H = 6980 ft; f = 6.000 in.; h = 1004 ft

(c) H = 2610 m; f = 152.4 mm; h = 324 m

27.9 The length of a football field from goal post to goal post scales 49.15 mm on a vertical photograph. Find the approximate dimensions (in meters) of a large rectangular building that also appears on this photo and whose sides measure 20.5 mm by 6.8 mm. (Hint: football goal posts are 120 yards apart.)

27.10* Compute the area in acres of a triangular parcel of land whose sides measure 48.78 mm, 84.05 mm, and 69.36 mm on a vertical photograph taken from 6050 ft above average ground with a 152.4 mm focal length camera.

27.11 Calculate the flight height above average terrain that is required to obtain vertical photographs at an average scale of S if the camera focal length is f for the following data:

(a) S = 1:8000; f = 152.4 mm

(b) S = 1:6000; f = 88.9 mm

27.12 Determine the horizontal distance between two points A and B whose elevations above datum are $h_A = 1560$ ft and $h_B = 1425$ ft, and whose images a and b on a vertical photograph have photo coordinates $x_a = 2.95$ in., $y_a = 2.32$ in., $x_b = -1.64$ in., and $y_b = -2.66$ in. The camera focal length was 152.4 mm and the flying height above datum 7500 ft.

27.13* Similar to Problem 27.12, except that the camera focal length was 3-1/2 in., the flying height above datum 4075 ft, and elevations h_A and h_B, 983 ft and 1079 ft, respectively. Photo coordinates of images a and b were $x_a = 108.81$ mm, $y_a = -73.73$ mm, $x_b = -87.05$ mm, and $y_b = 52.14$ mm.

27.14 On the photograph of Problem 27.12, the image c of a third point C appears. Its elevation $h_C = 1365$ ft, and its photo coordinates are $x_c = 2.96$ in. and $y_c = -3.02$ in. Compute the horizontal angles in triangle ABC.

27.15 On the photograph of Problem 27.12, the image d of a third point D appears. Its elevation is $h_D = 1195$ ft, and its photo coordinates are $x_d = 56.86$ mm and $y_d = 63.12$ mm. Calculate the area, in acres, of triangle ABD.

27.16 Determine the height of a radio tower, which appears on a vertical photograph for the following conditions of flying height above the tower base H, distance on the photograph from principal point to tower base r_b, and distance from principal point to tower top r_t.

*__(a)__ $H = 2425$ ft; $r_b = 3.18$ in.; $r_t = 3.34$ in.

(b) $H = 6600$ ft; $r_b = 96.83$ mm; $r_t = 98.07$ mm

27.17 On a vertical photograph, images a and b of ground points A and B have photographic coordinates $x_a = 3.27$ in., $y_a = 2.28$ in., $x_b = -1.95$ in., and $y_b = -2.50$ in. The horizontal distance between A and B is 5283 ft, and the elevations of A and B above datum are 646 ft and 756 ft, respectively. Using Equation (27.9), calculate the flying height above datum for a camera having a focal length of 152.4 mm.

27.18 Similar to Problem 27.17, except $x_a = -52.53$ mm, $y_a = 69.67$ mm, $x_b = 26.30$ mm, $y_b = -59.29$ mm, line length $AB = 4706$ ft, and elevations of points A and B are 925 and 875 ft, respectively.

27.19* An air base of 3205 ft exists for a pair of overlapping vertical photographs taken at a flying height of 5500 ft above MSL with a camera having a focal length of 152.4 mm. Photo coordinates of points A and B on the left photograph are $x_a = 40.50$ mm, $y_a = 42.80$ mm, $x_b = 23.59$ mm, and $y_b = -59.15$ mm. The x photo coordinates on the right photograph are $x_a = -60.68$ mm and $x_b = -70.29$ mm. Using the parallax equations, calculate horizontal length AB.

27.20 Similar to Problem 27.19, except the air base is 6940 ft, the flying height above mean sea level is 12,520 ft, the x and y photo coordinates on the left photo are $x_a = 37.98$ mm, $y_a = 50.45$ mm, $x_b = 24.60$ mm, and $y_b = -46.89$ mm, and the x photo coordinates on the right photo are $x_a = -52.17$ mm and $x_b = -63.88$ mm.

27.21 Calculate the elevations of points A and B in Problem 27.19.

27.22 Compute the elevations of points A and B in Problem 27.20.

27.23 List and briefly describe the four different categories of stereoscopic plotting instruments.

27.24 Name the three stages in stereoplotter orientation, and briefly explain the objectives of each.

27.25 What advantages does a softcopy plotter have over an analytical plotter?

27.26 What kind of images do softcopy stereoplotters require? Describe two different ways they can be obtained.

27.27 Compare an orthophoto with a conventional line and symbol map.
27.28 Discuss the advantages of orthophotos as compared to maps.

Aerial photography is to be taken of a tract of land that is X mi square. Flying height will be H ft above average terrain, and the camera has focal length f. If the focal plane opening is 9×9 in., and minimum sidelap is 30%, how many flight lines will be needed to cover the tract for the data given in Problems 27.29 and 27.30?

27.29* $X = 8$; $H = 4000$; $f = 152.4$ mm.
27.30 $X = 30$; $H = 10{,}000$; $f = 6$ in.

Aerial photography was taken at a flying height H ft above average terrain. If the camera focal plane dimensions are 9×9 in, the focal length is f and the spacing between adjacent flight lines is X ft, what is the percent sidelap for the data given in Problems 27.31 and 27.32?

27.31* $H = 4500$; $f = 152.4$ mm; $X = 4700$.
27.32 $H = 6800$; $f = 88.9$ mm; $X = 13{,}500$.

Photographs at a scale of S are required to cover an area X mi square. The camera has a focal length f and focal plane dimensions of 9×9 in. If endlap is 60% and sidelap 30%, how many photos will be required to cover the area for the data given in Problems 27.33 and 27.34?

27.33 $S = 1{:}6000$; $X = 6$; $f = 152.4$ mm
27.34 $S = 1{:}14{,}400$; $X = 40$; $f = 89.0$ mm.
27.35 Describe a system that employs GPS and which can reduce or eliminate ground control surveys in photogrammetry?
27.36 To what wavelengths of electromagnetic energy is the human eye sensitive? What wavelengths produce the colors blue, green, and red?
27.37 Discuss the uses and advantages of satellite imagery.

Problems 27.38 through 27.42 involve using WOLFPACK with images 5 and 6 on the companion website. The ground coordinates of the paneled points are listed in the file "ground.crd." The coordinates of the fiducials are listed in the file "camera.fid." To do these problems, digitize the eight fiducials and paneled points 21002, 4, 41, GYM, WIL1A, WIL1B, and RD on both images. After digitizing the points, perform an interior orientation to compute photo coordinates for the points on images 5 and 6. The focal length of the camera is 153.742 mm.

27.38 Using photo coordinates for points 4 and GYM on image 5, determine the scale of the photo.
27.39 Using photo coordinates for points 4 and GYM on image 5, determine the flying height of the camera at the time of exposure.
27.40 Using photo coordinates for points 4 and GYM on images 5 and 6, determine the ground coordinates of points WIL1A and WIL1B using Equation (27.12) and Equation (27.13).
27.41 Using the exterior orientation option in WOLFPACK, determine the exterior orientation elements for image 5.
27.42 Using the exterior orientation option in WOLFPACK, determine the exterior orientation elements for image 6.

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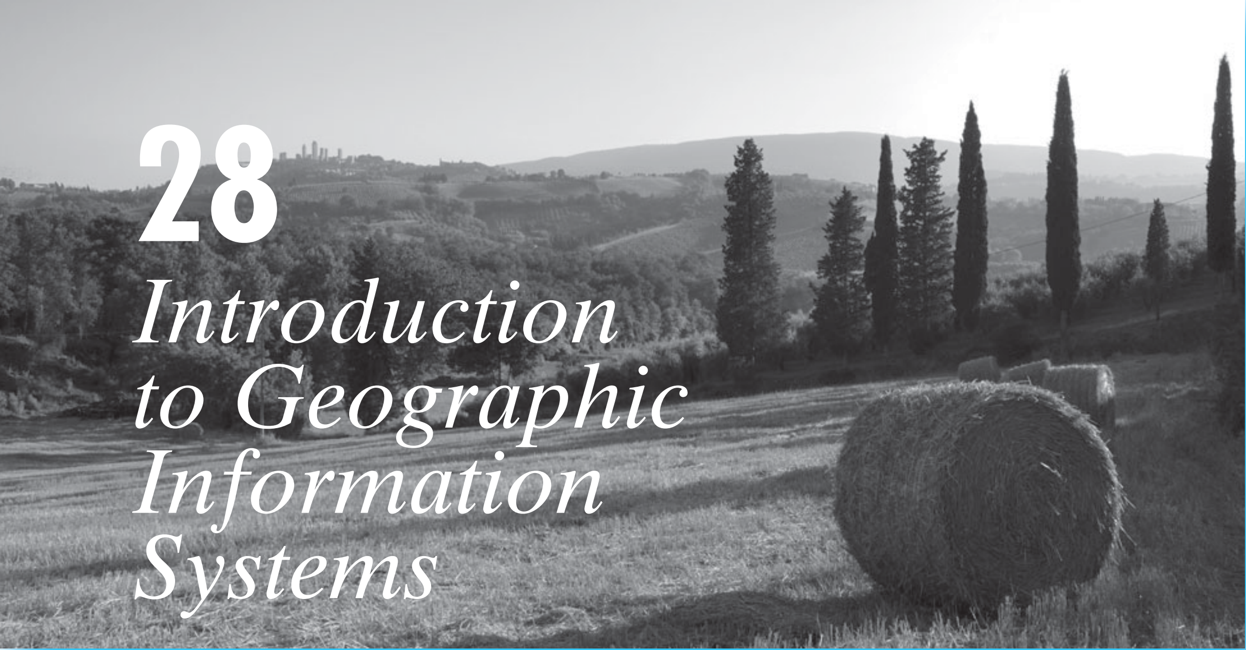


28 Introduction to Geographic Information Systems

■ 28.1 INTRODUCTION

The term *Geographic Information System* (GIS) first appeared in published literature in the mid-1960s. But although the term is relatively new, many of its concepts have long been in existence. For example, the *map overlay* concept, which is one of the important tools used in GIS spatial analysis (see Section 28.9), was used by French cartographer Louis-Alexandre Berthier more than 200 years ago. He prepared and overlaid a series of maps to analyze troop movements during the American Revolution. In 1854, Dr. John Snow demonstrated another early example illustrating the use and value of the overlay concept. He overlaid a map of London showing where cholera deaths had occurred with another, giving locations of wells in that city to demonstrate the relationship between those two data sets. These early examples illustrate fundamentals that still comprise the basis of our modern GIS, that is, making decisions based on the simultaneous analysis of data of differing types, all located spatially in a common geographic reference system. However, the full capabilities and benefits of our modern GISs could not occur until the advent of the computer.

In general, a geographic information system can be defined as a system of hardware, software, data, and organizational structure for collecting, storing, manipulating, and spatially analyzing "geo-referenced" data and displaying information resulting from those processes. A more detailed definition (Hanigan, 1988) describes a GIS as "any information management system that can:

1. Collect, store, and retrieve information based on its spatial location;
2. Identify locations within a targeted environment that meet specific criteria;

3. Explore relationships among data sets within that environment;
4. Analyze the related data spatially as an aid to making decisions about that environment;
5. Facilitate selecting and passing data to application-specific analytical models capable of assessing the impact of alternatives on the chosen environment; and
6. Display the selected environment both graphically and numerically either before or after analysis."

The thread that is common to both definitions just given is that in a GIS, decisions are made based on spatial analyses performed on data sets that are referenced in a common geographical system. The geographic referencing system used could be a state plane or UTM coordinate system, latitude and longitude, or other suitable local coordinate system. In any GIS, the accuracy of the spatial analyses and, hence, the validity of decisions reached as a result of those analyses are directly dependent on the quality of the spatially related data used. It is therefore important to realize at the outset of this chapter that the surveyor's role in developing accurately positioned data sets is a critical one in GIS activity.

A generalized concept of how data of different types or "layers" are collected and overlaid in a GIS is illustrated in Figure 28.1. In that figure, maps A through G represent some of the different layers of spatially related information that can be digitally recorded and incorporated into a GIS database and include parcels of different landownership A, zoning B, floodplains C, wetlands D, land

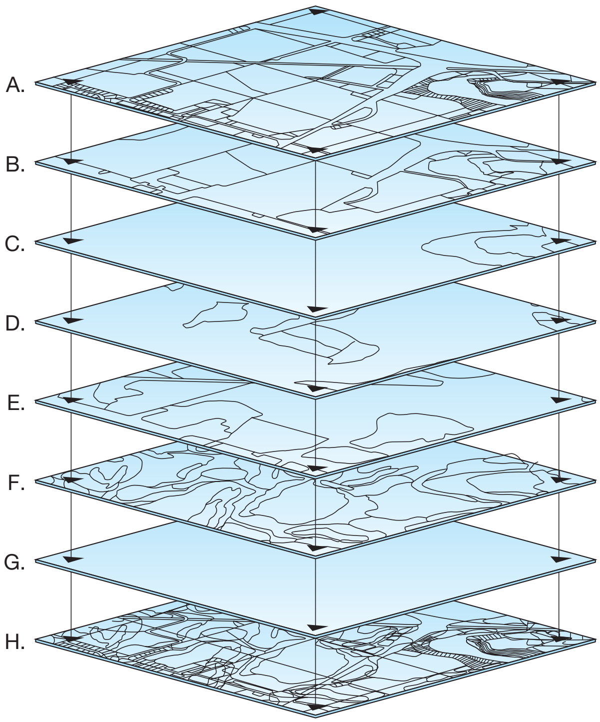


Figure 28.1
Concept of layers in a geographic information system. Map layers shown are (A) parcels; (B) zoning; (C) floodplains; (D) wetlands; (E) land cover; (F) soils; (G) reference framework; and (H) composite overlay. (From Land Information and Computer Graphics facility, College of Agricultural and Life Sciences, University of Wisconsin-Madison. Courtesy University of Wisconsin-Madison, College of Agricultural and Life Sciences.)

cover E, and soil types F. Map G is the geodetic reference framework, consisting of the network of survey control points in the area. Note that these control points are found in each of the other layers, thereby providing the means for spatially locating all data in a common reference system. Thus, composite maps that merge two or more different data sets can be accurately created. For example in Figure 28.1, bottom map H is the composite of all layers.

A GIS merges conventional database management software with software for manipulating spatial data. This combination enables the simultaneous storage, retrieval, overlay, and display of many different spatially related data sets in the manner illustrated by Figure 28.1. These capabilities, coupled with sophisticated GIS software to analyze and query the data sets that result from these different overlay and display combinations, provide answers to questions that never before were possible to obtain. As a result, GISs have become extremely important in planning, design, impact assessment, predictive modeling, and many other applications.

GISs have been applied in virtually every imaginable field of activity, from engineering to agriculture, and from the medical science of epidemiology to wildlife management. Flood forecasting on a large regional basis, such as statewide, is one particular example that illustrates some of the benefits that can be derived from using GIS. Critical location-related data entered into the GIS to support statewide flood forecasting would include the state's topography; soil; land cover; number, sizes, and locations of drainage basins; existing stream network with streamgaging records; locations and sizes, of existing bridges, culverts, and other drainage structures; data on existing dams and the water impoundment capacities of their associated reservoirs; and records of past rainfall intensity and duration. Given these and other data sets, together with a model to estimate runoff, a computer can be used to perform an analysis and predict locations of potential floods and their severity. In addition, experiments can be conducted in which certain input can be varied. Examples may include (1) input of an extremely intense rainfall for a lengthy duration in a given area to assess the magnitude of the resulting flood and (2) the addition of dams and other flood-detention structures of varying sizes at specific locations, to analyze their impact on mitigating the disaster. Many other similar examples could be given. Obviously, GISs are very powerful tools, and their use will increase significantly in the future.

The successful implementation of GISs relies on people with backgrounds and skills in many different disciplines, but none are more important than the contributions of those engaged in surveying (geomatics). Virtually every aspect of surveying, and thus all material presented in the preceding chapters of this book, bear upon GIS development, management, and use. However, of special importance are the global positioning system (Chapters 13 through 15), mapping surveys (Chapter 17), mapping (Chapter 18), control surveys and reference frames (Chapter 19), coordinate systems (Chapter 20), boundary or cadastral surveys (Chapter 21), the U.S. Public Land Survey System (Chapter 22), and photogrammetry and remote sensing (Chapter 27). In addition to surveying (geomatics) specialists, personnel in the fields of computer science, geography, soil science, forestry, landscape architecture, and many others play important roles in GIS development.

28.2 LAND INFORMATION SYSTEMS

The terms *Geographic Information System* (GIS) and *Land Information System* (LIS) are sometimes used interchangeably. They do have many similarities, but the distinguishing characteristic between the two is that a LIS has its focus directed primarily toward land records data. Information stored within a LIS for a given locality would include a spatial database of land parcel information derived from property descriptions in the U.S. Public Land system; other types of legal descriptions such as metes and bounds or block and lot that apply to parcels in the area; and other cadastral data. It might include the actual deeds and other records linked to the spatial data. Information on improvements and parcel values would also be included.

Land information systems and geographic information systems can share data sources such as control networks, parcel ownership information, and municipal boundaries. However, a GIS will usually incorporate data over a broader range and might include layers such as topography, soil types, land cover, hydrography, depth to ground water, and so on. Because of its narrower focus, there is a tendency to consider a LIS as a subset of a GIS.

LISs are used to obtain answers to questions about who has ownership or interests in the land in a certain area, the particular nature of those interests, and the specific land affected by them. They can also provide information about what resources and improvements exist in a given area and give their values. Answers to these questions are essential in making property assessments for taxation, transferring title to property, mortgaging, making investment decisions, resolving boundary disputes, and developing roads, utilities, and other services on the land that require land appraisal and property acquisitions. The data are also critical in policy development and land-use planning.

28.3 GIS DATA SOURCES AND CLASSIFICATIONS

As noted earlier, the capabilities and benefits of any GIS are directly related to the content and integrity of its database. Data that are entered into a GIS come from many sources and may be of varying quality. To support a specific GIS, a substantial amount of new information will generally need to be gathered expressly for its database. More than likely, however, some of the data will be obtained from existing sources such as maps, engineering plans, aerial photos, satellite images, and other documents and files that were developed for other purposes. Building the database is one of the most expensive and challenging aspects of developing a GIS. In fact, it has been estimated that this activity may represent about 60–80% of the total cost of implementing a GIS.

Two basic data classifications are used in GISs, (1) *spatial* and (2) *nonspatial*. These are described in the sections that follow.

28.4 SPATIAL DATA

Spatial data, sometimes interchangeably called *graphic data,* consists in general of natural and cultural features that can be shown with lines or symbols on maps, or seen as images on photographs. In a GIS these data must be represented and spatially located, in digital form, using a combination of fundamental elements called "simple spatial objects." The formats used in this representation are either *vector* or *raster*. The "relative spatial relationships" of the simple spatial objects are given by their *topology*.

These important GIS topics: (1) simple spatial objects, (2) data formats, and (3) topology are described in the following subsections.

28.4.1 Simple Spatial Objects

The simple spatial objects most commonly used in spatially locating data are illustrated in Figure 28.2 and described as follows:

1. *Points* define single geometric locations. They are used to locate features such as houses, wells, mines, or bridges [see Figure 28.2(a)]. Their coordinates give the spatial locations of points, commonly in state plane or UTM systems (see Chapter 20).
2. *Lines* and *strings* are obtained by connecting points. A line connects two points, and a string is a sequence of two or more connected lines. Lines and strings are used to represent and locate roads, streams, fences, property lines, etc. [see Figure 28.2(b)].
3. *Interior areas* consist of the continuous space within three or more connected lines or strings that form a closed loop [see Figure 28.2(c)]. For example, interior areas are used to represent and locate the limits of governmental jurisdictions, parcels of landownership, different types of land cover, or large buildings.
4. *Pixels* are usually tiny squares that represent the smallest elements into which a digital image is divided [see Figure 28.2(d)]. Continuous arrays of pixels, arranged in rows and columns, are used to enter data from aerial photos, orthophotos, satellite images, etc. Assigning a numerical value to each pixel specifies the distributions of colors or tones throughout the image. Pixel size can be varied and is usually specified by the number of *dots per inch* (dpi). As an example, 100 dpi would correspond to squares having dimensions of 1/100 in. on each side. Thus, 100 dpi yields 10,000 pixels per square inch.

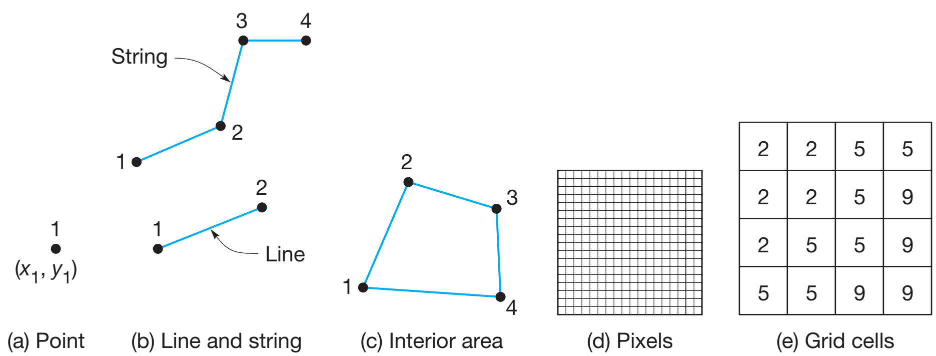


Figure 28.2 Simple spatial objects used to represent data digitally in a GIS.

5. *Grid cells* are single elements, usually square, within a continuous geographic variable. Similar to pixels, their sizes can be varied, with smaller cells yielding improved resolution. Grid cells may be used to represent slopes, soil types, land cover, water table depths, land values, population density, and so on. The distribution of a given data type within an area is indicated by assigning a numerical value to each cell; for example, showing soil types in an area using the number 2 to represent sand, 5 for loam, and 9 for clay, as illustrated in Figure 28.2(e).

28.4.2 Vector and Raster Formats

The simple spatial objects described in Section 28.4.1 give rise to two different formats for storing and manipulating spatial data in a GIS—*vector* and *raster*. When data are depicted in the vector format, a combination of points, lines, strings, and interior areas is used. The raster format uses pixels and grid cells.

In the vector format, points are used to specify locations of objects such as survey control monuments, utility poles, or manholes; lines and strings depict linear features such as roads, transmission lines, or boundaries; and interior areas show regions having common attributes; for example, governmental entities or areas of uniform land cover. An example illustrating the vector format is given with Figure 28.3 and Table 28.1. Figure 28.3 shows two adjacent land parcels, one designated parcel I, owned by Smith, and the other identified as parcel II, owned by Brown. As shown, the configuration consists of points, lines, and areas.

Vector representation of the data can be achieved by creating a set of tables, which list these points, lines, and areas (Table 28.1). Data within the tables are linked using *identifiers* and related spatially through the coordinates of points. As illustrated in Table 28.1 (a), all points in the area are identified by a reference number. Similarly, each line is described by its endpoints, as shown in

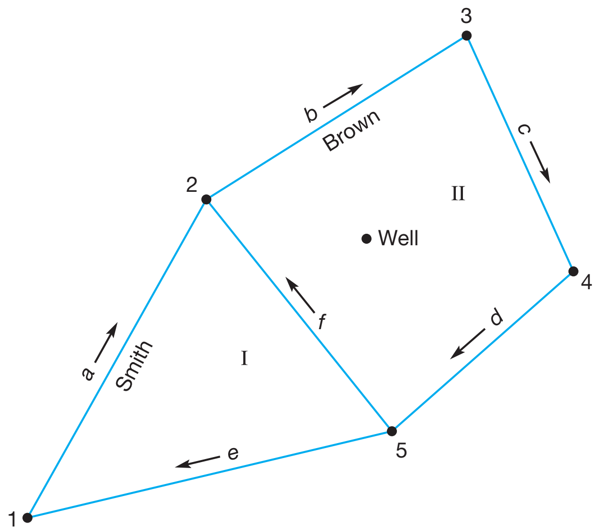


Figure 28.3
Vector representation of a simple graphic record.

TABLE 28.1 VECTOR REPRESENTATION OF FIGURE 28.3

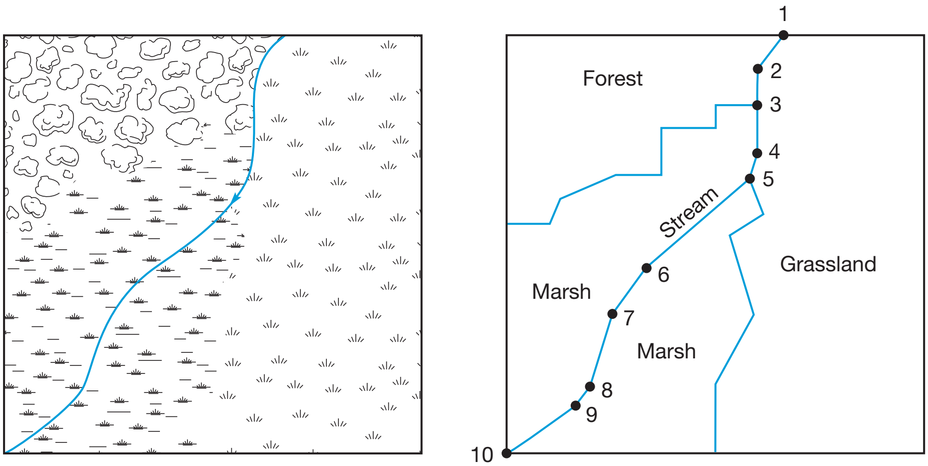
(a)		(b)		(c)	
Point Identifier	**Coordinates**	**Line Identifier**	**Points**	**Area Identifier**	**Lines**
1	$(X, Y)_1$	*a*	1,2	I	*a, f, e*
2	$(X, Y)_2$	*b*	2,3	II	*b, c, d, f*
3	$(X, Y)_3$	*c*	3,4		
4	$(X, Y)_4$	*d*	4,5		
5	$(X, Y)_5$	*e*	5,1		
Well	$(X, Y)_{well}$	*f*	5,2		

Table 28.1(b), and the endpoint coordinates locate the various lines spatially. As shown in Table 28.1(c), areas in Figure 28.3 are defined by the lines that enclose them. As before, coordinates of line endpoints locate the areas and enable determining their locations and computing their magnitudes.

Another data type can also be represented in vector format. For example, consider the simple case illustrated with the land cover map shown in Figure 28.4(a). In that figure, areas of different land cover (forest, marsh, etc.) are shown with standard topographic symbols (see Figure 18.5). A vector representation of this region is shown in Figure 28.4(b). Here lines and strings locate boundaries of regions having a common land cover. The stream consists of the string connecting points 1 through 10. By means of tables similar to Table 28.1 the data of this figure can also be entered into a GIS using the vector format. Having considered these simple vector representations, imagine the magnitude and complexity of entering data in vector format to cover a much larger area such as that shown in the map of Figure 17.2.

As an alternative to the vector approach, data can be depicted in the raster format using grid cells (or pixels if the data are derived from images). Each equal-sized cell (or pixel) is uniquely located by its row and column numbers and is coded with a numerical value or *code* that corresponds to the properties of the specific area it covers. In the raster format, a point would be indicated with a single grid cell, a line would be depicted as a sequence (linear array) of adjacent grid cells having the same code, and an area having common properties would be shown as a group of identically coded contiguous cells. Therefore, it should be appreciated that in general the raster method yields a coarser level of accuracy or definition of points, lines, and areas than the vector method.

In the raster format, the size of the individual cells defines the *resolution,* or precision, with which data are represented. Smaller the area covered by each cell, the higher the resolution for any given image. Examples illustrating raster representation and the degradation of resolution with increasing grid size are given in Figure 28.4(c) and (d). In these figures the land cover data from Figure 28.4(a) have been entered as two different raster data sets. Each cell has been assigned a



(a) (b)

F	F	F	S	G	G
F	F	M	S	G	G
F	M	S	S	G	G
M	S	S	G	G	G
M	S	M	G	G	G
S	S	M	G	G	G

(c)

F	F	F	F	F	F	F	S	G	G	G	G
F	F	F	F	F	F	F	S	G	G	G	G
F	F	F	F	F	F	M	S	G	G	G	G
F	F	F	F	F	M	M	S	G	G	G	G
F	F	M	M	M	M	S	S	G	G	G	G
F	M	M	M	M	S	S	G	G	G	G	G
M	M	M	M	S	S	M	G	G	G	G	G
M	M	M	S	M	M	M	G	G	G	G	G
M	M	S	S	M	M	M	G	G	G	G	G
M	M	S	M	M	M	G	G	G	G	G	G
M	S	S	M	M	M	G	G	G	G	G	G
S	S	M	M	M	M	G	G	G	G	G	G

(d)

Figure 28.4 Land cover maps of a region. (a) The region using standard topographic symbols. (b) Vector representation of the same region. (c) Raster representation of the region using a coarse-resolution grid cell. (d) Raster representation using a finer-resolution grid cell.

value representing one of the land cover classes, that is, *F* for forest, *G* for grassland, *M* for marsh, and *S* for stream. Figure 28.4(c) depicts the area with a relatively large-resolution grid and, as shown, it yields a coarse representation of the original points, lines, and areas. With a finer-resolution grid, such as in Figure 28.4(d), the points, lines, and areas are rendered with more precision. However, it is important to note that as grid resolution increases, so does the volume of data (number of grid cells) required to enter the data.

Despite the coarser resolution present in a raster depiction of spatial features, this format is still often used in GISs. One reason is that many data are available in raster format. Examples include aerial photos, orthophotos, and satellite images. Another reason for the popularity of the raster format is the ease with which it enables collection, storage, and manipulation of data using computers. Furthermore, various refinements of raster images are readily made using available "image processing" software programs. Finally, for many data sets such as wetlands and soil types, boundary locations are rather vague and the use of the raster format does not adversely affect the data's inherent accuracy.

28.4.3 Topology

Topology is a branch of mathematics that describes how spatial objects are related to each other. The unique sizes, dimensions, and shapes of the individual objects are not addressed by topology. Rather, it is only their *relative relationships* that are specified.

In discussing topology, it is necessary to first define *nodes*, *chains*, and *polygons*. These are some additional simple spatial objects that are commonly used for specifying the topological relationships of information entered into GIS databases. Nodes define the beginnings and endings of chains, or identify the junctions of intersecting chains. Chains are similar to lines (or strings) and are used to define the limits of certain areas or delineate specific boundaries. Polygons are closed loops similar to areas and are defined by a series of connected chains. Sometimes in topology, single nodes exist within polygons for labeling purposes.

In GISs, the most important topological relationships are:

1. *Connectivity*. Specifying which chains are connected at which nodes.
2. *Direction*. Defining a "from node" and a "to node" of a chain.
3. *Adjacency*. Indicating which polygons are adjacent on the left and which are adjacent on the right side of a chain.
4. *Nestedness*. Identifying what simple spatial objects are within a polygon. They could be nodes, chains, or other smaller polygons.

The topological relationships just described are illustrated and described by example with reference to Figure 28.3. For example in the figure, through connectivity, it is established that nodes 2 and 3 are connected to form the chain labeled *b*. Connectivity would also indicate that at node 2, chains *a*, *b*, and *f* are connected. Topological relationships are normally listed in tables and stored within the database of a GIS. Table 28.2(a) summarizes all of the connectivity relationships of Figure 28.3.

Directions of chains are also indicated topologically in Figure 28.3. For example, chain *b* proceeds from node 2 to node 3. Directions can be very important in a GIS for establishing such things as the flow of a river or the direction traffic moves on one-way streets. In a GIS, often a consistent direction convention is followed, that is, proceeding clockwise around polygons. Table 28.2(b) summarizes the directions of all chains within Figure 28.3.

The topology of Figure 28.3 would also describe, through adjacency, that Smith and Brown share a common boundary, which is chain *f* from node 5 to node 2, and that Smith is in the left side of the chain and Brown is on the right.

TABLE 28.2 TOPOLOGICAL RELATIONSHIPS OF ELEMENTS IN THE GRAPHIC RECORD OF FIGURE 28.3

(a) Connectivity		(b) Direction			(c) Adjacency			(d) Nestedness	
Nodes	Chain	Chain	From Node	To Node	Chain	Left Polygon	Right Polygon	Polygon	Nested Node
1–2	*a*	*a*	1	2	*a*	0	I	II	Well
2–3	*b*	*b*	2	3	*b*	0	II		
3–4	*c*	*c*	3	4	*c*	0	II		
4–5	*d*	*d*	4	5	*d*	0	II		
5–1	*e*	*e*	5	1	*e*	0	I		
5–2	*f*	*f*	5	2	*f*	I	II		

Obviously the chain's direction must be stated before left or right positions can be declared. Table 28.2(c) lists the adjacency relationships of Figure 28.3. Note that a zero has been used to designate regions outside of the polygons and beyond the area of interest.

Nestedness establishes that the well is contained within Brown's polygon. Table 28.2(d) lists that topological information.

The relationships expressed through the identifiers for points, lines, and areas of Table 28.1 and the topology in Table 28.2 conceptually yield a "map." With these types of information available to the computer, the analysis and query processes of a GIS are made possible.

28.5 NONSPATIAL DATA

Nonspatial data, also often called *attribute* or *descriptive data,* describe geographic regions or define characteristics of spatial features within geographic regions. Nonspatial data are usually alphanumeric and provide information such as color, texture, quantity, quality, and value of features. Smith and Brown as the property owners of parcels I and II of Figure 28.3 and the land cover classifications of forest, marsh, grassland, and stream in Figure 28.4 are examples. Other examples could include the addresses of the owners of land parcels, their types of zoning, dates purchased, and assessed values; or data regarding a particular highway, including its route number, pavement type, number of lanes, lane widths, and year of last resurfacing. Nonspatial data are often derived from sources such as documents, files, and tables.

In general, spatial data will have related nonspatial attributes, and thus some form of linkage must be established between these two different types of information. Usually this is achieved with a *common identifier* that is stored with both the graphic and the nongraphic data. Identifiers such as a unique parcel identification number, a grid cell label, or the specific mile point along a particular highway may be used.

28.6 DATA FORMAT CONVERSIONS

In manipulating information within a GIS database, it is often necessary to either integrate vector and raster data or convert from one form to the other. Integration of the two types of data, that is, using both types simultaneously, is usually accomplished by displaying vector data overlaid on a raster image background, as illustrated in Figure 28.5. In that figure, vector data representing the dwellings (points) that exist within the different subdivisions (areas) are overlaid on a satellite image of the same area. This graphic was developed as part of a population growth and distribution study being conducted for a municipality. The combination of vector and raster data is useful to provide a frame of reference and to assist GIS operators in interpreting displayed data. Sometimes it is necessary or desirable to convert raster data to vector format or vice versa. Procedures for accomplishing these conversions are described in subsections that follow.

28.6.1 Vector-to-Raster Conversion

Vector-to-raster conversion is also known as *coding* and can be accomplished in several ways, three of which are illustrated in Figure 28.6. Figure 28.6(a) is an overlay of the vector representation of Figure 28.4(b) with a coarse raster of grid cells. In one conversion method, called *predominant type coding,* each grid cell is

Figure 28.5 Vector data overlaid on a raster image background. (Courtesy Tom Pantages.)

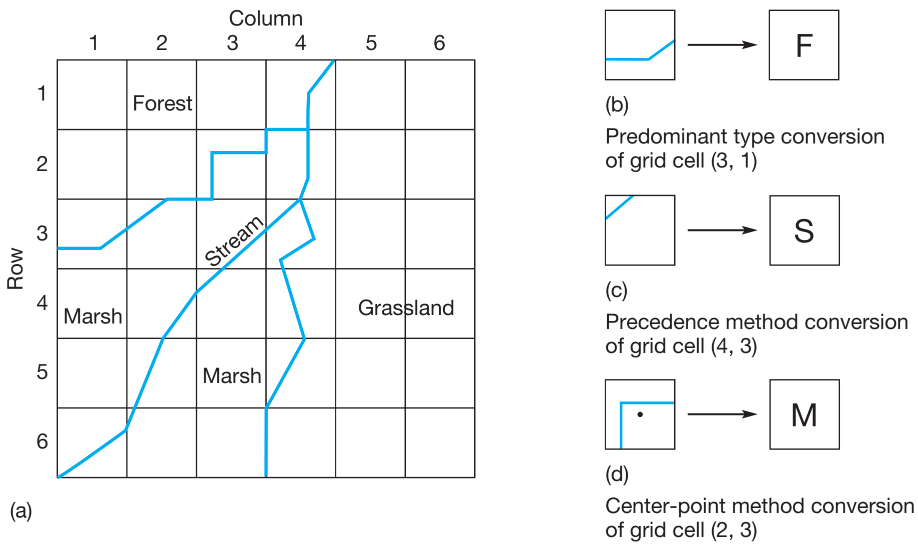


Figure 28.6 Methods for converting data from vector to raster format. (a) Vector representation of Figure 28.4(b) overlaid on a coarse raster format. (b), (c), and (d) vector-to-raster conversion by predominant type, precedence, and center-point method, respectively.

assigned the value corresponding to the predominant characteristic of the area it covers. For example, the cell located in row 3, column 1 of Figure 28.6(a) overlaps two polygons, one of forest (type *F*) and one of marsh (type *M*). As shown in Figure 28.6(b), since the largest portion of this cell lies in forest, the cell is assigned the value *F*—the predominant type.

In another coding method, called *precedence coding,* each category in the vector data is ranked according to its importance or "precedence" with respect to the other categories. In other words, each cell is assigned the value of the highest ranked category present in the corresponding area of the vector data. A common example involves water. While a stream channel may cover only a small portion of a cell area, it is arguably the most important feature in that area. Also, it is important to avoid breaking up the stream. Thus for the cell in row 4, column 3 of Figure 28.6(a), which is illustrated in Figure 28.6(c), water would be given the highest precedence and the cell coded *S* even though most of the cell is covered by marsh.

Center-point coding is the third technique for converting from vector-to-raster data. Here a cell is simply assigned the category value at the vector location corresponding to its center point. An example is shown in Figure 28.6(d), which represents the cell in row 2, column 3 of Figure 28.6(a). Here, since marsh exists at the cell's center, the entire cell is designated as category *M*. Note that the grid cell of row 3, column 4 would be classified by predominant type as grassland, by precedence as stream, and by center point as marsh. This illustrates how different conversion processes can yield different classifications for the same data.

The precisions of these vector-to-raster conversions depend on the size of the grid used. Obviously, using a raster of large cells would result in a relatively inaccurate representation of the original vector data. On the other hand, a fine-resolution grid can very closely represent the vector data, but would require a large amount of computer memory. Thus, the choice of grid resolution becomes a trade-off between computing efficiency and spatial precision.

28.6.2 Raster-to-Vector Conversion

Raster-to-vector conversions are more vaguely defined than vector to raster. The procedure involves extracting lines from raster data, which represent linear features such as roads, streams, or boundaries of common data types. Whereas the approach is basically a simple one and consists in identifying the pixels or cells through which vector lines pass, the resulting jagged- or "staircase"-type outlines are not indicative of the true lines. One raster-to-vector conversion example is illustrated in Figure 28.7(a), which shows the cells identifying the stream line of Figure 28.4(b). Having selected these cells, a problem that must be resolved is how to fit a line to these jagged forms. One solution consists in simply connecting adjacent cell centers [see Figure 28.7(b)] with line segments. Note, however, that the resulting line [see Figure 28.7(c)] does not agree very well with the original stream of Figure 28.4(a). This example illustrates that some type of "line smoothing" is usually necessary to properly represent the gently curved boundaries that normally occur in nature. However, fitting smooth lines to the jagged cell boundaries is a complicated mathematical problem that does not necessarily have a unique solution. The decision ultimately becomes a choice between accuracy of representation and cost of computation.

No matter which conversion is performed, errors are introduced during the process and some information from the original data is lost. Use of smaller grid cells improves the results. Nevertheless, as illustrated by the example of Figure 28.7, if a data set is converted from vector-to-raster and then back to vector (or vice versa), the final data set will not likely match the original. Thus, it is important for GIS operators to be aware of how their data have been manipulated and what can be expected if conversion is performed.

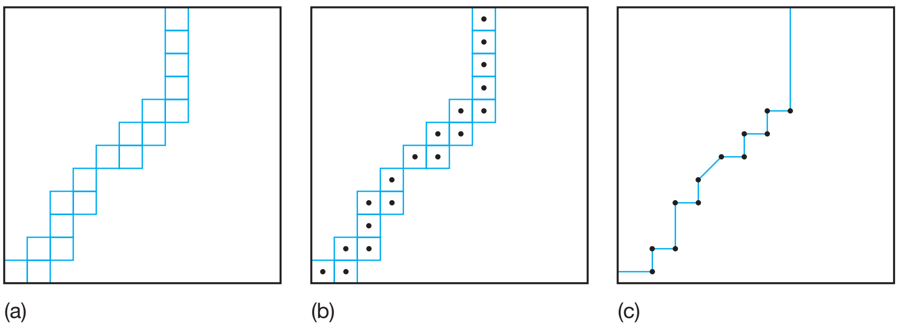


Figure 28.7 Conversion of data from raster-to-vector format. (a) Cells that identify the stream line of Figure 28.4(a). (b) Cell centers. (c) Vector representation of stream line recreated by connecting adjacent cell centers.

■ 28.7 CREATING GIS DATABASES

Several important factors must be considered prior to developing the database for a GIS. These include the types of data that need to be obtained, optimum formats for these data, the reference coordinate system that will be used for spatially relating all data, and the necessary accuracy of each data type. Provisions for updating the database must also be considered. Having made these decisions, the next step is to locate data sources. Depending on the situation, it may be possible to utilize existing data, in which case a significant cost savings could result. However, in many cases, it is necessary to collect new data to meet the needs of the GIS. Different methods are available for generating the digital data needed to support a GIS. These are discussed in the subsections that follow. Regardless of the method used, *metadata* (see Section 28.8) should be included with each data file to document its source, the instruments and procedures used to collect it, its reference coordinate system and elevation datum, its accuracy, and any other information needed to qualify the data or describe its character.

28.7.1 Generating Digital Data from Field Surveys

Spatially related information needed to support a GIS is often generated by conducting new field surveys expressly for that purpose. Any of the equipment and procedures described in preceding chapters that are capable of locating objects in space can be used for this work. However, total station instruments interfaced with data collectors and GNSS equipment are particularly convenient because they can rapidly and efficiently provide coordinates of points directly in a reference coordinate system that is suitable for the GIS, and because necessary identification codes can also be entered at the time the data is collected. Both planimetric positions and elevation data (preferably in the form of digital elevation models) can be obtained with these instruments. It must be remembered, however, that elevations obtained with GNSS equipment are related to the ellipsoid, and thus must be converted to orthometric heights (see Section 13.4.3). The field data can be downloaded into a computer, processed using COGO (see Chapter 11) or other software if necessary, and entered directly into the GIS database. The cost of generating digital data in this manner is relatively high, but the data are generally very accurate.

Code-based GNSS receivers offer several advantages over other instruments in collecting of mapping data for a GIS. These instruments are relatively inexpensive, and when the data is reduced using differential techniques, the positional accuracies obtained are often sufficient for many uses. Other advantages are speed, ease of use, and the ability to enter other ancillary data about a point. For instance, when collecting the position of a utility pole for inclusion into a GIS, additional information such as type (electric, cable, telephone, etc.), pole number, diameter, height, and condition can be entered while its position is being determined. These code-based units can also be used to collect the alignment of a feature such as a utility line, road, or sidewalk by simply entering the epoch rate and traversing the alignment either on foot or in a vehicle. Again, ancillary information about the alignment can be entered while the user traverses it. If additional accuracy is required, the surveyor can use carrier phase-shift GNSS receivers and kinematic reduction techniques to quickly obtain the data. However, as discussed in Chapter 15, in many situations kinematic surveys require project planning for successful completion.

The IP-S2 shown in Figure 1.4 and discussed in Section 17.9.6 provides an integrated solution to quickly and safely collecting georeferenced data for a GIS in urban areas where safety or speed are a major concern.

28.7.2 Digitizing from Aerial Photos with Stereoplotters

Information from aerial photographs can also be entered directly into a GIS database in digital form using an analytical or softcopy stereoplotter (see Section 27.14). In this procedure, both planimetric features and elevations can be recorded, and high accuracy can be achieved. The data are registered to the selected reference coordinate system and vertical datum by *orienting* the stereoplotter to ground control points prior to beginning the digitizing. Then to record planimetric features, an operator views the stereomodel, points to objects of interest, enters any necessary feature identifiers or codes, and depresses a key or foot pedal to transfer the information to a file in an interfaced computer. To digitize elevations, a DEM is read directly from the three-dimensional stereomodel and stored in a computer file.

Accuracies of data obtained using this procedure will depend mainly on the scale and quality of the aerial photography, the accuracy of the ground control points used to orient the stereoplotter, and the experience and capabilities of the stereoplotter operator. Other factors that may affect accuracy to a lesser degree include camera lens distortions, atmospheric refraction, differential shrinkage and expansion of the materials upon which the photographic products are printed, and optical and/or mechanical imperfections in the stereoplotting or digitizing equipment.

Data sets generated by digitizing stereomodels will usually need to be checked carefully to ensure that all desired features have been included. Also the data must be corrected or "cleaned" before being used in a GIS. In this process, unwanted points and line portions must be removed and "unclosed" polygons, which result from imprecise pointing when returning to a polygon's starting node, must be closed (see Section 18.8.2). Finally, thin polygons or "slivers" created by lines being inadvertently digitized twice, but not precisely in the same location, must be eliminated. This editing process can be performed by the operator, or with a program that can find and remove certain features that fall within a set of user-defined tolerances.

Digitizing from stereomodels and editing the data can usually be accomplished in less time and with lower cost than obtaining the data by field surveys, especially where relatively large areas must be covered. Of course, some field surveying is needed to establish the ground control needed to orient the stereomodels. If experienced people do the work carefully, the resulting accuracy of data obtained by digitizing stereomodels is usually very good.

28.7.3 Digitizing Existing Graphic Materials

If sources such as maps, orthophotos, plans, diagrams, or other graphic documents already exist that will meet the needs of the GIS database, these can be conveniently and economically converted to digital files using a tablet digitizer. Many

Figure 28.8 Tablet digitizer interfaced with personal computer. (Courtesy Tom Pantages.)

GIS software packages provide programs to support the procedure directly. A tablet digitizer, as shown in Figure 28.8, contains an electronic grid and an attached cursor. Movement of the cursor across the grid creates an electronic signal unique to the cursor's position. This signal is relayed to the computer, which records the digitizer's coordinates for the point. Data identifiers or attribute codes can be associated with each point through the computer's keyboard or by pressing numerical buttons on the cursor.

The digitizing process begins by securing the source document to the tablet digitizer. If the document is a map, the next step is to register its reference coordinate system to that of the digitizer. This is accomplished by digitizing a series of reference (tic) marks on the map for which geographic coordinates such as state plane, UTM, or latitude and longitude are precisely known. With both reference and digitizer coordinates known for these tic marks, a coordinate transformation (see Section 11.8) can be computed. This determines the parameters of scale change, rotation, and translation necessary to convert digitized coordinates into the reference geographic coordinate system. After this, any map features can be digitized, whereupon their coordinates are automatically converted to the selected system of reference, and they can then be entered directly into the database.

Both planimetric features and contours can be digitized from the map. Planimetric features are recorded by digitizing the individual points, lines, or areas that identify them. As described in Section 28.4.2, this process creates data in a vector format. Elevation data can be recorded as a digital elevation model (DEM) by digitizing critical points along contours. From these data, *triangulated irregular networks* (TIN models) can be derived using the computer (see Section 17.8). From the TIN models, point elevations, profiles, cross sections, slopes, aspects (slope directions), and contours having any specified contour interval can be derived automatically using the computer.

Data files generated in this manner can be obtained quickly and relatively inexpensively. Of course the accuracy of the resultant data can be no better than the accuracy of the document being digitized, and its accuracy is further diminished by differential shrinkages or expansions of the paper or materials upon which the document is printed and by inaccuracies in the digitizer and the digitizing process.

28.7.4 Keyboard Entry

Data can be entered into a GIS file directly using the keyboard on a computer. Often data input by this method are nonspatial, such as map annotations or numerical or tabular data. To facilitate keying in data, an intermediate file having a simple format is sometimes created. This file is then converted into a GIS-compatible format using special software.

Metes-and-bounds descriptions (see Section 21.4) can also be computed using coordinate geometry techniques (see Chapter 11). The resulting coordinates can be used to facilitate entry of the deed description into the GIS file.

28.7.5 Existing Digital Data Sets

Massive quantities of digital information are now being generated by a wide variety of offices and agencies involved in GIS activities. At the federal level, the U.S. Geological Survey, the National Oceanic and Atmospheric Administration, the Bureau of Land Management, the Environmental Protection Agency, and other organizations are developing digital information. The *digital line graphs* (DLGs) and *digital elevation models* (DEMs) produced by the U.S. Geological Survey (see Section 18.3) are examples of available digital files. In addition to federal agencies, offices of state governments, counties, and cities are involved in this work. As a result of this proliferation of information, an initiative known as the *National Spatial Data Infrastructure* (NSDI) has evolved at the federal level. The NSDI encompasses policies, standards, and procedures for organizations to cooperatively produce and share geographic data. The *National Geospatial Data Clearinghouse* is a component of the NSDI that provides a pathway to find information about available spatially referenced data.[1]

[1]Information about available geospatial data can be obtained by writing to the National Spatial Data Infrastructure, U. S. Geological Survey, 508 National Center, Reston, VA 20192, or by contacting them through the internet at http://nsdi.usgs.gov.

Of course before using existing data, information about its content, source, date, accuracy, and other characteristics must be scrutinized to determine if it is suitable for the GIS at hand. This requirement underscores the need for maintaining good quality metadata (see Section 28.8) for all digital files. Also, existing digital data must often undergo conversion of file structures and formats to be usable with specific GIS software. Because of differences in the way data are represented by different software, it is possible that information can be lost or that spurious data can creep in during this process.

28.7.6 Scanning

Scanners are instruments that automatically convert graphic documents into a digital format. As discussed in Section 27.14.4, they are used to digitize the contents of aerial images to support softcopy photogrammetry. In GIS work, scanners are used not only to digitize aerial photos but also to convert larger documents such as maps, plans, and other graphics into digital form. The principal advantages of using scanners for this work is that the tedious work of manual digitizing is eliminated and the process of converting graphic documents to digital form is greatly accelerated.

Scanners accomplish their objective by measuring the amount of light reflected from a document and assigning this information to pixels. This is possible because different areas of a document will reflect light in proportion to their tones, from a maximum for white through the various shades of gray to a minimum for black. For example, the scanner of Figure 28.9 uses a linear array of light sensors to capture the varying intensities of reflected light, line by line, as the document is fed through the system. This creates a raster data set. Its pixel size can be varied and made as small as $1/500$ in.^{2} (500 dpi). Large complicated documents can be scanned in a matter of a few minutes. The data are stored directly on the hard drive of an interfaced computer and can be viewed on a screen, edited, and manipulated. Editing is an important and necessary step in the process, because the scanner will record everything, including blemishes, stains, and creases.

Documents such as subdivision plats, topographic maps, engineering drawings and plans, aerial photos, and orthophotos can be digitized using scanners. Then, if necessary, the raster data can be converted to vector form using techniques described in Section 28.6.2.

Accuracy of the raster file obtained from scanning depends somewhat on the instrument's precision, but pixel size or resolution is generally the major factor. A smaller pixel size will normally yield superior resolution. However, there are certain trade-offs that must be considered. Whereas a large pixel size will result in a coarse representation of the original, it will require less scanning time and computer storage. Conversely, a fine resolution, which generates a precise depiction of the original, requires more scanning time and computer storage. An additional problem is that at very fine resolution, the scanner will record too much "noise," that is, impurities such as specks of dirt. For these reasons and others, this is the least preferred method of capturing data in a GIS.

Figure 28.9
Intergraph ANAtech Eagle 4050 scanner. (Courtesy Wisconsin Department of Transportation.)

28.7.7 Mobile Mapping

As discussed in Sections 17.9.5 and 27.18, LiDAR scanners can capture large amounts of data from mobile platforms. For example, the IP-S2 scanner shown in Figure 1.4 can capture three-dimensional, georeferenced points at a rate of 1.3 million per second. These devices provide a means of capturing data for entire scenes. Ground-based scanners can provide point clouds from which inventories of objects can be assessed and exported for use in a GIS. Aerial LiDAR systems can capture and provide inventories of objects such as vegetation, buildings, and other objects typically captured in aerial photography. Both systems can capture data to provide topographic maps for the system.

28.8 METADATA

Metadata, often simply defined as "data about data," describes the content, quality, condition, and other characteristics about geospatial data and provides a record of changes or modifications that have been made to that data. It normally includes information such as who originally created the data, when was it generated, what equipment and procedures were used in collecting the data, and what was its original scale and accuracy. Once created, data can travel almost instantaneously through a network and be transformed, modified, and used for many different kinds of spatial analyses. It can then be retransmitted to another user, and then to another, etc.[2] It is important that each change made to any data set be documented by updating its associated metadata.

Although generating the original metadata and updating it as changes are made may be burdensome and add cost, in the long run it is worth the effort because it preserves the value of the data and extends its useful life. If it is not done, prospective users may not trust the data and as a result they may fail to take advantage of it and incur the cost of duplicate data collection.

The *Federal Geographic Data Committee* (FGDC) has developed metadata standards that provide a common set of terms and definitions for describing geospatial data, and outline a consistent and systematic approach to documenting data characteristics.[3] The primary benefit to be realized by following these standards is that all users, regardless of their backgrounds or specialty areas, will have a common understanding of the source, nature, and quality of any data set.

28.9 GIS ANALYTICAL FUNCTIONS

Most GISs are equipped with a set of basic analytical functions that enable data to be manipulated, analyzed, and queried. These functions, coupled with appropriate databases, provide GISs with their powerful capabilities for supplying information that so significantly aids in planning, management, and decision making.

The specific functions available within the software of any particular GIS system will vary. They enable data to be stored, retrieved, viewed, analyzed spatially and computationally, and displayed. Some of the more common and useful spatial analysis and computational functions are (1) proximity analysis, (2) boundary operations, (3) spatial joins, and (4) logical operations. These are briefly described in the subsections that follow.

28.9.1 Proximity Analysis

This spatial analysis function creates new polygons that are geographically related to nodes, lines, or existing polygons, and usually involves processes called *buffering. Point buffering*, also known as *radius searching,* is illustrated in

[2] An example of a data exchange website, PASDA, with metadata on the Internet at http://www.pasda.psu.edu/.

[3] These metadata standards may be obtained from the FGDC Secretariat, U.S. Geological Survey, 590 National Center, Reston, VA 22092, or information can be obtained at the following website: http://www.fgdc.gov.

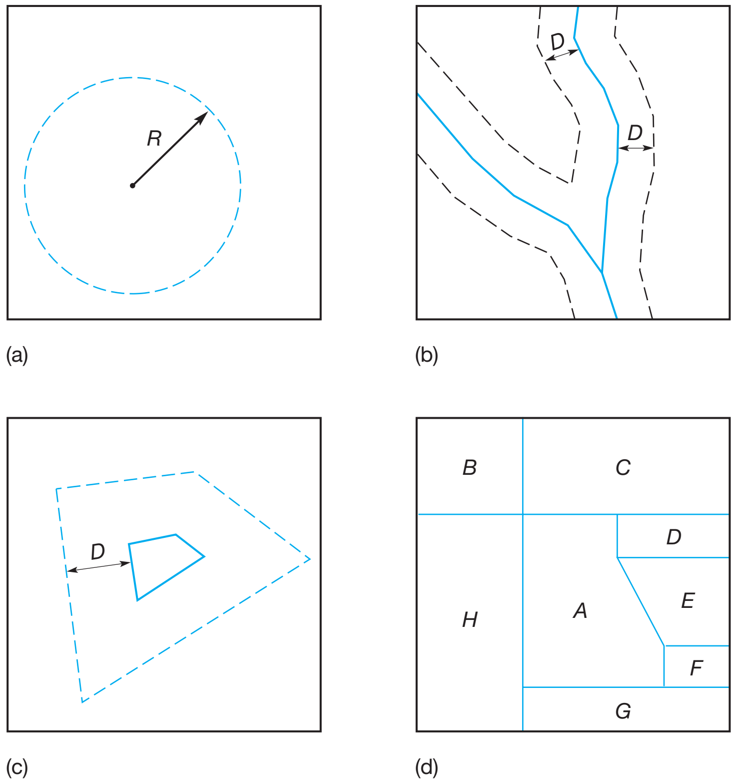


Figure 28.10 GIS spatial analysis functions. (a) Point buffering. (b) Line buffering. (c) Polygon buffering. (d) Adjacenty analysis.

Figure 28.10(a). It involves the creation of a circular buffer zone of radius R around a specific node. Information about the new zone can then be gathered and analyses made of the new data. A simple example illustrates its value. Assume that well water that was polluted by an accidental spill has just been discovered. With appropriate databases available, all dwellings within a specified radius of the well can be located, the names, addresses, and telephone numbers of all individuals living within the point buffer zone tabulated, and the people quickly alerted to the possibility of their water also being polluted.

Line buffering, illustrated in Figure 28.10(b), creates new polygons along established lines such as streams and roads. To illustrate the use of line buffering, assume that to preserve the natural stream bank and prevent erosion, a zoning commission has set the construction setback distance from a certain stream at D. Line buffering can quickly identify the areas within this zone. *Polygon buffering*, illustrated in Figure 28.10(c), creates new polygons around an existing polygon. An example of its use could occur in identifying those landowners whose property lies within a certain distance D of the proposed site of a new industrial facility. Many other examples could be given, which illustrate the value of buffering for rapidly extracting information to support management and decision making.

28.9.2 Boundary Operations

If the topological relationships discussed in Section 28.4.3 have been entered into a database, certain analyses regarding relative positioning of features, usually called boundary operations, can be performed. *Adjacency* and *connectivity* are two important boundary operations that often assist significantly in management and decision making. An example of adjacency is illustrated in Figure 28.10(d) and relates to a zoning change requested by the owner of parcel *A*. Before taking action on the request, the jurisdiction's zoning administrators are required to notify all owners of adjacent properties *B* through *H*. If the GIS database includes the parcel descriptions with topology and other appropriate attributes, an adjacency analysis will identify the abutting properties and provide the names and addresses of the owners.

Connectivity involves analyses of the intersections or connections of linear features. The need to repair a city's water main serves as an example to illustrate its value. Suppose that the decision has been made that these repairs will take place between the hours of 1:00 and 4:00 P.M. on a certain date. If infrastructure data are stored within the city's GIS database, all customers connected to this line whose water service will be interrupted by the repairs can be identified and their names and addresses tabulated. The GIS can even print a letter and address labels to facilitate a mailing announcing details of the planned interruption to all affected customers. Many similar examples could be given to illustrate benefits that can result from adjacency and connectivity.

28.9.3 Spatial Joins

Spatial joins, also called *overlaying*, is one of the most widely used spatial analysis functions of a GIS. As indicated in Figure 28.1, GIS graphic data are usually divided into layers, with each containing data in a single category of closely related features. Nonspatial data or "attributes" are often associated with each category. The individual layers are spatially registered to each other through a common reference network or coordinate system. Any number of layers can be entered into a GIS database, and could include parcels, municipal boundaries, public land survey system, zoning, soils, road networks, topography, land cover, hydrology, and many others.

Having these various data sets available in spatially related layers makes the overlay function possible. Its employment in a GIS can be compared to using a collection of Mylar overlays in traditional mapping. However, much greater efficiency and flexibility are possible when operating in the computer environment of a GIS, and not only can graphic data be overlaid, but attribute information can be combined as well.

Many examples could be given to illustrate the applications and benefits of the GIS spatial join or overlay process. Consider one case where the land in a particular area suitable for development must be identified. To perform an in-depth analysis of this situation, the evaluation would normally have to consider numerous variables within the area, including the topography (slope and aspect of the terrain), soil type, land cover, landownership, and others. Certain

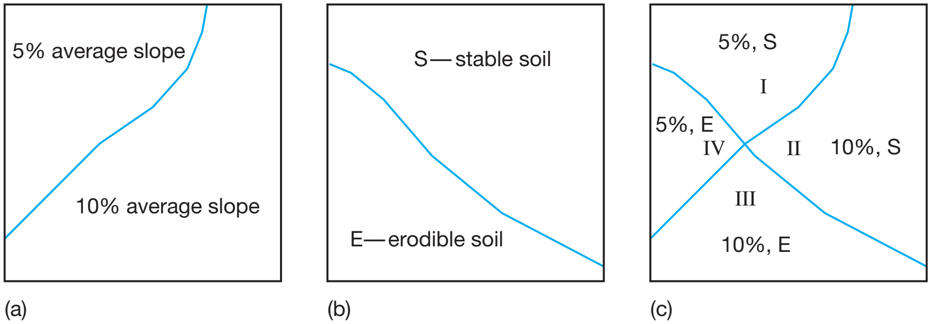


Figure 28.11 Example of GIS overlay function used to evaluate land suitability. (a) Polygons of differing slopes. (b) Varying soil types in the area. (c) Overlay of (a) and (b), identifying polygon I as an area combining lower slopes with stable soils that would be suitable for development.

combinations of these variables could make land unsuitable for development. Figure 28.11 illustrates the simple case of land suitability analysis involving only two variables, slope and soil type. Figure 28.11(a) shows polygons within which the average slope is either 5 or 10%. Figure 28.11(b) classifies the soils in the area as E (erodible) or S (stable). The composite of the two data sets, which results from a *polygon-on polygon* overlay, is shown in Figure 28.11(c). It identifies polygon I, which combines 5% slopes and low-erodibility class S soils. Since this combination does not present potential erosion problems, considering those variables, the area within polygon I is suitable for development, while areas II, III, and IV are not.

Another GIS overlay function is that of point in polygon. Here the question involves which point features are located in certain polygons where layers are combined. For example, to predict possible well contamination, a GIS operator may want to know which wells are located in an area of highly permeable soil. A similar overlay process, *line in polygon*, identifies specific linear features within polygons of interest. An example of its application would be the identification of all bituminous roads, paved more than 15 years ago, in townships whose roadway maintenance budgets are less than $250,000. Obviously such information would be valuable to support decisions concerning the allocation of state resources for local roadway maintenance.

The GIS functions just described can be used individually, as has been illustrated with the examples, or they can be employed in combination. The following example illustrates the simultaneous application of line buffering, adjacency, and overlay. The situation involves giving timely notice to affected persons of an impending flood that is predicted to crest at a specific stage above a particular river's normal elevation. Here it is first necessary to identify the lands that lie at or below the expected flood stage. This can be done using line buffering, where the thread of the river is the line of reference. However, the width of the buffer zone is variable and is determined by combining elevation data in the TIN model with the buffering process. The adjacency and overlay functions are then used to determine which landowners are next to or within the flood zone. Then the names, addresses, and telephone numbers of property owners and dwellers within and adjacent to the affected area can be tabulated. These people can then be notified of the impending situation, and emergency preparations such as construction of temporary levees can be performed, or, if necessary, the area can be evacuated. Should evacuation be required, the GIS may even be used to identify the best and safest escape routes. Data sets necessary for such analyses includes the topography in the area, including the river's location, normal stage, and floodplain cross sections; census data; property ownership; and the transportation network.

28.9.4 Logical Operations

Typically, attribute data that is related to the features present in the GIS are stored in a database. Thus, the database can be used to perform logical operations on the data. For instance, a city can construct a GIS database that contains the time streetlights are installed and their rated life. The manager can then query the system to show all lights that have passed their rated life cycle and schedule maintenance personnel to replace these lights. Today GISs are being used in large buildings to help managers keep records of maintenance and schedule routine maintenance jobs. The number of useful logical queries that can be performed in a GIS is limited only by the data contained in the GIS database and the imagination of the user.

28.9.5 Other GIS Functions

In addition to the spatial analysis functions described in preceding subsections, many other functions are available with most GIS software. Some of these include the capability of computing (1) the number of times a particular type of point occurs in a certain polygon; (2) the distances between selected points or from a selected line to a point; (3) areas within polygons; (4) locations of polygon centroids; and (5) volumes within polygons where depth or other conditions are specified. A variety of different mapping functions may be performed using GISs. These may include (a) performing map scale changes; (b) changing the reference coordinate system from, say, the state plane to the UTM system; (c) rotating the reference grid; and (d) changing the contour interval used to represent elevations.

Most GISs are also capable of performing several different digital terrain analysis functions. Some of these include (1) creating TIN models or other DEMs from randomly spaced *XYZ* terrain data; (2) calculating profiles along designated reference lines, and determining cross sections at specified points along the reference line; (3) generating perspective views where the viewpoint can be varied; (4) analyzing visibility to determine what can or cannot be seen from a given vantage point; (5) computing slopes and aspects; and (6) making sun intensity analyses.

Output from GISs can be provided in graphic form as charts, diagrams, and maps; in numerical form as statistical tabulations; or in other files that result from computations and manipulations of the geographic data. These materials can be supplied in either printed (hardcopy) form or on diskettes or tapes (softcopy).

■ 28.10 GIS APPLICATIONS

As stated earlier, and as indicated by the examples in preceding sections of this chapter, the areas of GIS applications are widespread. Further evidence of the diversity of GIS applications can be seen by reviewing the bibliography at the end of this chapter. The technology is being used worldwide, at all levels of government, in business and industry, by public utilities, and in private engineering and surveying offices. Some of the more common areas of application occur in (1) land-use planning; (2) natural resource mapping and management; (3) environmental impact assessment; (4) census, population distribution, and related demographic analyses; (5) route selection for highways, rapid-transit systems, pipelines, transmission lines, etc.; (6) displaying geographic distributions of events such as automobile accidents, fires, crimes, or facility failures; (7) routing buses or trucks in a fleet; (8) tax mapping and mapping for surveying and engineering purposes; (9) subdivision design; (10) infrastructure and utility mapping and management; (11) urban and regional planning; and many others.

As the use of GIS technology expands, there will be a growing need for trained individuals who understand the fundamentals of these systems. Users should be aware of the manner in which information is recorded, stored, managed, retrieved, analyzed, and displayed using a GIS. System users should also have a fundamental understanding of each of the GIS functions, including their bases for operation, their limits, and their capabilities. *Perhaps of most importance, users must realize that information obtained from a GIS can be no better than the quality of the data from which it was derived.*

From the perspective of those engaged in surveying (geomatics), it is important to underscore again the fact that the fundamental basis of GISs is a database of spatially related, digital data. Since accurate position determination and mapping are the surveyor's forte, in the future surveyors will continue to play key roles in designing, developing, implementing, and managing these systems. Their input will be particularly essential in establishing the necessary basic control frameworks, conducting ground and aerial surveys to locate geographic features and their attributes, compiling maps, and assembling the digital data files needed for these systems.

■ 28.11 DATA SOURCES

The surveyor will find many data sets useful to aid in tasks that they perform.[4] These data sets include digital raster graphics (DRGs), digital elevation models (DEMs) (see Section 17.8), digital line graphs (DLGs) (see Section 18.3), digital

[4]For a more detailed description of data useful to the surveyor, and where to find it, refer Chapter 10 of Watersheds: Processes, Assessment, and Management (DeBarry, 2004).

ortho quarter quadrangles (DOQQs) (see Section 27.15), LiDAR data (see Section 27.19), land-use and land-cover (LULC) data, soil survey geographic (SSURGO) data, national wetlands inventory (NWI) data, and Federal Emergency Management Agency (FEMA) flood map data. These data sets are described as follows.

Digital raster graphics (DRGs) are scanned digital images of U.S. Geological Survey (USGS) 7-1/2 min topographic quadrangle sheets and are in Tag Image File Format (TIFF). They contain all date that is located on the topographic maps only in digital format. Since they are images, they do not contain topology or elevation data. DRGs can be used as location maps for plans or to view topography, surrounding features, stream, and building locations before performing a field survey.

Digital elevation models (DEMs) are grid-based data where each grid cell has an average elevation of the area of the cell. The USGS has set the standard for DEM data with 10, 30, and 100 m cell size. DEMs are useful for spatial analyses and modeling. They can also be used to develop three-dimensional terrain models such as contours and triangulated irregular networks (see Section 17.8). From DEMs, flow direction, accumulation, and streams can be defined.

Digital line graphs are vector files containing the planimetric data such as U.S. Public Land Survey System (PLSS) corners, survey control and markers, transportation, hydrography, vegetative surface cover, and so on. These data can be useful to begin a base map.

Digital ortho quarter quadrangles (DOQQs) are rectified digital images of color or black and white aerial photos based upon one-quarter of the USGS 7-1/2 min quadrangles. As discussed in Section 27.15, rectification removes the edge distortion of typical aerial photographs. DOQQs are available in both GeoTIFF and native format, which consists of an ASCII keyword header followed by a series of 8-bit binary image lines for B/W and 24-bit band-interleaved-by-pixel (BIP) for color. DOQQs are in the Universal Transverse Mercator (UTM) map projection coordinate system (see Section 20.12) and are referenced to either the NAD27 or NAD83 (see Section 19.6.1).

LiDAR data (see Section 27.19) is used to produce various elevation data products including point-based digital terrain models (DTM), grid-based digital elevation models (DEM), and contours. In addition LiDAR processing generates raw point cloud, processed points, and breaklines. The accuracy of the data is much better than the data sets described above and can be utilized for accurate terrain and hydrologic modeling.

Land-use and land-cover data is polygon coverage of land cover based upon the Anderson (1976) method of land classification and describes water, vegetation, cultural, and natural surface features. These data files can be used to help in hydrologic modeling, analyzing land use trends, etc.

Soil survey geographic data has been developed by the Natural Resources Conservation Service (NRCS), formerly the Soil Conservation Service (SCS), and contains the spatial and tabular data contained in the county soil surveys in digital format. Utilizing the attributes, it can be used to bring in to a site plan and then determine the best location for on-lot septic systems, stormwater recharge areas, or used for erosion and sedimentation control plans.

The National Wetlands Inventory (NWI) data catalogued known wetlands on a USGS topographic quadrangle base map. The digital format can be brought into site plans to get a preliminary indication if wetlands are present on a particular property. They were developed by the U.S. Fish and Wildlife Service at the national level, and therefore do not contain all wetlands, and the boundaries are not specific to a particular site. Field delineation and boundary survey must be conducted for site planning initiatives.

The Federal Emergency Management Agency (FEMA) is responsible for delineating regulatory floodplains as part of the National Flood Insurance Program (NFIP). The original flood insurance rate maps (FIRMs) were paper copies of floodplains on simple base maps showing just roads and streams. FEMA put these floodplains into digital format called Q3 data, which can be brought into the GIS and overlaid onto aerial photographs. FEMA is currently working on the map modernization program and placing digital floodplains on accurate digital aerial photographs (typically flown during LiDAR data collection). The floodplains are corrected where known errors occurred utilizing the LiDAR elevation data.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

28.1 Describe the concept of layers in a geographic information system.

28.2 Discuss the role of a geographic reference framework in a GIS.

28.3 List the fundamental components of a GIS.

28.4 List the fields within surveying and mapping that are fundamental to the development and implementation of GISs.

28.5 Discuss the importance of metadata to a GIS.

28.6 Name and describe the different simple spatial objects used for representing graphic data in digital form. Which objects are used in raster format representations?

28.7 What are the primary differences between a GIS and a LIS?

28.8 How many pixels are required to convert the following documents to raster form for the conditions given:

(a)* A 384-in. square map scanned at 200 dpi.
(b) A 9-in. square aerial photo scanned at 1200 dpi.
(c) An orthophoto of 11×17 in. dimensions scanned at 300 dpi

28.9 Explain how data can be converted from:

(a) Vector to raster format
(b) Raster to vector format

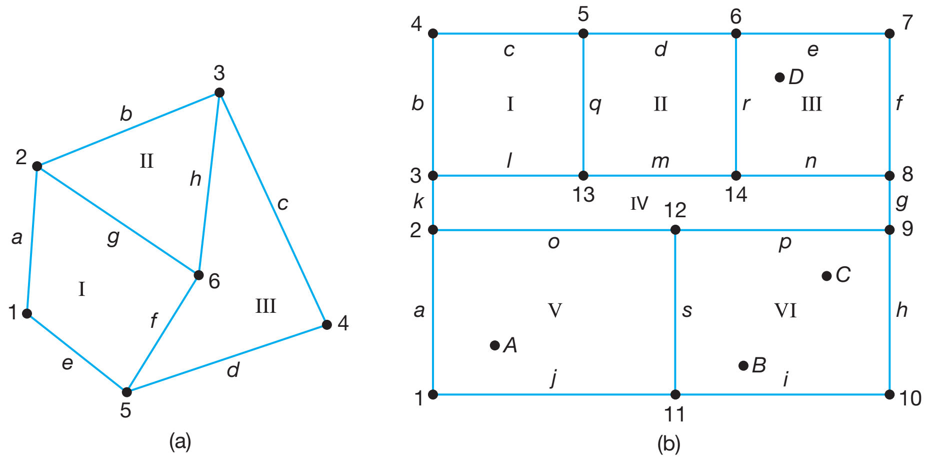
28.10 For what types of data is the vector format best suited?

28.11 Discuss the compromising relationships between grid cell size and resolution in raster data representation.

28.12 Define the term topology and discuss its importance in a GIS.

28.13 Develop identifier and topology tables similar to those of Tables 28.1 and 28.2 in the text for the vector representation of (see the following figures):

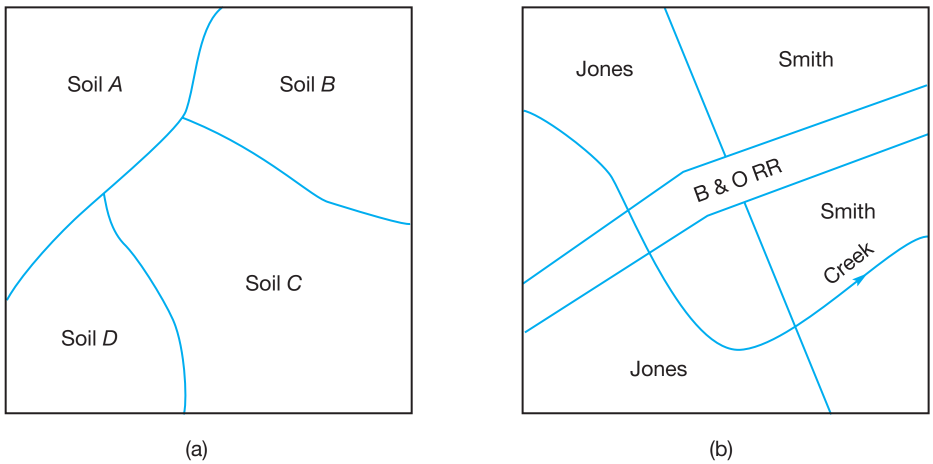
(a) Problem 28.13(a)
(b) Problem 28.13(b)



Problem 28.13

28.14 Compile a list of linear features for which the topological relationship of adjacency would be important.

28.15 Prepare a raster (grid cell) representation of the sample map of:
- **(a)** Problem 28.15(a), using a cell size of 0.10-in. square (see accompanying figure).
- **(b)** Problem 28.15(b), using a cell size of 0.20- in. square (see accompanying figure).



Problem 28.15

28.16 Discuss the advantages and disadvantages of using the following equipment for converting maps and other graphic data to digital form: (a) tablet digitizers and (b) scanners.

28.17 Explain the concepts of the following terms in GIS spatial analysis, and give an example illustrating the beneficial application of each: (a) adjacency and (b) connectivity.

28.18 If data were being represented in vector format, what simple spatial objects would be associated with each of the following topological properties?
(a) Connectivity
(b) Direction
(c) Adjacency
(d) Nestedness

28.19 Prepare a transparency having a 0.10-in grid, overlay it onto Figure 28.4(a), and indicate the grid cells that define the stream. Now convert this raster representation to vector using the method described in Section 28.6.2. Repeat the process using a 0.20-in grid. Compare the two resulting vector representations of the stream and explain any differences.

28.20 Discuss how spatial and nonspatial data are related in a GIS.

28.21 What are the actual ground dimensions of a pixel for the following conditions:
(a) A 1:10,000 scale orthophoto scanned at 500 dpi?
(b)*A 1:24,000 map scanned at 200 dpi?

28.22 Describe the following GIS functions, and give two examples where each would be valuable in analysis:
(a) line buffering, and
(b) spatial joins.

28.23 Go to the PASDA[2] website or a similar website in your state and download an example of:
(a) An orthophoto.
(b) Zoning.
(c) Floodplains and wetlands.
(d) Soil types.

28.24 Compile a list of data layers and attributes that would likely be included in an LIS.

28.25 Compile a list of data layers and attributes that would likely be included in a GIS for:
(a) Selecting the optimum corridor for constructing a new rapid-transit system to connect two major cities.
(b) Choosing the best location for a new airport in a large metropolitan area.
(c) Routing a fleet of school buses.
(d) Selecting the fastest routes for reaching locations of fires from various fire stations in a large city.

28.26 In Section 28.9.3, a flood-warning example is given to illustrate the value of simultaneously applying more than one GIS analytical function. Describe another example.

28.27 Consult the literature on GISs and, based on your research, describe an example that gives an application of a GIS in:
(a) Natural resource management.
(b) Agriculture.
(c) Engineering.
(d) Forestry.

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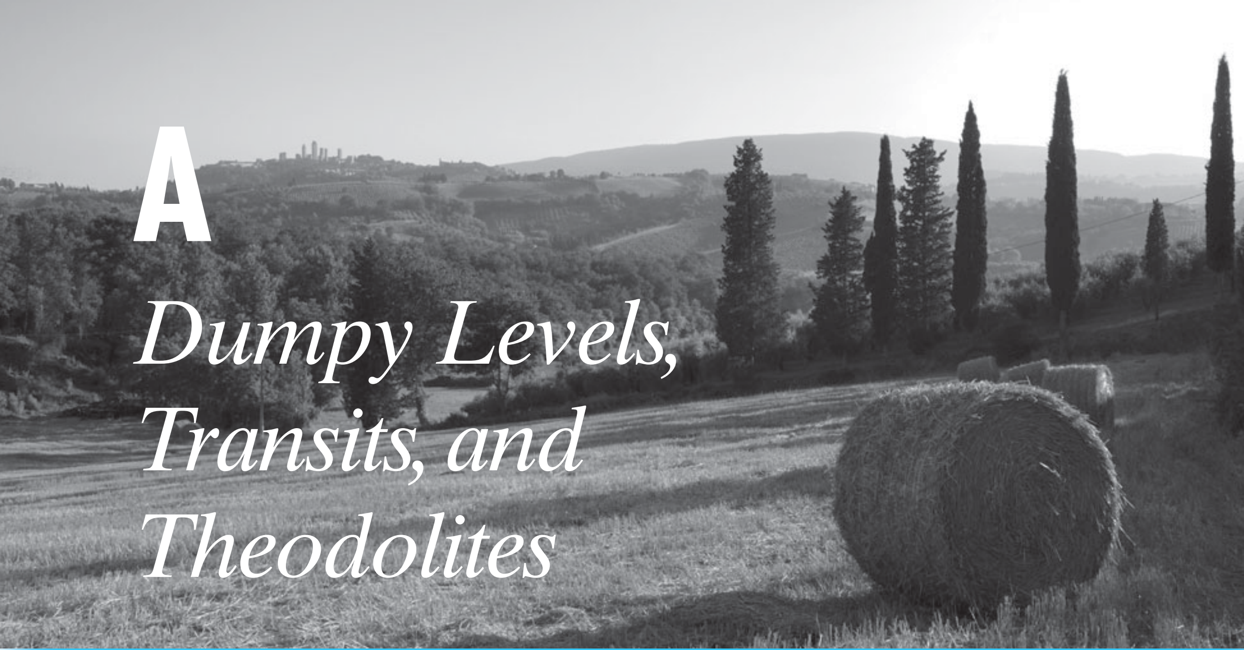


A

Dumpy Levels, Transits, and Theodolites

A.1 INTRODUCTION

Circa 1950, an important new development occurred in the evolution of leveling instruments—the automatic or "self-leveling" feature was introduced (see Section 4.10). This feature significantly reduced the time expended in setting up leveling instruments, and as a result leveling operations could be performed much more efficiently. Automatic levels therefore gradually replaced the "dumpy level," which up to that time had been the dominant leveling instrument. A few dumpy levels may still be in use and therefore these instruments are discussed in Section A.2.

Another important development in surveying instrumentation also occurred at about the midpoint of the 20th century—the first electronic distance measurement (EDM) instruments appeared. The earliest versions were bulky and required large power sources for operation. However, through research and development their sizes were reduced and their capabilities enhanced. Soon small EDM instruments were being attached to theodolites, so that both distances and angles could be measured from a single setup—a feature, which again greatly increased surveying efficiency. Later automatic angle-reading capabilities were incorporated into theodolites, and the theodolite and EDM instrument were interfaced with a microcomputer to create what is now known as the *total station instrument* (see Chapter 8). With their many advantages, total station instruments rapidly replaced their predecessors, the transit and theodolite. Again, a few transits and theodolites are still in use and thus these instruments are briefly described in Sections A.3 through A.5 in this Appendix.

A.2 THE DUMPY LEVEL

A dumpy level is shown in Figure A.1. As illustrated, the telescope (which as described in Section 4.7, consists of four main parts: the objective lens, the negative lens, the reticle, and the eyepiece) is rigidly fastened to the *dumpy bar,* also called the *level bar*. The bar is centered on an accurately machined vertical *spindle* that sits in a conical *socket* of the *leveling head*. This spindle and socket arrangement allows the dumpy bar to revolve in a horizontal plane when the instrument is properly adjusted. A *clamp screw* enables the telescope's line of sight to be locked in a given direction, but a *tangent screw* allows a fine adjustment to be made in its direction of pointing.

The dumpy level has its level vial (tube type) set in the dumpy bar (see Figure A.1) and it is thereby somewhat protected. The vial always remains in the same vertical plane as the telescope but screws at each end permit vertical adjustment or replacement of the vial. (Level vials are described in detail in Section 4.8.)

Four large leveling screws carry the conical socket into which the vertical spindle of the dumpy bar fits. They rest on the *base plate*, which is screwed onto the top of the tripod. The four leveling screws are in two pairs at right angles to each other and are used to center the bubble when leveling the instrument. To level the instrument, after the tripod has been firmly pressed into the ground, the telescope is rotated until it is over two opposite screws, as in the direction *AB* of Figure A.2. Using the thumb and first finger of each hand to adjust the opposite screws simultaneously "approximately" centers the bubble. The telescope is then turned so that it is aligned over the other two leveling screws and the process repeated. These two steps are repeated until the bubble stays centered in each direction. Working with each pair of screws about three times should complete the job. A simple useful rule in centering a bubble, illustrated in Figure A.2, is *A bubble follows the*

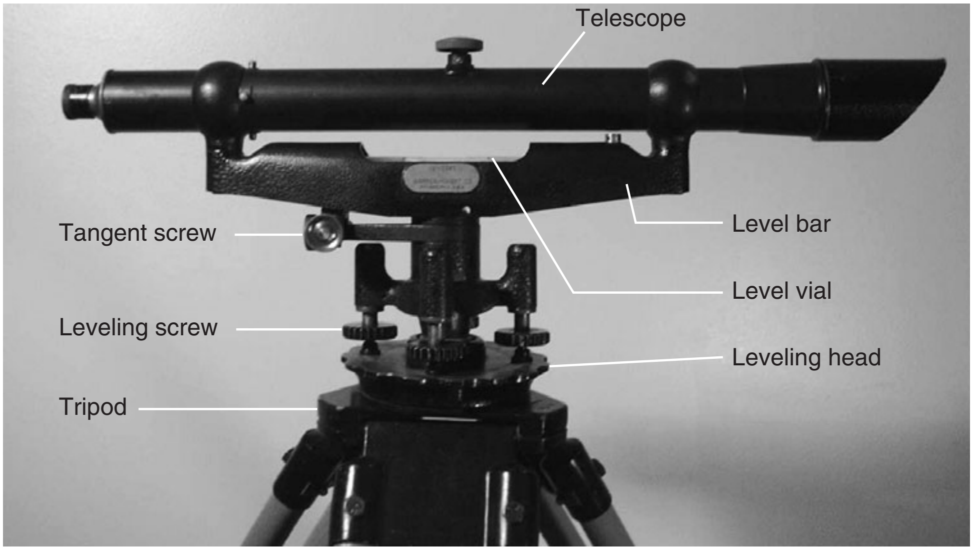


Figure A.1 Dumpy level.

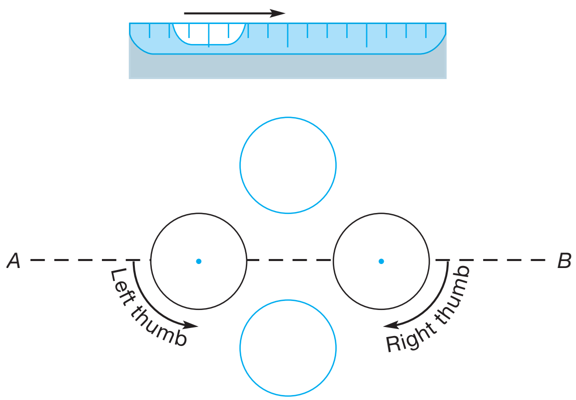


Figure A.2 Use of leveling screws with a four-screw leveling head.

direction of the left thumb when turning the leveling screws. Final precise adjustment can be made with one screw only. Leveling screws should be snug but not wrench tight to avoid damage to threads and/or base plate.

If the bubble runs when the telescope is turned 180° in azimuth after the instrument has been leveled, the level vial is out of adjustment. It can be adjusted by following the procedures described in Section 4.15.3. The adjustment is made by turning the *level adjusting nuts* on one end of the level vial (see Figure A.1). When the bubble is in adjustment and the leveling process is completed as described above, the line of sight generates a horizontal plane as the telescope is revolved about its vertical axis. The dumpy level can be used for all types of leveling including differential, profile, and construction leveling. Care should be exercised to ensure that the bubble is centered whenever a rod reading is taken.

Another older leveling instrument called the *wye level* is similar in many respects to the dumpy level. Its telescope rests in supports on the level bar called *wyes*. Its advantage is that the telescope can be removed from the wyes, a procedure which facilitates adjusting the instrument.

A.3 INTRODUCTION TO THE TRANSIT AND THEODOLITE

The transit and theodolite are predecessors of the total station instrument. These instruments are fundamentally equivalent and can accomplish basically the same tasks. The most important application of transits and theodolites was observing horizontal and vertical (or zenith) angles, but they could also be used to obtain horizontal distances and determine elevations of points by stadia, accomplish low-order differential leveling, and establish alignments.

The main components of a transit and theodolite include a sighting telescope and two graduated circles mounted in mutually perpendicular planes. Prior to measuring angles, the *horizontal* circle is oriented into a horizontal plane by means of level vials, which automatically puts the other circle in a vertical plane. Horizontal and zenith (or vertical) angles can then be observed directly in their respective planes of reference.

William Young of Philadelphia produced the first American transit in 1831. The term *transit* was adopted for the instrument because its telescope could be *transited,* or reversed in direction, by rotating it about a "horizontal axis." In Europe, the name *transiting theodolite* was adopted for this type of angle-measuring instrument. Europeans eventually dropped the adjective and retained the name *theodolite*.

There is no internationally accepted understanding among surveyors on the exact difference denoted by the terms *transit* and *theodolite*. The most commonly used criterion, however, is their general design, particularly their graduated circles and systems for reading them. Transits have an "open-circle" design (see Figures A.3 and A.4), in which their graduated circles, made of metal, are visible to an operator and are read with the aid of verniers (see Section A.4.2). Theodolites, on the other hand, feature an enclosed design (see Figures A.6 and A.7). Theodolites have graduated circles made of glass. They are not directly visible to

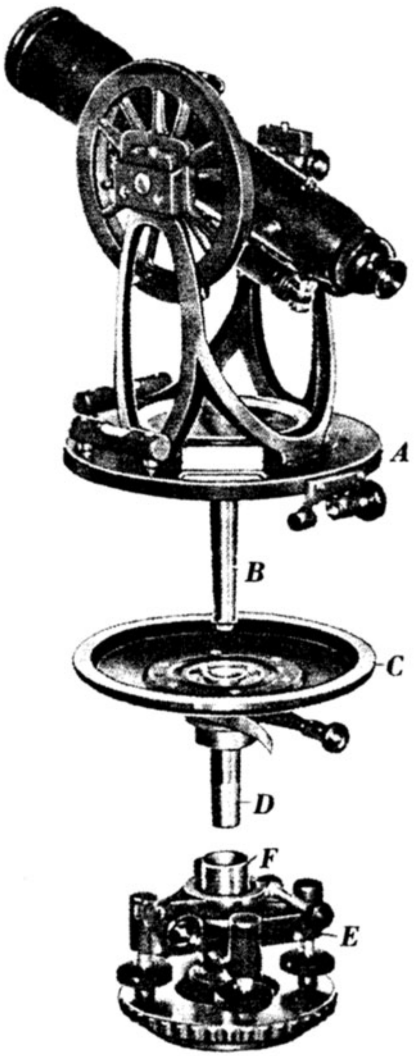


Figure A.3
Three main assemblies of the transit, from top to bottom: alidade, lower plate, and leveling head. Transit parts identified by letter are: (A) upper plate; (B) inner spindle; (C) lower plate; (D) outer spindle; (E) leveling head; and (F) socket. (Courtesy W. & L.E. Gurley.)

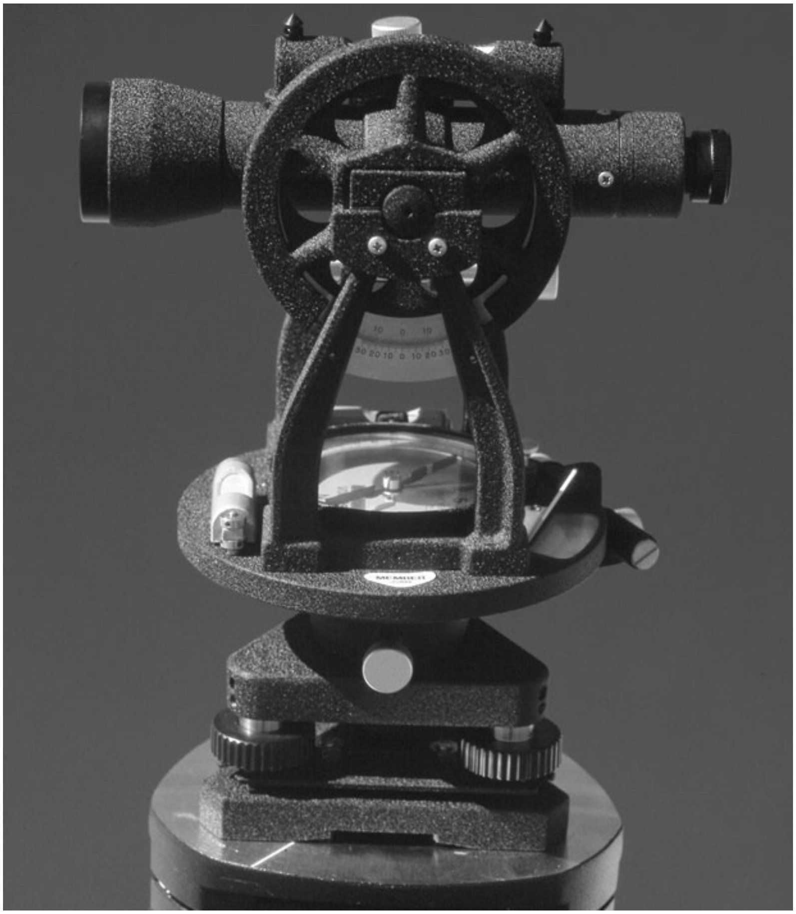


Figure A.4
Assembled transit.
(Courtesy Tom Pantages.)

an operator and must be read by means of an internal microscopic optical system; hence these instruments are called *optical-reading theodolites*. Later versions, called *digital theodolites,* could automatically resolve the circle readings using electronic systems and display them in digital form. Other distinctions between transits and theodolites are described subsequently in this section.

In general, theodolites are capable of greater precision and accuracy in angle observations than transits. Because of this and other advantages, during the period from the 1960s through the mid-1980s, theodolites were purchased much more frequently than transits in the United States. Since the mid-1980s, total station instruments with their automatic angle and distance readout capabilities and built-in microprocessors for real-time data processing have rapidly replaced both transits and theodolites.

■ A.4 THE TRANSIT

In the subsections that follow, the parts of a transit and their functions are described. Procedures for handling, setting up, and using the instrument are also discussed.

A.4.1 Parts of a Transit

Transits have three main assemblies: the (**1**) *alidade,* (**2**) *lower plate,* and (**3**) *leveling head*. These three assemblies are shown in their relative positions from top to bottom in Figure A.3 and assembled in the cutaway diagram of Figure A.4. Specific parts within the assemblies are identified in the figures. Following is a detailed description of each of these assemblies and parts. Reference to the figures will lead to a better understanding of the descriptions.

Alidade Assembly. The alidade assembly contains the *telescope, vertical circle,* and *upper plate* (A in Figure A.3). A vertical *spindle B* is attached to the upper plate, which enables the assembly to revolve about a vertical axis. The tapered design of the spindle assures that despite wear, unless damaged by dirt or an accident, it will still seat and center properly. Two level vials are attached to the upper plate: the *altitude bubble*, which is parallel with the telescope, and the *azimuth bubble*, which is at right angles to it (see Figure A.4). Also two *verniers,* referred to as *A* and *B*, are mounted on the upper plate and set 180° apart. Provisions are made for adjusting the verniers and level vials. Two vertical *standards* are cast as an integral part of the alidade to support the horizontal *crossarms* of the telescope in bearings. The telescope revolves in a vertical plane about the centerline through the arms, called the *horizontal axis*.

The telescope, similar to that of a dumpy level, contains an eyepiece, a reticle, and an objective lens system. A sensitive telescope level vial is attached to the telescope tube so the transit can be used as a leveling instrument on work where lower magnification and lesser sensitivity of the telescope vial are satisfactory. The telescope is said to be in the *normal* or *direct* position when the telescope level vial is below it. Turning the telescope 180° about its horizontal axis puts the level vial above, and the instrument is said to be in a *plunged, inverted,* or *reversed* mode. To permit use of the telescope for leveling in either the normal or inverted position, a *reversion vial* (curved and graduated on both its top and bottom so it is usable in both positions) is desirable.

The vertical circle is supported by the crossarm and turns with the telescope as it is revolved. The circle normally is divided into 1/2° spaces with readings to the nearest minute obtained from a vernier having 30 divisions. The vernier is mounted on one standard with provisions for adjustment. If properly set, it should read zero when the telescope bubble is centered. If out of adjustment, a constant *index error* is read from the circle with the bubble centered and must be applied to all vertical angles, with appropriate sign, to get correct values.

The alidade assembly also contains a *compass box* and holds the *upper-tangent screw*. A *vertical-circle clamp* (for the horizontal axis) is tightened to hold the telescope horizontal or at any desired inclination. After the clamp is set, a limited range of vertical movement is possible by manipulating the *vertical-circle tangent screw* (also called the vertical slow-motion screw).

Lower Plate Assembly. The *lower plate* (*C* in Figure A.3) is a horizontal circular plate graduated on its upper face. Its underside is attached to a vertical, hollow, tapered spindle *D* into which the alidade's spindle fits precisely. The upper plate completely covers the lower plate, except for two openings where the verniers exactly meet the graduated circle.

The *upper clamp* on the lower plate assembly fastens the upper and lower plates together. A small range of movement is possible after clamping by using the *upper-tangent screw* (located on the alidade assembly).

Leveling Head Assembly. The leveling head assembly (*E* in Figure A.3) consists of a bottom horizontal plate, which has a threaded collar to fit on a tripod, and a "spider" with four *leveling screws*. The leveling screws, set in cups to prevent scoring the bottom plate, are partly or completely enclosed for protection against dirt and damage. A socket (*F* in Figure A.3) of the leveling head includes a *lower clamp* to fasten the lower plate. The *lower tangent screw* (also on the leveling head) is used to make precise settings after the lower clamp is tightened. The base of the socket is fitted into a ball-and-socket joint resting on the bottom plate of the leveling head, on which it slides horizontally. A *plummet chain* attached to the center of the spindle holds a plumb-bob string. An *optical plummet,* which is a telescope through the vertical center (spindle), is available on some transits. Like those used on total station instruments, it points vertically when the instrument is level and is viewed at right angles (horizontally) by means of a prism for ease of observation.

A recapitulation of the use of the various clamps and tangent screws may be helpful to the beginner. The "vertical-circle clamp and tangent screw" on one standard controls movement of the telescope in the vertical plane. The "upper clamp" fastens the upper and lower plates together and an "upper-tangent screw" permits a small differential movement between them. A "lower clamp" fastens the lower plate to the socket, after which a "lower-tangent screw" turns the plate through a small angle. If the upper and lower plates are clamped together, they will, of course, move freely as a unit until the lower clamp is tightened.

Tripods used for transits may be either fixed- or adjustable-leg types.

A.4.2 Circle Scales and Verniers

The *horizontal circle* of the lower plate may be divided in various ways, but generally it is graduated into either 30′ or 20′ spaces. For convenience in measuring angles to the right or left, graduations are numbered from 0° to 360° both clockwise and counterclockwise. Figure A.5 shows these arrangements.

Vertical circles of most transits are graduated into 30′ spaces. They are usually numbered from zero at the bottom (for a horizontal sight) to 90° in both directions (for vertical sights), and then back to zero at the top. This facilitates reading either elevation or depression angles with the telescope in either a direct or an inverted position.

The horizontal and vertical circles of transits are read by means of *verniers*. A vernier is a short auxiliary scale set parallel to and beside a primary scale. It enables reading fractional parts of the smallest main-scale divisions without interpolation. Figures A.5(a) and (b) show two different vernier and angle scale combinations commonly used on transits. Both are double verniers; that is, they can be read clockwise or counterclockwise from the index (zero) mark. Considering the clockwise vernier scale of Figure A.5(a), it is constructed so that 30 of its

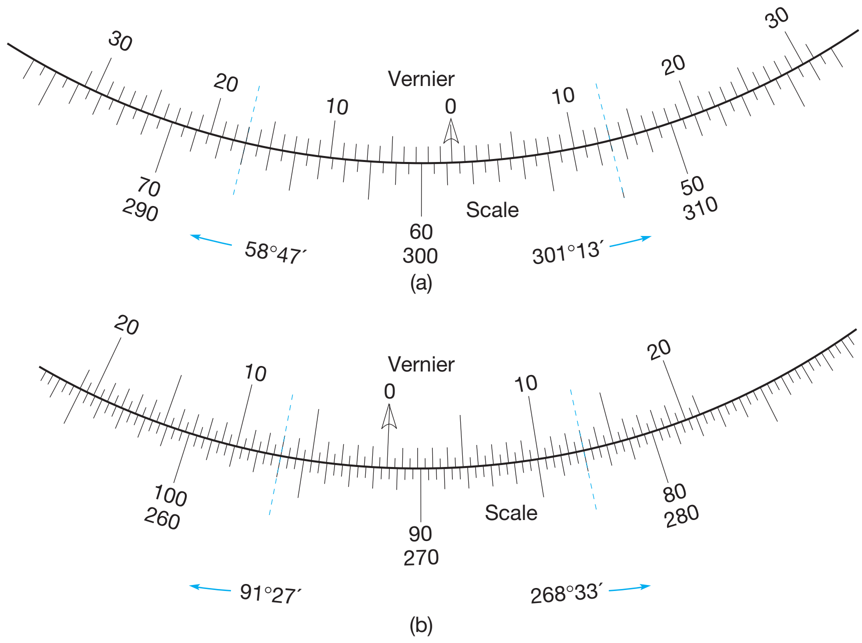


Figure A.5 Transit verniers. [Least count of (a) is 1′; least count of (b) is 1/2′.]

divisions cover 29 half-degree (30′) divisions on the graduated circle. Thus, each vernier division equals $29/30 \times 30' = 29'$. The difference between the length of one main-scale division and one vernier division is therefore 01′. This is the so-called *least count* of this vernier. In general, the least count of a vernier is given by

$$\text{least count} = d/n \tag{A.1}$$

where d is the value of the smallest main-scale division and n the number of vernier divisions that span $(n - 1)$ main-scale units. By Equation (A.1), the least count of the vernier of Figure A.5(a) is $30'/30 = 01'$. This verifies the intuitive determination given above. *An observer cannot make readings using a vernier without first determining its least count.*

For the vernier of Figure A.5(a), if the index mark were perfectly aligned with a certain main-scale division, say the 58°30′ mark (reading clockwise), and then the vernier advanced 01′ clockwise, the first vernier division would then be aligned with the first main-scale division to the left of the index mark and the reading would be 58°31′. If the circle were moved another 01′, the second vernier division would then be aligned with the second main-scale division left of the index mark and the reading would be 58°02′, etc. In Figure A.5(a) the seventeenth vernier division aligns with a main-scale division (shown by a dashed line); thus the circle has been advanced 17′ beyond 58°30′ and the clockwise reading is 58°47′.

With the double verniers shown in Figure A.5(a) and (b), an observer can measure angles both clockwise and counterclockwise. This is particularly convenient for measuring deflection angles, which can be right or left. With double verniers there will be two matching lines, one for the clockwise angle and the other for the counterclockwise angle. The counterclockwise circle reading in

Figure A.5(a) is $301°00' + 13' = 301°13'$. Note that the sum of the clockwise and counterclockwise angles should be $360°00'$.

In Figure A.5(b) the smallest circle graduation is $20'$, and 40 vernier divisions span 39 on the circle. Thus by Equation (A.1), the least count is $20'/40 = 1/2 \text{ min} = 30''$. The reading of the clockwise angle is $91°20' + 07' = 91°27'$; for the counterclockwise circle it is $268°20' + 13' = 268°33'$.

A.4.3 Properties of the Transit

Transits are designed to have a proper balance between magnification and resolution of the telescope, least count of the vernier, and sensitivity of the plate and telescope bubbles. An average length of sight of about 300 ft is assumed in design. Thus, a standard $1'$ instrument has the following properties:

Magnification, $18-30\times$
Field of view, $1'-1°30'$
Resolution, $3-5''$
Minimum focus, about 3–8 ft
Sensitivity of plate levels per 2-mm division, $60-100''$
Sensitivity of telescope vial per 2-mm division, $30-60''$
Weight of instrument head without tripod, 11–18 lb

Transit reticles usually include vertical and horizontal center hairs and two additional stadia hairs, one above and the other below the horizontal center hair. Short stadia lines, used on glass reticles, avoids confusion between the center hair and the stadia hairs.

A.4.4 Handling, Setting up, and Using a Transit

A transit is a precision instrument, and as such it should be handled carefully. Procedures for handling and caring for a total station instrument were discussed in Section 8.5, and these same precautions should be taken with a transit.

For setting a transit over a point, a plumb bob normally is used because most transits are not equipped with optical plummets. After the tripod has been placed so that the instrument is approximately over the point, the plumb-bob string is attached to a hook at the bottom of the spindle using a slipknot. This permits raising or lowering the bob without retying, and it also avoids knots. A small slide attachment is also useful in accomplishing this purpose. If the instrument is being set up for the purpose of measuring an angle, the plumb bob must be brought directly over a definite point such as a tack in a wooden stake, and the plates leveled. The tripod legs can be moved in, out, or sideways to approximately level the plates before the leveling screws are used. Shifting the legs affects the position of the plumb bob, however, which makes it more difficult to set up a transit than a level.

Two methods are used to bring the plumb bob within about 1/4 in. of the proper point. In the first method, the transit is set over the mark and one or more legs are moved to bring the plumb bob into position. One leg may be moved circumferentially to level the plates without greatly disturbing the plummet. Beginners sometimes have difficulty with this method because at the start the transit center is too far off the point, or the plates are badly out of level. Several movements of the tripod legs may then fail to both level the plates and center the plumb bob while maintaining a convenient height of instrument. If an adjustable-leg tripod is used, one or two legs can be lengthened or shortened to bring the bob directly over the point.

In the second method, which is particularly suited to level on uniformly sloping ground, the transit is set up near the point and the plates are approximately leveled by moving the tripod legs as necessary. Then, with one tripod leg held in the left hand, another under the left armpit, and the third supported in the right hand, the transit is lifted and placed over the mark. A slight shifting of one leg should bring the plumb bob within perhaps 1/4 in. of the proper position and leave the plates practically level.

Loosening all four leveling screws and sliding them on the bottom plate using the ball-and-socket shifting-head device, which permits a limited movement, precisely centers the plummet. To assure mobility in any direction, the shifting head should be approximately centered on the bottom plate before setting up the instrument and when boxing it.

A transit is accurately leveled by means of the four leveling screws in somewhat the same manner described in Section A.2 for a dumpy level. However, each level vial on the upper plate is first set over a pair of opposite screws, and because there are two vials available, the telescope position need not be changed in the initial leveling process. After both bubbles are carefully centered, the telescope is rotated 180°. As explained in Section 8.20 and illustrated in Figure 8.25, if the bubbles run, they will indicate *double* the dislevelment. They should therefore be brought back halfway, and the telescope rotated back to its original direction. If the bubbles remain in their halfway positions, even though not centered, the instrument is level; if not, the process is repeated until the bubbles stay in the same location no matter what direction the telescope is pointed. If after leveling the instrument, the bubbles are very far off center, it may be desirable to adjust the vials, which can be done using the procedure described in Section 8.19.1.

If the plumb bob is still over the mark after leveling, the instrument is ready for use. But if the plates were initially badly out of level, or the leveling screws were not uniformly set to begin with, the plummet will move off the mark during leveling. The screws must then be loosened and shifted again, and the transit releveled. It is evident that time can be saved by exercising reasonable care when first setting up the tripod so that its head is approximately level to avoid excessive manipulation and possible binding of the screws.

As noted earlier, transits can be used for observing horizontal and vertical angles, and also for setting out alignments and determining distances by stadia. Transits are so-called *repeating instruments* because horizontal angles can be measured by repetitively using them, that is, an angle can be repeated any number of times and the total added on the plates. To eliminate instrumental errors, equal numbers of the individual angle are measured in both the direct and reversed mode. The final angle is then taken as the average. Advantages of the repeating procedure are (1) better accuracy obtained through averaging and (2) disclosure of mistakes and errors by comparing values of the single and multiple readings. The procedure of observing horizontal angles by repetition using total station instruments was described in Section 8.8. The same procedure applies with transits. Methods for measuring vertical angles with total station instruments were discussed in Section 8.13, but again these also apply to transits. Finally, the method of determining distances by stadia was presented in Section 16.9.2.

A.5 THE THEODOLITE

The theodolite is discussed in the subsections that follow. In particular, two types of instruments are described: *repeating theodolites* (see Figure A.6) and *directional theodolites* (see Figure A.7). Characteristics of these instruments, and procedures for handling and setting them up, are discussed.

A.5.1 Characteristics of Theodolites

Theodolites differ from transits in appearance (they are generally more compact, lightweight, and "streamlined") and in design by a number of features, the more important of which are as follows:

1. The *telescopes* are short, have reticles etched on glass, and are equipped with rifle sights or collimators for rough pointing.
2. The *horizontal* and *vertical circles* are made of glass with graduation lines and numerals etched on the circles' surfaces (except for digital theodolites). The lines are very thin and more sharply defined than can be achieved by scribing them on the metal circles used on transits. Precisely graduated circles with small diameters can be obtained, and this is one reason the instruments are so compact. The circles are normally divided into conventional

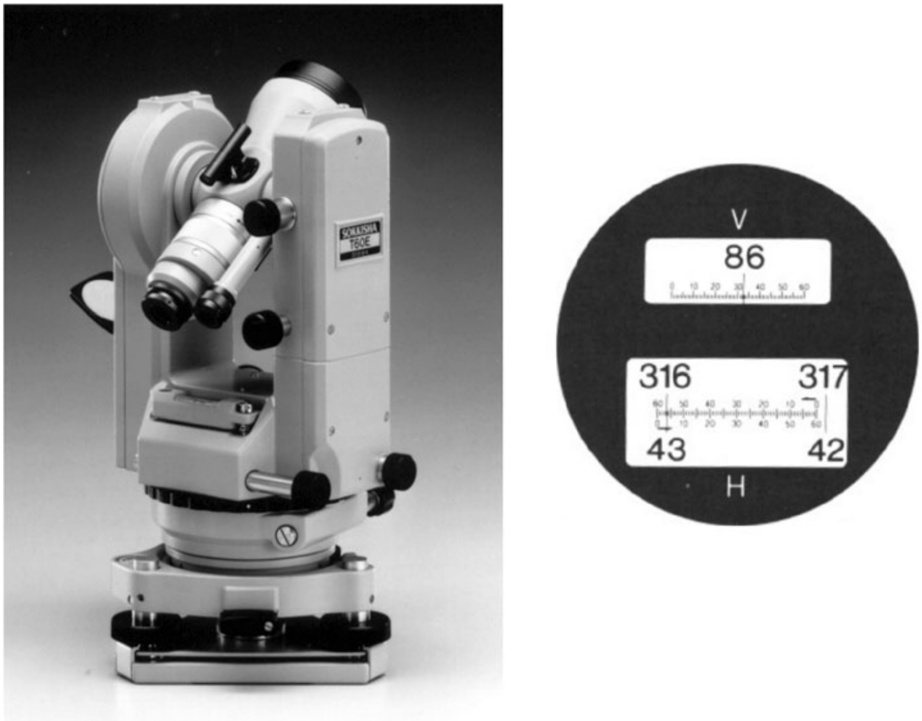


Figure A.6
(a) Lietz T60E optical-reading repeating theodolite and (b) reading system for optical-reading repeating theodolite. (Courtesy Sokkia Corporation.)

sexagesimal degrees and fractions (360°), but instruments can be obtained with graduations in centesimal *grads* or *gons* (full circle divided into 400^g).

3. The *vertical circle* of most theodolites is precisely indexed with respect to the direction of gravity in one of two ways: (a) by an *automatic compensator* or (b) by a *collimation level* or *index level,* usually the coincidence type connected to the reading system of the vertical circle. Both provide a more accurate plane of reference for measuring vertical angles than the plate levels used on transits. Vertical circle readings are *zenith angles*, that is, 0° occurs with the telescope pointing vertically, and either 90° (in the direct mode) or 270° (in the reversed mode) is read when it is horizontal.
4. The circle *reading systems* consist of a microscope with the optics inside the instrument (except for digital theodolites which have automatic angle reading capabilities). A reading eyepiece is generally adjacent to the telescope eyepiece or located on one of the standards. Some instruments have optical micrometers for fractional reading of circle intervals (the micrometer scale is visible through the microscope); others are direct-reading. With most theodolites a mirror located on one standard can be adjusted to reflect light into the instrument and brighten the circles for daytime use. They can be equipped with a battery-operated internal lighting system for night and underground operation. Some theodolites can also use the battery-operated system in lieu of mirrors for daytime work.
5. Rotation about the *vertical axis* occurs within a steel cylinder or on precision ball bearings, or a combination of both.
6. The *leveling head* consists of three screws or cams.
7. The *bases* or *tribrachs* of theodolites are often designed to permit interchange of the instrument with sighting targets, prisms, and EDM instruments without disturbing centering over the survey point.
8. An *optical plummet,* built into the base or alidade of most theodolites, replaces the plumb bob and permits centering with better accuracy.
9. A compass can be attached to a theodolite as an accessory, but it is not an integral part of the instrument, as it is with transits.
10. The tripods used with theodolites are the wide-frame type and most have adjustable legs. Some are all metal and feature devices for preliminary leveling of the tripod head and mechanical centering ("plumbing") to eliminate the need for a plumb bob or optical plummet.

A.5.2 Repeating Theodolites

As noted earlier, theodolites are divided into two basic categories: the *repeating* type and the *directional* model. Repeating theodolites are equipped with a double-vertical axis, usually cylindrical in shape, or a repetition clamp. The double-vertical axis is similar to the double-spindle arrangement used on transits. This design enables horizontal angles to be repeated any number of times and added directly on the instrument's circle.

Figure A.6(a) shows the Lietz T60E, which is an example of a repeating-type theodolite. The optical reading system, typical of these types of repeating theodolites, illustrated in Figure A.6(b), enables angles to be read directly to the nearest minute, with estimation possible to 0.1′. The instrument has a vertical-circle automatic compensator, a telescope with a standard eyepiece of 30° magnification, an optical plummet, and a plate bubble sensitivity of 30″/2-mm division. This instrument is representative of many others of this type.

The reading system of Lietz, and many other theodolites, consists of a graduated glass scale having a span of 1′ which appears superimposed on the degree divisions of the main circle. This scale is read directly by means of a microscope whose small eyepiece can be seen beside the main telescope in the figure. To take a reading, it is simply necessary to observe which degree number lies within the 1′ span of the glass scale and select the minute indicated by the index mark. The vertical- and clockwise horizontal-circle readings indicated in Figure A.6 are 86°32.5′ and 316°56.5′, respectively. (The counterclockwise horizontal circle reading is 43°03.5′.) Thus, on this instrument the horizontal and vertical circles can be viewed and read simultaneously through the reading microscope.

A.5.3 Directional Theodolites

A directional theodolite is used for reading *directions* rather than angles. With this type of instrument, after a sight has been taken on a point, the direction indicated on the circle is read. An observation on the next mark gives a new direction, so the angle between the lines can be found by subtracting the first direction from the second.

Directional theodolites have a single vertical axis and therefore cannot measure angles by the repetition method. They do, however, have a *circle-orienting drive* to make a rough setting of the horizontal circle at any desired position. On all directional theodolites each reading represents the *mean* of two diametrically opposed sides of the circle, made possible because the operator simultaneously views both sides of it through internal optics. This reading procedure, equivalent to averaging readings of the *A* and *B* verniers of a transit, automatically compensates for eccentricity errors (see Section 8.20.1).

A typical directional theodolite, the Wild T2, is shown in Figure A.7(a). It has a micrometer that permits reading the horizontal and vertical circles directly to 1″, with estimation possible to the nearest 0.1″. It also has a vertical control bubble for orienting the vertical circle, an optical plummet, and a plate bubble with 20″/2-mm division sensitivity. This instrument is representative of many other similar ones in the directional theodolite category.

Figure A.7(b) illustrates the reading system for the vertical circle of the Wild T-2 instrument of Figure A.7(a). The horizontal circle has a similar system. By means of internal prisms, an operator looking through the eyepiece of the microscope simultaneously sees the two diametrically opposed portions of either the vertical (or horizontal) circle. Only one circle can be viewed at a time, and an optical switch enables the choice to be made. The main circles are graduated in 5-min intervals. Within Figure A.7(b), there are three rectangles or "frames." The upper frame shows the 5′ graduations on the two diametrically opposed sides of the vertical circle, one side above the other and separated by a horizontal line. After completing the foresight in turning an angle, these

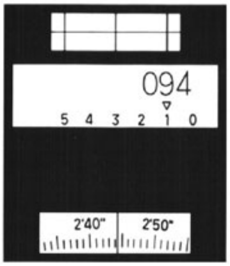


Figure A.7
(a) Wild T-2 optical-reading directional theodolite and (b) reading system for the Wild T-2. (Courtesy Leica Geosystems AG.)

opposite 5′ graduations in the upper frame will not coincide, but rather they will be offset. Coincidence is obtained by turning the micrometer knob until the top vertical lines match those on the bottom, as is shown in the upper frame of the figure. After setting the coincidence, the middle frame gives the degrees portion of the angle reading, plus the minutes part to the nearest 10′. The bottom frame enables the remaining minutes portion and seconds part of the reading to be made. In the example of Figure A.7(b), the middle frame provides a reading of 94°10′, and the bottom frame gives 2′44.3″. Thus the final value is 94°12′44.3″. The horizontal circle is read in the same manner. There are various other similar types of reading systems used in theodolites of different manufacture, and operators should become familiar with the system on their particular instrument.

A.5.4 Handling, Setting up, and Using a Theodolite

Theodolites should be carefully lifted from their carrying cases by grasping the standards (some instruments are equipped with handles for this purpose), and the instrument securely fastened to the tripod by means of a tribrach. Beginners can use a plumb bob to approximate the required setup position, but precise centering over the point is done by means of an optical plummet that provides a line of sight directed downward collinear with the theodolite's vertical axis. The instrument must be level for the optical plummet to define a vertical line. Most theodolite tribrachs have a relatively insensitive bull's-eye bubble to facilitate rough preliminary leveling before beginning final leveling with the plate bubble. Some tribrachs also contain an optical plummet.

The setup process using a theodolite is the same as that described in Section 8.5 for total station instruments. It was noted earlier that theodolites have a three-screw leveling head and a single plate bubble. The procedure to follow with this type of leveling head is also described in Section 8.5 and illustrated in Figure 8.4.

Theodolites can accomplish all of the tasks that can be done with transits. Procedures for measuring horizontal and vertical angles are described for total station instruments in Chapter 8, but the same methods apply with theodolites.

B

Example Noteforms

MEASURING DISTANCES WITH A STEEL TAPE

Sta.	Fwd. Dist.	Back Dist.	Mean	Error	Ratio
A					
	321.18	321.22	321.20	$\frac{.04}{321.20}$ =	$\frac{1}{8,000}$
B					
	276.54	276.60	276.57	$\frac{.06}{276.57}$ =	$\frac{1}{4,600}$
C					
	100.30	100.29	100.30	$\frac{.01}{100.30}$ =	$\frac{1}{10,000}$
D					
	306.77	306.81	306.79	$\frac{.04}{306.79}$ =	$\frac{1}{7,700}$
E					
	255.47	255.50	255.48	$\frac{.03}{255.48}$ =	$\frac{1}{8,500}$
A					

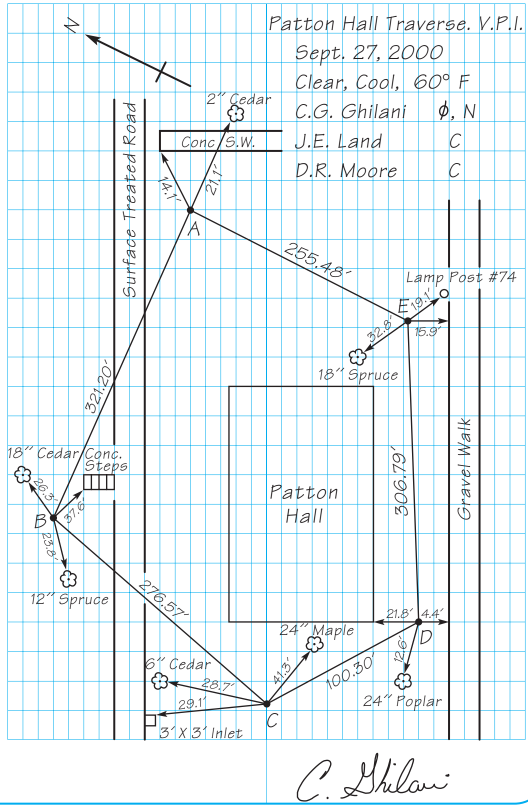


Plate B.1

BORROW-PIT LEVELING

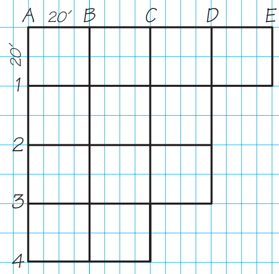
Point	+ Sight	HI	− Sight	Elev.	Cut
BM Road	4.22	364.70		360.48	
A,0			5.2	359.5	1.5
B,0			5.4	359.3	1.3
C,0			5.7	359.0	1.0
D,0			5.9	358.8	0.8
E,0			6.2	358.5	0.5
A,1			4.7	360.0	2.0
B,1			4.8	359.9	1.9
C,1			5.2	359.5	1.5
D,1			5.5	359.2	1.2
E,1			5.8	358.9	0.9
A,2			4.2	360.5	2.5
B,2			4.7	360.0	2.0
C,2			4.8	359.9	1.9
D,2			5.0	359.7	1.7
A,3			3.8	360.9	2.9
B,3			4.0	360.7	2.7
C,3			4.6	360.1	2.1
D,3			4.6	360.1	2.1
A,4			3.4	361.3	3.3
B,4			3.7	361.0	3.0
C,4			4.2	360.5	2.5
BM Road			4.22	360.48	

SECOND & OAK STREETS

Madison, WI
Cool, Cloudy, 60° F
B.A. Dewitt N
B.K. Harris ϕ
E.A. Custer ⊼
11 Oct. 2000
Kern Level #13

hn
BM Road-Description p.5
1.5
2.6
2.0
1.6
0.5
4.0
7.6
6.0
3.6
0.9
5.0
8.0
7.6
3.4
5.8
10.8
6.3
2.1
3.3
6.0
2.5
91.1 ⌊4

$22.8 \times \frac{400}{27} = 337$ cu.yd.



Grade elevation 358.0′

Volume = area of base × Σhn ÷ (4 × 27)

B.A. Dewitt

Plate B.2

TRAVERSING WITH A TOTAL STATION INSTRUMENT

Instrument at sta 101

$h_e = 5.8$ $h_r = 5.3$

Sta. Sighted	D/R	Horiz. Circle	Zenith Angle	Horiz. Dist.	Elev. Diff.
104	D	0°00′00″	86°30′01″	324.38	+19.84
102	D	82°18′19″	92°48′17″	216.02	−10.58
104	R	180°00′03″	273°30′00″		
102	R	262°18′18″	267°11′41″		

Instrument at sta 102

$h_e = 5.5$ $h_r = 5.5$

101	D	0°00′00″	87°11′19″	261.05	+10.61
103	D	95°32′10″	85°19′08″	371.65	+30.43
101	R	180°00′02″	272°48′43″		
103	R	275°32′08″	274°40′50″		

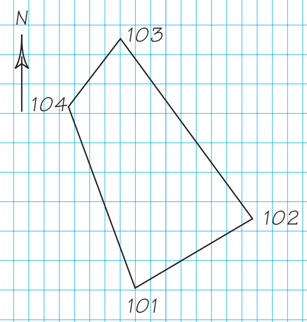
Instrument at sta 103

$h_e = 5.4$ $h_r = 5.4$

102	D	0°00′00″	94°40′48″	371.63	−30.42
104	D	49°33′46″	90°01′54″	145.03	− 0.08
102	R	180°00′00″	265°19′14″		
104	R	229°33′47″	269°58′00″		

Topo Control Survey
19 Oct. 2000
Cool, Sunny, 48° F
Pressure 29.5 in.
Total Station #7
Reflector #7A
M.R. Duckett —
N. Dahman — ϕ
T. Ruhren — N

Sketch



M. R. Duckett

Plate B.3

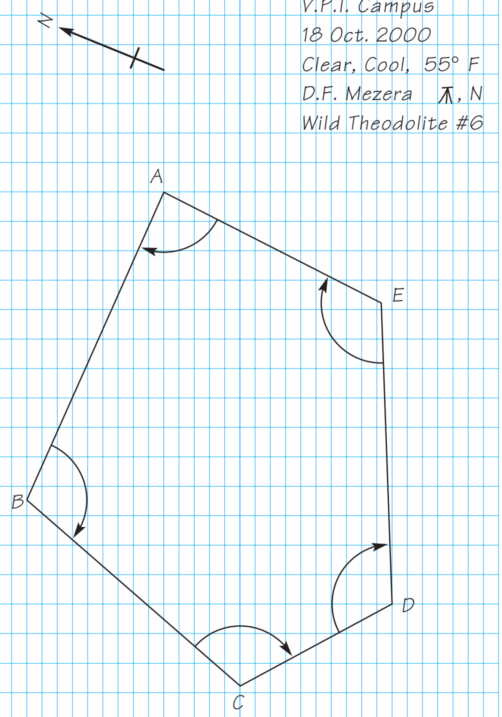
DOUBLE DIRECT ANGLES

Hub	Dist.	Single ∡	Double ∡	Avg. ∡
A		38°58.0′	77°56.8′	38°58.4′
	321.31′			
B		148°53.6′	297°47.0′	148°53.5′
	276.57′			
C		84°28.1′	168°56.2′	84°28.1′
	100.30′			
D		114°40.3′	229°20.9′	114°40.4′
	306.83′			
E		152°59.4′	305°58.6′	152°59.3′
	255.48′			
A				
Σ	1260.49′			539°59.7′
			Misclosure	0°00.3′

Σ interior ∡s = (N-2) 180°
= (5-2) 180°
= 540°00′

PATTON HALL TRAVERSE

V.P.I. Campus
18 Oct. 2000
Clear, Cool, 55° F
D.F. Mezera ⊼, N
Wild Theodolite #6



DFMezera

Plate B.4

CROSS-SECTION LEVELING

Sta.	+ Sight	H.I.	− Sight	Elev.	
5+00			9.5		
4+00			12.6		
TP 1	10.25	106.61	1.87	96.36	
3+00			2.1		
2+50			5.8		
2+00			7.4		
1+35			9.7		
1+00			5.6		
0+50			7.6		
0+00			8.5		
BM Pod	8.51	98.23		89.72	

HONOLULU-KAILUA HIGHWAY

99.2 $\frac{7.4}{52}$	101.5 $\frac{5.1}{30}$	97.4 $\frac{9.2}{10}$	97.1 9.5	95.8 $\frac{10.8}{12}$	97.0 $\frac{9.6}{28}$	103.8 $\frac{2.8}{45}$	
102.3 $\frac{4.3}{48}$	99.9 $\frac{6.7}{24}$	98.4 $\frac{8.2}{8}$	94.0 12.6	100.1 $\frac{6.5}{10}$	101.5 $\frac{5.1}{25}$	98.7 $\frac{7.9}{50}$	
	95.2 $\frac{3.0}{50}$	95.8 $\frac{2.4}{25}$	96.6 $\frac{1.6}{10}$	96.1 2.1	94.4 $\frac{3.8}{8}$	91.1 $\frac{7.1}{31}$	95.7 $\frac{2.5}{48}$
95.1 $\frac{3.1}{48}$	92.8 $\frac{5.4}{32}$	89.5 $\frac{8.7}{15}$	93.8 $\frac{4.4}{8}$	92.4 5.8	90.7 $\frac{7.5}{10}$	93.4 $\frac{4.8}{25}$	96.6 $\frac{1.6}{50}$
	92.3 $\frac{5.9}{54}$	90.0 $\frac{8.2}{30}$	90.8 $\frac{7.4}{10}$	90.8 7.4	91.3 $\frac{6.9}{9}$	93.2 $\frac{5.0}{25}$	95.9 $\frac{2.3}{40}$
	85.4 $\frac{12.8}{48}$	88.9 $\frac{9.3}{25}$	85.7 $\frac{12.5}{10}$	88.5 9.7	89.8 $\frac{8.4}{8}$	91.7 $\frac{6.5}{15}$	94.1 $\frac{4.1}{45}$
	88.6 $\frac{9.6}{52}$	97.2 $\frac{1.0}{28}$	92.2 $\frac{6.0}{12}$	92.6 5.6	95.8 $\frac{2.4}{10}$	93.6 $\frac{4.6}{28}$	95.4 $\frac{2.8}{50}$
	90.0 $\frac{8.2}{50}$	97.0 $\frac{1.2}{25}$	92.7 $\frac{5.5}{8}$	90.6 7.6	94.4 $\frac{3.8}{9}$	95.4 $\frac{2.8}{24}$	95.5 $\frac{12.7}{42}$
	88.6 $\frac{9.6}{50}$	96.1 $\frac{2.1}{25}$	92.0 $\frac{6.2}{10}$	89.7 8.5	93.5 $\frac{4.7}{8}$	97.0 $\frac{1.2}{25}$	91.5 $\frac{6.7}{50}$

Diamond Highway
25 Oct. 2000
Warm, Sunny 70°
A.C. Chun ⊼
R.E. Neilan N
W.E. Grube ϕ,C
M.L. Hagawa C
Lietz level #10

BM Pod-Kalini Valley, Oahu, Ewa-makai corner Hibiscus and Kiawe Drives, Spike in 30″ monkey pod tree, 2 ft. above ground.

Ruth E. Neilan

Plate B.5

8″ SEWER STAKEOUT

(1) Station	(2) +Sight	(3) HI	(4) −Sight	(5) Ground Elev.	(6) Pipe Flow Line	(7) Cut/Fill
BM 16	2.11	102.76		100.56		
0+00			6.21	96.55	96.55	Floor of Existing Catch Basin
+00℄			3.20	99.56	96.55	C 3.01
+50℄			3.91	98.85	95.95	C 2.90
1+00℄			4.07	98.69	95.34	C 3.35
+31℄			8.22	94.54	94.97	F 0.43
+50℄			4.01	98.75	94.74	C 4.02
2+00℄			4.52	98.24	94.14	C 4.09
+33.7℄			5.03	97.73	93.73	C 4.00
+33.7			9.03	93.73	93.73	Floor of Existing Manhole
BM 16			2.11√	100.65		

Flowline Calculations

Line drops 50′(1.206%) = 0.60′ per 50′

Example

Sta. 0+50 = 96.55 − 0.60 = 95.95

1+00 = 96.55 − 1.21 = 95.34

1+31 = 96.55 − 131 (1.206%) = 94.97

Third Street, Statesboro, GA

1 Nov. 2000

Cool, Clear, 65° F

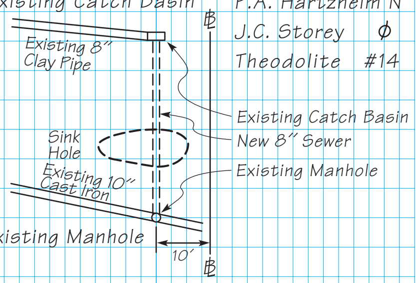
See page 23, Book 67 for description.

F.P. Barnes ⊼

P.A. Hartzheim N

J.C. Storey ϕ

Theodolite #14



$$\text{Grade} = \frac{\text{Fall}}{\text{Dist.}} = \frac{(96.55 - 93.73)(100)}{233.7} = 1.206\%$$

Paul Hartzheim

Plate B.6

C Astronomical Observations

■ C.1 INTRODUCTION

Astronomical observations in surveying (geomatics) consist of observing positions of the sun or certain stars. The principal purpose of astronomical observations in plane surveying was to determine the direction of the astronomic meridian (astronomic north). The resulting azimuth was needed to establish directions of new property lines so parcels could be adequately described; to retrace old property boundaries whose descriptions include bearings that were determined by astronomical methods; to specify directions of tangents on route surveys; to orient map sheets; and for many other purposes. These procedures have today been replaced with GNSS surveys where the coordinates of two points are established on the ground using either static and kinematic GNSS methods as discussed in Chapters 14 and 15. From these coordinates either geodetic or grid azimuths can be determined for the line.

The latitudes and longitudes of points can also be determined by making astronomical observations. However, this is seldom, if ever, done today for two reasons: (1) the field procedures and computations involved, especially for longitude, are quite difficult and time consuming especially if accurate results are expected and (2) the use of the global navigation satellite systems has now made the determination of latitudes and longitudes a rather routine operation. Thus in this appendix, only astronomical methods for determining azimuth are discussed. For a more thorough presentation of this subject, readers are directed to the 11th or earlier edition of this book.

To expand on the definition of the astronomic meridian given in Section 7.4, at any point it is a line tangent to, and in the plane of, the great circle which passes through the point and the Earth's north and south geographic poles. This is illustrated in Figure C.1, where P and P' are the poles located on the Earth's axis

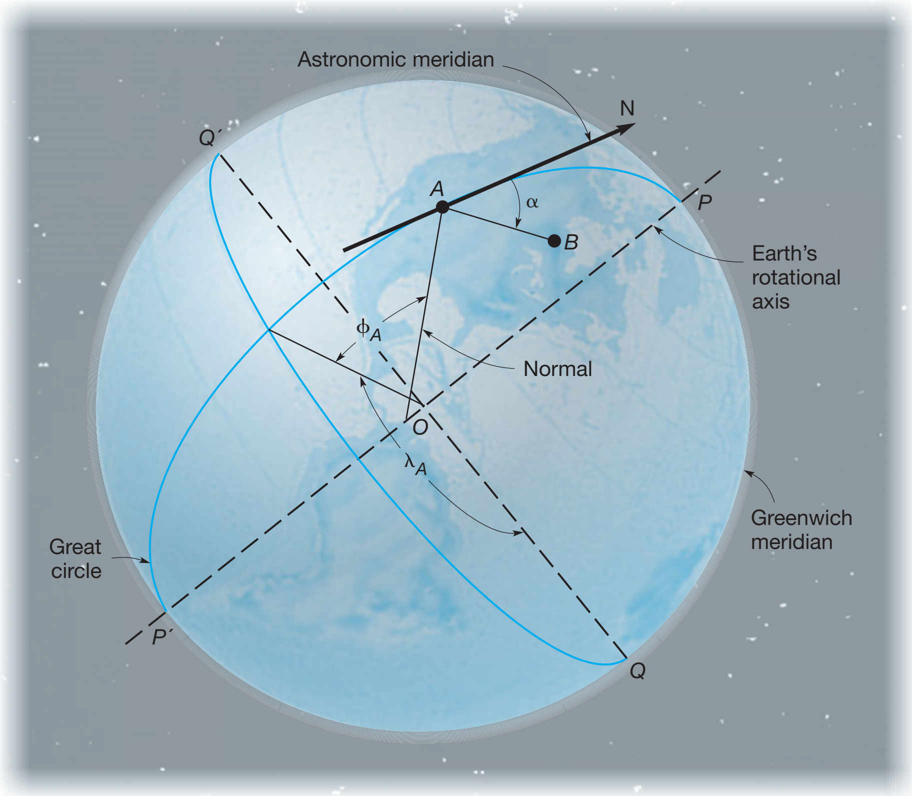


Figure C.1 Astronomic meridian, astronomic azimuth, latitude, and longitude.

of rotation. Arc PAP' is the great circle through A and line AN, the astronomic meridian (tangent to the great circle at A in plane $POP'A$). With an astronomic meridian established, the astronomic azimuth α of any line, such as AB of Figure C.1, can readily be obtained by determining horizontal angle NAB.

Astronomical observations are not necessarily required on every project where azimuths or bearings are needed. If a pair of intervisible control monuments from a previous survey exists in the area, and the azimuth or bearing is known for that line, new directions can be referenced to it. Also, as noted earlier, GNSS survey methods, as described in Chapters 13 through 15, can be used to establish the positions of the two points of a project line from which the azimuth is determined.

■ C.2 OVERVIEW OF USUAL PROCEDURES FOR ASTRONOMICAL AZIMUTH DETERMINATION

In Figure C.2, imagine that P is on the extension of the Earth's polar axis and that S is a star, which appears to rotate about P due to the Earth's rotation on its axis. Point N is on the horizon and vertically beneath P, and therefore line AN represents true north. For the situation shown in this figure, the general field procedures employed by surveyors to define the direction of astronomic north consist of the following steps: (1) a total station is set up and leveled at one end of the

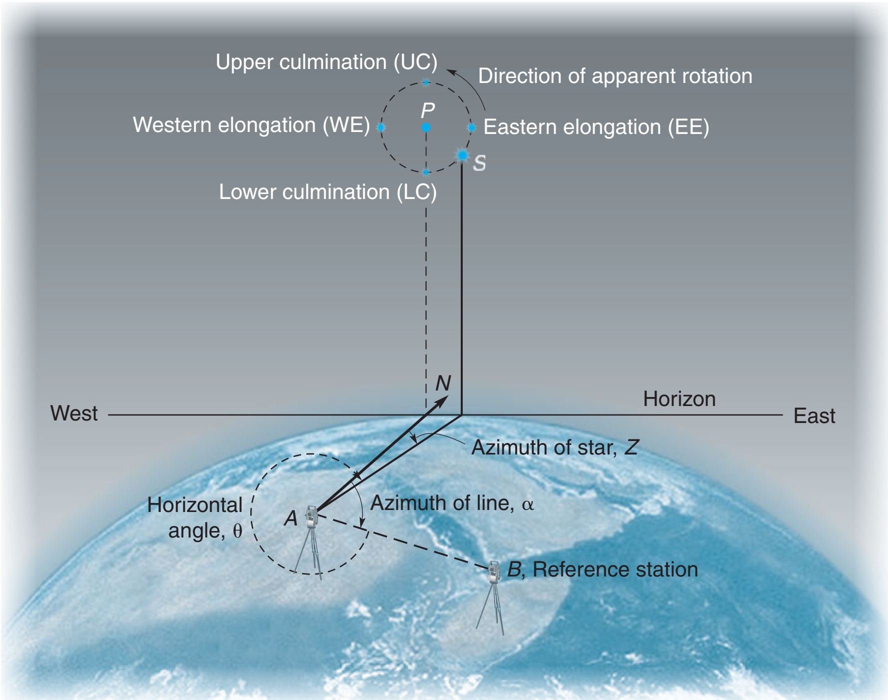


Figure C.2
Azimuth of a line on the ground from the azimuth of a star.

line whose azimuth is to be determined, like point A of Figure C.2; (2) the station at the line's other end, like B of Figure C.2, is carefully sighted and the instrument's horizontal circle indexed to 0°00′00″; (3) the telescope is turned clockwise and the star S carefully sighted; (4) the horizontal, and sometimes vertical, circles of the instrument are read at the instant of pointing on the star; (5) the precise time of pointing is recorded; and (6) the horizontal angle is recorded from the reference mark to the star, like angle θ of Figure C.2 from B to S. Office work involves (a) obtaining the precise location of the star in the heavens at the instant sighted from an ephemeris (almanac of celestial body positions); (b) computing the star's azimuth (angle Z in Figure C.2) based on the observed and ephemeris data; and (c) calculating the line's azimuth by applying the measured horizontal angle to the computed azimuth of the star as

$$\alpha = 360° + Z - \theta \qquad \text{(C.1)}$$

Any visible celestial body for which ephemeris data are available can be employed in the procedures outlined. However, the sun and, in the northern hemisphere, *Polaris* (the north star) are almost always selected.[1] The sun permits observations to be made in lighted conditions during normal daytime working hours; Polaris is preferred for higher-order accuracy. In the southern hemisphere a star in the *Southern Cross* is often used.

[1]In the southern hemisphere, the star *Sigmus Octantis* and the stars in the constellation *Southern Cross* are commonly used for astronomical observations.

Accuracies attainable in determining astronomical azimuths depend on many variables, including (1) the precision of the instrument used, (2) ability and experience of the observer, (3) weather conditions, (4) quality of the clock or chronometer used to measure the time of sighting, (5) celestial body sighted and its position when observed, and (6) accuracy of ephemeris and other data available. In the northern hemisphere Polaris observations provide the most accurate results and, with several repetitions of measurements utilizing first-order instruments, accuracies to within $\pm 1''$ are possible. Sun observations yield a lower order of accuracy but values accurate to within about $\pm 10''$ or better can be obtained if careful repeated measurements are made.

C.3 EPHEMERIDES

As noted previously, ephemerides are almanacs containing data on the positions of the sun and various stars versus time. Nowadays, ephemeris data are most conveniently obtained through the Internet. Jerry Wahl of the U.S. Bureau of Land Management, for example, maintains an ephemeris of the sun and Polaris on his website.[2] Table C.1, which applies to December, 2000, was taken from this website. The data in this table is used in connection with some of the example problems presented later in this chapter.

A variety of ephemerides are also published annually and are available to surveyors for astronomical work. One of those most useful to surveyors is the *The Sokkia Celestial Observation Handbook and Ephemeris*, published annually by Sokkia Corporation.[3] It contains tabulated data not only for the sun and Polaris but also for several other of the brighter stars in the heavens. This booklet also includes a substantial amount of explanatory material, plus worked examples that demonstrate the use of the tabulated data and the computational procedures. Other published ephemerides are *The Apparent Place of Polaris and Apparent Sidereal Time*, published by the U.S. Department of Commerce; *The Nautical Almanac*, published by the U.S. Naval Observatory; and *Apparent Places of Fundamental Stars*, published by Astronomisches Rechen-Institut, Heidelberg, Germany.

In addition to published ephemerides, computer programs are also available which solve for the positions of celestial bodies. Their major advantages are that they provide accurate results without tables and can be used year after year. However, they must occasionally be updated.

Values tabulated in ephemerides are given for *universal coordinated time* (UTC), which is also Greenwich civil time, so before extracting data from them, standard or daylight times normally recorded for observations must be converted. This topic is discussed further in Section C.5.

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[2]The Internet address for obtaining the ephemeris is http://www.cadastral.com/.

[3]The Sokkia ephemeris is authored by Drs. Richard Elgin, David Knowles, and Joseph Senne, and is available from the Sokkia Corporation, 9111 Barton, Box 2934, Overland Park, Kansas 66021; telephone: (800) 255-3913.

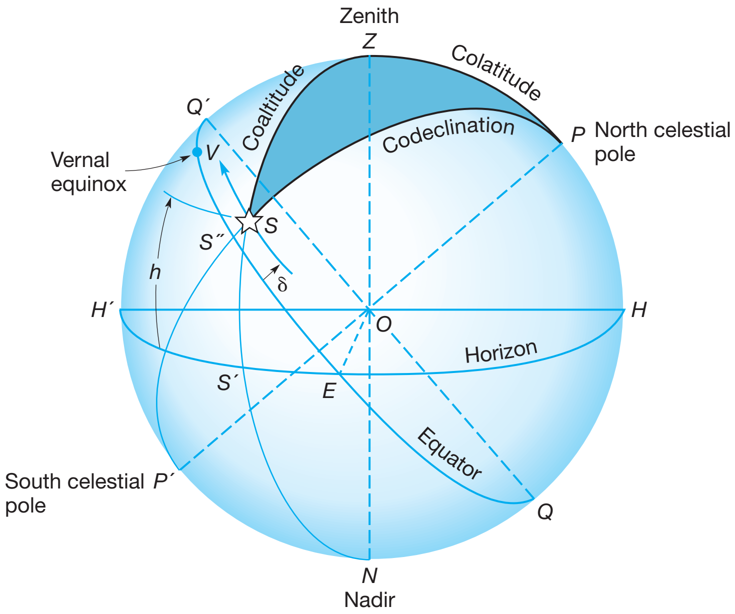


Figure C.3
Celestial sphere.

■ C.4 DEFINITIONS

In making and computing astronomical observations, the sun and stars are assumed to lie on the surface of a *celestial sphere* of infinite radius having the Earth as its center. Because of the Earth's rotation on its axis, all stars appear to move around centers that are on the extended Earth's rotational axis, which is also the axis of the celestial sphere. Figure C.3 is a sketch of the celestial sphere and illustrates some terms used in field astronomy. Here *O* represents the Earth and *S* a heavenly body, as the sun or a star whose apparent direction of movement is indicated by an arrow. Students may find it helpful to sketch the various features on a basketball or globe. Definitions of terms pertinent to the study of field astronomy follow.

The *zenith* is located where a plumb line projected upward meets the celestial sphere. In Figure C.3, Z designates it. Stated differently, it is the point on the celestial sphere vertically above the observer.

The *nadir* is the point on the celestial sphere vertically beneath the observer and exactly opposite the zenith. In Figure C.3, it is at *N*.

The *north celestial pole* is point *P* where the Earth's rotational axis, extended from the north geographic pole, intersects the celestial sphere.

The *south celestial pole* is point P' where the Earth's rotational axis, extended from the south geographic pole, intersects the celestial sphere.

A *great circle* is any circle on the celestial sphere whose plane passes through the center of the sphere.

A *vertical circle* is any great circle of the celestial sphere passing through the zenith and nadir and represents the line of intersection of a vertical plane with the celestial sphere. In Figure C.3, $ZSS'N$ is a vertical circle.

TABLE C.1 EPHEMERIS DATA FROM THE CADASTRAL SURVEY INTERNET SITE AT HTTP://WWW.CADASTRAL.COM/.

2000	SUN			. . . For 0 hrs Universal Time . . .							. . . Polaris . . .			0 hrs UT . . .					
	Declination			. . . GHA . . .			Eq of Tm		Semi-Di		Declination			. . . GHA . . .			. . . TUC*		
Date	d	m	s	d	m	s	m	s	m	s	d	m	s	d	m	s	h	m	s
Dec 1 FR	−21	48	47.9	182	45	19.8	+11	01.32	16	13.2	89	16	08.98	31	36	42.7	21	49	58.0
Dec 2 SA	−21	57	53.4	182	39	38.3	+10	38.55	16	13.4	89	16	09.35	32	36	02.1	21	46	01.3
Dec 3 SU	−22	06	33.5	182	33	47.8	+10	15.18	16	13.5	89	16	09.70	33	35	22.8	21	42	04.6
Dec 4 MO	−22	14	47.9	182	27	48.5	+09	51.23	16	13.7	89	16	10.04	34	34	44.4	21	38	07.8
Dec 5 TU	−22	22	36.4	182	21	40.9	+09	26.72	16	13.8	89	16	10.36	35	34	06.4	21	34	11.0
Dec 6 WE	−22	29	58.7	182	15	25.3	+09	01.69	16	14.0	89	16	10.67	36	33	28.2	21	30	14.2
Dec 7 TH	−22	36	54.7	182	09	02.1	+08	36.14	16	14.1	89	16	10.96	37	32	49.4	21	26	17.4
Dec 8 FR	−22	43	24.0	182	02	31.7	+08	10.12	16	14.3	89	16	11.24	38	32	09.4	21	22	20.7
Dec 9 SA	−22	49	26.5	181	55	54.5	+07	43.63	16	14.4	89	16	11.52	39	31	28.1	21	18	24.1
Dec 10 SU	−22	55	02.0	181	49	10.9	+07	16.72	16	14.5	89	16	11.81	40	30	45.7	21	14	27.6
Dec 11 MO	−23	00	10.4	181	42	21.1	+06	49.41	16	14.6	89	16	12.12	41	30	02.9	21	10	31.1
Dec 12 TU	−23	04	51.5	181	35	25.7	+06	21.71	16	14.7	89	16	12.45	42	29	20.6	21	06	34.6
Dec 13 WE	−23	09	05.2	181	28	24.9	+05	53.66	16	14.8	89	16	12.80	43	28	40.0	21	02	37.9
Dec 14 TH	−23	12	51.3	181	21	19.2	+05	25.28	16	14.9	89	16	13.16	44	28	01.7	20	58	41.1
Dec 15 FR	−23	16	09.7	181	14	08.9	+04	56.59	16	15.0	89	16	13.51	45	27	26.0	20	54	44.1

Dec 16 SA	−23	19	00.2	181	06	54.6	+04	27.64	16	15.1	89	16	13.84	46	26	52.2	20	50	47.0
Dec 17 SU	−23	21	22.9	180	59	36.5	+03	58.44	16	15.2	89	16	14.13	47	26	19.5	20	46	49.9
Dec 18 MO	−23	23	17.6	180	52	15.4	+03	29.02	16	15.3	89	16	14.43	48	25	46.9	20	42	52.7
Dec 19 TU	−23	24	44.1	180	44	51.5	+02	59.43	16	15.3	89	16	14.68	49	25	13.7	20	38	55.6
Dec 20 WE	−23	25	42.5	180	37	25.5	+02	29.70	16	15.4	89	16	14.92	50	24	39.5	20	34	58.5
Dec 21 TH	−23	26	12.7	180	29	57.8	+01	59.85	16	15.5	89	16	15.16	51	24	04.3	20	31	01.5
Dec 22 FR	−23	26	14.7	180	22	29.1	+01	29.94	16	15.5	89	16	15.39	52	23	28.2	20	27	04.5
Dec 23 SA	−23	25	48.4	180	14	59.9	+00	59.99	16	15.6	89	16	15.64	53	22	51.6	20	23	07.6
Dec 24 SU	−23	24	53.8	180	07	30.7	+00	30.05	16	15.6	89	16	15.89	54	22	15.2	20	19	10.7
Dec 25 MO	−23	23	31.1	180	00	02.2	+00	00.15	16	15.7	89	16	16.15	55	21	39.4	20	15	13.7
Dec 26 TU	−23	21	40.1	179	52	34.8	−00	29.68	16	15.7	89	16	16.42	56	21	04.8	20	11	16.7
Dec 27 WE	−23	19	21.0	179	45	09.3	−00	59.38	16	15.8	89	16	16.69	57	20	31.5	20	07	19.6
Dec 28 TH	−23	16	33.8	179	37	46.0	−01	28.93	16	15.8	89	16	16.96	58	19	59.7	20	03	22.3
Dec 29 FR	−23	13	18.5	179	30	25.7	−01	58.29	16	15.8	89	16	17.23	59	19	29.5	19	59	25.0
Dec 30 SA	−23	09	35.4	179	23	08.8	−02	27.41	16	15.9	89	16	17.47	60	19	00.4	19	55	27.6
Dec 31 SU	−23	05	24.4	179	15	55.8	−02	56.28	16	15.9	89	16	17.70	61	18	32.3	19	51	30.1

(Courtesy Cadastral Survey, U.S. Department of the Interior.)

*Universal time of upper culmination at Greenwich.

The *celestial equator* is the great circle on the celestial sphere whose plane is perpendicular to the axis of rotation of the Earth. It corresponds to the Earth's equator enlarged in diameter. Half of the celestial equator is represented by $Q'EQ$ in Figure C.3.

An *hour circle* is any great circle on the celestial sphere that passes through the north and south celestial poles. Therefore, hour circles are perpendicular to the plane of the celestial equator. They correspond to meridians (longitudinal lines) and are used to observe hour angles. In Figure C.3, $PSS''P'$ is an hour circle.

The *horizon* is a great circle on the celestial sphere whose plane is perpendicular to the direction of the plumb line. In surveying, the plane of the horizon is determined by a level vial. Half of the horizon is represented by $H'EH$ in Figure C.3.

A *celestial meridian*, interchangeably called *local meridian*, is that unique hour circle containing the observer's zenith. It is both an hour circle and a vertical circle. The intersection of the celestial meridian plane with the horizon plane is line $H'OH$ in Figure C.3, which defines the direction of true north. Thus, it is the astronomic meridian line used in plane surveying. Since east is 90° clockwise from true north, line OE in the horizon plane is a true east line. The celestial meridian is composed of two branches; the *upper branch* contains the zenith and is the semicircle $PZQ'H'P'$ in Figure C.3, and the *lower branch* includes the nadir and is arc $PHQNP'$.

A *diurnal circle* is the complete path of travel of the sun or a star in its apparent daily orbit about the Earth. Four terms describe specific positions of heavenly bodies in their diurnal circles (see Figure C.2): (1) *lower culmination*—the body's position when it is exactly on the lower branch of the celestial meridian; (2) *eastern elongation*—where the body is farthest east of the celestial meridian with its hour circle and vertical circle perpendicular; (3) *upper culmination*—when it is on the upper branch of the celestial meridian; and (4) *western elongation*— when the body is farthest west of the celestial meridian with its hour circle and vertical circle perpendicular.

An *hour angle* exists between a meridian of reference and the hour circle passing through a celestial body. It is measured by the angle at the pole between the meridian and hour circle, or by the arc of the equator intercepted by those circles. Hour angles are measured westward (in the direction of apparent travel of the sun or star) from the upper branch of the meridian of reference.

The *Greenwich hour angle (GHA)* of a heavenly body at any instant of time is the angle, measured westward, from the upper branch of the meridian of Greenwich to the meridian over which the body is located at that moment.[4] In the ephemeris of Table C.1, it is designated by GHA. Local hour angle (LHA) is similar to GHA, except it is observed from the upper branch of the observer's celestial meridian.

A *meridian angle* is like a local hour angle, except it is measured either eastward or westward from the observer's meridian, and thus its value is always between 0° and 180°.

[4]The meridian of Greenwich, England, is internationally accepted as the reference meridian for specifying longitudes of points on Earth and for giving positions of celestial bodies.

The *declination* of a heavenly body is the angular distance (measured along the hour circle) between the body and the equator; it is plus when the body is north of the equator and minus when south of it. Declination is usually denoted by δ in formulas and represented by arc $S''S$ in Figure C.3.

The *polar distance* or *codeclination* of a body is equal to 90° minus the declination. In Figure C.3, it is arc PS.

The *position* of a heavenly body with respect to the Earth at any moment may be given by its Greenwich hour angle and declination.

The *altitude* of a heavenly body is its angular distance measured along a vertical circle above the horizon, $S'S$ in Figure C.3. It is generally obtained by measuring a vertical angle with a total station (or theodolite), and correcting for refraction and parallax if the sun is observed. Altitude is usually denoted in formulas by h.

The *coaltitude* or *zenith distance* is arc ZS in Figure C.3 and equals 90° minus the altitude.

The *astronomical* or *PZS triangle* (darkened in the figure) is the spherical triangle whose vertices are the pole P, zenith Z, and astronomical body S. Because of the body's movement through its diurnal circle, the three angles in this triangle are constantly changing.

The *azimuth* of a heavenly body is the angle observed in the horizon plane, clockwise from either the north or south point, to the vertical circle through the body. An azimuth from north is represented by arc HS' in Figure C.3 and it equals the Z angle of the PZS triangle.

The *latitude* of an observer is the angular distance, measured along the meridian, from the equator to the observer's position. In Figure C.3, it is arc $Q'Z$. It is also the angular distance between the polar axis and horizon, or arc HP. Depending on the observer's position, latitude is measured north or south of the equator. Formulas in this book denote it as ϕ. *Colatitude* is $(90° - \phi)$.

The *vernal equinox* is the intersection point of the celestial equator and the hour circle through the sun at the instant it reaches zero declination and is proceeding into the northern hemisphere (about March 21 each year). For any calendar year, it is a fixed point on the celestial sphere (the astronomer's origin of coordinates in the sky) and moves with the celestial sphere just as the stars do. In Figure C.3, V designates it.

The *right ascension* of a heavenly body is the angular distance measured eastward from the hour circle through the vernal equinox to the hour circle of a celestial body. It is arc VS'' in Figure C.3. Right ascension frequently replaces Greenwich hour angle as a means of specifying the position of a star with respect to the Earth. In this system, however, the Greenwich hour angle of the vernal equinox must also be given.

■ C.5 TIME

Four kinds of time may be used in making and computing an astronomical observation.

1. *Sidereal time*. A sidereal day is the interval of time between two successive upper culminations of the vernal equinox over the same meridian. Sidereal

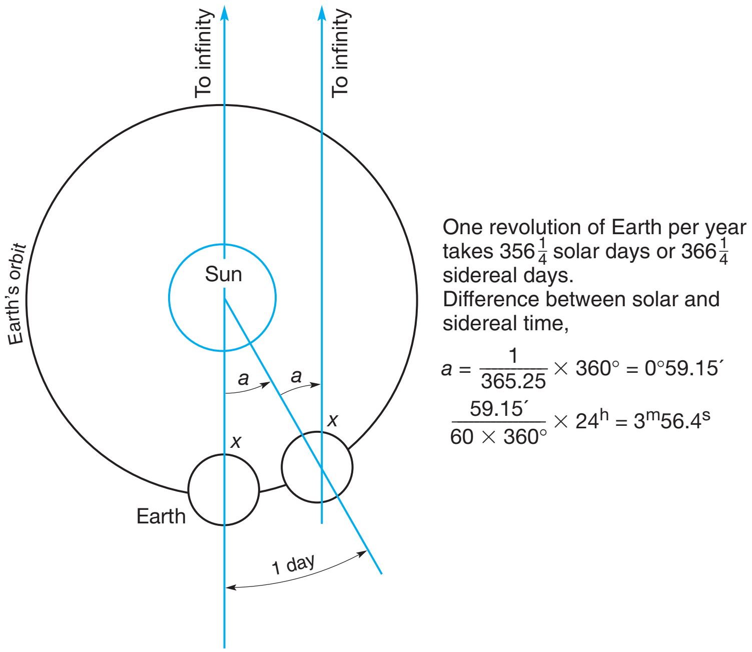


Figure C.4
Comparison of sidereal and solar time.

time is star time. At any location for any instant, it is equal to the local hour angle of the vernal equinox.

2. *Apparent solar time*. An apparent solar day is the interval of time between two successive lower culminations of the sun. Apparent solar time is sun time, and the length of a day varies somewhat because the rate of travel of the sun is not constant. Since the Earth revolves about the sun once a year, there is one less day of solar time in a year than sidereal time. Thus, the length of a sidereal day is shorter than a solar day by approximately 3 min 56 sec. The relationship between sidereal and solar time is illustrated in Figure C.4. (*Note:* The Earth's orbit is actually elliptical, but for simplicity it is shown circular in the figure.)
3. *Mean solar*, or *civil, time.* This time is related to a fictitious sun, called the "mean" sun, which is assumed to move at a uniform rate. It is the basis for watch time and the 24-h day.

   The *equation of time* is the difference between "apparent" solar and "mean" solar time. Its value changes continually as the true or apparent sun gets ahead of, and then falls behind, the mean sun. Values for each day of the year are given in ephemerides (see Table C.1). If the apparent sun is ahead of the mean sun, the equation is plus; if behind, it is minus. *Local apparent time is obtained by adding the equation of time to local civil time.*
4. *Standard time*. This is the mean time at meridians 15° or 1 h apart, measured eastward and westward from Greenwich. Eastern Standard Time (EST) at the 75th meridian differs from universal time (UT), or Greenwich civil time (GCT), by 5 h (earlier, since the sun has not yet traveled from the meridian of Greenwich to the United States). Standard time was adopted in the United

TABLE C.2 LONGITUDES OF STANDARD MERIDIANS IN THE UNITED STATES AND TIME DIFFERENCES FROM GREENWICH

		Corrections in Hours, to Add to Obtain UT	
Standard Time Zone (and Abbreviation)	**Longitude of Standard Meridian**	**Standard Time**	**Daylight Time**
Atlantic (AST)	60°	4	3
Eastern (EST)	75°	5	4
Central (CST)	90°	6	5
Mountain (MST)	105°	7	6
Pacific (PST)	120°	8	7
Yukon (YST)	135°	9	8
Alaska/Hawaii (AHST)	150°	10	9
Bering Sea (BST)	165°	11	10

States in 1883, replacing some 100 local times used previously. *Daylight saving time* (DST) in any zone is equal to the standard time in the adjacent zone to the *east*; thus, central daylight time is equivalent to Eastern Standard Time.[5]

As previously noted, sun and star positions tabulated in ephemerides are given in UT. Observation times, on the other hand, may be recorded in the standard or daylight times of an observer's location and must therefore be converted to UT. Conversion is based on the longitude of the standard meridian for the time zone. Table C.2 lists the different time zones in the United States, the longitudes of their standard meridians, and the number of hours to be added for converting standard and daylight time to UT.

In making civil time conversions based on longitude differences, the following relationships are helpful:

$$\begin{aligned} 360^\circ \text{ of longitude} &= 24 \text{ h} \\ 15^\circ \text{ of longitude} &= 1 \text{ h} \\ 1^\circ \text{ of longitude} &= 4 \text{ min (of time)} \end{aligned}$$

C.6 TIMING OBSERVATIONS

In the United States, the *National Institute of Standards and Technology* (NIST), formerly the National Bureau of Standards, broadcasts time signals from station WWV in Fort Collins, Colo., on frequencies of 2.5, 5, 10, 15, and 20 MHz. These signals can be received with short-wave radios, including inexpensive *time kubes* especially designed for this purpose. They can also be received over the telephone

[5]Daylight saving time officially begins at 2:00 A.M. on the first Sunday of April and ends at 2:00 A.M. on the last Sunday of October each year.

by dialing (303) 499-7111. To broaden coverage, signals are also transmitted from station WWVH in Hawaii on the same frequencies. These signals are broadcast as audible ticks with a computerized voice announcement of UT at each minute. In Canada, EST is broadcast from station CHU on frequencies of 3.33, 7.335, and 14.67 MHz. This can be converted to UT by adding 5 h.

The time that is broadcast by WWV is actually *coordinated universal time* (UTC), whereas the time used for tabulating sun and star positions in ephemerides is a corrected version known as UT1. UTC is a uniform time at Greenwich that, unlike UT1, does not vary with changing rates of rotation and other irregular motions of the Earth. Leap seconds are added to UTC as necessary to account for the gradual slowing of the Earth's rotation rate, and thus keep UTC within ± 0.7 sec of UT1 at all times. For precise astronomical work, a difference correction (DUT) can be added to UTC to obtain UT1. The required DUT correction is given by means of double ticks broadcast by WWV and CHU during the first 15 sec following each minute tone. Each double tick represents a 0.1 sec correction. A plus correction is applied for double ticks that occur during the first 7 sec after the minute tone, while a negative correction is made for double ticks that are heard during seconds 9 through 15. Thus, if double ticks occurred for the first 5 sec after the minute tone, $+0.5$ sec would be added to broadcast UTC to obtain UT1. If double ticks were heard during the ninth and tenth seconds, -0.2 sec would be added to UTC. Because the DUT correction is quite small, it can be ignored for most observations on Polaris or other stars with very high declinations. The correction should be considered, however, for more accurate observations on the sun and stars of lower declination. The current DUT correction is also available in Bulletin D on the International Earth Rotation and Reference System Service at http://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html on the Internet.

Digital watches, watches with sweep-second hands, and stopwatches are all suitable for timing most astronomical observations in surveying. Hand calculators and data collectors equipped with time modules are especially convenient since they can serve not only as timing devices but also for recording data and computing. Regardless of the timepiece used, it should be checked against WWV before starting observations and set to agree exactly with either UTC or the number of seconds it is fast or slow recorded. The time a check is made should also be recorded. When all observations are completed, the clock check should be repeated and any change recorded. Then intermediate observation times can be corrected in proportion to the elapsed time since the original check. With a stopwatch, checks can be made before and after each individual observation.

■ C.7 COMPUTATIONS FOR AZIMUTH FROM POLARIS OBSERVATIONS BY THE HOUR ANGLE METHOD

In this method, only the horizontal circle reading and precise time need to be recorded when the star is sighted. A vertical circle reading for at least one pointing is recommended, however, to ensure that the correct star has been sighted. To make the observations, the instrument is set up and leveled on one end of a line whose azimuth is to be determined. In the usual field procedure, the line's other

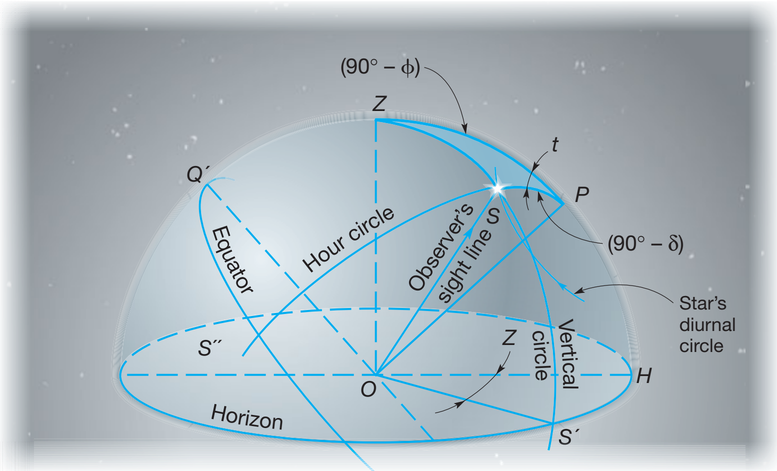


Figure C.5
The PZS triangle for Polaris at any hour angle.

end is first sighted and then the horizontal angle measured to the star. To eliminate the effects of instrumental errors, equal numbers of direct and reversed observations are taken and the results averaged.

Computations after fieldwork require the solution for angle Z in the astronomical (PZS) triangle (see Figure C.3). Two formulas for Z that apply in the hour angle method, derived from laws of spherical trigonometry, are

$$Z = \tan^{-1}\left(\frac{\sin t}{\cos\phi\tan\delta - \sin\phi\cos t}\right) \tag{C.2}$$

and

$$Z = \tan^{-1}\left(\frac{-\sin(LHA)}{\cos\phi\tan\delta - \sin\phi\cos(LHA)}\right) \tag{C.3}$$

The geometry upon which these equations are based is shown more clearly in Figure C.5, where the PZS triangle is again highlighted. The latitude ϕ of the observer's position is arc HP; thus arc PZ is $(90° - \phi)$, or *colatitude.* Declination δ of the star is arc $S''S$, so SP is $(90° - \delta)$, or *polar distance.* Angle ZPS in Figure C.5 is t, the *meridian angle,* which is related to the (LHA) of the star. Diagrams such as those of Figure C.6 are helpful in understanding and determining t angles and LHAs. These diagrams show the north celestial pole P at the center of the star's diurnal circle *as viewed from the observer's position within the sphere.* On the diagrams, west is to the left of the pole, east is to the right, and the apparent rotation of the stars is counterclockwise. Angle λ between the meridian of Greenwich (G) and the local meridian (L) through the observer's position is the longitude of the station occupied. The star's Greenwich hour angle (GHA) for the observation time is taken from an ephemeris. Sketching λ and GHA approximately to scale on diagrams such as those of Figure C.6 immediately makes clear the star's position. From Figure C.6, it can be seen that the LHA in the western hemisphere can be computed as

$$LHA = GHA - \lambda \tag{C.4a}$$

For the eastern hemisphere Equation (C.4a) becomes

$$LHA = GHA + \lambda \tag{C.4b}$$

As shown in Figure C.6(a), the LHA is between 0° and 180° when the star is west of north, and as seen in Figure C.6(b), it is between 180° and 360° if the star is east of north. Also, $t = LHA$ if the star is west of north and $t = (360° - LHA)$ if the star is east of north. The relationships between the LHA of a star, the sign of Z that is obtained using Equation (C.5), and the azimuth of the star are shown in Table C.3.

Note that the latitude of an observer's position is used directly in Equations (C.2) and (C.3) and that the station's longitude is also needed to compute either t or LHA. These values can both be scaled from a USGS quadrangle map and, with reasonable care, obtained to within ±2 sec. Declinations for use in these equations are extracted from an ephemeris for the instant of sighting.

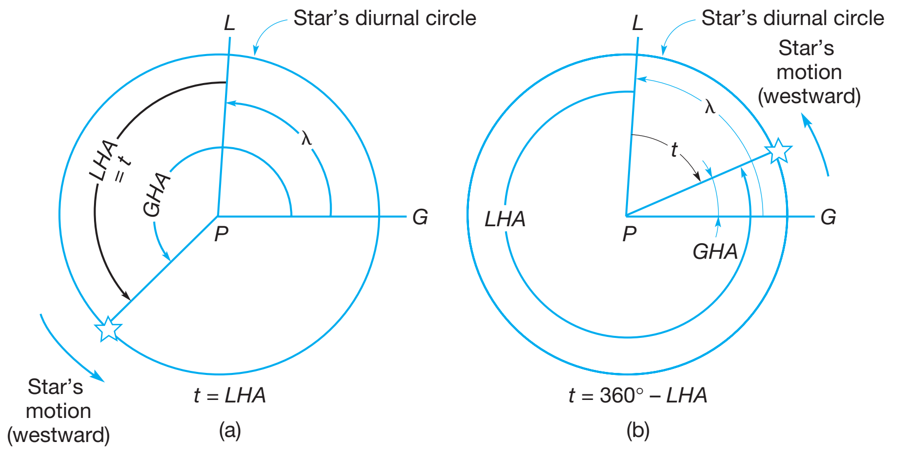


Figure C.6 Computation of the meridian angle *t*.

TABLE C.3 Relationship Between *LHA*, Sign of *Z*, and Azimuth of Star

LHA from	Z < 0	Z > 0
0° to 180°	Azimuth = 360° + Z	Azimuth = 180° + Z
180° to 360°	Azimuth = 180° + Z	Azimuth = Z

C.8 AZIMUTH FROM SOLAR OBSERVATIONS

Reduction of solar observations uses the same equations as star observations. However, at least one and possibly two major differences exist in the computations. Since the sun is relatively close to the Earth, the simple linear interpolation of the declination for stars is inadequate for the sun. The interpolation formula for the declination of the sun is

$$\delta_{\text{Sun}} = \delta_0 + (\delta_{24} - \delta_0)(\text{UT1}/24) + 0.0000395\,\delta_0 \sin(7.5 \times \text{UT1}) \qquad \text{(C.5)}$$

where δ_0 is the tabulated declination of the sun at 0-h UT1 on the day of the observation, δ_{24} is the tabulated declination of the sun at 24-h UT1 on the day of the observation (0-h UT1 of the following day), and UT1 is the universal time of the observation.

Observations on the sun can be made directly by placing a dark glass filter (designed for solar viewing) over the telescope objective lens. *A total station instrument should never be pointed directly at the sun without an objective lens solar filter. Failure to heed this warning may result in serious damage to sensitive electronic components in the total station. Additionally, the observer should never look at the sun without the solar filter in place, or permanent eye damage will occur.*

The second major difference depends on the method of pointing on the sun. Because the objective lens filter only allows the observer to see the instrument's crosshairs in the sun's illuminating circle, the most precise pointing will occur at the trailing edge of the sun as shown in Figure C.7. Thus, the observer should place the trailing edge of the sun near the vertical crosshair as in Figure C.7(a), then wait as the sun moves and record the time when the trailing edge of the sun just touches the vertical crosshair as shown in Figure C.7(b). When using this field procedure, or any field procedure that involves the sun's edges, a correction must be made to the horizontal angle for the sun's semi-diameter. The sun's semi-diameter varies with the distance from the Earth to the sun, and values are tabulated for each day in the ephemeris (see Table C.1). The correction that must be applied for semidiameter is computed as

$$C_{SD} = \frac{\text{Sun's semidiameter}}{\cos h} \qquad \text{(C.6)}$$

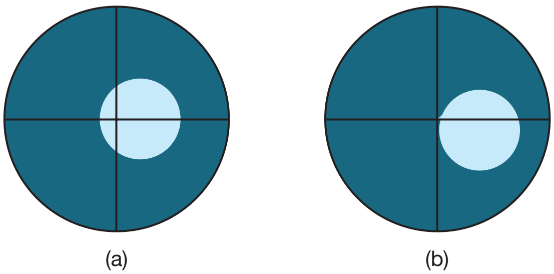


Figure C.7 View of the sun (a) just prior to coincidence of the vertical crosshair and (b) at coincidence.

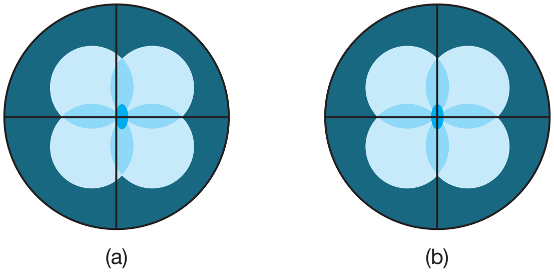


Figure C.8
View of the sun using a Roelof solar prism (a) just prior to and (b) at the time of the observation.

The correction for the sun's semidiameter can be avoided by using a *Roelof solar prism*. The Roelof solar prism is a device designed specifically for sighting the sun. It is easily mounted on the objective end of the telescope and by means of prisms produces four overlapping images of the sun in the pattern shown in Figure C.8. While viewing the sun, an observer can accurately center the vertical cross wire in the small diamond-shaped area in the middle of the field of view. Because of symmetry, this is equivalent to sighting the sun's center.

To compute the azimuth of a line from observations on the sun, the hour angle equation [either Equation (C.2) or Equation (C.3)] can be employed. These are the same ones used for Polaris observations. Again, latitude and longitude can be taken from a USGS quadrangle map, and declination extracted from an ephemeris for the time of observation.

■ C.9 IMPORTANCE OF PRECISE LEVELING

As discussed in Section 8.20.1, precise leveling is extremely important to horizontal directions whenever vertical angles are large. For example, at an altitude angle of 40° with a leveling error of 15″ (1/2 of division of a 30″ bubble), the estimated error in the horizontal direction observed by a total station is given by Equation (8.4) as

$$15'' \tan(40^\circ) = \pm 12.6''$$

Typically, the level on the vertical circle is more sensitive than the plate bubble. This fact can be used to precisely level a total station when performing an astronomical observation. The procedure involves aligning two leveling screws on the tripod in the direction of the astronomical body. After performing a typical leveling procedure, point the instrument in the general direction of the third leveling screw, clamp the vertical circle, read, and record the zenith angle. Now turn the instrument 180° opposite this position leaving the vertical circle clamped, read, and record the zenith angle again. Precise level is achieved by averaging the two zenith angles and slightly adjusting the third leveling screw to read this average on the vertical circle. This procedure only achieves precise level in the direction of the celestial body. However, precise leveling to the ground station is not as critical since the vertical angle to the target is small typically.

D
Using the Worksheets from the Companion Website

D.1 INTRODUCTION

The Mathcad worksheets on the companion website for this book at http://www.pearsonhighered.com/ghilani demonstrates many of the computational exercises presented in this book. The 42 worksheets allow you to modify the values of variables in the equations and see instantaneous changes in the results. Additionally, they further discuss topics presented in this book. These sheets require Mathcad version 14.0 or higher. For readers who do not own Mathcad 14.0 or higher, these worksheets have been converted to html files and can be viewed with a web browser. However, the html files are not computationally dynamic; that is, they can only display the equations at the time of creation and do not allow changes to the variables or equations. To use either the worksheets or html files, you must install them on your computer with the installation program provided on the companion website.

As shown in Figure D.1, if the Mathcad® worksheets are unzipped in the *handbook* directory under Mathcad, you will find the link to the electronic book (E-book) in the Mathcad help menu. Select the menu item entitled "Support files for Elementary Surveying: An Introduction to Geomatics" to open the electronic book. If the menu item is missing, use the "Open Book . . ." menu item to manually browse for the file "ElemSurv.hbk." Additionally, individual files in the *ElemSurv* subdirectory can be opened directly in Mathcad and modified as desired.

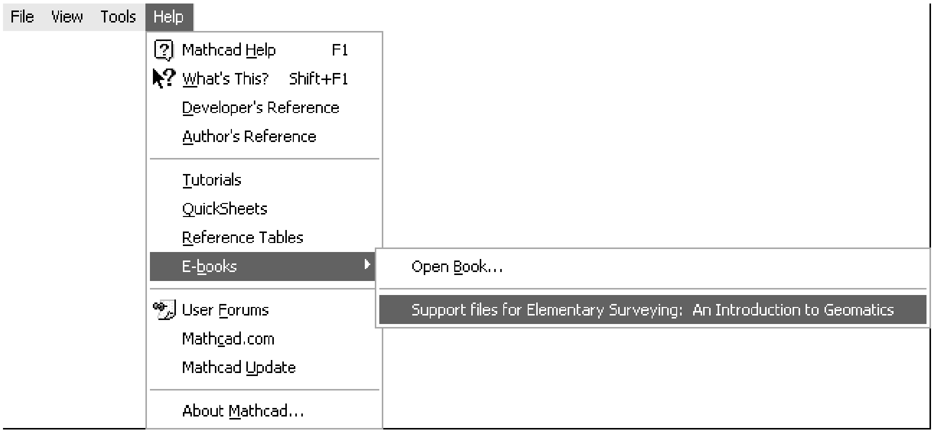


Figure D.1 Opening support files in Mathcad.

Figure D.2 Worksheets demonstrating map projection computations.

D.2 USING THE FILES

Sixteen of the 28 chapters in this book have associated Mathcad worksheets and html files. In some chapters, such as Chapter 20, there are several associated worksheets shown in Figure D.2. These additional worksheets provide further information on the map projections that are briefly mentioned in Section 20.13 of this book. They allow you to explore map projections that are not commonly used in the United States.

Other chapters with more than one worksheet include 11, 16, 19, and 27. In Chapter 11, besides demonstrating coordinate geometry problems presented in the chapter, the reader can view a least-squares solution of a two-dimensional conformal coordinate transformation, discussed in Section 11.8. In Chapter 16, several Mathcad worksheets demonstrate the least-squares method. The first worksheet shows the least-squares method for fitting points, first to a line and then to a circle using the equations discussed in Section 11.2 of the book. These adjustments, along with the two-dimensional conformal coordinate transformation, are not formerly covered in Chapter 16, but instead the theory of the least-squares method is applied. Additionally both the horizontal plane survey and differential-leveling network adjustments are demonstrated in separate worksheets. In Chapter 19, there are worksheets discussing the basics of geodesy, geodetic reductions of traditional observations, direct and inverse geodetic problems, three-dimensional geodetic computations, and transformation of ITRF 2000 coordinates with velocity vectors to NAD 83 coordinates for a particular epoch in time. Finally in Chapter 27, not only are the varying photogrammetric problems in the chapter demonstrated, but there are also two worksheets that cover the two-dimensional affine and projective coordinate transformations which are commonly used in photogrammetric computations.

While some worksheets obtain their data from values assigned to variables directly in the worksheet, others, such as the least-squares worksheets, obtain their data from text files generated using a text editor such as Notepad. For example, Figure D.3 shows the first screen in the support file for Chapter 3: Theory of Errors in Observations. Note the line which states "data := data3-1.txt" (there is an accompanying disk icon). This indicates that the contents of the file "data3-1.txt" are being read into the variable "data." The resultant variable data is partially listed on the right side of the window. (The values for this file come from Table 3.1 in this book.) The data file contains one observed value per line.

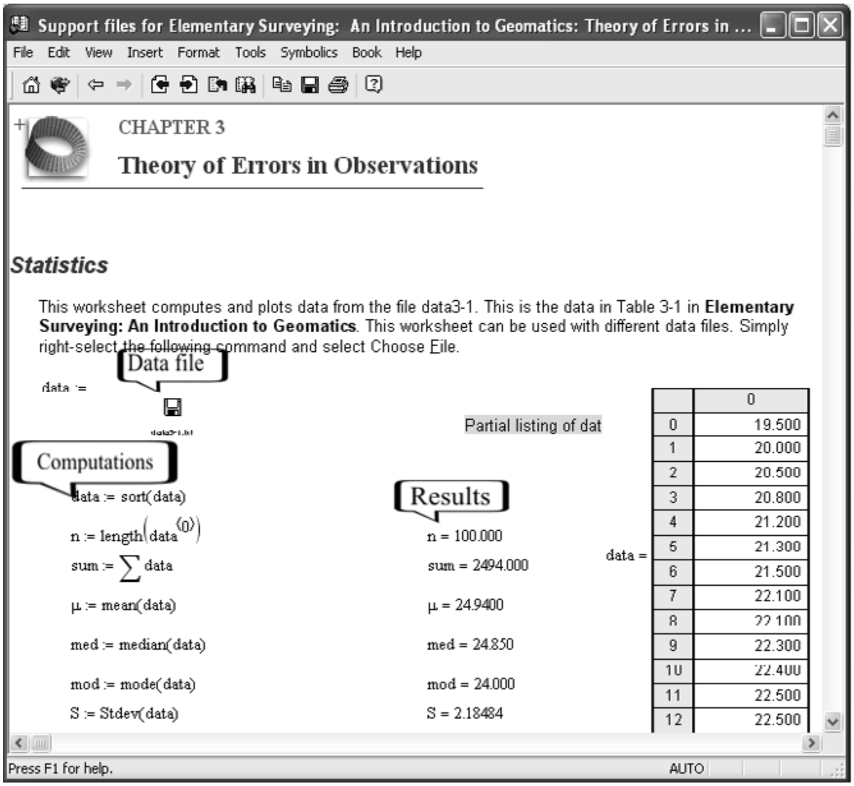


Figure D.3 Statistical computations for data from Table 3.1 of this book.

Figure D.4
Data entry pop-up menu.

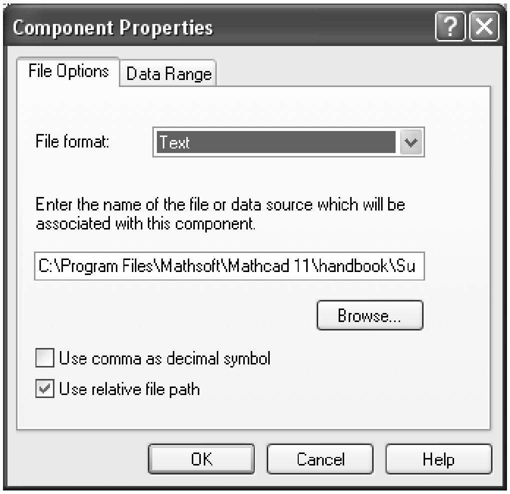
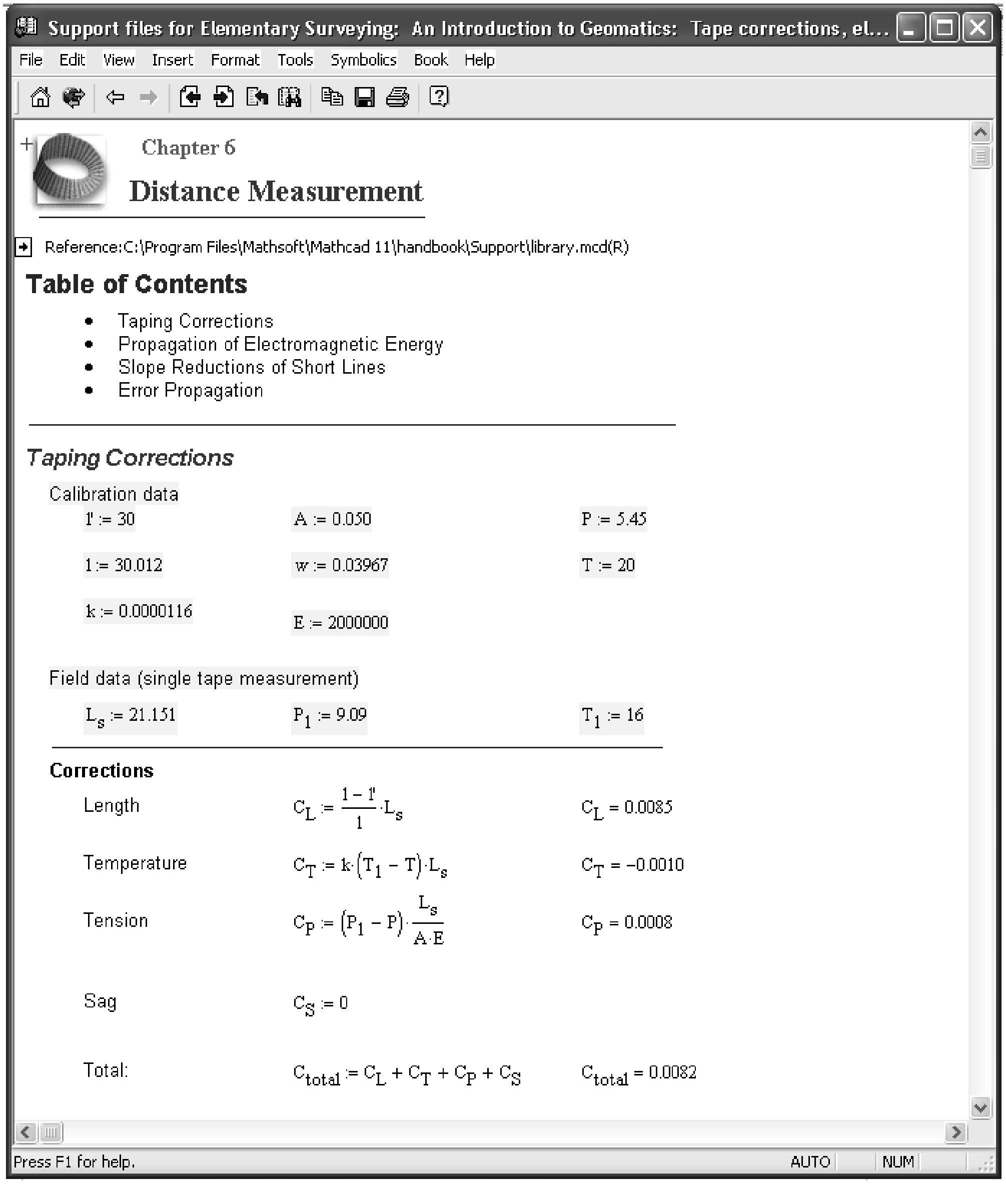


Figure D.5
Component Properties box displaying the file name in the middle of the box.

As previously stated, you can use a text editor such as Notepad or those in WOLFPACK, MATRIX, and STATS to create your own data files for other statistical problems. Once a file is created and saved to disk, you can change the worksheet's selected data file by right-clicking the "data" variable and selecting "Properties" in the resulting pull-down menu (see Figure D.4). This will display the "Component Properties" dialog box shown in Figure D.5. Click the "Browse" button to locate the desired data file and then click "OK." The worksheet will then automatically update its computations and graphical plots to match the data in the newly specified file.

In Figure D.3, the left side of the worksheet shows the calls to the statistical functions contained in Mathcad under the heading "Computations." Near the right side of the window, the column labeled "Results" displays the values that are assigned to each variable. For example, the mean of the data is 24.90, the median is 24.85, and the mode is 24.0. You can refer to the Mathcad Help system to learn more about Mathcad variables, functions, and their use in worksheets.

Figure D.6 shows the top of a worksheet that obtains its data from variables entered directly in the worksheet. This listing demonstrates the use of tape-correction formulas from Example 6.1. The variables near the top of the worksheet contain the calibration data for a 30-m tape as given in the example. Immediately



Support files for Elementary Surveying: An Introduction to Geomatics: Tape corrections, el...

File Edit View Insert Format Tools Symbolics Book Help

Chapter 6

Distance Measurement

Reference:C:\Program Files\Mathsoft\Mathcad 11\handbook\Support\library.mcd(R)

Table of Contents

- Taping Corrections
- Propagation of Electromagnetic Energy
- Slope Reductions of Short Lines
- Error Propagation

Taping Corrections

Calibration data

$l' := 30$	$A := 0.050$	$P := 5.45$
$l := 30.012$	$w := 0.03967$	$T := 20$
$k := 0.0000116$	$E := 2000000$	

Field data (single tape measurement)

$L_s := 21.151$	$P_1 := 9.09$	$T_1 := 16$

Corrections

Length	$C_L := \frac{l - l'}{l} \cdot L_s$	$C_L = 0.0085$
Temperature	$C_T := k \cdot (T_1 - T) \cdot L_s$	$C_T = -0.0010$
Tension	$C_P := (P_1 - P) \cdot \frac{L_s}{A \cdot E}$	$C_P = 0.0008$
Sag	$C_S := 0$	
Total:	$C_{total} := C_L + C_T + C_P + C_S$	$C_{total} = 0.0082$

Press F1 for help. AUTO NUM

Figure D.6 Tape corrections for last taped distance in Example 6.1 of book.

following the calibration data is the field data for the length of 21.151 m. Once the calibrated and field data are entered into the appropriate variables, corrections are computed using Equations (6.3) through (6.6) in the book. Finally, the sum of the corrections is determined in the variable C_{total}. Similar problems can be solved using this worksheet by modifying the field and calibration data as appropriate.

D.3 WORKSHEETS AS AN AID IN LEARNING

You should not use these worksheets to solve assigned homework problems, since you will only truly learn by solving problems on your own. Instead, you should use these worksheets as a method for testing your understanding and checking your computations. As can be seen in Figure D.6, a major advantage of using the worksheets is that intermediate calculations can be viewed and compared with hand-computed results. These comparisons allow you to determine the location of computational errors.

Another advantage of the worksheets is that they demonstrate some of the common programming routines used in surveying. For example, the worksheets on least-squares adjustments demonstrate the parsing of values from data files, computation of coefficients, building of matrices, and matrix methods used to solve the problem and determine post-adjustment statistics. Many of these routines can be emulated in higher-level programming languages such as Basic, C, Fortran, or Pascal. Additionally, many of these programming sheets can be modified to solve other problems that may be encountered in future studies.

E
Introduction to Matrices

■ E.1 INTRODUCTION

Matrix algebra enables users to express complicated systems of equations in a compact and easily manipulated form. It also provides a systematic mathematical method for solving systems of equations that can be easily programmed. Throughout this book, matrices have been used to solve systems of equations. Nowhere is this more evident than in Chapter 16 on least-squares adjustments. Matrices are frequently encountered in surveying, geodesy, and photogrammetry. This appendix provides readers with a basic understanding of matrices and their manipulations.

■ E.2 DEFINITION OF A MATRIX

A matrix is a set of number of symbols arranged in an array with m rows and n columns. This arrangement allows users to systematically express large systems of equations. For example, assume we have the three equations with three unknown parameters x, y, and z. The system of equations may appear as

$$\begin{aligned} 3x + 5y - 7z &= -24 \\ 2x - y + 6z &= 33 \\ 9x + 4y - 2z &= 12 \end{aligned} \tag{E.1}$$

This system of equations can be represented in matrix form as

$$\begin{bmatrix} 3 & 5 & -7 \\ 2 & -1 & 6 \\ 9 & 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -24 \\ 33 \\ 12 \end{bmatrix} \tag{E.2}$$

Equation (E.2) can be represented in compact matrix notation as

$$AX = L \tag{E.3}$$

As can be seen in Equation (E.2), each coefficient from Equation (E.1) has been placed in order in the first matrix called A; each unknown parameter has been placed in a single row of the second matrix called X; and similarly, each constant has been placed in a single row of the last matrix called L. Thus the A matrix is often called the *coefficient matrix*, the X matrix the *unknown matrix*, and the L matrix the *constants matrix*. Once in this form, a system of equations can be manipulated and solved algebraically using matrix methods.

E.3 THE DIMENSIONS OF A MATRIX

The number of rows and columns in each matrix expresses the dimensions or size of a matrix. For example, the A matrix in Equation (E.2) has three rows and three columns. It is said to have dimensions of 3 by 3 and is known as a *square matrix*. The X and L matrices have three rows, but only one column. Their dimensions are 3 by 1 and are also known as *vectors*. In general, a matrix can have m rows and n columns. When m is not equal to n, the matrix is known as a *rectangular matrix*. As previously stated by example, a *square matrix* is formed when the number of rows m equals the number of columns n.

Individual elements of a matrix can be designated by their row and column locations in the matrix. The row-column identifiers are known as *indices*. For example, the A matrix in Equation (E.2) has a value of 3 in row 1 and column 1. The index of 3 is thus 1,1 indicating that 3 is in the first row and first column of A. The elements of matrices are generally written in lower-case letters with subscripts representing their index. The indices of the elements are generally written without the intervening comma. For example, a_{11} has a value of 3. With this in mind, the entire matrix A can be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{E.4}$$

In Equation (E.4), each element of the A matrix from Equation (E.2) has been replaced by its elemental name. Thus, in reference to Equation (E.2), a_{11} is 3, a_{12} is 5, a_{13} is −7, and so on.

When a matrix is square such as Equation (E.4), the elements that have equal row and column indices are known as *diagonal elements*. Thus, a_{11}, a_{22}, and a_{33} are the diagonal elements of the A matrix in (E.4). In their entirety, they are known as the *diagonal* of matrix A. Their sum is known as the trace of matrix A. Only square matrices have diagonals. Matrices that are not square in dimensions, such as X and L in Equation (E.2), do not have diagonals.

E.4 THE TRANSPOSE OF A MATRIX

The *transpose* of a matrix is a process where each column of the transpose matrix is a row in the original matrix. That is, column 1 of the transpose matrix is row 1 of the original matrix, column 2 of the transpose matrix is row 2 of the original matrix, and so on. The transpose of the A matrix in Equation (E.2) is

$$A^{\mathrm{T}} = \begin{bmatrix} 3 & 2 & 9 \\ 5 & -1 & 4 \\ -7 & 6 & -2 \end{bmatrix} \tag{E.5}$$

Notice in Equation (E.5) that placing a "T" as a superscript indicates the transpose of A. Also note that the first column of A^{T} is the first row of the A matrix in Equation (E.2). Similarly, the second column is the second row and the third column is the third row. As seen in Chapter 16, the transpose of the coefficient matrix is used to create the *normal equations*.

E.5 MATRIX ADDITION

Two matrices can be added or subtracted when they have the same dimensions. As an example, assume that we have two matrices, A and B, which have dimensions of 3 by 2. The addition or subtraction of the two matrices is performed element by element. The following example illustrates this procedure.

$$A + B = \begin{bmatrix} -1 & 4 \\ 2 & 3 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 5 & -3 \\ 6 & 7 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -1 + 5 & 4 - 3 \\ 2 + 6 & 3 + 7 \\ 4 - 2 & 8 - 5 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 8 & 10 \\ 2 & 3 \end{bmatrix} = C \tag{E.6}$$

Notice in Equation (E.6) that the resulting matrix has the same dimensions as the original two matrices, A and B. Also note that addition is performed element by element. The difference between A and B is written as

$$A - B = \begin{bmatrix} -1 & 4 \\ 2 & 3 \\ 4 & 8 \end{bmatrix} - \begin{bmatrix} 5 & -3 \\ 6 & 7 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -1 - 5 & 4 - (-3) \\ 2 - 6 & 3 - 7 \\ 4 - (-2) & 8 - (-5) \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ -4 & -4 \\ 6 & 13 \end{bmatrix} = C \tag{E.7}$$

In Equation (E.7), each element of the C matrix is found by subtracting the individual elements of the B matrix from the A matrix.

E.6 MATRIX MULTIPLICATION

Matrix multiplication requires that the two matrices being multiplied have the same *inner dimensions*. That is, if A has dimensions of m rows by i columns and is to be multiplied by B, then B must have dimensions of i by n. Notice that A has

i columns and B has i rows. These are the inner dimensions of the product AB. Their resulting product, AB, will have dimensions of m rows and n columns. The *outer dimensions* of the product AB are m and n. This can be expressed as

$$_{m}A^{i}\,_{i}B^{n} = {}_{m}P^{n} \tag{E.8}$$

where P is the product of AB having dimensions of m by n.

When the number of rows of A does not equal the number of columns in B, then the product BA cannot be performed. Thus, matrix multiplication is not commutative. That is, the product of AB does not necessarily equal BA. In fact, when m does not equal n in Equation (E.8), it can't even be performed. The following matrix multiplications are possible.

$$_{3}A^{2}\,_{2}B^{4} = {}_{3}P^{4}$$

$$_{1}A^{3}\,_{3}B^{2} = {}_{1}P^{2}$$

$$_{6}A^{2}\,_{2}B^{6} = {}_{6}P^{6}$$

The following matrix multiplications are not possible.

$$_{6}A^{3}\,_{6}B^{3}$$
$$_{2}A^{3}\,_{4}B^{2}$$

The reason why these multiplications are not possible is best explained by understanding how matrix multiplication is performed. To obtain the first element of the product matrix P, we must multiply the first row of A by the first column of B. This is best demonstrated with an example. Suppose we wanted to find the product AB using the following two matrices

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix}$$

where P is the product of AB. Then p_{11} is computed as

$$p_{11} = 1 \times 1 - 2 \times 4 = -7$$

Note how each element in the first row of matrix A is multiplied by each element in the first column of matrix B and their sums accumulated. The remaining elements of P are computed as

$$p_{12} = 1 \times 2 - 2 \times 5 = -8$$
$$p_{13} = 1 \times 3 - 2 \times 6 = -9$$
$$p_{21} = 3 \times 1 + 4 \times 4 = 19$$
$$p_{22} = 3 \times 2 + 4 \times 5 = 26$$
$$p_{23} = 3 \times 3 + 4 \times 6 = 33$$

Thus, the product of AB is

$$\begin{bmatrix} -7 & -8 & -9 \\ 19 & 26 & 33 \end{bmatrix}$$

It should be understandable now that the product of BA can't be performed since there are not enough elements in the first column of A to pair with the elements in the first row of B.

Using matrix multiplication, the representation of Equation (E.1) as Equation (E.2) can now be verified. That is, the product of the first row of the A matrix in Equation (E.2) with the elements of the X matrix results in the first equation of Equation (E.1). Similarly, the product of the second row of the A matrix in Equation (E.2) with the X matrix results in the second equation, and the use of the third row of the A matrix results in the third equation.

E.7 MATRIX INVERSE

A *matrix inverse* is similar to division when working with numbers. When A is multiplied by its inverse, the resulting matrix is known as the *identity matrix* I. The identity matrix has values of 1 for the diagonal elements and zeros for all other elements. Thus, an identity matrix with dimensions of 3 by 3 is

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

When the identity matrix of appropriate dimension is multiplied by another matrix, say B, the resulting product is the same as matrix B. Thus, the identity matrix shares the properties of 1 in simple arithmetic multiplication. To solve Equation (E.2), we need to determine the inverse of A and multiply it by L to obtain X, or

$$X = A^{-1}L \tag{E.9}$$

There are several methods of inversing a matrix. While these methods are beyond the scope of this book, these methods often employ the same *elementary row transformations* that are used in mathematics to solve a system of equations. Software is readily available that can perform this operation. Once the inverse of the matrix is known, it can be used to solve a system of equations. For example, the inverse of the A matrix in Equation (E.2) expressed to five decimal places is

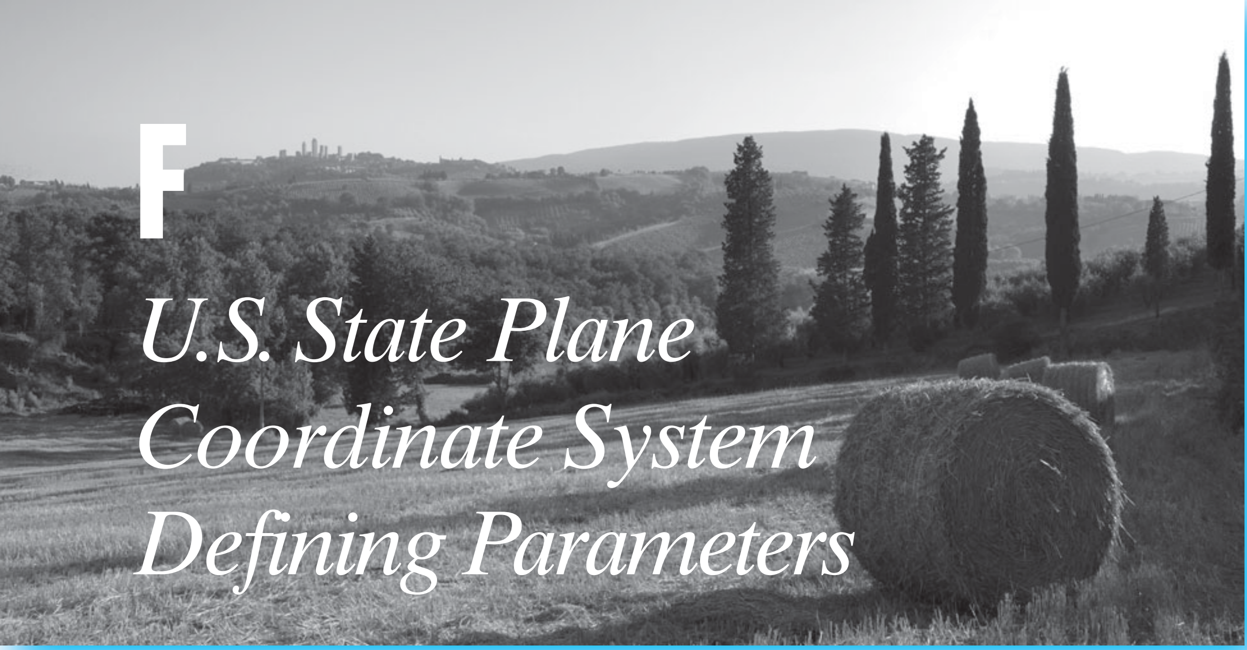
$$A^{-1} = \begin{bmatrix} -0.20952 & -0.17143 & 0.21905 \\ 0.55238 & 0.54286 & -0.30476 \\ 0.16190 & 0.31429 & -0.12381 \end{bmatrix}$$

When the inverse of A is multiplied times L, the resulting matrix X is

$$X = A^{-1}L = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

Thus, solution of Equation (E.2) is where x is 2, y is 1, and z is 5. The reader should check this by inserting these values into Equation (E.1) and confirming that the constants on the right side of the equation are determined.

The software program MATRIX is available for download on the companion website for this book at http://www.pearsonhighered.com/ghilani. The reader can use software to solve other matrix problems presented in this book.



F

U.S. State Plane Coordinate System Defining Parameters

F.1 INTRODUCTION

As discussed in several chapters, the U.S. State Plane Coordinate System (SPCS) is used in many applications in surveying/geomatics. In Chapter 20, the equations necessary to compute SPCS map projection coordinates are presented. This appendix provides the defining SPCS parameters for use in the United States. Section F.2 provides the defining parameters for states using the Lambert conformal conic map projection. Section F.3 provides the defining parameters for states using the Transverse Mercator map projection.

F.2 DEFINING PARAMETERS FOR STATES USING THE LAMBERT CONFORMAL CONIC MAP PROJECTION

State Zone	AR North	AR South	CA 401	CA 402	CA 403	CA 404	CA 405	CA 406
φ_S	34°56′	33°18′	40°00′	38°20′	37°04′	36°00′	34°02′	32°47′
φ_N	36°14′	34°46′	41°40′	39°50′	38°26′	37°15′	35°28′	33°53′
φ_0	34°20′	32°40′	39°20′	37°40′	36°30′	35°20′	33°30′	32°10′
λ_0	92°00′W	92°00′W	122°00′W	122°00′W	120°30′W	119°00′W	118°00′W	116°15′W
N_b	0.000	400,000.000	500,000.0	500,000.0	500,000.0	500,000.0	500,000.0	500,000.0
E_0	400,000.000	400,000.000	2,000,000.0	2,000,000.0	2,000,000.0	2,000,000.0	2,000,000.0	2,000,000.0

State	**CO**	**CO**	**CO**	**CT**	**FL**	**IA**	**IA**	**KS**
Zone	**North**	**Central**	**South**	**600**	**North**	**North**	**South**	**North**
φ_S	39°43′	38°27′	37°14′	41°12′	29°35′	42°04′	40°37′	38°43′
φ_N	40°47′	39°45′	38°26′	41°52′	30°45′	43°16′	41°47′	39°47′
φ_0	39°20′	37°50′	36°40′	40°50′	29°00′	41°30′	40°00′	38°20′
λ_0	105°30′W	105°30′W	105°30′W	72°45′W	84°30′W	93°30′W	93°30′W	98°00′W
N_b	304,800.6096	304,800.6096	304,800.6096	152,400.3048	0.0	1,000,000.0	0.0	0.0
E_0	914,401.8289	914,401.8289	914,401.8289	304,800.6096	600,000.0	1,500,000.0	500,000.0	400,000.0

State	**KS**	**KY**	**KY**	**LA**	**LA**	**LA**	**MD**	**MA**
Zone	**South**	**North**	**South**	**North**	**South**	**Offshore**	**1900**	**Mainland**
φ_S	37°16′	37°58′	36°44′	31°10′	29°18′	26°10′	38°18′	41°43′
φ_N	38°34′	38°58′	37°56′	32°40′	30°42′	27°50′	39°27′	42°41′
φ_0	36°40′	37°30′	36°20′	30°30′	28°30′	25°30	37°40′	41°00′
λ_0	98°30′W	84°15′W	85°45′W	92°30′W	91°20′W	91°20′W	77°00′W	71°30W
N_b	400,000.0	0.0	500,000.0	0.0	0.0	0.0	0.0	750,000.0
E_0	400,000.0	500,000.0	500,000.0	1,000,000.0	1,000,000.0	1,000,000.0	400,000.0	200,000.0

State	**MA**	**MI**	**MI**	**MI**	**MN**	**MN**	**MN**	**MT**
Zone	**Island**	**North**	**Central**	**South**	**North**	**Central**	**South**	**2500**
φ_S	41°17′	45°29′	44°11′	42°06′	47°02′	45°37′	43°47′	45°00′
φ_N	41°29′	47°05′	45°42′	43°40′	48°38′	47°03′	45°13′	49°00′
φ_0	41°00′	44°47′	43°19′	41°30′	46°30′	45°00′	43°00′	44°15′
λ_0	70°30′W	87°00′W	84°22′W	84°22′W	93°06′W	94°15′W	94°00′W	109°30′W
N_b	0.0	0.0	0.0	0.0	100,000.0	100,000.0	100,000.0	0.0
E_0	500,000.0	8,000,000.0	6,000,000.0	4,000,000.0	800,000.0	800,000.0	800,000.0	600,000.0

State	**NE**	**NY**	**NC**	**ND**	**ND**	**OH**	**OH**	**OK**
Zone	**2600**	**Long Island**	**3200**	**North**	**South**	**North**	**South**	**North**
φ_S	40°00′	40°40′	34°20′	47°26′	46°11′	40°26′	38°44′	35°34′
φ_N	43°00′	41°02′	36°10′	48°44′	47°29′	41°42′	40°02′	36°46′
φ_0	39°50′	40°10′	33°45′	47°00′	45°40′	39°40′	38°00′	35°00′
λ_0	100°00′W	74°00′W	79°00′W	100°30′W	100°30′W	82°30′W	82°30′W	98°00′W
N_b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E_0	500,000.0	300,000.0	609,601.2199	600,000.0	600,000.0	600,000.0	600,000.0	600,000.0

State	**OK**	**OR**	**OR**	**PA**	**PA**	**SC**	**SD**	**SD**
Zone	**South**	**North**	**South**	**North**	**South**	**3900**	**North**	**South**
φ_S	33°56′	44°20′	42°20′	40°53′	39°56′	32°30′	44°25′	42°50′
φ_N	35°14′	46°00	44°00′	41°57′	40°58′	34°50′	45°41′	44°24′
φ_0	33°20′	43°40′	41°40′	40°10′	39°20′	31°50′	43°50′	42°20′
λ_0	98°00′W	120°30′W	120°30′W	77°45′W	77°45′W	81°00′W	100°00W	100°20W
N_b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E_0	600,000.0	2,500,000.0	1,500,000.0	600,000.0	600,000.0	609,600.0	600,000.0	600,000.0

State Zone	TN 4100	TX North	TX North Central	TX Central	TX South Central	TX South	UT North	UT Central
φ_S	35°15′	34°39′	32°08′	30°07′	28°23′	26°10′	40°43′	39°01′
φ_N	36°25′	36°11′	33°58′	31°53′	30°17′	27°50′	41°47′	40°39′
φ_0	34°20′	34°00′	31°40′	29°40′	27°50′	25°40′	40°20′	38°20′
λ_0	86°00′W	101°30′W	98°30′W	100°20′W	99°00′W	98°30′W	111°30′W	111°30′W
N_b	0.0	1,000,000.0	2,000,000.0	3,000,000.0	4,000,000.0	5,000,000.0	1,000,000.0	2,000,000.0
E_0	600,000.0	200,000.0	600,000.0	700,000.0	600,000.0	300,000.0	500,000.0	500,000.0

State Zone	UT South	VA North	VA South	WA North	WA South	WV North	WV South	WI North
φ_S	37°13′	38°02′	36°46′	47°30′	45°50′	39°00′	37°29′	45°34′
φ_N	38°21′	39°12′	37°58′	48°44′	47°20′	40°15′	38°53′	46°46′
φ_0	36°40′	37°40′	36°20′	47°00′	45°20′	38°30′	37°00′	45°10′
λ_0	111°30′W	78°30′W	78°30′W	120°50′W	120°30′W	79°30′W	81°00′W	90°00′W
N_b	3,000,000.0	2,000,000.0	1,000,000.0	0.0	0.0	0.0	0.0	0.0
E_0	500,000.0	3,500,000.0	3,500,000.0	500,000.0	500,000.0	600,000.0	600,000.0	600,000.0

State Zone	WI Central	WI South	PR VI 5200
φ_S	44°15′	42°44′	18°02′
φ_N	45°30′	44°04′	18°26′
φ_0	43°50′	42°00′	17°50′
λ_0	90°00′W	90°00′W	66°26′W
N_b	0.0	0.0	200,000.0
E_0	600,000.0	600,000.0	200,000.0

F.3 DEFINING PARAMETERS FOR STATES USING THE TRANSVERSE MERCATOR MAP PROJECTION

State Zone	AL East	AL West	AK 5001/O.M.	AK 5002	AK 5003	AK 5004	AK 5005	AK 5006
$1{:}k_0$	25,000	15,000	10,000	10,000	10,000	10,000	10,000	10,000
φ_b	30°30′	30°00′	57°00′	54°00′	54°00′	54°00′	54°00′	54°00′
λ_b	85°50′W	85°50′W	133°40′W	142°00′W	146°00′W	150°00′W	154°00′W	158°00′W
E_0	200,000.0	600,000.0	5,000,000.0	500,000.0	500,000.0	500,000.0	500,000.0	500,000.0
N_b	0.0	0.0	5,000,000.0	0.0	0.0	0.0	0.0	0.0

State Zone	AK 5007	AK 5008	AK 5009	AZ East	AZ Central	AZ West	DE 700	FL East
$1{:}k_0$	10,000	10,000	10,000	10,000	10,000	15,000	200,000	17,000
φ_b	54°00′	54°00′	54°00′	31°00′	31°00′	31°00′	38°00′	24°20′
λ_b	162°00′W	166°00′W	170°00′W	110°10′W	111°55′W	113°45′W	75°25′W	81°00′W
E_0	500,000.0	500,000.0	500,000.0	213,360.0	213,360.0	213,360.0	200,000.0	200,000.0
N_b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

State Zone	FL West	GA East	GA West	HI 5101	HI 5102	HI 5103	HI 5104	HI 5105
1:k_0	17,000	10,000	10,000	30,000	30,000	100,000	100,000	0
φ_b	24°20′	30°00′	30°00′	18°50′	20°20′	21°10′	21°50′	21°40′
λ_b	82°00′W	82°10′W	82°10′W	155°30′W	156°40′W	158°00′W	155°30′W	155°30′W
E_0	200,000.0	200,000.0	700,000.0	500,000.0	500,000.0	500,000.0	500,000.0	500,000.0
N_b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

State Zone	ID East	ID Central	ID West	IL East	IL West	IN East	IN West	ME East
1:k_0	19,000	19,000	15,000	40,000	17,000	30,000	30,000	10,000
φ_b	41°40′	41°40′	41°40′	36°40′	36°40′	37°30′	37°30′	43°40′
λ_b	112°10′W	114°00′W	115°45′W	88°20′W	90°10′W	85°40′W	87°05′W	68°30′W
E_0	200,000.0	500,000.0	800,000.0	300,000.0	700,000.0	100,000.0	900,000.0	300,000.0
N_b	0.0	0.0	0.0	0.0	0.0	250,000.0	250,000.0	0.0

State Zone	ME West	MS East	MS West	MO East	MO Central	MO West	NV East	NV Central
1:k_0	10,000	20,000	20,000	15,000	15,000	17,000	10,000	10,000
φ_b	42°50′	29°30′	29°30′	35°50′	35°50′	36°10′	34°45′	34°45′
λ_b	70°10′W	88°50′W	90°20′W	90°30′W	92°30′W	94°30′W	115°35′W	116°40′W
E_0	900,000.0	700,000.0	700,000.0	250,000.0	250,000.0	250,000.0	200,000.0	500,000.0
N_b	0.0	0.0	0.0	0.0	0.0	0.0	8,000,000.0	6,000,000.0

State Zone	NV West	NH 2800	NJ/NY East 2900	NM East	NM Central	NM West	NY East	NY Central
1:k_0	10,000	30,000	10,000	11,000	10,000	12,000	10,000	16,000
φ_b	34°45′	42°30′	38°50′	31°00′	31°00′	31°00′	38°50′	40°00
λ_b	118°35′W	71°40′W	74°30′W	104°20′W	106°15′W	107°50′W	74°30′W	76°35′W
E_0	800,000.0	300,000.0	150,000.0	165,000.0	500,000.0	830,000.0	150,000.0	250,000.0
N_b	4,000,000.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

State Zone	NY West	RI 3800	VT 4400	WY East	WY East Central	WY West Central	WY West
1:k_0	16,000	160,000	28,000	16,000	16,000	16,000	16,000
φ_b	40°00	41°05′	42°30′	40°30′	40°30′	40°30′	40°30′
λ_b	78°35′W	71°30′W	72°30′W	105°10′W	107°20′W	108°45′W	110°05′W
E_0	350,000.0	100,000.0	500,000.0	200,000.0	400,000.0	600,000.0	800,000.0
N_b	0.0	0.0	0.0	0.0	100,000.0	0.0	100,000.0

G

Answers to Selected Problems

CHAPTER 2

2.4(a) 13,548.44 ft
2.5(a) 163.836 m
2.6(a) 668.6 ft
2.7(a) 2.1981 ac
2.10(a) 9.76 ac.
2.12(a) 125,845.89 ft
2.13(a) 122°24′
2.14(a) 160
2.16(a) 0.692867, 1.59705, and 0.851672 where sum = π rad.
2.19 From Section 2.7, to prevent loss of data

CHAPTER 3

3.6(a) 65.401 **(b)** ±0.003 **(c)** ±0.001
3.11(a) 65.396.4676 $\leq$ m $\leq$ 65.407, 100%
3.15(a) 23°29′56″ **(b)** ±14.9″ **(c)** ±7.5″
3.18 ±0.014 ft
3.22(a) 146.13 ± 0.023 ft
3.24(a) 236.378
3.27(a) 50,887 ± 14 ft^2
3.28(a) A = 49°24′28″; B = 39°02′28″; C = 91°33′04″

CHAPTER 4

4.2 0.068 m; 1.688 m; 6.750 m
4.7 38,160 ft or 7.23 mi
4.12 −0.003 ft
4.15 6.114 m
4.21 0.024 ft
4.28 854.02 ft; 846.18 ft

CHAPTER 5

5.8	0.000015 m	**5.19**	1.62 ft; 6.80 ft
5.21	16.7 m	**5.23**	2259.694 m
5.31	−1.4%	**5.35**	±13.7 mm

CHAPTER 6

6.2(a)	2.18 ft/pace, **(b)** 186 ft	**6.5**	See Section 6.14
6.8	86.05 ft	**6.19**	236.87 ft
6.23	0.0000027 s	**6.25(a)**	9.991 m
6.29	2273.68 ft	**6.32**	1653.860 m
6.36	±8.7 mm		

CHAPTER 7

7.5(a)	226.0518 grads		
7.10	150°00′28″		
7.13	S60°20′57″E		
7.16	Az_{CD}: 212°01′13″; Brg_{CD}: S32°01′13″W		
7.26(a)	11.9° W		
7.30	2°00′W	**7.33**	N22°03′E

CHAPTER 8

8.12(a)	41″	**8.14**	124″
8.20	75°46′04″	**8.24**	27″
8.26(a)	56″	**8.29**	16″

CHAPTER 9

9.9(a)	720°	**9.15**	13″
9.16	10″	**9.19**	±9.5″
9.22	21″	**9.24**	−15″; Third Order, Class I

CHAPTER 10

10.2	−21″; +3″
10.5	−3″ per angle; $C = 93°48′52″$
10.16	Distance for line *BC*
10.23	*BC*

CHAPTER 11

11.3	$m = 2.62783$; $b = -177.124$ m		
11.5	51°18′26″	**11.7**	0.044 m

11.9	(6932.18, 4868.39)	**11.13**	(3560.56, 2791.19)
11.15	(4330.13, 2998.69) or (3026.28, 2232.83)		
11.19	(4538.67, 2940.13)	**11.21(a)**	0.31006

CHAPTER 12

12.1	87,970 sq. units	**12.5**	2790 sq. ft.
12.10	66,810 m^2 or 6.681 ha	**12.16**	8,868,600 m^2 or 886.86 ha

CHAPTER 13

13.4 See Section 13.3, paragraphs 3 and 8
13.21 (320559.446, −4921314.168, 4031328.395)
13.24 (40°26′29.65168″N, 88°23′42.09876″W, 182.974 m)
13.28 314.149 m
13.30 95.888 m

CHAPTER 14

14.2(a)	20 min	**14.9(b)**	5 seconds
14.11	3	**14.20**	b
14.24	fourth order, class II	**14.30(e)**	0.79 ppm
14.34(a)	0.76 ppm		

CHAPTER 15

15.1 1 sec
15.6 5.8 mm
15.9 See Section 15.3
15.18 450 – 470 MHz
15.28 4800

CHAPTER 16

16.4 532.686 **16.9** $x = 135.469$ and $y = 158.609$
16.10 ±0.003, ±0.003
16.27 Ray: (37°48′16.20675″N, 108°33′19.20792″W, 501.250 m)
16.30 $-0.55119dx_{Steve} - 0.83438dy_{Steve} + 0.551191dx_{Frank} + 0.834379dy_{Frank} = 0.023$
16.33 $t = 31°57'43''$; $S_u = \pm 0.025$ ft; $S_v = \pm 0.016$ ft

CHAPTER 17

17.6	6 ft	**17.9**	2 in.
17.10	6%	**17.15**	No overhead obstructions
17.28	(10429.514, 5066.684, 590.026)	**17.34**	0.47 in.

CHAPTER 18

18.2	200 in.	**18.10**	20 ft
18.15	See Section 18.4, paragraph 2	**18.18**	1 in./30 ft or 1:360
18.24	2 cm	**18.25**	AB: (5431.493, 4472.792)

CHAPTER 19

19.4 436 m
19.9 6,364,725.399 m; 6,387,949.711 m
19.11 272.624 m
19.15 42°36′54.2″; 18°52′46.3″
19.18 −33.880 m
19.21 2456.310 m; 2458.868 m
19.25 268°19′43.2″
19.31 85°56′00.1″; 204°32′47.3″

CHAPTER 20

20.11 592.304 m, 242°36′12″
20.14 (329339.936, 124121.992) −2°08′11.2512″
20.17 (170227.750, 222784.094)
20.21 (41°12′23.2037″N, 78°26′30.3340″W)
20.23 (39°00′447.2423″N, 74°39′42.1526″W)
20.25(a) 2834.449 ft
20.32 0.99976777
20.40 205°39′03.1″

CHAPTER 21

21.12	e, c, b, d, a	**21.20**	11,700 ft^2; 10,220 ft^2

CHAPTER 22

22.1	4319 ft	**22.6(a)**	291.4 ft
22.8(a)	30 mi		
22.14(a)	240 rods		
22.21	Single proportion; Single proportion		

CHAPTER 23

23.10	0.88%	**23.16**	4000 ft^2
23.17	2.86 ft		

CHAPTER 24

24.2(a) 11°27′33″; 11°28′42″

24.3 $R = 1432.68$ ft; $T = 304.53$ ft; $E = 32.01$ ft; $M = 31.31$ ft; $LC = 595.74$ ft; $L = 1432.68$ ft; PC = 33 + 40.47
$\text{PT}_{Back} = 42 + 45.00$; $\text{PT}_{Forward} = 39 + 49.53$

24.24 1392.04 ft

24.25(a) $I/2$

24.31 349 ft

CHAPTER 25

25.10 685.714 ft

25.19 612.26 ft

25.26 781.61 ft

25.30 1108.21 ft

25.33 1.80 ft

CHAPTER 26

26.4 708 yds^3

26.9 6168.5 yd^3

26.14 3 yd^3; 705 yd^3

26.21 728.7 yd^3; 734.4 yd^3

26.25 3.114 ac-ft

26.29 419 ft^3/s

CHAPTER 27

27.5(a) 1/2000 in./ft

27.8(a) 1 in./1010 ft

27.10 24.76 ha

27.13 7988 ft

27.16(a) 116 ft

27.19 3409 ft

27.29 12 flight lines

27.31 30%

CHAPTER 28

28.8(a) 5,595,040,000 pixels

28.21(b) 6.23 in.

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Conversion Factors

Length

1 millimeter (mm) = 1000 micrometers (μm)
1 centimeter (cm) = 10 mm
1 meter (m) = 100 cm
1 m = 39.37 inches (in) [U.S. Survey Foot]
1 kilometer (km) = 1000 m
1 km = 0.62137 miles
1 in = 25.4 mm *exactly* [International Foot]
1 ft = 304.8 mm *exactly* [International Foot]
1 mile = 5280 ft
1 nautical mile = 6076.10 ft = 1852 m
1 rod = 1 pole = 1 perch = 16.5 ft
1 Gunter's chain (ch) = 66 ft = 4 rods
1 mile = 80 ch
1 vara = about 33 inches in Mexico and California and 33-1/3 inches in Texas
1 fathom = 6 ft

Volume

1 m^3 = 35.31 ft^3
1 yd^3 = 27 ft^3 = 0.7646 m^3
1 litre = 0.264 gal [U.S.]
1 litre = 0.001^3
1 gal [U.S.] = 3.785 litres
1 ft^3 = 7.481 gal [U.S.]
1 gal [Imperial] = 4.546 litres = 1.201 gal [U.S.]

Area

1 mm^2 = 0.00155 in^2
1 m^2 = 10.76 ft^2
1 km^2 = 247.1 acres
1 hectare (ha) = 2.471 acres
1 acre = 43,560 ft^2
1 acre = 10 ch^2, i.e., 10 (66 ft $\times$ 66 ft)
1 acre = 4046.9 m^2
1 ft^2 = 0.09290 m^2
1 ft^2 = 144 in^2
1 in^2 = 6.452 cm^2
1 mile^2 = 640 acres (normal section)

Angles

1 revolution = 360 degrees = 2π radians
1° (degree) = 60′ (minutes)
1′ = 60″ (seconds)
1° = 0.017453292 radians
1 radian = 57.29577951° = 57°17′44.806″
1 radian = 206,264.8062″
1 revolution = 400 grads (also called gons)
tan 1″ = sin 1″ = 0.000004848
π = 3.141592654

Other Conversions

1 gram (g) = 0.035 oz
1 kilogram (kg) = 1000 g = 2.20 lb
1 ton = 2000 lb = 2 kips = 907 kg
1 m/sec = 3.28 ft/sec
1 km/hr = 0.911 ft/sec = 0.621 mi/hr

GPS SIGNAL FREQUENCIES

Code	Frequency (MHz)
C/A	1.023
P	10.23
L1	1575.42
L2	1227.60
L5	1176.45

ELLIPSOID PARAMETERS

Ellipsoid	Semimajor Axis (a)	Semiminor Axis (b)	Flattening (1/f)
Clarke, 1866	6,378,206.4	6,356,583.8	294.97870
GRS80	6,378,137.000	6,356,752.314	298.257222101
WGS84	6,378,137.000	6,356,752.314	298.257223563

Some Other Important Numbers in Surveying (Geomatics)

Errors and Error Analysis

68.3 = percent of observations that are expected within the limits of one standard deviation
0.6745 = coefficient of standard deviation for 50% error *(probable error)*
1.6449 = coefficient of standard deviation for 90% error
1.9599 = coefficient of standard deviation for 95% error *(two-sigma error)*

Electronic Distance Measurement

299,792,458 m/sec = speed of light or electromagnetic energy in a vacuum
1 Hertz (Hz) = 1 cycle per second
1 kilohertz (kHz) = 1000 Hz
1 megahertz (MHz) = 1000 kHz
1 gigahertz (GHz) = 1000 MHz
1.0003 = approximate index of atmospheric refraction (varies from 1.0001 to 1.0005)
760 mm of mercury = standard atmospheric pressure

Taping

0.00000645 = coefficient of expansion of steel tape, per 1°F
0.0000116 = coefficient of expansion of steel tape, per 1°C
29,000,000 lb/in^2 = 2,000,000 kg/cm^2 = Young's modulus of elasticity for steel
490 lb/ft^3 = density of steel for tape weight computations
15°F = change in temperature to produce a 0.01 ft length change in a 100 ft steel tape
68°F = 20°C = standard temperature for taping

Leveling

0.574 = coefficient of combined curvature and refraction (ft/miles^2)
0.0675 = coefficient of combined curvature and refraction (m/km^2)
20.6 m = 68 ft = approximate radius of a level vial having a 20″ sensitivity

Miscellaneous

6,371,000 m = 20,902,000 ft = approximate mean radius of the earth
1.15 miles = approximately 1 minute of latitude = approximately 1 nautical mile
69.1 miles = approximately 1 degree of latitude
101 ft = approximately 1 second of latitude
24 hours = 360° of longitude
15° longitude = width of one time zone, i.e., 360°/24 hr
23°26.5′ = approximate maximum declination of the sun at the solstaces
$23^h56^m04.091^s$ = length of sidereal day in mean solar time, which is $3m55.909^s$ of mean solar time short of one solar day
5,729.578 ft = radius of 1° curve, arc definition
5,729.651 ft = radius of 1° curve, chord definition
100 ft = 1 station, English system
1000 m = 1 station, metric system
6 miles = length and width of a normal township
36 = number of sections in a normal township
10,000 km = distance from equator to pole and original basis for the length of the meter

Abbreviations

Construction Surveys

Bb	batter boards
BL	building line
CB	catch basin
CG	centerline of grade
CL	centerline
C	cut
CS	curve to spiral
esmt	easement
F	fill
FG	finish grade
FH	fire hydrant
FL	fence line
FS	finished surface
GC	grade change
GP	grade point
GR	grade rod (ss notes)
L, Lt	left (X-sect notes)
MH	manhole
PC	point of curvature
PI	point of intersection
PL	property line
PP	power pole
PT	point of tangency
pvmt	pavement
R, Rt	right (X-sect notes)
R/W	right-of-way
SC	spiral to curve
SD	storm drain
SG	subgrade
spec	specifications
Sq	square
ss	slope stake; side slope
Std	standard
Str Gr	straight grade
X sect	cross section

Control Surveys; GPS Surveys

BM	benchmark
BS	backsight
CORS	continuously operating reference station
DGPS	differential GPS
EDM	electronic distance measurement
FS	foresight
GPS	global positioning system
HARN	high accuracy reference network
HDOP	horizontal dilution of precision
NGRS	National Geodetic Reference System
OTF	on-the-fly initialization (kinematic GPS)
PDOP	positional dilution of precision
RTDGPS	real-time differential GPS
RTK	real-time kinematic (GPS)
SNR	signal-to-noise ratio
VDOP	vertical dilution of precision

Property Surveys

A	area
CF	curb face
ch "X"	chiseled cross
CI	cast iron
diam	diameter
Dr	drive
ER	end of return
Ex	existing
H & T	hub and tack
HC	house connection sewer
IB	iron bolt; iron bar
IP	iron pipe; iron pin
L & T	lead and tack
MHW	mean high water
MLLW	mean low low water
MLW	mean low water
Mon	monument
P	pipe; pin
PLS	professional land surveyor
Rec	record
St	street
Std Surv Mon	standard survey monument
$2'' \times 2''$	two-inch square stake
"X"	crosscut in stone
yd	yard

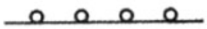
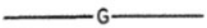
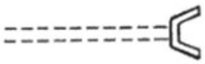
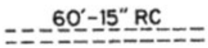
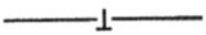
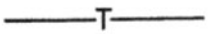
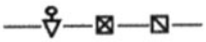
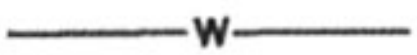
Public Lands Surveys

AMC	auxiliary meander corner
bdy, bdys	boundary; boundaries
BT	bearing tree
CC	closing corner
ch, chs	chain; chains
cor, cors	corner; corners
corr	correction
decl	declination
dist	distance
frac	fractional (sec, etc.)
GM	guide meridian
lk, lks	link; links (Gunter's chain)
mer	meridian
mkd	marked
Mi Cor	mile corner
MC	meander corner
MS	mineral survey
Prin Mer, PM	principal meridian
R, Rs	range; ranges
R 1 W	range 1 west
SC	standard corner
Sec, Secs	section; sections
SMC	special meander corner
Stan Par, SP	standard parallel
T, Tp, Tps	township; townships
T 2 N	township 2 north
USMM	U.S. mineral monument
WC	witness corner

Miscellaneous

alt	altitude
Chf	chief of party
CI	cast iron
Con Mon	concrete monument
CDT	central daylight time
CS	corrugated steel
CST	central standard time
Delta (Δ)	central angle (of curve)
dep	departure
Dir, D	direct
EDT	eastern daylight time
Elev	elevation
EST	eastern standard time
FB	field book
FL	face left
FR	face right
GHA	Greenwich hour angle
GIS	geographic information system
GPS	global positioning system
HA	hour angle
hi	height of instrument above station
HI	height of instrument above datum
hor	horizontal
IS	intermediate sight
IFS	intermediate foresight
ISS	inertial surveying system
lat	latitude
LHA	local hour angle
LIS	land information system
long	longitude
MDT	mountain daylight time
MST	mountain standard time
N	nadir point
NAD27	North American Datum of 1927
NAD83	North American Datum of 1983
NAVD88	North American Vertical Datum of 1988
NGVD29	National Geodetic Vertical Datum of 1929
obs	observer
obsn	observation
orig	original
PDT	Pacific daylight time
PST	Pacific standard time
red	reduction
RP	reference point
rev, R	reversed
sta	station
stk	stake
TBM	temporary benchmark
TIN	triangulated irregular network
TP	turning point
tel	telescope
temp	temperature
UTC	universal coordinated time
Z	zenith

Some Commonly Used Surveying Symbols

	Chiseled monument
	Concrete monument
	Curb inlet
	Fence, chain link
	Fence, wood w/posts
	Gas line
	Guy wire
	Headwall
	Hydrant on water line
	Iron pipe
	Power line
	Power pole
	Property line
	Stake (or hub) with tack
	Storm sewer (length, size, type)
	Telephone line (buried)
	Telephone line (suspended)
	Telephone pole
	Traffic signal
	Valves on water line
	Water line