Basic Inequality Proofs

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Example 1. *Prove that for all real numbers a and b:*

$$(a+b)^2 \geq 0.$$

Example 2. *Prove that for any positive real numbers x and y:*

$$\frac{x+y}{2} \geq \sqrt{xy}.$$

Example 3. *Show that for all real numbers a, b, and c:*

$$a^2 + b^2 + c^2 \geq ab + bc + ca.$$

Example 4. *Prove that for any positive real number x:*

$$x + \frac{1}{x} \geq 2.$$

Example 5. *For positive real numbers a and b, prove:*

$$\left(\frac{a+b}{2}\right)^2 \leq \frac{a^2+b^2}{2}.$$